NSF/CEE-82076

Bulletin of the Seismological Society of America, Vol. 72, No. 5, pp. 1663-1676, October 1982

# $b$ VALUES FOR FORESHOCKS AND AFTERSHOCKS IN REAL AND SIMULATED EARTHQUAKE SEQUENCES

BY L. KNOPOFF, Y. Y. KAGAN, AND R. KNOPOFF

ABSTRACT

**The differences in $b$ values between foreshock and aftershock sequences can be shown to be a statistically significant property of real earthquake sequences if a sufficiently large number of cases is considered, i.e., if the catalogs are long enough. These differences depend on the particulars of the data processing procedures used to define the sequences, such as space-time windowing and definition of cutoff magnitude thresholds for completeness of catalogs, as well as other magnitude cutoffs. They are also a statistically significant property of long artificial, or synthetic catalogs in which all subsequent earthquakes are modeled as resulting from a stochastic time delay applied to predecessor earthquakes. As a consequence, we make the conjecture that the difference in $b$ values is due to subtle asymmetries in data processing procedures rather than differences in the physical environment before and after a large earthquake.**

## INTRODUCTION

It has been proposed that the $b$ values of the earthquake magnitude-frequency distribution are different for foreshock and aftershock sequences of strong earthquakes (Utsu, 1970, 1971; Wyss and Lee, 1973), a proposal which has recently had renewed interest (von Seggern, 1980; Caputo, 1981; Smith, 1981). If the proposal is valid, it has important implications for earthquake prediction. The sign of the difference varies from author to author. To date, no systematic study has been made of the statistical significance of these differences. The number of studies of this type to date has been sufficiently small that those that have been reported take on the aura of a series of case histories, rather than a population of dimensions suitable for statistical analysis. Case histories, on occasion, give the impression of having been examples chosen *a posteriori* to illustrate a postulate; this may be a valid evolutionary process leading to discovery but, alternatively, statistical substantiation requires that an unbiased selection of cases, similarly processed, must be made to determine the percentage of cases in which the effect is present. Further, we must determine the uncertainties in the $b$ values of the two populations to see whether the two sets of $b$ values are statistically resolvable from one another.

Although aftershocks and foreshocks in general can be taken to be causally interactive with the main shock, precise definitions of these events have not been made consistently. Foreshocks and aftershocks can be defined according to some windowing algorithm which identifies as a foreshock or aftershock any event falling within specified intervals of time and distance from a main shock; the intervals are set larger for larger main events. To date, there has been no agreement concerning the use of a consistent windowing algorithm; in some cases, the use of a formal windowing procedure has been eschewed, and intuitive definitions of dependent events have been made.

A statistical analysis of dependent events depends on the quality of the earthquake catalog for those events that are weaker than the main shock. Although well-defined, or definable, procedures exist for determining the threshold magnitude of reliability of an earthquake catalog, no uniform definition has been applied to $b$-value studies to date. Evidently, failure to report the full complement of small earthquakes can lead to bias in the estimates of $b$ values. An equally important consideration, especially for the case of aftershocks, is that the cataloging of shocks with magnitudes slightly greater than the threshold of the networks is inadequate for short times after a large earthquake.

In this paper, we apply systematic and consistent data analyses to a number of the "standard" earthquake catalogs to determine whether differences between the two sets of $b$ values are statistically significant. We use several windows to define aftershocks and foreshocks and make several consistent truncations of catalogs at various magnitude thresholds.

We have had some success at simulating catalogs of earthquakes; the end product has considerable similarity to the real prototypes and includes the properties of foreshocks and aftershocks; the model used was a branching model for interaction among earthquakes. In this paper, we apply conventional techniques for catalog analysis to our synthetic catalogs as well; it is our hope that, since we have insights into the way in which the synthetic catalogs are formed, we will be able to make a contribution to understanding the reasons for the differences in the $b$ values.

Two cases of investigations of this type can be identified. In the first problem, we investigate the $b$ values of the distributions of all events identified as foreshocks and all events identified as aftershocks in a catalog. The second study involves the properties of individual sequences. In the second case, we inquire into the mean $b$ value of individual foreshock sequences and the corresponding mean for aftershock sequences; these means can be compared. We can also compute the differences between the two $b$ values for each case in which both sequences are defined for a given main earthquake and determine the statistical properties of these differences. The statistical quality of the result of the second problem is poorer than that of the first because each individual sequence has a smaller number of shocks than the full catalog. Even in the first cases, in which we use the full catalog, the uncertainties in the $b$ values are sufficiently large that we will find difficulty in obtaining significant differences for the two sets of earthquake sequences.

## Catalog Processing

We have already attacked the first problem and have shown that the differences in $b$ values for the full catalog are real (Kagan and Knopoff, 1978, 1980b). Although the method described in our earlier work provides a powerful significance test, nevertheless, the algorithm we have used is not computationally accessible to many readers since it involves a rather complex process of fitting to a stochastic model of interaction among earthquakes. Our model in the papers cited above takes into account aftershocks of aftershocks, a family tree that is not convenient for rapid analysis. In this paper, we perform a direct systematic evaluation of the $b$ values of the full catalogs of aftershocks and foreshocks by more conventional techniques.

We have used all of the catalogs of shallow earthquakes ($h \leq 70$ km) available to us and processed them by identical methods (Table 1). Except for the catalog compiled by the International Seismological Centre (ISC), all of the catalogs have been described by us previously (Kagan and Knopoff, 1980a, b). We have used two versions of the NOAA catalog in this study: in the first instance, the magnitudes were averaged and corrected for the saturation of the $m_b$ and $M_S$ scales (Kagan and Knopoff, 1980b), and in the second, the magnitudes were taken to be the nominal values of $m_b$ without taking saturation effects into account; the latter catalog is identified as NOAAB. In NOAAB, aftershocks at short time and distance intervals were not removed from the catalog (Kagan and Knopoff, 1978). The catalogs we

