IDENTIFICATION OF HYSTERETIC BEHAVIOR FOR EXISTING STRUCTURES

By

Sassan Toussi

and

James T. P. Yao

Technical Report No. CE-STR-80-19

Supported by

The National Science Foundation

through

Grant No. PFR 7906296

December 1980

School of Civil Engineering
Purdue University
West Lafayette, IN 47907

TABLE OF CONTENTS

Page

1. INTRODUCTION

1.1 General Remarks

During this past decade, there have been increasing applications of system identification techniques in structural engineering [4,12,15,16,23]. The pilot phase of this research project began in 1977, and the general problem was described in a conference paper [33]. A comprehensive literature review of this subject area was presented and published in 1979 [34]. Results of the pilot project are summarized in a technical report [35].

In recent years, sophisticated electrical and electronical instruments have been placed in existing building structures to record their response to test loads as well as natural loading conditions such as earthquakes and winds. The analysis of available earthquake data is an important task in structural engineering. A significant benefit resulting from the evaluation of earthquake data is the interpretation of the structural behavior at large response levels. In addition, it can be helpful in the detection and identification of the seismic damage in existing structures.

There have been several attempts to compare the recorded response of a building with the response of a synthesized linear model subjected to the same base excitation. The comparison has been followed by certain trial-and-error adjustment of the model parameters to achieve better fit between the calcuated and recorded responses. In reality, structures might undergo nonlinear behavior and appreciable amount of permanent deformations.

To-date, most structural engineering applications of system identification methods deal with the estimation of parameters for a given mathematical form representing the structure. In reality, the nonlinear and hysteretic behavior of complex structures are not well understood at present [34]. Available data on such behavior are obtained from mostly ad hoc and static tests, results of which are applicable mainly for those special details.

It is believed that the presence of plastic deformations can be used as an indicator of structural damage [36]. Therefore, it is important to indentify the time-history of plastic deformations from recorded earthquake motions. Preferably, this task should be performed without the use of any specific form for the hysteretic behavior, which remains unknown for each and every existing civil engineering structure.

1.2 Objective and Scope

The objective of this investigation is to develop methods, with which the hysteretic behavior of structures can be estimated from recorded data. Relevant literature is reviewed.

The present study consists of two alternative but compatible methods. Certain common assumptions are made for both methods. For example, because (a) the earthquake data are normally available from only a small number of locations in structure, and (b) there exist input as well as output noise in the records, it is easier to estimate the parameters of the dominant modes in the record rather than all the elements in the stiffness and the damping matrices of the complex structure as a whole. This assumption makes it possible to consider the model as a single-degree-of-freedom (SDOF) system even when the response spectrum shows the presence of the other modes. The development of two such methods is then presented with numerical examples. Results are discussed and future research activities are outlined.

2. LITERATURE REVIEW

A simple definition of system identification as given by Sage [24] refers to those techniques with which mathematical descriptions or models for a system can be found with the use of a set of known inputs and corresponding outputs. For most of the system identification techniques, a mathematical form or model for the structural system is assumed. Then the parameters of the model are estimated by using the known inputs and outputs. Therefore, system identification consists of two parts, namely the mathematical formulation of the structural system and the determination of the system parameters which describe the behavior of the system under consideration [25]. Recently, available literature on the application of system identification in structural engineering has been reviewed by several investigators including Rodeman and Yao [23], Hart and Yao [12], and Collins, Young, and Kiefling [8].

The system identification techniques can be classified according to the type of methods used as follows: modal methods, frequency domain, and estimation methods [4]. In modal methods, the damping matrix is assumed to be diagonalized. Therefore, the differential equations of motion are decoupled. Finally, it is simplified for the identification of a viscously damped SDOF system [5,10,22]. A linear and time-invariant system is frequently assumed in methods applied in the frequency domain. The transfer function is obtained with the use of Laplace transform or Fourier transform [9,20]. Finally, the estimation methods use the probabilistic and statistical methods in the identification of linear systems. The least-square method, the maximum-likelihood estimate, etc., are frequently used in the estimation methods [5,18,28].

As Ibanez [15] pointed out, almost all the available data reveal some nonlinear phenomena. Any attempt to fit a linear model to a highly nonlinear structure should fail. Consequently, the techniques used to deal with strong-motion earthquake data should consider the presence of nonlinear behavior. The existing methods which assume a non-linear model can also be classified as either parametric or non-parametric. Parametric methods usually require either the solution of matrix Ricatti equations or non-linear programming techniques, and non-parametric methods usually employ the Volterra series or the Weiner Kernel approach [29]. However, both approaches are computationally expensive and often do not provide adequate characterizations of the types of nonlinearities met within mechanical structural systems [1,13,18,30,32]. In the following, one example of each is reviewed and one difficulty in dealing with earthquake data is discussed.

Udwadia and Kuo [29] presented a non-parametric identification technique for the identification of an arbitrary and memoryless close-coupled nonlinear multidegree-of-freedom system. The nonlinearity is given as a series expansion in terms of orthogonal functions. Their model is a lumped-mass system with masses M_ℓ, $\ell = 1, \ldots N$, which are assumed to be either known or fairly well estimated from design drawings. Furthermore, they assume that the restoring force can be separated into two independent forces. One of them is only a function of relative displacement (spring force) and the other is in terms of relative velocity (damping force) between the masses M_ℓ and $M_{\ell+1}$. This enables them to expand the forces in terms of two different sets of orthogonal functions. Therefore, they can estimate the coefficients of each set at those instants of time that the other one is zero. Finally, the coefficients of this series is estimated by minimizing an error function. Although the method seems to be insensitive to noise, it is ineffective for the identification of the structures which have experienced permanent deformation.

Matzen and McNiven [19] presented a method with which the seismic behavior of a single story steel structure can be identified by using a nonlinear mathematical method. They assume a structural system with linear viscous damping and Ramberg-Osgood type hysteretic force-deformation relationship. An error function is minimized to obtain the best match between their model's response and the structure's response, when both are subjected to the same excitation. The criterion function is an integral squared error function that includes errors in both acceleration and displacement. However, when real data are used, it is observed that it is difficult to apply the method to match both response quantities at the same time.

For deteriorating structures, attempts are being made to apply linear system identification technqiues for those portions of records which exhibit linear behavior in between relatively few cycles of large inelastic excursions. The changes of parameters in the linear model can then be correlated to the damage level [6].

It is generally agreed that certain difficulties exist with the application of earthquake data. In addition, it should be mentioned that the ground motion acceleration is coupled to the system response. Such problems have been considered by Pilkey and Kalinowski [21] and they have called it instrumentation problem. One of their examples includes the identification of an earthquake acceleration record from a shock spectrum. They consider the observed response as a function of the applied load. Furthermore, some restrictions are developed on the assumed relationship. Finally, a curve-fitting process is tried to obtain the closest similarity between the observed and the calculated values from the assumed function.

3. HYSTERESIS IDENTIFICATION METHOD

In this part, a relatively simple parametric approach for the identification of the behavior of a SDOF non-linear system is presented. This method has the advantage of not being restricted to any type of inputs.

3.1 System Model

Suppose that the model is a SDOF system as shown in Figure 1 with mass m which is considered to be time-invariant. It is assumed that the base-acceleration, $\ddot{z}(t)$, and the relative acceleration (with respect to the base), $\ddot{x}(t)$, have been measured by the instruments located on the structure. It is further assumed that the restoring force K_R depends on the relative displacement and the relative velocity. Thus, we have

$$K_R(t) = K_R(x(t), \dot{x}(t)) \qquad (1)$$

Later it will be assumed that the restoring force is the summation of two separable forces, namely damping and spring forces. Furthermore, it will be assumed that the damping force is a function of relative velocity while the spring force is a function of relative displacement.

To formulate the problem, suppose that the system as shown in Figure 1 behaves nonlinearly in spring force and quasi-linearly in viscoelastic force. Figure 1-c represents the response of the system to the applied load. The solid line is used to denote the relative displacement and the dashed line is used to denote the velocity response. Points x_2, x_6 and x_{10} are not only the ending points of the loading paths but also the starting points of the very next unloading paths. On the other hand, points x_4, x_8, and x_{12} are the ending points of the unloading paths as well as the starting points of the next loading paths. The apparent feature of each segment of $(f_S - x)$ curve between the successive two points (e.g., x_2 and x_4) is that they are monotonically increasing or monotonically decreasing. Similarly if the points of the maximum or minimum values of velocity response are marked, the same conclusion from the $(f_D - \dot{x})$ curve can be drawn. Therefore, it is inferred that both force functions in each interval (i.e., t_1 to t_2, t_2 to t_3, ...) are monotonically changing. If N_i is the number of points within interval i, the restoring force, $f_R(t)$, can be written in terms of two polynomials which one corresponds to spring force and the other to damping force function, i.e.,

$$f_R(t) = f_S(t) + f_D(t) \qquad (2)$$

where

$$f_S(t) = a_0 + a_1 x + a_2 x^2 + \ldots + a_q x^q \quad (3)$$

$$f_D(t) = b_0 + b_1 \dot{x} + b_2 \dot{x}^2 + \ldots + b_p \dot{x}^p \quad (4)$$

and

$$p + q = N_i - 1 = M \quad (5)$$

Substitution of Equations 3 and 4 into Equation 2 yields:

$$f_R(t) = c + a_1 x + a_2 x^2 + \ldots + a_q x^q + b_1 \dot{x} + \ldots + b_p \dot{x}^p \quad (6)$$

where

$$c = a_0 + b_0 \quad (7)$$

3.2 Identification Procedure

Let N_i denote the number of points within each interval, i. Because that relative acceleration response, $\ddot{x}(t)$, has been measured, numerical integrations will yield the relative velocity response as well as the relative displacement time history. Thus acceleration, velocity, and displacement for each one of N_i points are known. Writing Equation 6 in terms of the response of each point yields N_i equations with N_i unknowns as follows:

$$\begin{aligned} f_R(t_j+0) &= c + a_1 x_1 + \ldots + a_q x_1^q + b_1 \dot{x}_1 + \ldots + b_p \dot{x}^p \\ f_R(t_j+\Delta t) &= c + a_1 x_2 + \ldots + a_q x_2^q + b_1 \dot{x}_2 + \ldots + b_p \dot{x}^p \\ &\;\;\vdots \\ f_R(t_j+M\Delta t) &= c + a_1 x_{N_i} + \ldots + a_q x_{N_i}^q + b_1 \dot{x}_{N_i} + \ldots + b_p \dot{x}_{N_i}^p \end{aligned} \quad (8)$$

where $M = N_i - 1$ as given in Equation 5. Equation 8 can be rewritten in matrix form as follows:

$$\begin{bmatrix} f_R(t_j+0) \\ f_R(t_j+\Delta t) \\ \cdot \\ \cdot \\ \cdot \\ f_R(t_j+K\Delta t) \\ \cdot \\ \cdot \\ f_R(t_j+M\Delta t) \end{bmatrix} = \begin{bmatrix} 1 & x_1 & & x_1^q \; \dot{x}_1 & & \dot{x}_1^p \\ 1 & x_2 & & x_2^q \; \dot{x}_2 & & \dot{x}_2^p \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 1 & x_{K+1} & \cdots & x_{K+1}^q \dot{x}_{K+1} & \cdots & \dot{x}_{K+1}^p \\ \cdot & & & & & \\ \cdot & & & & & \\ 1 & x_{N_i} & \cdots & x_{N_i}^q \dot{x}_{N_i} & \cdots & \dot{x}_{N_i}^p \end{bmatrix} \begin{bmatrix} c \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_q \\ b_1 \\ \vdots \\ b_p \end{bmatrix} \tag{9}$$

Equation 9 can also be written symbolically as

$$F_R = XA \tag{10}$$

In this equation, elements of X are known and F_R can be calculated from the following equation of motion for the SDOF system as shown in Figure 1-a.

$$m\ddot{x}(t) + f_R(t) = -m\ddot{z}(t) \tag{11}$$

where $m\ddot{x}(t)$, $f_R(t)$ and $-m\ddot{z}(t)$ are respectively the inertia force, restoring force, and the excitation force. In this equation, $\ddot{x}(t)$ and $\ddot{z}(t)$ are known and only m (mass) and $f_R(t)$ are unknown. One of the assumptions herein is that m is time-invariant. Therefore, it can be estimated from the linear part of system response by using any standard linear system identification method. However, to simplify the identification procedure, Equation 11 was divided by m and the following relation was assumed for the restoring force.

$$f_R(t) = f_R(t)/m = -\ddot{z}(t) - \ddot{x}(t) \tag{12}$$

Therefore, the restoring forces are also known since the base acceleration, $\ddot{z}(t)$, and the relative acceleration, $\ddot{x}(t)$, are the available information from the structure response. Equation 10 can now be solved for the parametric vector, A, by inverting response vector, X,

$$A = X^{-1}F_R \tag{13}$$

Having the parametric vector, A, calculated, the spring and damping forces at each point can be determined. However, to distinguish each force from the other one, the constant c in Equation 7 should be correctly divided to two subconstants a_0 and b_0 which are each a part of the spring and damping forces, respectively. So far the estimation of the parameters of each interval was independent of the estimates or the information available from the other intervals. However, dividing the constant c into a_0 and b_0 contradicts the advantage of interval independency unless the damping force has a linear relationship with velocity. Because there won't be any constant parameter in the damping force function, it enables us to equate a_0 to c and b_0 to zero. If this property doesn't exist for the damping force, then one should make use of the previously-obtained forces. In formulating the problem, it was mentioned that the points which are at the end of any interval will be used as the starting point of the very next interval. Therefore, having the forces of the last point of any interval calculated, one can make use of them as the initial values for the next interval. For example, suppose that the parametric vector A of interval i has been calculated. Now each one of the forces, spring and damping forces, for the starting point of interval i is obtained as follows:

$$f_R(t+0\Delta t) = \begin{bmatrix} f_S(t+0\Delta t) \\ \\ f_D(t+0\Delta t) \end{bmatrix} = \begin{bmatrix} 1 & x_1 & \cdots & x_1^q & 0 & \cdots & 0 \\ \\ 0 & \cdots & & 0 & \dot{x}_1 & \cdots & \dot{x}_1^p \end{bmatrix} \begin{bmatrix} c \\ a_1 \\ \vdots \\ a_q \\ b_1 \\ \vdots \\ b_p \end{bmatrix} \tag{14}$$

which results in the following expressions for the forces

$$f_S(t+0\ t) = c + a_1 x_1 + a_2 x_1^2 + \ldots + a_q x_1^q$$

and (15)

$$f_D(t+0\ t) = b_1 \dot{x}_1 + b_2 \dot{x}_1^2 + \ldots + b_p \dot{x}_1^p$$

We also have these forces from the previous interval. Therefore, the difference between the two values of one force will give us the constant term of the other force. For example, assume that the difference between the spring forces is h. It means that the spring force in the present interval has been over or under estimated. In order to correct it, one should subtract h from the present values of spring forces. Indeed, h is actually the same as b_0 which we were trying to distinguish from a_0.

$$b_0 = c - a_0 = f_S(x_1)_i - f_S(x_1)_{i-1} = h \tag{16}$$

It should be noticed that b_0 could have been obtained from the difference between the values of the other force (i.e. damping force). In other words, a known datum was left unused.

Equation 16 introduces the last stage of estimation of the unkown parameter vector A because each of the two forces can now be determined as

$$f_R(t+n\Delta t) = \begin{bmatrix} f_S(t+n\Delta t) \\ \\ f_D(t+n\Delta t) \end{bmatrix} = \begin{bmatrix} 1 & x_{n+1} & x^2_{n+1} & \cdot\cdot & x^q_{n+1} & 0 & 0 & \cdots\cdots & 0 \\ \\ 0 & 0 & 0 & \cdot\cdot & 0 & 1 & \dot{x}_{n+1} & \dot{x}^1_{n+1} \cdots & x^p_{n+1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_q \\ b_0 \\ b_1 \\ \cdot \\ \cdot \\ b_p \end{bmatrix} \qquad (17)$$

Clough and Penzien apply a numerical technique to a SDOF system which is nonlinear in spring force in order to get the response of the structure to an arbitrary load [7]. To apply the present method, it was decided to identify the properties of their example frame (i.e. damping coeff. and elastic force). Therefore, the response calculated by them and their selected load were chosen as the known output and input, respectively. Figure 2-a shows the system properties and the applied load (input). Figure 2-b shows the response of the system (output). Table 1 represents the estimated parameters for each one of the intervals.

