

SEISMIC SOIL-PILE-STRUCTURE INTERACTION PILE GROUPS

Report to
NATIONAL SCIENCE FOUNDATION



McClelland engineers

REPORT DOCUMENTATION PAGE	1. REPORT NO. NSF/CEE-81081	2.	3. Recipient's Accession No. P082 171471
Title and Subtitle Seismic Soil-Pile-Structure Interaction, Pile Groups		5. Report Date August 1981	
Author(s) T. Kagawa		6. 063236	
Performing Organization Name and Address McClelland Engineers, Inc. P.O. Box 37321 Houston, TX 77026		8. Performing Organization Rept. No.	
10. Project/Task/Work Unit No.		11. Contract(C) or Grant(G) No. (C) (G) PFR8001503	
12. Sponsoring Organization Name and Address W. Hakala, CEE Directorate for Engineering (ENG) National Science Foundation Washington, DC 20550		13. Type of Report & Period Covered	
14.		15. Supplementary Notes Submitted by: Communications Program (OPRM) National Science Foundation Washington, DC 20550	
16. Abstract (Limit: 200 words) A parametric study is presented of dynamic pile-group effects for an idealized soil-pile-structure system consisting of a lumped mass model of a superstructure and elastic piles fully embedded in a homogeneous, linearly elastic soil layer. The results are used to develop an approximate method, based upon a beam-on-Winkler foundation model, that can be used for dynamic response analyses of pile-groups in layered elastic soils. The approximate pile-group method is evaluated by comparing computed and observed seismic response data of a building supported by a group of piles. The comparison shows that the dynamic stiffness and damping characteristics of a pile group can be evaluated correctly using the procedure. The pile method was found to provide a rational tool to quantify and evaluate pile-group effects for the seismic response of complex pile-supported structures.			
17. Document Analysis a. Descriptors			
Earthquakes		Pile foundations	
Earthquake resistant structures		Pile structures	
Structural analysis		Soils	
Seismic response		Mathematical models	
b. Identifiers/Open-Ended Terms Ground motion T. Kagawa, /PI			
c. COSATI Field/Group			
Availability Statement NTIS		19. Security Class (This Report)	21. No. of Pages
		20. Security Class (This Page)	22. Price

SEISMIC SOIL-PILE-STRUCTURE INTERACTION
-PILE GROUPS-

by

Takaaki Kagawa, Ph.D., P.E.
Engineering Consultant
McClelland Engineers, Inc.

This material is based upon work supported by
the National Science Foundation under Grant
No. PFR 80-01503.

Any opinions, findings, and conclusions, or
recommendations expressed in the publication
are those of the author and do not necessarily
reflect the views of the National Science
Foundation.

SEISMIC SOIL-PILE-STRUCTURE INTERACTION
-PILE GROUPS-

* * *

by
Takaaki Kagawa

A Report on Research Sponsored by
the National Science Foundation

Grant No. PFR 80-01503

* * *

M C C L E L L A N D E N G I N E E R S , I N C .
Geotechnical Consultants
Houston, Texas
August 1981

C O N T E N T S

	<u>Page</u>
ABSTRACT	v
INTRODUCTION	1
Summary of Previous Work	1
Scope of Study	2
Report Format	3
THEORETICAL PILE-GROUP EFFECTS IN IDEALIZED PILE GROUPS	3
Analytic Model	3
System Parameters	4
Solution Procedure	5
Response of Soil Layer	5
Responses of Piles	7
Soil-Pile Compatibility Conditions	8
Boundary Conditions	11
Structural Response	12
Soil-Pile Interface Stress Patterns	13
Evaluation of Analytic Method	14
Appropriate Soil-Pile Stress Patterns	14
Comparison with Other Solution	16
Pile-Group Effects	16
Major Factors Affecting Pile-Group Effects	16
Directionality	18
Spring and Damping Constants of Pile Group	18
Shear and Moment Distributions	20
Seismic Pile-Group Effects	20
Soil-Pile Springs and Damping	21
Summary	22
SIMPLIFIED METHOD FOR PILE GROUPS	23
Introduction	23
Analytic Model	24
Soil-Pile Spring for a Single Pile	24
Soil-Pile Springs for a Pile Group	25
Correction Factors for Pile-Group Effects	26
Evaluation of Correction Factors	27
Numerical Procedure	28

C O N T E N T S (Cont.)

	<u>Page</u>
CASE STUDY	28
Introduction	28
Earthquake Response Observation	29
Analysis Procedure	29
Analysis Results	31
Comparison for the 1974 Izu Event	31
Comparison for the 1975 Ibaraki Event	31
Summary	32
CONCLUDING COMMENTS	33

I L L U S T R A T I O N S

	<u>Plate</u>
Analytic Model of a Pile-Supported Structure	1
Ranges of Nondimensional Parameters for Parameter Study	2
Soil-Pile Interface Stress	3
Soil-Pile Interface Stresses for Analysis	4
Representation of a Pile Section by Discrete Elements ...	5
Single-Pile Stiffness and Damping from Several Methods ..	6
Single-Pile Soil-Pile Spring Coefficients from Several Methods	7
2-Pile Interaction for Different Soil-Pile Stress Patterns	8
Comparison with Poulos' Group Deflection Factors	9
Soil Motions due to Pile Vibration	10
Effects of $s/2r_0$ and Frequency on 2-Pile Interaction	11
Effects of $H/2r_0$ on 2-Pile Interaction for Lateral Mode	12
Effects of $H/2r_0$ on 2-Pile Interaction for Lateral Mode	13
Effects of $H/2r_0$ on 2-Pile Interaction for Vertical Mode	14
Effects of \bar{K}_R on 2-Pile Interaction for Lateral Mode	15
Effects of K_C on 2-Pile Interaction for Vertical Mode ...	16
Directional Angle for 2-Piles	17
Directionality of 2-Pile Interaction	18

I L L U S T R A T I O N S (Cont.)

	<u>Plate</u>
Pile-Group Effects for 2x2 Group	19
Pile-Group Effects for 3x3 Group	20
Comparison of Damping for Pile Groups and Surface Footings	21
Distribution of Pile-Head Shear	22
Distribution of Pile-Head Moment	23
Pile-Group Effects on Soil-Pile Spring Coefficients	24
Analytic Model of a Pile-Supported Structure	25
Soil-Pile Spring and Dashpot	26
Average Soil-Pile Spring Coefficient for Lateral Mode ...	27
Average Soil-Pile Spring Coefficient for Vertical Mode ..	28
Performance of Approximate Method	29
Numerical Schemes for Program PILES	30
14-Story Building for Case Study	31
Soil Conditions and Seismic Observational Points	32
Structural Properties for Case Study	33
Earthquake Motions at Pile-Tip Level for Case Study	34
Computed and Observed Acceleration Time Histories for Izu Event	35
Computed and Observed Response Spectra for Izu Event	36
Computed and Observed Response Spectra for Ibaraki Event	37

APPENDIX A: REFERENCES

ABSTRACT

Dynamic pile-group effects were studied parametrically for an idealized soil-pile-structure system. The system consisted of a lumped mass model of a superstructure and elastic piles fully embedded in a homogeneous, linearly elastic soil layer. The results were used to develop an approximate method, based on a beam-on-Winkler foundation model, that can be used for dynamic response analyses of pile-groups in layered elastic soils.

Major findings from the parameter study for an idealized soil-pile-structure system are itemized below.

- (1) Correct pile-soil-pile interaction can be assessed using uniform soil-pile stress patterns over the width of a pile for a wide frequency range, although this assumption fails for a single pile at high frequencies.
- (2) Pile-group effects are affected primarily by the nondimensional frequency $\omega r_0 / V_s$ and the spacing ratio $s/2r_0$ for flexible and compressible piles. Other factors have only a minor influence on the pile-group effects.
- (3) 2-pile interaction is pronounced at spacing ratios less than 30 for the lateral vibration mode and about 20 for the vertical vibration mode.
- (4) Group stiffness varies strongly with frequency. For a clustered pile group, the group stiffness can be negative and can be larger than a simple summation of static single pile stiffnesses.
- (5) Damping of a pile group also is frequency-dependent, and it can be approximated by that of an equivalent surface footing at low frequencies, where the seismic soil-pile interaction is important.
- (6) Distribution patterns of shear and moment among piles depend largely on frequency. Center piles, for example, can be stiffer and carry a greater load than corner piles under dynamic loading conditions than they do under static.
- (7) Pile-group effects are strongly frequency-dependent. Pile-group efficiency from static theory is not necessarily appropriate for designing a dynamically loaded pile foundation.

The approximate pile-group method, "PILES", was evaluated by comparing computed and observed seismic response data of a building supported by a group of piles. The comparison showed that the dynamic stiffness and damping characteristics of a pile group can be evaluated correctly using the procedure. Thus, the method provides a rational tool to quantify and evaluate pile-group effects for the seismic response of complex pile-supported structures.

INTRODUCTIONSummary of Previous Work

Pile-supported structures have experienced significant damage and failure during major earthquakes. For example, a number of pile foundations for bridge structures suffered severe damage during the Niigata and Alaska Earthquakes of 1964 (Fukuoka, 1966; Kachadoorian, 1968), and many prestressed and reinforced concrete piles for buildings were damaged during the 1978 Miyagi-Oki Earthquake. The damage was caused by the failure of the foundation soil due to liquefaction or by the soil-pile interaction generated by earthquake excitations.

The dynamic response analysis of soil-pile-structure systems has been the subject of considerable interest and research in recent years. Design of pile-supported structures against seismic loading requires characterization of soil-pile springs that can be combined into the structural response analysis. The state-of-the-art on characterization of soil-pile springs under seismic loading is in a formative stage of development, and extensive work is needed to study the dynamic characteristics of the lateral load-deflection relationships of piles.

Soil-pile springs may be represented analytically by the discrete models such as the subgrade reaction theory (Ogata and Kotsubo, 1966; Yoshida and Yoshinaka, 1972; and Prakash and Chandrasekaran, 1973) and the Minlin's static solution (Parmelee, Penzien, Scheffey, Seed, and Thiers, 1964; and Liou and Pensien, 1977), by the continuous models such as the elastic solutions (Tajimi, 1969; Nogami and Novak, 1977; Kobori, Minai, and Baba, 1977; and Kagawa and Kraft, 1981), or by the semi-continuous models such as the finite element method (Blaney, Kausel, and Roesset, 1976; Kuhlemeyer, 1979; Roesset and Angelides, 1979; and Kagawa and Kraft, 1980a). Most of these studies are based on single pile systems.

Studies on pile-group effects have been based mostly on static considerations (Poulos, 1971 and 1979). Dynamic soil-pile

interaction effects were represented by the concept of "effective soil mass" (Parmelee, Penzien, Scheffey, Seed, and Thiers, 1964). In this procedure, a certain portion of the soil around piles is assumed to move together with piles, and the inertia effects of soil during pile vibration are approximated. No established criteria to evaluate the effective mass and radiation damping associated with soil-pile interaction, however, is available in this method. Therefore, a rational procedure is needed to evaluate the pile-group effects obtained from this procedure.

Recently, Wolf and von Arx (1978) solved a problem of a footing supported by up to 100 vertical piles by first solving the single pile problem for a visoelastic system using an axisymmetric finite element method. They constructed from this solution, by superposition, the total complex flexibility matrix of the soil-pile system, and finally the corresponding impedance matrix by inversion of the flexibility matrix. They illustrated the significance of the pile-soil-pile interaction effects; due to pile group effects, the stiffness of an average pile in a pile group may be substantially reduced and the radiation damping may increase considerably compared to an isolated pile. The possible wave scattering and generating of standing waves within a group of piles, however, were not fully accounted for in this study. The effect of the lack of standing waves in the solution on the pile-group effect needs to be examined.

Scope of Study

Considering the lack of our knowledge of dynamic pile-group effects, we first investigated parametrically the fundamental aspects of lateral pile-soil-pile interaction. A pile-supported structure was represented by an idealized soil-pile structure system and theoretical solutions were derived. For simplicity, the soil layer was assumed homogeneous and elastic. The effects of variations of soil-pile interface stress patterns on pile responses were examined to study the significance of the possible wave scattering and generating of standing waves within a group of piles. Pile-group effects were studied in terms of: (1) dyna-

mic group efficiency for horizontal stiffness of the pile group, (2) dynamic load-deflection relationships of piles represented by soil-pile springs and energy dissipation due to material and radiation damping, and (3) shear and moment distributions among the piles.

An approximate, economical method was constructed, using the results of the theoretical analysis, to study the pile-group effects of complex soil-pile-structure systems encountered in practice. The method is based on a linearly elastic beam-on-Winkler foundation model of a pile group. Discrete soil-pile springs and dashpots for the method were determined using the equivalent soil-pile springs for single piles and correction factors for group effects.

The approximate method was evaluated by comparing computed and observed seismic response data of an apartment building supported by a group of piles.

Report Format

The report first presents a theoretical development of soil-pile-structure interaction of an idealized pile group and the results of a parameter study. This is followed by descriptions of the approximate method and the results of the comparison between computed and observed seismic response data. All illustrations follow the text, and citations of all reference material are included in Appendix A.

THEORETICAL PILE-GROUP EFFECTS IN IDEALIZED PILE GROUPS

Analytic Model

A pile-supported structure was modeled by a linearly elastic soil-pile-structure system, Plate 1. The superstructure is represented by a lumped mass model that has translational degrees of freedom only. Each of the identical, linearly elastic piles has a uniform cross section and is fully embedded into the soil layer. The soil layer of uniform thickness rests on a rigid base at which a seismic excitation is applied. The pile-structure system is allowed to translate in the x-direction only and rotate

around the y axis. The rigid pile-cap does not contact the soil layer.

The major assumptions in our analytic model are: (1) the piles are fully embedded into the soil layer and are supported on the rigid base, (2) the soil layer is homogeneous, linearly elastic with hysteretic material damping, (3) vertical soil motion generated by the horizontal vibration of a pile has only a minor effect on the pile response and can be neglected, (4) horizontal soil motion generated by the vertical vibration of a pile can be neglected, (5) piles are perfectly bonded to soil, and (6) vertical shear forces at soil-pile interface do not influence the lateral vibration of the pile. These assumptions are reasonable for studying the fundamental response characteristics of most single piles (Kagawa and Kraft, 1980a), and are good also for evaluating most pile-group effects.

System Parameters

Factors affecting the pile-group response in the present analytic model include

- (1) Spacing ratio ($=s/2r_o$).
- (2) Frequency.
- (3) Pile slenderness ratio ($=H/2r_o$).
- (4) Pile flexibility factor ($K_R = EI/E_s H^4$).
- (5) Pile compressibility factor ($K_C = EA/E_s H^2$).
- (6) Poisson's ratio of soil.
- (7) Pile-head fixity condition, and loading condition.

Effects of variations of these factors were studied. Plate 2 summarizes the ranges of nondimensional parameters used in this study. Frequency is represented as the ratio between the excitation frequency f and the fundamental resonance frequency of the soil layer f_r .

Our study showed that the spacing ratio, excitation frequency, and pile slenderness ratio had a large influence on the dynamic pile-group effects. Variations of other nondimensional parameters on Plate 2 had a small impact on the pile-group effects. The pile-head fixity condition and loading condition had minor influences on the pile-group effects.

Solution Procedure

Response of Soil Layer. When the structural base translates in the x direction and rotates at the ground level around the y axis, the piles deform both in the lateral and axial directions. Thus, the behavior of piles can be described by the superposition of lateral and axial vibration modes, and the piles transmit lateral and vertical forces into the soil. When piles exert soil-pile forces $F^i(z)$ and $H^i(z)$, response of the homogeneous, linearly elastic soil layer can be described as

$$(\lambda_s^* + G_s^*) \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \Delta + G_s^* \nabla^2 \{U, V, W\} = \rho \frac{\partial^2}{\partial t^2} \{U, V, W\} - \sum_{i=1}^N \{f^i(x, y, z), 0, h^i(x, y, z)\} e^{i\omega t} \quad \dots \quad (1)$$

where $\Delta = \partial U / \partial x + \partial V / \partial y + \partial W / \partial z$, $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2$, U, V, W , = soil displacements in the x, y, and z directions, $G_s^* = G_s(1 + i2D)$, G_s = shear modulus of soil, D = material damping of soil, $i = \sqrt{-1}$, $\lambda_s^* = 2\nu G_s^* / (1 - 2\nu)$, ν = Poisson's ratio of soil, ρ = mass density of soil, N = number of piles in the group, ω = circular frequency ($= 2\pi f$), and the soil-pile interface stresses $f^i(x, y, z)$ and $h^i(x, y, z)$ are defined as

$$F^i(z) = \int f^i(x, y, z) \, dx \, dy \quad \dots \quad (2)$$

$$H^i(z) = \int h^i(x, y, z) \, dx \, dy$$

The relation between $F^i(z)$ and f^i is illustrated on Plate 3, and $H^i(z)$ and h^i are defined in a similar manner. The integrals are performed over the circumference of the pile at z.

To solve Eq. 1, we decoupled horizontal and vertical vibration modes: no vertical soil motion is involved in the horizontal vibration, and no horizontal soil motion is considered for the vertical vibration. We applied a horizontal rigid base motion $u_0 e^{i\omega t}$ to the system. Soil displacements and soil-pile forces were expanded into discrete Fourier series as

$$\{U, V, W\} = \sum_n \left\{ u_n + \frac{4u_0}{n\pi}, v_n, w_n \right\} \sin a_n z e^{i\omega t}$$

$$\{f^i, 0, h^i\} = \sum_n \{f_n^i, 0, h_n^i\} \sin a_n z \quad (n=1, 3, 5, \dots) \dots (3)$$

where $a_n = n\pi/2H$, and $H =$ height of the soil layer. The summation is taken for odd integer numbers of n . With Eq. 3, Eq. 1 can be reduced to

$$\begin{aligned} & \frac{\partial^2}{\partial x^2} \{h^2 u_n, v_n, w_n\} + \frac{\partial^2}{\partial y^2} \{u_n, h^2 v_n, w_n\} \\ & - \{(a_n^2 - k^2)u_n, (a_n^2 - k^2)v_n, (h^2 a_n^2 - k^2)w_n\} + (h^2 - 1) \frac{\partial^2}{\partial x \partial y} \{v_n, u_n, 0\} \\ & = - \frac{4u_0}{n\pi} \{k^2, 0, 0\} - \sum_{i=1}^N \frac{1}{G_s^*} \{f_n^i, 0, h_n^i\} \end{aligned} \quad \dots (4)$$

where $h^2 = (v_p/v_s)^2$, v_p and $v_s =$ compression and shear wave velocities of the soil, and the wave number is defined as $k = \omega/v_s$.

To solve Eq. 4, we applied Fourier transforms to the soil displacements and the soil-pile forces as

$$\begin{aligned} \{\bar{u}_n, \bar{v}_n, \bar{w}_n, \bar{f}_n^i, 0, \bar{h}_n^i\} &= \mathcal{F}[u_n, v_n, w_n, f_n^i, 0, h_n^i] \\ &= \frac{1}{2\pi} \iint_{-\infty}^{\infty} \{u_n, v_n, w_n, f_n^i, 0, h_n^i\} e^{i(\alpha x + \beta y)} dx dy \end{aligned} \quad \dots (5)$$

Also, we applied Fourier transforms to Eq. 4, and obtained the solutions to Eq. 4 as

$$\begin{aligned} \bar{u}_n &= \frac{1}{\psi_n} (\alpha^2 + \beta^2 h^2 + a_n^2 - k^2) \left\{ \frac{4u_0 k^2}{n\pi} (2\pi) \delta(\alpha) \delta(\beta) + \frac{1}{G_s^*} \sum_{i=1}^N \bar{f}_n^i \right\} \\ \bar{v}_n &= \frac{1}{\psi_n} (h^2 - 1) \alpha \beta \left\{ \frac{4u_0 k^2}{n\pi} (2\pi) \delta(\alpha) \delta(\beta) + \frac{1}{G_s^*} \sum_{i=1}^N \bar{f}_n^i \right\} \end{aligned}$$

$$\bar{w}_n = \sum_{i=1}^N \bar{h}_n^i / \{G_s^* (\alpha^2 + \beta^2 + h^2 a_n^2 - k^2)\} \dots\dots (6)$$

where $\psi_n = (\alpha^2 + \beta^2 + a_n^2 - k^2) \{h^2 (\alpha^2 + \beta^2) + a_n^2 - k^2\}$, and $\delta(\)$ is the Dirac's delta function. Soil displacements are then solved by applying inverse Fourier transforms to Eq. 6 as

$$U = \sum_n \left[\frac{4u_o}{n\pi} + \frac{4u_o}{n\pi} \frac{k^2}{a_n^2 - k^2} + \frac{1}{2\pi G_s^*} \int_{-\infty}^{\infty} \frac{1}{\psi_n} (\alpha^2 + \beta^2 h^2 + a_n^2 - k^2) \sum_{i=1}^N \bar{f}_n^i e^{-i(\alpha x + \beta y)} d\alpha d\beta \right] x \sin a_n z e^{i\omega t}$$

$$V = -\sum_n \frac{1}{2\pi G_s^*} \int_{-\infty}^{\infty} \frac{(h^2 - 1)}{\psi_n} \alpha \beta \sum_{i=1}^N \bar{f}_n^i e^{-i(\alpha x + \beta y)} d\alpha d\beta \sin a_n z e^{i\omega t}$$

$$W = \sum_n \frac{1}{2\pi G_s^*} \int_{-\infty}^{\infty} \frac{\sum_{i=1}^N \bar{h}_n^i}{\alpha^2 + \beta^2 + h^2 a_n^2 - k^2} e^{-i(\alpha x + \beta y)} d\alpha d\beta \sin a_n z e^{i\omega t} \dots\dots (7)$$

Soil response can be obtained once the soil-pile forces are specified in Eq. 7.

Responses of Piles. When the i-th pile in a group receives the lateral soil reaction $F^i(z)$ and the vertical soil reaction $H^i(z)$, the equations of motion of the pile can be represented as

$$m_p \frac{\partial^2}{\partial t^2} (U^i e^{i\omega t}) + EI \frac{\partial^4}{\partial z^4} (U^i e^{i\omega t}) = -F^i(z) e^{i\omega t} + \omega^2 m_p u_o e^{i\omega t}$$

$$m_p \frac{\partial^2}{\partial t^2} (W^i e^{i\omega t}) - EA \frac{\partial^2}{\partial z^2} (W^i e^{i\omega t}) = -H^i(z) e^{i\omega t} \dots\dots (8)$$

where U^i and W^i = horizontal and vertical pile displacements relative to the horizontal rigid base motion $u_o e^{i\omega t}$, m_p = mass per unit length of the pile, EI = flexural rigidity of the pile, and EA = axial stiffness of the pile. Reponse of the pile can be determined once the unknown soil reactions $F^i(z)$ and $H^i(z)$ and boundary conditions are specified. To solve Eq. 8, we expanded the soil-pile forces into discrete Fourier series as

$$\{F^i, H^i\} = \sum_n \{F_n^i, H_n^i\} \sin a_n z \quad \dots \quad (9)$$

With Eq. 9, general solutions to Eq. 8 are obtained as

$$U^i = A^i \sin \mu z + B^i \cos \mu z + C^i \sinh \mu z + D^i \cosh \mu z \\ + \sum_n \frac{4u_o \omega^2 m_p / n\pi - F_n^i}{EI a_n^4 - \omega^2 m_p} \sin a_n z \\ W^i = K^i \sin \tilde{\mu} z + L^i \cos \tilde{\mu} z - \sum_n \frac{H_n^i}{EA a_n^2 - \omega^2 m_p} \sin a_n z \quad \dots \quad (10)$$

where $\mu^4 = \omega^2 m_p / EI$, $\tilde{\mu}^2 = \omega^2 m_p / EA$, and A^i, B^i, C^i, D^i, K^i , and L^i are integration constants.

