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RELIABILITY ASPECTS OF STRUCTURAL CONTROL

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16. Abstract (Limit: 200 words)

Active control systems can be used to reduce the effect of lateral earthquake ground motion on structures. The basic assumptions and limitations which must be considered when applying an active control system are noted and several nonlinear control systems, including dead-zone, saturation, and a combination of the two, are studied. Through the use of a simulated earthquake, it is shown that the probability of failure may be reduced by using a perfect control system. However, there is always a time lag between the time when a control force is needed and the time when the required control force can be applied. The effect of such delay time on the reliability of structural control is examined.

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Abstract

To reduce the effect of lateral earthquake ground motion on structures, active control systems can be used. Several different types of nonlinearities such as dead-zone, saturation, and combination of both are considered in this report. Using simulated earthquake, it is shown that the probability of failure may be reduced by using a perfect control system. However, there is always a time lag between the time when a control force is needed and the time when the required control force can be applied. The effect of such delay time on the reliability of structural control is studied herein.

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RELIABILITY ASPECTS OF STRUCTURAL CONTROL*

by

M. A. Basharkhah and J. T. P. Yao

1. Introduction

Generally speaking, many different types of nonlinearities exist in engineering systems. Depending upon whether the nonlinearity is inherent in the system or intentionally inserted into the system, these nonlinearities affect the performance of the system in various ways. For example, some types of nonlinearity may cause instability in the system. However, certain nonlinearities are introduced into the system to improve the performance of the system. Although these intentional nonlinearities may improve the performance of the system under specified conditions, they may not improve the performance of the system under other conditions.

In civil engineering structures, we usually do not have much stability problems. Nevertheless, when one wants to use active control force to reduce the displacement of the response ; the stability of the system may become a major problem. In practical cases, we should be concerned with the stability of the nonlinear control systems.

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Frequently, it is not necessary to obtain analytical solutions of nonlinear differential equations. By using describing-function method, we may study the stability of simple nonlinear control systems. This method provides stability information for systems of any order, but it will not give exact information as to time-response characteristics. Although the describing-function method is quite useful in predicting the stability of unforced systems, it is not practical to use this method for nonlinear structures mostly encountered in civil engineering; because the input of the nonlinear element is not usually sinusoidal. Therefore, it is not practical to use this method for this type of structures.

To analyze and design highly nonlinear systems [6,9,12,21], actual or simplified nonlinear differential equation can be solved by using the phase-plane method or the second method of Liapunov. The phase-plane method provides information about time-response and stability of the system. However, it is only applicable to first and second-order system. In the phase-plane analysis of second order systems, trajectories may be constructed analytically or graphically. However, it is difficult to construct the trajectory of the civil engineering structures analytically or graphically because the main inputs of these systems are earthquakes. Therefore, the differential equations of the system must be solved by using numerical methods. The Runge-Kutta method can be used to solve nonlinear differential equations. The error in the Runge-Kutta method is of order $h^5 = \Delta t^5$. Also, the method avoids the necessity of calculating derivatives and hence excellent accuracy is obtained. Then, using the results of the solution, the trajectory of the system may be constructed.

The second method of Liapunov (see Appendix A) may be applied to any nonlinear system, but it is difficult to find Liapunov function for certain complicated nonlinear systems[8,16,19]. If we are to examine asymptotic stability of equilibrium states of nonlinear systems, stability analysis of linearized models of nonlinear system is completely inadequate. We must investigate nonlinear systems without linearization. Several methods based on the second method of Liapunov are available for this purpose. They include Krasovskii's method (see Appendix B) for testing sufficient conditions for asymptotic stability of the nonlinear systems and Lur'e's method applicable to stability analysis of certain nonlinear control systems. Suppose a system is described by $\dot{X} = f(X)$. To use Krasovskii's method, $f(X)$ must be differentiable with respect to X_i where X_i are the states of the systems. Therefore, this method may not be applicable to elastic-perfectly plastic systems. To use Lur'e's method, the nonlinearities of the system must be continuous. In applying these method to investigate the stability of the structures, one must keep in mind the basic assumptions and all limitations.

2. Nonlinear Systems

In discussing the dynamic behavior of single degree-of-freedom systems, we assume that in the model representing the structures, the resorting force is proportional to the displacement and the energy dissipation is proportional to the velocity. However, a linear model does not always adequately represent the dynamic behavior of the structure. Therefore, the spring force and damping force may not remain proportional to the displacement and the velocity. In such cases, the equation of motion is no longer linear. Such nonlinearities are called inherent nonlinearities. There is no general method for dealing with all nonlinear systems. Exact solutions can be found only for certain types of nonlinear differential equations[10]. Numerical procedure can be used for integration of nonlinear equation. As a first example, we consider a single degree-of-freedom system which has only inherent nonlinearity. Then, a nonlinear time invariant single degree-of-freedom system under different types of intentional nonlinearities will be discussed later.

Consider the model for a single degree-of-freedom system as shown in Fig. 1. The dynamic equilibrium in the system without active control force is established by setting to zero the sum of the inertia force, the damping force, the spring force , and the external force. The equilibrium of these forces at time t_i is expressed as follows:

$$F_I(t_i) + F_D(t_i) + F_S(t_i) = F(t_i) \qquad (1)$$

and the equilibrium of these forces at time t_{i+1} is expressed in the following:

$$F_I(t_{i+1}) + F_D(t_{i+1}) + F_S(t_{i+1}) = F(t_{i+1}) \qquad (2)$$

Where $\Delta t=t_{i+1} - t_i$ is called time increment. The differential equation of motion in terms of increments can be found by subtracting Equation 2 from Equation 1 and is given by:

$$\Delta F_I + \Delta F_D + \Delta F_S = \Delta F \quad (3)$$

To solve a nonlinear differential equation by using a numerical method, one can use either linear acceleration step-by-step technique or Runge-Kutta method. In this report, the energy dissipation is assumed to be proportional to the velocity. Therefore, the only inherent nonlinearity is the spring force. If any structure modeled as a single degree-of-freedom system is allowed to yield plastically, then the resorting force is likely to be the form shown in Fig. 2. There is a portion of the curve in which linear elastic behavior occurs, whereupon for any further deformation, plastic yielding takes place. When the structure is unloaded, the behavior is again elastic until further reverse loading produces compressive plastic yielding. This behavior is often simplified by assuming an elastic- perfectly plastic behavior. Therefore, the elastoplastic behavior of the system is the only inherent nonlinearity that it is considered in this report. The elastoplastic behavior of the system is shown in Fig. 3. The maximum displacement response of the system without any active control force to the 1940 El-Centro earthquake is shown as case 1 in Table 1. To reduce the displacement of this system, one can use a linear or nonlinear control system. As we mentioned earlier, base excitation can be assumed as a disturbance force. Therefore, the general form of the nonlinear equation of motion can be written by using the state space variable concept as follows:

$$\dot{X} = f(X,U,t) \tag{4}$$

Let us assume the control force to be a linear combination of the displacement and the velocity of the system, then the control law is given as follows:

$$U=-KX \tag{5}$$

Where X is a 2x1 state vector and K is a 1x2 gain matrix. Case 2 in table 1 shows the maximum displacement of the same system under the same earthquake when it is subjected to an active control as described by Equation 5. It can be seen that the maximum displacement of the system without active control force is greater than the maximum displacement response of the same system with active control force. The block diagram and numerical properties of the system is also shown in Fig. 4 and Fig. 5.

The stability of the open-loop system is not a major problem. If we take displacement and velocity as the coordinates of a plane, to each state of the system there corresponds a point in this plane. As time varies, this point describes a curve in this plane. Such a curve is called trajectory. The system is stable if trajectory of the system tend to one or more stable equilibrium points.

Now, a nonlinear time invariant single degree-of-freedom system is considered under different types of intentional nonlinearities. The inherent nonlinearity of the open-loop system is an elastic-perfectly plastic spring. Three different types of intentionally nonlinearities are examined in this study. First, a dead-zone intentional nonlinearity is inserted into the system. The block diagram of the closed-loop system is shown in Fig. 6. The output of a dead-zone nonlinearity is equal to zero when the absolute value of the displacement is less than D1 . Otherwise, it is a linear function of the displacement. The properties of the dead-zone nonlinearity element which it is used are shown in Fig. 7. These properties can be chosen such that the system performs in the linear range. Thus, the displacement of the system is always less than the displacement of the yield point. Second, a saturation nonlinearity element is used for the system. The block diagram of the system with saturation nonlinearity is shown in Fig. 8. The properties of the nonlinearity element can be assumed such that the behavior of the system remains in elastic range. The output of the saturation nonlinearity is a linear function of the input as long as the absolute value of the displacement is less than D2 . When the absolute value of the displacement is greater than D2 the output of the saturation nonlinearity element remains constant. Fig.9 shows the properties of the saturation nonlinearity which they are used in this example. Finally, the combination of a dead-zone and a saturation nonlinearities is inserted into the system. The block diagram of the closed-loop system with this kind of nonlinearity is shown in Fig. 10. In this case, the output of the nonlinearity is equal to zero as long as the absolute value of the displacement is less than D1 . When the absolute value of the displacement is greater than D2, the output of the nonlinearity becomes constant. Otherwise, the output of the nonlinearity is proportional to its input. Fig. 11 shows the properties of this kind of nonlinearity. These constant can be chosen in such away that the induced spring force will not increase the yield point. The maximum displacement, velocity, acceleration, spring force, and active control force for each case are listed in Table 1 respectively. It can be seen that the maximum displacement of the controlled system is always less than the maximum displacement of the uncontrolled system. The displacement , velocity, and active control force of these different cases are plotted in Fig. 14 through Fig. 18 respectively. For all cases, the trajectory and elastoplastic force-displacement are plotted. The phase-plane for each case shows that the closed-loop system is stable because the trajectory of each system converged to the equilibrium position. The elastoplastic force-displacement for each case shows that the closed-loop system has a nonlinear behavior. Moreover, Fig. 14 shows the trajectory of the open-loop system and the elastoplastic behavior of the system. It can be seen that the open-loop system is always stable.

3. Reliability of Control Systems

In this part of this report, the reliability aspect of control system is considered. In recent years, the idea of using active devices to reduce the displacement of civil engineering structure has been explored. Although such devices have been used in space- vehicle, missile-guidance, and aircraft-piloting; the application of these devices to control the motion of the civil engineering structures is still new. Many papers have been written about the use of the active devices in civil engineering structures. However, few, if any, of them dealt with the reliability of these control systems. In other control problems, the disturbance forces are not the major input of the system. Because the disturbance forces can be the major input to civil engineering structures, it is important to design a control system such that the probability of the failure due to any excitation should not increase as a result of using control devices. Earthquake is the major disturbance force under consideration, and it is assumed as a nonstationary random process in this study [18,22].

In control systems, if one knows the properties of the disturbance force, a feedforward system can be designed such that the effect of these disturbance forces would vanish. In the real world, no one can predict the exact properties of the next earthquake. Therefore, one has to assume these external forces as a random process. In this report, a linear time-invariant single-degree-of-freedom system as shown in Fig. 12 is considered. The properties of the system are assumed to be constant. To reduce the displacement of this single-degree- freedom system, a linear control law, $U=-KX$, is used. Note that the elements of the gain matrix, K, are determined by solving the Riccati equation for the system.

As a first example, the elements of the gain matrix are chosen as a deterministic elements. The block diagram of the closed-loop system and the properties of the system are shown in Fig. 13, where $G_p(s)$ is the transfer function of the plant, and $G_c(s)$ is the transfer function of the controller, and it can be assumed as a constant gain or variable. In this part of the study, the transfer function of the controller is assumed as a nondeterministic but can be either constant gain or variable. Let the transfer function of the controller be in the form of:

$$G_c(s) = \frac{K_p}{1+Ts} \tag{6}$$

Where Kp is a deterministic constant gain which it is determined by the control law, and T is a random variable representing the time constant of the controller. Let us assume an uniform probability density function [7,11] between interval zero and 0.1 for the time constant, the effect of this time constant is very important because by increasing its value the displacement of the controlled system will increase. Due to a value of the time constant, we have a cancellation of pole and zero in the system. The cancellation pole and zero will decrease the order of the system and will cause a higher displacement for the system. Therefore, the probability of the failure will be increased.

Using a standard program [15] (PSEQGN), twenty artificial earthquake records are generated. Then, the absolute value of the maximum displacement response of the system without any control force to each earthquake is computed. For the purpose of this study, failure is said to occur when the maximum displacement of the system exceeds a certain level. In real world, this failure level ,x_f, can be defined as the deformation at which the material yields. For the purpose of comparison, the reference failure level ,x_f, is defined as the median value, for which the probability of the failure for uncontrolled structure is equal 0.5 . To find the value of ,x_f, for the given system, one can use the extreme-values probability distribution. To use the extreme-values probability distribution, the maximum displacement of the response of the structure to each earthquake is computed. These values are listed in the first column of Table 2 . The analytical form of the extreme-values distribution function [5] is given as follows:

$$F_X(x) = \mathrm{Exp}(-\mathrm{Exp}(ax+b)) \tag{7}$$

Where a and b are the distribution parameters. Data sampled from this distribution function can be plotted as a straight line on extreme value probability paper. Shown in Fig. 19 is the empirical distribution function of the maximum displacement of the response without any active control system. A smooth curve was drawn through the points. The parameters of this distribution for the system without control are estimated by using the least-square method. These values are listed as case 1 in Table 3 . The least-square method has been used for all cases in this investigation. The resulting median value of x_f for this system is estimated as 1.53.

To find out how much the probability of failure can be decreased by using control system, a perfect controller for the system is studied first. In this case, the time constant of the transfer function for the controller is set to zero. Now, one can find the probability distribution function for the controlled system. To do this, the maximum displacement of the controlled response due to all the artificial earthquakes are computed and listed in the second column of Table 2. These data are plotted as a straight line on extreme value probability paper. Shown in Fig. 20 is the empirical distribution function of the maximum displacement of the controlled response. A smooth curve was drawn through the points. The parameters of the distribution are estimated by using least-square method and listed as case 2 in Table 3. By using this probability distribution function, the probability of the failure for the system is computed as follows:

$$p_f = 1.-F(1.53)=.088 \tag{8}$$

To show the effect of the time constant, 12 values for time constant have been chosen randomly from an uniform distribution in the interval (0.,0.1). Using each value of these time constants, the maximum displacement of the response to these 20 artificial earthquakes are computed as listed in Table 2. For each case, the empirical distribution function of the maximum displacement of the response has been plotted as shown in Fig. 21 through Fig. 32. A smooth curve was drawn through the points for each case separately. By using the least-square method the parameters of the distribution are calculated. These parameters are listed as case 3 through case 14 in Table 3. To find a conditional distribution function for the maximum displacement of the response given that the time constant of the system is given as a known value, one has to find a relationship between the time constant and "a" as well as the time constant and "b". Finally, a least-square curve-fitting program is used to obtain the best functional relationship between parameters "a", "b", and the time constant as follows:

$$a(t)=-4.8737+10.912t+121.21t^2$$
$$b(t)=4.9169+18.036t-227.71t^2 \tag{9}$$

Therefore, the conditional probability distribution of the maximum displacement of the response is given as following:

$$F_{X|T}(x|t) = \mathrm{Exp}(-\mathrm{Exp}(a(t)x+b(t))) \tag{10}$$

And the probability distribution function [14,17] of the maximum displacement is given as follows:

$$F_X(x) = \int_{0.}^{0.1} F_{X|T}(x|t)\, f_T(t)\, dt \tag{11}$$

Where $f_T(t)$ is the probability density function for the time constant which it is assume as an uniform distribution with interval of (0. ,0.1). Therefore, one can find the probability that the maximum peak of the response is less than the failure level, x_f ,and the probability of the failure as follows:

$$p_f = 1.-F(1.53)=.375 \tag{12}$$

It can be seen that the probability of the failure for a system with random time constant is greater than the probability of failure for the same system when the time constant of the controller is equal to zero. But the probability of failure is less than 0.5. It means that the maximum displacement of the controlled system is less than the maximum displacement of the same system without active control force. However, this is not always true. Because by increasing the interval of the time constant or by changing the shape of the probability density function for the time constant, we may obtain a higher probability of failure for the controlled system than that for uncontrolled systems. Therefore, one must be careful in applying an active control system for civil engineering structures.

