

DAMPING MEASUREMENTS OF TALL STRUCTURES

by

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DAMPING MEASUREMENTS OF TALL STRUCTURES

George T. Taoka*, Member, ASCE

ABSTRACT

The results of an investigation of damping measurements of five tall steel structures are presented. The structures include four tall steel buildings ranging in height from 103 to 170 meters, and a four-legged square steel tower of total height 333 meters.

For each structure, ambient vibration records were mechanically digitized and analyzed by three system identification methods: the correlation method, the spectral moments method, and the power spectral density method. A trapezoidal filter was used to isolate individual modes before the records were subjected to analysis.

In addition to estimates from random vibration data, damping estimates were also obtained under forced vibration rotating shaker tests. These estimates appeared consistent with those obtained from ambient data for fundamental modes of vibration. However, considerable variations were present when damping ratios for higher modes were compared.

I. THE CORRELATION METHOD

The theoretical basis for the use of the correlation function to estimate structural response parameters has been reviewed by Cherry and Brady [1]. A detailed discussion of the use of this method to estimate damping is available in a report by Taoka and Scanlan [10]. The following brief discussion of the general theory involved is taken from Cherry and Brady [1], and is included here for completeness.

If $h(\tau)$ is the system response function due to a unit impulse applied at $\tau = 0$, then the relation between an input function $x(t)$ and the resulting output response $y(t)$ is given by

$$y(t) = \int_0^{\infty} h(\tau) x(t-\tau) d\tau \quad (1)$$

For a lightly damped single degree of freedom oscillator having a natural circular frequency ω and a small critical damping ratio ζ the system response function is defined as

$$h(\tau) = \frac{e^{-\zeta\omega\tau}}{\omega\sqrt{1-\zeta^2}} \sin[\sqrt{1-\zeta^2} \omega\tau], \quad \tau \geq 0 \quad (2)$$

It is assumed that $h(\tau) = 0$ for $\tau < 0$.

The autocovariance function for a function $y(t)$ is defined by

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$$C_y(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} y(t) y(t+\tau) dt \quad (3)$$

It can be shown that if the input function $x(t)$ is composed of "white noise" with constant spectral density G_0 , that the output autocovariance function is given by

$$C_y(\tau) = \frac{\pi G_0}{2\zeta\omega^3} \left[e^{-\zeta\omega\tau} (\cos \sqrt{1-\zeta^2} \omega\tau + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \sqrt{1-\zeta^2} \omega\tau) \right] \quad (4)$$

Equation (4) represents a cosinusoidal function with exponential decay. The decay of the envelope of the autocovariance estimate of the output response can thus be used to estimate the critical damping ratio ζ of the system. The unbiased autocovariance estimate of a time series $\{y_n\}$ where n varies from 1 to N is given by

$$C_x(k) = \frac{1}{N-k} \sum_{n=1}^{N-k} y_n y_{n+k}, \quad 0 \leq k \leq M \quad (5)$$

The corresponding autocorrelogram estimate is given by

$$R_y(k) = \frac{C_y(k)}{C_y(0)} \quad 0 \leq k \leq M \quad (6)$$

In both Eqs. (5) and (6), the integer M , the maximum lag number, is kept small compared to N the number of points in the series.

The logarithmic decrement method [10] was used to estimate the critical damping ratio in each mode. The value is determined by

$$\zeta = \frac{1}{2\pi q} \ln \left[\frac{A_p}{A_{p+q}} \right] \quad (7)$$

where A_p is the peak amplitude at cycle p and A_{p+q} is the peak amplitude q cycles later.

II. SPECTRAL MOMENTS METHOD

In 1972, Vanmarcke [11] proposed a spectral moments method for estimating frequency and damping parameters of a randomly excited system. The zeroeth order moment gives the area under the power spectral density function. The first order moment is a function of its centroid, and the second order moment gives a measure of dispersion about a spectral peak indicating a central frequency.

If $y(t)$ is a stationary random process with zero mean value, its autocorrelation function $R(\tau)$ is defined by

$$R(\tau) = E [y(t) y(t+\tau)] \quad (8)$$

The "one-sided" Fourier Transform pair of formulas relating $G(\omega)$ and $R(\tau)$ is given by

$$G(\omega) = \frac{2}{\pi} \int_0^{\infty} R(\tau) \cos(\omega\tau) d\tau \quad (9)$$

$$R(\tau) = \int_0^{\infty} G(\omega) \cos(\omega\tau) d\omega \quad (10)$$

where $G(\omega)$ is the "one-sided" power spectral density function of the autocorrelation function of the zero-mean process $y(t)$. The mean square value $\langle y(t)^2 \rangle$ is obtained by setting $\tau = 0$, in Eq. (10), giving

$$R(0) = \int_0^{\infty} G(\omega) d\omega \quad (11)$$

The spectral moments are

$$\lambda_0 = \int_0^{\infty} \omega^0 G(\omega) d\omega = \int_0^{\infty} G(\omega) d\omega \quad (12)$$

$$\lambda_1 = \int_0^{\infty} \omega^1 G(\omega) d\omega \quad (13)$$

$$\lambda_2 = \int_0^{\infty} \omega^2 G(\omega) d\omega \quad (14)$$

Vanmarcke has introduced the following quantities related to the spectral moments above.

$$\omega_1 = \frac{\lambda_1}{\lambda_2} \quad (15)$$

$$\omega_2 = \left[\frac{\lambda_2}{\lambda_0} \right]^{1/2} \quad (16)$$

Note that ω_1 and ω_2 as defined have dimensions of circular frequency. The parameter ω_2 will be directly related to ω_n , the natural circular frequency of vibration. The following dimensionless parameter

$$q = \left[1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2} \right]^{1/2} \quad (17)$$

will be directly related to the percentage of critical damping ζ of the vibrating system.

