DYNAMIC RESPONSE OF JOINTED ROCK MASSES<br>by<br>Gregg Alan Scott B.S., University of Colorado, 1976

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| 16. Abstract (Limit: 200 words) <br> The static behavior of clean rock joints is reviewed and it is determined that the criteria of Patton, Jaeger, Ladanyi and Archambault, and Barton are appropriate for the analysis of rock masses under static loading conditions. An analytical method for three-dimensional limit equilibrium analysis of rock mass bounded by planar discontinuities is discussed, and the means of adapting the method for computer solution are noted. The dynamic shear behavior of clean rock joints is examined and a dynamic limit equilibrium analysis, including provisions for calculating permanent displacements and accounting for dynamic material behavior, is described. |  |  |  |
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Dynamic Response of Jointed Rock Masses
Thesis directed by Associate Professor Stein Sture

Growing public awareness and demand for safe design and construction of potentially hazardous projects in earthquake prone regions has prompted extensive work in earthquake engineering and research toward determining dynamic material response. However, only recently has research been directed toward determining the dynamic behavior of rock joints.

A servocontrolled dynamic direct shear apparatus has been developed at the University of Colorado, for testing rock joint specimens. Independent normal and shear load actuators are capable of dynamic testing up to a frequency of 10 Hz under load or displacement control. Artificial rough and smooth clean sandstone joint specimens, with nominal shear areas larger than $325 \mathrm{~cm}^{2}\left(50 \mathrm{in}^{2}\right)$, were tested under dry displacement controlled conditions. The response of the samples to dynamic shear excitations under constant normal stress was measured utilizing sensitive instrumentation. Shearing velocity was found to have a significant effect on the strength of clean rock joints.

A computer program was developed for performing dynamic three-dimensional limit equilibrium analyses of potentially unstable rock masses. When evaluating the effects of earthquakes is considered to be important, response history rigid block analyses for rock masses can be performed. Time-varying forces
are combined with static forces to determine time-varying resultant forces acting on potentially unstable rock masses. The potential mode of instability and factor of safety are determined at each time step during an earthquake analysis. When the factor of safety drops below 1.0, cumulative permanent displacements can be estimated by double integration of the relative acceleration between the rock mass and its underlying support. Provisions have also been made for including in the analysis dynamic material behavior as determined from the laboratory testing. Accounting for a change in strength with increasing velocity is shown to have a significant effect on cumulative permanent displacements for an example problem of a dam foundation.

Further research is needed to develop comprehensive models for the dynamic behavior of rock joints. Different rock $t y p e s$, effects of water and changes in water pressure, load controlled testing, testing of filled joints, and testing under dynamic normal loads warrant future consideration.

The form and content of this abstract are approved.

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## INTRODUCTION

### 1.1 Background

Growing public awareness and demand for the safe design and construction of potentially hazardous projects, such as large dams in earthquake prone regions, has prompted extensive work in earthquake eng ineering and research toward determining dynamic material response. It has been established that saturated cohesionless soils exhibit significant changes in pore pressure, deformation, and strength characteristics when subjected to cyclic or dynamic excitations. This behavior has been observed both in the field and under laboratory conditions, as summarized by Seed [25], Hardin [14], Woods [32], Finn [9], and Silver [26]. As a result of this behavior, earth structures have failed when subjected to earthquake ground motions, although pseudostatic analyses show them to be stable. The Madison Canyon (Montana) landslide, triggered by the 1959 Hebgen Lake earthquake may represent similar phenomena in rock. A buttress of jointed dolomite collapsed due to the shaking and allowed a landslide containing $33000000 \mathrm{~m}^{3}$ (43000 $000 \mathrm{yd}^{3}$ ) of material to occur. Sliding occurred along the foliation of a schist above the dolomite dipping at $50^{\circ}$ toward the canyon. Twenty-seven people lost their lives [1]. No major engineering accidents are known to have resulted from failure of a
rock.mass subjected to earthquake loading. However, many large dams are founded on nearly saturated rock in seismically active areas. It is therefore important to understand potential foundation response during earthquake conditions.

It is generally accepted that the behavior of a rock mass is controlled to a large extent by the presence of discontinuities $[11,15]$. These discontinuities may include bedding planes, foliation, geologic contacts, joints, seams, shear zones, and faults. The catastrophic abutment failure of the $61-m(200-\mathrm{ft})$ high arch Malpasset Dam in southern France occurred by sliding of a rock wedge on an upstream dipping fault plane. The disaster, which occurred in 1959, resulted in the deaths of over 420 people [17]. Similarly, the catastrophic landslide which occurred at Vajont Dam in Italy resulted in $240000000 \mathrm{~m}^{3}$ (314 $000000 \mathrm{yd}^{3}$ ) of material sliding along the limestone bedding of the reservoir rim. The slide, which occurred in 1963, resulted in a wave of water overtopping the $265-\mathrm{m}(870-\mathrm{ft})$ high arch dam by 100 m (330 ft). About 2,600 people were killed downstream. The arch dam, however, did not fail [17]. The shear strength and shear deformation characteristics of discontinuities are, therefore, often critical to the stability of a rock mass. State-of-the-art determination of these characteristics usually involves extensive in situ and laboratory tests conducted at loading rates appropriate for static conditions. Stability analyses are conducted incorporating static loads, material properties, and effective
stresses computed utilizing a steady-state seepage water pressure distribution [7]. Response of the rock mass to design earthquakes is considered by estimating appropriate ground-motion parameters and external dynamic loads. Stability analyses may be pseudostatic or include estimation of two-dimensional permanent cumulative displacements occurring when the strengths of rock discontinuities are exceeded for short periods of time during a response history earthquake analysis $[22,13,30]$. However, because little is known about the dynamic behavior of rock discontinuities, the material characteristics obtained from static tests are utilized, and no change to the steady-state water pressure distribution is considered.

Only recently has research been directed towards determining dynamic properties of rock discontinuities. Crawford and Curran [5] developed a servocontrolled direct shear apparatus for dynamically testing rock joints. Test results on saw cut joint specimens indicate changes in shear strength occur with changing shear velocity. This has been shown to be an important consideration when estimating permanent displacements of rock masses during earthquakes [6].

Similar research was conducted at the University of Colorado laboratories under the sponsorship of the U.S. Bureau of Reclamation and the National Science Foundation. A servocontrolled dynamic direct shear apparatus was developed, capable of testing specimens in load or displacement control. Artificial rough and smooth clean sandstone joint specimens were tested under dry
displacement controlled conditions. The response of the samples to the dynamic excitations was measured utilizing sensitive instrumentation and recorded using a data acquisition system. Details of the direct shear apparatus and testing are described by Gould [12].

### 1.2 Scope of Work

The results of the testing performed by Gould [12] are examined and compared to the work of Crawford and Curran [5]. Changes in the shear strength of rock joints as the result of dynamic loading are discussed. The development of a computer program for performing dynamic three-dimensional limit equilibrium earthquake analyses of rock masses is also discussed. Provisions for calculating cumulative permanent displacements of a rock mass where the factor of safety drops below 1.0 for short periods of time during an earthquake are included. The effects of dynamic material behavior as determined from the laboratory testing may also be included. The stability of a dam foundation during an earthquake is examined, including the effects of dynamic material behavior on calculated permanent displacements.

Chapter II contains a review of the static behavior of clean rock joints. Chapter III discusses an analytical method for three-dimensional limit equilibrium analysis of a rock mass bounded by planar discontinuities. The dynamic shear behavior of clean rock joints is discussed in Chapter IV. Chapter V contains a description of dynamic limit equilibrium analys is including
provisions for calculating permanent displacements and accounting for dynamic material behavior. Conclusions, suggestions for further study, and a summary are contained in Chapter VI.

CHAPTER II

REVIEW OF STATIC SHEAR BEHAVIOR OF CLEAN ROCK JOINTS

### 2.1 Introduction

The shear strength of rock discontinuities is the most important consideration in determining the stability of a rock mass for near surface projects where a kinematically possible mode of instability exists. Many investigators have proposed empirical criteria for the shear strength of clean joints. However, only the most widely accepted models will be reviewed here.

### 2.2 Smooth Planar Discontinuities

Consider a smooth and planar joint, resulting from a preferred orientation of the rock structure, subjected to a uniform normal stress, $\sigma$, and shear stress, $\tau$, as shown in Figure 2.1. If the incipient joint is initially cemented and the shear stress is increased until the rock slides along the joint, the resulting shear stress vs. Shear deformation curve would be similar to that shown in Figure 2.2. The peak strength occurs just prior to the point where bond is broken, after which the shear stress falls to residual strength. The residual and peak strengths would have been the same if the sample had been unbonded prior to testing. Identical samples tested at different normal stresses will exhibit increasing peak and residual strength with


Figure 2.1 Smooth Joint Under Direct Shear Conditions.


Figure 2.2 Typical Shear Stress vs. Shear Deformation Curves.
increasing normal stress. The resulting strength envelopes, developed by plotting shear strength vs. normal stress, would be similar to those shown in Figure 2.3. The peak strength representative of bonded joints may be defined by the linear equation:

$$
\begin{equation*}
\tau=c+\sigma \tan \emptyset_{p} \tag{2.1}
\end{equation*}
$$

in which $C$ is the cohesion or strength of the sample under zero normal stress, defined by the shear stress intercept, and tan $\emptyset_{p}$ is the slope of the peak strength envelope. The strength of unbonded smooth joints is given by:


Figure 2.3 Typical Shear Strength Envelopes.

$$
\begin{equation*}
\tau=\sigma \tan \emptyset_{r} \tag{2.2}
\end{equation*}
$$

in which $\tan \emptyset_{r}$ is the slope of the residual strength envelope, which indicates zero strength at zero normal stress. Equations (2.1) and (2.2) are often referred to as the Mohr-Coulomb failure criterion.

### 2.3 Rough Discontinuities

The shear stress vs. shear displacement curve for an unbonded rough joint specimen tested under constant normal stress might look similar to that shown in Figure 2.2 for bonded smooth joints. However, since the joint is open, it has essentially zero shear strength at zero normal stress. Tests at different normal stresses would indicate generally increasing shear strength with increasing normal stress. However, the resulting failure envelope would be nonlinear. Intuitively, the effects of roughness on shear strength should be normal stress dependent. At low normal stresses, dilation or riding up on asperities occurs during shearing of a rough joint. At high normal stresses, shearing through asperities occurs, since the work required to shear through the asperities is less than the work required to override them.

### 2.3.1 Patton's Criterion

Patton [23] is often credited as being the first to quant ify the effects of roughness on the shear strength of open joints. Figure 2.4 shows an idealized schematic of a rough open
joint in direct shear. For asperities uniformly inclined at an angle $i$ to the direction of shearing, it $c$ an be shown trigonometrically that the following relationship applies, assuming that the asperities are rigid and not sheared [15].

$$
\begin{equation*}
\tau=\sigma \tan (\emptyset+i) \tag{2.3}
\end{equation*}
$$

Patton verified that this equation is valid at low normal stresses with laboratory tests on artificial joints whose roughness was


Figure 2.4 Rough Joint Under Direct Shear Conditions.
simulated using regularly inclined teeth. At higher normal stresses, the resulting failure envelopes became nonlinear, with the slope of the line approaching that corresponding to the residual friction angle of the material. Patton suggested that a nonlinear, or bilinear envelope such as that shown in Figure 2.5, is appropriate for the shear strength of rough joints.

