EFFECTS OF GRAIN SIZE DISTRIBUTION ON DYNAMIC PROPERTIES AND LIQUEFACTION POTENTIAL OF GRANULAR SOILS

prepared by
Nien-Yin Chang and Hon-Yim Ko

for
NATIONAL SCIENCE FOUNDATION

under
Grant No. PFR 78-23094

Geotechnical Engineering Division
Department of Civil and Urban Engineering
University fo Colorado at Denver

Research Report R82-103
Geotechnical Publication 111

March 1982

Any opinions, findings, conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.
Research was undertaken to investigate the effects of grain-size distribution gradation on dynamic properties and on liquefaction potential of granular soils. Resonant column tests were performed to determine the dynamic shear moduli and damping ratios of granular soil samples prepared from a Denver sand. Cyclic triaxial test cells and an MTS closed-loop servo control material testing machine were used to determine liquefaction potential. Results indicate that the shear modulus of soils at small strains is strongly dependent on the uniformity coefficient. The liquefaction potential was found to be strongly affected by the mean diameter.
ABSTRACT

The objective of this study is to investigate the effects of the grain-size distribution (gradation) characteristics on the following two properties:

- Dynamic properties (shear moduli and damping ratios) and
- Liquefaction potential of granular soils

A granular soil, Denver sand, with constituents ranging from fine silts to gravels was adopted as the test sand. Samples of the Denver Sand with different gradation characteristics: Mean diameters ($D_{50}$: 0.149 mm to 1.68 mm) and uniformity coefficients ($C_u$: 2 to 16) etc., were tested.

Resonant column test apparatus was used to evaluate the dynamic property of six-inch diameter samples. Cyclic triaxial test cells and MTS closed-loop servo control material testing machine were used to determine the liquefaction potential of two-inch diameter samples.

The result of this investigation showed that the shear modulus of the sand at small strains was strongly dependent on the uniformity coefficient. The liquefaction potential of the sand was found to be strongly affected, instead, by the mean diameter. Thus, statistical analyses were performed to express the shear modulus and the liquefaction potential of the sand as the functions of its uniformity coefficient and mean diameter, respectively.
ACKNOWLEDGEMENT

This report presents the result of the investigation on "Effects of Grain Size Distribution on Dynamic Properties and Liquefaction Potential of Granular Soils" through the Research Initiation Grant No. PFR 78-23094 awarded to the University of Colorado at Denver (UCD) by the National Science Foundation (NSF). The contract was monitored by Dr. William W. Hakala, Program Manager of the Division of Problem-Focused Research Applications at NSF. The above funding from the National Science Foundation and the dedicated support of the University authorities are greatly appreciated.

The research has generated great interest in dynamic behaviors and dynamic properties of soils among the professionals in Denver. Several related research activities have been initiated to provide some preliminary information for the subjects of further studies.

Graduate students, current or former, who worked on this research project are: Shan-Tai Yeh, Houman Makarechi, Nan-Ping Hsieh, Majid Derakhshandeh and David Lee Samuelson. The Colorado Department of Highways provided facilities for an extensive sieve analysis program. The U.S. Bureau of Reclamation (Denver Research Center) provided some silt samples used as an admixture to Denver Sand. The Mine Safety and Health Administration of the U.S. Department of Labor provided some samples of granular coal waste
from the Eastern United States.

The successful conclusion of this research is the result of a synchronized and cooperative effort of multiple parties. This effort is gratefully acknowledged.

The opinions expressed in this report are those of the authors and not necessarily of the National Science Foundation.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Part I GENERAL INFORMATION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>TEST SANDS AND TEST PROGRAMS</td>
<td>2</td>
</tr>
<tr>
<td>2.1</td>
<td>Denver Sand</td>
<td>6</td>
</tr>
<tr>
<td>2.2</td>
<td>Monterey No. 0 Sand</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>Test Programs</td>
<td>10</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Dynamic Properties at Small Strains</td>
<td>12</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Liquefaction Potential</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>STATISTICAL ANALYSES</td>
<td>14</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>14</td>
</tr>
<tr>
<td>3.2</td>
<td>General Statistics</td>
<td>15</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Sample Mean</td>
<td>15</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Sample Standard Deviation</td>
<td>15</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Coefficient of Variation</td>
<td>16</td>
</tr>
<tr>
<td>3.2.4</td>
<td>Confidence Interval</td>
<td>16</td>
</tr>
<tr>
<td>3.3</td>
<td>Scatter Diagrams</td>
<td>17</td>
</tr>
<tr>
<td>3.4</td>
<td>Analysis of Correlation Coefficients</td>
<td>17</td>
</tr>
<tr>
<td>3.5</td>
<td>Regress Analysis</td>
<td>18</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Bivariate Regression Analysis</td>
<td>18</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Nonlinear Equations</td>
<td>19</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>3.6</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>PART II</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>5.2</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>5.2.1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>5.2.2</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>5.2.3</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>5.2.4</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>5.3</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>5.3.1</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>5.3.2</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>5.3.3</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>6.1</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>6.2</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>6.3</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>6.4</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7</td>
<td>SPECIMEN PREPARATION, TEST PROCEDURES AND DATA REDUCTION</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>7.1 Material and Sample Preparation</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>7.2 Specimen Preparation</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>7.3 Testing Procedures</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>7.4 Data Reduction</td>
<td>77</td>
</tr>
<tr>
<td>8</td>
<td>TESTING PROGRAM AND RESULTS</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>8.1 Testing Program</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>8.2 Test Results</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>8.3 Discussion</td>
<td>97</td>
</tr>
<tr>
<td>9</td>
<td>EVALUATION OF FUNCTIONAL RELATIONSHIP BETWEEN SHEAR MODULUS, SHEAR STRAIN AND GRAIN-SIZE DISTRIBUTION CHARACTERISTICS</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>9.1 Data Selection</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>9.2 Statistical Analyses</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>9.2.1 Introduction</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>9.2.2 Scatter Diagrams</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>9.2.3 Statistical Correlation</td>
<td>121</td>
</tr>
<tr>
<td></td>
<td>9.2.4 Regression Analyses</td>
<td>123</td>
</tr>
<tr>
<td></td>
<td>9.3 Summary of Results</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>9.4 Discussion</td>
<td>133</td>
</tr>
<tr>
<td>10</td>
<td>CONCLUSIONS AND RECOMMENDATIONS</td>
<td>139</td>
</tr>
<tr>
<td></td>
<td>10.1 Summary and Conclusions</td>
<td>139</td>
</tr>
<tr>
<td></td>
<td>10.2 Recommendations</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td>BIBLIOGRAPHY</td>
<td>143</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>PART III LIQUEFACTION POTENTIAL</td>
<td>146</td>
<td></td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>11 INTRODUCTION</td>
<td>149</td>
<td></td>
</tr>
<tr>
<td>11.1 General</td>
<td>149</td>
<td></td>
</tr>
<tr>
<td>11.2 Purpose</td>
<td>149</td>
<td></td>
</tr>
<tr>
<td>11.3 Scope</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>12 REVIEW OF LITERATURE</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>12.1 Behaviors of Saturated Granular Soils under Static and Dynamic Loads</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>12.1.1 Behaviors of Saturated Sands under Static Loads</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>12.1.2 Behavior of Saturated Sands under Cyclic Loads</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>12.2 Case Histories of Liquefaction Induced Failures</td>
<td>164</td>
<td></td>
</tr>
<tr>
<td>12.2.1 Introduction</td>
<td>164</td>
<td></td>
</tr>
<tr>
<td>12.2.2 Liquefaction Induced by Monotonically Changing Stresses</td>
<td>166</td>
<td></td>
</tr>
<tr>
<td>12.2.2.1 Zeeland Slides</td>
<td>166</td>
<td></td>
</tr>
<tr>
<td>12.2.2.2 Mississippi River Flow Slides</td>
<td>167</td>
<td></td>
</tr>
<tr>
<td>12.2.2.3 Partial Failure at Fort Peck Dam</td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>12.2.2.4 Flow Slide at Aberfan Village in Wales</td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>12.2.3 Liquefaction Induced by Cyclic Vibrations</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td>12.2.3.1 Liquefaction of Railway Embankment Near Weesp, Holland</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>12.2.3.2 Vibrations Induced by Pile Driving</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td>12.2.4 Flow Failures Induced by Earthquakes, Blasts, and Wave Action</td>
<td>173</td>
<td></td>
</tr>
<tr>
<td>12.2.4.1 One of the Earliest Recorded Landslides Resulting from Liquefaction</td>
<td>173</td>
<td></td>
</tr>
<tr>
<td>12.2.4.2 Examples of Fluidation Flow Induced by Earthquakes</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td>12.2.4.3 Kanto Earthquakes</td>
<td>177</td>
<td></td>
</tr>
<tr>
<td>12.2.4.4 Liquefaction During the Niigata Earthquake of 1964</td>
<td>178</td>
<td></td>
</tr>
<tr>
<td>12.2.4.5 Valdez Slide, 1964 Alaska Earthquake</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>12.2.4.6 Turnagain Heights Slide, 1964 Alaska Earthquake</td>
<td>181</td>
<td></td>
</tr>
<tr>
<td>12.2.4.7 Slides in the San Fernando Dams</td>
<td>182</td>
<td></td>
</tr>
<tr>
<td>12.2.4.8 Ekofisk Tank in the North Sea</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>12.2.4.9 Failure of the Swir III Dam</td>
<td>187</td>
<td></td>
</tr>
<tr>
<td>12.2.4.10 Sand Boils at Operation Snowball</td>
<td>188</td>
<td></td>
</tr>
<tr>
<td>12.2.5 Discussion of Case Histories</td>
<td>188</td>
<td></td>
</tr>
<tr>
<td>12.3 Methods for Evaluating Liquefaction Potential of Soils</td>
<td>191</td>
<td></td>
</tr>
<tr>
<td>12.3.1 Simplified Procedures</td>
<td>191</td>
<td></td>
</tr>
<tr>
<td>12.3.2 Ground Response Analysis</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>12.3.3</td>
<td>Empirical Method</td>
<td>193</td>
</tr>
<tr>
<td>12.4</td>
<td>Factors Affecting Liquefaction Potential</td>
<td>193</td>
</tr>
<tr>
<td>12.4.1</td>
<td>Effect of Relative Density</td>
<td>194</td>
</tr>
<tr>
<td>12.4.2</td>
<td>Grain and Gradation Characteristics</td>
<td>196</td>
</tr>
<tr>
<td>12.4.3</td>
<td>Effect of Consolidation Pressure</td>
<td>205</td>
</tr>
<tr>
<td>12.4.4</td>
<td>Effect of Anisotropy or Consolidation Ratio, $K_c$</td>
<td>208</td>
</tr>
<tr>
<td>12.4.5</td>
<td>Effect of Overconsolidation</td>
<td>208</td>
</tr>
<tr>
<td>12.4.6</td>
<td>Effect of Strain History</td>
<td>209</td>
</tr>
<tr>
<td>12.4.7</td>
<td>Effect of Reconstituted Versus Intact Specimens</td>
<td>210</td>
</tr>
<tr>
<td>12.5</td>
<td>Experimental Methods for Evaluating Liquefaction Potential</td>
<td>212</td>
</tr>
<tr>
<td>12.5.1</td>
<td>Cyclic Triaxial Test</td>
<td>213</td>
</tr>
<tr>
<td>12.5.2</td>
<td>Cyclic Simple Shear Test</td>
<td>213</td>
</tr>
<tr>
<td>12.5.3</td>
<td>Torsional Simple Shear Test</td>
<td>214</td>
</tr>
<tr>
<td>12.5.4</td>
<td>Shaking Table Test</td>
<td>215</td>
</tr>
<tr>
<td>12.6</td>
<td>Factor Affecting Cyclic Triaxial Test Results</td>
<td>216</td>
</tr>
<tr>
<td>12.6.1</td>
<td>Sample Preparation Methods</td>
<td>216</td>
</tr>
<tr>
<td>12.6.2</td>
<td>Membrane Penetration</td>
<td>220</td>
</tr>
<tr>
<td>12.6.3</td>
<td>Frictionless Caps and Bases</td>
<td>225</td>
</tr>
<tr>
<td>12.6.4</td>
<td>Effect of Sample Size</td>
<td>227</td>
</tr>
<tr>
<td>12.6.5</td>
<td>Effect of Porewater Pressure Parameter &quot;B&quot;</td>
<td>232</td>
</tr>
<tr>
<td>12.6.6</td>
<td>Loading Wave Form</td>
<td>235</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>12.6.7</td>
<td>237</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>TEST EQUIPMENT, SAMPLE PREPARATION AND EXPERIMENTAL PROCEDURES</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>13.1</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>Test Equipment</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>13.1.1</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>MTS Closed-Loop Servo Electro-hydraulic System</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>13.1.2</td>
<td>242</td>
<td></td>
</tr>
<tr>
<td>Data Acquisition System</td>
<td>242</td>
<td></td>
</tr>
<tr>
<td>13.1.3</td>
<td>243</td>
<td></td>
</tr>
<tr>
<td>Triaxial Testing System</td>
<td>243</td>
<td></td>
</tr>
<tr>
<td>13.2</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td>Sample Preparation</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td>13.2.1</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td>Test Sand and Test Program</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td>13.2.2</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>Forming the Test Specimen</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>13.2.3</td>
<td>264</td>
<td></td>
</tr>
<tr>
<td>Measuring the Diameter and Length of a Specimen</td>
<td>264</td>
<td></td>
</tr>
<tr>
<td>13.2.4</td>
<td>269</td>
<td></td>
</tr>
<tr>
<td>Calculation of Specimen Relative Density</td>
<td>269</td>
<td></td>
</tr>
<tr>
<td>13.3</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>Assembling the Triaxial Cell</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>13.4</td>
<td>271</td>
<td></td>
</tr>
<tr>
<td>Filling the Triaxial Cell with Water</td>
<td>271</td>
<td></td>
</tr>
<tr>
<td>13.5</td>
<td>272</td>
<td></td>
</tr>
<tr>
<td>Specimen Saturation</td>
<td>272</td>
<td></td>
</tr>
<tr>
<td>13.5.1</td>
<td>272</td>
<td></td>
</tr>
<tr>
<td>Flushing the Specimen with Water</td>
<td>272</td>
<td></td>
</tr>
<tr>
<td>13.5.2</td>
<td>273</td>
<td></td>
</tr>
<tr>
<td>Connecting the Pore Pressure Transducer</td>
<td>273</td>
<td></td>
</tr>
<tr>
<td>13.5.3</td>
<td>275</td>
<td></td>
</tr>
<tr>
<td>Raising the Cell and Back Pressure</td>
<td>275</td>
<td></td>
</tr>
<tr>
<td>13.5.4</td>
<td>276</td>
<td></td>
</tr>
<tr>
<td>Checking the B-Value</td>
<td>276</td>
<td></td>
</tr>
<tr>
<td>13.6</td>
<td>276</td>
<td></td>
</tr>
<tr>
<td>Consolidation of the Test Specimen</td>
<td>276</td>
<td></td>
</tr>
<tr>
<td>13.7</td>
<td>279</td>
<td></td>
</tr>
<tr>
<td>Placing the Triaxial Cell Within the MTS Loading Frame</td>
<td>279</td>
<td></td>
</tr>
<tr>
<td>13.8</td>
<td>284</td>
<td></td>
</tr>
<tr>
<td>Connecting X-Y and X-Y-Y' Recorders</td>
<td>284</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>13.9</td>
<td>286</td>
</tr>
<tr>
<td></td>
<td>13.10</td>
<td>289</td>
</tr>
<tr>
<td></td>
<td>13.10.1</td>
<td>289</td>
</tr>
<tr>
<td></td>
<td>13.10.2</td>
<td>290</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>292</td>
</tr>
<tr>
<td></td>
<td>14.1</td>
<td>292</td>
</tr>
<tr>
<td></td>
<td>14.2</td>
<td>297</td>
</tr>
<tr>
<td></td>
<td>14.3</td>
<td>299</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>309</td>
</tr>
<tr>
<td></td>
<td>14.5</td>
<td>317</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>324</td>
</tr>
<tr>
<td></td>
<td>15.1</td>
<td>324</td>
</tr>
<tr>
<td></td>
<td>15.2</td>
<td>325</td>
</tr>
<tr>
<td></td>
<td>15.3</td>
<td>326</td>
</tr>
<tr>
<td></td>
<td>REFERENCES</td>
<td>328</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Sample Specifications and Index Properties</td>
<td>69</td>
</tr>
<tr>
<td>9.1</td>
<td>Data Sets Selected for Statistical Analyses</td>
<td>115</td>
</tr>
<tr>
<td>9.2</td>
<td>Correlation Coefficient between Shear Modulus and Shear Strain in Part (1)</td>
<td>122</td>
</tr>
<tr>
<td>9.3</td>
<td>Correlation Coefficient between Variables in Part (2)</td>
<td>124</td>
</tr>
<tr>
<td>9.4</td>
<td>Correlation Coefficients between $G/G_{\text{max}}$ and $\gamma$</td>
<td>126</td>
</tr>
<tr>
<td>9.5</td>
<td>Regression Coefficients between $G/G_{\text{max}}$ and $\gamma$ for Each Data Set</td>
<td>127</td>
</tr>
<tr>
<td>9.6</td>
<td>Correlation Coefficients between $G_{\text{max}}$ and Other Variables</td>
<td>129</td>
</tr>
<tr>
<td>9.7</td>
<td>Regression Coefficients for $G_{\text{max}}$ for Each Data Set</td>
<td>130</td>
</tr>
<tr>
<td>9.8</td>
<td>Correlation Coefficients for Constants $a$ and $b$</td>
<td>132</td>
</tr>
<tr>
<td>9.9</td>
<td>Correlation Coefficients Between $a$ and $b$</td>
<td>133</td>
</tr>
<tr>
<td>9.10</td>
<td>Statistics of Variables used in the Development of the Functional Relationships</td>
<td>134</td>
</tr>
<tr>
<td>12.1</td>
<td>Difference between Liquefaction and Cyclic Mobility</td>
<td>160</td>
</tr>
<tr>
<td>12.2</td>
<td>Soils Tested by Lee and Fitton</td>
<td>200</td>
</tr>
<tr>
<td>13.1</td>
<td>The Index Properties of Test Samples</td>
<td>248</td>
</tr>
<tr>
<td>13.2</td>
<td>Mesh Openings of Zero Raining Devices</td>
<td>249</td>
</tr>
<tr>
<td>14.1</td>
<td>Values of Coefficients $a$, $b$ and $c$ in Equation 14.2</td>
<td>321</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Grain Shape and Size of Original Denver Sand</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>Grain Shape and Size of Sample DM-d₂ Retained in Sieve No. 40</td>
<td>7</td>
</tr>
<tr>
<td>2.3</td>
<td>Grain Shape and Size of Monterey No. 0 Sand</td>
<td>8</td>
</tr>
<tr>
<td>2.4</td>
<td>Gradation of Denver Sand Samples</td>
<td>9</td>
</tr>
<tr>
<td>2.5</td>
<td>Gradation of Monterey No. 0 Sand</td>
<td>11</td>
</tr>
<tr>
<td>5.1</td>
<td>Hysteretic Stress-Strain Relationship</td>
<td>29</td>
</tr>
<tr>
<td>5.2</td>
<td>Shear Modulus Versus Shear Strain</td>
<td>36</td>
</tr>
<tr>
<td>5.3</td>
<td>Damping Ratio Versus Shear Strain</td>
<td>37</td>
</tr>
<tr>
<td>6.1</td>
<td>Free-Free Torsional Resonant Column Schematic</td>
<td>52</td>
</tr>
<tr>
<td>6.2</td>
<td>Drnevich Free-Free Resonant Column Apparatus</td>
<td>54</td>
</tr>
<tr>
<td>6.3</td>
<td>Pressure and Vacuum System</td>
<td>56</td>
</tr>
<tr>
<td>6.4</td>
<td>Electronic Monitoring Devices</td>
<td>57</td>
</tr>
<tr>
<td>6.5</td>
<td>Wiring Diagram</td>
<td>58</td>
</tr>
<tr>
<td>6.6</td>
<td>Zero Height Raining Device</td>
<td>60</td>
</tr>
<tr>
<td>6.7</td>
<td>Leveling Device</td>
<td>61</td>
</tr>
<tr>
<td>6.8</td>
<td>Temporary Supporting Device</td>
<td>62</td>
</tr>
<tr>
<td>6.9</td>
<td>Complete Resonant Column Test System</td>
<td>64</td>
</tr>
<tr>
<td>6.10</td>
<td>Concrete Pad Around the Base of the Apparatus</td>
<td>65</td>
</tr>
<tr>
<td>7.1</td>
<td>Measuring the Diameter of the Specimen with a Pi-Tape</td>
<td>75</td>
</tr>
<tr>
<td>7.2</td>
<td>Tightening the Nuts with a Torque-Wrench</td>
<td>75</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>7.3</td>
<td>Dimensionless Frequency Factor (after Drnevich)</td>
<td>81</td>
</tr>
<tr>
<td>7.4</td>
<td>Dimensionless Frequency Factor (after Drnevich)</td>
<td>82</td>
</tr>
<tr>
<td>7.5</td>
<td>Strain Factor for Resonance Determined by Phase Measurement between Input Force and Motion at the Passive End (after Drnevich)</td>
<td>85</td>
</tr>
<tr>
<td>7.6</td>
<td>Coefficient for Manual Calculation of Damping Ratio for Resonance Determined by Phase Measurement Between Input Force and Motion at the Passive End (after Drnevich)</td>
<td>86</td>
</tr>
<tr>
<td>7.7</td>
<td>Amplitude Decay Curve</td>
<td>88</td>
</tr>
<tr>
<td>8.1</td>
<td>Shear Modulus Versus Shear Strain (Monterey No. 0 Sand)</td>
<td>92</td>
</tr>
<tr>
<td>8.2</td>
<td>Damping Ratio Versus Shear Strain (Monterey No. 0 Sand)</td>
<td>93</td>
</tr>
<tr>
<td>8.3</td>
<td>Shear Modulus Versus Shear Strain (Monterey No. 0 Sand)</td>
<td>94</td>
</tr>
<tr>
<td>8.4</td>
<td>Damping Ratio Versus Shear Strain (Monterey No. 0 Sand)</td>
<td>95</td>
</tr>
<tr>
<td>8.5</td>
<td>The Effect of Confining Pressure on Maximum Shear Modulus (Monterey No. 0 Sand)</td>
<td>96</td>
</tr>
<tr>
<td>8.6</td>
<td>Shear Modulus Versus Shear Strain (Denver Sand, DC-a)</td>
<td>98</td>
</tr>
<tr>
<td>8.7</td>
<td>Damping Ratio Versus Shear Strain (Denver Sand, DC-a)</td>
<td>99</td>
</tr>
<tr>
<td>8.8</td>
<td>Shear Modulus Versus Shear Strain (Denver Sand, DC-b)</td>
<td>100</td>
</tr>
<tr>
<td>8.9</td>
<td>Damping Ratio Versus Shear Strain (Denver Sand, DC-b)</td>
<td>101</td>
</tr>
<tr>
<td>8.10</td>
<td>Shear Modulus Versus Shear Strain (Denver Sand, DM-c)</td>
<td>102</td>
</tr>
<tr>
<td>8.11</td>
<td>Damping Ratio Versus Shear Strain (Denver Sand, DM-c)</td>
<td>103</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>8.12</td>
<td>Shear Modulus Versus Shear Strain (Denver Sand, DM-d)</td>
<td>104</td>
</tr>
<tr>
<td>8.13</td>
<td>Damping Ratio Versus Shear Strain (Denver Sand, DM-d)</td>
<td>105</td>
</tr>
<tr>
<td>8.14</td>
<td>Shear Modulus Versus Shear Strain (Denver Sand, DM-e)</td>
<td>106</td>
</tr>
<tr>
<td>8.15</td>
<td>Damping Ratio Versus Shear Strain (Denver Sand, DM-e)</td>
<td>107</td>
</tr>
<tr>
<td>8.16</td>
<td>Shear Modulus Versus Shear Strain (Denver Sand, DF-f)</td>
<td>108</td>
</tr>
<tr>
<td>8.17</td>
<td>Damping Ratio Versus Shear Strain (Denver Sand, DF-f)</td>
<td>109</td>
</tr>
<tr>
<td>8.18</td>
<td>Shear Modulus Versus Shear Strain (DM-d_2)</td>
<td>112</td>
</tr>
<tr>
<td>8.19</td>
<td>Damping Ratio Versus Shear Strain (DM-d_2)</td>
<td>113</td>
</tr>
<tr>
<td>9.1</td>
<td>The Effect of Void Ratio on Maximum Shear Modulus</td>
<td>117</td>
</tr>
<tr>
<td>9.2</td>
<td>The Effect of Uniformity Coefficient on Maximum Shear Modulus</td>
<td>118</td>
</tr>
<tr>
<td>9.3</td>
<td>The Effect of Mean Grain Size on Maximum Shear Modulus</td>
<td>118</td>
</tr>
<tr>
<td>9.4</td>
<td>The Effect of Effective Grain Size on Maximum Shear Modulus</td>
<td>119</td>
</tr>
<tr>
<td>9.5</td>
<td>Maximum Shear Modulus Versus Relative Density</td>
<td>120</td>
</tr>
<tr>
<td>9.6</td>
<td>Comparison of Measured and Computed Values of Shear Modulus</td>
<td>136</td>
</tr>
<tr>
<td>9.7</td>
<td>Comparison of Relationships for Maximum Shear Modulus</td>
<td>137</td>
</tr>
<tr>
<td>12.1</td>
<td>Critical Density Line (From Baladi and Rohani, 1978)</td>
<td>153</td>
</tr>
<tr>
<td>12.2</td>
<td>Typical Behavior of Saturated Soils Tested under Drained Triaxial Test Conditions</td>
<td>154</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>--------------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>12.3</td>
<td>Typical Behavior of Saturated Soils Tested under Undrained Triaxial Test Conditions (From Baladi and Rohani, 1978)</td>
<td>156</td>
</tr>
<tr>
<td>12.4</td>
<td>Critical Confining Pressure Versus Relative Density of Monterey No. 0 Sand (From Lee and Vernese, 1978)</td>
<td>158</td>
</tr>
<tr>
<td>12.5</td>
<td>Comparison of Axial Compression and Extension Consolidated-Undrained Triaxial Tests (Castro, 1975)</td>
<td>158</td>
</tr>
<tr>
<td>12.6</td>
<td>Liquefaction and Cyclic Mobility (Castro and Polous, 1977)</td>
<td>161</td>
</tr>
<tr>
<td>12.7</td>
<td>Typical Result of an Isotropically Consolidated Stress-Controlled Cyclic Triaxial Test</td>
<td>163</td>
</tr>
<tr>
<td>12.8</td>
<td>Phase Transformation Lines for Liquefying Sand</td>
<td>165</td>
</tr>
<tr>
<td>12.9</td>
<td>Dutch Hypothesis for Progressive Flow Failures (From Hvorslev, 1956)</td>
<td>168</td>
</tr>
<tr>
<td>12.10</td>
<td>Cyclic Stresses Causing Different Axial Strains, $\sigma_3 = 1 \text{ kg/cm}^2$ (From Seed and Lee, 1967)</td>
<td>195</td>
</tr>
<tr>
<td>12.11</td>
<td>Comparison of Shaking Table and Triaxial Test Result (after DeAlba, et al., 1976)</td>
<td>197</td>
</tr>
<tr>
<td>12.12</td>
<td>Effect of Density on Number of Square Wave Loading Cycle to Cause Initial Liquefaction (after Mulilis, et al., 1978)</td>
<td>198</td>
</tr>
<tr>
<td>12.13</td>
<td>Gradations of Soils Tested by Lee and Fitton (From Lee and Fitton, 1969)</td>
<td>201</td>
</tr>
<tr>
<td>12.14</td>
<td>Typical Load-Deflection and Pore Pressure Change Curves for Different Grain Sizes (From Lee and Fitton, 1969)</td>
<td>201</td>
</tr>
<tr>
<td>12.15</td>
<td>Comparison of Pulsating Loading Strengths of Different Grain-Size Soils (From Lee and Fitton, 1969)</td>
<td>203</td>
</tr>
<tr>
<td>12.16</td>
<td>Cyclic Triaxial Test Results Showing Influence of Membrane Penetration Effects (after Martin, et al., 1978)</td>
<td>206</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>12.17</td>
<td>Cyclic Stresses Causing Initial Liquefaction in 100 Cycles (From Seed and Lee, 1967)</td>
<td>207</td>
</tr>
<tr>
<td>12.18</td>
<td>Summary Curve Comparing the Cyclic Strength at 50 Percent Relative Density of Reconstituted and Undisturbed Sand Specimens from Niigata, Japan (From Silver, 1978)</td>
<td>211</td>
</tr>
<tr>
<td>12.19</td>
<td>Cyclic Stress Ratio Versus Number of Cycles to Initial Liquefaction for Different Compaction Procedures (From Seed, 1975)</td>
<td>218</td>
</tr>
<tr>
<td>12.20</td>
<td>Applied Stress Ratio Versus Number of Loading Cycles Required to Obtain Peak-to-Peak Axial Strain of 10 Percent (From Ladd, 1974)</td>
<td>221</td>
</tr>
<tr>
<td>12.21</td>
<td>Membrane Penetration into the Interstices of a Sample of Sand Due to Changes of Excess Pore-Water Pressure, ( \Delta u )</td>
<td>222</td>
</tr>
<tr>
<td>12.22</td>
<td>Frictionless Caps and Bases used in UCLA Study (From Vernese, 1977)</td>
<td>226</td>
</tr>
<tr>
<td>12.23</td>
<td>Comparison of Cyclic Strengths of Loose Monterey Sand using Frictionless and Regular Ends (From Vernese, 1977)</td>
<td>228</td>
</tr>
<tr>
<td>12.24</td>
<td>Comparison of Cyclic Strengths of Dense Monterey Sand using Frictionless and Regular Ends (From Vernese, 1977)</td>
<td>229</td>
</tr>
<tr>
<td>12.25</td>
<td>Comparison of Strength Data from Cyclic Triaxial Tests on Loose Monterey Sand (From Vernese, 1977)</td>
<td>231</td>
</tr>
<tr>
<td>12.26</td>
<td>Effect of Sample Size on Cyclic Stresses Causing Initial Liquefaction on Monterey Sand (From Seed, Wong, and Chan, 1975)</td>
<td>233</td>
</tr>
<tr>
<td>12.27</td>
<td>Effect of B-Value on Number of Load Cycles to Achieve ( \varepsilon_{da} = 5% ) Strain for Loose Sand ( D_r = 53.5% ) (after Chaney, 1978)</td>
<td>234</td>
</tr>
<tr>
<td>12.28</td>
<td>Effect of Wave Shape on the Cyclic Triaxial Strength of the Test Sand (after Silver, 1976)</td>
<td>236</td>
</tr>
<tr>
<td>13.1</td>
<td>Closed Loop Electro-Hydraulic Materials Test System Applying a Sinusoidal Loading to a Monterey No. 0 Sand Specimen</td>
<td>239</td>
</tr>
</tbody>
</table>
13.2 Triaxial Cell, Closed Loop Electro-Hydraulic Materials Test System, and Data Acquisition System ........................................ 244

13.3 Test Specimen Placed within the Triaxial Cell ........ 244

13.4 Pore Pressure Transducer and a 4-way Valve Used to Monitor Cell Pressure, Back Pressure and Specimen Pore Water Pressure ............... 245

13.5 Pressure Control Panel used to Apply Cell and Back Pressure to the Triaxial Cell. Graduated Burettes were used to Determine Specimen Volume Change ........ 245

13.6 Looking Down on the Base of the Triaxial Cell .......... 251

13.7 Checking for Leaks in the Rubber Membrane .......... 251

13.8 Vernier Caliper used to Measure the Thickness of the Membrane and Diameter of the Sand Specimen .......... 253

13.9 Vernier Caliper to Measure the Height of the Test Specimen ................................................................. 253

13.10 Rubber Membrane placed on the Base of the Triaxial Cell .......................................................... 255

13.11 Rubber O-Ring and O-Ring Expander ................. 255

13.12 Cylindrical Mold used in Forming the Sand Specimens ................................................................. 256

13.13 Cylindrical Mold Placed on the Base of the Triaxial Cell ................................................................. 256

13.14 Scale used to Weigh the Monterey No. 0 Sand ........ 258

13.15 Zero Raining Device used to Place the Monterey No. 0 Sand ................................................................. 258

13.16 Monterey No. 0 Sand Placed within the Zero Raining Device ................................................................. 259

13.17 Porous Stone Placed on Top of the Monterey No. 0 Sand Specimen ................................................................. 259

13.18 Requirements for Flatness and Parallelism of Specimen Ends (From Silver, 1977) ................................................................. 260
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.19</td>
<td>Leveling the Top Porous Stone so That the Top Porous Stone is Parallel to the Base of the Triaxial Cell</td>
<td>262</td>
</tr>
<tr>
<td>13.20</td>
<td>Top Cap Placed on Top of the Specimen with the Rubber Membrane Pulled up Around the Top Cap</td>
<td>262</td>
</tr>
<tr>
<td>13.21</td>
<td>Positioning of the Top Surface of the Loading Cap Parallel to the Base of the Triaxial Cell</td>
<td>263</td>
</tr>
<tr>
<td>13.22</td>
<td>Measuring the Length of the Sample using a Vernier Caliper</td>
<td>263</td>
</tr>
<tr>
<td>13.23</td>
<td>Effect of Specimen Density on Cyclic Triaxial Strength of Monterey No. 0 Sand (From Mulilis, Horz and Townsend, 1976)</td>
<td>266</td>
</tr>
<tr>
<td>13.24</td>
<td>Volume Change Measured in a 14.8 Foot Length of 1/4 Inch Diameter Polyethylene Tubing</td>
<td>278</td>
</tr>
<tr>
<td>13.25</td>
<td>Triaxial Cell Placed Within the MTS Loading Frame</td>
<td>280</td>
</tr>
<tr>
<td>13.26</td>
<td>Controls for the X-Y Recorder</td>
<td>282</td>
</tr>
<tr>
<td>14.1</td>
<td>Effect of Relative Denisty on Denver Sand</td>
<td>294</td>
</tr>
<tr>
<td>14.2</td>
<td>Effect of Relative Density on Liquefaction Potential of Denver Sand at Different Stress Cycles</td>
<td>295</td>
</tr>
<tr>
<td>14.3</td>
<td>Correction Factors Versus Number of Cycles Causing Initial Liquefaction</td>
<td>296</td>
</tr>
<tr>
<td>14.4</td>
<td>Typical Recorded Data</td>
<td>298</td>
</tr>
<tr>
<td>14.5</td>
<td>Effect of Mean Grain Size on Initial Liquefaction</td>
<td>300</td>
</tr>
<tr>
<td>14.6</td>
<td>Effect of Mean Grain Size on Initial Liquefaction</td>
<td>301</td>
</tr>
<tr>
<td>14.7</td>
<td>Effect of Mean Grain Size on Initial Liquefaction</td>
<td>302</td>
</tr>
<tr>
<td>14.8</td>
<td>Effect of Mean Grain Size on Initial Liquefaction</td>
<td>303</td>
</tr>
<tr>
<td>14.9</td>
<td>Effect of Mean Grain Size on Initial Liquefaction</td>
<td>304</td>
</tr>
<tr>
<td>14.10</td>
<td>Effect of Mean Grain Size on Stress Ratio Causing Initial Liquefaction in 10 Cycles</td>
<td>305</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>14.11</td>
<td>Effect of Mean Grain Size on Stress Ratio Causing Initial Liquefaction in 10 Cycles</td>
<td>306</td>
</tr>
<tr>
<td>14.12</td>
<td>Typical Recorded Test Data for Coarse Sand</td>
<td>308</td>
</tr>
<tr>
<td>14.13</td>
<td>Effect of Uniformity Coefficient on Liquefaction</td>
<td>310</td>
</tr>
<tr>
<td>14.14</td>
<td>Effect of Uniformity Coefficient on Liquefaction</td>
<td>311</td>
</tr>
<tr>
<td>14.15</td>
<td>Effect of Uniformity Coefficient on Liquefaction</td>
<td>312</td>
</tr>
<tr>
<td>14.16</td>
<td>Effect of Uniformity Coefficient on Liquefaction</td>
<td>313</td>
</tr>
<tr>
<td>14.17</td>
<td>Effect of Uniformity Coefficient on Liquefaction</td>
<td>314</td>
</tr>
<tr>
<td>14.18</td>
<td>Effect of Uniformity Coefficient on Liquefaction</td>
<td>315</td>
</tr>
<tr>
<td>14.19</td>
<td>Effect of Uniformity Coefficient on Stress Ratio Causing Initial Liquefaction in 10 Cycles</td>
<td>318</td>
</tr>
<tr>
<td>14.20</td>
<td>Effect of Uniformity Coefficient on Stress Ratio Causing Initial Liquefaction in 10 Cycles</td>
<td>319</td>
</tr>
</tbody>
</table>
Part I

GENERAL INFORMATION
CHAPTER 1

INTRODUCTION

During the past few decades, the dynamic behavior of soils has received considerable attention, mainly because of the significant growth in sizes and importance of engineering projects, such as nuclear power plants, earth embankment dams, pipelines, machine foundations and off-shore structures. Some of the projects are located near or in areas which are heavily populated and seismically active. These projects are very costly and their failures could endanger the public safety or cause significant environmental damages and/or property loss. Hence, there exists some great concern about their safe designs which involve the rigorous consideration of the static and dynamic behaviors of soils.

For a sound dynamic design of a structure, it is essential to know the dynamic properties and behaviors of soils which support the structure or of which it is constructed. Research on earthquakes and the resulting catastrophic events has focused much attention on the behavior of granular soils under dynamic loads. In order to evaluate the behavior of soil deposits subjected to earthquakes, ocean waves and other nonstatic loads, it is necessary to evaluate the dynamic properties, behaviors and strengths of underlying soils.
Knowledge of the strain-dependent dynamic properties (shear modulus and damping ratio) is essential for predicting the performance of an earth structure, a soil deposit, and the soil-structure interaction during earthquakes, etc. Dynamic properties at small strains (less than $10^{-4}$ radian) are also important in assessing the behavior of large machine foundations under vibration-induced motions.

Catastrophic ground failures due to earthquakes may result in either partial or total destruction of earth-embankment dams, mine-tailings dams, slopes and other super structures. Very often these ground failures are the direct result of the liquefaction of saturated granular soils. To achieve safe designs of earth structures, etc. constructed of or supported by saturated granular soils, one must have sound knowledge of their liquefaction potential under various conditions.

The Alaska and Niigata earthquakes of 1964 which caused considerable damage to earth structures and other super structures have accelerated the research in the field of the dynamic properties and the liquefaction potential of granular soils on factors, such as the relative density, void ratio, mean effective stress and cyclic strain and/or deviatoric stress amplitudes, etc. However, the effects of grain-size distribution characteristics on the dynamic properties and the liquefaction, while known to be significant, have not been systematically studied over a wide range of grain sizes. This study is to investigate such effects.

Most data on the dynamic properties and the liquefaction potential are for uniform clean sands. However, natural granular
soil deposits are neither uniform nor clean. They usually contain particles over a broad spectrum of grain size. Therefore, samples prepared from Denver Sand were tested. The samples have the following grain shape and grain-size distribution characteristics:

- Grain shape: Subrounded
- Specific gravity: 2.6
- Grain size: 0.038 mm to 9.5 mm
- Mean grain size ($D_{50}$): 0.149 mm to 1.68 mm
- Uniformity coefficient ($C_u$): 2 to 15

The free-free resonant column apparatus designed by Professor Vincent P. Drnevich at the University of Kentucky was used to determine the dynamic properties of 6-inch diameter dry sand samples. Dynamic shear moduli and damping ratios at strains smaller than $5 \times 10^{-4}$ radian were determined. The closed-loop servo control universal dynamic material testing system and the triaxial cells manufactured by MTS and Geotest, Inc. were used to assess the liquefaction potential of saturated sand samples of 2-inch diameters through cyclic triaxial tests.

The shear modulus of the dry Denver Sand was found to be strongly dependent on its uniformity. The shear modulus of sand increases with the increasing uniformity coefficient. Thus a well graded sand is stiffer than the uniformly graded one.

The liquefaction potential of sand was found to be strongly affected by its mean grain size, $D_{50}$. It increases with the decreasing mean grain size. In other words, the fine sand is easier to liquefy than the coarse one.

Finally, regression analyses were performed to formulate the
functional relationships between the shear modulus and the uniformity coefficient and between the liquefaction potential and the mean diameter, D_{50}. 
CHAPTER 2

TEST SANDS AND TEST PROGRAMS

2.1 Denver Sand

The primary material used in this study was a well-graded river sand purchased locally from the Denver Sand and Gravel Company in Denver, Colorado. Both coarse and fine particles of Denver Sand are subangular in shape. For comparison, Figs. 2.1, 2.2 and 2.3 show the grain size and shape of Denver Sand, a sample (DM-d_2) prepared from Denver Sand and Monterey No. 0 Sand. It is generally known that Monterey No. 0 Sand is a uniform sand with subrounded grain shape (Lee and Fitton, 1967 and Silver, et al., 1976). Denver Sand is a well-graded sand with subangular grain shape.

