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Estimation of Structural Strains In Underground Lifeline Pipes

by

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FOREWORD

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ESTIMATION OF STRUCTURAL STRAINS

IN UNDERGROUND LIFELINE PIPES

BY

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ABSTRACT

The risk analysis methodology previously developed for underground pipeline systems by the senior author has made use of a conversion factor β in order to estimate the structural strain in underground pipelines induced by the propagating seismic wave. The purpose of this study is (a) to derive a practical procedure to estimate the conversion factor that can be used not only for straight and bent pipes but also for structural details of more complex geometry and (b) to determine the conversion factor for a number of typical cases in order to provide the analyst with the numerical insight as to its values consistent with the physical conditions to which the pipeline system is subjected.

NOMENCLATURE

A	= cross-sectional area
A,B,C	= soil conditions
c	= cohesion stress or seismic wave velocity
D	= outer diameter of pipe
d	= wall thickness
E	= modulus of elasticity of steel
$G = \rho_G c^2$	= equivalent modulus of rigidity of soil
g	= acceleration due to gravity
I	= area moment of inertia of a cross-section
$K_G = 2\pi G$	= equivalent spring constant to reflect soil-structural interaction (force per unit area, Eq. 1)
$k = \pi D k_0$	= coefficient of soil reaction (force per unit area)
k_0	= coefficient of soil reaction (force per unit volume)
L	= seismic wave length
ℓ^*	= quantity indicating a point of slip initiation
M	= bending moment
$Q = LD\lambda^2/(4\pi)$	= parameter
q, q^*	= coefficients
S	= shear force
$T = L/c$	= period of seismic wave
$U(x)$	= displacement profile function
u_G	= free field displacement

u_S = displacement of pipe in the longitudinal direction

$W = 3Lk/(16AE\lambda)$ = parameter

β = conversion factor

β_0 = quasi-static conversion factor to be applied to straight pipes

$\hat{\beta}_0$ = dynamic conversion factor to be applied to straight pipes

β_{cr} = conversion factor for structural strain when the pipe is in slippage

β^*_{cr} = conversion factor for maximum relative displacement between straight pipe and soil when slippage occurs

β_{1B}, β_{2B} = conversion factors to be applied to elements (1) and (2) of bent pipe (Fig. 5)

β_{1T}, β_{2T} = conversion factors to be applied to elements (1) and (2) of a tee-junction (Fig. 6)

β_I = conversion factor to obtain resisting capacity in the I-th failure mode

γ_{cr} = critical shear strain of soil at which slippage initiates

γ_G = shear strain in soil

γ_0 = maximum shear strain in soil at the interface

Δ = relative displacement between pipe and soil

ε_{cr} = critical structural strain corresponding to γ_{cr}

ε_G = free field strain

ε_S = structural strain in the longitudinal direction

ε^* = resisting strain

$\zeta = (2\pi/L)Ed/G$ = parameter

θ = angle in bending

$\lambda = \sqrt[4]{k/(4EI)}$ = relative rigidity of pipe and soil

μ = mean value

ρ = mass density of steel

ρ_G = mass density of soil

σ = normal stress or standard deviation

τ = shear stress of soil

τ_{cr} = critical shear stress corresponding to γ_{cr}

τ_G = shear stress in soil

ϕ = angle of friction

$\omega = 2\pi/T$ = frequency of seismic wave

$\hat{\omega} = (2\pi/L)\sqrt{E/\rho}$ = frequency of wave of length L propagating longitudinally through the pipe

$\hat{\omega}_k = (2\pi k/L)\sqrt{E/\rho}$ = frequency of wave of length L/k propagating longitudinally through the pipe

$\omega_0 = \sqrt{K_G/\rho A}$ = frequency of simplified pipe-soil system model (Eq. 1)

INTRODUCTION

The risk analysis methodology developed for underground pipeline systems in (1) and (2) makes use of a conversion factor β in order to estimate the structural strain ε_S in the underground pipelines induced by the propagating seismic wave. The conversion is accomplished by multiplying the free field strain ε_G by β: $\varepsilon_S = \beta\varepsilon_G$. Whether or not the risk analysis methodology can be implemented to produce a credible assessment of siesmic risk of the pipeline system depends largely, although not exclusively, on the availability of accurate estimates of such a factor which in turn depends on structural details, pipe materials, properties of the surrounding soil, the nature of propagating waves, etc.

The purpose of this study, therefore, is (a) to derive a practical procedure to estimate the conversion factor that can be used not only for straight and bent pipes but also for structural details of more complex geometry such as tee-junctions and (b) to determine the conversion factor for a number of typical cases in order to provide the analyst with the numerical insight as to its values consistent with the physical conditions to which the pipeline system is subjected.

The analytical results of previous studies, e.g. (3), are usually based on the simplified model of a straight pipe imbedded (in the z-direction) in an infinite elastic (soil) medium for which the familiar differential equation can be established invoking D'Alembert's principle with respect to the inertia force, the internal force within the pipe and the force proportional to the displacement u_S of the pipe relative to that of the free field u_G:

$$\rho\partial^2 u_S/\partial t - E\partial^2 u_S/\partial z^2 = K_G(u_G - u_S) \qquad (1)$$

where ρ = mass density of the pipe material, E = Young's modulus of the pipe material and K_G = equivalent spring constant to reflect the soil-structure interaction. The analytical results derived from Eq. 1 indicate that the structural strain is in approximation equal to the free field strain and the inertia effects are negligible provided that no slip takes place between the pipe and the surrounding soil. A field measurement performed by Sakurai et al (3) at the time of the Matsushiro earthquake supports these observations.

It is important to note in this regard that these observations appear to be valid only when the earthquake intensity is mild and the free field strain is as small as 10^{-4} or less so that no slip is generally expected. It also appears that, when the earthquake intensity is severe and the free field strain reaches the order of magnitude of $10^{-3} \sim 10^{-2}$, the chances of a slip taking place between the pipe and the surrounding soil significantly increases causing large strain concentrations at various joints and connections in the pipeline system, thus resulting in significant structural damages at these structural locations. Therefore, for the proposed conversion factor to be useful particularly when the free field strain is large and hence significant pipe damages are expected, we must take the effect of possible slip between the pipe and the surrounding soil into consideration in the analysis. The simplified mechanism of slip we assume in this study involves more directly the shear stress τ acting on the pipe surface rather than the force proportional to the relative displacement between the pipe and the free field displacement $K_G(u_S - u_G)$ (see Eq. 1). In particular, the slip initiates when the shear stress τ exceeds $\tau_{cr} = c + \sigma \tan\phi$ where c = cohesion stress, σ = normal stress at the interface between the pipe and the soil and ϕ = angle of friction between the pipe and the soil. The normal stress σ and therefore τ_{cr} increase as the depth of soil cover increases. Therefore, the slip initiates at a larger value of τ when the depth of cover is larger. In the present analysis, we assume for numerical purposes, that the slip takes place when τ_{cr} reaches a value corresponding to the shear strain $\gamma_{cr} = 10^{-3}$ in the immediate vicinity of the pipe surface and that τ_{cr} is given by $\tau_{cr} = G\gamma_{cr}$ where G is the shear modulus of the soil.

