SEISMIC DESIGN RESPONSE BY AN ALTERNATIVE SRSS RULE

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16. Abstracts

The Square Root of the Sum of the Squares (SRSS) procedures are often used to obtain seismic design response. The design inputs for such procedures are usually defined in terms of psuedo velocity or acceleration response spectra. Erroneous results have been obtained with these existing SRSS procedures, especially in the calculation of response where high frequency effects dominate. Here an alternative SRSS procedure is developed using the so-called mode acceleration approach of structural dynamics. The design input in this procedure is defined in terms of, though, primarily relative acceleration spectra but also relative velocity spectra. The relative spectra can be related to psuedo spectra. For a given number of modes to be used in the analysis the new SRSS rule proposed here will predict a more accurate response value than the rules which use psuedo spectra as input, for systems with or without dominant high frequency mode effects.

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Dynamics	Relative Velocity Spectra	MODAL CORRELATION
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Structural Analysis	Psuedo Acceleration Spectra	DESIGN RESPONSE
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1. INTRODUCTION

For the evaluation of seismic design response of a linearly behaving structural system, the modal analysis approach with pseudo-acceleration spectra[1-3] as seismic design input is commonly used. The eigenvalue analysis of an analytical model of the structure is performed to obtain dynamic characteristics like mode shapes, natural frequencies and participation factors. These quantities are used with pseudo-acceleration spectra to obtain the maximum responses in each mode which are then combined by the commonly used procedure of the square-root-of the-sum-of-the-squares, usually abbreviated as SRSS. Several other forms[4-9] of this procedure are available in the literature, which mainly deal with the special problem of the combination of responses of the so-called closely spaced modes. Herein, these modified procedures will also be referred to as SRSS rules of different types.

One of the attributes associated with the modal analysis approach is that only a first few modes are necessary in the calculation of response as the contribution of high frequency modes to the response is usually considered small. It is probably true for some regular multistory buildings which are on flexible side. However, there are also many situations of practical interest where high frequency modes may contribute significantly to the total response. Though the displacement response of a structural system may be unaffected by higher modes, it is known that responses like support forces, bending and torsional moments, axial forces, etc., may often have significant contributions from such modes[10,11]. In such cases the use of only a first few modes in the calculation of response can possibly introduce large numerical errors[11]. This error is due to the so-called "missing mass" effect and can, at least theoretically, be removed by inclusion of higher modes. However, evaluation of higher modes is often beset with large numerical errors and thus inclusion of all modes may not necessarily improve the accuracy of the results in the normal course (unless eigensolution algorithms are carefully chosen).

The second problem associated with the high frequency modes is due to their combination with other modes in the calculation of design response. The modal responses of high frequency modes with frequencies higher than the forcing function frequency are rather known to be strongly correlated even though their frequencies may be fairly well separated. Therefore, as pointed out by Kennedy[11], the higher modes require a different mode combination rule than what are normally used. Thus some improved mode response combination rules[11-14] have recently been proposed. Conceivably another situation could also occur where the responses in two modes with widely different frequencies should be combined as absolute sum. For example, when the low frequency mode response is near its peak, several oscillations of the higher frequency mode may occur to produce an additive effect. Such is the case in the combination of the two cyclic thermal stress components which are caused by seasonal and diurnal temperature changes[15].

It is shown here that in the aforementioned situations, the SRSS rules[5,8] which implicitly assume the input to be white noise to develop the expressions for modal correlation coefficient provide inaccurate results. The SRSS approach proposed by the writer and his colleagues[6] earlier which does not make such an assumption about the input, however, can provide mathematically exact results (at least for the mean square response) as long as all the modes are calculated accurately and used in the approach. To avoid calculation of all modes, especially high frequency modes, an alternative method is necessary. A new SRSS formulation based on the method of "mode acceleration" approach[16-18] is, therefore, developed here.

2. ANALYTICAL FORMULATION

For a multi-degrees-of-freedom structural system excited by ground motion, say, in one direction, the equations of motion can be written in the following form:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{r\}\ddot{x}_g(t) \tag{1}$$

where [M], [C] and [K], respectively, are the mass, damping and stiffness matrices of the system; $\{x\}$ = response vector of relative displacements of the structure with respect to ground; $\{r\}$ = ground displacement influence vector[19]; and $\ddot{x}_g(t)$ = ground acceleration. It becomes necessary to use modal analysis approach for the calculation of design response if the design ground motions are characterized by ground response spectra[1-3]. In the modal analysis approach, the solution vector $\{x\}$ is expressed as a linear combination of normal modes using the expansion theorem as follows:

$$\{x\} = [\Phi]\{q\} \tag{2}$$

where $[\Phi]$ = modal matrix with its columns representing the normal modes of the system and $\{q\}$ = the vector of the principal coordinates. Substitution of equation (2) in (1) and with some standard manipulations involving orthogonal properties of the normal modes, the following decoupled equation in terms of jth principal coordinate is obtained:

$$\ddot{q}_j + 2\beta_j\omega_j\dot{q}_j + \omega_j^2 q_j = -\gamma_j\ddot{x}_g(t), \quad j = 1,2..n \tag{3}$$

where the suffix j is associated with the mode number; ω_j = modal frequency; γ_j = modal participation factor defined as $\{\phi_j\}^T[M][r\}/\{\phi_j\}^T[M]\{\phi_j\}$; $\beta_j = \{\phi_j\}'[C]\{\phi_j\}/(2\omega_j\{\phi_j\}^T[M]\{\phi_j\})$ is the so-called modal damping ratio; and T over a vector quantity represents its transpose. Equation (3) assumes that $\{\phi_j\}^T[C]\{\phi_k\} = 0$; that is, matrix [C] can be diagonalized by the normal modes and that it is, what is commonly called as, a proportional damping matrix. A more general case where the matrix [C] is not of this type can also be treated[20] but will not be persued any further here.

For a given excitation $\ddot{x}_g(t)$, equation (3) can be solved to define $\{x\}$ by equation (2). For a linear structure, other response quantities, which are linearly related to $\{x\}$ can also be obtained as

$$S(t) = \sum_{j=1}^{N} \zeta_j q_j(t) \tag{4}$$

where ζ_j = value of the response quantity S(t) in the jth mode of vibration; N = the number of degrees-of-freedom or the total number of modes of the system. ζ_j is also called the mode shape of the response quantity which can be obtained from the displacement mode shape $\{\phi_j\}$ by a simple linear transformation for a linearly behaving structure.

To obtain the design response, one should collectively consider a large number of ground motions in equation (1) or (3). Thus, it is appropriate to model the ground acceleration $\ddot{x}_g(t)$ as a random process. The design response is then related to the autocorrelation function (or more commonly, to the root mean square value) of the response. Algebra is considerably simplified if $\ddot{x}_g(t)$ is assumed to be a stationary random process. Although earthquake motions are inherently nonstationary, the assumption of stationarity has been extensively used to develop some acceptable analytical procedures in structural dynamics for earthquake induced ground motion. The commonly used SRSS procedure has its basic roots in such an assumption. Furthermore some corrections can be applied to the response calculated with stationary assumptions to incorporate the nonstationarity effects. For a zero mean stationary $\ddot{x}_g(t)$ characterized by a spectral density function $\Phi_g(\omega)$, the stationary mean square response of S(t) can be shown to be defined as[6]:

$$E[S^2(t)] = \sum_{j=1}^{N} \zeta_j^2 \gamma_j^2 I_1(\omega_j) + 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} \zeta_j \zeta_k \gamma_j \gamma_k$$

$$[A_1 I_1(\omega_j) + A_2 I_2(\omega_j)/\omega_j^2 + A_3 I_1(\omega_k) + A_4 I_2(\omega_k)/\omega_k^2] \qquad (5)$$

in which

$$I_1(\omega_j) = \int_{-\infty}^{\infty} \Phi_g(\omega) |H_j|^2 \, d\omega \qquad (6)$$

$$I_2(\omega_j) = \int_{-\infty}^{\infty} \Phi_g(\omega) \omega^2 |H_j|^2 \, d\omega \qquad (7)$$

where H_j is the complex frequency response function defined as = $1/(\omega_j^2 - \omega^2 + 2\beta_j \omega_j \omega i)$. The coefficients A_1, A_2, A_3 and A_4 are obtained as the solution of the following simultaneous equation:

$$[G]\{A\} = \{P_a\} \qquad (8)$$

where the elements of the 4x4 matrix [G] and vector $\{P_a\}$ are defined in Appendix I.

