STRUCTURAL ENGINEERING

CE-STR-82-38

SOME RECENT DEVELOPMENTS IN STRUCTURAL CONTROL

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Abstract

To reduce the effect of lateral earthquake ground motion on structures, active control systems can be used. During the last several decades, many different approaches have been applied to find a suitable gain matrix for the control law. Several recent investigations have been summerized and reviewed herein. In addition, a technique on basis of the system identification methods is developed and presented.

Table of Contents

1.	Introduction	1
2.	System Identification	5
3.	Approximate Method	10
4.	Numerical Examples	12
5.	Conclusion	14
	Acknowledgement	15
	Appendix A: Root Perturbations	16
	Appendix B: Additional Numerical Examples	17
	References	22
	Figures	24

1. Introduction

During these past several decades, modern civil engineering structures are becoming more flexible because of (a) the availability of better methods of analysis and computations, (b) the increasing cost of construction materials, and (c) the desire to build taller and longer structures. The dynamic response of such flexible structures to wind and earthquake excitations may exceed limit states for human comfort or structural integrity. Consequently, the possibility of applying feedback control to civil engineering structures was discussed in an earlier paper [1].

To control the behavior of a given structure, one can use either passive and/or active control systems. The main idea of using a control system is that flexible structures such as extremely tall buildings or long bridges can be designed to resist essentially the operational gravity loads and the active control system can be used to minimize side-sway motions resulting from lateral loads. Recently, the relevant literature on the interrelationship among structural identification, control, and reliability was reviewed [2].

To effectively control the motions of a given structure, it is necessary to describe the characteristics of this particular structure. Currently available mathematical representations result from generalizations of existing knowledge in the structural engineering profession. Following the completion of the construction process, each civil engineering structure possess its own characteristics, the precise description of which is difficult to obtain with the use of any general mathematical model [3]. In recent years, applications of system identification techniques includes mathematical modeling, damage assessment, and reliability evaluation of existing structures on the basis of field observations and test data [4,5]. Several approaches are being explored for the solution of this important problem [6-8].

A general approach to active structural control to satisfy simultaneously the requirement for safety, serviceability, and human comfort considerations was presented by Rohman and Leipholz [9]. The feasibility of using such a control was also considered. In another paper, Rohman and Leipholz [10] studied the vibration of a single-span bridge with the use of a control mechanism. They showed some advantages of using a closed-loop control in flexible civil engineering structures.

Tuned-mass dampers have been installed in two tall buildings to reduce motions during high winds [11,12]. These systems are presently designed to operate as passive tuned-mass dampers. However, questions remain as to the effectiveness of such systems [13]. Meanwhile, possible modifications to make these devices into active control systems have been studied by Lund [14].

Yao and Tang [15] and Masri and Beckey [16] proposed the application of servo-controlled pulse generators to mitigate the earthquake induced motions of tall buildings. Numerical results show that such control systems can be effective in controlling building motions during strong earthquakes.

The active control of structures by modal synthesis was presented by Meirovitch and Öz [17]. Their control scheme consists of independent modal control providing active damping for the controlled modes of the structure. Soong and Chang [18] studied an optimal control configuration using the theory of modal control. For tall buildings, the application of modern control theory introduces a number of difficult problems. An important problem is that of obtaining optimal control configuration of appropriate locations of controllers. This topic has been studied by Soong and Chang. Recently, optimal open-loop control of structures under earthquake excitation was studied by Yang and Lin [19]. They used an active tendon control system and an active mass damper system.

The concept of active feedback control was studied by Roorda [20]. Results of these experiments demonstrate in a simple way the essential ingredients of an active feedback control system. It involves the control of the midspan deflection of a king-post truss by selectively lengthening or shortening the under-slung cable in a controlled way. In addition, a vertical cantilever is controlled with a pair of vertical steel tendons fixed to a cross arm attached at the column and to a yoke which pivots about the column center line near the base.

Although many investigations have been reported concerning the application of active control systems in civil engineering structures, none is dealing with the reliability of these systems. The reliability aspect of control system was studied by Yao and Basharkhah [21], who showed that the system reliability depends a great deal on the time constant of the controller.

A classical design method in control theory consists of relocating the dominant closed-loop poles further away from the origin in the s-plane. The new location of these poles depends upon design specifications concerning factors such as relative stability, response times, and desirable accuracy. In this report, the design of feedback compensators for linear, constant coefficient multivariable system is considered. One of the design objectives is to obtain suitable pole locations in order to ensure satisfactory transient response. This problem is analyzed, first under the assumption that all state variables can be used in forming feedback signals. Output feedback, i.e. incomplete state feedback is also possible. To obtain a suitable set of pole locations, one may use the pole assignment method [22]. Although the pole assignment method is straight-forward and there exists at least one solution for control law whenever the open-loop system is completely controllable, one may not find the suitable pole locations or it is not always possible to relocate them to suitable locations for civil engineering structures. In this report, a method on the basis of system identification concept is developed. Results of numerical examples are presented to illustrate several advantages of using this new method when it is compared with the pole assignment method.

2. System Identification

In previous work of authors [21,22], the equation of motion and its parameters are assumed to be known. Then, a mathematical representation of the system is obtained by applying laws of physics. In reality, not all information of the system is available. Therefore, a model for the system must be found by using experimental measurements of available inputs and outputs. This subject is called system identification. In civil engineering, this concept has been used to obtain "realistic" equation of motion and to evaluate damage of the structures. Various system identification techniques have been developed in other branches of engineering to-date. Nevertheless, the application of this concept in controlling the response of civil engineering structures is new herein. To find the gain matrix of the control law, one may use different methods. It is noted that the gain matrix of the control law does not have an unique solution in many cases. To use these different approaches, there is no direct relationship between displacement or velocity response and the gain matrix. Therefore, by using the system identification concept, the gain matrix will be found such that the controlled response of the system becomes almost as closed as possible to the desired response.

