## CALIFORNIA INSTITUTE OF TECHNOLOGY

## SOIL MECHANICS LABORATORY

## DYNAMIC CENTRIFUGE TESTING OF CANTILEVER RETAINING WALLS

by<br>L. Alexander Ortiz

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# DYNAMIC CENTRIFUGE TESTING OF CANTILEVER RETAINING WALLS 

## Thesis by

L. Alexander Ortiz

## In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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California Institute of Technology
    Pasadena, California
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## ABSTRACT


#### Abstract

An investigation was made into the behavior of flexible cantilever walls retaining a cohesionless soil backfill and subjected to earthquake-type dynamic excitations using the centrifuge modeling technique. The study was motivated by the abundant observations of earth retaining structure damage and failures documented in earthquake damage reports.

The "prototype" typical walls were designed using the traditional Mononobe-Okabe dynamic lateral earth pressure theory, were properly scaled for use in the centrifuge at 50 g 's, and were subjected to lateral earthquake-like motions which were considered to be of realistic levels. The walls were amply instrumented with pressure and displacement transducers, accelerometers, and strain gages. Moment, pressure, shear, and displacement distributions (static, dynamic, and residual) were obtained.

From the test data, some empirical curves for relating the upper bound responses of the retaining walls to the strong motion characteristics of the applied earthquakes were obtained.


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## CHAPTER I

## INRRODUCTION

In this study, an investigation was made into the behavior of cantilever retaining walls, with a cohesionless soil backfill, subjected to earthquake-type dynamic excitations.

Interest in this problem arose from the fact that in virtually every earthquake damage report there is documentation of damage or failure of bridge abutments, sea walls, quay walls, canals, dikes, retaining walls, etc.; in other words, earth retaining structures. This is further enhanced by the fact that in most seismically active areas, there are absolutely no code provisions for some aseismic design of retaining structures. Where seismic considerations are taken into account, a design with the 60 year old pseudo-static Mononobe-Okabe theory with reduced design accelerations is usually accomplished.

Even though many experimental (model) and analytical studies have been done on the subject in the 1 ast 60 years, many have been improperly formulated, oversimplified, or simply inadequate. To this day there seems to be no general agreement as to what seismic method of design should be used or even if one should be used at all.

In recent years, the centrifuge has become a more accepted and aseful tool in the modeliing of soil mechanics problems. It was therefore decided to nse this device in order to try to develop some empiricaltype design guidelines for at least one type of retaining structure, namely cantilever retaining walls. In order to do this an
"earthquake generating" mechanism, simple and light enough not to take up a substantial portion of the centrifuge payload, was developed in order to provide properly scaled earthquake-type excitations to the properly scaled and designed wall-soil system.

A series of fourteen tests was performed on two properly scaled retaining walls which were designed according to the traditional seismic theory. Since these walls are bending beams, bending moments were measured directly. This appears to be unprecedented since model studies have generally been done only on rigid walls. In addition, earth pressures behind the walls were measured and these results integrated to determine the shear forces. With the aid of accelerometers and displacement transducers, deflection shapes were also determined.

Although model tests were performed, they provided the response of a real (not idealized) retaining structure system subjected to a real earthquake excitation. This was afforded by using the artificial gravity field provided by the centrifuge.

### 1.1. Mononobe-0kabe Method

During the $1920^{\prime} \mathrm{s}$, N. Mononobe and N. Matsuo [31], and S. Okabe [39], developed an approximate method for determining the dynamic lateral earth pressure on a retaining structure. The method was based on the traditional Coulomb lateral earth pressure theory where inertial forces of the soil due to the earthquake. were treated as additional static forces, through the use of horizontal and vertical accelerations. The observed failure mechanisms of gravity walls which had displaced
under lateral acceleration provided a physical basis for this approach. The method, therefore, does not incorporate a calculation of the pressures which may develop between wall and soil prior to wall failure.

The Mononobe-0kabe method set a standard with which most future research in the field would be compared. Ensuing research has been concerned with refinement of the method or tests of its validity by model studies. Only a few countries have building codes that specify earthquake provisions for wall structures [17,55], but in general, when specified, these provisions are based on the Mononobe-0kabe method. Even in localities where no specific code requirements exist, it appears that the Mononobe-Okabe method is used in design when a dynamic analysis is desired.

Details of the Mononobe-Okabe method and suggestions regarding its application to design problems are given by Seed and Whitman [55].

1. The wall is assumed to displace laterally a sufficient amount to generate minimum active pressure.
2. The soil is assumed to satisfy the Mohr-Coulomb failure criterion.
3. Failure in the soil is assumed to occur along a plane surface through the toe of the wall and inclined at some angle to the horizontal.
4. The wedge of soil between the wall and the failure plane is assumed to be in equilibrium at the point of incipient failure, under gravity, earthquake, and the boundary forces along the


MONONOBE-OKABE ANALYSIS

FIGURE 1.1
wall and failure surface. The forces acting on the soil wedge of weight $W$ are shown in Figure 1.1 for the case of a cohesionless soil.
5. Equivalent static horizontal and vertical forces $k_{h} W$ and $k_{V} W$, applied to the center of gravity of the wedge, represent the earthquake effect. The parameters $k_{h}$ and $k_{v}$ are the horizontal and vertical earthquake coefficients expressed as fractions of g, the gravitational acceleration.
6. The method gives the magnitude of the total acting force on the wall, but does not give the point of application or the pressure distribution. The method apparently was developed with the assumption that the total force acted $1 / 3 H$ above the base of the wall of height $H$. Based on more recent refinements to the method, as well as model test results, Seed and Whitman [55] recommended that the dynamic force should be assumed to act 0.6 H above the base. The total active wall force, due to gravity and earthquake, is determined by a force and moment equilibrium analysis of the soil wedge behind the wall (Figure 1.1).