Table 1. Estimated Restoring Force Function

Interval	Restoring Force Function
0-0.3	$52.935x - 3.585x^2 + 1.901\dot{x}$
0.3-0.6	$59.778 + 0.524x - 0.170x^2 + 1.984\dot{x}$
0.6-0.8	$25.317 + 13.414x + 7.002\dot{x}$

The functions are plotted in Figure 3-b. As seen in this plot, the estimated behavior of structure is almost identical to the actual behavior of structure seen in Figure 3a.

3.3 Problem of Change in Slope

One of the questions is how to deal with the sudden changes in the properties of structure. This sudden change may be considered as a slope change in the mathematical representation of elastic or damping forces. For example, fitting a low order function to an elasto-plastic material results in the failure of the method. Three following possible ways have been tried in dealing with this problem so far; (1) least square estimates, (2) curve-matching, (3) initial values which will be respectively discussed.

3.3.1 Least Square Estimates

Hudson [14] represents a least square solution for the case when a complete curve to be fitted consists of two or more submodels, and those have to be joined at points whose abscissa have to be estimated. Here, from the practical point of view his technique is described and summarized as some operations which can be easily programmed.

The technique can be briefly stated for the case of two submodels $f_1(x;a_1)$ and $f_2(x;a_2)$, joined together at $x=\alpha$. It is desired to find vectors a_1, a_2 and real values α and I which minimize

$$R(a_1,a_2,\alpha,I) = \sum_{K=1}^{I} w_k[y_k-f_1(x_k;a_1)]^2 + \sum_{K=I+1}^{n} w_k[y_k-f_2(x_k;a_2)]^2 \quad (18)$$

subject to the following relationships among the parameters:

$$f_1(\alpha;a_1) = f_2(\alpha\ a_2)$$
$$x_I \leq \alpha \leq x_{I+1} \quad (19)$$

The following example will clarify the statement of the problem and the parameters involved.

Example: Assume that two straight lines joined together at $\alpha = \alpha_1$. Therefore

$$\begin{aligned} f(x) &= a_{10} + a_{11}x = [a_{10}\ a_{11}] \{^1_x\}, \quad A_0 \le x \le \alpha_1 \\ &= a_{20} + a_{21}x = [a_{20}\ a_{21}] \{^1_x\}, \quad \alpha_1 \le x \le A_2 \end{aligned} \tag{20}$$

and they should be equal at $x = \alpha_1$

$$a_{10} + a_{11}\,\alpha_1 = a_{20} + a_{21}\,\alpha_1 \tag{21}$$

A_0 and A_2 define the limits of an interval consisting of n pairs (x_i, y_i) where the $\{x_i\}$ are distinct and the $\{y_i\}$ have known weights $\{w_i\}$. For our case the weight vector consists of equal numbers or in other words, $\{w_i\}$ is a unit vector. The overall residual sum of squares is

$$R = \sum_{i=1}^{n} [y_i - f(x_i)]^2 \tag{22}$$

and this is to be minimized with respect to both the $\{a_i\}$ and α_1.

3.3.1.1 Types of Join*

Hudson introduces four types of join of which only two types are involved in our problem [14]. Within this framework we are only interested in whether or not α_1 coincides with an x_i. Therefore the following classification conveniently demonstrates the two different types of join.

$\alpha_1 \neq x_i$
Type One

$\alpha_2 = x_i$
Type Two

As the classification shows the type-one join, α_1, lies strictly between two successive x values, while, the type-two join coincides with x_i.

*Throughout this section, "Join" is an abbreviation for "abscissa of the join point".

3.3.1.2 Estimation of Parameters When the Type of Join is Known

If the type of join is known, the overall least square solution can be applied whether or not the subscript i is initially unknown. Suppose that i is known and the join is of type one. Then, the overall least square solution (Appendix A) can be obtained. More specifically, we obtain vectors $\hat{a}_1$ and $\hat{a}_2$ of Equation 20 using the data with abscissas $(x_1, \ldots, x_i)$ and $(x_{i+1}, \ldots, x_n)$, respectively. Note that the hat (^) appearing over the vector a_i stands for the estimated values, not the real values.

Having vectors a_1 and a_2 estimated, we can solve for the join by setting

$$f_1[\alpha;\hat{a}_1] = f_2[\alpha;\hat{a}_2] \tag{23}$$

and find (by hypothesis) that there is at least one solution for α in Equation 23, say

$$\alpha = \hat{\alpha}(i) \tag{24}$$

Then we have automatically

$$[\hat{a}_1,\hat{a}_2,\hat{\alpha}] = [\hat{a}_1(i),\hat{a}_2(i),\hat{\alpha}(i)] \tag{25}$$

More generally, suppose we know that the join is of type one, but that the subscript i is unknown. Now the subsequent steps can be followed:

First choose a value of i, and estimate $a_j(i)$ as before. Find if the curves have at least one join a*(i) in the right place, i.e.,

$$x_i < \alpha^*(i) < x_{i+1} \tag{26}$$

where the superscript * stands for the "chosen value". If Equation 26 holds, put

$$T(i) = R^*(i) = R_1^*(i) + R_2^*(i) \tag{27}$$

where $R_1^*(i)$ and $R_2^*(i)$ are the local residual sum of squares.

If the curves do not join, or if Equation 26 is not satisfied, let

$$T(i) \to \infty \tag{28}$$

Now repeat the computation with all the other relevant values of i. Finally, choose the critical value of i from which T(i) is minimized. For convenience, this set of calculations is called step (1).

3.3.1.3 Join is of Type Two

Now we consider the case where the join is of type two. We have

$$\hat{\alpha} = x_i, \tag{29}$$

for some i. Suppose first that i is known. The remaining parameters can be easily found since the model is linear in them. The least square fit should be performed while it is subjected to the linear constraint

$$f_1(x_i;a_1) = f_2(x_i;a_2) \tag{30}$$

One way of doing the above estimation is described by Williams [32] which is in Appendix A. Then the resulting overall residual sum of squares is denoted by S(i).

More generally, if i is not known, we simply repeat the above calculations and record S(i); we then choose the critical value of i for which S(i) is minimized. For convenience, this set of calculations is called step (2).

3.3.1.4 Estimation of Parameters When the Type of Join is Unknown

In this case, the overall solution is based on that type of estimation which yields the minimum residual sum of squares. Therefore it is necessary to try the both types of estimation. We can start with step (1) (i.e.

searching for solutions with a join of type one). Then we start the search for the solutions with a join of type (2) (i.e. step (2)).

The last word on this method is that it doesn't give a unique answer. Furthermore, it might not give a satisfactory answer if the number of data points is insufficient. Finally, the steps should be repeated for the different assumed values of the other force which is computationally expensive.

3.3.2 Curve-Matching Method

Figure 4 represents the slope-change problem which was experienced during identifying an elasto-plastic type structure (described later). Although the plot is the exaggerated display of the problem, it should be mentioned that the early plots of the spring-force displacement were clearly displaying such incompatabilities between the neighboring intervals. Thus the current method which is a kind of trial-and-error was applied. The essence of the method is to plot the grossly estimated forces while the record of each interval estimate is preserved. Then the locations of the undesirable portions are marked and are given to the computer programs as a set of new input. For the second time the method is applied, but only to the in-trouble intervals. Subdividing the interval is a kind of remedy that can be used. However, it was sometimes experienced that the preceding operation should be repeated a few more times. Figure 4 clearly indicates why the operation might be repeated (i.e. the curves (I), (II), and (III) which are using different numbers and sequences of points can not solve the problem, the only way of solving the problem is to subdivide the whole interval to (1-5) and (5-9) subintervals).

Although, the results were satisfactory, the method appeared to be very time-inefficient. Moreover, a part of the method requires the engineer's judgement which might result in different estimates from engineer to engineer.

3.3.3 Initial-Values Method

The reason for choosing such a name for the method was the use of the knowledge of the previously calculated intervals to overcome the slope-change problem. In this approach, it is assumed while one of the forces, in-trouble force, faces the slope-change problem, the other one, helping force, is experiencing a very mild change in its behavior. This assumption allows us to extend the pattern of the latter force from the previous interval. Therefore, one shouldn't go through finding matrix X in Equation 10 and inverting it in order to find vector A (in the same equation). Instead, we use the estimated parameters of the helping force from the last interval to calculate the values of the forces in the present interval; then substracting these values from the corresponding restoring forces yields us the values for the in-trouble forces. On the other hand, we try to use as few points as possible for the estimation procedure in order to lower the probability of having any sharp change in the helping force. Furthermore, if the assumption was not satisfied within the present interval, a significant discrepancy between the values of the helpoing force in the current interval and the next-to estimate interval would be expected. Let's suppose this discrepancy is experienced by our detecting subroutine. Therefore, we should return to the in-trouble interval and try to experience a new arranement within the points chosen for each submodel. It should be noticed that the worst case will occur when both forces experience the slope-change problem at the same time.

The suggested method happened to be the most time-efficient and successful one among the three recommended ones.

3.4 Identification Using Stimulated Data

A single-degree-of-freedom system, shown in Figure 5, was subjected to the 1971 San Fernando earthquake. An elasto-plastic stiffness was used for the system while a linear function was assumed for the damping force vs.

velocity. A linear function was assumed for the acceleration increment in order to calculate the response of the frame to the applied load. Figure 6 shows the calculated response. A considerable feature in the response histories is the permanent deformation which is present in the displacement history after 14 seconds have elapsed. This permanent deformation is one of the worst situations for many of the linear and nonlinear methods. In fact they are not able to identify the structure parameters, although the structure has behaved linearly. A good point about the current method as well as the second one (described later) is their ability in dealing with such problems too. Figure 7 shows the estimated behavior of the frame.

In order to investigate the sensitivity of the method to measurement noise, the relative acceleration was added with an independent random variable. The noise (random variable) generated was from a normal distribution function with zero mean and σ standard deviation. The level of the noise, σ, was increased from one percent of the maximum acceleration amplitude to ten percent. Figures 8 and 9 represent the response and estimated behavior of the structure under one percent noise, respectively. Similarly Figures 10 and 11 are for the noise level of ten percent. It is seen that the method is not as sensitive to measurement noise for high amplitude response as it is for low amplitude response. However, it should be mentioned that as the noise level was increased to values higher than 10%, the method started to overesimate the behavior of the structure.

4. DELTA METHOD

Clough and Penzien [7] presented a method of analysis which can be applied for the development of a system identification technqiue for the non-linear system. They assume that the structure properties remain constant during a short time increment Δt, and then establish the condition of dynamic equilibrium at the beginning and at the end of each interval. The motion of the system during the time increment is evaluated approximately on the basis of an assumed response mechanism. The constant parameters and the incremental form of equation of motion during a short period of time were two main features which were used to develop the current method.

4.1 Formulation of Equation

The model is considered to be a SDOF system as shown in Figure 12. Nonlinear characteristics are assumed for the spring and damping forces (Figure 12-b). Figure 12-c shows an arbitrary applied load and, at last, Figure 12-d indicates what forces are acting on the mass m.

At any instant of time t the condition of the dynamic equilibrium is expressed as

$$f_I(t) + f_D(t) + f_S(t) = p(t) \tag{31}$$

where $f_I(t)$, $f_D(t)$, $f_S(t)$ and $p(t)$ introduce inertia, damping, spring, and applied forces, respectively. After a short time Δt the equation becomes

$$f_I(t+\Delta t) + f_D(t+\Delta t) + f_S(t+\Delta t) = p(t+\Delta t) \tag{32}$$

The incremental form of equilibrium equation is derived by subtracting Equation 31 from Equation 32, i.e.,

$$\Delta f_I(t) + \Delta f_D(t) + \Delta f_S(t) = \Delta p(t) \tag{33}$$

Therefore, this equilibrium is assumed to exist during the time interval t. Recalling the definition of each force yields the incremental forces as follows:

$$\Delta f_I(t) = f_I(t+\Delta t) - f_I(t) = m(t)\Delta\ddot{v}(t)$$

$$\Delta f_D(t) = f_D(t+\Delta t) - f_I(t) = c(t)\Delta\dot{v}(t) \qquad (34)$$

$$\Delta f_S(t) = f_S(t+\Delta t) - f_S(t) = K(t)\Delta v(t)$$

$$\Delta p(t) = p(t+\Delta t) - p(t)$$

In which m(t), c(t), K(t) represent the mass, damping coefficient, and stiffness coefficient at the time interval t, respectively. These structural properties are assumed to remain constant during Δt. Moreover it is assumed that the mass m is unchangeable throughout the entire motion of the structure.

Substituting the force expressions from Equation 33 leads to the final form of the incremental equilibrium equation for an arbitrary loading.

$$m\Delta\ddot{v}(t) + c(t)\ \Delta\dot{v}(t) + K(t)\ \Delta v(t) = \Delta p(t) \qquad (34)$$

Since the main object of this study which is the analysis of the earthquake data should be followed, the applied load is specified to be a ground-motion acceleration

$$p(t) = -m\ddot{v}_g(t) \qquad (36)$$

Hence

$$\Delta p(t) = -m\Delta\ddot{v}_g(t) \qquad (37)$$

substituting Equation 37 into Equation 35 and dividing both sides by m yields

$$\Delta\ddot{v}(t) + \frac{c(t)}{m}\ \Delta\dot{v}(t) + \frac{K(t)}{m}\ \Delta v(t) = -\Delta\ddot{v}_g(t) \qquad (38)$$

Recalling the definitions of damping ratio and natural frequency Equation 38 is reduced to the following

$$\Delta\ddot{v}(t) + 2\zeta(t)\ \omega(t)\ \Delta\dot{v}(t) + \omega^2(t)\ \Delta v(t) = -\Delta\ddot{v}_g(t) \qquad (39)$$

which is a simple second-order equation in terms of $\omega(t)$. However, this equation can not be solved for $\omega(t)$ since $\zeta(t)$ is also an unknown. The solution to this equation plays a significant role in the accuracy of the finally resulting estimates.

In the following, two simple approaches are presented and their applications to the simulated and actual data are evaluated.

4.2 Approach I

In this approach Equation 39 is reduced to a simple second-order equation in terms of $\omega(t)$. Therefore, a reasonable range of change for ξ, damping ratio, should be considered. One fact which helps in simplyfing Equation 39 is that most large structures exhibit light damping [26]. In addition, the effect of the damping force on equilibrium equation is not as important as those of the other forces for this kind of loading. Therefore, Equation 39 is solved for $\omega(t)$ while ξ is assumed to be linear and remain constant during the interval of interest. This results in a set of estimates for $\omega(t)$ for any specific value of ξ during that described interval. The response of the structure is, then, calculated for different couples of $\omega(t)$ and ξ. That couple is finally selected which better resembles the measured response for the interval. It should be mentioned that the following simplified form as given in Equation 40, is expected to have two real roots.

$$\omega^2(t)\ \Delta v(t) + 2\omega(t)\ \hat{\xi}\ \Delta\dot{v}(t) + \Delta\ddot{v}(t) + \Delta\ddot{v}_g(t) = 0 \qquad (40)$$

Choosing one of the roots needs a prior information about the natural frequency. This prior information and an estimate of the range of change of ξ can be calculated by using any linear system identification technqiues. The range should include the estimated $\hat{\xi}$.

4.2.1 Example 4.1: Real Data

The computer program developed for the first approach was used to estimate the parameters of the Union-Bank Building, Los Angeles, from the records obtained during the 1971 San Fernando earthquake. The Union-Bank building is a 42-story steel-frame structure in downtown Los Angeles which experienced peak acceleration at mid-height of 20% g (transverse direction) and 13% g (longitudinal direction) during this earthquake. At the time of the San Fernando earthquake, the measuring instruments were installed in the sub-basement, on the 19th floor and on the 39th floor, but the instrument on the 39th floor failed to record. The components of the digitized relative acceleration, velocity, and displacement at the 19th floor were used as the response data in the analysis. The sub-basement absolute acceleration was used as the input to the system. Several assumptions were made on the basis of the conclusions that Martin [17] and Beck [1] had drawn from analyzing the data. (I) Data belonged to a single-degree-of-freedom system. Although this assumption was not in agreement with the Fourier amplitude spectra of the components of the building response which showed the existence of first four lower modes, the response amplitudes of the second to the last modes were much smaller than that of the first mode. (II) System had behaved linearly. This assumption was actually made due to the fact that the Union-bank building had experienced minor damage such as plaster cracking and tile damage during the earthquake. Therefore, the analysis was limited to the analysis of a linear SDOF system. An attempt was made to find whether there was any change in the natural frequency of the structure. At the first time, the time step was set to 0.02 seconds, and the damping ratio was assumed to be within the 0.02-0.1 range. For five different values of damping ratio (i.e. 0.02, 0.04, 0.06, 0.08, and 0.1) Equation 40 was solved for w(t), natural frequency. Unfortunately, it was found that Equation 40 failed to be solved for some points. This problem was mostly experienced in the first ten-second period. The number of the unused points for the rest of the response was about 10% in comparison to 40% for the first period.