ASSUMPTIONS IN DETERMINING THE CONVERSION FACTOR

The conversion factor β is defined as the ratio of the structural strain ε_S to the free field strain ε_G.

$$\beta \equiv \varepsilon_S/\varepsilon_G \qquad (2)$$

Numerical evaluation of this factor can be made on the basis of the following assumptions:

(i) The free field strain is sinusoidally distributed with wave length L in the direction (z) of the pipe element.

(ii) Shear stresses on the pipe surface are transmitted directly from the surrounding soil medium (see Fig. 1).

(iii) The relationship between τ and γ within the surrounding soil is of the type as shown in Fig. 2.

(iv) Slippage initiates when the shear strain of the soil near the pipe surface exceeds the critical shear strain γ_{cr}.

(v) With respect to lateral motions, pipe elements are assumed to be embedded in an infinite elastic medium which has an average radial resistance, k, per unit area (e.g. $kgwt/cm^2$).

STRAIN IN STRAIGHT PIPES

Structural Strain and Free Field Strain

Shear Stress

Assume that the ground displacement near the pipe is given by

$$u_G(x,z) = U(x)\cdot\varepsilon_G\cdot L/(2\pi)\cdot\sin\{\omega t - (2\pi/L)\cdot z\} \tag{3}$$

in which L = wave length, ω = circular frequency of propagating wave, ε_G = free field strain and U(x) = displacement profile along the x axis perpendicular to the pipe axis, where

$$\lim U(x) = 1 \qquad \text{as } x \to \infty \tag{4}$$

Throughout the paper, the variable t is suppressed as much as possible. For the discussion of spatial distribution of stress, strain, etc., use $t = \pi/\omega$. The displacement profile U(x) is a function of the difference in the stiffnesses of the soil and the pipe, and also, of τ_{cr} among other physical and environmental conditions at the pipe surface.

The shear strain distribution $\gamma_G(x,z)$ in the soil medium follows from Eq. 3 as

$$\gamma_G(x,z) = \partial u_G/\partial x = \partial U(x)/\partial x\cdot\varepsilon_G\cdot L/(2\pi)\cdot\sin\{\omega t - (2\pi/L)\cdot z\} \tag{5}$$

If the shear strain γ_G remains within the elastic range, the shear stress of the soil, τ_G, at x = D/2 is given by

$$\tau_G = \tau_G(z) = G\gamma_G(D/2,z) = G\gamma_0\sin\{\omega t - (2\pi/L)\cdot z\} \tag{6}$$

This is the shear stress acting on the pipe surface where G = shear modulus of the soil, D = outer diameter of the pipe and γ_0 = maximum shear strain acting on the pipe surface, which is given by

$$\gamma_0 = [\partial U(x)/\partial x]_{x=D/2}\cdot\varepsilon_G\cdot L/(2\pi) \tag{7}$$

Equation of Motion

Consider the free body diagram of a segment of pipe dz shown in Fig. 3 subjected to an acceleration $\partial^2 u_S/\partial t^2$. Applying D'Alembert's principle to the surface, internal and inertia forces acting on this segment, we obtain

$$A\{\sigma_S + (\partial\sigma_S/\partial z)dz\} + \pi D\tau_G dz = A\sigma_S + \rho A dz\,\partial^2 u_S/\partial t^2 \tag{8}$$

in which d = pipe wall thickness, σ_S = axial stress in the pipe and A = cross-sectional area of pipe given by

$$A = \pi d(D - d) \doteq \pi D d \tag{9}$$

Noting that

$$\sigma_S = E\partial u_S/\partial z \tag{10}$$

we can rewrite Eq. 8 in the form

$$\rho\partial^2 u_S/\partial t^2 - E\partial^2 u_S/\partial z^2 = \tau_G/d \tag{11}$$

which reduces to the familiar one-dimensional wave equation in a homogeneous (ρ, E = constant) elastic rod as it should if $U(x) = 1$ and hence $\gamma_0 = \tau_G = 0$.

Structural Strain

We assume the structural displacement u_S is given by

$$u_S(z) = B\sin\{\omega t - (2\pi/L)\cdot z\} \tag{12}$$

Noting that

$$\partial^2 u_S/\partial^2 t = -\omega^2 u_S \tag{13}$$

$$\partial^2 u_S/\partial z^2 = -(2\pi/L)^2 u_S \tag{14}$$

and substituting Eqs. 6, 13 and 14 into 11, we obtain

$$-\rho\omega^2 B + E(2\pi/L)^2 B = G\gamma_0/d$$

Hence

$$B = \frac{(L/2\pi)^2\cdot G\gamma_0/(Ed)}{1 - (\omega/\hat{\omega})^2} \tag{15}$$

in which

$$\hat{\omega} = (2\pi/L)\sqrt{E/\rho} \tag{16}$$

The last quantity is the frequency of a longitudinal elastic wave of length L propagating through an infinite straight pipe with velocity $c = \sqrt{E/\rho}$.

Combining Eqs. 7, 12 and 15, and also differentiating, the structural displacement u_S and strain ε_S are obtained as follows:

$$u_S(z) = (L/2\pi)\cdot\varepsilon_G\cdot\beta_0 \sin\{\omega t - (2\pi/L)\cdot z\}/\{1 - (\omega/\hat{\omega})^2\} \tag{17}$$

$$\varepsilon_S(z) = -\varepsilon_G\cdot\beta_0\cos\{\omega t - (2\pi/L)\cdot z\}/\{1 - (\omega/\hat{\omega})^2\} \tag{18}$$

in which

$$\beta_0 = (L/2\pi)^2\cdot G\cdot[\partial U(x)/\partial x]_{x=D/2}/(Ed) \tag{19}$$

Conversion Factor and Maximum Relative Displacement

Following the general definition given by Eq. 2, and evaluating the maximum ratio of ε_S to ε_G, the dynamic conversion factor for a straight pipe (taking the inertia effect into consideration) may be defined as

$$\hat{\beta}_0 = \beta_0/\{1 - (\omega/\hat{\omega})^2\} \tag{20}$$

Using $\hat{\beta}_0$, we can rewrite Eqs. 17 and 18 in the following form;

$$u_S(z) = \hat{\beta}_0 \cdot u_G(\infty, z) \tag{21}$$

$$\varepsilon_S(z) = -\hat{\beta}_0 \cdot \varepsilon_G \cos\{\omega t - (2\pi/L)\cdot z\} \tag{22}$$

Since the seismic wave usually propagates with a much smaller frequency than $\hat{\omega}$ defined in Eq. 16, if follows that $\omega/\hat{\omega} << 1$. We can therefore conclude from Eq. 20 that the dynamic conversion factor is practically identical to the conversion factor based on the quasi-static consideration.

