NSF/RA-800330

PB81-148561

PRACTICAL RELIABILITY ANALYSIS OF LINEAR LIFELINES UNDER NATURAL HAZARDS

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Sponsored by the

NATIONAL SCIENCE FOUNDATION

Division of Problem-Focused Research

Grant No.: PFR-7724727

EDAC

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EDAC-103-140.01

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Any opinions, findings, conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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November 25, 1980

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TABLE OF CONTENTS

		Page
1.	INTRODUCTION	1-1
2.	GENERAL METHODOLOGY AND PROBABILISTIC MODELS	2-1
3.	RELIABILITY ANALYSIS OF A BRIDGE	3-1
4.	RELIABILITY ANALYSIS OF A BURIED PIPELINE	4-1
5.	SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FUTURE STUDIES	5-1

REFERENCES

APPENDICES

A - EVALUATION OF DEPENDENCY OF MECHANISMS FOR THE BRIDGE EXAMPLE

B - GRAPHICAL AIDS FOR CALCULATION OF BIVARIATE NORMAL DISTRIBUTIONS

ABSTRACT

A methodology for the assessment of the reliability of linear lifelines subjected to natural hazards was developed and applied to bridge and underground pipeline examples. Included were the assessments of probabilities of achieving various levels of functional goals for the lifelines and the evaluation of a damage probability matrix. Pairwise dependency between adjacent sections (or spans) was considered using binomial and bivariate normal models. For the bridge example, the failure of the lifeline was supposed to have occurred when a complete failure mechanism developed. For the pipeline example, the failure was supposed to have occurred when the applied axial strains, computed including the effects of vibratory motion and fault displacement, exceeded the yield strains. The loadings were considered to be both probabilistic and deterministic. Reliability analysis was performed for horizontal and vertical earthquake motions.

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1. INTRODUCTION

This report presents the results of an investigation, conducted at Engineering Decision Analysis Company, Inc. (EDAC), on the practical reliability analysis of linear lifelines under natural hazards, such as earthquakes. The purpose of this research was to develop and apply a practical methodology for reliability assessment of linear lifelines under the influence of seismic loads. Linear lifelines include a large category of lifelines that usually do not have any major branches, such as pipelines for the supply of oil, gas and water, transmission lines, highways and railroads, and bridges, etc.

The major emphasis of the research was on the development and use of reliable and practical techniques for the assessment of reliabilities of lifelines. The reliability assessments employed in this investigation consisted of estimating probabilities of attainment of various levels of functional goals. The types of lifelines of interest considered in this investigation included lifelines supported at intervals by piers, such as bridges, and lifelines supported continuously on a subgrade, such as underground pipelines.

A salient feature of the research work described in this report was the consideration of dependency between the properties and the behaviors of adjacent elements of a lifeline. It is usual in reliability analysis of lifelines to assume that the properties and loads at different locations of a lifeline are independent, random variables. This is not true in many cases, such as in a bridge where the capacities and loads for adjacent spans are usually statistically correlated, or in an underground pipeline where the properties of the surrounding soil are correlated from one region to the other. This is usually due to the uncertainty in the properties of structural materials and construction and fabrication procedures for structures, such as bridges, and the uncertainty in soil properties because of their non-uniform and non-homogeneous character resulting from their mode of formation. Furthermore, natural hazards, such as earthquakes, which are random in nature, produce forces which are also random. The loads at two points of a lifeline due to earthquake effects are, therefore, also usually correlated.

Chapter 2 of this report describes the general methodology for reliability assessment used in this study and its extension using two types of probabilistic models describing the behavior of dependency between the properties and loads for adjacent elements of a lifeline. The first model employs a joint binomial probability distribution for the description of the correlation in the behaviors of two adjacent elements. The capacities at critical sections of the lifeline are considered to be normally distributed and probabilistically uncorrelated, but the behavior of each critical section itself is considered to be binary and correlated to the behavior of the adjacent section. The second model employs a bivariate normal distribution for correlation of capacities and loads in adjacent spans. In both cases, a pairwise dependency is assumed.

Two types of examples of lifelines were selected for this study, after a very careful survey and discussions with numerous engineers in-house, as well as professional engineers in the industry. The first example consisted of a three-span concrete bridge. The geometry and properties for this example were based on an existing ten-span bridge in the City of Oakland in California. Chapter 3 describes the reliability assessment for this lifeline. The second example consisted of an underground pipeline. This example was based on the Trans-Alaska pipeline located in a seismically active area with the possibility of experiencing vibratory motions resulting from dynamic wave propagation effects, as well as fault displacements. The reliability assessment for this example is presented in Chapter 4. This is followed by Summary, Conclusions, and Recommendations for Future Studies, presented in Chapter 5.

The study described in this report was conducted for National Science Foundation under NSF Grant No. PFR77-24727, and was carried out under EDAC Project No. 103-140.

2. GENERAL METHODOLOGY AND PROBABILISTIC MODELS

INTRODUCTION

The reliability of a lifeline under natural hazards is defined as the probability of survival of that lifeline under such hazards. The survival (or success) occurs when there is no failure. The failure is a general terminology that requires accurate definition for individual cases. In Chapters 3 and 4, definitions of failures and damage levels are discussed for the bridge and the underground pipeline examples. Reliability analysis, as performed in this study, consisted of decomposing the overall system into an assemblage of elements which corresponded realistically to the functional characteristics of the overall system. Since failure cannot occur in zero length, the basic analysis methodology consisted of studies of discrete segments of finite dimensions.

In real lifelines, as mentioned earlier, the properties and loadings between adjacent elements are usually statistically dependent. For consideration of complete dependency between all elements, the knowledge of the joint probability distribution between these elements is required, leading to the necessity of solution of a very complex problem with no available easy solution techniques. For this reason, an assumption of pairwise dependency was considered to be a suitable model for most linear lifelines, since the knowledge of the coefficient of correlation described first order conditional reliabilities between elements which could lead to the calculation of overall reliabilities of the system.

Two types of probabilistic models were employed in this study for consideration of pairwise dependencies of capacities, loads, or element behaviors. These models consisted of binomial and bivariate normal distributions, as described below.

BINOMIAL MODEL

A binomial model based on the dependent binary behavior is one of the simplest models that can be assumed in order to consider the dependency of adjacent elements of a linear lifeline (Ref. 1). In this model, the behavior of an element is assumed to be binary, i.e., having the status of success or failure. Each element with a binary behavior is assumed to be dependent on the success and failure of its adjacent element according to the binomial distribution law. The failure of each element or section would occur, by definition, when the loads at the specified section of the element would exceed the corresponding capacities at that section.

It must be pointed out that, in using this model, the dependency is actually assumed between the behaviors of adjacent elements and not the loads or the capacities. The capacities are assumed to be independent, normal random variables, while the loads are assumed to be perfectly correlated normal random variables.

In using this model, the linear lifeline is first discretized into N finite size elements (or critical sections) that are connected in series. The overall reliability of the lifeline with a pairwise dependency can then be written as:

$$R_L = R_1 \cdot R_{2|1} \cdot R_{3|2} \cdots R_{i|i-1} \cdots R_{N|N-1} = R_1 \prod_{i=2}^{N} R_{i|i-1} \tag{2-1}$$

where

$R_{i|i-1}$ = Conditional Reliability of element i, given the survival of element i-1

= P (success of i/success of i-1) (2-2)

If the behaviors of elements i and i-1 are represented by two binary random variables X_i and X_{i-1}, where

$$F_i = P\ [X_i = 0] = P\ [\text{failure of } i] \tag{2-3}$$

$$R_i = P\ [X_i = 1] = P\ [\text{success of } i] = 1 - F_i$$

and, if the joint distribution between the random variables X_i and X_{i-1} is considered to be binomial, so that

$$P_{X_i X_{i-1}} = R_i \cdot R_{i-1} \tag{2-4}$$

Then, one can obtain the conditional reliability $R_{i|i-1}$ as follows:

Let the coefficient of correlation between X_i and X_{i-1} be given by

$$\rho_{i,i-1} = \frac{COV[X_i, X_{i-1}]}{\sigma_{X_i}\ \sigma_{X_{i-1}}} \tag{2-5}$$

For binomial distribution $P_{X_i X_{i-1}}$

$$\sigma^2_{X_i} = R_i F_i = R_i(1-R_i) \tag{2-6}$$

$$\sigma^2_{X_{i-1}} = R_{i-1}F_{i-1} = R_{i-1}(1-R_{i-1}) \tag{2-7}$$

$$\begin{aligned} COV[X_i, X_{i-1}] &= E\ [(X_i - m_i)(X_{i-1} - m_{i-1})] \\ &= E\ [X_i X_{i-1}] - m_i E[X_{i-1}] - m_{i-1}\ E[X_i] + m_i m_{i-1} \end{aligned} \tag{2-8}$$

where

m_i = mean of $X_i = R_i$ $\quad$ m_{i-1} = mean of $X_{i-1} = R_{i-1}$

By definition

$$R_{i,i-1} = P[\text{success of } i \text{ and } i-1] = R_{i|i-1} \cdot R_{i-1} = P[X_i=1,\ X_{i-1} = 1] = P_{X_i X_{i-1}} \tag{2-9}$$

But

$$E[X_i,\ X_{i-1}] = \sum_{X_i=0}^{1} \sum_{X_{i-1}=0}^{1} P_{X_i,X_{i-1}}\ X_i\ X_{i-1} = P_{X_i,X_{i-1}} \tag{2-10}$$

Therefore

$$R_{i,i-1} = E[X_i,X_{i-1}] \tag{2-11}$$

Inserting (2-6), (2-7) and (2-8) into (2-11)

$$R_{i,i-1} = E[X_i,X_{i-1}] = \text{COV}\ [X_i,X_{i-1}] + m_i m_{i-1} = \text{COV}\ [X_i,X_{i-1}] + R_i \cdot R_{i-1} \tag{2-12}$$

Inserting (2-5) into (2-12)

$$R_{i,i-1} = \rho_{i,i-1}\sqrt{R_i(1-R_i)\ R_{i-1}\ (1-R_{i-1})} + R_i \cdot R_{i-1} \qquad (2\text{-}13)$$

Inserting (2-9) into (2-13), and solving for $R_{i|i-1}$, we obtain

$$R_{i|i-1} = \frac{R_{i,i-1}}{R_{i-1}} = R_i + \rho_{i,i-1}\frac{\sqrt{R_i(1-R_i)R_{i-1}(1-R_{i-1})}}{R_{i-1}} \qquad (2\text{-}14)$$

The conditional reliability $R_{i|i-1}$ is calculated for all the elements and then the overall reliability of the linear lifeline is obtained from (2-1). The marginal reliability R_i is calculated from the distribution of load L_i and capacity C_i at the critical section (or element) i:

$$R_i = P\ [\text{success of section } i]$$

$$R = P\ [L_i \leqslant C_i] = 1 - P\ [L_i \geqslant C_i] = 1 - P\ [\text{failure of section } i]$$

Marginal Reliability for Different Conditions of Loads and Capacities

1. If L_i and C_i are independent normal random variables with means m_{ℓ_i} and m_{c_i}, and variances $\sigma_{\ell_i}^2$ and $\sigma_{c_i}^2$, then R_i can be obtained as follows:

Let,

$$Z_i = \frac{m_{\ell i} - m_{ci}}{\sqrt{\sigma^2_{\ell_i} + \sigma^2_{ci}}}$$

Then, (2-15)

$$R_i = P[L_i \leq C_i] = F\ [z \leq Z_i] = \int_{-\infty}^{Z_i} \frac{1}{\sqrt{2\pi}} \exp\ [-\tfrac{1}{2} z^2]\ dz$$

The value of R_i is equivalent to the shaded area shown in Fig. 2-1a.

The values of $F[z \leq Z_i]$ are obtained from standard normal density tables.

2. If L_i is a normal random variable and C_i is deterministic, then R_i is obtained from:

Let,

$$Z_i = \frac{m_{\ell i} - C_i}{\sigma_{\ell_i}} \tag{2-16}$$

Then,

$$R_i = P[L_i \leq C_i] = F[z \leq Z_i] \tag{2-17}$$

3. If L_i is deterministic and C_i is a normal random variable, then R_i is calculated from:

Let,

$$Z_i = \frac{L_i - m_{\ell i}}{\sigma_{c_i}} \tag{2-18}$$

Then,

$$R_i = P[L_i \leq C_i] = F[z \geq Z_i] \tag{2-19}$$

4. If L_i and L_{i-1} are probabilistic and perfectly correlated, then they can be discretized into M deterministic values with weighting factors equal to the probability levels at those values. For example, load L_i can be discretized into M levels, L_{ij} (j=1 to M) (Fig. 2-2) with probability values P_j. Each L_{ij} can be treated as a deterministic load similar to 3 above.

$$Z_{ij} = \frac{L_{ij} - m_{c_i}}{\sigma_{c_i}} \tag{2-20}$$

$$R_{ij} = P[\text{success } i | \text{load } j] \tag{2-21}$$

$$R_i = \sum_j R_{ij} P_j \tag{2-22}$$

The condition of perfect correlation is consistent with many practical linear lifeline examples and the advantage of this condition is that this assumption simplifies the calculation of reliability when the capacities, as well as the loadings, are correlated. This procedure is used throughout this study whenever probabilistic loadings are encountered.

BIVARIATE NORMAL MODEL

In this model, the assumption of dependency is considered between capacities of adjacent sections. The loads are assumed to be either deterministic or probabilistic with normal distribution, similar to the previous case. The capacities are assumed to be normal random variables, with a joint bivariate normal distribution between a pair of adjacent capacities.

The failure occurs, as before, when the load at a critical section or element exceeds the capacity. The success or failure of a section is therefore dependent on the behavior of its adjacent section, i.e.,

$$R_{i|i-1} = P[L_i \leq C_i | L_{i-1} \leq C_{i-1}] \tag{2-23}$$

If the capacities C_i and C_{i-1} are random variables, jointly dependent with a bivariate normal distribution, the joint probability distribution between C_i and C_{i-1} would be

$$P_{i,i-1}(C_i, C_{i-1}, \rho_{i,i-1}) = \frac{1}{2\pi\, \sigma_i \sigma_{i-1} \sqrt{1 - \rho^2_{i,i-1}}} \exp\left\{- \frac{1}{2(1-\rho^2_{i,i-1})} \left[\left(\frac{C_i - m_i}{\sigma_i}\right)^2 - 2\rho_{i,i-1} \frac{(C_i - m_i)(C_{i-1} - m_{i-1})}{\sigma_i \sigma_{i-1}} + \left(\frac{C_{i-1} - m_{i-1}}{\sigma_{i-1}}\right)^2\right]\right\} \tag{2-24}$$

where $\rho_{i,i-1}$ = Coefficient of Correlation between C_i and C_{i-1}

m_i, m_{i-1} = Means of C_i and C_{i-1}

$\sigma_i^2, \sigma_{i-1}^2$ = Variances of C_i and C_{i-1}

The marginal distributions for C_i and C_{i-1} are also normal, as shown below:

$$P_i(C_i) = \int_{-\infty}^{\infty} P_{i,i-1}\,(C_i,\ C_{i-1},\ \rho_{i,i-1})\ d\ C_{i-1}$$

$$= \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left[-1/2\left(\frac{C_i - m_i}{\sigma_i}\right)^2\right] \tag{2-25}$$

$$P_{i-1}(C_{i-1}) = \int_{-\infty}^{\infty} P_{i,i-1}(C_i, C_{i-1}, \rho_{i,i-1})\ dC_i \tag{2-26}$$

Deterministic Loads and Probabilistic Capacities

If the loads L_i and L_{i-1} are deterministic and the capacities C_i and C_{i-1} are jointly dependent through a bivariate normal distribution $P_{i,i-1}$, then the conditional reliability $R_{i|i-1}$ is derived from

$$R_{i|i-1} = \frac{R_{i,i-1}}{R_{i-1}} = \frac{P[L_i \le C_i \cap L_{i-1} \le C_{i-1}]}{P[L_{i-1} \le C_{i-1}]} \tag{2-27}$$

where

$$P[L_i \le C_i \cap L_{i-1} \le C_{i-1}] = \int_{L_i}^{\infty} dC_i$$

$$\int_{L_{i-1}}^{\infty} P_{i,i-1}\,(C_i, C_{i-1},\ \rho_{i,i-1})\ dC_{i-1} \tag{2-28}$$

$$P[L_{i-1} \le C_{i-1}] = \int_{-\infty}^{\infty} dC_i \quad \int_{L_{i-1}}^{\infty} P_{i,i-1}\,(C_i,\ C_{i-1},\ \rho_{i,i-1})\ dC_{i-1} \tag{2-29}$$

Then, the total reliability is obtained from

$$R_L = R_1 \cdot R_{2|1} \cdots R_{i|i-1} \cdots R_{N|N-1} = R_1 \prod_{i=2}^{N} R_{i|i-1} \tag{2-30}$$

The values of integrals in equation (2-29) are obtained from standard normal density tables. The values of integrals in equation (2-28) are obtained as follows:

$$L(h,k,\rho) = \int_h^\infty dx \int_k^\infty dy \cdot P(x,y,\rho) \tag{2-31}$$

where

$P(x,y,\rho)$ = Bivariate normal density between random variables x and y with zero means and unit standard deviations

$L(h,k,\rho)$ can be expanded as follows (Ref. 2):

$$L(h,k,\rho) = L(h,o,a) + L(h,o,b) - \begin{cases} 0 & \text{if } hk > 0 \text{ or } hk = 0 \text{ and } h + k \geq 0 \\ \frac{1}{2} & \text{Otherwise} \end{cases} \tag{2-32}$$

$$a = \frac{(\rho h-k)(\text{sgn } h)}{\sqrt{h^2-2\rho hk+k^2}} \qquad b = \frac{(\rho k-h)(\text{sgn } k)}{\sqrt{h^2-2\rho hk+k^2}} \tag{2-33}$$

$$\operatorname{sgn} h = 1 \text{ if } h > 0 \qquad = -1 \text{ if } h < 0 \tag{2-34}$$

The probability values in equation (2-28) are calculated by a transformation

$$P[L_i \leq C_i \cap L_{i-1} \leq C_{i-1}] = L\left(\frac{L_i - m_i}{\sigma_i}, \frac{L_{i-1} - m_{i-1}}{\sigma_{i-1}}, \rho_{i,i-1}\right) \tag{2-35}$$

The values of L(h,o,a) are obtained from bivariate normal graphs given in Appendix B.

