STRUCTURAL ENGINEERING

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APPLICATION OF MODERN CONTROL THEORY FOR BUILDING STRUCTURES

by

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1. Introduction:

Automatic control has played an important role in the advancement of engineering and science. In addition to its extreme importance in space-vehicle, missile-guidance, and aircraft-piloting systems, the application of automatic control to ensure comfort and safety of civil engineering structures has been explored in recent years. In earthquake-prone areas, all structures must be designed to withstand a certain intensity of ground motion without collapse. To-date, no well-designed tall building has suffered collapse. However, large displacements due to strong-motion earthquake can cause severe damage to these structures. To reduce the displacement of the structures, one can use either passive control devices and/or active control devices. Although the idea of active control is not new, the application of control theory to reduce motions of civil engineering structures is still being developed.

2. General Approach for n Degree-of-Freedom Systems:

A classical control technique was described in a recent report [1]. A fundamental method of classical design of the control system consists of forcing the dominant closed-loop poles to be suitably located in the s-plane. In this report, the design of feedback compensators is considered for linear and constant-coefficient multivariable systems. One of the basic design objective is to ensure satisfactory transient response by obtaining suitable pole locations. This problem is analyzed, first under the assumption that all state variables can be used in forming feedback signals. An additional design objective is to obtain a decoupled or noninteracting system. This means that each input component affects just one output component or some prescribed subset of output components.

Consider a system with following equations:

$$\dot{X} = AX + BU \tag{1}$$
$$Y = CX + DU$$

Where X is a nx1 state vector, U is the rx1 input vector, and Y is the mx1 output vector. The feedback gain matrix K is rxn and is assumed to consist of constant elements. The input V is assumed to be 1x1 vector and feedback matrix F is also assumed to be constant and is of dimension rx1. In civil engineering structures, we do not usually have input vector because the set point of the system is fixed. In the case of earthquakes, we have a base excitation, V. The block diagram of the modified system is shown in Fig. 2, in which V is the disturbance force vector (1x1) and F is a feedback matrix (nx1) for the system. It is assumed, at this point, that all state variables are measurable. Therefore, the C matrix becomes an identity matrix, and the D matrix becomes a null matrix. In this case, the system is represented with the following equations:

$$\dot{X} = AX + BU + FV \tag{2}$$
$$Y = X$$

Note also that several optimal control laws take the form of a state feedback control. Optimal control laws will be discussed later.

Let the control force be U=-KX,then the general form of equation becomes:

$$\dot{X} = AX - BKX + FV \tag{3}$$

or

$$\dot{X} = (A - BK)X + FV \tag{4}$$

The stability of the open-loop system depends on the eigenvalues of the A matrix. A system is stable if all eigenvalues of the open-loop system have a negative real part. By introducing the negative feedback system, we will have a closed-loop system and the stability of the system depends on the eigenvalues of the (A-BK) matrix. It means that to have a stable system, all poles of (A-BK) must have a negative real part. The stability of a linear time-invariant system depends only on the location of the eigenvalues in the s-plane. For a open-loop system, a system is stable if and only if all poles of the system be located on the left-hand side of the s-plane. This is also true for a closed-loop system. Relocating the poles of a system is called pole-assignment problem. In this method, we will find a K matrix such that the closed-loop system would have a desired eigenvalues and performance. The pole-assignment method is reviewed in detail in the Appendix. It has been proved that if and only if the open-loop system (A,B) is completely controllable, then any set of desired closed-loop eigenvalues can be achieved using a constant state feedback matrix K. To synthesize the system with real hardware, all elements of K must be real. It has been assumed that all state variables are available in our study. However, it is possible that we need to estimate these states. This method is called state reconstruction. The main goal of this method is to obtain a good estimate of the states $X_{(t)}$, given a knowledge of the output. In this case, the C matrix is not a square matrix. In this method, we will construct another system which is called observer. The input of the observer will depend on Y, U, and its output should be a good estimate of X. The second system has the form:

$$\dot{\bar{X}} = A_c \bar{X} + B_c Y + Z \qquad (5)$$

Where $\bar{X}$ is the nx1 vector estimation to X and A_c and Z are selected as:

$$A_c = A - B_c \cdot C \qquad (6)$$
$$Z = B U$$

B_c is a nxm matrix that its elements are unknown. If all eigenvalues of the A_c matrix have a negative real part, then as $t \to \infty$ we will have $\bar{X}_{(t)} \to X_{(t)}$. In this case, we must find B_c such that all eigenvalues of the A_c matrix have a negative real part. This problem is almost like pole-assignment method and one can use the same approach to find the elements of B_c matrix. It is always possible to find a B_c matrix which will yield any set of desired poles for A_c if the original open-loop system be completely observable. Now, we can use the observer system together with a constant state feedback matrix K. The input of the state feedback matrix is $\hat{X}$ rather than X, but we know that as time goes to infinity, the error of the system becomes smaller or even approaches to zero. Fig. 3 shows the block diagram of the composite system, the order of the combined system is equal to 2n. By proper selection of K and B_c, the system will have a desired performance. It is obvious that selections of K and B_c matrices are separate from each other. It means a separation principle. This approach is described as follow: First, we select n desired closed-loop eigenvalues for the system. By using the pole-assignment method, one can find the K matrix. Second, we choose n desired poles for the observer system and by the same method, one is able to determine the elements of the B_c matrix. The complete controllability and observability of the open-loop system guarantees to find at least one pair of K and B_c matrices.

The observer method described herein is called an identity observer because we constructed a total state vector. It is possible to design a lower order system, but we will not discuss it in this study.

Experience shows that to have a good design, poles of the observer should have a bigger real part than poles of closed-loop system. We will show later that in civil engineering structures, most eigenvalues of the closed-loop system will have a very small real part when we neglect the effect of the damping during the earthquake.

Consider a n-degree-of-freedom system as shown in Fig. 4, the equation of motion can be written as follows:

$$m\ddot{x} + c\dot{x} + kx = N + f \qquad (7)$$

Where m, c, and k are nxn mass, damping, and stiffness matrices of the system respectively; N is a nx1 external force induced by the ground acceleration and f is a nx1 control force. By using the state space variable concept, Equation 7 can be written as:

$$\dot{X} = AX + BU + FV \qquad (8)$$

To use the pole-assignment technique, the open-loop system must be completely controllable. Then, to find a constant feedback matrix, K, one should select the closed-loop poles of the system such that the performance of the system becomes desirable. The solution of the problem is only unique when the B matrix has only one column. It means that there is only one active control force which has been applied at one of the nodes of the system. The total number of the active control force are r. Because the duration of earthquake is short enough, one can neglect the effect of the damping. Then, Equation 7 can be written as follows:

$$m\ddot{x} + kx = N + f \qquad (10)$$

In this case, it is possible to find a similarity transformation, x=Tq, such that both mass and stiffness matrices become diagonal, then Equation 10 can be written as following:

$$m^{*}\ddot{q} + k^{*}q = N^{*} + f^{*} \qquad (11)$$

The matrices m and k are real symmetric and positive definite. Since the eigenvalues are real and positive the square root of them are called the natural frequencies of the system. The eigen-vectors of the system are orthogonal and they may be normalized. The left-hand side of Equation 11 is completely uncoupled but the right-hand side of the equation may or may not be uncoupled. The right-hand side of the equation is only uncoupled when the control force of each mode be a function of displacement and velocity of that mode, $f_i^* = f(q_i, \dot{q}_i, t)$. On the other hand, if the control force is assumed as a function of all states, $f_i^* = f(q_1, \dot{q}_1, \ldots, q_n, \dot{q}_n, t)$ then the right-hand side of the equation is coupled. When the right-hand side of the equation 11 is coupled; the equation of motion can be written as following:

$$\dot{X} = A\,X + B\,U + F\,V \qquad (12)$$

By using a state feedback system, U=-KX, one can use the pole-assignment technique and find the elements of the K matrix. Note that it is not necessary to neglect the damping of the system when the pole-assignment method has been used. Because there is no need to have a diagonal mass and stiffness matrices. This is one of the advantages of the pole-assignment method compare to another method that we will discuss later.

To have a completely uncoupled system, externally and internally, one can assume that the control force for each mode is a function of the displacement and velocity of that mode. In this case, we will have n single-degree freedom systems and each mode can be controlled separately. It is possible to control few modes of the system rather than all modes. The ith equation of Equation 11 can be written in the following:

$$m_i^* \ddot{q}_i + k_i^* q_i = N_i^* + f_i^* \tag{13}$$

where $f_i^* = f(q_i, \dot{q}_i, t)$ and by using the state space variable concept for each equation, we will have:

$$\dot{X}_i = A_i X_i + B_i U_i + F_i V_i \tag{14}$$

where:

$$X_i = \begin{bmatrix} q_i \\ \dot{q}_i \end{bmatrix} \quad A_i = \begin{bmatrix} 0 & 1 \\ -w_i^2 & 0 \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ \frac{1}{m_i^*} \end{bmatrix}$$

$$F_i = \begin{bmatrix} 0 \\ \frac{1}{m_i^*} \end{bmatrix} \quad u_i = -[K_i] X_i \quad V_i = N_i^* \tag{15}$$

Fig. 5 shows the block diagram of the ith mode. $G_c(s)$ is the transfer function of the controller. It may be assumed as a constant gain or variable. The transfer function of controller can be shown as:

$$G_c(s) = \frac{K_p}{1 + \tau_a S} \tag{16}$$

In this case, one can consider the effect of time constant, τ_a, on the system. $G_p(s)$ is the transfer function of the plant for the ith mode and may be written as following:

$$G_p(s) = \frac{1}{m_i^* S^2 + k_i^*} \tag{17}$$

H(s) is the state feedback element and it has been shown as optimal gain for each mode.

Note that it is not necessary to control all modes of a system because first few modes of the civil engineering structures are the most critical modes of the system. Equation 11 can be divided into two parts; first those modes that we want to control and second those we do not want to control. Then Equation 11 can be written as follows:

$$\begin{bmatrix} m_c^* & 0 \\ 0 & m_u^* \end{bmatrix} \begin{bmatrix} \ddot{q}_c \\ \ddot{q}_u \end{bmatrix} + \begin{bmatrix} k_c^* & 0 \\ 0 & k_u^* \end{bmatrix} \begin{bmatrix} q_c \\ q_u \end{bmatrix} = \begin{bmatrix} N_c^* \\ N_u^* \end{bmatrix} + \begin{bmatrix} f_c^* \\ f_u^* \end{bmatrix} \tag{18}$$

or

$$\begin{aligned} m_c^* \ddot{q}_c + k_c^* q_c &= N_c^* + f_c^* \\ m_u^* \ddot{q}_u + k_u^* q_u &= N_u^* + f_u^* \end{aligned} \tag{19}$$

and the similarity transformaion can be written as $[T] = [T_c | T_u]$. It means that the motion is almost a linear combination of the controlled modes and may be some important uncontrolled modes.

In Equation 15, the K matrix for each mode has only two elements. The elements of this matrix are shown as $[\bar{k}^* | \bar{c}^*]_i = [B]_i \ [K]_i$, then Equations 13 and 11 become respectively as follow:

$$m_i^* \, q_i^* + k_i^* \, q_i = N_i^* - \bar{c}_i^* \, \dot{q}_i - \bar{k}_i^* \, q_i \tag{20}$$

$$m^* \, \ddot{q} + k^* \, q = N^* - \bar{c}^* \, \dot{q} - \bar{k}^* \, q \tag{21}$$

where $\bar{c}^*$ and $\bar{k}^*$ are nxn diagonal matrices that each one has some zero elements for those uncontrolled modes and some nonzero elements corresponding to controlled modes of the system. By using the same similarity transformation, $q = T^{-1} x$, Equation 21 can be written as follows:

$$m\ddot{x} + \overline{c}\,\dot{x} + \overline{(k} + k)x = N \qquad (22)$$

where $\overline{c}$ and $(\overline{k}+k)$ are nxn equivalent damping and stiffness of the closed-loop system. In this case, the K matrix for the control law, U=-KX, can be found by using state space variable. To use the state space variable both Equation 10 and 22 can be shown by Equations 23 and 24 respectively:

$$\dot{X} = AX + BU + FV \qquad (23)$$

$$\dot{X} = \overline{A}X + FV \qquad (24)$$

Where $\overline{A} = A - BK$ and the K matrix can be found by using Equation 25.

$$K = (B^T B)^{-1} B^T (A-\overline{A}) \qquad (25)$$

In next chapter, a four degree-of-freedom system has been controlled by using this method and pole-assignment method. Note that the modal superposition method is not used in this approach;it is only used as a tool to find the [K] matrix for the control law. It is seen that this method is more applicable to civil engineering structures than pole- assignment method. To apply this method, the damping of the open-loop system must be neglected.But this method can be applied to underdamped structures although it has some approximate for these kind of structures.

Linear Optimal Control

We shall now consider the optimal problem that, given the system equation $\dot{X} = AX + BU$ determine the matrix K of the control vector $U=-KX$ so as to minimize the performance index:

$$J = \int_0^\infty (X^T Q X + U^T R U)dt \qquad (26)$$

The matrices Q and R determine the relative importance of the error and expenditure of this energy. In this case, we assume that the control vector U is unconstrained. Without proof, the linear control law given by $U=-KX$ is the optimal control law. Therefore, if the unknown elements of the matrix K are determined so as to minimize the performance index, then $U=-KX$ is optimal for any initial state. The optimal K matrix is given as follows:

$$K = R^{-1} B^T P \qquad (27)$$

where P is the solution of the Riccati equation [1].

The problem becomes easier when the system is internally and externally decoupled because in this case, P is a 2x2 matrix. Let the number of controlled modes be r, then for each mode, there is one optimal gain matrix K_i which satisfies the Riccati equation.

3. Numerical Examples:

In this chapter, we illustrate the application of the pole assignment method with several numerical examples. First, consider a two-story shear-type building which is subjected to earthquakes loads. All possible combinations of active control forces are considered herein. In the first case (System I), two active control forces are applied at each node of the system. In the second case (System II), an active control force is applied at node one; and in the third case (System III), an active control force is applied at node two. All three cases are shown in Fig. 6, and the properties of the system are given as follows:

$$m_1 = m_2 = 0.3 \frac{\text{lb-sec}^2}{\text{in}} \quad c_1 = c_2 = 0.5 \frac{\text{lb-sec}}{\text{in}} \quad k_1 = k_2 = 2.5 \frac{\text{lb}}{\text{in}}$$

The equation of motion for this system can be written as $\dot{x} = AX + BU + FV$. In this example, the effect of the damping is not neglected. Therefore, decoupling of the system is not always possible. The active control forces are function of the all states. It can be shown that all these three cases are completely controllable and observable. Therefore, the pole assignment method is applicable. The K matrix and control energy are computed for each case. The control energy is a measure of the control effort expanded and it is a part of the performance index, $\int_0^{t_f} u^T u \, dt$. Note that all elements of the K matrix must be real. The solution of pole assignment method is unique for cases II and III. However, it is not unique for case I. The eigenvalues of the open-loop system are found to be $\lambda_{1,2} = 2.18 \pm 4.13i$, and $\lambda_{3,4} = -.32 \pm 1.76i$ It is known that the open-loop system is always stable. To obtain a closed-loop system with desired eigenvalues, many different closed-loop eigenvalues have been chosen and for each case both the K matrix and the control energy are calculated. Then, the displacements and velocities of both nodes are plotted. The maximum displacements and velocities and control energy for the Case I are listed in Table 1. The closed-loop eigenvalues are given along with computed K matrix as follows:

Case 1-A: $\lambda_{1,2} = -5. \pm 8.0i$ $\lambda_{3,4} = -2.0 \pm 4.0i$

$$[K] = \begin{bmatrix} 25.59 & 2.27 & 9.37 & 2.57 \\ .20 & .07 & 3.03 & .43 \end{bmatrix}$$

Case 1-B: $\lambda_{1,2} = -5. \pm 8.i$ $\lambda_{3,4} = -2.0 \pm 4.0i$

$$[K] = \begin{bmatrix} 4.47 & 0.00 & 1.05 & .05 \\ 18.02 & 2.08 & 25.23 & 2.81 \end{bmatrix}$$

Case 2-A: $\lambda_{1,2} = -5. \pm 5.i$ $\lambda_{3,4} = -2. \pm 2.i$

$$[K] = \begin{bmatrix} 18.55 & 3.03 & 22.81 & 5.67 \\ -.83 & -.07 & -4.16 & -.33 \end{bmatrix}$$

Case 2-B: $\lambda_{1,2} = -5. \pm 5.i$ $\lambda_{3,4} = -2. \pm 2.i$

$$[K] = \begin{bmatrix} 3.40 & 1.04 & .05 & .18 \\ -9.17 & -4.92 & 14.06 & 1.66 \end{bmatrix}$$

Case 3-A: $\lambda_{1,2} = -7. \pm 5.i$ $\lambda_{3,4} = -3.0 \pm 2.0i$

$$[K] = \begin{bmatrix} 23.44 & 3.32 & 14.83 & 1.29 \\ -.18 & .15 & -.26 & 1.18 \end{bmatrix}$$

Case 3-B: $\lambda_{1,2} = -7. \pm 5.i$ $\lambda_{3,4} = -3.0 \pm 2.i$

$$[K] = \begin{bmatrix} 3.88 & 1.06 & .05 & .13 \\ -3.07 & -1.95 & 20.40 & 3.44 \end{bmatrix}$$

Case 4-A: $\lambda_{1,2} = -9. \pm 5.i$ $\lambda_{3,4} = -5. \pm 2.i$

$$[K] = \begin{bmatrix} 31.67 & 4.40 & 5.15 & .46 \\ 31.57 & 4.40 & 5.15 & .46 \end{bmatrix}$$

Case 4-B: $\lambda_{1,2} = -9. \pm 5.i$ $\lambda_{3,4} = -5. \pm 2.i$

$$[K] = \begin{bmatrix} 8.18 & 2.02 & -.13 & .18 \\ 1.98 & .13 & 29.53 & 4.88 \end{bmatrix}$$

Case 5-A: $\lambda_{1,2} = -9. \pm 8i$ $\lambda_{3,4} = -5. \pm 4.i$

$$[K] = \begin{bmatrix} 43.62 & 4.49 & 10.83 & 1.53 \\ -.47 & .19 & 9.07 & 2.41 \end{bmatrix}$$

Case 5-B: $\lambda_{1,2} = -9. \pm 8i$ $\lambda_{3,4} = -5. \pm 4i$

$$[K] = \begin{bmatrix} 11.28 & 1.95 & -.31 & .16 \\ 7.7 & .71 & 41.7 & 4.95 \end{bmatrix}$$

Case 6-A: $\lambda_{1,2} = -12. \pm 5.i$ $\lambda_{3,4} = -6. \pm 2.i$

$$[K] = \begin{bmatrix} 50.68 & 6.23 & 7.44 & .80 \\ -.57 & .18 & 9.09 & 3.07 \end{bmatrix}$$

Case 6-B: $\lambda_{1,2} = -12. \pm 5.i$ $\lambda_{3,4} = -6. \pm 2.i$

$$[K] = \begin{bmatrix} 11.34 & 2.60 & -.69 & .15 \\ 3.73 & .39 & 48.54 & 6.70 \end{bmatrix}$$

Case 7-A: $\lambda_{1,2} = -18. \pm 5.i$ $\lambda_{3,4} = -9. \pm 2.i$

$$[K] = \begin{bmatrix} 104.78 & 9.85 & 14.25 & 1.37 \\ -1.85 & .17 & 22.38 & 4.85 \end{bmatrix}$$

Case 7-B: $\lambda_{1,2} = -18. \pm 5.i$ $\lambda_{3,4} = -9. \pm 2.i$

$$[K] = \begin{bmatrix} 24.58 & 4.37 & -2.01 & .14 \\ 9.80 & .98 & 102.65 & 10.33 \end{bmatrix}$$

It can be seen that the closed-loop system is always stable because each pole has a negative real part. It is known that some changes in the K matrix may give a better result for the control of the displacement. Because the main goal of this investigation is to control the displacements of the system and the other performances of the system is not as important as the displacement, one can choose the best closed-loop poles for the system. For example, setting the negative elements of the K matrix equal to zero in both Cases 2-A and 2-B will give a better result. Cases 2-A-I and 2-B-I show the maximum displacements and velocities of the Cases 2-A and 2-B when those negative elements of the K matrix are set equal to zero. It is shown that the maximum displacements of these cases are smaller than the displacements of the original system. When the active control force has been applied at node one, System II, these closed-loop eigenvalues have been assumed for the system:

Case 1: $\lambda_{1,2} = -5. \pm 8.i \quad \lambda_{3,4} = -2. \pm 4.i$

Case 2: $\lambda_{1,2} = -5. \pm 5.i \quad \lambda_{3,4} = -2. \pm 2.i$

Case 3: $\lambda_{1,2} = -7. \pm 5.i \quad \lambda_{3,4} = -3. \pm 2.i$

For each case, the K matrix has been computed and the displacements and velocities of both nodes have been plotted. The K matrices are shown as:

Case 1: $[K] = [29.69 \quad 2.70 \quad 36.38 \quad 3.99]$

Case 2: $[K] = [14.40 \quad 2.70 \quad 2.00 \quad 4.00]$

Case 3: $[K] = [22.28 \quad 4.50 \quad 14.35 \quad 10.61]$

The maximum displacements, velocities and control energy for this case are shown in Table 2. For system III, when the active control force has been applied at node two rather than node one; these following closed-loop poles have been assumed for the system:

Case 1: $\lambda_{1,2} = -4.46 \pm 12.i \quad \lambda_{3,4} = 2.54 \pm 4.64i$

Case 2: $\lambda_{1,2} = -5. \pm 5.i \quad \lambda_{3,4} = -2. \pm 2.i$

Case 3: $\lambda_{1,2} = -7. \pm 5.i \quad \lambda_{3,4} = -3. \pm 2.i$

The K matrix has been computed for each case. The results are given as follows:

Case 1: $[K] = [55.28 \quad .00 \quad 53.99 \quad 2.70]$

Case 2: $[K] = [13.25 \quad .49 \quad 15.75 \quad 2.70]$

Case 3: $[K] = [32.24 \quad 6.30 \quad 21.96 \quad 4.5]$

Table 3 shows the maximum displacements, velocities, and control energy for system III due to these different K matrices. There is another point that we should mention here. The selection of desired closed-loop eigenvalues for the system is very important. For example, shifting the closed-loop poles of the system too far to the left of the s plane would not cause any problem for system I but it would cause some problems for the other two cases. Cases 4 thru 7 in both Tables 2 and 3 show that by shifting the poles of the system farther to the left of the s plane, the displacement of one node becomes larger than displacement of the system without active control force, even though the displacement of the other node is much smaller than the displacement of the node without active control force. Therefore, to design a control law, u = -KX, one has to be careful that the open-loop poles of the system should not shift too far to the left of the s plane. Of course, the problem can be solved by using more active control force. To show this problem, these following closed-loop eigenvalues have been chosen for Cases 4 thru 7 for both systems II and III.

Case 4: $\lambda_{1,2} = -9. \pm 5.i \quad \lambda_{3,4} = -5. \pm 2.i$

Case 5: $\lambda_{1,2} = -9. \pm 8.i \quad \lambda_{3,4} = -5. \pm 4.i$

Case 6: $\lambda_{1,2} = -12. \pm 5.i \quad \lambda_{3,4} = -6. \pm 2.i$

Case 7: $\lambda_{1,2,} = -18. \pm 5.i \quad \lambda_{3,4} = -9. \pm 2.i$

The [K] matrix for Systems II and III are given respectively as follows:

Case 4:

[K] = [33.47 6.90 79.20 27.42]

[K] = [106.29 21.45 31.92 6.90]

Case 5:

[K] = [46.85 6.90 169.13 28.56]

[K] = [202.93 10.54 65.40 6.90]

Case 6:

[K] = [47.94 9.30 197.39 49.09]

[K] = [235.30 38.49 50.09 9.30]

Case 7:

[K] = [117.13 14.70 953.49 107.51]

[K] = [1034.08 53.54 182.58 14.70]

The maximum displacements, velocities and control energy of these cases are shown in Table 2 and 3.

Consider a four degree freedom system shown in Fig. 7. The maximum displacements and velocities of each floor without active control force due to an artificial earthquake is shown in Table 4. Then, an active control force like a jet engine has been applied at top of the system. It has been assumed that the control force is a function of all states, therefore, it is not possible to have a completely internal and external decoupled system. In this case, the equation of motion can be written as $\dot{X} = AX + BU + FV$. It can be shown that there is a similarity transformation, [T], such that the A matrix becomes diagonal. It can also be shown that the system with one active control force at the top is completely controllable. Because there is no zero row in the $T^{-1}B$ matrix where T is the similarity transformation of the system. In this case, the damping of the system is neglected and the properties of the system are given as follows:

$$m_1 = m_2 = m_3 = m_4 = 0.3 \frac{lb\text{-}sec^2}{in} \quad k_1 = k_2 = k_3 = k_4 = 2.5 \frac{lb}{in}$$

The open-loop eigenvalues of the system are:

$$\lambda_{1,2} = \pm 5.43i, \ \lambda_{3,4} = \pm 4.42i, \ \lambda_{5,6} = \pm 2.89i \text{ and } \lambda_{7,8} = \pm i$$

Neglecting the damping of the system, the open-loop system is marginally stable. To have a stable closed-loop system, the closed-loop eigenvalues of the system are assumed as follows:

$$\lambda_{1,2} = -1. \pm 5.43i, \quad \lambda_{3,4} = -1. \pm 4.42i$$
$$\lambda_{5,6} = -1. \pm 2.89i \text{ and } \lambda_{7,8} = -1. \pm i$$

It means that we have shifted all open-loop eigenvalues to the left of the s plane. The control law has been assumed as $u = -KX$. The pole assignment method is applicable because the open-loop system, (A,B), is completely controllable. Then, the computed K matrix is

$$[K] = [-19.10 \quad 4.23 \quad 41.36 \quad -5.31 \quad -54.63 \quad .75 \quad 28.04 \quad 8.00]$$

Case 2 in Table 4 shows the maximum displacements and velocities of the system with active control force. It is seen that the system with active control force has smaller displacement than the system without active control force. As we mentioned earlier the elements of K matrix must be always real.