It is noted that $I_1(\omega_j)$ and $I_2(\omega_j)$, respectively, represent the mean square values of the relative displacement and velocity responses of an oscillator with frequency = ω_j and damping ratio = β_j excited by the stationary ground excitation $\ddot{x}_g(t)$. They can be related to the relative displacement (or psuedo-acceleration) and relative velocity spectra through their respective peak factors[21]. (Here, the peak factor is defined as a factor by which root mean square response should be multiplied to obtain a characteristic value of maximum response, like response spectrum value). Assuming that the peak factors relating the mean square values of the relative displacement and relative velocity responses to their respective response spectra are the same and represented by C_f, $I_1(\omega_j)$ and $I_2(\omega_j)$ can be written in terms of response spectrum values as:

$$I_1(\omega_j) = R_a^2(\omega_j)/\omega_j^4 c_f^2 \tag{9}$$

$$I_2(\omega_j) = R_v^2(\omega_j)/c_f^2 \tag{10}$$

where $R_a(\omega_j)$ and $R_v(\omega_j)$, respectively, are the psuedo acceleration and relative velocity response spectrum values at frequency ω_j and damping ratio β_j.

To obtain design response, the root mean square response of S(t) must be multiplied by its peak factor. This peak factor will depend upon the characteristics of the random process S(t) and could be somewhat different from that used in equations (9) and (10). Analytical procedure developed by Vanmarcke[21] and others can be used to define these peak factors. However, for simplicity and without loosing any significant accuracy the peak factor for S(t) can be assumed to the same as for relative displacement and relative velocity, i.e. c_f. Thus multiplying equation (5) by c_f^2 and using equations (9) and (10), the design response, S_d, can be expressed in terms of psuedo-acceleration and relative velocity spectra as follows:

$$S_d^2 = \sum_{j=1}^{N} \zeta_j^2 \gamma_j^2 R_a^2(\omega_j)/\omega_j^4 + 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} \zeta_k \zeta_j \gamma_j \gamma_k$$

$$[\{A_1 R_a^2(\omega_j) + A_2 \omega_j^2 R_v^2(\omega_j)\}/\omega_j^4 + \{A_3 R_a^2(\omega_k) + A_4 R_v^2(\omega_k)\omega_k^2\}\omega_k^4] \quad (11)a$$

$$S_d^2 = \sum_{j=1}^{N} S_j^2 + 2 \sum_{j=1}^{N} \sum_{k=j+1}^{N} S_{jk} \quad (11)b$$

where S_d is the design response of quantity S(t); $S_j = \gamma_j \zeta_j R_a(\omega_j)/\omega_j^2$ = maximum modal response in jth mode; and S_{jk} = the cross-modal response.

Equation (11) defines an SRSS rule for the combination of modal response. The single summation terms define the normally used SRSS rule. The double summation terms account for interaction of various modes and must be considered if the frequencies are closely spaced and **also when the high frequency modes contribute significantly.** A quantative measure of modal interaction is the modal correlation coefficient[4] defined as:

$$\rho_{jk} = \frac{S_{jk}}{S_j S_k} \quad (12)$$

which in view of equation (11)a is clearly seen to depend not only on the two modal frequencies (or their ratio) but also on the response characteristics of the input. Response independent expressions for ρ_{jk} have also been obtained[8] under a rather restrictive assumption of white noise input, wherein, besides damping, this coefficient is shown to depend only on the frequency ratio, $r_j = \omega_j/\omega_k$. It is high or very nearly equal to 1.0 when the frequencies are close i.e. when the frequency ratio is nearly equal to 1.0. It has also been shown[8] that this coefficient calculated on the basis of white noise input assumption is not very different from the coefficient calculated with nonwhite filtered Kanai-Tajimi type of spectral density function extending over the frequency domain of 0 to ∞.

A more close inspection, however, shows that the character of this correlation is significantly altered if the input spectral density functions with cut off frequency limits are considered or if it is evaluated at very high frequencies for a filtered white noise (Kanai-Tajimi) spectral density function. A seismic input with an upper limit on its constituent frequencies is more realistic than the input defined by a Kanai-Tajimi type of spectral density function with rather slowly diminishing tail which imply the existence of some very high frequency components in the input motion. Ground response spectra often prescribed for design clearly indicate absence of such high frequencies in the input motion. For example, the spectra as defined in Reference 22, do seem to suggest an absence of motion with frequencies higher than about 20 to 25 Hz, as no amplification beyond 33 Hz is seen.

Figure 1 shows the variation in ρ_{jk} with frequency ratio $r_j = \omega_j/\omega_k$ and damping ratios $\beta_j = \beta_k = .02$ for a broad band Kanai-Tajimi type of spectral density function with a cut-off frequency of 20 Hz. The graphs are plotted for ω_j (herein referred to as central frequency) values of 2,5,10,15,20,30,50 and 100 Hz. Also plotted is the curve obtained for the white noise input. (In this case only one curve is obtained for all frequencies as it only depends on the frequency ratio.) The difference in the curves obtained for the two different types of inputs is significant. Of special significance to the work presented later in this paper is the magnitude of correlation between two high frequency modes, (curves for $\omega_j > 30$ cps). This is erroneously predicted by the white noise formulation[5,8] Figure 1. This, as shown later, also results into significant errors in the calculated responses especially when the high frequency modes are dominant. On the other hand, however, equation (11) can still be used for an accurate calculation of response of structures with or without high frequency modes as long as all such modes are considered in the summation processs in the equation. Thus in many cases where high frequency modes are also important, a large number of modes may have to be evaluated by eigenvalue analyses to obtain accurate evaluation of design response. The situation can, however, be significantly improved by an alternative formulation where only a first few modes falling within the frequency range of the input will be adequate for an accurate evaluation of design response. This is based on the so-called "mode acceleration"[16-18] approach of structural dynamics. A preliminary account of this was presented by the writer in Reference 23.

3. ALTERNATIVE SRSS APPROACH

In the previous section equation (11) was developed directly from equation (4), where the principal coordinates $q_j(t)$s represent the modal displacements. Thus the preceeding approach is also referred to as the "mode displacement" approach. Using equation (3), equation (4) can also be written in an alternative form as

$$S(t) = -\sum_{j=1}^{N} \zeta_j \{\gamma_j \ddot{x}_g(t) + 2\beta_j \omega_j \dot{q}_j + \ddot{q}_j\}/\omega_j^2 \qquad (13)$$

where now the response S(t) is expressed as a function of $\ddot{q}_j(t)$ which is the jth mode acceleration, and thus the name "mode acceleration" approach. (Modal velocity $\dot{q}_j$ also enters equation (13), but being associated with damping term it usually has a much smaller contribution.) Using equation (13), the autocorrelation function of S(t) is obtained as

$$E[S(t_1)S(t_2)] = \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{\zeta_j \zeta_k}{\omega_j^2 \omega_k^2} [\gamma_j \gamma_k \; E\{\ddot{x}_g(t_1)\ddot{x}_g(t_2)\}$$

$$+ \gamma_j (2\beta_k \omega_k E\{\ddot{x}_g(t_1)\dot{q}_k(t_2)\} + E\{\ddot{x}_g(t_1)\ddot{q}_k(t_2)\})$$

$$+ \gamma_k (2\beta_j \omega_j E\{\ddot{x}_g(t_2)\dot{q}_j(t_1)\} + E\{\ddot{x}_g(t_2)\ddot{q}_j(t_1)\})$$

$$+ 4\beta_j \beta_k \omega_k E\{\dot{q}_j(t_1)\dot{q}_k(t_2)\} + E\{\ddot{q}_j(t_1)\ddot{q}_k(t_2)\}$$

$$+ 2\beta_j \omega_j E\{\dot{q}_j(t_1)\ddot{q}_k(t_2)\} + 2\beta_k \omega_k E\{\dot{q}_k(t_2)\ddot{q}_j(t_1)\}] \qquad (14)$$

where E(•) denotes the expected value of (•).