Denver Sand was oven dried first and then sixteen standard U.S. sieves with openings ranging from 38 microns (U.S. standard sieve No. 400) to 3/8 inch were used to segregate different size particles. As shown in the design gradation curves in Fig. 2.4, six different mean grain sizes (D_{50}) and uniformity coefficients (C_u) were adopted in sample preparation. The characteristics of samples for the determination of the dynamic property and the liquefaction potential are shown in Tables 7.1 and 14.1 respectively.

Minimum and maximum dry density of each sample were determined prior to testing. The procedures given in Appendix E-12
FIG. 2.1 GRAIN SHAPE AND SIZE OF ORIGINAL DENVER SAND

FIG. 2.2 GRAIN SHAPE AND SIZE OF SAMPLE DM-d₂ RETAINED IN SIEVE NO. 40
FIG. 2.3 GRAIN SHAPE AND SIZE OF MONTEREY NO. 0 SAND
FIG. 2.4  GRADATION OF DENVER SAND SAMPLES
on "Relative Density of Cohesionless Soils" of the Earth Manual (2nd Edition, 1974) published by the U.S. Bureau of Reclamation were used in evaluating them. The resulting minimum and maximum densities are included in Tables 7.1 and 14.1.

2.2 Monterey No. 0 Sand

In addition, Monterey No. 0 Sand was also used in the resonant column tests for the purpose of comparing the dynamic shear modulus and damping ratio with the published data. The sand is a uniform medium beach sand from Monterey, California with the following index properties:

- Mean grain size ($D_{50}$): 0.4 mm
- Uniformity coefficient ($C_u$): 1.5
- Specific gravity: 2.655
- Maximum density: 105.92 lbs/ft$^3$
- Minimum density: 91.96 lbs/ft$^3$
- Grain shape: subrounded
- Mineral composition: mostly quartz and some feldspar

Fig. 2.3 shows that its grain shape is subrounded and somewhat elongated. Fig. 2.5 gives the gradation curve of Monterey No. 0 Sand.

2.3 Test Programs

Detailed test programs are presented in Part II on "Dynamic Properties at Small Strains" and Part III on "Liquefaction Potential." Only a brief summary is given here.
FIG. 2.5 GRADATION OF MONTEREY NO. 0 SAND
2.3.1 Dynamic Properties at Small Strains

To determine the dynamic properties (shear modulus and damping ratio) of six-inch diameter dry Denver Sand samples, the free-free resonant column apparatus manufactured by Professor Vincent P. Drnevich at the University of Kentucky was used. Twenty-four samples prepared from Denver Sand were tested. To fill the goal of assessing the effect of gradation characteristics on the dynamic properties, all parameters other than those describing the gradation characteristics were kept constant. All samples were prepared at the relative density of about 30% and tested under 30 psi confining pressure.

Three dry Monterey No. 0 Sand samples were also tested. The resulting properties were to be compared with the published data.

2.3.2 Liquefaction Potential

Undrained cyclic triaxial tests were conducted to determine the liquefaction potential of samples prepared from Denver Sand. Four samples were prepared from each of the twenty-four samples of different gradation characteristics. All samples were prepared to achieve the relative density of approximately 50% and subjected to the effective confining pressure of 30 psi and the back pressure and the Skempton porewater pressure parameter B of no less than 60 psi and 0.95 respectively before tests. Cyclic stress ratios, \( R_s \), ranging from 0.1 to 0.45 were used, where the cyclic stress ratio is defined as:

\[
R_s = \frac{\sigma_{dc}}{2\sigma_3}
\]
where $\sigma_{dc} =$ cyclic deviatoric stress

$= (\sigma_1 - \sigma_3)$ cyclic

$\sigma_3 =$ effective confining pressure

A total of 123 cyclic triaxial tests were conducted.
3.1 Introduction

Statistical analyses were performed to assess the strength of correlation between the dependent and independent variables. Regression analyses (bivariate or multivariate) were then conducted to formulate, for variables with strong correlation, their functional relation. In this study, the dependent and independent variables are listed as follows:

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>Independent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum shear modulus, ( G_{\text{max}} ) ((\gamma &lt; 10^{-5} \text{ radian}))</td>
<td>Shear strain, ( \gamma )</td>
</tr>
<tr>
<td>Shear modulus, ( G )</td>
<td>Mean grain size, ( D_{50} )</td>
</tr>
<tr>
<td>Cyclic stress ratio, ( R_s )</td>
<td>Effective diameter, ( D_{10} )</td>
</tr>
<tr>
<td>Cyclic stress ratios required to cause liquefaction in 10 and 30 cycles, etc.</td>
<td>Uniformity coefficient, ( C_u )</td>
</tr>
<tr>
<td></td>
<td>Void ratio, ( e )</td>
</tr>
<tr>
<td></td>
<td>Number of stress cycles, ( N )</td>
</tr>
</tbody>
</table>

For the variables with nonlinear relationship, the variable(s) was transformed before it entered the regression analysis. By doing so, nonlinear functions, such as power, logarithmic, and exponential functions, can be easily formulated through the linear regression analysis.
The above statistical analyses can be performed by using the Statistical Package for the Social Sciences (SPSS) computer programs described by Nie, et al. (1975). These programs were available on the CDC 6400 computer system at the University of Colorado. Pertinent details regarding revisions to these programs have been presented by Hohlen (1978 a, 1978 b).

3.2 General Statistics

General statistics including the mean, standard deviation, coefficient of variation, range of values, and the 95-percent confidence interval about the mean can be computed by using SPSS programs. These statistics are defined in the following paragraphs.

3.2.1 Sample Mean

The arithmetic sample mean is the most representative parameter for the magnitude of a random physical quantity. The mean is defined as follows:

\[ \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \]

where \( \bar{x} \) = the mean of the sample,
\( x_i \) = the ith value of a random variable \( X \), and
\( n \) = the sample size, i.e., number of observations.

3.2.2 Sample Standard Deviation

The standard deviations serve as an absolute measure of the dispersion, or variation of the data. A large standard deviation
implies large dispersion and vice versa. The sample standard deviation is defined in the following equations:

\[
s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}
\]

where variables were defined in the previous section.

3.2.3 Coefficient of Variation

The coefficient of variation, \( V \), is defined as the sample standard deviation divided by the sample mean:

\[
V = \frac{s}{\bar{x}}
\]

This coefficient provides a relative measure of the dispersion of the data which enables one to compare the variation in several sets of data.

3.2.4 Confidence Interval

The sample mean and sample standard deviation, an interval which contains the true mean \( \mu \) can be calculated with an associated probability. The 95-percent confidence interval (C.I.) for the true mean, for a normal population with unknown population standard deviation \( \sigma \), is computed by using the following equation (Miller and Freund, 1965, p. 149):

\[
0.95 \text{ C.I.} = \bar{x} \pm t \left( \frac{s}{\sqrt{n}} \right)
\]

where \( t = 0.025 \) t-statistic for \((n-1)\) degrees of freedom and other variables are as previously defined.
3.3 Scatter Diagrams

Scatter diagrams show the dispersion of the experimental data points. The diagram provides a visual aid for the assessment of the strength of correlation and for the observation of the possible functional relationships (linear or nonlinear) between a pair of parameters. It also enables one to look for piecewise relationships. If such a relationship exists for any variable pair, then a separate, piecewise analysis must be performed for each of the various regions of the diagram.

3.4 Analysis of Correlation Coefficients

The correlation coefficient measures the strength of correlation between two variables. Bivariate correlation coefficients are computed by using the following equation:

\[
R = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left[ \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \right] \left[ \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n} \right]^{1/2}}}
\]

where \( R \) = estimate of the true correlation coefficient between \( X \) and \( Y \),
\( x_i \) = ith value of the random variable \( X \),
\( y_i \) = ith value of the random variable \( Y \),
\( n \) = number of pairs of observations,
\( \bar{x} \) = sample mean of variable \( X \), and
\( \bar{y} \) = sample mean of variable \( Y \).

When a straight line is a perfect fit to the data, the correlation coefficient \( R \) is equal to -1.0 or 1.0. A negative value of \( R \) implies that the regression line has a negative slope, which
indicates an inverse relationship. When a straight line is a poor fit to the data, the absolute value of R is close to zero and the variables are said to be uncorrelated. It is generally considered that if the absolute value of the correlation coefficient is equal to or greater than 0.7, then one variable can be predicted from the other by using a linear bivariate regression equation. A small correlation coefficient implies only that no linear relationship exists between two variables and the equation will not give a reliable prediction of the dependent variable. It does not imply that the variables are totally unassociated and have no relationship whatsoever. Indeed, a very strong nonlinear relationship between two variables can yield a correlation coefficient close to zero.

3.5 Regress Analyses

3.5.1 Bivariate Regression Analysis

In a bivariate regression analysis, a regression line is formulated by using a set of data involving two variables. An equation of the form \( y = b_0 + b_1x \) is developed, where \( y \) is the dependent variable, \( x \) is the independent variable, and the regression coefficients \( b_0 \) and \( b_1 \) are estimates of the \( y \)-intercept and slope, respectively, of the true regression line. The bivariate regression coefficients are computed from the data by using the following equations:
The techniques of bivariate regression analysis have been described by Draper and Smith (1966), Walpole and Myers (1972), and in numerous other books.

3.5.2 Nonlinear Equations

In any regression analysis, the quality and usefulness of the developed prediction equation is limited by the form of the equation which was assumed. Therefore, in order to investigate many different models, nonlinear as well as linear relationships are often studied, and these nonlinear relationships can involve numerous transformed variables.

Furthermore, a nonlinear equation (nonlinear in the parameters to be estimated), which can be expressed by a suitable transformation of the variables in the standard linear form, is said to be intrinsically linear. For example, \( y = \exp(b_0 + b_1x_1) \) can be expressed in the standard linear form \( \ln y = b_0 + b_1x_1 \), and therefore is an intrinsically linear equation.

It is generally considered that a linear bivariate regression equation can be used for engineering predictions if the absolute value of the correlation coefficient between the two variables is equal to or greater than 0.7. Furthermore, this correlation
coefficient should be significant, and thus based on a reasonable number of data pairs (cases). Finally, it must be noted that computed correlation coefficients are often based on data exhibiting a limited range of values. Furthermore, any prediction equation is valid only for the range of values of the independent variable used in its development.

3.6 Multiple Regression Analysis

In multiple regression analysis, an equation which involves more than one independent variable is formulated by using a set of data comprised of multivariate observations. An equation of the form $y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k$ is developed, where $y$ is the dependent variable, $x_1, \ldots, x_k$ are independent variables and $b_0, \ldots, b_k$ are regression coefficients computed from the data. In multiple regression, a multiple correlation coefficient $R$ can be computed, which is similar to the bivariate correlation coefficient. The coefficient of multiple determination ($R^2$) measures the amount of variation of the dependent variable accounted for by the regression equation. The methods and techniques of multiple regression analysis have been described by Draper and Smith (1966), Walpole and Myers (1972), and in numerous other books.

As in bivariate regression analysis, the quality and usefulness of a developed multiple regression equation is limited by the form of the equation which was assumed. Therefore, in order to investigate many different models, transformed variables are often included in the analysis.

It is generally considered that a multiple regression equation
can be used for engineering predictions if the multiple correlation coefficient \( R \) is equal to or greater than 0.7. Furthermore, the equation should be developed by using a reasonable number of data sets and it is valid only for the ranges of values of the independent variables used in its development.
Part II

DYNAMIC PROPERTIES AT SMALL STRAINS
ABSTRACT

Knowledge of strain-dependent dynamic properties, such as shear modulus and shear damping ratio of soils is essential for predicting the performance of earth structures, the behavior of soil deposits, and the soil-structure interaction during earthquakes. These dynamic properties at low strains are also important in predicting the behavior of machine foundations under vibration-induced motions. Previous researchers have studied and demonstrated the strong dependence of dynamic properties of granular soils on several factors, such as the void ratio, relative density, shear strain amplitude and mean effective stress. However, the effects of grain size distribution on dynamic properties, while known to be significant, have not been systematically studied.

The purpose of this study is to investigate such effects and to formulate the functional relationships between the dynamic properties at low strain levels and the grain size distribution characteristics: Uniformity coefficient ($C_u$), mean diameter ($D_{50}$), effective diameter ($D_{10}$), etc. To study such effects, sand samples of various gradations were prepared. The uniformity coefficients and the mean diameters of these samples range from 2 to 15 and from 0.149 to 1.68 mm, respectively.

Resonant column tests were performed to determine their
dynamic shear moduli and damping ratios. Analysis of statistical correlation was performed on all dependent variables (shear modulus) and independent variables (the indices specifying the grain size distribution) to evaluate the strength of their correlation. Functional relations were then formulated among variables which show strong correlation through regression analyses. The functional relationship so obtained can then be used to determine the shear modulus of granular soils of known gradations at small strains.
CHAPTER 4

INTRODUCTION

Large engineering projects, such as large earth embankment dams, nuclear power plants and off-shore structures located near or in earthquake prone zones require some extensive dynamic analyses. The analysis would need the dynamic properties of soils over a wide range of strains. There has been a substantial increase in the number of such important engineering projects in the last two to three decades. This leads to an increasing importance of the subject of soil dynamics. The failure of these large, costly structures could endanger the safety of the general public, cause significant environmental damage and incur great property loss. Hence, there is great concern over the safe design of these engineering projects which require rigorous consideration of both static and dynamic loading conditions. To evaluate the behavior of structures under seismic loads, it is necessary to evaluate the dynamic property and strength of soils which support the structure or of which they are constructed.

Knowledge of the strain-dependent dynamic properties (shear modulus and damping ratio) is essential for predicting the performance of an earth structure, the behavior of a soil deposit, and the soil-structure interaction during earthquakes. The
dynamic properties of soils at small strains are also important in assessing the behavior of machine-foundations under vibration-induced motions.

Many factors affect the dynamic property of granular soils. Previous researchers have demonstrated the strong dependence of the dynamic property of granular soils on factors such as the relative density, void ratio, mean effective stress and shear strain amplitude, etc. These factors and their effects on shear modulus and damping ratio are discussed in Chapter 5. However, the effects of grain size distribution on dynamic properties, while known to be significant, have not been systematically studied over a wide range of grain sizes. This study aims at providing this information and the functional relationships between grain size distribution characteristics and dynamic properties at low strain levels.

Most available data on dynamic properties of granular soils are for uniform clean sands. Natural granular soil deposits are neither clean nor uniform, they usually contain particles over a broad spectrum of grain size. Therefore, samples ranging from uniform to well-graded were tested. The uniformity coefficients, $C_u$, and mean grain sizes, $D_{50}$, of the samples range from 2 to 15 and 0.149 to 1.68 mm, respectively.

Resonant column tests were conducted to determine the dynamic properties of soils at low strain levels (below $10^{-4}$). The Drnevich Free-Free resonant column device was used to evaluate the shear modulus and damping ratio of granular soils at shear strain amplitudes below $10^{-4}$. 
This study concluded that the shear modulus of granular soils is strongly affected by shear strain amplitude, void ratio and uniformity coefficient. Maximum shear modulus, $G_{\text{max}}$, is also affected by mean grain size, $D_{50}$, and effective grain size, $D_{10}$. In general, the shear modulus of granular soils increases with the increasing uniformity coefficient. No relationship was obtained for damping ratios because their values were found to be unusually large and their relationships with respect to the confining pressure and density are contrary to some expected trends.
CHAPTER 5

DYNAMIC PROPERTIES OF SOILS

5.1 Introduction

In order to analyze the dynamic behavior of soils and soil-supported structures under dynamic loadings, the shear stress versus strain relationship of the soil must be established. A general dynamic stress-strain relation of soils is very complex because of its dependence on a large number of parameters. However, by studying special cases and understanding them, one might be able to link them together to establish a generalized constitutive relation. In this study the dynamic shear modulus $G$ and damping ratio $D$ at shear strains less than $10^{-4}$ were determined for twenty-four samples of various gradations. The properties were determined by using Drnevich free-free resonant column test device.

Because of the stress versus strain behavior of soils being, in general, nonlinear and nonelastic, a loading-unloading cycle generally produces a hysteresis loop which represents the energy consumed during the process. A hysteresis loop shown in Fig. 5.1 gives the stress versus strain relationship of a soil for a complete cycle of loading and unloading. The slope of the secant $B'B$ defines the shear modulus $G$. The shear damping ratio $D$ is proportional to the ratio between the adsorbed energy (the area
SHEAR MODULUS

\[ G = \frac{1}{\gamma} \]

DAMPING

\[ D = \frac{1}{2\pi} \frac{\text{AREA (LOOP)}}{\text{AREA (OAB + OAB')}} \]

FIG. 5.1 HYSTERETIC STRESS-STRAIN RELATIONSHIP
of the loop) and the applied strain energy (the area OAB + OA'B') and is defined as follows:

\[ D = \frac{1}{2\pi} \frac{\text{Area of loop}}{\text{Area} (OAB + OA'B')} \]  

(5.1)

It should be noted, however, in defining G and D as above, the nonlinear behavior of the soil is represented by an equivalent linear viscoelastic model.

5.2 Measurement of Dynamic Properties

To obtain the representative shear modulus and damping ratio for analyzing the performance of a soil-structure system or an earth structure under a dynamic load, a dynamic test must cover the expected strain spectrum. Three common types of laboratory tests used in obtaining such dynamic properties are: Cyclic triaxial tests, cyclic simple shear tests, and resonant column tests. The in-situ geophysical test is often used in evaluating \( G_{\text{max}} \), the shear modulus at a very low strain level. These tests are discussed in the following sections.

5.2.1 Geophysical Tests

This test is used for the in-situ determination of the maximum shear modulus, \( G_{\text{max}} \), at strain levels below or at \( 10^{-7} \). Damping ratio cannot be easily measured in-situ due to the fact that there is geometric damping besides the in-situ material damping. Shear wave velocity of a soil layer is measured in this test. The shear modulus of the soil layer, G, is then related to the shear wave velocity as follows:

\[ G = \rho \, v_s^2 \]  

(5.2)
where $\rho$ is the mass density and $v_s$ is the in-situ shear wave velocity measured in the field.

There are several methods for the measurement of the in-situ shear wave velocity such as: Refraction method, down-hole method, cross-hole method, and up-hole method (Richart, et al., 1970).

5.2.2 Cyclic Triaxial Tests

The cyclic triaxial test is the most popular laboratory dynamic test. This test can be used for determination of shear modulus and damping ratio at high strain levels ($\gamma$ larger than $10^{-4}$). After consolidation, isotropic or anisotropic, a cylindrical specimen is subjected to a series of repetitive axial compression and extension loads. The load can be either strain-controlled or stress-controlled. Although several different wave forms can be used, the most popular one is the sine wave with the frequency of half a Hertz (cycle per second). Vertical deformation is monitored during a test. Hysteresis loop of the tenth cycle is used in calculating the shear modulus $G$ and the damping ratio $D$. The Poisson's ratio can also be determined if the diametrical deformation is also monitored.

Cyclic triaxial tests can also be used for liquefaction potential or cyclic strength investigation. In conducting a liquefaction test, in addition to the deformation measurement, the excess pore pressure should also be monitored. The purpose of conducting cyclic triaxial tests for dynamic properties is to observe the changes in hysteresis loop; namely, the changes in shear modulus and damping ratio. However, the purpose of conducting a cyclic
test for dynamic strength is two-fold: (1) to determine the number of cycles required to achieve a certain percent strain (sands and clays) and (2) to determine the number of cycles required to reach initial liquefaction.

5.2.3 Cyclic Simple Shear Tests

The loading conditions and the resulting strain of cyclic simple shear tests are believed to closely simulate the stress and strain in a soil mass subjected to a strong motion earthquake (Ishibashi, et al., 1974). In 1968, Peacock and Seed introduced a box-type simple shear device which more closely duplicated the in-situ stress-strain conditions in the laboratory and was capable of applying cyclic loads to a soil sample. In 1971, Finn reported an improved simple shear instrument. In 1974, Ishibashi and Sherif developed a torsional simple shear device, which eliminated the effects of wall friction. All stress components during a test are always known.

The test can be either stress-controlled or strain-controlled. A sinusoidal stress wave of 0.5 Hz is frequently used. Shear deformation and stress are monitored during a test. The hysteresis loop of the tenth cycle is used in calculating the shear modulus $G$ and shear damping $D$. Similar to a cyclic triaxial test, a cyclic simple shear test can be used in determining both dynamic properties at strains larger than $10^{-4}$ and liquefaction potential.

5.2.4 Resonant Column Tests

According to Richart (1970) the resonant column test is the most convenient laboratory method for determination of dynamic
properties at a low strain level. There are various types of resonant column apparatus. The test is based on the principle of wave propagation in a soil column. In this test after consolidation, isotropic or anisotropic, a specimen is forced to vibrate either longitudinally and/or torsionally in one of its normal modes by the use of a vertical and/or torsional oscillator. The shear modulus is determined from the resonant frequency and the dimensions of a specimen. The damping ratio of a specimen can be determined from steady-state vibration and/or free vibration as discussed in Chapter 7.

Various types of resonant column apparatus give similar results. They differ mainly in the specimen boundary conditions. A torsional wave is applied either from the specimen top with its base fixed such as Hardin oscillator or from the specimen base with its top free to vibrate, such as Drnevich free-free resonant column apparatus. Specimen geometry (i.e., solid or hollow) and dimensions may also vary from one apparatus to the other. A group of researchers (Skoglund, et al., 1976) made the comparison of test results from six different types of resonant column test devices. Three of the devices had the capability of exciting a specimen in either the longitudinal mode or the torsional mode and were driven from their bases. The other three were for the torsional mode only, had fixed bases, and were driven from the specimen top cap. Solid samples were used. Their diameters ranged from 1.40 inches to 4.09 inches. The result of this investigation showed that no systematic or consistent differences could be associated with the different apparatus used.
Wave velocity in a soil column is measured at the top of a specimen with a velocity transducer. In some resonant column devices instead of the velocity transducer, an accelerometer is used to measure the acceleration at the top of a specimen. Almost all resonant column devices are designed to operate at small strain amplitudes (less than $10^{-5}$ or sometimes up to $10^{-4}$). Due to its low strain level, the test is commonly known as being nondestructive. However, studies of dynamic soil behavior at relatively high strain levels (greater than $10^{-4}$) were also conducted by Drnevich (1967) using a torsional resonant column apparatus developed at the University of Michigan. This device was also capable of monitoring the change in length of a soil specimen during a test.

Drnevich free-free resonant column apparatus, as described in Chapter 6, was used in this study. Torsional vibration is fed from the base. Two velocity transducers are located at the top of a specimen.

5.3 Factors Affecting the Dynamic Properties of Granular Soils

In general shear modulus $G$ and damping ratio $D$ are affected by many factors. Some of these factors have very strong effects on dynamic properties, some have minor or insignificant effects, and the effects of others are not known very well, mainly due to the fact that they have not been studied systematically. In the following sections the very important factors (primary factors) and less important factors (secondary factors) are discussed. This classification is based on the results of studies of previous
researchers. However, it is important to note that this is not a general classification, because some factors might have very strong effects on shear modulus while they are insignificant with respect to damping ratio or vice-versa. Also all investigators do not agree with the importance or insignificance of some factors.

5.3.1 Primary Factors

Almost all researchers agree that strain amplitude $\gamma$, void ratio $e$, relative density $D_r$, and effective mean principal stress $\sigma_0$ are the most important factors affecting the shear modulus $G$ of all soils. However, according to some researchers $\sigma_0$ does not have a strong effect on damping ratio $D$.

Shear modulus decreases very rapidly with shear strain $\gamma$. At very small strains ($\gamma < 10^{-7}$) shear modulus $G$ has a maximum value, $G_{\text{max}}$, and it will remain relatively constant until strain amplitudes reach about $10^{-5}$. At shear strains greater than $10^{-5}$ the rate of decrease of shear modulus with increasing strain depends primarily on the value of $G_{\text{max}}$ and on the shear strength of the soil. Shear modulus seems to reach a steady magnitude at strains of about $10^{-2}$ to $5 \times 10^{-2}$ depending on soil types (Richart, et al., 1970). Fig. 5.2 shows a typical variation of shear modulus with shear strain.

Damping ratio $D$ increases very rapidly with strain amplitude $\gamma$ and is equal to zero at zero strain. At a large strain damping ratio appears to approach a maximum value, $D_{\text{max}}$, asymptotically (Hardin, et al., 1972). Fig. 5.3 shows a typical variation of
FIG. 5.2 SHEAR MODULUS VERSUS SHEAR STRAIN
FIG. 5.3 DAMPING RATIO VERSUS SHEAR STRAIN

Shear Strain Amplitude, $\gamma$

Damping Ratio, $\xi$

$10^{-6}$ $10^{-5}$ $10^{-4}$

$10^{-7}$ $10^{-8}$ $10^{-9}$ $10^{-10}$

Damping Ratio $\xi$

Shear Strain Amplitude $\gamma$
damping ratio with shear strain.

Shear modulus decreases with increasing void ratio, \( e \), in undisturbed cohesive soils. Hardin, et al. (1969) suggested an empirical function for this effect as given by:

\[
F(e) = \frac{(2.973-e)^2}{(1+e)}
\]  

(5.3a)

where \( e \) is void ratio. Drnevich (1976) suggested the following continuous function \( F(e) \) for a quartz sand:

\[
F(e) = 8.839 \exp(-1.51e)
\]  

(5.3b)

Richart (1970) used equation (5.3a) for angular-grained materials and suggested another function for round-grained sands (with void ratios less than 0.80) as follows:

\[
F(e) = \frac{(2.17-e)^2}{(1+e)}
\]  

(5.3c)

According to Hardin and Drnevich (1972) and Edil and Luh (1978), void ratio is a very important factor affecting damping ratio. Damping ratio increases with increasing void ratio in undisturbed cohesive soils. However, according to Sherif, et al. (1977) and Seed, et al. (1970) void ratio has a very insignificant influence on damping ratio for void ratios in the range of 0.50 to 1.185.

Since relative density, \( D_r \), is a function of void ratio, \( e \) usually the effect of either \( D_r \) or \( e \) is considered. Relative density is known to have important influence on the densification and hysteresis of sands. When subjected to cyclic shear, sands exhibit dilatancy or densification and a highly nonlinear stress-strain relationship. The extent of these phenomena depends on
the relative density of sand, the amplitude of the shear strain, and the state of stress. Sands with a low relative density decrease in volume with the application of cyclic shear, whereas the opposite is true for sands with a high relative density. If the induced strains are very small, little relative movement will take place between individual sand grains and the resulting volume change will be small. In addition, the stress-strain response of sands will be largely linear and elastic with relatively little damping. However, at higher strain levels the stress-strain behavior becomes distinctly nonlinear and inelastic with significant hysteretic damping. As a consequence of grain rearrangements that occur during successive cycles of shear, shear modulus increases as the number of cycles increases and decreases as strain amplitude increases. A group of researchers (Cuellar, et al., 1977) conducted some strain-controlled cyclic simple shear tests on a crystal silica No. 20 sand and concluded that the densification of sands subjected to cyclic shear and the shape of the hysteresis loop are strongly dependent on shear strain, number of cycles, confining stress, and relative density over a broad range of these parameters. The densification rate increases with the shear strain amplitude and decreases with the number of cycles and the relative density.

Shear modulus is strongly affected by effective mean principal stress, $\bar{\sigma}_0$, and increases with increasing $\bar{\sigma}_0$. At very small strains shear modulus varies with the square root of $\bar{\sigma}_0$, but at large strains the modulus depends primarily on the strength of soil and varies linearly with $\bar{\sigma}_0$ (Hardin and Drnevich, 1972).
Damping ratio at a given strain decreases with $\sigma_o$. Hardin and Drnevich (1972) suggested that damping ratio decreases approximately with the square root of $\sigma_o$. Sherif and Ishibashi (1977) indicated that damping is significantly affected by the confining pressure and it decreases with increasing confining pressure.

5.3.2 Secondary Factors

Hardin and Drnevich (1972) presented a list of less important or secondary factors which includes: Number of cycles of loading $N_c$, overconsolidation ratio O.C.R., effective strength parameters $c$ and $\phi$, octahedral shear stress $\tau_0$, grain characteristics, time effect, frequency of loading $f$, soil structures, temperature, degree of saturation $S$, and stress history. The effects of some of these factors are discussed in this section.

A history of initial shear stress causes the modulus to be lower for the first few cycles of loading than it would be for the same soil without such a history of loading. However, Hardin and Drnevich (1972) conducted cyclic simple shear tests on clean sands and found that the effect of history of initial shear stress is removed by the tenth cycle, and the tenth and the 100th cycle data are similar. According to Wu (1975), for granular soils the effect of stress history is negligible on shear modulus. However, shear modulus is higher for overconsolidated soils than for normally consolidated soils.

Damping ratio is increased by the initial shear stress. The increase is more significant at a small strain. However, the deviatoric component, $\tau_0$, (octahedral shear stress) of the initial
state of stress is much less important to the modulus and damping in soils than the isotropic component, $\bar{\sigma}_0$, effective mean principal stress (Hardin and Drnevich, 1972).

Hardin and Black (1969) reported that the number of loading cycles, $N_c$, has very little effect on maximum shear modulus, $G_{\text{max}}$. However, for a sand, shear modulus increases slightly with the number of cycles and for cohesive soils the modulus decreases somewhat with the number of cycles (Hardin and Drnevich, 1972). This behavior is dependent on stress ratio and test conditions and might not be valid in all tests. Edil and Luh (1978) suggested that the number of cycles of loading is relatively unimportant with respect to the shear modulus. However, it exerts a significant influence on the damping ratio. Sherif and Ishibashi (1976) reported that shear modulus increases with an increasing number of shear cycles, $N_c$, with the increase equal to approximately 28% for $2 < N_c < 25$ and remains nearly constant thereafter. However, they could not draw any definite conclusions with regard to the effects of number of loading cycles on the damping ratio. Hardin and Drnevich (1972) found that the damping ratio for both cohesive and cohesionless soils decreases approximately with the logarithm of the number of loading cycles. However, there is evidence that damping begins to increase with $N_c > 50,000$, which is due to some fatigue mechanism (Hardin, et al., 1972). Silver and Seed (1971) have also reported the tendency of damping to decrease with increasing number of cycles. From the above information one can conclude that the number of cycles, $N_c$ is a secondary or less important factor for modulus and a primary or more important
factor with respect to the damping ratio.

The degree of saturation, $S$ has a negligible effect on the modulus and damping for non-plastic soils but a strong effect for plastic soils. According to Hardin and Drnevich (1972), the effect of degree of saturation on shear modulus in cohesionless soils is small, but the modulus of cohesive soils increases rapidly with decreasing degree of saturation. The effect on damping in cohesionless soils is small.

Hardin and Black (1969) reported that frequencies up to 300 Hz have very little effect on maximum shear modulus for dry cohesionless soils. For undisturbed cohesive soils, damping ratio $D$ only increases slightly with frequency within the range of 25 to 38 Hz. Although frequencies above 0.1 Hz have a relatively minor effect on $G$ and $D$ in cohesive soils, the behavior should be expected to change drastically for much lower frequencies where creep phenomena are involved (Hardin and Drnevich, 1972).

The effect of overconsolidation ratio, O.C.R. on maximum shear modulus, $G_{\text{max}}$ has been reviewed by many researchers. The effect of preconsolidation is to increase $G_{\text{max}}$ over its normally consolidated value, depending on the plasticity index, $PI$, of a soil. Overconsolidation ratio almost has no effect on $G_{\text{max}}$ for soils with $PI = 0$ (Hardin, et al., 1972). Hardin and Black (1969) and Seed and Idriss (1970) have developed relationships for $G_{\text{max}}$ considering the effect of O.C.R. These relationships are discussed in Section 5.4.

Beniwal (1977) reported that the modulus, $G$ of most
particulate materials (coarse silty soil to gravelly boulders) increases with time (at least, at low strain amplitude). This behavior may be due to progressive increase in junction growth with or without oscillations. Final results may be more bonding (adhesion) between the particles, and the specimen will be more rigid. In the case of fine grained (uncemented) soils, inter-particle bond strength increases more rapidly than that of coarse grained soils. This could be a reason for higher gain in shear modulus of fine grained soil. But in coarse grained soil, the initial value of shear modulus is much higher than that in fine grained soil.

5.3.3 Grain Characteristics and Gradation

Grain characteristics and grain size distribution of soils can be characterized by many parameters such as: Grain size and shape, mineral types, and indices such as the uniformity coefficient $C_u$, the effective grain size $D_{10}$, the percent finer than 74 microns, the mean grain size $D_{50}$, and the coefficient of gradation $C_g$. A number of researchers have studied the effects of some of the above parameters on dynamic properties and the results of their findings are presented herein.

Seed and Idriss (1970) reported that at small strain levels, shear moduli for gravelly soils are between 1.25 and 2.5 times greater than those for dense sands. Afifi and Richart (1973) showed that the increase in $G_{\text{max}}$ varied with grain size of the soil. Hardin and Drnevich (1972) explained that the grain characteristics will affect both the void ratio, a very important
parameter, and effective strength envelope, a less important parameter. But if the void ratio and the effective strength envelope of the soil are accounted for, then these two parameters will also account for most of the effects of grain characteristics, making it relatively unimportant in this context. Hardin (1973) (experimental data) found that shear modulus for coarse material (gravel size) is about 2.5 times of that for fine material (sand size). Krizek, et al. (1974) and Kuribayashi, et al. (1975) mentioned that for practical purposes, the particle size and distribution exert no influence on the wave propagation velocity (and hence the shear modulus). Based on the range of particle sizes and distribution used in their studies, results were inconclusive (Beniwal, 1977). Sherif, et al. (1977) observed that the damping is influenced by the coefficient of gradation, \( C_g \) (defined in Section 5.4). Iwasaki and Tatsuoka (1977) conducted a series of resonant column tests on clean sands (with \( C_u < 1.8 \) and no grains finer than 0.074 mm in diameter), natural sands which are well-graded and contain fine soils, and artificially graded sands which are not clean. The results of their tests show that, in general, shear moduli of sands decrease with the increase in uniformity coefficient, \( C_u \) and also decrease with the increase in the content of fine soils for the same values of mean principal stress, void ratio, and shear strain amplitude. They concluded that shear moduli of normally consolidated disturbed sands are greatly affected by grain size distribution characteristics. However, the effects of grain size distribution on damping properties of the sands tested are not so great as the effects on
shear moduli. Beniwal (1977) reported that the maximum shear modulus of a granular soil increases with increasing $D_5$ (grain diameter at 5% finer), whereas gradation, particle shape and particle size range do not show any effect on the shear modulus at strain amplitude, below $10^{-6}$. However at higher strain amplitude the value of shear modulus is greatly influenced by the gradation of soils. Particle shape and particle orientation seems to have some influence on shear modulus at higher strain amplitudes. The mineral to mineral friction between grains also has some (minor) influence on the shear modulus of a granular soil. He used a resonant column device for his tests and most of his tests were conducted on gravelly soils. Yet other researchers (Edil and Luh, 1978) suggested that "reference shear modulus," $G_o$ (value of G at $0.25 \times 10^{-4}$ shear strain amplitude) decreases with increasing mean roundness or decreasing angularity and is virtually independent of mean grain size within the range of $0.1 < D_{50} < 0.7$ mm and uniformity coefficient within the range of $1.2 < C_u < 3.4$. The sands used in their tests consist, mineralogically, of 80% or more quartz grains. Their final conclusion was that the effect of mean grain size, $D_{50}$ and uniformity coefficient, $C_u$ on dynamic modulus and damping is relatively unimportant.

5.4 Functional Relationships for Shear Modulus and Damping Ratio

Many researchers have studied the dynamic properties of soils, and in the past twenty years numerous relationships have been developed for shear modulus and damping ratio. In this
section some of these relationships are discussed.

Hardin and Richart (1963) suggested an empirical equation for maximum shear modulus which can be written as:

$$ G_{\text{max}} = A F(e) \frac{\sigma}{\sigma_0}^n $$  \hspace{1cm} (5.4)

where $G_{\text{max}}$ is the shear modulus at low strains less than $10^{-7}$, $F(e)$ is the void ratio effect and was explained in Section 5.3.1 (equations 5.3a, b, c), $\sigma_0$ is the effective mean stress and $n$ varies from 0.5 to 0.6. The value of $A$ for crushed quartz sand is 1230 (psi)$^{1/2}$, $\sigma_0$ and $G_{\text{max}}$ are both expressed in psi. For round-grained sands ($e < 0.80$), $n = 0.5$, $A = 2630$ (psi)$^{1/2}$, and $F(e)$ is from equation (5.3c).

Hardin and Black (1969) used the equation (5.4) for cohesive soils and modified it for stress history by including a factor for O.C.R. in the form of,

$$ G_{\text{max}} = A F(e)(O.C.R.)^k \frac{\sigma}{\sigma_0}^n $$  \hspace{1cm} (5.5)

where $k$ varies from 0 to 1/2 depending on the plasticity index and O.C.R. is the overconsolidation ratio.

Hardin and Drnevich (1970, 1972) presented a comprehensive study of nonlinear response and defined modulus and damping in terms of hyperbolic strain, $\gamma_h$. The modulus $G$, at a strain level $\gamma$ is given by:

$$ G = \frac{G_{\text{max}}}{(1 + \gamma_h)} $$  \hspace{1cm} (5.6)

where $G_{\text{max}}$ is the maximum shear modulus and $\gamma_h$ is defined by:

$$ \gamma_h = \gamma \left[ 1 + a \exp \left( -b \left( \frac{\gamma}{\gamma_r} \right)^c \right) \right] $$  \hspace{1cm} (5.7)
where $\gamma_r = \tau_{\text{max}}/G_{\text{max}}$ and $\gamma$ is the shear strain amplitude

$$\tau_{\text{max}} = \left[ \frac{(1 + K_0)}{2} \bar{\sigma}_v \left( \sin \phi + \bar{c} \cos \phi \right)^2 - \left( \frac{(1 - K_0)}{2} \bar{\sigma}_v \right)^2 \right]^{1/2}$$

(5.8)

where $K_0$ = coefficient of lateral earth pressure at rest, $\bar{\sigma}_v$ = vertical effective stress, $\bar{c}$ and $\bar{\phi}$ are strength parameters, and the values of $a$, $b$, and $c$ depend on loading types and soil types. For clean sands in the case of complete stress reversal, $a = -0.5$, $b = 0.16$ and $c = 1.0$.

The damping ratio $D$, at a strain level $\gamma$ is given by:

$$D = \frac{\gamma_h D_{\text{max}}}{1 + \gamma_h}$$

(5.9)

where $\gamma_h$ is the hyperbolic strain defined as above and

$$D_{\text{max}} = 33 - 1.5 \log N_c$$

(5.10)

for clean dry sands ($N_c$ is the number of cycles), and in the case of complete stress reversal, $a = 0.6(N_c^{1/6}) - 1$, $b = 1 - N_c^{1/12}$ and $c = 1.0$.

Beniwal (1977) suggested a modified and generalized form of the following empirical expression for shear modulus of granular soils that includes the effect of grain size distribution:

$$G = \frac{G_{\text{max}}}{1 + \frac{\gamma}{\gamma_r} \left[ 1 + (1.75 \pm 0.5) C_u^{0.35} \exp \left( -\frac{Q_{\text{A}}}{\gamma_r} \right) \right]}$$

(5.11)

where $\gamma_r = \tau_{\text{max}}/G_{\text{max}}$, $\gamma$ is the shear strain amplitude, $\tau_{\text{max}}$ is defined by equation (5.8) and $C_u$ is the uniformity coefficient. (+) sign is for feldspar (clean) gravels and (-) sign is for natural gravels. As it was stated earlier most of the tests in his study were conducted on gravelly soils.
Sherif, et al. (1976, 1977) proposed relationships for modulus and damping for dry sands. In their studies the equivalent shear modulus, $G_{eq}$ is defined as the slope of the line that connects the two ends of a nonsymmetric hysteresis loop. $G_{eq}$ at a strain level $\gamma$ is given by:

$$G_{eq} = [2.8 \phi (\sigma_c)^{0.85}]^{-0.6} \gamma \quad \text{for } \gamma > 0.0003 \quad (5.12a)$$

$$G_{eq} = [2.8 \phi (\sigma_c)^{11.67} \gamma + 0.5]^{40 (0.205)} \gamma/0.03 \quad \text{for } \gamma < 0.0003$$

(5.12b)

where $\sigma_c$ is the effective confining pressure and $\phi$ is the angle of internal friction. Equations (5.12a) and (5.12b) are valid for the second cycle only, in order to obtain $G_{eq}$ for other cycles, one should use the correlation factors given by:

$$F_n(N_c) = 0.253 \log N_c + 0.924 \quad \text{for } 2 < N_c < 25 \quad (5.13a)$$

$$F_n(N_c) = 1.28 \quad \text{for } 25 < N_c \quad (5.13b)$$

where $N_c$ is the number of cycles of loading and $F_n(N_c)$ is the correlation factor, when multiplied by the $G_{eq}$ values for the second cycle (Eq. 5.12a and 5.12b), will yield the corresponding $G_{eq}$ value for a specific number of cycles.

