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RESPONSE OF STRUCTURES TO NONVERTICALLY INCIDENT SEISMIC WAVES

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TABLE OF CONTENTS

ABSTRACT

A study of the earthquake response of symmetric elastic structures subjected to SH wave excitation with different angles of incidence and to Rayleigh waves is presented. For SH wave excitation, particular emphasis is given to the study of the possible reduction of the response due to filtering by the foundation and to the torsional response. For Rayleigh wave excitation, the effects of the additional rocking associated with the vertical component of the excitation are investigated. The results obtained for models of a ten story reinforced concrete building and of the containment structure of a nuclear power plant reveal that the response for nonvertically incident waves is significantly different from that obtained on the basis of the usual assumption of vertically incident SH waves.

INTRODUCTION

Most of the presently available methods to evaluate the seismic response of structures including the soil-structure interaction effects are based on the assumption that the seismic excitation can be represented by plane vertically incident compressional or shear waves. As a result of such assumption, the input motion is considered to be equal for all points along the base of the foundation and would consist of a pure vertical or horizontal translation. In terms of the structural response, the implication of the assumption of vertically incident waves is that torsional response will occur only if the superstructure or foundation are not symmetric and that the rocking response will be associated to a large degree with the mass distribution of the structure in height.

The present strong motion instrumentation of structures is such that one typically finds only one accelerograph per floor. Under these conditions, it is difficult to find unambiguous evidence for nonvertically incident seismic excitation. In particular, it is not possible to separate the possible torsional response of symmetric structures, or to identify the rocking response. This situation will remain until buildings are properly instrumented with several accelerographs per floor.

In spite of the conditions just described, there is some experimental evidence for the existence of nonvertically incident seismic excitation. Housner (1957) in a study of the response of the Hollywood Storage Building to the Kern County earthquake in 1952 found a marked reduction of the intermediate and high frequency components of the recorded basement motion as compared with the motion recorded in the free-field and advanced the hypothesis that such reductions could be explained by nonvertically incident waves. Crouse (1973) noted similar effects on the same structure for the San Fernando earthquake of 1971. Duke, et. al. (1971) have shown that the reduction of the high-frequency components could be explained in part by scattering of waves by the embedded foundation. This study, however, does not rule out the possibility of nonvertically incident seismic excitation. Analysis of the roof response of the Hollywood Storage Building suggests the existence of a significant torsional response that could only be generated by nonvertically incident waves given the symmetry of the structure. Reductions of the peak accelerations recorded on the base of structures as compared with free-field values have also been noted by Yamahara (1970). These reductions could be associated with nonvertically incident waves.

A different type of experimental information results from analyses of the El Centro 1940 records made by Trifunac (1971). Trifunac found that the arrival times of many of the significant pulses coincided with the arrival times of Rayleigh and Love waves and concluded that surface waves may be responsible for major contributions to the recorded motion.

The experimental evidence as well as theoretical analyses indicate that nonvertically incident seismic waves may have important effects on the response of structures. The most significant implications are: (i) Love waves and nonvertically incident SH waves will generate a marked torsional response even in the case of symmetric structures and foundations, (ii) Rayleigh waves and nonvertically incident P and SV waves will induce additional contributions to the rocking response, and (iii) the spatial variations of the free-field motion may lead to significant reductions of the high-frequency components of the translational response.

These implications have lead to an increased interest on the analysis of the response of structures to obliquely incident seismic waves. Trifunac (1972) and Wong and Trifunac (1974) have studied the two-dimensional response of a shear wall excited by nonvertically incident antiplane SH waves and found that the angle of incidence of the shear waves has a marked effect on the structural response if the vertical cross-section of the embedded foundation is not semi-circular. For three-dimensional structures, Newmark (1969) found that nonvertically incident SH waves may induce a large torsional response even in symmetric buildings. In Newmarks's pioneering work, the rotational input for oblique SH waves was evaluated approximately and the soil-structure interaction effects were not included. Iguchi (1973) studied the response of a one-story structure to nonvertically incident SH excitation including the effects of soil-structure interaction and confirmed Newmark's observations. The response of nuclear power plant structures to obliquely incident SH waves including the effects of soil-structure interaction has been studied by Lee and Welsey (1975) by using an approximate expression for the torsional input. In this work, it was found that the torsional response of nuclear power plants induced by obliquely incident SH waves is larger than the response associated with typical eccentricities. Kobori and Shinozaki (1975) have analyzed the torsional response of a one-story structure to obliquely incident SH waves. In this work, the exact torsional input for a circular foundation was used. Finally, Luco studied the torsional response of continuous elastic and symmetrical structures supported on a flat circular foundation (1976a) and an embedded hemispherical foundation (1976b) when excited by obliquely incident SH waves.

Studies of the response of three-dimensional structures subjected to obliquely incident waves have been limited by the lack of efficient and accurate techniques to obtain the response of foundations to different types of seismic waves. Recently, the authors have developed a procedure to obtain the response of arbitrarily shaped flat foundations supported on a layered visco-elastic half-space and subjected to different types of seismic waves (Wong and Luco, 1978a,b; Luco and Wong, 1977). The solution of this basic problem opens the door to complete analyses of the response of three-dimensional structures to different types of seismic excitations. As a first step in that direction, this study is addressed at the analysis of the response of symmetric elastic structures supported on flat rigid rectangular foundations placed on a uniform viscoelastic half-space and excited by nonvertically incident SH waves and Rayleigh waves. The model under consideration is illustrated in Figure 1.

One of the objectives of the study is to determine the relative importance of the torsional response produced by SH waves with different

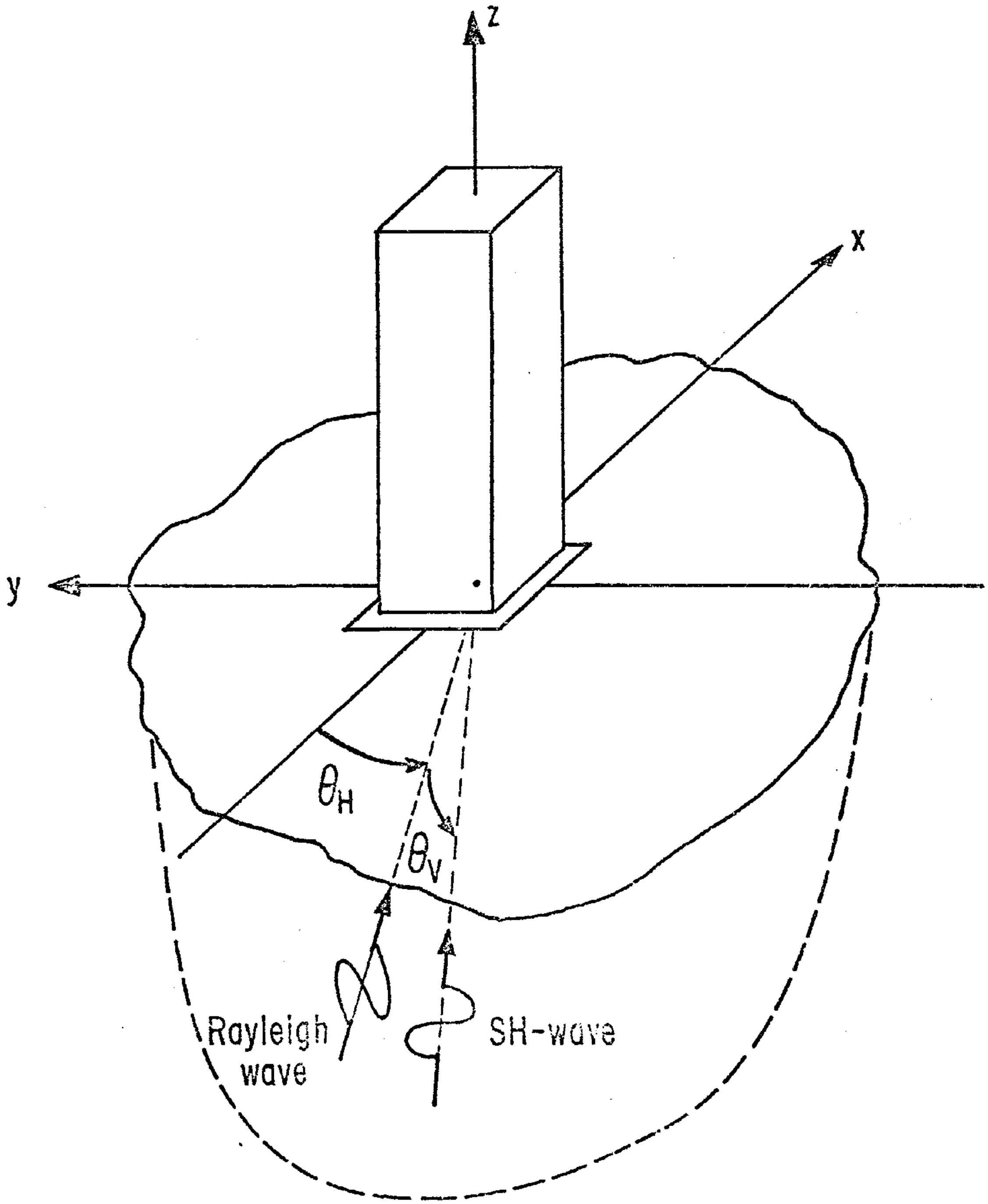

Figure 1. Description of the Model

angles of incidence. It is also of interest to establish the extent of the reduction of the high-frequency components of the response associated with scattering of seismic waves by the rigid foundation. Another objective is to determine the importance of the rocking response for Rayleigh wave excitation and its effects on the translational response at the higher levels of the superstructure. Finally, it is important to establish whether the conventional assumption of vertically incident seismic waves leads to conservative or unconservative estimates of the response for more general seismic excitation.

INTERACTION EQUATIONS

Based on the assumed linearity of the model, the response of the soil-structure system may be obtained in two stages. In the first stage, the frequency response of the system for harmonic excitation with time dependence of the type $\exp(i\omega t)$ is obtained. The second stage corresponds to the evaluation of the response in time by means of the Fourier synthesis given by

$$u(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(\omega) e^{i\omega t} d\omega \qquad (1)$$

In this equation, $u(t)$ denotes the response in time, while $U(\omega)$ represents the frequency response. The Fourier synthesis described by equation (1) is readily calculated by use of the Fast Fourier Transform algorithm (Liu and Fagel, 1971).

The approach used herein to obtain the frequency response for the complete soil-structure system is illustrated in Figure 2 and it consists of subdividing the complete problem into a set of simpler problems that may be solved independently. Once the solution for each of the basic problems is known, the response of the interacting soil-structure system may be easily obtained.

The first basic problem that needs to be considered corresponds to the evaluation of the free-field motion, i.e., to the determination of the response of the soil deposit for a given incident seismic wave in absence of the foundation. For uniform or horizontally layered soil deposits and for plane incident waves, the free-field motion on the soil surface may be characterized by the three-component displacement

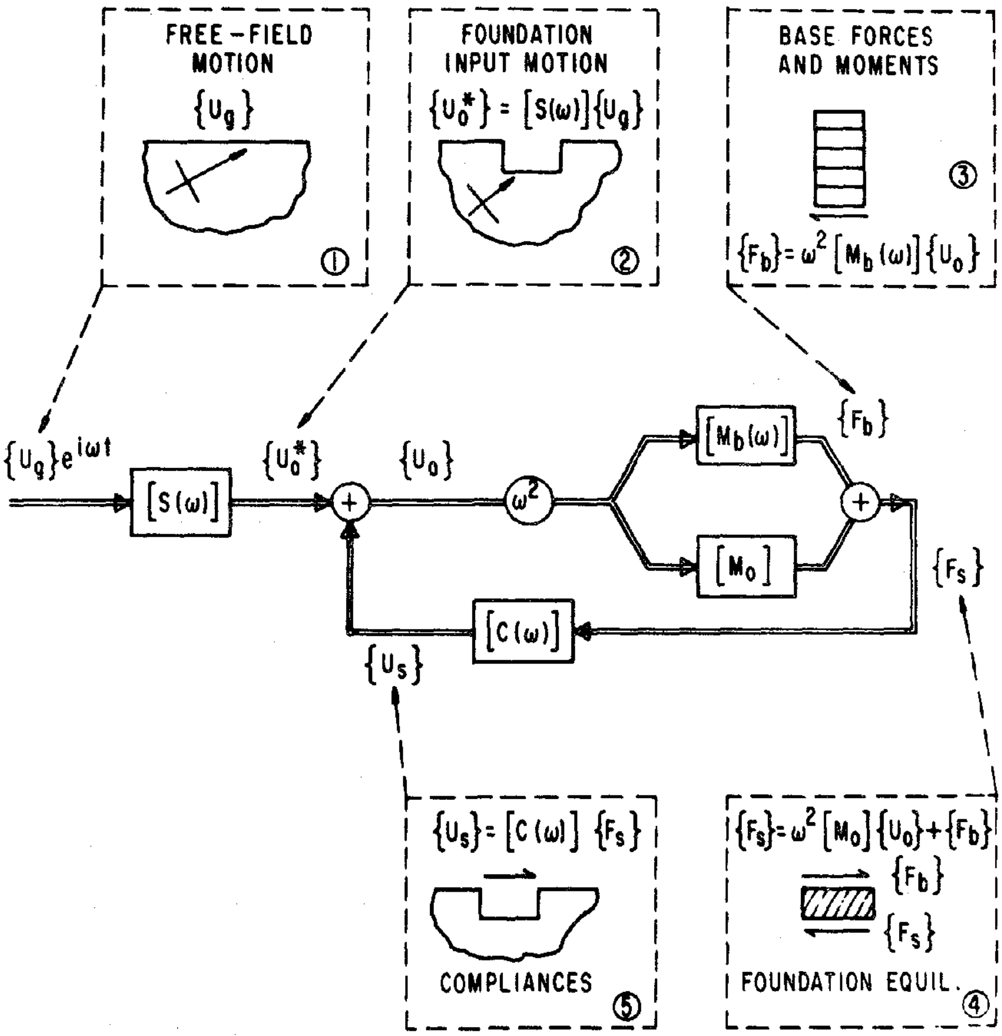

Figure 2. Interaction Block Diagram

vector $\{U_g\}e^{i\omega t}$ at a reference point, by the horizontal direction of propagation of the incident wave, and by the apparent horizontal velocity c of the incident wave in the direction of propagation. For a given seismic excitation the three components of the free-field motion vector of the soil surface $\{U_g\}$ are not independent. In addition, for surface wave excitation in a layered soil model, the apparent horizontal velocity c depends on frequency ω. The evaluation of the free-field motion for plane body waves and for surface waves can be accomplished by standard techniques (Ewing, Jardetsky and Press, 1957) and will not be discussed here.

The second basic problem is associated with the evaluation of the harmonic response of the rigid foundation bonded to the soil and subjected to the incident seismic wave in absence of the superstructure. In this step, the foundation is assumed massless, the inertia of the foundation being incorporated at a later stage. The presence of the rigid foundation modifies the free-field motion and the resulting response of the massless foundation, called here foundation input motion, may be represented by a six-component vector

$$\{U_0^*\} = (\Delta_x^*, \theta_y^*, \Delta_y^*, \theta_x^*, \Delta_z^*, \theta_z^*)^T \quad ,$$

in which Δ_x^*, Δ_y^* and Δ_z^* represent the translational components of the response about the x, y and z axis, respectively. Techniques are available (Wong and Luco, 1978a,b; Luco and Wong, 1977) to determine the foundation input motion for flat rigid foundations of arbitrary shape supported on layered viscoelastic media. The foundation input motion $\{U_0^*\}$ depends on the frequency of the excitation, geometry of the foundation, characteristics of the soil deposit and on the type of seismic excitation.