Consider the same four degree-of-freedom system shown in Fig. 7. The equation of motion without active control force can be written as $m\ddot{x} + kx = N$ and by using the similarity transformation, the equation of motion may be written as: $m^{*}\ddot{q} + k^{*}q = N^{*}$. Both m^{*} and k^{*} matrices are diagonal, then the right hand side of the equation is completely uncoupled. In this case, active control is only used to control the first node. The active control force for this mode is $f_4 = -[1.0 \quad 0.] \begin{matrix} q_4 \\ \dot{q}_4 \end{matrix}$.

Therefore, all elements of $\bar{k}^{*}$ is equal to zero except the last element which it is equal to one. Knowing that $\bar{k}$ is equal to $T\bar{k}^{*}T^{-1}$ where T matrix is the similarity transformation; one can find the $\bar{k}$ matrix which it is equal to:

$$[\bar{k}] = \begin{bmatrix} .05199 & .09771 & .13164 & .14970 \\ .09771 & .18363 & .24746 & .28134 \\ .13165 & .24741 & .33333 & .37906 \\ .14970 & .28134 & .37905 & .43104 \end{bmatrix}$$

knowing that the control law is equal to $U = -KX$, by using Equation 25 the K matrix can be found as follows:

$$[K] = \begin{bmatrix} .05199 & 0. & .09771 & 0. & .13164 & 0. & .14970 & .0 \\ .09771 & 0. & .18363 & 0. & .24740 & 0. & .28134 & .0 \\ .13165 & 0. & .24741 & 0. & .33333 & 0. & .37906 & .0 \\ .14970 & 0. & .28134 & 0. & .37905 & 0. & .43104 & .0 \end{bmatrix}$$

The maximum displacements and velocities of this system without active control force is shown by Case 1 in Table 4 and the maximum displacements and velocities of this system by using the above active control force is shown by Case 3 in Table 4. Note that it maybe useful to know that the natural frequencies of this system without active control force are $w_1 = 5.43$, $w_2 = 4.42$, $w_3 = 2.89$ and $w_4 = 1$. By using active control force the natural frequencies of the system are $w_1 = 5.43$, $w_2 = 4.42$, $w_3 = 2.89$ and $w_4 = 2.0$. It can be seen that the lowest natural frequency has been increased, and there is no change in the other natural frequencies of the system. It means that only two lowest open-loop poles of the system have been shifted to the left of the S plane and the other open-loop poles have not been shifted. This is one of the advantages of using this method rather than pole assignment method. It can also be seen that the dimension of K matrix in control law, U = -KX, is 4x8 then the number of control forces are equal to four. It means that we need an active control force at each node of the structure. It is shown that the maximum displacements of Case 3 in Table 4 are greater than the maximum displacements of Case 2 in the same Table. It is possible to reduce the displacements of system in Case 3 either by controlling more modes of the system or by using higher gain for each mode or both at the same time. For example, in the Case 4 in Table 4, two lower modes of the same system are controlled. The nonzero elements of $[\bar{k}^*]$ matrix in this case, are $\bar{k}^*_{33} = .5$ and $\bar{k}^*_{44} = 1$. The $[\bar{k}]$ is computed for this case are follows:

$$[\bar{k}] = \begin{bmatrix} .21866 & .26438 & .13164 & -.01697 \\ .26438 & .35030 & .24740 & .11468 \\ .13165 & .24741 & .33333 & .37906 \\ -.01697 & .11468 & .37905 & .59771 \end{bmatrix}$$

The $[\overline{c}^*]$ matrix is assumed as a null matrix, then the final [K] matrix of the control law, u = -KX, for this case is computed as follows:

$$[K] = \begin{bmatrix} .21866 & 0.00 & .26438 & 0. & .13164 & 0. & -.01697 & 0. \\ .26438 & 0.00 & .35030 & 0. & .24740 & 0. & .11468 & 0. \\ .13165 & 0.00 & .24741 & 0. & .33333 & 0. & .37906 & 0. \\ -.01697 & 0.00 & .11468 & 0. & .37905 & 0. & .59771 & 0. \end{bmatrix}$$

The maximum displacement and velocity of each node for this case is shown in Case 4 of Table 4. It is shown that maximum displacements of each node is less than the maximum displacements of each node without any active control force. To reduce the displacement of each node, one can use higher gains for the system. As an example, let consider the previous case again. In this case, both nonzero elements of the $[\overline{k}^*]$ matrix have been increased. These nonzero elements are given as $\overline{k}^*_{33}$ = 1. and $\overline{k}^*_{44}$ = 2. then the $[\overline{k}]$ matrix is computed as follows:

$$[\overline{k}] = \begin{bmatrix} .43732 & .52875 & .26329 & -.03393 \\ .52875 & .70060 & .49481 & .22935 \\ .26329 & .49482 & .66667 & .75812 \\ -.03393 & .22935 & .75809 & 1.19542 \end{bmatrix}$$

Again, the $[\overline{c}^*]$ matrix is assumed as a null matrix and the final [K] matrix for the control law is found as follows:

$$[K] = \begin{bmatrix} .43732 & 0.00 & .52875 & 0. & .26329 & 0. & -.03393 & 0. \\ .52875 & 0.00 & .70060 & 0. & .49481 & 0. & .22935 & 0. \\ .26329 & 0.00 & .49482 & 0. & .66667 & 0. & .75812 & 0. \\ -.03393 & 0.00 & .22935 & 0. & .75809 & 0. & 1.19542 & 0. \end{bmatrix}$$

The maximum displacements and velocities of this system are shown in Case 5 of Table 4. It is seen that the displacement of each node is less than the displacement of that node without active control force and it can be seen that that the maximum displacement of each node of this case is less than the maximum displacement of Case 4. Note that the advantage of this method compare to pole assignment method is obvious. Because by using higher gains for each individual modes of the system, one can control the displacement of the structure better than using the pole assignment method. Shifting the open-loop poles of the system to the left of the s plane will not give an unique solution for our problem. The application of pole assignment method for civil engineering structure is a trial and error method, but the other method will give an unique solution. It means to reduce the displacement of the system, one can control more mode or use higher gains or both at the same time. Note that in this report the space-state variable method is always used and the modal superposition method has been used as a tool. Because by introducing the control law, we may not have an uncoupled closed-loop system. Note also that the final control law for the system is a function of all states. The displacements and velocities for all cases in Tables 1, 2, 3 and 4 are shown in Figures 9 through 42.

4. Concluding Remarks

In this report, an attempt has been made to find a suitable gain matrix for active control of structures. The application of the pole-assignment method is discussed. Although it is easy to apply this method easily to any system which is completely controllabl and observable, it does not provide a physical understanding of civil engineering problems. Because the pole-assignment method is based on the shifting of the open-loop poles further to the left-hand side of the s-plane, this method is most applicable for mechanical or electrical systems. The second method which is developed in this study has certain advantages over the pole assignment method. The main advantages of this second method is to provide physical understanding about the active control of civil engineering structures, because several modes of the system are controlled. The K matrix of the control law can be found such that the first few modes of the system have smaller displacements than the corresponding displacements of the original system. Therefore, one can choose more modes or higher gain or both to reduce substantially displacements of the system.

Results of numerical examples show the use of the pole-assignment method requires the shifting of the open-loop poles of the system to the far left side of the s-plane and one still will not always obtain much smaller displacements. Therefore, the second method is the better one to use for the cntrol of civil engineering structures.

Acknowledgment

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Appendix A:

Pole Assignment Method:

Consider the system with the following equations:

$$\dot{X} = AX + BU + FV$$

$$U = -KX \qquad \text{(A-1)}$$

The eigenvalues of the closed-loop system are the root of

$$\Delta'(\lambda) = \left| \lambda I - A + BK \right| = 0 \qquad \text{(A-2)}$$

Equation (A-2) can be written as:

$$\Delta'(\lambda) = (\lambda I_n - A)\ [I_n + (\lambda I_n - A)^{-1}\ BK]$$
$$= \lambda I_n - A\ .\ I_n + (\lambda I_n - A)^{-1}\ BK \qquad \text{(A-3)}$$

The eigenvalues of the open loop system are the root of

$$\Delta(\lambda) = \lambda I_n - A \qquad \text{(A-4)}$$

The Laplace transform of the open-loop transition matrix is:

$$\phi(\lambda) = (\lambda I_n - A)^{-1} \qquad \text{(A-5)}$$

Now, the equation (A-3) can be written as:

$$\Delta\ (\lambda) = \Delta(\lambda) I_n + \phi(\lambda)\ BK = \Delta(\lambda) I_r + K\phi(\lambda)B \qquad \text{(A-6)}$$

The matrix K must be selected so that $\dot{\Delta}(\lambda_i) = 0$ for each λ_i i = 1,2,...n. It means that the rxr determinant of the equation (A-6) must be equal to zero for each λ_i because $\Delta(\lambda)$ can not be equal to zero. It has been assumed that the desired closed-loop eigenvalues are different from the open-loop eigenvalue of the system. The determinant of $I_r + K\ \phi(\lambda)B$ will be zero if any one row or column is zero. Define the jth column of I_r as 1_j and define $\Psi(\lambda_i) = \phi(\lambda_i)B$, with the jth column being Ψ_j. Then λ_i is a root of $\Delta(\lambda)$ if K is selected to satisfy $e_j + K\Psi_j(\lambda_i) = 0$, since this forces column j to be zero. Then,

$$K\Psi_j(\lambda_i) = -e_j \tag{A-7}$$

To determine K matrix, we must find an independent equation of type (A-7) for each λ_i. Because the open loop system, (A,B), is completely controllable we will have at least one K matrix. If all the desired λ_i are distinct, it will always possible to find n linearly independent columns $\Psi_{j_1}(\lambda_1)$, $\Psi_{j2}(\lambda_2)\ldots,\Psi_{jn}(\lambda_n)$ from the columns of the nxnr matrix $[PSI(\lambda_1)\ \Psi(\lambda_2)\ \ldots\ \Psi(\lambda_n)]$, then

$$K[\Psi_{j1}(\lambda_1)\ \Psi_{j2}(\lambda_2)\ldots\Psi_{jn}(\lambda_n)] = -[e_{j1}\ e_{j2}\ldots e_{jn}] \tag{A-8}$$

or

$$K = -[e_{j1}\ e_{j2}\ldots e_{jn}][\Psi_{j1}(\lambda_1)\Psi_{j2}(\lambda_2)\ldots\Psi_{jn}(\lambda_n)]^{-1} \tag{A-9}$$

This method can be modified for the repeated eigenvalues, but will not discuss it in this paper.

The gain matrix, K, is not unique, and the remaining freedom of choice may be useful in meeting other system specifications.

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Table 1: Maximum Displacements, Velocities, and Control Energy for System I.

Case No.	X1 MAX	V1 MAX	X2 MAX	V2 MAX	J
Open-Loop	4.34	20.04	5.17	22.84	-
1-A	1.89	23.57	3.08	23.78	13,825.
1-B	3.08	25.73	2.40	28.05	16,719.
2-A-	4.15	38.12	3.43	25.02	29,276.
2-A-I*	4.38	33.99	3.20	25.32	30,416.
2-B-	2.45	22.53	6.36	43.89	7,662.
2-B-I*	2.23	19.43	2.29	21.48	3512.
3-A	1.48	20.74	2.05	20.06	8,819.
3-B	2.29	19.48	2.33	21.49	3,496.
4-A	.91	15.86	.99	16.01	12,542.
4-B	1.69	18.60	1.02	16.39	8,625.
5-A	.99	15.47	1.72	18.74	12,262.
5-B	1.70	20.11	.92	15.67	11,599.
6-A	.74	13.11	1.42	17.58	11.774.
6-B	1.46	18.62	.72	12.99	11,390.
7-A	.47	8.69	1.01	15.77	16,211.
7-B	.98	16.06	.49	9.04	15,781.

Table 2. Maximum Displacements, Velocities and Control Energy for System II.

Case No.	X1 MAX	V1 MAX	X2 MAX	V2 MAX	J
Open-Loop	4.34	20.04	6.17	22.84	-
1-	4.84	38.47	3.46	26.41	31,596.
2-	2.44	22.15	3.02	24.19	15,453.
3-	4.32	38.69	2.43	22.79	51,046.
4-	7.23	79.55	2.27	23.74	168,725.
5-	8.49	96.08	2.28	25.03	223,853.
6-	8.76	116.75	1.85	23.90	314,765.
7-	9.82	159.14	1.30	21.91	652,149.

Table 3: Maximum Displacements, Velocities, and Control Energy for System III.

Case No.	X1 MAX	V1 MAX	X2 MAX	V2 MAX	J
Open-Loop	4.34	20.04	6.17	22.84	-
1	2.23	25.72	4.54	47.89	44,745.
2	3.45	26.02	2.67	26.03	7,073.
3	2.86	22.82	3.59	30.45	33,920.
4	2.23	24.63	6.53	67.91	139,277.
5	2.59	27.57	7.11	69.04	119,040.
6	1.86	25.06	7.91	103.58	251,385.
7	1.59	24.34	8.34	120.95	353,657.

Table 4: Maximum Displacements, and velocities of a four-degree -of-freedom System.

	X_{1max}	V_{1max}	X_{2max}	V_{2max}	X_{3max}	V_{3max}	X_{4max}	V_{4max}
1	8.55	27.53	11.46	27.48	16.90	34.92	21.85	31.53
2	4.45	21.89	7.08	21.66	6.97	22.25	7.49	19.91
3	8.87	26.68	10.77	26.33	12.42	33.97	15.52	46.06
4	6.57	28.97	10.33	27.01	12.34	33.23	16.07	46.56
5	5.99	32.88	8.08	27.53	11.25	35.73	12.82	38.85

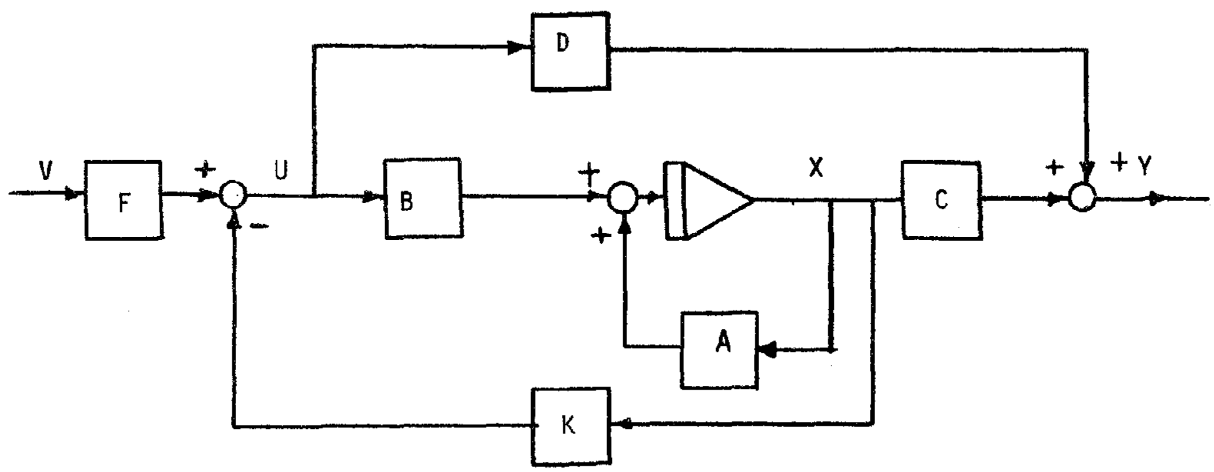

FIG. 1 State Variable Feedback System.

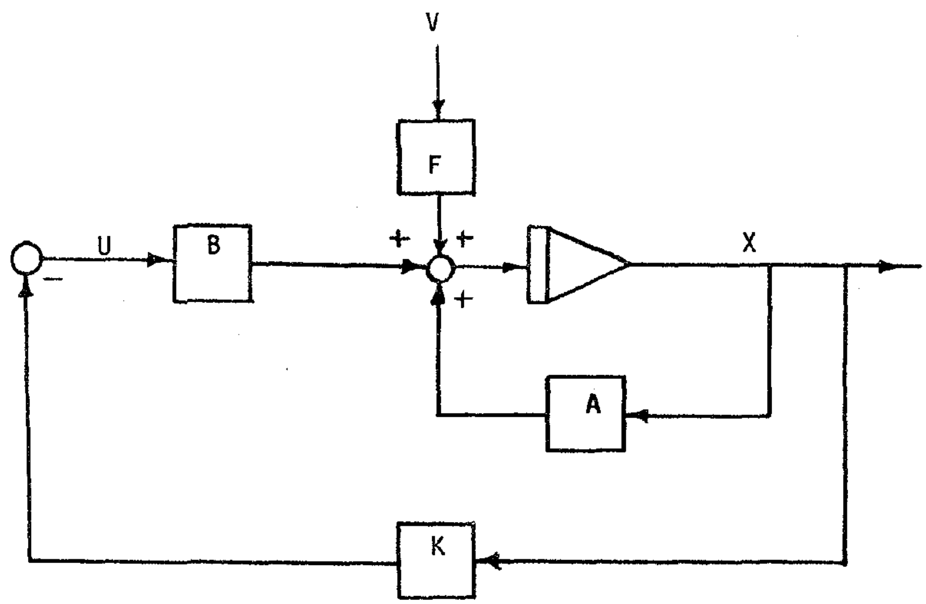

FIG. 2 State Variable Feedback for Civil Engineering Structures.

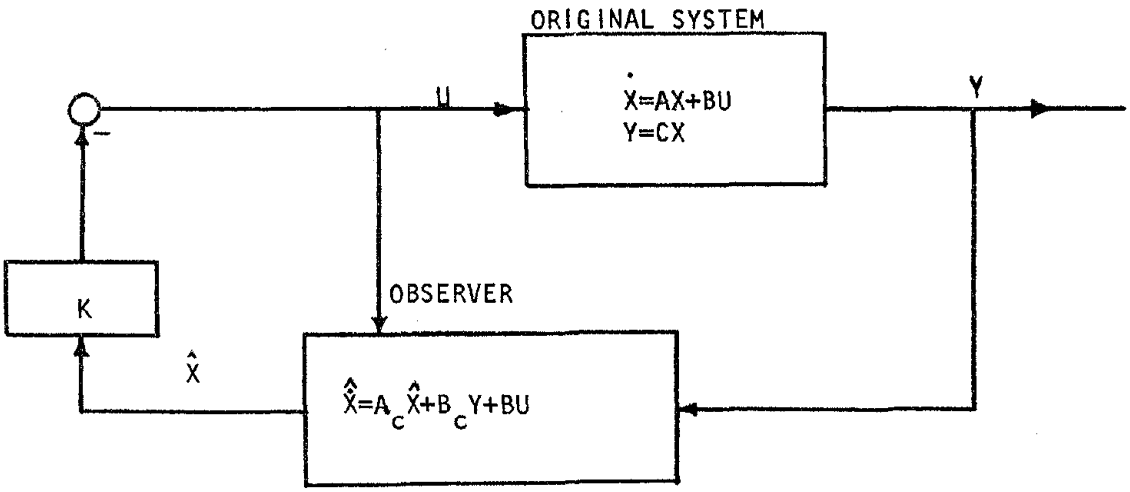

FIG. 3 Combined Observer and Stste Feedback System.

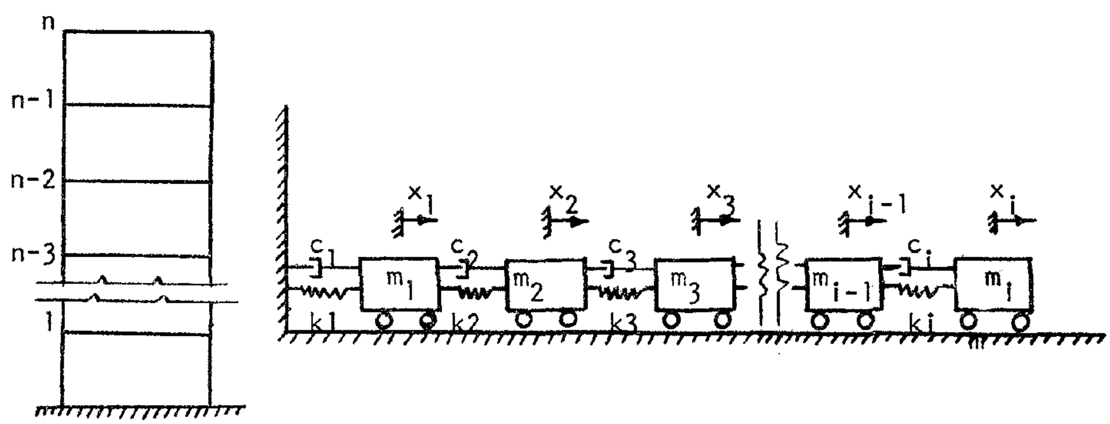

FIG.4 n Degree Freedom System and Its Mechanical Modal.

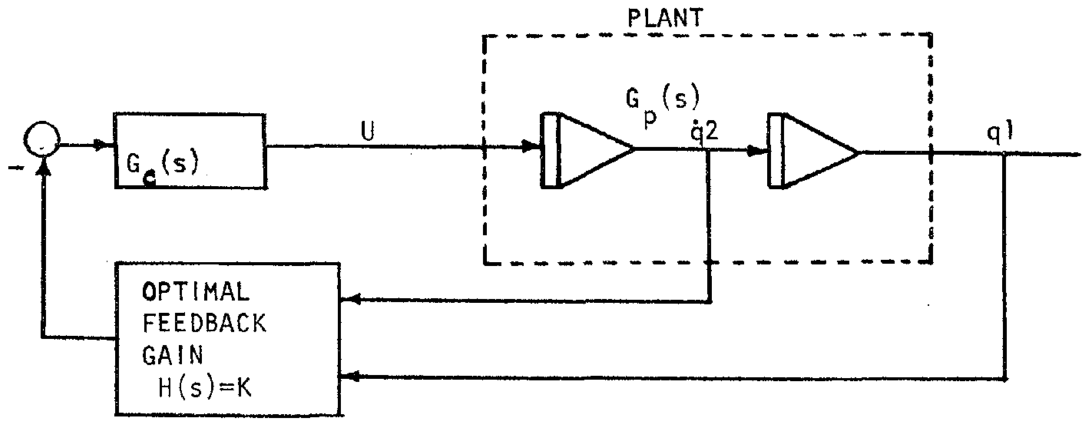

FIG. 5 The Block Diagram for i^{th} Mode of the System.

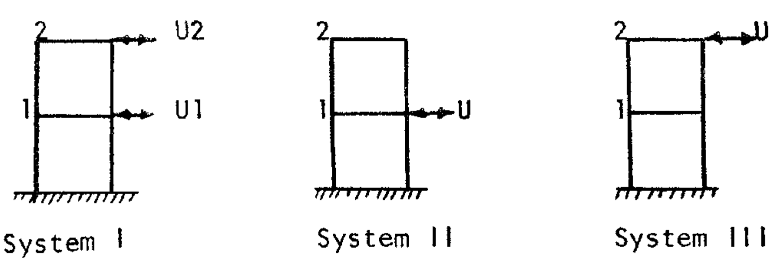

FIG. 6 Two Degree Freedom Shear-Type Building.

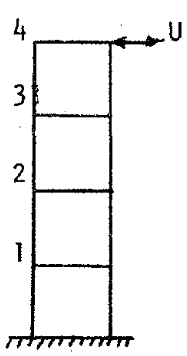

FIG. 7 Four Degree Freedom System Shear-Type Building.