As in a Coulomb analysis, the angle of the failure plane is varied to give a maximam value of the wall force per unit width $P_{A E}$, and under the critical condition it can be shown that

$$
\begin{equation*}
P_{A E}=1 / 2 \gamma H^{2}\left(1-k_{v}\right) K_{A E} \tag{1.1}
\end{equation*}
$$

in which:

$$
\begin{aligned}
& K_{A E}=\frac{\cos ^{2}(\phi-\theta-\beta)}{\cos \theta \cos ^{2} \beta \cos (\delta+\beta+\theta)}\left[1+\left(\frac{\sin (\phi+\delta) \sin (\phi-\theta-i)}{\cos (\delta+\beta+\theta) \cos (i-\beta)}\right)^{1 / 2}\right]^{-2} \\
& \text { (coefficient of lateral earth pressure) } \\
& \theta=\tan ^{-1} \frac{k_{h}}{1-k_{v}} \\
& \gamma=\text { unit weight of soil } \\
& \phi=\text { angle of internal friction of soil } \\
& \delta=\text { angle of wall-soil friction } \\
& i=\text { angle of backfill slope } \\
& \beta=\text { angle of wall slope } \\
& k_{h}=\text { horizontal earthquake coefficient (fraction of g) } \\
& k_{v}=\text { vertical earthquake coefficient (fraction of } g \text { ) } \\
& \text { Figure } 1.2 \text { illustrates the variation of } K_{A E} \text { with } k_{h} \text { with changes in } \\
& \text { the various soil/wall/1ateral acceleration parameters. The } \\
& \text { Mononobe-Okabe method can be readily extended to encompass cohesive } \\
& \text { soils by considering the equilibrium of cohesive forces acting along the } \\
& \text { wall boundary and the failure surface. } \\
& \text { Some limitations on the method are given by Wood [67]. A brief } \\
& \text { summary follows: }
\end{aligned}
$$

1. For full active pressure (full plastic state) to develop in the soil behind the wall, it is necessary for the top of the wall to deflect latexally about $0.5 \%$ of the wall height. This


FIGURE 1.2 - FROM (55)
condition probably occurs readily in gravity and cantilever walls, but may not always occur in channel sections or anchored sheet-pile walls. It was shown by Food that for a rigid wall on a rigid foundation the earthquake force component computed by elastic theory was likely to be greater than twice the force computed by the Mononobe-Okabe method. This result was based on a static solution of identical horizontal earthquake coefficients for each case. Thus failure of a rigid structure designed using the Mononobe-Okabe criterion is a great possibility.

Unlike design procedures which allow yielding of structural members of building frames during strong earthquakes, it is generally undesirable to allow excessive yielding in retaining structures. This is because yielding of the structure generally tends to occur in only one direction away from the backfill. Unidirectional yielding may lead to excessive wall displacements with severe cracking to both wall and backfill. It is thus considered desirable to prevent yielding of the retaining structure during an earthquake.
2. Although the assumption of a plane failure surface appears reasonable, its validity has been based on a very limited number of test and field observations.
3. The Mononobe-Okabe Method is a pseudo-static method. Inertia forces are included by use of the earthquake coefficients $k_{h}$ and $k_{v}$. These are generally chosen without any uniform basis
and are generally well below the values for expected peak accelerations. This is basically due to the assumption that some permanent movement of the wall due to shaking can be tolerated.
4. In the Mononobe-Okabe method no account is taken of resonance effects or the amplification of earthquake motions that might occur as a result of the propagation of the motion through the relatively soft soil behind the wall.
5. The Mononobe-Okabe method neglects the influence of the dynamic behavior of the wall structure itself on the earth pressures. Richards and Elms [43] (see section 1.3) have performed a study taking wall parameters into consideration.

### 1.2. Experimental Studies

In order to verify the Mononobe-Okabe theory, experiments on smallscale laboratory models subjected to sinusoidal excitation on shaking tables have been performed by a number of researchers: Mononobe and Matsuo, 1929 [31]; Jacobsen, 1939 [19]; Ishii, Arai, and Tsuchida, 1960 [18]; Matsuo and Ohara, 1960, [28]; Murphy, 1960 [33]; Niwa, 1960 [36]; Ohara, 1960 [38].

Mononobe and Matsuo used a 4 ft high, 4 ft wide, and 9 ft long sandbox which was subjected to horizontal excitations with vibration periods of 0.42 to 0.48 seconds. The end-walls of the box were hinged at the base and restrained by pressure measuring devices at the top. Total end-wall forces were measured and were found to be in reasonable
agreement to those given by the Mononobe-Okabe method. Although no details were given, the wall was presumably allowed to displace sufficiently to allow full active pressure to develop.

Jacobsen performed tests on a sandbox using a shaking table and a 3 ft high layer of sand. Although no other details as to size of the box, flexibility of the wall, or type of excitation are given, it was concluded that the tests were in reasonable agreement with the Mononobe-Okabe method, and that the dynamic component of the force acted at about two thirds of the height of the sand layer above the base.