When the inertia of the foundation and the presence of the superstructure are taken into account, the total response $\{U_0\} = (\Delta_x, \theta_y, \Delta_y, \theta_x, \Delta_z, \theta_z)^T$ at the reference point in the rigid foundation may be written in the form

$$\{U_0\} = \{U_0^*\} + \{U_s\} \tag{2}$$

in which the 6 x 1 vector $\{U_s\}$ corresponds to the additional motion of the foundation associated with deformation of the soil caused by the forces and moments that the foundation exerts on the soil. The forces and moments that the foundation exerts on the soil can be represented by the 6 x 1 vector $\{F_s\} = (F_{xs}, M_{ys}, F_{ys}, M_{xs}, F_{zs}, M_{zs})^T$, in which F_{xs}, F_{ys} and F_{zs} represent the components of the resultant force, while M_{xs}, M_{ys} and M_{zs} denote the components of the resultant moment about the point of reference in the foundation. The motion $\{U_s\}$ of the foundation caused by the generalized force $\{F_s\}$ is given by

$$\{U_s\} = [C(\omega)]\{F_s\} \tag{3}$$

where $[C(\omega)]$ is the 6 x 6 compliance matrix for the rigid foundation. The third basic problem corresponds then to the evaluation of the compliance matrix for the rigid foundation. A variety of techniques are available to evaluate the compliance matrix for different types of foundations, in particular, the authors (Wong and Luco, 1976, 1978b) have developed a procedure to determine the compliance matrix for flat rigid foundations of arbitrary shape supported on a layered viscoelastic half-space. The compliance matrix depends on the frequency of the excitation, the geometry of the foundation and on the characteristics of the underlying soil deposits.

If $\{F_b\} = (F_{xb}, M_{yb}, F_{yb}, M_{xb}, F_{zb}, M_{zb})^T$ represents the resultant forces and moments that the superstructure exerts on the foundation, then the equation of motion for the rigid foundation can be written in the form

$$\{F_s\} = \omega^2[M_0]\{U_0\} + \{F_b\} \tag{4}$$

where $[M_0]$ is the 6 x 6 mass matrix for the rigid foundation.

It can be shown that the generalized force $\{F_b\}$ which the superstructure exerts on the foundation can be expressed in terms of the total motion of the foundation $\{U_0\}$ through the relation

$$\{F_b\} = \omega^2[M_b(\omega)]\{U_0\} \tag{5}$$

where $[M_b(\omega)]$ plays the role of a 6 x 6 frequency-dependent equivalent mass matrix. This matrix depends on the geometry, mass distribution and elastic properties of the superstructure. The fourth basic problem entails the construction of the equivalent mass matrix $[M_b(\omega)]$. An effective technique to construct the equivalent mass matrix has been presented by Lee and Wesley (1971) and will be described in the following section.

Once the basic sub-problems have been solved, the total motion $\{U_0\}$ of the foundation including the soil-structure interaction effects can be obtained by eliminating $\{F_b\}$, $\{F_s\}$ and $\{U_s\}$ from equations (2) through (5). The resulting expression is

$$\{U_0\} = ([I] - \omega^2[C(\omega)]([M_0] + [M_b(\omega)]))^{-1}\{U_0^*\} \tag{6}$$

in which [I] denotes the 6 x 6 identity matrix. Equation (6) clearly separates the different interaction effects. The effects of the scattering of the incident seismic waves by the rigid foundation are included in the foundation input motion vector $\{U_0^*\}$ which, in the case of non-vertically incident waves, involves translational and rotational components. The interaction effects between the superstructure, foundation and soil are represented in equation (6) by the term $[C(\omega)]([M_0] + [M_b(\omega)])$. The total motion of the foundation $\{U_0\}$ results from a combination of both types of effects as shown in the feedback block diagram of Figure 2.

Once the total motion at foundation level has been obtained, the response in the frequency and time domains at any level of the superstructure can be easily obtained by standard techniques.

EQUIVALENT MASS MATRIX

To evaluate the equivalent mass matrix $[M_b(\omega)]$ appearing in equations (5) and (6), it is convenient to follow the procedure described by Lee and Welsey (1971) for a model of the superstructure consisting of N rigid bodies interconnected by elastic members. In general, the relative motion of the j-th rigid body with respect to a frame of reference attached to the moving rigid foundation can be described by six generalized coordinates $(U_{xj}, U_{yj}, U_{zj}, \theta_{xj}, \theta_{yj}, \theta_{zj})$ corresponding to the components of the relative displacement of the center of mass and to the components of the relative rotation vector. The generalized relative displacement vector for the superstructure {U} is defined by the (6N x 1) vector

$$\{U\} = (U_{x1},\ldots,U_{xN},\theta_{y1},\ldots,\theta_{yN},U_{y1},\ldots,U_{yN},\theta_{x1},\ldots,\theta_{xN}, U_{z1},\ldots,U_{zN},\theta_{z1},\ldots,\theta_{zN})^T \quad . \tag{7}$$

The generalized total displacement vector $\{U_t\}$ is defined similarly except that the translational and rotational components are referred to a fixed frame of reference.

For small vibrations, the total displacement vector $\{U_t\}$ is given by

$$\{U_t\} = [\alpha]\{U_0\} + \{U\} \tag{8}$$

where $\{U_0\} = (\Delta_x, \theta_y, \Delta_y, \theta_x, \Delta_z, \theta_z)^T$ is the total foundation motion and $[\alpha]$ is the (6N x 6) rigid displacement matrix

$$[\alpha] = \begin{bmatrix} \{1\} & \{z\} & 0 & 0 & 0 & -\{y\} \\ 0 & \{1\} & 0 & 0 & 0 & 0 \\ 0 & 0 & \{1\} & -\{z\} & 0 & \{x\} \\ 0 & 0 & 0 & \{1\} & 0 & 0 \\ 0 & -\{x\} & 0 & \{y\} & \{1\} & 0 \\ 0 & 0 & 0 & 0 & 0 & \{1\} \end{bmatrix} \qquad (9)$$

in which

$$\{1\} = (1,\ldots,1)^T \qquad (10)$$
$$\{x\} = (x_1,\ldots,x_N)^T \quad , \quad \{y\} = (y_1,\ldots,y_N)^T \quad , \quad \{z\} = (z_1,\ldots,z_N)^T$$

where (x_j, y_j, z_j) correspond to the coordinates of the j-th rigid body.

By writing the equations of motion for the superstructure and using the fixed base modes of vibration, it may be shown that

$$\{U\} = [\Phi][\ulcorner D(\omega) \lrcorner][\beta]^T\{U_0\} \qquad (11)$$

where $[\Phi]$ is the (6N x 6N) fixed-base modal matrix for the superstructure normalized with respect to the mass matrix of the superstructure, i.e.,

$$[\Phi]^T[M][\Phi] = [I] \quad , \qquad (12)$$

the (6N x 6N) matrix $[\beta]$ corresponds to the modal participation matrix

$$[\beta] = [\alpha]^T[M][\Phi] \quad , \qquad (13)$$

and $[\ulcorner D(\omega) \lrcorner]$ is a diagonal (6N x 6N) modal amplification matrix having for elements

$$D_r(\omega) = \frac{(\omega/\omega_r)^2}{1 - (\omega/\omega_r)^2 + 2i\xi_r(\omega/\omega_r)} \quad , \quad (r=1,6N) \qquad (14)$$

in which ω_r and ξ_r correspond to the r-th fixed-base natural frequency and modal damping ratio, respectively.

From equations (8) and (11), it is found that the total displacement $\{U_t\}$ can be calculated by

$$\{U_t\} = ([\alpha] + [\Phi][{}^{\backslash}D(\omega)_{\backslash}][\beta]^T)\{U_0\} \tag{15}$$

once the total motion of the foundation $\{U_0\}$ is known.

The generalized force $\{F_b\} = (F_{xb}, M_{yb}, F_{yb}, M_{xb}, F_{zb}, M_{zb})^T$ that the superstructure exerts on the foundation (referred to the point of reference in the foundation) may be obtained by considering the total linear and angular moments. For small vibrations,

$$\{F_b\} = \omega^2[\alpha]^T[M]\{U_t\} \quad . \tag{16}$$

By substitution from equation (15) into equation (16), it is found that $\{F_b\}$ can be written in the form indicated by equation (5) where the equivalent mass matrix $[M_b(\omega)]$ is given by

$$[M_b(\omega)] = [M_{b0}] + [\beta][{}^{\backslash}D(\omega)_{\backslash}][\beta]^T \tag{17}$$

in which

$$[M_{b0}] = [\alpha]^T[M][\alpha] \tag{18}$$

corresponds to the 6×6 mass matrix of the superstructure for rigid translations and rotations about the point of reference on the foundation.

It is of interest to discuss the behavior of the equivalent mass matrix $[M_b(\omega)]$ as the frequency ω tends to infinity. From equation (14), it can be seen that $[{}^{\backslash}D(\omega)_{\backslash}] \to -[I]$ as $\omega \to \infty$. Equation (17) indicates, then, that

$$[M_b(\omega)] \rightarrow [M_{b0}] - [\beta][\beta]^T \quad \text{as} \quad \omega \rightarrow \infty \quad . \tag{19}$$

If the set of fixed-base modes is complete, it can be shown that

$$[\alpha] = [\Phi][\beta]^T \tag{20}$$

and

$$[M_{b0}] = [\beta][\beta]^T \tag{21}$$

Equations (19) and (21) reveal that if the set of fixed base modes is complete, $[M_b(\omega)] \rightarrow 0$ as $\omega \rightarrow \infty$. Referring to equation (6), it may be seen that at sufficiently high frequencies, the total response of the foundation $\{U_0\}$ becomes independent of the properties of the superstructure.

If the set of modes is not complete, $[M_b(\omega)]$ will not tend to zero as $\omega \rightarrow \infty$, but to a residual mass matrix which represents the contribution to the base forces and moments of the modes excluded.

For most practical applications, not all fixed-base modes need to be considered. If the first $\bar{N}$ modes of vibration are included, then the matrices $[\Phi]$, $[\beta]$ and $[\text{`}D(\omega)\text{.}]$ will have dimensions $(6N \times \bar{N})$, $(6 \times \bar{N})$ and $(\bar{N},\bar{N})$, respectively. One of the advantages of the formulation is that for frequencies lower than the fixed-base natrual frequencies of the modes excluded, the contributions of these modes to the base forces and moments are still approximately represented in the equivalent mass matrix $[M_b(\omega)]$ through the matrix $[M_{b0}]$.

Another advantage of the procedure described to obtain the equivalent mass matrix is that the fixed-base mode shapes, natural frequencies, modal damping ratios and participation factors can be obtained by the method most appropriate to the particular structure being analyzed. For complex structures, these quantities may be obtained by use of a finite element model (Wong and Luco, 1977); in other cases, simplified lumped mass or continuous representations may be adequate.

FOUNDATION INPUT MOTION AND IMPEDANCE FUNCTIONS

Two types of seismic excitations are considered in this study. The first type corresponds to obliquely incident SH waves with particle motion parallel to one of the sides of the rectangular foundation ($\theta_H = 0°$ or $\theta_H = 90°$ in Figure 1). In particular, for $\theta_H = 0$, the free-field motion can be represented by

$$\begin{Bmatrix} u_x(x,y,0) \\ u_y(x,y,0) \\ u_z(x,y,0) \end{Bmatrix} = U_g \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \exp\left[i\omega\left(t - \frac{x\cos\theta_v}{\beta(1+2i\xi)^{\frac{1}{2}}}\right)\right] \tag{22}$$

where $U_g(\omega)$ is the amplitude of the free-field motion at the point where the center of the foundation will be located, β is the shear wave velocity in the soil and θ_v is the vertical angle of incidence shown in Figure 1 ($\theta_v = 0$ for horizontally propagating SH waves).

The response of a massless rectangular foundation to this excitation consists of a translation U_y^* along the y-axis, a rotation θ_z^* about the vertical z-axis and a small rotation θ_x^* about the x-axis. The components $(U_y^*, \theta_z^*, \theta_x^*)$ of the foundation input motion for SH wave excitation and for a soil characterized by a Poisson's ratio $\nu = 0.33$ and a hysteretic damping ratio $\xi = 0.05$ have been calculated using the procedure described by the authors (Wong and Luco, 1978a,b). The normalized amplitudes $|U_y^*/U_g|$ and $|a\theta_z^*/U_g|$ are shown Figure 3 as a function of the dimensionless frequency $a_0 = \omega a/\beta$ (where a is the half-width of the foundation) for three values of the angle of incidence θ_v. The results presented in Figure 3 show a marked reduction of the translational response of the

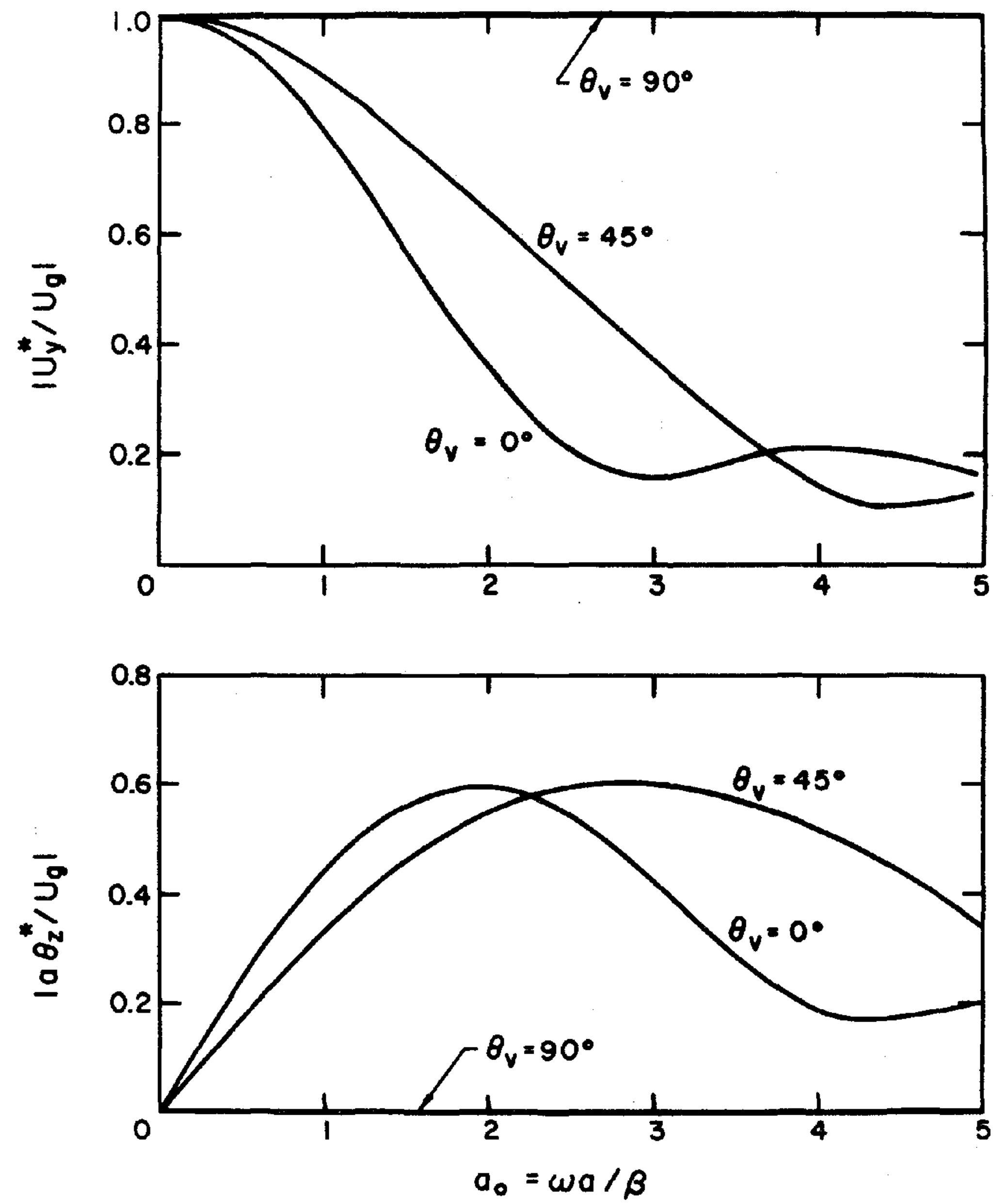

Figure 3. Normalized Amplitudes of the Horizontal and Torsional Components of the Foundation Input Motion for SH Wave Excitation

foundation at intermediate and high frequencies for nonvertically incident SH waves. On the other hand, a large torsional response component is obtained. For flat foundations, the rocking component of the input motion for SH waves, θ_x^*, is extremely small and can be neglected for all practical purposes (Wong and Luco, 1978a).