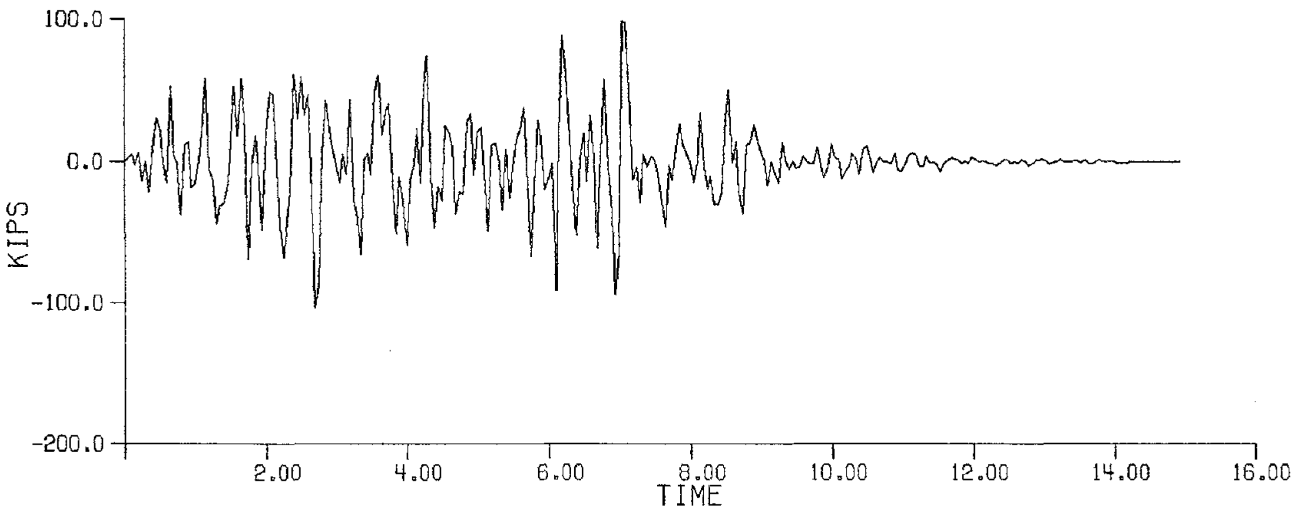

Fig. 8 External Force

An artificial earthquake which it is used as an external force for all cases in this report. The properties of the earthquke can be found in the reference 1.

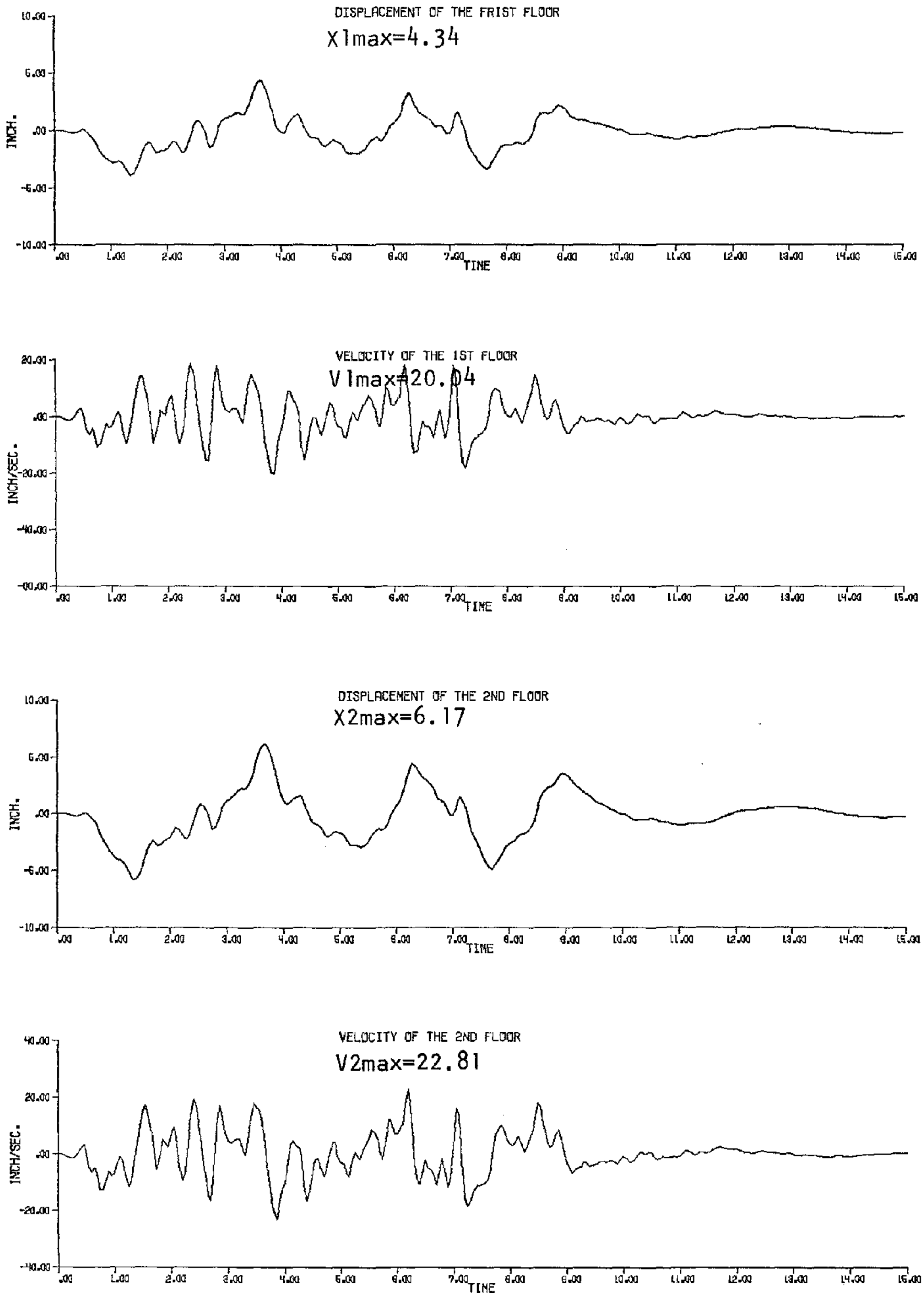

Fig. 9 The Maximum Displacements and Velocity of the Open-Loop System for System I, II, and III .