They also proposed a similar equation for damping ratio, $D$ as:

$$D = \frac{50 - 0.6 \sigma_c (73.3F - 53.3) \gamma^{0.3}}{38} \quad R(N_c) \quad (5.14)$$

where $\sigma_c$ is the effective confining pressure (psi), $F$ is the soil gradation and sphericity factor given by:

$$F = \frac{1}{\psi^2 C_g} \quad (5.15)$$
\[ \psi = \text{soil sphericity and } C_g = \text{coefficient of gradation} = \frac{D_{30}^2}{D_{10} \times D_{60}}, \]
\[ D_{10}, D_{30}, \text{ and } D_{60} \text{ denote soil grain diameters corresponding to 10, 30 and 60 percent finer by weight on the gradation curve, } \gamma \text{ is the shear strain in the range of } 5 \times 10^{-5} \text{ to } 10^{-2} \text{ and } R(N_c) \text{ is the correlation factor given by:} \]
\[ R(N_c) = 1.01 - 0.046 \log N_c \] (5.16)

where \( N_c \) is the number of cycles of loading. Equation (5.16) is based on the results reported by Hardin, et al. (1972).

Iwasaki and Tatsuoka (1977) from Japan suggested an empirical expression for shear moduli \( G \) of various clean sands given by:

\[ G = A(\gamma) \cdot B \cdot \frac{(2.17 - e)^2}{(1+e)} \cdot p^m(\gamma) \] (5.17)

where \( A(\gamma) \) is a function of shear strain \( \gamma \) and \( A \) (at \( \gamma = 10^{-5} \)) = 900, \( A \) (at \( \gamma = 10^{-5} \)) = 850 and \( A \) (at \( \gamma = 10^{-4} \)) = 700; \( m(\gamma) \) is also a function of \( \gamma \): \( m \) (at \( \gamma = 10^{-5} \)) = 0.4, \( m \) (at \( \gamma = 10^{-5} \)) = 0.44 and \( m \) (at \( \gamma = 10^{-4} \)) = 0.5, \( B \) is a parameter dependent on the variety of a normally consolidated sand and for uniform clean sands without fine soils is equal to 1.0, \( e \) is void ratio and \( p \) is mean principal stress.

Edil and Luh (1978) proposed relationships for shear modulus and damping ratio of sands for a certain range of strains given by:

\[ G = G_o \left( 1.004 - 345.4 \gamma \right) \] (5.18)

where \( G_o \) is the reference shear modulus (\( G \) at \( 0.25 \times 10^{-4} \) strain) and is given by:
\[ G_0 = [0.769 + \ln \left( \frac{\overline{\sigma}}{98.07} + 1 \right) (2.793 - 2.195 e) - 0.801 R] \times 10^{-5} \]  

(5.19)

\( G, \overline{G}_0, \overline{\sigma}_0 \) (effective mean principal stress) all in \((\text{KN/m}^2)\), \( R \) is mean roundness and \( e \) is void ratio.

The relationship for damping \( \nu \) is expressed as:

\[ \nu = \nu_0 \left( 1.131 - 0.0453 \log N_c \right) \]  

(5.20)

where \( \nu_0 \) is reference damping ratio \((\nu \text{ at } N_c = 1000 \text{ cycles})\) and is given by:

\[ \nu_0 = 0.88 + 6592 \gamma e^{1/2} - 0.28 \gamma^{1/3} \left( \frac{\overline{\sigma}_0}{98.07} \right)^3 - 73.55 \gamma \frac{\overline{\sigma}_0}{98.07}^{1/2} \]  

(5.21)

\( D \) and \( \nu_0 \) are in \%, \( \overline{\sigma}_0 \) in \((\text{KN/m}^2)\), \( N_c \) is the number of cycles and \( e \) is void ratio.

All equations suggested by Edil and Luh are only valid for strains less than \( 2 \times 10^{-3} \) and the sands used contain 80\% or more of quartz grains.
CHAPTER 6

TEST EQUIPMENTS

6.1 Drnevich Free-Free Torsional Resonant Column Apparatus

In using a torsional resonant column device to measure dynamic properties of soils, a torsional excitation is applied to one end of a specimen. The other end of the specimen can either be free or fixed.

Usually a sinusoidal wave is used to excite a specimen. However, theoretically any wave forms are acceptable. During a test the wave frequency is continuously varied, usually decreased, in an attempt to obtain a resonant frequency. The resonance is evidenced by a straight line Lissajous figure. A shear modulus can then be determined from the corresponding resonant frequency.

A damping ratio can be determined from the steady state vibration or free vibration as shown in Step 15 of the section on data reduction in Chapter 7. One of these methods utilizes the curve of the logarithmic decay of torsional vibration amplitude. A typical logarithmic decay curve is shown in Fig. 7.12.

The schematic diagram of the free-free torsional resonant column device is shown in Fig. 6.1. The excitation torsional...
FIG. 6.1 FREE-FREE TORSIONAL RESONANT COLUMN SCHEMATIC
wave is applied from the lower end of a specimen and the other end is free to vibrate. The excitation is applied by means of two electric coils and magnets as shown in Fig. 6.2. The device is capable of testing 2.8-, 4- and 6-inch diameter specimens. Either vacuum or confining pressure can be used to supply effective stresses to a specimen. Vacuum can be applied from a quick connector which leads to the inside of a specimen. The connector is located on the base of the apparatus. Confining pressure can be applied from the top of the pressure chamber. Specimens can be either dry or saturated. In testing saturated specimens pore pressure can be measured by means of a transducer located at the base of the apparatus. Confining medium can be either air or water. Water can be filled into the chamber from a quick connector located at the base.

Two velocity transducers (active) located under the base of the chamber are used to determine the resonant frequency of the apparatus which includes the base, an active (or bottom) platen, 8 rods and nuts, chamber and top lid. During a test the resonant frequency can be determined for the system which includes the apparatus including a passive (or top) platen and a specimen. Hence, there are two resonant frequencies, one for the "apparatus" \(f_{0T}\) and the other for the "system" \(f_T\). Procedures for determining these frequencies are outlined in Chapter 7.

6.2 Pressure and Vacuum System

The pressure and vacuum system which provides an access
to vacuum, chamber pressure and back pressure is shown in
Fig. 6.3. It consists of one vacuum gauge and regulator
connected to a vacuum pump; two pressure gauges and regulators
(one pair each for confining and back pressures respectively)
connected to a house pressure line. A burette for monitoring
the volume change of a saturated specimen during a test is also
provided. A pair of quick connecters and shut-off valve is
provided for each vacuum and pressure line.

6.3 Electronic Monitoring System

The electronic devices used in the test and the wiring
diagram are shown in Figs. 6.4 and 6.5 respectively. The
electronics include a function generator, a frequency counter,
a power amplifier, a digital voltmeter and two storage oscillo-
scopes.

The sine wave generated by the function generator is
amplified by the power amplifier and then fed into the driving
coils which are located on the resonant column device. The
frequency of the input wave can be adjusted by a frequency dial
and read by a digital counter. The input wave induces vibra-
tion in the coils and the resulting torsional wave is fed into
a soil sample. The wave travels upward in the sample and the
output is picked up by the two velocity transducers at the top
of the specimen (passive transducers). The voltage of the
input and output waves is read through the digital voltmeter.
If the switch is on the "Power" position the voltmeter reads
the input voltage (torque) and if it is on the "Passive"
FIG. 6.3 PRESSURE AND VACUUM SYSTEM
FIG. 6.4 ELECTRONIC MONITORING DEVICES
FIG. 6.5 WIRING DIAGRAM
position the voltmeter reads the output (passive) voltage. The Lissajous figure can be observed on the oscilloscope. The "Active" transducers located at the base of the chamber are for measuring the resonant frequency of the apparatus without a sample. The procedures for this measurement are described in Chapter 7.

6.4 Additional Supporting Equipment

To improve the quality of artificially prepared specimens, three additional devices were designed and manufactured. A "zero-raining" device, shown in Fig. 6.6, was designed to improve the repeatability and uniformity of artificially prepared specimens. It is a hollow cylinder with a screen attached to its end. Procedures for its use are outlined in Chapter 7. End screens with different openings can be used. The selection of screens is guided by the particle size of a specimen.

A leveling device shown in Fig. 6.7 was designed to level the top of a specimen in a mold. The device has a plate with two holes for mounting the device on two supporting rods, a circular rod with a leveling blade and a handle attached to its ends. To level a specimen, set the blade on its top and rotate the handle. Detailed procedures are given in Chapter 7.

Finally a temporary supporting device shown in Fig. 6.8 is used to support the passive (or top) platen. After a specimen is prepared and leveled, the passive platen is placed on its top. This heavy platen needs to be supported by a fixed temporary support to prevent sample disturbance. Therefore, the
FIG. 6.6 ZERO HEIGHT RAINING DEVICE
FIG. 6.7 LEVELING DEVICE
FIG. 6.8 TEMPORARY SUPPORTING DEVICE
temporary supporting device is used to support the platen during the placement of the membrane and O-ring. The support can be removed after the membrane and O-ring are in place as described in Chapter 7. Fig. 6.9 shows a complete resonant column test facility.

6.5 Improvements to the Testing Equipment

In order to achieve better accuracy and conduct more consistent tests, several modifications were adopted. After performing numerous tests it was found that the flexibility of the floor and the friction between the base of the equipment and the floor have important effects on the test results. Therefore, the apparatus was bolted to a 10 inch concrete floor by means of four anchor bolts and a concrete pad (2 inches thick) was constructed around the base of the apparatus in order to prevent any movements (Fig. 6.10). It was also observed that the tightness of the nuts at the top of the chamber influences the test results. Hence, the round aluminum nuts were replaced with hexagon steel nuts and each nut was tightened to 600 (lb-in) with a torque wrench. Eight more nuts were attached to the bottom of the rods to prevent their rotation.

Originally the apparatus was equipped with two boxes called the "Control" box and the "Switch" box, which were used to connect the equipment to the electronic devices. These boxes were replaced by a panel that accommodated all of the components of the boxes and was mounted on an instrument rack (Fig. 6.4). Therefore, the number of long wires were reduced and, as a
FIG. 6.9 COMPLETE RESONANT COLUMN TEST SYSTEM
result, the noise level in the circuit was lowered.

In order to provide faster and easier pressure and vacuum connections all of the valves and fittings were replaced by quick connecters.
CHAPTER 7

SPECIMEN PREPARATION, TEST PROCEDURES AND DATA REDUCTION

7.1 Material and Sample Preparation

The primary material used in this study was a well-graded river sand obtained locally in Denver, Colorado. Figs. 2.1 and 2.2 show the grain size and shape of the Denver sand, and one of the samples, respectively. Some of the samples were prepared by adding gravels to the above sand. Gravel sizes ranged from 3/8 inch to No. 10 sieve. After the sand was oven-dried, it was sieved by sieves ranging from No. 10 to No. 400. Six different mean grain sizes \(D_{50}\) were chosen as shown on the grain size distribution chart (Fig. 2.4). At each mean diameter grain size distribution curves with uniformity coefficients \(C_u\) ranging from 2 to 15 were plotted as shown in Fig. 2.4. Then a sample was mixed according to each grain size distribution curve. A total of twenty-four samples were obtained in this manner. Each sample was labeled according to its mean grain size and uniformity coefficient. Samples DC-a2, DC-a4, DC-a6, and DC-a8 have the same mean grain size, \(a\) \(D_{50} = 1.68\) mm with uniformity coefficients of 2, 4, 6, and 8 respectively. Symbol DC stands for Denver Coarse sand. Similarly samples DC-b4, DC-b6, DC-b8, DC-b10, and DC-b15 have the same mean grain size \(b\) \(D_{50} = 0.84\) mm and different uniformity coefficients. The other samples
were prepared in the same manner with mean grain sizes: c
\(D_{50} = 0.59\) mm, d \(D_{50} = 0.42\) mm, e \(D_{50} = 0.25\) mm and f
\(D_{50} = 0.149\) mm and different uniformity coefficients. Symbols
DM and DF represent Denver Medium and Fine sands respectively.

In addition to the above samples, Monterey No. 0 sand was
also used in some tests. It is a uniform \(C_u = 1.5\), medium
grain size beach sand from California, composed mostly of quartz
and some feldspar. The grains are subrounded and somewhat
elongated (Fig. 2.3). Fig. 2.5 shows the grain size distribution
of Monterey No. 0 sand.

Minimum and maximum densities of each sample were measured
prior to testing. The procedures in the Earth Manual were used
to obtain these densities. The procedures are given in the
Appendix E-12 on "Relative Density of Cohesionless Soils" in the
Resources Service (formerly the Bureau of Reclamation). The
values of minimum and maximum densities and other material proper-
ties are given in Table 7.1.

7.2 Specimen Preparation

The preparation of a specimen is the most important and
critical part of the resonant column test. Extreme caution must
be exercised in order to prepare a desired specimen without any
grain segregation. Detailed procedures for preparing specimens
of granular soils are as follows:

1. Measure the inside diameter and height of the specimen
   mold.


<table>
<thead>
<tr>
<th>Sample*</th>
<th>( \gamma_{\text{max}} ) lb/(ft)(^3)</th>
<th>( \gamma_{\text{min}} ) lb/(ft)(^3)</th>
<th>( D_r ) (%)</th>
<th>( e )</th>
<th>( G_s )</th>
<th>( C_u )</th>
<th>( D_{50} ) (mm)</th>
<th>( D_{10} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-a(_2)</td>
<td>104.89</td>
<td>87.03</td>
<td>29.9</td>
<td>0.811</td>
<td>2.66</td>
<td>2</td>
<td>1.68</td>
<td>0.97</td>
</tr>
<tr>
<td>DC-a(_4)</td>
<td>114.80</td>
<td>94.46</td>
<td>29.5</td>
<td>0.667</td>
<td>2.66</td>
<td>4</td>
<td>1.68</td>
<td>0.55</td>
</tr>
<tr>
<td>DC-a(_6)</td>
<td>121.24</td>
<td>98.91</td>
<td>29.8</td>
<td>0.587</td>
<td>2.66</td>
<td>6</td>
<td>1.68</td>
<td>0.40</td>
</tr>
<tr>
<td>DC-a(_8)</td>
<td>125.03</td>
<td>101.88</td>
<td>27.9</td>
<td>0.549</td>
<td>2.66</td>
<td>8</td>
<td>1.68</td>
<td>0.32</td>
</tr>
<tr>
<td>DC-b(_4)</td>
<td>116.00</td>
<td>95.45</td>
<td>27.4</td>
<td>0.656</td>
<td>2.66</td>
<td>4</td>
<td>0.84</td>
<td>0.28</td>
</tr>
<tr>
<td>DC-b(_6)</td>
<td>121.33</td>
<td>99.41</td>
<td>29.5</td>
<td>0.582</td>
<td>2.66</td>
<td>6</td>
<td>0.84</td>
<td>0.20</td>
</tr>
<tr>
<td>DC-b(_8)</td>
<td>123.96</td>
<td>102.38</td>
<td>29.8</td>
<td>0.537</td>
<td>2.66</td>
<td>8</td>
<td>0.84</td>
<td>0.16</td>
</tr>
<tr>
<td>DC-b(_{10})</td>
<td>126.23</td>
<td>103.86</td>
<td>29.9</td>
<td>0.514</td>
<td>2.66</td>
<td>10</td>
<td>0.84</td>
<td>0.135</td>
</tr>
<tr>
<td>DC-b(_{15})</td>
<td>129.58</td>
<td>106.83</td>
<td>29.2</td>
<td>0.475</td>
<td>2.66</td>
<td>15</td>
<td>0.84</td>
<td>0.10</td>
</tr>
<tr>
<td>DM-c(_2)</td>
<td>105.74</td>
<td>87.03</td>
<td>29.5</td>
<td>0.808</td>
<td>2.66</td>
<td>2</td>
<td>0.59</td>
<td>0.34</td>
</tr>
<tr>
<td>DM-c(_4)</td>
<td>118.36</td>
<td>93.96</td>
<td>28.8</td>
<td>0.663</td>
<td>2.66</td>
<td>4</td>
<td>0.59</td>
<td>0.195</td>
</tr>
<tr>
<td>DM-c(_8)</td>
<td>125.64</td>
<td>101.39</td>
<td>28.8</td>
<td>0.547</td>
<td>2.66</td>
<td>8</td>
<td>0.59</td>
<td>0.11</td>
</tr>
<tr>
<td>DM-c(_{10})</td>
<td>127.41</td>
<td>103.47</td>
<td>30.9</td>
<td>0.511</td>
<td>2.66</td>
<td>10</td>
<td>0.59</td>
<td>0.09</td>
</tr>
<tr>
<td>DM-c(_{15})</td>
<td>129.27</td>
<td>105.35</td>
<td>30.0</td>
<td>0.489</td>
<td>2.66</td>
<td>15</td>
<td>0.59</td>
<td>0.07</td>
</tr>
</tbody>
</table>

* DC = Denver Coarse Sand; DM = Denver Medium Sand; DF = Denver Fine Sand; M = Monterey No. 0 Sand
### TABLE 7.1 (Continued)

SAMPLE SPECIFICATIONS AND INDEX PROPERTIES

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\gamma_{\text{max}}$ lb/(ft)$^3$</th>
<th>$\gamma_{\text{min}}$ lb/(ft)$^3$</th>
<th>$D_r$ (%)</th>
<th>e</th>
<th>$G_s$</th>
<th>$C_u$</th>
<th>$D_{50}$ (mm)</th>
<th>$D_{10}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM-d$_2$</td>
<td>106.08</td>
<td>85.05</td>
<td>31.0</td>
<td>0.832</td>
<td>2.66</td>
<td>2</td>
<td>0.42</td>
<td>0.24</td>
</tr>
<tr>
<td>DM-d$_4$</td>
<td>115.75</td>
<td>91.98</td>
<td>30.5</td>
<td>0.692</td>
<td>2.66</td>
<td>4</td>
<td>0.42</td>
<td>0.14</td>
</tr>
<tr>
<td>DM-d$_6$</td>
<td>121.78</td>
<td>96.93</td>
<td>28.6</td>
<td>0.613</td>
<td>2.66</td>
<td>6</td>
<td>0.42</td>
<td>0.10</td>
</tr>
<tr>
<td>DM-d$_8$</td>
<td>123.35</td>
<td>98.91</td>
<td>31.2</td>
<td>0.575</td>
<td>2.66</td>
<td>8</td>
<td>0.42</td>
<td>0.08</td>
</tr>
<tr>
<td>DM-d$_{10}$</td>
<td>125.94</td>
<td>101.88</td>
<td>30.5</td>
<td>0.535</td>
<td>2.66</td>
<td>10</td>
<td>0.42</td>
<td>0.07</td>
</tr>
<tr>
<td>DM-e$_2$</td>
<td>106.38</td>
<td>82.67</td>
<td>34.3</td>
<td>0.854</td>
<td>2.66</td>
<td>2</td>
<td>0.25</td>
<td>0.145</td>
</tr>
<tr>
<td>DM-e$_4$</td>
<td>116.92</td>
<td>90.495</td>
<td>26.7</td>
<td>0.725</td>
<td>2.66</td>
<td>4</td>
<td>0.25</td>
<td>0.08</td>
</tr>
<tr>
<td>DM-e$_6$</td>
<td>119.67</td>
<td>94.55</td>
<td>30.5</td>
<td>0.644</td>
<td>2.66</td>
<td>6</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>DF-f$_2$</td>
<td>102.95</td>
<td>79.60</td>
<td>29.6</td>
<td>0.946</td>
<td>2.66</td>
<td>2</td>
<td>0.149</td>
<td>0.09</td>
</tr>
<tr>
<td>DF-f$_3$</td>
<td>108.86</td>
<td>84.06</td>
<td>30.2</td>
<td>0.839</td>
<td>2.66</td>
<td>3</td>
<td>0.149</td>
<td>0.06</td>
</tr>
<tr>
<td>M-1-1</td>
<td>105.92</td>
<td>91.96</td>
<td>11.8</td>
<td>0.774</td>
<td>2.655</td>
<td>1.5</td>
<td>0.40</td>
<td>0.27</td>
</tr>
<tr>
<td>M-2-1</td>
<td>105.92</td>
<td>91.96</td>
<td>45.1</td>
<td>0.696</td>
<td>2.655</td>
<td>1.5</td>
<td>0.40</td>
<td>0.27</td>
</tr>
<tr>
<td>M-3-1</td>
<td>105.92</td>
<td>91.96</td>
<td>73.0</td>
<td>0.629</td>
<td>2.655</td>
<td>1.5</td>
<td>0.40</td>
<td>0.27</td>
</tr>
</tbody>
</table>

* DC = Denver Coarse Sand; DM = Denver Medium Sand; DF = Denver Fine Sand; M = Monterey No. O Sand
2. Measure the height of the lower platen. The difference between the height of the mold and platen is termed "effective height."

3. Measure the thickness of the membrane. The difference between the inside diameter of the mold and two times the thickness of the membrane is "effective diameter."

4. Calculate the "effective volume" of the mold. This volume indicates the maximum volume of soil that can be placed in the mold.

\[ V_e = \frac{\pi(d_e)^2}{4} \times (H_e - h) \]

where
- \( V_e \) = effective volume,
- \( d_e \) = effective diameter,
- \( H_e \) = effective height, and
- \( h \) = the distance between the top of the specimen and the top of the mold.

The distance \( h \) can be adjusted to obtain a specimen of a specific height or density. For example:

\( H_e - h = 12 \) inches for a 6-inch diameter specimen of aspect ratio 2.

5. To prepare a specimen of a desired density, the required weight of soil can be calculated as follows:

\[ W_{\text{req}} = V_e \times \gamma_d \]

where
- \( W_{\text{req}} \) = required weight of the sample,
- \( V_e \) = effective volume, and
- \( \gamma_d \) = desired dry density of the sample.
Usually a specimen is prepared to achieve a certain relative density \(D_r\). Hence, one must convert this relative density to a corresponding dry density \(\gamma_d\) and then determine the required weight of the soil. All the specimens were prepared to obtain a relative density of 30%.

6. Weigh the required amount of soil.

7. Apply a thin layer of vacuum grease to the side of the lower platen.

8. Place the membrane over the lower platen and seal it on the lower platen by an O-ring. The top surface of the platen has sharp edges. Extreme caution must be exercised so that the membrane will not be torn.

9. Place the mold over the lower platen. Pull the membrane through the mold and wrap it around the upper edge of the mold.

10. Apply vacuum to the mold so that the membrane sticks to the inner wall of the mold without any wrinkles. Seal the membrane on the top of the mold with an O-ring.

11. Place a small piece of filter paper over the center hole of the lower platen.

12. Place the "zero-raining" device in the mold. Be careful not to tear the membrane.

13. Pour the desired amount of soil in the "zero-raining" device, mix it thoroughly and then raise the device so that the soil can be uniformly deposited in the mold. Occasionally, it is necessary to tap the mold by a
plastic-head hammer while raising the device in order to achieve the desired relative density. Therefore, several trials should be performed to obtain the desired density.

14. Attach four of the locking rods to the apparatus for supporting the leveling system. Do not vibrate the sample.

15. Attach the leveling system to two of the rods and lower the lever arm so that the blade of the lever arm just touches the soil in the mold. Rotate the lever arm until the top of the specimen is completely leveled. Remove the leveling system.

16. Thinly grease the top platen and fix it in its temporary support by means of four screws.

17. Lower the temporary support with the four supporting rods slowly sliding inside its four holes until the top platen sits on top of the specimen and then carefully remove the temporary support.

18. Use a level to make sure that the top platen is properly leveled.

19. Lower the O-ring on the mold top to release the membrane, place the membrane over the top platen and seal it with the O-ring.

20. Apply a vacuum of 10 inches of mercury to the specimen and wait for 10 to 15 minutes.

21. Carefully remove the mold by removing the screws on the mold.
22. Measure the diameter of the specimen by using a "Pi-tape" as shown in Fig. 7.1. When wrapped around the specimen, "Pi-tape" directly reads the specimen diameter. Also measure the height of the specimen.

23. Back calculate the density of the specimen by using the measured values of diameter, height and weight to make sure that it is the same as the desired density.

24. Screw the nuts and washers on the bottom of the rods and place the rods on the apparatus. Tighten the nuts in order to prevent the rods from rotating and touching the bottom plate.

25. Place the plastic chamber over the specimen.

26. Connect the lead from the passive transducers to its receptacle on the lid of the apparatus. Place the lid on the apparatus and carefully hand tighten it onto the apparatus by eight nuts and washers. Then tighten each nut with a torque-wrench up to 600 lb-in as shown in Fig. 7.2.

27. Connect the pressure line to the top of the apparatus and gradually apply a confining pressure of 3 to 4 (psi) and lower the vacuum by the same magnitude.

28. Repeat the procedures until zero vacuum. Adjust the confining pressure to the desired magnitude.

29. For dry sands, the specimen will consolidate within the first 15 to 30 minutes. However, all the specimens were consolidated for one hour prior to testing and the pore pressure line was left open during consolidation and
FIG. 7.1 MEASURING THE DIAMETER OF THE SPECIMEN WITH A PI-TAPE

FIG. 7.2 TIGHTING THE NUTS WITH A TORQUE-WRENCH
7.3 Testing Procedures

Before preparing the specimen and testing, the apparatus resonant frequency should be determined in the following manner:

1. Apparatus in normal position, i.e., lower platen attached, chamber, rods, lid, and nuts in place and tightly fastened.

2. Wiring should be like the wiring diagram except that the lead from the control panel which is marked "voltmeter" must be connected to the Y-axis of the oscilloscope instead of the voltmeter. Switch B must be in the "Active" position.

3. Apply a small amount of power to the coils by gradually turning up the "Volume" knob on the amplifier.

4. Turn switch A on the control panel to "On" position.

5. Adjust the frequency until the Lissajous figure closes to a straight sloping line. The lowest frequency at which the straight line is observed is the resonant frequency of the apparatus \( f_{OT} \).

After the specimen is consolidated, testing can begin according to the following procedures:

1. Connect all the electrical connections according to the wiring diagram shown in Fig. 6.5.

2. Turn switch A on the control panel to the "Standby" position and switch B to the "Power" position.

3. Select a desirable wave form. Sine wave was used in
this study.

4. Set frequency slightly above the apparatus resonant frequency ($f_{oT}$).

5. Apply a small amount of power so that the voltmeter reads about 5 mV rms. Record this power (torque) reading.

6. Turn switch A on the control panel to "On" position and adjust the frequency until the shape of Lissajous figure is close to a straight sloping line. The frequency is the system resonant frequency ($f_T$). Record the frequency.

7. Turn switch B on the control panel to "Passive" position and record the passive transducer reading with the voltmeter.

8. Turn switch A on the control panel to "Off" position and switch B to "Power" position.

9. Turn switch A on the control panel to "Standby" position and increase the power by 50% and record this power (torque) reading.

10. Repeat the steps 5 to 9 until the maximum allowable power (50 watts) is reached.

Readings are taken at equally timed intervals (2 minutes).

7.4 Data Reduction

The procedures given in this section are for the manual calculation of shear modulus, shear strain amplitude and damping ratio. These parameters are calculated by using the wave propagation theory and the schematic shown in Fig. 6.1 (Drnevich, 1978).
The computer program RESCOL 7 (Drnevich and Lorenz, May 1980) was used for the calculation of shear modulus, shear strain amplitude and damping ratio.

Following are the procedures:

(Subscript T denotes torsional vibration.)

1. Record the specimen weight measured before a test.

2. The volume of the specimen should be the same as the "effective volume." Calculate the volume of the specimen from its measured diameter and height. The calculated volume should be close to the "effective volume."

3. Calculate density ($\gamma_d$):

$$\gamma_d = \frac{W}{V}$$

where $W$ = weight of the specimen (lb. or gr.),
$V$ = volume of the specimen (ft$^3$ or cm$^3$), and
$\gamma_d$ = density of the specimen (lb/ft$^3$ or g/cm$^3$).

$\gamma_d$ should be the same as the "desired" density.

4. Calculate mass density ($\rho$):

$$\rho = \frac{\gamma_d}{g}$$

where $\gamma_d$ = density (g/cm$^3$),
$g$ = 980.66 (cm/sec$^2$) + gravitational acceleration, and
$\rho$ = mass density (g-sec$^2$/cm$^4$).

5. Calculate specimen rotational inertia (J):

$$J = \frac{Wd^2}{8g}$$

where $W$ = weight (g),
d = diameter (cm),
g = 980.66 (cm/sec^2), and
J = specimen rotational inertia (g-cm-sec^2).

6. Calculate passive end inertia ratio (P_T):

\[ P_T = \frac{J_p}{J} \]

where \( J_p = J_{\text{top platen}} + J_{\text{top cap}} = 363.2 \text{ (g-cm-sec}^2\text{)} \)
for 6" dia. platens,
J = specimen rotational inertia (g-cm-sec^2), and
P_T = passive end inertia ratio.

7. Apparatus damping constant (ADC_T):

\[ ADC_T = 2 f_{\text{oT}} J_A \delta_T \]

where \( f_{\text{oT}} = \) apparatus resonant frequency (Hz),
\( J_A = \) rotational inertia of the active platen (g-cm-sec^2),
\( \delta_T = \) logarithmic decrement, and
ADC_T = damping constant (g-cm-sec^2-Hz).

8. Apparatus damping factor (ADF_T):

\[ ADF_T = \frac{ADC_T}{2\pi f_T J} \]

where ADC_T = apparatus damping constant (g-cm-sec^2-Hz),
\( f_T = \) system resonant frequency (Hz),
J = specimen rotational inertia (g-cm-sec^2), and
ADF_T = damping factor.
ADF_T should be calculated for every value of system resonant frequency.

9. Active end inertia factor (T):
$$T = \frac{J_A}{J} \left[ 1 - \left( \frac{f_{0T}}{f_T} \right)^2 \right]$$

where $J_A = \text{active end platen system rotational inertia (g-cm-sec}^2\text{)},$

$J = \text{rotational inertia of the specimen (g-cm-sec}^2\text{),}$

$f_{0T} = \text{apparatus resonant frequency (Hz),}$

$f_T = \text{system resonant frequency (Hz), and}$

$T = \text{active end inertia factor.}$

10. Dimensionless frequency factor ($F$):

$F$ is a function of system factors $T$, $P_T$, and $ADF$ and of specimen damping ratio, $D$.

For cases where $ADF$ is zero (or very small) and specimen damping ratio is less than 10%, values of $F$ can be obtained from Figs. 7.3 and 7.4.

For cases where $ADF$ and specimen damping ratio ($D$) are small and where $T$ and $P_T$ are both greater than 10, dimensionless frequency factor ($F$) may be approximated by:

$$F = \sqrt{\frac{1}{T} + \frac{1}{P_T}}$$

where $T = \text{active end inertia factor, and}$

$P_T = \text{passive end inertia ratio.}$

This equation is also useful for checking results obtained from Figs. 7.3 and 7.4. $F$ should be determined for every corresponding pair of values of $T$ and $P_T$. But in most cases values of $T$ for different resonant
FIG. 7.3 DIMENSIONLESS FREQUENCY FACTOR
(AFTER DRNEVICH)
FIG. 7.4 DIMENSIONLESS FREQUENCY FACTOR
(AFTER DRNEVICH)
frequencies are close to each other and it is sufficient to use one value of $F$.

11. Shear modulus ($G$):

$$G = \rho \left(2\pi L\right)^2 \left[\frac{f_T}{F}\right]^2$$

where $\rho = \text{mass density (g-sec}^2/\text{cm}^4)$,
$L = \text{length of specimen (cm)}$,
$f_T = \text{system resonant frequency (Hz) or (cycle/sec)}$,
$F = \text{dimensionless frequency factor}$, and
$G = \text{shear modulus (g/cm}^2)$.

$G$ should be calculated for each value of system resonant frequency ($f_T$).

12. Shear strain amplitude ($\gamma$):

$$\gamma = (\text{RCF})_p (\text{RTO})_p (\text{SF}) \left[\frac{d}{3L}\right] \times 100\%$$

where \((\text{RCF})_p = \text{rotation calibration factor for passive transducer} \frac{\text{PK.rad}}{\text{mv rms}}\),
\((\text{RTO})_p = \text{rotational passive transducer output (mv rms)},\)
$\text{SF} = \text{strain factor}$,
$d = \text{diameter of specimen (cm)}$,
$L = \text{length of specimen (cm)}$, and
$\gamma = \text{shear strain amplitude (})$.

\begin{align*}
(\text{RCF})_p &= \frac{8.42 \times 10^{-5}}{f_T} \frac{\text{PK.rad}}{\text{mv rms}} \\
\text{where} \quad f_T &= \text{system resonant frequency (Hz)}. 
\end{align*}
(RCF)_p should be calculated for every f_T. (RTO)_p is recorded on the data sheet. SF is a function of T and P_T and can be determined from Fig. 7.5.

13. Magnification factor (MMF_T):

\[ \text{MMF}_T = \frac{(RCF)_p(RTO)_p}{(TCF)(CR_T)}J(2\pi f_T)^2 \]

where

\[ (RCF)_p = \frac{8.42 \times 10^{-5}}{f_T} \text{PK.rad/mv}_{\text{rms}} \]

- rotation calibration factor for passive transducer,

\[ (RTO)_p = \text{rotational passive transducer output} \]

- (mv_rms) recorded during the test,

\[ (TCF) = \text{Torque/current constant} \frac{(g \cdot cm)(\text{PK.rad}^3)}{\text{mv}_{\text{rms}}} \]

\[ (CR_T) = \text{Current reading to torsional excitation system = torque reading (mv_rms) which is voltage drop across a fixed 5\Omega resistor,} \]

\[ J = \text{specimen rotational inertia (g \cdot cm \cdot sec}^2)\text{, and} \]

\[ f_T = \text{system resonant frequency (cycle/sec).} \]

14. Amplification coefficient (A):

This coefficient may be obtained from Fig. 7.6

For cases where ADF is close to zero and D is less than 10%. For other values of ADF, the value of A is only an approximation.

For cases where values of both T and P_T are greater than 10, the following equation may be used to approximate A.

\[ A = 2(T + P_T) \]
FIG. 7.5 STRAIN FACTOR FOR RESONANCE DETERMINED BY PHASE MEASUREMENT BETWEEN INPUT FORCE AND MOTION AT THE PASSIVE END (AFTER DRHEVICH)
FIG. 7.6 COEFFICIENT FOR MANUAL CALCULATION OF DAMPING RATIO FOR RESONANCE DETERMINED BY PHASE MEASUREMENT BETWEEN INPUT FORCE AND MOTION AT THE PASSIVE END (AFTER ORNEVICH)
15. Damping ratio (D):

A. Steady State Vibration:

\[ D = \frac{100}{[A(MMF)]} \]

where \( A = \) amplification coefficient, \( MMF = \) magnification factor, and \( D = \) specimen damping ratio (%).

B. Free Vibration:

This procedure is theoretically exact for apparatus where the passive end can be assumed to be rigidly fixed. For other cases this method is approximate. The passive transducer must be used to obtain the amplitude decay curve (Fig. 7.7). Values of \( T \) and \( P \) should both be greater than 10 when amplitude decay is used.

(1) System energy ratio (S)

\[ S = \frac{J_A}{J} \left[ \frac{f_{oT} F}{f_T} \right]^2 \]

where \( J_A = \) active end platen system rotational inertia \((g\cdotcm\cdotsec^2)\),

\( J = \) specimen rotational inertia \((g\cdotcm\cdotsec^2)\),

\( f_{oT} = \) apparatus resonant frequency \((Hz)\),

\( F = \) dimensionless frequency factor,

\( f_T = \) system resonant frequency \((Hz)\), and

\( S = \) system energy ratio.

S should be calculated for each value of \( f_T \).
(2) System logarithmic decrement from the free vibration ($\delta_s$):

$$\delta_s = \frac{1}{n} \log \frac{A_1}{A_{n+1}}$$

where $A_1 = \text{amplitude of vibration for first cycle after power is cut off}$,
$A_{n+1} = \text{amplitude of vibration for } (n+1)\text{th cycle of free vibration}$,
$n = \text{number of free vibration cycles which must be 10 or less}$, and
$\delta_s = \text{system logarithmic decrement}$.

(3) Damping ratio (D):

$$D = \left[ \delta_s (1+S) - S\delta_T \right] \times \frac{100\%}{(2\pi)}$$

where $\delta_s = \text{system logarithmic decrement}$,
$S = \text{system energy ratio}$,
$\delta_T = \text{apparatus logarithmic decrement}$,
and
$D = \text{damping ratio (})$. 

8.1 Testing Program

The Drnevich free-free torsional resonant column device was used to determine shear moduli (G) and damping ratio (D) of Denver sand and Monterey No. 0 sand from California.

Twenty-four Denver sand specimens with different grain size distribution characteristics were tested under 30 (psi) confining pressure. All the specimens were prepared to have an initial relative density of 30%. In addition to the above specimens, specimen DM-d_2 was also tested at the Water and Power Resources Service. The apparatus used in testing specimen DM-d_2 was also a Drnevich free-free resonant column device. However, it was not the same model as the one used for testing the other twenty-four specimens. The purpose of this additional test was to compare the test results from the two different models of free-free torsional resonant column devices.

Three Monterey No. 0 sand specimens with different relative densities were also tested. Two of the specimens were tested under 30 (psi) confining pressure and the third one was tested under 15, 30 and 45 (psi) confining pressures.

The specifications and index properties for all the specimens
are given in Table 7.1. It should be noted that the values of relative density \((D_r)\) and void ratio \((e)\) given in Table 7.1 are the initial values (prior to consolidation). After applying confining pressure and during the test, the relative density of the specimens changes by around 2%.

8.2 Test Results

Test results for Monterey No. 0 sand are shown in Figs. 8.1 to 8.5. Fig. 8.1 shows the shear modulus \((G)\) versus shear strain \((\gamma)\) curves for three specimens with three different relative densities \((D_r = 12, 45, 73\%)\). All specimens were tested under 30 (psi) confining pressure. Their damping ratio \((\zeta)\) versus shear strain \((\gamma)\) relationships are shown in Fig. 8.2. Figs. 8.3 and 8.4 show the effect of confining pressure on shear modulus and damping ratio, respectively. In this test specimen M-3 was subjected to three different confining pressures. First, it was consolidated under 15 (psi) and tested up to strains of about \(10^{-5}\) (M-3-1), then the test was stopped and confining pressure was raised to 30 (psi). After consolidation at 30 (psi), the specimen was tested again and strain amplitude was kept below \(10^{-5}\) (M-3-2). Finally, the specimen was tested again under 45 (psi) and this time the strain amplitudes were as high as \(6 \times 10^{-5}\) (M-3-3). The reason for testing the specimen at strain levels below \(10^{-5}\) was to prevent it from becoming disturbed so that it may be tested at different confining pressures. Fig. 8.5 illustrates the effect of confining pressure on maximum shear modulus for the specimen M-3.
FIG. 8.1 SHEAR MODULUS VERSUS SHEAR STRAIN
(MONTEREY NO. O SAND)

- H-3-2, Dr = 79%
- M-2-1, Dr = 45%
- M-1-1, Dr = 12%
- q = 30 psi

Shear Modulus, G (MPa)
Shear Strain Amplitude, \( \gamma \)
FIG. 8.2 DAMPING RATIO VERSUS SHEAR STRAIN
(MONTEREY NO. 0 SAND)
Fig. 8.3 Shear modulus versus shear strain
(Monterey No. 0 sand)
FIG. 8.4 DAMPING RATIO VERSUS SHEAR STRAIN
(MONTEREY NO. 0 SAND)
FIG. 8.5 THE EFFECT OF CONFINING PRESSURE ON MAXIMUM SHEAR MODULUS (MONTEREY NO. 0 SAND)
The results of the tests on Monterey No. 0 sand indicated that the shear modulus (G) increases with increasing relative density (or decreasing void ratio) and also increases with confining pressure. The damping ratio (D) demonstrated a trend similar to shear modulus. However, damping ratio should have decreased instead with increasing relative density (D_r) and increasing confining pressure (σ_3).

Shear modulus versus shear strain curves and damping ratio versus shear strain curves of Denver sand specimens are presented in Figs. 8.6 to 8.17. All of the specimens were consolidated at 30 (psi) confining pressure and their initial relative densities were about 30%. Figs. 8.6 and 8.7 illustrate the shear modulus (G) and the damping ratio (D) for specimens DC-a_2, DC-a_4, DC-a_6 and DC-a_8. These specimens have the same mean grain size (a_50 = 1.68 mm). Shear modulus and damping ratio curves for specimens DC-b_4, DC-b_6, DC-b_8, DC-b_10 and DC-b_15 with the same median grain size are shown in Figs. 8.8 and 8.9, respectively. Similarly, Figs. 8.10 to 8.17 show the shear modulus and the damping ratio curves for the rest of the specimens.