Then as an attempt to reduce the effect of noise, thought to be the largest factor in creating the problem, the time step was increased. Although, this remedy appeared to be effective for the last forty-second period of the response it didn't lead to a considerable improvement for the first ten-second period. These attempts left only the conclusion that the method is very sensitive to noise. It was further concluded that if the higher modes' participation in the response was not filtered out, it should be expected that the method experience difficulties. These conclusions seemed to be more meaningful by paying attention to a report by Hanks [11]. He showed that there were significant errors in some records at periods less than 16 seconds. Thus, these records should be high-pass filtered with a cut-off frequency corresponding to a period of about 16 seconds. However, as it was earlier mentioned the study of filtering would be left to the future research.

Figure 13 represents the effect of increasing the length of the time increment on the reduction of the number of unused points. This effect had been tested over a period of ten seconds (i.e., 30-40 second period). The figure reveals that for a linear system a time step of about 30% of the structure period results in the most use of the available data. Figure 14 indicates the estimation of the natural frequency. It shows a good agreement with those results obtained by Beck [1]. The figure also shows that damping ratio is not that much effective in the estimation approach.

Results of this example show that, for a set of real data, the method is sensitive to noise. In addition, the method is not reliable if the participation of the other modes is not ignorable. The reason for this, as discussed earlier in this section, is that the method doesn't work for the first phase (the first ten seconds) of the data which is corrupted with a high level of noise as well as a considerable participation of the other modes. However, for the second phase (the rest of the data), the estimated parameters were in a good agreement with those obtained by the probabilistic and the statistic methods. Although, a unique set of parameters was not obtained for the uncorrupted data, the results proved that for the large structure damping force has a very small effect.

The question which still remains is how the damping force can be estimated when its effect should not be ignored. This question might be answered by the second approach, however, how effective the method is in dealing with the nonlinear systems as well as the elastic-force estimation occupy the next section.

4.4.2 Example 4-2: Simulated Data

The base-ment acceleration and the response of the non-linear SDOF system of Section 3.4 were used as the input and the output, respectively. Damping ratio was assumed to be constant and estimated by using the method developed by Chen and Yao [5,6]. Then it was set to be a constant during the estimation of spring coefficients. Finally, the elastic forces at any instant were calculated as follows:

$$(1) \quad f_S(0) = 0 \tag{41}$$

Although this assumption fits this specific example, it should be considered that the initial value is not always known.

$$\begin{aligned}(2) \quad f_S(\Delta t) &= f_S(0) + \omega(0) \; * \; (x(\Delta t) - x(0)) \\ &= \omega(0) * x(\Delta t) \end{aligned} \tag{42}$$

$$(3) \quad f_S(n\Delta t) = f_S((n-1)\Delta t) + \omega((n-1)\Delta t) * [x(n\Delta t) - x((n-1)\Delta t)] \quad (43)$$

Therefore, each $f_S(n\Delta t)$ corresponding to $x(n\Delta t)$ was obtained. The plot of elastic force versus displacement was then drawn.

Figure 15 represents the estimated behavior of the structure. The apparent origin-shift in hysteretic behavior explains the common difficulty associated with the methods using the previous estimates in estimation of the present time values. Overstimulation and understimulation of the parameters are also due to the same problem (past-dependent estimation method). However, when a prior knowledge about the structure behavior was used in solving Equation 40, an improved plot was resolved. Figure 16 displays this improvement. However, there still exists an origin-shift in the hysteretic behavior.

4.3 Approach II

There are three unsatisfactory problems associated with the 1st approach. First, damping ratio should be estimated by any technique but the present method itself. Second, the second-order equation, Equation 40, is very sensitive to noise and to the small values of the stiffness coefficient. Finally, the elastic forces are calculated in such a way that the previously-estimated elastic forces are involved. Therefore, the error associated with any instant of the estimation is carried over the rest of the estimation. The present approach tires to overcome the first two shortcomings of the first approach. The solution to the last problem will be discussed later.

In this approach, it is assumed that the structural properties remain constant during the n-th and (n+1)-th steps (i.e., during two successive time steps). Equation 40 now is repeated twice while two conditions should be considered

$$K(t)\Delta v(t) + C(t)\Delta\dot{v}(t) + \Delta\ddot{v}(t) + \Delta\ddot{v}_g(t) = 0 \quad (45)$$

$$K(t+\Delta t)\Delta v(t+\Delta t) + C(t+\Delta t)\Delta\dot{v}(t+\Delta t) + \Delta\ddot{v}(t+\Delta t) + \Delta\ddot{v}_g(t+\Delta t) = 0$$

where

$$K(t) = K(t+\Delta t)$$
$$C(t) = C(t+\Delta t) \qquad (46)$$

It should be noticed that K(t) and C(t) are respectively representing stiffness and damping properties. Heretofore these were represented by $\omega^2(t)$ and $2\zeta(t)\omega(t)$ in Equation 40. Now there are two linear equations with two unknowns. Having these parameters estimated, elastic forces are estimated by following the steps indicated by Equation 41 through Equation 43.

The first problem which might be experienced is the slope-change problem. Unlike the first method, this problem is overcome very easily. Having the parameters estimated at each instant, the detecting subroutine will reveal what couple points don't follow the pattern of their preceding and following points. The first point of any couple is re-estimated in such a way that it is in harmony with the foregoing points, and the second point in such a way which is in harmony with the coming points.

As a numerical example, the data as given in section 3.4 were used. Figure 18 represents the estimated behavior of the SDOF nonlinear system. It is much more imporved than that one obtained using the first approach. However, the dependence of the present value to the previous values remains to be a problem.

4.4 Present-to-Past Dependence Problem

Equation 43 indicates that the present estimated value of the spring force depends on the force estimates of the previous instances. Therefore, if there is an error associated to the previously-estimated forces, this error will be automatically carried over to the future estimates of the force. To overcome this problem, the condition of dynamic equilibrium can be imposed to the estimated forces at any instant of time. If the estimated forces couldn't satisfy the equilibrium condition, the existing discrepancy must be in somehow divided into the two forces. A point-by-point search revealed that this problem initiates from those intervals which one of the forces experiences a sharp change in its property. Furthermore, it was found that the mentioned discrepancy belonged only to the force in question. The improved (modified) computer program based on the derived conclusion resulted in a stimated behavior, Figure 18, for the simulated-data frame which is similar to as that obtained by the first method.

5. DISCUSSION AND CONCLUDING REMARKS

It is well understood that difficulties exist when these two methods are applied to noise-corrupted data on the response of an existing structure. At present, there is no error criterion associated with these methods. One possible extension is to develop suitable error criteria in order to minimize the difference between the responses of the real system and the model.

Two improvements can be made for the first method as given herein. Having measuring instruments which operate in a faster sampling rate will result in more points within each interval. There are two reasons why having more points is helpful. First, the sum of the degrees of two polynomials chosen for the forces (i.e. damping and spring forces) might be set to be much fewer than the number of the points within any interval. This enables us to use a least square type of error function. Therefore, the optimal estimates of the parameters are obtained. Second, the possibility of having a polynomial whose degree is larger than the real force function is increased. Now we can apply certain averaging techniques to reduce the effect of noise.

The last suggestion is more related to the laboratory experimental models. Because the displacement and acceleration of them are measurable, both quantities are exposed to measurement noise. Therefore, it is suggested that the acceleration time history be used as the only available output. Then the velocity and displacement components are obtained by integrating the acceleration record. This integration process will help to smooth or reduce the effect of noise.

The second method (Delta method) as mentioned earlier is also unable of dealing with noise of high level if it is present in the data. However, this method is better in the sense that the averaging or smoothing techniques are applicable to it. For instance, Figure 14 indicates the averaged natural frequencies of five successive intervals. In spite of this advantage, we should be careful in using the smoothing techniques for non-linear structures. Because the averaging procedure shouldn't destroy the pattern of the change of the forces. Moving average [3] and the exponential smoothing [3] are the kinds of smoothing techniques that could be used in this case.

It is noted earlier that these methods are developed by assuming predominant model responses. Schiff [26] has made certain comments which seem to be compatible with this assumption. He states that, because most large structures exhibit light damping, their identification can be reduced to the finding of the system's lower natural frequencies and modal damping. He further indicates that most identification schemes are based on the property that a lightly damped multi-degree-of-freedom system can be closely approximated by a SDOF system of appropriate natural frequency and damping in the region near each of the system's natural frequencies. In reality, existing structures are very complex systems and many modes are present in their response records. Therefore, a natural extension of these methods is to deal with multi-degree-of-freedom systems.

Sozen [27] believes that the inter-story drift is an important design consideration. In fact, any inter-story permanent deformations can be significant indicators of structural damage. It is expected that the present study can be extended to obtain the inter-story hysteretic behavior of multi-story buildings. The experimental results of Sozen and his associates [27] will be invaluable in such an extension.

ACKNOWLEDGEMENT

This investigation is supported in part by the National Science Foundation through Grant No. PFR 7906296. Molly Harrington capably typed this report.

Appendix A:

FITTING TWO CURVES CONSTRAINED TO JOIN AT $x = \alpha$

The model is

$$f(x) = \begin{cases} f_1(x,\beta_1) = a_1 + a_2x + \ldots + a_nx^{n-1}, & x_1 \leq x \leq \alpha \\ f_2(x,\beta_2) = b_1 + b_2x + \ldots + b_mx^{m-1}, & \alpha \leq x \leq x_N \end{cases} \qquad (A.1)$$

or in a matrix form

$$f(x) = \begin{cases} f_1(x,\beta_1) = [1 \ x \ .. \ x^{n-1}] \begin{bmatrix} a_1 \\ \vdots \\ a_{n-1} \end{bmatrix} = \{x_1\}^T \beta_1, & x_1 \leq x \leq \alpha \\ f_2(x,\beta_2) = [1 \ x \ .. \ x^{m-1}] \begin{bmatrix} b_1 \\ \vdots \\ b_{m-1} \end{bmatrix} = \{x_2\}^T \beta_2, & \alpha \leq x \leq x_N \end{cases}$$

To estimate β_1 and β_2, the overall residual sum of squares should be set and minimized with respect to β_1 and β_2.

(a) Residual sum of squares (RSS)

$$R = \sum_{j=1}^{N} [y_j - f(x_j)]^2$$

$$= \sum_{j=1}^{i} [y_j - f_1(x_j,\beta_1)]^2 + \sum_{j=i+1}^{N} [y_j - f_2(x_j,\beta_2)]^2$$

$$= \qquad R_1 \qquad + \qquad R_2 \qquad (A.3)$$

to minimize R, its derivatives with respect to each element of β_1 and β_2 should be equated to zero. However, as far as the model has not been subjected to any restriction, R_1 and R_2 would be independent of each other and the following relations stand for R_1 and R_2

$$\frac{\partial R_1}{\partial \beta_2} = \frac{\partial R_2}{\partial \beta_1} = 0 \qquad \text{(A.4)}$$

Eq. (A.4) simplifies the operation of taking derivatives of R with respect to β_1 and β_2. Because $\frac{\partial R}{\partial \beta_1}$ and $\frac{\partial R}{\partial \beta_2}$ can be respectively replaced by $\frac{\partial R_1}{\partial \beta_1}$ and $\frac{\partial R_2}{\partial \beta_2}$. Finally, each one can be carried out independently from the other.

(b) <u>Derivative of R_1 with respect to β_1</u>

$$R_1 = \sum_{j=1}^{i} [u_j - f_1(x_j, \beta_1)]^2 \qquad \text{(A.5)}$$

or

$$R_1 = Y_1^T Y_1 - 2Y^T [x_1] \, \beta_1 + \beta_1^T [x_1]^T [x_1] \, \beta_1 \qquad \text{(A.6)}$$

where

$$Y_1 = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_i \end{bmatrix} \qquad [X_1] = \begin{bmatrix} 1 & x_1 & \cdots & x_1^n \\ 1 & x_2 & \cdots & x_2^n \\ \vdots & \vdots & & \vdots \\ 1 & x_i & \cdots & x_i^n \end{bmatrix} \qquad \text{(A.7)}$$

set

$$C_1 = [x]^T [x] \qquad \text{(A.8)}$$

It is apparent that C_1 is symmetric. Eq. (A.6) is now written as

$$R_1 = Y_1^T \, Y_1 - 2Y_1^T [x_1] \beta_1 + \beta_1^T C_1 \beta_1 \qquad \text{(A.9)}$$

Now the derivative of R_1 with respect to β_1 can be simply obtained.

$$\frac{\sigma R_1}{\sigma \beta_1} = -2Y_1^T [x_1] + 2\beta_1^T C_1 \qquad \text{(A.10)}$$

Equating $\frac{\sigma R_1}{\sigma \beta_1}$ to zero yields the estimation of β_1.

$$\beta_1^* = C_1^{-1} [x_1]^T Y_1 \qquad (A.11)$$

[Here superscript * stands for the unconstrained estimate]. Similarly, β_2 can be calculated as follows:

$$\beta_2^* = C_2^{-1} [x_1]^T Y_2 \qquad (A.12)$$

Therefore, the unconstrained estimates of β_1 and β_2 should now be restricted to the constraint that the two curves join at α.

$$f_1(\alpha,\beta_1) = f_2(\alpha,\beta_2) \qquad (A.13)$$

(c) <u>Unconstrained estimates of β_1 and β_2 [14,32]</u>

The unconstrained R.S.S. is $(\rho_1^* + \rho_2^*)$, where

$$\rho_j^* = Y_j^T Y_j - Y_j^T [x_j] \beta_j^*$$

Since f_1 and f_2 are linear in β_1 and β_2, respectively, the constraint is also linear in them and can be written

$$a_1 + a_2\alpha + \ldots + a_n\alpha^{n-1} = b_1 + b_2\alpha + \ldots + b_n\alpha^{m-1} \qquad (A.14)$$

or

$$(\beta_1^T, \beta_2^T) \,.\, Q = 0 \qquad (A.15)$$

where

$$Q^T = (1, \alpha, \alpha^2, \ldots, \alpha^{n-1}, -1, -\alpha, \ldots, -\alpha^{m-1}) \qquad (A.16)$$

Since the unconstrained estimates do not in general satisfy this equation, there would be

$$(\beta_1^{*T}, \beta_2^{*T}) \cdot Q = S \tag{A.17}$$

Therefore, β_1^* and β_2^* should be replaced by $\hat{\beta}_1$ and $\hat{\beta}_2$ such that

$$f_1(\alpha,\hat{\beta}_1) = f_2(\alpha,\hat{\beta}_2) \tag{A.18}$$

Substituting Eq. (A.11) and (A.12) into Eq. (A.17) yields

$$\begin{bmatrix} \hat{\beta}_1 \\ \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} \beta_1^* \\ \\ \beta_2^* \end{bmatrix} - \frac{S}{t} C^{-1}Q \tag{A.19}$$

where

$$C^{-1} = \begin{bmatrix} C_1^{-1} & 0 \\ & \\ 0 & C_2^{-1} \end{bmatrix} \text{ and } \quad t = Q^T C^{-1} Q \tag{A.20}$$

Appendix B: REFERENCES

[1] Beck, J. L., Determining Models of Structures from Earthquake Records, EERRL 78-01, Caltech, Pasadena, 1978.

[2] Bekey, G. A., "System Identification - An Introduction and a Survey", Simulation, October 1970, pp. 151-166.

[3] Chatfield, C., The Analysis of Time Series: Theory and Practice, Chapman and Hall (London) 1978, pp. 41-44 and 85-89.