Soil-Pile Compatibility Conditions. According to Eq. 7, the horizontal and vertical soil movements at the center line of the i -th pile, (x^i, y^i) , can be represented as

$$U(x^i, y^i, z) = \sum_n \left\{ \frac{4u_o}{n\pi} + \frac{4u_o k^2}{n\pi(a_n^2 - k^2)} + \sum_{j=1}^N \sigma^{ij} F_n^j \right\} \sin a_n z e^{i\omega t} \\ W(x^i, y^i, z) = \sum_n \left\{ \sum_{j=1}^N \tau_n^{ij} H_n^j \right\} \sin a_n z e^{i\omega t} \quad \dots \quad (11)$$

These soil displacements equal the i -th pile displacements to maintain the soil-pile compatibility condition. Using Eqs. 10 and 11, we represented the condition as

$$\begin{aligned}
 & A^i \sin \mu z + B^i \cos \mu z + C^i \sinh \mu z + D^i \cosh \mu z \\
 & + \sum_n \frac{4u_o \omega^2 m_p / n\pi - F_n^i}{EI a_n^4 - \omega^2 m_p} \sin a_n z \\
 & = \sum_n \left\{ \frac{4u_o}{n\pi} + \frac{4u_o k^2}{n\pi(a_n^2 - k^2)} + \sum_{j=1}^N \sigma_n^{ij} F_n^j \right\} \sin a_n z \dots\dots\dots (12)
 \end{aligned}$$

$$\begin{aligned}
 & K^i \sin \bar{\mu} z + L^i \cos \bar{\mu} z - \sum_n \frac{H_n^i}{EA a_n^2 - \omega^2 m_p} \sin a_n z \\
 & = \sum_n \left\{ \sum_{j=1}^N \tau_n^{ij} H_n^j \right\} \sin a_n z
 \end{aligned}$$

When we expand $\sin \mu z$, $\cos \mu z$, $\sinh \mu z$, $\cosh \mu z$, $\sin \bar{\mu} z$, and $\cos \bar{\mu} z$ into discrete Fourier series, Eq. 12 reduces to the following matrix form.

$$\begin{aligned}
 [\Phi]_n \begin{bmatrix} F_n^1 \\ \vdots \\ F_n^N \end{bmatrix} &= \frac{4u_o}{n\pi} \left(\frac{\omega^2 m_p}{EI a_n^4 - \omega^2 m_p} - \frac{k^2}{a_n^2 - k^2} \right) \begin{bmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{bmatrix} + E_n \begin{bmatrix} A^1 \\ \vdots \\ A^N \end{bmatrix} \\
 & \quad + F_n \begin{bmatrix} B^1 \\ \vdots \\ B^N \end{bmatrix} + I_n \begin{bmatrix} C^1 \\ \vdots \\ C^N \end{bmatrix} + J_n \begin{bmatrix} D^1 \\ \vdots \\ D^N \end{bmatrix} \\
 [\Psi]_n \begin{bmatrix} H_n^1 \\ \vdots \\ H_n^N \end{bmatrix} &= M_n \begin{bmatrix} K^1 \\ \vdots \\ K^N \end{bmatrix} + N_n \begin{bmatrix} L^1 \\ \vdots \\ L^N \end{bmatrix} \dots\dots\dots (13a)
 \end{aligned}$$

where the matrices $[\Phi]_n$ and $[\Psi]_n$, and the Fourier coefficients E_n , F_n , I_n , J_n , M_n , and N_n are given as

$$\begin{aligned}
 [\Phi]_n &= [\sigma_n^{ij}] + \frac{1}{EIa_n^4 - \omega^2 m_p} \begin{bmatrix} 1 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix} \\
 [\Psi]_n &= [\tau_n^{ij}] + \frac{1}{EAa_n^2 - \omega^2 m_p} \begin{bmatrix} 1 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix}
 \end{aligned}
 \dots\dots (13b)$$

$$\sin \mu z = \sum_n E_n \sin a_n z, \quad \cos \mu z = \sum_n F_n \sin a_n z$$

$$\sinh \mu z = \sum_n I_n \sin a_n z, \quad \cosh \mu z = \sum_n J_n \sin a_n z$$

$$\sin \bar{\mu} z = \sum_n M_n \sin a_n z, \quad \cos \bar{\mu} z = \sum_n N_n \sin a_n z$$

Eq. 13a can be solved for F_n^i and H_n^i as

$$\begin{aligned}
 F_n^i &= \frac{4u_o}{n\pi} \left(\frac{\omega^2 m_p}{EIa_n^4 - \omega^2 m_p} - \frac{k^2}{a_n^2 - k^2} \right) \sum_{j=1}^N \bar{\sigma}_n^{ij} + E_n \sum_{j=1}^N \bar{\sigma}_n^{ij} A^j \\
 &\quad + F_n \sum_{j=1}^N \bar{\sigma}_n^{ij} B^j + I_n \sum_{j=1}^N \bar{\sigma}_n^{ij} C^j + J_n \sum_{j=1}^N \bar{\sigma}_n^{ij} D^j \\
 H_n^i &= M_n \sum_{j=1}^N \bar{\tau}_n^{ij} K^j + N_n \sum_{j=1}^N \bar{\tau}_n^{ij} L^j
 \end{aligned}
 \dots\dots (14)$$

where $\bar{\sigma}_n^{ij}$ and $\bar{\tau}_n^{ij}$ are the (i,j) element of the inverse matrices of $[\Phi]_n$ and $[\Psi]_n$. With Eqs. 10 and 14, the unknown soil-pile forces can be eliminated, and the pile displacements are

$$\begin{aligned}
U^i &= A^i \sin \mu z + B^i \cos \mu z + C^i \sinh \mu z + D^i \cosh \mu z \\
&+ \sum_n \frac{4u_o/n\pi}{EIa_n^4 - \omega^2 m_p} \left\{ \omega^2 m_p - \left(\frac{\omega^2 m_p}{EIa_n^4 - \omega^2 m_p} - \frac{k^2}{a_n^2 - k^2} \right) \sum_{j=1}^N \bar{\sigma}_n^{ij} \right\} \sin a_n z \\
&- \sum_{j=1}^N \left\{ \sum_n \frac{E_n \bar{\sigma}_n^{ij}}{EIa_n^4 - \omega^2 m_p} \sin a_n z \right\} A^j - \sum_{j=1}^N \left\{ \sum_n \frac{F_n \bar{\sigma}_n^{ij}}{EIa_n^4 - \omega^2 m_p} \sin a_n z \right\} B^j \\
&- \sum_{j=1}^N \left\{ \sum_n \frac{I_n \bar{\sigma}_n^{ij}}{EIa_n^4 - \omega^2 m_p} \sin a_n z \right\} C^j - \sum_{j=1}^N \left\{ \sum_n \frac{J_n \bar{\sigma}_n^{ij}}{EIa_n^4 - \omega^2 m_p} \sin a_n z \right\} D^j
\end{aligned}$$

$$W^i = K^i \sin \mu z + L^i \cos \mu z$$

$$\begin{aligned}
&- \sum_{j=1}^N \left\{ \sum_n \frac{M_n \bar{\tau}_n^{ij}}{EAa_n^2 - \omega^2 m_p} \sin a_n z \right\} K^j \\
&- \sum_{j=1}^N \left\{ \sum_n \frac{N_n \bar{\tau}_n^{ij}}{EAa_n^2 - \omega^2 m_p} \sin a_n z \right\} L^j
\end{aligned}$$

..... (15)

Thus, the pile-group response can be obtained by determining the unknown integration constants $(A^i, B^i, C^i, D^i, K^i, L^i; i=1, 2, \dots, N)$ using appropriate boundary conditions for the piles.

Boundary Conditions. The most general boundary conditions for the present analytic model are: (1) piles are pinned at the rigid base, (2) pile heads are connected rigidly to the pile cap, and (3) moment as well as shear is applied at the pile cap. Under seismic loading conditions, the moment and shear at the pile cap represent the inertia loads from the superstructure. The pinned tip condition yields $B^i, D^i,$ and L^i to be zeroes. The assumption of the rigid connection at the pile head gives the following $2N-1$ equations.

$$U^1(H) = U^2(H) = \dots = U^N(H) = u_b \quad \dots \quad (16)$$

$$\left. \frac{dU^1}{dz} \right|_{z=H} = \left. \frac{dU^2}{dz} \right|_{z=H} = \dots = \left. \frac{dU^N}{dz} \right|_{z=H} = \zeta \quad \dots \quad (17)$$

where u_b = pile-cap lateral displacement relative to the rigid base, and ζ = rotation of the pile cap. Also, the geometric constraint for the pile-cap rotation calls for additional N equations

$$W^i(H) = -\zeta x^i \quad (i=1,2,\dots,N) \quad \dots \quad (18)$$

where x^i is the arm-length of the i -th pile measured from the axis of rotation, Plate 3.

Finally the boundary conditions that specify the loading on the pile cap yield

$$EI\mu^3 \left\{ \sum_{i=1}^N A^i \cos\mu H + \sum_{i=1}^N C^i \cosh\mu H \right\} = L \quad \dots \quad (19)$$

$$EA\bar{\mu} \cos\bar{\mu} H \sum_{i=1}^N (x^i)^2 = -M$$

Eqs. 16 through 19 give $3N+1$ number of equations. Since we also have $3N+1$ unknowns ($A^1, \dots, A^N, C^1, \dots, C^N, K^1, \dots, K^N$, and ζ), all unknowns can be determined to obtain the pile-group response.

Structural Response. Detailed procedures to evaluate the response of the superstructure on Plate 1 were presented by Kagawa and Kraft (1981). The method is based on the mode superposition procedure. Base shear and moment are obtained as a function of pile-cap displacement and rotation as

$$L = K_H (u_o + u_b) + K_{HR} \zeta \quad \dots \quad (20)$$

$$M = K_{RH} (u_o - u_b) + K_R \zeta$$

where $K_H = 12EI_n \{A_m\}_n / h_n^3$, $K_{HR} = 12EI_n \{A_r\}_n / (\omega^2 h_n^3) - 6EI_n / h_n^2$, $K_{RH} = 6EI_n \{A_m\}_n / h_n^2$, and $K_R = 6EI_n \{A_r\}_n / (\omega^2 h_n^2) - 4EI_n / h_n$.

$\{A_m\}$ and $\{A_r\}$ are given as

$$\{A_m\} = [\phi] \{T_j(\omega) ([\phi]^T \{m_s\})_j\} \quad \dots \quad (21)$$

$$\{A_r\} = [\phi] \{T_j(\omega) ([\phi]^T \{r\})_j\}$$

where $[\phi]$ = modal matrix, $T_j(\omega)$ = dynamic magnification of the j -th mode defined by $\omega^2 / [K_j^* \{1 - (\omega/\omega_j)^2 + i2\beta_j(\omega/\omega_j)\}]$, K_j^* = generalized stiffness of the j -th mode, ω_j = circular natural frequency for the j -th mode, β_j = damping ratio for the j -th mode, $\{m_s\}^T = (m_1, \dots, m_n)$, and $\{r\}^T = (0, \dots, 0, 6EI_n/h_n^2)$. Eq. 20 can be solved with Eqs. 16 through 19 to obtain the seismic response of the pile groups.

Soil-Pile Interface Stress Patterns. To study the pile-group effects, we considered two types of soil-pile interface stress patterns for the lateral pile vibration: uniform and Boussinesq types, Plate 4. These stress patterns at x^i and y^i are described by

$$\begin{aligned} f^i(x, y, z) &= \sum_n \bar{f}_n^i \delta(x-x^i) S(y-y^i) \sin a_n z \quad (\text{uniform}) \\ &= \sum_n \bar{f}_n^i \delta(x-x^i) \frac{2/\pi}{\sqrt{1-(y-y^i)^2/r_o^2}} \sin a_n z \quad (\text{Boussinesq}) \\ &\dots\dots\dots (22) \end{aligned}$$

where $S(y)$ = box function defined in the interval (y^i-r_o, y^i+r_o) . Using Eq. 22, we can represent Eq. 2 as

$$\begin{aligned} F^i(z) &= \int f^i(x, y, z) dx dy \\ &= 2r_o \sum_n \bar{f}_n^i \sin a_n z \quad (\text{uniform and Boussinesq}) \\ &\dots\dots\dots (23) \end{aligned}$$

Also, the Fourier transform of Eq. 22 yields

$$\begin{aligned} \mathcal{F}[f^i] &= \sum_n \frac{\bar{f}_n^i}{\pi} \frac{\sin(\beta r_o)}{\beta} e^{i(\alpha x^i + \beta y^i)} \sin a_n z \quad (\text{uniform}) \\ &= \sum_n \frac{\bar{f}_n^i}{\pi} r_o J_0(\beta r_o) e^{i(\alpha x^i + \beta y^i)} \sin a_n z \quad (\text{Boussinesq}) \\ &\dots\dots\dots (24) \end{aligned}$$

where $J_0(\)$ is the 0-th order Bessel function of the first kind.

For vertical vibration, we considered a "cross-blade" type soil-pile interface stress pattern where each blade has a uniform stress pattern.

$$h^i(x, y, z) = \sum_n \bar{h}_n^i \{ \delta(x-x^i)S(y-y^i) + S(x-x^i)\delta(y-y^i) \} \sin a_n z \quad \dots\dots (25)$$

For this stress pattern, $H^i(z)$ and the Fourier transform of h^i are given as

$$H^i(z) = 4r_o \sum_n \bar{h}_n^i \sin a_n z$$

$$\mathcal{F}[h^i] = \sum_n \frac{\bar{h}_n^i}{\pi} \left\{ \frac{\sin(\alpha r_o)}{\alpha} + \frac{\sin(\beta r_o)}{\beta} \right\} e^{i(\alpha x^i + \beta y^i)} \sin a_n z \quad \dots\dots (26)$$

Eqs. 23, 24, and 26 can be incorporated into Eqs. 7 and 10 to determine the pile-group response. The integrations in Eq. 7 were evaluated numerically.

Evaluation of Analytic Method

Appropriate Soil-Pile Stress Patterns. Soil stress distribution patterns around piles depend largely on exciting frequency and pile spacing ratios, among other factors. Use of a soil-pile stress pattern that is independent of these factors may result in erroneous pile response. Therefore, the effects of variations of soil-pile stress patterns on pile response were evaluated in detail for the lateral vibration mode to select a simple but reasonably accurate soil-pile stress pattern.

To generate exact pile-soil-pile interaction, we discretized a pile into several strip segments as shown on Plate 5. Uniform stress acts over the width of each segment, but its magnitude is determined using the condition that all segments within a pile move together. Six to eight strip segments usually are required to approximate the correct stress variation over the width of a pile. Although the method may be attractive from an academic view point, the method is prohibitively expensive for a large number of piles. Therefore, the pile-soil-pile interaction was obtained using simple soil-pile stress patterns, and the results were evaluated in the light of solutions from this rigorous

method. The simple stress distribution patterns used in this study included the uniform and Boussinesq types as described by Eqs. 22 and 25.

Plate 6 shows the lateral pile-head spring and damping constants of a pile for both uniform and Boussinesq type soil-pile stress patterns. Also shown on Plate 6 are the mathematical solutions for a pile with a circular cross section by Kagawa and Kraft (1981). For the uniform and Boussinesq type stress patterns, the soil displacements at the center of a loaded area were considered to be representative of the soil motion due to the soil-pile stresses. The soil displacements caused by these soil-pile stresses, however, vary from the center to the edge of the loaded area, and the soil near the center moves more than that at the edge. Therefore, the pile stiffnesses and dampings from these stress patterns are less than the theoretical solutions. The Boussinesq type stress pattern, however, provided good approximation to the theoretical solutions at low frequency ratios, although it resulted in pile stiffnesses similar to that of the uniform stress pattern at high frequencies. Pile responses with the uniform stress distribution can be improved by averaging soil displacements, as frequently done for footing vibration problems (e.g. Housner and Castellani, 1969). A simple alternative to the weighted average concept is to reduce the computed soil displacement. Thus, a reduction factor of 0.85 was applied in our study to compute pile responses. Plate 6 shows that the pile stiffness using the uniform stress pattern with a 0.85 factor provides a good match of the rigorous solution up to frequency ratios of about 40. The damping, however, is still underestimated. A similar tendency is observed also for the weighted average soil-pile spring coefficient $\bar{\delta} = \bar{\delta}_1 + i\bar{\delta}_2$, Plate 7. Thus, the impact of this deficiency on the response of pile groups will be evaluated below.

Plate 8 shows the effects of the difference in assumed soil-pile stress patterns on 2-pile responses. The two piles are located in-line with loading, with the directional angle θ being equal to zero. Pile responses were obtained using the three

different soil-pile stress patterns: (1) uniform stress pattern with a factor of 0.85, (2) Boussinesq type stress pattern, and (3) variable stress pattern that simulates exact solutions (six strip segments were used). Plate 8 shows clearly that the uniform stress pattern approximates closely the exact solutions and that the Boussinesq type stress pattern results in erroneous pile-group effects for closely-spaced piles. Thus, the uniform stress pattern with a factor of 0.85 was used in the remaining study to evaluate pile-group effects.

The vertical vibration mode is secondary to the objective of the present study. Therefore, the validity of the "cross-blade" type distribution was confirmed by comparing the single pile responses obtained from this distribution and the theoretical solution derived by the author previously. The two solutions compared favorably.

Comparison with Other Solution. The lateral pile-group effects from our method were compared with those by Poulos (1979) for static pile-head loading conditions, Plate 9. Our results are based on the piles fully embedded into the soil layer, whereas Poulos' results are for a floating pile group. The group deflection factor here is defined, according to Poulos (1979), as the ratio of the pile-group displacement to the displacement of a free-head pile carrying average load per pile in the group. Our results are in general agreement with those by Poulos (1979), although the present solutions are rather insensitive to the variation of K_R for the fixed-head condition.

Pile-Group Effects

Major Factors Affecting Pile-Group Effects. Pile-soil-pile interaction is due to the soil motion generated by the vibration of piles. Typical soil motion attenuations around a pile for the lateral vibration mode are shown on Plate 10 for frequency ratios of 10 and 30. The soil motion generated by the pile vibration depends strongly on frequency and involves phase changes along the distance from the pile. Thus, the soil motion at the second pile generated by the first pile is not in phase with the motion

of the second pile, and effective stiffness and damping of the second pile are altered due to the vibration of the first pile. The spacing ratio and the excitation frequency, therefore, have large effects on pile-group responses.

The effects of spacing ratio and frequency on two pile interaction are shown on Plate 11. Since the soil motion generated by the pile vibration vanishes at a large distance from the pile, both spring and damping constants of a pile in a group approach those of a single pile for large pile spacing. Under the static condition, this distance is about 30 pile diameters for the lateral mode and about 20 pile diameters for the vertical mode. As the frequency increases, the ratios of dynamic to static spring and damping constants exhibit wavy variations with the spacing ratio. These ratios, however, are nearly unity at spacing ratios of about 50.

The influence of the pile slenderness ratio $H/2r_o$ on the lateral pile-group effects are shown on Plate 12. The pile slenderness ratio, $H/2r_o$, has a similar influence on the vertical pile-group effects. The influence of $H/2r_o$ on lateral as well as vertical pile-group effects, however, can be masked if we adopt a nondimensional frequency $\omega r_o/V_s$ as the frequency scale instead of the frequency ratio f/f_r , Plates 13 and 14. In these figures, the pile radius was varied while the pile length, H , was kept constant. With the effects of the pile slenderness ratio, $H/2r_o$, masked, we can redefine the relative stiffnesses between the soil and pile as in Eq. 27 that were kept constant in these results.

$$\tilde{K}_R = EI/(E_s r_o^4) \quad (\text{local pile flexibility}) \quad \dots\dots (27)$$

$$\tilde{K}_C = EA/(E_s r_o^2) \quad (\text{local pile compressibility})$$

The results on Plates 13 and 14 indicate that the pile-group effects depend primarily on the spacing ratio and the nondimensional frequency for given soil-pile relative stiffness. The local pile flexibility and compressibility factors \tilde{K}_R and \tilde{K}_C in Eq. 27 have relatively minor effects on the

pile-soil-pile interaction for the range of flexible and compressible piles examined in this study, Plates 15 and 16. The spacing ratio and the nondimensional frequency have larger impact on dynamic pile-group effects than \bar{K}_R and \bar{K}_C .

Directionality. 2-pile interaction is affected by the directional angle as well as absolute distance between the piles. The directional angle θ is defined on Plate 17 where the angle is measured from the x axis counter clockwise. On Plate 17, the two piles have the same directional angle θ . Our study showed that the 2-pile interaction is about the same when the first pile is at the origin of the local coordinates (\bar{x}, \bar{y}) and the second pile is located on an ellipse

$$\bar{x}^2 + (2\bar{y})^2 = (\text{constant})^2 \quad \dots\dots (28)$$

This observation was examined for two piles with directional angles of 0 and 90 degrees. Plate 18 demonstrates the 2-pile interaction results for the cases with the constant in Eq. 28 being equal to 4 and 16 pile diameters. This observation is nearly exact under static loading conditions and only approximate at high frequencies.

Spring and Damping Constants of Pile Group. Spring and damping constants of a pile group can be evaluated using 2-pile interaction results. Our analytic model is linearly elastic. Therefore, 2-pile interaction results can be superposed to obtain the pile-group effects for any number of piles. A superposition procedure to obtain the spring and damping constants of a pile group is summarized below.

When d_{ij}^H is the lateral complex pile-head displacement of the i-th pile due to the unit lateral pile-head forces on the i-th and j-th piles, the complex lateral displacement of the i-th pile d_i^H , is given as

$$P_1(d_{i1}^H - d_{ii}^H) + \dots + P_{i-1}(d_{i,i-1}^H - d_{ii}^H) + P_i d_{ii}^H + P_{i+1}(d_{i,i+1}^H - d_{ii}^H) + \dots \\ + P_N(d_{iN}^H - d_{ii}^H) = d_i^H \quad \dots\dots (29)$$

where P_i = lateral complex pile-head force on the i-th pile. d_{ij}^H equals d_{ji}^H due to reciprocity. When we set d_i^H to unity, the

pile-head forces P_1, \dots, P_N represent the complex pile stiffnesses that include the pile-group effects. The sum of these forces gives the complex stiffness of the pile group. Eq. 29 can be simplified and represented also as

$$P_1(\Delta_{i1}^H - 1) + \dots + P_{i-1}(\Delta_{i,i-1}^H - 1) + P_i + P_{i+1}(\Delta_{i,i+1}^H - 1) + \dots + P_N(\Delta_{iN}^H - 1) = d_i^H / d_{ii}^H \quad \dots \quad (30)$$

where $\Delta_{ij}^H = d_{ij}^H / d_{ii}^H = k_{ij}^H / k_{ii}^H$. k_{ij} is the complex lateral stiffness of the i -th pile when the i -th and j -th piles carry the same pile-head loads. Spring and damping values for a group of piles in the rocking vibration mode can be evaluated similarly with vertical 2-pile interaction results. With a complex pile stiffness k computed, equivalent spring and damping coefficients of the pile are obtained as

$$k = K + i\omega C \quad \dots \quad (31)$$

Dynamic stiffness and damping of pile groups are shown on Plates 19 and 20 for 2x2 and 3x3 groups. Dynamic stiffness of a pile group is strongly frequency-dependent. The dynamic stiffness of a pile group can be larger than the stiffness estimated from the static stiffness without the pile-group effects. For clustered pile groups, the dynamic stiffness can be negative. Also, the damping for a pile group shows a wavy variation with frequency. The damping has several peaks. The large peaks tend to occur at higher frequencies for piles with closer spacings.

The major difference between the damping characteristics of a surface footing and a pile foundation was studied. Plate 21 compares the damping of pile groups with that of surface footings. The damping is essentially due to radiation damping except below frequency ratio of 1.0 where material damping is dominant. Three foundation sizes are considered with the plan dimensions of the surface footings equal to the outer dimensions of the pile groups. Plate 21 shows that the damping of a pile group can be larger than that of an equivalent surface footing at low frequencies when the piles are closely spaced. The damping of a pile group at high frequencies, however, does not increase lin-

early with frequency, and can be considerably lower than that of an equivalent surface footing for widely spaced piles. For most seismic response analyses of pile foundations that involve low frequency responses, damping may be approximated by that of a surface footing. For high frequency machine foundation problems, it appears appropriate not to account for radiation damping.