If $x(t)$ is the input and $y(t)$ the output of a linear system, the relationship between $x(t)$ and $y(t)$ is given by Eq.(1), where $h(\tau)$ is the impulse response function of the system. The system transfer function $H(\omega)$ is defined to be the Fourier transform of the impulse response function $h(\tau)$.

If $G_0(\omega)$ is the power spectral density function of the stationary "white noise" input $x(t)$, and $G(\omega)$ is the output spectral density function, the equation defining these relationship is

$$G(\omega) = |H(\omega)|^2 G_0(\omega) . \quad (18)$$

For a single degree of freedom system with natural circular frequency ω_n and damping ratio coefficient ζ , the square of the absolute value of the system transfer function is given by

$$|H(\omega)|^2 = \frac{1}{(\omega_n^2 - \omega^2)^2 + 4\zeta^2 \omega_n^2 \omega^2} \quad (19)$$

It can then be shown that the following relationships exist:

$$\lambda_0 = \frac{\pi G_0}{4\zeta \omega_n^3} \quad (20)$$

$$\omega_2 = \omega_n \quad (21)$$

$$q^2 = \frac{4\zeta}{\pi} [1 - 1.1\zeta + 0(\zeta^2)] . \quad (22)$$

When $|\zeta| \ll 1$, it can be seen that

$$q^2 \approx \frac{4\zeta}{\pi} \quad (23)$$

for very light damping [$\zeta \leq 0.15$]. Thus the natural circular frequency and critical damping ratio is given by

$$\omega_n = \left[\frac{\lambda_2}{\lambda_0} \right]^{1/2} \quad (24)$$

$$\zeta = \frac{\pi}{4} q^2 = \frac{\pi}{4} \left(1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2} \right) . \quad (25)$$

Since the output function $y(t)$ is filtered by a band-limited window function between circular frequencies ω_a and ω_b , the effect of excluding frequencies outside of (ω_a, ω_b) will now be discussed. It will be convenient to define dimensionless band limits

$$\Omega_a = \frac{\omega_a}{\omega_n}, \quad \Omega_b = \frac{\omega_b}{\omega_n} . \quad (26)$$

Vanmarcke has demonstrated that the natural frequency is still given by

$$\omega_n = \left[\frac{\lambda_2}{\lambda_0} \right]^{1/2} \quad (27)$$

However, the critical damping ratio is corrected according to the relationship

$$\zeta = \left[\frac{1+\Omega_a}{1-\Omega_b} \right] \frac{\pi}{4} \left[1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2} \right] \quad (28)$$

for the special case when $\Omega_a = \Omega_b$. This condition was utilized in this investigation.

III. POWER SPECTRAL DENSITY METHOD

This method is based on analyzing the power spectral density function of the response spectra. In the output power spectrum, the spectral density becomes maximum at a frequency $\omega\sqrt{1-2\zeta^2}$, and the damping coefficient ζ can be calculated as follows, knowing the frequencies f_1 and f_2 where the spectral densities become $1/\lambda$ of its maximum.

$$\zeta \approx \frac{A}{2} \left(1 - \frac{3}{8} A^2\right) \quad (29)$$

where

$$A = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2} \cdot \frac{1}{\sqrt{\lambda - 1}} \quad (30)$$

In practice, $\lambda = 2$ is usually used for simplicity.

Equation (30) is derived from the power spectral density function

$$S(\omega) = \frac{S_0}{(\omega_0^2 - \omega^2)^2 + 4\zeta^2 \omega_0^2 \omega^2} \quad (31)$$

This method has been used by Tanaka, Yoshizawa, Osawa, and Morishita [8] to estimate period and damping parameters of some buildings in Japan.

The power spectral density function is defined as the Fourier transform of the true autocorrelation function derived from a vibration record of infinite length of time. When the length of the record is finite, we cannot estimate the true autocorrelation function for arbitrarily long lags. In practical cases, therefore, we can only compute the so-called apparent autocorrelation function from a record of relatively short length. However, a good estimation of smoothed values of the true power spectrum can be obtained from the Fourier transform of a modified apparent autocorrelation function, which is the product of the apparent autocorrelation and a suitable even function called the lag window.

In the present investigation, the so-called 'hamming' type lag window expressed by Eq. (32) was used.

$$\begin{aligned} \omega(\tau) &= 0.54 + 0.46 \cos \frac{\pi\tau}{T_m} & \text{for } |\tau| < T_m \\ &= 0 & \text{for } |\tau| > T_m \end{aligned} \quad (32)$$

where, τ is a time lag and T_m is the maximum lag which we desire to use.