The other useful relationships used in this study are

$$L(-h,-k,\rho) = \int_{-\infty}^{h} dx \int_{-\infty}^{k} dy \cdot P(x,y,\rho) \tag{2-36}$$

$$L(h,k,\rho) = L(k,h,\rho) \tag{2-37}$$

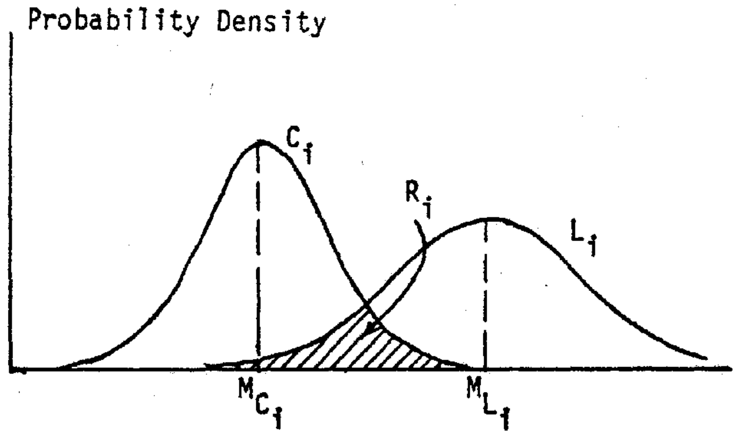
$$L(-h,o,\rho) = 1/2 - L(h,o,-\rho) \tag{2-38}$$

Probabilistic Loads and Probabilistic Capacities

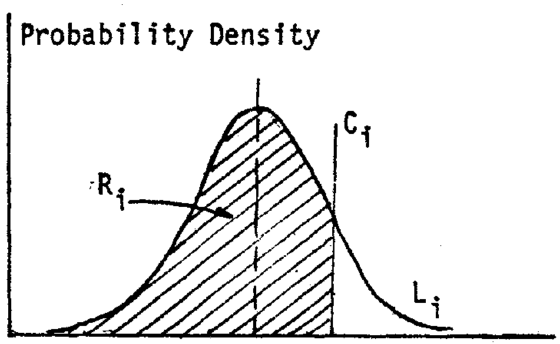
If the loads L_i and L_{i-1} are probabilistic, for most practical purposes they can be assumed to be perfectly correlated. This means that if the loads L_i and L_{i-1} are perfectly correlated, they can be discretized into M load values L_{ij}, each with a weighting factor equal to the probability P_j of the load at that level. Then the total reliability can be obtained similar to (2-22)

$$R_L = \sum_{j=1}^{M} R_1 \prod_{i=2}^{N} R_{i|i-1,j} \cdot P_j \tag{2-39}$$

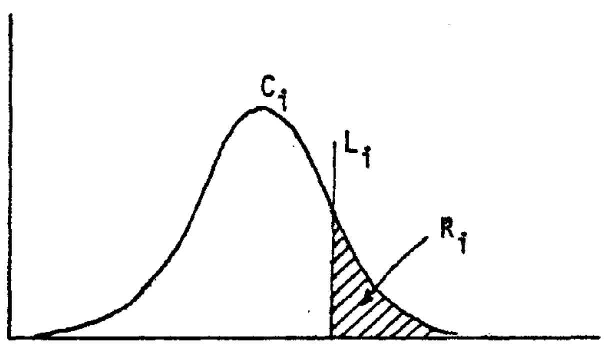
$$\begin{aligned} R_{i|i-1,j} &= \frac{P\,[\text{success } i \text{ and success } i-1 | \text{load } j]}{P\,[\text{success } i-1 | \text{load } j]} \\ &= \frac{R_{i,i-1|j}}{R_{i-1|j}} \end{aligned} \tag{2-40}$$



(a) Probabilistic Load L_i and Capacity C_i



(b) Probabilistic Load L_i and Deterministic Capacity C_i



(c) Deterministic L_i and Probabilistic C_i

FIGURE 2-1 DEFINITION OF RELIABILITY AT CROSS-SECTION i

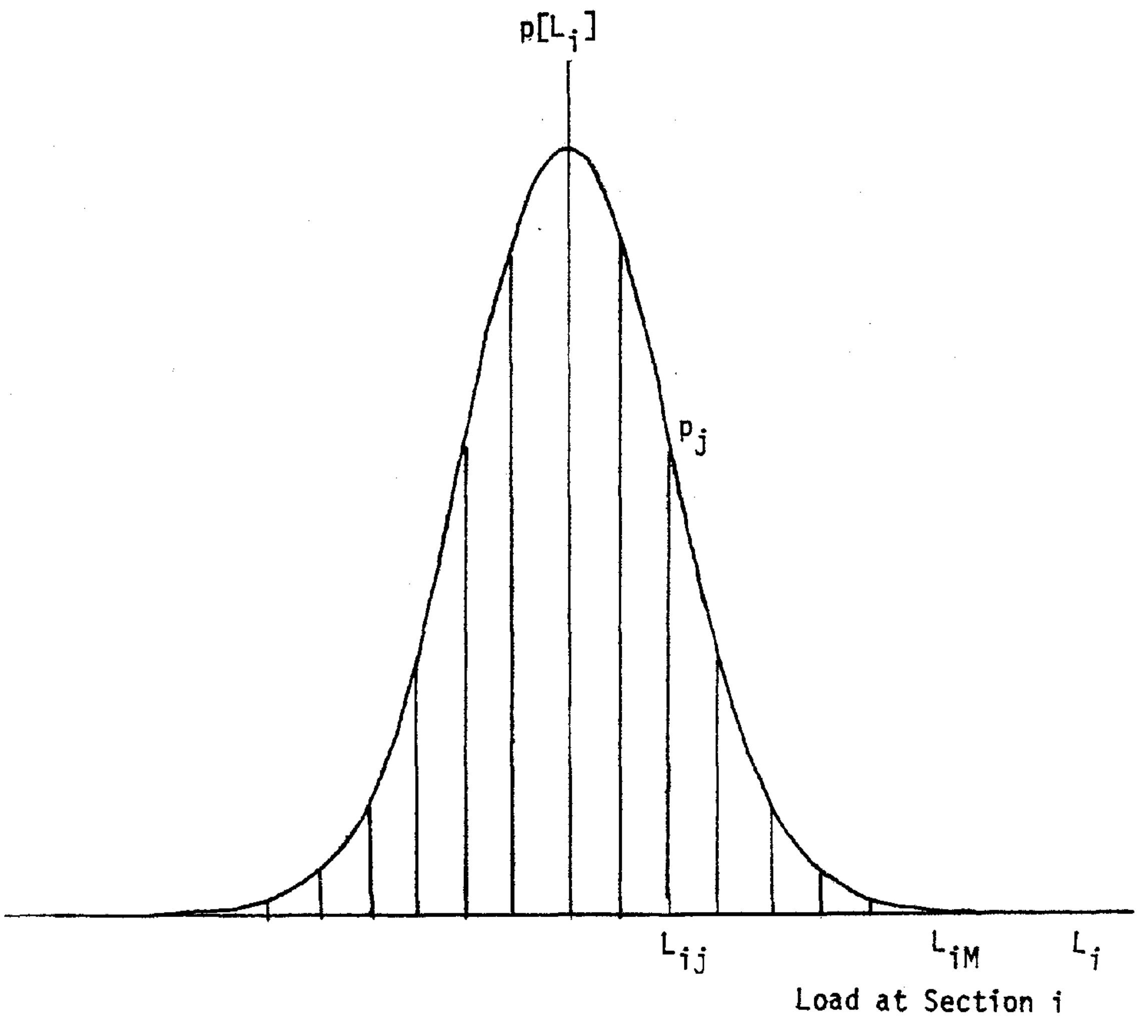


FIGURE 2-2 DISCRETIZATION OF PROBABILISTIC LOADING

3. RELIABILITY ANALYSIS OF A BRIDGE

INTRODUCTION

This chapter describes the reliability analysis of a three-span, reinforced concrete bridge. The methodology presented in Chapter 2 was applied to this bridge example, using both the binomial and the bivariate normal models. The original structure is an existing ten-span, reinforced concrete bridge, located in the City of Oakland in California. For simplicity, only three central spans of the bridge were used for the reliability assessment. The extension of the methodology to ten spans should be obvious.

The reliability assessment was performed for both horizontal and vertical components of earthquake. The failure at a critical section of the bridge was assumed to occur when the applied moment exceeded the ultimate moment capacity computed using the recommendations of the ACI Code (Ref. 3). The reliability assessments were performed considering the applied moment loads to be both deterministic and probabilistic. In addition, the statistical dependency between the behaviors of adjacent elements was modeled using a binomial model, as well as a joint bivariate normal model. The influence of the correlation between the capacities of adjacent elements was thus also investigated.

Finally, a damage probability matrix was also developed for the case of the bridge subjected to vertical ground motion. The resulting damage matrix was then used in a decision analysis example to demonstrate the application of methodology developed in this study to a more general seismic risk problem.

DESCRIPTION OF THE BRIDGE EXAMPLE

A three-span box girder bridge example was selected for the reliability analysis described in this chapter. The dimensions of the bridge are shown in Fig. 3-1. The bridge consists of two 85 ft end spans and one 108 ft central span. The reinforcement details for the columns of the bridge are shown in Fig. 3-2. The cross-sections assumed for the reliability analysis in this study are shown in Fig. 3-3.

The material and section capacities of the girders, also taken from the original design, were as follows:

f'_c = compressive strength of concrete = 4 ksi

f_y = yield strength of steel = 60 ksi

w_1 = weight per unit length of spans AB and CD = 21 k/ft

w_2 = weight per unit length of span BC = 22.6 k/ft

C_1^+ = positive moment capacity of spans AB and CD = 10261 k-ft

C_2^+ = positive moment capacity of span BC = 23089 k-ft

$C_1^- = C_2^-$ = negative moment capacity of spans AB, BC and CD = 29,296 k-ft

C_a, C_b, C_c, C_d = Random moment capacities of columns a, b, c, and d

RELIABILITY ANALYSIS - HORIZONTAL GROUND MOTION

For horizontal ground motion, the reinforced concrete bridge considered in this example would primarily act as a rigid girder supported on flexible columns, since girders were much stiffer than columns. This means that failure would occur mainly at the top and bottom of the columns. For this example, the failure was assumed to occur when plastic hinges formed at critical cross-sections, i.e., when the applied moments exceeded the ultimate moment capacities at the critical cross-sections, thus forming a mechanism. The failure mechanism for the bridge is shown in Fig. 3-4. The resulting forces and moments due to dead and earthquake loads at the ends of each column are shown in Figs. 3-5 and 3-6.

To account for the randomness of the material properties of the columns, a normal distribution was assumed for the reinforcement ratio, r, for each column, with mean $\bar{r}$ equal to the design reinforcement ratio r_o = .035. Two types of distributions were used, a narrow band distribution with standard deviation, σ_{r_1} = 0.005, and a wide band distribution with a standard deviation, σ_{r_2} = 0.015. Since the ultimate moment capacities of the columns were linearly dependent on the reinforcement ratio, they would also have normal distributions. The capacities were calculated using column interaction diagrams shown in Fig. 3-7.

The applied loads (moments and axial loads) were calculated at the two ends of the columns for various levels of earthquake accelerations, $\ddot{u}g$ varying from 0 to 2.0g. (The influence of axial loads was found to be insignificant for this bridge example.) The applied loads were considered to be deterministic in this example, while the capacities were assumed to be probabilistic, as mentioned earlier.

The reliability was then calculated as follows:

For each value of $\ddot{u}g$, one can write the probability of failure at A (or plastic hinge forming at A), as follows:

$$P_A\,(\ddot{u}g) = P\left[C_a \leqslant S_1 \middle| \ddot{u}g\right] \qquad (3\text{-}1)$$

$$\text{Therefore, } P_A(\ddot{u}g) = \frac{S_1(\ddot{u}g) - \bar{C}_a}{\sigma_{C_a}} \qquad (3\text{-}2)$$

where $\bar{C}_a$ and σ_{C_a} are mean and standard deviations of C_a. Similar expressions were used for other columns.

The loads for different columns were considered to be perfectly correlated. However, the column behaviors were considered to be independent, since there was no indication that their capacities were statistically correlated. For this reason, the probability of total failure for the bridge was computed as follows:

$$P_L(\ddot{u}_g) = P_A(\ddot{u}_g)\ P_B(\ddot{u}_g)\ P_C(\ddot{u}_g)\ P_D(\ddot{u}_g) \quad (3\text{-}3)$$

and the total reliability of the bridge, for a ground acceleration $\ddot{u}g$, was therefore given by:

$$R_L(\ddot{u}_g) = 1 - P_L(\ddot{u}_g) \quad (3\text{-}4)$$

The values of the reliabilities for horizontal earthquake for the two types of normal distributions (narrow and wide) are presented in Table 3-1. The corresponding curves are shown in Figure 3-8.

RELIABILITY ANALYSIS - VERTICAL GROUND MOTION

Since the axial stiffness of the columns was very high, for vertical excitation the columns would behave almost as rigid bodies in the vertical direction. The bridge structure was therefore modeled as a continuous girder shown in Figure 3-9b. Since the flexural stiffness of columns was much smaller than that of girders, the end supports were assumed to be simple hinges. The loads on the three girders due to earthquake excitation were obtained by using equivalent static loads applied at the center of each span (Fig. 3-9c). The resulting moments from the dead and earthquake loads are shown in Figures 3-10a and b. In Figure 3-10c, the total moments due to dead and earthquake loads and the capacities at critical sections of the girders are illustrated. Due to symmetry, only C_1^+, C_1^- and C_2^+ were required in the analysis. The capacities were assumed to be probabilistic. A normal distribution was used to represent them, with their mean values equal to the design values. Two types of normal distributions were used, namely, narrow band and wide band. The parameters for these distributions are given in Table 3-2.

The failure was assumed to occur when a mechanism was formed. Since the behavior of the third span was exactly similar to the first span because of the symmetry, the analysis was reduced to computing the reliability of two spans, AB and BC only. The reliability of the bridge was then considered to be the probability of survival of all three spans.

Two types of models were studied to represent the statistical dependency of adjacent spans: a binomial model and a bivariate normal model. In the binomial model, the dependency was assumed between the behaviors (or mechanisms) of adjacent spans, whereas in the bivariate normal model, the dependency was assumed between the capacities of the girders. The reliability analyses were performed for deterministic as well as probabilistic loads.

Use of Binomial Model for Statistical Dependence of Adjacent Span

For this model, it was assumed that the success or failure of any span would be dependent on the success or failure of its adjacent spans. For example, it was assumed that the mechanism 2 (for span 2) was dependent on mechanism 1 (for span 1) (see Figure 3-9d). (Mechanisms 3 and 1 are the same.) The total reliability, R_L, of the bridge can therefore be considered to be equal to the probability of success of all spans.

$$R_L = R_{1,2} = R_1 \cdot R_{2|1} \tag{3-5}$$

where the Marginal Reliability for Span 1, R_1, is given by

$$\begin{aligned} R_1 &= P(\text{success of span AB against mechanism AEB}) \\ &= 1-P[\text{plastic hinges at E and B}] \qquad (3\text{-}6) \\ &= 1-P[\text{plastic hinge at E}].\ P[\text{plastic hinge at B}] \\ &= 1-p_1 p_2 \end{aligned}$$

and

$R_{2|1}$ = Conditional reliability of span 2, given the survival of span 1

The assumption has been made that the formations of plastic hinges at E and B were independent.

The Marginal Reliability for span BC was given by:

$$R_2 = 1\text{-}P[\text{plastic hinge at B}].\ P[\text{plastic hinge at F}] = 1 - p_2 p_3 \quad (3\text{-}7)$$

The values of p_1, p_2 and p_3 were obtained from the standard normal density tables, using the following expressions:

$$p_1 = P[S_1^+ \geqslant C_1^+] = P[\text{plastic hinge at E}] \quad (3\text{-}8)$$

$$p_2 = P[S_1^- \geqslant C_1^-] = P[\text{plastic hinge at B}] \quad (3\text{-}9)$$

$$p_3 = P[S_2^+ \geqslant C_2^+] = P[\text{plastic hinge at F}] \quad (3\text{-}10)$$

According to equation (2-14), the conditional reliability of span BC can then be written as

$$R_{2|1} = R_2 + \rho \frac{\sqrt{R_2(1-R_2)R_1(1-R_1)}}{R_1} \quad (3\text{-}11)$$

where R_1 and R_2 are marginal reliabilities of spans AB and BC, respectively

and ρ = coefficient of correlation between behaviors of spans AB and BC

Deterministic Loading

A procedure similar to that employed for horizontal earthquake motion was used for the evaluation of reliability of the system for vertical earthquake motion. The reliability values corresponding to different levels of vertical earthquake motion, $\ddot{V}g$, are presented in Tables 3-3 and 3-4 for the two types of normal distributions for the capacities of the girders. The corresponding reliability curves are shown in Figure 3-11.

Probabilistic Loading

The real earthquake loading is probabilistic in nature. For example, for each level of earthquake ground acceleration, $\ddot{V}g$, a probability level $P(\ddot{V}g)$ can be assigned which can be determined from seismic risk studies or statistical analyses of past earthquake history. In addition, the loadings at two points of a lifeline might be statistically correlated.

The earthquake loading, in general, can be assumed to be perfectly correlated from one point of a lifeline to the other. A probabilistic loading with a distribution $P(\ddot{V}g)$ can thus be discretized into deterministic load levels $\ddot{V}g_i$ with weighting factors equal to $P(\ddot{V}g_i) = P_i$. The overall reliability can be obtained using equation (2-22), as follows:

$$R_L = \sum_i R_i P_i \qquad (3\text{-}12)$$

where R_i = reliability of the bridge for the load level $\ddot{V}g_i$
= P(survival of the bridge $|\ddot{V}g_i$)

For this case study, the probability distribution of vertical ground acceleration $\ddot{V}g$ was assumed to be normal, as shown in Figure 3-12. The overall reliability of the 3-span bridge was thus obtained from Tables 3-3 and 3-4, using equation (3-12). These values are shown in Table 3-5 for narrow-band and wide-band distributions on capacities, and for different values of coefficient of correlation ρ.

Use of Bivariate Normal Model for Statistical Dependence of Adjacent Spans

For this case study, the statistical dependency criteria for adjacent spans was assumed to be a joint bivariate normal distribution between the capacities C_1^+, C_1^- and C_2^+ at critical cross-sections of the bridge. The earthquake loading was considered to be both deterministic and probabilistic as discussed earlier for the binomial case. The failure was also supposed to occur in a manner similar to the binomial case, namely when a mechanism was formed.

In the binomial model, it was assumed that the behavior of the two spans AB and BC, and therefore the mechanisms 1 and 2, were dependent on each other by a coefficient of correlation ρ_{12}. In Appendix A, the value of ρ_{12} is calculated and compared with a threshold value ρ_0. By showing that $\rho_{12} < \rho_0$, one can reasonably assume that the mechanisms 1 and 2, as well as all the mechanisms, are uncorrelated by virtue of symmetry. This approach has been used by other authors for the study of the reliability of structural frames (Ref. 4).

The total reliability of the bridge was therefore computed as follows

$$R_L = 1-P[Z_1 \leq 0 \text{ or } Z_2 \leq 0] = 1-P[Z_1 \leq 0]-P[Z_2 \leq 0] + P[Z_1 \leq 0].\ P[Z_2 \leq 0] \qquad (3\text{-}13)$$

where

$$P(Z_1 < 0) = P(\text{mechanism AEB in span AB}) = P(\text{plastic hinges formed at E and B}) = p_{11} \qquad (3\text{-}14)$$

The formation of plastic hinges at points E and B would be dependent in contrast to the binomial case, since the capacities at E and B were assumed to be jointly distributed through a bivariate normal probability density function.

Similarly

$$P[Z_2 \leq 0] = P(\text{mechanism BFC in span BC}) = P(\text{plastic hinges formed at B and F}) = p_{12} \quad (3\text{-}15)$$

The value of p_{11} was obtained from

$$p_{11} = \int_{-\infty}^{S_1^+} dC_1^+ \int_{-\infty}^{S_1^-} p_{C_{11}} (C_1^+, C_1^-, \rho)\, dC_1^- \quad (3\text{-}16)$$

where $p(C_1^+, C_1^-, \rho)$ is the joint probability density function between the capacities C_1^+ and C_1^-. The distribution is bivariate normal with a coefficient of correlation ρ between C_1^+ and C_1^-. S_1^+ and S_1^- are the load levels at the critical sections E and B.

Similarly, for span BC:

$$p_{12} = \int_{-\infty}^{S_1^-} dC_1^- \int_{-\infty}^{S_2^+} p_{C_{12}} (C_1^-, C_2^+, \rho)\, dC_2^+ \quad (3\text{-}17)$$

with a joint bivariate normal density function $p_{C_{12}}$ between capacities C_1^- and C_2^+ at the sections B and F. The load levels are S_1^- and S_2^+ at these sections.

The total reliability, R_L, was computed from (3-13), using bivariate normal curves. The resulting reliability values for different values of $\ddot{V}g$ are summarized in Table 3-6, and the reliability curve $R_L(\ddot{V}g)$ is shown in Figure 3-13 for the narrow-band normal distribution.

DAMAGE ANALYSIS

A damage analysis was performed for the three-span bridge example for vertical ground motion. A damage probability matrix was also developed for different damage levels of the bridge. The resulting damage matrix was then used in a decision analysis example. The following scenario was used in the damage analysis.

Before any plastic hinge is developed in the girders, the bridge would undergo elastic deformations. Under this condition, the resulting deformations are quite small, so that almost no damage occurs to the bridge.

However, as soon as a plastic hinge is formed, the change of slopes could cause some damage to the bridge. The differential rotation $\Delta\theta_B$ is shown in Fig. 3-14a where

$$\Delta\theta_B = |\theta_1| + |\theta_2| \qquad (3\text{-}18)$$

For the earthquake loadings P_1 and P_2, the rotations θ_1 and θ_2, after the formation of plastic hinges can be estimated from standard slope deflection equations.

It was found that, for the vertical ground acceleration $\ddot{V}g$ of approximately 7 ft/sec^2, the value of $\Delta\theta_B$ was equal to 0.5^o. This value was considered to be small and was expected to cause no damage.

