PB81-218497

REPORT NO.
UCB/EERC-81/05
APRIL 1981

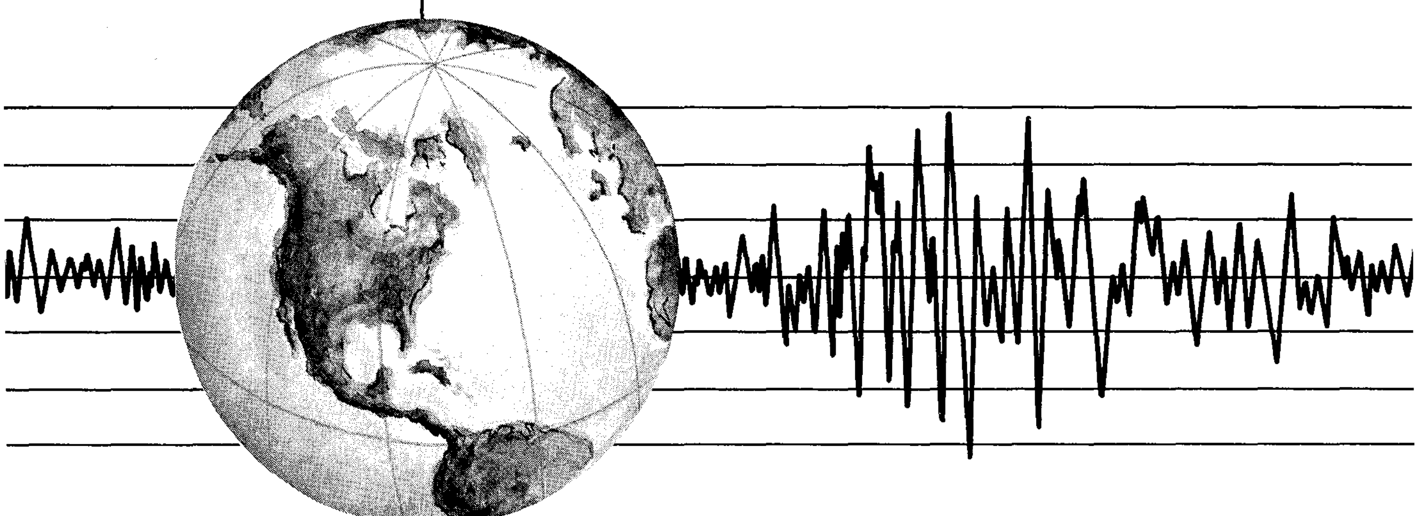
EARTHQUAKE ENGINEERING RESEARCH CENTER

DYNAMIC RESPONSE OF LIGHT EQUIPMENT IN STRUCTURES

by

ARMEN DER KIUREGHIAN
JEROME L. SACKMAN
BAHRAM NOUR-OMID

Report to the National Science Foundation



COLLEGE OF ENGINEERING
UNIVERSITY OF CALIFORNIA · Berkeley, California

For sale by the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22161.

See back of report for up to date listing of EERC reports.

DISCLAIMER

Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation or the Earthquake Engineering Research Center, University of California, Berkeley

50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO. NSF/ENG-81005	2.	3. Recipient's Accession No. PB81 218497

4. Title and Subtitle Dynamic Response of Light Equipment in Structures	5. Report Date April 1981
	6.
7. Author(s) A. Der Kiureghian, J.L. Sackman and B. Nour-Omid	8. Performing Organization Rept. No. UCB/EERC-81/05
9. Performing Organization Name and Address Earthquake Engineering Research Center University of California, Berkeley 47th Street and Hoffman Blvd. Richmond, California 94804	10. Project/Task/Work Unit No. 11. Contract(C) or Grant(G) No. (C) (G) CME-7923651 ENG-7905906
12. Sponsoring Organization Name and Address National Science Foundation 1800 G Street, N.W. Washington, D. C. 20550	13. Type of Report & Period Covered 14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

Perturbation methods are employed to determine the dynamic properties of a combined system composed of a multi-degree-of-freedom structure to which is attached a light, single-degree-of-freedom equipment item. Closed form expressions are derived for the natural frequencies, mode shapes, modal dampings and other modal properties of the combined system in terms of the dynamic properties of the structure alone and the equipment alone. The derived properties are used to determine the response of the equipment when the structure is subjected to a random input excitation. For earthquake-type excitations, a response spectrum method is developed whereby various statistical measures of the equipment response are given directly in terms of the input ground response spectrum. Tuning and equipment-structure interaction are included. The method efficiently generates floor spectra which include the effect of interaction. The conventional floor spectra, which ignore interaction, are obtained simply by setting the equipment mass equal to zero. For an example structure, the results based on the method developed demonstrate remarkable agreement with a numerical analysis employing a set of 20 simulated ground motions.

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
Release Unlimited	20. Security Class (This Page)	22. Price

(See ANSI–Z39.18) — See Instructions on Reverse — OPTIONAL FORM 272 (4–77) (Formerly NTIS–35) Department of Commerce

NOTICE

THIS DOCUMENT HAS BEEN REPRODUCED FROM THE BEST COPY FURNISHED US BY THE SPONSORING AGENCY. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE.

DYNAMIC RESPONSE OF LIGHT EQUIPMENT IN STRUCTURES

By

Armen Der Kiureghian
Assistant Professor of Civil Engineering
University of California, Berkeley

Jerome L. Sackman
Professor of Engineering Science
University of California, Berkeley

Bahram Nour-Omid
Graduate Student in Civil Engineering
University of California, Berkeley

A report on research sponsored by
the National Science Foundation

Report No. UCB/EERC-81/05
Earthquake Engineering Research Center
College of Engineering
University of California, Berkeley

April 1981

ABSTRACT

Perturbation methods are employed to determine the dynamic properties of a combined system composed of a multi-degree-of-freedom structure to which is attached a light, single-degree-of-freedom equipment item. Closed form expressions are derived for the natural frequencies, mode shapes, modal dampings and other modal properties of the combined system in terms of the dynamic properties of the structure alone and the equipment alone. The derived properties are used to determine the response of the equipment when the structure is subjected to a random input excitation. For earthquake-type excitations, a response spectrum method is developed whereby various statistical measures of the equipment response are given directly in terms of the input ground response spectrum. Tuning and equipment-structure interaction are included. The method efficiently generates floor spectra which include the effect of interaction. The conventional floor spectra, which ignore interaction, are obtained simply by setting the equipment mass equal to zero. For an example structure, the results based on the method developed demonstrate remarkable agreement with a numerical analysis employing a set of 20 simulated ground motions.

ACKNOWLEDGMENT

This research work was sponsored by the National Science Foundation under Grants No. CME-7923651 and ENG-7905906. This sponsorship is gratefully acknowledged.

TABLE OF CONTENTS

INTRODUCTION

The dynamic analysis of light equipment in structures is of wide engineering interest. Control equipment, pressure vessels, pumps, etc., in power plants, communication and control items in transportation vehicles, and vital equipment in certain lifeline systems are common examples. In theory, the equipment can be incorporated in a dynamic model of the combined system as a part of the structure and analyzed by a conventional method. However, this approach in practice leads to two difficulties. First, for light equipment, the mass and stiffness matrices of the combined system will have elements with vastly different magnitudes, resulting in numerical difficulties in the dynamic analysis. Second, in design situations, it is often required to study many equipment items attached to a structure or several alternative attachment configurations or locations of a single item. Clearly, such an approach can be prohibitively costly because a multitude of dynamic models of the combined system must be constructed and analyzed. These difficulties can be overcome if, instead of using a conventional approach, an alternative procedure is followed taking advantage of the mismatch between the properties of the light equipment and the structure.

In the first part of this report, closed form expressions are derived for the dynamic properties of the combined equipment-structure system in terms of those of the structure alone and of the equipment alone. These are obtained using a single-degree-of-freedom model for the equipment and a multi-degree-of-freedom one for the structure. This development is based on a perturbation method that takes advantage of the intuitively obvious fact that, for light equipment, the dynamic properties of the combined system are not too different from those of the original structure. The effect of interaction between the equipment and the structure, which is especially significant when the equipment frequency coincides with one of the structure frequencies, is included in this analysis. By this approach, the numerical difficulties alluded to above are avoided and the computational effort becomes trivial.

Once the modal properties of the combined equipment-structure system are determined, any dynamic analysis procedure employing the mode-superposition method, e.g., the time-history integration method for a deterministic input or the random vibration method with a power spectral density or a response spectrum description of the input, can be used to evaluate the response of the equipment. However, in using the random vibration method, a peculiar problem arises when the equipment is light and has a frequency close to one of the structure frequencies. Then, closely spaced modes occur in the combined system, the responses of which are known to be highly correlated (3). The central issue for an accurate evaluation of the equipment response then focuses on a proper accounting of this correlation in the superposition of modal responses.

In the second part of this report, a mode-superposition procedure for the response of the equipment including the effect of closely spaced modes is developed. First, the random vibration approach for response to a stationary input described through a power spectral density function is presented. This forms the basis for development of the response spectrum method in the second portion which is applicable under a set of broad conditions. In this method, which is of great interest in earthquake engineering, the equipment response is obtained directly in terms of information readily available to the designer; namely, the dynamic properties of the structure alone, the dynamic properties of the equipment alone, and the design response spectrum describing the input into the structure. These results should prove to be especially effective in design applications where a multitude of analyses is required.

The method presented here is more comprehensive and at the same time is computationally more efficient than any of the methods currently used. The existing methods (e.g., Refs. 6, 8, 9 and 12) invariably ignore one or more of the problems associated with the response of light equipment in structures, e.g., interaction between equipment and structure, effect of coincidence of frequencies, and correlation between closely spaced frequencies. Most common among the existing methods is the floor spectrum procedure where the response spectrum associated with the equipment attachment point in the structure is first determined, ignoring the equipment-structure interaction, and then used as the input into the equipment (e.g., Ref. 8). With the present results, the conventional floor spectrum is produced simply by setting the mass of the equipment to zero, resulting in an algorithm simpler than any current one. Furthermore, with the present method it becomes possible to efficiently generate floor spectra which include the effect of equipment-structure interaction.

In the final part of this report, numerical examples are presented in order to illustrate the accuracy and the simplicity of the method. These include comparisons between the dynamic properties of the combined equipment-structure system obtained by an exact analysis and by the present method, and comparisons between the equipment response as obtained from time-history integrations and from the response spectrum formulation developed herein. In general, excellent agreement is obtained between the results from the two analyses.

DYNAMIC PROPERTIES OF EQUIPMENT-STRUCTURE SYSTEM

Consider an n-degree-of-freedom structure with mass, damping, and stiffness matrices **M**, **C**, and **K** having elements m_{ij}, c_{ij}, and k_{ij}, $i,j=1,2,...,n$, respectively. Assume the structure has classical modes with natural frequencies ω_i, modal vectors $\mathbf{\Phi}_i = \left[\phi_{1i}\ \phi_{2i}\ \ldots\ \phi_{ni}\right]^T$, and modal damping coefficients ζ_i, where a superposed T denotes a transpose. Consider an equipment item, modeled as a single-degree-of-freedom oscillator of mass m_e, natural frequency ω_e, and damping coefficient ζ_e, attached to the structure such that it is only affected by motion in the k-th structure degree of freedom. The combined system, see Fig. 1, is an n+1-degree sys-

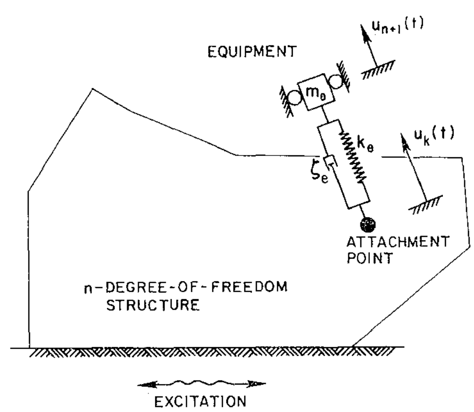


Figure 1. Equipment-Structure System

tem with $n+1$ modes. In general, the equipment frequency can be close or equal, i.e., nearly or perfectly tuned, to a natural frequency of the structure, say ω_l. Then, two closely spaced modes will occur in the combined system which require special treatment. It is the objective here to determine the dynamic properties of the combined system, including the case where tuning occurs, in terms of the properties of the two subsystems when the equipment mass is small compared to that of the structure. It is subsequently shown that for such conditions the modes of the combined system are closely related to those of the structure alone. Each structure mode, with contributions from other modes, produces a corresponding mode in the combined system. In addition, all the structure modes together with the equipment generate a "new" mode of the combined system. For convenience, the "new" mode will be identified by the subscript zero with the other modes retaining their original numbers. This process is conceptually illustrated in Fig. 2.

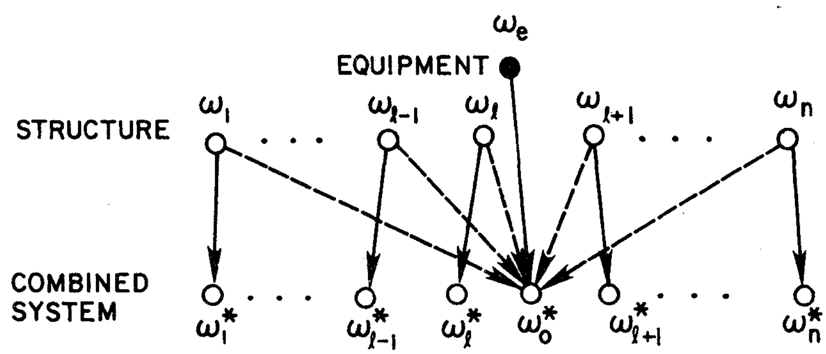


Figure 2. Schematic Representation of Modal Frequencies

The system of equations which determine the i-th $(i=0,1,...,n)$ frequency and mode shape of the combined system may be obtained from the known properties of the two subsystems as

$$\mathbf{K}^*\mathbf{\Phi}_i^* = \omega_i^{*2}\mathbf{M}^*\mathbf{\Phi}_i^* \tag{1}$$

where a superscript asterisk has been used to denote values associated with the combined system. In the above, $\mathbf{K}^*$ and $\mathbf{M}^*$ are the stiffness and mass matrices for the combined system, and ω_i^* and $\mathbf{\Phi}_i^*$ are the i-th natural frequency and mode shape, given by

$$\mathbf{K}^* = \left[\begin{array}{ccccc|c} k_{11} & \cdots & k_{1k} & \cdots & k_{1n} & 0 \\ \vdots & & \vdots & & \vdots & \vdots \\ k_{k1} & \cdots & k_{kk}+m_e\omega_e^2 & \cdots & k_{kn} & -m_e\omega_e^2 \\ \vdots & & \vdots & & \vdots & \vdots \\ k_{n1} & \cdots & k_{nk} & \cdots & k_{nn} & 0 \\ \hline 0 & \cdots & -m_e\omega_e^2 & \cdots & 0 & m_e\omega_e^2 \end{array}\right]$$

$$\mathbf{M}^* = \left[\begin{array}{ccccc|c} m_{11} & \cdots & m_{1k} & \cdots & m_{1n} & 0 \\ \vdots & & \vdots & & \vdots & \vdots \\ m_{k1} & \cdots & m_{kk} & \cdots & m_{kn} & 0 \\ \vdots & & \vdots & & \vdots & \vdots \\ m_{n1} & \cdots & m_{nk} & \cdots & m_{nn} & 0 \\ \hline 0 & \cdots & 0 & \cdots & 0 & m_e \end{array}\right]$$

and

$$\mathbf{\Phi}_i^* = \left\{\begin{array}{c} \phi_{1i}^* \\ \vdots \\ \phi_{ki}^* \\ \vdots \\ \phi_{ni}^* \\ \hline \phi_{n+1,i}^* \end{array}\right\}$$

where the $n+1$-th degree of freedom has been assigned to the equipment motion. Partitioning the matrices as shown and expanding Eq. 1, one obtains

$$\mathbf{K}\left\{\begin{array}{c} \phi_{1i}^* \\ \vdots \\ \phi_{ki}^* \\ \vdots \\ \phi_{ni}^* \end{array}\right\} + \left\{\begin{array}{c} 0 \\ \vdots \\ m_e\omega_e^2(\phi_{ki}^* - \phi_{n+1,i}^*) \\ \vdots \\ 0 \end{array}\right\} = \omega_i^{*2}\mathbf{M}\left\{\begin{array}{c} \phi_{1i}^* \\ \vdots \\ \phi_{ki}^* \\ \vdots \\ \phi_{ni}^* \end{array}\right\} \qquad (2a)$$

and

$$- m_e \omega_e^2 (\phi_{ki}^* - \phi_{n+1,i}^*) = \omega_i^{*2} m_e \phi_{n+1,i}^* \tag{2b}$$

From Eq. 2b,

$$\phi_{n+1,i}^* = - \frac{\omega_e^2}{\omega_i^{*2} - \omega_e^2} \phi_{ki}^* = \alpha_i \phi_{ki}^* \tag{3}$$

where $\alpha_i = \phi_{n+1,i}^* / \phi_{ki}^*$ is the modal amplification factor of the equipment relative to the attachment point. Substituting this in Eq. 2a, yields

$$\mathbf{K} \begin{Bmatrix} \phi_{1i}^* \\ \vdots \\ \phi_{ki}^* \\ \vdots \\ \phi_{ni}^* \end{Bmatrix} + \begin{Bmatrix} 0 \\ \vdots \\ m_e \phi_{ki}^* \omega_i^{*2} \omega_e^2 / (\omega_i^{*2} - \omega_e^2) \\ \vdots \\ 0 \end{Bmatrix} = \omega_i^{*2} \mathbf{M} \begin{Bmatrix} \phi_{1i}^* \\ \vdots \\ \phi_{ki}^* \\ \vdots \\ \phi_{ni}^* \end{Bmatrix} \tag{4}$$

The single nonzero term in the second vector on the left hand side is small for light equipment and, therefore, will only slightly modify the frequencies and mode shapes of the original structure. The well known Rayliegh's quotient establishes that first-order errors in mode shapes result in second-order errors in frequencies (5). Thus, as a first approximation, it is assumed here that the portions of modal vectors corresponding to structural degrees of freedom retain their shapes after the equipment is attached; i.e. it is assumed that $\phi_{mi}^* = \phi_{mi}$, for $m = 1,2,...,n$ and for $i \neq 0$. This will lead to a second-order approximation in the frequencies which is desirable for the computation of the response, since it can be shown that accuracy in frequencies is more critical than that in mode shapes. It should be pointed out that this approximation scheme is only appropriate when the major contribution to $\mathbf{\Phi}_i^*$, the i-th mode shape of the combined equipment-structure system, comes from $\mathbf{\Phi}_i$, the i-th mode shape of the structure. This precludes applying this approximation to cases where the equipment is tuned to a cluster of closely spaced structure modes. In such situations, all of the modes of the cluster will participate in a significant way in generating the mode shape of the combined system. The method presented here can be extended to deal with this problem.

Premultiplying Eq. 4 by $\mathbf{\Phi}_i^T$, one obtains

$$\omega_i^2 M_i + m_e \phi_{ki}^2 \frac{\omega_i^{*2} \omega_e^2}{\omega_i^{*2} - \omega_e^2} = \omega_i^{*2} M_i \tag{5}$$

where $M_i = \Phi_i^T \mathbf{M} \Phi_i = \omega_i^{-2} \Phi_i^T \mathbf{K} \Phi_i$ is the i-th modal mass of the structure. This yields

$$(1+\beta_i)\left(\frac{\omega_i^*}{\omega_i}\right)^4 - 2\left(1+\frac{\beta_i+\gamma_i}{2}\right)\left(\frac{\omega_i^*}{\omega_i}\right)^2 + 1 = 0 \tag{6}$$

where $\beta_i = (\omega_i^2 - \omega_e^2)/\omega_e^2$ and $\gamma_i = m_e/(M_i/\phi_{ki}^2)$ are the *detuning parameter* and the *effective mass ratio*, respectively, for mode i. These two parameters play important roles in the subsequent analysis. The solution of Eq. 6 is

$$\left(\frac{\omega_i^*}{\omega_i}\right)^2 = \begin{cases} \dfrac{1+\dfrac{\beta_i+\gamma_i}{2} - \left[\left(1+\dfrac{\beta_i+\gamma_i}{2}\right)^2 - (1+\beta_i)\right]^{1/2}}{1+\beta_i}, & \beta_i < 0 \\[2ex] \dfrac{1+\dfrac{\beta_i+\gamma_i}{2} + \left[\left(1+\dfrac{\beta_i+\gamma_i}{2}\right)^2 - (1+\beta_i)\right]^{1/2}}{1+\beta_i}, & \beta_i \geqslant 0 \end{cases} \tag{7}$$

where the proper root has been chosen according to the sign of β_i so as to produce ω_i^* near ω_i. (The other root, which yields ω_i^* near ω_e, is the contribution to the "new" mode. This contribution is subsequently retrieved by imposition of orthogonality between the modes.) For $\beta_i = 0$, either root is acceptable; here, the second one is chosen for definiteness. Substituting Eq. 7 in Eq. 3, the modal amplification factor is obtained as

$$\alpha_i = \begin{cases} -\dfrac{1}{\dfrac{\beta_i+\gamma_i}{2} - \left[\left(1+\dfrac{\beta_i+\gamma_i}{2}\right)^2 - (1+\beta_i)\right]^{1/2}}, & \beta_i < 0 \\[2ex] -\dfrac{1}{\dfrac{\beta_i+\gamma_i}{2} + \left[\left(1+\dfrac{\beta_i+\gamma_i}{2}\right)^2 - (1+\beta_i)\right]^{1/2}}, & \beta_i \geqslant 0 \end{cases} \tag{8}$$

It is noted that the magnitude of α_i can be very large for small β_i which occurs in the case of near or perfect tuning.