Various auto and cross correlation terms in equation (14) can be evaluated in terms of input autocorrelation through equation (3). See Appendix II for details. Considering the stationary value of response, as done in the derivation of equation (11), each term can be expressed in terms of the input spectral density function and frequency response functions H_j and H_k. The mean square response is then obtained by the substitution $t = t_1 = t_2$ and is given as follows:

$E[S^2(t)] =$

$$\Big(\sum_{j=}^{N} \frac{\gamma_j \zeta_j}{\omega_j^2}\Big)^2 E(\ddot{x}_g^2) + \sum_{j=1}^{N} (\zeta_j \gamma_j/\omega_j^2)^2 \int_{-\infty}^{\infty} [2\omega_j^2 \omega^2 (1-2\beta_j^2) - \omega^4] |H_j|^2 \Phi_g(\omega) d\omega$$

$$+ \sum_{j=1}^{N} \sum_{k=j+1}^{N} \frac{\zeta_j \zeta_k \gamma_j \gamma_k}{\omega_j^2 \omega_k^2} \int_{-\infty}^{\infty} \omega^2 [(4\beta_j \beta_k \omega_j \omega_k + \omega^2)(H_j H_k^* + H_j^* H_k)$$

$$+ 2\omega_j^2(1-4\beta_j^2)|H_j|^2 + 2\omega_k^2(1-4\beta_k^2)|H_k|^2 - 2\omega^2(|H_j|^2 + |H_k|^2)$$

$$+ 8\omega^2(\omega_j \beta_j - \omega_k \beta_k)\{\omega_j \omega_k(\beta_j \omega_k - \beta_k \omega_j) - \omega^2(\beta_j \omega_j - \beta_k \omega_k)\}|H_j|^2 |H_k|^2] \Phi_g(\omega) d\omega \tag{15}$$

where in equation (15) several terms of equation (14) have been combined. To obtain the design response the mean square value defined by equation (15) is to be multiplied by c_f^2. With relative velocity spectrum as defined by equation (10) and the relative acceleration spectrum, $R_r(\omega_j)$, and maximum ground acceleration, A_g, defined as

$$R_r^2(\omega_j) = c_f^2 \int_{-\infty}^{\infty} \omega^4 |H_j|^2 \Phi_g(\omega) d\omega \tag{16}$$

$$A_g^2 = c_f^2 E[\ddot{x}_g^2(t)] \tag{17}$$

and using equation (15), the design response S_d can be expressed as follows:

$$S_d^2 = c_f^2 E[S^2(t)] \tag{18}$$

$$S_d^2 = (\Sigma \frac{\gamma_j \zeta_j}{\omega_j^2})^2 A_g^2 + \sum_{j=1}^{N} [2\omega_j^2(1-2\beta_j^2) R_v^2(\omega_j) - R_r^2(\omega_j)](\zeta_j \gamma_j / \omega_j)^2$$

$$+ \sum_{j=1}^{N} \sum_{k=j+1}^{N} 2 \frac{\zeta_j \zeta_k \gamma_j \gamma_k}{\omega_j^2 \omega_k^2} [\omega_j^2 (1-4\beta_j^2) R_v^2(\omega_j) + \omega_k^2 (1-4\beta_k^2) R_v^2(\omega_k)$$

$$- R_r^2(\omega_j) - R_r^2(\omega_k) + B_1 \omega_k^2 R_v^2(\omega_j) + B_2 R_r^2(\omega_j) + B_3 \omega_k^2 R_v^2(\omega_k) + B_4 R_r^2(\omega_k) \}] \tag{19}$$

where $B_1 \ldots B_4$ are obtained from the solution of the following simultaneous equations:

$$[G]\{B\} = \{P_b\} \tag{20}$$

where the elements of matrix [G] and vector $\{P_b\}$ are defined in Appendix I.

It will now be shown that the first term on the right hand side of equation (18) is the squared value of the static response induced by the inertia forces corresponding to the maximum value of the ground acceleration applied statically, with no regard to any dynamic effects. That is, it can be obtained from the solution of the following statics problem

$$[K]\{x_s\} = [M]\{r\}A_g \tag{21}$$

where the vector $\{r\}$ on the right hand side of equation (21) ensures that inertia forces are applied only in the direction of excitation. The vector $\{x_s\}$ defines the displacements of this static problem. The static values, $\overline{s}$, of a response quantity for the displacement vector $\{x_s\}$ is obtained by a simple linear transformation of the following form:

$$\overline{S} = \{k_i\}^T \{x_s\} \tag{22}$$

where $\{K_i\}^T$, defined in terms of stiffness coefficients, transforms the displacement $\{x_s\}$ into the response $\overline{S}$. To show that $\overline{S}^2$ obtained from equation (21) and (22) is the same as in the first term of equation (19), $\{x_s\}$ is expanded in terms of N independent modal vectors of the system as follows:

$$\{x_s\} = \sum_{k}^{N} q_{sk}\{\phi_k\} \tag{23}$$

where q_{sk} are the coefficients of expansion. Substitution of equation (23) in (21) and premultiplication by $\{\phi_j\}^T$, gives

$$\{\phi_j\}^T[K]\sum_{k=1}^{N} q_{sk}\{\phi_k\} = \{\phi_j\}^T[M]\{r\}A_g \tag{24}$$

Invoking orthogonality of modes,

$$q_{sj} = \frac{\{\phi_j\}^T[M]\{r\}}{\{\phi_j\}^T[K]\{\phi_j\}} A_g \tag{25}$$

$$q_{sj} = \frac{\{\phi_j\}^T[M]\{r\}}{\omega_j^2\{\phi_j\}^T[M]\{\phi_j\}} A_g = \frac{\gamma_j}{\omega_j^2} A_g \tag{26}$$

Thus from equation (23)

$$\{x_s\} = A_g \sum_{j=1}^{N} \frac{\gamma_j}{\omega_j^2}\{\phi_j\} \tag{27}$$

Substituting in equation (22),

$$\overline{S} = A_g \sum_{j=1}^{N} \frac{\gamma_j}{\omega_j^2}\{k_i\}^T\{\phi_j\} \tag{28}$$

$$\overline{S} = A_g \sum_{j=1}^{N} \frac{\gamma_j \zeta_j}{\omega_j^2} \tag{29}$$

Thus, the first term on the right hand side of equation (18) can be replaced by $\overline{S}^2$ which can be obtained by a static analysis of the system with inertia forces applied statically.

It is noted that in equation (19), the input is required to be defined in terms of relative acceleration and relative velocity spectra, rather than psuedo acceleration and relative velocity spectra as in equation (11). It is noted here that Hadjian[14] was probably the first one to advocate the use of relative acceleration spectra as input. However, the mode combination rules proposed herein is different from the one proposed by Hadjian and, also, it has been developed on the basis of an entirely different formulation. No further comparison of these two rules has been reported here.

If any two types of spectra are known, the third one can be calculated fairly accurately by the following expression

$$R_a^2(\omega_j) \cong A_g^2 - R_r^2(\omega_j) + 2(1-2\beta_j^2)\omega_j^2 R_v^2(\omega_j) \tag{30}$$

Equation (30) is approximate in as much as it assumes that the peak factors applied to the mean square values of the relative and psuedo response quantities to obtain their respective maximum values are the same at the same oscillator frequency and damping values. The expression is exact, however, if the response spectrum terms in equation (30) as replaced by their respective root mean square values.

Here it is seen that the two SRSS rules, Equation (11) and (19), are similar in as much as both require two forms of seismic inputs. Also they must give the same numerical value of a response quantity as they use the same equation in its two different but equivalent forms. Indeed, it has been verified that these two approaches provide exactly the same numerical results as long as the seismic inputs used in the two approaches are consistent as per equation (30) and a complete set of modes are used in the analysis. Thus no special advantage seems to be immediately apparent in the use of the new SRSS rule, equation (19), over the old SRSS rule of equation (11).

The advantages in the use of equation (19) over (11) are, however, numerical and become apparent when a first few modes only are used in the evaluation of design response. In such a case, equation (19) provides a very accurate value of the design response whereas the use of equation (11), the old rule based on mode displacement approach, can lead to significant errors especially if the high frequency mode contribute significantly.

In Table 2 are shown some numerical results obtained for the bending moments in the columns of a 3-story, nine degrees-of-freedom system shown in Figure 2. The seismic input to the system has been defined by a spectral density function of the following form with a cut-off frequency, ω_c = 20 cps.

$$\Phi_g(\omega) = \sum_{i=1}^{3} S_i \frac{\omega_i^4 + 4\omega_i^2\beta_i^2\omega^2}{(\omega_i^2 - \omega^2)^2 + 4\beta_i^2\omega_i^2\omega^2} \; ; \; -\omega_c \leq \omega \leq \omega_c \qquad (31)$$

The numerical values of the parameters S_i, ω_i and β_i for this density function are shown in Table 1. This density function has been used earlier in earthquake engineering studies and represents a Kanai-Tajimi type broad-band input equivalent to the broad-band response spectrum defined in Reference 22. To make the frequency content of such an input more realistic, the frequencies higher than ω_c = 20 cps have been eliminated.