Ishii, Arai and Tsuchida performed tests with which they concluded that, in general, their results were in agreement with the Mononobe-Okabe analysis. They conducted tests on a sandbox with fixed and movable end-walls. Model gravity walls were also used in the box. A 2.3 ft depth of sand was used behind the walls. The entire box was subjected to sinusoidal excitations of approximately 3 Hz and 0.1 g to 0.7 g amplitude. Observations on wall displacement, sand settlement, residual earth pressures, and phase relationships between the earth pressures and base motion were made.

Matsuo and Ohara performed tests on dry and saturated sands in a shaking box $3.28 \mathrm{ft} \times 1.97 \mathrm{ft} \times 1.31 \mathrm{ft}$ high. Conditions were similar to the tests of Ishii, et al. The box was subjected to 3 Hz sinusoidal excitations with an amplitude of 0.1 g to 0.4 g . Tests were conducted for both a fixed end-wall (essentially rigid) and a movable end-wall that was permitted to rotate about its base. Shaking was allowed to vary during the tests. For the rigid case the earth pressures were
significantly higher than predicted by Mononobe-0kabe. The earth pressure distributions were also found to deviate considerably from linear.

Based on elasticity theory, Matsuo and Ohara also derived analytical expressions for pressure distributions for the fixed and rotating wall. The experimental pressures were significantly less for the rigid wall than those predicted by their theory. They attribute the discrepancies to influences of the side walls of the box and to the elasticity of the pressure cells used.

Murphy conducted tests to determine the mode of failure of wallsoil systems. A $1 / 64$ scale wall model was placed in a shaking sand box and subjected to a 5.4 Hz excitation with maximum acceleration of 0.25 g . No pressures or displacements were recorded. It was found that failure occurred by outward rotation of the wall about the toe with a failure surface in the soil inclined at $35^{\circ}$ to the horizontal. The results were considered consistent with the failure plane in the Mononobe-Okabe method.

Niwa performed tests on a large-scale gravity-type quay wall model. The wall was 9.8 ft high and 13.1 ft wide with a 16.4 ft 1 ong sand backfill. In addition, a $6.6 \mathrm{ft} \times 6.6 \mathrm{ft} \times 13.1 \mathrm{ft}$ surcharge of sand was placed right behind the wall. A large vibration generator was used. It was capable of delivering frequencies of 3 Hz to 6 Hz with a 1 ateral force of 35 tons @ 6 Hz and a lateral accelexation of $0.3 \mathrm{~g} @ 6 \mathrm{~Hz}$. The generator was placed 34.8 ft behind the wall. A sizeable number of transducers were used to instrument the wall. These included pressure cells, as well as displacement, velocity and acceleration transducers.

Unfortunately, results were very sketchy. Pressures recorded were zero at the top and increased fairly linearly towards the bottom. No comparison with the Mononobe-Okabe method was given.

Ohara conducted experiments on a 12 in deep, 22 in wide and 39 in long sandbox which was harmonically forced with accelerations of up to 0.4 g . The end wall was given controlled displacements and the results were found to be consistent with those predicted by the Mononobe-Okabe method.

From the shaking table experiments it is generally concluded that the Mononobe-Okabe method gives the total resultant force reasonably we11, but not the pressure distribution, and hence, does not predict the point of application of the force or the magnitude of the overturning moment correctly. Overall, the results of the shaking table experiments are questionable. The tests were performed under fairly unreal conditions. They generally had externally controlled restricted displacements and rotations of the wall. The tests were performed in the 1aboratory at earth gravity, using scaled harmonic forcing, which was not random as seismic forcing is and may not adequately represent transient earthquake stresses. The rationale for these tests is based on the following argument (Wood[67]). A similarity condition for an elastic soil and a rigid wall under the assumption that both the model and prototype have the same Poisson's ratio is given by the dimensionless equation for the frequency of a vibrating elastic system:

$$
\begin{equation*}
\frac{f_{p}^{2} \rho_{p} H p^{2}}{G_{p}}=\frac{f_{m m}^{2} \rho_{m} H_{m}^{2}}{G_{m}} \tag{1.3}
\end{equation*}
$$

where:

$\rho_{m, p}=$ soil mass density
$H_{m, p}=$ height
$G_{m, p}=$ shear modulus
and both model and prototype tests are performed at the same gravitational acceleration.

The equation is usually employed to determine the frequency at which the model is to be vibrated to simulate the full-scale behavior. If the ratio of length scale in prototype to model is denoted as $N$, the equation can be rearranged in terms of frequency to give

$$
\begin{equation*}
\frac{f_{m}}{f_{p}}=\left[\left(\frac{H_{p}}{H_{m}}\right)^{2}\left(\frac{G_{m}}{G_{p}}\right)\left(\frac{\rho_{p}}{\rho_{m}}\right)\right]^{1 / 2} \tag{1.4}
\end{equation*}
$$

However, $H_{p} / H_{m}=N$, and, since the same soil is generally used in model and prototype, $\rho_{p} / \rho_{m}$ is close to unity so that

$$
\begin{equation*}
\frac{f_{m}}{f_{p}}=N\left(\frac{G_{m}}{G_{p}}\right)^{1 / 2} \tag{1.5}
\end{equation*}
$$

In a clay, a laboratory model can be prepared with $G_{m}$ essentially any desired value, from a low level, appropriate in some way to the model dimension, to a value the same as the prototype. In sands, the shear modulus $G$ varies with the effective stress, which depends directly
on the gravitational field. As a consequence $G_{m}$ in model sand is considerably smaller over the wall depth than $G_{p}$ in the full-scale domain. The choice of $f_{m}$ therefore, depends on the relationship adopted between $G$ and the effective stress in the sand. If $G$ is taken to vary linearly with effective stress, then $f_{m}$ is approximately equal to $N^{1 / 2} f_{p}$. Alternative if $G$ is taken to vary as some power of effective stress, say $1 / 2$ (Seed and Idriss [54]), $f_{m}$ would be given as $N^{3 / 4} f_{p}$. Given this uncertainty about the variation of $G$ with effective stress, no clear approach is indicated, nor do the experiments clarify the effect on the dynamic pressure distributions obtained by the use of different model exciting frequencies. It can be concluded that it is difficult or impossible to achieve a pressure distribution in a (one g) model on a shaking table similar to that found in the full-scale field situation. Therefore, true modelling of the prototype soil cannot be attained in a (one g) shaking table experiment.