TABLE 1

CONDITIONAL *b* VALUES FOR CATALOGS OF EARTHQUAKES

| Catalog No. | Catalog Name | Years | Time-Distance Window | Number of Earthquakes | | | | | Magnitude Threshold | | *b* Value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Total | Main | Dependent | Foreshocks | Aftershocks | All Events | Main Events | Foreshocks | Aftershocks | Weighted Difference |
| 1 | USGS | 1971–1976 | I | 7340 | 3896 | 3444 | 702 | 1365 | 1.5 | 3.3 | 0.786 ± 0.045 | 0.828 ± 0.033 | 0.753 |
| 2 | USGS | 1971–1976 | I | 2683 | 1409 | 1274 | 313 | 552 | 2.0 | 3.3 | 0.917 ± 0.070 | 0.939 ± 0.053 | 0.251 |
| 3 | USGS | 1971–1976 | II | 7340 | 5661 | 1679 | 229 | 823 | 1.5 | 3.3 | 0.734 ± 0.078 | 0.783 ± 0.042 | 0.553 |
| 4 | USGS | 1971–1976 | II | 2683 | 1982 | 701 | 93 | 357 | 2.0 | 3.3 | 0.711 ± 0.122 | 0.899 ± 0.065 | 1.360 |
| 5 | USGS | 1971–1976 | III | 7340 | 6591 | 749 | 59 | 440 | 1.5 | 3.3 | 0.541 ± 0.146 | 0.750 ± 0.057 | 1.333 |
| 6 | USGS | 1971–1976 | III | 2683 | 2321 | 362 | 35 | 205 | 2.0 | 3.3 | 0.616 ± 0.194 | 0.903 ± 0.086 | 1.352 |
| 7 | USGS[h] | 1971–1976 | II | 7340 | 6034 | 1306 | 161 | 748 | 1.5 | 3.3 | 0.695 ± 0.093 | 0.770 ± 0.044 | 0.729 |
| 8 | USGS[h] | 1971–1976 | II | 2683 | 2090 | 593 | 67 | 330 | 2.0 | 3.3 | 0.655 ± 0.142 | 0.886 ± 0.068 | 1.467 |
| 9 | USGS[h] | 1971–1976 | III | 7340 | 6905 | 435 | 50 | 320 | 1.5 | 3.3 | 0.509 ± 0.157 | 0.703 ± 0.066 | 1.139 |
| 10 | USGS[h] | 1971–1976 | III | 2683 | 2444 | 239 | 33 | 165 | 2.0 | 3.3 | 0.623 ± 0.200 | 0.914 ± 0.096 | 1.312 |
| 11 | USGS[a] | 1971–1976 | I | 7340 | 3896 | 3444 | 1951 | 3183 | 1.5 | 3.3 | 0.757 ± 0.027 | 0.801 ± 0.021 | 1.286 |
| 12 | USGS | 1971–1976 | I | 7340 | 3896 | 3444 | 616 | 1200 | 1.5 | 3.5 | 0.780 ± 0.048 | 0.823 ± 0.035 | 0.724 |
| 13 | USGS | 1971–1976 | I | 7340 | 3896 | 3444 | 602 | 1186 | 1.5 | 4.0 | 0.779 ± 0.049 | 0.823 ± 0.035 | 0.731 |
| 14 | USGS | 1971–1976 | II | 7340 | 5661 | 1679 | 159 | 692 | 1.5 | 4.0 | 0.823 ± 0.096 | 0.796 ± 0.046 | −0.254 |
| 15 | USGS | 1971–1976 | II | 2683 | 1982 | 701 | 54 | 293 | 2.0 | 4.0 | 0.721 ± 0.161 | 0.901 ± 0.072 | 1.021 |
| 16 | CACA | 1944–1977 | II | 1581 | 759 | 822 | 42 | 681 | 3.5 | 5.0 | 0.915 ± 0.191 | 0.758 ± 0.046 | −0.799 |
| 17 | CACA | 1944–1977 | II | 631 | 258 | 373 | 15 | 331 | 4.0 | 5.0 | 0.877 ± 0.316 | 0.954 ± 0.069 | 0.238 |
| 18 | ALE | 1971–1973 | I | 604 | 98 | 506 | 220 | 185 | 3.5 | 5.0 | 0.988 ± 0.085 | 0.879 ± 0.090 | −0.880 |
| 19 | ALE | 1971–1973 | II | 604 | 462 | 142 | 24 | 62 | 3.5 | 5.0 | 0.726 ± 0.241 | 0.683 ± 0.149 | −0.152 |
| 20 | NZ | 1940–1966 | I | 212 | 99 | 113 | 47 | 51 | 5.0 | 6.0 | 1.083 ± 0.188 | 1.078 ± 0.180 | −0.019 |
| 21 | NZ | 1967–1977 | I | 418 | 210 | 208 | 37 | 83 | 4.5 | 6.0 | 1.018 ± 0.208 | 1.207 ± 0.145 | 0.745 |
| 22 | JP | 1926–1964 | I | 594 | 10 | 584 | 412 | 327 | 5.5 | 7.0 | 0.786 ± 0.059 | 0.901 ± 0.068 | 1.277 |
| 23 | JP | 1926–1964 | II | 594 | 300 | 294 | 90 | 205 | 5.5 | 7.0 | 0.815 ± 0.127 | 0.826 ± 0.085 | 0.072 |
| 24 | JP | 1926–1964 | II | 258 | 142 | 116 | 37 | 85 | 6.0 | 7.0 | 0.856 ± 0.201 | 0.902 ± 0.134 | 0.190 |
| 25 | JP | 1965–1977 | I | 430 | 25 | 405 | 97 | 267 | 5.0 | 6.5 | 1.076 ± 0.130 | 0.849 ± 0.075 | −1.512 |
| 26 | JP | 1965–1977 | II | 430 | 237 | 193 | 21 | 123 | 5.0 | 7.0 | 1.200 ± 0.289 | 0.786 ± 0.108 | −1.342 |
| 27 | NOAA | 1965–1977 | I | 5808 | 1679 | 4129 | 1463 | 1777 | 5.3 | 7.0 | 1.309 ± 0.036 | 1.245 ± 0.032 | −1.329 |
| 28 | NOAA | 1965–1977 | I | 1354 | 479 | 875 | 322 | 413 | 5.8 | 7.0 | 1.226 ± 0.074 | 1.142 ± 0.064 | −0.859 |
| 29 | NOAA | 1965–1977 | II | 5808 | 3849 | 1959 | 337 | 864 | 5.3 | 7.0 | 1.260 ± 0.073 | 1.322 ± 0.046 | 0.719 |
| 30 | NOAA | 1965–1977 | II | 1354 | 986 | 368 | 72 | 182 | 5.8 | 7.0 | 0.972 ± 0.148 | 1.187 ± 0.100 | 1.204 |
| 31 | NOAA | 1965–1977 | III | 5808 | 5144 | 664 | 28 | 310 | 5.3 | 7.0 | 0.904 ± 0.233 | 1.285 ± 0.077 | 1.553 |
| 32 | NOAAB | 1965–1977 | I | 6347 | 3081 | 3266 | 367 | 681 | 5.3 | 6.5 | 1.415 ± 0.073 | 1.538 ± 0.055 | 1.346 |
| 33 | NOAAB | 1965–1977 | II | 6347 | 4690 | 1657 | 61 | 263 | 5.3 | 6.5 | 1.067 ± 0.164 | 1.550 ± 0.089 | 2.589 |
| 34 | ISC | 1964–1977 | I | 4518 | 2474 | 2044 | 123 | 398 | 5.3 | 6.5 | 1.292 ± 0.122 | 1.589 ± 0.073 | 2.089 |
| 35 | ISC | 1964–1977 | II | 4518 | 3479 | 1039 | 16 | 153 | 5.3 | 6.5 | 0.861 ± 0.306 | 1.524 ± 0.115 | 2.028 |
| 36 | DUDA | 1897–1977 | II | 1078 | 822 | 256 | 68 | 120 | 7.0 | 8.0 | 0.814 ± 0.146 | 1.110 ± 0.118 | 1.577 |
| 37 | DUDA | 1921–1977 | II | 820 | 622 | 198 | 42 | 93 | 7.0 | 8.0 | 1.176 ± 0.203 | 1.383 ± 0.143 | 0.834 |

