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EFFECTS OF GAIN AND DELAY-TIME OF CONTROL DEVICES TO STRUCTURAL SAFETY

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16. Abstract (Limit: 200 words)

The effects of several combinations of gain and delay time on the control of such flexible structures as tall buildings and long bridges are studied. Investigations of active structural control, active damping of large structures in winds, anti-earthquake application of pulse generators, active feedback control, modal synthesis, and modal control are reviewed. To reduce undesirable effects of disturbances, the feedforward control system, the feedback control system, or a combination of the two may be used. Ways to minimize the effects of the disturbance force are examined. An example is provided of a single-degree-of-freedom structural system subjected to an artificially generated earthquake. It is found that the larger the gain and the smaller the delay time, the more effective the control system will be.

17. Document Analysis a. Descriptors

Structural design	Buildings	Safety
Earthquakes	Skyscrapers	Structures
Pulse generators	Bridges	Earthquake resistant structures

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Structural control

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1. Introduction:

The useful life of a flexible structure, such as a ship, a tall building, or a large airplane, depends on the possibility and probability of fatigue failures due to the vibration of such structures. Each vibrational mode can be described with an equation of motion of the form: $m\ddot{x} + b\dot{x} + kx = U(t)$, where $U(t)$ is the input or excitation force. Theoretically, it is possible to find an input which will drive both the deflection, $x(t)$, and velocity, $\dot{x}(t)$, to zero in finite time for arbitrary initial conditions because such a linear system is completely controllable.

In the real-world, one is concerned with limited supply of energy which is required to produce control forces. It is well known that more energy is required for higher gains. Moreover, there always exists a delay time in generating the desired control force. To study the reliability aspects of the structural control problem [1], it is necessary to consider the effects of the gain and delay time which are inherent in applying control systems to ensure structural safety.

The objective of this study is to find the effects of several combinations of gain and delay time on structural control. To solve the problem, one is able to use the following two approaches: the classical control theory in the frequency domain, and the modern control theory in the time domain.

2. Literature Review:

During these past several decades, active control has been extensively applied in aerospace and mechanical engineering practices. However, applications to civil engineering structures began only in recent years. One reason is that civil engineering structures tend to be more complex as well as bulkier and thus more difficult to control. While active control is not advocated for every structure, there are certain structures for which the use of active control can result in a more efficient and reliable design.

To control the behavior of a given structure, one can use either passive and/or active control systems. The main idea of using a control system is that flexible structures such as extremely tall buildings or long bridges can be designed to resist essentially the operational gravity loads and the active control system can take care of any side-sway motions resulting from lateral loads. Recently, the relevant literature and the interrelationship among structural identification, control, and reliability in wind engineering were discussed [2]. In the following, additional literature on this subject matter is reviewed briefly.

A general approach to active structural control was discussed by M. Abdel Rohman and H. H. E. Leipholz [3]. They proposed a unified approach to be used in the active control of structures to satisfy simultaneously the requirements for safety, serviceability, and human comfort considerations. The feasibility of using such a control was also considered. In another paper, Rohman and Leipholz [4] studied the vibration of a single-span bridge. The control mechanism has been used to control the vibration of the bridge. They also showed some of the benefits from using a closed-loop control in controlling flexible civil engineering structures.

Active damping of large structures in winds was studied by Richard A. Lund [5]. Mass dampers have been installed in large buildings to reduce building motion during high winds. These systems are presently designed to operate as passive tuned mass dampers. An investigation of the benefits of active control in such systems was presented by Lund.

Anti-earthquake application of pulse generators was studied by Sami F. Masri and George A. Bekey [6]. They used servo-controlled gas pulse generators to mitigate the earthquake induced motions of tall buildings.

The active control of structures by modal synthesis was presented by L. Meirovitch and H. Öz [7]. Their control scheme consists of independent modal control providing active damping for the controlled modes of the structure.

The concept of active feedback control is studied by John Roorda [8]. The first experiment demonstrates in a simple way the essential ingredients of an active feedback control system. It involves the control of the midspan deflection of a king-post truss by selectively lengthening or shortening the under-slung cable in a controlled way. In addition, a vertical cantilever is controlled with a pair of vertical steel tendons fixed to a cross arm attached at the column and to a yoke which pivots about the column center line near the base.

Soong and Chang [9] studied an optimal control configuration using the theory of modal control. For tall buildings, the application of modern control theory introduces a number of difficult problems. An important problem is that of obtaining optimal control configuration or the determination of appropriate locations of controllers. This topic has been studied by Soong and Chang [9].

Recently, optimal open-loop control of structures under earthquake excitation was studied by J. N. Yang and M. J. Lin [10]. They used an active tendon control system and an active mass damper system.

3. Formulation of the Problem

A lumped-mass one-story building with active control devices is shown in Figure (1). The particular control devices as considered herein consist of a "jet-engine-like impulse force generator" attached to the top of the building. The equation of motion for such a structure can be written in the following form:

$$m\ddot{x} + b\dot{x} + kx = N + u \tag{1}$$

where $N = N(t)$ is the external excitation such as earthquakes. For example, $N = -m\ddot{x}_g$, where $\ddot{x}_g$ denotes the ground acceleration. In Equation 1, the term

$u \equiv u(x,\dot{x};t)$ denotes the control force generated with a control device which is located at the top of the system. The Laplace transfer function of the plant (structure) can be written as follows:

$$G_p(s) = \frac{1}{ms^2+bs+k} \tag{2}$$

where:

m = Mass of the system
b = Damping of the system
k = Stiffness of the spring

Without loss of generality, the external force can be disregarded for the subsequent discussions because the external force can be assumed as a part of u in Equation (1) becomes:

$$m\ddot{x} + b\dot{x} + kx = u \tag{3}$$

For a single-degree-of-freedom system, Equation (3) can be written in the following form:

$$\dot{X} = AX + Bu$$
$$Y = CX \tag{4}$$

where:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \qquad C = [1 \quad 0]$$

Now, the matrix P can be defined in the following form:

$$P = [B \mid AB] \tag{5}$$

Because the rank of the P matrix is equal to two, therefore, this system is controllable (see Appendix A). The system is also observable because the rank of Q matrix is also equal to two. Where Q matrix is defined as:

$$Q = [c^T, A^TC^T] \tag{6}$$

The block diagram of the system without any control device is shown in Fig. 2. A generalized closed loop system schematics is shown in Fig. 3.

In civil engineering structures, the disturbance force is the major input force of the system. To reduce the undersirable effects of disturbances, one can use either the feedforward control system or feedback control system. It is also possible to use both systems at the same time.

For example, consider the system as shown in Fig. 4 where K_p is an adjustable gain and $G_c(s)$ and $H(s)$ are fixed components. The closed-loop transfer function for the disturbance is:

$$\frac{X_N(s)}{N(s)} = \frac{G_p(s)}{1+K_pG_c(s)G_p(s)H(s)} \tag{7}$$

To minimize the effect of the disturbance force, the adjustable gain K_p should be chosen as large as possible. One can also reduce the undersirable disturbance force by using a feedforward control. Fig. 5 shows a system with feedforward control. A disturbance feedforward is an open-loop and it depends on the constancy of the parameters. In Fig. 5 both open-loop and closed-loop control systems are used simultaneously. In this system, errors from all causes can be reduced without requiring a large loop gain. A feedforward system can be used if and only if one can measure the disturbance forces.

Consider the system as shown in Fig. 5. It has been assumed that both plant transfer function, $G_p(s)$, and disturbance transfer function, $G_2(s)$, are known. One can easily find a suitable controller transfer function, $G_C(s)$, then, the disturbance feedforward transfer function, $G_1(s)$ can be found as follows:

$$G_1(s) = -\frac{G_2(s)}{G_C(s)G_p(s)} \tag{8}$$

Consider the system shown in Fig. 4. The transfer function of the system can be obtained as

$$\frac{X_N(s)}{N(s)} = \frac{G_p(s)}{1+K_pG_c(s)G_p(s)H(s)} \tag{9}$$

The effect of the controller is seen by the presence of K_p in the denominator of the transfer function. From this equation, the response $X_N(s)$ to the disturbance force, N(s), can be found. On the other hand, in considering the response to the reference input R(s), we may assume that the disturbance is zero. Then the response $X_R(s)$ to the reference input R(s) can be obtained from

$$\frac{X_R(s)}{R(s)} = \frac{K_pG_c(s)G_p(s)}{1+K_pG_c(s)G_p(s)H(s)} \tag{10}$$

The response to the simultaneous application of the reference input and disturbance can be obtained by adding the two individual responses. In other words, the total response X(s) due to the simultaneous application of the reference input R(s) and disturbance N(s) is given by

$$X(s) = \frac{G_p(s)}{1+K_pG_c(s).G_p(s).H(s)} [K_pG_c(s)R(s) + N(s)] \tag{11}$$

The reference input R(s) can be assumed to be zero because the set point of the controller is fixed. Consider now the case where $|K_pG_c(s)H(s)| >> 1$ and $|KG_c(s)G_p(s)H(s)| >> 1$. In this case, the closed-loop transfer function $\frac{X_N(s)}{N(s)}$ becomes almost zero, or as small as possible. This is an advantage of using the closed loop system. If the reference input R(s) not to be equal to zero, then the closed-loop transfer function $\frac{X_R(s)}{R(s)}$ approaches $\frac{1}{H(s)}$ as the gain of $KG_c(s)G_p(s)H(s)$ increases. This means that the closed-loop transfer function $\frac{X_R(s)}{R(s)}$ becomes independent of $K_pG_c(s)$ and $G_p(s)$ and becomes inverseley proportional to H(s) so that the varia ions of $G_p(s)$ and $K_pG_c(s)$ do not affect the closed-loop transfer function $\frac{X_R(s)}{R(s)}$. This is another advantage of the closed-loop system. It can easily be seen that any closed-loop system with unity feedback tends to equalize the input and output.

A lumped mass n-story building with active control devices is shown in Fig. (6). The particular control devices considered herein consist of jet-engine attached to each floor. The equations of motion can be written in the following form:

$$m\ddot{y} + c\dot{y} + ky = N + bu \tag{12}$$

in which y is the relative displacement vector, N is the external excitation vector that it is called disturbance force, u is the vector of control force. Each element of u represents either the pushing or pulling force generated by a control device located at each floor. b is a nxr matrix whose elements depend on the arrangement of the controller. m, c, and k are the mass matrix, the damping matrix, and the stiffness matrix respectively and they are nxn matrices.

By using state space concept, Equation 12 can be converted into a set of 2n first order differential equation. Note also that without loss of generality, the external forces or disturbance forces can be disregarded and hence Equation 12 becomes:

$$\dot{X} = Ax + Bu \tag{13}$$

where: $Y = cx$

$$X = \begin{Bmatrix} Y \\ \dot{Y} \end{Bmatrix} \qquad A = \begin{bmatrix} 0 & I \\ -m^{-1}k & -m^{-1}c \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ m^{-1}b \end{bmatrix}$$

Because the main objective of this investigation is earthquake; one can neglect the damping of the system only during the time of the earthquake.

By using the similarity transformation, Equation 12 can be written in the following form:

$$\dot{X} = \overline{A}X + \overline{B}u \tag{14}$$

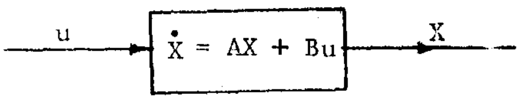
$$Y = \overline{c}x$$

where

$$\bar{A} = T^{-1}AT , \quad \bar{B} = T^{-1}B , \text{ and } \bar{c} = Tc$$

It can be shown that the eigenvectors corresponding to distinct eigenvalues are independent. Hence if A has 2n distinct eigenvalues, one can find a T matrix that diagonalizes it. In this case the T matrix is called the modal matrix of the system and all differential equations are uncoupled and the system can be solved like a single input single output.

If, however, an eigenvalue is repeated, the situation is more complicated. For example, if λ is an eigenvalue of multiplicity k, then it is possible that there are anywhere from 1 to k independent eigenvectors associated with it. The actual number depends on the particular matrix A. Note that Equation 13 represents the open-loop system. It is also assumed that the open-loop system has 2n distinct eigenvalue. It means that there exists a nonsingular modal matrix. By using block diagram, Equation 13 can be shown as follows:



The solution of this equation is:

$$X(t) = e^{A(t-t_1)} X(t_1) + \int_{t_1}^{t} e^{A(t-\tau)} Bu(\tau)d\tau \tag{15}$$

The application of the modal matrix is useful in solving these equations.

To find out whether the system is controllable one needs to form the P matrix. The rank of the P matrix must be equal to 2n. To have a stable system, it is necessary that all eigenvalues of the system must be located in left half plane.

4. Optimal Control

Suppose a controller input is to be manipulated in such a way that the performance index

$$J = \int_{0}^{\infty} (\langle y,\overline{Q}y\rangle + \langle u,Ru\rangle)\,dt \tag{16}$$

is minimised for a plant having state-space equations

$$\dot{X} = AX + Bu$$
$$Y = CX \tag{17}$$

where (A,C) is an observable pair. The corresponding optimal control action is given by

$$u = -R^{-1}B^{T}\overline{P}X, \quad Q = C^{T}\overline{Q}\,C \tag{18}$$

where $\overline{P}$ is the unique positive definite solution of the steady-state matrix Riccati equation.

$$-\overline{P}A-A^{T}\overline{P} + \overline{P}BR^{-1}B^{T}\overline{P} = C^{T}\overline{Q}C \tag{19}$$

This is the negative feedback convention form of the Riccati equation.

5. Numerical Example:

Consider the control system shown in Fig. 1. The equation for the plant is:

$$\dot{X} = AX + Bu \tag{20}$$

where:

$$X = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \qquad A = \begin{bmatrix} 0 & 1 \\ -\frac{25}{3} & -\frac{5}{3} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{10}{3} \end{bmatrix}$$

$$\overline{P}A + A^{T}\overline{P} + Q - \overline{P}BR^{-1}B^{T}\overline{P} = 0 \tag{21}$$

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -\frac{25}{3} & -\frac{5}{3} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{25}{3} \\ 1 & -\frac{5}{3} \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 \\ \frac{10}{3} \end{bmatrix}[0 \quad \tfrac{10}{3}]\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = 0 \tag{22}$$

The solution of the Riccati equation is:

$$\begin{bmatrix} .1681 & .0578 \\ .0578 & .008 \end{bmatrix} \tag{23}$$

and the optimal control force is:

$$u = -[.1926 \quad .0267]X \tag{24}$$

6. Discussion and Conclusions

Consider a single-degree-of-freedom structural system subjected to an artificially generated earthquake. The properties of this earthquake are given as follows:

(1) The duration of the record is 15 seconds;

(2) The uniform time interval is .05 sec.;

(3) The exponential decay constant is .62;

(4) The duration of the parabolic buildup time is .5 sec.;

(5) The time at start of exponential decay is 7.5 sec.;

(6) The lowest input spectrum frequency is .1 Hz;

(7) The highest input spectrum frequency is 10 Hz.

(8) The maximum acceleration is 1.g;

(9) The Maximum velocity is 24 in/sec.;

(10) The Maximum displacement is 5 in.

Moreover, the properties of the system are given as follows:

$m = .3 \quad \text{Kip} \ \frac{\text{sec}^2}{\text{in}}$

$c = .5 \quad \text{Kip} \ \frac{\text{sec}}{\text{in}}$

$K = 2.5 \quad \text{Kip/in}$

Considering the information as listed in Table (1), it is observed that by increasing the gain K_p, one can control the displacement of the system. It is also clear that by reducing the displacement the control force will increase. The same discussion is valid for the feedback gain. It means that by increasing the feedback gain, H, the displacement of the response will decrease. If the feedback gain is set to be .20, a nearly optimal solution can be obtained. However, the reduction of the displacement is not very much. Of course, by changing the "$\tilde{Q}$" we may find another gain that it may give a desire displacement. Result as summarized in Table (1) are shown in Fig. (8) through 15.