Consider a n-degree-of-freedom flexible structure. The equation of motion may be written as follows:

$$m\ddot{x}+c\dot{x}+kx=N+f \tag{1}$$

Where m, c, and k are mass, damping, and stiffness matrices of the system respectively. N is a nxl external force vector induced by ground acceleration and f is a nxl control force vector.

By using state space variable concept, Equation 1 may be written as follows:

$$\mathring{X} = AX+BU+FV \tag{2}$$

Where A is known as the plant matrix; B as the control matrix; and F as the disturbance matrix. X, U, and V are state vector, control vector, and external or disturbance force vector respectively. Let us assume a linear state feedback for the control law as follows:

$$U=-KX \tag{3}$$

Where K is the gain matrix of the control law. It is possible to use the pole assignment technique to find the gain matrix, but one has to specify the locations of the closed-loop poles in the s-plane. In most civil engineering structures, it is not possible to predict the suitable location for the poles of the system. Because either we do not know where they should be relocated exactly or it is not practical to shift them far enough to the left of the s-plane. By using system identification method, there is no need to know the exact location of the closed-loop poles. This is one of the advantages of using this method rather than the pole assignment method. In this approach, there is a direct relationship between the elements of the gain matrix and the desired displacement or velocity response. To show this method, the closed-loop equation of motion may be written as following:

$$\dot{X} = \bar{A}X + FV \qquad (4)$$

Where $\bar{A}$ is known as closed-loop plant matrix and is given as follows:

$$\bar{A} = A-BK \qquad (5)$$

The final goal of this method is to find the elements of the gain matrix such that the displacement of the system not to exceed a certain level. By using Equation 1, one can find the displacement, velocity, and acceleration of the open-loop system without any control force due to external forces which are assumed as base excitation in this study. Knowing the displacement, velocity, and acceleration of each node, one may prespecify a new set of displacement, velocity, and acceleration for the closed-loop system. It is easy to assume these new set as a factor of the open-loop outputs. Because the main objective of this study is to reduce the displacement response of the system, it is obvious that the above factor should be less than one. Therefore, a new set of outputs may be assumed for closed-loop system according with the desired response. Now, the inputs of the system are the same as the open-loop case, but the outputs of the system are reduced by some prespecified factor. Let us again consider Equation 4. The elements of the closed-loop plant matrix are the only unknowns in this equation. Because both inputs and outputs of the system are known. To find the elements of $\bar{A}$ matrix, let us use the transpose of Equation 4 which may be written as follows:

$$\dot{X}^T = X^T\bar{A}^T + [FV]^T \qquad (6)$$

or

$$az=y \tag{7}$$

where

$$y=[\dot{X}-FV]^T, a=X^T, z = \bar{A}^T \tag{8}$$

In Equation 6, V is a 2nx1 external force which it is assumed as a input for the system; X is a 2nx1 state vector which it is assumed according with the desired performance of the system. $\dot{X}$ is the derivative of X with respect to the time and it is known. Therefore, the only unknown in Equation 6 is the $\bar{A}$ matrix. Either X is assumed to be continuous or discrete function; we should discretize the equation. In this case, Equation 7 becomes a simple simultaneous linear algebraic equations which usually the number of equations are greater than the number of unknowns. Since no one $\bar{A}$ can satisfy all the simultaneous equations, it is inappropriate to write the equality $X^T\bar{A}^T=[\dot{X}-FV]^T$. Rather, an 2nx1 error vector e is introduced:

$$e=y-az \tag{9}$$

The least-square method yields that one $\bar{A}$ which minimizes the sum of the squares of the e_i components. By using least-square, the closed-loop plant matrix may be found as follows:

$$z=[a^T a]^{-1} a^T y \tag{10}$$

Usually, there are more equations than the number of unknowns. It is more efficient to use the recursive least-square method. Assume that a set of p equations $y_k=az+e$ has been used to obtain a least-square estimate for z, denoted by z_k which may be written as

$$z_k = [a^T a]^{-1} a^T y_k \tag{11}$$

As is often the case, assume that one additional set of relations

$$y_{k+1} = H_{k+1} z + e_{k+1} \tag{12}$$

is available. Then the new estimate for z is given as follows:

$$z_{k+1} = z_k + K_k [y_{k+1} - H_{k+1} z_k] \tag{13}$$

where

$$K_k = P_k H_{k+1}^T [H_{k+1} P_k H_{K+1}^T + I]^{-1} \tag{14}$$

$$P_k = [a^T a]^{-1} \tag{15}$$

$$P_{k+1} = P_k - P_k H_{K+1}^T [H_{K+1} P_k H_{k+1}^T + I]^{-1} H_{k+1} P_K \tag{16}$$

Therefore, it is possible to find the best estimation of the closed-loop plant matrix. Now, let us assume that the $\bar{A}$ matrix is given as z_{k+1}^T by using the least-square method. Then, the gain matrix for the control law may be found as follows:

$$BK = A - \bar{A} \tag{17}$$

or

$$K = [B^T B]^{-1} \; B^T \; \Delta A \tag{18}$$

Where $\Delta A = BK$. There is a solution for the gain matrix if the inverse of BB^T matrix exists. Otherwise, by using the least-square method, one may find the best solution for the algebraic equation $BK = \Delta A$. To use the least-square method, the rank of [B] matrix must be equal to the rank of $[B|\Delta A]$ matrix. Otherwise, there is no solution for the gain matrix. In this case, one has to change the desired performance of the system. In most civil engineering structures, it is not possible to change the open-loop plant matrix very much. Therefore, it is assumed that ΔA is small enough to use some approximation (see Appendix A). These approximate methods are called root perturbations.