[4] Chen, S. J. Hong, Methods of System Identification in Structural Engineering, Thesis, School of Civil Engineering, Purdue University, West Lafayette, IN, August 1976.

[5] Chen, S. J. Hong, and Yao, J. T. P., "Identification of Structural Damage Using Earthquake Response Data", Proceeding, Third Engineering Mechanics Conference, September 17-19, 1979, The University of Texas at Austin, Austin, Texas, pp. 661-664.

[6] Chen, S. J. Hong, and Yao, J. T. P., Data Analyses For Damage Assessment of Existing Structures, Report No. CE-STR-80-18, School of Civil Engineering, Purdue University, West Lafayette, IN, December 1980.

[7] Clough, R. W. and Penzien, J., Dynamics of Structures, McGraw-Hill, 1975, pp. 124-128.

[8] Collins, J. D., Young, J. P., and Kiefling, L., "Methods of Application of System Identification in Shock and Vibration", System Identification of Vibrating Structures; Mathematical Models from Test Data, D. Pilkey and R. Cohen, eds., ASME, 1972, pp. 75-71.

[9] Gersch, W., Nielsen, N. N. and Akaike, H., "Maximum Likelihood Estimation of Structural Parameters from Random Vibration Data", Journal of Sound and Vibration Data, v. 311, n. 3, 1973, pp. 295-308.

[10] Hall, B. M., Calkin, E. D., and Sholar, M. S., "Linear Estimation of Structural Parameters from Dynamic Test Data", AIAA/ASME 11th Structures Structural Dynamic and Materials Conference, April 1970, pp. 193-197.

[11] Hanks, T. C., Current Assessment of Long Period Errors, In Strong Motion Earthquake Accelerograms, Part G, II. Report No. 73-52, California Institute of Technology, Pasadena, California, 1973.

[12] Hart, G. C. and Yao, J. T. P., "System Identification in Structural Dynamics", Journal of the Engineering Mechanics Division, ASCE, v. 103, n. EM6, December 1977, pp. 1089-1104.

[13] Hays, C., "Inelastic Materials Models in Earthquake Response", Proceedings 3rd Engineering Mechanics Specialty Conference, September 17-19, 1979, pp. 634-637.

[14] Hudson, J. D., "Fitting Segmented Surveys Whose Join Points Have to be Eliminated", Journal of American Statistical Association, Vol. 61, 1966, pp. 1097-1129.

[15] Ibanez, Paul, Review of Analytical and Experimental Techniques for Improving Structural Dynamic Models, Report sponsored by the Subcommittee on Dynamic Analysis of Pressure Components of the Pressure Vessel Research Committee on the Welding Research Council, June 1979.

[16] Liu, S. C., and Yao, J. T. P., "Structural Identification Concept", Journal of the Structural Division, ASCE, v. 104, n. ST12, December 1978, pp. 1845-1858.

[17] Martin, A. C. and Associates, "Union Bank Square, in San Fernando, California, Earthquake of February 9, 1971", L. M. Murphy (ed.), v. 1, part B, pp. 575-596, U. S. Dept. of Commerce, Washington, D.C.

[18] Masri, S. F. and Caughey, T. K., "A Nonparametric Identification Technique for Non-Linear Dynamic Problems", Journal of Applied Mechanics, ASME, Vol. 46, June 1979, pp. 433-447.

[19] Matzen, V. C. and McNiven, H. D., Investigation of the Inelastic Characteristics of a Single Story Steel Structure Using System Identification and Shaking Table Experiments, Report No. ERR 76-20, August 1976, Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, California.

[20] Mehra, R. K., Stepener, D. E. and Tyler, J. S., "Maximum Likelihood Identification of Aircraft Stability and Control Derivatives", Journal of Aircraft, Vol. 11, No. 2, February 1974, pp. 81-89.

[21] Pilkey, W. D., and Kalinawski, J., "Identification of Shock and Vibration Forces", System Identification of Vibrating Structure, Mathematical Models from Test Data, D. Pilkey and R. Cohen, eds., ASME, 1972, pp. 73-86.

[22] Raney, J. P., "Identification of Complex Structures Using Near-Response Testing", The Shock and Vibration Bulletin, No. 38, Part 2, August 1968, pp. 23-32.

[23] Rodeman, R. and Yao, J. T. P., Structural Identification - Literature Review, Report No. CE-STR-73-3, Civil Engineering, Purdue University, West Lafayette, IN, December 1973.

[24] Sage, A. P., "System Identification - History, Methodology, Future Prospects", System Identification of Vibrating Structures; Mathematical Models from Test Data, D. Pilkey & R. Cohen, eds., ASME, 1972, pp. 1-22.

[25] Sage, A. P., and Melsa, J. L., System Identification, Academic Press, Inc., New York, 1971.

[26] Schiff, A. J., "Identification of Large Structures Using Data from Ambient and Low Level Excitations", System Identification of Vibrating Structures: Mathematical Models from Test Data, D. Pilkey and R. Cohen eds., ASME, 1972, pp. 87-115.

[27] Sozen, M. A., "Review of Earthquake Response of R/C Buildings with a View to Drift Control", Private Communications, 11 August 1980.

[28] Sweet, A. L., Schiff, A. J. and Kelley, J. W., "Identification of Structural Parameters Using Low-Amplitude Impulsive Loading", Journal of Acoustical Society of America, Vol. 57, No. 5, May 1975, pp. 1128-1137.

[29] Udwadia, F. S. and Kuo, Chin-Po, "Non-Parametric Identification of a Class of Nonlinear Close-Coupled Dynamic System", University of Southern California, Los Angeles, CA.

[30] Udwadia, F. E. and Shah, P. C., "Identification of Structures Through Records Obtained During Strong Earthquake Ground Motion", Journal of Engineering for Industry, ASME, Vol. 98, No. 4, 1976, pp. 1347-1362.

[31] Wiener, N., Nonlinear Problems in Random Theory, Cambridge, MA: M.I.T. Press, 1958.

[32] Williams, E. J., Regression Analysis, New York: John Wiley and Sons, 1959.

[33] Yao, J. T. P., "Assessment of Seismic Damage in Existing Structures", Proceedings, U.S.-S.E. Asia Joint Symposium on Engineering for Natural Hazard Protection, Manila, Philippines, 28 September 1977, A. H-S. Ang, Ed., University of Illinois, Urbana, IL, pp.

[34] Yao, J. T. P., "Damage Assessment and Reliability Evaluation of Existing Structures", Engineering Structures, v. 1, n. 5, October 1979, pp. 245-251.

[35] Yao, J. T. P., Reliability of Existing Buildings in Earthquake Zones - Final Report, Technical Report No. CE-STR-79-6, School of Civil Engineering, Purdue University, West Lafayette, IN, December 1979.

[36] Yao, J. T. P., and Munse, W. H., "Low-Cycle Axial Fatigue Behavior of Mild Steel", Special Technical Publication, No. 338, ASTM, 1962, pp. 5-24.

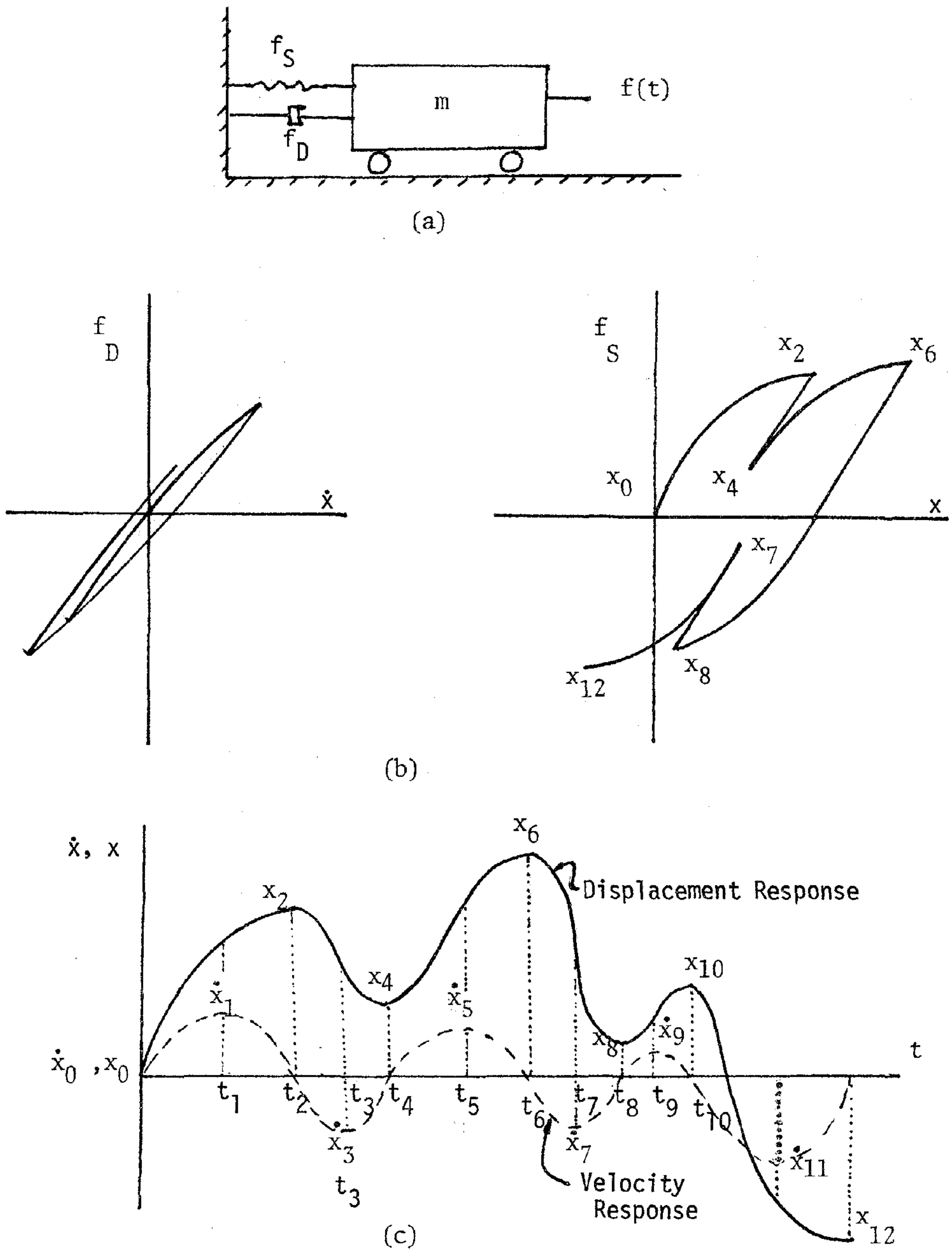


Figure 1: System model

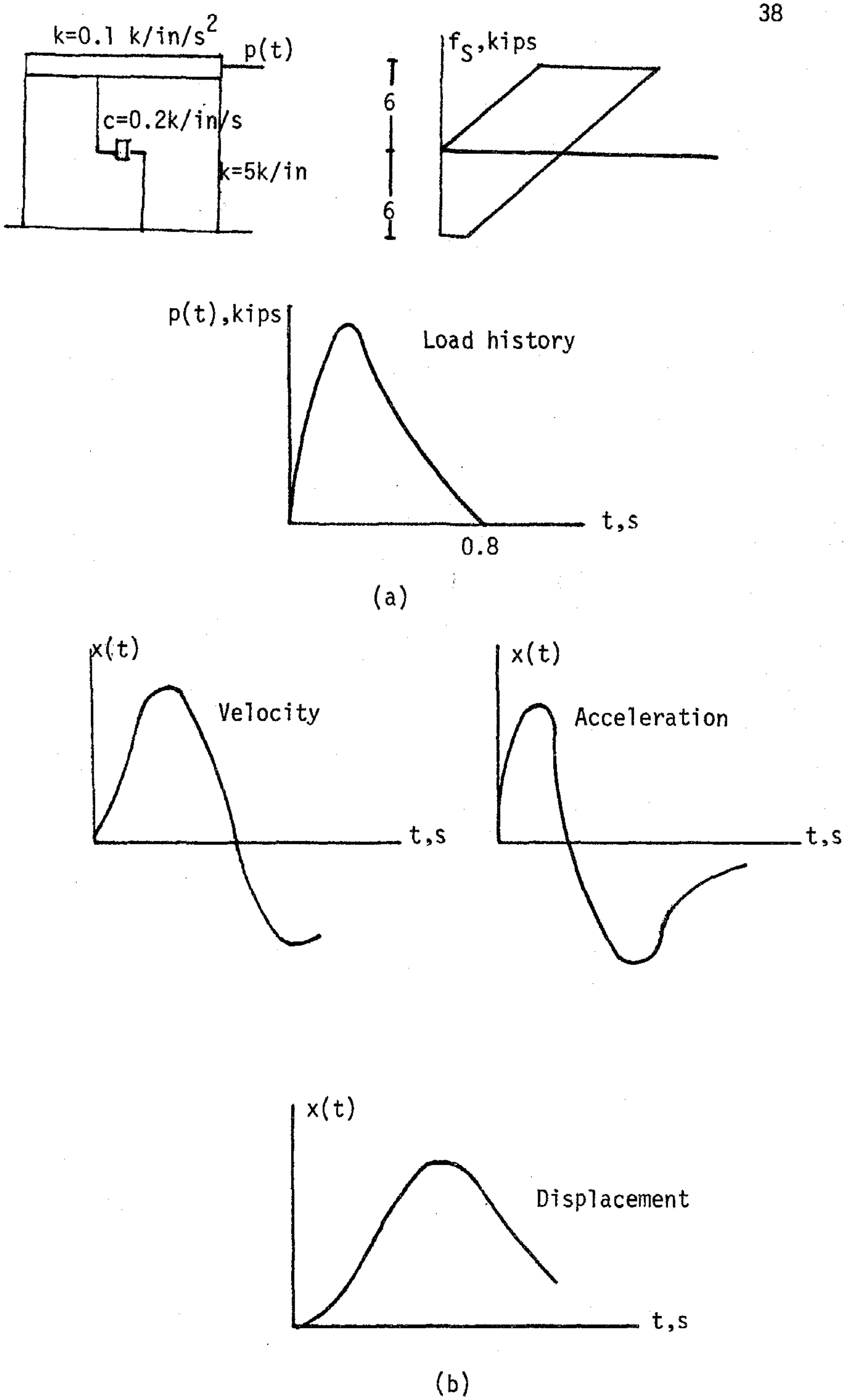
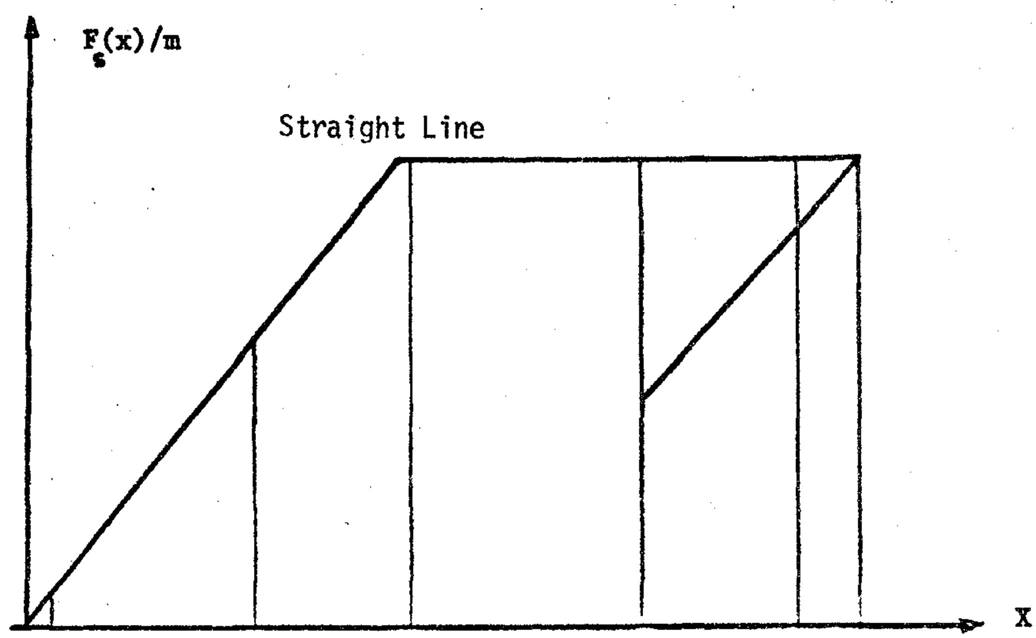
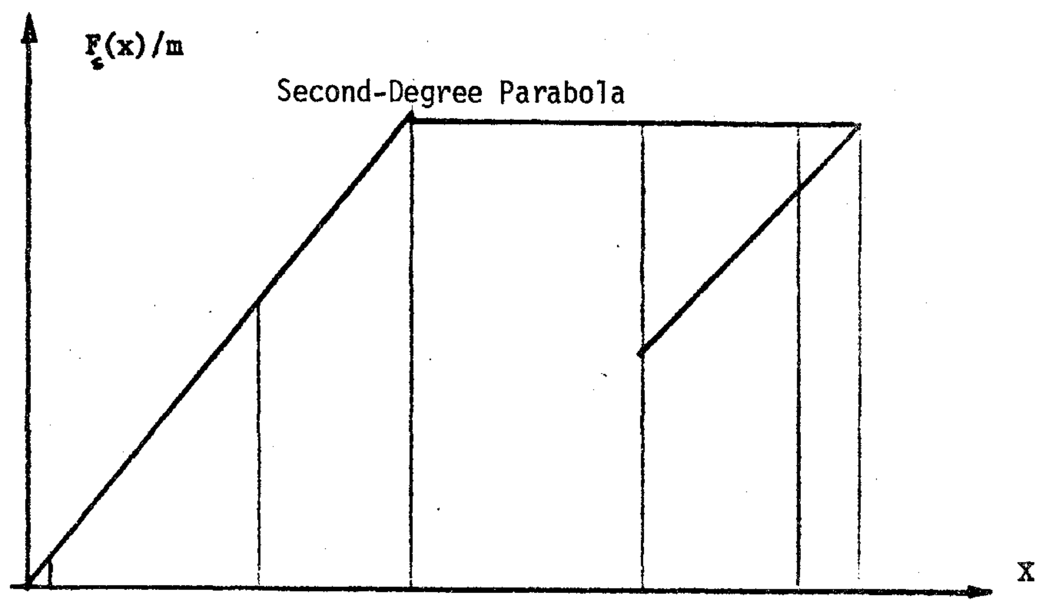