Fig. (8) shows the behavior of the system without active control force. The top plot is the external force, which is the product of mass and the ground acceleration. The active control force and displacement of the response have been plotted. It can be seen that the control force is equal to zero and the maximum displacement is equal to -3.32 inches. By using the active control force, the response displacement becomes smaller than the displacement of the system without active control system. Note that by increasing either the gain K_p or the feedback gain, H, or both, the response of the system becomes smaller. In Table (1) an unity feedback control system is used, and only the behavior of the system with respect to changing of the gain K_p is shown. Fig. (9) shows the active control force and displacement of the response when K_p is equal to .20. The maximum displacement in this case is -3.30 in. It is obvious that the required active control force is small and also the displacement does not change very much.

Fig. (10) shows the same system K_p is equal to one and the maximum displacement is -3.06 inches. The active control force has been increased in this case. The maximum displacement in Fig. (11) is equal to 3.22 inches and the value of the active control force has been increased.

By choosing K_p to be equal to 5.0, the maximum displacement is found to be +3.14 inches. When K_p is equal to 10, the maximum displacement is +2.9 inches. Due to higher gains such as K_p = 25 or K_p = 100, the maximum displacements became smaller. In these last two cases the maximum displacements are respectively -2.32 inches and +1.27 inches. These results are shown in Fig. (12) thru (15). Note that the transfer function of the controller has been assumed as a constant gain, K_p,. If we assume that the transfer function of the jet engine to be $\frac{1}{T_a s+1}$; we may have the same results but it is obvious that the system may or may not be stable. By changing T_a, we may have an unstable system. Table (2) shows the result of the same system when the transfer function of the controller has been assumed as follows: $G_c(s) = \frac{1}{T_a s+1}$. The value of H, K_p have been chosen as 5. and 1. respectively. The only variable of the system is T_a and the rest of the parameters are constant. Assuming T_a equal to zero, we will have a maximum displacement equal to .179". By increasing the value T_a, we will have a higher value for the displacement. For example, Fig. (18) shows the displacement of the response when T_a is equal to .001. In this case, the maximum displacement is equal to .180" and it is greater than the maximum displacement with T_a equal to zero. Fig. (19) shows the plot of the active control force for this case. Note also that the system is stable because all eigenvalues have a negative real part. Fig. (20) shows the behavior of the same system when T_a has been chosed as .005. The displacement of the system is higher than both previous cases with maximum value of .181". The active control force for this case has been shown in Fig. (21). Fig. (22) thru (30) show the behavior of the system by increasing the value of T_a. The displacement of the system has become larger. Due to some value of T_a, we may have an unstable system. For example, by choosing T_a equal to .5 we have an unstable system. The displacement of the system at time t = 20 sec. is equal to .966". Fig. (28) shows the response of this sytem. In this case, we have two eigenvalue with positive real part. By increasing the value of T_a to one, the system becomes stable and all eigenvalues have a negative real part. Note also that the maximum displacement of the system without any active control force due to the same external force, sin2t, is equal to .60". By increasing the value of T_a, the displacement becomes larger. Notice that in most cases the value of the displacement is smaller than the value of displacement without any active control force unless the system becomes unstable. For example, by chossing T_a equal to 1. although we have a stable system, but the displacement of the system with active control system is greater than the displacement without any active control system. It means that by using active control system the performance of the system is much worse than performance of the system without active control force, and note also that the stability of the system is important.

In this report, the effects of gain and delay time are studied. Generally, the larger the gain and the smaller the delay time, the more effective will be the control system. In the real world, there exist practical limitations as to the upper bound of the gain and the lower bound of the delay time. The structural reliability as a function of these factors is being studied in more detail and will be presented in the next technical report.

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APPENDIX: Controllability and Observability of Linear Systems

The linear system as represented by the following equation of motion,

$$\dot{X} = AX + Bu \tag{A-1}$$

with the following solution:

$$X(t_1) = \Phi(t_1,t_0)X(t_0) + \int_{t_0}^{t_1} \Phi(t_1,\sigma)B(\sigma)u(\sigma)d\sigma \tag{A-2}$$

is said to be controllable at time t_0 if there exists $u(\sigma)$, $t_0 \leq \sigma \leq t_1$, which satisfies A-2 for arbitrarily specified vectors, $X(t_1)$, $X(t_0)$.

One can show that $u(\sigma)$ is a control which satisfies A-2 for arbitrary $X(t_1)$ and $X(t_0)$. $u(\sigma)$ is given by following equations:

$$u(\sigma) = B^*(\sigma)\Phi^*(t_1,\sigma)X(t_1,t_0)^{-1}\ (X(t_1) - \Phi(t_1,t_0)X(t_0)) \tag{A-3}$$

and

$$X(t_1,t_0) \triangleq \int_{t_0}^{t_1} \Phi(t_1,\sigma)B(\sigma)B^*(\sigma)\Phi^*(t_1,\sigma)d\sigma \tag{A-4}$$

Theorem:

The linear system $\dot{X} = AX + Bu$ is controllable at t_0 if and only if there exists a finite $t_1 > t_0$ such that

$$X(t_1,t_0) > 0 \tag{A-5}$$

$X(t_1,t_0)$ is called the controllability matrix, and one can show that $X(t,t_0)$ satisfies the following equation:

$$\dot{X}(t,t_0) = X(t,t_0)A^*(t) + A(t)X(t,t_0) + B(t)B^*(t) \tag{A-6}$$

$$X(t_0,t_0) = 0$$

For a time-invariant system, both matrices, A, B, are constants. Using Cayley-Hamilton theorem we have:

Theorem:

A linear time invariant system is controllable only if rank P = n where P is a matrix as follows:

$$[P] = [B, AB, A^2G, \text{--------}, A^{n-1}B] \qquad \text{(A-7)}$$

There is another theorem for controllability of a linear-time-invariant system. If A, B are constants, then the linear system is controllable if and only if $X(\infty,0)>0$, or

$$X(\infty,0) = \int_0^\infty e^{At}BB^*e^{A^*t}dt>0 \qquad \text{(A-8)}$$

and if A is stable $X(\infty,0)$ satisfies

$$X(\infty,0)A^* + AX(\infty,0) + BB^* = 0 \qquad \text{(A-9)}$$

Therefore there are two controllability criteria as follows:

Controllability Criterion 1

The constant coefficient system, for which A has distinct eigenvalues, is completely controllable if and only if there are no zero rows of $B_n = T^{-1}B$, where T is the modal matrix.

Controllability Criterion 2

A constant coefficient linear system with the representation $\{A,B,C,D\}$ is completely controllable if and only if the matrix of

$$P \triangleq [B|AB|A^2B|\text{----}|A^{n-1}B] \text{ has rank n.} \qquad \text{(A-10)}$$

Consider the following single-degree-of-freedom mechanical system:

$$M\ddot{X} + B\dot{X} + KX = u$$

$$\begin{Bmatrix}\dot{X}_1 \\ \dot{X}_2\end{Bmatrix} = \begin{bmatrix}0 & 1 \\ a & b\end{bmatrix}\begin{Bmatrix}X_1 \\ X_2\end{Bmatrix} + \begin{bmatrix}0 \\ c\end{bmatrix} u$$

where:

$$a = \frac{-K}{M} \quad b = \frac{-B}{M} \quad c = \frac{1}{M}$$

$$A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ c \end{bmatrix}$$

$$P \triangleq \begin{bmatrix} 0 & c \\ c & bc \end{bmatrix}$$

It is obvious that rank $P = 2$. Therefore, the system is controllable. In this example the disturbance force has been ignored.

Observability of Linear Systems:

The linear system $\dot{X} = AX$, $y = cX$ with solution

$$y(\sigma) = c(\sigma)\Phi(\sigma,t)X(t), \quad \sigma>t \tag{A-11}$$

is observable at time t if there exists some $t_1>t$ such that $X(t)$ can uniquely be determined by measuring $y(\sigma)$.

The linear system is observable at time, t, if and only if there exists a finite $t_1>t$ such that $K(t_1,t)>°$ where $K(t_1,t)$ is given by:

$$K(t_1,t) \triangleq \int_t^{t_1} \Phi^*(\sigma,t)c^*(\sigma)c(\sigma)\Phi(\sigma,t)\,d\sigma \tag{A-12}$$

$$X(t) = K(t_1,t)^{-1} \int_t^{t_1} \Phi^*(\sigma,t)c^*(\sigma)y(\sigma)\,d\sigma \tag{A-13}$$

$K(t_1,t)$ is called the observability matrix and it must satisfy the following differential equation.

$$-\dot{K}(t_1,t) = K(t_1,t)A(t) + A^*(t)K(t_1,t) + c^*(t)c(t)$$

$$K(t,t) = 0 \tag{A-14}$$

Observability for time-invariant systems depend only on the constant matrices A and c. No reference need be made to a particular interval $[t_0,t_1]$.

Observability Criterion 1

The constant coefficient system, for which A has distinct eigenvalues, is completely observable if and only if there are no zero columns of $c_n = cT$. where T is the modal matrix.

Observability Criterion 2

A constant coefficient linear system is completely observable if and only if the nxmn matrix, Q, has rank n:

$$Q \triangleq [c^* | A^* c^* | \text{------} A^{n-1^*} c^*] \tag{A-15}$$

If both matrices A and c are constant then the linear time-invariant system is observable if and only if

$$K(\infty,0) \triangleq \int_0^\infty e^{A^* t} . c^* . c . e^{At} dt > 0 \tag{A-16}$$

If A is stable then $K(\infty,0)$ satisfies

$$K(\infty,0)A + A^* K(\infty,0) + c^* c = 0 \tag{A-17}$$

The controllability as well as observability of a linear time-invariant system is invariant under andy similarity transformation.

As an example, consider the single degree freedom system. The equation of motion is given by:

$$\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ c \end{bmatrix} u$$

$$\{Y\} = [1 \quad 0] \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

In this case the output of the system is displacement.

$$Q \triangleq [c^T, A^T c^T, \text{----} \quad]$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{rank } Q = 2$$

It is obvious that the rank Q is two, therefore the system is observable.

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TABLE (1)

Case	Fig.	T_a	H	K_p	Max. Displacement	Max. Velocity	Max. Acc.
1	8	.00	0.00	0.00	-3.32	23.14	-372.24
2	9		1.	.20	-3.3	23.40	-373.84
3	10	.00	1.	1.0	-3.06	-24.68	-377.67
4	11	.00	1.	2.0	3.22	-25.66	387.55
5	12	.00	1.	5.0	3.14	-24.13	429.17
6	13	.00	1.	10.	2.9	-24.78	451.57
7	14	.00	1.	25.	-2.32	-28.83	463.93
8	15	.00	1.	100.	1.27	22.74	525.89

TABLE (2)

Case	Fig.	N	T_a	H	K_p	Max. Displacement
9	16	Sin2t	0.	5.	1.	.17987
10	18	Sin2t	.001	5.	1.	.18017
11	20	Sin2t	.005	5.	1.	.18137
12	22	Sin2t	.01	5.	1.	.18291
13	24	Sin2t	.05	5.	1.	.19654
14	26	Sin2t	.1	5.	1.	.2143
15	28	Sin2t	.5	5.	1.	.86574
16	30	Sin2t	1.	5.	1.	.6056

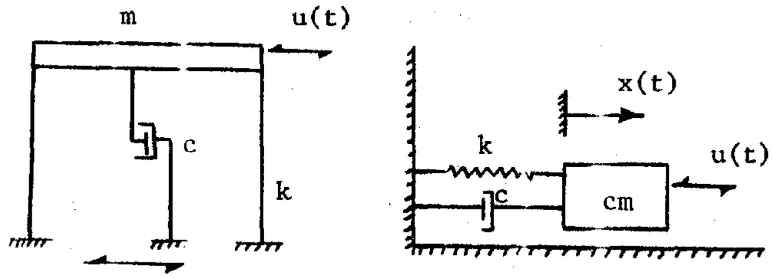


Fig. 1

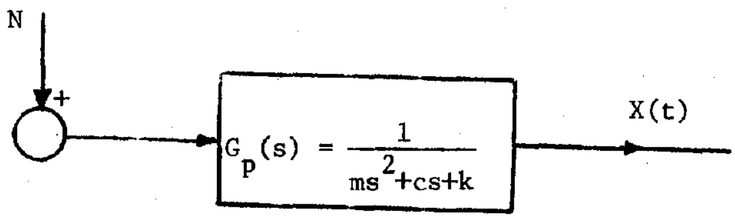
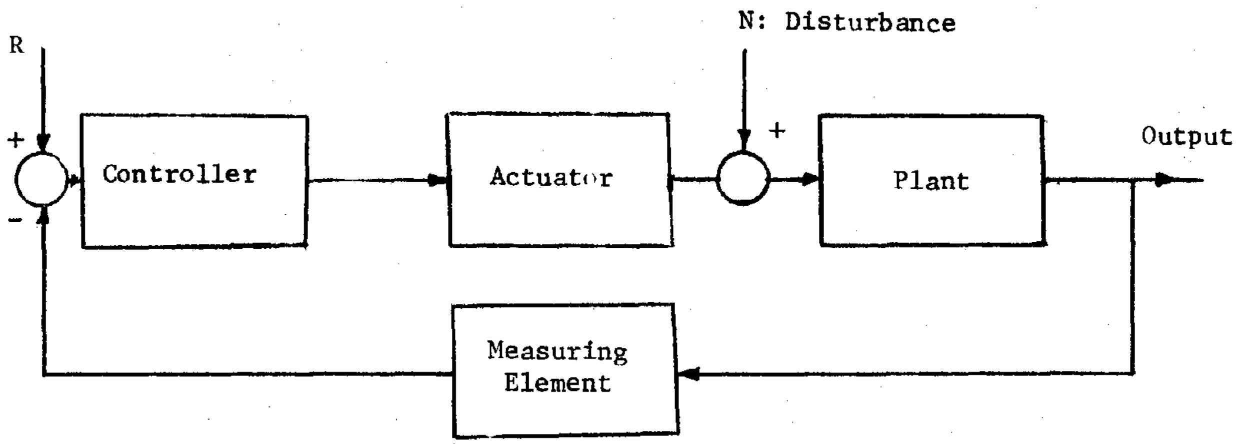


Fig. 2



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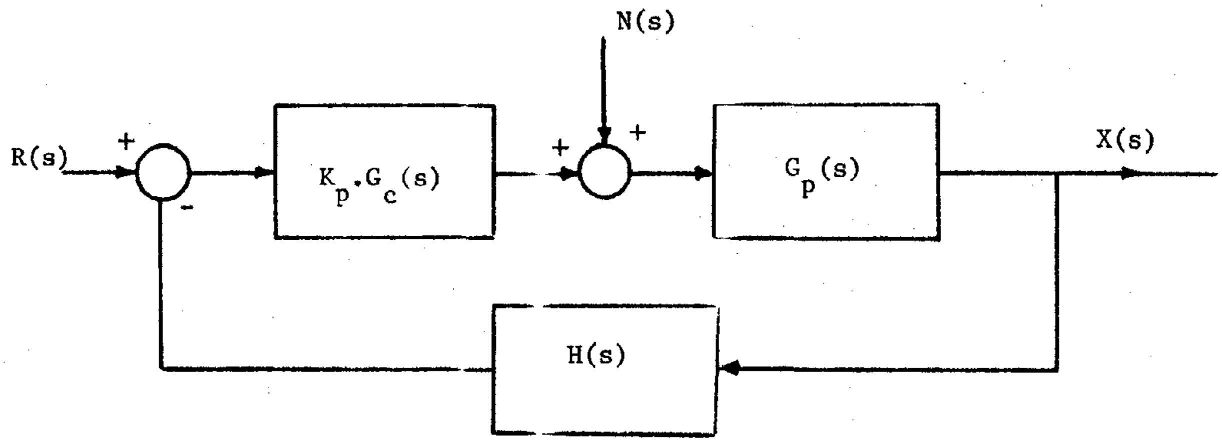

