UNIVERSITY OF CALIFORNIA, SAN DIEGO

DEPARTMENT OF APPLIED MECHANICS AND ENGINEERING SCIENCES

DYNAMIC RESPONSE OF FLEXIBLE RECTANGULAR FOUNDATIONS ON AN ELASTIC HALF-SPACE

by

M. IGUCHI

Department of Architectural Engineering, Faculty of Science and Engineering, Science University of Tokyo, Noda City, Japan

and

J. E. LUCO

Department of Applied Mechanics and Engineering Sciences University of California, San Diego, La Jolla, California

La Jolla, California

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M. Iguchi, J. E. Luco

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University of California, San Diego
Department of Applied Mechanics and Engineering Sciences
La Jolla, CA 92110

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SUMMARY

An approximate method for the analysis of the dynamic interaction between a flexible rectangular foundation and the soil with consideration of the out-of-plane deformation of the foundation is presented. The procedure is based on an extension of the subdivision method developed by Wong and Luco for rigid foundations.

Numerical results describing the influence of the flexibility of the foundation on the vertical and rocking impedance functions and on the contact stresses between the foundation and the soil are presented. The possibility of representing a flexible foundation by an equivalent rigid foundation having the same force-displacement relationships is also discussed. The results obtained indicate that at low frequencies, the dynamic stiffness coefficients for flexible foundations are lower than those for a rigid foundation of the same area. At higher frequencies the opposite behavior is observed. The radiation damping coefficients for flexible foundations are significantly lower than those for a rigid foundation of the same area.

INTRODUCTION

Most studies of the dynamic interaction between foundations and the soil are based on the assumption of a rigid foundation. The assumption of a rigid foundation may not always be valid. In fact, significant out-of-plane deformations of foundations have been observed in dynamic tests of actual buildings.[3, 11, 13] In spite of this situation, few studies have been addressed to the analysis of the effects of the flexibility of the foundation.

Oien has analyzed the response of a flexible strip supported on a homogeneous elastic half-plane when excited by obliquely incident harmonic waves.[9] Iguchi has presented an approximate method to evaluate the response of a slender foundation resting on an elastic half space when excited by obliquely incident seismic motions.[4] A second method to evaluate the dynamic response of flexible foundations has been presented by Iguchi.[5] In this method, the foundation is subdivided into strips in one of the directions of the plate. Lin has analyzed the impedance functions of flexible circular plates with a rigid perimeter supported on a homogeneous visco-elastic half space.[6] Matsui and Seya have studied the seismic response of a shell-type structure supported on a ring foundation with special consideration for the in-plane deformation of the ring.[7] Recently, the dynamic response of flexible rectangular foundations resting on a homogeneous elastic half space has been studied by researchers at the Muto Institute[8] and by Savidis and Richter.[10] In the first study, numerical results are limited to one example and the effects of the flexibility of foundation are not described in detail. In Savidis and Richter's work,[10] the dynamic response of uniform plates subjected to vertical excitation has been presented only for low values of the non-dimensional frequency.

This paper is addressed to the study of the vertical and rocking response of flexible plates supported on a homogeneous elastic half-space. The problem may be formulated in the form of integral equations in terms of the contact stresses between the foundation and the soil. Such approach, however, is not sufficiently flexible to cover foundation plates of arbitrary geometry. In this study, a hybrid approach is followed in which the finite element method is used to obtain the stiffness matrix for the foundation plate while integral equations are used to analyze the underlying half-space. Following the work of Wong and Luco,[12, 14] the dynamic flexibility matrix for the ground is obtained through discretization of these integral equations by dividing the contact region between the ground and the foundation into small rectangular subregions and by assuming that the contact stresses are uniformly distributed within each subregion. In the next step, the dynamic stiffness matrix for the ground, defined at the intersections of the subregions, is obtained by inversion of the dynamic flexibility matrix. Combining the stiffness matrices of the foundation and ground leads to a set of linear algebraic equations for the soil-foundation system in terms of the nodal displacements. Once the nodal displacements are obtained, the contact stresses for each subregion may be easily evaluated. The analytical procedure described is applicable to foundation plates of arbitrary plan geometry and variable cross-sections. In the analysis presented here, only out-of-plane deformation of the foundation is considered and the shearing contact stresses and inertia forces of the foundation are neglected. Also, although slippage between the foundation and the soil is allowed, it is assumed that the foundation remains in contact with the ground.

FORMULATION OF THE PROBLEM

The dynamic response of flexible and massless rectangular foundations supported on a homogeneous, elastic half-space and subjected to harmonic excitations will be studied. Since steady-state vibrations are considered here, the time factor $e^{i\omega t}$ will be omitted in what follows.

The geometry of the problem and the co-ordinate system employed are shown in Fig. 1 in which S denotes the contact

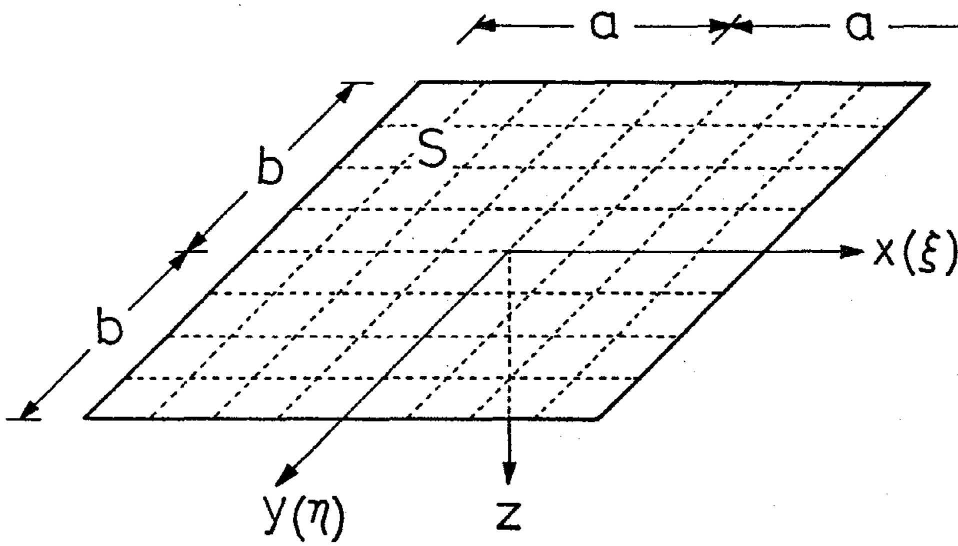

Fig. 1: Description of the soil-foundation system and co-ordinates.