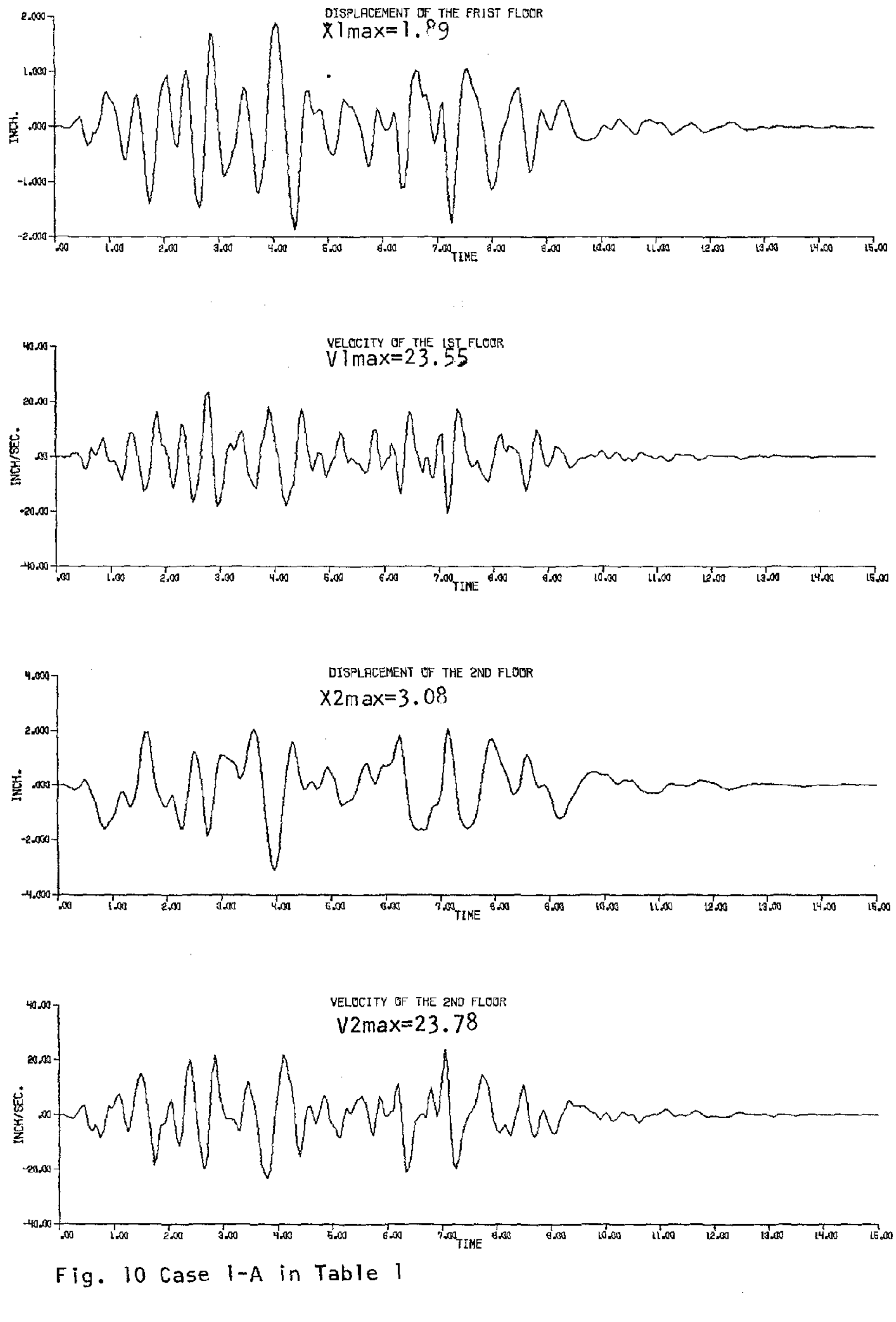

Fig. 10 Case 1-A in Table 1

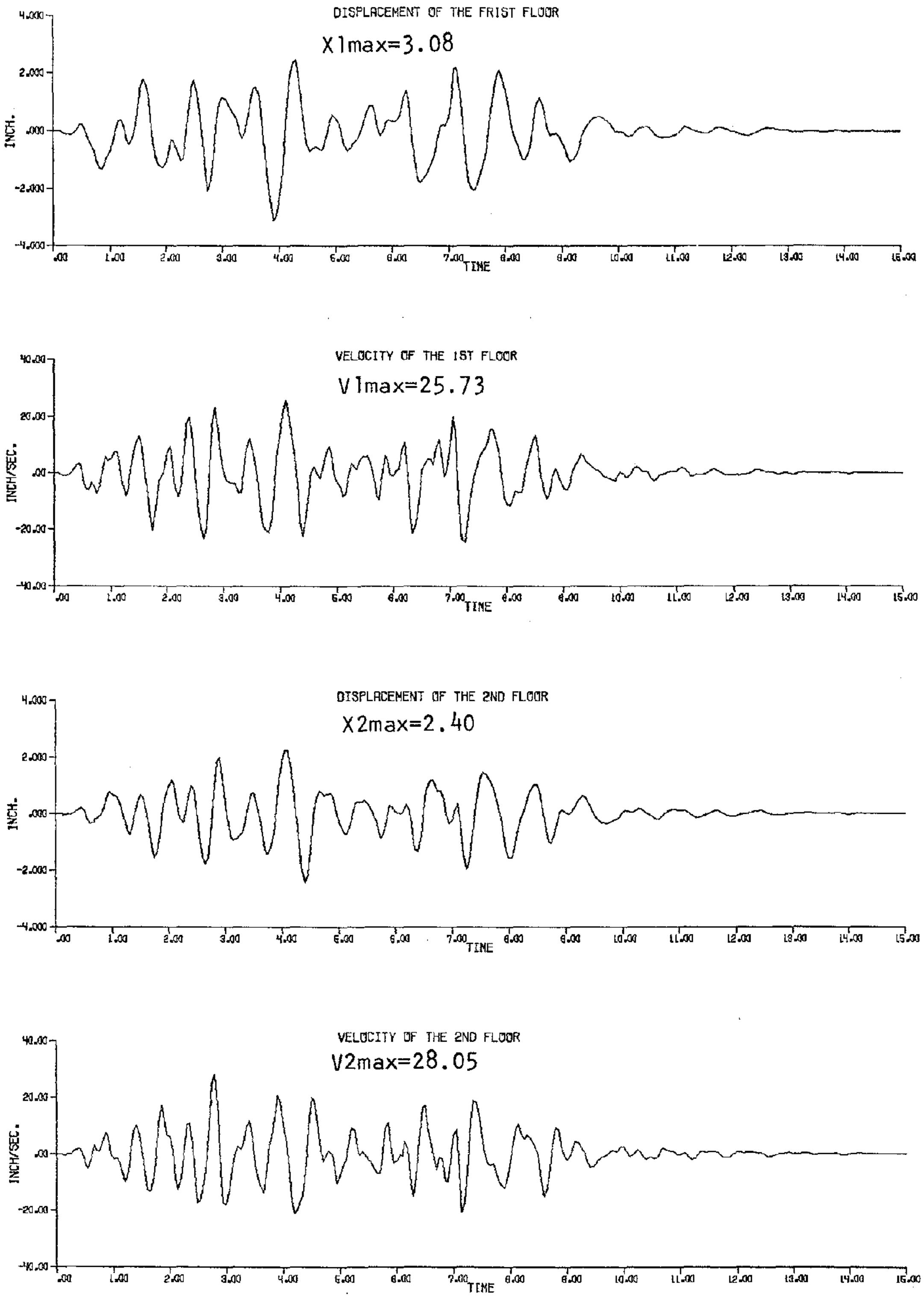

Fig. 11 Case 1-B in Table 1

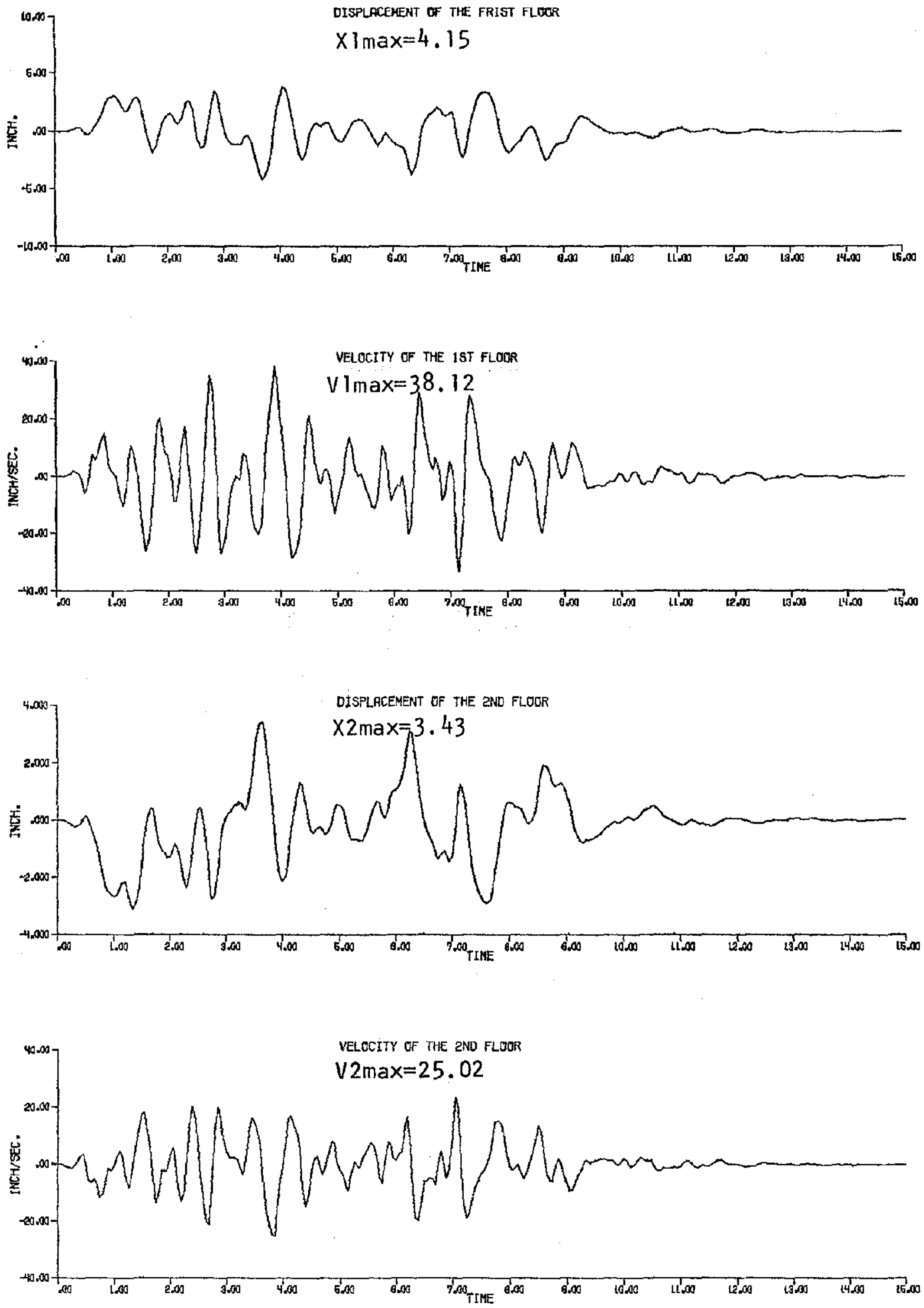

Fig. 12 Case 2-A in Table 1

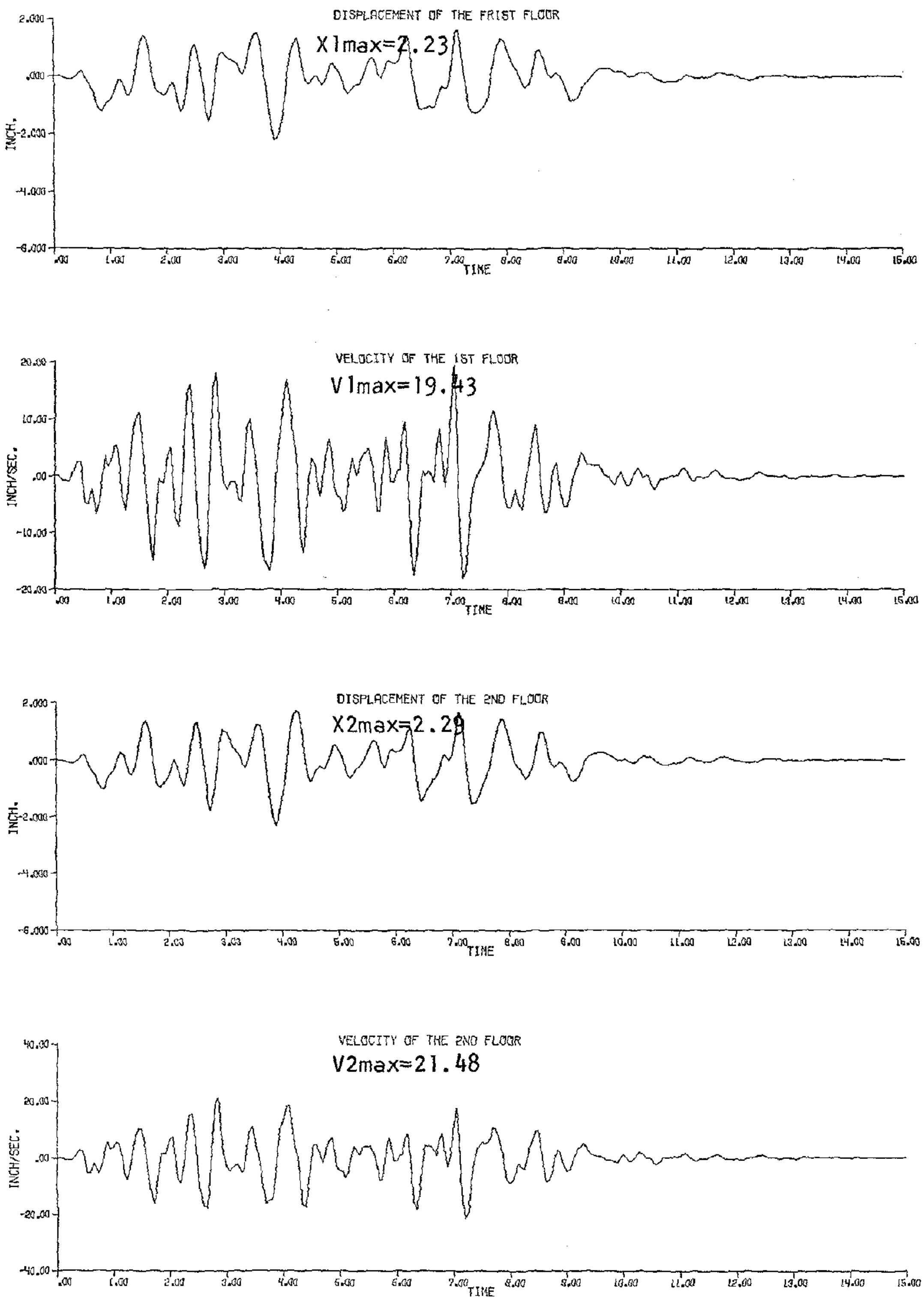

Fig. 13 Case 2-B-1 in Table 1

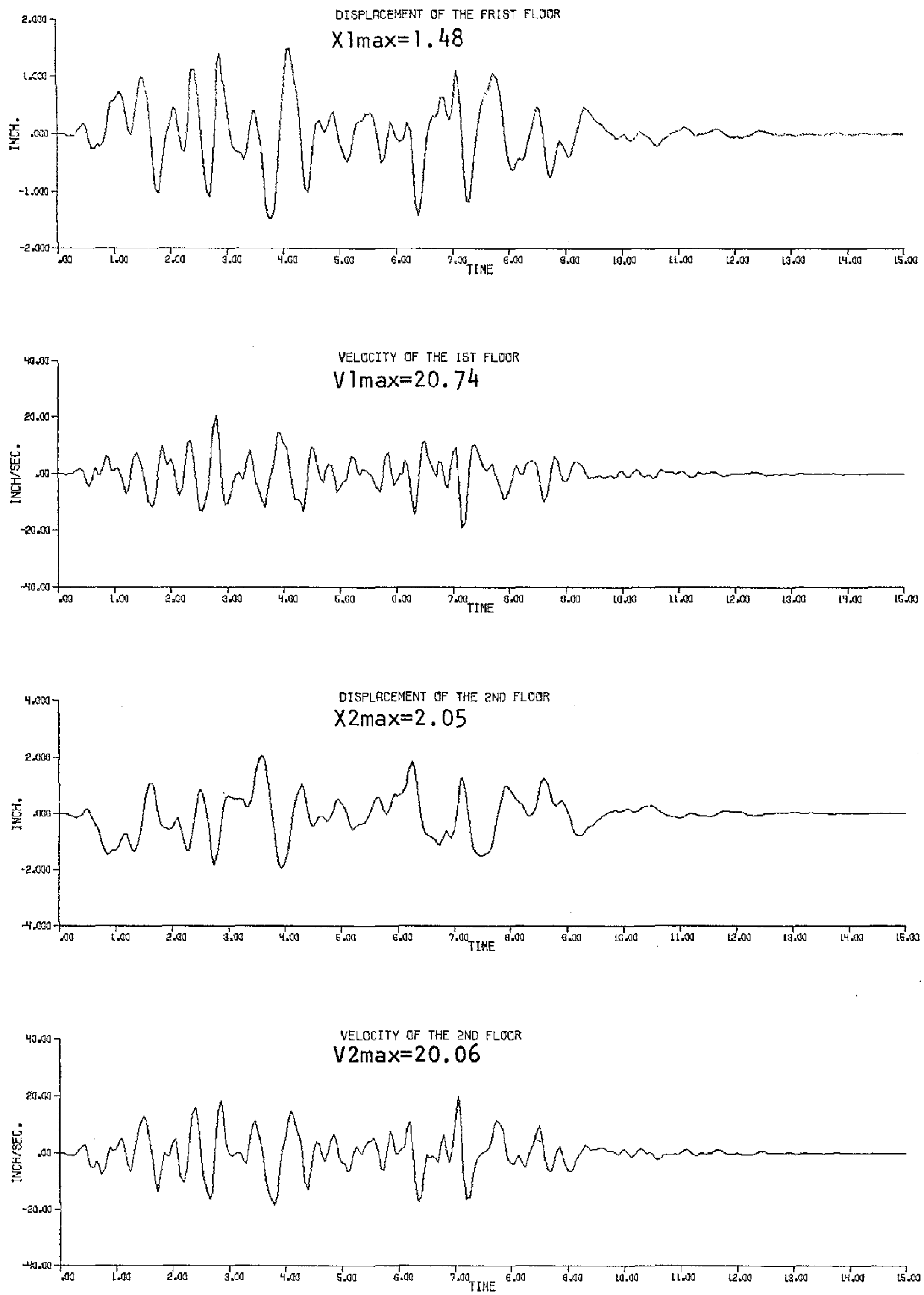

Fig. 14 Case 3-A in Table 1

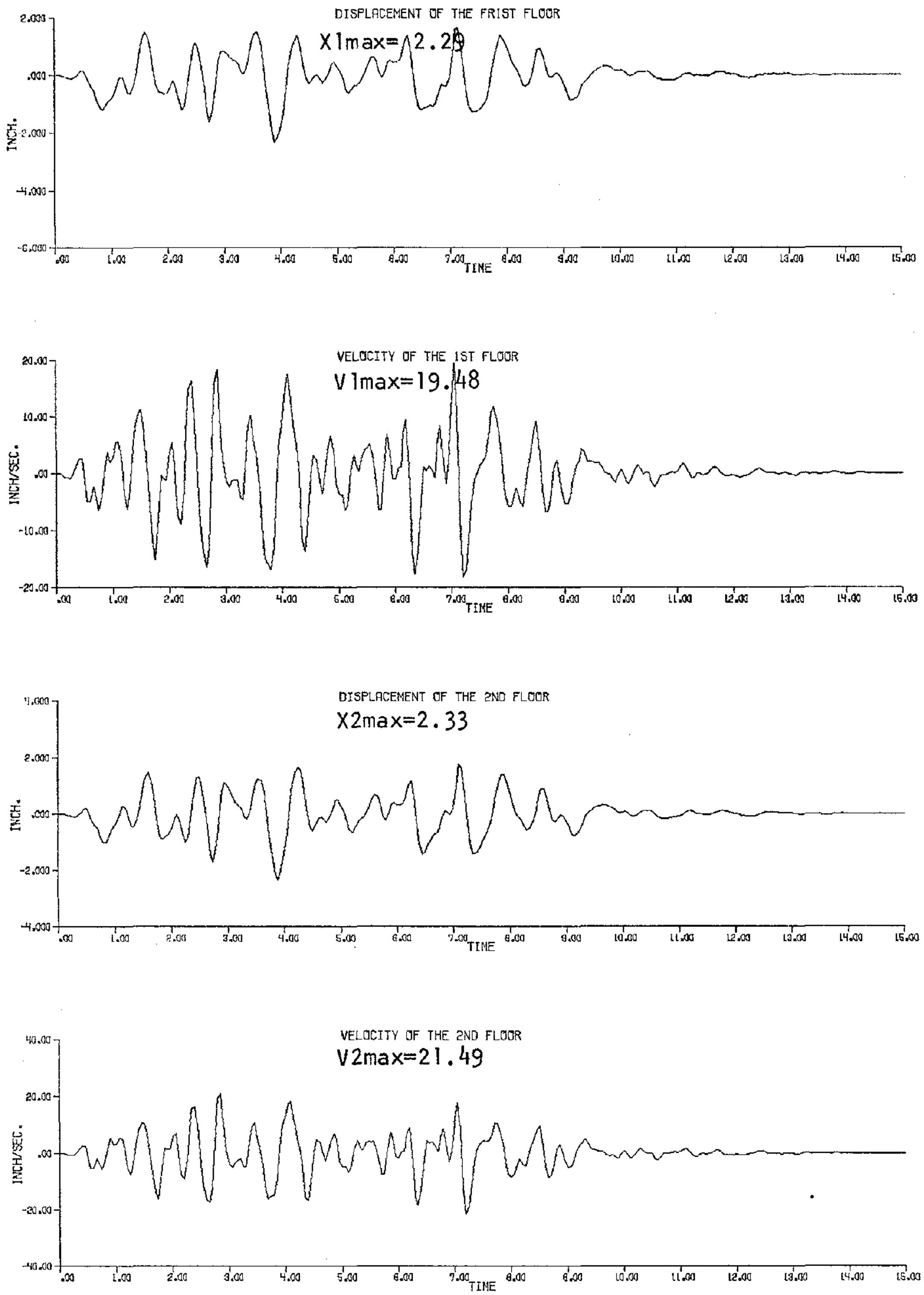

Fig. 15 Case 3-B in Table 1

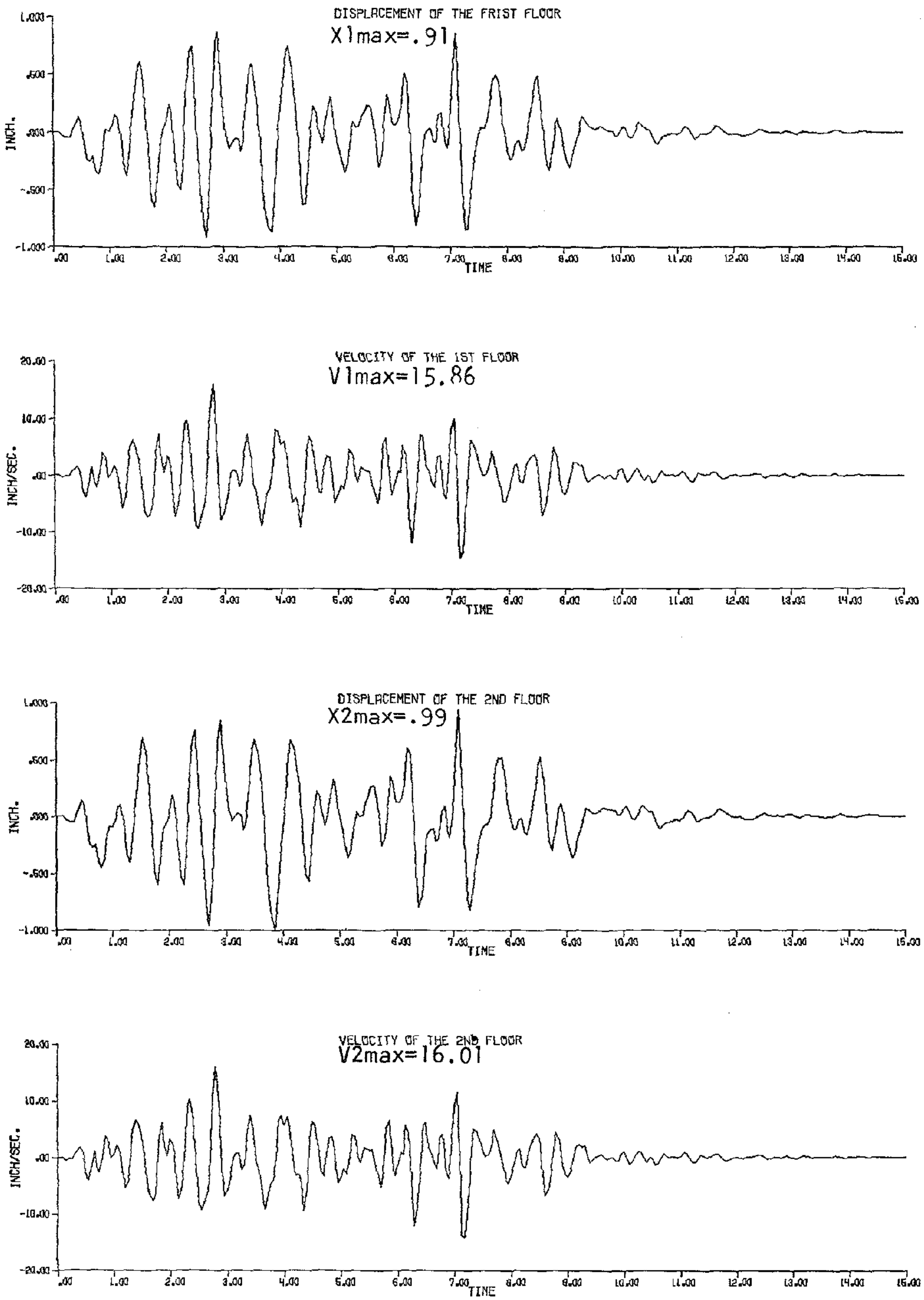

Fig. 16 Case 4-A in Table 1

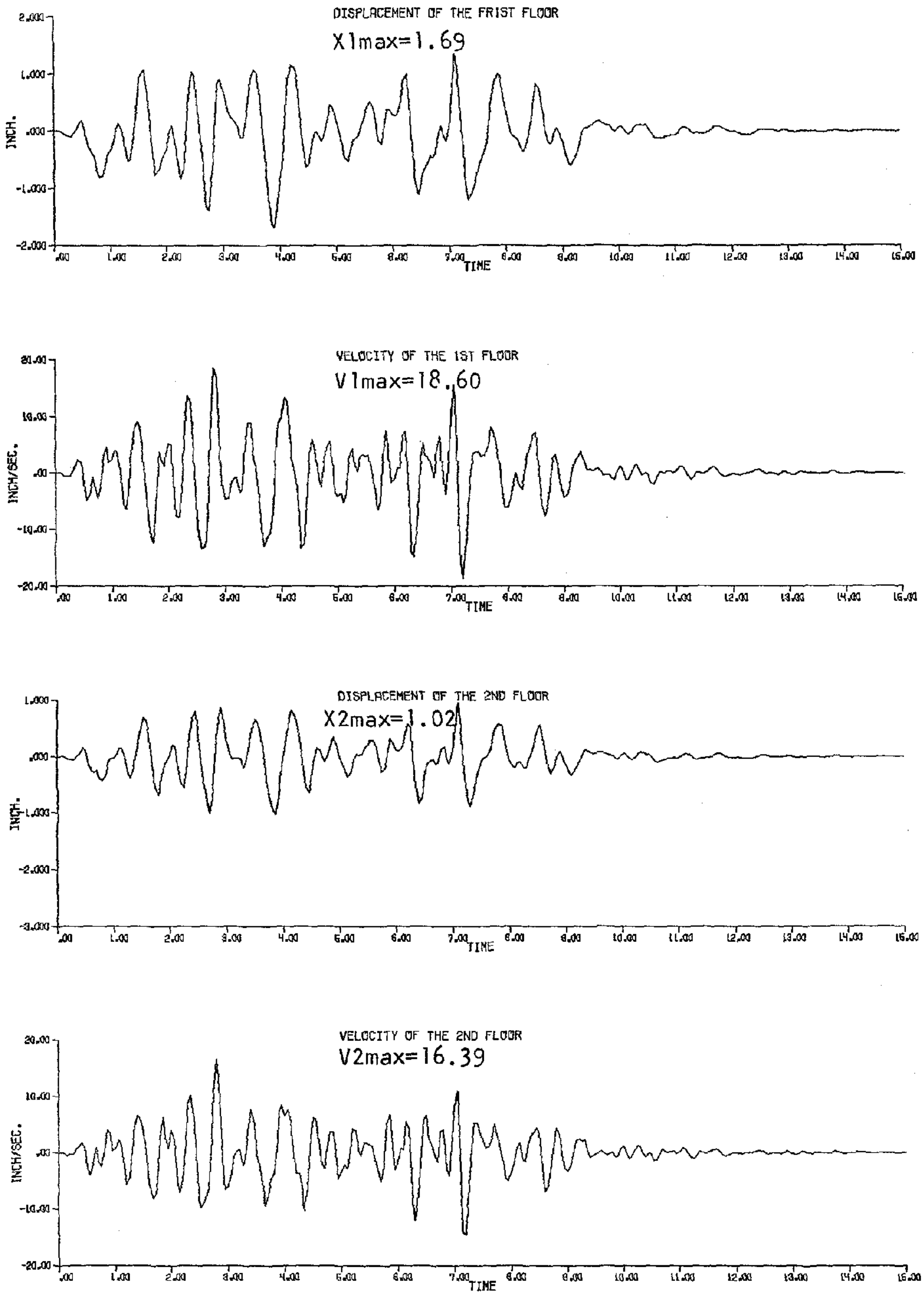

Fig. 17 Case 4-B in Table 1

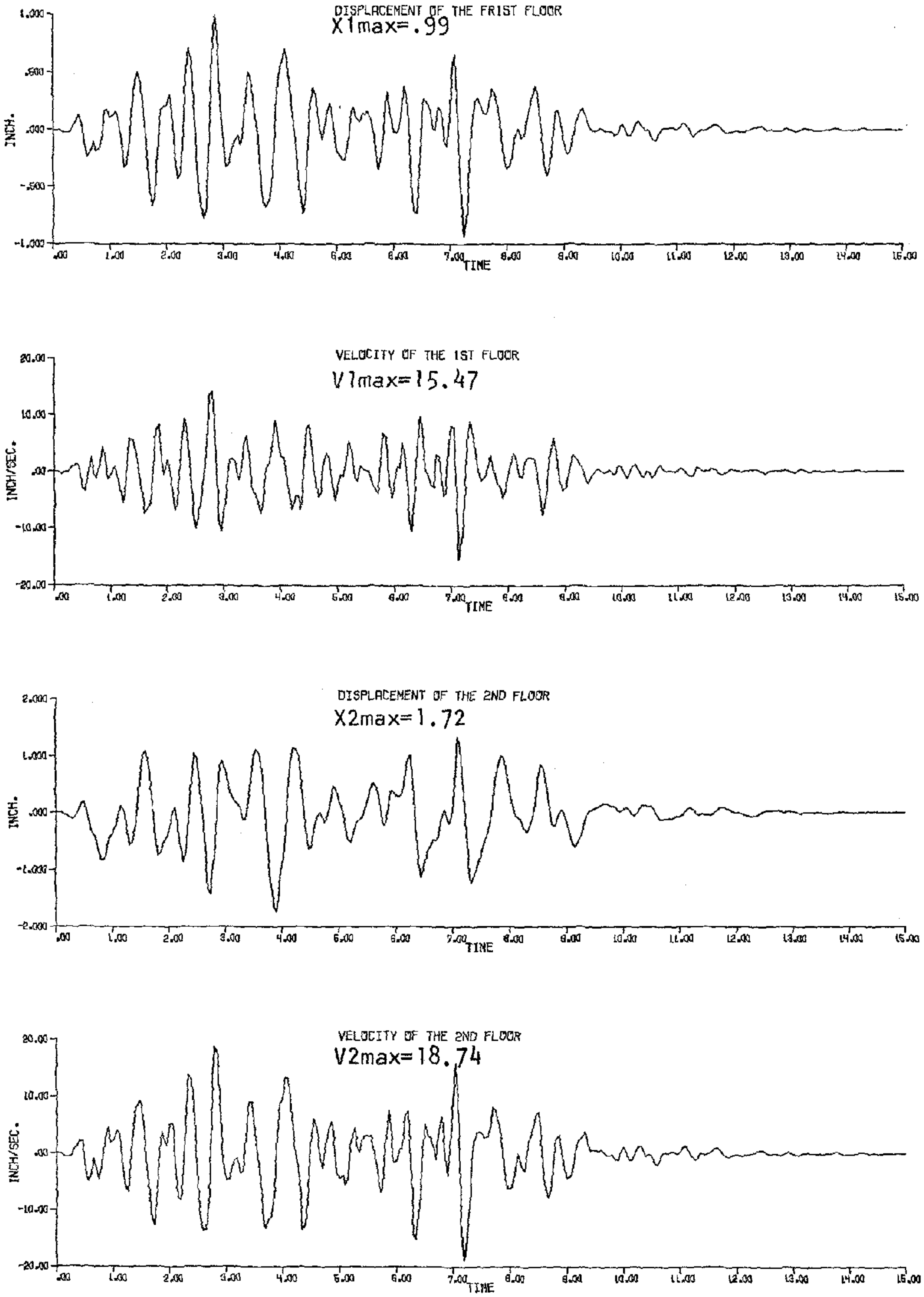

Fig. 18 Case 5-A in Table 1