The results indicated that, in general, the shear modulus and the damping ratio of specimens increases with increasing uniformity coefficient (C_u) for the same median grain size (a_50) and same relative density and confining pressure.

8.3 Discussion

The results of tests on Monterey No. 0 sand show the effects of relative density, void ratio and confining pressure on shear
FIG. 8.6  SHEAR MODULUS VERSUS SHEAR STRAIN  
(DEMVER SAND; DC-a)
FIG. 8.7 DAMPING RATIO VERSUS SHEAR STRAIN
(DEMVER SAND, DC-a)
FIG. 8.8 SHEAR MODULUS VERSUS SHEAR STRAIN
(DENVER SAND, DC-b)
FIG. 8-9 DAMPING RATIO VERSUS SHEAR STRAIN (DENVER SAND, DC-b)
FIG. 8.10 SHEAR MODULUS VERSUS SHEAR STRAIN
(DENVER SAND, DM-c)

$\sigma_3 = \text{psi}$
FIG. 8.11 DAMPING RATIO VERSUS SHEAR STRAIN (DENVER SAND, DM-c)
FIG. 8.12 SHEAR MODULUS VERSUS SHEAR STRAIN
(DENVER SAND, DM-d)

Shear Strain Amplitude, $\gamma$

Shear Modulus, $G$ (MPa)

Symbols:
+ DM-$d_{10}$
○ DM-$d_{8}$
○ DM-$d_{6}$
□ DM-$d_{4}$
△ DM-$d_{2}$

$\sigma_3 = 30$ psi
FIG. 8.13 DAMPING RATIO VERSUS SHEAR STRAIN
(DENVER SAND, DM-d)

Shear Strain Amplitude, γ

Damping Ratio, D (%)
Fig. 8.14 Shear Modulus versus Shear Strain (Denver Sand, DM-e)

Shear Strain Amplitude, $\gamma$

Shear Modulus, $G$ (MPa)

$\sigma_3 = 30$ psi

- $\square$ DM-e$_6$
- $\triangle$ DM-e$_4$
- $\odot$ DM-e$_2$
FIG. 8.15 DAMPING RATIO VERSUS SHEAR STRAIN
(DENVER SAND, DM-e)

Damping Ratio, D (%)

Shear Strain Amplitude, $\gamma$

$\sigma_3 = 30$ psi
Shear Modulus, G (MPa)

Shear Strain Amplitude, \( \gamma \)

FIG. 8.16 SHEAR MODULUS VERSUS SHEAR STRAIN (DENVER SAND, DF-f)

\( \sigma = 30 \text{ psi} \)
FIG. 8.17 DAMPING RATIO VERSUS SHEAR STRAIN
(DENVER SAND, DF-f)

Shear Strain Amplitude, $\gamma$

Damping Ratio, $D$ (%)
modulus (G) and damping ratio (D). The effects of these factors on G are in general agreement with previous studies. However, the behavior of damping ratio shown in Figs. 8.2 and 8.4 is contrary to the expected trend.

Test results from Denver sand indicated that shear modulus of the specimens is affected by the uniformity coefficient ($C_u$) and increases with increasing $C_u$ for the same mean grain size ($D_{50}$) and the same relative density ($D_r$) and confining pressure ($\sigma_3$). However, the results also showed that damping ratio (D) is affected in the same manner. This behavior is contrary to the expected result. In general, the values of damping ratio for all of the tests were larger than expected values. In particular, the initial values of damping ratio (at strain levels below $10^{-6}$) for some of the specimens were very large (3.5% for DF-f$_3$ at $\gamma = 4 \times 10^{-7}$). In general, the specimens with larger uniformity coefficients ($C_u$) and small mean grain sizes ($D_{50}$) had larger initial damping ratios. A careful inspection of the apparatus showed that during testing some of the vibration which is generated by the driving coils is transferred to the base of the apparatus and the surrounding area. Therefore, all of the vibration generated by the coils is not applied to the specimen. The vibration at the base also causes some rocking motion and the specimen will no longer be under pure torsion. These effects reduce the passive transducers output and hence the values for damping ratio will be large. For finer and more well-graded specimens the vibration at the base was larger and therefore
the values for damping ratio were great.

Figs. 8.18 and 8.19 show a comparison between the test results of the two models of Drnevich free-free resonant column devices. As shown in Fig. 8.18 the values of shear modulus obtained from the device at the Water and Power Resources Service (WPRS) are larger than the ones obtained from the device used in this study (UCD). The difference between the values of the maximum shear moduli is about 11%. Fig. 8.19 shows that the values of damping ratio obtained from the device at WPRS are about one-tenth of the damping ratios obtained from the UCD device at strains below $10^{-5}$. At higher strain amplitudes the difference is even more. The damping ratios obtained from the WPRS device are very small and remain nearly the same for the whole range of strain amplitudes.

After reviewing all the results from all the specimens, it was decided to perform a statistical analysis and obtain a functional relationship for the shear modulus ($G$). This analysis and corresponding relationship are presented in Chapter 9. Because of the unusually large values of damping ratio and the unusual trend of the increase of damping with confining pressure and relative density, the damping ratios in this study were considered incorrect and it was decided not to develop a functional relationship for damping ratio.
FIG. 8.18 SHEAR MODULUS VERSUS SHEAR STRAIN
(DM-d2)

Shear Modulus, G (MPa)

Shear Strain Amplitude, \( \gamma \)

- DM-d2, \( D_r = 31\% \), \( e = 0.833 \) (WPRS)
- DM-d2, \( D_r = 31\% \), \( e = 0.832 \) (UCD)

\( \sigma_3 = 30 \) psi
FIG. 9.19  DAMPING RATIO VERSUS SHEAR STRAIN

Shear Strain Amplitude, y

Damping Ratio, D (%)
CHAPTER 9

EVALUATION OF FUNCTIONAL RELATIONSHIP BETWEEN SHEAR MODULUS, SHEAR STRAIN AND GRAIN-SIZE DISTRIBUTION CHARACTERISTICS

9.1 Data Selection

In conducting the statistical analyses, the results from all of the tests on Denver sand were used. The test results indicated that for each soil the value of shear modulus, \( G \) is approximately constant at strains smaller than \( 10^{-5} \). These values were designated as \( G_{\text{max}} \), the maximum shear modulus. For each test four points at strains greater than \( 10^{-5} \) were chosen from the shear modulus \( G \) versus shear strain \( \gamma \) curve to formulate the functional relationship between \( G \) and \( \gamma \). Therefore, the result of each test was characterized by one value of \( G_{\text{max}} \) for strains less than \( 10^{-5} \) and four values of \( G \) for strains greater than \( 10^{-5} \). This gives 96 pairs (24 tests x 4 points for each test) of \( G \) versus \( \gamma \) and 24 values of \( G_{\text{max}} \).

In order to study the effects of gradation characteristics on shear modulus, the data were divided into six sets. Each data set contained the specimens with the same mean grain size, \( D_{50} \). Table 9.1 summarizes these six data sets.

9.2 Statistical Analyses

9.2.1 Introduction

Statistical analyses, described in Chapter 3, were performed
TABLE 9.1:

DATA SETS SELECTED FOR STATISTICAL ANALYSES

<table>
<thead>
<tr>
<th>Data Set No.</th>
<th>Specimens in the Set</th>
<th>$D_{50}$ (mm)</th>
<th>No. of Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DC-$a_2$, 4', 6', 8'</td>
<td>1.68</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>DC-$b_4$, 6', 8', 10', 15</td>
<td>0.84</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>DM-$c_2$, 4', 8', 10', 15</td>
<td>0.59</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>DM-$d_2$, 4', 6', 8', 10</td>
<td>0.42</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>DM-$e_2$, 4', 6</td>
<td>0.25</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>DF-$f_2$, 3</td>
<td>0.149</td>
<td>2</td>
</tr>
</tbody>
</table>
to determine the correlation coefficient and formulate the empirical relationship between the shear modulus of the twenty-four samples tested and strain or grain size distribution characteristics. These statistical analyses were performed by using the statistical programs from the SPSS, Statistical Package for the Social Sciences (Nie, et al., 1975).

9.2.2 Scatter Diagrams

Scatter diagrams were plotted for different dependent and independent variables to assist the observation of correlation and to investigate the dispersion of data points. Figs. 9.1 to 9.5 are examples of scatter diagrams.

Fig. 9.1 shows that maximum shear modulus $G_{\text{max}}$ and void ratio $e$ are strongly related and a linear relationship can be found between $G_{\text{max}}$ and $e$ or $(\log e)$ or $(\log e)^2$ or a combination of these terms. Fig. 9.2 shows that $G_{\text{max}}$ is also strongly related to uniformity coefficient $C_u$ and again a linear relationship can be found between $G_{\text{max}}$ and different functions of $C_u$. Fig. 9.3 indicates that $G_{\text{max}}$ and mean grain size $D_{50}$ are not strongly related and there is a wide dispersion. Fig. 9.4 shows that, for specimens with the same $D_{50}$, strong correlation exists between $G_{\text{max}}$ and effective grain size $D_{10}$. However, the correlation between $G_{\text{max}}$ and $D_{10}$ for all data of various $D_{50}$ is not strong as indicated by the low value of the correlation coefficient $R$ in the next section. Fig. 9.5 shows the scatter of data with respect to relative density $D_r$. As stated in the earlier
FIG. 9.2 THE EFFECT OF UNIFORMITY COEFFICIENT ON MAXIMUM SHEAR MODULUS

FIG. 9.3 THE EFFECT OF MEAN GRAIN SIZE ON MAXIMUM SHEAR MODULUS

Uniformity Coefficient, \( C_u \)

Median Grain Size, \( D_{50} \) (mm)

\( \sigma_3 = 30 \) psi

\( \sigma_3 = 30 \) psi
FIG. 9.4 THE EFFECT OF EFFECTIVE GRAIN SIZE ON MAXIMUM SHEAR MODULUS
FIG. 9.5 MAXIMUM SHEAR MODULUS VERSUS RELATIVE DENSITY
chapters the aim was to prepare the specimens with the same relative density ($D_r = 30\%$). However, it is difficult to prepare identical specimens. Therefore, some of the specimens did not have the relative density of 30\% and Fig. 9.5 shows the range of densities. Since the relative densities of most of the specimens are close to each other and the void ratio of these specimens is already considered as one of the variables, it was decided to exclude $D_r$ as a variable.

9.2.3 **Statistical Correlation**

After a careful study of the test results, it was decided to perform the statistical analyses in two parts. In Part (1) the objective was to obtain a relationship between normalized shear modulus $G/G_{\text{max}}$ and the shear strain amplitude $\gamma$. In Part (2) the strength of correlation between $G_{\text{max}}$ and various independent variables were evaluated. Bivariate correlation coefficients, $R$, between various dependent and independent variables were obtained by using the Pearson Correlation subprogram (Nie, et al., 1975).

**Part (1):** $G/G_{\text{max}}$ versus $\gamma$

In this part the correlation coefficients, $R$, between shear modulus, $G$, maximum shear modulus, $G_{\text{max}}$, normalized shear modulus, $G/G_{\text{max}}$ and shear strain, $\gamma$, were obtained for the whole data population. Their correlation coefficients are shown in Table 9.2. It becomes obvious that $G/G_{\text{max}}$ and shear strain, $\gamma$ have a very strong correlation. Therefore, bivariate and multiple regression analyses were performed to obtain a functional relation
### TABLE 9.2

**CORRELATION COEFFICIENT BETWEEN SHEAR MODULUS AND SHEAR STRAIN IN PART (1)**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Correlation Coefficient R</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\text{max}}$, $\gamma$</td>
<td>-0.276</td>
</tr>
<tr>
<td>$G$, $\gamma$</td>
<td>-0.657</td>
</tr>
<tr>
<td>$G/G_{\text{max}}$, $\gamma$</td>
<td>-0.949</td>
</tr>
<tr>
<td>$G/G_{\text{max}}$, ($\log \gamma$)</td>
<td>-0.891</td>
</tr>
<tr>
<td>$G/G_{\text{max}}$, ($\log \gamma)^2$</td>
<td>0.861</td>
</tr>
</tbody>
</table>
between $G/G_{\text{max}}$ and $\gamma$ (Section 9.2.4).

Part (2): $G_{\text{max}}$ versus $C_u$, $e$, $D_{10}$, and $D_{50}$

Table 9.3 shows all correlation coefficients obtained in Part (2). In obtaining these coefficients, all of the data points were included for each analysis. The correlation coefficients indicate that $G_{\text{max}}$ is strongly related to uniformity coefficient, $C_u$, and void ratio, $e$. Also $e$ and $C_u$ show strong correlation.

9.2.4 Regression Analyses

Regression analyses were also performed in two parts. In Part (1) bivariate and multiple regressions were conducted to obtain a relationship between normalized shear modulus, $G/G_{\text{max}}$ and shear strain, $\gamma$. In Part (2) an empirical relationship was obtained for $G_{\text{max}}$ through the multiple regression analysis. In each part correlation coefficient, $R$, and coefficient of determination, $R^2$, between different variables were obtained and the variables which showed strong correlation ($R$ greater than $+0.7$) were considered in the formulation of functional relationships.

Part (1): $G/G_{\text{max}}$ versus $\gamma$

In order to obtain a functional relationship for $G/G_{\text{max}}$, first bivariate coefficients were obtained between $G/G_{\text{max}}$ and shear strain, $\gamma$, and then multiple correlation coefficients were obtained between $G/G_{\text{max}}$ and $\gamma$ and log $\gamma$, by including all data points. Table 9.4 presents the result of these analyses. Since the two coefficients are nearly identical, it was decided to relate $G/G_{\text{max}}$ to $\gamma$ as follows:
### TABLE 9.3

**CORRELATION COEFFICIENT BETWEEN VARIABLES IN PART (2)**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Correlation Coefficient R</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\text{max}} \cdot C_u$</td>
<td>0.947</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot (C_u)^2$</td>
<td>0.859</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot \log C_u$</td>
<td>0.963</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot (\log C_u)^2$</td>
<td>0.965</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot e$</td>
<td>-0.922</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot \log e$</td>
<td>-0.943</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot (\log e)^2$</td>
<td>0.962</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot D_{50}$</td>
<td>0.104</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot (D_{50})^2$</td>
<td>-0.020</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot \log D_{50}$</td>
<td>0.290</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot (\log D_{50})^2$</td>
<td>-0.487</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot D_{10}$</td>
<td>-0.388</td>
</tr>
<tr>
<td>$G_{\text{max}} \cdot \log D_{10}$</td>
<td>-0.389</td>
</tr>
</tbody>
</table>
TABLE 9.3 (Continued)

CORRELATION COEFFICIENT BETWEEN VARIABLES IN PART (2)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Correlation Coefficient R</th>
</tr>
</thead>
<tbody>
<tr>
<td>e, Cu</td>
<td>-0.873</td>
</tr>
<tr>
<td>e, log Cu</td>
<td>-0.958</td>
</tr>
<tr>
<td>log e, log Cu</td>
<td>0.972</td>
</tr>
<tr>
<td>e, log D_{10}</td>
<td>0.203</td>
</tr>
<tr>
<td>e, (log D_{50})^2</td>
<td>0.670</td>
</tr>
<tr>
<td>Cu, log D_{10}</td>
<td>-0.419</td>
</tr>
<tr>
<td>Cu, (log D_{50})^2</td>
<td>-0.427</td>
</tr>
</tbody>
</table>
where $a$ and $b$ are the regression coefficients and $G$ and $G_{\text{max}}$ are in MPa.

The next step was to obtain a relationship similar to equation (9.3) for each of the data sets in Table 9.1. Therefore, regression was performed for each set and Table 9.5 gives the resulting regression coefficients.

Since the correlation coefficients for all data sets are very strong, it was decided to perform one regression analysis for all data. The following equation was obtained:

$$G_{\text{G max}} = -1417 \gamma + 0.996$$

(9.4)

where $G$ and $G_{\text{max}}$ are in MPa. The correlation coefficient, $R$, for this equation is 0.949 and the coefficient of determination, $R^2$, is 0.901. Hence, the equation represents the sound functional relationship between $G_{\text{G max}}$ and $\gamma$. The results of the regression analysis for the entire data set are also shown in Table 9.5.

In order for equation (9.4) to satisfy the condition of $G_{G_{\text{max}}} = 1$ at $\gamma = 0$ the constant term in the equation was changed to 1 instead of 0.996. Therefore, the following equation was
TABLE 9.5

REGRESSION COEFFICIENTS BETWEEN $G/G_{\text{max}}$ AND $\gamma$ FOR EACH DATA SET

<table>
<thead>
<tr>
<th>Data Set No.</th>
<th>No. of Data Points in Set*</th>
<th>a</th>
<th>b</th>
<th>R</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>-1450</td>
<td>0.986</td>
<td>0.949</td>
<td>0.901</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>-1764</td>
<td>0.996</td>
<td>0.983</td>
<td>0.966</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>-1338</td>
<td>0.989</td>
<td>0.975</td>
<td>0.951</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>-1390</td>
<td>0.997</td>
<td>0.986</td>
<td>0.972</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>-1303</td>
<td>1.006</td>
<td>0.999</td>
<td>0.998</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>-963</td>
<td>1.006</td>
<td>0.999</td>
<td>0.998</td>
</tr>
<tr>
<td>All Data</td>
<td>96</td>
<td>-1417</td>
<td>0.996</td>
<td>0.949</td>
<td>0.901</td>
</tr>
</tbody>
</table>

where a and b are regression coefficients,

R is the correlation coefficient, and

$R^2$ is the coefficient of determination.

* These numbers all apply to similar tables to be presented later.
chosen to represent the relationship between \( G/G_{\text{max}} \) and \( \gamma \) for all data.

\[
G/G_{\text{max}} = -1417 \gamma + 1.00, \quad R = 0.949 \quad (9.5)
\]

It should be noted that equation (9.5) was obtained from the data with strains less than \( 2 \times 10^{-4} \).

Part (2): \( G_{\text{max}} \) versus \( C_u \), \( e \), \( D_{10} \) and \( D_{50} \)

In this part an empirical relationship was obtained for \( G_{\text{max}} \) in terms of the other variables, void ratio, \( e \), effective grain size, \( D_{10} \), uniformity coefficient, \( C_u \) and mean grain size, \( D_{50} \). As in Part (1), the multiple correlation coefficient, \( R \) and the coefficient of multiple determination, \( R^2 \), were obtained. Table 9.6 gives these coefficients for all data.

After studying the coefficients, for simplicity, it was decided to use the following regression equation:

\[
G_{\text{max}} = a \log C_u + b \quad (9.6)
\]

where \( G_{\text{max}} \) is in MPa, \( C_u \) is the uniformity coefficient and \( a \) and \( b \) are the regression coefficients. Although a combination of functions of \( C_u \) like \( \log C_u \) and \( (\log C_u)^2 \) gives a slightly larger correlation coefficient, but for all practical purposes the simpler form (Eq. 9.6) is adequate.

After selecting the type of the equation for \( G_{\text{max}} \), regression was performed for the six data sets as shown in Table 9.1. Table 9.7 presents the regression coefficients for each set.

Another regression analysis was performed to obtain a relationship between \( G_{\text{max}} \) and \( C_u \) by including all data as follows:

\[
G_{\text{max}} = 54.71 \log C_u + 80.46, \quad R = 0.963 \quad (9.7)
\]
TABLE 9.6

CORRELATION COEFFICIENTS BETWEEN $G_{\text{max}}$ AND OTHER VARIABLES

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Independent Variable</th>
<th>R</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\text{max}}$</td>
<td>$\log C_u$</td>
<td>0.963</td>
<td>0.927</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>$(\log C_u)^2$</td>
<td>0.965</td>
<td>0.931</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>$\log C_u, (\log C_u)^2$</td>
<td>0.968</td>
<td>0.938</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>$\log e$</td>
<td>-0.943</td>
<td>0.889</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>$(\log e)^2$</td>
<td>0.962</td>
<td>0.925</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>$\log e, (\log e)^2$</td>
<td>0.962</td>
<td>0.925</td>
</tr>
<tr>
<td>e</td>
<td>$\log C_u$</td>
<td>0.958</td>
<td>0.918</td>
</tr>
<tr>
<td>$\log e$</td>
<td>$\log C_u$</td>
<td>0.972</td>
<td>0.944</td>
</tr>
</tbody>
</table>
### TABLE 9.7

**REGRESSION COEFFICIENTS FOR \( G_{\text{max}} \)**
**FOR EACH DATA SET**

<table>
<thead>
<tr>
<th>Data Set No.</th>
<th>a</th>
<th>b</th>
<th>R</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43.1</td>
<td>87.0</td>
<td>0.945</td>
<td>0.893</td>
</tr>
<tr>
<td>2</td>
<td>63.3</td>
<td>73.3</td>
<td>0.924</td>
<td>0.854</td>
</tr>
<tr>
<td>3</td>
<td>60.6</td>
<td>76.9</td>
<td>0.999</td>
<td>0.998</td>
</tr>
<tr>
<td>4</td>
<td>52.8</td>
<td>82.6</td>
<td>0.995</td>
<td>0.990</td>
</tr>
<tr>
<td>5</td>
<td>21.5</td>
<td>97.5</td>
<td>0.985</td>
<td>0.970</td>
</tr>
<tr>
<td>6</td>
<td>21.5</td>
<td>91.9</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>All Data</td>
<td>54.7</td>
<td>80.5</td>
<td>0.963</td>
<td>0.927</td>
</tr>
</tbody>
</table>
where \( G_{\text{max}} \) is in MPa. \( R = 0.963 \) and \( R^2 = 0.927 \) for this equation.

In order to study the effects of void ratio, \( e \), effective grain size, \( D_{10} \) and mean grain size, \( D_{50} \) on \( G_{\text{max}} \), the constants \( a's \) and \( b's \) shown in Table 9.7 were correlated to the above variables. Table 9.8 presents the bivariate and multiple correlation coefficients.

After reviewing the correlation coefficients, it was decided to formulate constant \( a \) as a function of \((\log D_{50})^2\), \( \log D_{10} \) and \( \log e \). Equation (9.8) was obtained through the multiple regression analysis:

\[
a = -103.83 (\log D_{50})^2 - 24.83 (\log D_{10}) + 60.51 (\log e) + 55.79
\]

where \( D_{50} \) and \( D_{10} \) are in mm. \( R = 0.898 \) and \( R^2 = 0.806 \) for this equation.

The next step was to formulate the constant \( b \) as a function of the constant \( a \). Therefore, regression analyses were performed between \( b \) and \( a \) by using the values of these constants in Table 9.7. Table 9.9 summarizes the correlation coefficients that were obtained.

After reviewing the correlation coefficients, equation (9.9) was chosen for constant \( b \):

\[
b = -0.006 a^2 + 98.26
\]

\( R = 0.984 \) and \( R^2 = 0.968 \) for this equation.

9.3 Summary of Results

Empirical relationships for \( G/G_{\text{max}} \) and \( G_{\text{max}} \) were obtained
TABLE 9.8

CORRELATION COEFFICIENTS FOR CONSTANTS $a$ AND $b$

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$(\log D_{50})^2$</td>
<td>0.849</td>
</tr>
<tr>
<td>$a$</td>
<td>$(\log D_{50})^2, \log D_{10}$</td>
<td>0.884</td>
</tr>
<tr>
<td>$a$</td>
<td>$e, \log D_{10}$</td>
<td>0.704</td>
</tr>
<tr>
<td>$a$</td>
<td>$\log e, \log D_{10}$</td>
<td>0.711</td>
</tr>
<tr>
<td>$a$</td>
<td>$(\log e)^2, \log D_{10}$</td>
<td>0.708</td>
</tr>
<tr>
<td>$a$</td>
<td>$(\log D_{50})^2, \log D_{10}, \log e$</td>
<td>0.898</td>
</tr>
<tr>
<td>$b$</td>
<td>$(\log D_{50})^2$</td>
<td>0.740</td>
</tr>
<tr>
<td>$b$</td>
<td>$(\log D_{50})^2, \log D_{10}$</td>
<td>0.783</td>
</tr>
<tr>
<td>$b$</td>
<td>$(\log e)^2, (\log D_{10})^2$</td>
<td>0.704</td>
</tr>
<tr>
<td>$b$</td>
<td>$(\log D_{50})^2, \log D_{10}, \log e$</td>
<td>0.789</td>
</tr>
</tbody>
</table>
TABLE 9.9
CORRELATION COEFFICIENTS BETWEEN a AND b

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Independent Variable</th>
<th>R</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>a</td>
<td>0.969</td>
<td>0.939</td>
</tr>
<tr>
<td>b</td>
<td>a²</td>
<td>0.984</td>
<td>0.968</td>
</tr>
<tr>
<td>b</td>
<td>a, a²</td>
<td>0.985</td>
<td>0.970</td>
</tr>
</tbody>
</table>

through statistical correlation and regression analyses. The following equations are the result of these analyses.

\[
\frac{G}{G_{\text{max}}} = -1417 \gamma + 1.00 \quad (9.10)
\]

\[
G_{\text{max}} = a \log C_u + b \quad (9.11)
\]

where

\[
a = -103.83 (\log D_{50})^2 - 24.83 (\log D_{10}) + 60.51 (\log e) + 55.79 \quad (9.11a)
\]

\[
b = 0.006 a^2 + 98.26 \quad (9.11b)
\]

In all of the above equations, G and G_{\text{max}} are in MPa and D_{50} and D_{10} are in mm.

These are empirical equations based on the results of resonant column tests on twenty-four different gradations of Denver sand. The strain for these tests is less than 2 \times 10^{-4}. Table 9.10 presents the range of variables, their mean, \(\bar{X}\), and standard deviation, S, that were used in obtaining the above equations.

9.4 Discussion

The equations developed in the previous section were used
### TABLE 9.10

**STATISTICS OF VARIABLES USED IN THE DEVELOPMENT OF THE FUNCTIONAL RELATIONSHIPS**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range of Values</th>
<th>Mean ($\bar{X}$)</th>
<th>Standard Deviation ($S$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>$71.36 - 147.82$ (MPa)</td>
<td>110.42 (MPa)</td>
<td>17.20</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>$96.04 - 149.20$ (MPa)</td>
<td>119.37 (MPa)</td>
<td>15.65</td>
</tr>
<tr>
<td>$G/G_{\text{max}}$</td>
<td>$0.743 - 0.998$</td>
<td>0.924</td>
<td>0.069</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$5.54 \times 10^{-6} - 2.1 \times 10^{-4}$</td>
<td>$5 \times 10^{-5}$</td>
<td>0.005</td>
</tr>
<tr>
<td>$C_u$</td>
<td>$2.0 - 15.0$</td>
<td>6.21</td>
<td>3.75</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>$0.149 - 1.68$ (mm)</td>
<td>0.709 (mm)</td>
<td>0.494</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>$0.060 - 0.97$ (mm)</td>
<td>0.208 (mm)</td>
<td>0.205</td>
</tr>
<tr>
<td>$e$</td>
<td>$0.475 - 0.946$</td>
<td>0.652</td>
<td>0.134</td>
</tr>
<tr>
<td>$a$</td>
<td>$21.47 - 63.30$</td>
<td>48.45</td>
<td>15.72</td>
</tr>
<tr>
<td>$b$</td>
<td>$73.34 - 97.46$</td>
<td>82.84</td>
<td>8.09</td>
</tr>
</tbody>
</table>
to calculate the values of shear modulus at different strain levels. Fig. 9.6 shows that the measured values of shear modulus are nearly equal to the computed values. The regression line has a coefficient of determination, $R^2$, equal to 0.95.

Several researchers have proposed relationships for shear modulus of sands at small strains. Fig. 9.7 shows a comparison between the relationships for maximum shear modulus $G_{\text{max}}$. Hardin and Richart (1963) developed an empirical relationship for $G_{\text{max}}$ for angular-grained soils such as crushed quartz sand as follows:

$$G_{\text{max}} = 1230 \frac{(2.973-e)^2}{1+e} \left(\overline{\sigma}_0\right)^{1/2}$$

(9.12)

where $G_{\text{max}}$ is the maximum shear modulus in Psi, $e$ is the void ratio and $\overline{\sigma}_0$ is the effective mean principal stress in Psi.

Iwasaki and Tatsuoka (1977) suggested an empirical relationship for $G_{\text{max}}$ based on tests on uniform and well-graded sands as follows:

$$G_{\text{max}} = 900 \frac{(2.17-e)^2}{1+e} \left(\overline{\sigma}_0\right)^{0.40} B$$

(9.13)

where $G_{\text{max}}$ and $\overline{\sigma}_0$ are in KSC and $B$ is a parameter dependent on the uniformity coefficient and the content of fine particles in the soil sample.

Equations (9.12) and (9.13) were used to calculate the maximum shear modulus, $G_{\text{max}}$, for the Denver sand samples. $G_{\text{max}}$ for these samples were also computed by using the relationships obtained in this study (Section 9.3). Then, the measured values of $G_{\text{max}}$ from the test on Denver sand samples were plotted against the computed values obtained from the equations suggested by
FIG. 9.6 COMPARISON OF MEASURED AND COMPUTED VALUES OF SHEAR MODULUS
FIG. 9.7 COMPARISON OF RELATIONSHIPS FOR MAXIMUM SHEAR MODULUS
Hardin and Richart, Iwasaki and Tatsuoka and the one derived in this study. Fig. 9.7 shows the comparison between the three relationships. The values of $G_{\text{max}}$ obtained from the Hardin and Richart equation are generally larger than the measured values, the difference being less for uniform samples. The values obtained from the Iwasaki and Tatsuoka equation are closer to the measured values than the ones obtained by the Hardin and Richart equation. Computed values of $G_{\text{max}}$ from the equation derived in this study are nearly the same as the measured values as shown in Fig. 9.7. Therefore, the empirical relationship obtained in this study estimates the values of maximum shear modulus for Denver sand better than the other relationships proposed by Hardin and Richart and Iwasaki and Tatsuoka.
CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

10.1 Summary and Conclusions

The purpose of this study was to investigate the effects of grain size distribution characteristics on the dynamic properties of granular soils at small strains. To accomplish this goal, twenty-four samples of a Denver sand were prepared. These samples had a wide range of grain size distribution characteristics and contained particles from gravels to coarse silts.

Resonant column tests were performed to obtain the shear moduli and damping ratios of the samples. The specimens were prepared at an initial relative density of 30% and were consolidated for one hour at a confining pressure of 30 (Psi) prior to testing.

Shear moduli and damping ratios were then plotted against strain amplitudes as shown in Figs. 8.6 to 8.17. The results showed unusually large damping ratios for some specimens and the incorrect trend of the increase of damping with confining pressure and relative density. The damping ratios in this study were considered incorrect. The reason for these large initial values of damping ratio were explained in Chapter 8. The values
of shear modulus seemed reasonable and the relationship between shear modulus and factors, such as void ratio, relative density and confining pressure were correct.

Statistical analyses were performed on the results of these twenty-four tests. First, correlation coefficients were obtained between dependent variables (shear modulus) and independent variables \((e, C_u, D_{10}, D_{50}, \gamma)\) to evaluate the strength of their correlation. Then, functional relations were formulated for normalized shear modulus \((G/G_{max})\) and maximum shear modulus \((G_{max})\) through bivariate and multiple regression analyses. The results show that normalized shear modulus is strongly affected by shear strain amplitude, \(\gamma\), and a linear relation exists between them. Also, maximum shear modulus \((G \text{ at } \gamma < 10^{-5})\) is strongly affected by uniformity coefficient, \(C_u\), and void ratio, \(e\), and is related to effective grain size, \(D_{10}\), and mean grain size, \(D_{50}\). In general, the shear modulus of the specimens increased with increasing uniformity coefficient and decreased with increasing void ratio.

It should be noted again, that the relationships given in this study are empirical relations based on the results of resonant column tests on various samples of Denver sand with diverse grain size distribution characteristics. All specimens in these tests had an initial relative density near 30\% and the tests were conducted under a confining pressure of 30 (Psi). The formulated equations are for strain amplitudes less than \(2 \times 10^{-4}\). The most significant achievement of this study is
the formulation of functional relationships for the shear modulus of sands in terms of their grain size distribution characteristics: Uniformity coefficient, $C_u$, mean grain size, $D_{50}$, and effective grain size, $D_{10}$.

10.2 Recommendations

As stated in earlier chapters, damping ratios obtained in this study are incorrect and the equipment used in this study needs to be extensively studied to determine the exact cause of such unusually large damping ratios. After this is accomplished and the problems are corrected, tests should be conducted to evaluate damping ratios of sands used in this study. Functional relationships of the damping ratio in terms of the grain size distribution characteristics should then be formulated.

In addition to the improvements to the testing equipment that were described in Chapter 6 (Section 6.5), several other modifications should be adopted to achieve better accuracy and conduct more consistent tests. These modifications include: (1) replacement of the aluminum rods with steel rods because of their flexibility and (2) the apparatus should be bolted to a large isolated foundation pad. The isolated foundation pad would eliminate effects of floor vibration.

In order to be able to establish a general relationship for dynamic properties of granular soils, several samples of sands and gravels from different origins and mechanical properties should be tested with various relative densities, void ratios
and under different confining pressures.

A new apparatus should be designed to handle dynamic tests over a wide spectrum of strain amplitudes.


Drnevich, V.P. "Resonant Column Test." Geotechnical Laboratory, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, Paper S-78-6, Final Report (July, 1978).


Part III

LIQUEFACTION POTENTIAL
ABSTRACT

Catastrophic earthquake-induced ground failures may result in either partial or total destruction of earth-embankment dams, mine-tailings dams, slopes, and other super-structures. These ground failures may also cause great loss of both lives and properties. Earthquake-induced ground liquefaction is most detrimental to any super or substructures. The research on the quake-induced liquefaction has been greatly accelerated after the considerable damage caused by the Alaska and Niigata earthquakes of 1964.

The liquefaction potential of granular soils was found to be strongly dependent on the relative density, void ratio, mean effective stress, and cyclic deviatoric-stress amplitude. The effect of grain-size distribution on liquefaction potential, while known to be significant, has not been systematically studied over a broad spectrum of grain size.

The parameters which characterize the grain-size distribution of soils are: Uniformity coefficient (Cu), mean diameter (D_{50}) and effective diameter (D_{10}). The effect of these parameters on the liquefaction potential of a granular soil was investigated. A series of isotropically consolidated undrained cyclic triaxial tests were conducted to evaluate the liquefaction potential of samples prepared from Denver Sand. Twenty-four samples with
different grain-size distribution characteristics were tested. The samples have the following gradation characteristics: (a) grain size: 0.038 mm to 9.53 mm; (b) mean grain size: 0.14 mm to 1.50 mm; and (c) uniformity coefficient: 1.89 to 16. The particle shape of the sand is subangular.

The result of this investigation indicated that the liquefaction potential is strongly dependent on the mean diameter of Denver sand. Thus, the functional relationship between the liquefaction potential and the mean diameter was formulated by the use of regression analysis.
CHAPTER 11

INTRODUCTION

11.1 General

The quake-induced liquefaction of cohesionless soils, because of its disastrous damages of extensive nature has earned the attention of worldwide researchers in the field of dynamic behaviors and properties of soils, since the catastrophic earthquakes of Alaska and Niigata of 1964. The liquefaction induced in these earthquakes resulted in massive flow slides, serious ground subsidence, large differential settlement and/or bearing capacity failures of super structures (earth and non-earth), waterfront structures, and bridge abutments and piers, etc. To mitigate these hazards caused by the quake-induced liquefaction requires a clear understanding of the dynamic behaviors of saturated cohesionless soils and the factors affecting them.

11.2 Purpose

Research studies have shown that cohesionless soil grains when loose and subjected to cyclic stresses tend to compact and reorient. This causes the generation of pore pressure if drainage is restricted. This transfer of load from soil grains to pore water results in the reduction of the effective stress. When a cohesionless soil is subjected to a sufficient number of stress
cycles, the pore water pressure may become equal to the initial confining pressure on the soil. At this time the soil, if of low relative density, may undergo large deformations with minimal resistance and is said to have liquefied.

The liquefaction potential of a granular soil is strongly dependent on its relative density, void ratio, mean effective stress, and cyclic deviatoric-stress amplitude. All these factors have been extensively investigated, but the effect of grain-size distribution on liquefaction potential, while known to be significant, has not been systematically studied.

The purpose of this study is to investigate the effects of grain-size distribution (gradation) characteristics on the liquefaction potential of cohesionless soils. Gradation of soils can be characterized by indices, such as the uniformity coefficient, \( C_u \), median diameter, \( D_{50} \), and effective grain size, \( D_{10} \), etc. Functional relationships were formulated between the liquefaction potential and gradation characteristics.

11.3 Scope

To investigate the effect of grain-size distribution on the liquefaction potential of granular soils, undrained cyclic triaxial tests were performed. Granular soil samples prepared from a Denver sand were tested. These samples were graded to have different uniformity coefficients at six selected mean grain sizes. The test sand has grain particle sizes ranging from silt to small gravels.

After completion of the laboratory work, all cyclic triaxial
test results were systematically studied and statistically analyzed. The analysis included statistical correlation and regression analysis. The liquefaction potential of granular soils was formulated as functions of gradation indices with the strong correlation. It is hoped that the result of this research will assist in the assessment of the safety of earth structures containing saturated granular soils and the foundation system on them was founded during potentially damaging earthquakes.

An extensive literature review is presented in Chapter 12. The review includes: 1) Behaviors of saturated granular soils under static and cyclic loads; 2) case histories of liquefaction induced ground failures; 3) methods for evaluating liquefaction potential of soils; 4) experimental methods for evaluating liquefaction potential; 5) factors affecting liquefaction potential; and 6) factors affecting cyclic triaxial test results. Test equipment, sample preparation and experimental procedures are detailed in Chapter 13. The test results were analyzed and discussed in Chapter 14. Summary and conclusions are presented in Chapter 15.
CHAPTER 12

REVIEW OF LITERATURE

12.1 Behaviors of Saturated Granular Soils under Static and Dynamic Loads

12.1.1 Behaviors of Saturated Sands under Static Loads

The shear deformation of sand is accompanied by volume changes (Reynolds, 1885). The volume decrease of a loose sand and the volume increase of a dense sand during shearing would tend to produce the same "critical void ratio" or "critical density" at which a cohesionless soil can undergo any amount of deformation without volume change (Casagrande, 1936). The critical void ratio (or density) is not constant for a soil. As shown in Fig. 12.1, the critical density increases with the increase of confining pressure (Seed and Lee, 1967). The critical void ratio is also a function of the type of test used to determine it (Perloff and Baron, 1976).

The critical void ratio can be reached from either a loose or dense state. Fig. 12.2 shows typical (qualitative) behavior of soils sheared under drained triaxial compression condition, i.e., the curves marked "1" represent the behavior of a dense sand, while the curves marked "2" represent that of a loose sand. If a saturated sand is not allowed to change its volume, then the tendency to change its volume will result in the change in porewater pressure.
Effective Stress $p'$

FIG. 12.1 CRITICAL DENSITY LINE (From Baladi and Rohani, 1978)
FIG. 12.2 TYPICAL BEHAVIOR OF SATURATED SOILS TESTED UNDER DRAINED TRIAXIAL TEST CONDITIONS
Thus a sand in a looser state than that of the critical void ratio will experience an increase in pore-water pressure, and a corresponding decrease in shearing resistance. On the other hand, a sand in a denser state than that of the critical void ratio will experience a decrease in pore-water pressure and a corresponding increase in shearing resistance. Fig. 12.3 shows a typical variety of stress-strain-pore-water pressure response curves manifested by a saturated soil tested in undrained shear in a triaxial compression device. The three specimens were first isotropically consolidated to the same effective mean normal stress level (point 2), then subjected to undrained shearing. The curves marked "2→3" show the typical response of a very loose sand to undrained shearing. The curves marked "2→5" show typical behaviors of a very dense sand. Between the extreme responses of these very loose and dense soils, there exist a gradual change in response exemplified herein by the curves marked "2→4". The actual response depends on the state of compactness of the material. It is clear from this figure that only the effective stress will affect the shear strength of soil.

Theoretically speaking, the critical void ratio does exist for a sand, however, within the usual effective stress range in laboratory testing (say <150 psi), sand samples on the critical state line will be extremely loose. They are often looser than the loosest state that can be achieved by any laboratory sample preparation method. Very often an extremely loose state for a sand can only be reached through dilation during shear (Atkinson and Bransby, 1978; K.H. Roscoe, 1970). In other words, the behavior shown by
FIG. 12.3 TYPICAL BEHAVIOR OF SATURATED SOIL TESTED UNDER UNEARRED TRIAXIAL TEST CONDITIONS
(From Baladi and Rohani, 1978)
a loose sand in Figs. 12.2 and 12.3 were selected observed in the laboratory. For Monterey No. 0 sand, the critical confining pressure versus the relative density of samples is shown in Fig. 12.4 (Lee and Vernese, 1978). The critical confining pressure is 8 KSC (kg/cm²) for relative density of 60 percent and 13 KSC for relative density of 80 percent. Lade's test results (1972) indicated that the Monterey No. 0 sand shows a strong tendency of dilation under shear even for very loose sample relative density = 21%. In the literature, little work was done on the behavior of a saturated sand under a triaxial extension test in which the axial load is reduced while the radial stress is kept constant. According to the study by Castro (1975), the sample tested in triaxial extension yields considerably and eventually fails by necking under a considerably smaller deviatoric stress than that tested in triaxial compression. Fig. 12.5 shows the comparison of axial compression and extension consolidated-undrained triaxial tests of a dense clean fine to medium sand. It is imperative to know the behavior of sand under extension loads if one were to predict its behavior under a cyclic load with stress reversal.