Alternatively, one can use another approach to find the probability distribution function for the maximum displacement of the response. For each value of the time constants, which have been already generated randomly, one value of the maximum displacement is picked at random from Table 2. In this case, 12 different values of the maximum displacement corresponding to 12 different time constant are picked. The empirical distribution function is plotted on probability paper. A least-square method has been used to find the parameters of this probability distribution function. These parameters are shown as case 15 in Table 3. By using this probability distribution function, the probability that the maximum displacement of the response not exceed the failure level is given as follows:

$$p_f = 1.-F(1.53)=.460 \qquad (13)$$

The probability of failure for this approach is found to be higher than the previous method. It is possible to use a larger sample for the empirical distribution. For example, 240 points corresponding to all 12 different time constant have been chosen. Then, the empirical distribution function has been plotted and the parameters of the distribution function are computed by using the least-square method. These parameters are shown as case 16 in Table 3. The probability of failure for the system by using this approach is given as follows:

$$p_f = 1.-F(1.53)=.497 \tag{14}$$

In this part of this study, another probability distribution function is considered. For many cases, the extreme-values probability function will not yield a smooth curve through the empirical distribution function. In this case, another function is introduced here and is given as follows:

$$F_X(X) = \mathrm{Exp}(1.-\mathrm{Exp}(\mathrm{Exp}(ax+b))) \tag{15}$$

To use this probability distribution function, we have used the same sample of 20 artificial earthquakes and the same values for time constant. Therefore, all properties of the system remain the same for the purposes of comparison. The maximum displacement response for the system with these different values of the time constant are given in Table 2. For each case, the empirical distribution function has been plotted on a probability paper and the parameters corresponding to that particular time constant have been computed as listed in Table 4. Fig. 33 through Fig. 46 show the empirical distribution for the uncontrolled system, the controlled system with zero time constant, and all different time constants. Finally, a least-square curve-fitting program is used to obtain the best functional relationship between parameters "a", "b", and the time constant. The curve-fitting program yields the coefficient of the best polynominal, and they are given as follows:

$$a(t)=-3.864+7.619t+98.128t^2$$
$$b(t)=3.467+16.129t-183.19t^2 \tag{16}$$

Therefore, the conditional distribution function for the maximum displacement of the response given that the time constant is equal to a certain value is given as following:

$$F_{X|T}(x|t) = Exp(1.-Exp(Exp(a(t)x+b(t)))) \tag{17}$$

The probability distribution function of the maximum displacement can be found as follows:

$$F_X(x) = \int_{0.}^{0.1} F_{X|T}(x|t)\ f_T(t)\ dt \tag{18}$$

Where $f_T(t)$ is the probability density function of the time constant which it is assumed as an uniform distribution with interval of (0. ,0.1). Again, by using this last distribution, the probability that the maximum displacement of the response not to exceed the failure level and the probability of failure are calculated as follows:

$$p_f = 1.-F(1.53)=.366 \tag{19}$$

It can be seen that these values are almost the same as those values that they are computed by using the extreme-values distribution.

In this part of this report, a Markov process [1-4,20] has been used to find the reliability of the system. By using this process, a cumulative distribution function will be fitted through the points of the empirical distribution function. First, let us use a stationary B model to find a cumulative distribution function. To use this method, those data which they have been already generated are used. As we mentioned earlier, 12 different values for time constant have been found randomly. The maximum displacement of the response due to twenty artificial earthquakes has been computed for each random time constant. These values are listed in Table 2. Fig. 47 shows the empirical distribution function for these data. The mean and variance of this distribution are given as follows:

mean=1.53 variance=.1374

By using a stationary B model for this process, the parameters of the process are calculated as following:

b=18 r=89.17

Fig. 48 shows the cumulative distribution function for those data which they are used in this process. It can be seen that the cumulative distribution function is fitted perfectly to the empirical distribution function. By using the cumulative distribution function which it is found by the B model, the mean and variance of the process are computed, and they are given as following:

mean=1.53 variance=.1367

It can be seen that these calculated values of the mean and variance of the cumulative distribution function are matched to corresponding values of the empirical distribution function. By using this approach, one can find the probability that the maximum displacement of the response not to exceed the failure level. This value is computed as 0.5 which it is less than those values that they are found by using first method, extreme-values distribution, 0.625, or the third approach, 0.634, but it is almost equal to the value which it is found by the second procedure,0.503,.

Finally, let us as a final note describe the effect of the time constant on the system. For this case, the maximum displacement of the response due to the El-Centro 1940 earthquake has been computed for all different values of the time constant. These values are shown in Table 5. It can be seen that by increasing the value of the time constant, the maximum displacement of the system will be increased. Fig. 49 shows the variation of the maximum displacement with respect to the time constant.

4. Conclusion

Although it is a good idea to apply active control for the reduction of displacement response of civil engineering structures, one must keep in mind all the basic assumption and limitations involved in such applications. In this report, the time constant of the system is considered as a random variable. It has been shown that by using a perfect control system (where the time constant is zero), the probability of failure of the system is minimum. In real world, however, there is no perfect control system. Results of this investigation show that probabilities of failure for real world systems are higher than those for idealized ones; which have been subjected by other investigators.

In the first part of this report, several nonlinear control systems were studied. It is found that the use of those intentional nonlinearities can be beneficial. For each type of nonlinearity, there are several parameters which can be assumed as random variables. In addition, the elements of the gain matrix for the controller can be assumed as a random variable. It is not always easy to consider all parameters of the system as random variables at the same time. Nevertheless, one can examine the effect of each variable or a few of them at a time. In this manner, a joint distribution function may be found for the system. Another important subject is the shape of the probability density function for the random variable. For example, the probability density function for the time constant in this report is assumed to follow an uniform distribution. Changing the shape of this distribution or even the size of the interval may alter the probability of failure. Writers believe that it is very important to consider the reliability aspect of structural control before such systems can become practical. Because of the complexity of real-world systems, more studies of this important problem are needed.

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Appendix A: Concepts of Stability:

There are many definitions of stability, but they all basically are concerned with whether neighboring solutions of the differential equation remain neighboring. Since there are an infinite number of solutions, some of which may qualify as "stable" and some as "unstable". We must recognize that "stability" is a solution of a set of differential equations. It is not precise to say that "a system is stable or unstable", but we are precise to say a particular solution of a differential equation is stable or unstable.

Second Method of Liapunov:

The second method of Liapunov attempts to give information on the stability of equilibrium states of linear and nonlinear systems without any knowledge of their solutions. The stability information obtained by this method is precise and involves no approximation. In this section we first present Liapunov's main stability theorem. We assume that the equilibrium state under consideration is at the origin of the state space. The essence of the second method of Liapunov is given in the following main theorem:

Theorem:

Let the system be defined as follows:

$$\dot{X}=f(X,t) \tag{A-1}$$

Suppose that $f(0,t)=0$ for all t and there exists a scalar function V(X,t) which has continuous first partial derivatives. If V(X,t) satisfies the following conditions:

1. V(X,t) is positive definite, namely, V(0,t)=0 and

2. $V(X,t) \geq \alpha(||X||) > 0$ for all $X \neq 0$ and t where α is a continuous, nondecreasing scalar function such that $\alpha(0)=0$.

3. The total derivative $\dot{V}$ is negative for all $X \neq 0$ and all t, or $\dot{V}(X,t) \leq -\gamma(||X||) < 0$ for all $X \neq 0$ and all t where γ is a continuous nondecreasing scalar function such that $\gamma(0)=0$.

4. There exists a continuous, nondecreasing scalar function such that $\beta(0)=0$ and, for all t, $V(X,t) \leq \beta(||X||)$

5. $\alpha(||X||)$ approaches infinity as $||X||$ increases indefinitely, or $\alpha(||X||) \to \infty$ as $||X|| \to \infty$

then the origin of the system, X=0, is uniformly asymptotically stable in the large.

Theorem

If there exist a scalar function V(X,t), with continuous first partial derivatives, satisfying the following conditions:

(a) $V(X,t) > 0$ for all $X \neq 0$ in Ω and all t and $V(0,t)=0$ for all t

(b) $\dot{V}(X,t) < 0$ for all $X \neq 0$ in Ω and all t $\dot{V}(0,t) = 0$ for all t

then the origin of the system is uniformly asymptotically stable. If the foregoing conditions are satisfied in the whole state space and either $V(X,t) \to \infty$ as $||X|| \to \infty$, or $\dot{V}(X,t) \leq -m < 0$ for all $||X|| \Rightarrow M$ and some m, M >0, then the origin is uniformly asymptotically stable in the large.

Theorem

If there exists a scalar function $V(X,t)$, with continuous first partial derivatives, satisfying the following conditions

(a) $V(X,t) > 0$ for all $X \neq 0$ in Ω and all t

$V(0,t)=0$ for all t

(b) $\dot{V}(0,t) \leq 0$ for all $X \neq 0$ in Ω and all t

$\dot{V}(0,t)=0$ for all t

then the origin of the system is uniformly stable.

Theorem

If there exists a scalar function $V(X,t)$, with continuous first partial derivatives, satisfying the following conditions:

(a) $V(X,t) > 0$ for all $X \neq 0$ in Ω and all t

$V(0,t) = 0$ for all t

(b) $\dot{V}(X,t) > 0$ for all $X \neq 0$ in Ω and all t

$\dot{V}(0,t)=0$ for all t or

(a′) $V(X,t) < 0$ for all $X \neq 0$ in Ω and all t

$V(0,t) = 0$ for all t

(b′) $\dot{V}(X,t) < 0$ for all $X \neq 0$ in Ω and all t

$\dot{V}(0,t)=0$ for all t then the origin of the system is unstable. This theorem states that if both $V(X,t)$ and $\dot{V}(X,t)$ are definite

and have the same sign, then the origin is unstable and the trajectories near the origin diverge to a limiting value or infinity.

Theorem

Suppose that there exist a scalar function $V(X,t)$, with continuous first partial derivatives, satisfying the following conditions:

(a) $V(X,t) > 0$ for all $X \neq 0$ and all t

$V(0,t) = 0$ for all t

(b) $\dot{V}(X,t) \leq 0$ for all $X \neq 0$ and all t

$\dot{V}(0,t) = 0$ for all t

Let E be the set of all points where $\dot{V}(X,t)=0$, and let M be the maximum invariant set contained in E. Then every solution bounded for $t \geq 0$ approaches M as $t \to \infty$. If, in addition, $V(X,t) \to \infty$ as $||X|| \to \infty$, then each solution is bounded for $t \geq 0$, and all solutions approaches M as $t \to \infty$.

We say that $X(t)$ approaches a set M as t approaches infinity if for each $\varepsilon > 0$ there is a $T > 0$ with the property that, for each $t > T$ there is a state p in M with $||X(t)-p|| < \varepsilon$; that is, for all $t > T$ the states $X(t)$ are within a distance ε of M. If the maximum invariant set contained in E is the origin, $X = 0$, then we may state this theorem as follows:

Theorem

If there exists a scalar function $V(X,t)$, with continuous first partial derivatives, satisfying the following conditions:

(a) $V(X,t) > 0$ for all $X \neq 0$ and all t

$V(0,t) = 0$ for all t

(b) $\dot{V}(X,t) \leq 0$ for all $X \neq 0$ and all t

$\dot{V}(0,t) = 0$ for all t

(c) $\dot{V}(\Phi(t;X_0,t_0),t)$ does not vanish identically in $t \geq t_0$ for any t_0 and any $X_0 \neq 0$, where $\Phi(t;X_0,t_0)$ denotes the solution starting from X_0 at t_0 then the origin of the system is uniformly asymptotically stable in the large.

This may be seen as follows: If $\dot{V}(X,t)$ is not negative definite but only negative semidefinite, then the trajectory of the representative point can become tangent to some particular surface $V(X,t) = C$. Since $\dot{V}(\Phi(t;X_0,t_0),t)$ does not vanish identically in $t \geq t_0$ for any t_0 and any $X_0 \neq 0$, the representative point cannot remain at the tangent point and therefore must move toward the origin.

Appendix B: Krasovskii's Method:

Consider the nonlinear system

$$\dot{X} = f(X) \qquad \text{(B-1)}$$

Where X is nxl state vector and f(X) is a vector whose elements are nonlinear functions of X_1, X_2, ..., X_n the Jacobian matrix for the system is shown as F(X). In this nonlinear system there may be more than one equilibrium state. It is, however, possible to transfer the equilibrium state under consideration to the origin of the state space by an appropriate transformation of coordinates. We shall, therefore, consider the equilibrium state under consideration to be at the origin.

We shall now present Krasovskii's theorem.

Theorem:

Consider the system described by Equation B-1. Assume that f(0)=0 and that f(X) is differentiable with respect to X_i, i=1,2,..., n. Define

$$\overline{F}(X) = F^{*}(X) + F(X) \tag{B-2}$$

Where F(X) is the jacobian matrix and $F^{*}(X)$ is the conjugate transpose of F(X). If f(X) is real, then F(X) is real and $F^{*}(X)$ can be written as $F^{T}(X)$. $\overline{F}(X)$ is clearly Hermitian. If $\overline{F}(X)$ is negative definite, then the equilibrium state X=0 is asymptotically stable. A liapunov function for this system is:

$$V(X) = f^{*}(X)f(X) \tag{B-3}$$

If, in addition $f^{*}(X)f(X)$ goes to infinity as $||X||$ goes to infinity, then the equilibrium state is asymptotically stable in the large.

Theorem

If there exists a scalar function $V(X,t)$, with continuous first partial derivatives, satisfying the following conditions:

(a) $V(X,t) > 0$ for all $X \neq 0$ and all t

$V(0,t) = 0$ for all t

(b) $\dot{V}(X,t) < 0$ for all $X \neq 0$ and all t

$\dot{V}(0,t) = 0$ for all t

(c) $\dot{V}(\Phi(t;X_0,t_0),t)$ does not vanish identically in $t > t_0$ for any t_0 and any $X_0 \neq 0$, where $\Phi(t;X_0,t_0)$ denotes the solution starting from X_0 at t_0 then the origin of the system is uniformly asymptotically stable in the large.

This may be seen as follows: If $\dot{V}(X,t)$ is not negative definite but only negative semidefinite, then the trajectory of the representative point can become tangent to some particular surface $V(X,t) = C$. Since $\dot{V}(\Phi(t;X_0,t_0),t)$ does not vanish identically in $t > t_0$ for any t_0 and any $X_0 \neq 0$, the representative point cannot remain at the tangent point and therefore must move toward the origin.

Appendix B: Krasovskii's Method:

Consider the nonlinear system

$$\dot{X} = f(X) \tag{B-1}$$

Where X is nx1 state vector and f(X) is a vector whose elements are nonlinear functions of $X_1, X_2, \ldots, X_n$ the Jacobian matrix for the system is shown as F(X). In this nonlinear system there may be more than one equilibrium state. It is, however, possible to transfer the equilibrium state under consideration to the origin of the state space by an appropriate transformation of coordinates. We shall, therefore, consider the equilibrium state under consideration to be at the origin.

We shall now present Krasovskii's theorem.

Theorem:

Consider the system described by Equation B-1. Assume that f(0)=0 and that f(X) is differentiable with respect to X_i, i=1,2,..., n. Define

$$\overline{F}(X) = F^*(X) + F(X) \tag{B-2}$$

Where F(X) is the jacobian matrix and $F^*(X)$ is the conjugate transpose of F(X). If f(X) is real, then F(X) is real and $F^*(X)$ can be written as $F^T(X)$. $\overline{F}(X)$ is clearly Hermitian. If $\overline{F}(X)$ is negative definite, then the equilibrium state X=0 is asymptotically stable. A liapunov function for this system is:

$$V(X)=f^*(X)f(X) \tag{B-3}$$

If, in addition $f^*(X)f(X)$ goes to infinity as $||X||$ goes to infinity, then the equilibrium state is asymptotically stable in the large.

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Table 1: Maximum Displacement, Velocity, Acceleration, Spring Force, and Active Control Force.

Case No.	X_{max}	V_{max}	Acc_{max}	$F_{S_{max}}$	U_{max}
1	3.36	12.11	127.86	5.00	0.00
2	2.58	12.96	133.44	5.00	2.58
3	2.49	13.06	131.98	5.00	3.72
4	2.60	13.30	138.39	5.00	3.75
5	2.54	13.06	131.98	5.00	2.50

Table 2: Maximum Displacement Response of the System with Different values of Time Constant Due to Twenty Artificial Earthquakes.