(a) Failure Envelopes.

(b) Graphical Construction for Jaeger's Criterion.

Figure 2.5 Patton's [23] and Jaeger's [16] Failure Criteria for Rough Joints.

### 2.3.2 Jaeger's Criterion

Jaeger [16] proposed a continuously variable empirical shear strength equation as a more appropriate model for actual rock joints. The equation is written as:

$$
\begin{equation*}
\tau=C_{j}\left(1-e^{-b \sigma}\right)+\sigma \tan \emptyset_{r} \tag{2.4}
\end{equation*}
$$

where $C_{j}$ is the shear strength (cohesion) derived from the asperities, $\emptyset_{r}$ is the residual friction angle of the wall rock, and $b$ is an empirical curve fitting parameter. The parameter $b$ can be determined from test data by sketching lines asymptotic to the peak strength envelope as shown in Figure 2.5. A parameter, $p$, can be calculated as:

$$
\begin{equation*}
p=C_{j}+\sigma \tan \emptyset_{r}-\tau \tag{2.5}
\end{equation*}
$$

The slope of the line $\ln (p)$ plotted against normal stress, $\sigma$, is then equal to $b$ as shown in Figure 2.5. It should be noted that at high normal stress, equation (2.4) reduces to the form:

$$
\begin{equation*}
\tau=C_{j}+\sigma \tan \emptyset_{r} \tag{2.6}
\end{equation*}
$$

### 2.3.3 Ladanyi and Archambault's Criterion

Ladanyi and Archambault [18] also recognized the shortcomings of the bilinear failure envelope when applied to joints other than those with regularly inclined teeth. They suggested that a curved envelope is more appropriate and proposed a gradual transition from dilation at low normal stresses to shearing through
asperities at high normal stresses. The following equation for peak shear strength, shown graphically in Figure 2.6, was proposed:

$$
\begin{equation*}
\tau=\frac{\sigma\left(1-a_{s}\right)\left(\dot{v}+\tan \emptyset_{m}\right)+a_{s} \tau_{r}}{1-\left(1-a_{s}\right) \dot{v} \tan \emptyset_{m}} \tag{2.7}
\end{equation*}
$$

in which $a_{s}$ is the fraction of the joint surface sheared through asperities, $\dot{v}$ is the dilation rate at peak shear strength (change in normal displacement/change in shear displacement), $\tau_{r}$ is the shear strength of intact rock material, and $\emptyset_{\mathrm{m}}$ is the angle of frictional sliding resistance along the contact surfaces of the asperities. It was suggested that the shear strength for intact rock be represented by the criterion proposed by Fairhurst [8].

$$
\begin{equation*}
\tau_{r}=c_{0} \frac{m-1}{n}\left(1+n \frac{\sigma}{c_{0}}\right)^{1 / 2} \tag{2.8}
\end{equation*}
$$

in which $C_{0}$ is the uniaxial compressive strength of the asperities, $n$ is the ratio of uniaxial compressive to uniaxial tensile strength of the asperities, and $m=(1+n) 1 / 2$. Hoek and Bray [15, p. 87] indicate that n is approx imately 10 for most hard rocks. The values of $a_{s}$ and $\dot{v}$ are not always readily available. Therefore, Ladanyi and Archambault proposed the following empirical relationships based on laboratory test data:

$$
\begin{align*}
& \dot{v}=\left(1-\frac{\sigma}{C_{0}}\right)^{k_{2}}(\tan i)  \tag{2.9}\\
& a_{s}=1-\left(1-\frac{\sigma}{C_{0}}\right)^{k_{1}} \tag{2.10}
\end{align*}
$$



Figure 2.6 Ladanyi and Archambault's [18], and Barton's [3] Failure Criteria for Rough Joints.
in which $k_{1}$ is approximately equal to 1.5 and $k_{2}$ is approximately equal to 4 for their laboratory data. It should be noted that at very low normal stresses, when no shearing through asperities occurs ( $\dot{v}=\tan i, a_{s}=0$ ), equation (2.7) reduces to the form of equation (2.3). Similarly, at very high normal stresses, when the asperities are completely sheared ( $a_{\mathrm{S}}=1$ ), equation (2.7) reduces to the shear strength of intact rock.

### 2.3.4 Barton's Criterion

Barton [2] and Barton and Choubey [3] proposed an alternate method for predicting the curved shear strength envelope for rough joints. The following empirical equation, shown graphically in Figure 2.6, was determined from laboratory tests:

$$
\begin{equation*}
\tau=\sigma \tan \left[J R C \log _{10}(J C S / \sigma)+\emptyset_{r}\right] \tag{2.11}
\end{equation*}
$$

in which JCS is the joint wall compressive strength, usually determined from Schmidt rebound hammer tests for natural joints, and JRC is an empirical curve fitting coefficient related to the joint roughness. JRC varies from about 5 for smooth nearly planar joints to 20 for rough undulating joints. Barton and Bandis [4] recommend that JRC be determined from tilt or push tests, where the normal stress is induced by the self-weight of the specimen. The tests should be performed on joints of the natural block size, or block size formed by all intersecting joint sets, of a given rock mass. Barton and Choubey suggest that equation (2.11) is valid as long as the calculated strength does not exceed $\sigma$ tan $70^{\circ}$.

### 2.4 Water Pressure and Effective Stress

The effects of water pressure on the strength of a rock joint are usually accounted for by the effective stress 1 aw [7, 11, 15]. The total normal stress acting across the joint is reduced by the water pressure, $u$, to yield the effective normal stress. For most hard rocks, the material strength characteristics are not significantly affected by the presence of water and
the reduction in shear strength is almost entirely a result of reduced effective normal stress [15]. Goodman and Onishi [10] performed undrained direct shear tests on laboratory joint samples. At high normal stresses during shearing, the joint water pressure was found to increase in the compressing joint, with a net loss in strength according to the effective stress law until slip. At the point of slip, dilation occurred and the water pressure dropped. At low normal stresses, the joints dilated practically from the onset of shearing, in some cases inducing negative water pressures.

### 2.5 Conclusions

The shear strength criteria reviewed in previous sections are appropriate for the analysis of rock masses under static loading conditions. The limit equilibrium approach for the analys is of rock masses bounded by planar discontinuities, described in the next chapter, utilizes shear strengths of the discontinuities and water forces acting normal to them.

Rather than developing new criteria for the behavior of rock joints subjected to dynamic loading, a first approach would be to study the deviations from static behavior that occur. This approach will be pursued in Chapter IV.

## CHAPTER III

## THREE-DIMENSIONAL LIMIT EQUILIBRIUM ANALYSIS

### 3.1 Introduction

Three-dimensional limit equilibrium analyses are widely accepted as one means of studying the stability of rock masses such as slopes and dam foundations [7, 11, 15, 20]. The general approach is to first identify discontinuities forming potentially unstable blocks or wedges. Forces acting on a potentially unstable rock mass are then determined. Normal forces acting on potential sliding planes are computed and the possibility of sliding along a discontinuity or the intersection of two discontinuities under the applied loads is checked. The factor of safety against sliding is computed by dividing the resistance that may ultimately be developed on planes with compressive normal forces, by the driving component of the resultant force. Thus, the factor of safety reflects the resistance that could be developed if failure was imminent and not the mobilized resistance under nonfailure conditions. The assumption is made that the rock block or wedge is rigid and does not deform under the applied loads. As a result of this restriction, no shear resistance is considered to develop in the resisting discontinuities transverse to the potential direction of sliding (see Figure 3.1). Therefore, the maximum normal force, and thus the maximum shear resistance, is considered to develop on


Figure 3.1 Shear Resistance Transverse to Direction of Potential Sliding for Two Plane Wedge.
potential sliding planes. Mantab and Goodman [21] performed three-dimensional finite element studies of rock slopes, representing discontinuities with planar joint elements. It was found that for large ratios of constant joint normal stiffness to joint
shear stiffness, the calculated normal stresses approach those given by the rigid block method as shown in Figure 3.2. Since rock discontinuities usually exhibit large normal stiffness relative to their shear stiffness, the rigid block method of analysis is appropriate in most cases.

Three-dimensional analytical solutions to the rigid block limit equilibrium problem have been proposed [13, 19, 31]. However, most have been adapted to only three potential sliding planes. Von Thun [29] suggested a generalized solution to handle any number of potential sliding planes. The method, which has not been previously documented in detail, will be presented here.

### 3.2 Three-Dimensional Vector Description of a Plane

The orientation of a planar discontinuity can be defined by two angles, strike and dip, as shown in Figure 3.3. The strike angle, $S$, is defined by the azimuth angle measured clockwise from north to the horizontal projection of the plane [15]. The dip angle, $D$, is measured from horizontal at a right angle to the strike, down to the right looking in the direction of the strike. For the discussion that follows, the strike and dip vectors are defined such that their cross product, defining a vector normal to the plane, is directed into the potentially unstable rock block. A right hand coordinate system is established such that the $x$ and $y$ axes define a horizontal plane and the $+z$ axis is directed vertically downward (see Figure 3.3). If $\theta$ is the clockwise angle



Figure 3.3 Definition of Strike and Dip of a Planar Discontinuity, and Coordinate System.
from north to the $+x$ axis, the direction cosines of the strike unit vector (see Figure 3.3) are given by:

$$
\begin{equation*}
\left\{C_{S}\right\} ; C_{S x}=\cos (S-\theta), C_{S y}=\sin (S-\theta), C_{S z}=0.0 \tag{3.1}
\end{equation*}
$$

The direction cosines of the dip unit vector (see Figure 3.3) are given by:

$$
\begin{equation*}
\left\{C_{d}\right\} ; C_{d x}=-C_{s y} \cos (D), C_{d y}=C_{s x} \cos (D), C_{d z}=\sin (D) \tag{3.2}
\end{equation*}
$$

The cross product of the strike and dip unit vectors results in a unit vector normal to the plane:

$$
\begin{align*}
& C_{n x}=\left(C_{s y}\right)\left(C_{d z}\right)-\left(C_{s z}\right)\left(C_{d y}\right) \\
& C_{n y}=\left(C_{s z}\right)\left(C_{d x}\right)-\left(C_{s x}\right)\left(C_{d z}\right) \\
& C_{n z}=\left(C_{s x}\right)\left(C_{d y}\right)-\left(C_{s y}\right)\left(C_{d x}\right) \\
& \quad \text { or }\left\{C_{n}\right\}=\left[C_{s}\right] x\left[C_{d}\right] \tag{3.3}
\end{align*}
$$

A matrix [C] containing the direction cosines of the strike, dip, and normal unit vectors is defined as:

$$
[C]=\left[\begin{array}{ccc}
C_{s x} & C_{s y} & C_{s z}  \tag{3.4}\\
C_{d x} & C_{d y} & C_{d z} \\
C_{n x} & C_{n y} & C_{n z}
\end{array}\right]
$$

### 3.3 Potential Sliding on a Plane

The components of water force acting on each plane are determined by:

$$
\left\{\begin{array}{c}
F_{w x}  \tag{3.5}\\
F_{w y} \\
F_{w z}
\end{array}\right\}=u\left\{\begin{array}{l}
c_{n x} \\
c_{n y} \\
c_{n z}
\end{array}\right\}
$$

where $U$ is the water force acting normal to the plane. The total force components acting on the potentially unstable block, including dead load, water loads, and external forces, are summed and represented by .a vector:

$$
\left\{F_{r}\right\}=\left\{\begin{array}{l}
F_{r x}  \tag{3.6}\\
F_{r y} \\
F_{r z}
\end{array}\right\}
$$