In the above calculations, it was assumed that a plastic hinge would form when the probability of occurrence of a plastic hinge is greater than a threshold probability level p_o equal to 0.10.

Using the above criterion, for a higher acceleration level of $\ddot{V}g$ = 9 ft/sec^2, it was found that

$$p_3 = P(\text{failure at F}) = .125 > p_o$$
$$p_2 = P(\text{failure at B}) = .37 > p_o$$
$$p_1 = P(\text{failure at E}) = .004 < p_o$$

The hinges would therefore develop at F and B and the mechanism BFC would form (Figure 3-14b).

Similarly, as the acceleration levels were increased, it was found that, at $\ddot{V}g$ = 22 ft/sec^2, the second collapse mechanism would occur, resulting in possible collapse.

Three distinct damage states were assumed for the bridge under vertical ground motion, namely, minor, moderate and major damage, as shown in Figures 3-14a, b and c. These damage levels are defined below:

Minor Damage:

Formation of plastic hinges at the supports where negative moments occur, resulting in possible minor cracks in sidewalks and pavement of the bridge.

Moderate Damage:

Formation of first mechanism (in the middle span). This could involve major cracks in the sidewalks and pavement of the bridge and could cause disruption of traffic.

Major Damage:

Formation of second mechanism - collapse in all the three spans.

Using the above approach, a damage probability matrix was developed, and is shown in Table 3-7.

Decision Analysis

The damage probability matrix calculated in the previous section was actually the conditional probability $P(\text{damage}|\ddot{V}g)$ of each damage level for a given ground acceleration. These values can be used directly in decision analysis when a decision has to be made between different alternatives of an action.

As an example, suppose the example bridge was already subjected to an earthquake that caused minor damage. If it was to be investigated whether one should repair the bridge for full serviceability or tolerate the probable economic losses due to future earthquakes, a damage probability matrix could be developed, given that the bridge has already suffered minor damage, as shown in Table 3-8.

The decision would then have to be made between the expected costs associated with no repair and the expected cost associated with performing repair (Fig. 3-15). The decision criteria are therefore reduced to costs and monetary gains and losses here. A detailed decision analysis could therefore be performed readily if utilities and the actual costs were provided using a decision tree similar to the one shown in Fig. 3-15.

TABLE 3-1

Reliability of the Three-Span Bridge Under Horizontal Ground Acceleration $\ddot{u}g$ for Two Types of Normal Distributions for Column Reinforcement Ratio, r

Ground Acceleration	Narrow-Band Distribution of r σ_{r_1} = .005					Wide-Band Distribution of r σ_{r_2} = .015				
$\ddot{u}g$(ft/sec^2)	P_A	P_B	P_C	P_D	R_L	P_A	P_B	P_C	P_D	R_L
16	.045	0	.08	.045	1.0	.28	.087	.319	.284	.998
20	.35	0	.36	.36	1.0	.448	.156	.452	.452	.986
24	.83	.02	.18	.83	.997	.63	.255	.382	.63	.961
32	1.0	.603	.997	1.0	.40	.89	.54	.82	.89	.65
40	1.0	.992	1.0	1.0	.008	.98	.79	.95	.98	.28
48	1.0	1.0	1.0	1.0	0	.999	.937	.990	.999	.07

Notes: (1) r = Reinforcement ratio for the Columns

(2) σ_r = Standard deviation of r

(3) P_A, P_B, P_C, and P_D = Probabilities of formation of plastic hinges in Columns a, b, c, and d, respectively

(4) R_L = Reliability of the bridge

TABLE 3-2

Parameters for Distributions of Capacities of Girder Cross-Sections of the Bridge (Vertical Ground Motion Example)

Moment Capacity	Narrow-Band Distribution of Capacities			Wide-Band Distribution of Capacities		
	Mean m (k-ft)	Standard Deviation σ_c (k-ft)	Coeff. of Variation σ_c/m	Mean m (k-ft)	Standard Deviation σ_c (k-ft)	Coeff. of Variation σ_c/m
C_1^+	14547	1250	.09	14547	3750	.27
C_1^-	29296	1250	.04	29296	3750	.12
C_2^+	23089	1250	.05	23089	3750	.15

TABLE 3-3

Reliability of the Bridge Under Deterministic Vertical Ground Acceleration, $\ddot{V}g$, for Narrow-Band Distribution of Capacities - Binomial Model

(σ_c = 1250 k-ft)

Ground Acceleration	Failure Probabilities			Marginal Reliability		Conditional Reliability $R_{2\|1}$						Total Reliability of the Bridge, R_L					
$\ddot{V}g$ (ft/sec^2)	p_1	p_2	p_3	R_1	R_2	ρ=0	.2	.4	.6	.8	1.0	ρ=0	.2	.4	.6	.8	1.0
1	0	0	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
4	0	0	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
8	.003	.33	.022	.999	.993	.993	.994	.994	.995	.995	.996	.922	.993	.993	.994	.994	.995
10	.006	.869	.386	.995	.665	.665	.672	.678	.685	.691	.698	.66	.67	.67	.68	.69	.69
12	.011	.996	.922	.989	.081	.081	.087	.929	.987	.105	.11	.08	.086	.092	.098	.103	.109

Notes: (1) p_1, p_2, p_3 = Probabilities of formation of plastic hinges at points E, B, and F of the girders

(2) R_1, R_2 = Reliability for spans 1 and 2, respectively

(3) ρ = Coefficient of correlation between behaviors of adjacent spans

(4) $R_{2|1}$ = Conditional reliability for span 2, given survival of span 1

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TABLE 3-4

Reliability of the Bridge Under Deterministic Vertical Ground Acceleration, $\ddot{V}g$, for Wide-Band Distribution of Capacities - Binomial Model

(σ_c = 3750 k-ft)

Ground Acceleration	Failure Probabilities			Marginal Reliability		Conditional Reliability $R_{2\|1}$						Total Reliability of the Bridge, R_L					
$\ddot{V}g$ (ft/sec^2)	p_1	p_2	p_3	R_1	R_2	ρ=0	.2	.4	.6	.8	1.0	ρ=0	.2	.4	.6	.8	1.0
4	.14	.117	.035	.983	.996	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
8	.179	.44	.25	.92	.89	.89	.91	.93	.94	.96	.98	.82	.84	.86	.87	.88	.90
10	.20	.644	.460	.87	.70	.70	.74	.77	.81	.84	.88	.61	.64	.67	.70	.73	.77
12	.224	.813	.681	.82	.45	.45	.50	.54	.59	.63	.68	.37	.41	.44	.48	.52	.56
16	.274	.973	.947	.73	.08	.08	.11	.14	.18	.21	.24	.06	.08	.10	.13	.15	.18

Notes: (1) p_1, p_2, p_3 = Probability of formation of plastic hinges at points E, B, and F of the girders

(2) R_1, R_2 = Reliability for spans 1 and 2, respectively

(3) ρ = Coefficient of correlation between behaviors of adjacent spans

(4) $R_{2|1}$ = Conditional reliability for span 2, given survival of span 1

TABLE 3-5

Reliability of the Bridge Under Probabilistic Vertical Ground Acceleration with the Distribution of Fig. 3-12

Binomial Model

Coefficient of Correlation	Reliability of Bridge $R_L = \sum_i R_i P_i$	
ρ	Narrow-Band Distribution of Capacities	Wide-Band Distribution of Capacities
0	.862	.777
.2	.863	.796
.4	.863	.815
.6	.867	.831
.8	.870	.847
1.0	.870	.870

TABLE 3-6

Reliability Calculations for the Bridge Under Vertical Ground Motions, $\ddot{V}g$ - Bivariate Normal Model

Coeff. of Corr.	$\ddot{V}g = 6\ ft/sec^2$			$\ddot{V}g = 8.56\ ft/sec^2$			$\ddot{V}g = 10.34\ ft/sec^2$			$\ddot{V}g = 12\ ft/sec^2$		
ρ	p_{11}	p_{12}	R_L	p_{11}	p_{12}	R_L	p_{11}	p_{12}	R_L	p_{11}	p_{12}	R_L
0	.001	.01	.99	.001	.03	.97	.006	.46	.54	.01	.92	.08
.2	.001	.01	.99	.001	.04	.96	.006	.47	.53	.01	.92	.08
.4	.001	.01	.99	.001	.05	.95	.006	.48	.52	.01	.92	.08
.6	.001	.01	.99	.001	.055	.945	.006	.49	.51	.01	.93	.07
.8	.001	.01	.99	.001	.067	.93	.006	.50	.50	.01	.93	.07
1.0	.001	.01	.99	.001	.067	.093	.006	.50	.50	.01	.94	.06

Notes:

(1) ρ = Correlation coefficient between capacities of adjacent girders

(2) p_{11} = Probability of formation of plastic hinges at E and B

(3) p_{12} = Probability of formation of plastic hinges at B and F

(4) R_L = Reliability of the bridge

TABLE 3-7

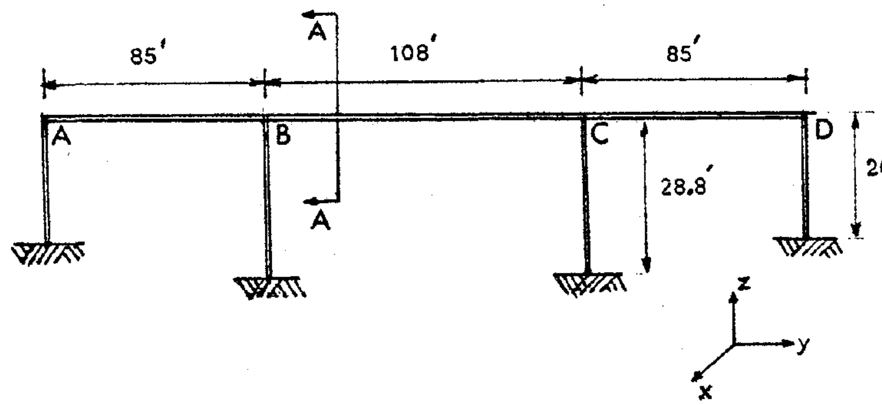
Damage Probability Matrix for Vertical Ground Motions, $\ddot{V}g$

Ground Acc.	Probability of Damage			
$\ddot{V}g$	No Damage	Minor	Moderate	Major
<.22g	1.0			
.22g	.89	.11		
.25g	.66	.33	.01	
.28g	.58	.37	.05	
.31g		.66	.33	.01
.68g			.86	.14

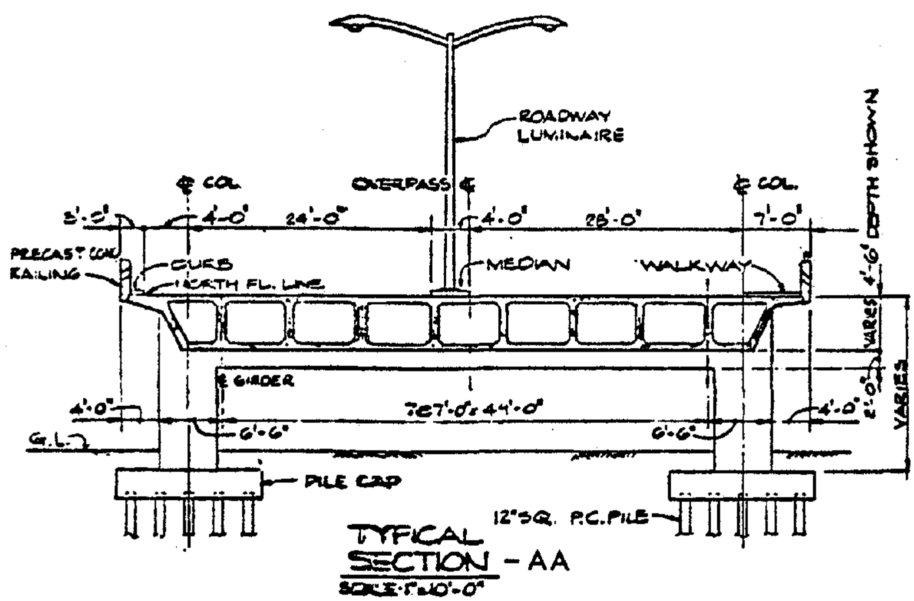
TABLE 3-8

Damage Probability Matrix for Vertical Ground Motions, $\ddot{V}g$, Given Minor Damage

Ground Acc.	Probability of Damage			
$\ddot{V}g$	No Damage	Minor	Moderate	Major
<.22g		1.0		
.22g		.998	.002	
.25g		.96	.03	.01
.28g		.875	.125	
.31g		.48	.50	.02
.68g			.86	.14



(a) Two-Dimensinal Bridge Frame



(b) Typical Cross-Section

FIGURE 3-1 Dimensions and Typical Cross-Section of the Three-Span Bridge

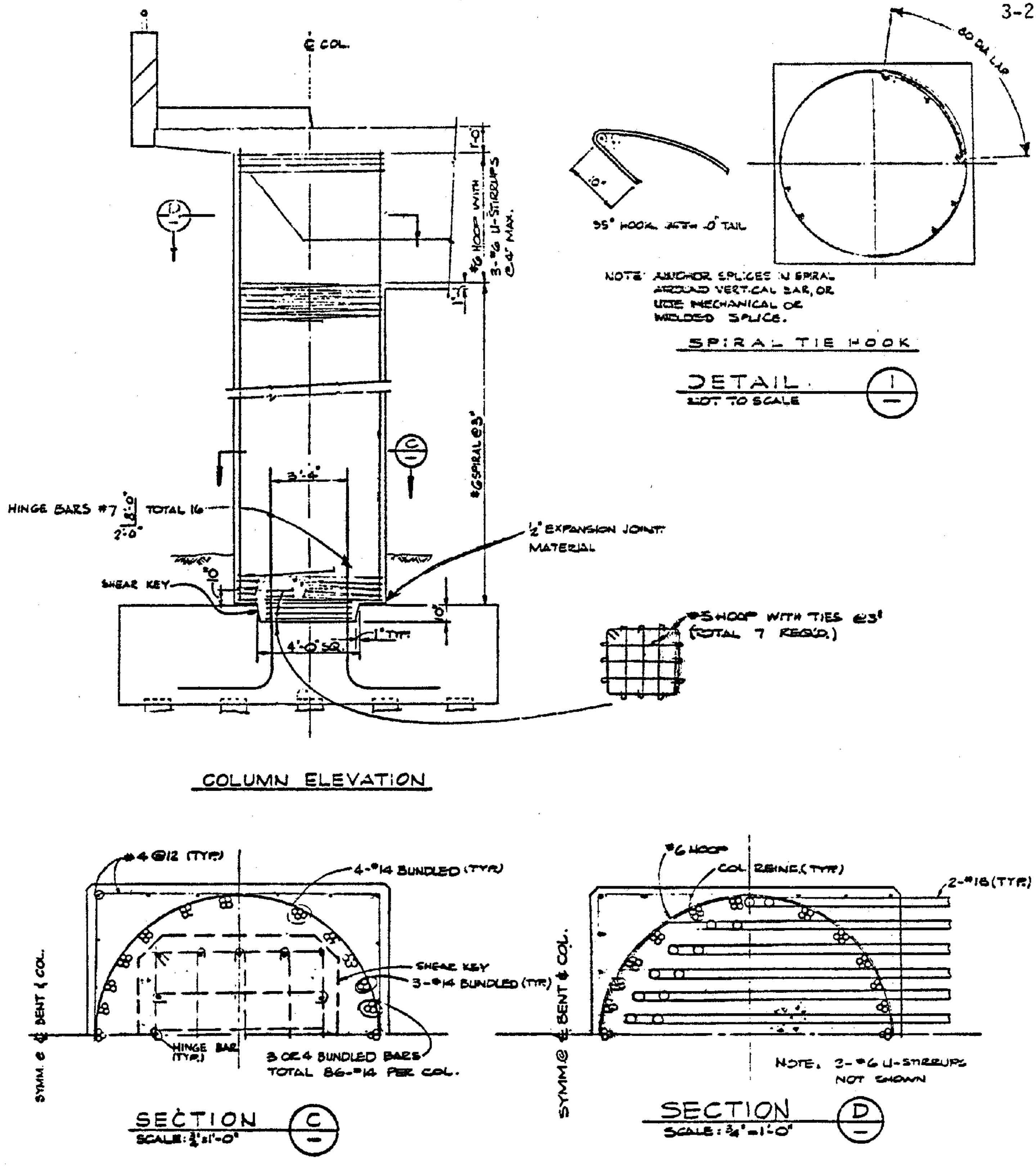


FIGURE 3-2 REINFORCEMENT DETAILS FOR COLUMNS OF THE BRIDGE

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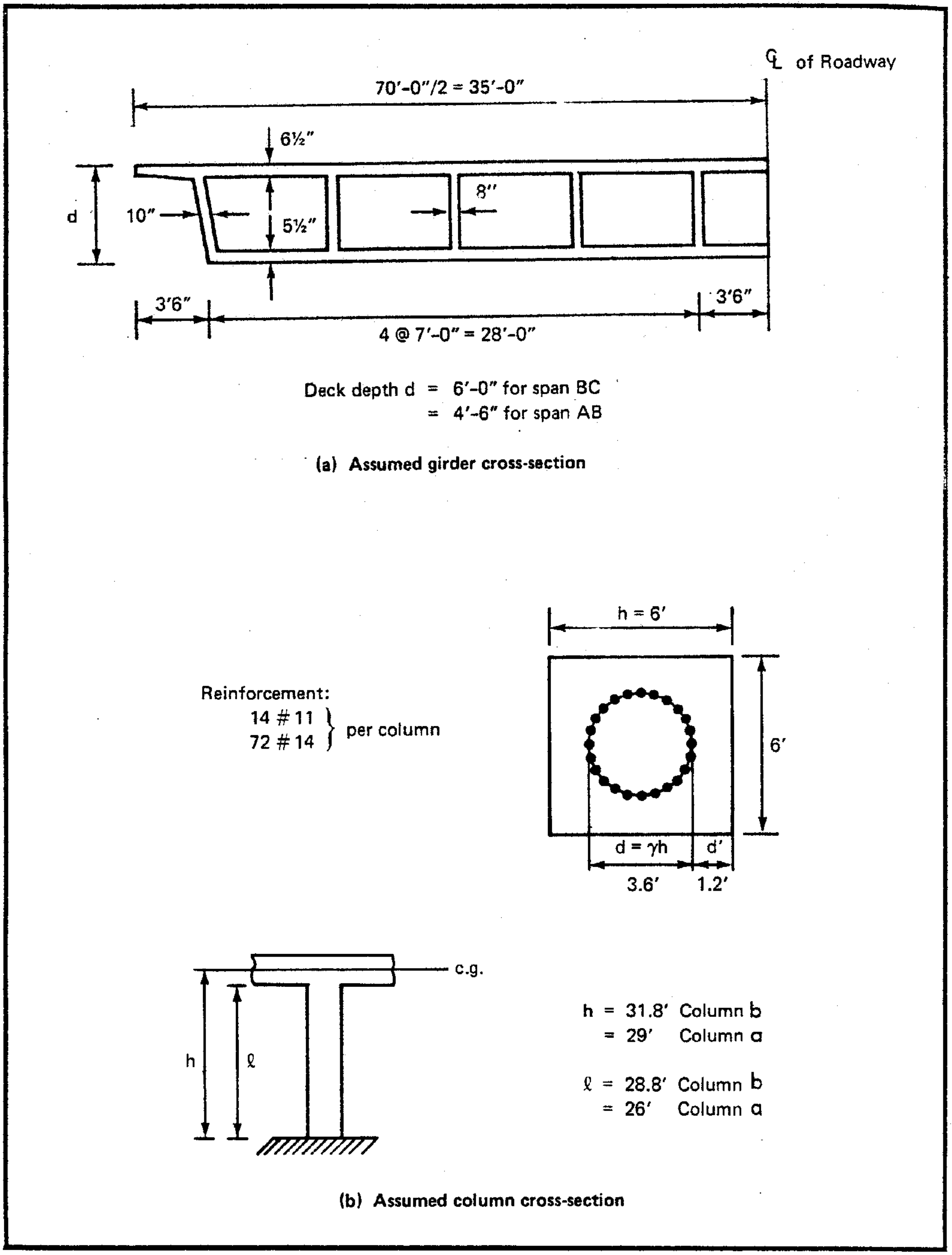