The maximum relative displacement Δ between the pipe and the soil is observed at $z = L/4$ (point of maximum amplitude) and equal to

$$\Delta = u_G(\infty, L/4) - u_S(L/4) = (1 - \hat{\beta}_0)\cdot\varepsilon_G\cdot L/(2\pi) \tag{23}$$

Displacement Profile Function

For an explicit analytical expression for the conversion factor β_0 and $\hat{\beta}_0$ the functional form of $U(x)$ must be assumed. In this respect, it appears reasonable and mathematically expedient to assume that

$$U(x) = 1 - \exp\{-\zeta\cdot(2\pi/L)\cdot x\} \tag{24}$$

in which

$$\zeta = (2\pi/L)\cdot Ed/G \tag{25}$$

Use of Eqs. 19 and 24 then produces the conversion factor β_0 given by

$$\beta_0 = \exp\{-\zeta\cdot(2\pi/L)\cdot D/2\} = 1/\exp\{\zeta\cdot(2\pi/L)\cdot D/2\} \tag{26}$$

Since the argument $\zeta\cdot(2\pi/L)\cdot D/2$ is usually much smaller than unity, β_0 can be approximated by

$$\beta_0 \doteq 1/\{1 + (2\pi/L)^2\cdot(AE/K_G)\} \tag{27}$$

and

$$\hat{\beta}_0 \doteq \beta_0/\{1 - (\omega/\hat{\omega})^2\} \doteq 1/\{1 + (2\pi/L)^2\cdot(AE/K_G) - (\omega/\omega_0)^2 - (\omega/\hat{\omega})^2\} \tag{28}$$

in which $A \doteq \pi Dd$, $K_G = 2\pi G$ = equivalent soil spring constant per unit area (e.g. kgwt/cm^2) and $\omega_0 = \sqrt{K_G/\rho A}$ = frequency of simplified pipe-soil model represented by Eq. 1.

Similarly, γ_0 in Eq. 7 can be shown to be

$$\gamma_0 = \zeta\varepsilon_G\beta_0 = (2\pi/L)\cdot(Ed/G)\varepsilon_G\beta_0 \tag{29}$$

Numerical Examples of $\hat{\beta}_0$ and β_0

For each of the three geological site conditions A, B and C, conversion factors $\hat{\beta}_0$ and β_0 are computed, where A represents unconsolidated to poorly consolidated sediments, B semi-consolidated to moderately consolidated sediments, and C very dense ioneous and metamorphic rocks. The results are shown in Table 1 under the assumption that the wave lengths are L = 400 m and 150 m respectively. In both cases, we observe that the straight pipes deform almost in unison with the soil under the conditions of no slippage and that the dynamic effect is negligible in the sense that the values of $\hat{\beta}_0$ and β_0 are practically identical.

Structural Strain When Slippage Occurs

Structural Strain in Slippage

As described in the Introduction, the slippage along the interface between the buried pipe and the surrounding soil can take place when the earthquake intensity is severe enough so that the shear stress τ produced in the interface reaches the value $\tau_{cr} = c + \sigma \tan \phi$. In the present study, we assume, as also indicated in the Introduction, that the value of τ_{cr} is such that $\gamma_{cr} = \tau_{cr}/G$ is equal to 10^{-3}.

Noting that γ_0 in Eq. 7 is the maximum shear strain in the soil at the interface, the following criteria can be used to determine whether the slippage will or will not take place at least in some portion along the interface:

If $\gamma_0 < \gamma_{cr}$, slippage will not take place

If $\gamma_0 \geqq \gamma_{cr}$, slippage will take place.

Using Eq. 7 together with Eqs. 18 and 19, we can show under the conditions of no slippage that the maximum structural strain ε_{cr} can be expressed as

$$\varepsilon_{cr} = L/(2\pi)\cdot G/(Ed)\cdot\gamma_0/\{1 - (\omega/\hat{\omega})^2\} = \gamma_0/\zeta\cdot 1/\{1 - (\omega/\hat{\omega})^2\} = \varepsilon_G\hat{\beta}_0 \tag{30}$$

Since Eq. 30 is still valid when γ_0 becomes γ_{cr}, the maximum structural strain ε_{cr} at the onset of slippage is

$$\varepsilon_{cr} = \gamma_{cr}/\zeta\cdot 1/\{1 - (\omega/\hat{\omega})^2\} = \hat{\beta}_0\varepsilon_G \tag{31}$$

Focusing our attention only on a pipe segment of length L (one seismic wave length) as shown in Fig. 4, and assuming the symmetric distribution of shear strain in the soil along the interface (symmetric with respect to z = L/4 and 3L/4), we observe that the slippage between the pipe and the surrounding soil occurs in the interval (ℓ^*, L/2 - ℓ^*) and (L/2 + ℓ^*, L - ℓ^*) along the axial direction of the pipe, where ℓ^* is related to γ_{cr} through the following expression:

$$\ell^* = (L/2\pi)\text{arc}\sin(\gamma_{cr}/\gamma_0) \tag{32}$$

The distribution of shear stress in the soil at the interface is then given by

$$\tau_G(z) = G\gamma_G(D/2,z) \tag{33}$$

in the intervals $(0,\ell^*)$, $(L/2 - \ell^*, L/2 + \ell^*)$ and $(L - \ell^*, L)$ where no slippage has occurred and

$$\tau_G(z) = G\gamma_{cr} \tag{34}$$

in the intervals $(\ell^*, L/2 - \ell^*)$ and $(L/2 + \ell^*, L - \ell^*)$ where the slippage has occurred. This distribution of shear stress is schematically shown also in Fig. 4 by solid lines and can be expanded into a Fourier sine series as below:

$$\tau_G(z) = G\gamma_{cr} \sum_{k=1}^{\infty} A_k \sin\{\omega t - (2\pi/L)kz\} \tag{35}$$