Fig. 4

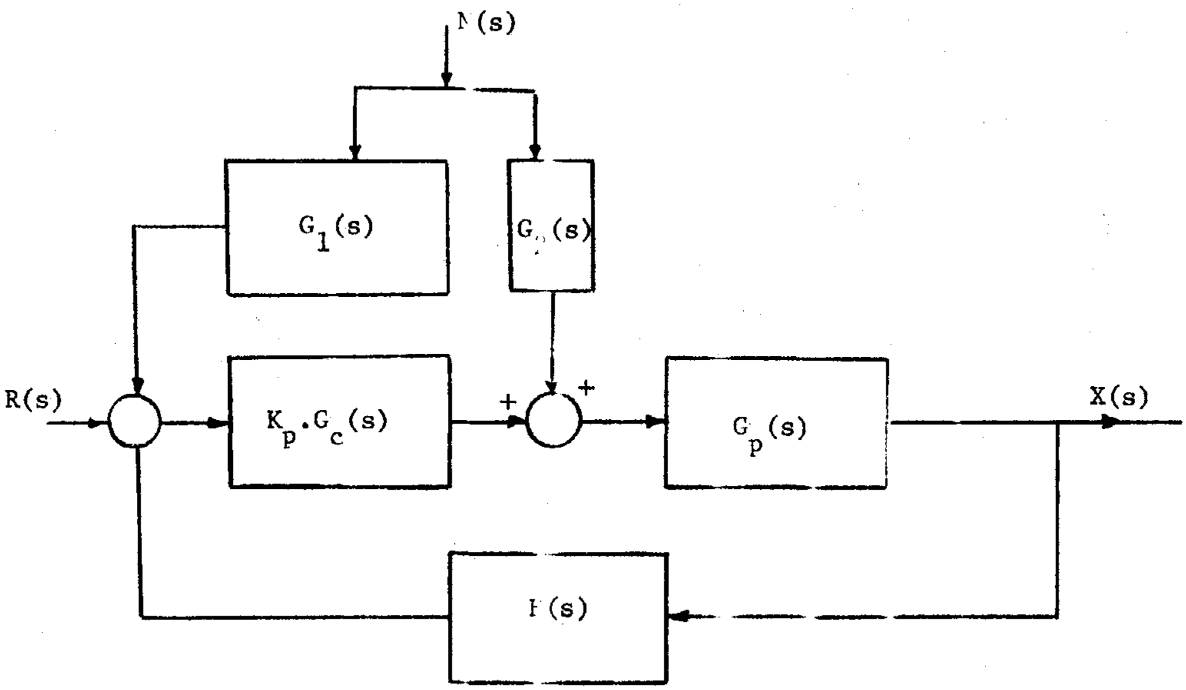

Fig. 5

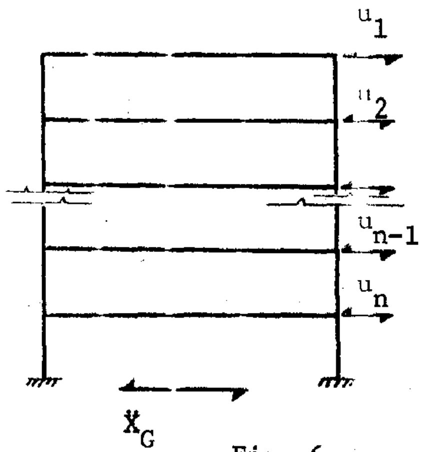

Fig. 6

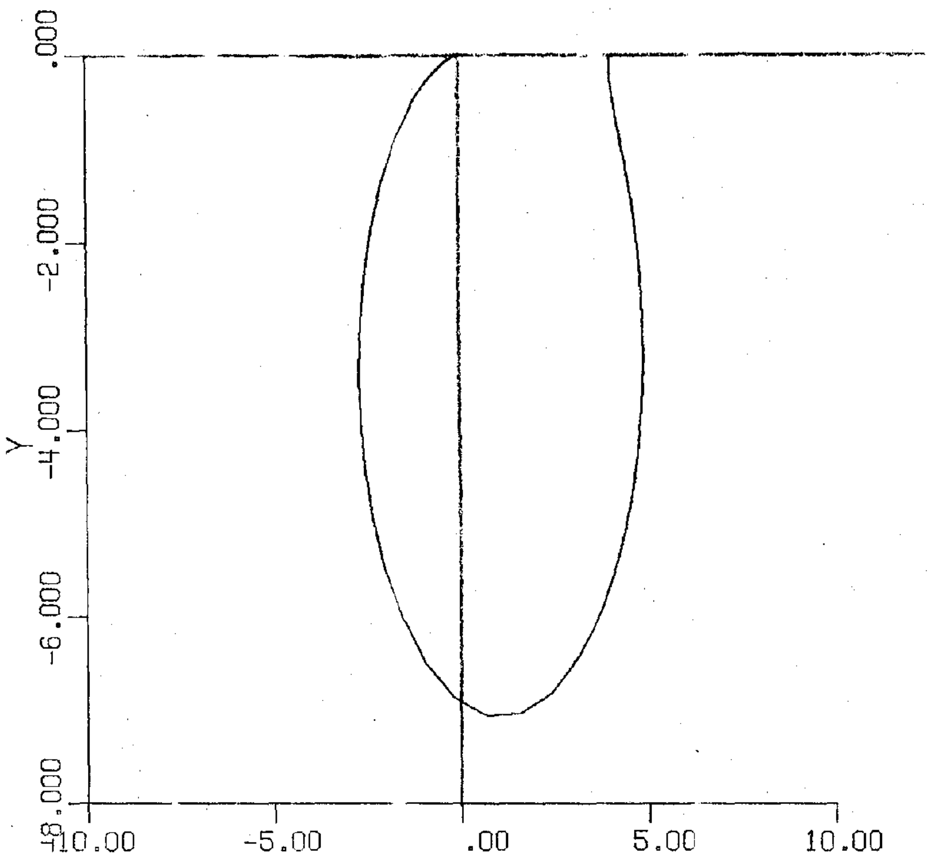

Fig. 7. Nyguist Plot of the Open-Loop System.

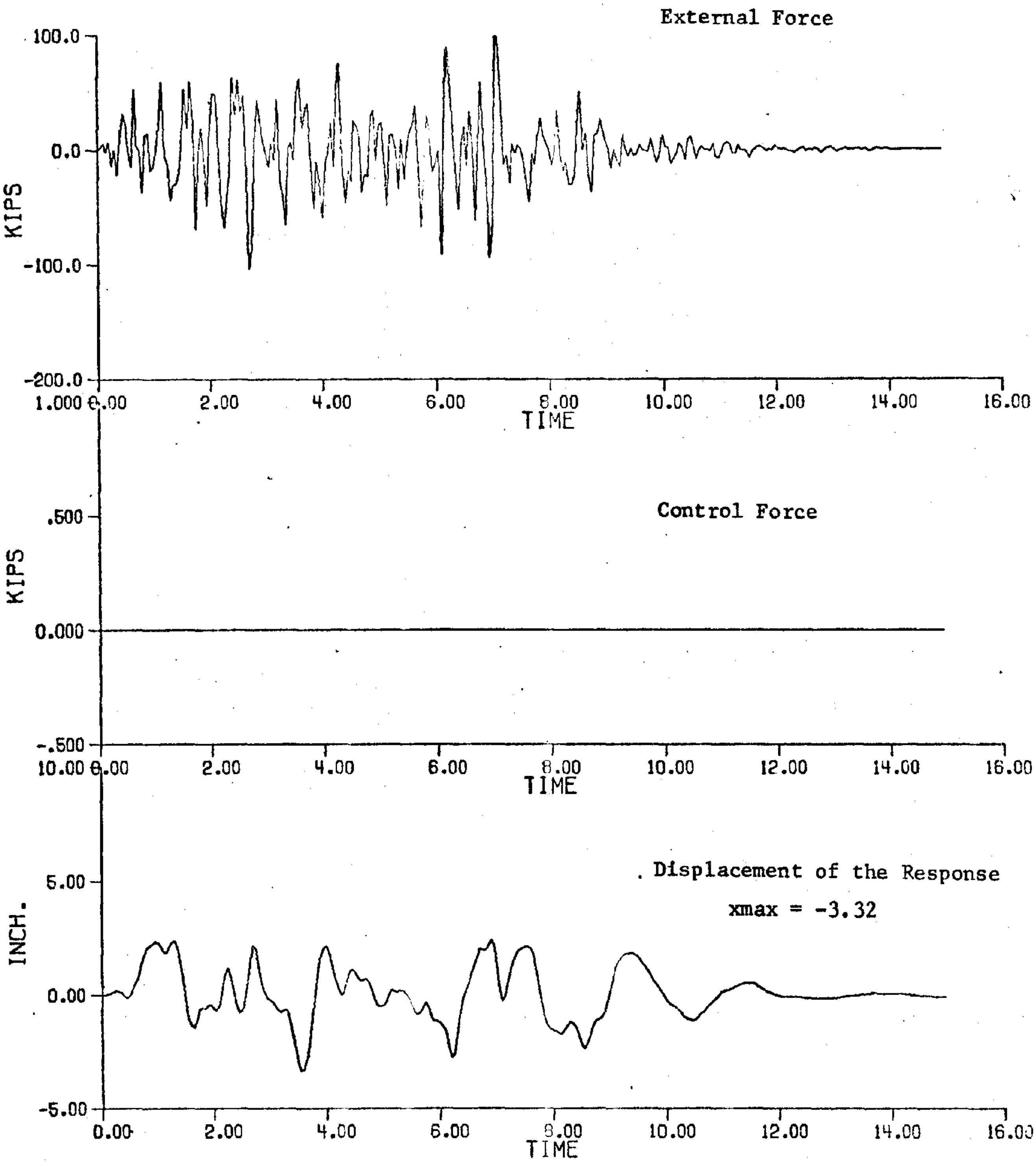

Fig. 8

External Force, Control Force, and Displacement Response for Case 1 in Table 1.

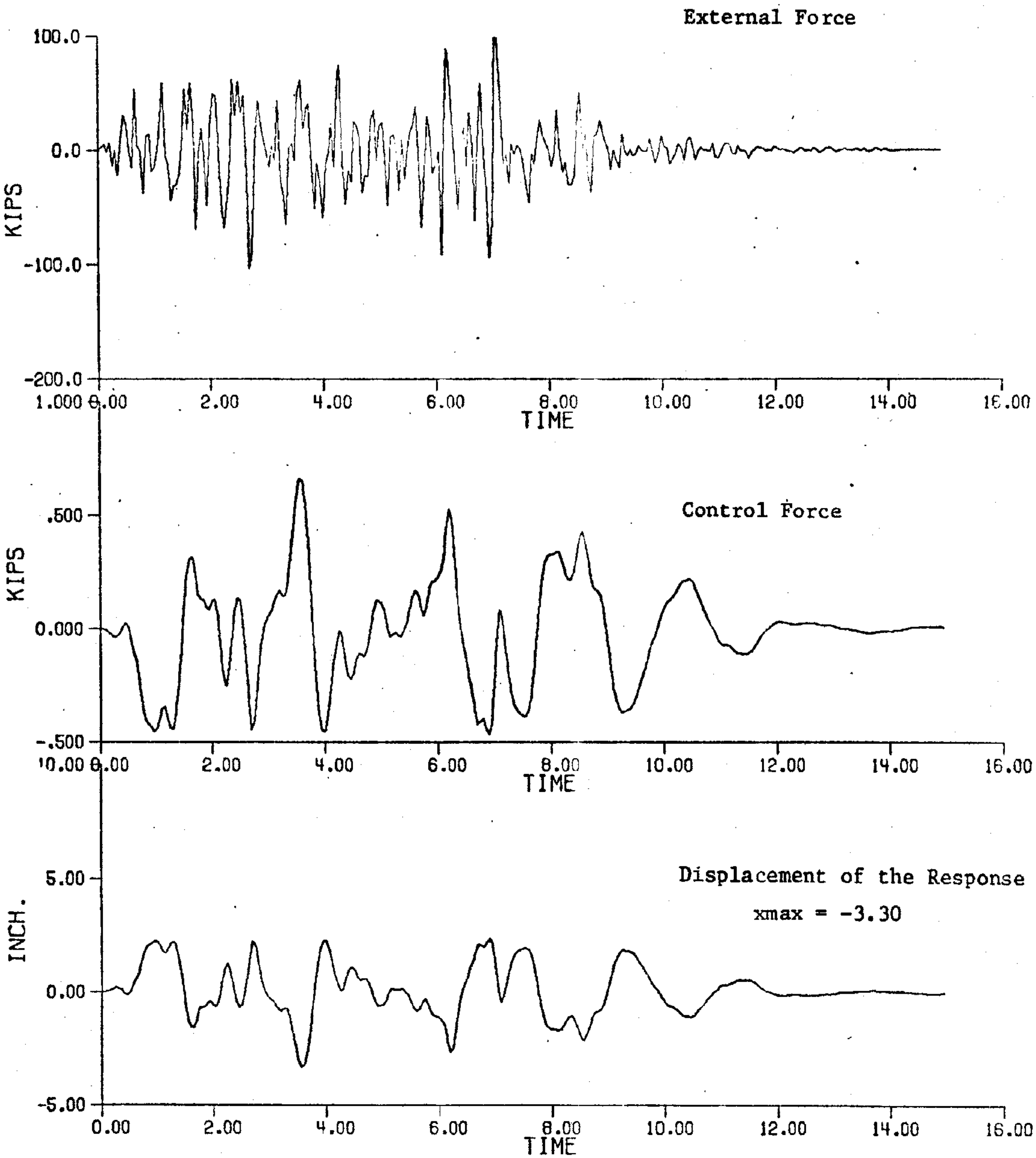

Fig. 9

External Force, Control Force, and Displacement Response for Case 2 in Table 1.

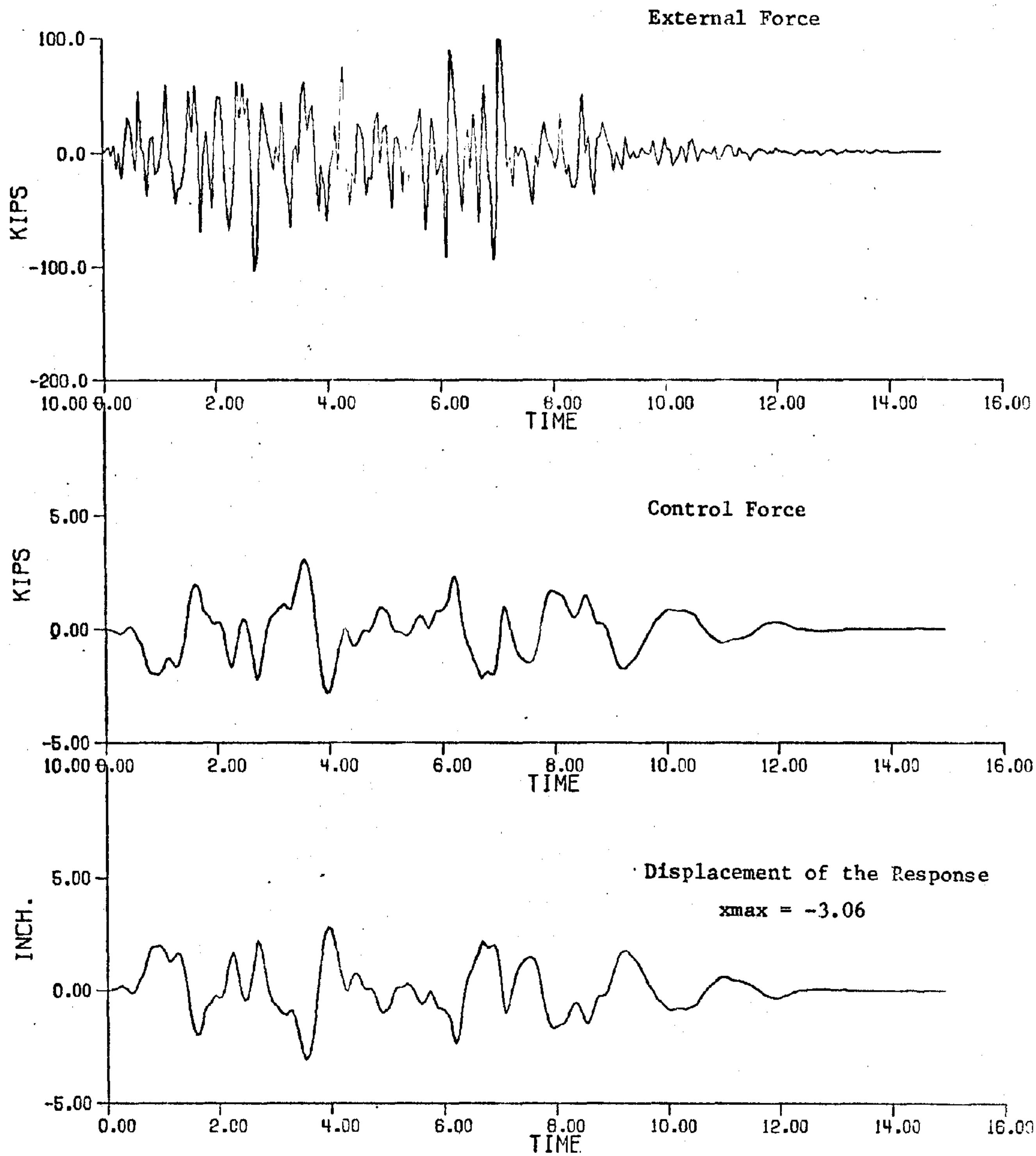

Fig. 10

External Force, Control Force, and Displacement Response for Case 3 in Table 1.

