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DEPARTMENT OF CIVIL ENGINEERING

SHORE - IV

Finite Element Program for Dynamic and Static Analysis
of Shells of Revolution

USERS' MANUAL

by

O. M. El-Shafee, P. K. Basu, B. J. Lee

and

P. L. Gould

Research Report No. 56 Structural Division October 1981

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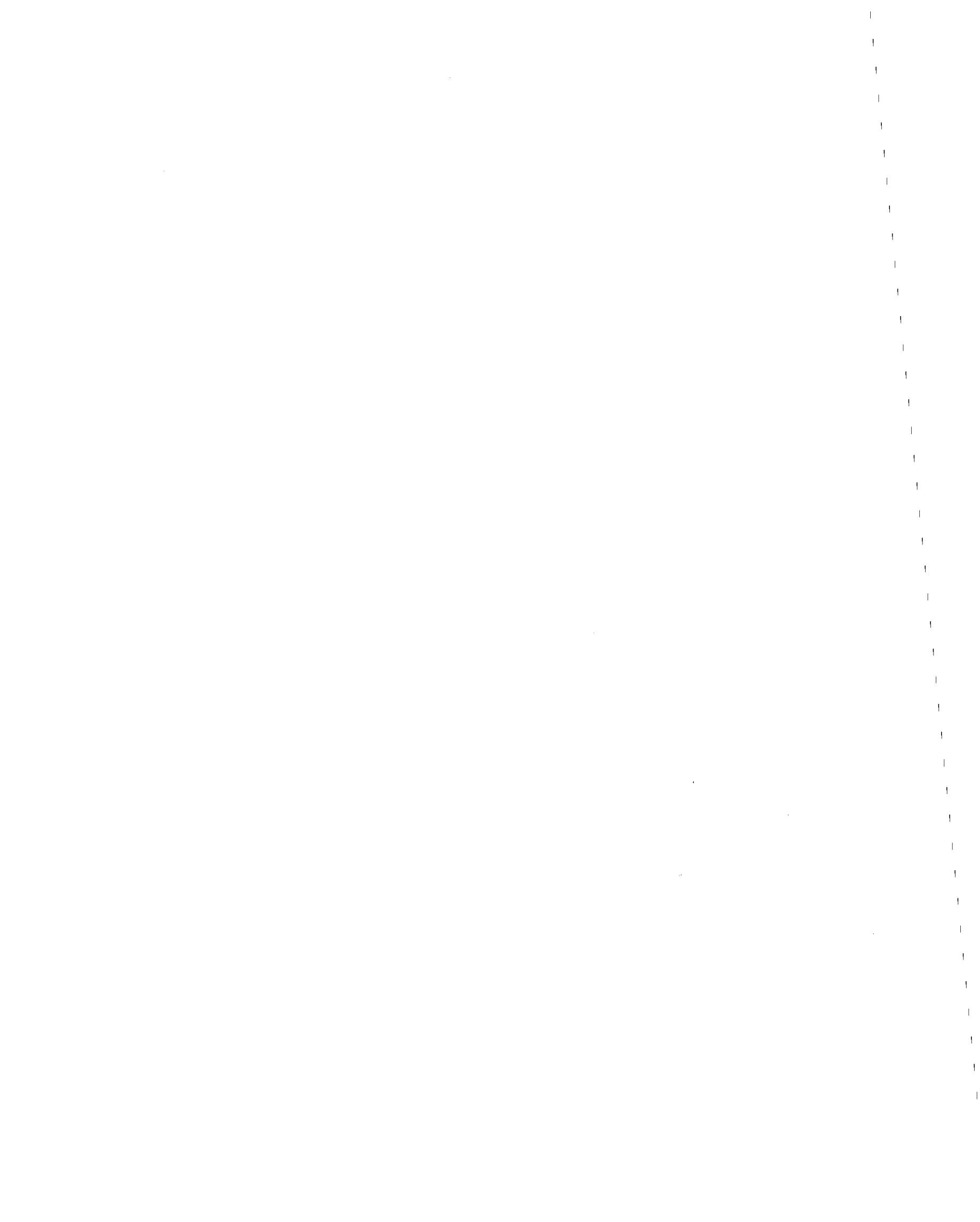
Department of Civil Engineering

School of Engineering and Applied Science

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October 1981

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or recommendations expressed in this
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PREFACE

The SHORE-IV program can be used for the static and dynamic analysis of arbitrarily loaded thin elastic shells of revolution with or without column supports, in the elastic regime. The soil effect on the dynamic behaviour can be considered. This manual describes the procedure to be followed in preparing the input data for this program and also assists in the interpretation of the output. A number of sample inputs and outputs utilizing the various options of the program are included. This and the accompanying Theoretical Manual comprise the documentation of SHORE-IV program.

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INTRODUCTION

The SHORE-IV program is designed for the linear static and dynamic analysis of arbitrarily loaded thin to moderately thick elastic shells of revolution. The meridional curve of the shell may have any quadratic shape including the closed end case. The shell may be isotropic, or single or multi-layer orthotropic, with the two principal material directions at any point coinciding with the two principal directions of the middle surface. The shell may have discrete supports in the form of a framework of linear members with various end conditions and arrangements. Such a framework may also be located at any other level except at the top. Also, complete framed structures having the form of a surface of revolution with the linear members running along the principal directions of the middle surface can be analyzed. As a special case, flat axisymmetric plates may also be considered.

Axisymmetric shells founded on footing foundations may be analyzed dynamically including the soil-structure interaction effect. The soil model consists of isoparametric quadratic solid axisymmetric elements with transmitting boundaries to account for the far-field effect. The soil may be an elastic half-space or horizontally layered strata underlied by bedrock at an actual or assumed depth. Cross-anisotropy for the soil material is assumed. Element to element variation of material properties is admissible. The user may supply detailed information for the soil data or may supply only as few as three cards for the soil data, with automatic data generation by the program. In the case of deep foundations the soil effect can not be considered.

The shell is discretized by a series of curved rotational elements and, if necessary, cap elements. Discontinuous meridian curves are permissible, provided a nodal point is located at such discontinuity. The thickness of an element may vary linearly along the meridian. Element to element variation of material properties is admissible. Figure (1) shows the system model.

In the case of static analysis the following loading conditions can be considered:

1. Distributed pressure loading acting in the u , v , and w directions (for sign convention see Figures 2, 3, and 4)
2. Concentrated line loads applied at designated nodal points in the u , v , w , β_ϕ , and β_θ directions.
3. Gravity loads due to self-weight (or a fraction thereof) acting in the Z , or R directions.
4. Thermal loads.

In addition, non-zero nodal displacements in the u , v , w , β_ϕ , and β_θ directions can be prescribed. If desired, the base constraints may be prescribed with reference to the global Cartesian coordinate system (R, Z).

In the case of dynamic analysis, apart from the above loading cases, the effect of base accelerations due to earthquake can be considered. It is possible to use either a consistent or a lumped mass matrix; but, for better results, it is preferable to use the former. The program is capable of carrying out time-history analysis by direct integration. For this purpose, the user has the option of choosing any single step higher derivative scheme (e.g., Newmark's, Wilson's, etc.) or any multi-step scheme (e.g. Houbolt's method). Otherwise, Wilson's θ -method (with

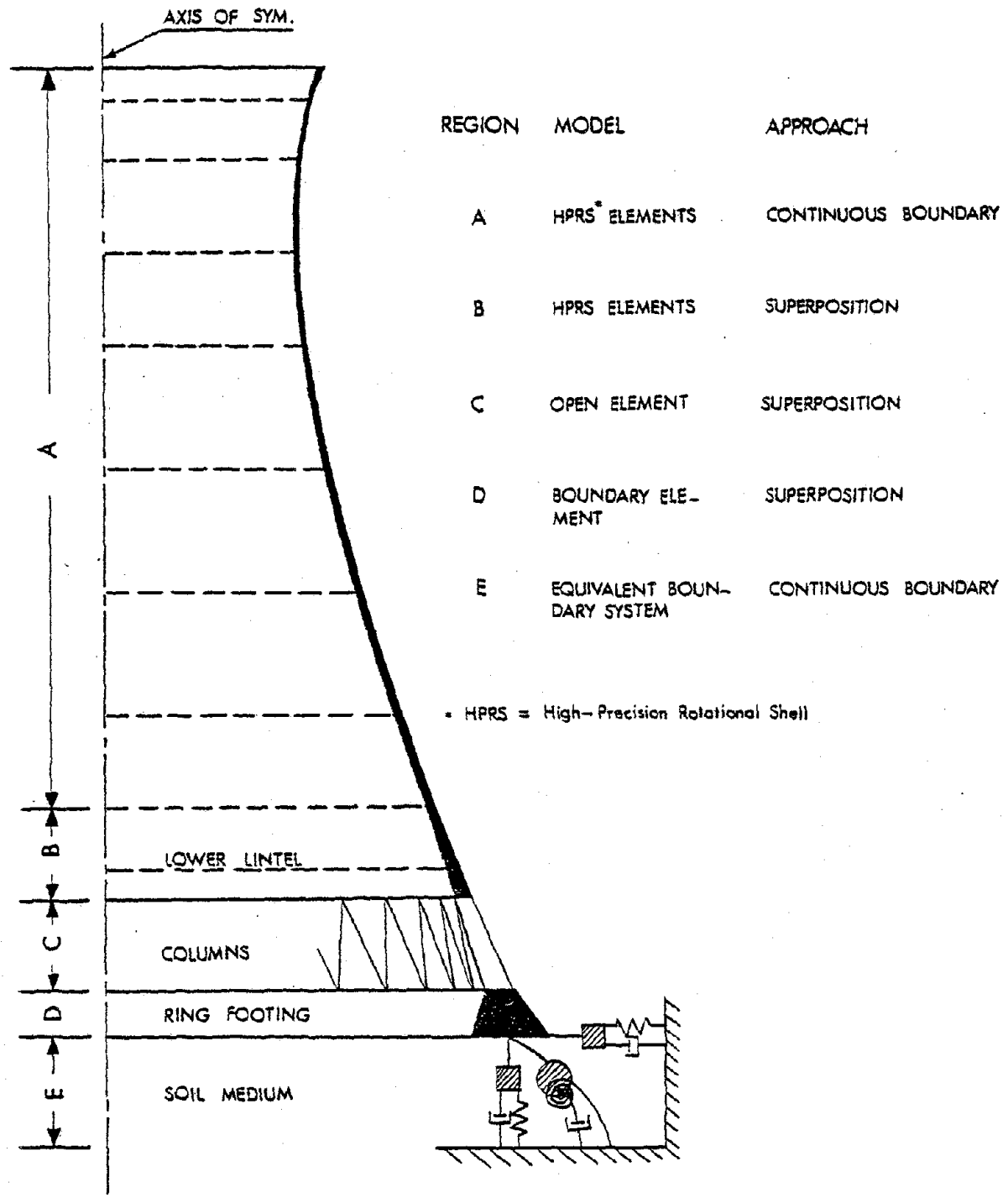
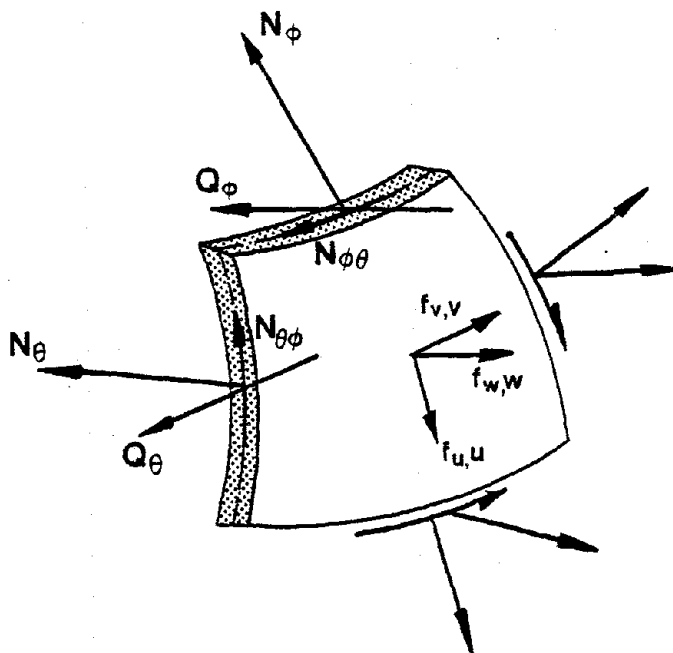
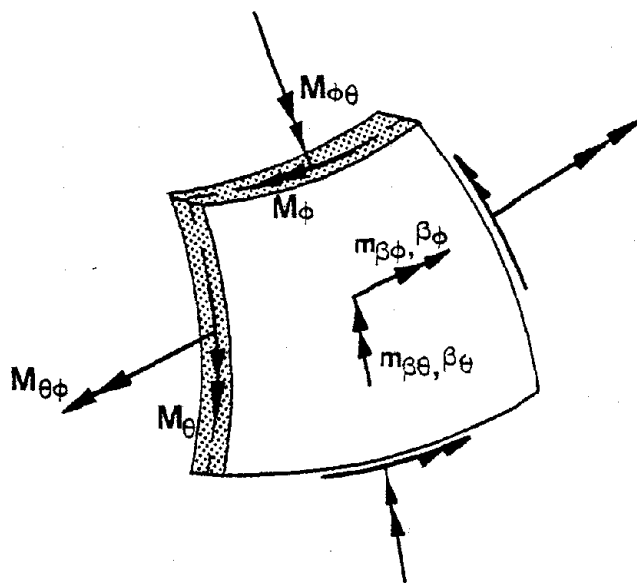


Figure 1. Proposed Model for a Cooling Tower Shell

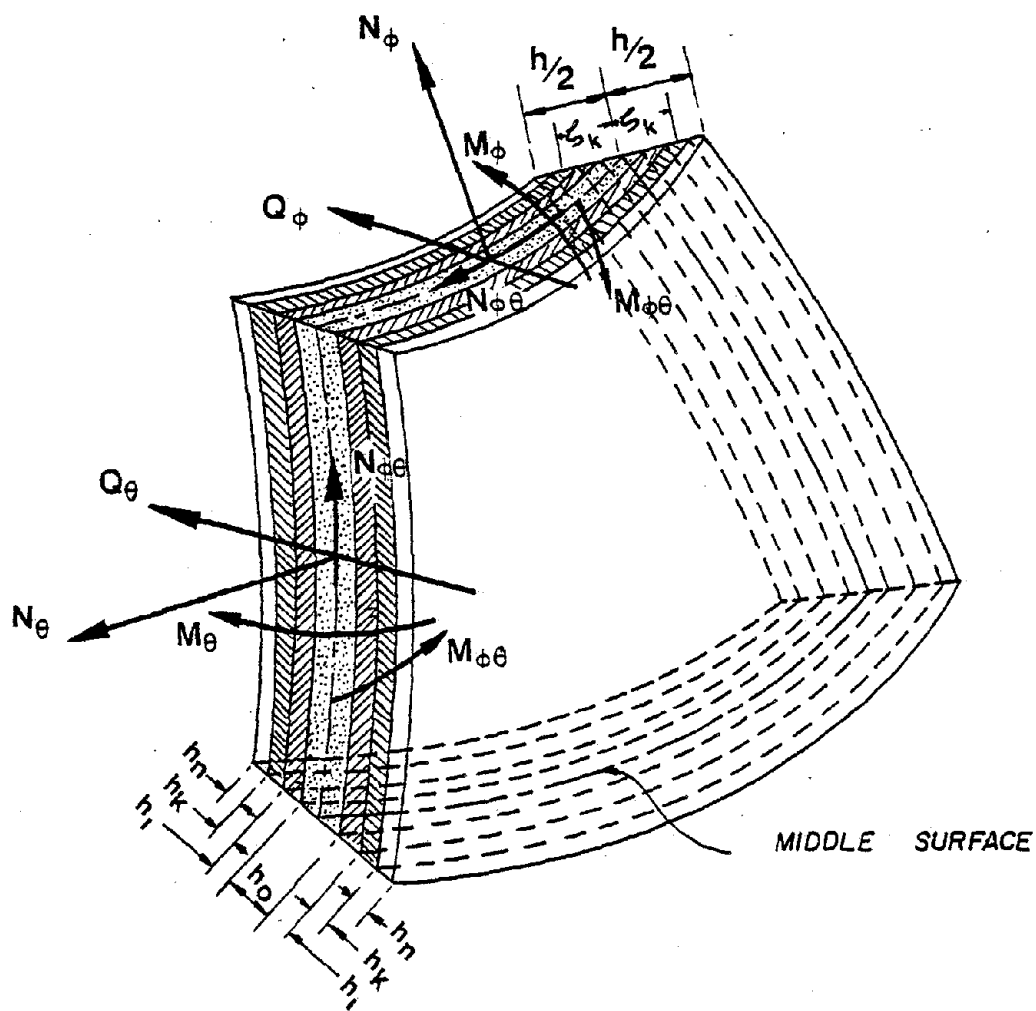


Stress Resultants



Stress-Couple Resultants

Figure 2. Sign Conventions



h_n — THICKNESS OF OUTER LAYER
 h_k — THICKNESS OF INTERMEDIATE LAYER
 h_o — THICKNESS OF MIDDLE LAYER

Figure 3. Multilayer Shell Element

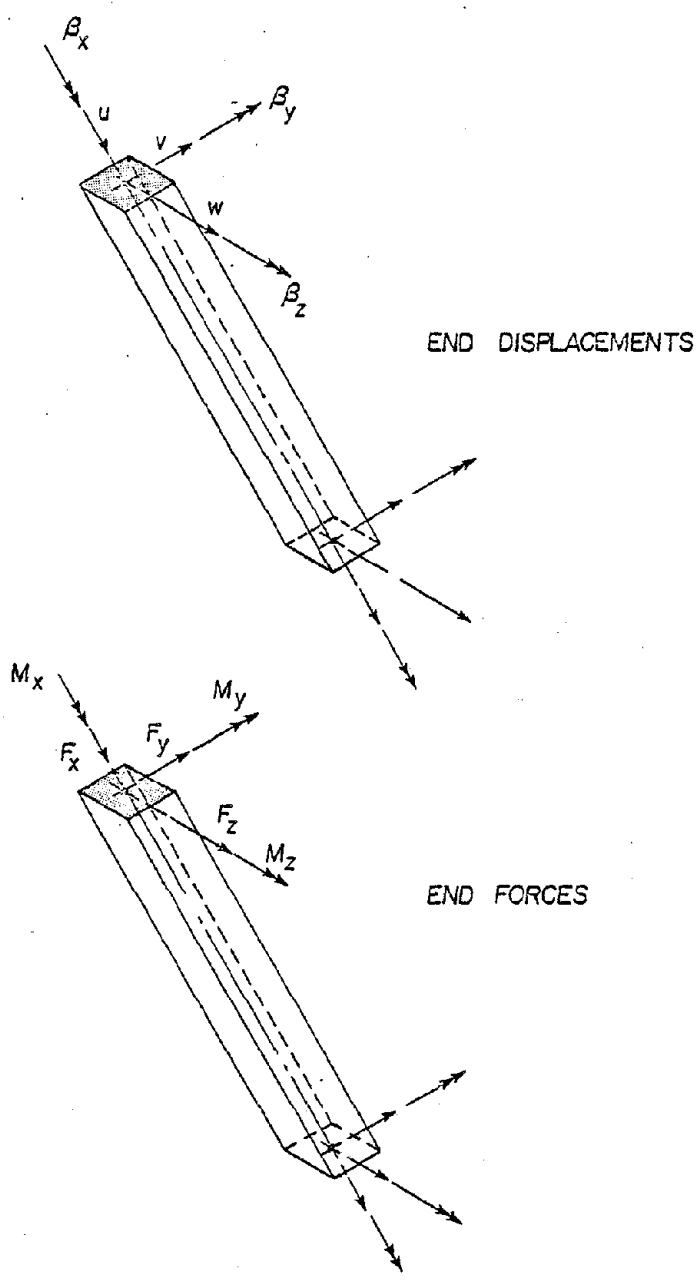


Figure 4. Sign Convention for Members in Open Type Elements

$\theta = 1.4$) can be used as the default option. If desired, it is possible to specify more than one time step. This may sometimes be helpful in saving computer time. Earthquake analysis can also be carried out by the response spectrum method. The free vibration analysis is carried out by the combined Sturm sequence and inverse iteration technique [1].

All loads which are not axisymmetric are required to be expanded in Fourier harmonics; for this purpose, a separate program package (FOHARM) may be used. The loading need not be symmetric about the $\theta = 0^\circ$ line, i.e. both sine and cosine harmonics can be considered simultaneously. The number of harmonics to be considered will depend upon the nature of the loading and the accuracy desired. The distributed loadings and the temperature distribution may vary linearly along the meridian of each element.

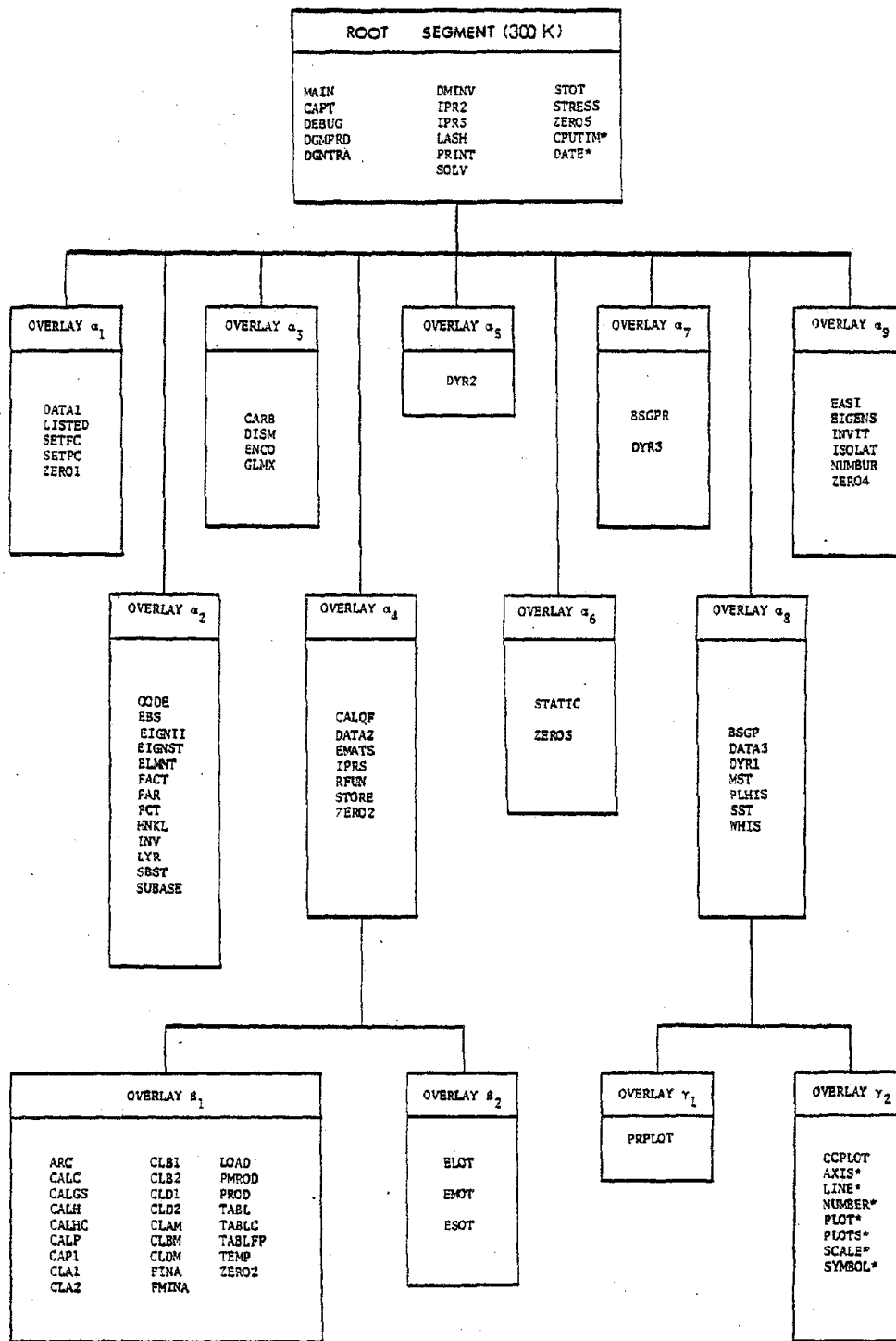
The input data format is such that repetitive data is reduced to minimum. Moreover, in the event of some specific errors in input data, the run is terminated before the problem is executed and the corresponding error code, or message is printed out.

Various printout options for displacements, stress-resultants and stresses are available. The time history plot for displacements and stress resultants can be obtained on the printer. Alternatively, for offline plotting on a 760/563 Calcomp plotter system, a plot tape can be created. Also, the time history results can be obtained on punched cards for further uses.