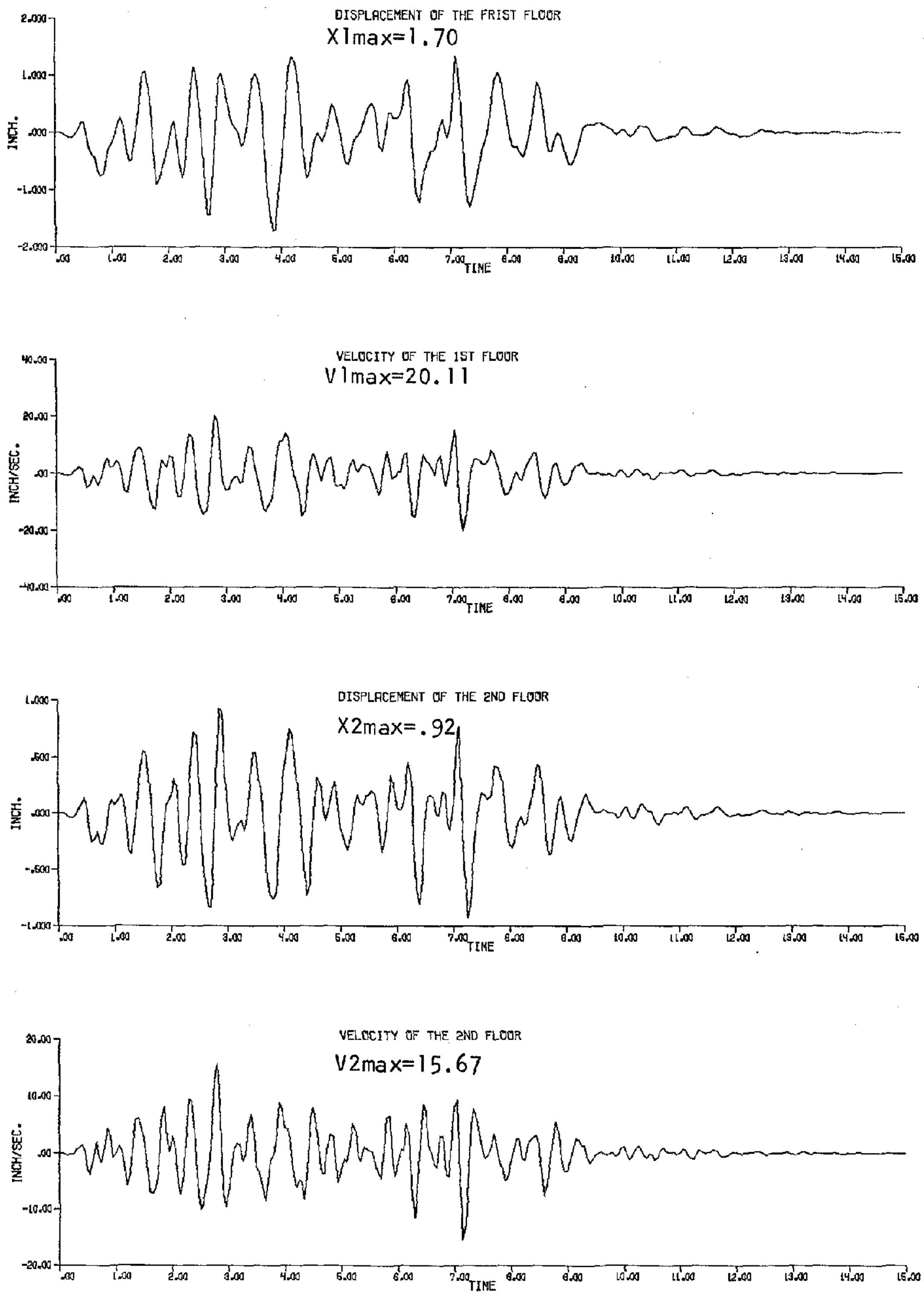

Fig. 19 Case 5-B in Table 1

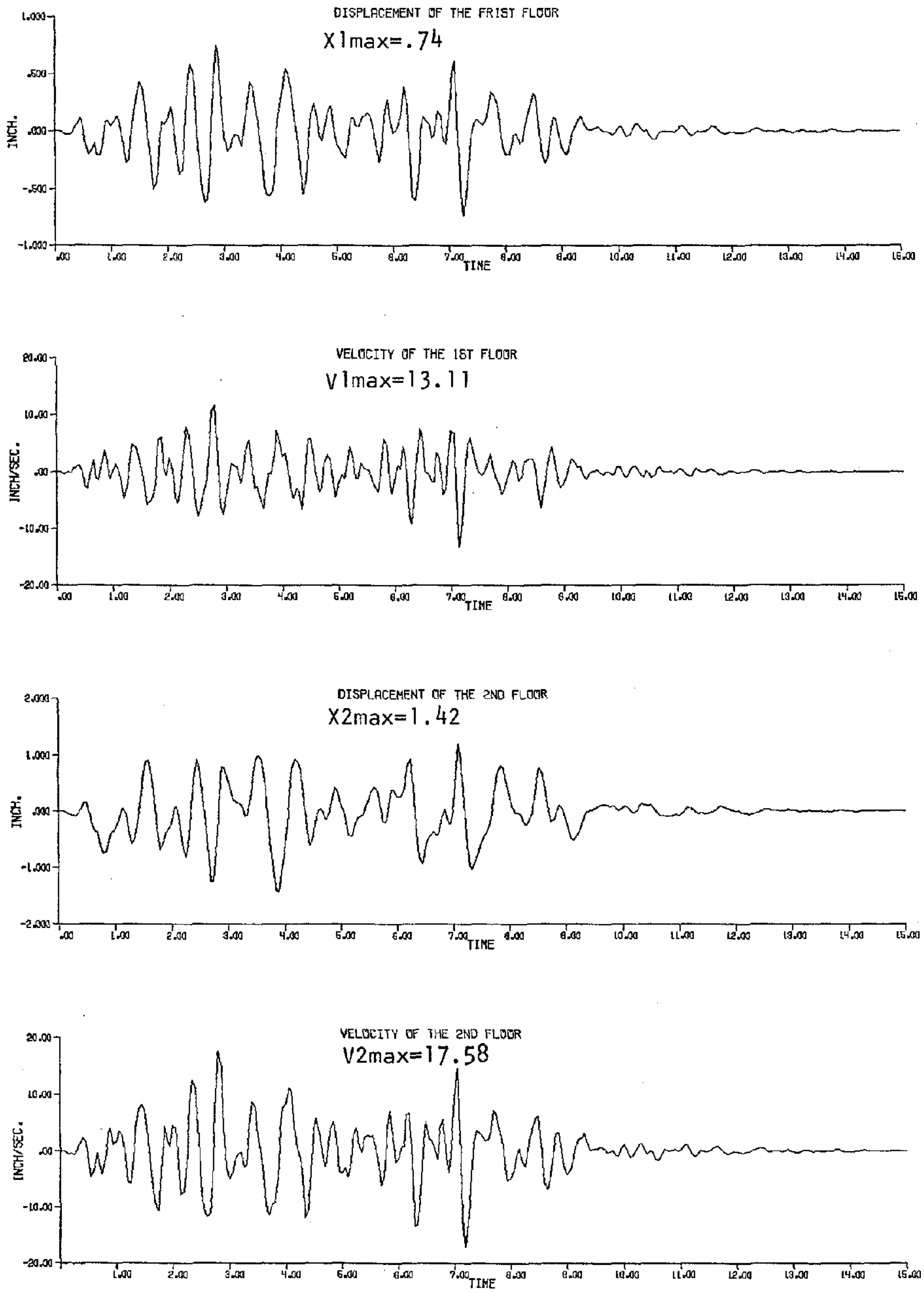

Fig. 20 Case 6-A in Table 1

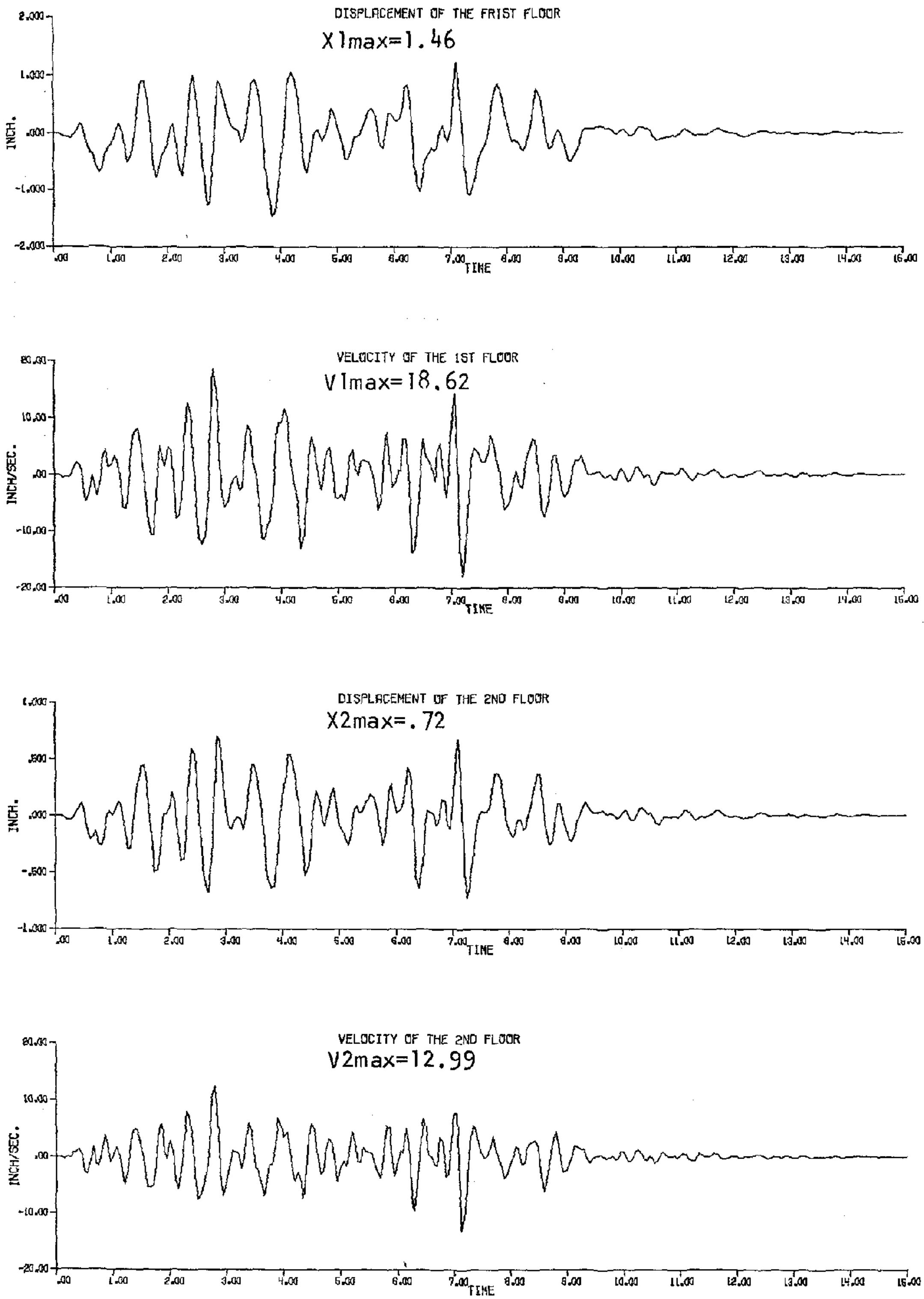

Fig. 21 Case 6-B in Table 1

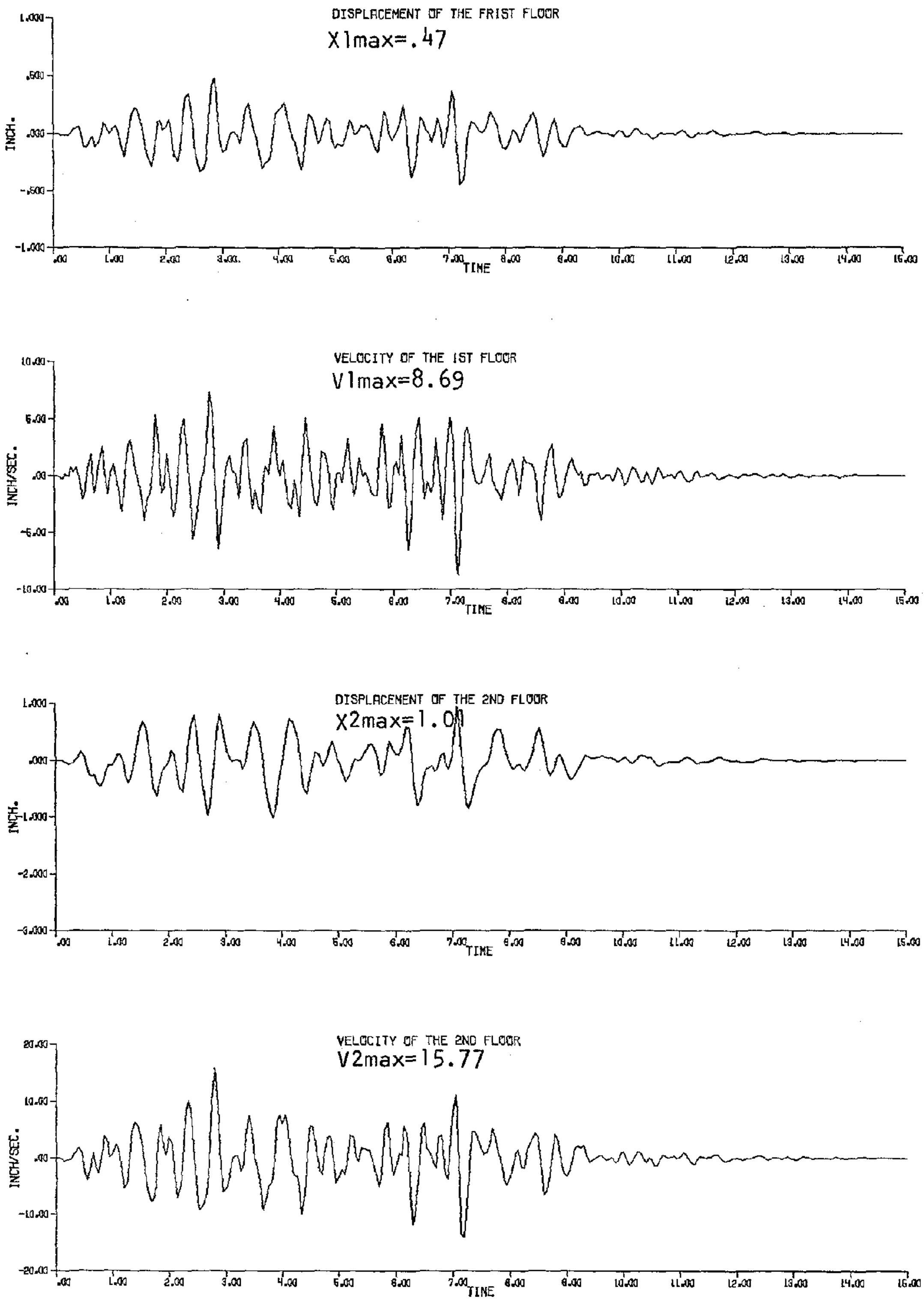

Fig. 22 Case 7-A in Table 1

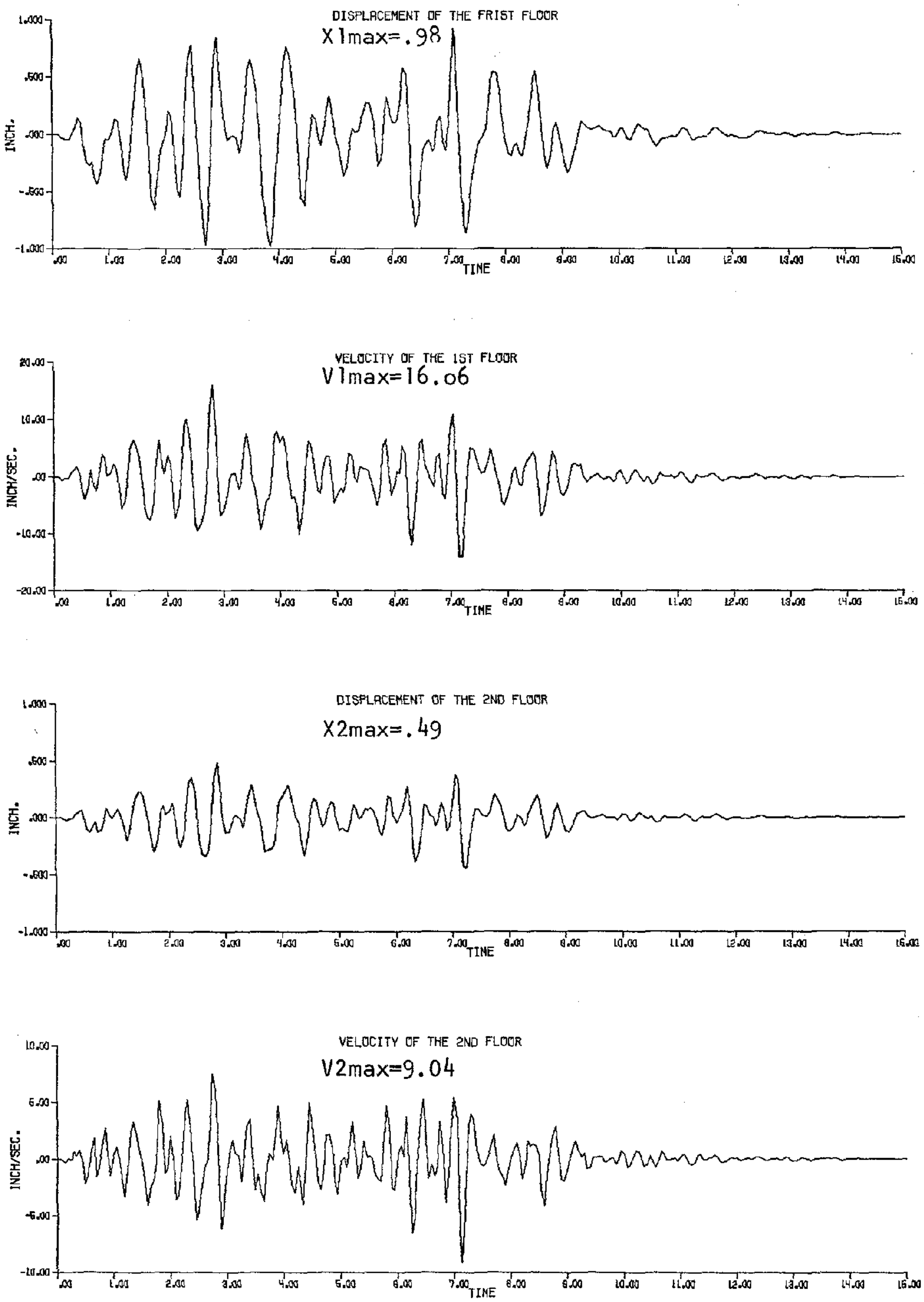

Fig. 23 Case 7-B in Table 1

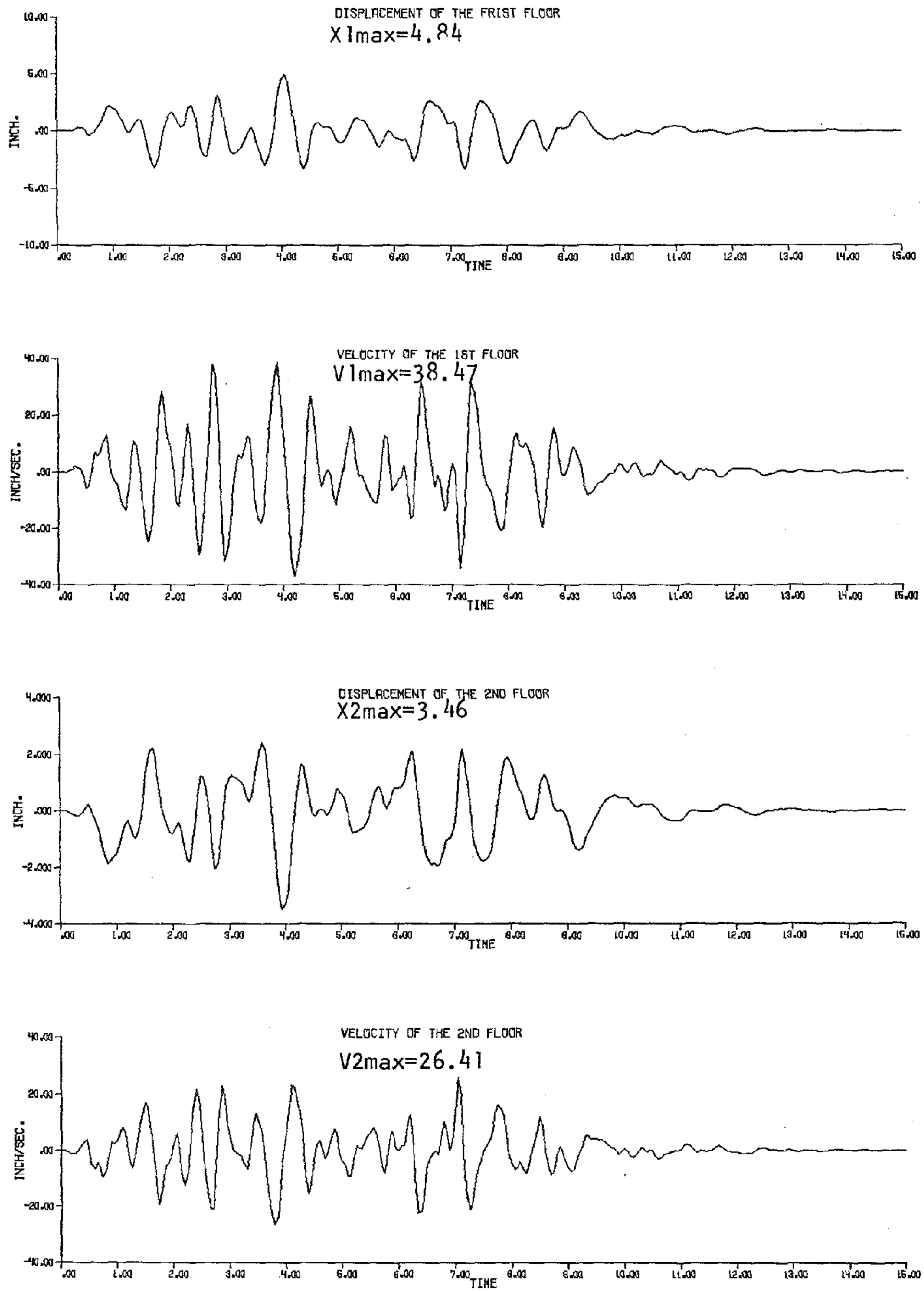

Fig. 24 Case 1 in Table 2

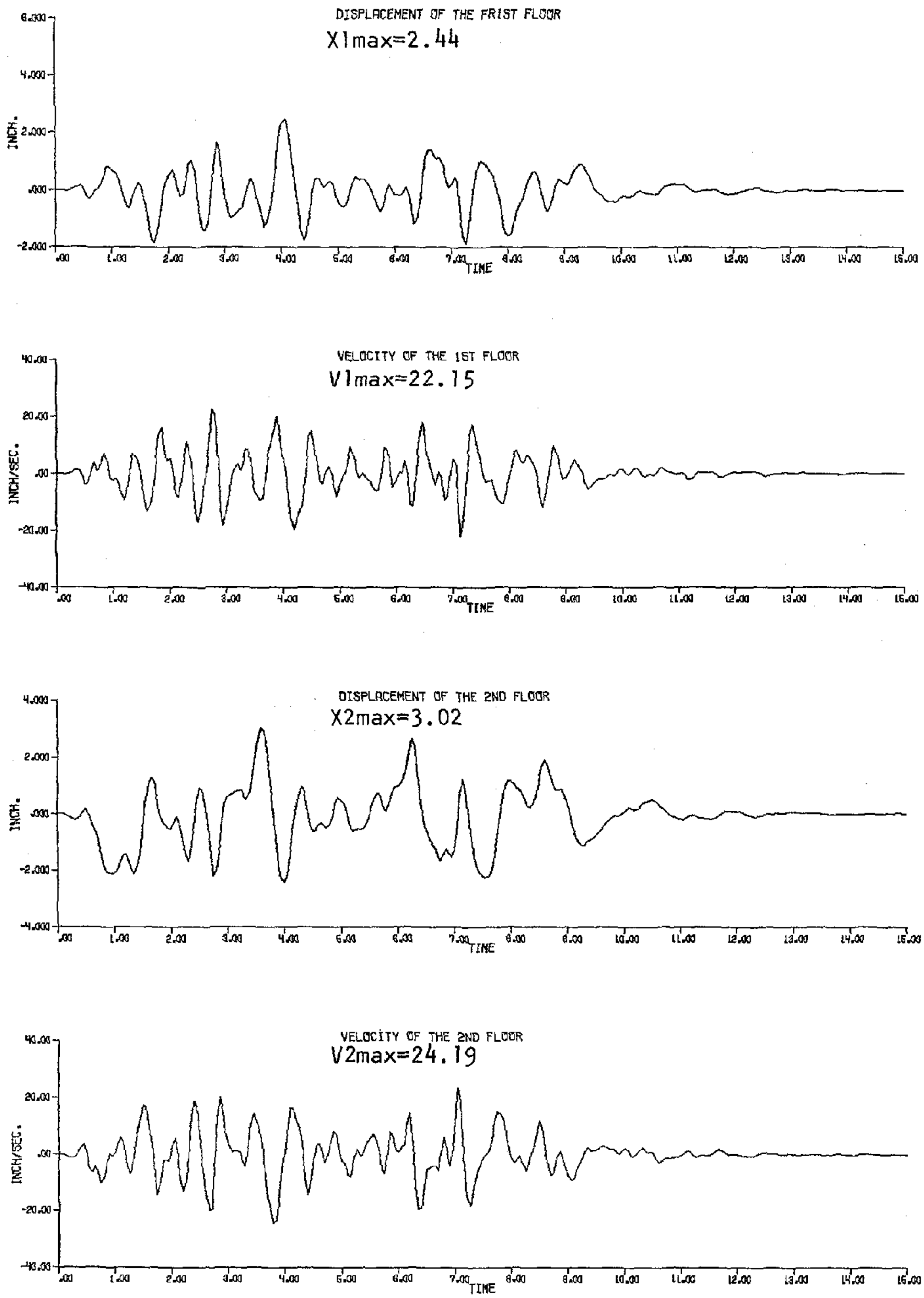

Fig.25 Case 2 in Table 2

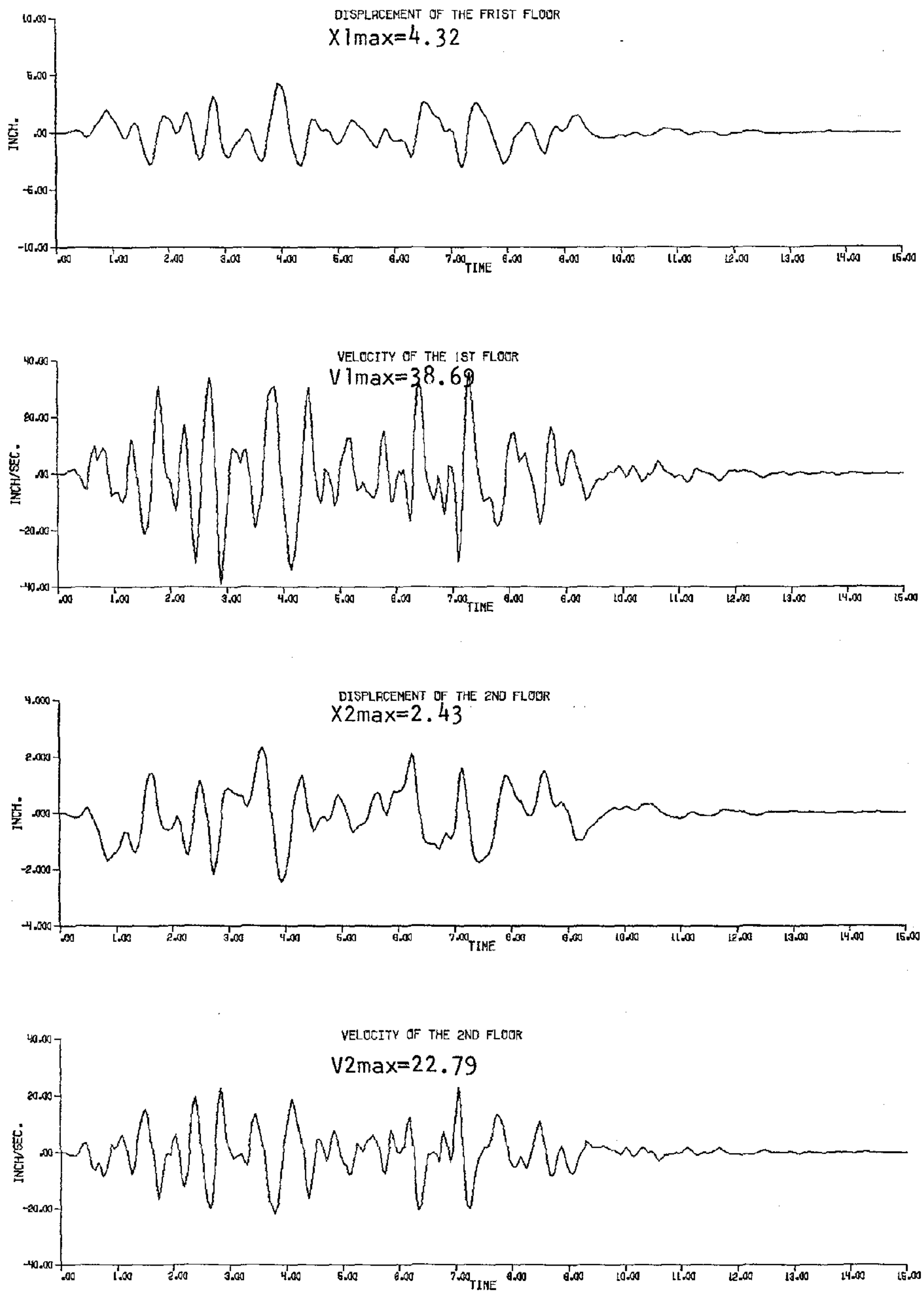

Fig. 26 Case 3 in Table 2

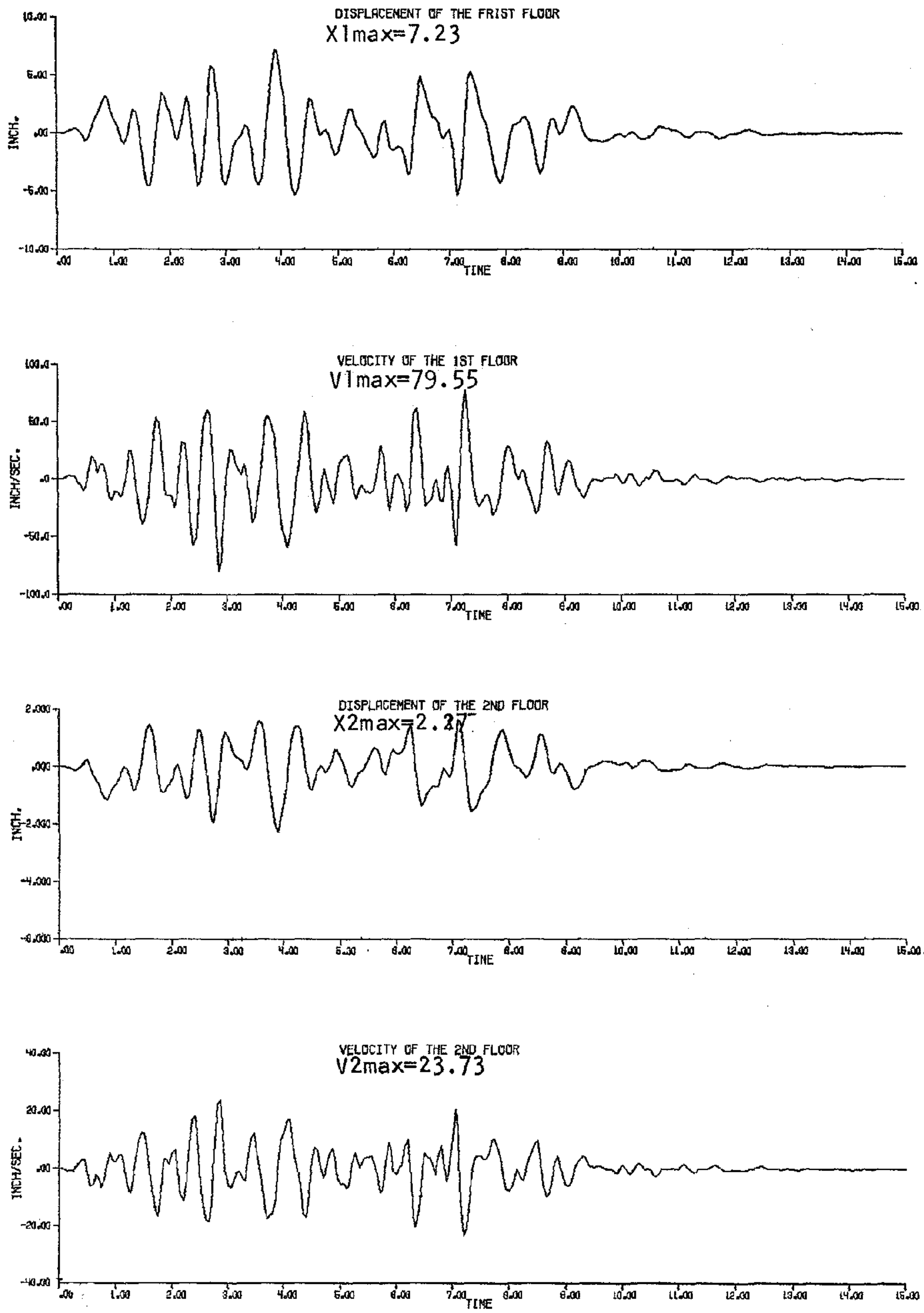