The resultant force is resolved into components normal and parallel to each plane that defines a potentially unstable rock wedge by:

$$
\{T\}=\left\{\begin{array}{l}
T_{1}  \tag{3.7}\\
T_{2} \\
T_{3}
\end{array}\right\}=-[C] \quad\left\{F_{r}\right\}
$$

where $T_{1}$ is the force parallel to the strike, $T_{2}$ is the force parallel to the dip, and $T_{3}$ is the force parallel to the normal unit vector as shown in Figure 3.4 (a). The negative sign in equation (2.6) results in a positive force being directed out of the block, and a compressive normal force is then positive. The total shear force acting on the plane is given by:

$$
\begin{equation*}
F_{s t}=\left(T_{1}^{2}+T_{2}^{2}\right)^{1 / 2} \tag{3.8}
\end{equation*}
$$

The global components of the shear force, shown in Figure 3.4 (b), are then determined as:

$$
\left\{F_{s}\right\}=\left\{\begin{array}{lll}
F_{s x} & F_{s y} & \left.F_{s z}\right\}
\end{array}\right\}=\left\{\begin{array}{ll}
T_{1} & T_{2}
\end{array}\right\}\left[\begin{array}{lll}
C_{s x} & C_{s y} & C_{s z}  \tag{3.9}\\
C_{d x} & C_{d y} & C_{d z}
\end{array}\right]
$$

When the normal force on a plane is compressive, potential sliding along that plane is considered, providing the orientation of the shear force does not direct movement into another plane. This is checked by solving the following equation:


Figure 3.4 Resolution of Forces on a Plane.

$$
K=\left\{F_{s}\right\}\left\{\begin{array}{c}
C_{n x}^{\prime}  \tag{3.10}\\
C_{n y}^{\prime} \\
C_{n z}^{\prime}
\end{array}\right\}=\left\{F_{s}\right\}\left\{C_{n}^{\prime}\right\}
$$

in which $C_{n}^{1}$ contains the direction cosines for the normal of a plane potentially blocking movement. If $K$ is greater than zero, movement is blocked. However, movement along the intersection of two planes is possible. If movement is not blocked the factor of safety against sliding is computed as:

$$
\begin{equation*}
S_{f}=\frac{R_{n}}{F_{s t}} \tag{3.11}
\end{equation*}
$$

where $R_{n}$ is the maximum resisting force based on the shear strength, and normal force or average normal stress acting across the plane.

### 3.4 Potential Sliding on the Intersection of Two Planes

The intersection of two planes is defined by the cross product of their normals.

$$
\begin{align*}
& J_{x}=\left(C_{n y}\right)\left(C_{n z}^{\prime}\right)-\left(C_{n z}\right)\left(C_{n y}^{\prime}\right) \\
& J_{y}=\left(C_{n z}\right)\left(C_{n x}^{\prime}\right)-\left(C_{n x}\right)\left(C_{n z}^{\prime}\right) \\
& J_{z}=\left(C_{n x}\right)\left(C_{n y}^{\prime}\right)-\left(C_{n y}\right)\left(C_{n x}^{\prime}\right) \\
& \quad \text { or }\{J\}=\left[C_{n}\right] \times\left[C_{n}^{\prime}\right] \tag{3.12}
\end{align*}
$$

Since the normals may not be orthogonal, the direction cosines of a unit vector in the direction of the intersection are computed by normalizng the intersection vector.

$$
\begin{align*}
& J_{x y z}=\left(J_{x}^{2}+J_{y}^{2}+J_{z}^{2}\right)^{I / 2}  \tag{3.13}\\
& C_{i x}=J_{x} / J_{x y z} \\
& C_{i y}=J_{y} / J_{x y z} \\
& C_{i z}=J_{z} / J_{x y z} \\
& \text { or }\left\{C_{i}\right\}=\{J\} / J_{x y z} \tag{3.14}
\end{align*}
$$

A matrix [N] containing the direction cosines of the normals to the two planes and their intersection is defined as:

$$
[N]=\left[\begin{array}{ccc}
C_{n x} & C_{n y} & C_{n z}  \tag{3.15}\\
C_{n x}^{\prime} & C_{n y}^{\prime} & C_{n z}^{\prime} \\
C_{i x} & C_{i y} & C_{i z}
\end{array}\right]
$$

The components of force perpendicular to each plane and along their intersection are calculated as:

$$
\{Q\}=\left\{\begin{array}{l}
Q_{1}  \tag{3.16}\\
Q_{2} \\
Q_{3}
\end{array}\right\}=-\left[N^{\top}\right]-1\left\{F_{r}\right\}
$$

where $Q_{1}$ is the force normal to the first plane, $Q_{2}$ is the force normal to the second plane, and $Q_{3}$ is the force along their intersection as shown in Figure 3.5. Restraint against movement is checked as in the case of sliding along one plane, except that:

$$
\begin{equation*}
\left\{F_{S}\right\}=Q_{3}\left\{C_{i}\right\} \tag{3.17}
\end{equation*}
$$

If movement is not blocked, the factor of safety is computed as:

$$
\begin{equation*}
S_{f}=\frac{R_{n}+R_{n}^{\prime}}{Q_{3}} \tag{3.18}
\end{equation*}
$$

in which $R_{n}$ and $R_{n}^{\prime}$ are the max imum resisting forces for the two planes, based on the shear strengths, and normal forces or average normal stresses acting across the planes.


Figure 3.5 Resolution of Forces on Two Planes.

If a compressive normal force is not found to exist across any plane, the rock wedge is considered to be lifted. Similarly, if movement is completely blocked, the rock wedge is considered to be stable.

### 3.5 Displacement Compatibility for Shear Strengths

Von Thun [28] demonstrated the importance of determining shear strengths at compatible shear displacements when considering composite sliding planes. The same consideration applies for sliding on the intersection of two planes. Consider the shear stress vs. shear deformation curves for two discontinuities, shown in Figure 3.6. The peak shear strength of the first discontinuity occurs at a much smaller shear displacement than the second. Thus, simple addition of peak shear strengths is not appropriate for the case of sliding along the intersection of the two discontinuities. The shear strengths utilized in an analysis must, therefore, be developed in conjunction with compatible shear displacements.

### 3.6 Conclusions

The method of analysis described in this chapter can be readily adapted to computer solution. If the appropriate forces acting on a rigid block are digitized for each time step during an earthquake, a response history analys is can be performed for that block. The potential mode of instability and factor of safety $c a n$ be computed for each time step during the earthquake, utilizing a
computer program. Additional details of this type of dynamic analysis will be discussed in Chapter $V$.


Figure 3.6 Shear Stress vs. Shear Deformation for Two Joints.

CHAPTER IV

DYNAMIC SHEAR BEHAVIOR OF CLEAN ROCK JOINTS

### 4.1 Experimental Program

Crawford and Curran [5] have studied the effects of relative velocity on the shear resistance of rock discontinuities in the laboratory. A servocontrolled dynamic direct shear machine was developed for the research, capable of testing specimens approximately 200 mm ( 8 in ) square. The machine has three degrees of freedom (two translations and one rotation) with independently controlled horizontal and vertical actuators, each having a capacity of $250 \mathrm{kN}(56 \mathrm{kips})$. Artificial saw cut joints in four rock types, summarized in Table 4.1, were lapped with silicon carbide grit and displaced at various constant shear velocities under various constant normal loads. Similar research was initiated at the University of Colorado, Boulder, Colorado, prior to the publication of Crawford and Curran's results. The experimental program, sponsored by the National Science Foundation and the U.S. Bureau of Reclamation, is discussed in the following sections.

### 4.1.1 Test Apparatus

A dynamic direct shear apparatus has been developed in the 1 aboratories of the Department of Civil, Environmental, and Architectural Engineering at the University of Colorado, Boulder,

Table 4.1 Summary of Rocks Tested by Crawford and Curran [5]

| Rock Type | Uniaxial Com- <br> pressive Strength <br> $(M P a)$ | Shore Hardness |
| :--- | :---: | :--- |
| Black syenite | 97 | 68 |
| Grey dolomite | 142 | 47 |
| Buff sandstone | 198 | 80 |
| Pink granite | 160 | 62 |

Colorado. A detailed description of the apparatus and its performance characteristics is given by Gould [12]. The apparatus consists of independent servocontrolled horizontal and vertical loading actuators, reaction frames, and shear box fixtures as shown schematically in Figures 4.1 through 4.3. A sample with maximum dimensions of about $200 \times 200 \times 100 \mathrm{~mm}(8 \times 8 \times 4 \mathrm{in})$, and with a maximum shear area of about $400 \mathrm{~cm}^{2}\left(64 \mathrm{in}^{2}\right)$, is potted in the upper and lower shear box compartments using a sulphur capping compound. The shear box compartments are then bolted to support plates and the actuators. The upper shear box compartment is restrained against horizontal motion by roller bearings on the top support plate which rest against the specimen reaction frames. The specimen reaction frames are connected to a stiff and strong structural floor with oversize stiff bolts. The bottom support plate rests on two rows of roller bearings with seven bearings in each row. The applied shear force thus causes horizontal movement
of the lower part of the specimen, relative to the top part, in the direction of the roller bearings.

Closed-loop servocontrolled MTS (MTS Systems Corporation) actuators are used to apply the shear and normal force. Stiff load reaction frames, bolted to the structural floor, permit application of the forces to the specimen. The normal force


Figure 4.1 Dynamic Direct Shear Apparatus (Side View Schematic).


Figure 4.2 Dynamic Direct Shear Apparatus (Schematic Overhead View).


Figure 4.3 Dynamic Direct Shear Apparatus (Assembled with Control Unit).
actuator has a capacity of 736 kN ( 165 kips ) and the shear force actuator has a capacity of 156 kN ( 35 kips ). Each actuator is equipped with a load cell for measuring the applied force and an internal LVDT (linearly varying differential transducer) for monitoring stroke. Several loading patterns are possible through the use of a digital function generator. Each actuator can be operated in the load controlled or displacement controlled mode. Realistically, frequencies up to 10 Hz can be applied. With the normal actuator in load control, the apparatus has four degrees of freedom. These include vertical translation, horizontal translation in the direction of shearing, rotation about a horizontal axis perpendicular to the direction of shearing, and rotation
about a horizontal axis parallel to the direction of shearing. In addition, slight horizontal translation perpendicular to the direction of shearing is allowed. The shearing load and specimen reaction are applied co-linear with the shearing plane, thus reducing moments applied to the specimen.