Shear and Moment Distributions. Pile-head shear and moment distributions among piles can be affected considerably by the pile-group effects. Distribution patterns of the pile-head shear and moment vary with frequency. The pile-head shear and moment distribution patterns computed for static loading conditions may be invalid for dynamic loading conditions. Plate 22 shows the distributions of shear forces for a 3x3 group with spacing ratios of 2, 4, and 8. For the case with a spacing ratio of 2, the center pile carries the least load under the static condition and the maximum load at a frequency ratio of 60. Thus, the effective stiffness of the center pile increased by a factor of about 3 due to the pile-group effects as the frequency ratio increased from 0 to 60. On the other hand, the corner pile became less stiff as the frequency increased. For pile groups with large spacings, the variations become less pronounced. Similar phenomena are observed for the pile-head moment distribution patterns on Plate 23.

Seismic Pile-Group Effects. Effective stiffness of a pile is different theoretically when the pile is loaded either at the pile head or seismically. The difference is due to the pile deformation due to the free-field excitation. To study the effect of the difference in loading condition on the pile-group response, we computed the effective stiffnesses of pile groups by applying sinusoidal lateral excitation at the rigid base. Our study indicated that the pile-group effects are essentially independent of loading conditions for flexible piles. Thus, the effective stiffness and damping of a pile group for a seismic response analysis can be determined using single-pile responses under the seismic loading condition and the pile group effects obtained for the pile-head loading condition.

Soil-Pile Springs and Damping. Equivalent soil-pile springs provide useful information to the determination of the soil-pile springs for a beam-on-Winkler foundation analysis of piles. Equivalent soil-pile springs were evaluated for the present analytic model following the method by Kagawa and Kraft (1980a, 1980b, 1981) for single piles. The lateral load-displacement relation (p-y) was defined as

$$p = E_s^* \delta^H y \quad \dots\dots (32)$$

where p = lateral soil reaction on a unit pile length, E_s^* = complex Young's modulus of soil ($=2(1+\nu) G_s^*$). δ^H = lateral soil-pile spring coefficient ($=\delta_1^H + i\delta_2^H$), and y = pile displacement relative to the free field. All quantities in Eq. 32 are complex numbers. The real part of δ^H , δ_1^H , is the "true" soil-pile spring coefficient, and the imaginary part represents the material as well as the radiation damping associated with the soil-pile interaction. The soil-pile spring coefficient δ^H varies with depth even for homogeneous soil conditions. Equivalent average of δ^H with depth, however, can be defined as (Kagawa and Kraft, 1980b).

$$\bar{\delta}^H = \int_0^H \delta^H y^2 dz / \int_0^H y^2 dz \quad \dots\dots (33)$$

which may be used with a beam-on-Winkler foundation model for layered soil conditions (Kagawa and Kraft, 1980b).

The axial load-displacement relation (f-z) can be defined in a similar manner as

$$f = E_s^* \delta^V z \quad \dots\dots (34)$$

where f = axial soil reaction on a unit pile length, δ^V = soil-pile spring coefficient ($=\delta_1^V + i\delta_2^V$), and z = axial pile displacement. Equivalent average of δ^V with depth can be defined similar to Eq. 33.

The average soil-pile spring coefficients are related uniquely to pile-head spring constants. When we consider a pile loaded with a unit lateral force at the pile head, we can equate the work done at the pile head to the work done along the embedded portion of the pile as

$$(1) d = \int_0^H py dz \quad \dots\dots (35)$$

where d = pile-head lateral displacement. With Eqs. 32 and 33, Eq. 35 can be reduced to

$$d = E_s^* \bar{\delta}^H \int_0^H y^2 dz \quad \dots\dots (36)$$

Thus, the pile-head spring constant is given as

$$k^H = 1/d = 1.0 / \{ E_s^* \bar{\delta}^H \int_0^H y^2 dz \} \quad \dots\dots (37)$$

Eq. 37 shows that the average soil-pile spring coefficient is related to the pile-head spring constant through the integration of y^2 . A similar relation holds for the vertical vibration mode. Our study showed that the behavior of $\bar{\delta}^H$ and $\bar{\delta}^V$ for pile groups is very similar to that of pile-head spring and damping constants. Typical behavior of the average lateral soil-pile spring coefficient for 2 piles is illustrated on Plate 24.

Summary

Pile-group effects were studied parametrically for an idealized analytic model of a pile group. Major findings are summarized below.

- (1) Correct pile-soil-pile interaction can be assessed using uniform soil-pile stress patterns over the width of a pile for a wide frequency range, although this assumption fails for a single pile at high frequencies.
- (2) Pile-group effects are affected primarily by the non-dimensional frequency $\omega r_o / V_s$ and the spacing ratio $s/2r_o$ for flexible and compressible piles. Other factors have only minor influence on the pile-group effects.
- (3) 2-pile interaction is pronounced at the spacing ratio less than 30 for the lateral vibration mode and about 20 for the vertical vibration mode.

- (4) Stiffness of a pile group varies strongly with frequency. For a clustered pile group, the group stiffness can be negative and can be larger than a simple summation of static single pile stiffnesses.
- (5) Damping of a pile group also is frequency-dependent. Damping of a pile group can be approximated by that of an equivalent surface footing at low frequencies, where the seismic soil-pile interaction is important.
- (6) Distribution patterns of shear and moment among piles depend largely on frequency. Center piles, for example, can be stiffer and carry a greater load than corner piles under dynamic loading conditions than they do under static conditions.
- (7) Pile-group effects are strongly frequency-dependent. Pile-group efficiency from a static theory is not necessarily appropriate for designing a dynamically loaded pile foundation.

SIMPLIFIED METHOD FOR PILE GROUPS

Introduction

Pile-group effects were studied in the last section for a homogeneous, elastic soil. To study the pile-soil-pile interaction in layered soil conditions that are found in practice, we extended the results in the last section to include soil layering effects and an approximate, but efficient method was developed using the beam-on-Winkler foundation. Discrete soil-pile springs and dashpots were used to represent the continuous nature of lateral as well as axial soil resistance. These discrete parameters were determined from the equivalent dynamic soil-pile springs for single piles presented by Kagawa and Kraft (1980b) and the pile-group effects described in the last section.

Description of the analytic model for the method and the procedures to determine the discrete soil-pile springs and dashpots for a pile group will be discussed below. The approximate method

will be evaluated by comparing computed and observed seismic response data of an apartment building supported by a group of piles.

Analytic Model

The method is based on a beam-on-Winkler foundation model of a soil-pile system, Plate 25. The superstructure is modeled in the same manner as on Plate 1. The rigid foundation block is connected to the free-field surface with a sliding spring and a rotational spring to simulate the interaction between the soil and pile-cap. The foundation also can have a translational mass and a rotational mass. The soil deposit is layered with hysteretic soil damping. All vertical piles are identical. Piles are discretized following the layering of soils, and their properties can vary with depth from one element to another. Although the bottom soil layer rests on a rigid base in this analytic model and pile elements extend down to this base, floating pile groups can be analyzed in an approximate manner by assigning appropriate soil moduli to "pile elements" below pile tips. Each pile element has bending as well as axial stiffnesses, but the contribution of shear deformation to the pile stiffness is neglected. Each pile element is connected to the free-field soil through the soil-pile springs described below.

Excitation to the model can be applied either to the pile-cap or seismically. The pile-cap excitation is limited to sinusoidal steady-state shear force in the x direction and moment around the y axis. A free-field control motion can be specified at any layer boundary. The seismic excitation also is limited to the x direction.

Soil-Pile Spring for a Single Pile

Dynamic characteristics of equivalent soil-pile springs were studied in detail by Kagawa and Kraft (1980b, 1981) for a pile in a homogeneous, elastic soil layer. The study showed that the lateral soil reaction, p , at some depth on a unit pile length can be approximated by

$$p = E_s \bar{\delta}_{11}^H y + 2\rho B(V_s + V_p) \frac{dy}{dt} \quad \dots\dots (38)$$

where E_s = Young's modulus of soil, $\bar{\delta}_{11}^H$ = average soil-pile spring coefficient that is invariant with depth, ρ = mass density of soil, and B = equivalent width of a pile. The soil-pile element in Eq. 38 is shown on Plate 26. The soil-pile spring coefficient $\bar{\delta}_{11}^H$ for homogeneous, elastic soil conditions can be related to the "local pile flexibility factor," \bar{K}_R , as shown on Plate 27. $\bar{\delta}_{11}^H$ for layered, elastic soil conditions may be approximated by those for homogeneous, elastic soil profiles if a weighted soil modulus determined by Eq. 39 is used to represent the soil (Kagawa and Kraft, 1980b).

$$\bar{E}_s = \int_0^H E_s y^2 dz / \int_0^H y^2 dz \quad \dots\dots (39)$$

Similarly, the vertical soil reaction f for a homogeneous soil condition at some depth on a unit pile length can be approximated by

$$f = E_s \bar{\delta}_{11}^V z + 4 \rho B V_s \frac{dz}{dt} \quad \dots\dots (40)$$

where $\bar{\delta}_{11}^V$ = average soil-pile spring coefficient. The soil-pile spring coefficient $\bar{\delta}_{11}^V$ is related to the "local pile compressibility factor," \bar{K}_C , as shown on Plate 28. $\bar{\delta}_{11}^V$ for a layered soil condition and the average soil modulus \bar{E}_s are defined similar to the p - y relation.

Soil-Pile Springs for a Pile Group

The soil-pile springs for piles in a group are affected by the pile-group effects. The p - y and f - z relations for a single pile cannot be used directly in a pile-group analysis. An approximate simple method based on the results of homogeneous soil conditions is presented below to determine the soil-pile springs for a pile group. In this approximate method, we considered that the pile-group effects for layered soil conditions could be approximated by those of homogeneous soil conditions with the weighted moduli in Eq. 39. For this equivalent pile-group, we can relate easily the discrete soil-pile springs

and dashpots to pile-head stiffness and damping, and the pile-group effects can be determined from the equilibrium conditions at pile heads.

Correction Factors for Pile-Group Effects. For the lateral vibration mode, the complex lateral pile-head displacement of the i -th pile can be written as Eq. 30. We set the ratio between the i -th pile-head displacement in the group and the pile-head displacement for a single pile condition, d_i^H/d_{ii}^H , to be unity and solve for pile-head forces P_i ($i=1,2,\dots,N$). These pile-head quantities can be related to the lateral soil reaction on the i -th pile, p_i , and the pile displacement relative to the free field, y_i , by equating the work done at the pile head and along the pile depth.

$$\begin{aligned} P_i/d_{ii}^H &= \int_0^H p_i y_i dz \\ &= E_s^* \bar{\delta}_i^H \int_0^H y_i^2 dz \end{aligned} \quad \dots\dots (41)$$

where $\bar{\delta}_i^H$ = average soil-pile spring coefficient for the i -th pile with the pile-group effects. Our study showed that the integral in Eq. 41 can be approximated with good accuracy by

$$\int_0^H y_i^2 dz = \left[\sum_j^N P_j (\bar{y}_{ij}/d_{ii}^H) - \sum_{j \neq i}^N P_j (\bar{y}_{ii}/d_{ii}^H) \right]^2 \quad \dots\dots (42)$$

where the second summation excludes the i -th term. The root mean square integral of the pile displacement \bar{y}_{ij} and d_{ii}^H are given as

$$\begin{aligned} \bar{y}_{ij} &= \left[\int_0^H y_{ij}^2 dz \right]^{0.5} \\ d_{ii}^H &= d_{11}^H = E_s^* \bar{\delta}_{11}^H \bar{y}_{11}^2 \end{aligned} \quad \dots\dots (43)$$

where y_{ij} is the pile displacement of a 2-pile group with unit lateral loads on both piles.

With Eqs. 41 through 43, we can express the ratio between $\bar{\delta}_i^H$ and $\bar{\delta}_{11}^H$ as

$$\bar{\delta}_i^H / \bar{\delta}_{11}^H = P_i / \left[\sum_j^N P_j (\bar{y}_{ij} / \bar{y}_{ii}) - \sum_{j \neq i}^N P_j j^2 \right] \dots \dots \dots (44)$$

which represents the correction factor for the i-th pile on the p-y relation in Eq. 38 to include the pile-group effects.

For the rocking mode, d_{ij}^H in the above procedure is replaced by that of the vertical mode and d_i^H is represented as ζx^i . x^i is the x coordinate of the i-th pile when the origin of the x coordinate is located at the center of gravity of the pile group. ζ is the rotation of the pile-cap. The correction factor for the i-th pile on the f-z relation in Eq. 40 is obtained as

$$\bar{\delta}_i^V / \bar{\delta}_{11}^V = P_i x^i / \left[\sum_j^N P_j (\bar{y}_{ij} / \bar{y}_{ii}) - \sum_{j \neq i}^N P_j j^2 \right] \dots \dots \dots (45)$$

where y_{ij} is defined for the pile displacement for the vertical vibration mode.

The correction factors on the p-y and f-z relations are determined using Eqs. 44 and 45 and 2-pile interaction results. Lateral 2-pile interaction, however, is influenced by the positions of the piles relative to the load direction and the pile spacing. Thus, an extensive amount of data on the lateral 2-pile interaction is required to obtain the correction factors for any arrangement of piles. To simplify the procedure, we adopted an assumption that the 2-pile interaction is about the same when the second pile is located on an ellipse represented by Eq. 28. This assumption is nearly exact under static loading conditions, and it provides a reasonable assumption at low frequencies where the seismic soil-pile interaction is important. With this assumption, we determine the 2-pile interaction at any relative position using the results at a specific directional angle.

Evaluation of Correction Factors. Using the approximate procedure, we predicted the correction factors on the p-y and f-z relations for homogeneous soil conditions. The results were compared with those from the theoretical solutions in the last section. An example of this comparison for the lateral vibration

mode is shown on Plate 29 for the corner piles of 2x2 and 3x3 groups. The approximate procedure is shown to be very good at low frequencies. Even at high frequencies, the procedure provides reasonable estimates of soil-pile spring coefficients.

Numerical Procedure

The procedure was automated into a computer program "PILES" to perform dynamic response analyses of pile groups loaded either at pile caps or seismically. The major steps of computation are illustrated on Plate 30.

Analysis is done in the frequency domain using the complex response method. Seismic loading is represented as a superposition of sinusoidal waves using the Fast Fourier Transform technique, and responses of the soil-pile-structure system are computed at discrete frequencies. The computed responses are summed using the inverse Fast Fourier Transform to generate time histories of responses of the system.

Responses of the structure and the piles are solved independently with unknown pile-cap movements and loads. Complete responses of the structure and the piles are determined by eliminating the unknown pile-cap quantities using pertinent boundary conditions at the pile cap.

CASE STUDY

Introduction

General validity of the pile-group method "PILES" presented in the last section was evaluated with a case study of the seismic response of a pile-supported apartment building. Computed results were compared with field observational data to evaluate if stiffness as well as damping characteristics of a pile group can be represented correctly by the program PILES. Observed data were of low seismic intensity. Therefore, the predictions were limited to the linear range, although the program PILES may be modified to incorporate soil nonlinearity effects using the equivalent-linear parameter concept (Seed and Idriss, 1969).

Earthquake Response Observation

The field observational data that we used in our study were obtained by the Building Research Institute of Japan. Details of the observational data and analyses were published, for example, by Sugimura (1975), the Building Research Institute (1976), and the Japan Residence Corporation (1979). Major points are extracted from these publications and are described below.

Earthquake response measurements have been made since 1972 for a 14-story reinforced concrete, apartment building located at Toshima 5-Chome, Kita-ku, Tokyo. Plate 31 shows the plan views of the building and the foundation pile layout. The building is supported by 44, 4-pile groups (prestressed concrete piles, length=25 m, diameter=0.6-0.7 m, wall thickness=0.09-0.10 m). Typical spacing ratio within a 4-pile group is 2.3. The soil conditions at the site and the points of acceleration measurement at which seismic response data were available for this study are shown on Plate 32. The piles are driven through the soft layers and are supported by the underlying stiff sandy gravel layer with SPT values exceeding 50. Seismic response data recorded were mostly of low intensity. The seismic events used in this study had maximum acceleration amplitudes at the pile tip on the order of 0.002 to 0.003g. Thus, the responses of the soil-pile-structure system were essentially within the linear range.

Analysis Procedure

The apartment building was modeled by five lumped masses connected by beams. The mass and spring data for the building were published in the report by the Japan Residence Corporation (1979), Plate 33. These data were used in our analysis with the analytic model on Plate 25. Modal damping was assumed 4 percent for all modes. The mass of the soil enclosed by the embedded portion of the foundation frame was computed and treated as a base mass in our analysis. The lateral soil resistance to the embedded portion of the foundation frame was estimated using a subgrade reaction theory and was modeled by a base spring. The

values of the base mass and the base spring used in our analysis are shown also on Plate 33.

The foundation soil above -25.5 m was discretized into ten layers and the variation of soil properties was included in the analysis. Free-field soil motion was computed for this soil column of 25.5-m thickness by specifying observed seismic records at -25.5-m depth. Only vertically propagating shear waves were included in the free-field analysis. Material damping of the soil layers was assumed 3 percent.

The piles also were discretized into ten segments. The pile foundation has more than 170 piles. To simplify our analysis, we replaced each group of four piles by an equivalent pile. Bending as well as axial stiffnesses of the four piles were assigned for this equivalent pile. But, the diameter of the equivalent pile was assumed 100 cm, from our experience, that equals 75 percent of the center-to-center distance of the 4-pile group. The flexural and axial rigidities of the equivalent pile were 7.6×10^{10} kgf-cm² and 3.0×10^9 kgf. Unit weight of the equivalent pile was 1.7 gf/cm³. No damping was considered for the piles. Although the building is supported by 44, 4-pile groups, these piles were replaced by regularly spaced 42 equivalent piles.

The observational data used here were for seismic events near Tokyo with epicenters at the south coast of the Izu Peninsula (1974) and the southwest of the Ibaraki Prefecture (1975). The epicentral distances, from the observational point, were about 155 km for the Izu event and 55 km for the Ibaraki event. Thus, the observed data for the Izu event had considerably more low frequency components than the Ibaraki event. Acceleration time histories for these events at the pile tip level are shown on Plate 34. Although the observed records included about 30-second shaking, major shaking was within the first 15 seconds. Thus, we used only the first 16-second data for our analysis. The maximum frequency used in our analysis was 4 Hz, since major dynamic amplification of the system occurs below this frequency.

Analysis Results

Comparison for the 1974 Izu Event. Plate 35 compares the observed and predicted acceleration time histories at the 14th floor and the base of the apartment building. Slight difference in the predicted and observed motions may be due to the following reasons. The observed motions do not include the very beginning of the responses. The observed motions include the frequency components up to 12.5 Hz, whereas the predicted motions include frequency components up to 4.0 Hz. Estimated damping values for the structure and the soil layers can be slightly different from those of the field condition. The predicted motions, however, are in reasonable agreement with the observed motions.

To evaluate the predicted motions in more detail, we computed acceleration response spectra of the predicted and observed motions. The results are shown on Plate 36. The predicted and observed results are in good agreement. This indicates that the predicted motions have essentially the same frequency and amplitude characteristics as those of the observed motions. The acceleration spectra of the motions at the top of the building have a pronounced peak at 1.45 Hz that corresponds to the fundamental natural frequency of the building itself, and do not possess peaks at around 1.7 Hz that is the fundamental natural frequency of the soil layer. The building base moved essentially with the free field and a strong peak occurs at the fundamental natural frequency of the soil layer. A slight input power loss is observed at the building base level: the predicted response spectrum of the building base is slightly lower than that of the free-field surface. The effects, however, are not pronounced. A similar phenomenon was observed for the pile motions at 16 m below the ground level.

Comparison for the 1975 Ibaraki Event. Similar comparisons were made for the seismic records of the Ibaraki Event. Response spectra computed for the predicted and observed motions are compared on Plate 37. The motion at the 14th floor is well reproduced by the program PILES. The predicted motions at the building base level and of the pile, however, appear somewhat

different in nature from the observed motions. The response spectra of the observed motion at the building base at frequencies less than 1.0 Hz are about four times larger than those of the motion within the pile. On the other hand, we see only a slight difference, at this frequency range, in the response spectra of the computed motions at these two points. Therefore, the differences between the predicted and the observed responses at the building base and of the pile are due to the difference in the predicted and observed free-field behavior. The observed ground surface motion involves more low frequency components than the predicted motion, and it does not show clearly the peak corresponding to the fundamental natural frequency of the soil layer at 1.7 Hz. The assumption of the vertically propagating shear waves in our analysis appears inappropriate to evaluate the free-field behavior for this event. The large dynamic amplification near the ground surface indicates that, surface waves, such as Rayleigh and Love waves, might be dominating the site response. In spite of the difference of the motions due to the difference in the free-field behavior, the predicted motions did explain the general response characteristics of the soil-pile-structure system.

Summary

Performance of the pile-group method "PILES" was evaluated in the light of seismic response observational data. The study showed that the program PILES reproduced satisfactorily the seismic response characteristics of the 14-story building supported on piles. This indicates that the dynamic stiffness and damping characteristics of the pile group were evaluated correctly using the procedure. Thus, the method may be effective for practical problems to evaluate the seismic response characteristics of pile-supported structures.

Through the analysis of the Ibaraki event, we encountered a difficulty of reproducing correctly the free-field behavior. Since most piles move together with soil except probably near the ground surface, a correct assessment of the free-field motion is a key to the seismic response evaluation of pile-supported

structure. The difference between the predicted and observed free-field behavior for the Ibaraki event might be due to the effects of surface waves. The proposed method can be modified to incorporate any type of seismic environments when such data become available.

CONCLUDING COMMENTS

The present study illustrated that pile-group effects are strongly frequency-dependent. Pile-group stiffness can be larger than the direct summation of single pile stiffnesses, and it also can be negative. Damping of a pile group is generally smaller than that of an equivalent surface footing whose dimensions are equivalent to the outer dimensions of the pile group. These results cannot be derived from static theories that have been often used to predict the dynamic pile-group effects. Use of the pile-group effects from a static theory can lead to a serious error in assessing the dynamic pile-group effects. A rational procedure is required to assess correctly the dynamic pile-group effects.