The structural displacement $u_S(z)$ consistent with the shear stress given by Eq. 35 can then be assumed to be of the following form:

$$u_S(z) = \sum_{k=1}^{\infty} B_k \sin\{\omega t - (2\pi/L)kz\} \tag{36}$$

Substituting Eqs. 35 and 36 into Eq. 11, we obtain

$$B_k = \{L/(2\pi k)^2\} \cdot G\gamma_{cr}/(Ed) \cdot A_k/\{1 - (\omega/\hat{\omega}_k)^2\} \tag{37}$$

where $k = 1,2,3,\ldots$ and $\hat{\omega}_k = (2\pi k/L) \cdot \sqrt{E/\rho} = k\hat{\omega}$. Hence, the structural displacement under the conditions of slippage is given by

$$u_S(z) = (L/2\pi)\hat{\beta}_0 \varepsilon_G (\gamma_{cr}/\gamma_0) \sum_{k=1}^{\infty} [\{1 - (\omega/\hat{\omega})^2\}/\{1 - (\omega/\hat{\omega}_k)^2\}]$$

$$\times (A_k/k^2) \sin\{\omega t - (2\pi/L)kz\} \tag{38}$$

where use of Eqs. 28 and 29 has been made. Differentiating Eq. 38 with respect to z, we obtain the corresponding structural strain as

$$\varepsilon_S(z) = -\hat{\beta}_0 \varepsilon_G (\gamma_{cr}/\gamma_0) \sum_{k=1}^{\infty} [\{1 - (\omega/\hat{\omega})^2\}/\{1 - (\omega/\hat{\omega}_k)^2\}]$$

$$\times (A_k/k) \cos\{\omega t - (2\pi/L)kz\} \tag{39}$$

At this point, we recall our assumption that, under the condition of no slippage, the structural displacement, strain and stress are all travelling waves of sinusoidal shape with the same frequency ω and wave length L as those of the propagating seismic wave. For the cases where the slippage is taking place, we are using essentially the same assumption. The only difference lies in the fact that the shapes of these travelling waves are no longer sinusoidal since they must be consistent with the shape of $\tau_G(z)$ as shown in Fig. 4. While we believe that the assumption is reasonable and useful, it remains to be seen exactly to what degree and with what accuracy the assumption is valid.

Conversion Factors and Maximum Relative Displacement

We now introduce the conversion factor β_{cr} which converts the free field strain ε_G into the maximum structural strain ε_{cr} in the straight pipe under the conditions of slippage:

$$\beta_{cr} = \varepsilon_{cr}/\varepsilon_G \tag{40}$$

While the maximum structural strain can, in principle, be obtained from Eq. 39 by differentiation, it depends on the value γ_0 ($\geqq \gamma_{cr}$). Also, such a detailed analysis may not really be justified in view of the simplifying assumptions on the basis of which Eq. 39 has been derived. We, therefore, first consider the limiting case where $\ell^* = 0$ or the slippage takes place throughout the interface along the pipe and hence the distribution of the shear stress $\pm\tau_{cr}$ is uniform over each interval of length L/2 (e.g., $0 < z < L/2$ in Fig. 4);

$$\tau_G(z) = \tau_{cr}\ f\{\omega t - (2\pi/L)z\} \tag{41}$$

where $f(\xi)$ is a periodic function such that $f(\xi) = 1$ for $0 < \xi < \pi$ and $f(\xi) = -1$ for $-\pi < \xi < 0$. Solving Eq. 11 with τ_G given by Eq. 41, we obtain

$$\begin{aligned} u_S(z) &= -L^2/(2\pi)^2/\{1 - (\omega/\hat{\omega})^2\}\cdot\tau_{cr}/(Ed)\cdot g\{\omega t - (2\pi/L)z\} \\ &= -L/(2\pi)\cdot\hat{\beta}_0\cdot\varepsilon_G\cdot(\gamma_{cr}/\gamma_0)\cdot g\{\omega t - (2\pi/L)z\} \end{aligned} \tag{42}$$

where $g(\xi)$ is a periodic function such that $g(\xi) = -\pi\xi/2 + \xi^2/2$ for $0 < \xi < \pi$ and $g(\xi) = -\pi\xi/2 - \xi^2/2$ for $-\pi < \xi < 0$. The corresponding maximum structural strain ε_{cr} is then obtained as $\max|\partial u_S(0,t)/\partial z|$ and is equal to

$$\varepsilon_{cr} = (\pi/2)\cdot(\gamma_{cr}/\zeta)\cdot 1/\{1 - (\omega/\hat{\omega})^2\} = (\pi/2)(\gamma_{cr}/\gamma_0)\hat{\beta}_0\varepsilon_G \tag{43}$$

When the pipe is partially in slippage or $0 < \ell^* < L/4$, it is expected that ε_{cr} is

$$\varepsilon_{cr} = q(\gamma_{cr}/\zeta)\cdot 1/\{1 - (\omega/\hat{\omega})^2\} = q(\gamma_{cr}/\gamma_0)\hat{\beta}_0\varepsilon_G \tag{44}$$

and hence, the conversion factor β_{cr} can in general be written as

$$\beta_{cr} = \varepsilon_{cr}/\varepsilon_G = q(\gamma_{cr}/\zeta)\cdot[1/\{1 - (\omega/\hat{\omega})^2\}]/\varepsilon_G = q(\gamma_{cr}/\gamma_0)\hat{\beta}_0 \tag{45}$$

where $1 \leqq q \leqq \pi/2$.

The maximum relative displacement Δ between the pipe and the ground occurs at $z \doteq L/4$ and is expected to be of the following form:

$$\Delta = u_G(\infty, L/4) - u_S(L/4) = (1 - \beta^*_{cr})\cdot\varepsilon_G\cdot L/(2\pi) \tag{46}$$

with

$$\beta^*_{cr} = q^*\ (\gamma_{cr}/\gamma_0)\hat{\beta}_0 \tag{47}$$

where q^* is a quantity that depends on the extent of the slippage. With the aid of Eq. 45, Eq. 47 becomes

$$\beta^*_{cr} = (q^*/q)\beta_{cr} \tag{48}$$

If the slippage is taking place throughout the pipe, we can show that $q^* = \pi^2/8$ from Eq. 42 and $q = \pi/2$ from Eq. 43, and therefore $q^*/q = \pi/4$. At the onset of slippage, on the other hand, $\gamma_0 = \gamma_{cr}$ and $q^* = q = 1.0$. Hence, we assume in approximation that $\beta^*_{cr} = \beta_{cr}$ regardless of the extent of the slippage.

Numerical Examples

Using the data indicated in Table 1, assuming $\gamma_{cr} = 10^{-3}$ as mentioned earlier, and considering ε_G between 10^{-4} and 10^{-2}, we evaluated and listed in Table 2 the conversion factors β_0, $\hat{\beta}_0$, β_{cr} and β^*_{cr} as appropriate at the sites with the soil conditions A, B and C, respectively under the free field strains of 10^{-2} and 10^{-3}, 5.0×10^{-4} and 10^{-4}. For the evaluation of β_{cr} and β^*_{cr}, we have used q = 1.0 in Eq. 45 for reasons to be mentioned later. Table 2 indicates that the free field strains of 10^{-4} and 5.0×10^{-4} considered respectively for C and B are too small to initiate slippage so that the values of β_0 and β_0 are identical to those listed in Table 1. It further indicates that the pipes buried in site A and subjected to free field strains of 10^{-3} and 10^{-2} undergo slippage with β_{cr} and β^*_{cr} drastically decreasing as the free field strain increases from 10^{-3} to 10^{-2}.