### 4.1.2 Sample Preparation

Dynamic direct shear tests were performed on homogeneous Loveland Sandstone. The uniaxial compressive strength of the rock, measured from five NX core specimens according to ASTM (American Society for Testing Materials) standards ranged from 136 to $168 \mathrm{MPa}\left(19700\right.$ to $24400 \mathrm{lb} / \mathrm{in}^{2}$ ) and averaged 156 MPa (23 $000 \mathrm{lb} / \mathrm{in}^{2}$ ). Artificial joints were created in the laboratory for testing under controlled conditions. Six-inch-diameter cores were fractured by the Brazilian split cylinder method to create rough tension joints. The fractured cores were then cut into 4-inch cubical shaped blocks with the fracture surface parallel to one set of the cube's faces. Four cubical blocks were glued together with epoxy to form a $200 \times 200 \times 100 \mathrm{~mm}(8 \times 8 \times 4 \mathrm{in})$ specimen with a nominal shear area of $400 \mathrm{~cm}^{2}$ ( $64 \mathrm{in}^{2}$ ) as shown in Figure 4.4. Care was taken when joining the cubical shaped blocks to assure a common shearing plane with perfectly mated upper and lower surfaces. Smooth joint surfaces were prepared by gluing four unfractured cubical blocks together and subsequently sawing and grinding smooth a $400-\mathrm{cm}^{2}\left(64-i n^{2}\right)$ shear surface. The lower edges of the shear plane perpendicular to direction of


Figure 4.4 Typical Direct Shear Sample.
shearing were beveled for all samples to prevent tensile failure near the edge and keep the nominal normal stress constant. However, a nominal shear area greater than $325 \mathrm{~cm}^{2}$ ( $50 \mathrm{in}^{2}$ ) was maintained. Additional details of the samples are discussed by Gould [12].

### 4.1.3 Monitoring System and Data Acquisition

The normal load and shear load are monitored during a dynamic test using the load cells on the MTS actuators. The vertical translation is monitored with the internal LVDT of the normal load actuator. However, because of the small flexibility of the shear force transmission system, LVDT's are attached
directly to the top and bottom of the sample, as shown in Figure 4.5, to measure the relative shear deformation.

The normal load, normal displacement, shear load, and relative shear displacement are recorded with an HP (Hewlett Packard) data acquisition system controlled by an HP desk top computer. The equipment, shown in Figure 4.6, is capable of scanning and recording at the rate of 400 readings per second. The desk top computer records the data on magnetic cassette tape for postprocessing data reduction. Additional details of the monitoring and data acquisition system are discussed by Gould [12].


Figure 4.5 LVDT's for Measuring. Shear Deformation.


Figure 4.6 Data Acquisition System.

### 4.1.4 Test Program

Initial tests were conducted in shear displacement control under relatively constant normal stresses on dry joint samples. Sinusoidal shear displacements were applied to the samples at various amplitudes and frequencies under various normal stresses. The test schedule for each specimen is illustrated in Table 4.2. The sequence of testing is shown by the numbers in parentheses. Five rough samples labeled A, B, C, D, and E were tested in the main test program. Typical plots of shear stress vs. shear displacement are shown in Figures 4.7 through 4.11. Small fluctuations in normal stress were accounted for by the working assumption that the shear strength varies linearly with normal stress,
Table 4.2 Schedule for Ory Displacment Controlled Tests

| Frequency | Shear Displacement Stress | $\begin{aligned} & 10 \mathrm{lb} / \mathrm{in}^{2} \\ & 0.069 \mathrm{MPa} \end{aligned}$ | $30 \mathrm{Ib} / \mathrm{in}^{2}$ 0.207 MPa | $10010 / i n^{2}$ 0.690 MPa | $10 \mathrm{lb} / \mathrm{in}^{2}$ 0.069 MPa | $\begin{aligned} & 30 \mathrm{lb} / \mathrm{in}^{2} \\ & 0.207 \mathrm{MPa} \end{aligned}$ | $100 \mathrm{lb} / \mathrm{in}^{2}$ 0.690 MPa | $\begin{aligned} & 10 \mathrm{lb} / \mathrm{in}^{2} \\ & 0.069 \mathrm{MPa} \end{aligned}$ | $\begin{aligned} & 30 \mathrm{lb} / \mathrm{in}^{2} \\ & 0.207 \mathrm{MPa} \end{aligned}$ | $\begin{aligned} & 100 \mathrm{ib} / \mathrm{in}^{2} \\ & 0.690 \mathrm{MPa} \end{aligned}$ | $\begin{gathered} 500 \mathrm{lb} / \mathrm{in}^{2} \\ 3.45 \mathrm{MPa} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.01 Hz | $\pm 0.05 \mathrm{in}$ $( \pm 0.127 \mathrm{~cm})$ | $\begin{aligned} & 1-1 \\ & (1) \end{aligned}$ | $\begin{aligned} & 1-2 \\ & (8) \end{aligned}$ | $\begin{aligned} & 1-3 \\ & (15) \end{aligned}$ | $\begin{aligned} & 1-4 \\ & (22) \end{aligned}$ | $\begin{aligned} & 1-5 \\ & (29) \end{aligned}$ | $\begin{aligned} & 1-6 \\ & (36) \end{aligned}$ | $\begin{aligned} & 1-7 \\ & (43) \end{aligned}$ | $\begin{aligned} & 1-8 \\ & (50) \end{aligned}$ | $\begin{aligned} & 1-9 \\ & (57) \end{aligned}$ | $\begin{aligned} & 1-10 \\ & (64) \end{aligned}$ |
| 0.1 Hz | $\pm 0.05 \mathrm{in}$ ( $\pm 0.127 \mathrm{~cm}$ ) | $\begin{aligned} & 2-1 \\ & (2) \end{aligned}$ | $\begin{aligned} & 2-2 \\ & (9) \end{aligned}$ | $\begin{aligned} & 2-3 \\ & (16) \end{aligned}$ | $\begin{aligned} & 2-4 \\ & (23) \end{aligned}$ | $\begin{aligned} & 2-5 \\ & (30) \end{aligned}$ | $\begin{aligned} & 2-6 \\ & (37) \end{aligned}$ | $\begin{aligned} & 2-7 \\ & (44) \end{aligned}$ | $\begin{aligned} & 2-8 \\ & (51) \end{aligned}$ | $\begin{aligned} & 2-9 \\ & (58) \end{aligned}$ | $\begin{aligned} & 2-10 \\ & (65) \end{aligned}$ |
| 1.0 Hz | $\pm 0.05$ in $( \pm 0.127 \mathrm{~cm})$ | $\begin{aligned} & 3-1 \\ & (3) \end{aligned}$ | $\begin{aligned} & 3-2 \\ & (10) \end{aligned}$ | $\begin{aligned} & 3-3 \\ & (17) \end{aligned}$ | $\begin{aligned} & 3-4 \\ & (24) \end{aligned}$ | $\begin{aligned} & 3-5 \\ & (31) \end{aligned}$ | $\begin{aligned} & 3-6 \\ & (38) \end{aligned}$ | $\begin{aligned} & 3-7 \\ & (45) \end{aligned}$ | $\begin{aligned} & 3-8 \\ & (52) \end{aligned}$ | $\begin{aligned} & 3-9 \\ & (59) \end{aligned}$ | $\begin{aligned} & 3-10 \\ & (66) \end{aligned}$ |
| 10.0 Hz | $\pm 0.05$ in ( $\pm 0.127 \mathrm{~cm}$ ) | $\begin{aligned} & 4-1 \\ & (4) \end{aligned}$ | $\begin{aligned} & 4-2 \\ & (11) \end{aligned}$ | $\begin{aligned} & 4-3 \\ & (18) \end{aligned}$ | $\begin{aligned} & 4-4 \\ & (25) \end{aligned}$ | $\begin{aligned} & 4-5 \\ & (32) \end{aligned}$ | $\begin{aligned} & 4-6 \\ & (39) \end{aligned}$ | $\begin{aligned} & 4-7 \\ & (46) \end{aligned}$ | $\begin{aligned} & 4-8 \\ & (53) \end{aligned}$ | $\begin{aligned} & 4-9 \\ & (60) \end{aligned}$ | $\begin{aligned} & 4-10 \\ & (67) \end{aligned}$ |
| 0.01 Hz | $\begin{aligned} & \pm 0.1 \mathrm{in} \\ & ( \pm 0.254 \mathrm{~cm}) \end{aligned}$ | $\begin{aligned} & 5-1 \\ & (5) \end{aligned}$ | $\begin{aligned} & 5-2 \\ & (12) \end{aligned}$ | $\begin{aligned} & 5-3 \\ & (19) \end{aligned}$ | $\begin{aligned} & 5-4 \\ & (26) \end{aligned}$ | $\begin{aligned} & 5-5 \\ & (33) \end{aligned}$ | $\begin{aligned} & 5-6 \\ & (40) \end{aligned}$ | $\begin{aligned} & 5-7 \\ & (47) \end{aligned}$ | $\begin{aligned} & 5-8 \\ & (54) \end{aligned}$ | $\begin{aligned} & 5-9 \\ & (61) \end{aligned}$ | $\begin{aligned} & 5-10 \\ & (68) \end{aligned}$ |
| 0.1 Hz | $\begin{aligned} & \pm 0.1 \mathrm{in} \\ & ( \pm 0.254 \mathrm{~cm}) \end{aligned}$ | 6-1 <br> (6) | $\begin{aligned} & 6-2 \\ & (13) \end{aligned}$ | $\begin{aligned} & 6-3 \\ & (20) \end{aligned}$ | $\begin{aligned} & 6-4 \\ & (27) \end{aligned}$ | $\begin{aligned} & 6-5 \\ & (34) \end{aligned}$ | $\begin{aligned} & 6-6 \\ & (41) \end{aligned}$ | $\begin{aligned} & 6-7 \\ & (48) \end{aligned}$ | $\begin{aligned} & 6-8 \\ & (55) \end{aligned}$ | $\begin{aligned} & 6-9 \\ & (62) \end{aligned}$ | $\begin{aligned} & 6-10 \\ & (69) \end{aligned}$ |
| 1.0 Hz | $\begin{aligned} & \pm 0.1 \mathrm{in} \\ & ( \pm 0.254 \mathrm{~cm}) \end{aligned}$ | $\begin{aligned} & 7-1 \\ & (7) \end{aligned}$ | $\begin{aligned} & 7-2 \\ & (14) \end{aligned}$ | $\begin{aligned} & 7-3 \\ & (21) \end{aligned}$ | $\begin{aligned} & 7-4 \\ & (28) \end{aligned}$ | $\begin{aligned} & 7-5 \\ & (35) \end{aligned}$ | $\begin{aligned} & 7-6 \\ & (42) \end{aligned}$ | $\begin{aligned} & 7-7 \\ & (49) \end{aligned}$ | $\begin{aligned} & 7-8 \\ & (56) \end{aligned}$ | $\begin{aligned} & 7-9 \\ & (63) \end{aligned}$ | $\begin{aligned} & 7-10 \\ & (70) \end{aligned}$ |




Figure 4.7 Typical Shear Stress vs. Shear Displacement Curves for Sample $A$.


Figure 4.8 Typical Shear Stress vs. Shear Displacement Curves for Sample B.


Figure 4.9 Typical Shear Stress vs. Shear Displacement Curves for Sample C.


Figure 4.10 Typical Shear Stress vs. Shear Displacement Curves for Sample D.