The new mode, denoted by Φ_0^*, is obtained through the requirement of orthogonality with the previous modes. Let

$$\mathbf{\Phi}_0^* = \begin{Bmatrix} \phi_{10}^* \\ \cdot \\ \cdot \\ \cdot \\ \phi_{n0}^* \\ 1 \end{Bmatrix} = \begin{Bmatrix} \mathbf{\Phi}_0 \\ 1 \end{Bmatrix} \tag{9}$$

be the new mode shape, scaled so that $\phi_{n+1,0}^* = 1$. Orthogonality requires that $\mathbf{\Phi}_i^{*T}\mathbf{M}^*\mathbf{\Phi}_0^* = 0$, for $i \neq 0$. Using $\mathbf{M}^*$, $\mathbf{\Phi}_i^*$, and $\mathbf{\Phi}_0^*$ as described above, one then obtains

$$\mathbf{\Phi}_i^{*T}\mathbf{M}^*\mathbf{\Phi}_0^* = \mathbf{\Phi}_i^T\mathbf{M}\mathbf{\Phi}_0 + m_e\alpha_i\phi_{ki} = 0, \quad i = 1,2,\ldots,n \tag{10}$$

which gives

$$\mathbf{\Phi}_i^T\mathbf{M}\mathbf{\Phi}_0 = -m_e\alpha_i\phi_{ki}, \quad i = 1,2,\ldots,n \tag{11}$$

Collecting all such equations in a matrix form,

$$\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}_0 = -\begin{Bmatrix} m_e\alpha_1\phi_{k1} \\ \cdot \\ \cdot \\ \cdot \\ m_e\alpha_n\phi_{kn} \end{Bmatrix} \tag{12}$$

where $\mathbf{\Phi} = \begin{bmatrix}\mathbf{\Phi}_1 \ \mathbf{\Phi}_2 \ldots \mathbf{\Phi}_n\end{bmatrix}$ is the modal matrix of the structure alone. It is easy to show that $\mathbf{\Phi}\mathbf{D}^{-1}$ is the inverse of $\mathbf{\Phi}^T\mathbf{M}$, where $\mathbf{D}$ is a diagonal matrix with elements M_i. Premultiplying Eq. 12 by $\mathbf{\Phi}\mathbf{D}^{-1}$, therefore, yields

$$\mathbf{\Phi}_0 = -\mathbf{\Phi}\mathbf{D}^{-1}\begin{Bmatrix} m_e\alpha_1\phi_{k1} \\ \cdot \\ \cdot \\ \cdot \\ m_e\alpha_n\phi_{kn} \end{Bmatrix} = -\begin{Bmatrix} \sum_i \alpha_i\gamma_i\phi_{1i}/\phi_{ki} \\ \cdot \\ \cdot \\ \cdot \\ \sum_i \alpha_i\gamma_i\phi_{ni}/\phi_{ki} \end{Bmatrix} \tag{13}$$

Note that all structure modes contribute to the generation of the new mode. However, observe that generally the shape of the new mode will be dominated by that structure mode which has a frequency closest to the equipment frequency, i.e. the l-th mode, because its modal amplification factor, α_l, is the largest. To determine the frequency of the new mode, Eqs. 9 and 13 are used in conjunction with Eq. 2(b) giving

$$\omega_0^* = \left[1+\sum_i \alpha_i \gamma_i\right]^{1/2} \omega_e \tag{14}$$

Observe that the frequency of the new mode is only slightly removed from the equipment frequency, with the difference being contributed by all structure modes. Clearly, the largest contribution to this difference comes from the l-th mode.

To investigate the nature of the error in the mode shapes and frequencies, a norm of the second vector on the left-hand side of Eq. 4 is examined in comparison with the corresponding norms of the two remaining vectors. This is accomplished by premultiplying Eq. 4 by Φ_i^T. For light equipment, the scalars thus obtained from the two remaining vectors are of order $\omega_i^2 M_i$, whereas that from the former vector is of order $\omega_i^2 M_i \gamma_i \omega_e^2/(\omega_i^{*2}-\omega_e^2)$. The ratio of these norms, r, which is a measure of the modification of frequencies and mode shapes of the structure resulting from the addition of the light equipment, can be written as

$$r = \frac{\gamma_i}{\left[\frac{\omega_i^*}{\omega_i}\right]^2 (1+\beta_i)-1} \tag{15}$$

where the ratio of frequencies is given in terms of β_i and γ_i in Eqs. 7. It is easy to show that in the extreme cases of gross detuning, i.e. when $|\beta_i|$ is large, and of perfect tuning, i.e. when β_i is zero, the modification factor r is of order γ_i and $\sqrt{\gamma_i}$, respectively. A plot of $|r/\sqrt{\gamma_i}|$ versus β_i is shown in Fig. 3 for $\gamma_i = 0.001, 0.01$ and 0.05. Observe that for small γ_i, the modification term rapidly increases very near tuning, i.e. when $\beta_i \rightarrow 0$. This leads to a formal definition of gross detuning as values of β_i for which $|r/\sqrt{\gamma_i}| << 1$. From Eq. 15, this definition can be expressed as $|\beta_i| >> \sqrt{\gamma_i}$.

It is observed from Eq. 15 that errors in the estimated mode shapes and frequencies are largest for modes with frequencies closest to the equipment frequency. To achieve a uniform accuracy, it is necessary to improve the results for modes having frequencies near the equipment frequency. For simplicity, it is assumed herein that all but the l-th mode are well spaced from the equipment frequency. Therefore, a refinement of only mode l need be considered.

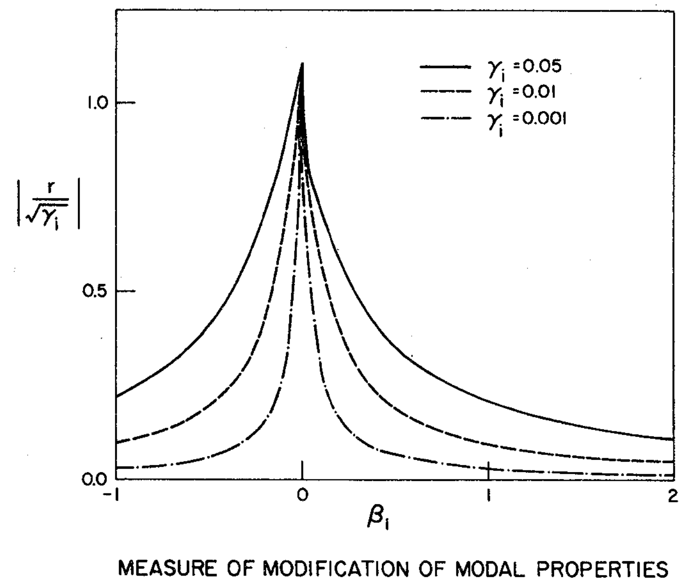


Figure 3. Measure of Modification of Modal Properties

To construct a refined shape for the l-th mode, the same procedure is used as for the new mode above, i.e. the refined l-th mode is obtained by imposition of orthogonality with all other modes, including the new mode. This results in

$$\Phi_l^* = \begin{Bmatrix} \sum_{i \neq l} \alpha_i \gamma_i \phi_{1i}/\phi_{ki} - \frac{1}{\alpha_l}\phi_{1l}/\phi_{kl} \\ \cdot \\ \cdot \\ \cdot \\ \sum_{i \neq l} \alpha_i \gamma_i \phi_{ni}/\phi_{ki} - \frac{1}{\alpha_l}\phi_{nl}/\phi_{kl} \\ -1 \end{Bmatrix} \tag{16}$$

where, for the sake of uniformity with Φ_0^*, this mode has been scaled such that $\phi_{n+1,l}^* = -1$. In this derivation, only the dominant terms in Φ_0^* , i.e. those arising from the l-th mode, have been included. This is consistent with the order of accuracy in other modes.

The refined shape of the l-th mode could be used to obtain a more refined estimate of ω_l^* from Eq. 2(b). However, it is clear from Rayliegh's quotient that the second-order correction in the mode shape will result in only a third-order correction in the frequency, which is neglected for uniformity in approximation.

It is assumed herein that the damping in the equipment can be modeled by a viscous damper connecting the equipment to the attachment point, see Fig. 1. Since the structure is assumed to have modal damping, intuitively it would appear that for light damping the combined system will also very nearly have modal damping. The preceding perturbation scheme can be used to determine the damping coefficients for the combined system in terms of those of the two subsystems. To this end, the modal damping relation

$$2\zeta_i^*\omega_i^*M_i^* = \mathbf{\Phi}_i^{*T}\mathbf{C}^*\mathbf{\Phi}_i^* \tag{17}$$

is used, where $\mathbf{C}^*$, the damping matrix of the combined system, is given by

$$\mathbf{C}^* = \left[\begin{array}{ccccc|c} c_{11} & \cdots & c_{1k} & \cdots & c_{1n} & 0 \\ \vdots & & \vdots & & \vdots & \vdots \\ c_{k1} & \cdots & c_{kk}+2\zeta_e\omega_e m_e & \cdots & c_{kn} & -2\zeta_e\omega_e m_e \\ \vdots & & \vdots & & \vdots & \vdots \\ c_{n1} & \cdots & c_{nk} & \cdots & c_{nn} & 0 \\ \hline 0 & \cdots & -2\zeta_e\omega_e m_e & \cdots & 0 & 2\zeta_e\omega_e m_e \end{array}\right]$$

Using the preceding in conjunction with the expressions of β_i and γ_i as well as the identity $2\zeta_i\omega_i M_i = \mathbf{\Phi}_i^T\mathbf{C}\mathbf{\Phi}_i$, Eq. 17 gives

$$\zeta_i^* = \begin{cases} \dfrac{\sqrt{1+\beta_i}\,\zeta_i + (1-\alpha_i)^2\gamma_i\zeta_e}{\sqrt{1+\beta_i}\,(1+\alpha_i^2\gamma_i)}\,\dfrac{\omega_i}{\omega_i^*}, & i=1,2,\ldots,n \\[2ex] \dfrac{\sum_j \sqrt{1+\beta_j}\,\alpha_j^2\gamma_j\zeta_j + \left[1+\sum_j \alpha_j\gamma_j\right]^2\zeta_e}{1+\sum_j \alpha_j^2\gamma_j}\,\dfrac{\omega_e}{\omega_i^*}, & i=0 \end{cases} \tag{18}$$

In arriving at the result for ζ_i^*, the unrefined shape of the i-th mode was used for reasons of simplicity and consistency in approximation. Note that the ratios of frequencies in the preceding relations are close to unity and can be discarded.

To obtain the response of the combined system to specified input excitations, it is necessary to compute modal masses and participation factors. Using the mode shapes previously obtained, these are determined in closed form as

$$M_i^* = \mathbf{\Phi}_i^{*T}\mathbf{M}^*\mathbf{\Phi}_i^*$$
$$= \begin{cases} (1+\alpha_i^2\gamma_i)M_i, & i \neq l,0 \\ \left[1+\alpha_l^2\gamma_l\left(1+\sum_{j\neq l}\alpha_j^2\gamma_j\right)\right]\dfrac{M_l}{(\alpha_l\phi_{kl})^2}, & i=l \\ \left(1+\sum_j \alpha_j^2\gamma_j\right)m_e, & i=0 \end{cases} \tag{19}$$

and

$$\Gamma_i^* = \frac{\mathbf{\Phi}_i^{*T}\mathbf{M}^*\mathbf{R}^*}{M_i^*}$$

$$= \begin{cases} \dfrac{\Gamma_i+\alpha_i\gamma_i r_{n+1}^*/\phi_{ki}}{1+\alpha_i^2\gamma_i}, & i\neq l,0 \\ -\dfrac{\alpha_l\phi_{kl}\Gamma_l-\alpha_l^2\gamma_l\left(\sum_{j\neq l}\alpha_j\phi_{kj}\Gamma_j-r_{n+1}^*\right)}{1+\alpha_l^2\gamma_l\left(1+\sum_{j\neq l}\alpha_j^2\gamma_j\right)}, & i=l \\ -\dfrac{\sum_j\alpha_j\phi_{kj}\Gamma_j-r_{n+1}^*}{1+\sum_j\alpha_j^2\gamma_j}, & i=0 \end{cases} \tag{20}$$

where $\Gamma_i = \mathbf{\Phi}_i^T\mathbf{MR}/M_i$ is the participation factor associated with the i-th mode of the structure, in which $\mathbf{R}=[r_1\ r_2\ \cdots\ r_n]^T$ and $\mathbf{R}^*=[r_1\ r_2\ \cdots\ r_n\ r_{n+1}^*]^T$ are the conventional influence vectors coupling the input to the degrees of freedom of the structure and of the combined system, respectively.

PERFECTLY TUNED EQUIPMENT

It is instructive to examine the preceding results in the special case when the equipment is perfectly tuned to the l-th mode and grossly detuned with respect to all other modes. Using Eqs. 7 and 14, the modal frequencies in this case reduce to

$$\omega_i^* \approx \begin{cases} \omega_i, & i \neq l,0 \\ \left(1+\dfrac{\sqrt{\gamma_l}}{2}\right)\omega_l, & i=l \\ \left(1-\dfrac{\sqrt{\gamma_l}}{2}\right)\omega_e, & i=0 \end{cases} \tag{21}$$

where $\omega_l = \omega_e$ and terms only up to order $\sqrt{\gamma_i}$ have been retained. The corresponding results for modal amplification factors, obtained from Eqs. 8, are

$$\alpha_i \approx \begin{cases} -\dfrac{1}{\beta_i}, & i \neq l,0 \\ -\dfrac{1}{\sqrt{\gamma_l}}, & i=l \end{cases} \tag{22}$$

These factors together with γ_i determine the mode shapes as

$$\mathbf{\Phi}_i^* \approx \begin{Bmatrix} \phi_{1i} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{ni} \\ -\dfrac{\phi_{ki}}{\beta_i} \end{Bmatrix}, \; i \neq l,0; \quad \mathbf{\Phi}_l^* \approx \begin{Bmatrix} \sqrt{\gamma_l}\dfrac{\phi_{1l}}{\phi_{kl}} - \sum\limits_{i \neq l} \dfrac{\gamma_i}{\beta_i}\dfrac{\phi_{1i}}{\phi_{ki}} \\ \cdot \\ \cdot \\ \cdot \\ \sqrt{\gamma_l}\dfrac{\phi_{nl}}{\phi_{kl}} - \sum\limits_{i \neq l} \dfrac{\gamma_i}{\beta_i}\dfrac{\phi_{ni}}{\phi_{ki}} \\ -1 \end{Bmatrix}; \quad \mathbf{\Phi}_0^* \approx \begin{Bmatrix} \sqrt{\gamma_l}\dfrac{\phi_{1l}}{\phi_{kl}} + \sum\limits_{i \neq l} \dfrac{\gamma_i}{\beta_i}\dfrac{\phi_{1i}}{\phi_{ki}} \\ \cdot \\ \cdot \\ \cdot \\ \sqrt{\gamma_l}\dfrac{\phi_{nl}}{\phi_{kl}} + \sum\limits_{i \neq l} \dfrac{\gamma_i}{\beta_i}\dfrac{\phi_{ni}}{\phi_{ki}} \\ +1 \end{Bmatrix} \tag{23}$$

The corresponding expressions for damping coefficients from Eqs. 18 are

$$\zeta_i^* \approx \begin{cases} \zeta_i, & i \neq l,0 \\ \dfrac{\zeta_l + \zeta_e}{2}, & i = l,0 \end{cases} \tag{24}$$

The modal masses from Eqs. 19 reduce to

$$M_i^* \approx \begin{cases} M_i, & i \neq l,0 \\ 2m_e, & i = l,0 \end{cases} \tag{25}$$

Note that the small modal masses associated with the l-th and 0-th modes result from the scaling used for these modes, i.e. from the scaling of their last components to 1 and -1, respectively. The expressions for the participation factors from Eqs. 20 are

$$\Gamma_i^* \approx \begin{cases} \Gamma_i, & i \neq l,0 \\ \frac{1}{2}\left\{\frac{\Gamma_l}{\sqrt{\gamma_l}}\phi_{kl} - \sum_{j \neq l}\frac{\Gamma_j}{\beta_j}\phi_{kj} - r_{n+1}^*\right\}, & i = l \\ \frac{1}{2}\left\{\frac{\Gamma_l}{\sqrt{\gamma_l}}\phi_{kl} + \sum_{j \neq l}\frac{\Gamma_j}{\beta_j}\phi_{kj} + r_{n+1}^*\right\}, & i = 0 \end{cases} \tag{26}$$

Observe that the participation factors for the l-th and 0-th modes are much larger than those for other modes as a consequence of the scaling mentioned above.

The preceding results for the case of perfect tuning to the l-th structure mode and gross detuning to all other structure modes exhibit a remarkable symmetry between the l-th and the 0-th modes of the combined system. Observe that their frequencies are equally spaced on each side of the tuning frequency and their damping coefficients and modal masses are equal. Furthermore, note that the expressions for their modal shapes and participation factors are symmetrical in form. It is also seen that the frequencies, damping coefficients, modal masses, and participation factors of the grossly detuned modes are unaffected by the addition of the light equipment.

RESPONSE OF EQUIPMENT-STRUCTURE SYSTEM

It is well known (e.g., see Ref. 5) that any response of a linear MDOF system to a prescribed input excitation can be obtained as a superposition of modal contributions. In particular, any response quantity of the equipment in the combined system under consideration can be written as

$$R(t) = \sum_{i=0}^{n} R_i(t) = \sum_{i=0}^{n} \Psi_i^* S_i(t) \tag{27}$$

where $R_i(t) = \Psi_i^* S_i(t)$ is the contribution of mode i to the response, Ψ_i^* is the i-th effective participation factor, and $S_i(t)$ is the i-th normal coordinate representing the response of an oscillator of frequency ω_i^* and damping coefficient ζ_i^* to the given excitation. The effective participation factor, Ψ_i^*, depends on the particular response sought and in general is given as a product of the i-th participation factor, Γ_i^*, and a linear combination of the components of the i-th modal vector. For example, for the displacement response of the equipment, it is simply

$\Gamma_i^* \phi_{n+1,i}^*$, and for its displacement relative to the attachment point, it is $\Gamma_i^*(\phi_{n+1,i}^* - \phi_{ki}^*)$.

It is interesting to examine the values of the effective participation factors for the equipment displacement in the case of perfect tuning. Using Eqs. 26, these become

$$\Psi_i^* \approx \begin{cases} -\dfrac{\Psi_i}{\beta_i}, & i \neq l,0 \\ -\dfrac{1}{2}\left(\dfrac{\Psi_l}{\sqrt{\gamma_l}} - \sum\limits_{j \neq l} \dfrac{\Psi_j}{\beta_j} - r_{n+1}^*\right), & i = l \\ +\dfrac{1}{2}\left(\dfrac{\Psi_l}{\sqrt{\gamma_l}} + \sum\limits_{j \neq l} \dfrac{\Psi_j}{\beta_j} + r_{n+1}^*\right), & i = 0 \end{cases} \tag{28}$$

where $\Psi_i = \Gamma_i \phi_{ki}$, $i = 1,2,...,n$, is the effective participation factor for the i-th mode for the displacement of the attachment point in the structure alone. It is important to observe the presence of $\sqrt{\gamma_l}$ in the denominators of the leading terms of Ψ_l^* and Ψ_0^*. Because of this term, the effective participation factors for these modes are two large numbers of opposite signs and almost equal magnitudes. This fact has an important bearing on the manner in which these two modes combine to produce the equipment response, as will be shown subsequently.

Depending on the type of input excitation, a deterministic or a probabilistic method is required to evaluate Eq. 27. For a deterministic input time history, any suitable time-integration method could be used with no problems anticipated. On the other hand, if the excitation is stochastic, particular care must be exercised to account for the correlation between normal coordinates, $S_i(t)$, which can be significant for modes with closely spaced frequencies, as would occur for the case of tuned or nearly tuned equipment. Because of this, and since the probabilistic approach is of such current interest (e.g., in earthquake engineering, offshore structures, flight structures), this problem is addressed in detail in the next two sections. Specifically, in the following section, the response of the equipment to a stationary excitation of the combined system described through an input power spectral density function is discussed. This provides the basis for the development of a response spectrum method, presented in the subsequent section, which properly accounts for closely spaced modes. These developments make use of new results presented in Refs. 2 and 3 , which are especially pertinent to the problem of closely spaced modes.

RESPONSE TO STOCHASTIC INPUT: POWER SPECTRAL DENSITY APPROACH

Consider the stationary response of the combined equipment-structure system to a zero-mean stationary input excitation, $F(t)$, with a one-sided power spectral density $G_F(\omega)$. Using mode superposition, the power spectral density of the equipment response is

$$G_R(\omega) = \sum_{i=0}^{n}\sum_{j=0}^{n}\Psi_i^*\Psi_j^* G_F(\omega) H_i(\omega)\overline{H}_j(\omega) \tag{29}$$

where, for base input excitation, $H_i(\omega) = (\omega_i^{*2} - \omega^2 + 2i\zeta_i^*\omega_i^*\omega)^{-1}$ is the complex frequency-response function for mode i, and a superposed bar denotes the complex conjugate.