----	.0000	.0068	.0178	.0399	.0439	.0441	.0517	.0680	.0794	.0811	.0876	.0906	.0985
1.04	.64	.66	.71	.84	.87	.87	.91	1.01	1.09	1.11	1.17	1.21	1.31
1.16	.82	.81	.87	.98	1.01	1.01	1.05	1.17	1.20	1.21	1.27	1.29	1.35
1.22	.91	.93	.98	1.15	1.17	1.17	1.20	1.22	1.25	1.27	1.31	1.32	1.36
1.28	.96	.99	1.05	1.16	1.18	1.18	1.24	1.37	1.49	1.51	1.55	1.57	1.62
1.32	.97	1.01	1.06	1.17	1.20	1.20	1.24	1.43	1.51	1.51	1.59	1.64	1.68
1.33	.99	1.01	1.07	1.21	1.23	1.23	1.26	1.45	1.51	1.52	1.60	1.64	1.69
1.38	1.06	1.08	1.11	1.22	1.25	1.25	1.27	1.46	1.57	1.59	1.64	1.66	1.72
1.44	1.08	1.12	1.19	1.26	1.28	1.28	1.28	1.46	1.59	1.61	1.65	1.67	1.79
1.46	1.13	1.14	1.20	1.29	1.29	1.29	1.30	1.54	1.60	1.62	1.68	1.69	1.79
1.51	1.15	1.15	1.26	1.40	1.41	1.41	1.30	1.54	1.66	1.67	1.70	1.74	1.82
1.52	1.19	1.20	1.28	1.43	1.46	1.46	1.43	1.55	1.73	1.76	1.81	1.82	1.86
1.53	1.22	1.23	1.30	1.44	1.47	1.47	1.48	1.61	1.73	1.76	1.83	1.84	1.89
1.62	1.24	1.23	1.32	1.45	1.47	1.47	1.56	1.70	1.78	1.79	1.84	1.85	1.90
1.62	1.27	1.27	1.39	1.47	1.51	1.51	1.58	1.71	1.78	1.79	1.87	1.89	1.91
1.81	1.31	1.32	1.40	1.50	1.53	1.53	1.58	1.72	1.79	1.80	1.87	1.91	2.02
1.85	1.31	1.34	1.40	1.56	1.58	1.58	1.63	1.73	1.83	1.84	1.87	1.92	2.03
1.87	1.34	1.34	1.40	1.57	1.60	1.61	1.67	1.83	1.95	1.97	2.00	2.02	2.06
2.03	1.34	1.37	1.43	1.60	1.65	1.65	1.76	1.92	1.97	1.98	2.05	2.07	2.14
2.08	1.45	1.47	1.54	1.77	1.80	1.80	1.85	2.04	2.26	2.30	2.43	2.49	2.67
2.62	1.47	1.52	1.60	1.80	1.86	1.86	1.99	2.29	2.52	2.56	2.69	2.74	2.89

Table 3: Parameters of the probability Distribution Function.

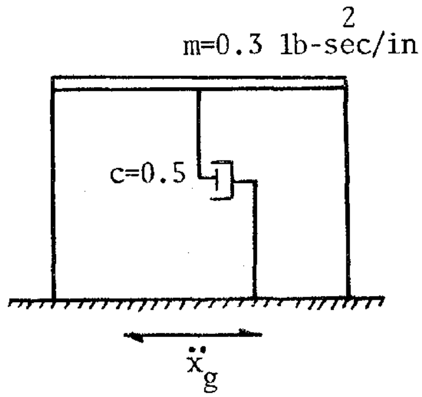
Case No.	Time constant	a	b
1	-----	-2.88	4.04
2	0.0000	-4.80	4.96
3	0.0068	-4.80	5.04
4	0.0178	-4.57	5.09
5	0.0399	-4.29	5.33
6	0.0439	-4.23	5.35
7	0.0441	-4.23	5.36
8	0.0517	-3.93	5.10
9	0.0680	-3.58	5.16
10	0.0794	-3.17	4.84
11	0.0811	-3.12	4.80
12	0.0876	-2.96	4.72
13	0.0906	-2.90	4.70
14	0.0985	-2.71	4.56
15	-----	-3.18	4.38
16	-----	-3.20	4.52

Table 4: Parameters of the Probability Distribution Function.

Case No.	Time Constant	a	b
1	------	-2.37	2.92
2	0.0000	-3.81	3.51
3	0.0068	-3.82	3.59
4	0.0178	-3.63	3.62
5	0.0399	-3.44	3.85
6	0.0439	-3.39	3.89
7	0.0441	-3.39	3.89
8	0.0517	-3.18	3.72
9	0.0680	-2.90	3.78
10	0.0794	-2.58	3.53
11	0.0811	-2.54	3.50
12	0.0876	-2.42	3.45
13	0.0906	-2.38	3.44
14	0.0985	-2.23	3.34

Table 5: Maximum Displacement of the system Due to the El-Centro Earthquake.

Case No.	Time Constant	Maximum Displacement
1	0.0000	1.79
2	0.0068	2.01
3	0.0178	2.38
4	0.0399	3.12
5	0.0439	3.25
6	0.0441	3.26
7	0.0517	3.51
8	0.0680	4.02
9	0.0794	4.35
10	0.0811	4.40
11	0.0876	4.59
12	0.0906	4.67
13	0.0985	4.89



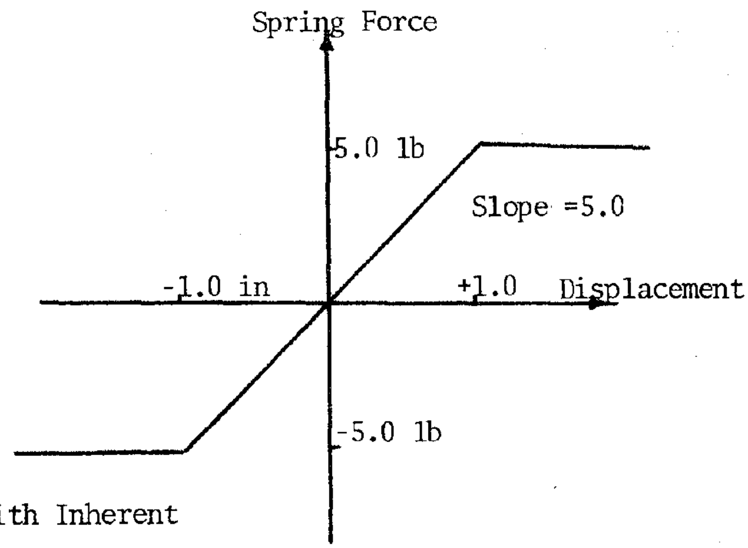


Fig. 1 : A Single Degree-of-Freedom System with Inherent Nonlinearity.

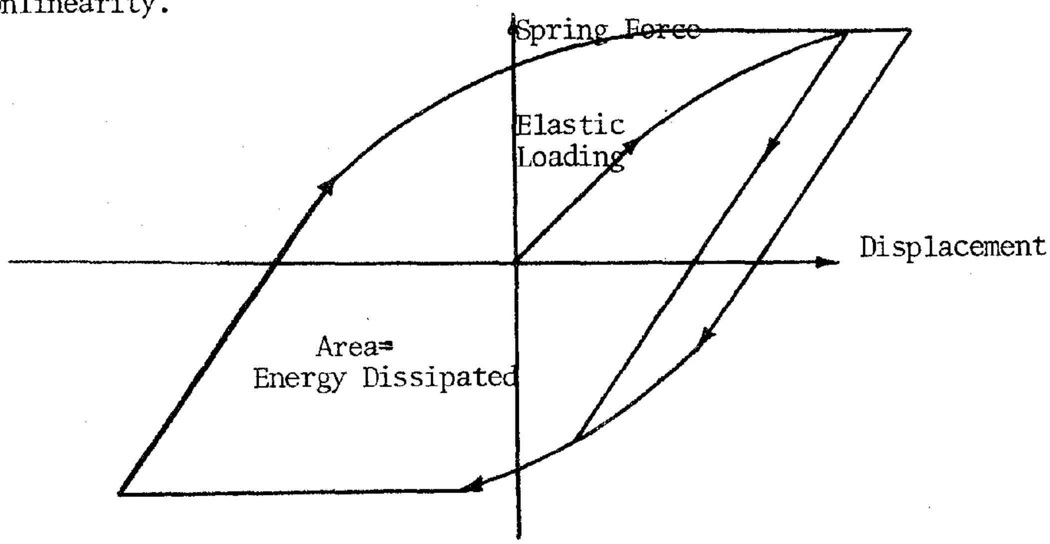


Fig. 2 :General Plastic Behavior of the Structural Model.

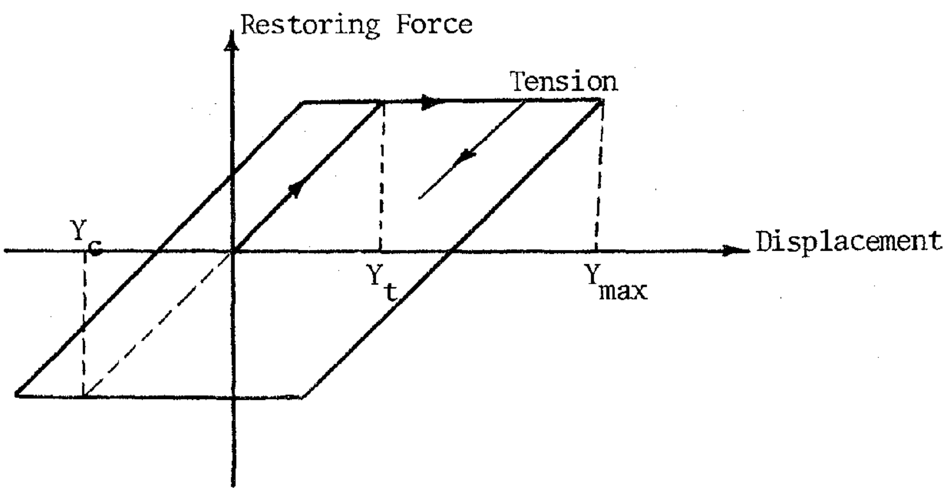


Fig. 3 : Elastoplastic Behavior of the Structural Model.

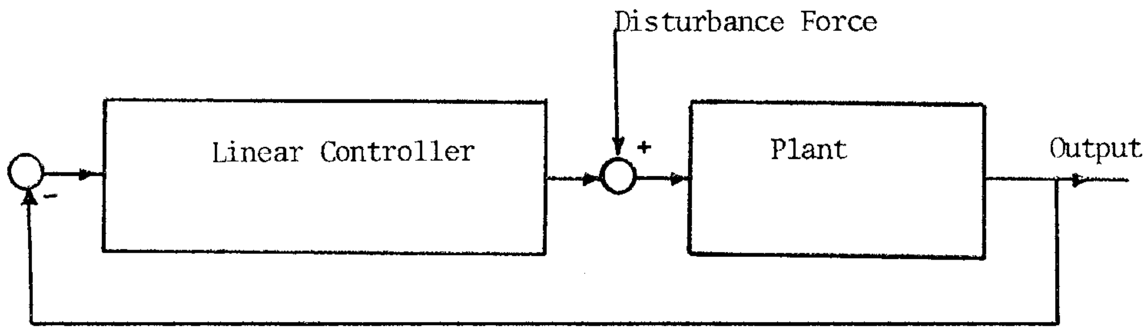


Fig. 4: Block Diagram of the Closed-loop System with Linear Controller.

$U=-(K_{11} \quad K_{12})\ X$

$K_{11}=1.0$

$K_{12}=0.$

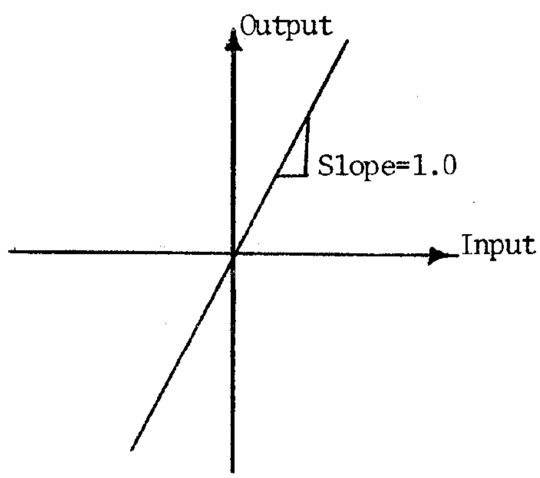


Fig. 5: Properties of the Linear Control Law.

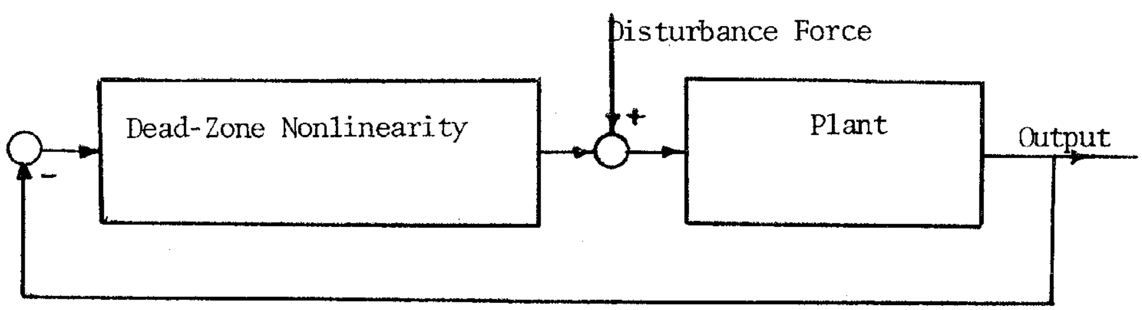


Fig. 6: Block Diagram of the Closed-Loop System with Dead-Zone Nonlinearity.

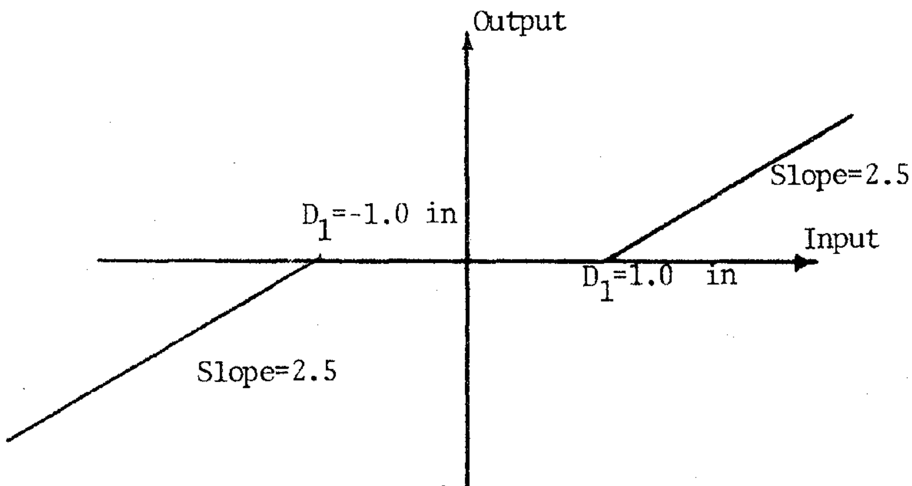


Fig. 7: Properties of the Dead-Zone Nonlinearity

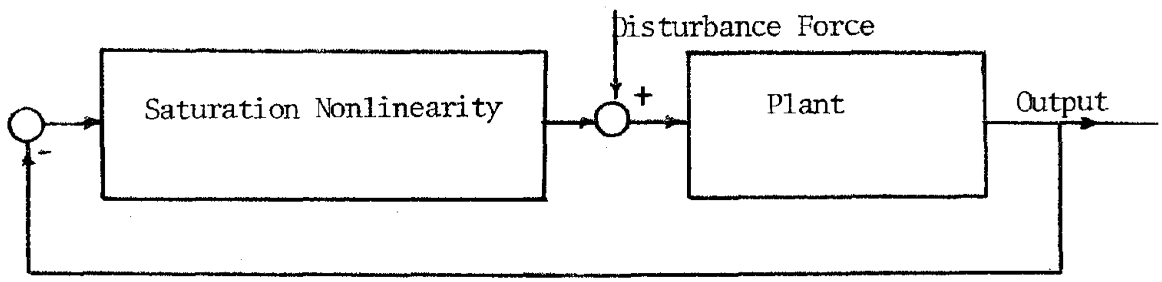


Fig. 8: Block Diagram of the Closed-Loop System with Saturation Nonlinearity.

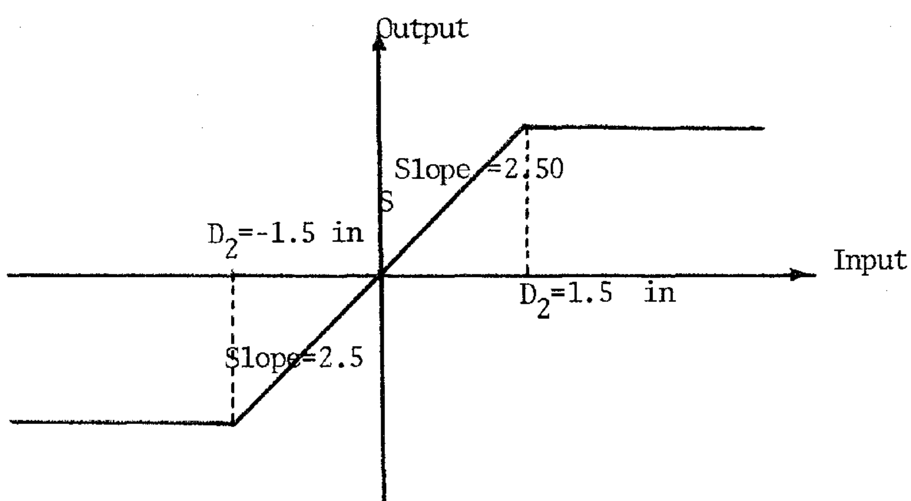


Fig. 9: Properties of the Saturation Nonlinearity.