Fig. 27 Case 4 in Table 2

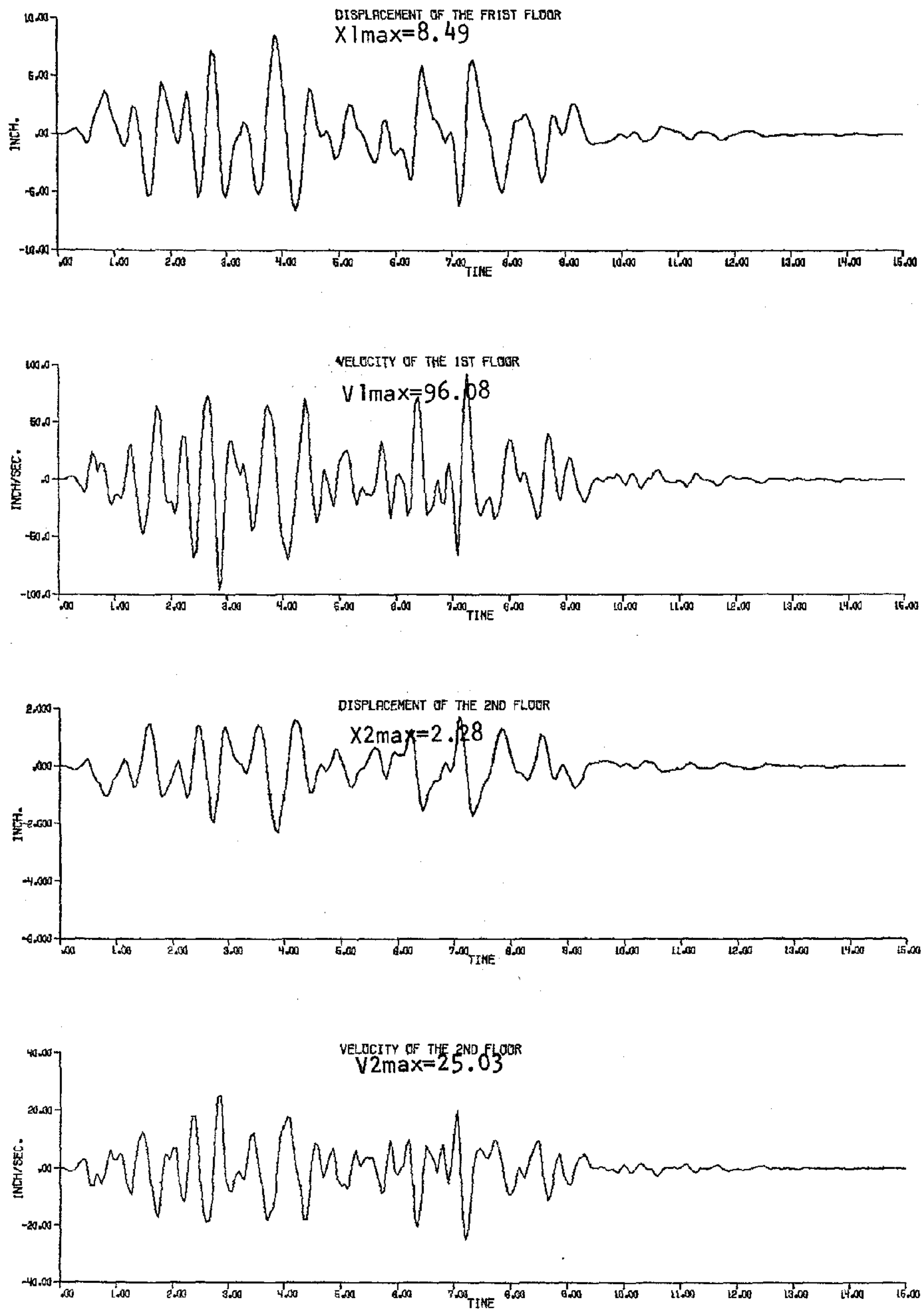

Fig. 28 Case 5 in Table 2

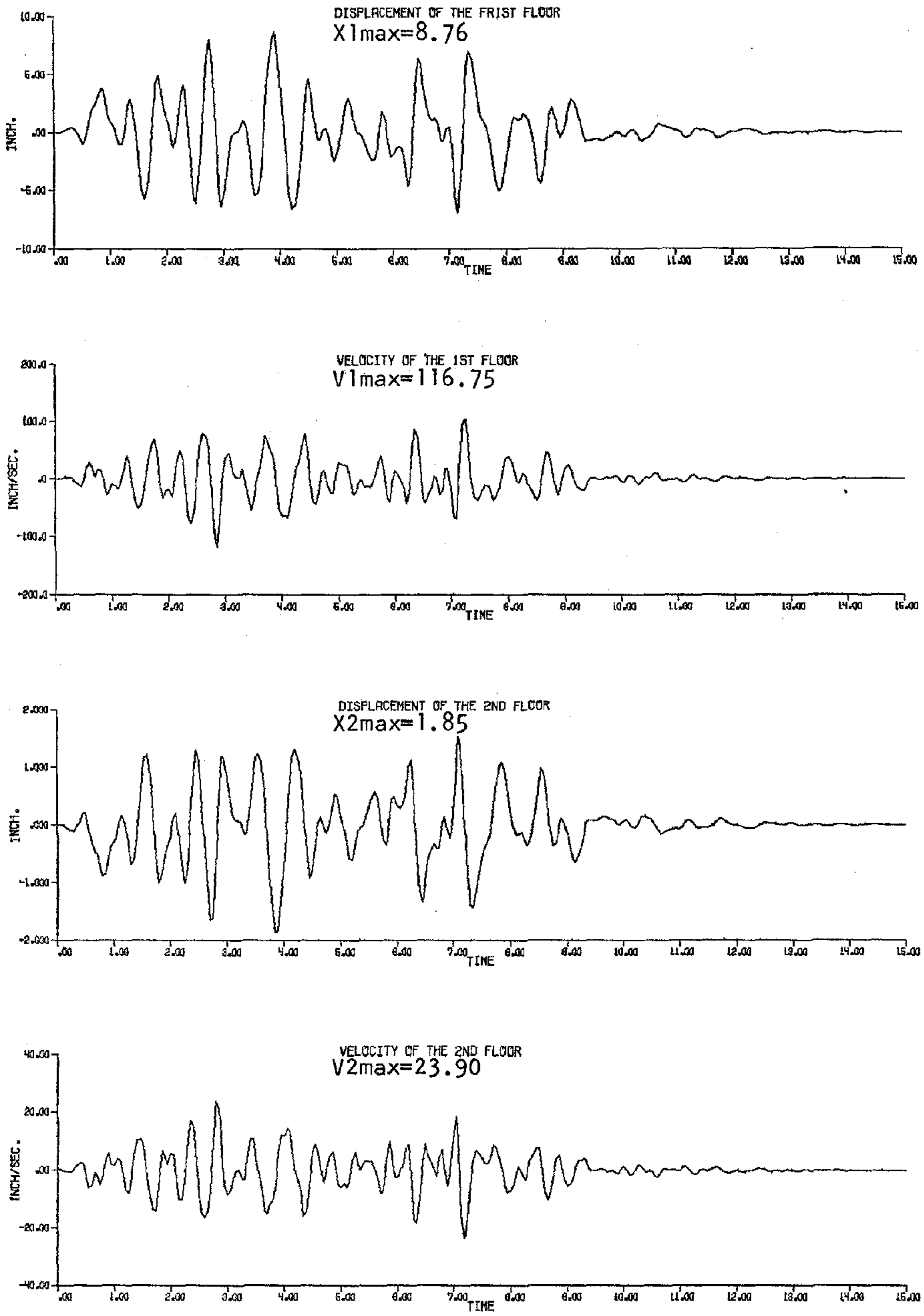

Fig. 29 Case 6 in Table 2

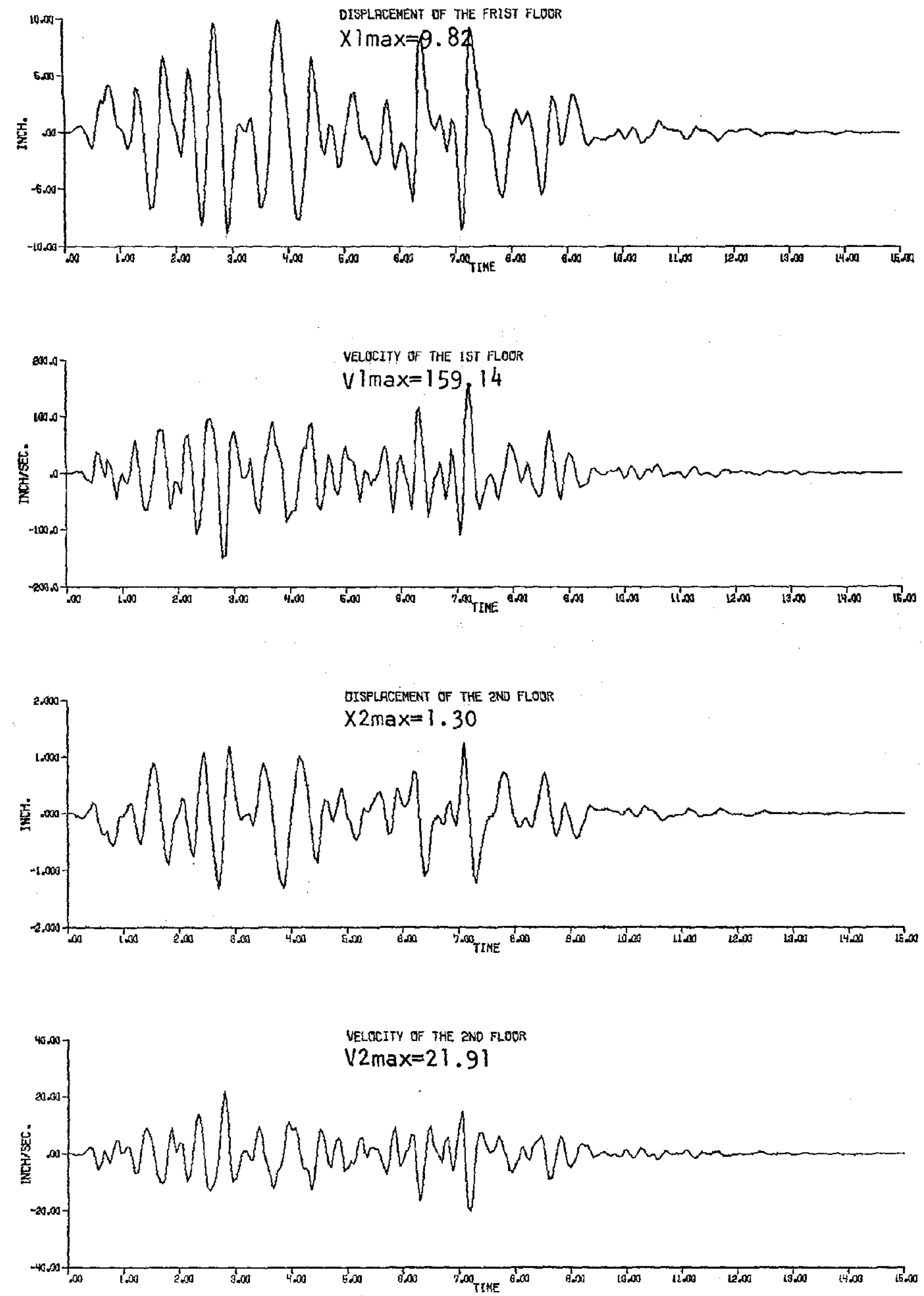

Fig. 30 Case 7 in Table 2

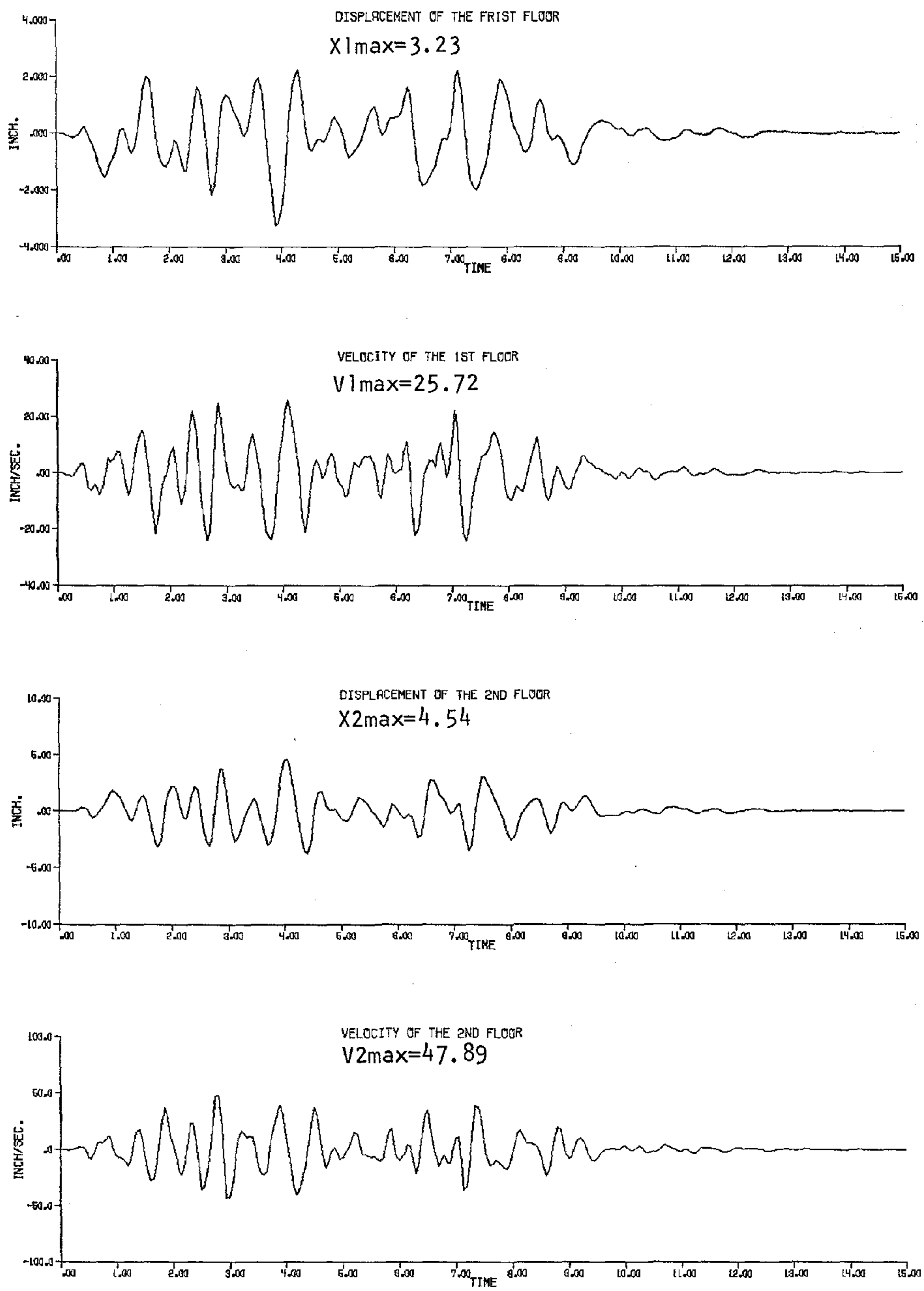

Fig. 31 Case 1 in Table 3

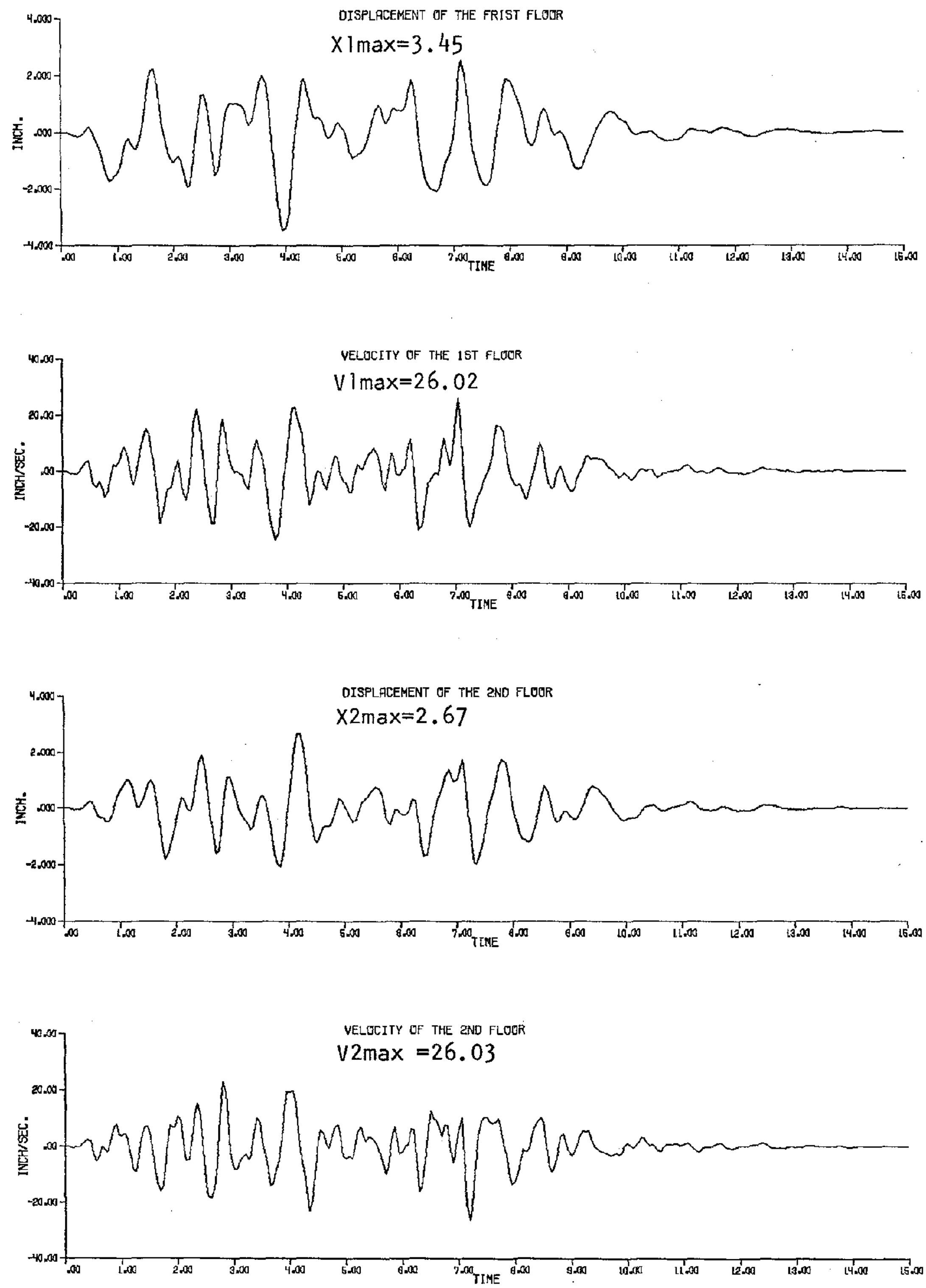

Fig. 32 Case 2 in Table 3

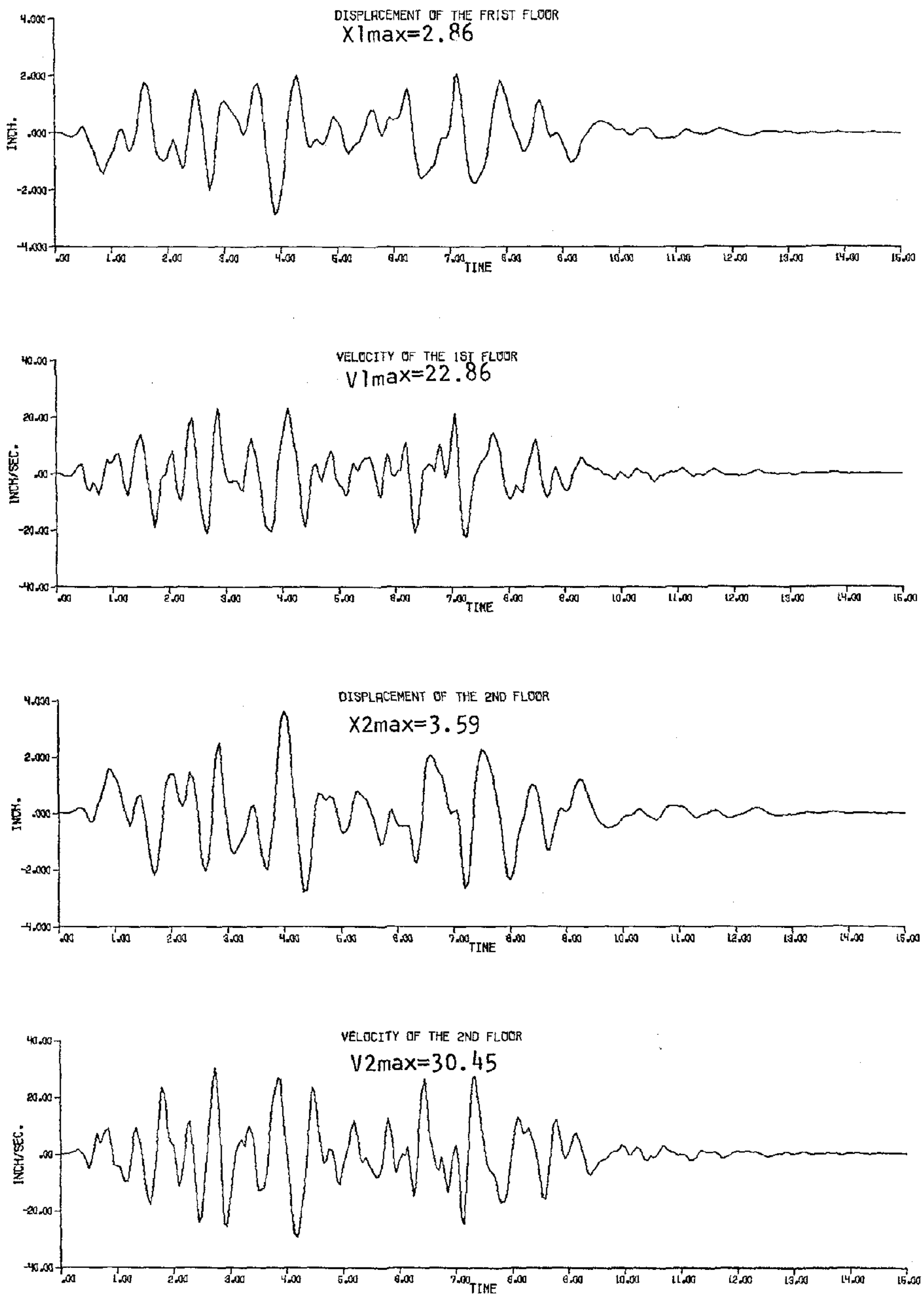

Fig. 33 Case 3 in Table 3

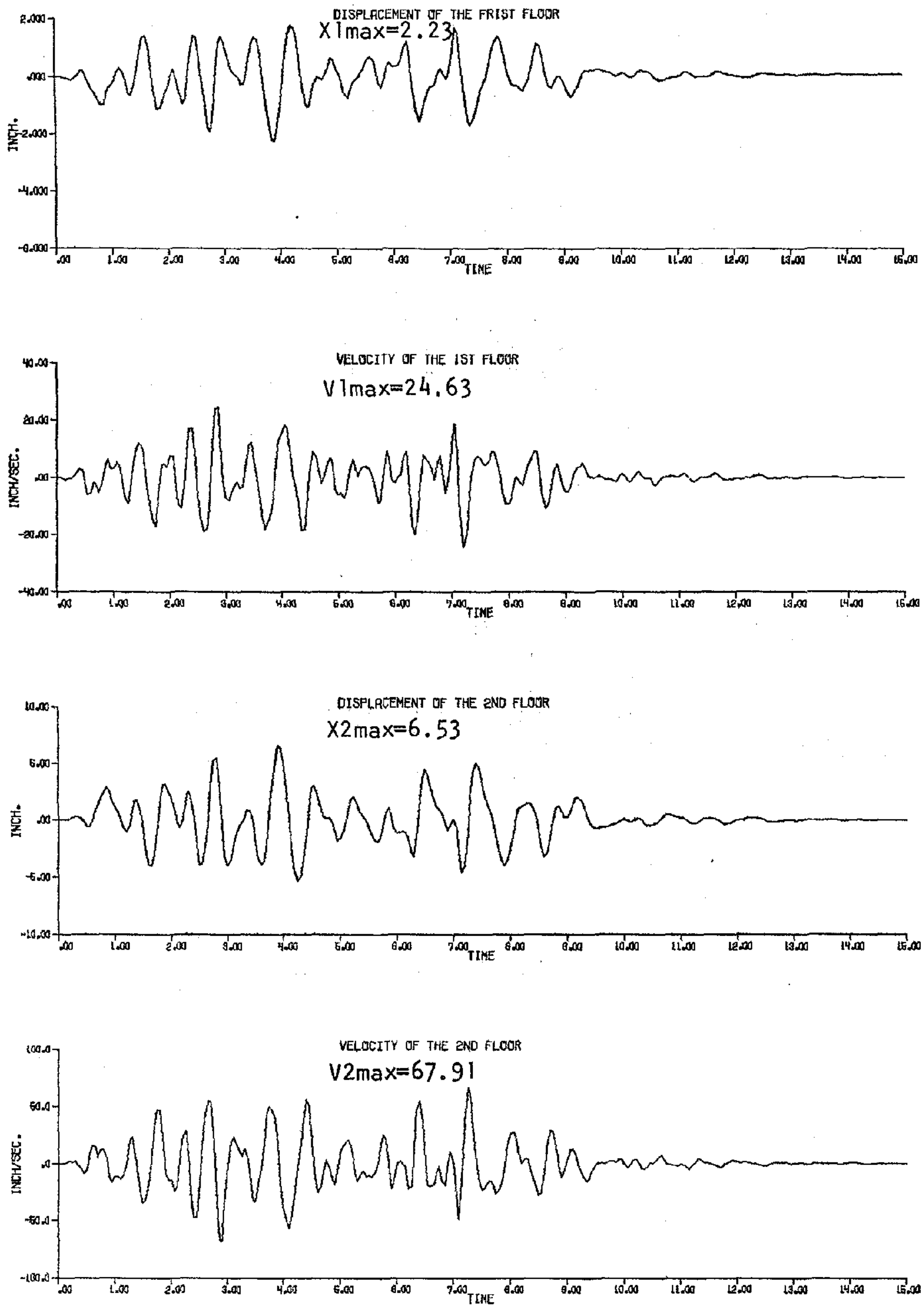

Fig. 34 Case 4 in Table 3

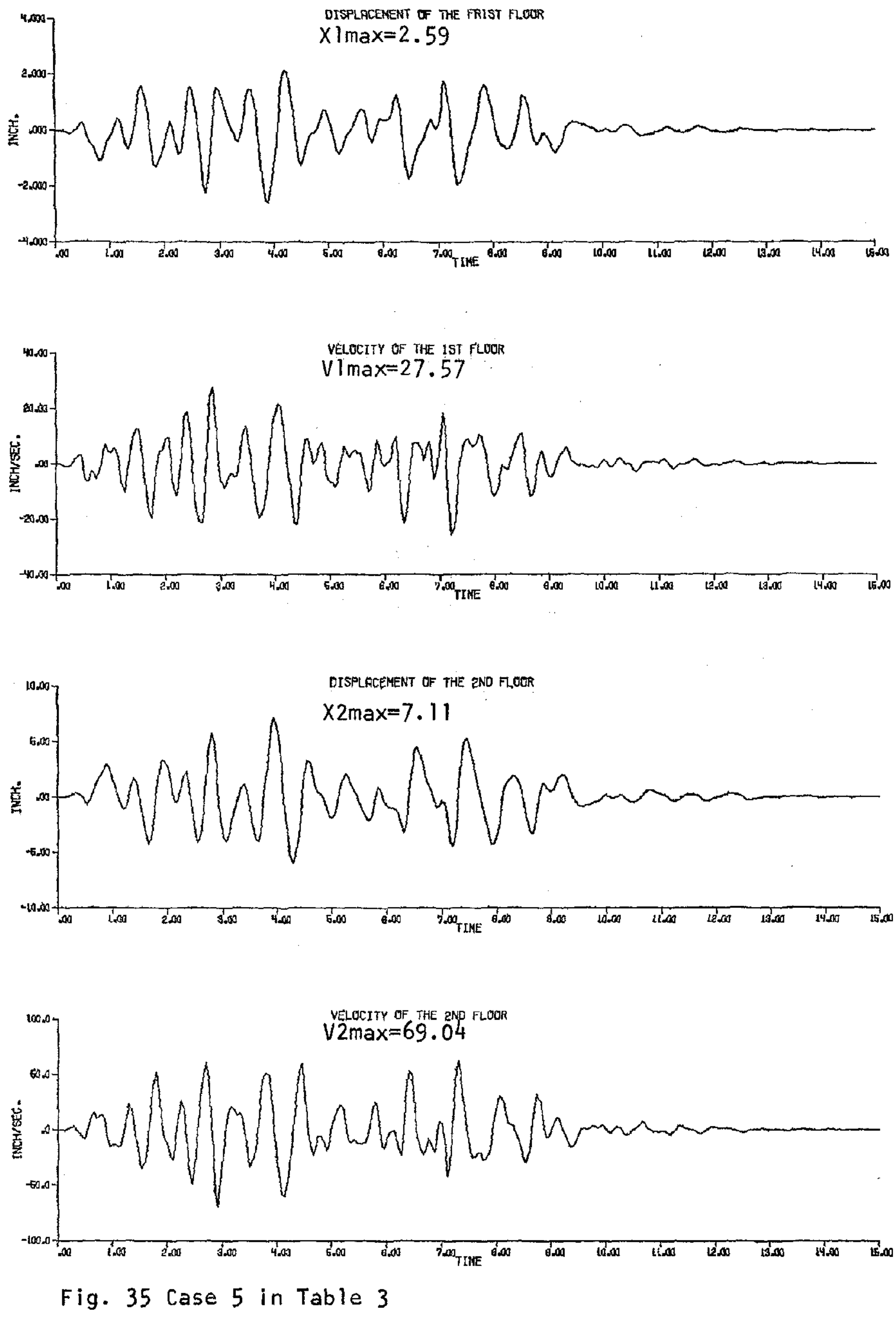

Fig. 35 Case 5 in Table 3

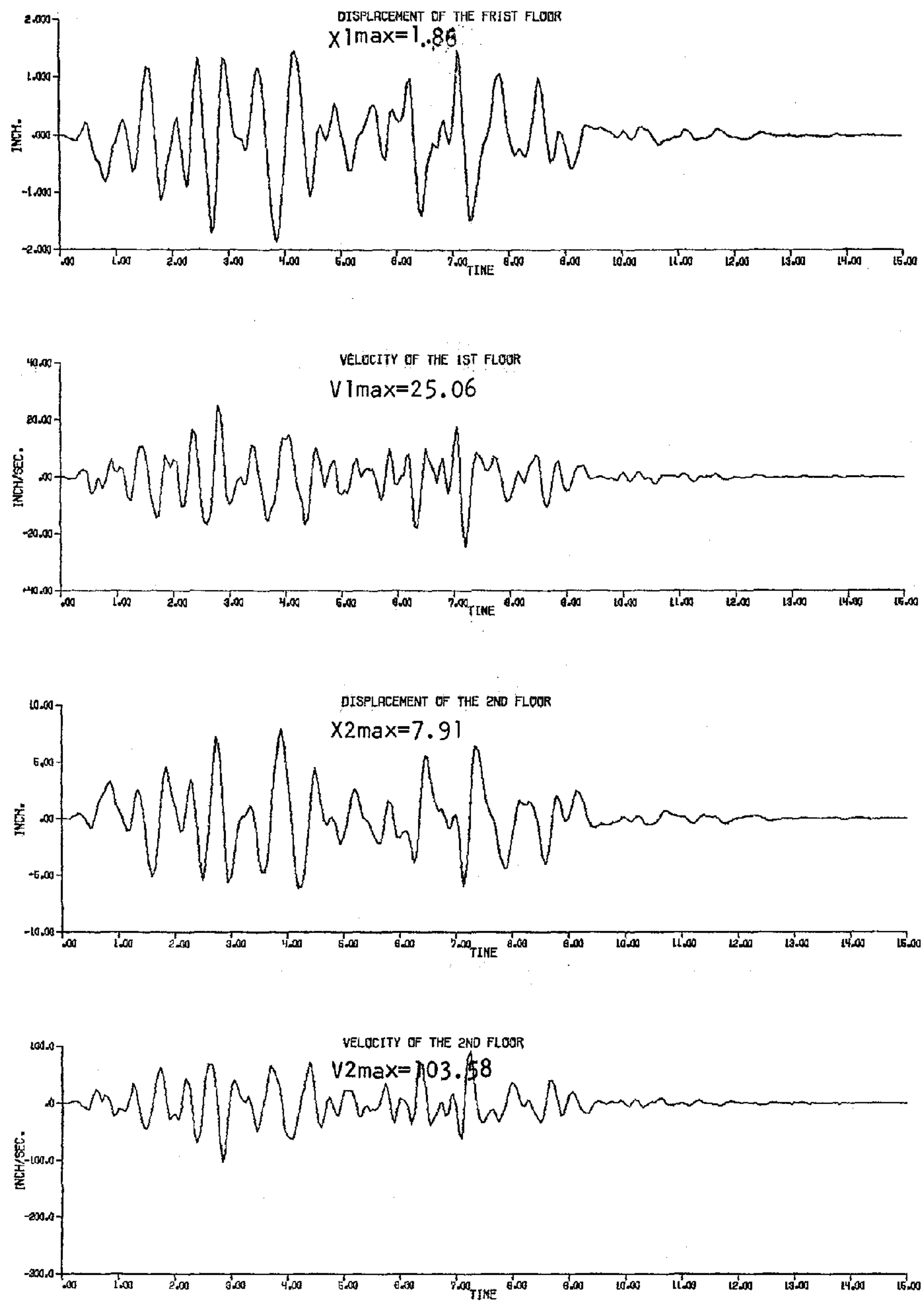

Fig. 36 Case 6 in Table 3