CONVERSION FACTORS FOR BENT PIPES AND TEE-JUNCTIONS

Bent Pipes With a Right Angle

When a seismic wave excites the ground, bent pipes experience additional stresses. In particular, if a bent pipe with a right angle is subjected to a seismic wave of wave length L propagating in the direction (z) of one of the straight legs, element (1), the largest relative displacement Δ will occur at the corner (z = L/4) when one of the nodes of the wave is passing the point (z = 0) a distance L/4 from the corner (see Fig. 5). The other leg, element (2), is assumed to be infinitely long and imbedded in the elastic soil with a coefficient of lateral reaction k (4).

The relative displacement Δ between the soil and the straight pipe (of infinite length) is given by Eq. 23 under the conditions of no slippage and by Eq. 46 under the conditions of slippage. Since, however, the shear force S_2 of element (2) acts as an axial force on element (1), an elongation Δ_S is developed in element (1):

$$\Delta_S = (L/4)\cdot S_2/(AE) \tag{49}$$

Then, the resultant relative displacement Δ' as shown in Fig. 5 is approximately

$$\Delta' = \Delta - \Delta_S \tag{50}$$

This is an approximation since no exact boundary conditions involving the bent pipe has been used for solutions.

Using the theory of structural analysis, we can obtain shear forces S_1 and S_2, bending moments M_1 and M_2 and angles of rotation θ_1 and θ_2 (see Fig. 5) as

$$S_1 = (1/4)\cdot(k/\lambda)\cdot\Delta' , \quad S_2 = 3S_1$$
$$M_1 = M_2 = (1/4)\cdot(k/\lambda^2)\cdot\Delta' , \quad \theta_1 = \theta_2 = (1/2)\cdot\lambda\cdot\Delta' \tag{51}$$

where $\lambda = \{k/(4EI)\}^{1/4}$. It then follow that

$$\Delta' = (L/2\pi)\cdot(1 - \beta)\cdot\varepsilon_G/(1 + W) \tag{52}$$

$$\varepsilon_{S1} = (M_1/EI)\cdot(D/2) + S_2/(AE) = \{Q + (2/\pi)W\}\cdot(1 - \beta)\cdot\varepsilon_G/(1 + W) \tag{53}$$

$$\varepsilon_{S2} = (M_2/EI)\cdot(D/2) = Q\cdot(1 - \beta)\cdot\varepsilon_G/(1 + W) \tag{54}$$

$$\beta_{1B} = \{Q + (2/\pi)W\}\cdot(1 - \beta)/(1 + W) \tag{55}$$

$$\beta_{2B} = Q\cdot(1 - \beta)/(1 + W) \tag{56}$$

with $\beta = \hat{\beta}_0$ under the conditions of no slippage and $\beta = \beta^*_{cr}$ under the conditions of slippage,

$$Q = LD\lambda^2/(4\pi) \tag{57}$$

$$W = (3/16)\cdot kL/(AE\lambda) \tag{58}$$

where ε_{S1} and ε_{S2} = axial strains induced by the bending moments and shear forces in elements (1) and (2) respectively and β_{1B} and β_{2B} = conversion factors to convert the free field strain ε_G into axial strains ε_{S1} and ε_{S2} respectively. These conversion factors are listed in Table 3.

Tee-Junction Pipes

A typical tee-junction pipe configuration is shown in Fig. 6. The direction of the seismic wave is assumed to be parallel to the pipe element (1). Then, because of the symmetry, we observe that

$$M_1 = S_1 = \theta_1 = \theta_2 = 0 \tag{59}$$

Shear force S_2 and bending moment M_2 are found to be

$$S_2 = (1/2)\cdot(k/\lambda)\cdot\Delta' \ , \quad M_2 = (1/2)\cdot(k/\lambda^2)\cdot\Delta' \tag{60}$$

and the relative displacement Δ' is given by

$$\Delta' = (L/2\pi)\cdot(1 - \beta)\cdot\varepsilon_G/\{1 + (4/3)W\} \tag{61}$$

The resulting conversion factors β_{1T} for element (1) and β_{2T} for element (2) are shown below as well as in Table 3.

$$\beta_{1T} = \{8/(3\pi)\}\cdot W\cdot(1 - \beta)/\{1 + (4/3)W\} \tag{62}$$

$$\beta_{2T} = 2Q\cdot(1 - \beta)/\{1 + (4/3)W\} \tag{63}$$

Since, as in the case of bent pipes, these factors convert the free field strain ε_G into the axial strains in elements (1) and (2), the expressions for the axial strains similar to those in Eqs. 53 and 54 can obviously be obtained by multiplying β_{1T} and β_{2T} by ε_G.

Numerical Examples

The conversion factors derived above for bent pipes and tee-junctions are evaluated with the same data as used for straight pipes under the conditions that $\beta^*_{cr} = \hat{\beta}_0 = 0.1$, 0.5 and 0.9 and L = 400 m and 150 m. The results are shown in Table 4 which indicates that the conversion factors for bent pipes and tee-junctions are larger for smaller values of $\hat{\beta}_0 = \beta^*_{cr}$. This is physically expected since a smaller value of $\hat{\beta}_0$ or $\beta^*_{cr} = \beta_{cr}$ indicates a larger extent of slippage producing a larger relative displacement. It is for this reason that we have assumed q to be equal to unity in Eq. 45. Table 4 also shows that the conversion factors are generally larger for L = 400 m than for L = 150 m.

DISCUSSION AND SUMMARY

To summarize our findings above, Fig. 7 is constructed in which conversion factors $\hat{\beta}_0$, β^*_{cr} and β_{1B} and β_{2T} are plotted as functions of ε_G for the seismic wave lengths of L = 150, 170, 200, 250, 300 and 400 m. For this purpose, the data listed in Table 1 are used with $\gamma_{cr} = 10^{-3}$. In Fig. 7, the conversion factors $\hat{\beta}_0$ and β^*_{cr} are plotted in log-scale on the ordinate and ε_G is also in log-scale on the abscissa. Three horizontal bars along the abscissa indicate estimated ranges of free field strain values respectively on the sites with soil conditions A, B and C that may result from earthquakes of intensities VI - IX (2).

We evaluate $\hat{\beta}_0$ using Eqs. 27 and 28 when $\gamma_0 \leq \gamma_{cr}$ while β^*_{cr} using Eq. 45 with q = 1.0 when $\gamma_0 \geq \gamma_{cr}$. The conversion factor $\hat{\beta}_0$ depends on the soil condition and the wave length L but not on the free field strain ε_G. For example, $\hat{\beta}_0$ is constant as indicated by the straight line PP' for L = 150 m under soil condition A with ε_G producing γ_{cr} when it reaches 3.4×10^{-4}. When ε_G exceeds this value, the slippage takes place and the conversion factor to be used is β^*_{cr} which decreases as an inverse function of ε_G as indicated by the straight line P'P".