SHEAR STRESS VS SHEAR DISPLACEMENT


SHEAR STRESS VS SHEAR DISPLACEMENT SAMPLE E (ROUGH) - TEST NO. 6-2
0.207 MPa NORMAL STRESS - $0.1 \mathrm{HZ}-2$ CYCLES (53-54)


SHEAR STRESS VS SHEAR DISPLACEMENT SAMPLE E (ROUGH) - TEST NO. 7-2 0.207 MPa NORMAL STRESS - 1.0 HZ - 4 CYCLES (55-58)


Figure 4.11 Typical Shear Stress vs. Shear Displacement Curves for Sample E.
within the range of normal stress fluctuation. The shear stress was normalized to the assumed constant normal stress by the relationship:

$$
\begin{equation*}
\tau=\tau_{\mathrm{m}} \frac{\sigma_{\text {test }}}{\sigma_{\text {actual }}} \tag{4.1}
\end{equation*}
$$

where $\sigma_{\text {test }}$ is the assumed constant normal stress, $\sigma_{\text {actual }}$ is the actual normal stress, and $\tau_{m}$ is the measured shear stress. The tests at $0.069 \mathrm{MPa}\left(10 \mathrm{lb} / \mathrm{in}^{2}\right)$ normal stress had very large percentages of normal load fluctuation and may be of 1 imited value. Similarly, tests at 10 Hz could not be completely recorded by the data acquisition system. One group of tests at 10 Hz was recorded with a Honeywell visicorder strip chart recorder, and the data were reduced manually.

### 4.2 Velocity Effects

Crawford and Curran [5] concluded that in general the shear resistance of harder rocks decreases with increasing velocity greater than a variable critical velocity, and the shear resistance of softer rocks increases with increasing shear velocity up to a critical velocity. Data from the testing of Crawford and Curran [5] are shown in Figures 4.12 and 4.13. The average strength at a given velocity is normalized with respect to the "static" strength, or strength at the lowest velocity for a given sample and normal stress. The normalized strength is then plotted against shearing velocity. Only the tests on dolomite indicate that the shearing velocity effects are normal stress


Figure 4.12 Normalized Shear Strength vs. Shear Velocity for Tests on Syenite and Dolomite [5].


Figure 4.13 Normalized Shear Strength vs. Shear Velocity for Tests on Sandstone and Granite [5].
dependent. The other tests indicate an increase or decrease in strength with increasing shear velocity that is relatively independent of the normal stress. Crawford and Curran [6] propose a velocity dependent shear strength model that is independent of normal stress. Changes in shear strength are assumed to occur only beyond a critical velocity. The shear strength is assumed to vary linearly with the logarithm of velocity beyond this critical velocity.

The relative shear displacement for the samples tested at the University of Colorado under displacement control is given by:

$$
\begin{equation*}
d=d_{\max } \sin (2 \pi f t+B) \tag{4.2}
\end{equation*}
$$

where $d_{\text {max }}$ is the displacenent amplitude, $f$ is the test frequency, $t$ is time, and $\beta$ is the phase angle. The relative velocity is given by:

$$
\begin{equation*}
v=\frac{d d}{d t}=d_{\max } 2 \pi f \cos (2 \pi f t+\beta) \tag{4.3}
\end{equation*}
$$

The maximum velocity occurs when $\cos (2 \pi f t+\beta)=1$ or where the slope of the displacement vs. time curve (see Figure 4.14) is steepest at displacement $=0$. Thus:

$$
\begin{equation*}
v_{\max }=d_{\max } 2 \pi f \tag{4.4}
\end{equation*}
$$

The average shear strength at maximum velocity, normalized with respect to the average "static" shear strength which occurs at


Figure 4.14 Displacement vs. Time for Displacement Controlled Tests (Conceptual).
maximum velocity and a frequency of 0.01 Hz , is plotted in Figures 4.15 through 4.17 for the rough samples [12]. Normalization was performed independently for each displacement amplitude and normal stress. Results presented by Gould [12] indicate that there is essentially no loss in strength of the samples due to


Figure 4.15 Normalized Shear Strength vs. Shear Velocity for Sample C.


Figure 4.16 Normalized Shear Strength vs. Shear Velocity for Sample D.

Figure 4.17 Normalized Shear Strength vs. Shear Velocity for Samples A Through E
(Excluding Tests at 0.069 MPa Normal Stress).
degradation. This allows a close examination of the shearing rate or velocity effects.

The results from the testing at the University of Colorado indicate deviation from the "static" strength at somewhat lower velocities than those presented by Crawford and Curran [5]. Excluding tests at $0.069 \mathrm{MPa}\left(10 \mathrm{lb} / \mathrm{in}^{2}\right.$ ) normal stress, samples C and $D$ show the most consistent results. The results from sample $C$ indicate that the velocity effects may be normal stress dependent (Figure 4.15). However, if the two tests showing increasing strength at $0.207 \mathrm{MPa}\left(30 \mathrm{lb} / \mathrm{in}^{2}\right)$ normal stress are considered, the dependency on normal stress is less obvious. In addition, the results from sample $D$ indicate that velocity effects are relatively independent of normal stress (Figure 4.16). Trends are not apparent in samples $A, B$, and E. The data from all five rough samples are shown in Figure 4.17. Although, in general, the strength of the samples seems to increase with increasing shear velocity, the scatter in the data is relatively large. For tests conducted at approximately 2.54 mm ( 0.1 in ) displacement amplitude, there are about the same number of tests showing decreasing strength as increasing strength, as the shear velocity increases. These tests should be the most consistent since the samples have been displaced under the same normal stress but a smaller displacement amplitude beforehand, and the samples should approach a steady-state condition. The indicated changes in strength with velocity, therefore, are probably also influenced by properties of the test and sample geometry. This can be further substantiated by examining the
results from a smooth sample. Although a complete series of tests was not performed on a smooth sample, some results are plotted in Figures 4.18 and 4.19. They indicate that the shear strength increases slightly with increasing shear velocity. This is similar to the average response of the rough joints and is not inconsistent with the results of Crawford and Curran. Based on uniaxial compressive strength, the Loveland Sandstone used in the University of Colorado tests is in the middle range of hardness of the rocks tested by Crawford and Curran.

The velocity dependent model proposed by Crawford and Curran [6] therefore seems to be appropriate for the average response of clean rock joints. An increase or decrease in shear strength with respect to the static shear strength c an be made according to:

$$
\begin{align*}
& \text { for } v \leq v_{c} \quad \tau_{d}=\tau_{s}  \tag{4.5}\\
& \text { for } v>v_{c} \quad \tau_{d}=\tau_{s}\left[1+m\left(\log _{10} v-\log _{10} v_{c}\right)\right]
\end{align*}
$$

in which $v$ is the relative velocity across a sliding joint, $v_{C}$ is the critical velocity, $\tau_{d}$ is the dynamic shear strength of the joints, $\tau_{s}$ is the static shear strength of the joint, and $m$ is the change in increase or decrease of shear strength divided by the change in $\log _{10}$ (velocity), defined by the slope of a line similar to those shown in Figures 4.15 and 4.16.


Figure 4.18 Typical Shear Stress vs. Shear Displacement Curves for Smooth Sample 3.

Figure 4.19 Normalized Shear Strength vs. Shear Velocity for Smooth Sample 3.

## CHAPTER V

## DYNAMIC LIMIT EQUILIBRIUM ANALYSIS

### 5.1 Response History Analysis

The three-dimensional limit equilibrium approach described in Chapter III can be readily extended to study the stability of rock masses subjected to time-varying forces, such as those created by an earthquake, if these forces are digitized at discrete time steps. At each time step during the earthquake, threedimensional time-varying forces, such as dynamic loads from a dam and inertia forces acting opposite to the ground acceleration, are added to the static forces. Force resolution is performed and a factor of safety computed at each time step as appropriate. The large number of time steps usually encountered in a dynamic analysis necessitates the use of a computer program, such as that described in the Appendix.

As an example, consider the potential localized instability created by a powerplant excavation at the base of an arch dam as shown in Figure 5.1. Two diverging fault planes cross the foundation from upstream to downstream. A prominent and continuous joint set dips slightly downstream and daylights in the powerplant excavation between the fault planes. A potentially unstable rock wedge including a plane contacting the heel of the


Figure 5.1 Example of a Potentially Unstable Rock Mass.
dam and daylighting at the base of the powerplant excavation can be considered as a worst case. An isometric view of the resulting potentially unstable rock mass is shown in Figure 5.2. The orientations of the planes forming the rock mass are summarized in Table 5.1 along with shear strengths and water forces. The static forces acting on the block are summarized in Table 5.2. The shear strength of the fault planes is approximated by the nonlinear failure envelope shown in Figure 5.3. In reality, the water forces acting normal to each plane are dependent on the stresses transmitted by the dam. An increase in stress closes the discontinuities and changes the seepage characteristics of the rock mass. However, coupled stress-flow problems are difficult to solve, particularly when there are a large number of discontinuities where flow can occur, such as a jointed rock mass. Therefore, a more practical approach utilizes an equivalent continuum seepage analysis with anisotropic permeability to represent preferred directions of flow. The water forces shown in Table 5.1 were determined in this manner.

In addition to the static forces, the dam is subjected to a Richter Magnitude 6.5 earthquake represented by three synthetic accelerograms shown in Figure 5.4 [27]. Three-dimensional forces acting on the potentially unstable rock mass are computed at each time step during the earthquake from a finite element analysis of the dam utilizing the method described by Scott and Dreher [24]. The resulting histories of forces are shown in Figure 5.5. Inertia forces acting on the rock wedge opposite to the directions


Figure 5.2 Isometric View of Example Wedge.


Figure 5.3 Nonlinear Shear Strength Envelope.

Table 5.1 Summary of Planes for Example Rock Mass (See Figure 5.2)

| Plane | Strike | Dip | Water Force <br> $(\mathrm{MN})$ | Area <br> $\left(\mathrm{m}^{2}\right)$ | Shear <br> Strength |
| :--- | :--- | :--- | :---: | :---: | :--- |
| 1 | $\mathrm{~N} 57^{\circ} \mathrm{E}$ | $90^{\circ}$ | 225 | 111 | $\emptyset=45^{\circ}$ |
| 2 | $\mathrm{~N} 30^{\circ} \mathrm{W}$ | $90^{\circ}$ | 202 | 408 | Figure 5.3 |
| 3 | $\mathrm{~N} 57^{\circ} \mathrm{E}$ | $8^{\circ} \mathrm{SE}$ | 1606 | 4617 | $\mathrm{JRC}=15$ |
|  |  |  |  | . | $\mathrm{JCS}=86 \mathrm{MPa}$ <br>  <br> 4 |
| $\mathrm{~N} 58^{\circ} \mathrm{E}$ | $90^{\circ}$ | 0 | - | 0 |  |
| 5 | $\mathrm{~N} 35^{\circ} \mathrm{W}$ | $90^{\circ}$ | 94 | 552 | Figure 5.3 |
| 6 | Horizontal |  | 0 | - | 0 |

Table 5.2 Static Forces Acting on Example Rock Mass

|  | Cross Canyon* <br> $(M N)$ | Upstream/Downstream* <br> (MN) | Vertica1 <br> (MN) |
| :--- | :---: | :---: | :---: |
| Dam | -574 | 4893 | 8363 |
| Weight |  |  | 1134 |

* NOTE: +x is directed in the cross canyon direction toward the left abutment (looking downstream) and $+y$ is directed downstream.


Figure 5.5 Loads From Dam During Richter M6.5 Earthquake [24].
of ground acceleration are also computed for each time step by multiplying the values of acceleration by the mass of the rock block and changing the sign of the resulting force.