The corresponding frequency function is

$$W(\omega) = 0.54 W_0(\omega) + 0.23 \left[W_0\left(\omega + \frac{\pi}{T_m}\right) + W_0\left(\omega - \frac{\pi}{T_m}\right) \right] \quad (33)$$

$$W(\omega) = 2T_m \frac{\sin \omega T_m}{\omega T_m} \quad (34)$$

The smoothed spectral function is then expressed by

$$P(u) = K \int_{-\infty}^{\infty} \frac{1}{(1 - u'^2)^2 + 4\zeta^2 u'^2} [Y_0(u - u') + \frac{23}{54} \{Y_0(u - u' + \frac{1}{2f_0 T_m}) + Y_0(u - u' - \frac{1}{2f_0 T_m})\}] du' \quad (35)$$

where,

$$K = \frac{1.08 T_m G_0}{\omega_n^4}, \quad Y_0(u-u') = \frac{\sin \omega_n T_m (u-u')}{\omega_n T_m (u-u')}, \quad u = \frac{\omega}{\omega_n}, \quad u' = \frac{\omega'}{\omega_n}.$$

This method has been used by Kobayashi and Sugiyama [3] to estimate the structural dynamic parameters of the five structures investigated in this report. The results of this analysis are presented later.

IV. RECORD DATA AND BUILDING DETAILS

Data Collection Procedure

The vibration sensors used in this investigation were three horizontal velocity meters manufactured by the Hosaka Instrument Company of Japan. The analog signals were subjected to low pass filtering effectively eliminating frequencies greater than 10 Hz. The resulting signals were then digitized at a constant interval of 0.04 seconds, which gives a Nyquist frequency of 12.5 Hz. For the World Trade Center Building, a record length for each sensor of approximately 6,000 points, or four minutes, was recorded on each floor. For the other three buildings, as well as for the Tokyo Tower, 4,500 points, or three minutes of data, were continuously recorded.

The method for recording the ambient vibration data was the same for all the structures, so the method will only be described in detail for the World Trade Center Building. Data were collected on the 40th floor (1st mode), 28th floor (3rd mode), and 16th floor (2nd mode) for this building. On each floor level, the three vibration sensors, were placed as shown in Fig. 2. Sensor #1 was placed at the geometric center of the cross section, facing North-South. Sensor #2 was placed at the Northern tip of the cross section, facing East-West. Sensor #3 also faced East-West, but was located at the Southern tip of the cross section. Thus, if $y_1(t)$, $y_2(t)$, and $y_3(t)$ were the output from the three sensors, the North-South, East-West, and Torsional responses were given by

$$y_{N-S}(t) = y_1(t) \quad (36)$$

$$y_{E-W}(t) = 1/2[y_2(t) + y_3(t)] \quad (37)$$

$$y_T(t) = 1/2[y_2(t) - y_3(t)] \quad (38)$$

The ambient responses for this building were subjected to a Trapezoidal Filter with passbands of 0.363 Hz, 0.394 Hz, and 0.607 Hz for the first three modes, respectively. Figure 1 shows the details of the Trapezoidal Filter.

Tokyo World Trade Center Building (WTC)

This building rises 40 stories to a height of 152.2 meters at roof level. A small structure on the roof raises the overall height to 156.0 meters. The tower is a steel frame, almost symmetrical in plan view, being 51.4 m (E-W) by 48.8 m (N-S) in cross section. The area of a typical floor is 2,458 m². Details of the building are given in Figs. 2 and 3. Vibration tests on this structure have been reported by Muto [4].

International Tele-Communications Center (ITC)

This building is a steel frame which rises 32 stories to a height of 170 meters. It is almost square in plan view, with plan dimensions 51 m by 54 m. Its structural framing consists of closely spaced columns lying on the perimeter of the cross section. Its vibrational characteristics and design features have also been reported by Muto [5]. Records were taken from the 32nd floor for the fundamental modes and the 17th floor for the other modes. Its details are shown in Fig. 4.

Asahi Tokai Building (ATB)

The Asahi Tokai Building is a steel frame 30 story high-rise structure with a total height of 119 m. There is a concrete foundation of three floors below ground level. It is essentially square in cross section, with plan dimensions 36 m by 35 m. Each typical floor has a cross sectional area of 1,249 m². Its dynamic characteristics are described by Ichinose [2]. Its structural details are shown in Fig. 5. Data for this building were taken on the 22nd and 14th floors.

Yokohama Terri Building (YTB)

The last building studied in this investigation is also a steel frame whose cross section is perfectly square, being 27 meters on each side. Its structural framing system consists of a "tube in tube" design. It consists of 27 stories and rises to a height of 103 m. There are also three basement floors below ground level. Its structural details are shown in Fig. 6. Vibration test for this building are described in a report by Tamano [7]. Ambient data were recorded on the 27th, 20th, and 13th floors.

Tokyo Tower (TT)

As can be seen in Fig. 7, the Tokyo Tower is an isolated free-standing steel-framed tower composed of two parts; a primary tower with a height of 253 m and another one of a height of 80 m being added on the top of the former. The latter tower may be divided into two parts, i.e., the super-gain-antenna of 60 m in height and the super-turn-antenna of 20 m in height. Thus, the total height of the tower is 333 m above ground level. The design of this tower, as well as its dynamic response to vibration test and under typhoon conditions, has been reported by Naito, Nasu, Takeuchi, and Kubota [6].

