CIVIL ENGINEERING STUDY
STRUCTURAL SERIES 81-10

ODSEWS-2D
OPTIMUM DESIGN OF STATIC, EARTHQUAKE, AND WIND STEEL STRUCTURES

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January, 1981

Prepared for the National Science Foundation under Grant No.
NSF PFR 7805694

BIBLIOGRAPHIC DATA SHEET	1. Report No. Structural Series 81-10	2.	3. Recipient's Accession No. PB81 232738
4. Title and Subtitle ODSEWS-2D Optimum Design of Static, Earthquake, and Wind Steel Structures			5. Report Date January, 1981
			6.
7. Author(s) Franklin Y. Cheng, D. Srifuengfung, and L.H. Sheng			8. Performing Organization Rept. No Structural Series
9. Performing Organization Name and Address Civil Engineering Department University of Missouri-Rolla Rolla, Missouri 65401			10. Project/Task/Work Unit No.
			11. Contract/Grant No. NSF PFR 7805694
12. Sponsoring Organization Name and Address Division of Problem-Focused Research Application National Science Foundation Washington, D.C. 20550			13. Type of Report & Period Covered
			14.

15. Supplementary Notes

16. Abstracts An optimum design technique is presented for trusses and braced and unbraced frames with consideration of various constraints of stresses, displacements, natural frequencies, member sizes, and upper- and lower-bounds of member stiffnesses. A computer program ODSEWS (Optimum Design of Static, Earthquake, and Wind Structures) has been developed for which the dynamic input may be response spectra or code provisions. More than 80 cases were designed in which the frames were varied from one story to 30 stories for examining various response parameters. Among them the particular interests are in the effect of the interacting ground motions including P-Δ on the stiffness distribution at critical regions and at the entire system as well as in the assessment of structural systems and their serviceability. The method is considered to be superior to the conventional design technique because it can simultaneously provide the required stiffnesses and satisfy the given constraints which cannot be expected from the conventional design process.

17. Key Words and Document Analysis. 17a. Descriptors

beams, bracings, columns, consistent mass, damping, design, displacement method, frames, modal, optimization, second-order, steel, stiffness, truss.

17b. Identifiers/Open-Ended Terms

17c. COSATI Field Group

18. Availability Statement	19. Security Class (This Report) UNCLASSIFIED	21. No. of Pages 314
	20. Security Class (This Page) UNCLASSIFIED	22. Price

Civil Engineering Study

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ABSTRACT

This report presents the optimum design for various plane steel structures, a component of 3-D systems, subjected to the multicomponent input of static loads, dynamic forces, and seismic excitations for the purpose of: 1) examining the effect of the interaction of ground motions on their relative stiffness requirements, overall stiffness distribution at critical regions, and on the entire system, 2) selecting suitable structural systems for certain types of loads, and 3) providing member properties for detailed design. The structural systems that have been studied are trusses, unbraced, single-braced, double-braced, and K-braced frameworks in which the constituent members are bar elements for bracings and truss-members and beam-column elements for columns and girders. The beam-column elements are either the built-up sections or the hot-rolled wide flange sections available in the AISC Steel Construction Manual. The structures can be subjected to static loads, dynamic forces, and horizontal and vertical ground motions. The dynamic forces and the seismic excitations can be used on the basis of either direct integrations or response spectra. In addition, the equivalent lateral seismic force recommended by the Uniform Building Code can also be used for the design.

The recommended design method is based on an optimal criterion and a recursion relation for which the behavior constraints of static and dynamic displacements and stiffnesses as well as the constraints of natural frequencies are presented. Other constraints are the desirable sizes of the members and the limitation on the difference between the maximum and minimum moments of inertia of any given system.

The structural formulation, which is based on the displacement method, takes into consideration the consistent mass formulation and the second-order effect resulting from the static and dynamic forces that act axially on the columns. The columns and girders are considered to have axial and bending deformations, thus each node of a structural system has three degrees of freedom. Various displacement constraints can be applied to individual nodes with any specific numbers. A sophisticated computer program designated as ODSEWS (Optimum Design of Static, Earthquake, and Wind Structures) has been developed for the design of static loads, dynamic forces, and seismic excitations, as well as for any combination of these.

Fifteen-story, one-bay unbraced, single-braced, double-braced, and K-braced framed structures have been designed for the multicomponent input of static loads and seismic excitations. It has been found that the K-braced system provides better serviceability than any other system for multicomponent ground motions. It also has been found that the first three modes are needed for a structural design subjected to one horizontal ground motion only; however, the first five modes are necessary for designing a structure subjected to the interaction of horizontal and vertical motions.

The Uniform Building Code underestimates the total shear forces, and the design based on the equivalent seismic forces provided by the Code yields a much lighter weight than that obtained by using either actual earthquake records or the average response spectra. A heavier sturctural design is needed to withstand the effect of a vertical ground motion that is combined with a horizontal earthquake horizontal earthquake and a second-order P-Δ effect than for a horizontal earthquake acting alone. Apparently the material savings in the design are of great interest; however, the most important point is the scientific approach of determining the stiffness distribution of various structural systems subjected to different loading conditions. The scientific design method, which can always provide the required stiffnesses and satisfy the designer's given constraints, is considered much better than the conventional design technique based on the trial and trial process by using either desk calculators or analysis computer programs.

ACKNOWLEDGMENTS

This report is based on partial results obtained from a research project sponsored by the National Science Foundation under Grant No. PFR 7805694. The authors deeply appreciate the financial support and the continuous encouragement and advice that Dr. S.C. Liu, Program Manager, provided during the course of the investigation. They also wish to thank Dr. J.H. Senne, Chairman of Civil Engineering, for providing facilities and a substantial amount of computer time for this work. Finally, Mr. F.H. Niu's capable assistance in the drawings and Mrs. Margot Lewis' expert typing are gratefully acknowledged.

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LIST OF SYMBOLS

A_o = cross sectional area

$\vec{A}_{mi}$ = vector of fixed-end actions (moments and shears) caused by uniform load on member i

$\underline{a}_i$ = compatibility matrix connecting the generalized coordinates of a structure and those of the members

a_o = coefficient in damping

$\underline{B}_i$ = cross sectional property matrix

b = width of the wide-flange section

b_j = limitation imposed on the j^{th} behavior constraint

b_o = coefficient in damping

C = seismic coefficient

$\underline{C}$ = viscous damping matrix of a structure in system coordinates

$\underline{c}$ = viscous damping matrix of a structure in normal coordinates

c_i = i^{th} diagonal term of the viscous damping matrix $\underline{c}$

CN_i = total number of structural masses lumped on member i

$\underline{D}$ = diagonal matrix containing the Duhamel integrals

D_i = i^{th} diagonal term of the matrix $\underline{D}$

d = depth of the wide-flange section

d_{min} = lower bound of the depth d

d_{max} = upper bound of the depth d

E = modulus of elasticity

$\vec{e}$ = unit vector

e = exponential number

F_i = design lateral force at level i

F_t = design lateral force at top level

F_x = design lateral force at level x

$f(t)$ = time function

g = gravity acceleration

$g_j(x)$ = j^{th} constraint function

h_i = height of level i from the ground level

h_x = height of level x from the ground level

I = coefficient depending on the importance of the structure

I_o = cross sectional moment of inertia

J = set of design variables unconstrained by the lower bounds

J_o = set of design variables constrained by the lower bounds

K = factor depending on the type of structural system

$\underline{K}$ = $\underline{K}_s + \underline{K}_g$, stiffness matrix of a structure in system coordinates

$\underline{K}_g$ = geometric stiffness matrix of a structure in system coordinates

$\underline{K}_s$ = structural stiffness matrix of a structure in system coordinates

$\underline{k}$ = stiffness matrix of a structure in normal coordinates

k_i = i^{th} diagonal term of the matrix $\underline{k}$

L = length of a typical beam member

$L(x)$ = auxiliary function

L_i, ℓ_i = length of member i

$\underline{M}$ = $\underline{M}_n + \underline{M}_s$, total mass matrix of a structure in system coordinates

$\underline{M}_n$ = nonstructural mass matrix resulting from the superimposed weight in system coordinates

$\underline{M}_s$ = structural mass matrix resulting from the weight of the structural members in system coordinates

M = bending moment

m = number of members

$\underline{m}$ = mass matrix of a structure in normal coordinates

m_i = i^{th} diagonal term of the matrix $\underline{m}$

N = number of the active behavior constraints

N_i = weight of nonstructural masses acting axially on member i

n = number of the behavior constraints

n_s = number of story levels

P_i = total axial force on member i

p = forcing frequency based on Rayleigh quotient formula

$\vec{Q}_j$ = vector of unit load in the j^{th} direction and zero values for others

$\vec{q}(t)$ = displacement vector in normal coordinates

$q_i(t)$ = i^{th} term of the vector $\vec{q}(t)$

$\vec{R}$ = vector of static load or fixed-end moments and forces at the structural nodes

$\vec{R}_c$ = vector of actual joint loads

$\vec{R}_e$ = vector of equivalent joint loads

$\vec{R}_o$ = vector of the magnitude of dynamic loads

$\vec{R}(t)$ = vector of dynamic load referred to system coordinates

$\vec{r}$ = vector of combined displacements referred to system coordinates

$\vec{r}(x)$ = vector of static displacements measured from the undeformed position to the equilibrium position

$\vec{r}(x,t)$ = vector of dynamic displacements measured from the equilibrium position after deformation caused by static loads

$\dot{\vec{r}}(x,t)$ = velocity vector associated with $\vec{r}(x,t)$

$\ddot{\vec{r}}(x,t)$ = acceleration vector associated with $\vec{r}(x,t)$

$\ddot{\vec{r}}_g(t)$ = vector of ground accelerations

S = site resonance factor

S_o = section modulus

$\vec{S}_i$ = vector of the internal forces on member i

s_d = spectral displacement

s_i = Lagrange multiplier associated to the i^{th} design variable

T = building period

T_s = site period

t = time

t_f = flange thickness

t_w = web thickness

U = strain energy

$u_j(x)$ = static displacement constraint function for the j^{th} direction

$u_j(x,t)$ = dynamic displacement constraint function for the j^{th} direction

V = shear

v_o = shear flow

$\vec{v}_i(x)$ = displacement vector of member i referred to member coordinates

W = total weight of a building

$W(x)$ = objective function

w = uniform load

w_i = weight at level i

w_x = weight at level x

x_i = i^{th} design variable

x_i° = lower bound on the i^{th} design variable

$x_h(t)$ = specific horizontal ground acceleration

$x_v(t)$ = specific vertical ground acceleration

$y_j(x)$ = j^{th} behavior constraint

Z = a zone coefficient

$z(x)$ = measuring function of static stiffness of a structure

$z(x,t)$ = measuring function of dynamic stiffness of a structure

α_i = i^{th} relative design variable

β_i = relative damping ratio for the i^{th} mode

δ = deflection at member end

η_i = ratio of cross sectional area to moment of inertia for member i

θ = rotation at member end

γ_i = i^{th} modal participation factor

Λ = scaling factor

λ_j = j^{th} Lagrange multiplier

$\vec{\lambda}$ = vector of Lagrange multipliers

ρ_i = mass density of member i

$\vec{\sigma}_i$ = vector of stresses on member i

τ = time variable for integration

$\vec{\phi}_i$ = i^{th} normal mode vector

$\underline{\Phi}$ = matrix of normal modes

Ψ_i = $1/m_i\omega_i$

$\underline{\Omega}$ = diagonal matrix containing the square of the natural frequencies

ω_i = i^{th} undamped natural frequency

ω_{di} = i^{th} damped natural frequency

I. INTRODUCTION

A. OBJECTIVES

It has long been recognized that there is an urgent need for the development of more efficient computer programs for seismic structural design. The methods for these programs should be more rational and reliable than those that are now commonly used and based on conventional trial and error processes. Recent progress in computer technology has, however, led to the development of some sophisticated computer programs that can be used to analyze complex structures. When these programs are used for design, the relative stiffnesses of the constituent members must first be assumed. If these preliminary stiffnesses are not correctly determined, in spite of the program's sophistication, repeated analyses can only yield an improved design. Apparently, such use does not guarantee an efficient design; this is particularly true in the case of aseismic designs. Current electronic applications are actually based on the conventional designs, and more art than science is required to use them.

Recent studies of the response behavior of plane structures subjected to combined horizontal and vertical earthquake motions indicate that the inclusion of the vertical ground motion can significantly cause an increase in the ductility requirement of some members and the displacement response of an entire system.[8,9] The most recent analytical studies of the response behavior of three-dimensional braced and unbraced structures with and without floor slabs subjected to three-dimensional components of ground motions reveal that the interaction of three earthquake components remarkably increases both the internal forces and moments and that the increase in some members of a system can be several times greater than that resulting from just one horizontal earthquake component.[12,13]

The multibay and multistory plane and space building systems, which were used in the study, were based on the conventional design of selecting the relative stiffnesses of the constituent members of the individual systems. The various stiffness distributions of a system can induce different response behavior. The effect of the interaction of ground motions on the structural response is definitely critical, but the influence on the structural parameters is not necessarily unique.

In this report, primary objectives are 1) to develop an efficient optimization method and a computer program for an automated design of typical bays of 3-dimensional symmetric structural systems (plane structures) with and without bracings that can be used for both academic and industrial purposes, 2) to determine the influence of interacting earthquake on relative stiffness requirements and overall stiffness distributions at critical regions as well as on the entire system of various structures, 3) to evaluate the effectiveness of various bracing systems in a seismic structural design by observing the final design results of the structural weight, 4) to study the adequacy of seismic force requirement in the Uniform Building Code (UBC), and 5) to establish pilot work for further studying the optimum design and assessment of three-dimensional unsymmetric building systems for code provisions and parametric multicomponent earthquake motion.

B. LITERATURE REVIEW

In the past decade, a considerable amount of literature has been published in the area of optimum structural design. The increasing number of publications corresponds closely to the rapid demand for economical and reliable structural designs in virtually all fields of endeavor. Most of the designs have been focused on minimum weight structures for static loads, much less has been published on optimum structural design for dynamic forces, and very little has been done on the optimum design of seismic structures. This is due to the fact that dynamic stresses and responses are parametric in time and can have discontinuous first partial derivatives with respect to the design variables. In addition, the seismic excitations are random in nature. Thus, the optimum design of dynamic and seismic structures can become extremely complex and difficult.

A literature review of structural optimization can be at least classified into three groups on the basis of 1) loadings and constraints, 2) types of structures, and 3) optimization methods. The boundaries of these classes are quite nebulous because there is some overlap. Nevertheless, a brief review of each is given, and typical references are cited.

The first group can be divided into four categories of a) static loads and constraints,[2,10,24,31,44] b) dynamic, seismic loads and constraints,[3,7,10,11,14,17,25,37,40,46,47,49,51] c) aeroelastic requirements,[48] and d) probabilistic approach and nonconservative systems.[5] The second group can be divided into a) trusses,[31,38,39,41] b) frames,[2,4,5,7,31,45] c) plates and shells.[5,19,48] The third group includes a) mathematical programming,[1,2,3,4,17,26] b) optimization criteria,[10,11,15] c) optimal control theory,[48] and d) dynamic programming.[5,48] These references are typical but not exclusive.

In general, most of the optimization techniques have some limitations and are best suited for certain types of problems. The technique that is based on energy distribution has been proven to be effective for large structural systems in aerospace engineering.[46] Recently, Venkayya and Cheng[47] and Cheng and his associates[10,11] extended the optimization algorithm for structures subjected to earthquake motions. Previous studies of optimum seismic structural design are mostly based on the linearization technique and static equivalent seismic forces for simple structures and shear buildings.[37,49] Cheng and Botkin[4,7] studied the feasible direction technique for the design of tall buildings and large frameworks. They included the geometric nonlinearity of the P-Δ effect in their work. The technique was also studied by Ray et al.[29] and Walker and Pister[50] for optimal elastic structures. These references among others mainly deal with feasible seismic structural designs for which the feasible domain is an expression of the standards or code requirements; however, another distinct branch in seismic resistant design is associated with the problem of minimizing the total cost with a design, that is maximization of the total benefit minus the total cost; the merit function includes the building cost and the expected damage,[20,21,27,32] but the constraints do not include engineering code requirements.

The development of optimality criterion methods[28,44] in the early 70's is considered to be a great contribution in the field of engineering optimization, because they offer major improvements over any other optimization methods currently in vogue. The significant advantage of the methods is that the number of iterations required to converge on an optimum (or pseudo-optimum) design is largely independent of the number of variables in the problem, that is, in fact, a downfall of pure mathematical programming. Thus, the optimality criteria have been extended to this research work.

C. SCOPE OF THE REPORT

A computer program ODSEWS (Optimum Design of Static, Earthquake, and Wind Structures), based on the IBM 370/168 computer has been developed for six static and six dynamic loading conditions for multibay and multistory plane steel structures. The static loads can be uniform or concentrated, and the dynamic excitations can be applied as time-dependent forces, the ground motions of seismic or shock waves, and the equivalent seismic lateral forces recommended by the Uniformed Building Code. The second-order P-Δ effect resulting from structural and nonstructural weight as well as vertical ground motion is included for the design of braced and unbraced structural systems. The constituent members of a system are made of either built-up sections or hot-rolled wide flange sections. The constraints considered herein are stresses (axial, bending, and shear), displacements, natural frequencies, maximum differences between relative stiffnesses, and upper and lower bounds of cross sections. The objective is to obtain the minimum weight of a structural system.

The contents of the following chapters of this report are outlined below.

Chapter II presents a detailed description of the method used to analyze a multi-degree-of-freedom system subjected to multicomponent ground motions and static loads. A closed form of integration for seismic and wind excitations is also included.

Chapter III describes the techniques used for transforming uniform loads into fixed-end action and of formulating the strain energy of uniformly loaded members. The use of the response spectrum method and the UBC equivalent seismic forces is also presented.

In Chapter IV, the general problem of optimization and the Kuhn-Tucker conditions of optimality are briefly introduced, and then the optimality criteria and the calculation of the Lagrange multipliers for multiple constraints are discussed in detail.

Chapter V contains the detailed derivations of the behavior constraints of stress and displacement for both static and dynamic cases and the constraint of natural frequencies.

The details of the cross-sectional properties of both built-up sections and hot-rolled wide-flange sections are discussed in Chapter VI.

Chapter VII includes the numerical procedure for deriving static and seismic structural designs.

Numerical design results are given in Chapters VIII and IX. Extensive studies of 15-story building systems are presented in Chapter IX.

Chapter X reviews the work and lists the conclusions based on the results.

II. MATHEMATICAL FORMULATION OF A STRUCTURAL SYSTEM

General structural systems of plane frameworks with and without bracing members are formulated on the basis of the displacement method and the consistant mass technique.[6] The constituent members are prismatic between nodes and may have bending and axial deformations. Although the structures considered herein are linear, the second-order effect of the axial loads on the columns is included. The structural girders are modeled so that they have either three nodes at the mid-point and both ends of a member or two nodes at both ends of a member. The typical stiffness, mass, and the second-order geometric stiffness matrices of a constituent member, i, are given in Appendix A.

In this chapter, the motion equation for the combined action of the static load and multicomponent ground motions is established first, then the solutions of the motion equation are found in a closed form as well as in a piece-wise linear form, and the responses of displacements and internal stresses and the eigensolutions are finally noted.

A. EQUATIONS OF MOTION FOR STATIC LOADS AND MULTICOMPONENT GROUND EXCITATIONS

The equations of motion for a multidegree-of-freedom structural system that is subjected to both static load and multicomponent ground motions can be expressed as

$$(\underline{M}_n + \underline{M}_s)\ddot{\vec{r}}(x,t) + \underline{C}\,\dot{\vec{r}}(x,t) + (\underline{K}_s - \underline{K}_g)(\vec{r}(x,t) + \vec{r}(x))$$

$$= -(\underline{M}_n + \underline{M}_s)\ddot{\vec{r}}_g(t) + \vec{R}, \qquad (2.1)$$

in which

$\underline{M}_n$ = nonstructural mass matrix resulting from the superimposed weight,

$\underline{M}_s$ = structural mass matrix resulting from the weight of the structural members,

$\underline{C}$ = viscous damping matrix,

$\underline{K}_s$ = structural stiffness matrix,

$\underline{K}_g$ = geometric stiffness matrix,

$\ddot{\vec{r}}(x,t), \dot{\vec{r}}(x,t), \vec{r}(x,t)$ = acceleration, velocity, and displacement vectors of system coordinates measured from the equilibrium position after deformation caused by static loads,

$\vec{r}(x)$ = static displacement vector measured from the undeformed position to the equilibrium position,

$\ddot{\vec{r}}_g(t)$ = vector of ground accelerations, and

$\vec{R}$ = vector of static loads or fixed-end moments and forces at the structural nodes.

Let the generalized coordinates of the i^{th} member and the structure be related in terms of

$$\vec{v}_i(x) = \underline{a}_i \vec{r}(x), \tag{2.1a}$$

in which $\vec{v}_i(x)$ is the displacement vector of the i^{th} member, and $\underline{a}_i$ is the compatibility matrix connecting the generalized coordinates of the i^{th} member and those of the structural system. Then the system matrices in Eq. (2.1) can be obtained by assembling the member matrices as follows:

$$\underline{M_s} = \sum_{i=1}^{m} \underline{a_i^T} \; \underline{M_{si}} \; \underline{a_i} \tag{2.1b}$$

$$\underline{K_s} = \sum_{i=1}^{m} \underline{a_i^T} \; \underline{K_{si}} \; \underline{a_i} \tag{2.1c}$$

$$\underline{K_g} = \sum_{i=1}^{m} P_i \; \underline{a_i^T} \; \underline{K_{gi}} \; \underline{a_i}, \tag{2.1d}$$

in which m is the total number of the constituent members of a system, and $\underline{M_{si}}$, $\underline{K_{si}}$, and $\underline{K_{gi}}$ are the mass, structural stiffness, and geometric stiffness matrices of the i^{th} member respectively, and

$$P_i = N_i + \sum_{k=1}^{CN_i} \rho_k \; A_k \; \ell_k \tag{2.1e}$$

in which P_i is the total axial force, N_i is the weight of nonstructural masses acting axially on the i^{th} member, and CN_i is the total number of structural masses lumped on the i^{th} member. A typical structure is sketched in Fig. 1 for which typical P's are shown below:

$$P_5 = \frac{1}{2}[w_1\ell_1 + w_3\ell_3] + \frac{1}{2}\,\rho[A_1\ell_1 + A_5\ell_5 + A_8\ell_8 + A_3\ell_3 + A_8\ell_8],\; CN_5 = 5,$$

$$P_6 = \frac{1}{2}[w_1\ell_1 + w_2\ell_2 + w_3\ell_3 + w_4\ell_4]$$

$$+ \frac{1}{2}\,\rho[A_1\ell_1 + A_2\ell_2 + A_6\ell_5 + A_9\ell_9 + A_3\ell_3 + A_4\ell_4 + A_9\ell_9],\; CN_6 = 7,$$

$$P_7 = \frac{1}{2}[w_2\ell_2 + w_4\ell_4] + \frac{1}{2}\,\rho[A_2\ell_2 + A_7\ell_7 + A_{10}\ell_{10} + A_4\ell_4 + A_{10}\ell_{10}],$$

$$CN_7 = 5,$$

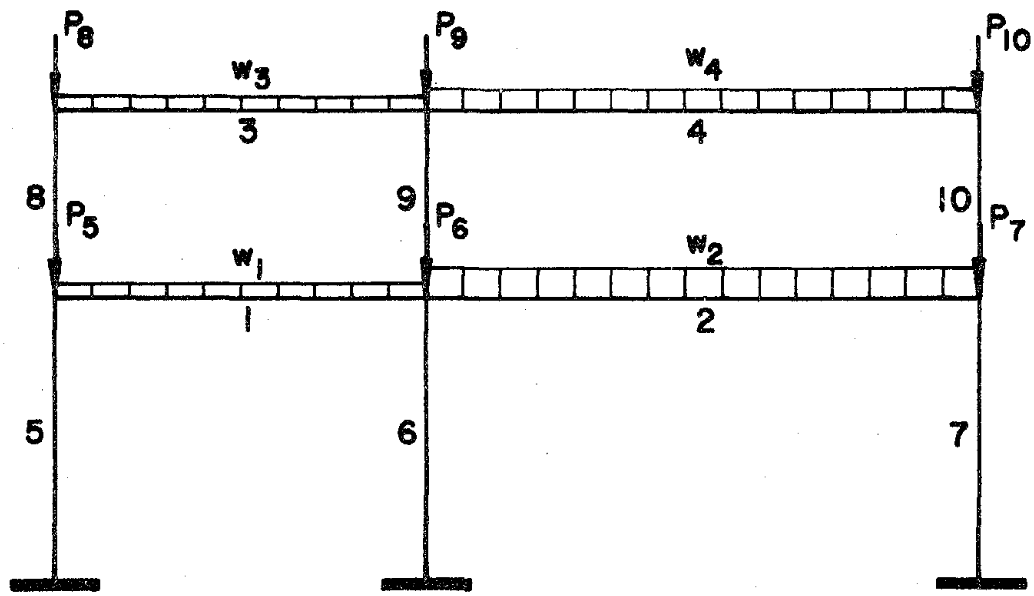


Figure 1. Typical Diagram for the Axial Forces of P-Δ Effect.

$$P_8 = \frac{1}{2} w_3\ell_3 + \frac{1}{2} \rho[A_3\ell_3 + A_5\ell_5],\ CN_8 = 2,$$

$$P_9 = \frac{1}{2}[w_3\ell_3 + w_4\ell_4] + \frac{1}{2} \rho[A_3\ell_3 + A_4\ell_4 + A_9\ell_9],\ CN_9 = 3,$$

$$P_{10} = \frac{1}{2} w_4\ell_4 + \frac{1}{2} \rho[A_4\ell_4 + A_{10}\ell_{10}],\ CN_{10} = 2.$$

$\underline{M_n}$ can be similarly established as $\underline{M_s}$ for uniformly distributed non-structural masses and may include lumped masses associated with the concentrated superimposed weight.

The static displacements, $\vec{r}(x)$, can be obtained directly from the static equilibrium equation

$$\vec{r}(x) = \underline{K}^{-1}\vec{R}, \tag{2.2}$$

in which

$$\underline{K} = \underline{K_s} - \underline{K_g}. \tag{2.3}$$

In Eq. (2.3), the negative sign indicates that the geometric stiffness matrix is due to the compressive forces only. The dynamic displacements, $\vec{r}(x,t)$, must be solved by employing the dynamic equilibrium equations, which can be derived from Eq. (2.1) by using

$$\underline{M}\ddot{\vec{r}}(x,t) + \underline{C}\dot{\vec{r}}(x,t) + \underline{K}\vec{r}(x,t) = -\underline{M}\ddot{\vec{r}}_g(t), \tag{2.4}$$

in which

$$\underline{M} = \underline{M_n} + \underline{M_s}. \tag{2.5}$$

Let the system coordinates in Eq. (2.4) be expressed separately in terms of horizontal, vertical, and rotational directions, then

$$\underline{M}\begin{Bmatrix}\ddot{\vec{r}}_h(x,t)\\ \ddot{\vec{r}}_v(x,t)\\ \ddot{\vec{r}}_\theta(x,t)\end{Bmatrix} + \underline{C}\begin{Bmatrix}\dot{\vec{r}}_h(x,t)\\ \dot{\vec{r}}_v(x,t)\\ \dot{\vec{r}}_\theta(x,t)\end{Bmatrix} + \underline{K}\begin{Bmatrix}\vec{r}_h(x,t)\\ \vec{r}_v(x,t)\\ \vec{r}_\theta(x,t)\end{Bmatrix} = -\,\underline{M}\begin{Bmatrix}\ddot{\vec{r}}_{gh}(t)\\ \ddot{\vec{r}}_{gv}(t)\\ \ddot{\vec{r}}_{g\theta}(t)\end{Bmatrix}, \qquad (2.6)$$

in which the subscripts h, v, and θ indicate the degree-of-freedom corresponding to horizontal, vertical, and rotational movement respectively.

By neglecting the effect of rotational ground motion, one may rewrite the horizontal and vertical ground accelerations in the following forms:

$$\ddot{\vec{r}}_{gh}(t) = \vec{e}x_h(t) \qquad (2.7a)$$

and

$$\ddot{\vec{r}}_{gv}(t) = \vec{e}x_v(t), \qquad (2.7b)$$

in which $x_h(t)$ and $x_v(t)$ are the functions of the specific horizontal and vertical ground accelerations, respectively, and $\vec{e}$ is a unit vector.

The substitution of Eqs. (2.7a,b) for the right side of Eq. (2.6) yields

$$\underline{M}\ddot{\vec{r}}(x,t) + \underline{C}\dot{\vec{r}}(x,t) + \underline{K}\vec{r}(x,t) = -\underline{M}\begin{Bmatrix}\vec{e}\\0\\0\end{Bmatrix}x_h(t) - \underline{M}\begin{Bmatrix}0\\\vec{e}\\0\end{Bmatrix}x_v(t). \qquad (2.8)$$

Equation (2.8) represents a set of coupling second-order differential equations. These equations can be uncoupled by using the modal superposition technique, for which the damping matrix $\underline{C}$ can be expressed in terms of mass and stiffness matrices as

$$\underline{C} = a_o\underline{M} + b_o\underline{K}, \qquad (2.9)$$

in which a_o and b_o are determined by employing the damping coefficient and the natural frequency of each mode to be used.

The uncoupling procedures require coordinate transformations. This can be done by changing the system coordinates to normal coordinates and the eigensolutions of the eigenvalues and eigenvectors. The eigensolutions can be obtained first by employing Eq. (2.10),

$$\underline{K}\vec{\phi}_i = \omega_i^2\underline{M}\vec{\phi}_i, \qquad (2.10)$$

in which ω_i is the i^{th} undamped natural frequency associated to the mode shape $\vec{\phi}_i$. Then the coordinate transformation can be expressed as

$$\vec{r}(x,t) = \underline{\Phi}\vec{q}(t), \qquad (2.11)$$

in which $\underline{\Phi}$ represents the eigenvector of each mode arranged in columns, and $\vec{q}(t)$ denotes the displacement vector associated with the normal coordinates. Thus, Eq. (2.8) becomes

$$\underline{m}\ddot{\vec{q}}(t) + \underline{c}\dot{\vec{q}}(t) + \underline{k}\vec{q}(t) = -\underline{\Phi}^T\underline{M}\begin{Bmatrix}\vec{e}\\0\\0\end{Bmatrix}x_h(t) - \underline{\Phi}^T\underline{M}\begin{Bmatrix}0\\\vec{e}\\0\end{Bmatrix}x_v(t). \tag{2.12}$$

In Eq. (2.12), $\underline{m}$, $\underline{c}$, and $\underline{k}$ are diagonal matrices and are represented by

$$\underline{m} = \underline{\Phi}^T\underline{M}\underline{\Phi}, \tag{2.13a}$$

$$\underline{c} = \underline{\Phi}^T\underline{C}\underline{\Phi}, \tag{2.13b}$$

and

$$\underline{k} = \underline{\Phi}^T\underline{K}\underline{\Phi}, \tag{2.13c}$$

By employing Eqs. (2.13a) and (2.13c), one can rewrite Eq. (2.10) as

$$\underline{k} = \underline{m}\underline{\Omega}, \tag{2.14}$$

in which $\underline{\Omega} = [\omega_i^2]$ as a diagonal matrix containing the square of the natural frequencies.

The i^{th} equation of the uncoupled system of Eq. (2.12) can now be expressed as

$$m_i\ddot{q}_i(t) + c_i\dot{q}_i(t) + k_iq_i(t) =$$

$$-\vec{\phi}_i^T\underline{M}\begin{Bmatrix}\vec{e}\\0\\0\end{Bmatrix}x_h(t) - \vec{\phi}_i^T\underline{M}\begin{Bmatrix}0\\\vec{e}\\0\end{Bmatrix}x_v(t). \tag{2.15}$$

From Eq. (2.14),

$$k_i = m_i \omega_i^2. \tag{2.16}$$

Also, let

$$\beta_i = \frac{c_i}{2m_i \omega_i} . \tag{2.17}$$

Then Eq. (2.15) becomes

$$\ddot{q}_i(t) + 2\beta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) =$$

$$- \frac{1}{m_i} \vec{\phi}_i^T \underline{M} \begin{Bmatrix} \vec{e} \\ 0 \\ 0 \end{Bmatrix} x_h(t) - \frac{1}{m_i} \vec{\phi}_i^T \underline{M} \begin{Bmatrix} 0 \\ \vec{e} \\ 0 \end{Bmatrix} x_v(t). \tag{2.18}$$

Equation (2.18) represents the motion equation for a single degree-of-freedom system having a frequency, ω_i, a mass, m_i, and a relative viscous damping ratio, β_i, and which is subject to multi-component horizontal and vertical ground motions.

B. CLOSED FORM SOLUTION FOR GROUND MOTION OR WIND EXCITATIONS

The solution of the motion equation shown in Eq. (2.18) for multicomponent ground motions can be obtained by using the Laplace transformation. Let Eq. (2.18) be rewritten as

$$\ddot{q}_i(t) + 2\beta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = - \frac{\gamma_{hi}}{m_i} x_h(t) - \frac{\gamma_{vi}}{m_i} x_v(t), \tag{2.19}$$

in which

$$\gamma_{hi} = \vec{\phi}_i^T \underline{M} \begin{Bmatrix} \vec{e} \\ 0 \\ 0 \end{Bmatrix} \tag{2.20a}$$

$$\gamma_{vi} = \vec{\phi}_i^T \underline{M} \begin{Bmatrix} 0 \\ \vec{e} \\ 0 \end{Bmatrix}. \tag{2.20b}$$

The terms γ_{hi} and γ_{vi} are called modal participation factors and are associated with the i^{th} mode.

By using the Laplace transformations with the initial conditions of $q_i(0) = 0$ and $\dot{q}_i(0) = 0$, one may write

$$\ddot{q}_i(t) = s^2\bar{q}_i(s) - sq_i(0) - \dot{q}_i(0) = s^2\bar{q}_i(s), \tag{2.21a}$$

$$\dot{q}_i(t) = s\bar{q}_i(s) - q_i(0) \qquad = s\bar{q}_i(s), \tag{2.21b}$$

$$q_i(t) = \bar{q}_i(s), \tag{2.21c}$$

and

$$c_1x_h(t) + c_2x_v(t) = c_1F_h(s) + c_2F_v(s) \tag{2.21d}$$

in which c_1 and c_2 are constants and F's are functions in the Laplace transformation coordinates. Thus, Eq. (2.19) becomes

$$(s^2 + 2\beta_i\omega_i s + \omega_i^2)\bar{q}_i(s) = -\frac{\gamma_{hi}}{m_i} F_h(s) - \frac{\gamma_{vi}}{m_i} F_v(s). \tag{2.22}$$

Let

$$a = -\beta_i\omega_i - \omega_i\sqrt{1-\beta_i^2}\,\bar{i} \tag{2.23a}$$

and

$$b = -\beta_i\omega_i + \omega_i\sqrt{1-\beta_i^2}\,\bar{i}, \tag{2.23b}$$

in which $\bar{i}$ is an imaginary number.

Substitute Eqs. (2.23a,b) for their equivalents in Eq. (2.22) and solve for $\bar{q}_i(s)$ as follows:

$$\bar{q}_i(s) = \frac{1}{(s-a)(s-b)}\left[-\frac{\gamma_{hi}}{m_i}F_h(s) - \frac{\gamma_{hi}}{m_i}F_v(s)\right]. \tag{2.24}$$

By employing

$$\frac{1}{(s-a)(s-b)} = \frac{e^{bt} - e^{at}}{b-a} \qquad a \neq b \tag{2.25}$$

and

$$F(s)G(s) = \int_0^t F(\tau)G(t-\tau)d\tau, \tag{2.26}$$

in which F and G are functions and τ is the time variable for integration, Eq. (2.24) can now be transformed back to its former condition as follows:

$$q_i(t) = \frac{1}{b-a}\left[-\frac{\gamma_{hi}}{m_i}\left\{\int_0^t x_h(\tau)e^{b(t-\tau)}d\tau - \int_0^t x_h(\tau)e^{a(t-\tau)}d\tau\right\}\right.$$

$$\left. - \frac{\gamma_{vi}}{m_i}\left\{\int_0^t x_v(\tau)e^{b(t-\tau)}d\tau - \int_0^t x_v(\tau)e^{a(t-\tau)}d\tau\right\}\right]. \tag{2.27}$$

By substituting a and b of Eqs. (2.23a,b) for their equivalents in Eq. (2.27) and using

$$\omega_{di} = \omega_i \sqrt{1 - \beta_i^2}\ , \tag{2.28a}$$

$$e^{\bar{i}\omega_{di}(t-\tau)} = \cos\omega_{di}(t-\tau) + \bar{i}\ \sin\omega_{di}(t-\tau), \tag{2.28b}$$

and

$$e^{-\bar{i}\omega_{di}(t-\tau)} = \cos\omega_{di}(t-\tau) - \bar{i}\ \sin\omega_{di}(t-\tau), \tag{2.28c}$$

the closed form solution of Eq. (2.19) can finally be obtained:

$$q_i(t) = -\frac{\gamma_{hi}}{m_i\omega_{di}}\int_0^t x_h(\tau)e^{-\beta_i\omega_i(t-\tau)}\sin\omega_{di}(t-\tau)d\tau$$

$$-\frac{\gamma_{vi}}{m_i\omega_{di}}\int_0^t x_v(\tau)e^{-\beta_i\omega_i(t-\tau)}\sin\omega_{di}(t-\tau)d\tau. \tag{2.29}$$

This proves that the dynamic response of a structural system subject to multicomponent earthquake motions can be obtained, without any second-order effect, by using the superposition of the response associated with each individual component.

If the structure is subjected to both static loads and dynamic forces, such as wind excitations, $\vec{R}(t)$, the motion equations can be similarly established as

$$\underline{M}\ddot{\vec{r}}(x,t) + \underline{C}\dot{\vec{r}}(x,t) + \underline{K}(\vec{r}(x,t) + \vec{r}(x)) = \vec{R}(t) + \vec{R}, \tag{2.30}$$

in which

$$\vec{R}(t) = \vec{R}_0 f(t). \tag{2.31}$$

Apparently, the solution associated with the dynamic response of Eq. (2.30) becomes

$$q_i(t) = \frac{\gamma_i}{m_i \omega_{di}} \int_0^t f(\tau) e^{-\beta_i \omega_i (t-\tau)} \sin\omega_{di}(t-\tau) d\tau. \tag{2.32}$$

For this case, the modal participation factor in Eq. (2.32) is

$$\gamma_i = \vec{\phi}_i^T \vec{R}_0. \tag{2.33}$$

C. <u>NUMERICAL SOLUTION FOR SEISMIC OR WIND EXCITATIONS</u>

It is apparent that the integration of Eqs. (2.29) and (2.32) can be evaluated exactly if the functions, $x_h(\tau)$, $x_v(\tau)$, and $f(\tau)$, are the mathematical expressions suitable for integration. For seismic or wind excitations, these functions are not continuous expressions, for which the integration must be performed numerically.

Let us consider a typical discontinuous forcing function of an earthquake record. For a small portion of a time interval, $\Delta t = t_2 - t_1$, the seismic excitation can be represented linearly as shown in Fig. 2. For simplicity, let Eq. (2.19) consist of one earthquake component, $x_g(t)$, and be expressed as

$$\ddot{q}_i(t) + 2\beta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = -\frac{\gamma_i}{m_i} x_g(t). \tag{2.34}$$

In order to include the effect of the initial conditions at any time, t, the solution associated with the initial conditions can be

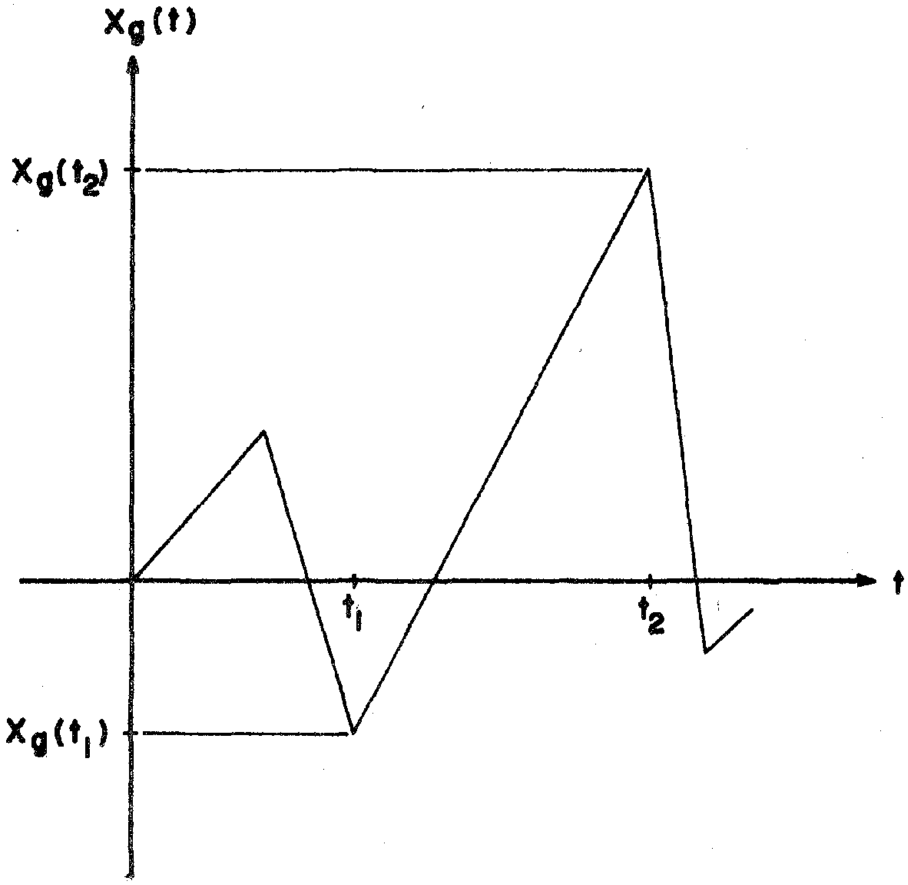


Figure 2. Arbitrary Forcing Function for Dynamic and Seismic Excitations.

added to the solution of Eq. (2.29) as shown in Eq. (2.35), which can be obtained directly by using the Laplace transforms:

$$q_i(t) = \frac{\gamma_i}{m_i} [e^{-\beta_i \omega_i t} (c_1 \cos\omega_{di} t + c_2 \sin\omega_{di} t)$$

$$+ \frac{1}{\omega_{di}} \int_0^t x_g(\tau) e^{-\beta_i \omega_i (t-\tau)} \sin\omega_{di}(t-\tau) d\tau], \tag{2.35}$$

in which c_1 and c_2 are constants to be evaluated from the initial conditions at a given time.

Because the function of the ground excitation is assumed to be linear during any time interval, t_1 and t_2, as shown in Fig. 2, it can be expressed as

$$x_g(t) = A + Bt, \tag{2.36}$$

in which

$$A = x_g(t_1) \tag{2.37}$$

and

$$B = \frac{x_g(t_2) - x_g(t_1)}{t_2 - t_1} . \tag{2.38}$$

By substituting Eq. (2.36) for its equivalent in Eq. (2.35) and considering only the second term in the brackets on the right side of the equation, one can obtain

$$\frac{1}{\omega_{di}}\int_0^t x_g(\tau)e^{-\beta_i\omega_i(t-\tau)}\sin\omega_{di}(t-\tau)d\tau = \frac{e^{-\beta_i\omega_i t}\sin\omega_{di}t}{\omega_{di}}$$

$$\int_0^t (A + B\tau)e^{\beta_i\omega_i\tau}\cos\omega_{di}\tau d\tau - \frac{e^{-\beta_i\omega_i t}\cos\omega_{di}t}{\omega_{di}}$$

$$\int_0^t (A + B\tau)e^{\beta_i\omega_i\tau}\sin\omega_{di}\tau d\tau. \tag{2.39}$$

The integrals of Eq. (2.39) become

$$\frac{1}{\omega_{di}}\int_0^t x_g(\tau)e^{-\beta_i\omega_i(t-\tau)}\sin\omega_{di}(t-\tau)d\tau = \frac{A + Bt}{\omega_i^2} - \frac{2\beta_i B}{\omega_i^3}. \tag{2.40}$$

Thus, the solution of Eq. (2.35) is

$$q_i(t) = \frac{\gamma_i}{m_i}[e^{-\beta_i\omega_i t}(c_1 \cos\omega_{di}t + c_2 \sin\omega_{di}t)$$

$$+ \frac{A + Bt}{\omega_i^2} - \frac{2\beta_i B}{\omega_i^3}]. \tag{2.41}$$

Differentiating Eq. (2.41) with respect to time yields the velocity, $\dot{q}_i(t)$, and acceleration, $\ddot{q}_i(t)$, at any time, t, during the linear forcing function; $q_i(t)$ and $\dot{q}_i(t)$ are then used to evaluate c_1 and c_2 by employing the following initial conditions at time t_1:

$$q_i(0) = q_i(t_1) \tag{2.42a}$$

and

$$\dot{q}_i(0) = \dot{q}_i(t_1). \tag{2.42b}$$

By substituting Eqs. (2.42a,b) respectively into Eq. (2.41) and its associated velocity equation and then solving for c_1 and c_2, one can find that

$$c_1 = q_i(t_1) - \frac{A}{\omega_i^2} + \frac{2\beta_i B}{\omega_i^3} \tag{2.43}$$

and

$$c_2 = \frac{1}{\omega_{di}} \{\dot{q}_i(t_1) + \beta_i\omega_i q_i(t_1) - \frac{\beta_i A}{\omega_i} + \frac{(2\beta_i^2 - 1)B}{\omega_i^2}\}. \tag{2.44}$$

Consequently, the modal response at any time, t, within the interval of time, t_1 and t_2, can be computed by substituting c_1 and c_2 in Eqs. (2.43) and (2.44) for their equivalents in Eq. (2.41) as follows:

$$q_i(t) = \frac{\gamma_i e^{-\beta_i\omega_i t}}{m_i} [\{q_i(t_1) - \frac{A}{\omega_1^2} + \frac{2\beta_i B}{\omega_i^3}\}\cos\omega_{di}t + \frac{1}{\omega_{di}} \{\dot{q}_i(t_1)$$

$$+ \beta_i\omega_i q_i(t_1) - \frac{\beta_i A}{\omega_i} + \frac{(2\beta_i^2 - 1)B}{\omega_i^2}\}\sin\omega_{di}t]$$

$$+ \frac{\gamma_i}{m_i} [\frac{A + Bt}{\omega_i^2} - \frac{2\beta_i B}{\omega_i^3}]. \tag{2.45}$$

The velocity and the acceleration associated with Eq. (2.45) are

$$\dot{q}_i(t) = \frac{\gamma_i e^{-\beta_i\omega_i t}}{m_i} [\{\dot{q}_i(t_1) - \frac{B}{\omega_i^2}\} \cos\omega_{di}t + \frac{1}{\omega_{di}} \{A - \omega_i^2 q_i(t_1)$$

$$- \beta_i\omega_i(\dot{q}_i(t_1) + \frac{B}{\omega_i^2})\} \sin\omega_{di}t] + \frac{\gamma_i B}{m_i\omega_i^2} \tag{2.46}$$

and

$$\ddot{q}_i(t) = \frac{\gamma_i e^{-\beta_i\omega_i t}}{m_i} [\{A - \omega_i^2 q_i(t_1) - 2\beta_i\omega_i\dot{q}_i(t_1)\}\cos\omega_{di}t$$

$$+ \frac{1}{\omega_{di}} \{-\beta_i\omega_i A + B + \beta_i\omega_i^3 q_i(t_1) + \omega_i^2(2\beta_i^2 - 1)\dot{q}_i(t_1)\}$$

$$\sin\omega_{di}t]. \tag{2.47}$$

Equations (2.45) and (2.46) are used to compute the end values, which become the initial conditions for the next linear portion. The complete solution of the modal response over the required time-history can be obtained by repeating these equations.

D. DISPLACEMENT AND STRESS RESPONSES TO STATIC AND DYNAMIC EXCITATIONS

Once $q_i(t)$ is computed for all significant modes, the displacements associated with the system coordinates can be obtained by means of Eq. (2.11) as follows:

$$\vec{r}(x,t) = \underline{\Phi}\ \underline{\Psi}\ \underline{D}\ \underline{\Phi}^T\ \vec{R}_0. \tag{2.48}$$

In this equation, $\underline{\Psi}$ and $\underline{D}$ are diagonal matrices, and their elements are given by

$$\Psi_i = \frac{1}{m_i \omega_i} \tag{2.49}$$

and

$$D_i = \int_0^t f(\tau) e^{-\beta_i \omega_i (t-\tau)} \sin\omega_{di}(t-\tau) d\tau. \tag{2.50}$$

The total displacements, $\vec{r}$, resulting from both the static loading and dynamic loading can be computed as follows:

$$\vec{r} = \vec{r}(x) \pm \vec{r}(x,t), \tag{2.51}$$

in which $\vec{r}(x)$ and $\vec{r}(x,t)$ are taken from Eqs. (2.2) and (2.48) respectively.

The nodal stresses of the constituent members can be computed in a manner similar to that used for the displacement response. On the basis of Eq. (2.1a), the internal forces, $\vec{S}_i$, of the i^{th} member can be determined by using

$$\vec{S}_i = \underline{K_i} \vec{v}_i(x), \tag{2.52}$$

in which $\underline{K_i} = \underline{K_{si}} - \underline{K_{gi}}$ of the member. Then the stresses, $\vec{\sigma}_i$, at the nodes of any member, i, can be computed from

$$\vec{\sigma}_i = \underline{B_i}\, \vec{S}_i, \tag{2.53}$$

in which $\underline{B_i}$ contains the cross-sectional properties of member i as shown in Appendix A.

The substitution of Eqs. (2.1a), (2.51), and (2.52) for their equivalents in Eq. (2.53) yields Eq. (2.54), in which the first term is due to static loading, and the second to dynamic loading;

$$\vec{\sigma}_i = \underline{B_i}\ \underline{K_i}\ \underline{a_i}\ \vec{r}(x) \pm \underline{B_i}\ \underline{K_i}\ \underline{a_i}\ \vec{r}(x,t). \qquad (2.54)$$

It is apparent that Eq. (2.54) gives the upper bound solutions for the individual modes. An adjustment of the stress and displacement responses on the basis of root-mean-square is also employed in the computer program in Appendix C and is discussed in Chapter III.

It is also apparent that a considerable amount of computer time is needed for the analysis of the eigenvalue problems. It has been found, however, that only certain eigenvalues and their associated eigenvectors are significant to the optimum design. A method particularly suitable for finding a limited number of eigenvalues and eigenvectors is described in Appendix B. The method is based on the Sturm sequence property in conjunction with a simple bisection procedure with which any eigenvalue can be determined without having to find any of the other eigenvalues.

III. STATIC LOADS AND SEISMIC EXCITATIONS

For the purpose of developing a computer program for a general optimum design, a structural system can be subjected to both static loads and seismic excitations. The static loads should be uniform and concentrated and should consist of the superimposed weight as well as the weight of the structure itself. For the seismic excitations, one of three forms should be used: 1) a response spectrum of any specific earthquake record, 2) an averaged response spectrum that is based on several earthquake records, or 3) the equivalent lateral forces recommended by the Uniform Building Code[43]. These three forms of seismic excitation that are discussed in this chapter can not only serve as an efficient technique for optimum design but can also be used for comparative studies of the optimum solutions resulting from a use of the Code and the response spectrum techniques.

A. JOINT FORCES ATTRIBUTABLE TO STATIC CONCENTRATED AND UNIFORM LOADS

The static concentrated loads applied at a structural joint for which generalized coordinates are designated can be directly formulated as the loading vector, $\vec{R}$, shown in Eq. (2.1). The uniform loads on the constituent members can also be directly formulated in $\vec{R}$, if they are lumped as various concentrated loads to which the structural joints are introduced. Consequently, the constituent members are divided into many segments, and the number of the degrees-of-freedom of a system can be considerably increased.

The most efficient approach for taking uniform loads into account is to use fixed-end actions (moments and shears) that can be transformed into equivalent joint loads. The formulation of equivalent joint loads from fixed-end actions on members can be expressed as

$$\vec{R}_e = - \sum_{i=1}^{m} \underline{a}_i^T \vec{A}_{mi} \quad , \tag{3.1}$$

in which

$\vec{R}_e$ = vector of equivalent joint loads,

$\underline{a}_i$ = compatibility matrix connecting the generalized coordinates of a system and those of the i^{th} member, and

$\vec{A}_{mi}$ = vector of fixed-end actions (moments and shears) on the i^{th} member.

The equivalent joint loads are then combined with the actual joint loads, $\vec{R}_c$, to form the loading vector, $\vec{R}$, in Eq. (3.2):

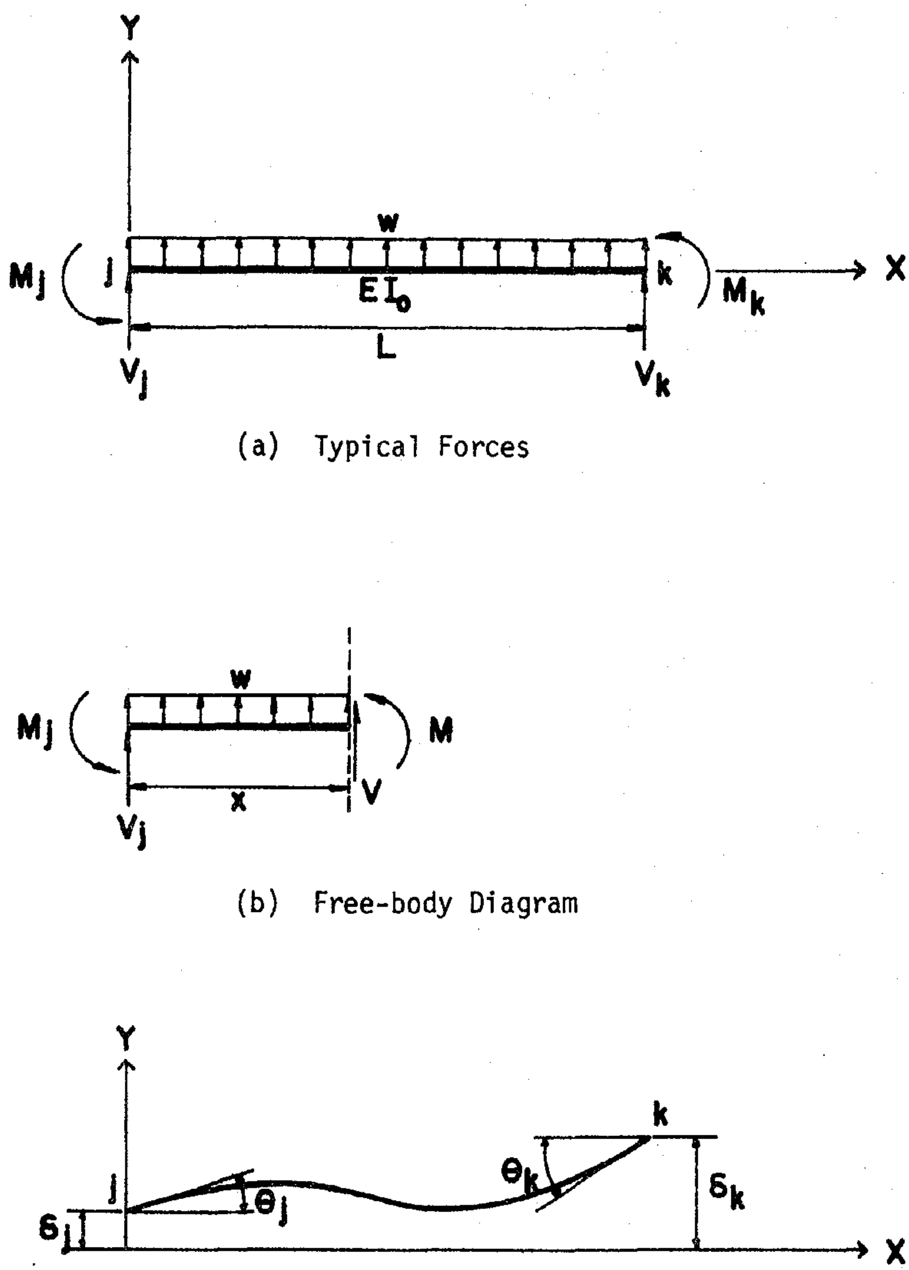
$$\vec{R} = \vec{R}_c + \vec{R}_e \quad . \tag{3.2}$$

The final joint forces for a member having a uniform load can be calculated by using

$$\vec{S}_i = \underline{K}_i \vec{v}_i(x) + \vec{A}_{mi} \quad . \tag{3.3}$$

B. STRAIN ENERGY OF A UNIFORMLY LOADED MEMBER

As explained in Chapter IV, the strain energy of the members of a system is needed for a design based on optimality criteria. For a case in which the loads are applied to the structural nodes, the strain energy can be obtained by using the nodal deformations directly, but the strain energy of a uniformly loaded member must be calculated on the basis of the elastic curve of the member. Consider a typical member, Fig. 3 , for which the positive end-forces and end-deformation are shown. In such a case, the strain energy



(a) Typical Forces

(b) Free-body Diagram

(c) Typical Deformations

Figure 3. Uniform Load on a Typical Girder.

attributable to the bending deformation can be generally expressed as

$$U = \int_0^L \frac{M^2}{2EI_0} dx \quad , \tag{3.4}$$

in which M is the bending moment at x, and EI_0 the flexural rigidity of the prismatic bar.

Because the elastic curve of the deformed bar can be related to the moment,

$$EI_0 \frac{d^2y}{dx^2} = M \quad . \tag{3.5}$$

The substitution of Eq. (3.5) for its equivalent in Eq. (3.4) yields

$$U = \frac{EI_0}{2} \int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx \quad . \tag{3.6}$$

By using the moment at any point x, one can obtain

$$M = - M_j + V_j x + \frac{wx^2}{2} \quad . \tag{3.7}$$

Then, Eq. (3.5) becomes

$$\frac{d^2y}{dx^2} = - \frac{M_j}{EI_0} + \frac{V_j x}{EI_0} + \frac{wx^2}{2EI_0} \quad , \tag{3.8}$$

for which the integration yields

$$\frac{dy}{dx} = - \frac{M_j x}{EI_0} + \frac{V_j x^2}{2EI_0} + \frac{wx^3}{6EI_0} + c_1 \tag{3.9}$$

and

$$y = - \frac{M_j x^2}{2EI_0} + \frac{V_j x^3}{6EI_0} + \frac{wx^4}{24EI_0} + c_1 x + c_2 \quad . \tag{3.10}$$

In the last two equations, c_1 and c_2 are evaluated by using the following boundary conditions:

$$\frac{dy}{dx} = \theta_j, \qquad y = \delta_j \qquad \text{at } x = 0 \tag{3.11}$$

and

$$\frac{dy}{dx} = \theta_k, \qquad y = \delta_k \qquad \text{at } x = L \quad . \tag{3.12}$$

Employment of Eqs. (3.11) and (3.12) yields

$$c_1 = \theta_j \quad , \tag{3.13}$$

$$c_2 = \delta_j \quad , \tag{3.14}$$

$$M_j = \frac{EI_0}{L} (4\theta_j + 2\theta_k + 6 \frac{\delta_j}{L} - 6 \frac{\delta_k}{L}) - \frac{wL^2}{12} , \tag{3.15}$$

and

$$V_j = \frac{6EI_0}{L^2} (\theta_j + \theta_k + 2 \frac{\delta_j}{L} - 2 \frac{\delta_k}{L}) - \frac{wL}{2} \quad . \tag{3.16}$$

The elastic curve of the member can finally be expressed as

$$y = \delta_j + \theta_j x - \frac{x^2}{L}\left(2\theta_j + \theta_k + \frac{3\delta_j}{L} - \frac{3\delta_k}{L}\right) + \frac{x^3}{L^2}\left(\theta_j + \theta_k\right.$$

$$\left. + \frac{2\delta_j}{L} - \frac{2\delta_k}{L}\right) + \frac{wx^2}{24EI_0}(L - x)^2 \quad . \tag{3.17}$$

In order to formulate the strain energy equation, differentiate Eq. (3.17) twice as follows:

$$y'' = -\frac{2}{L}\left(2\theta_j + \theta_k + 3\frac{\delta_j}{L} - 3\frac{\delta_k}{L}\right) + 6\frac{x}{L^2}\left(\theta_j + \theta_k + 2\frac{\delta_j}{L} - 2\frac{\delta_k}{L}\right)$$

$$+ \frac{w}{12EI_0}(L^2 - 6Lx + 6x^2) \quad . \tag{3.18}$$

By defining the terms on the right-hand side of Eq. (3.18) as

$$y''_d = -\frac{2}{L}\left(2\theta_j + \theta_k + 3\frac{\delta_j}{L} - 3\frac{\delta_k}{L}\right)$$

$$+ 6\frac{x}{L^2}\left(\theta_j + \theta_k + 2\frac{\delta_j}{L} - 2\frac{\delta_k}{L}\right) \tag{3.19}$$

and

$$y''_e = \frac{w}{12EI_0}(L^2 - 6Lx + 6x^2) \quad , \tag{3.20}$$

the term y'' can be expressed as

$$y'' = y''_d + y''_e \quad . \tag{3.21}$$

The substitution of Eq. (3.21) for its equivalent in Eq. (3.6) gives

$$U = \frac{EI_0}{2}\int_0^L (y''_d)^2 dx + \frac{EI_0}{2}\int_0^L (y''_e)^2 dx + EI_0\int_0^L y''_d y''_e dx \quad . \tag{3.22}$$

Integrating each term on the right side of Eq. (3.22) yields

$$\int_0^L (y_d'')^2 dx = \frac{4}{L} \{\theta_j^2 + \theta_j\theta_k + \theta_k^2 + \frac{3}{L} (\theta_j + \theta_k)(\delta_j - \delta_k)$$

$$+ \frac{3}{L^2} (\delta_j - \delta_k)^2 \} \quad , \tag{3.23}$$

$$\int_0^L (y_e'')^2 dx = \frac{w^2 L^5}{720(EI_0)^2} \quad , \tag{3.24}$$

and

$$\int_0^L y_d'' y_e'' dx = \quad y_d'' y_e' \Big]_0^L - y_d''' y_e \Big]_0^L + \int_0^L y_d^{iv} y_e dx = 0 \quad . \tag{3.25}$$

Equation (3.25) is based on the physical interpretation of Eqs. (3.19) and (3.20) for the former $y_d^{iv} = 0$ and for the latter $y_e' = y_e = 0$ at $x = 0$ and $x = L$. Substitution of Eqs. (3.23) through (3.25) for their equivalents in Eq. (3.22) gives the following strain energy of a uniformly loaded member:

$$U = \frac{2EI_0}{L} \{\theta_j^2 + \theta_j\theta_k + \theta_k^2 + \frac{3}{L} (\theta_j + \theta_k)(\delta_j - \delta_k)$$

$$+ \frac{3}{L^2} (\delta_j - \delta_k)^2\} + \frac{w^2 L^5}{1440 EI_0} \quad . \tag{3.26}$$

C. SEISMIC EXCITATIONS

The response of a structure subject to seismic excitation is parametric with time. To avoid using excessive computing time to solve for the dynamic response, the time parameter is eliminated by using three methods: the response spectrum, the average response spectrum, and the equivalent static lateral forces recommended by the Uniform Building Code. These three methods are used in this report for the purpose of showing how the individual loading consideration affects the optimum results.

1. Response Spectrum. As discussed in Chapter II, the response of a multidegree-of-freedom system can be simplified to that of a single-degree-of-freedom system for simultaneous horizontal and vertical ground motions, for which the result is given in Eq. (2.29). The solution of Eq. (2.29), which requires integration at different time intervals, demands a considerable amount of computational effort. In order to avoid the time dependency, one can define the spectral displacement as

$$s_d(\omega,\beta) = \left|\frac{1}{\omega_d}\int_0^t x_g(\tau)e^{-\beta\omega(t-\tau)}\sin\omega_d(t-\tau)d\tau\right|_{max} , \qquad (3.27)$$

in which $x_g(t)$ is a ground excitation as a function of time. By using the integration technique shown in Eq. (2.45), one can eliminate the time dependency by finding the maximum values of the integral of Eq. (3.27) for various values of the natural frequencies, ω, and the damping ratio, β.

Therefore, the maximum response as expressed in Eq. (2.29) can be found as follows:

$$|q_i(t)|_{max} = -\frac{\gamma_{hi}}{m_i}s_{dh}(\omega_i,\beta_i) - \frac{\gamma_{vi}}{m_i}s_{dv}(\omega_i,\beta_i) \qquad (3.28)$$

in which

$$s_{dh}(\omega_i,\beta_i) = \left|\frac{1}{\omega_{di}}\int_0^t x_h(\tau)e^{-\beta_i\omega_i(t-\tau)}\sin\,\omega_{di}(t-\tau)d\tau\right|_{max} , \quad (3.29a)$$

and

$$s_{dv}(\omega_i,\beta_i) = \left|\frac{1}{\omega_{di}}\int_0^t x_v(\tau)e^{-\beta_i\omega_i(t-\tau)}\sin\,\omega_{di}(t-\tau)d\tau\right|_{max} . \quad (3.29b)$$

The spectral accelerations are

$$s_{ah}(\omega_i,\beta_i) = \omega_{di}^2\ s_{dh}(\omega_i,\beta_i) , \quad (3.30a)$$

and

$$s_{av}(\omega_i,\beta_i) = \omega_{di}^2\ s_{dv}(\omega_i,\beta_i) . \quad (3.30b)$$

The spectral accelerations of the 1940 El Centro earthquake are shown in Fig. 4.

It should be noted that Eq. (3.28) always gives the conservative value of the response, because the maximum values for the individual modes do not occur at the same time. Consequently, the method provides a safe design result. A technique of leveling off the maximum responses is discussed in Section D of this chapter.

2. Average Response Spectrum. The average response spectrum which is based on Housner's work[16], can be obtained by computing the response spectra for two components of each of four different earthquakes and then normalizing, averaging, and smoothing the resulting curves. The four strong ground motions that are used in the average spectrum are El Centro 1934, El Centro 1940, Olympia 1949,

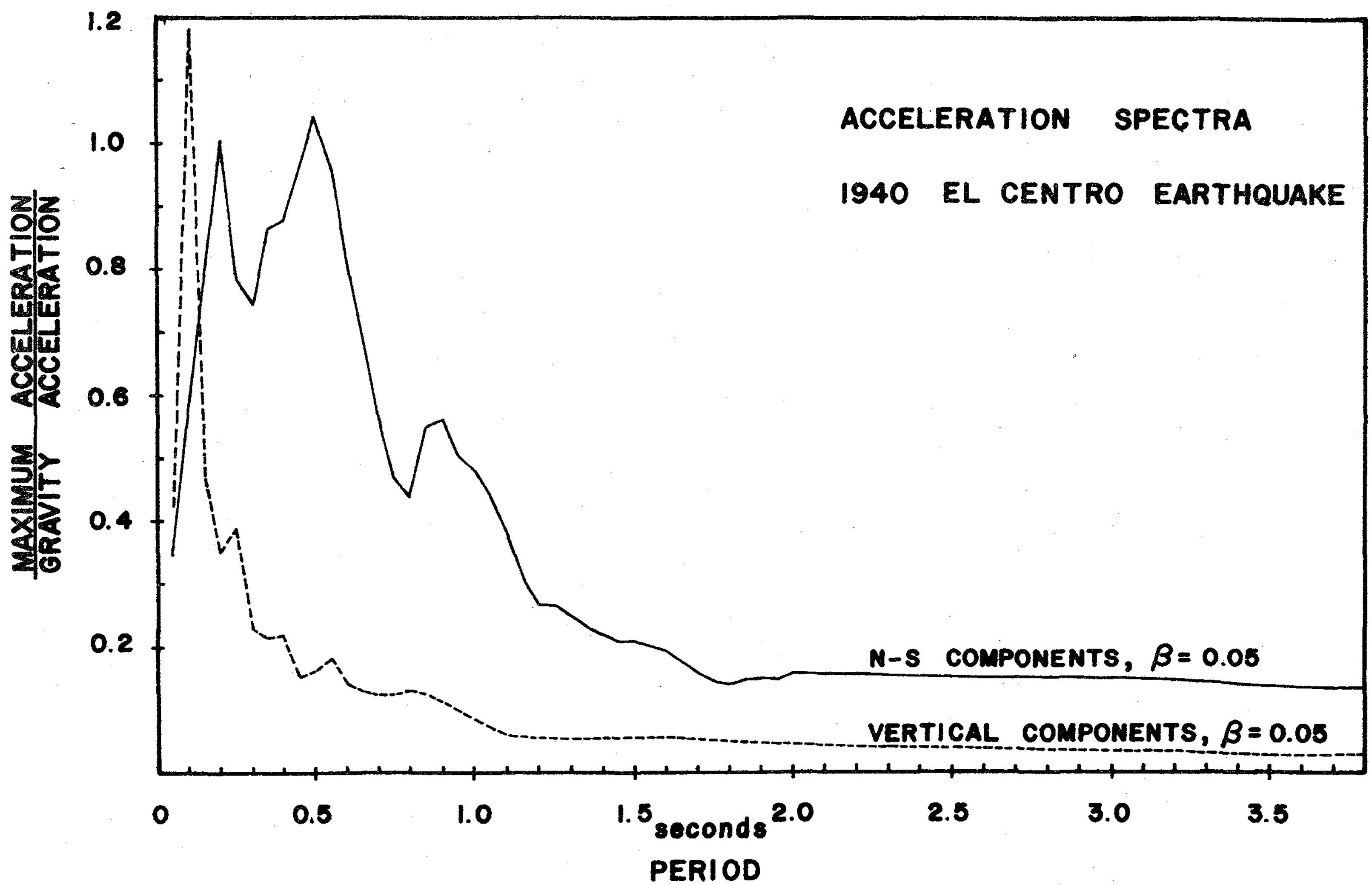


Figure 4. Acceleration Spectrum of El Centro Earthquake, May 18, 1940

and Tehachapi 1952. A typical average acceleration spectrum is shown in Fig. 5, for which the ordinates should be multiplied by scale factors to bring them into agreement with the specific ground motion, for instance 2.7, 1.9, and 1.6 for El Centro 18 May 1940, El Centro 30 December 1940, and Taft 21 July 1952 respectively.

3. Code Provisions of Equivalent Static Lateral Forces. The Uniform Building Code considers earthquake excitations as the equivalent of static lateral forces that are applied along the floors of a structure. The basic provisions of the Code provide a design base shear, V, which is defined as

$$V = ZIKCSW, \tag{3.31}$$

in which

Z = a zone coefficient having a value anywhere from 0 to 1 depending on the earthquake zone,

I = a coefficient that depends on the importance of the structure and ranges from 1 for ordinary buildings to 1.5 for essential buildings,

K = a factor that defines the type of structural system,

C = the seismic coefficient,

S = the site resonance factor, which depends on the ratio of building period, T, to the site period, T_s, and

W = the total weight of the building.

Methods are provided for determining the site period through the use of geotechnical investigations. However, the site period used for designing must lie within the interval of 0.5 sec. $\leq T_s \leq$ 2.5 sec.

The factor S may be as little as 1, if the building period and site period are widely separated. It can range up to a maximum of

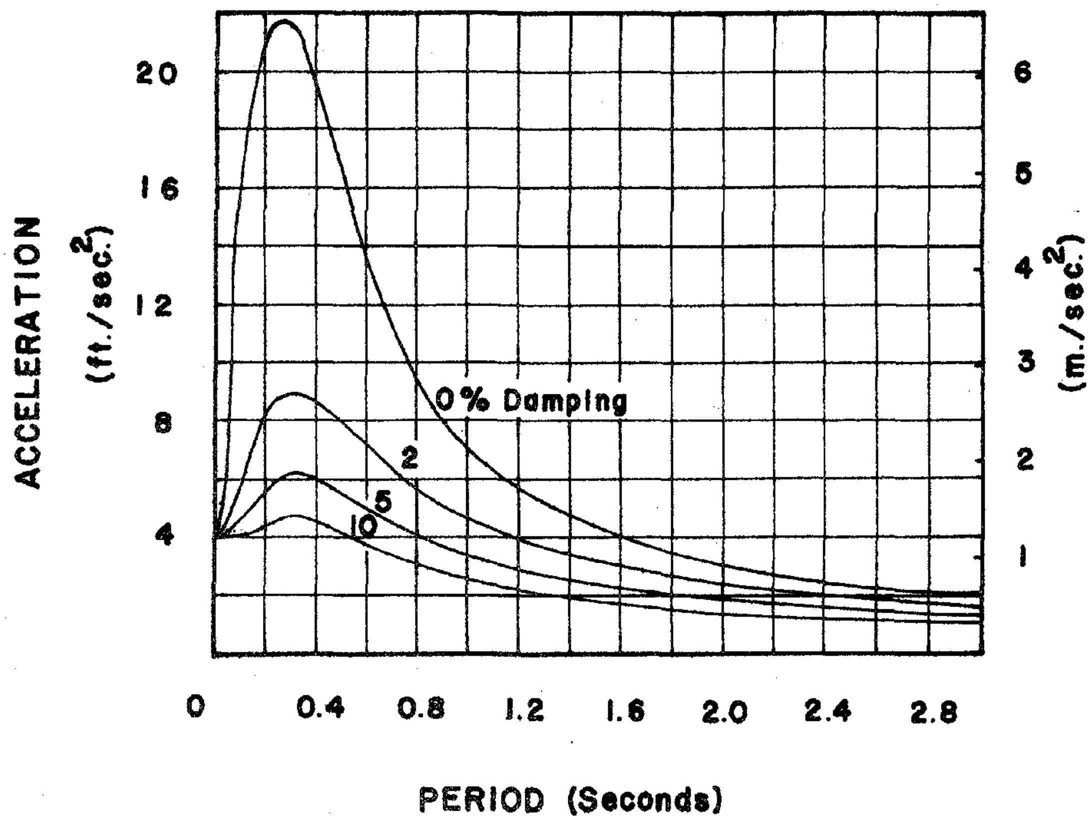


Figure 5. Average Acceleration Spectrum.

1.5, if the building period and site period are the same. Two transition formulas are specified for determining the value of S.

$$\text{For } \frac{T}{T_s} > 1, \qquad S = 1.2 + 0.6\,\frac{T}{T_s} - 0.3\,\left(\frac{T}{T_s}\right)^2 , \tag{3.32}$$

and

$$\text{for } \frac{T}{T_s} \leq 1, \qquad S = 1 + \frac{T}{T_s} - 0.5\,\left(\frac{T}{T_s}\right)^2 , \tag{3.33}$$

but T may not be taken as less than 0.3 second for determining S.

The building period can be determined by using the Rayleigh quotient formula given by

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^{n_s} w_i \delta_i^2}{g[F_t \delta_{ns} + \sum F_i \delta_i]}} \tag{3.34}$$

in which

w_i = weight at level i,

F_i = design lateral force at level i,

δ_i = computed lateral displacement at level i,

n_s = total number of levels, and

g = gravity acceleration.

Equation (3.34) is available in the computer program. One may also find the exact natural frequencies by using the eigenvalue subroutine.

The seismic coefficient is specified to be

$$C = \frac{1}{15\sqrt{T}} \quad , \tag{3.35}$$

and the upper limits are specified for both C and the product CS as

$$C \leq 0.12 \tag{3.36}$$

and

$$CS \leq 0.14 \quad . \tag{3.37}$$

The force assigned to the top is a function of the building period.

$$\text{For } T \leq 0.7 \text{ sec.,} \qquad F_t = 0 \quad , \tag{3.38}$$

$$\text{for } T > 0.7 \text{ sec.,} \qquad F_t = 0.07\ V \quad , \tag{3.39}$$

and for any case, $F_t \leq 0.25$ V. The remainder of the base shear is distributed linearly according to the following formula:

$$F_x = (V - F_t) \frac{w_x h_x}{\sum_{i=1}^{n_s} w_i h_i} \tag{3.40}$$

in which

F_x = design lateral force at level x,

w_x = weight at level x, and

h_x = height of level x from the ground level.

D. MODIFICATION OF SPECTRAL VALUES

The dynamic responses can be obtained by using any of the three options: a) direct superposition of desired modes, b) superposition of absolute quantities of desired modes, and c) the root-mean-square method that is the square root of the sum of the squares of the desired modal responses. Methods (a) and (b) are quite straightforward. Method (c) can be illustrated by showing the maximum total displacement vector, $|\vec{r}(x,t)|_{max}$, as

$$|\vec{r}(x,t)|_{max} = [\sum_{k=1}^{DM} |\vec{r}_k(x,t)|^2_{max}]^{1/2} \quad , \tag{3.41}$$

in which k is the number of desired modes (DM) to be used. Similarly, the maximum internal forces of member i can be approximated by using

$$|\vec{S}_i|_{max} = [\sum_{k=1}^{DM} \{(\underline{K}_{si} - \underline{K}_{gi})\underline{a}_i|\vec{r}_k(x,t)|_{max}\}^2]^{1/2} \quad . \tag{3.42}$$

Note that the portion of the energy density, μ'_{ji}, associated with the dynamic response must be consistently calculated according to the displacements obtained by one of these three methods.

Although it has been recognized that the correlation between modal responses for modes with closely spaced frequencies may lead to overly conservative or unsafe results, yet the superposition techniques discussed above do not present any difficulties in the design because the frequencies are far spaced. Nevertheless, the new technique of complete quadratic combination (CQC) method can be easily implemented in the program.

IV. OPTIMALITY CRITERIA AND RECURSION RELATION

The optimization technique is based on optimality criteria that are derived on the basis of Kuhn-Tucker conditions. The recursion relation is used as an iterative procedure to satisfy the optimality requirement for an optimum structural design.

A. FORMULATION OF A GENERAL PROBLEM IN OPTIMUM STRUCTURAL DESIGN

A general problem in optimum structural design can be stated as follows:

$$\left.\begin{array}{l} \text{Minimize } W(x) \\ \text{subject to } g_j(x) \leq 0 \;, \qquad j = 1,2,\ldots,n \\ \text{and } x \geq 0 \end{array}\right\} \tag{4.1}$$

in which

$W(x)$ = objective function,

x = vector of design variables,

$g_j(x)$ = the j^{th} constraint, and

n = total number of constraints.

In an optimum structural design, $W(x)$ can represent the total cost of the structure; however the total weight of the structure has been widely used in most publications and is used as the objective function in this work.

The constraints, $g_j(x)$, may represent bounds on frequencies and the behavior constraints of stress and displacement limitations imposed on a structure. These behavior constraints may be required for both static loads and seismic excitations. The problem shown in Eq. (4.1) is an inequality form of constrained minimization. To develop an algorithm for solving this problem, the characteristics of an optimization known as the Kuhn-Tucker conditions must be considered.

B. KUHN-TUCKER CONDITIONS OF OPTIMALITY

Kuhn-Tucker's conditions for optimality may be stated as follows:

Assume that $W(x)$ is to be minimized and is subject to the constraints $g_j(x) \leq 0$, $j = 1,2,\ldots,n$, $x \geq 0$ and that $W(x)$ and $g_j(x)$ are differentiable. Then x^* can be an optimal solution to the above nonlinear optimization problem if and only if there exist n numbers, $\lambda_1^*, \lambda_2^*, \ldots, \lambda_n^*$ such that the following conditions are satisfied:

$$\frac{\partial W(x)}{\partial x_i} + \sum_{j=1}^{n} \lambda_j \frac{\partial g_j(x)}{\partial x_i} = 0, \qquad x = x^*, \text{ for } x^* > 0, \tag{4.2}$$

$$\frac{\partial W(x)}{\partial x_i} + \sum_{j=1}^{n} \lambda_j \frac{\partial g_j(x)}{\partial x_i} \geq 0, \qquad x = x^*, \text{ for } x^* = 0, \tag{4.3}$$

$$g_j(x^*) = 0, \text{ for } \lambda_j^* > 0 \quad , \tag{4.4}$$

$$g_j(x^*) \leq 0, \text{ for } \lambda_j^* = 0 \quad , \tag{4.5}$$

$$x^* \geq 0 \quad , \tag{4.6}$$

and

$$\lambda_j^* \geq 0 \quad . \tag{4.7}$$

Equations (4.2) through (4.7) are the necessary conditions for the objective function $W(x)$ to be a relative or local minimum. It should be noted that these conditions are necessary but not sufficient.

Only if additional conditions are imposed, it can be claimed that they are sufficient to guarantee an optimum. For instance, if $W(x)$ is concave and the constraints form a convex set, the conditions are sufficient for x^* to be the optimal solution.

C. RECURSION RELATION RELATED TO OPTIMALITY CRITERIA

The optimization technique considered in this project can be applied to trusses and frameworks that have or do not have bracing members. The trusses are composed of two-force members. The members of frameworks that do not have bracings are subjected to combined bending and axial forces as beam-columns, and the frameworks having bracings include beam-columns and two-force members. Thus the formulation for the braced frames can be a generalized presentation for all three types of structures. Based on the Kuhn-Tucker necessary conditions, the problem in the optimum design of a braced plane frame can be expressed as follows:

$$\text{Minimize } W(x) = \sum_{i=1}^{m} \rho_i \eta_i x_i \ell_i \tag{4.8}$$

$$\text{subject to } g_j(x) = y_j(x) - b_j \le 0, \qquad j = 1, 2, \ldots, n \tag{4.9}$$

$$x_i \ge x_i^0, \qquad i = 1, 2, \ldots, m \tag{4.10}$$

in which

$W(x)$ = total weight of a structure composed of columns, girders, and bracing members,

ρ_i = mass density of the i^{th} member,

η_i = ratio of the cross-sectional area, A_i, to the moment of inertia, x_i, for the i^{th} member,

x_i = primary design variable of the i^{th} member,

ℓ_i = length of the i^{th} member,

$y_j(x)$ = expression of the j^{th} behavior constraint,

b_j = limitations imposed on the j^{th} behavior constraint,

x_i^o = lower bound on the i^{th} design variable,

m = total number of members, and

n = total number of behavior constraints.

The primary design variable, x_i, are the moments of inertia for the columns and girders and represent the cross-sectional areas for bracings and truss members. Apparently, the ratio of the cross-sectional area to the moment of inertia for the i^{th} bracing member is unity. The functions, $y_j(x)$, may represent the stress, displacement, and natural frequency of a structure.

By using the necessary conditions of Kuhn-Tucker to characterize any local minimum, Eqs. (4.8) through (4.10) can be written for each design variable as

$$\frac{\partial W(x)}{\partial x_i} + \sum_{j=1}^{n} \lambda_j \frac{\partial y_j(x)}{\partial x_i} - s_i = 0, \qquad i = 1, 2, \ldots, m \tag{4.11}$$

with

$$\lambda_j [y_j(x) - b_j] = 0, \qquad j = 1, 2, \ldots, n \ , \tag{4.12}$$

$$s_i [x_i^o - x_i] = 0, \qquad i = 1, 2, \ldots, m \ , \tag{4.13}$$

$$\lambda_j \geq 0, \qquad j = 1, 2, \ldots, n \ , \tag{4.14}$$

and

$$s_i \geq 0, \qquad i = 1, 2, \ldots, m \ , \tag{4.15}$$

in which the variables λ_j and s_i are the Lagrange multipliers associated with the constraints in Eqs. (4.9) and (4.10) respectively. By examining Eqs. (4.12) and (4.14), one can observe that when the j^{th} constraint is active $\lambda_j > 0$, and when the constraint is not active, the corresponding $\lambda_j = 0$. Similar observations can be applied to the side constraints expressed in Eqs. (4.13) and (4.15).

For simplicity, the active side constraints can be separated from the active behavior constraints. Let N denote the number of active behavior constraints, then the separated form of the Kuhn-Tucker necessary conditions from Eq. (4.11) can be simplified to

$$\frac{\partial W(x)}{\partial x_i} + \sum_{j=1}^{N} \lambda_j \frac{\partial y_j(x)}{\partial x_i} = 0, \qquad i \in J \tag{4.16}$$

and

$$\frac{\partial W(x)}{\partial x_i} + \sum_{j=1}^{N} \lambda_j \frac{\partial y_j(x)}{\partial x_i} \geq 0, \qquad i \in J_o , \tag{4.17}$$

in which J is the set of design variables for the conditions of $x_i > x_i^o$, and J_o is the set of design variables constrained by the lower bounds, x_i^o.

The separated form of the Kuhn-Tucker necessary conditions in Eqs. (4.16) and (4.17) can be combined into a single equation as follows:

$$-\frac{\sum_{j=1}^{N} \lambda_j \mu_{ji}}{\tau_i} = I_i, \qquad i = 1, 2, \ldots, m \tag{4.18}$$

in which

$$I_i \begin{cases} = 1, & i \in J \\ \leq 1, & i \in J_o \end{cases} \tag{4.19}$$

$$\mu_{ji} = \frac{\partial y_j(x)}{\partial x_i} \tag{4.20}$$

and

$$\tau_i = \frac{\partial W(x)}{\partial x_i} . \tag{4.21}$$

Equations (4.18) and (4.19) are the optimality criteria. Any design that satisfies these criteria is a relative minimum. However, it should be pointed out that these criteria are necessary but not sufficient to guarantee global optimum as a general case of mathematical nonlinear programming.

The recursion relation can now be derived by using the optimality criteria in Eq. (4.18) with all $x_i > x_i^o$, that is $I_i = 1$, $i = 1, 2, \ldots, m$. Therefore, Eq. (4.18) becomes

$$- \frac{\sum_{j=1}^{N} \lambda_j \mu_{ji}}{\tau_i} = 1, \qquad i = 1, 2, \ldots, m. \tag{4.22}$$

For the convenience of computer programming, let

$$x_i = \Lambda \alpha_i, \qquad i = 1, 2, \ldots, m \tag{4.23}$$

in which α_i are the relative design variables corresponding to the design variables, x_i; Λ is a scaling factor. By multiplying both sides of Eq. (4.22) by $(\Lambda \alpha_i)^2$, rearranging the terms, and taking the square root, one obtains

$$\Lambda\alpha_i = \alpha_i \left[- \frac{\sum_{j=1}^{N} \lambda_j \mu'_{ji}}{\tau'_i} \right]^{1/2} , \qquad i = 1, 2, \ldots, m \tag{4.24}$$

in which

$$\mu'_{ji} = \Lambda\mu_{ji} , \tag{4.25}$$

$$\tau'_i = \frac{\tau_i}{\Lambda} , \tag{4.26}$$

and $\Lambda\alpha_i$ is the i^{th} design variable, which is expressed as a function of α_i.

The form of Eq. (4.24) suggests the following recursion relation for determining the design variable in each cycle:

$$(\Lambda\alpha_i)_{v+1} = (\alpha_i)_v \left[- \frac{\sum_{j=1}^{N} \lambda_j \mu'_{ji}}{\tau'_i} \right]_v^{1/2} , \qquad i = 1, 2, \ldots, m \tag{4.27}$$

in which the subscripts v and v+1 denote the cycles of iteration. The use of the recursion relation expressed in Eq. (4.27) is the same as forcing the final design to satisfy the Kuhn-Tucker necessary conditions given in Eqs. (4.18) and (4.19).

D. CALCULATION OF LAGRANGE MULTIPLIERS FOR MULTIPLE ACTIVE CONSTRAINTS

At any stage in the design process in which Eq. (4.27) is used, it is possible that there is more than one active constraint. Therefore, it is necessary to find the Lagrange multipliers that correspond to the active constraints of the current design variables. The calculation of the multipliers can be accomplished from Eq. (4.22) by forming an auxiliary function, $L(\lambda)$, as follows

$$L(\lambda) = \sum_{i=1}^{J} \left[1 + \frac{\sum_{j=1}^{N} \lambda_j \mu_{ji}}{\tau_i} \right]^2 . \tag{4.28}$$

This is minimized by solving the set of linear equations,

$$\frac{\partial L(\lambda)}{\partial \lambda_j} = 0, \qquad j = 1, 2, \ldots, N \tag{4.29}$$

for the Lagrange multipliers, λ_j. Equation (4.29) can then be written in an expanded form as follows:

$$\begin{aligned}
\frac{\partial L(\lambda)}{\partial \lambda_1} = 0 = {} & (E_{11} + E_{12} + \ldots + E_{1J}) + (E_{11}^2 + E_{12}^2 + \ldots \\
& + E_{1J}^2)\lambda_1 + (E_{11}E_{21} + E_{12}E_{22} + \ldots + E_{1J}E_{2J})\lambda_2 \\
& + \ldots + (E_{11}E_{N1} + E_{12}E_{N2} + \ldots + E_{1J}E_{NJ})\lambda_N \\
\frac{\partial L(\lambda)}{\partial \lambda_2} = 0 = {} & (E_{21} + E_{22} + \ldots + E_{2J}) + (E_{21}E_{11} \\
& + E_{22}E_{12} + \ldots + E_{2J}E_{1J})\lambda_1 + (E_{21}^2 + E_{22}^2 + \ldots \\
& + E_{2J}^2)\lambda_2 + \ldots + (E_{21}E_{N1} + E_{22}E_{N2} + \ldots \\
& + E_{2J}E_{NJ})\lambda_N \\
& \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
& \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
& \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\frac{\partial L(\lambda)}{\partial \lambda_N} = 0 = {} & (E_{N1} + E_{N2} + \ldots + E_{NJ}) + (E_{N1}E_{11} + E_{N2}E_{12} \\
& + \ldots + E_{NJ}E_{1J})\lambda_1 + (E_{N1}E_{21} + E_{N2}E_{22} + \ldots \\
& + E_{NJ}E_{2J})\lambda_2 + \ldots + (E_{N1}^2 + E_{N2}^2 + \ldots + E_{NJ}^2)\lambda_N \quad .
\end{aligned} \tag{4.30}$$

in which

$$E_{ji} = \frac{\mu_{ji}}{\tau_i} . \tag{4.31}$$

In matrix form, Eq. (4.30) becomes

$$\underline{H}\vec{\lambda} = \vec{G} \tag{4.32}$$

in which

$$H_{jk} = \sum_{i=1}^{J} E_{ji}E_{ki}, \tag{4.33}$$

and

$$G_k = - \sum_{i=1}^{J} E_{ki}. \tag{4.34}$$

The matrix, $\underline{H}$, has the property of being symmetrical. By solving Eq. (4.32), the Lagrange multipliers corresponding to the active constraints can be determined from the current design variables. These multipliers are then used in Eq. (4.27) for determining the design variables of the next cycle in the design process.