Several empirical procedures have been proposed by other investigators and used to perform design analyses of pile foundations under dynamic loading. The most commonly used procedure is based on the concept of the effective soil mass (Parmelee, Penzien, Scheffey, Seed, and Thiers, 1964) that is assumed to vibrate with piles. The effective mass accounts for the inertia force generated in soil due to pile vibration, and it represents the dynamic nature of pile-group effects. No procedure, however, is present to account for the radiation damping associated with soil-pile interaction. Based on this concept, many investigators studied seismic response problems of pile groups. Seismic responses of pile-supported structures obtained from this concept were in general agreement with observed responses (e.g. Japan Residence Corporation, 1979, and Ohta, Niwa, and Ueno, 1978). These studies suggested that the stiffness of a pile group is about the same as static pile-group stiffness and the radiation damping may be neglected for most seismic response

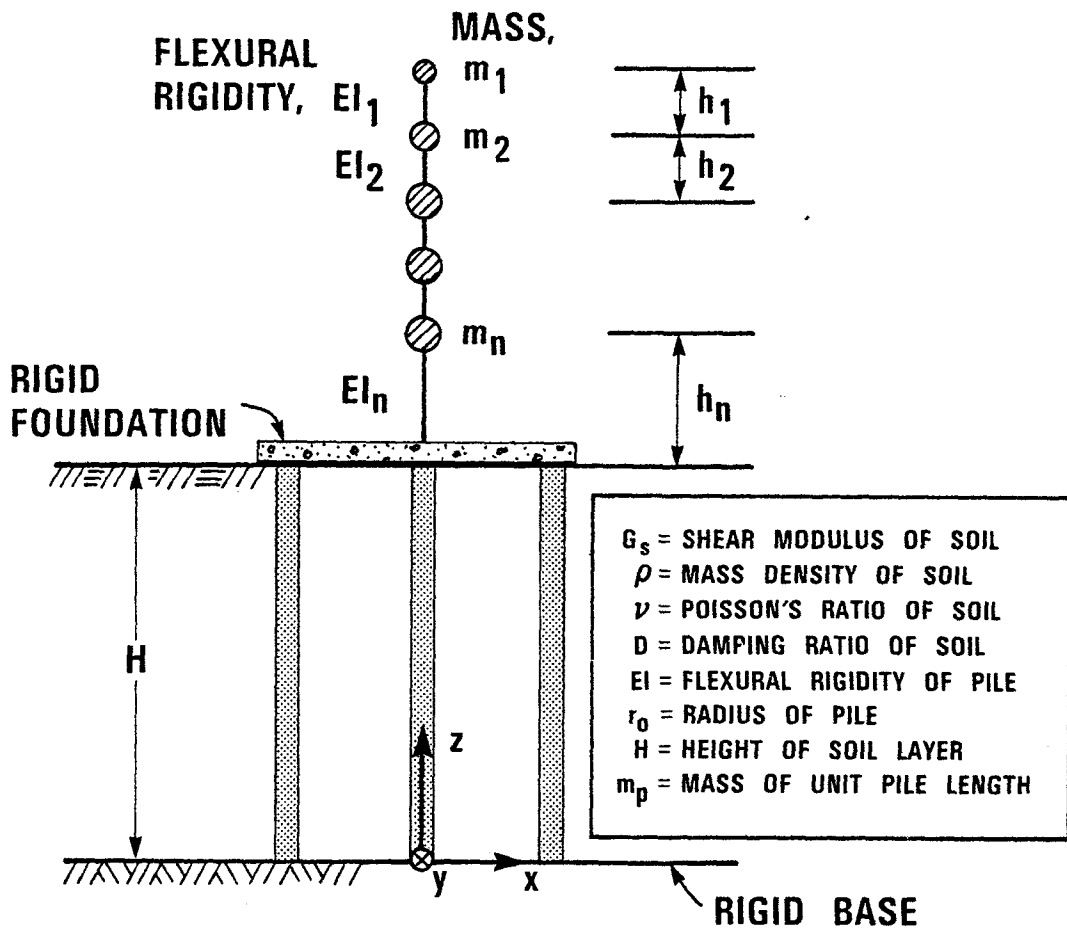
analyses. From this point of view, the case study presented in this report may not fully justify the value of the study. The value of the study is in the findings of potential significance of dynamic pile-group effects and in the development of the rational procedure that can be used to quantify the dynamic pile-group effects. The method can be used to evaluate or confirm the results obtained from rather simple empirical approaches.

Although the present study clarified the fundamental aspects of the pile-group effects and provided a method to analyze complex soil-pile-structure interaction, several important topics remain unsolved. The theories in the present study are limited to the linear range. Also, the pile-group effects are strongly frequency-dependent. Thus, the results cannot be extended directly to the time-domain analyses of pile groups that involve several forms of nonlinearity. Development of a procedure to determine soil-pile springs for nonlinear beam-on-Winkler models of pile groups may be an important subject of future research.

Most piles are known to follow the deformation of free-field soils. Correct assessment of free-field soil response is a key factor to the successful prediction of seismic soil-pile-structure interaction. Future effort should be directed to develop numerical procedures that can incorporate any form of seismic environment including surface waves.

Finally an extensive effort, of course, is required to collect and generate field observational data that can be used to calibrate numerical methods currently available.

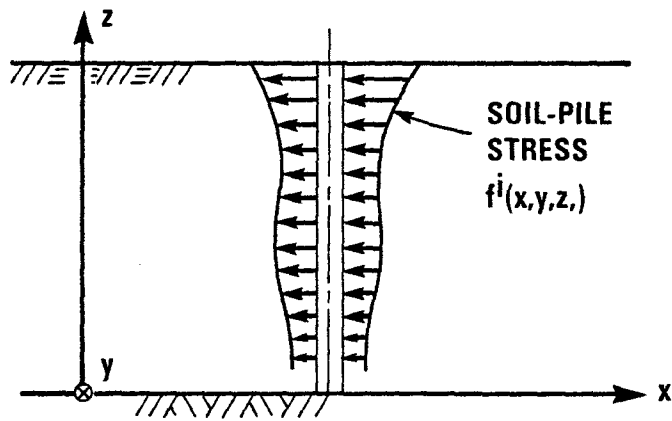
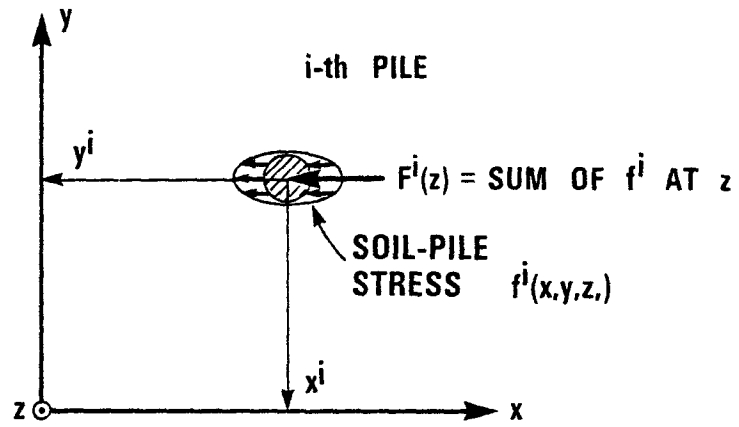
I L L U S T R A T I O N S



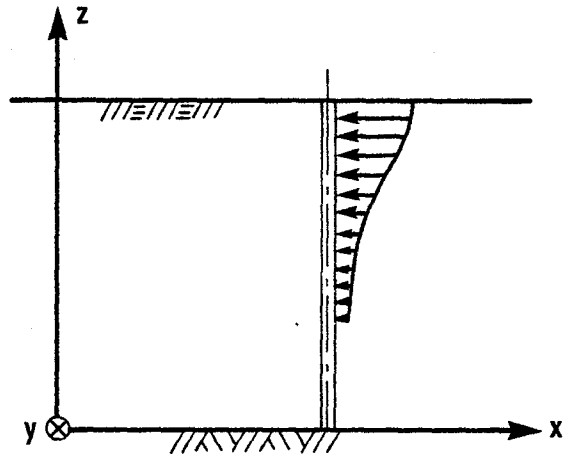
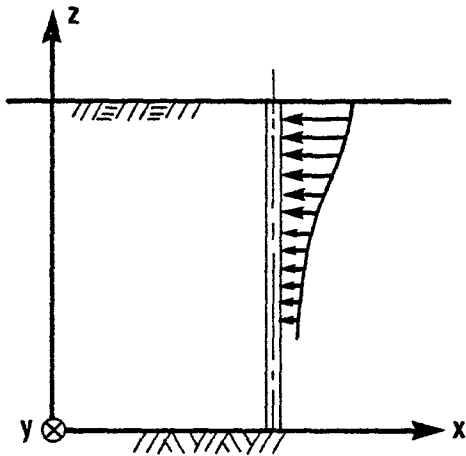
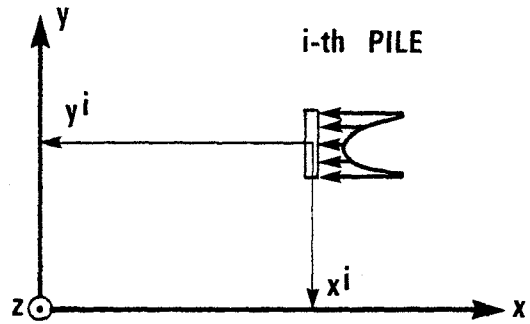
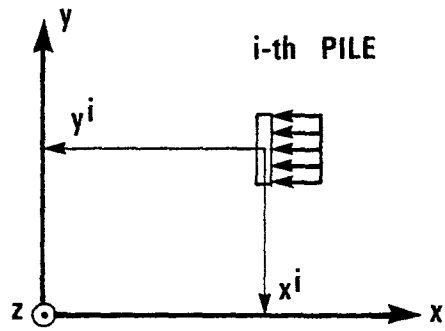
ANALYTIC MODEL OF A PILE-SUPPORTED STRUCTURE

	RANGE OF VALUES
PILE SLENDERNESS RATIO ($H/2r_0$)	33-133
SPACING RATIO ($S/2r_0$)	2-100
FREQUENCY RATIO (f/f_r)	0-80
PILE FLEXIBILITY FACTOR ($EI/E_s H^4$)	10^{-6} - 10^{-3}
PILE COMPRESSIBILITY FACTOR ($EA/E_s H^2$)	10^{-2} -1
POISSON'S RATIO OF SOIL (ν)	0.30-0.45

RANGES OF NONDIMENSIONAL PARAMETERS
FOR PARAMETER STUDY



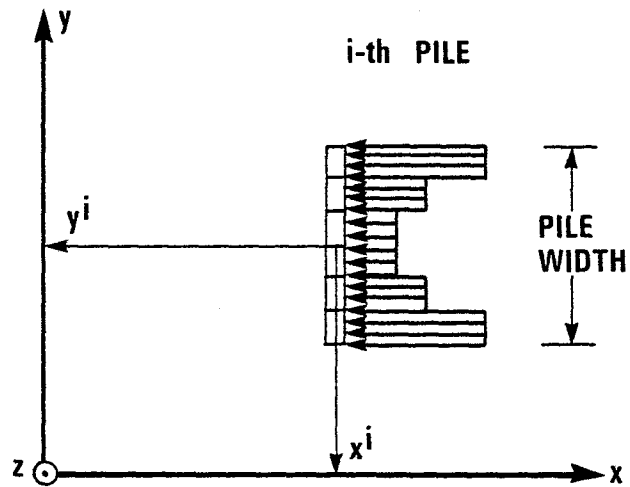
SOIL-PILE INTERFACE STRESS



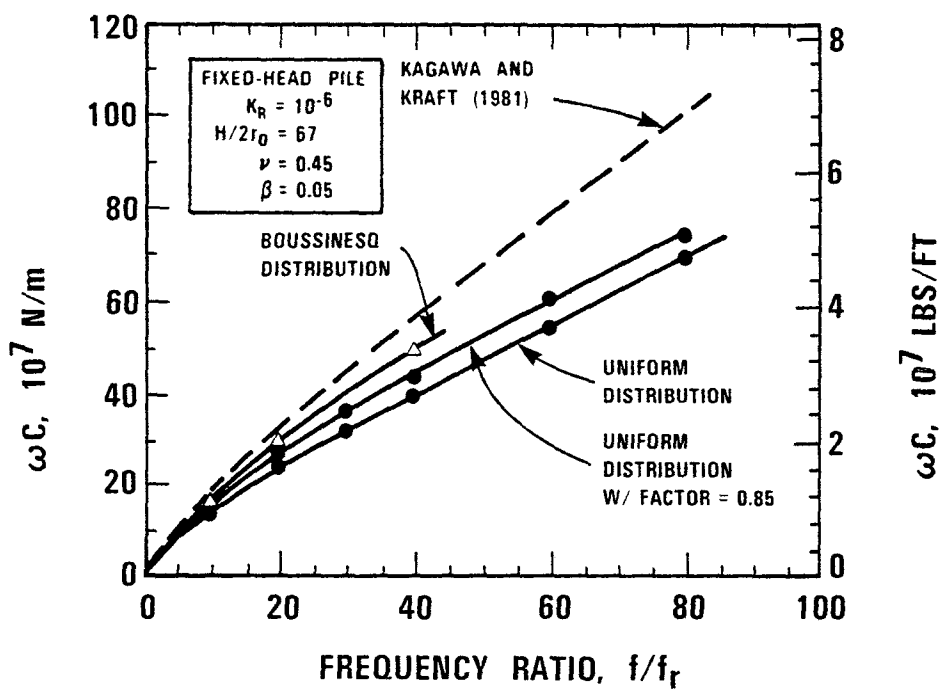
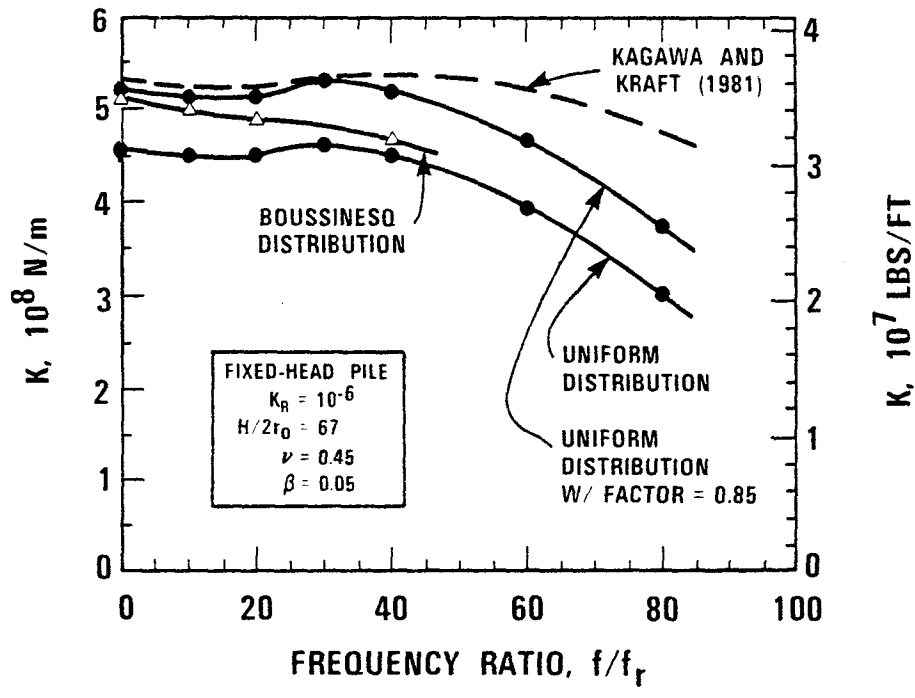
(a) UNIFORM SOIL-PILE
STRESS PATTERN

(b) BOUSSINESQ SOIL-PILE
STRESS PATTERN

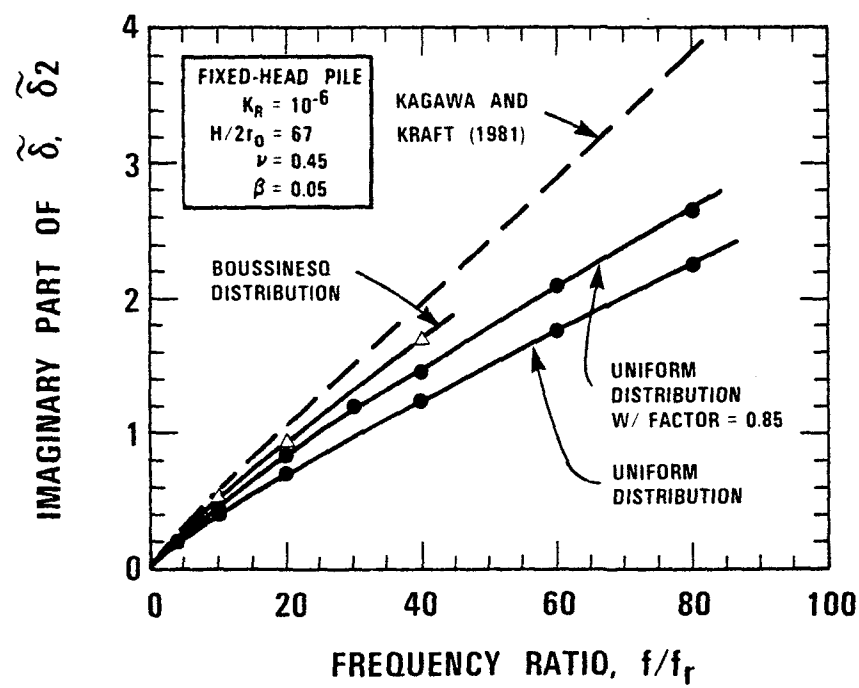
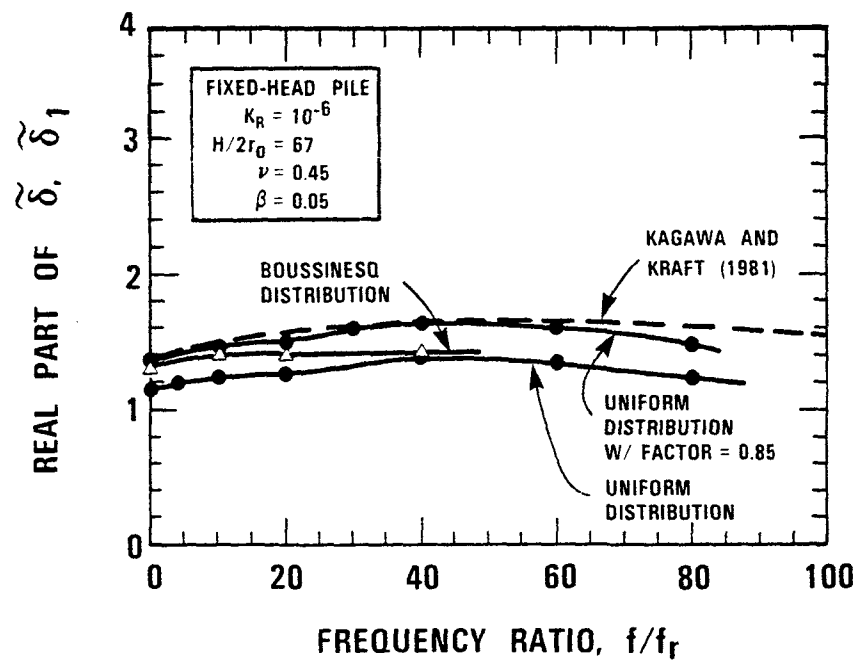
SOIL-PILE INTERFACE STRESSES FOR ANALYSIS



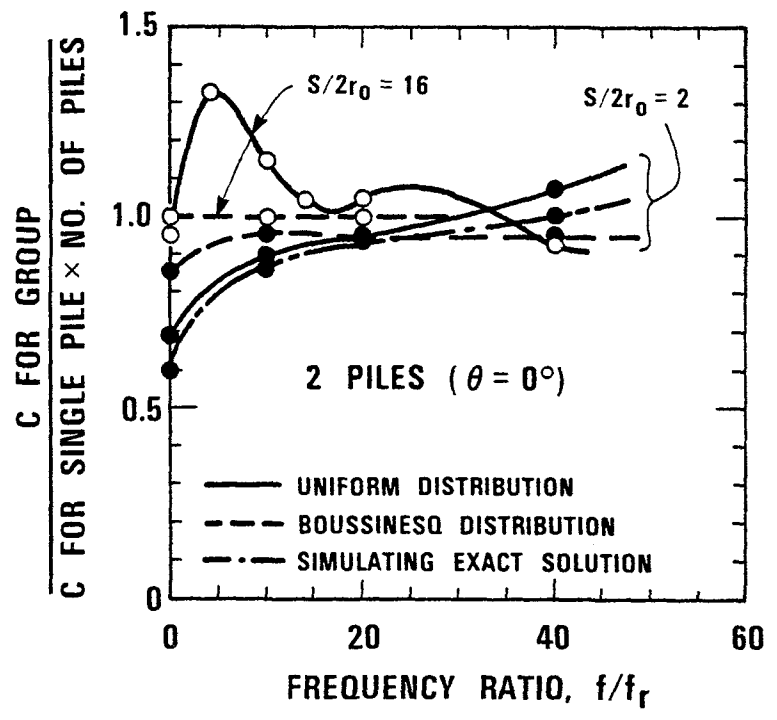
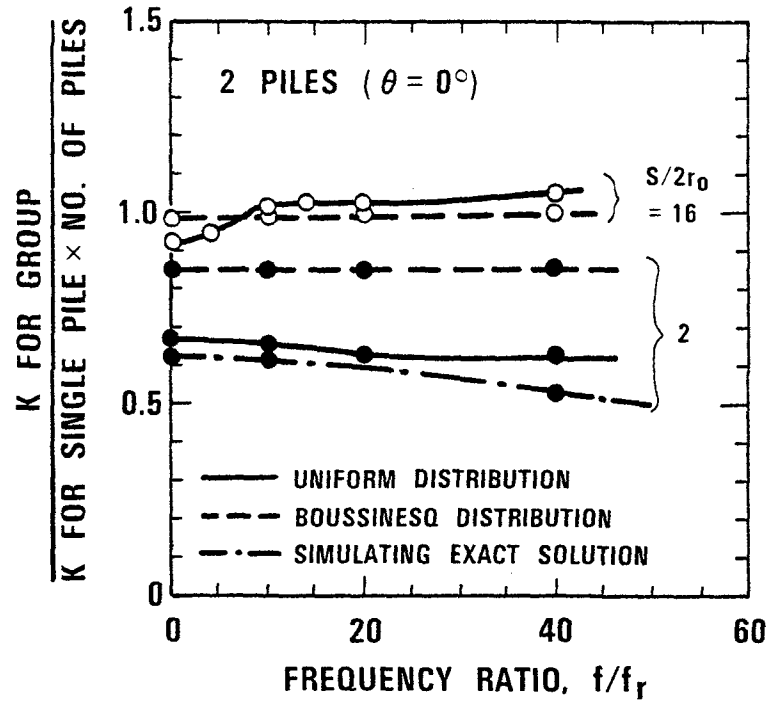
REPRESENTATION OF A PILE SECTION
BY DISCRETE ELEMENTS



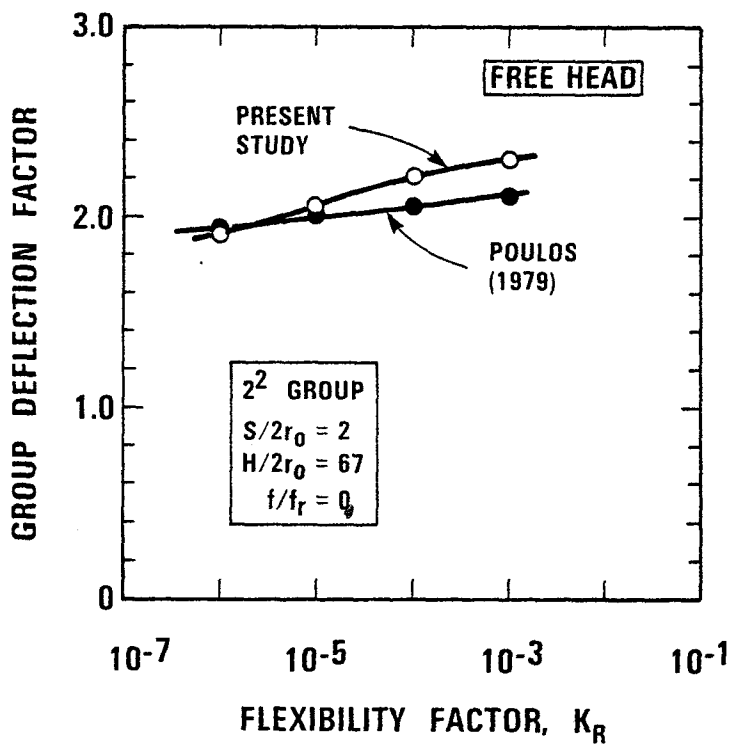
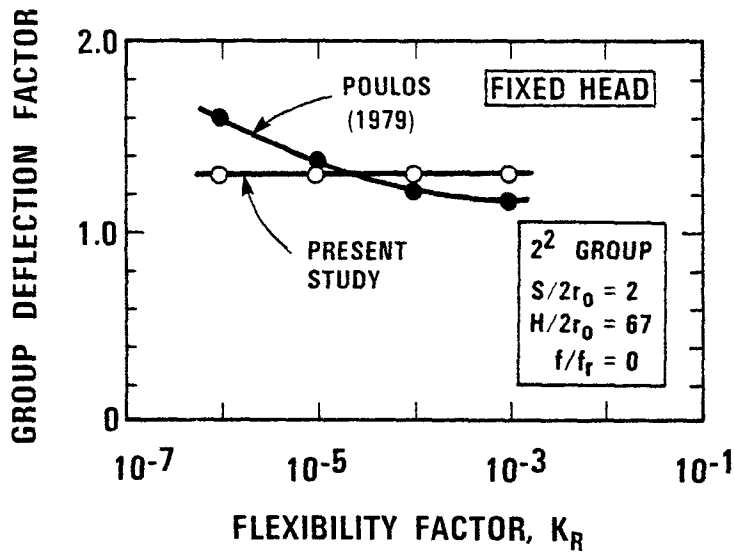
SINGLE-PILE STIFFNESS AND DAMPING FROM SEVERAL METHODS