have listed in Table 1 are those for Central California (USGS), combined Northern and Southern California (CACA), Aleutian Islands (ALE), New Zealand (NZ), Japan (JP), and the catalog of large events (DUDA).

We have deleted from the catalogs all events with magnitudes less than some cutoff value in order to avoid questions of the uniformity of sampling in time, space, or magnitude due to presumed limitations of the seismographic network (Kagan and Knopoff, 1978, 1980a). To ensure that our cutoff thresholds are appropriate, in almost all cases we have also computed the $b$ values with the magnitude cutoff threshold increased by 0.5 (see Table 1).

Three standard aftershock identification windows were used. The limits for the first window (I) are indicated in Table 2; the window increases in size three-fold for each unit increase in magnitude. The entries for the second window (II) are a factor of $10^{1/2}$ less than those of the first window. The entries for the third window (III) are a factor of 10 less than those for the first window.

We surround each earthquake in the catalog by one of the three time-space windows (Gardner and Knopoff, 1974). All events that occur in the window and have a magnitude smaller than the parent earthquake are identified as dependent events. The same window is used for both foreshocks and aftershocks. We study

TABLE 2
Time and Distance Limits for Windows Used to Identify Foreshocks and Aftershocks

| Magnitude of Main Event | Window Limits | |
|---|---|---|
| | Distance (km) | Time (days) |
| 8.0 | 900.00 | 2805.00 |
| 7.0 | 300.00 | 935.00 |
| 6.0 | 100.00 | 311.67 |
| 5.0 | 33.33 | 103.89 |
| 4.0 | 11.11 | 34.63 |
| 3.0 | 3.70 | 11.54 |

aftershock/foreshock sequences for independent earthquakes that have a magnitude greater than or equal to the "magnitude threshold, main event" (column 11, Table 1). Dependent shocks (Table 1) include all events identified as foreshocks or aftershocks in the catalogs and having magnitudes greater than the cutoff imposed for completeness. The numbers of foreshocks and aftershocks identified in the table are those that correspond to main events only. Because the time-distance windows of different earthquakes may overlap, especially if the windows are very wide, some earthquakes may be counted several times as members of different clusters; as a consequence, the sum of the numbers of foreshocks and aftershocks may exceed the total number of dependent events, as in the case of the Japanese catalog. In most cases, spatial windowing was performed two-dimensionally using the epicenter of each event as the center of the window; in a few cases, catalogs identified in Table 1 by a superscript h were windowed three-dimensionally with one of the three scaled versions of the window given in Table 2, using the hypocenter of each event as the center of the window. The application of windows and the setting of thresholds of catalogs in some cases yielded numbers of dependent events that were too small to give statistically significant results; these cases are not reported in Table 1. Our threshold for acceptance of an aftershock or foreshock sequence for subsequent processing was that it should have at least 10 events and that there be at least one event in each of seven different magnitude intervals; the magnitude intervals differ from one another by 0.1. In most cases, the maximum likelihood procedure we used would not converge if the above conditions were not satisfied.

From the composite magnitude distributions for aftershocks and foreshocks of all independent earthquakes, we determine the $b$ value and the uncertainty of its estimate by a maximum likelihood procedure (Aki, 1965). The standard deviations were obtained by inverting the matrix of second derivatives of the likelihood function with respect to the $a$ and $b$ values of the magnitude-frequency relation. In the table, these uncertainties are indicated as though the estimates of the $b$ values are distributed according to a Gaussian distribution. This is the usual assumption and is valid provided that the number of events used to estimate the $b$ value is large enough.

From the values of $b$ for the composite foreshock and aftershock distributions, we test whether the difference between these quantities is statistically significant. To do this, we form the weighted difference $w$, between the two values,

$$w = \frac{b_+ - b_-}{\sqrt{\sigma_+^2 + \sigma_-^2}}$$

where the subscripts + and − refer to aftershocks and foreshocks respectively; the quantities $\sigma_+$ are the corresponding standard deviations of the estimates of the $b$ values. For large samples, $w$ is asymptotically distributed normally with zero mean and unit variance (Wilks, 1962, chapter 13). The null hypothesis that the two $b$ values are equal can be rejected at the 95 per cent confidence level when $|w| > 2$.