It has been shown (10) that most response quantities of interest can be expressed in terms of the first three moments, λ_0, λ_1, and λ_2, of the response power spectral density

$$\lambda_m = \int_0^\infty \omega^m G_R(\omega)\, d\omega\,, \quad m = 0,1,2 \tag{30}$$

Using the modal description of Eq. 29 and following Ref. 3, the spectral moments can be expressed as

$$\lambda_m = \sum_{i=0}^{n}\sum_{j=0}^{n}\Psi_i^*\Psi_j^*\lambda_{m,ij}^*\,, \quad m = 0,1,2 \tag{31}$$

where

$$\lambda_{m,ij}^* = \mathrm{Re}\left[\int_0^\infty \omega^m G_F(\omega) H_i(\omega)\overline{H}_j(\omega)\, d\omega\right], \quad m = 0,1,2 \tag{32}$$

are defined as cross-spectral moments of normal coordinates for modes i and j of the combined system. As in Ref. 3, it is useful to introduce coefficients $\rho_{m,ij}$ as

$$\rho_{m,ij} = \frac{\lambda_{m,ij}^*}{\sqrt{\lambda_{m,ii}^*\lambda_{m,jj}^*}}\,, \quad m = 0,1,2 \tag{33}$$

(It is noted that $\rho_{0,ij}$ and $\rho_{2,ij}$ are correlation coefficients between the normal coordinates for modes i and j of the combined system and between their time derivatives, respectively; see Ref. 3.) Introducing these relations in Eq. 31, spectral moments of response can be obtained in

Ref. 3.) Introducing these relations in Eq. 31, spectral moments of response can be obtained in terms of spectral moments of individual normal coordinates as

$$\lambda_m = \sum_i \sum_j \Psi_i^* \Psi_j^* \rho_{m,ij} \sqrt{\lambda_{m,ii}^* \lambda_{m,jj}^*}, \quad m = 0,1,2 \tag{34}$$

Closed form expressions for $\lambda_{m,ij}^*$ and $\rho_{m,ij}$ for response to white-noise and filtered white-noise inputs have been given in Ref. 3. For completeness of this presentation, the results for white-noise input are given in Appendix I. In the case of wide-band excitations, $\rho_{m,ij}$ rapidly diminish as the spacing between the frequencies of the two modes i and j grows, especially when damping is small. As an example, $\rho_{m,ij}$ for response to white-noise input are illustrated in Fig. 4. Because of this behavior, cross-modal terms in Eqs. 31 and 34 need only be retained for modes with closely spaced frequencies. It should be noted from Fig. 4 that in the case of closely spaced modes the coefficients $\rho_{m,ij}$ critically depend on the modal frequencies and damping ratios. Therefore, in the case of tuned equipment when closely spaced modes occur in the combined system, the accurate estimation of modal frequencies and damping ratios is essential, as was alluded to before.

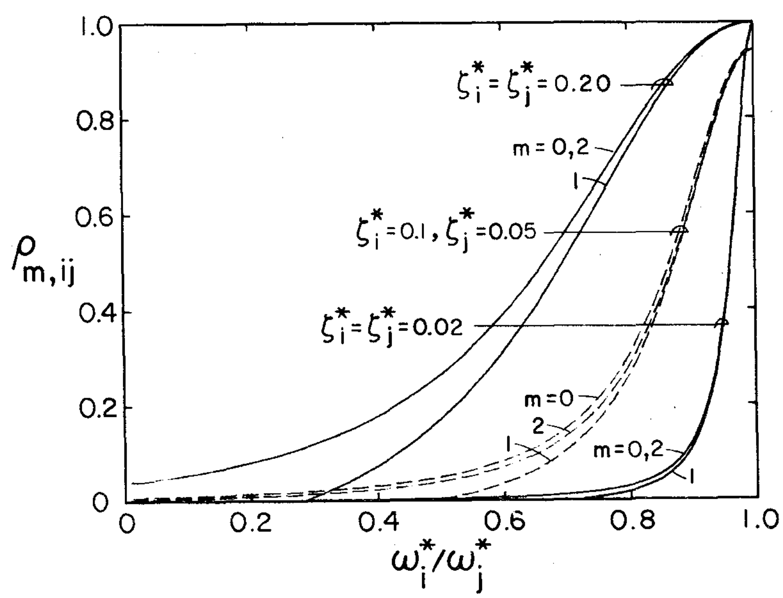


Figure 4. **Modal Cross-Correlation Coefficients for Response to White-Noise Input**

In terms of the spectral moments, the equipment response quantities are the root-mean-square response, $\sigma_R = \sqrt{\lambda_0}$, and the root-mean-square of response rate, $\sigma_{\dot{R}} = \sqrt{\lambda_2}$. In addition, when $F(t)$ and, hence, $R(t)$ are Gaussian, the mean zero-crossing rate of the response process is obtained as $\nu = \overline{\omega}/\pi = \sigma_{\dot{R}}/\pi\sigma_R = \sqrt{\lambda_2/\lambda_0}/\pi$, where $\overline{\omega}$ is the response mean frequency. Furthermore, in this case, the distribution as well as the mean and variance of the peak response over a specified duration can be expressed in terms of the spectral moments. Specifically, for the maximum absolute response over duration τ, denoted by $R_\tau = \max_\tau |R(t)|$, the cumulative distribution as given in Ref. 11 is

$$F_{R_\tau}(r) = \left[1 - \exp(-a^2/2)\right] \exp\left[-\nu\tau \frac{1 - \exp(-\sqrt{\pi/2}\,\delta_e a)}{\exp(a^2/2) - 1}\right], \quad r > 0 \tag{35}$$

where $a = r/\sigma_R$, $\delta_e = \delta^{1.2}$, and $\delta = \sqrt{1 - \lambda_1^2/\lambda_0\lambda_2}$. The parameter δ has a value between zero and one and has been shown to be a measure of dispersion of the shape of the power spectral density about its centroid (10). For this reason, it will be referred to herein as the shape factor. The mean and standard deviation associated with the above distribution can be obtained as $\overline{R}_\tau = p\sigma_R$ and $\sigma_{R_\tau} = q\sigma_R$, respectively, where p and q are peak factors. For $10 \leqslant \nu\tau \leqslant 1000$, which is the usual range of interest in earthquake engineering, approximate expressions for p and q based on results in Ref. 3 are

$$p = \sqrt{2\ln\nu_e\tau} + \frac{0.5772}{\sqrt{2\ln\nu_e\tau}} \tag{36}$$

$$q = \frac{1.2}{\sqrt{2\ln\nu_e\tau}} - \frac{5.4}{13 + (2\ln\nu_e\tau)^{3.2}} \tag{37}$$

in which

$$\nu_e\tau = \begin{cases} \max(2.1,\ 2\delta\nu\tau), & 0 < \delta \leqslant 0.1 \\ (1.63\delta^{0.45} - 0.38)\nu\tau, & 0.1 < \delta < 0.69 \\ \nu\tau, & 0.69 \leqslant \delta < 1 \end{cases} \tag{38}$$

is a reduced mean crossing rate signifying an effective portion of zero-crossings which may be considered as statistically independent.

It is interesting to examine the case where the equipment is perfectly tuned to the l-th structure mode with the other modes having frequencies well spaced from the tuning frequency. In this case, the cross terms in Eq. 34 can all be neglected except those between modes l and 0. Using Eqs. 28, the spectral moments then become

$$\lambda_m \approx \sum_{i\neq l}\frac{\Psi_i}{\beta_i}^2 \lambda_{m,ii} + \left[-\frac{\Psi_l}{2\sqrt{\gamma_l}}+\frac{1}{2}\left(\sum_{i\neq l}\frac{\Psi_i}{\beta_i}+r_{n+1}^*\right)\right]^2 \lambda_{m,ll}^*$$
$$+\left[+\frac{\Psi_l}{2\sqrt{\gamma_l}}+\frac{1}{2}\left(\sum_{i\neq l}\frac{\Psi_i}{\beta_i}+r_{n+1}^*\right)\right]^2 \lambda_{m,00}^*$$
$$+2\rho_{m,l0}\left[-\frac{\Psi_l^2}{4\gamma_l}+\frac{1}{4}\left(\sum_{i\neq l}\frac{\Psi_i}{\beta_i}+r_{n+1}^*\right)^2\right]\sqrt{\lambda_{m,ll}^*\lambda_{m,00}^*}\,, \quad m=0,1,2 \tag{39}$$

where $\lambda_{m,ii}$ are the spectral moments for the structure alone. For wide-band inputs, on the basis of response to white noise (see Appendix I), it is reasonable to assume that $\lambda_{m,ii}$ is proportional to ω_i^{-m}. Thus, for small γ_l, noting that for perfect tuning $\zeta_l^* \approx \zeta_0^* \approx (\zeta_l+\zeta_e)/2$, and using the approximate expressions for frequencies given in Eqs. 21, the spectral moments for modes l and 0 become, after some algebraic manipulation

$$\lambda_{m,ll}^* \approx \left(1-\frac{3-m}{2}\sqrt{\gamma_l}\right)\lambda_{m,aa}\,, \quad m=0,1,2 \tag{40}$$

$$\lambda_{m,00}^* \approx \left(1+\frac{3-m}{2}\sqrt{\gamma_l}\right)\lambda_{m,aa}\,, \quad m=0,1,2 \tag{41}$$

where $\lambda_{m,aa}$ are spectral moments evaluated at frequency $\omega_a=(\omega_l+\omega_e)/2=\omega_l=\omega_e$, and damping coefficient $\zeta_a=(\zeta_l+\zeta_e)/2$. In this case (i.e. perfect tuning and a wide-band input), approximate expressions for the coefficients $\rho_{m,l0}$ can be developed from the results given in Appendix I as

$$\rho_{0,l0} \approx \rho_{2,l0} \approx \frac{4\zeta_a^2}{\gamma_l+4\zeta_a^2} \tag{42}$$

$$\rho_{1,l0} \approx \rho_{0,l0}\left(1-\frac{\gamma_l}{2\pi\zeta_a}\right) \tag{43}$$

Using Eqs. 40-43 in Eqs. 39, spectral moments of the response reduce to

$$\lambda_m \approx \sum_{i\neq l}\left(\frac{\Psi_i}{\beta_i}\right)^2\lambda_{m,ii}+\left\{\left[\frac{1}{2(\gamma_l+4\zeta_a^2)}+\frac{\epsilon_m}{4\pi\zeta_a}\right]\Psi_l^2\right.$$
$$+\left[1-\frac{\gamma_l}{2(\gamma_l+4\zeta_a^2)}-\frac{\epsilon_m\gamma_l}{4\pi\zeta_a}\right]\left[\sum_{i\neq l}\frac{\Psi_i}{\beta_i}+r_{n+1}^*\right]^2$$
$$\left.+\frac{3-m}{2}\left[\sum_{i\neq l}\frac{\Psi_i}{\beta_i}+r_{n+1}^*\right]\Psi_l\right\}\lambda_{m,aa}\,, \quad m=0,1,2 \tag{44}$$

where $\epsilon_0=\epsilon_2=0$, and $\epsilon_1=1$.

A careful examination of the preceding expression is helpful in understanding the relative significance of the various terms. Clearly, the first term inside the braces is the most important because of its small denominator. It should be sufficient in many cases to retain only this term as a simple first approximation. However, under certain circumstances, other terms in the expression may also become significant. For example, when the effective participation factor of the tuned structure mode, Ψ_l, is much smaller than those of some of the other structure modes (which will occur if the equipment is attached to the structure at or near a node of the tuned mode, or if it is tuned to a very high structure mode), then contributions from the other modes may dominate. Also, when the input excitation is poor in frequencies around the tuning mode but rich in frequencies around some of the nontuning modes, then $\lambda_{m,aa}$ may be sufficiently small compared to some of the $\lambda_{m,ii}$ such that the entire term it multiplies can be neglected. The second most important term is the first summation expression on the right-hand side. Note that in that expression contributions from modes much higher than the tuning mode can be neglected, because not only are Ψ_i small but also β_i are large for such modes. The last two terms inside the braces would generally be insignificant because of cancellations in the summations.

The result in Eq. 44 can be compared with an exact expression given in Ref. 1 for a single-degree-of-freedom structure for $m=0$. After retaining the dominant terms in the exact result, Eq. 44 differs only in the presence of $\zeta_g=\sqrt{\zeta_l\zeta_e}$, the geometric mean, in place of ζ_a, the arithmetic mean, in the denominator of the dominant term on the right-hand side. This difference is a consequence of assuming modal damping for the combined equipment-structure system. Note that for all but extreme cases, the difference between ζ_g and ζ_a is small.

Another special case of interest is that of gross detuning, i.e. when the structure frequencies are well spaced from the equipment frequency. If, in addition, the structure frequencies are also well spaced, then all cross terms in Eqs. 34 can be neglected. Using the first and last of Eqs. 20 together with the general expressions for the mode shapes specialized for the case of gross detuning, the formulae for spectral moments then simplify to

$$\lambda_m = \sum_i \left(\frac{\Psi_i}{\beta_i}\right)^2 \lambda_{m,ii} + \left(\sum_i \frac{\Psi_i}{\beta_i} + r^*_{n+1}\right)^2 \lambda_{m,ee}\,, \quad m=0,1,2 \tag{45}$$

where $\lambda_{m,ee}$ are spectral moments evaluated at the equipment frequency and damping. It is observed that the major contributions to the equipment response in this case come from low structure modes, which have larger Ψ_i values, and from structure modes with frequencies closest to the equipment frequency, which have smaller β_i values.

RESPONSE TO STOCHASTIC INPUT: RESPONSE SPECTRUM APPROACH

In many engineering applications, characterization of the input excitation through the power spectral density function is not convenient. A widely used alternative, which is of special interest in earthquake engineering, is a description in terms of a response spectrum. Under certain conditions, based on the developments in the preceding section, it is possible to construct a rational method whereby the peak response of a system over a specified duration can be obtained in terms of the response spectrum of the input excitation. These conditions are (2): (*a*) that the input excitation be wide-banded, i.e. it have a smoothly varying power spectral density over a wide range of frequencies covering the significant modes of vibration of the combined system, (*b*) that the input excitation be a stationary Gaussian process, and (*c*) that the response of the system over the duration of interest be stationary. In practice, the latter two requirements can be considerably relaxed with little loss of accuracy.

The response spectrum used in this study is defined as a function, $\bar{S}_\tau(\omega,\zeta)$, representing the mean peak response over a duration τ of an oscillator of frequency ω and damping coefficient ζ to a prescribed ensemble of excitations. This definition is consistent with that of the "smooth" response spectrum commonly employed in earthquake engineering, with an additional refinement here to account for the duration of excitation.

To compute the spectral moments of the response in this approach, it is necessary to interpret the spectral moments of the normal coordinates, $\lambda^*_{m,ii}$, in Eqs. 34 in terms of the response spectrum. For a wide-band input, it is reasonable to assume (see Ref. 2) that the mean zero-crossing rate and the shape factor associated with the i-th normal coordinate, denoted as ν_i and δ_i, respectively, can be approximated by their values for the response of a single-degree-of-freedom oscillator to white-noise input, i.e., by

$$\nu_i = \frac{\omega_i^*}{\pi} \tag{46}$$

and

$$\delta_i = \left[1 - \frac{1}{1-\zeta_i^{*2}}\left(1 - \frac{2}{\pi}\tan^{-1}\frac{\zeta_i^*}{\sqrt{1-\zeta_i^{*2}}}\right)^2\right]^{1/2} \approx 2\left(\frac{\zeta_i^*}{\pi}\right)^{1/2} \tag{47}$$

where the approximation in Eq. 47 is valid for small damping. Noting that $\sqrt{\lambda_{0,ii}^*}$ is the root-mean-square of the i-th normal coordinate, and using Eqs. 36 and 38 to compute the associated peak factor p_i in terms of τ, ν_i, and δ_i, $\lambda_{0,ii}^*$ can be obtained as

$$\lambda_{0,ii}^* = \left[\frac{1}{p_i}\bar{S}_\tau(\omega_i^*,\zeta_i^*)\right]^2 \tag{48}$$

Using the relation $\nu_i = \sqrt{\lambda_{2,ii}^*/\lambda_{0,ii}^*}/\pi$ together with Eq. 46, this gives

$$\lambda_{2,ii}^* = \omega_i^{*2}\lambda_{0,ii}^* = \omega_i^{*2}\left[\frac{1}{p_i}\bar{S}_\tau(\omega_i^*,\zeta_i^*)\right]^2 \tag{49}$$

Finally, using the relation $\delta_i = \sqrt{1-\lambda_{1,ii}^{*2}/\lambda_{0,ii}^*\lambda_{2,ii}^*}$ and Eq. 47, $\lambda_{1,ii}^*$ becomes

$$\lambda_{1,ii}^* = \omega_i^*\left(1 - \frac{4\zeta_i^*}{\pi}\right)^{1/2}\lambda_{0,ii}^* = \omega_i^*\left(1 - \frac{4\zeta_i^*}{\pi}\right)^{1/2}\left[\frac{1}{p_i}\bar{S}_\tau(\omega_i^*,\zeta_i^*)\right]^2 \tag{50}$$

With the three spectral moments determined, the various quantities of response can be evaluated as previously demonstrated. Specifically, using Eqs. 34, the parameters $\nu = \sqrt{\lambda_2/\lambda_0}/\pi$ and $\delta = \sqrt{1-\lambda_1^2/\lambda_0\lambda_2}$ are first determined. For this calculation, since the input is wide band, it is reasonable to use expressions of $\rho_{m,ij}$ which are based on white-noise input (see Appendix I and Ref. 2). These parameters, together with τ and $\sigma_R = \sqrt{\lambda_0}$, are then used to evaluate the distribution of the peak response from Eq. 35 and the mean and standard deviation through the peak factors p and q given in Eqs. 36-38.

It is useful to further examine the expression for the mean of the peak response. From the relation $\bar{R}_\tau = p\sqrt{\lambda_0}$, one obtains

$$\bar{R}_\tau = \left[\sum_{i=0}^{n} \sum_{j=0}^{n} \frac{p^2}{p_i p_j} \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \right]^{1/2} \tag{51}$$

where $\bar{R}_{i\tau} = \Psi_i^* \bar{S}_\tau(\omega_i^*, \zeta_i^*)$ is the mean value of the peak contribution from mode i. Note that because of the presence of p, the above relation implicitly depends on τ, λ_1 and λ_2, in addition to λ_0. It is easy to show (2) that the ratios p/p_i are usually close to unity and slowly decrease with increasing mode number. When the frequencies of modes making dominant contributions to the response are not very widely spaced, and when the response itself is not extremely narrow banded, for all practical purposes these ratios can be replaced by unity. In that case, Eq. 51 reduces to

$$\bar{R}_\tau = \left[\sum_{i=0}^{n} \sum_{j=0}^{n} \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \right]^{1/2} \tag{52}$$

In the special case of perfect tuning to the fundamental mode with very small mass ratio and light damping, the equipment response can be extremely narrow banded, e.g., $\delta < 0.1$. In that case the peak factor for the response process, p, can be considerably smaller than the peak factor for the fundamental mode, p_1. Numerical tests have shown that for $\gamma_1 = 0.001$, $\zeta_1 = 0.05$, and $\zeta_e = 0.02$, the ratio p/p_1 can be as small as 0.7. It should be clear that in such cases Eq. 52 would tend to overestimate the equipment response. In spite of its possible conservatism, Eq. 52 can be very useful in practice since it gives the mean of the peak response directly in terms of the response spectrum ordinates through the modal contributions, $\bar{R}_{i\tau}$, without the necessity of computing the spectral moments, λ_m. Furthermore, this result is independent of the duration τ, except for the dependence implied through the specified input response spectrum. Finally, it is noted that cross terms in Eqs. 51 and 52 can be neglected if the frequencies of the combined system are well spaced, since $\rho_{0,ij}$ vanish for such modes. In this case, Eq. 52 reduces to the well known square-root-of-sum-of-squares (SRSS) method for modal combination.

It is important to realize that in most equipment-structure systems closely spaced modes do occur. Thus, the use of the SRSS method for equipment-structure systems is in general inappropriate and should be avoided. In particular, for tuned and nearly tuned equipment, the effective participation factors Ψ_l^* and Ψ_0^*, associated with the resulting two closely spaced modes, are large numbers of opposite sign and nearly equal magnitude. Because of this, the cross terms associated with these modes are negative and of the same magnitude as the squared terms associated with the individual modes. Hence, it is clear that the SRSS rule, which neglects the cross terms, has the potential for severely overestimating the equipment response. This has been demonstrated in several numerical studies which have indicated that the overestimation can be as much as a factor of 10 or more (4). For these reasons, general use of the SRSS method for equipment-structure systems is inappropriate.

In a manner similar to that used for determining the mean of the peak response, employing the relation $\sigma_{R_\tau} = q\sigma_R$, one obtains for the standard deviation of the peak response

$$\sigma_{R_\tau} = \frac{q}{p}\,\overline{R}_\tau = \left(\sum_{i=0}^{n}\sum_{j=0}^{n}\frac{q^2}{p_i p_j}\rho_{0,ij}\overline{R}_{i\tau}\overline{R}_{j\tau}\right)^{1/2} \tag{53}$$

The ratios q/p_i in this expression can not be set equal to unity and, hence, all three spectral moments must be computed to evaluate the standard deviation.