This program is an extension of the Shell of Revolution Finite Element Program SHORE-III [2,3]. It is written in FORTRAN IV language and has been developed on an IBM 360/65 computer.

SHORE-IV is an in-core solver requiring less than 500 K in high speed storage. For running the program it is necessary to use the overlay structure shown in Figure 5.

The input data and intermediate results are required to be stored on seventeen scratch disk files and three tapes. For Calcomp plotting a 800 bpi, 9-track tape is also required.



NOTE: Subroutines Marked with an Asterisk (*) are Installation Dependent.

Figure 5. Overlay Structure of SHORE-IV

INPUT FOR SHORE-IV PROGRAM

The input data for the SHORE-IV program consists of complete data sets for each problem to be run, placed sequentially. The data for each problem is organized into a number of groups associated with card types A through L. Some of these groups, namely those corresponding to card types E, G, H, etc., are subdivided into various optional blocks of input data. There is no limit to the number of problems that can be executed per run. The schematics of input data decks for various types of problems are shown in Figures 6 to 10.

The program uses seventeen temporary disk files, namely 9, 10, 11, ... 25, and three temporary tapes, namely 26, 27 and 28. In the case of dynamic analysis when Calcomp plotting of time history response is desired, a permanent tape, the PLOTTAPE, (9 track, 800 bpi), is required. Therefore, the above mentioned deck should be preceded by suitable system control cards allocating the above mentioned spaces. For example, on an IBM 360/65 computing system, the JCL statements shown in Appendix A may be used.

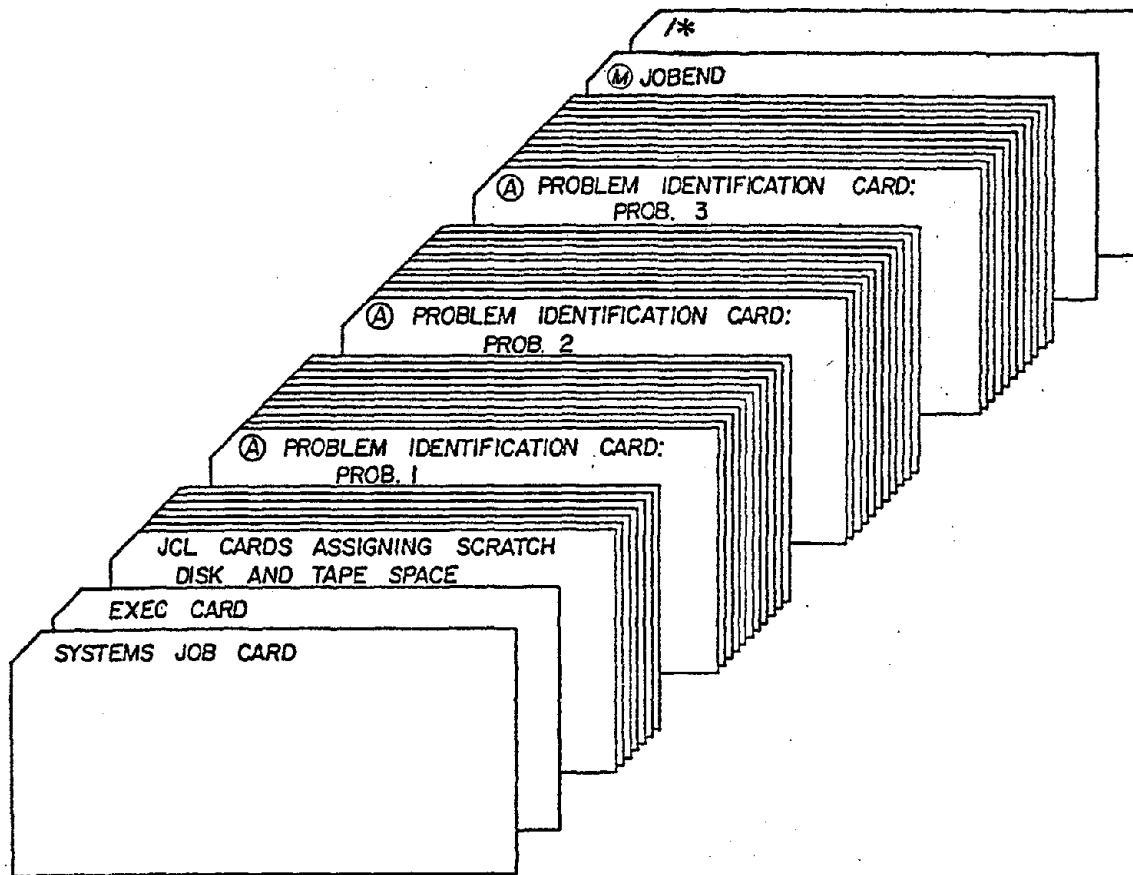


Figure 6. Input Data Stream for a Job

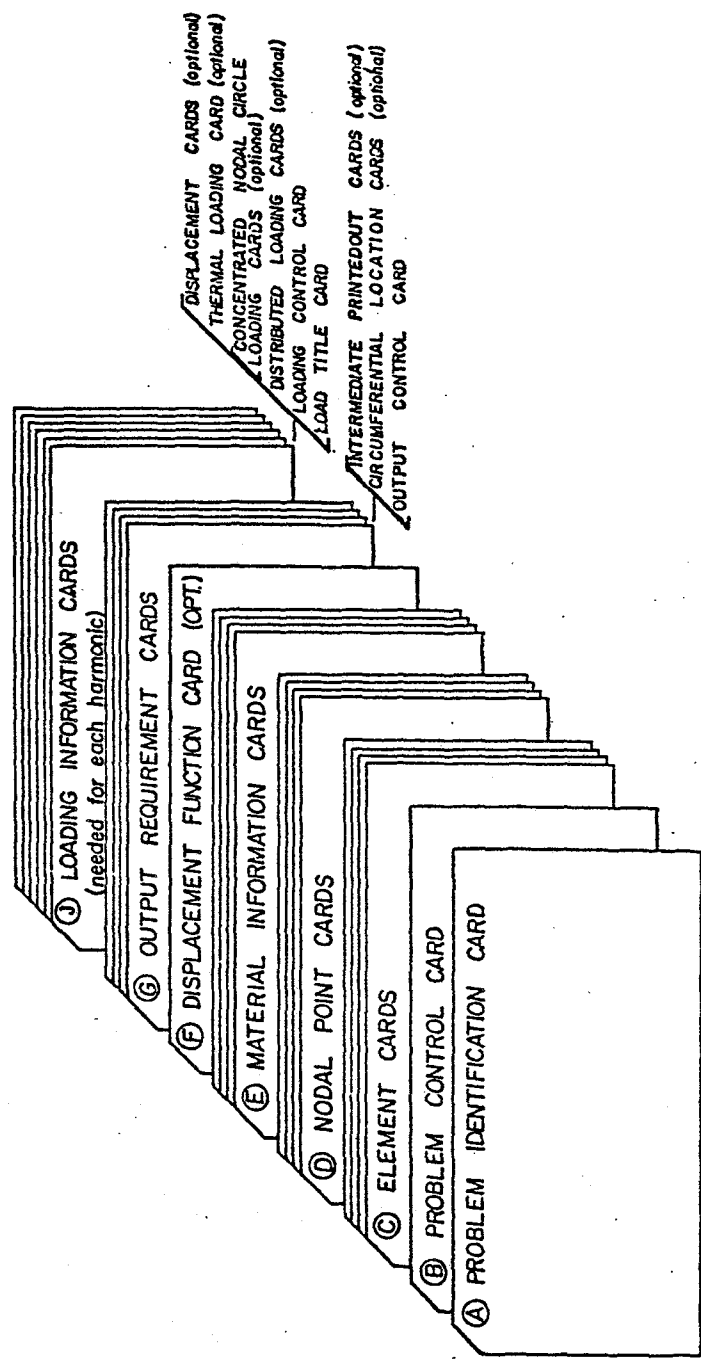


Figure 7. Input Data Deck for Static Analysis

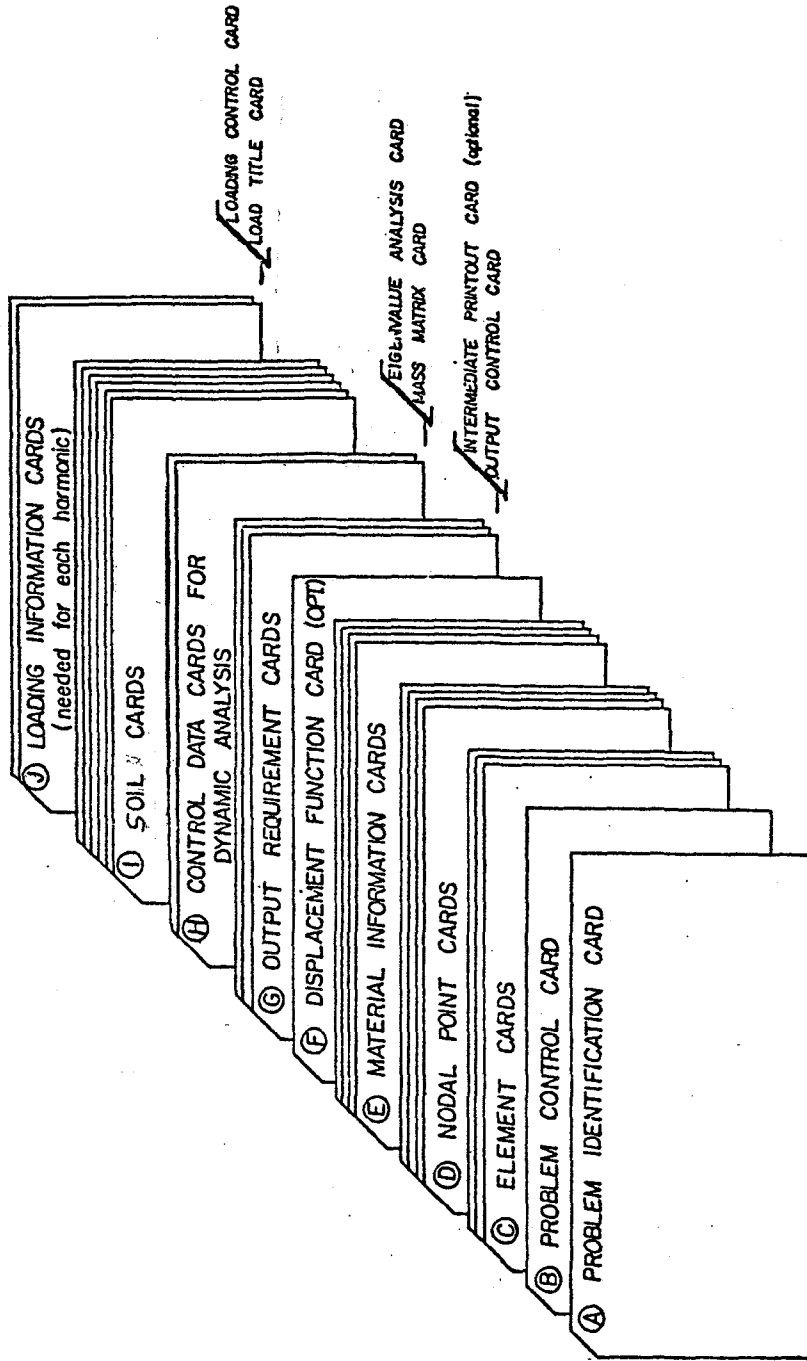


Figure 8. Input Data Deck for Free Vibration Analysis

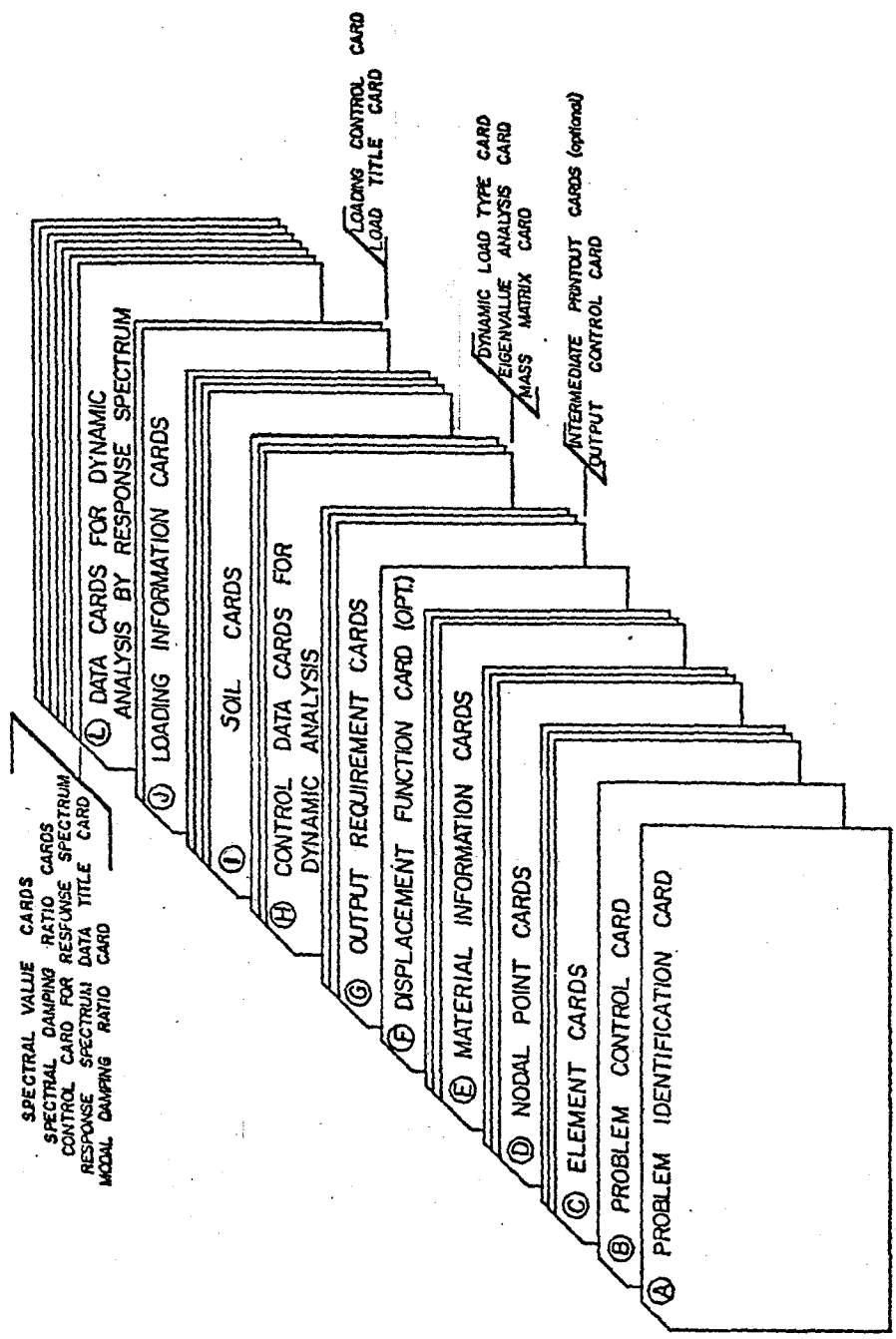
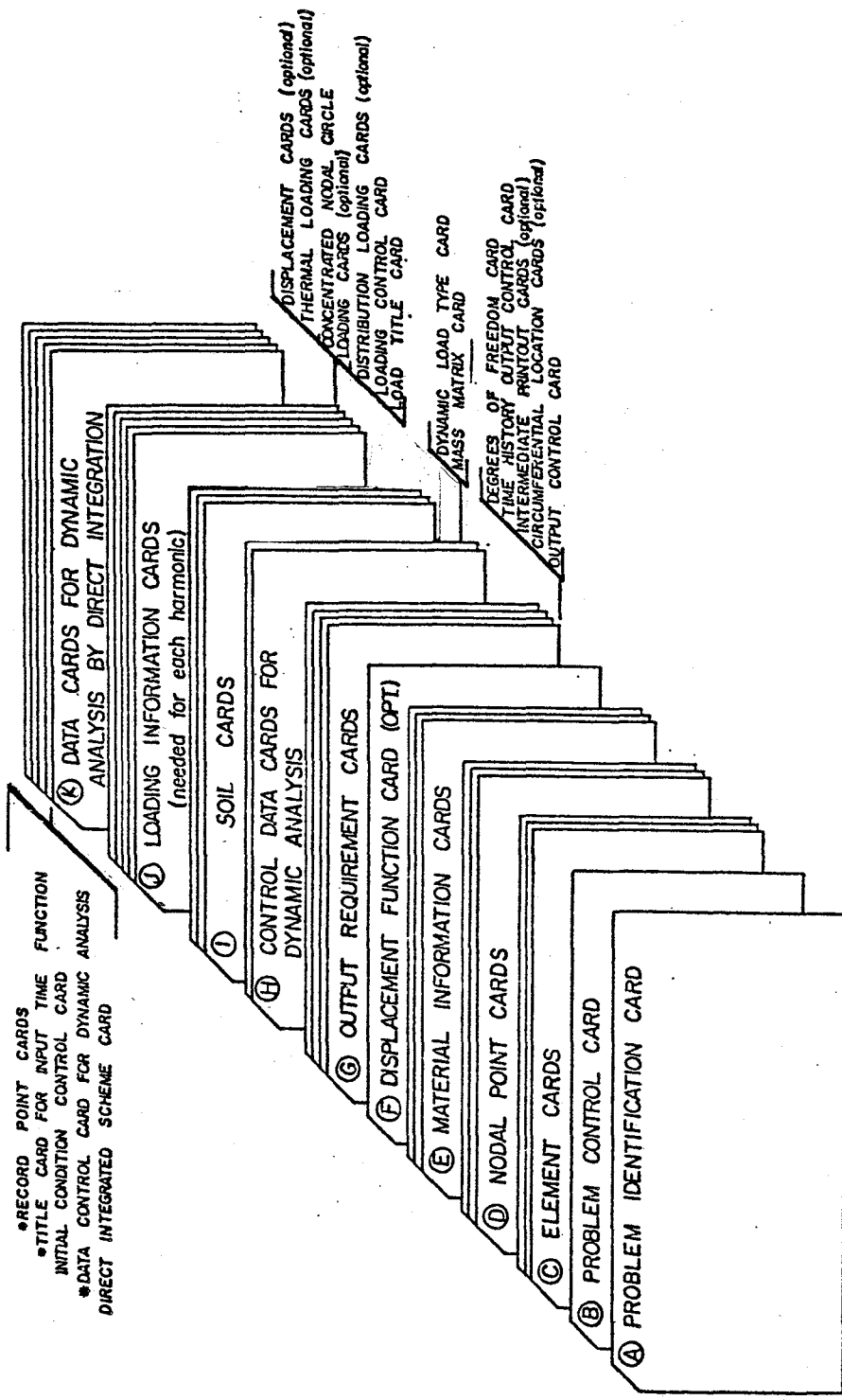


Figure 9. Input Data Deck for Response Spectrum Analysis



Note: Cards marked with an asterisk (*) may be repeated for each harmonic depending upon the option code specified in problem control card.

Figure 10. Input Data Deck for Time History Analysis by Direct Integration

Card Formats

The integer field is expressed as I2, I3, I5, etc., meaning thereby integer fields of 2, 3, 5, etc. characters. The floating point numbers are indicated mostly by F8, F9, F10, etc., meaning floating point numbers with fields of 8, 9, 10, etc. characters. In a few cases the floating point numbers are expressed as E9, E10, meaning floating point numbers with exponents consisting of 9, 10 characters. Floating point numbers shall be entered with a decimal point, and those in E-format and the integers must be right justified. For each record (or card) of 80 columns, the formats shall be as stated in subsequent sections.

Units

Any consistent units may be used. Of course, the force and distance units for all input quantities should be the same. For convenience the units used may be stated on any of the title cards.

DATA CARDS FOR EACH PROBLEM

A. Problem Identification Card:

The information contained on this card is the first print out for each problem and is usually the title assigned to the problem. In the last eight columns of this card the code words for input data echo options are stated.

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
2 - 72	A	Problem Title to be output with results
73 - 79	A	NO ECHO - if the problem is to be run without printing the echo of input data

If the columns 73 - 80 are left blank the input data echo is printed and also the problem is run.

B. Problem Control Card:

The information contained on this card is as follows:

1. The number of rotational finite elements used for discretizing the structure. The maximum number of elements allowed is 48.
2. The number of harmonic loading cases to be considered.
3. The type of structural cross-section with the following code numbers:

isotropic	0 (or blank)
single layer orthotropic	1
framed	2
multilayer (Each layer may be orthotropic or isotropic.)	number of layers (must be odd). The minimum is 3 and maximum 7.

In the case of multilayer shells, even if the number of layers is not odd and/or the layers are unsymmetrical, the problem can be solved to a point by specifying 2 as the code number; the output results will be stress resultants and displacements only.

4. The code number for the type of analysis:

static analysis	0
free vibration analysis	1
time history analysis	
mode superposition	2
response spectrum	3
direct integration	4

- 5. The maximum degree of polynomial approximation to be used in the stiffness and mass matrices. The maximum is 6, and the default value is 3. It is recommended that 6 be used for static analysis and 3 for dynamic analysis [3].
- 6. The code number to avoid inputting of repetitive data required for the time history analysis by direct integration:

Control data for dynamic analysis, or time history data to be supplied for each harmonic	1
The above data to be supplied for the first harmonic only	0

The format of Problem Control Card will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Number of finite elements to be used
6 - 10	I5	Number of harmonic loading cases
11 - 15	I5	Relevant code number for shell material
16 - 20	I5	Code number for type of analysis
21 - 25	I5	Maximum degree of polynomial approximation
30 - 35	I5	1 or 0, refers to control data for dynamic analysis by direct integration
36 - 40	I5	1 or 0, refers to time history data
41 - 45	I5	1 or 0, refers to soil-structure interaction effect

The flag 1 in the sixth field signifies that the control data card for dynamic analysis, J(b), is input separately for each harmonic. If left blank, it signifies that this card is input for the first harmonic only. Similarly, the flag 1 in the seventh field signifies that the input time function cards, J(e) and J(f), are input separately for each harmonic. Otherwise, the same are input for the first harmonic only.

If the columns 41 to 45 are left blank the soil-structure interaction is not considered.

If the flag 1 appears in the column 45 the interaction effect is considered and the soil data of section "I" must be supplied.

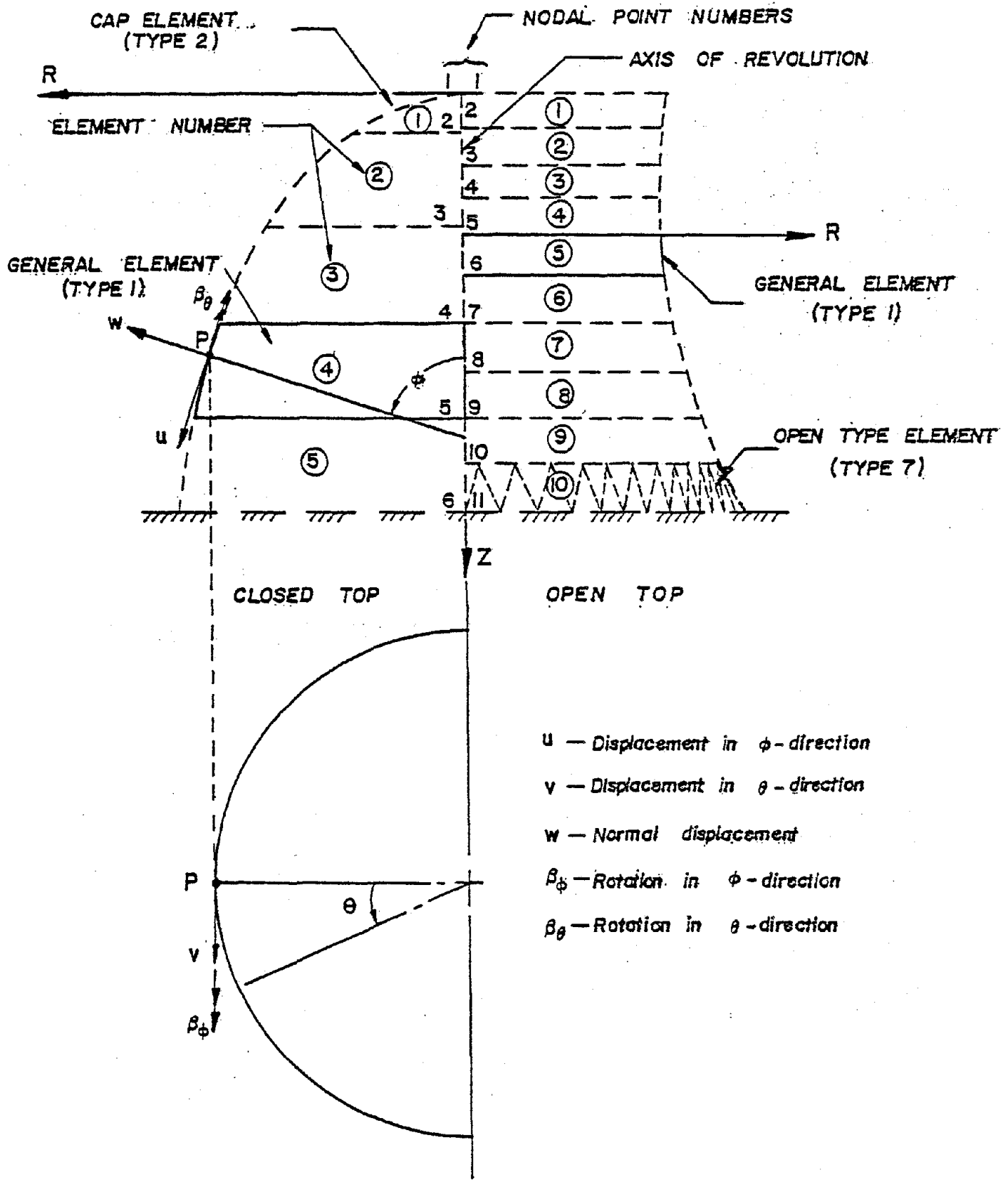
For static analysis columns 41 to 45 must always be left blank.