The second type of seismic excitation considered corresponds to Rayleigh waves impinging normal to one of the sides of the foundation ($\theta_H = 0°$ or $\theta_H = 90°$ in Figure 1). The free-field motion for Rayleigh waves propagating along the y-axis ($\theta_H = 90°$) is given by

$$\begin{Bmatrix} u_x(x,y,0) \\ u_y(x,y,0) \\ u_z(x,y,0) \end{Bmatrix} = U_g \begin{Bmatrix} 0 \\ 1 \\ R_v \end{Bmatrix} \exp\left[i\omega\left(t - \frac{y}{c_R(1+2i\xi)^{\frac{1}{2}}}\right)\right] \tag{23}$$

where c_R is the Rayleigh wave velocity, U_g is the amplitude of the horizontal component, and R_v is the ratio of the vertical to the horizontal component. For a soil characterized by a Poisson's ratio of $\nu = 0.33$, $c_R = 0.9325\beta$ and $R_v = -1.565i$ if the effects of material attenuation on c_R and R_v are neglected.

The response of a massless rectangular foundation to the excitation described by equation (23) consists of horizontal translation U_y^* rocking θ_x^* and vertical translation U_z^* (Luco and Wong, 1977). The normalized amplitudes $|U_y^*/U_g|$, $|a\theta_x^*/U_g|$ and $|U_z^*/U_g|$ for the foundation and soil model under consideration are shown in Figure 4 versus the dimensionless frequency a_0. The results shown in Figure 4 indicate that scattering by the foundation leads to significant reductions in the translational components at intermediate and high frequencies. At the same time, a

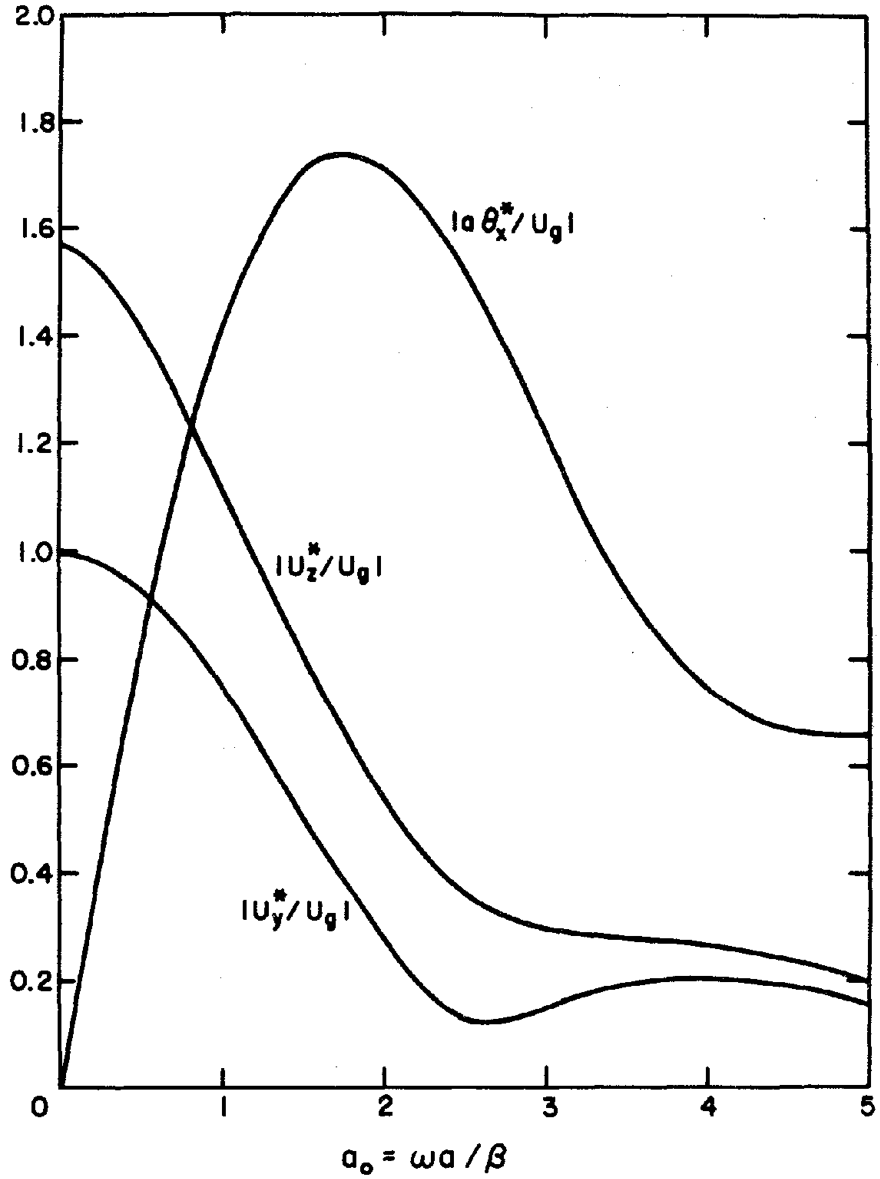

Figure 4. Normalized Amplitudes of the Horizontal, Vertical, and Rocking Components of the Foundation Input Motion for Rayleigh Wave Excitation

large rocking component is obtained.

Once the horizontal component of $U_g(\omega)$ of the free-field motion has been specified, the foundation input motion can be obtained by multiplying the transfer functions U_y^*/U_g, $a\theta_x^*/U_g$, etc., by $U_g(\omega)$.

Finally, to complete the discussion of the terms appearing in equation (6), reference must be made to the compliance matrix $[C(\omega)]$. A standard procedure is now available (Wong and Luco, 1976; 1978b) to calculate the compliance, or, its inverse, the impedance matrix for rectangular foundations. In the particular case of a square foundation of width 2a, the impedance matrix can be written in the form

$$[K] = [C]^{-1} = Ga \begin{bmatrix} K_{HH} & aK_{HM} & 0 & 0 & 0 & 0 \\ aK_{MH} & a^2K_{MM} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{HH} & -aK_{HM} & 0 & 0 \\ 0 & 0 & -aK_{MH} & a^2K_{MM} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{VV} & 0 \\ 0 & 0 & 0 & 0 & 0 & a^2K_{TT} \end{bmatrix} \qquad (24)$$

where G is the shear modulus of the soil. Each one of the elements appearing in the stiffness matrix can be written in turn in the form $K = k + ia_0c$, where k and c are designated normalized stiffness and damping coefficients. The numerical values of these coefficients for the foundation and soil model under consideration are shown in Figure 5 versus the dimensionless frequency a_0.

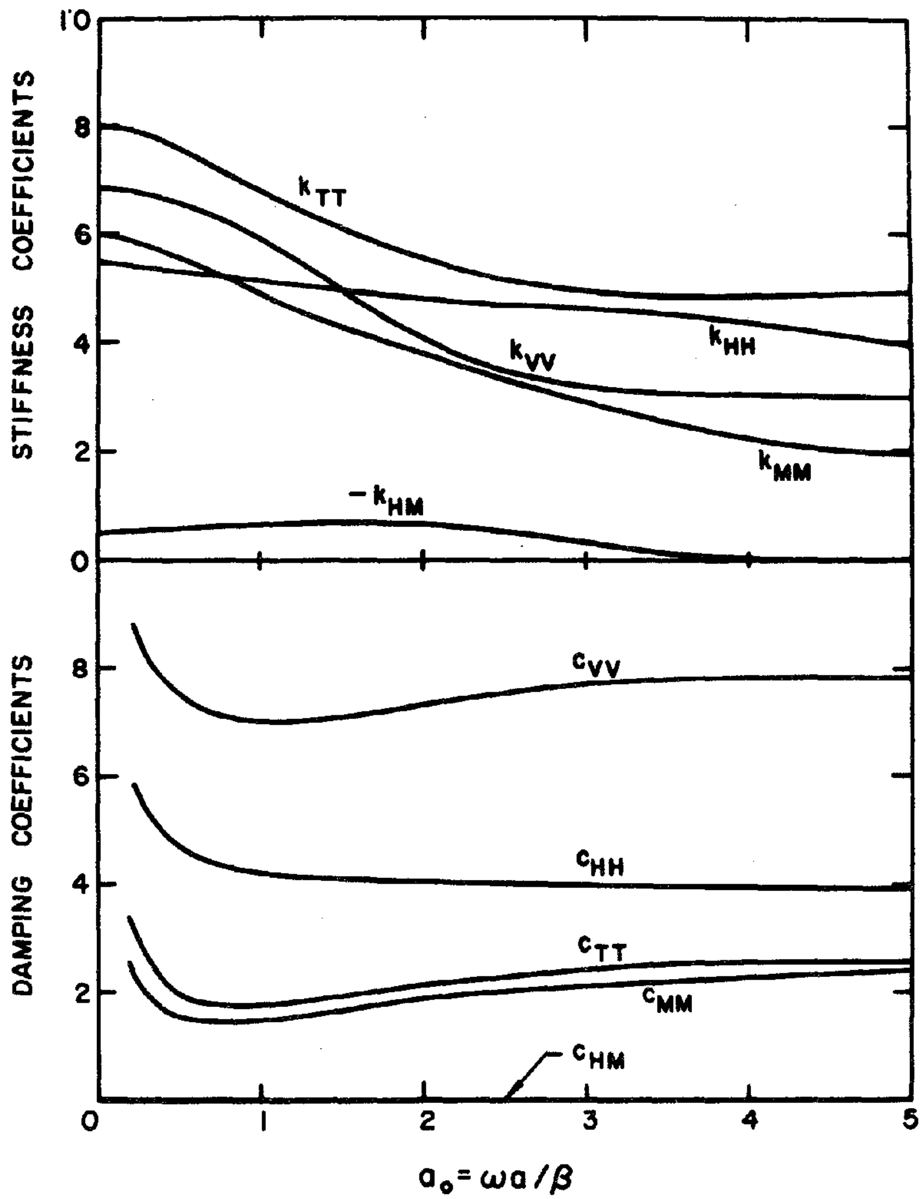

Figure 5. Dynamic Stiffness and Damping Coefficients for a Square Foundation (ν = 0.33, ξ = 0.05)

DESCRIPTION OF THE MODELS CONSIDERED

To illustrate the effects of nonvertically incident seismic waves on the response of structures, it is convenient to consider first the case of symmetric structures and foundations. In this case, the mass matrix for the rigid foundation can be written in the form

$$[M_0] = \left[\begin{array}{cc|cc|c|c} M_0' & S_{0y} & 0 & & 0 & 0 \\ S_{0y} & I_{0y} & & & & \\ \hline 0 & & M_0' & -S_{0x} & 0 & 0 \\ & & -S_{0x} & I_{0x} & & \\ \hline 0 & & 0 & & M_0' & 0 \\ \hline 0 & & 0 & & 0 & I_{0z} \end{array}\right] \qquad (25)$$

where M_0' represents the mass of the foundation, while I_{0x}, I_{0y} and I_{0z} represent the mass moments of inertia of the foundation with respect to the x, y and z axes, respectively. In particular, for a flat square foundation of mass M_0' and half-width a, $S_{0x} = S_{0y} = 0$ and $I_{0z} = 2I_{0x} = 2I_{0y} = 2M_0'a^2/3$). For symmetric structures, the matrix $[M_{b0}]$ defined by equation (18) can be written in the same form except that the quantities M_0', S_{0y}, I_{0y}, etc., should be substituted by M_b', S_{by}, I_{by}, etc. The term M_b' represents the total mass of the superstructure while the quantities I_{bx}, I_{by} and I_{bz} correspond to the mass moments of inertia of the superstructure with respect to axes parallel to the x, y and z axes passing through the point of reference in the foundation (in particular, for a uniform structure of total mass M_b' and height H, $I_{bx} = I_{by} = M_0'H^2/3$ if the rotatory inertia of each floor is neglected, and $S_{bx} = S_{by} = M_0'H/2$).

If the fixed-base modes of vibration of the superstructure are grouped into modes of vibration in the xz plane, yz plane, vertical and torsional modes, the matrix of participation factors [β] reduces to

$$[\beta]^T = \left[\begin{array}{cc|cc|c|c} \{\beta_1\} & \{\beta_2\} & \multicolumn{2}{c|}{0} & 0 & 0 \\ \hline \multicolumn{2}{c|}{0} & \{\beta_3\} & -\{\beta_4\} & 0 & 0 \\ \hline \multicolumn{2}{c|}{0} & \multicolumn{2}{c|}{0} & \{\beta_5\} & 0 \\ \hline \multicolumn{2}{c|}{0} & \multicolumn{2}{c|}{0} & 0 & \{\beta_6\} \end{array}\right] \qquad (26)$$

where the participation factors $\{\beta_1\}$ and $\{\beta_2\}$ are associated with vibrations in the xz plane, while $\{\beta_3\}$ and $\{\beta_4\}$ are associated with vibrations in the yz plane, $\{\beta_5\}$ is connected with vertical vibrations, and $\{\beta_6\}$ with torsional vibrations. In particular, if for vibrations in the xz and yz planes, the structure is modeled as a uniform shear wall, while for vertical and torsional vibrations it is modeled as a uniform bar, the normalized fixed-base translational modes of vibration are given by

$$\phi_r(z) = \left(\frac{2}{M_b'}\right)^{\frac{1}{2}} \sin[(2r-1)\pi z/2H] \quad , \quad r=1,2,3, \ldots \qquad (27)$$

The normalized fixed-base torsional modes of vibration are also given by equation (27) after the total mass M_b' is substituted by the total moment of inertia I_{bz}. In this case, the components of the participation factor vectors appearing in equation (26) are

$$\beta_{r1} = \beta_{r3} = \beta_{r5} = \frac{2\sqrt{2}}{(2r-1)\pi} (M_b')^{\frac{1}{2}}$$

$$\beta_{r2} = \beta_{r4} = (-1)^{r-1}\sqrt{2} \left[\frac{2}{(2r-1)\pi}\right]^2 H(M_b')^{\frac{1}{2}}$$

$$\beta_{r6} = \frac{2\sqrt{2}}{(2r-1)\pi} (I_{bz})^{\frac{1}{2}} \qquad , \quad r=1,2,3, \ldots \qquad (28)$$

From equations (17) and (26) it can be found that the equivalent mass matrix $[M_b(\omega)]$ is block diagonal can be partitioned in the same form as $[M_o]$ given by equation (25). Since the compliance matrix $[C(\omega)]$ for symmetric foundations is also block diagonal, equation (16) reveals that the components of the response involving vibrations in the xz plane, yz plane and vertical and torsional vibrations can be calculated independently.