Figure 2: Example on Page 11



(a) Actual Behavior of Structure



(b) EStimated Behavior of Structure

Figure 3: Estimated behavior of Example frame on Page 11

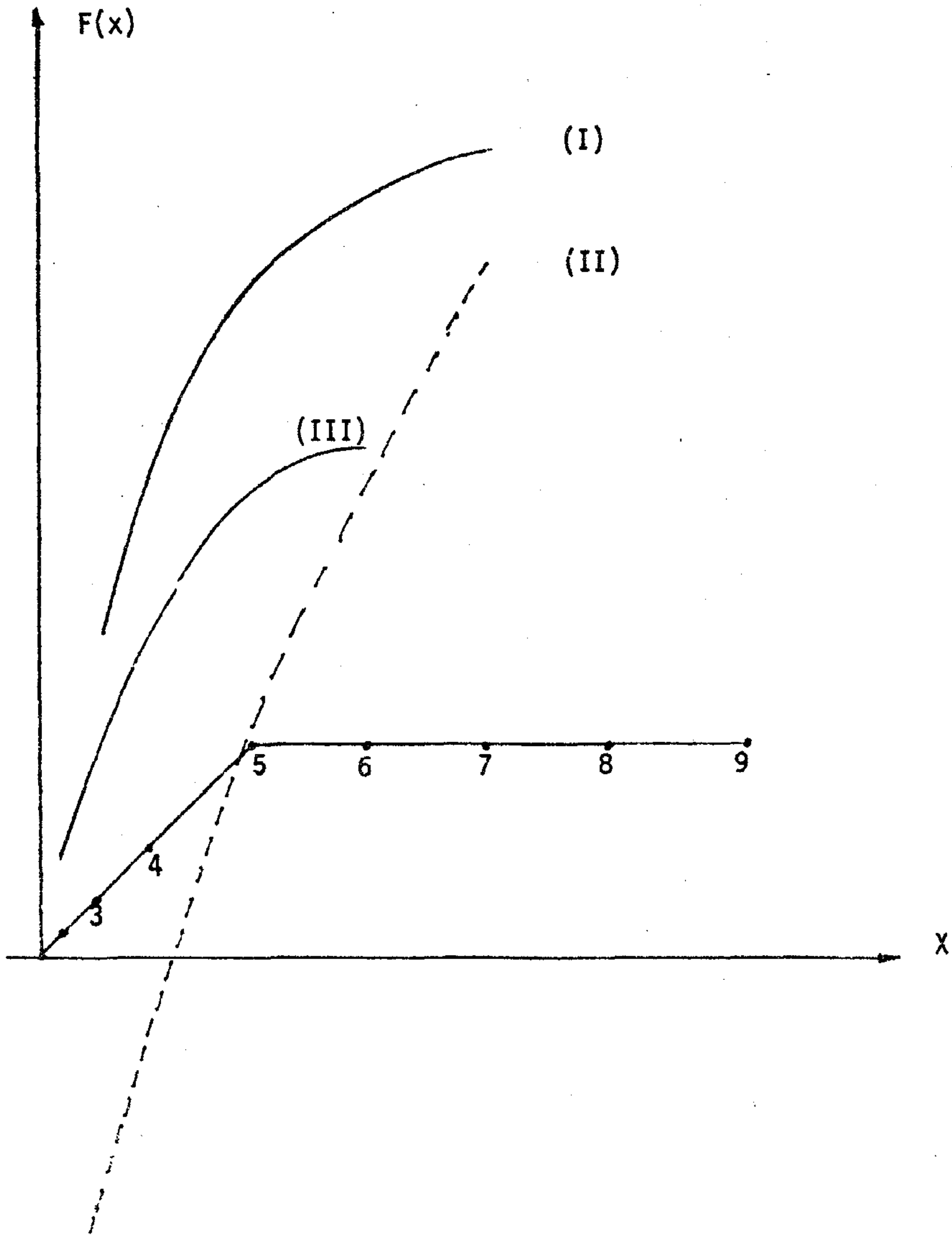


Figure 4: Slope-change problem

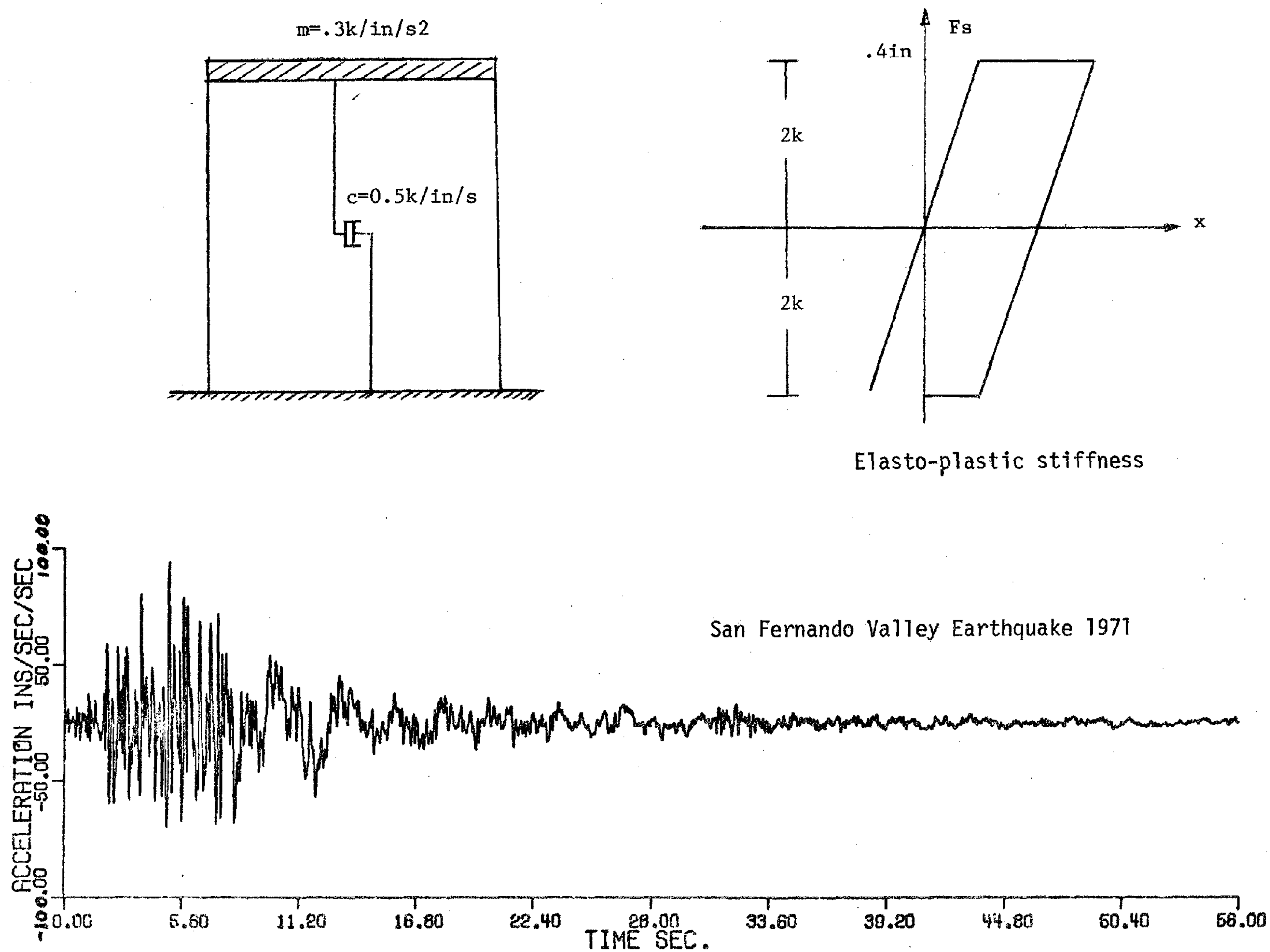


Figure 5: Elastic-plastic Frame and Dynamic loading

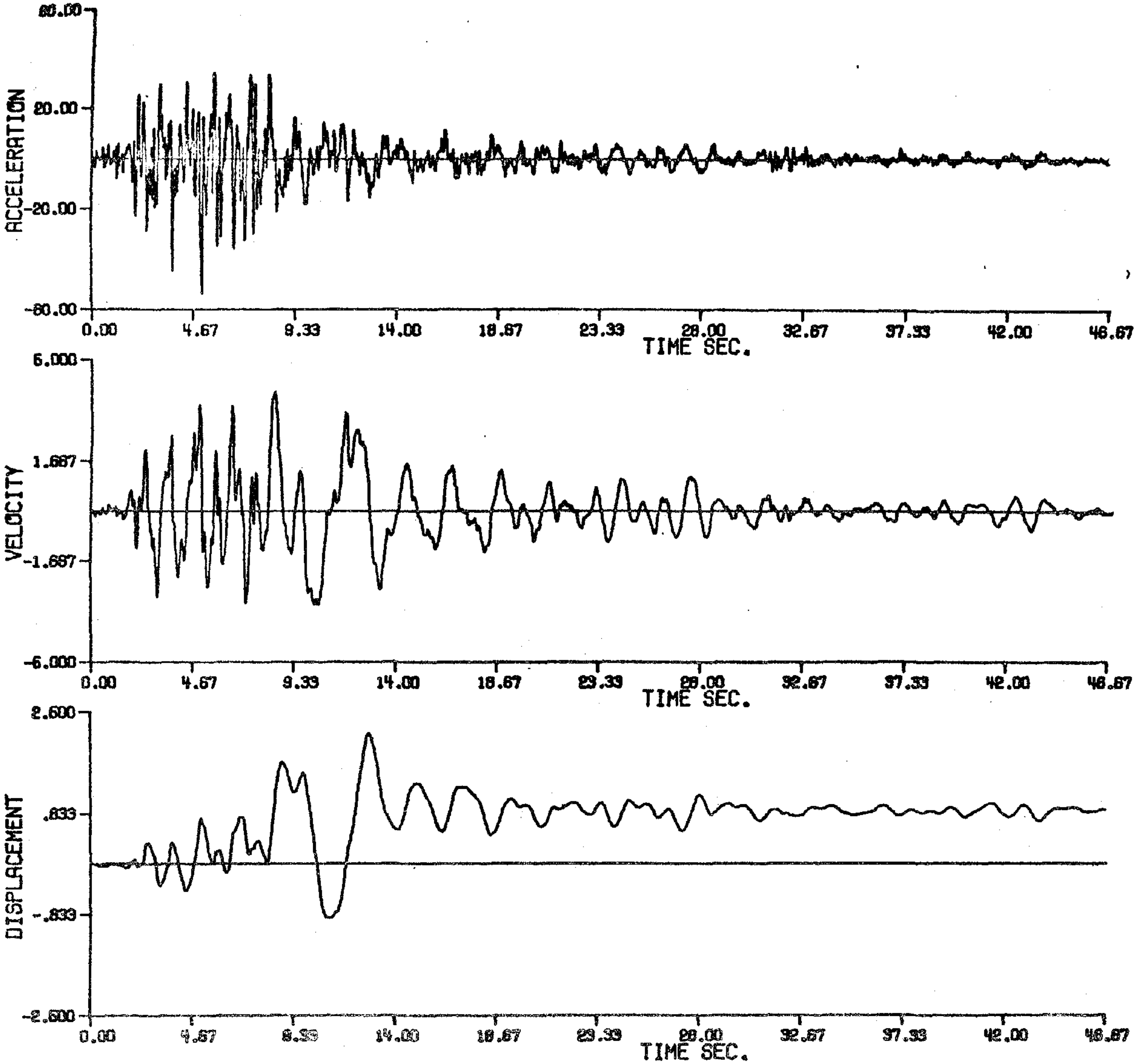


Figure 6: Structure response (0 per-cent noise)

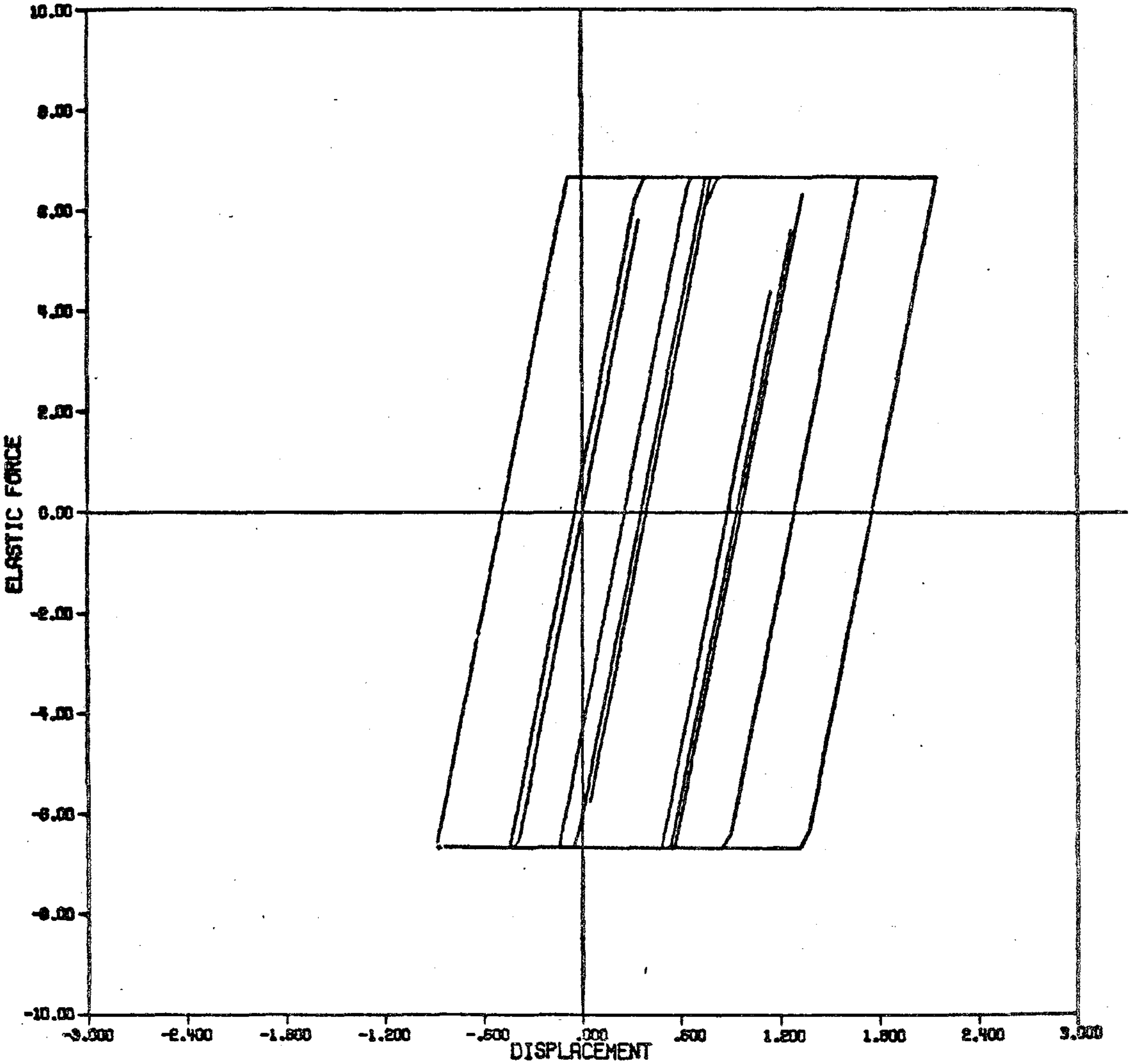


Figure 7: Estimated behavior of Structure (0 per-cent noise)