### 1.3. Analytical Studies

In addition to the experimental research, analytical models have been proposed to describe the dynamic earth pressures acting on walls: Tajimi, 1969-73 [59-61]; Prakash and Basavanna, 1969 [42]; Scott, 1973 [50]; Wood, 1973 [67]; Richards and Elms, 1977 [43]; Chang and Chen, 1981 [6,7].

Tajimi obtained the solution for earthquake-induced soil pressures on a cylindrical structure embedded in an elastic soil. He also obtained the solution for a harmonically forced rigid wall of finite height at the corner of a quarter-infinite elastic medium (Figure 1.3). The analysis was based on elastic wave propagation theory. Although the boundary conditions are not very realistic, the solntion can be used as an approximation for some dynamic problems.

Prakash and Basavanna computed an approximate wall pressure distribution on a wall under similar assumptions to those of the Mononobe-Okabe method. It was determined that the pressure distribation was essentially parabolic although the resultant was virtually of the same magnitude as give by Mononobe-Okabe. The resultant, however, acts at a height above the base $H_{a}$ given by:

$$
\begin{equation*}
H_{a}=C_{a} H / 3 \tag{1.6}
\end{equation*}
$$

where $C_{a}$ is a very complicated expression dependent on all the Mononobe-Okabe wall-soil parameters. $H$ is the height of the wall. $C_{a}$ is greater than one. For $k_{h}=0.3, H_{a}$ is approximately midheight and continues to rise with higher lateral acceleration.

Scott performed an analysis on a simple yet useful model (Figure 1.4). It basically consists of a rigid wall with the soil modelled as a simple shear beam on a Winkler foundation. He also performed an analysis where a wall flexibility was included. Closed form solutions were obtained for varions cases that include variations of the elastic constants with depth and certain types of wall


TAJIMI'S PROBLEM

FIGURE 1.3


## SCOTT'S MODEL

FIGURE 1.4
deformations. Because of simplicity the solutions are quite useful in preliminary design applications. Scott concluded that what happens in an earthquake to a wall designed by the Mononobe-Okabe method is that "elastic", transient forces much higher than those predicted by Mononobe-0kabe act on the wall, causing it to displace and rotate. When the wall reaches a displacement of $1 / 2 \%$ or so of the height, the soil reaches failure. The wall continues to displace and rotate due to inertia and when it stops what is observed is the failure (Mononobe-Okabe) mechanism - not the stresses that caused failure. This is why all the experiments involving failure end up by concluding that Mononobe-Okabe is the right analysis. If the earthquake force only reached Mononobe-Okabe levels of stress, then the wall designed to M-0 should not fail.

Wood, using elastic and elastic wave propagation theories developed solutions for an elastic soil stratum of finite or infinite length and finite depth on rigid base with a rigid wall under various and forcing conditions. For a perfectly rigid wall (Figure 1.5), supporting a relatively long layer of soil, he determined that the earthquake force component computed was likely to be greater than twice that estimated by the Mononobe-Okabe method (Figure 1.6). Identical horizontal earthquake coefficients $k_{n}$ were used in the comparison. It was thus recommended that for a rigid wall embedded in rock or very firm soil, restrained by piles or deeply buried, an elastic analysis should be used in 1 ieu of the Mononobe-Okabe method.


WOOD'S RIGID WALL PROBLEM

FIGURE 1.5
LENGTH/HEIGHT $=10.0$

Pressure distributions for one-g static horizontal body force. Comparison between elastic theory
FIGURE 1.6 - FROM (67)

Richards and Elms extended the Mononobe-Okabe method to include the influence of the dynamic behavior of the wall structare itself (Figure 1.7). It was concluded that for gravity retaining walls the Mononobe-Okabe analysis is satisfactory provided that the inertia of the wall is taken into consideration. In addition, Richards and Elms give a description of the significance of each of the Mononobe-Okabe parameters which is useful in a further understanding of the method.

Chang and Chen developed an upper bound technique of 1 imit analysis and then applied it the earthquake problem. It is basically an approach similar to Mononobe-Okabe with the main difference being that more refined failure surfaces (Figure 1.8) are used. The seismic coefficient of active earth pressure $K_{A E}$ was found to be practically the same as that obtained by a Mononobe-Okabe analysis.

### 1.4. Earthquake Damage to Retaining Structures

Failures in retaining structures due to earthquakes occur very frequently. These are documented in virtually every earthquake-damage report. It should be noted that in most reports, unless failures are dramatic, retaining-structure damage is given secondary importance. This is generally due to the fact that failure of these structures does not entail severe loss of 1 ife and limb. The damage done by the earthquake can, however, be very costly in terms of repair and replacement as well as economic setbacks to a community. A few examples of damage to retaining structures follow.