V. CONSTRAINTS IN THE OPTIMUM STRUCTURAL DESIGN

The behavior constraints, $y_j(x)$, considered in optimum structural design are the limitations of displacements (flexibility) and stresses (stiffness) that are imposed on both static and seismic structures. However, for seismic structures, an additional behavior constraint of natural frequencies is taken into account. The side constraints are the limitations on the size and moment of inertia or cross-sectional area of the constituent members. This chapter presents the derivations of various behavior constraints.

A. FLEXIBILITY CONSTRAINTS FOR STATIC LOADS

The flexibility constraints of a system result from the displacement limitations of either certain nodes or all the nodes. The displacement constraint function may be expressed by using the following virtual work at any nodal point:

$$u_j(x) = \vec{Q}_j^T \vec{r}(x), \tag{5.1}$$

in which

$\vec{Q}_j$ = load vector with unit value for the j^{th} direction and zero values for others, and

$\vec{r}(x)$ = vector of generalized displacements attributable to the static load, $\vec{R}$.

Differentiating $u_j(x)$ of Eq. (5.1) with respect to the design variable, x_i, yields

$$\frac{\partial u_j(x)}{\partial x_i} = \vec{Q}_j^T \frac{\partial \vec{r}(x)}{\partial x_i}, \tag{5.2}$$

in which $\partial\vec{r}(x)/\partial x_i$ can be obtained by differentiating Eq. (2.2) with respect to x_i and considering $\vec{R}$ independent of the design variables. Thus,

$$\frac{\partial\vec{r}(x)}{\partial x_i} = - \underline{K}^{-1} \frac{\partial\underline{K}}{\partial x_i} \vec{r}(x). \tag{5.3}$$

The substitution of Eq. (5.3) for its equivalent in Eq. (5.2) yields

$$\frac{\partial u_j(x)}{\partial x_i} = - \vec{Q}_j^T \underline{K}^{-1} \frac{\partial\underline{K}}{\partial x_i} \vec{r}(x). \tag{5.4}$$

Let $\vec{q}_j(x)$ be a vector of the generalized displacements attributable to load $\vec{Q}_j$, then

$$\vec{q}_j^{\,T}(x) = (\underline{K}^{-1}\vec{Q}_j)^T = \vec{Q}_j^T\underline{K}^{-1} \tag{5.5}$$

and Eq. (5.4) becomes

$$\frac{\partial u_j(x)}{\partial x_i} = - \vec{q}_j^{\,T}(x) \frac{\partial\underline{K}}{\partial x_i} \vec{r}(x). \tag{5.6}$$

From Eqs. (2.1c), (2.1d), and (2.3), one can write

$$\frac{\partial\underline{K}}{\partial x_i} = \underline{a}_i^T \frac{\partial\underline{K}_{si}}{\partial x_i} \underline{a}_i - \frac{\partial P_i}{\partial x_i} \underline{a}_i^T \underline{K}_{gi} \underline{a}_i. \tag{5.7}$$

Because the elements in $\underline{K}_{si}$ and $\partial P_i/\partial x_i$ have a linear relationship with the design variable, x_i, Eq. (5.7) can be written in the following form:

$$\frac{\partial \underline{K}}{\partial x_i} = \frac{1}{x_i}\, \underline{a_i^T}\, \underline{K_{si}}\, \underline{a_i} - \frac{P_i'}{x_i}\, \underline{a_i^T}\, \underline{K_{gi}}\, \underline{a_i}, \tag{5.8a}$$

in which

$$P_i' = \frac{\rho_i n_i \ell_i}{2} . \tag{5.8b}$$

Note that the second term of the right-hand side of Eq. (5.8a) can be applied only to the column. The substitution of Eq. (5.8) for its equivalent in (5.6) yields

$$\frac{\partial u_j(x)}{\partial x_i} = - \frac{1}{x_i}\, \vec{q}_j^{\,T}(x) \underline{a_i^T}\, \underline{K_{si}}\, \underline{a_i} \vec{r}(x) + \frac{P_i'}{x_i}\, \vec{q}_j^{\,T}(x) \underline{a_i^T}\, \underline{K_{gi}}\, \underline{a_i} \vec{r}(x). \tag{5.9}$$

Now the terms μ_{ji}' and τ_i' in the recursion relation of Eq. (4.27) can be physically expressed as

$$\mu_{ji}' = \Lambda \frac{\partial u_j(x)}{\partial x_i} \tag{5.10}$$

and

$$\tau_i' = \frac{1}{\Lambda} \frac{\partial W(x)}{\partial x_i} . \tag{5.11}$$

In Eq. (5.11), $W(x)$ is given in Eq. (4.8). In a case where there is only a single displacement constraint in the j^{th} direction, the optimality criteria of Eq. (4.18) has the following form:

$$\frac{\vec{q}_j^{\,T}(x)\, \underline{a_i^T}\, \underline{K_{si}}\, \underline{a_i}\, \vec{r}(x) - P_i'\, \vec{q}_j^{\,T}(x)\, \underline{a_i^T}\, \underline{K_{gi}}\, \underline{a_i}\, \vec{r}(x)}{\rho_i n_i x_i \ell_i} = \text{constant}, \quad i=1,2,\ldots,m. \tag{5.12}$$

Equation (5.12) may reveal some physical meaning that the optimum structure for the specified displacement is the one in which the ratio of the average virtual strain energy density to the mass density is the same for all its members.

B. STIFFNESS CONSTRAINTS FOR STATIC LOADS

The stiffness constraints are used to measure the limitations of the allowable shear stress and the allowable combined stress of axial and bending.

The stiffness of the structure can be described by the work caused by the static load, $\vec{R}$, multiplied by the generalized displacement, $\vec{r}(x)$, in the form of

$$z(x) = \frac{1}{2} \vec{R}^T \vec{r}(x), \tag{5.13}$$

because the product, $\vec{R}^T \vec{r}(x)$, is an inverse measure of the stiffness. Thus, $z(x)$ may be called a measuring function of static stiffness. Differentiating Eq. (5.13) with respect to the design variables, x_i, yields

$$\frac{\partial z(x)}{\partial x_i} = \frac{\vec{R}^T}{2} \frac{\partial \vec{r}(x)}{\partial x_i} . \tag{5.14}$$

After substituting Eqs. (5.3) and (5.8) for their equivalents in Eq. (5.14), one obtains either

$$\begin{aligned} \frac{\partial z(x)}{\partial x_i} = & - \frac{1}{2x_i} \vec{R}^T \underline{K}^{-1} \underline{a_i^T} \, \underline{K_{si}} \, \underline{a_i} \, \vec{r}(x) \\ & + \frac{P_i'}{2x_i} \vec{R}^T \underline{K}^{-1} \underline{a_i^T} \, \underline{K_{gi}} \, \underline{a_i} \, \vec{r}(x) \end{aligned} \tag{5.15}$$

or

$$\frac{\partial z(x)}{\partial x_i} = -\frac{1}{2x_i}\vec{r}^T(x)\ \underline{a}_i^T\ \underline{K}_{si}\ \underline{a}_i\ \vec{r}(x)$$

$$+\frac{P_i'}{2x_i}\vec{r}^T(x)\ \underline{a}_i^T\ \underline{K}_{gi}\ \underline{a}_i\ \vec{r}(x). \tag{5.16}$$

Consequently, μ_{ji}' in the recursion relation of Eq. (4.27) becomes

$$\mu_{ji}' = \Lambda \frac{\partial z(x)}{\partial x_i}, \tag{5.17}$$

The optimality criteria of Eq. (4.18) for a single loading condition can be expressed as

$$\frac{1}{2}\ \frac{\vec{r}(x)\ \underline{a}_i^T\ \underline{K}_{si}\ \underline{a}_i\ \vec{r}(x) - P_i'\vec{r}(x)\ \underline{a}_i^T\ \underline{K}_{gi}\ \underline{a}_i\ \vec{r}(x)}{\rho_i \eta_i x_i \ell_i} = \text{constant},$$

$$i = 1, 2, \ldots, m. \tag{5.18}$$

Note that the numerator of Eq. (5.18) represents the average strain energy in which the second term is associated with the column members only.

C. FLEXIBILITY CONSTRAINTS FOR DYNAMIC LOADS

The dynamic displacement constraint function can be expressed in a form similar to that of Eq. (5.1) in terms of virtual work as follows:

$$u_j(x,t) = \vec{Q}_j^T \vec{r}(x,t), \tag{5.19}$$

in which

$\vec{Q}_j$ = load vector with unit force for the j^{th} direction only, and the time function is also a unit value, and

$\vec{r}(x,t)$ = vector of generalized displacements attributable to the dynamic load, $\vec{R}(t)$.

Differentiating Eq. (5.19) with respect to the design variables, x_i, yields

$$\frac{\partial u_j(x,t)}{\partial x_i} = \vec{Q}_j^T \frac{\partial \vec{r}(x,t)}{\partial x_i} , \tag{5.20}$$

in which $\partial\vec{r}(x,t)/\partial x_i$ can be found by using the following motion equation:

$$\underline{M}\ddot{\vec{r}}(x,t) + \underline{K}\vec{r}(x,t) = \vec{R}(t). \tag{5.21}$$

Damping is included in the response analysis and therefore is not considered in the above equation.

In order to find $\partial\vec{r}(x,t)/\partial x_i$ in Eq. (5.20), take the first variation of Eq. (5.21), which is

$$\underline{M}\,\frac{\partial \ddot{\vec{r}}(x,t)}{\partial x_i} + \underline{K}\,\frac{\partial \vec{r}(x,t)}{\partial x_i} = -\frac{\partial \underline{M}}{\partial x_i}\,\ddot{\vec{r}}(x,t) - \frac{\partial \underline{K}}{\partial x_i}\,\vec{r}(x,t). \tag{5.22}$$

After substituting Eqs. (2.1b), (2.5), and (5.7), Eq. (5.22) becomes

$$\underline{M}\,\frac{\partial \ddot{\vec{r}}(x,t)}{\partial x_i} + \underline{K}\,\frac{\partial \vec{r}(x,t)}{\partial x_i} = -\underline{a}_i^T\,\frac{\partial \underline{M}_{si}}{\partial x_i}\,\underline{a}_i\,\ddot{\vec{r}}(x,t)$$

$$-\underline{a}_i^T\,\frac{\partial \underline{K}_{si}}{\partial x_i}\,\underline{a}_i\,\vec{r}(x,t) + \frac{\partial P_i}{\partial x_i}\,\underline{a}_i^T\,\underline{K}_{gi}\,\underline{a}_i\,\vec{r}(x,t). \tag{5.23}$$

The elements in the matrices, M_{si}, K_{si}, and $\partial P_i/\partial x_i$, are linear with the design variable, x_i; therefore, Eq. (5.23) can be rewritten in the following form:

$$\underset{\sim}{M} \frac{\partial \ddot{\vec{r}}(x,t)}{\partial x_i} + \underset{\sim}{K} \frac{\partial \vec{r}(x,t)}{\partial x_i} = - \frac{1}{x_i} [\underline{a}_i^T \underline{M}_{si} \underline{a}_i \ddot{\vec{r}}(x,t)$$

$$+ \underline{a}_i^T \underline{K}_{si} \underline{a}_i \vec{r}(x,t) - P_i' \underline{a}_i^T \underline{K}_{gi} \underline{a}_i \vec{r}(x,t)]. \qquad (5.24)$$

Solutions of Eq. (5.24) can be obtained by using the modal analysis based on the same eigenvectors employed in the response analysis as shown in the following equation:

$$\frac{\partial \vec{r}(x,t)}{\partial x_i} = - \frac{1}{x_i} \underline{\Phi}\, \underline{\Psi}\, \underline{D}\, \underline{\Phi}^T [\underline{a}_i^T \underline{M}_{si} \underline{a}_i \ddot{\vec{r}}(x,t)$$

$$+ \underline{a}_i^T \underline{K}_{si} \underline{a}_i \vec{r}(x,t) - P_i' \underline{a}_i^T \underline{K}_{gi} \underline{a}_i \vec{r}(x,t)]. \qquad (5.25)$$

The terms Φ_i, Ψ_i, and D_i have been defined in Eqs. (2.10), (2.49), and (2.50) respectively.

Because the displacements resulting from the applied load can be expressed by a finite number of harmonic motions, the frequency p, of a vibrating system may be approximately obtained by using the Rayleigh quotient,

$$p^2 = \frac{\vec{r}^T(x,t)\, \underset{\sim}{K}\, \vec{r}(x,t)}{\vec{r}^T(x,t)\, \underset{\sim}{M}\, \vec{r}(x,t)} \,. \qquad (5.26)$$

Therefore,

$$\ddot{\vec{r}}(x,t) = - p^2 \vec{r}(x,t), \qquad (5.27)$$

and Eq. (5.25) becomes

$$\frac{\partial \vec{r}(x,t)}{\partial x_i} = - \frac{1}{x_i} \underline{\Phi}\ \underline{\Psi}\ \underline{D}\ \underline{\Phi}^T [- p^2\ \underline{a_i^T}\ \underline{M_{si}}\ \underline{a_i}\ \vec{r}(x,t)$$

$$+ \underline{a_i^T}\ \underline{K_{si}}\ \underline{a_i}\ \vec{r}(x,t) - P_i'\ \underline{a_i^T}\ \underline{K_{gi}}\ \underline{a_i}\ \vec{r}(x,t)] \ . \qquad (5.28)$$

Substitution of Eq. (5.28) for its equivalent in Eq. (5.20) yields

$$\frac{\partial u_j(x,t)}{\partial x_i} = - \frac{1}{x_i} \vec{Q}_j^T\ \underline{\Phi}\ \underline{\Psi}\ \underline{D}\ \underline{\Phi}^T [- p^2\ \underline{a_i^T}\ \underline{M_{si}}\ \underline{a_i}\ \vec{r}(x,t)$$

$$+ \underline{a_i^T}\ \underline{K_{si}}\ \underline{a_i}\ \vec{r}(x,t) - P_i'\ \underline{a_i^T}\ \underline{K_{gi}}\ \underline{a_i}\ \vec{r}(x,t)] \ . \qquad (5.29)$$

The application of $\vec{Q}_j f(t)$ induces $\vec{q}_j(x,t)$ of which the motion equations are

$$\underline{M}\ddot{\vec{q}}_j(x,t) + \underline{K}\vec{q}_j(x,t) = \vec{Q}_j f(t) \qquad (5.30)$$

and

$$\vec{q}_j(x,t) = \underline{\Phi}\ \underline{\Psi}\ \underline{D}\ \underline{\Phi}^T\ \vec{Q}_j. \qquad (5.31)$$

Thus, Eq. (5.29) can be simplified to

$$\frac{\partial u_j(x,t)}{\partial x_i} = - \frac{1}{x_i} [\vec{q}_j^T(x,t)\ \underline{a_i^T}\ \underline{K_{si}}\ \underline{a_i}\ \vec{r}(x,t)$$

$$- P_i'\ \vec{q}_j^T(x,t)\ \underline{a_i^T}\ \underline{K_{gi}}\ \underline{a_i}\ \vec{r}(x,t)$$

$$- p^2\ \vec{q}_j^T(x,t)\ \underline{a_i^T}\ \underline{M_{si}}\ \underline{a_i}\ \vec{r}(x,t)], \qquad (5.32)$$

and the term μ'_{ji} in the recursion equation of Eq. (4.27) can be based on Eq. (5.32) as follows:

$$\mu'_{ji} = \frac{\Lambda \partial u_j(x,t)}{\partial x_i} . \qquad (5.33)$$

As noted before, the second term of the right-hand side of Eq. (5.32) can be applied to columns only. Then p is determined from Eq. (5.26) for any number of desired modes.

D. STIFFNESS CONSTRAINTS FOR DYNAMIC LOADS

The measuring function of dynamic stiffness may be established in a manner similar to that of Eq. (5.13) as follows:

$$z(x,t) = \frac{1}{2} \vec{R}_o^T \vec{r}(x,t), \qquad (5.34)$$

in which $\vec{R}_o$ represents the magnitude of the dynamic load vector.

Differentiating Eq. (5.34) with respect to the design variable, x_i, yields

$$\frac{\partial z(x,t)}{\partial x_i} = \frac{\vec{R}_o^T}{2} \frac{\partial \vec{r}(x,t)}{\partial x_i} . \qquad (5.35)$$

After substituting Eq. (5.28),

$$\begin{aligned} \frac{\partial z(x,t)}{\partial x_i} = - \frac{1}{2x_i} \vec{R}_o^T \underline{\Phi}\, \underline{\Psi}\, \underline{D}\, \underline{\Phi}^T \, [- p^2 \, \underline{a_i^T} \, \underline{M_{si}} \, \underline{a_i} \, \vec{r}(x,t) \\ + \underline{a_i^T} \, \underline{K_{si}} \, \underline{a_i} \, \vec{r}(x,t) \\ - P'_i \, \underline{a_i^T} \, \underline{K_{gi}} \, \underline{a_i} \, \vec{r}(x,t)] . \end{aligned} \qquad (5.36)$$

From Eq. (2.48) of $\vec{r}(x,t) = \underline{\Phi}\ \underline{\Psi}\ \underline{D}\ \underline{\Phi}^T\ \vec{R}_0$, Eq. (5.36) finally becomes

$$\frac{\partial z(x,t)}{\partial x_i} = -\frac{1}{2x_i}\,[\vec{r}^T(x,t)\ \underline{a}_i^T\ \underline{K}_{si}\ \underline{a}_i\ \vec{r}(x,t)$$

$$+ \vec{r}^T(x,t)\ \underline{a}_i^T\ \underline{K}_{gi}\ \underline{a}_i\ \vec{r}(x,t)$$

$$- p^2\ \vec{r}^T(x,t)\ \underline{a}_i^T\ \underline{M}_{si}\ \underline{a}_i\ \vec{r}(x,t)]\ . \tag{5.37}$$

Consequently, the term μ'_{ji} in Eq. (4.27) can be expressed in terms of Eq. (5.37) as

$$\mu'_{ji} = \frac{\Lambda\partial z(x,t)}{\partial x_i}\ . \tag{5.38}$$

Equation (5.37) or (5.38) represents the average strain energy density combined with kinetic energy density of any member, i.

E. CONSTRAINTS IMPOSED ON THE NATURAL FREQUENCIES OF A STRUCTURE

The natural frequency of any mode, ω_j, of a structure can be obtained by using the Rayleigh quotient

$$\omega_j^2 = \frac{\vec{\phi}_j^T\ \underline{K}\ \vec{\phi}_j}{\vec{\phi}_j^T\ \underline{M}\ \vec{\phi}_j}\ , \tag{5.39}$$

in which ω_j represents not only the natural frequency but also a mathematical function. Differentiation of Eq. (5.39) with respect to the design variable, x_i, yields

$$\frac{\partial\omega_j^2}{\partial x_i} = \frac{2(\partial\vec{\phi}_j^T/\partial x_i)(\underline{K} - \omega_j^2\ \underline{M})\vec{\phi}_j + \vec{\phi}_j^T(\partial\underline{K}/\partial x_i)\vec{\phi}_j - \omega_j^2\vec{\phi}_j^T(\partial\underline{M}/\partial x_i)\vec{\phi}_j}{\vec{\phi}_j^T\ \underline{M}\ \vec{\phi}_j}\ . \tag{5.40}$$

By observing Eq. (2.10), one may find that $\underline{K} = \omega_j^2 \underline{M}$, thus Eq. (5.40) may be reduced to

$$\frac{\partial \omega_j^2}{\partial x_i} = \frac{\vec{\phi}_j^T \underline{a}_i^T \underline{K}_{si} \underline{a}_i \vec{\phi}_j - P_i' \vec{\phi}_j^T \underline{a}_i^T \underline{K}_{gi} \underline{a}_i \vec{\phi}_j - \omega_j^2 \vec{\phi}_j^T \underline{a}_i^T \underline{M}_{si} \underline{a}_i \vec{\phi}_j}{x_i \vec{\phi}_j^T \underline{M} \vec{\phi}_j} . \quad (5.41)$$

Thus, the term μ_{ji}' in Eq. (4.27) becomes

$$\mu_{ji}' = \frac{\Lambda \partial \omega_j^2}{\partial x_i} . \quad (5.42)$$

Because $\phi_j^T \underline{M} \phi_j$ is a constant, the recursion relation of Eq. (4.18) for the specified j^{th} natural frequency can be expressed as

$$\frac{\vec{\phi}_j^T \underline{a}_i^T \underline{K}_{si} \underline{a}_i \vec{\phi}_j - P_i' \vec{\phi}_j^T \underline{a}_i^T \underline{K}_{gi} \underline{a}_i \vec{\phi}_j - \omega_j^2 \vec{\phi}_j^T \underline{a}_i^T \underline{M}_{si} \underline{a}_i \vec{\phi}_j}{\rho_i \eta_i x_i \ell_i}$$

$$= \text{constant}, \ i = 1, 2, \ldots, m. \quad (5.43)$$

Note that the numerator of the above equation represents the difference between the strain energy density and the kinetic energy density of the j^{th} mode and that the second term of the numerator can be established for columns only.

VI. CROSS SECTIONAL PROPERTIES AND DESIGN VARIABLES

The columns and girders of a framework are assumed to be wide-flange steel sections. The bracing members can be solid bars of rectangular or circular shape. Because the bending behavior of the members is of primary importance, the moment of inertia, I_o, is selected as the design variable of the member. The wide-flange cross sections can be either selected from the AISC Manual[22] or designed for built-up sections.

A. AISC WF SECTIONS

In the AISC Construction Manual, the cross sectional properties of area, section modulus, and moment of inertia are given for each WF section; however, these properties do not have any simple linear relationship. Because the moment of inertia, I_o, is considered to be a primary design variable, the section modulus, S_o, and the cross sectional area, A_o, must be related to the moment of inertia by a mathematical expression such that S_o and A_o are functions of I_o. These approximate relations are suggested by Brown and Ang[2] for most of the economical WF sections available in the Manual. The properties of these sections are first plotted, and then the approximate relations are obtained by curve fitting with selected algebraic expressions, which are

$$S_o = \begin{cases} \sqrt{60.6\, I_o + 84100} - 290, & 0 \leq I_o \leq 9000 \\ \dfrac{I_o - 8056.3}{1.876}, & 9000 \leq I_o \leq 20300 \end{cases} \tag{6.1}$$

and

$$A_o = \begin{cases} 0.465\sqrt{I_o}, & 0 \leq I_o \leq 9000 \\ \dfrac{I_o + 2300}{256}, & 9000 \leq I_o \leq 20300 , \end{cases} \tag{6.2}$$

in which the units are in^4, in^3, and in^2 for I_o, S_o, and A_o respectively.

B. BUILT-UP WF SECTIONS

The wide flange sections in the AISC Manual are not adequate for all design problems, particularly when large cross sections are needed for tall and heavy buildings. Thus the built-up sections can be used in the computer program. Figure 6 shows a typical built-up section of steel for which the moment of inertia, I_o, is considered as a primary design variable, and the depth, d, flange thickness, t_f, and web thickness, t_w, are the secondary design variables.

The expression of the cross sectional area, A_o, the moment of inertia, I_o, the section modulus, S_o, and the shear flow, v_o, of the section are

$$A_o = d^2\left[\frac{t_w}{d} + 2\,\frac{t_f}{d}\left(\frac{b}{d} - \frac{t_w}{d}\right)\right] \tag{6.3}$$

$$I_o = d^4\left[\frac{b}{2d}\left(\frac{t_f}{d}\right)\left(1 - \frac{t_f}{d}\right)^2 + \frac{1}{12}\,\frac{t_w}{d}\left(1 - \frac{2t_f}{d}\right)^3\right] \tag{6.4}$$

$$S_o = d^3\left[\frac{b}{d}\left(\frac{t_f}{d}\right)\left(1 - \frac{t_f}{d}\right)^2 + \frac{1}{6}\,\frac{t_w}{d}\left(1 - \frac{2t_f}{d}\right)^3\right] \tag{6.5}$$

and

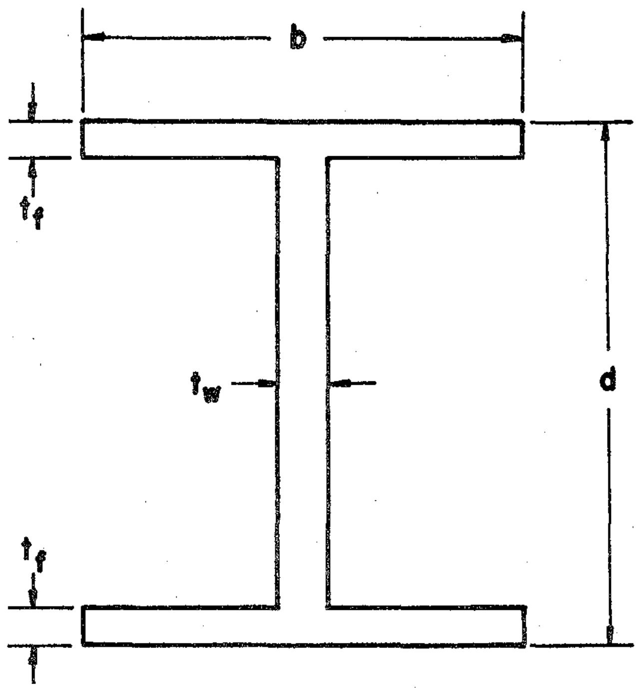


Figure 6. Typical Built-Up WF Section.

$$v_o = \frac{d^2[\frac{b}{2d}(\frac{t_f}{d})(1 - \frac{t_f}{d})^2 + \frac{1}{12}\frac{t_w}{d}(1 - 2\frac{t_f}{d})^3]\frac{t_w}{d}}{[\frac{b}{2d}(\frac{t_f}{d})(1 - \frac{t_f}{d}) + \frac{1}{8}\frac{t_w}{d}(1 - \frac{2t_f}{d})^2]}, \qquad (6.6)$$

in which

v_o = shear flow of WF section (in^2),

b = width of WF section (in),

d = depth of WF section (in),

t_f = flange thickness of WF section (in), and

t_w = web thickness of WF section (in).

The upper and lower bounds of the design variables are imposed on the secondary design variables instead of the primary design variable, and they are

$$d_{min} \leq d \leq d_{max} \qquad (6.7)$$

and

$$(\frac{t_f}{d})_{min} \leq \frac{t_f}{d} \leq (\frac{t_f}{d})_{max} . \qquad (6.8)$$

However, the ratios of the minimum moment of inertia to the maximum moment of inertia of both girders and columns can be specified. The width, b, and the ratio of web thickness to depth, t_w/d, of the WF section are kept constant for each design variable.

VII. NUMERICAL PROCEDURES

An outline of the design procedure for using an optimality criterion algorithm is given below. The design steps correspond with the numbers shown in the flow chart (Fig. 7). Explanatory notes for the computer program can be found in Appendix C.

1. All the input must be supplied according to the input data format.

2. The design begins with the solution of equal sizes for all the design variables. The stiffness and mass matrices are then assembled.

3. If it is a dynamic design, the operator can proceed to Step 6.

4. The structure is analyzed for static loads. The static displacements are computed by solving Eq. (2.2).

5. In the case of static design only, the operator can proceed to Step 12.

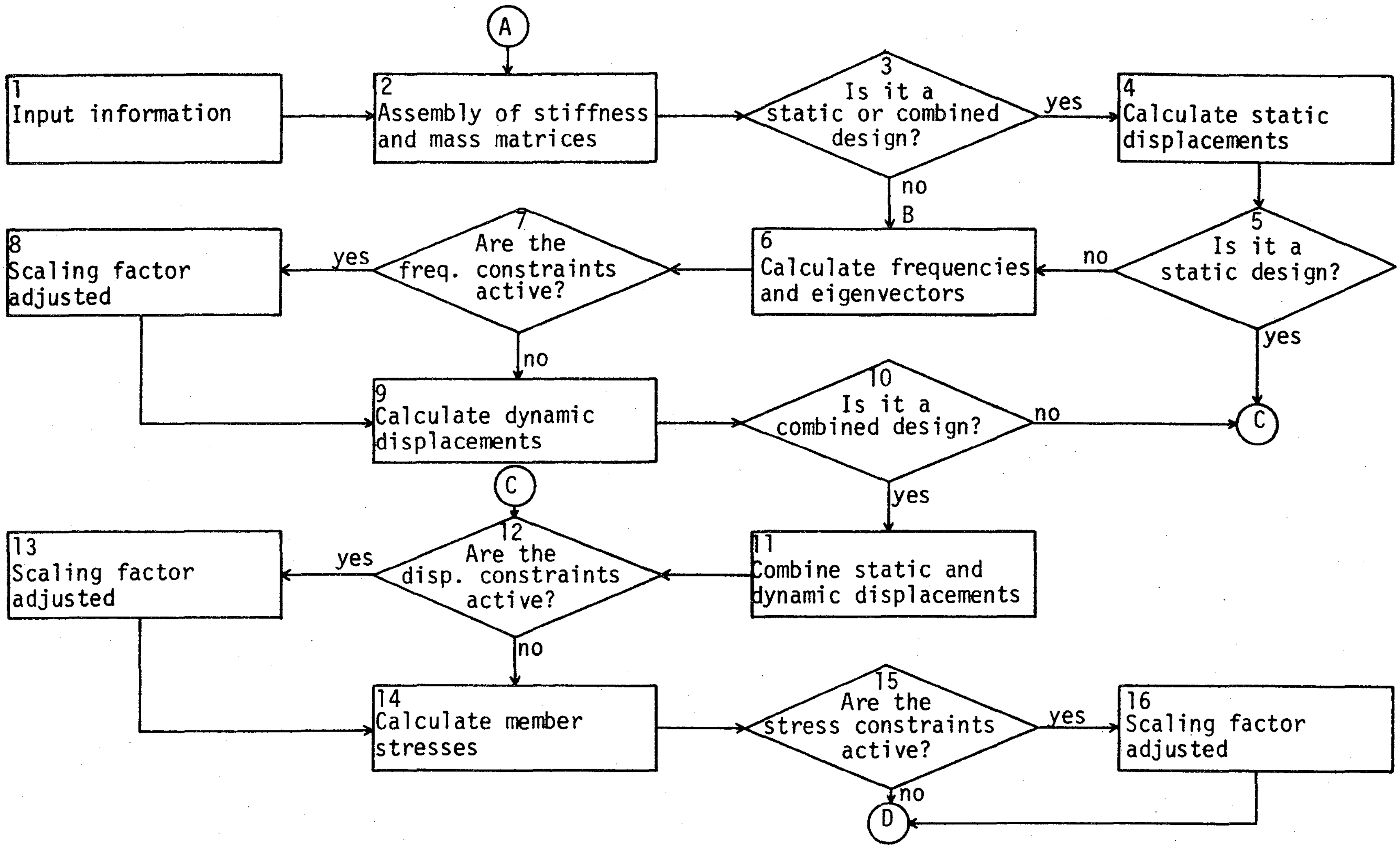
6. The eigenvalues and eigenvectors are determined by using the Sturm sequence property in conjunction with the bisection procedure as described in Appendix B.

7. A determination is made as to whether the frequency requirements are violated. The design can proceed to Step 9, if they are within the limits.

8. The structure is scaled to satisfy the frequency requirements. The most violated constraint is recorded.

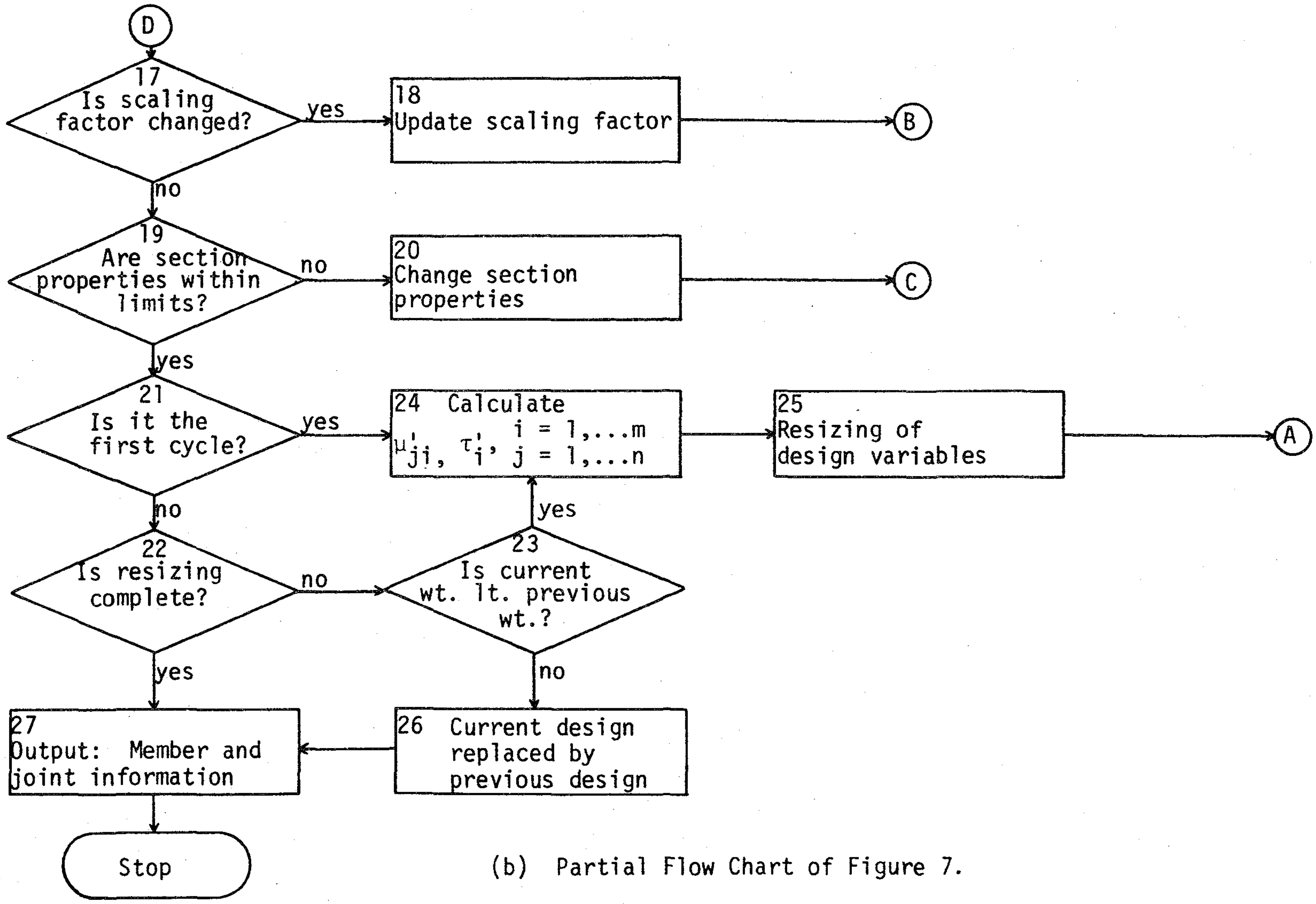
9. The dynamic displacements are computed by using Eq. (2.48).

10. The operator can proceed to Step 12 if the design is for a dynamic case only.



(a) Partial Flow Chart of Figure 7.

Figure 7. Flow Chart of Design Procedures



(b) Partial Flow Chart of Figure 7.

Figure 7. Flow Chart of Design Procedures (continued)

11. The dynamic displacements are then added to the static displacements determined in Step 4 by Eq. (2.51).

12. A determination is made as to whether the displacement requirements are within limits. The operator can proceed to Step 14, if the requirements are not violated.

13. The structure is scaled to satisfy the combined static and dynamic requirements. The most violated constraint is also recorded.

14. The member stresses resulting from the combined static and dynamic loads can be determined by using Eq. (2.54).

15. The operator can proceed to Step 17, if all the stresses are within the limits.

16. The structure is again scaled to satisfy the combined static and dynamic requirements. The most violated constraint is then recorded.

17. The change in the scaling factor in Steps 8, 13, and 16 is insignificant, if

$$\left| \frac{\Lambda_{new} - \Lambda_{old}}{\Lambda_{new}} \right| \leq \varepsilon \tag{7.1}$$

in which Λ_{old} and Λ_{new} are the scaling factors at the beginning and at the end of a design cycle, and ε is a prescribed tolerance. The operator can proceed to Step 19, if the change in the scaling factor is insignificant.

18. If the change in the scaling factor is significant, the proportion of the structural to nonstructural mass determines the effect of this scaling on eigenvalues and eigenvectors. There are

three possible cases:

Case 1: Structural mass only.

The eigenvalues and eigenvectors are uneffected by the scaling.

Case 2: The structural mass is insignificant.

The eigenvalues can be scaled in the same proportion as the design variables but the eigenvectors have to be modified.

Case 3. The structural and nonstructural masses are of similar proportions.

Both eigenvalues and eigenvectors have to be updated. In this case, Steps 6 through 16 have to be repeated.

19. In this step, the limits on section properties of the built-up sections as defined in Chapter VI are checked.

20. For a desired design variable (moment of inertia), the ratio of t_f/d can be determined by keeping the depth of the built-up section constant as follows:

$$\left(\frac{t_f}{d}\right)_{v+1} = \frac{2\left\{\frac{I_o}{d^4} - \frac{1}{12}\frac{t_w}{d}\left[1 - 2\left(\frac{t_f}{d}\right)_v\right]^3\right\}}{\frac{b}{d}\left[1 - \left(\frac{t_f}{d}\right)_v\right]^2}, \tag{7.2}$$

in which the subscript v refers to the cycle of iteration. Equation (7.2) is derived from Eq. (6.4) and can be used to determine the required t_f/d with a few cycles of iteration.

If the t_f/d ratio exceeds the upper limit, the depth of the built-up section can be increased while maintaining the ratio at the upper bound by using the following relation:

$$d = \left[\frac{I_o}{\{\frac{b}{2d}(\frac{t_f}{d})_{max}[1-(\frac{t_f}{d})_{max}]^2 + \frac{1}{12}\frac{t_w}{d}[1-2(\frac{t_f}{d})_{max}]^3\}}\right]^{0.25}. \qquad (7.3)$$

Equation (7.3) is also derived from the Eq. (6.4) with t_f/d replaced by $(t_f/d)_{max}$.

An equation similar to Eq. (7.3) can be used to calculate d, if t_f/d is less than its lower limit;

$$d = \left[\frac{I_o}{\{\frac{b}{2d}(\frac{t_f}{d})_{min}[1-(\frac{t_f}{d})_{min}]^2 + \frac{1}{12}\frac{t_w}{d}[1-2(\frac{t_f}{d})_{min}]^3\}}\right]^{0.25}. \qquad (7.4)$$

If d determined from Eq. (7.4) is less than d_{min}, then d_{min} should be used.

The operator proceeds to Step 12, when the section properties are changed.

21. The feasible weight of the structure is determined. If it is the first cycle, the operator can proceed to Step 24.

22. The design is stopped when the number of iteration exceeds the maximum allowance.

23. The design is also terminated when the current weight is more than the previous weight. Then the operator proceeds to Step 26.

24. Either μ_{ji} or μ'_{ji}, i = 1,2, ... m, given in Eq. (4.20) or (4.25) is determined for each constraint j. When the constraints are the displacements of a structure, μ' is expressed in Eqs. (5.10) and (5.33) for the static and dynamic loads respectively. For a combined static and dynamic design, the combination of both equations should be used for μ'_{ji}. Similarly, if the constraints are stresses,

Eqs. (5.17) and (5.38) should be respectively employed for static and dynamic loads. Apparently, μ'_{ji} in Eq. (5.42) is for the frequency constraint. The term τ'_1 is computed from Eq. (5.11).

25. The Lagrange multipliers associated with the active constraints are determined by solving Eq. (4.32). Therefore, a set of resizing design variables can be calculated by using the recursion relation of Eq. (4.27). The Lagrange multiplier associated with nonactive constraints can also be determined by solving Eq. (4.32). Hence, there are (n-N) sets of resizing design variables that correspond to the remaining nonactive constraints. The maximum value of each resizing design variable from these sets is to be used for the next design cycle. The operator then proceeds to Step 2.

26. When the current weight of the structure is more than the one given in the problem, the current design variables are replaced by the previous design variables.

27. The member stresses and the displacements corresponding to the final design are calculated, and the results of the program are printed.

VIII. NUMERICAL EXAMPLES OF TRUSSES AND FRAMEWORKS

The ODSEWS (Optimum Design of Static, Earthquake, and Wind Structures) computer program has been developed for the design of trusses and framed structures subjected to the multicomponent input of static loads and dynamic excitations. Dynamic excitations are wind forces and horizontal and vertical earthquake motions. The response for these can be obtained by using either direct numerical integrations or response spectra. The members of the trusses and bracings are bar elements, and the girders and columns of the frameworks are beam-column elements that can be either built-up sections or hot-rolled wide flange sections available in the AISC Steel Construction Manual. In this chapter, various applications of the computer program and comparisons of the design results are discussed, and in Chapter IX, comparative studies of the effect of multicomponent motions on 30-story unbraced, single-braced, double-braced, and K-braced structural systems are reviewed.

A. DESIGN OF TRUSSES

1. Design of a Cantilever Truss for Static Loads--Example 1. The ten-bar truss shown in Fig. 8 illustrates the conditions for the iteration cycles versus the structural weight at each design cycle. The structure has ten design variables and eight degrees of freedom. The static loads of 100 kips (445 kN) are applied at nodes 2 and 4. The member properties of the design problem are the modulus of elasticity, $E = 10 \times 10^6$ lbs/in.^{2} (69.0 GN/m^2), the mass density, $\rho = 0.10$ lb/in.^{3} (2.764 g/cm^3), and the allowable stress,

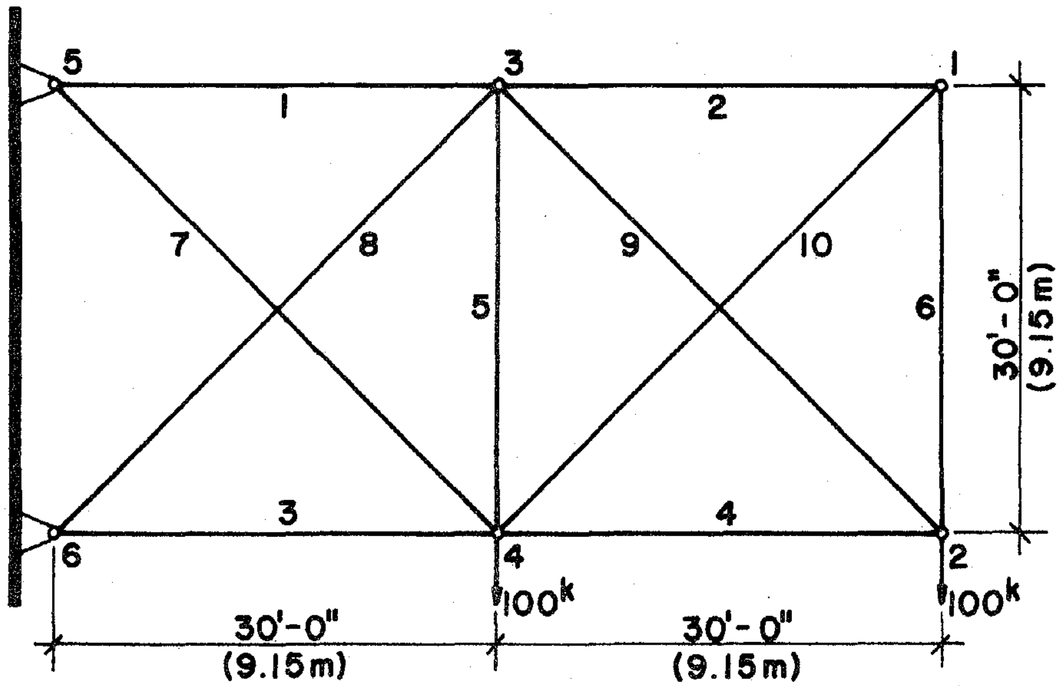


Figure 8. Cantilever Truss for Static Loads of Example 1.

$\sigma_{all} = \pm$ 25,000 lbs/in.^{2} (172.5 MN/m^2).

The two behavior constraints considered in this example are Case (a), stress constraint only, and Case (b), both stress and displacement constraints. The displacement constraints are $\pm$ 2.0 in. (5.08 cm) for all nodes in both the x and y directions. The lower bound of the design variables is assumed to be 0.10 in.^{2} (6.45 cm^2) which is the side constraint in both cases.

A plot of the number of design iterations versus the structural weight corresponding to each iteration is shown in Fig. 9. Table I reveals the final design results of the cross-sectional areas and the stresses of the individual members. The optimum results of Case (a) are due to the stress constraints active in members 1, 3, 4, 7, 8, and 9, and those of Case (b) are due to the displacement constraints active in the vertical direction at nodes 1 and 2. These two displacement constraints, which are simultaneously active under the given loading condition, represent a typical case of the multiple constraint problems discussed in Chapter IV.

2. Design of a Twenty-Bar Tower for Static Loads and Ground Motions--Example 2. Figure 10 shows the configuration of a 20-bar tower, which has a concentrated dead load of 38.64 kips (171.95 kN) acting respectively at nodes 9 and 10. The structure has 20 design variables and 16 degrees of freedom and was designed to withstand a dead load and a horizontal ground motion. The earthquake excitation is represented by the average spectral acceleration shown in Fig. 5 with 0.33 g and 5% damping. The member properties of the structure are modulus of elasticity, E = 10 x 10^6 lbs/in.^{2} (69.0 GN/m^2), mass density, ρ = 0.10 lb/in.^{2} (2.764 g/cm^3), and

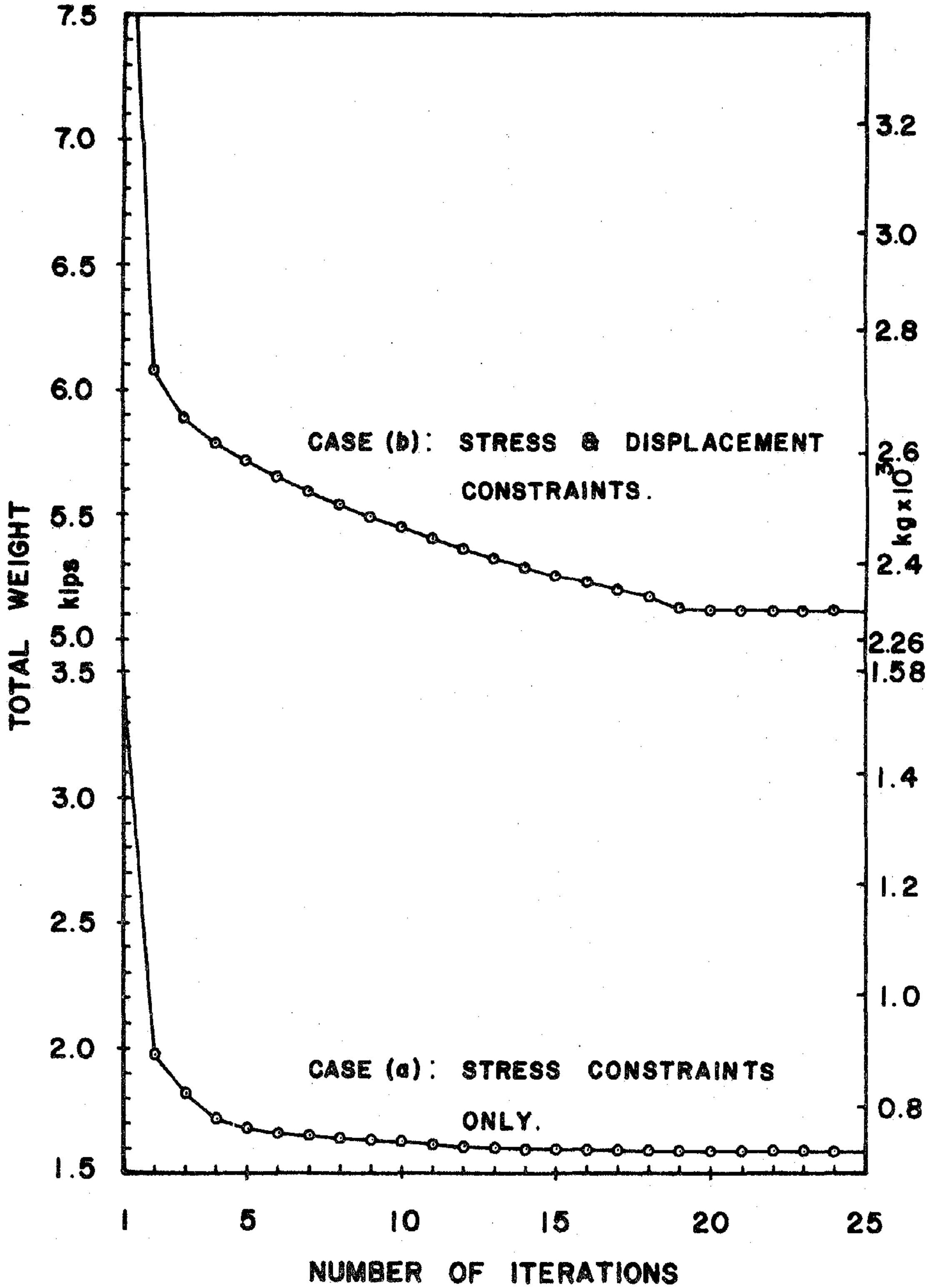


Figure 9. Optimum Weights of Cantilever Truss of Example 1.

TABLE I. DESIGN RESULTS OF EXAMPLE 1 (1 in.^{2} = 6.452 cm^3, 1 lb/in.^{2} = 6.9 kN/m^2 , 1 lb = 0.453 kg)

	Case (a)		Case (b)	
Member numbers	Area (in.^{2})	Stress (lb/in.^{2})	Area (in.^{2})	Stress (lb/in.^{2})
1	7.938	25000	30.512	6621
2	0.100	15533	0.100	52
3	8.062	25000	21.901	9040
4	3.938	25000	15.154	6599
5	0.100	0	0.101	20047
6	0.100	15533	0.100	52
7	5.745	25000	8.924	15526
8	5.569	25000	21.496	6712
9	5.569	25000	21.431	6599
10	0.100	21967	0.100	73
Final weight (lb)	1593.2		5088.2	
No. of iterations	20		20	

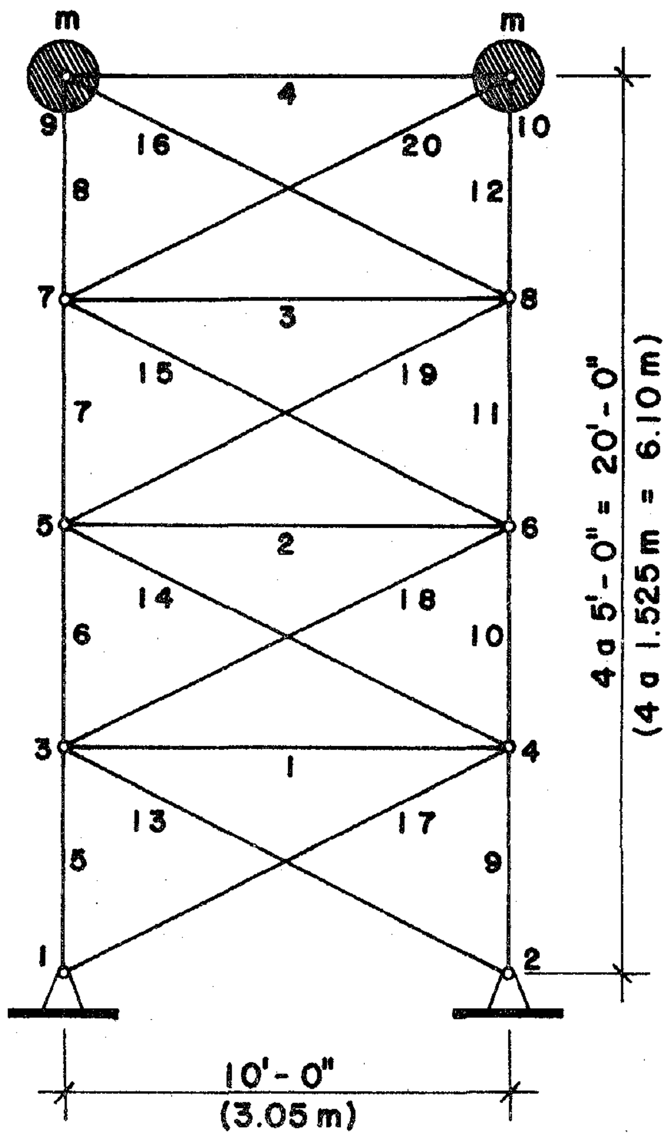


Figure 10. Truss Tower for Static Loads and Ground Motion of Example 2.

the allowable stresses, $\sigma_{all} = \pm$ 25,000 lbs/in.^{2} (172.5 MN/m^2). The displacement constraints of 0.25 in. (0.635 cm) are imposed on all the nodes in both the x and y directions. The lower bound of the design variables is 0.10 in.^{2} (0.645 cm^2).

The tower is designed for two different cases: Case (a) is for a horizontal ground motion only, and Case (b) is for both the dead load and the ground motion. For the dynamic design, the first five modes are used. The design results are given in Table II.

B. DESIGN OF FRAMED STRUCTURES

1. Design of a Two-Story, One-Bay Frame for Gravity and Wind Loads--

Example 3. The two-story, one-bay frame of Fig. 11 with six design variables and 12 degrees of freedom is designed for gravity and wind loads. The nonstructural masses of the floor are w_1 = 2 kips/ft (2970 kg/m) for the first story and w_2 = 1 kip/ft (1485 kg/m) for the second. The displacement constraints are 2.4 in. (6.10 cm) and 3.0 in. (7.62 cm) for the first and the second story respectively. The members of the frame are the hot-rolled wide flange sections for which the relationships of the cross-sectional properties are expressed in Eqs. (6.1) and (6.2). The material properties of the members are modulus of elasticity, E = 29 x 10^6 lb/in.^{2} (200.1 GN/m^2), the mass density, ρ = 0.283 lb/in.^{3} (7.823 g/cm^3), and the allowable stress, σ_{all} = 29,000 lb/in.^{2} (200.1 MN/m^2). The lower bound of all the design variables is 10 in.^{4} (416.23 cm^4).

The wind force has a magnitude of 35.7 lbs/ft^2 (1.71 kN/m^2) and a half-sine time function, which is shown in Fig. 12. The response spectrum of the function is given in Fig. 13. The force is distributed on the frames spacing at a distance of 20 ft (6.1 m) center to center.

TABLE II. DESIGN RESULTS OF EXAMPLE 2 (1 in.^{2} = 6.452 cm,^{4} 1 lb/in.^{2} = 6.9 kN/m^2, 1 lb = 0.453 kg)

	Case (a)		Case (b)	
Member numbers	Area (in.^{2})	Stress (lb/in.^{2})	Area (in.^{2})	Stress (lb/in.^{2})
1	0.100	8.6883E-03	0.100	7.3781E+02
2	0.100	1.8568E-02	0.100	1.5301E+01
3	0.138	2.7816E-02	0.100	1.4308E+03
4	0.302	4.0701E-02	0.839	5.0411E+02
5,9	27.581	2.4553E+03	28.961	3.6767E+03
6,10	20.011	2.4769E+03	23.626	3.7186E+03
7,11	12.533	2.5080E+03	14.654	4.7641E+03
8,12	5.451	2.5048E+03	9.274	5.5345E+03
13,17	8.949	2.2734E+03	8.546	2.4367E+03
14,18	8.985	2.2616E+03	8.554	2.4407E+03
15,19	9.211	2.2013E+03	8.599	2.4222E+03
16,20	9.922	2.0370E+03	8.633	2.4229E+03
Final weight (lb)	1789		1853	
No. of iterations	3		5	

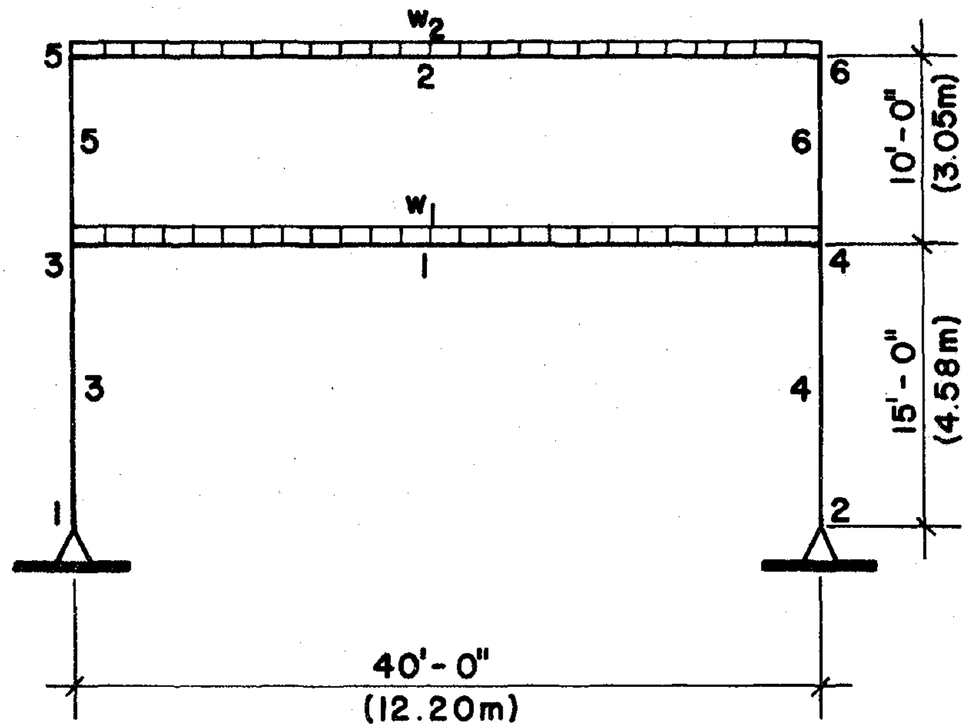


Figure 11. Two-Story, One-Bay Frame for Wind Load of Example 3.

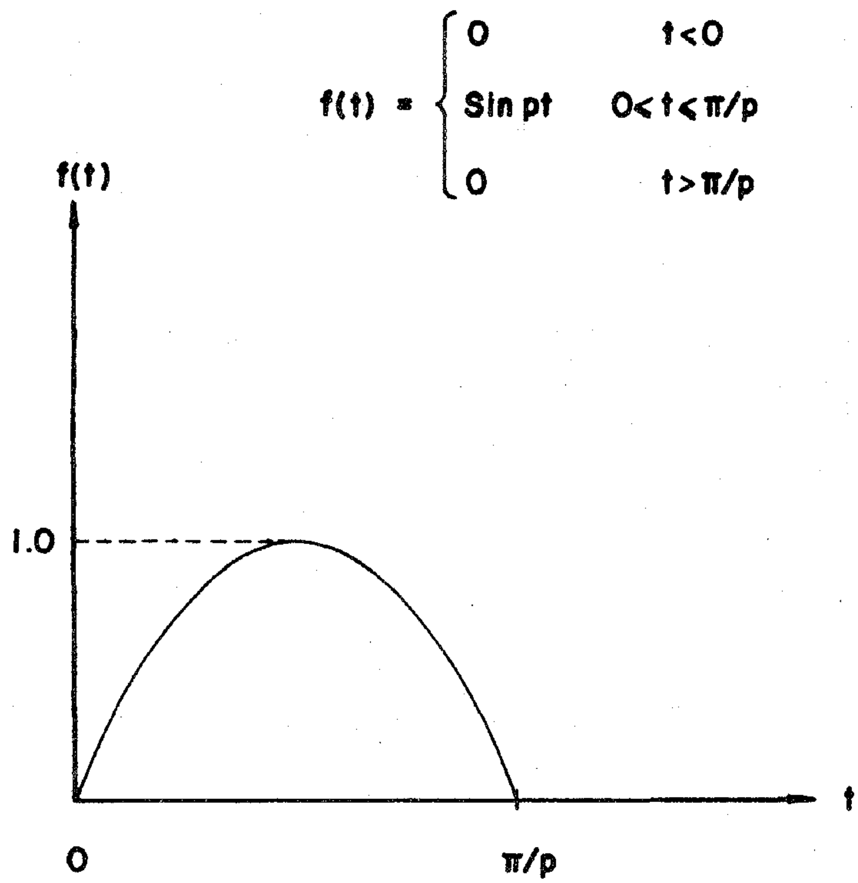


Figure 12. Forcing Function of Wind Load of Example 3.

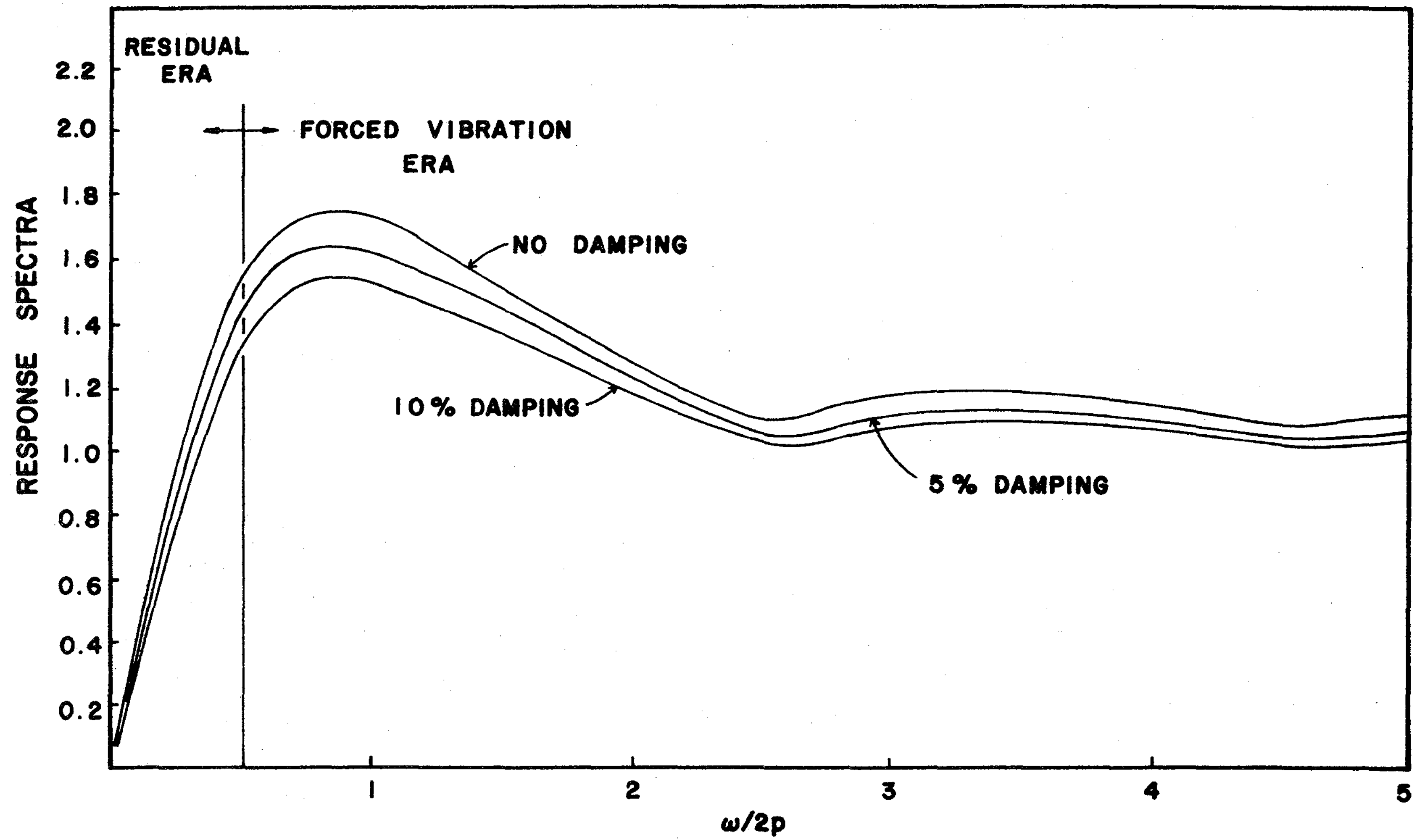


Figure 13. Response Spectrum for the Half Sine Wave of Figure 12.

The structure is designed without consideration of the P-Δ effect for the following three cases:

Case (a): i) The frame is designed to resist only a wind force for which the uniform load of 35.7 lbs/ft^2 (1.71 kN/m^2) is treated as a static load. ii) In addition to the static load considered in i), the gravity load of the slab weights is included in the design.

Case (b): i) The frame is designed for a wind force, which is treated as an undamped dynamic excitation for which the forcing function is shown in Fig. 12. ii) The design is due to a combined load of the dynamic force and the slab weights. The frequency, p, of the wind force is assumed to be 3 rad/sec. For computational convenience, the minimum fundamental frequency, ω, of the structure is chosen to be 3 rad/sec., because the response spectrum indicates that when $\omega/2p \geq 0.5$ the maximum responses always occur in the forced vibration era. Let the time interval, Δt, be 0.05 sec, then the total number of time steps for the design is 25. At any time step, i, the maximum responses of the internal forces occurring between $t = 0$ and $t = t_i$ are recorded. Thus, the maximum internal forces at $t = 0$ through the end of the forcing function are used in this design for which the response analysis is apparently exact but time consuming. The example is used only for comparison of this design result with the one that is based on the approximate approach of the response spectrum in Case (c).

Case (c): i) The structure is designed for a wind load for which the response spectrum of Fig. 13 and the root-mean-square technique are used. ii) The design takes into account the combined load of the slab weights and the response spectrum of the wind force. Use of the response spectrum eliminates the time parameter and thus saves computing time. Because the maximum responses of individual modes do not necessarily occur at the same time, the root-mean-square technique is employed to level off the peak response of all the modes involved.

The structural weights and the iteration cycles of these three cases are given in Table III in which one can observe the convergence of the recursion process and the differences between the design results. The moment of inertia, the combined stress of the axial and bending forces, the total number of iterations, the floor displacements, and the violated constraints are listed in Table IV. The cross-sectional areas, A_0, and the section modulus, S_0, are not shown and can be obtained by using Eqs. (6.1) and (6.2). Case (a)i has multiple active constraints that are the stress of member 2 and the displacement at the top floor. However, Case (a)ii has one behavior constraint violated as the stress of member 2. The violated constraints of (i) and (ii) of Case (b) are the displacement at the top floor and the stress of member 1 respectively. Case (c)i has the multiple constraints violated as the top floor displacement and the stresses of members 4 and 6, whereas Case (c)ii has a single constraint violated. Note that the structural weight resulting from the design for the static wind force is about 74% of that based on the dynamic force design, which in turn yields almost the same amount as that resulting from the response spectrum approach.

TABLE III. STRUCTURAL WEIGHT AT EACH DESIGN ITERATION OF EXAMPLE 3 (1 lb = 0.453 kg)

		Weight (lb)									
Cases	No. of Iterations	1	2	3	4	5	6	7	8	9	10
(a)	i	4136	3626	3381	3200	3136	3116	3107	3102	3099	3096
	ii	8053	6635	6486	--	--	--	--	--	--	--
(b)	i	5562	4818	4477	4269	4194	--	--	--	--	--
	ii	8961	7837	7411	--	--	--	--	--	--	--
(c)	i	5492	4782	4448	4257	4212	--	--	--	--	--
	ii	8973	7820	7406	--	--	--	--	--	--	--

TABLE IV. DESIGN RESULTS OF EXAMPLE 3 (1 lb/in.^{2} = 6.9 kN/m^2, 1 in.^{4} = 41.623 cm^4, 1 lb = 0.453 kg, 1 in. = 2.54 cm)

	Case (a)				Case (b)				Case (c)			
	i		ii		i		ii		i		ii	
Member numbers	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})
1	604	22364	1679	26909	958	24813	2013	28999	972	24155	2033	28782
2	27	28978	644	28996	94	23364	781	28800	98	24957	782	29000
3	441	26780	981	28275	784	26936	1390	27506	765	26422	1374	27303
4	441	26780	981	28275	784	26935	1390	27546	765	26422	1374	27303
5	51	25800	677	26989	90	28460	1096	21245	98	29000	1089	21409
6	51	25800	677	26989	90	28460	1096	21192	98	29000	1089	21409
Final weight (lb)	3096		6486		4194		7411		4212		7406	
No. of iterations	10		3		5		3		5		3	

TABLE IV. DESIGN RESULTS OF EXAMPLE 3 (continued)

	Case (a)				Case (b)				Case (c)			
	i		ii		i		ii		i		ii	
Member numbers	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})	I_o (in.^{4})	σ (lb/in.^{2})
Constraints violated	σ_2, u_2		σ_2		u_2		σ_1		σ_4, σ_6, u_2		σ_2	
Displ. u_1	1.99 in.		0.76 in.		2.02 in.		0.92 in.		1.98 in.		0.91 in.	
u_2	3.00 in.		0.96 in.		2.99 in.		1.18 in.		2.99 in.		1.18 in.	

2. Design of a Two-Story, Two-Bay Frame for Equivalent Seismic Forces--Example 4. Figure 14 of a two-story, two-bay frame is designed to resist the equivalent seismic forces specified by the Uniform Building Code. This example has ten design variables and 21 degrees of freedom for which the nonstructural masses of the slabs are 2 kips/ft (2970 kg/m) and 1 kip/ft (1485 kg/m) for the first and top floor respectively. The displacement constraints are u_1 = 2.4 in. (6.10 cm) and u_2 = 3.0 in. (7.62 cm). They correspond respectively to the first and top story. The properties of the constituent members, the allowable stresses, and the lower bound of the design variables are the same as those given in Example 3.

The lateral equivalent seismic loads are computed according to the recommendation of the 1976 Uniform Building Code as the earthquake-zone coefficient, Z = 1.0, the important factor of the building, I = 1.5, the stiffness coefficient of the building, K = 1.0, and the estimated site period, T_s = 1.0.

The frame is designed without a P-Δ effect for two cases: Case (a) is the design resulting from equivalent lateral seismic loads, whereas Case (b) is due to the gravity loads of the slabs combined with these lateral forces. The final design results are shown in Table V, which includes the moments of inertia and combined stresses of the individual members, the final weight, number of design iterations, and the actual displacements. It is apparent that the critical constraints at the final design are the stresses in members 1 and 3 for Cases (a) and (b) respectively. Because the members are based on the AISC wide flange sections, the section modulus, S_o, and the area, A_o, of the final design results are

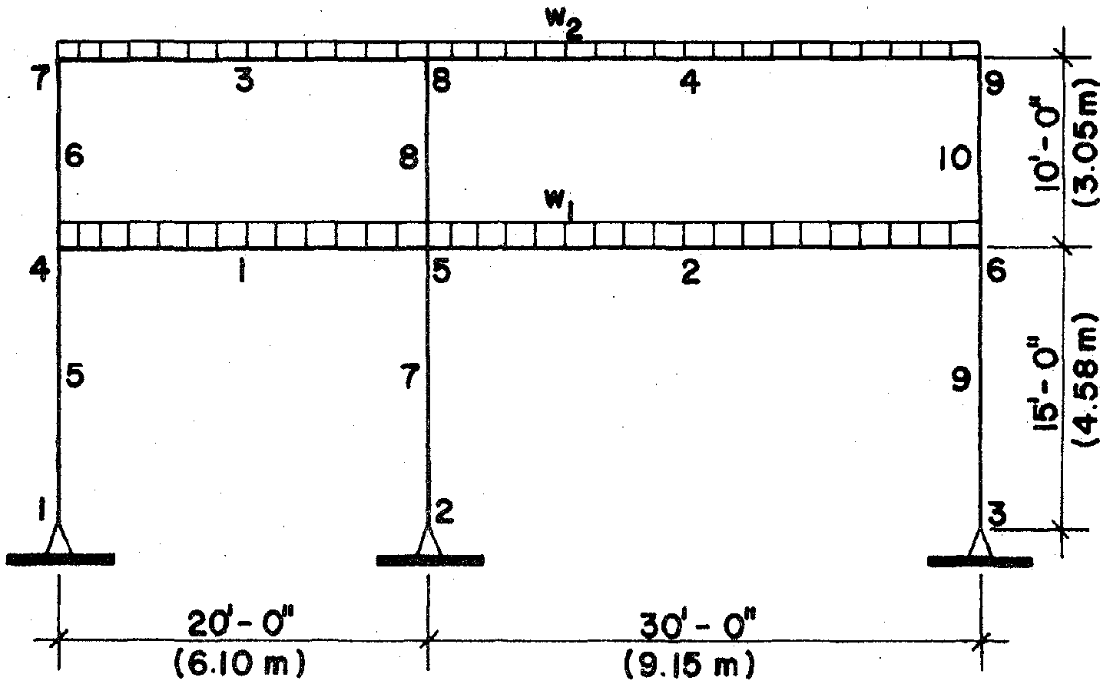


Figure 14. Two-Story, Two-Bay Frame for UBC Equivalent Seismic Forces of Example 4.

TABLE V. FINAL DESIGN RESULTS OF EXAMPLE 4 (1 in.^{4} = 41.623 cm,^{4} 1 lb/in.^{2} = 6.9 kN/m^2, 1 lb = 0.453 kg, 1 in. = 2.54 cm)

Member numbers	Case (a) Moment of inertia I_o(in.^{4})	Case (a) Stress σ(lb/in.^{2})	Case (b) Moment of inertia I_o(in.^{4})	Case (b) Stress σ(lb/in.^{2})
1	573	29000	1113	24239
2	294	24536	1118	28592
3	105	24313	263	28999
4	60	20057	398	26914
5	566	26154	604	14224
6	94	25285	166	18366
7	768	26469	1399	18263
8	152	22753	255	26361
9	392	17148	1556	19464
10	91	11847	538	20849
Final weight (lb)	4460		7331	
No. of iterations	3		3	
Disp: u_1	1.57 in.		0.81 in.	
u_2	2.18 in.		1.04 in.	

calculated by the computer according to Eqs. (6.1) and (6.2) and then printed in the output.

3. Design of a Five-Story, One-Bay Frame for Dynamic Loads and Seismic Excitations--Example 5. The five-story, one-bay frame shown in Fig. 15 is designed for two loadings. The first loading is the dynamic wind loads that result from a uniform load of 35.7 psf (1.71 kN/m^2) that is distributed on the frames at 20 ft (6.1 m) spacing. The shock spectrum and 5% damping shown in Fig. 13 are taken into consideration. The second loading is due to a horizontal ground motion that is based on the average spectral acceleration, 5% damping, and 0.27 g as shown in Fig. 5. The uniform loads on girders, w, are 180 lb/in. (3210.2 kg/m).

Two design cases are considered: Case (a) is for the frame to resist the wind load and the seismic excitations that act independently as two loading conditions, and Case (b) is for the frame to resist these two loadings, each of which is combined with the static gravity load. The first three natural modes are used to approximate the dynamic response on the basis of the root-mean-square technique. The member properties, the allowable stress, and the lower bound of the design variables are the same as given in Example 3. No P-Δ effect is considered for this design. The displacement constraints are based on the allowable deflection relative to the ground which is equal to 0.005 times the floor height. The final design results are shown in Table VI. Note that the columns at each floor are assumed to be identical and are treated as one design variable, which is selected according to Eqs. (6.1) and (6.2). This example is designed to show that the ODSEWS (Optimum Design of Static, Earthquake, and Wind Structures) computer program is

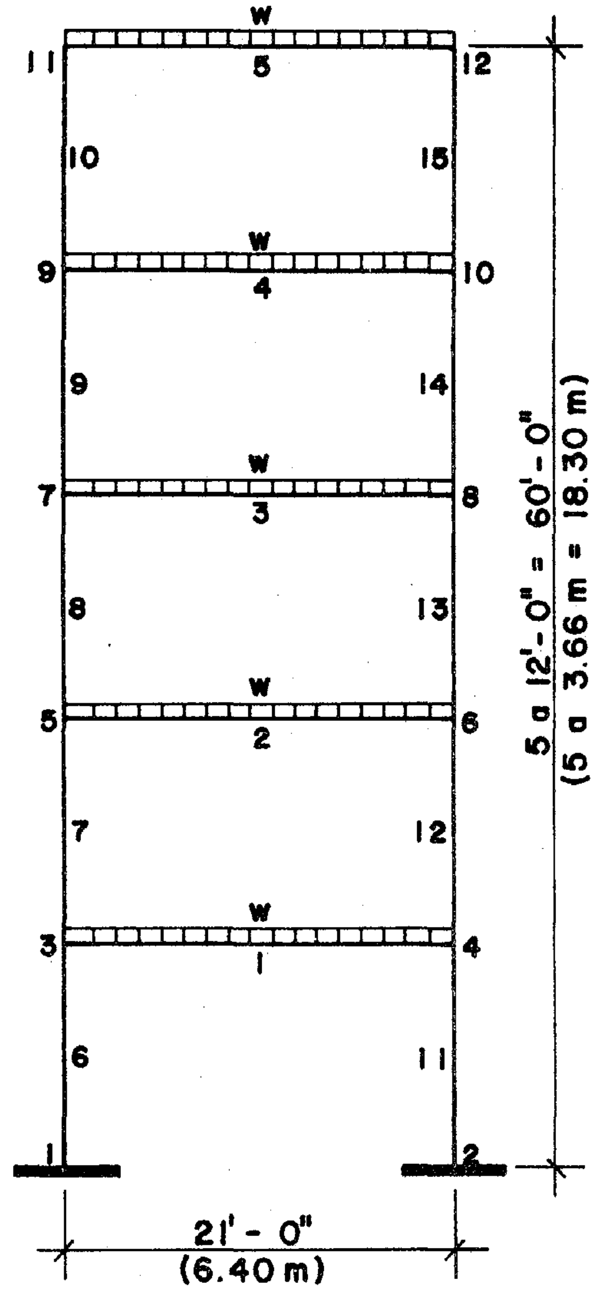


Figure 15. Five-Story, One-Bay Frame for Wind Loads and Seismic Excitations of Example 5.

TABLE VI. FINAL DESIGN RESULTS OF EXAMPLE 5(1 $in.^4$ = 41.623 $cm,^4$ 1 $lb/in.^2$ = 6.9 kN/m^2, 1 lb = 0.453 kg)

Member numbers	Case (a) Moment of inertia $I_0(in.^4)$	Case (a) Stress $\sigma(lb/in.^2)$	Case (b) Moment of inertia $I_0(in.^4)$	Case (b) Stress $\sigma(lb/in.^2)$
1	1879	17076	2693	19375
2	2437	16702	2865	20708
3	1871	16934	2352	20955
4	1125	18931	1528	22889
5	251	26764	671	25926
6,11	1983	20713	3067	23443
7,12	1275	19208	2311	21595
8,13	961	20235	1785	20711
8,14	634	22177	1191	21155
10,15	304	29000	635	29000
Final weight (lb)	11966		15031	
No. of iterations	6		3	
Constraints violated	σ_{10},σ_{15}		σ_{10},σ_{15}	
Disp: u_1, u_2, u_3, u_4, u_5 (in.)	0.27,0.74, 1.25,1.83,2.58		0.22,0.61, 1.04,1.50,1.97	

versatile enough for the optimum design of various steel structures subjected to various types of loading.

4. <u>Comparison of Design Results Based on the UBC Equivalent Lateral Forces and the Average Seismic Acceleration Spectrum for a One-Bay Frame Varies from One-Stroy to 30-Stories--Example 6</u>. One-story to 30-story one-bay frames are designed according to the 1976 Uniform Building Code (UBC) and the average acceleration spectrum of 0.27 g and 5% damping as shown in Fig. 5. The unbraced single-bay frames have a typical span length of 21 ft (6.40 m) and a typical floor height of 12 ft (3.66 m). The dead load on each floor is 180 lb/in. (3210.2 kg/m). The allowable combined stress for the bending and axial force is σ = 29 ksi (200.1 MN/m^2), the allowable shear stress is 0.65 σ, and the allowable deflections at any floor are equal to 0.005 times the height of the story from the ground level. Neither the static stress nor the P-Δ effect occasioned by the superimposed masses on the floors are considered. The first five modes are used in conjunction with the root-mean-square technique in the design. The natural periods are obtained by using the eigensolution subroutine. The properties of the built-up sections described in Chapter VI are b = 10 in. (25.4 cm), d_{max} = 36 in. (91.2 cm), d_{min} = 8 in. (20.3 cm), $(t_f/d)_{max}$ = 0.045, $(t_f/d)_{min}$ = 0.023, and t_w/d = 0.020.

For the solution associated with the equivalent seismic forces as specified by the Uniform Building Code, the values of Z, K, S, and I are assumed to be one which corresponds to Zone 4 of the highest seismic risk. The natural period is obtained by using the Rayleigh quotient equation in the Code.

Figure 16 shows the comparison of the design results of the total weight of each of the 30 unbraced, single-bay frames. The figure indicates that the Uniform Building Code requires a much lighter structural weight than the average response spectrum for which only a moderate magnitude of 0.27 g is used. The base shear factor, CS, which corresponds to the final design of Fig. 16, is illustrated in Fig. 17 for both design cases. It is apparent that the Uniform Building Code underestimates the total shear forces as compared to the average acceleration spectrum of 0.27 g in magnitude.

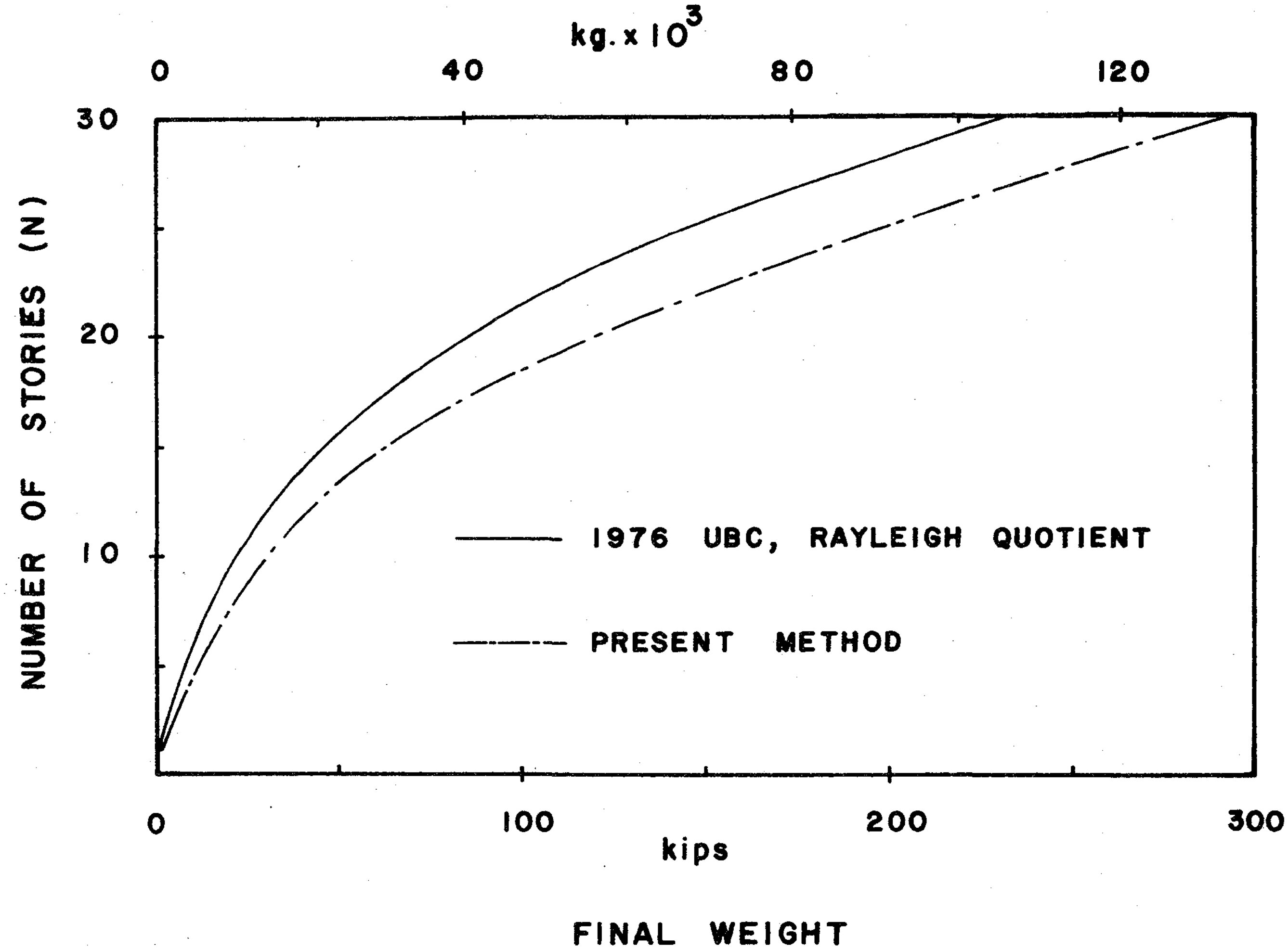


Figure 16. Comparison of Optimal Weights of Example 6.

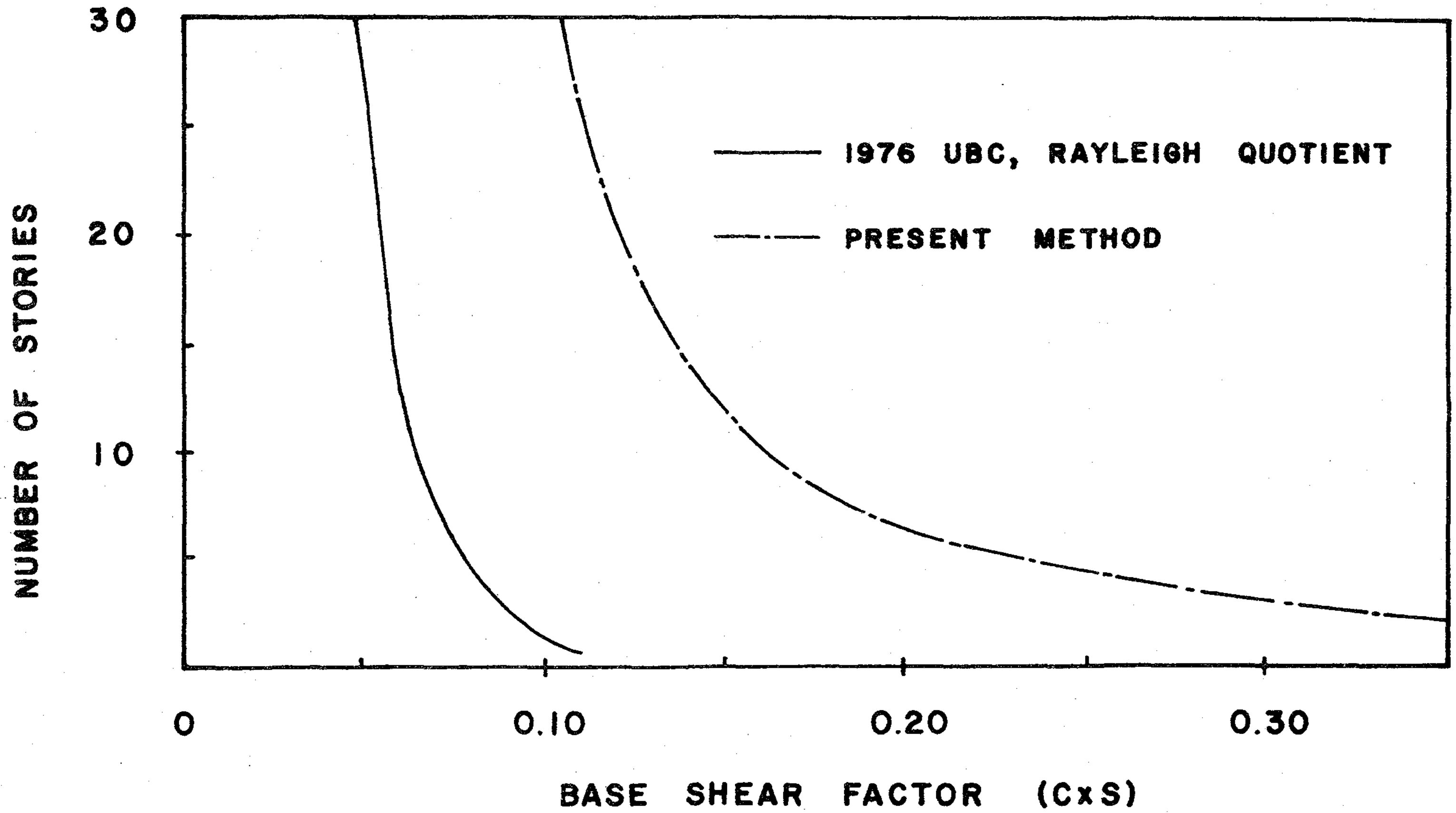


Figure 17. Comparison of Base Shear of Example 6.