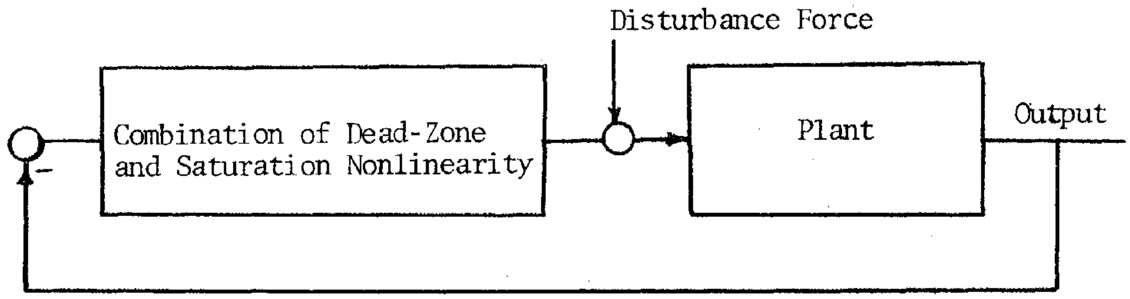


Fig. 10: Block Diagram of the Closed-Loop System with Combination of Dead-Zone and Saturation Nonlinearity.

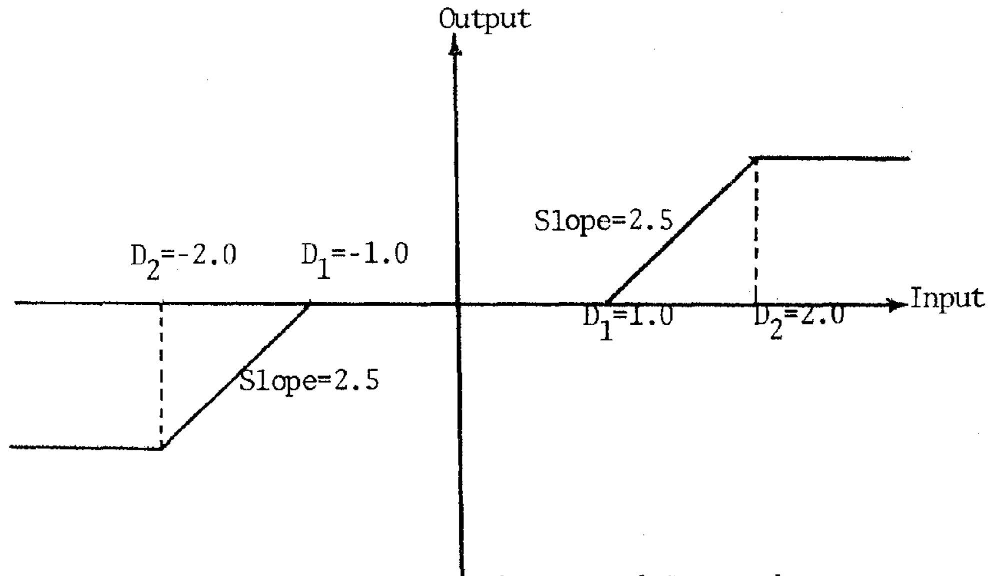


Fig. 11: Properties of the Combination of Dead-zone and Saturation Nonlinearity.

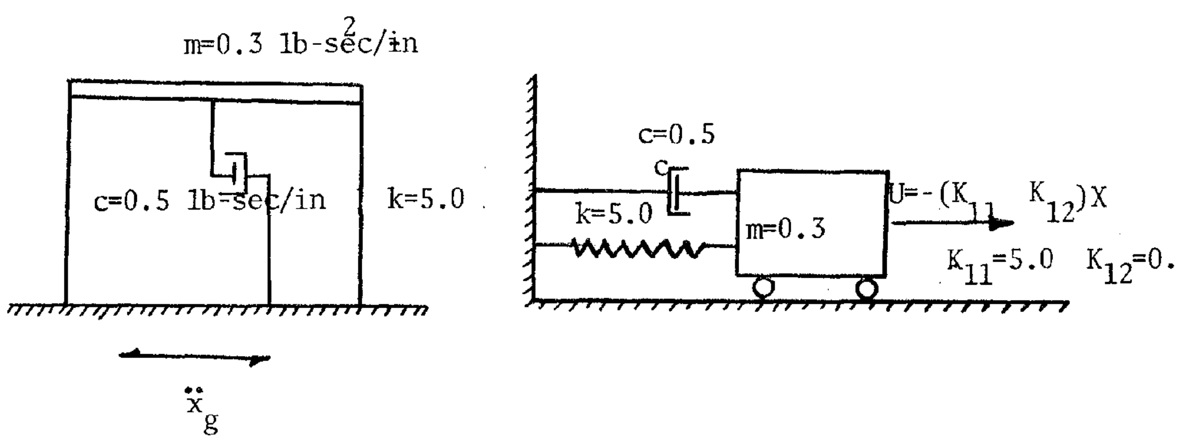


Fig. 12: A Linear Time Invariant Single-Degree-of-Freedom System.

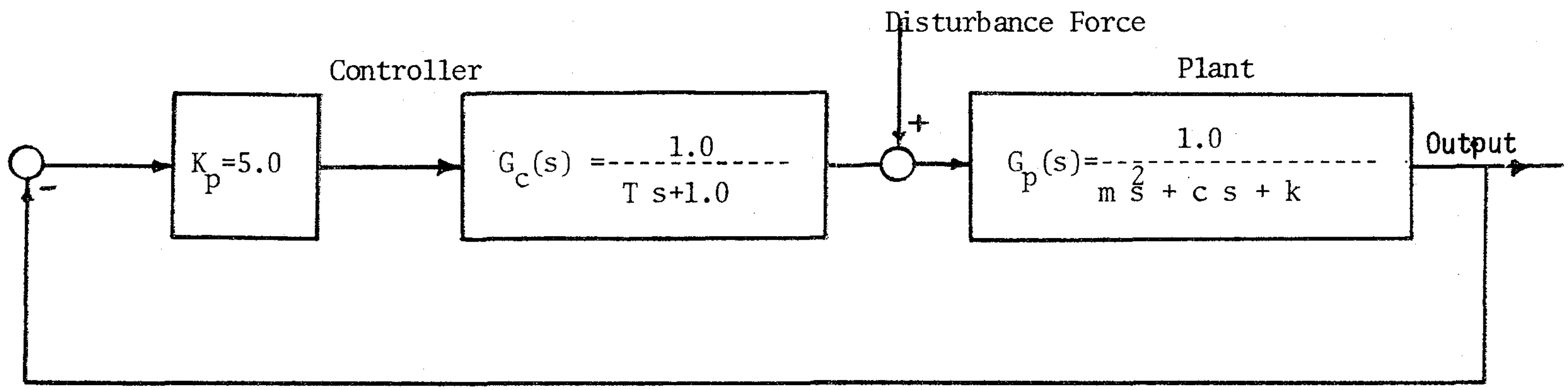


Fig. 13: Block Diagram of the Closed-Loop System.

$m = 0.3$ lb-sec^2/in

$c = 0.5$ lb-sec/in
$k = 5.0$ lb/in
T: Time Constant which it is assumed as a random variable.

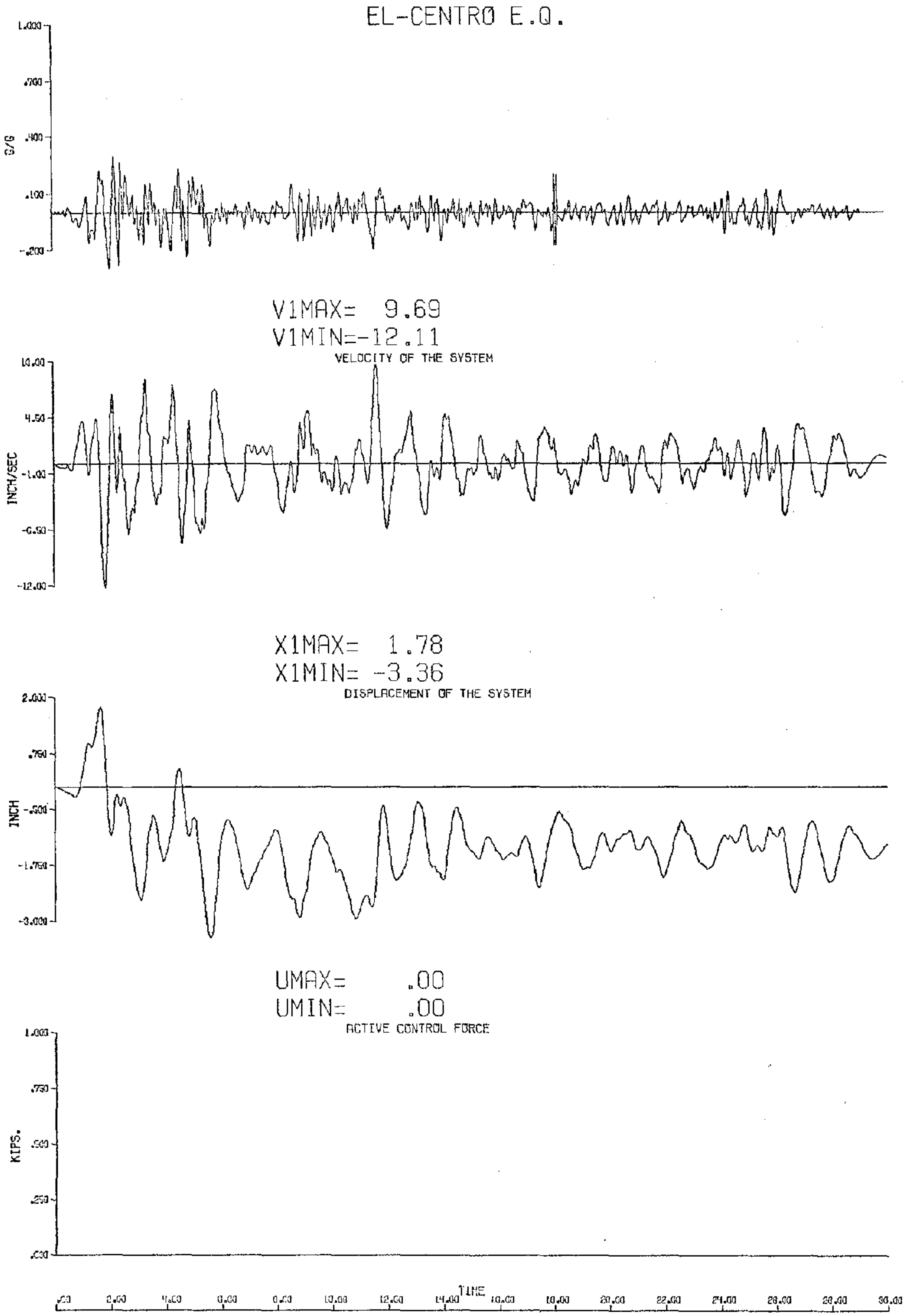
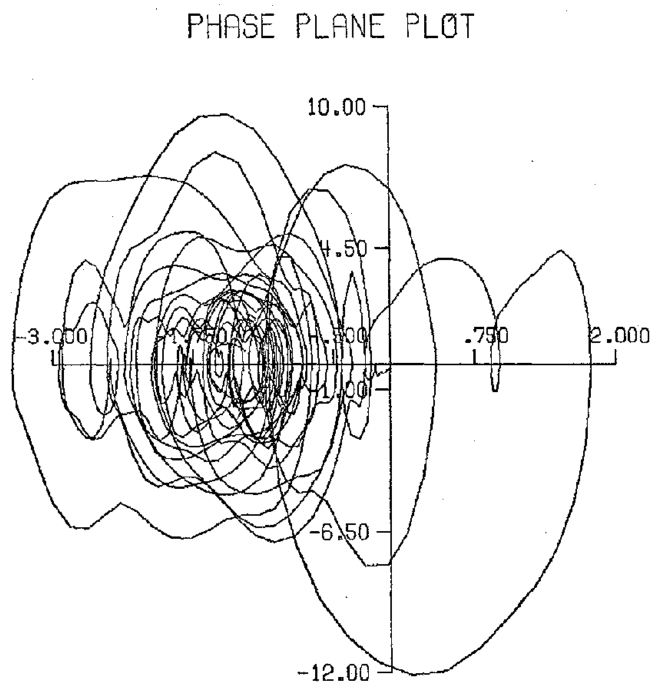


Fig. 14;Case 1 in Table 1.



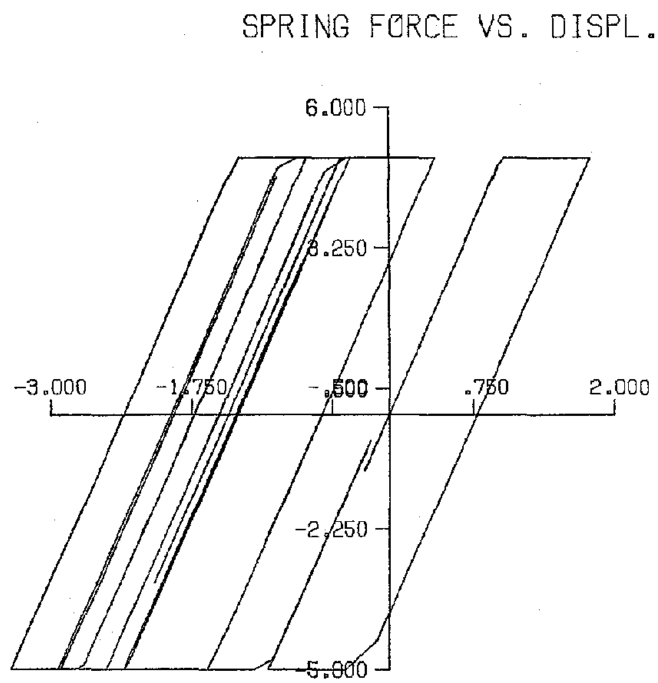


Fig. 14(con.): Case 1 in Table 1.