V. COMPARISON OF DAMPING RATIO ESTIMATES FROM DIFFERENT METHODS OF ANALYSES

In this section, parameter estimates for the four buildings obtained from the Correlation, Spectral moments, and Spectral Density methods of analysis will be compared. The estimates from the Spectral Density Method were calculated by Professor Hiroyoshi Kobayashi, with the aid of graduate student Nao Sugiyama [3]. It should be noted that each estimate was obtained from the same ambient vibration record, subjected to a Trapezoidal Filter. Thus any differences in these values would be solely due to the different method used to analyze the data. Forced vibration estimates from three structures are also included in the results.

World Trade Center (WTC)

The average values for the natural frequencies obtained from these analytical methods are 0.281, 0.861, and 1.60 Hz for the North-South direction, and 0.284, 0.870, and 1.61 Hz for the East-West direction. The first two Torsional frequencies are 0.350 and 0.977 Hz. The corresponding forced vibration natural frequency estimates for the translational modes are 0.318, 0.980 and 1.82 Hz for the North-South direction, and 0.315, 0.990, and 1.85 Hz for the East-West direction.

The critical damping ratio estimates are shown in Table 1. The last column in Tables 1 through 5, labeled S/N Ratio, are the signal-to-noise ratios present in the data record from which the modal damping estimates were calculated.

Table 1
CRITICAL DAMPING RATIO (ζ_n) FOR WTC BUILDING

Mode	Filtered Correlogram	Spectral Moments	Spectral Density	Forced Vibration	S/N Ratio
N-S First	0.0094	0.0070	0.008	0.007	32.8
N-S Second	0.0116	0.0072	0.008	0.013	17.3
N-S Third	0.0183	0.0152	0.016	0.014	6.9
E-W First	0.0100	0.0096	0.014	0.009	19.9
E-W Second	0.0096	0.0104	0.014	0.013	16.4
E-W Third	0.0108	0.0094	0.025	0.015	17.3
Torsion First	0.0128	0.0119	---	0.008	22.2
Torsion Second	0.0138	0.0116	---	---	14.5

The correlation curves for the first three North-South modes are shown in Fig. 11. The corresponding forced vibration frequency curves for the same three modes are shown in Figs. 8, 9, and 10. Further discussion of these curves are presented in an NSF report by Taoka [9].

International Telecommunications Center (ITC)

The system identification estimates are 0.324 and 0.955 Hz for North-South, and 0.314 and 0.929 Hz for the East-West directions, respectively. The first two torsional estimates are 0.413 and 1.05 Hz. The corresponding critical damping ratio estimates are listed in Table 2.

Table 2
CRITICAL DAMPING RATIO (ζ_n) OF ITC BUILDING

Mode	Filtered Correlogram	Spectral Moments	Spectral Density	S/N Ratio
N-S First	0.0050	0.0048	0.004	47.2
N-S Second	0.0070	0.0057	0.007	22.3
E-W First	0.0112	0.0115	0.009	23.0
E-W Second	0.0060	0.0060	0.006	22.8
Torsion First	0.0076	0.0091	0.005	26.3
Torsion Second	0.0144	0.0118	0.013	9.9

Asahi Tokai Building (ATB)

The system identification estimates are 0.380 and 1.14 Hz for the first two North-South directions. The fundamental East-West estimate is 0.387 Hz, and the fundamental Torsional estimate is 0.389 Hz. The corresponding forced vibration estimates are 0.434, 1.27, 0.436, and 0.562 Hz. The critical damping ratios for ATB Building are shown in Table 3. The signal-to-noise ratios are generally small, with the exception of the fundamental North-South mode.

Table 3
CRITICAL DAMPING RATIO (ζ_n) FOR ATB BUILDING

Mode	Filtered Correlogram	Spectral Moments	Spectral Density	Forced Vibration	S/N Ratio
N-S First	0.0052	0.0072	0.026	0.009	26.7
N-S Second	0.0209	0.0165	0.013	0.012	7.8
E-W First	0.0455	0.0287	0.024	0.009	7.8
Torsion	0.0473	0.0271	---	0.0073	7.7

Yokohama Tenri Building (YTB)

The system identification estimates are 0.460 and 1.32 Hz for North-South, 0.461 and 1.33 Hz for East-West, and 0.602 and 1.57 Hz for Torsional modes. The damping ratios are listed in Table 4.

Table 4
CRITICAL DAMPING RATIO (ζ_n) OF YTB BUILDING

Mode	Filtered Correlogram	Spectral Moments	Spectral Density	S/N Ratio
N-S First	0.0074	0.0075	0.005	24.5
N-S Second	0.0073	0.0059	0.005	14.5
E-W First	0.0129	0.0103	0.006	19.1
E-W Second	0.0091	0.0074	0.013	10.5
Torsion First	0.0125	0.0090	0.003	13.8
Torsion Second	0.0175	0.0173	0.014	6.5

The Tokyo Tower (TT)

Since the Tokyo Tower is symmetric in both North-South and East-West directions, only the North-South and Torsional modes were analyzed. The natural frequency estimates for the three North-South modes are 0.358, 0.594, and 1.25 Hz, with Torsional frequency estimates of 1.44 and 2.04 Hz. The forced vibration estimates for the translational

modes were 0.377, 0.645, and 1.28 Hz. The damping estimates are shown in Table 5.