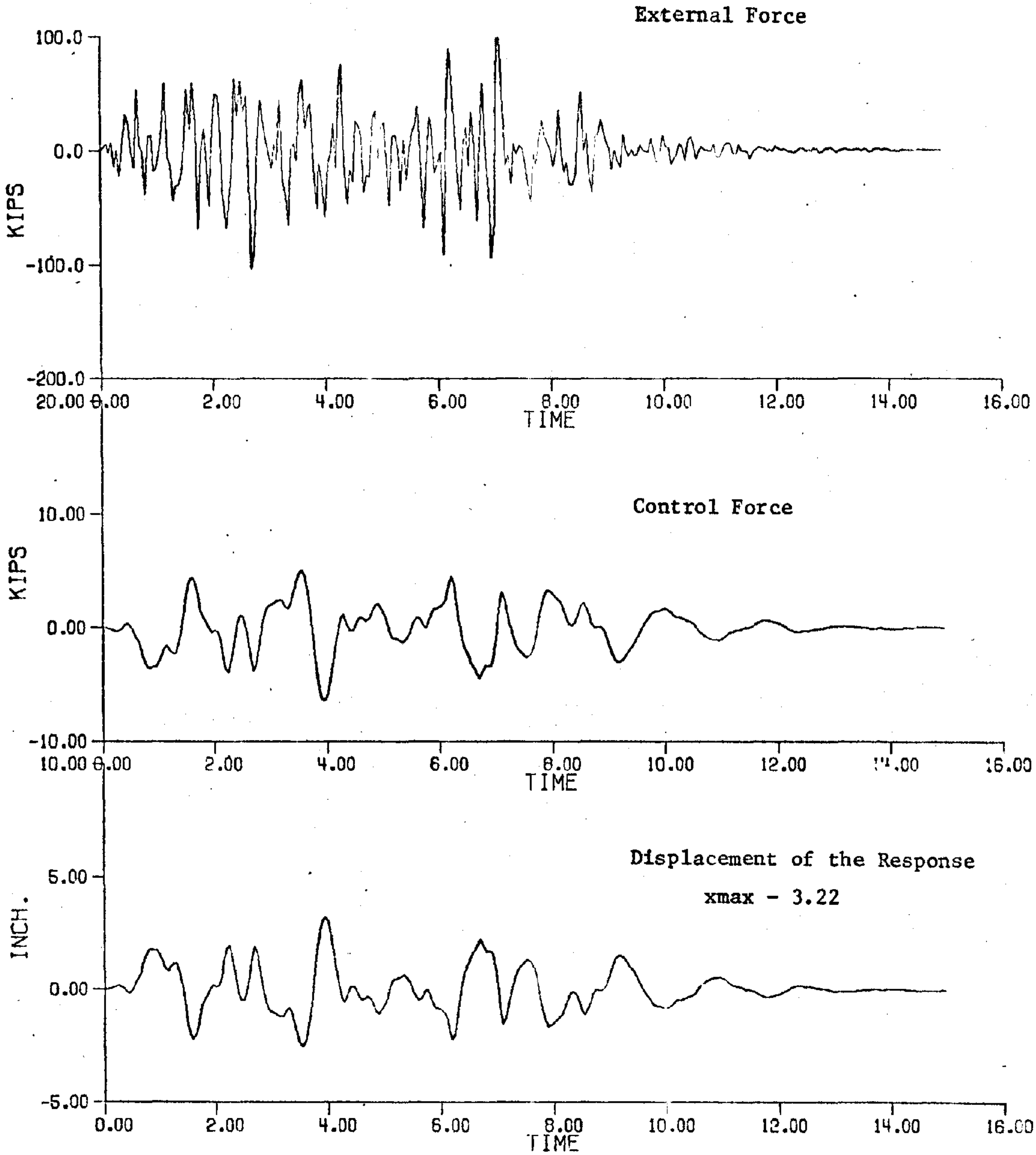

Fig. 11

External Force, Control Force, and Displacement Response for Case 4 in Table 1.

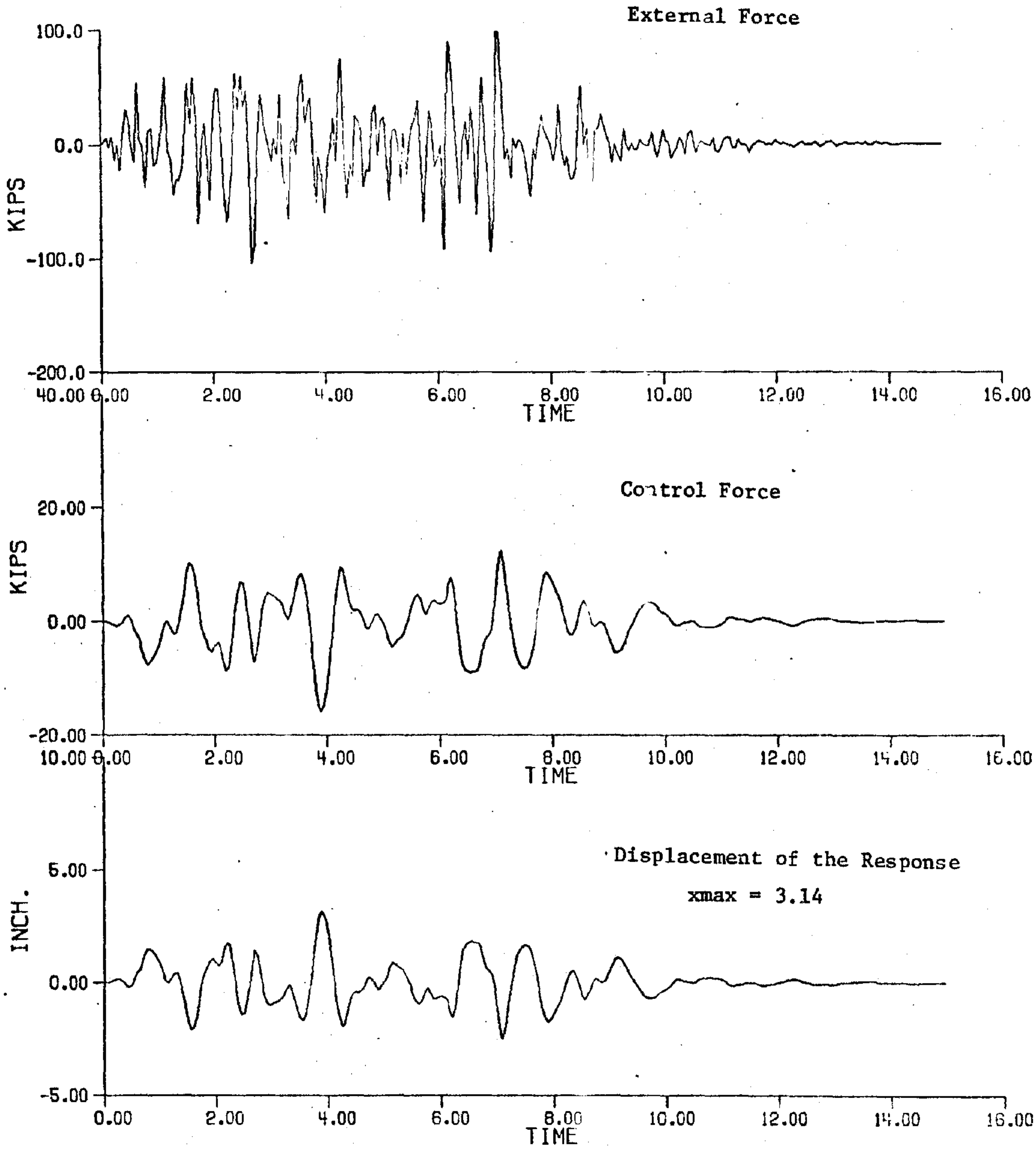

Fig. 12

External Force, Control Force, and Displacement Response
for Case 5 in Table 1.

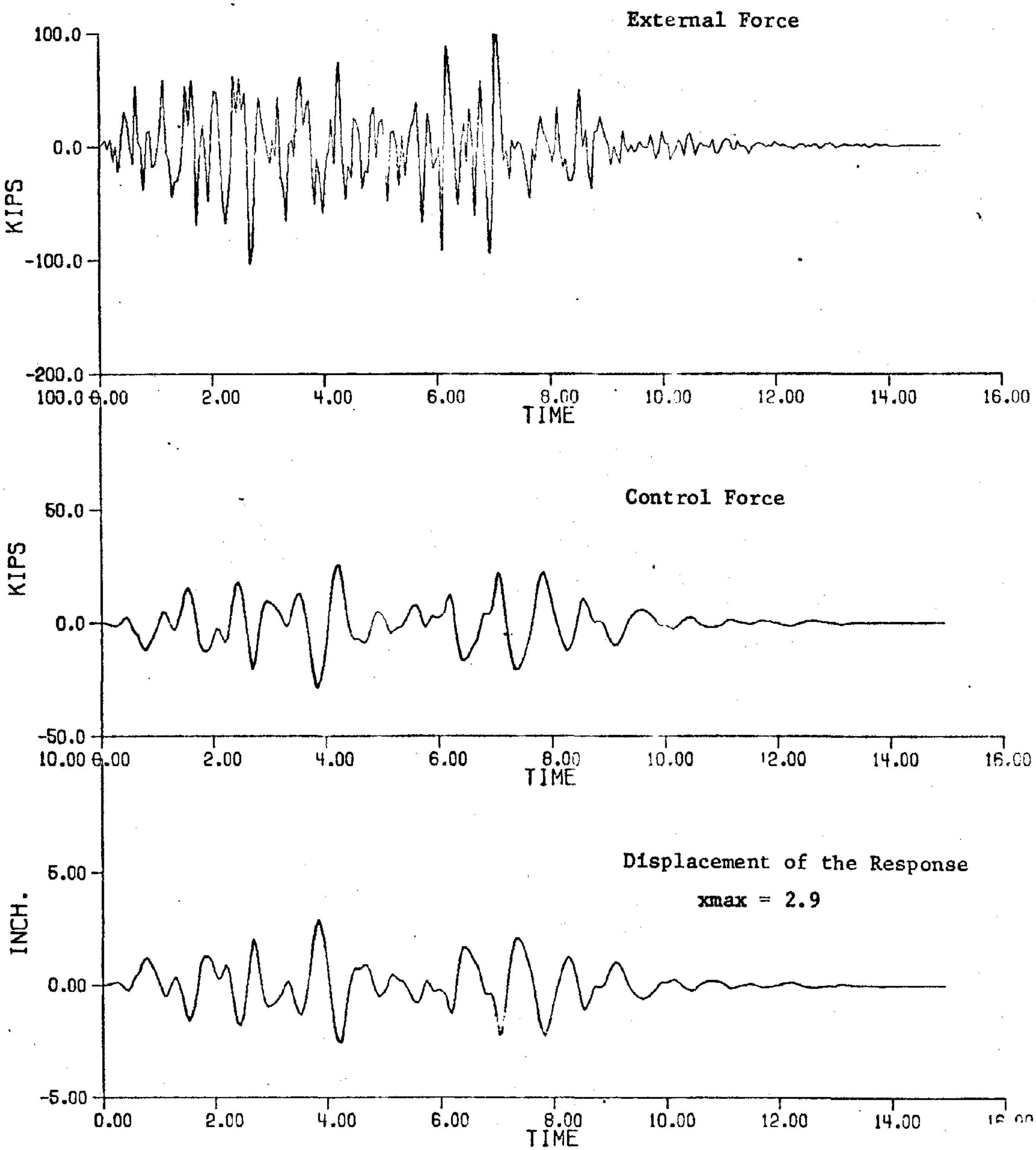

Fig. 13

External Force, Control Force, and Displacement Response for Case 6 in Table 1.

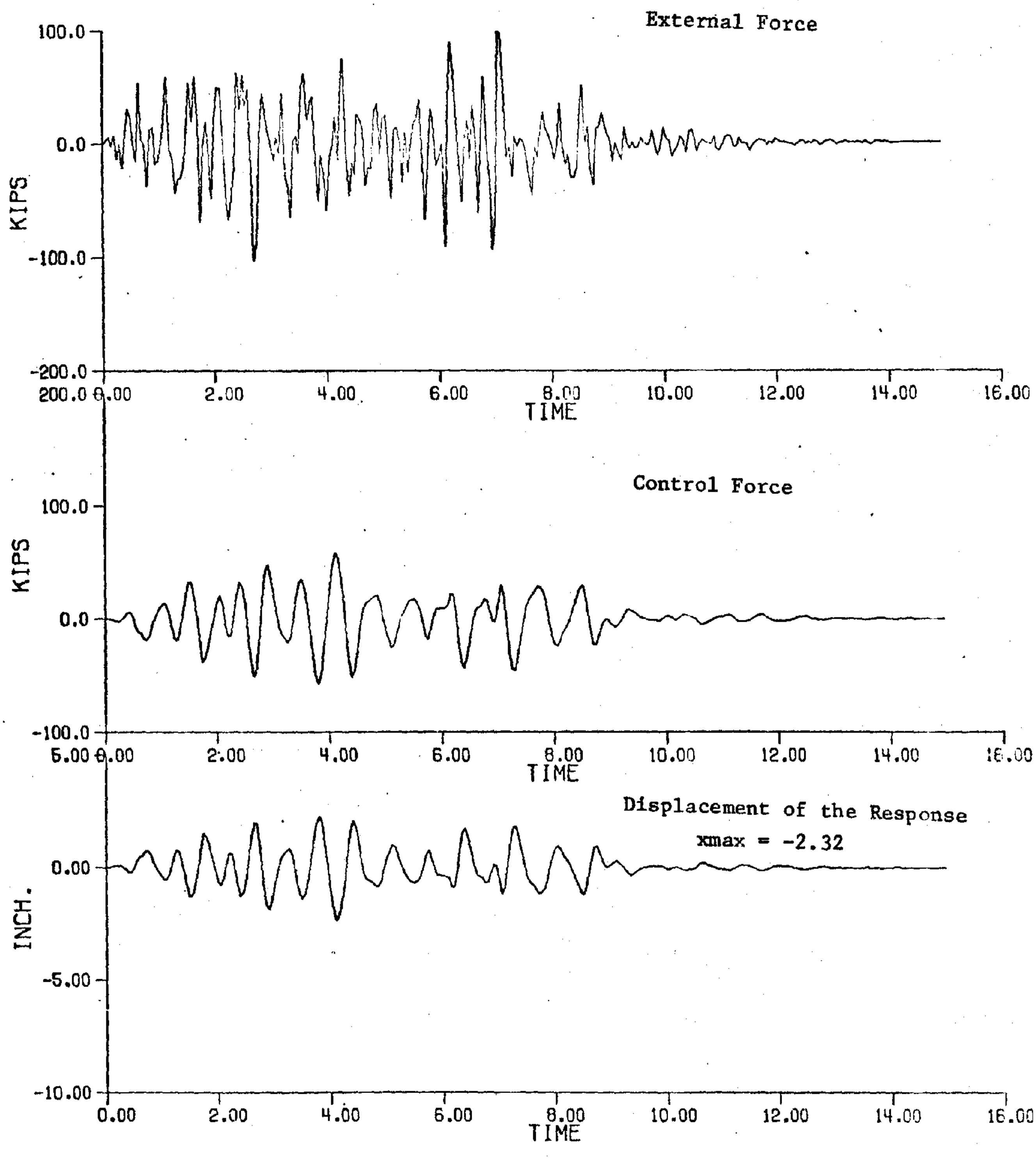

Fig. 14

External Force, Control Force, and Displacement Response for Case 7 in Table 1.