The system shown in Figure 2 represents a three story building with rigid floors connected by four corner columns. Relative values of the masses and stiffnesses of the system are shown in the figure. The system properties have been adjusted such that the frequencies are on high side, and thus the contribution from higher frequencies is significant. Also, the system has some very small eccentricity between the centers of mass and stiffness, which tend to create some closely spaced frequencies.

Table 2 shows the root mean square (RMS) values of the bending moment in two different columns at the ground floor obtained by various mode combination rules. As the root mean square value is proportional to the maximum value the results shown in Table 2 could be considered as the representative values of design response. The results under Case A have been calculated using all 9 modes of the system shown in Fig. 2. The fundamental frequency of the system is 20.8 cps, a little higher than the highest frequency in the ground motion. As the system has closely spaced frequencies, double summation terms have also been used. The results in Col. 2 are obtained using equations (11) or (19) with all modes taken in the calculations. Thus the RMS values in this column are called the exact values. The bending moment values in next five columns are shown in terms of the ratios of the RMS values obtained by various mode response combination rules to the exact value shown in Col. 2. Thus the closer a ratio is to 1.0 the more accurate is the calculated value by a rule. In Col. 3 are shown the values obtained by equation (19), the alternative SRSS rule (also called the new SRSS rule). A value = 1.00, of course, shows that the two approaches are equivalent. The value in Col.4 is obtained from equation (11), and thus it obviously also gives a ratio of 1.0 if all the modes of the system are used. Also shown are the results obtained by Ref. 5 (and 9), conventional SRSS and absolute sum procedures, respectively, in Col. 5,6 and 7 of the table. The values corresonding to Ref. 5 were obtained for the earthquake duration parameter of 10 seconds. In References 5 and 9 the evaluation of the cross modal response terms is based on the assumption that the input is white noise. Though there are some procedural differences between these two approaches, they provide almost identical values which are only about 75% of the correct RMS values. Thus even though the complete set of modes are included, the values calculated by these two approaches are inaccurate mainly because the correlation between the high frequency modes, developed on the basis of white noise as input, is imporperly accounted for. The conventional SRSS rule, on the other hand, completely ignores the modal interaction. Thus the calculated response can be grossly inaccurate, as is evident from the results in Col. 6. Furthermore, the absolute sum method is seen to provide a very good estimate of the true response. It is because all the modes having frequencies higher than the highest frequency in the input motion are in phase with each other and have strong positive correlation.

As alluded to earlier, the main advantage of the new SRSS rule is that it can provide an accurate value of design response even with a first few modes. The results shown under Case B in Table 2 clearly demonstrate this. The value in Col. 2 is again the exact RMS value obtained with the complete set of modes. Whereas the values in remaining columns have been calculated with only the first three modes. The Col. 3 values have been calculated with equation (19) and are shown to provide a very accurate estimation of the correct values. Equation (11), which even though considers the correlation between modes correctly, does not provide a good estimate of correct values; this will in general be the case when the response is affected by the higher modes. Other mode combination rules, References 5 and 9 and also conventional SRSS and absolute sum procedures, are again seen to provide inaccurate values of response. The inaccuracies in the values are directly related to the missing mass effect and *are* seen to become worse if the system is more rigid as is shown by the results for Case C. Here the system has been made significantly stiffer with the lowest frequency = 41.6 cps, for which again the new SRSS rule gives an excellant estimate of correct response value even with just a first few modes whereas the rules based on the mode displacement formulations, and also the other rules, provide grossly inaccurate responses. For the case when the high frequency modes are not dominant, i.e. when the system is primarily a flexible system, as it is for Case D, the approaches of equation (11) and References 5 and 8, will provide a fairly accurate estimate of response. As the frequencies in this case are fairly closely spaced, the conventional SRSS rule will tend to provide inaccurate response. The absolute sum method results are closer to the correct values and it is probably just by chance. In this case also, the superiority of the new SRSS rule over the old SRSS rules is clearly seen. The Case E is again for the flexible system, but here all nine modes have been included in the calculation of response. The overconservativeness of the absolute sum method is clearly seen in this case.

4. CHARACTERISTICS OF RELATIVE ACCELERATION AND VELOCITY SPECTRA

The aforementioned effectiveness of the new SRSS rule is due to a very desirable characteristic of relative acceleration and velocity spectra which are used in the proposed SRSS rule. In Figure 3 is shown the variation of the root mean square (r.m.s.) values of the psuedo and relative accelerations with the frequency and damping of an oscillator. The oscillator is excited by the ground motions with spectral density function as in equation (31). The ratio of the relative to psuedo acceleration r.m.s. value is plotted in Figure 4 for two damping values. Similar plots are also shown for the relative and psuedo velocities in Figures 5 and 6. The trend in the variations of the psuedo and relative response spectrum values will be similar to those of their r.m.s. values in Figures 3 and 5. Therefore, the plots in Figures 3-6 are also fairly good representations of the corresponding curves for the response spectrum values. The r.m.s. curves of the psuedo and relative acceleration responses are conspicuously different in the high and low frequency ranges and similar differences will also be observed in their response spectrum curves. Such differences are also there in velocity curves[24], with relatively larger differences in the high frequency range. In general, however, it is seen that for high oscillator frequencies, especially higher than the highest frequency in the input, the relative spectrum values will always be less than the corresponding psuedo spectrum values. In fact for the high frequencies the psuedo-accleration spectrum approaches a constant value equal to the maximum ground acceleration whereas as the relative acceleration spectrum value apporaches zero, Figure 3.

The observation made in the preceding paragraph about the psuedo and relative response of an oscillator have a special relevance here. In the SRSS rules of equation (11) and References 5 and 8 which primarily use the psuedo acceleration spectra as inputs, the contribution of a mode can be measured, among other things, by the product of terms $(\gamma_j \zeta_j/\omega_j^2)$and $R_a(\omega_j)$. This contribution decreases because the term $(\gamma_j \zeta_j/\omega_j^2)$ usually decreases for higher modes. On the other hand the term $R_a(\omega_j)$ approaches a constant value equal to that of the maximum ground acceleration. However, in the proposed SRSS rule, which employs relative spectra, a mode contribution is affected, of course, by $(\zeta_j \gamma_j/\omega_j^2)$ but in addition, also by $R_r(\omega_j)$ which continuously decreases for higher modes. Thus, as a result of neglecting the higher modes a smaller error will be introduced in the calculation of response by the proposed SRSS rule than by the SRSS rule which employ psuedo spectra. This is clearly shown by the results of Table 2. Thus the proposed SRSS rule can be used to obtain an accurate value of design response even with a first few modes.

The question which naturally arises is then: how many modes should one consider to obtain an accurate value of design response? It is felt that the modes with periods less than the zero-acceleration period can be omitted from the analysis. (The zero-period is the period of an oscillator below which no amplification in psuedo acceleration is obtained. For design spectra prescribed in Referene 22, the zero period is .03 secs. which corresponds to the oscillator frequency of 33 cps.).

Of course, more research effort is necessary to establish a more specific rule.

Parallel results were also obtained for the absolute acceleration response of a floor using the old SRSS rule and the new rule based on the mode acceleration approach which again indicated the superiority of the mode acceleration formulation. These latter results have special relevance in the generation of floor response spectra.

5. CONCLUSIONS

Several SRSS rules are available for the combination of maximum modal response to obtain the seismic design response of a linearly behaving structural system. Most of these rules use psuedo-acceleration spectra as seismic design input. In the evaluation of design response for structures with closely-spaced frequencies, modal correlation effects are considered in these SRSS rules. Based on the assumption of white noise as input, some methods for the evaluation of the modal correlation, and its effects on the design response, have been proposed. Here it is observed that these correlations are misrepresented, especially when two high frequency modes are concerned. It is shown that these correlations are important in the evaluation of response not only when the modal frequencies are close, but also when a response has a significant contribution from the high frequency modes. To obtain an accurate evaluation of response, especially in such cases, it is necessary that all modes (a complete set) calculated with high precision be used in the analysis. It is also necessary that SRSS formulation like equation (11), where modal correlation terms depend upon the input response characteristic be used; the formulations where these correlations are considered independent of input, such as References 5 and 8, can lead to erroneous results. If, however, the system is flexible relative to the frequency of the input, the formulations based on white noise as input can also provide accurate value of design response.

A new SRSS rule is developed here for the calculation of design response. In its development certain assumptions about the peak factor values and characteristics of input (i.e., stationarity of ground motion) have been made. These assumptions, however, do not distract the applicability of the proposed SRSS rule anymore than they do for the existing SRSS rules, which are also based on similar assumptions and the applicability of which have been verified in several earlier studies.