C. Element Cards

One card is required for each element, with the cards placed in numerical sequence of element numbers. The elements are required to be numbered consecutively from the top to the bottom of the shell beginning with element number 1. Flat plates should be numbered consecutively from inside to outside. Only one cap element may be used for the analysis of a closed shell of revolution and this must be numbered element 1. (See Figures 11 and 12).

If some cards are omitted, the element information for the omitted elements is set equal to the element information on the preceding element card. However, element cards for the first and last elements must always be supplied.

Each element card should state the element number, the element type, the meridian definition code and certain constants defining the meridian. For element types, refer to the library of elements in Figure 13. In the case of open type elements, the element type field also states the number and end conditions of the members comprising the open type element. For end conditions of open type elements, refer to Figure 14. The meridian definition code defines how the meridian curve of the element is specified. For a global coordinate system the code number is zero (or blank) and for a local coordinate system it is the nodal point number where $Z = 0$. For a type 2 cap element, the Z-coordinate of the pole must be zero. The meridian definition code is not applicable in the case of type 4 and type 5 elements.



- u — Displacement in ϕ -direction
- v — Displacement in θ -direction
- w — Normal displacement
- β_ϕ — Rotation in ϕ -direction
- β_θ — Rotation in θ -direction

Figure 11. Finite Element Discretization of Shell of Revolution

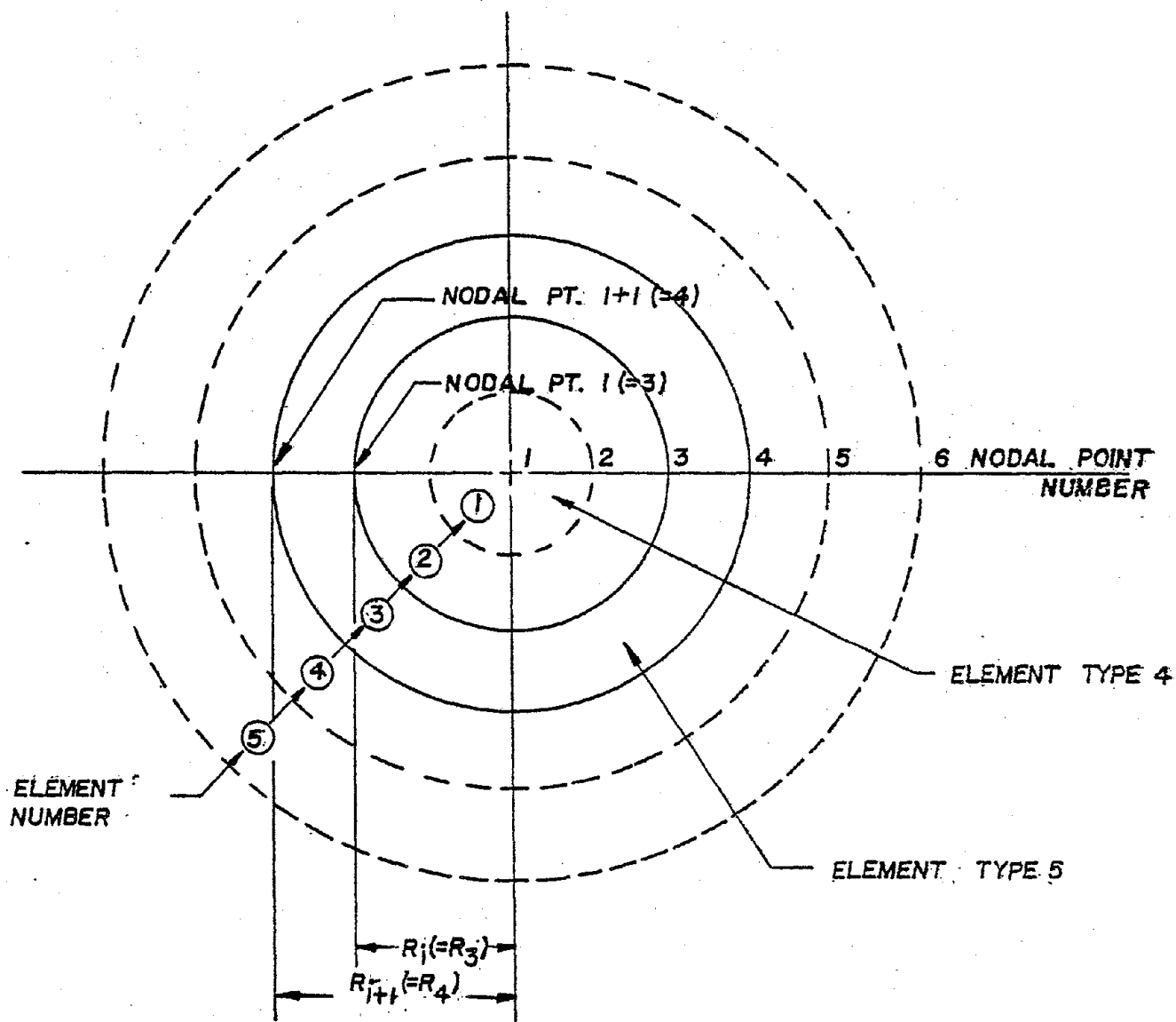


Figure 12. Finite Element Discretization of an Axisymmetric Plate

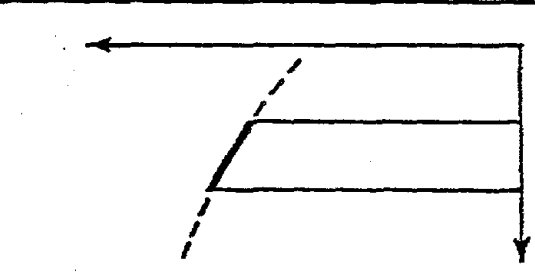
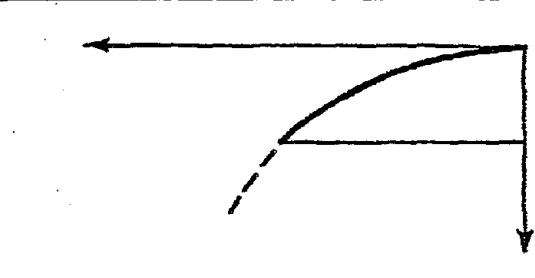
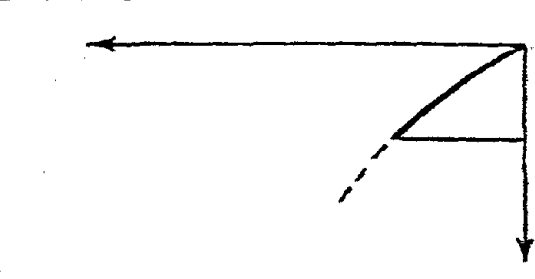
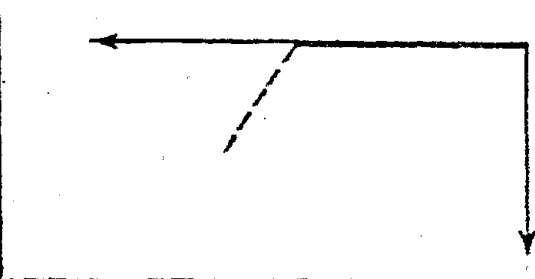
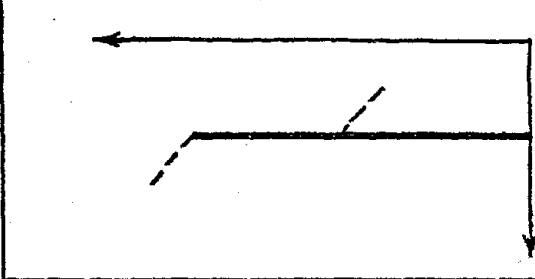
CODE	TYPE	SHAPE
1	GENERAL (CURVED)	
2	CAP (INFINITE SLOPE)	
3	CAP (FINITE SLOPE)	
4	CAP (FLAT PLATE)	
5	GENERAL (FLAT PLATE)	

Figure 13. Library of Elements

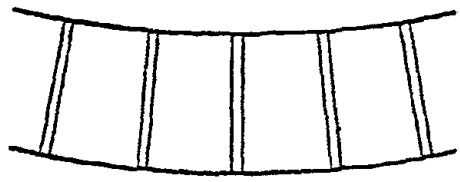
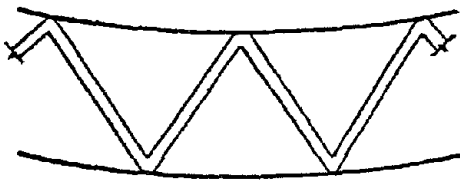
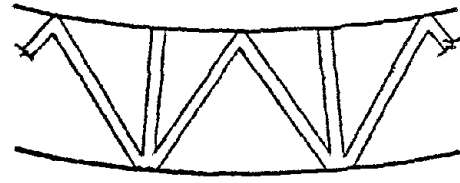
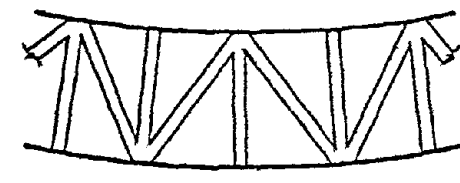
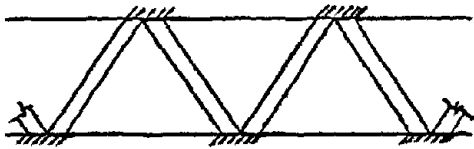
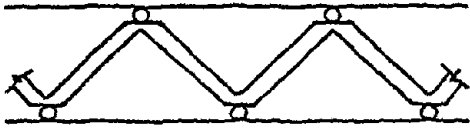
CODE	TYPE	SHAPE
6	OPEN	 A diagram of a curved truss structure. It consists of two curved lines, one at the top and one at the bottom, connected by five vertical members. The top curve is slightly concave down, and the bottom curve is slightly concave up.
7	OPEN	 A diagram of a curved truss structure. It consists of two curved lines, one at the top and one at the bottom, connected by two diagonal members forming a zigzag pattern. The top curve is slightly concave down, and the bottom curve is slightly concave up.
8	OPEN	 A diagram of a curved truss structure. It consists of two curved lines, one at the top and one at the bottom, connected by two diagonal members forming a zigzag pattern and two vertical members. The top curve is slightly concave down, and the bottom curve is slightly concave up.
9	OPEN	 A diagram of a curved truss structure. It consists of two curved lines, one at the top and one at the bottom, connected by two diagonal members forming a zigzag pattern and two vertical members. The top curve is slightly concave down, and the bottom curve is slightly concave up.

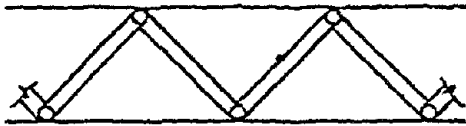
Figure 13. (Contd.)



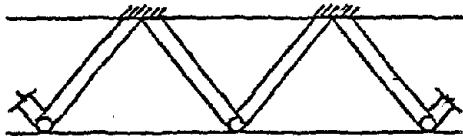
FULL CONTINUITY AT
BOTH ENDS
CODE NUMBER: 1



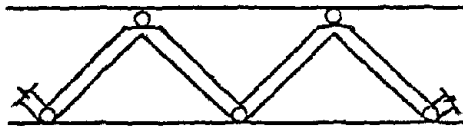
MEMBERS MONOLITHIC
PINNED ENDS
CODE NUMBER: 2



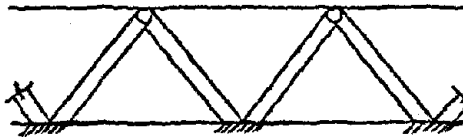
MEMBERS AND ENDS
PINNED
CODE NUMBER: 3



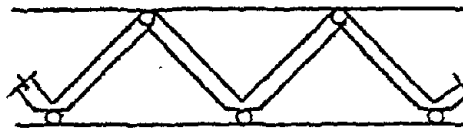
TOP ENDS LIKE CODE NO.1
BOT. ENDS LIKE CODE NO.3
CODE NUMBER: 4



TOP ENDS LIKE CODE NO.2
BOT. ENDS LIKE CODE NO.3
CODE NUMBER: 5



TOP ENDS LIKE CODE NO.3
BOT. ENDS LIKE CODE NO.1
CODE NUMBER: 6



TOP ENDS LIKE CODE NO.3
BOT. ENDS LIKE CODE NO.2
CODE NUMBER: 7

Figure 14. End Conditions of Open Type Elements

In the case of element types 1 to 3, the constants are actually the six coefficients of the following equation of the meridian curve:

$$AZ^2 + BRZ + CR^2 + DZ + ER + F = 0$$

in which

R and Z are the coordinate defining points on the meridian (see Figure 11), and A, B, C, D, E, and F are constants for the meridian curve with the requirement that C should always be positive. In the case of element types 4 and 5, it is required to specify the coefficients A and B only, A being equal to the R-location of nodal point i and B equal to the R-location of nodal point (i+1), as shown in Figure 12. In the case of element types 6 to 9, with the theoretical middle surface following the shape of the frustum of a cone, it is necessary to specify the coefficients D, E, and F only.

The format of Element Cards will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 3	I3	Element number
4 - 10	I7	Element type (in the case of open type elements see next page)
11 - 13	I3	Meridian definition code
14 - 24	F11	A coefficient
25 - 35	F11	B coefficient
36 - 46	F11	C coefficient
47 - 57	F11	D coefficient
58 - 68	F11	E coefficient
69 - 79	F11	F coefficient

In the case of open type elements the entry in Columns 4 to 10 shall be

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
4 - 8	I5	Number of members in the element.
9	I1	Code number for end conditions (see Figure 14)
10	I1	Element type

D. Nodal Point Cards

One card is required for each nodal point, placed in numerical sequence of nodal numbers. Nodal points must be numbered consecutively from the top to the bottom of a shell or from the innermost part to the outer part of a plate, beginning with nodal point number 1 (see Figures 11, 12).

If some cards are omitted, the omitted nodal points are generated at equal intervals between the defined nodal points. However, nodal point cards for the first and last nodal points must always be supplied.

Each nodal point card states the nodal point number, Z-coordinate of the nodal point in global coordinate system and the geometric constraints corresponding to the displacement components u , v , w , β_ϕ , and β_θ at the nodal point. When a displacement component is zero, the constraint code number corresponding to that displacement component will be 1. When a non-zero displacement component is specified, the constraint code number will be 2. If, however, the displacement component is not constrained, the code number will be zero (or blank). Constraint codes for all omitted nodal point cards are set equal to zero. Thus, it is necessary to provide data for all constrained nodal points. The maximum number of prescribed non-zero constraints is limited to 10.

The format of Nodal Point Cards will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Nodal point number
6 - 15	F10	Nodal point location (Z-coordinate in global coordinate system)
16 - 20	I5	Constraint Code for u-displacement

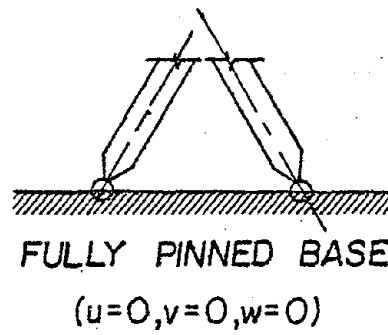
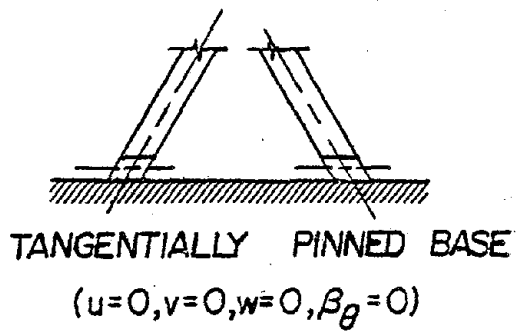
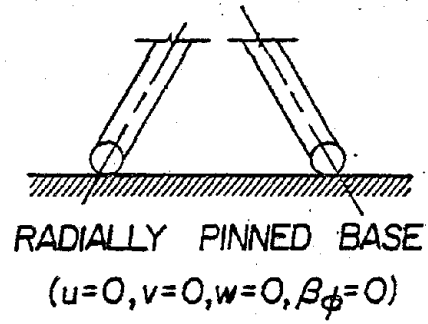
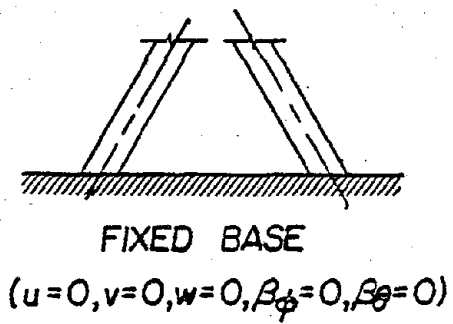


Figure 15. Support Constraints for Open Type Elements

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
21 - 25	I5	Constraint Code for v-displacement
26 - 30	I5	Constraint Code for w-displacement
31 - 35	I5	Constraint Code for β_ϕ -rotation
36 - 40	I5	Constraint Code for β_θ -rotation
41 - 45	I5	Blank or 1, the former signify that the <u>support constraints</u> refer to the <u>curvilinear reference frame</u> and the later signifies that the same refer to the <u>global cartesian reference frame</u>

For the case of open type elements the support constraints for various end conditions are shown in Figure 15.

E. Material Information Cards:

In the case of isotropic material, only one material information card placed in numerical sequence of element numbers, is required for each element. If some cards are omitted, the material information for the omitted elements are set equal to those on the preceding card. However material information cards for the first and last elements must be supplied. Each material information card states the element number, thickness of element, modulus of elasticity, Poisson's ratio, shear factor, and mass density for the element. In the case of open type elements, each material information card states the cross-sectional area, torsional constant (Saint-Venant's), modulus of elasticity, moments of inertia about the circumferential and normal axes, mass density, width and depth of the typical member, and Poisson's ratio. (See Figure. 16)

The format of Material Information Cards for isotropic material will be as follows:

<u>Column</u>	<u>Format</u>	<u>Entry</u>
1 - 3	I3	Element number
4 - 12	E9	Thickness of element at node with smaller no. (h_i)
13 - 21	E9	Thickness of element at node with larger no. (h_{i+1})
22 - 30	E9	Modulus of elasticity
31 - 39	E9	Poisson's ratio
40 - 48	E9	Shear factor (default = 5/6)
49 - 57	E9	Mass density of material

The format of Material Information Cards for open type elements will be as follows:

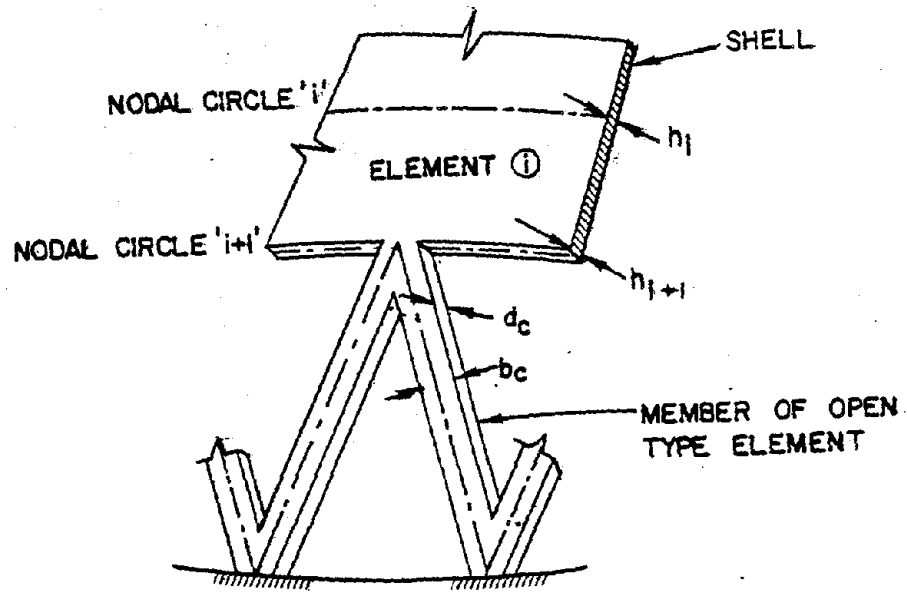


Figure 16.

<u>Column</u>	<u>Format</u>	<u>Entry</u>	
1 - 3	I3	Element number	
4 - 12	E9	Cross-sectional area of member	
13 - 21	E9	Torsional constant ($I_0 = I + I_z$)	
22 - 30	E9	Modulus of elasticity	
31 - 39	E9	Moment of inertia about the circumferential axis	
40 - 48	E9	Moment of inertia about the normal axis	
49 - 57	E9	Mass density of material (γ/g)	
58 - 66	E9	Width of member (b_c)	For wind effect For cols. effect on the shell For temp. levels
67 - 75	E9	Depth of member (d_c)	
76 - 80	F5	Poisson's ratio	

Note: Maximum number of different values for Poisson's ratio for open-type elements is 16.

In the case of single layer orthotropic or multi-layer materials or framed structures, a set of three material information cards are required for each element, placed in numerical sequence of element numbers. If such card sets are omitted, the material information for the omitted elements are set equal to those on the preceding set. However, material property cards for the first and last elements must always be supplied. In this case, the first material information card for each element contains the element number, and thickness of element. The second and third cards contain the ten elements of the constitutive matrix [C], Appendix B.

First Card:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 3	I3	Element number
4 - 12	E9	Thickness of element at node with smaller no.
13 - 21	E9	Thickness of element at node with larger no.

Second Card:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	C ₁₁
11 - 20	F10	C ₁₂
21 - 30	F10	C ₂₂
31 - 40	F10	C ₃₃
41 - 50	F10	C ₄₄
51 - 60	F10	C ₄₅
61 - 70	F10	C ₅₅
71 - 80	F10	C ₆₆

Third Card:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	C ₇₇
11 - 20	F10	C ₈₈

In the case of multi-layer shells, these three cards should be followed by another group of four material information cards. For each element there will then be a total of seven cards. All such sets shall be arranged in numerical sequence of element numbers and, as above, intermediate sets may be omitted. The last four cards in each set contain information regarding thicknesses and elastic constants for each layer of the element (see Figure 2). The maximum allowable number of layers is seven and the layers should be symmetrical about the middle

surface in all respects. It may, however, be noted that if the number of layers is even and/or the layers are unsymmetrical, the last four cards in the set need not be provided since stresses cannot be computed.

The format of the additional sets of Material Information Cards for multilayer materials will be as follows:

Fourth Card:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Element number
6 - 15	F10	Thickness of middle layer (h_0)
16 - 25	F10	Thickness of layer next to middle (h_1), i.e. the outermost layer of a three layer system
26 - 35	F10	Thickness of layer next to h_1 (h_2), i.e. the outermost layer of a five layer system
36 - 45	F10	Thickness of layer next to h_2 (h_3), i.e. the outermost layer of a seven layer system

Fifth Card:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5		To be left blank
6 - 15	F10	<u>Modulus of elasticity in meridional direction (E_ϕ) for middle layer</u>
16 - 25	F10	- do - for layer '1'
26 - 35	F10	- do - for layer '2'
36 - 45	F10	- do - for layer '3'

Sixth Card:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5		To be left blank
6 - 15	F10	<u>Modulus of elasticity in circumferential direction (E_θ) for middle layer</u>
16 - 25	F10	- do - for layer '1'
26 - 35	F10	- do - for layer '2'
36 - 45	F10	- do - for layer '3'

Seventh Card:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5		To be left blank
6 - 15	F10	<u>Tangential shear modulus</u> ($G_{\phi\theta}$) for middle layer
16 - 25	F10	- do - for layer '1'
26 - 35	F10	- do - for layer '2'
36 - 45	F10	- do - for layer '3'

When the number of layers is three, the last two entries in each of the above four cards should be left blank. Similarly, in the case of five layers the last entry in each of these cards should be left blank.