For bent pipes, the conversion factor β_{1B} for element (1) is plotted in Fig. 7. For example, if $\varepsilon_G = 6.8 \times 10^{-4}$ (point Q") under soil condition A and under the seismic wave of wave length L = 150 m, the value of $\beta^*_{cr} = \beta_{cr}$ can be shown (from Eq. 45 with q = 1.0) to be 0.48 (point Q). Using Eq. 55, we can then show that β_{1B} is equal to 0.5. Point Q' on the straight line P'P" then indicates the fact that if $\varepsilon_G = 6.8 \times 10^{-4}$, $\beta_{1B} = 0.5$ for soil condition A and L = 150 m. Point R on the straight line corresponding to L = 400 m and soil condition A also represents a combination of ε_G and β^*_{cr} that produces $\beta_{1B} = 0.5$. In fact, the curve stretching between Q' and R is obtained by interpolating these similar points at which $\beta_{1B} = 0.5$. Curves for β_{1B} = 0.2, 0.6, 0.7, 0.8 and 0.9 are also constructed and shown in Fig. 7. For soil conditions B and C, the estimated free field strains are confined to the range where the corresponding values of γ_0 are always less than γ_{cr} and the conversion factor $\hat{\beta}_0$ is practically unity. The diagram indicating the conversion factor β_{2T} for element (2) of tee-junctions is constructed in Fig. 7 using the same procedure.

We observe from Fig. 7 that the conversion factors $\hat{\beta}_0$ and β^*_{cr} are smaller under seismic waves of shorter wave lengths. No such straightforward trend can be observed for the conversion factors β_{1B} and β_{2T}, however. We believe that these diagrams provide us with a practical and useful numerical insight to this rather difficult problem of estimating the strains in buried pipes under seismic conditions. Further studies are particularly suggested to examine and improve our assumptions on the displacement profile function and on the spatial distribution of the shear stress acting on the pipe in the state of slippage.

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TABLE 1 Numerical Example of Conversion Factor

$L = 400$ m

Parameters		Geological Site Conditions		
		A	B	C
c	m/s	150	300	500
G	kgwt/cm^2	344.4	1377.6	3826.5
K_G	kgwt/cm^2	2164	8656	24043
ζ	-	1.15	0.287	0.103
T	sec	2.67	1.33	0.8
ω	rad/s	2.356	4.712	7.85
ω_0	rad/s	844.8	1689.7	2816.0
$\hat{\omega}$	rad/s	80.4	80.4	80.4
$(\omega/\omega_0)^2$	-	0.08×10^{-4}	0.08×10^{-4}	0.08×10^{-4}
$(\omega/\hat{\omega})^2$	-	8.65×10^{-4}	34.35×10^{-4}	95.33×10^{-4}
$(2\pi/L)^2\ AE/K_G$	-	90.63×10^{-4}	22.66×10^{-4}	8.16×10^{-4}
β_0	-	0.991	0.998	0.999
$\hat{\beta}_0$	-	0.992	1.001	1.009

$L = 150$ m

Parameters		Geological Site Conditions		
		A	B	C
c	m/s	150	300	500
G	kgwt/cm^2	344.4	1377.6	3826.5
K_G	kgwt/cm^2	2164	8656	24043
ζ	-	3.07	0.765	0.275
T	sec	1.0	0.5	0.3
ω	rad/s	6.28	12.57	20.95
ω_0	rad/s	844.8	1689.7	2816.0
$\hat{\omega}$	rad/s	214.5	214.5	214.5
$(\omega/\omega_0)^2$	-	0.55×10^{-4}	0.55×10^{-4}	0.55×10^{-4}
$(\omega/\hat{\omega})^2$	-	8.57×10^{-4}	34.34×10^{-4}	95.39×10^{-4}
$(2\pi/L)^2 \cdot AE/K_G$	-	644.48×10^{-4}	161.14×10^{-4}	58.03×10^{-4}
β_0	-	0.939	0.984	0.994
$\hat{\beta}_0$	-	0.940	0.988	1.004

TABLE 2 Conversion Factor for Straight Pipes
($\gamma_{cr} = 10^{-3}$)

Site Conditions		Strains			Conversion Factors			
Site	Seismic wave Length	Free Field Strain	Shear Srain	Critical Axial Strain	Static	Dynamic	For Stain	For Dispacement
X	L m	ε_G $\times 10^{-4}$	γ_0 $\times 10^{-4}$	ε_{cr} $\times 10^{-4}$	β_0	$\hat{\beta}_0$	β_{cr}	β^*_{cr}
C	400	1.0	0.103	97.1 ∿ 152.5	0.999	1.009	-	-
	150	1.0	0.275	36.4 ∿ 57.2	0.994	1.004	-	-
B	400	5.0	1.44	34.8 ∿ 54.7	0.998	1.001	-	-
	150	5.0	3.38	13.6 ∿ 21.4	0.984	0.988	-	-
A	400	10.0	11.5	8.8 ∿ 13.8	-	-	0.88 ∿ 1.0	0.88 ∿ 1.0
	150	10.0	30.7	3.25 ∿ 5.1	-	-	0.33 ∿ 0.51	0.33 ∿ 0.4
A	400	100.0	115.0	8.8 ∿ 13.8	-	-	0.09 ∿ 0.14	0.09 ∿ 0.1
	150	100.0	307.0	3.25 ∿ 5.1	-	-	0.03 ∿ 0.05	0.03 ∿ 0.04

TABLE 3 Conversion Factors for Bent Pipes and Tee-Junctions

Structure		No slippage $\gamma_{cr} > \gamma_0$	Slippage $\gamma_{cr} \leq \gamma_0$
Straight		β_0 or $\hat{\beta}_0$	β_{cr}
Bent	(1)	$\frac{Q + (2/\pi)\ W}{1 + W}(1 - \hat{\beta}_0)$	$\frac{Q + (2/\pi)\ W}{1 + W}(1 - \beta^*_{cr})$
	(2)	$\frac{Q}{1 + W}(1 - \hat{\beta}_0)$	$\frac{Q}{1 + W}(1 - \beta^*_{cr})$
Tee-junction	(1)	$\frac{8W / (3\pi)}{1 + (4/3)W}(1 - \hat{\beta}_0)$	$\frac{8W / (3\pi)}{1 + (4/3)W}(1 - \beta^*_{cr})$
	(2)	$\frac{2Q}{1 + (4/3)W}(1 - \hat{\beta}_0)$	$\frac{2Q}{1+ (4/3)W}(1 - \beta^*_{cr})$