The new SRSS rule proposed here provides a better alternative for the calculation of design response. In this approach the effect of high frequency modes (or often referred to as missing mass) is included through a static analysis. The additional computational effort spent in static analysis, which requires the solution for a set of linear simultaneous equations, constitutes only a small part of the effort spent in the evaluation of a high frequency modes in eigenvalue analysis. For a given number of modes, this alternative rule is expected to provide a more accurate value of response than the existing SRSS rules for all cases of structural systems, with or without dominant high frequency modes.

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TABLE 1: PARAMETERS OF SPECTRAL DENSITY FUNCTION, $\phi_g(\omega)$, EQUATION (31)

i	S_i ft^2-sec/rad	ω_i rad/sec.	β_i
1	0.0015	13.5	0.3925
2	0.000495	23.5	0.3600
3	0.000375	39.0	0.3350

TABLE 2: MEAN SQUARE COLUMN BENDING MOMENT RESPONSE FOR VARIOUS SYSTEMS OBTAINED BY DIFFERENT MODE COMBINATION RULES

Column	B.M. Exact Value	B.M. RATIO FOR VARIOUS MODE COMBINATION RULES				
		Eq. 19	Eq. 11	Ref. 5 or 9	Conventional SRSS	Absolute Sum
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A. System in Fig. 3 with all nine modes (ω_1 = 20.8, ω_9 = 123.4 cps)						
1.	1.365	1.000	1.000	0.754	0.528	1.010
2.	1.379	1.000	1.000	0.758	0.531	1.013
B. System in Fig. 3 with first three modes (ω_1 = 20.8, ω_9 = 123.4 cps)						
1.	1.365	0.980	0.722	0.719	0.508	0.723
2.	1.379	0.980	0.722	0.723	0.511	0.724
C. Stiffer System with first three modes (ω_1 = 41.6, ω_9 = 246.7 cps)						
1.	0.315	0.996	0.692	0.685	0.484	0.692
2.	0.318	0.996	0.691	0.691	0.489	0.691
D. Flexible System with first three modes (ω_1 = 5.2, ω_9 = 30.8 cps)						
1.	39.89	0.999	0.931	0.931	0.657	0.940
2.	40.58	0.999	0.931	0.932	0.659	0.932
E. Flexible System with all nine modes (ω_1 = 5.2, ω_9 = 30.8)						
1.	39.89	1.000	1.000	0.952	0.669	1.190
2.	40.58	1.000	1.000	0.953	0.671	1.183

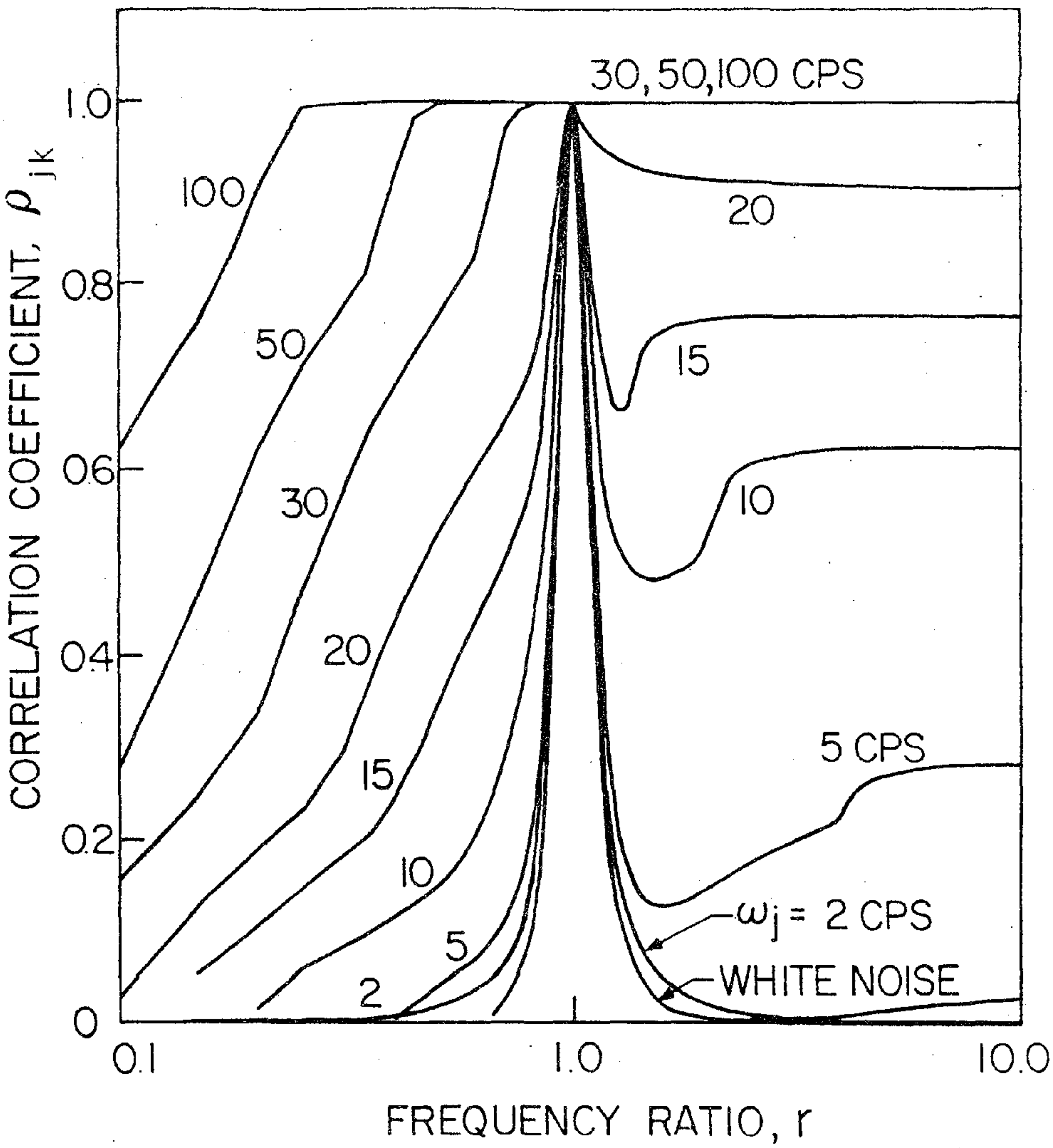

Fig. 1 Correlation Coefficient ρ_{jk} at Various Central Frequencies for White Noise and Band-Limited Filtered White Noise Spectral Density Functions.

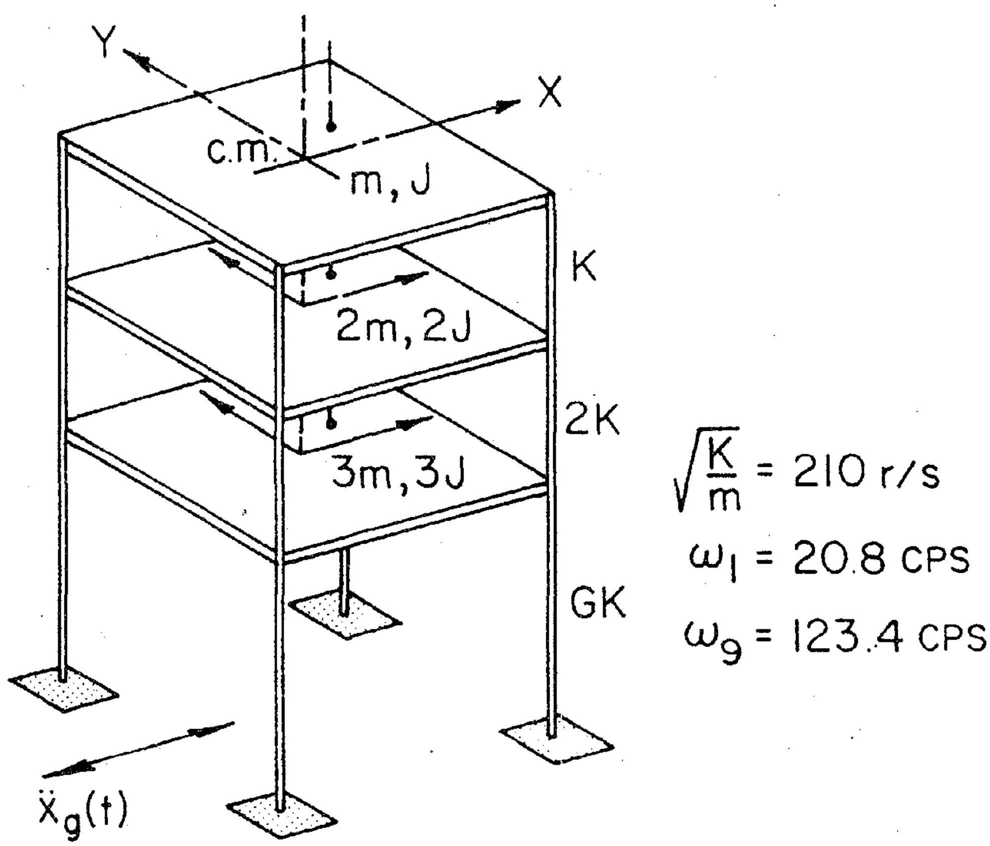

Fig. 2 A Structural Model with Nine Degrees-of-Freedom.