## Results of Statistical Analyses

In Table 1, we have listed the values of the criterion function $w$ obtained from the analyses of all foreshocks or aftershocks in a catalog, as well as the values of $b$ and their standard deviations. The values of $w$ indicate that most of the differences between the $b$ values are not statistically significant; in general, the differences between the $b$ values have large and seemingly random fluctuations. Some general trends can be observed: the use of a smaller window to define dependent shocks corresponds to larger values of $w$ in most cases, i.e., there is greater statistical significance to the differences between the $b$ values; if the cutoff magnitude for small shocks is increased, the differences between the $b$ values also increases; the larger catalogs generally have larger values of $w$. Statistically, significant differences in the $b$ values are obtained for only two catalogs, namely the NOAAB and the ISC catalogs; these are two of the three largest catalogs.

The failure of most of the catalogs to yield significant differences between $b_+$ and $b_-$ may be due more to the mode of analysis than to the absence of the effect in the catalogs. If we apply the same $w$ test to the parameters $c_+$ that have been derived for several standard catalogs (Kagan and Knopoff, 1980b, Table 1), we obtain the following results

| | |
|---|---|
| USGS | $w = 9.8$ |
| CACA | $w = 4.2$ |
| DUDA | $w = 1.3$ |
| NOAA | $w = 7.5.$ |

The parameters $c_\pm$ (Kagan and Knopoff, 1978, 1980b) correspond in a rough sense to the parameters $b_\pm$; the model is more sophisticated and takes into account all the possible interactions among the events, rather than the simple description in which all aftershocks or foreshocks are first generation offspring of a single parent. If we had been able to include genuine foreshocks and aftershocks in the present analysis, our $b$ values would have corresponded to $10 \log_{10} c$. In reality, our data, especially foreshock sequences, are heavily "contaminated" by other events, so this comparison cannot be used. The four values of $w$ above, indicate that the aftershock and foreshock magnitude-frequency distributions are genuinely and significantly different from each other in at least three of the cases. We are unable to obtain an unambiguous answer only in the case of the DUDA catalog which has a relatively small number of events in the rather narrow magnitude range $7.0 \leqq M_S \leqq 8.9$.

As we have implied, the conventional method of analysis, in which we designate one dominant parent event, is not normally able to provide appropriate resolution between the $b$ values. To appreciate some of the reasons why the general trends in the results of the conventional analysis summarized above, arise, we make the following comments. If we use large windows, the foreshocks or aftershocks of a given earthquake may be increasingly contaminated by earthquakes belonging to other clusters. When the size of the window is reduced, some dependent events are no longer identified as such; in some extreme cases, the total number of events may become too small for the reliable determination of the $b$ value, and thus the uncertainty in $b$ may become large. As we have remarked, the way to avoid these problems is to evaluate higher order interactions in the cluster; the higher order interactions are characterized by the stochastic model described by Kagan and Knopoff (1978, 1980b).

We have noted that many small aftershocks immediately after a strong earthquake may have been excluded from the catalog. This effect should cause the value of $b_+$ to decrease from its "true" value; the value of $w$ decreases correspondingly. Thus, the value of $w$ increases with increase in the cutoff magnitude as noted in Table 1.

The use of hypocenters instead of epicenters in a three-dimensional windowing procedure does not influence the results significantly, as indicated in Table 1. Nor do changes in the magnitude limits that define main earthquakes influence our conclusion significantly. In one case (item 11, Table 1), we refrained from subdividing the USGS catalog into dependent and independent sets of events; in this case, any event that is a dependent event of a nearby, larger event can also serve as a main event for other nearby dependent events. Comparison of the results with the case in which the same catalog is subdivided into dependent/independent events (item 1, Table 1) yields no important differences. Apparently, our present analysis does not possess sufficient discriminating power to yield a significant result.

Most publications on this subject report differences $b_+ - b_-$ that are greater than those quoted in Table 1. We have already suggested that one reason for this may be that the windows that have been used to define aftershocks and foreshocks are at variance with one another; there are other reasons. To investigate the effect of the window in greater detail, we have considered nine windows, each of which is a scaled version of window I of Table 2; the scaling factors for the nine cases are the multiplicative factors at the *left* and *top* of Table 3. We identify dependent shocks in the USGS catalog for the example of the three-dimensional case in which hypocenters are used to determine the distances between pairs of events. In Table 3, we display $w$ values for the cases in which the cutoff magnitude thresholds for reliability of the catalog are taken to be 1.5 and 2.0, respectively. The values of $w$ in Table 3 do not vary systematically. We infer that if the size of the window or if the threshold magnitude of the catalog is varied, the value of $w$ can be changed by as much as almost two units. We suppose that the other basic window configurations, not simply scaled from our prototype, will also give similar results. By changing the sizes of windows or thresholds, additional degrees of freedom are introduced into the model which are difficult to take into account in evaluating the quality of the results. An *a posteriori* selection of windows and thresholds could yield results that fit well some preconceptions regarding the differences in $b$ values.

## CONDITIONAL $b$ VALUES FOR SYNTHETIC CATALOGS

We have shown elsewhere that it is possible to reproduce most of the known statistical properties of earthquake sequences by a rather simple branching model. We repeat the analysis for differences $b_+ - b$ for the simulated catalogs (Kagan and Knopoff, 1981). Briefly, the catalogs are created as follows. An earthquake sequence is considered as a number of identical infinitesimal subevents. Each of these subevents can beget a Poissonian number of offspring, i.e., dependent events; the mean number of offspring of any parent is one. All offspring are born after the parent; the time difference between the birth of a parent and its clone offspring is determined from the probability density $t^{-3/2}$. Each new subevent can generate its own offspring according to the same law. Thus the process continues until all events

TABLE 3

$w$ VALUE FOR USGS[h] CATALOG

| Time Window | Distance Window | | | | | |
|---|---|---|---|---|---|---|
| | $M_{co} = 1.5$ | | | $M_{co} = 2.0$ | | |
| | $10^{-1}$ | $10^{-0.5}$ | 1 | $10^{-1}$ | $10^{-0.5}$ | 1 |
| $10^{-1}$ | 1.139 | 0.781 | 0.200 | 1.312 | 1.419 | 0.991 |
| $10^{-0.5}$ | 1.805 | 0.729 | 0.174 | 1.919 | 1.467 | 0.264 |
| 1 | 1.915 | 0.467 | 0.815 | 1.955 | 1.292 | 0.301 |

have been "born." All parents and all offspring have the same size. When these events cluster in time, they are presumed to be identified as individual earthquakes by some seismographic network that can only record the existence of sufficient seismic activity above a certain background noise threshold.