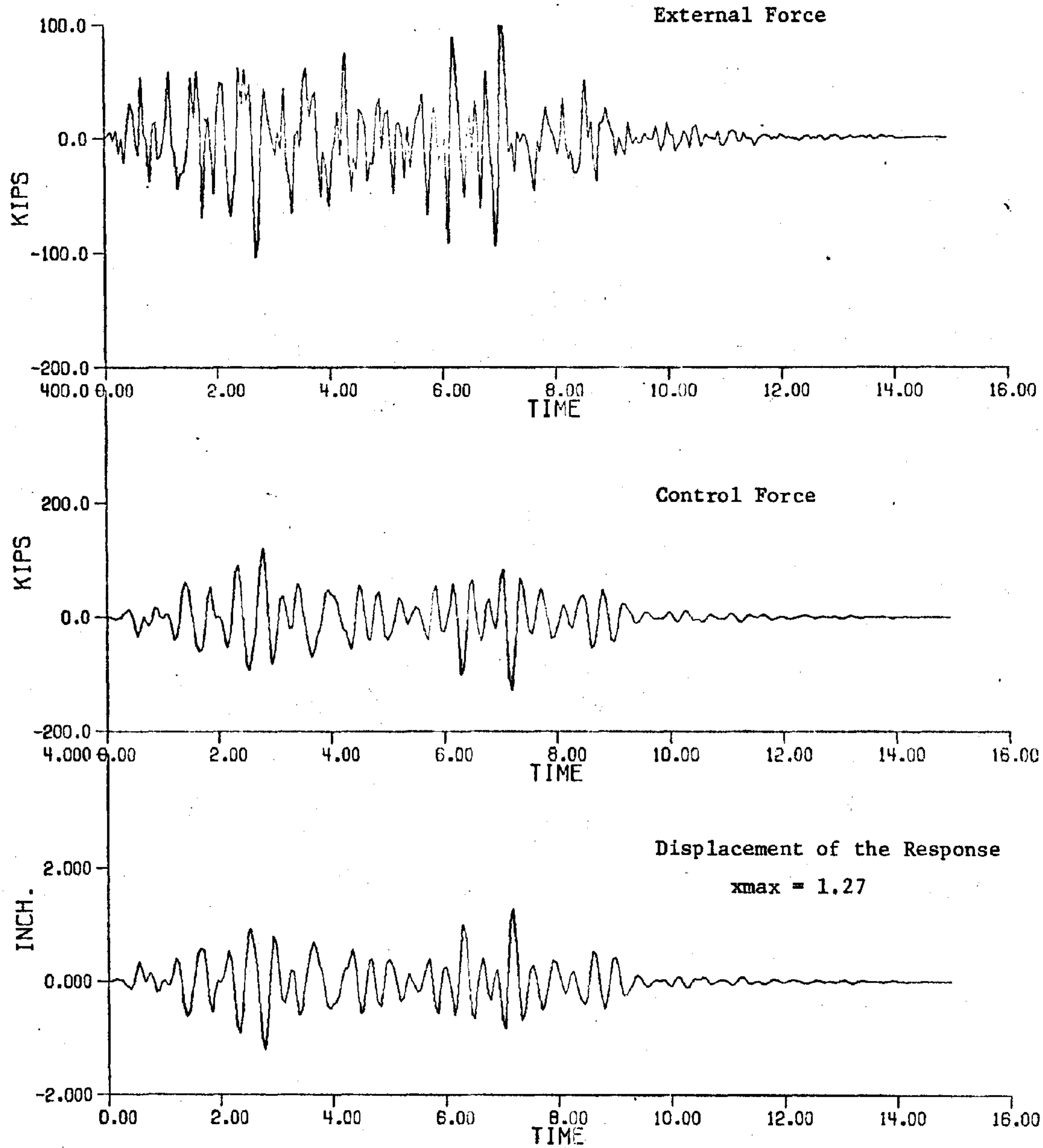

Fig. 15

External Force, Control Force, and Displacement Response for Case 8 in Table 1.

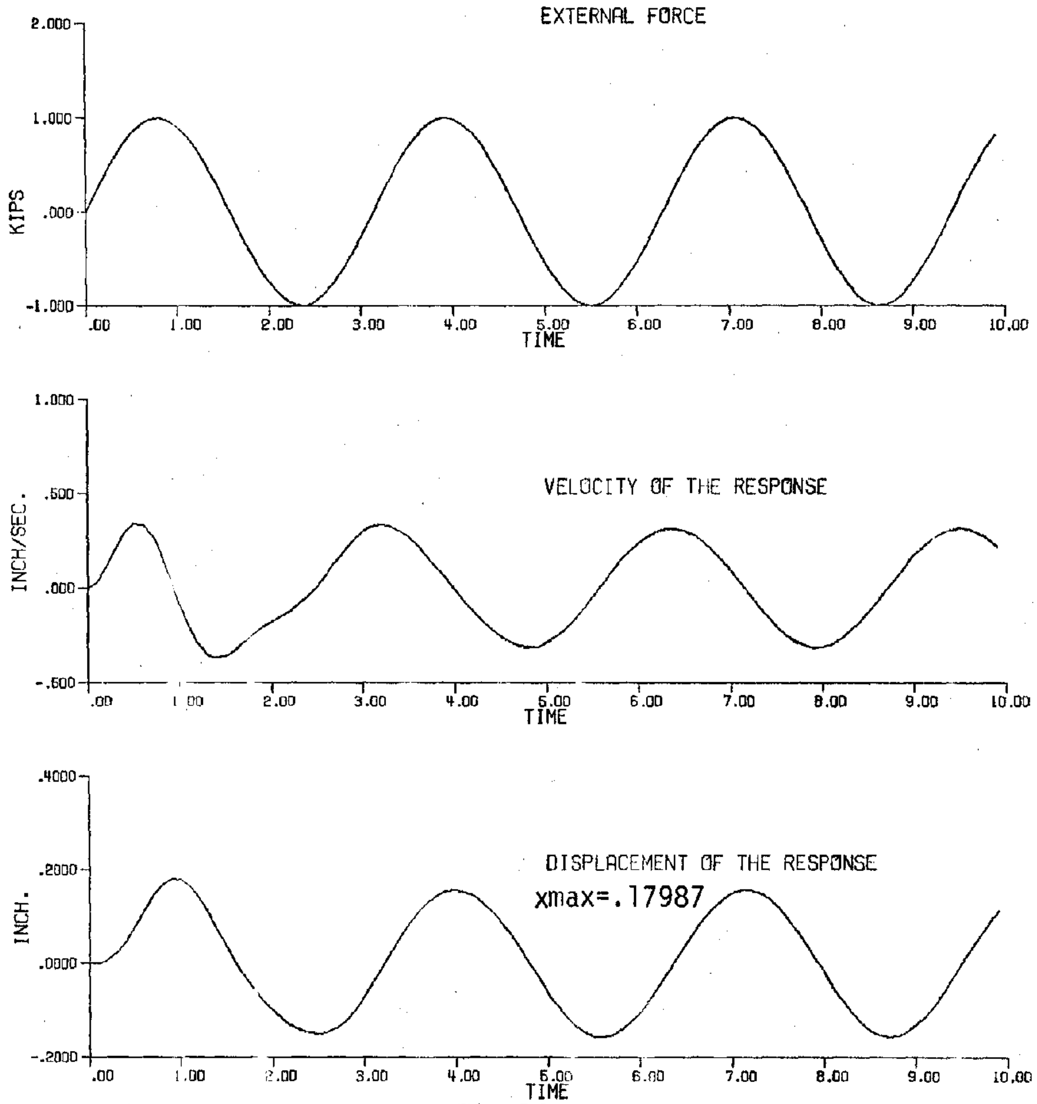

Fig.16

External Force, Velocity, and Displacement Response
for Case 9 in Table 2.

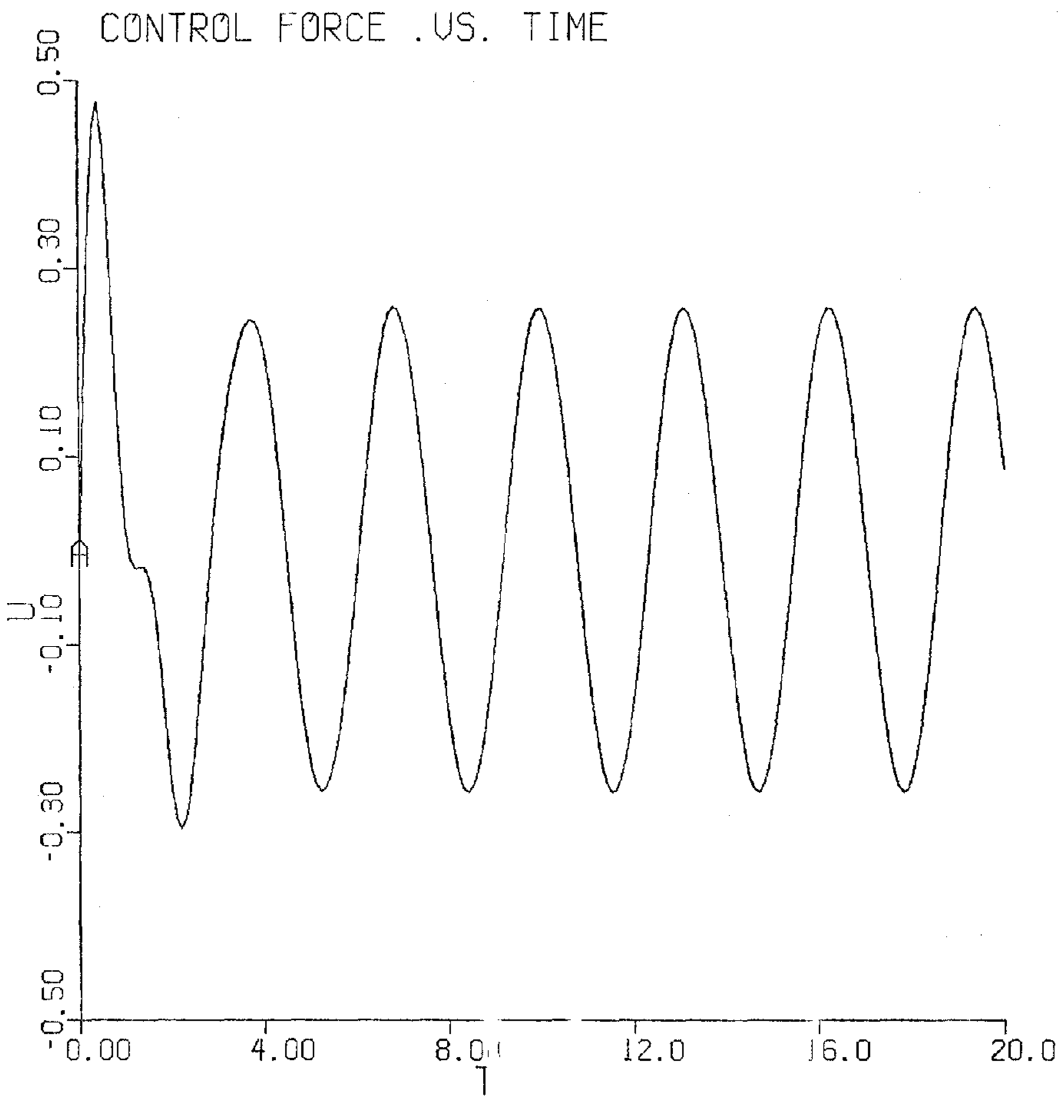

Fig. 17

Case 9 in Table 2.

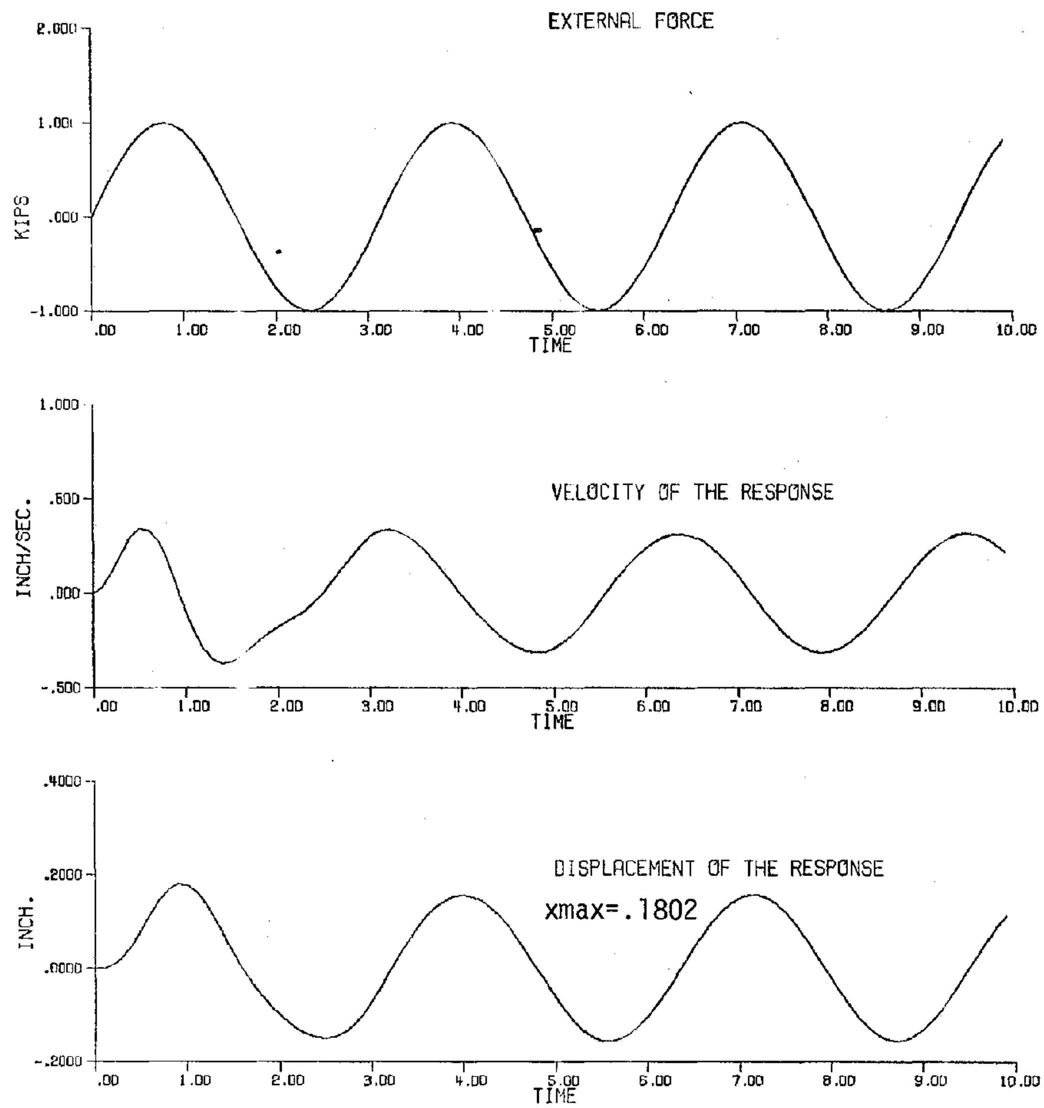

Fig. 18

External Force, Velocity, and Displacement Response
for Case 10 in Table 2.