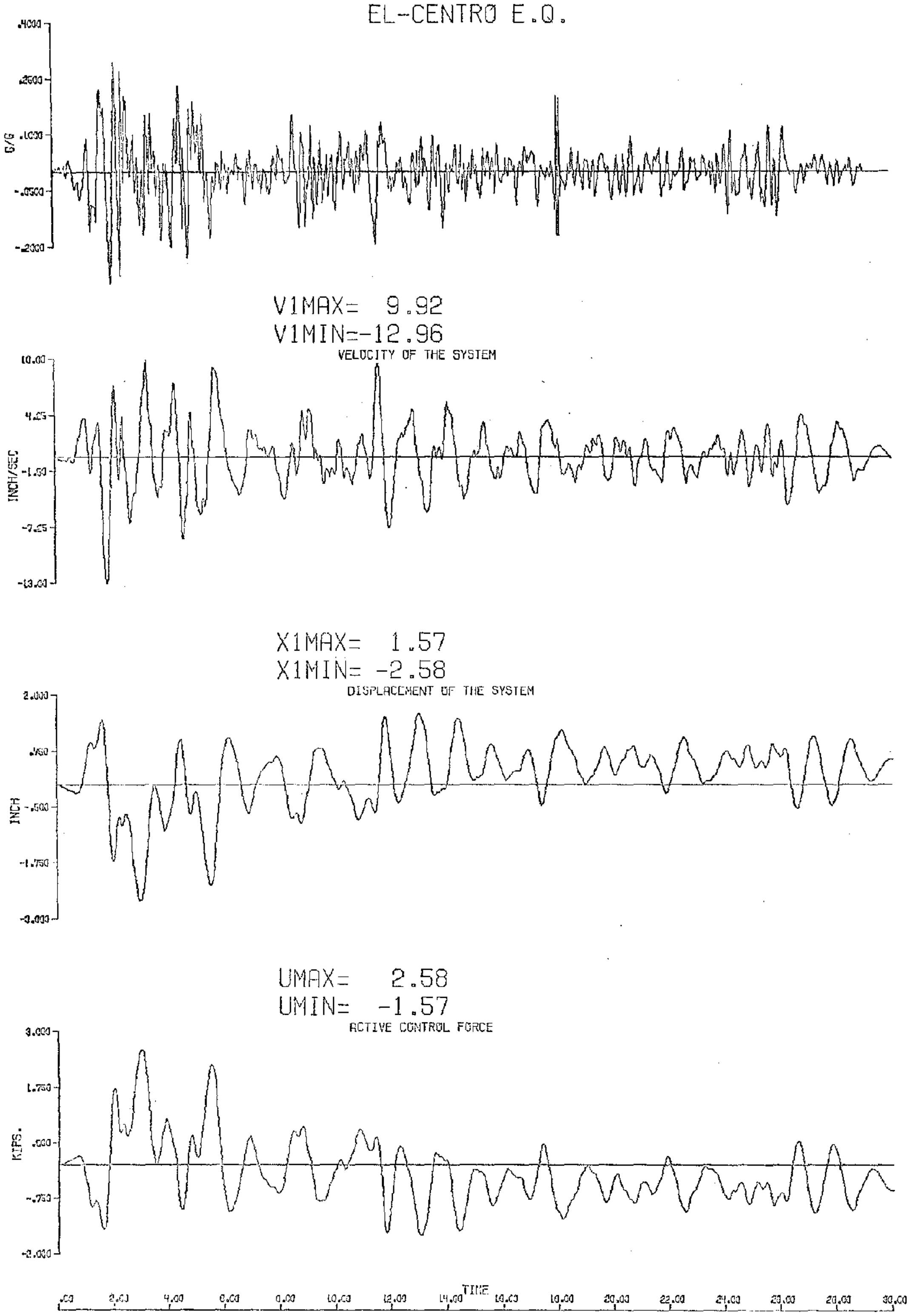
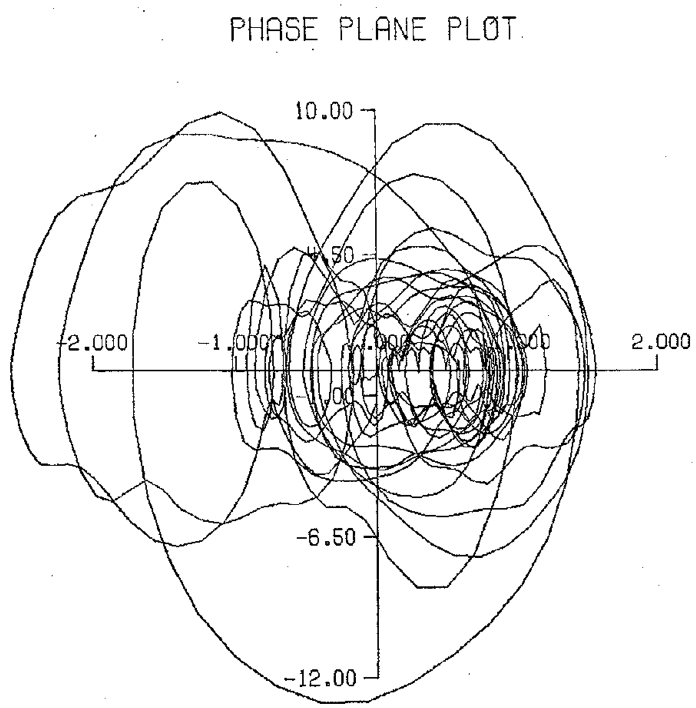


Fig. 15: Case 2 in Table 1.



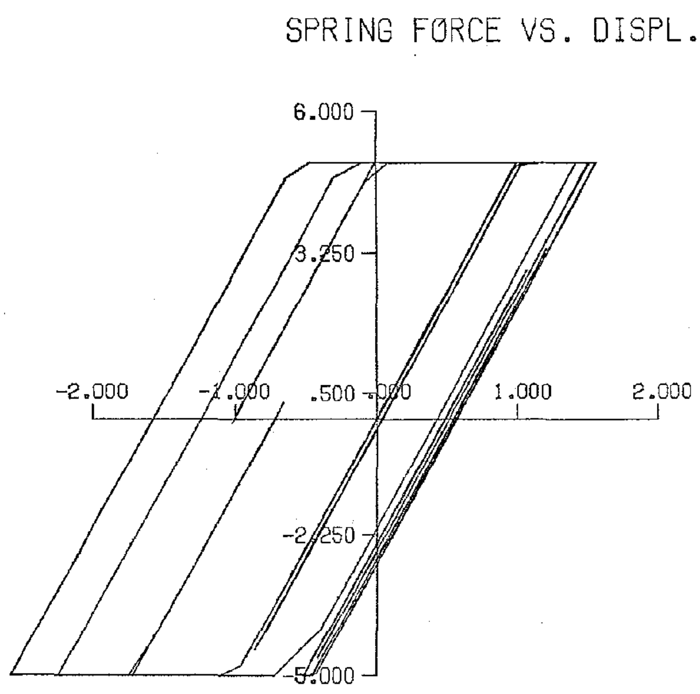


Fig. 15(con.): Case 2 in Table 1.

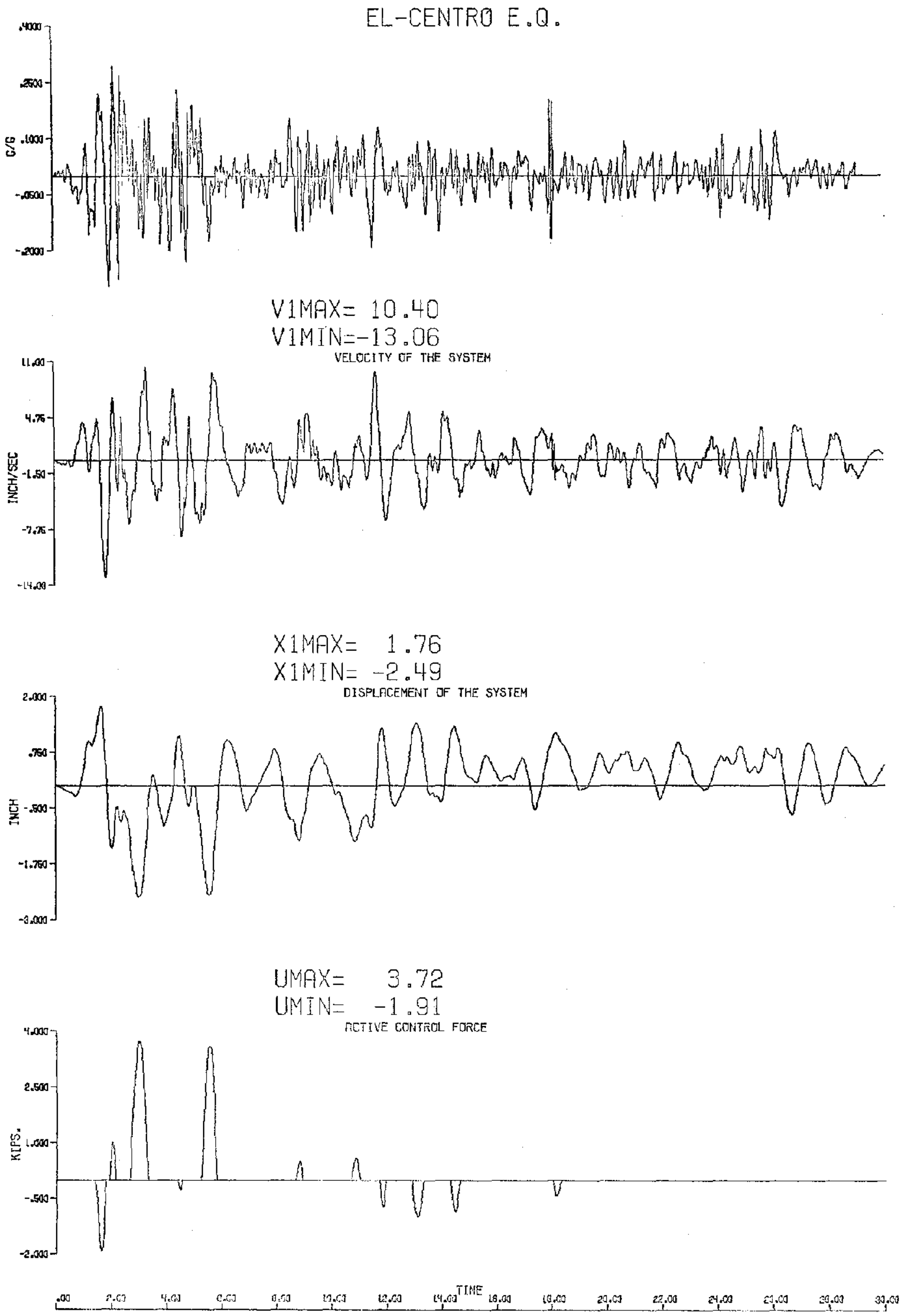
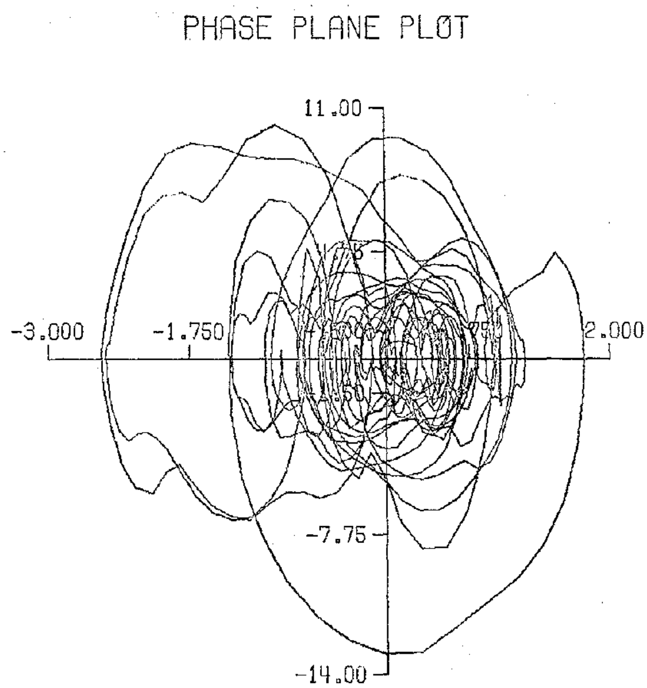


Fig. 16: Case 3 in Table 1.



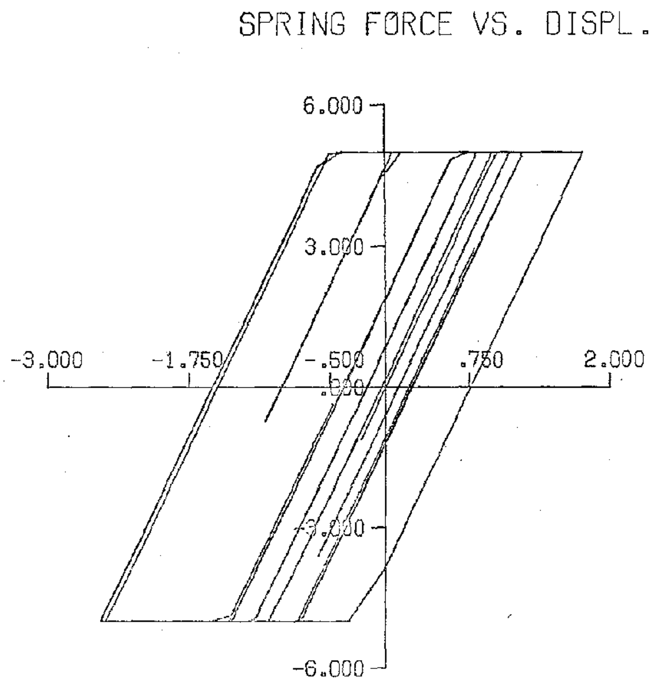


Fig. 16(con.): Case 3 in Table 1.

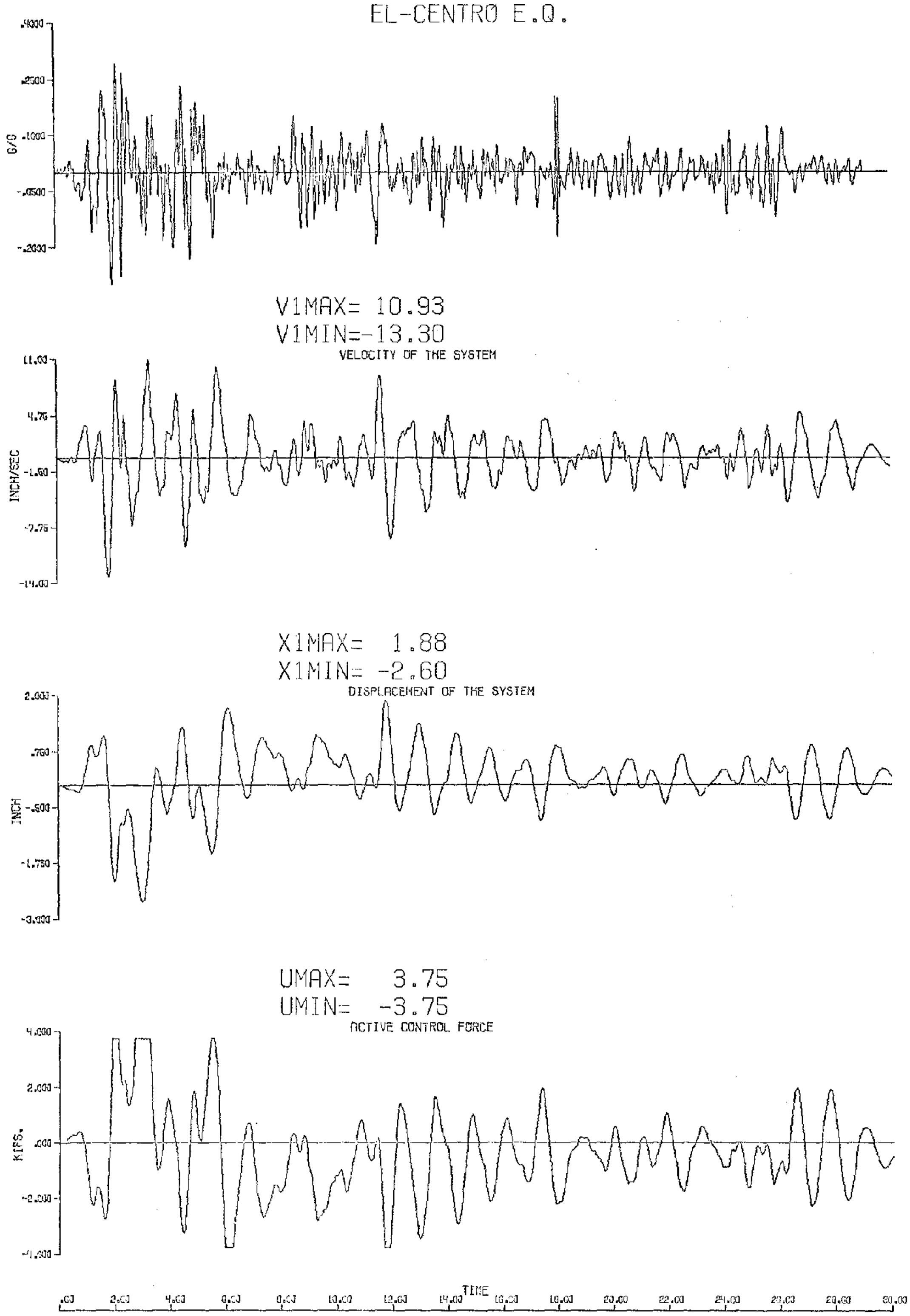
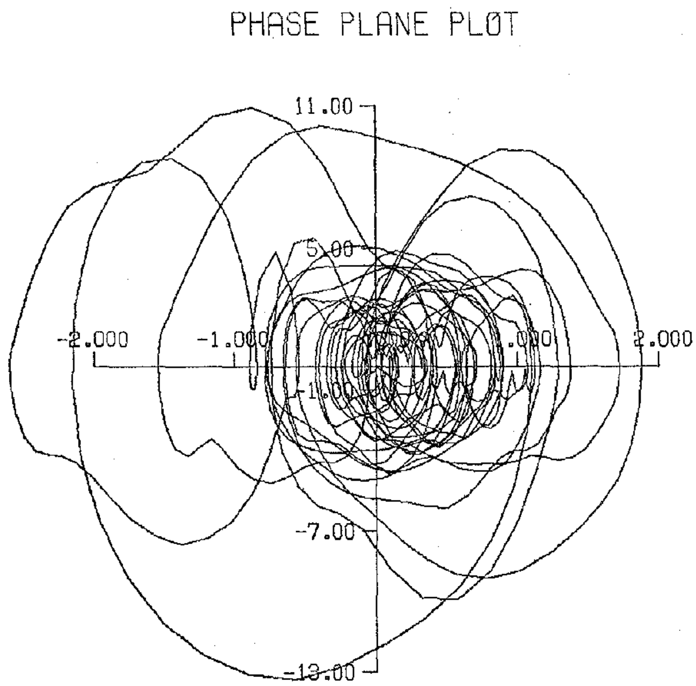


Fig. 17: Case 4 in Table 1.