12.1.2 Behavior of Saturated Sands under Cyclic Loads

The most important and critical phenomenon of saturated sands under cyclic loads is liquefaction. If a saturated sand is subjected to cyclic loads, it has a tendency to compact and decrease in volume. If the drainage line is closed, then the tendency to decrease in volume results in an increase in pore-water pressure. If the pore-water pressure equalizes the overburden (or the confining)
FIG. 12.4 CRITICAL CONFINING PRESSURE VERSUS RELATIVE DENSITY
OF MONTEREY NO. 0 SAND (From Lee and Vernese, 1978)

FIG. 12.5 COMPARISON OF AXIAL COMPRESSION AND EXTENSION
CONSOLIDATED-UNDRAINED TRIAXIAL TESTS (Castro, 1975)
pressure and the effective confining stress becomes zero, the sand
loses its strength completely, and is in a liquefied state (Casagrande, 1936).

A quantitative approach to the prediction of the onset of
liquefaction was presented by Seed and Lee (1966). In this approach
the liquefaction potential was evaluated by results of cyclic
triaxial tests. The main conclusion derived from this approach is
that the larger the cyclic stress ratio and the looser the samples,
the smaller the number of cycles of the cyclic stress are required
to cause liquefaction. Test results by Seed and Lee (1966) showed
that the liquefaction can be produced by cyclic loading also in
dense sand and is not limited to loose sands. This behavior contra­
dicts the sand behavior implied by the critical void ratio (or
critical state) concept, which states that dense sands will only
dilate under shearing. Seed and Lee (1966) concluded the critical
state approach which can explain the behavior of sand under static
compression loads very well cannot be used to explain the behavior
of a sand under cyclic loads.

Further studies conducted by Casagrande (1975), Castro (1975),
and Castro and Poulos (1977) introduced the following terms to
describe the behavior of saturated sands under cyclic loads: (1)
the "liquefaction" of a loose sand and (2) the "cyclic mobility" of
a dense sand. The detail differences between these two terms were
summarized by Holtz and Kovacs (1981) in Table 12.1. Fig. 12.6
also shows the difference of these two terms in describing the
similar behavior of sands. In general, the liquefaction can take
### TABLE 12.1
DIFFERENCE BETWEEN LIQUEFACTION AND CYCLIC MOBILITY*

<table>
<thead>
<tr>
<th></th>
<th>Liquefaction</th>
<th>Cyclic Mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>Most likely in uniform, fine, clean, loose sand. Static load can cause liquefaction. Cyclic loads causing shear stresses larger than the steady-state strength also can cause liquefaction.</td>
<td>Any soil in any state can develop cyclic mobility in the laboratory if the cyclic stresses are large enough.</td>
</tr>
<tr>
<td>Effect of $\sigma'_c$ at constant void ratio for $\sigma'_c/\sigma'_e$</td>
<td>Increased $\sigma'_c$ means larger deformations if liquefaction is induced. The magnitude and/or number of cyclic loads needed to cause liquefaction increases with $\sigma'_c$. Cyclic loads smaller than the steady-state strength cannot cause liquefaction but may cause cyclic mobility.</td>
<td>Increased $\sigma'_c$, means increased cyclic load to cause cyclic mobility. But the cyclic mobility ratio $\dfrac{\sigma'_c}{\sigma'_e}$ usually decreases with increasing $\sigma'_e$.</td>
</tr>
<tr>
<td>Effect of $\sigma'_c/\sigma'_e$ at constant void ratio and $\sigma'_e$</td>
<td>Smaller additional loads are needed to cause liquefaction as $\sigma'_c/\sigma'_e$ increases. When $\sigma'_c/\sigma'_e$ is large, a soil is more unstable and may, in the extreme, be susceptible to &quot;spontaneous liquefaction.&quot;</td>
<td>In soils that have low permeability, increased $\sigma'_c/\sigma'_e$ seems to result in somewhat smaller cyclic mobility stresses, which is a reasonable trend. In clean sands, cyclic mobility stress increases with $\sigma'_c/\sigma'_e$. This unusual result for clean sands is postulated to be due to the substantial test error due to redistribution of void ratio in the laboratory specimens.</td>
</tr>
</tbody>
</table>

---


$\dfrac{(\sigma_1 - \sigma_3)/2}{\sigma'_e}$, where $(\sigma_1 - \sigma_3)$ is the dynamic principal stress difference, or the cyclic mobility stress.

(After Holtz and Kovacs, 1981)
CONTRACTIVE SOILS (LOOSE)

MONOTONIC LOADING

CYCLIC MOBILITY

DILATIVE SOILS (DENSE)

EFFECTIVE NORMAL PRINCIPAL STRESS

FIG. 12.6 LIQUEFACTION AND CYCLIC MOBILITY
(Castro and Polous, 1977)
place in a sand looser than its critical state either under a static or a cyclic load. The liquefaction of a loose sand still can be explained by the critical state concept. The cyclic mobility of a dense sand (denser than its critical state) cannot use the critical state concept to explain it. Liquefaction consists of a loss in shear strength and cyclic mobility consists of gradually increasing cyclic strains but does not entail a loss in shear strength.

Casagrande (1975) concluded that (1) the cyclic mobility in test specimens is caused by redistribution of water content which is generated by the mechanism that normally are absent in-situ, and (2) the cyclic mobility normally cannot develop in a dense sand in-situ. Because it is unlikely that laboratory tests can be devised to eliminate the severe stress gradients in test specimens and to reproduce the uniform stresses that exist in a typical element in-situ (Casagrande, 1975). Since the cyclic triaxial tests cannot simulate the in-situ loading condition (Casagrande, 1975; Finn, Pickering and Bransby, 1971), correction factors are necessary to be established for cyclic triaxial test results so that it can be more representative of the in-situ condition (Pedro, Seed and Chan, 1976).

Typical test results of an isotropically consolidated stress controlled cyclic triaxial strength test are shown in Fig. 12.7. The important feature of the behavior depicted in Fig. 12.7 is that the accumulated strain during cyclic loads prior to liquefaction is small compared with the strain after liquefaction, whereas the
FIG. 12.7 TYPICAL RESULT OF AN ISOTROPICALLY CONSOLIDATED STRESS-CONTROLLED CYCLIC TRIAXIAL TEST
pore pressure increases rapidly prior to liquefaction. The other important behavior is that the liquefaction takes place suddenly during the test (Seed and Lee, 1966).

The sudden initiation of liquefaction has been studied by many researchers and different conclusions have been drawn. According to the study by Ishihara, Tatsuoka and Yasuda (1975), liquefaction will be initiated when the effective stress touches the "phase transformation line" which is defined as the loci of the points at which the effective stress paths undergo abrupt changes in curvature and direction. Fig. 12.8 shows the phase transformation line for liquefying sands. Some other studies concluded that the liquefaction is initiated when the excess pore-water pressure reaches the magnitude of 60% of the confining pressure (Singh, Donovan and Park, 1980; Finn, 1981). Another conclusion offered by Baladi and Rohani (1978) indicates that the liquefaction is initiated when the effective stress path touches the failure envelope of a soil.

The results of cyclic triaxial tests (Seed and Lee, 1966) indicated that the liquefaction takes place when the deviatoric stress is equal to zero. The same conclusion was also reached about the state of liquefaction by Wang (1981) from this theoretical analysis.

12.2 Case Histories of Liquefaction Induced Failures

12.2.1 Introduction

Although experience has shown that the greatest damage to civil engineering structures due to liquefaction occurs as a
FIG. 12.8 PHASE TRANSFORMATION LINES FOR LIQUEFYING SAND
result of earthquakes, cases of failures triggered by other mechanisms will also be discussed. This chapter presents case histories of liquefaction failures grouped according to the triggering mechanism.

12.2.2 Liquefaction Induced by Monotonically Changing Stresses

12.2.2.1 Zeeland Slides

The Dutch province of Zeeland consists of a large group of islands separated by wide estuaries. On the shores of the islands in that province, numerous flow slides have occurred. From 1881 to 1946, 229 such slides were reported to involve 32.7 million cubic yards of soil and an area loss of 660 acres. The soils involved in these slides were sands deposited in shallow water by the rivers Rhine, Meuse, and Scheldt. Grain-size curves reported by Koppejan, Van Wamelen and Weinberg (1943) showed these sands to be mostly fine and uniform, with a typical $D_{10}$ of 0.08 mm and uniformity coefficients, typically, of less than 2.

Erosion and seepage pressures resulting from tidal fluctuations of up to 4.6 m were believed to have triggered the slides. According to the description by Koppejan, these slides progressed inland as a succession of small slides starting when the toe of a slope was scoured away by tidal current, releasing lateral support in the slope. Under reduced lateral stress, the soil tended to expand, generating negative pore water pressure which caused inward seepage. Water seeping into the soil relieved
the negative pore pressure, which became positive as shear strains occurred and finally became large enough to cause liquefaction flow in that portion of the slope. After a certain amount of material had flowed away in one small slide, the process would begin again and continue in this manner until denser, more stable material was reached at which time the flow process would stop. Fig. 12.9 taken from Hvorslev's paper (1956) clearly illustrates the above process. According to Koppejan, the slides progressed inland at a rate of about 50 m per hour.

12.2.2.2 Mississippi River Flow Slides

Numerous flow slides have occurred along the banks of the Mississippi River (Hvorslev, 1956). The materials involved in these slides were uniform fine sands with rounded to subrounded particles and uniformity coefficients from 1.5 to 1.8. Those slides occurring in revetted point bar deposits have been carefully studied, but slides have occurred in unrevetted reaches of the river as well. The slides have damaged revetments and levees built for flood protection. On March 24, 1949 a slide involving the loss of a portion of a levee occurred at Free Nigger Point near Baton Rouge, Louisiana. This single slide involved the volume displacement of 4.4 million cubic yards of material and occurred in less than 12 hours. The slides occurring on the river are described by M.J. Hvorslev (1956) to be similar to those occurring in Zeeland, Holland, in regard to form, dimensions, slopes, duration, material, and groundwater conditions.
SECTION B TENDS TO EXPAND CAUSING NEGATIVE PORE WATER PRESSURE AND INWARD SEEPAGE AS TEMPORARY STABILIZING FORCES

AFTER INITIAL INWARD SEEPAGE, LATERAL EXPANSION AND SHEARING STRAINS OCCUR, CAUSING GRADUAL CHANGE OF NEGATIVE TO POSITIVE PORE WATER PRESSURES IN SECTION B

THE POSITIVE OR EXCESS PORE WATER PRESSURES INCREASE WITH INCREASING STRAINS AND FINALLY BECOME LARGE ENOUGH TO CAUSE FAILURE OF SECTION B

FIG. 12.9 DUTCH HYPOTHESIS FOR PROGRESSIVE FLOW FAILURES (From Hvorslev, 1956)
12.2.2.3 Partial Failure at Fort Peck Dam

A granular shell of a dam constructed by the hydraulic fill method is likely to be deposited in a loose condition, and therefore susceptible to liquefaction failure. The Fort Peck Dam in Montana is an embankment with shells of this type. On September 22, 1938, a partial failure occurred involving a 1700-foot-long section of the upstream shell, portions of which moved some 1500 feet in a period of about 4 minutes.

The dam was constructed of river sands and finer grained alluvial soils on a foundation of alluvial sands, gravels, and clays with a total thickness of up to 130 feet. Beneath the river alluvium Bearpaw shale was present which contained layers of bentonite. In a discussion of a paper by Middlebrooks (1942), Gilboy expressed the opinion, which was shared by Casagrande, that liquefaction was the predominant failure mechanism which was triggered by shear strains in the foundation shale. This failure involved the movement of about 10 million cubic yards of material.

12.2.2.4 Flow Slide at Aberfan Village in Wales

Aberfan is a mining village, situated at the foot of a low mountain, the side of which had been used to deposit waste materials from coal mining operations. The waste materials were deposited in hillocks which were locally called "tips." From 1915 to October 1966, seven tips had been formed, and on October 21, 1966, the most recently constructed tip failed by liquefaction
sending about 200,000 cubic yards of material flowing down the mountainside and into the village. In this catastrophe, 146 children lost their lives in an elementary school in the flow path. The tip that failed had been deposited over natural springs issuing from outcrops of the underlying sandstone. At the time of failure, material was still being deposited on the tip. Heavy rainfall, common to this area in October may have eroded the base of the tip, reducing lateral stress and causing internal shear strains. This condition, coupled with the natural groundwater conditions of the mountainside and the continued loading of the tip, was believed to have triggered the liquefaction failure (Hutchinson, 1967).

Hutchinson (1967) described the material involved in the flow slide as consisting chiefly of shale fragments and mudstones ranging from clay size particles to large boulders as much as 3 feet in diameter. For the grain sizes small enough to be sieved, the average $D_{10}$ size was between 0.1 and 0.8 mm and the coefficient of uniformity was approximately 18. Hutchinson pointed out that the Aberfan material was neither grained nor poorly graded. He described tailing materials from other sites which failed in a similar manner as having high uniformity coefficients and questions the uniformity of soils as a criterion for liquefaction susceptibility.
12.2.3 Liquefaction Induced by Cyclic Vibrations

12.2.3.1 Liquefaction of Railway Embankment Near Weesp, Holland

Koppejan (1943) described a flow slide in the approach to a railway bridge over the Merwede Canal near Weesp, Holland, in 1918. The embankment which failed had been formed by dumping moist sand. The sand had consequently "bulked," forming a very loose and unstable structure. After the water level in the canal rose and saturated the sand, the embankment was subjected to vibrations from a passing train and collapsed with the train causing heavy casualties.

It is probable that capillary forces present in the moist sand served to give the embankment initial stability. As saturation occurred, the capillary forces disappeared and the stability of the embankment decreased to the extent that vibrations from the passing train were sufficient to cause liquefaction. The coefficient of uniformity of the sand involved in the slide was about 2 and the $D_{10}$ size was about 0.14 mm. According to Koppejan (1943) liquefaction triggered by vibrations occurs much more rapidly than liquefaction by monotonic stress changes because the entire mass subjected to vibrations liquefies almost simultaneously. This railway accident was so serious that as a result, the formal practice of soil mechanics was begun immediately in the Netherlands.

12.2.3.2 Vibrations Induced by Pile Driving

In an effort to develop a field test to evaluate earthquake potential, Ishihara and Mitsui (1972) drove piles into a sand
foundation and measured the induced pore pressure in the vicinity of the pile. According to Ishihara, this approach is superior to blasting tests since vibrations caused by blasts are very short in duration compared to the duration of an earthquake, and blasting cannot be performed in congested urban areas.

The site selected for the investigation was near Tokyo. The foundation material was a hydraulically placed, fine uniform sand about 8 meters in thickness. The uniformity coefficient of the material was between 2 and 5 and the D₁₀ size between 0.05 and 0.20 mm. The standard penetration resistance blow count was approximately N = 5.

To measure the induced pore pressure and acceleration, piezometers and accelerometers were installed 4 or 8 meters deep in the foundation in a circular configuration about the point at which the pile would be driven. The pile was driven at the pre-selected spot by a vibratory generator capable of producing a variable oscillating force up to 35 tons at a frequency of 17 Hz. The oscillating driving force was adjusted to produce a penetration rate of 1-2 meters per minute. The pile was a closed-end steel pipe with an outside diameter of 40 centimeters. Measurements during the driving of the pile indicated that during some tests, the pore pressure became temporarily equal to the overburden pressure, but quickly dissipated. Acceleration was measured only in the vertical direction. Measurements showed that there was a unique relationship between sand densisty and acceleration level which produced a given pore pressure near the pile. The test results show a significant reduction of the strength of the
foundation soils resulted from the slow dissipation of excess pore water pressure and the remolding of soils during pile driving. Such a strength reduction, even if not sufficient to cause liquefaction, could result in large settlements or even bearing capacity failures in adjacent structures.

12.2.4 Flow Failures Induced by Earthquakes, Blasts, and Wave Action

Liquefaction induced by shock waves and the resulting random vibrations in potentially a more serious threat to civil engineering structures than that induced by other disturbing causes. Shock waves from earthquakes and blast loads or explosive charges are included in this general category. Earthquakes present probably the most serious liquefaction threat because the tremendous amount of energy released during an earthquake can disturb very large areas almost simultaneously. Consequently, structures built on or with cohesionless soils in seismically active areas must be designed considering the probability of earthquake shaking.

12.2.4.1 One of the Earliest Recorded Landslides Resulting from Liquefaction

The destruction of Helice, Greece described by Marinatos (1960) may possibly record the earliest known case of a major landslide resulting from soil liquefaction induced by an earthquake.

"In the year 373 B.C., during a disastrous winter night, a strange thing happened in central Greece. Helice, a great and prosperous town on the north coast of the Peloponnesus, was engulfed by the waves after being levelled by a great earthquake. Not a single soul survived.... The next day two thousand men hastened to the spot to bury the dead,
but they found none, for the people of Helice had been buried under the ruins and subsequently carried to the bottom of the sea, where they now lie."

Helice was located on deltaic deposits of alluvial sand between the mouths of the Selinus and Cerynites rivers and about a mile and a half from the coast. Marinatos indicated that the city is now completely covered and no trace of it exists, neither on the ground surface nor on the bottom of the sea.

The events leading to the disappearance of Helice and its inhabitants are not immediately clear. A general subsidence of the land area during the earthquake undoubtedly occurred and this alone could have led to flooding of the city.

Marinatos (1960) who made detailed studies of the event, concluded that in addition to destruction of buildings by the ground shaking and flooding due to land subsidence, the ground slipped towards the sea possibly as much as half a mile. Marinatos noted that ordinarily it would be expected that building destruction and flooding would lead to some of the dead floating to the surface where they would have been picked up by the Achaens for burial. He concludes that only the entrapment of the inhabitants in collapsed buildings, and temporarily liquefied and flowing soils could have led to the recorded facts that no one survived and no dead were found.

Additional evidence in support of this concept is provided by the fact that

"The phenomenon was repeated, in exactly the same place though to a lesser degree, during the earthquake of December 26, 1861.... Again the soil slipped to the northeast (toward the sea) in the following way: A crack about eight miles
long and six feet wide appeared in the earth along the foot of the mountain. A strip of plain 325 to 425 feet wide disappeared slowly under the sea along the whole eight-mile length, while the remaining part of the plain sank about six feet and showed many minor cracks and small chasms."

In view of the fact that the earthquake of 373 B.C. is estimated to have been about 10 times greater in intensity than the 1861 earthquake, it is very likely that the disappearance of Helice can be attributed to liquefaction of the underlying sand foundation.

12.2.4.2 Examples of Fluidation Flow Induced by Earthquakes

Fluidation, i.e., flow of an essentially dry, cohesionless material with air as the pore fluid, can also be induced by shock waves. One of the most dramatic series of flow slides in recorded history occurred in the Chinese province of Kansu on December 16, 1920. Close and McCormick (1922) described the events in the following terms:

"Of that most remarkable series of seismic disturbances which occurred throughout the world in November and December, 1920, the most phenomenal was undoubtedly the great Kansu earthquake of the late evening of December 16... Landslides that eddied like waterfalls, crevasses that swallowed houses and camel trains, and villages that were swept away under a rising sea of loose earth, were a few of the subsidiary occurrences that made the earthquake one of the most appalling catastrophes in history."

"The area of greatest destruction, 100 miles by 300 miles in extent, contains ten large cities besides numerous villages. In it is the heart of the loess country...where the loose earth cascaded down the valleys and buried every object in its path."

"It is in the loess area that the immense slides out of the terraced hills occurred, burying or carrying away villages...damming stream beds and turning valleys into lakes, and accomplishing those hardly believable freaks which the natives name 'the footsteps of the gods.' The loss
of nearly two hundred thousand lives and the total destruction of hundreds of towns and cities calls for reconstruction work on a staggering scale."

The authors described the scene in one valley as follows:

"Our route through this larger valley led us past three lakes formed through the blocking of the stream by five enormous slides.... Some of the scooped-out places left by these slides were half a mile in width at the mouth, extending back into the hills for a mile and furnished enough dirt to cover several square miles of valley floor.... In each case the earth which came down bore the appearance of having shaken loose, clod from clod and grain from grain, and then cascading like water forming vortices, swirls, and all the convulsions into which a torrent might shape itself."

Fluidization of the loess material is evident from the above description. Obviously the vibrations resulting from the earthquake were sufficient to break down the connections between the slightly cemented particles of loess.

Casagrande (1971) cites three examples of fluidization probably induced by seismic motions. The first example given was a prehistoric 400 million cubic yard slide of marble breccia which occurred at Blackhawk Canyon in the San Bernardino Mountains of Southern California. The slide mass was 5 miles long, 2 miles wide, and between 30 and 50 feet thick. The second fluidization example cited was a 40 million cubic yard slide across the Sherman Glacier in Alaska triggered by the 1964 earthquake. The slide formed a blanket of broken rock 2 miles wide and 10-20 feet thick. The third example was a slide in volcanic ash in a high mountain desert in Chile. The slide was roughly circular in shape with an effective diameter of 1500 feet. Pore air pressure during this slide dropped quickly enough that the slide was arrested before it had moved very far.
12.2.4.3 **Kanto Earthquake**

On September 1, 1923 the Kanto region of Japan was hit with one of the most disastrous earthquakes in Japanese history. Kuribayashi and Tatsuoka (1975) indicated that of the 44 major earthquakes which occurred in Japan from 1872 to 1968 the Kanto earthquake was by far the most damaging in terms of human life. It has been estimated that 99,331 lives were lost to this earthquake having a magnitude of 7.9.

A majority of the sites which liquefied were located on the original winding river course of the existing Tone river located immediately to the northeast of Tokyo. Approximately three hundred and fifty years ago, the estuary of the Tone river was linked to the Pacific Ocean by human power. The original Tone river had previously flowed into Tokyo Bay during which time the river deposited very deep and soft deposits of sediment and interwoven sand layers. These deposits retained a high potential for liquefaction, as witnessed by liquefaction failure during the Tone-Karyu (January 18, 1895), Nishi-Saitama (September 21, 1931), and Kanto earthquakes.

In the plains, numerous fissures and mud volcanos spurted intermittently with water and sand; the ejection would stop and then resume action a few seconds later, continuing long after the earthquake.

A rather surprising event described by Ambraseys and Sarma (1969) occurred in a paddy field near the Sagami River. During the earthquake seven vertical wooden piles, 60 centimeters in diameter, suddenly emerged above ground level. Successive
aftershocks caused these piles to emerge more, finally reaching a height of 110 centimeters above ground level. It is interesting to note that these piles were unknown to the local people before the earthquake. Later, it was found that they belonged to a bridge built in 1182 over the old course of the Sagami River.

12.2.4.4 Liquefaction During the Niigata Earthquake of 1964

On June 16, 1964 a violent earthquake shook Niigata, Japan inflicting severe damage to the city. The recorded magnitude of this earthquake was 7.3 on the Richter scale, and the epicenter was about 35 miles north of the city (Seed and Idriss, 1967). Niigata is located on the west coast of Japan where the Shinano River enters the Sea of Japan. The city is underlain by about 100 feet of fine subangular alluvial sand with a uniformity coefficient between 2 and 10 and a \( D_{10} \) size between 0.04 and 0.2 mm. Damage due to the earthquake resulted primarily from liquefaction of the loose sand deposits in low-lying areas. Buildings not imbedded deeply in firm material sank in the resulting quicksand or tilted toward the direction of the center of gravity. One apartment building is reported to have tilted 80 degrees. Underground structures such as septic tanks, storage tanks, sewage conduits, and manholes floated upward to extend as much as 10 feet above the ground surface. In low-lying areas, mud volcanos began ejecting water and sand 2-3 minutes after the shaking stopped and continued for as much as 20 minutes. Sand deposits 20-30 centimeters thick covered the entire city as if it had been hit by a sand flood. Five simply supported girders of the Showa
Bridge across the Shinano River fell when pier foundation piles deflected due to the loss of lateral support.

As illustrated in other cases of liquefaction, little or no damage to buildings and engineering structures occurred as the direct result of shaking (Ambraseys and Sarma, 1969). Damage which did occur appeared to be entirely related to foundation failure. Ground movements recorded by instruments in Niigata indicated that shortly after the earthquake started ground motion had begun to subside. Once an underlying strata had liquefied it was no longer able to transfer earthquake vibrations to overlying deposits.

The greatest devastation by the quake occurred in the low-lying areas near the river. An extensive soil survey after the quake revealed that the standard penetration blow count was between \( N = 6 \) and \( N = 12 \) in the upper 30 feet in the areas suffering the greatest damage. The penetration resistance was about the same over the entire area in the upper 15 feet; but in areas where the resistance was measurable higher in the lower 15 feet, notably lighter damage occurred. The recorded peak acceleration during the quake was 0.16 g. The water table was about 5 feet below the ground surface in areas of heaviest damage and 9 feet or more in areas of lighter damage.

It was reported by Seed and Idriss (1967) that Niigata had been shaken by an earthquake of the same intensity 130 years before the event of 1964. This indicates that areas which have been shaken by an earthquake are not immune to further damage unless remedial measures are taken.
12.2.4.5 Valdez Slide, 1964 Alaska Earthquake

During the 1964 Alaska earthquake an extensive landslide believed to be due to liquefaction occurred in Valdez (Seed, 1968). The city is located on a deltaic deposit of silt, fine sand, and gravel, with the silt and fine sand occurring in layers within the coarse sand and gravel. The sand particles were generally subangular in shape with the uniformity coefficient greater than 10. The standard penetration resistance of the Valdez material varied between $N = 7$ and $N = 25$. The magnitude of the earthquake was 8.3 on the Richter scale with the epicenter about 40 miles away. The landslide resulting from the quake involved approximately 98 million cubic yards of material and extended about 500 feet inland from the coastline, destroying the harbor facilities and near-shore installations. This slide was reported to have occurred so rapidly and with such violence that eye witnesses were unsure of what actually happened directly beneath their feet. The dock structure at Valdez supporting several large buildings was reported to have broken off and disappeared into the sea. The slide was responsible for the loss of 30 lives. About 40 percent of the buildings of Valdez were seriously damaged. Lateral movement toward the sea occurred as much as 3600 feet inland, causing settlement, fissuring, and ground breakage. In addition to buildings, the water and sewage systems were severely damaged by ground movement. In many instances, fissures occurring in the foundation of buildings resulted in basements being pumped full of sand and water. Open cracks spurting sand and water along the streets substantial distances inland indicated that liquefaction
was not limited to the coast. Since the year 1899 submarine slide damage was reported to have occurred at this location on at least five other occasions, probably caused by seismic activity.

12.2.4.6 Turnagain Heights Slide, 1964 Alaska Earthquake

A dramatic event that occurred during the earthquake of 1964 was the large landslide along the coastline in the Turnagain Heights area of Anchorage (Seed, 1968; Seed and Wilson, 1967). Bluffs up to 70 feet high sloping at about 1-1/2:1 down to the bay were prominent along the coastline. The slide extended about 8500 feet from west to east along the bluff-line and retrogressed inland a distance of about 1200 feet at its west end and about 600 feet at its east end. A surface area of about 130 acres was affected by the slide.

Lateral movement of the soil mass was extensive with portions of the slide moving as much as 2000 feet out into the bay. The ground at the west end of the slide area was undeveloped but at the east end had been developed as a residential area. Approximately 75 homes located on the east end of the slide were destroyed.

The Turnagain slide area was covered with a surface layer of sand and gravel varying in thickness from 15 to 20 feet at the east end of the slide area to about 5 to 10 feet at the west end. A deep bed of Bootlegger Cove clay, about 100 to 150 feet in thickness, underlaid the sand and gravel. Shear strength of the clay decreased from about 1 ton/ft$^2$ at its surface (El. 50 approx.) to about 0.45 ton/ft$^2$ at E. 0 and then increasing to about 0.6 ton/ft$^2$ at El. -30.
The clay deposit contained numerous lenses of silt and fine sand, particularly near the surface on which sliding occurred. These lenses varied in thickness from a fraction of an inch to several feet. Below the sliding surface the sand lenses were only occasionally encountered.

Seed (1968) performed a number of cyclic loading tests on representative samples of the sand and clay to determine, for each of these materials, the relationship between the magnitudes of the cyclic stress and the number of cycles required to cause failure of the clay or liquefaction of the sand. This analysis indicated that the sand could be expected to liquefy after about 12 stress cycles, corresponding to about 45 seconds of ground motion in the analysis, while the clay would not be expected to fail until after about 22 cycles or 1-1/2 minutes of ground shaking.

From the above information physical features on the displaced slide mass (sand boils, intermixing of sand and clay) and eyewitness reports, it appears very likely that liquefaction of sand lenses was an important factor in the development of the slide.

12.2.4.7 Slides in the San Fernando Dams

On February 9, 1971 an earthquake shook southern California resulting in a major landslide in the upstream slope of the lower San Fernando Dam (Seed, et al., 1973). This earthen dam is about 140 feet high and provides 20,000 acre-feet of reservoir capacity. The water level in the reservoir was about 35 feet below the crest of the dam at the time of the event, and the resulting slides on both
the upstream and downstream slopes left about five feet of freeboard. Because of the possibility that aftershocks might cause further sliding and possible failure of the dam, 80,000 people living downstream were evacuated until the reservoir could be drawn down to a safe level.

Upstream from the lower dam is the upper San Fernando Dam which is about 80 feet high and provides a reservoir of 1850 acre-feet. During the earthquake, the crest of this dam moved five feet downstream and settled three feet, but no water was lost from the reservoir. Had water from the upper dam been released, the lower dam would have been overtopped and consequently would have failed.

This near catastrophe prompted an immediate investigation of the safety of earth dams with respect to earthquake loading. The event of February 9, 1971 has been assigned a Richter magnitude of 6.6 with the epicenter located about eight and one half miles northeast of the dams. The maximum acceleration in the vicinity of the dams was of the order of 0.55-0.60 g.

Both the San Fernando dams were old structures, having been constructed in the period between 1915 and 1925 by the hydraulic-fill method. The dams were constructed directly on alluvial soil with no stripping prior to the placement of the embankment fill. Field investigation after the quake revealed that penetration resistance (N values) was very low in the silt and clay core of the hydraulic fill, but was somewhat higher in the outer sands and gravels of the shell. The underlying alluvium was very heterogenous, exhibiting an erratic blow count in most
of the drill holes. Laboratory investigations showed that the hydraulic-fill material was finer, more uniform, and less dense than the alluvium. The hydraulic-fill lower dam sands were coarser than the upper dam sands, $D_{50}$ being between 0.05 and 0.8 mm for the upper dam sands and between 0.05 and 1.0 mm for the lower dam sands. The average relative density for all the hydraulic-fill material investigated was about 54 percent. The uniformity coefficients ranged from four to six for the upper dam materials and from seven to ten for the lower dam materials.

As a result of the 1971 earthquake, the upstream embankment and about 30 feet of the crest of the lower San Fernando Dam moved about 70 feet into the reservoir, the movement being due to liquefaction of the hydraulic fill near the base of the embankment. Liquefaction was evidenced by observed increases in pore pressure in the embankment, the occurrence of large horizontal displacements, and the formation of sand boils in the slide debris.

H.B. Seed, et al. (1973) concluded that a major catastrophe very nearly occurred. If any of a number of conditions had been less favorable, the lower dam could have failed resulting in the sudden release of 10,000 acre-feet of water over a heavily populated urban residential area. Laboratory tests and analytical techniques indicate that the instability of the lower dam was due to insufficient density in the water-saturated shell materials.

Other dams in the area were subjected to shaking of lesser intensity (0.2 g maximum acceleration) with no detrimental structural damage. Another hydraulic-fill dam with no water in the reservoir (unsaturated) was subjected to shaking of the same
intensity as the San Fernando dams with no detrimental damage. These observations indicate that hydraulic-fill dams are not inherently unstable but become so if the embankments are saturated and experience shaking of sufficient intensity and duration.

12.2.4.8 Ekofisk Tank in the North Sea

In June 1973 the 490,000-ton Ekofisk oil storage tank was installed in the North Sea (Lee and Focht, 1965). The structure is a reinforced concrete cylinder roughly circular in shape, about 93 meters in diameter, and 90 meters tall on a mat foundation in 70 meters of water. The tank imposes a vertical stress of about 2.72 kg/cm² on the foundation which is a 26-meter-thick stratum of medium-to-dense, fine, uniform sand. The sand particles were mostly angular in shape with a D50 of 0.11 millimeters and a uniformity coefficient of two. There was concern that the sand under the tank might liquefy due to stresses induced in the foundation by the action of storm waves against the walls of the tank.

The loading that the foundation of the tank would be subjected to was thought to be similar to earthquake loading with the following notable differences:

1. Storm waves have periods considerably longer than earthquake loadings.
2. The duration of ocean storms is much longer than that of earthquakes.
3. It is probable that a structure in the ocean would be subjected to a number of minor storms with intermittent
periods of calm before the occurrence of the design storm.

The structure was designed for a 100-year storm producing a wave 23.8 meters from crest to trough. The design wave would exert a lateral force of 35 million pounds on the tank.

Cyclic load triaxial tests of the type described by Seed and Lee (1965) were performed on the foundation material and the procedure described by Seed and Idriss (1970) used to evaluate liquefaction potential. Because loading from wave action was of a considerably longer period than for earthquakes, the rate of loading was reduced from 1 Hz, typical for earthquake tests, to 1/12 Hz. The laboratory test procedures were modified to account for the fact that during long ocean storms and during periods between storms, pore pressure dissipation would occur in the permeable sand. This was done by subjecting the test specimen to several cycles of undrained cyclic loading until some desired pore pressure response was reached, then stopping the loading and reconsolidating the specimens. After several cycles of such loading and reconsolidation, the test specimen was cyclically loaded to 100 percent pore pressure response (zero effective stress). It was found that the stability of the specimen increased substantially after several cycles of loading and reconsolidation. The increase in relative density due to reconsolidation was from one to four percent and would not explain the drastic increase in stability. Bjerrum (1973) mentioned that in addition to an increase in density, there was a change in the packing structure of the sand resulting from cyclic loading and reconsolidation.
which may explain the disproportionate increase in stability.

This study revealed that the supporting sands of this tank could withstand a 100-year storm without liquefaction. During the ten-month period following the installation of the tank, storms approaching the design storm occurred on at least three occasions with both the storage tank and foundation performing satisfactorily.

12.2.4.9 Failure of the Swir III Dam

During the spring of 1935 construction of the Swir III Dam in the Union of Soviet Socialist Republic was completed and the reservoir filled for the first time. The Swir III Dam was a concrete dam with a sand embankment next to the concrete section. The embankment, containing a till core, was formed by dumping moist sand on both sides of the core.

With the reservoir filled, blasting operations were being performed about 600 feet upstream of the dam when the sand embankment started to liquefy and flow away. According to Terzaghi (1956) the failure started at the contact surface between the concrete section of the dam and the embankment. In less than one minute, liquefaction had spread more than 1000 feet to the outer end of the embankment. The failure reduced the embankment slope from 1V on 2H before failure to 1V on 10H after failure. The embankment sand had been deposited in a moist state and was probably in a bulked and very loose condition. The embankment became saturated as the reservoir filled, and the blasting was evidently sufficient to trigger liquefaction.
12.2.4.10 Sand Boils at Operation Snowball

On July 17, 1964, a 500-ton hemispherical TNT charge on the ground was detonated over a glacially deposited silt with underlying beds of clay, sand, and gravel (Rooke, et al., 1968). The deposit consisted of a friable silty clay to a depth of 12 feet, a brown silty clay from a depth of 12-27 feet, and interbedded stratum of coarse sand and gravel ($D_{10} = 0.15 \text{ mm}$ and $C_u = 3-4$) from 27-32 feet, and gray silty clay from a depth of 32-67 feet. The water table was located 23 feet below the ground surface at the time of detonation.

Postshot surveys and reconnaissance reports (Rooke, et al., 1968) showed that the detonation produced a crater immediately after the shot measuring 14 feet deep, with a radius of 140 feet. High pore pressures due to compressive stresses were indicated by water flowing both inside and outside the crater. Sand boils were also observed to occur in and around the crater. The outflow of the water, sand, and silt caused subsidence of the area and circumferential tension cracks as far as 509 feet from the center of the crater. These descriptions and the occurrence of sand boils suggest that the underlying layers of sand and silt liquefied and flowed to the surface.

12.2.5 Discussion of Case Histories

An examination of the case histories reveals that with the exception of the Aberfan and Kansu flow slides, the materials liquefied were fine, uniform sands. Typically, $D_{10}$ sizes ranged from 0.05 to 1.0 mm and the coefficient of uniformity ranged
from less than 2 to 10. These observations are in agreement with, but broader than, the criteria of D'Appolonia (1968) and others (Seed and Idriss, 1970; Whitman, 1970; Terzaghi and Peck, 1948). D'Appolonia's criteria state that liquefaction-prone soils are cohesionless with less than ten percent fines, fine grained ($D_{50}$ between 0.2 and 1.0 mm), and uniform ($C_u$ between 2 and 5). In all cases the soils were loosely deposited, and in many cases they were water-saturated. However, water saturation is not absolutely necessary as soils with air as a pore fluid were reported to flow (Close and McCormick, 1922).

The case history review also shows that a variety of triggering mechanisms can cause liquefaction. These mechanisms cause collapse of the grain structure, resulting in high pore pressure and subsequent loss in strength and flow. They range from monotonically changing stresses caused by changes in externally applied loads to vibrations and shock waves caused by reciprocating machinery, earthquakes, or blast loadings. However, the most common triggering mechanisms are (1) monotonically changing stresses, such as those causing the Mississippi River and Fort Peck flow slides; and (2) earthquake-induced flow slides, such as those at Niigata and the San Fernando dams. Liquefaction flow slides induced by blasting or cyclic vibrations are less commonly observed and reported. Earthquake-induced liquefaction failures are generally more catastrophic than those induced by other mechanisms due to the tremendous energy released during an earthquake causing almost instant liquefaction over a large area. Conversely, flow slides induced by monotonically changing stresses generally are more limited in area and occur
less rapidly.

Sands which are initially loose and liquefy when subjected to earthquake shaking do not become significantly more stable against liquefaction. Ambraseys and Sarma (1969) suggested that deposits which are susceptible to liquefaction, e.g., 10-20 meters thick, have been accumulating in seismically active areas for thousands of years. It is highly improbable that such deposits have never been disturbed by shocks at some stage during their deposition. It is more probable that these deposits have been subjected to a number of earthquakes and have liquefied a number of times and are still susceptible to liquefaction. Historical records for Niigata show three occasions where liquefaction was reported in or near the city (Seed and Idriss, 1970). This would indicate that earthquakes are incapable of densifying originally loose deposits into a stable mass. This is pointed out by the fact that strong earthquake aftershocks have caused granular deposits to reliquefy after initial liquefaction during the earthquake. It seems then that the existence of deposits in seismically active areas which have been stabilized by earthquake shocks would be the exception rather than the rule. Apparently, after liquefaction occurs, the material subsides again to an unstable structure still quite susceptible to liquefaction.

Conversely, soils which are subjected to vibrations and experience an increase in pore pressure but not complete liquefaction gain stability against liquefaction far beyond what the increase in density after drainage occurs would suggest. Bjerrum (1973) suggests that the disproportionate increase in stability
stems from the fact that the structure of a sand is altered by vibrations.

12.3 Methods for Evaluating Liquefaction Potential of Soils

Newmark (1965) first proposed the basic concept and procedures for evaluating the potential deformation of an embankment dam due to earthquakes shaking. He assumed slope failure would not be developed and slip would not occur if the driving force caused by the earthquake is less than the resistance of the sliding mass. This resistance to earthquake shock motion is a function of the shearing resistance of material under an earthquake.

Newmark used this theoretical procedure to analyze three important cases of sliding which were modeled after a rock filled dam. The model tests show a fair agreement with the calculation for comparable conditions. However, the method has been proved to be useful only in cases where the yield resistance of the soil can be reliably determined (Seed, 1979). Newmark's method did not use the term "liquefaction," although it was obvious the yield resistance is affected by liquefaction potential of dam materials.

Current literature indicates that the most practical and broadly used methods for evaluating soil liquefaction potential of level grounds can be categorized into three groups: (1) Simplified procedures, (2) ground response analysis, and (3) empirical method. These will be described in the following sections.

12.3.1 Simplified Procedures

This method was developed in 1971 by Seed and Idriss (1971)
and also by Donovan (1971). The difference between these two approaches is mainly in the manner of using the distribution of stress cycles.