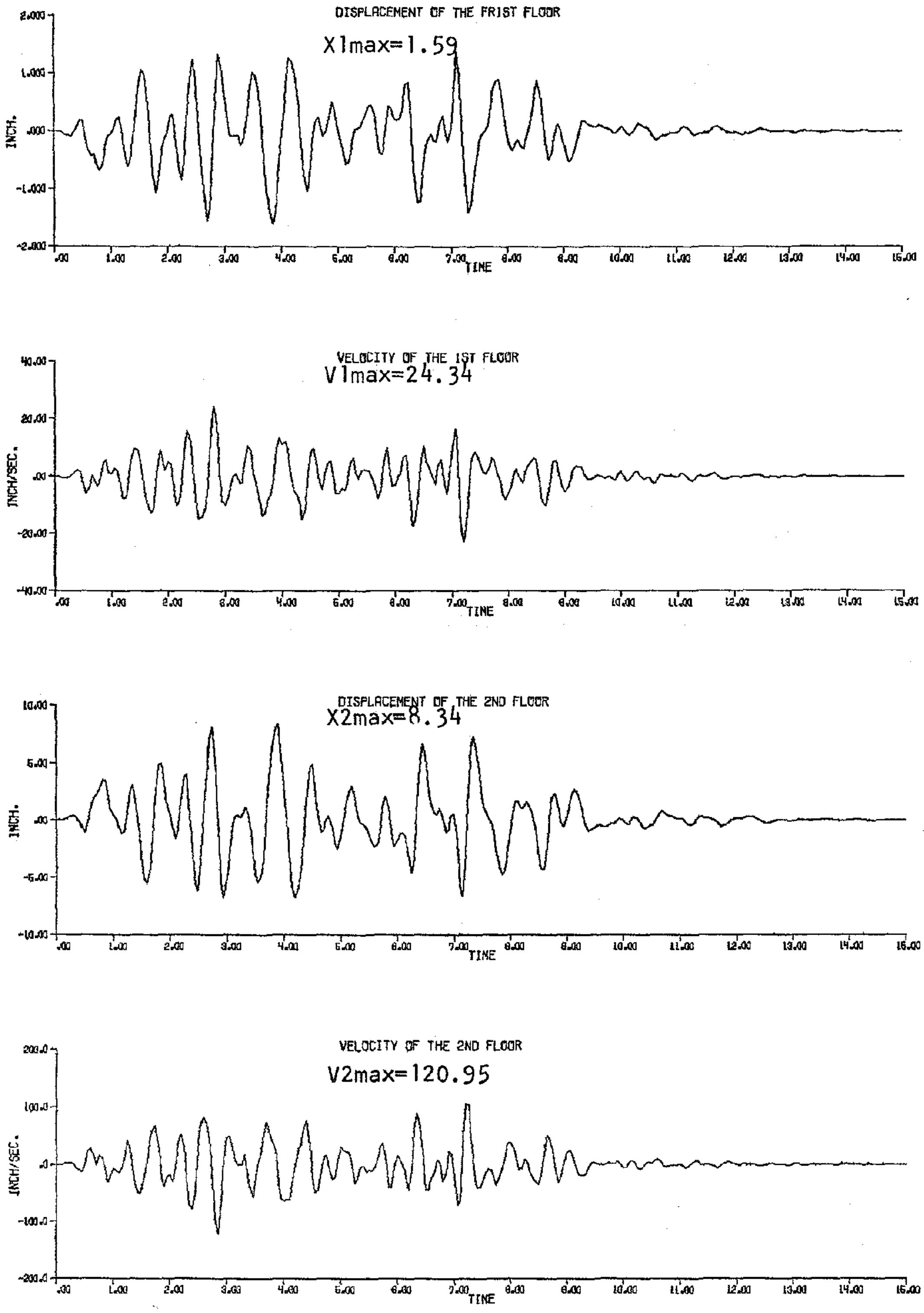

Fig. 37 Case 7 in Table 3

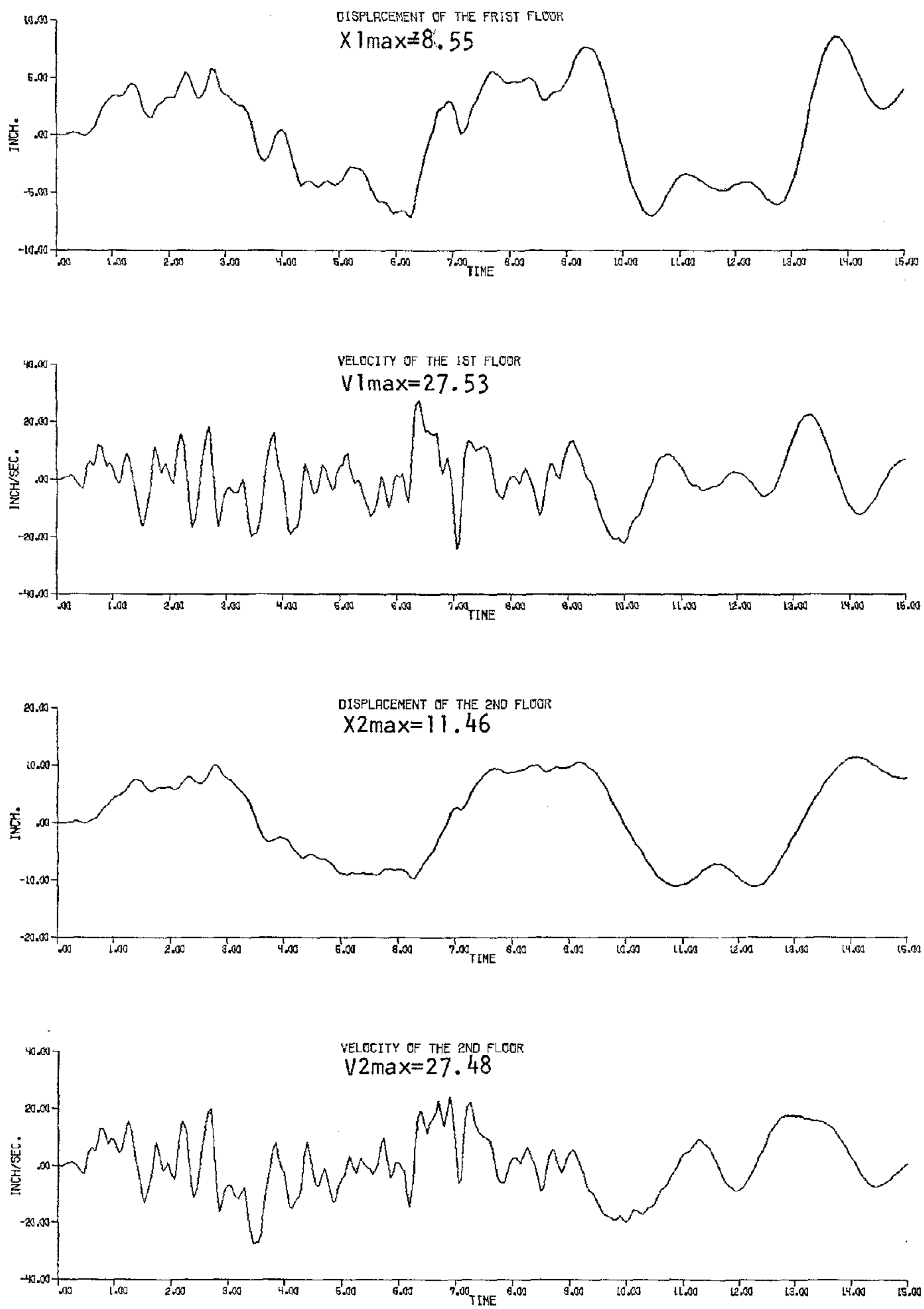

Fig. 38-a Case 1 in Table 4
The Maximum Displacements and Velocities of the Open-Loop System.

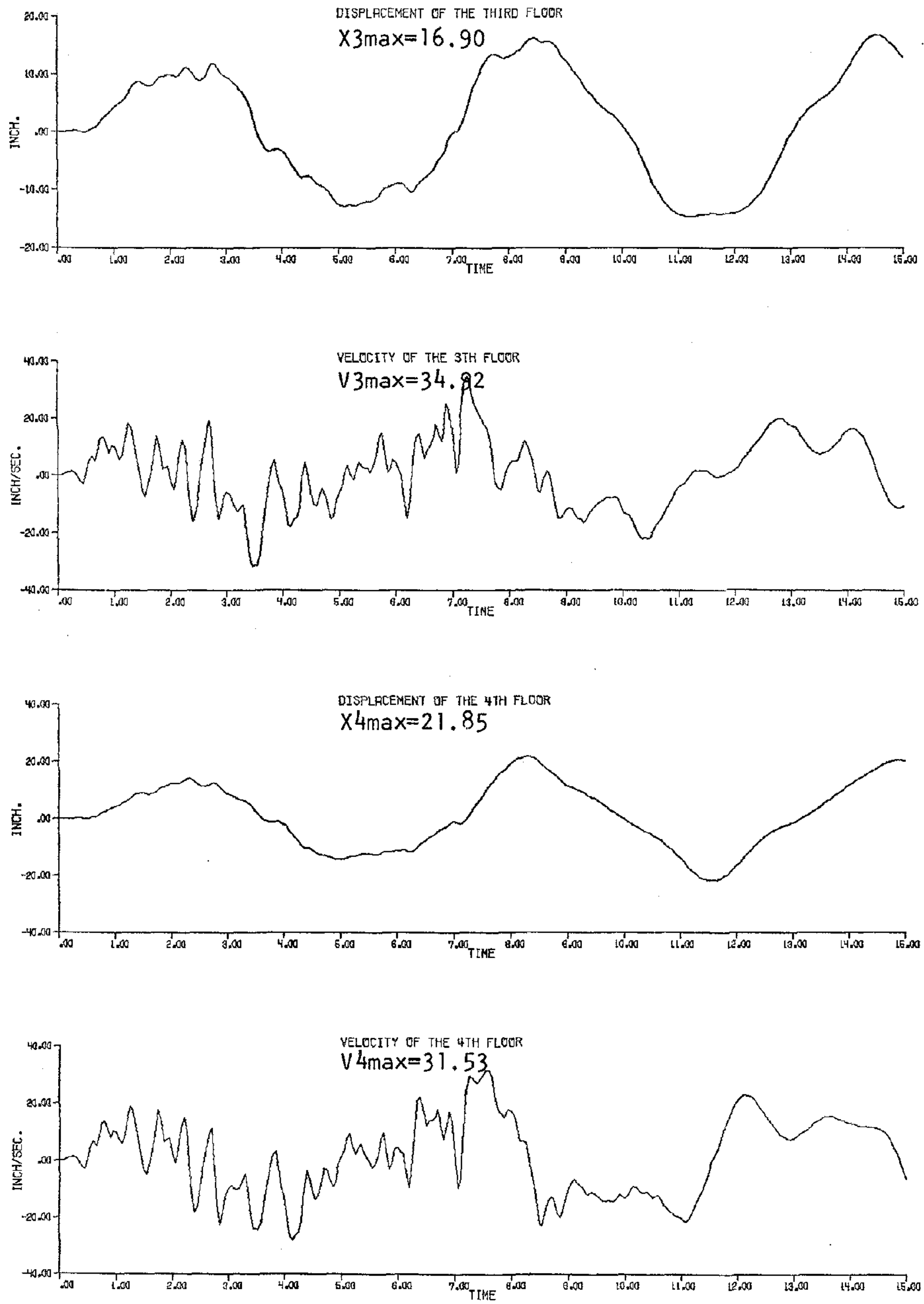

Fig. 38-b Case 1 in Table 4
The Maximum Displacements and velocities of the Open-Loop System.

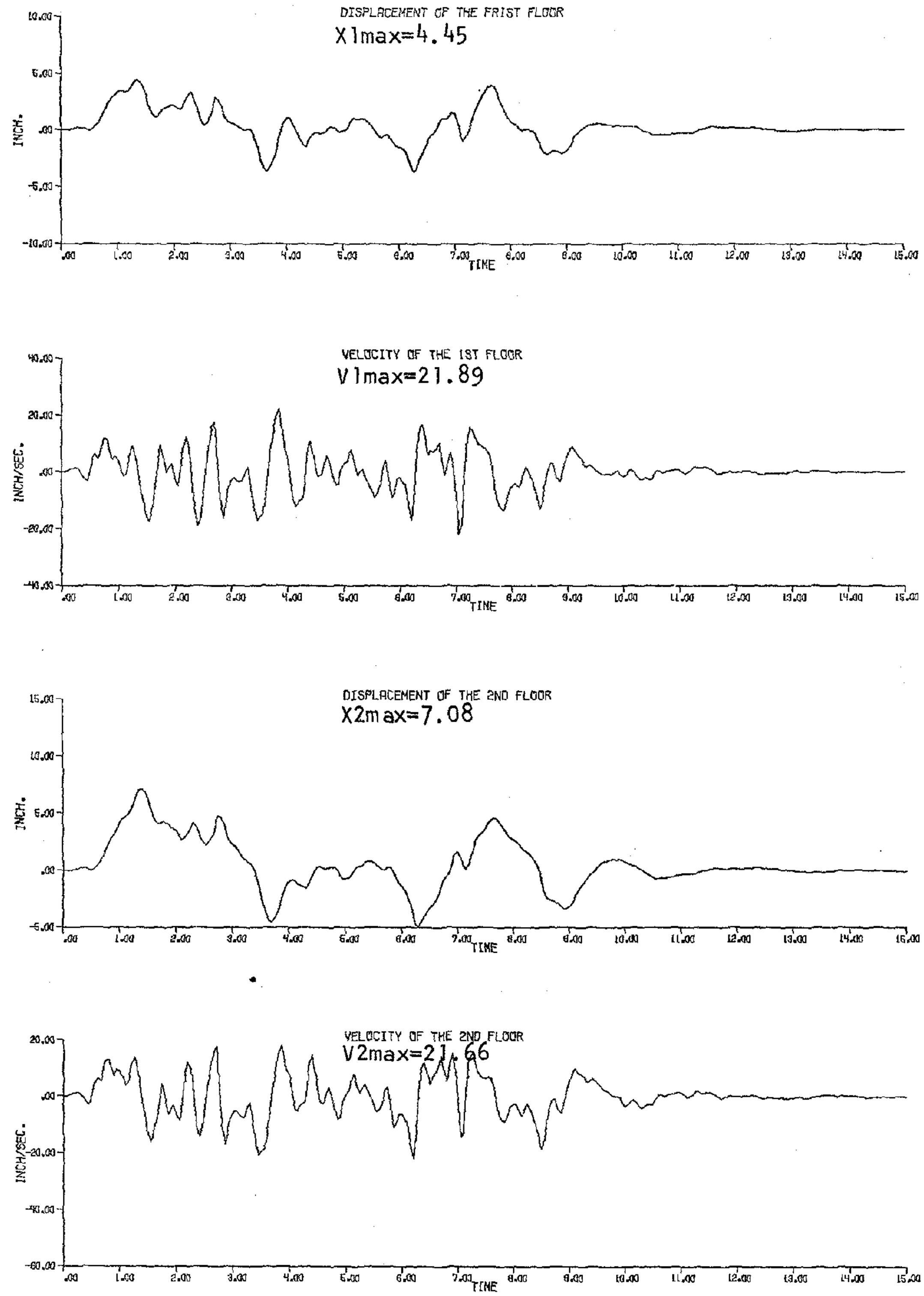

Fig. 39-a Case 2 in Table 4
The Maximum Displacements and Velocities of the Closed-Loop System.

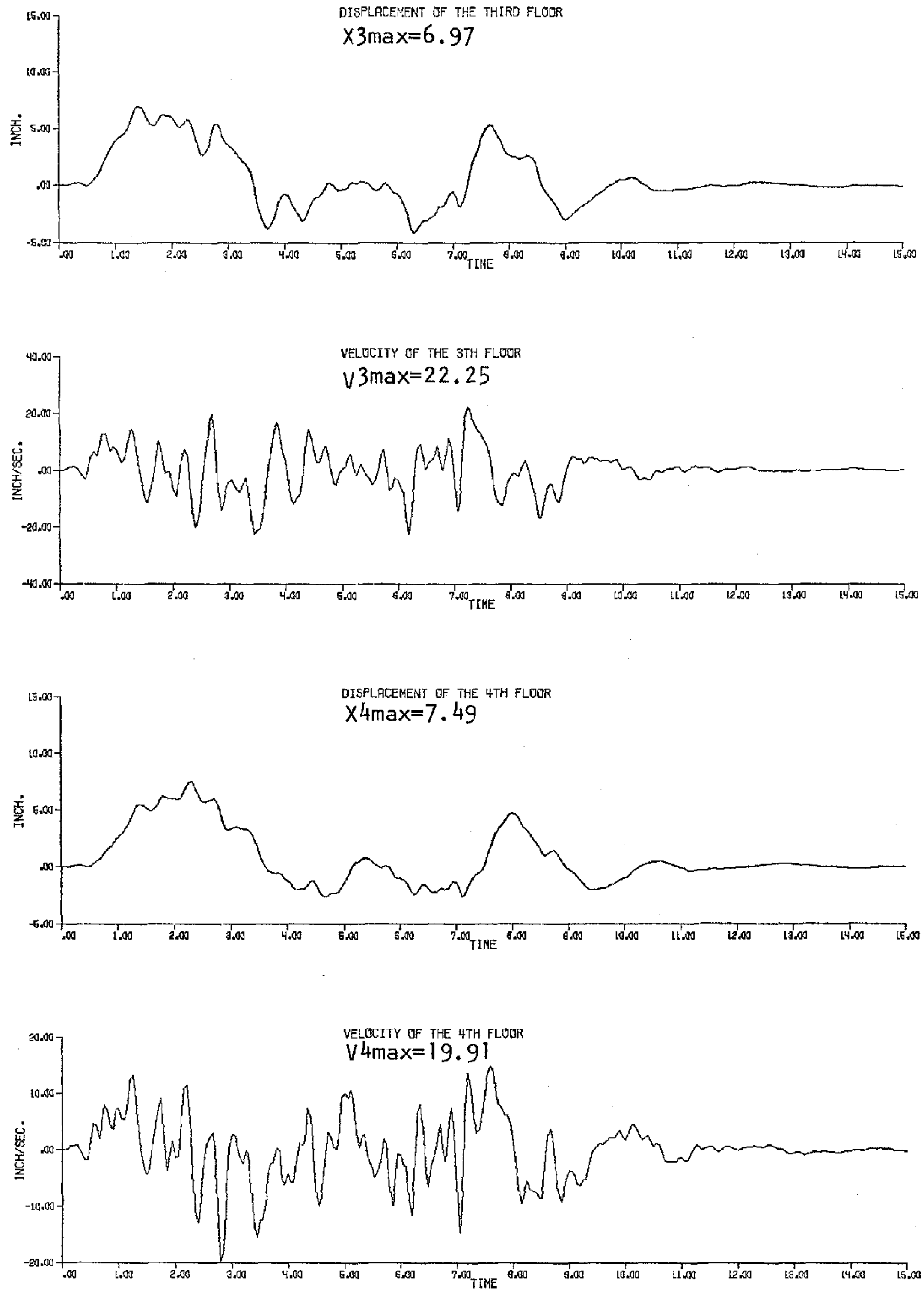

Fig. 39-b Case 2 in Table 4
The Maximum Displacements and Velocities of the Closed-Loop System.