FIGURE 3-3 ASSUMED DIMENSIONS FOR THE RELIABILITY ANALYSIS

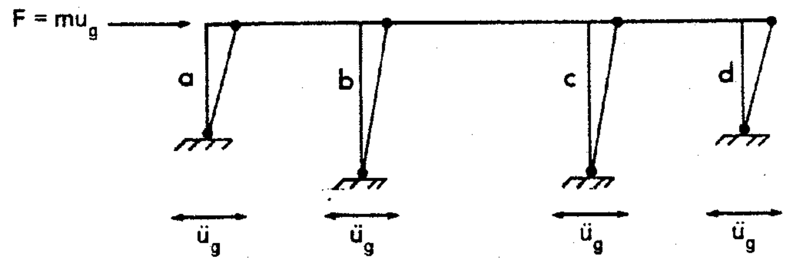


FIGURE 3-4 FAILURE MECHANISM DUE TO HORIZONTAL GROUND ACCELERATION $\ddot{u}_g$

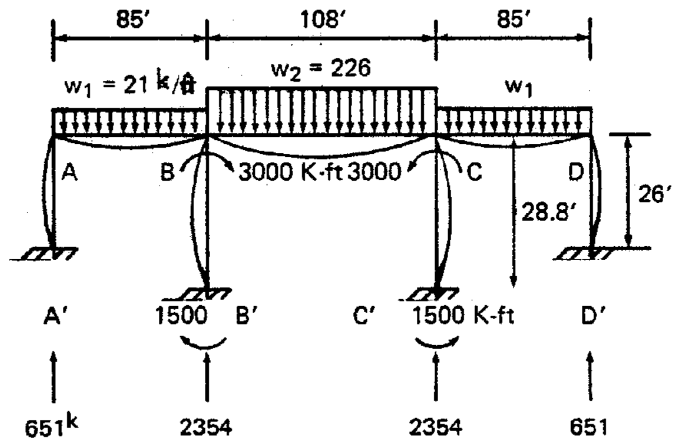


FIGURE 3-5 SUMMARY OF DEAD LOADS AND RESULTING FORCES IN THE COLUMNS

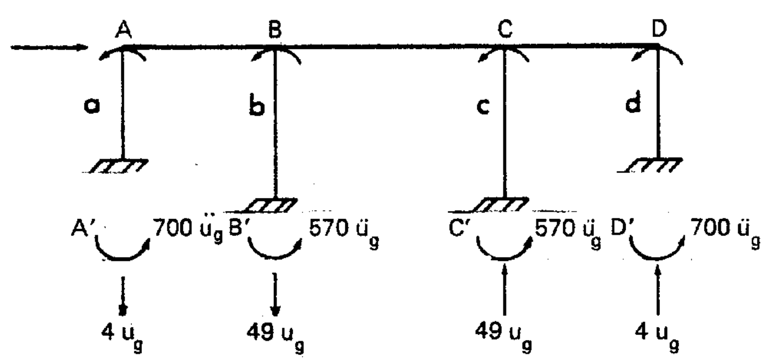


FIGURE 3-6 SUMMARY OF EARTHQUAKE LOADS AND RESULTING FORCES IN THE COLUMNS

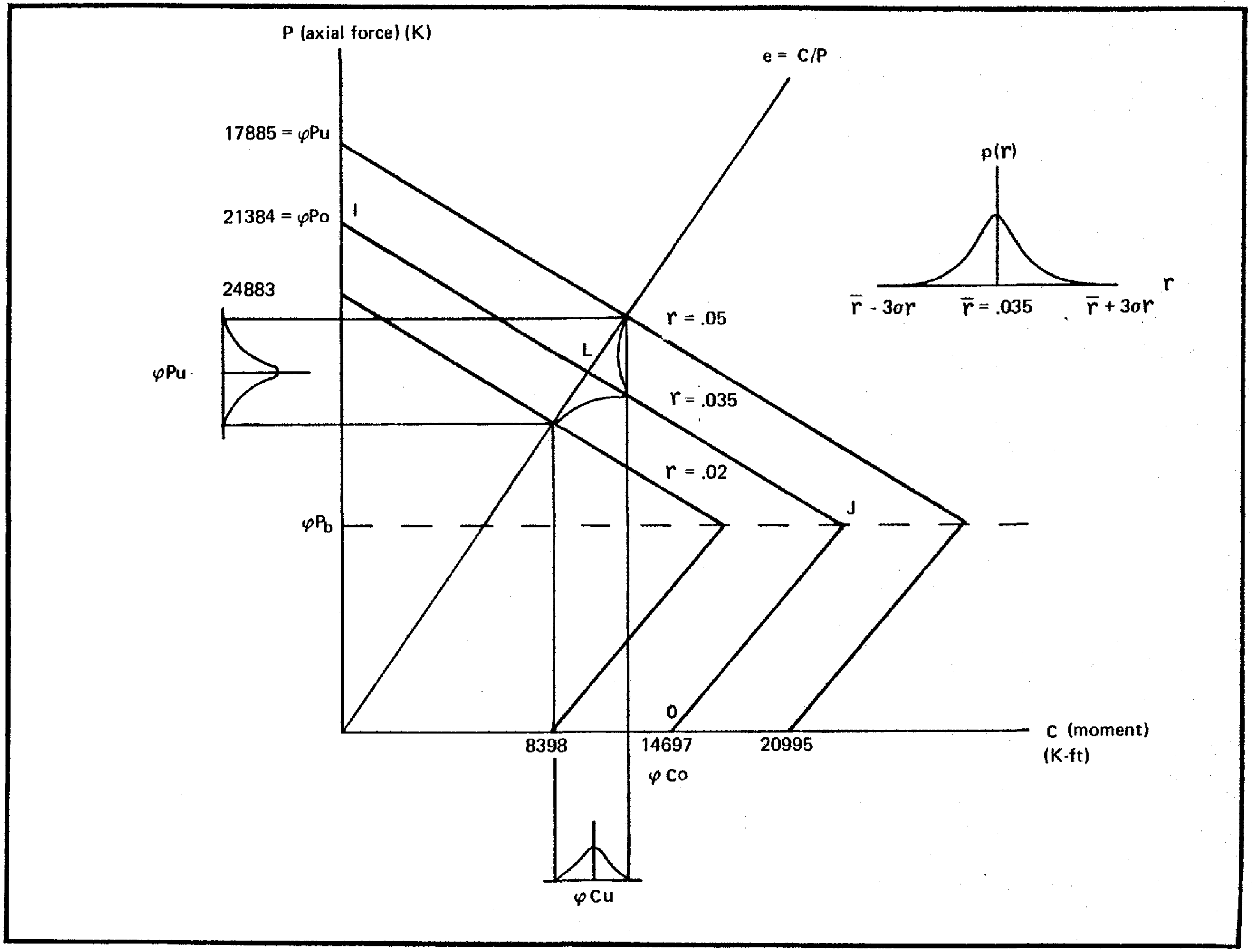


FIGURE 3-7 INTERACTION DIAGRAM FOR A COLUMN ALONG WITH PROBABILITY DISTRIBUTIONS OF φC_u, φP_u and r

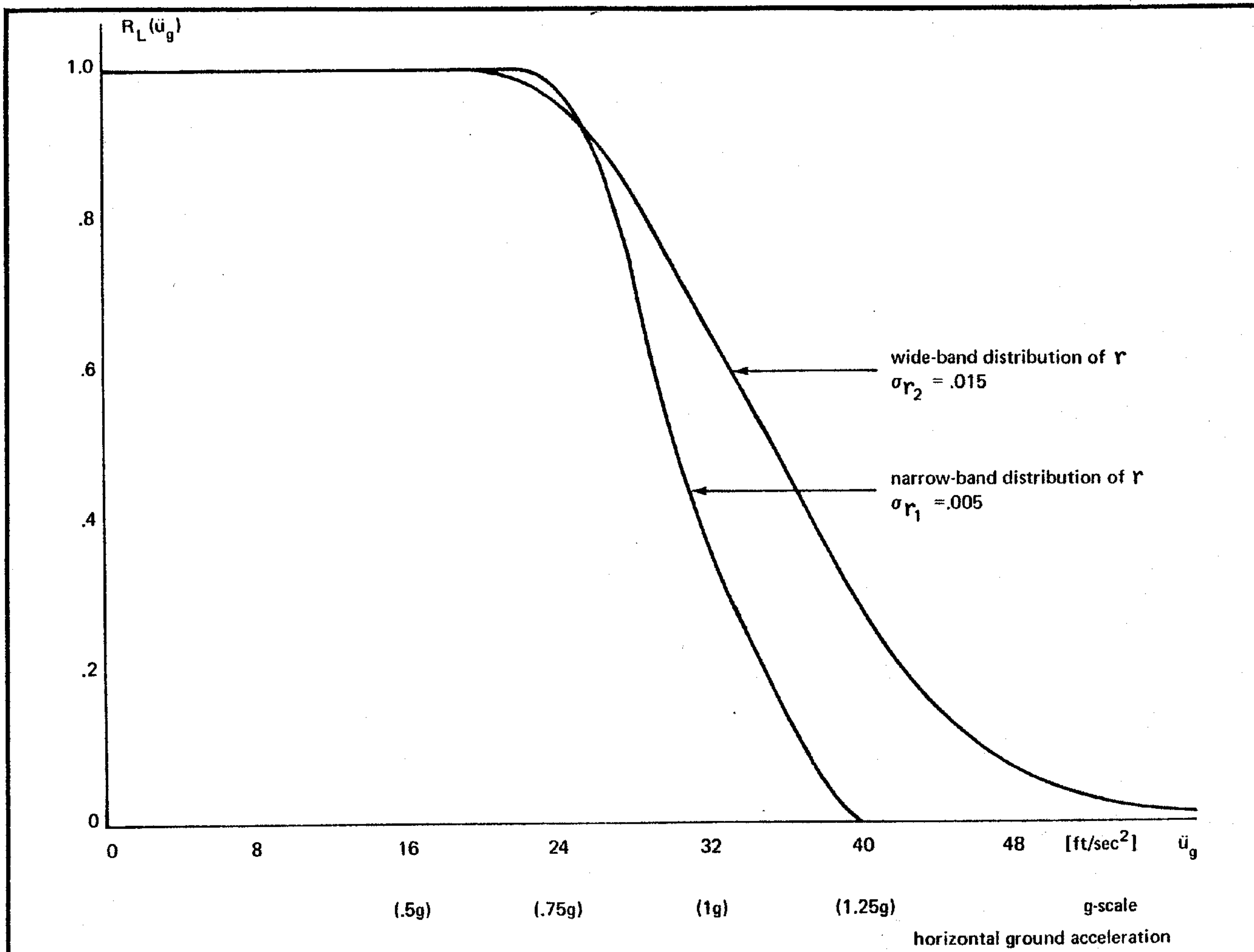
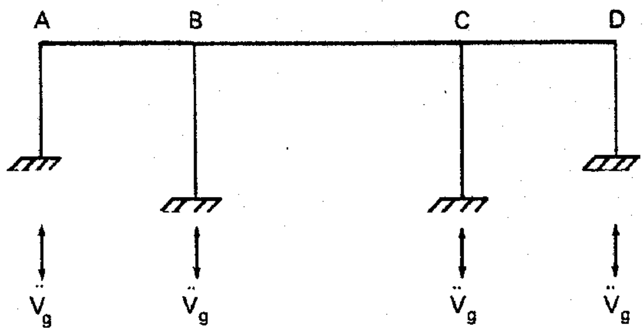
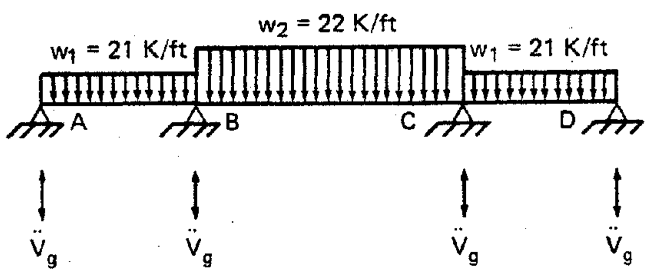


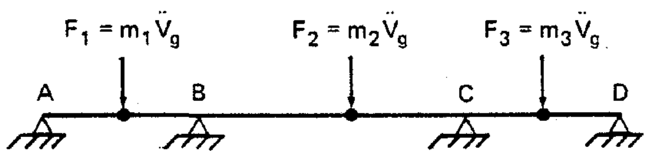
FIGURE 3-8 RELIABILITY OF THE 3-SPAN BRIDGE SUBJECTED TO HORIZONTAL GROUND ACCELERATION



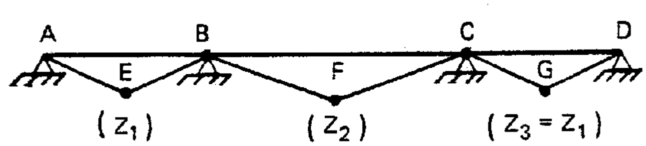
(a) The vertical ground motion on the 3-span bridge