A useful parameter characterizing the response is the response mean frequency, $\overline{\omega} = \sqrt{\lambda_2/\lambda_0}$. Using Eqs. 48 and 49 in Eqs. 34,

$$\overline{\omega} = \left[\frac{\displaystyle\sum_{i=0}^{n}\sum_{j=0}^{n}\frac{p^2}{p_i p_j}\rho_{2,ij}\overline{R}_{i\tau}\overline{R}_{j\tau}\omega_i^*\omega_j^*}{\displaystyle\sum_{i=0}^{n}\sum_{j=0}^{n}\frac{p^2}{p_i p_j}\rho_{0,ij}\overline{R}_{i\tau}\overline{R}_{j\tau}}\right]^{1/2} \tag{54}$$

where, for convenience, the numerator and the denominator in the expression have been multiplied by p. As noted above, in many instances, the ratios p/p_i are near unity; then, the preceding simplifies to

$$\bar{\omega} = \left\{ \frac{\sum_{i=0}^{n}\sum_{j=0}^{n} \rho_{2,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \omega_i^* \omega_j^*}{\sum_{i=0}^{n}\sum_{j=0}^{n} \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau}} \right\}^{1/2} \tag{55}$$

Observe that in this case the response mean frequency is the average of the modal frequencies of the equipment-structure system as weighted by the modal contributions to the response. This response parameter should be useful in studies concerned with equipment fatigue.

It is observed from Eqs. 51-55 that the equipment response quantities are obtained directly in terms of the ordinates of the response spectrum describing the input into the combined system. Note that the other quantities appearing in these equations are given in terms of the dynamic properties of the structure alone and of the equipment alone. It is important to point out that these results are computationally simple and far more comprehensive than any of the current response spectrum methods for equipment response. Furthermore, these results are superior to those currently available because they are based on a more realistic model of the combined system, since they include the equipment-structure interaction, and because they properly account for the correlation between modal responses. Some of these aspects have invariably been neglected in the literature on the equipment-structure problem.

It is useful to examine the mean of the peak response for the special cases discussed in the previous section in terms of the response spectrum formulation. For simplicity, the ratios p/p_i are discarded in this study. In the special case of perfect tuning to structure mode l, using Eqs. 44 for $m=0$, the mean peak response is

$$\bar{R}_\tau = \left\{ \sum_{i \neq l} \left(\frac{\Psi_i}{\beta_i} \right)^2 \bar{S}_\tau^2(\omega_i, \zeta_i) + \left[\left(\frac{\Psi_l}{\sqrt{2(\gamma_l + 4\zeta_a^2)}} \right)^2 + \left(1 - \frac{\gamma_l}{2(\gamma_l + 4\zeta_a^2)} \right) \left(\sum_{i \neq l} \frac{\Psi_i}{\beta_i} + r_{n+1}^* \right)^2 + \frac{\Psi_l}{2/3} \left(\sum_{i \neq l} \frac{\Psi_i}{\beta_i} + r_{n+1}^* \right) \right] \bar{S}_\tau^2(\omega_a, \zeta_a) \right\}^{1/2} \tag{56}$$

where it is recalled that $\omega_a = (\omega_l + \omega_e)/2 = \omega_l = \omega_e$ and $\zeta_a = (\zeta_l + \zeta_e)/2$. Based on the comparison of Eq. 44 with the exact results for a single-degree-of-freedom structure (1), it is conjectured that the more appropriate expression for perfect tuning that would account for nonmodal damping of the combined equipment-structure system would be obtained by replacing ζ_a by $\zeta_g = \sqrt{\zeta_l \zeta_e}$ in the denominators on the right-hand side of Eq. 56.

For the case of gross detuning, the corresponding result is

$$\bar{R}_\tau = \left\{ \sum_i \left(\frac{\Psi_i}{\beta_i} \right)^2 \bar{S}_\tau^2(\omega_i,\zeta_i) + \left(\sum_i \frac{\Psi_i}{\beta_i} + r_{n+1}^* \right)^2 \bar{S}_\tau^2(\omega_e,\zeta_e) \right\}^{1/2} \tag{57}$$

The preceding relation for the case of gross detuning is equivalent to results obtained in Refs. 6 and 7. However, to the writers' knowledge, the result in Eq. 56 for the case of perfect tuning has no equivalent in the literature.

It should be pointed out that although the results developed here correspond to the expected value of the responses to an ensemble of input excitations, the simplified expression for the mean peak response in Eq. 52 should also provide a good approximation for the peak response to an individual deterministic input because the dispersion in the peak response is generally small (i.e. 0.1-0.3 coefficient of variation, depending on the response frequency). Expected errors associated with such an approximation are anticipated to range at most between 10-40 percent.

As indicated in the introduction, the conventional floor spectrum associated with the k-th degree of freedom (i.e., the k-th floor), which is equivalent to the peak equipment response ignoring interaction, can easily be generated with the present method. For this purpose, it is sufficient to set all $\gamma_i = 0$ in Eq. 51 or 52 to obtain the ordinate of the mean floor spectrum for selected values of ω_e and ζ_e. At tuning, when $\omega_e = \omega_l$, Eqs 51 and 52 assume an indefinite form because in that case $\Psi_0^*/\Psi_l = -\Psi_l^*/\Psi_l = \infty$. The proper limit for this special case is obtained from Eq. 56 by letting $\gamma_l = 0$

$$\bar{R}_\tau = \left\{ \sum_{i \neq l} \left(\frac{\Psi_i}{\beta_i} \right)^2 \bar{S}_\tau^2(\omega_i,\zeta_i) + \left[\left(\frac{\Psi_l}{2\sqrt{2}\zeta_a} \right)^2 \right. \right.$$

$$+\left[\sum_{i\neq l}\frac{\Psi_i}{\beta_i}+r^*_{n+1}\right]^2+\frac{\Psi_l}{2/3}\left[\sum_{i\neq l}\frac{\Psi_i}{\beta_i}+r^*_{n+1}\right]\Bigg]\bar{S}^2_\tau(\omega_a,\zeta_a)\Bigg\}^{1/2} \tag{58}$$

As was indicated after Eq. 56, it would be appropriate to replace ζ_a appearing in the denominator of the right-hand side of the above equation by ζ_g to account for the effect of nonmodal damping. In the case of gross detuning, Eq. 57 is the appropriate expression for the floor spectrum under all conditions. This equation is equivalent to floor spectrum results for gross detuning in Refs. 6 and 7. However, Eq. 58 for perfect tuning is new.

It should be observed that with the present method the conventional floor spectrum is given directly in terms of the modal properties of the structure and the mean response spectrum associated with an ensemble of input excitations. This clearly is far more efficient than the traditional method of generating floor spectra which would involve a multitude of time history computations for any selected values of the frequency and the damping coefficient of the floor spectrum.

In light of the results developed in this study, there is in principle no point in using the conventional floor spectrum method for a single-degree-of-freedom equipment since it ignores equipment-structure interaction. Nevertheless, in certain practical applications where it is desired to develop equipment design criteria which is independent of an explicit description of the structure and the input excitation (e.g., when two different groups perform the structural design and the equipment evaluation), the floor spectrum method may still be useful. For this purpose, using the present method, floor spectra including the effect of interaction can be generated. Thus, for a typical floor, a family of spectra corresponding to a sequence of equipment masses can be developed. The equipment designer would then employ that spectrum which corresponds to the mass of the equipment under consideration. The floor spectrum might also be useful for a light multi-degree-of-freedom equipment item that is attached to a single floor. When the interaction can be neglected, the equipment response can be obtained by modal superposition using the conventional floor spectrum as the base input in the usual way. When interaction is significant, it might also be possible to determine the equipment response using the family of spectra previously described. However, at this time it is not apparent as to how to assign to each mode of the equipment the appropriate spectrum from the family. Furthermore, it is not clear how accurate the response spectrum method will be for a multi-degree-of-freedom equipment, since the floor response is generally not a wide-band process, as was assumed in the development of the response spectrum method. These matters require further research.

NUMERICAL EXAMPLES

As an example structure, a 10-story uniform shear building is considered. Mass, stiffness, and modal damping coefficients are shown in Fig. 5. Two alternative attachment positions of the equipment are studied, i.e., one on the fifth floor and one on the tenth floor. The modal frequencies and the elements of the modal vectors, normalized to give unit modal masses, for the two attachment points are shown in Table 1. These values are used to determine the

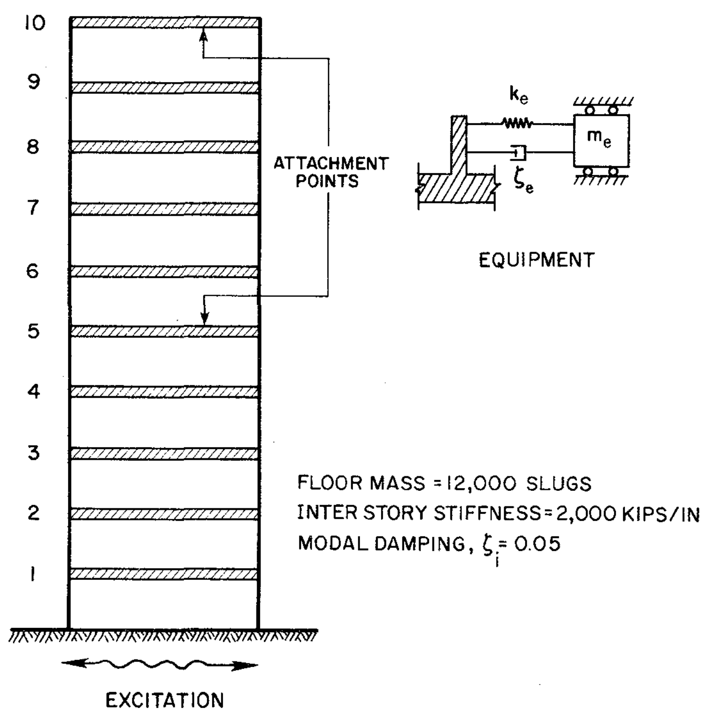


Figure 5. Example Structure and Equipment

effective mass ratios, γ_i, for each of the structure modes for any given equipment mass. In this study, three values of equipment mass were chosen so as to produce an effective mass ratio based on the first mode of $\gamma_1 =$ 0.001, 0.01, and 0.05. These correspond to equipment masses of $m_e =$ 63.4, 634, 3170 slugs, respectively, and to equipment to floor mass ratios of 0.005, 0.053, and 0.264, respectively.

Table 1. Structure Natural Frequencies and Components of Mode Shapes for Attachment Degrees of Freedom

Mode i	Freq., rad/sec ω_i	Elements of Mode Shapes ϕ_{5i}	Elements of Mode Shapes ϕ_{10i}
1	6.684	0.297	0.435
2	19.903	−0.341	0.425
3	32.677	0.246	−0.406
4	44.721	−0.378	−0.378
5	55.767	−0.189	−0.341
6	65.566	−0.406	0.297
7	73.901	0.128	−0.246
8	80.585	0.425	0.189
9	85.469	−0.065	−0.129
10	88.444	−0.435	0.065

In order to examine the accuracy of the natural frequencies and mode shapes of the equipment-structure system as given by Eqs. 7, 8, 9, 13, 14, and 16, comparisons are made in Tables 2, 3, and 4 with exact results for the cases of perfect tuning to structure modes 1, 2, and 3, respectively. The exact results were obtained by a direct solution of the eigenvalue problem for the combined (11-degree-of-freedom) equipment-structure system. To describe the error in the i-th mode shape, the difference vector $\mathbf{\Delta}_i = \mathbf{\Phi}_i^{*E} - \mathbf{\Phi}_i^*$ was considered where $\mathbf{\Phi}_i^{*E}$ is the exact mode shape. Rather than listing the elements of this vector, for brevity, only the ratio of its length to that of the modal vector as normalized by the mass matrix, i.e. the ratio $(\mathbf{\Delta}_i^T\mathbf{M}^*\mathbf{\Delta}_i/\mathbf{\Phi}_i^{*T}\mathbf{M}^*\mathbf{\Phi}_i^*)^{1/2}$, is listed in Tables 2 to 4 for each of the modes.

A perusal of Tables 2 to 4 demonstrates that the perturbation procedure generates very accurate estimates of eigenproperties of the combined equipment-structure system for light equipment. Even for heavy equipment, e.g., $\gamma_1 = 0.05$ when the equipment mass is more than a quarter of the floor mass, acceptable accuracy is obtained. Note that errors in both frequencies and mode shapes tend to increase as the equipment of fixed mass is tuned to higher structure modes.

Table 2. Comparison of Natural Frequencies and Mode Shapes of Equipment-Structure System with Exact Values: Equipment Tuned to 1st Structure Mode

Mass Ratio γ_1	Mode	Equipment Attached to 5th Floor			Equipment Attached to 10th Floor		
		Exact Frequency rad/sec	Error in Frequency %	Error in Mode Shape %	Exact Frequency rad/sec	Error in Frequency %	Error in Mode Shape %
0.001	0	6.579	−0.00	0.23	6.580	−0.00	0.16
	1	6.790	0.00	0.23	6.790	0.01	0.16
	2	19.905	0.00	0.02	19.904	0.00	0.02
	3	32.678	0.00	0.00	32.678	0.00	0.00
	4	44.722	0.00	0.00	44.722	0.00	0.00
	5	55.767	0.00	0.00	55.767	0.00	0.00
	6	65.567	0.00	0.00	65.566	0.00	0.00
	7	73.901	0.00	0.00	73.901	0.00	0.00
	8	80.586	0.00	0.00	80.585	0.00	0.00
	9	85.469	0.00	0.00	85.469	0.00	0.00
	10	88.444	0.00	0.00	88.444	0.00	0.00
0.01	0	6.354	−0.09	0.72	6.355	−0.06	0.49
	1	7.021	0.08	0.74	7.023	0.05	0.50
	2	19.920	0.00	0.19	19.915	0.00	0.16
	3	32.682	0.00	0.10	32.683	0.00	0.09
	4	44.730	0.00	0.09	44.725	0.00	0.06
	5	55.768	0.00	0.05	55.769	0.00	0.04
	6	65.573	0.00	0.08	65.568	0.00	0.03
	7	73.902	0.00	0.04	73.902	0.00	0.02
	8	80.591	0.00	0.09	80.586	0.00	0.02
	9	85.469	0.00	0.03	85.469	0.00	0.01
	10	88.449	0.00	0.08	88.444	0.00	0.00
0.05	0	5.959	−0.59	1.65	5.966	−0.40	1.15
	1	7.443	0.40	1.73	7.453	0.27	1.22
	2	19.986	0.00	0.93	19.963	0.00	0.79
	3	32.702	0.00	0.48	32.708	0.00	0.44
	4	44.763	0.00	0.45	44.741	0.00	0.30
	5	55.775	0.00	0.27	55.779	0.00	0.22
	6	65.598	0.00	0.38	65.574	0.00	0.16
	7	73.904	0.00	0.20	73.906	0.00	0.12
	8	80.614	0.00	0.46	80.588	0.00	0.08
	9	85.470	0.00	0.17	85.470	0.00	0.05
	10	88.471	0.00	0.42	88.444	0.00	0.03

Table 3. Comparison of Natural Frequencies and Mode Shapes of Equipment-Structure System with Exact Values: Equipment Tuned to 2nd Structure Mode

Mass Ratio γ_1	Mode	Equipment Attached to 5th Floor			Equipment Attached to 10th Floor		
		Exact Frequency rad/sec	Error in Frequency %	Error in Mode Shape %	Exact Frequency rad/sec	Error in Frequency %	Error in Mode Shape %
0.001	1	6.680	0.00	0.02	6.680	0.00	0.01
	0	19.544	−0.01	0.11	19.599	0.01	0.25
	2	20.267	0.01	0.19	20.214	−0.00	0.17
	3	32.684	0.00	0.13	32.685	0.00	0.12
	4	44.730	0.00	0.10	44.726	0.00	0.06
	5	55.768	0.00	0.05	55.769	0.00	0.04
	6	65.572	0.00	0.07	65.568	0.00	0.03
	7	73.902	0.00	0.04	73.902	0.00	0.02
	8	80.590	0.00	0.09	80.586	0.00	0.02
	9	85.469	0.00	0.03	85.469	0.00	0.00
	10	88.449	0.00	0.08	88.444	0.00	0.00
0.01	1	6.647	0.00	0.18	6.647	0.00	0.15
	0	18.792	−0.11	0.66	18.972	0.04	1.19
	2	21.059	0.10	1.12	20.904	−0.02	0.78
	3	32.743	0.00	1.28	32.761	0.00	1.20
	4	44.811	0.00	0.98	44.763	0.00	0.65
	5	55.783	0.00	0.53	55.792	0.00	0.43
	6	65.628	0.00	0.74	65.582	0.00	0.31
	7	73.906	0.00	0.37	73.910	0.00	0.22
	8	80.639	0.00	0.87	80.590	0.00	0.16
	9	85.470	0.00	0.31	85.471	0.00	0.10
	10	88.495	0.00	0.79	88.444	0.00	0.05
0.05	1	6.503	0.02	1.67	6.503	0.03	0.73
	0	17.585	−1.00	4.49	17.964	0.15	4.17
	2	22.475	0.67	1.64	22.168	0.13	4.05
	3	32.999	0.01	6.17	33.100	−0.04	5.92
	4	45.164	0.00	4.77	44.934	−0.01	3.28
	5	55.849	0.00	2.66	55.895	−0.01	2.20
	6	65.877	0.00	3.70	65.646	0.00	1.57
	7	73.928	0.00	1.86	73.948	0.00	1.13
	8	80.855	−0.00	4.41	80.611	0.00	0.79
	9	85.475	−0.00	1.56	85.480	0.00	0.51
	10	88.713	−0.02	4.03	88.446	0.00	0.25

Table 4. Comparison of Natural Frequencies and Mode Shapes of Equipment-Structure System with Exact Values: Equipment Tuned to 3rd Structure Mode

Mass Ratio γ_1	Mode	Equipment Attached to 5th Floor			Equipment Attached to 10th Floor		
		Exact Frequency rad/sec	Error in Frequency %	Error in Mode Shape %	Exact Frequency rad/sec	Error in Frequency %	Error in Mode Shape %
0.001	1	6.681	0.00	0.02	6.681	0.00	0.01
	2	19.882	0.00	0.23	19.888	0.00	0.01
	0	32.251	−0.01	0.24	32.208	0.02	1.05
	3	33.103	0.01	0.54	33.171	−0.02	0.82
	4	44.763	0.00	0.45	44.741	0.00	0.30
	5	55.773	0.00	0.19	55.776	0.00	0.15
	6	65.586	0.00	0.24	65.571	0.00	0.10
	7	73.903	0.00	0.12	73.904	0.00	0.07
	8	80.601	0.00	0.26	80.586	0.00	0.05
	9	85.469	0.00	0.09	85.470	0.00	0.03
	10	88.459	0.00	0.23	88.444	0.00	0.01
0.01	1	6.650	0.00	0.16	6.650	0.00	0.14
	2	19.699	0.00	2.28	19.755	−0.00	1.94
	0	31.370	−0.21	1.70	31.296	0.26	4.25
	3	33.981	0.23	3.39	34.296	−0.17	2.66
	4	45.126	0.01	4.35	44.917	−0.01	3.01
	5	55.826	0.00	1.91	55.858	−0.00	1.57
	6	65.769	−0.00	2.41	65.617	−0.00	1.03
	7	73.918	0.00	1.16	73.930	−0.00	0.70
	8	80.748	−0.00	2.65	80.600	−0.00	0.48
	9	85.473	0.00	0.93	85.476	−0.00	0.30
	10	88.600	−0.01	2.37	88.445	−0.00	0.15
0.05	1	6.514	0.04	0.82	6.515	0.02	0.68
	2	18.978	0.01	10.19	19.229	−0.08	8.76
	0	30.075	−1.32	7.60	30.038	1.32	12.21
	3	35.231	1.70	12.88	36.310	−0.12	11.25
	4	46.521	0.14	18.58	45.734	−0.21	15.03
	5	56.056	0.01	9.29	56.252	−0.08	8.26
	6	66.557	0.01	11.80	65.839	−0.03	5.40
	7	73.985	0.00	5.74	74.056	−0.02	3.72
	8	81.395	−0.00	13.69	80.666	−0.01	2.54
	9	85.486	0.00	4.64	85.504	−0.00	1.60
	10	89.344	−0.17	12.46	88.452	−0.00	0.79

To examine the accuracy of the response spectrum method for equipment response developed herein, an ensemble of 20 simulated ground acceleration time histories with a mean peak ground acceleration of 0.5g and a strong motion duration of 11 seconds were employed. The bases for generation of these time histories and the computation of their associated mean ground response spectra are discussed in Ref. 2. A typical member of the ensemble and the mean ground response spectra are shown in Figs. 6 and 7, respectively. Using these spectra,