region between the foundation plate and the ground. Denoting the vertical displacement of the foundation by w(x, y), the normal contact stress distribution between the foundation and the soil by $\tau_{zz}(x, y)$, and the vertical excitation on the foundation by p(x, y), the fundamental equations within the contact region S may be expressed in the form;

$$\int_S G_{zz}(r, k_\beta, \nu)\ \tau_{zz}(\xi, \eta)\ d\xi\ d\eta = w(x, y) \tag{1}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w(x, y) = \frac{1}{D} \left\{p(x, y) - \tau_{zz}(x,y)\right\}. \tag{2}$$

In Eq. (1), G_{zz} is the Green's function expressing the vertical displacement at (x, y) due to unit vertical point force at (ξ, η) on the surface of the homogeneous elastic ground. As is well known, the function G_{zz} depends on $r = \sqrt{(x - \xi)^2 + (y - \eta)^2}$, wavenumber $k_\beta = \omega/V_s$ where V_s is the shear wave velocity of the ground, and on the Poisson's ratio of the soil ν.(2) In Eq. (2), $D = E\ t_f^3/[12\ (1 - \nu_f^2)]$ represents the flexural rigidity of the foundation plate, where E stands for Young's modulus, t_f for the thickness and ν_f for Poisson's ratio.

The integral equation expressed in Eq. (1) can be discretized by dividing the contact region S into n rectangular subregions s_k (k = 1, 2,....n) as shown by broken lines in Fig. 1, and by assuming uniform contact stress within each subregion. It is also assumed that the displacement of each subregion may be represented by the average value over the subdivision. Under these assumptions, Eq. (1) may be reduced to the set of algebraic equations given by

$$\sum_{j=1}^{n} \alpha_{ij}(k_\beta, \nu)\, \bar{R}_j = \bar{w}_i \quad , \quad (i = 1, 2, \ldots. n) \tag{3}$$

where, $\bar{w}_i$ is the average displacement of the i-th subregion of the ground and $\bar{R}_j$ is the resultant force of the contact stress on the j-th subregion. In Eq. (3), α_{ij} represents the flexibility coefficient of the ground, which may be expressed by

$$\alpha_{ij}(k_\beta, \nu) = \frac{1}{A_i A_j} \int_{s_i} \left[\int_{s_j} G_{zz}(r, k_\beta, \nu)\, d\xi\, d\eta \right] dx\, dy$$

$$(i, j = 1, 2, \ldots. n) \tag{4}$$

where, A_i is the area of the i-th subregion. The values of the flexibility coefficients of the ground must be evaluated numerically. Using matrix notation, Eq. (3) may be expressed in the form:

$$_{g}A\, \bar{R} = \bar{W} \tag{5}$$

where, $_{g}A$ represents the flexibility matrix of the ground with elements α_{ij}. The vectors $\bar{R}$ and $\bar{W}$ are the force and displacement vectors having for elements $\bar{R}_i$ and $\bar{w}_i$, respectively.

Inspection of Eq. (4) reveals that $\alpha_{ij} = \alpha_{ji}$ indicating that the matrix ${}_gA$ is symmetric.

The partial differential equation for the foundation plate given by Eq. (2) can be discretized by use of standard finite element techniques.(1) In particular, the foundation plate is subdivided into rectangular elements as shown in Fig. 1 and a shape function of the type

$$w(x,y) = \left(1,\ x,\ y,\ x^2,\ xy,\ y^2,\ x^3,\ x^2y,\ xy^2,\ y^3,\ x^3y,\ x^2y^2,\ xy^3,\ x^3y^2,\ x^2y^3,\ x^3y^3\right)C \quad , \quad (6)$$

where C is a (16 x 1) coefficient vector, is used within each element. In this representation, each node has four degrees of freedom: a vertical displacement, two rotational angles about the x and y axes, and a twisting component. Following the conventional procedure, Eq. (2) is reduced to the set algebraic equations

$$\begin{bmatrix} {}_fk_{11} & {}_fk_{12} & {}_fk_{13} & {}_fk_{14} \\ & {}_fk_{22} & {}_fk_{23} & {}_fk_{24} \\ & & {}_fk_{33} & {}_fk_{34} \\ \text{sym.} & & & {}_fk_{44} \end{bmatrix} \begin{bmatrix} W \\ \theta_y \\ \theta_x \\ \theta_{xy} \end{bmatrix} = \begin{bmatrix} P_z \\ M_y \\ M_x \\ M_{xy} \end{bmatrix} - \begin{bmatrix} R \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

where , W, θ_y, θ_x and θ_{xy} are the nodal displacement vectors which correspond to the values of w, $-\partial w/\partial x$, $\partial w/\partial y$ and $\partial^2 w/\partial x \partial y$ at each node, respectively, and P_z, M_y, M_x and M_{xy} are the corresponding nodal load vectors of external forces acting on the foundation. The vector R corresponds to the nodal force vector of the contact stress.

The discretized force-displacement relationship for the ground given by Eq. (5) involves the average displacement and the resultant force of the contact stress on each subregion of the soil-foundation interface. On the other hand, the discretized force-displacement relationship for the foundation plate given by Eq. (7) involves the nodal displacements and the nodal forces associated with the contact stress at the corners of the different subregions. The compatibility between the out-of-plane deflection of the foundation plate and the motion of the ground is introduced by noting that the average vertical displacement of the i-th subregion $\bar{w}_i$ $(i = 1, 2, \ldots n)$ can be approximated by the mean value of the displacements at the four corners of the subregion. Denoting the displacements at the corner points by w_{i1}, w_{i2}, w_{i3}, and w_{i4}, the above described relationship may be expressed by

$$\bar{w}_i = \frac{1}{4} (w_{i1} + w_{i2} + w_{i3} + w_{i4}) \ , \ (i = 1, 2, \ldots n) \qquad (8)$$

or, in matrix form, by

$$\overline{W} = T\,W \tag{9}$$

in which $\overline{W}$ is the vector of average displacements over each subregion of the ground and W is the vector of nodal displacements of the foundation plate. Eq. (8) indicates that compatibility between motion of the ground and of the foundation is satisfied within each subregion in the sense of average value. If the number of subregions is n and the number of nodes is m the matrix T is of order (n x m). Proceeding in a similar fashion, the vector of nodal forces R associated with the contact stress can be expressed in terms of the vector $\overline{R}$ of resultant forces within each subregion by equally distributing the resultant force on each subregion among its four corners. Under this simplification, the nodal force vector R can be written in the form

$$R = T^T\,\overline{R} \tag{10}$$

where the super-script T indicates matrix transposition.