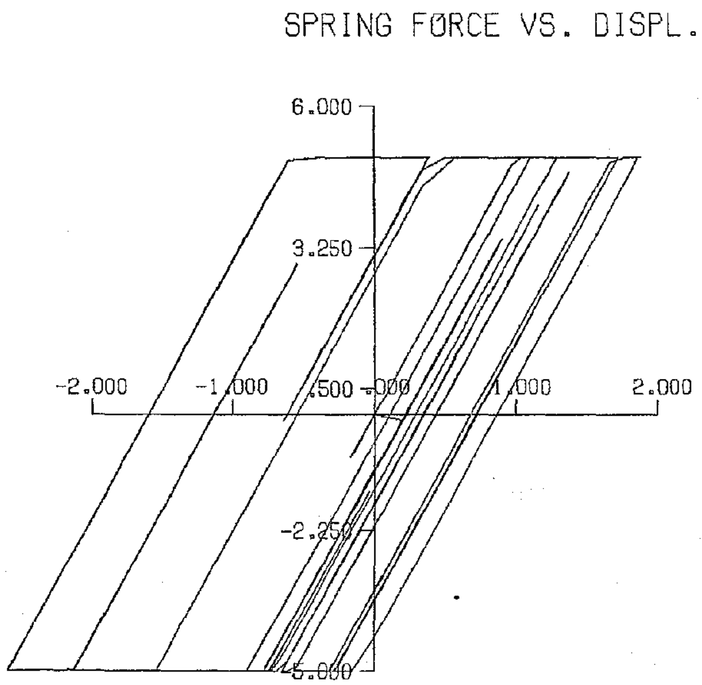


Fig. 17(con.): Case 4 in Table 1.

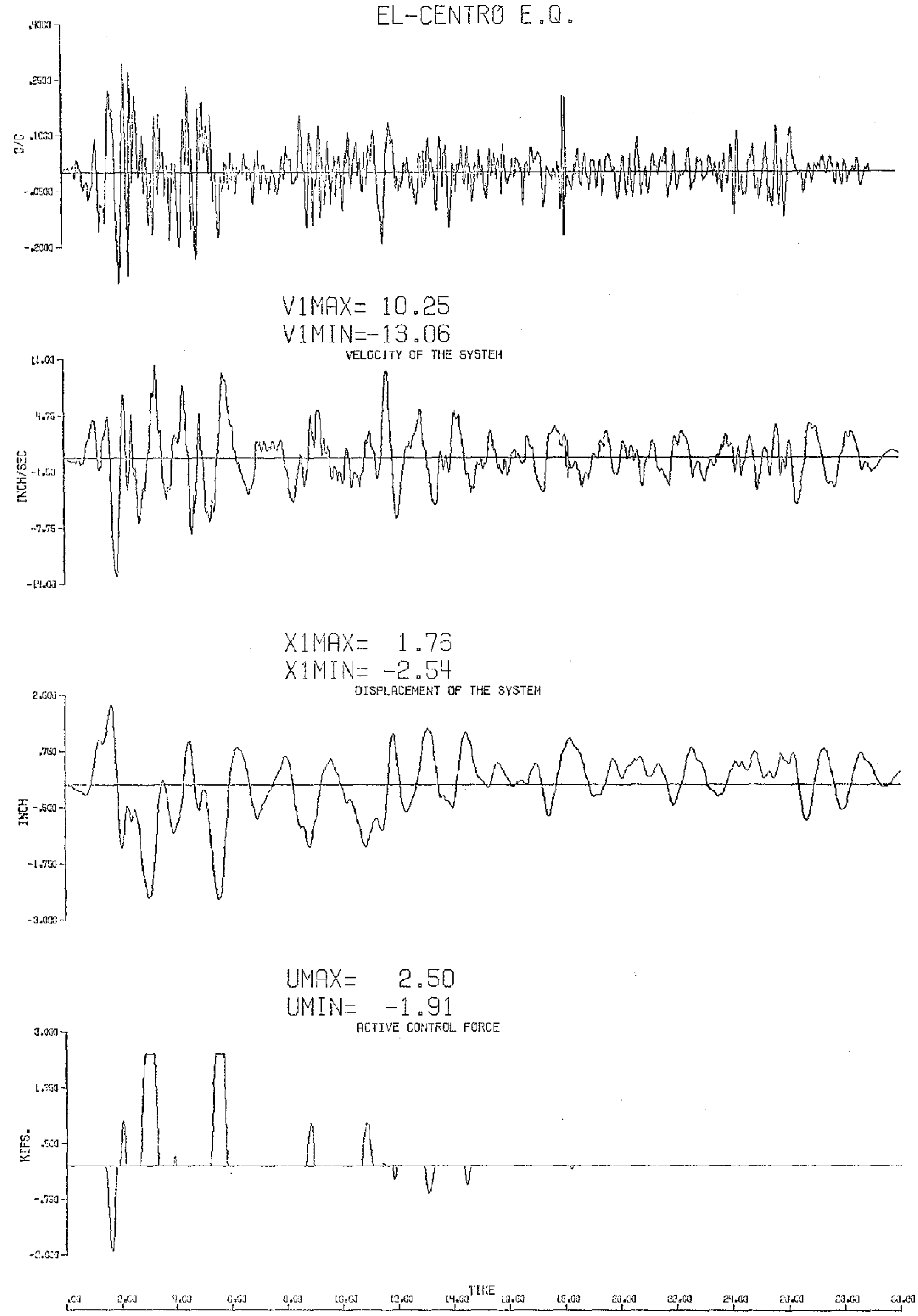
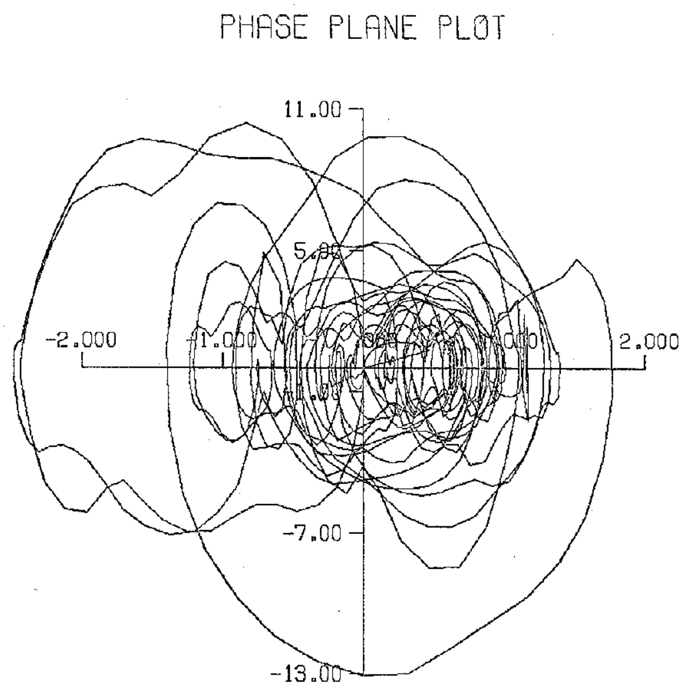


Fig. 18: Case 5 in Table 1.



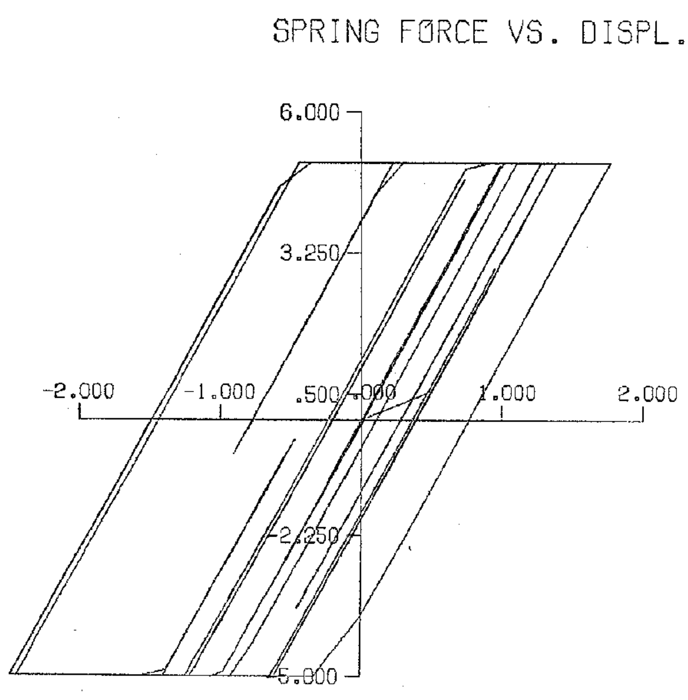


Fig. 18(con.): Case 5 in Table 1.

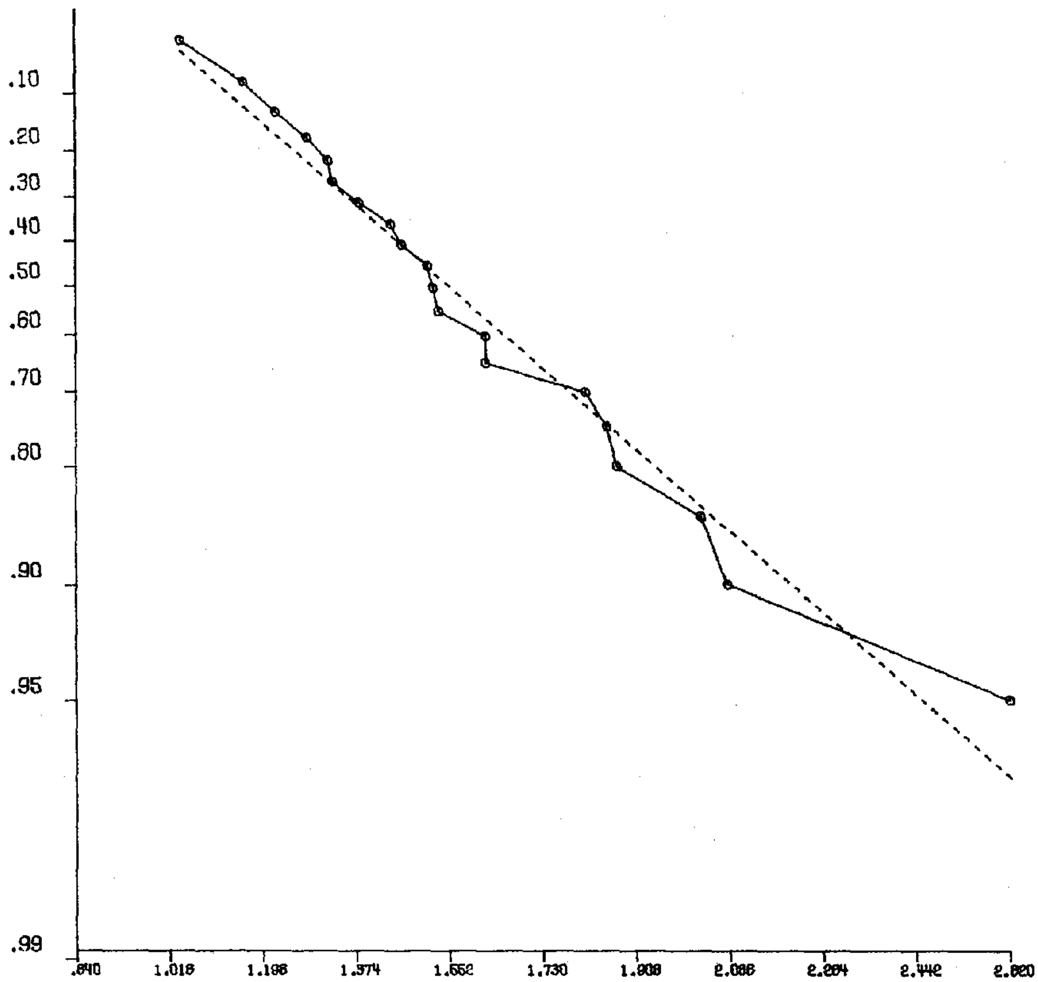


Fig. 19: Empirical Distribution of the Maximum Displacement of the Uncontrolled Response.

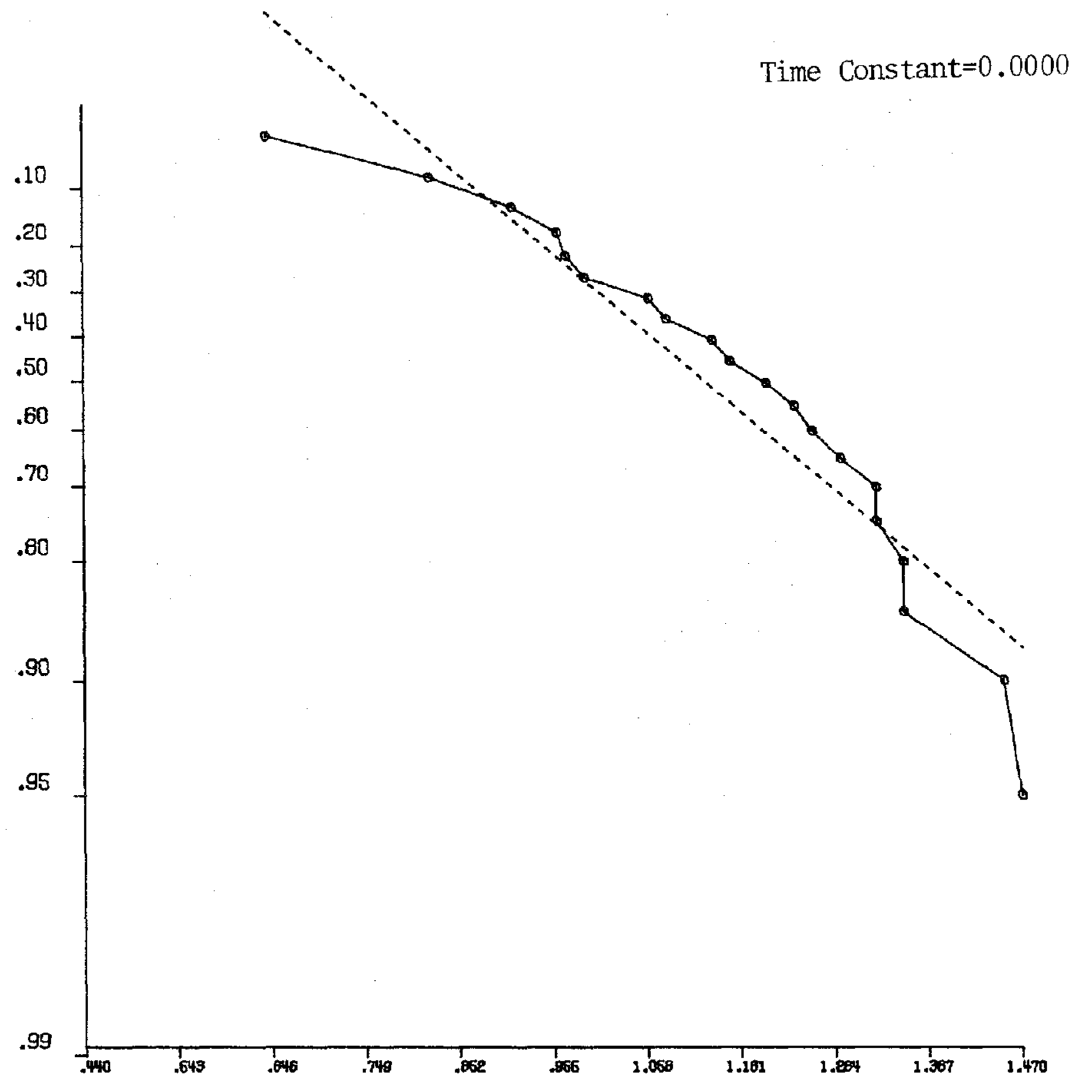


Fig. 20: Empirical Distribution of the Maximum Displacement of the Controlled Response.

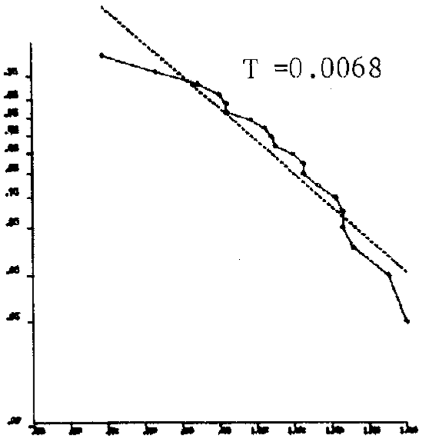


Fig. 21: Case 3 in Table 3.