IX. THE EFFECT OF MULTICOMPONENT SEISMIC MOTIONS ON 15-STORY STRUCTURAL SYSTEMS

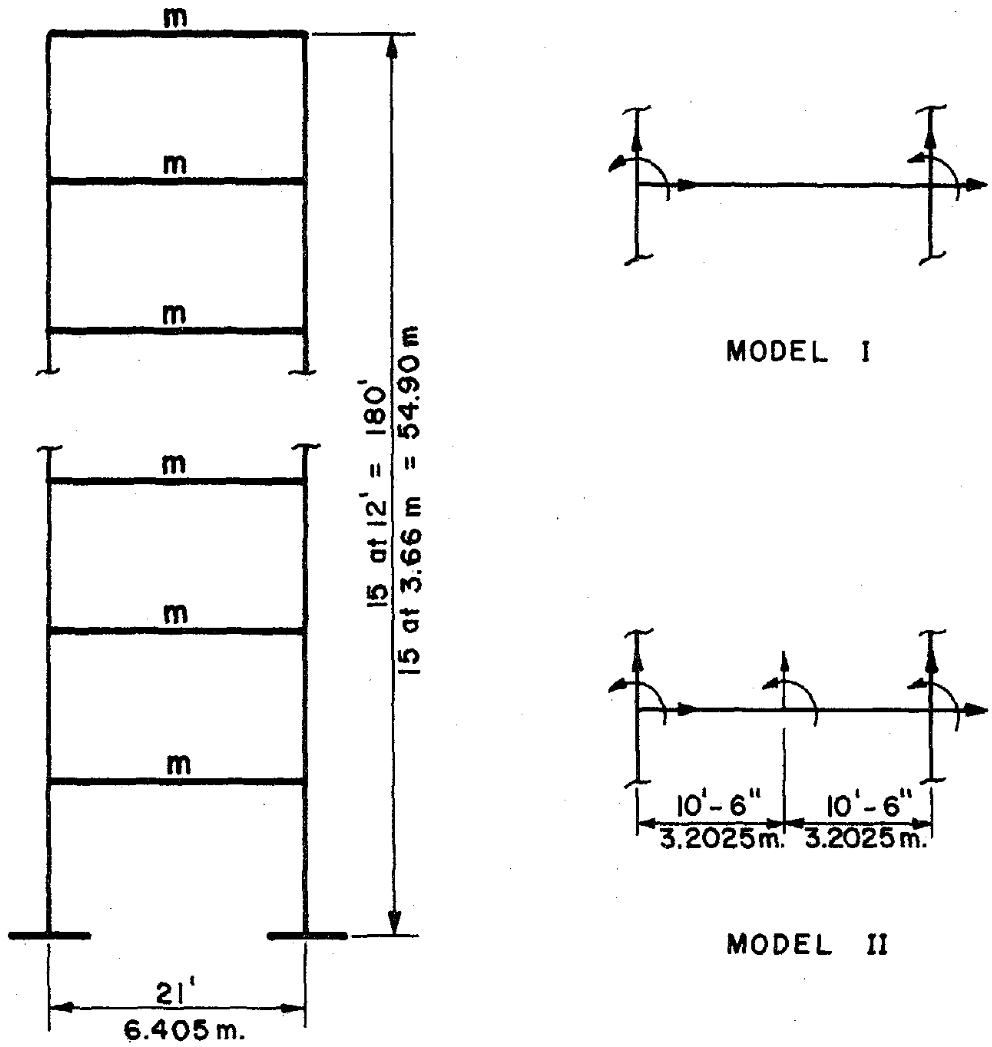
In this chapter, the design results for 15-story braced and unbraced buildings are compared. These buildings are considered to have been subjected to the horizontal and vertical ground motions of the 1940 El Centro earthquake for which the acceleration spectra with 5% damping are given in Fig. 4. In addition to the consideration of vertical component of the ground motion, the second-order P-Δ effect on the design is also included. The P-Δ effect is a result of all the axial forces exerted on the columns. These forces are composed of the dead loads of the structural and nonstructural masses and the associated inertial forces occasioned by vertical acceleration. The structures are assumed to have two models: Model I has two nodes at both ends of a girder, and Model II has three nodes at both ends and at the mid-span of a girder.

The 15-story buildings discussed in this chapter have the following structural dimensions, weights, member properties, and allowable stresses: The span length and floor height are 21 ft (6.405 m) and 12 ft (3.66 m) respectively. The dead load (nonstructural mass) on each floor is 180 lbs/in. (3210.2 kg/m). The mass density and the modulus of elasticity of the construction material are 0.283 lbs/in.^{3} (7.823 g/cm^3) and 29,000 ksi (200.1 GN/m^2) respectively. The allowable stress, σ, for bending combined with the axial forces of the columns and girders is 29 ksi (200.1 MN/m^2), and the allowable shear stress, σ_v, is less than or equal to 0.65 σ. The allowable axial stress of the bracings is assumed to be 20 ksi (138 MN/m^2).

Although different allowable deflections may be imposed at any particular node, the allowable deflection of each floor is limited to 0.005 times the height of that floor from the ground level. The columns and girders are made of the built-up sections described in Chapter VI and have the following properties: $b = 25$ in. (63.50 cm), $d_{max} = 75$ in. (190.50 cm), $d_{min} = 15$ in. (38.10 cm), $(t_f/d)_{max} = 0.045$, $(t_f/d)_{min} = 0.023$, and $t_w/d = 0.020$.

A. 15-STORY UNBRACED FRAMES

The 15-story unbraced frame shown in Fig. 18(a) is used to study the significant effect of multicomponent ground motion and of various constraints on the optimal design results. The optimal design results of the structure are observed by considering the stress constraints only, the stress and displacement constraints, and the constraints on the relative stiffness of the constituent members. Four design cases are considered by using Models I and II shown in Fig. 18(b): Case (a) is designed for horizontal ground motion only, Case (b) for a horizontal ground motion and the P-Δ effect of the dead load associated with the structural and non-structural masses, Case (c) for horizontal and vertical earthquake components but no P-Δ effect, and Case (d) for horizontal and vertical earthquake components as well as the P-Δ effect occasioned by the dead load and the vertical inertial forces associated with the structural and nonstructural masses. In this chapter, the design results are identified by H, H + PΔ(DL), H + V, H + V + PΔ(DL + V), which correspond to Cases (a) through (d) respectively. The stresses caused by gravity loads on girders are not included in



(a) 15-story Unbraced Frame

(b) Structural Models

Figure 18. 15-Story, Single-Bay Unbraced Frame of Examples 7 through 9.

the design of Examples 7 through 9. Thus the design results are based on the dynamic excitations only.

1. Stress Constraints Only -- Example 7. The plot of the total weights versus the number of iterations for Case (a) through Case (d) is shown in Fig. 19. Table VII lists the final weights of these four cases, which are identified as Group A. The table also includes the final displacement at the top floor, the number of modes, and the associated natural periods used in the design. It is apparent that the multicomponent ground motion when combined with the P-Δ effect of the structural and nonstructural masses can yield nearly 3% of an increase in the structural weight over that which would be required for one horizontal component only.

Figure 20 shows the ratio of the energy (kinematic and strain), W, of the individual modes to the total energy, W_T, associated with the total number of natural modes included in the design. This plot signifies that the first mode is the most significant for all cases. The modes beyond the third have little effect on a structure subject to a horizontal motion such as in Cases (a) and (b) of Model I. However, when a structure is subjected to combined horizontal and vertical ground motions as in Cases (c) and (d) of Model II, the first five modes are needed for an adequate design.

The moments of columns and girders are shown in Figs. 21(a) and (b). The shear envelopes at individual floor levels are plotted in Fig. 22, which shows that the shear at each level is uniformly increased from the top floor to the bottom. The inclusion of the vertical ground motion and the P-Δ effect significantly increases

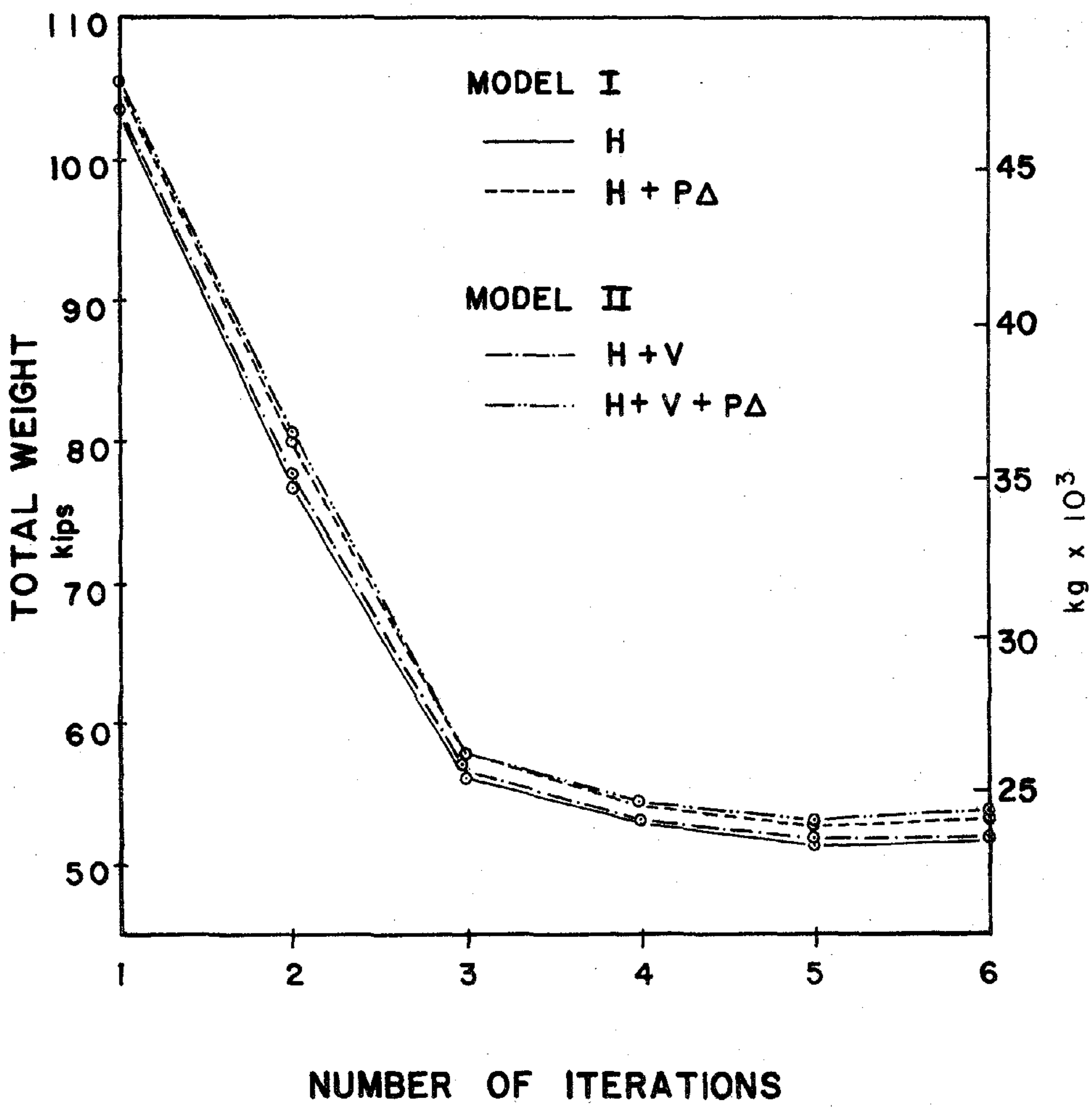


Figure 19. Optimum Weights of Unbraced Frame of Example 7.

TABLE VII. FINAL WEIGHTS, NATURAL PERIODS, AND DISPLACEMENTS OF 15-STORY FRAMES (A = UNBRACED WITH STRESS CONSTRAINTS, B = UNBRACED WITH STRESS AND DISPLACEMENT CONSTRAINTS, C = SINGLE-BRACED, D = DOUBLE-BRACED, E = K-BRACED) (1 kip = 453 kg, 1 in. = 2.54 cm)

Group	Case	Final Weight (kips)	Natural Period (sec.)					Disp. at top floor (in.)
			1	2	3	4	5	
A	a	51.88	2.409	0.796	0.462	0.327	0.245	13.19
	b	53.03	2.412	0.792	0.461	0.323	0.243	13.21
	c	52.15	2.413	0.792	0.461	0.326	0.245	13.22
	d	53.32	2.409	0.786	0.458	0.322	0.242	13.22
B	a	60.44	2.152	0.694	0.404	0.282	0.212	10.80
	b	62.21	2.179	0.671	0.386	0.268	0.202	10.80
	c	60.77	2.158	0.692	0.402	0.281	0.217	10.80
	d	62.21	2.152	0.677	0.395	0.276	0.212	10.80
C	a	45.02	1.879	0.490	0.256	0.247	0.187	9.00
	b	46.48	1.876	0.479	0.274	0.252	0.209	9.05
	c	45.04	1.874	0.488	0.291	0.255	0.224	9.02
	d	46.75	1.874	0.485	0.288	0.253	0.221	9.02
D	a	30.67	2.136	0.510	0.316	0.265	0.263	10.75
	b	32.02	2.120	0.502	0.313	0.261	0.260	10.63
	c	30.87	2.140	0.531	0.402	0.362	0.337	10.80
	d	31.55	2.134	0.509	0.398	0.361	0.335	10.80

TABLE VII. FINAL WEIGHTS, NATURAL PERIODS, AND DISPLACEMENTS OF 15-STORY FRAMES (continued)

Group	Case	Final weight (kips)	Natural Period (sec.)					Disp. at top floor (in.)
			1	2	3	4	5	
E	c	28.13	2.104	0.522	0.277	0.271	0.179	10.47
	d	28.61	2.111	0.519	0.274	0.269	0.178	10.55

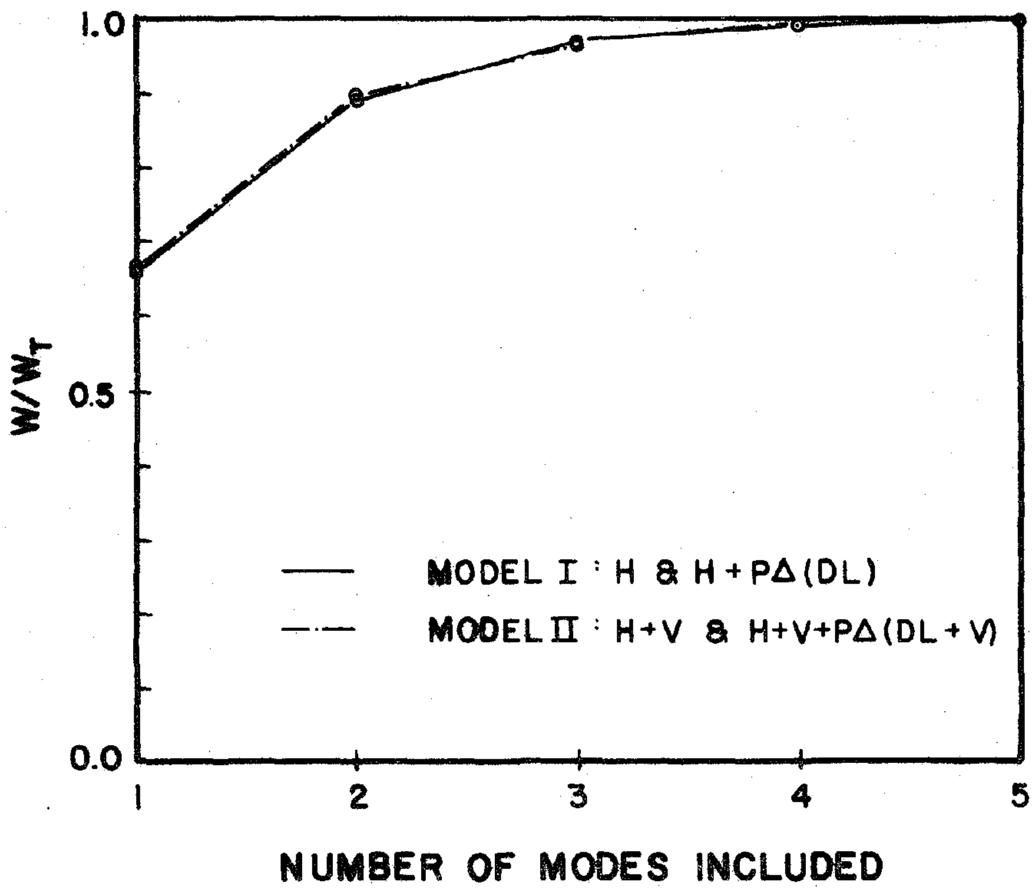


Figure 20. Contribution of Energy of Individual Modes to the Design of Example 7.

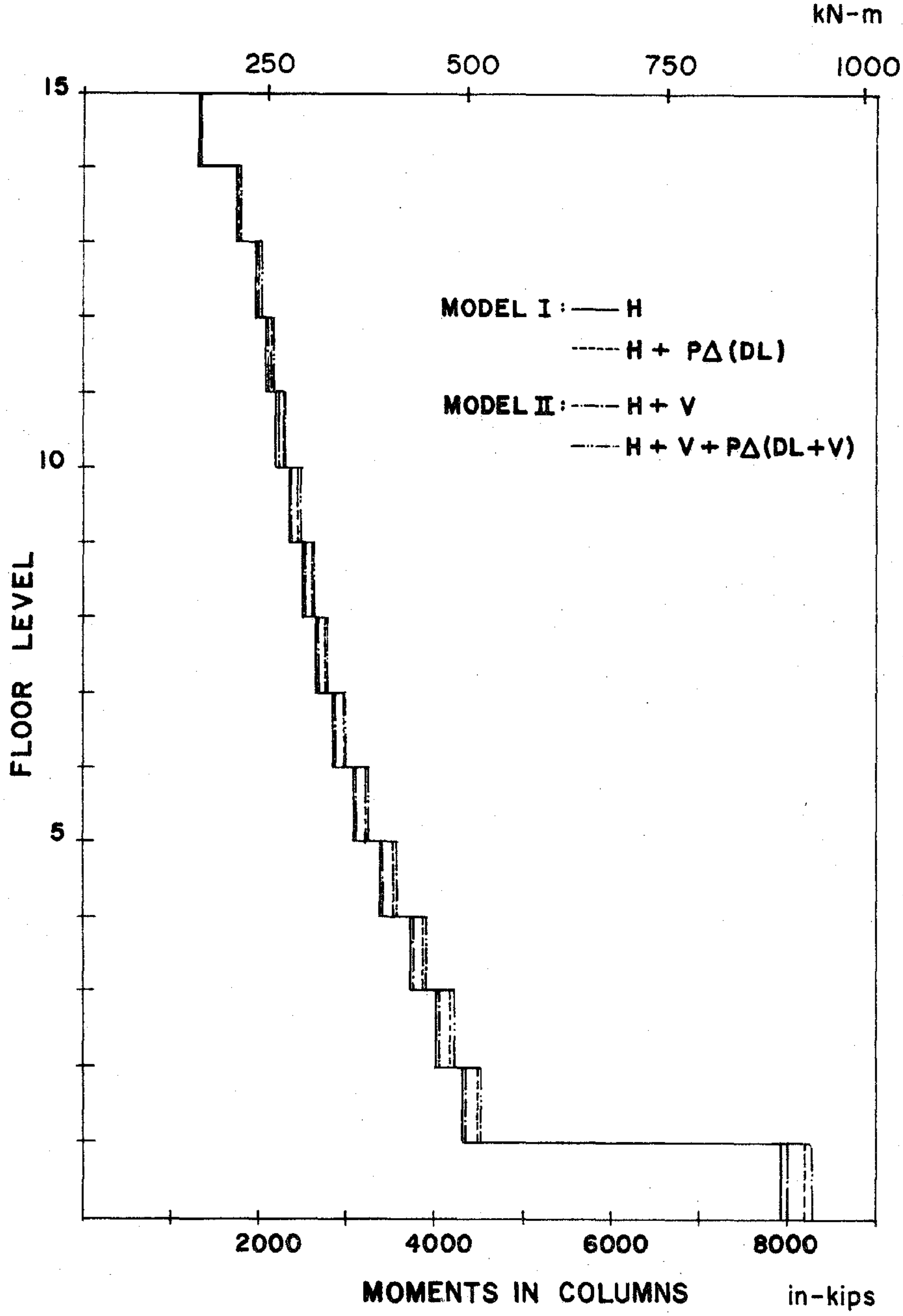


Figure 21 (a). Column Moments of Example 7.

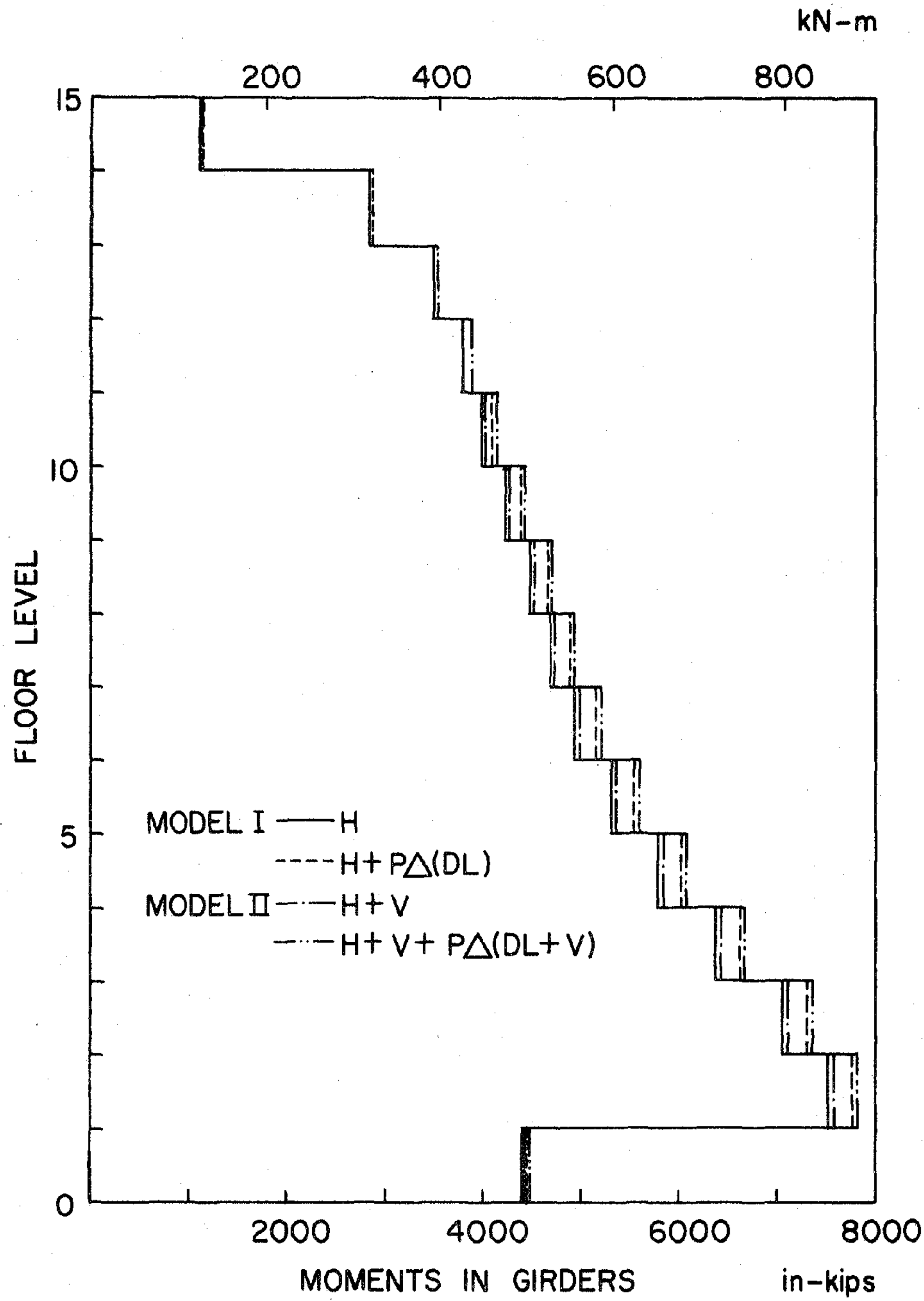


Figure 21 (b). Girder Moments of Example 7.

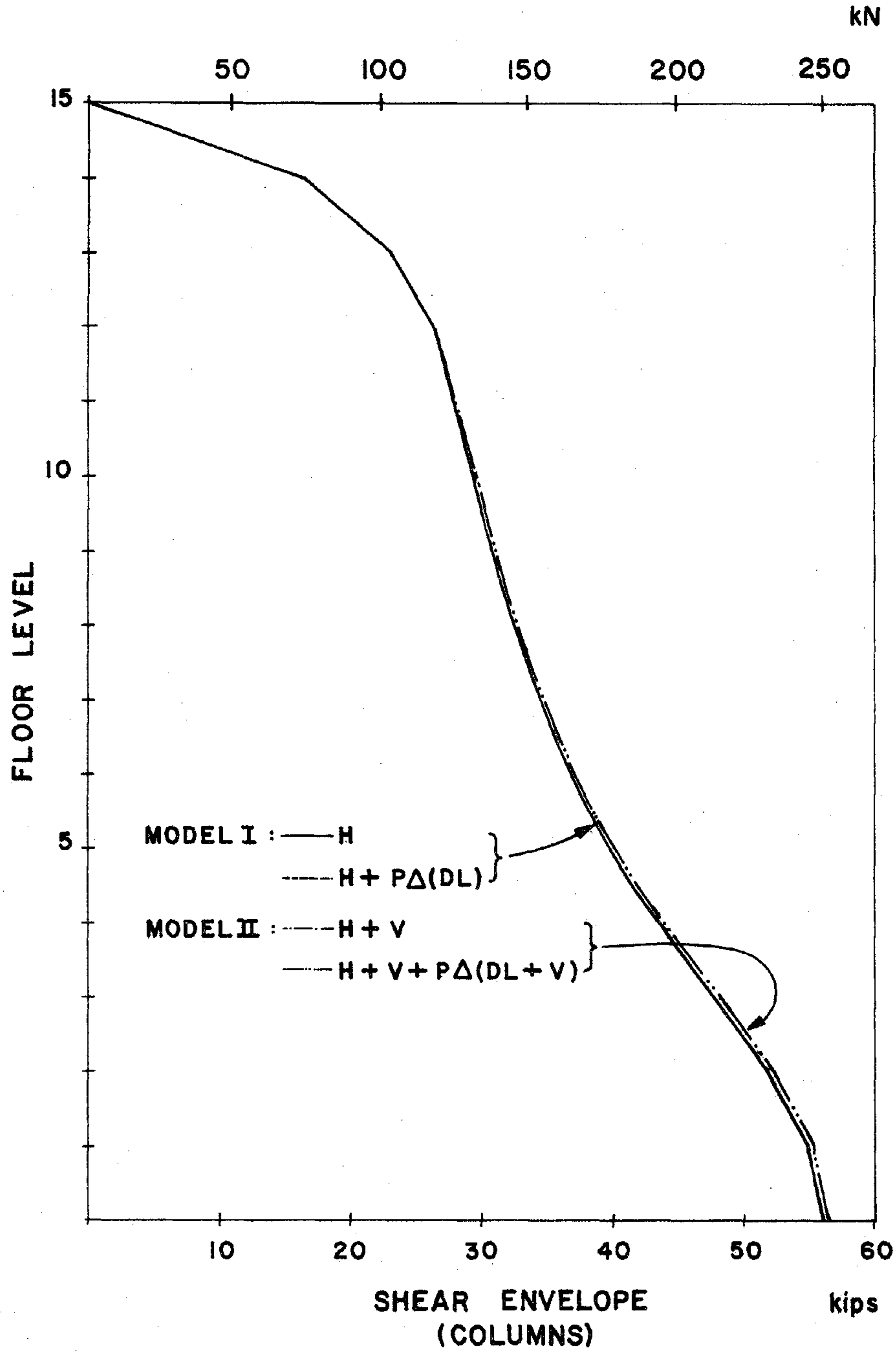


Figure 22. Shear Envelopes of Example 7.

increases the shear forces at all the floor levels.

Figures 23 and 24 show the distribution of the moments of inertia of the columns and girders respectively. The moment of inertia for the columns is the smallest at the top floor and increases almost linearly from the top floor to the second floor and abrubtly increases at the first floor. The moment of inertia for the girders is relatively small at the first floor and increases suddenly at the second floor. It decreases almost linearly from the second to the 13th floor and abruptly decreases from the 14th to the top. If the moments of inertia of all the four cases are divided by the corresponding moments of inertia of Case (a), the normalized moments of inertia of the columns and girders corresponding to Fig. 23 and 24 can be shown as they appear in Figs. 25 and 26, respectively. Apparently the moment of inertia associated with Case (a) is normalized to one, and the normalized moments of inertia of the other cases can reflect how much the multicomponent ground motion affects the stiffness requirements. From Figs. 25 and 26, one can observe that the vertical ground motion when combined with the horizontal ground excitation and the P-Δ effect can significantly increase the moments of inertia of both columns and girders.

The ratios of the combined stresses of the axial and bending forces to the allowable stresses for the columns and girders are respectively plotted in Figs. 27 and 28 in which the variations of the actual stresses in the final design can be observed. The

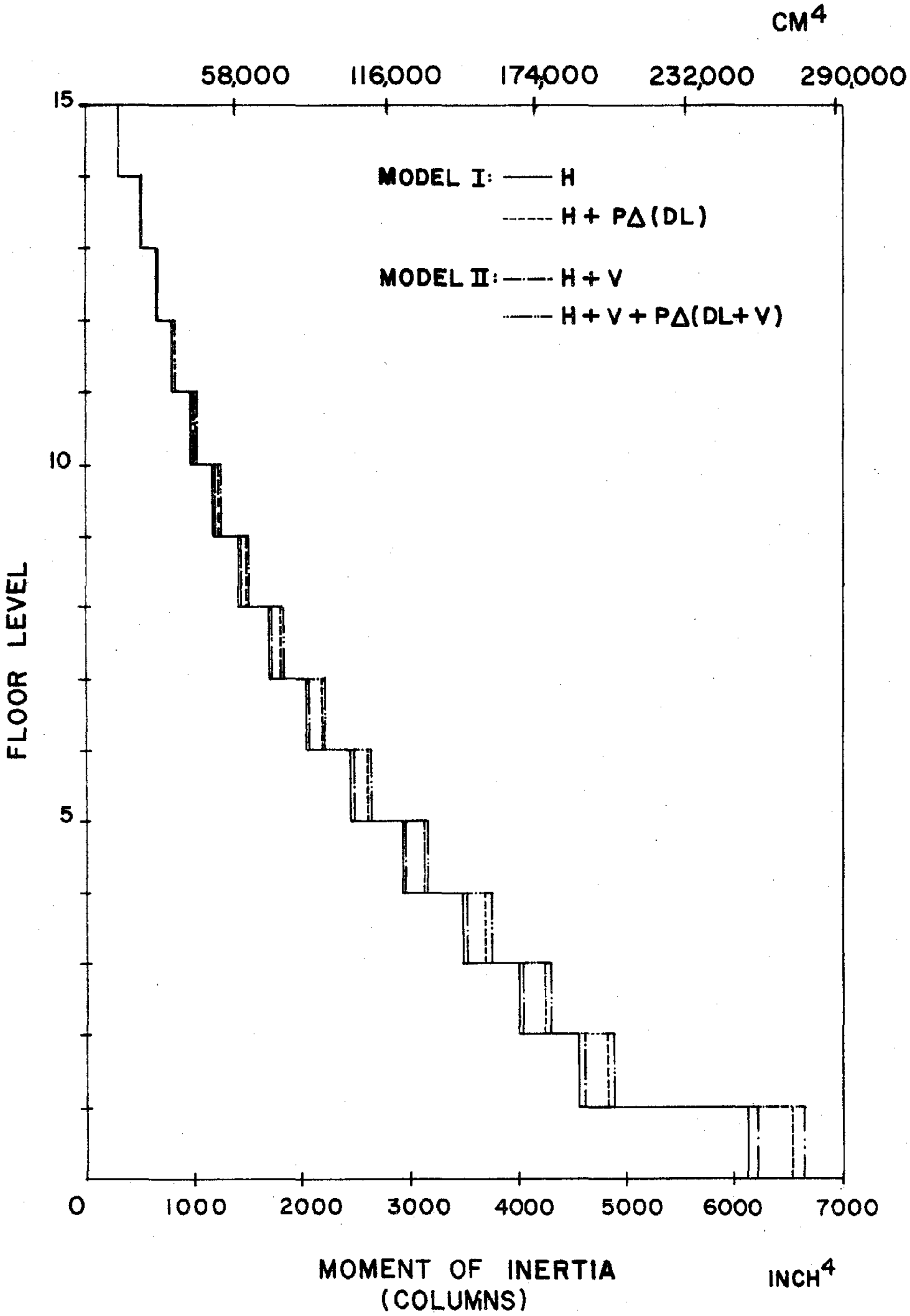


Figure 23. Distribution of Moment of Inertia of Columns of Example 7.

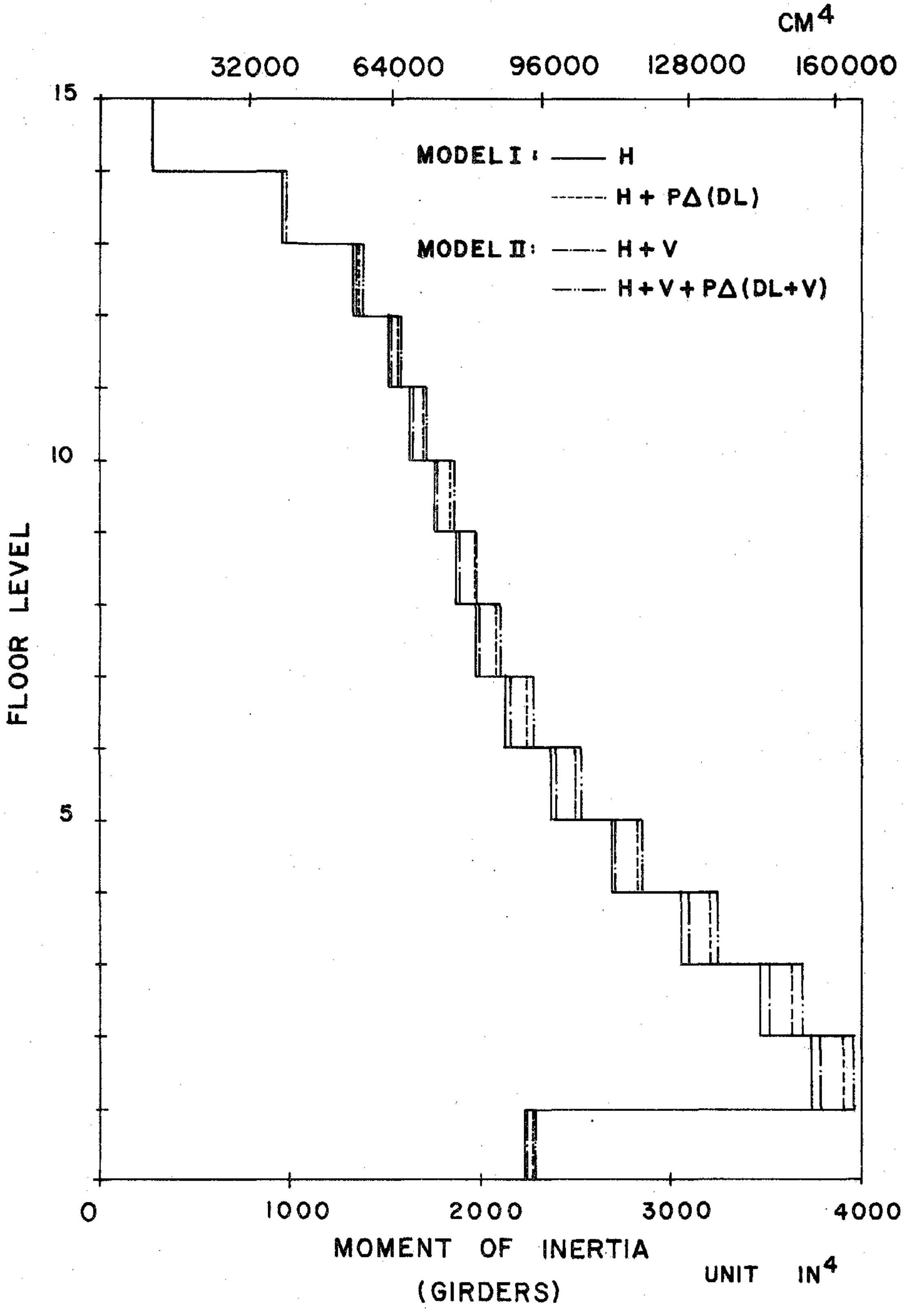


Figure 24. Distribution of Moment of Inertia of Girders of Example 7.

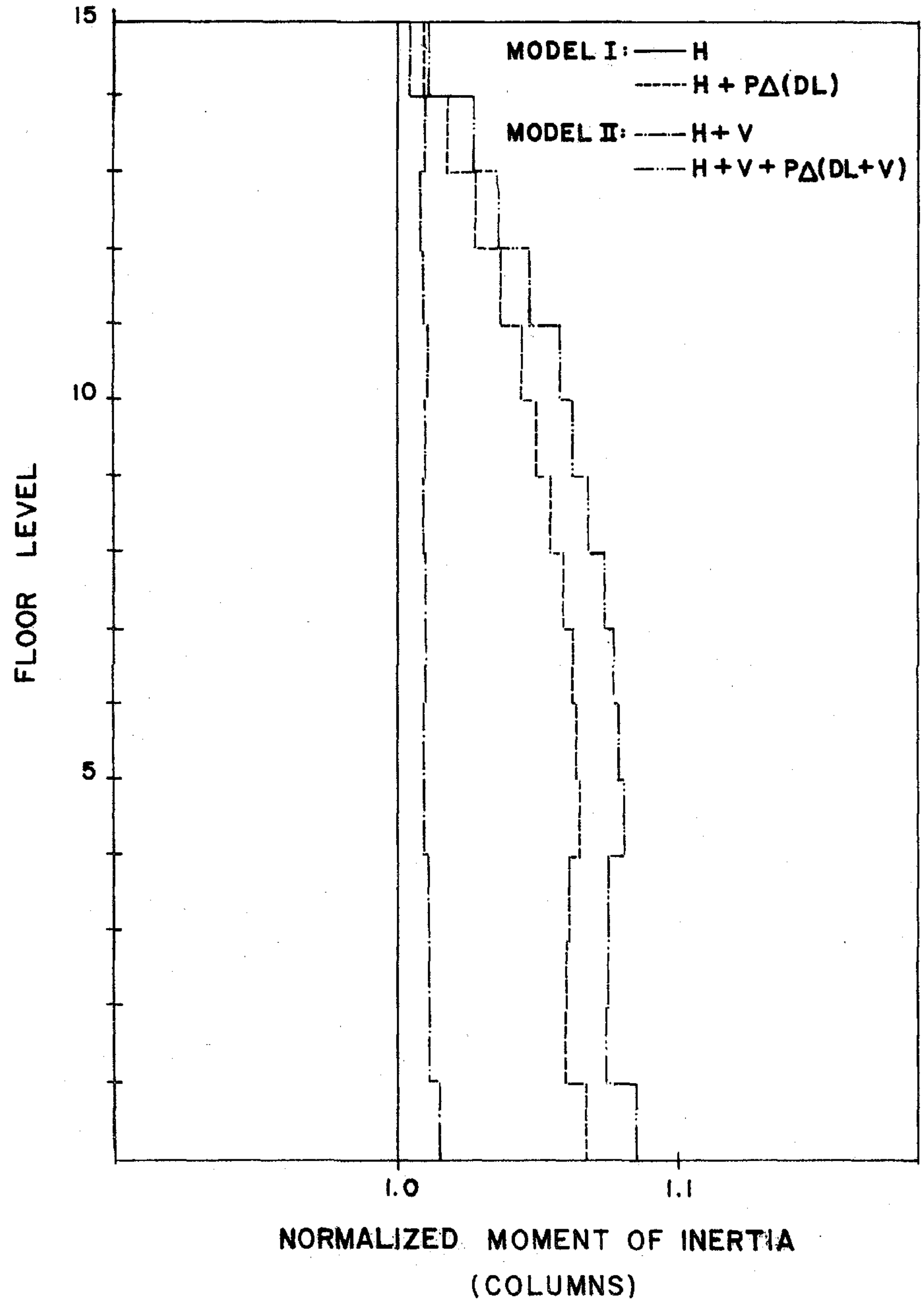


Figure 25. Normalized Moment of Inertia of Columns of Example 7.

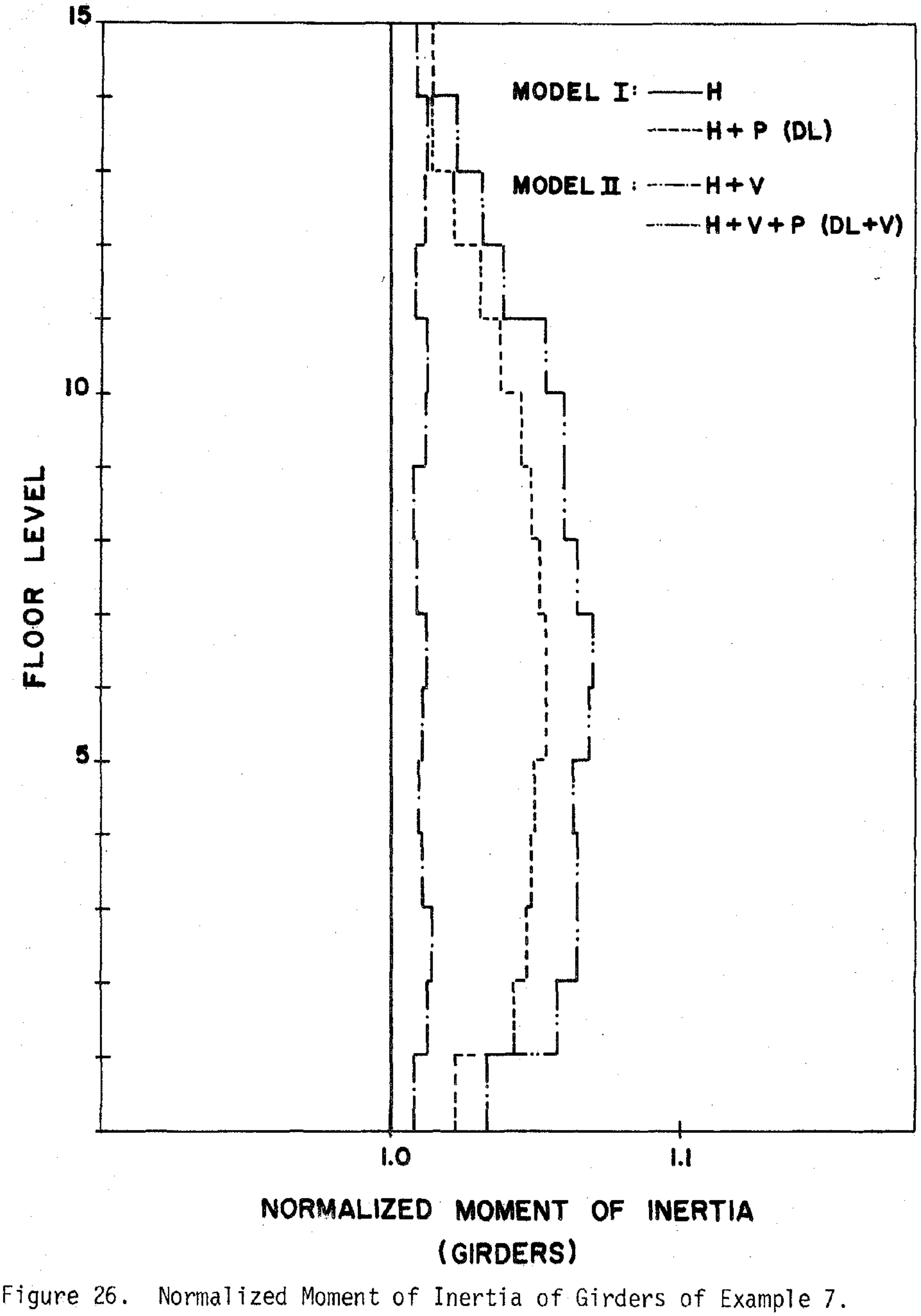


Figure 26. Normalized Moment of Inertia of Girders of Example 7.

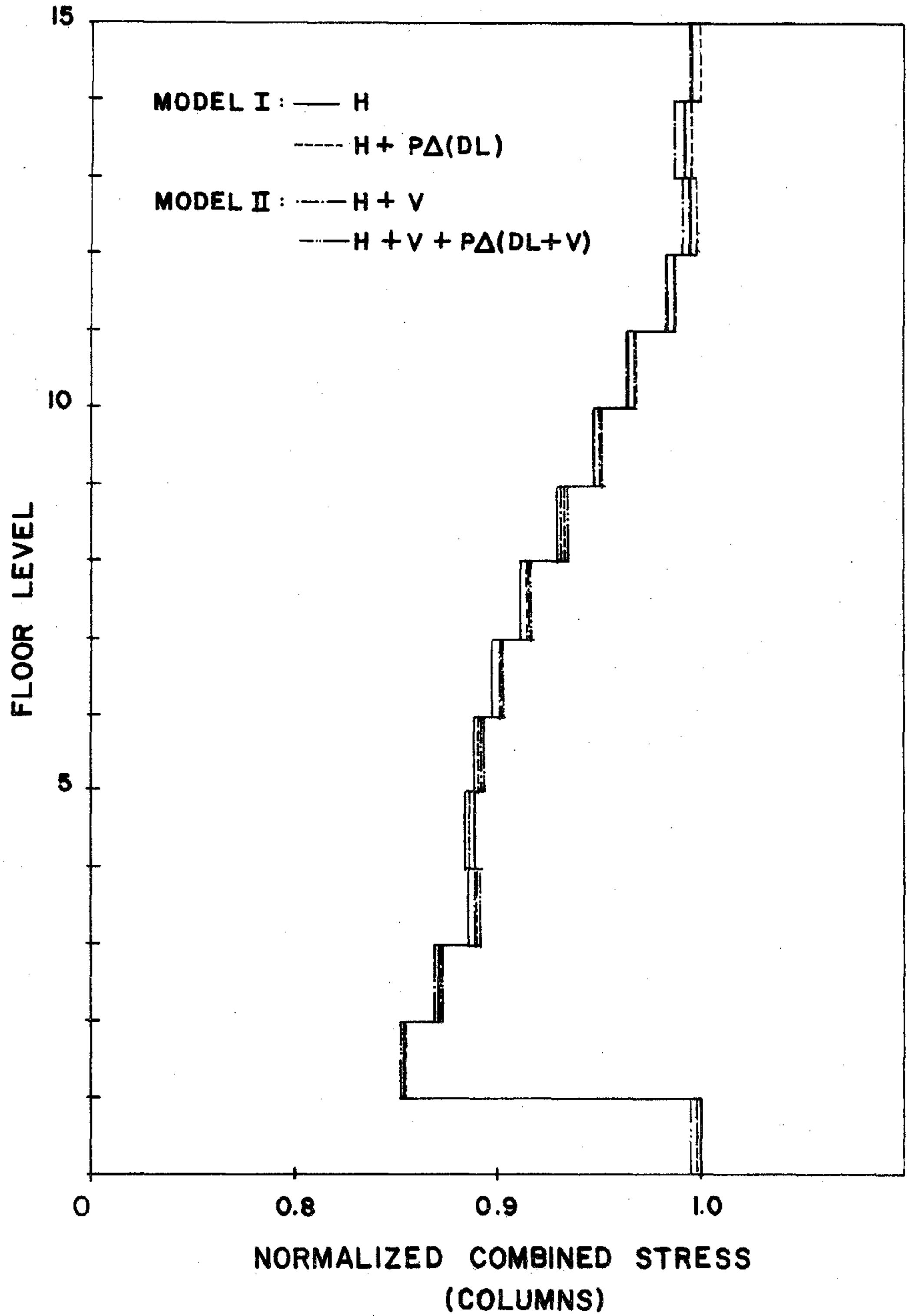


Figure 27. Ratio of Stress to Allowable Stress of Columns of Example 7.

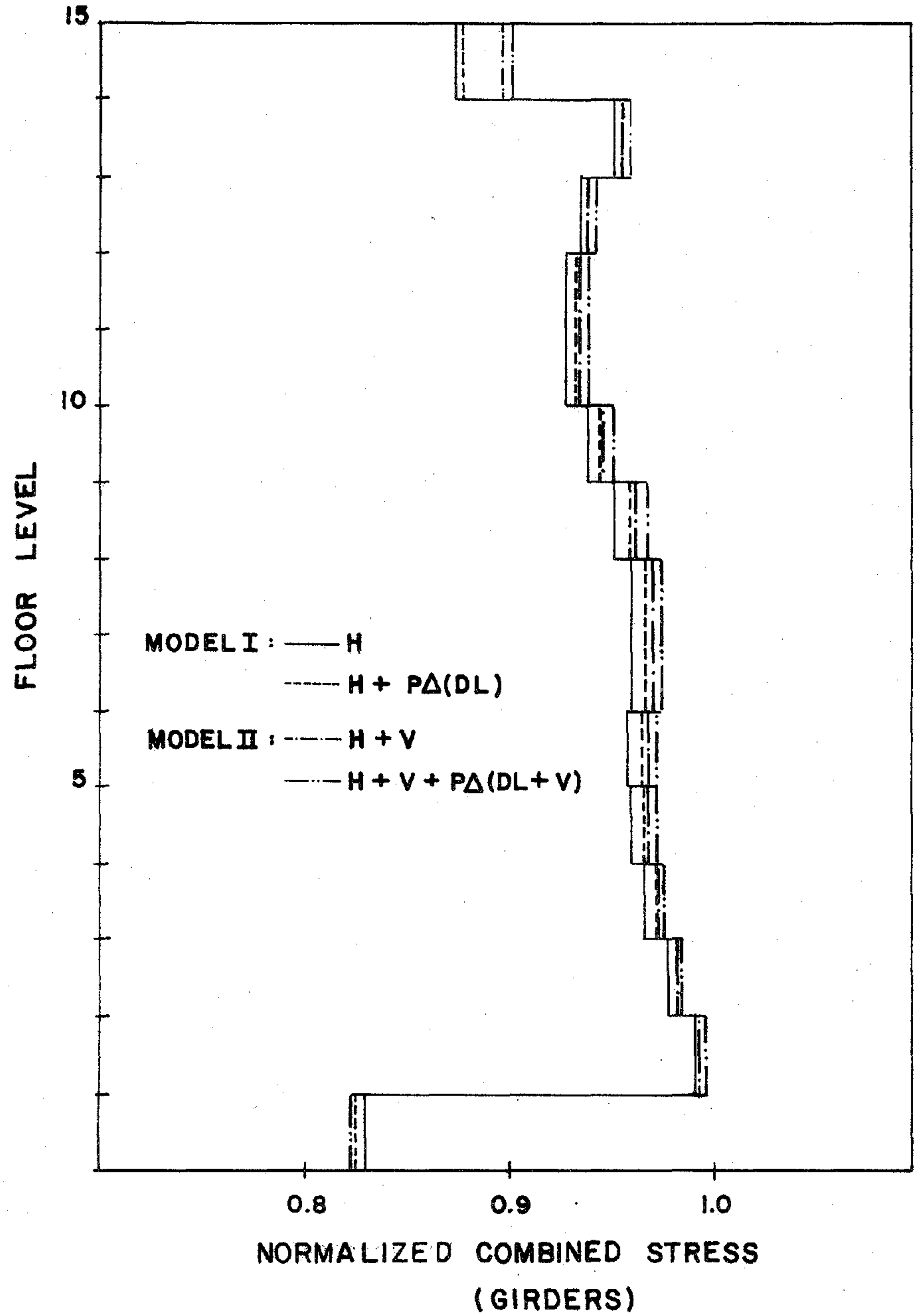


Figure 28. Ratio of Stress to Allowable Stress of Girders of Example 7.

stresses of the columns at the first floor and top three floors are close to the allowable stress. The stresses of girders reach the allowable stress for all floors except the top and the first floor for which the stresses at the first floor are the smallest.

Figure 29 shows a plot of the displacements at the different floor levels relative to the ground for the four cases. On the basis of the final design weight listed in Table VII, the displacements are obviously inversely proportional to the overall stiffness of the structure.

2. Stress and Displacement Constraints--Example 8. In addition to the allowable stress of 29 ksi (200.1 MN/m^2) that is imposed on all the members, the allowable displacement of 0.005 times the story height from the ground is imposed on each floor. Figure 30 shows the interation history and the corresponding structural weights for the four cases of (a) through (d). The final weights, periods, and displacements at the top story for all cases are listed in Table VII in Group B. The design result shows that the structural weight, which is needed to withstand the effects of a vertical ground motion and a P-Δ effect, is about 3.0% greater than the structural weight needed for a horizontal ground motion alone. It is apparent that the displacement constraints cause the structure of Group B to be more rigid than those in Group A; thus, the final structural weight of Group B is much heavier than that in Group A.

The distribution of the moments in columns and girders is given in Figs. 31(a) and (b). The shear envelopes are illustrated in Fig 32. The moments of the columns are the largest at the first floor and gradually decrease from the second floor to the top. The moments of the girders are remarkably small relative to those at the second floor which are in fact the largest. Some variations in the shear envelopes are

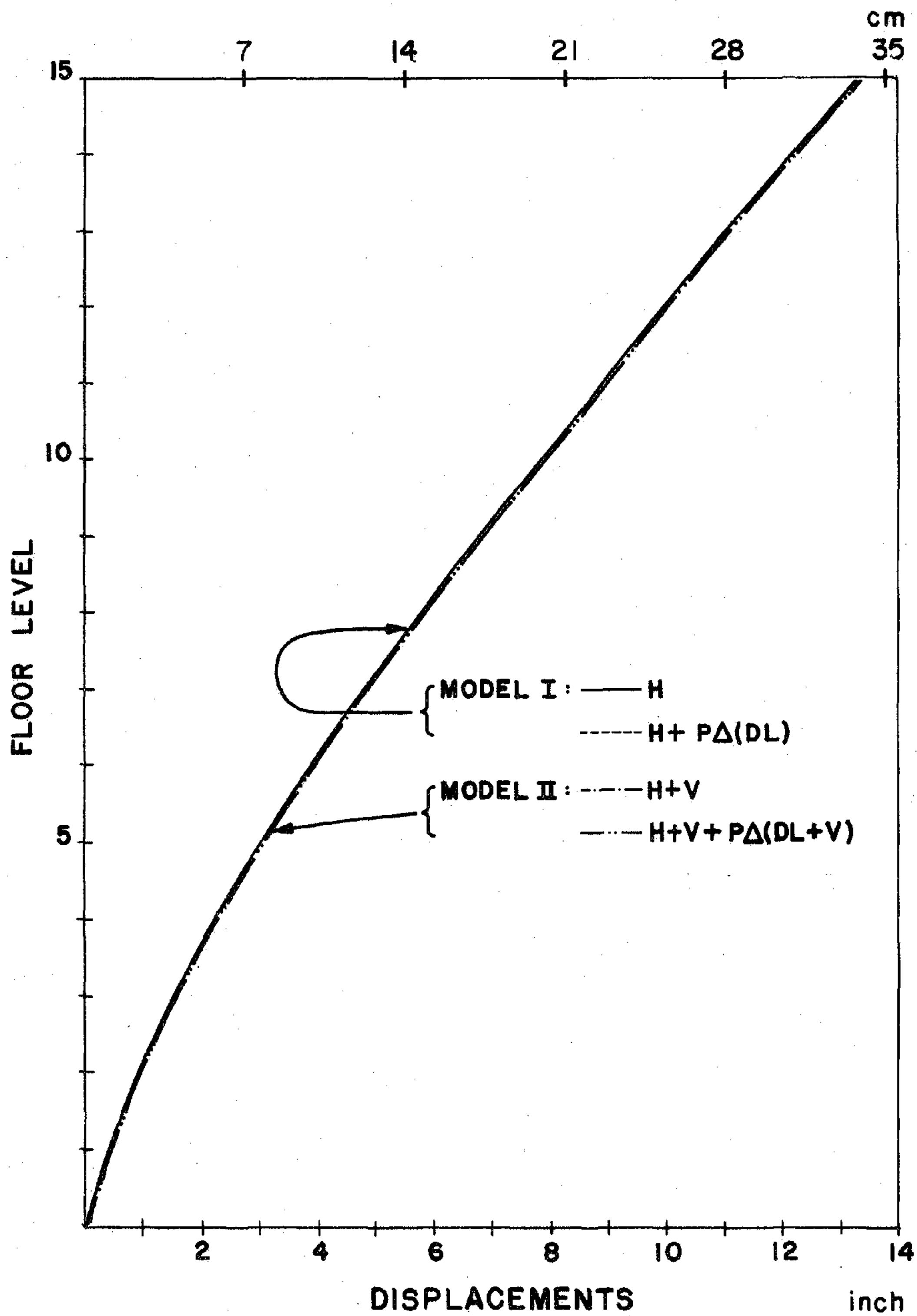


Figure 29. Displacements at Floor Levels of Example 7.

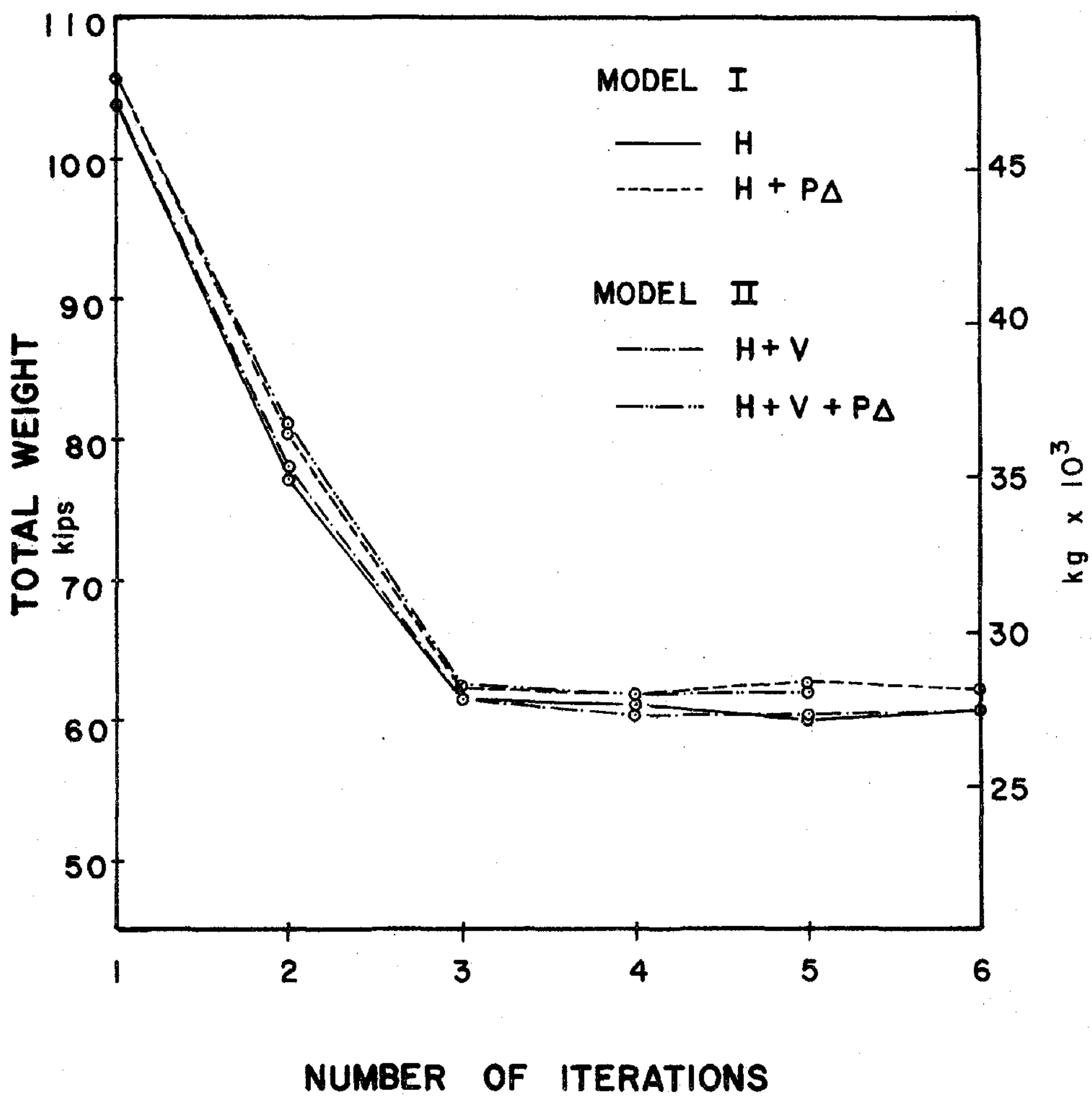


Figure 30. Optimum Weight of Unbraced Frame of Example 8.

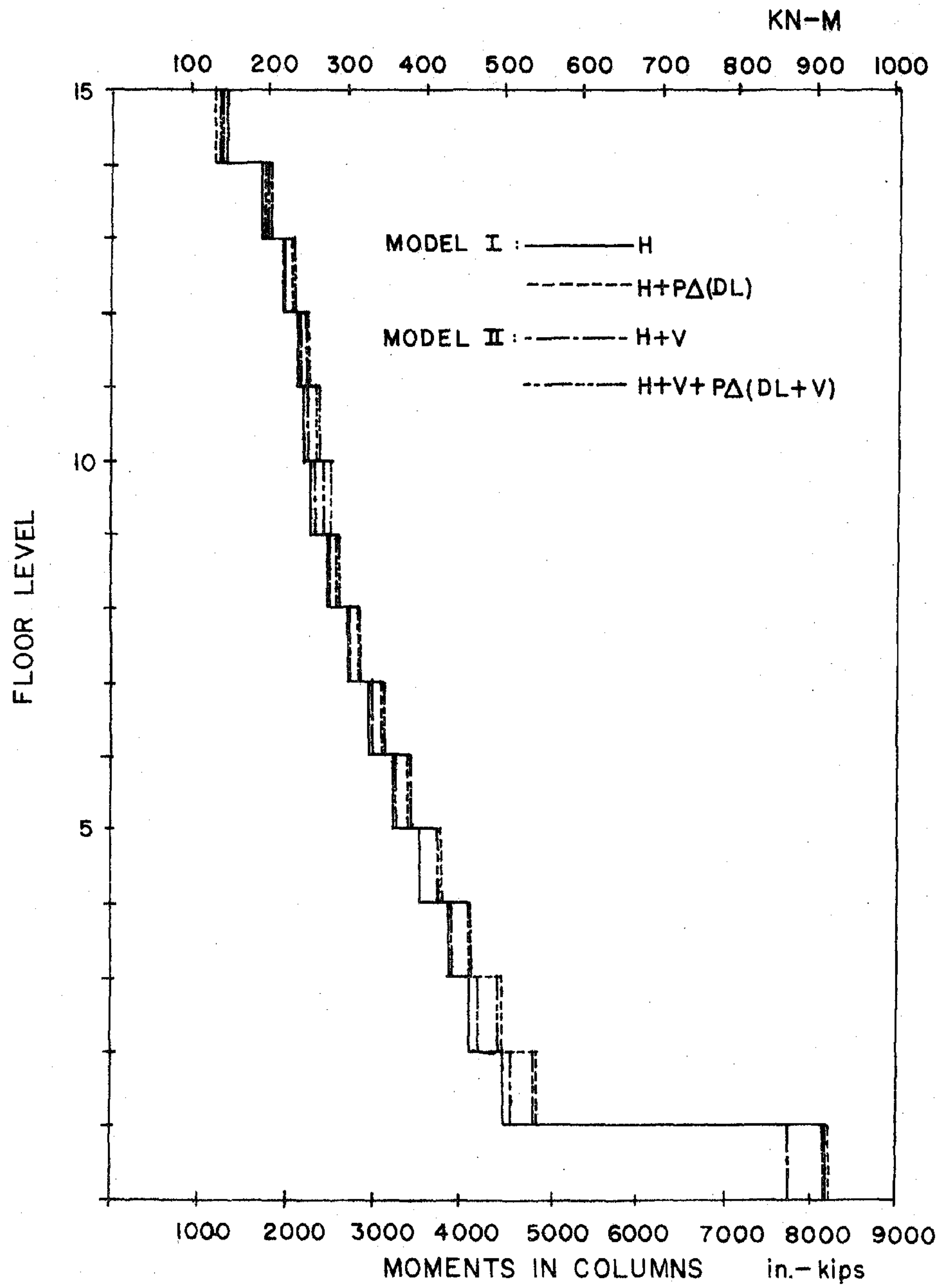


Figure 31 (a). Column Moments of Example 8.

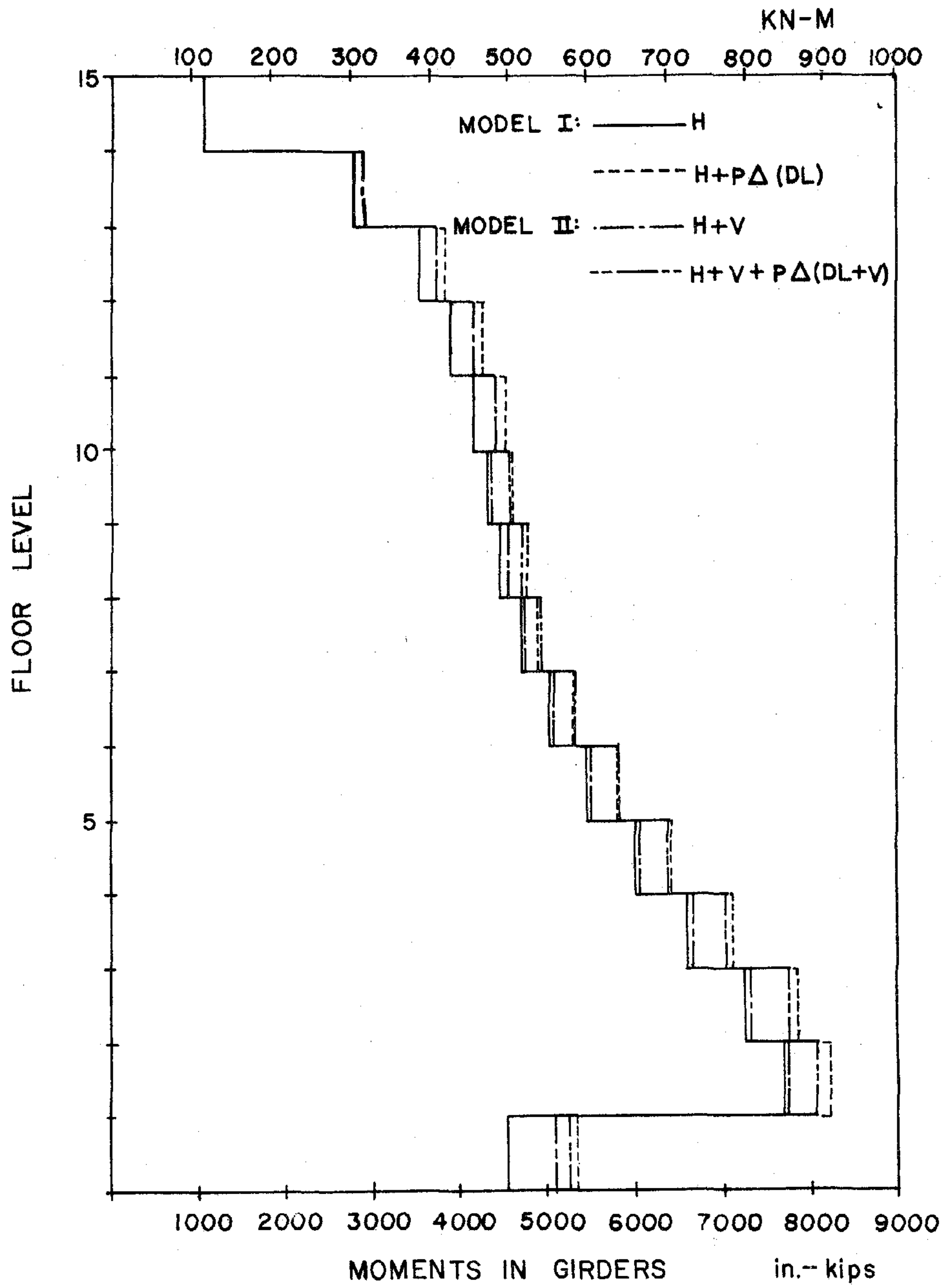


Figure 31 (b). Girder Moments of Example 8.

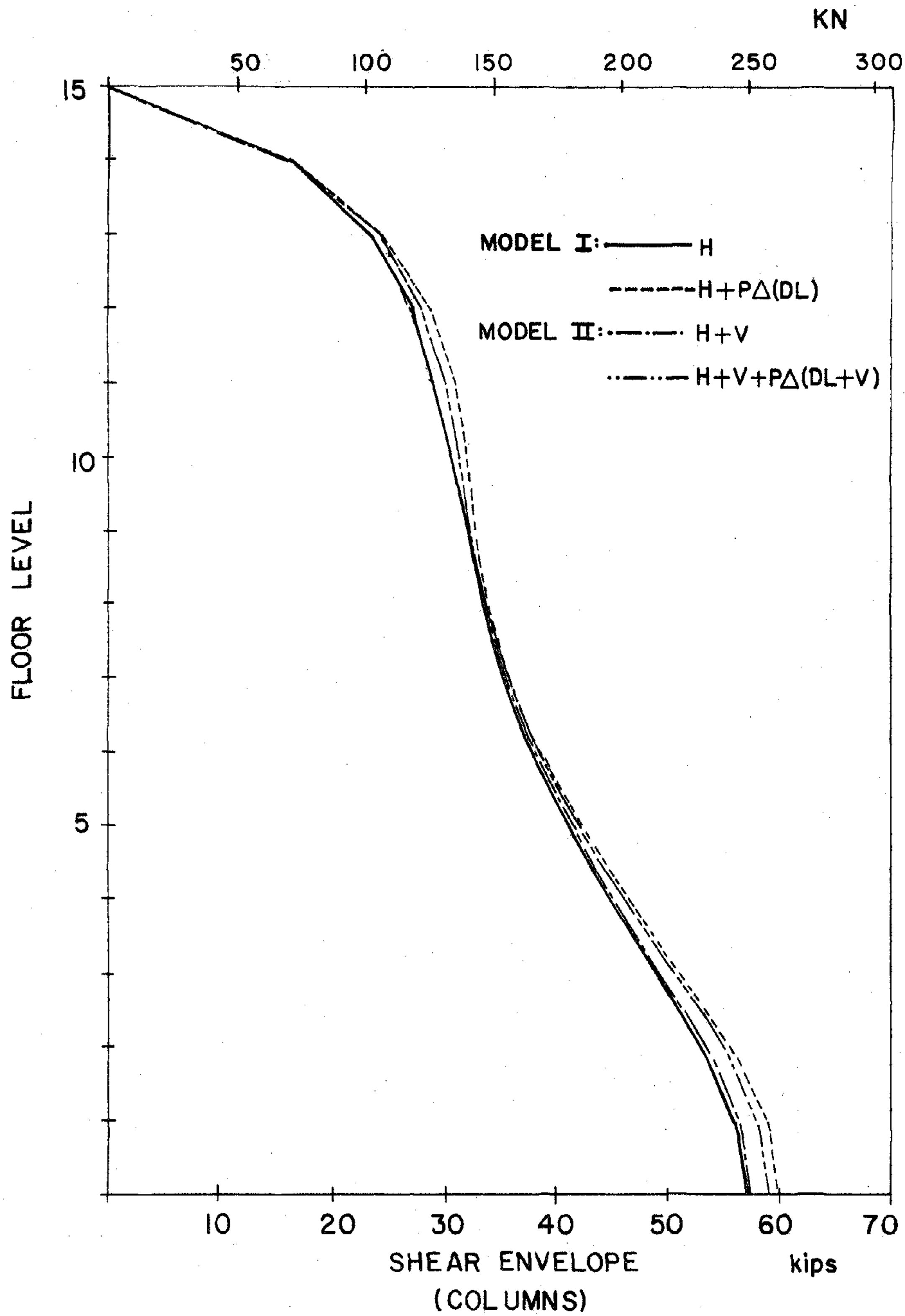


Figure 32. Shear Envelopes of Example 8.

shown for cases of (a) through (d) at certain floors.

Figures 33 and 34 reveal the distribution of the moments of inertia of the columns and girders respectively. The moment of inertia of the columns is the smallest at the top, the largest at the supports, and almost linearly increases from the second floor to the top floor. The moment of inertia of the girders is small at the first floor, suddenly increases at the second floor, and gradually decreases from the second to the sixth floor. It becomes practically constant from the sixth to the 12th and significantly decreases from the 12th to the top floor. The normalized moments of inertia of the columns and girders are shown in Figs. 35 and 36 respectively. Note that the moments of inertia of Case (a) is greater than that of other cases for the columns above the eighth floor and for the girders above the tenth floor.

The ratios of the actual combined stresses to the allowable stress of both the columns and girders are shown in Figs. 37 and 38 respectively. Note that all the design cases have been terminated, because the allowable deflection at the top floor is violated as shown in Table VII. The displacements at the floor levels are sketched in Fig. 39 and the significance of the individual modes for the design is shown in Fig. 40.

3. Constraints on the Relative Stiffnesses of the Constituent Members --Example 9. As shown in the previous design examples, the final moments of inertia of the columns and girders of a system can have a great deal of difference in magnitude. In engineering practice, the

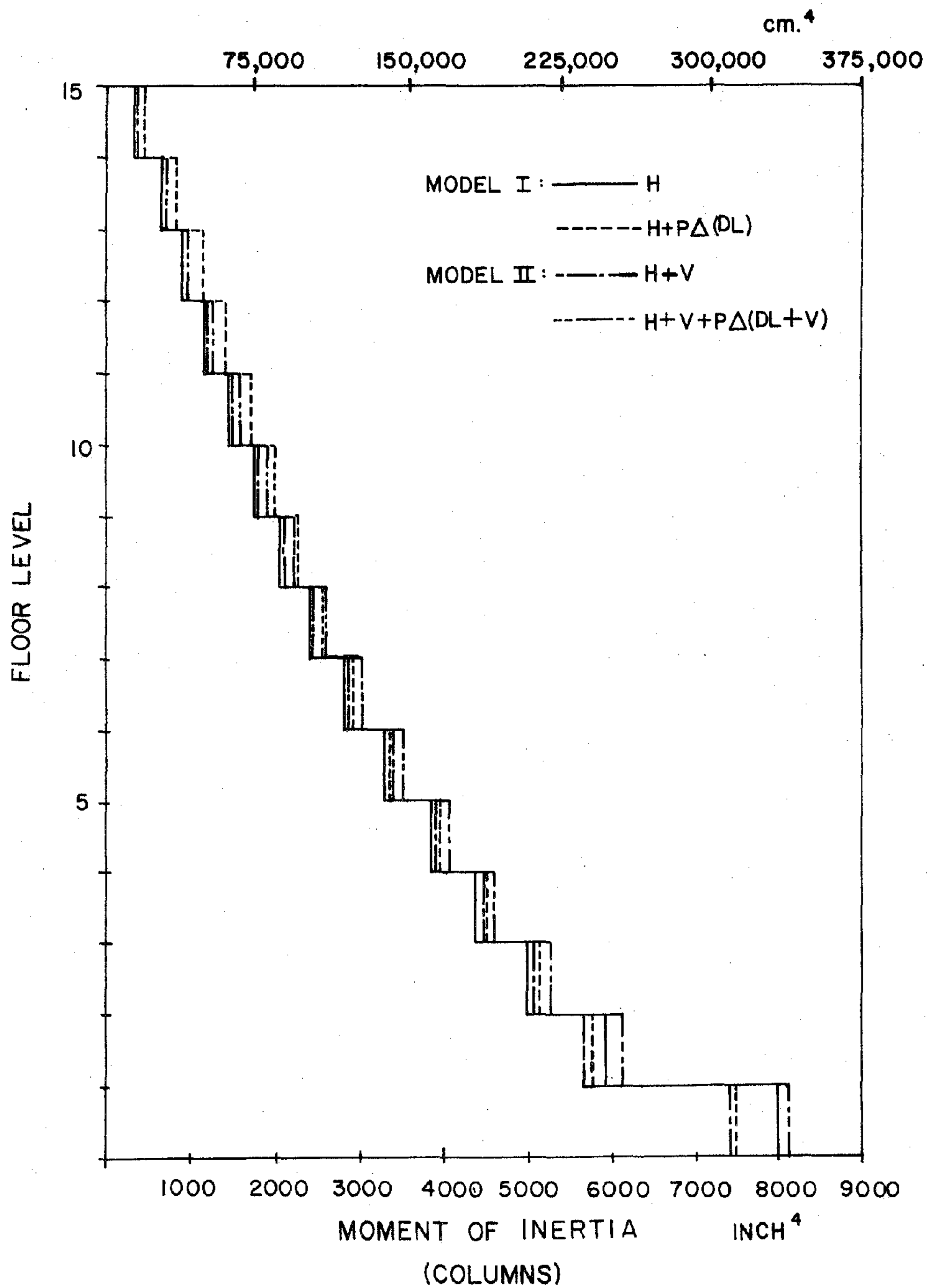


Figure 33. Distribution of Moment of Inertia of Columns of Example 8.

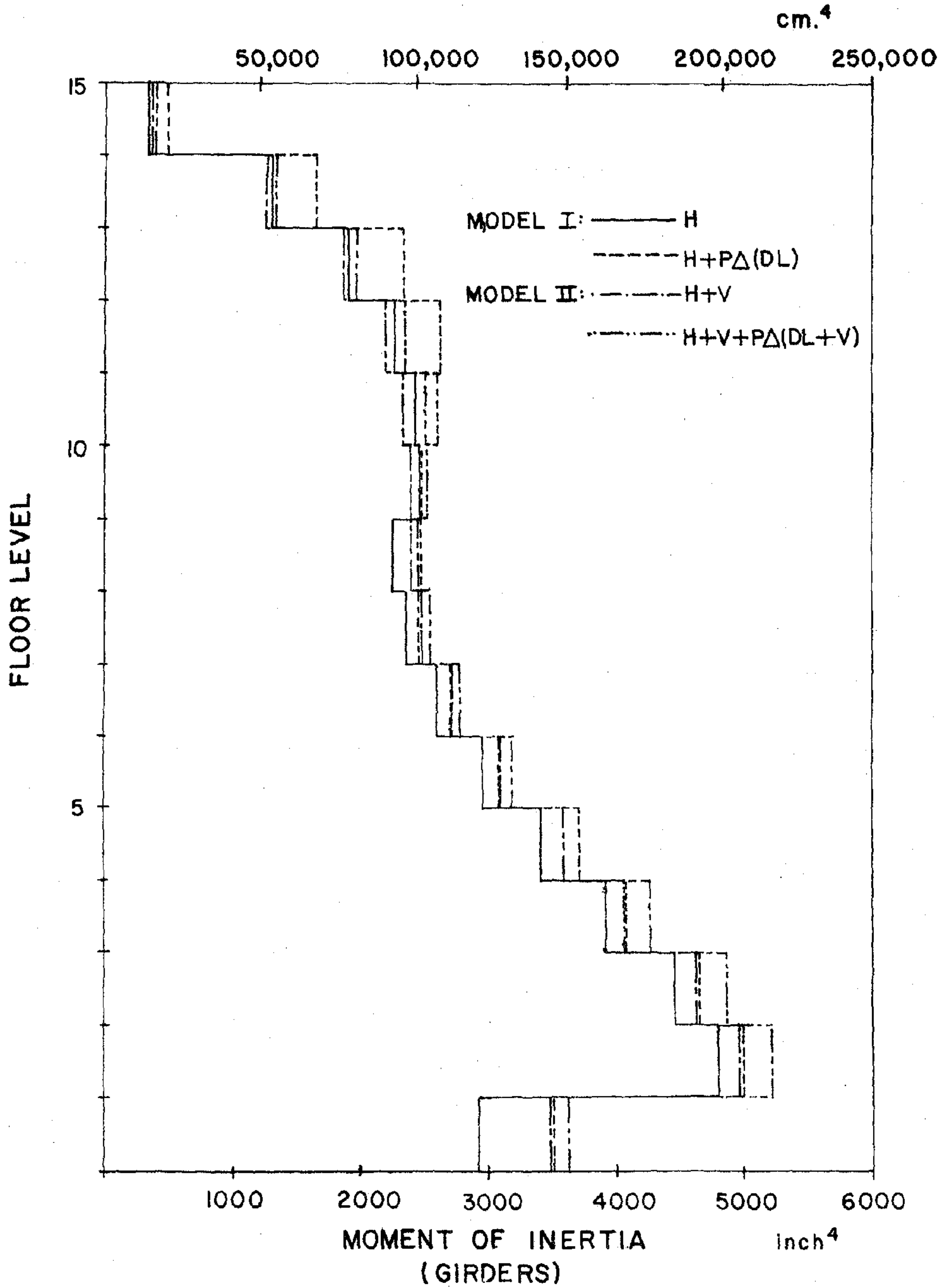


Figure 34. Distribution of Moment of Girders of Example 8.

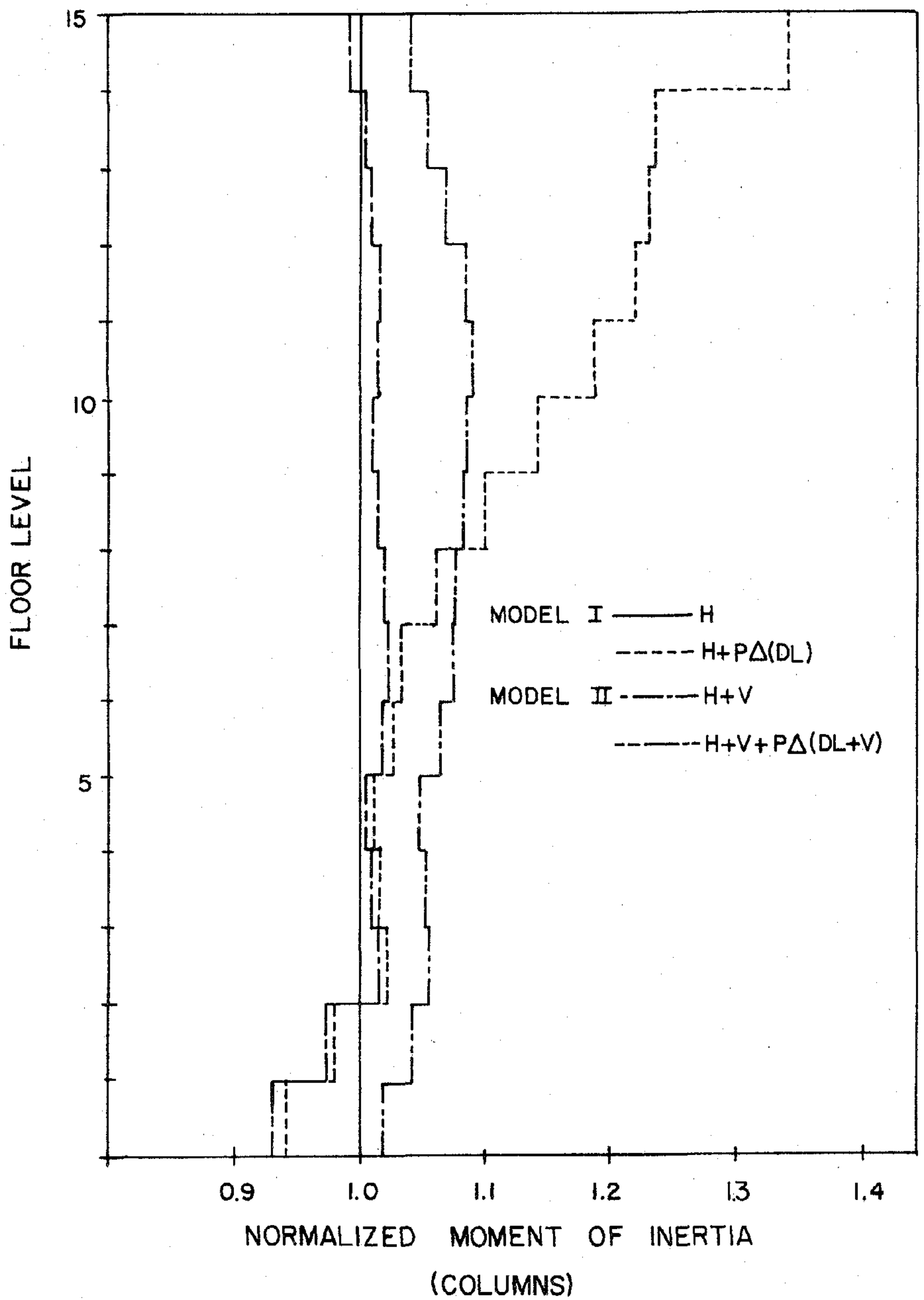


Figure 35. Normalized Moment of Inertia of Columns of Example 8.

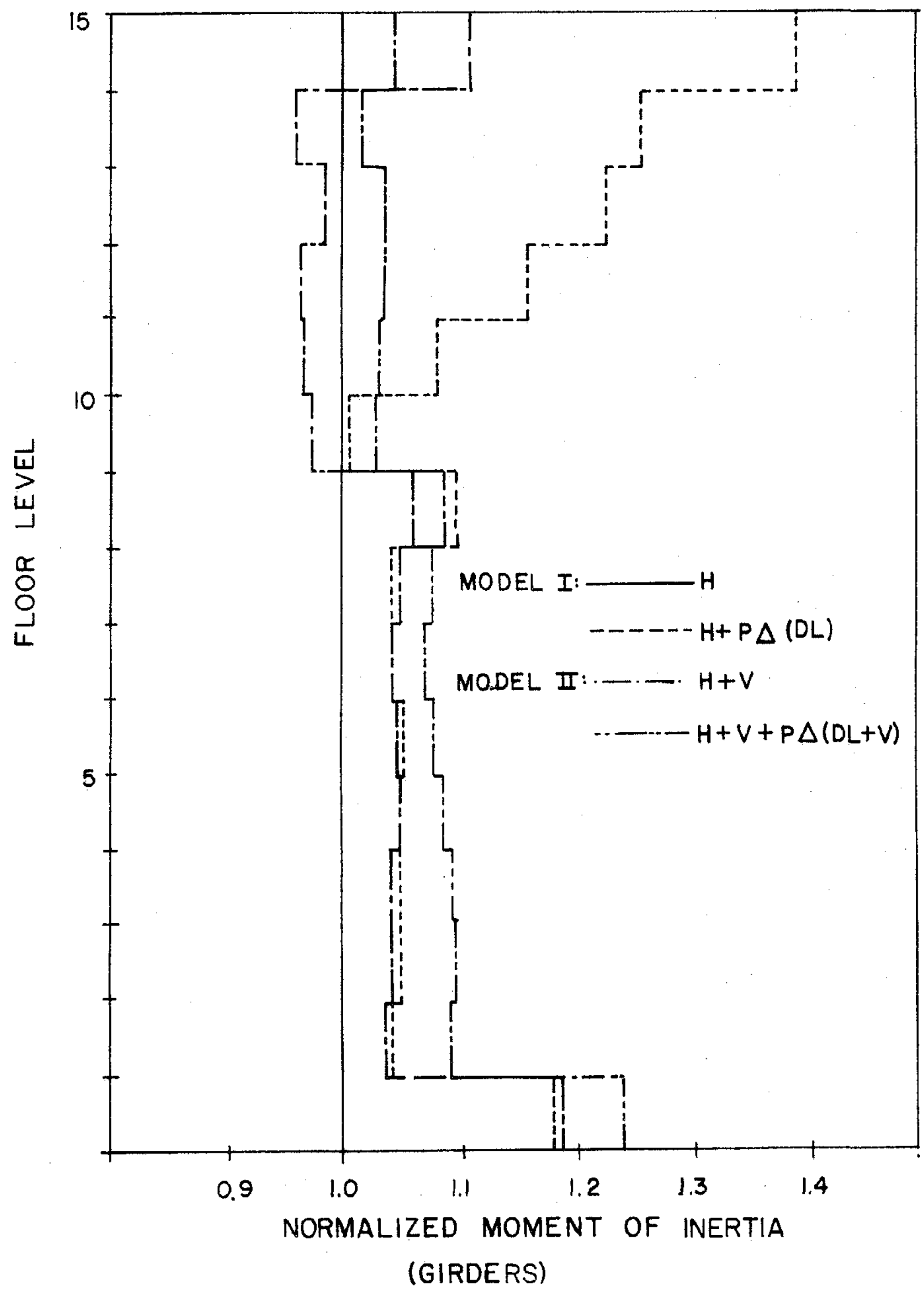


Figure 36. Normalized Moment of Inertia of Girders of Example 8.