We have shown that the above model generates temporal seismicity patterns that are virtually identical to many of those of real earthquakes. In particular, although this model introduces no anticipatory time relationships, foreshocks appear as a natural consequence of the random processes built into the model, and these foreshock sequences have time and magnitude distributions that are similar to those of reach earthquake catalogs. This means that there is no basic difference among foreshocks, main shocks, and aftershocks: the main shock is simply the largest in a series of shocks; in one sense all shocks are aftershocks of the first infinitesimal subevent in the series. Further details are given by Kagan and Knopoff (1981).

By use of the above procedure, 11 catalogs were simulated, all intended to imitate the USGS catalog as a prototype. These catalogs were then processed as above. Only the smallest window (III) was used, i.e., that of Table 2 reduced by a factor of 10 in both time and distance. The cutoff magnitude was taken to be 1.5 while the magnitude of main events was taken to be 3.5 and greater. Unfortunately, the generation of such catalogs is limited by computational costs, so that the number of dependent events in each catalog is too small to obtain a statistically significant rejection of the null hypothesis $(b_+ - b_-) = 0$. However, the values of $w$ are all positive (Table 4) and have a much smaller range of variation in comparison with the catalogs in Table 1. Most likely, the favorable result for synthetic catalogs is due to their greater consistency: for example, they do not contain errors of location of hypocenters or of determinations of magnitude. Since all 11 realizations are independent, we can chain them into a single, long catalog; for the chained case, the result is given in the last row of Table 4. For the chained catalog, the null hypothesis is rejected at a significance level of better than 98 per cent. This means that, for a long synthetic catalog, constructed without the inclusion of any anticipatory effects, $(b_+ - b_-) > 0$. This result was also obtained for the values of $(c_+ - c_-)$ derived by maximum likelihood methods for similar catalogs (Kagan and Knopoff, 1981).

## Discussion of Magnitude-Frequency Relations for the Full Catalog of Dependent Events

We have shown that the difference $(b_+ - b_-)$ for the full catalog of foreshocks and aftershocks is significantly positive for simulated and for some real catalogs. However, the difference is small and, if we use the rather inefficient technique of estimating the $b$ values directly as described in "Results of Statistical Analyses," we will have to process catalogs with several thousand entries to obtain statistically significant results. A more efficient way to test the hypothesis that $b_+ - b_- > 0$ is to use a stochastic model of the complete process of interactive earthquake occurrence (Kagan and Knopoff, 1978, 1980b) and then derive all of the parameters of the model, and especially the quantities $c_+$ and $c_-$, via a maximum likelihood optimization procedure. In the stochastic model, the possibility that one earthquake could belong to several different clusters is taken into account automatically. Hence, we do not have to cope with the problem of "contamination" by earthquakes that are isolated in the time-distance windows but belong to the tails of other aftershock sequences, or with the problem of clusters of clusters, such as the ambiguities of considering aftershocks of large foreshocks. These latter effects are automatically

TABLE 4

Conditional $b$ Values for Synthetic Catalogs of Earthquakes

| Catalog No. | Number of Earthquakes: Total | Main | Dependent | Foreshocks | Aftershocks | $b$ Value: Foreshocks | Aftershocks | Weighted Difference |
|---|---|---|---|---|---|---|---|---|
| 1 | 335 | 22 | 313 | 122 | 317 | 0.533 ± 0.102 | 0.569 ± 0.064 | 0.306 |
| 2 | 564 | 59 | 505 | 138 | 639 | 0.448 ± 0.092 | 0.561 ± 0.045 | 1.099 |
| 3 | 381 | 50 | 331 | 55 | 341 | 0.395 ± 0.142 | 0.564 ± 0.061 | 1.088 |
| 4 | 1280 | 116 | 1164 | 423 | 1269 | 0.449 ± 0.053 | 0.521 ± 0.031 | 1.175 |
| 5 | 194 | 9 | 185 | 91 | 75 | 0.581 ± 0.119 | 0.701 ± 0.136 | 0.664 |
| 6 | 333 | 50 | 283 | 33 | 286 | 0.486 ± 0.192 | 0.558 ± 0.067 | 0.356 |
| 7 | 274 | 40 | 234 | 15 | 205 | 0.410 ± 0.275 | 0.583 ± 0.080 | 0.752 |
| 8 | 478 | 70 | 408 | 116 | 191 | 0.501 ± 0.103 | 0.560 ± 0.082 | 0.452 |
| 9 | 564 | 92 | 472 | 63 | 329 | 0.471 ± 0.138 | 0.610 ± 0.063 | 0.916 |
| 10 | 280 | 46 | 234 | 27 | 190 | 0.464 ± 0.210 | 0.597 ± 0.083 | 0.589 |
| 11 | 274 | 62 | 212 | 50 | 197 | 0.410 ± 0.151 | 0.538 ± 0.080 | 0.749 |
| All | 4957 | 616 | 4341 | 1133 | 4039 | 0.467 ± 0.033 | 0.555 ± 0.018 | 2.388 |

reproduced in our synthetic catalogs; this allows us to conclude that the differences in $b$ values are real effects, and the result removes any doubts we might have regarding their evaluation in real catalogs, doubts that arise due to unknown differences of the methods of data processing.