RICHARDS \& ELMS ANALYSIS

FIGURE 1.7

fIgURE 1.8-from (7) Log-Sandwich Failure Mechanism for Seismic

### 1.4.1. Chile

Duke and Leeds [11] provide an extensive account of damage to retaining structures in the 1960 Chilean Earthquakes, the most severe of which had a Richter magnitude of 8.5. At Puerto Montt (Figure 1.9), the Modified Mercalii Intensity (MMI) was estimated to be between VIII and IX. There was essentially total failure of the harbor gravity-type quay walls (Figs. 1.10, 1.11, 1.12, 1.13). Both walls completely overturned. Sheet pile sea walls (Figs. 1.11, 1.14) were severely damaged. The piles had approximately $5^{\prime \prime} \times 15^{\prime \prime}$ hat-shaped cross-sections with $5 / 16^{\prime \prime}$ thick webs and were made in Germany. Since the walls were about 30 years old at the time of the earthquake, failure principally occurred when the corroded rods broke due to the added tension resulting from the added soil pressure.

Most of the above-mentioned structures were founded on fill consisting of gravel, sand, silt, some masonry fragments, and organic matter. In general, it was placed by dumping although some was placed hydrodynamically by dredging from the harbor bottom. The disastrous damage to structures retaining this material was largely due to 1iquefaction as a result of earthquake motion.

Figure 1.15 shows distortion of the Is1a Teja bridge in Valdivia (MMI X) due to the added soil pressure on the abutment whose excessive movement caused damage to the bridge superstructure. Unlike the Puerto Montt failures, damage to this structure was not due to liquefaction, but solely to the added inertia from the shaking.


FIGURE 1.9 - FROM (11)


FIGURE 1.10-FAILURE OF QUAY WALL AT PUERTO MONTT - FROM (55)


FIGURE 1.11 - PUERTO MONTT,WATERFRONT WALLS,DESIGN FEATURES - FROM (I 1)


FIGURE 1.12 - FAILURE OF GRAVITY WALL AT PUERTO MONTT - FROM (11)

-
FIGURE 1.13 - PUERTO MONTT.GRAVITY WALL FAILURE - FROM (I I)


FIGURE 1.14 - FAILURE OF SHEET-PILE SEA WALL AT PUERTO MONTT - FROM (1 1)


FIGURE 1.15 - DISTORTION OF ISLA TEJA BRIDGE DUE TO SOIL PRESSURE ON ABUTMENT - FROM (55)

Seed and Whitman [55] also report on a gravity retaining wall failure at Frutillar (MMI VIII) where dry material was encountered (Fig. 1.16).

### 1.4.2. Alaska

Ross, Seed, and Migliaccio [45] report on extensive bridge damage due to the 8.4 magnitude Alaska earthquake of 1964 . Most of the bridges which suffered damage were 50 to 80 miles away from the cone of major energy release. The most severe damage occurred on the Seward, Sterling, and Copper River Highways (Fig. 1.17). Table 1.1 gives a foundation damage classification reduced from reports of the Alaska Department of Highways.

Most of the bridges were founded on alluvial deposits composed of granular materials which ranged anywhere from coarse gravels to fine sands and silts depending on location. The deposits ranged in depth from 50 to 150 ft and were generally underlain by clays or bedrock. A few bridges were founded on bedrock.

Damage was due completely or in part to the lateral displacement of the bridge abutments toward the channels cansing tilting of piers and buckling of superstructures (Figs. $1.18,1.19,1.20$ ). There was also spreading and settlement of abutment fills. The greatest concentrations of severe damage occurred in regions characterized by thick deposits of saturated cohesionless soils. There was ample evidence of liquefaction of these materials during the earthquake. This phenomenon probably played a major role in the development of foundation displacements and


FIGURE 1.16 - FRUTILLAR,RETAINING WALL FAILURE - FROM (55)

TABLE 1.1
Classification of Damage to Highway Bridge Foundations During the Alaska Earthquake (from Ross et al. [45])

114 Bridges Classified

| \|C1assification | Description | Percentage |
| :---: | :---: | :---: |
| Severe | Abutments moved streamward and/or markedly subsided; piers shifted, tilted, or settled; substructure rendered unsalvageable | 28 |
| Moderate | Distinct and measurable net displacements as in previous category, but to a lesser degree, so that substructure could perhaps be repaired and used to support a new superstructure | 22 |
| Minor | Evidence of foundation movements (such as cracked backwalls, split piles, closed expansion devices), but net displacements small and substructure serviceable. | 18 |
| Nil | No evidence of foundation displacements | 32 |

bridge damage. Even where damage was moderate or minor, there was evidence of bridge joints closing indicating lateral displacement of the abutments.

It should be noted that where bridges were founded completely on bedrock there was virtually no damage. However, severe to moderate displacements were reported for bridges founded partly on bedrock and partly in soils.

Highway routes of seven main bridge-damage lncations in earthquake-damaged region, south central Alaska FIGURE 1.17 - FROM (45)


FIGURE 1.18 - SUPERSTRUCTURE BUCKLING OF SNOW RIVER BRIDGE 605 - FROM (45)


FIGURE 1.19 - SUPERSTRUCTURE BUCKLING OF SNOW RIVER BRIDGE 604 - FROM (45)


FIGURE 1.20 - SUPERSTRUCTURE DRIVEN THROUGH ABUTMENT
BACKW/LL,COPPER RIVER BRIDGE 345 - FROM (45)

### 1.4.3. Niigata, Japan

The 7.5 magnitude, 1964 Niigata, Japan earthquake caused severe damage to waterfront structures and virtually paralyzed operations at the port of Niigata, one of Japan's most important. Accounts of the damage are given by Hayashi, Kubo, and Nakase [14], and by Kawasumi [22].