(b) Continuous beam model for the bridge

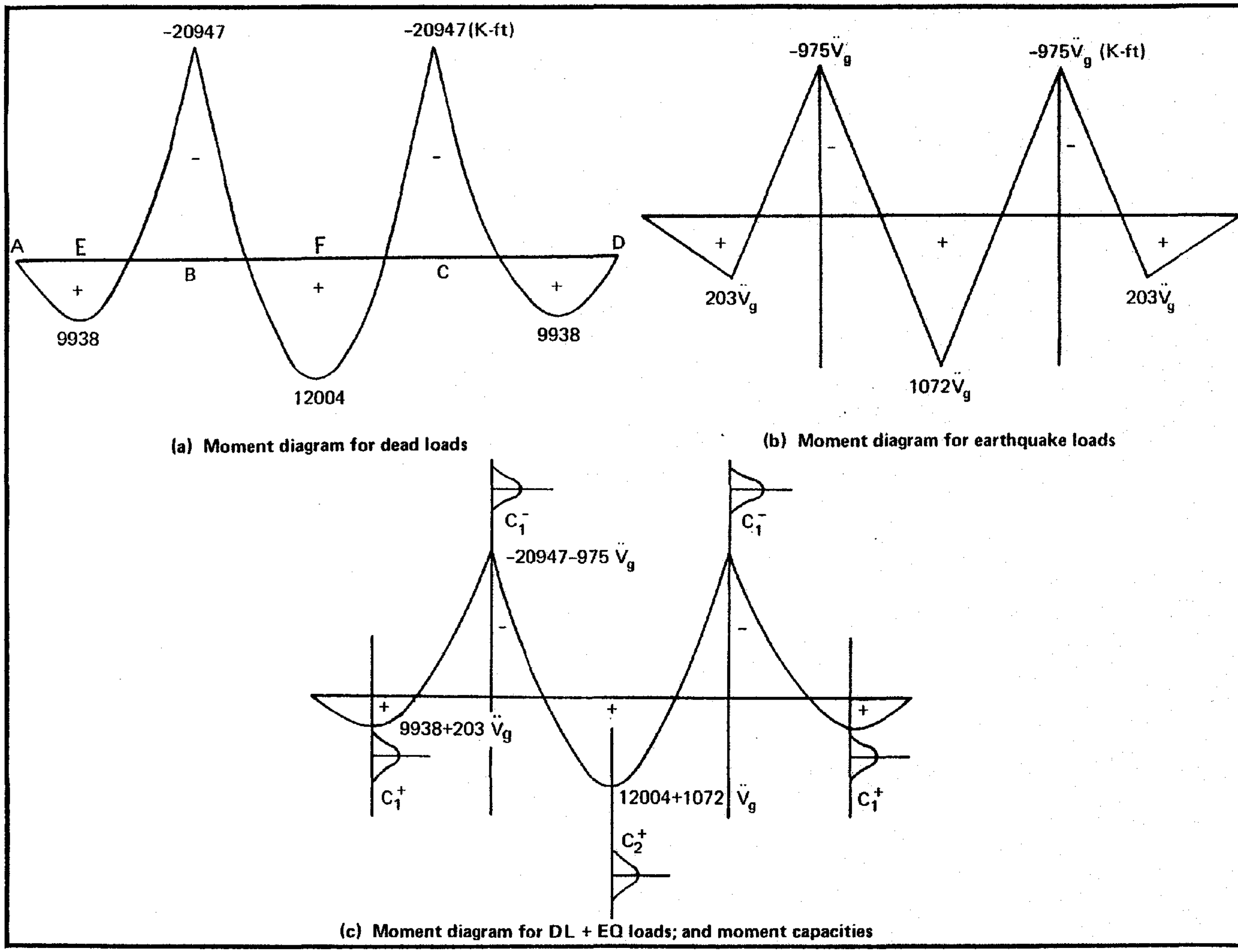


(c) Equivalent static model



(d) Failure mechanisms

FIGURE 3-9 MODEL, FAILURE MECHANISMS, AND FORCES FOR VERTICAL GROUND MOTION



(a) Moment diagram for dead loads

(b) Moment diagram for earthquake loads

(c) Moment diagram for DL + EQ loads; and moment capacities

FIGURE 3-10 MOMENT DIAGRAMS FOR DEAD AND EARTHQUAKE LOADS

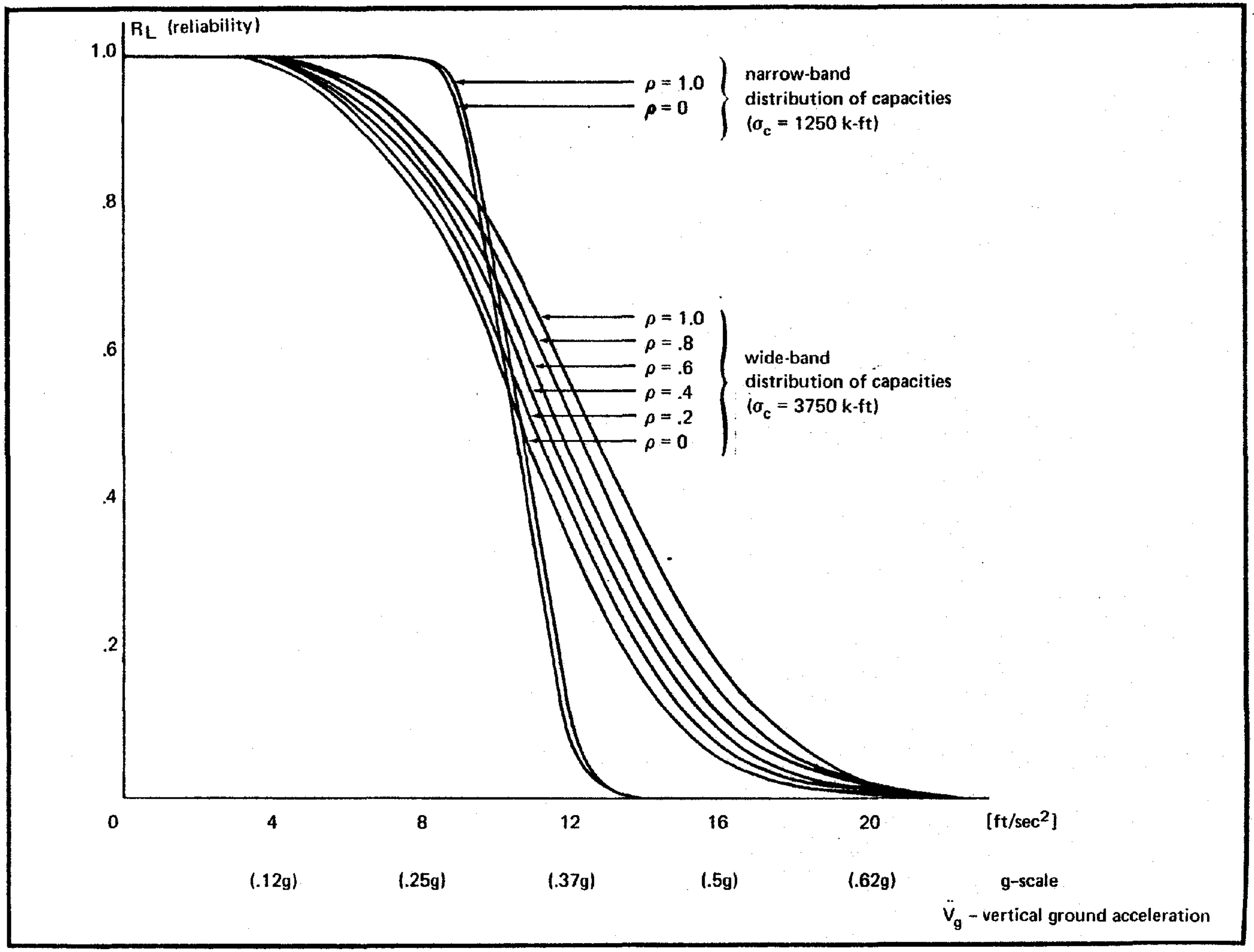


FIGURE 3-11 RELIABILITY OF THE 3-SPAN BRIDGE UNDER VERTICAL GROUND ACCELERATION

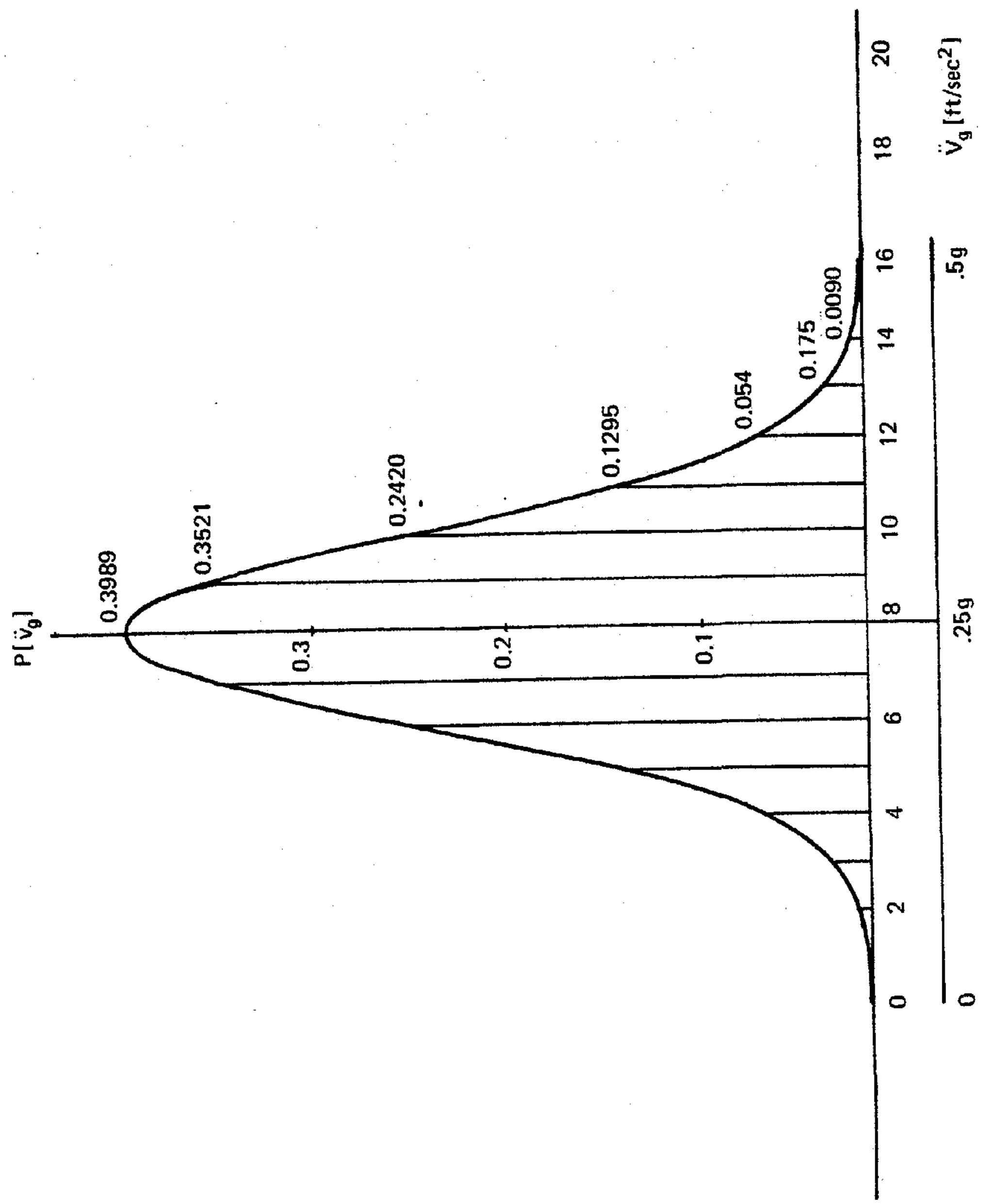


FIGURE 3-12 ASSUMED DISTRIBUTION OF GROUND ACCELERATION, $\ddot{V}_g$

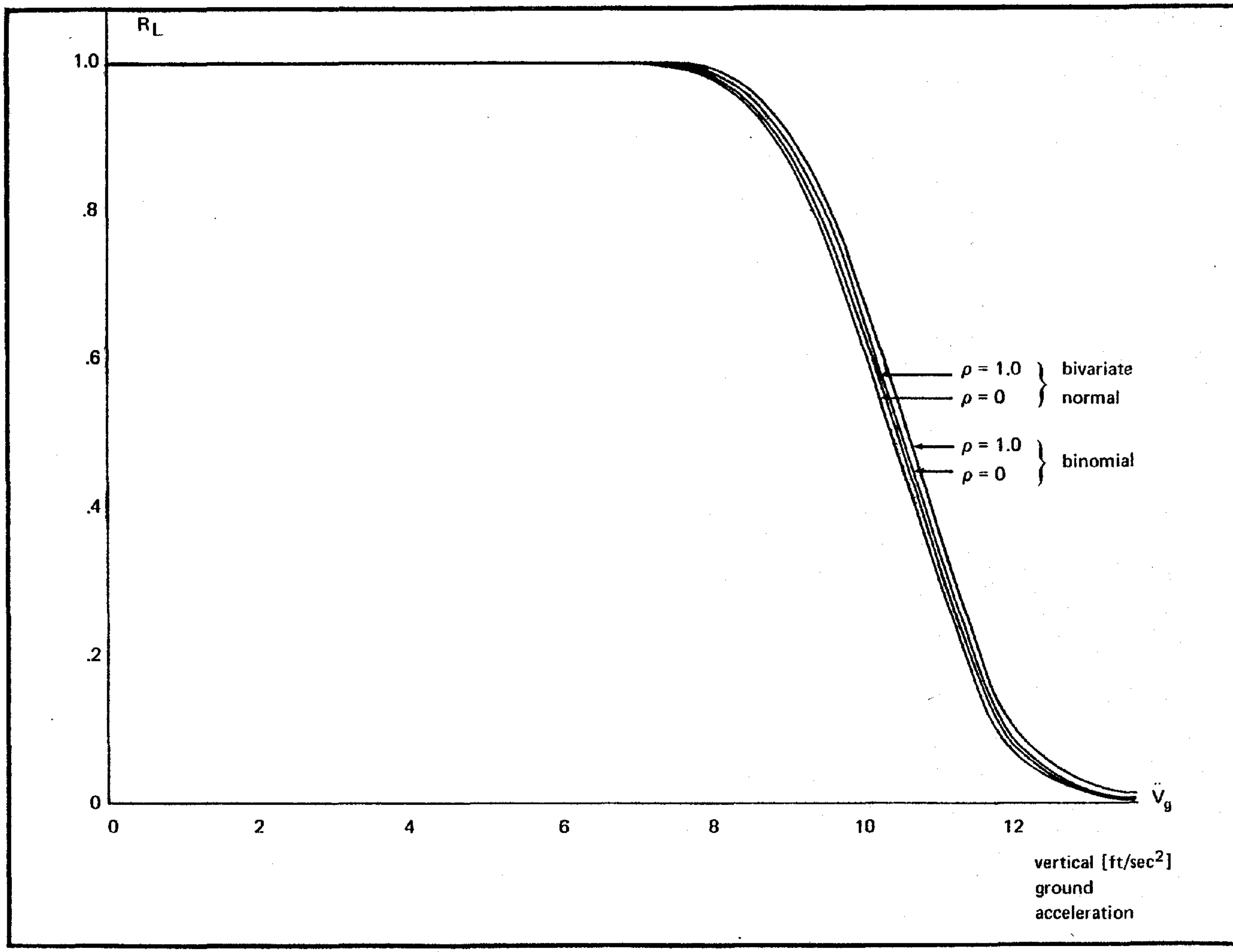
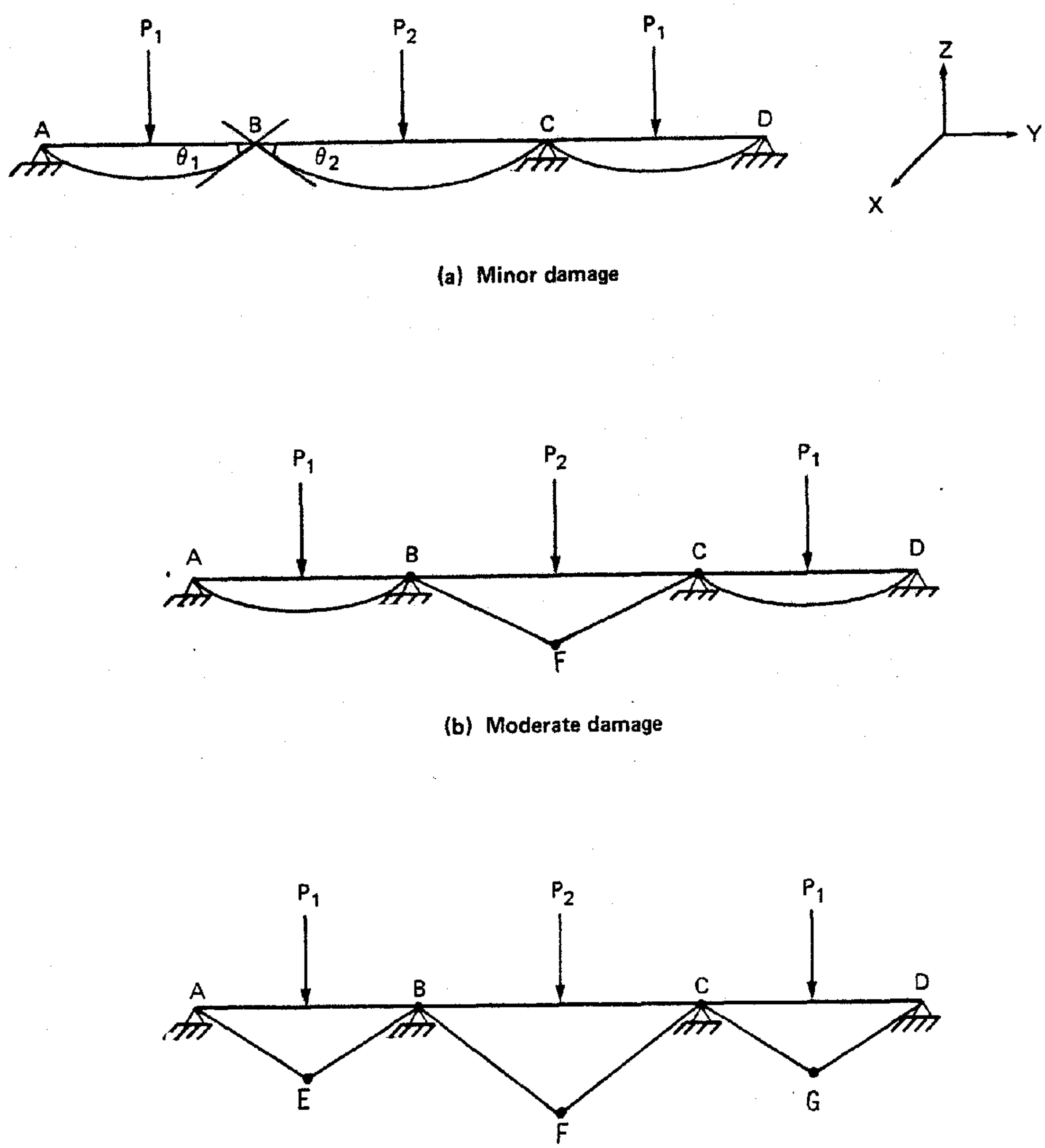


FIGURE 3-13 RELIABILITY OF THE 3-SPAN BRIDGE SUBJECTED TO VERTICAL GROUND ACCELERATION FOR BIVARIATE NORMAL DISTRIBUTION BETWEEN ADJACENT MOMENT CAPACITIES (RELIABILITY OF THE BRIDGE FOR BINOMIAL DISTRIBUTION IS ALSO SHOWN FOR COMPARISON)



(a) Minor damage

(b) Moderate damage

(c) Major damage

FIGURE 3-14 DAMAGE CONDITIONS

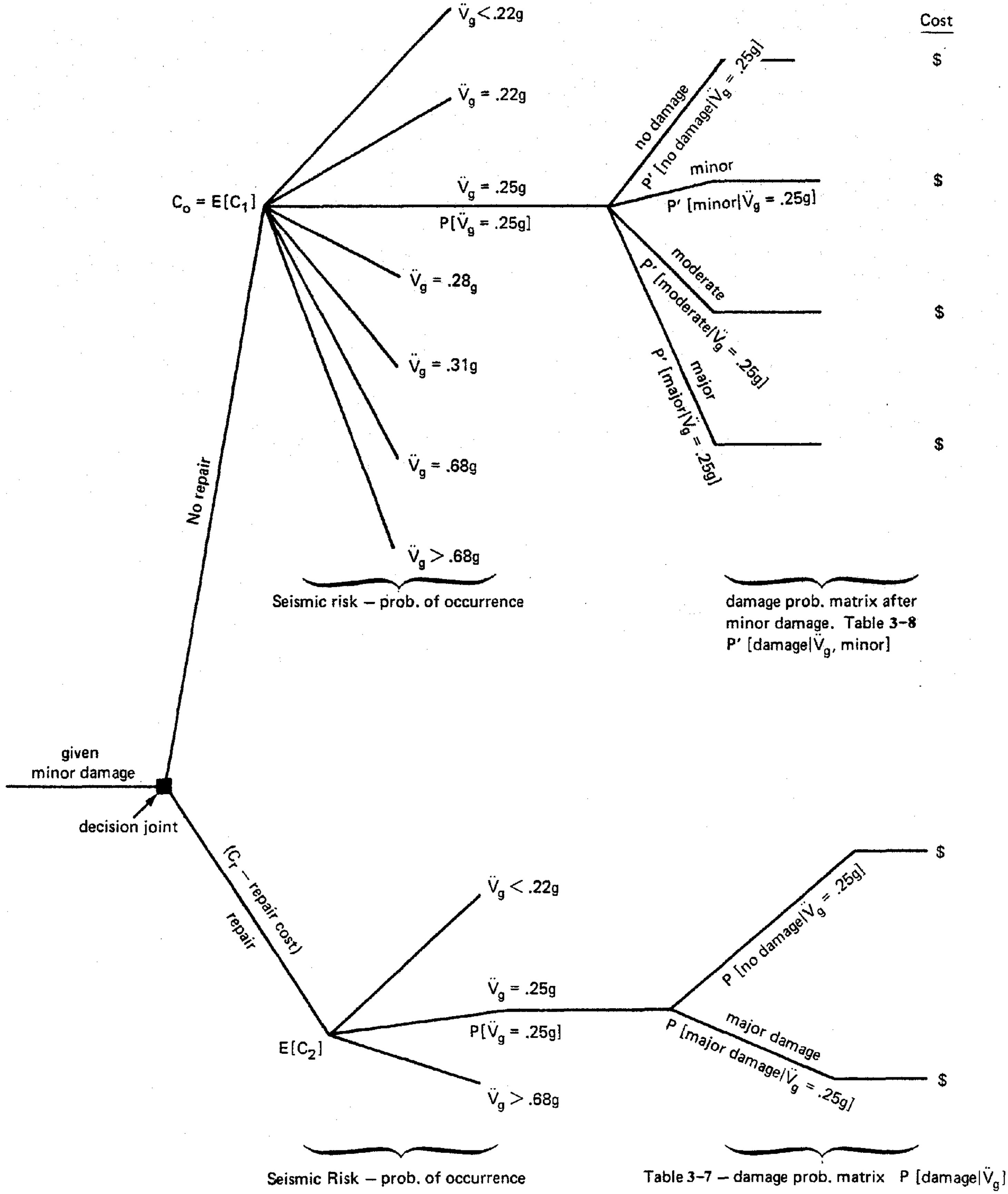


FIGURE 3-15 DECISION ANALYSIS EXAMPLE

4. RELIABILITY ANALYSIS OF A BURIED PIPELINE

INTRODUCTION

In this chapter, a description is presented of a study of the reliability of a long, buried pipeline, similar to the Trans-Alaska pipeline, subjected to seismic loadings. The general methodology used for the determination of the reliability of the pipeline was similar to that used in the previous chapter for the determination of the reliability of a bridge, especially in regard to the dependence between adjacent spans.

Past studies (Ref. 5, 6, 7, 8) have shown that the following conditions are usually true for buried pipelines:

- A majority of the steel and concrete pipelines are flexible with respect to the surrounding soil so that the pipeline follows the soil motions. This suggests that the seismic interaction effects between the pipeline and the surrounding soil may usually be considered to be negligible.

- Major failures in buried pipelines have been observed to be caused by axial deformations rather than flexural, shear, and torsional deformations.

- Larger axial strains are usually found to be developed in continuous pipelines than in segmented pipelines.

- Section properties of the pipelines, except at bends, tees, and knees, etc. do not have significant effect on the developed strains. This is due to the fact that the pipelines are very flexible and conform to the displacements of the surrounding soil.

The reliability analyses described in this chapter were based on the above considerations. In addition, the following assumptions were also made:

- The seismically induced stresses in buried pipelines are caused by either landslides or soil failures, large displacements due to fault crossings, or transient stresses due to ground shakings. (In this study, the effects of landslides and soil failures were not considered.)

- The pipeline essentially follows the displacements of the surrounding soil. There are no relative displacements between the soil and the pipeline, so that the axial strains induced in the pipeline are equal to the axial strains induced in the soil, due to seismic loads.

FAILURE CRITERIA

Two types of seismically induced strains were studied in this report: axial strains due to compressional wave vibrations and axial strains due to fault displacements, as discussed below:

(1) Axial Strains Due to Compressional Wave Vibrations Only

The axial strain, ϵ_p, in the pipeline resulting from a compressional wave can be written as follows (Ref. 5):

$$\epsilon_p = \frac{V_m}{V_p} \qquad (4\text{-}1)$$

where V_m = maximum ground velocity
V_p = compressional wave velocity

It can be assumed in the use of the above equation that the compressional wave velocity, V_p, is a normal random variable with parameters depending on the soil property. The probability distribution of ϵ_p can then be derived from equation (4-1), using the assumption that V_m is deterministic. This can be justified by the fact that the uncertainty in ϵ_p is mainly caused by the randomness in the soil properties, rather than V_m.

If the probability distribution of V_p is denoted by $p(V_p)$, then the probability distribution of ϵ_p, $p(\epsilon_p)$ can be obtained from:

$$p(\epsilon_p) = p(V_p)\frac{dV_p}{d\epsilon_p} \tag{4-2}$$

but $\epsilon_p = \frac{V_m}{V_p}$

and $\frac{d\epsilon_p}{dV_p} = -\frac{V_m}{V_p^2}$

V_p is assumed as a normal random variable with parameters μ_p and σ_p.

Therefore,

$$p(V_p) = \frac{1}{\sigma_p\sqrt{2\pi}} \exp\left[-1/2\left(\frac{V_p-\mu_p}{\sigma_p}\right)^2\right] \tag{4-3}$$

Making appropriate substitutions, we obtain

$$p(\varepsilon_p) = \frac{1}{\sigma_p\sqrt{2\pi}}\ \frac{\varepsilon_p^2}{V_m} \exp\left[-1/2\left(\frac{V_m/\varepsilon_p - \mu_p}{\sigma_p}\right)^2\right] \tag{4-4}$$

In general, the random variable, ϵ_p, is not normally distributed according to equation (4-4); however, for a wide range of typical values of V_m, μ_p and σ_p, the probability density, $p(\epsilon_p)$, can be closely approximated by a normal distribution.