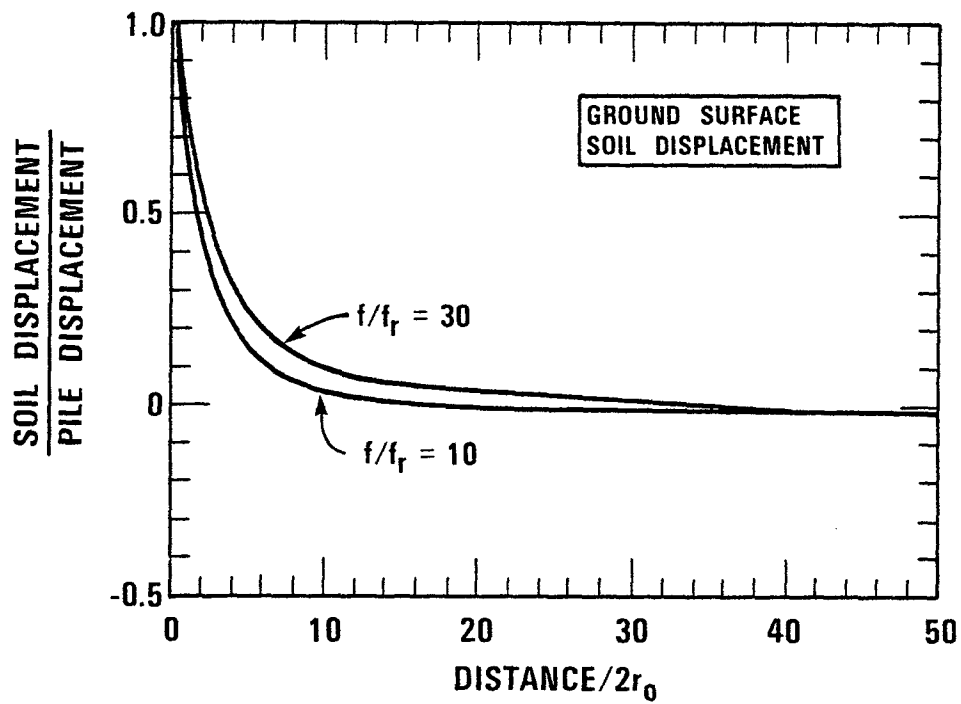
SINGLE-PILE SOIL-PILE SPRING COEFFICIENTS FROM SEVERAL METHODS



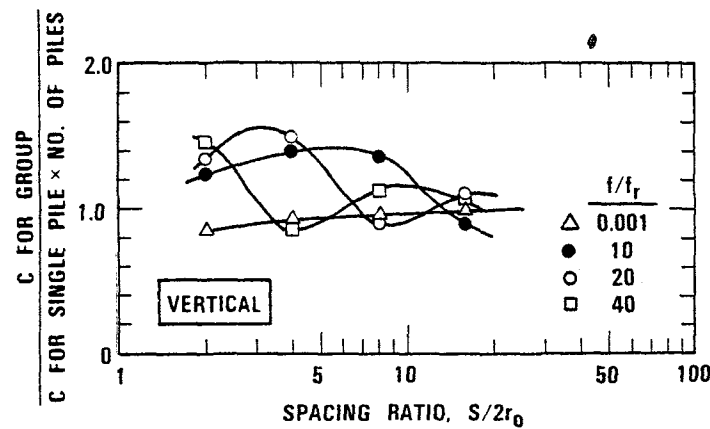
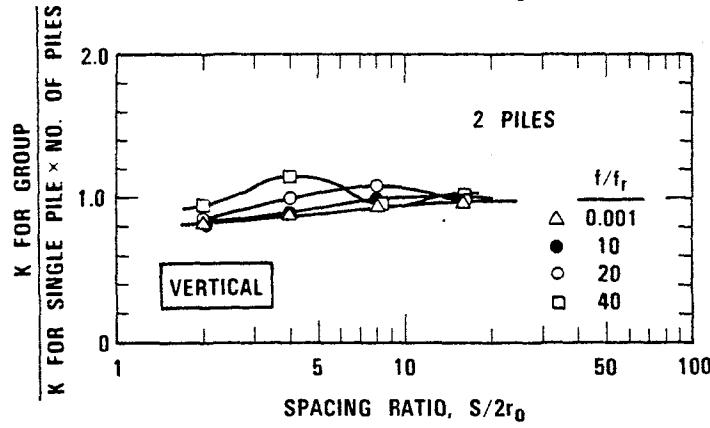
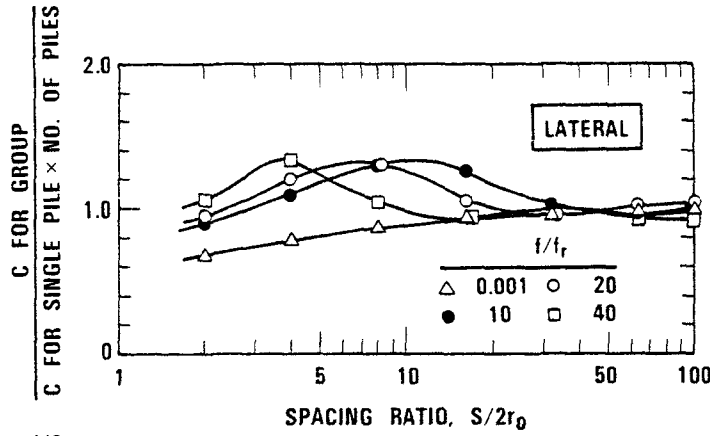
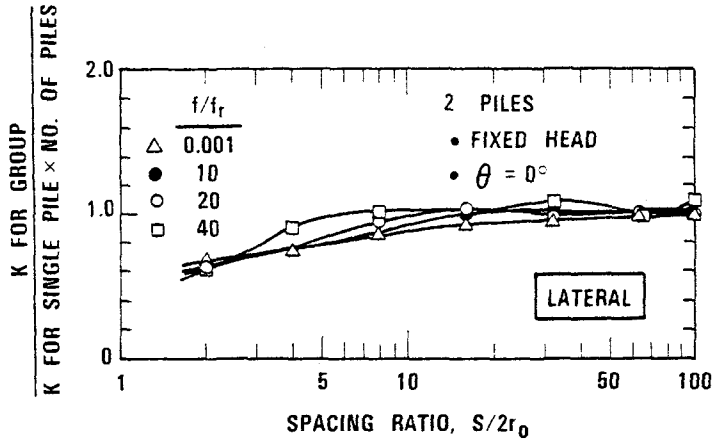
2-PILE INTERACTION FOR DIFFERENT SOIL-PILE STRESS PATTERNS



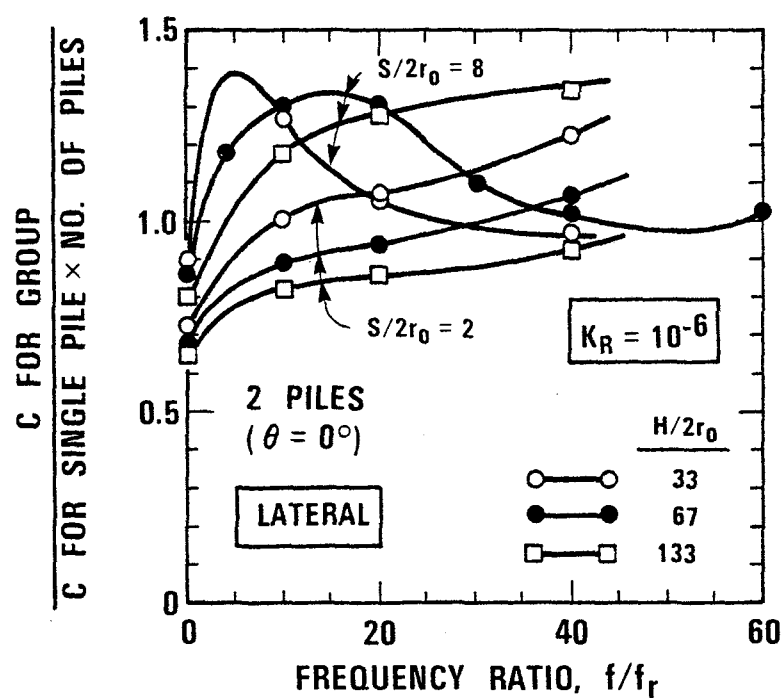
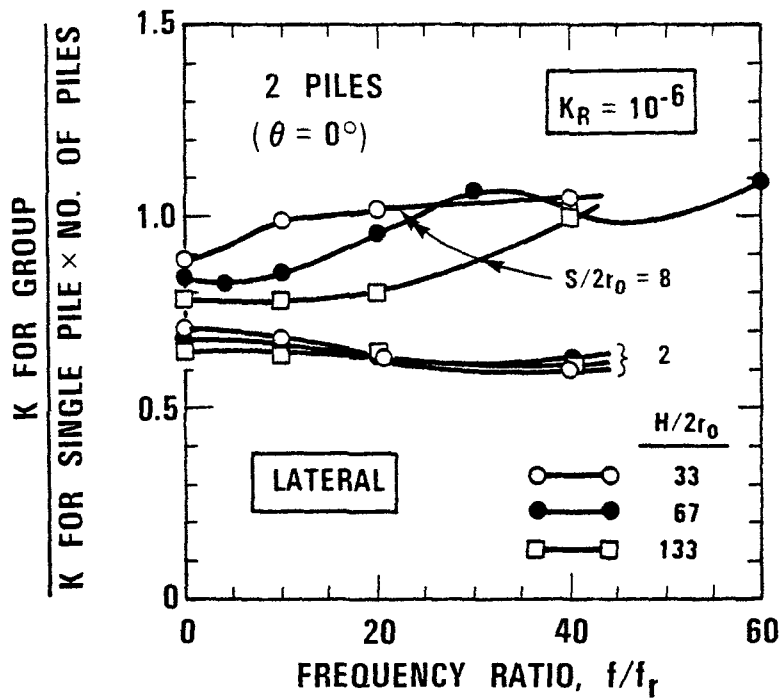
COMPARISON WITH POULOS' GROUP DEFLECTION FACTORS



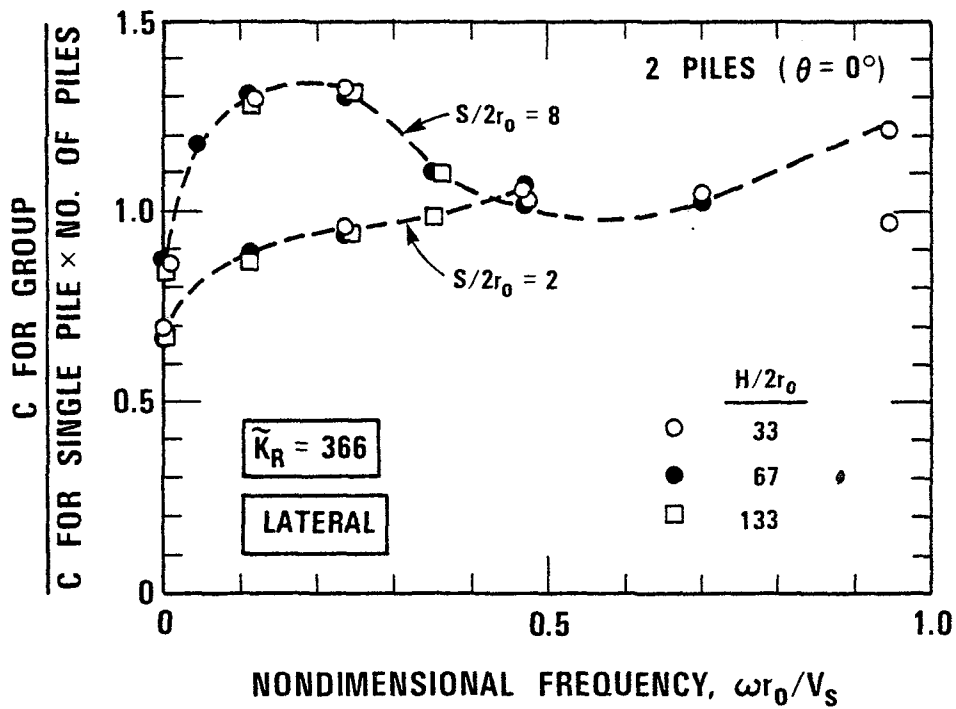
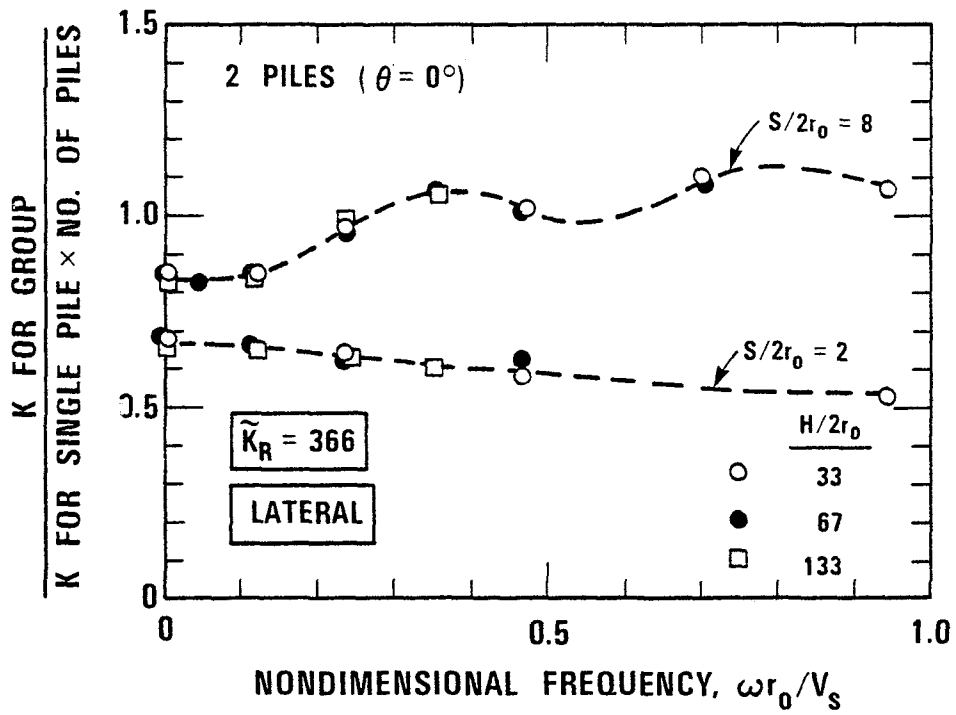
SOIL MOTIONS DUE TO PILE VIBRATION



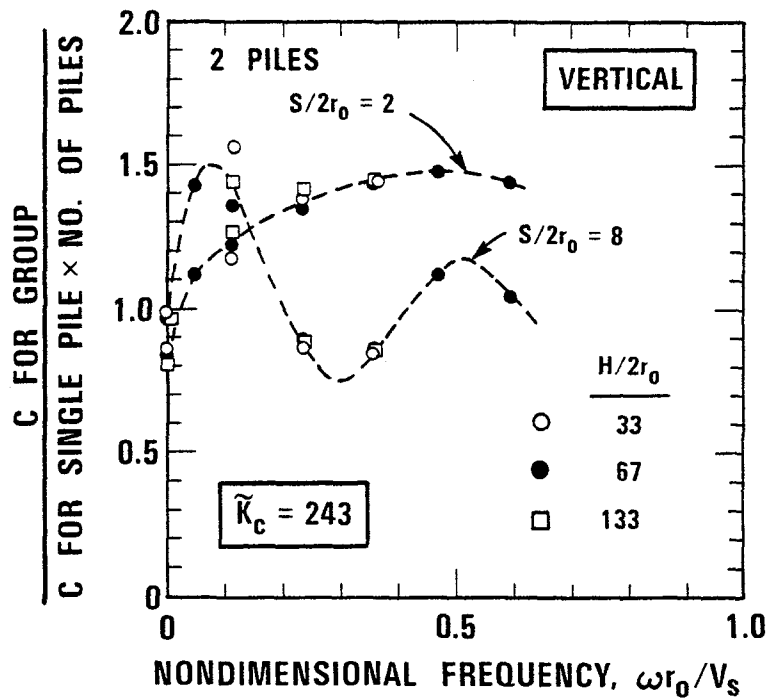
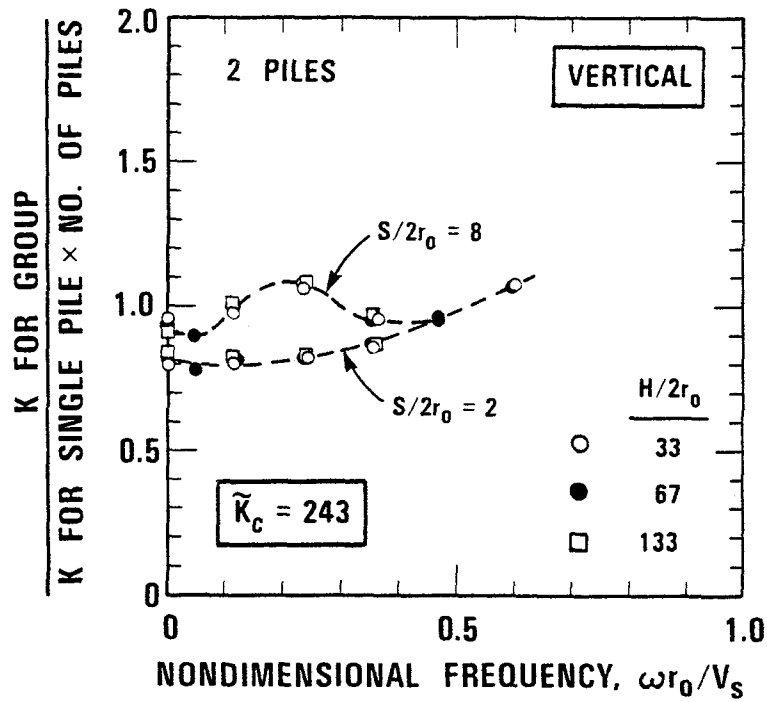
EFFECTS OF $S/2r_0$ AND FREQUENCY ON 2-PILE INTERACTION



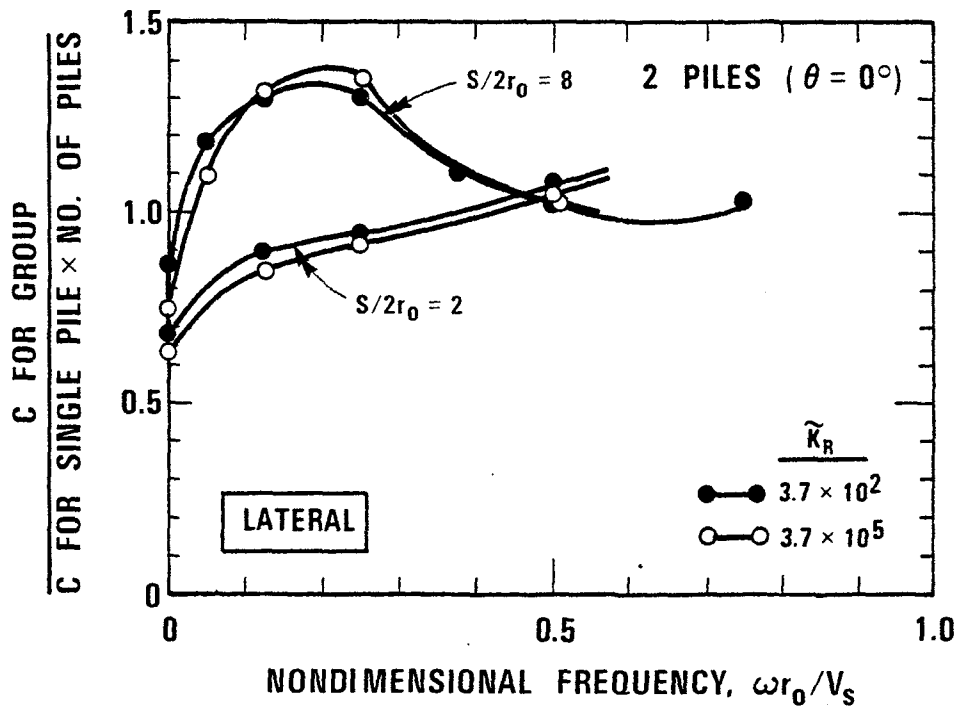
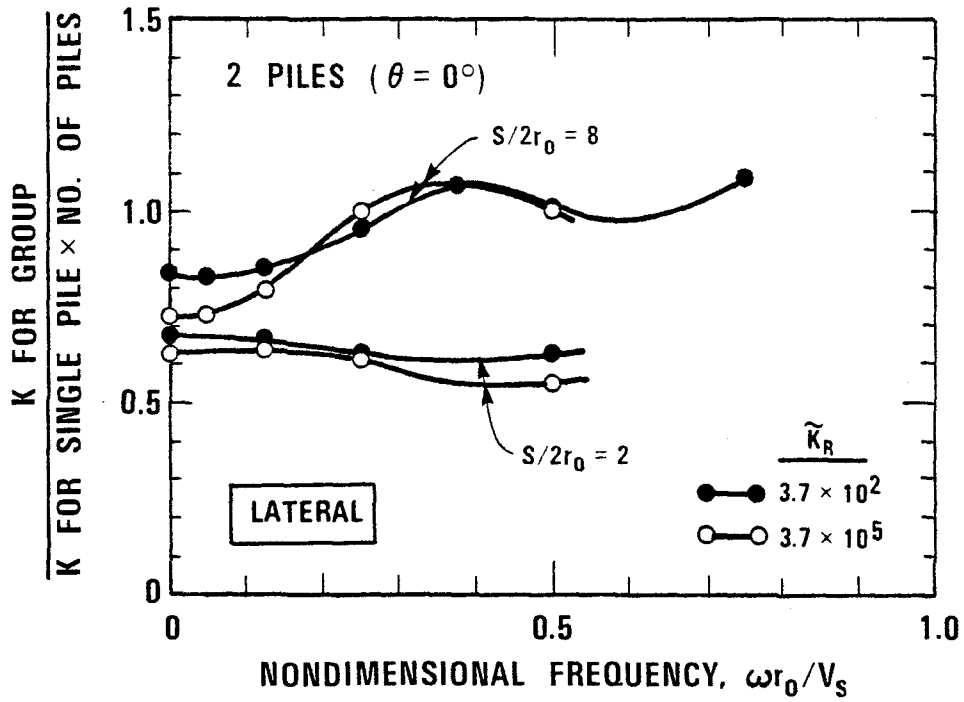
EFFECTS OF $H/2r_0$ ON 2-PILE INTERACTION FOR LATERAL MODE