For the purpose of this study, two symmetric structures with the properties listed in Table 1 were considered. The first structure (Model 1) corresponds approximately to a ten-story reinforced concrete building with a height of 40m and a square floor plan of 20 x 20m. The structure is assumed to have significantly different stiffness in the x and y directions. The 20 x 20m square foundation is assumed to be rigid and resting on a uniform soil characterized by a shear wave velocity of 400m/sec. The second structure considered (Model 2) corresponds to an idealized model of a containment building in a nuclear power plant. The model, in this case, has the same stiffness in the x and y directions. The foundation is represented by an equivalent rigid 40 x 40m square mat resting on a uniform soil with a shear wave velocity of 600m/sec. For simplicity, the fixed-base natural frequencies were assumed to be in the ratios 1 : 3 : 5 : 7 ..., and the mode

TABLE 1

Properties of the Models Considered

	Model 1	Model 2
H (m)	40	60
a (m)	10	20
M_b' (kg)	10^7	$3\text{x}10^7$
$S_{bx}/HM_b' = S_{by}/HM_b'$	0.50	0.50
$I_{bx}/H^2M_b' = I_{by}/H^2M_b'$	0.33	0.38
I_{bz}/H^2M_b'	0.04	0.10
M_0'/M_b'	0.15	0.45
$S_{0x}/HM_b' = S_{0y}/HM_b'$	0.000	0.000
$I_{0x}/H^2M_b' = I_{0y}/H^2M_b'$	0.003	0.017
I_{0z}/H^2M_b'	0.006	0.034
Fixed-base natural frequencies (Hz):		
vibrations in xz-plane	1, 3, 5, 7, 9	5, 15, 25
vibrations in yz-plane	2, 6, 10, 14, 18	5, 15, 25
vertical vibrations	3, 9, 15, 21, 27	6, 18, 30
torsional vibrations	3, 9, 15, 21, 27	8, 24, 40
Modal damping coefficients (all modes)	0.02	0.02
Shear wave velocity in soil (m/sec)	400	600
Shear modulus in soil (N/m^2)	$3\text{x}10^8$	$6.75\text{x}10^8$
Poisson's ratio in soil	0.333	0.333
Damping coefficients in soil	0.05	0.05

shapes and modal participation factors were taken as given by equations (27) and (28). Modal damping coefficients of 2 percent were used for both structures and all modes considered.

Calculations of the response in the time domain and of floor response spectra were obtained by setting the horizontal component of the free-field motion equal to the NS acceleration time-history recorded at El Centro for the 1940 Imperial Valley earthquake. For Model 1, calculations were performed separately for excitations impinging normal to both sides of the foundation as shown in Figure 6. For the symmetric Model 2, it is sufficient to consider excitation leading to response in only one direction.

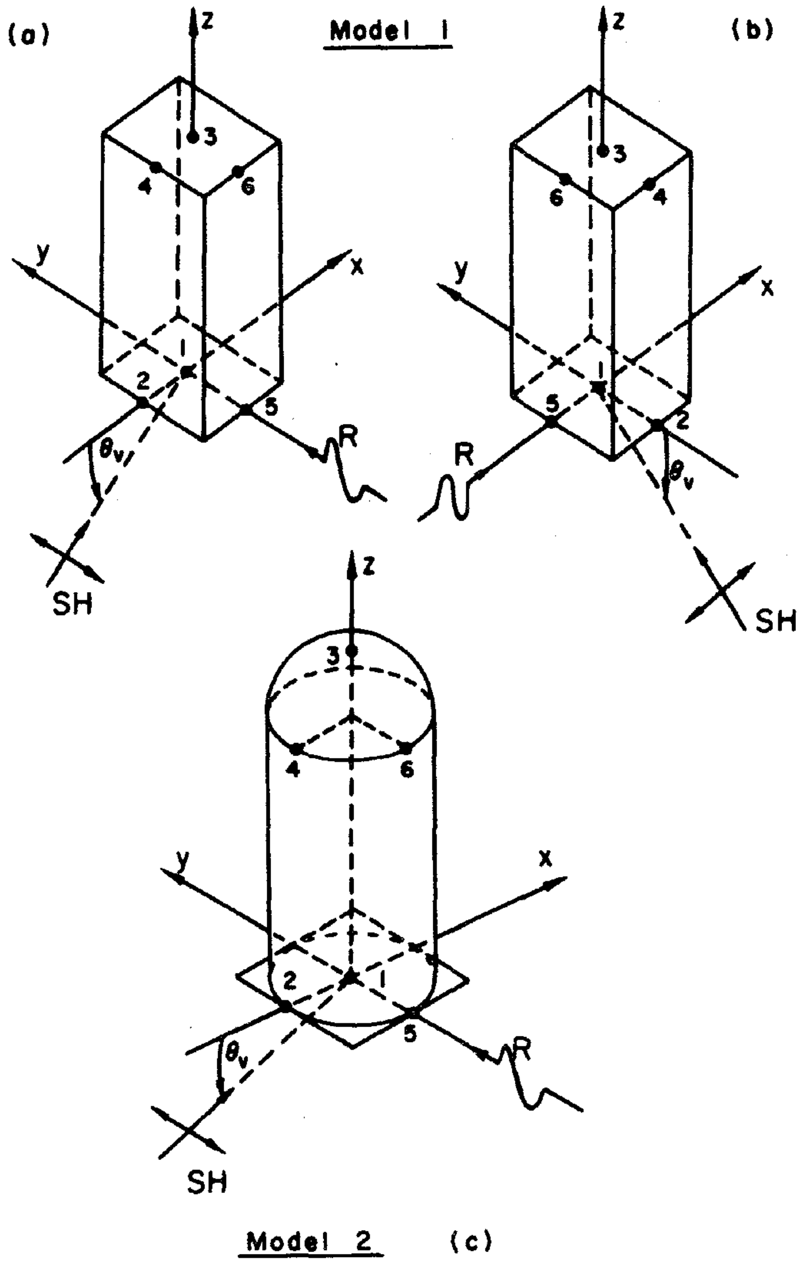

Figure 6. Description of the Types of Seismic Excitation Considered and Locations at Which the Response is Calculated

RESPONSE TO NONVERTICALLY INCIDENT SH WAVES

The evaluation of the response of the two models under study was conducted in two steps. In the first step, the response in the frequency domain (transfer functions) for a free-field motion U_g of unit amplitude was calculated at 2,049 equally-spaced frequencies covering the range from 0 to 25 cps. The amplitudes of the resulting transfer functions at several locations in both models (points 1, 2, 3 and 4; Figure 6) are shown in Figures 9, 11 and 13. The second step leading to the response in the time domain involved multiplying the transfer functions by the Fourier transform of the acceleration time history for the NS component of the El Centro 1940 record followed by inversion to the time domain. An example of the acceleration time histories obtained at different locations of Model 1 is shown in Figures 7 and 8. At each location, floor (in-structure) absolute acceleration response spectra were also calculated as shown in Figures 10, 12 and 14.

The transfer functions and floor response spectra at several locations on Model 1 when excited by SH waves impinging on the foundation along the x axis (Figure 6a) are shown in Figures 9 and 10, respectively, for angles of incidence $\theta_v = 0°$ (horizontal), 45° and 90° (vertical incidence). The translational response (U_y) at the center of the base, shown in Figures 9a and 10a, illustrates the effects of soil-structure in the neighborhood of the fixed-base natural frequencies (2, 6, 10, 14, 18 cps). A marked reduction of the response associated with scattering by the rigid foundation can be noticed at frequencies higher than 6 cps. Figures 9b and 10b illustrate the translational response U_y at a

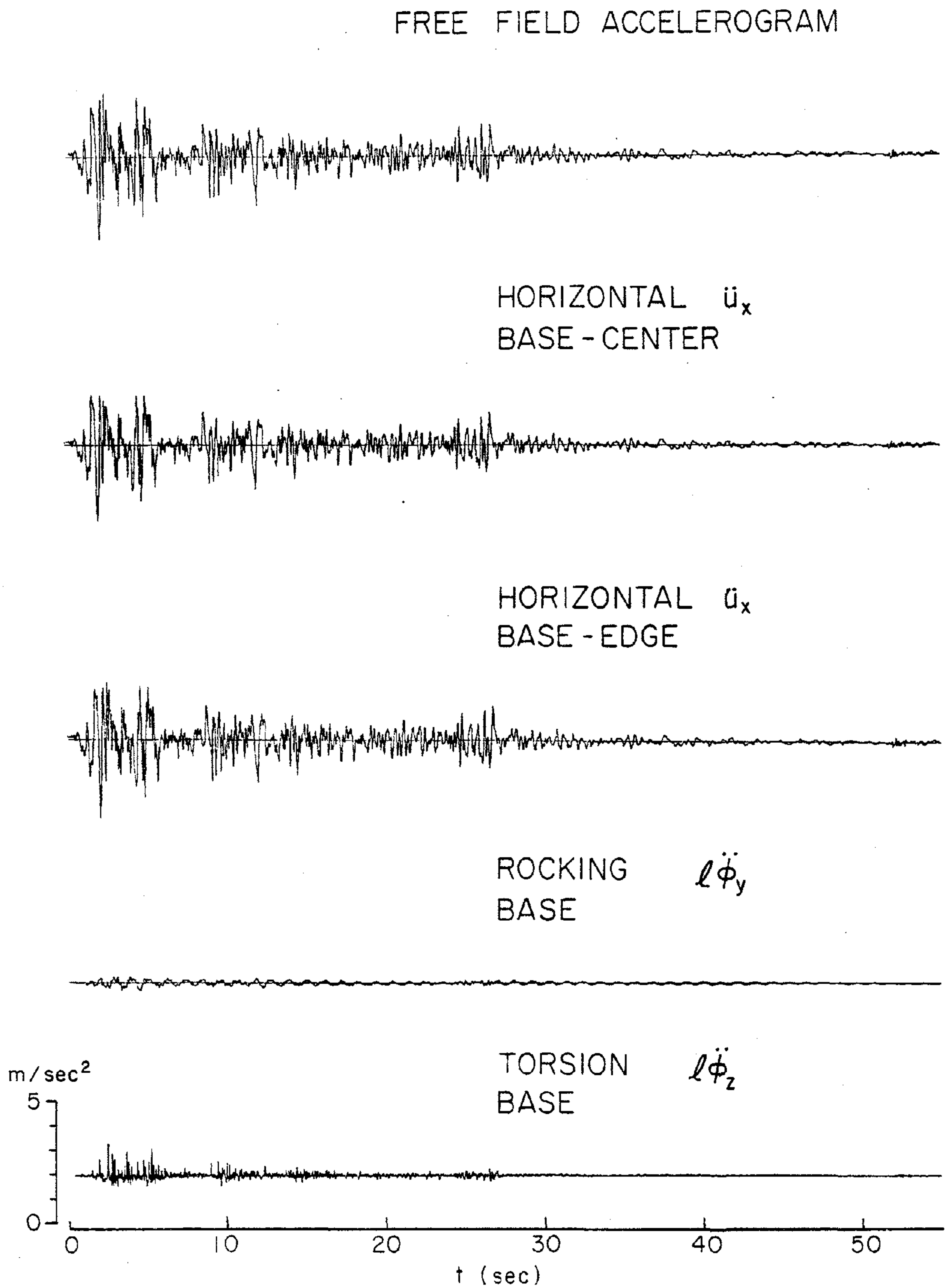

Figure 7. Free-Field and Calculated Acceleration Time Histories for SH Wave Excitation (θ_v = 45°, Model 1)

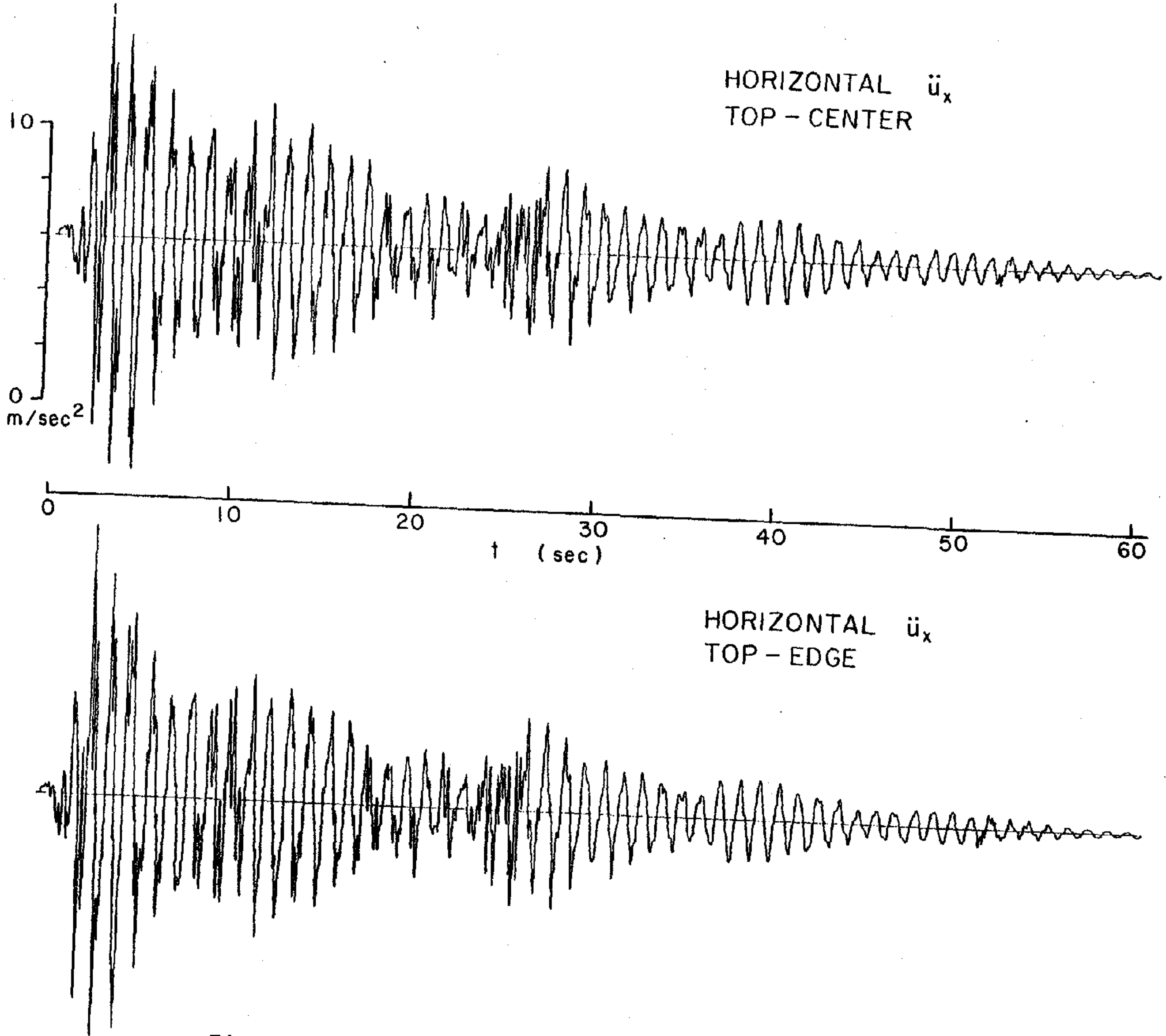

Figure 8. Calculated Accelerations at the Top of Model 1 for SH Wave Excitation ($\theta_v = 45°$)

point at the edge of the base (location 2 in Figure 6a). For nonvertically incident SH waves, the contribution of the torsional response of the structure can be seen in the neighborhood of the fixed-base natural frequencies in torsion (3, 9, 15, 21 and 27 cps). In addition, due to the torsional response of the foundation, the reduction of the response at high frequencies is not as marked as at the center of the foundation.