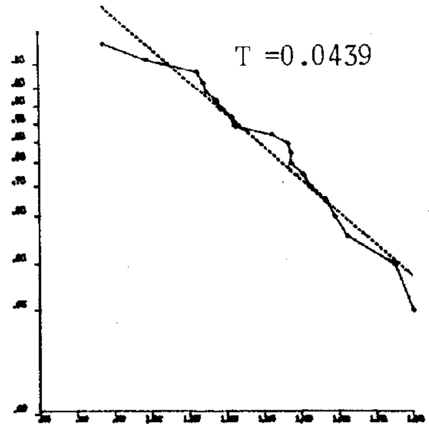


Fig. 24: Case 6 in Table 3.

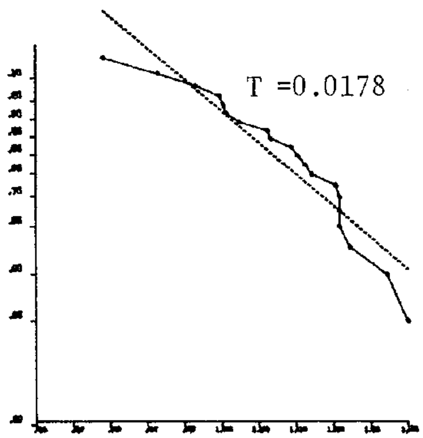


Fig. 22: Case 4 in Table 3.

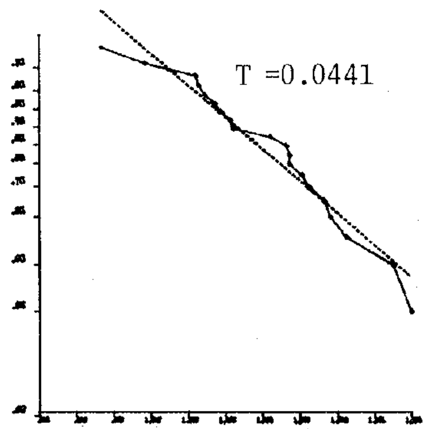


Fig. 25: Case 7 in Table 3.

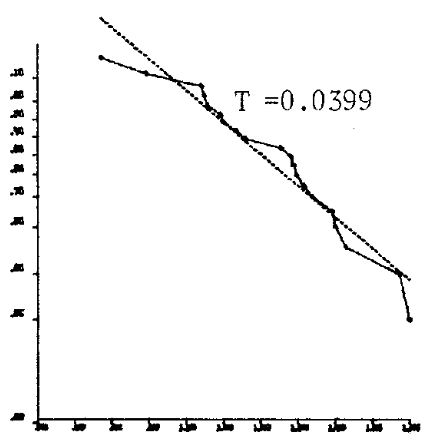


Fig. 23: Case 5 in Table 3.

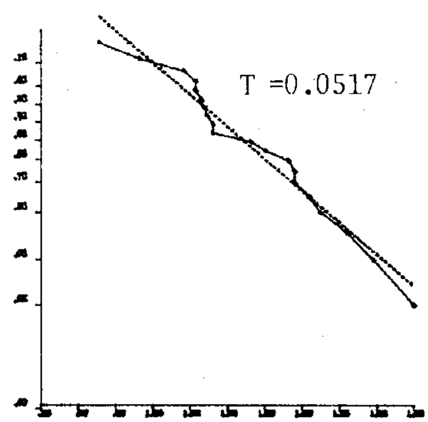


Fig. 26: Case 8 in Table 3.

Fig. 21 Through Fig. 26; Empirical Distribution of the Maximum Displacement of the Controlled Response.

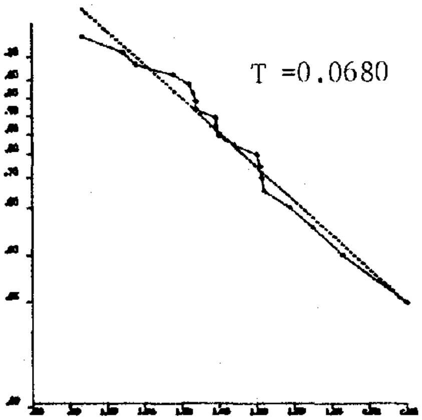


Fig. 27: Case 9 in Table 3.

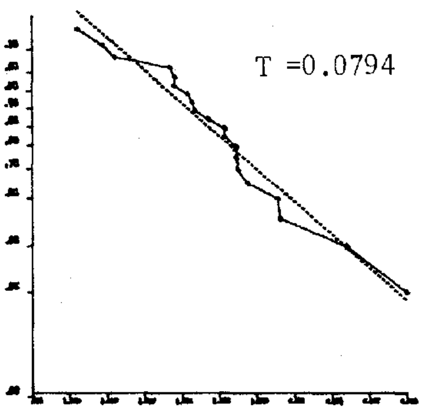


Fig. 28: Case 10 in Table 3.

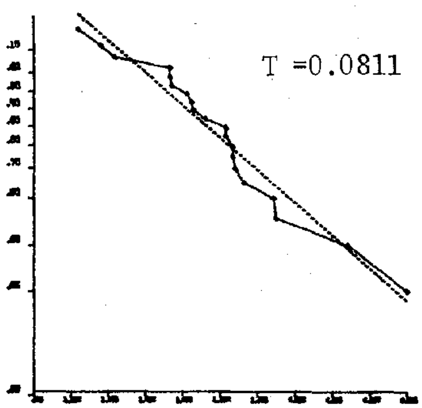


Fig. 29: Case 11 in Table 3.

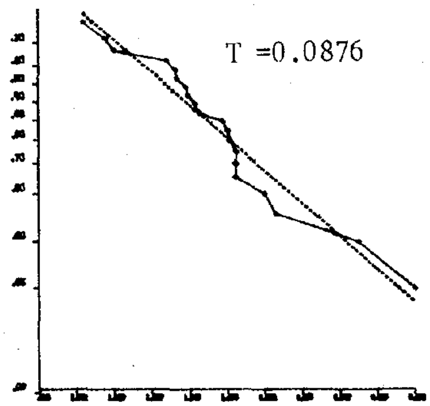


Fig. 30: Case 12 in Table 3.

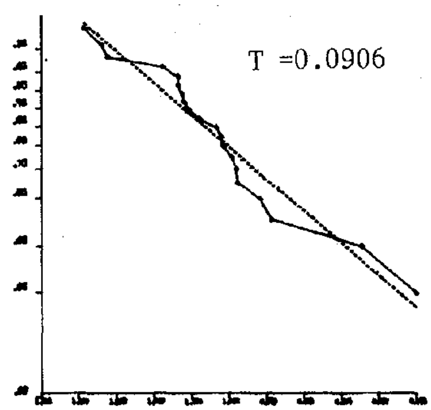


Fig. 31: Case 13 in Table 3.

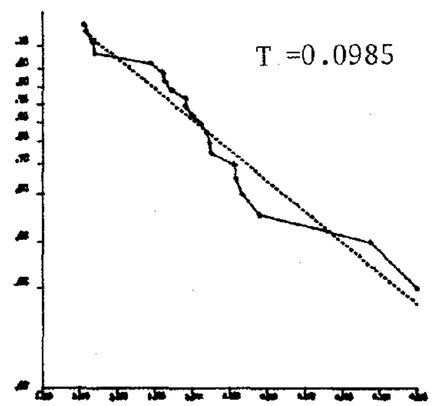


Fig. 32: Case 14 in Table 3.

Fig. 27 Through Fig. 32: Empirical Distribution of the Maximum Displacement of the Controlled Response.

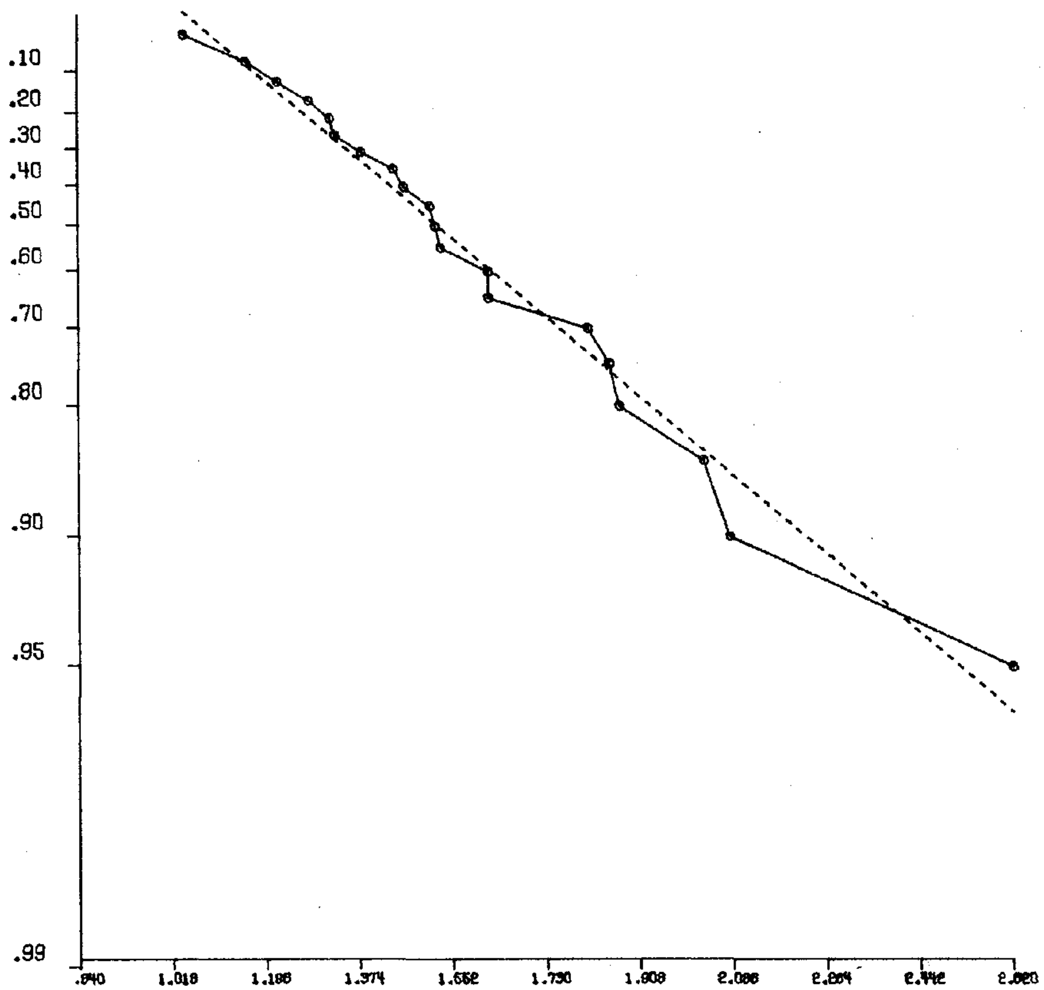


Fig. 33: Empirical Distribution of the Maximum Displacement of the Uncontrolled Response.

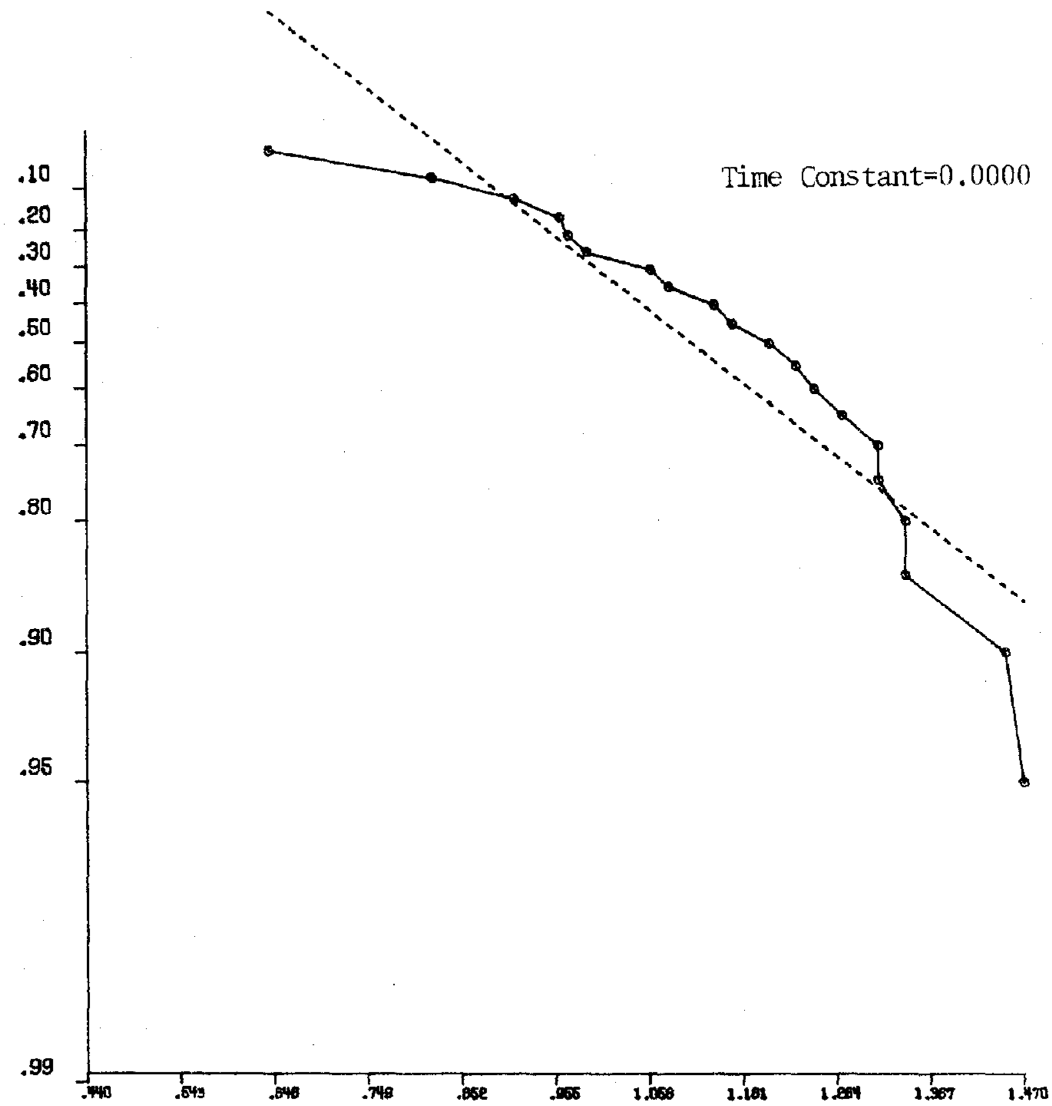


Fig. 34: Empirical Distribution of the Maximum Displacement of the Controlled Response.

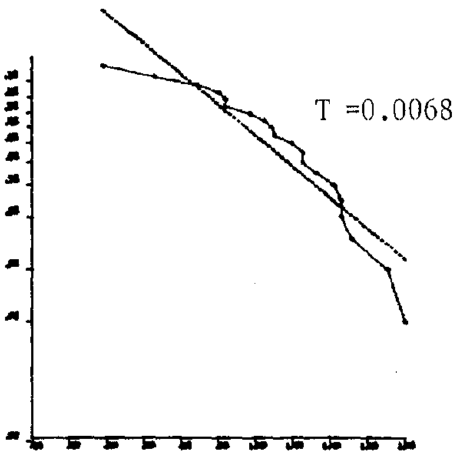


Fig. 35: Case 3 in Table 4.

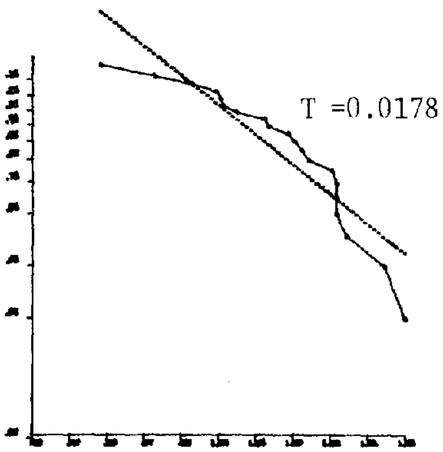


Fig. 36: Case 4 in Table 4.

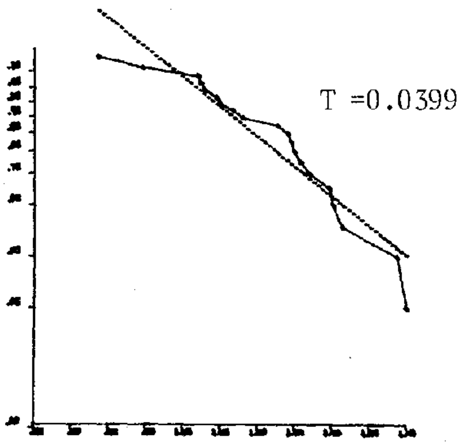


Fig. 37: Case 5 in Table 4.