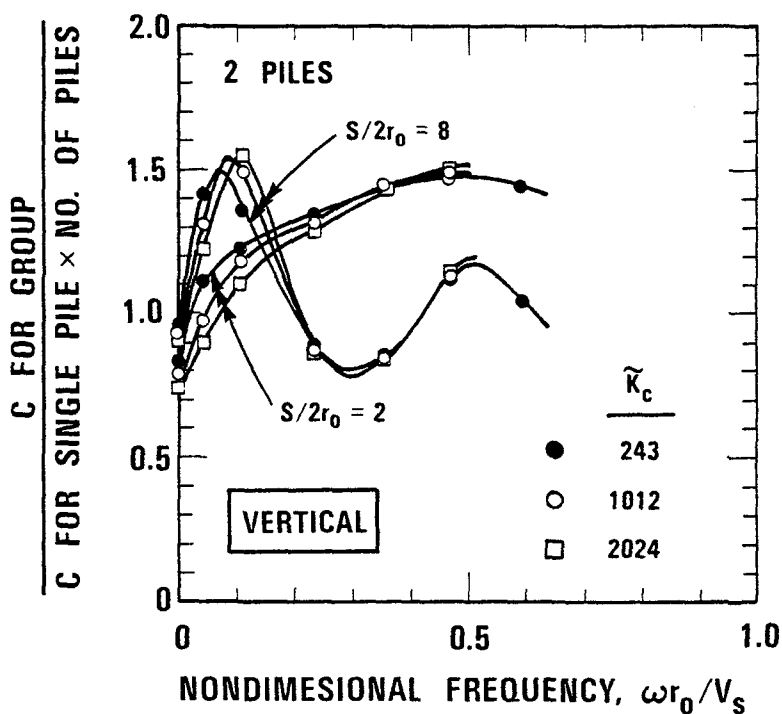
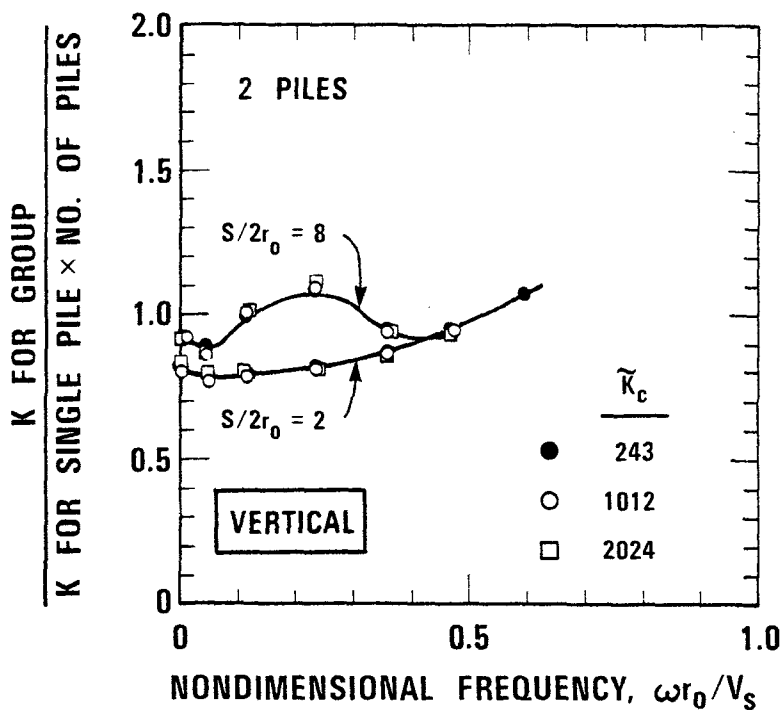
EFFECTS OF $H/2r_0$ ON 2-PILE INTERACTION
FOR LATERAL MODE



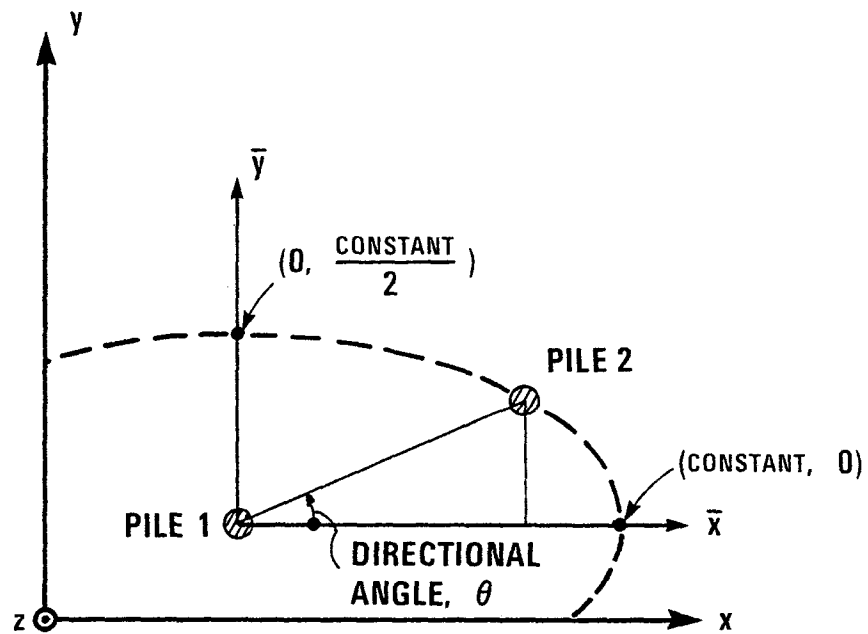
EFFECTS OF $H/2r_0$ ON 2-PILE INTERACTION FOR VERTICAL MODE



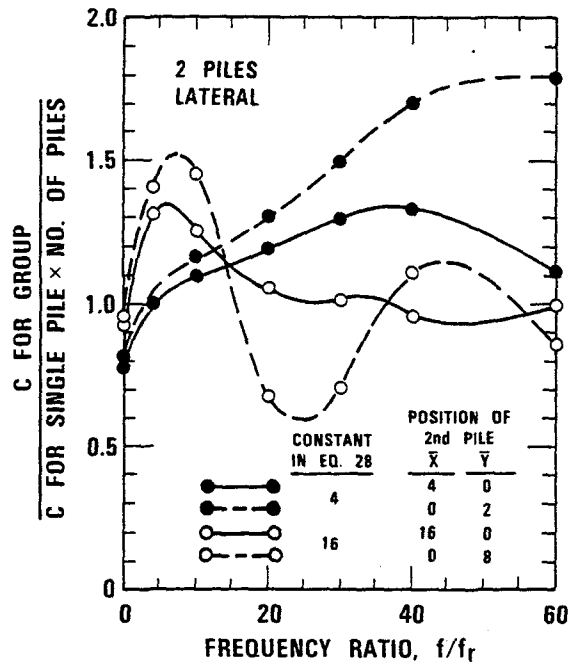
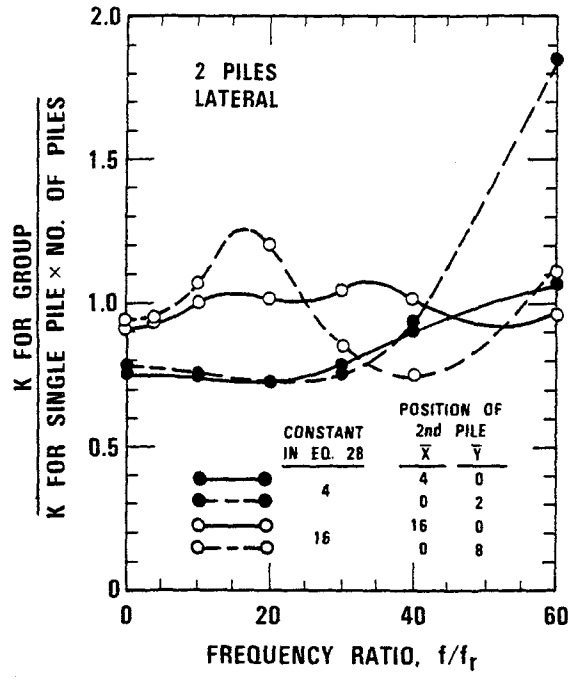
EFFECTS OF \tilde{K}_R ON 2-PILE INTERACTION FOR LATERAL MODE



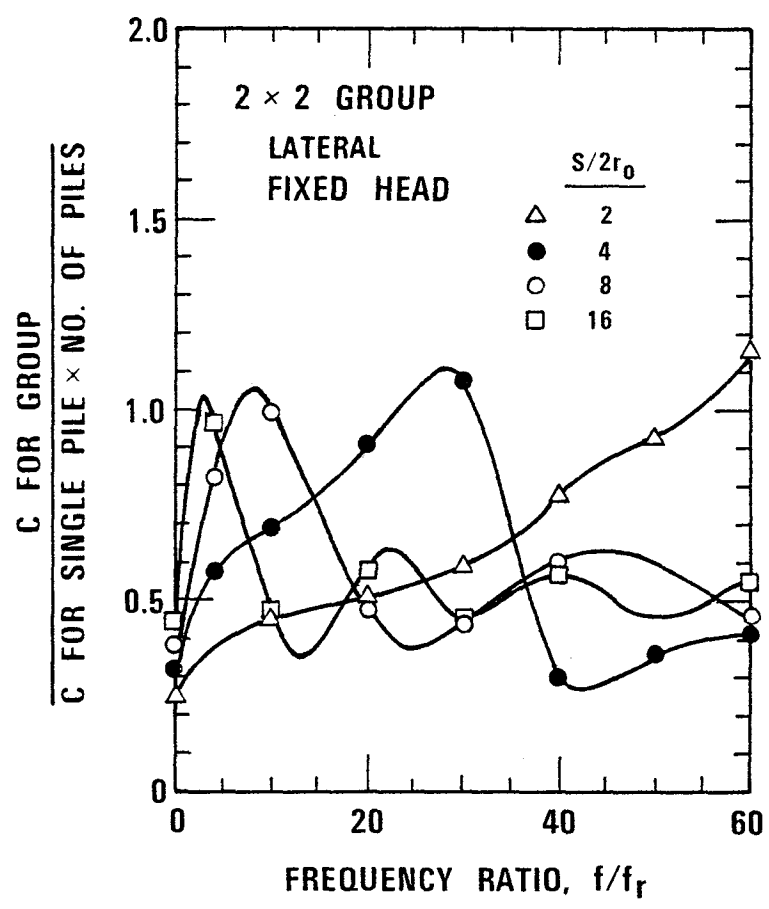
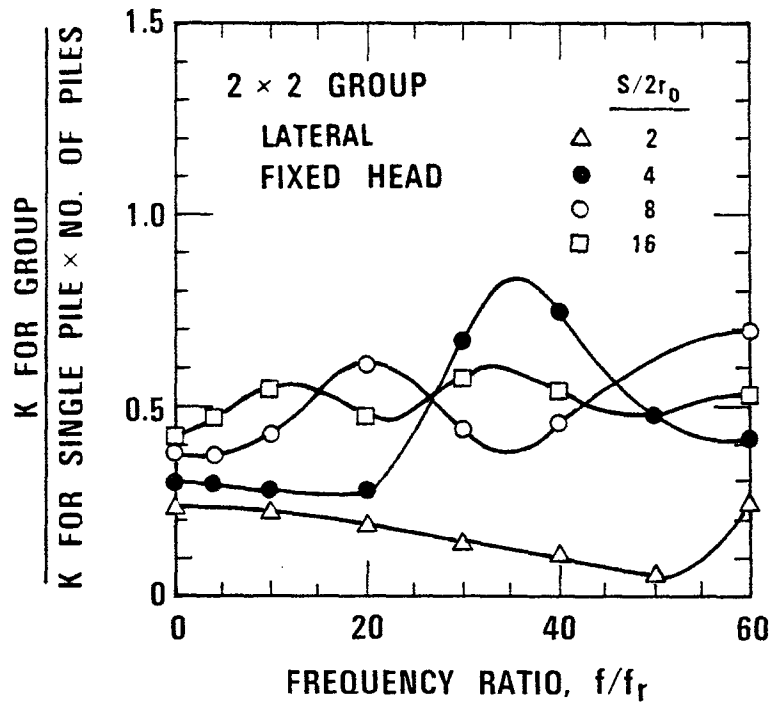
EFFECTS OF $\tilde{\kappa}_c$ ON 2-PILE INTERACTION FOR VERTICAL MODE



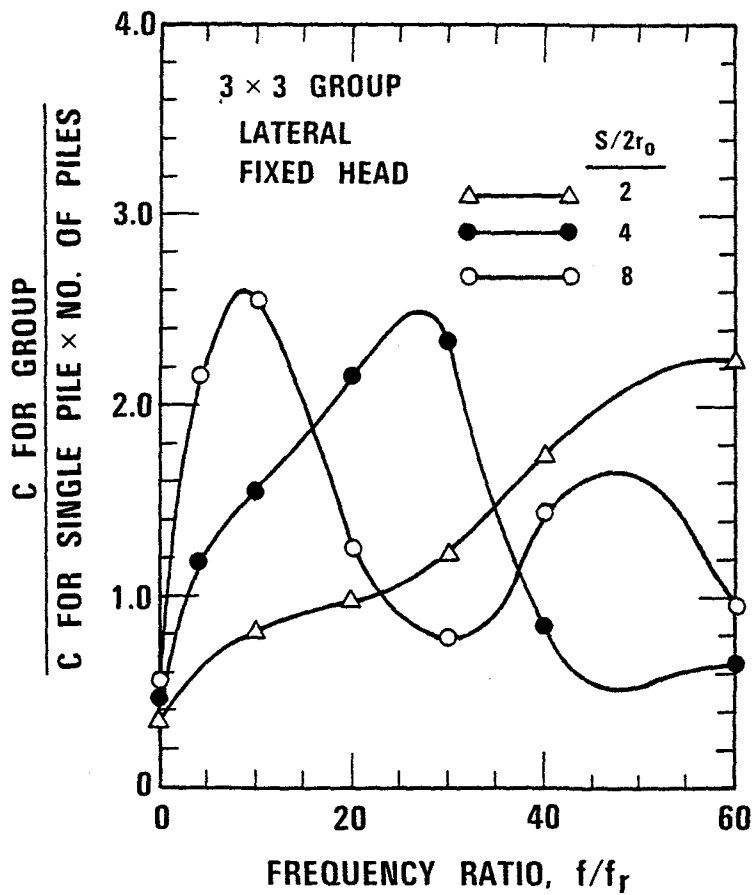
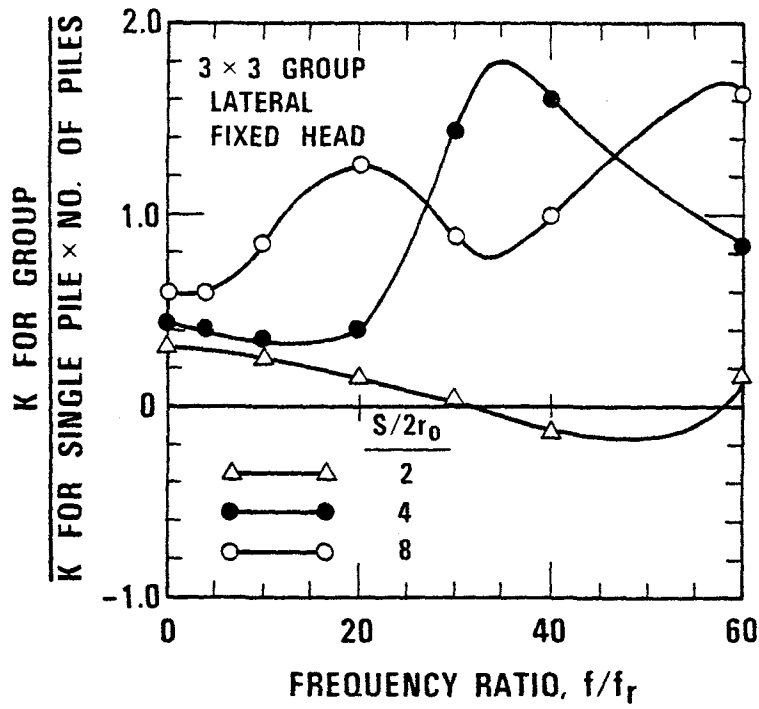
DIRECTIONAL ANGLE FOR 2-PILES



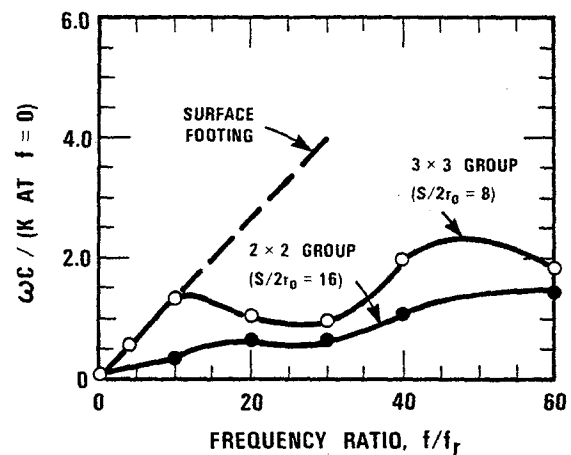
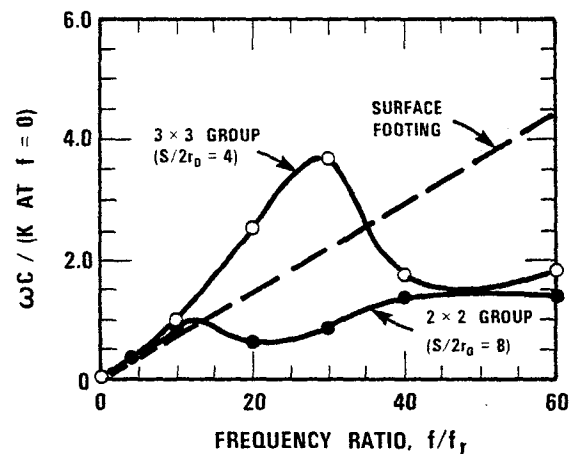
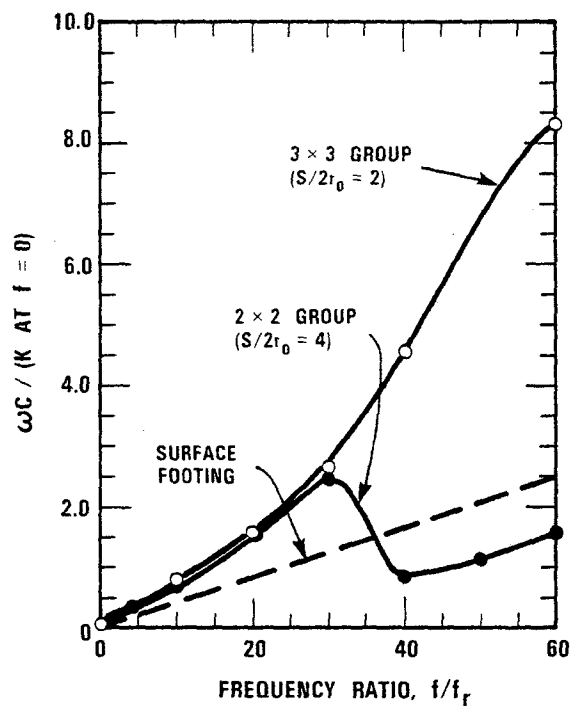
DIRECTIONALITY OF 2-PILE INTERACTION



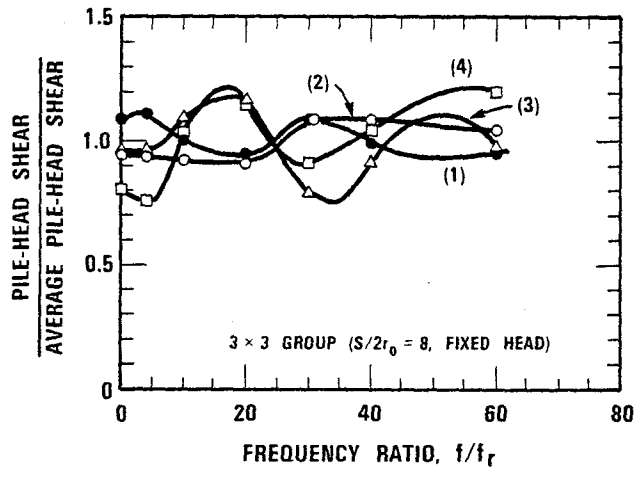
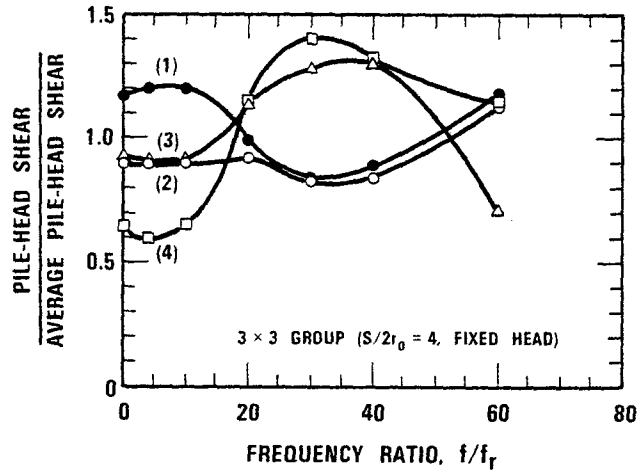
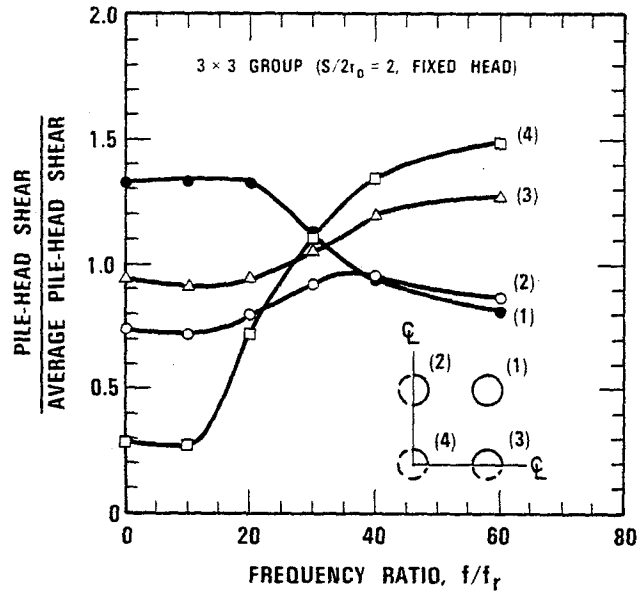
PILE-GROUP EFFECTS FOR 2 × 2 GROUP



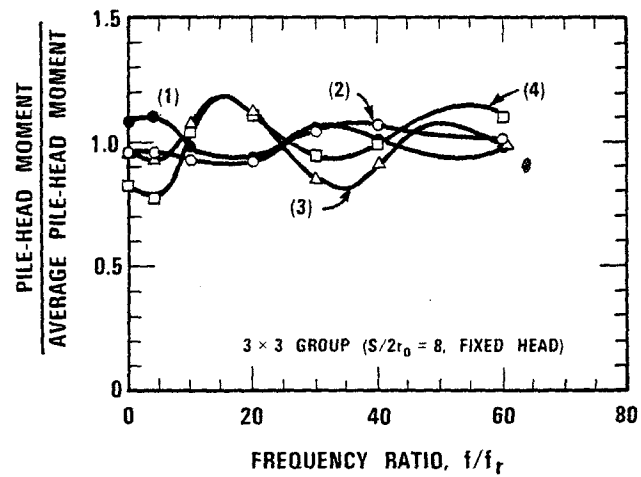
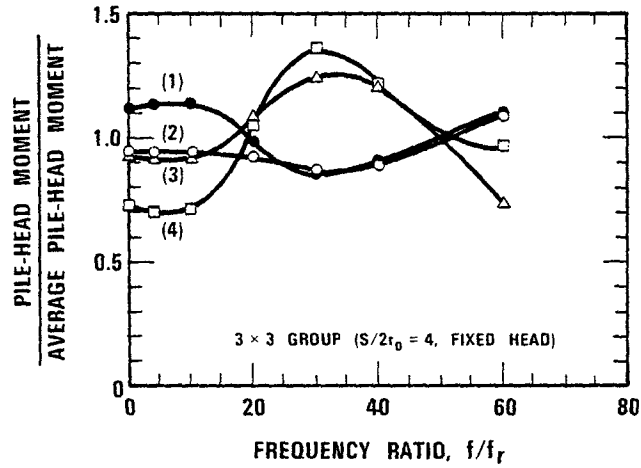
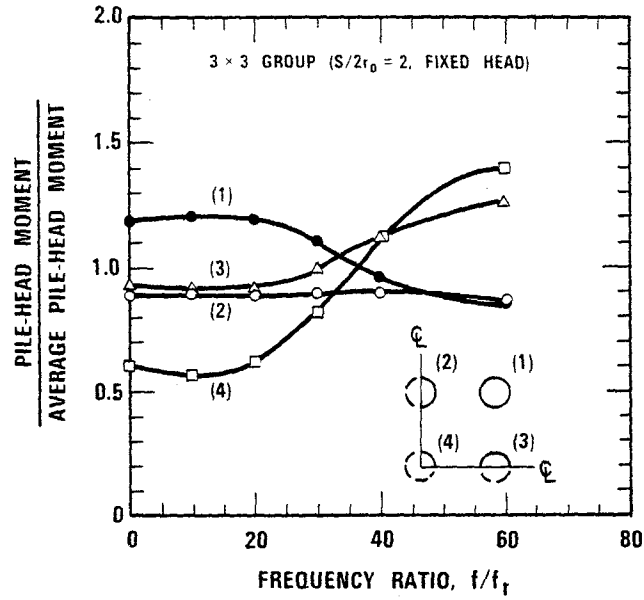
PILE-GROUP EFFECTS FOR 3 × 3 GROUP