3. Approximate Method

Consider Equation 1. By using a similarity transformation, $x=Tq$, it is possible to diagonalize both mass and stiffness matrices. Then, Equation 1 may be rewritten as follows:

$$m^*\ddot{q}+c^*\dot{q}+k^*q=N^* + f^* \tag{19}$$

Where

$$m^* = T^TmT \quad ,c^* = T^TcT \quad ,k^* = T^TkT \tag{20}$$

It is possible to choose a similarity transformation such that the new mass matrix becomes identity matrix. In this case, the square root of the diagonal elements of the k^* represent the natural frequencies of the structure. Equation 20 may be rewritten and used as follows:

$$\ddot{q}+c^*\dot{q}+k^*q=N^* + f^* \tag{21}$$

Because the main external force in this study is due to earthquake; the damping of the structure does not have a major effect. In this case, it is possible to neglect the damping matrix. In any event, if one wants to include the damping in the system the c^* matrix may be rewritten as a diagonal matrix as follows:

$$c^* = \text{diagonal}[2\xi_1\omega_1,\ldots\ldots.2\xi_n\omega_n] \tag{22}$$

Where ξ_i is the damping ratio of the ith mode. There is some approximation when one use an approximated damping matrix. There is no need to neglect the damping matrix when the previous method has been used. This is one of the advantages of the previous method. By using state space variable, Equation 21 may be written as follows:

$$\dot{X} = AX+BU+FV \tag{23}$$

Where

$$A = \begin{matrix} 0 & I \\ -k^* & -c^* \end{matrix} \tag{24}$$

Let us neglect the damping matrix. Since c^* is a null matrix,therefore, the damping ratio, ξ_i, for each mode is zero and the open-loop eigenvalues of the system are given as:

$$\lambda_i = \pm j\omega_i \tag{25}$$

To find ΔA matrix by using this method, one can change the c^* and k^* matrices. Changing these matrices will cause some changes in ξ_i and ω_i. Let $\Delta\xi_i$ and $\Delta\omega_i$ be the change in damping ratio and natural frequency of the ith mode of the system providing that both increments are small enough. Knowing that the ith eigenvalue of the system has the following form:

$$\lambda_i = -\xi_i \pm j\omega_i \sqrt{1-\xi_i^2} \tag{26}$$

Then the change of the each pole is given by:

$$\Delta\lambda_i = -\Delta\xi_i\,\omega_i + j\Delta\omega_i \tag{27}$$

The changes in c^* and k^* matrices are given as follows:

$$\begin{aligned} \Delta c^*_{ii} &= 2\Delta\xi_i\,\omega_i \\ \Delta k^*_{ii} &= 2\Delta\omega_i\,\omega_i \end{aligned} \tag{28}$$

By adding these incremental matrices to the c^* and k^*, one can find a new set of values for damping and stiffness matrices. Substituting these new matrices into Equation 21 and by using state variable concept , one can find the $\overline{A}$ matrix. Knowing $\overline{A}$ and A matrices, the ΔA matrix may be computed. Finally, the gain matrix can be found exactly by the same procedure which we use for previous case.

4. Numerical Examples

Consider a four-degree-of-freedom shear typed structure which its parameters are given as follows:

$$m_1 = m_2 = m_3 = m_4 = 0.3 \quad \frac{\text{lb-sec}^2}{\text{in}}$$
$$c_1 = c_2 = c_3 = c_4 = 0.5 \quad \frac{\text{lb-sec}}{\text{in}}$$
$$k_1 = k_2 = k^3 = k^4 = 2.5 \quad \frac{\text{lb}}{\text{in}}$$

The open-loop plant matrix for this system is given as follows:

$$\begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -16.67 & -3.33 & 8.33 & 1.67 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 8.33 & 1.67 & -16.67 & -3.33 & 8.33 & 1.67 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 8.33 & 1.67 & -16.67 & -3.33 & 8.33 & 1.67 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 8.33 & 1.67 & -8.33 & -1.67 \end{bmatrix}$$

The displacement, velocity, and acceleration of the structure without active control force due to an simulated earthquake are computed. The maximum displacement and velocity of each node are found as following:

$$X_{1_{max}} = 5.22 \quad X_{2_{max}} = 8.23 \quad X_{3_{max}} = 10.24 \quad X_{4_{max}} = 11.47$$

$$V_{1_{max}} = 19.25 \quad V_{2_{max}} = 23.17 \quad V_{3_{max}} = 25.70 \quad V_{4_{max}} = 26.65$$

To reduce the displacement response of the system, a linear state feedback control system is used. It is desired to reduce the displacement response of the each node by a factor 0.4. To find the elements of the closed-loop plant matrix, the displacement and velocity of each node is reduced by a factor 0.2, but the acceleration of each node and the inputs of the system are assumed the same as the open-loop system. The closed-loop plant matrix, $\bar{A}$, may be found by using the transpose of the equation 13 as follows:

$$\begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -83.33 & -16.67 & 41.67 & 8.33 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 41.67 & 8.33 & -83.33 & -16.67 & 41.67 & 8.33 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 41.67 & 8.33 & -83.33 & -16.67 & 41.67 & 8.33 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 41.67 & 8.33 & -41.67 & -8.33 \end{bmatrix}$$