The rocking response $a\theta_x$ (a = 10m) about the x-axis at the base of the structure is shown in Figures 9c and 10c in which it can be seen that a lower response is obtained for horizontally incident waves. Figures 9d and 10d illustrate the torsional response $a\theta_z$ at the base of the structure for angles of incidence $\theta_v = 0°$ and 45° (for vertical incidence, the torsional response is zero under the assumption of a symmetric structure). Figure 9d shows that the torsional response at the base follows the torsional input motion $a\theta_z^*$ (Figure 3) except for oscillations near the fixed-base torsional frequencies.

The translational response at the center of the top of the structure (point 3 in Figure 6a) is shown in Figures 9e and 10e. The transfer function at the top shows a significant reduction at frequencies higher than 6 cps for nonvertically incident waves. The floor response spectrum at the top, however (Figure 10e), does not show the same behavior. The reduction at high frequencies in this case is less than 5%. The difference must be attributed to the fact that the response in the fundamental mode which is affected only slightly by scattering of waves controls the time response. Finally, Figures 9f and 10f illustrate the translational response at a point on the edge of the top (point 4 in Figure 6a). The effects of the torsional response can be seen

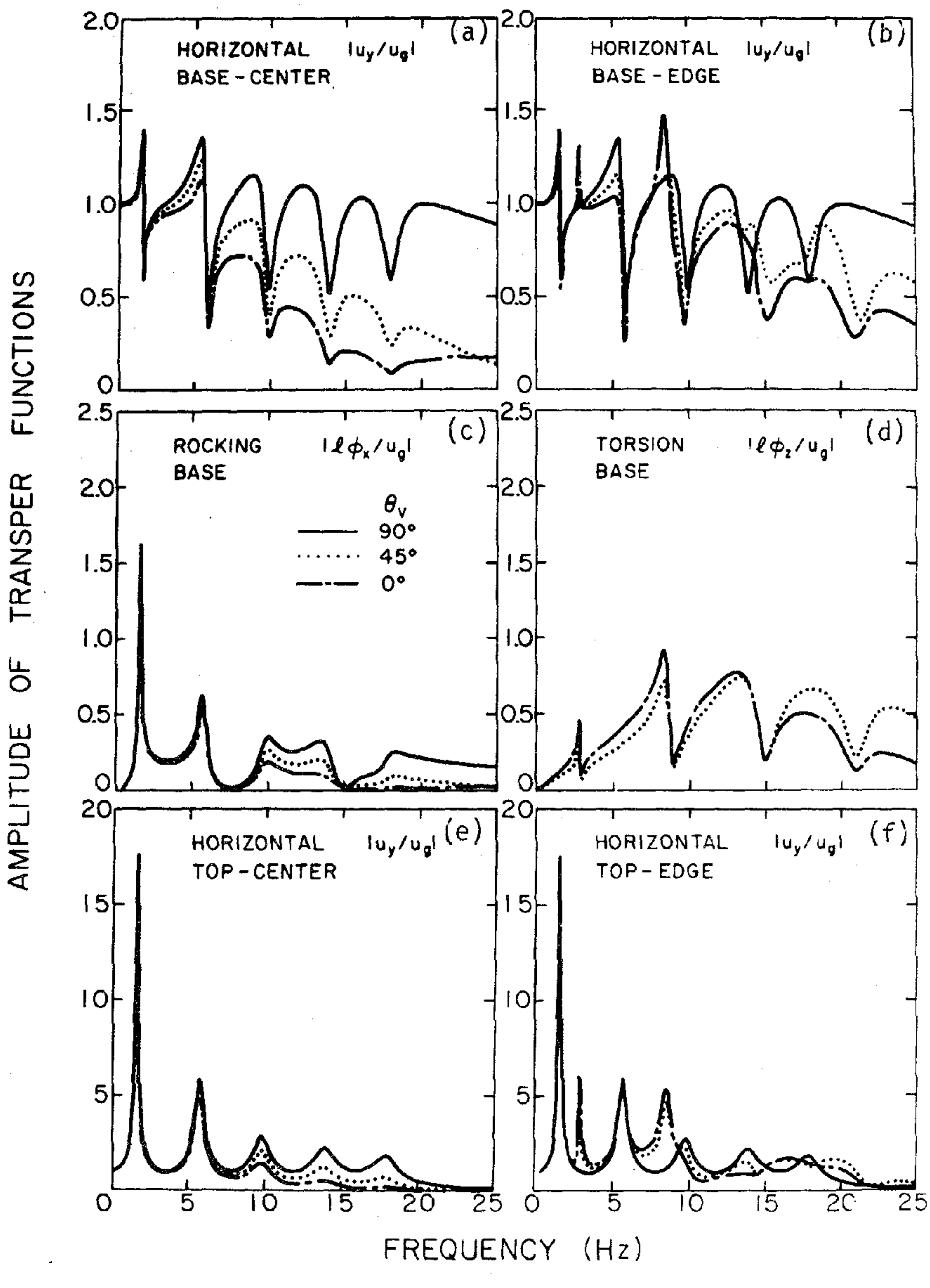

Figure 9. Transfer Functions for Model 1 Excited by SH Waves with Particle Motion Along y-Axis

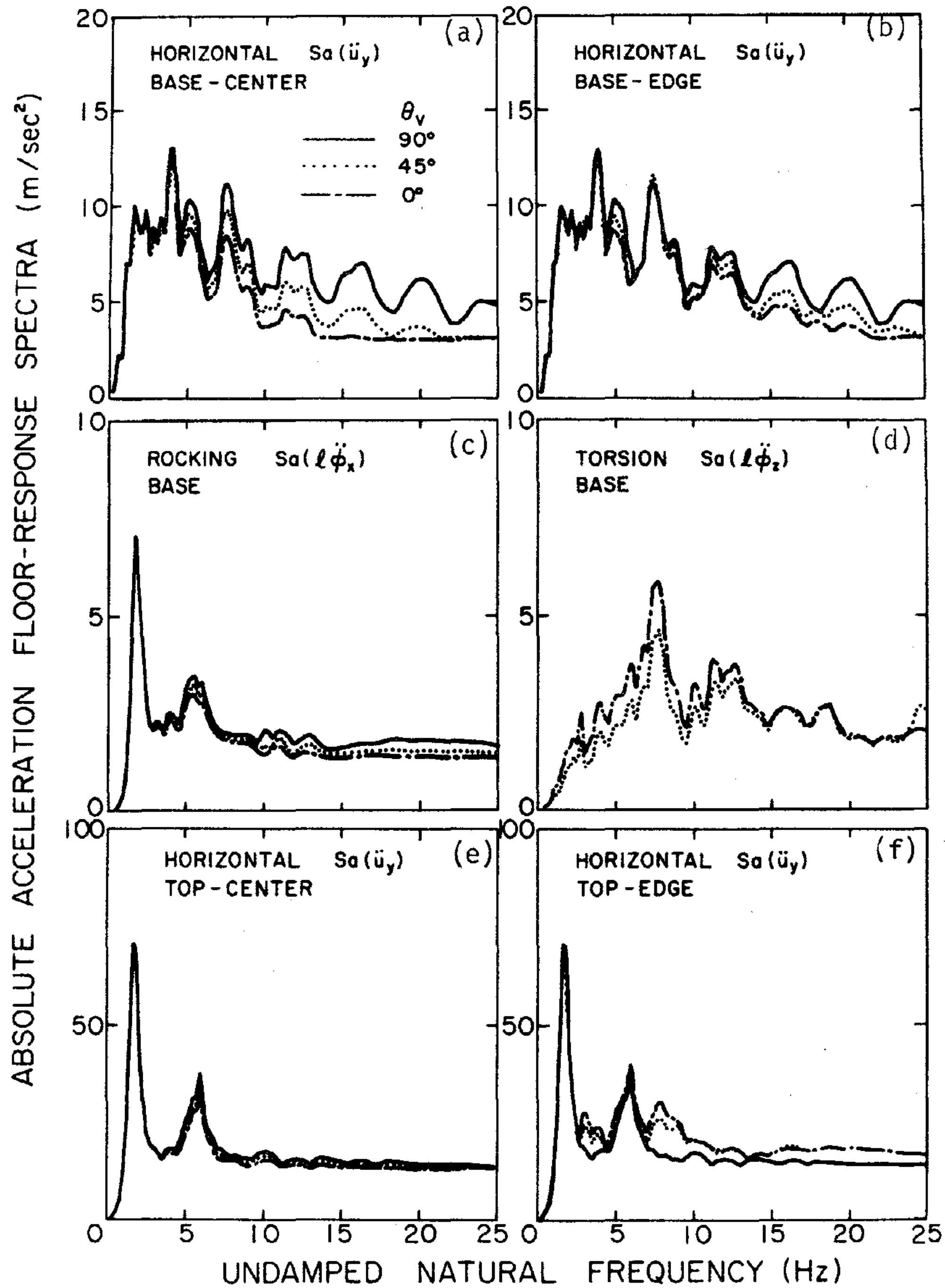

Figure 10. Absolute Acceleration Floor Response Spectra for Model 1 Excited by SH Waves with Particle Motion Along y-Axis (2% Damping)

clearly at 3 and 9 cps. The floor response spectrum (Figure 10f) shows that the response at the high frequency end of the spectrum is 20% higher for nonvertically incident waves than for vertically incident excitation.

The transfer functions and floor response spectra at several locations on Model 1 when excited by SH waves impinging on the foundation along the y-axis (Figure 6b) are shown in Figures 11 and 12. The results are similar to those just described for excitation along the x-axis. In this case, it is more difficult to visualize the torsional effects since the fixed-base torsional frequencies (3, 9, 15, 21 and 27 cps) coincide with some of the fixed-base translational frequencies. One significant difference corresponds to the much lower rocking (Figures 11c and 12c) response for vibrations in the more flexible direction of the structure.

The response of Model 2 to SH waves with particle motion along the y-axis and with different angles of incidence in shown in Figures 13 and 14. In this case, due to the higher mass and stiffness of the superstructure, the effects of soil-structure interaction are, in general, more pronounced than for Model 1. The translational response at the center of the base presented in Figures 13a and 14a exhibits a pronounced filtering of nonvertically incident SH waves by the foundation in the frequency range from 5 to 20 cps. The translational response at the edge of the base (point 2 in Figure 6c) shown in Figures 13b and 14b is clearly affected by torsion of the foundation (Figures 13d and 14d) for frequencies in the vicinity of 6 and 16 cps. These torsional effects can also be seen in Figure 13e illustrating the transfer function for a point on the edge of the structure at elevation 40m (point 4 in Figure 6c). The response at the top of Model 2 shown in Figures 13f and 14f exhibits a marked filtering effect for nonvertically incident waves. Due to the strong interaction effects, large shifts in resonant frequencies take place. The peak translational response at the top occurs at a frequency of approximately 3 cps while the fundamental fixed-base translational frequency for this model in 5 cps. Similarly, the peak torsional response occurs at 6 cps while the fundamental fixed-base torsional frequency is 8 cps.

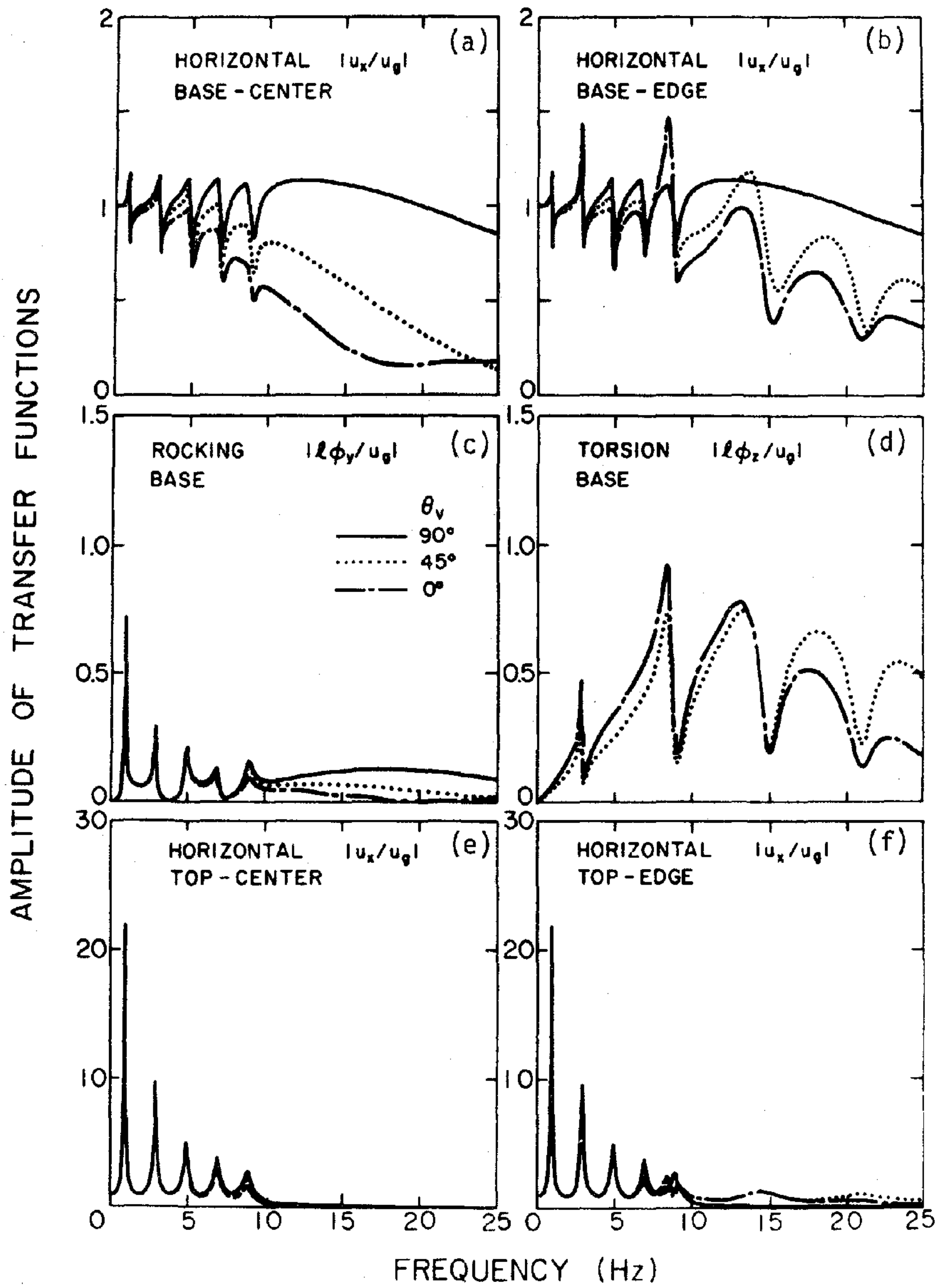

Figure 11. Transfer Functions for Model 1 Excited by SH Waves with Particle Motion Along x-Axis