The Seed and Idriss' simplified procedure involves the computation of the equivalent uniform cyclic shear stress induced by the earthquake at any point in a soil deposit. The equivalent uniform cycles can be related to the duration of ground shaking and magnitude of the earthquake. On the other hand, the cyclic shear stresses required to cause liquefaction of a given soil in a given number of stress cycles can be determined by means of an appropriate test program. By comparing these two stresses, the liquefaction potential of a soil deposit can, therefore, be evaluated.

The Donovan's simplified procedure is described by the cumulative damage concept. The safety factor against liquefaction can be obtained based on the ratio of the shear stress required to cause liquefaction, or significant strain, to the shear stresses induced by the earthquake for the same equivalent number of cycles. With this method, the stress distribution developed during an earthquake can be scaled until a damage of unity is obtained.

12.3.2 Ground Response Analysis

Ground response analysis is based on the one-dimensional wave propagation theory used to obtain the induced dynamic shear stresses and their distribution and variation with time (Valera, 1977; Seed and Idriss, 1967). In this method, the potential for liquefaction of a soil deposit can be estimated by comparing the equivalent uniform cyclic shear stress obtained from the ground
response analysis to the laboratory test resistance.

12.3.3 Empirical Method

The empirical method involving the estimate of potential liquefaction is based on the past histories of sites where liquefaction did or did not occur during earthquakes. The standard penetration resistance of the soil is used to differentiate the liquefiable soils from non-liquefiable ones. This method was first developed in Japan after the severe structural damages caused by the Niigata earthquake in 1964.

Seed and Peacock (1971) presented the condition of various sites and their liquefaction histories and formed the most useful empirical correlation for the prediction of liquefaction potential of saturated sands. Generally speaking, this method relates the standard penetration resistance (the resistance of a soil) to the critical stress which causes the soil to liquefy in a number of cycles under an earthquake of a certain intensity. The standard penetration resistance of soils is affected by grain size and relative density (Ishihara, etc., 1978 and Tasuoka, etc., 1980).

12.4 Factors Affecting Liquefaction Potential

In the past two decades, numerous laboratory studies have been directed toward the investigation of the effect of a number of factors, such as relative density, gradation characteristics, strain history, overconsolidation, consolidation pressure, and anisotropy, etc. The effects of each individual factor are discussed in the following subsections.
12.4.1 Effect of Relative Density

Seed and Lee in 1967 first reported the significant effect of density on liquefaction. They concluded that initial liquefaction and complete liquefaction occurred almost simultaneously at relative density below approximately 50%. However, at relative density above approximately 70%, a considerable number of stress cycles was required after the initial liquefaction to develop a large strain amplitude. The stresses required to cause initial liquefaction increased linearly to about 60% relative density. In the simplified procedures of assessing liquefaction potentials, this linear relationship was used as a basis to evaluate the stresses causing liquefaction at the relative density below 80% (Seed and Idriss, 1971).

Fig. 12.10 clearly shows the importance of density in preventing large strains when cyclic stress loading is applied. For example, it may be noted that the cyclic stress required to cause a double amplitude of strain of 20% at a relative density of 75% (1.18 dg/cm²) is about four times that required at a relative density of 25% (0.29 kg/cm²). It is important to note that the most beneficial effects of densification in increasing the cyclic stress required to induce large strain amplitudes only develop above a relative density of about 70% for this sand. Thus, whereas an increase in relative density from 25% to 75% will increase the cyclic stress required to cause 20% strain amplitude in a given number of stress cycles by about 400%, an increase in relative density to 85% would increase the cyclic stresses required by
FIG. 12:70 CYCLIC STRESSES CAUSING DIFFERENT AXIAL STRAINS, $\sigma_3 = 1 \text{ kg/cm}^2$
(From Seed and Lee, 1967)
550% to 800% depending on the number of stress cycles involved. Additional densification has proportionately more beneficial effects.

Seed and Lee (1967) ran the previously discussed set of tests at higher confining pressures of 5 kg/cm² and 15 kg/cm² with results quite similar to a confining pressure of 1 kg/cm². One difference noted at a confining pressure of 15 kg/cm² was the fact that the difference between the stresses required to cause initial liquefaction and the stresses required to cause 20% strain amplitude becomes increasingly smaller. This difference in behavior may be attributed to the reduced ability of the sand to dilate and thereby stabilize itself at the higher confining pressures.

Other investigations on the effect of relative density also indicated that the stress ratio required to cause liquefaction in 10 cycles has a linear relationship with the relative density up to 60-70% (Peakcock and Seed, 1968). A comparison of the effect of relative density by using two types of testing methods is shown in Fig. 12.11.

Mulilis, et al. (1978) have presented data showing the cyclic strength of soils (Fig. 12.12). Slightly denser specimens were found to be significantly stronger. For Monterey No. 0 sand, the 12% increase in relative density can cause an increase in dynamic strength of approximately 22-30%.

12.4.2 Grain and Gradation Characteristics

Lee and Fitton (1969) performed a series of cyclic triaxial tests on alluvial sand and gravel deposits located at El Monte,
FIG. 12.11 COMPARISON OF SHAKING TABLE AND TRIAXIAL TEST RESULT (after DeAlba, et al., 1976)
FIG. 12.12 EFFECT OF DENSITY ON NUMBER OF SQUARE WAVE LOADING CYCLE TO CAUSE INITIAL LIQUEFACTION (after Mullins, et al., 1978)

\[ \Delta = 99.55 \text{ pcf} \quad D_r = 66.4\% \]

\[ \bullet = 98.39 \text{ pcf} \quad D_r = 59.5\% \]
California. Samples were remolded and compacted to either 50 or 75 percent relative density. The granular material obtained from the deposit was sieved in six separate uniform gradations with particle sizes ranging from 3/4 to 3/8 inch-diameter down to the finest gradation containing particles between the No. 100 and No. 200 sieve. Particles of El Monte sand were angular to sub-angular. Table 12.2 shows the types of soils tested and Fig. 12.13 their gradations.

Results of the cyclic triaxial tests were presented in the form of double amplitude axial strain and peak pore water pressure versus number of cycles. Fig. 12.14 shows that the fine soil (C) had virtually no strain during the first three loading cycles but on the fourth cycle rapid straining of the sample occurred with a corresponding rapid rise in the pore water pressure. By the fifth cycle soil (C) had liquefied. Soil (B) behaves similarly to soil (C) with the exception that soil (B) does not develop a rapid increase in pore pressure and large double amplitude strains until approximately the tenth loading cycle. Soil (A) on the other hand does not build up pore pressure rapidly following a certain amount of cycles but instead has a uniform build up of pore pressure. It also should be noted that sample (A) did not reach liquefaction and following approximately 150 cycles, the sample developed severe necking. Necking of the sample created a zone of concentrated stresses dissimilar to any field loading condition thereby making results from additional loading meaningless.

Several important facts which can be gained by examination
TABLE 12.2
SOILS TESTED BY LEE AND FITTON

<table>
<thead>
<tr>
<th>Code</th>
<th>Name and Description</th>
<th>Size Range</th>
<th>Cu</th>
<th>D&lt;sub&gt;50&lt;/sub&gt;</th>
<th>LL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>U.S. Sieve No.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AA</td>
<td>El Monte, medium gravel fraction</td>
<td>3/4 to 3/8</td>
<td>1.4</td>
<td>13.4</td>
<td>--</td>
</tr>
<tr>
<td>A</td>
<td>El Monte, fine fraction</td>
<td>3/8 to 4</td>
<td>1.4</td>
<td>6.5</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>El Monte, coarse sand fraction</td>
<td>4.76 to 2</td>
<td>1.5</td>
<td>3.0</td>
<td>--</td>
</tr>
<tr>
<td>C</td>
<td>El Monte, medium sand fraction</td>
<td>0.84 to 0.59</td>
<td>1.2</td>
<td>0.68</td>
<td>--</td>
</tr>
<tr>
<td>D</td>
<td>El Monte, fine sand fraction</td>
<td>0.297 to 0.149</td>
<td>1.4</td>
<td>0.22</td>
<td>--</td>
</tr>
<tr>
<td>E</td>
<td>El Monte, very fine sand fraction</td>
<td>0.148 to 0.074</td>
<td>1.5</td>
<td>0.10</td>
<td>--</td>
</tr>
<tr>
<td>F</td>
<td>Silt</td>
<td>-0.074</td>
<td>1.9</td>
<td>0.018</td>
<td>26</td>
</tr>
<tr>
<td>G</td>
<td>Silty clay</td>
<td>-0.149</td>
<td>23</td>
<td>0.0053</td>
<td>38</td>
</tr>
<tr>
<td>H</td>
<td>Silty sand, hydraulic fill from Puerto Montt, Chile</td>
<td>-10 to -2</td>
<td>10</td>
<td>0.065</td>
<td>NP</td>
</tr>
<tr>
<td>I</td>
<td>El Monte, pit run coarse silty sand</td>
<td>-4 to -4.76</td>
<td>7.5</td>
<td>0.40</td>
<td>--</td>
</tr>
<tr>
<td>J</td>
<td>Ottawa standard sand</td>
<td>0.84 to 0.59</td>
<td>1.2</td>
<td>0.68</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Cu = D<sub>60</sub>/D<sub>10</sub>. The coefficient of uniformity. D<sub>60</sub>, D<sub>50</sub>, and D<sub>10</sub> are the grain sizes of the 60, 50 and 10 percent sizes respectively.

<sup>b</sup> LL and Pi are the liquid and plastic limits respectively.
FIG. 12.13  GRADATIONS OF SOILS TESTED
BY LEE AND FITTON
(From Lee and Fitton, 1969)

FIG. 12.14  TYPICAL LOAD-DEFLECTION AND PORE
PRESSURE CHANGE CURVES FOR
DIFFERENT GRAIN SIZES
(From Lee and Fitton, 1969)
of Fig. 12.14 are (1) medium to coarse sands reach liquefaction in far less cycles than gravels and (2) once the pore pressure starts to build up in a medium coarse sand, sudden large strains can occur within several additional loading cycles as compared to gravel soils which have a relatively slow rate of straining and generally do not reach liquefaction.

Fig. 12.15 shows the cyclic deviator stress required to cause 5 and 20 percent strain in 30 cycles. Mean (or median) grain size \( D_{50} \) was used as an index for comparison. It should be noted that the cyclic strength decreased with decreasing mean grain size for both relative densities until the grain size was less than the No. 200 sieve. The effect of grain size distribution and grain shape was considered to be less significant than the grain size. The effect of segregation of soil particles on the liquefaction potential was also investigated (Lee and Fitton, 1969). When a well-graded soil was poured through water into the forming mold, severe segregation resulted with the larger sized particles rapidly settling through the water and finer sized particles slowly settling. As a result the segregated soil samples were approximately 10 percent stronger than the uniform sample.

Wong, et al. (1975) examined the liquefaction potential of gravelly soils. Since the grain particle for some specimens is greater than 1 in., cylindrical specimens with a diameter of 12 in. and a height of 29 in. were used. Their results showed that as the mean grain size increased from about 0.1 mm to 30 mm, the stress required to cause ±10% strain increases about 60%. However,
FIG. 12.15 COMPARISON OF PULSATING LOADING STRENGTHS OF DIFFERENT GRAIN-SIZE SOILS
(From Lee and Fitton, 1969)
contrary to their expectations, the well graded material was somewhat weaker than the uniform material. These surprising results were ascribed to a greater densification tendency at the small strain in the well graded material as fine particles move into the spaces between large particles. Samples with large uniform particles have little tendency to densify under small strain amplitudes. In addition, the greater effect of membrane compliance in the tests on uniformly graded gravels is also a contributing factor to their higher dynamic resistance.

Ishihara, et al. (1978) also examined the effect of grain size and percentage of fines in the reconstituted and undisturbed samples. They concluded that the pore pressure does not build up fully to become equal to the initial effective confining pressure for soils containing a large percentage of fines, even though the considerable axial strain may develop during cyclic loading. The maximum pore water pressure that developed as a result of the application of an infinite number of loading cycles will decrease with decreasing mean grain size. Therefore, they suggested that the adoption of "initial liquefaction" as a failure criterion for liquefaction is not quite suitable for a soil containing some fines.

Martin, et al. (1978) re-examined the effect of grain size on liquefaction by considering membrane compliance. They found that the membrane penetration effect for given size samples is dependent on the particle size and independent on the sample density. The test results of Lee and Fitton (1969) and Wong, et al. (1975) were
corrected based on their finding. The corrected results indicated that the relationship between the stress ratio and the number of cycles to the initial liquefaction is almost identical for 2.8 in. and 12 in. samples. Fig. 12.16 indicates that for the mean grain diameters greater than 0.1 mm, the effect of grain size on liquefaction does not seem to be significant. Tatsuoka, et al. (1980) used this relationship to assess the liquefaction potential of two sites located in Japan. There is, however, some deviation in their finding. The approach which takes into consideration of the effect of grain size produces a more reasonable solution.

12.4.3 Effect of Consolidation Pressure

In the 1964 Niigata earthquake, field observations showed that liquefied zones are usually located at fairly shallow depths. Only a few cases are below 50 to 60 feet. Kishida (1969) also reported that in this earthquake, liquefaction did not occur wherever the effective overburden stress exceeded 2 kg/cm² (27 psi).

The effect of consolidation pressure was compiled by Peacock, et al. (1968) and Finn, et al. (1971) from the earlier studies conducted by Lee and Seed (1966, 1967). Fig. 12.17 shows that the shear stress required to cause liquefaction in remolded sand specimens at relative density less than 80% varies linearly with consolidation pressure. "Stress ratio," the ratio of the shear stress causing liquefaction to the initial consolidation pressure, is used to represent the liquefaction potential of a soil.

Contrary to their expectations, Mulilis (1975) and Castro
FIG. 2.3 CYCLIC TRIAXIAL TEST RESULTS SHOWING INFLUENCE OF MEMBRANE PENETRATION EFFECTS (after Martin, et al., 1978)

FIG. 12.16 CYCLIC TRIAXIAL TEST RESULTS SHOWING INFLUENCE OF MEMBRAND PENETRATION EFFECTS (after Martin, et al., 1978)
FIG. 12.17 CYCLIC STRESSES CAUSING INITIAL LIQUEFACTION IN 100 CYCLES (From Seed and Lee, 1967)
and Poulos (1976) presented data showing that the stress ratio required to cause liquefaction decreases with increasing confining pressure. The magnitude of such decrease is dependent on the relative density, soil type and specimen preparation procedures (Castro and Poulos, 1976).

12.4.4 Effect of Anisotropy or Consolidation Ratio, $K_c$

Seed, et al. (1975) investigated the effect of anisotropy on the slide of the lower San Fernando Dam during the 1971 earthquake. They concluded that at a given radial pressure, the maximum deviatoric stress needed to produce a critical strain at a specified number of cycles increases with the increase of the consolidation ratio, $K_c$.

However, Castro and Poulos (1976) presented data showing opposite results. They found that samples consolidated under higher $K_c$ would require a smaller increment in stresses to cause liquefaction, because at a higher $K_c$, the specimen is closer to failure.

12.4.5 Effect of Overconsolidation

Seed and Peacock (1971) used a simple shear test to assess the effect of $K_o$ and overconsolidation ratio on the liquefaction potential of sands. It was found that, under the $K_o$ condition, when the overconsolidation ratio increases from 1 to 8, the stress ratio required to cause initial liquefaction also increases.

Ishihara, et al. (1978) present the results of liquefaction tests on reconstituted and undisturbed samples of Koto Sand.
Despite the content of fines in the test samples, the resistance of soil against liquefaction was found to increase with increasing overconsolidation ratio. For a specimen with a higher percentage of fines, the increase tends to be larger.

12.4.6 Effect of Strain History

Finn, et al. (1970) used a simple shear apparatus to examine the effect of pre-strain on the dynamic strength of Ottawa Sand. A specimen that has previously liquefied is more susceptible to reliquefaction despite its potential density change. Conversely, when a specimen subjected to small strain disturbance is reconsolidated before it has liquefied, and resubjected to cyclic loading, it appears to have more resistance to liquefaction. This phenomenon can be ascribed to the densification of the specimen which results in the rearrangement of soil particles. On the other hand, the specimen of reliquefied soil loses its strength because of the destruction of the stable soil structure and soil fabric.

Seed, et al. (1977) investigated the influence of seismic history on liquefaction of sands both analytically and experimentally. They emphasized that when soil is subjected to small strains prior to liquefaction, the resistance increases even though there is no significant change in density. Mori, et al. (1977) also examined the influence of sample disturbance on the dynamic strength of soils. The conclusion was the same, but the effects of prior strain history were lost when the samples were slightly disturbed during sampling.
12.4.7 Effect of Reconstituted Versus Intact Specimens

The reconstitution of a sample can duplicate the in-situ density but not the in-situ fabric. Fabric considerations are important because research has shown that different specimen preparation procedures and resulting differences in soil fabric can significantly influence measured soil strength values both under static and dynamic loading conditions (Mulilis, et al, 1976; Lade, 1976). Thus, specimens at the same density but prepared with different specimen preparation techniques may show different strength values. Correction factors should be applied to the strength of the reconstituted samples to obtain the cyclic strength of in-situ cohesionless soils.

Marshall L. Silver (1978) performed an extensive study of the differences in strength between undisturbed and reconstituted sand samples from Niigata, Japan. Undisturbed samples were obtained from the bank of the Shinano River, an area in which severe liquefaction occurred during the 1964 earthquake. This area was reclaimed by deposition of sand into the river channel approximately five years before the 1964 earthquake. Examination of undisturbed samples obtained from this area revealed that the horizontal layering of the samples was kept intact and lenses of coarse sand, fine sand, and silt were undisturbed, clear, and sharp.

Reconstituted specimens were prepared by pluviating sand through water and tamping the sides of the mold during filling to obtain the required density. Fig. 12.18 shows that at a given relative density, undisturbed specimens were stronger than reconstituted specimens.
FIG. 12.18 SUMMARY CURVE COMPARING THE CYCLIC STRENGTH AT 50 PERCENT RELATIVE DENSITY OF RECONSTITUTED AND UNDISTURBED SAND SPECIMENS FROM NIIGATA, JAPAN (From Silver, 1978)
The U.S. Army Corps of Engineers Waterways Experiment Station following a slide failure on Fort Peck Dam obtained undisturbed samples in the vicinity of the slide and compared their cyclic triaxial strength with those of the reconstituted sample (Marcuson, et al., 1976). In place samples were obtained by using a 3-inch Hvorslev fixed-piston sampler with the aid of drilling mud.

Test results of the shell and foundation material from Fort Peck Dam indicated that reconstitution of samples changes their cyclic triaxial strengths. The results show that this effect was dependent on the sampling procedure, the remolded sample preparation procedure and the soil characteristics, i.e. cementation and stratigraphy, and grain-size distribution. Undisturbed specimens of shell material were as much as 80% stronger than reconstituted specimens. The undisturbed foundation material was approximately 70% more stable than the same material in a reconstituted state.

In summary, cyclic triaxial tests show that the "undisturbed" samples are significantly stronger than the remolded samples. The resulting difference in strength between reconstituted and "undisturbed" samples will depend on the sampling procedure, the sample preparation procedure and the soil characteristics.

12.5 Experimental Methods for Evaluating Liquefaction Potential

Where the in-situ assessment of liquefaction potential is impossible, it is necessary to simulate the field conditions under seismic shaking by a laboratory apparatus. Currently, there are four experimental devices available for such purpose:

1) Cyclic triaxial test,
2) Cyclic simple shear test,
3) Torsional shear test, and
4) Shaking table test.

12.5.1 Cyclic Triaxial Test

The use of the cyclic triaxial test to predict the resistance of a soil to liquefaction when subjected to earthquake loadings was initiated in 1966 by Seed and Lee who produced in the laboratory, liquefaction of sand by cyclic loading. Since this time the cyclic triaxial test has grown to be the predominant test in evaluating the earthquake response of various soils (Seed, 1979). Most of the factors affecting the liquefaction potential of soil, as discussed in Section 12.4, were examined by using this test.

Similar to the static triaxial test, samples are first consolidated under a confining pressure and then subjected to a cyclic deviatoric stress under an undrained condition. The factors affecting cyclic triaxial test results are discussed in detail in Section 12.6.

12.5.2 Cyclic Simple Shear Test

Due to the limitation and deficiency of the cyclic triaxial test, Peacock and Seed (1968) developed the cyclic simple shear test device. A specimen which uses a rectangular sample is consolidated under $K_0$ condition by a vertical stress. A cyclic horizontal shear stress is then applied to the sample with drainage prohibited.

Early simple shear devices (Peacock and Seed, 1968) all
confronted with different degrees of difficulty, such as boundary friction, etc. These difficulties were partially removed by Seed and Peacock (1971) and Finn, et al. (1971).

Cyclic triaxial test and cyclic simple shear results were compared by Seed and Peacock (1979) and Park and Silver (1975) and De Alba, et al. (1976). Generally speaking, the cyclic simple shear strength was about 35 to 50 percent less than the comparable cyclic triaxial strength.

Although the cyclic simple shear test can more closely simulate the in-situ condition, it is not readily available. Besides, Peacock and Seed (1968) pointed out that the accurate simulation of field stress condition by a simple shear device is also limited by several factors. The cyclic triaxial test is more often used to evaluate the cyclic strength under the simple shear stress condition by reducing 35 to 50 percent from the cyclic triaxial strength.

12.5.3 Torsional Simple Shear Test

A torsional simple shear test was introduced by Ishibashi and Sherif (1974). A "donut-like" soil sample was used in the test. A detailed schematic of this apparatus can be found in their publications. From their experience, this torsional simple shear device has the following advantages:

1) It introduces a uniform shear stress and strain throughout a soil sample,

2) It is possible to apply vertical and horizontal stresses independently,
3) Lateral strain during a test can be prevented,
4) It enables the measurement of the pore-water pressure parameter B prior to the application of the cyclic loading, and
5) It closely simulates the in-situ stress-strain conditions during an earthquake.

A comparison was also made between the conventional simple shear test and the torsional simple shear test. It was concluded that the torsional simple shear test can reduce the frictional stresses along the sides of a conventional simple shear test device. And, thus, it appears to give a lower cyclic strength than the cyclic simple shear apparatus.

De Alba, et al. (1976) also indicated that data comparable to the representative data of simple shear field condition can be obtained with a carefully conducted small-scale torsional shear test by using good quality equipment.

12.5.4 Shaking Table Test

A shaking table provides a direct method for creating liquefaction under simple shear loading. It is widely used in Japan and Europe. However, such factors as sample placement, sample confinement, realistic overburden and deformation characteristics of the container do complicate the procedure (Ferrito and Forrest, 1976).

A comprehensive comparison of the results of the shaking table test with the simple shear test was presented by De Alba, et al. (1976). A very good agreement was found among the results reported by Seed and Peacock (1971), Finn (1972) and Yoshimi and Oh-Oka (1973).
12.6 Factor Affecting Cyclic Triaxial Test Results

Because of its convenience and efficiency a cyclic triaxial test has been widely used, in both research and consulting, for investigating the dynamic behavior of soils. Much effort has also been devoted to the investigation of the limitation and the effects of some factors on the result of the cyclic triaxial test. Since the cyclic triaxial test was used in this study, an extensive review of the effects of various factors on the cyclic triaxial test results was conducted and reported in the following subsections.

12.6.1 Sample Preparation Methods

A number of researchers (Seed, et al., 1975; Ladd, 1974, 1976; Mulilis, et al., 1977, 1978; ect.) have investigated the effects of sample preparation methods on liquefaction potential of sand.

An extensive study was performed by H.B. Seed, J.P. Mulilis, and C.K. Chan (1977) wherein undrained stress-controlled cyclic triaxial tests were performed on saturated samples of Monterey No. 0 sand compacted to a relative density of 50% by 11 different procedures listed as follows:

1) Pluviation through the air,
2) Pluviation through water,
3) High frequency (120 Hz) vibrations applied horizontally to dry samples formed in one seven-inch layer,
4) High frequency (120 Hz) vibrations applied horizontally to dry samples formed in seven one-inch layers,
5) High frequency (120 Hz) vibrations applied vertically to dry samples formed in seven one-inch layers,
6) High frequency (120 Hz) vibrations applied horizontally to moist samples \((w = 8\ \text{percent})\) formed in seven one-inch layers,

7) Low frequency (20 Hz) vibrations applied horizontally to dry samples formed in seven one-inch layers,

8) Low frequency (20 Hz) vibrations applied vertically to dry samples formed in seven one-inch layers,

9) Tamping moist soil \((w = 8\ \text{percent})\) with a 1.4-inch compaction foot to form samples in seven one-inch layers,

10) Rodding moist soil \((w = 8\ \text{percent})\) with a 3/8-inch compaction foot to form samples in seven one-inch layers, and

11) Rodding dry soil with a 3/8-inch compaction foot to form samples in seven one-inch layers.

Following its preparation, the specimen was saturated, consolidated under an effective confining pressure of 8 psi, and subjected to a sinusoidal cyclic deviatoric stress. Axial deformations, the number of cycles to reach liquefaction, and axial strains of \(\pm 2.5\), \(\pm 5.0\), and \(\pm 10.0\) percent were recorded. The resulting curves of the cyclic stress ratios versus number of cycles to initial liquefaction are shown in Fig. 12.19.

Fig. 12.19 shows that the weakest samples were formed by pluviating the soil through air with the strongest samples being formed by vibrating the soil in a moist condition. The difference in the cyclic stress ratio causing liquefaction is approximately 110% at 10 cycles and about 61% at 100 cycles. Using the same
FIG. 12.19 CYCLIC STRESS RATIO VERSUS NUMBER OF CYCLES TO INITIAL LIQUEFACTION FOR DIFFERENT COMPACTATION PROCEDURES (From Seed, 1975)
compaction methods, samples prepared in the moist condition were stronger than samples prepared in the dry condition. This is evidenced by the fact that samples formed by moist rodding were approximately 20% stronger than those formed by dry rodding at 10 and 100 cycles. Samples formed by moist vibration were between 55% and 75% stronger than those formed by dry vibration at 10 cycles.

In an attempt to investigate the reason for the effect of sample preparation methods on liquefaction potential, Mulilis, et al. (1977) also performed an investigation to determine the spatial distribution of the density within samples. Results of this study indicated that samples produced by air pluviation and low-frequency vibrations were very uniform; samples prepared in seven one-inch layers compacted by high-frequency vibrations were slightly less uniform; and samples placed in one seven-inch layer compacted by high-frequency vibrations were relatively nonuniform, and with a dense layer near the top and a loose layer at the middle of the samples. The loose layer may explain why the specimens prepared in one seven-inch layer had the lowest dynamic strength among those prepared by the three methods.

Mulilis, et al. (1978) presented the data obtained on specimens of Monterey No. 0 sand prepared by the moist-rodding and the dry-rodding methods. The result showed that the cyclic strength of the moist-rodding samples is approximately 58 percent stronger than that of the dry-rodding samples. The effect of rodding foot size was found insignificant.
Richard S. Ladd (1974) performed cyclic triaxial tests on three different sands prepared by either dry vibration or wet tamping. Results of this study shown in Fig. 12.20 also indicated that samples prepared by dry vibration were significantly weaker than those prepared by wet-tamping. At a specific number of cycles causing a peak-to-peak strain of 10%, the cyclic strength of samples prepared by dry vibration were approximately one-half that of samples prepared by wet tamping. In a similar study, Silver, et al. (1976) also reached the same conclusion.

Ladd (1977) compared the cyclic strengths of the samples prepared by moist vibration, moist tamping, dry tamping and dry vibration. The same conclusion as that of Mulilis, et al. (1977) was found.

12.6.2 Membrane Penetration

Cyclic triaxial test specimens are enclosed in thin cylindrical rubber membranes. When a confining stress is applied, the rubber membrane is forced into any interstices at the surface of the specimen as shown in Fig. 12.21. During an undrained cyclic triaxial test on a fully saturated sample, the volume of the specimen, i.e., the volume of the grain structure and the voids enclosed by the grains, is assumed to be constant. This assumption has been shown by several investigators (Manfred, et al., 1977; Lade, et al., 1977; etc.) to be violated for the following reason. At the beginning of an undrained cyclic triaxial test, when the excess pore pressure equals zero, the rubber membrane completely transmits the consolidation pressure to the grain
FIG. 12.20 APPLIED STRESS RATIO VERSUS NUMBER OF LOADING CYCLES REQUIRED TO OBTAIN PEAK-TO-PEAK AXIAL STRAIN OF 10 PERCENT (From Ladd, 1974)
FIG. 12.21 MEMBRANE PENETRATION INTO THE INTERSTICES OF A SAMPLE OF SAND DUE TO CHANGES OF EXCESS PORE-WATER PRESSURE, $\Delta u$
structure as shown in Fig. 12.21. During the test, a part of the consolidation pressure is transferred to the water as excess pore-water pressure, \( \Delta u \), and a change in the membrane penetration occurs. To satisfy volumetric compatibility this increased void volume along the surface of the sample due to membrane de-penetration must be accompanied by a corresponding volume decrease of the grain structure. In situ, however, the deformation of the grain structure during rapid loading may occur without volume change. Therefore the excess pore-water pressure and effective stress measured in a laboratory test where membrane penetration occurs may differ from those in situ. The volume change of the grain structure per square centimeter of membrane surface due to the membrane penetration is commonly referred to as membrane penetration.

Newland and Allely (1959) who tested samples composed of lead shot discovered the importance of the membrane effect. Assuming that the sample is isotropic, they suggested that the membrane penetration can be evaluated for the sample under an isotropic pressure as the difference between the observed volumetric strain and three times the measured axial strain. This method is questionable, however, due to possible anisotropy of sand.

Roscoe, Schofield, and Thurairajah (1963) investigated the membrane penetration by testing, under ambient pressure conditions, 1-1/2-inch diameter triaxial specimens containing central brass rods throughout the height of the sample. Rod diameters varying between 1/4 inch and 1-3/8 inch were used. The membrane penetration was obtained for each value of ambient pressure by plotting
volume change measured against rod diameter and extrapolating the resulting straight line to obtain the volume change for a 1.5-inch diameter sample.

Frydman, Zeitlen, and Alpan (1973) measured the membrane penetration in an undrained triaxial test where a membrane with a thickness of 0.2 mm was used. They found that while at a constant cell pressure of 1.0 kg/cm², when the excess pore-water pressure rose from zero to 0.5 kg/cm² the "volume of voids" along the surface of the specimen increased by $5 \times 10^{-3} \text{cm}^3/\text{cm}^2$ of membrane surface.

Wong, et al. (1975) raised the question of membrane penetration on the liquefaction potential of gravelly soils. To minimize this effect, a relatively thick membrane was used to reduce the amount of initial penetration into the irregular sample surface.

Manfred and Schuppener (1977) showed that, due to volumetric compatibility, the horizontal deformation of the grain structure of a specimen of 7.5-cm diameter will be $\varepsilon_h = 2 \times 2 \times 10^{-3}/7.5 = 5 \times 10^{-4} = 0.05\%$. If elastic behavior and a Young's modulus for sand of $E_s = 1000 \text{kg/cm}^2$ are assumed, the stresses of the grain structure corresponding to this deformation which will not be observed in situ, are $\sigma_h = \varepsilon_h \times E_s = 5 \times 10^{-4} \times 1000 = 0.5 \text{kg/cm}^2$. Therefore the error in pore pressure measurement will be in the order of 0.5 kg/cm².

Martin, et al. (1978) investigated the effect of system compliance on uniform sands. They concluded that the effect of membrane compliance for well graded sand samples would be less than that of uniformly graded samples. In addition, samples
containing a small proportion of gravel would produce a relatively large void on the sample surface, leading to a large increase in the apparent resistance to liquefaction.

In conclusion, in soils which contract during a cyclic loading test, test results not corrected for membrane penetration lead to an overestimation of undrained shear strength or an underestimation of the excess pore-water pressure. Also, the major factor influencing the membrane penetration for any applied cell pressure is particle size. The particle shape and distribution and the specimen density have minor effects.

12.6.3 Frictionless Caps and Bases

Vernese and Lee (1977) conducted an extensive research on the effect of "frictionless" caps and bases (ends) on the cyclic strength of Monterey No. 0 sand. The frictionless ends reduce the lateral end constraint and provide more uniform stresses throughout a sample.

"Frictionless" cap and base, as shown in Fig. 12.22, were placed on the top and bottom of a test specimen. Each "frictionless" end consisted of two layers of rubber, each 0.012-inch thick and each separated by a generous smear of high vacuum silicone grease. Long prongs, 1/8 inch in diameter and 0.5-inch long, were fixed in the center of the cap and base to keep the sample from sliding off the ends. The prongs were found to have no effect on the cyclic strength of Monterey No. 0 sand.

Samples of Monterey No. 0 sand placed at a relative density of 60 percent when tested using "frictionless" ends were
FIG. 12.22  FRICTIONLESS CAPS AND BASES USED IN UCLA STUDY 
(From Vernese, 1977)
approximately 10 to 20 percent stronger than those tested using regular ends as shown in Fig. 12.23. Fig. 12.24 shows that samples of Monterey No. 0 sand prepared to a relative density of 80 percent were 30 to 35 percent stronger when tested with "frictionless" ends as compared to regular ends. This is probably due to the fact that the samples with "frictionless" ends had more of a tendency to dilate because of the reduced lateral constraint.

Silver (1977) pointed out the fact that "frictionless" top and bottom platens constitute a special test condition and, therefore, recommended that "frictionless" top and bottom platens not be used in professional practice.

Vernese (1977) also suggested that there appeared to be no advantage in adopting the "frictionless" ends when performing tests to solve routine engineering design problems. In special research, however, Vernese suggested that it might be desirable to use "frictionless" ends.

12.6.4 Effect of Sample Size

The cap and base friction of a triaxial specimen might be different for samples of different diameters. Bishop and Green (1965) recommended a height-to-diameter ratio of 2. However, for samples with large-size particles, it is also necessary to use a large-size specimen. A sample diameter of six times the maximum particle size is recommended to avoid possible effects of sample geometry (Wong, Seed and Chan, 1975).

Lee and Fitton (1969) studied the difference in the cyclic
FIG. 12.23 COMPARISON OF CYCLIC STRENGTHS OF LOOSE MONTEREY SAND USING FRICTIONLESS AND REGULAR ENDS (From Vernese, 1977)

- Key:
  - Ends With Prongs: $K_c=1.0$, $D_i=60\%$, $\omega=1.0$ Hz
  - Ends Without Prongs: $K_c=1.0$, $D_i=60\%$, $\omega=1.0$ Hz

- Equation:
  - $K_v = 14.5$ psi
  - $D_i = 1.4$ in.

- Frictionless Ends

- Regular Ends Without Prongs

- Regular Ends With Prongs

- Numbers of Cycles to Failure:
  - 10
  - 30
  - 100
  - 300

- Cyclic Stress Ratio (C/S)*

* C/S = \( \frac{S_C}{S_0} \)
FIG. 12.24 COMPARISON OF CYCLIC STRENGTHS OF DENSE MONTEREY SAND USING FRICTIONLESS AND REGULAR ENDS (From Vernese, 1977)
strength of 1.4-inch and 2.8-inch diameter specimens of a coarse sand (No. 4 to No. 10 sieve size) and a fine sand (No. 50 to No. 100 sieve size). They concluded that the difference is insignificant with the 2.8-inch diameter samples being somewhat weaker.

Vernese and Lee (1977) performed a series of tests on Monterey Sand specimens having diameters of 1.4 and 2.8 inches. Results shown in Fig. 12.25 also indicated that there was no appreciable difference between the cyclic strengths of 1.4 and 2.8-inch diameter specimens.

Wong, Seed, and Chan (1975) performed a series of tests on Monterey Sand samples of 2.8-inch and 12-inch diameters to determine the effect of variations in sample size on the cyclic strength of sand.

Wong, Seed and Chan pointed out two things which might change the cyclic strength of sand as the specimen diameter is varied. First, the concentration of stress near the cap and base of a triaxial cell might vary with differing specimen diameters. This effect can be minimized by using a constant height/diameter ratio and the specimen diameter of at least six times the maximum particle size. Secondly, membrane penetration will vary as the diameter and thickness of the membrane changes.

Membrane penetration is caused by the small change in volume which occurs when the pore pressure within a specimen builds up. This forces the membrane to move outwards from the crevices among soil particles. This results in a decreased rate of pore pressure generation and, therefore, the sample becomes more resistant to
FIG. 12.25 COMPARISON OF STRENGTH DATA FROM CYCLIC TRIAXIAL TESTS ON LOOSE MONTEREY SAND (FROM VERNES, 1977)
cyclic loading. Effects of membrane penetration increase, for a
given membrane thickness, as the particle size increases. Corre-
spondingly its effect decreases, for a constant particle size
and membrane thickness, as the diameter of the specimen increases.
A detailed discussion of the effects of membrane compliance is
found in Section 12.6.2.

Results shown in Fig. 12.26 indicated that the cyclic devia-
toric stress required to cause the initial liquefaction in any
given number of stress cycles were approximately ten percent less
for the 12-inch diameter specimens than for the 2.8-inch diameter
specimens. This difference may largely be due to the effects of
membrane compliance, which, as previously discussed, would be
greatest for small-diameter specimens if the particle size and
membrane thickness remained constant.

12.6.5 Effect of Porewater Pressure Parameter "B"

The effect of Skempton's "B" parameter on the cyclic triaxial
strength was studied by Chaney (1978). As shown in Fig. 12.27,
"B" parameter ranging from 0.91 to 0.99 could significantly affect
the triaxial strength. The magnitude of such difference is
influenced by the density, the consolidation pressure and other
soil characteristics.

Conversely, Mulilis, et al. (1978) examined the effect of the
"B" parameter on the cyclic strength of Monterey No. 0 Sand. The
result of testing specimens of 60-percent relative density and
subjected to 14.5 psi initial confining pressure suggested that the
effect was minor compared with the effect of the sample preparation
FIG. 12.26 EFFECT OF SAMPLE SIZE ON CYCLIC STRESSES CAUSING INITIAL LIQUEFACTION ON MONTEREY SAND (From Wong, Seed and Chan, 1975)

Specimen diameter 12 inches
Relative Density = 60%
Initial Eff. Confining Pressure = 2.0 kg cm

Specimen diameter 2.8 inches

Number of Cycles

Cyclic Deviator Stress, \( \sigma_{dp} \) - kg/cm²

100

30

3

10

10

0.4

1.2

1.6

0.8

0.4
FIG. 12.27 EFFECT OF B-VALUE ON NUMBER OF LOAD CYCLES TO ACHIEVE $\varepsilon_{da} = 5\%$

STRAIN FOR LOOSE SAND $D_r = 53.5\%$ (after Chaney, 1978)
procedure.

12.6.6 Loading Wave Form

Available loading wave forms for the cyclic triaxial test of soils are: Sine, severe square, degraded square, and triangular waves, etc. The effect of the loading wave form on the cyclic strength of soils has been extensively studied (Lee and Fitton, 1969; Mulilis, Townsend and Horz, 1978, 1976; Silver, 1977). Results of the study by Silver (1977) shown in Fig. 12.28 indicated that the cyclic strength of a saturated sand is significantly affected by the loading wave form. Specimens tested with severe loading waves with a fast rise time showed the cyclic strength approximately 15% less than those tested with sine or degraded loading wave without fast rise time. The pore water pressure recorded during the test with severe square loading waves indicated that a stress wave was propagated through the specimen because of the instantaneous velocity change. This stress wave reflected in the form of pore pressure spikes. The more rapid generation of excess pore-water pressure caused the sample to liquefy in less number of cycles than that subjected to other loading wave forms.

Because of the rapid jump in excess pore-water pressure associated with severe square wave loading, Silver (1977) recommended that a sine wave loading or a degraded square wave with the rise time approximately 10% of the loading period be used instead.

Mulilis, et al. (1978) reported that the specimens tested using a nearly triangular loading wave and a sine loading wave were approximately 13 and 30 percent, respectively, stronger than
FIG. 12.28 EFFECT OF WAVE SHAPE ON THE CYCLIC TRIAXIAL STRENGTH OF THE TEST SAND (after Silver, 1976)
specimens tested using a severe square loading wave at high stress ratios.

12.6.7 Frequency

Minor effect in the frequency ranging from 0.01 Hz to 1 Hz has been reported (Lee and Fitton, 1969; Wong, et al., 1975). Lee and Fitton (1969) pointed out that the slower loading frequency appeared to have a slightly lower strength. Conversely, Wong presented data indicating that although it has a negligible effect in the frequency range of 20 cpm to 1 cpm, the slower frequency loading does give a slightly higher strength.