Substitution from Eqs. (9) and (10) into Eq. (5) leads to

$$R = {}_gK\,W \tag{11}$$

where

$${}_gK = T^T\,{}_gA^{-1}\,T \tag{12}$$

${}_gK$ represents the dynamic stiffness matrix of the ground defined at the nodes of the ground surface. The stiffness matrix thus obtained is symmetric of order (mxm), complex and frequency-dependent.

Finally, substitution from Eq. (11) into Eq. (7) leads to the desired equation for the soil-foundation system:

$$\begin{bmatrix} {}_gK + {}_fk_{11} & {}_fk_{12} & {}_fk_{13} & {}_fk_{14} \\ & {}_fk_{22} & {}_fk_{23} & {}_fk_{24} \\ & & {}_fk_{33} & {}_fk_{34} \\ \text{sym.} & & & {}_fk_{44} \end{bmatrix} \begin{bmatrix} W \\ \theta_y \\ \theta_x \\ \theta_{xy} \end{bmatrix} = \begin{bmatrix} P_z \\ M_y \\ M_x \\ M_{xy} \end{bmatrix} \qquad (13)$$

Once the displacement vector W for a set of external forces is obtained from Eq. (13), the contact stress of each subregion may be evaluated from Eqs. (5) and (9).

The procedure described above for the analysis of the dynamic behavior of the soil-foundation system when excited by external forces is also applicable with minor modifications to the study of the response of the system to seismic waves.[5,15]

EFFECTS OF FLEXIBILITY OF THE FOUNDATION ON THE IMPEDANCE FUNCTIONS

The technique described above has been employed to study the effects of the flexibility of the foundation on its response to external forces and moments. For this purpose, the two foundation models illustrated in Fig. 2 have been selected. Both models correspond to a square flexible plate of thickness t_f and plan dimension 2a x 2a. In Model (a) the perimeter of the plate is assumed rigid while in Model (b) the plate has a rigid zone of dimension a x a in the central part. External harmonic excitations corresponding to vertical forces $Pe^{i\omega t}$ and rocking moments $Me^{i\omega t}$ are applied on the rigid portions of both models. The plates, characterized by Young's modulus E and Poisson's ratio ν_f, rest on an elastic half-space characterized by the shear modulus μ and Poisson's ratio ν.

The vertical displacement Δ and the rocking angle Φ of the rigid portion of each foundation when excited by a vertical force P and a rocking moment M also applied on the rigid portion can be described by the relations

$$\mu a\Delta/P = C_V(a_o) \tag{14}$$

$$\mu a^3\Phi/M = C_M(a_o) \tag{15}$$

where $C_V(a_o)$ and $C_M(a_o)$ are the dimensionless vertical and rocking compliance functions for the foundation. The compliance functions depends on the non-dimensional frequency $a_o = \omega a/V_s$, on Poisson's ratio of the soil ν, and on relative stiffness δ defined by

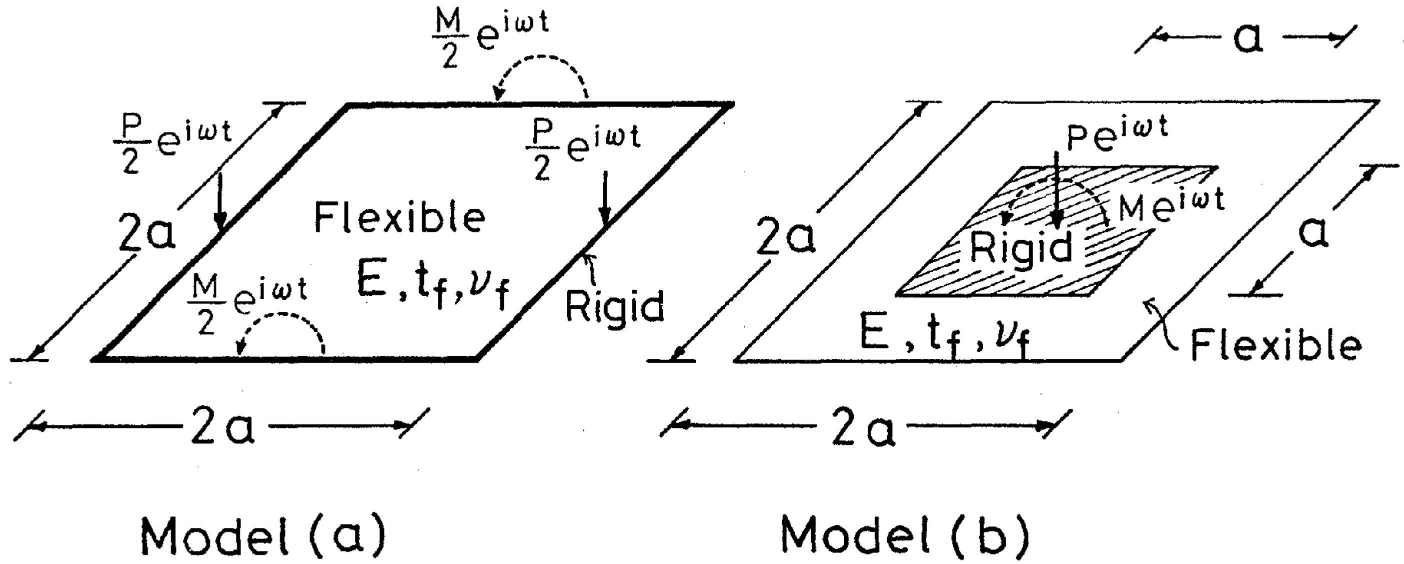

Fig. 2: Models of flexible foundations resting on an homogeneous elastic soil. Model (a): Flexible plate with a rigid perimeter, Model (b): Flexible plate with a rigid central region.