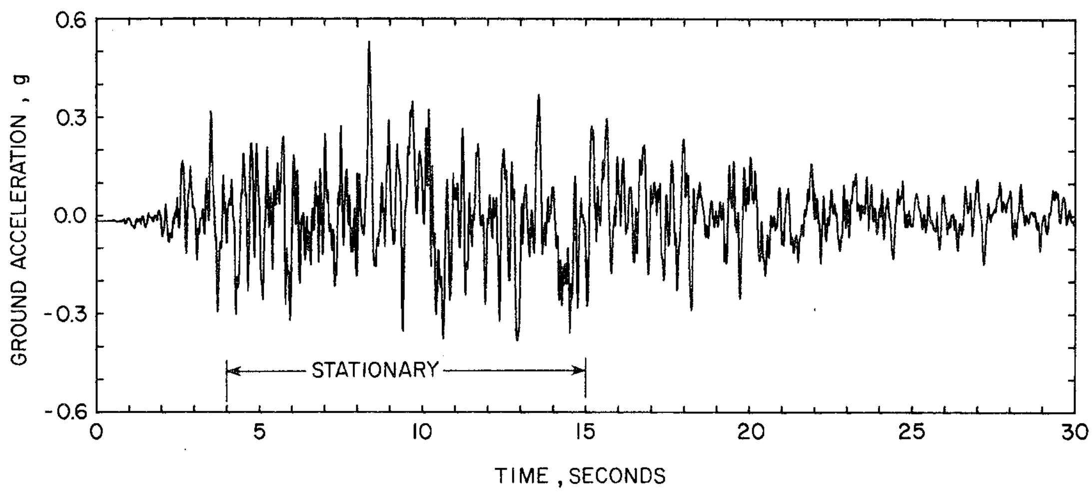


Figure 6. Sample of Simulated Earthquake Ground Motion

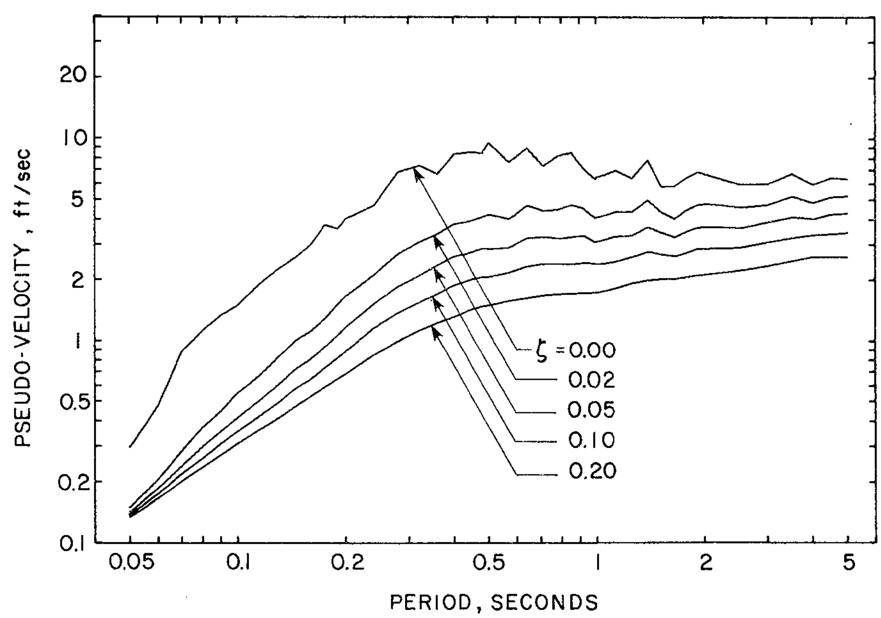


Figure 7. Mean Pseudo-Velocity Spectra for 20 Simulated Earthquakes

statistical measures of the equipment acceleration response (i.e., the root-mean-square, the mean of the peak, and the standard deviation of the peak) were computed for a continuous range of equipment frequencies, for the three values of γ_1, and for two values of equipment damping, $\zeta_e = 0.02$ (Fig. 8) and $\zeta_e = 0.05$ (Fig. 9). For brevity, the results are presented only

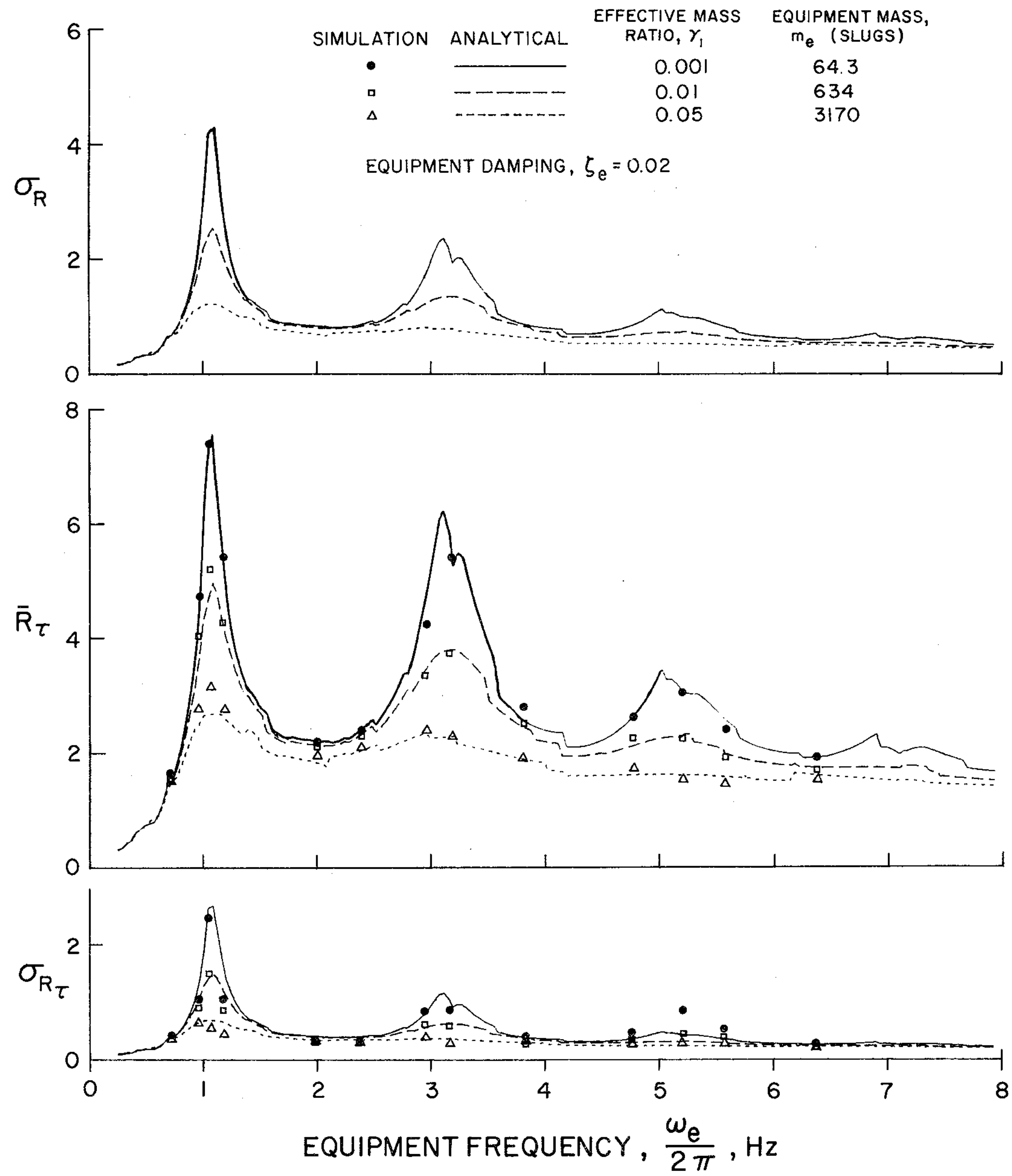


Figure 8. Acceleration Response Quantities for Equipment Attached to 10-th Floor with $\zeta_e = 0.02$: σ_R = root-mean-square; $\bar{R}_\tau$ = mean of peak; σ_{R_τ} = standard deviation of peak

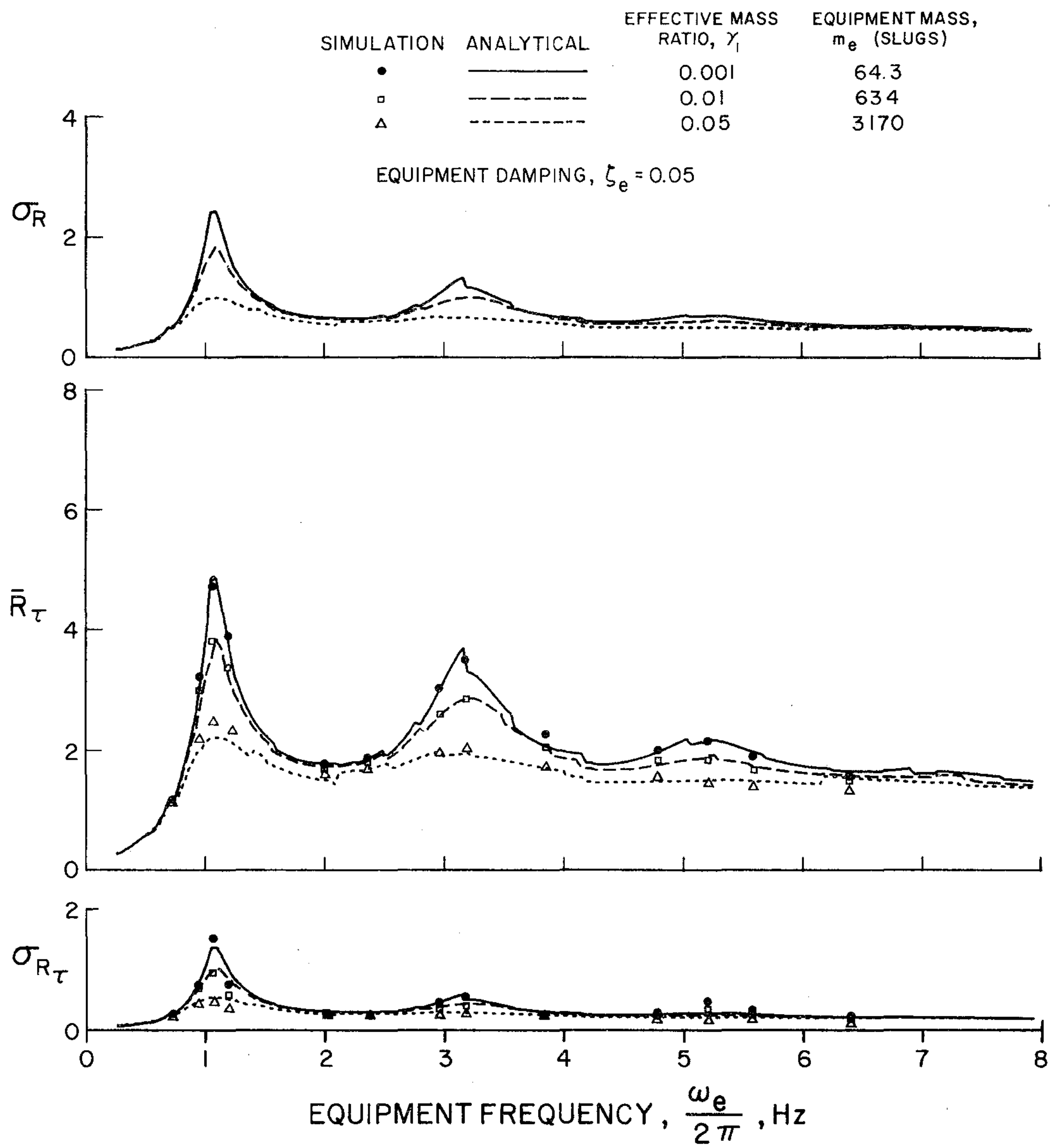


Figure 9. Acceleration Response Quantities for Equipment Attached to 10-th Floor with $\zeta_e = 0.05$: σ_R = root-mean-square; $\bar{R}_\tau$ = mean of peak; σ_{R_τ} = standard deviation of peak

for the case of the equipment attached to the 10-th floor of the structure. Results for attachment to the 5-th floor are of the same character. The computation involved determining the modal spectral moments from Eqs. 48-50 which were then used in Eq. 34 to give the spectral moments of the response. The response measures were then computed in terms of these spectral moments as described in the text following Eq. 34. In particular, the mean and standard deviation of the peak response were computed from Eqs. 51 and 53, respectively. It is noted that the simplified expression for the mean, Eq. 52, gave more or less the same results as Eq. 51 except at tuning to the first mode of the structure when up to 30 percent overestimation was observed for the lightest equipment mass. As indicated before, this is a consequence of the narrow-bandedness of the equipment response process for that case.

To provide a basis for comparison, ensembles of "exact" peak responses were computed for selected values of the equipment frequency by means of time-history analyses of the combined equipment-structure system for the simulated ground motions. A matrix time marching algorithm rather than a modal approach was used in this computation so as not to introduce any error due to nonmodal damping. Means and standard deviations of these ensembles were computed and are shown in Figs. 8 and 9 by various symbols for the three values of γ_1. The agreement between the analytical and the simulated results is remarkable.

From Figs. 8 and 9 the effect of equipment-structure interaction on the equipment response is seen to be significant. For example, with the equipment damping equal to 0.02, there is about 30 percent reduction in the mean peak equipment response at tuning when γ_1 is increased from 0.001 to 0.01. For the damping values considered, further reduction in the mass ratio below 0.001 did not cause any significant increase in the mean peak response. Thus, the mean peak response curves in Figs. 8 and 9 associated with $\gamma_1 = 0.001$ ($m_e = 63.4$ slugs) essentially represent the conventional floor spectra. The other curves, i.e. those associated with $\gamma_1 = 0.01$ ($m_e = 634$ slugs) and $\gamma_1 = 0.05$ ($m_e = 3170$ slugs) may be considered as floor spectra which include the effect of equipment-structure interaction. These spectra could be used directly by the equipment analyst to design any equipment item having a prescribed mass, frequency, and damping.

Another quantity of the equipment response that would be of interest to the analyst is the response mean frequency, $\bar{\omega}$, which provides an estimate of the number of cycles of oscillation. As indicated before, this quantity would be useful in studies concerning equipment fatigue. Using Eq. 54, this quantity for the acceleration response has been computed in terms of the input response spectrum for the equipment attached to the 10-th floor of the structure and is plotted in Fig. 10 for two damping values of the equipment. It is seen that the response mean frequency, $\bar{\omega}$, is slightly higher than the equipment frequency, ω_e, when ω_e is less than the fundamental frequency of the structure and tends to be less than ω_e when ω_e is greater than the fundamental frequency of the structure. An interesting observation is that $\bar{\omega}$ tends to

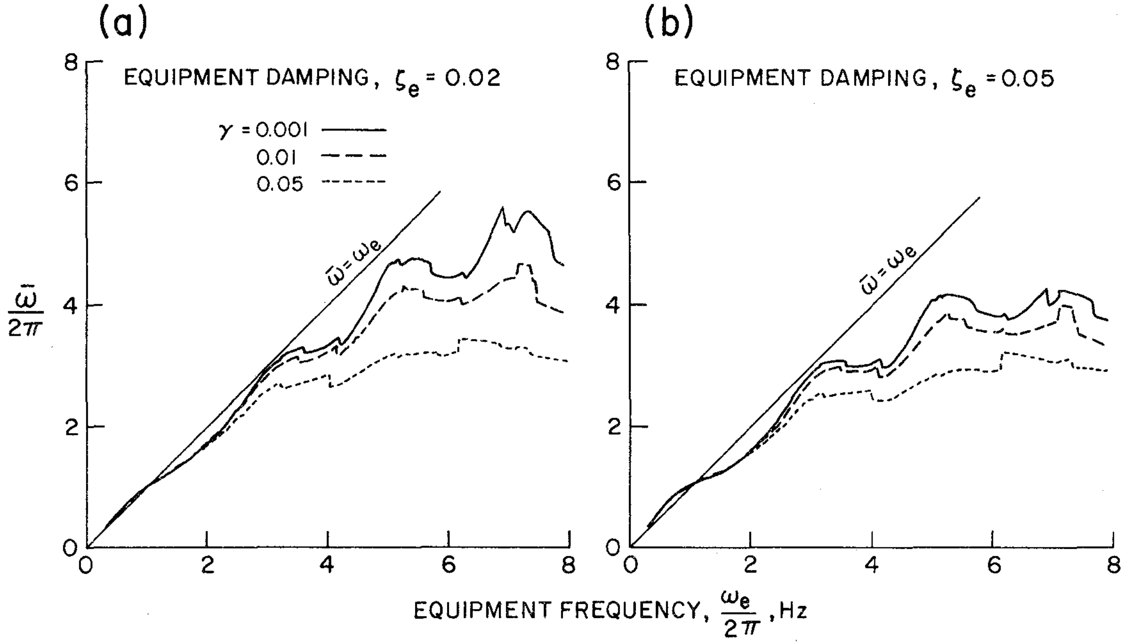


Figure 10. Mean Frequency of Acceleration Response for Equipment Attached to 10-th Floor

approach closer to ω_e when tuning occurs. This is a consequence of the fact that $\bar{\omega}$ is an average of the modal frequencies as weighted by the modal contributions to the response, and at tuning the tuning modes, which have frequencies close to ω_e, make dominant contributions.

SUMMARY AND CONCLUSIONS

A simple and accurate method is developed to estimate the response of light equipment in structures subjected to random excitations, such as earthquake ground motions. The method employs perturbation techniques to determine the dynamic properties of the combined equipment-structure system in terms of those of the structure alone and of the equipment alone. These derived properties are used to determine the equipment response by a modal superposition technique which accounts for cross-correlation between modal responses. The results include the effect of equipment-structure interaction, which is particularly significant when the equipment is tuned to a natural frequency of the structure. Two characterizations of the input excitation are employed; namely, the power spectral density approach for a stationary input and the response spectrum approach for earthquake-type excitations. Various statistical measures of the equipment response including its root-mean-square and that of its time derivative, and the mean, the standard deviation and the cumulative distribution function of the peak response over a specified duration, are obtained in terms of the dynamic properties of the two subsystems separately and the description of the input excitation. In the case of the response spectrum approach, which is of particular interest in practical applications in earthquake engineering, the equipment response is given in terms of the response spectrum associated with the input ground motion.

The method developed generates as a special case the conventional floor spectrum by simply setting the equipment mass equal to zero, which is equivalent to neglecting equipment-structure interaction. More generally, the method produces floor spectra which include the effect of interaction and are as easily applicable in practice as is the conventional spectra.

A comprehensive numerical study employing a 10-degree-of-freedom example structure is presented. The results demonstrate the accuracy of the methodology in generating the eigenproperties of the combined equipment-structure system and in estimating the response of the equipment. It is found that in a variety of situations, the equipment-structure interaction is significant and must be taken into account.

APPENDIX I.-CROSS-SPECTRAL MOMENTS FOR RESPONSE TO WHITE NOISE

Cross-spectral moments of normal coordinates corresponding to response to a stationary excitation can be expressed as

$$\lambda_{m,ij} = \rho_{m,ij}\sqrt{\lambda_{m,ii}\lambda_{m,jj}}, \quad m=0,1,2 \tag{59}$$

For the case of response to white noise input, from Ref. 3 or 10

$$\lambda_{0,ii} = \frac{\pi G_0}{4\zeta_i\omega_i^3} \tag{60}$$

$$\lambda_{1,ii} = \frac{\pi G_0}{4\zeta_i\omega_i^2}\,\frac{1-\dfrac{2}{\pi}\tan^{-1}(\zeta_i/\sqrt{1-\zeta_i^2})}{\sqrt{1-\zeta_i^2}} \tag{61}$$

$$\lambda_{2,ii} = \frac{\pi G_0}{4\zeta_i\omega_i} \tag{62}$$

where G_0 is the constant amplitude of the power spectral density. Exact expressions for $\rho_{m,ij}$ for this case were given in Ref. 3, which for small damping were shown to reduce to

$$\rho_{0,ij} = \frac{2\sqrt{\zeta_i\zeta_j}\left[(\omega_i+\omega_j)^2(\zeta_i+\zeta_j) + (\omega_i^2-\omega_j^2)(\zeta_i-\zeta_j)\right]}{4(\omega_i-\omega_j)^2+(\zeta_i+\zeta_j)^2(\omega_i+\omega_j)^2} \tag{63}$$

$$\rho_{1,ij} = \frac{2\sqrt{\zeta_i\zeta_j}\left[(\omega_i+\omega_j)^2(\zeta_i+\zeta_j) - \dfrac{4}{\pi}(\omega_i-\omega_j)^2\right]}{4(\omega_i-\omega_j)^2+(\zeta_i+\zeta_j)^2(\omega_i+\omega_j)^2} \tag{64}$$

$$\rho_{2,ij} = \frac{2\sqrt{\zeta_i\zeta_j}\left[(\omega_i+\omega_j)^2(\zeta_i+\zeta_j) - (\omega_i^2-\omega_j^2)(\zeta_i-\zeta_j)\right]}{4(\omega_i-\omega_j)^2+(\zeta_i+\zeta_j)^2(\omega_i+\omega_j)^2} \tag{65}$$

APPENDIX II.-REFERENCES

[1] Crandall, S.H., and Mark, W.D., *Random Vibration of Mechanical Systems*, Academic Press, New York, N.Y., 1963.

[2] Der Kiureghian, A., "A Response Spectrum Method for Random Vibrations," *Report No. UCB/EERC-80/15*, Earthquake Engineering Research Center, University of California, Berkeley, CA., June, 1980.

[3] Der Kiureghian, A., "Structural Response to Stationary Excitation," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 106, No. EM6, Proc. Paper 15898, December, 1980, pp. 1195-1213.

[4] Kelly, J.M., and Sackman, J.L., "Conservatism in Summation Rules for Closely Spaced Modes," *Earthquake Engineering and Structural Dynamics*, Vol. 8, 1980, pp. 63-74.

[5] Meirovitch, L., *Analytical Methods in Vibration*, The Macmillan Co., New York, N.Y., 1967.

[6] Peters, K.A., Schmitz, D., and Wagner, U., "Determination of Floor Response Spectra on the Basis of the Response Spectrum Method," *Nuclear Engineering and Design*, Vol. 44, 1977, pp. 255-262.