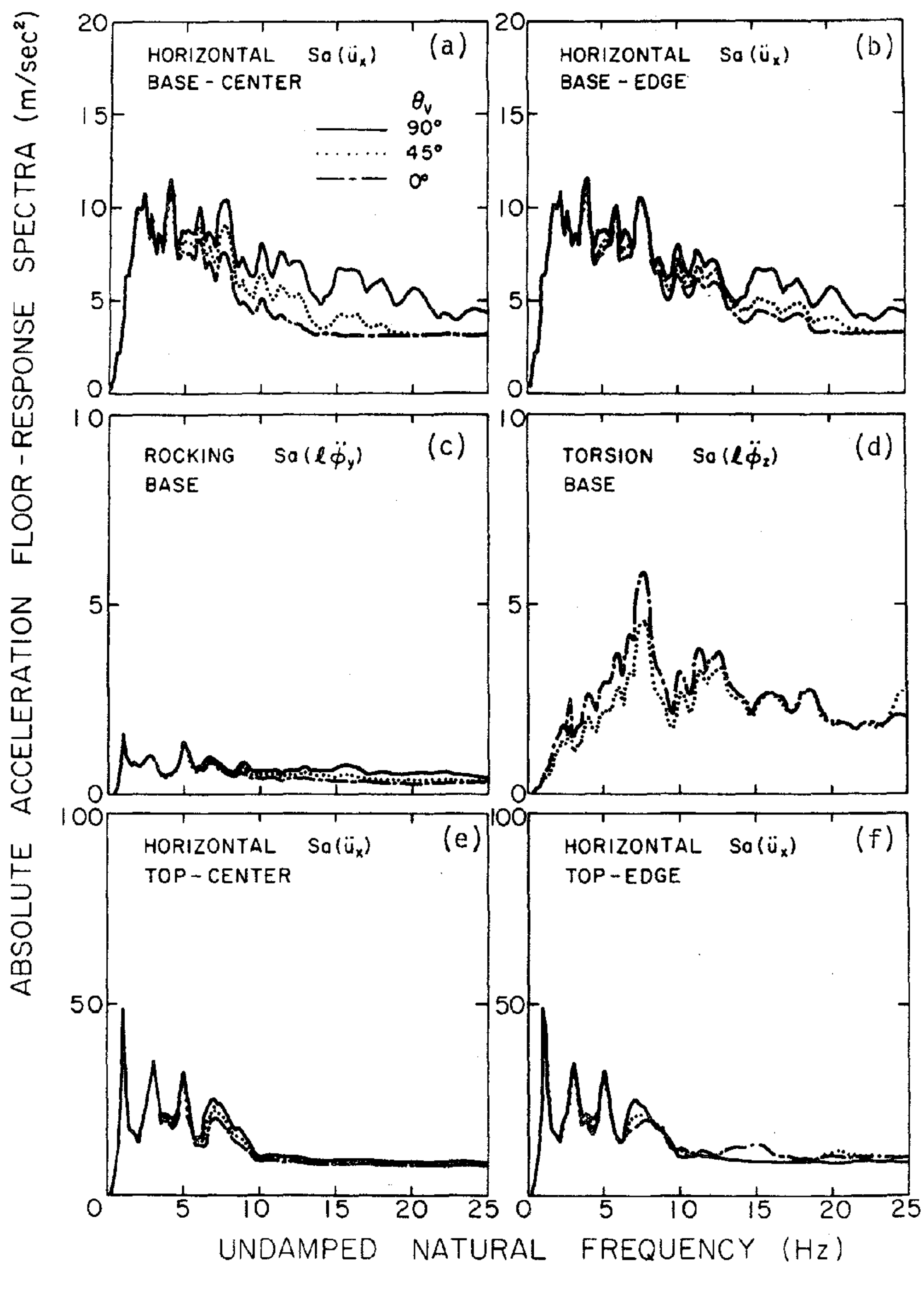

Figure 12. Absolute Acceleration Floor Response Spectra for Model 1 Excited by SH Waves with Particle Motion Along x-Axis (2% Damping)

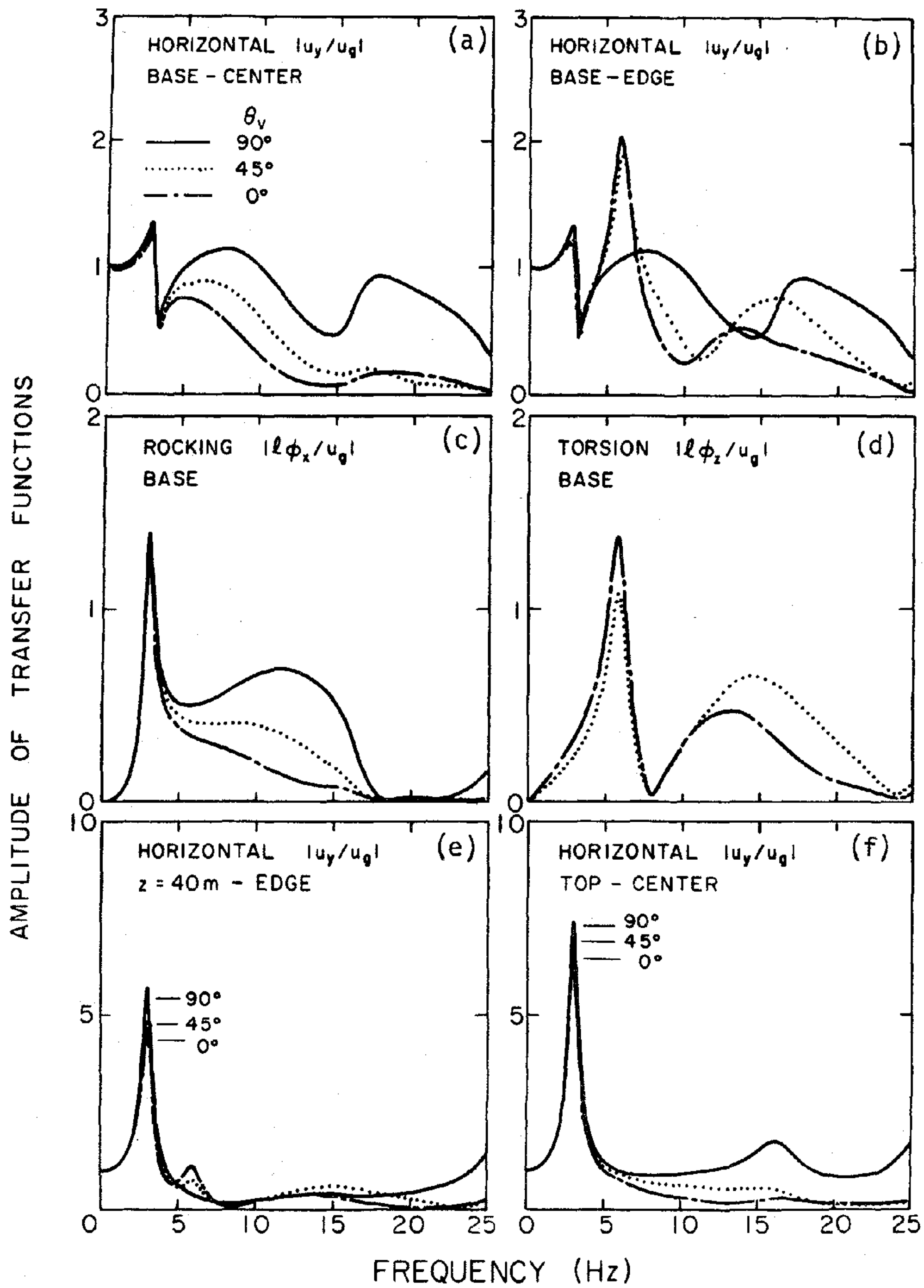

Figure 13. Transfer Functions for Model 2 Excited by SH Waves with Particle Motion Along y-Axis

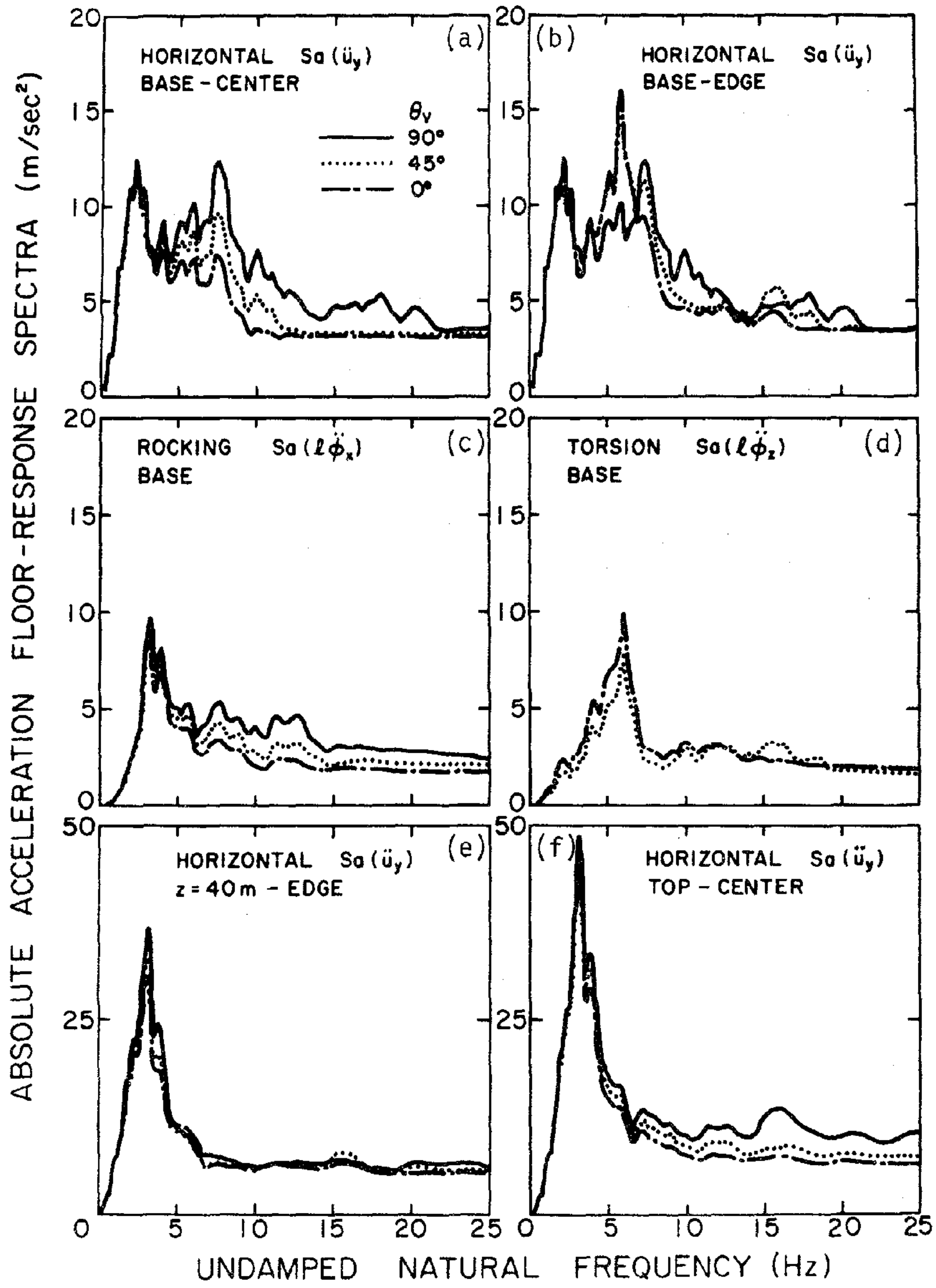

Figure 14. Absolute Acceleration Floor Response Spectra for Model 2 Excited by SH Waves with Particle Motion Along y-Axis (2% Damping)

RESPONSE TO RAYLEIGH WAVES

The response of Models 1 and 2 to Rayleigh wave excitation was obtained using the same procedure already described for SH wave excitation. Transfer functions for the frequency response at locations 1, 3, 5 and 6 (Figure 6) are shown in Figures 15, 17 and 19. Floor (in-structure) absolute acceleration response spectra at the same locations for Rayleigh wave input with horizontal amplitude equal to the acceleration time history of the NS component of the El Centro 1940 earthquake are shown in Figures 16, 18 and 20.

The transfer functions and floor response spectra for Model 1 excited by a Rayleigh propagating in the direction of the y axis are shown in Figures 15 and 16, respectively. The transfer function for the horizontal response at the center of the base (Figure 15a) shows a very pronounced filtering for frequencies higher than 2 cps in agreement with the foundation input motion shown in Figure 4. The floor response spectrum at the same location (Figure 16a) is characterized by lower amplitudes than the corresponding spectrum for vertically incident SH waves shown in Figure 10a. The rocking transfer function at the base, shown in Figure 15b, generally follows the rocking input motion described in Figure 4 except for oscillations in the vicinity of the fixed-base natural frequencies (2, 6, 10, 14 and 18 cps). As expected, the rocking response for Rayleigh (Figures 15b and 16b) is significantly higher than the corresponding response for vertically incident SH waves (Figures 10c and 11c).

The vertical response at the center and edge of the base (locations 1 and 5 in Figure 6a) shown in Figures 15c and 16c are twice as large as the horizontal response. In general, the vertical response at the edge is larger than at the center due to the contributions of rocking. The horizontal response at the top (Figures 15d and 16d) is almost twice as large than the corresponding response for vertically incident SH waves (Figures 10e and 11e). This amplification results from the large rocking input motion associated with Rayleigh waves. The vertical response at the center and edge of the top of the structures (Figures 15e and 16e) is not as large as the horizontal response due to the higher stiffness of the structure in the vertical direction and to the large radiation damping associated with vertical vibrations (Figure 5).

The results obtained for the response of Model 1 to Rayleigh waves propagating along the x axis (shown in Figures 17 and 18), as well as those for Model 2 (shown in Figures 19 and 20) are not significantly different from those already described for Model 1.