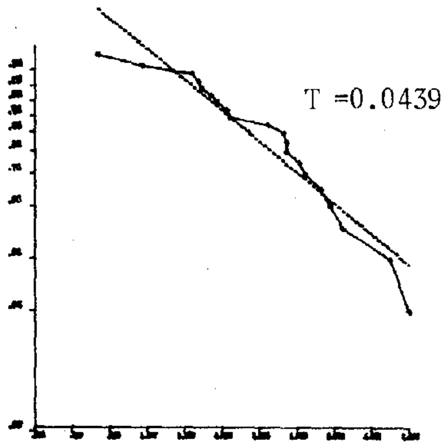


Fig. 38: Case 6 in Table 4.

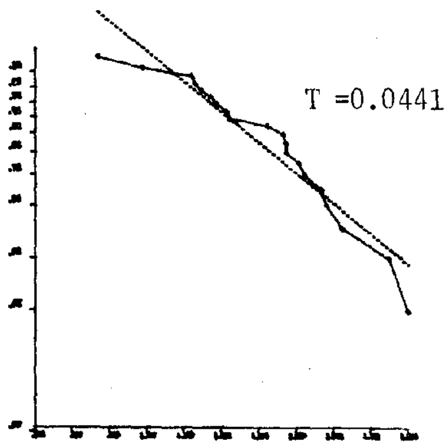


Fig. 39; Case 7 in Table 4.

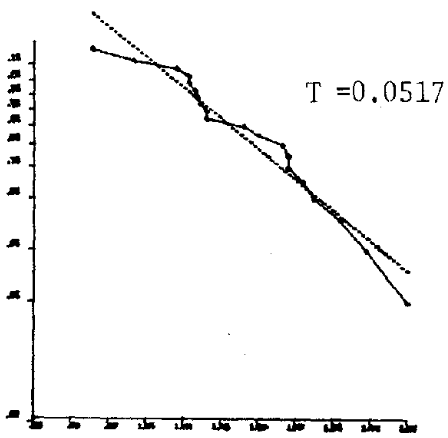


Fig. 40: Case 8 in Table 4.

Fig. 35 Through Fig. 40: Empirical Distribution of the Maximum Displacement of the Controlled Response.

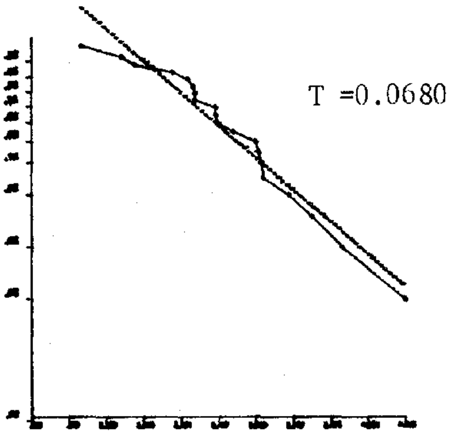


Fig. 41: Case 9 in Table 4.

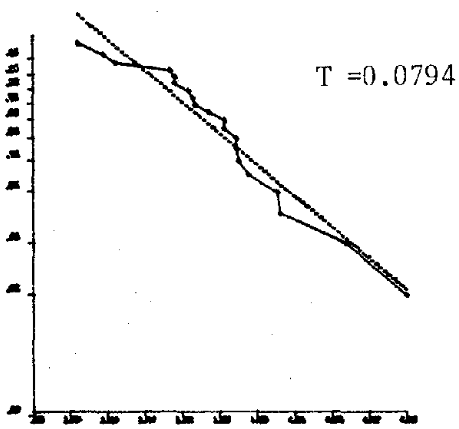


Fig. 42: Case 10 in Table 4.

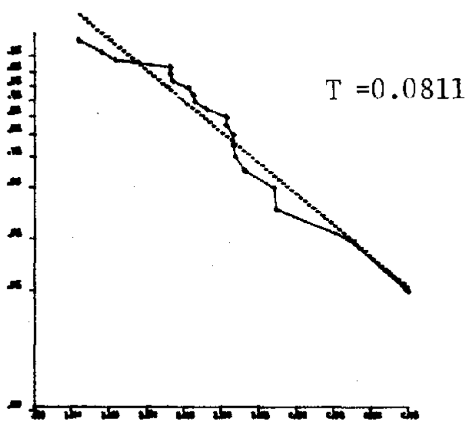


Fig. 43: Case 11 in Table 4.

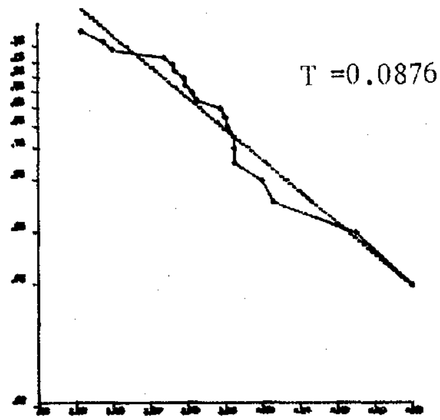


Fig. 44: Case 12 in Table 4.

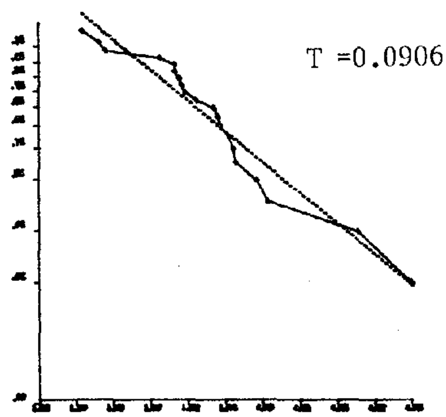


Fig. 45: Case 13 in Table 4.

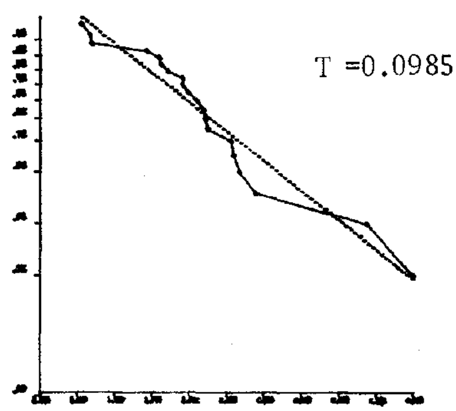


Fig. 46: Case 14 in Table 4.

Fig. 41 Through Fig. 46: Empirical Distribution of the Maximum Displacement of the Controlled Response.

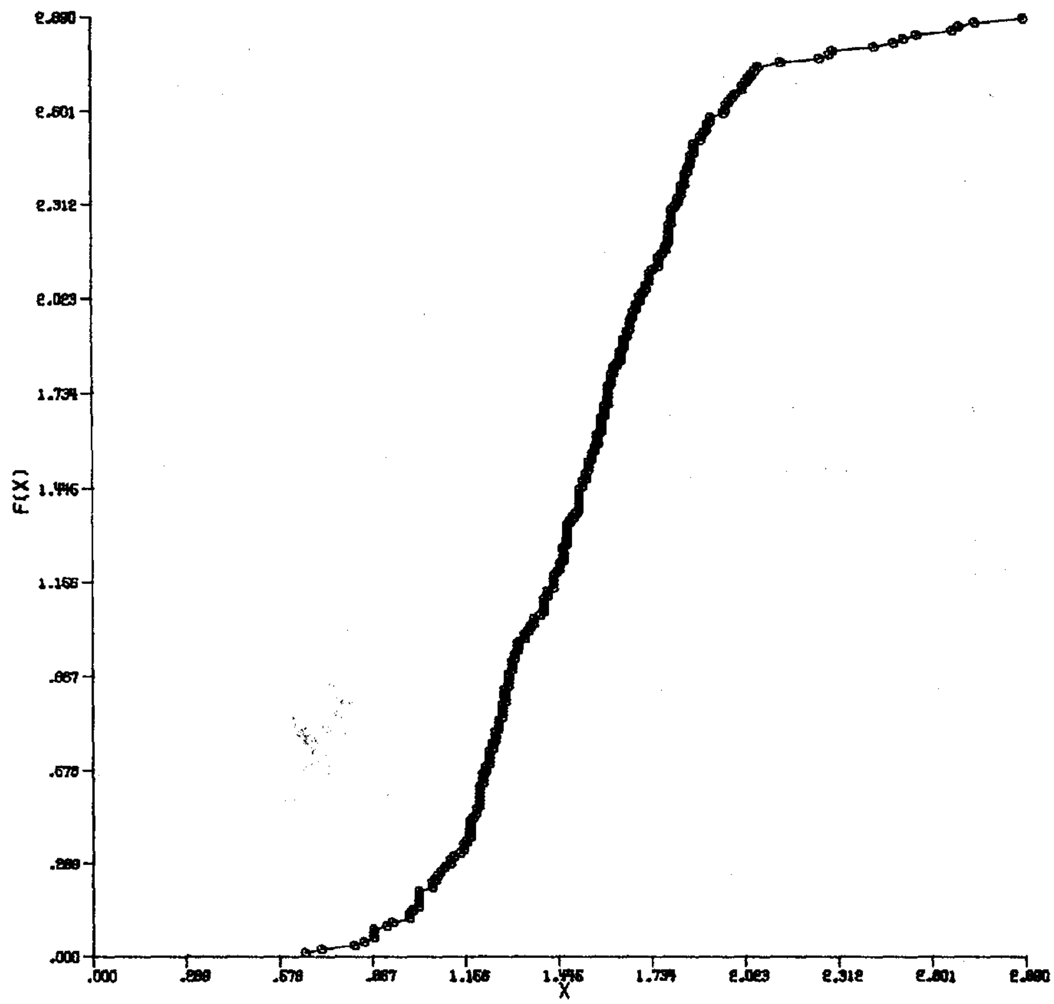


Fig. 47: Empirical Distribution Function.

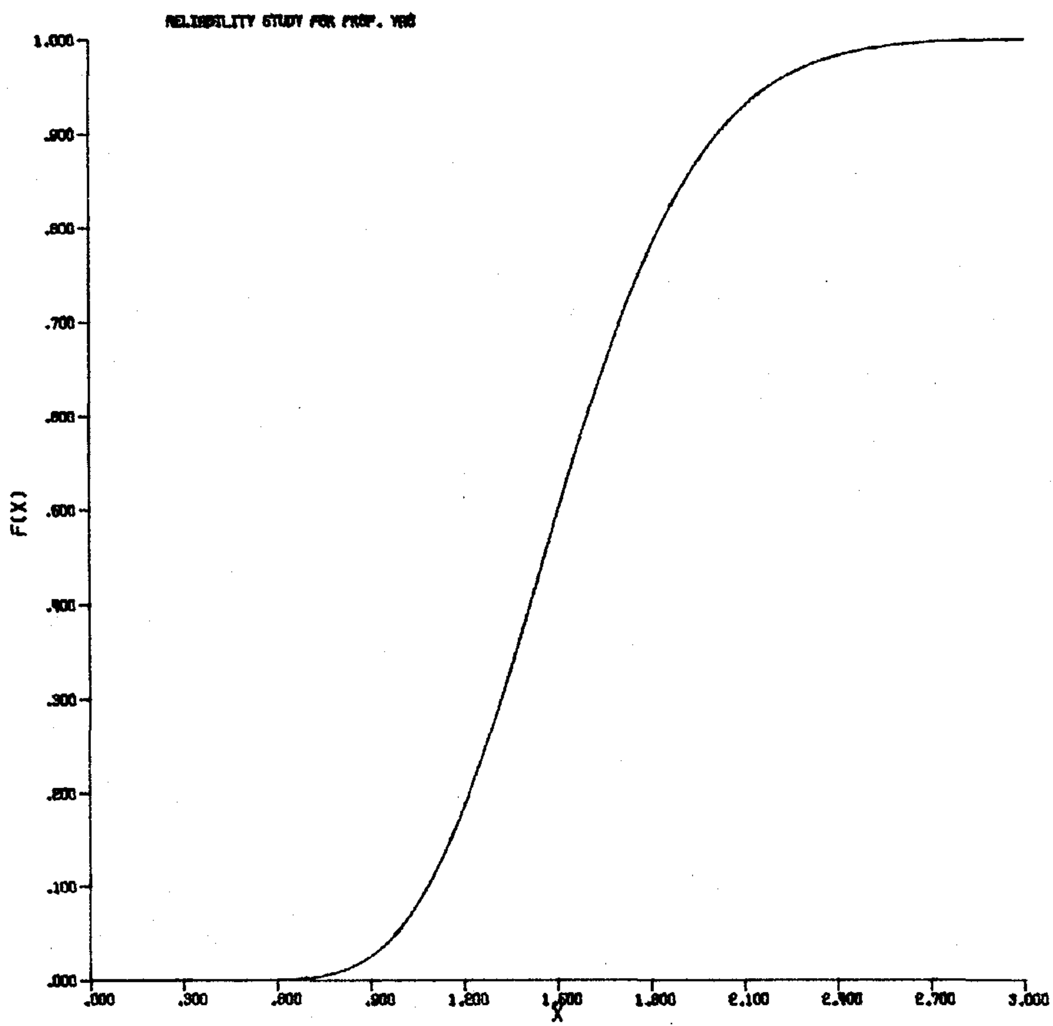


Fig. 48: Cumulative Distributon Function.

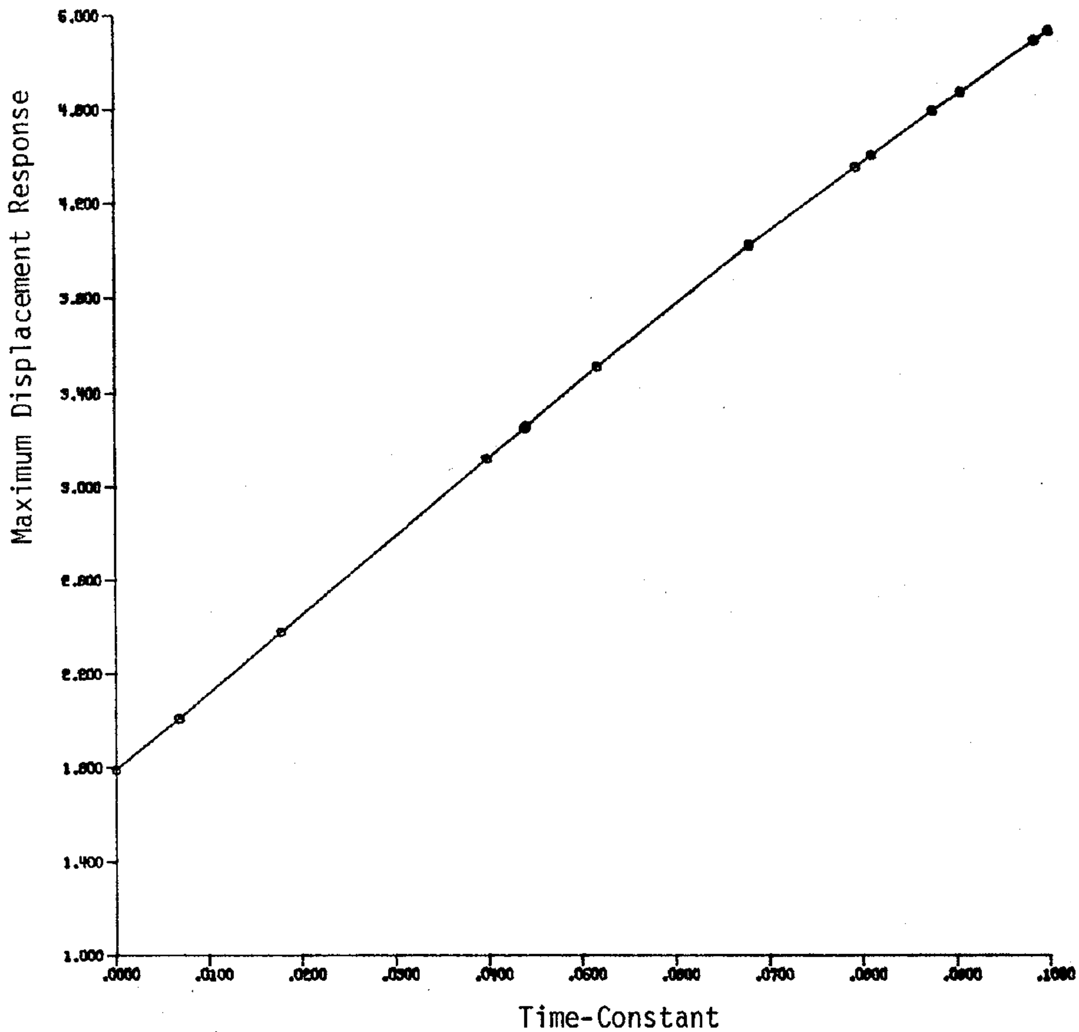


Fig. 49: Time Constant vs. Maximum Displacement Response.