$$\delta = Et_f^3 \,/\, [\mu a^3 \, (1-\nu_f^2)] \tag{16}$$

The force-displacement relationships for the foundation can also be described in terms of the dimensionless impedance functions:

$$P/(\mu a \Delta) \;=\; {}_eK_V(a_o) + ia_o \; {}_eC_V(a_o) \tag{17}$$

$$M/(\mu a^3 \Phi) = {}_eK_M(a_o) + ia_o \; {}_eC_M(a_o) \tag{18}$$

where ${}_eK_V$ and ${}_eK_M$ can be interpreted as dynamic stiffness coefficients, while ${}_eC_V$ and ${}_eC_M$ can be interpreted as equivalent damping coefficients for the foundation-soil system.

Before proceeding with the study of the effects of flexibility of the foundation plate on the compliance and impedance functions, it is necessary to determine the minimum number of subregions required to achieve sufficiently accurate results. Computations were conducted dividing the contact region and the foundation plate into 16, 64, and 144 equal square subregions. The real and imaginary parts of the resulting vertical compliance functions for models (a) and (b) with $\delta = 0.05$ and $\nu = 0.4$ are shown in Figs. 3(a) and 3(b), respectively. The results obtained indicate that at least 64 subregions are required to obtain reliable results in the non-dimensional frequency range $0 \leq a_o < 4$.

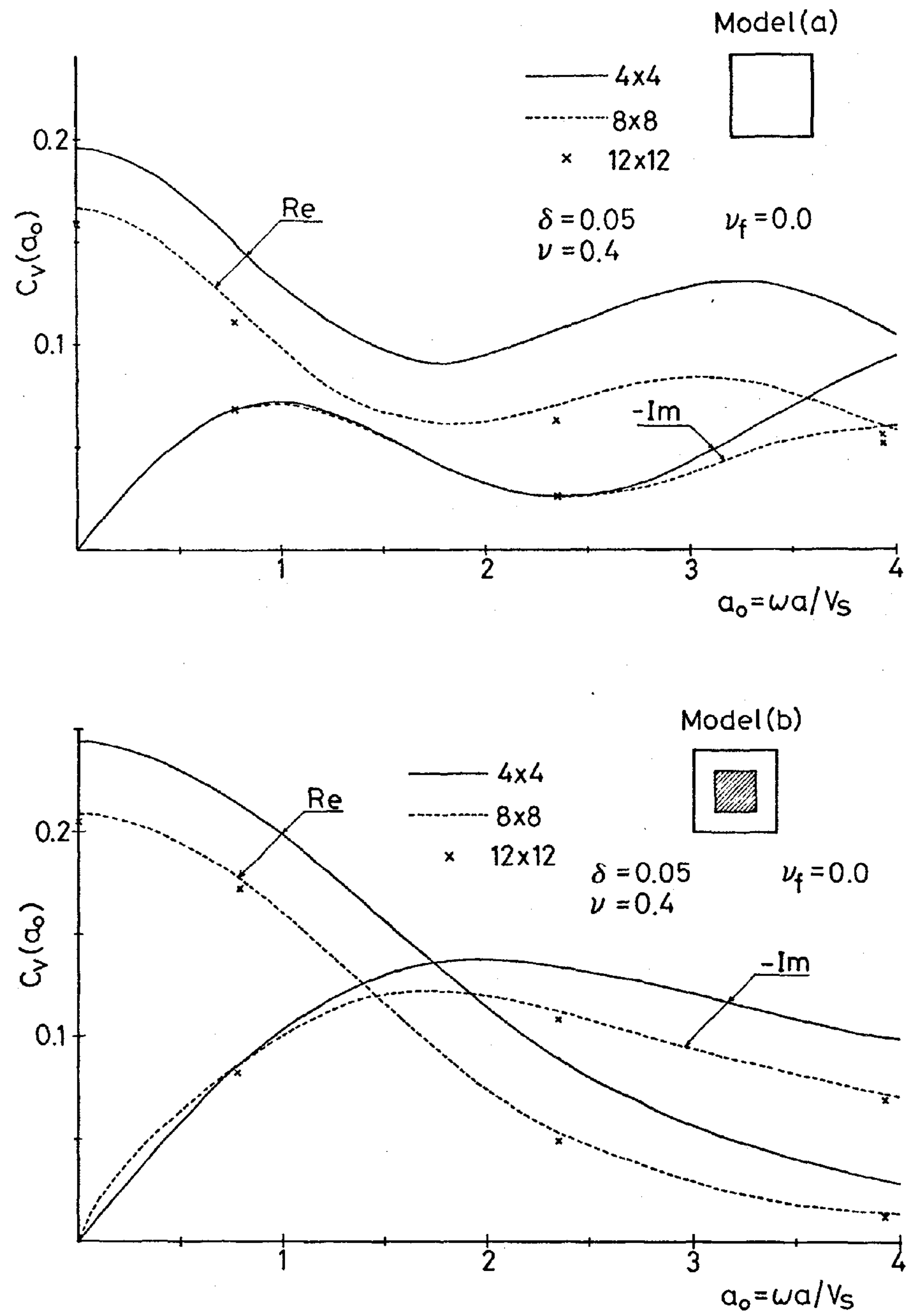

Fig. 3: Effect of the number of subregions on the vertical compliance functions for Models (a) and (b).

Comparing these results with those obtained by Wong and Luco[12] for a rigid foundation plate indicates that a larger number of subregions is necessary in the case of flexible foundation plates.