Knowing the open-loop plant matrix, A, and the closed-loop plant matrix, $\bar{A}$, the gain matrix of the control law ,U=-KX, may be found by using Equation 18 and is given as follows:

$$\begin{bmatrix} 20.00 & 4.00 & -10.00 & -2.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -10.00 & -2.00 & 20.00 & 4.00 & -10.00 & -2.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -10.00 & -2.00 & 20.00 & 4.00 & -10.00 & -2.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -10.00 & -2.00 & 10.00 & 2.00 \end{bmatrix}$$

The displacement and velocity of the closed-loop system for each node are found as follows:

$$X_{1_{max}} = 2.13 \quad X_{2_{max}} = 3.49 \quad X_{3_{max}} = 4.45 \quad X_{4_{max}} = 4.96$$
$$V_{1_{max}} = 15.65 \quad V_{2_{max}} = 21.70 \quad V_{3_{max}} = 24.91 \quad V_{4_{max}} = 27.78$$

The average reduction in the displacement of each node is found to be about 58 percent. Thus it is seen that the displacements of the closed-loop system are almost equal to the desired ones. The displacement and velocity of each node for both open-loop and closed-loop system are plotted in Fig. 1 and Fig. 2. It is shown that the controlled displacement response of each node is less than its corresponding value of the uncontrolled displacement response. Appendix B shows more numerical examples. It is shown that the controlled displacement response is always less than the uncontrolled displacement response. To have smaller displacement, we must have some bigger elements for the gain matrix. It is not always possible to have a desired gain matrix. Therefore, one should find the best possible gain matrix for the linear control law. This subject is called linear optimal control theory and will be discussed in later report.

5. Conclusion

There are many different methods which the gain matrix of the control law can be found. Nevertheless, none yields a direct relationship between the displacement or velocity response and the elements of the gain matrix. A method is developed in this report, and results of numerical examples show that it is easier to use this method than the pole assignment method.

Acknowledgement

This study was supported in part by the National Science Foundation through Grant No. CME-8018963. Authors wish to thank Drs. M. P. Gaus and J. E. Goldberg for their continued interest and considerations.

Appendix A: Root Perturbation Method

If λ_i and e_i are respectively the ith eigenvalue and eigenvector of A and $(\lambda_i+\Delta\lambda_i)$ and $(e_i+\Delta e_i)$ are the ith eigenvalue and eigenvector of $(A+\Delta A)$ we will have:

$$\Delta\lambda_i = l_i^* \Delta A e_i \tag{A-1}$$

and

$$\Delta E = EH \tag{A-2}$$

Where H has elements:

$$H_{ij} = l_i^* \Delta A e_j \quad \frac{1.-\delta_{ij}}{\lambda_j - \lambda_i} \tag{A-3}$$

We know that $AE=E\Lambda$ where Λ is a diagonal matrix. We also have:

$$(A+\Delta A)(E+\Delta E)=(E+\Delta E)(\Lambda+\Delta\Lambda) \tag{A-4}$$

$$AE+A\Delta E+\Delta AE+\Delta A\Delta E=E\Lambda+E\Delta\Lambda+\Delta E\Lambda+\Delta E\Delta\Lambda \tag{A-5}$$

$$\Delta\Lambda=E^{-1}A\Delta E+E^{-1}\Delta AE-E^{-1}\Delta E\Lambda \tag{A-6}$$

$$\Delta\Lambda=E^{-1}AEH+E^{-1}\Delta AE-E^{-1}EH\Lambda \tag{A-7}$$

$$\Delta\Lambda=E^{-1}\Delta AE+\Lambda H-H\Lambda \tag{A-8}$$

$$\Delta\lambda_i = l_i^* \Delta A e_i \tag{A-9}$$

$$0_{ij} = l_i^* \Delta A e_j + \lambda subi H_{ij} - H_{ij}\lambda_j \tag{A-10}$$

$$H_{ij} = \frac{l_i^* \Delta A e_j}{\lambda_j - \lambda_i} \tag{A-11}$$

Appendix B: Additional Numerical Examples

Case 1: The displacement of each node is reduced by factor 0.8, but the velocity and acceleration of each node are held fixed. The closed-loop plant matrix, $\bar{A}$, is given as:

$$\begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -20.83 & -3.33 & 10.42 & 1.67 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 10.42 & 1.67 & -20.83 & -3.33 & 10.42 & 1.67 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 10.42 & 1.67 & -20.83 & -3.33 & 10.42 & 1.67 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 10.42 & 1.67 & -10.42 & -1.67 \end{bmatrix}$$

The gain matrix is given as:

$$\begin{bmatrix} 1.25 & 0.00 & -0.62 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -0.62 & 0.00 & 1.25 & 0.00 & -0.62 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -0.62 & 0.00 & 1.25 & 0.00 & -0.62 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -0.62 & 0.00 & 0.62 & 0.00 \end{bmatrix}$$

The maximum displacement and velocity of each node is given as:

$$X_{1_{max}} = 4.54 \quad X_{2_{max}} = 7.68 \quad X_{3_{max}} = 9.77 \quad X_{4_{max}} = 10.93$$

$$V_{1_{max}} = 20.61 \quad V_{2_{max}} = 23.22 \quad V_{3_{max}} = 26.09 \quad V_{4_{max}} = 27.18$$