Table 5
CRITICAL DAMPING RATIO (ζ_n) OF TOKYO TOWER

<u>Mode</u>	<u>Filtered Correlogram</u>	<u>Spectral Moments</u>	<u>S/N Ratio</u>
N-S First	0.0078	0.0109	29.1
N-S Second	0.0082	0.0062	13.7
N-S Third	0.0010	0.0011	78.4
Torsion First	0.0025	0.0021	41.3
Torsion Second	0.0020	0.0019	29.1

VI. CONCLUSIONS

The signal-noise (S/N) ratio was found to have significant effect on the resulting accuracy of damping ratio estimates obtained in this study. Therefore, these ratios were listed in Tables 1 through 5. They measured the ratio of peak amplitude to the average noise amplitude in a region of 0.05 Hz on both sides of a natural frequency estimate. Generally speaking, damping estimates were accurate when the S/N ratio exceeded 15, but were unreliable when the S/N was below 10. The region $10 < S/N < 15$ was a "gray area" where no definitive statement could be made about damping accuracy.

For fundamental modes of vibration, all three methods of analysis gave reasonably close damping estimates, consistent with forced vibration measurements. For higher modes, however, considerable variations exist in the estimates obtained by different methods. With the exception of the ATB Building, whose records exhibited unfavorable signal-to-noise ratios, most damping estimates were in the range of about 0.5% to 1.5% of critical, under both ambient and forced vibration conditions.

VII. ACKNOWLEDGMENTS

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f_c = CENTER FREQUENCY
 B = HALF - POWER BANDWIDTH
 C = CUTOFF BANDWIDTH

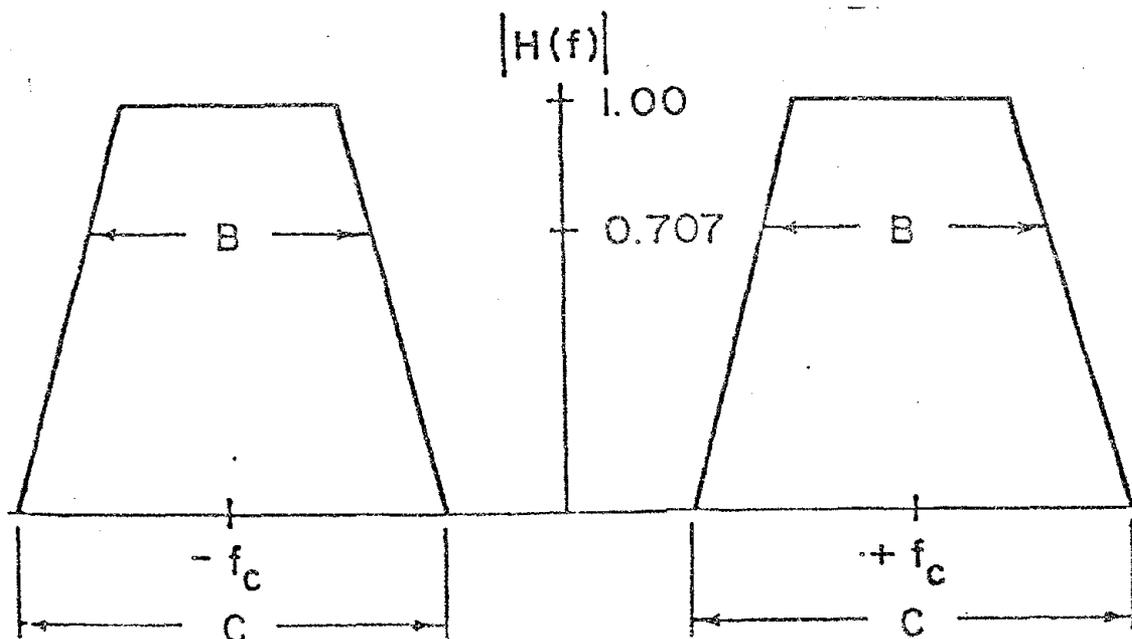


FIGURE 1 TRAPEZOIDAL FILTER

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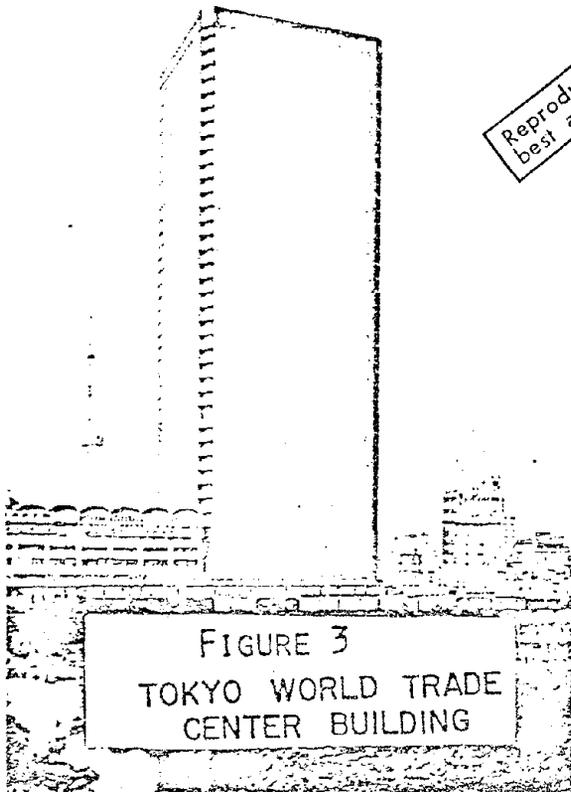