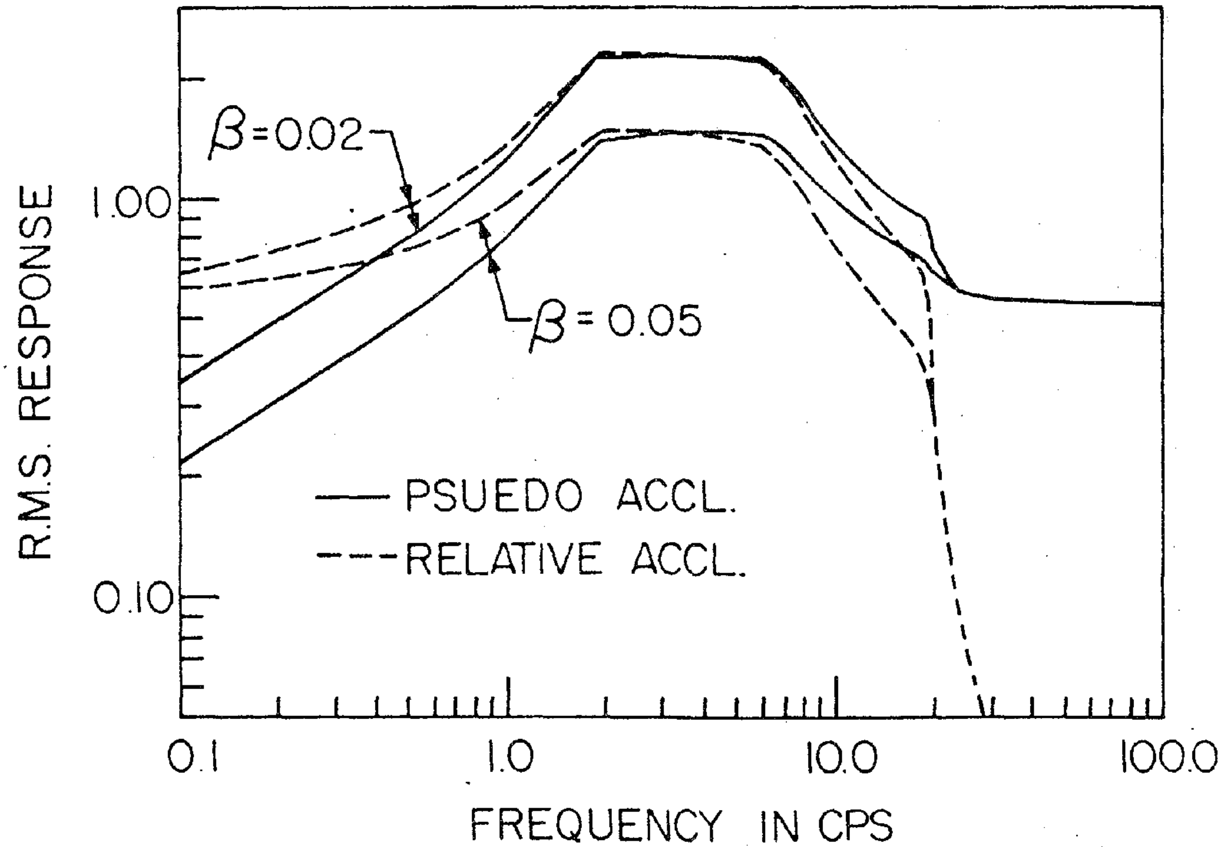

Fig. 3 R.M.S. Values of Psuedo and Relative Acceleration Response, in Ft/Sec^2, of Oscillators Excited by a Band Limited Filtered White Noise Input.

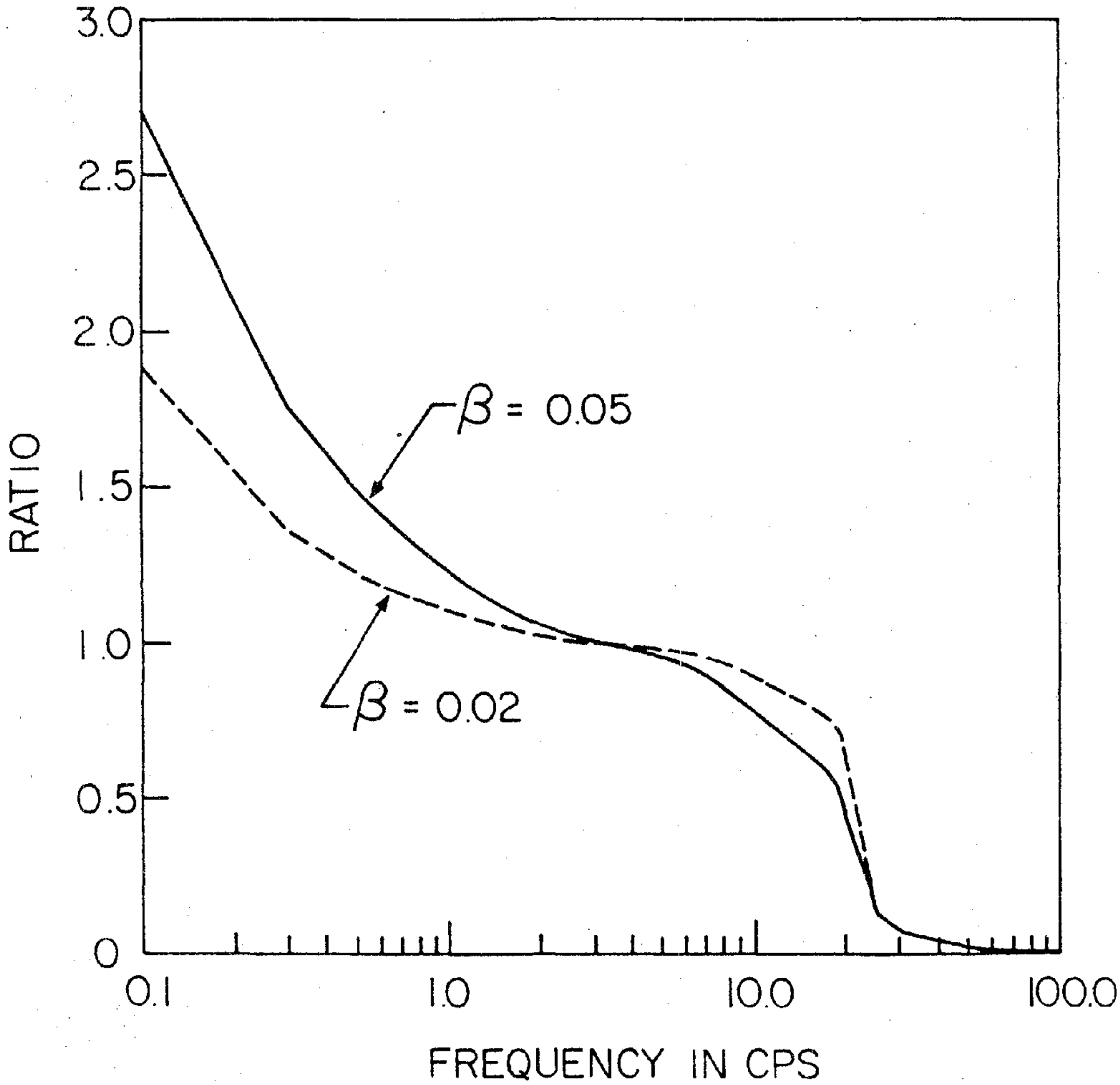

Fig. 4 Variation of the Ratio of Relative to Psuedo Acceleration R.M.S. Response with Frequency.