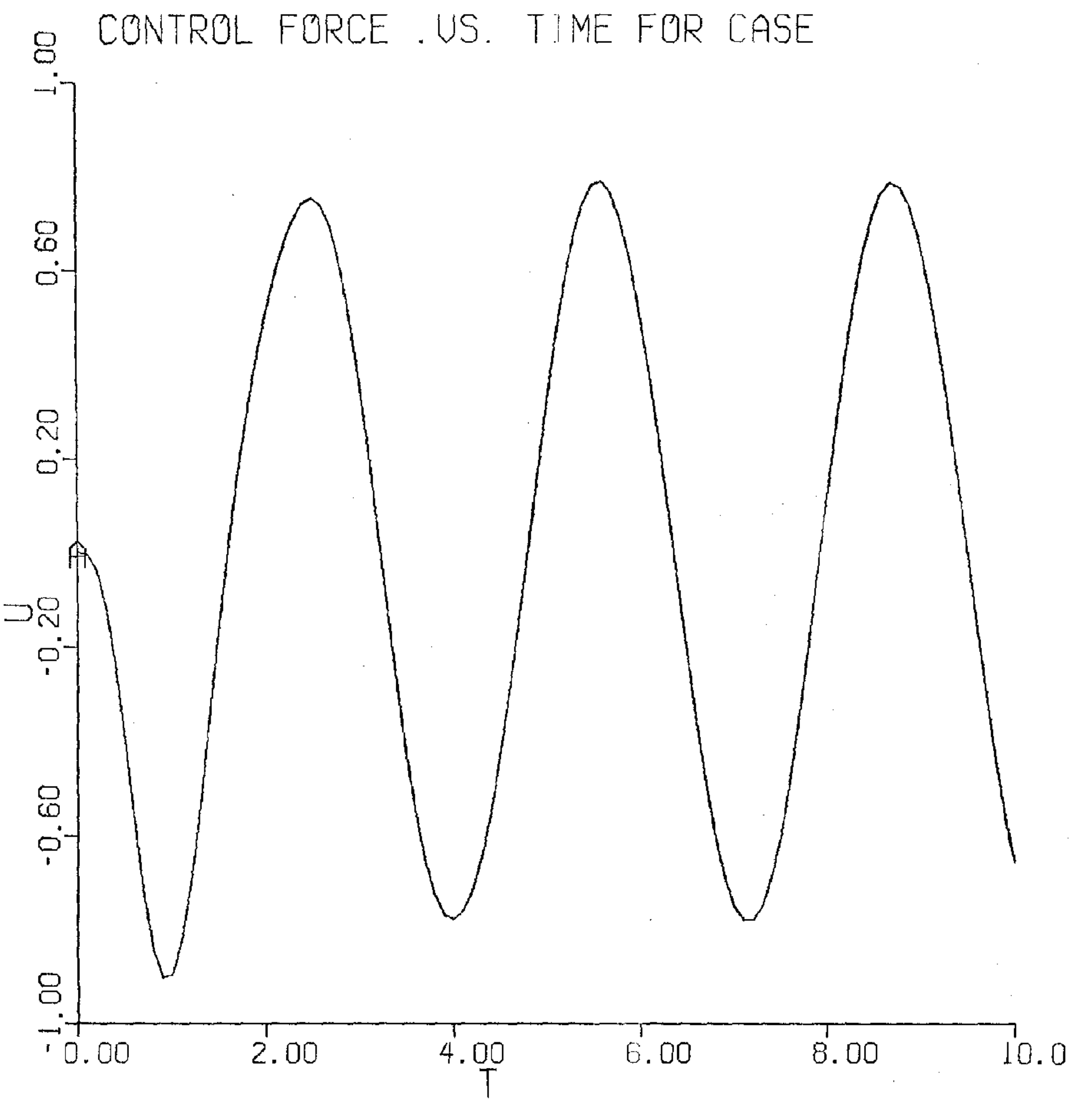

Fig. 19

Case 10 in Table 2.

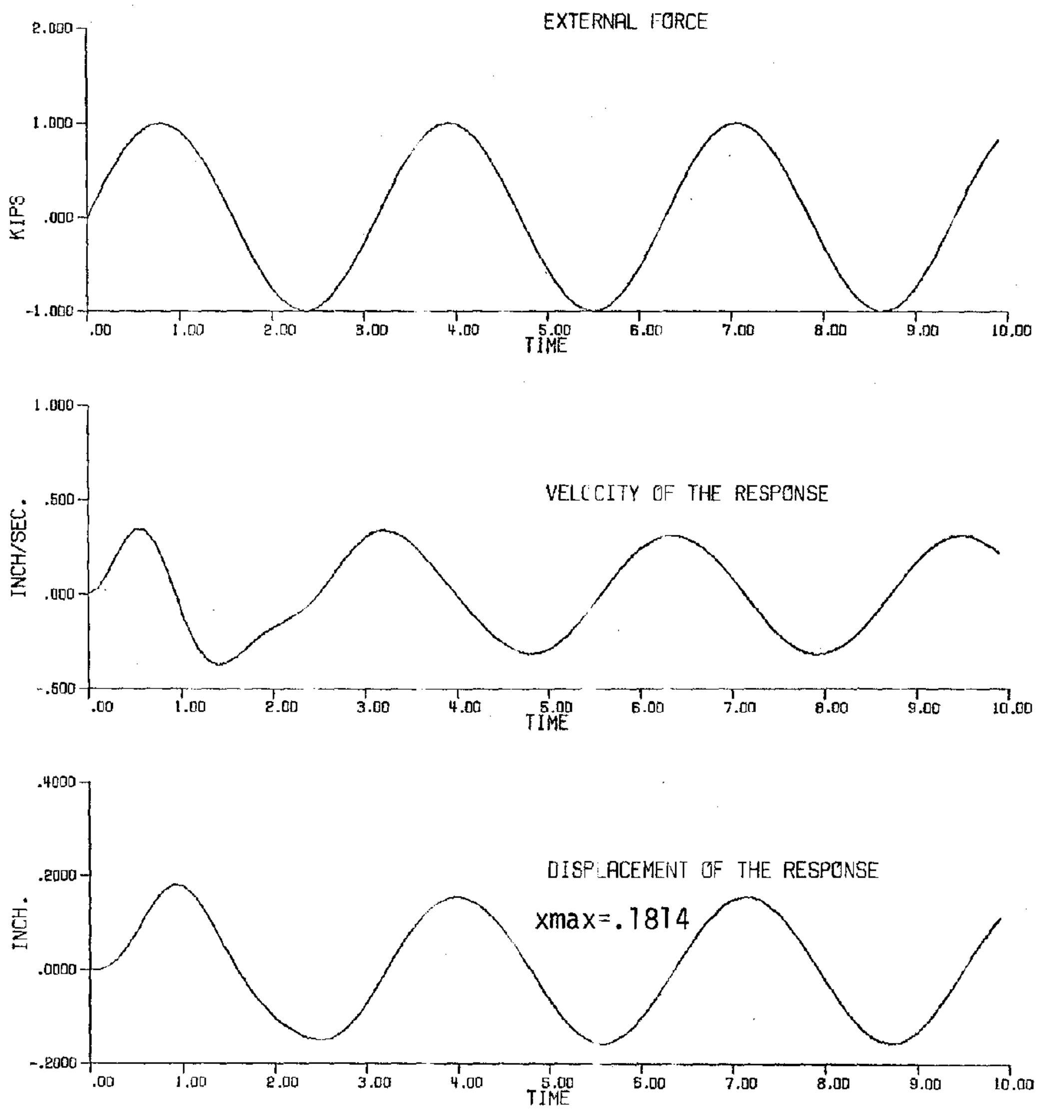

Fig. 20

External Force, Velocity, and Displacement Response for Case 11 in Table 2.

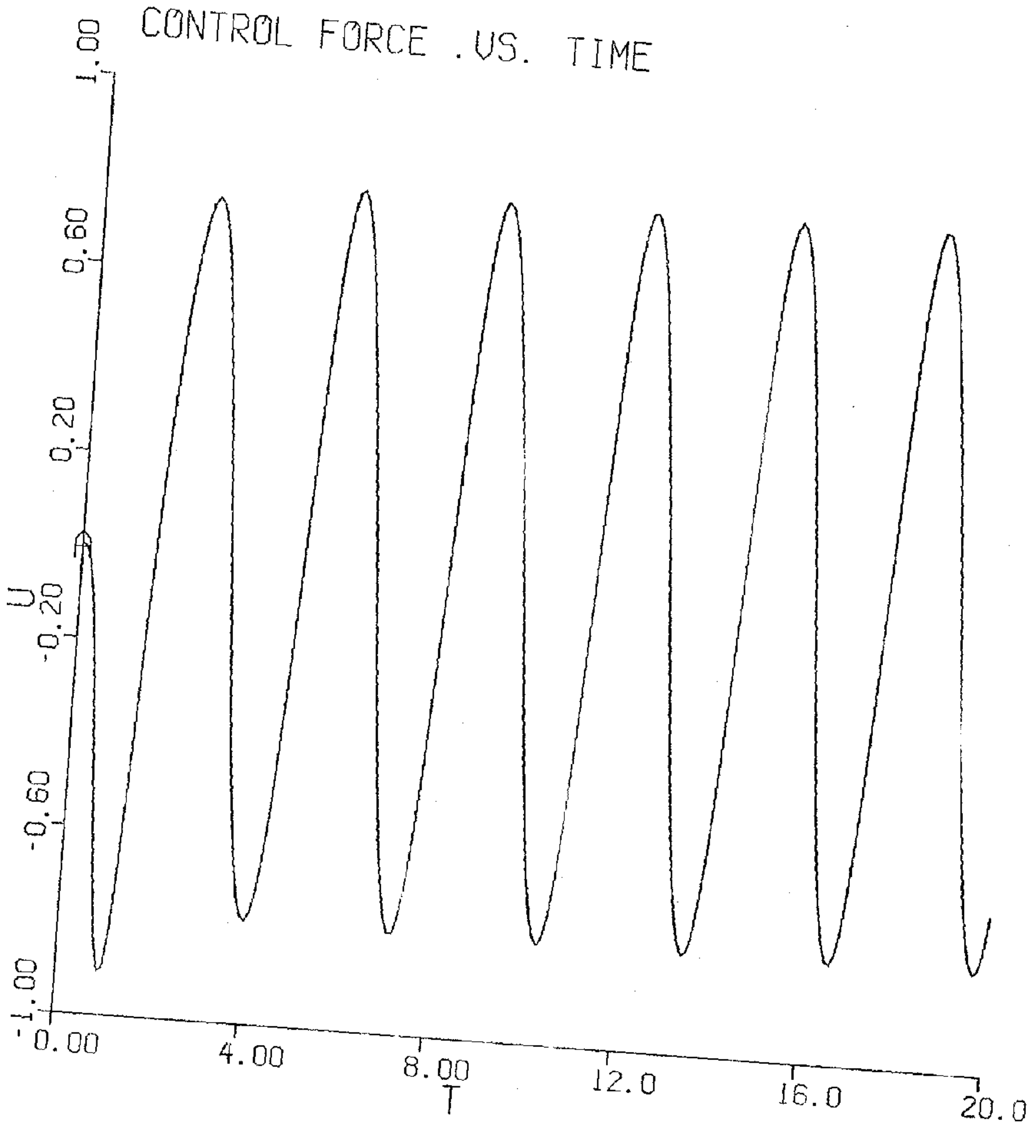

Fig. 21

Case 11 in Table 2.

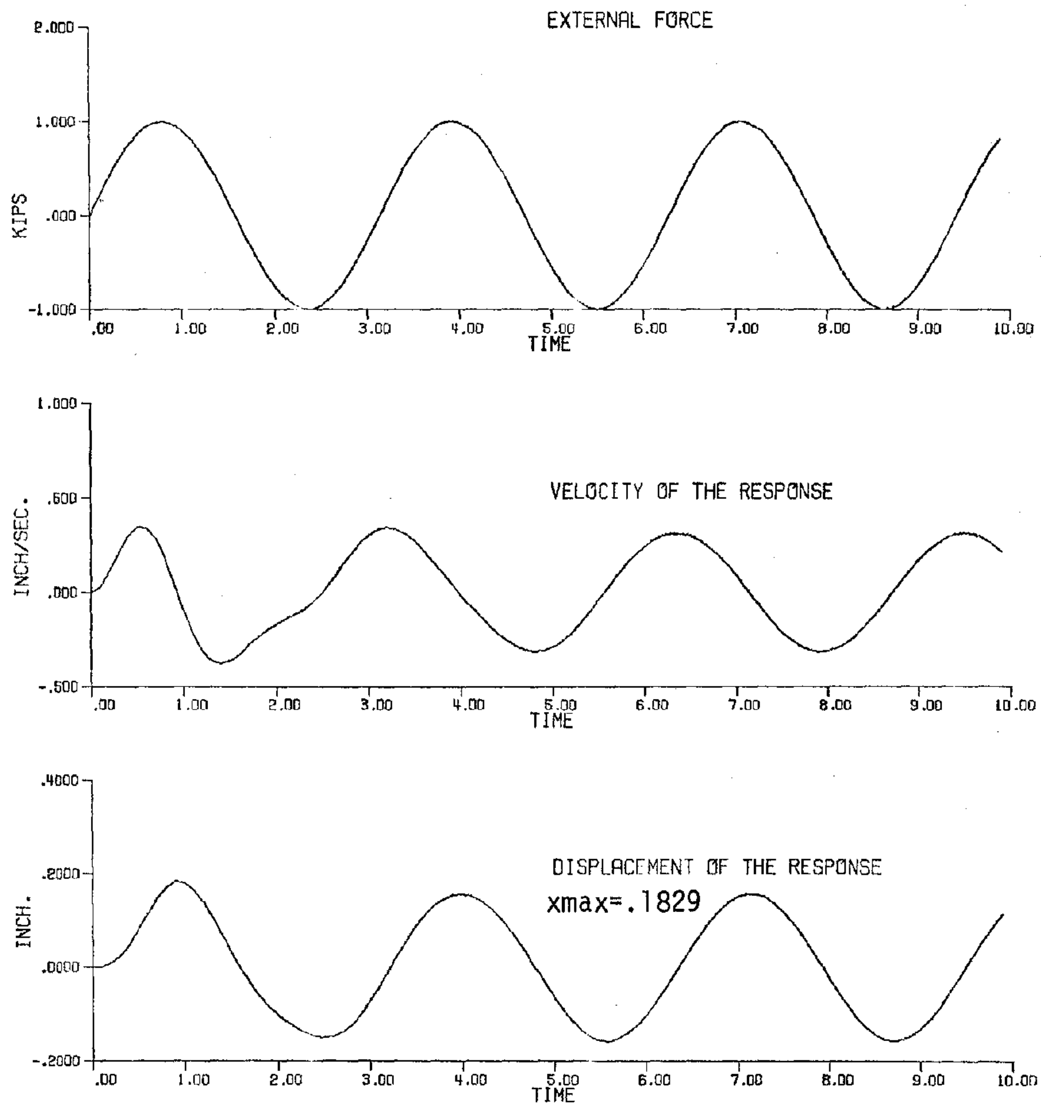

Fig. 22

External Force, Velocity, and Displacement Response for Case 12 in Table 2.