Equation (4-1) can be approximated to be linear around the mean μ_p, so that

$$\varepsilon_p(V_p) = \varepsilon_p(\mu_p) + \varepsilon_p'(\mu_p)(V_p - \mu_p) \tag{4-5}$$

where,

$$\varepsilon_p(\mu_p) = \frac{V_m}{\mu_p}$$

$$\varepsilon_p'(\mu_p) = \frac{-V_m}{\mu_p^2}$$

Therefore,

$$\varepsilon_p(V_p) = \frac{V_m}{\mu_p} - \frac{V_m}{\mu_p^2}(V_p - \mu_p) \tag{4-6}$$

(2) <u>Axial Strains Due to Fault Displacement Only</u>

The axial strain, ϵ_f, induced in the pipeline resulting from displacements at an intersecting fault, can be written as follows (Ref. 4).

$$\epsilon_f = \frac{D}{2L}\cos\phi \tag{4-7}$$

where ϕ = angle of the fault with respect to the pipeline

D = fault displacement

L = anchored length of the pipeline (length between two anchored points)

(3) Combined Axial Strains

If both the above types of axial strains exist, the total axial strain in the pipeline is given by:

$$\varepsilon_t = \varepsilon_p + \varepsilon_f \tag{4-8}$$

(4) Failure Criteria

The axial failure is considered to occur when the total axial strain, ϵ_t, in the pipeline exceeds the yield strain, ϵ_{yp}. In addition, various levels of failure can be defined in terms of levels of damage. For example, the total strain may exceed strains ϵ_{y_1} or ϵ_{y_2}, which define two levels of damage, as follows:

minor damage: when $\epsilon_t > \epsilon_{y_1}$

moderate damage: when $\epsilon_t > \epsilon_{y_2}$

RELIABILITY ANALYSIS

Reliability analyses of pipelines subjected to seismic loads were demonstrated for two examples. In the first example, the pipeline was assumed to pass through four types of soft soils with different properties. The failure was considered to occur when the seismically induced strains exceeded the yield strain of the pipeline in each soil region (Fig. 4-3a). In the second example, the same pipeline was assumed to pass through regions of the soil ranging from soft to very hard (rock) (Fig. 4-3b).

The material properties used for the pipelines corresponded to the X-65 grade steel used for Trans-Alaska pipeline. The axial stress-strain relationship for this material is shown in Figure 4-1.

The reliability analyses were performed for axial strains due to vibratory compressional waves, as well as those due to fault displacements, as described below.

(1) Reliability Analysis Based on Axial Strains Due to Vibratory Compressional Waves

The reliability analysis of a straight pipeline, shown in Fig. 4-3a, was first performed based on axial strains induced in the pipeline due to vibratory compressional waves. The axial strain in the pipe was obtained from equation (4-1).

Four kinds of soft soils were considered in this example. The parameters of the compressional wave velocity and the corresponding parameters of axial strains in all the four types of soils are given in Table 4-1.

(a) Binomial Model

As discussed earlier, for each value of the maximum ground velocity, V_m, probability distributions can be readily obtained for the corresponding induced strains for different soil conditions. Levels of failure were defined in these reliability analyses for a continuous pipeline corresponding to the induced axial strains which just exceeded the strains ϵ_{y_1} and ϵ_{y_2}, corresponding to minor and moderate damage levels, respectively. In the reliability analysis, using the binomial model described here, the success or failure of a segment of the pipeline in a soil region was assumed to be dependent on the success or failure of its adjacent segment in the adjacent soil region. The dependency factor was determined by a coefficient of correlation ρ between the two segments. ρ was assumed to be the same for all the pairs of adjacent segments throughout the pipeline.

The marginal reliability of a region i was computed as shown below

$$R_i = P\left[\epsilon_{p_i} \leqslant \epsilon_{y_1} (\text{or } \epsilon_{y_2})\right] = 1-P\left[\epsilon_{p_i} \geqslant \epsilon_{y_1} (\text{or } \epsilon_{y_2})\right] = 1-p_i \qquad (4\text{-}9)$$

The conditional reliablity $R_{i|i-1}$ of segment i based on the success of segment i-1 was obtained from equation (2-14) on the basis of pairwise dependency. The total reliability of the buried pipeline, shown in Fig. 4-3a, under the effect of compressional seismic wave, was therefore expressed by:

$$R_L = R_1 \cdot R_{2|1} \cdot R_{3|2} \cdot R_{4|3} \qquad (4\text{-}10)$$

The results of the reliability analyses described above are presented in Table 4-2 and are plotted in Fig. 4-4. The reliability values are shown for different levels of earthquake motions, expressed in terms of ground velocities, V_m.

(b) Bivariate Normal Model

In the reliability analysis using the bivariate normal model, the compressional wave velocities V_{p_1}, V_{p_2}, V_{p_3} and V_{p_4} in the four soil regions were assumed to be random variables that were pairwise-dependent through a joint bivariate normal density function expressed by equation (2-24).

The parameters used for these random variables were similar to those used for the binomial model. The coefficient of correlation, ρ, was defined for compressional wave velocities for a pair of adjacent spans, and it was assumed to be equal for all the pairs.

Using a linear relationship between ϵ_p and V_p, a bivariate normal distribution between ϵ_p's of adjacent segments was obtained from a bivariate normal distribution between V_p's of adjacent spans.

Therefore, the joint reliability of two adjacent segments i and i-1 was expressed as follows:

$$R_{i,i-1}(V_m) = \int_{-\infty}^{\varepsilon_{y_1}} d\varepsilon_{p_i} \int_{-\infty}^{\varepsilon_{y_1}} p_{\varepsilon_{i,i-1}} (\rho, \varepsilon_{p_i}, \varepsilon_{p_{i-1}}, V_m)\, d\varepsilon_{p_{i-1}} \quad (4\text{-}11)$$

where

$p_{\epsilon_{i,i-1}}$ = joint probability between ϵ_{p_i} and $\epsilon_{p_{i-1}}$

The total reliability of the pipeline was therefore expressed as shown below:

$$R_L(\rho, V_m) = R_1 \cdot R_{2|1} \cdot R_{3|2} \cdot R_{4|3} \quad (4\text{-}12)$$

but

$$R_{i|i-1}(\rho, V_m) = \frac{R_{i,i-1}(\rho, V_m)}{R_{i-1}(\rho, V_m)} \quad (4\text{-}13)$$

Inserting (4-13) into (4-12), the following expression was obtained:

$$R_L(\rho, V_m) = \frac{R_{21} \cdot R_{32} \cdot R_{43}}{R_2 \cdot R_3} \quad (4\text{-}14)$$

The above expression was used to compute reliability values corresponding to different values of ρ and V_m. The results are plotted in Fig. 4-5.

(2) <u>Reliability Analysis Based on Axial Strains Due to Combined Effects of Vibratory Compressional Waves and Fault Displacements</u>

This section describes the reliability analysis for combined effects of vibratory compressional waves and fault displacement for the pipeline example shown in Fig. 4-2b.

Four kinds of soils, from soft to very stiff, were considered. The pipeline was assumed to be anchored at the very stiff soils. The total anchored length L (length between anchors) was equal to

$$L = L_1 + L_2 \tag{4-15}$$

where L_1 and L_2 were the lengths to the right and left of the point of intersection of the pipeline and the fault line.

The total axial strains, ϵ_t, induced in the pipeline on the right and left of the fault were a direct summation of the axial strains due to compressional wave and the fault displacement effects as shown in equation (4-8), and were expressed as follows:

$$\varepsilon_t = \varepsilon_p + \varepsilon_f \tag{4-16}$$

The axial strains due to fault displacements were computed for the two spans L_1 and L_2 as follows:

$$\varepsilon_{f_1} = \frac{D}{L_1} \cos \phi \approx \left[\frac{D}{\mu_{L_1}} - \frac{D}{\mu^2_{L_1}} \; (L_1 - \mu_{L_1}) \right] \cos \phi \tag{4-17}$$

$$\epsilon_{f_2} = \frac{D}{L_2} \cos\phi \approx \left[\frac{D}{\mu_{L_2}} - \frac{D}{\mu_{L_2}^2} (L_2 - \mu_{L_2})\right] \cos\phi \qquad (4\text{-}18)$$

where μ_{L_1} and μ_{L_2} were the mean values of random variables L_1 and L_2, as discussed below.

The reliability analysis was performed using the following assumptions:

(a) The location of the point of intersection of the pipeline and the fault line was assumed to be random; the spans L_1 and L_2 were therefore random variables. A normal distribution was assumed for the spans L_1 and L_2, while $L = (L_1 + L_2)$ was assumed to be constant. Assuming normal distributions for L_1 and L_2, and using linear relationships, the distributions of ϵ_{f_1} and ϵ_{f_2} were computed to be normally distributed.

(b) The compressional wave velocity, V_p, was considered to be a normal, random variable, similar to the previous section. Similarly, the axial strain, ϵ_p, induced in the pipeline due to vibratory compressional wave was considered to be normally distributed, assuming a linear relationship between ϵ_p and V_p.

(c) The maximum ground velocity, V_m, and the fault displacement, D, were assumed to be deterministic. The value of V_m was conservatively chosen to be equal to 1 ft/sec. The effect of the variation in D was then examined on the reliability of the pipeline for the combined loading.

(d) The angle between the pipeline and the fault line was assumed to be zero to be conservative.

(e) The following values were assumed for various parameters.

$L = L_1 + L_2 = 1200$ ft

$\mu_{L_1} = 330$ ft

$\sigma_{L_1} = 90$ ft

$\mu_{L_2} = 870$ ft

$\sigma_{L_2} = 90$ ft

The parameters for the axial strains for the segments of pipeline on either side of the fault were computed as shown in equations (4-17) and (4-18).

For failure criteria, levels of damage corresponding to minor and moderate damages were defined by yield strains of $\epsilon_{y_1} = 0.0024$ and $\epsilon_{y_2} = 0.04$, respectively.

Using these parameters, the above assumptions and the procedures described previously, a reliability analysis was performed for the example pipeline. The results are shown in Table 4-4 and Figure 4-6 for different values of the fault displacement, D. As the results indicate, the dependency criteria, i.e., the coefficient of correlation between the compressional wave velocities in adjacent soil regions, did not have significant effect on the overall reliability. This was mainly due to the fact that the relative effect of the fault displacement on the system reliability was much more significant than that of the vibratory compressional wave. The variation in the properties of soil for various regions, therefore, did not have any significant effect on the reliability of the system. It was also found that, although minor damage ($\epsilon_{y_1} = 0.0024$) occurred at low values of fault displacements (≤ 1.0 ft), moderate damage ($\epsilon_{y_2} = 0.04$) occurred at significantly high values of fault displacements (approximately 10.0 ft). As a general conclusion, it was also found, similar to the bridge example, that the correlation coefficient, ρ, did not have any significant effect on the overall system reliability.

TABLE 4-1

Normalized Axial Strains for Vibratory Compressional Wave for Various Soil Types

Soil Type	Compressional Wave Velocity (V_p)		Normalized Axial Strain (ϵ_p/V_m)	
	Mean	Standard Deviation	Mean	Standard Deviation
	(ft/sec)	(ft/sec)	(ft^{-1})	(ft^{-1})
S_1	1300	200	7.69×10^{-4}	1.18×10^{-4}
S_2	1200	200	8.33×10^{-4}	1.39×10^{-4}
S_3	1100	200	9.04×10^{-4}	1.65×10^{-4}
S_4	1000	200	10.00×10^{-4}	2.00×10^{-4}

TABLE 4-2

Reliability of the Pipeline Based on Axial Strains Due to Vibratory Compressional Wave - Binomial Model

Ground Velocity	Probabilities of Failure				Total Reliability of Pipeline					
V_m					R_L					
(ft/sec)	p_1	p_2	p_3	p_4	$\rho=0$	.2	.4	.6	.8	1.0
1	0	0	0	0	1.0	1.0	1.0	1.0	1.0	1.0
1.8	0	0	.003	.02	.997	.979	.980	.982	.983	.985
2	0	.004	.03	.16	.81	.82	.83	.85	.86	.88
2.5	.057	.181	.353	.60	.20	.26	.32	.40	.48	.57
3	.40	.59	.74	.93	0	.02	.04	.07	.12	.19

Notes: (1) p_1 to p_4 = Probabilities of axial failure of the pipeline in the soil regions S_1 to S_4, respectively.

(2) ρ = Coefficient of correlation between behaviors of pipeline segments in adjacent soil regions.

TABLE 4-3

Reliability of the Pipeline Based on Axial Strains

Due to Vibratory Compressional Wave - Bivariate Normal Model

Ground Velocity	Correlation Coefficient	Reliabilities							
V_m (ft/sec)	ρ	R_1	R_2	R_3	R_4	R_{21}	R_{32}	R_{43}	R_L
1.0	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2.0	0	1.0	0.96	0.97	0.84	1.0	0.99	0.93	0.96
	0.2	1.0	0.96	0.97	0.84	1.0	0.99	0.94	0.97
	0.4	1.0	0.96	0.97	0.84	1.0	0.99	0.95	0.98
	0.6	1.0	0.96	0.97	0.84	1.0	0.99	0.95	0.98
	0.8	1.0	0.96	0.97	0.84	1.0	0.99	0.96	0.99
	1.0	1.0	0.96	0.97	0.84	1.0	0.99	0.96	0.99
2.5	0	0.94	0.82	0.65	0.40	0.93	0.65	0.23	0.26
	0.2	0.94	0.82	0.65	0.40	0.94	0.69	0.29	0.35
	0.4	0.94	0.82	0.65	0.40	0.95	0.73	0.30	0.39
	0.6	0.94	0.82	0.65	0.40	0.96	0.76	0.33	0.45
	0.8	0.94	0.82	0.65	0.40	0.97	0.78	0.36	0.51
	1.0	0.94	0.82	0.65	0.40	0.98	0.79	0.40	0.58
3.0	0	0.60	0.41	0.26	0.07	0.12	0.20	0.13	0.02
	0.2	0.60	0.41	0.26	0.07	0.13	0.22	0.13	0.03
	0.4	0.60	0.41	0.26	0.07	0.14	0.24	0.14	0.04
	0.6	0.60	0.41	0.26	0.07	0.15	0.26	0.14	0.05
	0.8	0.60	0.41	0.26	0.07	0.17	0.27	0.15	0.06
	1.0	0.60	0.41	0.26	0.07	0.19	0.28	0.16	0.08

Notes: (1) R_i = Reliability of the pipeline in the soil region S_i

(2) R_{ij} = Probability of survival of the pipeline in the soil regions S_i and S_j

(3) R_L = Overall reliability of the pipeline

(4) ρ = Coefficient of correlation between compressional wave velocities of adjacent soil regions.

TABLE 4-4

Reliability of the Pipeline Based on Axial Strains

Due to Vibratory Compressional Wave and Fault Displacement

Binomial Model

Fault Displacement D (ft)	Failure Probabilities						Reliability R_L	
	Soil Region S_1		Soil Region S_2		Soil Region S_3			
	Minor Damage	Moderate Damage	Minor Damage	Moderate Damage	Minor Damage	Moderate Damage	Minor Damage	Moderate Damage
.5	.6	0	0	0	0	0	.4	1.0
1.0	.99	0	.14	0	0	0	.01	1.0
5.0	1.0	0	1.0	0	1.0	0	0	1.0
10	1.0	.09	1.0	0	1.0	0	0	.91
15	1.0	.54	1.0	0	1.0	0	0	.46
20	1.0	.94	1.0	0	1.0	0	0	.06

Notes: (1) The Maximum Ground Velocity, V_m = 1.0 ft/sec.