The effects of the flexibility of the foundation plate on the vertical and rocking impedance functions are illustrated in Figs. 4 and 5, respectively. The results shown in these figures were calculated by subdividing the foundation plate and the contact region into 64 equal square subregions. A value of $\nu = 0.4$ was used for Poisson's ratio in the soil. The results presented for values of the relative stiffness $\delta = 0.005$, 0.05, 0.5 and ∞ (rigid plate) indicate that, at low frequencies, the dynamic stiffness coefficients (${}_eK_V$, ${}_eK_M$) for a flexible foundation plate can be significantly lower than those for a rigid plate. At high frequencies, however, the dynamic stiffness coefficients for flexible foundation plates can be higher than those for a rigid plate. For Model (a), the effects of flexibility of the foundation on the vertical and rocking stiffness coefficients are similar. For Model (b), the effects on the rocking stiffness coefficients are more pronounced.

Perhaps the most significant effect shown in Figs. 4 and 5 corresponds to the reduction of the damping coefficients

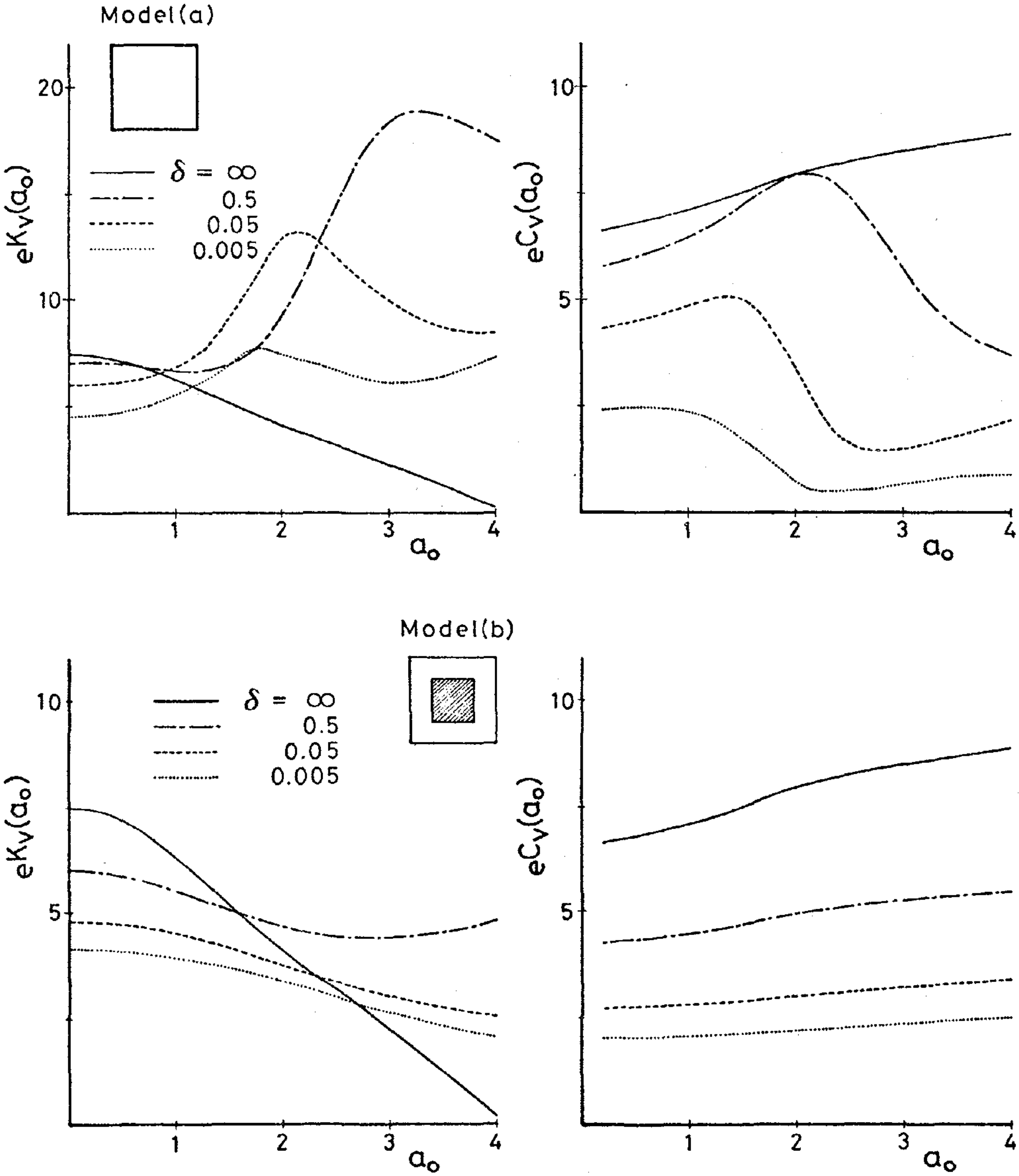

Fig. 4: Effects of flexibility of the foundations on the vertical impedance functions (ν = 0.4).