The values of the absolute acceleration floor response spectra for a frequency of 25 Hz are listed in Table 2 for SH and Rayleigh wave excitation. The results presented in Table 2 summarize the effect of obliquely incident seismic waves on the high frequency response components at the different locations shown in Figure 6. A comparison of the floor response spectra at the top of Model 2 for vertically incident ($\theta_v = 90°$) SH waves and for Rayleigh waves is shown in Figure 21. The corresponding response for the same structure supported on a rigid soil is also shown in Figure 21. The results presented in Figure 21 indicate that the

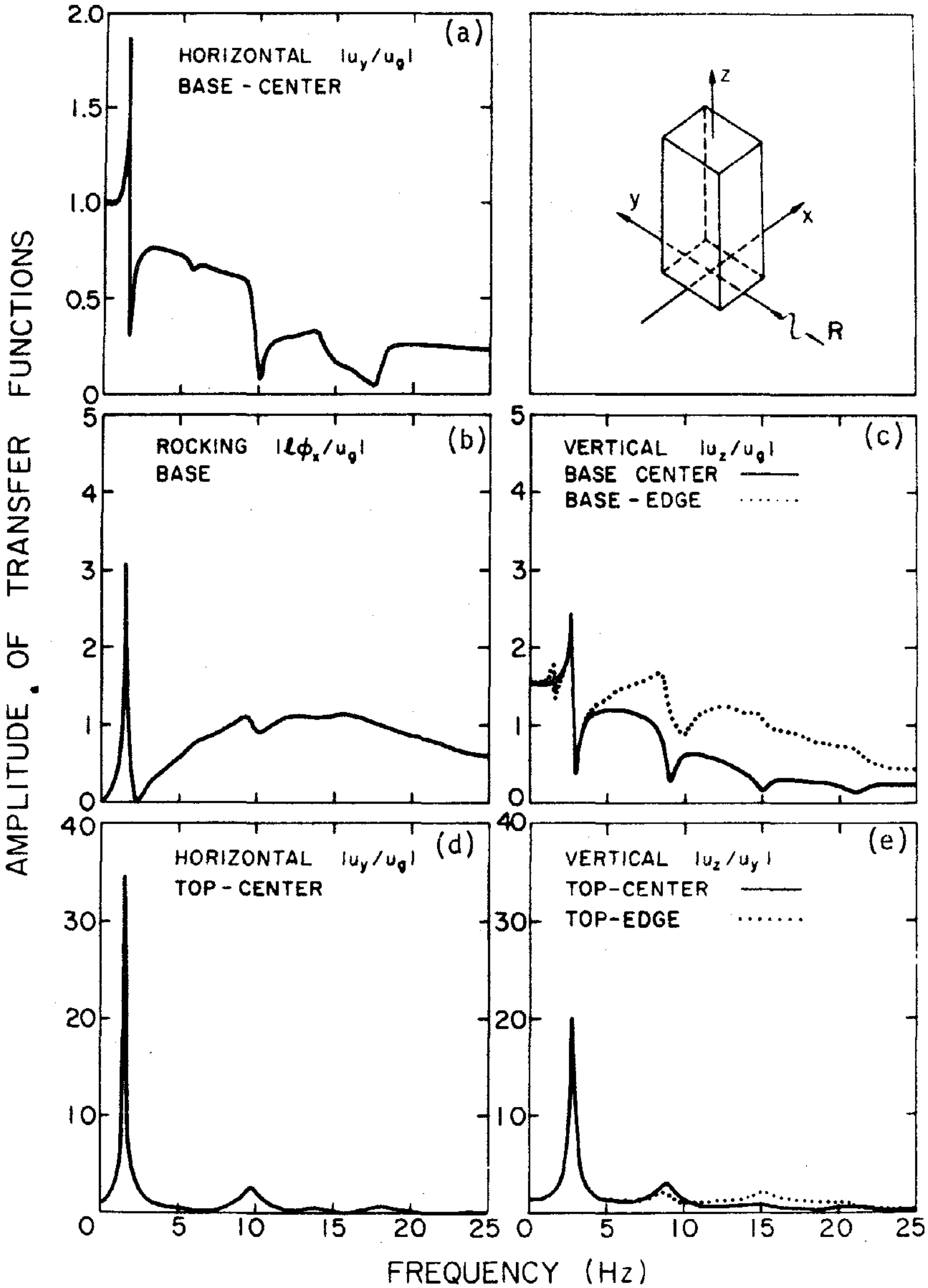

Figure 15. Transfer Functions for Model 1 Excited by Rayleigh Waves Propagating Along the y-Axis

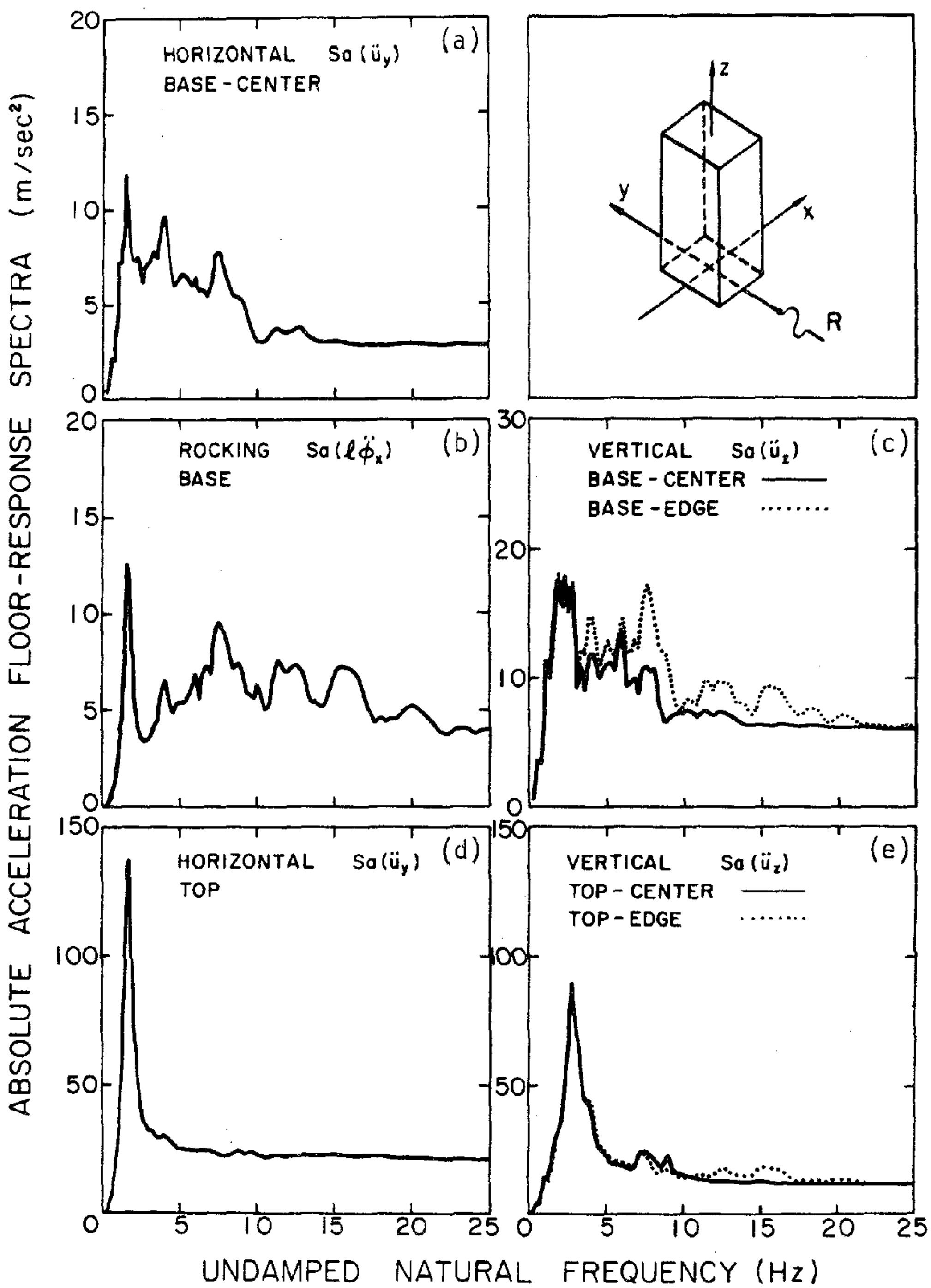

Figure 16. Absolute Acceleration Floor Response Spectra for Model 1 Excited by Rayleigh Waves Propagating Along the y-Axis (2% Damping)

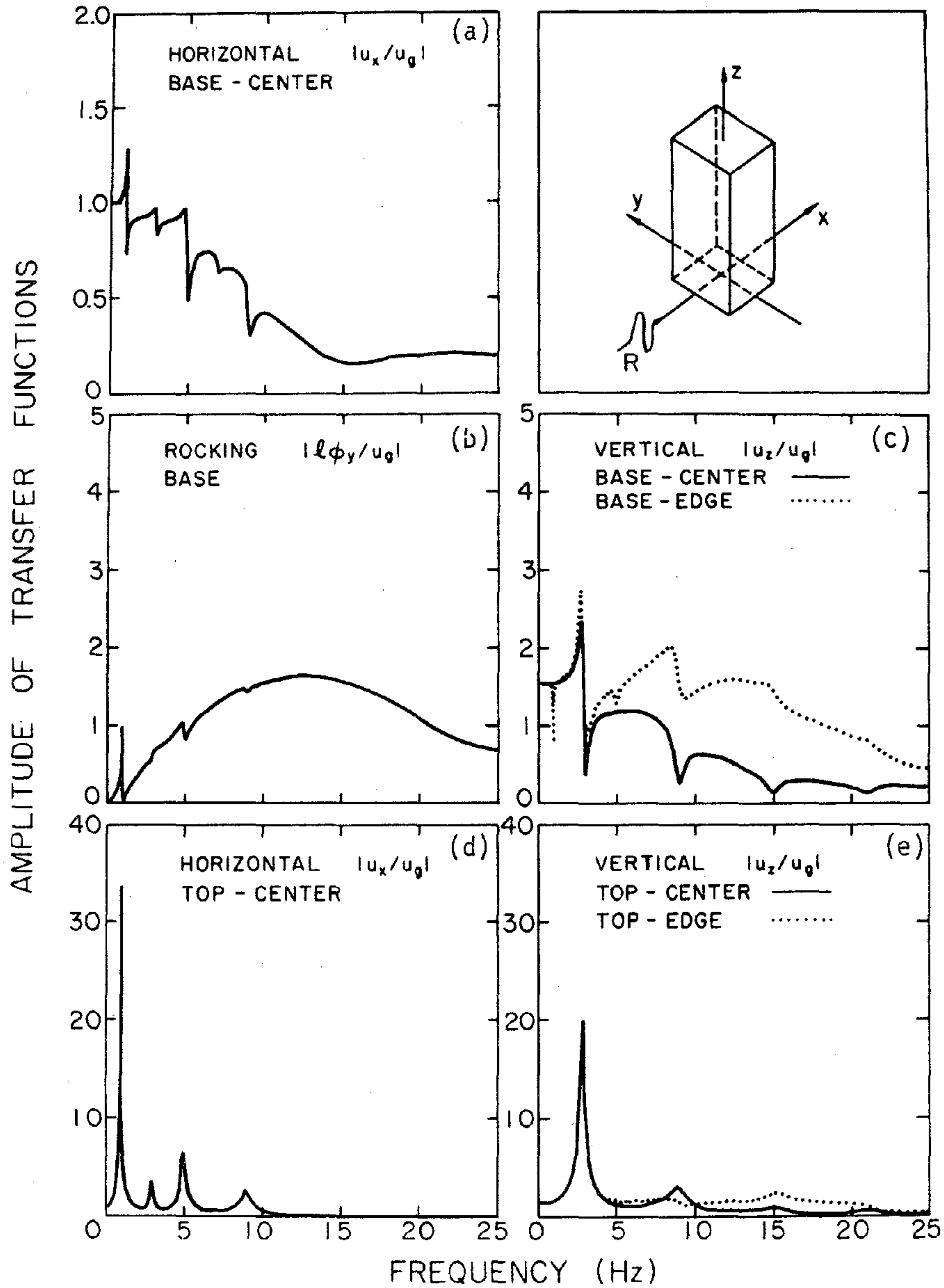

Figure 17. Transfer Functions for Model 1 Excited by Rayleigh Waves Propagating Along the x-Axis

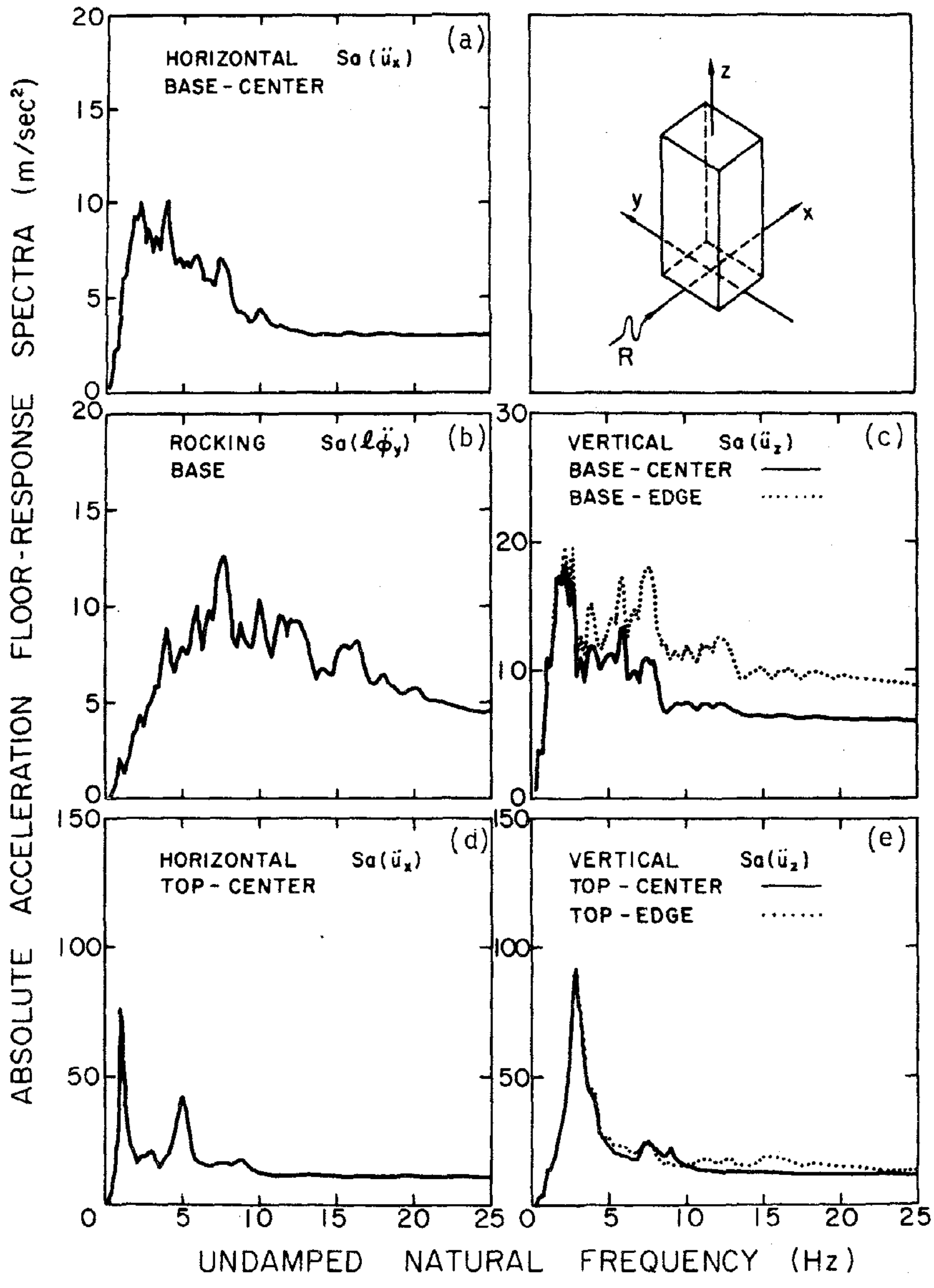

Figure 18. Absolute Acceleration Floor Response Spectra for Model 1 Excited by Rayleigh Waves Propagating Along the x-Axis (2% Damping)