FIGURE 3
TOKYO WORLD TRADE
CENTER BUILDING

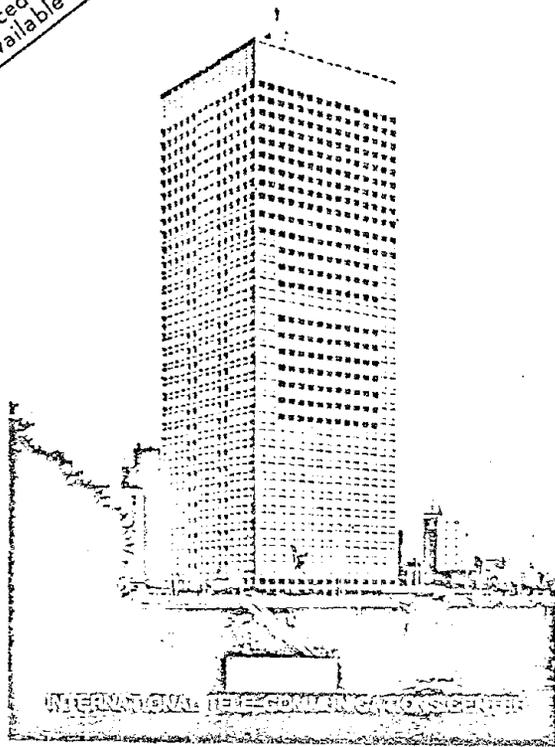


FIGURE 4

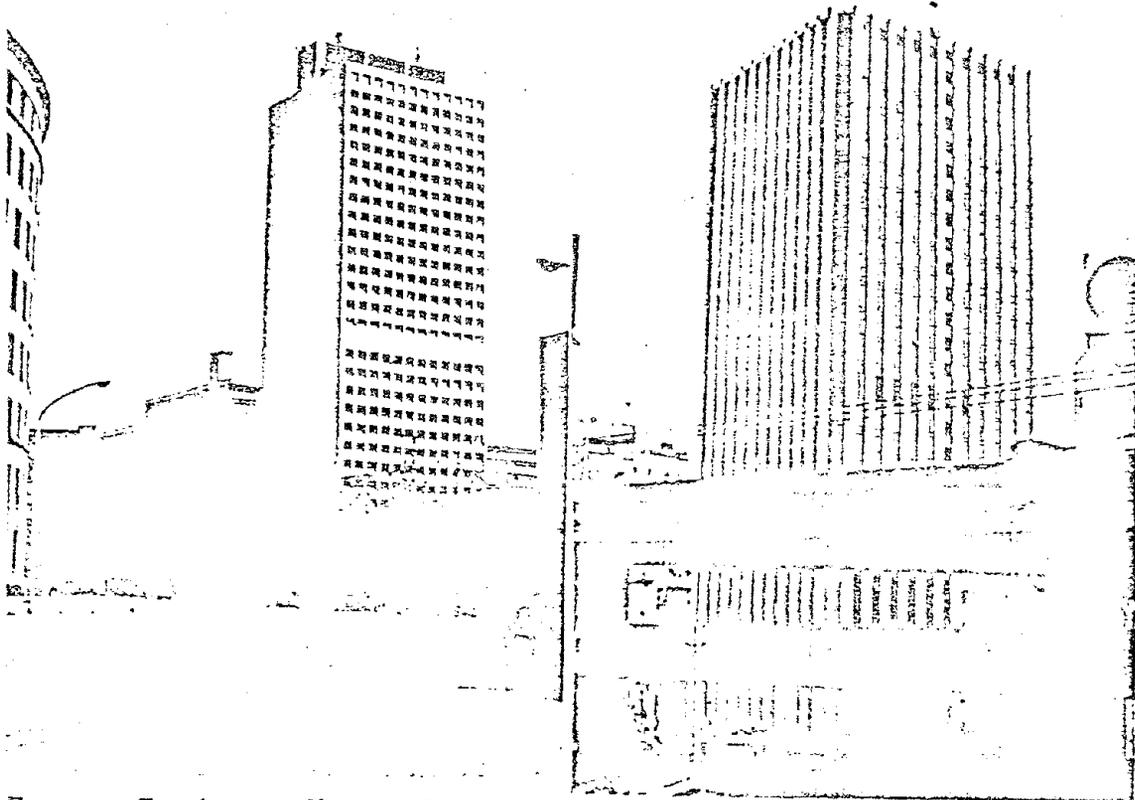


FIGURE 5 ASAHI TOKAI BUILDING FIG. 6 YOKOHAMA TENRI BUILDING

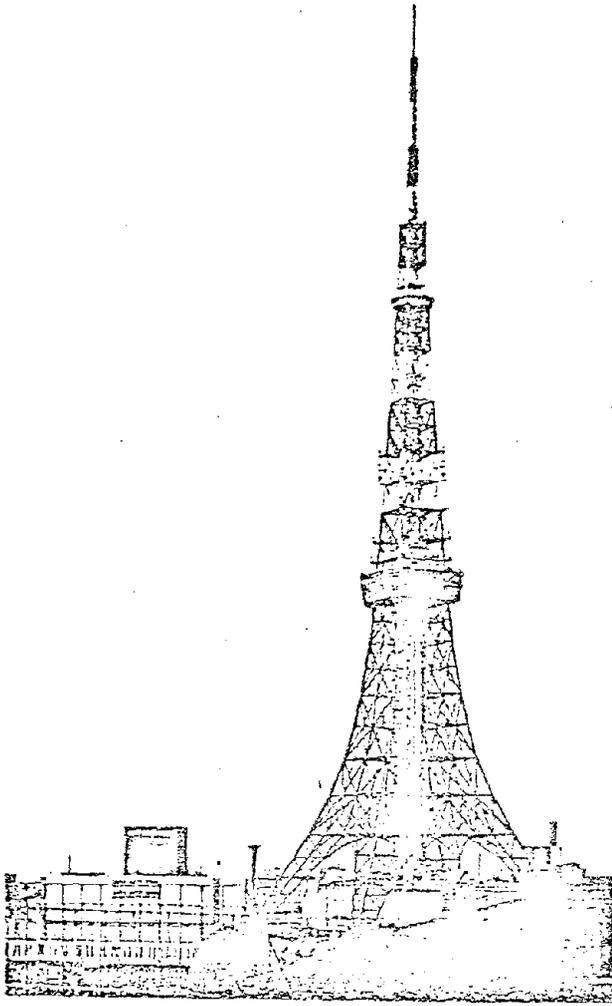


FIGURE 7 TOKYO TOWER

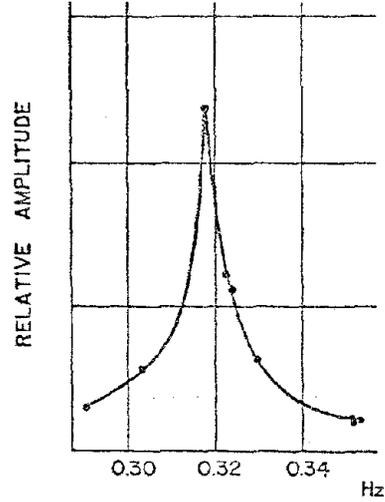