To investigate the effect of frequency on the cyclic strength, Samuelson (1981) tested Monterey No. 0 Sand samples with the relative density of 30 percent and under the confining pressure of 30 psi and the cyclic stress ration $R_s$ of 0.25. Frequencies ranging from 0.0001 Hz to 1.0 Hz were used. It was found that there was no significant effect at frequencies less than 0.01 Hz. However, the effect became increasingly significant as the frequency increases with the specimens tested with a high frequency loading waves being stronger.
CHAPTER 13

TEST EQUIPMENT, SAMPLE PREPARATION AND EXPERIMENTAL PROCEDURES

13.1 Test Equipment

13.1.1 MTS Closed-Loop Servo Electrohydraulic System

Three different types of cyclic loading systems are available: Electrohydraulic system, pneumatic system and mechanical system. An MTS closed-loop electrohydraulic system, Model 810 as shown in Fig. 13.1, was used for performing stress-controlled cyclic triaxial tests at various frequencies. The system has a hydraulic pump with a pump capacity of 3 gpm and a loading capacity of 20,000 pounds.

This system performs very well until the onset of liquefaction at which time an abrupt change in the specimen rigidity occurs. A large quantity of the hydraulic fluid (oil) must be furnished at a fast rate during liquefaction to allow the loading piston to follow the sudden and rapid specimen deformation and maintain a constant stress amplitude. Such a rapid rate of fluid flow might exceed the hydraulic pump capacity. For this reason the amplitude of the cyclic load decreases as the specimen approaches liquefaction. Besides, it seems irrational to demand the machine actuator to follow the behavior of a soil specimen which is essentially failed.
FIG. 13.1 CLOSED LOOP ELECTRO-HYDRAULIC MATERIALS TEST SYSTEM APPLYING A SINUSOIDAL LOADING TO A MONTEREY NO. 0 SAND SPECIMEN
Two levels of operation are provided: An output pressure of 300 psi for the low or bypass condition and an output pressure of 3000 psi for the high condition.

A fluid-to-water heat exchanger is used to dissipate the temperature built up in the hydraulic power supply system. A temperature-sensitive switch will prevent the hydraulic fluid temperature from exceeding a predetermined limit.

The hydraulic actuator is the force-generating and/or positioning device in the system. Movement of the loading piston is the direct result of the application of fluid pressure to one side of the piston. A load applied to some external reaction point by the piston is equal to the effective piston area times the activating pressure.

The hydraulic actuator is controlled by the opening and closing of a servovalve in response to a control signal from the valve driver or controller. The servovalve can open in either of two positions, thereby permitting high pressure fluid to enter into either side of the piston. This alternating application of hydraulic pressure to either side of the piston makes it possible to apply smooth cyclic tension and compressive loads to a test specimen. When the servovalve is opened to allow fluid to flow into one end of the cylinder, the valve on the opposite end of the cylinder is opened to provide a path for fluid to flow back to the hydraulic power supply.

The rate of fluid flow through the servovalve is in direct proportion to the magnitude of the control signal. The polarity
of the control signal determines which end of the actuator cylinder will receive additional fluid thereby determining the direction of the piston stroke.

The load cell is a force-measuring transducer that provides an output voltage directly proportional to the applied load.

The linear displacement of the loading ram is measured by a linear variable differential transformer (LVDT). The LVDT requires a-c excitation and provides an a-c output. The amplitude of the output varies in direct proportion to the amount of displacement of the LVDT core.

Transducer conditioners supply excitation voltages to their respective transducers and control the transducer output voltages to d-c levels suitable for use in the control portion of the system. Output of each transducer conditioner is ±10 volts at 100% of the operating range which is ±20,000 pounds or ±5 inches under the stress and stroke control modes, respectively.

The servo controller compares the command and feedback inputs and provides an error signal which opens the servovalve in a direction and by an amount which causes the hydraulic actuator to correct the error. When the error signal is reduced to zero, the servovalve closes, and the system is in a state of equilibrium. Cyclic wave forms such as sine, haversine, and haversquare and various ramp functions are available. The inversion of the cyclic wave forms are also available. Frequencies range from 0.00001 to 990 Hz. The ramp functions include ramp, dual slope, triangle, saw tooth, and trapezoid. The ramp functions are adjustable from 0.001 second to 11.45 days.
Electro-mechanical counters can be programmed to stop the test after a predetermined number of these events.

The counter has a capacity of 999,999 counts at a maximum frequency of 25 Hz. A $x_{10}$ and $x_{100}$ multiplier can be utilized to increase the counters capacity wherein only every 10th or 100th event is counted.

13.1.2 Data Acquisition System

The axial load on the specimen, pore pressure within the specimen, and vertical deformation of the specimen were recorded during a cyclic triaxial test.

An X-Y recorder was utilized for recording the axial load versus vertical deformation relation of a specimen and the associated hysterisis loop.

The pore water pressure within the specimen and the vertical deformation of the specimen were recorded on an Esterline Angus X-Y-Y' 540 recorder as shown in Fig. 13.1. The X axis was established as the sweep axis. The rate of sweep is dependent upon the estimated amount of time required to complete a test. It should give adequate resolution to high frequency tests and be set to a slower rate of sweep during low frequency tests.

A Tektronix Power Supply and Digital Multimeter were used to supply power and measure the output voltage from the pore pressure transducer which is also manufactured by Tektronix. The Digital Multimeter in connection with the pore water pressure transducer was also used extensively to measure the cell pressure and back pressure during saturation and consolidation of a test specimen.
Fig. 13.2 shows the triaxial cell, MTS loading equipment, and data acquisition system as seen during a typical cyclic triaxial test.

13.1.3 Triaxial Testing System

The triaxial cell shown in Fig. 13.3 was used in performing all cyclic triaxial tests. A pore pressure transducer attached to a four-way valve plugged into the right side of the triaxial cell base was used to measure the cell pressure, back pressure, and pore water pressure at the top and/or the bottom of a specimen. The pore pressure transducer was attached to a four-way valve such that by changing the position of the valve the transducer could be subjected to either the cell pressure, back pressure, or pore water pressure. Fig. 13.4 shows the transducer and four-way valve disconnected from the triaxial cell.

On the left side of Fig. 13.3, four pressure lines are seen. The upper horizontal line entering the base of the triaxial cell applied the confining cell pressure, with the line passing behind the cell and over to the four-way valve used for transmitting the cell pressure to the pore pressure transducer. The middle line entering the base of the triaxial cell applied the back pressure to the specimen with the lower line used for transmitting the pore pressure from the top of the specimen to the four-way valve and pressure transducer.

The pressure control panel shown in Fig. 13.5 could be used to apply cell and back pressures simultaneously to three triaxial cells. An air compressor connected to the pressure control panel furnished a maximum air pressure of 175 psi for the pressure
FIG. 13.2 TRIAXIAL CELL, CLOSED LOOP ELECTRO-HYDRAULIC MATERIALS TEST SYSTEM, AND DATA ACQUISITION SYSTEM

FIG. 13.3 TEST SPECIMEN PLACED WITHIN THE TRIAXIAL CELL
FIG. 13.4 Pore pressure transducer and a 4-way valve used to monitor cell pressure, back pressure and specimen pore water pressure.

FIG. 13.5 Pressure control panel used to apply cell and back pressure to the triaxial cell. Graduated burettes were used to determine specimen volume change.
control panel. When the pressure in the air tank dropped below a specified minimum value a pressure switch would turn on the compressor.

The graduated burettes (vertical standpipes) shown in Fig. 13.5 were used to measure specimen volume changes within an accuracy of 0.1 cubic centimeter. Specimen volume change measurements were made after consolidation of the test specimen and after attaching the loading ram to the specimen top cap.

13.2 Sample Preparation

Numerous researchers (Seed, 1975; Lade, 1976 and Mulilis, 1977) have shown that sand samples prepared by different sample preparation methods to the same relative density have different liquefaction potentials. Studies, including the use of qualitative x-ray analysis, indicate differences in dynamic strength of test specimens might be associated with the difference in the orientation of the sand grain contact and the uniformity of compaction received by each specimen.

13.2.1 Test Sand and Test Program

A commercially available Denver sand, a river sand, purchased from the Denver Sand and Gravel Company was selected as the test material. Both coarse and fine grains of Denver sand are subangular in shape as shown in Figs. 2.1 and 2.2. Sixteen U.S. standard sieves with opening from 3/8 in. to 38 microns (U.S. standard sieve No. 400) were used to segregate different size particles. Following the pre-selected gradation curves shown in Fig. 2.4, samples with
different mean grain diameters and/or different uniformity coefficients were prepared accordingly. To assure each sample was well mixed, a splitter was used. Samples of twenty-four different gradations were prepared. Two-inch diameter specimens were used in this study. To satisfy the rule of thumb of the maximum grain size not being greater than one-sixth of the specimen diameter, the maximum grain size is limited to 3/8". In addition, no samples contain more than 20% silt. Under these restrictions, the mean grain size of samples ranges from 0.149 mm to 1.68 mm. Their coefficients of uniformity range from 2 to 15. Each individual specimen, after completion of a cyclic test, was subjected to sieve analysis to check its grain size distribution. The index properties of the test samples were given in Table 13.1, in which the maximum and minimum densities were evaluated by following the procedure given in Earth Manual (E-12, 2nd Edition, 1974) published by the U.S. Bureau of Reclamation.

After reviewing various sample preparation methods, a zero raining device was adopted to place the sand. As shown in Table 13.2, the mesh openings of zero-raining devices increase with the increase in the maximum grain size of a specimen. It is believed that specimens prepared by using the zero raining device were uniform and without serious segregation. This is evidenced by the visual examination of each test specimen and by the repeatability of the cyclic triaxial test results. Although the desired sample relative density after consolidation was 50%, the relative density achieved for the test specimen ranged from 49.6% to 59.3%.
<table>
<thead>
<tr>
<th>Sample No.*</th>
<th>Max. Density (pcf)</th>
<th>Min. Density (pcf)</th>
<th>Specific Gravity**</th>
<th>(D_{50}), in.</th>
<th>Uniformity Coefficient</th>
<th>(e_{\text{max}})</th>
<th>(e_{\text{min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-a₂</td>
<td>104.89</td>
<td>87.03</td>
<td>2.66</td>
<td>1.50</td>
<td>1.89</td>
<td>0.91</td>
<td>0.58</td>
</tr>
<tr>
<td>DC-b₂</td>
<td>104.78</td>
<td>88.51</td>
<td>2.66</td>
<td>0.82</td>
<td>2.17</td>
<td>0.87</td>
<td>0.58</td>
</tr>
<tr>
<td>DM-c₂</td>
<td>105.74</td>
<td>87.03</td>
<td>2.66</td>
<td>0.53</td>
<td>2.16</td>
<td>0.91</td>
<td>0.57</td>
</tr>
<tr>
<td>DM-d₂</td>
<td>106.08</td>
<td>85.05</td>
<td>2.66</td>
<td>0.38</td>
<td>2.20</td>
<td>0.95</td>
<td>0.56</td>
</tr>
<tr>
<td>DM-e₂</td>
<td>106.38</td>
<td>82.67</td>
<td>2.66</td>
<td>0.24</td>
<td>2.88</td>
<td>1.01</td>
<td>0.56</td>
</tr>
<tr>
<td>DF-f₂</td>
<td>102.95</td>
<td>79.60</td>
<td>2.66</td>
<td>0.15</td>
<td>2.70</td>
<td>1.09</td>
<td>0.61</td>
</tr>
<tr>
<td>DC-a₄</td>
<td>114.80</td>
<td>94.46</td>
<td>2.66</td>
<td>1.44</td>
<td>3.73</td>
<td>0.76</td>
<td>0.45</td>
</tr>
<tr>
<td>DC-b₄</td>
<td>116.00</td>
<td>95.45</td>
<td>2.66</td>
<td>0.68</td>
<td>3.71</td>
<td>0.74</td>
<td>0.43</td>
</tr>
<tr>
<td>DM-c₄</td>
<td>118.36</td>
<td>93.96</td>
<td>2.66</td>
<td>0.51</td>
<td>4.31</td>
<td>0.77</td>
<td>0.40</td>
</tr>
<tr>
<td>DM-d₄</td>
<td>115.75</td>
<td>91.98</td>
<td>2.66</td>
<td>0.37</td>
<td>4.93</td>
<td>0.80</td>
<td>0.43</td>
</tr>
<tr>
<td>DM-e₄</td>
<td>116.92</td>
<td>90.50</td>
<td>2.66</td>
<td>0.23</td>
<td>5.15</td>
<td>0.77</td>
<td>0.42</td>
</tr>
<tr>
<td>DF-f₃</td>
<td>108.87</td>
<td>84.06</td>
<td>2.66</td>
<td>0.14</td>
<td>3.40</td>
<td>0.97</td>
<td>0.52</td>
</tr>
<tr>
<td>DC-a₆</td>
<td>121.24</td>
<td>98.91</td>
<td>2.66</td>
<td>1.17</td>
<td>5.50</td>
<td>0.68</td>
<td>0.37</td>
</tr>
<tr>
<td>DC-b₆</td>
<td>121.33</td>
<td>99.41</td>
<td>2.66</td>
<td>0.62</td>
<td>5.70</td>
<td>0.67</td>
<td>0.37</td>
</tr>
<tr>
<td>DM-c₆</td>
<td>122.59</td>
<td>99.40</td>
<td>2.66</td>
<td>0.48</td>
<td>7.20</td>
<td>0.67</td>
<td>0.35</td>
</tr>
<tr>
<td>DM-d₆</td>
<td>121.78</td>
<td>96.93</td>
<td>2.66</td>
<td>0.40</td>
<td>7.00</td>
<td>0.71</td>
<td>0.36</td>
</tr>
<tr>
<td>DM-e₆</td>
<td>119.67</td>
<td>94.55</td>
<td>2.66</td>
<td>0.25</td>
<td>7.50</td>
<td>0.76</td>
<td>0.39</td>
</tr>
<tr>
<td>DC-b₈</td>
<td>123.96</td>
<td>102.38</td>
<td>2.66</td>
<td>0.69</td>
<td>9.50</td>
<td>0.62</td>
<td>0.34</td>
</tr>
<tr>
<td>DM-c₈</td>
<td>125.64</td>
<td>101.39</td>
<td>2.66</td>
<td>0.52</td>
<td>9.00</td>
<td>0.64</td>
<td>0.32</td>
</tr>
<tr>
<td>DM-d₈</td>
<td>123.35</td>
<td>98.91</td>
<td>2.66</td>
<td>0.37</td>
<td>9.70</td>
<td>0.68</td>
<td>0.35</td>
</tr>
<tr>
<td>DC-b₁₀</td>
<td>126.23</td>
<td>103.86</td>
<td>2.66</td>
<td>0.62</td>
<td>10.30</td>
<td>0.60</td>
<td>0.31</td>
</tr>
<tr>
<td>DM-c₁₀</td>
<td>127.41</td>
<td>103.47</td>
<td>2.66</td>
<td>0.57</td>
<td>11.30</td>
<td>0.60</td>
<td>0.30</td>
</tr>
<tr>
<td>DM-d₁₀</td>
<td>125.94</td>
<td>101.88</td>
<td>2.66</td>
<td>0.37</td>
<td>12.00</td>
<td>0.63</td>
<td>0.32</td>
</tr>
<tr>
<td>DM-c₁₅</td>
<td>129.21</td>
<td>105.35</td>
<td>2.66</td>
<td>0.35</td>
<td>16.00</td>
<td>0.58</td>
<td>0.28</td>
</tr>
</tbody>
</table>

*Sample No. provides information on the gradation characteristics. For instance, a₂ denotes sample with median grain size of "a" (1.50 mm) and a designed uniformity coefficient of 2 as shown in Figure 4.2.

**Specific gravity was assumed to be the same based on test results on c₂, c₄, c₆, c₁₀.
<table>
<thead>
<tr>
<th>Mesh No.</th>
<th>Diameter (in)</th>
<th>Opening (in)</th>
<th>Test Samples**</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.080</td>
<td>0.2533</td>
<td>*a_2, b_4, b_6, c_6, c_8, c_{10}, d_6, d_8, d_{10}</td>
</tr>
<tr>
<td>6</td>
<td>0.035</td>
<td>0.1317</td>
<td>b_2, c_2, c_4, d_4, e_6</td>
</tr>
<tr>
<td>10</td>
<td>0.028</td>
<td>0.0720</td>
<td>d_2, e_2, e_4</td>
</tr>
<tr>
<td>16</td>
<td>0.018</td>
<td>0.0445</td>
<td>f_2, f_3</td>
</tr>
</tbody>
</table>

*Sample No. provides information on its gradation characteristics. For instance, \*a_2 implies the sample with mean grain size of "a" (1.50 mm) and a designed uniformity coefficient of 2 as shown in Figure 2.4.

**For samples not listed in this table, zero raining device was not used.
To nicely define the curve of the stress ratio versus number of cycles to reach the initial liquefaction requires a minimum of four samples for each gradation shown in Fig. 2.4. A total of 123 cyclic triaxial test specimens were prepared and tested in this study.

13.2.2 Forming the Test Specimen

Before placing the lower porous stone and filter paper on the base of the triaxial cell shown in Fig. 13.6, the base of the triaxial cell was carefully wiped off with a clean rag to remove any standing water, particularly on the pedestal on which the lower porous stone was placed. This prevented the possibility of water from being raised up by capillary forces through the bottom porous stone and wetting the filter paper and sand placed on top of the porous stone.

The quick coupler and line in the base of the triaxial cell, which were connected to the top cap via a small diameter polyethylene flexible tube were also dried. This allows samples to remain dry under vacuum during the specimen preparation.

To save time, it is very important to make sure that the rubber membrane is flawless before use. To detect holes in a rubber membrane before it is placed around a specimen, the rubber membrane can be inflated with water as shown in Fig. 13.7. The triaxial cell back pressure line was connected to a water supply line and the membrane was slowly inflated with water. When the membrane had been inflated with water, the outside surface of the membrane was wiped off with a dry cloth so that newly formed drops of water passing through the membrane could be detected.
FIG. 13.6 LOOKING DOWN ON THE BASE OF THE TRIAXIAL CELL

FIG. 13.7 CHECKING FOR LEAKS IN THE RUBBER MEMBRANE
After a membrane has been tested and found to contain no holes, the inside and outside surfaces of the membrane should be wiped off with a cloth. The membrane should be completely dry before the sand specimen is formed to prevent sand from adhering to the inside of the membrane.

A dry porous stone was placed on the base pedestal of the triaxial cell and a circular filter paper placed on top of the porous stone. The disc of filter paper must be cut to the right size, neither larger nor smaller than the porous stone. If the filter paper disc is larger than the porous stone, the edge of the filter paper will be lifted up by the surrounding rubber membrane resulting in a reduction in sample diameter at its contact with the porous stone. Conversely, if the paper disc is smaller in diameter than the porous stone, sand can come in direct contact with the porous stone around the perimeter of the filter paper disc.

Thickness of the rubber membrane was measured at each end of the membrane using the vernier caliper shown in Fig. 13.8. An average was taken of the thicknesses measured at both ends of the membrane. The total thickness of the top porous stone, filter paper, and top cap was also measured with the vernier caliper. This was later used in calculating the length of the specimen.

The height of the lower filter paper disc above the bottom of the triaxial cell base was measured to within 0.001 inch using the vernier caliper shown in Fig. 13.9. Measurements were taken
FIG. 13.8 VERNIER CALIPER USED TO MEASURE THE THICKNESS OF THE MEMBRANE AND DIAMETER OF THE SAND SPECIMEN

FIG. 13.9 VERNIER CALIPER TO MEASURE THE HEIGHT OF THE TEST SPECIMEN
from three sides of the triaxial cell and their average was calculated and recorded.

The rubber membrane was carefully placed over the pedestal of the triaxial cell base as shown in Fig. 13.10 with any excess length of rubber membrane carefully folded up around the pedestal of the triaxial cell. Care was exercised such that the sides of the membrane were as vertical as possible with no unequal stretching of the membrane.

An O-ring was placed over the rubber membrane using the O-ring expander shown in Fig. 13.11. The O-ring was seated in the lower of the two grooves formed in the pedestal of the triaxial cell. This provided a water tight seal between the membrane and the pedestal of the triaxial cell.

Forming of the sand specimen was facilitated by use of the 2.020-inch diameter by 5-inch long split mold shown in Fig. 13.12. The split mold was positioned on the base of the triaxial cell with a clamp placed around the mold to hold the two halves of the mold tightly in place. A 20-inch mercury vacuum was applied to the void between the mold and the membrane. By placing four fingers within the top of the rubber membrane and equally stretching outward, the membrane was stretched over the top of the mold as shown in Fig. 13.13. With the membrane supported by the cylindrical mold a smooth inner surface of approximately 2 inches in diameter was provided.

Based on the required relative density of the specimen and the known volume of the mold, it was possible to calculate the
FIG. 13.10 RUBBER MEMBRANE PLACED ON THE BASE OF THE TRIAXIAL CELL

FIG. 13.11 RUBBER O-RING AND O-RING EXPANDER
FIG. 13.12 CYLINDRICAL MOLD USED IN FORMING THE SAND SPECIMENS

FIG. 13.13 CYLINDRICAL MOLD PlACED ON THE BASE OF THE TRIAXIAL CELL
required weight of sand to be placed in the mold. The required amount of sand was weighed in the Ohaus Dial a Gram Scale shown in Fig. 13.14. This scale had a maximum capacity of 2610 grams and an accuracy of ±0.1 gram.

A zero raining device shown in Fig. 13.15 was used in placing the sand within the mold. The zero raining device was placed in the mold and the sand previously weighed was poured into the zero raining device as shown in Fig. 13.16. By slowly and smoothly lifting up the zero raining device, the sand particles fell through the screened opening on the end of the zero raining device.

To ensure a direct and firm contact between the filter paper and the bottom porous stone, a vacuum was applied to the porous stone prior to placing any sand within the mold. After approximately 1/16-inch layer of sand was poured over the lower filter paper disc and the edges of the filter paper disc gently tamped with a screw driver, the filter paper disc was in all cases directly in contact with the porous stone. The vacuum on the lower porous stone was shut off with the remaining portion of the sand specimen placed with the zero raining device as previously described.

Once the sand had been placed within the mold, it was necessary to level off the top surface of the specimen so that the top porous stone could be placed perpendicular to the longitudinal axis of the specimen. Silver (1977) recommended that the maximum tilting of the specimen top be limited to 0.002 times the diameter of the specimen as shown in Fig. 13.18. Leveling of the top of the specimen was performed by placing the top porous stone upon the
FIG. 13.14 SCALE USED TO WEIGH THE MONTEREY NO. 0 SAND

FIG. 13.15 ZERO RAINING DEVICE USED TO PLACE THE MONTEREY NO. 0 SAND
FIG. 13.16 MONTEREY NO. 0 SAND PLACED WITHIN THE ZERO RAINING DEVICE

FIG. 13.17 POROUS STONE PLACED ON TOP OF THE MONTEREY NO. 0 SAND SPECIMEN
Fig. 13.18 Requirements for flatness and parallelism of specimen ends (From Silver, 1977)
sand surface, and gently rotating the porous stone in a clockwise-counterclockwise direction. A filter paper disc was then placed in direct contact with the top of the sand specimen. The top porous stone was then placed on top of the filter paper disc as shown in Fig. 13.17, with a level shown in Fig. 13.19 used in positioning the porous stone parallel to the base of the triaxial cell.

Prior to placing the top cap on the specimen, the threaded stud on the top cap was lubricated with grease along with the female connection on the loading ram. Lubricating these connections facilitated the later attachment of the loading ram to the loading cap.

The top cap was positioned over the porous stone with the rubber membrane carefully pulled up around the top cap. Generally there was an excess length of rubber membrane extending above the top cap. This excess amount of membrane was folded back over itself such that there was no membrane extending above the surface of the top cap as shown in Fig. 13.20.

The top cap was checked as shown in Fig. 13.21 to assure that the top surface of the cap was parallel to the base of the triaxial cell. The plastic tubing (1/8-inch diameter) leading from the top cap was connected to the base of the triaxial cell and a 10-inch mercury vacuum (-5 psi) was applied through the back pressure line to the sand specimen. Once the vacuum had been applied to the specimen it was possible to remove the mold from around the specimen.
FIG. 13.19 LEVELING THE TOP POROUS STONE SO THAT THE TOP POROUS STONE IS PARALLEL TO THE BASE OF THE TRIAXIAL CELL

FIG. 13.20 TOP CAP PLACED ON TOP OF THE SPECIMEN WITH THE RUBBER MEMBRANE PULLED UP AROUND THE TOP CAP
FIG. 13.21 POSITIONING OF THE TOP SURFACE OF THE LOADING CAP PARALLEL TO THE BASE OF THE TRIAXIAL CELL

FIG. 13.22 MEASURING THE LENGTH OF THE SAMPLE USING A VERNIER CALIPER
13.2.3: Measuring the Diameter and Length of a Specimen

The accurate measurement of the physical dimensions of the specimen is very critical as minor errors may significantly affect the calculated dry unit weight of the specimen. Extra care should be used during this step to make sure that the initial diameter and height of the specimen, \( D_0 \) and \( H_0 \), respectively, are measured to an accuracy no less than \( \pm 0.001 \) inch.

An average diameter of the specimen must be determined following the measurement of diameters at various locations. In this research the diameter was measured twice at 90° apart between readings at the top 1/3 point, middle, and bottom 1/3 point. The average diameter was then obtained by dividing the sum of the six diameter measurements by six.

The U.S. Army Corps of Engineers (MacIver, et al., 1970) used the following equation for evaluating the average diameter:

\[
D_0 = \frac{D_t + 2D_c + D_b}{4}
\]

where

- \( D_0 \) = average diameter
- \( D_t \) = diameter at the top
- \( D_c \) = diameter at the center
- \( D_b \) = diameter at the bottom

A \( \pi \)-tape, a thin steel circumferential tape, was initially used to measure specimen diameters. It was found, though, that with the low specimen density, small specimen diameter, and small vacuum, the \( \pi \)-tape formed a groove in the specimen when it was wrapped around the specimen. Care had to be exercised so that the \( \pi \)-tape was placed perfectly parallel to the top cap and the
base pedestal of a triaxial cell and directly in contact with the specimen or erroneous measurements were obtained.

Due to the difficulty in obtaining accurate diameter measurements with the π-tape, a vernier caliper was used instead. While several investigators (Silver, 1977; Mulilis, 1978) have recommended against the use of a vernier caliper, our experience indicated that accurate measurements can be obtained if proper care is exercised when using a vernier caliper.

The importance of a properly measured specimen diameter on the evaluation of dry unit weight can be illustrated by the difference in dry unit weights resulting from an error in specimen diameter of only 0.01 inch. If the diameter of a 2.000-inch diameter by 4.000-inch high Monterey Number 0 sand specimen prepared to a relative density of 50% is erroneously measured as 2.01 inch an error in the dry unit weight of approximately 0.98 lb/ft³ results. The calculated relative density of the specimen (42.9%) in this example will be approximately 7.1 percent less than the true relative density (50.0%). When this specimen is cyclically loaded it will have a greater cyclic strength than would have been estimated based on the erroneously calculated relative density.

Mulilis, et al. (1976) found that a difference in the dry unit weight of Monterey Number 0 sand of only 1.2 lb/ft³ could result in a difference of 22-33 percent in the cyclic stress ratio required to reach initial liquefaction as shown in Fig. 13.23. It can be expected that an error in dry unit weight of 0.98 lb/ft³ would correspondingly result in a difference of 18-27 percent in the
Fig. 13.23 Effect of specimen density on cyclic triaxial strength of Monterey No. 0 Sand

(From Mulilis, Horz and Townsend, 1976)
cyclic stress ratio causing initial liquefaction. Errors of this magnitude severely distort the cyclic triaxial test results and must be reduced by accurate measurements of the specimen diameter.

Careful measurements of the specimen height, $H_o$, should be made at four locations, 90 degrees from one another. Silver (1977) recommends that a dial gage and gage stand be used when measuring the specimen height and measurements be made to an accuracy of 0.001 inch. While a normal vernier caliper shown in Fig. 13.8 may be used to measure the specimen height, a vernier caliper with a supporting base and extension arm shown in Fig. 13.22 was used. It is believed that the measurements obtained with this instrument were of equal or greater accuracy than those obtained using a dial gage and stand.

This instrument had several advantages over the normal vernier caliper:

1. The extension arm only had to be placed in contact with the specimen cap when making a measurement as compared to a vernier caliper where both the top and bottom jaws (or arms) had to be adjusted visually.

2. Measurements had a much higher degree of accuracy as it eliminated guessing where the top and bottom edges of the porous stones were located. With the rubber membrane placed over the sample it is difficult to see through the membrane and accurately locate the porous stone edges.

3. It is very difficult to adjust each jaw of a vernier caliper such that it is lined up horizontally with the top and
bottom edges of the two porous stones. If your eyes are not looking horizontally across the jaws of the vernier caliper, an incorrect measurement will be made. In comparison with the vernier caliper shown in Fig. 13.22, alignment of the instrument is constantly maintained.

4. It is very easy to poke holes in the thin rubber membrane with the very sharp points on the jaws of the vernier caliper shown in Fig. 13.8 when measuring the specimen length. This potential is completely eliminated with the vernier caliper shown in Fig. 13.22.

Three measurements were obtained at 90° from one another with the base of the vernier caliper positioned in the same location that measurements from the top of the lower filter disc to the base of the triaxial cell were obtained as described in Section 13.2.2. By knowing the distance from the top of the lower filter paper disc to the base of the triaxial cell; thickness of top porous stone, filter disc, and top cap; and distance from the top of the top cap to the base of the triaxial cell it was possible to calculate the specimen length.

To evaluate the effect on the relative density of errors in length measurements, a calculation was made whereby the length was erroneously measured as 4.010 inch instead of the correct length of 4.000 inch. If the specimen is Monterey Number 0 sand placed at a relative density of 50%, the mistake in measuring the specimen length results in a calculated relative density of 48.2%. The error of +0.01 inch in specimen length will result in underestimating
the relative density by about 2%. It can be seen the effect is not as serious as the erroneous measurements in specimen diameter.

13.2.4 Calculation of Specimen Relative Density

The true diameter of the specimen was determined by subtracting from the average diameter of the specimen and membrane the average thickness of the membrane. Knowing the true diameter of the specimen and the measured specimen length, the specimen volume was determined. The unit weight of the specimen was determined by dividing the weight of the Denver sand placed within the mold by the specimen volume. The following equation was then used to calculate the relative density of the specimen:

\[
D_r(\%) = \frac{\gamma_d - \gamma_{d_{\text{min}}}}{\gamma_{d_{\text{max}}}} \times 100
\]

where

- \(D_r\) = relative density;
- \(\gamma_{d_{\text{max}}}, \gamma_{d_{\text{min}}}\) = maximum and minimum dry densities of the test sand; and
- \(\gamma_d\) = dry density of the specimen.

If the relative density of the specimen was less than the required relative density, the mold was placed back around the specimen and the back pressure vacuum shut off. The side of the mold was gently tapped with the screw driver shown in Fig. 13.12 causing the sand to densify. By reapplying the vacuum, removing the mold, and remeasuring the specimen dimensions, the new relative density of the specimen could be determined. If the specimen was
still too loose, the above described procedure was repeated until
the required relative density was achieved. On a number of instances,
when the relative density of the specimen was greater than the
required relative density by more than 1%, the sand was poured out
of the mold and a new specimen was reformed. When the relative
density of the specimen was within ±1% of the required relative
density, no attempt was made to modify the specimen density.

13.3 Assembling the Triaxial Cell

Once the specimen had been placed to the required relative
density, the triaxial cell was assembled. The connections on the
line leading from the top cap to the base of the triaxial cell were
tightened if loose.

The base of the triaxial cell was brushed and/or wiped clean
of all sand particles. The rubber O-ring was removed from the
base and carefully wiped off with a cloth rag including the cleaning
of the portion of the triaxial cell base previously covered by the
O-ring. The O-ring was uniformly coated with a covering of high
viscosity vacuum grease and the O-ring was reseated back on the
base of the triaxial cell. It was necessary to clean and grease
the chamber and base interface. The above indicated steps on
cleaning and lubricating the triaxial cell base, chamber, and O-ring
are very important to prevent the formation of leaks between the
triaxial cell base and chamber when the cell pressure is applied.
It is equally important that the seats between the base and chamber
of the triaxial cell be cleaned of sand, to prevent sand grains from
scratching and damaging the smooth metal seal surfaces.
Before lifting the triaxial cell chamber onto the base, a check was made to make sure that the loading ram was lifted fully and secured adequately. If the loading ram is not lifted before placing the triaxial cell chamber onto the base, the loading ram could come in contact with the top loading cap and might severely disturb the specimen. Once all seals between the base and chamber of the triaxial cell had been cleaned and the loading ram lifted, the chamber of the triaxial cell was carefully placed on the triaxial cell base. The chamber was gently rocked to make sure that the chamber was tightly seated on the base and then a base clamp was placed around the base of the triaxial cell. This compression ring securely held the triaxial chamber to the base of the triaxial cell.

To help steady the top cap placed on the test specimen, the loading ram was lowered so that the ram was just in contact with the threaded stud connection on the loading cap.

13.4 Filling the Triaxial Cell with Water

Before moving the triaxial cell from the location where the specimen was formed, the back pressure valve was closed. This maintained a -5 psi effective stress on the sand specimen making it possible to disconnect the vacuum line.

A water line was plugged into the cell pressure coupling and the cell was slowly filled with water. In order for the air to escape during filling, the air vent valve at the top of the triaxial cell chamber had to be opened. Some air void was left on
top of the triaxial cell chamber when filling it with water to prevent fluctuations in the cell pressure during cyclic loading. Once the chamber was filled with water with the exception of the previously mentioned air void, the air vent valve was closed.

13.5 Specimen Saturation

A cyclic triaxial specimen should be completely saturated prior to consolidation to allow the measurement of volume changes during consolidation. Commonly the soil which liquefies is in a saturated condition below the water table. The soil specimen is saturated with water to model this field condition.

13.5.1 Flushing the Specimen with Water

A small elevated container located above the level of the sand specimen was used to saturate the sand specimen. The hydraulic gradient across the sand specimen during saturation should not exceed approximately 5 to insure that fines will not be washed out of the specimen (Silver, 1977).

Before connecting the back pressure and cell pressure lines to the triaxial cell, these lines should be free of air bubbles. This air was easily removed by connecting each line to a de-aired water supply and running de-aired water through each line expelling all air bubbles. The cell pressure line prior to entering the cell pressure valve was saturated along with the connection which was used to transmit the cell pressure to the pressure transducer.

Both the back pressure and cell pressure valves were closed when the pressure in the cell pressure line was increased to 5 psi.
Pressure in the back pressure line was 0 psi, although the effective stress in the specimen remained at 5 psi vacuum. By applying the cell and back pressures simultaneously (cell pressure valve was opened just slightly sooner than the back pressure), the cell pressure in the triaxial cell was increased to 5 psi with the back pressure in the specimen equal to 0 psi. It is desirable to minimize consolidation of the specimen during saturation; therefore, the difference between the triaxial cell pressure and back pressure did not exceed 5 psi at any time.

The back pressure line was hooked up to a small elevated water container and the back pressure valve was opened to allow de-aired water to flush through the sand specimen under a small hydraulic gradient. In order to expel the air within the specimen, a quick connection with an attached short length of tubing was plugged into the line leading to the top of the specimen. De-aired water was allowed to flush through the specimen until air bubbles were no longer observed leaving the specimen.

13.5.2 Connecting the Pore Pressure Transducer

Once the specimen had been flushed with de-aired water, it was necessary to hook up the pore pressure transducer. A four-way valve with the pressure transducer disconnected was plugged into the triaxial cell. The electrical lead on the pore pressure transducer was connected to the Tektronix Power Supply and Digital Multimeter.

Before connecting the pressure transducer to the triaxial cell, the pressure transducer output signal was zeroed. This was
accomplished by saturating the pressure transducer and then holding the pressure transducer at the same elevation as the transducer would be at when connected to the triaxial cell. If the output voltage was not zero, a fine-blade screw driver was used to adjust the Digital Multimeter until a zero reading was obtained.

All lines in the triaxial cell base had to be saturated. The valve on the triaxial cell base controlling the line leading from the top of the specimen was closed. By turning the four-way valve to the back pressure position and opening the back pressure valve, de-aired water was forced through the back pressure line in the base of the triaxial cell. To permit full saturation of the four-way valve, the four-way valve was rotated so that the air bubbles flowed up through the four-way valve.

To saturate the line leading from the top of the specimen to the four-way valve, the four-way valve was turned to this line. With the valve in the line leading from the top of the specimen open, de-aired water flowed through the specimen, out or the line attached to the top cap, and through the line connected to the four-way valve until no bubbles were observed to be coming from the sand specimen.

With all lines attached to the four-way valve fully saturated, it was possible to connect the pressure transducer to the four-way valve. Extreme care was required in this operation to make sure that the transducer was not damaged. The four-way valve was opened to the back pressure with the result that de-aired water started to flow out of the four-way valve. The pressure transducer was completely saturated with de-aired water flowing from the four-way
valve and then was carefully connected to the four-way valve.

Before tightening the coupling between the pressure transducer and four-way valve, a check was always made to make sure that the four-way valve was turned to the back pressure or cell pressure position. If the four-way valve is not turned to either of the previously mentioned positions, but say is turned halfway between the cell pressure and back pressure position, water flow from the four-way valve when connecting the transducer may be prevented. Due to the incompressibility of water when the pressure transducer is connected to the four-way valve, the resulting high water pressure during the tightening of the transducer may rupture the diaphragm in the pore pressure transducer.

13.5.3 Raising the Cell and Back Pressure

A check was made of the water levels in the burettes connected to the cell and back pressures to make sure that the burettes were at least two-thirds full of water. When the cell pressure was raised, the air at the top of the triaxial cell chamber was compressed with the result that additional water from the cell pressure burette entered the cell. For this reason, I generally filled the cell pressure burette fully with water knowing that upon raising the cell pressure that a significant quantity of water would flow from this burette.

To assure that the specimen was not overstressed during the saturation phase, the back pressure and cell pressure were slowly increased simultaneously maintaining the cell pressure at 5 psi greater than the back pressure. It is important to slowly raise
the cell and back pressures to maintain the difference between the cell and back pressures not to exceed 5 psi. The back and cell pressures were raised to 65 and 70 psi, respectively, and the specimen was allowed to sit overnight.

13.5.4 Checking the B-Value

With the back pressure drainage lines closed, the cell pressure was increased by 9.437 psi (1 mv for the pressure transducer used). By observing the corresponding rise in the specimen's pore water pressure, $\Delta u$, it was possible to calculate the B parameter using the following equation:

$$B = \frac{\Delta u}{\Delta \sigma_3}$$

where

- $B$ = Skempton's pore pressure parameter
- $\Delta u$ = rise in pore water pressure
- $\Delta \sigma_3$ = change in cell pressure

The specimen was required to have a B parameter of greater than or equal to 0.95 before the specimen was considered satisfactorily saturated. If it turned out that the B parameter was less than 0.95, the cell and back pressures were raised while maintaining the 5 psi difference, and the B parameter rechecked after a few hours.

13.6 Consolidation of the Test Specimen

When the B parameter was greater than or equal to 0.95, the specimen was consolidated under an isotropic consolidation pressure of 30 psi. Care was used when applying the consolidation pressure
to make sure that the pore water could be gradually drained from the specimen under a slow increase in consolidation pressure as versus subjecting the specimen to a sudden jump in consolidation pressure.

The burette reading of the water table in the back pressure burette tube was noted before and a short period after application of the additional 35 psi of confining pressure.

The change in volume shown in the burette was equal to the change in volume of the saturated specimen plus the volumetric expansion of the triaxial cell, back pressure line, and pore water transducer. It was assumed that the volumetric expansion of the triaxial cell and pore water transducer was negligible.

Volumetric expansion of the back pressure line was measured by plugging the back pressure line into the pressure control panel and incrementally noting the volume change as the pressure increased. A plot of volumetric expansion versus internal pressure in the back pressure line is shown in Fig. 13.24. By subtracting the amount of volumetric expansion of the back pressure line from the measured volume change in the back pressure burette it was possible to determine the volume of the specimen after consolidation.

As can be seen from Fig. 13.24 the volumetric expansion of the polyethylene back pressure line was 0.48 cm$^3$ for a 30 psi increase in pressure (0.000976 in$^3$/psi). Volumetric expansion of this magnitude, unless corrected for, will result in significant errors in the measured volume change of the specimen during consolidation.

Silver (1977) indicated that the change in volume for all
FIG. 13.24 VOLUME CHANGE MEASURED IN A 14.8 FOOT LENGTH OF 1/4 INCH DIAMETER POLYETHYLENE TUBING.
assembled components of the pore water pressure measurement system should not exceed 0.0001 in³/psi. Short runs of stiff, small diameter metal tubing should be used to reduce volumetric expansion of the pore pressure measurement system. Rigidity of the pressure transducer and triaxial cell should also be determined.

13.7 Placing the Triaxial Cell Within the MTS Loading Frame

Following the consolidation of a test specimen, the triaxial cell was placed on the loading frame of the closed loop electro-hydraulic Materials Test System (MTS) as shown in Fig. 13.25.

The MTS machine was turned on with all reset buttons cleared. Generally this was done approximately 30 minutes before any cyclic loading was applied to the triaxial specimen, thereby permitting the MTS machine time to warm up.