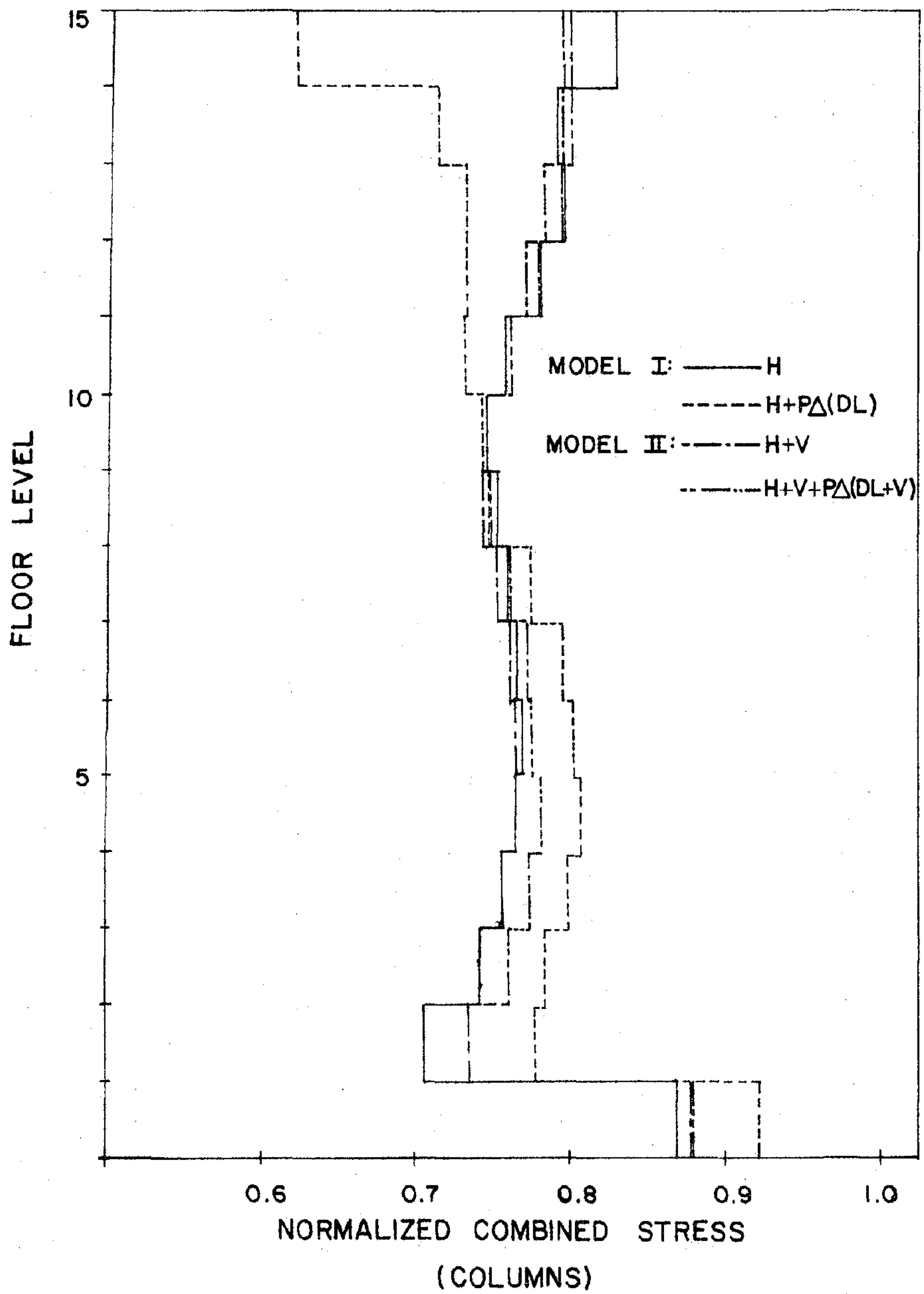


Figure 37. Ratio of Stress to Allowable Stress of Columns of Example 8.

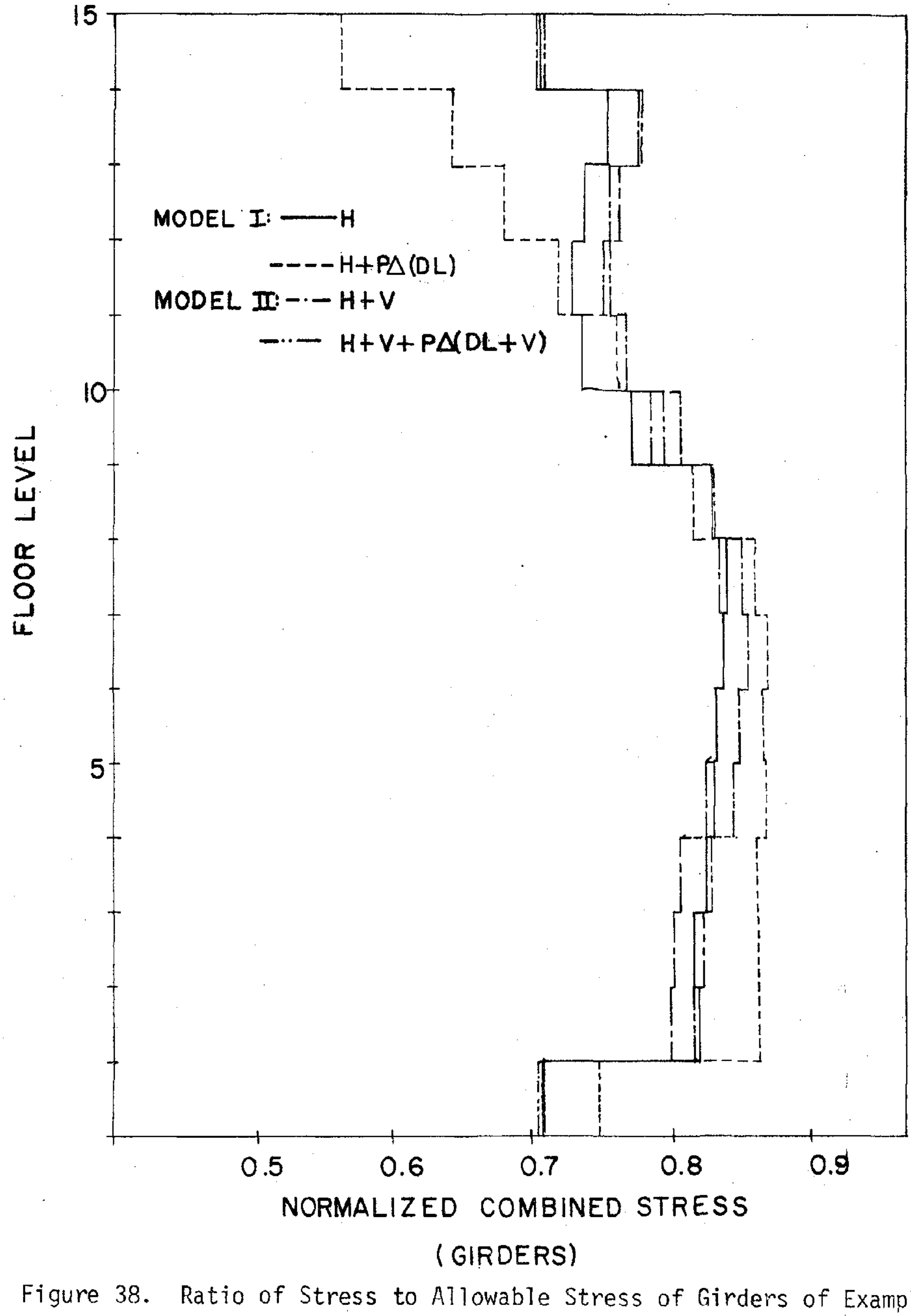


Figure 38. Ratio of Stress to Allowable Stress of Girders of Example 8.

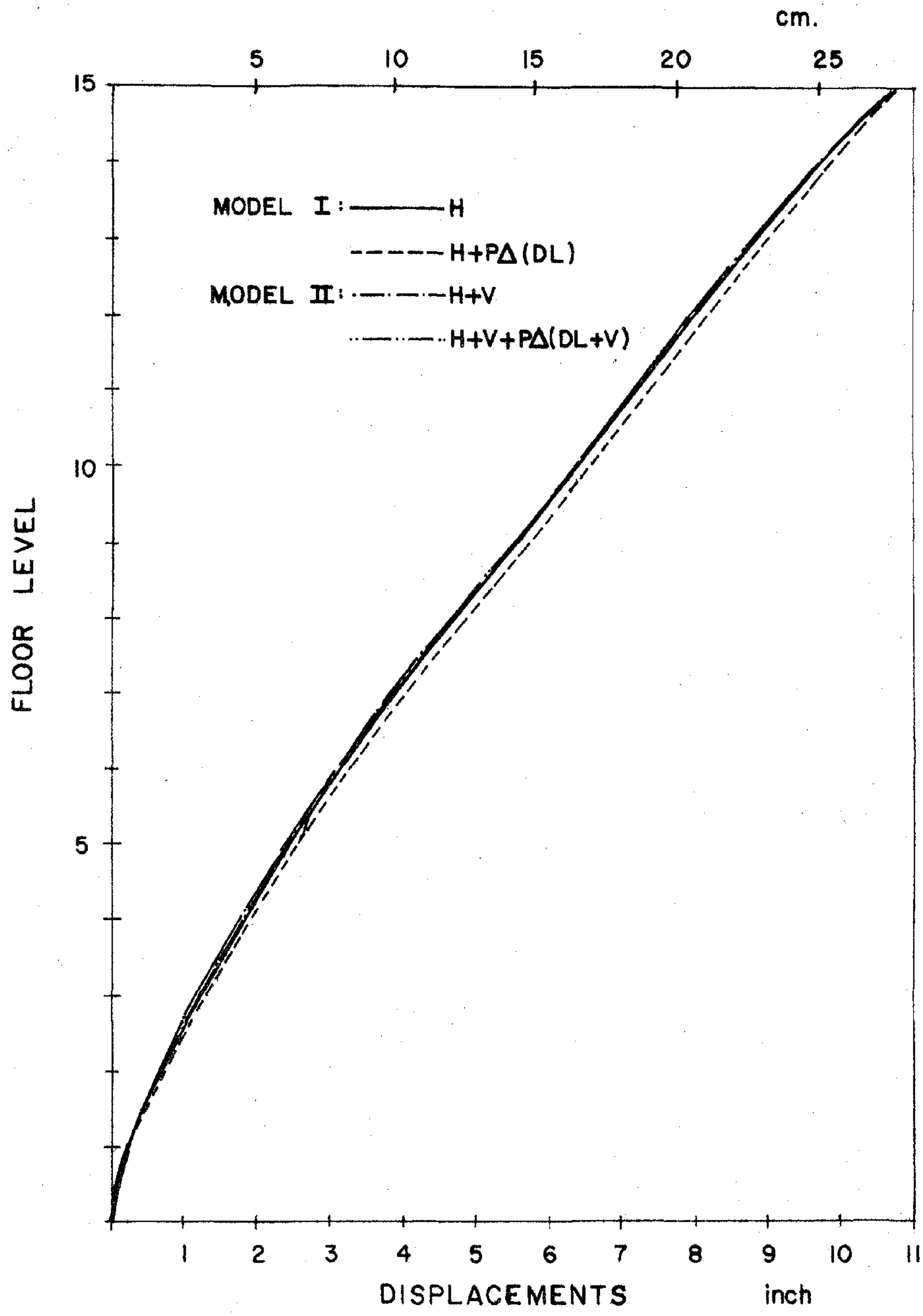


Figure 39. Displacements at Floor Levels of Example 8.

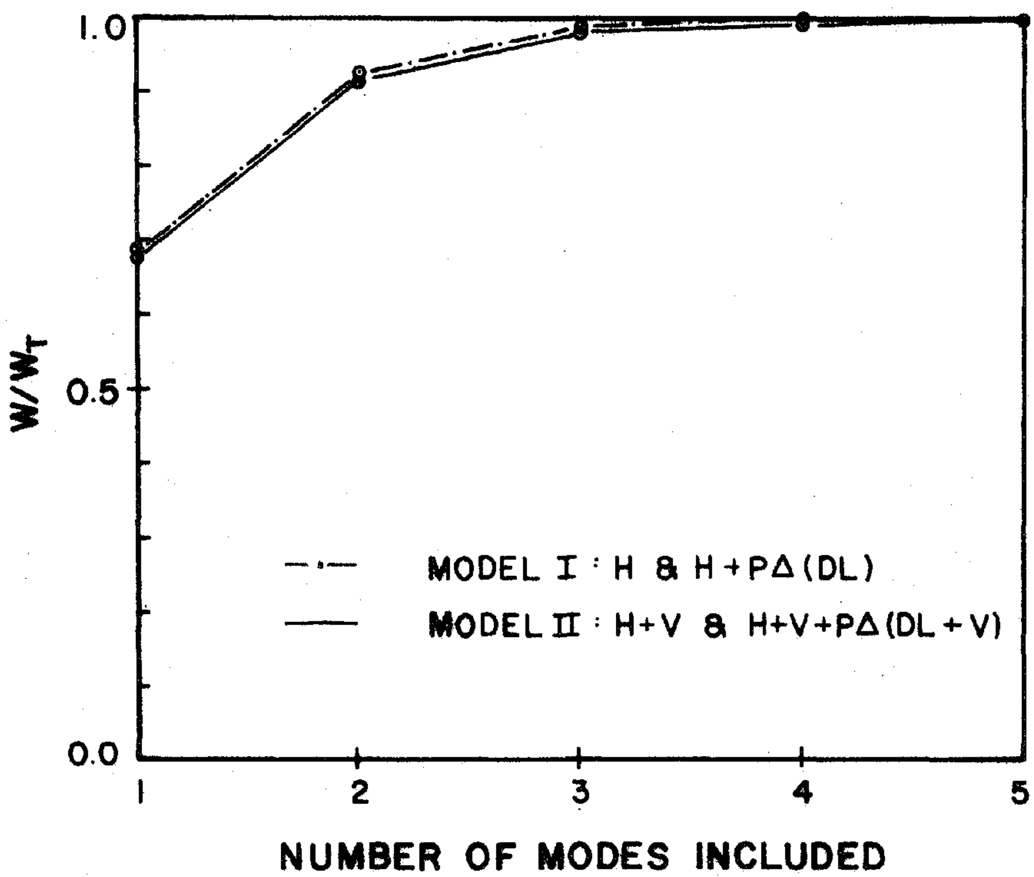


Figure 40. Contribution of Energy of Individual Modes to the Design of Example 8.

cross-sectional properties of the columns and girders of a structure are limited for constructional as well as architectual reasons; thus, the ratios of the maximum moments of inertia to the minimum moments of inertia are usually restrained. For the purpose of studying the effect of relative stiffness on optimum results, the 15-story building depicted in Fig. 18 has been redesigned for the following three cases: Case (i), the ratios of the maximum to the minimum moments of inertia of the columns and girders are equal to or less than 10; Case (ii), the ratios are equal to or less than 5; and Case (iii), the moments of inertia of the columns and girders for every two floor levels are assumed to be the same; however the ratios of the maximum moments of inertia to those of the minimum for the system cannot be more than 10. The design results of these three cases are compared with those of Case (iv), which is actually duplicated from Case (d) in Example 8.

Figures 41 and 42 illustrate the distribution of the moments of inertia of the columns and girders respectively for the four cases. The additional constraint of the ratios of the maximum to minimum moments of inertia somewhat changes the final design but not the distribution pattern from the first floor to the top. Other comparisons of the design results including the normalized moment of inertia, combined stresses, moments, shear envelopes, displacements, and design weights are shown from Figs. 43 through 51.

Table VIII lists the final weights, periods, and displacements at the top floor for the four cases. All the design cases have been terminated, because the allowable deflection at the top floor is

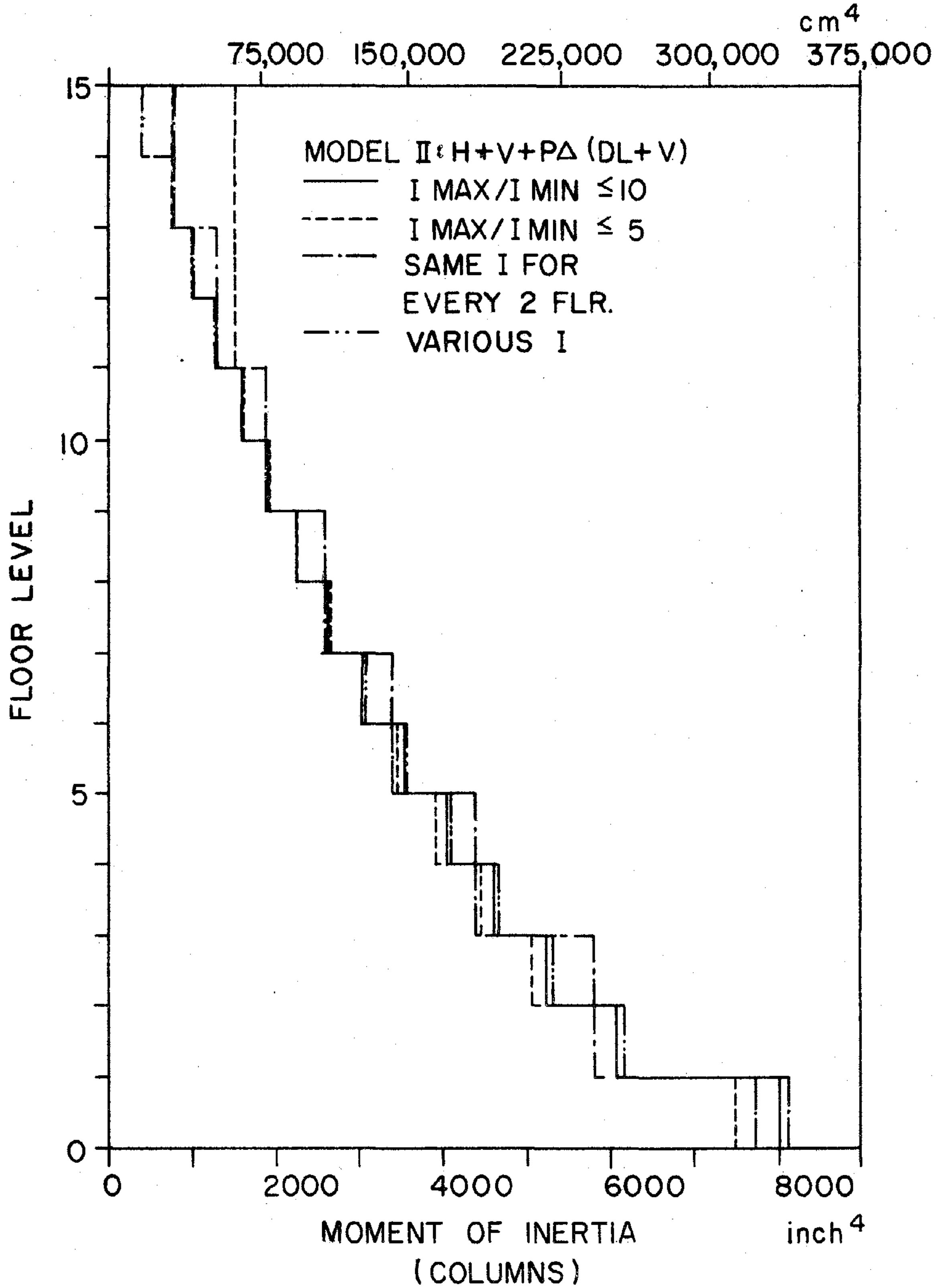


Figure 41. Distribution of Moment of Inertia of Columns of Example 9.

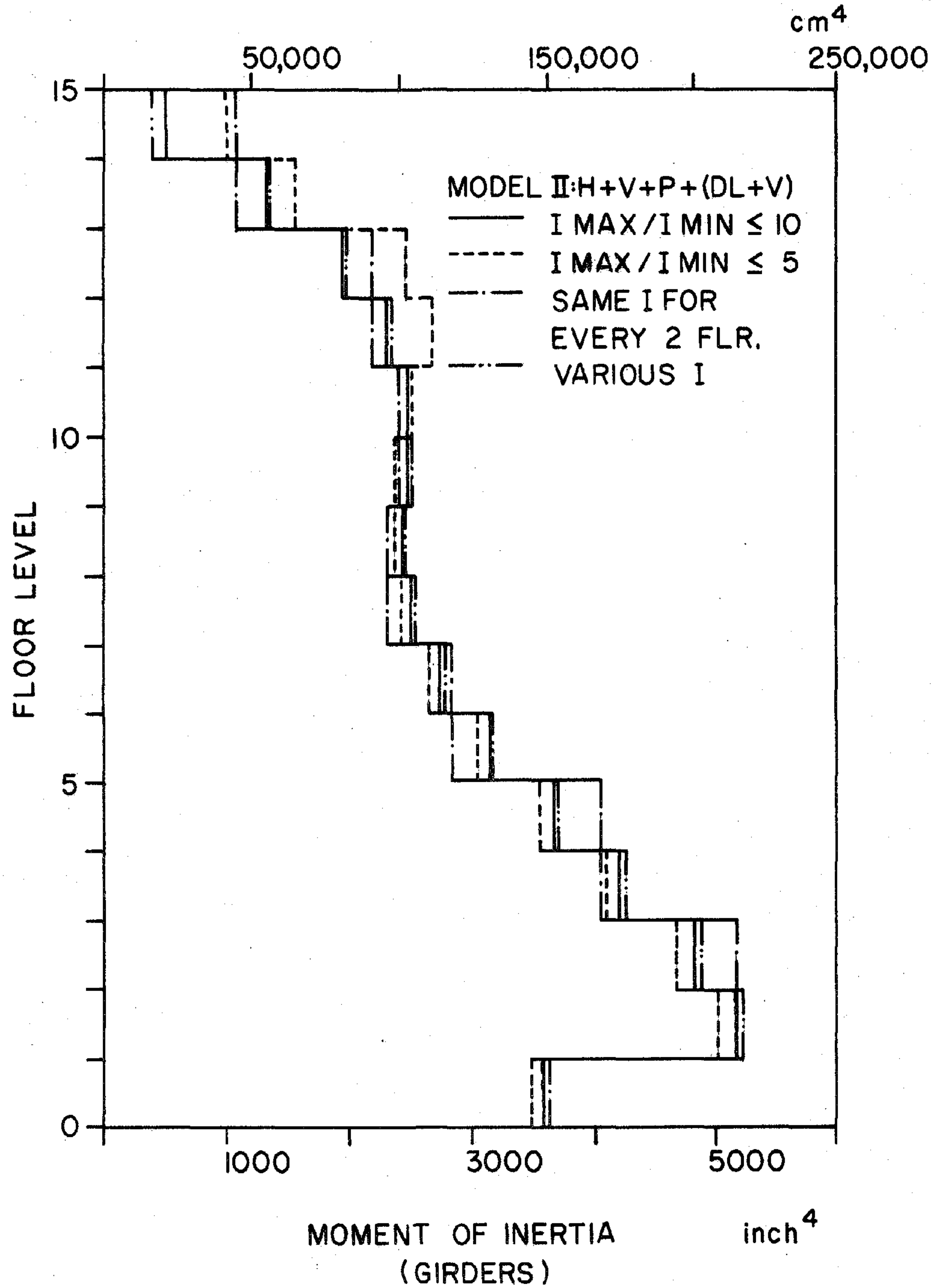


Figure 42. Distribution of Moment of Inertia of Girders of Example 9.

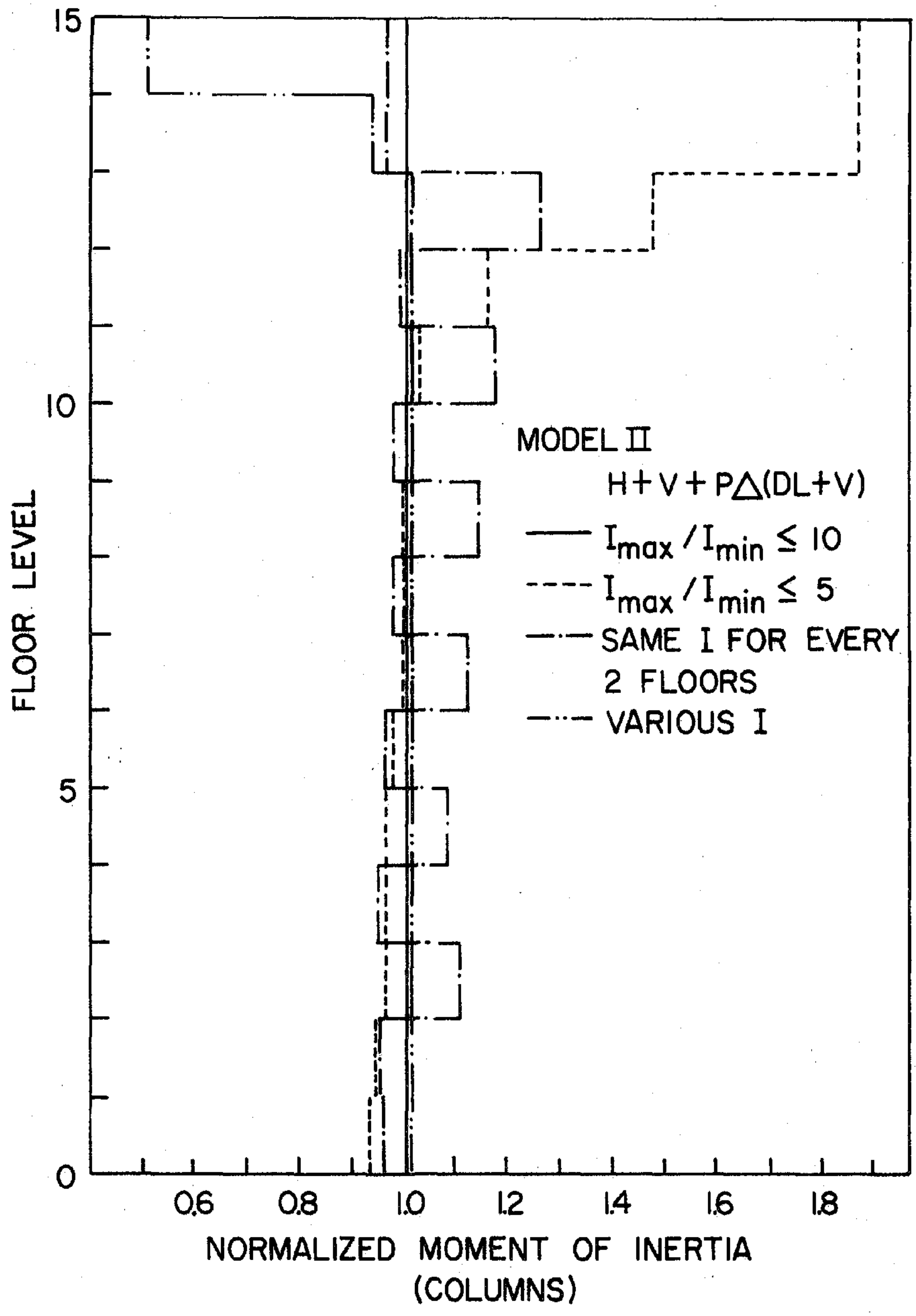


Figure 43. Normalized Moment of Inertia of Columns of Example 9.

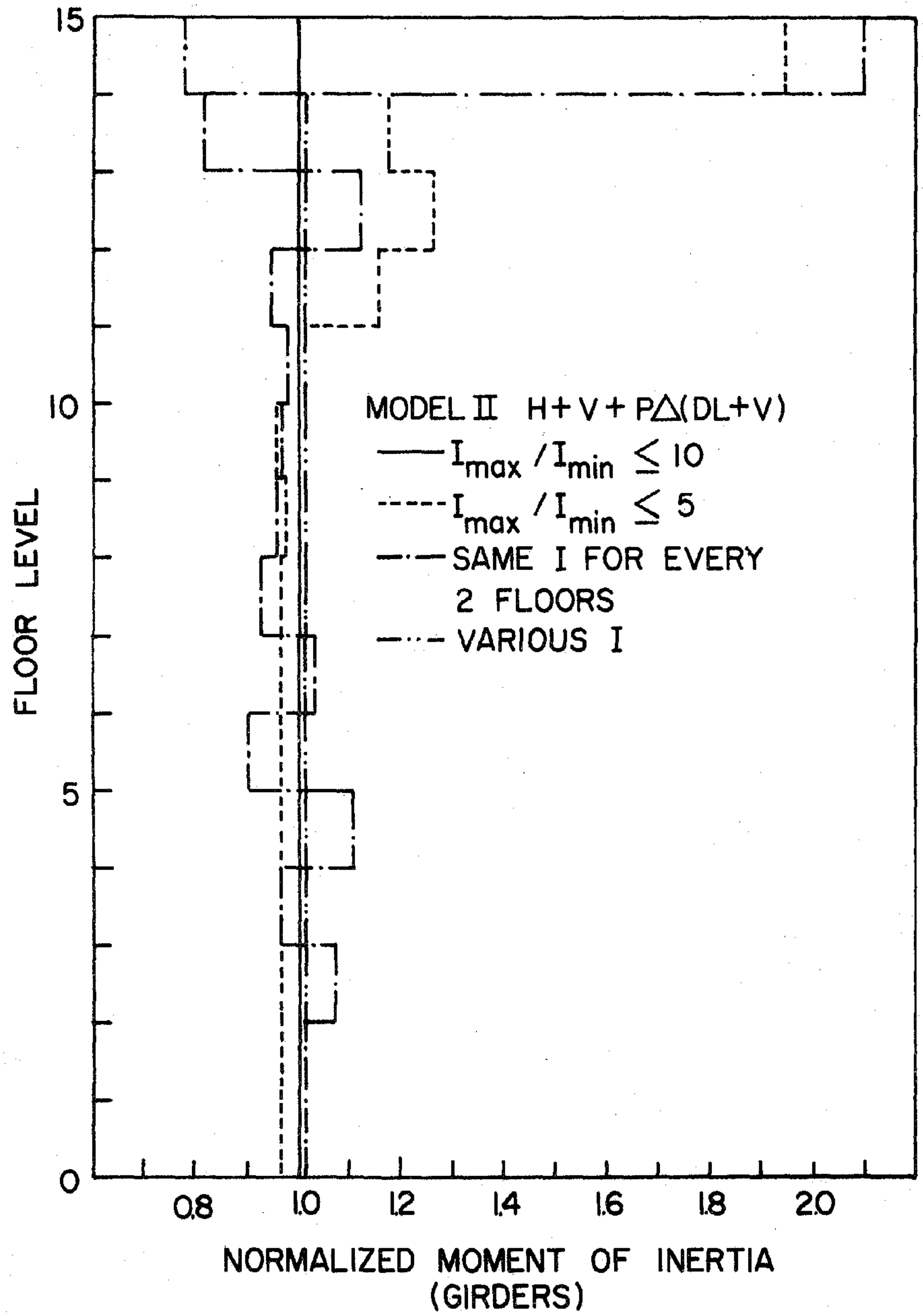


Figure 44. Normalized Moment of Inertia of Girders of Example 9.

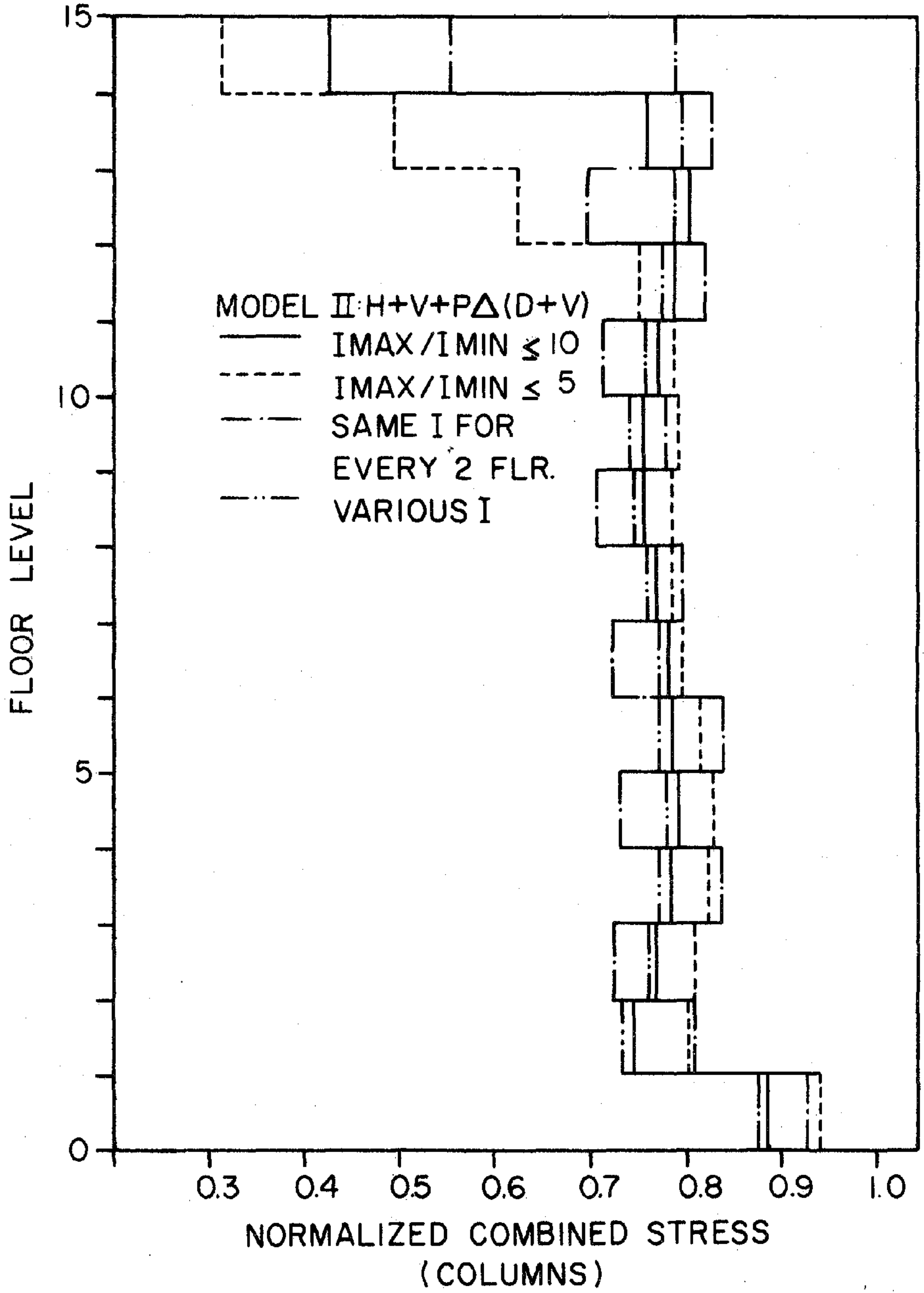


Figure 45. Ratio of Stress to Allowable Stress of Columns of Example 9.

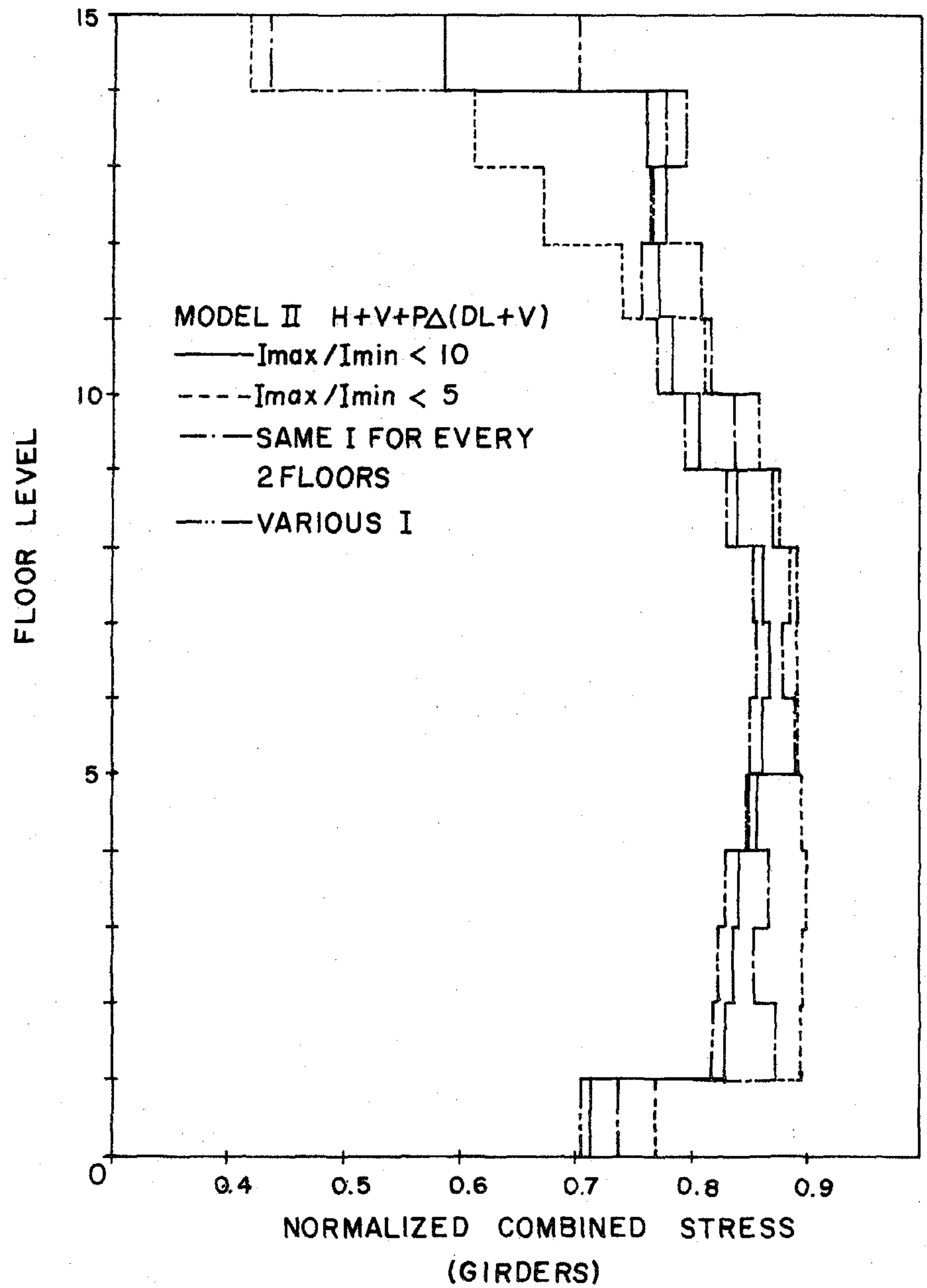


Figure 46. Ratio of Stress to Allowable Stress of Girders of Example 9.

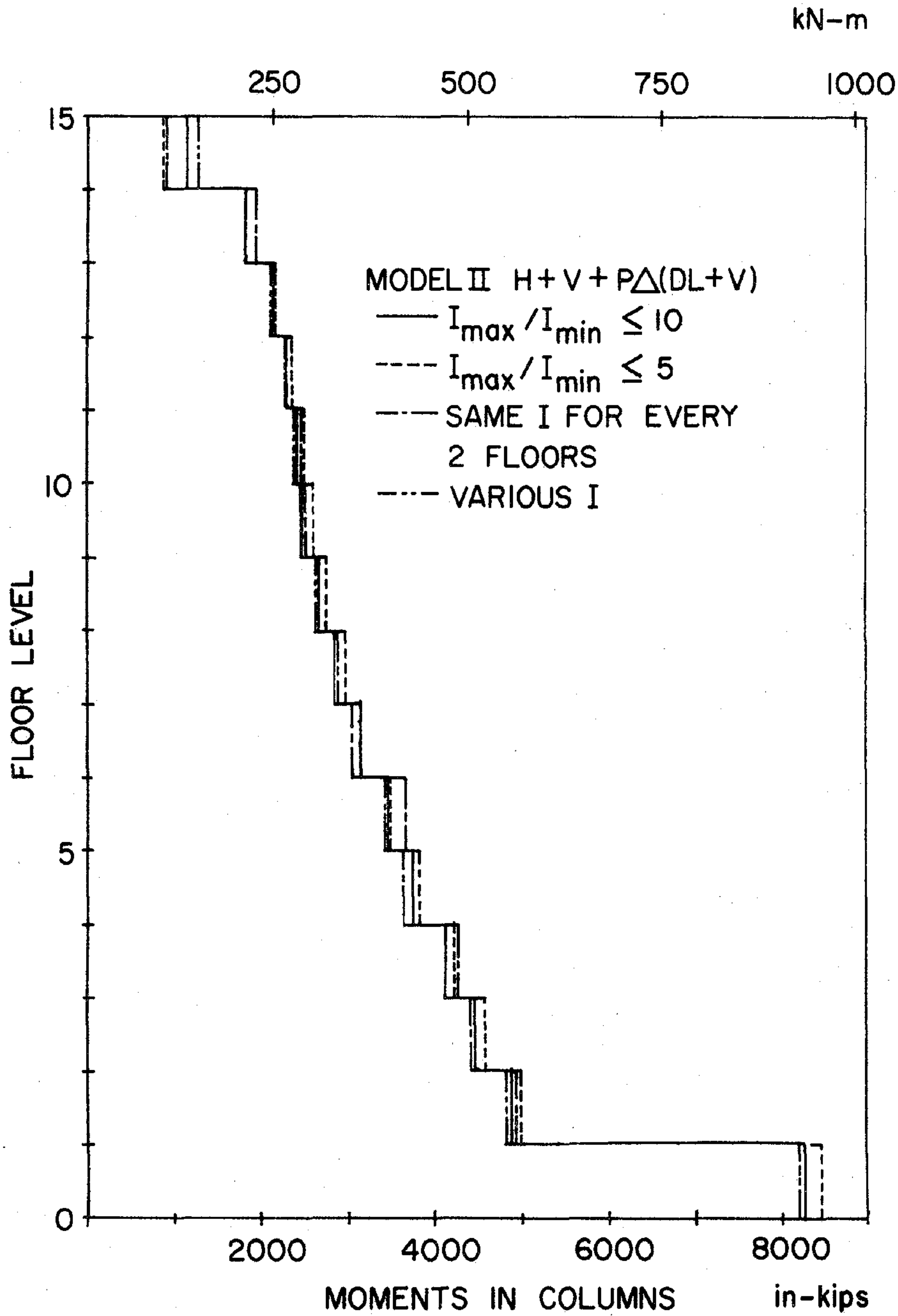


Figure 47. Moments in Columns of Example 9.

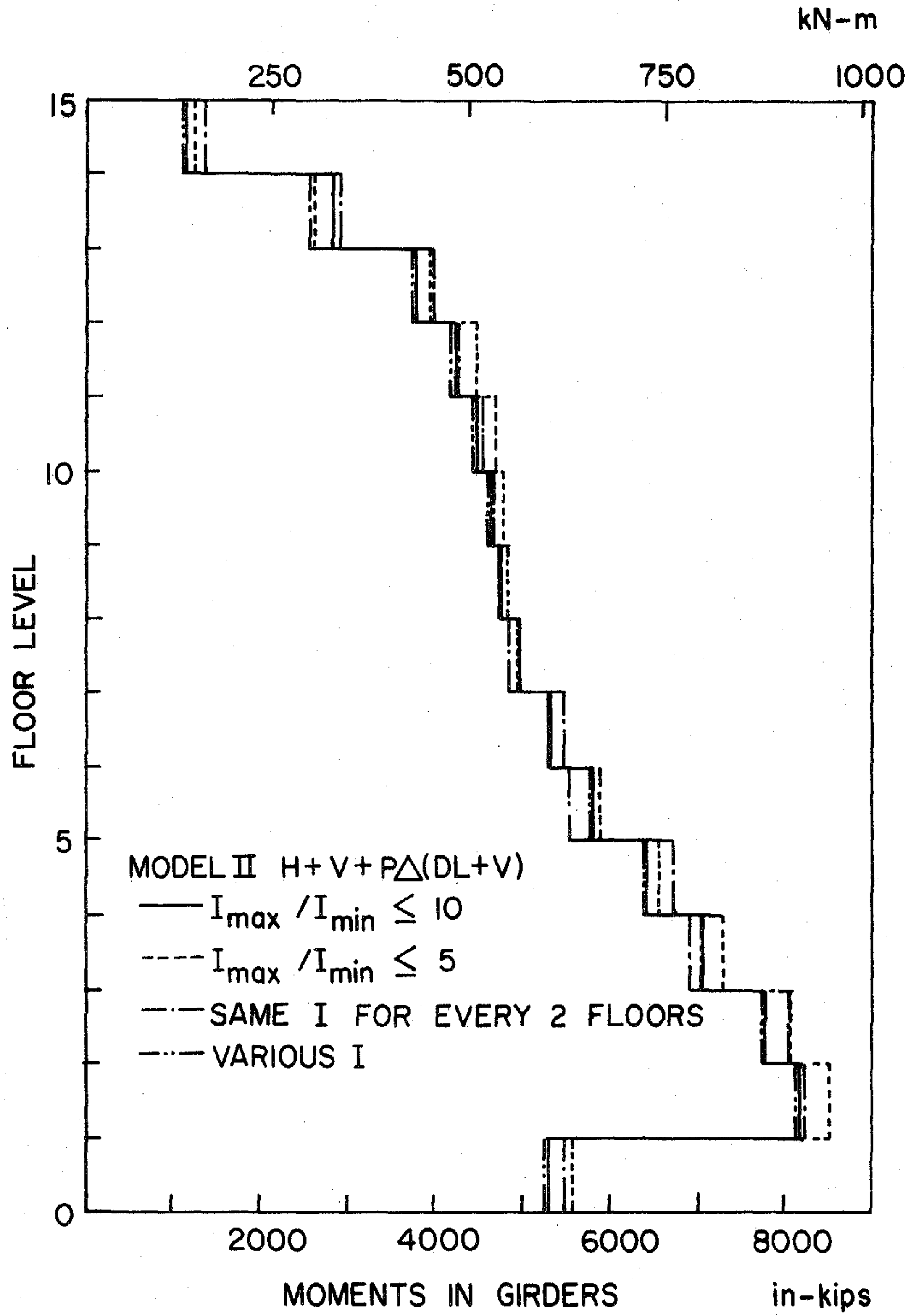


Figure 48. Moments in Girders of Example 9.

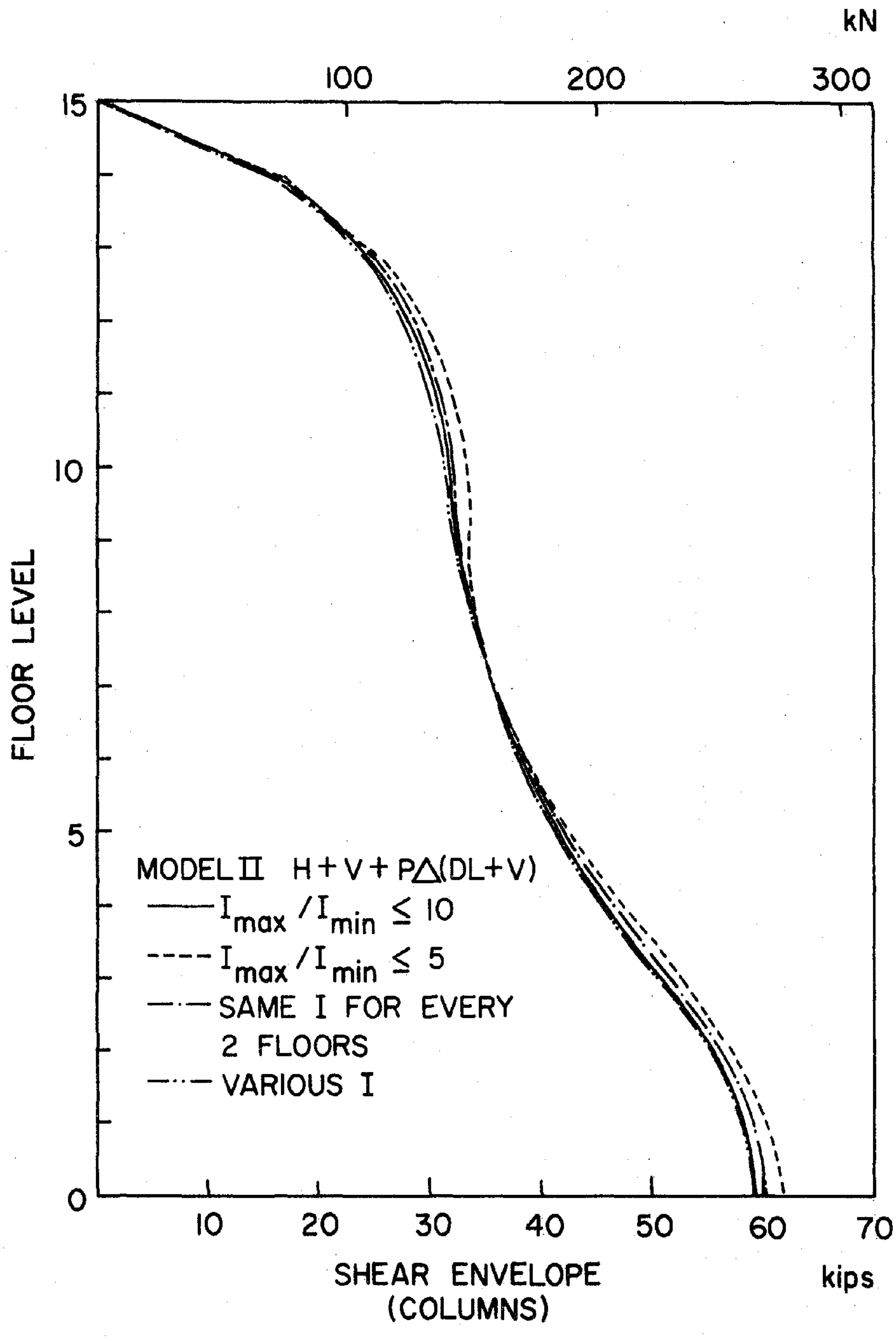


Figure 49. Shear Envelopes of Example 9.

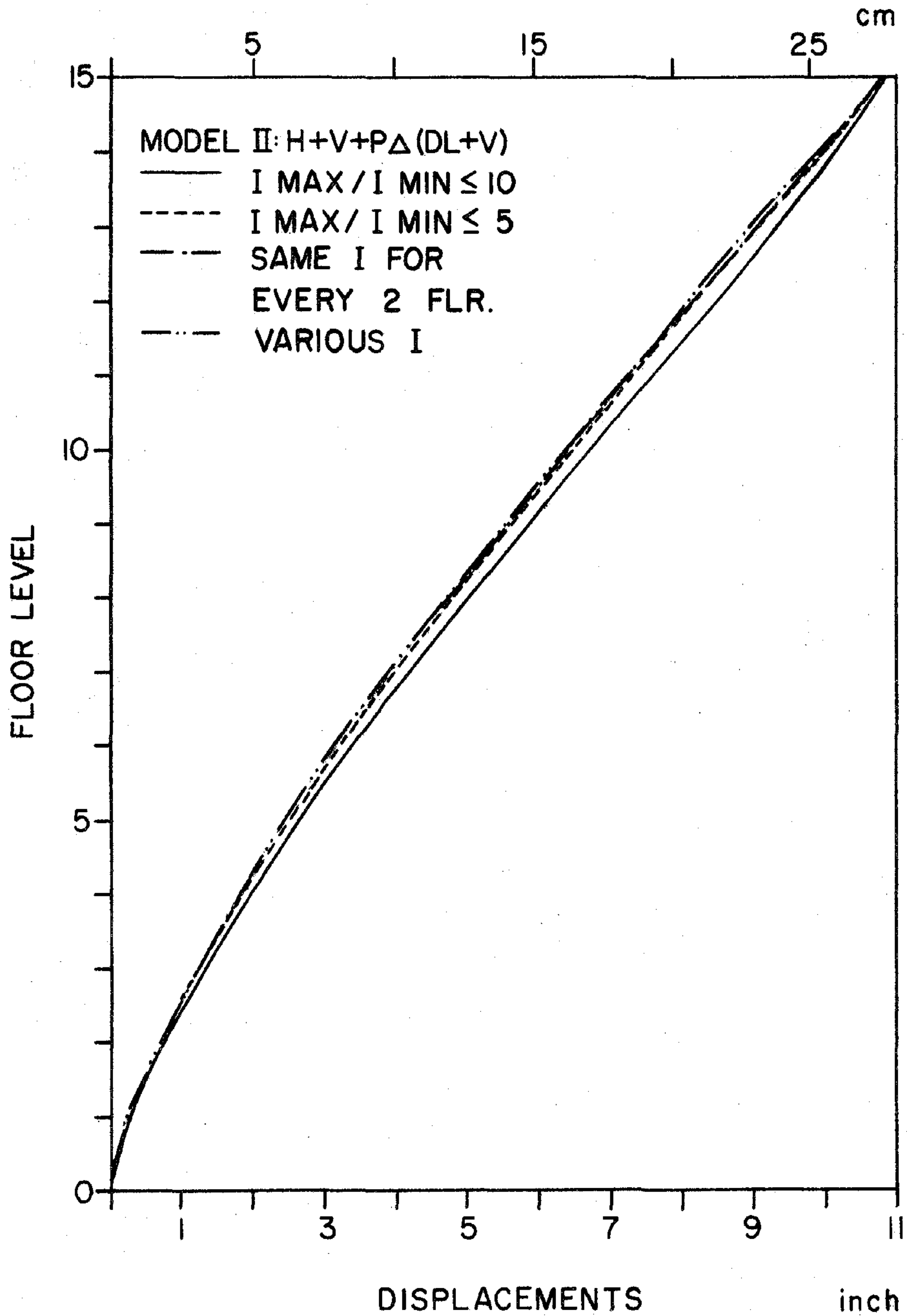


Figure 50. Displacements at Floor Levels of Example 9.

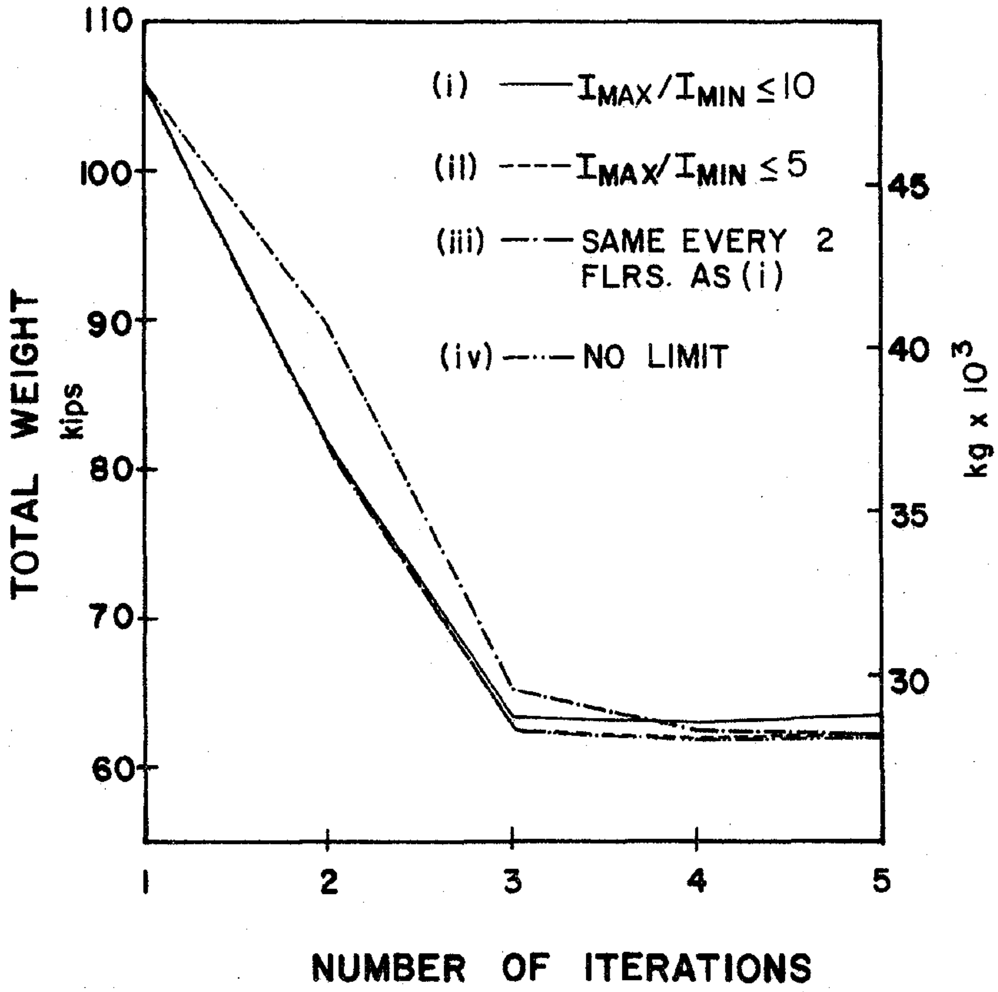


Figure 51. Comparison of Optimum Design Weight of Example 9.

TABLE VIII. FINAL WEIGHTS, NATURAL PERIODS, AND DISPLACEMENTS OF EXAMPLE 9 ($i = I_{max}/I_{min} = 10$, $ii = I_{max}/I_{min} = 5$, iii = SAME MOMENT OF INERTIA OF COLUMNS AND THAT OF GIRDERS FOR EVERY TWO FLOORS AND $I_{max}/I_{min} = 10$, iv = NO LIMIT (SAME AS CASE (d) OF EXAMPLE 8)) (1 kip = 453 kg, 1 in. = 2.54 cm)

Case	Final Weight (kips)	Natural Period (sec.)					Disp. at top floor (in.)
		1	2	3	4	5	
i	63.42	2.186	0.658	0.367	0.253	0.204	10.80
ii	62.35	2.174	0.677	0.388	0.267	0.207	10.80
iii	62.58	2.166	0.676	0.386	0.266	0.206	10.80
iv	62.21	2.152	0.677	0.396	0.276	0.212	10.80

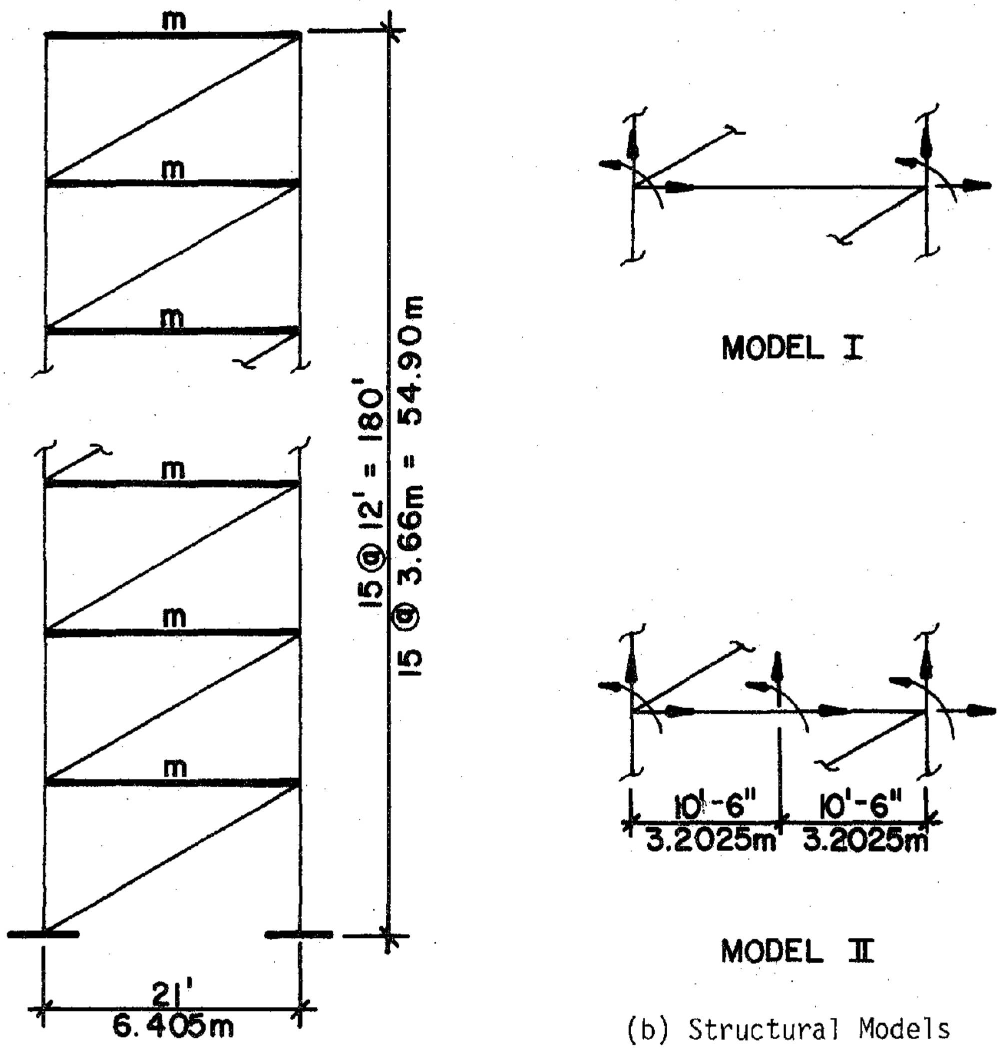
violated. As expected, the final weight of Case (iv) without the restraint on the relative stiffness has the least weight.

B. 15-STORY BRACED FRAMES

Three types of braced frames, namely the single-braced, double-braced, and K-braced, are designed so that the effect of the interaction of ground motions on the optimum solutions can be studied. All of the design information is the same as that given at the beginning of this chapter with the stress and displacement constraints except that the minimum depth for the columns and girders, d_{min}, is 10 in. (25.40 cm). The maximum and minimum areas for the bracings are assumed to be 23 in.^{2} (148.39 cm^2) and 1 in.^{2} (6.45 cm^2) respectively. All of the design examples of the braced systems include the static stresses resulting from the weight of the nonstructural masses on the girders.

1. 15-Story Single-Braced Frame--Example 10. Figure 52 shows the configuration of the 15-story, single-braced frame for which the design variables of the columns on each of the floor levels are assumed to be the same. Figure 53 illustrates the total weights versus the number of iterations for the four cases of (a) through (d). Other design results are given in Table VII of Group C. Examination of the figure reveals that the multicomponent ground motions when combined with the P-Δ effect can increase the structural weight by 40% over the structural weight for the horizontal component only.

The ratios of the strain and kinematic energies of the individual modes to those of all the modes in the design are plotted in Fig. 54. The plot indicates that the first three modes and the first four modes are adequately accurate for the design of Models I and II of the single-braced systems respectively.



(b) Structural Models

(a) 15-Story Single-Braced Frame

Figure 52. 15-Story, Single-Bay, Single-Braced Frame of Example 10.

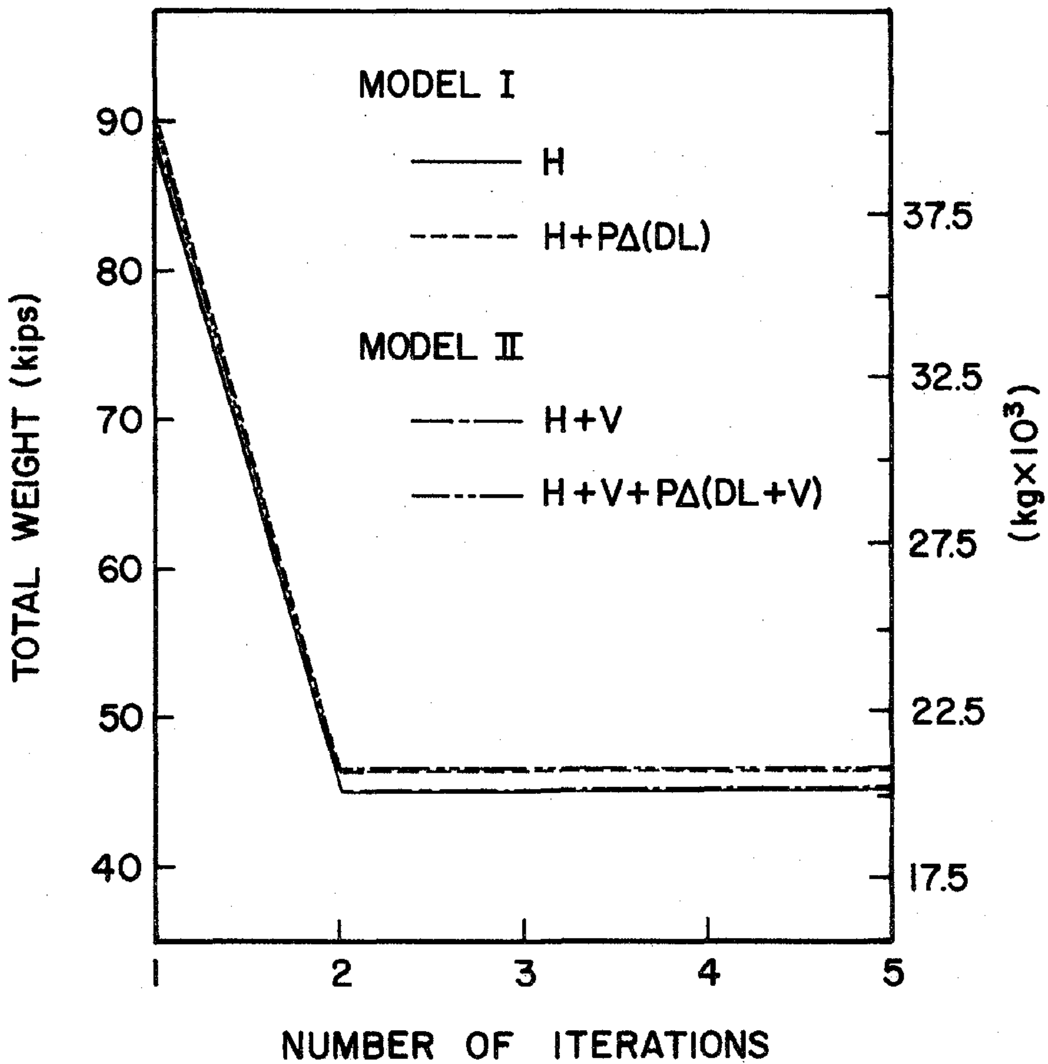


Figure 53. Optimum Weights of Single-Braced Frame of Example 10.

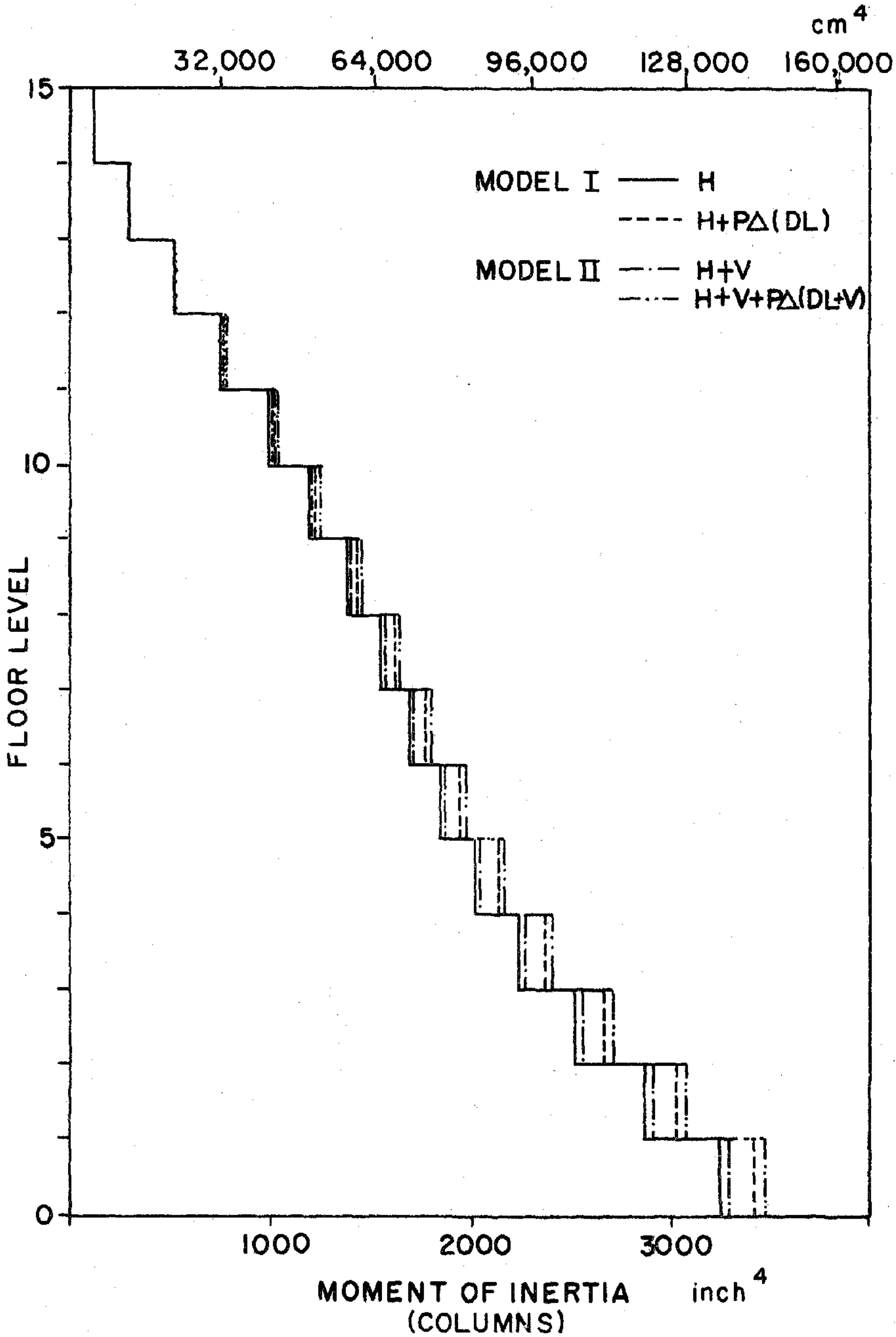


Figure 55. Distribution of Moment of Inertia of Columns of Example 10.

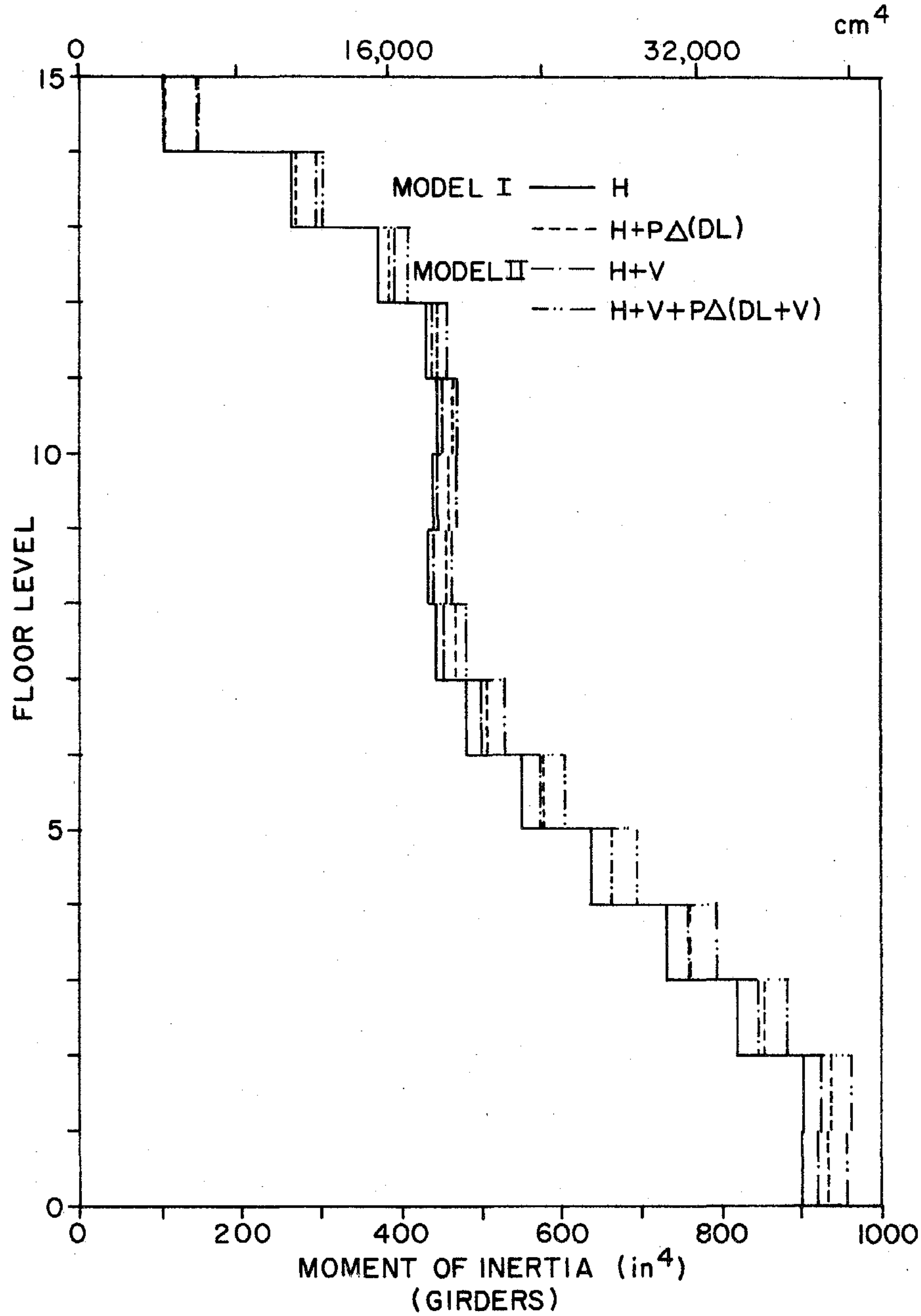


Figure 56. Distribution of Moment of Inertia of Girders of Example 10.

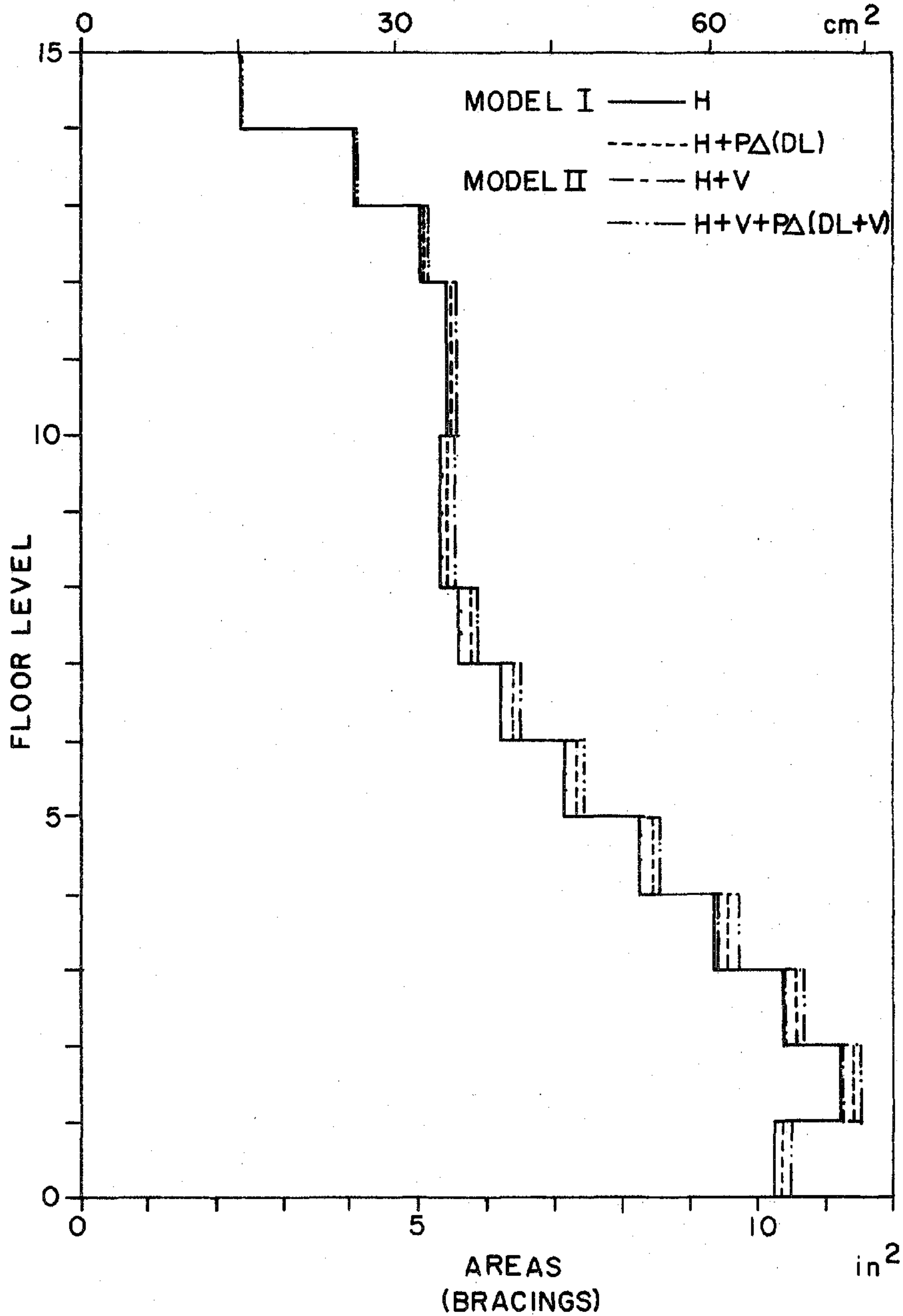


Figure 57. Distribution of Areas of Bracings of Example 10.

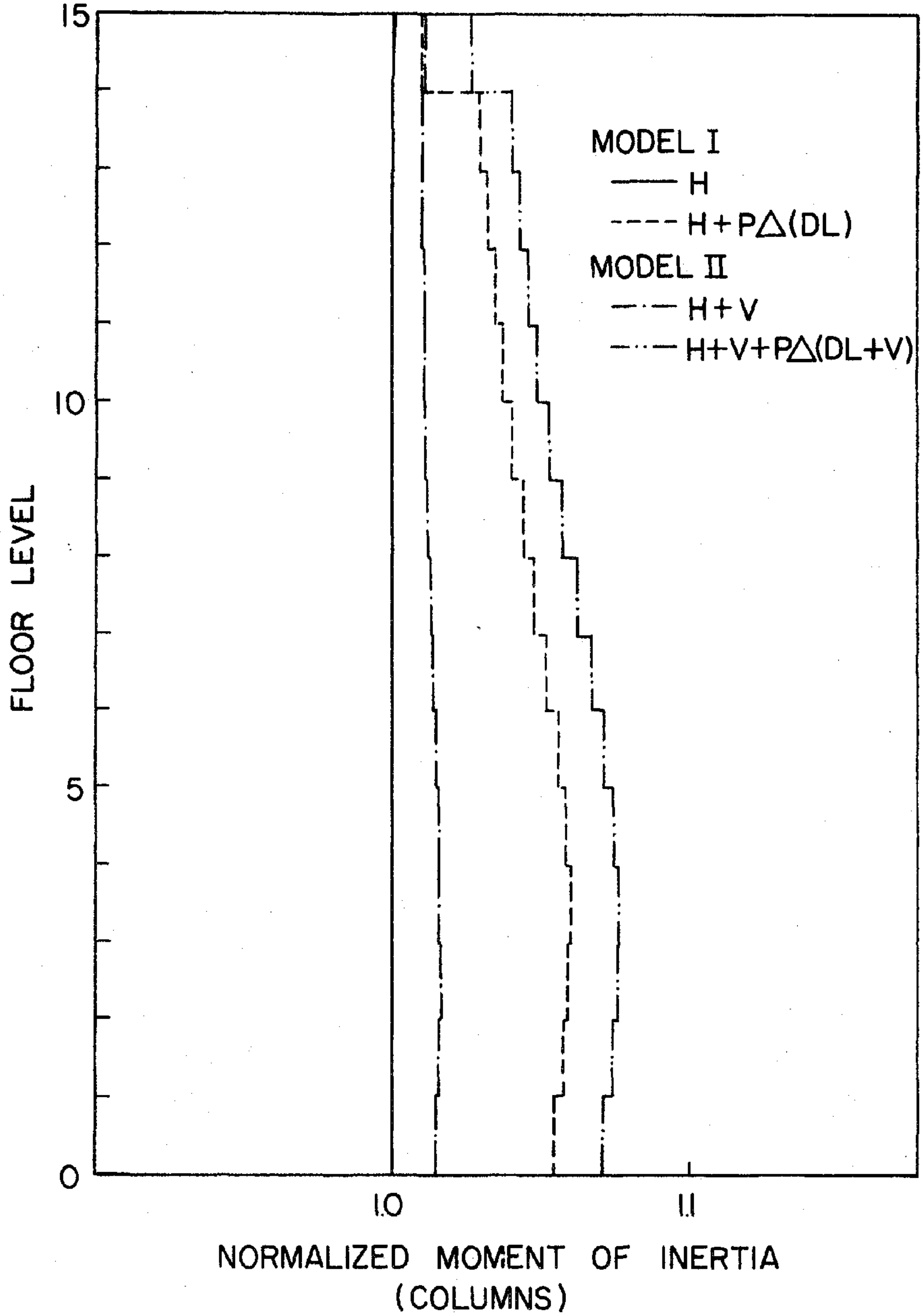


Figure 58. Normalized Moment of Inertia of Columns of Example 10.

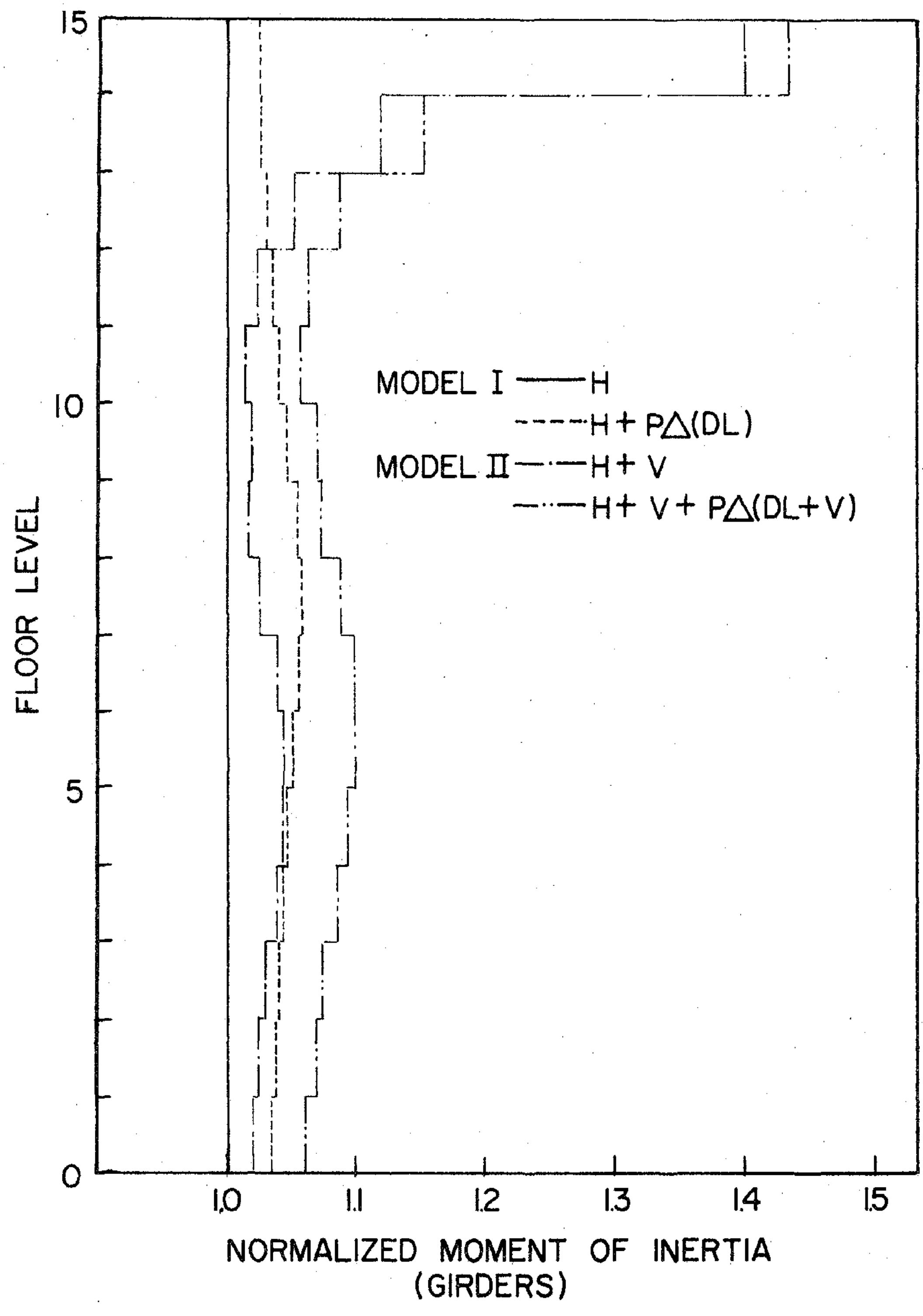


Figure 59. Normalized Moment of Inertia of Girders of Example 10.

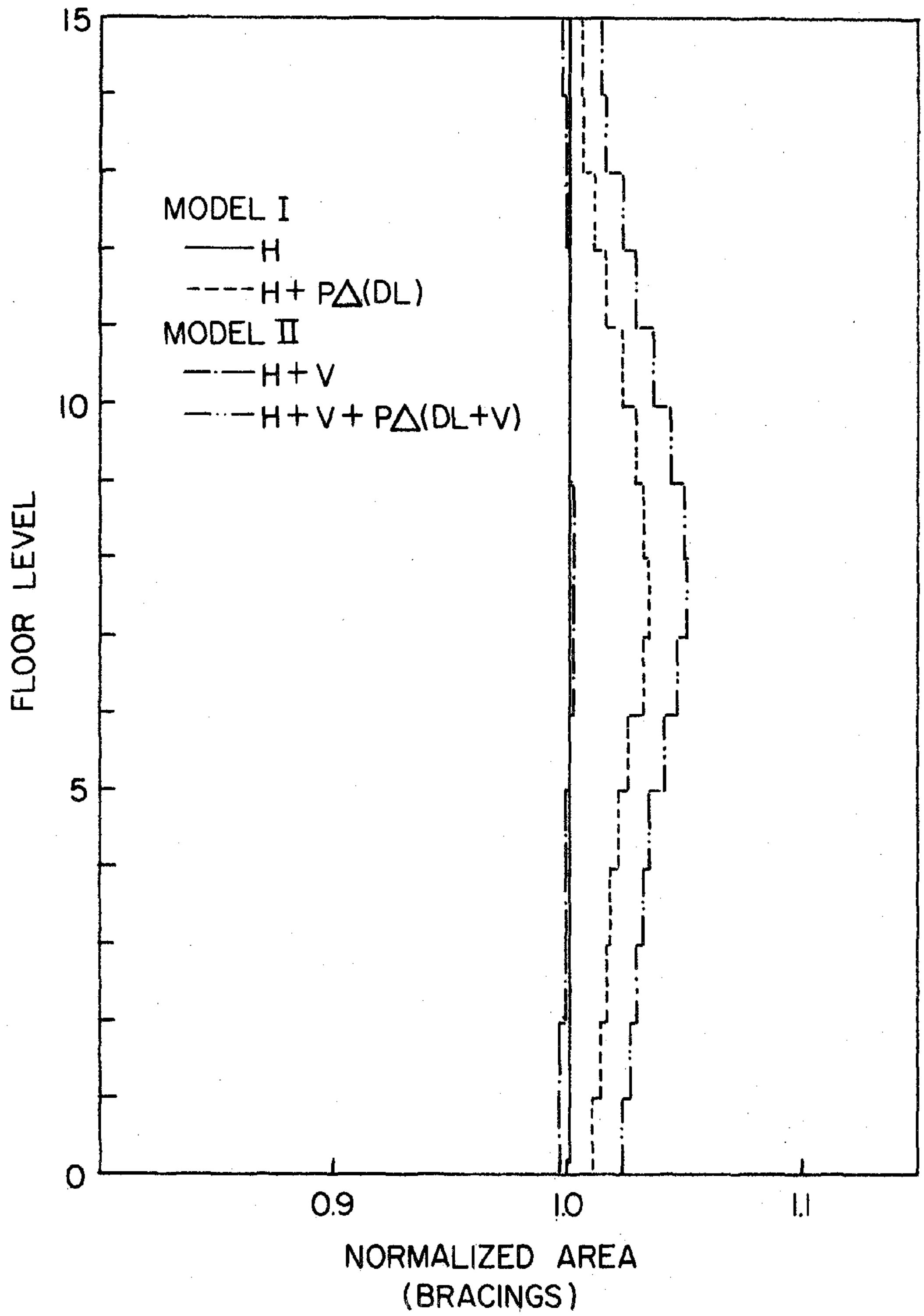


Figure 60. Normalized Cross-sectional Areas of Bracings of Example 10.