Case 2: The velocity of each node is reduced by factor 0.8, but the displacement and acceleration of each node are the same as open-loop system. The closed-loop plant matrix is given as:

$$\begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -16.67 & -4.17 & 8.33 & 2.08 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 8.33 & 2.08 & -16.67 & -4.17 & 8.33 & 2.08 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 8.33 & 2.08 & -16.67 & -4.17 & 8.33 & 2.08 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 8.33 & 2.08 & -8.33 & -2.08 \end{bmatrix}$$

The gain matrix is given as:

$$\begin{bmatrix} 0.00 & 0.25 & 0.00 & -0.12 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.12 & 0.00 & 0.25 & 0.00 & -0.12 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -0.12 & 0.00 & 0.25 & 0.00 & -0.12 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -0.12 & 0.00 & 0.12 \end{bmatrix}$$

The maximum displacement and velocity of each node is given as:

$$X_{1_{max}} = 5.04 \quad X_{2_{max}} = 7.95 \quad X_{3_{max}} = 9.90 \quad X_{4_{max}} = 11.03$$

$$V_{1_{max}} = 19.00 \quad V_{2_{max}} = 22.60 \quad V_{3_{max}} = 25.16 \quad V_{4_{max}} = 26.19$$

Case 3: Both displacement and velocity of each node are reduced by factor 0.8, but the acceleration of the closed-loop system is the same as acceleration of the open-loop system. The closed-loop plant matrix, $\overline{A}$ is given as follows:

$$
\begin{bmatrix}
0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
-20.83 & -4.17 & 10.42 & 2.08 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
10.42 & 2.08 & -20.83 & -4.17 & 10.42 & 2.08 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 10.42 & 2.08 & -20.83 & -4.17 & 10.42 & 2.08 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 10.42 & 2.08 & -10.42 & -2.08
\end{bmatrix}
$$

The gain matrix is given as:

$$
\begin{bmatrix}
1.25 & 0.25 & -0.62 & -0.12 & 0.00 & 0.00 & 0.00 & 0.00 \\
-0.62 & -0.12 & 1.25 & 0.25 & -0.62 & -0.12 & 0.00 & 0.00 \\
0.00 & 0.00 & -0.62 & -0.12 & 1.25 & 0.25 & -0.62 & -0.12 \\
0.00 & 0.00 & 0.00 & 0.00 & -0.62 & -0.12 & 0.62 & 0.12
\end{bmatrix}
$$

The maximum displacement and velocity is given as:

$$X_{1_{max}} = 4.30 \quad X_{2_{max}} = 7.06 \quad X_{3_{max}} = 8.92 \quad X_{4_{max}} = 9.88$$

$$V_{1_{max}} = 20.16 \quad V_{2_{max}} = 22.63 \quad V_{3_{max}} = 25.49 \quad V_{4_{max}} = 26.73$$

Case 4: The displacement of each node is reduced by factor 0.4, and the velocity of each node is reduced by factor 0.8. The acceleration of each node for both open-loop and closed-loop is the same. The closed- loop plant matrix is given as follows:

$$\begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -41.67 & -4.17 & 20.83 & 2.08 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 20.83 & 2.08 & -41.67 & -4.17 & 20.83 & 2.08 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 20.83 & 2.08 & -41.67 & -4.17 & 20.83 & 2.08 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 20.83 & 2.08 & -20.83 & -2.08 \end{bmatrix}$$

The gain matrix is given as:

$$\begin{bmatrix} 7.50 & 0.25 & -3.75 & -0.12 & 0.00 & 0.00 & 0.00 & 0.00 \\ -3.75 & -0.12 & 7.50 & 0.25 & -3.75 & -0.12 & 0.00 & 0.00 \\ 0.00 & 0.00 & -3.75 & -0.12 & 7.50 & 0.25 & -3.75 & -0.12 \\ 0.00 & 0.00 & 0.00 & 0.00 & -3.75 & -0.12 & 3.75 & 0.12 \end{bmatrix}$$

The maximum displacement and velocity of each node is given as:

$$X_{1_{max}} = 4.04 \quad X_{2_{max}} = 6.62 \quad X_{3_{max}} = 8.28 \quad X_{4_{max}} = 9.12$$
$$V_{1_{max}} = 22.93 \quad V_{2_{max}} = 25.67 \quad V_{3_{max}} = 25.94 \quad V_{4_{max}} = 28.10$$

Case 5: Both displacement and velocity of each node are reduced by factor 0.4, but the acceleration of each node for both open-loop and closed-loop system is the same. The closed-loop plant matrix, $\overline{A}$, is given as following:

$$
\begin{bmatrix}
0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
-41.67 & -8.33 & 20.83 & 4.17 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
20.83 & 4.17 & -41.67 & -8.33 & 20.83 & 4.17 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 20.83 & 4.17 & -41.67 & -8.33 & 20.83 & 4.17 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 20.83 & 4.17 & -20.83 & -4.17
\end{bmatrix}
$$

The gain matrix is given as:

$$
\begin{bmatrix}
7.50 & 1.50 & -3.75 & -0.75 & 0.00 & 0.00 & 0.00 & 0.00 \\
-3.75 & -0.75 & 7.50 & 1.50 & -3.75 & -0.75 & 0.00 & 0.00 \\
0.00 & 0.00 & -3.75 & -0.75 & 7.50 & 1.50 & -3.75 & -0.75 \\
0.00 & 0.00 & 0.00 & 0.00 & -3.75 & -0.75 & 3.75 & 0.75
\end{bmatrix}
$$