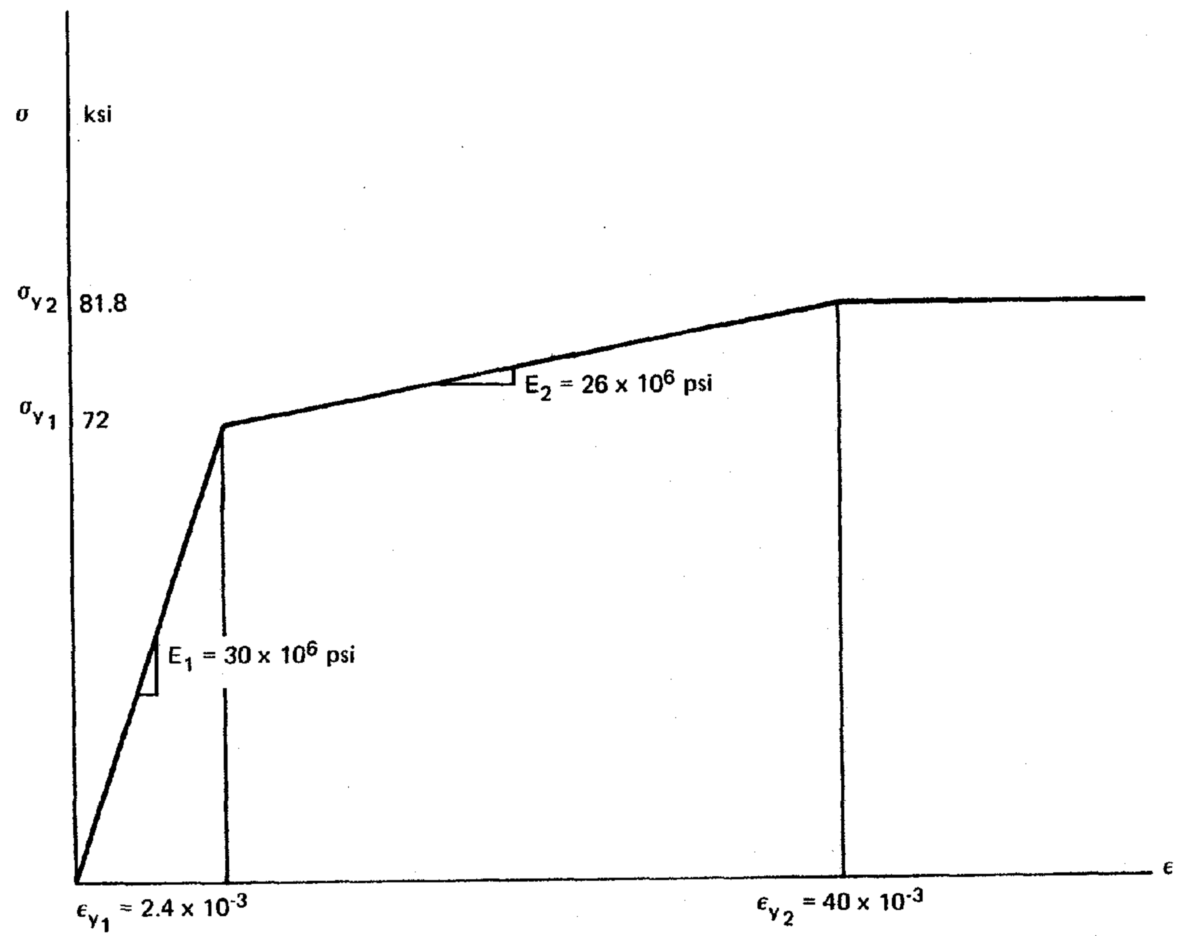


FIGURE 4-1 UNIAXIAL STRESS-STRAIN RELATIONSHIP FOR THE X-65 GRADE PIPELINE

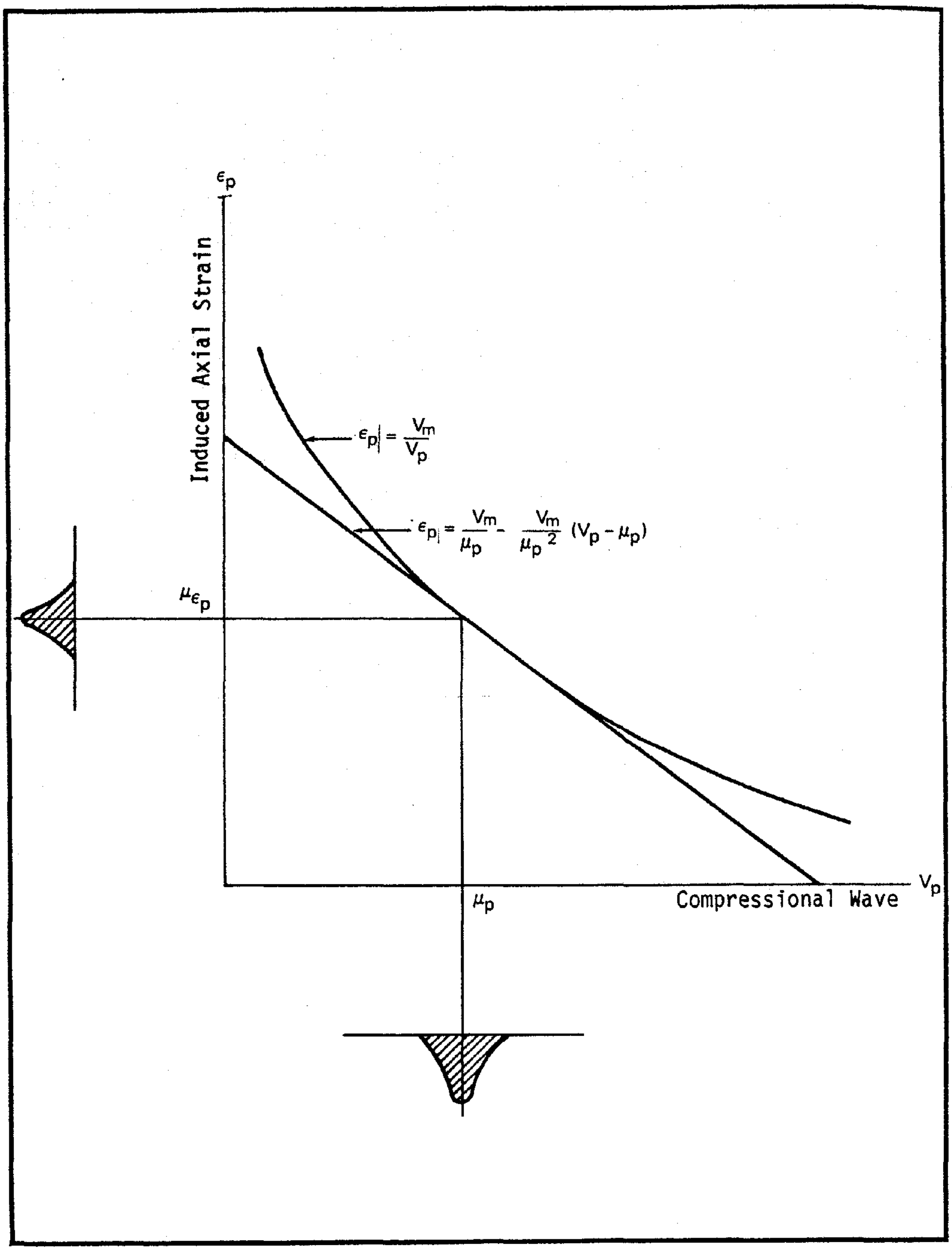


FIGURE 4-2 LINEAR APPROXIMATION BETWEEN ε_p and V_p

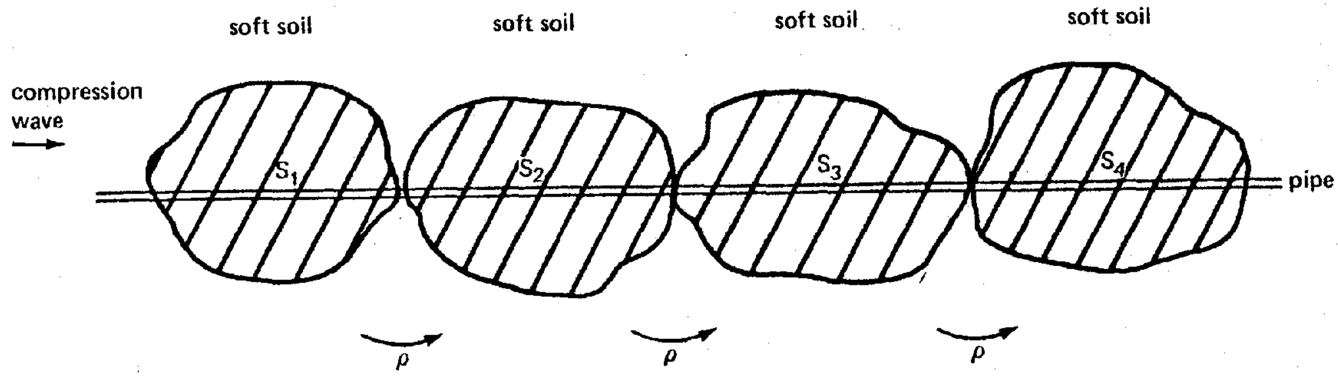


FIGURE 4-3a BURIED PIPELINE SUBJECTED TO VIBRATORY COMPRESSIONAL WAVE

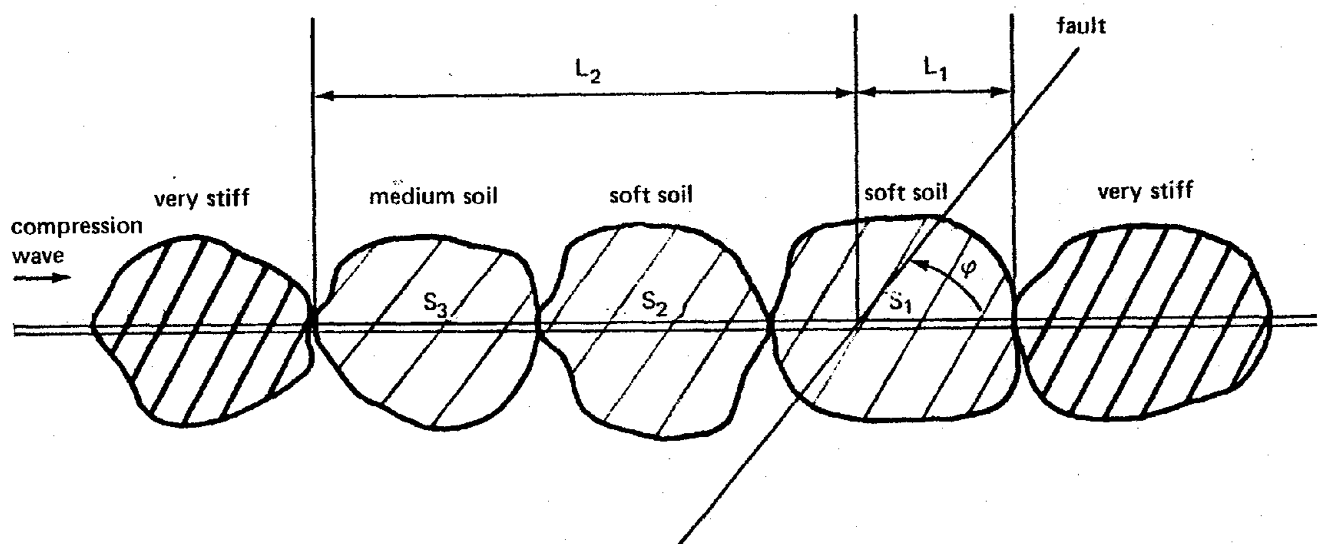


FIGURE 4-3b BURIED PIPELINE SUBJECTED TO VIBRATORY COMPRESSIONAL WAVE AND FAULT DISPLACEMENT

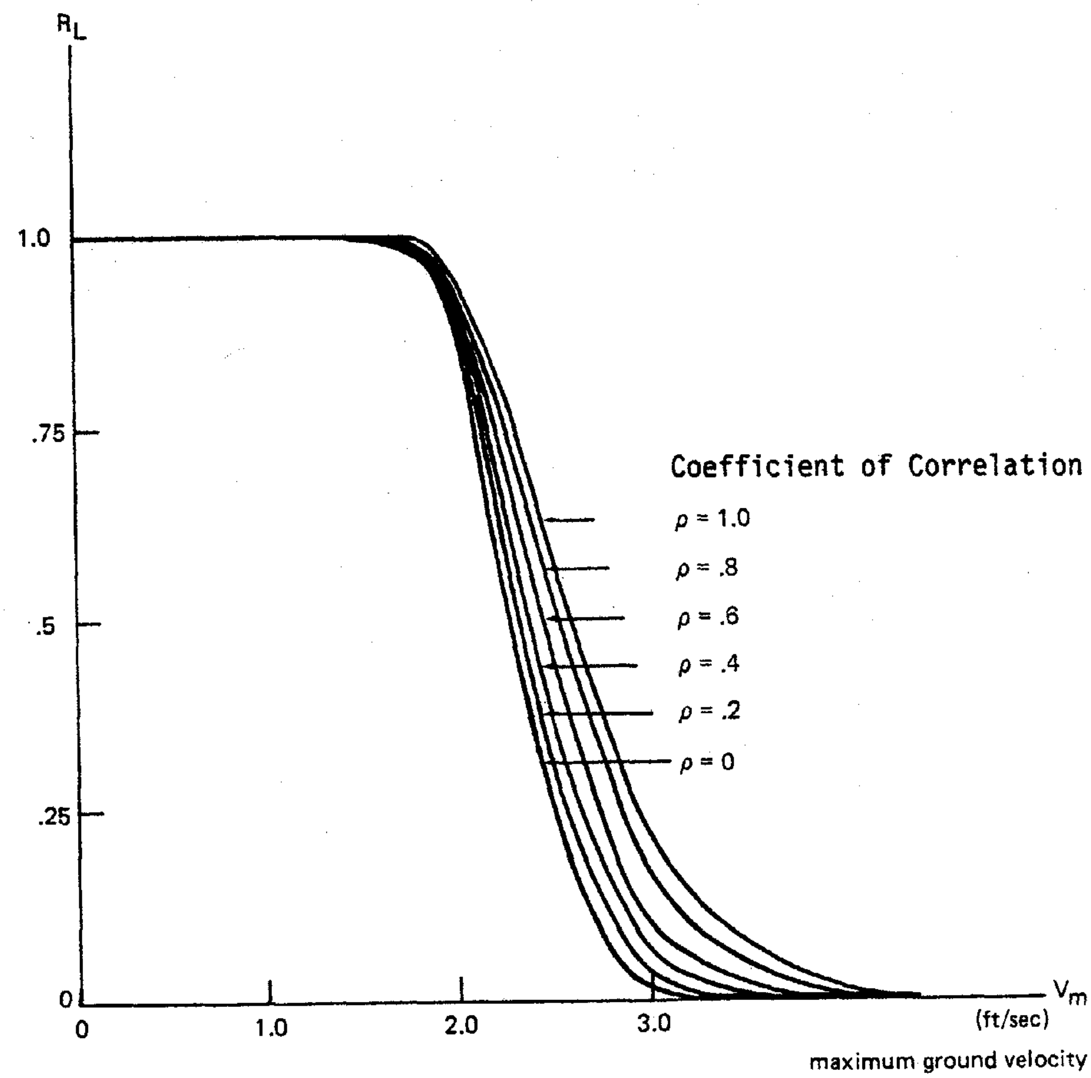


FIGURE 4-4 RELIABILITY OF PIPELINE BASED ON AXIAL STRAINS DUE TO VIBRATORY COMPRESSION WAVE - BINOMIAL MODEL

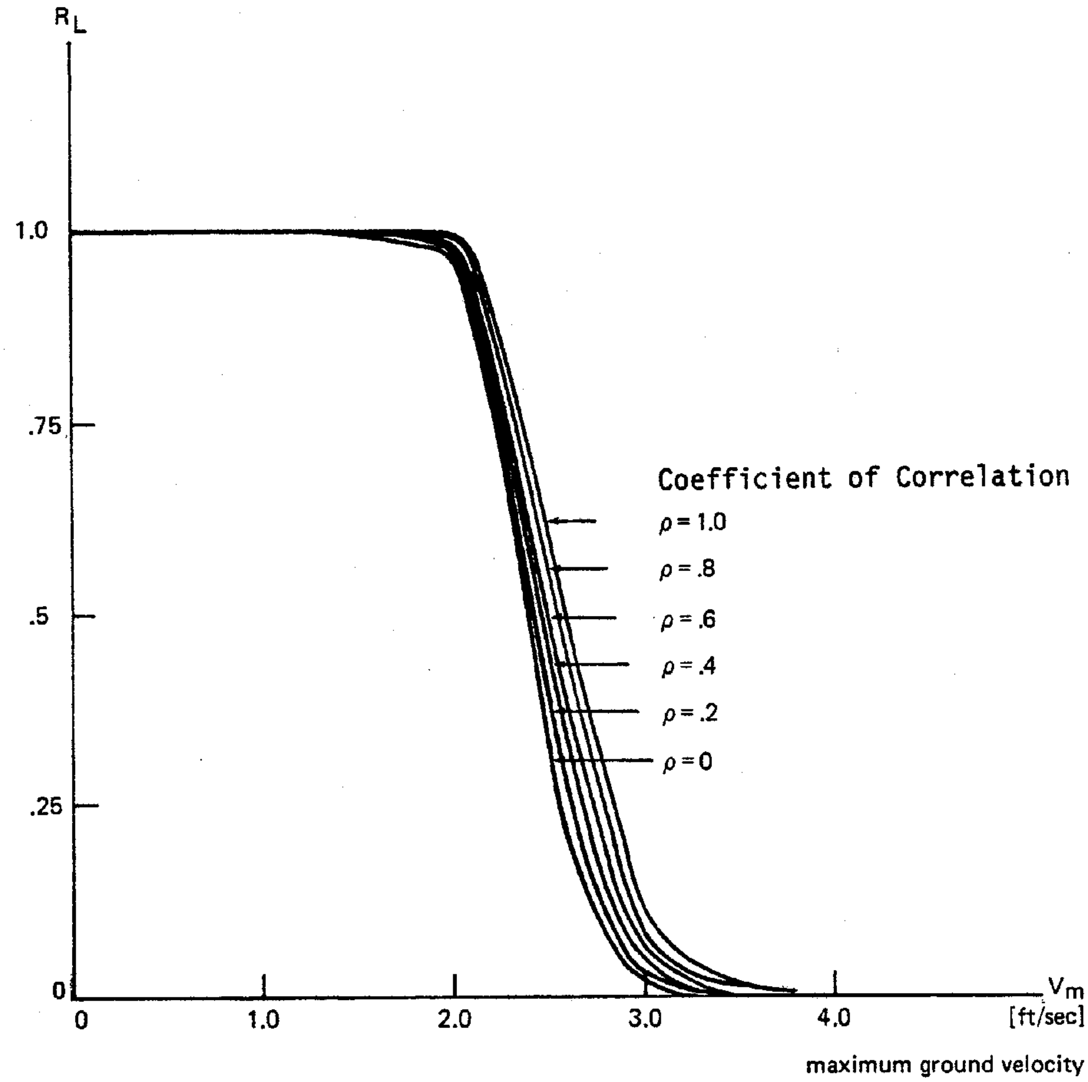


FIGURE 4-5 RELIABILITY OF PIPELINE BASED ON AXIAL STRAINS DUE TO VIBRATORY COMPRESSION WAVE - BIVARIATE JOINT NORMAL DISTRIBUTION

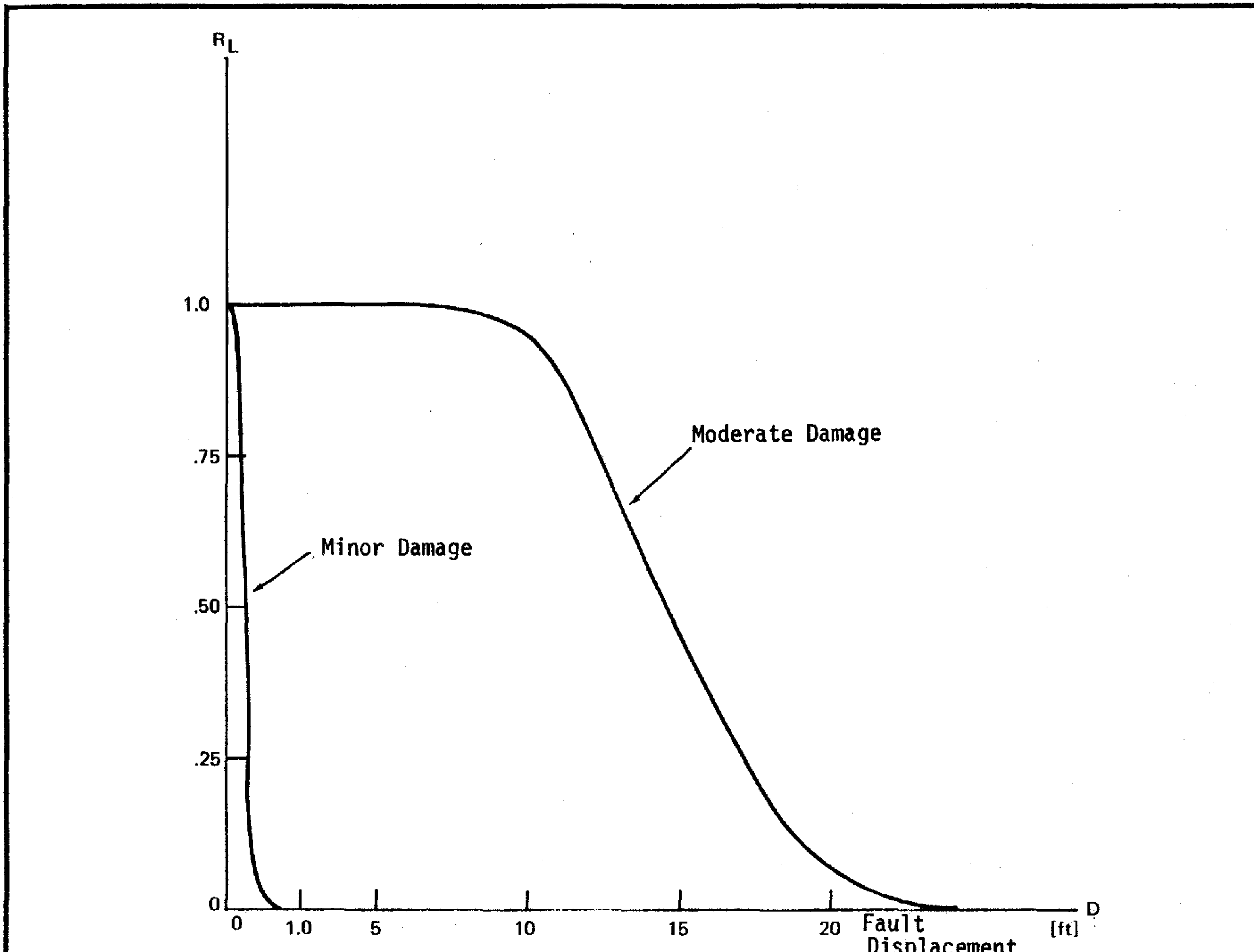


FIGURE 4-6 RELIABILITY OF THE PIPELINE BASED ON AXIAL STRAINS DUE TO COMBINED VIBRATORY COMPRESSION WAVE AND FAULT DISPLACEMENT, Vm = 1.0 ft/sec

5. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FUTURE STUDIES

SUMMARY

The main objective of this investigation was to develop and apply to practical problems, a methodology for the assessment of the reliability (defined as the probability of survival) of linear lifelines subjected to natural hazards, such as earthquakes. The methodology was applied to two sample problems, namely, a three-span bridge and an underground pipeline.

The emphasis of the investigation was on the development of practical and reliable techniques for the assessment of reliabilities of lifelines. This included assessment of probabilities of achieving various levels of functional goals for the lifelines and the evaluation of a damage probability matrix leading to a decision analysis.

One of the important features of the investigation was the consideration of dependency between adjacent sections (or spans) of a lifeline. The usual practice in the reliability analysis of lifelines is to assume that the loads, the behaviors, and the capacities of adjacent sections of a lifeline are statistically independent. This assumption is not true in most cases. Two types of probabilistic models were used in this investigation, namely, binomial and bivariate normal models.

For the binomial model, it was assumed that the success or failure of an element of a lifeline was a discrete random variable jointly dependent on the behavior of its adjacent elements. A coefficient of correlation was used for dependency between the binary random variables representing the behavior of adjacent elements. For the bivariate normal model, the loads and capacities for adjacent spans of the lifeline were assumed to be continuous random variables that were jointly dependent from one section to the other, with a bivariate normal distribution, leading to indirect dependency between the behaviors of adjacent sections (or spans). A pairwise dependency was considered between adjacent elements for both models.