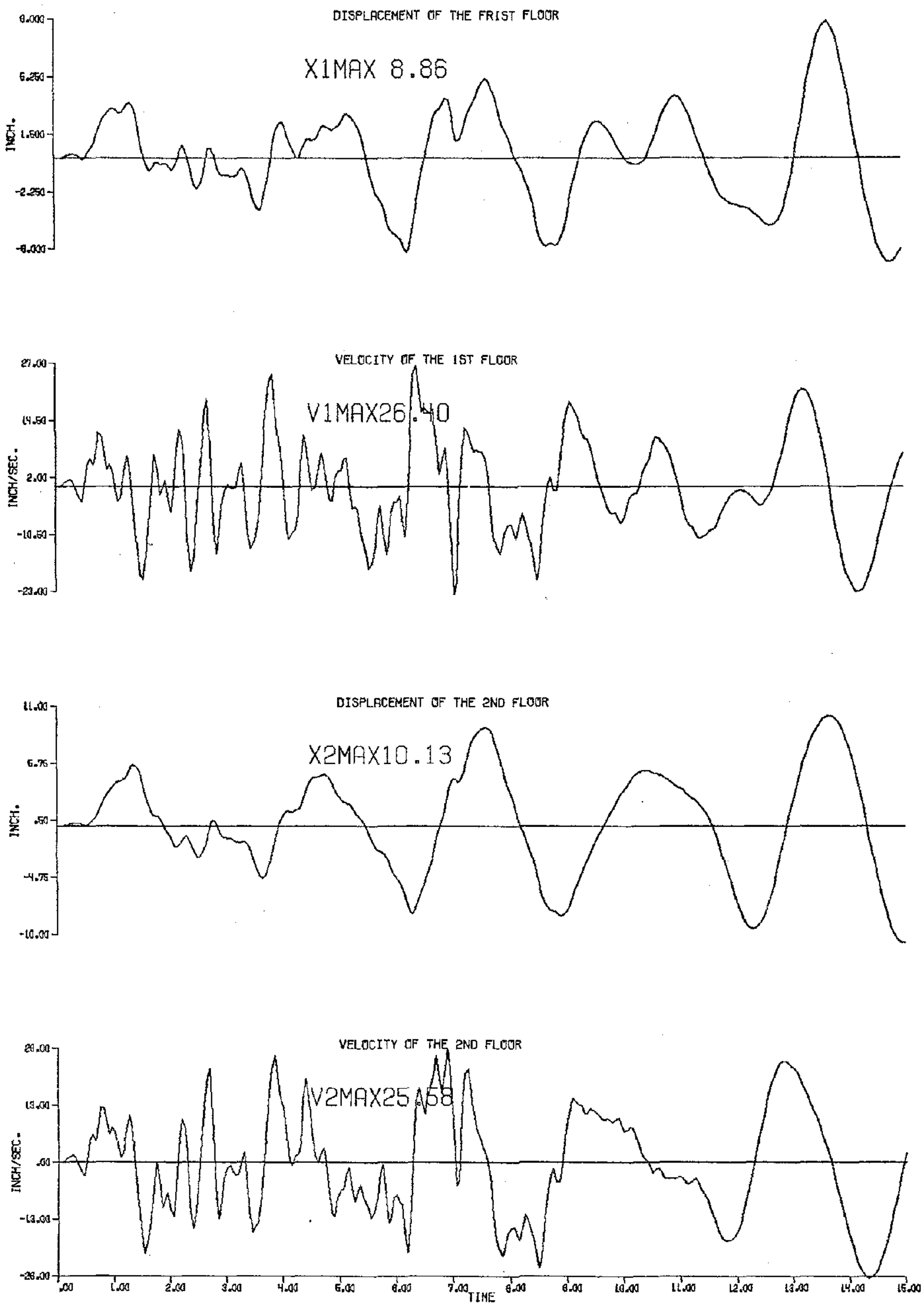

Fig. 40-a Case 3 in Table 4

The Maximum Displacements and Velocities of the Closed-Loop System.

Note: Values Shown on the Plots are Maximum Positive Value.

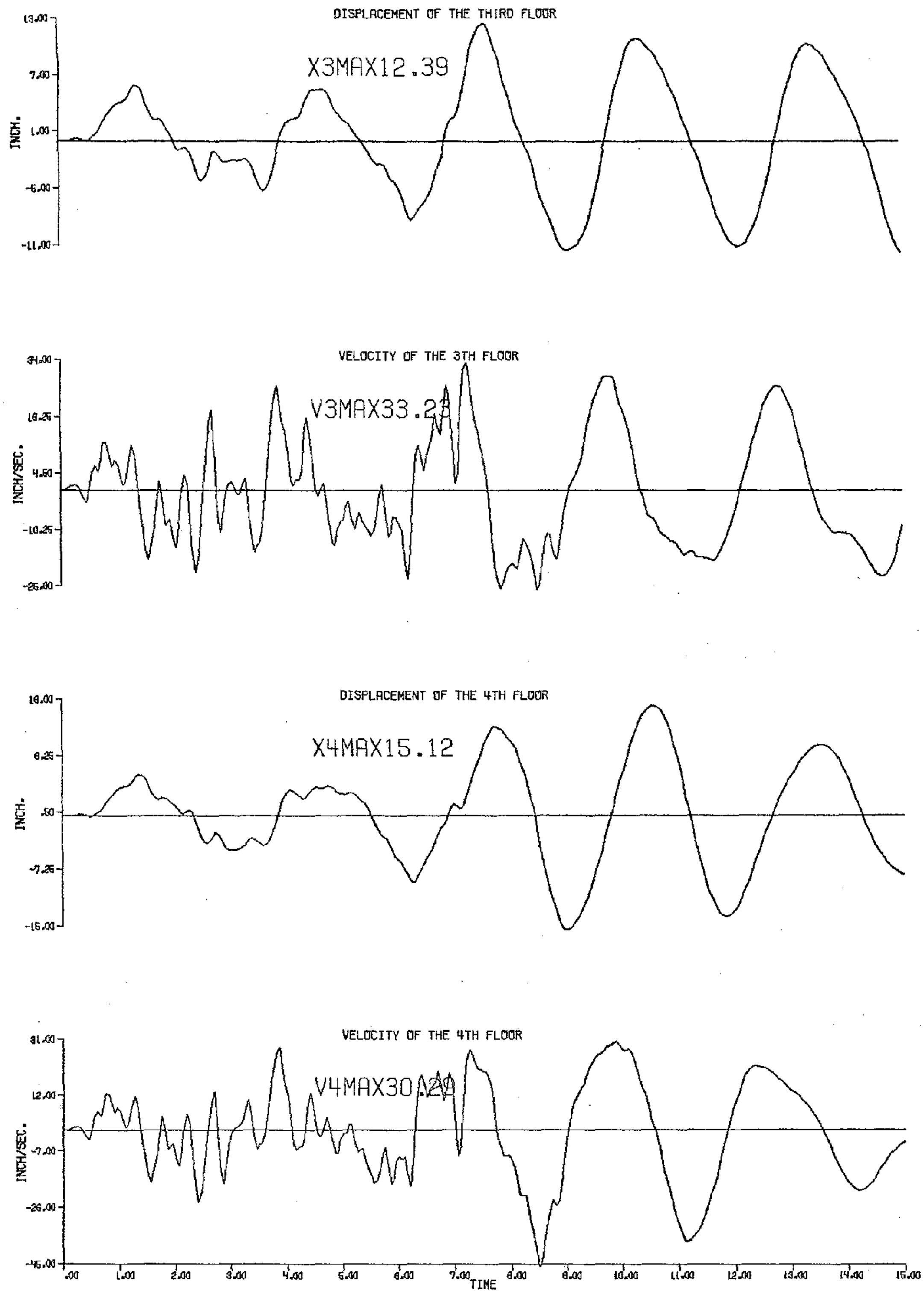

Fig. 40-b Case 3 in Table 4

The Maximum Displacements and Velocities of the Closed-Loop System.

Note: Values Shown on the Plots are Maximum Positive Value.

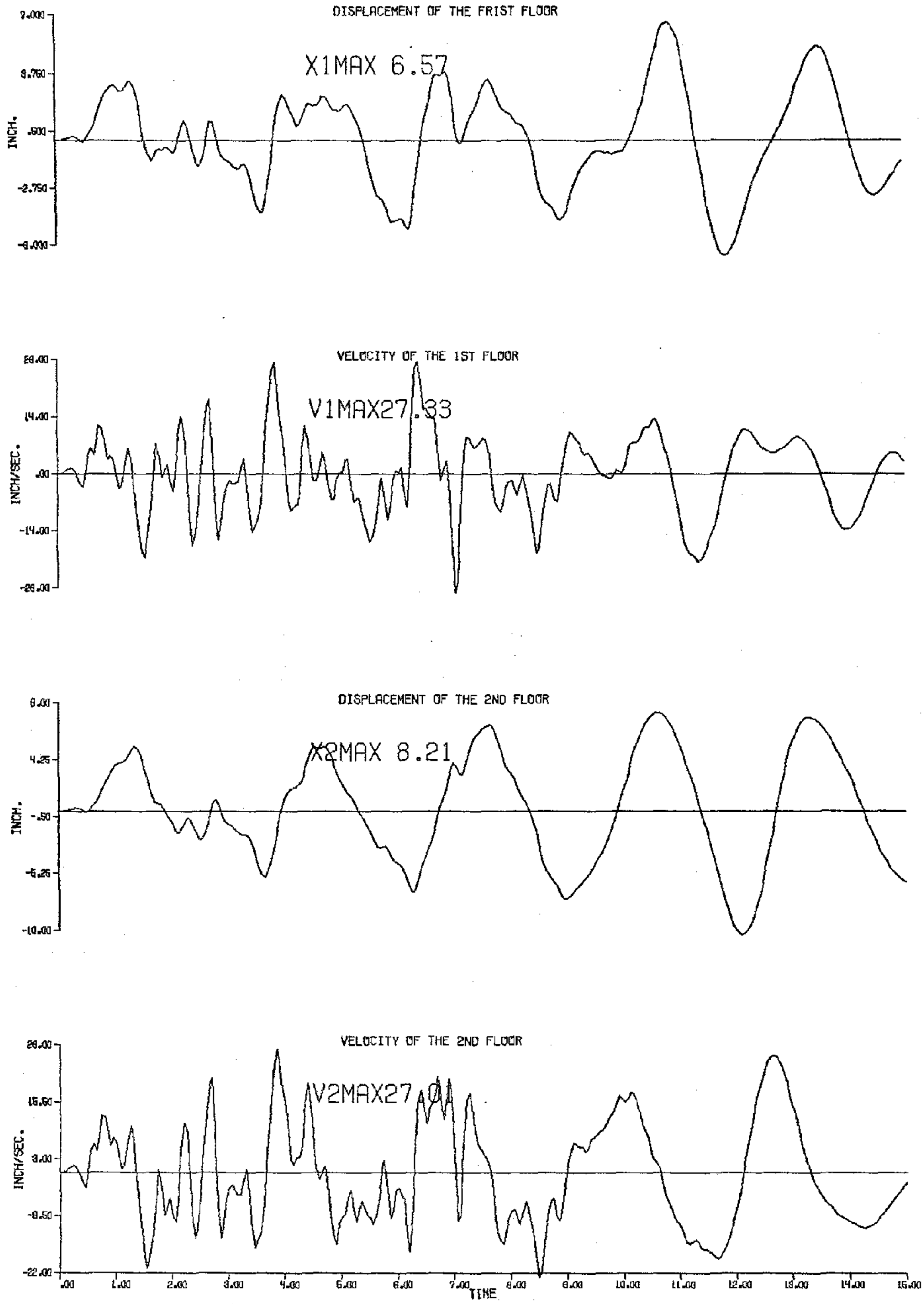

Fig. 41-a Case 4 in Table 4

The Maximum Displacements and Velocities of the Closed-Loop System.

Note: Values Shown on the Plots are Maximum Positive Value.

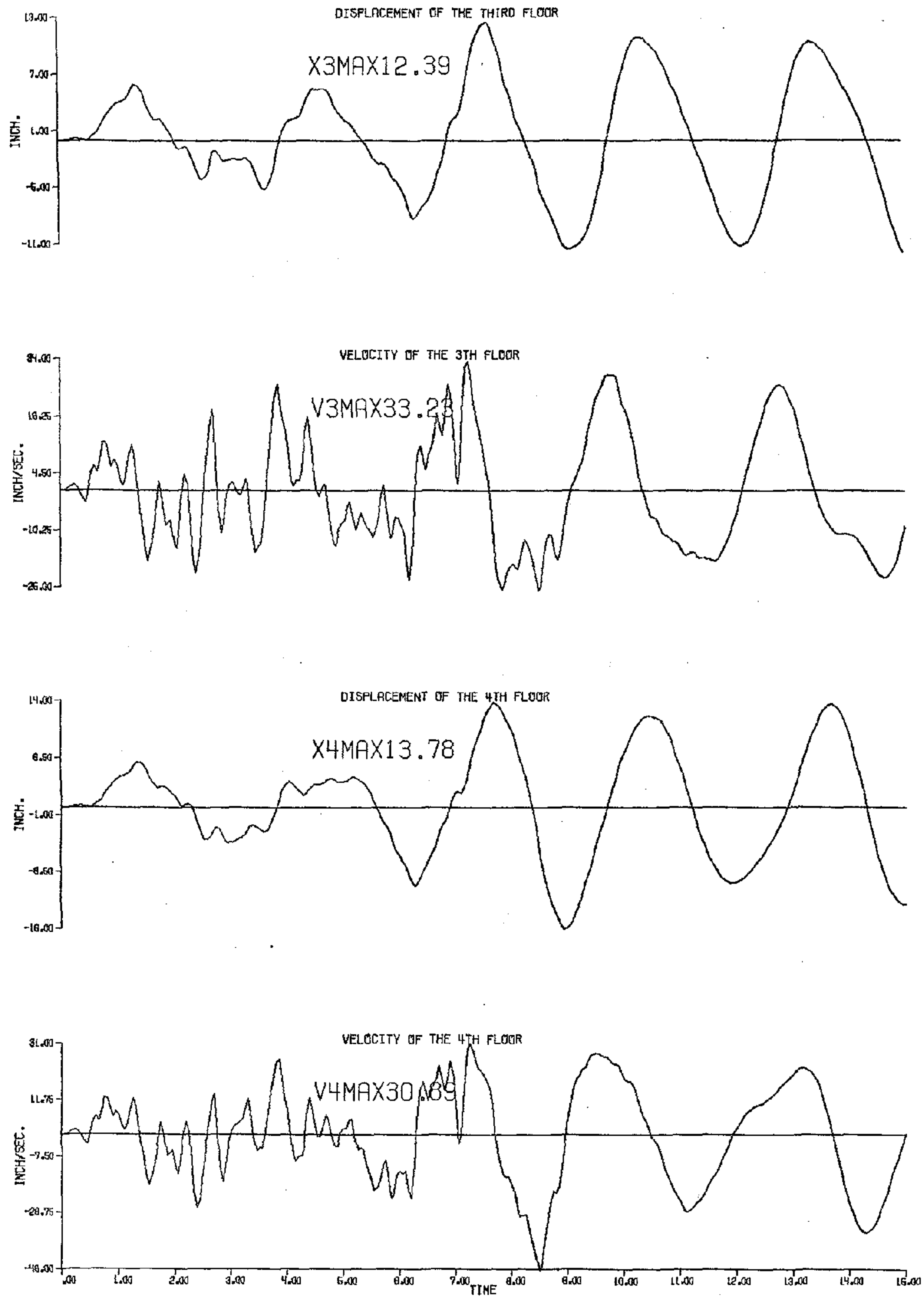

Fig. 41-b Case 4 in Table 4

The Maximum Displacements and Velocities of the Closed-Loop System.

Note: Values Shown on the Plots are Maximum Positive Value.

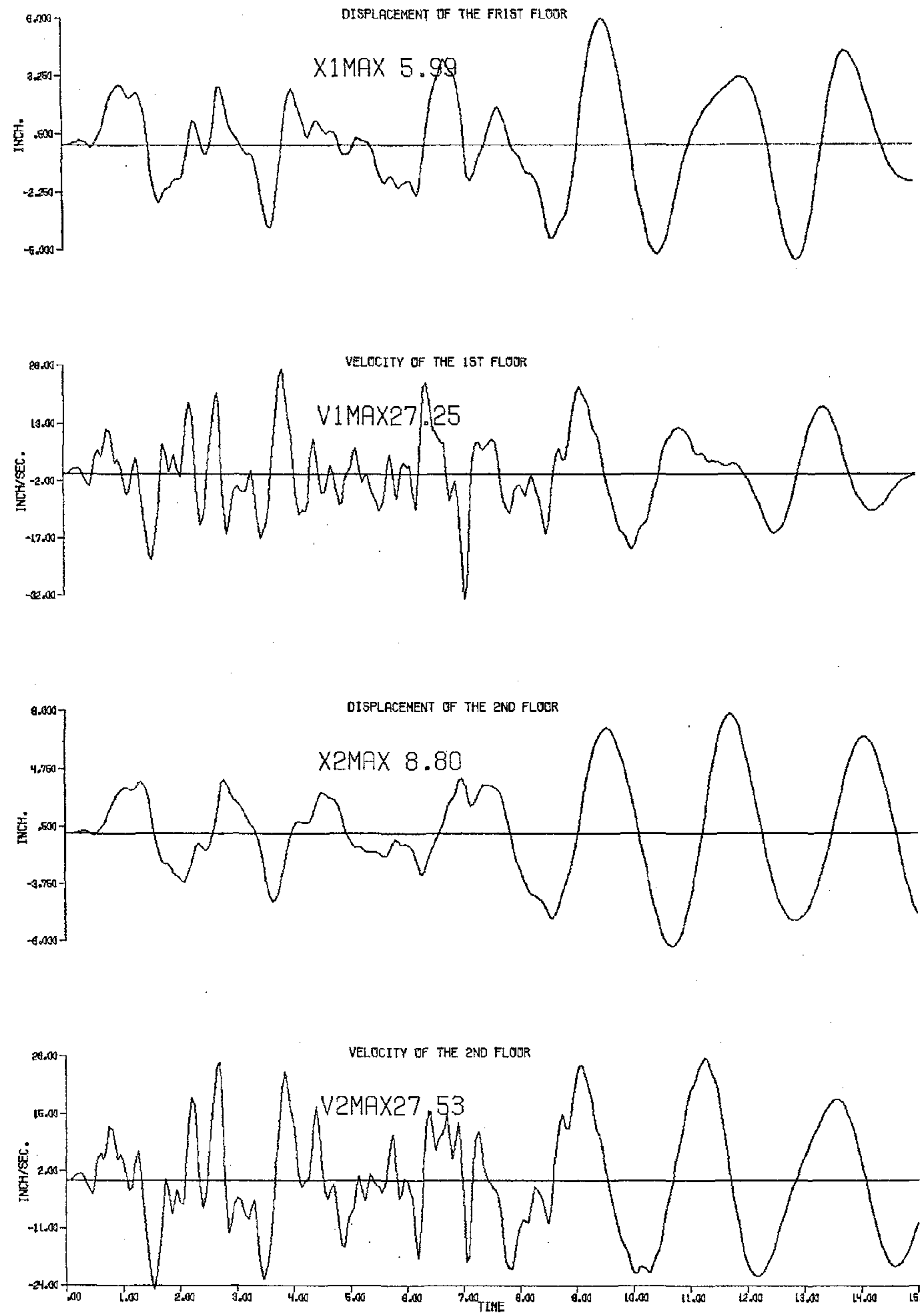

Fig. 42-a Case 5-a in Table 4

The Maximum Displacements and Velocities of the Closed-Loop System.

Note: Values Shown on the Plots are Maximum Positive Value.

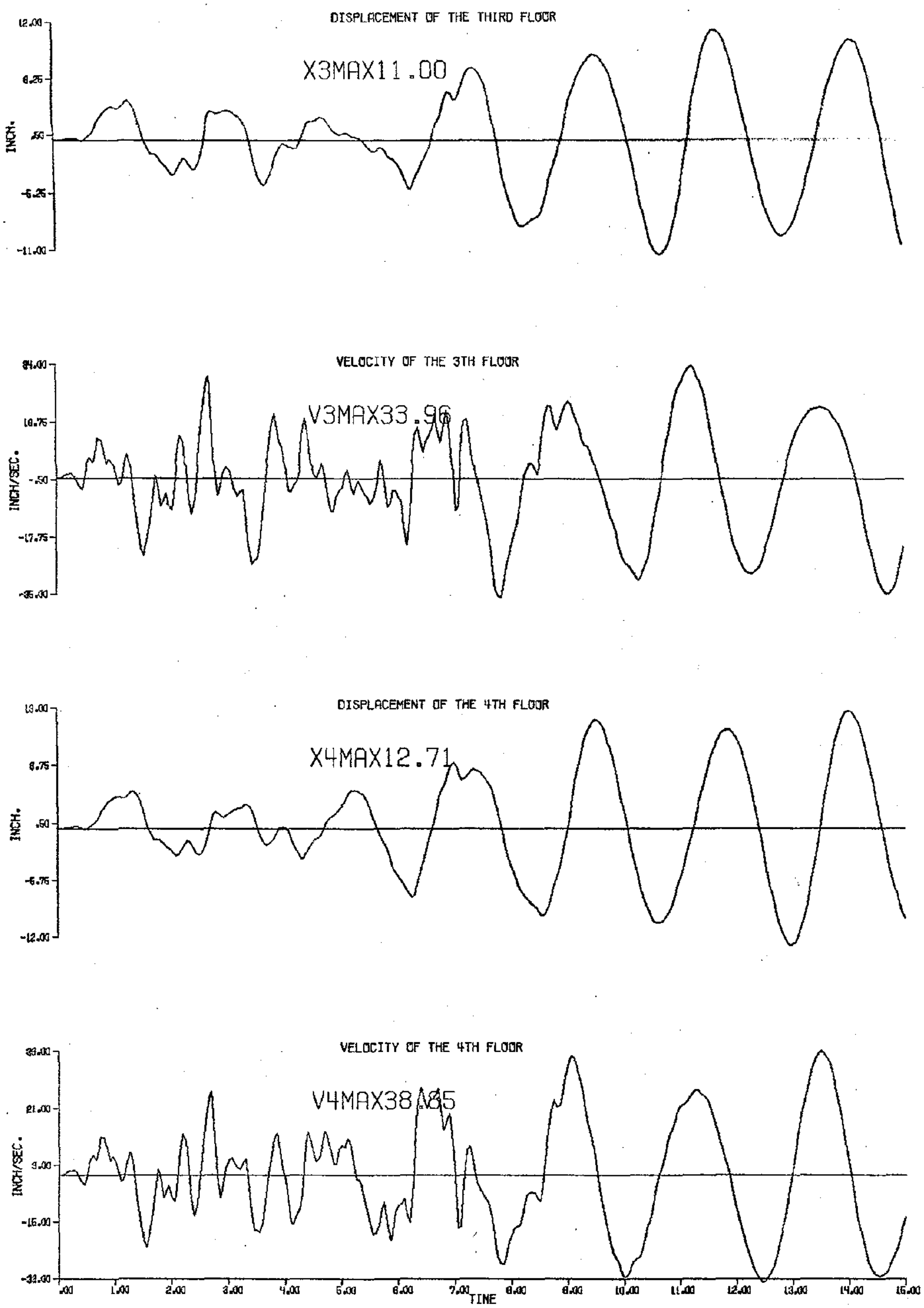

Fig. 42-b Case 5 in Table 4

The Maximum Displacements and Velocities of the Closed-Loop System.

Note: Values Shown on the Plots are Maximum Positive Value.