We can suggest a heuristic argument in favor of the hypothesis $b_+ > b$ . Generally, foreshocks, main events, and aftershocks are distinguished on the basis of time-magnitude relations in earthquake sequences, i.e., an earthquake is considered a foreshock or aftershock if its magnitude is smaller than some other event which is

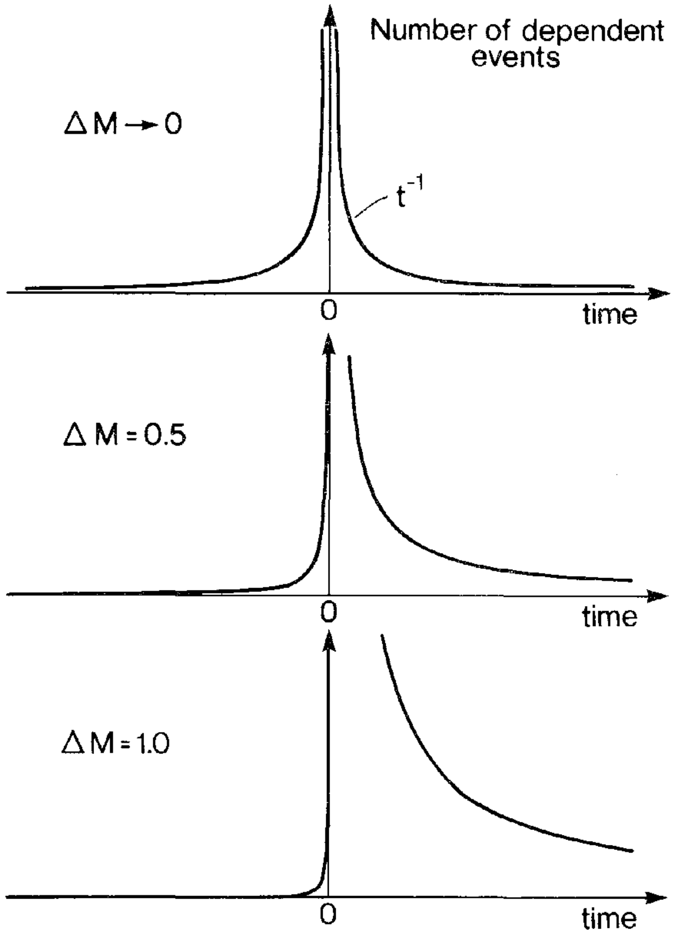


FIG. 1. Schematic rate of occurrence of foreshocks and aftershocks as a function of the time ($t$) to a main event and the magnitude difference ($\Delta M$) between the main shock and dependent events.

situated close to it in space and time. Let $\Delta M$ be the magnitude difference between the main shock and any foreshock or aftershock. Suppose, for the moment, that all earthquakes are equal in magnitude; then the numbers of foreshocks and aftershocks in all time intervals should be equal because of the time symmetry of the earthquake process as $\Delta M \rightarrow 0$, i.e., there would be no difference between foreshocks, main events, and aftershocks (Figure 1). The resulting group of events would be called a *cluster* of earthquakes. However, in most earthquake sequences, the number of foreshocks is usually much smaller than the number of aftershocks. This means that the number of aftershocks per infinitesimal magnitude interval increases as $\Delta M$ increases, whereas the number of foreshocks should remain constant or decrease. Since these numbers should be the same in the case $\Delta M = 0$, the only feasible way to explain the above disparity is to postulate a large difference in $b$ values for both sequences. Hence, the $b$ value for foreshocks should be much smaller than $b_+$. Furthermore, since aftershocks constitute a major part of typical earthquake catalogs, $b_+$ cannot differ considerably from the $b$ value for all earthquakes. We found $c_+ \approx 1/c_-$ (Kagan and Knopoff, 1978, 1980b) i.e., $b_+ \approx -b_-$. The average number of foreshocks per main earthquake is less than one (Kagan and Knopoff, 1978). Thus, the usual determinations of $b$ values for foreshocks are obtained from samples in which genuine foreshocks constitute only an insignificant part. This explains why we have been unable, in the main, to reject the null hypothesis $b_+ = b_-$ in this study; we can assume the same conclusion applies to other investigators who use conventional techniques of $b$ estimation.

We illustrate the above ideas by means of a schematic magnitude-log-frequency plot (Figure 2). The magnitude-frequency curves for foreshocks and aftershocks

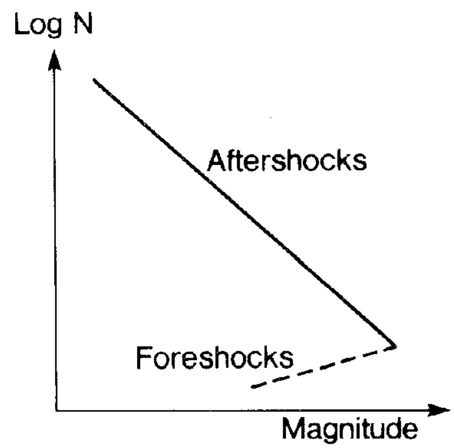


FIG. 2. Schematic differential magnitude-frequency distribution for dependent earthquakes. The curves can be interpreted as corresponding to the average numbers of foreshocks and aftershocks for a main earthquake of fixed magnitude.

intersect where they have a common magnitude, i.e., where it is not possible to discriminate among the several types of events. As indicated above, the curve for aftershocks should be similar to a standard Gutenberg-Richter relation, since the numbers of aftershocks overwhelm the numbers of foreshocks. However, unless we postulate an abnormal nonlinear foreshock curve, for example, one that is peaked, the aftershock curve should have a steeper slope than that of foreshocks. In the figure, we plot the curves according to the above results, $b_+ \approx -b_-$.

Several authors have proposed that the $b$ values are controlled by either stress and stress heterogeneities or by the complexity of fault geometries (Hanks, 1979; Andrews, 1980; von Seggern, 1980). Laboratory experiments support the view that the stress field has an influence on the $b$ values (Scholz, 1968). One of the difficulties we have with attributing an influence on $b$ values to geometry is that the spatial distribution of intermediate and deep focus earthquakes seems to be quite different from the spatial distribution of shallow events (Kagan and Knopoff, 1980a); however, the $b$ values are essentially the same for earthquakes in all these depth ranges (Utsu, 1970, 1971; Kagan and Knopoff, 1980b). Even if the $b$ values were to vary with depth, as suggested by Gibowicz (1974), for example, the changes in $b$ are so small that they cannot be established with statistical confidence with available data. Thus, if geometrical effects have an influence on the $b$ values, this influence is probably small. Our stochastic model for earthquake occurrence (Kagan and Knopoff, 1981) is in support of the view that geometrical effects are not important in influencing the $b$ values; our model is a purely temporal one and invokes no influences of the spatial characteristics of faults to reproduce the features of what is essentially a time-dependent property of an earthquake sequence.