The maximum displacement and velocity of each node is given as:

$$X_{1_{max}} = 3.42 \qquad X_{2_{max}} = 5.61 \qquad X_{3_{max}} = 6.91 \qquad X_{4_{max}} = 7.53$$

$$V_{1_{max}} = 19.55 \qquad V_{2_{max}} = 22.58 \qquad V_{3_{max}} = 23.17 \qquad V_{4_{max}} = 24.43$$

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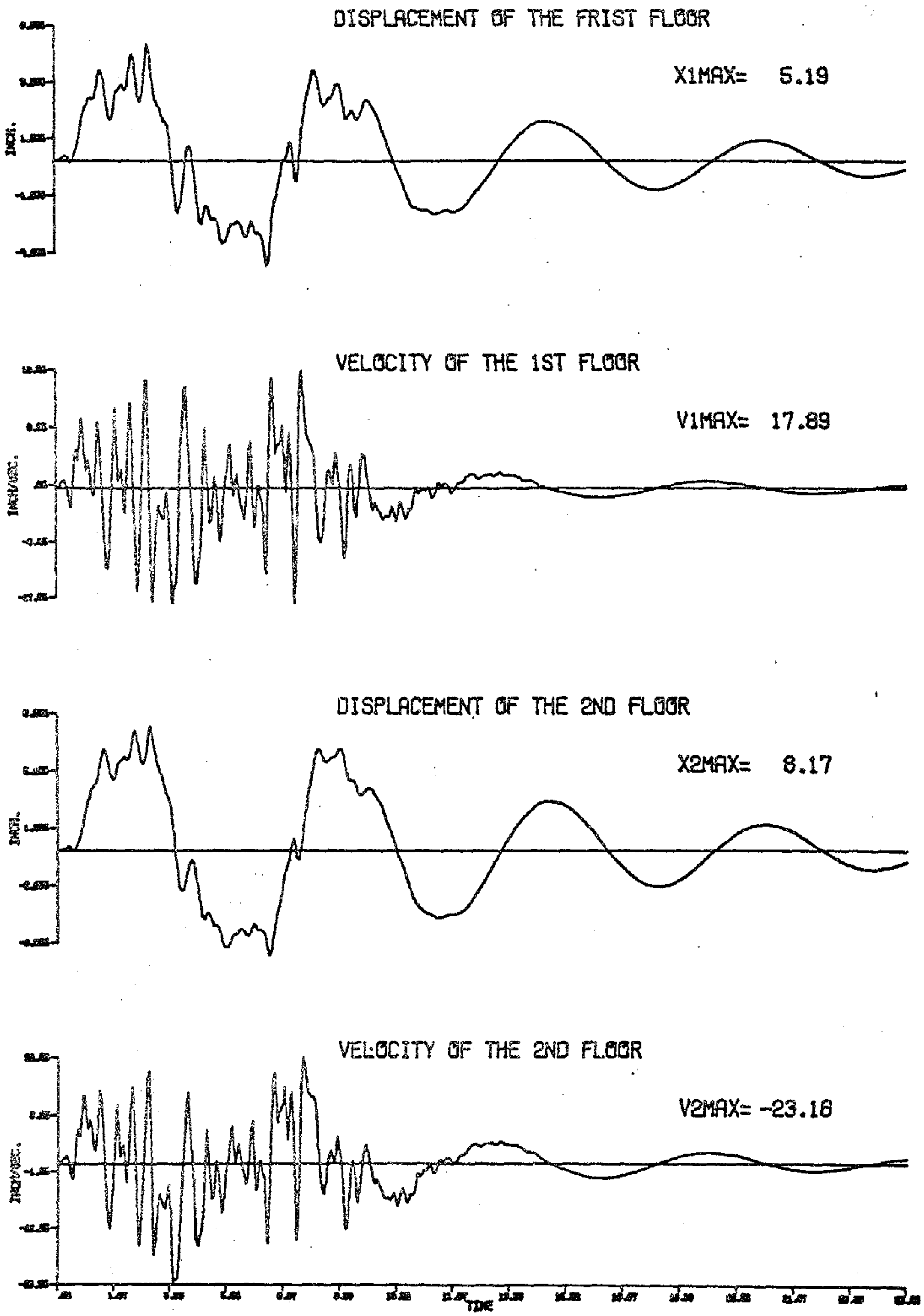

Fig. 1: Displacement and Velocity Response of the Uncontrolled System.