FIGURE 8.
FORCED VIBRATION AMPLITUDE
RESPONSE CURVE, WTC 1st MODE NS.

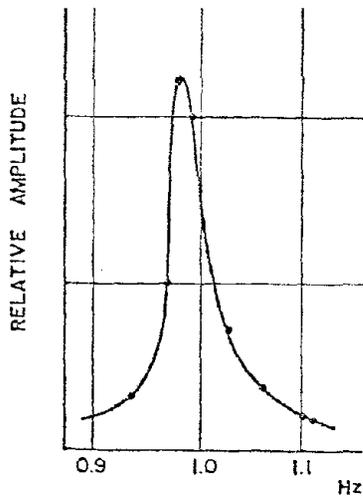


FIGURE 9.
FORCED VIBRATION AMPLITUDE
RESPONSE CURVE, WTC 2nd MODE NS.

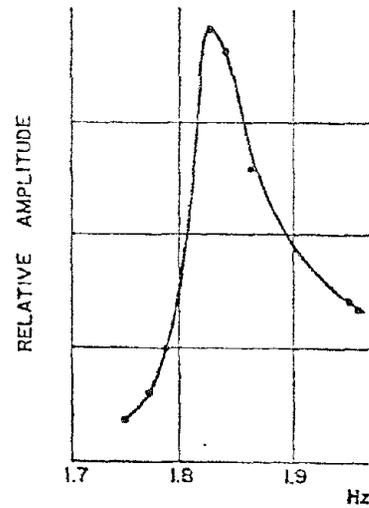


FIGURE 10.
FORCED VIBRATION AMPLITUDE
RESPONSE CURVE, WTC 3rd MODE NS.