For the bridge example, the failure of the lifeline was considered to have occurred when a complete failure mechanism developed. A plastic hinge was supposed to have occurred in a span of the bridge when the applied moment at that section exceeded the moment capacity. The effect of axial load on the moment capacity was also considered. The reliability analysis was performed for horizontal and vertical earthquakes. Equivalent static analyses were used to compute the applied loads. The loading was considered to be both deterministic and probabilistic. A damage analysis leading to decision analysis was also performed.

For the pipeline example, it was assumed that the pipeline followed the motions of the surrounding soil and there was no soil-pipeline interaction. The failure of the pipeline was considered to have occurred when the applied axial strains exceeded the yield strains. The applied axial strains were computed, including the effects of compressional vibratory motion and displacements due to an intersecting fault. The dependency between adjacent soil regions through which the pipeline passed were considered in a manner similar to the bridge example. The deterministic as well as probabilistic loadings were considered. Reliability analysis for earthquake loadings was then performed.

CONCLUSIONS

The following conclusions were reached on the basis of the work performed in this investigation. These conclusions are based on only two examples used in this investigation and should not be generalized for all types of lifelines.

- It was demonstrated that the methodology for the assessment of reliability of lifelines developed in this investigation can be successfully applied to a variety of lifelines.

- It was demonstrated that the methodology can be easily used for performing damage analysis, and can therefore be used for prediction of damage to lifelines for different levels of earthquake motions.

- It was also demonstrated that the methodology can be readily extended to performing decision analysis, and can therefore be used as a tool for making major decisions in regard to alternatives between making major repairs or replacing a lifeline damaged by an earthquake. The methodology can therefore also be used for earthquake insurance purposes.

- It was shown that the lifeline reliabilities were not sensitive to the use of different probabilistic models (viz., binomial versus bivariate normal). For practical reliability analysis, it could therefore be sufficient to use simpler probabilistic models which are representative of the physical situations.

- It was shown that variations in the coefficient of correlation for dependency between adjacent spans did not have any significant effect on the reliability of the lifelines considered in this study. The effect of dependency was found to be less pronounced for the reliability of the bridge example than for the pipeline example.

RECOMMENDATIONS FOR FUTURE STUDIES

The following are the recommendations for future studies based on the results of this investigation:

- For linear lifelines, similar to those studied in this investigation, the spatial variation of earthquake motions should be considered.

- For linear lifelines, similar to those studied in this investigation, the applied earthquake forces in the lifelines should be computed using more sophisticated dynamic analysis procedures, including material and geometric nonlinearities, and soil-structure interaction effects, etc.

- The reliability methodology developed and demonstrated in this study for linear lifelines should be used in close conjunction with the available seismic risk methodologies.

- The reliability methodology developed for linear lifelines in this study should be extended to networks and other lifelines of more complex configuration.

- The treatment of dependency for adjacent sections (spans) in the methodology used in this study (based on pairwise dependency) should be extended to take into consideration the dependency of all sections (spans) of lifelines.

- The reliability methodology developed in this study should be applied to other practical lifelines to ensure that it is applicable to all types of lifelines, and the conclusions reached in this study can be generalized.

- The reliability methodology developed and applied to earthquake hazards in this study should be tested for other hazards, such as tornadoes, floods, explosions, etc.

- A study should be performed to survey and collect available data on the failures of lifelines subjected to earthquake motions for comparison against the results of this study.

- A study should be performed to collect data on testing and failure of concrete, soil, and other materials and structural elements and systems for the development of better probabilistic models to represent the behavior of lifelines and the dependency of sections (spans) of lifelines.

- A study should be performed to develop detailed damage and decision analysis methodologies based on the reliability methodology of this study using actual cost data and realistic utility functions.

- A computer program should be developed to automate the reliability analysis methodology developed in this study for everyday use of this methodology in the industry.

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REFERENCES

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APPENDIX A

EVALUATION OF DEPENDENCY OF MECHANISMS FOR THE BRIDGE EXAMPLE

APPENDIX A

EVALUATION OF DEPENDENCY OF MECHANISMS FOR THE BRIDGE EXAMPLE

In this Appendix, a coefficient of correlation, ρ_{12}, is calculated between the mechanisms 1 and 2 of spans AB and BC for the bridge example of Chapter 3 for bivariate normal model. It is further assumed that the two mechanisms are independent whenever ρ_{12} is smaller than or equal to a threshold value, ρ_0. This approach is taken by several authors, such as Ang (Ref. 4) for the study of the reliability of structural frames. According to this approach

Z_i = performance function of mechanism i

$$Z_i = \sum_j a_{ij} C_{ij} + \sum_k b_{ik} S_{ik} \qquad \text{(A-1)}$$

where C_{ij} = moment capacity at the plastic hinge j which participates in mechanism i

S_{ik} = load k that is active in producing mechanism i.

a_{ij} and b_{ik} are capacity and load coefficients, respectively.

By definition, a mechanism i occurs when $Z_i < 0$. The coefficient of correlation ρ_{ij} between mechanisms i and j is obtained from

$$\rho_{ij} = \frac{\Sigma(a_i a_j \sigma_C^2 + b_i b_j \sigma_S^2)}{\sigma_{Z_i} \sigma_{Z_j}} \tag{A-2}$$

where σ_C^2, σ_S^2 = variances of those C and S that are common to Z_i and Z_j

$\sigma_{Z_i}^2$, $\sigma_{Z_j}^2$ = variances of random variables Z_i and Z_j

a_i, a_j = resistance coefficients of moment capacities common to Z_i and Z_j

b_i, b_j = load coefficients of loads common to Z_i and Z_j

According to this approach, if the value of ρ_{ij} is smaller than a threshold value ρ_0, then the two mechanisms Z_i and Z_j can be assumed to be independent. At the present time, the value of ρ_0 is subjective, and it is largely determined by engineering judgments. The value of ρ_0 is chosen equal to 0.6 in Ref. 4.

For the 3-span bridge example:

$$Z_1 = a_{11}\, C_1^+ + a_{12}\, C_1^- + b_{11}\, F_1 \tag{A-3}$$

$$Z_2 = a_{22}\, C_1^- + a_{23}\, C_2^+ + b_{22}\, F_2 \tag{A-4}$$

where F_1 and F_2 are the loads in spans AB and BC

$a_{11} = 4$ $a_{22} = 2$ $b_{11} = -85/2$

$a_{12} = 2$ $a_{23} = 4$ $b_{22} = -108/2$

b_i = load coefficient common to Z_1 and $Z_2 = 0$

For the narrow type normal distribution of capacities, we have

$$\sigma C_1^+ = \sigma C_1^- = \sigma C_2^+ = 1{,}250 \text{ k-ft}$$

if all the random variables C_1^+, C_1^-, C_2^+, F_1 and F_2 are normally distributed, Z_1 and Z_2 are also normally distributed with variances equal to

$$\sigma_{Z_1}^2 = 16\sigma_{C_1^+}^2 + 4\sigma_{C_1^-}^2 + (85/2)^2\,\sigma_{F1}^2 \qquad \text{(A-5)}$$

$$\sigma_{Z_2}^2 = 4\sigma_{C_1^+}^2 + 16\sigma_{C_2^+}^2 + (108/2)^2\,\sigma_{F_2}^2 \qquad \text{(A-6)}$$

For deterministic loading

$$\sigma_{F_1} = \sigma_{F_2} = 0$$

Then

$$\sigma_{Z_1} = \sigma_{Z_2} = 5{,}590 \text{ k-ft}$$

and

$$\rho_{12} = 0.10$$

For this low value of ρ_{12}, it can be fairly assumed that mechanisms 1 and 2 are independent.

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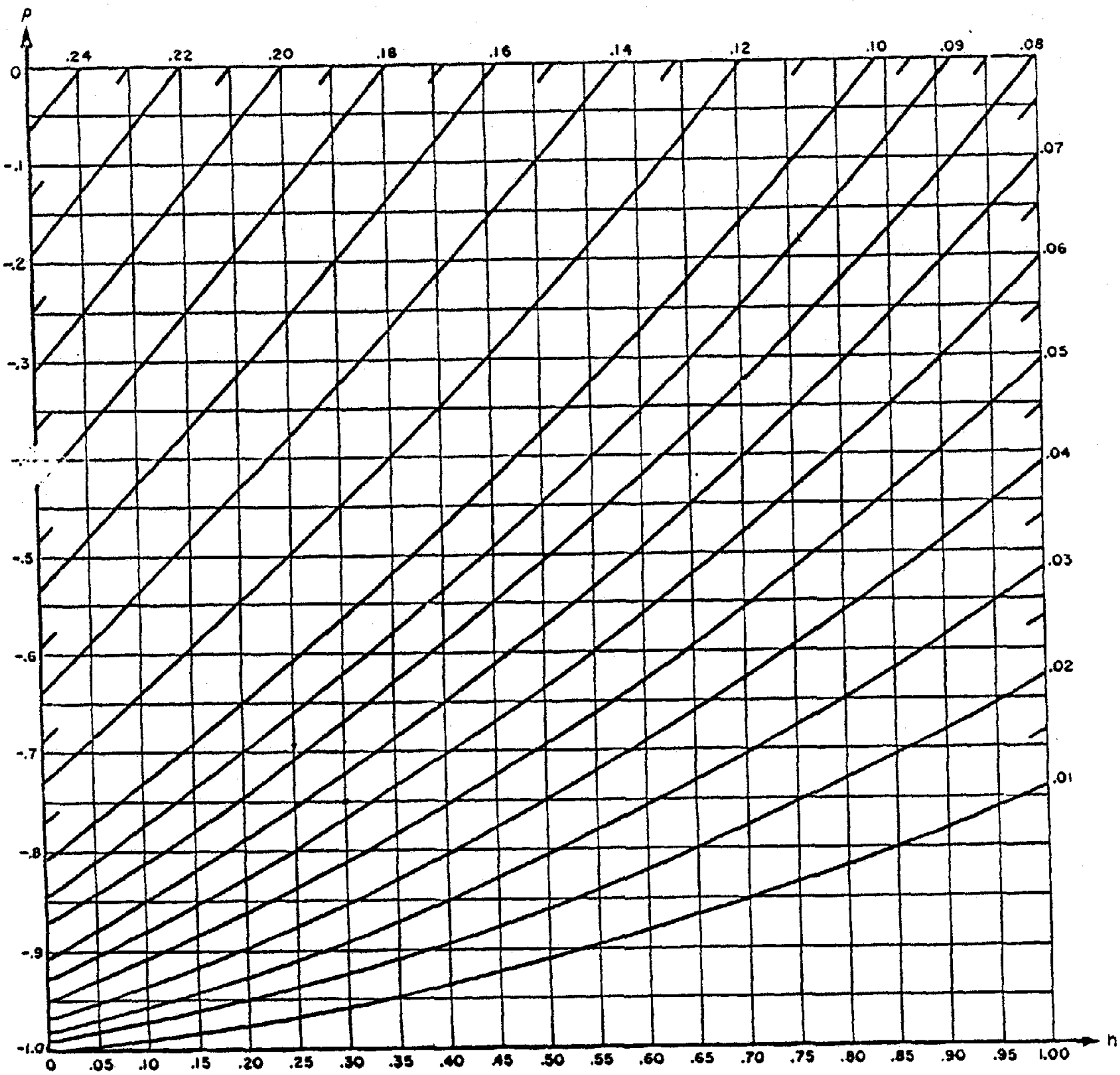
APPENDIX B

GRAPHICAL AIDS FOR CALCULATION OF BIVARIATE NORMAL DISTRIBUTIONS



$$L(h,k,\rho)=L\left(h,0,\frac{(\rho h-k)(\mathrm{sgn}\ h)}{\sqrt{h^2-2\rho hk+k^2}}\right)+L\left(k,0,\frac{(\rho k-h)(\mathrm{sgn}\ k)}{\sqrt{h^2-2\rho hk+k^2}}\right)-\begin{cases}0 & \text{if } hk>0 \text{ or } hk=0 \text{ and } h+k\geq 0\\ \frac{1}{2} & \text{otherwise}\end{cases}$$

where sgn $h=1$ if $h\geq 0$ and sgn $h=-1$ if $h<0$.

Values for $h<0$ can be obtained using $L(h,0,-\rho)=\frac{1}{2}-L(-h,0,\rho)$

FIGURE B-1 $L(h, 0, \rho)$ for $0\leq h\leq 1$ and $-1\leq \rho\leq 0$

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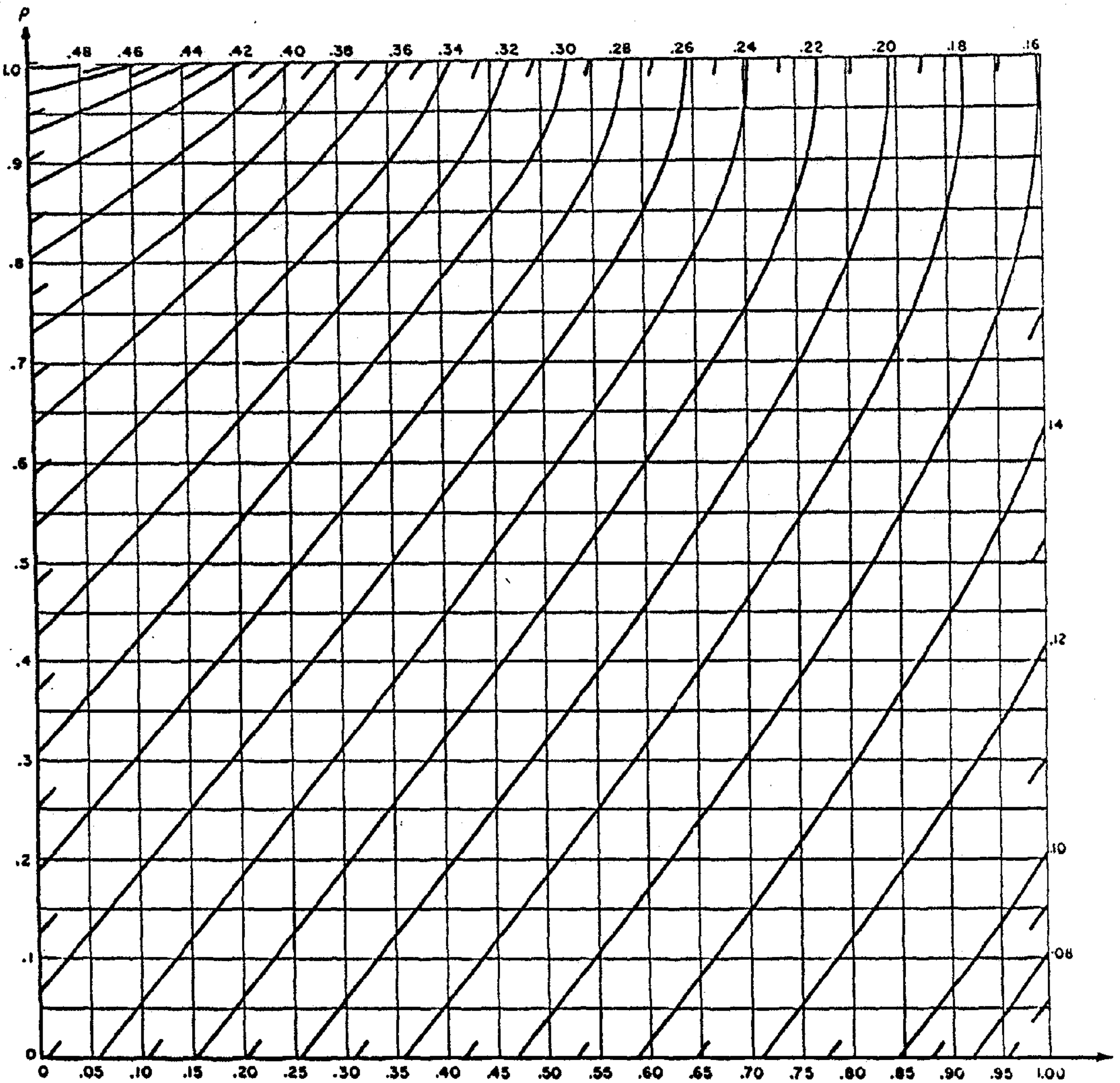

Values for h<0 can be obtained using $L(h,0,-\rho)=\frac{1}{2}-L(-h,0,\rho)$.

FIGURE B-2 $L(h, 0, \rho)$ for $0 \leq h \leq 1$ and $0 \leq \rho \leq 1$

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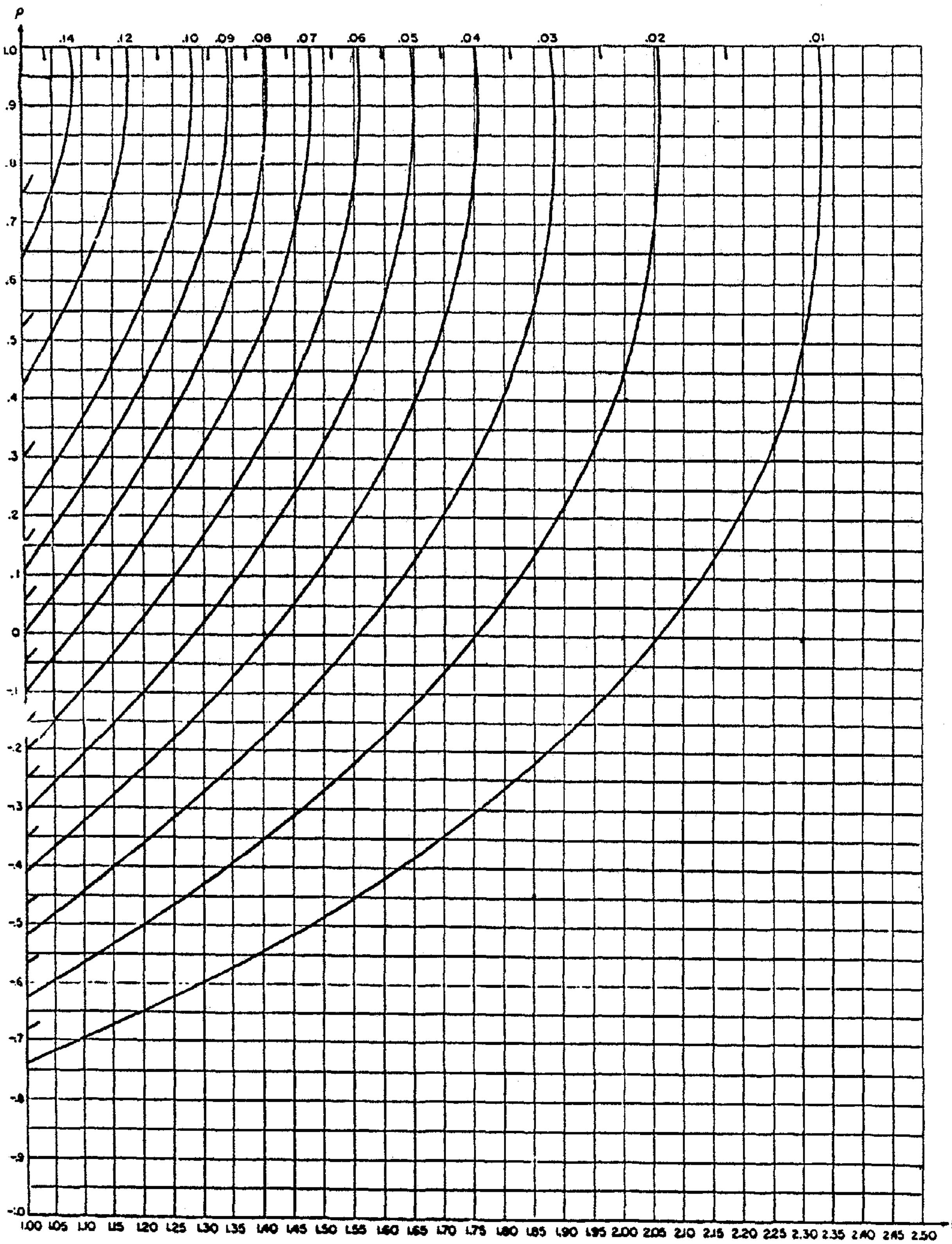

Values for h<0 can be obtained using $L(h,0,-\rho) = \frac{1}{2} - L(-h,0,\rho)$.

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FIGURE B-3 $L(h, 0, \rho)$ for $h \geq 1$ and $-1 \leq \rho \leq 1$