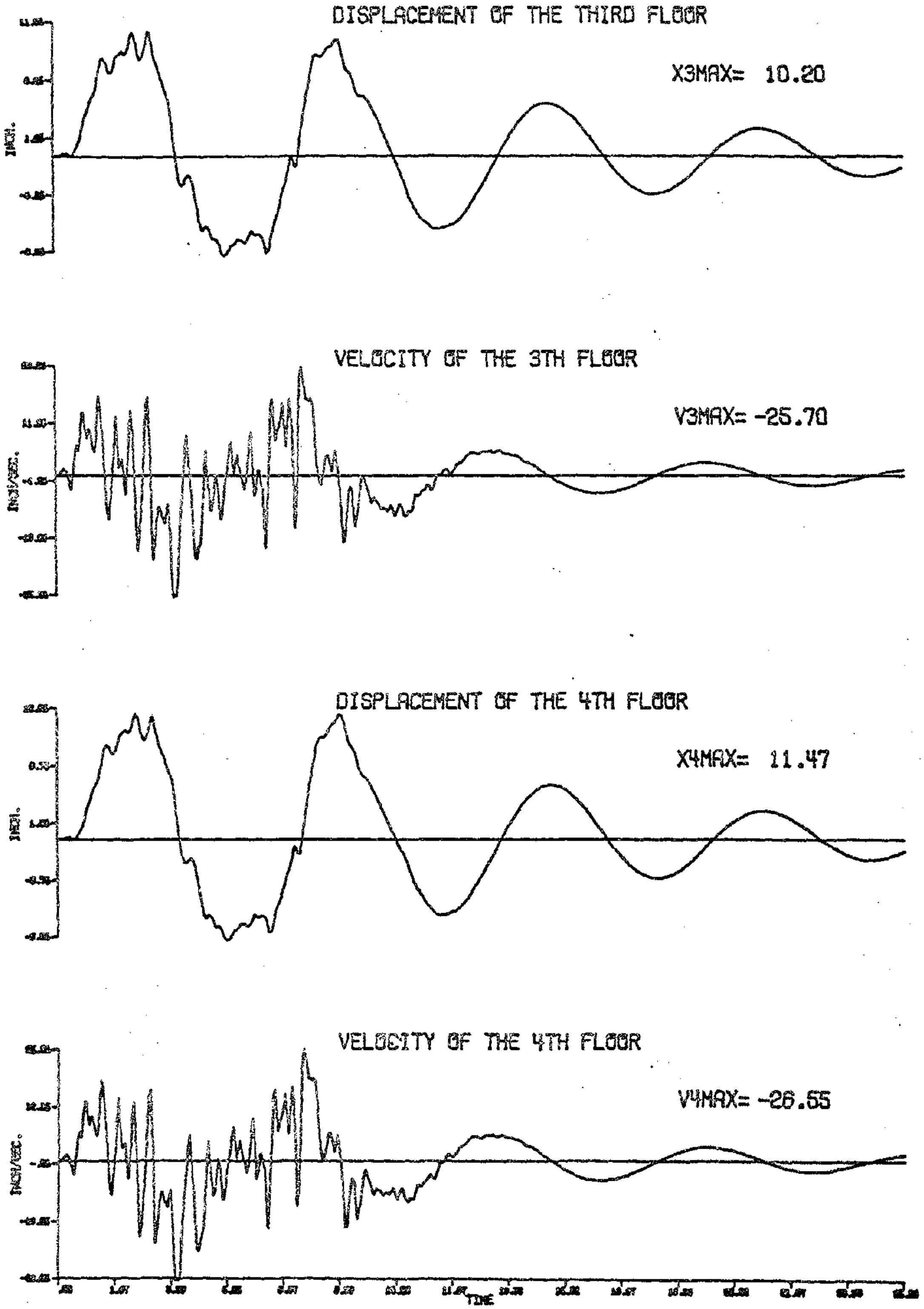

Fig. 1 (con.): Displacement and Velocity Response of the Uncontrolled System.