F. Displacement Function Card:

Omit this card if columns 21-25 of the Problem Control Card are left blank. For static analysis, it is recommended that 6 be used in each field from columns 1-25. At the present time, it is recommended that the default value be used for dynamic analysis [4]. Then, this card is not required. Otherwise, the degree of polynomial approximation for the displacement functions to be used in forming the stiffness matrix (col. 1-25) and the mass matrices (col. 26-50) may be specified shown below. However, the maximum permissible degree is 6. In the case of static analysis the columns 26 to 50 relating to the mass matrix should be left blank.

<u>Column</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Degree of polynomial approximation for u
6 - 10	I5	Degree of polynomial approximation for v
11 - 15	I5	Degree of polynomial approximation for w
16 - 20	I5	Degree of polynomial approximation for β_ϕ
21 - 25	I5	Degree of polynomial approximation for β_θ
26 - 30	I5	Degree of polynomial approximation for u
31 - 35	I5	Degree of polynomial approximation for v
36 - 40	I5	Degree of polynomial approximation for w
41 - 45	I5	Degree of polynomial approximation for β_ϕ
46 - 50	I5	Degree of polynomial approximation for β_θ

G. Output Requirement Cards:

a) The first card, the 'output control card', states the number of circumferential locations where displacements, stresses, etc. in each element are desired to be printed out, and also the number of intermediate printout options desired. For available options see Appendix C. The third field in this card represents the number of intermediate stations within each element at which it is desired to compute displacements and stresses. The default number is 5 and maximum is 9. The fourth field states the number of modes to be considered in the case of time history analysis by modal superposition (analysis type 2) or response spectrum analysis (analysis type 3) and the sixth field indicates if it is necessary to print out the input time history data for each time step or the input response spectrum data. The seventh field to the sixteenth field indicates the nodal points for detailed output and the corresponding circumferential angle step for each node.

It should be noted that further output control information is specified on the 'Loading Control Card', which is described later.

The format of Output Control Card will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	<u>Number of circumferential locations</u> where displacements, stresses, etc. are desired (Maximum = 16)
6 - 10	I5	Number of intermediate printout code numbers specified (Maximum = 8)
11 - 15	I5	<u>Number of intermediate stations per element</u> where displacements, stresses, etc. are desired (Maximum = 9; Default = 5)
16 - 20	I5	1, if eigenvalues and eigenvectors are not to be output; otherwise leave blank
21 - 25	I5	<u>Number of modes</u> to be considered in analysis types 2 and 3

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
26 - 30	I5	1, if <u>input</u> time history, or response spectrum data are to be <u>printed out</u> ; otherwise leave blank
31 - 32	I2	Node number at which detailed output is required (1st detailed node)
33 - 40	F8	Angle step for this node
41 - 42	I2	Node number at which detailed output is required (2nd detailed node)
43 - 50	F8	Angle step for this node
.		
.		
.		
71 - 72	I2	Node number at which detailed output is required (5th detailed node)
73 - 80	F8	Angle step for this node

Note: Maximum number of nodes for detailed output is five.

The "Output Control Card" should be followed by "the nodal thickness card" if columns 31 → 80 are not blank.

The format of the nodal thickness card shall be as follows: (5F10.0)

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	Thickness of 1st detailed node
11 - 20	F10	Thickness of 2nd detailed node
21 - 30	F10	Thickness of 3rd detailed node
31 - 40	F10	Thickness of 4th detailed node
41 - 50	F10	Thickness of 5th detailed node
51 - 60	F10	Last circumferential location at which detailed output is required (default = 180°)

If the first field of the output control card is not left blank, the above card (cards) should be followed by the stated number of circumferential location cards, one card for each location stating the

circumferential location (in degrees) where displacements, stresses, etc., are to be output. The maximum number of locations is 16. If, however, the first field of the output control card is left blank, no circumferential location cards are required and results are output for the location = 0° only.

If the second field in the output control card is not blank, the circumferential location cards (if any) should be followed by the stated number of intermediate printout option code cards, one card for each option code. There are eight option code numbers, namely 86 to 93. For a description of intermediate printouts corresponding to these option codes, see Appendix C. If, however, columns 6 - 10 in the output control card are left blank, no intermediate printout option code cards are required.

The format of Circumferential Location Cards shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	Circumferential location in degrees

The format of Intermediate Printout Option Cards shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Intermediate printout option code

b) In the case of time history analysis only, i.e. analysis types 2 and 4, one or more of the following cards are necessary. The first card is called the 'time history output control card'. This card states both the form and the extent of output desired. Any subsequent cards are necessary only if the plotting option is utilized. It is possible to plot the displacement history, the stress resultant history and the acceleration history. The maximum number of components allowed

for each case is ten. The response due to any number of harmonics beginning with the first may be plotted. At the end, the sum total results of all the stated harmonics at $\theta = 0^\circ$ are plotted. Plots can be obtained either on the line printer or off-line using a 760/563 Calcomp Plotter System.

The format of Time History Output Control Card will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	1, if <u>absolute maximum</u> displacements are to be printed; otherwise leave blank.
6 - 10	I5	1, if <u>maximum</u> displacements are to be printed (at given time step interval); otherwise leave blank.
11 - 15	I5	1, if <u>displacements</u> are to be printed at <u>given time step</u> interval; otherwise leave blank.
16 - 20	I5	The <u>number</u> of displacement components for which <u>time histories</u> are to be plotted (Maximum = 10).
21 - 25	I5	1, if <u>absolute maximum</u> stress resultants are to be printed; otherwise leave blank.
26 - 30	I5	1, if <u>maximum stress</u> resultants are to be printed (at given time step interval); otherwise leave blank.
31 - 35	I5	1, if <u>stress resultants</u> and <u>stress components</u> are to be printed at a given time step interval; otherwise leave blank.
36 - 40	I5	The <u>number</u> of stress resultant components for which <u>time histories</u> are to be plotted (Maximum = 10).
41 - 45	I5	1, if absolute maximum <u>relative</u> accelerations are to be printed; otherwise leave blank.
46 - 50	I5	1, if maximum <u>relative</u> accelerations at given time step interval are to be printed; otherwise leave blank.

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
51 - 55	I5	1, if maximum <u>relative</u> acceleration at given time step intervals are to be printed; otherwise leave blank.
56 - 60	I5	The number of <u>total</u> acceleration components for which time histories are to be plotted (Maximum = 10).
61 - 65	I5	1, if maximum <u>total</u> accelerations at given time step intervals are to be printed; otherwise leave blank.

Relative accelerations can be output in the case of base acceleration only and hence should not be specified for other loading cases. Moreover, even in the case of base acceleration, either relative acceleration or total acceleration can be output, not both. If the columns 16-20, 36-40, and 56-60 of the above card are not blank, an additional data card is required for each, placed in the same order. On each of these cards, the global degrees of freedom (in increasing order) corresponding to which plots are desired, shall be stated. The maximum number of degrees of freedom in each case is limited to ten. In the case of displacements and accelerations, the five degrees of freedom at each node correspond to u , v , w , β_ϕ and β_θ respectively. In the case of stress resultants, the five degrees of freedom at each node will refer to N_ϕ , N_θ , M_ϕ , M_θ , and Q_ϕ , respectively, for the purpose of plots only. For instance, for node number N , the global degrees of freedom for these displacements (or stress resultants) will be $(5N - 4)$, $(5N - 3)$, $(5N - 2)$, $(5N - 1)$, $5N$ respectively.

The format of these Degrees of Freedom Cards shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Global degree of freedom
6 - 10	I5	Global degree of freedom
etc.	etc.	etc.

H. Control Data Cards for Dynamic Analysis:

Omit this section in the case of static analysis.

a) The first card in this section, called the 'mass matrix card', defines whether a lumped mass or a consistent mass matrix is to be used. However, the consistent mass matrix is always preferable in the opinion of the authors.

The format of Mass Matrix Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	F5	1.0, if lumped mass matrix is to be used; otherwise leave blank

b) Omit this card if the analysis type is equal to 4. This second card is called the 'eigenvalue analysis card'. Here the eigenvalue P_i is the square of the inverse of the circular frequency ω_i .

The format of Eigenvalue Analysis Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	F5	1.0, if eigenvectors are not desired; otherwise leave blank
6 - 15	F10	Upper limit for range of eigenvalue (P_U)
16 - 25	F10	Lower limit for range of eigenvalue (P_L)
26 - 30	F5	Blank, if <u>all roots</u> over the range P_U, P_L are <u>desired</u> ; otherwise, state the <u>desired number of first roots</u> over the range (P_U, P_L)
31 - 35	F5	Blank if the eigenvector is to be <u>normalized</u> with respect to the <u>largest</u> component; otherwise, state the degree of freedom with respect to which the normalization is to be done

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
36 - 46	E10	Precision of root separation during the isolation of individual roots (Default value = 1.0E-10)
47 - 56	E10	Convergence tolerance factor for the eigenvalues (Default value = 0.001)

Here the eigenvalue, λ , is equal to $1/\omega^2$, where ω is the circular frequency in rad./sec. The assigned value of P_U may be based on a rough estimate [2.], or a suitable guess. In most cases, the value of P_L will be set to zero. The output will be helpful in determining if a proper choice of P_U has been made with respect to the fundamental frequency, as discussed in the next section.

The parameter described as the 'precision of root separation' is required for isolating the desired number of roots to this accuracy, over the given range, by the Sturm sequence procedure.

The parameter described as the 'convergence tolerance factor' is used for checking the accuracy of root convergence during the location of roots by the inverse iteration technique. Thus, at the end of the r^{th} step $|\lambda_r - \lambda_{r-1}|/|\lambda_r|$ will be the measure of convergence, and if this value happens to be less than the convergence tolerance factor (usually set between 0.001 and 0.0001) then λ_r is taken to be equal to λ_{r+1} .

c) Omit this card if the analysis type is equal to 1. This card, the 'dynamic load type card', states one of the following code numbers to describe the nature of loading,

- External dynamic loading (mechanical or thermal) 2
- Horizontal base acceleration 3
- Vertical base acceleration 4

The format of Dynamic Load Type Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Relevant code number for load type

I. Soil Cards:

Omit this section if columns 41 - 45 of the Problem Control Card are left blank.

a) The first card, the 'Soil Control Card', states the number of horizontal soil layers beneath the ring footing, the type of element data generation, the upper limit harmonic number for which the soil formulation is to be applied, the output of the soil analysis, the ring footing data, and, finally, the driving frequency of the soil analysis as well as the damping ratio for such analysis. The format for the 'Soil Control Card' shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Total number of layers. If the total number is 1, elastic half space to be assumed.
6 - 10	I5	Flag for elements data. If 0, data to be generated. If 1, data to be supplied.
11 - 15	I5	*Limit for the number of harmonics for the EBS. If 0, all harmonics to be considered.
16 - 20	I5	Flag for the intermediate soil analysis results output. If 0, no intermediate results to be printed out.
21 - 30	F10.0	Outer radius of the ring footing.
31 - 40	F10.0	Width of the ring footing.
41 - 50	F10.0	*Driving frequency (rad/sec)
51 - 60	F10.0	Soil damping (percentage of critical damping).

b) The 'Soil Control Card' shall be followed by the 'layers information cards' which consist of a card for each layer containing the material properties and thickness of the layer.

*For explanation see Theoretical Manual and the Example Problems.

The format of each card placed in order shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	E10.4	Shear modulus of the layer
11 - 20	E10.4	Poisson's ratio of the layer
21 - 30	E10.4	Weight density of the layer
31 - 40	E10.4	Thickness of the layer

c) If the columns 6 - 10 in the 'Soil Control Card' are not left blank, the 'Soil Elements Data Cards' must be supplied. These cards consist of one control card and material and geometry cards.

The first card in the 'Soil Elements Data Cards' shall contain the number of different materials in the core region. Its format shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Number of different materials in the Finite Element region of the soil model

Note: the total number of different materials input may be greater than the true number of different materials due to the fact that each material has to be considered as a new material for a group of elements in succession (see example number 3 in Appendix G).

The following cards are the material information cards and they consist of a number of cards equal to the number of different materials preceding them. The format is as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	The number of first element for the given material property
6 - 10	I5	The number of last element for the given material property
11 - 20	E10.4	Shear modulus
21 - 30	E10.4	Poisson's ratio
31 - 40	E10.4	Weight density

The following cards are the nodal point cards for the soil elements and they consist of NE cards, where NE equals twice the total number of elements in the FE zone (see Figure 17). The total number of elements must be 4 NL where NL is the total number of layers (if NL=1, for the elastic half space case, the total number of elements shall be 24 elements). For each element there shall be two cards, the first containing the radial coordinates of the eight nodes and the second containing the vertical coordinates of the nodes (see Figure 18).

The format of either card shall be as follows (starting with element number one and finishing with element number NE):

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10.0	Radial (vertical) coordinate of 1st node
11 - 20	F10.0	DO ... for 2nd node
21 - 30	F10.0	DO ... for 3rd node
.		
.		
.		
71 - 80	F10.0	DO ... for 8th node

d) The last group of cards in this section is the 'Fourier Harmonic Numbers Cards', one card for each harmonic. The total number of these data cards group is equal to the total number of harmonics specified in the problem control card. However if columns 11 - 15 in the 'Soil Control Card' are not blank, only the limit number of harmonics must be supplied.

The format of the 'Fourier Harmonic Numbers Cards' are as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Harmonic number

Note: For soil analysis, only harmonics zero and 1 are available.

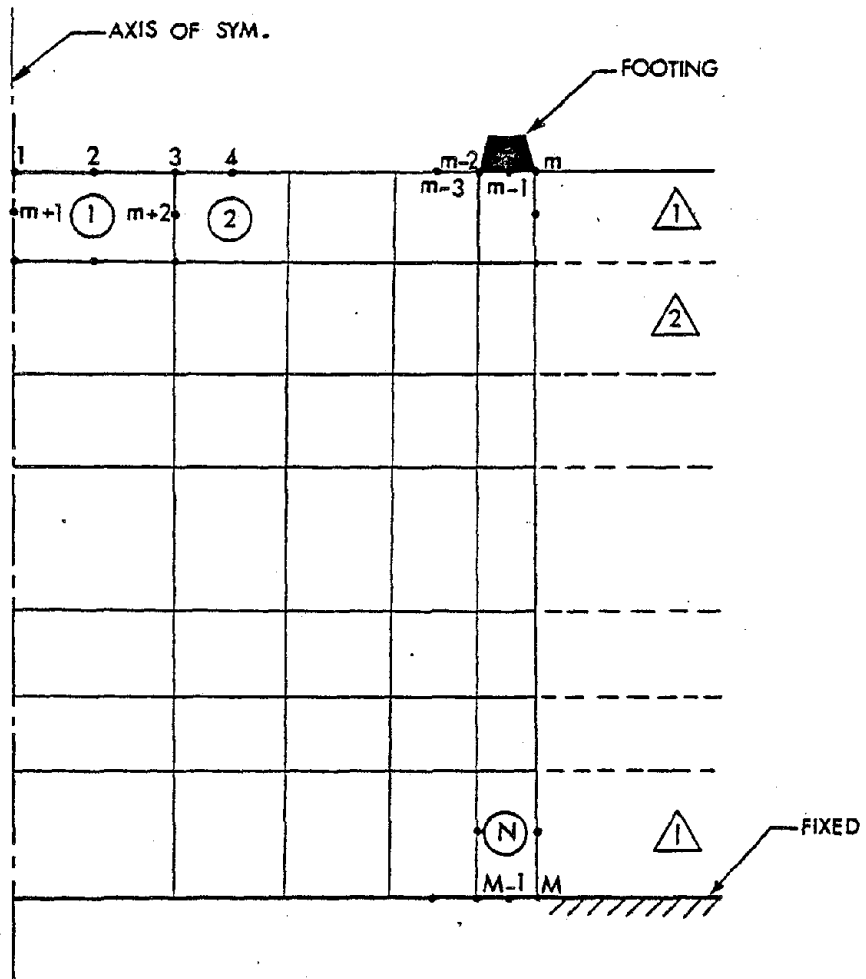


Figure 17: Soil Mesh with the Ring Footing

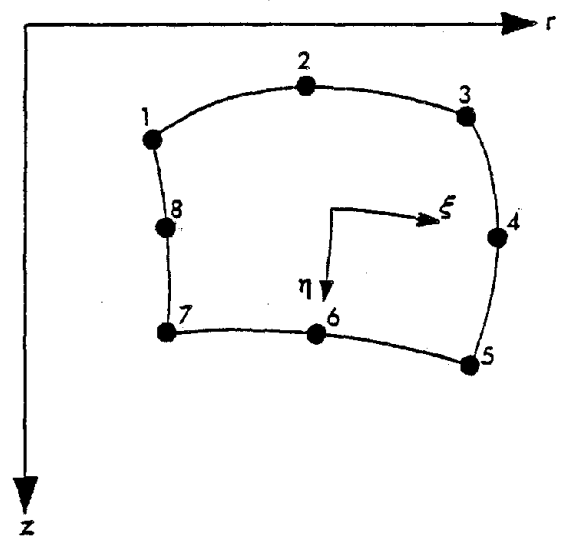


Figure 18. Isoparametric Quadratic Solid Element

J. Loading Information Cards:

For each harmonic, there shall be one title card, the 'load title card', followed by a control card, the 'loading control card'. These cards are further followed by one or all of the following sets of additional cards, excepting in the case of analysis types 1 and 3.

a) Distributed Loading Cards - For each region, extending over one or more elements, one card is required with the distributed loading Fourier coefficients being either constant or linearly varying between the first and last nodal points of that region.

b) Concentrated Loading Cards - One card is required for each nodal point.

c) Thermal Loading Cards - For each temperature region, extending over one or more elements, one card is required with the thermal Fourier coefficients being either constant or linearly varying between the first and last nodal points of that region.

d) Displacement Cards - For each node with specified displacements, one card is required. The maximum permissible number of specified displacement components happens to be ten.

The format for the Load Title Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
2 - 80	A	Any alphanumeric information identifying the load

The information to be supplied on the Loading Control Card are the harmonic numbers (0,1,2,3, etc.), the numbers of distributed loading cards,

concentrated nodal loading cards, thermal loading cards and displacement cards to follow, printout option code (0, or 1, or 2, or 3), gravity loading option, reference temperature (i.e. stress free temperature, and the angle which the line of symmetry for loading makes with the $\theta = 0^\circ$ line).

The gravity loading option (valid for static analysis only) is applicable to harmonics 0 and 1 only. If it is desired to include the effect of gravity loading (or a fraction thereof), it is necessary to set the gravity loading option equal to the weight density of the shell material (or an appropriate fraction thereof). This option applies to the zero harmonic when the gravity loading is in Z-direction and to harmonic number 1 when it is in the R-direction. If static analysis due to gravity loading is not desired, the corresponding columns in the loading control card should be left blank. No distributed loading cards are required for gravity loading.

Printout option codes control the output in the following manner.

<u>Printout Option Code</u>	<u>Description of Printout</u>
0	- No result will be printed for the current harmonic loading case
1	- Printout will be provided for the <u>current</u> harmonic loading only
2	- Printout will be provided for the <u>subtotal</u> results of all harmonic loadings up to and including the current harmonic loading
3	- Printout will be provided for the <u>current</u> harmonic loading and also the <u>subtotal</u> results of all harmonic loadings up to and including the current harmonic loading

Regardless of the printout option codes specified for different harmonic loading cases, a printout of the total results is automatically provided after the final harmonic loading case is completed. The user is cautioned that codes 1 and 2 may produce very voluminous output if several harmonics are present.

The format of Loading Control Card for each harmonic will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Harmonic number
6 - 10	I5	Number of distributed loading cards to follow
11 - 15	I5	Number of concentrated loading cards to follow
16 - 20	I5	Number of thermal loading cards to follow
21 - 25	I5	Number of displacement cards to follow (Maximum = 10)
26 - 30	I5	Print-out option code
31 - 40	F10	Gravity loading option (Blank or weight density)
41 - 50	F10	Reference temperature
51 - 60	F10	Blank, if loading is symmetrical about $\theta = 0^\circ$ 90.0, if loading is symmetrical about $\theta = 90^\circ$

With respect to the data in columns 51-60, it may further be stated that if, for instance, for a particular harmonic the loading in the u-direction and w-direction consists of a cosine term and that in v-direction a sine term, these columns should be left blank. On the other hand, if the case is reversed, i.e., the sine term is associated with the u-direction and w-direction and the cosine term with v-direction, the value in these columns should be 90.0. In the following cards the parenthetical values of θ refer to the latter case.

The format of Distributed Loading Cards for each harmonic will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Nodal point where loading begins
6 - 10	I5	Nodal point where loading ends
11 - 20	F10	Magnitude of distributed loading Fourier coefficient in u-direction at the <u>beginning</u> of nodal point at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
21 - 30	F10	Magnitude of distributed loading Fourier coefficient in u-direction at the <u>end</u> nodal point at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
31 - 40	F10	Magnitude of distributed loading Fourier coefficient in v-direction at the <u>beginning</u> nodal point at $\theta = 90^\circ$ (or $\theta = 0^\circ$)
41 - 50	F10	Magnitude of distributed loading Fourier coefficient in v-direction at the <u>end</u> nodal point at $\theta = 90^\circ$ (or $\theta = 0^\circ$)
51 - 60	F10	Magnitude of distributed loading Fourier coefficient in w-direction at the <u>beginning</u> nodal point at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
61 - 70	F10	Magnitude of distributed loading Fourier coefficient in w-direction at the <u>end</u> nodal point at $\theta = 0^\circ$ (or $\theta = 90^\circ$)

The format of Concentrated Nodal Circle Loading Cards for each harmonic will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Nodal point number
6 - 15	F10	Fourier coefficient of line load in u-direction at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
16 - 25	F10	Fourier coefficient of line load in v-direction at $\theta = 90^\circ$ (or $\theta = 0^\circ$)
26 - 35	F10	Fourier coefficient of line load in w-direction at $\theta = 0^\circ$ (or $\theta = 90^\circ$)

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
36 - 45	F10	Fourier coefficient of line load in β_ϕ direction at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
46 - 55	F10	Fourier coefficient of line load in β_θ direction at $\theta = 90^\circ$ (or $\theta = 0^\circ$)

The format of Thermal Loading Cards for each harmonic shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Nodal point where the thermal load begins
6 - 10	I5	Nodal point where the thermal load ends
11 - 20	F10	Magnitude of the <u>outer</u> surface temperature Fourier coefficient at the <u>beginning</u> node point
21 - 30	F10	Magnitude of the <u>inner</u> surface temperature Fourier coefficient at the <u>beginning</u> node point
31 - 40	F10	Magnitude of the <u>outer</u> surface temperature Fourier coefficient at the <u>end</u> node point
41 - 50	F10	Magnitude of the <u>inner</u> surface temperature Fourier coefficient at the <u>end</u> node point
51 - 60	F10	Coefficient of thermal expansion for the region

The format of Displacement Cards for each harmonic will be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Node number
6 - 15	F10	Fourier coefficients for u-displacement at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
16 - 25	F10	Fourier coefficient for v-displacement at $\theta = 90^\circ$ (or $\theta = 0^\circ$)
26 - 35	F10	Fourier coefficient for w-displacement at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
36 - 45	F10	Fourier coefficient for β_ϕ -rotation at $\theta = 0^\circ$ (or $\theta = 90^\circ$)
46 - 55	F10	Fourier coefficient for β_θ -rotation at $\theta = 90^\circ$ (or $\theta = 0^\circ$)

K. Control Cards for Dynamic Analysis by Direct Integration:

Omit this section if the analysis type is not equal to 4. In most cases, it is very helpful to run a free vibration analysis (Type 1) before carrying out a Type 4 analysis. The following cards are needed under this group.

a) The first card, called the 'direct integration scheme card', states the integration scheme to be used and the values of control parameters for the scheme. The details of three alternative schemes are given in Appendix D.

The code numbers for integration schemes,

Single step higher derivation scheme	1
Modified single step higher derivation scheme	2
Four-step scheme	3

The format of Direct Integration Scheme Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Code number for integration scheme (1, or 2, or 3) Default scheme: 2
6 - 13	F8	*Value parameter P1
14 - 21	F8	Value parameter P2
22 - 29	F8	Value parameter P3
30 - 37	F8	Value parameter P4
38 - 45	F8	Value parameter P5
46 - 53	F8	Value parameter P6
54 - 61	F8	Value parameter P7
62 - 69	F8	Value parameter P8

* For values of parameters P1 to P8 see Appendix D. In the case of the default scheme (Code number = 2), P1 is taken as 1.4.

b) The next card is called the 'data card for dynamic analysis'. It contains the coefficients (C_0 and C_1) of the proportional damping matrix - $[C_0M+C_1K]$, scalar multiplier for time function, total number of records in the input time function, number of different time step lengths to be used, number of time steps between the printing and the plotting of stress resultants, displacements, etc., number of harmonics for which the time histories are to be plotted, the initial and final step numbers for plotting histories, the plotting device to be used (line printer or the Calcomp plotter) or the punched output cards, printer plot spacing, and code number which signifies whether or not the analysis is being carried out for base acceleration.