[7] Sackman J.L., and Kelly, J.M., "Seismic Analysis of Internal Equipment and Components in Structures," *Engineering Structures*, Vol. 1, No. 4, July, 1979, pp. 179-190.

[8] Singh, M.P., "Generation of Seismic Floor Spectra," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 101, No. EM5, Proc. Paper 11651, October, 1975, pp. 593-607.

[9] Singh, M.P., "Seismic Design Input for Secondary Structures," *Journal of the Structural Division*, ASCE, Vol. 106, No. ST2, Proc. Paper 15207, February, 1980, pp. 505-517.

[10] Vanmarcke, E.H., "Properties of Spectral Moments with Application to Random Vibration," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 98, No. EM2, Proc. Paper 8822, April, 1972, 425-446.

[11] Vanmarcke, E.H., "On the Distribution of the First-Passage Time for Normal Stationary Random Processes," *Journal of Applied Mechanics*, Vol 42, March, 1975, pp. 215-220.

[12] Vanmarcke, E.H., "Structural Response to Earthquakes," in *Seismic Risk and Engineering Decision*, C. Lomnitz and E. Rosenblueth, Editors, Elsevier, New York, 1976.

APPENDIX III.- NOTATION

The following symbols are used in this report:

$a = r/\sigma_R$	normalized response level;
$\mathbf{C}$	damping matrix of structure alone;
$\mathbf{C}^*$	damping matrix of combined system;
c_{ij}	element of $\mathbf{C}$;
$\mathbf{D}$	diagonal matrix with elements M_i;
$F(t)$	zero-mean stationary input process;
$F_{R_\tau}(r)$	cumulative probability distribution function of R_τ;
$G_F(\omega)$	one-sided power spectral density of input process;
$G_R(\omega)$	one-sided power spectral density of equipment response;
$H_i(\omega)$	complex frequency-response function for mode i;
$\mathbf{K}$	stiffness matrix of structure alone;
$\mathbf{K}^*$	stiffness matrix of combined system;
k_{ij}	element of $\mathbf{K}$;
l	mode number of structure to which equipment is tuned;
$\mathbf{M}$	mass matrix of structure alone;
$\mathbf{M}^*$	mass matrix of combined system;
M_i	i-th modal mass of structure alone;
M_i^*	i-th modal mass of combined system;
m_{ij}	element of $\mathbf{M}$;
m_e	mass of equipment;
n	number of degrees of freedom of structure;
p,q	peak factors for equipment response;
p_i,q_i	peak factors for response in mode i;
$R(t)$	equipment response;
$R_i(t)$	contribution of mode i to equipment response;
R_τ	peak of $R(t)$ over τ;
$\bar{R}_\tau$	mean of R_τ;
$\bar{R}_{i\tau}$	mean of peak contribution of mode i to equipment response;
$\mathbf{R}$	influence vector for structure alone;
$\mathbf{R}^*$	influence vector for combined system;
r	measure of modification of frequencies and mode shapes;
r_i	element of $\mathbf{R}$ and $\mathbf{R}^*$;
r_{n+1}^*	element of $\mathbf{R}^*$ associated with equipment degree of freedom;
$S_i(t)$	i-th normal coordinate of combined system;
$\bar{S}_\tau(\omega,\zeta)$	response spectrum ordinate at frequency ω and damping ζ;
α_i	modal amplification factor relative to attachment point for mode i;
β_i	detuning parameter associated with mode i;
Γ_i	participation factor associated with i-th mode of structure;
Γ_i^*	participation factor associated with i-th mode of combined system;
γ_i	effective mass ratio associated with mode i;
$\mathbf{\Delta}_i$	difference vector associated with mode i;
δ	power spectral density shape factor;
δ_e	an effective value of δ;
δ_i	shape factor for response in mode i;
ϵ_m	constant associated with λ_m;
ζ_a	arithmetic mean of ζ_e and ζ_l;
ζ_e	damping coefficient of equipment;
ζ_g	geometric mean of ζ_e and ζ_l;
ζ_i	i-th modal damping coefficient of structure alone;

ζ_i^*	i-th modal damping coefficient of combined system;
λ_m	m-th spectral moment of equipment response;
$\lambda_{m,aa}$	m-th spectral moment associated with ω_a and ζ_a;
$\lambda_{m,ee}$	m-th spectral moment associated with ω_e and ζ_e;
$\lambda_{m,ij}$	m-th cross spectral moment associated with modes i and j of structure alone;
$\lambda_{m,ij}^*$	m-th cross spectral moment associated with modes i and j of combined system;
ν	mean zero-crossing rate of $R(t)$;
ν_e	reduced mean zero-crossing rate;
ν_i	mean zero-crossing rate associated with mode i;
$\rho_{m,ij}$	cross-correlation coefficient between modal responses of combined system;
σ_R	root-mean-square of $R(t)$;
$\sigma_{\dot{R}}$	root-mean-square of time-derivative of $R(t)$;
σ_{R_τ}	standard deviation of R_τ;
τ	duration of stationary excitation;
$\mathbf{\Phi}$	modal matrix of structure alone;
$\mathbf{\Phi}^*$	modal matrix of combined system;
$\mathbf{\Phi}_i, \mathbf{\Phi}_i^*$	i-th columns of $\mathbf{\Phi}$ and $\mathbf{\Phi}^*$, respectively;
$\mathbf{\Phi}_i^{*E}$	exact value of $\mathbf{\Phi}_i^{*E}$;
ϕ_{mi}, ϕ_{mi}^*	m-th elements of $\mathbf{\Phi}_i$ and $\mathbf{\Phi}_i^*$, respectively;
Ψ_i	effective participation factor for attachment point for i-th mode of structure alone;
Ψ_i^*	effective participation factor for i-th mode of combined system;
ω	frequency;
$\bar{\omega}$	response mean frequency;
ω_a	arithmetic mean of ω_e and ω_l;
ω_e	equipment frequency;
ω_i	i-th modal frequency of structure alone;
ω_i^*	i-th modal frequency of combined system;

EARTHQUAKE ENGINEERING RESEARCH CENTER REPORTS

NOTE: Numbers in parenthesis are Accession Numbers assigned by the National Technical Information Service; these are followed by a price code. Copies of the reports may be ordered from the National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia, 22161. Accession Numbers should be quoted on orders for reports (PB --- ---) and remittance must accompany each order. Reports without this information were not available at time of printing. Upon request, EERC will mail inquirers this information when it becomes available.

EERC 67-1 "Feasibility Study Large-Scale Earthquake Simulator Facility," by J. Penzien, J.G. Bouwkamp, R.W. Clough and D. Rea - 1967 (PB 187 905)A07

EERC 68-1 Unassigned

EERC 68-2 "Inelastic Behavior of Beam-to-Column Subassemblages Under Repeated Loading," by V.V. Bertero - 1968 (PB 184 888)A05

EERC 68-3 "A Graphical Method for Solving the Wave Reflection-Refraction Problem," by H.D. McNiven and Y. Mengi - 1968 (PB 187 943)A03

EERC 68-4 "Dynamic Properties of McKinley School Buildings," by D. Rea, J.G. Bouwkamp and R.W. Clough - 1968 (PB 187 902)A07

EERC 68-5 "Characteristics of Rock Motions During Earthquakes," by H.B. Seed, I.M. Idriss and F.W. Kiefer - 1968 (PB 188 338)A03

EERC 69-1 "Earthquake Engineering Research at Berkeley," - 1969 (PB 187 906)All

EERC 69-2 "Nonlinear Seismic Response of Earth Structures," by M. Dibaj and J. Penzien - 1969 (PB 187 904)A08

EERC 69-3 "Probabilistic Study of the Behavior of Structures During Earthquakes," by R. Ruiz and J. Penzien - 1969 (PB 187 886)A06

EERC 69-4 "Numerical Solution of Boundary Value Problems in Structural Mechanics by Reduction to an Initial Value Formulation," by N. Distefano and J. Schujman - 1969 (PB 187 942)A02

EERC 69-5 "Dynamic Programming and the Solution of the Biharmonic Equation," by N. Distefano - 1969 (PB 187 941)A03

EERC 69-6 "Stochastic Analysis of Offshore Tower Structures," by A.K. Malhotra and J. Penzien - 1969 (PB 187 903)A09

EERC 69-7 "Rock Motion Accelerograms for High Magnitude Earthquakes," by H.B. Seed and I.M. Idriss - 1969 (PB 187 940)A02

EERC 69-8 "Structural Dynamics Testing Facilities at the University of California, Berkeley," by R.M. Stephen, J.G. Bouwkamp, R.W. Clough and J. Penzien - 1969 (PB 189 111)A04

EERC 69-9 "Seismic Response of Soil Deposits Underlain by Sloping Rock Boundaries," by H. Dezfulian and H.B. Seed 1969 (PB 189 114)A03

EERC 69-10 "Dynamic Stress Analysis of Axisymmetric Structures Under Arbitrary Loading," by S. Ghosh and E.L. Wilson 1969 (PB 189 026)A10

EERC 69-11 "Seismic Behavior of Multistory Frames Designed by Different Philosophies," by J.C. Anderson and V. V. Bertero - 1969 (PB 190 662)A10

EERC 69-12 "Stiffness Degradation of Reinforcing Concrete Members Subjected to Cyclic Flexural Moments," by V.V. Bertero, B. Bresler and H. Ming Liao - 1969 (PB 202 942)A07

EERC 69-13 "Response of Non-Uniform Soil Deposits to Travelling Seismic Waves," by H. Dezfulian and H.B. Seed - 1969 (PB 191 023)A03

EERC 69-14 "Damping Capacity of a Model Steel Structure," by D. Rea, R.W. Clough and J.G. Bouwkamp - 1969 (PB 190 663)A06

EERC 69-15 "Influence of Local Soil Conditions on Building Damage Potential during Earthquakes," by H.B. Seed and I.M. Idriss - 1969 (PB 191 036)A03

EERC 69-16 "The Behavior of Sands Under Seismic Loading Conditions," by M.L. Silver and H.B. Seed - 1969 (AD 714 982)A07

EERC 70-1 "Earthquake Response of Gravity Dams," by A.K. Chopra - 1970 (AD 709 640)A03

EERC 70-2 "Relationships between Soil Conditions and Building Damage in the Caracas Earthquake of July 29, 1967," by H.B. Seed, I.M. Idriss and H. Dezfulian - 1970 (PB 195 762)A05

EERC 70-3 "Cyclic Loading of Full Size Steel Connections," by E.P. Popov and R.M. Stephen - 1970 (PB 213 545)A04

EERC 70-4 "Seismic Analysis of the Charaima Building, Caraballeda, Venezuela," by Subcommittee of the SEAONC Research Committee: V.V. Bertero, P.F. Fratessa, S.A. Mahin, J.H. Sexton, A.C. Scordelis, E.L. Wilson, L.A. Wyllie, H.B. Seed and J. Penzien, Chairman - 1970 (PB 201 455)A06

EERC 70-5 "A Computer Program for Earthquake Analysis of Dams," by A.K. Chopra and P. Chakrabarti - 1970 (AD 723 994)A05

EERC 70-6 "The Propagation of Love Waves Across Non-Horizontally Layered Structures," by J. Lysmer and L.A. Drake 1970 (PB 197 896)A03

EERC 70-7 "Influence of Base Rock Characteristics on Ground Response," by J. Lysmer, H.B. Seed and P.B. Schnabel 1970 (PB 197 897)A03

EERC 70-8 "Applicability of Laboratory Test Procedures for Measuring Soil Liquefaction Characteristics under Cyclic Loading," by H.B. Seed and W.H. Peacock - 1970 (PB 198 016)A03

EERC 70-9 "A Simplified Procedure for Evaluating Soil Liquefaction Potential," by H.B. Seed and I.M. Idriss - 1970 (PB 198 009)A03

EERC 70-10 "Soil Moduli and Damping Factors for Dynamic Response Analysis," by H.B. Seed and I.M. Idriss - 1970 (PB 197 869)A03

EERC 71-1 "Koyna Earthquake of December 11, 1967 and the Performance of Koyna Dam," by A.K. Chopra and P. Chakrabarti 1971 (AD 731 496)A06

EERC 71-2 "Preliminary In-Situ Measurements of Anelastic Absorption in Soils Using a Prototype Earthquake Simulator," by R.D. Borcherdt and P.W. Rodgers - 1971 (PB 201 454)A03

EERC 71-3 "Static and Dynamic Analysis of Inelastic Frame Structures," by F.L. Porter and G.H. Powell - 1971 (PB 210 135)A06

EERC 71-4 "Research Needs in Limit Design of Reinforced Concrete Structures," by V.V. Bertero - 1971 (PB 202 943)A04

EERC 71-5 "Dynamic Behavior of a High-Rise Diagonally Braced Steel Building," by D. Rea, A.A. Shah and J.G. Bouwkamp 1971 (PB 203 584)A06

EERC 71-6 "Dynamic Stress Analysis of Porous Elastic Solids Saturated with Compressible Fluids," by J. Ghaboussi and E. L. Wilson - 1971 (PB 211 396)A06

EERC 71-7 "Inelastic Behavior of Steel Beam-to-Column Subassemblages," by H. Krawinkler, V.V. Bertero and E.P. Popov 1971 (PB 211 335)A14

EERC 71-8 "Modification of Seismograph Records for Effects of Local Soil Conditions," by P. Schnabel, H.B. Seed and J. Lysmer - 1971 (PB 214 450)A03

EERC 72-1 "Static and Earthquake Analysis of Three Dimensional Frame and Shear Wall Buildings," by E.L. Wilson and H.H. Dovey - 1972 (PB 212 904)A05

EERC 72-2 "Accelerations in Rock for Earthquakes in the Western United States," by P.B. Schnabel and H.B. Seed - 1972 (PB 213 100)A03

EERC 72-3 "Elastic-Plastic Earthquake Response of Soil-Building Systems," by T. Minami - 1972 (PB 214 868)A08

EERC 72-4 "Stochastic Inelastic Response of Offshore Towers to Strong Motion Earthquakes," by M.K. Kaul - 1972 (PB 215 713)A05

EERC 72-5 "Cyclic Behavior of Three Reinforced Concrete Flexural Members with High Shear," by E.P. Popov, V.V. Bertero and H. Krawinkler - 1972 (PB 214 555)A05

EERC 72-6 "Earthquake Response of Gravity Dams Including Reservoir Interaction Effects," by P. Chakrabarti and A.K. Chopra - 1972 (AD 762 330)A08

EERC 72-7 "Dynamic Properties of Pine Flat Dam," by D. Rea, C.Y. Liaw and A.K. Chopra - 1972 (AD 763 928)A05

EERC 72-8 "Three Dimensional Analysis of Building Systems," by E.L. Wilson and H.H. Dovey - 1972 (PB 222 438)A06

EERC 72-9 "Rate of Loading Effects on Uncracked and Repaired Reinforced Concrete Members," by S. Mahin, V.V. Bertero, D. Rea and M. Atalay - 1972 (PB 224 520)A08

EERC 72-10 "Computer Program for Static and Dynamic Analysis of Linear Structural Systems," by E.L. Wilson, K.-J. Bathe, J.E. Peterson and H.H.Dovey - 1972 (PB 220 437)A04

EERC 72-11 "Literature Survey - Seismic Effects on Highway Bridges," by T. Iwasaki, J. Penzien and R.W. Clough - 1972 (PB 215 613)A19

EERC 72-12 "SHAKE-A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites," by P.B. Schnabel and J. Lysmer - 1972 (PB 220 207)A06

EERC 73-1 "Optimal Seismic Design of Multistory Frames," by V.V. Bertero and H. Kamil - 1973

EERC 73-2 "Analysis of the Slides in the San Fernando Dams During the Earthquake of February 9, 1971," by H.B. Seed, K.L. Lee, I.M. Idriss and F. Makdisi - 1973 (PB 223 402)A14

EERC 73-3 "Computer Aided Ultimate Load Design of Unbraced Multistory Steel Frames," by M.B. El-Hafez and G.H. Powell 1973 (PB 248 315)A09

EERC 73-4 "Experimental Investigation into the Seismic Behavior of Critical Regions of Reinforced Concrete Components as Influenced by Moment and Shear," by M. Celebi and J. Penzien - 1973 (PB 215 884)A09

EERC 73-5 "Hysteretic Behavior of Epoxy-Repaired Reinforced Concrete Beams," by M. Celebi and J. Penzien - 1973 (PB 239 568)A03

EERC 73-6 "General Purpose Computer Program for Inelastic Dynamic Response of Plane Structures," by A. Kanaan and G.H. Powell - 1973 (PB 221 260)A08

EERC 73-7 "A Computer Program for Earthquake Analysis of Gravity Dams Including Reservoir Interaction," by P. Chakrabarti and A.K. Chopra - 1973 (AD 766 271)A04

EERC 73-8 "Behavior of Reinforced Concrete Deep Beam-Column Subassemblages Under Cyclic Loads," by O. Küstü and J.G. Bouwkamp - 1973 (PB 246 117)A12

EERC 73-9 "Earthquake Analysis of Structure-Foundation Systems," by A.K. Vaish and A.K. Chopra - 1973 (AD 766 272)A07

EERC 73-10 "Deconvolution of Seismic Response for Linear Systems," by R.B. Reimer - 1973 (PB 227 179)A08

EERC 73-11 "SAP IV: A Structural Analysis Program for Static and Dynamic Response of Linear Systems," by K.-J. Bathe, E.L. Wilson and F.E. Peterson - 1973 (PB 221 967)A09

EERC 73-12 "Analytical Investigations of the Seismic Response of Long, Multiple Span Highway Bridges," by W.S. Tseng and J. Penzien - 1973 (PB 227 816)A10

EERC 73-13 "Earthquake Analysis of Multi-Story Buildings Including Foundation Interaction," by A.K. Chopra and J.A. Gutierrez - 1973 (PB 222 970)A03

EERC 73-14 "ADAP: A Computer Program for Static and Dynamic Analysis of Arch Dams," by R.W. Clough, J.M. Raphael and S. Mojtahedi - 1973 (PB 223 763)A09

EERC 73-15 "Cyclic Plastic Analysis of Structural Steel Joints," by R.B. Pinkney and R.W. Clough - 1973 (PB 226 843)A08

EERC 73-16 "QUAD-4: A Computer Program for Evaluating the Seismic Response of Soil Structures by Variable Damping Finite Element Procedures," by I.M. Idriss, J. Lysmer, R. Hwang and H.B. Seed - 1973 (PB 229 424)A05

EERC 73-17 "Dynamic Behavior of a Multi-Story Pyramid Shaped Building," by R.M. Stephen, J.P. Hollings and J.G. Bouwkamp - 1973 (PB 240 718)A06

EERC 73-18 "Effect of Different Types of Reinforcing on Seismic Behavior of Short Concrete Columns," by V.V. Bertero, J. Hollings, O. Küstü, R.M. Stephen and J.G. Bouwkamp - 1973

EERC 73-19 "Olive View Medical Center Materials Studies, Phase I," by B. Bresler and V.V. Bertero - 1973 (PB 235 986)A06

EERC 73-20 "Linear and Nonlinear Seismic Analysis Computer Programs for Long Multiple-Span Highway Bridges," by W.S. Tseng and J. Penzien - 1973

EERC 73-21 "Constitutive Models for Cyclic Plastic Deformation of Engineering Materials," by J.M. Kelly and P.P. Gillis 1973 (PB 226 024)A03

EERC 73-22 "DRAIN - 2D User's Guide," by G.H. Powell - 1973 (PB 227 016)A05

EERC 73-23 "Earthquake Engineering at Berkeley - 1973," (PB 226 033)A11

EERC 73-24 Unassigned

EERC 73-25 "Earthquake Response of Axisymmetric Tower Structures Surrounded by Water," by C.Y. Liaw and A.K. Chopra 1973 (AD 773 052)A09

EERC 73-26 "Investigation of the Failures of the Olive View Stairtowers During the San Fernando Earthquake and Their Implications on Seismic Design," by V.V. Bertero and R.G. Collins - 1973 (PB 235 106)A13

EERC 73-27 "Further Studies on Seismic Behavior of Steel Beam-Column Subassemblages," by V.V. Bertero, H. Krawinkler and E.P. Popov - 1973 (PB 234 172)A06

EERC 74-1 "Seismic Risk Analysis," by C.S. Oliveira - 1974 (PB 235 920)A06

EERC 74-2 "Settlement and Liquefaction of Sands Under Multi-Directional Shaking," by R. Pyke, C.K. Chan and H.B. Seed 1974

EERC 74-3 "Optimum Design of Earthquake Resistant Shear Buildings," by D. Ray, K.S. Pister and A.K. Chopra - 1974 (PB 231 172)A06

EERC 74-4 "LUSH - A Computer Program for Complex Response Analysis of Soil-Structure Systems," by J. Lysmer, T. Udaka, H.B. Seed and R. Hwang - 1974 (PB 236 796)A05

EERC 74-5 "Sensitivity Analysis for Hysteretic Dynamic Systems: Applications to Earthquake Engineering," by D. Ray 1974 (PB 233 213)A06

EERC 74-6 "Soil Structure Interaction Analyses for Evaluating Seismic Response," by H.B. Seed, J. Lysmer and R. Hwang 1974 (PB 236 519)A04

EERC 74-7 Unassigned

EERC 74-8 "Shaking Table Tests of a Steel Frame - A Progress Report," by R.W. Clough and D. Tang - 1974 (PB 240 869)A03