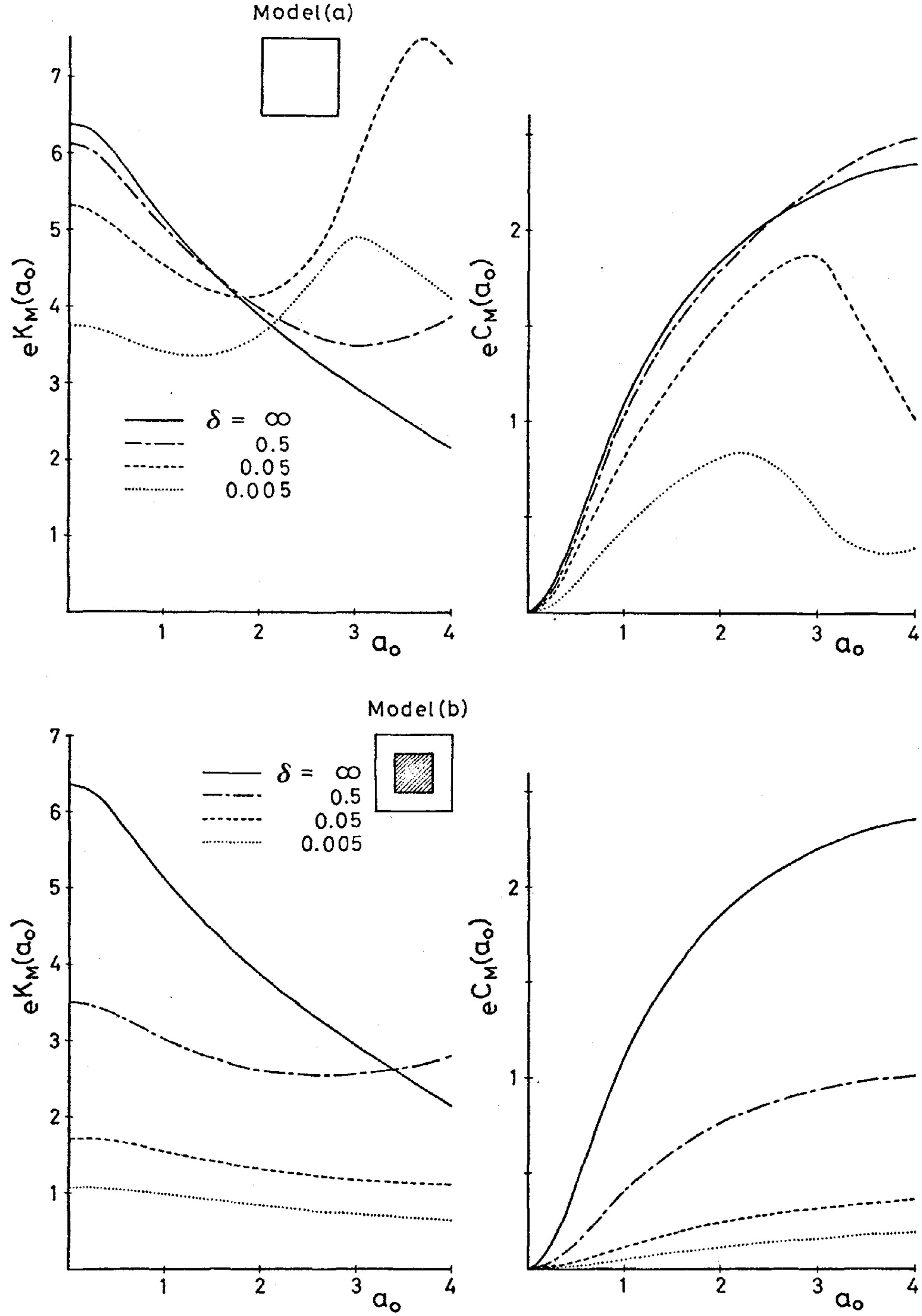

Fig. 5: Effects of flexibility of the foundations on the rocking impedance functions ($\nu = 0.4$).

($_eC_V$, $_eC_M$) associated with flexibility of the foundation. It is apparent that a flexible foundation plate is less efficient in radiating energy into the ground than a rigid foundation. For Model (a), the reduction of the vertical damping coefficient is more pronounced than that of the rocking damping coefficient. For Model (b), the reduction of the rocking damping coefficient is more pronounced.

EQUIVALENT RIGID FOUNDATION

Since a considerable amount of information on the dynamic response of rigid foundations is available, it is of interest to explore the possibility of representing a flexible foundation by an equivalent rigid foundation. One possibility is to define the dimensions of the equivalent rigid foundation in such a way that the static stiffness coefficients (real parts of the impedance function at $a_o = 0$) coincide with those for the flexible foundation. As example, the lengths $2\bar{a}_V$ of rigid square foundations having the same static vertical stiffness coefficients as flexible foundations corresponding to Model (b) with $\delta = 0.5$, 0.05 and 0.005 are defined by $a_V/a = 0.798$, 0.641 and 0.551, respectively. The corresponding equivalent lengths $2\bar{a}_M$ obtained by equating the static rocking stiffness coefficients are given by $\bar{a}_M/a = 0.820$, 0.646 and 0.552.

For Model (b) values of the ratios $\bar{a}_V/a$ and $\bar{a}_M/a$ range from 1.0 for $\delta = \infty$ to 0.5 for $\delta = 0$. It is interesting to notice that the equivalent length for rocking excitation is slightly higher than that for vertical excitation.

The equivalent rigid foundation described above is based on the static response of the flexible foundation. It is, then, necessary to test the adequacy of the equivalent representation at different frequencies. If the flexible foundation and its equivalent rigid representation give the same force-displacement relationships, the following equations would be satisfied:

$$C_V(a_o) = (a/\bar{a}_V)\ \bar{C}_V(\bar{a}_{oV}) \tag{19}$$

$$C_M(a_o) = (a/\bar{a}_M)^3\ \bar{C}_M(\bar{a}_{oM}) \tag{20}$$

where $\bar{C}_V$ and $\bar{C}_M$ denote the dimensionless compliance functions for the equivalent rigid foundation. Also, $\bar{a}_{oV} = \omega\bar{a}_V/V_S$ and $\bar{a}_{oM} = \omega\bar{a}_M/V_S$ represent the non-dimensional frequencies associated with the equivalent lengths $\bar{a}_V$ and $\bar{a}_M$, respectively. Comparisons of the compliance functions for flexible foundations (Model (b)) with those defined by Eqs. (19) and (20) are presented in Tables 1 and 2 for values of $\delta = 0.5$ and 0.05. The results listed in Table 1 indicate that the dynamic vertical response of flexible foundations of the type described by Model (b) can be assimilated to the response of an equivalent rigid foundation. The results presented in Table 2 for rocking excitation indicate that the equivalent rigid foundation can be used as a first approximation. In this case, however, the imaginary part of the compliance function for the equivalent rigid foundation can be somewhat larger than that for the flexible foundation.

The same type of equivalence discussed for Model (b) could be used for Model (a). In the case of Model (a), the central part of the plate is not very effective suggesting that a flexible foundation could be represented by a rigid foundation with an internal hole. Based on the results of Wong and Luco for this type of foundation[12], it is found that a flexible foundation of the type described by Model (a) with $\delta = 0.05$ could be represented by a square rigid foundation of the same length 2a with a square internal hole of half-length d = 0.8a.

Table 1. Comparison of the Vertical Compliance Functions for Model (b) with those for an Equivalent Rigid Foundation (ν = 0.4).