We have observed that the $b$ values for foreshocks and aftershocks in our synthetic catalogs have the same property as their real counterparts, namely that $b_+ - b_- > 0$. Both foreshocks and aftershocks are produced by the same time-delay process in our model; there is no inclusion of any anticipatory ingredient in the modeling. Thus, we surmise that the different magnitude distributions of foreshocks and aftershocks probably can be attributed to the procedures that have been used to identify and sample these events. Therefore, we are led to suspect that the difference of the $b$ values for simulated earthquakes is a statistical artifact of the data processing procedure. Under the rubric of "the data processing procedure," we include the basic idea that individual earthquakes exist in an earthquake sequence and subsequently one can identify individual foreshocks, main events, and aftershocks. As shown previously (Kagan and Knopoff, 1981), even the identification that individual earthquakes exist cannot be made unambiguously—what appears to be an earthquake on the usual level of analysis, becomes a series of events on an alternative model, i.e., a multiple event. What appears to be a sequence of individual earthquakes in the usual model, may be a single event with occasional subdetection threshold rumblings amid the outbursts of activity, on an alternative model.

In particular, we believe the method of selection of foreshocks is the source of the difficulty. There is a time-asymmetry that has been introduced into the typical analysis of an earthquake sequence that is connected with the *a posteriori* designation of a particular shock as the main shock. The same window that was used in the forward direction from the main shock to identify aftershocks was, in our case, also applied in the reverse direction from the main shock to identify foreshocks. If we had applied the window in the forward direction to the foreshocks, most of the events would not have been identified as foreshocks of a later large event; the $b$ value for foreshocks would have been thereby decreased.

In consequence of the above remarks, we offer the following conjecture: *a difference in the b values for the full catalog of aftershocks and foreshocks is an expected and real property of an earthquake sequence, but is an artifact of the data processing procedure, rather than of any differences between anticipatory and postparticipatory physical processes.* In addition, we suspect there are components of *a posteriori* reasoning connected with many of the reports of differences in the literature; it is impossible to assess the statistical significance of such "wishful thinking."

## $b$ VALUES FOR INDIVIDUAL SEQUENCES

We turn to the second problem indicated at the end of the "Introduction." To study individual sequences of earthquakes, we have isolated identifiable sequences from selected catalogs listed in Table 1. We use the case of item 13, Table 1, as an example (Table 5). There are 12 independent events with $M \geqq 4.0$. Of these, 9 have abundant enough foreshocks and 11 have abundant enough aftershocks that maximum likelihood methods can be applied to the determination of $b$ and its standard

TABLE 5

CONDITIONAL $b$ VALUES FOR MAIN SHOCKS IN USGS CATALOG WITH MAGNITUDES GREATER THAN OR EQUAL TO 4.0 (SEE TABLE 1, ENTRY 13)

| No. | Date | Time | Latitude | Longitude | $M$ | Foreshocks | | Aftershocks | | $b_+ - b_-$ | $w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $b$ Value | No. | $b$ Value | No. | | |
| 1 | 02-24-1972 | 15h56m51s | 36.578 | −121.209 | 5.0 | 0.724 ± 0.086 | 189 | 0.830 ± 0.056 | 471 | 0.106 ± 0.103 | 1.035 |
| 2 | 09-04-1972 | 18h04m41s | 36.625 | −121.274 | 4.6 | 0.765 ± 0.122 | 96 | 0.946 ± 0.085 | 213 | 0.181 ± 0.149 | 1.217 |
| 3 | 10-03-1972 | 06h30m02s | 36.802 | −121.529 | 4.8 | 0.741 ± 0.137 | 75 | 0.627 ± 0.142 | 66 | −0.114 ± 0.197 | −0.579 |
| 4 | 01-15-1973 | 09h43m30s | 36.672 | −121.334 | 4.1 | 0.739 ± 0.247 | 23 | 0.474 ± 0.137 | 64 | −0.265 ± 0.282 | −0.938 |
| 5 | 06-22-1973 | 01h29m12s | 36.563 | −121.205 | 4.2 | 1.179 ± 0.244 | 29 | 0.812 ± 0.182 | 44 | −0.367 ± 0.304 | −1.203 |
| 6 | 10-03-1973 | 10h07m27s | 37.204 | −121.585 | 4.6 | — | 4 | 0.792 ± 0.380 | 10 | — | — |
| 7 | 11-12-1973 | 18h17m13s | 37.226 | −121.975 | 4.5 | — | 3 | — | 3 | — | — |
| 8 | 01-10-1974 | 11h22m25s | 36.951 | −121.595 | 4.3 | 0.715 ± 0.286 | 17 | 0.861 ± 0.305 | 16 | 0.146 ± 0.418 | 0.349 |
| 9 | 11-28-1974 | 23h01m25s | 36.916 | −121.478 | 5.1 | 0.830 ± 0.109 | 124 | 0.875 ± 0.086 | 204 | 0.045 ± 0.139 | 0.326 |
| 10 | 01-06-1975 | 11h17m12s | 35.925 | −120.537 | 4.5 | 1.053 ± 0.341 | 14 | 1.021 ± 0.352 | 13 | −0.032 ± 0.490 | −0.064 |
| 11 | 09-13-1975 | 21h20m59s | 35.978 | −120.569 | 4.8 | 0.822 ± 0.270 | 20 | 1.098 ± 0.154 | 70 | 0.276 ± 0.311 | 0.887 |
| 12 | 03-17-1976 | 04h01m53s | 36.819 | −121.133 | 4.3 | — | 8 | 1.092 ± 0.372 | 12 | — | — |
| Average values | | | | | | 0.784 ± 0.050 | | 0.839 ± 0.035 | | 0.053 ± 0.062 | 0.542 |

deviation; we have chosen the cutoff for abundancy at 10 or more dependent events. The $w$ values vary widely. In some cases, the $b$ values for aftershocks are greater than for foreshocks and in some cases, the reverse is true. If we generalize from these cases, we can suppose that, if we were allowed to use selected sequences from a catalog and then be allowed to choose the windowing limits in an *a posteriori* manner, it might be possible, in principle, to display not only that $(b_+ - b_-)$ is greater than zero, but also the opposite result as well.