TABLE 4 Numerical Examples of Conversion Factor

L= 400 m

	$\hat{\beta}_0$, β^*_{cr}	Element (1)	Element (2)
Straight	0.1	0.1	---
	0.5	0.5	---
	0.9	0.9	---
Bent	0.1	1.87	1.29
	0.5	1.04	0.72
	0.9	0.20	0.14
Tee-junction	0.1	1.14	2.22
	0.5	0.63	1.23
	0.9	0.13	0.25

L= 150 m

	$\hat{\beta}_0$, β^*_{cr}	Element (1)	Element (2)
Straight	0.1	0.1	---
	0.5	0.5	---
	0.9	0.9	---
Bent	0.1	0.86	0.71
	0.5	0.48	0.39
	0.9	0.10	0.08
Tee-junction	0.1	0.19	1.29
	0.5	0.11	0.72
	0.9	0.02	0.14

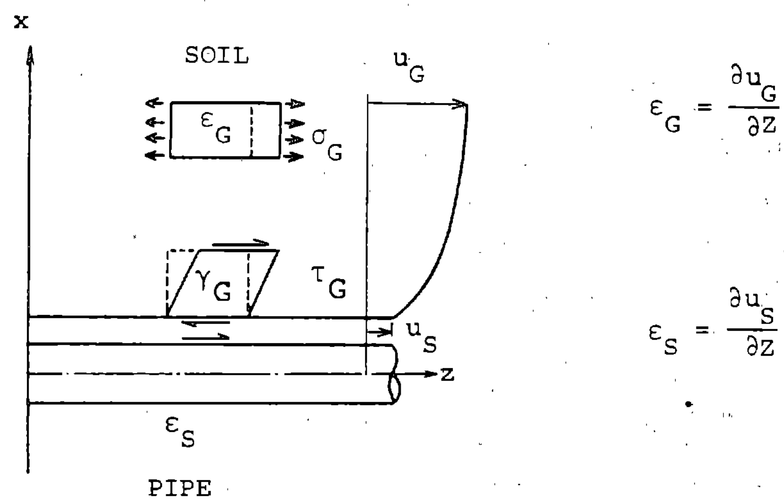

Fig. 1 Pipe and Soil Stresses and Strains