	δ = 0.5, $\bar{a}_v/a$ = 0.798					δ = 0.05, $\bar{a}_v/a$ = 0.641				
	$C_v(a_o)$			$(a/\bar{a}_v)\bar{C}_v(\bar{a}_{ov})$		$C_v(a_o)$			$(a/\bar{a}_v)\bar{C}_v(\bar{a}_{ov})$	
a_o	Re	-Im	$\bar{a}_{ov}$	Re	-Im	Re	-Im	$\bar{a}_{ov}$	Re	-Im
0.0	0.168	0.0	0.0	0.168	0.0	0.209	0.0	0.0	0.209	0.0
1.0	0.110	0.091	0.80	0.110	0.091	0.160	0.099	0.64	0.159	0.100
2.0	0.039	0.082	1.60	0.036	0.088	0.074	0.120	1.28	0.073	0.122
3.0	0.016	0.059	2.40	0.012	0.062	0.029	0.094	1.92	0.027	0.095
4.0	0.010	0.044	3.20	0.004	0.046	0.013	0.071	2.56	0.011	0.072

Table 2. Comparison of the Rocking Compliance Functions for Model (b) with those for an Equivalent Rigid Foundation (ν = 0.4).

	$\delta = 0.5,\ \bar{a}_M/a = 0.820$					$\delta = 0.05,\ \bar{a}_M/a = 0.646$				
	$C_M(a_o)$			$(a/\bar{a}_M)^3\bar{C}_M(\bar{a}_{oM})$		$C_M(a_o)$			$(a/\bar{a}_M)^3\bar{M}(\bar{a}_{oM})$	
a_o	Re	-Im	$\bar{a}_{oM}$	Re	-Im	Re	-Im	$\bar{a}_{oM}$	Re	-Im
0.0	0.285	0.0	0.0	0.285	0.0	0.582	0.0	0.0	0.582	0.0
1.0	0.335	0.042	0.84	0.332	0.045	0.644	0.045	0.65	0.653	0.048
2.0	0.283	0.165	1.68	0.299	0.200	0.667	0.249	1.29	0.686	0.267
3.0	0.179	0.194	2.52	0.160	0.247	0.501	0.416	1.94	0.538	0.467
4.0	0.112	0.170	3.36	0.074	0.214	0.322	0.437	2.58	0.308	0.501

CONTACT STRESSES

The effects of the flexibility of the foundation on the contact stresses between the foundation and the ground have been studied for the two foundation models shown in Fig. 2. The vertical contact stresses induced by a harmonic vertical force acting on Models (a) and (b) are shown in Figs. 6 and 7, respectively. The real part of the contact stress is shown on the left of each figure while the imaginary part is shown on the right. The results are presented for three values of the dimensionless frequency a_o and for two values of the relative stiffness δ. The results shown correspond to the average value of the vertical contact stress within each of the subregions considered and are normalized by $P/4a^2$. Given the symmetry of the foundation, the contact stress distribution is presented for only a quadrant of the contact region.

For a very flexible foundation, viz. $\delta = 0.005$, the higher contact stresses are concentrated along the rigid perimeter of Model (a) and around or beneath the central rigid region of Model (b). As the relative stiffness of the plate increases the distribution of contact stresses tends to become more uniform. The subregions employed are too large to permit any conclusion as to the singularity of the contact stresses along the perimeter of the foundation. The results obtained indicate that the contact stress distribution