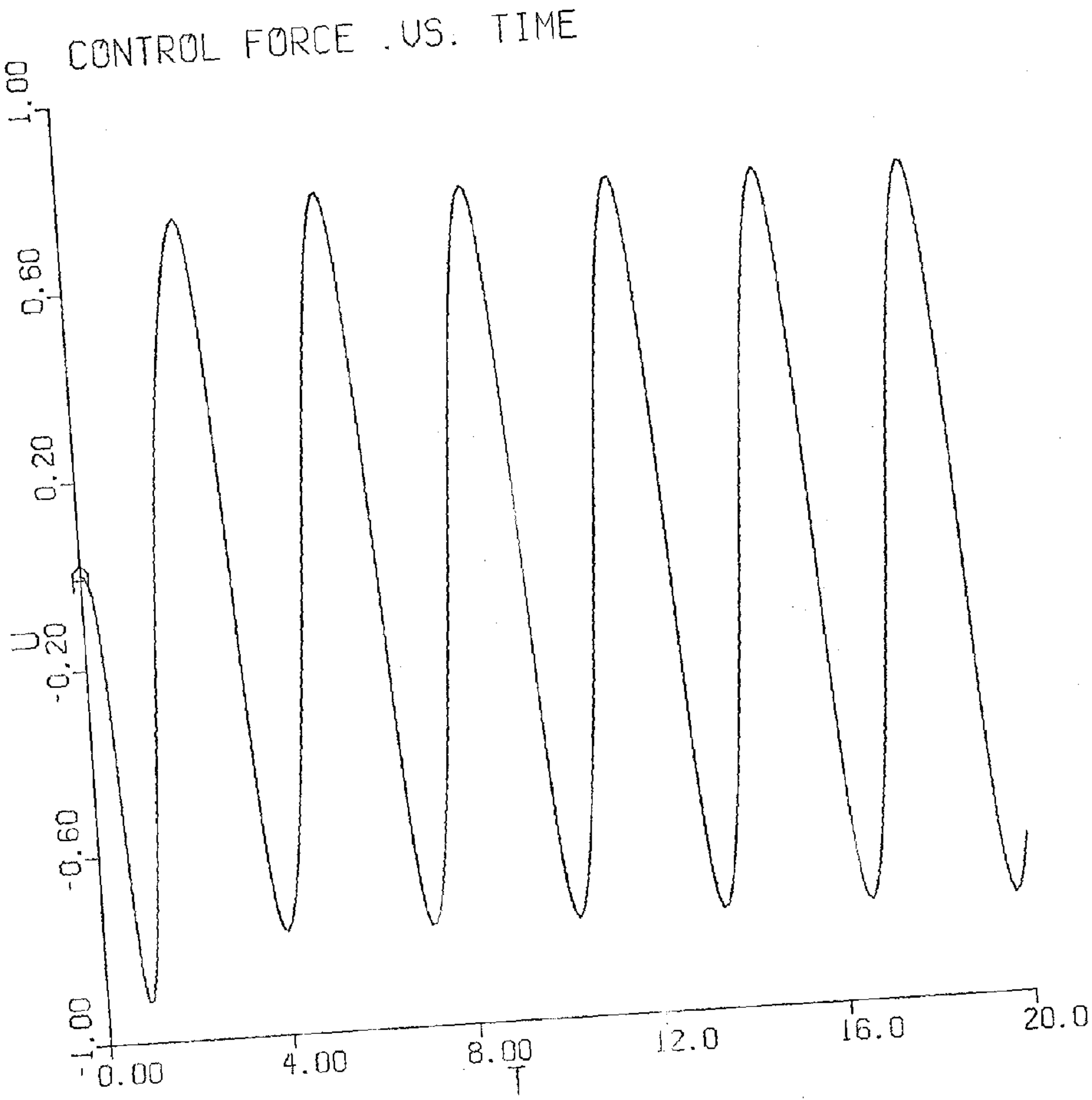

Fig. 23

Case 12 in Table 2.

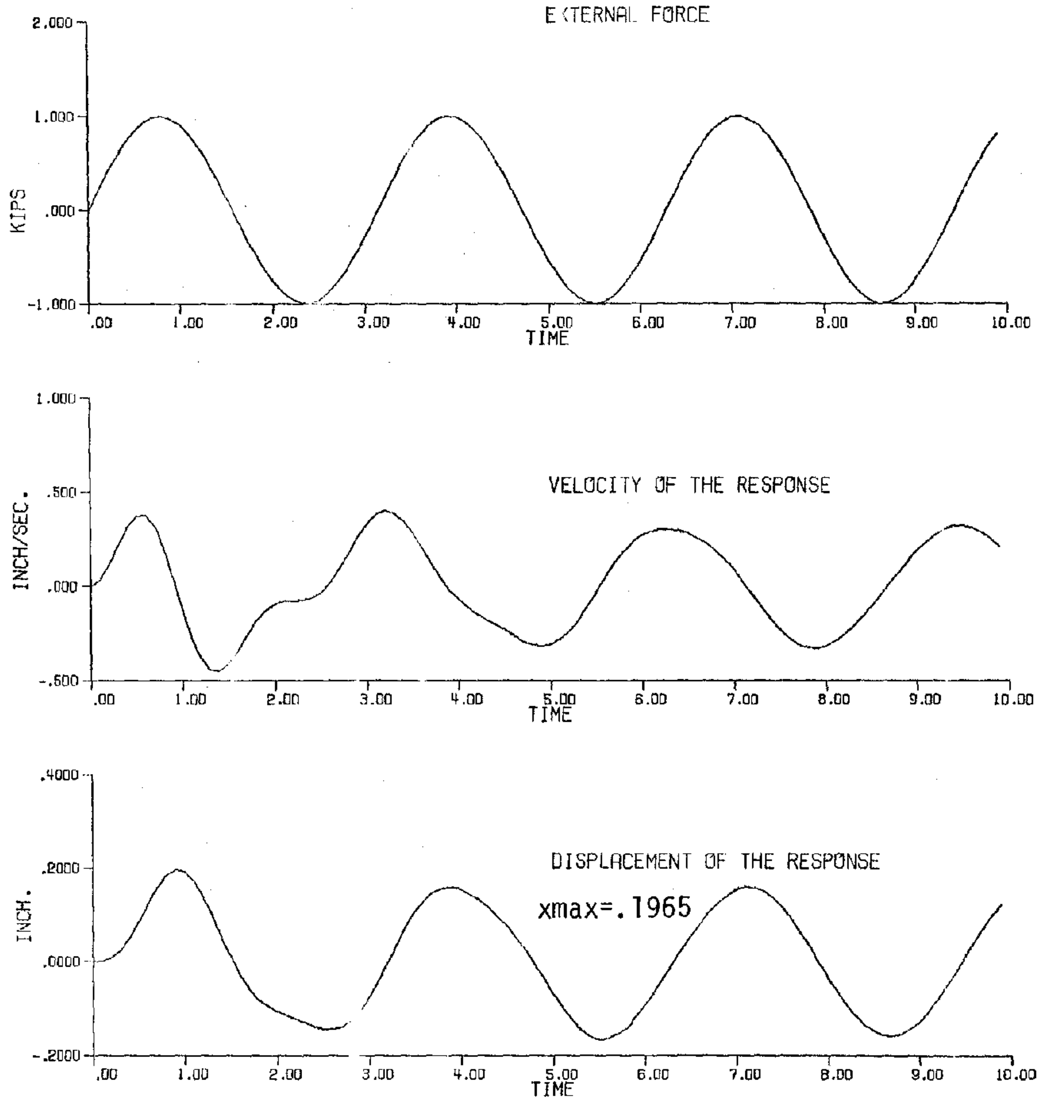

Fig. 24

External Force, Velocity, and Displacement Response for Case 13 in Table 2.

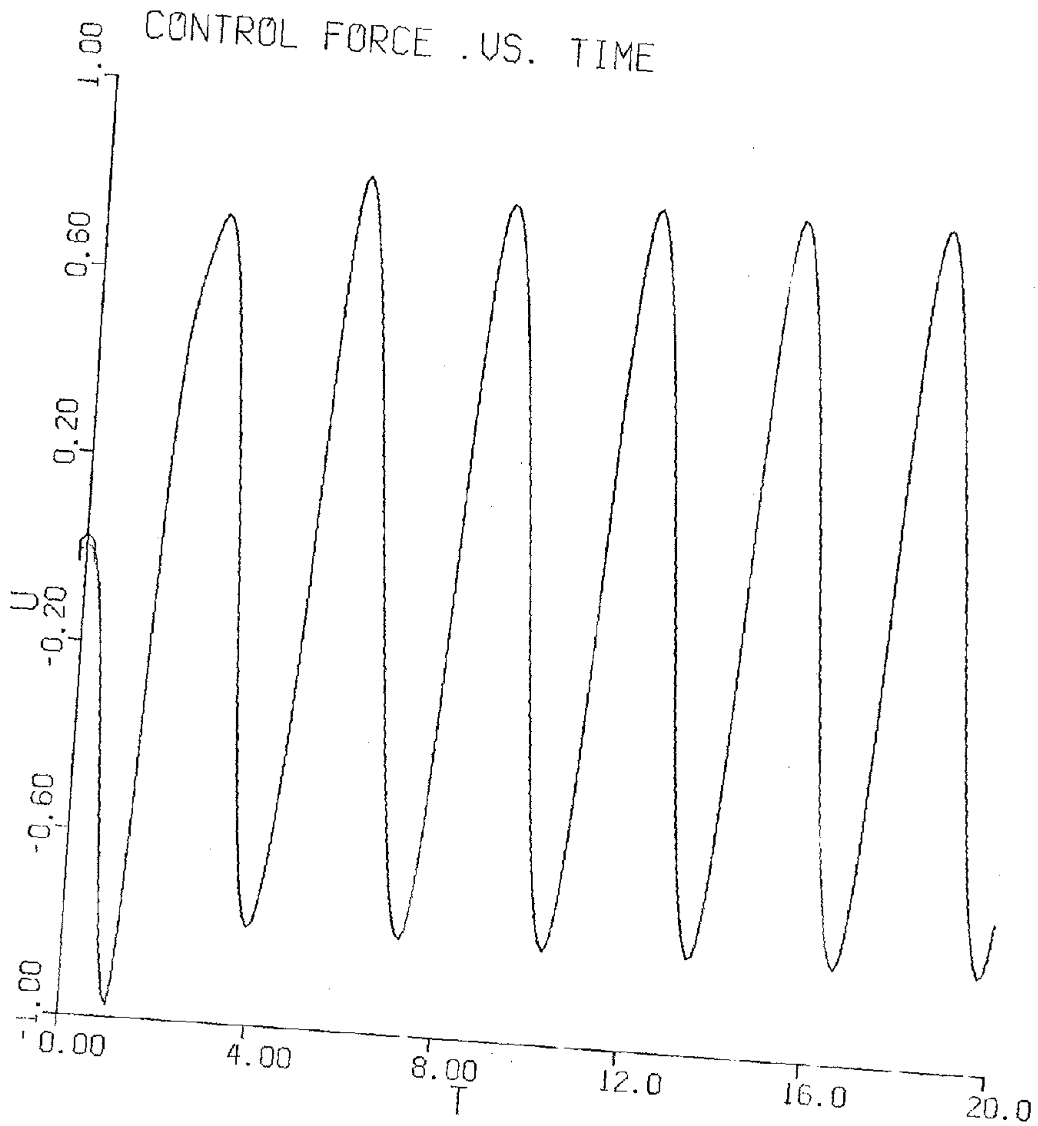

Fig. 25

Case 13 in Table 2.

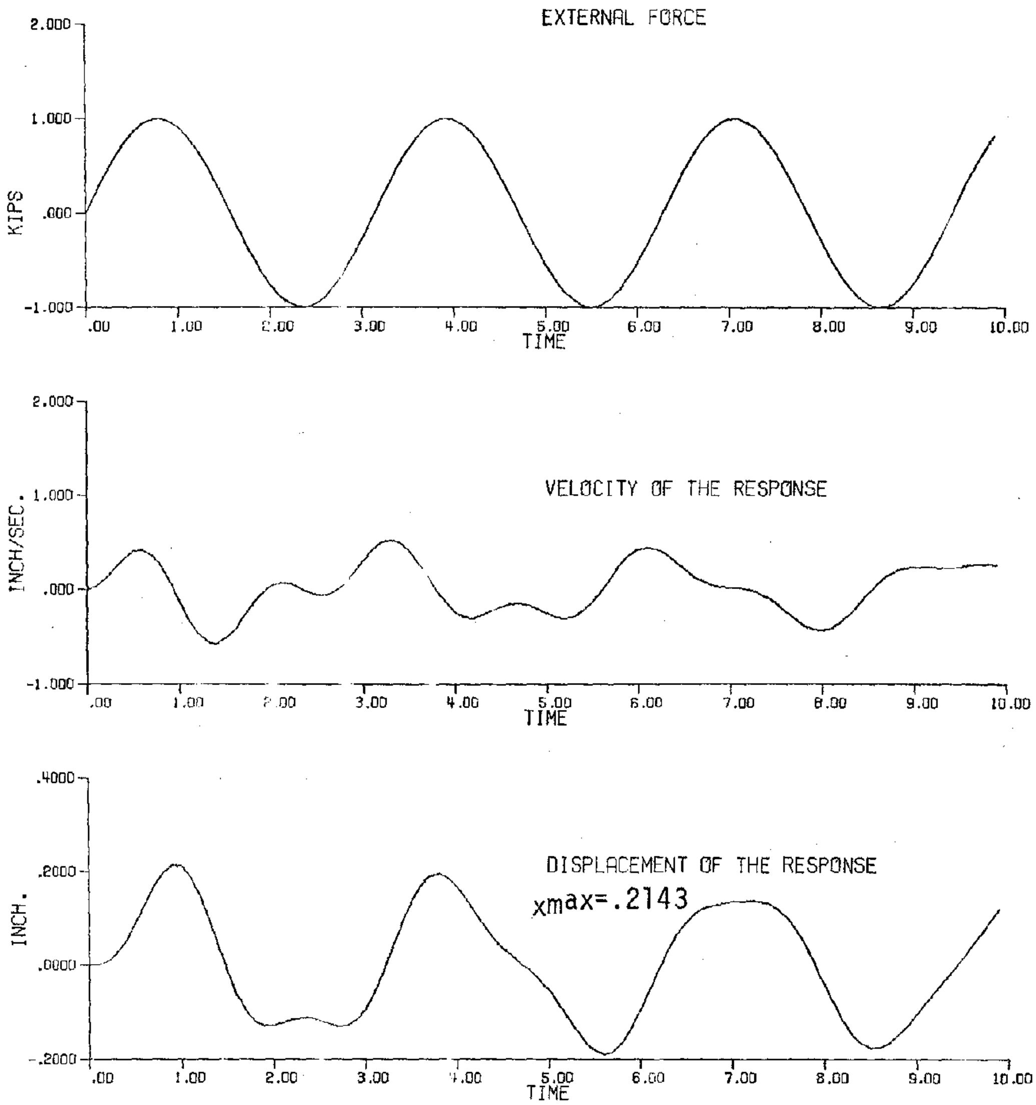

Fig. 26

External Force, Velocity, and Displacement Response for Case 14 in Table 2.

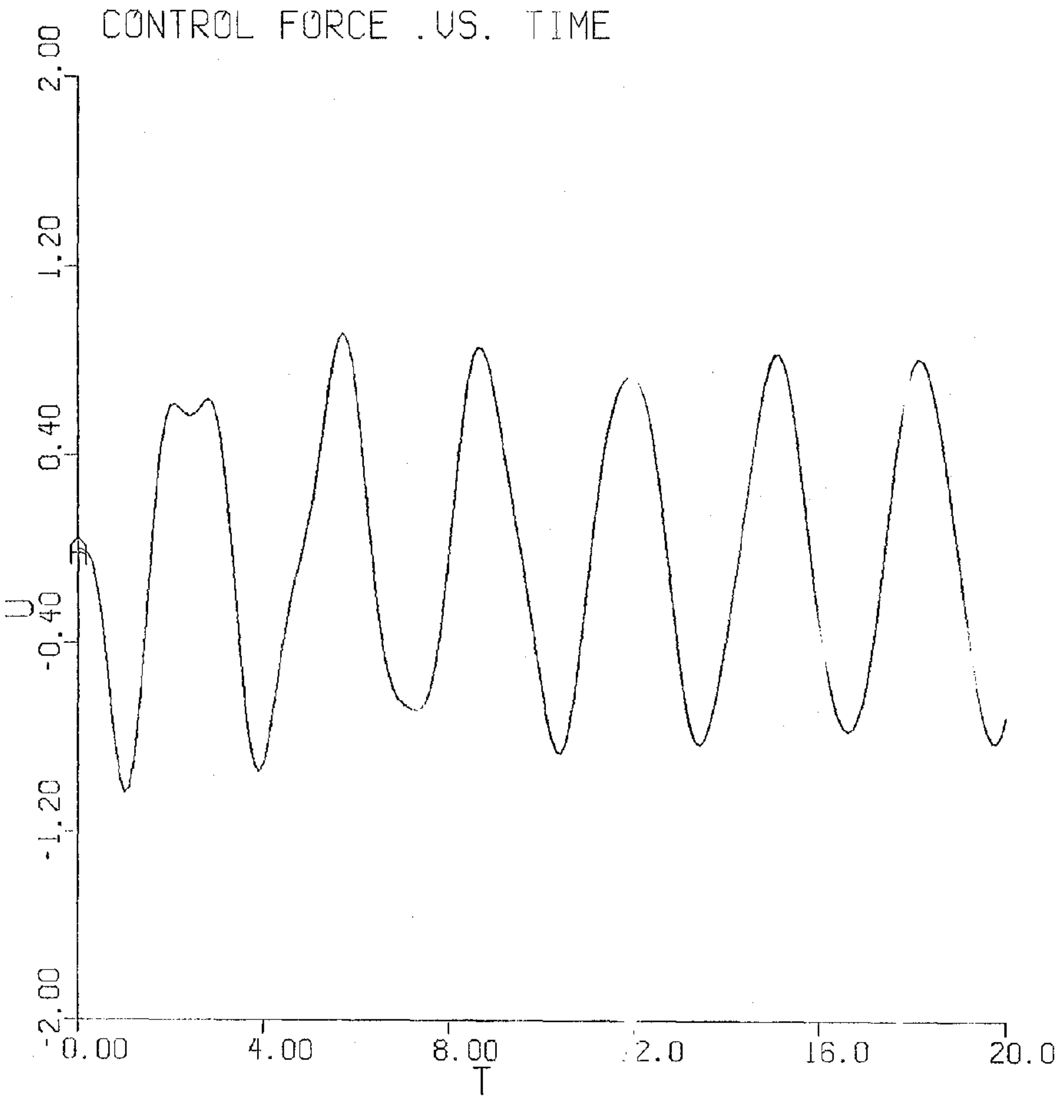

Fig. 27

Case 14 in Table 2.