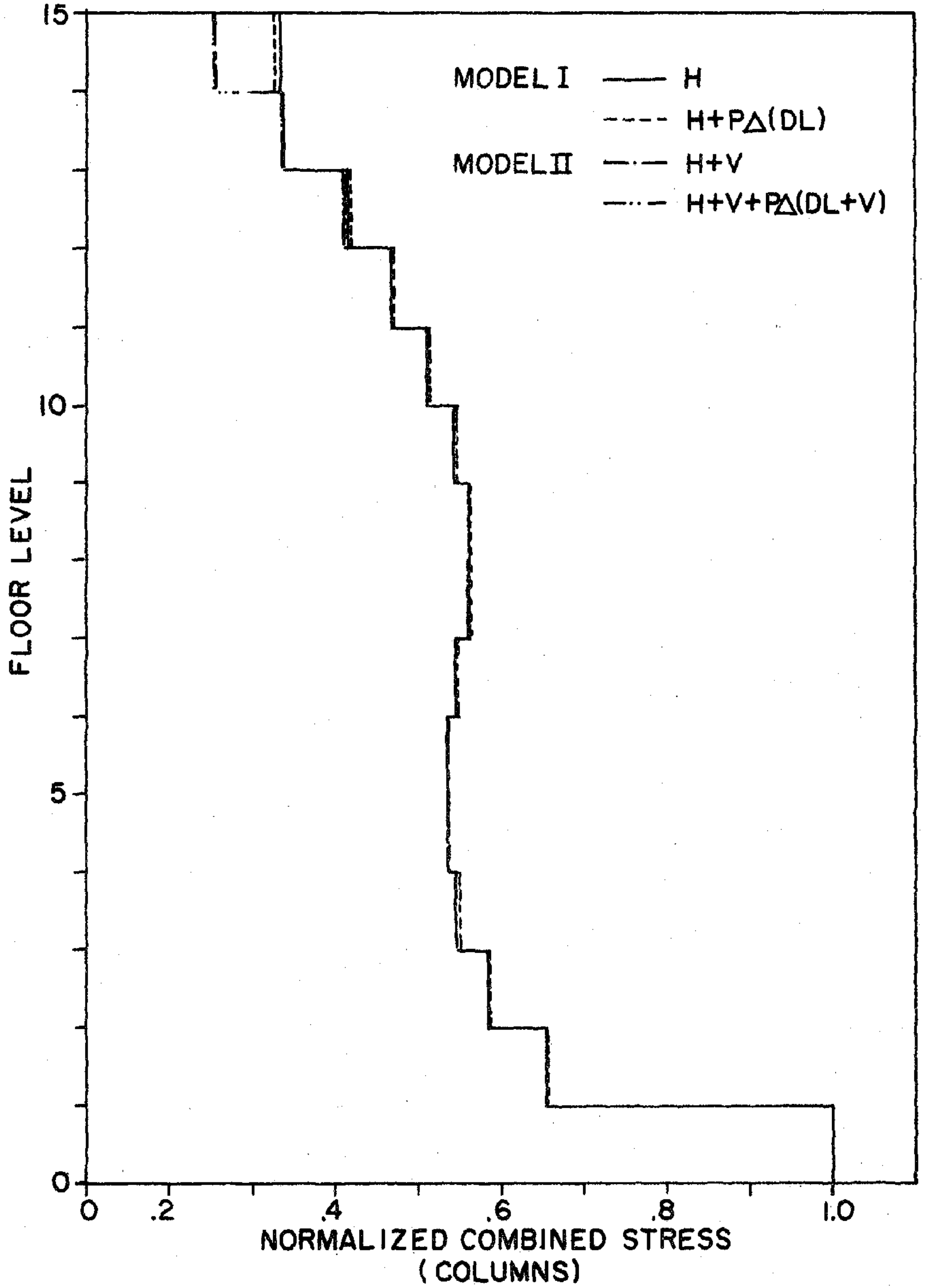


Figure 61. Ratio of Stress to Allowable Stress of Columns of Example 10.

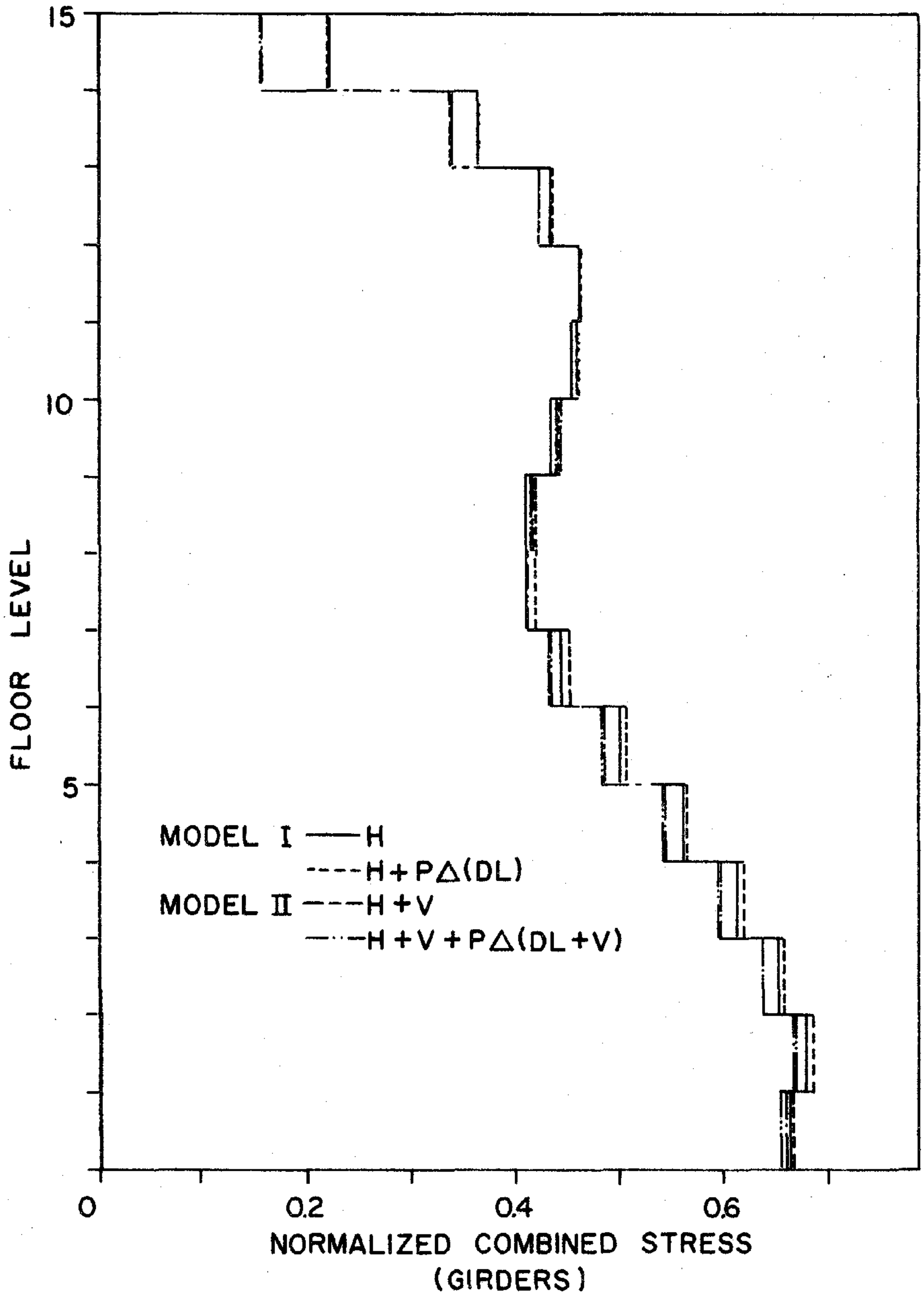


Figure 62. Ratio of Stress to Allowable Stress of Girders of Example 10.

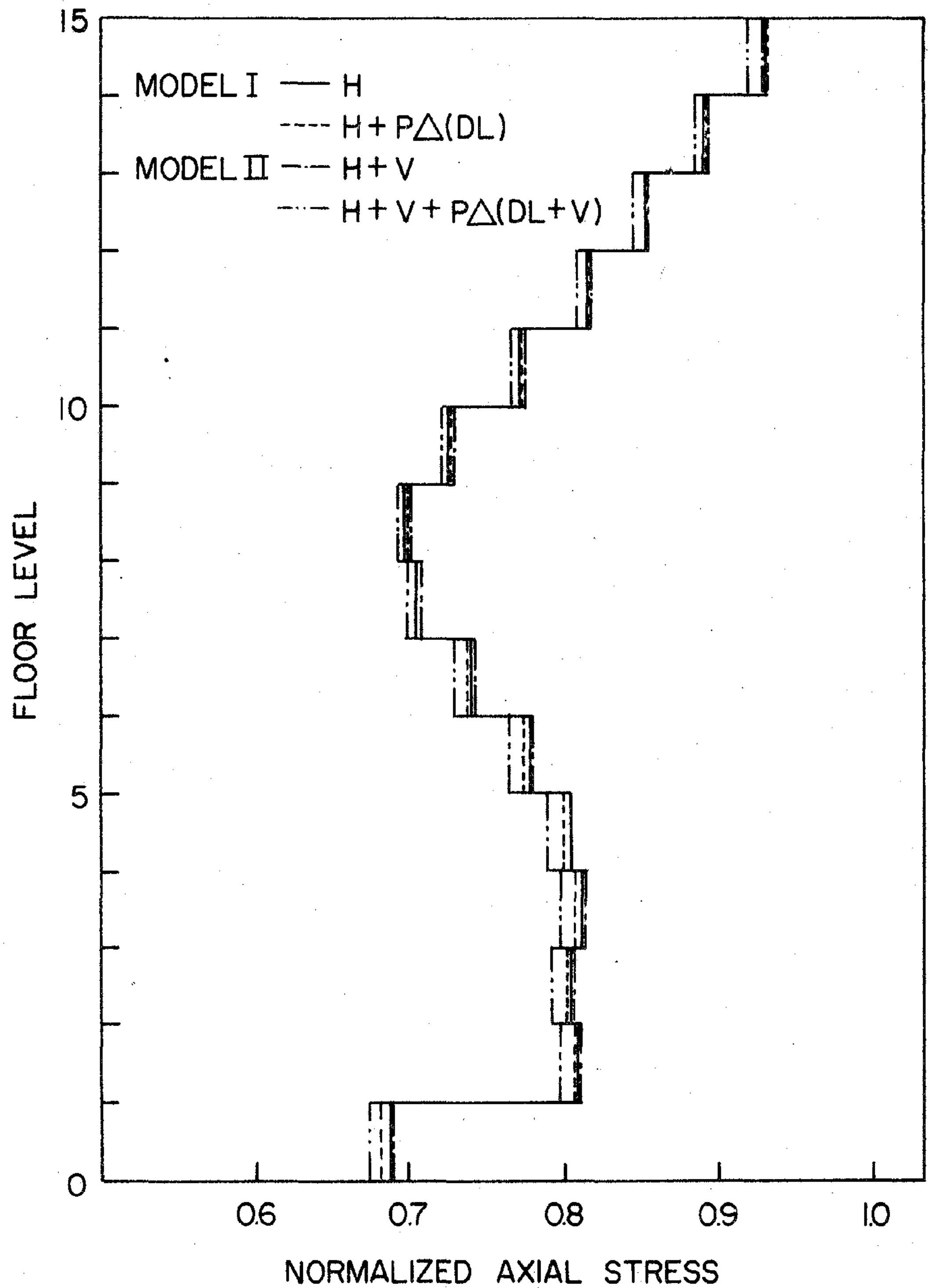


Figure 63. Ratio of Stress to Allowable Stress of Bracings of Example 10.

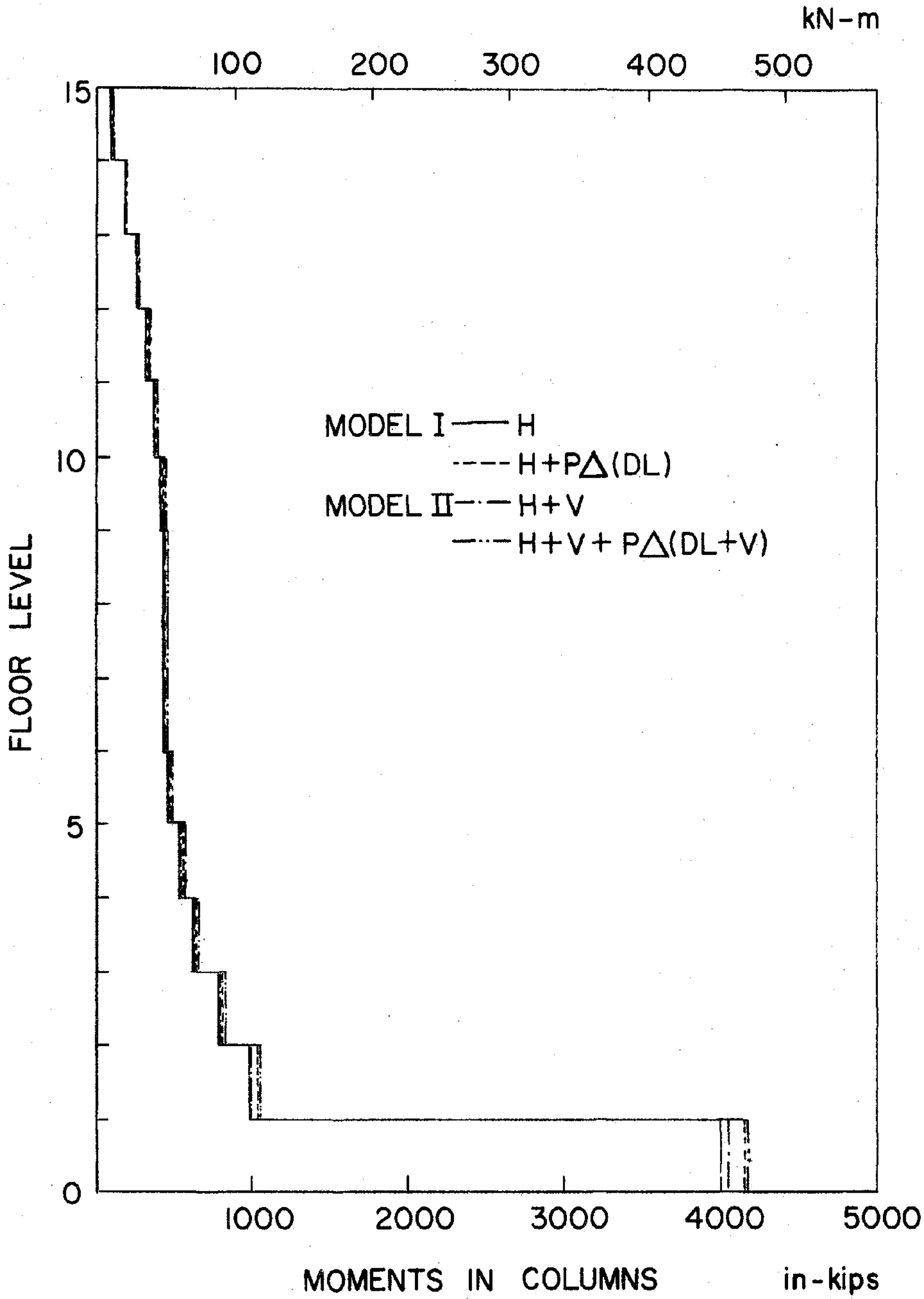


Figure 64. Moments in Columns of Example 10.

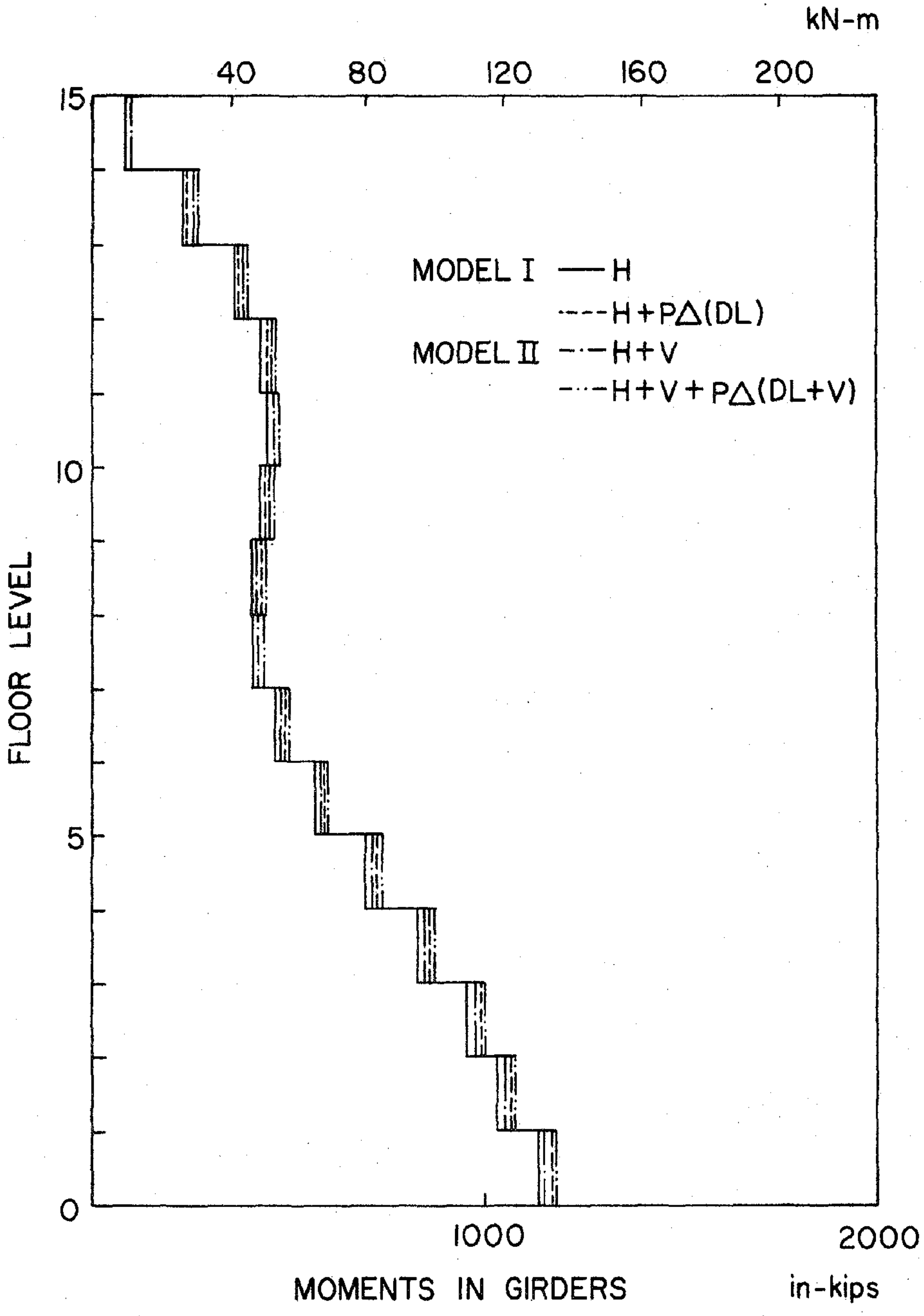


Figure 65. Moments in Girders of Example 10.

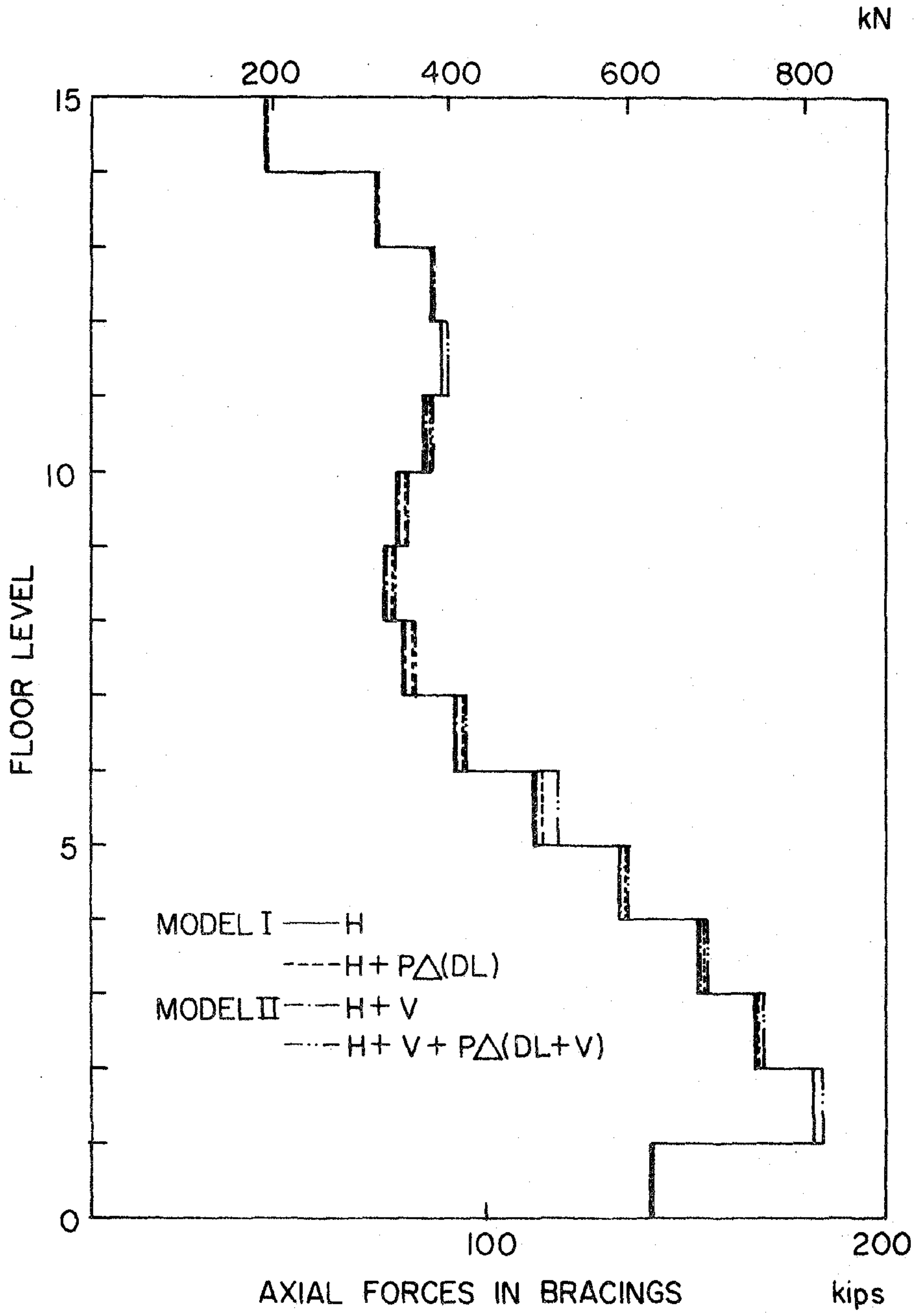


Figure 66. Axial Forces in Bracings of Example 10.

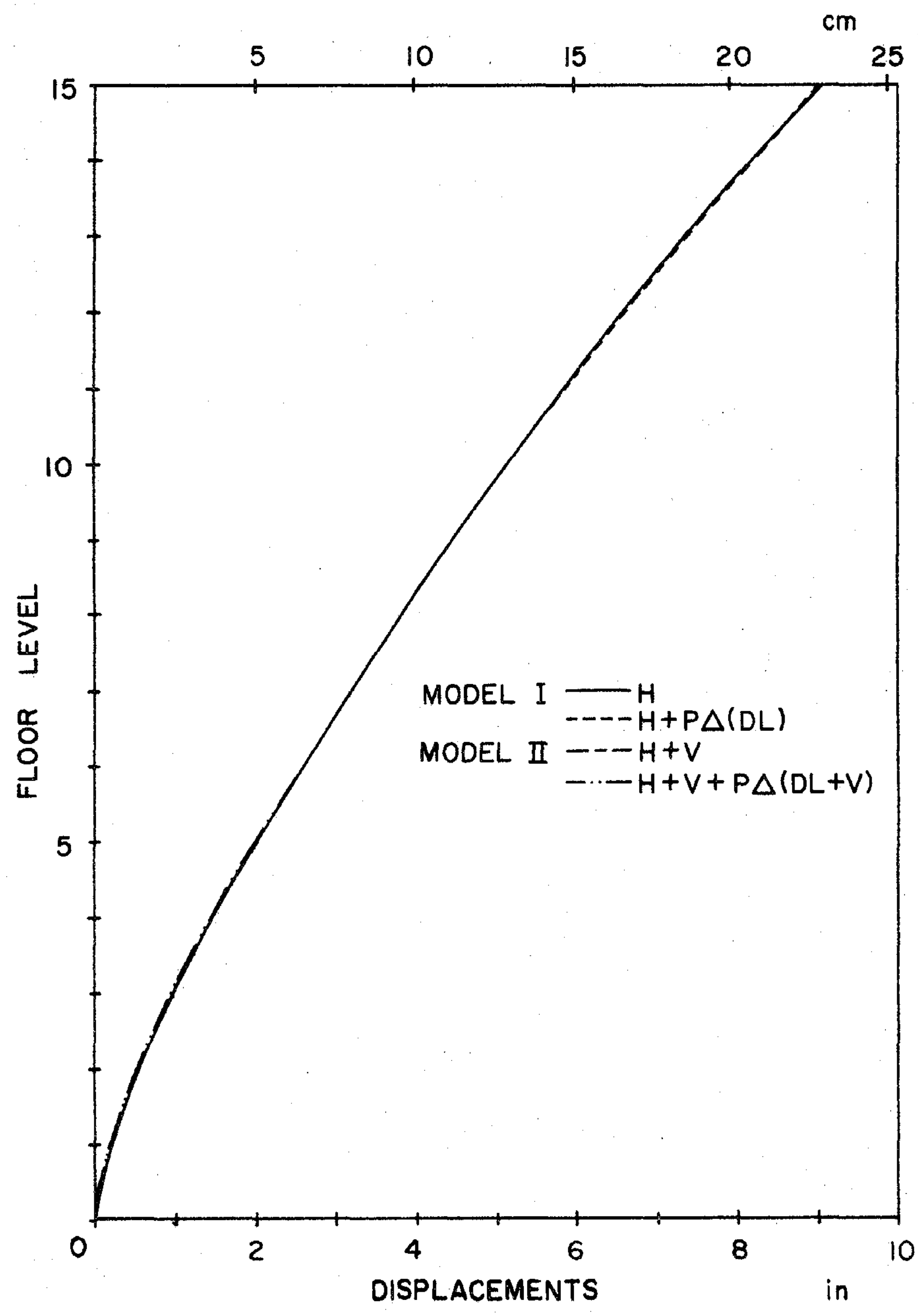
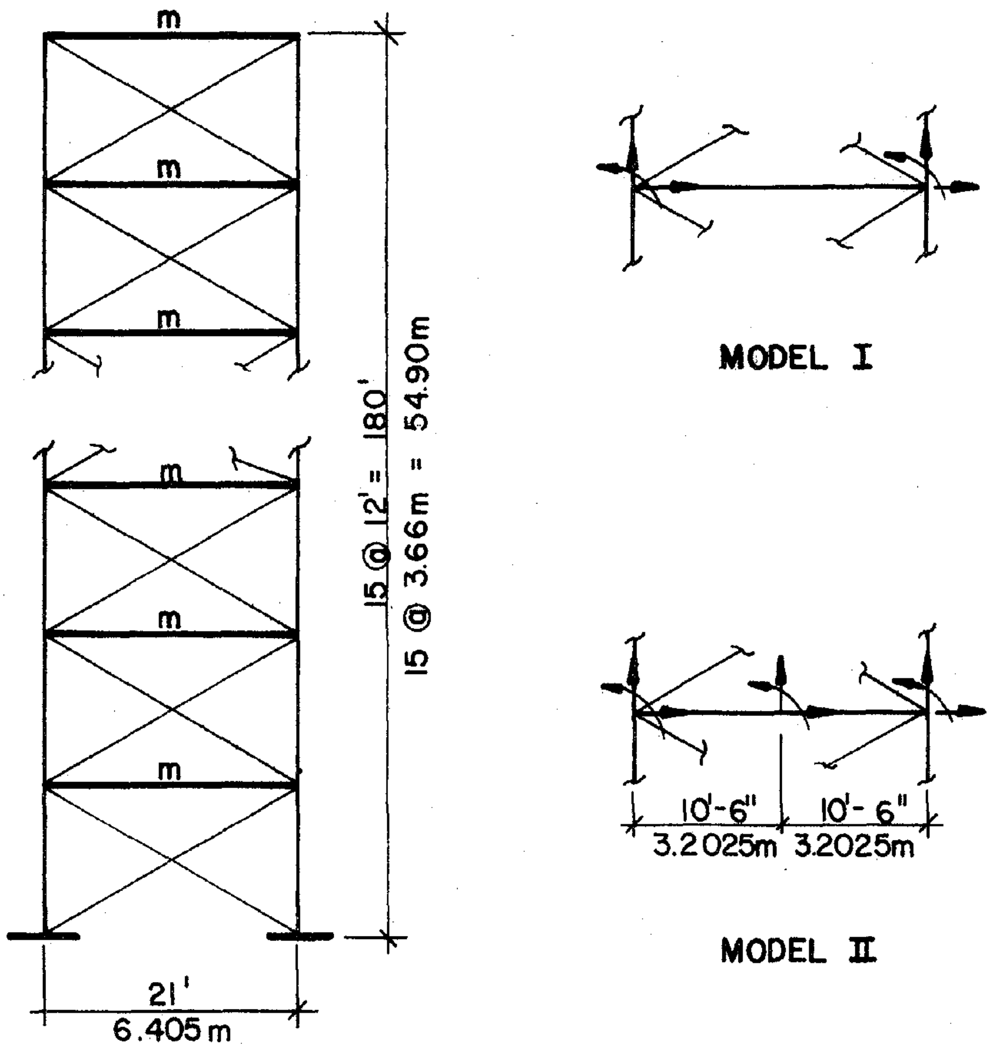


Figure 67. Displacements at Floor Levels of Example 10.



(a) 15-Story Double-Braced Frame

(b) Structural Models

Figure 68. 15-Story, Single-Bay, Double-Braced Frame of Example 11.

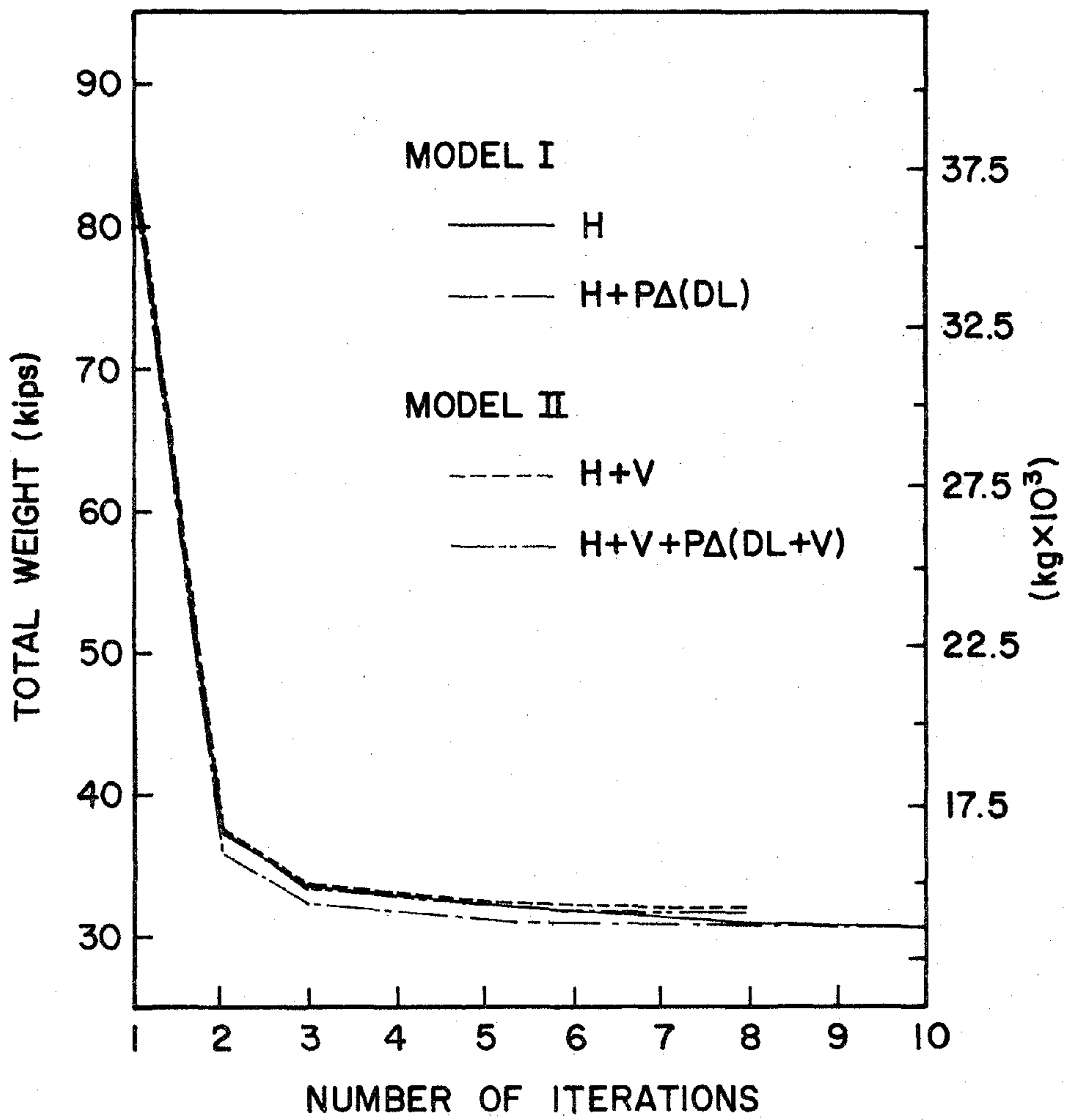


Figure 69. Optimum Weights of Double-Braced Frame of Example 11.

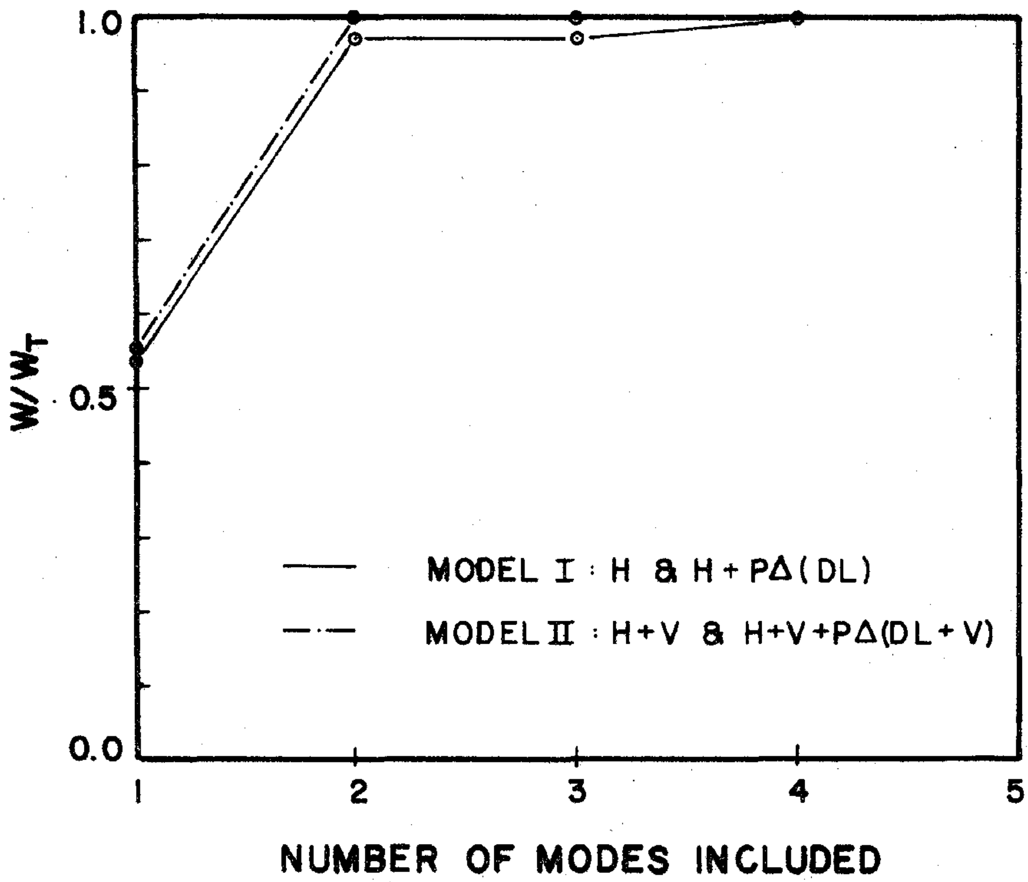


Figure 70. Contribution of Energy of Individual Modes to the Design of Example 11.

necessary for an accurate design based on Model I, but the design based on Model II requires at least the first four modes.

Figures 71 through 73 respectively indicate the distribution of the moments of inertia of the columns and girders as well as the bracing areas. These figures show that the frames with double bracings demand smaller moments of inertia for the columns and girders than single-braced frames. Note that the moments of inertia for all the girders and those of columns between the 13th and 15th floor are a minimal 289 in.^{4} (12029 cm^4) and that the distribution of the bracing areas follows a pattern similar to that of the single-braced frame; however, the areas required for the double-bracings are about half those for single-bracings. The normalized moments of inertia of the columns and girders and the normalized areas of the bracing members are shown in Figs. 74 through 76 respectively.

The ratios of the actual combined stresses to the allowable stress of the columns and girders are practically similar for all the floors except the columns at the top three floors as illustrated in Figs. 77 and 78. Figure 79 includes the ratio of axial stresses to the allowable stress of the bracings for all four cases. The maximum moments of the columns and girders and the axial forces of the bracings are shown in Figs. 80, 81, and 82 respectively. The displacements are shown in Fig. 83.

3. <u>15-Story K-Braced Frame--Example 12</u>. For the 15-story K-braced frame shown in Fig. 84(a), only Model II of Fig. 84(b) can be employed for the design because the bracing members intersect at the mid-span of the girders. The configuration of K-bracing can increase the longitudinal

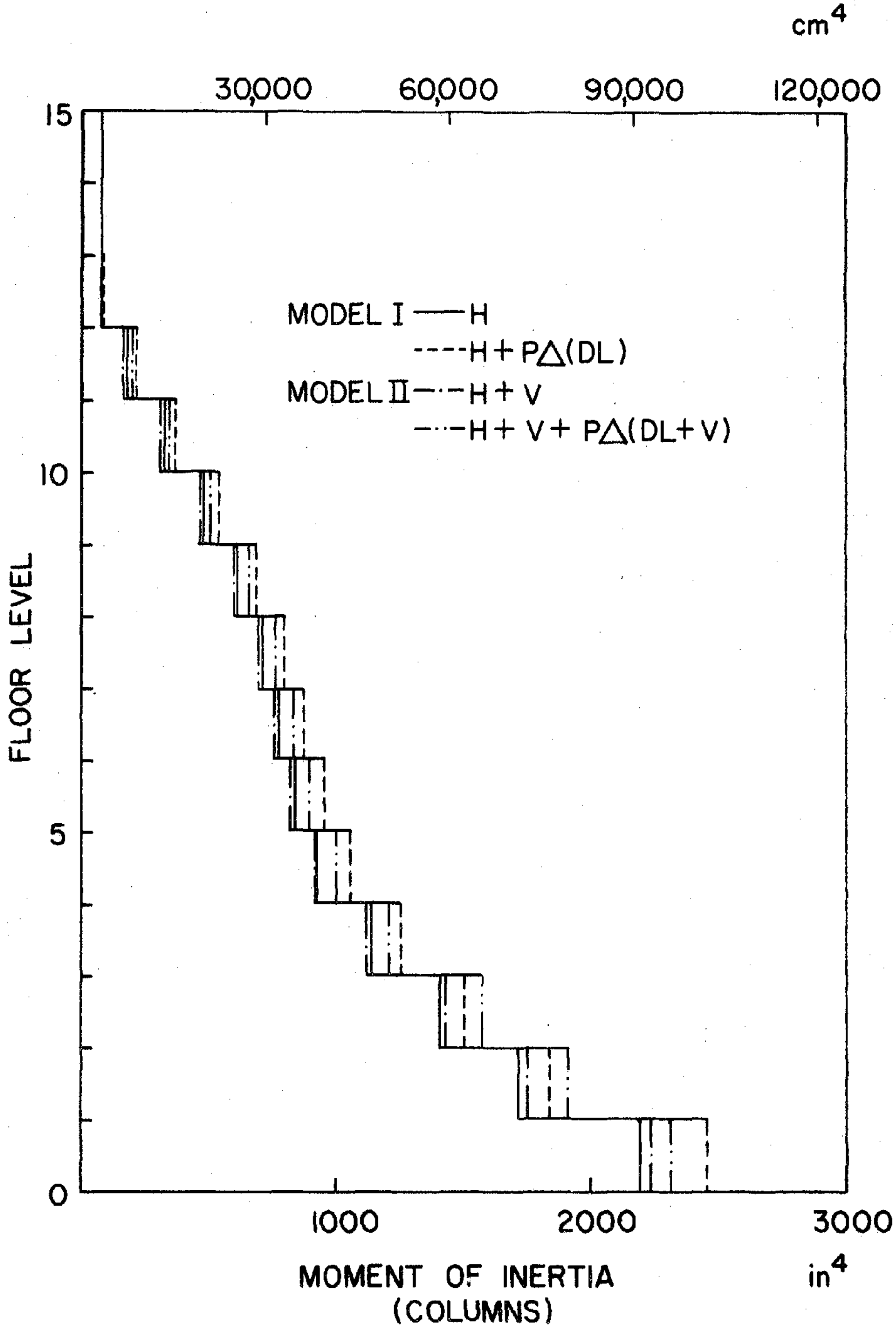


Figure 71. Distribution of Moment of Inertia of Columns of Example 11.

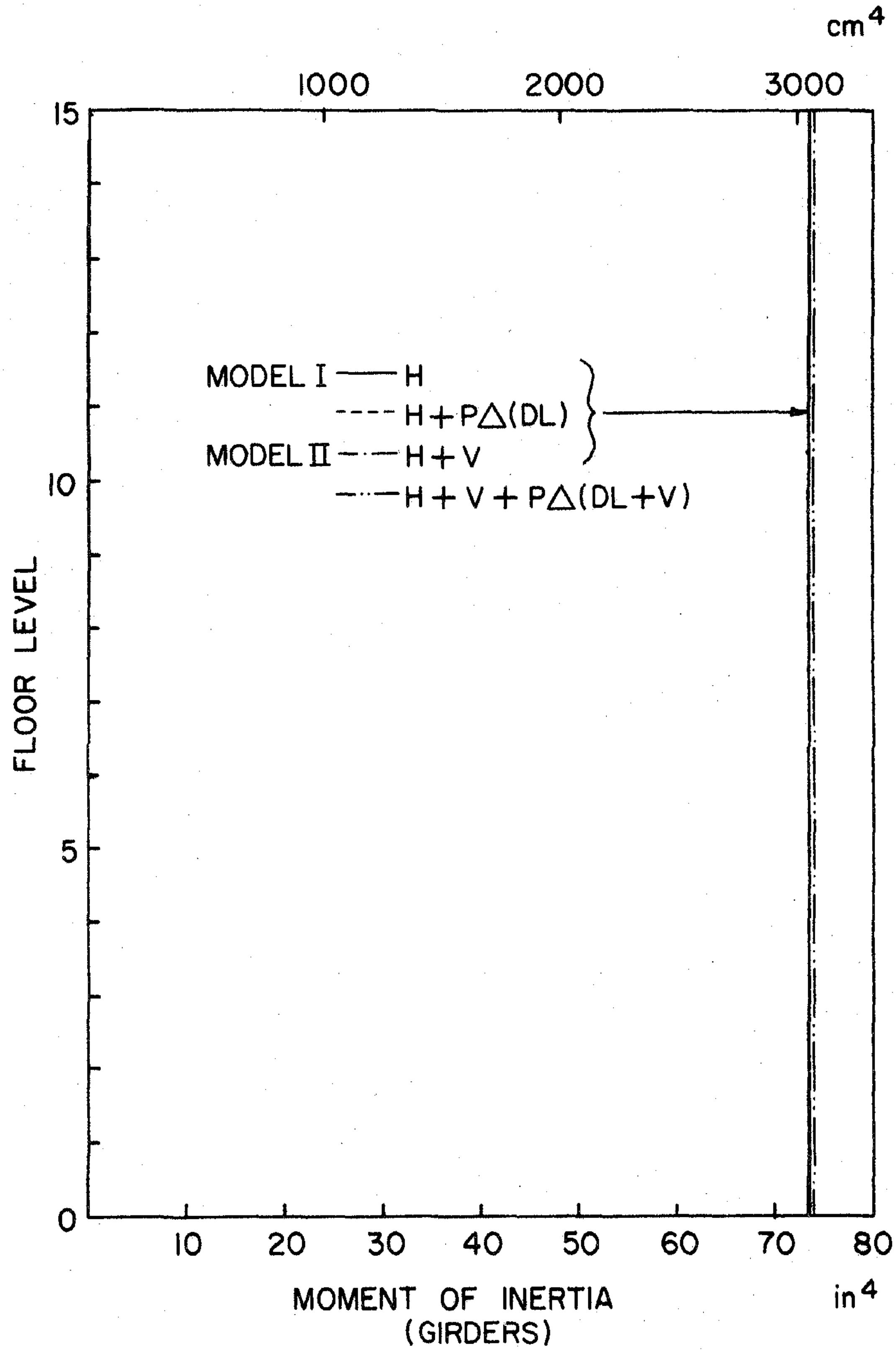


Figure 72. Distribution of Moment of Inertia of Girders of Example 11.

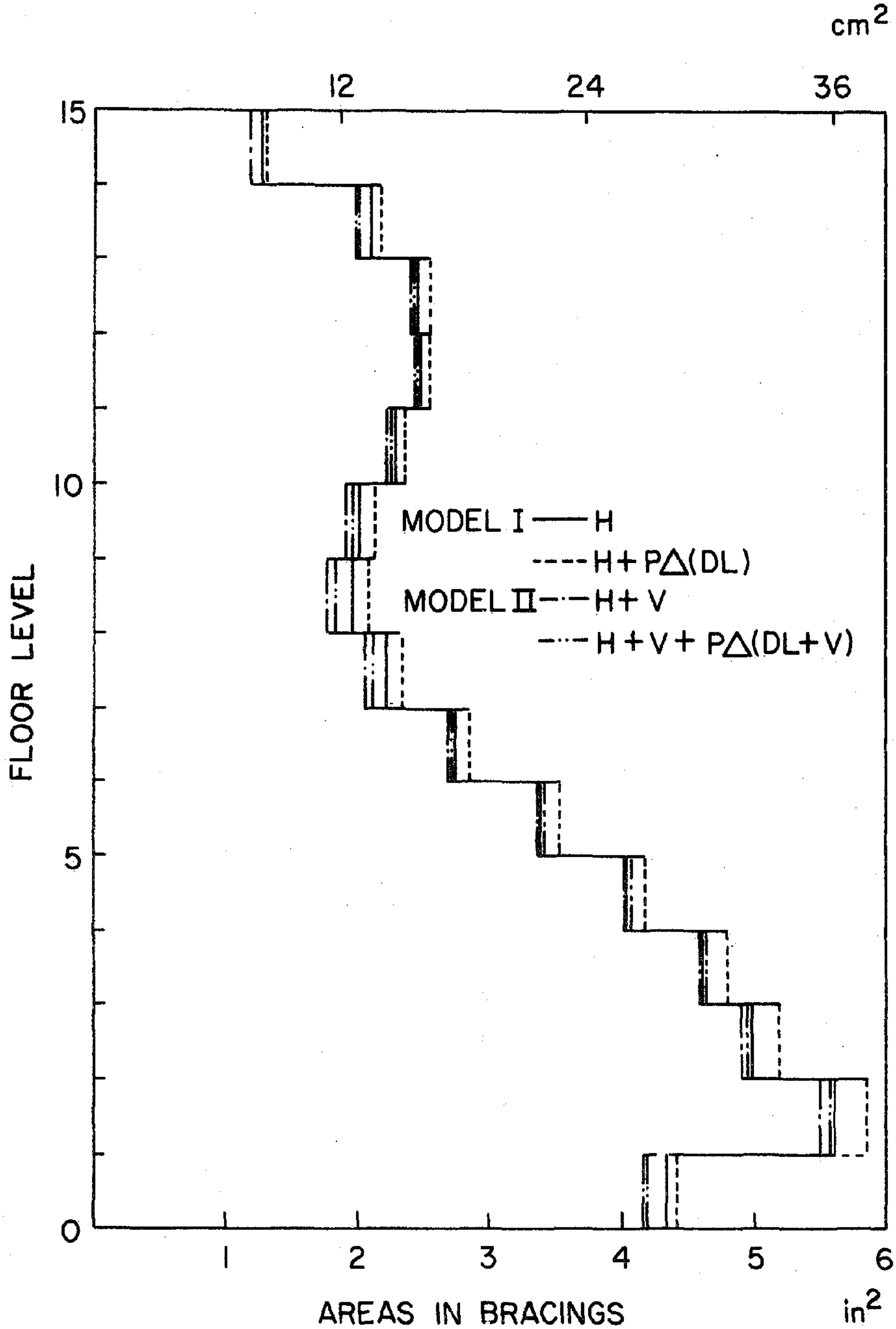


Figure 73. Distribution of Areas of Bracings of Example 11.

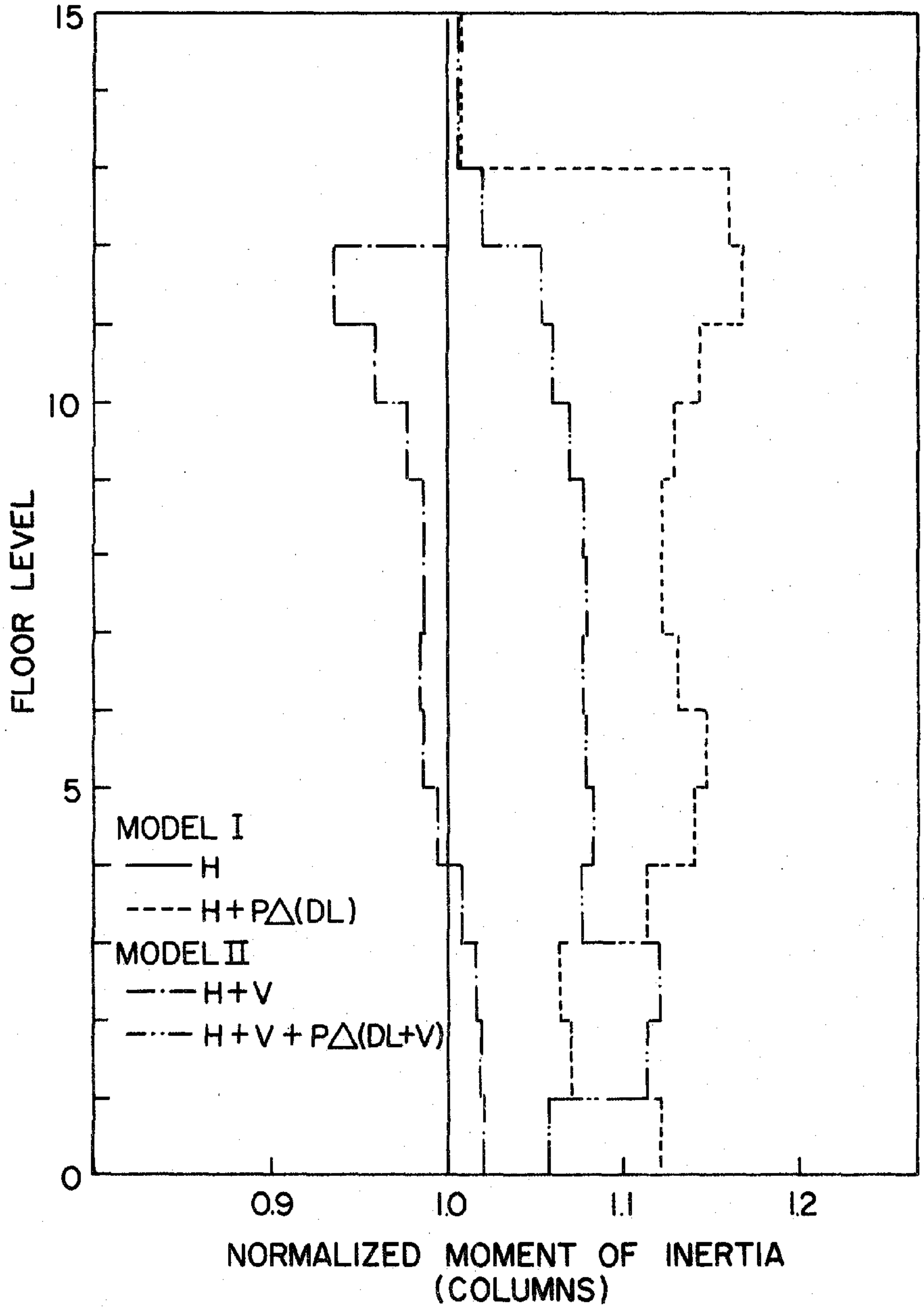


Figure 74. Normalized Moment of Inertia of Columns of Example 11.

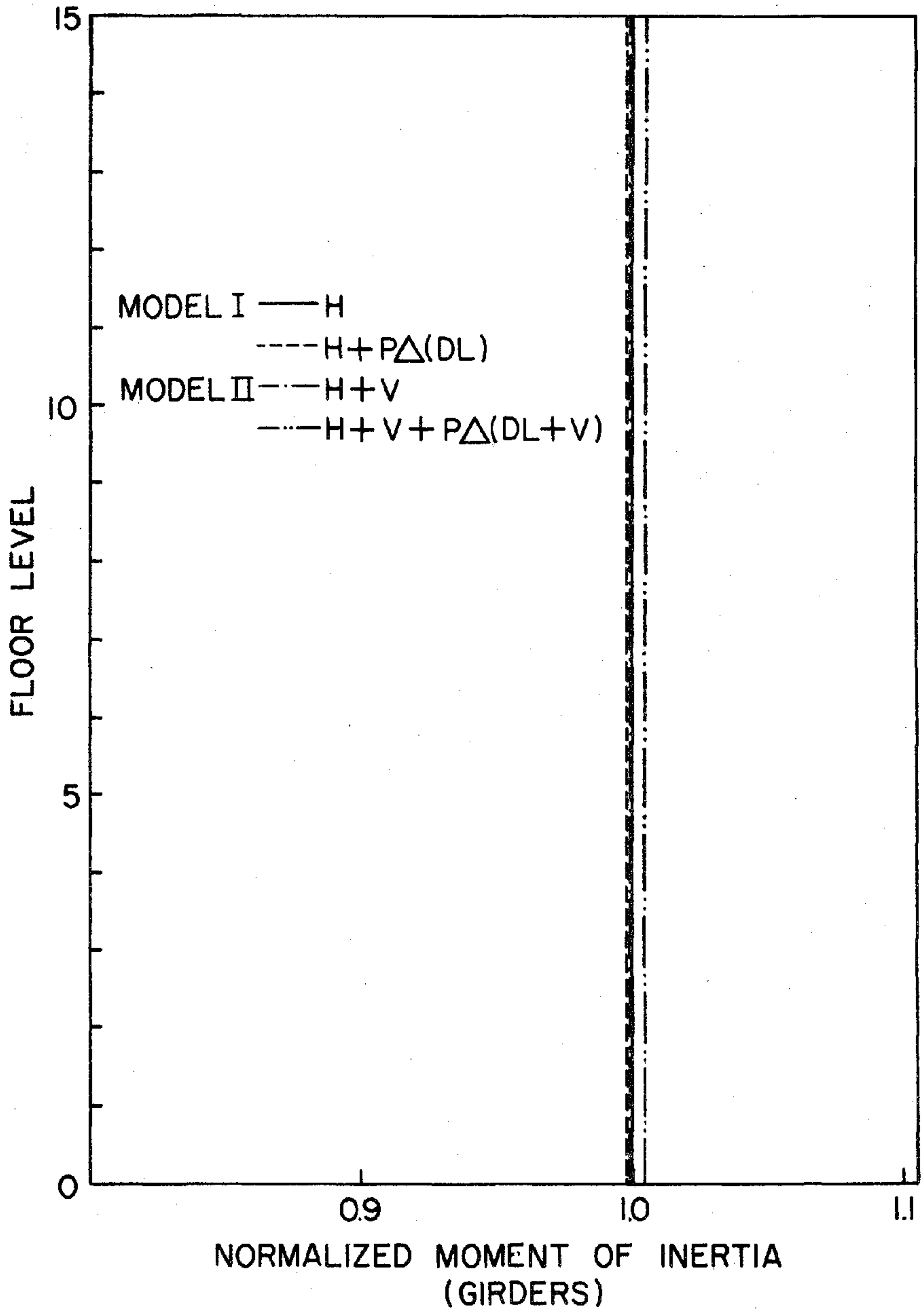


Figure 75. Normalized Moment of Inertia of Girders of Example 11.

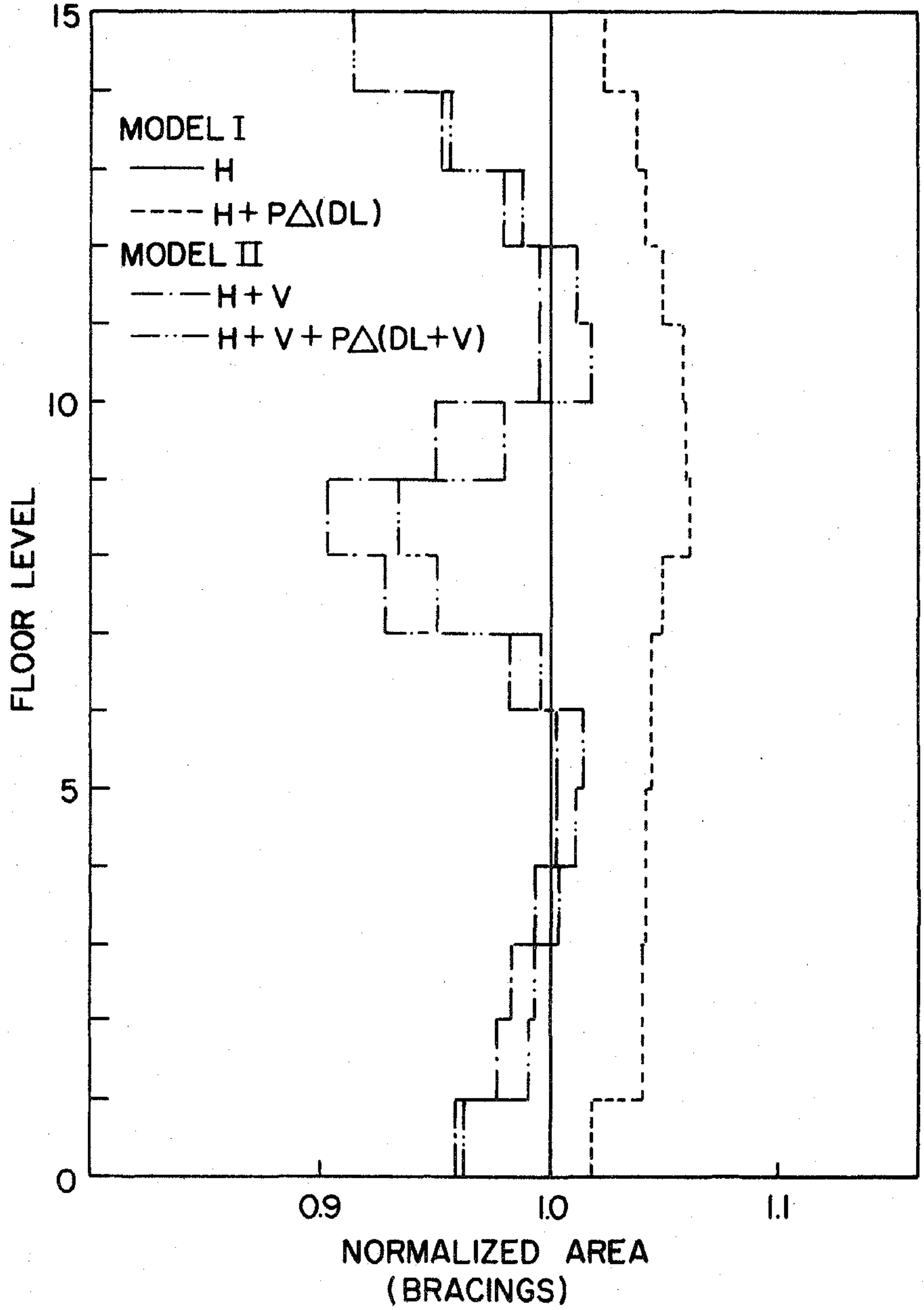


Figure 76. Normalized Cross-sectional Areas of Bracings of Example 11.

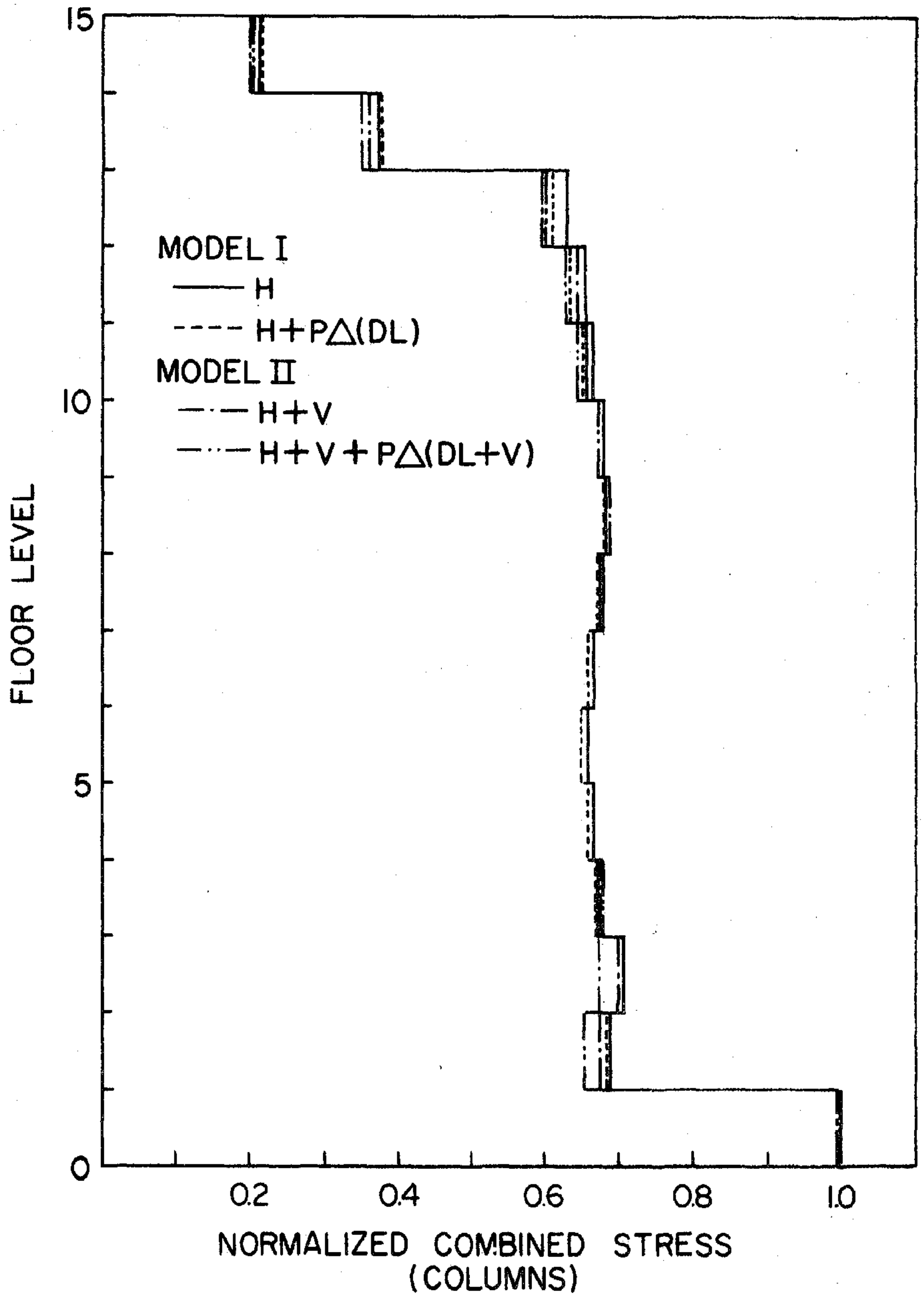


Figure 77. Ratio of Stress to Allowable Stress of Columns of Example 11.

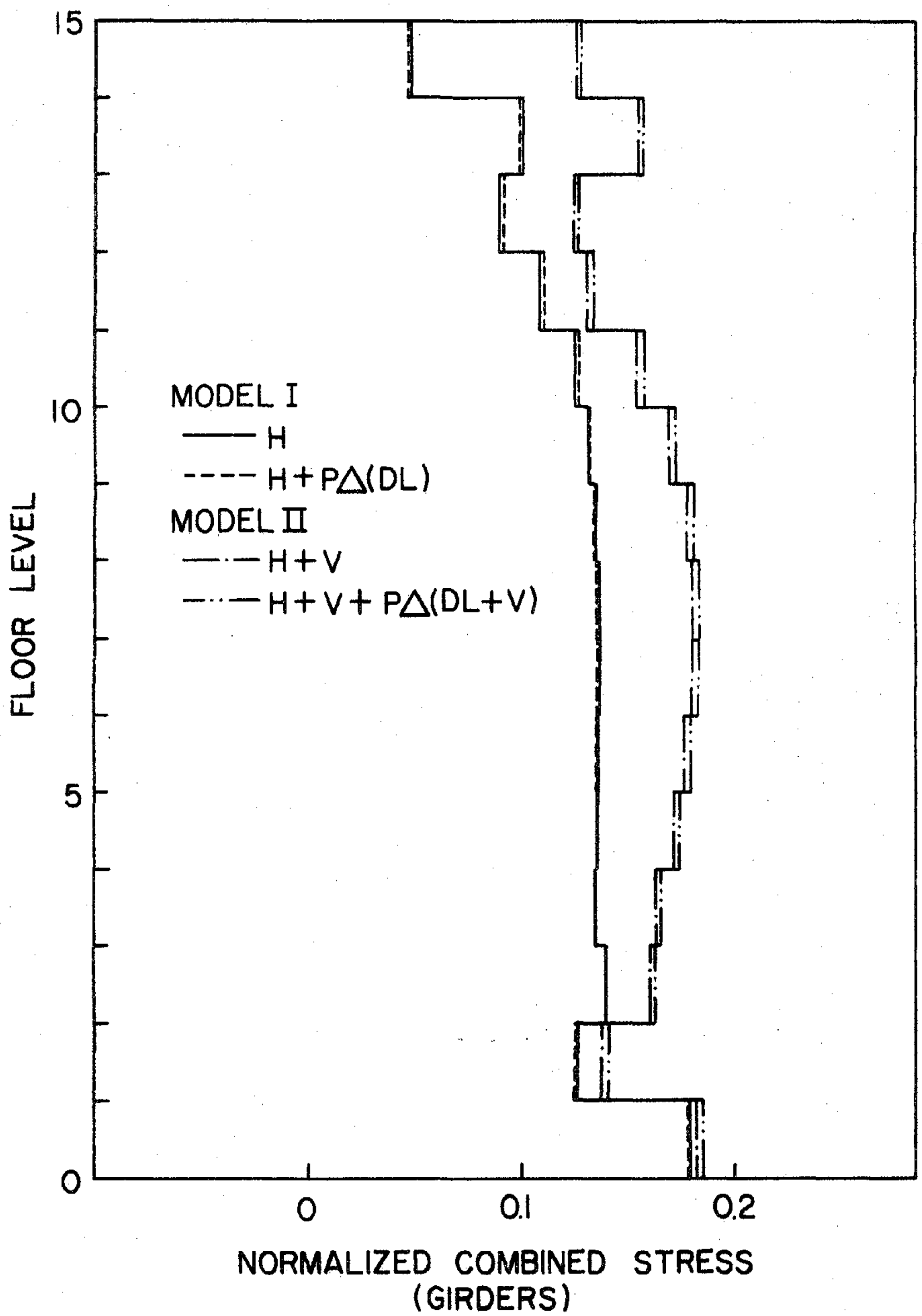


Figure 78. Ratio of Stress to Allowable Stress of Girders of Example 11.

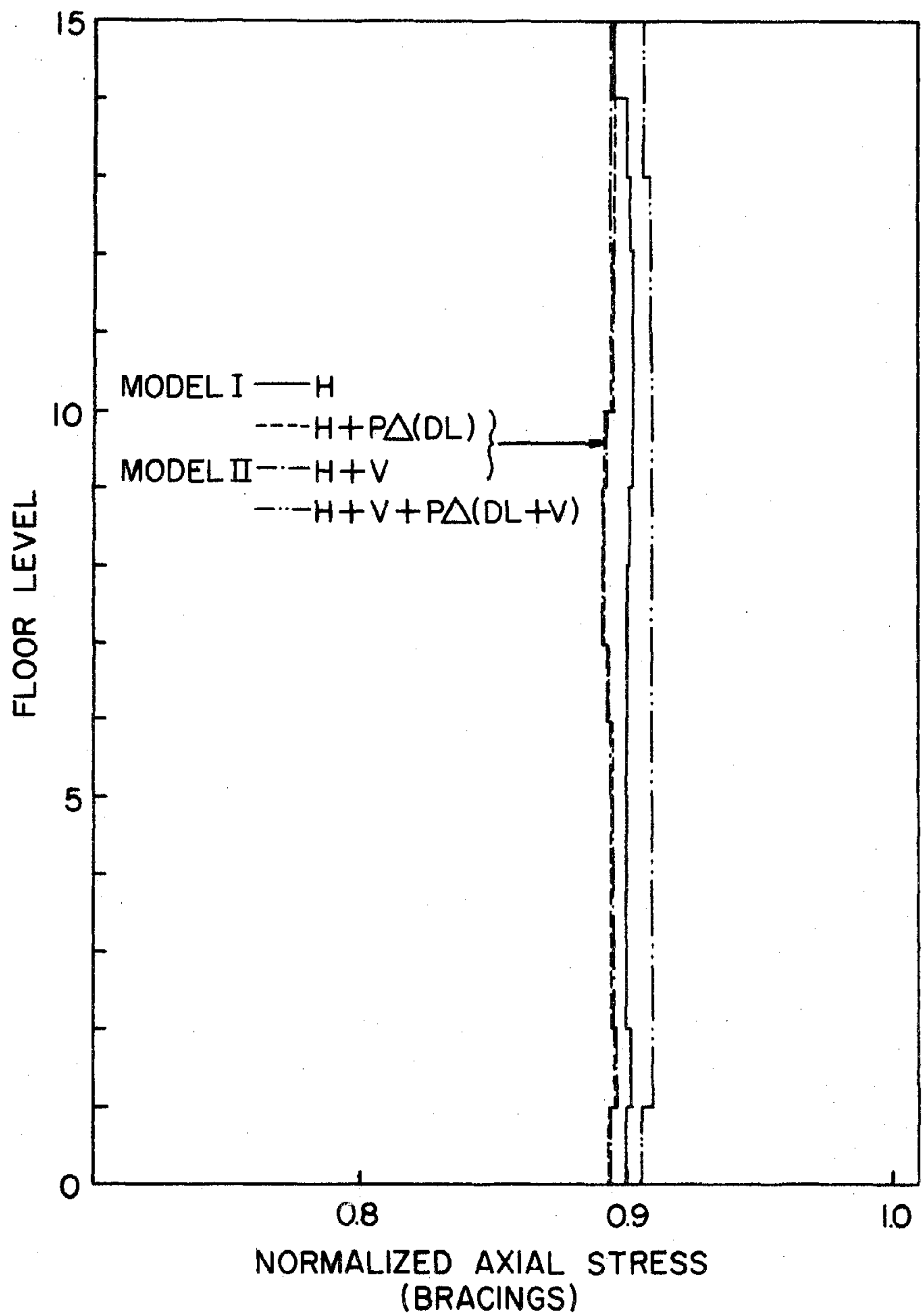


Figure 79. Ratio of Stress to Allowable Stress of Bracings of Example 11.

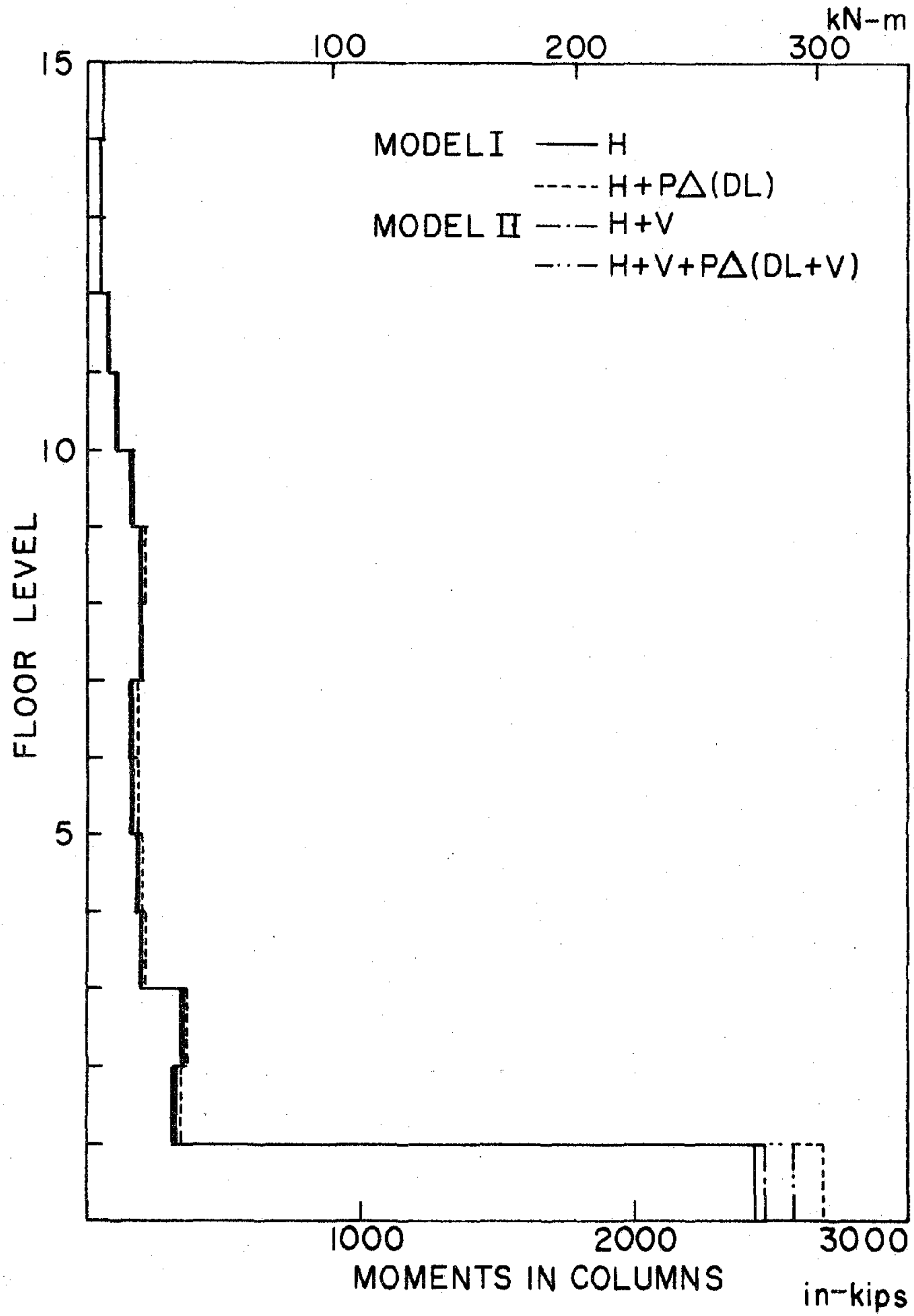


Figure 80. Moments in Columns of Example 11.

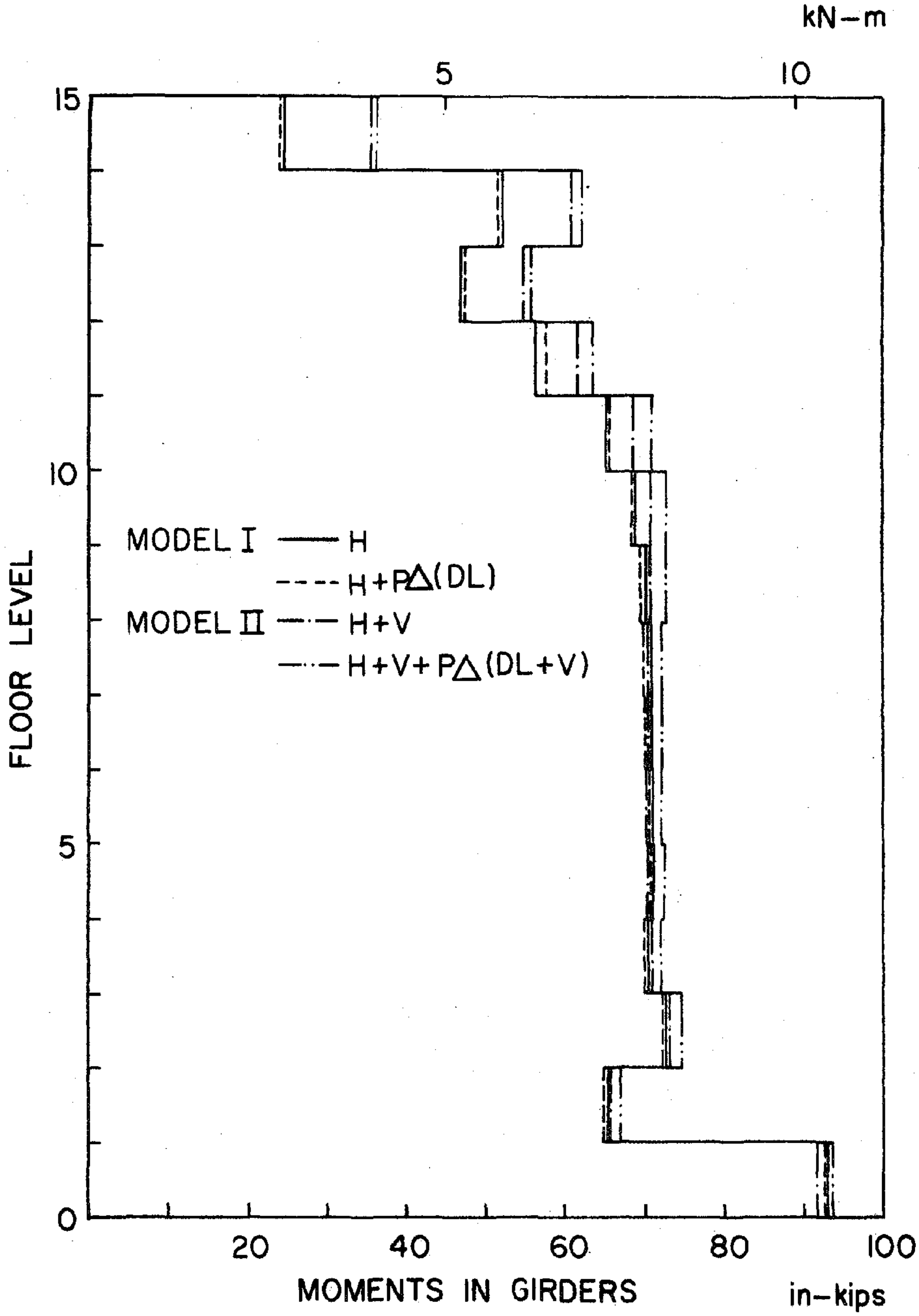


Figure 81. Moments in Girders of Example 11.

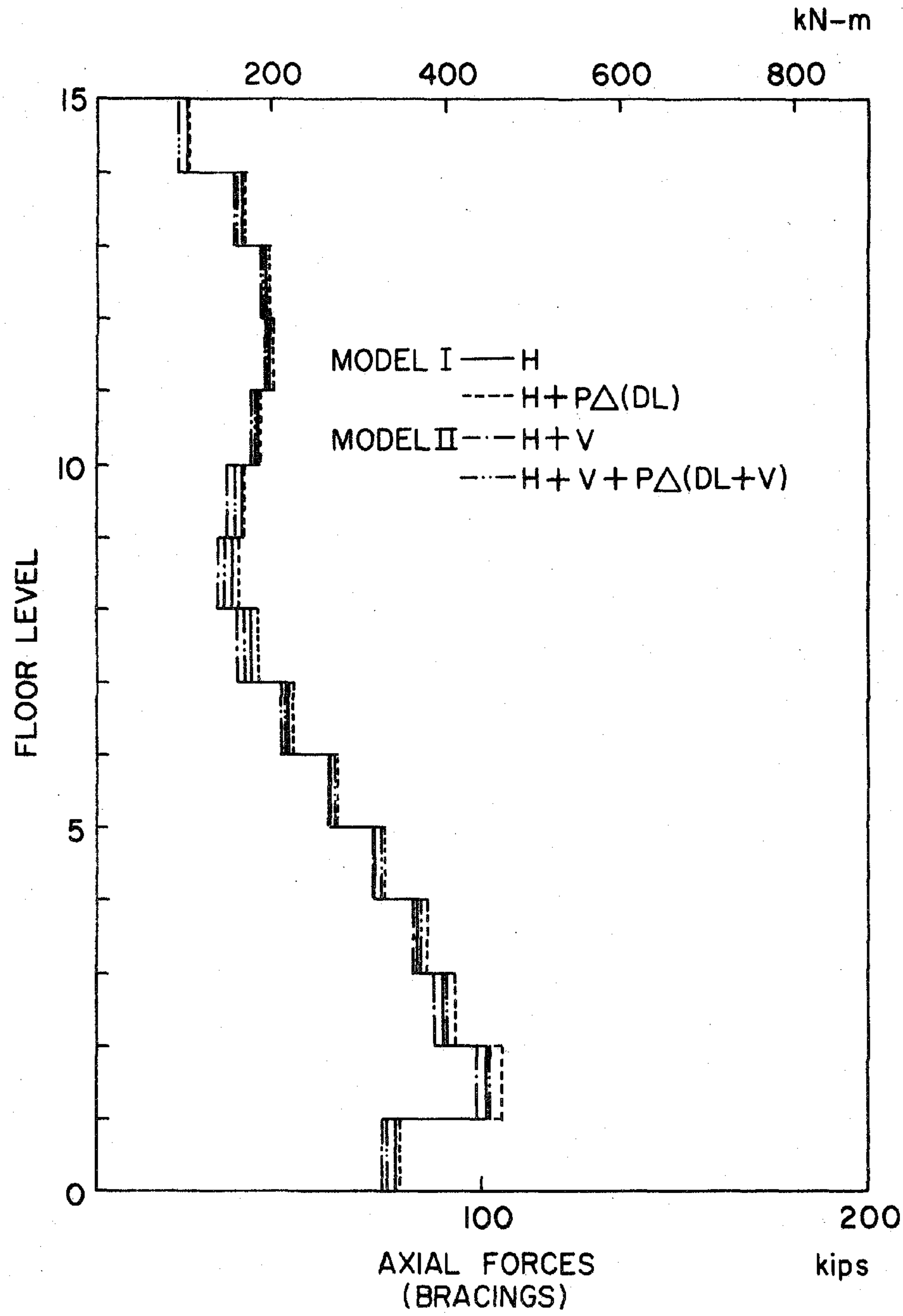


Figure 82. Axial Forces in Bracings of Example 11.

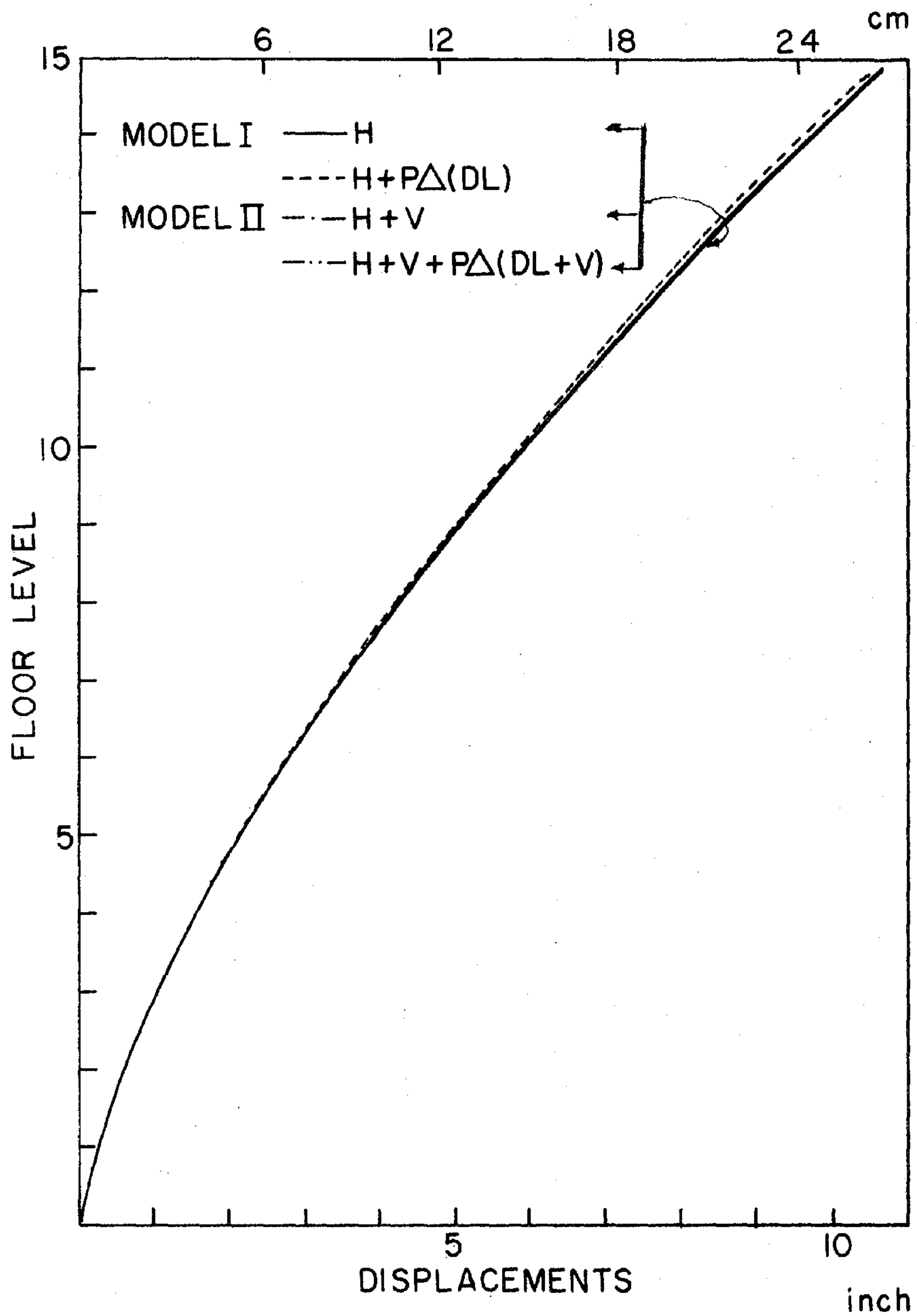
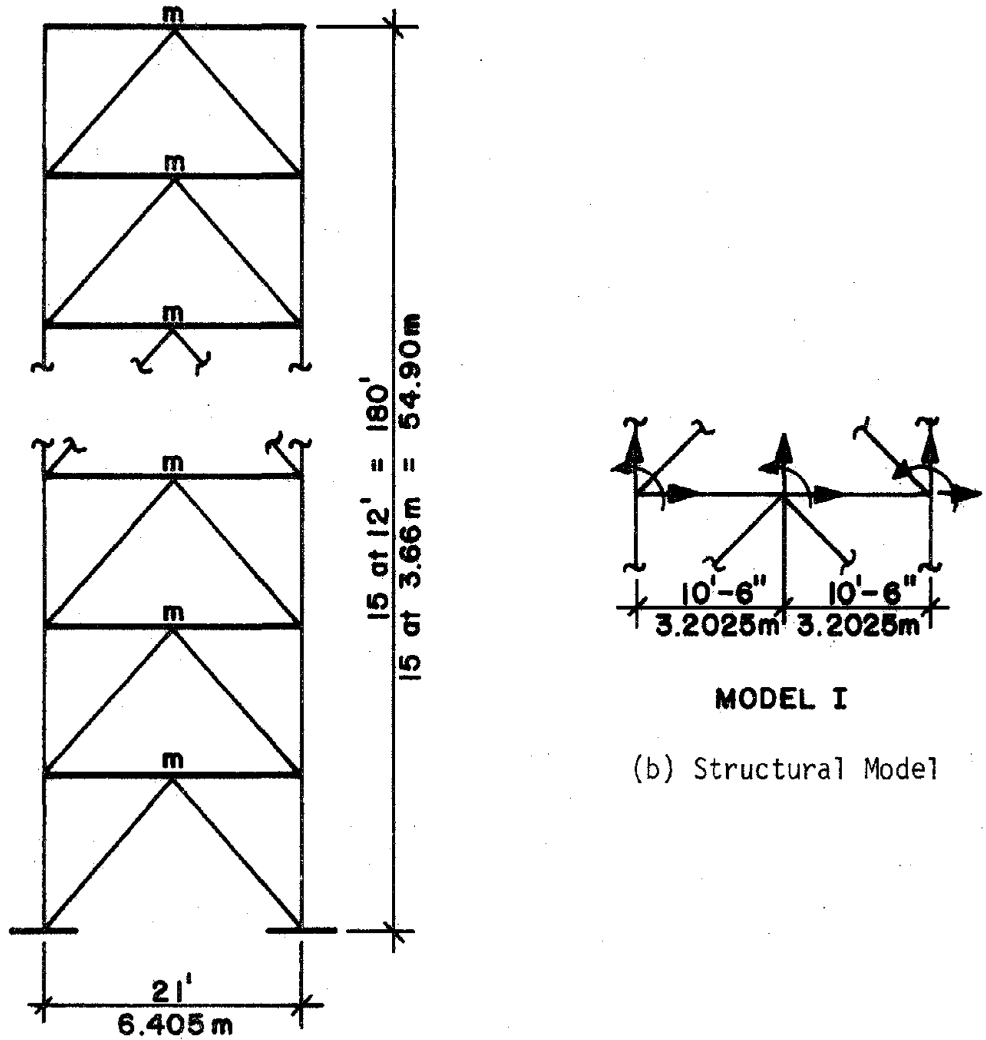


Figure 83. Displacements at Floor Levels of Example 11.