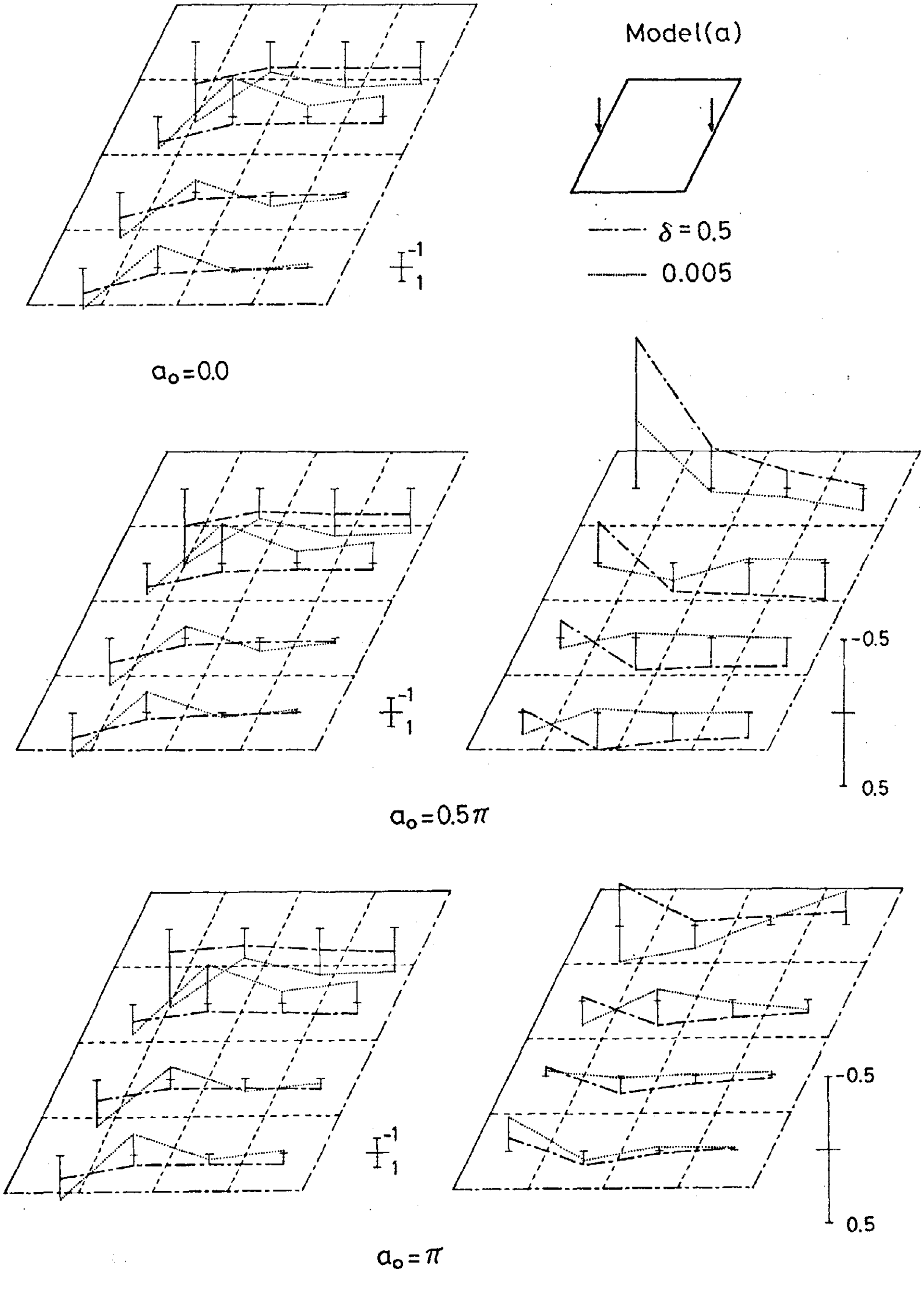

Fig. 6: Contact stress between the soil and foundation: Model (a), vertical excitation.

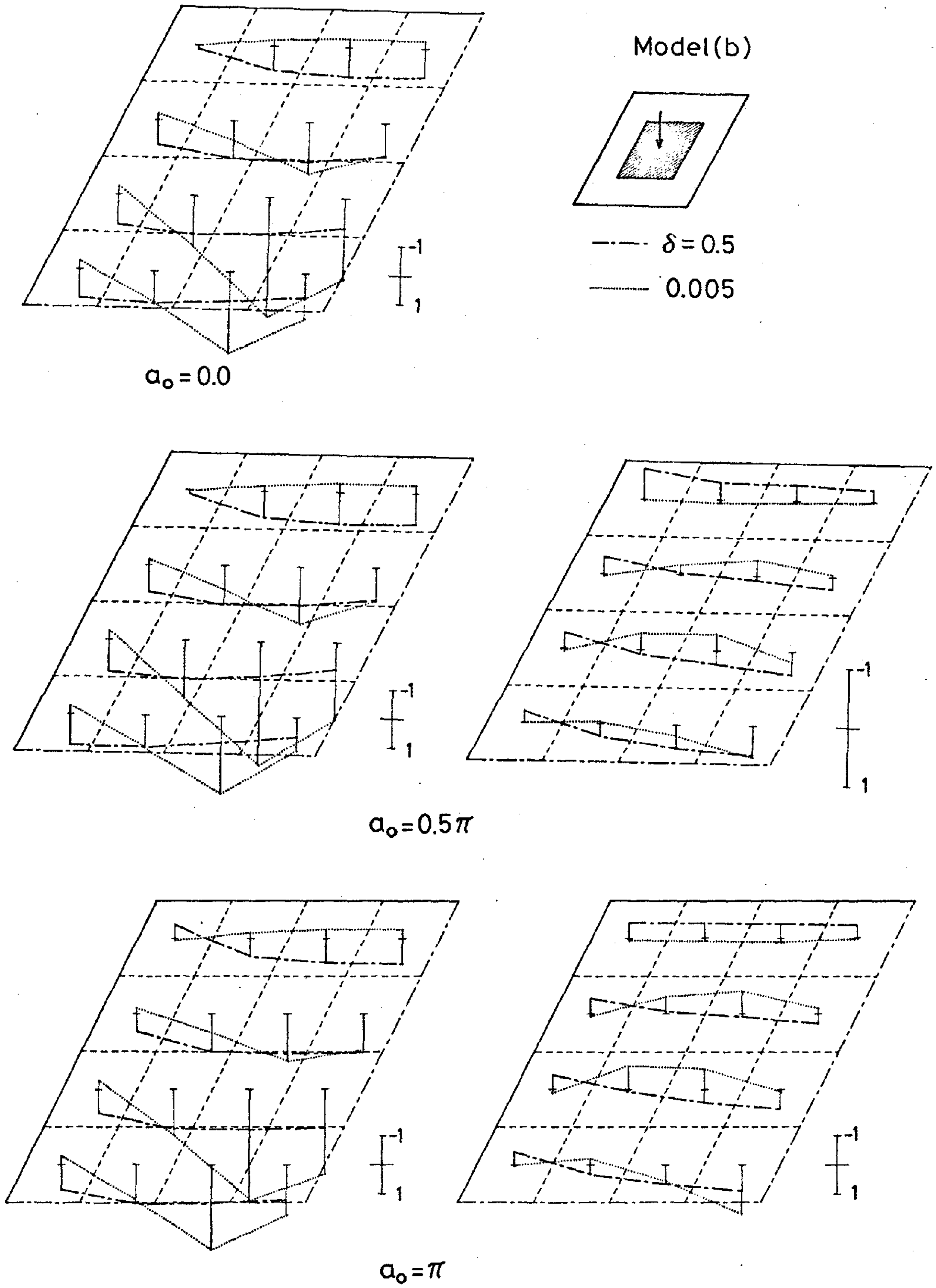

Fig. 7: Contact stress between the soil and foundation: Model (b), vertical excitation.

is highly dependent on the flexibility of the foundation. The contact stress distributions for rocking excitation exhibit similar characteristics to those shown in Figs. 6 and 7.

CONCLUSIONS

A numerical procedure for the evaluation of the dynamic response of flexible foundations supported on an elastic half space and excited by harmonic external forces has been presented. Although not shown in this paper, this procedure is also applicable to the study of the response of flexible foundations subjected to seismic waves.

The effects of the flexibility of the foundation on the vertical and rocking impedance functions for flexible square plates with a rigid perimeter or a rigid central region have been studied for different values of the relative stiffness of the plate. The results obtained indicate, in particular, a marked reduction of the vertical and rocking radiation damping coefficients as the flexibility of the plate increases. The possibility of representing a flexible foundation by an equivalent rigid foundation having the same compliance function has been explored. It has been shown that this representation leads to adequate results for vertical excitation and also for low-frequency rocking excitation if the effective length of the equivalent rigid foundation is properly selected.

The contact stresses between the soil and the foundation for two types of flexible foundations subjected to vertical and rocking excitations have been evaluated. It has been confirmed that the higher contact stresses concentrate around or beneath the stiffer portions of the foundation plate.

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