EERC 74-9 "Hysteretic Behavior of Reinforced Concrete Flexural Members with Special Web Reinforcement," by V.V. Bertero, E.P. Popov and T.Y. Wang - 1974 (PB 236 797)A07

EERC 74-10 "Applications of Reliability-Based, Global Cost Optimization to Design of Earthquake Resistant Structures," by E. Vitiello and K.S. Pister - 1974 (PB 237 231)A06

EERC 74-11 "Liquefaction of Gravelly Soils Under Cyclic Loading Conditions," by R.T. Wong, H.B. Seed and C.K. Chan 1974 (PB 242 042)A03

EERC 74-12 "Site-Dependent Spectra for Earthquake-Resistant Design," by H.B. Seed, C. Ugas and J. Lysmer - 1974 (PB 240 953)A03

EERC 74-13 "Earthquake Simulator Study of a Reinforced Concrete Frame," by P. Hidalgo and R.W. Clough - 1974 (PB 241 944)A13

EERC 74-14 "Nonlinear Earthquake Response of Concrete Gravity Dams," by N. Pal - 1974 (AD/A 006 583)A06

EERC 74-15 "Modeling and Identification in Nonlinear Structural Dynamics - I. One Degree of Freedom Models," by N. Distefano and A. Rath - 1974 (PB 241 548)A06

EERC 75-1 "Determination of Seismic Design Criteria for the Dumbarton Bridge Replacement Structure, Vol. I: Description, Theory and Analytical Modeling of Bridge and Parameters," by F. Baron and S.-H. Pang - 1975 (PB 259 407)A15

EERC 75-2 "Determination of Seismic Design Criteria for the Dumbarton Bridge Replacement Structure, Vol. II: Numerical Studies and Establishment of Seismic Design Criteria," by F. Baron and S.-H. Pang - 1975 (PB 259 408)A11 (For set of EERC 75-1 and 75-2 (PB 259 406))

EERC 75-3 "Seismic Risk Analysis for a Site and a Metropolitan Area," by C.S. Oliveira - 1975 (PB 248 134)A09

EERC 75-4 "Analytical Investigations of Seismic Response of Short, Single or Multiple-Span Highway Bridges," by M.-C. Chen and J. Penzien - 1975 (PB 241 454)A09

EERC 75-5 "An Evaluation of Some Methods for Predicting Seismic Behavior of Reinforced Concrete Buildings," by S.A. Mahin and V.V. Bertero - 1975 (PB 246 306)A16

EERC 75-6 "Earthquake Simulator Study of a Steel Frame Structure, Vol. I: Experimental Results," by R.W. Clough and D.T. Tang - 1975 (PB 243 981)A13

EERC 75-7 "Dynamic Properties of San Bernardino Intake Tower," by D. Rea, C.-Y. Liaw and A.K. Chopra - 1975 (AD/A008 406) A05

EERC 75-8 "Seismic Studies of the Articulation for the Dumbarton Bridge Replacement Structure, Vol. I: Description, Theory and Analytical Modeling of Bridge Components," by F. Baron and R.E. Hamati - 1975 (PB 251 539)A07

EERC 75-9 "Seismic Studies of the Articulation for the Dumbarton Bridge Replacement Structure, Vol. 2: Numerical Studies of Steel and Concrete Girder Alternates," by F. Baron and R.E. Hamati - 1975 (PB 251 540)A10

EERC 75-10 "Static and Dynamic Analysis of Nonlinear Structures," by D.P. Mondkar and G.H. Powell - 1975 (PB 242 434)A08

EERC 75-11 "Hysteretic Behavior of Steel Columns," by E.P. Popov, V.V. Bertero and S. Chandramouli - 1975 (PB 252 365)A11

EERC 75-12 "Earthquake Engineering Research Center Library Printed Catalog," - 1975 (PB 243 711)A26

EERC 75-13 "Three Dimensional Analysis of Building Systems (Extended Version)," by E.L. Wilson, J.P. Hollings and H.H. Dovey - 1975 (PB 243 989)A07

EERC 75-14 "Determination of Soil Liquefaction Characteristics by Large-Scale Laboratory Tests," by P. De Alba, C.K. Chan and H.B. Seed - 1975 (NUREG 0027)A08

EERC 75-15 "A Literature Survey - Compressive, Tensile, Bond and Shear Strength of Masonry," by R.L. Mayes and R.W. Clough - 1975 (PB 246 292)A10

EERC 75-16 "Hysteretic Behavior of Ductile Moment Resisting Reinforced Concrete Frame Components," by V.V. Bertero and E.P. Popov - 1975 (PB 246 388)A05

EERC 75-17 "Relationships Between Maximum Acceleration, Maximum Velocity, Distance from Source, Local Site Conditions for Moderately Strong Earthquakes," by H.B. Seed, R. Murarka, J. Lysmer and I.M. Idriss - 1975 (PB 248 172)A03

EERC 75-18 "The Effects of Method of Sample Preparation on the Cyclic Stress-Strain Behavior of Sands," by J. Mulilis, C.K. Chan and H.B. Seed - 1975 (Summarized in EERC 75-28)

EERC 75-19 "The Seismic Behavior of Critical Regions of Reinforced Concrete Components as Influenced by Moment, Shear and Axial Force," by M.B. Atalay and J. Penzien - 1975 (PB 258 842)A11

EERC 75-20 "Dynamic Properties of an Eleven Story Masonry Building," by R.M. Stephen, J.P. Hollings, J.G. Bouwkamp and D. Jurukovski - 1975 (PB 246 945)A04

EERC 75-21 "State-of-the-Art in Seismic Strength of Masonry - An Evaluation and Review," by R.L. Mayes and R.W. Clough 1975 (PB 249 040)A07

EERC 75-22 "Frequency Dependent Stiffness Matrices for Viscoelastic Half-Plane Foundations," by A.K. Chopra, P. Chakrabarti and G. Dasgupta - 1975 (PB 248 121)A07

EERC 75-23 "Hysteretic Behavior of Reinforced Concrete Framed Walls," by T.Y. Wong, V.V. Bertero and E.P. Popov - 1975

EERC 75-24 "Testing Facility for Subassemblages of Frame-Wall Structural Systems," by V.V. Bertero, E.P. Popov and T. Endo - 1975

EERC 75-25 "Influence of Seismic History on the Liquefaction Characteristics of Sands," by H.B. Seed, K. Mori and C.K. Chan - 1975 (Summarized in EERC 75-28)

EERC 75-26 "The Generation and Dissipation of Pore Water Pressures during Soil Liquefaction," by H.B. Seed, P.P. Martin and J. Lysmer - 1975 (PB 252 648)A03

EERC 75-27 "Identification of Research Needs for Improving Aseismic Design of Building Structures," by V.V. Bertero 1975 (PB 248 136)A05

EERC 75-28 "Evaluation of Soil Liquefaction Potential during Earthquakes," by H.B. Seed, I. Arango and C.K. Chan - 1975 (NUREG 0026)A13

EERC 75-29 "Representation of Irregular Stress Time Histories by Equivalent Uniform Stress Series in Liquefaction Analyses," by H.B. Seed, I.M. Idriss, F. Makdisi and N. Banerjee - 1975 (PB 252 635)A03

EERC 75-30 "FLUSH - A Computer Program for Approximate 3-D Analysis of Soil-Structure Interaction Problems," by J. Lysmer, T. Udaka, C.-F. Tsai and H.B. Seed - 1975 (PB 259 332)A07

EERC 75-31 "ALUSH - A Computer Program for Seismic Response Analysis of Axisymmetric Soil-Structure Systems," by E. Berger, J. Lysmer and H.B. Seed - 1975

EERC 75-32 "TRIP and TRAVEL - Computer Programs for Soil-Structure Interaction Analysis with Horizontally Travelling Waves," by T. Udaka, J. Lysmer and H.B. Seed - 1975

EERC 75-33 "Predicting the Performance of Structures in Regions of High Seismicity," by J. Penzien - 1975 (PB 248 130)A03

EERC 75-34 "Efficient Finite Element Analysis of Seismic Structure - Soil - Direction," by J. Lysmer, H.B. Seed, T. Udaka, R.N. Hwang and C.-F. Tsai - 1975 (PB 253 570)A03

EERC 75-35 "The Dynamic Behavior of a First Story Girder of a Three-Story Steel Frame Subjected to Earthquake Loading," by R.W. Clough and L.-Y. Li - 1975 (PB 248 841)A05

EERC 75-36 "Earthquake Simulator Study of a Steel Frame Structure, Volume II - Analytical Results," by D.T. Tang - 1975 (PB 252 926)A10

EERC 75-37 "ANSR-I General Purpose Computer Program for Analysis of Non-Linear Structural Response," by D.P. Mondkar and G.H. Powell - 1975 (PB 252 386)A08

EERC 75-38 "Nonlinear Response Spectra for Probabilistic Seismic Design and Damage Assessment of Reinforced Concrete Structures," by M. Murakami and J. Penzien - 1975 (PB 259 530)A05

EERC 75-39 "Study of a Method of Feasible Directions for Optimal Elastic Design of Frame Structures Subjected to Earthquake Loading," by N.D. Walker and K.S. Pister - 1975 (PB 257 781)A06

EERC 75-40 "An Alternative Representation of the Elastic-Viscoelastic Analogy," by G. Dasgupta and J.L. Sackman - 1975 (PB 252 173)A03

EERC 75-41 "Effect of Multi-Directional Shaking on Liquefaction of Sands," by H.B. Seed, R. Pyke and G.R. Martin - 1975 (PB 258 781)A03

EERC 76-1 "Strength and Ductility Evaluation of Existing Low-Rise Reinforced Concrete Buildings - Screening Method," by T. Okada and B. Bresler - 1976 (PB 257 906)A11

EERC 76-2 "Experimental and Analytical Studies on the Hysteretic Behavior of Reinforced Concrete Rectangular and T-Beams," by S.-Y.M. Ma, E.P. Popov and V.V. Bertero - 1976 (PB 260 843)A12

EERC 76-3 "Dynamic Behavior of a Multistory Triangular-Shaped Building," by J. Petrovski, R.M. Stephen, E. Gartenbaum and J.G. Bouwkamp - 1976 (PB 273 279)A07

EERC 76-4 "Earthquake Induced Deformations of Earth Dams," by N. Serff, H.B. Seed, F.I. Makdisi & C.-Y. Chang - 1976 (PB 292 065)A08

EERC 76-5 "Analysis and Design of Tube-Type Tall Building Structures," by H. de Clercq and G.H. Powell - 1976 (PB 252 220) A10

EERC 76-6 "Time and Frequency Domain Analysis of Three-Dimensional Ground Motions, San Fernando Earthquake," by T. Kubo and J. Penzien (PB 260 556)A11

EERC 76-7 "Expected Performance of Uniform Building Code Design Masonry Structures," by R.L. Mayes, Y. Omote, S.W. Chen and R.W. Clough - 1976 (PB 270 098)A05

EERC 76-8 "Cyclic Shear Tests of Masonry Piers, Volume 1 - Test Results," by R.L. Mayes, Y. Omote, R.W. Clough - 1976 (PB 264 424)A06

EERC 76-9 "A Substructure Method for Earthquake Analysis of Structure - Soil Interaction," by J.A. Gutierrez and A.K. Chopra - 1976 (PB 257 783)A08

EERC 76-10 "Stabilization of Potentially Liquefiable Sand Deposits using Gravel Drain Systems," by H.B. Seed and J.R. Booker - 1976 (PB 258 820)A04

EERC 76-11 "Influence of Design and Analysis Assumptions on Computed Inelastic Response of Moderately Tall Frames," by G.H. Powell and D.G. Row - 1976 (PB 271 409)A06

EERC 76-12 "Sensitivity Analysis for Hysteretic Dynamic Systems: Theory and Applications," by D. Ray, K.S. Pister and E. Polak - 1976 (PB 262 859)A04

EERC 76-13 "Coupled Lateral Torsional Response of Buildings to Ground Shaking," by C.L. Kan and A.K. Chopra - 1976 (PB 257 907)A09

EERC 76-14 "Seismic Analyses of the Banco de America," by V.V. Bertero, S.A. Mahin and J.A. Hollings - 1976

EERC 76-15 "Reinforced Concrete Frame 2: Seismic Testing and Analytical Correlation," by R.W. Clough and J. Gidwani - 1976 (PB 261 323)A08

EERC 76-16 "Cyclic Shear Tests of Masonry Piers, Volume 2 - Analysis of Test Results," by R.L. Mayes, Y. Omote and R.W. Clough - 1976

EERC 76-17 "Structural Steel Bracing Systems: Behavior Under Cyclic Loading," by E.P. Popov, K. Takanashi and C.W. Roeder - 1976 (PB 260 715)A05

EERC 76-18 "Experimental Model Studies on Seismic Response of High Curved Overcrossings," by D. Williams and W.G. Godden - 1976 (PB 269 548)A08

EERC 76-19 "Effects of Non-Uniform Seismic Disturbances on the Dumbarton Bridge Replacement Structure," by F. Baron and R.E. Hamati - 1976 (PB 282 981)A16

EERC 76-20 "Investigation of the Inelastic Characteristics of a Single Story Steel Structure Using System Identification and Shaking Table Experiments," by V.C. Matzen and H.D. McNiven - 1976 (PB 258 453)A07

EERC 76-21 "Capacity of Columns with Splice Imperfections," by E.P. Popov, R.M. Stephen and R. Philbrick - 1976 (PB 260 378)A04

EERC 76-22 "Response of the Olive View Hospital Main Building during the San Fernando Earthquake," by S. A. Mahin, V.V. Bertero, A.K. Chopra and R. Collins - 1976 (PB 271 425)A14

EERC 76-23 "A Study on the Major Factors Influencing the Strength of Masonry Prisms," by N.M. Mostaghel, R.L. Mayes, R. W. Clough and S.W. Chen - 1976 (Not published)

EERC 76-24 "GADFLEA - A Computer Program for the Analysis of Pore Pressure Generation and Dissipation during Cyclic or Earthquake Loading," by J.R. Booker, M.S. Rahman and H.B. Seed - 1976 (PB 263 947)A04

EERC 76-25 "Seismic Safety Evaluation of a R/C School Building," by B. Bresler and J. Axley - 1976

EERC 76-26 "Correlative Investigations on Theoretical and Experimental Dynamic Behavior of a Model Bridge Structure," by K. Kawashima and J. Penzien - 1976 (PB 263 388)A11

EERC 76-27 "Earthquake Response of Coupled Shear Wall Buildings," by T. Srichatrapimuk - 1976 (PB 265 157)A07

EERC 76-28 "Tensile Capacity of Partial Penetration Welds," by E.P. Popov and R.M. Stephen - 1976 (PB 262 899)A03

EERC 76-29 "Analysis and Design of Numerical Integration Methods in Structural Dynamics," by H.M. Hilber - 1976 (PB 264 410)A06

EERC 76-30 "Contribution of a Floor System to the Dynamic Characteristics of Reinforced Concrete Buildings," by L.E. Malik and V.V. Bertero - 1976 (PB 272 247)A13

EERC 76-31 "The Effects of Seismic Disturbances on the Golden Gate Bridge," by F. Baron, M. Arikan and R.E. Hamati - 1976 (PB 272 279)A09

EERC 76-32 "Infilled Frames in Earthquake Resistant Construction," by R.E. Klingner and V.V. Bertero - 1976 (PB 265 892)A13

UCB/EERC-77/01 "PLUSH - A Computer Program for Probabilistic Finite Element Analysis of Seismic Soil-Structure Interaction," by M.P. Romo Organista, J. Lysmer and H.B. Seed - 1977

UCB/EERC-77/02 "Soil-Structure Interaction Effects at the Humboldt Bay Power Plant in the Ferndale Earthquake of June 7, 1975," by J.E. Valera, H.B. Seed, C.F. Tsai and J. Lysmer - 1977 (PB 265 795)A04

UCB/EERC-77/03 "Influence of Sample Disturbance on Sand Response to Cyclic Loading," by K. Mori, H.B. Seed and C.K. Chan - 1977 (PB 267 352)A04

UCB/EERC-77/04 "Seismological Studies of Strong Motion Records," by J. Shoja-Taheri - 1977 (PB 269 655)A10

UCB/EERC-77/05 "Testing Facility for Coupled-Shear Walls," by L. Li-Hyung, V.V. Bertero and E.P. Popov - 1977

UCB/EERC-77/06 "Developing Methodologies for Evaluating the Earthquake Safety of Existing Buildings," by No. 1 - B. Bresler; No. 2 - B. Bresler, T. Okada and D. Zisling; No. 3 - T. Okada and B. Bresler; No. 4 - V.V. Bertero and B. Bresler - 1977 (PB 267 354)A08

UCB/EERC-77/07 "A Literature Survey - Transverse Strength of Masonry Walls," by Y. Omote, R.L. Mayes, S.W. Chen and R.W. Clough - 1977 (PB 277 933)A07

UCB/EERC-77/08 "DRAIN-TABS: A Computer Program for Inelastic Earthquake Response of Three Dimensional Buildings," by R. Guendelman-Israel and G.H. Powell - 1977 (PB 270 693)A07

UCB/EERC-77/09 "SUBWALL: A Special Purpose Finite Element Computer Program for Practical Elastic Analysis and Design of Structural Walls with Substructure Option," by D.Q. Le, H. Peterson and E.P. Popov - 1977 (PB 270 567)A05

UCB/EERC-77/10 "Experimental Evaluation of Seismic Design Methods for Broad Cylindrical Tanks," by D.P. Clough (PB 272 280)A13

UCB/EERC-77/11 "Earthquake Engineering Research at Berkeley - 1976," - 1977 (PB 273 507)A09

UCB/EERC-77/12 "Automated Design of Earthquake Resistant Multistory Steel Building Frames," by N.D. Walker, Jr. - 1977 (PB 276 526)A09

UCB/EERC-77/13 "Concrete Confined by Rectangular Hoops Subjected to Axial Loads," by J. Vallenas, V.V. Bertero and E.P. Popov - 1977 (PB 275 165)A06

UCB/EERC-77/14 "Seismic Strain Induced in the Ground During Earthquakes," by Y. Sugimura - 1977 (PB 284 201)A04

UCB/EERC-77/15 "Bond Deterioration under Generalized Loading," by V.V. Bertero, E.P. Popov and S. Viwathanatepa - 1977

UCB/EERC-77/16 "Computer Aided Optimum Design of Ductile Reinforced Concrete Moment Resisting Frames," by S.W. Zagajeski and V.V. Bertero - 1977 (PB 280 137)A07

UCB/EERC-77/17 "Earthquake Simulation Testing of a Stepping Frame with Energy-Absorbing Devices," by J.M. Kelly and D.F. Tsztoo - 1977 (PB 273 506)A04

UCB/EERC-77/18 "Inelastic Behavior of Eccentrically Braced Steel Frames under Cyclic Loadings," by C.W. Roeder and E.P. Popov - 1977 (PB 275 526)A15

UCB/EERC-77/19 "A Simplified Procedure for Estimating Earthquake-Induced Deformations in Dams and Embankments," by F.I. Makdisi and H.B. Seed - 1977 (PB 276 820)A04

UCB/EERC-77/20 "The Performance of Earth Dams during Earthquakes," by H.B. Seed, F.I. Makdisi and P. de Alba - 1977 (PB 276 821)A04

UCB/EERC-77/21 "Dynamic Plastic Analysis Using Stress Resultant Finite Element Formulation," by P. Lukkunapvasit and J.M. Kelly - 1977 (PB 275 453)A04

UCB/EERC-77/22 "Preliminary Experimental Study of Seismic Uplift of a Steel Frame," by R.W. Clough and A.A. Huckelbridge 1977 (PB 278 769)A08

UCB/EERC-77/23 "Earthquake Simulator Tests of a Nine-Story Steel Frame with Columns Allowed to Uplift," by A.A. Huckelbridge - 1977 (PB 277 944)A09

UCB/EERC-77/24 "Nonlinear Soil-Structure Interaction of Skew Highway Bridges," by M.-C. Chen and J. Penzien - 1977 (PB 276 176)A07

UCB/EERC-77/25 "Seismic Analysis of an Offshore Structure Supported on Pile Foundations," by D.D.-N. Liou and J. Penzien 1977 (PB 283 180)A06

UCB/EERC-77/26 "Dynamic Stiffness Matrices for Homogeneous Viscoelastic Half-Planes," by G. Dasgupta and A.K. Chopra - 1977 (PB 279 654)A06

UCB/EERC-77/27 "A Practical Soft Story Earthquake Isolation System," by J.M. Kelly, J.M. Eidinger and C.J. Derham - 1977 (PB 276 814)A07

UCB/EERC-77/28 "Seismic Safety of Existing Buildings and Incentives for Hazard Mitigation in San Francisco: An Exploratory Study," by A.J. Meltsner - 1977 (PB 281 970)A05

UCB/EERC-77/29 "Dynamic Analysis of Electrohydraulic Shaking Tables," by D. Rea, S. Abedi-Hayati and Y. Takahashi 1977 (PB 282 569)A04

UCB/EERC-77/30 "An Approach for Improving Seismic - Resistant Behavior of Reinforced Concrete Interior Joints," by B. Galunic, V.V. Bertero and E.P. Popov - 1977 (PB 290 870)A06

UCB/EERC-78/01 "The Development of Energy-Absorbing Devices for Aseismic Base Isolation Systems," by J.M. Kelly and D.F. Tsztoo - 1978 (PB 284 978)A04