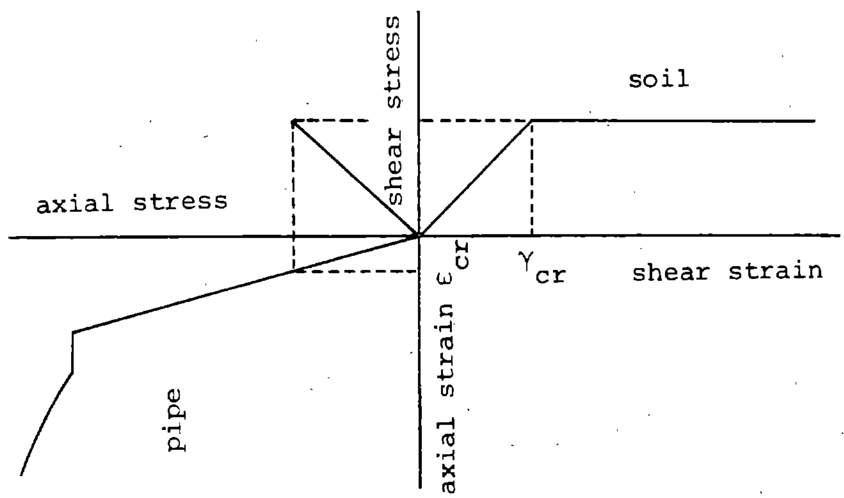

Fig. 2 Schematical Stress-strain Relationship for Soil and Pipe

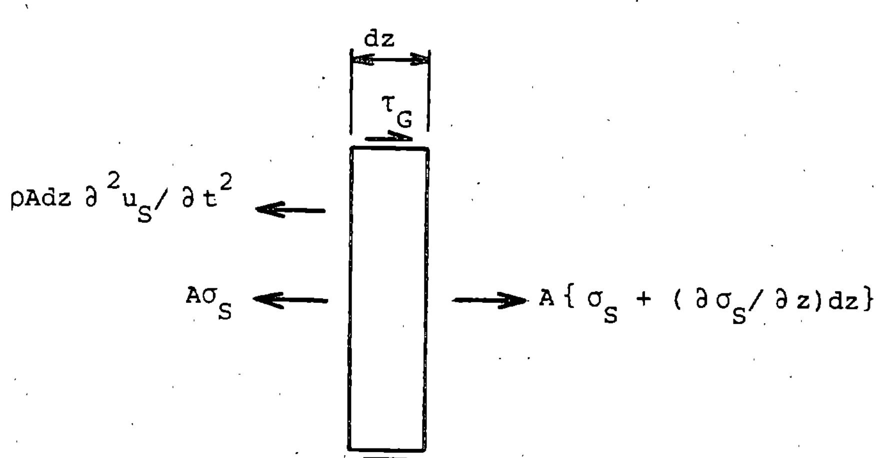

Fig.3 Free Body Diagram of a Pipe Segment

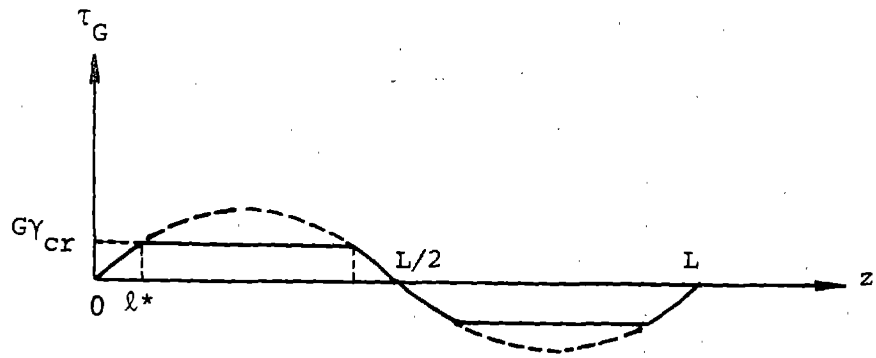

Fig.4 Shear Stress Distribution along the Pipe at the Interface

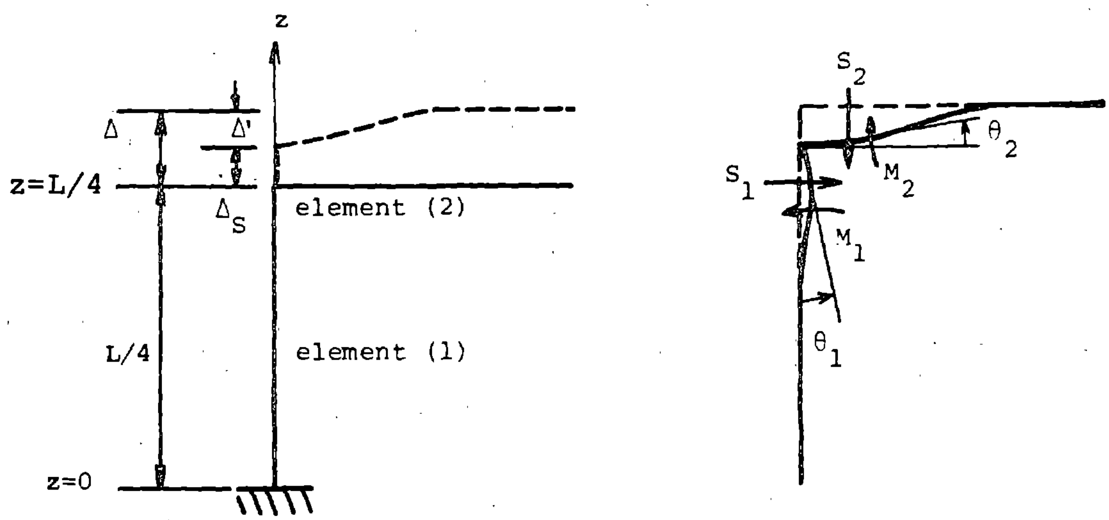

Fig. 5 Internal Forces in a Bent Pipe

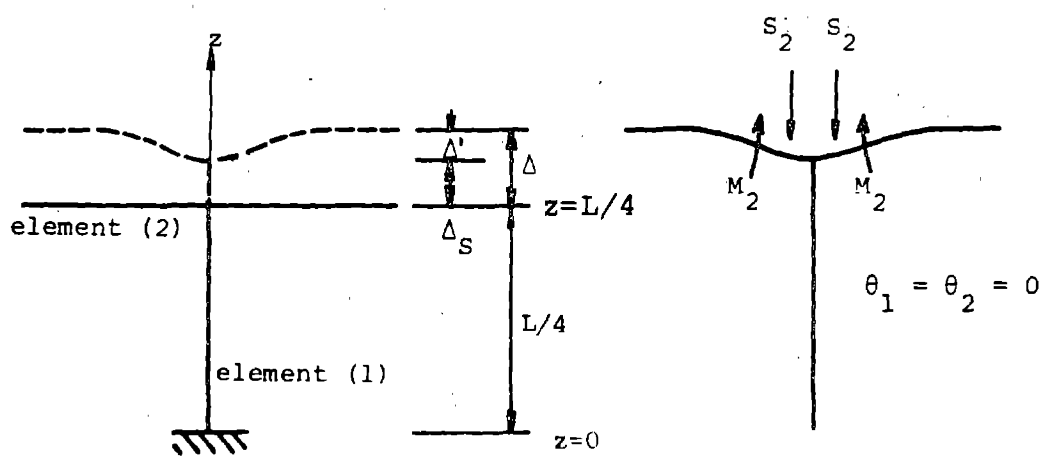

Fig. 6 Internal Forces in a Tee-junction

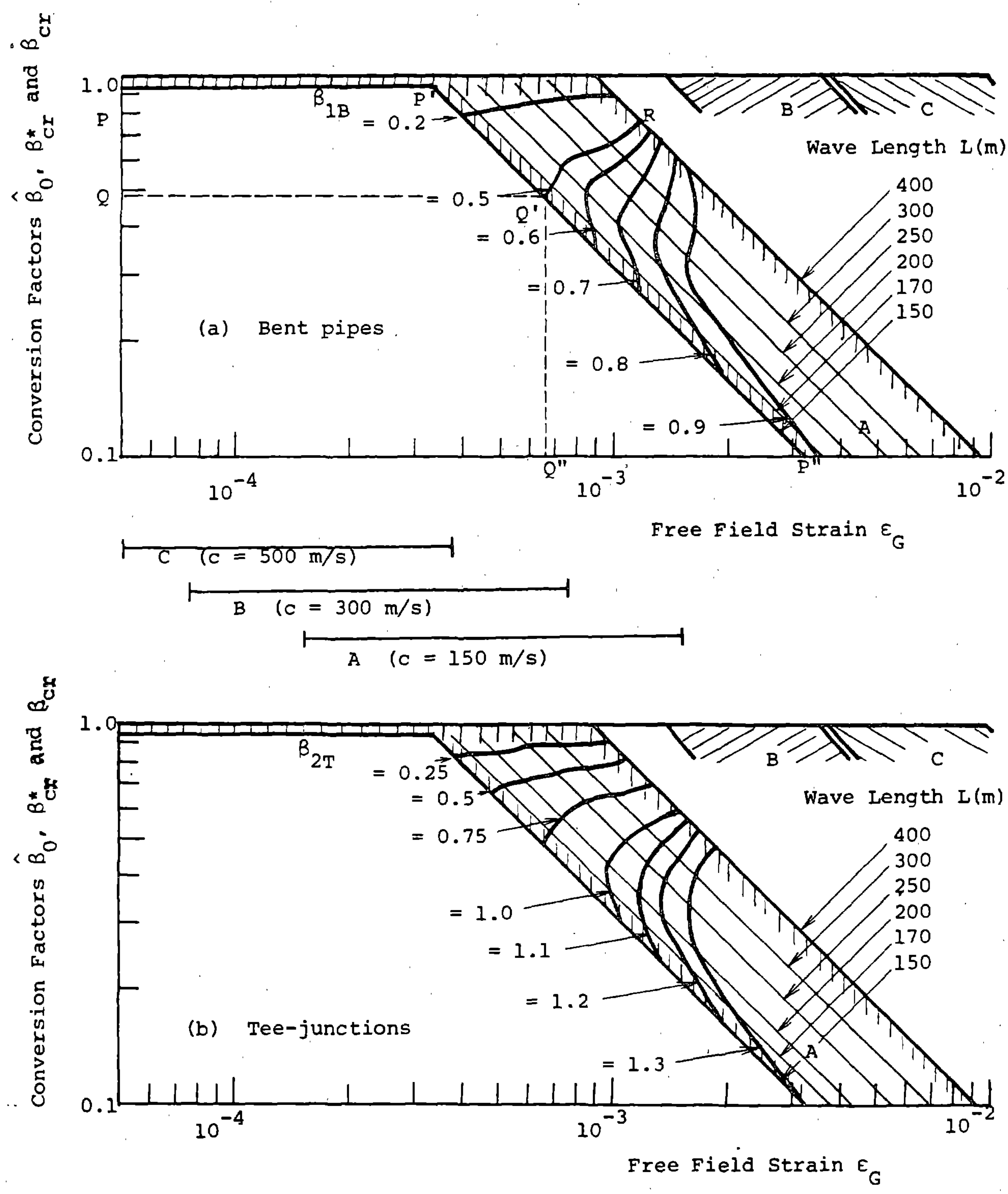

Fig. 7 Conversion Factors $\left(\gamma_{cr} = 10^{-3}\right)$