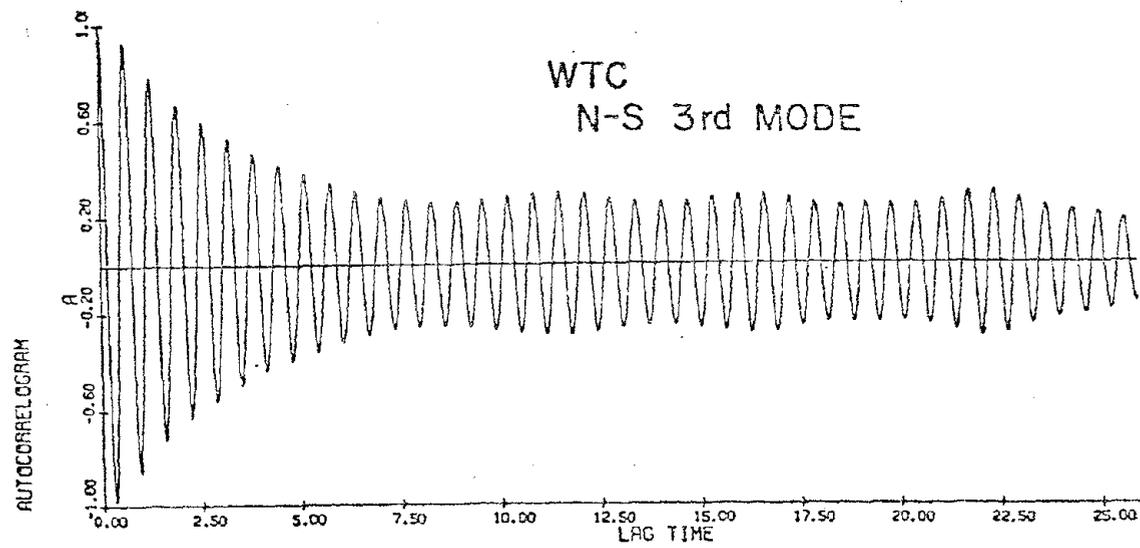
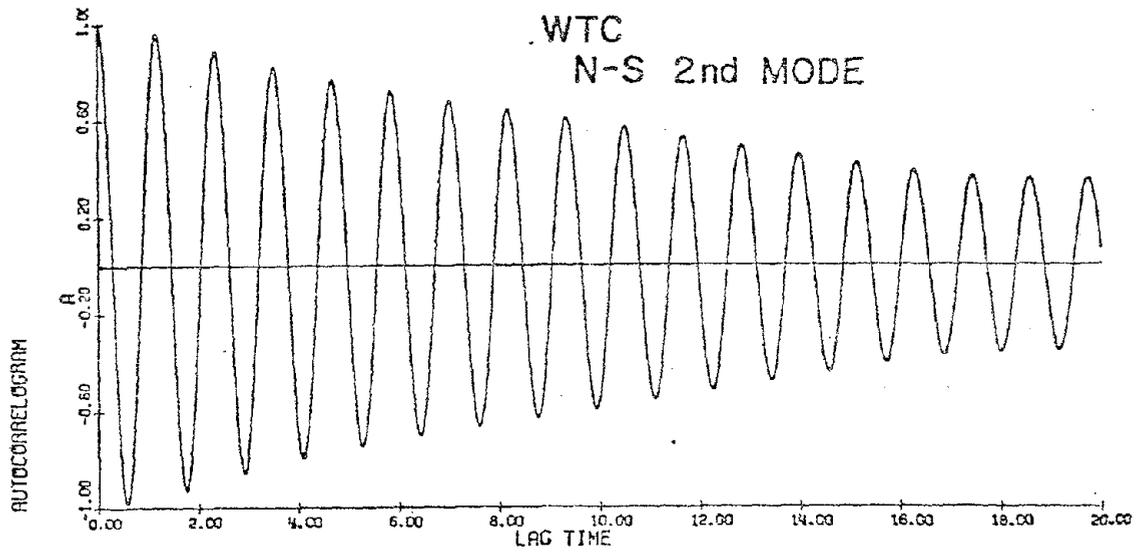
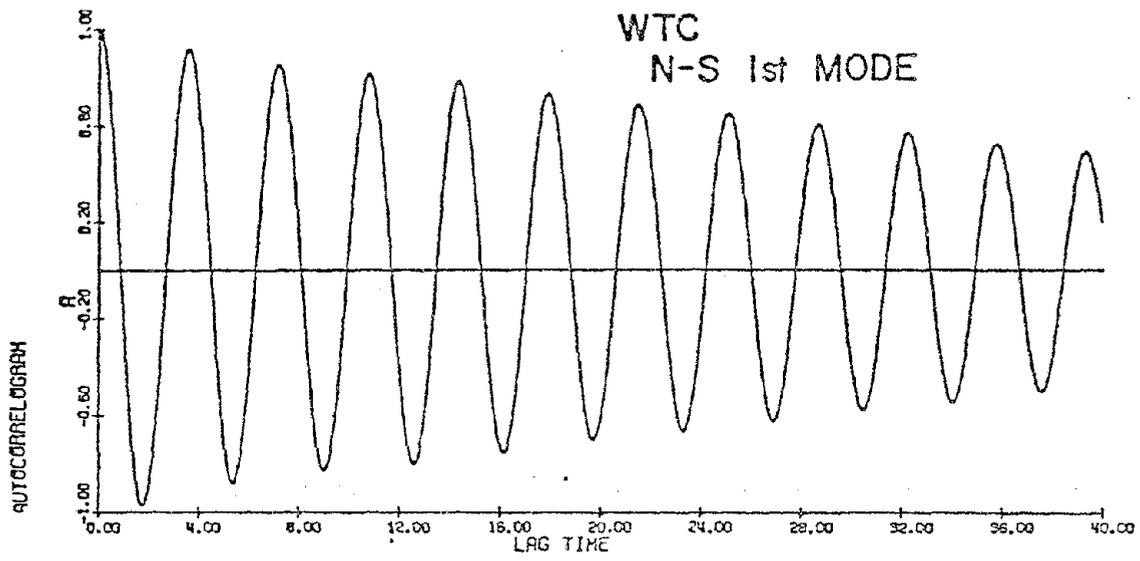


FIGURE 11