UCB/EERC-78/02 "Effect of Tensile Prestrain on the Cyclic Response of Structural Steel Connections, by J.G. Bouwkamp and A. Mukhopadhyay - 1978

UCB/EERC-78/03 "Experimental Results of an Earthquake Isolation System using Natural Rubber Bearings," by J.M. Eidinger and J.M. Kelly - 1978 (PB 281 686)A04

UCB/EERC-78/04 "Seismic Behavior of Tall Liquid Storage Tanks," by A. Niwa - 1978 (PB 284 017)A14

UCB/EERC-78/05 "Hysteretic Behavior of Reinforced Concrete Columns Subjected to High Axial and Cyclic Shear Forces," by S.W. Zagajeski, V.V. Bertero and J.G. Bouwkamp - 1978 (PB 283 858)A13

UCB/EERC-78/06 "Inelastic Beam-Column Elements for the ANSR-I Program," by A. Riahi, D.G. Row and G.H. Powell - 1978

UCB/EERC-78/07 "Studies of Structural Response to Earthquake Ground Motion," by O.A. Lopez and A.K. Chopra - 1978 (PB 282 790)A05

UCB/EERC-78/08 "A Laboratory Study of the Fluid-Structure Interaction of Submerged Tanks and Caissons in Earthquakes," by R.C. Byrd - 1978 (PB 284 957)A08

UCB/EERC-78/09 "Model for Evaluating Damageability of Structures," by I. Sakamoto and B. Bresler - 1978

UCB/EERC-78/10 "Seismic Performance of Nonstructural and Secondary Structural Elements," by I. Sakamoto - 1978

UCB/EERC-78/11 "Mathematical Modelling of Hysteresis Loops for Reinforced Concrete Columns," by S. Nakata, T. Sproul and J. Penzien - 1978

UCB/EERC-78/12 "Damageability in Existing Buildings," by T. Blejwas and B. Bresler - 1978

UCB/EERC-78/13 "Dynamic Behavior of a Pedestal Base Multistory Building," by R.M. Stephen, E.L. Wilson, J.G. Bouwkamp and M. Button - 1978 (PB 286 650)A08

UCB/EERC-78/14 "Seismic Response of Bridges - Case Studies," by R.A. Imbsen, V. Nutt and J. Penzien - 1978 (PB 286 503)A10

UCB/EERC-78/15 "A Substructure Technique for Nonlinear Static and Dynamic Analysis," by D.G. Row and G.H. Powell - 1978 (PB 288 077)A10

UCB/EERC-78/16 "Seismic Risk Studies for San Francisco and for the Greater San Francisco Bay Area," by C.S. Oliveira - 1978

UCB/EERC-78/17 "Strength of Timber Roof Connections Subjected to Cyclic Loads," by P. Gülkan, R.L. Mayes and R.W. Clough - 1978

UCB/EERC-78/18 "Response of K-Braced Steel Frame Models to Lateral Loads," by J.G. Bouwkamp, R.M. Stephen and E.P. Popov - 1978

UCB/EERC-78/19 "Rational Design Methods for Light Equipment in Structures Subjected to Ground Motion," by J.L. Sackman and J.M. Kelly - 1978 (PB 292 357)A04

UCB/EERC-78/20 "Testing of a Wind Restraint for Aseismic Base Isolation," by J.M. Kelly and D.E. Chitty - 1978 (PB 292 833)A03

UCB/EERC-78/21 "APOLLO - A Computer Program for the Analysis of Pore Pressure Generation and Dissipation in Horizontal Sand Layers During Cyclic or Earthquake Loading," by P.P. Martin and H.B. Seed - 1978 (PB 292 835)A04

UCB/EERC-78/22 "Optimal Design of an Earthquake Isolation System," by M.A. Bhatti, K.S. Pister and E. Polak - 1978 (PB 294 735)A06

UCB/EERC-78/23 "MASH - A Computer Program for the Non-Linear Analysis of Vertically Propagating Shear Waves in Horizontally Layered Deposits," by P.P. Martin and H.B. Seed - 1978 (PB 293 101)A05

UCB/EERC-78/24 "Investigation of the Elastic Characteristics of a Three Story Steel Frame Using System Identification," by I. Kaya and H.D. McNiven - 1978

UCB/EERC-78/25 "Investigation of the Nonlinear Characteristics of a Three-Story Steel Frame Using System Identification," by I. Kaya and H.D. McNiven - 1978

UCB/EERC-78/26 "Studies of Strong Ground Motion in Taiwan," by Y.M. Hsiung, B.A. Bolt and J. Penzien - 1978

UCB/EERC-78/27 "Cyclic Loading Tests of Masonry Single Piers: Volume 1 - Height to Width Ratio of 2," by P.A. Hidalgo, R.L. Mayes, H.D. McNiven and R.W. Clough - 1978

UCB/EERC-78/28 "Cyclic Loading Tests of Masonry Single Piers: Volume 2 - Height to Width Ratio of 1," by S.-W.J. Chen, P.A. Hidalgo, R.L. Mayes, R.W. Clough and H.D. McNiven - 1978

UCB/EERC-78/29 "Analytical Procedures in Soil Dynamics," by J. Lysmer - 1978

UCB/EERC-79/01 "Hysteretic Behavior of Lightweight Reinforced Concrete Beam-Column Subassemblages," by B. Forzani, E.P. Popov and V.V. Bertero - April 1979(PB 298 267)A06

UCB/EERC-79/02 "The Development of a Mathematical Model to Predict the Flexural Response of Reinforced Concrete Beams to Cyclic Loads, Using System Identification," by J. Stanton & H. McNiven - Jan. 1979(PB 295 875)A10

UCB/EERC-79/03 "Linear and Nonlinear Earthquake Response of Simple Torsionally Coupled Systems," by C.L. Kan and A.K. Chopra - Feb. 1979(PB 298 262)A06

UCB/EERC-79/04 "A Mathematical Model of Masonry for Predicting its Linear Seismic Response Characteristics," by Y. Mengi and H.D. McNiven - Feb. 1979(PB 298 266)A06

UCB/EERC-79/05 "Mechanical Behavior of Lightweight Concrete Confined by Different Types of Lateral Reinforcement," by M.A. Manrique, V.V. Bertero and E.P. Popov - May 1979(PB 301 114)A06

UCB/EERC-79/06 "Static Tilt Tests of a Tall Cylindrical Liquid Storage Tank," by R.W. Clough and A. Niwa - Feb. 1979 (PB 301 167)A06

UCB/EERC-79/07 "The Design of Steel Energy Absorbing Restrainers and Their Incorporation into Nuclear Power Plants for Enhanced Safety: Volume 1 - Summary Report," by P.N. Spencer, V.F. Zackay, and E.R. Parker - Feb. 1979(UCB/EERC-79/07)A09

UCB/EERC-79/08 "The Design of Steel Energy Absorbing Restrainers and Their Incorporation into Nuclear Power Plants for Enhanced Safety: Volume 2 - The Development of Analyses for Reactor System Piping,""Simple Systems" by M.C. Lee, J. Penzien, A.K. Chopra and K, Suzuki "Complex Systems" by G.H. Powell, E.L. Wilson, R.W. Clough and D.G. Row - Feb. 1979(UCB/EERC-79/08)A10

UCB/EERC-79/09 "The Design of Steel Energy Absorbing Restrainers and Their Incorporation into Nuclear Power Plants for Enhanced Safety: Volume 3 - Evaluation of Commercial Steels," by W.S. Owen, R.M.N. Pelloux, R.O. Ritchie, M. Faral, T. Ohhashi, J. Toplosky, S.J. Hartman, V.F. Zackay and E.R. Parker - Feb. 1979(UCB/EERC-79/09)A04

UCB/EERC-79/10 "The Design of Steel Energy Absorbing Restrainers and Their Incorporation into Nuclear Power Plants for Enhanced Safety: Volume 4 - A Review of Energy-Absorbing Devices," by J.M. Kelly and M.S. Skinner - Feb. 1979(UCB/EERC-79/10)A04

UCB/EERC-79/11 "Conservatism In Summation Rules for Closely Spaced Modes," by J.M. Kelly and J.L. Sackman - May 1979(PB 301 328)A03

UCB/EERC-79/12 "Cyclic Loading Tests of Masonry Single Piers; Volume 3 - Height to Width Ratio of 0.5," by P.A. Hidalgo, R.L. Mayes, H.D. McNiven and R.W. Clough - May 1979(PB 301 321)A08

UCB/EERC-79/13 "Cyclic Behavior of Dense Course-Grained Materials in Relation to the Seismic Stability of Dams," by N.G. Banerjee, H.B. Seed and C.K. Chan - June 1979(PB 301 373)A13

UCB/EERC-79/14 "Seismic Behavior of Reinforced Concrete Interior Beam-Column Subassemblages," by S. Viwathanatepa, E.P. Popov and V.V. Bertero - June 1979(PB 301 326)A10

UCB/EERC-79/15 "Optimal Design of Localized Nonlinear Systems with Dual Performance Criteria Under Earthquake Excitations," by M.A. Bhatti - July 1979(PB 80 167 109)A06

UCB/EERC-79/16 "OPTDYN - A General Purpose Optimization Program for Problems with or without Dynamic Constraints," by M.A. Bhatti, E. Polak and K.S. Pister - July 1979(PB 80 167 091)A05

UCB/EERC-79/17 "ANSR-II, Analysis of Nonlinear Structural Response, Users Manual," by D.P. Mondkar and G.H. Powell - July 1979(PB 80 113 301)A05

UCB/EERC-79/18 "Soil Structure Interaction in Different Seismic Environments," A. Gomez-Masso, J. Lysmer, J.-C. Chen and H.B. Seed - August 1979(PB 80 101 520)A04

UCB/EERC-79/19 "ARMA Models for Earthquake Ground Motions," by M.K. Chang, J.W. Kwiatkowski, R.F. Nau, R.M. Oliver and K.S. Pister - July 1979(PB 301 166)A05

UCB/EERC-79/20 "Hysteretic Behavior of Reinforced Concrete Structural Walls," by J.M. Vallenas, V.V. Bertero and E.P. Popov - August 1979(PB 80 165 905)A12

UCB/EERC-79/21 "Studies on High-Frequency Vibrations of Buildings - 1: The Column Effect," by J. Lubliner - August 1979 (PB 80 158 553)A03

UCB/EERC-79/22 "Effects of Generalized Loadings on Bond Reinforcing Bars Embedded in Confined Concrete Blocks," by S. Viwathanatepa, E.P. Popov and V.V. Bertero - August 1979

UCB/EERC-79/23 "Shaking Table Study of Single-Story Masonry Houses, Volume 1: Test Structures 1 and 2," by P. Gülkan, R.L. Mayes and R.W. Clough - Sept. 1979

UCB/EERC-79/24 "Shaking Table Study of Single-Story Masonry Houses, Volume 2: Test Structures 3 and 4," by P. Gülkan, R.L. Mayes and R.W. Clough - Sept. 1979

UCB/EERC-79/25 "Shaking Table Study of Single-Story Masonry Houses, Volume 3: Summary, Conclusions and Recommendations," by R.W. Clough, R.L. Mayes and P. Gülkan - Sept. 1979

UCB/EERC-79/26 "Recommendations for a U.S.-Japan Cooperative Research Program Utilizing Large-Scale Testing Facilities," by U.S.-Japan Planning Group - Sept. 1979(PB 301 407)A06

UCB/EERC-79/27 "Earthquake-Induced Liquefaction Near Lake Amatitlan, Guatemala," by H.B. Seed, I. Arango, C.K. Chan, A. Gomez-Masso and R. Grant de Ascoli - Sept. 1979(NUREG-CR1341)A03

UCB/EERC-79/28 "Infill Panels: Their Influence on Seismic Response of Buildings," by J.W. Axley and V.V. Bertero - Sept. 1979(PB 80 163 371)A10

UCB/EERC-79/29 "3D Truss Bar Element (Type 1) for the ANSR-II Program," by D.P. Mondkar and G.H. Powell - Nov. 1979 (PB 80 169 709)A02

UCB/EERC-79/30 "2D Beam-Column Element (Type 5 - Parallel Element Theory) for the ANSR-II Program," by D.G. Row, G.H. Powell and D.P. Mondkar - Dec. 1979(PB 80 167 224)A03

UCB/EERC-79/31 "3D Beam-Column Element (Type 2 - Parallel Element Theory) for the ANSR-II Program," by A. Riahi, G.H. Powell and D.P. Mondkar - Dec. 1979(PB 80 167 216)A03

UCB/EERC-79/32 "On Response of Structures to Stationary Excitation," by A. Der Kiureghian - Dec. 1979(PB 80166 929)A03

UCB/EERC-79/33 "Undisturbed Sampling and Cyclic Load Testing of Sands," by S. Singh, H.B. Seed and C.K. Chan - Dec. 1979(

UCB/EERC-79/34 "Interaction Effects of Simultaneous Torsional and Compressional Cyclic Loading of Sand," by P.M. Griffin and W.N. Houston - Dec. 1979

UCB/EERC-80/01 "Earthquake Response of Concrete Gravity Dams Including Hydrodynamic and Foundation Interaction Effects," by A.K. Chopra, P. Chakrabarti and S. Gupta - 1980

UCB/EERC-80/02 "Rocking Response of Rigid Blocks to Earthquakes," by C.S. Yim, A.K. Chopra and J. Penzien - 1980

UCB/EERC-80/03 "Optimum Inelastic Design of Seismic-Resistant Reinforced Concrete Frame Structures," by S.W. Zagajeski and V.V. Bertero - 1980

UCB/EERC-80/04 "Effects of Amount and Arrangement of Wall-Panel Reinforcement on Hysteretic Behavior of Reinforced Concrete Walls," by R. Iliya and V.V. Bertero - 1980

UCB/EERC-80/05 "Shaking Table Research on Concrete Dam Models," by A. Niwa and R.W. Clough - 1980

UCB/EERC-80/06 "Piping With Energy Absorbing Restrainers: Parameter Study on Small Systems," by G.H. Powell, C. Oughourlian and J. Simons - 1980

UCB/EERC-80/07 "Inelastic Torsional Response of Structures Subjected to Earthquake Ground Motions," by Y. Yamazaki - 1980

UCB/EERC-80/08 "Study of X-Braced Steel Frame Structures Under Earthquake Simulation," by Y. Ghanaat - 1980

UCB/EERC-80/09 "Hybrid Modelling of Soil-Structure Interaction," by S. Gupta, T.W. Lin, J. Penzien and C.S. Yeh - 1980

UCB/EERC-80/10 "General Applicability of a Nonlinear Model of a One Story Steel Frame," by B.I. Sveinsson and H. McNiven - 1980

UCB/EERC-80/11 "A Green-Function Method for Wave Interaction with a Submerged Body," by W. Kioka - 1980

UCB/EERC-80/12 "Hydrodynamic Pressure and Added Mass for Axisymmetric Bodies," by F. Nilrat - 1980

UCB/EERC-80/13 "Treatment of Non-Linear Drag Forces Acting on Offshore Platforms," by B.V. Dao and J. Penzien - 1980

UCB/EERC-80/14 "2D Plane/Axisymmetric Solid Element (Type 3 - Elastic or Elastic-Perfectly Plastic) for the ANSR-II Program," by D.P. Mondkar and G.H. Powell - 1980

UCB/EERC-80/15 "A Response Spectrum Method for Random Vibrations," by A. Der Kiureghian - 1980

UCB/EERC-80/16 "Cyclic Inelastic Buckling of Tubular Steel Braces," by V.A. Zayas, E.P. Popov and S.A. Mahin - 1980

UCB/EERC-80/17 "Dynamic Response of Simple Arch Dams Including Hydrodynamic Interaction," by C.S. Porter and A.K. Chopra - 1980.

UCB/EERC-80/18 "Experimental Testing of a Friction Damped Aseismic Base Isolation System with Fail-Safe Characteristics," by J.M. Kelly, K.E. Beucke and M.S. Skinner - 1980

UCB/EERC-80/19 "The Design of Steel Energy-Absorbing Restrainers and their Incorporation into Nuclear Power Plants for Enhanced Safety (Vol 1B): Stochastic Seismic Analyses of Nuclear Power Plant Structures and Piping Systems Subjected to Multiple Support Excitations," by M.C. Lee and J. Penzien - 1980

UCB/EERC-80/20 "The Design of Steel Energy-Absorbing Restrainers and their Incorporation into Nuclear Power Plants for Enhanced Safety (Vol 1C): Numerical Method for Dynamic Substructure Analysis," by J.M. Dickens and E.L. Wilson 1980

UCB/EERC-80/21 "The Design of Steel Energy-Absorbing Restrainers and their Incorporation into Nuclear Power Plants for Enhanced Safety (Vol 2): Development and Testing of Restraints for Nuclear Piping Systems," by J.M. Kelly and M.S. Skinner - 1980

UCB/EERC-80/22 "3D Solid Element (Type 4-Elastic or Elastic-Perfectly-Plastic) for the ANSR-II Program," by D.P. Mondkar and G.H. Powell - 1980

UCB/EERC-80/23 "Gap-Friction Element (Type 5) for the ANSR-II Program," by D.P. Mondkar and G.H. Powell - 1980

UCB/EERC-80/24 "U-Bar Restraint Element (Type 11) for the ANSR-II Program," by C. Oughourlian and G.H. Powell - 1980

UCB/EERC-80/25 "Testing of a Natural Rubber Base Isolation System by an Explosively Simulated Earthquake," by J.M. Kelly 1980

UCB/EERC-80/26 "Input Identification from Structural Vibrational Response," by Y. Hu - 1980

UCB/EERC-80/27 "Cyclic Inelastic Behavior of Steel Offshore Structures," by V.A. Zayas, S.A. Mahin and E.P. Popov - 1980

UCB/EERC-80/28 "Shaking Table Testing of a Reinforced Concrete Frame with Biaxial Response," by M.G. Oliva and R.W. Clough 1980

UCB/EERC-80/29 "Dynamic Properties of a Twelve-Story Prefabricated Panel Building," by J.G. Bouwkamp, J.P. Kollegger and R.M. Stephen - 1980

UCB/EERC-80/30 "Dynamic Properties of a Eight-Story Prefabricated Panel Building," by J.G. Bouwkamp, J.P. Kollegger and R.M. Stephen - 1980

UCB/EERC-80/31 "Predictive Dynamic Response of Panel Type Structures Under Earthquakes," by J.P. Kollegger and J.G. Bouwkamp 1980

UCB/EERC-80/32 "The Design of Steel Energy-Absorbing Restrainers and their Incorporation into Nuclear Power Plants for Enhanced Safety: Vol 3, Testing of Commercial Steels in Low-Cycle Torsional Fatigue," by P. Spencer, E.R. Parker, E. Jongewaard and M. Drory - 1980

UCB/EERC-80/33 "The Design of Steel Energy-Absorbing Restrainers and their Incorporation into Nuclear Power Plants for Enhanced Safety: Vol 4, Shaking Table Tests of Piping Systems with Energy-Absorbing Restrainers," by S.F. Stiemer and W.G. Godden - 1980

UCB/EERC-80/34 "The Design of Steel Energy-Absorbing Restrainers and their Incorporation into Nuclear Power Plants for Enhanced Safety: Vol 5, Summary Report," by P. Spencer 1980

UCB/EERC-80/35 "Experimental Testing of an Energy Absorbing Base Isolation System," by J. Kelly, M.S. Skinner and K.E. Beucke - 1980

UCB/EERC-80/36 "Simulating and Analyzing Artificial Non-Stationary Earthquake Ground Motions," by R.F. Nau, R.M. Oliver and K.S. Pister - 1980

UCB/EERC-80/37 "Earthquake Engineering at Berkeley," - 1980

UCB/EERC-80/38 "Inelastic Seismic Analysis of Large Panel Buildings," by V. Schricker and G.H. Powell - 1980

UCB/EERC-80/39 "Dynamic Response of Embankment, Concrete-Gravity and Arch Dams Including Hydrodynamic Interaction," by J.F. Hall and A.K. Chopra - 1980

UCB/EERC-80/40 "Inelastic Buckling of Steel Strut Under Cyclic Load Reversal," by R.G. Black, W.A. Wenger and E.P. Popov 1980

UCB/EERC-80/41 "Influence of Site Characteristics on Building Damage During the October 3, 1974 Lima Earthquake," by P. Repetto, I. Arango and H.B. Seed - 1980

UCB/EERC-80/42 "Evaluation of a Shaking Table Test Program on Response Behavior of a Two Story Reinforced Concrete Frame," by J.M. Blondet, R.W. Clough and S.A. Mahin - 1980.

UCB/EERC-80/43 "Modelling of Soil-Structure Interaction by Finite and Infinite Element," by F. Medina - 1980

UCB/EERC-81/01 "Control of Seismic Response of Piping Systems and Other Structures by Base Isolation," edited by J.M. Kelly - 1981

UCB/EERC-81/02 "OPTNSR - An Interactive Software System for Optimal Design of Statically and Dynamically Loaded Structures with Nonlinear Response," by M.A. Bhatti, V. Ciampi and K.S. Pister - 1981

UCB/EERC-81/03 "Analysis of Local Variations in Free Field Seismic Ground Motion," by J.-C. Chen, J. Lysmer and H.B. Seed 1981

UCB/EERC-81/04 "Inelastic Offshore Platforms for Seismic Loading," by V.A. Zayas, P. Shum, B. Shing, S.A. Mahin and E.P. Popov 1981

UCB/EERC-81/05 "Dynamic Response of Light Equipment in Structures," by A. Der Kiureghian, J.L. Sackman and B. Nour-Omid - 1981