The total length of waterfront structures including jetties and dikes at the port of Niigata was 10.25 miles. About $76 \%$ of this length was composed of earth retaining structures. Sixty-nine percent of these were steel sheetpile balkheads, $8 \%$ were concrete sheetpile walls and $6 \%$ were concrete gravity walls. The severity of damage to harbor structures is outlined in Table 1.2 .

TABLE 1.2
(from Hayashi, et al. [14])

| Grade of Damage | Description | $\begin{gathered} \text { Total Length* } \\ \text { (mi.) } \end{gathered}$ | Proportion to the Overall Length* (\%) |
| :---: | :---: | :---: | :---: |
| 4 | Complete failure of whole structure | $\begin{gathered} 5.43 \\ (4.43) \end{gathered}$ | $\begin{gathered} 52.8 \\ (57.1) \end{gathered}$ |
| 3 | Failure in main part of structure | $\begin{gathered} 2.32 \\ (2.32) \end{gathered}$ | $\begin{gathered} 22.6 \\ (30.0) \end{gathered}$ |
| 2 | Appreciable Deformation to main part of structure | $\begin{gathered} 0.07 \\ (0.02) \\ \hline \end{gathered}$ | $\begin{gathered} 0.7 \\ (0.3) \\ \hline \end{gathered}$ |
| 1 | Failure in sub-part of structure | $\begin{gathered} 3.98 \\ (0.39) \end{gathered}$ | $\begin{aligned} & 14.5 \\ & (5.0) \end{aligned}$ |
| 0 | No damage | $\begin{gathered} 0.97 \\ (0.59) \\ \hline \end{gathered}$ | $\begin{gathered} 9.4 \\ (7.6) \end{gathered}$ |

*Figures in parentheses refer to earth retaining structures only.

It should be noted that due to the failure of earth retaining structures, 61 warehonses and sheds, $676,600 \mathrm{ft}^{2}$ in total area, fell down completely, and $92,691,500 \mathrm{ft}^{2}$, were seriously damaged (Figs. 1.21, 1.22).

Most of the sheet pile structures in Niigata Harbor underwent damage and a large number were completely destroyed or submerged. A common feature of the damage was a swelling of the backfill and inclination of the wall toward the sea. This type of damage was typically observed in bulkheads with poor anchor resistance. Tie rods were severed in some cases. In others there was a shear failure in the concrete anchor blocks due to the stress concentration created by the tie rods. The sheetpile bulkheads were designed employing a Mononobe-Okabe Analysis and a seismic coefficient of 0.10 . Actual horizontal ground accelerations were around 0.2 g amplitude.

The brand new Yamanoshita wharf (completed 1963) which had been Mononobe-Okabe designed with a seismic coefficient of 0.12 suffered no appreciable damage, except for local sinking of the fill behind the anchor plate.

Concrete sheetpile walls, which formed a small part of the waterfront, were completely destroyed by the earthquake.

The gravity retaining walls were generally composed of several concrete blocks stacked up on top of each other and then assumed to act as a monolithic structure. A seismic coefficient of 0.10 was used in design, but it was later found that when the seismic coefficient reaches


FIGURE 1.22 - WAREHOUSE COLLAPSE DUE TO SHEET-PILE BULKHEAD FAILURE,NIIGATA - FROM (14)
0.12 or 0.13 the structure cannot any longer be assumed to act monolithically. As a result, damage was characterized by blocks falling forward, slippage, and sinking of blocks, and general inclination and sliding of the structures. Damage was severe.

The general soil profile of the Niigata area consists of a layer of sand about 130 ft deep underlain by clays and containing pockets of fine silty soil in the top 60 ft . The soil was generally saturated and much of the damage was due to the occurrence of 1 iquefaction. Before the earthquake the top 30 ft of soil was characterized by an average blowcount of from 4 to 8 using the Standard Penetration Test. Between 30 and 60 feet, it varied 1 inearly from about 8 at 30 ft to about 30 at 60 ft . These figures were reduced by one third after the earthquake. In general, the deeper the structure was embedded in the soil the less severe the damage.

Based on the damage caused by the 1964 earthquakes, replacement structures have been designed and built based on a seismic coefficient of 0.2 .

### 1.4.4. San Fernando, California

The 1971 San Fernando, California, earthquake, which had a magnitude of 6.2 , severely damaged, in some cases, earth retaining structures including flood control channels, bridge abutments, and underground water storage tanks and tunnels.

Murphy [32], Scott in Reference [21], Lew, Leyendecker and Dickers [24], and Wood [67] provide descriptions of damage to the Wilson Canyon
and Mansfield Street Flood Control Channels and to the Lopez Canyon Diversion Channel which were located in an area where transient lateral accelerations may have been as high as $50 \% \mathrm{~g}$.

The Wilson Canyon Channel is partially an open, rectangular, reinforced concrete channe1, with a width of about 15 feet and wall heights which vary from 9 to 11.5 feet (Figure 1.23) and partially a covered, rectangular, box section with widths varying from 15 to 22 feet and wall heights ranging from 105 to 16 feet; it is about 3 miles long.

The Lopez Canyon Diversion Channel is an open, rectangular reinforced concrete channel about 1.8 miles long, with widths varying from 12 to 16 feet and wall heights ranging from 7 to 12 feet.