The potential mode of sliding as determined from force resolution at each time step, is plotted against time in Figure 5.6. The potential mode of instability changes during the earthquake due to the time-varying forces. The factor of safety against sliding of the example rock mass is plotted against time in Figure 5.7. The factor of safety drops below 1.0 twice during the earthquake. However, the total time the factor of safety is below 1.0 is less than 0.1 second, and it may not be reasonable to assume complete failure.

### 5.2 Permanent Cumulative Displacement

The permanent displacement of a rock mass can be estimated when the factor of safety against sliding is less than 1.0 for a period of time. Von Thun and Harris [30] describe a method for estimating cumulative permanent displacements of slopes subjected to time-varying forces in two directions. The material shear strength is assumed to follow rigid-perfect plastic behavior, ignoring elastic shear deformation. Movement is considered to occur only when the shear strength of the resisting plane is exceeded.

This method can be extended to three dimensions to estimate permanent deformations of rock mâsses. Displacement occurs


Figure 5.6 Modes of Potential Instability vs. Time - I23 Represents Sliding on the Intersection of Planes 2 and 3, P3 Represents Sliding on Plane 3, and I35 Represents Sliding on the Intersection of Planes 3 and 5 (See Figure 5.2).


Figure 5.7 Factor of Safety Against Sliding vs. Time.
only when the factor of safety drops below 1.0. Linear interpolation between input time steps may be utilized to find the exact time at which movement initiates, if it does not occur at an even time step, as shown in Figure 5.8. The time of impending motion,


Figure 5.8 Linear Interpolation Between Input Time Steps.
when the factor of safety is just equal to 1.0 , is determined by:

$$
\begin{gather*}
S_{f}\left(T_{1}+\Delta t_{1}\right)=S_{f}\left(T_{1}\right)+\frac{S_{f}\left(T_{2}\right)-S_{f}\left(T_{1}\right)}{\Delta T} \Delta t_{1}=1.0  \tag{5.1}\\
\Delta t_{1}=\left[1.0-S_{f}\left(T_{1}\right)\right] \frac{\Delta T}{S_{f}\left(T_{2}\right)-S_{f}\left(T_{1}\right)} \tag{5.2}
\end{gather*}
$$

in which the factor of safety at time $T_{1}$ is greater than 1.0 , the factor of safety at time $T_{2}$ is less than $1.0, \Delta t_{1}$ is defined in

Figure 5.8, and $\Delta T$ is the time between $T_{1}$ and $T_{2}$. When the safety factor drops below 1.0, the unbalanced force acting on the unstable rock mass at any time step is given by:

$$
\begin{equation*}
F_{u}=D-R \tag{5.3}
\end{equation*}
$$

in which $D$ is the driving force and $R$ is the resisting force, both of which vary with time. Movement occurs in the direction of the driving force. For the case of sliding on a single plane, the resisting force is a function of the normal stress and shear strength of the single plane, and the driving force is represented by the magnitude and direction of the shear force in the plane. In the case of sliding on an intersection, the resisting force is a function of the normal stress and shear strength for both planes forming the intersection, and the driving force is the component of the resultant force in the direction of the intersection. In case of lifting, the resisting force is zero and the driving force is the total resultant acting on the wedge.

When sliding occurs, the effects of post-peak shear strength behavior must be considered. Thus, residual strengths should be used to estimate resisting forces once movement begins. In addition, if sliding occurs along the intersection of two planes, the shear strengths utilized in the analysis must be developed at compatible displacements.

The relative acceleration between the unstable mass and the underlying rock at a particular instant in time is given by:

$$
\begin{equation*}
a=\frac{F_{u}}{M} \tag{5.4}
\end{equation*}
$$

where $M$ is the mass of the unstable rock. Assuming the acceleration varies linearly between input time steps:

$$
\begin{equation*}
a(t)=a\left(T_{n-1}\right)+\frac{a\left(T_{n}\right)-a\left(T_{n-1}\right)}{\Delta T} t \tag{5.5}
\end{equation*}
$$

where $\Delta T$ is the time step between $T_{n-1}$ and $T_{n}$, and $t$ varies from 0 to $\Delta T$. The relative velocity for each time step, calculated by integration of the relative acceleration is:

$$
\begin{align*}
v\left(T_{n}\right) & =v\left(T_{n-1}\right)+\int_{0}^{\Delta T} a(t) d t \\
& =v\left(T_{n-1}\right)+\left[a\left(T_{n-1}\right) t+\frac{a\left(T_{n}\right)-a\left(T_{n-1}\right)}{\Delta T} \frac{t^{2}}{2}\right] \Delta T \\
& =v\left(T_{n-1}\right)+\frac{\Delta T}{2}\left[a\left(T_{n-1}\right)+a\left(T_{n}\right)\right] \tag{5.6}
\end{align*}
$$

Another integration yields the relative displacement for the time step.

$$
\begin{align*}
d\left(T_{n}\right) & =d\left(T_{n-1}\right)+\int_{0}^{\Delta T} \int_{0}^{\Delta T} a(t) d t \\
& =d\left(T_{n-1}\right)+\left[v\left(T_{n-1}\right) t+a\left(T_{n-1}\right) \frac{t^{2}}{2}+\frac{a\left(T_{n}\right)-a\left(T_{n-1}\right)}{\Delta T} \frac{t^{3}}{6}\right] \frac{\Delta T}{0} \\
& =d\left(T_{n-1}\right)+v\left(T_{n-1}\right) \Delta T+\left[2 a\left(T_{n-1}\right)+a\left(T_{n}\right)\right] \frac{\Delta T^{2}}{6} \tag{5.7}
\end{align*}
$$

Movement stops when the relative velocity becomes zero. The exact time that movement stops, as shown in Figure 5.9, may be determined
by quadratic interpolation between input time steps, since velocity is a quadratic function of time. Inserting $\Delta t_{2}$ (see Figure 5.9) into equation (5.6), setting the equation equal to zero, and solving by the quadratic formula yields:
$\Delta t_{2}=\frac{-a\left(T_{n-1}\right) \pm\left[a\left(T_{n-1}\right)^{2}-\frac{2}{\Delta T}\left[a\left(T_{n}\right)-a\left(T_{n-1}\right)\right] v\left(T_{n-1}\right)\right]^{1 / 2}}{\frac{1}{\Delta T}\left[a\left(T_{n}\right)-a\left(T_{n-1}\right)\right]}$

The solution which lies in the interval between 0 and $\Delta T$ is used to calculate the time at which movement stops. The total displacement magnitude and direction is determined by vector addition of


Figure 5.9. - Quadratic Interpolation Between Input Time Steps.
the displacements at each time step. The direction of movement at each time step is determined by the direction cosines of the shear force. In the case of sliding on a single plane they are given by:

$$
\left\{C_{m}\right\}=\left\{\begin{array}{l}
C_{m x}  \tag{5.9}\\
C_{m y} \\
C_{m z}
\end{array}\right\}=\frac{1}{\left(F_{s x}^{2}+F_{s y}^{2}+F_{s z}^{2}\right)^{1 / 2}}\left\{\begin{array}{l}
F_{s x} \\
F_{s y} \\
F_{s z}
\end{array}\right\}
$$

In the case of sliding along the intersection of two planes the direction cosines of the movement are given by (see Chapter III):

$$
\begin{equation*}
\left\{c_{m}\right\}=\left\{c_{i}\right\} \tag{5.10}
\end{equation*}
$$

The process is repeated for each cycle in which the factor of safety drops below 1.0, and displacements are accumulated vectorally. The displacement calculated in this manner is approximate and only appropriate for small displacements, since the mode of instability is assumed to change instantaneously without repositioning of the block.

### 5.3 Effects of Dynamic Material Behavior

The method described in the previous section results in calculation of the relative velocity for sliding planes at each time step during a dynamic analysis. Shear strengths can, therefore, be revised during the analysis according to the relative velocity calculated for the previous time step, and the velocity dependent shear strength criteria described by equation (4.5).

For the example problem described in previous sections, cumulative displacements were calculated considering no change in strength with velocity. Values of critical velocity and $m$ which envelope the results from the University of Colorado dynamic direct shear tests conducted at displacement amplitudes of 2.54 mm ( 0.1 in ) were also considered for plane 3 only. The results, shown in Table 5.3, indicate that velocity effects can be an important consideration in estimating cumulative permanent displacements of potentially unstable rock masses.

Table 5.3 Velocity Effects on Cumulative Permanent Displacements of Example Rock Mass

| Critical Velocity <br> $(\mathrm{mm} / \mathrm{s})$ | m | Cumulative Permanent <br> Displacement $(\mathrm{mm})$ |
| :---: | :---: | :---: |
| - | 0 | 10.6 |
| 0.3 | 0.119 | 0.5 |
| 0.2 | -0.052 | 51.1 |

The example problem is based on work performed for Auburn Dam. However, details were modified for illustration purposes.

## CHAPTER VI

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FURTHER STUDY

The behavior of rock discontinuities under static loading conditions has received much attention and research in recent years. Research into the behavior of such discontinuities subjected to dynamic loading is just beginning. Crawford and Curran [5] studied the effects of shear velocity on the shear strength of clean rock joints utilizing a sophisticated servocontrolled dynamic direct shear apparatus. A similar machine was developed at the University of Colorado as described by Gould [12]. The results from testing in both machines indicate that shearing velocity can have a pronounced effect on discontinuity shear strength. The effects seem to be independent of the normal stress across the discontinuity. The strength appears to increase or decrease linearly with the logarithm of velocity beyond a critical threshold velocity. However, there is a relatively large amount of scatter in the data, and further verification work is warranted. The approach adopted in this research has been to examine changes from static behavior of rock joints, when they are subjected to dynamic loads. Although this is probably a good first approach, further research may lead to comprehensive material models for dynamic loading.

When evaluating the effects of earthquakes is important, response history analyses for rock masses can be performed. The three-dimensional limit equilibrium approach is a cost effective means of performing such analyses for masses bounded by planar discontinuities. If the dynamic loads and accelerations are digitized for each time step during an earthquake, analytical computer techniques $c a n$ be utilized. At each time step during the earthquake, an analysis is performed. Dynamic and static forces, including water forces acting normal to the discontinuities, are summed and resolved. The potential mode of instability and factor of safety against sliding are determined. If the factor of safety drops below 1.0 for short periods of time during the earthquake, cumulative permanent displacements can be estimated by double integration of the relative acceleration. The effects of relative shearing velocity can be included and appear to have a significant effect on the estimated displacement.

The rigid block analysis is appropriate when the potentially unstable rock mass is bounded by planar discontinuities. The ratio of normal stiffness to shear stiffness of the discontinuities must also be large, which is usually the case in nature. Localized stress concentrations cannot be considered using this method. The permanent displacement calculated for dynamic analyses is approximate and only appropriate for small displacements, since the mode of instability is assumed to change instantaneously without repositioning of the block. The major weakness in the analytical approach is the treatment of water forces. Currently
they are assumed to be constant throughout the earthquake. However, it is reasonable to expect that the dynamic loads may cause an increase in stress in certain areas of the rock, resulting in closing of discontinuities and an increase in water pressure. Similarly, once movement starts, the discontinuities can be expected to dilate, thus reducing water pressures.

Research is continuing at the University of Colorado to examine changes in water pressures and effects of water during dynamic shearing of sandstone joints subjected to constant normal stress. The effects of cycling in load control rather than displacement control are also being studied. Different rock types may behave differently, and dynamic studies of joints in several rock types are warranted. Joints containing infilling may also behave differently and should be studied under dynamic loading. In reality, the normal load acting across a discontinuity is not constant during an earthquake. Further testing where the normal load is cycled at various phase angles to the shear load are also justified.