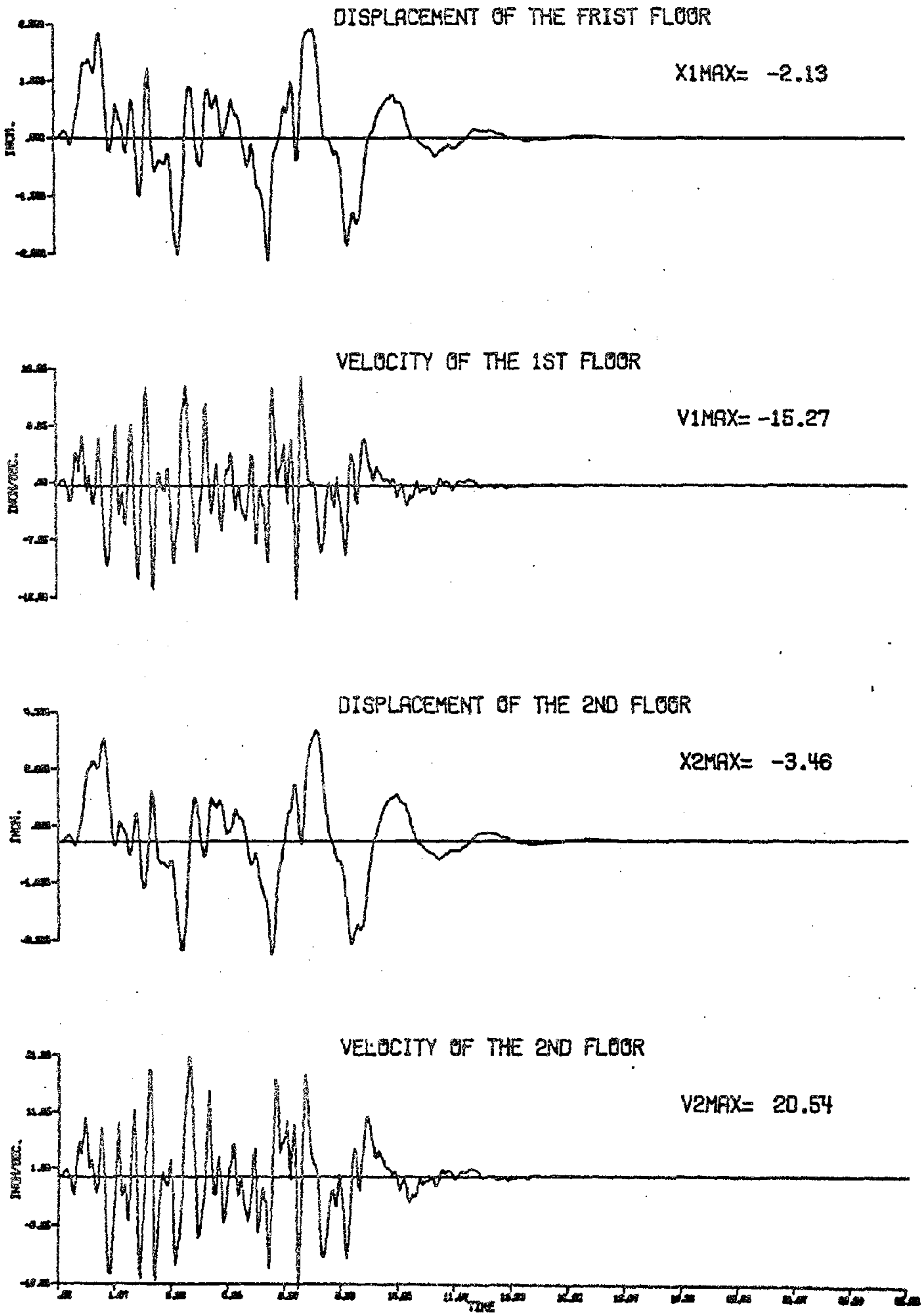

Fig.2; Displacement and Velocity Response of the Controlled System.

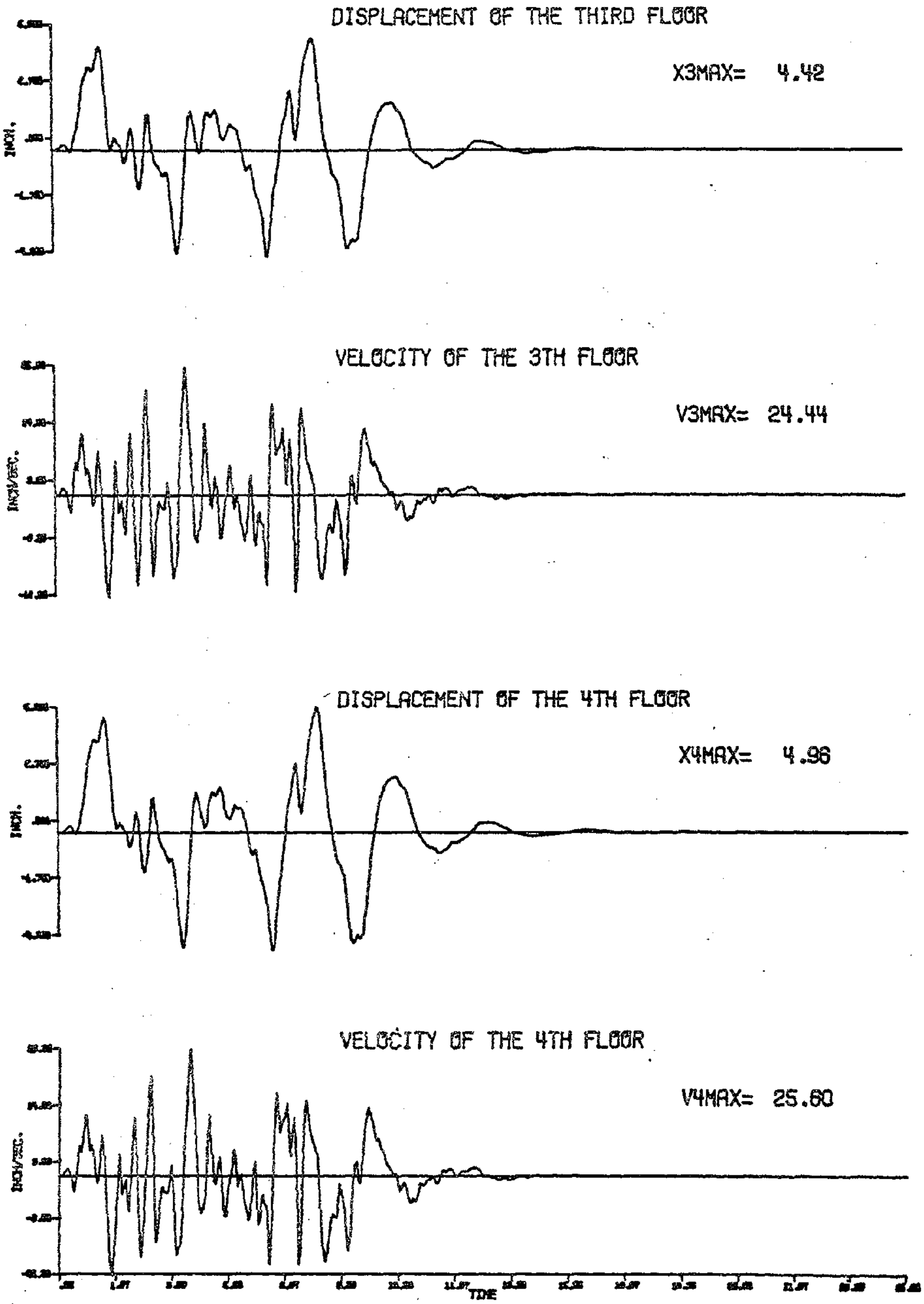

Fig. 2 (con.); Displacement and Velocity Response of the Controlled System.