Since stress controlled cyclic triaxial tests were used in this study, it is necessary to make sure that MTS was stress control tests. If the controller happened to be set on strain control and the set point control was not zeroed but instead set to apply a compressive strain, the hydraulic piston on the MTS loading frame could suddenly come down and disturb the triaxial specimen upon pressurization of the hydraulic system. For this reason, it was always necessary to make sure that the controller was on stress control and the set point control was set to lift the piston up (apply a tensile load) before starting the hydraulic pump.

"Low" pressure was initially applied to the hydraulic system for several minutes before the "High" pressure control button was
FIG. 13.25 TRIAXIAL CELL PLACED WITHIN THE MTS LOADING FRAME
pushed to raise the hydraulic pressure from 300 to 3000 psi. By adjusting the set point control the hydraulic piston on the load frame was brought down and connected to the triaxial cell loading ram.

A universal joint was used in connecting the piston with the triaxial cell loading ram to make sure that the loading ram was not bent in case the triaxial cell was slightly off-center. By watching the ease with which the universal joint slid onto the loading ram and by close "eye-ball ing," it was possible to closely center the triaxial cell beneath the piston.

It was necessary to zero the SET POINT control so that the upward force on the loading ram and weight of the triaxial cell were balanced. The SET POINT control with no weight on it is zeroed at a value of 5.000. Values larger than 5.000 apply tensile loads (lift the piston up) and values less than 5.000 apply compressive loads (force the piston down). When the piston was zeroed it neither moved up nor down. To help determine if the piston was gradually moving when attempting to zero it, the X-Y recorder was utilized. Graph paper 11 x 16-1/2 inches was placed in the X-Y recorder and aligned with the recorder axes. With the POWER switch to ON and the PAPER switch to LOAD, the paper was held in contact with the recorder. Fig. 13. shows the controls on the X-Y recorder.

With the PAPER switch set on RECORD and the INPUT switch of both axes to ZERO, the pen was positioned to the desired "zero" reference point. The INPUT switches were then set to NORMAL with
**FIG. 13.21 CONTROLS FOR THE X-Y RECORDER**

- **PAPER LOAD**: Moves the carriage to the right for paper replacement.
- **RECORD**: Conditions the recorder for normal operation.
- **INPUT**: Selects
  - NORMAL for external signal
  - ZERO for a check of pen position without input
  - SWEEP for time base signal
- **CM** for metric scale
- **IN** for English scale
- **HIGH**, **LOW**
- **GUARD & GROUND**: Input connectors extend through case for either top or bottom use.
- **SWEEP**: Puts the sweep circuit to the "zero" position.
- **START**: Actuates the sweep circuit.
- **PEN**: UP lifts the pen off the paper, DOWN lowers the pen to the paper.
- **ZERO**: Adjusts no input position of pen. Controls offset voltage for selected portion of external signal. Locking ring prevents change of adjustment.
- **SWEET**: For time base signal.
- **LOW**
- **GUARD & GROUND**: Input connectors extend through case for either top or bottom use.
- **POWER**: OFF removes ac power. ON applies ac power and illuminates switch lamp.
- **LOCAL/REMOTE**: Selects pen mode of operation.
- **RATE**: Allows the selection of sweep rate in INCHES per SECOND.

---

For a detailed explanation of each control, please refer to the diagram and text provided.
the RANGE switches set so that the load versus deformation hysteresis could be recorded as large as possible using the 11 x 16-1/2 inch sheet of recording paper. Deformation was recorded on the X axis with load being recorded on the Y axis. By watching the horizontal movement of the pen, it was possible to determine if the hydraulic piston on the loading frame was moving.

The SET POINT control was adjusted so that the piston just started to move up and then was adjusted so that the piston just started to move down. Noting the previous two SET POINT readings and dividing the sum of the two readings by two, it was possible to determine the zero setting for the SET POINT control. With the SET POINT control set at the zero value setting, the loading ram stays at a neutralized position and would move neither upward nor downward.

When connecting the loading ram to the triaxial specimen top cap, the back pressure valves were closed to permit the observation of the specimen pore water pressure fluctuation. By turning the SET POINT control to a value slightly less than the zero setting the loading ram was pushed down toward the loading cap. When the loading ram came in contact with the top cap, the loading ram was screwed onto the top cap until the loading ram was securely attached. An attempt was always made to limit the generated pore pressure produced while attaching the loading ram to a maximum value of 1 psi. Following attachment of the loading ram to the top cap the SET POINT control was zeroed and the piston was held in a stationary position. A pin was used to secure the universal joint to the hydraulic piston.
The back pressure valve was opened to dissipate the very small pore pressure generated during the process of engaging the loading ram and the top cap.

Three C-clamps shown in Fig. 13.25 were used to attach the triaxial cell to the pedestal of the MTS loading frame. Each of the three C-clamps were placed at 120 degrees from one another and tightened uniformly to prevent possible tilting of the triaxial cell.

13.8 Connecting X-Y and X-Y-Y' Recorders

The X-Y recorder was used for recording the load versus deformation relationship. The pen was zeroed at the center of the paper and the PEN switch was placed in the DOWN position. Vertical load was recorded on the Y axis and axial deformation recorded on the X axis. With the load control set at 10% of the range (20,000 pounds \( \times 10\% = 2,000 \) pounds) and the range of Y axis set at 1% of the range/inch, the pen moved vertically one inch for each 20 pounds (1%/inch \( \times 2,000 \) pounds = 20 pounds/inch). The range of X axis was set at 2% of the range/inch. With the strain controller set at 100% of the range, the pen moved horizontally one inch for each 0.100 inch of axial deformation (2% \( \times 5 \) inch = 0.10 inch of axial deformation/inch).

The specimen pore pressure with time was recorded on the X-Y-Y' recorder utilizing the X-Y' channel. Electric wires from the pore pressure transducer were connected to the X-Y-Y' recorder. Paper was placed in the recorder and the POWER switch was turned to LOAD to hold the paper in place.

By adjusting the X ZERO control, the pen was moved to the
far left of the paper with the pen lined up with the left edge of the graph paper. The X RANGE switch was set to SWEEP with the SWEEP RATE switch set based on the anticipated amount of time required to run the cyclic triaxial test.

The Y' RANGE switch was set at 1 mv/inch. Maximum change in pore water pressure to occur in the specimen during cyclic loading would be 30 psi because of the confining pressure being 30 psi. Knowing that the particular pressure transducer used has a change in output voltage of 1 mv/9.437 psi, the output voltage would change 3.18 mv for a 30 psi rise in the specimen's pore water pressure. This change in voltage of 3.18 mv would move the Y' pen vertically 3.18 inches. Knowing that the Y' pen could move a maximum of 3.18 inches vertically, the pen was zeroed four inches from the top of the paper with the four-way valve on the triaxial cell turned to the back pressure line.

A small tick mark was made to show the zero pore pressure reference line by moving the POWER switch to PEN (moves pen down) and the pen moved in the X direction by utilizing the START switch. When a short tick line had been drawn the pen was moved back to its original starting point by flipping the switch from START to RESET.

To show the vertical location of 30 psi on the Y' recorder, the four-way valve was turned from the back pressure line to the cell pressure, an increase in pressure of 30 psi. A tick mark was made on the graph paper in the same manner as described in the previous paragraph and then the four-way valve was turned back to monitor the specimen back pressure.
The X-Y-Y' recorder was used to record the vertical deformation of the specimen with time. Electrical leads plugged into the MTS controller chassis transferred the voltage output from the transducer conditioner connected to the LVDT to the Y-axis of the recorder. One of these leads was plugged into the controller chassis immediately to the right of the STROKE LOWER LIMIT light with the second lead plugged in a ground. The lead other than the ground was plugged into the HIGH connection on the Y axis of the X-Y-Y' recorder with the ground lead plugged into the LOW connection.

The Y pen was zeroed over a horizontal line on the graph paper and a tick mark was made as previously described for the Y' pen. If the tick mark was not perfectly centered on the line a minor adjustment was made in the pen and a new tick mark was drawn.

13.9 Determination of Cyclic Load to be Applied to the Specimen

All cyclic triaxial tests were performed at controlled stress ratios with the stress ratio being defined in the following equation:

\[ S.R. = \frac{\sigma_{dc}}{2\sigma_3} \]

where

- \( S.R. \) = stress ratio;
- \( \sigma_{dc} \) = average single amplitude cyclic deviatoric stress = \((\sigma_1 - \sigma_3)\) cyclic; and
- \( \sigma_3 \) = initial effective confining pressure.

By knowing the required stress ratio and the effective confining pressure on the specimen (30 psi) it was possible to calculate the required average single amplitude cyclic deviatoric
stress (15 psi). The cyclic single amplitude load was determined by multiplying the consolidated area of the specimen by the required single amplitude cyclic axial stress.

The SPAN control, located immediately to the right of the SET POINT control on the MTS control panel, was used to adjust the amplitude of the axial load command signal originating in the function generator.

The SPAN control on the MTS machine is calibrated so that the peaks of the input function drive the controlled load amplitude to 100% of the operating range, i.e., 20,000 pounds tension and 20,000 pounds compression, when the control is set at 1000. The dial setting for the SPAN control to provide a desired dynamic loading amplitude can be determined by either of the following two methods:

1. The MTS Instruction Manual gives the following equation for determining the SPAN control setting:

   \[
   \text{SPAN} = \frac{(P_{\text{max}} - P_{\text{min}})f}{\text{OR}}
   \]

   where
   \( P_{\text{max}} \) = maximum value of the controlled variable (most positive or least negative);
   \( P_{\text{min}} \) = minimum value of the controlled variable (least positive or most negative);
   \( f \) = scale factor of the input function = 500, for functions varying between ±100% = 1000, for functions varying between zero and ±100% or -100%; and
OR = Operating Range.

The following example illustrates the use of this equation in calculating the SPAN setting:

With the consolidated area of the specimen, \( A_c = 3.141 \text{ in}^2 \), S.R. = 0.25, \( \sigma_3 = 30 \text{ psi} \), the load control set at 10% of the operating range, then the SPAN setting for the cyclic sinusoidal load to be applied to the specimen can be determined as follows:

\[
\sigma_{dc} = \text{S.R.} \times 2\sigma_3 = 0.25 \times 2 \times 30 \text{ psi} = 15 \text{ psi};
\]

\[
P_{\text{max}} = \sigma_{dc} \times A_c = 15 \text{ psi} \times 3.141 \text{ in}^2 = 47 \text{ lbs};
\]

\[
P_{\text{min}} = -P_{\text{max}} = -47 \text{ lbs};
\]

\[
f = 500;
\]

\[
\text{OR} = 20,000 \times 10\% = 2,000; \text{ and}
\]

\[
\text{SPAN} = \frac{47 - (-47)}{2000} \times 500 = 23.5.
\]

Therefore, the SPAN dial when set at a scale reading of 23.5 would apply a cyclic loading of ±47 pounds.

2. The second method for determining the SPAN setting is to divide the programmed operating range of the controlled variable by the number of dial divisions on the SPAN control. For example if the operating range of the controlled variable is ±2000 pounds as in the above example and the number of SPAN dial divisions is 500, each SPAN dial division equals four pounds of single amplitude cyclic load. Thus, to apply a single amplitude cyclic load of 47 pounds, the SPAN dial would be set on 11.75 divisions (47 lbs/4 lbs per division = 11.75 divisions).
Since there are only 500 divisions on the SPAN control for the full scale reading of 1000, each scale division is equal to a scale reading of 2. The SPAN Dial would be set at a scale reading of 23.5 (11.75 divisions) which is the same SPAN setting as determined by the first method.

13.10 Data Reduction

13.10.1 Specimen Length and Diameter after Consolidation

Before the magnitude of cyclic load to be applied to each specimen could be determined, it was necessary to determine the consolidated diameter and length of the specimen. When calculating the specimen consolidated diameter and length, an assumption was made that the sand specimen was isotropic; that is the stress-strain relations were the same regardless of the orientation of the axes on which the stress-strain relations were measured. If this assumption is accepted, it is possible to write the following two simultaneous equations from which the consolidated diameter and length can be determined:

\[ L_i \cdot \frac{\pi D_i^2}{4} \cdot \Delta V = L_c \cdot \frac{\pi D_c^2}{4} \]

where

- \( \Delta V \) = change in volume during consolidation, \( V_i - V_c \),
- \( V_i \) = initial specimen volume,
- \( V_c \) = specimen volume after consolidation,
- \( L_i \) = initial specimen length,
- \( L_c \) = length of specimen after consolidation,
\[ D_i = \text{initial specimen diameter}, \quad \text{and} \]
\[ D_c = \text{specimen diameter after consolidation}. \]

Knowing the diameter after consolidation, and the selected stress ratio as defined in Section 13.9, the single amplitude for a cyclic load test can be determined. From the volume change at the completion of consolidation, the specimen volume and relative density before the commencement of the cyclic loading test can be calculated.

13.10.2 Stress Ratio versus Number of Stress Cycles Causing Initial Liquefaction

Each sample was subjected to the cyclic triaxial test under a selected stress ratio \( R_s \) after full consolidation and saturation. The cyclic stress versus deformation, excess pore water pressure and axial deformation versus number of stress cycles were recorded throughout a test. The records are shown in Figs. 14.4.a, b, and c. From this record, the number of stress cycles causing initial liquefaction, \( N_L \), is determined. \( N_L \) is defined as the number of stress cycles required to cause the excess pore water pressure to be equal to the initial effective confining pressure.

As can be observed in Fig. 14.4.a, the cyclic stress amplitude decreases somewhat at the onset of liquefaction where the sample axial deformation increases. This occurred before the initial liquefaction and when the excess pore water pressure equals approximately two thirds of the initial effective confining pressure. This stress amplitude reduction necessitates an empirical adjustment of \( N_L \) to a number smaller than the number of stress cycles causing
the excess pore water pressure to equalize the initial effective confining pressure. This adjustment was based on the degree of load amplitude reduction as the sample approached the initial liquefaction.

The number of stress cycles causing initial liquefaction, $N_L$, as evaluated in the previous paragraph paired with the selected stress ratio, $R_s$, under which the cyclic triaxial test was conducted formed a data point on the $R_s$ versus $N_L$ space. Four data points are usually required to satisfactorily define the $R_s$ versus $N_L$ curve. After a minimum of four cyclic triaxial tests on samples of a certain gradation were conducted, $R_s$ versus $N_L$ curve was then constructed.
14.1 Correction of Relative Density

A total of 123 cyclic triaxial tests were performed using the sample preparation and testing procedures described in Chapter 13. From the experience of previous works, it is known that to prepare the sample at an exact relative density was extremely difficult. Although a relative density of 50% was targeted, the relative density achieved for the specimen used in this study ranged from 49.6% to 59.3%. To investigate the effect of grain size distribution characteristics on liquefaction, the same base relative density must be used. A test program was then carried out to investigate the effect of small differences in relative density on the cyclic strength of a saturated sand. The test results are discussed in the following paragraphs. This resulting information was then used to adjust the cyclic stress ratio of each specimen to the magnitude corresponding to that of the base relative density of 50%.

The medium sand samples with the median grain size, $D_{50} = 0.38$ mm and $C_u = 2.2$, were chosen for this correction study. This soil was designated DM-d_2 in Table 13.2. Three groups of samples were prepared to achieve the relative density of 50.3%, 55.7% and 60.7%,
respectively. Each group contains five samples. All samples were back pressured to achieve the Skempton pore water pressure parameter B of no less than 0.95 and consolidated under a 30 psi consolidation pressure before cyclic triaxial tests. The test results are shown in Fig. 14.1. From Fig. 14.1 the stress ratios causing the initial liquefaction in 5, 10, 20, 40 and 100 cycles were determined. The resulting stress ratios were then plotted against the relative density in Fig. 14.2. This relationship was found to be linear. However, these straight lines do not pass through the origin (zero stress ratio and zero relative density) as reported by Seed, et al. (1967). A similar conclusion was also reported by Mulilis, et al. (1978).

To correct the cyclic strength of samples with the relative density ranging from 50% to 60%, correction factors plotted with respect to the number of cycles causing initial liquefaction are shown in Fig. 14.3. These factors basically are the slopes of the straight lines in Fig. 14.2 at each stress cycle to cause initial liquefaction. It was assumed that the results obtained for soil DM-d_2 are applicable to all other samples. The cyclic stress ratio ($R_s$) at the relative density higher than 50% was, therefore, corrected by using the following equation:

$$(R_s)_{50\%} = R_s - (D_r - 0.5) \times R$$

where

$$(R_s)_{50\%} = \text{stress ratio to cause initial liquefaction at 50\% relative density;}$$

$R_s = \text{stress ratio to cause initial liquefaction in the test;}$$
FIG 14.1 EFFECT OF RELATIVE DENSITY ON DENVER SAND

$D_50 = 0.30$ mm
$D_r = 2.20$ mm
$D_f = 50.3$
$D_r = 55.7$
$D_f = 60.7$

$\sigma_0 = 30$ psi

Number of Cycles Causing Initial Liquefaction ($N_L$)

Stress Ratio ($\sigma_0 \sigma_0^* )$
FIG. 14.2 EFFECT OF RELATIVE DENSITY ON LIQUEFACTION POTENTIAL
OF DENVER SAND AT DIFFERENT STRESS CYCLES

Relative Density, \( D_F (\%) \)

\( \sigma_0 = 30 \text{ psi} \)
\( D_{50} = 0.38 \text{ mm} \)
\( C_u = 2.2 \)

\( N_L = 5 \)
\( N_L = 10 \)
\( N_L = 20 \)
\( N_L = 40 \)
\( N_L = 100 \)

Stress Ratio, \( \frac{\sigma'}{\sigma} \)
FIG. 14.3  CORRECTION FACTORS VERSUS NUMBER OF CYCLES CAUSING INITIAL LIQUEFACTION

Number of Cycles Causing Initial Liquefaction ($N_L$) vs. Correction Factors, $R (10^{-3})$.

0 10 20 30 40 50 60 70 80 90 100

10 20 30 40 50 60 70 80 90 100

Correlation Factors, $R (10^{-3})$. (Not visible in the image.)
\[ D_r = \text{relative density, in decimal, of the sample tested; and} \]

\[ R = \text{correction factor from Fig. 14.3.} \]

14.2 General Test Results

After the completion of a cyclic triaxial test, the specimen was subjected to sieve analysis to check its grain size distribution characteristics. Compared with the designed gradation shown in Fig. 2.4, some changes in distribution characteristics were observed. The gradation curves all shifted somewhat toward fine side. This could be attributed to the washing of samples.

Records obtained from a typical cyclic triaxial test on a medium uniform sand at a low stress ratio are shown in Fig. 14.4. The same general shape of test record was observed from all cyclic triaxial tests. From the figure, it was observed that a hysteresis loop was formed for each loading cycle and that as the number of cycles of loading increased, the shear modulus decreased, the damping ratio increased, and the hysteresis loop increased in size. However, the study of the general dynamic behavior of the saturated sand with different grain size distribution characteristics is beyond the scope of this study.

The pore pressure generated during the application of cyclic load increased steadily until it reached a value of approximately two-thirds of the initial effective confining pressure. The rate of pore water pressure generation became increasingly large afterward. The sample deformation was also found to increase rapidly.
FIG. 14.4 TYPICAL RECORDED DATA
The upper bound of the pore pressure curve as shown in Fig. 14.4 is interestingly found to be similar to a creep curve in shape. A pore pressure generation similar to tertiary creep was initiated after the excess pore pressure reached a value of two-thirds of the initial effective confining pressure. Therefore, an empirical equation similar to a creep function can be formulated to evaluate excess pore pressure generation under cyclic loading.

14.3 Effect of Mean Grain Size

Results of the cyclic triaxial tests performed on the samples with different mean grain sizes and uniformity coefficients ranging from 1.9 to 16.0 are presented in Figs. 14.5 to 14.9. In each graph, the test results of samples with similar uniformity coefficients were presented. As the data indicated, if the small variation in uniformity coefficients of the samples is disregarded in each figure, the effect of mean grain size on the liquefaction potential of sands was found to be significant. The dynamic strength of the samples with large mean grain size, $D_{50}$, was found to be stronger than those of the samples with small $D_{50}$. Samples with $D_{50}$ within the range of fine sands are the weakest among the soils investigated.

Stress ratios needed to cause the initial liquefaction at the tenth and thirtieth cycles were plotted against the logarithm of the mean grain size. As shown in Figs. 14.10 and 14.11, with the uniformity coefficient ranging from 1.9 to 16.0, the overall cyclic strength of samples containing a small amount of fine gravel size particles was found to be about two times as strong as the cyclic
FIG. 14.5 EFFECT OF MEAN GRAIN SIZE ON INITIAL LIQUEFACTION

\[ \sigma_0 = 30 \text{ psi} \]
\[ D_r = 50\% \]

<table>
<thead>
<tr>
<th>( D_{50} ) (mm)</th>
<th>( C_u )</th>
<th>( e )</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.89</td>
<td>0.74</td>
<td>( a_2 )</td>
</tr>
<tr>
<td>0.82</td>
<td>2.17</td>
<td>0.73</td>
<td>( b_2 )</td>
</tr>
<tr>
<td>0.53</td>
<td>2.16</td>
<td>0.74</td>
<td>( c_2 )</td>
</tr>
<tr>
<td>0.38</td>
<td>2.20</td>
<td>0.76</td>
<td>( d_2 )</td>
</tr>
<tr>
<td>0.24</td>
<td>2.88</td>
<td>0.78</td>
<td>( e_2 )</td>
</tr>
<tr>
<td>0.15</td>
<td>2.70</td>
<td>0.85</td>
<td>( f_2 )</td>
</tr>
</tbody>
</table>

Number of Cycles Causing Initial Liquefaction \( (N_L) \)
FIG. 14.6 EFFECT OF MEAN GRAIN SIZE ON INITIAL LIQUEFACTION
FIG. 14.7 EFFECT OF MEAN GRAIN SIZE ON INITIAL LIQUEFACTION
**FIG. 14.8 EFFECT OF MEAN GRAIN SIZE ON INITIAL LIQUEFACTION**

- \( D_0 = 30 \text{ psi} \)
- \( D_{50} = 50\% \)
- \( D_{50} \text{ (mm) Cu e} \)
- \( d_b \text{ Designation} \)
- \( 0.69 \ 9.5 \ 0.48 \)
- \( 0.52 \ 9.0 \ 0.48 \)
- \( 0.37 \ 9.7 \ 0.51 \)

**Number of Cycles Causing Initial Liquefaction \( (N_L) \)**

- \( 0 \)
- \( 30 \)
- \( 100 \)
- \( 300 \)
- \( 1000 \)

**Stress Ratio \( (\sigma_0 / \sigma)^{1/2} \)**

- \( 0.5 \)
- \( 0.4 \)
- \( 0.3 \)
- \( 0.2 \)
- \( 0.7 \)
- \( 0 \)
$\sigma_0 = 30$ psi
$D_r = 50$

\begin{tabular}{ccc}
\hline
$D_{50}$ & Cu & e & Designation \\
\hline
0.62 & 10.3 & 0.46 & \textbf{b}_{10} \\
0.57 & 11.3 & 0.45 & \textbf{c}_{10} \\
0.37 & 12.0 & 0.47 & $d_{10}$ \\
0.53 & 16.0 & 0.43 & \textbf{c}_{15} \\
\hline
\end{tabular}

FIG. 14.9 EFFECT OF MEAN GRAIN SIZE ON INITIAL LIQUEFACTION
FIG. 14.10 EFFECT OF MEAN GRAIN SIZE ON STRESS RATIO CAUSING INITIAL LIQUEFACTION IN 10 CYCLES
FIG. 14.11 EFFECT OF MEAN GRAIN SIZE ON STRESS RATIO CAUSING INITIAL LIQUEFACTION IN 10 CYCLES
strength of very fine sands. This is larger than the values obtained by Lee and Fitton (1969) and Wong, et al. (1975). This can be ascribed to the differences in test sands, test sample density, and initial stress conditions.

Membrane compliance and fine particle migration during a test might also contribute to the false dynamic strength of coarse sands as discussed below.

Membrane Effect

As described by Wong, et al. (1975) and Martin, et al. (1978), the effect of membrane compliance can be significant. The following two possible effects of membrane compliance can be observed in the test:

During the consolidation phase:

The relative density of the testing sample can be exaggerated by the penetration of membrane due to application of the effective confining pressure; and

During the cyclic test phase:

The generation of excess pore pressure results in a decrease of effective confining pressure. As a result of this decrease, the membrane penetration caused by application of the initial effective confining pressure reduces. Therefore, the sample appears to show more resistance. Typical test results affected by the membrane compliance are shown in Fig. 14.12. From this figure, the rate of pore pressure generation decreases as the excess pore pressure approaches the initial confining pressure, and the number of cycles
FIG. 14.12 TYPICAL RECORDED TEST DATA FOR COARSE SAND

\( \sigma_0 = 30 \) psi

Excess Pore Pressure (psi)

Deformation (in.)
required to cause initial liquefaction seems to be somewhat too high.

Particle Migration

Another source of error may be caused by the segregation of fine grain soils in the specimen prior to initial liquefaction. In fact, it was visually observed that some fine particles migrated downward in the voids created by coarse particles as a result of excess porewater pressure generation during undrained cyclic tri-axial tests. As shown in Figs. 14.6 and 14.7, the cyclic strength of samples with mean grain size ranging from 1.17 mm to 1.44 mm was found to be significantly stronger than another sample. It is believed that the downward migration of fine particles might have resulted in increase in the membrane penetration effect in the upper portion of the sample and caused either or both aforementioned errors. This may result in over-estimation of the cyclic strength.

No attempt was made to correct these errors in this investigation. Despite the above inevitable errors, the test results in this study for liquefaction indicated that fine sands tend to possess less resistance to liquefaction than coarse sands.

14.4 Effect of Uniformity Coefficient

In an attempt to investigate the effect of uniformity coefficient, \( C_u \), on the liquefaction potential of sands, the data presented in Figs. 14.5 through 14.9 were rearranged. The results of samples with similar \( D_{50} \) and different uniformity coefficient, \( C_u \) were plotted in the same figure as shown in Figs. 14.13 through 14.18. It was observed that the effect of the uniformity coefficient
\[ \sigma_0 = 30 \text{ psi} \]
\[ D_r = 50\% \]

\[ \begin{array}{ccc}
D_{50} & C_i & \epsilon \\
1.50 & 1.89 & 0.74 \\
1.44 & 3.73 & 0.60 \\
1.17 & 5.50 & 0.52 \\
\end{array} \]

Designation: \( e \)

FIG. 14.13 EFFECT OF UNIFORMITY COEFFICIENT ON LIQUEFACTION

Number of Cycles Causing Initial Liquefaction \( (N_L) \)
FIG. 14.14 EFFECT OF UNIFORMITY COEFFICIENT ON LIQUEFACTION
FIG. 14.15 EFFECT OF UNIFORMITY COEFFICIENT ON LIQUEFACTION
FIG. 14.16 EFFECT OF UNIFORMITY COEFFICIENT ON LIQUEFACTION
FIG. 14.17 EFFECT OF UNIFORMITY COEFFICIENT ON LIQUEFACTION
FIG. 14.18 EFFECT OF UNIFORMITY COEFFICIENT ON LIQUEFACTION
was found to be less important than that of the mean grain size. As shown in Figs. 14.10 and 14.11, the stress ratios required to cause initial liquefaction at the tenth and the thirtieth cycles increase monotonically with mean grain size. However, the effect of the uniformity coefficient on the stress ratio required to cause initial liquefaction was found to be relatively small. One must be cautioned that the data in each figure also contain the variation in void ratios. Therefore, a definite conclusion cannot be drawn on the effect of uniformity coefficient. However, the data indicates that the effect of uniformity coefficient on the liquefaction potential of sand is negligible.

By comparing the band width of data points in Figs. 14.13 through 14.18, it was observed that the effect of membrane penetration and uniformity coefficient seems to become smaller as the mean grain decreases. Hence, the effect of uniformity coefficient and membrane penetration is smaller for the fine sand than the coarse one. At the same $D_{50}$, the samples with smaller $C_u$ seem to possess a higher cyclic strength. However, compared with the effect of mean grain size, the effect of sample uniformity on the cyclic strength was found to be negligible. As shown in Figs. 14.15 to 14.17, the same conclusion can be drawn for medium sand and fine sand. However, the cyclic strength of very fine sands, as shown in Fig. 14.18, was found to be somewhat stronger for the fine sand samples with higher $C_u$. The specimen DF-f3 with 20% fines (minus 0.074 mm), coarse silt, is somewhat stronger than the specimen without any fine. It implies that the presence of
silt might have some effect on cyclic strength of soils. This effect is currently being investigated by the authors.

Based on the summary of all test results, as shown in Figs. 14.19 and 14.20, the effect of sample uniformity on initial liquefaction at the tenth and the thirtieth cycles is negligible for samples with mean grain size smaller than 1.5mm. However, the effect of mean grain size on liquefaction is strongly affected by the uniformity of the sample. The dispersion of data points is larger for uniform soils than for well graded soils. At the same uniformity coefficient, the cyclic strength of the sample with a large mean grain size is higher than the sample with a small mean grain size. But for samples with uniformity coefficient greater than 8, the stress ratios required to cause initial liquefaction at the tenth and the thirtieth cycles are almost constant despite the difference in the mean grain size.

14.5 Development of Functional Relationship

One of the major purposes of this study is to obtain a functional relationship between the stress ratio causing initial liquefaction and the gradation characteristics of sand. The result may aid in assessing liquefaction potential of sands before laboratory cyclic tests are performed. The statistical correlation analysis was performed to assess the strength of the relationship between the dependent and independent variables. The regression analysis was then used to formulate the functional relationship between the dependent variable and the independent variable(s). The SPSS (Statistical Packages for the Social Sciences) programs
FIG. 14.19 EFFECT OF UNIFORMITY COEFFICIENT ON STRESS RATIO CAUSING INITIAL LIQUEFACTION IN 10 CYCLES

- Uniformity Coefficient, Cu
- Liquefaction in 10 Cycle
- Stress Ratio to Cause Initial Liquefaction

- $\sigma = 30$ psf
- $D_0 = 50$(mm)
- Table: D0 = 1.17 - 1.50, $0.62 - 0.82$
- 0.48 - 0.57
- 0.37 - 0.40
- 0.23 - 0.25
- 0.14 - 0.15
FIG. 14.20 EFFECT OF UNIFORMITY COEFFICIENT ON STRESS RATIO CAUSING INITIAL LIQUEFACTION IN 10 CYCLES
were used in performing these analyses.

The curves of Fig. 14.5 to Fig. 14.9 will first be formulated to relate the cyclic strength (expressed by stress ratio) to number of cycles to cause initial liquefaction. From statistical correlation and regression analyses, these curves can be fitted well by the general form of the following equation:

$$R_s = a + b \log_{10} N_L + c(\log_{10} N_L)^2$$

where

$$R_s = \frac{\sigma_{dc}}{2\sigma_3} \left( \frac{\sigma}{\sigma_0} \right) = \text{cyclic stress ratio;}$$

$$N_L = \text{number of cycles to cause initial liquefaction; and}$$

$$a, b, c = \text{coefficients determined by regression analysis and } a \text{ is equal to the cyclic stress ratio at one cycle.}$$

The $R$ value, the correlation coefficient between dependent and independent variables, was found to be higher than 0.97 in most of the cases. Twenty-four sets of coefficients, as shown in Table 14.1, were determined by this analysis. The variation of the coefficients is believed to be related to the grain size distribution characteristics.

By the same technique, the relationships between the coefficients and the grain size distribution characteristics were obtained as follows:

$$a = 0.5781 - 0.1383 D_{10} + 0.3729 \log_{10} D_{50}$$

$$R = 0.783$$
### Table 14.1

VALUES OF COEFFICIENTS $a$, $b$ AND $c$ IN EQUATION 14.2

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4995</td>
<td>-0.3268</td>
<td>0.0778</td>
<td>0.9985</td>
</tr>
<tr>
<td>2</td>
<td>0.4781</td>
<td>-0.3496</td>
<td>0.1000</td>
<td>0.9676</td>
</tr>
<tr>
<td>3</td>
<td>0.4041</td>
<td>-0.2347</td>
<td>0.0555</td>
<td>0.9999</td>
</tr>
<tr>
<td>4</td>
<td>0.4657</td>
<td>-0.3821</td>
<td>0.1281</td>
<td>0.9761</td>
</tr>
<tr>
<td>5</td>
<td>0.5041</td>
<td>-0.2468</td>
<td>0.0508</td>
<td>0.9920</td>
</tr>
<tr>
<td>6</td>
<td>0.4065</td>
<td>-0.3950</td>
<td>0.1118</td>
<td>0.9845</td>
</tr>
<tr>
<td>7</td>
<td>0.4468</td>
<td>-0.3067</td>
<td>0.0820</td>
<td>0.9999</td>
</tr>
<tr>
<td>8</td>
<td>0.5199</td>
<td>-0.5704</td>
<td>0.2050</td>
<td>0.9887</td>
</tr>
<tr>
<td>9</td>
<td>0.4633</td>
<td>-0.4726</td>
<td>0.1199</td>
<td>0.9974</td>
</tr>
<tr>
<td>10</td>
<td>0.5047</td>
<td>-0.2514</td>
<td>0.0752</td>
<td>0.9704</td>
</tr>
<tr>
<td>11</td>
<td>0.4096</td>
<td>-0.2392</td>
<td>0.0555</td>
<td>0.9972</td>
</tr>
<tr>
<td>12</td>
<td>0.5066</td>
<td>-0.5353</td>
<td>0.2169</td>
<td>0.9992</td>
</tr>
<tr>
<td>13</td>
<td>0.4301</td>
<td>-0.3263</td>
<td>0.0928</td>
<td>0.9725</td>
</tr>
<tr>
<td>14</td>
<td>0.4605</td>
<td>-0.3950</td>
<td>0.1118</td>
<td>0.9845</td>
</tr>
<tr>
<td>15</td>
<td>0.6912</td>
<td>-0.5423</td>
<td>0.1541</td>
<td>0.9999</td>
</tr>
<tr>
<td>16</td>
<td>0.5430</td>
<td>-0.5365</td>
<td>0.1711</td>
<td>0.9926</td>
</tr>
<tr>
<td>17</td>
<td>0.4051</td>
<td>-0.3470</td>
<td>0.1109</td>
<td>0.9264</td>
</tr>
<tr>
<td>18</td>
<td>0.3763</td>
<td>-0.2985</td>
<td>0.0898</td>
<td>0.9966</td>
</tr>
<tr>
<td>19</td>
<td>0.2621</td>
<td>-0.1859</td>
<td>0.0696</td>
<td>0.8173</td>
</tr>
<tr>
<td>20</td>
<td>0.3367</td>
<td>-0.2994</td>
<td>0.1033</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

*Sample No. provides information on its gradation characteristics. For instance, $a_2$ denotes sample with mean grain size of $d_50$ (1.50 mm) and a designed uniformity coefficient of 2.
\[ b = -0.3096 + 0.785 \log_{10} e + 0.145 \log_{10} Cu \\
+ 0.1307 \left( \log_{10} D_{50} \right)^2 \]

\((R = 0.422)\)

\[ c = 0.306 - 0.0492 \log_{10} Cu - 0.258 e \]

\((R = 0.364)\)

where

\[ D_{50} = \text{mean grain size in millimeters}; \]

\[ Cu = \text{uniformity coefficient of the sand}; \]

\[ e = \text{void ratio of the soil}; \]

\[ D_{10} = \text{effective grain size in millimeters}. \]

From the discussion in Section 14.4, the effect of uniformity is negligible compared with the effect of mean diameter. If the effect of mean grain size is the only governing factor among the grain size distribution characteristics, the stress ratio required to cause initial liquefaction at the tenth cycle and the thirtieth cycle can be expressed by the following equations:

At the tenth cycle

\[ R_s = 0.249 + 0.2374 \log_{10} D_{50} \\
+ 0.111 \left( \log_{10} D_{50} \right)^2 \]

\((R = 0.936)\)

At the thirtieth cycle

\[ R_s = 0.1979 + 0.1783 \log_{10} D_{50} \\
+ 0.096 \left( \log_{10} D_{50} \right)^2 \]

\((R = 0.908)\)

The above equations are similar to the following equations which were obtained by Finn (1972). But the latter equations give higher
stress ratios:

At the tenth cycle

\[ \frac{\tau}{\sigma_0} = 0.285 + 0.068 \log_{10} D_{50} \]

At the thirtieth cycle

\[ \frac{\tau}{\sigma_0} = 0.259 + 0.0814 \log_{10} D_{50} \]

where

\( \tau = \text{cyclic shear stress} ; \)

\( \sigma_0 = \text{confining pressure} ; \) and

\( \frac{\tau}{\sigma_0} = \text{cyclic stress ratio}. \)
15.1 Summary

The purpose of this study was to investigate the effect of grain size distribution on the liquefaction potential of granular soils. Laboratory cyclic triaxial tests were performed on samples prepared from Denver sand. The mean grain size and the uniformity coefficient of these samples range from 0.1 mm to 1.5 mm and 2 to 16, respectively. At each uniformity coefficient, samples of several different mean grain sizes were prepared.

All samples were consolidated under 30 psi initial confining pressure, and prepared to achieve the relative density of approximately 50%. Due to the difficulty of achieving exactly 50% relative density, a test program was performed to investigate the effect of the small variation in relative density on liquefaction. The result of this investigation was then used to adjust the cyclic stress ratio of samples with relative density other than 50% to the stress ratio corresponding to 50% relative density. A slow frequency of 0.05 Hz was used to reduce the effect of deficiency in the hydraulic power.

The effect of mean grain size on liquefaction potential was studied by using the test results on samples with the same uniformity
coefficient and different mean grain sizes. Although the actual uniformity of samples varied slightly, the result was found to be consistent to that of other researchers. The effect of uniformity coefficient was investigated by using the data from samples with the same mean grain size and varying uniformity coefficients.

Functional relationships between the liquefaction potential and the mean grain size were formulated by using regression analyses.

15.2 Conclusions

The liquefaction potential was found to be significantly affected by the mean grain size. However, its effect becomes less significant when the uniformity is greater than 10. Since the test results were not corrected for the membrane penetration effect, the effect might have been overestimated. Because of the strong correlation between the liquefaction potential and the grain size, the cyclic stress ratios causing initial liquefaction at the 10th and 30th cycles were formulated as the function of the mean grain size.

The uniformity coefficient has much less effect on the liquefaction potential than the mean grain size. However, its effect is more significant for the coarse-grained soil than the fine-grained one. For soils with their mean grain sizes smaller than 0.40 mm, the effect of the uniformity coefficient on the liquefaction potential was found to be insignificant.

The effect of membrane penetration on the triaxial test result has received very much less attention than it really
deserves. The effect is believed to be more significant for coarse and uniform soils.

15.3 Recommendation for Future Study

Many factors have various degrees of effect on the liquefaction potential of sands. To have a preliminary assessment without an elaborate laboratory testing program of the liquefaction potential of a sand of a given gradation, one must be equipped with the knowledge of the degree of the effect of various factors. However, no priority of importance and a definite rule for assessing the liquefaction potential were given to facilitate the preliminary assessment of the cyclic strength of sands by a geotechnical engineer. Thus, a sensitivity study on the factors affecting the liquefaction potential and cyclic strength of sands is now overdue.

It is extremely important to develop a proper mathematical model and implement it in a finite element analysis program. This program can then be used to investigate the progressive deterioration of bearing capacity of saturated sands, the interaction between saturated sands and structures, and the behavior of a saturated sand deposit under a cyclic or seismic load. The mathematical model should handle the problems involving a compression wave and a wave with the stress reversal. To be able to predict the behavior of a saturated sand subjected to the latter wave, the model should also be capable of simulating the behavior under an extension wave.

From a practical standpoint, the use of the in-situ SPT (standard penetration test) result in predicting the potential of
liquefaction of a saturated sand deposit has a great practical value. This is mainly due to the availability of the massive SPT data in both the United States and Japan and the SPT being a part of basic subsoil investigation. However, to achieve this goal requires the study on the relation between the standard penetration resistance and the standardization of SPT.

A natural sand deposit frequently is not clean but contains some fines (minus 60 mm). Thus, it is important to study the effect of fines on the cyclic strength and the liquefaction potential of sands.

In a lacustrine deposit, alluvial deposit and hydraulic fill, sands often exist as isolated lenses or layer within low permeable media, such as clay. Under a dynamic load, the sand would be essentially undrained. Therefore, the study on the undrained cyclic behavior of a saturated sand and its effect on the global behavior of the above deposits becomes worthwhile.

Finally, an extensive study on the effect of membrane penetration on the dynamic properties, the pore water generation mechanism and the liquefaction potential of sands under nonstatic loads is required to provide a necessary correction factor for cyclic triaxial test results.
REFERENCES


Casagrande, A. "Characteristics of Cohesionless Soils Affecting the Stability of Slopes and Earth Fills." Journal of Boston Society of Civil Engineers." (June, 1936)


Wang, W.S. "Mechanism of Soil Liquefaction." International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, St. Louis, Missouri (April, 1981).