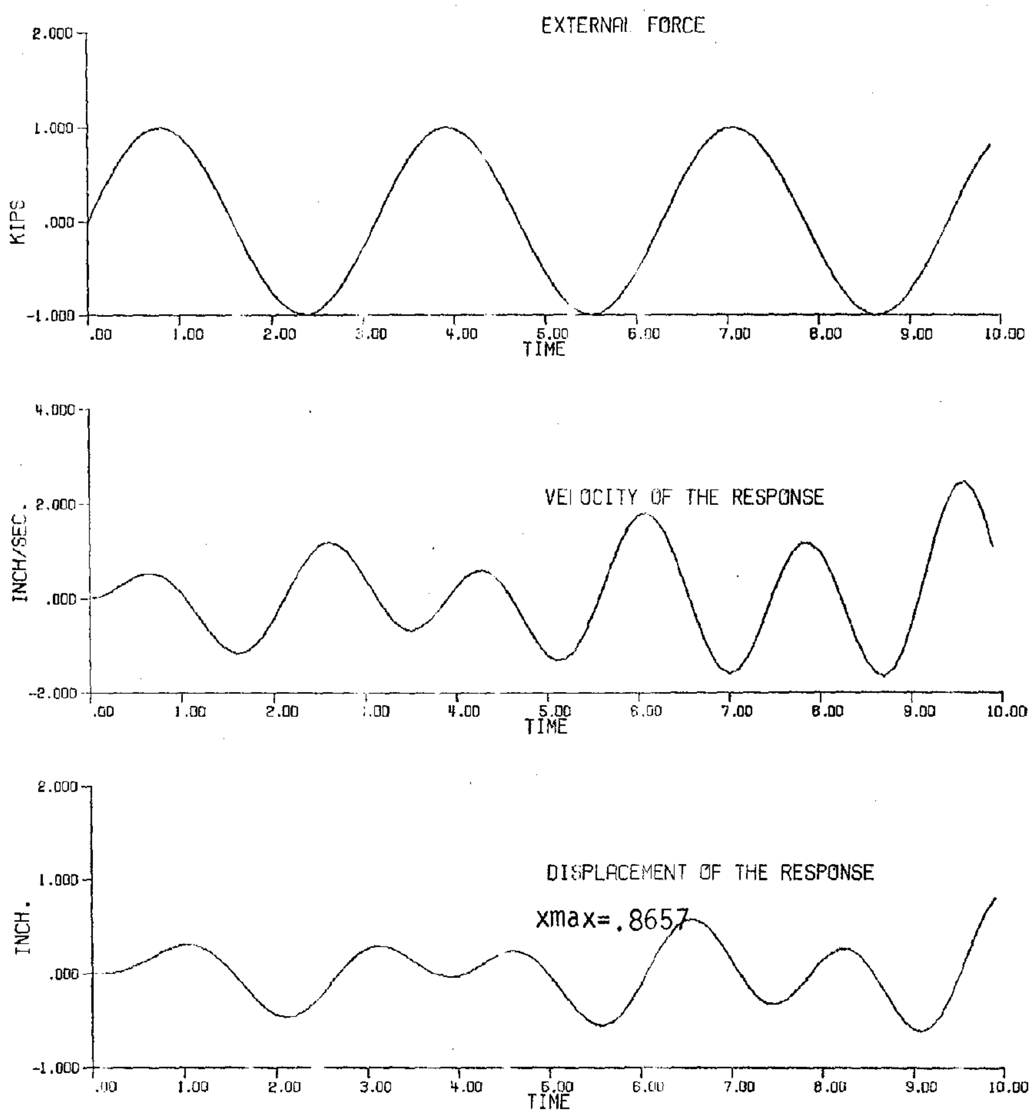

Fig. 28

External Force, Velocity, and Displacement Response
for Case 15 in Table 2.

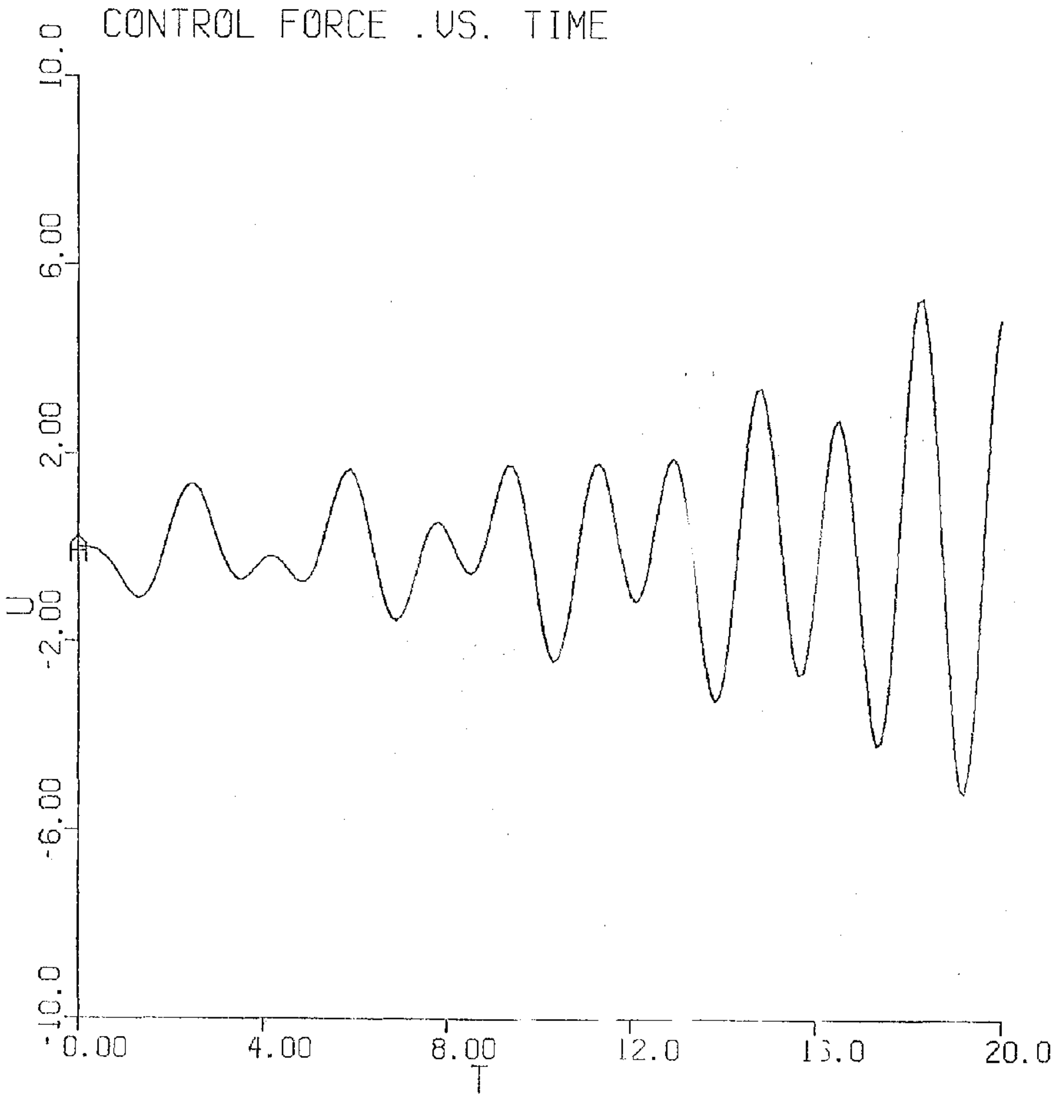

Fig. 29

Case 15 in Table 2.

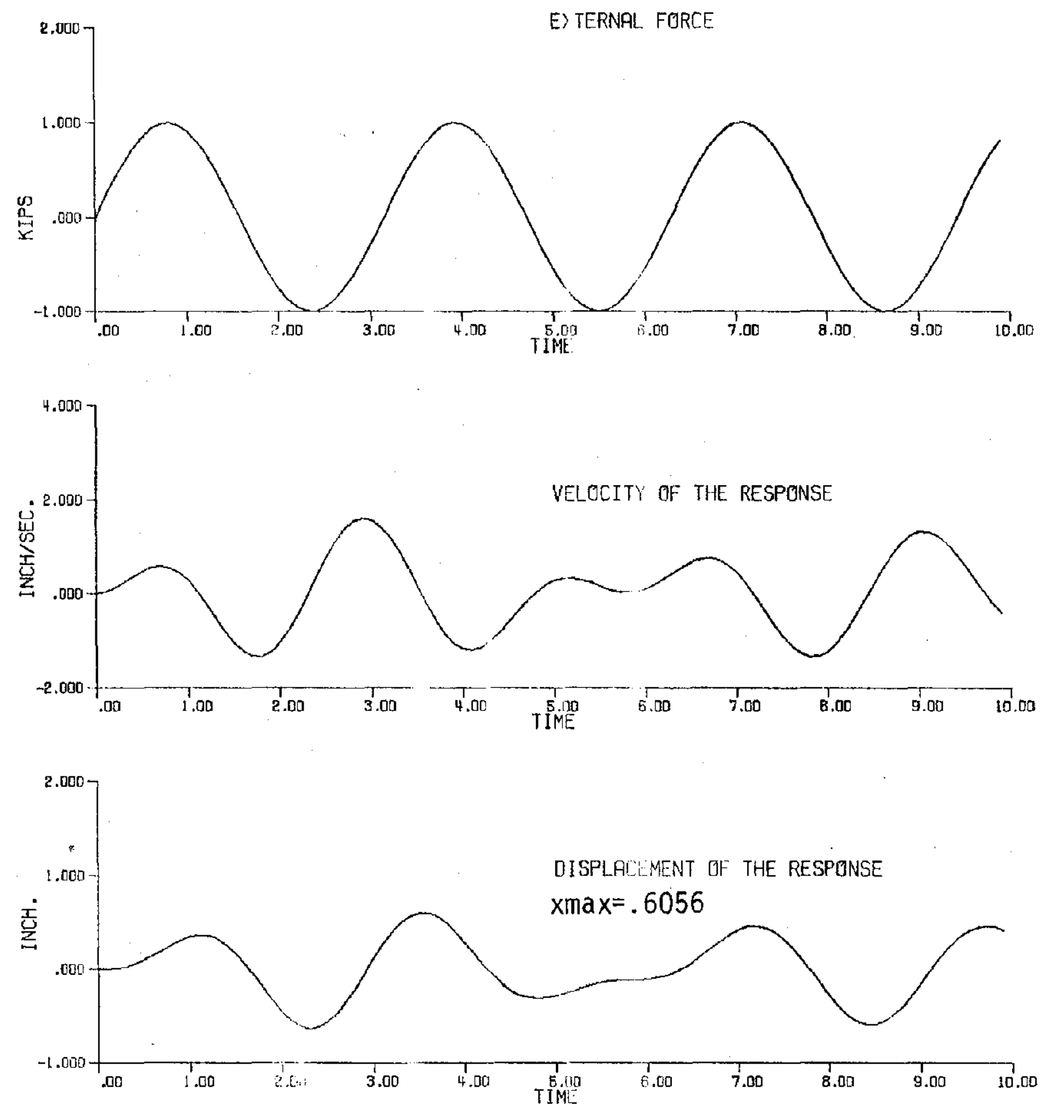

Fig. 30

External Force,Velocity, and Displacement Response for Case 16 in Table 2.

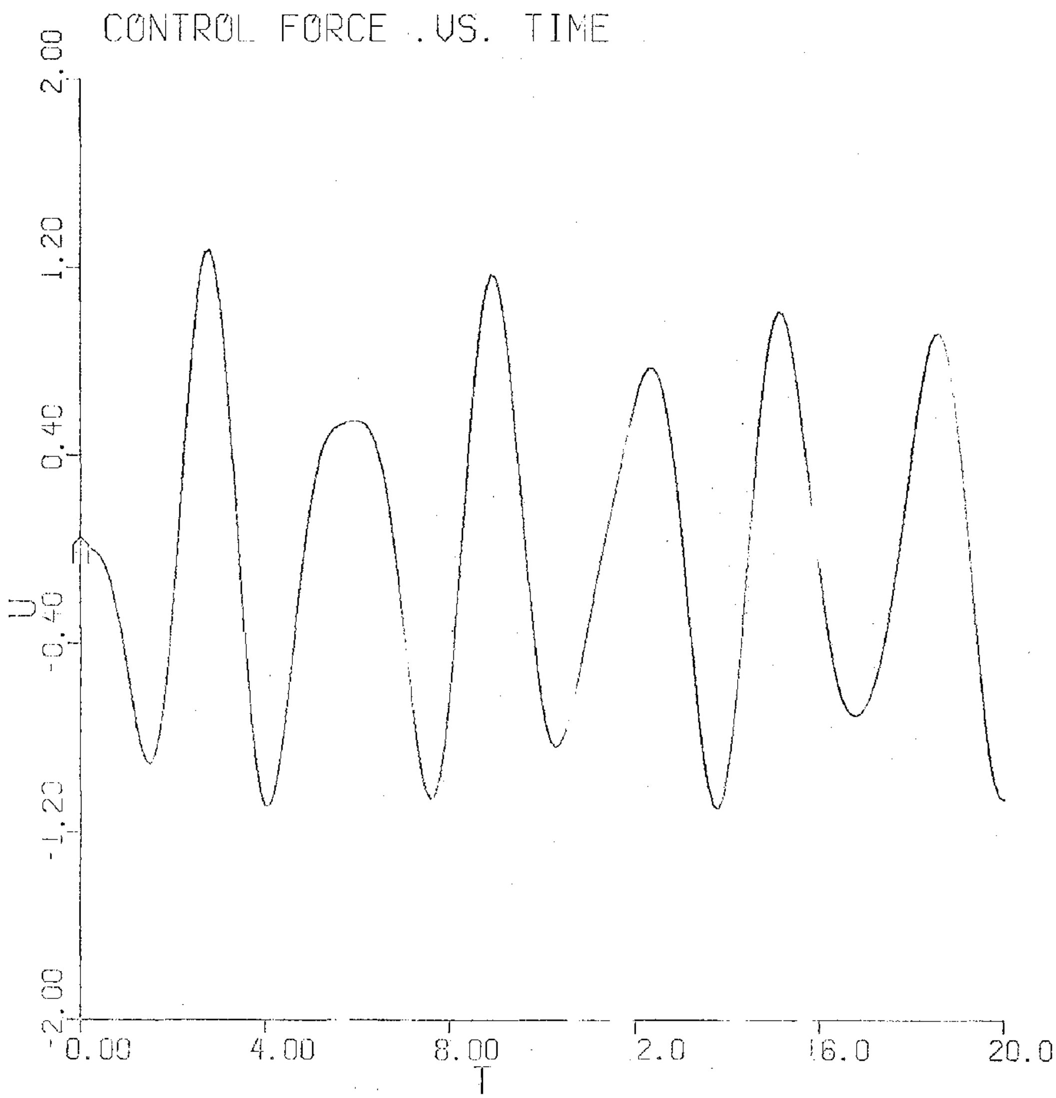

Fig. 31

Case 16 in Table 2.

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