Research into the behavior of jointed rock masses during dynamic earthquake loading is just beginning. It is hoped that the need for such research is realized and that further work can be performed.

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A. User's Guide

Consistent units must be used throughout the input tapes.
Input for PROGRAM RIGID from TAPE5
Card 1 FORMAT (5A10, $5 \mathrm{X}, \mathrm{A} 5, \mathrm{I} 5)$
Column Variable Description

1-50 JOB Title information to be printed on output.

56-60 IBLOCK Identification for rigid block being anal yzed.

61-65 IPROBN Nonzero problem identification number.

Card 2 FORMAT (30I1)

| Column | Variable | Description |
| :---: | :---: | :---: |
| 1 | IOC(1) | Input 0 for static analysis. Input 1 for dynamic analysis with input time-varying loads and accelerations on TAPE4. |
| 2 | IOC(2) | Input 1 for extra output. Input 0 for standard output (0 should be specified for large dynamic runs). |
| 3 | IOC(3) | Input 1 if cumulative permanent displacements of unstable rock masses are to be estimated. This can only be specified for dynamic analyses. Input 0 to suppress this option. |

Card 3 FORMAT (5F10.0)

| Column | Variable | $\frac{1}{c}$ Description |
| :--- | :--- | :--- |
| $1-10$ | WT | Weight of rigid block (positive). |
| $21-30$ | BMASS | Mass of rigid block. |
| $31-40$ | QY | External static force acting on <br> rigid block in x-direction. |
| $41-50$ | QZ | External static force acting on <br> rigid block in y-direction. |
| External static force acting on <br> rigid block in z-direction. |  |  |

NOTE: The program uses a right hand cartesian coordinate system with $x$ and $y$ forming a horizontal plane and $+z$ directed vertically downward.

Card 4 FORMAT (2F10.0)

| Column | Variable | Description |
| ---: | :--- | :--- |
| $1-10$ | CC | Cohesion of treatment concrete. |
| $11-20$ | TPHIC | Tangent of the friction angle of <br> treatment concrete in the planes of <br> discont inuities. |

NOTE: Treatment concrete can only be considered if the area of the plane is also specified (see card 6A). Resistance due to concrete treatment will only be considered if a positive normal force acts on the treated plane. In dynamic analyses, the concrete cohesion will be set to zero once the factor of safety drops below 1.0 .

Card 5 FORMAT (F5.1)
Column Variable Description
1-5 XAXIS Clockwise angle (degrees) from north to $+x$ axis.

One set of cards 6A through 6H for each plane forming the block (up to 19 planes may be input).

Card 6A FORMAT (I5, 2F10.0, I5, 4F10.0)

| Column | Variable | Description |
| :---: | :---: | :---: |
| 1-5 | I | Plane identification number (sequential - no missing numbers). |
| 6-15 | STRIKE (I) | Clockwise angle (degrees) from north to strike direction vector. |
| 16-25 | DIP(I) | Dip angle (degrees) measured down to the right from horizontal looking in the direction of the strike vector. |
| 26-30 | IPHI ( I ) | Shear strength code. Input 1 if shear strength for plane I is to be represented by cohesion and a friction angle. |
|  |  | Input 2 if the shear strength is to be represented by a normal stress vs. shear strength curve. |
|  |  | Input 3 if Barton's shear strength criterion is to be used. |
|  |  | Input 4 to use Ladanyi and Archambault's criterion for clean rough joints. |
|  |  | Input 5 to use Ladanyi and Archambault's criteria for filled joints ${ }^{1}$. |
|  |  | Input 6 to use Jaeger's criterion. |
| 31-40 | UP(I) | Water uplift force acting normal to plane I. |
| 41-50 | AREA(I) | Area of planar face of plane I (needed if cohesion, Barton's parammeters, Ladanyi and Archambault's parameters, Jaeger's criterion, or shear strength curve is specified) including concrete treatment area. |

1Ladanyi, B., and G. Archambauit, "Shear Strength and Deformability of Filled Indented Joints," Int. Symposium on the Geotechnics of Structurally Complex Formations, Capri, Vol. 1, 1977, pp. 317-326.

| Column | Variable |
| :---: | :---: |
| 51-60 | Description |
| AREAC(I) | Area of treatment concrete in plane I <br> (may be specified only if AREA(I) is <br> also specified). |
| $71-75$ | PHIR(I) |
| Residual friction angle (degrees) to <br> be used once movement initiates. <br> Input only if IOC(1) and IOC(3) are |  |
| 1. Input 0 if strength is to remain |  |
| unchanged. |  |

NOTE: The strike and dip must be described such that the plane normal is directed into the block. The normal direction is found by crossing the strike vector into the dip vector through an angle of $90^{\circ}$. The strike must be changed by $180^{\circ}$ if the normal is not directed into the block, and the dip must be changed appropriately.

Skip card 6B if IPHI(I) is not equal to 1.
Card 6B FORMAT (2F10.0)

| Column | $\frac{\text { Variable }}{1-10}$ | PHI(I) |
| ---: | :--- | :--- |$\quad$| Friction angle (degrees) for plane I. |
| :--- |
| $11-20$ |$\quad$ COHESN(I) $\quad$| Cohesion value for plane I (units |
| :--- |
| must be compatible with area units). |

Skip cards 6C and 6D if IPHI(I) is not equal to 2.
Card 6C FORMAT (I5)

| Column |  |
| :--- | :--- |
| $1-5$ | Variable |$\quad$| Nescription |
| :--- |
| Nhear of points used to describe |
| shear sth curve for plane I. |

Cards 6D FORMAT (2F10.0)
One card for each of NSSP points.

| Column | Variable |
| :---: | :---: |
| 1-10 | Description |
| $11-20$ | $\operatorname{TAU}(J, I)$ | | Normal stress for shear strength |
| :--- |
| point J, plane I (units must be |
| compatible with area units). |

Skip card 6E if IPHI(I) is not equal to 3.
Card 6E FORMAT (3F10.0)

| Column | Variable | Description |
| :---: | :---: | :---: |
| $1-10$ | FJRC(I) | Joint roughness coefficient, JRC, <br> for Barton's shear strength criterion. |
| 21-30 | FJCS(I) | Unconfined compressive strength of <br> joint wall rock, JCS, for Barton's <br> shear strength criterion. |
|  | Residual friction angle (degrees) of <br> joint for Barton's shear strength <br> criterion. |  |

Skip card 6F if IPHI(I) is not equal to 4.
Card 6F FORMAT (3F10.0)

| Column | Variable | Description |
| :---: | :---: | :---: |
| 1-10 | PHI(I) | Friction angle (degrees) of asperi- <br> ties for Ladanyi and Archambault's <br> shear strength criterion. |
| 21-30 | FJCS(I) | Compressive strength of joint wall <br> rock for Ladanyi and Archambault's <br> shear strength criterion. |
|  | "i" angle (degrees) of asperities |  |
| for Ladanyi and Archambault's shear |  |  |
| strength criterion. |  |  |

Skip card 6 G if $\mathrm{IPHI}(\mathrm{I})$ is not equal to 5.

Card 6G FORMAT (4F10.0)

| Column | Variable | Description |
| :---: | :---: | :---: |
| 1-10 | FJRC(I) | Thickness of filled joint for <br> Ladanyi and Archambault's shear <br> strength criteria. |
| 21-20 | AM(I) | Amplitude of roughness of filled <br> joint for Ladanyi and Archambault's <br> shear strength criteria. |
| $31-40$ | COHESN(I) | Undrained cohesion of infilling <br> material for Ladanyi and Archambault's <br> shear strength criteria. |
|  | Undrained friction angle (degrees) <br> of infilling material for Ladanyi <br> and Archambault's shear strength <br> criteria. |  |

Skip card 6 H if $\operatorname{IPHI}(\mathrm{I})$ is not equal to 6.
Card 6H FORMAT (3F10.0)

| Column | Variable | $\frac{\text { Description }}{1-10}$ |
| :---: | :---: | :---: |
| COHESN(I) | Cohesion of asperities for Jaeger's <br> criterion. |  |
| 11-20 | PHI(I) | Residual friction angle (degrees) of <br> wall rock for Jaeger's criterion. |
| $21-30$ | AM(I) | Exponent b for Jaeger's criterion <br> (positive value). |

Skip card 7 if IVEL(I) is not equal to 1.
Card 7 FORMAT (2F10.0)

| Column | Variable | Description |
| :---: | :---: | :---: |
| CRITV(I) | Critical velocity. No change in <br> shear strength occurs below this <br> relative velocity. |  |
|  | SLOG(I) | Slope of line defining change in <br> shear strength with increasing <br> velocity, defined as (change from <br> static strength $) /(\log 10$ velocity $)$. |

The input for plane description cards must end with 20 in columns 4 and 5.

Card 8 FORMAT (20I1).
Enter a 1 in each column corresponding to the plane identification number of any plane that can contribute no resistance to sliding.

Another problem may be started here. Otherwise terminate the run with a blank card.

Input for PROGRAM RIGID from TAPE4
TAPE4 is used to supply time-varying forces and ground accelerations for response history dynamic analyses.

Card 1 FORMAT (215)

| Column | Variable | Description |
| :---: | :---: | :---: |
| 1-5 | NTS4 | Number of time steps with forces and accelerations on TAPE4 (It is recommended that each time step be less than or equal to about 0.02 second). |
| 6-10 | IB | Block identification number. |
| Card(s) 2 | FORMAT (7E14.6) | Need NTS4 cards |
| Column | Variable | Description |
| 1-14 | TIME | Time during the earthquake. |
| 15-28 | FX | $x$-direction force at time TIME. |
| 29-42 | FY | $y$-direction force at time TIME. |
| 43-56 | FZ | $z$-direction force at time TIME. |
| 57-70 | AX | $x$-direction ground acceleration at time TIME. |
| 71-84 | AY | $y$-direction ground acceleration at time TIME. |
| 85-98 | AZ | z-direction ground acceleration at time TIME. |

Files used by PROGRAM RIGID
TAPE4 - File containing time-varying forces and ground accelerations representing earthquake loading.

TAPE5 - File containing program input.
TAPE6 - Printer formatted ouput file.
TAPE7 - Formatted output file containing information that can be used for plotting. Consult listing of SUBROUTINE SRBFS for further details.