The format of Data Card for Dynamic Analysis shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	Coefficient ' C_0 ' of proportional damping matrix
11 - 20	F10	Coefficient ' C_1 ' of proportional damping matrix
21 - 30	F10	Scalar multiplier for the input time function
31 - 35	I5	Total number of record prints in the input time function
36 - 40	I5	Number of different time step lengths to be used (Maximum = 4)
41 - 45	I5	Number of time steps between the printing of maximum displacements, stress resultants, etc.
46 - 50	I5	Number of time steps between the printing and plotting of displacements, stress resultants, stress components, etc.

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
51 - 55	I5	Number of harmonics for which the histories are to be plotted
56 - 60	I5	The initial time step for plotting the histories
61 - 65	I5	The final time step for plotting the histories
66 - 70	I5	Code number of plotter (= 1, printer plot; = 2, Calcomp plot). If blank punched output will be generated.
71 - 75	I5	Printer plot spacing (Default = 1)
76 - 80	I5	1, loading is due to base acceleration; otherwise leave blank

The above card may be repeated for each harmonic provided that the column 35 in the problem control card is not left blank.

Note: The punched cards option can be very useful as the plotting sub-routines are installation dependent (see the overlay structure of Figure 4) and by choosing such option one may use the punched time history with the appropriate local plotting routine available. A Plotting Program suitable for a 760/563 Calcomp Plotter system is given in Appendix F (Program 'THPLOT').

c) The next card, called the 'initial condition control card', specifies whether the nodal displacements, or velocities or accelerations are non-zero at the starting time.

The format of Initial Condition Control Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	1, if the initial nodal displacements are non-zero; otherwise leave blank
6 - 10	I5	1, if the initial nodal velocities are non-zero; otherwise leave blank
11 - 15	I5	1, if the initial nodal accelerations are non-zero; otherwise leave blank

In the above card only one of the three responses may be taken as non-zero. In the case of earthquake loading it is preferable to use a blank card.

d) The following card, called the 'time step control card', gives the time step length and the record point number of the input time function up to which it is to be used. A maximum number of five time steps can be specified, provided that all time steps are integral multiples of the smallest time step specified. In order to reduce the effect of artificial damping introduced by the integration scheme it is desirable that the maximum time step length used be not more than, say, one-tenth the smallest natural period.

The format of Time Step Control Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	First time step
11 - 15	I5	The last record point number, for this step, in the input function
16 - 25	F10	Second time step
26 - 30	I5	The last record point number, for this step, in the input function
etc.	etc.	etc.

Note: In the case of a fixed foundation, one should be especially careful in choosing the time steps for the numerical integration. Relatively large time steps may cause numerical instability and the solution may diverge instead of converging (see Example number 8).

e) The next card is the 'title card for input time function'. This card is required only if the column 30 of the Output Control Card is not left blank.

The format of Title Card for Input Time Function shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
2 - 72	A	Alphanumeric information about the load function

f) The next card (or cards), called the 'record points card', provides the record points of the time history input function, which may be the multiplication factors for any mechanical or thermal loading or the base acceleration (vertical or horizontal) due to earthquake. In the case of multiple loading, the multiplication factors should reflect the net effect of all the components. It is necessary to provide this card (or cards) for each harmonic only if these factors are different for different harmonics and column 40 or Problem Control Card is not left blank.

The format of Record Points Card(s) shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	Time
11 - 20	F10	Value
21 - 30	F10	Time
31 - 40	F10	Value
etc.	etc.	etc.

There can be a maximum of four pairs of data per card.

L. Data Cards for Dynamic Analysis by Response Spectrum

Omit this section if the analysis type is not equal to 3.

a) The first card is the 'modal damping ratio card' and gives the damping ratio to be used for each mode.

The format of Modal Damping Ratio Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Mode number
6 - 15	F10	Damping ratio
16 - 20	I5	Mode number
21 - 30	F10	Damping ratio
etc.	etc.	etc.

Each damping ratio card may contain a maximum of five pairs of data. The maximum number of modes that can be considered has been limited to ten.

b) The next card, to be called as the 'response spectrum data title card', is needed only if the column 30 of output control card is not left blank.

The format of Response Spectrum Data Title Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
2 - 72	A	Any alphanumeric information describing the response spectrum data

c) The next card, called the 'control card for response spectrum', gives information such as the number of damping ratios for which the data is given, the number of record points for each damping ratio, whether the data consists of spectral velocity or spectral acceleration, and the scale factor for the spectral values.

The format of Control Card for Response Spectrum shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 5	I5	Number of damping ratios for which response spectrum data is given (Maximum = 8)
6 - 10	I5	Number of record points for each damping ratio
11 - 15	I5	1, if spectral velocity is given 2, if spectral acceleration is given
16 - 25	F10	Scale factor for spectral values (Default = 1)

d) The next card gives the values of damping ratios corresponding to which the response spectrum data is given.

The format of Spectral Damping Ratio Cards shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	Damping ratio no. 1
11 - 20	F10	Damping ratio no. 2
21 - 30	F10	Damping ratio no. 3
31 - 40	F10	Damping ratio no. 4
etc.	etc.	etc.

e) The following cards, called the 'spectral value cards', give the frequencies (starting from the highest and going towards the lowest) and the corresponding spectral values corresponding to each damping ratio at a time starting with damping ratio no. 1.

The format of Spectral Value Cards shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 10	F10	Frequency (Hz)
11 - 20	F10	Spectral value
21 - 30	F10	Frequency (Hz)
31 - 40	F10	Spectral value
etc.	etc.	etc.

There can be a maximum of four pairs of data per card.

M. Last Control Card:

This card signals termination of job and is the last data card to be provided in a job. Preceding this card the data cards for any number of problems can be placed.

The format of Last Control Card shall be as follows:

<u>Columns</u>	<u>Format</u>	<u>Entry</u>
1 - 6	A	JØBEND

SHORE-IV PROGRAM OUTPUT

Typical examples of actual output will be supplied on request.

Printouts for Input Data:

All input data for each problem are printed out in suitable formats in order that the user can verify them easily. Moreover, as mentioned earlier, an echo (i.e. card image) of the input data can also be printed out.

Various checks are performed by the program internally and obvious errors cause a run to terminate, after printing out the error messages, before execution of the problem takes place.

Printouts for Results:

In the case of Static Analysis, the results of computations printed out by the program are comprised of the following:

- 1) For each harmonic number, a table of nodal point displacement components (u , v , w , β_ϕ and β_θ , for sign convention see Figure 2) is printed out together with Z-location of all nodal points.
- 2) Depending on the printout option code specified in the Loading Control Card for a harmonic loading case, tables of displacements, stress resultants (N_ϕ , N_θ , $N_{\phi\theta}$, M_ϕ , M_θ , Q_ϕ , and Q_θ , for sign convention see Figure 1), and stress components (σ_ϕ , σ_θ , $\sigma_{\phi\theta}$, σ_1 , σ_2 and σ_{eq}) are printed out for each element at the nodal points and at the specified number of intermediate points. The stresses SIGMA(1) (i.e., σ_1) and SIGMA(2) (i.e., σ_2) are the principle stresses, while SIGMA(EQUI) (i.e., σ_{eq}) is an equivalent stress, corresponding to von Mises-yield criterion, defined by

the expression

$$\sigma_{eq} = \sqrt{(\sigma_1)^2 - \sigma_1 \sigma_2 + (\sigma_2)^2}$$

The meridional length of the element and its thickness at the top and bottom nodes are also given. The non-dimensionalized arc length variable s is used to define locations along each meridian curve and this variable ranges from 0 at the top to 1 at the bottom node of an element. In the case of open-type elements the displacements and forces F_x , F_y , F_z , M_x , M_y , and M_z (for sign convention see Figure 3) at the end of the members are printed out. Moreover, the length(s) of the member(s) is also printed out.

In the case of Free Vibration Analysis, the results printed out by the program may include the following:

- 1) A table giving information on isolated roots is printed out. If in the first line of this table, the number of roots with values less than the upper limit for P (i.e. P_U) happens to be less than the order of stiffness or mass matrix, it signifies that number of lowest frequencies equal to the difference has been missed. In that case, the problem should be rerun after suitably increasing the value of P_U .
- 2) The eigenvalues, circular frequencies (rad/sec), and cyclic frequency (Hz) for all the desired modes in each harmonic are printed out.
- 3) Depending upon the printout option used in the Eigenvalue Analysis Card, the normalized eigenvectors corresponding to each mode are printed out.

In the case of Response Spectrum Analysis, the results of computation printed out by the program may comprise of the following:

- 1) Results same as those for the Free Vibration Analysis are printed out.
- 2) Tables of maximum displacements, accelerations, and stress resultants for each mode are printed out.
- 3) Tables of root mean square values of maximum displacements, accelerations, and stress resultants for each mode are printed out.

In the case of Time History Analysis by Direct Integration, the results of computation printed out by the program may include the following:

1) Depending upon the printout option codes specified in the Time History Output Control Card, tables of absolute maximum displacements, and/or accelerations (relative or total), and/or stress resultants are printed out for each harmonic. The time instants at which such maxima occur are also printed out in the same table. If, however, the bottom element of the shell happens to be open-type, the stress resultants at the base of the shell are printed out as zero, irrespective of the support conditions of the members of the open type element. Again, if the maximum responses at a given number of time step intervals are desired, similar tables with the time range number stated therein are printed out.

2) If the relevant option code is specified in the Time History Control Card (and Loading Control Card), stress resultants, stress components, and displacements at a specified number of time step intervals in the form

of tables similar to those described for static analysis are printed out. The instant of time for which the values are valid are also printed out.

Plots for Time History Response:

Two alternative kinds of plots are possible, namely the printer plot and the Calcomp plot. The plots can be for the displacement Fourier coefficient history, and/or the total acceleration Fourier coefficient history, and/or the stress resultant Fourier coefficient history. Before actual plotting is done the components to be plotted, their maximum values, and the instant of time when the maxima occurs are tabulated. The components are defined by the node and the component numbers 1, 2, 3, 4, and 5, which, respectively, stand for the components along u , v , w , β_ϕ , and β_θ in the case of displacements and accelerations, and N_ϕ , N_θ , M_ϕ , M_θ , and Q_ϕ in the case of stress resultants. First the plotting is done for the individual harmonics and is then followed at the end by resultant plots for all the specified harmonics corresponding to the reference position $\theta = 0^\circ$.

When the Calcomp plot option is specified, a table of plotted values, for each harmonic only are printed out. In this case the program actually creates a plot tape to finally carry out off-line plotting on a 760/563 Calcomp plotter system.

Time Log Printout:

At the end of each problem a time-log for the problem is printed out. The time-log states the CPU time in seconds required to execute different sections of a problem.

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APPENDIX A

SYSTEM CONTROL CARDS

The image of the system control cards used to run SHORE-IV from permanent file (WU650E.SHORSS) at Washington University Computing Facility using an IBM 360/65 computer is shown below. If, in the case of time history analysis by direct integration, the printer plot option is used, the last four devices are to be replaced by three scratch tapes (Nos. 26, 27, 28 with data set names NTAP8, NTAP9, NTAP10, respectively).

```

//A EXEC FORTRAN.LIBRARY='WU650E.SHORSS',PROGRAM=SHORSS
//GO.FT09F001 DD UNIT=SYSDA,SPACE=(TRK,(25,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=84,BLKSIZE=84,8UFNO=1)
//GO.FT10F001 DD UNIT=SYSDA,SPACE=(72,(5,1)),DCB=(RECFM=VBS,LRECL=68,
//              BLKSIZE=72,BUFNO=3)
//GO.FT11F001 DD UNIT=SYSDA,SPACE=(TRK,(1,1)),DCB=(RECFM=VBS,LRECL=112,
//              BLKSIZE=788,BUFNO=1)
//GO.FT12F001 DD UNIT=SYSDA,SPACE=(TRK,(15,5)),DCB=(RECFM=VBS,
//              LRECL=1148,BLKSIZE=1152,BUFNO=1)
//GO.FT13F001 DD UNIT=SYSDA,SPACE=(TRK,(18,1)),DCB=(RECFM=VBS,
//              LRECL=3700,BLKSIZE=3704,BUFNO=1)
//GO.FT14F001 DD UNIT=SYSDA,SPACE=(TRK,(6,1)),DCB=(RECFM=VBS,
//              LRECL=5204,BLKSIZE=5208,BUFNO=1)
//GO.FT15F001 DD UNIT=SYSDA,SPACE=(TRK,(10,7),RLSE),DCB=(RECFM=VBS,
//              LRECL=524,BLKSIZE=528,BUFNO=1)
//GO.FT16F001 DD UNIT=SYSDA,SPACE=(TRK,(5,1),RLSE),DCB=(RECFM=VBS,
//              LRECL=1604,BLKSIZE=1608,BUFNO=1)
//GO.FT17F001 DD UNIT=SYSDA,SPACE=(TRK,(5,2)),DCB=(RECFM=VBS,
//              LRECL=1156,BLKSIZE=1160,BUFNO=1)
//GO.FT18F001 DD UNIT=SYSDA,SPACE=(TRK,(10,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=1148,BLKSIZE=1152,BUFNO=1)
//GO.FT19F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=164,BLKSIZE=168,BUFNO=1)
//GO.FT20F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=324,BLKSIZE=328,BUFNO=1)
//GO.FT21F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=324,BLKSIZE=328,BUFNO=1)
//GO.FT22F001 DD UNIT=SYSDA,SPACE=(TRK,(1,1),RLSE),DCB=(RECFM=VBS,
//              LRECL=84,BLKSIZE=84,8UFNO=1)
//GO.FT23F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=324,BLKSIZE=328,BUFNO=1)
//GO.FT24F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=324,BLKSIZE=328,BUFNO=1)
//GO.FT25F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=324,BLKSIZE=328,BUFNO=1)
//GO.FT26F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=804,BLKSIZE=808,BUFNO=1)
//GO.FT27F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=804,BLKSIZE=808,BUFNO=1)
//GO.FT28F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5),RLSE),DCB=(RECFM=VBS,
//              LRECL=804,BLKSIZE=808,BUFNO=1)
//GO.PLOTTAPE DD DSN=PL0TTAPE,UNIT=TAPE98,VOL=SER=420561,DCB=DEN=2,
//             DISP=(NEW,KEEP),LABEL=(1,SL)
//GO.SYSIN DD *

```


APPENDIX B

ELEMENTS OF CONSTITUTIVE MATRIX

The elements of constitutive matrix [C] for isotropic and single-layer orthotropic shells are given below.

	Isotropic	Single-layer Orthotropic
C_{11}	$A_i h$	$A_o h$
C_{12}	$\mu A_i h$	$\mu_{\phi\theta} A_o h$
C_{22}	$A_i h$	$\frac{\mu_{\phi\theta}}{\mu_{\theta\phi}} A_o h$
C_{33}	$2(1-\mu)A_i h$	$4 G_{\phi\theta} h$
C_{44}	$A_i h^3/12$	$A_o h^3/12$
C_{45}	$\mu A_i h^3/12$	$\mu_{\phi\theta} A_o h^3/12$
C_{55}	$A_i h^3/12$	$\frac{\mu_{\phi\theta}}{\mu_{\theta\phi}} A_o h^3/12$
C_{66}	$(1-\mu)A_i h^3/6$	$G_{\phi\theta} h^3/3$
C_{77}	$\lambda(1-\mu)A_i h/2$	$\lambda G_{\phi n} h$
C_{88}	$\lambda(1-\mu)A_i h/2$	$\lambda G_{\theta n} h$

$$A_i = E/(1-\mu^2); h = \text{total shell thickness}$$

λ = shear factor (to suppress transverse shearing strains, i.e. Kirchhoff hypothesis, set $\lambda=100$)

$$A_o = E_{\phi} / (1 - \mu_{\theta\phi} \cdot \mu_{\phi\theta})$$

E_{ϕ} = Young's modulus in ϕ -direction

$\mu_{\phi\theta}, \mu_{\theta\phi}$ = Poisson's ratios for ϕ or θ directions with respect to θ or ϕ directions

$G_{\phi\theta}, G_{\phi n}, G_{\theta n}$ = the shear moduli for ϕ - θ , ϕ - n , and θ - n planes

The elements of constitutive matrix [C] for multi-layer orthotropic and framed shells are given below.

	Multi-layer Orthotropic	Framed
C ₁₁	$\sum_h (A_o)_k h_k$	$\frac{EA_\phi}{d_\phi}$
C ₁₂	$\sum_h (\mu_{\phi\theta} A_o)_k h_k$	0
C ₂₂	$\sum_h \left(\frac{\mu_{\phi\theta}}{\mu_{\theta\phi}} A_o\right)_k h_k$	$\frac{EA_\theta}{d_\theta}$
C ₃₃	$4\sum_h (G_{\phi\theta})_k h_k$	$2G \left(\frac{A_\theta}{d_\phi} + \frac{A_\phi}{d_\theta}\right)$
C ₄₄	$\sum_h (A_o)_k \left(\zeta_k^2 + \frac{h_k^2}{12}\right) h_k$	$\frac{EI_\phi}{d_\phi}$
C ₄₅	$\sum_h (\mu_{\phi\theta} A_o)_k \left(\zeta_k^2 + \frac{h_k^2}{12}\right) h_k$	0
C ₅₅	$\sum_h \left(\frac{\mu_{\phi\theta}}{\mu_{\theta\phi}} A_o\right)_k \left(\zeta_k^2 + \frac{h_k^2}{12}\right) h_k$	$\frac{EI_\theta}{d_\theta}$
C ₆₆	$4\sum_h (G_{\phi\theta})_k \left(\zeta_k^2 + \frac{h_k^2}{12}\right) h_k$	$\frac{G}{4} \left(\frac{K_\phi}{d_\phi} + \frac{K_\theta}{d_\theta}\right)$
C ₇₇	$\lambda \sum_h (G_{\phi n})_k h_k$	$\lambda \frac{GA_\phi}{d_\phi}$
C ₈₈	$\lambda \sum_h (G_{\theta n})_k h_k$	$\lambda \frac{GA_\theta}{d_\theta}$

- $(A_o)_k$, etc. = the value of A_o , etc for layer 'k'
- h_k = thickness of layer 'k'
- ζ_k = the distance from the centroid of layer k to middle surface, assuming symmetrical layers (see Fig. 5)
- A_ϕ, A_θ = the area of members of frame in ϕ, θ directions
- K_ϕ, K_θ = St. Venant torsional constants in ϕ, θ directions
- d_ϕ, d_θ = the spacing of meridional and circumferential members
- E, G = Young's and Shear moduli

APPENDIX C

INTERMEDIATE PRINTOUT OPTION CODES

<u>Code No.</u>	<u>Ref. Subroutines</u>	<u>Description of Intermediate Printout</u>
86	GLMX	Upper diagonal part of structure stiffness and mass matrices in banded form as arrays GS (1 to N) and GM (1 to N). Where, N = 6x (no. of nodes) for j = 0 and 10x (No. of nodes) for j > 0. The elements of these matrices need to be multiplied by 2 π for j = 0, and by π for j > 0.
87	FINA FMINA ESOT EMOT	Upper diagonal part of improved element stiffness and mass matrices as arrays A and AM or BM. The elements of all these matrices need to be multiplied by 2 π for j = 0, and by π for j > 0. Moreover, the elements of A need to be multiplied by a factor 10 ⁶ .
88	GLMX	Structure loading vector as RF (1 to M) array. Where M = 5 x (No. of nodes) for j > 0, and 3 x (No. of nodes) for j = 0. The elements of all these vectors should be multiplied by 2 π for j = 0, and by π for j > 0.
89	CALQF	Element nodal loading vector as QF (1 to 6) and QFC (1 to 10) arrays corresponding to harmonic numbers j = 0, and j > 0, respectively.
90	ARC	Local of nodes and three intermediate points on the meridian of each element with respect to the global coordinate system.
91	TEMP	Temperature loading data for an element. Printout is in the form of an array TX (1 to 30)*.
92	LASH	Section properties of multi-layer orthotropic material as AS (1 to 7)** array.
93	SST MST	Coefficients of Direct Integration Scheme.

* TX(1) to TX(5) and TX(16) to TX(19) = Temperature data

TX(6), TX(12), TX(14) = Elastic and thermal properties

TX(7) to TX(11) and TX(15)	= Geometric data
TX(20) to TX(30)	= Thermal loading data. For further details see subroutine TEMP.
** AS(1)	= Overall thickness
AS(2)	= Equivalent area per unit length of middle surface in terms of outer layer (ϕ - direction)
AS(3)	= - do - (θ -direction)
AS(4)	= - do - ($\phi\theta$ -direction, for shear stress calculation due to $N_{\phi\theta}$)
AS(5)	= Equivalent moment of inertia per unit length of middle surface in terms of outer layer (ϕ -direction)
AS(6)	= - do - (θ -direction)
AS(7)	= - do - ($\phi\theta$ -direction, for shear stress calculation due to $M_{\phi\theta}$)

APPENDIX D

DIRECT INTEGRATION SCHEMES

1. Single Step Higher Derivative Schemes [4]:

After the stiffness matrix [K] and mass matrix [M] has been formed,

it is necessary to evaluate the following constants.

$$A_1 = \left(\frac{1}{P_1 \Delta t^2} + \frac{P_2 P_3 C_0}{P_1 \Delta t} \right) / \left(P_3^2 + \frac{P_2 P_3 C_1}{P_1 \Delta t} \right)$$

$$A_2 = P_3^2 + \frac{P_2 P_3 C_1}{P_1 \Delta t}$$

$$A_3 = \frac{1}{P_1 \Delta t^2} + \frac{P_2 P_3 (C_0 - A_1 C_1)}{P_1 \Delta t} + \left(\frac{1}{P_3} - P_3^2 \right) A_1$$

$$A_4 = \frac{1}{P_1 \Delta t} + \left(\frac{P_2 P_3}{P_1} - \frac{1}{P_3} \right) (C_1 - A_1 C_1) + (1 - P_3^2) A_1 \Delta t$$

$$A_5 = \left(\frac{1}{2P_1} - 1 \right) + \left(\frac{P_2 P_3}{2P_1} - 1 \right) (C_0 - A_1 C_1) \Delta t + \\ \left(P_3 - P_3^2 \right) A_1 \frac{\Delta t^2}{2} + \left(\frac{P_3 - 1}{P_3} \right)$$

$$A_6 = \left(A_2 - \frac{1}{P_3} \right)$$

$$A_7 = \left(A_2 - 1 \right) \Delta t - \frac{C_1}{P_3}$$

$$A_8 = \left(\frac{P_2 P_3}{2P_1} - 1 \right) C_1 \Delta t + \frac{P_3 (P_3 - 1)}{2} \Delta t^2$$

$$A_9 = \frac{1}{P_1 \Delta t^2}$$

$$A_{10} = \frac{1}{P_1 \Delta t}$$

$$A_{11} = \left(\frac{1}{2P_1} - 1 \right)$$

$$A_{12} = \frac{P_2}{P_1 \Delta t}$$

$$A_{13} = \left(\frac{P_2}{P_1} - 1 \right)$$

$$A_{14} = \left(\frac{P_2}{2P_1} - 1 \right) \Delta t$$

where

P_1, P_2, P_3 = the parameters of integration scheme

Δt = the time step

C_0, C_1 = the coefficients of Rayleigh damping matrix

Next, it is necessary to form the modified stiffness matrix. It is followed by forward elimination.

$$[\tilde{K}] = [K] + A_1 [M]$$

Then, the modified load vector is formed as below.

$$\begin{aligned} \{\tilde{F}(t+\Delta t)\} &= \{F(t+\Delta t)\} + \frac{1-P_3}{P_3} \{F(t)\} \\ &+ [M] (A_3 \{\Delta(t)\} + A_4 \{\dot{\Delta}(t)\} + A_5 \{\ddot{\Delta}(t)\}) \end{aligned}$$

Finally, the displacements $\{\bar{\Delta}(t+\Delta t)\}$ are calculated by back substitution.