All the above-mentioned structures were built in the early $1960^{\circ} s$ by the Corps of Engineers in accordance with the Chief of Engineers' design criteria with no seismic consideration. Allowable design stresses were $f_{c}^{\prime}=1.05 \mathrm{ksi}$ for concrete and $f_{s}=20 \mathrm{ksi}$ for steel. The channels were designed as L-type retaining walls where the wall heights were less than half the channel width, and as U-type channel sections otherwise.

No significant ground displacements seem to have occurred in the vicinity of the damaged sections of the Wilson Canyon and Mansfield Street Channels so the damage can be attributed to an increase in the Iateral earth pressure due to ground shaking. There were some inward displacements in the open sections which measured up to 6 inches at the top of the walls (Figs. 1.24, 1.25). Damage to the underground box sections varied from hairline cracks to major shear and moment failures in


FIGURE 1.23 - FROM (67)


FIGURE 1.24 - WILSON CANYON CHANNEL:WALL TOP DISPLACED 4" WITH RESPECT TO THE BRIDGE ABUTMENT AT LEFT - FROM (67)


FIGURE 1.25 - WILSON CANYON CHANNEL:CRACKING IN SOIL AS A RESULT OF WALL DISPLACEMENT - FROM (67)


#### Abstract

walls. Inward deflections of up to 12 inches at midheight were measured at the most severely damaged sections.

Complete failure occurred in sections of the Lopez Canyon Channel, but the failed sections were close to a surface expression of the faulting associated with the earthquake and probably permanent ground displacements contributed significantly to the damage.

It should be noted that the failure of the flood control structures did not create any danger to human 1 ife and since in the Los Angeles area these carry only infrequent flood flows, a need for seismic consideration in design and construction might not be economically warranted except for replacement costs.


### 1.4.5. Friuli, Italy

Similar in magnitude to the San Fernando Earthquake were the 1976 Friuli, Italy earthquakes. The May main shock had a magnitude of about 6.5 while two September aftershocks had magnitudes around 6.0. There was some damage to earth retaining structures [10,57].

Along the Ledra River a retaining wall was considerably damaged during the May shock (Figs. 1.26, 1.27). There were reports of water and sand gushing and evidence of severe cracking in the backfill indicating that liquefaction had occarred. After the September shocks water and sand gushing occurred again in lines parallel to the river course, and the damaged wall completely collapsed.

After the May event damage to the Udine-Carnia-Tarvisio highway due to movement by the retaining structures below it was observed


FIGURE 1.26 - WEIR ON THE LEDRA RIVER:DAMAGED RETAINING WALL - FROM (IO)


FIGURE 1.27 - WEIR ON THE LEDRA RIVER:DAMAGED RETAINING WALL - FROM (IO)
(Figs. 1.28, 1.29). This is where the highway runs between a canal and a mountainside. On the canal side the embankment is retained by a 33 ft high wall built on piles. Figure 1.30 illustrating a normal section of the road axis shows the relative positions of the canal, the retaining wall, and road embankment, with a rough representation of the supporting soil profile.

Perhaps the fact that the entire embankment was underlain by an inclined rock formation contributed to the slipping of the retaining wall towards the canal and probably to the failure of the foundation piles. Vertical displacements along the 660 yards of retaining wall ranged from 1.6 to 9.5 inches while horizontal movements were between 9.1 and 19.3 inches.

As a consequence of the September aftershocks the damage described above increased.

In addition, there was also some severe damage of several anto strada (freeway) bridges in the area, but these were due mainly to impact from the moving bridge superstructures as opposed to failure due to increase in lateral earth pressure.

### 1.4.6. Tangshan, China

Yuxian [70] reports bridge failure during the 1976 Tangshan (People's Republic of China) earthquake which had a Richter magnitude of 7.8. The failure came from falling of superstructures to the river, or more usually, from sliding and tilting of the abutments. Lateral movement of abutments is blamed for buckling in bridge decks which would


FIGURE 1.29 - DAMAGE TO EMBANKMENT RETAINING WALL AND
CANAL,UDINE-CARNIA-TARVISIO HIGHWAY - FROM (IO)


FIGURE 1.30 - UDINE-CARNIA-TARVISIO HIGHWAY,SECTION THROUGH EMBANKMENT RETAINING STRUCTURE ADJACENT TO CANAL -FROM (IO)
have otherwise remained standing. No details were given on design criteria or construction methods.

### 1.4.7. Miyagi-Ken-Oki, Japan

The 7.4 magnitude Miyagi-Ken-Oki, Japan earthquake of 1978 caused failures in several sites where earth retaining structures were in place due mainly to soil 1 iquefaction (Yanev [6] and E11ingwood [12]). A dike along the Natori River was contained by a concrete retaining wall. A section of wall several hundred yards long moved about one foot toward the river (Figure 1.31). Longitudinal fissures opened in the dike behind the wall and in some concrete pavement along part of the dike. The dike also settled as much as one foot. The site, which is at the mouth of the river, is underlain by at least 65 feet of sand.

In the port of Ishinomaki, a fine-sand fill liquefied, causing severe damage to anchored sheet-pile bulkheads. The fill material had been dredged from the seafloor and placed hydraulically with no compaction. It was placed next to old beach deposits, and the boundary of the liquefaction damage followed the contact very closely; the beach deposits were not involved in the 1 iquefaction.