PROGRAM RIGID was developed on a CDC CYBER/170 computer and is written in FORTRAN. Modifications may be required to run the program on other machines. The program can be run in timeshare or batch modes using the NOS operating system.

$$
\begin{aligned}
& \text { THE FOLLOWING IS AN EXAMPLE OF A CCL (CYBER CONTROL LANGUAGE) } \\
& \text { PROCEDURE FILE THAT MAY BE USED TO RUN PROGRAM RIGID. THE PROCEDURE } \\
& \text { FILE IS STORED UNDER THE NAME PRORIG ON A PERMANENT FILE. }
\end{aligned}
$$

. PROC, PRORIG2, INPUT, ABC, TAPE4, TAPE7
HEADING,ABC. NAME
HEADING,ABC. RIGID
GET.RIGID/UN=ERO2219.
GET. INPUT, TAPE4/NA.
HEADING,ABC. NAME
HEADING,ABC. RIGID
GET.RIGID/UN=ERO2219.
GET. INPUT, TAPE4/NA.
RIN. $\operatorname{RIGBIN,~INPUY,~ABC,~TAPE4.~TAPET.~}$
REPLACE, TAPE7/NA
REWIND, ABC.
ROUTE, ABC, DC=LP.
DAYFILE.OF

TO RUN THE PROGRAM TYPE


PROGRAM LISTING
$\infty$

COMMON /FOUR/ WT,BMASS, QX,OY,QZ,TPHIC,CC,FXU,FYU,FZU
COMMON /SIX/ FSMI,FACTOS,TIMM1,TINE,TFLAG3,ZCS(3) COMMON /FOUR/ WT, BMASS, QX,OY,QZ,TPHIC,CC,FXU,FYU,FZU
COMMON /SIX/ FSMI,FACTOS,TIMM1,TIME,IFLAG3,ZCS(3)
commence by reading in heading and control data
$\cup \cup$

PROGRAM RIGID
+(INPUT, OUTPUT, TAPE4,TAPE7,TAPE5 = INPUT, TAPEG=OUTPUT)

COMPUTES THE FACTOR OF SAFETY AGAINST MOVEMENT OF A RIGID BLOCK
COMNON NPL, IOC(30) , JOB(5), IPROBN, DATE, IELOCK, IPAGE, XAXIS

COMPUTES THE FACTOR OF SAFETY AGAINST MOVEMENT OF A RIGID BLOCK
COMNON NPL, IOC(30) , JOB(5), IPRDBN, DATE, IELOCK, IPAGE, XAXIS

0000 REWIND 4
$100 \operatorname{READ}(5,1020)(\operatorname{JOB}(I), I=1,5)$, IBLOCK, IPROBN
$\operatorname{IF}(E O F(5)) 150,110$
$110 \operatorname{IF}(\operatorname{IPROBN}, \mathrm{EO} .0)$ GO TO 150
$\operatorname{READ}(5,1030)(\operatorname{IOC}(I), \mathrm{I}=1.30)$







$\cup \cup$
CAIL DPAGE (4)
WRITE(6.1210) $\times$ AXIS

|  | continue reading input with plane description |
| :---: | :---: |
|  | NPL $=0$ |
|  | CALL DPAGE(3) WRITE (6, 1000) |
|  | READ(5,1200) I, Strike(i), dip(i), iPhi(f), UP(1), AREA(1) |
|  | +areac(i), Phir (1), ivel(i) |
|  | If(i.eq. 20 ) Go to 200 |
|  | call dpage (1) |
|  | WRITE(6, 1010)I, STRIKE(I), dip(i), IPHI(I), UP(I), AREA(I). |
|  | +areac (I) |
|  | GO TO ( $110,120,140,150,150,160$ ) IPHI (I) |
| 110 R |  |
|  | Call dpage (1) |
|  |  |
|  | PHI (I) $=$ PHI (I) +0.017453295 |
|  | go to 170 |
| 120 R | READ (5,1040)nssp |
|  | DO $130 \mathrm{~J}=1$, NSSP |
|  | Read (5, 1020)Sig(u, i), tau(u, i) |
|  | call dpage(1) |
|  | WRItE( 6,1050 ) Sig( J, i) , tau (J, I) |
| 130 | continue |
|  | go to 170 |
| 140 R | READ ( 5 , 1060) FJRC(I), FJCS(1).PHI(I) |
|  | Call dpage (4) |
|  | WRITE (6, 1070)FJRC(I).FJCS(I), PHI(I) |
|  | go to 170 |
| 150 R | Read (5, 1080) Phit(i), fucs(i), fiali) |
|  | Call dpage (4) |
|  | WRIte(6, 1090)PHI(I), FJCS(I), FIA (I) |
|  | PHI (I) $=$ PHIL (I) +0.0174532925 |
|  | FIA (I) =FIA(I) *0.0174532925 |
|  | IF(IPHII(I).EQ.4) Go to 170 |
|  |  |
|  | CALL DPAGE(5) |
|  | WRITE(6, 1120)FJRC(I), AM(I), COHESN(I), PhiU(I) |
|  | PHIU( I$)=\mathrm{PHIU}(\mathrm{I}) * 0.0174532925$ |
|  | go to 170 |
| 160 R |  |
|  | call opage (4) |



1130 FORMAT ( 76 X, , RESIDUAL PHI $=*$ F $10.2, *$ WILL BE*/


COMMON /SIX/ FSM1,FACTOS,TIMM1,TIME.IFLAG3, ZCS(3)
IF (IOC(2).EQ.1)CALL DPAGE (5)
IF (IOC(2).EQ.1)WRITE $(6,1090)$
$F X=A X=0$.
$F Y=A Y=0$
IF(IOC(1).EQ.O)GO TO 100



1060 FORMAT (2F10.2,5H 2,2I5,3E10.4)

IF(PHIR(I).EQ.O.)GO To 100
PHI (I) $=$ PHIR(I)

IF (COHESN(I).GT.O.O. AND.FIAC(I).GE. 15.0.AND.FIA(I).LE. 30.0

+ .AND.STRAT.LT O.5)GO TO 180
FIA(I)=FIA(I)*0.0174532925
$\mathrm{C}=\mathrm{COHESN}(\mathrm{I})+\operatorname{SIGMA}+\operatorname{TAN}(\operatorname{PHIU}(\mathrm{I}))$
$T A U R=F M *(T A U R-C)+C$
GD TO 200
180 GIA TO 200 FIA(I) +0.0174532925
FIE $=A \operatorname{TAN}(F M+T A N(F I A(I)))$
TAUR $=\operatorname{COH} \operatorname{ESN}(I) /(1 .-T A N(F I E) * T A N(P H I U(I)))$
+ +SIGMA +TAN(PHIU(I)+FIE)

IF (AREA 200
TAUR = COHE

TO 220
XP(-AM(I)
TO 210
IF (IVEL.(I). NE. 1)GO
IF (V.LE.CRITV(I)) GO TO 210
CALL RESVEL(RESIST, I)
RESIST=RESIST+(TPHIC*SIGMA+C $)$
CALL RESVEL (RESIST, I)
RESIST $=$ RESIST $+(T P H I C * S I G M A+C$



## IF(IOC(2).EQ. 1)WRITE 6,1010 )I, SIGMA, TAUR

$$
\begin{aligned}
& \text { GO TO } 230 \\
& \text { CALL DPAGE(2) } \\
& \text { WRITE } 6.1020)
\end{aligned}
$$

$$
\begin{aligned}
& \text { C } \\
& \text { C ROUTINE TO ACCOUNT FOR VELOCITY EFFECTS ON SHEAR STRENGTH } \\
& \text { C }
\end{aligned}
$$

IF (V.EQ.O. OR.CRITV(I).EQ.O.)GO TO 10
RESIST=RESIST*(SLOG(I)*(ALOG1O(V)-ALOG1O(CRITV(I)))+1.) RESIST

10 CALL DPAGE (2)
1000 FORMAT (/* STOP - ZERO VELOCITY IN RESVEL*)

00




[^0]



 000
N


000

0.00
$-100000 E-01$
$.200000 E-01$
$.300000 E-01$
$.400000 E-01$
$.500000 E-01$
$.600000 E-01$
$.700000 E-01$
$.800000 E-01$
$.900000 E-01$
$.100000 E+00$
$-110000 E+00$
$.120000 E+00$
$.130000 E+00$
$.140000 E+00$
$.150000 E+00$
$.160000 E+00$
$.170000 E+00$
$.180000 E+00$
$.190000 E+00$
$.200000 E+00$
$.210000 E+00$
$.220000 E+00$
$.230000 E+00$
$.240000 E+00$
$.250000 E+00$
$.260000 E+00$
$.270000 E+00$
$.280000 E+00$
$.290000 E+00$
$.300000 E+00$
$.310000 E+00$
$.320000 E+00$
$.330000 E+00$
$.340000 E+00$
$.350000 E+00$
$.360000 E+00$
$.370000 E+00$ $a$
$\sigma$
$\sigma$








THE fOLLOWING is a portion of the program output for the example
problem described by the previous input.

| PROBLEM | NUMBER | 1 |
| :---: | :---: | :---: |
|  | PAGE | 1 |
| DAT | E 82/1 |  |





##  <br> PRO

|  |  | $.9096 E+08 \cdot 1308 E+10.7851 E+09$ NORMAL $=.5341 E+08 \quad$ PLANE $E P \quad .8417 E-03.7388 E-02.1044 E$ | normal |  | 123 SHEAR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{TIME}_{\text {FACTOR }}=4.61$ SAFETV INCREMENTAL $X, y, z$ |  movements for time ste | $.2208 \mathrm{E}+09 \cdot 1206 \mathrm{E}+10 \cdot 7497 \mathrm{E}+09$ NORMAL $=.5718 E+O 9$ SHEAR $=$ $\qquad$ | $3 E+10$ | ALP | $18 \mathrm{E}+02$ |
| rime $=4.62$ FACTOR OE SAFETY | total X.y,z forees <br>  | -3226E+09.1128E+10-7560E+09 NORMAL $=.5859 E+09$ PLANE 5 EP $2265 E-03.1113 E-O 2.1596$ | mal | . $8838 \mathrm{E}+08$ | 135 SHEAR |
| MOVEMENT STOPS AT TOTAL X,Y,Z COMPONE TOTAL MOVEMENT FOR |  | 23E-02 . 1676E-01 . 2382E-O2 |  |  |  |
| ${ }_{\text {FACTOR }}^{\text {TIME }}$ OF SAFETY |  |  | norma | ¢ +0 | 135 |
|  |  |  |  | 2674E+09 | 135 SH |
| ${ }_{\text {fime }}^{\text {factor }}$ |  |  | normal | $3246 \mathrm{E}+09$ | 135 SHEA |
| $\underset{\text { FACT }}{\substack{\text { THE }}}$ |  |  | NORM | 3135+0 | 135 SHEA |
|  |  |  | NORM | 3751 E + 09 | 35 |
|  |  | $\begin{array}{rcc} .5950 E+O 9 & .9481 E+09 & .1487 E+10 \\ \text { NORMAL. }=.1326 E+10 \quad \text { PLANE } 5 \end{array}$ | norma | 3876 | 135 |
| $\underset{\text { cimet }}{\text { fime }}$ |  |  | Norma | $3612 \mathrm{t}+0$ | 135 SHEAR |
| $\mathrm{THME}_{\text {FACTOR }}^{\text {c }}$ |  |  | NORM | $3131{ }^{+}+$ | 135 |
|  |  |  | NORMAL | $2687 \mathrm{~F}+09$ | 135 Sti |
| ${ }_{\text {THAME }}$ |  |  | normal | $2154 \mathrm{E}+09$ | 135 SH |
| ${ }_{\text {fluctor }}^{\text {fine }}$ |  |  | normal | . $14206+09$ | 135 |
| ${ }_{\text {factior }}$ |  |  | NoRM | 5876E +08 | 135 SHEAR |
|  |  | ${ }^{2} \mathrm{~N}$ |  |  |  |

$\begin{array}{r}\text { PROBLEM NUMBER } \\ \text { PAGE } \\ \text { DATE } \\ \hline\end{array} \quad 82 / 10 / 15$


[^0]:    DATE
    RACt
    Extr
    
    