Thus,

$$\{\bar{\Delta}(t+\Delta t)\} = [\bar{K}]^{-1} \{\bar{F}(t+\Delta t)\}$$

and the true displacement vector at time 't+Δt' will then be

$$\{\Delta(t+\Delta t)\} = (\{\bar{\Delta}(t+\Delta t)\} + A_6\{\Delta(t)\} + A_7\{\dot{\Delta}(t)\} + A_8\{\ddot{\Delta}(t)\}) / A_2$$

the corresponding velocity and acceleration vectors are then calculated from

$$\{\dot{\Delta}(t+\Delta t)\} = A_{12}\{\Delta(t+\Delta t)\} - A_{12}\{\Delta(t)\} - A_{13}\{\dot{\Delta}(t)\} - A_{14}\{\ddot{\Delta}(t)\}$$

$$\{\ddot{\Delta}(t+\Delta t)\} = A_9\{\Delta(t+\Delta t)\} - A_9\{\Delta(t)\} - A_{10}\{\dot{\Delta}(t)\} - A_{11}\{\ddot{\Delta}(t)\}$$

In the case of Newmark's β-method, $P_3 = 1$. For Wilson's θ-method, $P_1 = 1/6$ and $P_2 = 1/2$. For an unconditionally stable scheme, in the first case, one should take $P_1 = 1/4$ and $P_2 = 1/2$, and in the later case $P_3 = 1.4$.

2. Multistep (3-step) Schemes [4]:

Here the constants to be evaluated are

$$A_1 = \left(\frac{P_5}{\Delta t} + P_1 C_0 \right) / (\Delta t + C_2 C_1)$$

$$A_2 = \left(1 + \frac{P_1 C_1}{\Delta t} \right)$$

$$A_3 = -\frac{P_6}{\Delta t^2} - \frac{P_2 (C_0 - C_1 A_1)}{\Delta t}$$

$$A_4 = -\frac{P_7}{\Delta t^2} - \frac{P_3 (C_0 - C_1 A_1)}{\Delta t}$$

$$A_5 = -\frac{P_8}{\Delta t^2} - \frac{P_4 (C_0 - C_1 A_1)}{\Delta t}$$

$$A_6 = -\frac{P_2 C_1}{\Delta t}$$

$$A_7 = \frac{P_3 C_1}{\Delta t}$$

$$A_8 = -\frac{P_4 C_1}{\Delta t}$$

where P_1, P_2, \dots are the parameters of integration scheme.

Here the modified stiffness matrix and load vector will be

$$[\tilde{K}] = [K] + A_1 [M]$$

$$\{\tilde{F}(t+\Delta t)\} = \{F(t+\Delta t)\} + [M] (A_3 \{\Delta(t)\} +$$

$$A_4 \{\Delta(t-\Delta t)\} + A_5 \{\Delta(t-2\Delta t)\})$$

As in scheme 1, after solving the following for $\{\bar{\Delta}(t+\Delta t)\}$

$$[\bar{K}] \{\bar{\Delta}(t+\Delta t)\} = \{\bar{F}(t+\Delta t)\}$$

the displacement vector is calculated from

$$\begin{aligned} \{\Delta(t+\Delta t)\} = & \{(\bar{\Delta}(t+\Delta t))\} + A_6\{\Delta(t)\} + A_7\{\Delta(t-\Delta t)\} \\ & + A_8\{\Delta(t-2\Delta t)\} / A_2 \end{aligned}$$

In the case of first time step, i.e. at $t=\Delta t$, it is necessary to modify some of the aforementioned constants as below.

$$\bar{C}_1 = (P_3 + 8P_4)G_1$$

$$\bar{C}_2 = (P_7 + 8P_8) + (P_3 + 8P_4)C_0$$

$$\bar{C}_3 = (\bar{C}_2 - A_1\bar{C}_1) / (A_2 - \bar{C}_1)$$

$$A_1^* = (A_1A_2 - \bar{C}_2) / (A_2 - \bar{C}_1)$$

$$A_3^* = (A_1A_2 - \bar{C}_2) / (A_2 - \bar{C}_1)$$

$$A_4^* = (A_3 - P_2\bar{C}_3)$$

$$A_5^* = (A_5 - P_4\bar{C}_3)$$

For this step, A_1 , A_3 , A_4 , and A_5 are to be replaced by A_1^* , A_3^* , A_4^* , and A_5^* . Moreover, it is necessary to evaluate the following

$$\{\Delta(-\Delta t)\} = \{\Delta(0)\} - \Delta t \{\dot{\Delta}(0)\} + \Delta t^2 \{\ddot{\Delta}(0)\}/2.0$$

$$\{\Delta(-2\Delta t)\} = -\{\Delta(0)\} + 2\{\Delta(-\Delta t)\} + \Delta t^2 \{\ddot{\Delta}(0)\}$$

In the case of Houbolt's scheme

$$P_1 = 11/6; P_2 = -3.0; P_3 = 1.5; P_4 = -1/3;$$

$$P_5 = 2.0; P_6 = -5.0; P_7 = 4.0; P_8 = -1.0$$

APPENDIX E

LISTING OF 'FORHAM' (Program for calculating Fourier Coefficients)

```

C *****MAIN0001
C                                     MAIN0002
C THIS IS THE MAIN CONTROL PROGRAM FOR CALCULATING FOURIER COEFS FORMAIN0003
C AN ARIBTRARY LOAD FUNCTION DISTRIBUTED AROUND THE CIRCUMFERENCE MAIN0004
C FOR TWO SITUATIONS MAIN0005
C   1) WHEN THE ORDINATES ARE GIVEN AT EQUAL INTERVALS MAIN0006
C     AND THE NUMBER OF ORDINATES ARE EQUAL TO THE MAIN0007
C     NUMBER OF FOURIER COEFFICIENTS (IFL=0) MAIN0008
C   2) WHEN EITHER THE NUMBER OF ORDINATES ARE DIFFERENTMAIN0009
C     FROM THE DESIRED NUMBER OF FOURIER COEFFICIENTS MAIN0010
C     OR THE INTERVALS BETWEEN THE ORDINATES ARE NOT MAIN0011
C     EQUAL OR BOTH (IFL>0) MAIN0012
C                                     MAIN0013
C PROGRAMMED BY: MAIN0014
C                                     PRODYOT KUMAR BASU MAIN0015
C                                     WASHINGTON UNIVERSITY MAIN0016
C                                     AUGUST, 1976 MAIN0017
C                                     MAIN0018
C *****MAIN0019
C IMPLICIT REAL*8 (A-H,P-Z) MAIN0020
C DIMENSION VAL(50),ARG(50),X(50),Y(50),H(450),A(25),B(25),P(50), MAIN0021
C X HEO(20),C(200,25),T(200),D(25,200) MAIN0022
C                                     MAIN0023
C NI = NUMBER OF DATA SETS FOR WHICH FOURIER COEFFICIENTS ARE MAIN0024
C   REQUIRED MAIN0025
C NDIM = NUMBER OF GIVEN ORDINATES (AT EQUAL INTERVALS OR OTHERWISE)MAIN0026
C , PROVIDED THE LAST ORDINATE FOR NS=0 CORRESPONDS TO THETA=360 MAIN0026
C DEGREES, AND THAT FOR NS>0 TO THETA=180 DEGREES. MAIN0026
C NO = 2 TIMES THE MAXIMUM ORDER OF HARMONICS TO BE MAIN0027
C   FITTED PLUS 1 (I.E. THE NUMBER OF COSINE TERMS MAIN0028
C   AND SINE TERMS TO BE CONSIDERED + 1) MAIN0029
C   IF IFL = 0 : NO < NDIM MAIN0030
C IFL = 0, NO INTERPOLATION USING CUBIC SPLINES MAIN0031
C   > 0, INTERPOLATION NECESSARY, IF THE ORDINATES ARE NOT AT MAIN0032
C   EQUAL INTERVAL AND 'NDIM-1' IS NOT ODD MAIN0033
C NDD = 0, NO PUNCHED OUTPUT OF TIME HISTORY DATA MAIN0034
C   1,PUNCHED OUTPUT OF TIME HISTORY DATA REQUIRED MAIN0035
C IF NDD = 0, PUT NHC = 0, AND NHS = 0 MAIN0036
C IF NDD = 1, NHC = NUMBER OF HARMONICS(COS) TO BE PUNCHED BEGINNINGMAIN0037
C   WITH THE ZEROETH; MAXIMUM VALUE = (NO-1)/2 + 1 MAIN0038
C   NHS = NUMBER OF HARMONICS(SIN) TO BE PUNCHED BEGINNINGMAIN0039
C   WITH THE FIRST; MAXIMUM = (NO-1)/2 MAIN0040
C IDF = 0, DATA IS INPUT TIME STEPWISE MAIN0041
C IDF = 1, DATA IS INPUT POINTWISE MAIN0042
C TII = INITIAL TIME (NEEDED IF NDD=1) MAIN0043
C TS = TIME STEP (NEEDED IF NDD=1) MAIN0044
C   TII=0.0 MAIN0045
C   TS=0.0 MAIN0046
C AMIN = 1.000 MAIN0047

```

	PYE = 4.000*DATAN(AMIN)	MAIN0048
	READ(5,1) HED,NI,NDIM,NO,IFL,NDD,NHC,NHS,IOF	MAIN0049
	1 FORMAT(20A4/8I5)	MAIN0050
	NHS = NHS + 1	MAIN0051
	IF(NDD.EQ.0) GO TO 23	MAIN0052
	READ(5,21) TII,TS	MAIN0053
	21 FORMAT(2F12.6)	MAIN0054
	TI = TII	MAIN0055
	DO 22 I=1,NI	MAIN0056
	T(I) = TI	MAIN0057
	22 TI = TI + TS	MAIN0058
	23 CONTINUE	MAIN0059
	WRITE(6,51) NI,NDIM,NO,IFL,NDD,NHC,NHS,TII,TS	MAIN0060
	51 FORMAT(7I5,2F12.6)	MAIN0061
	IF(IFL.GT.0) GO TO 60	MAIN0062
	NK = NDIM - 1	MAIN0063
C		MAIN0064
	IF(NO.LE.NK) GO TO 60	MAIN0065
C		MAIN0066
	WRITE(6,50)	MAIN0067
	50 FORMAT(1H1//77H*****ERROR IN INPUT DATA*****NUMBER OF FOURIER COEFF	MAIN0068
	XICIENTS SPECIFIED IS WRONG)	MAIN0069
C		MAIN0070
	GO TO 200	MAIN0071
	60 CONTINUE	MAIN0072
	NM = NO/2	MAIN0073
	NM = NO - NM*2	MAIN0074
	IF(NM.EQ.1) GO TO 4	MAIN0075
C		MAIN0076
	WRITE(6,40)	MAIN0077
	40 FORMAT(1H1//69H*****ERROR IN INPUT DATA*****NUMBER OF FOURIER COEFF	MAIN0078
	XICIENTS IS NOT ODD)	MAIN0079
	GO TO 200	MAIN0080
	4 CONTINUE	MAIN0081
	WRITE(6,2) HED	MAIN0082
	2 FORMAT(1H1/20A4//)	MAIN0083
	MO = NO	MAIN0083
	IF(IFL.NE.0.AND.NO.LT.25) MO=25	MAIN0083
	MOL1 = MO - 1	MAIN0083
	NOL1 = MO - 1	MAIN0084
	M = NOL1/2	MAIN0085
	X(1)=0.0	MAIN0086
	Y(1)=0.0	MAIN0087
	PI = 2.0*PYE/NO	MAIN0088
	PIE = 2.0*PYE/MO	MAIN0088
	DO 3 I=1,NOL1	MAIN0089
	3 X(I+1) = X(I) + PI	MAIN0090
	DO 24 I = 1,MOL1	MAIN0091
	24 Y(I+1) = Y(I) + PIE	MAIN0092

	READ(5,5) (ARG(I),I=1,NDIM)	MAIN0093
	5 FORMAT(8F10.6)	MAIN0094
C		MAIN0095
C		MAIN0096
C		MAIN0097
C		MAIN0098
	IF(IDF.EQ.0) GO TO 45	MAIN0099
	DO 49 K = 1,NDIM	MAIN0100
	READ(5,43) (C(L,K),L=1,NI)	MAIN0101
43	FORMAT(8F10.4)	MAIN0102
49	CONTINUE	MAIN0103
45	DO 100 NIP = 1,NI	MAIN0104
	IF(IDF.EQ.1) GO TO 46	MAIN0105
	READ(5,6) (VAL(I),I=1,NDIM)	MAIN0106
6	FORMAT(8F10.4)	MAIN0107
	GO TO 48	MAIN0108
46	DO 47 I=1,NDIM	MAIN0109
47	VAL(I) = C(NIP,I)	MAIN0110
48	WRITE(6,7) NIP	MAIN0111
7	FORMAT(/5X,20HINPUT DATA SET NO.= ,15//5X,16HTHETA IN DEGREES,5X,	MAIN0112
	X13HPRESSURE HEAD//)	MAIN0113
	DO 8 I=1,NDIM	MAIN0114
	ARGO = ARG(I)	MAIN0115
	ARG(I) = ARGO*PYE/180.0	MAIN0116
8	WRITE(6,9) ARGO,VAL(I)	MAIN0117
9	FORMAT(10X,F10.6,6X,E12.5/)	MAIN0118
	IF(IFL.EQ.0) GO TO 18	MAIN0119
	WRITE(6,10) NIP	MAIN0120
10	FORMAT (1H1,5X,66H INTERPOLATED VALUES AS PER CUBIC SPLINE SCHEME	MAIN0121
	X FOR DATA SET NO.=,15//5X,16HTHETA IN DEGREES,5X,26HINTERPOLATED P	MAIN0122
	XRESSURE HEAD//)	MAIN0123
C		MAIN0124
	CALL CUBSPL(VAL,ARG,NDIM,MO,Y,H,IER)	MAIN0125
C		MAIN0126
18	IF(IFL.EQ.0) NN = NDIM - 1	MAIN0127
	IF(IFL.NE.0) NN = MO	MAIN0128
	XD = 0.0	MAIN0129
	DO 11 I = 1,NN	MAIN0130
	IF(IFL.EQ.0) GO TO 25	MAIN0131
	P(I) = Y(I)	MAIN0132
	XD = (I-1)*360.0/MO	MAIN0133
	WRITE(6,12) XD,Y(I)	MAIN0134
	GO TO 11	MAIN0135
25	P(I) = VAL(I)	MAIN0136
11	CONTINUE	MAIN0137
12	FORMAT(10X,F10.6,6X,F10.6/)	MAIN0138
	WRITE(6,13) NIP	MAIN0139
13	FORMAT(1H1,5X,49H VALUES OF FOURIER COEFFICIENTS FOR DATA SET NO.=	MAIN0140
	X,15//5X,16H HARMONIC NUMBER,5X,25HMAIN0141	MAIN0141

```

X COEFFICIENTS FOR SIN(JX)//)
M1=M+1
CALL FOURIE(P,NN,M1,A,8,IER,PYE)
DO 15 J=1,M1
  LL = J - 1
  WRITE(6,14) LL,A(J),8(J)
14 FORMAT(10X,15,20X,F10.6,20X,F10.6)
15 CONTINUE
  IF(NIP.EQ.NI) GO TO 99
  DO 20 K=1,NDIM
20 ARG(K) = ARG(K)*180.0/PYE
  Y(1) = 0.0
  DO 30 I=1,MO
30 Y(I+1) = Y(I) + 2*PYE/MO
99 CONTINUE
  IF(NDD.EQ.0) GO TO 98
  DO 31 K=1,NHC
31 D(NIP,K) = A(K)
  IF(NS.GT.0) GO TO 98
  IF(NIP.EQ.1) GO TO 98
  DO 32 K=2,NHS
  KK = NHC + K - 1
32 D(NIP,KK) = B(K)
98 CONTINUE
100 CONTINUE
  IF(NDD.EQ.0) GO TO 200
  NHS = NHS - 1
  IF(NS.GT.0) NHS=0
  NH = NHC + NHS
  DO 150 K=1,NH
  WRITE(7,110) K
110 FORMAT(1H ,I3)
  IF(TII.LT.0.000001) GO TO 140
  WRITE(7,120) T(1),D(1,K),T(2),D(2,K),T(3),D(3,K)
120 FORMAT(20X,6F10.5)
  WRITE(7,130) (T(J),D(J,K),J=4,NI)
130 FORMAT(4(F10.5,F10.5))
  GO TO 150
140 CONTINUE
  WRITE(7,130) (T(J),D(J,K),J=1,NI)
150 CONTINUE
  WRITE(6,160)
160 FORMAT(1H1/40H ****MEAN VALUES OF FOURIER COEFFICIENTS//10X,
X34HFOURIER COEFF. NO.      MEAN VALUE//)
  DO 180 K=1,NH
  T(K) = 0.0
  DO 175 J=1,NI
  T(K) = T(K) + D(J,K)
  IF(IJ.LT.NI) GO TO 175

```

```

MAIN0142
MAIN0143
MAIN0144
MAIN0145
MAIN0146
MAIN0147
MAIN0148
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MAIN0154
MAIN0155
MAIN0156
MAIN0157
MAIN0158
MAIN0159
MAIN0160
MAIN0161
MAIN0162
MAIN0163
MAIN0164
MAIN0165
MAIN0166
MAIN0167
MAIN0168
MAIN0169
MAIN0170
MAIN0171
MAIN0172
MAIN0173
MAIN0174
MAIN0175
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MAIN0179
MAIN0180
MAIN0181
MAIN0182
MAIN0183
MAIN0184
MAIN0185
MAIN0186
MAIN0187
MAIN0188
MAIN0189

```

```

AFC = T(K)/NI
WRITE(6,170) K,AFC
170 FORMAT(18X,I4,11X,F12.6)
175 CONTINUE
180 CONTINUE
200 STOP

```

```

END
SUBROUTINE FOURIE(P,NO,M1,A,B,IER,PYE)

```

C
C
C
C

THIS SUBROUTINE CALCULATES THE FOURIER COEFFICIENTS

```

IMPLICIT REAL*8 (A-H,P-Z)
DIMENSION A(1),B(1),P(1)

```

C
C

CHECK FOR PARAMETER ERRORS

```

M = M1 - 1
IER=0
20 IF(M) 30,40,40
30 IER=2
RETURN
40 N = (NO - 1)/2
IF(M-N) 60,60,50
50 IER=1
RETURN

```

C
C
C

COMPUTE AND PRESET CONSTANTS

```

60 AN=N
CDEF=2.0/(2.0*AN+1.0)
CONST=PYE*CDEF
S1 = DSIN(CONST)
C1 = DCOS(CONST)
C=1.0
S=0.0
J=1
PZ=P(1)
70 U2=0.0
U1=0.0
I=2*N+1

```

C
C
C

FORM FOURIER COEFFICIENTS RECURSIVELY

```

75 U0=P(I)+2.0*C*U1-U2
U2=U1
U1=U0
I=I-1
IF(I-1) 80,80,75
80 A(J)=CDEF*(PZ+C*U1-U2)

```

MAIN0190
MAIN0191
MAIN0192
MAIN0193
MAIN0194
MAIN0195
MAIN0196
FOUR1001
FOUR1002
FOUR1003
FOUR1004
FOUR1005
FOUR1006
FOUR1007
FOUR1008
FOUR1009
FOUR1010
FOUR1011
FOUR1012
FOUR1013
FOUR1014
FOUR1015
FOUR1016
FOUR1017
FOUR1018
FOUR1019
FOUR1020
FOUR1021
FOUR1022
FOUR1023
FOUR1024
FOUR1025
FOUR1026
FOUR1027
FOUR1028
FOUR1029
FOUR1030
FOUR1031
FOUR1032
FOUR1033
FOUR1034
FOUR1035
FOUR1036
FOUR1037
FOUR1038
FOUR1039
FOUR1040
FOUR1041
FOUR1042

```

      B(J)=COEF*S*U1
      IF(J-(M+1)) 90,100,100
90    Q=C1*C-S1*S
      S=C1*S+S1*C
      C=Q
      J=J+1
      GO TO 70
100  A(1)=A(1)*0.5
      RETURN
      END
      SUBROUTINE CUBSPL(F,X,N,M,G,H,IER)

```

C
C
C
C

THIS SUBROUTINE CARRIES OUT CUBIC SPLINE INTERPOLATION

```

      DIMENSION      F(1),X(1),G(1),H(1)
      DOUBLE PRECISION F,X,H,G,EPSLN,OMEGA,ETA,W,HT1,HT2,PROD,DELSQS
      DATA           EPSLN,OMEGA/1.0-14,1.071796769724490800/

```

C

```

      I2=N
      I3=N+N
      I4=I3+N
      I5=I4+N
      I6=I5+N
      I7=I6+N
      I8=I7+N
      I9=I8+N
      NT=16
      IER = 0
      N1 = N-1

```

C

```

      DO 5 I=1,N1
          J2=I2+I
          H(I)=X(I+1)-X(I)
          H(J2)=(F(I+1)-F(I))/H(I)
5    CONTINUE
      DO 10 I=2,N1
          J2=I2+I
          J3=I3+I
          J4=I4+I
          J5=I5+I
          J6=I6+I
          J7=I7+I
          H(J3)=H(I-1)+H(I)
          H(J4)=.5*H(I-1)/H(J3)
          H(J5)=(H(J2)-H(J2-1))/H(J3)
          H(J6)=H(J5)+H(J5)
          H(J7)=H(J6)+H(J5)
10   CONTINUE

```

FOUR I043
 FOUR I044
 FOUR I045
 FOUR I046
 FOUR I047
 FOUR I048
 FOUR I049
 FOUR I050
 FOUR I051
 FOUR I052
 CUBSP001
 CUBSP002
 CUBSP003
 CUBSP004
 CUBSP005
 CUBSP006
 CUBSP007
 CUBSP008
 CUBSP009
 CUBSP010
 CUBSP011
 CUBSP012
 CUBSP013
 CUBSP014
 CUBSP015
 CUBSP016
 CUBSP017
 CUBSP018
 CUBSP019
 CUBSP020
 CUBSP021
 CUBSP022
 CUBSP023
 CUBSP024
 CUBSP025
 CUBSP026
 CUBSP027
 CUBSP028
 CUBSP029
 CUBSP030
 CUBSP031
 CUBSP032
 CUBSP033
 CUBSP034
 CUBSP035
 CUBSP036
 CUBSP037
 CUBSP038
 CUBSP039

	H(I6+1)=0.	CUBSP040
	J6=I6+N	CUBSP041
	H(J6)=0.	CUBSP042
C	KCOUNT=0	CUBSP043
15	ETA=0.	CUBSP044
	KCOUNT=KCOUNT+1	CUBSP045
	DO 25 I=2,N1	CUBSP046
	J4=I4+I	CUBSP047
	J6=I6+I	CUBSP048
	J7=I7+I	CUBSP049
	W=(H(J7)-H(J4)*H(J6-1)-(.5-H(J4))*H(J6+1)-H(J6))*OMEGA	CUBSP050
	IF(DABS(W).LE.ETA) GO TO 20	CUBSP051
	ETA=DABS(W)	CUBSP052
C	20 H(J6)=H(J6)+W	CUBSP053
	25 CONTINUE	CUBSP054
	IF(KCOUNT.GT.NT) GO TO 75	CUBSP055
	IF (ETA.GE.EPSLN) GO TO 15	CUBSP056
C	DO 30 I=1,N1	CUBSP057
	J6=I6+I	CUBSP058
	J8=I8+I	CUBSP059
	H(J8)=(H(J6+1)-H(J6))/H(I)	CUBSP060
30	CONTINUE	CUBSP061
	DO 65 J=1,M	CUBSP062
	I=1	CUBSP063
	J9=I9+J	CUBSP064
	IF (G(J)-X(I)) 70,60,35	CUBSP065
35	IF (G(J)-X(N)) 45,50,70	CUBSP066
40	IF (G(J)-X(I)) 55,60,45	CUBSP067
45	I=I+1	CUBSP068
	GO TO 40	CUBSP069
50	I=N	CUBSP070
55	I=I-1	CUBSP071
C	60 J6=I6+I	CUBSP072
	J2=I2+I	CUBSP073
	J8=I8+I	CUBSP074
	HT1=G(J)-X(I)	CUBSP075
	HT2=G(J)-X(I+1)	CUBSP076
	PROD=HT1*HT2	CUBSP077
	H(J9)=H(J6)+HT1*H(J8)	CUBSP078
	DELSQS=(H(J6)+H(J6+1)+H(J9))/6.	CUBSP079
	G(J)=F(I)+HT1* H(J2)+PROD*DELSQS	CUBSP080
65	CONTINUE	CUBSP081
	GO TO 9005	CUBSP082
70	IER=129	CUBSP083
	GO TO 9000	CUBSP084
		CUBSP085
		CUBSP086
		CUBSP087
		CUBSP088

APPENDIX G

DEMONSTRATION PROBLEMS

Example 1 - Free Vibration Analysis of a Cylindrical Shell

Free vibration analysis of the shell shown in Fig. 17 is considered. The shell is discretized with seven general elements. The elements are of third order. The mass density for the material of the shell is taken as 0.000736 lb.sec²/in⁴. The upper limit for the range of eigenvalues (P_U) is taken as 0.000001; whereas the lower limit (P_L) is taken as zero. The desired number of eigenvalues and eigenvectors per harmonic is specified as 3, the number of harmonics considered being two. The SHORE-III echo of the input data is given below. The results are presented in the Theoretical Manual.