(a) 15-Story K-Braced Frame

(b) Structural Model

Figure 84. 15-Story, Single-Bay, K-Braced Frame of Example 12.

stiffness of the structure and consequently reduces the effect of the vertical ground motion. Because a node must be assumed at the mid-point of girders, Model II is used for loading Cases of (c) and (d) of which the design results are shown in Table VII as well as Fig. 85.

Figure 86 represents the contribution of the energies of the individual modes to the design for all cases. The modes beyond the fourth contribute little to the design, because they correspond mainly to the vertical earthquake component.

The P-Δ effect requires somewhat larger columns and bracings as shown respectively in Figs. 87 and 89. The girders are designed on the basis of minimum sizes. The corresponding ratios of the normalized section properties of columns, girders, and bracings are illustrated in Figs. 90 through 91.

The ratios of the combined stresses to the allowable stresses of the columns, girders, and bracings are shown in Figs. 93 through 95 which indicate that the actual stresses of these two cases are very close and that the columns at the support are only the members reach the allowable stress. The stress behavior of the columns can be further observed from Fig. 96 where the moments of the support columns are very large. The moments of the girders, the axial forces in the bracings, and the displacements at the floor levels may be found in Fig. 97 through 99 respectively.

4. Comparison of Single-Braced, Double-Braced, and K-Braced Systems--

Example 13. Figure 100 represents the comparison of the design weights of Case (d) for the single-braced, double-braced, and K-braced

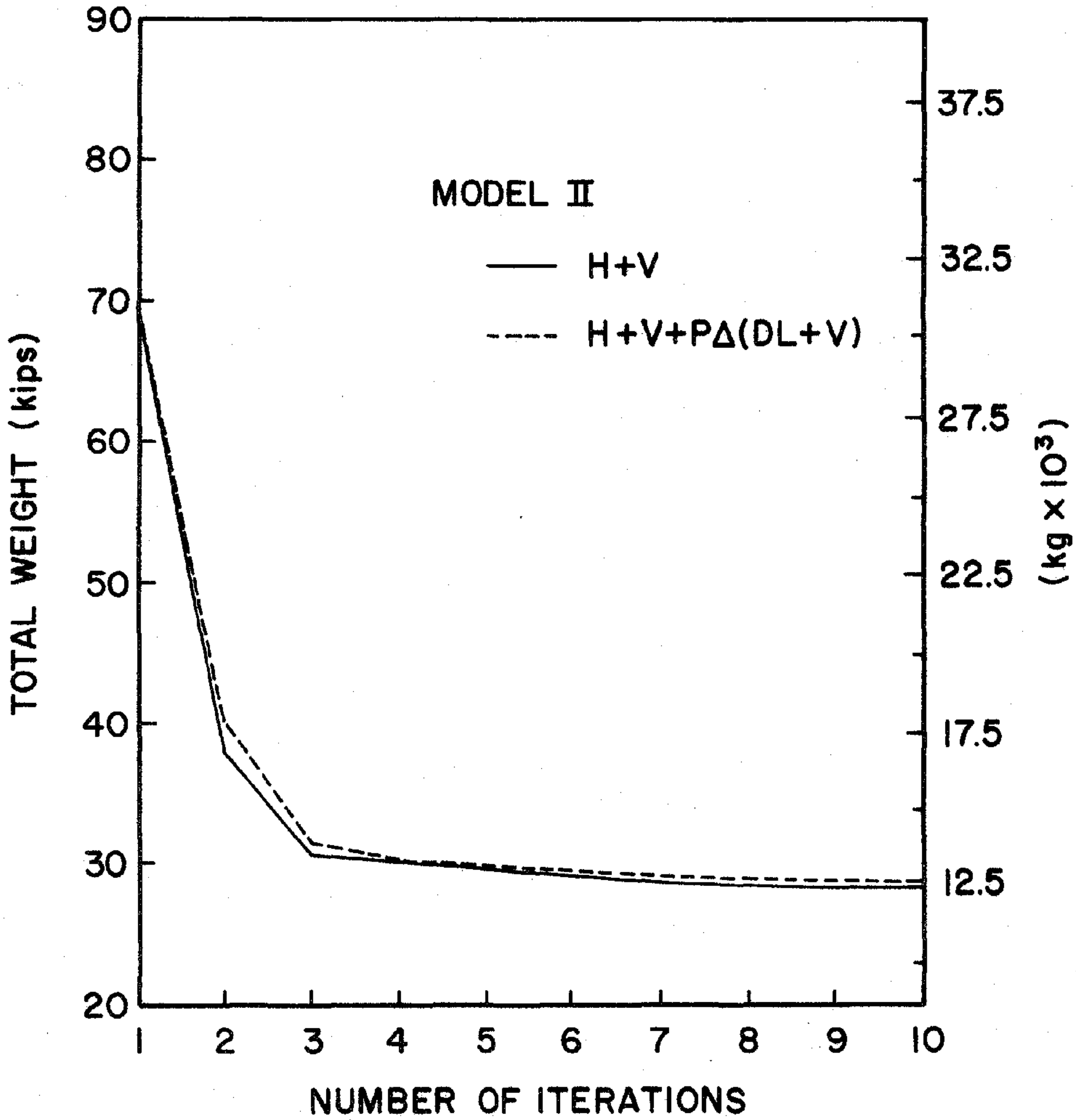


Figure 85. Optimum Weights of K-Braced Frame of Example 12.

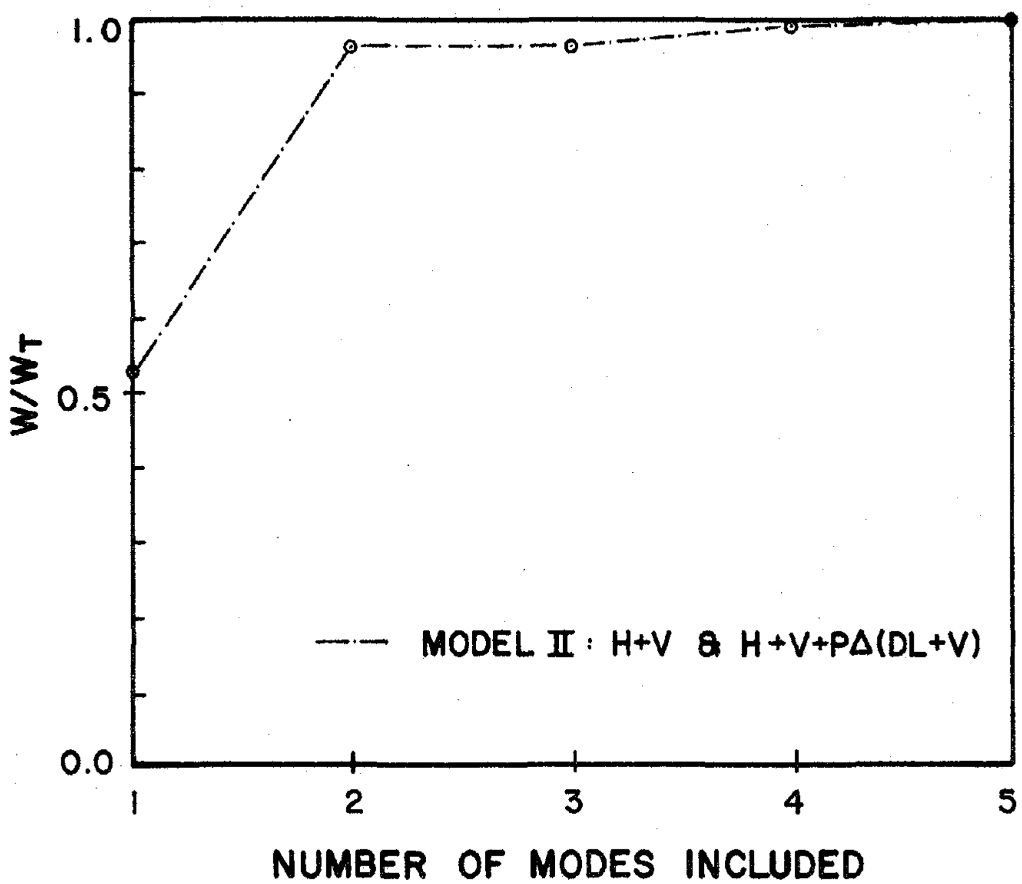


Figure 86. Contribution of Energy of Individual Modes to the Design of Example 12.

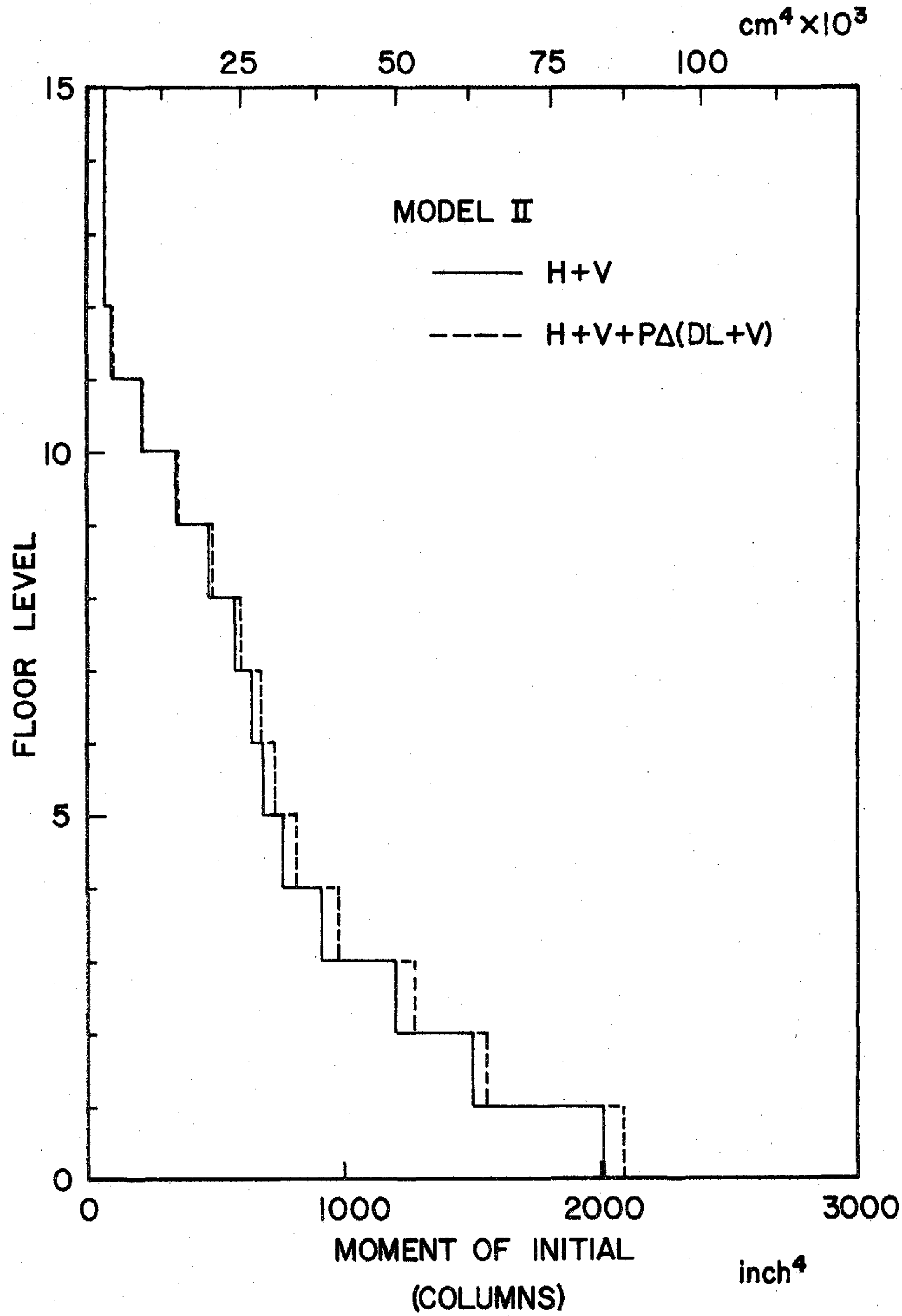


Figure 87. Distribution of Moment of Inertia of Columns of Example 12.

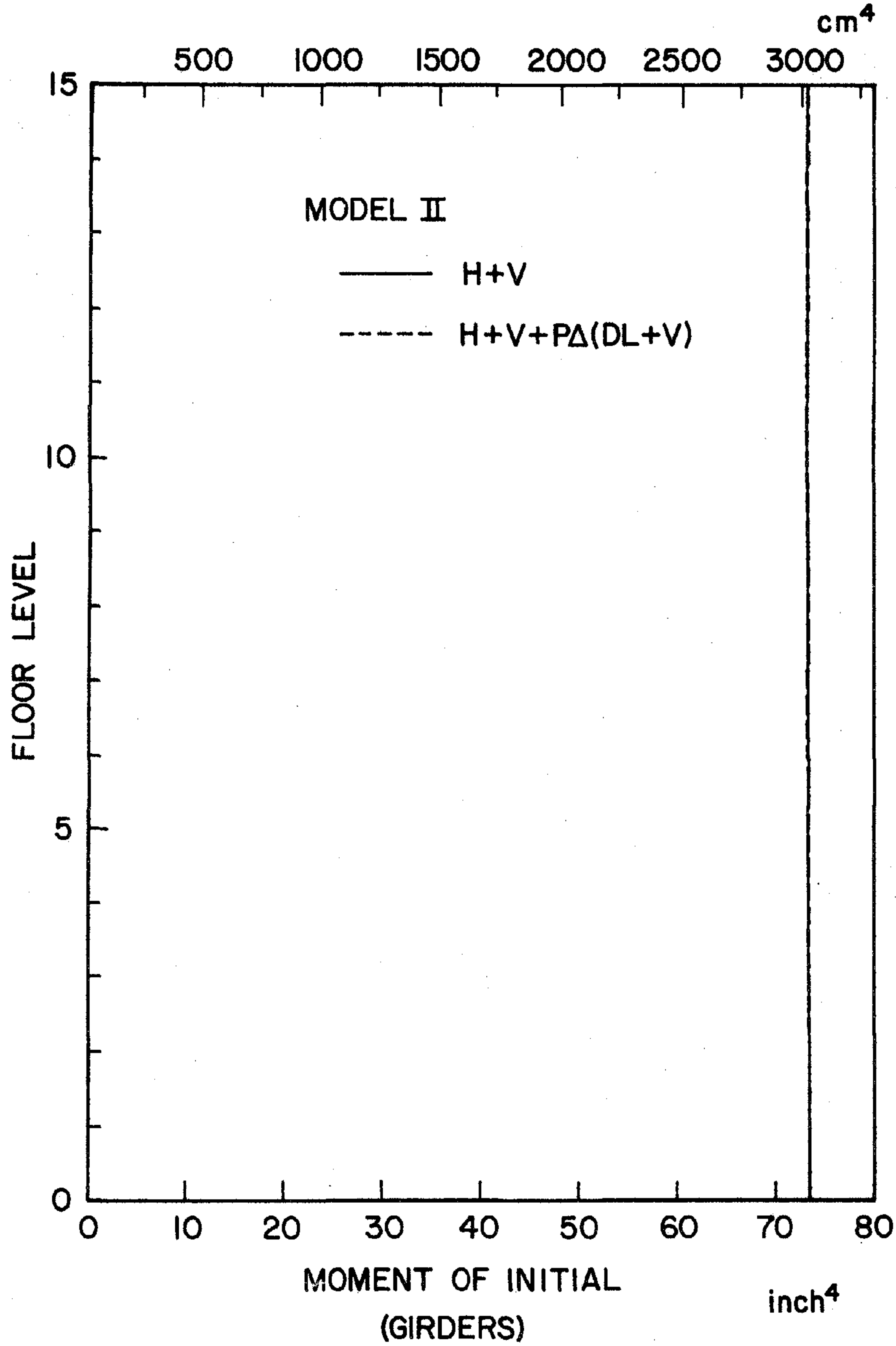


Figure 88. Distribution of Moment of Inertia of Girders of Example 12.

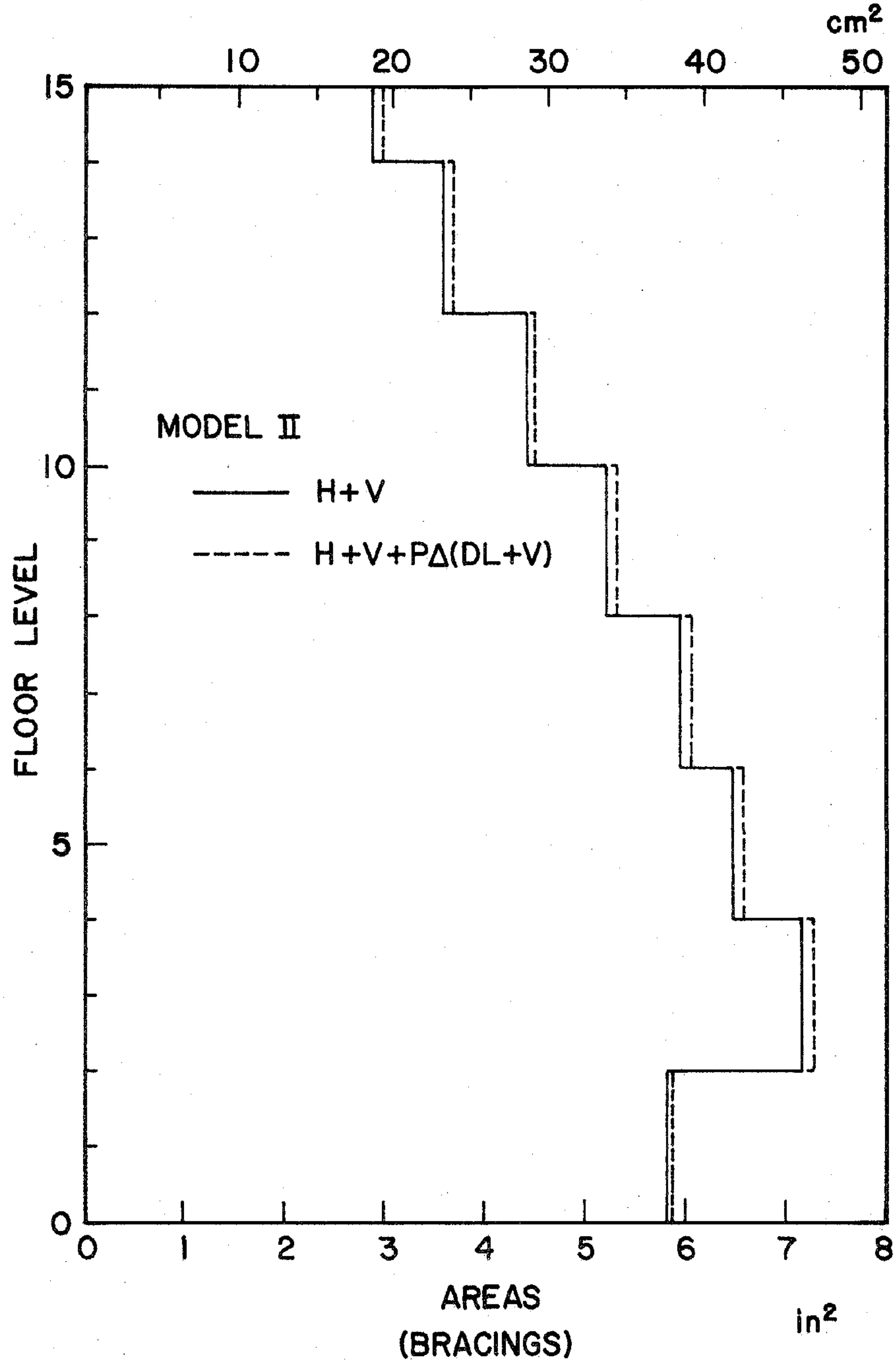


Figure 89. Distribution of Areas of Bracings of Example 12.

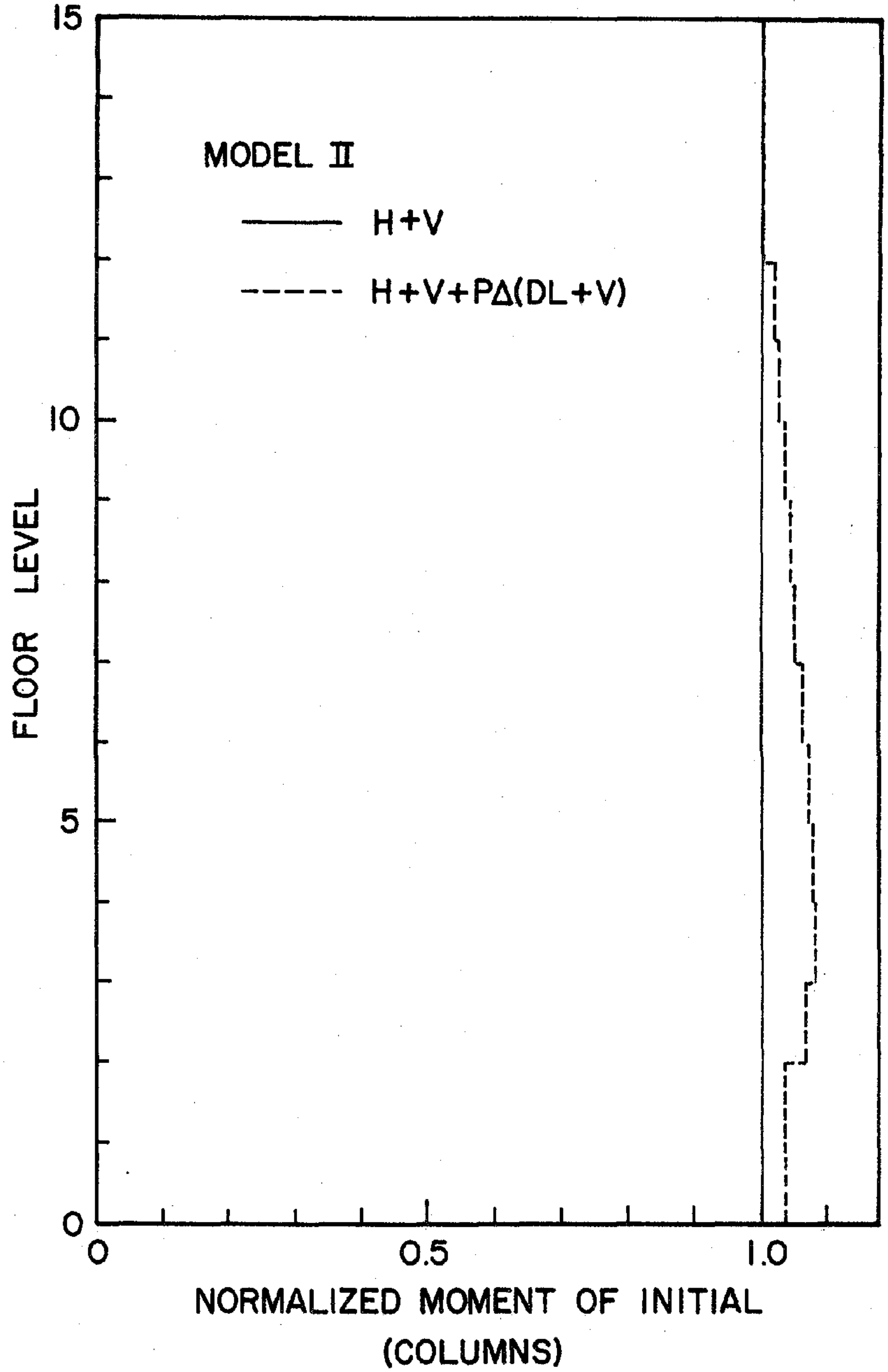


Figure 90. Normalized Moment of Inertia of Columns of Example 12.

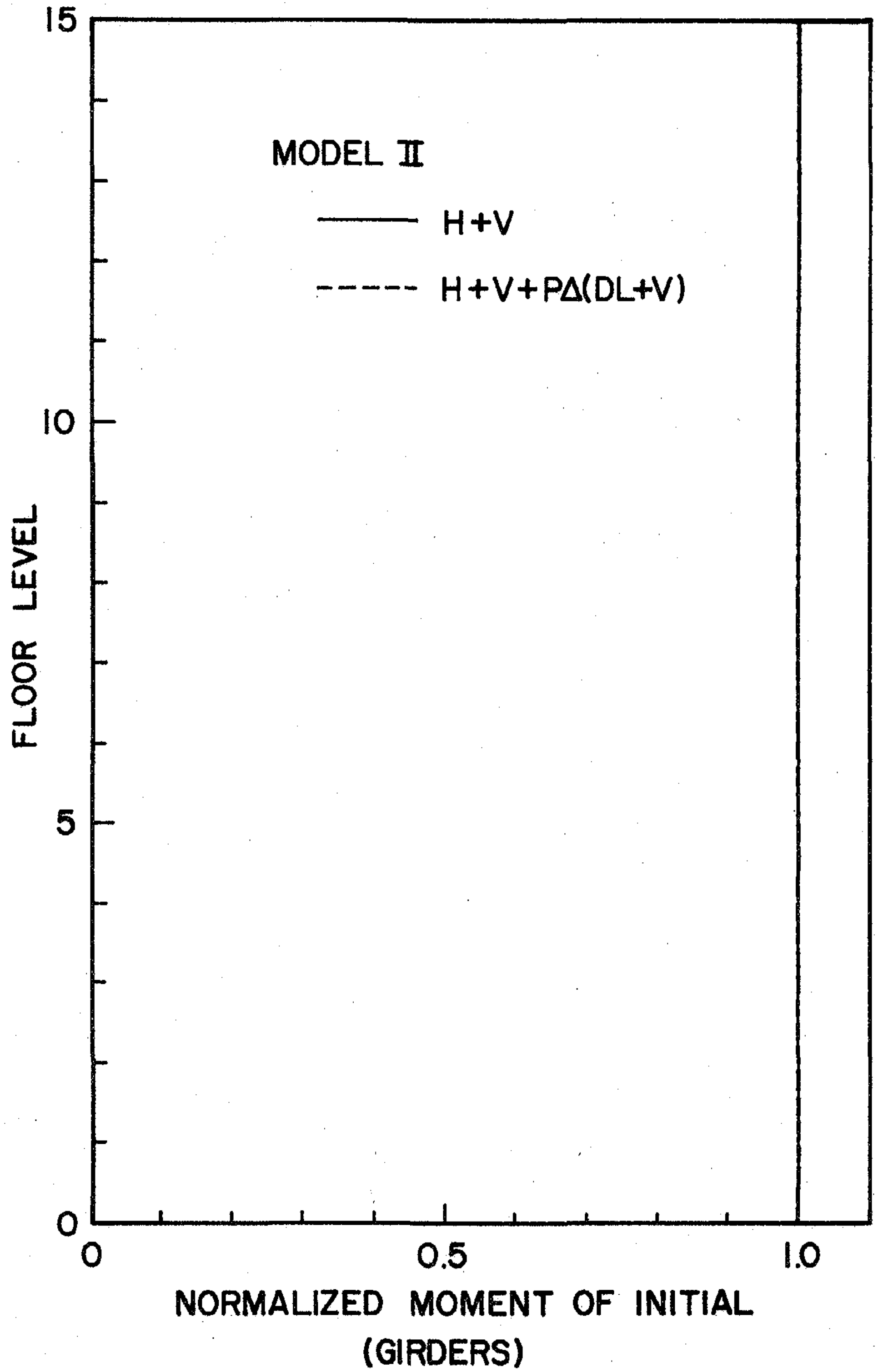


Figure 91. Normalized Moment of Inertia of Girders of Example 12.

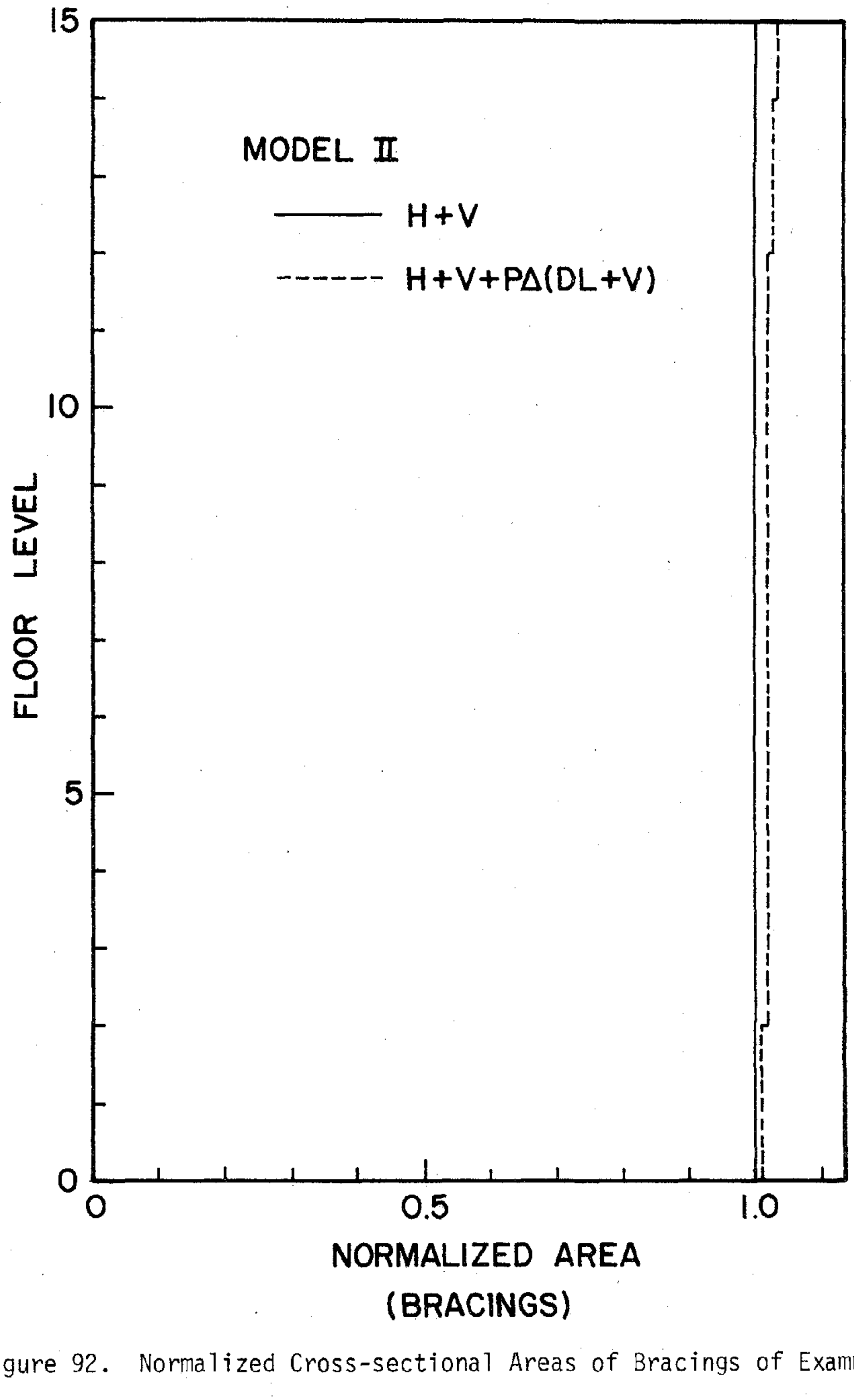


Figure 92. Normalized Cross-sectional Areas of Bracings of Example 12.

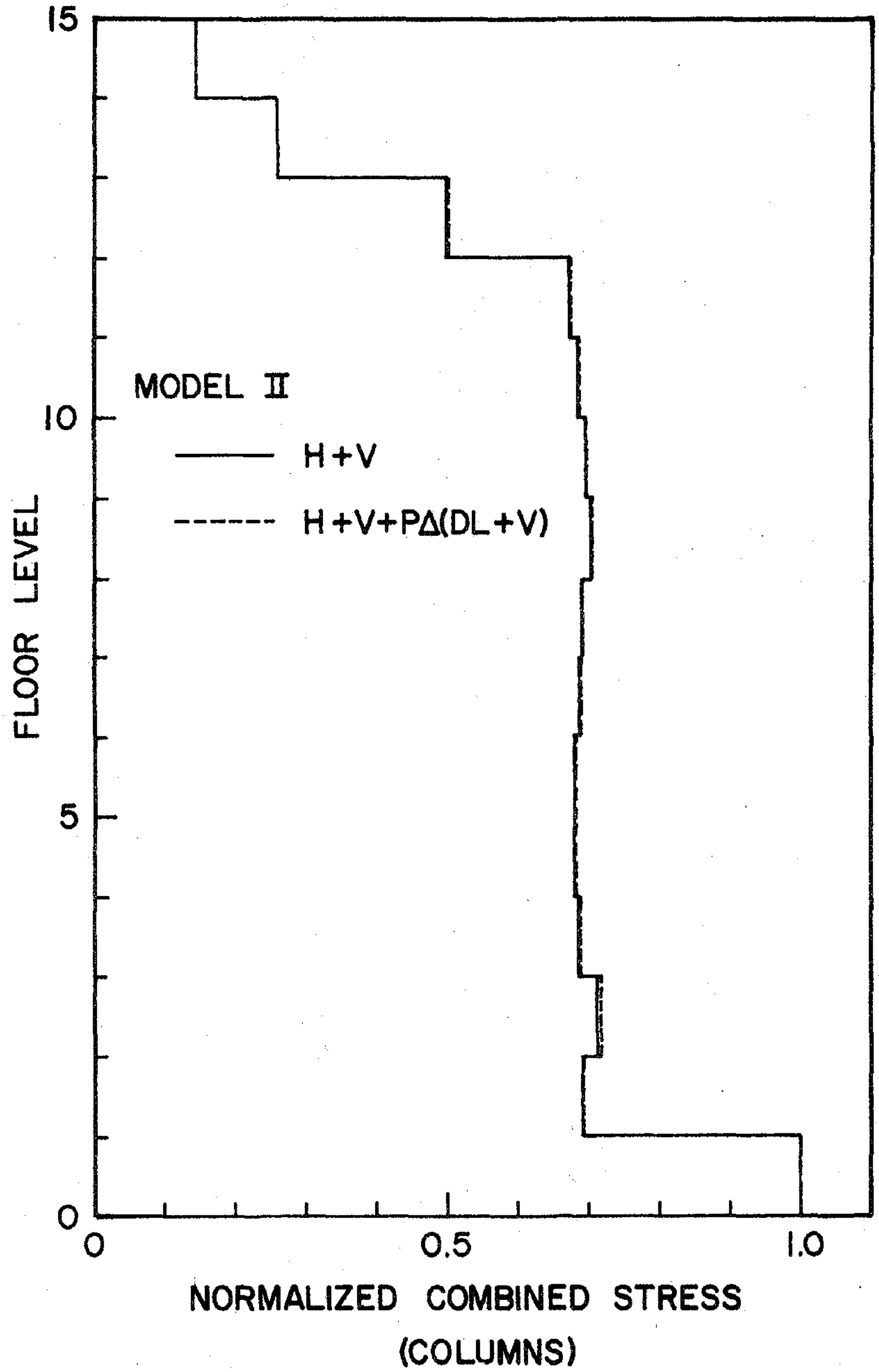


Figure 93. Ratio of Stress to Allowable Stress of Columns of Example 12.

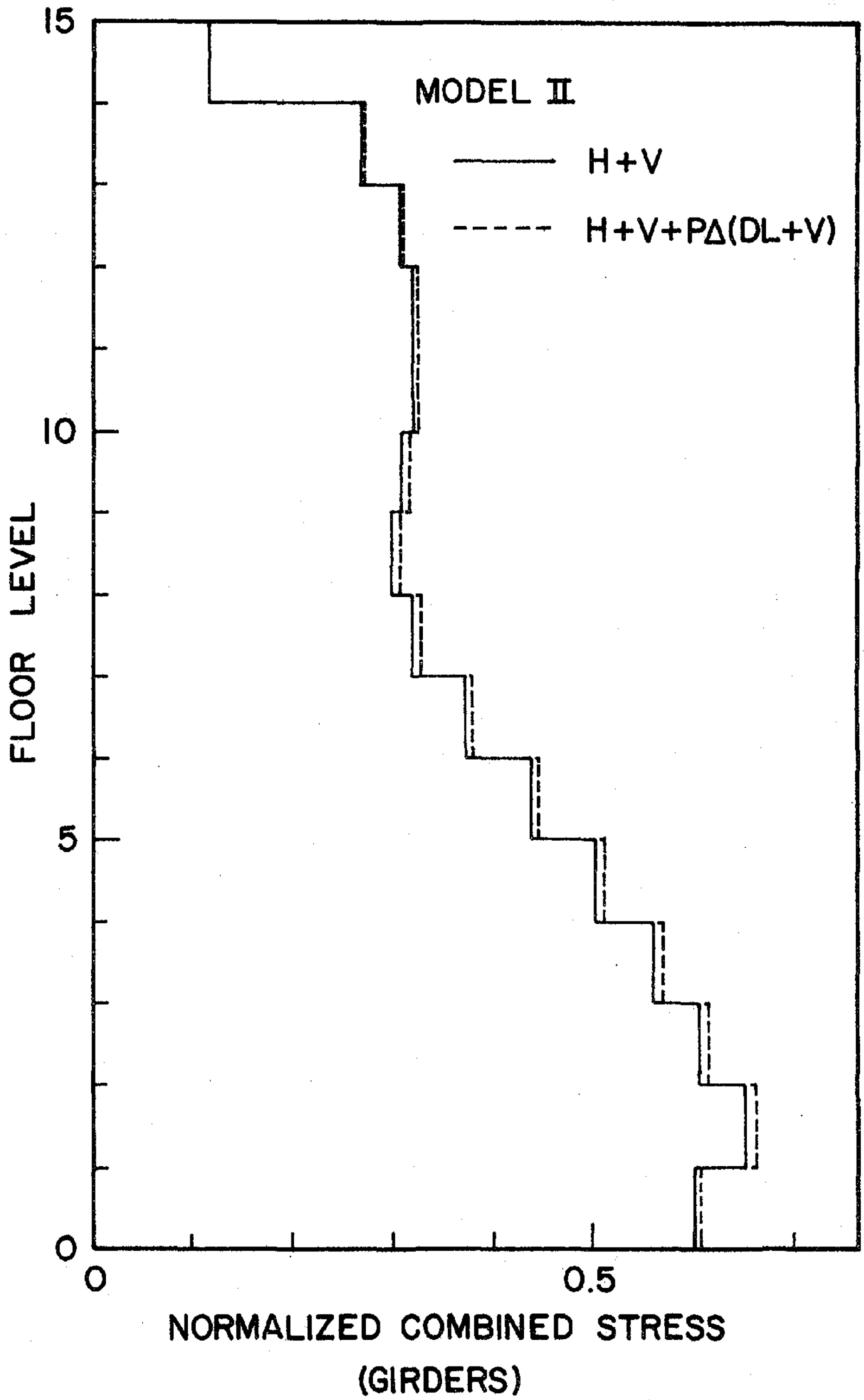


Figure 94. Ratio of Stress to Allowable Stress of Girders of Example 12.

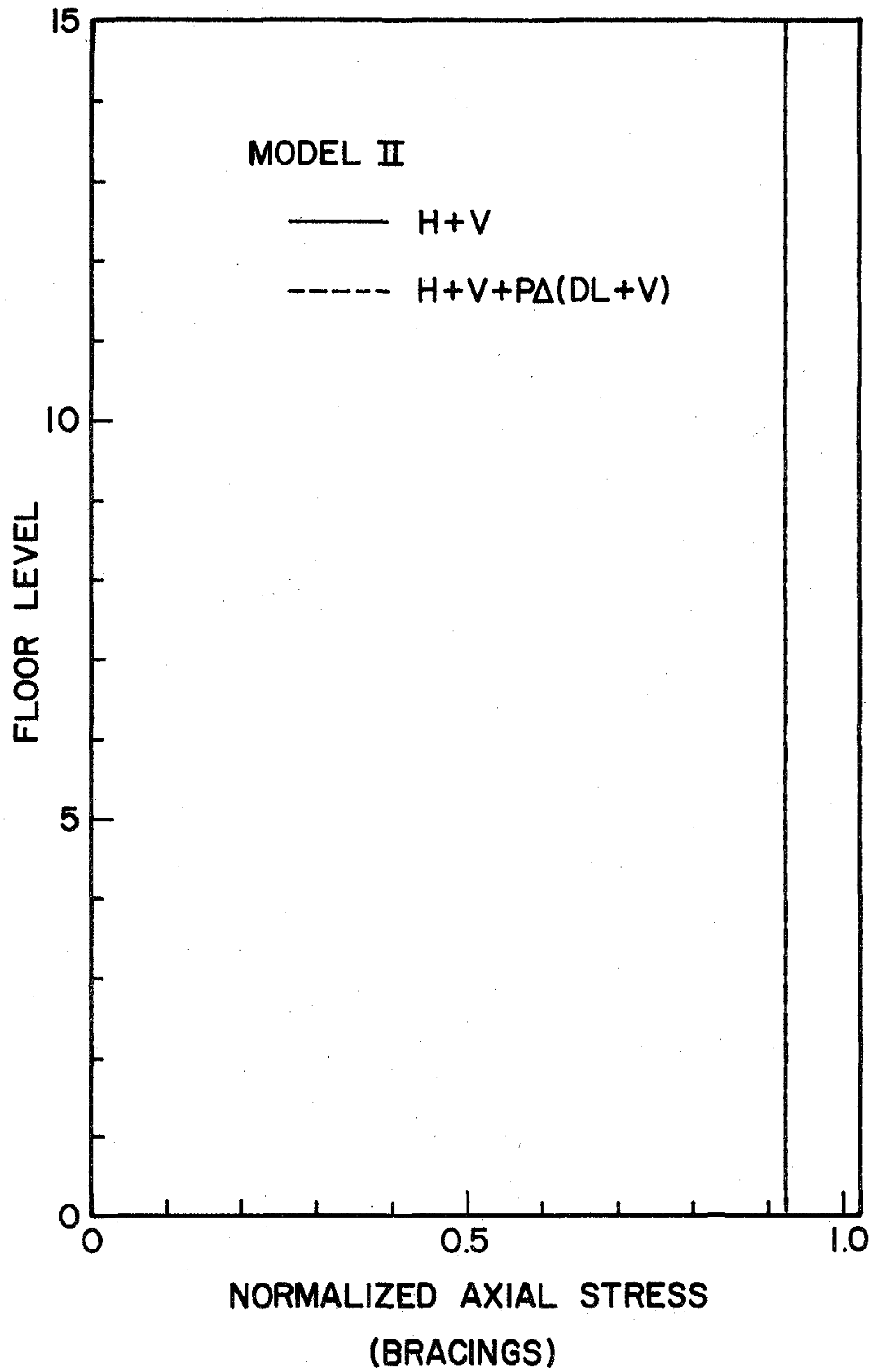


Figure 95. Ratio of Stress to Allowable Stress of Bracings of Example 12.

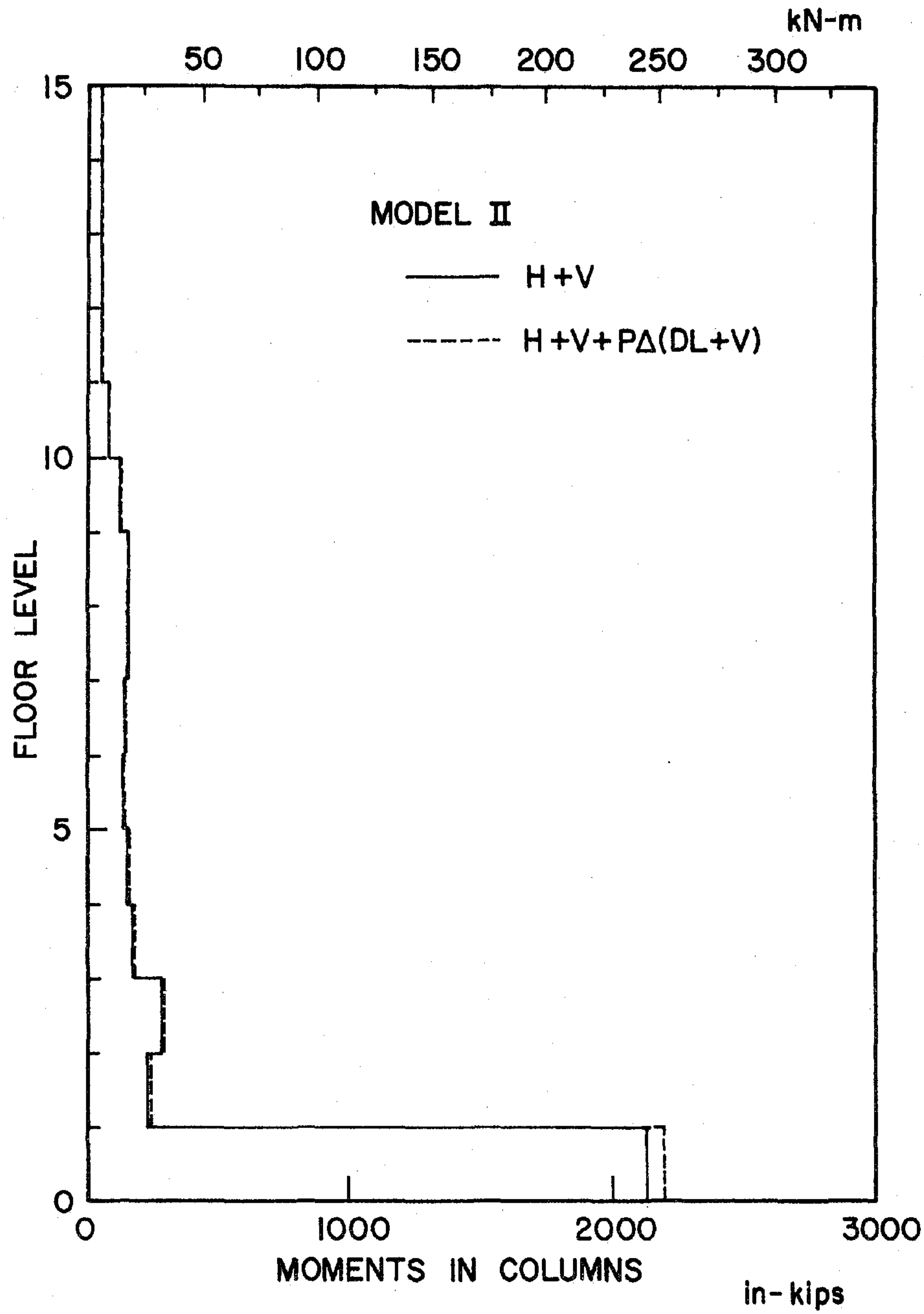


Figure 96. Moments in Columns of Example 12.

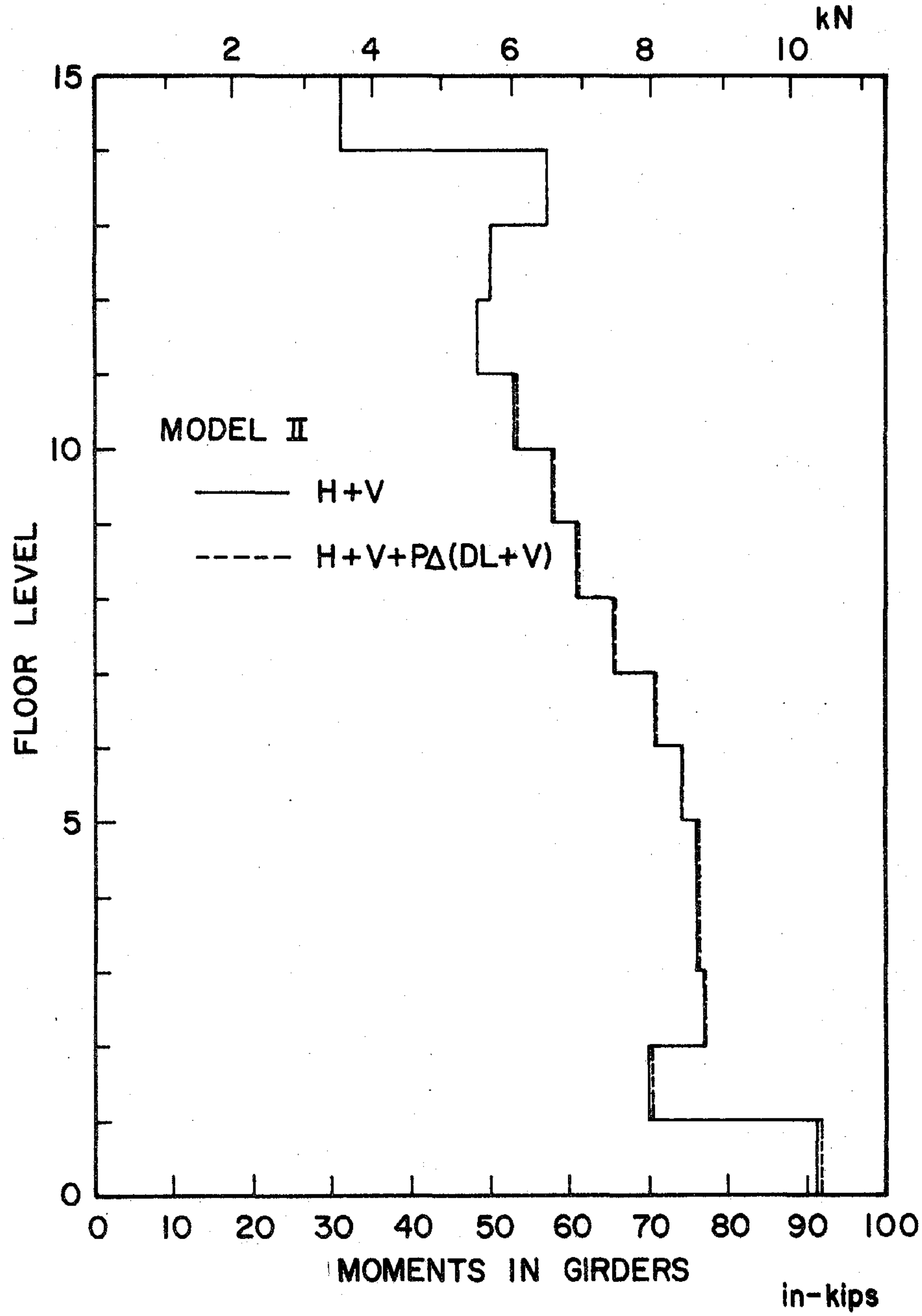


Figure 97. Moments in Girders of Example 12.

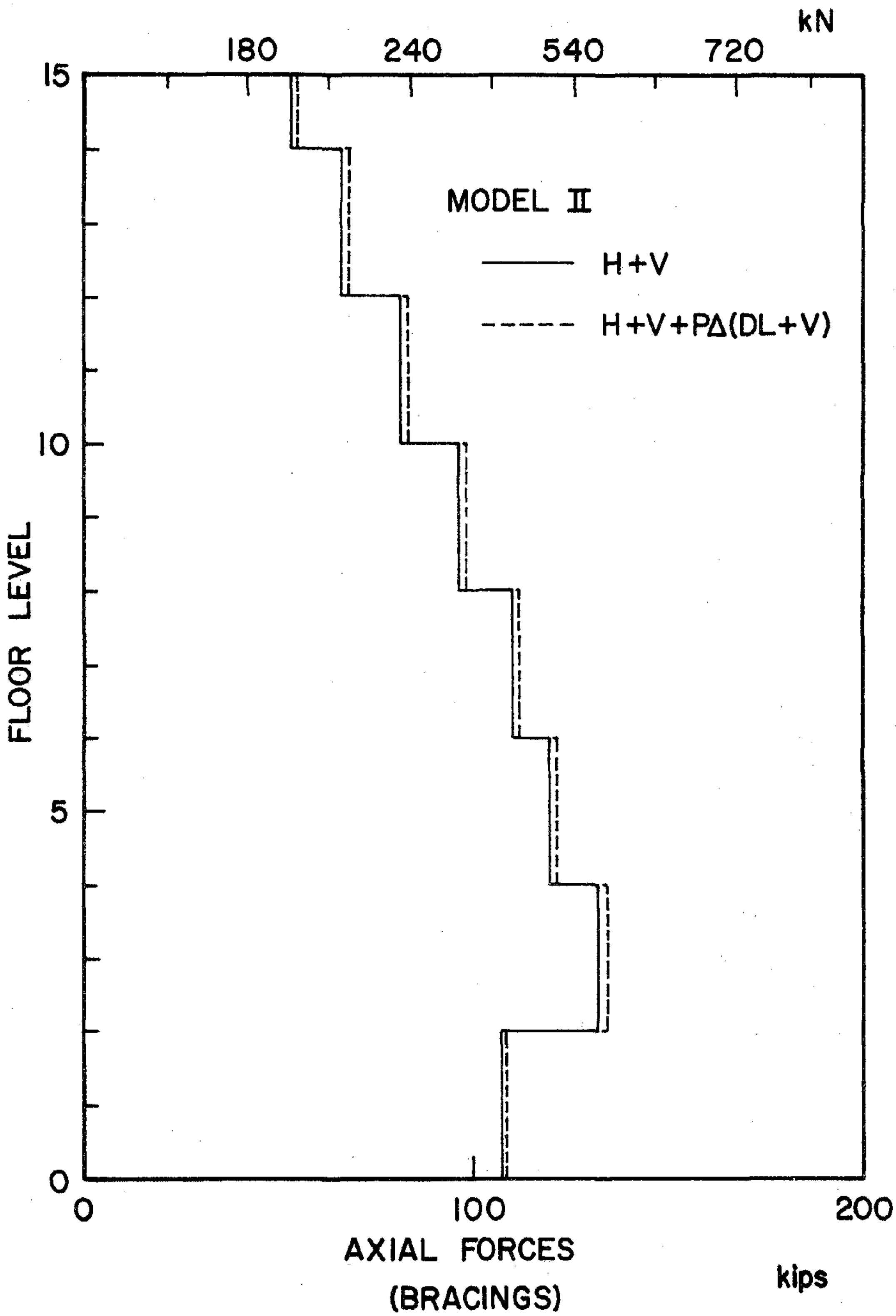


Figure 98. Axial Forces in Bracings of Example 12.

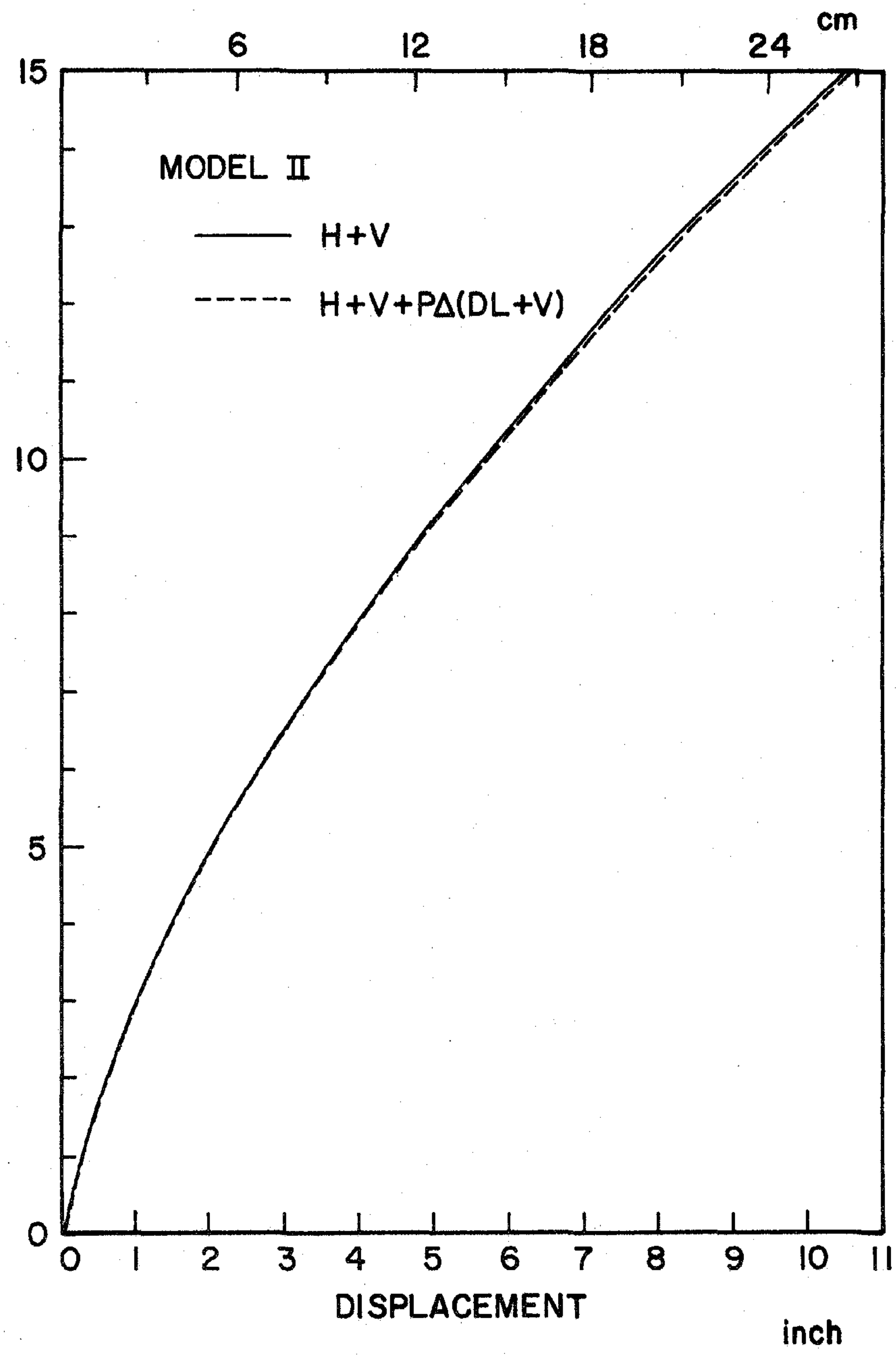


Figure 99. Displacements at Floor Levels of Example 12.

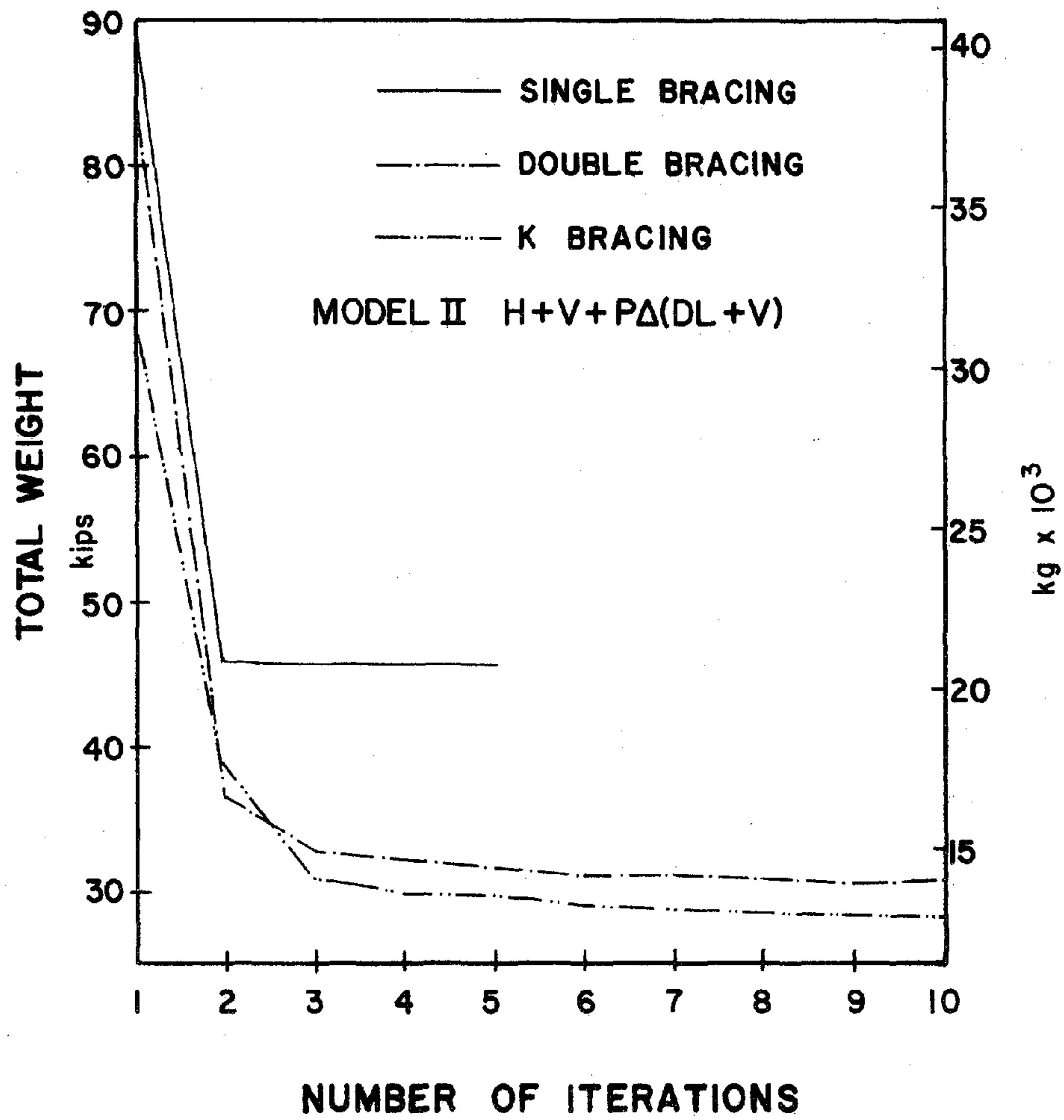


Figure 100. Comparison of Optimum Weights of Braced Systems of Example 13.

systems of Examples 10, 11, and 12. The K-braced system is the lightest structure, which is about 40 and 10% lower than the weights of single-braced and double-braced systems respectively.

Comparisons of the moments of inertia of the columns and girders as well as the areas of the bracings corresponding to the three systems have been replotted in Figs. 101 through 103. Other comparisons of normalized member properties of moment of inertia and area, normalized stresses, and moments and axial forces are shown in Figs. 104 through 112. Figure 113 shows the floor displacements of the three braced systems for which the displacement responses are almost identical and violate the allowable deflection at the top floor.

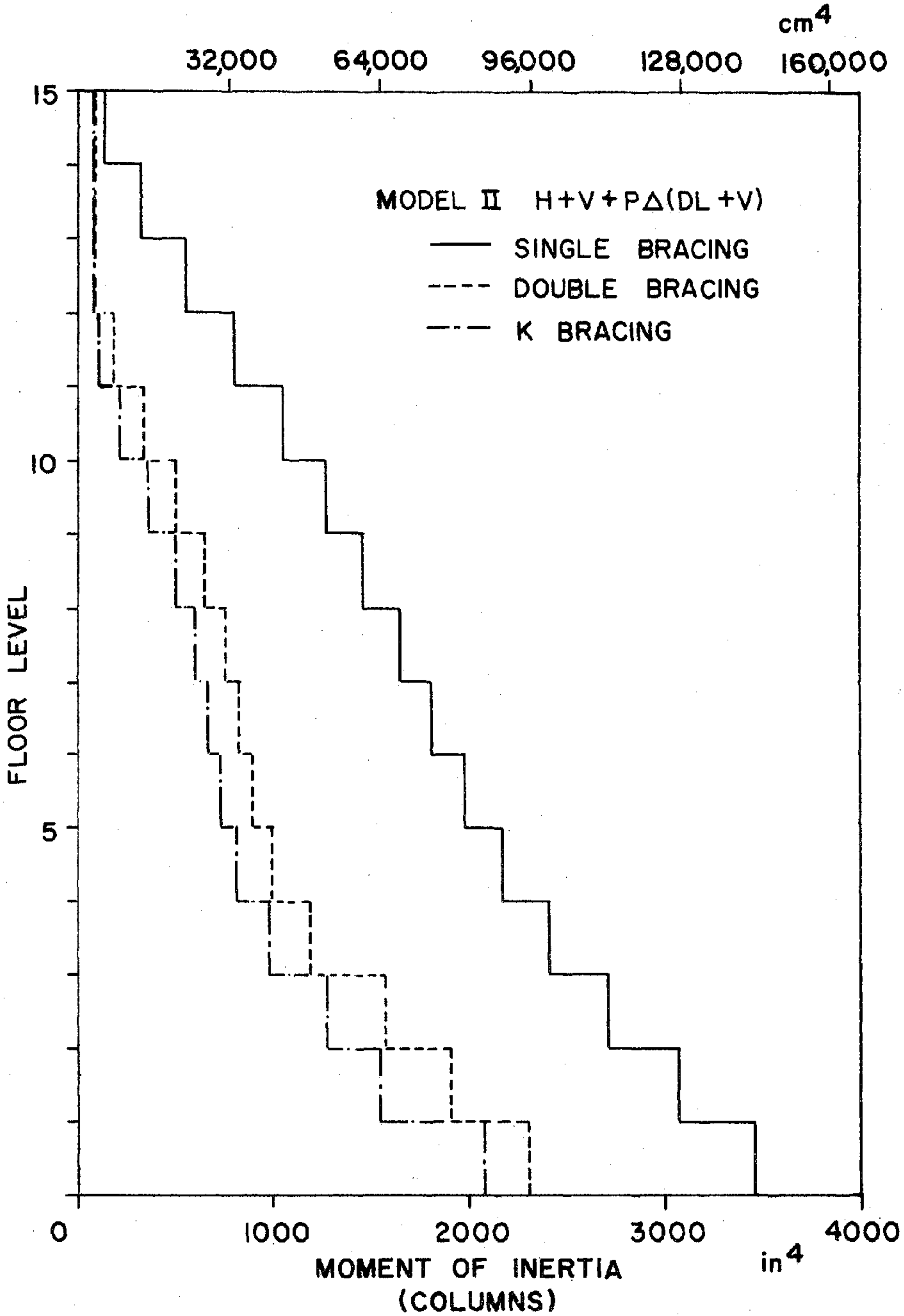


Figure 101. Comparison of Moment of Inertia of Columns of Braced Systems of Example 13.

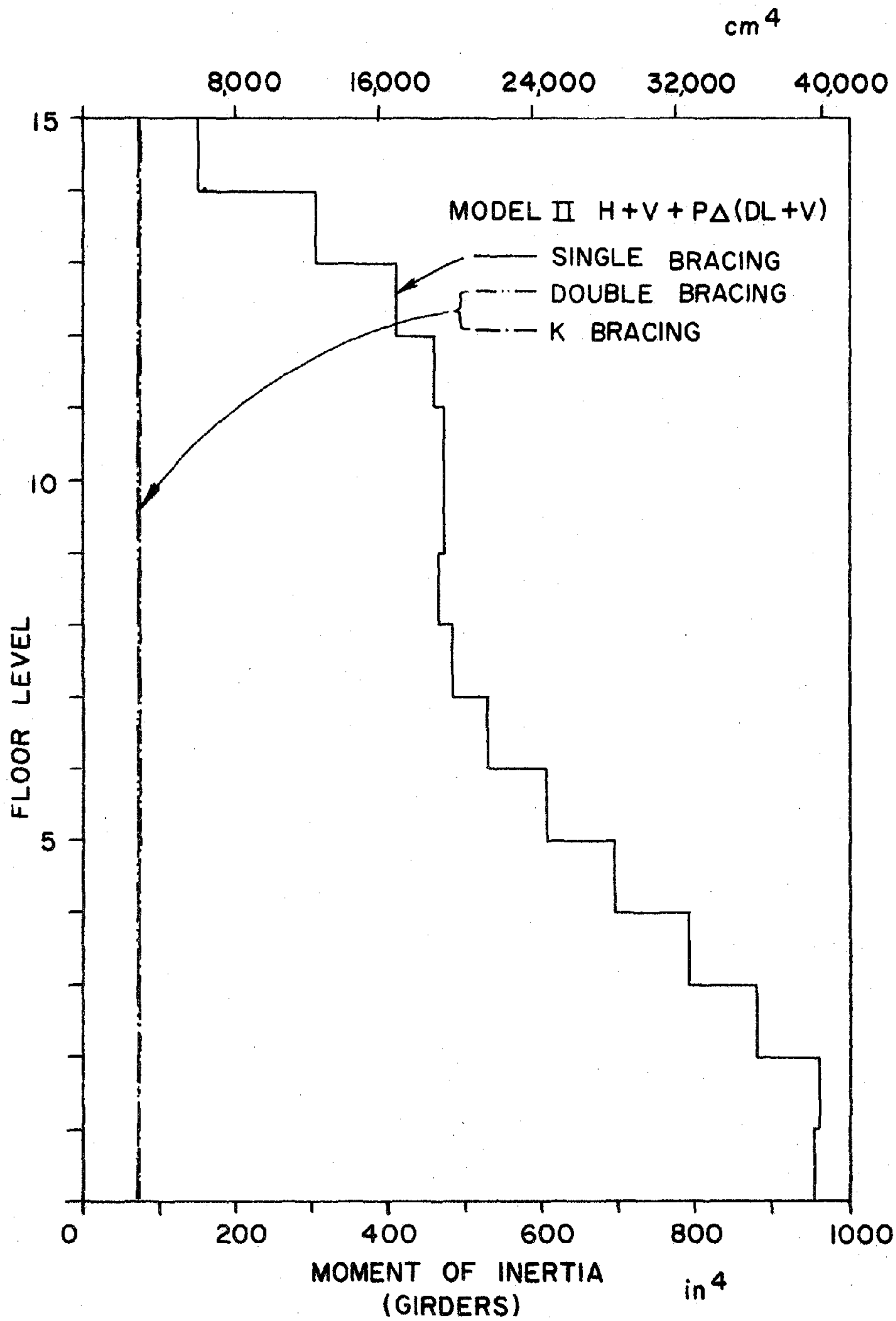


Figure 102. Comparison of Moment of Inertia of Girders of Braced Systems of Example 13.

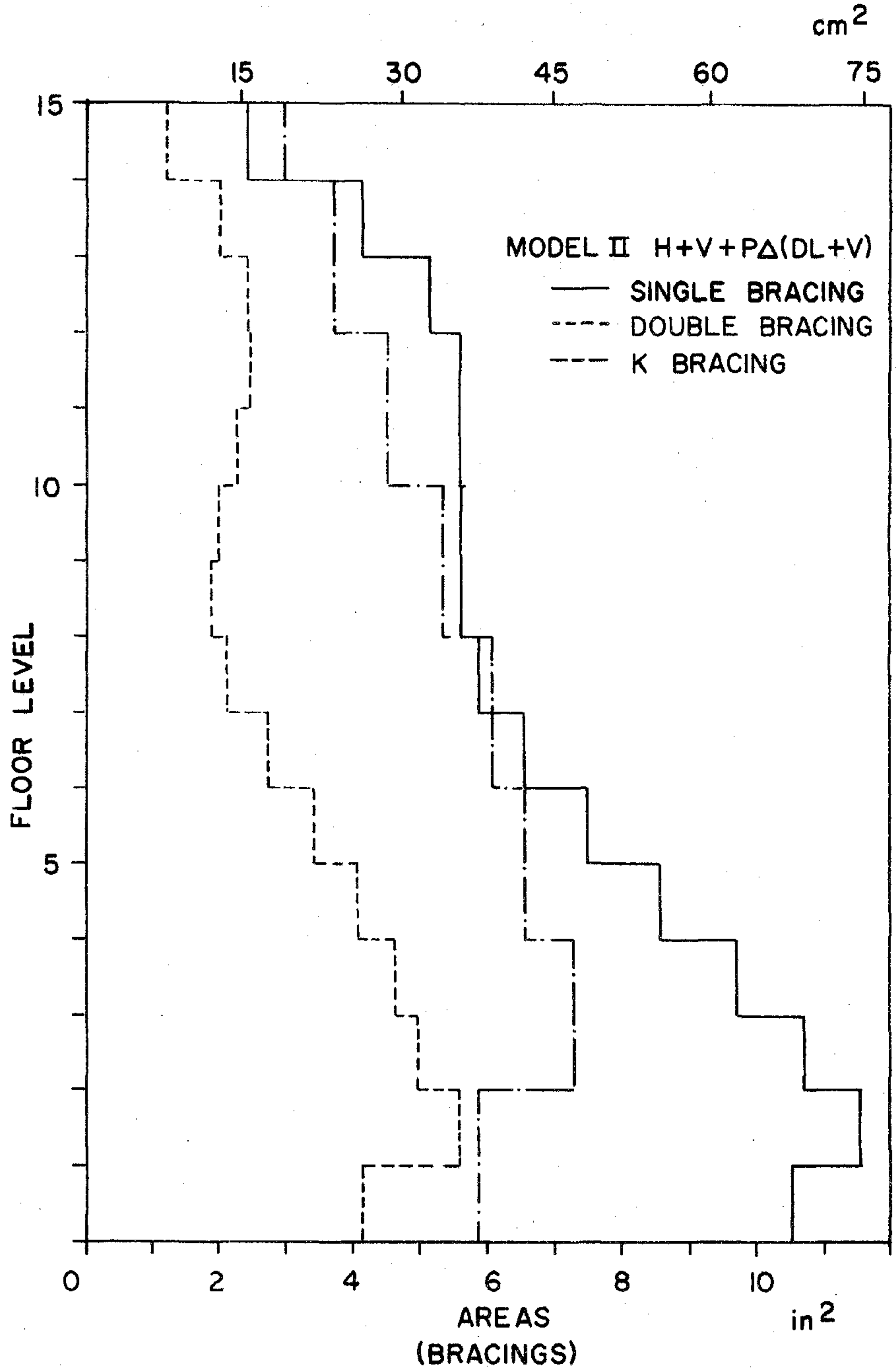


Figure 103. Comparison of Areas of Bracings of Braced Systems of Example 13.

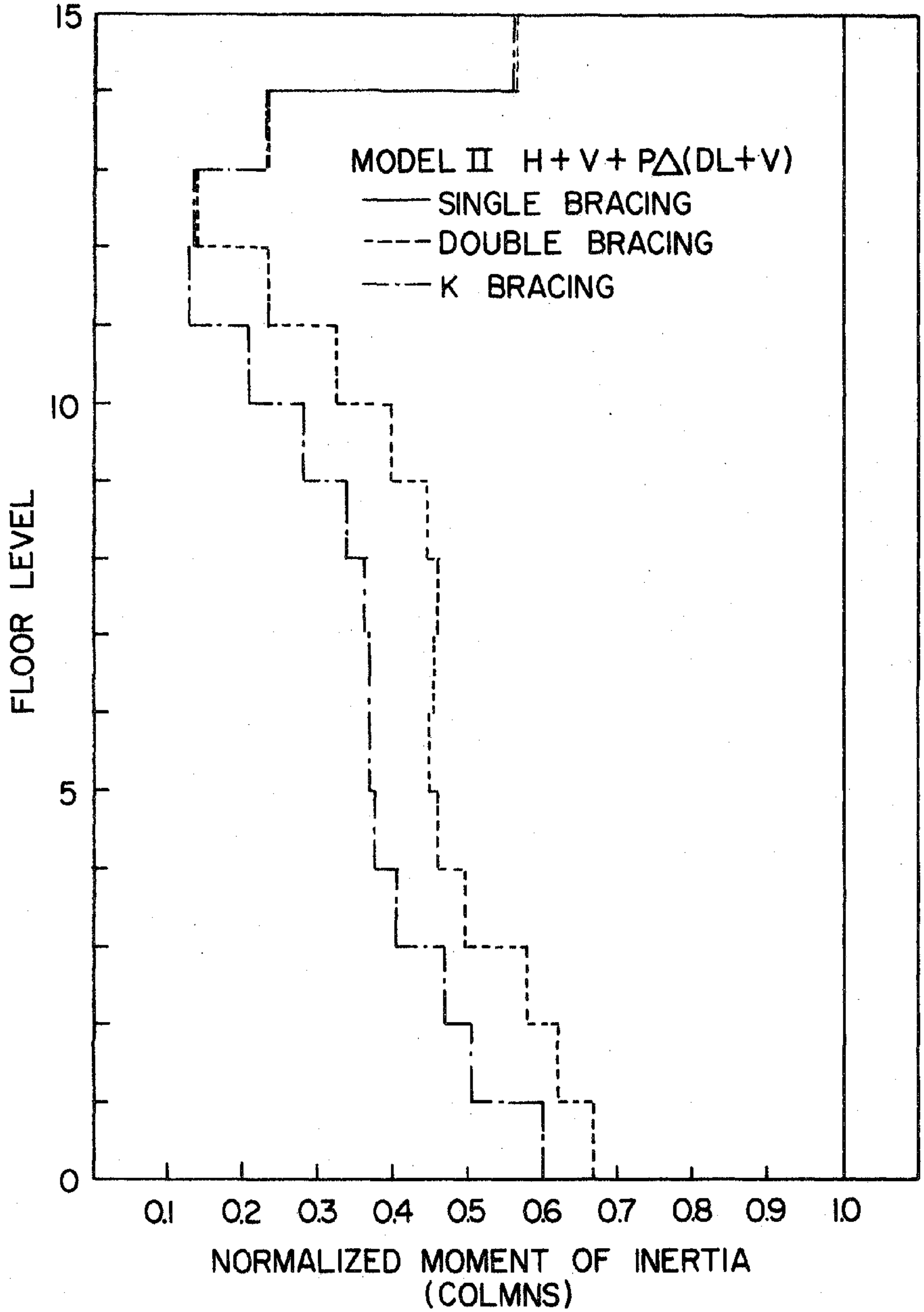


Figure 104. Comparison of Normalized Moment of Inertia of Columns of Example 13.

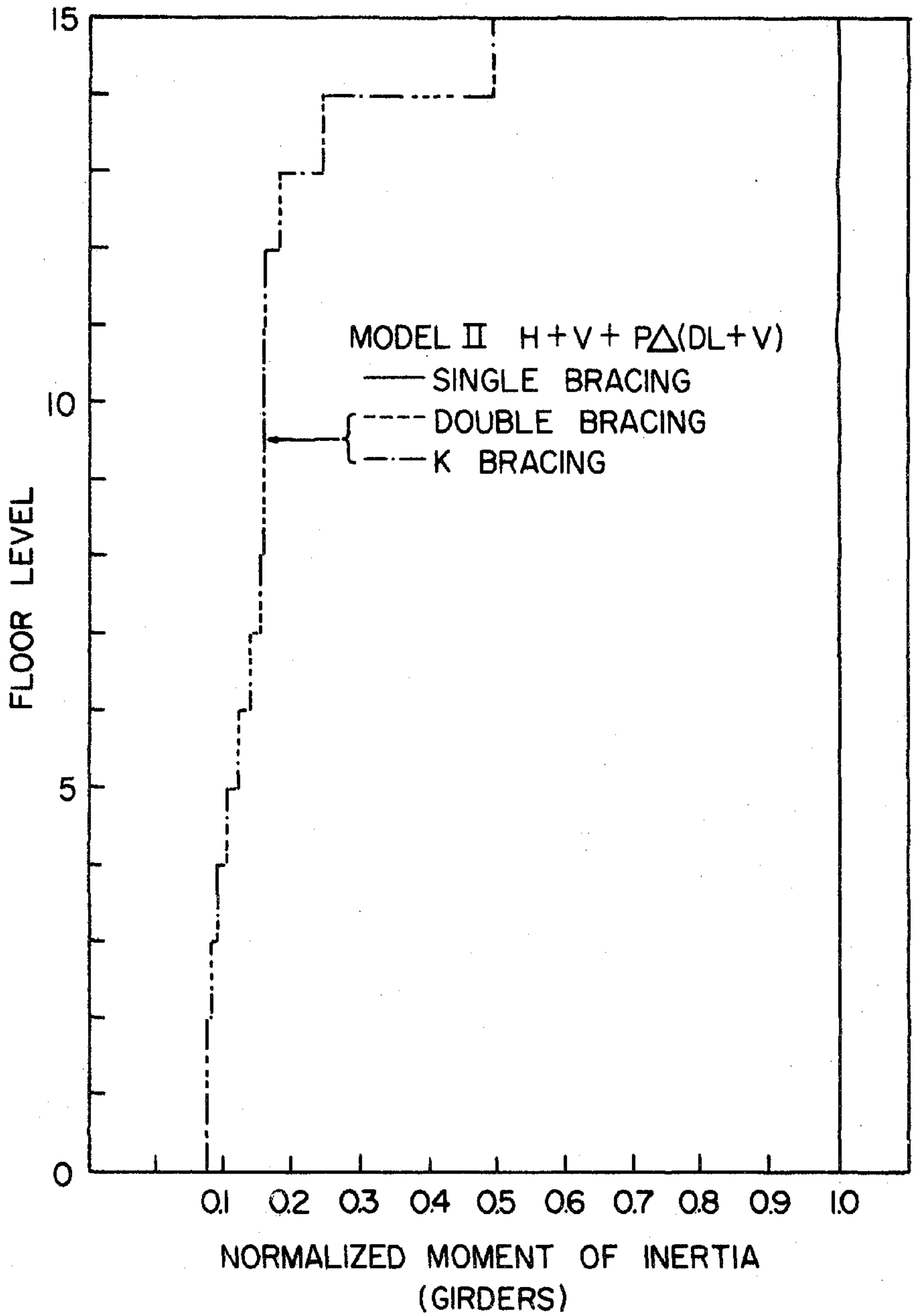


Figure 105. Comparison of Moment of Inertia of Girders of Example 13.

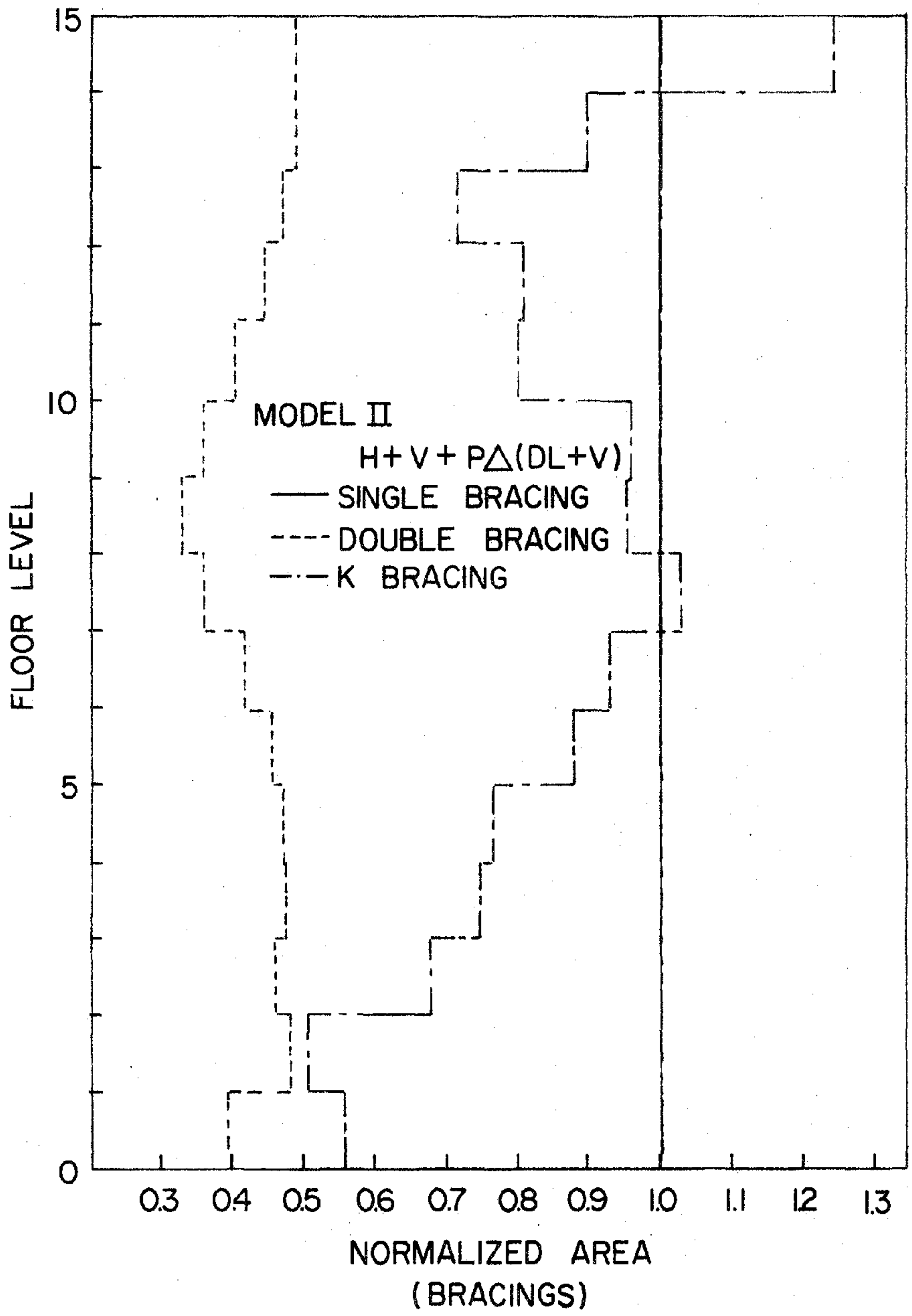


Figure 106. Comparison of Normalized Bracing Areas of Example 13.

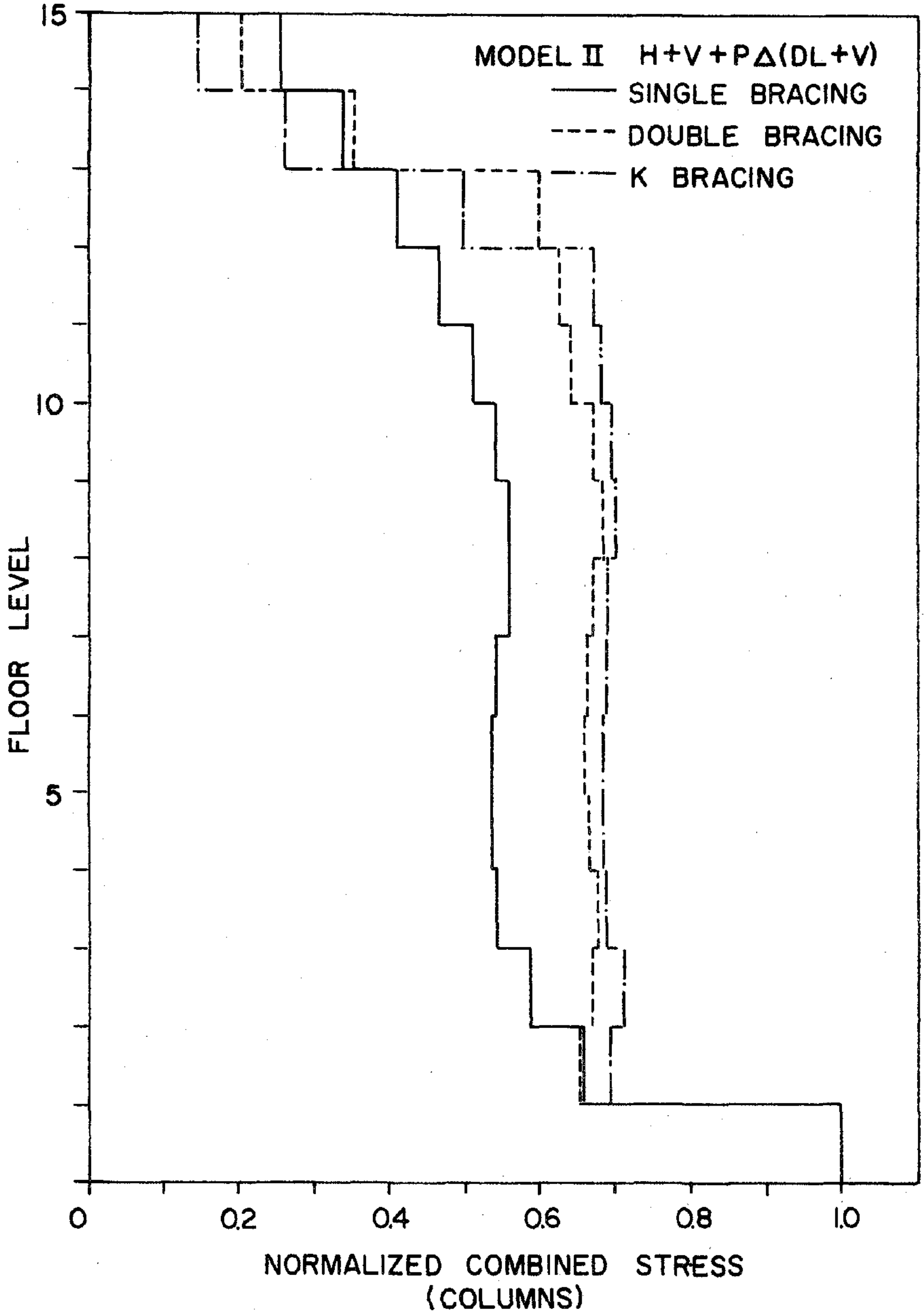


Figure 107. Comparison of Normalized Stress of Columns of Example 13.