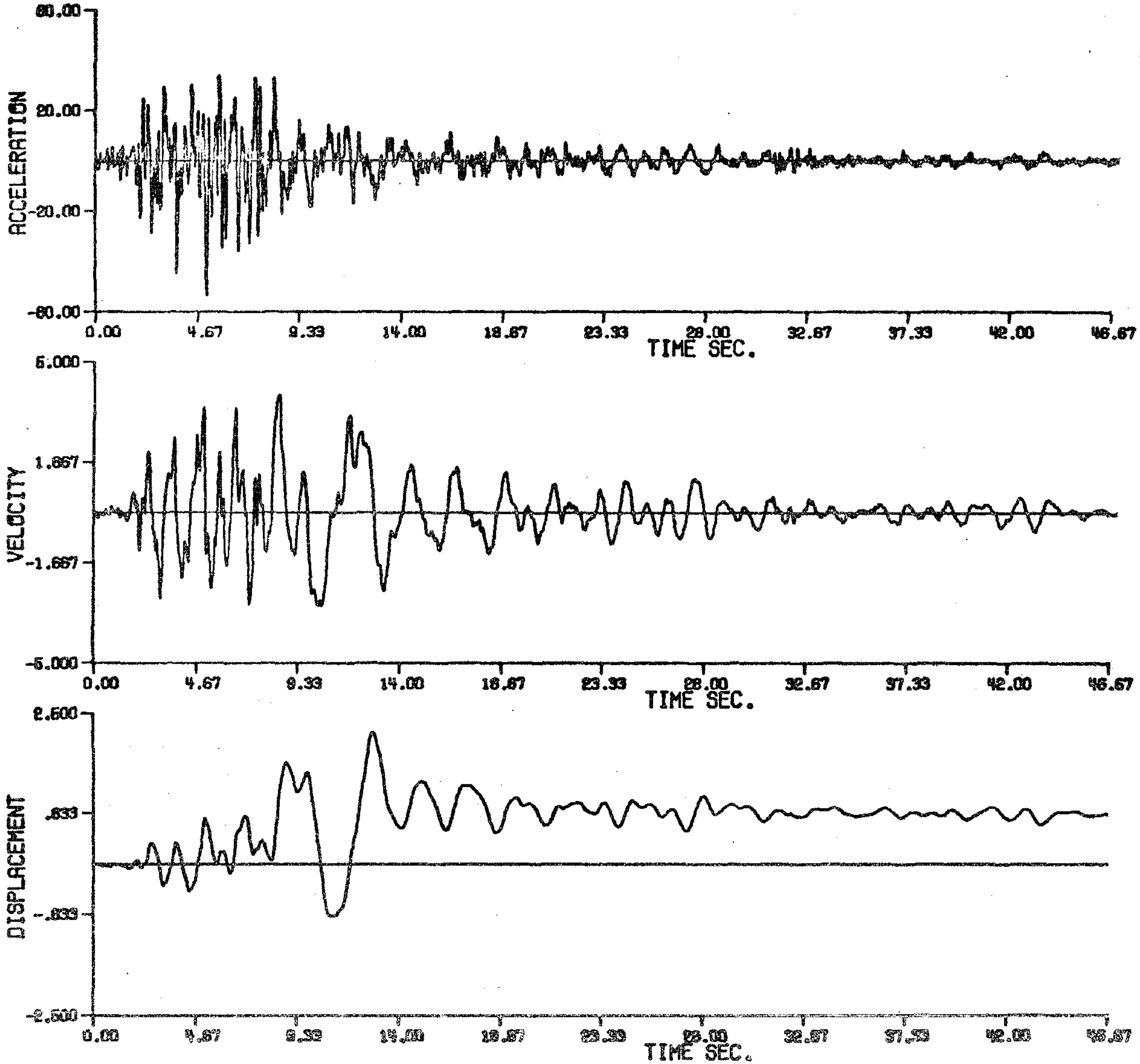


Figure 8: Structure response (1 per-cent noise)

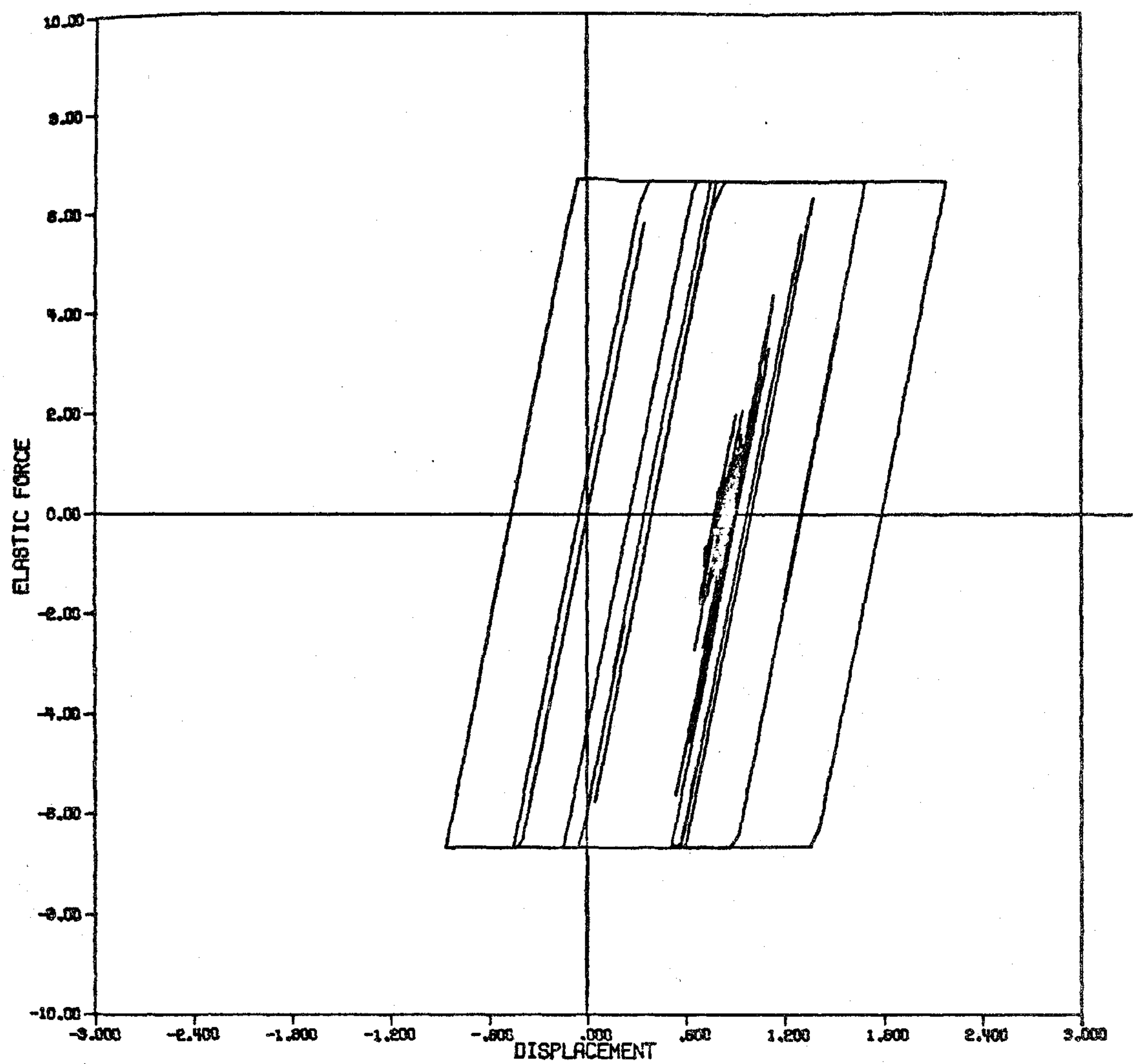


Figure 9: EStimated behavior of Structure (1 per-cent noise)

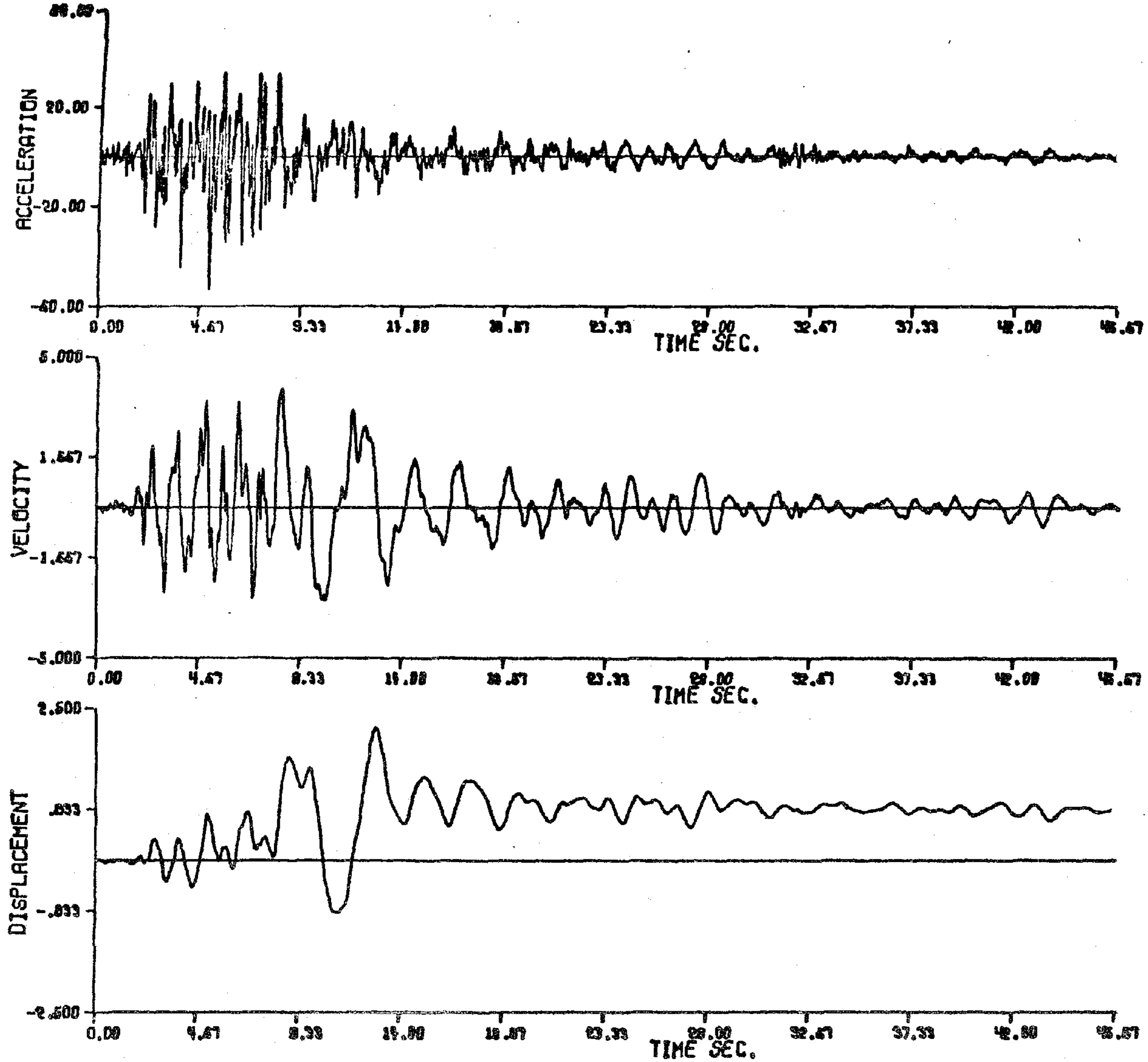


Figure 10: Structure response (10 per-cent noise)

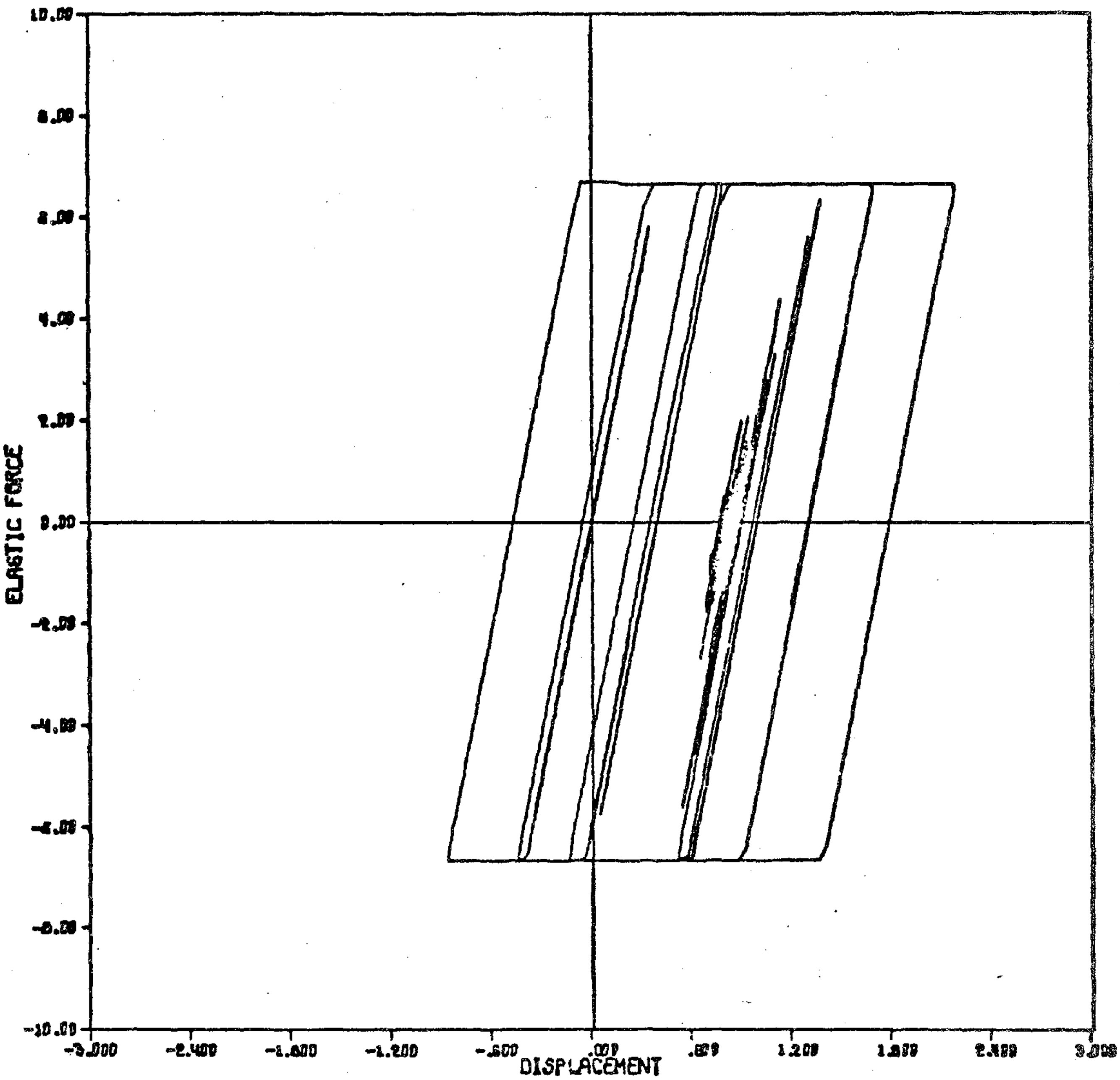


Figure 11: Estimated behavior of Structure (10 per-cent noise)