ECHO OF INPUT DATA DECK

0 1 1 2 2 3 3 4 4 5 5 6 6 7 7 8
.....
5.....

FREE VIBRATION ANALYSIS OF A FIXED BASE CYLINDRICAL SHELL. (Units:lb.,in.) Problem Title Card
7 2 1 3 Problem Control Card
1 1 1 1 Element Cards
7 1 1 Element Cards

10.0
20.1286
30.3886
40.9286
52.0286
88.9286

Nodal Point Cards

1 1 1 1 1
10.0400E00.0400E03.0000E070.3000E00 0.0736E-2
70.0400E00.0400E03.0000E070.3000E00 0.0736E-2
3 3 3 3 3 3 3 3 3 3

Material Information Cards
Displacement Function Card
Output Control Card (blank)
Mass Matrix Card (blank)
Eigenvalue Analysis Card

0.000001 3.0

HARMONIC NUMBER = 0

0

HARMONIC NUMBER = 1

1

Load Title & Loading Control
Cards for 2 Harmonics

JOBEND

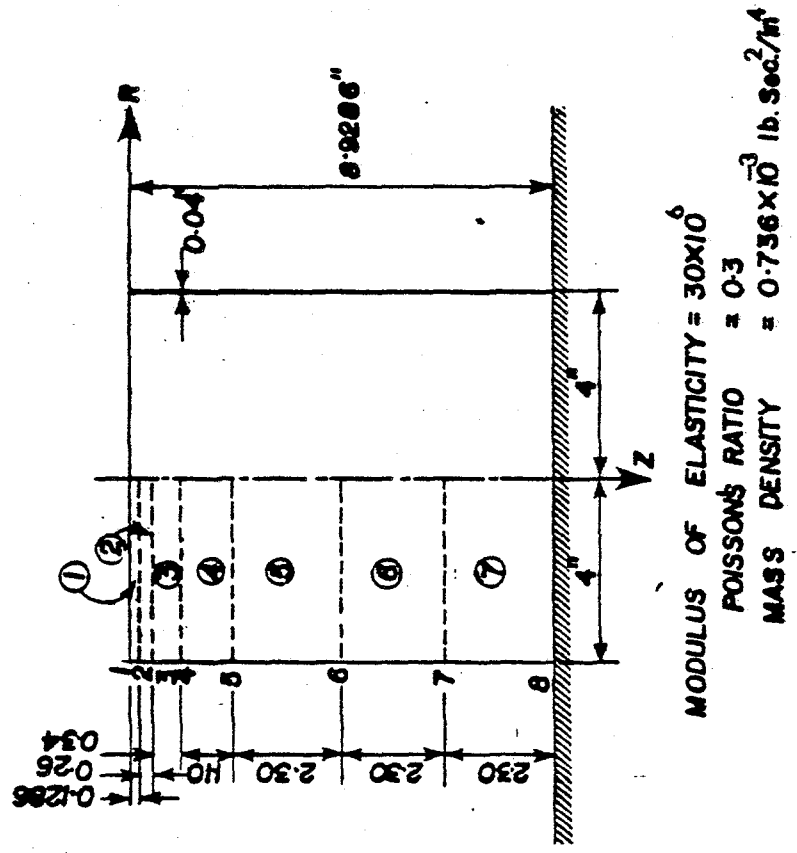


Figure 19. Circular Cylindrical Shell

Example 2 - Free Vibration Analysis of a Hemispherical Shell

The hemispherical shell shown in Fig. 18 is discretized with one sixth-order cap element and four sixth-order general elements. The echo of the input data deck for this problem is given below. The results are discussed in the Theoretical Manual.

```

ECHO OF INPUT DATA DECK
*****
0 1 1 2 2 3 3 4 4 5 5 6 6 7 7 8
...5...0...5...0...5...0...5...0...5...0...5...0...5...0
*****
FREE VIBRATION ANALYSIS OF HEMISPHERICAL SHELL - (6th ORDER ELEMENTS)
5 1 1 6
1 2 1.0 1.0 -50.0
2 1 1.0 1.0 -50.0
5 1 1.0 1.0 -50.0
10.0
625.0 1
10.5000E000.5000E003.0000E070.3000E00 0.0736E-2
50.5000E000.5000E003.0000E070.3000E00 0.0736E-2
6 6 6 6 6 6 6 6 6 6

```

```

0.0000001 4.0
HARMONIC NUMBER = 0
0
JOBEND

```

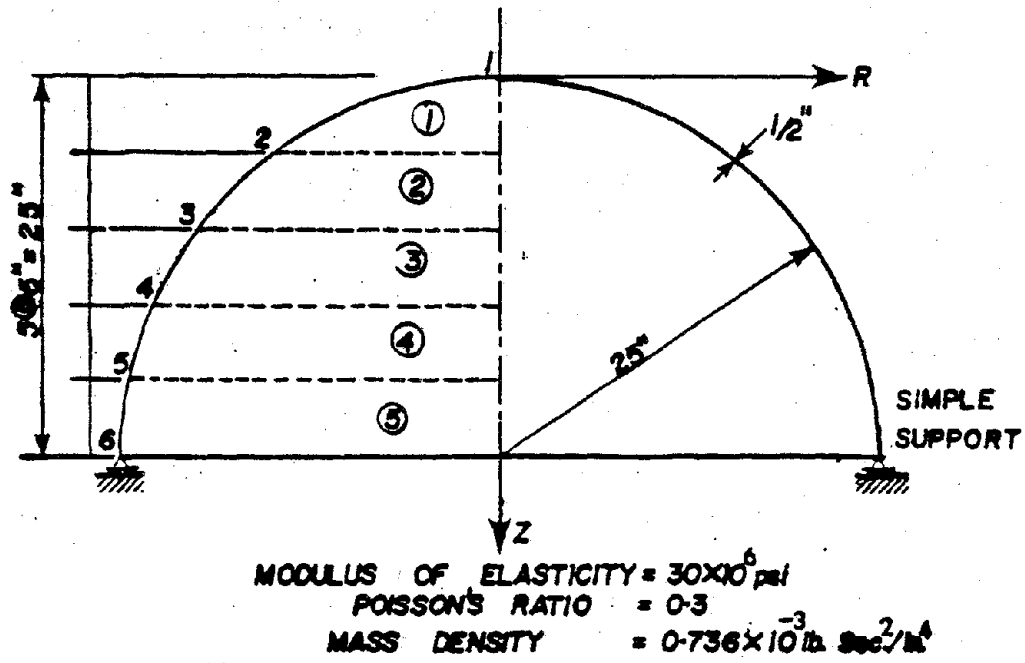


Figure 20. Hemispherical Shell

*Example 3 - Free Vibration Analysis of Empty Water Tank with Soil
Interaction Effect.*

The tank shown in Fig. 21 was analyzed using seven sixth-order general elements, the seventh element is the ring footing of the tank. The soil is modeled with sixteen isoparametric quadratic solid elements. The soil model is analyzed at a driving frequency of 358.8 rad/sec, the fundamental frequency of the tank on rigid foundation.

The input data echo of the problem is given in the following page. The results are presented in the Theoretical Manual.

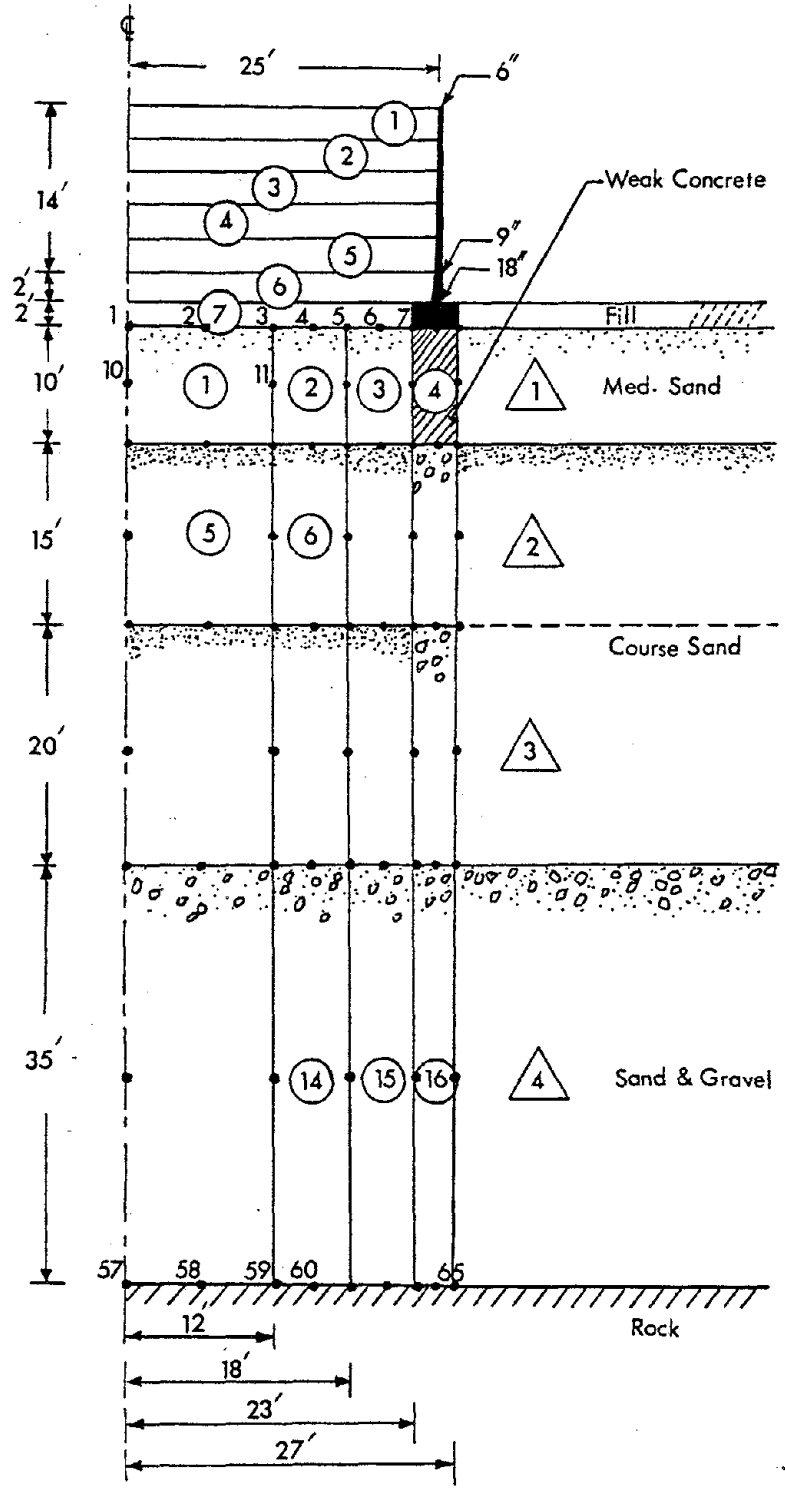


Figure 21. Cylindrical Water Tank on Flexible Foundation

Example 4 - Static Analysis of Column Supported Hyperboloidal Tower Under Wind Load

The tower shown in Fig. 22 was analyzed using ten sixth-order general elements and open-type element (Type 7) at the base. It was analyzed for a mean static wind pressure expressed in terms of the following six harmonics.

<u>Harmonic Number</u>	<u>Fourier Coefficient</u>
0	-0.064317
1	-2.072903
2	-3.846021
3	-3.404910
4	-3.962074
5	0.3289617

The input data echo of the problem is given in the following page. The results are presented in the Theoretical Manual.

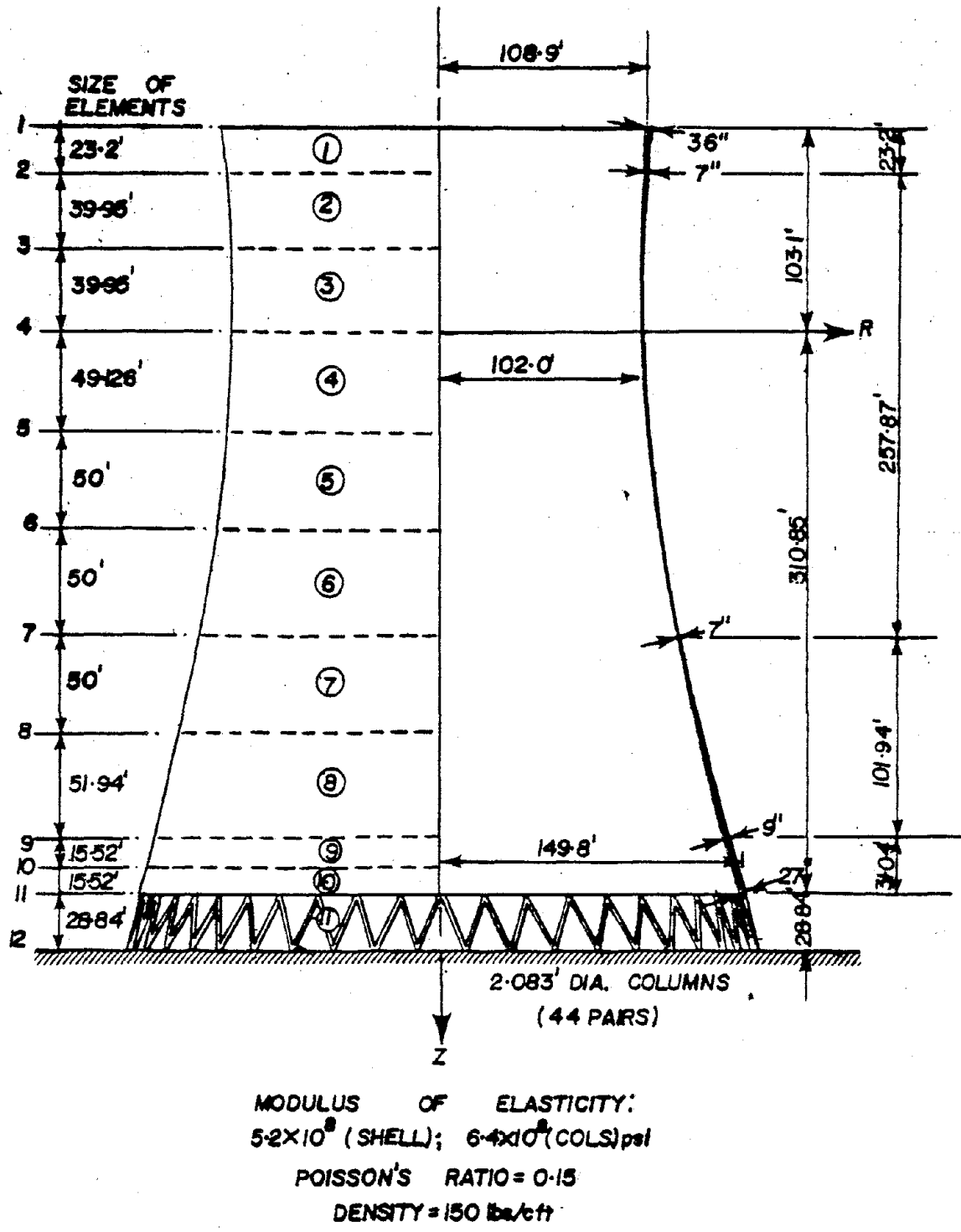
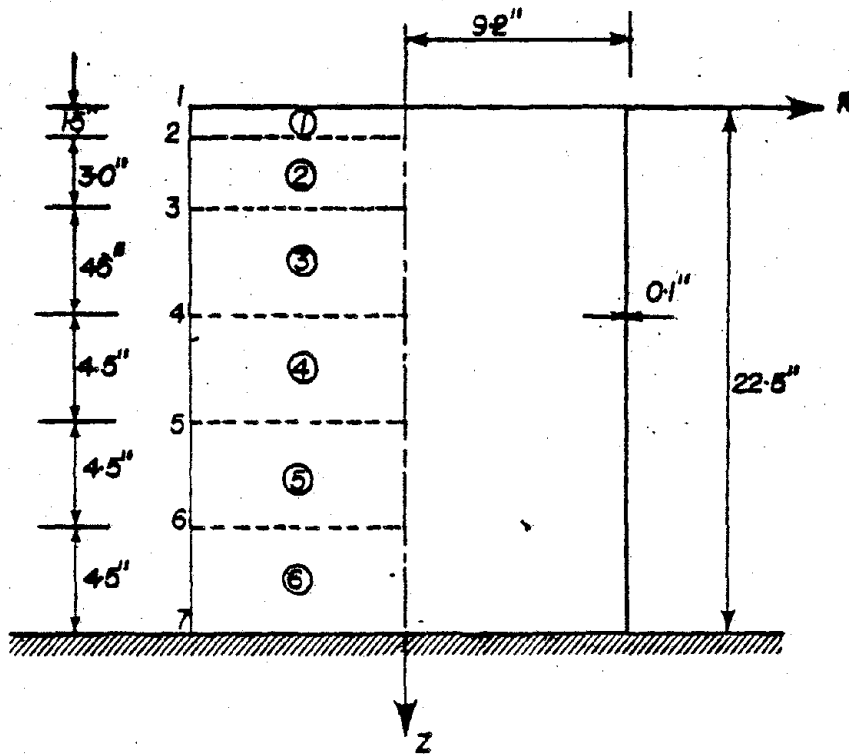


Figure 22. Discretization of Hyperboloidal Creek Tower



MODULUS OF ELASTICITY = 10.5×10^6 psi
POISSON'S RATIO = 0.3
MASS DENSITY = 2.4×10^{-4} lb. sec²/in⁴

Figure 23. Fixed Base Cylindrical Shell

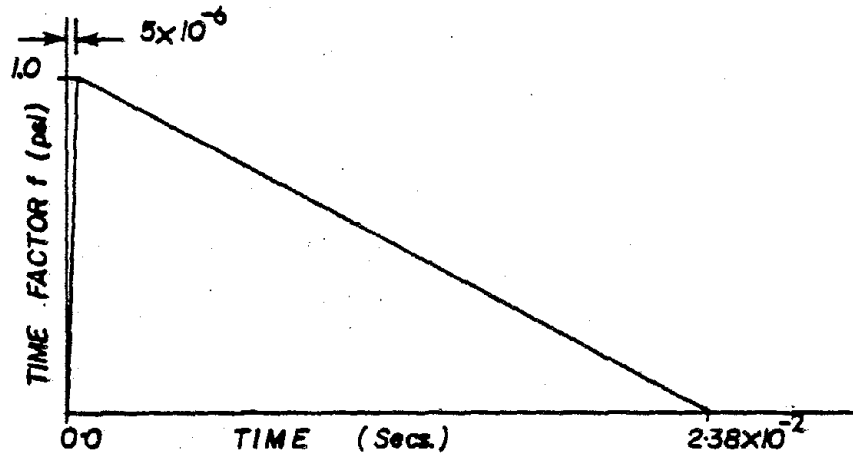
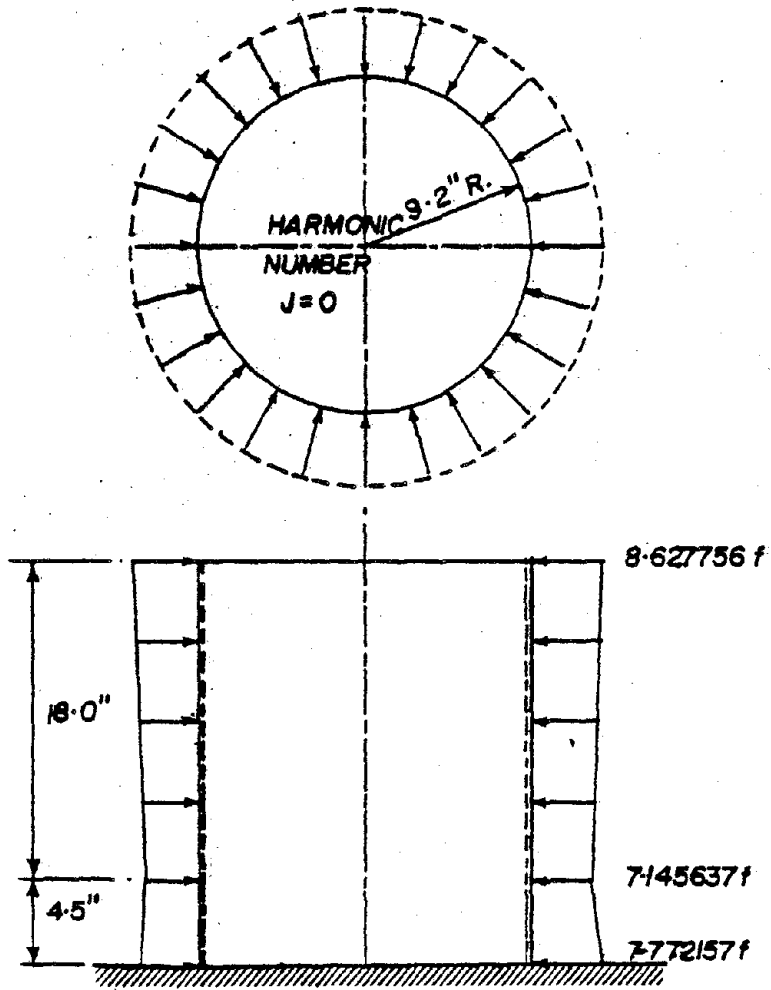


Figure 24: Zero Harmonic of the Blast Loading on Cylindrical Shell

Example 6 - Hyperboloidal Shell Under Dynamic Wind Load

The tower of Fig. 22 is analyzed for the wind loading, digitized at 0.5 sec. interval, shown in Fig. 25. This analysis was preceded by a free vibration analysis to determine the damping coefficients ($\alpha = 0.276$, $\beta = 0.0058$) and to select the time step ($\Delta t = 0.025$ sec.). The pressure was assumed constant over the height of the tower and eight harmonics were used. The input data echo for the problem is given in the following page. It may be noted that the control card for dynamic analysis for direct integration, data card for dynamic analysis, and the record points cards are provided for the first harmonic only. Accordingly the columns 30-35 and 36-40 of the problem control card were left blank. The results are presented and discussed in the Theoretical Manual.