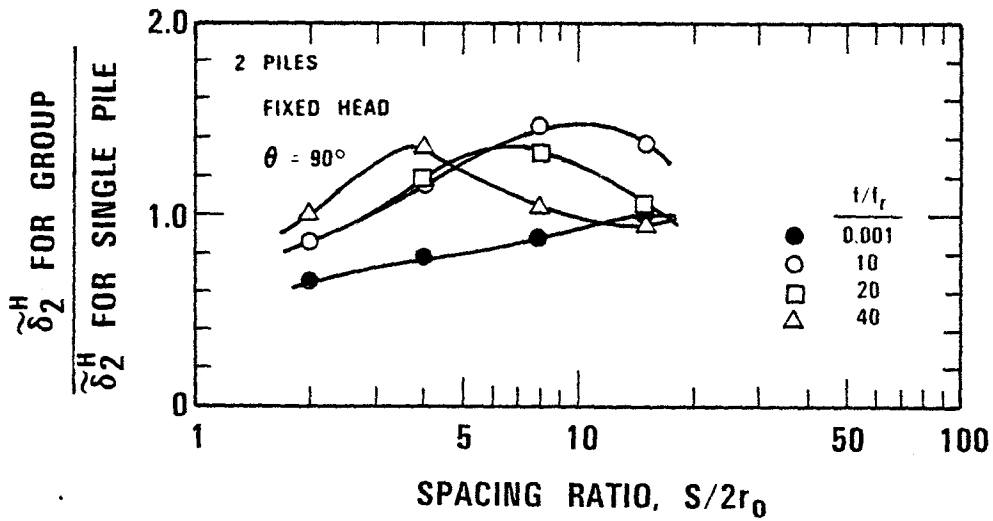
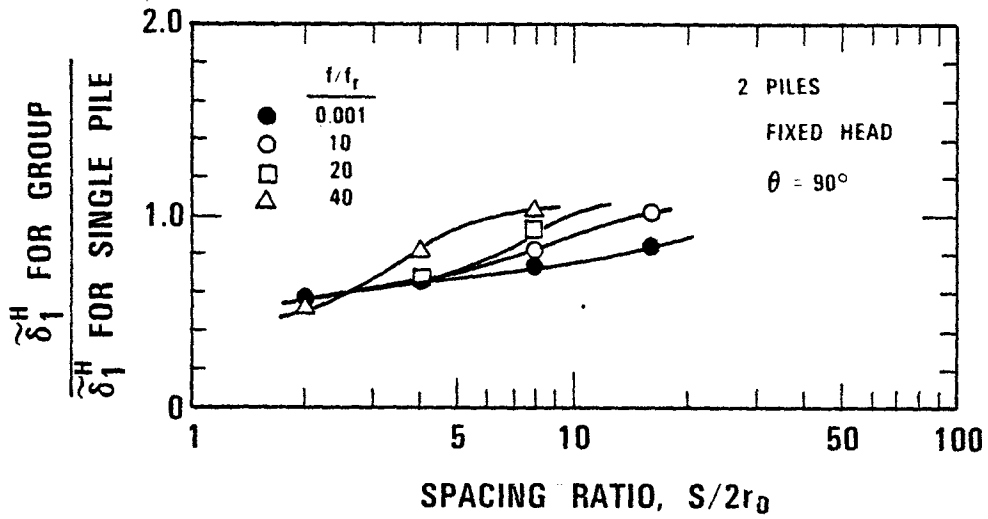
COMPARISON OF DAMPING FOR PILE GROUPS AND SURFACE FOOTINGS



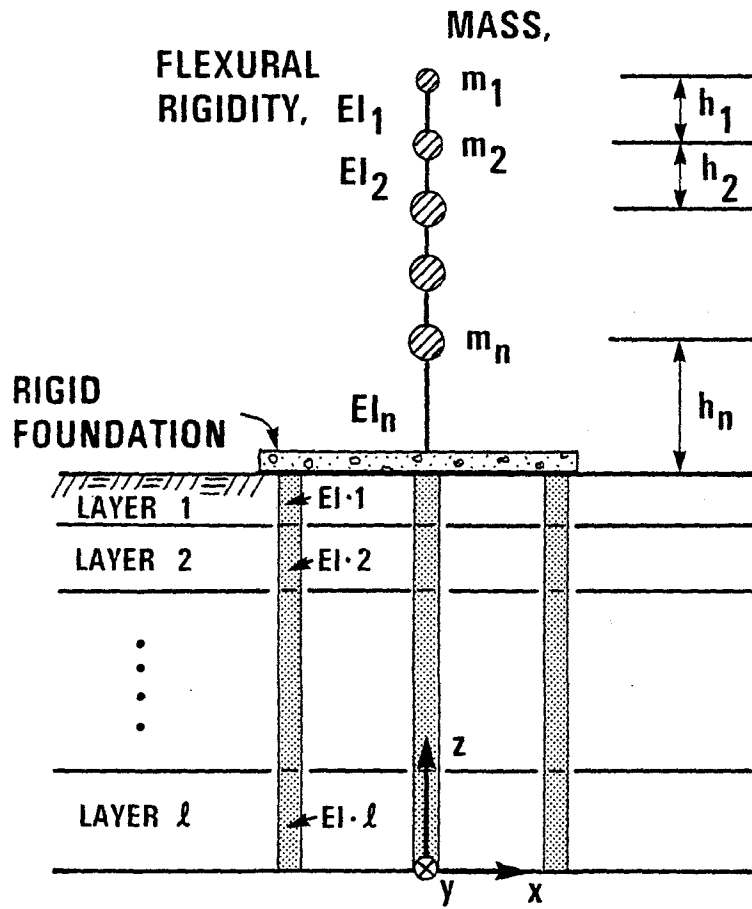
DISTRIBUTION OF PILE-HEAD SHEAR



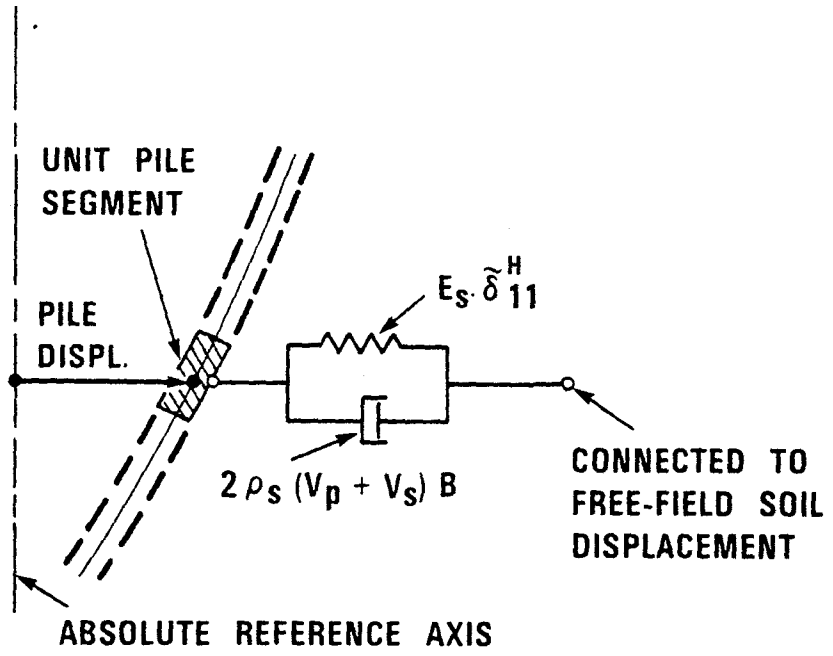
DISTRIBUTION OF PILE-HEAD MOMENT



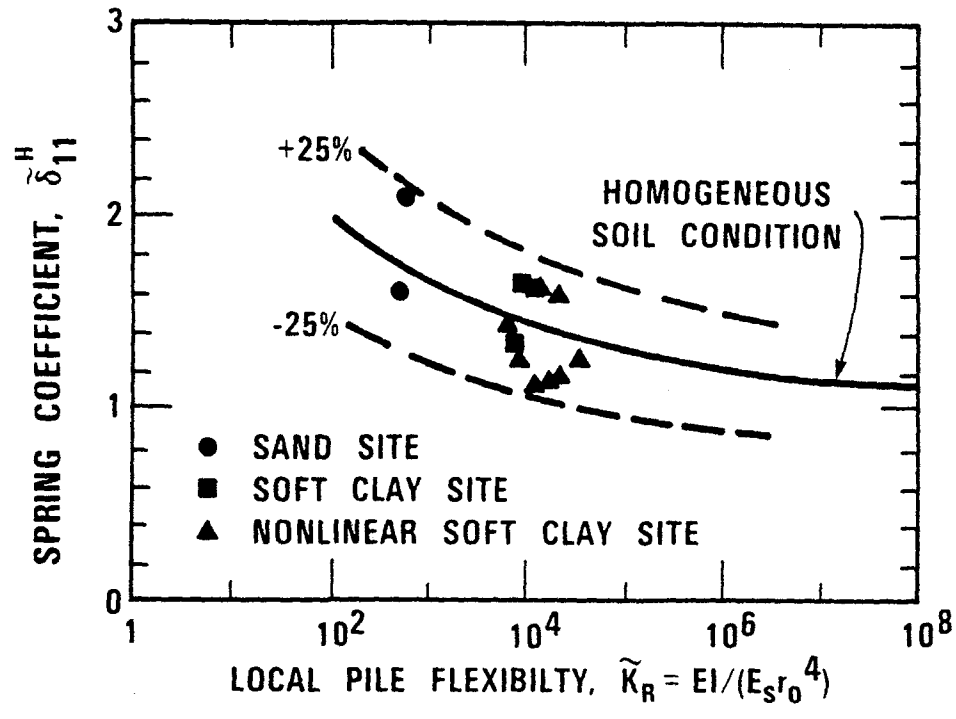
PILE-GROUP EFFECTS ON SOIL-PILE SPRING COEFFICIENTS



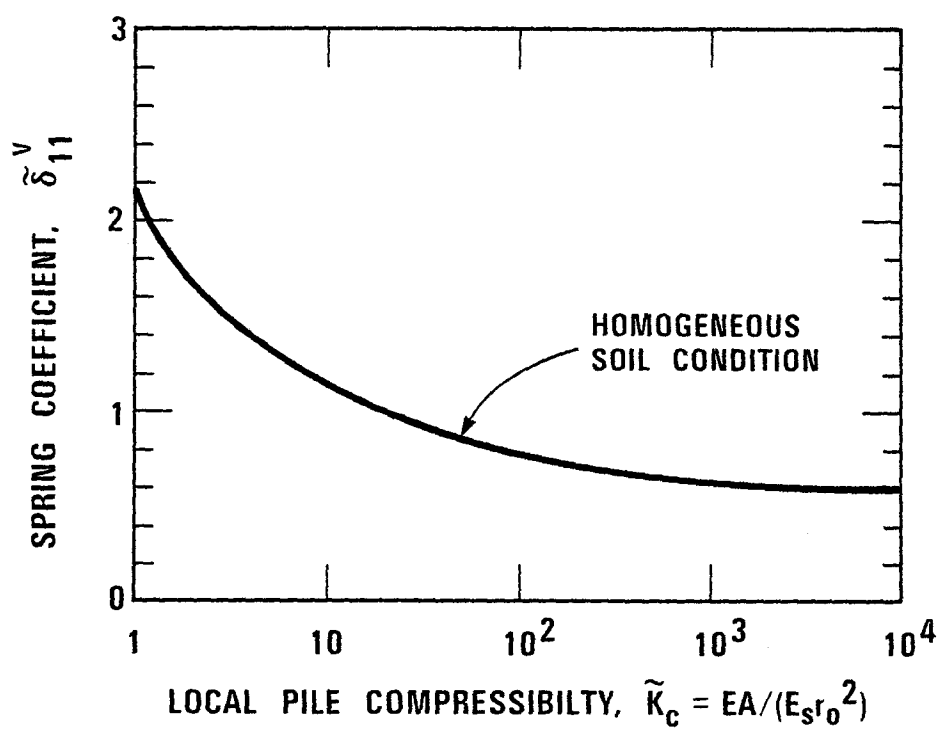
ANALYTIC MODEL OF A PILE-SUPPORTED STRUCTURE



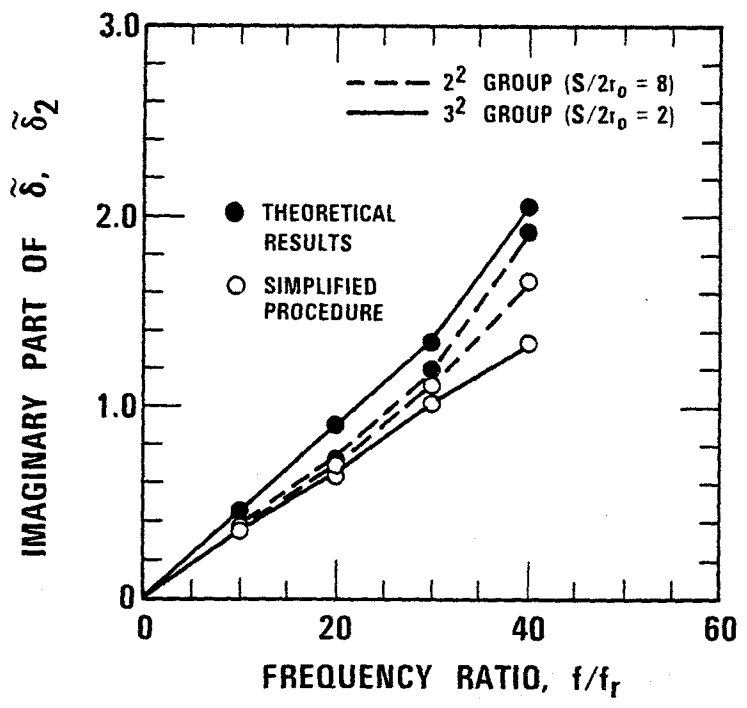
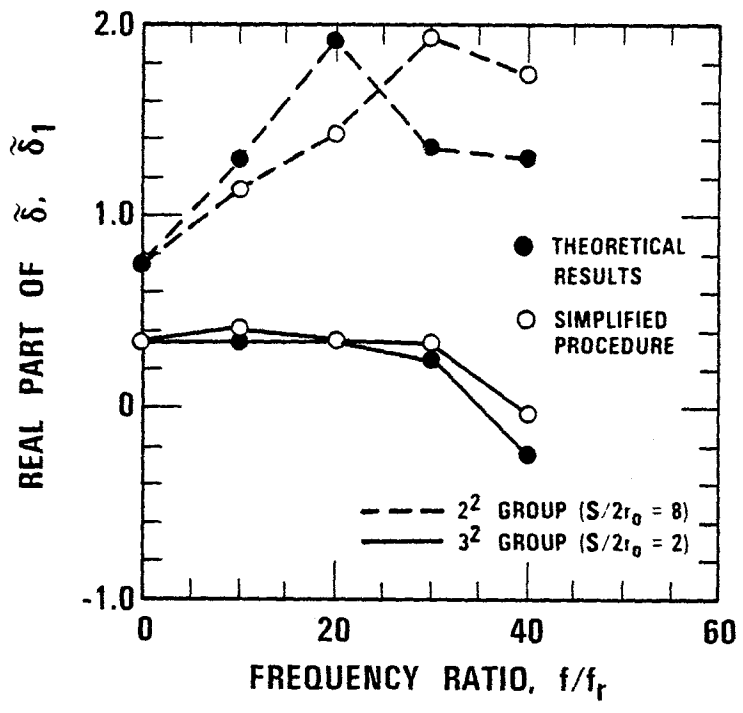
SOIL-PILE SPRING AND DASHPOT