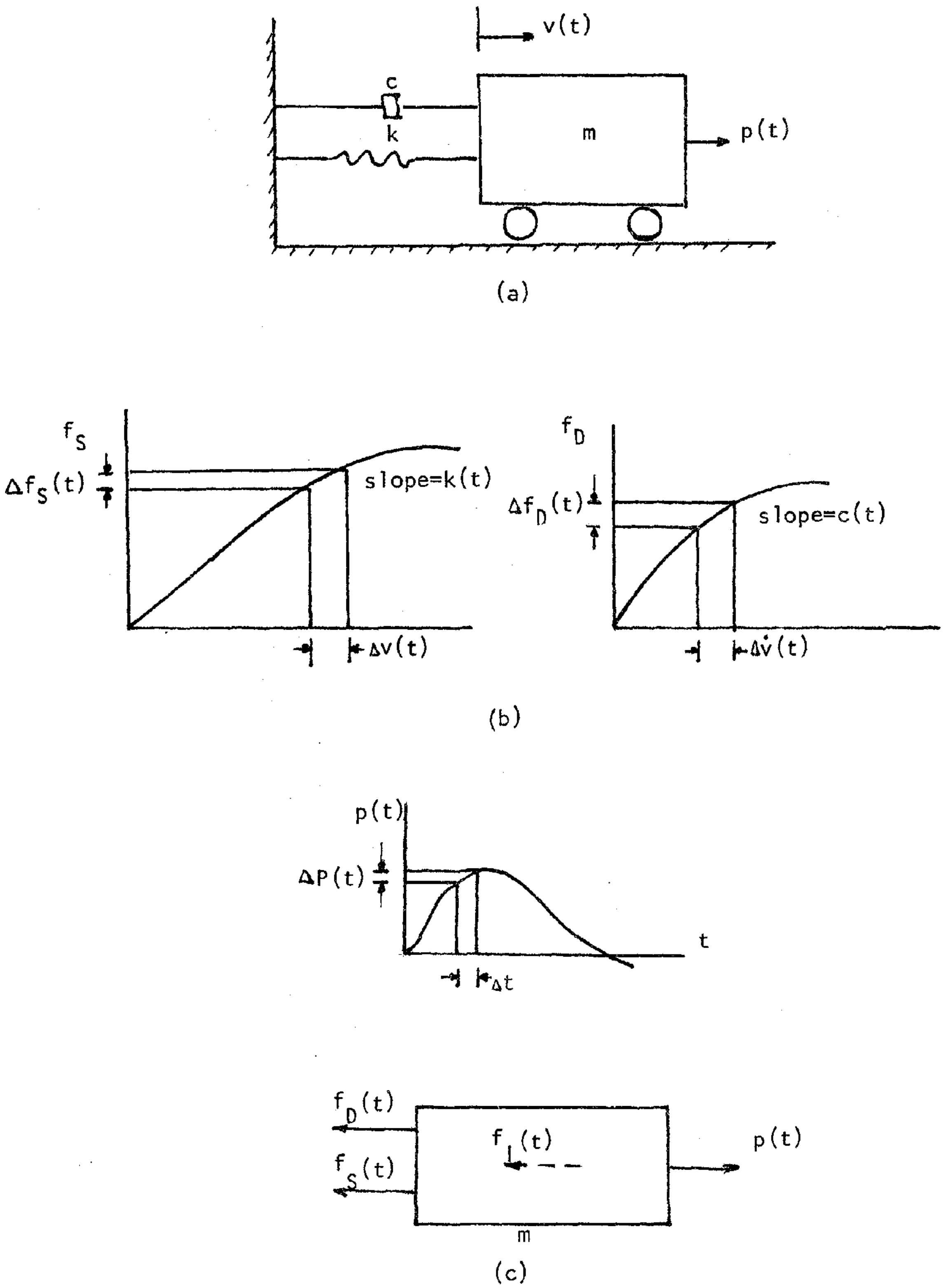


Figure 12:Non-linear dynamic system

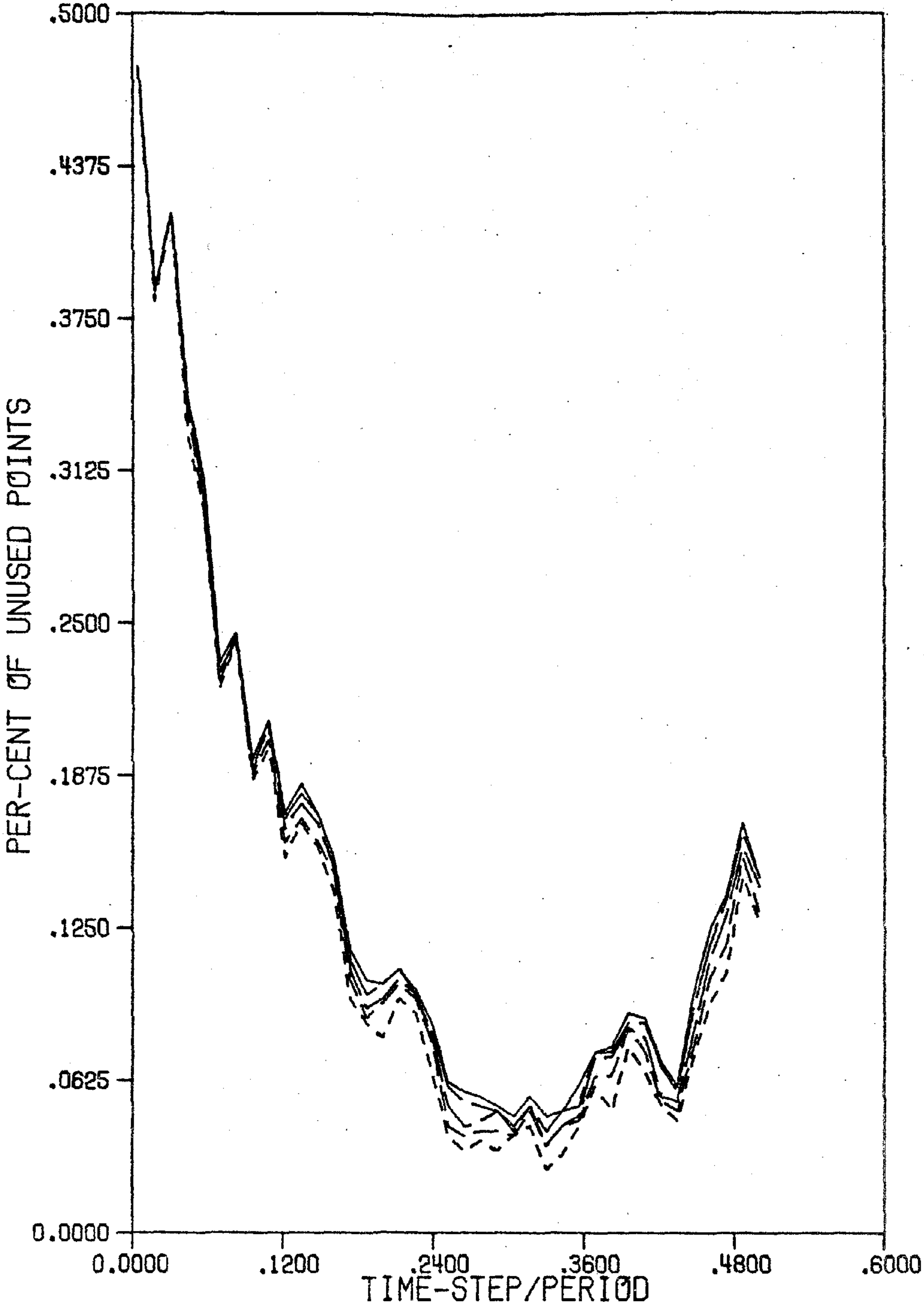


Figure 13: Effect of increasing the time increment on the reduction of unused points (linear system)