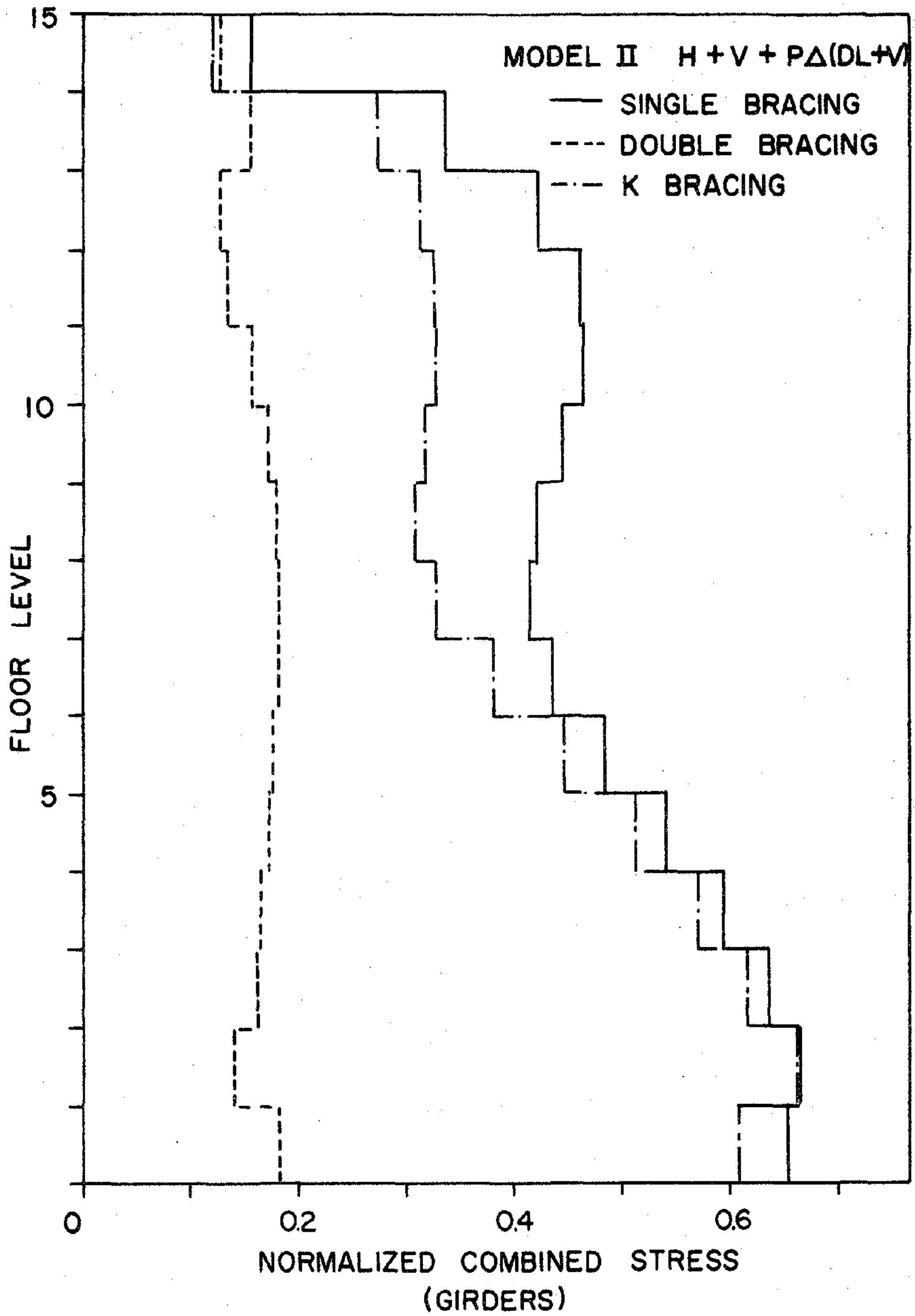


Figure 108. Comparison of Normalized Stress of Girders of Example 13.

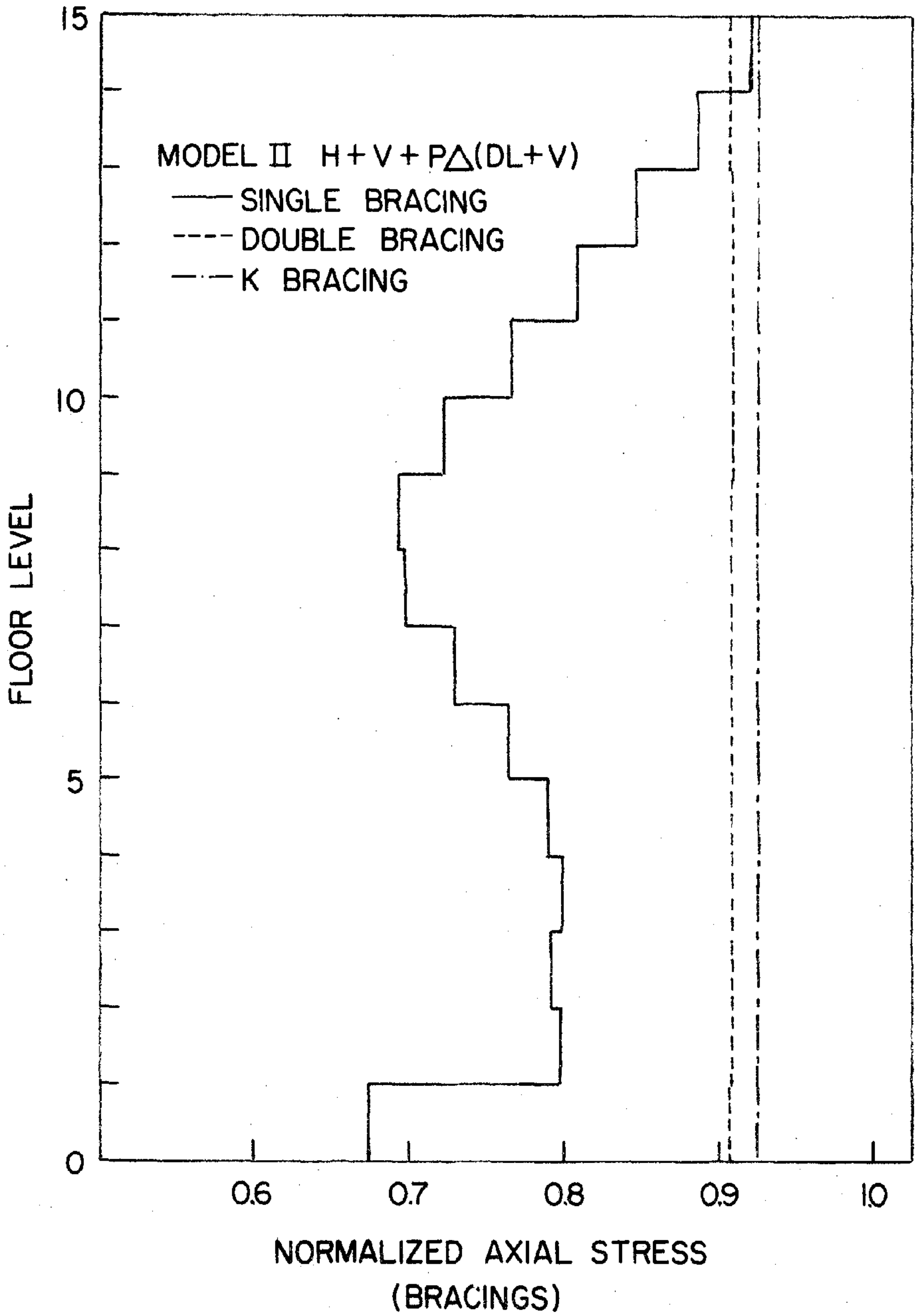


Figure 109. Comparison of Normalized Stress of Bracings of Example 13.

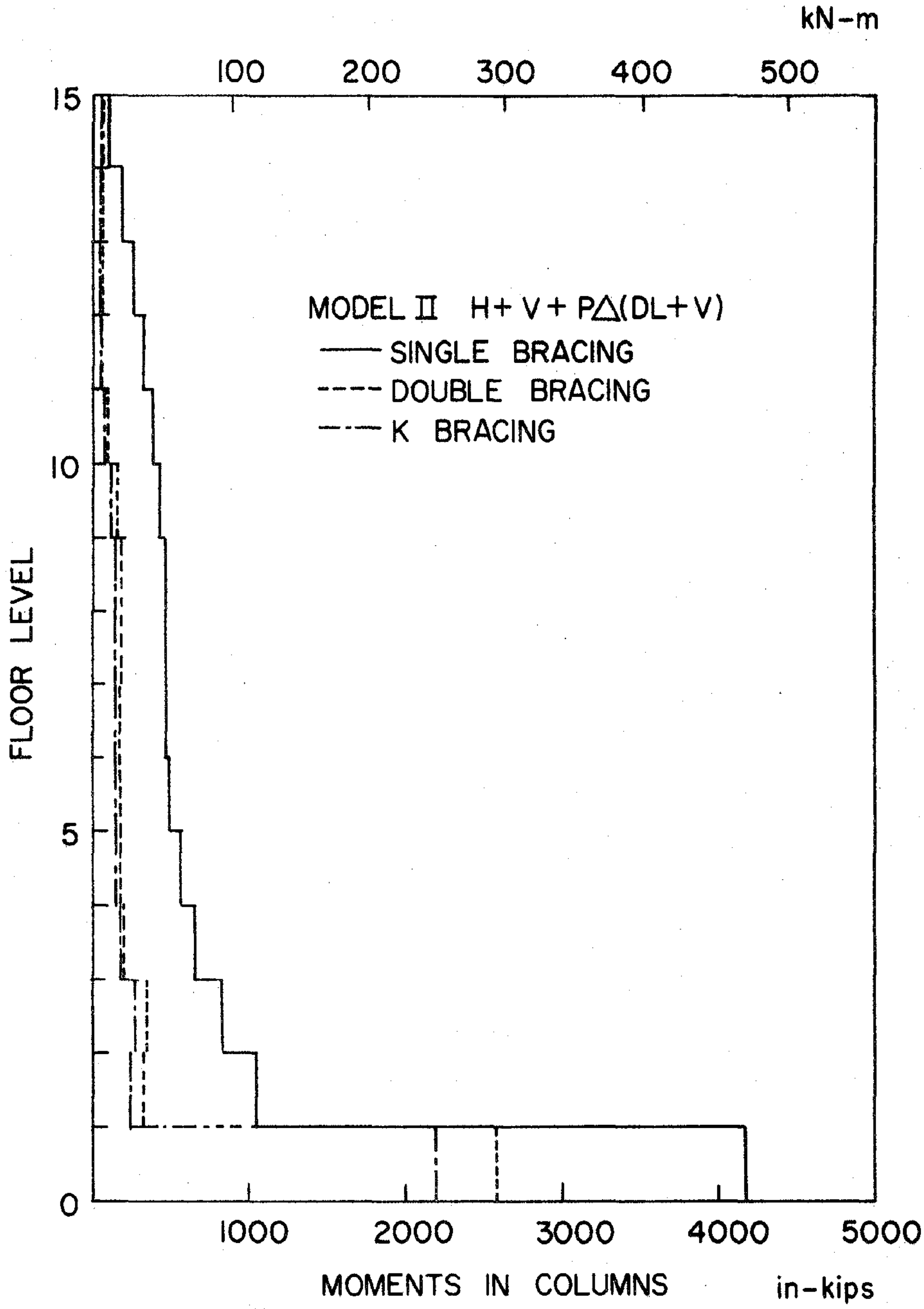


Figure 110. Comparison of Moments in Columns of Example 13.

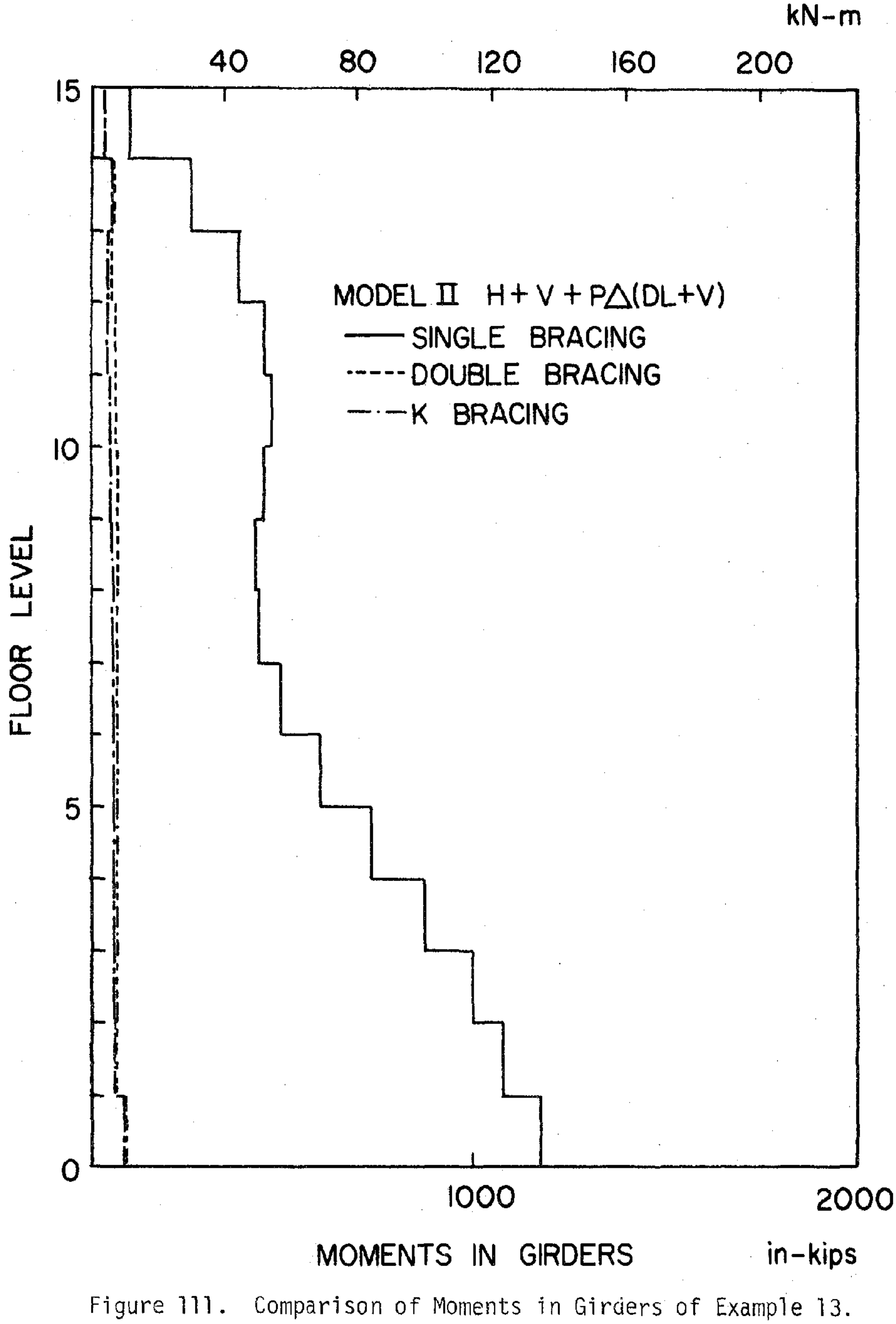


Figure 111. Comparison of Moments in Girders of Example 13.

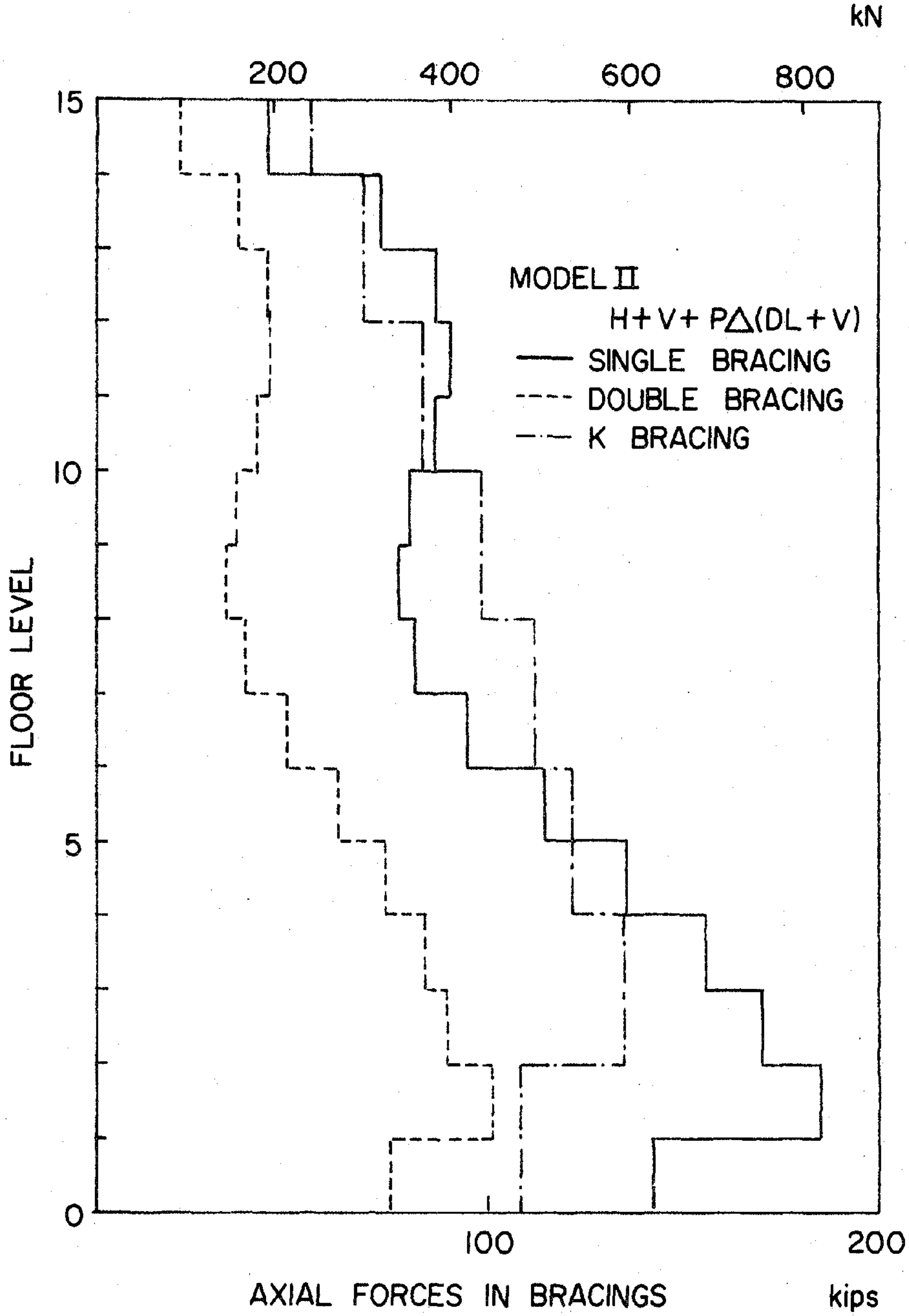


Figure 112. Comparison of Axial Forces of Bracings of Example 13.

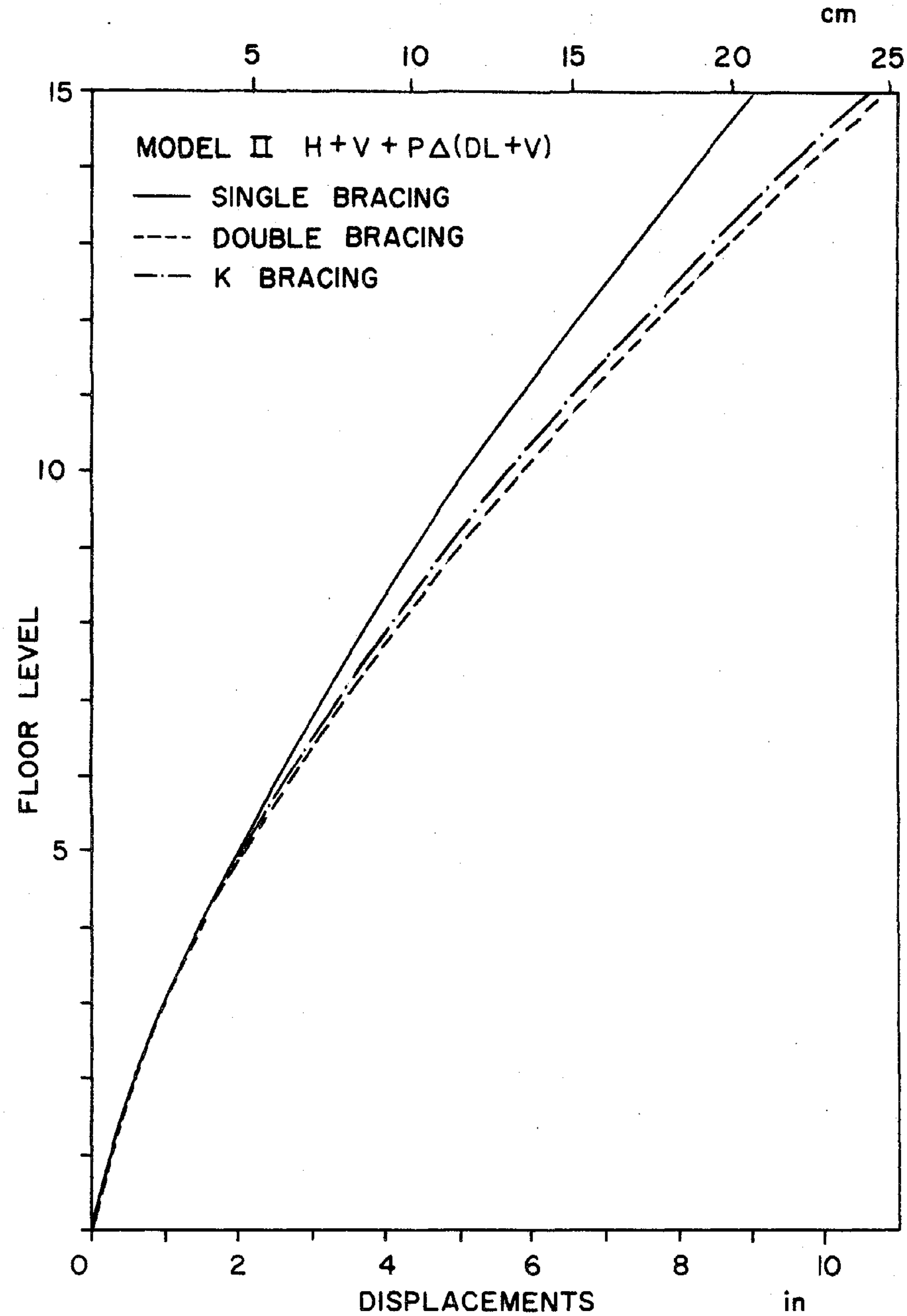


Figure 113. Comparison of Floor Displacements of Example 13.

X. REVIEW AND CONCLUSIONS

A. REVIEW

In this report, the effect that ground motions have on the relative stiffness requirements, on the overall stiffness distribution at critical regions, and on the entire system of various plane structures (a component of 3-D symmetric structures) has been examined. The structural systems studied are trusses, unbraced, single-braced, double-braced, and K-braced steel frameworks for which the constituent members are bar elements for bracings and truss-members and beam-column elements for columns and girders. The beam-column elements are either the built-up sections or the hot-rolled wide flange sections that are available in the AISC Steel Construction Manual. The structures can be subjected to static loads and dynamic forces as well as to horizontal and vertical ground motions. The dynamic forces and the seismic excitations can be used on a basis of either direct integrations or response spectra. In addition, the equivalent lateral seismic force recommended by the Uniform Building Code can also be used for the design purposes.

The optimum design method is based on an optimal criterion and a recursion relation for which the behavior constraints of static and dynamic displacements and stiffnesses as well as the constraints of natural frequencies are presented in detail. Other constraints are the desirable sizes of the members and the limitation on the difference between the maximum and minimum moments of inertia of a system. The method is considered to be more advantageous than any of the other optimization methods currently in vogue, because the number of iterations required to converge on an optimum design is independent of the number of variables in the problem. Thus, the method can be applied practically to the design of large structural systems.

The structural formulation is based on the displacement method, and consideration is given to the consistent mass formulation and the second-order effect resulting from static and dynamic forces acting axially on the columns. It is postulated that the columns and girders have axial and bending deformations, thus each node of a structural system has three degrees of freedom. Various displacement constraints can be applied to the individual nodes with any specific numbers. A sophisticated computer program, ODSEWS (Optimum Design of Static, Earthquake, and Wind Structures), has been developed for the design of static loads, dynamic forces, and seismic excitation as well as for any combination of these. The program is based on an IBM 370/168 for which the input and output formats are detailed in Appendix C.

B. CONCLUSIONS AND REMARKS

The following is a summary of the important conclusions determined from this investigation:

(1) The use of a response spectrum in conjunction with a root-mean-square technique can yield very satisfactory design results compared to those resulting from application of an exact response method, which is based on numerical integrations.

(2) The use of wind forces as static loads in the design can yield a much lighter structure than one that is based on actual wind forces.

(3) The distribution of the moments of inertia of the columns of an unbraced system shows that the moments of inertia are relatively small at the top floor and increase almost linearly from the top to the second floor and abruptly increases at the first floor. The distribution is consistent for both constraint conditions (stress constraints and stress displacement constraints) and the four loading of cases (a) through (d).

(4) The distribution of the moments of inertia of the girders of an unbraced system indicates that the moments of inertia of a rigid system (based on both stress and displacement constraints) are relatively small at the first floor, increase suddenly at the second and gradually decrease from the second to the sixth floor. It becomes practically constant from the sixth to the 12th and significantly decreases from the 12th to the top floor. The moments of inertia of the girders of a flexible system (based on the stress constraints only) are also small at the first floor and increase suddenly from the second to the fifth. They decrease almost linearly from the second to the 13th and then decrease abruptly from the 13th to the top.

(5) The restraint of the ratios of the maximum moments of inertia to the minimum of moments of inertia does not change the distribution pattern as discussed in (3) and (4) above but increases some moments of inertia to the members having small stresses because of the side constraint of I_{max}/I_{min}. The increase depends on the magnitude of the restraint.

(6) The distribution of the moments of inertia of the columns of single-braced and double-braced systems is similar to those of unbraced frames except that the braced-systems require much smaller moments of inertia because bracings are used in the frames. The moments of inertia of the supporting columns and those of the first floor girders of both braced systems are not abruptly changed as the case of unbraced systems. The

girders of a double-braced system from the first to the top are actually governed by the lower bound of the design variables.

(7) The bracings of a single-braced system are relatively large for the first floor and are considerably smaller at the top. The distribution of double-bracings is similar to that of singles, and the areas of the individual doubles are about half of those of corresponding single bracings.

(8) The distribution of the moments of inertia of columns and girders of a K-braced system is similar to that of single-braced and double-braced systems. Because the K-bracings intersect at the midspan of girders, the longitudinal structural stiffness is increased considerably and consequently reduces the effect of the vertical ground motions on the structural design. The K-braced system is the lightest structure among all the structures investigated.

(9) Even though the first mode is the most significant in dynamic response, the first three modes are needed to provide an adequate design for structures subject to a horizontal ground motion only. However, the first five modes are necessary for designing structures subject to multicomponent earthquake motions. Model II can describe the response behavior of multicomponent ground motions better than Model I.

(10) The Uniform Building Code underestimates the total shear forces compared to those resulting from the use of either actual earthquake records or the average acceleration spectrum. Therefore the design based on the equivalent seismic forces provided

by the Code yields a much lighter weight than that obtained by using the spectrum method.

(11) A heavier structural design is needed to withstand the effect of a vertical ground motion that is combined with a horizontal earthquake and a second-order P-Δ effect than for a horizontal movement acting alone.

(12) For tall buildings, the design is mostly controlled by the displacement constraint at the top floor and has a few members reaching fully stressed.

(13) In the design, the material savings are apparently of great interest. However, the most important point is the scientific approach of determining the stiffness distribution of various braced and unbraced systems. The scientific design method, which can always provide the required stiffnesses and satisfy the designer's given constraints, is considered to be much better than the conventional design technique based on the trial and error process by using either the desk calculators or the analysis computer programs.

(14) The ODSEWS program is sophisticated and versatile that can be used for the design of various systems subjected to static loads, wind forces, and multicomponent seismic input, as well as any combination of these.

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APPENDICES

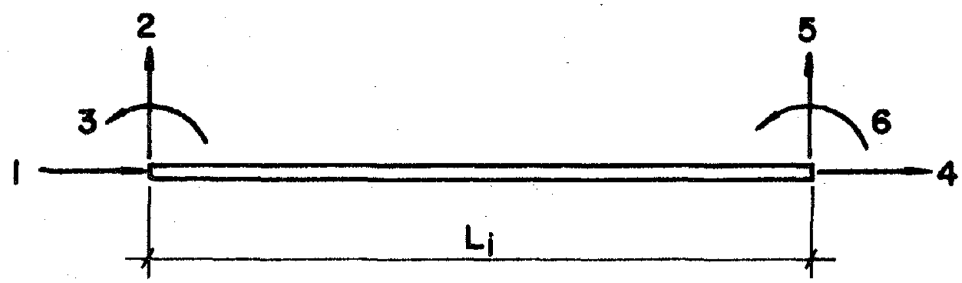
APPENDIX A. MATRICES OF STIFFNESS, MASS, GEOMETRIC STIFFNESS, AND CROSS SECTIONAL PROPERTIES OF TYPICAL CONSTITUENT MEMBERS

STIFFNESS, GEOMETRIC, AND MASS MATRICES

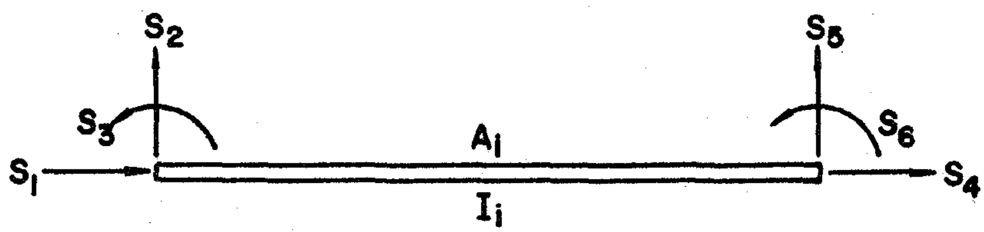
The coordinates and positive forces and deformations of the typical constituent members of a structural system are shown in Fig. 93. The typical members are beam-column elements of either girders or columns and two-force bar elements of bracings. The matrices of stiffness, $\underline{K_{si}}$, mass, $\underline{M_{si}}$, and geometric stiffness, $\underline{K_{gi}}$, of a girder or column are:

$$\underline{K_{si}} = \frac{EI_i}{L_i}\begin{bmatrix} A_i/I_i & 0 & 0 & -A_i/I_i & 0 & 0 \\ 0 & 12/L_i^2 & 6/L_i & 0 & -12/L_i^2 & 6/L_i \\ 0 & 6/L_i & 4 & 0 & -6/L_i & 2 \\ -A_i/L_i & 0 & 0 & A_i/I_i & 0 & 0 \\ 0 & -12/L_i^2 & -6/L_i & 0 & 12/L_i^2 & -6/L_i \\ 0 & 6/L_i & 2 & 0 & -6/L_i & 4 \end{bmatrix}$$

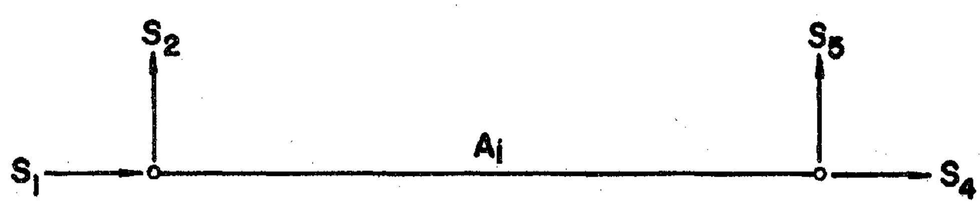
$$\underline{M_{si}} = \frac{\rho_i A_i L_i}{g}\begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 13/35 & 11L_i/210 & 0 & 9/70 & -13L_i/420 \\ 0 & 11L_i/210 & L_i/105 & 0 & 13L_i/420 & -L_i/140 \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 9/70 & 13L_i/420 & 0 & 13/35 & -11L_i/210 \\ 0 & -13L_i/420 & -L_i^2/140 & 0 & -11L_i/210 & L_i^2/105 \end{bmatrix}$$



(a) Coordinates of a Planar Member



(b) Positive Forces and Deformations for both Girders and Columns



(c) Positive Forces and Deformations for Bracings

Figure 93. Positive Forces and Deformations of Typical Member

$$
\underline{K}_{gi} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6/(5L_i) & 1/10 & 0 & -6/(5L_i) & 1/10 \\ 0 & 1/10 & 2L_i/15 & 0 & -1/10 & -L_i/130 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6/(5L_i) & -1/10 & 0 & 6/(5L_i) & -1/10 \\ 0 & 1/10 & -L_i/30 & 0 & -1/10 & 2L_i/15 \end{bmatrix}.
$$

The matrices of stiffness, $\underline{K_{si}}$, and mass, $\underline{M_{si}}$ of a bracing member are:

$$
\underline{K}_{si} = \frac{EA_i}{L_i} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\underline{M}_{si} = \frac{\rho_i A_i L_i}{g} \begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 1/6 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

The cross sectional property matrix, $\underline{B_i}$, of a beam-column element is

$$
\underline{B}_i = \begin{bmatrix} 1/A_i & 0 & -1/S_i & 0 & 0 & 0 \\ 0 & 1/v_i & 0 & 0 & 0 & 0 \\ 1/A_i & 0 & 1/S_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/A_i & 0 & -1/S_i \\ 0 & 0 & 0 & 0 & 1/v_i & 0 \\ 0 & 0 & 0 & 1/A_i & 0 & 1/S_i \end{bmatrix}.
$$

For bracings, the cross sectional property matrix becomes

$$
\underline{B}_i = \begin{bmatrix} 1/A_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/A_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
$$

The symbols used in this appendix are:

A_i = cross sectional area of member i,

E = modulus of elasticity,

g = gravity acceleration,

I_i = moment of inertia of member i,

L_i = length of member i,

S_i = section modulus of member i,

v_i = shear flow of member i for built-up sections only, and

ρ_i = mass density of member i.

APPENDIX B. EIGENSOLUTION TECHNIQUE

DESCRIPTION OF STURM SEQUENCE PROPERTY AND BISECTION PROCEDURE

The eigensolution technique employed in this research possesses two distinguishing characteristics: the eigenvalues and eigenvectors of any desired modes can be obtained directly without using any of the remaining modes, and the determination of any eigenvalues, whether fundamental or higher modes, does not require inverting the stiffness matrix. The method was presented early by Peters and Wilkinson* and has the Sturm sequence property for the following equation:

$$\underline{A}\vec{x} = \lambda\underline{I}\vec{x},$$

in which λ is the eigenvalue, $\vec{x}$ the eigenvector, and $\underline{I}$ the identity matrix.

When $\underline{A}$ is real and symmetric, the leading principal minors of $(\underline{A} - \lambda\underline{I})$ form a Sturm sequence; that is, the number of eigenvalues greater than λ is equal to the number of agreements in sign between consecutive members of the sequence, P_r, for $r = 0, 1, \ldots, n$. The term P_r is defined as

$$P_r = \det\,(\underline{A_r} - \lambda\underline{I}),$$

in which $\underline{A_r}$ is the leading principal submatrix of the order r of $\underline{A}$. The term P_o is assumed to be equal to one.

---

*Peters, G. and Wilkinson, J.H., "Eigenvalues of $Ax = \lambda Bx$ with Band Symmetric A and B," Computer Journal, Vol. 12, (1969), 398-404.

Peters and Wilkinson have shown that the Sturm sequence property is valid for the problem in the following form:

$$\underline{A}\vec{x} = \lambda\underline{B}\vec{x}.$$

When the matrix $\underline{B}$ is symmetric and positive definite, the sign of det $(\underline{A}_r - \lambda\underline{B}_r)$ is the same as that of det $(\underline{A}_r - \lambda\underline{I})$.

By using the Sturm sequence property in conjunction with a simple bisection procedure, the eigenvalues of any desired modes can be obtained, and the associated eigenvectors can be determined by using an iteration procedure based on the following recurrence relation:*

$$(\underline{A} - \lambda_i\underline{B})\vec{x}_i^{v+1} = \underline{B}\vec{x}_i^{v},$$

in which the subscript i signifies the mode number, and v refers to the cycle of iteration.

NUMERICAL ILLUSTRATION

Let $\underline{A}$ and $\underline{B}$ be symmetric and positive definite as shown below:

$$\underline{A} = \begin{bmatrix} 60 & -60 & 0 \\ -60 & 180 & -120 \\ 0 & -120 & 300 \end{bmatrix}$$

and

$$\underline{B} = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.0 \end{bmatrix}.$$

---

*Wilkinson, J.H., The Algebraic Eigenvalue Problem, Clarendon Press, Oxford, 1965.

Assume that it is necessary to find the second mode of the eigenvalue, and it is also assumed that all the eigenvalues are in the interval (-150, 150).

Starting with $\lambda = 0$, the leading principal minors of $(\underline{A} - \lambda\underline{B})$ are calculated as

$$P_o = 1,$$

$$P_1 = 60,$$

$$P_2 = 60(180) - (-60)(-60) = 7200,$$

and

$$P_3 = 60[180(300) - (-120)(-120)] - (-60)[-60(300)] = 1296000.$$

The number of agreements in sign between consecutive members of the sequence, P_r, $r = 0,1,2,3$, defined as $S(\lambda)$, is three. Therefore, there are three eigenvalues that are greater than $\lambda = 0$, which means that all the eigenvalues lie within (0, 150).

By employing a simple bisection procedure, one can find the new value of λ by using

$$\lambda = \frac{0 + 150}{2} = 75.$$

With the new value of $\lambda = 75$, one can again calculate the leading principal minors of $(\underline{A} - \lambda\underline{B})$ and then find $S(\lambda) = 2$, which indicates that there are two eigenvalues greater than 75. In order to obtain the second mode, one should use a λ greater than 75 for the next calculation. This process of calculating is continued until the difference between the upper and lower values of the interval is less than some appropriate value which is equal to 0.001 for this example. The solution of this example is $\lambda = 96.396$. The detailed calculations are given in Table IX.

It should be noted that only the signs of the leading principal minors of $(\underline{A} - \lambda\underline{I})$ are of interest and not their magnitudes. The computer program has a simple method* of evaluating the leading principal minors of a square matrix. It is a variation of Gaussian elimination with partial pivoting.

---

*Wilkinson, J.H., ibid.

TABLE IX. NUMERICAL ILLUSTRATION OF EIGENVALUES

λ	P_0	P_1	P_2	P_3	S( )	Interval
0.0	1.0	60.000	7200.000	1296000.000	3	0.0 -150.000
75.00	1.0	-15.000	-4612.500	-475875.000	2	75.000-150.000
112.500	1.0	-52.500	-4190.625	441703.125	1	75.000-112.500
93.750	1.0	-33.750	-4928.906	-68502.000	2	93.750-112.500
103.125	1.0	-43.125	-4691.602	181162.312	1	93.750-103.125
98.438	1.0	-38.438	-4843.211	54043.625	1	93.750- 98.438
96.094	1.0	-36.094	-4894.297	-7916.687	2	96.094- 98.438
97.266	1.0	-37.266	-4870.812	22906.000	1	96.094- 97.266
96.680	1.0	-36.680	-4883.070	7453.500	1	96.094- 96.680
96.387	1.0	-36.387	-4888.812	-242.125	2	96.387- 96.680
96.533	1.0	-36.533	-4885.974	3603.062	1	96.387- 96.533
96.460	1.0	-36.460	-4887.402	1679.875	1	96.387- 96.460
96.423	1.0	-36.423	-4888.109	718.750	1	96.387- 96.423
96.405	1.0	-36.405	-4888.461	238.062	1	96.387- 96.405
96.396	1.0	-36.396	-4888.637	-1.938	2	96.396- 96.405
96.400	1.0	-36.400	-4888.551	117.937	1	96.396- 96.400
96.398	1.0	-36.398	-4888.594	57.938	1	96.396- 96.398
96.397	1.0	-36.397	-4888.617	27.875	1	96.396- 96.397
96.396	1.0	-36.396	-4888.625	12.688	1	96.396- 96.396

APPENDIX C. ODSEWS COMPUTER PROGRAM

INPUT DATA

I. TOTAL NUMBER OF STRUCTURES (I5)

Columns	Entry
1- 5	Total number of structures to be designed.

II. CONTROL CARD (8I5, 10A4)

1- 5	Total number of members.
6- 10	Total number of nodal joints.
11- 15	Total number of restrained degrees of freedom.
16- 20	Total number of loading conditions.
21- 25	Displacement constraint code: Eq. 0; No displacement constraints, Eq. 1; Displacement constraint is the same for all nodal joints, and Eq. 2; Displacement constraints can vary per nodal joints.
26- 30	Design type code: Eq. -1; Static design, Eq. 0; Static + dynamic design, and Eq. 1; Dynamic design.
31- 35	Maximum number of iterations.
36- 40	Type of section Eq. 0; Built-up sections Eq. 1; Wide flange sections based on the AISC Manual
40- 80	User identification.

III. MEMBER-INFORMATION CARDS

a. Girder-Column Property Card

(i) BUILT-UP SECTION (2F10.0, 2F5.0, 5F10.0)

Columns	Entry
1- 10	Modulus of elasticity.
11- 20	Specific weight.
21- 25	Width.
26- 30	Initial depth.
31- 40	Maximum depth.
41- 50	Minimum depth.
51- 60	Maximum t_f/d.
61- 70	Minimum t_f/d.
71- 80	Ratio of the minimum moment of inertia to the maximum moment of inertia.

(ii) WF SECTION (6F10.0)

1- 10	Modulus of elasticity.
11- 20	Specific weight.
21- 30	Initial moment of inertia.
31- 40	Maximum moment of inertia.
41- 50	Minimum moment of inertia.
51- 60	Ratio of the minimum moment of inertia to the maximum moment of inertia.

b. Bracing Property Card (5F10.0)

1- 10	Modulus of elasticity.
11- 20	Specific weight.
21- 40	Maximum area.
41- 50	Minimum area.

c. Girder Identification Cards (4I5, 2F10.0, I5)

Columns	Entry
1- 5	Identification number of structural numbers (numbering the independent variables first, the subsequent members are then numbered in the order of the common design variables).
6- 10	First node number (smaller nodal number at both ends).
11- 15	Second node number (larger nodal number at both ends).
16- 20	Member-type code (2 for girder).
21- 30	Allowable stress.
31- 40	Uniform nonstructural weight/unit length.
41- 45	Design variable number (use the number of the independent variable if that is the common design variable for this member).

d. Column Identification Cards (4I5, 2F10.0, 8I5/(16I5))

1- 5	Identification number.
6- 10	First node number (see Girder Identification).
11- 15	Second node number (see Girder Identification).
16- 20	Member-type code (1 for column).
21- 30	Allowable stress.
31- 40	Additional axial force (PΔ force).
41- 45	Design variable number (see Girder Identification)
46- 50	Number of members contributing their weights to this column (see Fig. 1 and Eq. (2.1)).
51- 55	Identification number of the member contributing its weight.
	Repeat for additional members.

e. Bracing Identification Cards (4I5, F10.0, 10X, I5)

Columns	Entry
1- 5	Identification number.
6- 10	First node number.
11- 15	Second node number.
16- 20	Member-type code (0 for bracing).
21- 30	Allowable stress.
31- 40	Blank.
41- 45	Design variable number (see Girder Identification).

IV. JOINT INFORMATION CARDS (I5, 2F10.0)

1- 5	Joint identification number.
6- 15	X-coordinate.
16- 25	Y-coordinate.

V. BOUNDARY CONDITION CARDS (16I5)

1- 5	Number of the degree of freedom of the node which is restrained. For node K, the degree of freedom numbers are 3*K-2, 3*K-1, 3*K.
6- 10	Repeat for additional nodes.

VI. STATIC LOADING CARDS

The following set of data is for one loading case. Repeating of this set of data is necessary for multiple loading cases.

a. Control Card (I5)

1- 5 Loading Parameter:

GE.0; Number of load components in the i^{th} loading condition.

LT.0; Equivalent seismic loads by UBC.

b. Loading Cards

(i) LOADING PARAMETER $\geq$0(3(F10.0, 2I5))

Columns	Entry
1- 10	Magnitude of load.
11- 15	Direction of load (X-dir. = 1, Y-dir. = 2, Z-dir. = 3).
16- 20	Number of node where the load is applied.
	Repeat for additional nodes.

(ii) LOADING PARAMETER < 0

(ii-1) INFORMATION CARD (I5, 4F5.0)

1- 5	Total number of stories.
6- 10	Earthquake-zone coefficient.
11- 15	Coefficient depending on the importance of structure (1.0 - 1.5).
16- 20	Coefficient determined by type of structure (0.67 - 1.33).
21- 25	Site period (0.5 sec. - 2.5 sec.).
26- 35	Gravity acceleration.

(ii-2) STORY-INFORMATION CARDS (2I5, 2F10.0)

1- 5	Story-level.
6- 10	Node number where the equivalent story-force is applied.
11- 20	Height of the story from the ground level.
21- 30	Total nonstructural weight for each story.

VI. DYNAMIC PROPERTY CARDS

Enter a blank control card for a static design and skip (b) through (d).

a. Control Card (4I5, F10.0)

Columns	Entry
1- 5	Option code: Eq. 1; Design using only significant modes.
	Eq. 2; Design using all specified modes.
6- 10	Type of mode superposition:
	Eq. -1; Direct superposition.
	Eq. 0; Root-mean-square superposition.
	Eq. 1; Absolute superposition.
11- 15	The first of the desired modes.
16- 20	The last of the desired modes.
21- 30	Gravity acceleration.

b. Damping Ratio Cards (8F10.0)

1- 10	Damping ratio (modal damping/critical damping) of the first of the desired modes.
11- 20	

c. Additional Nodal Mass Cards (8F10.0)

1- 10	Additional nodal mass at the j^{th} node.
11- 20	(j = 1, ... , joints).

d. Dynamic Loading Information Cards

The following set of data is for one loading case. Repeating of this set of data is necessary for multiple loading cases.

(i) CONTROL CARD (4I5, 3F5.0, I5, 10A4)

1- 5	Loading Parameter:
	GE.0; Number of load components in the i^{th} loading condition.
	LT.0; Earthquake motion in the i^{th} loading condition.

(i) CONTROL CARD (continued)

Columns	Entry
6- 10	Direction of earthquake motion:
	Eq. 1; Horizontal direction.
	Eq. 2; Vertical direction.
	LT.0; Both horizontal and vertical directions.
11- 15	Type of input acceleration data:
	Eq. 0; Time-history data.
	Eq. 1; Spectrum data.
16- 20	Number of acceleration data.
21- 25	Load multiplier or scale factor for horizontal motion.
26- 30	Scale factor for vertical motion.
31- 35	Time increment.
36- 40	Number of time steps.
41- 80	User identification.

(ii) LOADING CARDS (3(F10.0, 2I5))

This set of cards is needed only when the loading parameter is greater than zero.

1- 10	Magnitude of load.
11- 15	Direction of load.
16- 20	Number of node where the load is applied.

(iii) ACCELERATION DATA CARDS

(iii-1) PERIOD OR TIME CARDS (8F10.0)

1- 10	First data of period or time.
11- 20	Repeat for additional data.

(iii-2) HORIZONTAL ACCELERATION CARDS (8F10.0)

Columns	Entry
1- 10	First data of acceleration spectra or ground acceleration in the horizontal direction.
11- 20	Repeat for additional data.

(iii-3) VERTICAL ACCELERATION CARDS (8F10.0)

This set of cards is needed for the multicomponent ground motions.

1- 10	First data of acceleration spectra or ground acceleration in the vertical direction.
11- 20	Repeat for additional data.

VII. DISPLACEMENT CONSTRAINT CARDS

Either (a) or (b) is necessary to be used in according to the displacement constraint code.

a. Same Displacement Constraint for All Nodes (3F10.0)

1- 10	Allowable displacement in X-direction.
11- 20	Allowable displacement in Y-direction.
21- 30	Allowable displacement in Z-direction.

b. Displacement Constraints Vary

(i) CONTROL CARD (I5)

1- 5	Number of allowable displacements.

(ii) ALLOWABLE DISPLACEMENTS (3(F10.0, 2I5))

1- 10	Allowable displacement.
11- 15	Direction code (1 for X-dir., 2 for Y-dir. and 3 for Z-dir.).
16- 20	Node Number.

Note that one card is needed for three allowable displacements.

VIII. <u>FREQUENCY CONSTRAINT CARDS</u> (8F10.0)

Skip these cards for static design.