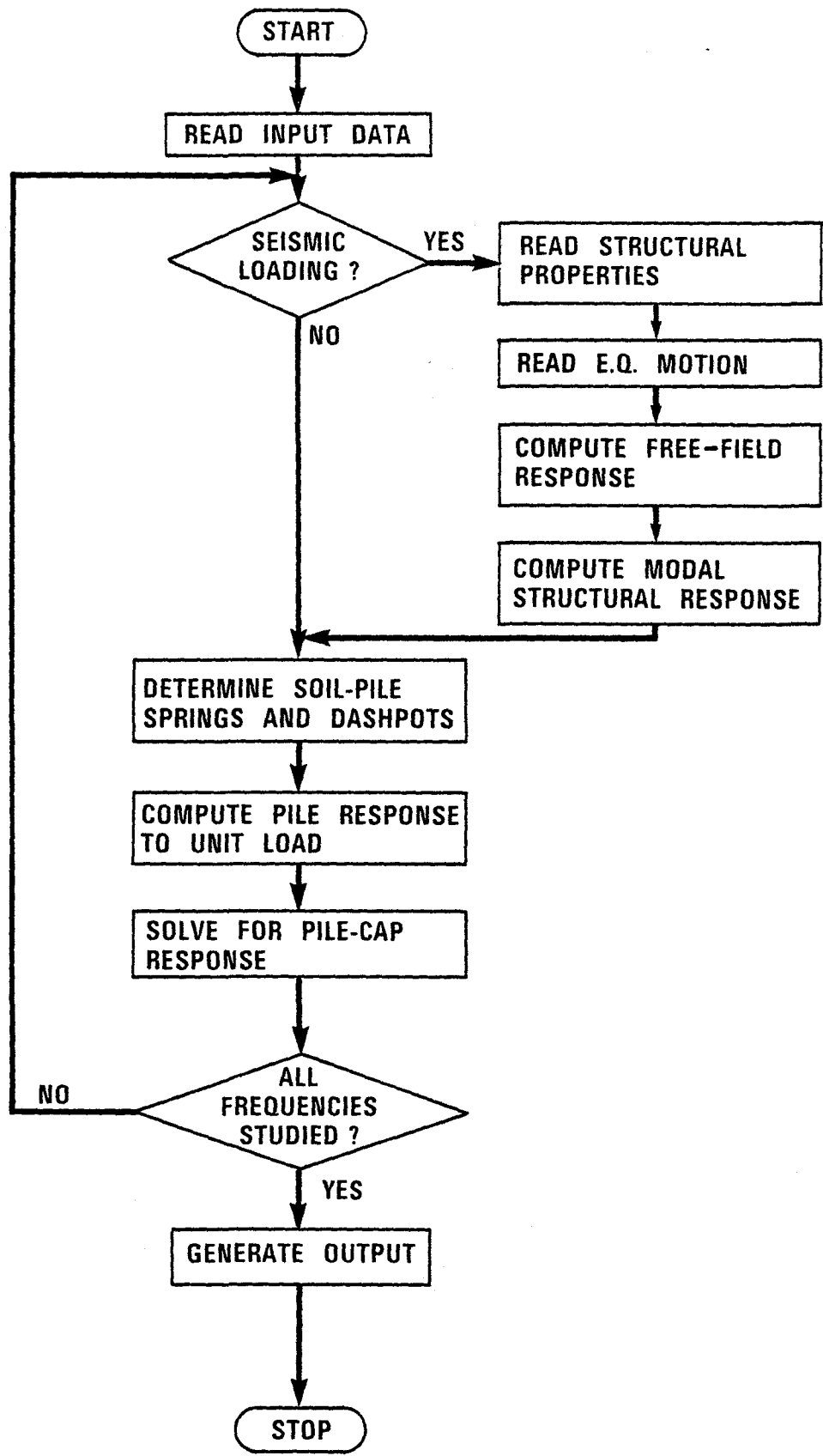
AVERAGE SOIL-PILE SPRING COEFFICIENT FOR LATERAL MODE



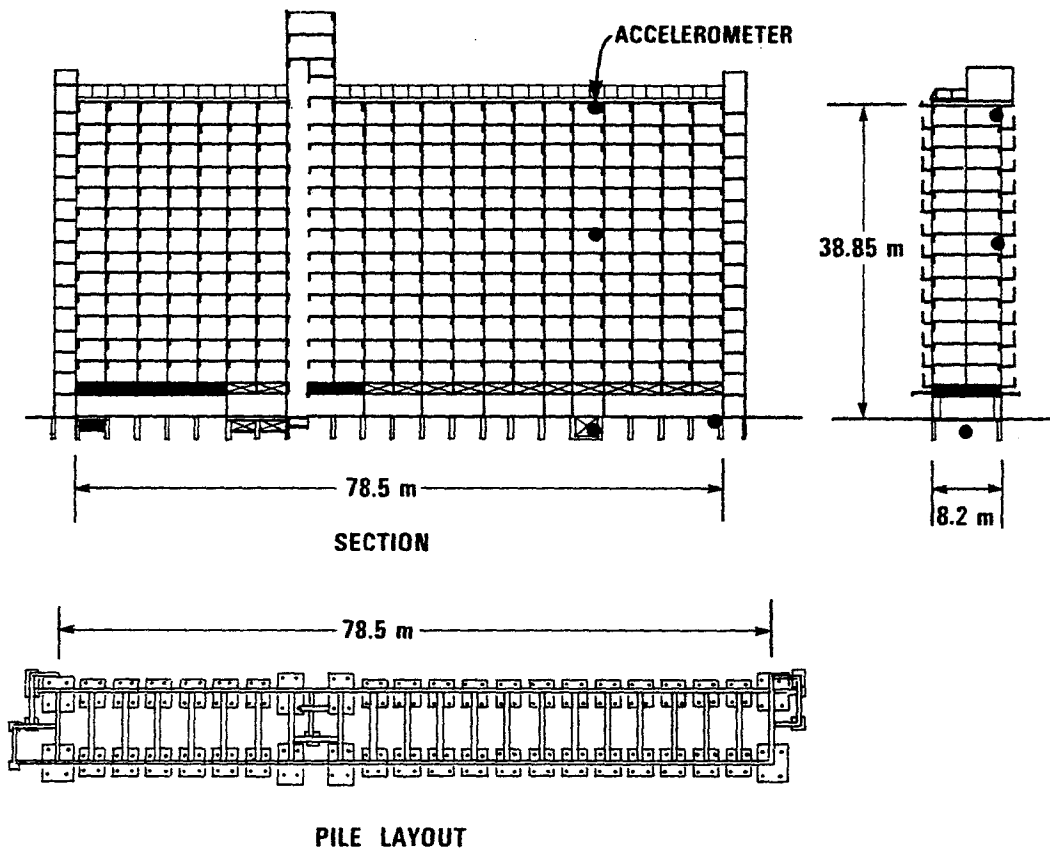
AVERAGE SOIL-PILE SPRING COEFFICIENT FOR VERTICAL MODE



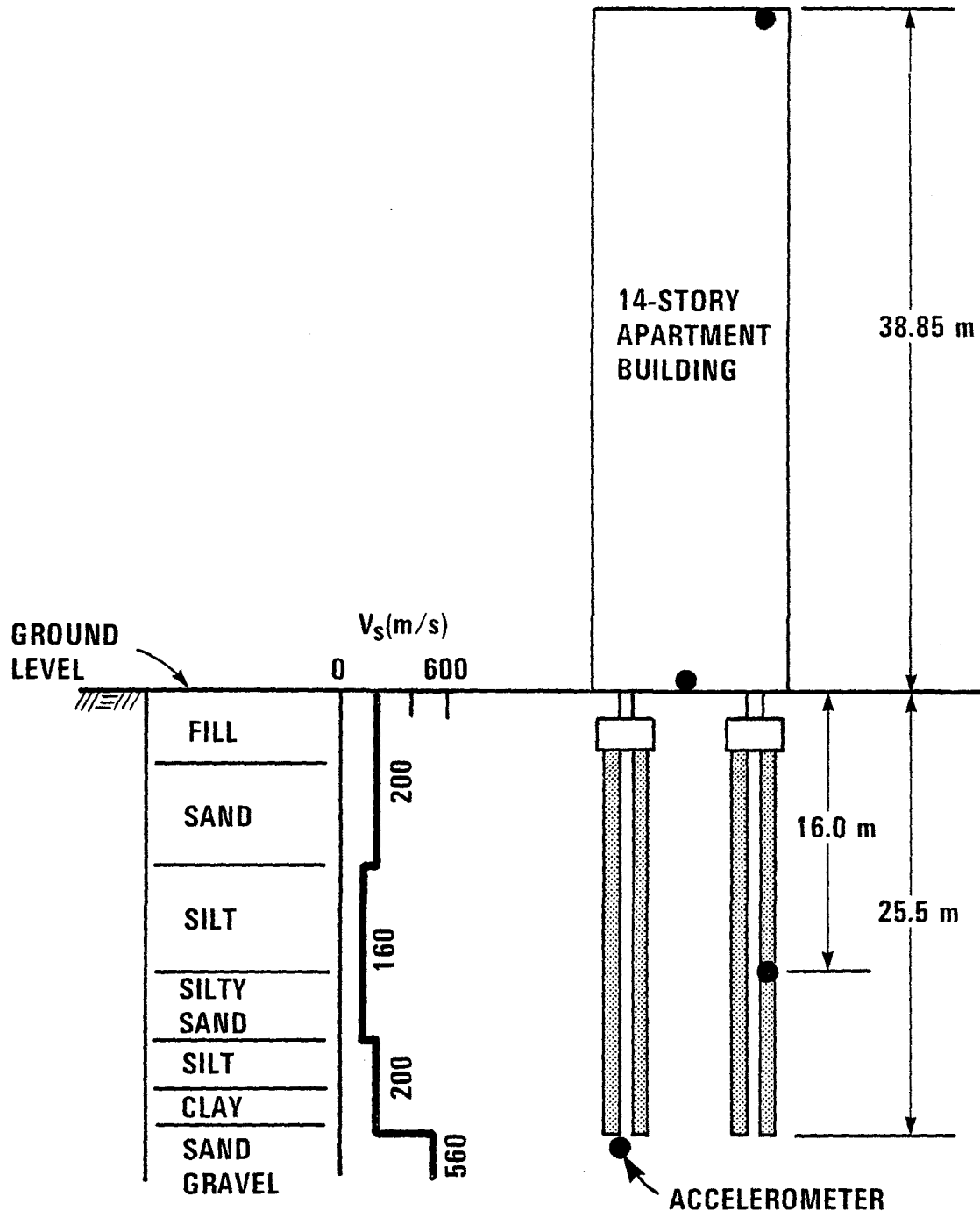
PERFORMANCE OF APPROXIMATE METHOD



NUMERICAL SCHEMES FOR PROGRAM PILES



14-STORY BUILDING FOR CASE STUDY



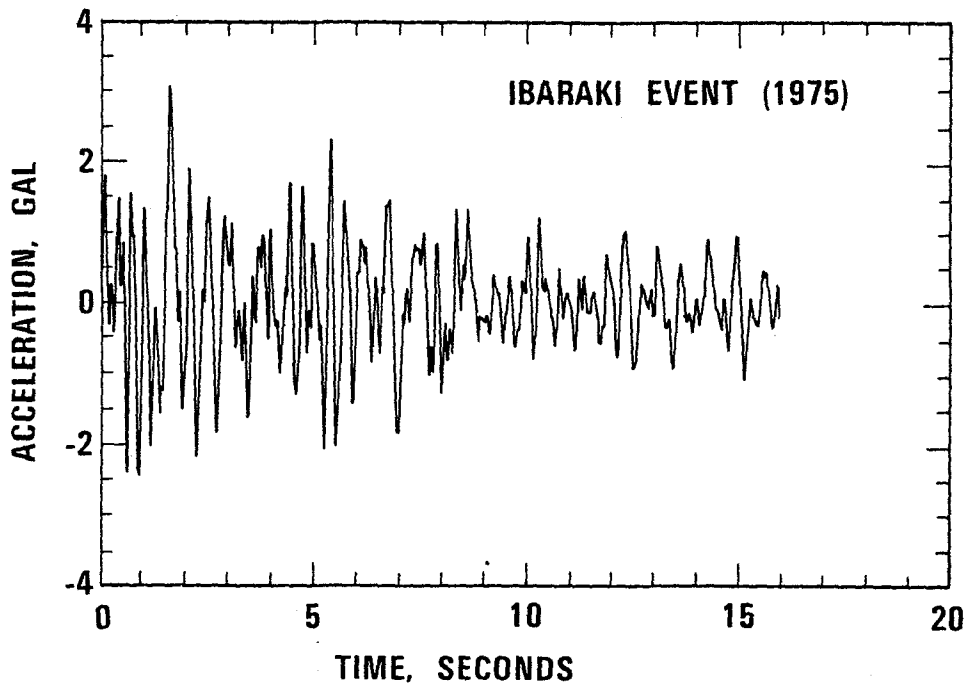
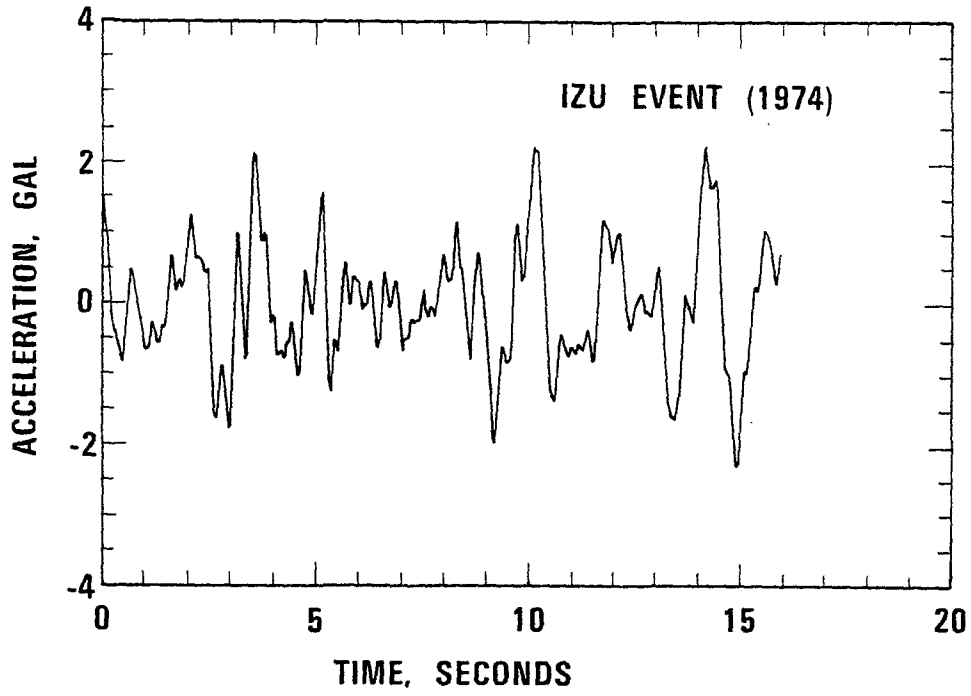
SOIL CONDITIONS AND SEISMIC OBSERVATIONAL POINTS

<u>STORY</u>	<u>MASS</u> <u>(kgf · s²/cm)</u>	<u>SPRING</u> <u>(10⁶ kgf /cm)</u>	<u>HEIGHT</u> <u>(cm)</u>
1	1967	0.66	1055
2	2691	0.61	785
3	2450	2.54	805
4	2502	6.13	810
5	3812	12.48	430

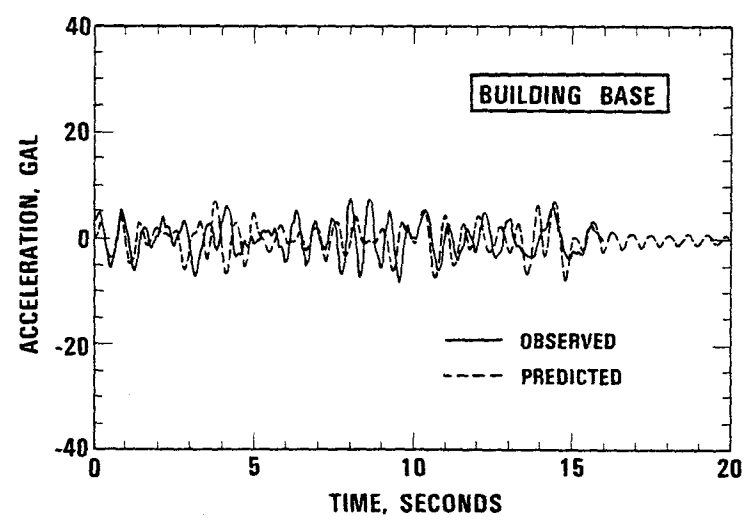
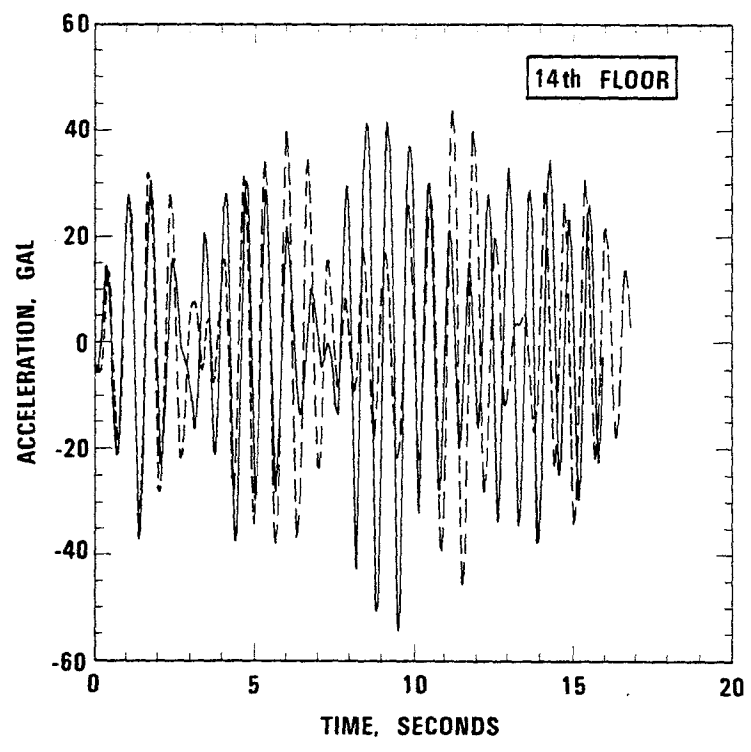
BASE MASS = 2903 kgf · s²/cm

BASE SPRING = 8.2 × 10⁶ kgf /cm

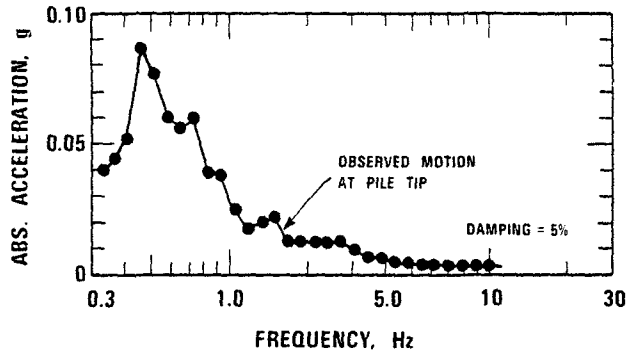
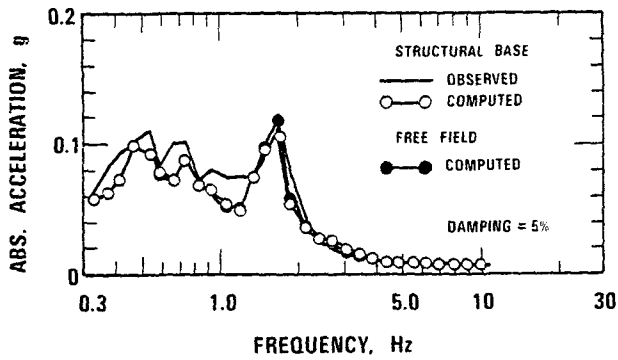
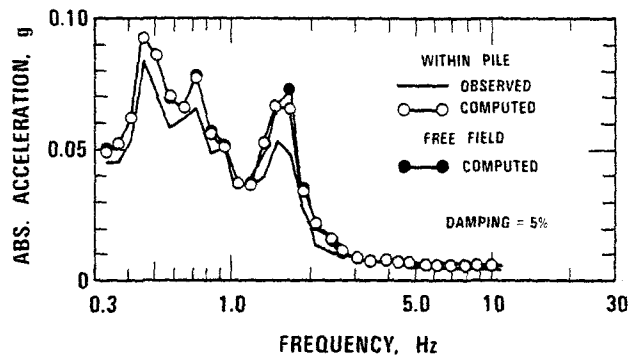
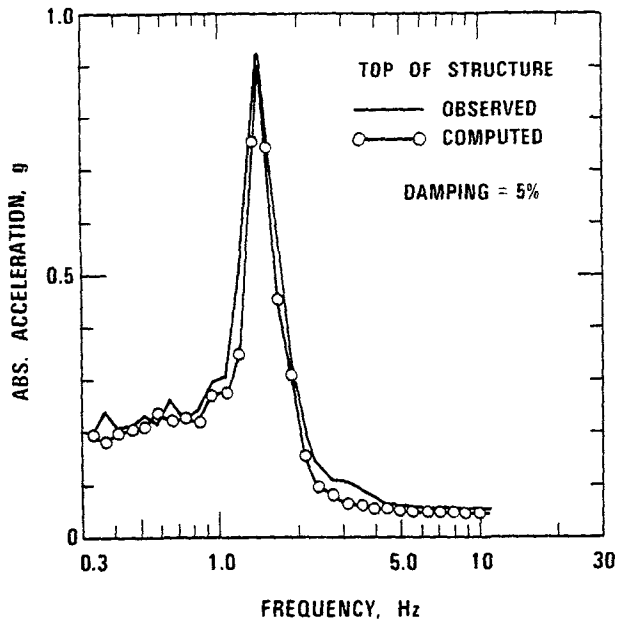
STRUCTURAL PROPERTIES FOR CASE STUDY



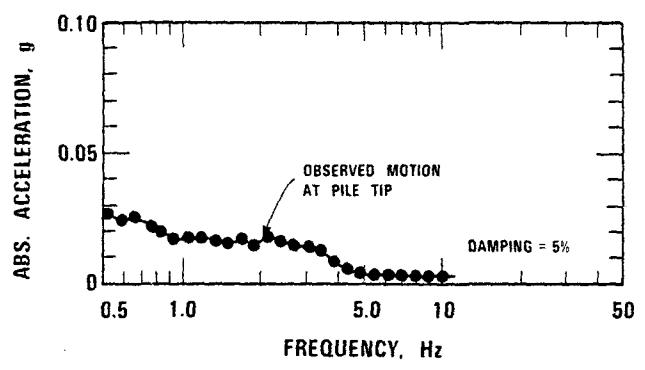
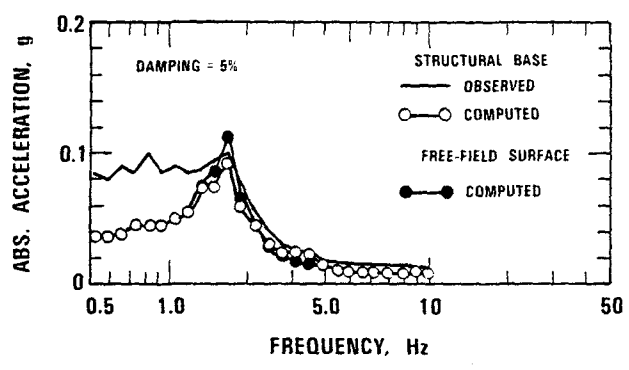
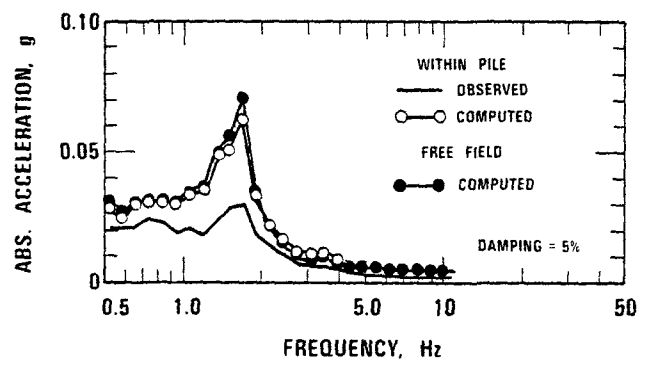
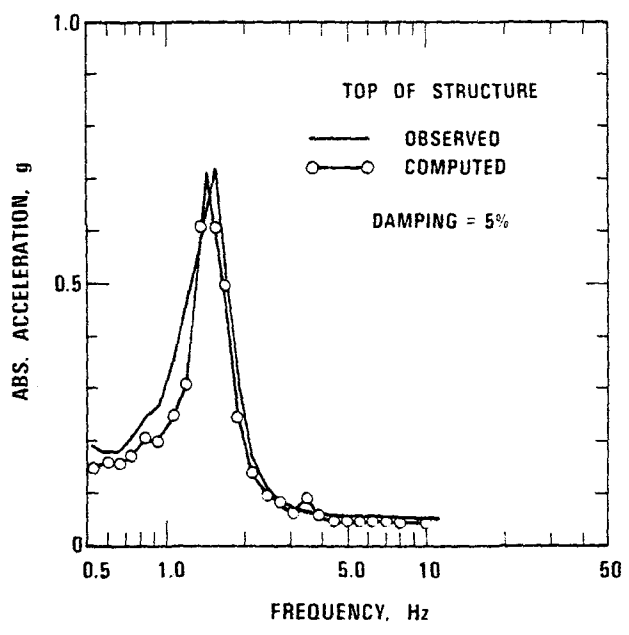
EARTHQUAKE MOTIONS AT PILE-TIP LEVEL
FOR CASE STUDY



COMPUTED AND OBSERVED ACCELERATION TIME HISTORIES FOR IZU EVENT



COMPUTED AND OBSERVED RESPONSE SPECTRA FOR IZU EVENT



COMPUTED AND OBSERVED RESPONSE SPECTRA FOR IBARAKI EVENT

A P P E N D I X A

APPENDIX A: REFERENCES

Blaney, G.W., Kausel, E., and Roesset, J.M. (1976), "Dynamic Stiffness of Piles," Numerical Methods in Geomechanics, edited by C.S. Desai, Vol. II, pp. 1001-1012.

Building Research Institute (1976), Digitized Earthquake: Accelerograms in a Soil-Structure System, Kenchiku Kenkyu Shiryo, Vol. 12, Ministry of Construction, Japan (in Japanese).

Fukuoka, M. (1966), "Damage to Civil Engineering Structure," Soils and Foundations, Vol. 6, No. 2, pp. 45-52.

Japan Residence Corporation (1979), Research on Seismic-Resistant Characteristics of High-Rise Apartment Buildings Based on Observational Data, Report No. 79-064.

Housner, G.W. and Castellani, A. (1969), Discussion of "Comparison of Footing Vibration Tests with Theory" by F.E. Richart, Jr. and R.V. Whitman, Journal, Soil Mechanics and Foundations Division, ASCE, Vol. 95, No. SM1, pp. 360-364.

Kachadoorian, R. (1968), "Effects of the Earthquake of March 27th, 1964 on the Alaska Highway System," Geological Survey Professional Paper 545-C, U.S. Department of Interior, Washington, D.C.

Kagawa, T. and Kraft, L.M., Jr. (1980a), "Seismic p-y Responses of Flexible Piles", Journal, Geotechnical Engineering Division, ASCE, Vol. 106, No. GT8, pp. 899-918.

Kagawa, T. and Kraft, L.M., Jr. (1980b), "Lateral Load-Deflection Relationships of Piles Subjected to Dynamic Loadings," Soils and Foundations, Vol. 20, No. 4, pp. 19-36.

Kagawa, T. and Kraft, L.M., Jr. (1981), "Dynamic Characteristics of Lateral Load-Deflection Relationships of Flexible Piles," Journal, Earthquake Engineering and Structural Dynamics, Vol. 9, No. 1, pp. 53-68.

Kobori, T., Minai, R., and Baba, K. (1977), "Dynamic Behavior of a Laterally Loaded Pile," Proceedings, the Specialty Session 10, 9th International Conference on Soil Mechanics and Foundation Engineering, Tokyo, pp. 175-180.

Kuhlemeyer, R. (1979), "Static and Dynamic Laterally Loaded Floating Piles," Journal, Geotechnical Engineering Division, ASCE, Vol. 105, No. GT2, pp. 289-304.

Liou, D.D. and Penzien, J. (1977), Seismic Analysis of an Off-shore Structure Supported on Pile Foundations, EERC Report 77-25, Earthquake Engineering Research Center, University of California, Berkeley.

Nogami, T. and Novak, M. (1977), "Resistance of Soil to a Horizontally Vibrating Pile," Journal, Earthquake Engineering and Structural Dynamics, Vol. 5, No. 3, pp. 249-261.

Ogata, N. and Kotsubo, S. (1966), "Seismic Force Effect on Pile Foundation," Proceedings, Japan Earthquake Engineering Symposium, Tokyo, Japan (in Japanese).

Ohta, T., Niwa, M., and Ueno, K. (1978), "Seismic Response Characteristics of Structure with Pile Foundation on Soft Subsoil Layer," Proceedings, Japan Earthquake Engineering Symposium (in Japanese).

Parmalee, R.A., Penzien, J., Scheffey, C.F., Seed, H.B., and Thiers, G.R. (1964), Seismic Effects on Structures Supported on Piles Extending Through Deep Sensitive Clays, Report submitted to the California State Division of Highways, SESM 64-2, University of California, Berkeley.

Poulos, H.G. (1971), "The Behavior of Laterally Loaded Piles: II-Pile Groups," Journal, Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM5, pp. 733-751.

Poulos, H.G. (1979), "Group Factors for Pile-Deflection Estimation," Journal, Geotechnical Engineering Division, ASCE, Vol. 105, No. GT12, pp. 1489-1509.

Prakash, S. and Chandrasekaran, V. (1973), "Pile Foundations under Dynamic Loads," Symposium on Behavior of Earth and Earth Structures Subjected to Earthquakes and other Dynamic Loads, Roorkee, India, March, Vol. 1, pp. 165-173.

Roesset, J.M. and Angelides, D. (1979), "Dynamic Stiffness of Piles," Proceedings, Numerical Methods in Offshore Piling, Institute of Civil Engineers, London, May, pp. 57-63.

Seed, H.B. and Idriss, I.M. (1969), "Influence of Soil Conditions on Ground Motions during Earthquakes," Journal, Soil Mechanics and Foundations Division, ASCE, Vol. 95, No. SM1, pp. 99-137.

Sugimura, Y. (1975), "Earthquake Observation and Dynamic Analysis of Pile-Supported Building," Proceedings, Japan Earthquake Engineering Symposium.

Tajimi, H. (1969), "Dynamic Analysis of a Structure Embedded in an Elastic Stratum," Proceedings, 4th World Conference on Earthquake Engineering, Santiago, Chile, Vol. 3, pp. 54-69.

Wolf, J.P. and von Arx, G.A. (1978), "Impedance Function of a Group of Vertical Piles," Proceedings, Conference on Soil Dynamics and Earthquake Engineering, ASCE, Pasadena, CA., Vol. 2, pp. 1024-1041.

Yoshida, I. and Yoshinaka, R. (1972), "A Method to Estimate Modulus of Horizontal Subgrade Reaction for Pile," Soils and Foundations, Vol. 12, No. 3, pp. 1-17.