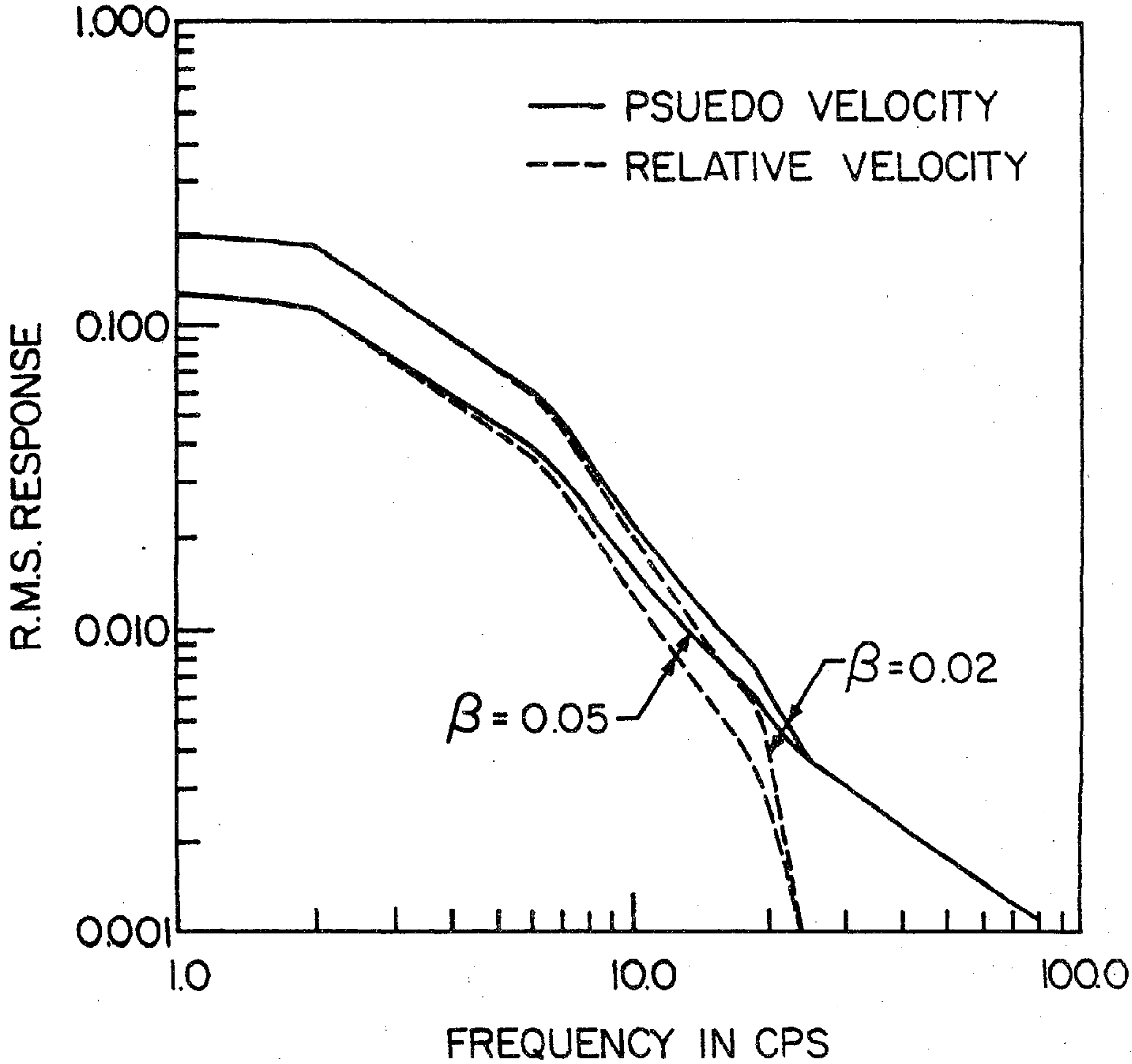

Fig. 5 R.M.S. Values of Psuedo and Relative Velocity Responses in Ft/Sec of Oscillators Excited by a Band Limited Filtered White Noise Input.

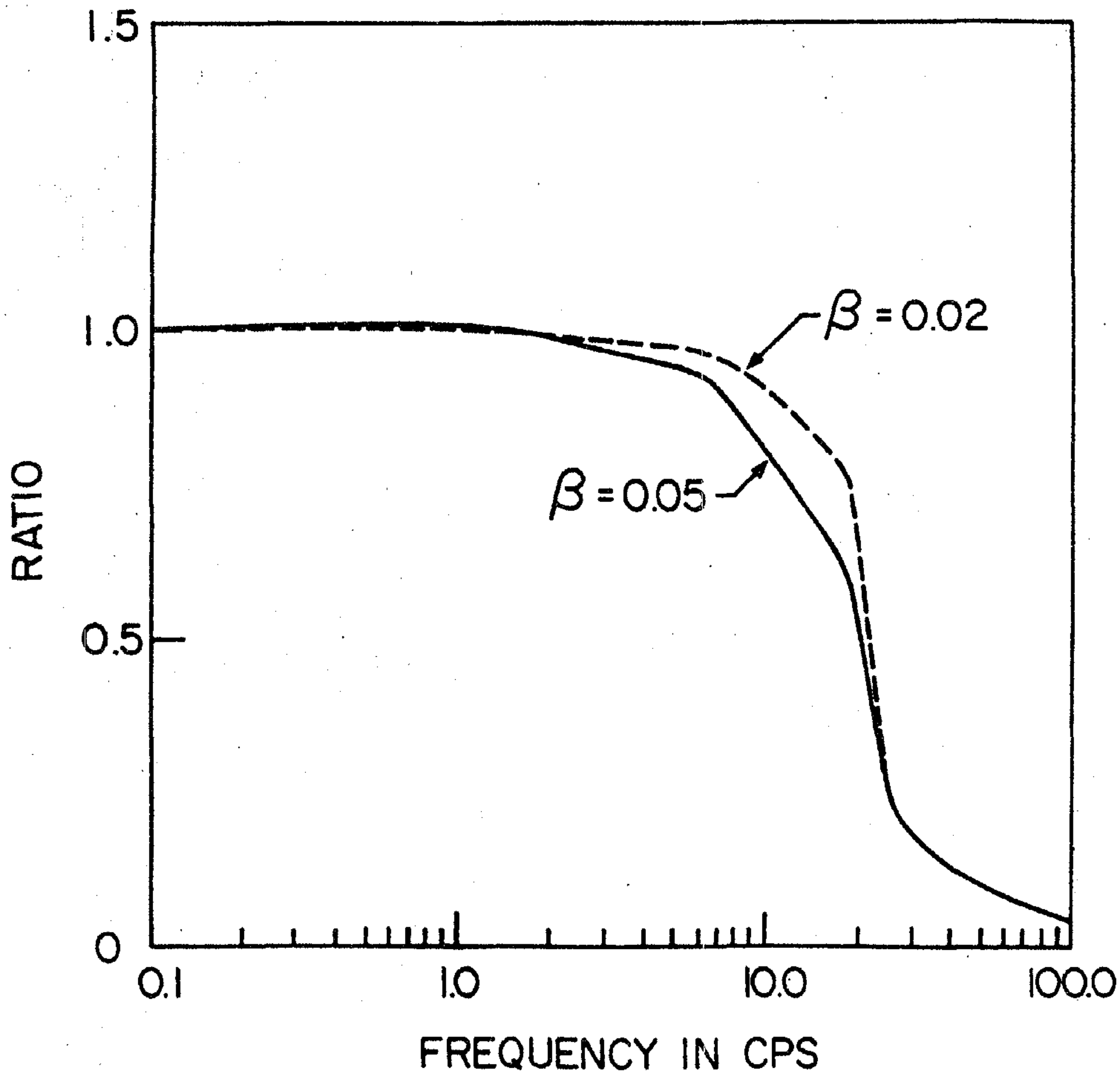

Fig. 6 Variation of the Ratio of Relative to Psuedo Velocity R.M.S. Responses with Frequency.

APPENDIX I - ELEMENTS OF MATRICES [G] $\{P_a\}$, $\{P_b\}$

The elements of matrix [G] and vectors $\{P_a\}$ and $\{P_b\}$ used in equations (8) and (20) are defined as follows

$$[G] = \begin{bmatrix} 1 & 0 & r^4 & 0 \\ -2(1-2\beta_k^2) & 1 & -2r^2(1-2\beta_k^2) & r^4 \\ 1 & -2(1-2\beta_k^2) & 1 & -2r^2(1-2\beta_j^2) \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\{P_a\}' = \{r^2,\ -(1+r^2-4\beta_j\beta_k r),\ 1,\ 0\}$$

$$\{P_b\}' = \{4\beta_j\beta_k r^3,\ r^2 - 4\beta_j\beta_k r(1+r^2-4\beta_j\beta_k r) + t(\beta_j - r\beta_k)r,$$

$$-(1+r^2 - 8r\beta_j\beta_k) - \frac{t^2}{4},\ 1\}$$

where $r = \omega_j/\omega_k$ and $t = 4(r\beta_j - \beta_k)$

APPENDIX II - EVALUATION OF CORRELATION TERMS IN EQ. 14

In this appendix, various auto and cross correlation terms required in Eq. 14 to obtain Eq. 15 are developed.