In the case of Table 5, the weighted mean $b$ value is $0.784 \pm 0.050$ for foreshocks and is $0.839 \pm 0.039$ for aftershocks; the weighting factors are proportional to the reciprocal variance, or what is almost the same thing, to the number of foreshocks or aftershocks. It is not coincidental that these averages are virtually the same as for the chained catalog (Table 1); most of the burden of the weighted entries for the aftershocks is carried by sequences 1, 2, 9 which have the smallest variances. The cases with $(b_+ - b_-) < 0$ are weighted far less in computing the average value of

TABLE 6

ANALYSIS OF INDIVIDUAL FORESHOCK AND AFTERSHOCK SERIES

| Catalog | USGS | NOAA | NOAAB |
|---|---|---|---|
| Table 1 reference | 13 | 27 | 32 |
| Window | I | I | I |
| No. of foreshock series | 9 | 13 | 9 |
| $b$ | $0.784 \pm 0.050$ | $1.322 \pm 0.037$ | $1.416 \pm 0.085$ |
| No. of aftershock series | 11 | 22 | 11 |
| $b_+$ | $0.839 \pm 0.035$ | $1.265 \pm 0.032$ | $1.525 \pm 0.063$ |
| No. of series with both aftershocks and foreshocks | 9 | 12 | 6 |
| $b_+ - b_-$ | $0.053 \pm 0.062$ | $-0.004 \pm 0.053$ | $0.112 \pm 0.134$ |

$(b_+ - b_-)$ than are the cases with positive values of this difference. The weighted mean value of the difference $(b_+ - b_-)$ is $0.053 \pm 0.062$ for the nine sequences, which cannot be considered as significant.

We have listed in Table 6, the analyses for two other catalogs. In no case do the $b_+ - b_-$ values for individual sequences show significant differences of either sign. This is to be expected, since the complete catalogs of dependent sequences described in Table 1 do not display significant differences. In none of the three cases of Table 1 in which the complete catalogs of dependent shocks show statistically significant differences, namely cases 33, 34, and 35, are individual dependent earthquake sequences produced of sufficient length or with sufficiently broad magnitude distributions to permit any statistical analysis whatsoever.

## SUMMARY

Given internally consistent definitions of aftershocks and foreshocks, made in an *a priori* manner, it is doubtful that individual earthquake sequences will show significant differences between the $b$ values for aftershocks and foreshocks. At least, no significant differences were found in the 12 cases we have investigated in Table 5. The effect, if present, is certainly small and is overwhelmed by the variances arising from the fact that the dependent sequences simply are too short. When all the dependent shocks in a catalog are considered, it is only in the rare case of catalogs of great length, that significant differences in $b$ values are found. We are forced to conclude that most reports of consistent differences in $b$ values to be found in the literature are based on "wishful thinking," i.e., an *a posteriori* selection of reportable instances of the phenomena.

Based on our work with synthetic catalogs, we nevertheless believe the effect is likely to be a real one in most earthquake catalogs, but at a very low level of difference. However, the likelihood that this result might be useful as a tool in earthquake prediction is not great since, it must be shown that the observations do not depend on a definition of foreshocks that does not require: (a) an *a posteriori* identification of the largest event in the sequence as the main shock, and (b) adequate reporting of small aftershocks at the same threshold after a large earthquake.

## Acknowledgments

This research was supported by Grant PFR80-08588 of the ASRA program of the National Science Foundation. We are grateful to an anonymous referee for his useful remarks.

## References

Aki, K. (1965). Maximum likelihood estimate of $b$ in the formula $\log N = a - bM$ and its confidence limits, *Bull. Earthquake Res. Inst., Tokyo Univ.* **43,** 237–239.

Andrews, D. J. (1980). A stochastic fault model. 1. Static case, *J. Geophys. Res.* **85,** 3867–3877.

Caputo, M. (1981). A note on a random stress model for seismicity statistics and earthquake prediction, *Geophys. Res. Letters* **8,** 485–488.

Gardner, J. K. and L. Knopoff (1974). Is the sequence of earthquakes in Southern California, with aftershocks removed, Poissonian? *Bull. Seism. Soc. Am.* **64,** 1363–1367.

Gibowicz, S. J. (1974). Frequency-magnitude, depth, and time relations for earthquakes in an island arc: North Island, New Zealand, *Tectonophysics* **23,** 283–297.

Hanks, T. C. (1979). $b$ values and $\omega^{-\gamma}$ seismic source models: implications for tectonic stress variations along active crustal zones and the estimation of high-frequency strong ground motion, *J. Geophys. Res.* **84,** 2235–2242.

Kagan, Y. and L. Knopoff (1978). Statistical study of the occurrence of shallow earthquakes, *Geophys. J.* **55,** 67–86.

Kagan, Y. Y. and L. Knopoff (1980a). Spatial distribution of earthquakes: the two-point correlation function, *Geophys. J.* **62,** 303–320.

Kagan, Y. Y. and L. Knopoff (1980b). Dependence of seismicity on depth, *Bull. Seism. Soc. Am.* **70,** 1811–1822.

Kagan, Y. Y. and L. Knopoff (1981). Stochastic synthesis of earthquake catalogs, *J. Geophys. Res.* **86,** 2853–2862.

Scholz, C. H. (1968). The frequency-magnitude relation of microfracturing in rock and its relation to earthquakes, *Bull. Seism. Soc. Am.* **58,** 399–415.

Smith, W. D. (1981). The $b$-value as an earthquake precursor, *Nature* **289,** 136–139.

von Seggern, D. (1980). A random stress model for seismicity statistics and earthquake prediction, *Geophys. Res. Letters* **7,** 647–650.

Utsu, T. (1970). Aftershocks and earthquake statistics, *J. Faculty Sci. Hokkaido Univ. Japan,* Ser. VII, **3** (Part II), 197–266.

Utsu, T. (1971). Aftershocks and earthquake statistics, *J. Faculty Sci. Hokkaido Univ. Japan,* Ser. VII, **3** (Part III), 379–441.

Wilks, S. S. (1962). *Mathematical Statistics,* John Wiley & Sons, New York, 644 pp.

Wyss, M. and W. H. K. Lee (1973). Time variation of the average earthquake magnitude in Central California in *Proc. Conf. on Tectonic Problems of the San Andreas Fault System,* (School of Earth Science, Stanford University), 24–42.

Institute of Geophysics and Planetary Physics
University of California
Los Angeles, California 90024
Contribution No. 2311

Manuscript received 26 January 1982