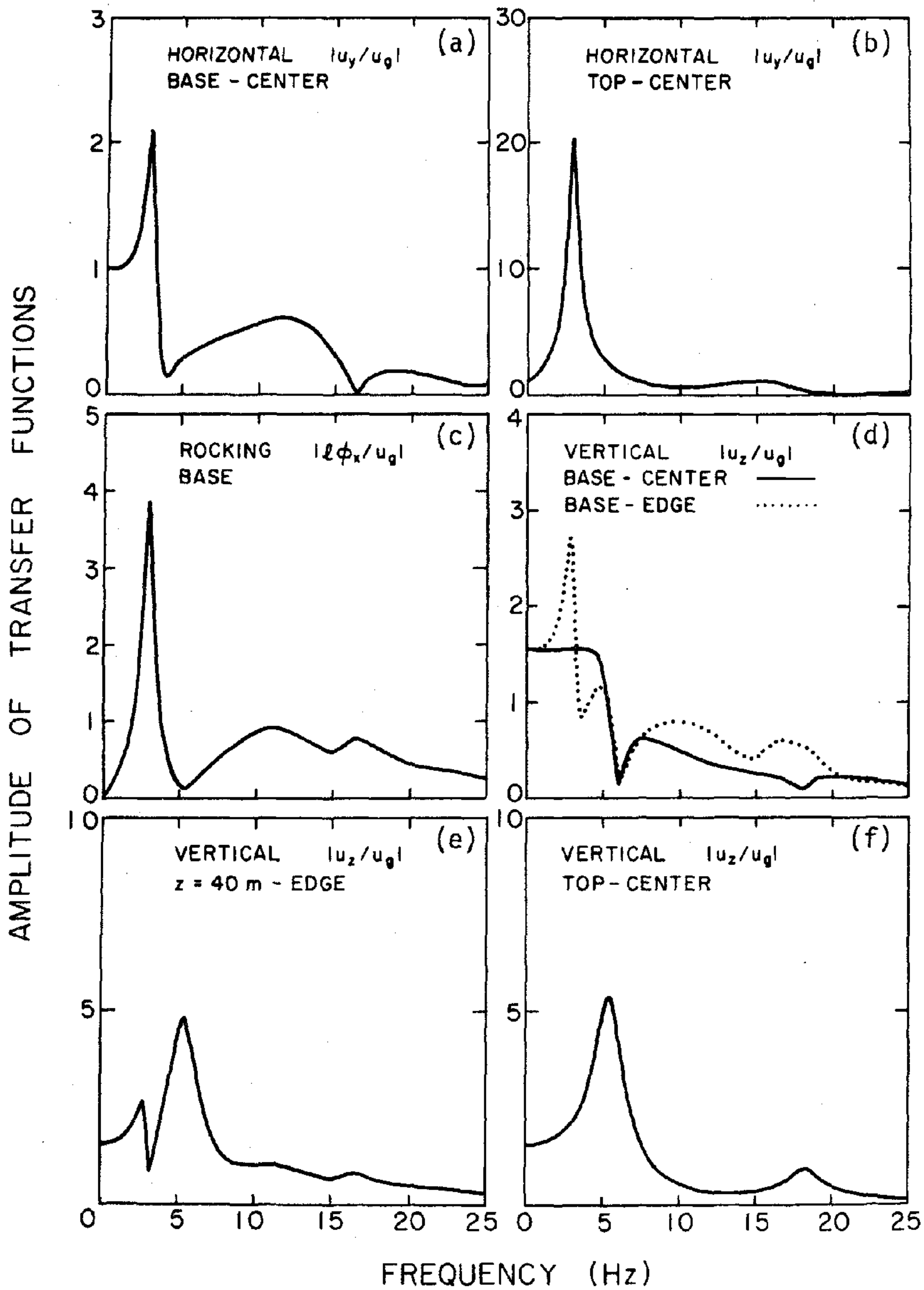

Figure 19. Transfer Functions for Model 2 Excited by Rayleigh Waves Propagating Along the y-Axis

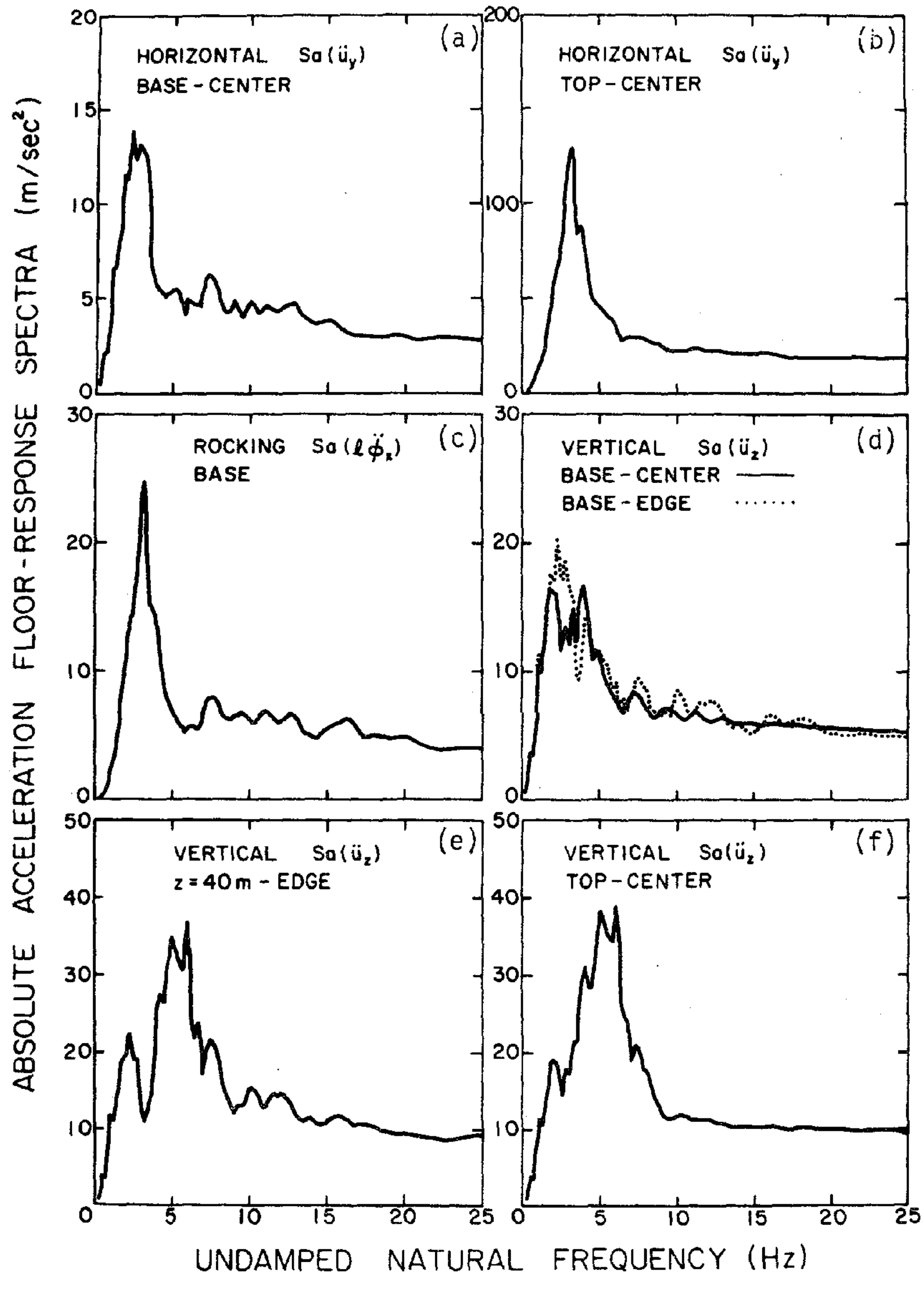

Figure 20. Absolute Acceleration Floor Response Spectra for Model 2 Excited by Rayleigh Waves Propagating Along the y-Axis (2% Damping)

response for Rayleigh wave excitation is significantly higher than that for vertically incident SH waves. Comparison with the response for a rigid soil model clearly shows the importance of the effect of soil-structure interaction for this case.

TABLE 2

Comparison of Spectral Amplitudes at 25 Hz (m/sec^2)

	Station	Component	SH Waves 90°	SH Waves 45°	SH Waves 0°	Rayleigh Waves
Model 1 (motion in yz-plane)	Free-Field	$\ddot{u}_y$	4.07	4.07	4.07	4.07
	1	$\ddot{u}_y$	4.89	3.18	3.12	2.94
	1	$a\ddot{\theta}_x$	1.66	1.46	1.38	4.04
	1	$a\ddot{\theta}_z$	0.00	2.58	2.04	0
	1	$\ddot{u}_z$	0	0	0	6.12
	2	$\ddot{u}_y$	4.89	3.37	3.27	3.94
	5	$\ddot{u}_z$	1.66	1.46	1.38	6.20
	3	$\ddot{u}_y$	14.0	13.7	13.5	21.8
	4	$\ddot{u}_y$	14.0	16.6	16.9	21.8
	3	$\ddot{u}_z$	0	0	0	11.9
	6	$\ddot{u}_z$	-	-	-	12.0
Model 1 (motion in xz-plane)	Free-Field	$\ddot{u}_x$	4.07	4.07	4.07	4.07
	1	$\ddot{u}_x$	4.39	3.18	3.24	3.11
	1	$a\ddot{\theta}_y$	0.433	0.330	0.315	4.62
	1	$a\ddot{\theta}_z$	0	2.58	2.04	0
	1	$\ddot{u}_z$	0	0	0	6.12
	2	$\ddot{u}_x$	4.39	3.41	3.43	3.11
	5	$\ddot{u}_z$	0.433	0.330	0.315	8.87
	3	$\ddot{u}_x$	8.81	8.58	8.32	11.5
	4	$\ddot{u}_x$	8.81	10.1	9.98	11.5
	3	$\ddot{u}_z$	0	0	0	11.9
	6	$\ddot{u}_z$	-	-	-	13.7

TABLE 2 (Continued)

	Station	Component	SH Waves 90°	45°	0°	Rayleigh Waves
Model 2	Free-Field	$\ddot{u}_y$	4.07	4.07	4.07	4.07
	1	$\ddot{u}_y$	3.67	3.33	3.23	2.95
	1	$a\ddot{\theta}_x$	2.43	2.08	1.75	3.90
	1	$a\ddot{\theta}_z$	0	1.61	1.88	0
	1	$\ddot{u}_z$	0	0	0	5.36
	2	$\ddot{u}_y$	3.67	3.59	3.50	2.59
	5	$\ddot{u}_z$	2.43	2.08	1.75	4.89
	3	$\ddot{u}_y$	10.6	7.36	6.55	19.5
	4	$\ddot{u}_y$	6.31	5.87	5.61	-
	3	$\ddot{u}_z$	0	0	0	9.42
	6	$\ddot{u}_z$	-	-	-	9.24

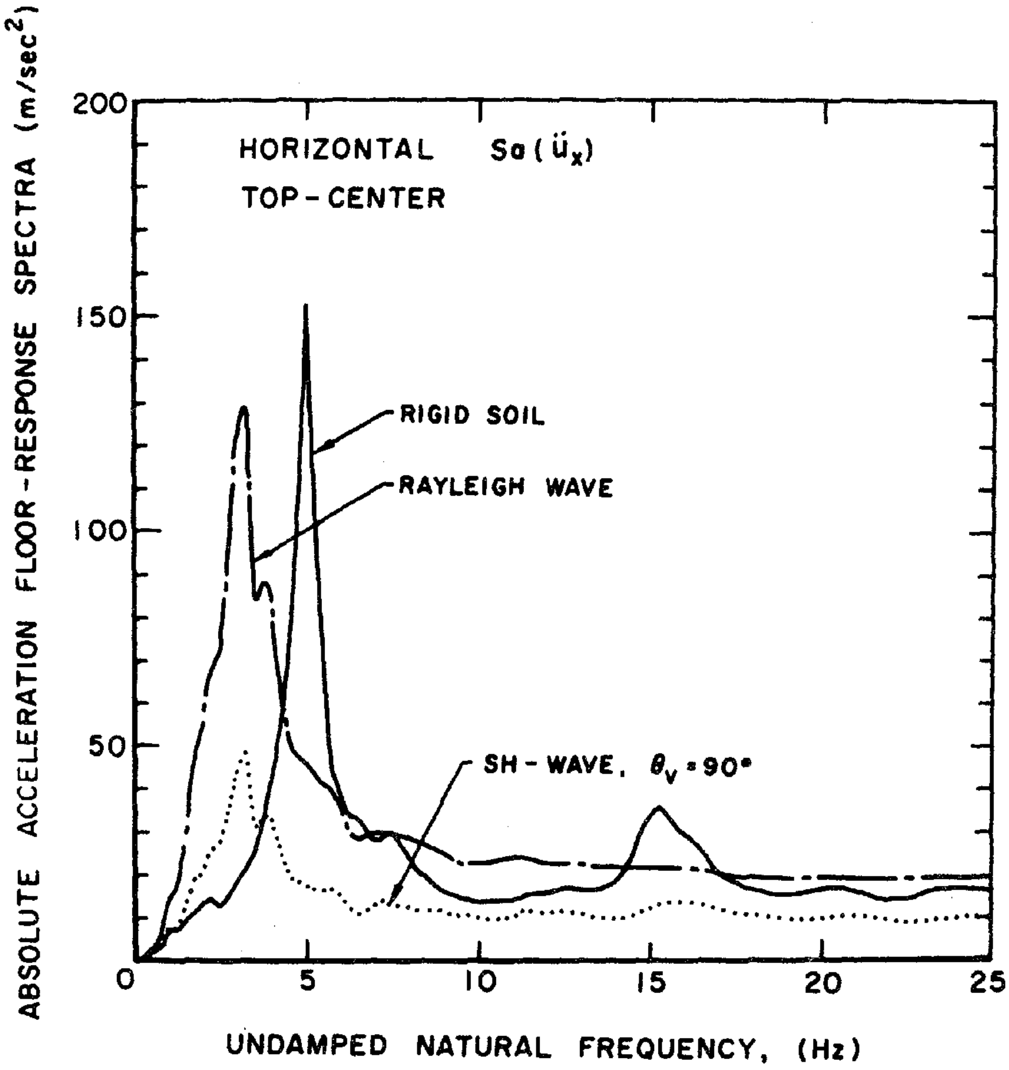

Figure 21. Comparison of Floor Response Spectra at the Top of Model 2 for Vertically Incident SH Waves, Rayleigh Waves and Rigid Soil Model (2% Damping)

CONCLUSIONS

A method to evaluate the earthquake response of three-dimensional structures to nonvertically incident waves including the effects of soil-structure interaction has been presented. The method relies on solving first the problems associated with radiation and scattering by the foundation and in combining the resulting compliance functions and foundation input motion with an equivalent dynamic mass matrix for the superstructure. The frequency response is then obtained by solving the interaction equation and the response in the time domain is evaluated by use of the Fast Fourier Transform algorithm for a specified motion on the free-field.

The response to nonvertically incident SH waves and to Rayleigh waves of simplified models of a ten-story reinforced concrete building with different stiffness in two orthogonal directions (Model 1), and of the containment structure of a nuclear power plant (Model 2) has been obtained. For obliquely incident SH wave excitation, it has been found that the translational response at the center of the foundation at frequencies higher than 5 cps is significantly lower than the response for vertically incident SH waves. The response at the center of the top of the structure is also lower for nonvertically incident SH waves, but the reduction depends on the characteristics of the structure: reductions of less than 6% were obtained for Model 1, while a reduction of 38% was obtained for Model 2 at high frequencies. Due to the contribution of the torsional components, the response at the edge of the top of the structure can be higher for nonvertically incident SH waves. The torsional response induced by nonvertically incident waves is significant as shown by the results obtained at the base of the structure.

For Rayleigh wave excitation, the horizontal response at the base of the structure is significantly lower than the corresponding response for vertically incident SH waves. The rocking response, on the other hand, is much higher for Rayleigh wave excitation leading to a much higher response at the top. For Model 2, the response at the top for Rayleigh waves is twice that for vertically incident SH waves. For Model 1, increases of 31% and 56% at high frequencies are obtained, depending on the direction of incidence of the wave.

The results obtained indicate the possible importance of the effects introduced by nonvertically incident seismic waves. Due to the simplifications introduced, generalization from these results must be viewed with caution. In particular, the assumption of a uniform half-space as a soil model may exaggerate the amplitude of the vertical component of the Rayleigh waves, leading to a large rocking response and a higher response at the top. Also, the assumption that the free-field motion consists entirely of SH waves with a common angle of incidence or to Rayleigh waves, is an important limitation. In addition, the phase velocities employed in this study may be lower than those which actually occur. For these reasons, the effects described can perhaps be considered upper bounds to what can be expected in a more realistic situation. The limitations mentioned emphasize the need for a more complete characterization of the free-field motion.

ACKNOWLEDGEMENT

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