$E[\ddot{X}_g(t_1)\dot{q}_k(t_2)]$:

From Eq. 3,

$$q_k(t_2) = -\gamma_k \int_0^{t_2} \ddot{X}_g(\tau)h_k(t_2-\tau)d\tau \qquad (II.1)$$

where $h_k(t)$ is the impulse response function of Eq. 3. The cross correlation function is

$$E[\ddot{X}_g(t_1)\dot{q}_k(t_2)] = \frac{\partial}{\partial t_2} E[\ddot{X}_g(t_1)q_k(t_2)]$$

$$= -\gamma_k \frac{\partial}{\partial t_2} \int_0^{t_2} E[\ddot{X}_g(t_1)\ddot{X}_g(\tau)]h_k(t_2-\tau)d\tau \qquad (II.2)$$

For stationary excitation,

$$E[\ddot{X}_g(t_1)\ddot{X}_g(\tau)] = \int_{-\infty}^{\infty} \Phi_g(\omega)e^{i\omega(t_1-\tau)}d\omega \qquad (II.3)$$

Substituting in Eq. II.2, the stationary value at $t_2 \to \infty$ is obtained as

$$E[\ddot{X}_g(t_1)\dot{q}_k(t_2)] = -\gamma_k \frac{\partial}{\partial t_2} \{\int_{-\infty}^{\infty} \Phi_g(\omega)H_k^* e^{i\omega(t_1-t_2)}d\omega\} \qquad (II.4)$$

where $H_j(\omega)$ = frequency response function = $1/(\omega_j^2-\omega^2+2i\beta_j\omega_j\omega)$. Thus

$$E[\ddot{X}_g(t_1)\dot{q}_k(t_2)] = \gamma_k \int_{-\infty}^{\infty} i\,\omega\Phi_g(\omega)H_k^* \, e^{i\,\omega(t_1-t_2)} d\omega \qquad (II.5)$$

Similarly, the stationary values of other related cross correlation terms are obtained as:

$$E[\ddot{X}_g(t_1)\ddot{q}_k(t_2)] = \gamma_k \int_{-\infty}^{\infty} \omega^2\Phi_g(\omega)H_k^* \, e^{i\,\omega(t_1-t_2)} d\omega \qquad (II.6)$$

$$E[\ddot{X}_g(t_2)\dot{q}_j(t_1)] = -\gamma_j \int_{-\infty}^{\infty} i\,\omega\phi_g(\omega)H_j \, e^{i\,\omega(t_1-t_2)} d\omega \qquad (II.7)$$

$$E[\ddot{X}_g(t_2)\ddot{q}_j(t_2)] = \gamma_j \int_{-\infty}^{\infty} \omega^2\Phi_g(\omega)H_j \, e^{i\,\omega(t_1-t_2)} d\omega \qquad (II.8)$$

$E[q_j(t_1)q_k(t_2)]$:

Substituting for q_j and q_k, we obtain

$$E[q_j(t_1)q_k(t_2)] = \gamma_j\gamma_k \int_0^{t_1}\int_0^{t_2} E[\ddot{X}_g(\tau_1)\ddot{X}_g(\tau_2)]h_j(t_1-\tau_1)h_k(t_2-\tau_2)d\tau_1 d\tau_2 \qquad (II.9)$$

Substituting for the autocorrelation function of the ground acceleration term in terms of its spectral density function, the stationary value when $t_1 \to \infty$, $t_2 \to \infty$ can be obtained as follows:

$$E[q_j(t_1)q_k(t_2)] = \gamma_j\gamma_k \int_{-\infty}^{\infty} \Phi_g(\omega)H_jH_k^* \, e^{i\,\omega(t_1-t_2)} d\omega \qquad (II.10)$$

Using this expression, other related correlation functionss can be obtained as follows:

$$E[\dot{q}_j(t_1)\dot{q}_k(t_2)] = \gamma_j \gamma_k \int_{-\infty}^{\infty} \omega^2 \Phi_g(\omega) H_j H_k^* \, e^{i\omega(t_1-t_2)} d\omega \qquad \text{(II.11)}$$

$$E[\ddot{q}_j(t_1)\ddot{q}_k(t_2)] = \gamma_j \gamma_k \int_{-\infty}^{\infty} \omega^4 \Phi_g(\omega) H_j H_k^* \, e^{i\omega(t_1-t_2)} d\omega \qquad \text{(II.12)}$$

$$E[\dot{q}_j(t_1)\ddot{q}_k(t_2)] = -\gamma_j \gamma_k \int_{-\infty}^{\infty} i\omega^3 \Phi_g(\omega) H_j H_k^* \, e^{i\omega(t_1-t_2)} \qquad \text{(II.13)}$$

$$E[\ddot{q}_j(t_1)\dot{q}_k(t_2)] = \gamma_j \gamma_k \int_{-\infty}^{\infty} i\omega^3 \Phi_g(\omega) H_j H_k^* \, e^{i\omega(t_1-t_2)} \qquad \text{(II.14)}$$

Substitution of these in Eq. 14, and then separation of terms with $j=k$ and $j \neq k$, gives Eq. 15. For $j=k$ terms, the terms with odd powers of ω which appear as complex conjugate, cancel out. However when $j \neq k$ these should be properly retained. Combining appropriate terms, the imaginary terms are eliminated. The Eq. 15 thus contains only the real terms.

NOTATIONS

The following symbols are used in this report:

A_g	maximum ground acceleration
A_1, A_2, A_3, A_4	elements of vector $\{A\}$ defined by solution of Eq. 8
B_1, B_2, B_3, B_4	elements of vector $\{B\}$ defined by solution of Eq. 20
$[C]$	system damping matrix
C_f	peak factor
$E[\cdot]$	expected value of $[\cdot]$
$[G]$	matrix defined in Appendix I
H_j	complex frequency response function of an oscilatior with frequency ω_j and damping ratio β_j
$h_j(t)$	impulse response quantity of Eq. 3
$I_1(\omega_j), I_2(\omega_j)$	mean square values of the relative displacement and velocity responses of an oscillator with frequency ω_j and damping ratio β_j excited by the stationary ground acceleration $\ddot{x}_g(t)$
$[K]$	system stiffness matrix
$\{K_i\}^T$	vector defined in terms of stiffness coefficient which transforms the displacement vector $\{x_s\}$ into response $\overline{S}$
$[M]$	system mass matrix
N	number of degress of freedom
$\{P_a\}, \{P_b\}$	vectors defined in Appendix I
$\{q\}$	vector of principle coordinate defined by Eq. 2
q_{sj}	coefficient of expansion defined by Eq. 26
$R_a(\omega_j)$	psuedo acceleration response spectrum value at frequency ω_j and damping ratio β_j

$R_r(\omega_j)$	relative acceleration response spectrum value at frequency ω_j and damping ratio β_j
$R_v(\omega_j)$	relative velocity response spectrum value at frequency ω_j and damping ratio β_j
$\{r\}$	ground displacement influence vector
r_j	frequency ratio defined by ω_j/ω_k
$S(t)$	response quantity
$\overline{S}$	the static value of a response quantity for the displacement vector defiend by Eq. 22
S_d	design response value quantity $S(t)$
S_i	a parameter of ground spectral density function
S_j	maximum modal response in the jth mode defined by $S_j = \gamma_j \zeta_j R_a(\omega_j)/\omega_j^2$
S_{jk}	cross modal response
$\{x\}$	response vector of relative displacement of the structure with respect to ground
$\ddot{x}_g(t)$	ground acceleration
β_j	damping ratio parameter used in ground spectral density function
β_j	modal damping ratio
γ_j	jth mode participation factor
ζ_j	jth mode shape of response quantity
ρ_{jk}	modal correlation coefficient defined by Eq. 12
$[\Phi]$	modal matrix with its column representing the normal modes
$\{\phi_j\}$	jth displacement mode shape
$\Phi_g(\omega)$	ground spectral density function

ω_c cut-off frequency

ω_i frequency parameter used in ground spectral density function

ω_j jth modal frequency