Columns	Entry
1- 10	The allowable natural frequency for the first of the desired modes.
11- 20	The allowable natural frequency for other nodes if any.

```
C
C     MAIN:    'ODSEWS', OPTIMUM DESIGN OF STATIC, EARTHQUAKE, AND WIND
C              STRUCTURES FOR STRESS, DISPLACEMENT, AND FREQUENCY
C              CONSTRAINTS.
C
      COMMON/DATA/FACTOR(3),XV(3),MTYPE(3),MOTYPE(3),NTYPE(3),NTIME(3),
     1          DEL(3),NNN
      COMMON/STIFF/BASEA,EE,E1,XG,AREAA
      COMMON/ABC/NZERO,NBEGIN,NDESIR,NITER,MDMAX,NNMAX
      COMMON/BBC/ISEC,TWD,DEPMAX,DEPMIN,TFDMAX,TFDMIN,RHO,RHO1
      COMMON/ACODE/Z(3),CI(3),CK(3),TS(3),KNODE(30,3),HT(30),WS(30,3),
     *          NSTORY
      INTEGER TITLE(10)
      DATA IMAX,IIMAX,INSIST,EPSI,EPS1,EPS2/1,10,0,0.02,0.01,0.01/
      DIMENSION EK(6,6),EM(6,6),A(3,3),B(6,6),C(6,6),D(6,6),E(6,6),
     1          IM(3),JM(3),TFR(3),DEFMAX(3),AML(6),IBND(50)
      DIMENSION TTT(3),CC(3),SS(3),TCTAL(3),FT(3)
      DIMENSION NDEFEQ(3),MDEFEQ(3),JDEFEQ(10,3),FQ(3),SUM1(3),SUM2(3),
     1          S(6,3),SIGMA(6,3),EDR(6,3),RATIO1(6)
      DIMENSION MA(200),MB(200),AE(200),AAE(200),DEP(200),TFD(200),
     1          ELENTH(200),JTYPE(200),WT(200),DDEP(200),TFFD(200),
     2          FRR(200),STRES(200),ENGLTA(200),NMB(200),MNO(200,200),
     3          SK(3500),GM(3500),SKK(3500),GN(3500),FRS(200),MBC(200),
     4          ICOL(300),IDIAG(300),ICOLS(300),IDIAGS(300),DEFLMT(300),
     5          FI(300,3),DI(300,3),FR(300,3),DR(300,3),FS(300,3),
     6          DS(300,3),FV(300,3),FD(300,3),DM(300,10),VAL(300),
     7          XH(300,3),XS(300,3),X(100),Y(100),WMASS(100),
     8          VALMIN(10),TT(10),VWORK(10),DAMP(10),
     9          DLF1(10,3),DLF2(10,3),HH(10,3),VV(10,3)
      DIMENSION RATIO(20),CONST(20),STRENG(20),ELENG(20),ENG(200,20),
     1          ENG1(14),ENG2(14),S1(6,14),S2(6,14),SS1(6,14),SS2(6,14),
     2          EDF(6,14),EFF(6,14),FF(300,14),DF(300,14),
     3          PNT(200,3),H1(200,3),H2(200,3)
C
C
C     BUILT-UP SECTION PROPERTIES
C
      FLANGE(B,D,TFD,T)=0.5*(B/D)*(TFD)*(1.0-TFD)**2
      WEB(B,D,TFD,T)=(1.0/12.0)*T*(1.0-2.0*TFD)**3
      ENRTIA(B,D,TFD,T)=(D**4)*(FLANGE(B,D,TFD,T)+WEB(B,D,TFD,T))
C
      NJMAX=100
      NMMAX=200
      NNMAX=300
      MAXSK=3500
      MAXNC=20
```

```
      LDMAX=3                                                           MAIN  37
      MCMAX=10                                                          MAIN  38
      NGMAX=200                                                         MAIN  39
      NDMAX=200                                                         MAIN  40
C
      READ(5,2) NSTRS                                                   MAIN  41
      KSTRS=1                                                           MAIN  42
C
C     GENERAL INFCRMATICN
C
    1 READ(5,2) MEMBS,JCINTS,NBNDRY,LOADS,LMTDSP,INDEX,NCOUNT,ISEC,     MAIN  43
     1          TITLE                                                   MAIN  44
    2 FCRMAT(8I5,10A4)                                                  MAIN  45
      WRITE(6,2000) KSTRS,TITLE,MEMBS,JOINTS,NBNCRY,LOADS,LMTDSP        MAIN  46
 2000 FORMAT(1H1,14H STRUCTURE NO.,I2,3H:   ,10A4/                      MAIN  47
     1 /2X,40H TCTAL NUMBER OF MEMBERS----------------,I5               MAIN  48
     2 /2X,40H TOTAL NUMBER CF NODES------------------,I5               MAIN  49
     3 /2X,40H TOTAL NUMBER OF RESTRAINED D.O.F.------,I5               MAIN  50
     4 /2X,40H TOTAL NUMBER CF LCADING CONDITIONS-----,I5               MAIN  51
     5 /2X,40H DISPLACEMENT CONSTRAINT CODE-----------,I5               MAIN  52
     6 /5X,36H EQ. 0:  NO DISPLACEMENT CONSTRAINTS/                     MAIN  53
     7  5X,58H EQ. 1:  DISPLACEMENT CCNSTRAINT IS THE SAME FOR ALL NCDESMAIN  54
     8 /5X,55H EQ. 2:  DISPLACEMENT CONSTRAINT CAN BE VARIED PER NCDE)  MAIN  55
      WRITE(6,2001) INDEX,NCCUNT,ISEC                                   MAIN  56
 2001 FORMAT(/2X,40H TYPE OF DESIGN------------------------,I5/         MAIN  57
     1 5X,22H EQ.-1:  STATIC DESIGN/                                    MAIN  58
     2 5X,34H EQ. 0:  STATIC ANC CYNAMIC CESIGN/                        MAIN  59
     3 5X,23H EQ. 1:  DYNAMIC DESIGN/                                   MAIN  60
     4 2X,40H MAXIMUM NUMBER OF ITERATIONS-----------,I5/               MAIN  61
     5 2X,40H TYPE OF SECTICNS----------------------,I5/                MAIN  62
     6 5X,40H EQ. 0:  BUILT-UP SECTICNS             /                   MAIN  63
     7 5X,40H GT. 0:  WF SECTICNS                   )                   MAIN  64
C
      MM=3                                                              MAIN  65
      NN=MM*JCINTS                                                      MAIN  66
      NB=NBNDRY                                                         MAIN  67
      NM=NN-NB                                                          MAIN  68
      MBAND=0                                                           MAIN  69
      KCCUNT=1                                                          MAIN  70
      NPAGE=0                                                           MAIN  71
      IL=1                                                              MAIN  72
      IF(LMTDSP .GT. 0) IL=2                                            MAIN  73
      LOAD=IL*LOADS                                                     MAIN  74
C
C     MEMBER INFCRMATICN
C
```

```
      IF(ISEC) 11,11,13                                                  MAIN  75
   11 READ(5,2004) EE,RHO,WIDTH,DEPTH,DEPMAX,DEPMIN,TFDMAX,TFDMIN,AENM   MAIN  76
 2004 FORMAT(2F10.0,2F5.0,5F10.0)                                        MAIN  77
      TWD=0.02                                                           MAIN  78
      TFDD=0.03                                                          MAIN  79
      DO 12 I=1,MEMBS                                                    MAIN  80
      TFD(I)=TFDD                                                        MAIN  81
   12 DEP(I)=DEPTH                                                       MAIN  82
      BASEA=ENRTIA(WIDTH,DEPTH,TFDD,TWD)                                 MAIN  83
      AEMNMX=ENRTIA(WIDTH,DEPMAX,TFDMAX,TWD)                             MAIN  84
      AEMNMM=ENRTIA(WIDTH,DEPMIN,TFDMIN,TWD)                             MAIN  85
      WRITE(6,2005) EE,RHO,WIDTH,DEPMAX,DEPMIN,TFDMAX,TFDMIN             MAIN  86
 2005 FORMAT(////2X,26H GIRDER-COLUMN PROPERTIES:/                       MAIN  87
     1 5X,40H MODULUS OF ELASTICITY------------------,F15.3/             MAIN  88
     2 5X,40H SPECIFIC WEIGHT------------------------,F15.3/             MAIN  89
     3 5X,40H WIDTH----------------------------------,F15.3/             MAIN  90
     4 5X,40H MAXIMUM DEPTH--------------------------,F15.3/             MAIN  91
     5 5X,40H MINIMUM DEPTH--------------------------,F15.3/             MAIN  92
     6 5X,40H MAXIMUM TF/D---------------------------,F15.3/             MAIN  93
     7 5X,40H MINIMUM TF/D---------------------------,F15.3)             MAIN  94
      GO TO 14                                                           MAIN  95
   13 READ(5,3) EE,RHO,BASEA,AEMNMX,AEMNMM,AENM                          MAIN  96
    3 FORMAT(8F10.0)                                                     MAIN  97
      WRITE(6,2010) EE,RHO                                               MAIN  98
 2010 FORMAT(////2X,26H GIRDER-COLUMN PROPERTIES:/                       MAIN  99
     1 5X,40H MODULUS OF ELASTICITY------------------,F15.3/             MAIN 100
     2 5X,40H SPECIFIC WEIGHT------------------------,F15.3)             MAIN 101
   14 WRITE(6,2011) BASEA,AEMNMX,AEMNMM,AENM                             MAIN 102
 2011 FORMAT(5X,40H INITIAL MOMENT OF INERTIA--------------,F15.3/       MAIN 103
     1 5X,40H MAXIMUM MOMENT OF INERTIA--------------,F15.3/             MAIN 104
     2 5X,40H MINIMUM MOMENT OF INERTIA--------------,F15.3/             MAIN 105
     3 5X,40H RATIO OF MINIMUM I TO MAXIMUM I--------,F15.3)             MAIN 106
      READ(5,3) E1,RHO1,ABRACE,ABRMAX,ABRMIN                             MAIN 107
      WRITE(6,2015) E1,RHO1,ABRMAX,ABRMIN                                MAIN 108
 2015 FORMAT(////2X,20H BRACING PROPERTIES:/                             MAIN 109
     1 5X,40H MODULUS OF ELASTICITY------------------,F15.3/             MAIN 110
     2 5X,40H SPECIFIC WEIGHT------------------------,F15.3/             MAIN 111
     3 5X,40H MAXIMUM BRACING AREA-------------------,F15.3/             MAIN 112
     4 5X,40H MINIMUM BRACING AREA-------------------,F15.3)             MAIN 113
      CALL INPUT(MA,MB,JTYPE,MBC,STRES,NMB,MNO,WT,FRR,MDIFF,MEMBS,NMMAX,MAIN 114
     *           NGMAX)                                                  MAIN 115
C
C     NODAL JOINT INFORMATION
C
      DO 7 I=1,JOINTS                                                    MAIN 116
      READ(5,4) JT,X(JT),Y(JT)                                           MAIN 117
```

```
    4 FORMAT(I5,2F10.0)                                                  MAIN 118
    7 CONTINUE                                                           MAIN 119
C
      IF(RHO .LE. 0.) EE=E1                                              MAIN 120
      IF(RHO .LE. 0.) BASEA=ABRACE                                       MAIN 121
      BASEAE=BASEA*EE                                                    MAIN 122
      AEMNMM=AEMNMM*EE                                                   MAIN 123
      ABRACE=ABRACE*EE                                                   MAIN 124
      ABRMAX=ABRMAX*EE                                                   MAIN 125
      ABRMIN=ABRMIN*EE                                                   MAIN 126
      DO 8 I=1,MEMBS                                                     MAIN 127
      AE(I)=1.0                                                          MAIN 128
      IF(JTYPE(I) .EQ. 0) AE(I)=ABRACE/BASEAE                            MAIN 129
    8 IF(MBAND .LT. (MB(I)-MA(I))) MBAND=MB(I)-MA(I)                     MAIN 130
      MBAND=(MBAND+1)*MM - 1                                             MAIN 131
      IF(MBANC .GE. NM) MBAND=NM-1                                       MAIN 132
      WRITE(6,9) MBAND                                                   MAIN 133
    9 FORMAT(///2X,8H MBAND =,I3)                                        MAIN 134
C
      READ(5,5) (IBND(I),I=1,NB)                                         MAIN 135
    5 FORMAT(16I5)                                                       MAIN 136
      WRITE(6,2020) (IBND(I),I=1,NB)                                     MAIN 137
 2020 FORMAT(////2X,31H RESTRAINED DEGREES CF FREEDOM:/(12I10))          MAIN 138
      CALL POP2(MEMBS,JOINTS,MM,MA,MB,ICOL,IDIAG,NONZRO)                 MAIN 139
      IF(NCNZRO .GT. MAXSK) GO TC 1000                                   MAIN 140
      DO 15 I=1,NN                                                       MAIN 141
      ICOLS(I)=ICOL(I)                                                   MAIN 142
   15 IDIAGS(I)=IDIAG(I)                                                 MAIN 143
      DO 16 I=1,NN                                                       MAIN 144
      DO 16 J=1,LCADS                                                    MAIN 145
      DS(I,J)=0                                                          MAIN 146
   16 FS(I,J)=0                                                          MAIN 147
      DO 17 I=1,MEMBS                                                    MAIN 148
   17 FRS(I)=C                                                           MAIN 149
      DO 18 J=1,LOADS                                                    MAIN 150
   18 CK(J)=0                                                            MAIN 151
      IF(INDEX) 20,20,31                                                 MAIN 152
C
C     STATIC LCADING INFCRMATION
C
   20 DO 21 L=1,MEMBS                                                    MAIN 153
      CALL FIXEND(FS,AML,WT(L),MA(L),MB(L),X,Y,AL,0,LOADS,NNMAX)         MAIN 154
      ELENTH(L)=AL                                                       MAIN 155
   21 CONTINUE                                                           MAIN 156
      WRITE(6,2025)                                                      MAIN 157
 2025 FORMAT(////2X,28H STATIC LCADING INFORMATICN:)                     MAIN 158
```

```
      CALL LOADIN(FS,FV,XG,ICOL,IDIAG,MM,LOADS,O,PNT,H1,H2,NDMAX,NN,
     *            NNMAX)
      CALL RECUCE(FS,IBNC,NN,NBNCRY,LCADS,NNMAX)
C
C     CYNAMIC PROPERTIES
C
   31 READ(5,6) KOPTON,NSIGN,NBEGIN,NDESIR,XG
    6 FORMAT(4I5,F10.0)
      WRITE(6,2030) NBEGIN,NDESIR,XG,NSIGN
 2030 FORMAT(/////2X,20H DYNAMIC PRCPERTIES:/
     1 5X,40H FIRST MODE NUMBER----------------------,I5/
     2 5X,40H LAST MODE NUMBER-----------------------,I5/
     3 5X,40H GRAVITY ACCELERATION-------------------,F7.2/
     4 5X,40H TYPE OF MODE SUPERPOSITION-------------,I5/
     5 5X,58H     EQ.-1:  CIRECT SUPERPOSITICN CF MODES              /
     6 5X,58H     EQ. 0:  ROOT-MEAN-SQUARE SUPERPOSITION CF MODES    /
     7 5X,58H     EQ. 1:  ABSOLUTE SUPERPOSITION CF MODES            )
      IF(KOPTON.LE.0 .OR. NBEGIN.LE.0) GO TC 50
      READ(5,3) (DAMP(I),I=NBEGIN,NDESIR)
      WRITE(6,2029) (DAMP(I),I=NBEGIN,NDESIR)
 2029 FORMAT(//5X,29H DAMPING RATIO FOR EACH MCDE:/(10F13.3))
      CO 32 I=1,NCNZRO
   32 GN(I)=0
      READ(5,3) (WMASS(I),I=1,JOINTS)
      WRITE(6,2031) (WMASS(I),I=1,JCINTS)
 2031 FORMAT(//5X,25H ADDITICNAL NODAL MASSES:/(10F13.3))
      CO 33 I=1,JOINTS
      KX=MM*(I-1) + 1
      DO 33 J=1,MM
      ID=ICIAG(KX)
      GN(ID)=WMASS(I)
   33 KX=KX+1
      CC 34 L=1,MEMBS
      CALL ELMENT(EK,EM,AE(L),MA(L),MB(L),JTYPE(L),0.0,WT(L),X,Y,A,AL,2)
      CALL TRNSFM(EK,EM,A,B,C,D,E,0)
      CALL ASMBL3(SK,GN,C,E,MA(L),MB(L),MM,ICOL,ICIAG,6)
      ELENTH(L)=AL
   34 CONTINUE
      IF(INDEX) 50,38,38
C
C     CYNAMIC LOADING INFORMATION
C
   38 WRITE(6,2032)
 2032 FORMAT(////2X,29H DYNAMIC LOACING INFCRMATICN:)
      NNN=0
      DC 35 I=1,NN
```

```
      DO 35 J=1,LOADS
      FV(I,J)=0
   35 FD(I,J)=0
   36 FORMAT(3(F10.0,2I5))
      CALL LOADIN(FD,FV,XG,ICOL,IDIAG,MM,LOADS,1,PNT,H1,H2,NDMAX,NN,
     *           NNMAX)
      CALL REDUCE(FD,IBNC,NN,NBNDRY,LOADS,NNMAX)
      CALL REDUCE(FV,IBND,NN,NBNDRY,LOADS,NNMAX)
      CC 37 I=1,NN
      DO 37 J=1,LCADS
      XH(I,J)=FD(I,J)
   37 XS(I,J)=FV(I,J)
      DO 39 L=1,MEMBS
      CALL BEAMWT(NMB(L),MNO,WT,ELENTH,FRS(L),L,NMMAX,NGMAX)
   39 CONTINUE
      IF(INDEX .EQ. 0) GC TC 50
      DO 40 L=1,MEMBS
      FRR(L)=FRR(L)+FRS(L)
   40 FRS(L)=0
   50 CONTINUE
C
C     INFORMATICN OF DISPLACEMENT CCNSTRAINTS
C
      IF(LMTDSP .GT. 0) WRITE(6,2040)
 2040 FORMAT(////2X,26H DISPLACEMENT CONSTRAINTS:)
   60 IF(LMTDSP-1) 70,61,63
   61 READ(5,3) (DEFMAX(I),I=1,MM)
      WRITE(6,2041) (DEFMAX(I),I=1,MM)
 2041 FORMAT(3X,12H X-DIRECTION,3X,12H Y-DIRECTION,3X,12H Z-DIRECTION/
     1 3F15.3)
      DO 62 I=1,JOINTS
      KX=MM*(I-1) + 1
      DO 62 J=1,MM
      CEFLMT(KX)=DEFMAX(J)
   62 KX=KX+1
      GO TO 70
   63 CC 64 I=1,NN
   64 DEFLMT(I)=1000.0
      READ(5,5) NLTDEF
      KH=NLTDEF
      WRITE(6,2042)
 2042 FORMAT(3X,5H NODE,10H DIRECTION,9X,6H VALUE)
   65 IF(KH-MM) 66,66,67
   66 KX=KH
      GO TO 68
   67 KX=MM
```

```
   68  READ(5,36) (TFR(I),IM(I),JM(I),I=1,KX)                          MAIN 242
       DO 69 I=1,KX                                                     MAIN 243
       KY=MM*(JM(I)-1) + IM(I)                                          MAIN 244
       DEFLMT(KY)=TFR(I)                                                MAIN 245
   69  WRITE(6,2043) JM(I),IM(I),TFR(I)                                 MAIN 246
 2043  FORMAT(3X,I5,I6,4X,F15.3)                                        MAIN 247
       KH=KH-KX                                                         MAIN 248
       IF(KH) 70,70,65                                                  MAIN 249
   70  CONTINUE                                                         MAIN 250
C
C      INFORMATION OF FREQUENCY CONSTRAINTS
C
       IF(KOPTON.LE.O .OR. NBEGIN.LE.O) GO TC 95                        MAIN 251
   75  READ(5,3) (VALMIN(I),I=NBEGIN,NDESIR)                            MAIN 252
       WRITE(6,2050)                                                    MAIN 253
 2050  FORMAT(////2X,23H FREQUENCY CCNSTRAINTS:/3X,5H MODE,7X,8H MINIMUM)MAIN 254
       CO 80 I=NBEGIN,NDESIR                                            MAIN 255
   80  WRITE(6,2051) I,VALMIN(I)                                        MAIN 256
 2051  FORMAT(3X,I5,F15.3)                                              MAIN 257
       CC 81 I=NBEGIN,NDESIR                                            MAIN 258
   81  VALMIN(I)=VALMIN(I)*VALMIN(I)                                     MAIN 259
C
   95  LAST=0                                                           MAIN 260
       KSAVE=1                                                          MAIN 261
       KSTEP=1                                                          MAIN 262
       WTLAST=10000000000.                                              MAIN 263
  100  CONTINUE                                                         MAIN 264
       IITER=1                                                          MAIN 265
  101  KCHANG=0                                                         MAIN 266
  102  KANLYZ=0                                                         MAIN 267
       KH=0                                                             MAIN 268
       FBASEA=BASEA                                                     MAIN 269
       ITER=1                                                           MAIN 270
       AP=BASEA                                                         MAIN 271
       BASEAE=BASEA*EE                                                  MAIN 272
       CO 103 I=1,NN                                                    MAIN 273
       ICOL(I)=ICOLS(I)                                                 MAIN 274
  103  IDIAG(I)=IDIAGS(I)                                               MAIN 275
       CO 104 I=1,NN                                                    MAIN 276
       DO 104 J=1,LOADS                                                 MAIN 277
  104  CR(I,J)=0                                                        MAIN 278
C
C      FORMULATE STIFFNESS AND MASS MATRICES
C
       DC 105 I=1,NCNZRO                                                MAIN 279
       GM(I)=GN(I)                                                      MAIN 280
```

```
  105 SK(I)=0
      DO 110 L=1,MEMBS
      CALL STRSWT(NMB(L),MNO,ELENTH,JTYPE,FRS(L),AE,BASEA,WIDTH,DEP,TFD,
     *            L,1,NMMAX,NGMAX)
      AXIAL=FRR(L)+FRS(L)
      CALL AREA(AE(L),BASEA,WIDTH,DEP(L),TFD(L),JTYPE(L),SPWT,AREAA,SEC,
     *            QUE)
      CALL ELMENT(EK,EM,AE(L),MA(L),MB(L),JTYPE(L),AXIAL,SPWT,X,Y,A,AL,
     *            INDEX)
      CALL TRNSFM(EK,EM,A,B,C,D,E,0)
      CALL ASMBL3(SK,GM,C,E,MA(L),MB(L),MM,ICOL,IDIAG,6)
  110 CONTINUE
      CALL BOUND2(SK,GM,IBND,NN,NBNDRY,IDIAG,ICOL)
      NZERO=IDIAG(NM)
      DO 111 I=1,NZERO
  111 SKK(I)=SK(I)
      NDCOMP=0
      IF(INDEX) 115,115,118
C
C     CALCULATE STATIC DISPLACEMENTS
C
  115 DO 117 J=1,LOADS
      IF(CK(J) .LT. 0.00001) GO TO 117
      IF(MTYPE(J)) 116,117,117
  116 CALL CODE(FS,DS,MM,KCOUNT,IITER,TTT(J),CC(J),SS(J),TOTAL(J),FT(J),
     *            BASEAE,IBND,NN,NBNDRY,XG,J,LOADS,NNMAX)
  117 CONTINUE
      CALL GAUSS(SKK,FS,DS,ICOL,IDIAG,LOADS,NM,NNMAX,0)
      CALL RESTOR(DS,IBND,NN,NBNDRY,LOADS,NNMAX)
      NDCOMP=1
C
  118 IF(KOPTON.LE.0 .OR. NBEGIN.LE.0) GO TO 119
      CALL EIGEN(SK,GM,ICOL,IDIAG,VAL,DM,NM,MBAND)
      IF(NITER .GE. 78) GO TO 1000
C
  119 IF(INDEX) 169,120,120
C
C     CALCULATE DYNAMIC DISPLACEMENTS
C
  120 DO 125 J=1,LOADS
      IF(MTYPE(J)) 121,125,125
  121 DO 122 I=1,NM
      DI(I,1)=XH(I,J)
  122 DI(I,2)=XS(I,J)
      CALL MLTPLY(GM,FI,DI,ICOL,IDIAG,2,NM,NNMAX)
      DO 123 I=1,NM
```

```
      FD(I,J)=FI(I,1)
123   FV(I,J)=FI(I,2)
125   CONTINUE
131   DO 132 I=1,NN
      CO 132 J=1,LCADS
132   DR(I,J)=0
      DO 150 II=NBEGIN,NDESIR
      DO 141 I=1,NM
      DO 141 J=1,LOADS
141   CI(I,J)=DM(I,II)
      CALL MLTPLY(GM,FI,CI,ICOL,IDIAG,1,NM,NNMAX)
      CALL ENERGY(ENG1,DI,FI,NM,1,NNMAX)
      ADR=ENG1(1)*VAL(II)
      CALL ENERGY(ENG1,DI,FD,NM,LCADS,NNMAX)
      CALL ENERGY(ENG2,DI,FV,NM,LCADS,NNMAX)
      TT(II)=6.283185307/SQRT(VAL(II)*BASEAE)
      DO 145 J=1,LCADS
      CALL SPTRUM(TT(II),DLF1(II,J),DAMP(II),J,PNT,H1,NDMAX,LOACS)
      HH(II,J)=ENG1(J)*FACTOR(J)*DLF1(II,J)/ADR
      VV(II,J)=0.
      IF(MOTYPE(J)) 143,144,144
143   CALL SPTRUM(TT(II),DLF2(II,J),DAMP(II),J,PNT,H2,NDMAX,LOADS)
      VV(II,J)=ENG2(J)*XV(J)*FACTOR(J)*DLF2(II,J)/ADR
144   CC 145 I=1,NM
145   DR(I,J)=DR(I,J) + CI(I,J)*(HH(II,J)+VV(II,J))
      IF(KSTEP .GT. 1) GC TO 150
      CALL MLTPLY(SK,FR,CR,ICOL,IDIAG,LOADS,NM,NNMAX)
      CALL ENERGY(ENG1,FR,DR,NM,LCACS,NNMAX)
      VWCRK(II)=0
      DO 149 J=1,LCADS
      IF(VWORK(II)-ENG1(J)) 148,149,149
148   VWORK(II)=ENG1(J)
149   CCNTINUE
150   CONTINUE
      IF(KSTEP .GT. 1) GC TC 153
      DO 151 I=1,NDESIR
151   VWORK(I)=VWORK(I)/VWORK(NDESIR)
      IF(KOPTON .GT. 1) GC TC 153
      MAXMDE=NDESIR
      NBEGIN=-1
      DO 152 I=1,MAXMDE
      ADR=VWORK(MAXMDE)-VWORK(I)
      IF(ADR .GT. 0.99) GC TC 152
      IF(NBEGIN .LT. 1) NBEGIN=I
      IF(ACR .GT. 0.01) GO TO 152
      IF(NDESIR .GT. I) NDESIR=I
```

```
  152   CONTINUE                                                          MAIN 365
        KSTEP=2                                                           MAIN 366
        GO TO 131                                                         MAIN 367
  153   CONTINUE                                                          MAIN 368
        IF(NSIGN) 162,156,159                                             MAIN 369
  156   DO 158 J=1,LOADS                                                  MAIN 370
        DO 158 I=1,NM                                                     MAIN 371
        SUM=0                                                             MAIN 372
        DO 157 II=NBEGIN,NDESIR                                           MAIN 373
  157   SUM=SUM + (DM(I,II)**2)*(HH(II,J)**2+VV(II,J)**2)                 MAIN 374
  158   CR(I,J)=SQRT(SUM)                                                 MAIN 375
        GO TO 162                                                         MAIN 376
  159   DO 161 J=1,LOADS                                                  MAIN 377
        DO 161 I=1,NM                                                     MAIN 378
        SUM=0                                                             MAIN 379
        DO 160 II=NBEGIN,NDESIR                                           MAIN 380
  160   SUM=SUM + DM(I,II)*(ABS(HH(II,J))+ABS(VV(II,J)))                  MAIN 381
  161   DR(I,J)=SUM                                                       MAIN 382
  162   CONTINUE                                                          MAIN 383
        CALL RESTOR(DR,IBNC,NN,NBNDRY,LOADS,NNMAX)                        MAIN 384
  169   CONTINUE                                                          MAIN 385
C
        CALL RESTOR(DM,IBNC,NN,NBNCRY,NDESIR,NNMAX)                       MAIN 386
        CALL ADD1(DS,DR,INDEX,LOADS,NN,NNMAX)                             MAIN 387
  170   NTOTAL=LOADS                                                      MAIN 388
        DO 171 J=1,LCAD                                                   MAIN 389
  171   RATIO1(J)=0                                                       MAIN 390
        CO 172 J=1,MAXNC                                                  MAIN 391
  172   RATIO(J)=0                                                        MAIN 392
        IF(LMTDSP .EQ. 0) GO TO 180                                       MAIN 393
C
C       CHECK DISPLACEMENTS WITH DISPLACEMENT CONSTRAINT
C
        RATIOA=0                                                          MAIN 394
        DO 174 I=1,NN                                                     MAIN 395
        CEFBAE=DEFLMT(I)*BASEAE                                           MAIN 396
        DO 174 J=1,LCADS                                                  MAIN 397
        AMAX=ABS(DR(I,J))                                                 MAIN 398
        ADR=AMAX/DEFBAE                                                   MAIN 399
        IF(RATIC1(J) .GT. ACR) GO TO 173                                  MAIN 400
        RATIO1(J)=ADR                                                     MAIN 401
  173   IF(RATIOA.GT.ACR) GO TO 174                                       MAIN 402
        RATIOA=ADR                                                        MAIN 403
  174   CONTINUE                                                          MAIN 404
  180   CONTINUE                                                          MAIN 405
C
```

```
C     CALCULATE ELEMENT STRESSES AND CHECK WITH STRESS LIMIT
C
      DC 195 L=1,MEMBS
      AXIAL=FRR(L)+FRS(L)
      CALL AREA(AE(L),BASEA,WICTH,DEP(L),TFD(L),JTYPE(L),SPWT,AREAA,SEC,
     *          QUE)
      CALL ELMENT(EK,EM,AE(L),MA(L),MB(L),JTYPE(L),AXIAL,O.,X,Y,A,AL,-1)
      CALL TRNSFM(EK,EM,A,B,C,D,E,1)
      DO 181 I=1,6
      DO 181 J=1,LOADS
      S1(I,J)=0
  181 S(I,J)=C
      IF(INDEX) 182,182,184
C
  182 CALL ELFORC(A,B,D,CS,EDR,S1,SS1,MA(L),MB(L),LOADS,MM,NNMAX)
      CALL FIXEND(FI,AML,WT(L),MA(L),MB(L),X,Y,AL,1,1,NNMAX)
      DC 183 I=1,6
      DO 183 J=1,LOADS
  183 S1(I,J)=AML(I)+S1(I,J)
      IF (INDEX) 189,184,184
C
  184 DC 186 II=NBEGIN,NCESIR
      CALL CHANGE(DI,DM,HH,VV,NSIGN,II,LOADS,NN,MDMAX,NNMAX)
      CALL ELFORC(A,B,D,DI,EDR,S2,SS2,MA(L),MB(L),LCADS,MM,NNMAX)
      CALL ADD2(S,S2,NSIGN,6,LOADS)
  186 CONTINUE
      IF(NSIGN) 189,187,189
  187 DO 188 I=1,6
      DO 188 J=1,LOADS
  188 S(I,J)=SQRT(S(I,J))
C
  189 CALL ADD1(S1,S,INDEX,LCADS,6,6)
      CALL PDELTA(S,FRS(L),NMB(L),LCADS)
  190 CALL WFBEAM(S,SIGMA,AE(L),BASEA,AREAA,SEC,QUE,LOADS)
      DO 195 J=1,LOADS
      ADR=0.
      DC 193 I=1,6
      STRSS=STRES(L)
      IF(I.EQ.2.OR.I.EQ.5) STRSS=0.65*STRSS
  191 ROSTRS=(ABS(SIGMA(I,J)))/STRSS
  192 IF(ROSTRS.GT.ADR) ADR=ROSTRS
  193 CONTINUE
  194 LL=LOADS*(IL-1) + J
      IF(RATIO1(LL).GT.ADR) GO TO 195
      RATIO1(LL)=ADR
      MDEFEQ(J)=L
```

```
      RATIO(J)=ADR                                                      MAIN 447
  195 CONTINUE                                                          MAIN 448
      RATIOA=0.                                                         MAIN 449
      DO 200 J=1,LOAD                                                   MAIN 450
      IF(RATIC1(J).LT.RATIOA) GO TO 200                                 MAIN 451
      RATIOA=RATIC1(J)                                                  MAIN 452
  200 CONTINUE                                                          MAIN 453
C
      IF(LMTDSP .EQ. 0) GC TC 214                                       MAIN 454
      CO 213 J=1,LOADS                                                  MAIN 455
      NDEFEQ(J)=0                                                       MAIN 456
      IF((RATIOA-RATIO1(J)) .GT. 0.20*RATIOA) GO TC 213                 MAIN 457
      CC 212 I=1,NN                                                     MAIN 458
      CEFBAE=DEFLMT(I)*BASEAE                                           MAIN 459
      AMAX=ABS(DR(I,J))                                                 MAIN 460
      ADR=AMAX/DEFBAE                                                   MAIN 461
      IF((RATIO1(J)-ADR) .GT. EPSI*RATIO1(J)) GC TO 212                 MAIN 462
      NDEFEQ(J)=NDEFEQ(J)+1                                             MAIN 463
      JDEFEQ(NDEFEQ(J),J)=I                                             MAIN 464
      NTOTAL=NTCTAL+1                                                   MAIN 465
      RATIC(NTOTAL)=ADR                                                 MAIN 466
  212 CONTINUE                                                          MAIN 467
  213 CONTINUE                                                          MAIN 468
      NDISPL=NTOTAL-LOADS                                               MAIN 469
  214 CONTINUE                                                          MAIN 470
C
C     CHECK FREQUENCIES WITH FREQUENCY LIMIT
C
      NFRQCY=0                                                          MAIN 471
      IF(KOPTON.LE.0 .OR. NBEGIN.LE.0) GO TO 217                        MAIN 472
  215 IF(VALMIN(NBEGIN) .LE. 0.0) GC TO 217                             MAIN 473
      DO 216 II=NBEGIN,NDESIR                                           MAIN 474
      NTOTAL=NTOTAL+1                                                   MAIN 475
      VALUE=VAL(II)*BASEAE                                              MAIN 476
      ADR=VALMIN(II)/VALUE                                              MAIN 477
      RATIC(NTCTAL)=ADR                                                 MAIN 478
  216 CONTINUE                                                          MAIN 479
      NFRQCY=(NDESIR-NBEGIN) + 1                                        MAIN 480
  217 CONTINUE                                                          MAIN 481
      NSUBT=NTOTAL-NFRQCY                                               MAIN 482
C
      IP=0                                                              MAIN 483
      RATIOA=0.                                                         MAIN 484
      CC 218 J=1,NTOTAL                                                 MAIN 485
      IF(RATIO(J) .LT. RATICA) GC TC 218                                MAIN 486
      RATIOA=RATIO(J)                                                   MAIN 487
```

```
      IP=J                                                              MAIN 488
  218 CONTINUE                                                          MAIN 489
C
      BASEAE=BASEAE*RATIOA                                              MAIN 490
      BASEA=BASEA*RATIOA                                                MAIN 491
      IF(ABS(FBASEA-BASEA).LE.EPS2*BASEA) GO TO 225                     MAIN 492
      IF(IITER.GE.IIMAX) GO TO 225                                      MAIN 493
      IITER=IITER+1                                                     MAIN 494
      AP=BASEA                                                          MAIN 495
      GO TO 102                                                         MAIN 496
  225 CONTINUE                                                          MAIN 497
      IF(ABS(AP-BASEA).LE.(EPS1*BASEA)) GO TO 224                       MAIN 498
      IF(ITER.GE.IMAX) GO TO 224                                        MAIN 499
      ITER=ITER+1                                                       MAIN 500
      AP=BASEA                                                          MAIN 501
      IF(INDEX) 221,222,222                                             MAIN 502
  221 GO TO 170                                                         MAIN 503
  222 CALL REDUCE(DM,IBNC,NN,NBNDRY,NDESIR,NNMAX)                       MAIN 504
      GO TO 131                                                         MAIN 505
  224 CONTINUE                                                          MAIN 506
C
C     CHECK SECTION PROPERTIES WITH SECTION LIMITS
C
      IF(ISEC.GT.0) GO TO 237                                           MAIN 507
      IF(KCHANG.GT.1.AND.KANLYZ.EQ.0) GO TO 237                         MAIN 508
      KDCHNG=0                                                          MAIN 509
      DO 227 I=1,MEMBS                                                  MAIN 510
      IF(JTYPE(I).EQ.0) GO TO 227                                       MAIN 511
      TFDD=0                                                            MAIN 512
      DO 226 K=1,5                                                      MAIN 513
  226 TFDD=2.0*((AE(I)*BASEA)/(DEP(I)**4)-WEB(WIDTH,DEP(I),TFDD,TWD))   MAIN 514
     1/((WIDTH/DEP(I))*(1.0-TFDD)**2)                                   MAIN 515
      TFD(I)=TFDD                                                       MAIN 516
      IF(TFD(I).GT.TFDMAX.AND.DEP(I).LE.DEPMAX) KDCHNG=1                MAIN 517
  227 IF(TFD(I).LT.TFDMIN) KDCHNG=1                                     MAIN 518
      IF(KDCHNG.EQ.0) GO TO 237                                         MAIN 519
      IF(KCHANG.GE.3) GO TO 237                                         MAIN 520
      DO 234 I=1,MEMBS                                                  MAIN 521
      IF(TFD(I)-TFDMAX) 231,231,228                                     MAIN 522
  228 DEP(I)=(AE(I)*BASEA)/(FLANGE(WIDTH,DEP(I),TFDMAX,TWD)+WEB(WIDTH,  MAIN 523
     1DEP(I),TFDMAX,TWD))                                               MAIN 524
      DEP(I)=SQRT(DEP(I))                                               MAIN 525
      DEP(I)=SQRT(DEP(I))                                               MAIN 526
      TFD(I)=TFDMAX                                                     MAIN 527
      IF(DEP(I) .LE. DEPMAX) GO TO 234                                  MAIN 528
      DEP(I)=DEPMAX                                                     MAIN 529
```

```
      TFDD=0
      DO 230 K=1,5
  230 TFDD=2.0*((AE(I)*BASEA)/(DEP(I)**4)-WEB(WIDTH,DEP(I),TFDD,TWD))
     1/((WIDTH/DEP(I))*(1.0-TFDD)**2)
      TFD(I)=TFDD
      GO TO 234
  231 IF(TFDMIN-TFD(I)) 234,234,233
  233 DEP(I)=(AE(I)*BASEA)/(FLANGE(WIDTH,DEP(I),TFDMIN,TWD)+WEB(WIDTH,
     1DEP(I),TFDMIN,TWD))
      DEP(I)=SQRT(DEP(I))
      DEP(I)=SQRT(DEP(I))
      TFD(I)=TFDMIN
      IF(DEP(I) .LT. DEPMIN) DEP(I)=DEPMIN
  234 CONTINUE
      KCHANG=KCHANG+1
      KANLYZ=1
      GO TO 170
  237 CONTINUE
C
  245 WRITE(6,3000)
 3000 FORMAT(1H1/2X,27H RELATIVE SIZES OF MEMBERS:)
      WRITE(6,3001) (AE(I),I=1,MEMBS)
 3001 FORMAT(10F13.6)
      DO 246 J=1,LOADS
      IF(CK(J) .LT. 0.00001) GO TO 246
      CS=CC(J)*SS(J)
      WRITE(6,3005) J,TTT(J),CC(J),SS(J),CS,FT(J),TCTAL(J)
 3005 FORMAT(///2X,50H DATA COMPUTED FROM 1976 UBC FCR LOADING CONDITION
     1,I2/5X,40H PERIOD OF STRUCTURE, T------------------,F13.5/
     2 5X,40H VALUE OF C-----------------------------,F13.5/
     3 5X,40H VALUE OF S-----------------------------,F13.5/
     4 5X,40H VALUE OF CS----------------------------,F13.5/
     5 5X,40H TOP-STOREY FORCE, FT-------------------,E13.5/
     6 5X,40H TOTAL SHEAR FORCE, V-------------------,E13.5)
  246 CONTINUE
      IF(KOPTON.LE.0 .OR. NBEGIN.LE.0) GO TO 255
      WRITE(6,3010)
 3010 FORMAT(///2X,21H NATURAL FREQUENCIES:)
      DO 247 I=NBEGIN,NDESIR
  247 VAL(I)=VAL(I)*AP*EE
      WRITE(6,3011) (VAL(I),I=NBEGIN,NDESIR)
 3011 FORMAT(2X,10E13.6)
      IF(INDEX) 255,248,248
  248 WRITE(6,3012)
 3012 FORMAT(//2X,5H MODE,6X,14H EIGENVECTORS:)
      CALL REDUCE(DM,IBND,NN,NBNDRY,NDESIR,NNMAX)
```

```
      DO 249 II=NBEGIN,NDESIR
  249 WRITE(6,3013) II,(DM(I,II),I=1,NM)
 3013 FORMAT(2X,I5,6X,9E13.6/(13X,9E13.6))
      WRITE(6,3014)
 3014 FORMAT(//2X,5H MODE,6X,7H PERIOD,5X,
     1 48H DYNAMIC LOAD FACTOR FOR EACH LOADING CONDITION:)
      DO 250 I=NBEGIN,NDESIR
  250 WRITE(6,3015) I,TT(I),(DLF1(I,J),J=1,LOADS)
 3015 FORMAT(2X,I5,9F13.5/(20X,8F13.5))
      MOTION=0
      DO 251 J=1,LOADS
  251 IF(MOTYPE(J) .LT. MOTION) MOTION=MOTYPE(J)
      IF(MOTION .GE. 0) GO TO 253
      WRITE(6,3018)
      DO 252 I=NBEGIN,NDESIR
  252 WRITE(6,3015) I,TT(I),(DLF2(I,J),J=1,LOADS)
  253 WRITE(6,3016)
 3016 FORMAT(//2X,50H TOTAL WORKDONE VS TOTAL NUMBER OF MODES INCLUDED:/
     1 2X,7H NUMBER,10X,24H WORKDONE/TOTAL WORKDONE)
      NUM=0
      DO 254 I=NBEGIN,NDESIR
      NUM=NUM+1
  254 WRITE(6,3017) NUM,VWORK(I)
 3017 FORMAT(2X,I5,13X,F15.6)
 3018 FORMAT(/2X,5H MODE,6X,7H PERIOD,5X,
     1 57H VERTICAL DYNAMIC LOAD FACTOR FOR EACH LOADING CONDITION:)
  255 CONTINUE
      WRITE(6,3019) FBASEA,BASEA
 3019 FORMAT(///2X,49H OLD SCALING FACTOR------------------------------,
     1 E13.6/    2X,49H NEW SCALING FACTOR------------------------------,
     2 E13.6)
 3020 FORMAT(///2X,10E13.6)
      DO 256 J=1,LOADS
  256 WRITE(6,3021) J,MDEFEQ(J)
 3021 FORMAT(2X,35H ACTIVE STRESS IN LOADING CONDITION,I2,'------------'
     1 ,I5)
      IF(LMTDSP) 259,259,257
  257 DO 258 J=1,LOADS
      KN=NDEFEQ(J)
  258 WRITE(6,3022) J,(JDEFEQ(I,J),I=1,KN)
 3022 FORMAT(2X,41H ACTIVE DISPLACEMENT IN LOADING CONDITION,I2,'------'
     1 ,10I5)
  259 WRITE(6,3023) IP
 3023 FORMAT(2X,49H MOST CRITICAL CONSTRAINT------------------------,I5)
 3025 FORMAT(I5,E13.6,2X,10F11.6/(20X,10F11.6))
      WRITE(6,3026) IITER
```

```
3026  FORMAT(2X,49H NUMBER CF ANALYSIS-------------------------------,I5)
 260  WEIGHT=0
      CO 265 I=1,MEMBS
      CALL AREA(AE(I),BASEA,WICTH,DEP(I),TFD(I),JTYPE(I),SPWT,AREAA,SEC,
     *          QUE)
 265  WEIGHT=WEIGHT + SPWT*AREAA*ELENTH(I)
      WRITE(6,3030) WEIGHT,BASEAE,KCCUNT
3030  FORMAT(///2X,42H ****************************************** ,
     1 28H TOTAL WEIGHT CF STRUCTURE =,1PE14.6,
     2 42H  ****************************************/
     3 2X,42H ****************************************** ,
     4 10H BASE AE =,1PE14.6,1H;,2X,12H CYCLE NC. =,I3,
     5 42H  ****************************************)
C
      IF(NCOUNT .EQ. 1) GC TC 505
      IF(KCOUNT .EQ. 1) GO TO 290
      IF(INSIST.EQ.1 .ANC. KCCUNT.LT.NCOUNT) GC TO 290
      IF(LMTDSP .GE. 1) GO TO 272
      IF(LAST .GE. 1) GO TO 271
      IF((WTLAST-WEIGHT).LT.0.0 .CR. KCOUNT.GE.NCOUNT) GO TO 273
      PCTWT=0.0001*WEIGHT
      IF((WTLAST-WEIGHT) .GT. PCTWT) GO TO 290
 271  IF(LMTDSP .EQ. 0) GC TC 500
      LAST=0
      GO TC 278
 272  CONTINUE
      IF(LAST .GE. 1) GC TO 500
      KSAVE=0
      IF(WTLAST .GT. WEIGHT) KSAVE=1
      IF(KCOUNT.LT.NCOUNT .AND. KSAVE.EQ.1) GO TC 290
      IF(WEIGHT .LE. WTLAST) GO TC 500
 273  CO 274 I=1,MEMBS
      DEP(I)=DDEP(I)
      TFD(I)=TFFD(I)
 274  AE(I)=AAE(I)
      BASEA=BALAST
      LAST=LAST+1
      GC TC 100
 278  CONTINUE
      IF(WEIGHT .LE. WTLAST) GO TO 290
      AMAXAE=0
      DO 279 L=1,MEMBS
      AE(L)=(AE(L)+AAE(L))/2.
      DEP(L)=(DEP(L)+DDEP(L))/2.
      TFD(L)=(TFD(L)+TFFD(L))/2.
      IF(AE(L).GT.AMAXAE) AMAXAE=AE(L)
```

```
 279  CONTINUE
      DO 285 L=1,MEMBS
      AE(L)=AE(L)/AMAXAE
      IF(JTYPE(L)) 281,281,283
 281  IF(AE(L)*BASEAE.LT.ABRMIN) AE(L)=ABRMIN/BASEAE
      IF(AE(L)*BASEAE.GT.ABRMAX) AE(L)=ABRMAX/BASEAE
      GO TO 285
 283  IF(AE(L)*BASEAE.LT.AEMNMM) AE(L)=AEMNMM/BASEAE
 285  CONTINUE
      WRITE(6,3040)
3040  FORMAT(//////2X,18H ...STEP REDUCE...)
      KCOUNT=KCOUNT+1
      GO TO 100
 290  CONTINUE
C
C     CALCULATE DISPLACEMENTS DUE TO UNIT LOAD IN THE DIRECTION CF THE
C     VIOLATED DISPLACEMENT CONSTRAINT
C
 300  IF(LMTDSP) 305,305,301
 301  KJ=0
      DO 303 J=1,LCADS
      KN=NDEFEQ(J)
      IF(KN.LE.0) GO TO 303
      DC 303 K=1,KN
      KJ=KJ+1
      KK=JDEFEQ(K,J)
      DO 302 I=1,NN
      CF(I,KJ)=0
 302  FF(I,KJ)=0
      FF(KK,KJ)=DR(KK,J)/ABS(DR(KK,J))
 303  CONTINUE
      CALL REDUCE(FF,IBND,NN,NBNDRY,NDISPL,NNMAX)
      CALL GAUSS(SKK,FF,CF,ICOL,IDIAG,NDISPL,NM,NNMAX,NDCOMP)
      CALL RESTOR(DF,IBNC,NN,NBNDRY,NDISPL,NNMAX)
 305  CONTINUE
C
C     CALCULATE FREQUENCIES CF STRUCTURE
C
      DC 311 J=1,LOACS
      FQ(J)=0
      SUM1(J)=0
 311  SUM2(J)=0
      IF(INDEX) 340,320,320
 320  DO 326 II=NBEGIN,NDESIR
      CALL CHANGE(DI,DM,HH,VV,NSIGN,II,LOADS,NN,MDMAX,NNMAX)
      CALL MLTPLY(SK,FI,CI,ICOL,IDIAG,LOADS,NM,NNMAX)
```

```
      CALL  ENERGY(ENG1,DI,FI,NM,LCADS,NNMAX)                           MAIN 705
      CALL  MLTPLY(GM,FI,DI,ICOL,IDIAG,LOADS,NM,NNMAX)                  MAIN 706
      CALL  ENERGY(ENG2,DI,FI,NM,LCADS,NNMAX)                           MAIN 707
      DO 326 J=1,LOADS                                                  MAIN 708
      IF(NSIGN) 323,324,325                                             MAIN 709
  323 SUM1(J)=SUM1(J)+ENG1(J)                                           MAIN 710
      SUM2(J)=SUM2(J)+ENG2(J)                                           MAIN 711
      GO TO 326                                                         MAIN 712
  324 SUM1(J)=SUM1(J)+(ENG1(J))**2                                      MAIN 713
      SUM2(J)=SUM2(J)+(ENG2(J))**2                                      MAIN 714
      GO TO 326                                                         MAIN 715
  325 SUM1(J)=SUM1(J)+ABS(ENG1(J))                                      MAIN 716
      SUM2(J)=SUM2(J)+ABS(ENG2(J))                                      MAIN 717
  326 CONTINUE                                                          MAIN 718
      DO 330 J=1,LOADS                                                  MAIN 719
      IF(NSIGN) 327,328,327                                             MAIN 720
  327 FQ(J)=(SUM1(J)/SUM2(J))*BASEAE                                    MAIN 721
      GO TO 330                                                         MAIN 722
  328 FQ(J)=(SQRT(SUM1(J)/SUM2(J)))*BASEAE                              MAIN 723
  330 CONTINUE                                                          MAIN 724
      WRITE(6,3300) (FQ(J),J=1,LOADS)                                   MAIN 725
 3300 FORMAT(///2X,52H RAYLEIGH'S QUOTIENT APPROXIMATION FOR EACH LOADINMAIN 726
     1G:/2X,10E13.6)                                                    MAIN 727
      CALL RESTOR(DM,IBNC,NN,NBNDRY,NDESIR,NNMAX)                       MAIN 728
  340 CONTINUE                                                          MAIN 729
C
C     CALCULATE ENERGY DENSITY IN EACH MEMBER
C
  345 DO 346 J=1,NTOTAL                                                 MAIN 730
  346 CONST(J)=1.0E+70                                                  MAIN 731
  350 DO 400 L=1,MEMBS                                                  MAIN 732
      AXIAL=0.                                                          MAIN 733
      CALL  STRSWT(NMB(L),MNO,ELENTH,JTYPE,AXIAL,AE,BASEA,WIDTH,DEP,TFC, MAIN 734
     *           L,0,NMMAX,NGMAX)                                       MAIN 735
      CALL  AREA(AE(L),BASEA,WIDTH,DEP(L),TFD(L),JTYPE(L),SPWT,AREAA,SEC,MAIN 736
     *           QUE)                                                   MAIN 737
      CALL  ELMENT(EK,EM,AE(L),MA(L),MB(L),JTYPE(L),AXIAL,SPWT,X,Y,A,AL, MAIN 738
     *            INDEX)                                                MAIN 739
      CALL  TRNSFM(EK,EM,A,B,C,C,E,1)                                   MAIN 740
      ENGLTA(L)=AREAA*SPWT*AL/BASEA                                     MAIN 741
      DO 355 J=1,NTOTAL                                                 MAIN 742
  355 STRENG(J)=0                                                       MAIN 743
C
      IF(INDEX) 395,370,370                                             MAIN 744
  370 DO 390 II=NBEGIN,NDESIR                                           MAIN 745
      CALL  CHANGE(DI,DM,HH,VV,NSIGN,II,LOADS,NN,MDMAX,NNMAX)           MAIN 746
```

```
      CALL ELFORC(A,B,C,CI,EDR,S2,SS2,MA(L),MB(L),LOADS,MM,NNMAX)
      IF(LMTDSP .LE. 0) GO TO 380
      KJ=0
      KK=LOADS
      DO 378 J=1,LOADS
      KN=NDEFEQ(J)
      IF(KN .LE. 0) GO TO 378
      DO 377 K=1,KN
      KJ=KJ+1
      DO 376 I=1,6
  376 EFF(I,K)=EDR(I,J)
      DO 377 I=1,NN
  377 FF(I,K)=DF(I,KJ)
      CALL ELFORC(A,B,D,FF,EDF,S1,SS1,MA(L),MB(L),KN,MM,NNMAX)
      CALL ENERGY(ENG1,S1,EFF,6,KN,6)
      CALL ENERGY(ENG2,SS1,EFF,6,KN,6)
      CALL ADD3(STRENG,ENG1,ENG2,FQ,BASEAE,NSIGN,KK,J,KN)
  378 CONTINUE
  380 CALL ENERGY(ENG1,S2,EDR,6,LOADS,6)
      CALL ENERGY(ENG2,SS2,EDR,6,LOADS,6)
      CALL ADD3(STRENG,ENG1,ENG2,FQ,BASEAE,NSIGN,0,0,LOADS)
  390 CONTINUE
      IF(NSIGN) 395,393,395
  393 DO 394 J=1,NSUBT
  394 STRENG(J)=SQRT(STRENG(J))
  395 CONTINUE
C
      IF(INDEX) 360,360,369
  360 CALL ELFORC(A,B,D,DS,EDR,S1,SS1,MA(L),MB(L),LOADS,MM,NNMAX)
      IF(LMTDSP.LE.0) GO TO 364
      KJ=0
      KK=LOADS
      DO 363 J=1,LOADS
      KN=NDEFEQ(J)
      IF(KN .LE. 0) GO TO 363
      DO 362 K=1,KN
      KJ=KJ+1
      DO 361 I=1,6
  361 EFF(I,K)=EDR(I,J)
      DO 362 I=1,NN
  362 FF(I,K)=DF(I,KJ)
      CALL ELFORC(A,B,D,FF,EDF,S2,SS2,MA(L),MB(L),KN,MM,NNMAX)
      CALL ENERGY(ENG1,S2,EFF,6,KN,6)
      DO 363 K=1,KN
      KK=KK+1
      STRENG(KK)=STRENG(KK)+ENG1(K)
```

```
 363   CONTINUE                                                          MAIN 792
 364   CALL ENERGY(ENG1,S1,EDR,6,LCADS,6)                                MAIN 793
 367   CC 368 J=1,LCADS                                                  MAIN 794
 368   STRENG(J)=STRENG(J)+ENG1(J)+WT(L)*WT(L)*(AL**5)/(720.0*AE(L))     MAIN 795
 369   CONTINUE                                                          MAIN 796
C
       IF(KOPTON.LE.0 .OR. NBEGIN.LE.0) GO TC 398                        MAIN 797
       IF(VALMIN(NBEGIN) .LE. 0.0) GO TO 398                             MAIN 798
       CALL ELFORC(A,B,D,CM,EDF,S1,SS1,MA(L),MB(L),NDESIR,MM,NNMAX)      MAIN 799
       CALL ENERGY(ENG1,S1,EDF,6,NDESIR,6)                               MAIN 800
       CALL ENERGY(ENG2,SS1,EDF,6,NDESIR,6)                              MAIN 801
       NI=NSUBT                                                          MAIN 802
       DO 396 II=NBEGIN,NDESIR                                           MAIN 803
       NI=NI+1                                                           MAIN 804
 396   STRENG(NI)=ENG1(II) - VAL(II)*ENG2(II)/BASEAE                     MAIN 805
C
 398   CC 399 J=1,NTOTAL                                                 MAIN 806
 399   ENG(L,J)=STRENG(J)                                                MAIN 807
       DO 400 J=1,NTOTAL                                                 MAIN 808
       ADR=ENG(L,J)/ENGLTA(L)                                            MAIN 809
       IF(ADR.LT.1.0E-35) ADR=1.0E-35                                    MAIN 810
       ALAMDA=((RATIO(J)/RATIOA)**2)/ADR                                 MAIN 811
       IF(ALAMDA.GT.CONST(J)) GO TC 400                                  MAIN 812
       CONST(J)=ALAMDA                                                   MAIN 813
 400   CONTINUE                                                          MAIN 814
C      WRITE(6,3020) (CONST(J),J=1,NTCTAL)
C
C      RESIZE RELATIVE SIZES OF MEMBERS
C
 410   AMAXAE=0                                                          MAIN 815
       WRITE(6,4010)                                                     MAIN 816
4010   FORMAT(///2X,3H NO,11H CCNSTRAINT,21H ENERGY DENSITY RATIO)       MAIN 817
       DO 460 L=1,MEMBS                                                  MAIN 818
       ADR=0                                                             MAIN 819
       JJ=1                                                              MAIN 820
       DO 450 J=1,NTOTAL                                                 MAIN 821
       ELEEN=ENG(L,J)*CONST(J)                                           MAIN 822
       IF(ADR.GT.ELEEN) GC TC 450                                        MAIN 823
       ADR=ELEEN                                                         MAIN 824
       JJ=J                                                              MAIN 825
 450   CONTINUE                                                          MAIN 826
       ADRR=ADR/ENGLTA(L)                                                MAIN 827
       WRITE(6,4020) L,JJ,ADRR                                           MAIN 828
4020   FORMAT(I5,I6,6X,E15.6)                                            MAIN 829
       IF(WTLAST.LE.WEIGHT.OR.KSAVE.EQ.0) GO TO 455                      MAIN 830
       AAE(L)=AE(L)                                                      MAIN 831
```

```
      DDEP(L)=DEP(L)                                                    MAIN 832
      TFFD(L)=TFD(L)                                                    MAIN 833
  455 CONTINUE                                                          MAIN 834
      AE(L)=1000.0*AE(L)*SQRT(ADRR)                                     MAIN 835
  460 IF(AMAXAE.LT.AE(L)) AMAXAE=AE(L)                                  MAIN 836
      AEMAXC=0.                                                         MAIN 837
      AEMAXG=0.                                                         MAIN 838
      DO 463 I=1,MEMBS                                                  MAIN 839
      IF(JTYPE(I)-1) 463,461,462                                        MAIN 840
  461 IF(AE(I) .LT. AEMAXC) GO TO 463                                   MAIN 841
      AEMAXC=AE(I)                                                      MAIN 842
      GO TO 463                                                         MAIN 843
  462 IF(AE(I) .LT. AEMAXG) GO TC 463                                   MAIN 844
      AEMAXG=AE(I)                                                      MAIN 845
  463 CONTINUE                                                          MAIN 846
      AEMAXC=AEMAXC/AMAXAE                                              MAIN 847
      AEMAXG=AEMAXG/AMAXAE                                              MAIN 848
      IP=0                                                              MAIN 849
      DO 470 I=1,MEMBS                                                  MAIN 850
      AE(I)=AE(I)/AMAXAE                                                MAIN 851
      IF(JTYPE(I)-1) 465,467,468                                        MAIN 852
  465 IF((AE(I)*BASEAE).LE.ABRMAX) GC TO 466                            MAIN 853
      IP=IP+1                                                           MAIN 854
      AE(I)=ABRMAX/BASEAE                                               MAIN 855
      GO TO 470                                                         MAIN 856
  466 IF((AE(I)*BASEAE).GE.ABRMIN) GO TO 470                            MAIN 857
      IP=IP+1                                                           MAIN 858
      AE(I)=ABRMIN/BASEAE                                               MAIN 859
      GO TC 470                                                         MAIN 860
  467 IF(AE(I) .LT. (AENM*AEMAXC)) AE(I)=AENM*AEMAXC                    MAIN 861
      GO TO 469                                                         MAIN 862
  468 IF(AE(I) .LT. (AENM*AEMAXG)) AE(I)=AENM*AEMAXG                    MAIN 863
  469 IF((AE(I)*BASEAE).GE.AEMNMM) GC TO 470                            MAIN 864
      IP=IP+1                                                           MAIN 865
      AE(I)=AEMNMM/BASEAE                                               MAIN 866
  470 CONTINUE                                                          MAIN 867
      WRITE(6,5) IP                                                     MAIN 868
      IF(KCOUNT.GE.NCOUNT.OR.NCCUNT.EQ.1) GO TO 505                     MAIN 869
      IF(WTLAST.GT.WEIGHT.OR.KSAVE.EQ.1) WTLAST=WEIGHT                  MAIN 870
      IF(WTLAST.GT.WEIGHT.OR.KSAVE.EQ.1) BALAST=FBASEA                  MAIN 871
      CALL SAME(AE,MBC,MDIFF,MEMBS)                                     MAIN 872
      KCOUNT=KCOUNT+1                                                   MAIN 873
      GC TC 100                                                         MAIN 874
  500 CONTINUE                                                          MAIN 875
      IF(KANLYZ.EQ.1) GO TO 102                                         MAIN 876
  505 CCNTINUE                                                          MAIN 877
```

```
C
C     PRINT OUTPUT RESULTS
C
      IF(INDEX) 507,506,506                                              MAIN 878
 506  CALL RESTOR(DM,IBND,NN,NBNDRY,NDESIR,NNMAX)                        MAIN 879
 507  CONTINUE                                                           MAIN 880
      NPAGE=NPAGE+1                                                      MAIN 881
      LINES=1                                                            MAIN 882
      DO 530 L=1,MEMBS                                                   MAIN 883
      IF((LINES+2*LOADS).LT.51.AND.L.GT.1) GO TO 510                     MAIN 884
      LINES=1                                                            MAIN 885
      WRITE(6,5011) NPAGE                                                MAIN 886
      NPAGE=NPAGE+1                                                      MAIN 887
      WRITE(6,5012)                                                      MAIN 888
 510  CONTINUE                                                           MAIN 889
      AXIAL=FRR(L)+FRS(L)                                                MAIN 890
      CALL AREA(AE(L),BASEA,WIDTH,DEP(L),TFD(L),JTYPE(L),SPWT,AREAA,SEC,MAIN 891
     *          QUE)                                                     MAIN 892
      CALL ELMENT(EK,EM,AE(L),MA(L),MB(L),JTYPE(L),AXIAL,0.,X,Y,A,AL,-1)MAIN 893
      CALL TRNSFM(EK,EM,A,B,C,D,E,1)                                     MAIN 894
      DO 511 I=1,6                                                       MAIN 895
      DO 511 J=1,LOADS                                                   MAIN 896
      S1(I,J)=0                                                          MAIN 897
 511  S(I,J)=0                                                           MAIN 898
      IF(INDEX) 512,512,514                                              MAIN 899
C
 512  CALL ELFORC(A,B,D,CS,EDR,S1,SS1,MA(L),MB(L),LOADS,MM,NNMAX)        MAIN 900
      CALL FIXEND(FI,AML,WT(L),MA(L),MB(L),X,Y,AL,1,1,NNMAX)             MAIN 901
      DO 513 I=1,6                                                       MAIN 902
      DO 513 J=1,LOADS                                                   MAIN 903
 513  S1(I,J)=AML(I)+S1(I,J)                                             MAIN 904
      IF(INDEX) 519,514,514                                              MAIN 905
C
 514  DO 516 II=NBEGIN,NDESIR                                            MAIN 906
      CALL CHANGE(DI,DM,HH,VV,NSIGN,II,LOADS,NN,MDMAX,NNMAX)             MAIN 907
      CALL ELFORC(A,B,D,DI,EDR,S2,SS2,MA(L),MB(L),LOADS,MM,NNMAX)        MAIN 908
      CALL ADD2(S,S2,NSIGN,6,LOADS)                                      MAIN 909
 516  CONTINUE                                                           MAIN 910
      IF(NSIGN) 519,517,519                                              MAIN 911
 517  DO 518 I=1,6                                                       MAIN 912
      DO 518 J=1,LOADS                                                   MAIN 913
 518  S(I,J)=SQRT(S(I,J))                                                MAIN 914
C
 519  CALL ADD1(S1,S,INDEX,LOADS,6,6)                                    MAIN 915
 520  CALL WFBEAM(S,SIGMA,AE(L),BASEA,AREAA,SEC,QUE,LOADS)               MAIN 916
      IF(JTYPE(L)-1) 521,522,522                                         MAIN 917
```

```
 521   WRITE(6,5015) L,AL,AE(L),AREAA,MA(L),MB(L),S(1,1)
       WRITE(6,5014) SIGMA(1,1)
       GO TO 523
 522   EI=AE(L)*BASEA
       ENGLTA(L)=SEC
       WRITE(6,5013) L,AL,AE(L),EI,AREAA,MA(L),MB(L),(S(I,1),I=1,6)
       WRITE(6,5014) (SIGMA(I,1),I=1,6)
 523   IF(LOADS.EQ.1) GO TO 530
       IF(JTYPE(L)-1) 524,526,526
 524   DO 525 K=2,LOADS
       WRITE(6,5014) S(1,K)
 525   WRITE(6,5014) SIGMA(1,K)
       GO TO 530
 526   DO 527 K=2,LOADS
       WRITE(6,5014) (S(I,K),I=1,6)
 527   WRITE(6,5014) (SIGMA(I,K),I=1,6)
 530   LINES=LINES+2*LOADS+1
5011   FORMAT(1H1,120X,5HPAGE ,I3////)
5012   FORMAT(//1X,6HMEMBER,1X,6HLENGTH,3X,2HAE,10X,1HI,6X,4HAREA,4X,
      12HMA,2X,2HMB,4X,11HAXIAL FORCE,4X,5HSHEAR,8X,6HMOMENT,6X,
      211HAXIAL FORCE,4X,5HSHEAR,8X,6HMOMENT//)
5013   FORMAT(/I5,F9.2,F7.3,F12.3,F8.3,I5,I4,1PE15.5,1PE13.5,
      11PE13.5,1PE14.5,1PE13.5,1PE13.5)
5014   FORMAT(50X,1PE15.5,1PE13.5,1PE13.5,1PE14.5,1PE13.5,1PE13.5)
5015   FORMAT(/I5,F9.2,F7.3,12X,F8.3,I5,I4,1PE15.5)
C
       WRITE(6,5011) NPAGE
       WRITE(6,5021)
5021   FORMAT(//34H GIRDER-COLUMN SECTION PROPERTIES:///
      1 3X,6HMEMBER,2X,4HTYPE,7X,15HSECTION MODULUS,5X,4HTF/D,
      2 5X,5HDEPTH,8X,2HTW,9X,2HTF)
       DO 535 L=1,MEMBS
       IF(JTYPE(L).EQ.0) GO TO 535
       TW=TWD*DEP(L)
       TF=TFD(L)*DEP(L)
       WRITE(6,5022) L,JTYPE(L),ENGLTA(L),TFD(L),DEP(L),TW,TF
5022   FORMAT(/2X,I5,1X,I5,8X,F12.3,5X,F9.5,2X,F7.3,4X,F8.4,3X,F8.4)
 535   CONTINUE
C
       DO 540 I=1,NN
       DO 540 J=1,LOADS
 540   DR(I,J)=DR(I,J)/BASEAE
       CALL PRNTDR(DR,X,Y,NN,MM,LOADS,JOINTS,NPAGE,NNMAX)
C
1000   CONTINUE
       IF(KSTRS.GE.NSTRS) GO TO 1001
```

```
      KSTRS=KSTRS+1                                                    MAIN 961
      GO TO 1                                                          MAIN 962
 1001 CONTINUE                                                         MAIN 963
      STOP                                                             MAIN 964
      END                                                              MAIN 965
C
C     SUBROUTINE ADD1:
C     ADD STATIC DISPLACEMENTS WITH DYNAMIC DISPLACEMENTS
C
      SUBROUTINE ADD1 (S,D,INC,L,N,NN)                                 ADD1   1
      DIMENSION S(NN,L),D(NN,L)                                        ADD1   2
      DO 30 I=1,N                                                      ADD1   3
      DO 30 J=1,L                                                      ADD1   4
      DMAX=S(I,J)+D(I,J)                                               ADD1   5
      DMIN=S(I,J)-D(I,J)                                               ADD1   6
      IF(INC) 20,10,20                                                 ADD1   7
   10 IF(ABS(DMAX)-ABS(DMIN)) 15,20,20                                 ADD1   8
   15 D(I,J)=DMIN                                                      ADD1   9
      GO TO 30                                                         ADD1  10
   20 D(I,J)=DMAX                                                      ADD1  11
   30 CONTINUE                                                         ADD1  12
      RETURN                                                           ADD1  13
      END                                                              ADD1  14
C
C     SUBROUTINE ADD2:
C     COMBINE DYNAMIC FORCES ACCORDING TO TYPE OF COMBINATION
C
      SUBROUTINE ADD2 (S,S2,NS,N,L)                                    ADD2   1
      DIMENSION S(N,L),S2(N,L)                                         ADD2   2
      DO 30 I=1,N                                                      ADD2   3
      DO 30 J=1,L                                                      ADD2   4
      IF (NS) 10,15,20                                                 ADD2   5
   10 S(I,J)=S(I,J)+S2(I,J)                                            ADD2   6
      GO TO 30                                                         ADD2   7
   15 S(I,J)=S(I,J)+(S2(I,J))**2                                       ADD2   8
      GO TO 30                                                         ADD2   9
   20 S(I,J)=S(I,J)+ABS(S2(I,J))                                       ADD2  10
   30 CONTINUE                                                         ADD2  11
      RETURN                                                           ADD2  12
      END                                                              ADD2  13
C
C     SUBROUTINE ADD3:
C     COMBINE ENERGIES ACCORDING TO TYPE OF COMBINATION
C
      SUBROUTINE ADD3 (S,E1,E2,FQ,BA,NS,KJ,JJ,L)                       ADD3   1
      DIMENSION S(1),E1(1),E2(1),FQ(1)                                 ADD3   2
```

```
      DO 40 J=1,L                                                       ADD3   3
      KJ=KJ+1                                                           ADD3   4
      IF(JJ) 10,10,20                                                   ADD3   5
   10 JK=J                                                              ADD3   6
      GO TO 30                                                          ADD3   7
   20 JK=JJ                                                             ADD3   8
   30 ADR=E1(J) - E2(J)*FQ(JK)/BA                                       ADD3   9
      IF(NS) 31,32,33                                                   ADD3  10
   31 S(KJ)=S(KJ)+ADR                                                   ADD3  11
      GO TO 40                                                          ADD3  12
   32 S(KJ)=S(KJ)+(ADR**2)                                              ADD3  13
      GO TO 40                                                          ADD3  14
   33 S(KJ)=S(KJ)+ABS(ADR)                                              ADD3  15
   40 CONTINUE                                                          ADD3  16
      RETURN                                                            ADD3  17
      END                                                               ADD3  18
C
C     SUBROUTINE AREA:
C     CALCULATE SECTION PROPERTIES
C
      SUBROUTINE AREA(AE,BA,B,D,TFD,JT,SPWT,AA,SS,QQ)                   AREA   1
      COMMON/BBC/IS,T,DEPMAX,DEPMIN,TFDMAX,TFDMIN,RHO,RHO1              AREA   2
      FLANGE(B,D,TFD,T)=0.5*(B/D)*(TFD)*(1.0-TFD)**2                    AREA   3
      WEB(B,D,TFD,T)=(1.0/12.0)*T*(1.0-2.0*TFD)**3                      AREA   4
      SECMOD(AE,D,BA,TFD)=(AE*BA)/(0.5*D*(1.0+TFD))                     AREA   5
      SHEMOD(AE,B,D,BA,TFD,T)=(1.0/(T*(D**2)))*((FLANGE(B,D,TFD,T)/     AREA   6
     1(1.0-TFD))+1.5*(WEB(B,D,TFD,T)/(1.0-2.0*TFD)))/(FLANGE(B,D,TFD,T) AREA   7
     2+WEB(B,D,TFD,T))                                                  AREA   8
      AREAA(B,D,TFD,T)=(D**2)*(T+2.0*TFD*(B/D-T))                       AREA   9
C
      IF(JT-1) 10,20,20                                                 AREA  10
C
C     BRACING-TYPE MEMBER
   10 SPWT=RHO1                                                         AREA  11
      AA=AE*BA                                                          AREA  12
      SS=1.0                                                            AREA  13
      QQ=0.                                                             AREA  14
      GO TO 100                                                         AREA  15
C
C     COLUMN-GIRDER TYPE MEMBER
   20 SPWT=RHO                                                          AREA  16
      IF(IS) 30,30,40                                                   AREA  17
C     ----- BUILT-UP SECTION -----
   30 KD=0                                                              AREA  18
      DO 1 K=1,5                                                        AREA  19
    1 TFD=2.0*((AE*BA)/(D**4)-WEB(B,D,TFD,T))/((B/D)*(1.0-TFD)**2)      AREA  20
```

```
      AA=AREAA(B,C,TFD,T)
      SS=SECMOD(AE,D,BA,TFD)
      QQ=SHEMOD(AE,B,D,BA,TFD,T)
      GO TO 100
C     ----- WF SECTION -----
   40 EI=AE*BA
      QQ=0.
      IF(EI-9000.) 11,11,12
   11 AA=SQRT(0.21583*EI)
      SS=SQRT(60.6*EI+84100.) - 290.
      GO TO 100
   12 IF(10000000.-EI) 13,14,14
   13 WRITE(6,50)
      STOP
   14 AA=EI/((EI+174498.)/12841.)**2
      SS=(EI-8056.3)/1.876
   50 FORMAT(//2X,56H ***PROGRAM STOP DUE TO SECTION EXCEEDS MAXIMUM LIM
     1IT***)
  100 CONTINUE
      RETURN
      END
C
C     SUBROUTINE ASMBL3:
C     FORM THE OVERALL STIFFNESS AND MASS MATRICES
C
      SUBROUTINE ASMBL3(A,G,B,C,MA,MB,MM,IC,ID,M)
      DIMENSION A(1),B(M,M) ,IC(1),ID(1),G(1),C(M,M)
      IX(I,J)=I*(J-1)+1
      M2=2*MM
      IA=IX(MM,MA)
      JA=IX(MM,MB)
      IAA=IA
      KH=MM
      KY=MM
      KX=JA-IA-MM
      DO 30 J=1,M2
      IF(J .LE. KH)GO TO 15
      IAA=JA
      KH=M2
   15 JX=ID(IAA)-IAA+IA
      KY=MM
      DO 25 I=1,J
      IF(J .LE. KY .OR. I .LE. MM) GO TO 20
      JX=JX+KX
      KY=M2
   20 A(JX)=A(JX)+B(I,J)
```

```
      G(JX)=G(JX)+C(I,J)                                                ASMB  22
   25 JX=JX+1                                                           ASMB  23
   30 IAA=IAA+1                                                         ASMB  24
      RETURN                                                            ASMB  25
      END                                                               ASMB  26
C
C     SUBROUTINE BEAMWT:
C     CALCULATE NCNSTRUCTURAL WEIGHTS ON CCLUMN
C
      SUBRCUTINE BEAMWT (NMB,MNO,WT,EL,FR,L,NM,NG)                      BEAM   1
      DIMENSION WT(1),EL(1),MNC(NM,NG)                                  BEAM   2
      SUM=FR                                                            BEAM   3
      IF(NMB) 20,20,10                                                  BEAM   4
   10 DO 15 J=1,NMB                                                     BEAM   5
      KK=MNO(L,J)                                                       BEAM   6
   15 SUM = SUM+WT(KK)*EL(KK)/2.0                                       BEAM   7
   20 FR=SUM                                                            BEAM   8
      RETURN                                                            BEAM   9
      END                                                               BEAM  10
C
C     SUBROUTINE BOUND2:
C     REARRANGE STIFFNESS AND MASS MATRICES ACCORDING TO BOUNDARY
C     CONDITION
C
      SUBRCUTINE BOUND2(A,B,IB,N,NB,ID,IC)                              BCUN   1
      DIMENSICN A(1),IB(1),ID(1),IC(1),B(1)                             BOUN   2
      IH=NB                                                             BCUN   3
      NH=N                                                              BCUN   4
      DC 30 JA=1,NB                                                     BOUN   5
      IA=IB(IH)                                                         BCUN   6
      IF(IA .GE. NH) GO TC 20                                           BCUN   7
      KH=IA+1                                                           BOUN   8
      IF(IA .GT. 1) GO TO 5                                             BCUN   9
      KX=1                                                              BCUN  10
      JX=1                                                              BCUN  11
      GO TO 6                                                           BCUN  12
    5 JX=ID(IA)-ID(IA-1)                                                BCUN  13
      KX=ID(IA-1)+1                                                     BCUN  14
    6 DO 10 I=KH,NH                                                     BCUN  15
      KY=1                                                              BOUN  16
      IF(IC(I) .LE. IA) GO TO 7                                         BCUN  17
      IC(I-1)=IC(I)-1                                                   BCUN  18
      I1=I                                                              BCUN  19
      KY=0                                                              BCUN  20
      GC TC 8                                                           BCUN  21
    7 IC(I-1)=IC(I)                                                     BOUN  22
```

```
      I1=I-1
    8 K=IC(I)
      ID(I-1)=ID(I)-JX-KY
      CO 10 J=K,I1
      IF(J .EQ. IA)  JX=JX+1
      KXX=KX+JX
      A(KX)=A(KXX)
      B(KX)=B(KXX)
   10 KX=KX+1
   20 NH=NH-1
      IH=IH-1
   30 CONTINUE
      RETURN
      END
C
C     SUBRCUTINE CHANGE:
C     COMBINE DYNAMIC DISPLACEMENTS ACCORDING TO TYPE OF COMBINATION
C
      SUBROUTINE CHANGE (DI,DM,HH,VV,NS,II,L,N,MD,NN)
      DIMENSION DI(NN,L),DM(NN,MD),HH(MD,L),VV(MD,L)
      DO 10 I=1,N
      DO 10 J=1,L
   10 DI(I,J)=0
      IF(NS) 20,30,40
   20 DO 25 I=1,N
      DO 25 J=1,L
   25 DI(I,J)=DM(I,II)*(HH(II,J)+VV(II,J))
      GO TO 50
   30 DO 35 I=1,N
      DO 35 J=1,L
   35 DI(I,J)=DM(I,II)*SQRT(HH(II,J)**2+VV(II,J)**2)
      GO TO 50
   40 DO 45 I=1,N
      DO 45 J=1,L
   45 DI(I,J)=DM(I,II)*(ABS(HH(II,J))+ABS(VV(II,J)))
   50 CONTINUE
      RETURN
      END
C
C     SUBROUTINE CODE:
C     CALCULATE LATERAL EQUIVALENT STATIC FORCES ACCORDING TO 1976 UBC
C
      SUBROUTINE CODE (FS,DS,MM,KCOUNT,IITER,T,C,S,TOTAL,FT,BA,IB,N,NB,
     1                 XG,J,L,NN)
      COMMON/ACODE/Z(3),CI(3),CK(3),TS(3),KNODE(30,3),H(30),W(30,3),
     1      NSTORY
```

```
      DIMENSION FS(NN,L),DS(NN,L),IB(1),FX(300,1),DX(300,1)
      DO 1 I=1,N
      DX(I,1)=DS(I,J)/BA
    1 FX(I,1)=FS(I,J)
    2 CALL RESTOR(FX,IB,N,NB,1,NN)
      IF(KCOUNT.GT.1 .OR. IITER.GT.1) GO TO 5
      DRIFT=0.005*H(1)
      FORCE=0.
      DO 4 I=1,NSTORY
      KK=MM*KNODE(I,J) - 2
      IF(I .EQ. 1) GO TO 3
      DRIFT=DRIFT+0.005*(H(I)-H(I-1))
    3 FORCE=FORCE+0.14*Z(J)*CI(J)*CK(J)*W(I,J)
      DX(KK,1)=DRIFT
    4 FX(KK,1)=FORCE
    5 UP=0.
      DOWN=0.
      DO 6 I=1,NSTORY
      KK=MM*KNODE(I,J) - 2
      UP=UP+W(I,J)*(DX(KK,1)**2)
    6 DOWN=DOWN+FX(KK,1)*DX(KK,1)
      T=6.283185307*SQRT(UP/(XG*DOWN))
      IF(T .LT. 0.3) T=0.3
      TTS=T/TS(J)
      IF(TTS-1.0) 10,10,11
   10 S=1.0+TTS-0.5*(TTS**2)
      GO TO 12
   11 S=1.2+0.6*TTS-0.3*(TTS**2)
      IF(S .GT. 1.5) S=1.5
   12 C=1.0/(15.0*SQRT(T))
      IF(C .GT. 0.12) C=0.12
      CS=C*S
      IF(CS .GT. 0.14) CS=0.14
      WEIGHT=0.
      WH=0.
      DO 15 I=1,NSTORY
      WEIGHT=WEIGHT+W(I,J)
   15 WH=WH+W(I,J)*H(I)
      TOTAL=Z(J)*CI(J)*CK(J)*CS*WEIGHT
      IF(T-0.7) 16,16,17
   16 FT=0.
      GO TO 18
   17 FT=0.07*T*TOTAL
      IF(FT .GT. (0.25*TOTAL)) FT=0.25*TOTAL
   18 SHEAR=TOTAL-FT
      DO 20 I=1,NSTORY
```

```
      KK=MM*KNODE(I,J) - 2                                              CCDE  51
   20 FX(KK,1)=SHEAR*W(I,J)*H(I)/WH                                     CODE  52
      FX(KK,1)=FX(KK,1)+FT                                              CODE  53
      CALL REDUCE(FX,IB,N,NB,1,NN)                                      CCCE  54
      DO 25 I=1,N                                                       CODE  55
   25 FS(I,J)=FX(I,1)                                                   CODE  56
      RETURN                                                            CCDE  57
      END                                                               CODE  58
C
C     SUBRCUTINE EIGEN:
C     CALCULATE EIGEN VALUES AND EIGENVECTORS
C
      SUBRCUTINE EIGEN (SK,GM,ICOL,IDIAG,VAL,DM,NM,MBAND)               EIGE   1
      COMMON/ABC/NZERO,NBEGIN,NDESIR,NITER,MD,NN                        EIGE   2
      DIMENSION SK(1),GM(1),ICOL(1),IDIAG(1),VAL(1),DM(NN,MD),          EIGE   3
     1          SKK(3500),FI(300,1),DI(300,1),DRP(300,1),VALUP(300)     EIGE   4
      DO 10 I=1,NM                                                      EIGE   5
      VALUP(I)=-1.0                                                     EIGE   6
   10 VAL(I)=0.0                                                        EIGE   7
      VALUE=1.0                                                         EIGE   8
      NITER=0                                                           EIGE   9
   11 NITER=NITER+1                                                     EIGE  10
      IF(NITER.GE.78) GO TO 100                                         EIGE  11
      CALL STURM (SK,GM,ICOL,IDIAG,MBAND,VALUE,NGREAT,NM)               EIGE  12
      KH=NM-NGREAT                                                      EIGE  13
      IF(KH.LT.NM)VAL(KH+1)=VALUE                                       EIGE  14
      IF(KH.EQ.0) GO TO 12                                              EIGE  15
      IF(VALUP(KH).LT.0.)VALUP(KH)=VALUE                                EIGE  16
      IF(VALUE.LT.VALUP(KH)) VALUP(KH)=VALUE                            EIGE  17
      VALUE=VALUE/10.0                                                  EIGE  18
      GO TO 11                                                          EIGE  19
   12 DO 13 I=2,NM                                                      EIGE  20
   13 IF(VAL(I).LT.VAL(I-1)) VAL(I)=VAL(I-1)                            EIGE  21
      VALUE=VAL(NM)                                                     EIGE  22
      NITER=0                                                           EIGE  23
   14 VALUE=VALUE*10.                                                   EIGE  24
      NITER=NITER+1                                                     EIGE  25
      IF(NITER.GE.78) GO TO 100                                         EIGE  26
      CALL STURM (SK,GM,ICOL,IDIAG,MBAND,VALUE,NGREAT,NM)               EIGE  27
      KH=NM-NGREAT                                                      EIGE  28
      IF(KH.GE.NM) GO TO 16                                             EIGE  29
      IF(VAL(KH+1).LT.VALUE)VAL(KH+1)=VALUE                             EIGE  30
      IF(VALUP(KH).LT.0.)VALUP(KH)=VALUE                                EIGE  31
      IF(VALUE.LT.VALUP(KH))VALUP(KH)=VALUE                             EIGE  32
      DO 15 I=2,NM                                                      EIGE  33
   15 IF(VAL(I).LT.VAL(I-1))VAL(I)=VAL(I-1)                             EIGE  34
```

```
      GO TO 14
   16 VALHIG=VALUE
      IF(VALUP(NM).LT.0.)VALUP(NM)=VALHIG
      IF(VALHIG.LT.VALUP(NM))VALUP(NM)=VALHIG
      KX=NM
      NM1=NM-1
      DO 29 I=1,NM1
      IF(VALUP(KX-1).LT.0.)VALUP(KX-1)=VALUP(KX)
   29 KX=KX-1
      DO 20 I=NBEGIN,NDESIR
   27 VALKY=VALUP(I)
      VALKX=VAL(I)
   18 VALUE=(VALKX+VALKY)/2.
   19 CALL STURM (SK,GM,ICOL,IDIAG,MBAND,VALUE,NGREAT,NM)
      KH=NM-NGREAT
      KX=KH+1
      DO 26 J=KX,NM
   26 IF(VAL(J).LT.VALUE)VAL(J)=VALUE
      IF(I.GE.KH) GO TO 24
      DO 28 J=1,KH
   28 IF(VALUP(J).GT.VALUE)VALUP(J)=VALUE
   24 IF(KH-I)21,22,23
   21 VALKX=VALUE
      GO TO 25
   22 VALKY=VALUE
      GO TO 25
   23 VALKY=VALUE
   25 IF((VALKY-VALKX).GT.(.005*VALUE)) GO TO 18
      VAL(I)=VALUE
   20 CONTINUE
      DO 40 II=NBEGIN,NDESIR
      DO 30 I=1,NM
      DI(I,1)=1.0
   30 FI(I,1)=0.0
      DO 31 I=1,NZERO
   31 SKK(I) = GM(I)-SK(I)/VAL(II)
      NDCOMP=0
      NITER=0
   32 DO 33 I=1,NM
   33 DRP(I,1)=DI(I,1)
      NITER=NITER+1
      IF(NITER.GT.30) GO TO 36
      CALL MLTPLY (SK,FI,DRP,ICOL,IDIAG,1,NM,NN)
      CALL GAUSS (SKK,FI,DI,ICOL,IDIAG,1,NM,NN,NDCOMP)
      NDCOMP=1
      ADR=0.0
```

```
      DO 34 J=1,NM
   34 IF(ABS(DI(J,1)).GT.ADR) ADR=ABS(DI(J,1))
      DO 35 J=1,NM
   35 DI(J,1)=DI(J,1)/ADR
      DO 36 I=1,NM
      IF((ABS(DRP(I,1))-ABS(DI(I,1))).GT.0.000001) GO TO 32
   36 CONTINUE
      DO 38 I=1,NM
   38 DM(I,II)=DI(I,1)
   40 CONTINUE
      IF(NITER-78) 200,100,100
  100 WRITE(6,101)
  101 FORMAT(/////46H ERROR: *****NO SOLUTIONS TO EIGENVALUES***** )
  200 RETURN
      END
C
C     SUBROUTINE ELFORC:
C     CALCULATE MEMBER FORCES
C
      SUBROUTINE  ELFORC(A,B,D,DR,EDR,S,SS,MA,MB,L,M,NN)
      DIMENSION A(3,3),B(6,6),DR(NN,L),S(6,L),EDR(6,L),D(6,6),SS(6,L)
      NL=3*(MA-1)+1
      NJ=M*(MB-1)+1
      MM=2*M
      DO 86 I=1,6
      JA=M
      NI=NL
      DO 88 K= 1,L
      EDR(I,K)=0
      SS(I,K) = 0.0
   88 S(I,K)=0
      IF(I-3)104, 104,105
  104 KH=NI
      JAA=0
      GO TO 106
  105 KH=NJ
      JAA=-M
  106 IAA=I+JAA
      DO 108 J=1,M
      DO 107 K=1,L
  107 EDR(I,K)=EDR(I,K)+A(IAA,J)*DR(KH,K)
  108 KH=KH+1
      DO   86 J=1,MM
      DO 89 K = 1,L
      SS(I,K) = SS(I,K) + D(I,J)* R(NI,K)
   89 S(I,K) =S(I,K)+B(I,J)*DR(NI,K)
```

```
      IF(J.LT.JA) GO TO  85
      NI=NJ-1
      JA=MM
   85 NI=NI+1
   86 CONTINUE
      RETURN
      ENC
C
C     SUBROUTINE ELMENT:
C     FORM ELEMENT STIFFNESS AND ELEMENT MASS
C
      SUBROUTINE ELMENT (EK,EM,AE,MA,MB,JT,FR,RHO,X,Y,A,AL,INC)
      COMMON/STIFF/FB,EE,E1,XG,AREAA
      DIMENSION EK(6,6),EM(6,6),X(1),Y(1),A(3,3)
      KX=MA
      KY=MB
      AM=X(KY) - X(KX)
      AN=Y(KY) - Y(KX)
      AL= SQRT(AM**2+AN**2)
      DO 4 J=1,6
      CO 4 I=1,6
      EM(I,J)=0
    4 EK(I,J)=0
      IF(INC .EQ. 2) GO TO 16
      IF(JT-1) 30,5,5
    5 AEK=AREAA/(FB*AL)
      BEK=AE/(AL**3)
      CEK=AE/(AL**2)
      DEK=AE/AL
      FO=FR/(FB*EE)
      EK(1,1)=AEK
      EK(4,1)=-AEK
      EK(2,2)=12.*BEK - 1.2*FO/AL
      EK(3,2)=6.*CEK - 0.1*FO
      EK(5,2)=-12.*BEK + 1.2*FO/AL
      EK(6,2)=EK(3,2)
      EK(3,3)=4.*DEK - FO*AL/7.5
      EK(5,3)=-EK(3,2)
      EK(6,3)=2.*DEK + FO*AL/30.0
      EK(4,4)=AEK
      EK(5,5)=EK(2,2)
      EK(6,5)=-EK(3,2)
      EK(6,6)=EK(3,3)
      IF(INC) 20,15,15
   15 AEM=(RHO/XG)*AREAA
      GO TO 18
```

```
16 IF(RHO .EQ. 0.0) GC TO 40                                   ELME  36
   AEM=RHO/XG                                                  ELME  37
18 BEM=AEM*AL                                                  ELME  38
   CEM=AEM*(AL**2)                                             ELME  39
   DEM=AEM*(AL**3)                                             ELME  40
   EM(1,1)=BEM/3.0                                             ELME  41
   EM(4,1)=BEM/6.0                                             ELME  42
   EM(2,2)=(13.0/35.0)*BEM                                     ELME  43
   EM(3,2)=(11.0/210.0)*CEM                                    ELME  44
   EM(5,2)=(9.0/70.0)*BEM                                      ELME  45
   EM(6,2)=(-13.0/420.0)*CEM                                   ELME  46
   EM(3,3)=DEM/105.0                                           ELME  47
   EM(5,3)=-EM(6,2)                                            ELME  48
   EM(6,3)= -DEM/140.0                                         ELME  49
   EM(4,4)=EM(1,1)                                             ELME  50
   EM(5,5)=EM(2,2)                                             ELME  51
   EM(6,5)=-EM(3,2)                                            ELME  52
   EM(6,6)=EM(3,3)                                             ELME  53
20 DO 25 I=1,5                                                 ELME  54
   IP=I+1                                                      ELME  55
   DO 25 J=IP,6                                                ELME  56
   EM(I,J)=EM(J,I)                                             ELME  57
25 EK(I,J)=EK(J,I)                                             ELME  58
   GO TO 40                                                    ELME  59
30 AEK=AE*EI/(AL*EE)                                           ELME  60
   EK(1,1)=AEK                                                 ELME  61
   EK(4,4)=EK(1,1)                                             ELME  62
   EK(1,4)=-EK(1,1)                                            ELME  63
   EK(4,1)=EK(1,4)                                             ELME  64
   FO=FR/(FB*EE)                                               ELME  65
   EK(2,2)=FO/AL                                               ELME  66
   EK(5,5)=EK(2,2)                                             ELME  67
   EK(2,5)=-EK(2,2)                                            ELME  68
   EK(5,2)=EK(2,5)                                             ELME  69
   IF(INC) 40,35,35                                            ELME  70
35 AEM=(RHO/XG)*AE*FB*AL/6.0                                   ELME  71
   EM(1,1)=2.0*AEM                                             ELME  72
   EM(1,4)=AEM                                                 ELME  73
   EM(2,2)=EM(1,1)                                             ELME  74
   EM(2,5)=EM(1,4)                                             ELME  75
   EM(4,4)=EM(1,1)                                             ELME  76
   EM(5,5)=EM(1,1)                                             ELME  77
   EM(4,1)=EM(1,4)                                             ELME  78
   EM(5,2)=EM(2,5)                                             ELME  79
40 AA=AM/AL                                                    ELME  80
   AB=AN/AL                                                    ELME  81
```

```
      A(1,1)=AA
      A(1,2)=AB
      A(1,3)=0
      A(2,1)=-AB
      A(2,2)=AA
      A(2,3)=0
      A(3,1)=0
      A(3,2)=0
      A(3,3)=1.0
      RETURN
      END
C
C     SUBROUTINE ENERGY:
C     MULTIPLICATION OF TWO VECTORS
C
      SUBROUTINE ENERGY (ENG,A,B,N,L,NN)
      DIMENSION ENG(L),A(NN,L),B(NN,L)
      DO 1 I = 1,L
      ENG(I) = 0.0
      DO 1 J = 1,N
    1 ENG(I) = ENG(I) + A(J,I)*B(J,I)
      RETURN
      END
C
C     SUBROUTINE FIXEND:
C     CALCULATE FIXEND ACTIONS AND FORM EQUIVALENT JOINT LOADS
C
      SUBROUTINE FIXEND (FR,AML,WT,MA,MB,X,Y,AL,INC,L,NN)
      DIMENSION FR(NN,L),AML(6),X(1),Y(1)
      AM=X(MB)-X(MA)
      AN=Y(MB)-Y(MA)
      AL=SQRT(AM**2+AN**2)
      AML(1)=0.
      AML(2)=0.5*WT*AL
      AML(3)=WT*(AL**2)/12.0
      AML(4)=AML(1)
      AML(5)=AML(2)
      AML(6)=-AML(3)
      IF(INC) 20,20,30
   20 CX=AM/AL
      CY=AN/AL
      DO 25 K=1,L
      FR(3*MA-2,K) = FR(3*MA-2,K)-AML(1)*CX+AML(2)*CY
      FR(3*MA-1,K) = FR(3*MA-1,K)-AML(1)*CY-AML(2)*CX
      FR(3*MA,K)   = FR(3*MA,K)-AML(3)
      FR(3*MB-2,K) = FR(3*MB-2,K)-AML(4)*CX+AML(5)*CY
```

```
      FR(3*MB-1,K) = FR(3*MB-1,K)-AML(4)*CY-AML(5)*CX
   25 FR(3*MB,K)   = FR(3*MB,K)-AML(6)
   30 CONTINUE
      RETURN
      END
C
C     SUBROUTINE GAUSS:
C     SOLVE LINEAR EQUATIONS
C
      SUBROUTINE  GAUSS(A,F,D,IC,ID,L,N,NN,NDCOMP)
      DIMENSION A(1),IC(1), ID(1),F(NN,L),D(NN,L)
      IF(NDCOMP .EQ. 1)GO TO 15
      DO 10 I=1,N
      I1=I-1
      DO  9 J=I,N
      IF(IC(J) .GT. I)GO TO 9
      IX=ID(J)-J+I
      IF(I1 .EQ. 0)GO TO 8
      DO  7 K=1,I1
      IF(IC(J) .GT. K .OR. IC(I) .GT. K)GO TO 7
      KX=ID(I)-I+K
      KY=ID(J)-J+K
      KZ=ID(K)
      A(IX)=A(IX)-(A(KX)*A(KZ)* A(KY))
    7 CONTINUE
    8 IF(I .EQ. J)GO TO 9
      KZ=ID(I)
      IF(A(KZ) .EQ. 0.)GO TO 100
      A(IX)=A(IX)/A(KZ)
    9 CONTINUE
   10 CONTINUE
   15 DO 40 K=1,L
      DO 30 I=1,N
      D(I,K)=F(I,K)
      I1=I-1
      IF(I1 .EQ. 0) GO TO 30
      DO 20 J=1,I1
      IF(IC(I) .GT. J)GO TO 20
      IX=ID(I)-I+J
      D(I,K)=D(I,K)-A(IX)*D(J,K)
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
      DO 70 I=1,N
      KX=ID(I)
      DO 70 K=1,L
```

```
   70 D(I,K)=D(I,K)/A(KX)
      DO 90 K=1,L
      IX=N
      DC 90 I=2,N
      IX=IX-1
      I1=I-1
      KX=IX
      DO 80 J=1,I1
      KX=KX+1
      IF(IC(KX) .GT. IX)GC TC 80
      KY=ID(KX)-KX+IX
      D(IX,K)=D(IX,K)-A(KY)*D(KX,K)
   80 CONTINUE
   90 CONTINUE
      GC TO 110
  100 WRITE(6,120)
  120 FORMAT(///2X,21HSTRUCTURE IS UNSTABLE///)
  110 RETURN
      END
C
C     SUBRCUTINE INPUT:
C     READ AND PRINT OF MEMBER IDENTIFICATION DATA
C
      SUBRCUTINE INPUT (MA,MB,JT,NT,ST,NMB,MNO,W,F,MD,MM,NM,NG)
      DIMENSION MA(1),MB(1),JT(1),NT(1),ST(1),W(1),F(1),NMB(1),
     1          MNO(NM,NG)
      MD=0
      WRITE(6,3)
      DO 30 L=1,MM
      READ(5,1) ID,MA(ID),MB(ID),JT(ID),ST(ID),WT,NT(ID),NB,
     1          (MNO(ID,J),J=1,NB)
      NMB(ID)=NB
      IF(NT(ID) .GT. MD) MD=NT(ID)
      W(ID)=0.0
      F(ID)=0.0
      IF(JT(ID)-1) 20,10,15
   10 F(ID)=WT
      GC TO 20
   15 W(ID)=WT
   20 WRITE(6,2) ID,MA(ID),MB(ID),JT(ID),ST(ID),WT,NT(ID),NB,
     1          (MNO(ID,J),J=1,NB)
   30 CONTINUE
    1 FORMAT(4I5,2F10.0,8I5/(16I5))
    2 FORMAT(4I5,2F15.3,16I5/(60X,14I5))
    3 FORMAT(////2X,29H MEMBER IDENTIFICATION DATA: /
     1 55X,44H IDENTIFICATION OF GIRDERS ABOVE EACH COLUMN/
```

```
  2 2X,3H NO,6H J-END,6H K-END,5H TYPE,13H STRESS LIMIT,2X,           INPU  24
  3 13H UNIFORM LOAD,1X,4H DVN,2X,3H NO,2X,                            INPU  25
  4 31H GIRDER IDENTIFICATICN NUMBERS:)                                INPU  26
    RETURN                                                             INPU  27
    END                                                                INPU  28
C
C   SUBRCUTINE LOADIN:
C   READ AND PRINT STATIC OR DYNAMIC LOADING INFORMATICN
C
    SUBROUTINE LOADIN (FR,FV,XG,IC,ID,MM,L,INC,PT,H1,H2,ND,N,NN)       LOAD   1
    COMMON/DATA/F(3),XV(3),MT(3),MO(3),NZ(3),NT(3),DEL(3),NNN          LCAD   2
    CCMMCN/ACODE/Z(3),CI(3),CK(3),TS(3),KNODE(30,3),HH(30),WS(30,3),   LCAD   3
   1      NSTORY                                                       LOAD   4
    INTEGER TITLE(10)                                                  LCAD   5
    DIMENSICN FR(NN,L),FV(NN,L),IC(1),ID(1),IM(3),JM(3),TFR(3),        LOAD   6
   1          PT(ND,L),H1(ND,L),H2(ND,L),XS(300,1)                     LCAD   7
    CO 40 J=1,L                                                        LCAD   8
    READ(5,4) NJ,MC,NZ(J),NNM,F(J),XV(J),DEL(J),NT(J),TITLE            LOAD   9
    MT(J)=NJ                                                           LCAD  10
    MO(J)=MC                                                           LCAD  11
    IF(NJ) 21,40,11                                                    LOAD  12
 11 KH=NJ                                                              LCAC  13
    WRITE(6,100) J,TITLE                                               LCAD  14
100 FORMAT(/3X,34H JOINT LCADS FCR LOADING CONDITION,I2,5H ... ,10A4/  LOAD  15
   1 4X,6H JOINT,4X,10H DIRECTION,15X,6H VALUE)                        LCAD  16
 12 IF(KH-3)13,13,14                                                   LOAD  17
 13 KX=KH                                                              LCAD  18
    GO TO 15                                                           LCAD  19
 14 KX=3                                                               LOAD  20
 15 READ(5,16) (TFR(I),IM(I),JM(I),I=1,KX)                             LCAD  21
    DO 20 I=1,KX                                                       LCAD  22
    WRITE(6,101) JM(I),IM(I),TFR(I)                                    LOAD  23
101 FORMAT(2I10,13X,F15.5)                                             LCAD  24
    KY=MM*JM(I)-MM+IM(I)                                               LCAD  25
 20 FR(KY,J)=FR(KY,J)+TFR(I)                                           LOAD  26
    KH=KH-KX                                                           LCAC  27
    IF(KH)21,21,12                                                     LCAD  28
 21 CONTINUE                                                           LOAD  29
    IF(INC) 36,36,22                                                   LCAD  30
 22 KSTEP=1                                                            LOAD  31
    M1=MO                                                              LCAD  32
    IF(MO.LE.0) M1=1                                                   LCAD  33
    IF(NJ) 23,30,30                                                    LOAD  34
 23 DO 24 I=1,N                                                        LOAD  35
 24 XS(I,1)=0                                                          LCAD  36
    DO 25 I=M1,N,MM                                                    LOAD  37
```

```
   25 XS(I,1)=1.0
      IF(KSTEP.GT.1) GO TO 27
      DO 26 I=1,N
   26 FR(I,J)=XS(I,1)
   27 IF(MO) 28,28,30
   28 KSTEP=KSTEP+1
      M1=M1+1
      IF(KSTEP.EQ.2) GO TO 23
      DO 29 I=1,N
   29 FV(I,J)=XS(I,1)
   30 READ(5,3) (PT(I,J),I=1,NNM)
      READ(5,3) (H1(I,J),I=1,NNM)
      WRITE(6,200) J,TITLE,MO,NNM,F(J),XV(J),NZ(J)
  200 FORMAT(/3X,27H DATA FOR LOADING CONDITION,I2,5H ... ,10A4/
     1 5X,40H MOTION TYPE CODE-----------------------,I5/
     2 5X,58H     LT. 0:  HORIZONTAL + VERTICAL MOTIONS              /
     3 5X,58H     GT. 0:  HORIZONTAL = 1 ;  VERTICAL = 2 .           /
     4 5X,40H NUMBER OF ACCELERATION DATA------------,I5/
     5 5X,40H LOAD MULTIPLIER------------------------,F7.3/
     6 5X,40H SCALE FACTOR FOR VERTICAL MOTION-------,F7.3/
     7 5X,40H TYPE OF INPUT ACCELERATION DATA--------,I5/
     8 5X,40H   EQ. 0:  TIME-HISTORY DATA             /
     9 5X,40H   EQ. 1:  SPECTRUM DATA                 )
      IF(NZ(J)) 31,31,32
   31 WRITE(6,201) DEL(J),NT(J)
  201 FORMAT(5X,40H TIME INCREMENT--------------------------,F7.3/
     1 5X,40H NUMBER OF TIME STEPS-------------------,I5)
   32 WRITE(6,202)
  202 FORMAT(/5X,19H ACCELERATION DATA:)
      IF(MO .LT. 0) GO TO 39
      DO 33 I=1,NNM
   33 WRITE(6,203) I,PT(I,J),H1(I,J)
  203 FORMAT(I10,3F15.5)
      GO TO 35
   39 READ(5,3) (H2(I,J),I=1,NNM)
      DO 34 I=1,NNM
   34 WRITE(6,203) I,PT(I,J),H1(I,J),H2(I,J)
   35 IF(NNN .LT.NNM) NNN=NNM
      IF(NJ .LT.0) F(J)=F(J)*XG
      GO TO 40
   36 IF(NJ) 37,40,40
   37 READ(5,5) NSTORY,Z(J),CI(J),CK(J),TS(J)
      WRITE(6,210) NSTORY,Z(J),CI(J),CK(J),TS(J)
  210 FORMAT(/5X,110H THE LATERAL EQUIVALENT STATIC LOADS ARE COMPUTED A
     1CCORDING TO THE REQUIREMENTS SPECIFIED IN THE 1976 UNIFORM /
     2 3X,54H BUILDING CODE WITH THE FOLLOWING INPUT INFORMATIONS:-/
```

```
    3 5X,40H TOTAL NUMBER OF STORIES, N--------------,I5/
    4 5X,40H EARTHQUAKE-ZONE COEFFICIENT, Z----------,F7.3/
    5 5X,40H IMPORTANT FACTOR OF BUILDING, I---------,F7.3/
    6 5X,40H STIFFNESS COEFFICIENT, K----------------,F7.3/
    7 5X,40H ESTIMATED SITE PERIOD, TS---------------,F7.3//
    8 3X,10H STORY NO.,3X,5H NODE,5X,13H STORY HEIGHT,5X,
    9 13H STORY WEIGHT)
      DO 38 I=1,NSTORY
      READ(5,6) II,KNODE(II,J),HH(II),WS(II,J)
   38 WRITE(6,211) II,KNODE(II,J),HH(II),WS(II,J)
  211 FORMAT(4X,I5,5X,I5,7X,F13.3,5X,F13.3)
   40 CONTINUE
    2 FORMAT(14I5)
    3 FORMAT(8F10.0)
    4 FORMAT(4I5,3F5.0,I5,10A4)
    5 FORMAT(I5,4F5.0)
    6 FORMAT(2I5,2F10.0)
   16 FORMAT(3(F10.0,2I5))
      RETURN
      END
C
C     SUBROUTINE MLTPLY:
C     MULTIPLICATION OF A MATRIX WITH A VECTOR
C
      SUBROUTINE MLTPLY(A,B,C,IC,ID,L,N,NN)
      DIMENSION A(1),B(NN,L),C(NN,L),IC(1),ID(1)
      DO 10 I = 1,N
      DO 4 K = 1,L
    4 B(I,K) = 0.0
      DO 9 J = 1,N
      IF (J - I) 6,6,7
    6 IF (IC(I) .GT. J) GO TO 9
      IX = ID(I) + J - I
      GO TO 8
    7 IF (IC(J) .GT. I) GO TO 9
      IX = ID(J) + I - J
    8 DO 5 K = 1,L
    5 B(I,K) = B(I,K) + A(IX)*C(J,K)
    9 CONTINUE
   10 CONTINUE
      RETURN
      END
C
C     SUBROUTINE PDELTA:
C     STORE AXIAL FORCE IN COLUMN
C
```

```
      SUBROUTINE PDELTA (S,FS,NMB,L)                                   PDEL
      DIMENSION S(6,L)                                                 PDEL
      IF(NMB.EQ.0) GO TO 20                                            PDEL
      ADR=0                                                            PDEL
      DO 10 J=1,L                                                      PDEL
      IF(ABS(S(1,J)).LE.ADR) GO TO 10                                  PDEL
      ADR=ABS(S(1,J))                                                  PDEL
   10 CONTINUE                                                         PDEL
      FS=ADR                                                           PDEL
   20 CONTINUE                                                         PDEL
      RETURN                                                           PDEL
      END                                                              PDEL
C
C     SUBROUTINE POP2:
C     CALCULATE TOTAL NUMBER OF ELEMENTS IN STIFFNESS MATRIX TO BE STOREC
C
      SUBROUTINE POP2(MMB,JN,MM,MA,MB,IC,ID,NZ)                        POP2
      DIMENSION MA(1),MB(1),IC(1),ID(1)                                POP2
      IX(I,J)=I*(J-1)+1                                                POP2
      NZ=0                                                             POP2
      NN=MM*JN                                                         POP2
      DO 10 I=1,NN                                                     POP2
   10 IC(I)=NN                                                         POP2
      DO 20 L=1,MMB                                                    POP2
      KX=IX(MM,MA(L))                                                  POP2
      KY=IX(MM,MB(L))                                                  POP2
      IF(IC(KY) .LT. KX) GO TO 20                                      POP2
      DO 19 I=1,MM                                                     POP2
      IC(KY)=KX                                                        POP2
   19 KY=KY+1                                                          POP2
   20 CONTINUE                                                         POP2
      DO 30 I=1,NN,MM                                                  POP2
      IF(IC(I) .LT. I)GO TO 30                                         POP2
      KX=I                                                             POP2
      DO 25 J=1,MM                                                     POP2
      IC(KX)=I                                                         POP2
   25 KX=KX+1                                                          POP2
   30 CONTINUE                                                         POP2
      DO 40 I=1,NN                                                     POP2
      NZ=NZ+(I-IC(I)+1)                                                POP2
   40 ID(I)=NZ                                                         POP2
      KX=(NN*(NN+1))/2                                                 POP2
      KY=((MM**2)*MMB)+(MM*(MM+1)*JN)/2                                POP2
      WRITE(6,2)                                                       POP2
      WRITE(6,3) KX,KY,NZ                                              POP2
      WRITE(6,4)                                                       POP2
```

```
      WRITE(6,5)(IC(I),I=1,NN)
      WRITE(6,6)
      WRITE(6,5)(ID(I),I=1,NN)
      WRITE(6,7)
    2 FORMAT(1H1,////22X,14HTOTAL ELEMENTS,5X,16HNCNZERO ELEMENTS,
     15X,15HSTORED ELEMENTS///)
    3 FORMAT(18X,I14,I18,I22//)
    4 FORMAT(//2X,36HSTARTING ROW NUMBERS FOR EACH COLUMN///)
    5 FORMAT(5X,10I12)
    6 FORMAT(//2X,61HNUMBERS OF DIAGCNAL ELEMENTS IN SINGLE ARRAY STIFFN
     1ESS MATRIX///)
    7 FCRMAT(///8X,120HTCTAL ELEMENTS MEAN THE TOTAL NUMBER OF ELEMENTS
     1IN THE STIFFNESS MATRIX, NONZERO ELEMENTS ARE THE TOTAL NUMBER OF
     2NON- /3X,124HZERO ELEMENTS IN THE STIFFNESS MATRIX, AND STOREC ELE
     3MENTS ARE THE ELEMENTS IN THE STIFFNESS MATRIX NEEDED TO BE STORED
     4 BY  /3X,125HUSING THIS NUMBERING SCHEME.  WHEN THE NUMBERING SCHE
     5ME IS AN EFFICIENT ONE, THE NUMBER OF NONZERO ELEMENTS IS CLOSED T
     6C THE /3X,26HNUMBER OF STCRED ELEMENTS.)
      RETURN
      END
C
C     SUBROUTINE PRNTDR:
C     PRINT THE OUTPUT OF DISPLACEMENTS
C
      SUBROUTINE PRNTDR(A,X,Y,N,M,L,NJ,NP,NN)
      CIMENSICN A(NN,L),X(1),Y(1)
      NP=NP+1
      LINES=1
      WRITE(6,1)NP
      WRITE(6,2)
      CO 10 I=1,NJ
      IF (LINES+L-51)4,3,3
    3 LINES=1
      WRITE(6, 1) NP
      WRITE(6, 2)
      NP=NP+1
    4 KH=3*I
      KHH=KH-2
      WRITE(6, 5)I,X(I),Y(I), ( A(J,1),J=KHH,KH)
    5 FCRMAT(/ 3X,I4,7X,F9.3,3X,F9.3,13X,1PE18.8,1PE17.8,1PE18.8)
      IF(L .EQ. 1) GCTC 8
      CC 7   K=2,L
      WRITE(6,6) (A(J,K),J=KHH,KH)
    6 FORMAT( 48X,1PE18.8,1PE17.8,1PE18.8)
    7 CONTINUE
    8 LINES =LINES +L+1
```

```
   10 CONTINUE                                                            PRNT
    1 FORMAT(1H1,120X,5HPAGE ,I3////)                                     PRNT
    2 FORMAT(//3X,5HJOINT,10X,2H-X,10X,2H-Y,21X,7HDISPL-X,10X,7HDISPL-Y,PRNT
     110X,8HROTATION//)                                                   PRNT
      RETURN                                                              PRNT
      END                                                                 PRNT
C
C     SUBROUTINE REDUCE:
C     REDUCE FORCE OR DISPLACEMENT VECTOR BY SUBSTITUTING BOUNDARY
C     CONDITION
C
      SUBROUTINE REDUCE (F,IB,N,NB,L,NN)                                  REDU
      DIMENSION F(NN,L) ,IB(1)                                            REDU
      DO 5 J=1,L                                                          REDU
      IH=NB                                                               REDU
      NH=N                                                                REDU
    1 I=IB(IH)                                                            REDU
      IF(I-NH) 2,4,4                                                      REDU
    2 NH1=NH-1                                                            REDU
      DO 3 K=I,NH1                                                        REDU
      K1=K+1                                                              REDU
    3 F(K,J) =F(K1,J)                                                     REDU
    4 IH=IH-1                                                             REDU
      NH=NH-1                                                             REDU
      IF(IH.EQ.0) GO TO 5                                                 REDU
      GO TO1                                                              REDU
    5 CONTINUE                                                            REDU
      RETURN                                                              REDU
      END                                                                 REDU
C
C     SUBROUTINE RESTOR:
C     RESTORE FORCE OR DISPLACEMENT VECTOR INTO GLOBAL SYSTEM
C
      SUBROUTINE RESTOR( D,IB,N,NB,L,NN)                                  REST
      DIMENSION D(NN,L),IB(1),TDR1(40),TDR2(40)                           REST
      NH=N-NB                                                             REST
      IH=1                                                                REST
    1 I=IB(IH)                                                            REST
      IF(I.GT.NH) GO TO 7                                                 REST
      DO 2 K=1,L                                                          REST
      TDR1(K)=D(I,K)                                                      REST
    2 D(I,K)=0.                                                           REST
    3 J=I+1                                                               REST
      IF(J.GT.NH) GO TO 5                                                 REST
      DO 4 K=1,L                                                          REST
    4 TDR2(K)=D(J,K)                                                      REST
```

```
5      DO 6 K=1,L                                                        REST  14
       D(J,K) =TDR1(K)                                                   REST  15
6      TDR1(K)=TDR2(K)                                                   REST  16
       IF(I.GE.NH) GO TO 9                                               REST  17
       I=I+1                                                             REST  18
       GO TC 3                                                           REST  19
7      DO 8 K=1,L                                                        REST  20
8      C(I,K)=0.                                                         REST  21
9      IF(IH.GE.NB) GC TC 10                                             REST  22
       IH=IH+1                                                           REST  23
       NH=NH+1                                                           REST  24
       GO TO 1                                                           REST  25
10     CONTINUE                                                          REST  26
       RETURN                                                            REST  27
       END                                                               REST  28
C
C      SUBRCUTINE SAME:
C      CONSTRAINTS ON COMMCN MEMBERS
C
       SUBROUTINE SAME(A,ITYPE,MTYPE,MEMBS)                              SAME   1
       DIMENSION A(1),ITYPE(1)                                           SAME   2
       IF(MTYPE.EQ.MEMBS) GO TO 10                                       SAME   3
     9 CO 4 I=1,MTYPE                                                    SAME   4
       L=0                                                               SAME   5
       CO 3 J=1,MEMBS                                                    SAME   6
       IF(ITYPE(J).NE.I) GC TC 3                                         SAME   7
       IF (L) 5,5,6                                                      SAME   8
     5 L=L+1                                                             SAME   9
       AREAA=A(J)                                                        SAME  10
     6 IF(AREAA-A(J)) 2,3,3                                              SAME  11
     2 AREAA=A(J)                                                        SAME  12
     3 CONTINUE                                                          SAME  13
       DO 4 K=1,MEMBS                                                    SAME  14
       IF(ITYPE(K).NE.I) GO TO 4                                         SAME  15
       A(K)=AREAA                                                        SAME  16
     4 CONTINUE                                                          SAME  17
    10 RETURN                                                            SAME  18
       END                                                               SAME  19
C
C      SUBROUTINE SPTRUM:
C      CALCULATE DYNAMIC LCAD FACTCR
C
       SUBROUTINE SPTRUM (X,Y,DAMP,NRU,PT,H,ND,LD)                       SPTR   1
       COMMON/DATA/F(3),XV(3),MT(3),MO(3),NZ(3),NT(3),DDT(3),NNN         SPTR   2
       DIMENSION PT(ND,LD),H(ND,LD)                                      SPTR   3
       XX=X                                                              SPTR   4
```

```
      IF(NZ(NRU)) 20,20,1                                               SPTR   5
C
C     SPECTRUM INTERPOLATION
    1 DO 2 L=2,NNN                                                      SPTR   6
      IF(XX .LT. PT(L,NRU)) GC TC 3                                     SPTR   7
    2 CONTINUE                                                          SPTR   8
    3 N=L-1                                                             SPTR   9
      FO=H(N,NRU)                                                       SPTR  10
      F1=H(N+1,NRU)                                                     SPTR  11
      HO=PT(N,NRU)                                                      SPTR  12
      H1=PT(N+1,NRU)                                                    SPTR  13
      Y = FO + (F1-FO)*(XX-HO)/(H1-HO)                                  SPTR  14
   10 GO TO 700                                                         SPTR  15
C
C     TIME-HISTORY ... USING EXPLICIT INTEGRATION
   20 W=6.283185307/XX                                                  SPTR  16
      WW=W*W                                                            SPTR  17
      ZW=DAMP*W                                                         SPTR  18
      TZW=2.0*ZW                                                        SPTR  19
      WD=W*SQRT(1.0-DAMP**2)                                            SPTR  20
      FB=TZW/(WW*WW)                                                    SPTR  21
      FA=ZW/WW                                                          SPTR  22
      FV=ZW*WW                                                          SPTR  23
      FVD=WW*(2.0*DAMP**2-1.0)                                          SPTR  24
      FBB=(2.0*DAMP**2-1.0)/WW                                          SPTR  25
C
      L=0                                                               SPTR  26
      II=1                                                              SPTR  27
      Y=0.                                                              SPTR  28
      VO=0.                                                             SPTR  29
      VDO=0.                                                            SPTR  30
      TC=PT(1,NRU)                                                      SPTR  31
   30 DT=DDT(NRU)                                                       SPTR  32
   50 B=(H(II+1,NRU)-H(II,NRU))/(PT(II+1,NRU)-PT(II,NRU))               SPTR  33
      A=H(II,NRU) + B*(TC-PT(II,NRU))                                   SPTR  34
      TT=TO+DT                                                          SPTR  35
      IF(PT(II+1,NRU) .GT. TT) GO TO 100                                SPTR  36
      DELT=PT(II+1,NRU)-TC                                              SPTR  37
      GO TO 200                                                         SPTR  38
  100 DELT=DT                                                           SPTR  39
C
  200 EX=EXP(-ZW*DELT)                                                  SPTR  40
      FT=WD*DELT                                                        SPTR  41
      CS=COS(FT)                                                        SPTR  42
      SN=SIN(FT)                                                        SPTR  43
      VT=(VDO+ZW*VO-FA*A+FBB*B)*SN/WD                                   SPTR  44
```

```
      VT=VT + (VO-A/WW+FB*B)*CS
      VT=VT*EX + (A/WW-FB*B+B*DELT/WW)
C
      VDT=(A-WW*VO-ZW*(VCC+B/WW))*SN/WD
      VCT=VDT + (VDO-B/WW)*CS
      VDT=VDT*EX + B/WW
C
      VO=VT
      VDC=VDT
      IF(PT(II+1,NRU) .GT. TT) GO TC 500
      CT=CT-DELT
      II=II+1
      TO=PT(II,NRU)
      IF(DT .EQ. 0.) GO TO 600
      GC TO 50
  500 TO=TO+DT
  600 L=L+1
      IF(Y .LT. ABS(VT)) Y=ABS(VT)
      IF(L .LT. NT(NRU)) GO TO 30
      Y=Y*WW
  700 CONTINUE
      RETURN
      END
C
C     SUBROUTINE STRSWT:
C     CALCULATE STRUCTURAL WEIGHTS ON COLUMN
C
      SUBROUTINE STRSWT (NMB,MNO,EL,JT,FS,AE,BA,B,D,TFD,I,INC,NM,NG)
      COMMON/BBC/IS,T,DEPMAX,DEPMIN,TFDMAX,TFDMIN,RHO,RHO1
      DIMENSICN MNC(NM,NG),EL(1),AE(1),JT(1),D(1),TFD(1)
      SUM=0.
      NB=NMB
      IF(NB .EQ. 0) GO TC 20
      DC 10 J=1,NB
      KK=MNO(I,J)
      IF(INC.EQ.0 .AND. KK.NE.I) GO TO 10
      CALL AREA(AE(KK),BA,B,D(KK),TFD(KK),JT(KK),SPWT,AA,SS,QQ)
      SUM=SUM + SPWT*AA*EL(KK)/2.0
   10 CONTINUE
   20 FS=FS+SUM
      RETURN
      END
C
C     SUBROUTINE STURM:
C     CALCULATE STURM SEQUENCE PRCPERTIES CF THE I MODE
C
```

```
      SUBROUTINE STURM (A,B,IC,IC,MB,VALUE,NGREAT,N)
      DIMENSION A(1),B(1),IC(1),ID(1),IREC(300),ICHNG(300)
     1          ,C(50,100),D(100)
      M1 = MB + 1
      M21 = 2*MB + 1
      IF(M21 .GT. N)M21=N
      MB2=M21-1
      DO 5 I = 1,M1
      DO 5 J=1,M21
    5 C(I,J)=0.
      KH = 1
      CC 10 I = 1,M21
      IF (I .GT. M1) KH = I - MB
      DO 9 J = KH,M1
      IF (J - I) 6,6,7
    6 IF (IC(I) .GT. J) GC TC 9
      IX = ID(I) + J - I
      GO TO 8
    7 IF (IC(J) .GT. I) GC TC 9
      IX = ID(J) + I - J
    8 C(J,I)=A(IX)-VALUE*B(IX)
    9 CONTINUE
   10 CONTINUE
      DC 12 I = 1,N
      IREC(I) = 1
   12 ICHNG(I) = 0
      KFR = 1
      IF (C(1,1) .LT. 0.) KFR = -1
      IREC(1) = KFR
      N1 = N - 1
      M221=M21
      CC 50 I = 1,N1
      IF (I .LE. MB) GO TC 20
      DO 13 J = 1,M21
   13 D(J) = 0.
      KR1 = I + 1
      IF (I .GE. (N-MB))  M221=M221-1
      KX=I-MB
      DC 17 J = 1,M221
      IF (J - M1) 14,14,15
   14 IF (IC(KR1) .GT. (KX+J)) GC TC 17
      IX = ID(KR1) - KR1 + (KX + J)
      GC TC 16
   15 KX = KR1 + J -M1
      IF (IC(KX) .GT. KR1) GO TO 17
      IX = ID(KX) - KX + KR1
```

```
 16   D(J) = A(IX)-VALUE*B(IX)                                    STUR
 17   CONTINUE                                                    STUR
      DO 19 J = 1,MB2                                             STUR
      DO 18 K = 1,MB                                              STUR
 18   C(K,J)=C(K+1,J+1)                                           STUR
 19   C(M1,J)=D(J)                                                STUR
      DO 4 K=1,MB                                                 STUR
  4   C(K,M21)=0.                                                 STUR
      C(M1,M21)=D(M21)                                            STUR
 20   IMB=MB                                                      STUR
      IF(I .LT. MB)IMB=I                                          STUR
      DO 40 J = 1,IMB                                             STUR
      IF(ABS(C(J,J)).GE. ABS(C(IMB+1,J)))GO TO 25                 STUR
      IF(C(J,J)/C(IMB+1,J) .GE. 0.)KFR=KFR*(-1)                   STUR
      DO 105 K=J,M21                                              STUR
      SAVE=C(IMB+1,K)                                             STUR
      C(IMB+1,K)=C(J,K)                                           STUR
105   C(J,K)=SAVE                                                 STUR
      ICHNG(J) = ICHNG(J) + 1                                     STUR
 25   C(IMB+1,J)=C(IMB+1,J)/C(J,J)                                STUR
      J1 = J + 1                                                  STUR
      DO 30 K = J1,M21                                            STUR
 30   C(IMB+1,K)=C(IMB+1,K)-C(IMB+1,J)*C(J,K)                     STUR
 40   CONTINUE                                                    STUR
      IF(C(IMB+1,IMB+1) .LT. 0.)KFR=KFR*(-1)                      STUR
      IREC(I+1) = KFR                                             STUR
 50   CONTINUE                                                    STUR
      NGREAT = 0                                                  STUR
      IF (IREC(1) .GT. 0) NGREAT = NGREAT + 1                     STUR
      DO 70 I = 2,N                                               STUR
 70   IF (IREC(I) .EQ. IREC(I-1)) NGREAT = NGREAT + 1             STUR
100   FORMAT (1HO,(12X,E13.6,5(5X,E13.6)/))                       STUR
101   FORMAT (1X,25I5)                                            STUR
      RETURN                                                      STUR
      END                                                         STUR
C
C     SUBROUTINE TRNSFM:
C     TRANSFORM MEMBER MATRICES INTO GLOBAL MATRICES
C
      SUBROUTINE TRNSFM(EK,EM,A,B,C,D,E,INC)                      TRNS
      DIMENSION A(3,3),B(6,6),C(6,6),D(6,6),E(6,6) ,EK(6,6),EM(6,6)   TRNS
      DO  30 I=1,6                                                TRNS
      MD=0                                                        TRNS
      DO  29 J=1,6                                                TRNS
      JA=J-MD                                                     TRNS
      B(I,J)=0                                                    TRNS
```

```
      D(I,J)=0
      DO   28 K=1,3
      KB=K+MD
      D(I,J)=D(I,J)+ EM(I,KB)*A(K,JA)
   28 B(I,J)= B(I,J) +EK(I,KB)*A(K,JA)
      IF(J .LT. 3) GO TO 29
      MD=3
   29 CONTINUE
   30 CONTINUE
      IF (INC .EQ. 1) RETURN
C
      MD=0
      DO   34 I=1,6
      DO   33 J=1,6
      IA= I-MD
      C(I,J)=0
      E( I,J)=0
      DO   32 K=1,3
      KB=K+MD
      E(I,J)= E(I,J)+A(K,IA)*D(KB,J)
   32 C(I,J)=C(I,J)+A(K,IA)*B(KB,J)
   33 CONTINUE
      IF(I.LT. 3) GO TO  34
      MD=3
   34 CONTINUE
      RETURN
      END
C
C     SUBROUTINE WFBEAM:
C     CALCULATE MEMBER STRESSES
C
      SUBROUTINE WFBEAM (S,SIGMA,A,AE,AREAA,SEC,SHE,L)
      DIMENSION S(6,L),BETA(3,6),SIGMA(6,L)
      DO 132 I=1,3
      DO 132 J=1,6
  132 BETA(I,J)=0.
   15 BETA(1,1)=1.0/AREAA
      BETA(1,3)=-1.0/SEC
      BETA(1,4)=-BETA(1,1)
      BETA(1,6)=-BETA(1,3)
      BETA(2,2)=SHE
      BETA(2,5)=BETA(2,2)
      BETA(3,1)=BETA(1,1)
      BETA(3,3)= -BETA(1,3)
      BETA(3,4)= -BETA(1,1)
      BETA(3,6)=BETA(1,3)
```

```
      DO 131  K=1,L
      DO 131  I=1,3
      SIGMA(I,K)=0
      SIGMA(I+3,K)=0
      DO 131  J=1,3
      SIGMA(I,K)=SIGMA(I,K)+BETA(I,J)*S(J,K)
  131 SIGMA(I+3,K)=SIGMA(I+3,K)+BETA(I,J+3)*S(J+3,K)
      RETURN
      END
```