ECHO OF INPUT DATA CHECK

0 1 2 3 4 5 6 7 8
5 0 0 0 0 0 0 0 0

TIME HISTORY ANALYSIS OF M.C. TOWER FOR WIND LOAD - P.U. DATA

11	1	-0.419	1.0	444.164	-55708.727
10	1	-0.419	1.0	444.164	-55708.727
11	1	0.30354682	-1.0		149.8
1	1	103.10			
2	1	70.90			
3	1	39.90			
4	1	0.0			
5	1	49.124			
6	1	98.124			
7	1	149.124			
8	1	199.124			
9	1	251.06			
10	1	299.90			
11	1	349.90			
12	1	310.0			
10	1	3000	1	4.6584	
25	1	3333	1	4.6584	
65	1	3333	1	4.6584	
75	1	3333	1	4.6584	
85	1	3333	1	4.6584	
90	1	3000	1	4.6584	
101	1	3000	1	4.6584	
113	1	4078	1	4.6584	
6	1	6.4	6	3.6465E	2.0830E 2.0830E 0.15

1 1 1 2
51 54

WIND LOADING - HARMONIC NUMBER = 0

1	12	-0.0067801	-0.0067801
0.2750	0.0098	1.0	14 1 1 8 200 440 2

0.025 HISTORY LOADING DATA FOR ALL HARMONICS - P.U. LAST 6 SECS.

2.0	0.0	5.0	8.8453	5.5	9.1441	6.0	9.3946
2.5	9.3672	7.0	9.3834	7.5	9.7878	8.0	10.1956
2.5	9.7668	9.0	11.0295	9.5	11.6456	10.0	10.7592
10.5	10.3728	11.0	10.0518				

WIND LOADING - HARMONIC NUMBER = 1	1 12	-0.2185178	-0.2185178
WIND LOADING - HARMONIC NUMBER = 2	1 12	-0.4054330	-0.4054330
WIND LOADING - HARMONIC NUMBER = 3	1 12	-0.3589330	-0.3589330
WIND LOADING - HARMONIC NUMBER = 4	1 12	-0.0417667	-0.0417667
WIND LOADING - HARMONIC NUMBER = 5	1 12	0.03467792	0.03467792
WIND LOADING - HARMONIC NUMBER = 6	1 12	0.01465779	0.01465779
WIND LOADING - HARMONIC NUMBER = 7	1 12	0.01669296	0.01669296

JR8EMO

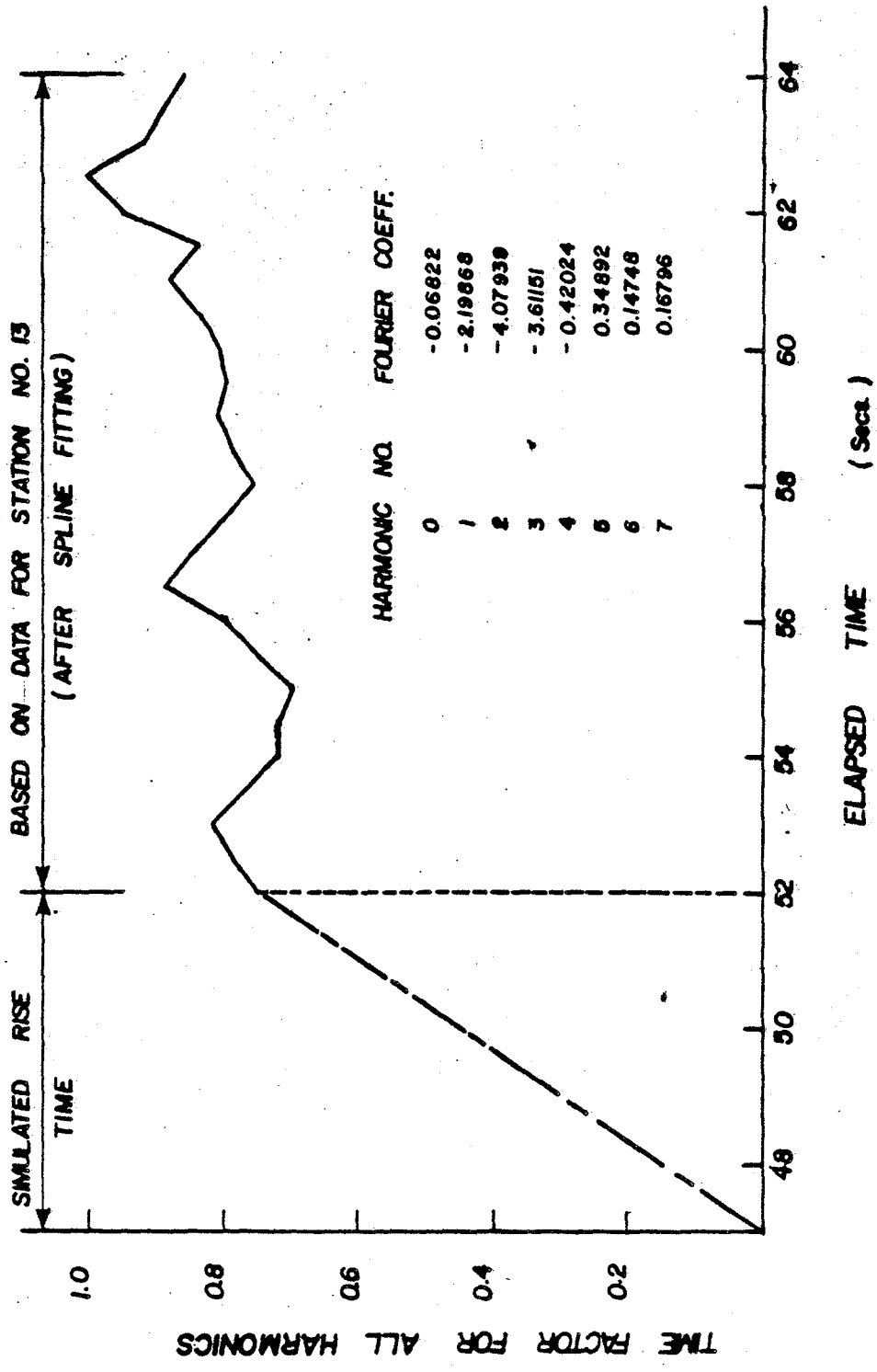


Figure 25. Time History of Wind Loading for Hyperboloidal Tower

*Example 7 - Response Spectrum Analysis of Hyperboloidal Shell With
Soil Interaction.*

The tower shown in Fig. 26 is analyzed under the ground motion given by the response spectrum of Fig. 27. The damping ratios for both the shell and the soil is 5% of the critical damping ratio. The analysis of the soil is carried out under driving frequency of 20 rad/sec. The soil medium is an elastic half space with shear modulus of 1600 k/sq. ft. and Poisson's ratio of 0.4. The input data echo for the problem is given in page (112). The results are presented and discussed in the Theoretical Manual.

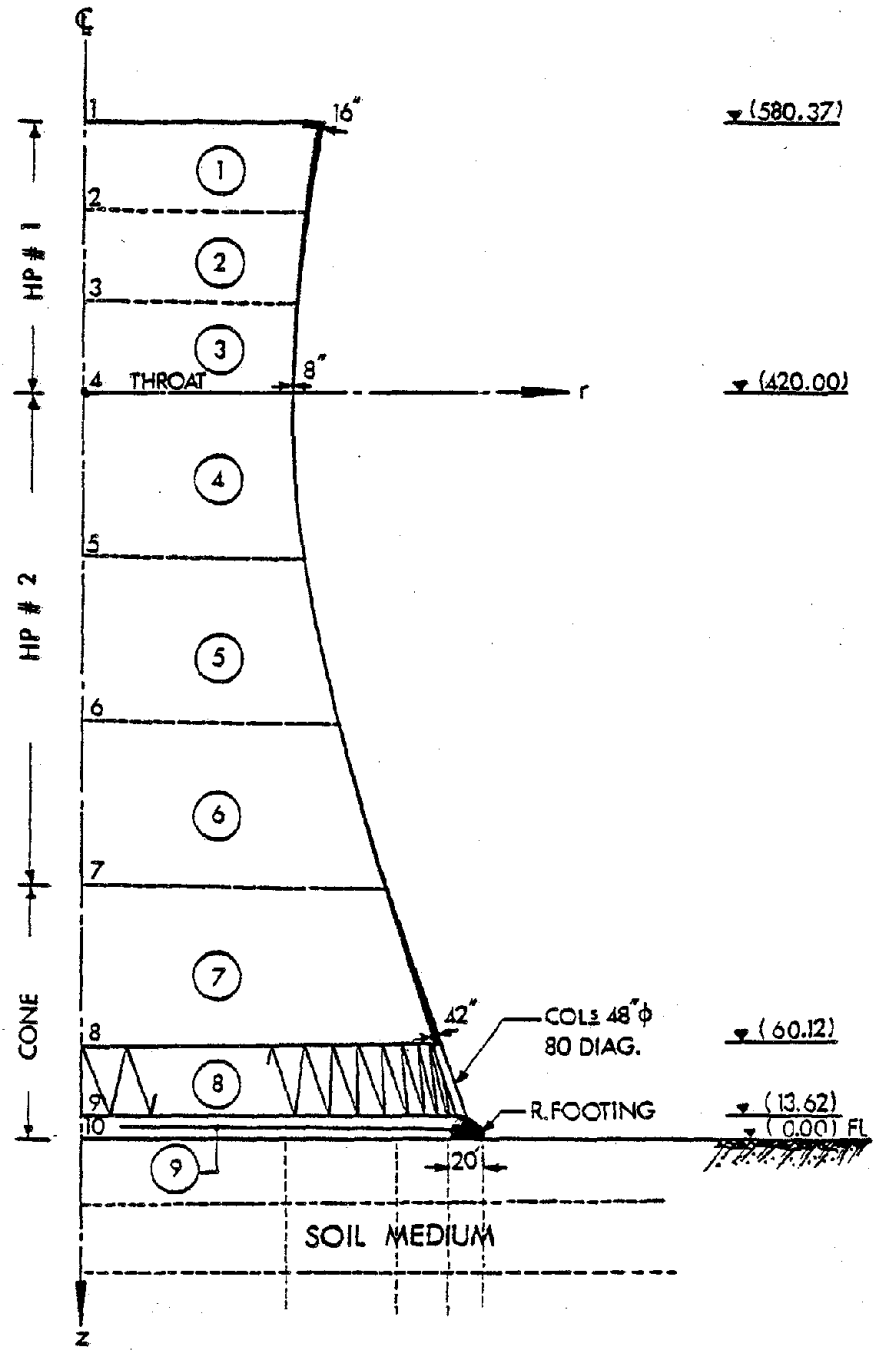


Figure 26. Cooling Tower on a Hypothetical Foundation

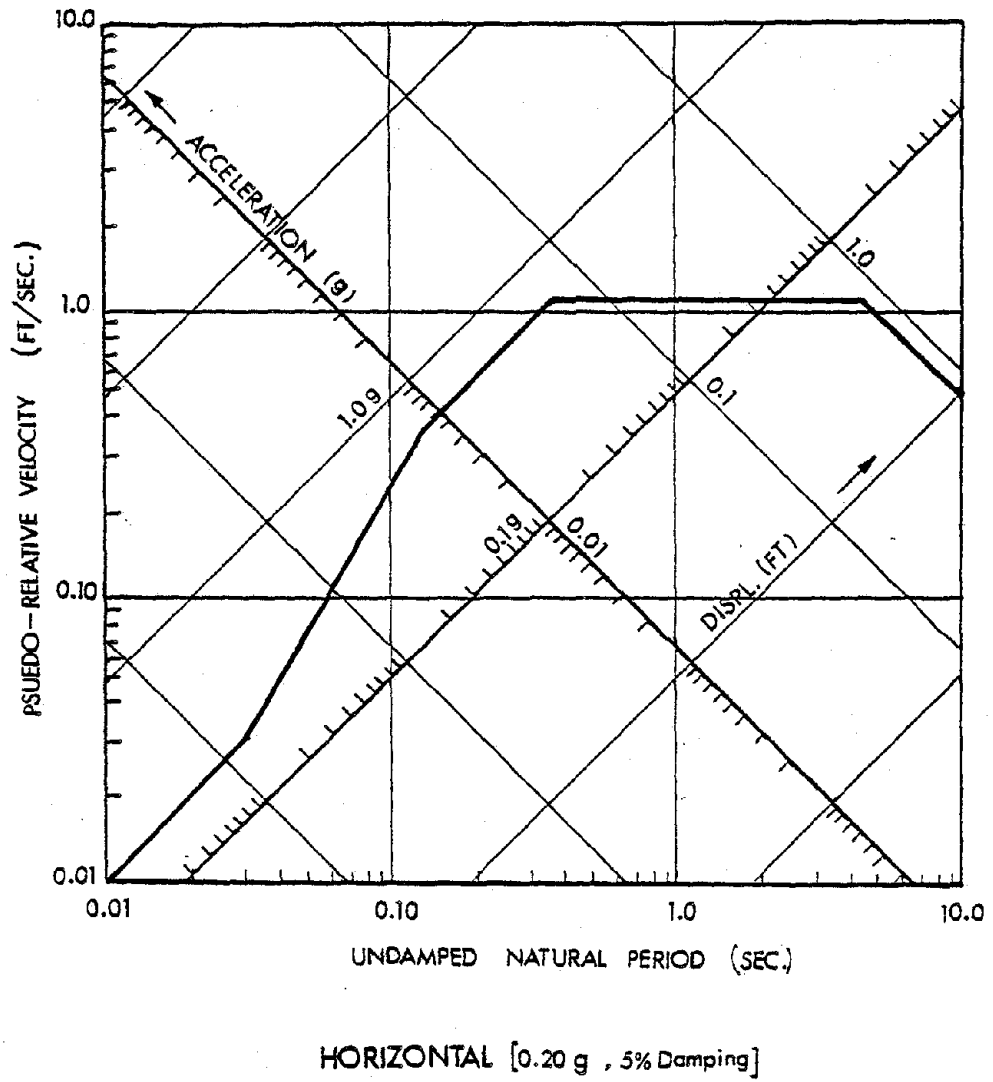


Figure 27. Horizontal Response Spectrum

***** ECHO OF INPUT DATA DECK *****

0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
5	5	5	5	5	5	5	5	5

TEST PROBLEM (RESPONSE SPECTRUM ANALYSIS) ... BASE WITH E.B.S.

1	0	123.68377	-27587.51651536846	.537
4	0	9.40153	-1302.5923225462	.88292
7	0			
8	8017		.2999997804	-1. 87.2111682
9	0		.2999997804	-1. 87.2111682

4	359.880			
9	406.380			
10	420.000			
11	333E	1.000E	5.191E+05.1667E	.4166E
21	000E	.6667E	5.191E+05.1667E	.4166E
3	6667E	.6667E	5.191E+05.1667E	.4166E
5	6667E	.8310E	5.191E+05.1667E	.4166E
6	8310E	.9580E	5.191E+05.1667E	.4166E
7	9580E	3.500E	5.191E+05.1667E	.4166E
8	12.57E	25.14E	5.804E+05.12.57E	12.57E
94	000E	20.00E	5.191E+05.1667E	.4166E

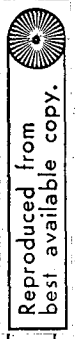
1	3333	0.6667	0.958	9.0 4	9.0 7	9.0 9	4.5
3	100.	0.	3.				
1	225.7	25.	20.		0.05		
1	1600E+04.	4000E+00	3500E-02				

HARMONIC NUMBER = 1

J = 1 (HORIZONTAL RESPONSE SPECTRUM) S M M P 0.05 DAMPING (SSE)

100.0000/	.010	33.3333	.032	17.8125	.350	2.5000	1.100
.2273	1.100	.1	.465				

JOBEND



Example 8 - Time History Analysis of Hyperboloidal Shell

- a. With Soil Effect
- b. Without Soil Effect

The tower of Fig. 26 is analyzed under El-Centro Earthquake (5-18-1940) EW Comp. The soil medium is considered as elastic half space as in Example 7 and then the analysis repeated with fixed lower boundary at node #10. The time of the analysis is taken as five seconds and the time step for Newmark β method integration is taken 0.02 second in case a (soil-structure interaction) and 0.005 second in case b (fixed base). The damping coefficients ($\alpha=0.715779$ and $\beta=0.003356$) are obtained based on 5% damping ratio for the first two modes of vibration (the modes of vibrations are obtained in Example 7). The input data echo for the problem is given in the next two pages. Fig. 28 shows the plot of M_{ϕ} at node #7 ($\theta=0^{\circ}$) for the soil-structure interaction case (case a). The rest of the results are presented in the Theoretical Manual.

ECHO OF INPUT DATA DECK

 0 1 2 3 4 5 6 7 8

TIME HISTORY ANAL. (COOLING TOWER WITH FIXED BASE) EL CINTRQ EW

1	0	-1.	123.00377	-27587.51651536040.537
4	0	-1.	9.40153	-1302.5923225402.448292
7	0			.2999997804 -1.47.2111602
9	0			.2999997804 -1.47.2111602
9	0			.2999997804 -1.47.2111602

11.333E	1.000E	5.191E+05.1007E	.4100E	.4658E-02
21.400E	1.000E	5.191E+05.1007E	.4100E	.4658E-02
3.0007E	1.000E	5.191E+05.1007E	.4100E	.4658E-02
5.6067E	1.000E	5.191E+05.1007E	.4100E	.4658E-02
6.0310E	1.000E	5.191E+05.1007E	.4100E	.4658E-02
7.9389E	1.000E	5.191E+05.1007E	.4100E	.4658E-02
412.457E	25.14E	5.804E+05.12.57E	12.57E	.4658E-02
94.000E	20.00E	5.191E+05.1007E	.4100E	.4658E-02

1	2	1	5
2	4	31	32 33

HARMONIC NUMBER 1 (HORIZONTAL GROUND MOTION)

1	.25	.5	1.	250	1	1	1	250	2	1
	.716779	.003356								

EL CINTRQ EARTHQUAKE (5-18-1940) EW COMP. 5 SEC.

0.00	0.00	0.04	0.00234	0.06	0.22310	0.00	0.09514
0.10	0.09514	0.12	0.17717	0.14	0.27231	0.10	0.17388
0.18	0.04593	0.20	0.17348	0.22	0.43463	0.24	0.40062
0.26	0.21654	0.28	0.20013	0.30	0.43635	0.32	0.16076
0.34	0.31490	0.36	0.01640	0.38	0.79724	0.40	0.43033
0.42	0.15092	0.44	-0.26575	0.46	-0.51181	0.48	-0.27887
0.50	-0.10171	0.52	0.13123	0.54	0.27837	0.56	0.43307
0.58	0.59055	0.60	0.53150	0.62	0.07546	0.64	0.38202
0.66	0.26247	0.68	0.25202	0.70	0.30009	0.72	0.42979
0.74	0.26575	0.76	0.04255	0.78	-0.49541	0.80	-1.08258
0.82	-1.74541	0.84	-2.11942	0.86	-1.72244	0.88	-1.38155
0.90	-1.31502	0.92	-1.08903	0.94	-1.36620	0.96	-1.87664
0.98	-1.73226	1.00	-1.63058	1.02	-1.63326	1.04	-1.80440
1.12	-1.61517	1.14	-1.21434	1.16	-1.18700	1.18	-0.91664
1.14	-0.90949	1.16	-0.44823	1.18	-0.69882	1.20	-0.15420
1.22	0.70538	1.24	1.57152	1.26	1.90289	1.28	3.61640
1.30	0.44026	1.32	-1.76509	1.34	-3.17535	1.36	-3.00493
1.38	-3.47341	1.40	-1.51050	1.42	-3.27756	1.44	-3.07087
1.46	-3.70079	1.48	-4.61950	1.50	-3.97966	1.52	-3.60503
1.54	-2.81496	1.56	-2.08601	1.58	-1.24672	1.60	-0.50853
1.62	0.35105	1.64	1.10892	1.66	1.96522	1.68	2.73950
1.70	3.58268	1.72	4.34712	1.74	4.87205	1.76	3.91076
1.78	3.62533	1.80	3.29068	1.82	3.54003	1.84	2.70341
1.86	1.21063	1.88	-0.73819	1.90	-4.65893	1.92	-5.88598
1.94	-5.21326	1.96	-5.55447	1.98	-4.99016	2.00	-4.74410
2.02	-4.42257	2.04	-4.15026	2.06	-1.86811	2.08	-3.59250
2.10	-3.27100	2.12	-2.50656	2.14	-1.95210	2.16	-1.31150
2.18	-1.77822	2.20	-1.47441	2.22	-2.52225	2.24	-1.91222
2.26	0.81037	2.28	-0.41995	2.30	-2.21957	2.32	-2.70994
2.34	-1.20332	2.36	-1.98325	2.38	-2.24278	2.40	-1.26638
2.42	2.79648	2.44	3.80798	2.46	3.02494	2.48	2.54205
2.50	1.73478	2.52	1.51141	2.54	-0.00039	2.56	-2.00737
2.58	-2.29659	2.60	-1.04331	2.62	-0.12467	2.64	0.43176
2.66	1.34882	2.68	3.01502	2.70	3.03845	2.72	2.13583
2.74	3.63663	2.76	-0.49541	2.78	0.47244	2.80	1.38268
2.82	2.53261	2.84	1.87407	2.86	5.22310	2.88	5.83990
2.90	4.87861	2.92	3.88124	2.94	2.38530	2.96	1.06955
2.98	1.15158	3.00	1.73257	3.02	2.26378	3.04	2.33268
3.06	2.17520	3.08	2.03740	3.10	1.77822	3.12	1.70932
3.14	2.08006	3.16	2.29003	3.18	2.65748	3.20	1.41404
3.22	-0.18045	3.24	-1.50591	3.26	-0.67545	3.28	-0.02297
3.30	0.85302	3.32	1.59447	3.34	2.50050	3.36	2.48348
3.38	2.00459	3.40	1.15158	3.42	0.64961	3.44	1.40740
3.46	1.89901	3.48	2.43405	3.50	3.10309	3.52	1.35427
3.54	-0.19357	3.56	-1.67051	3.58	-3.03400	3.60	-3.12564
3.62	-3.08431	3.64	-2.91333	3.66	-2.69240	3.68	-2.42742
3.70	-2.04725	3.72	-1.64370	3.74	-1.23360	3.76	-0.78740
3.78	-0.41011	3.80	-2.04421	3.82	-0.56102	3.84	-1.48123
3.86	-2.33268	3.88	-2.10958	3.90	-0.90879	3.92	-0.21325
3.94	0.55440	3.96	-0.75455	3.98	-1.43370	4.00	-1.87008
4.02	-0.21325	4.04	0.48228	4.06	0.67257	4.08	0.66570
4.10	1.44029	4.12	1.66995	4.14	1.58137	4.16	-2.12799
4.18	-3.23163	4.20	-3.52362	4.22	-2.20001	4.24	-0.83950
4.26	0.85630	4.28	2.01772	4.30	3.35302	4.32	3.58976
4.34	3.12992	4.36	2.87074	4.38	2.16804	4.40	1.71548
4.42	2.21785	4.44	3.54003	4.46	2.52023	4.48	2.85195
4.50	1.94226	4.52	2.11014	4.54	-3.35315	4.56	-3.05118
4.58	-3.50394	4.60	-3.33858	4.62	-3.39567	4.64	-3.31693
4.66	-5.02625	4.68	-4.08400	4.70	-3.66670	4.72	-2.86740
4.74	-2.01119	4.76	-0.44593	4.78	0.20374	4.80	1.43370
4.82	2.27090	4.84	2.32204	4.86	2.26373	4.88	2.29001
4.90	2.04390	4.92	2.60499	4.94	2.00093	4.96	1.72244
4.98	1.51903	5.00	1.74009				

UQAFEND

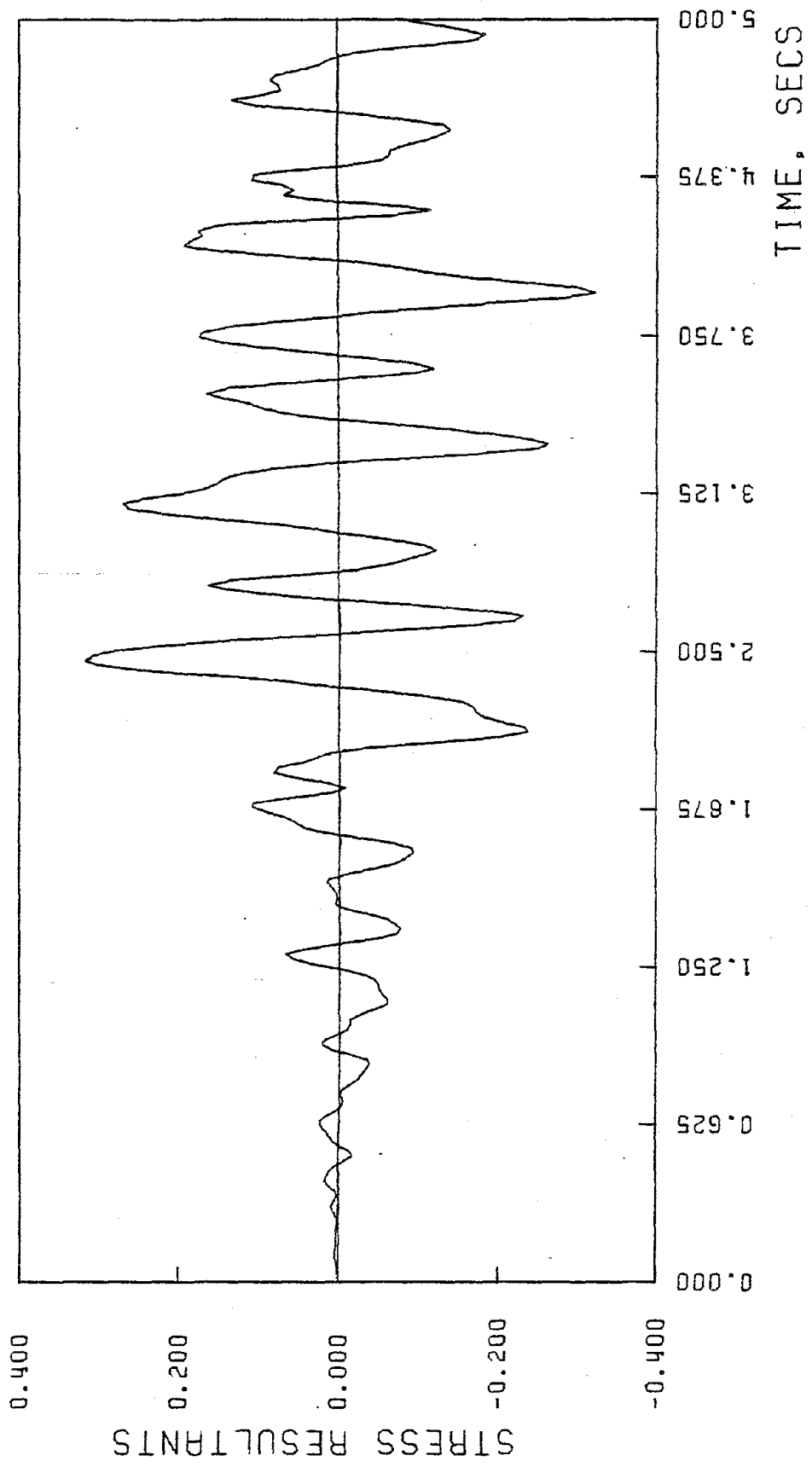


Figure 28. TIME HISTORY PLOT FOR HARMONIC NUMBER = 1
COMPONENT NO. 3 AT NODE NO. 7

REFERENCES

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