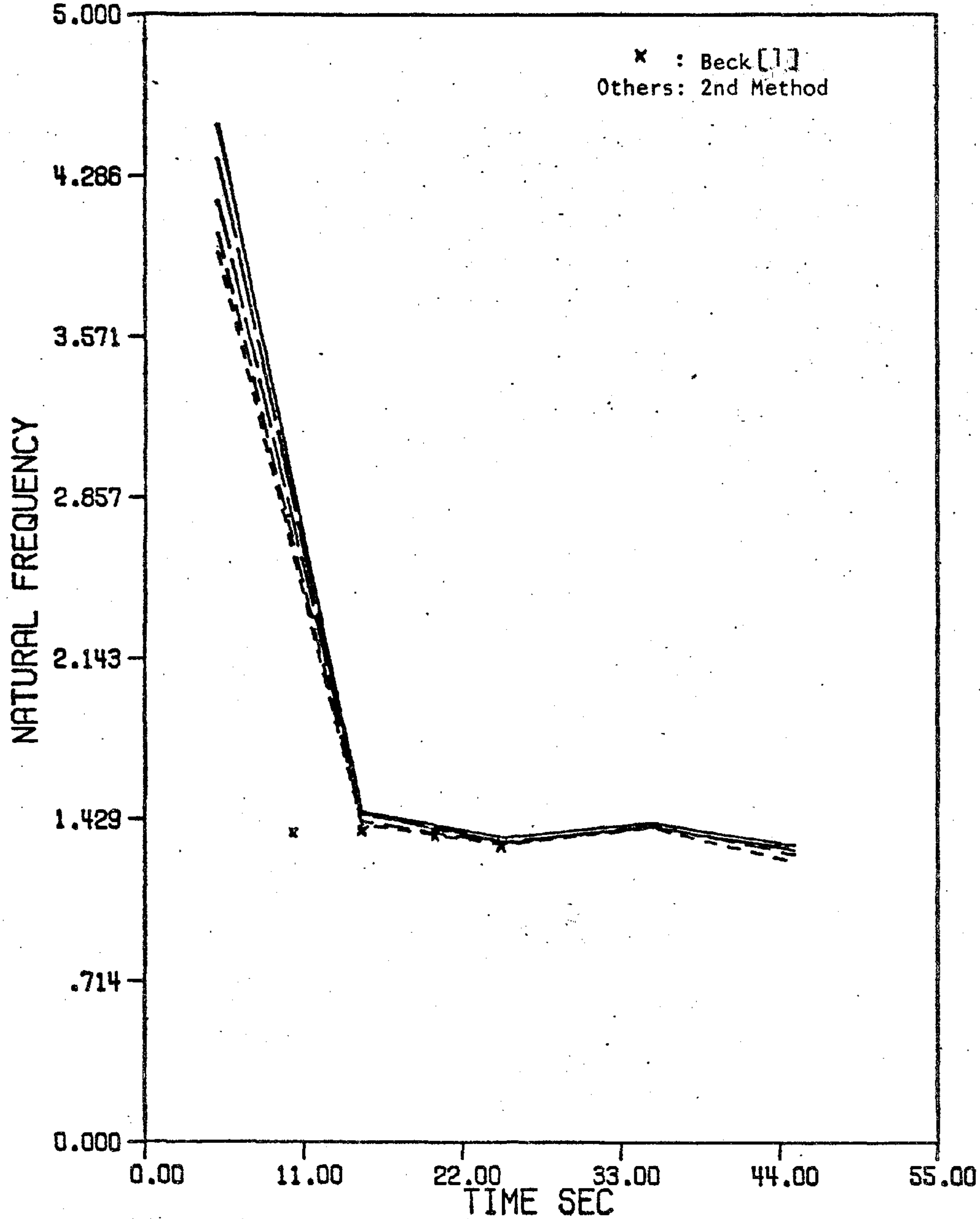


Figure 14: Estimation of Union-bank building's natural frequency

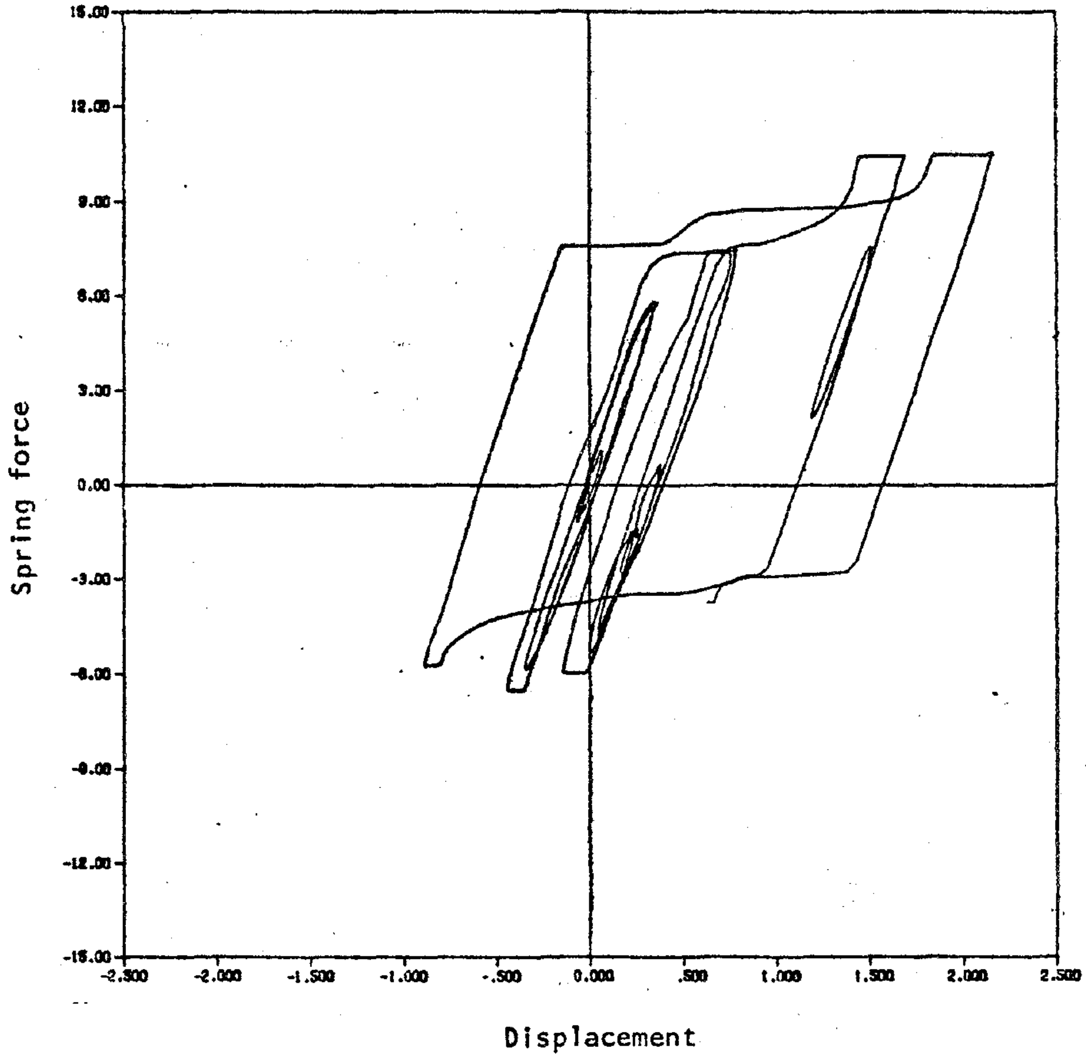


Figure 15: Estimated behavior of Example 4-2 frame

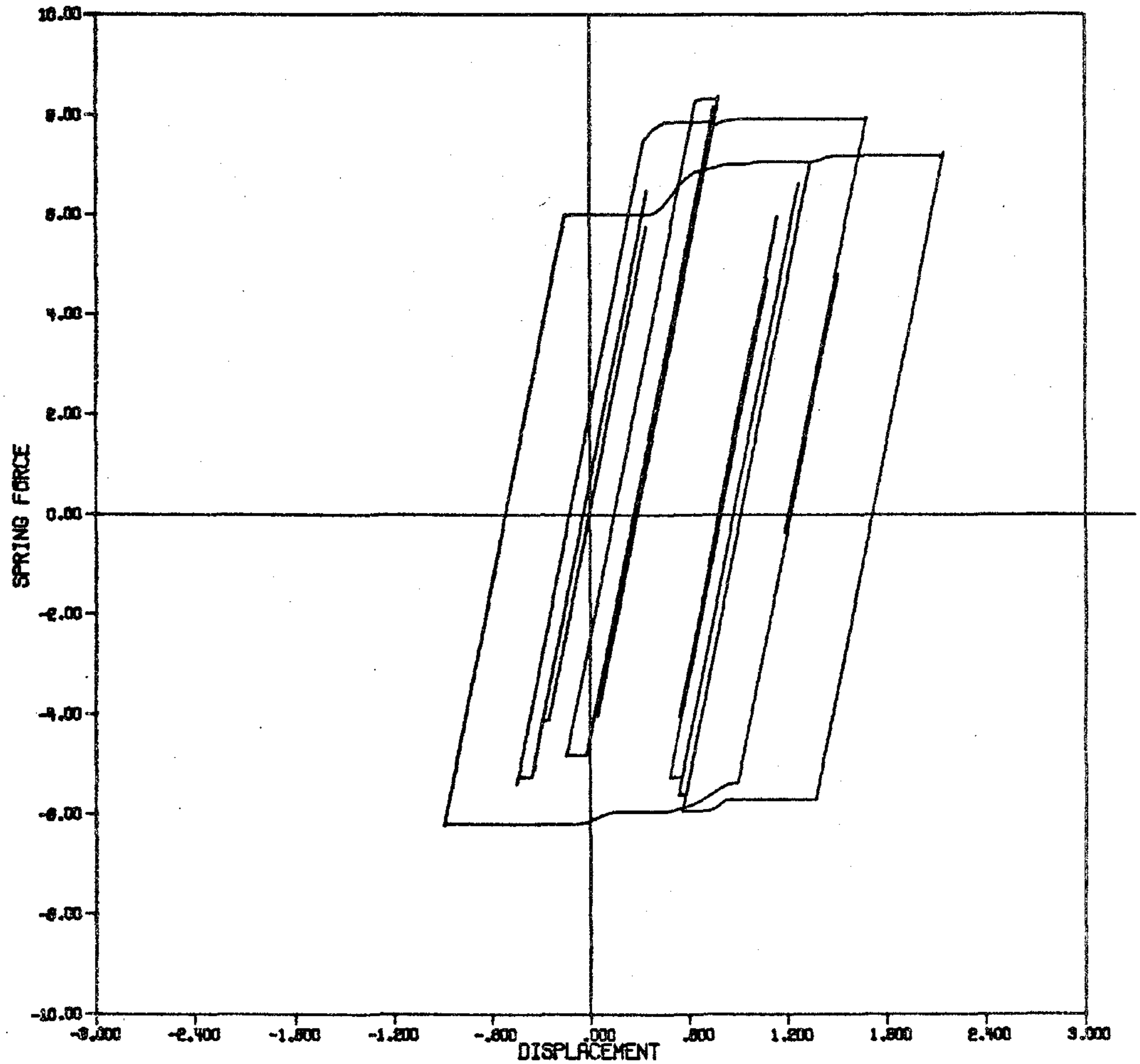


Figure 16 :Estimated behavior of Example 4-2 frame(using a prior knoledge)

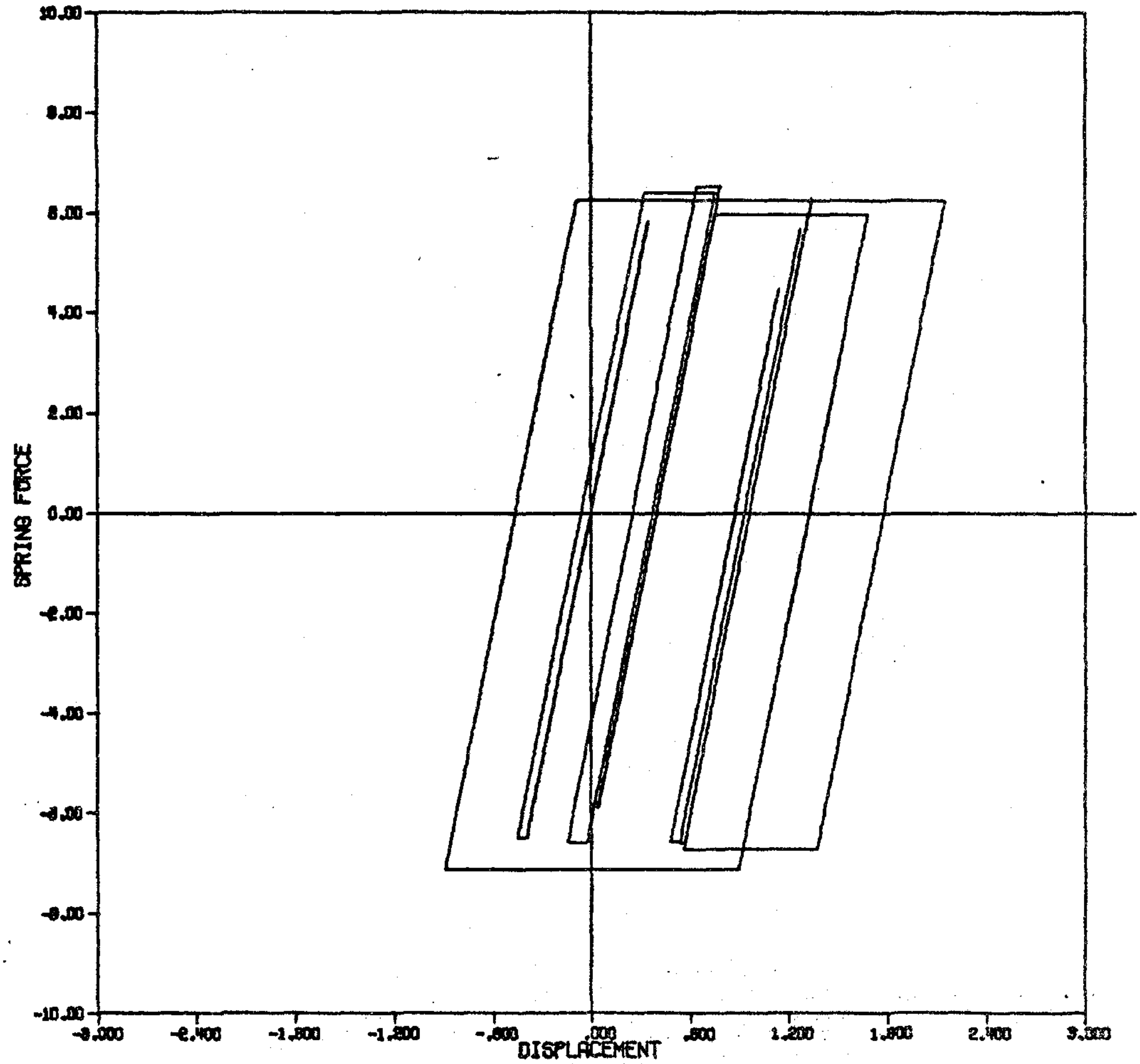


Figure 17: Estimated behavior of Example 4-3 frame

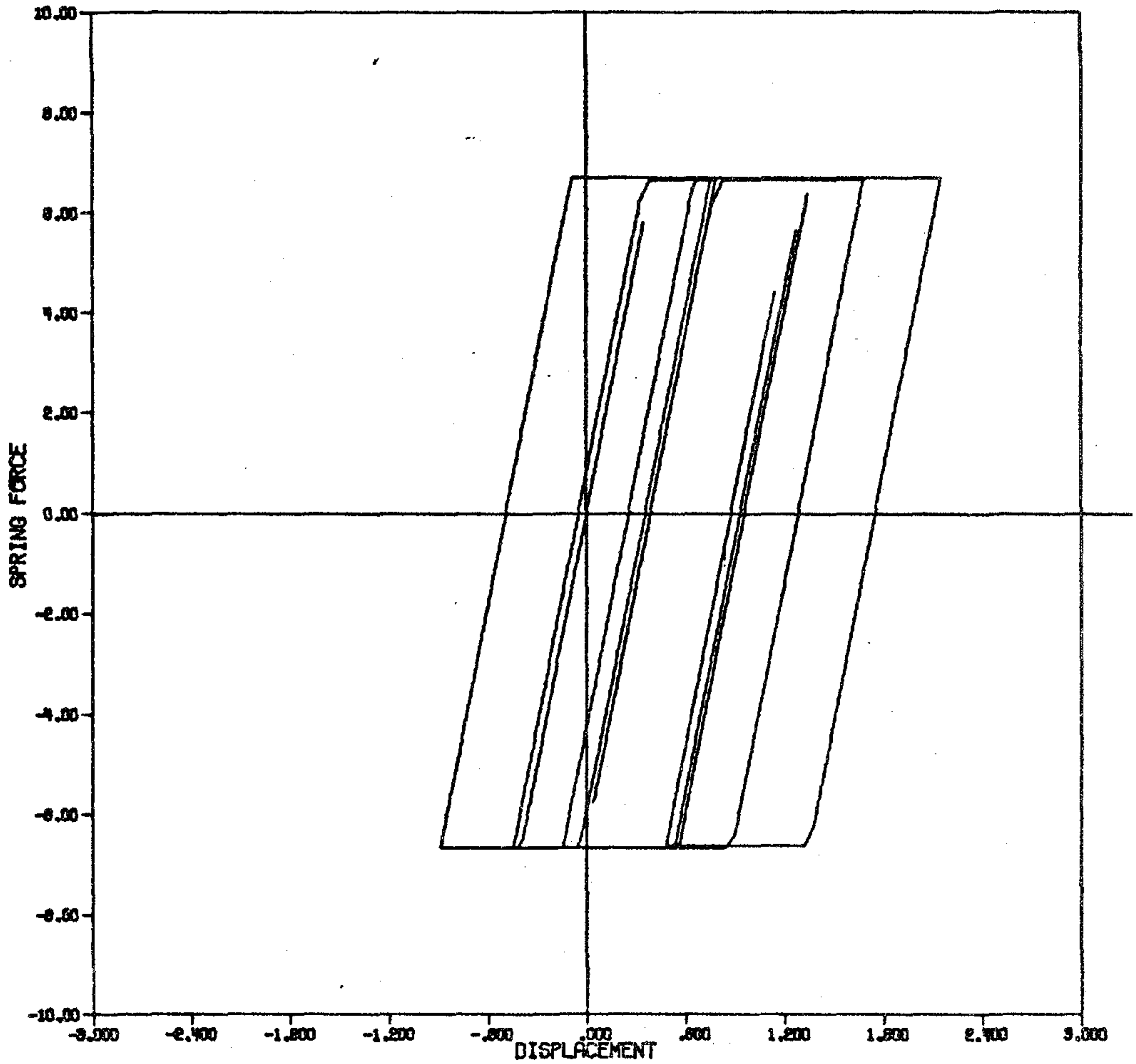


Figure 18: Estimated behavior of Example 4-2 frame after applying the solution of the present-to-past dependence problem

PURDUE UNIVERSITY

School of Civil Engineering

Structural Engineering Technical Reports

(Since 1973)

CE-STR-73-1 "Active Control of Civil Engineering Structures", by J. T. P. Yao and J. P. Tang (PB-222594)

CE-STR-73-2 "Probabilistic Analysis of Elasto-Plastic Structures", by T. L. Paez and J. T. P. Yao (PB-223328).

CE-STR-73-3 "Structural Identification - Literature Review", by R. Rodeman and J. T. P. Yao.

CE-STR-74-1 "Application of Decision Theory in Structural Engineering", by I. H. Chou and J. T. P. Yao.

CE-STR-74-2 "Formulation of Structural Control", by J. N. Yang and J. T. P. Yao (PB-238065/AS).

CE-STR-75-1 "Fatigue Behavior of Tall Buildings - Literature Review", by R. Yu, A. R. Rao and J. T. P. Yao.

CE-STR-75-2 "Active Control of Building Structures Subjected to Wind Loads", by S. Sae-Ung and J. T. P. Yao (PB-252414).

CE-STR-76-1 "Active Control of Building Structures", by S. Sae-Ung and J. T. P. Yao.

CE-STR-78-1 "System Identification, Damage Assessment, and Reliability Evaluation of Structures", by E. C. Ting, S. J. Hong Chen, and J. T. P. Yao.

CE-STR-78-2 "Analysis of Concrete Cylinder Structures Under Hydrostatic Loading", by W. F. Chen, H. Suzuki, and T. Y. Chang.

CE-STR-78-3 "Tests of Fabricated Tubular Columns", by W. F. Chen and D. A. Ross.

CE-STR-79-1 "Effect of External Pressure on the Axial Capacity of Fabricated Tubular Columns", by W. F. Chen and S. Toma.

CE-STR-79-2 "Influence of End Restraint on Column Stability", by W. F. Chen.

CE-STR-79-3 "Double-Punch Test for Tensile Strength of Concrete", by W. F. Chen and R. L. Yuan.

CE-STR-79-4 "An Approach to Damage Assessment of Existing Structures", by J. T. P. Yao (PB-80123482).

CE-STR-79-5 "Fixed Bottom-Supported Concrete Platforms", by W. J. Graff and W. F. Chen.

CE-STR-79-6 "Reliability of Existing Buildings in Earthquake Zones, Final Report", by J. T. P. Yao (PB-159692).

CE-STR-80-1 "Elastic-Plastic Behavior of Beam-Columns and Struts", by W. F. Chen and S. Toma

CE-STR-80-2 "Static Behavior of Beam-to-Column Moment Connections", by W. F. Chen and K. V. Patel.

CE-STR-80-3 "Tests of Beam-to-Column Web Connection Details", by G. P. Rentschler, W. F. Chen, and G. C. Driscoll.

CE-STR-80-4 "Elastic-Plastic Large Displacement Analysis of Pipes", by A. F. Saleeb and W. F. Chen.

CE-STR-80-5 "A Structural Analysis Program for Reinforced Concrete Columns Under Biaxial Bending", by S. Al-Noury and W. F. Chen.

CE-STR-80-6 "Reliability Considerations for Fatigue Analysis and Design of Structures", by J. T. P. Yao.

CE-STR-80-7 "System Identification in Earthquake Engineering", by J. T. P. Yao and A. J. Schiff.

CE-STR-80-8 "A Rule-Inference Method for Damage Assessment of Existing Structures", by M. Ishizuka, K. S. Fu, and J. T. P. Yao.

CE-STR-80-9 "Nonlinear Hyperelastic (Green) Constitutive Models for Soils", by A. F. Saleeb and W. F. Chen.

CE-STR-80-10 "Performance of Low-Rise Buildings -- Existing and New", by J. T. P. Yao.

CE-STR-80-11 "Application of Fuzzy Sets in Earthquake Engineering", by K. S. Fu and J. T. P. Yao.

CE-STR-80-12 "Plasticity Models for Soils", by E. Mizuno and W. F. Chen.

CE-STR-80-13 "Effect of Human Errors to Structural Reliability", by B. Randich and J. T. P. Yao.

CE-STR-80-14 "A Plastic-Fracture Model for Concrete - Part I: Theory", by S. Hsieh, E. C. Ting and W. F. Chen.

CE-STR-80-15 "Analysis of Soil Response with Different Plasticity Models", by E. Mizuno and W. F. Chen.

CE-STR-80-16 "Upper Bound Limit Analysis of the Stability of a Seismic-Infirmed Earthslope", by S. W. Chan, S. L. Koh, and W. F. Chen.

CE-STR-80-17 "Inference Method for Damage Assessment System of Existing Structures", by M. Ishizuka, K. S. Fu and J. T. P. Yao.

CE-STR-80-18 "Data Analyses for Damage Assessment of Existing Structures", S. J. Hong Chen and J. T. P. Yao.