In addition, there were reports of cracking and settlements of bridge abutments. A comparison was made between the Japanese and American criteria for bridge design under earthquake conditions. According to the 1971 Japan Road Association (JRA) bridge design code a provision is made for the inclusion of a design force for lateral seismic earth pressure, whereas the 1977 American Association of State


FIGURE 1.31 - REPAIRED PORTION OF DIKE,NATORI RIVER - FROM (I2)Highway and Transportation Officials (AASHTO) criteria, which is anadaptation of the criteria developed by the California Department ofTransportation in 1973, does not. From the earthquake damage descrip-tions above, it sems clear that even the seismic design criteria forearth retaining structures are inadequate. No country, whether wealthyor poor, where there is seismic activity seems to be immune from thistype of damage.

## CHAPTER II

CENTRIFUGE MODEL TESTING

In recent years, the centrifnge has become a more accepted and useful tool in the modelling of soil mechanics problems. Most soil properties are generally dependent on continuum stresses which are generally gravity-induced. Thus, it is very difficult and inconvenient, if not impossible, to find a model material which will exhibit correctly scaled properties if a test is to be performed at the same gravitational acceleration as the prototype. It would be convenient to use prototype material, but as demonstrated in Chapter $I$, it would obviously not behave in an appropriate manner at the reduced confining stresses in the model. In such a model, in order to develop the same stresses as in the prototype, it is necessary to increase the gravitational acceleration by the 1 ineal scale factor. Thus, if a $1 / 50$ th scale model, made of the same material as the prototype is subjected to a gravitational acceleration 50 times that of the prototype, the confining stresses, and thus the properties and behavior of the model are the same as in the prototype (an analytical description of scaling relations is found in Appendix A). A centrifuge is a machine that can provide model gravity as desired.

It must be realized that the model structure must be properly scaled to provide accurate results. The ratio of the accelerations in model and prototype structures is inversely proportional to the ratio of their lineal dimensions. If the ratio of linear prototype dimensions to
those of the centrifuge model is $N$, then the ratio of area is $N^{2}$ and volumes $N^{3}$. The scaling relations indicate that the forces in the prototype are $N^{2}$ times those in the model and moments $N^{3}$ times while the stresses (force per unit area) are unchanged. Deformation in the prototype is $N$ times larger than in the model, but strains (deformation per unit length) are the same. Thus, the pressure of the same material in both prototype and model results in identical stresses and strains at homologous points.

In the experiments, it was necessary to model the reinforced concrete walls by means of aluminum due to the difficulty in properly scaling down both the reinforcement bars and concrete to a small scale (see Chapter 3). Therefore, the model wall was designed to a similar stiffness per unit width, EI with the stiffness in the prototype being $N^{3}$ times that in the model.

Where dynamic problems are involved, it turns out that the prototype time scale is N times that in the model. As a consequence, model frequencies are higher by the factor $N$. Table 2.1 lists the relations between prototype and model (centrifuge) parameters when the centrifuge is employed $[15,46]$.

In the experiments described here, $N$ was chosen to be 50 , so that the model was $1 / 50$ of the prototype 1 inear dimension, and the model acceleration employed was 50 times normal terrestrial gravity. It was also considered desirable to subject the retaining wall and associated

TABLE 2.1
Scaling Relations

| Parameter | Full Scale <br> (Prototype) | Centrifugal <br> Modelat Ng's |
| :--- | :---: | :---: |
| Acceleration | 1 | N |
| Velocity | 1 | 1 |
| Linear Dimension | 1 | $1 / \mathrm{N}$ |
| Area | 1 | $1 / \mathrm{N}^{2}$ |
| Volume | 1 | $1 / \mathrm{N}^{3}$ |
| Stress | 1 | 1 |
| Strain | 1 | $1 / \mathrm{N}^{2}$ |
| Force | 1 | $1 / \mathrm{N}^{3}$ |
| Mass | 1 | 1 |
| Mass Derisity | 1 | N |
| Weight Density (Unit Weight) | 1 | $1 / \mathrm{N}$ |
| Time (dynamic) | 1 | $1 / \mathrm{N}^{2}$ |
| Time (consolidation) | 1 | N |
| Frequency | 1 | $1 / \mathrm{N}^{3}$ |
| Unit stiffness, EI |  |  |

soil mass as a passive system to essentially random, earthquake-1ike excitations at levels equivalent to strong earthquake motions.

As previously described by Scott [52], the attractiveness of the centrifugal method is that the stresses and strains in the model are identical to those in the prototype so that it avoids problems associated with testing, at earth gravity, small soil models involving
material with strongly nonlinear behavior. The disadvantages are associated with performing the tests on models which are rotating at rates of 100 to 500 rpm in a centrifuge. Power and signals have to be passed in and out through electric and hydraulic sliprings. There are problems associated with the addition of electrical noise in recording transducer output. The noise comes from ambient sources, the electric motor driving the centrifuge, as well as mundane sources such as local radio stations. Most noise can be effectively taken care of by proper amplification and filtering of output signals as well as numerical smoothing of the digitized data.

In initiating a program of centrifuge testing several questions must be asked concerning the proof or the accuracy of the technique. How we11 does a model test predict a prototype behavior? Do the scaling relations tell the whole story? In addition, particularly when models of particularly small dimensions such as retaining walls are considered for testing, there is a problem in deciding at what soil grain scale the applicability of continum constitutive laws to both model and prototype soils breaks down. For very fine grained soils, such as clays, there will be many particles per unit width in both model and prototype retaining wall; on the other hand, in a coarse sand, with grains one twentieth of an inch or so in diameter, there will be relatively few grains per model retaining wall unit width. It is likely that gravity scaling will apply to the constitutive laws, but not to the grain dimensions in the first example. In the second example, it seems possible that the stress-strain relations of model and prototype may not be the
relevant factors, but that the individual grains in the model represent the behavior of boulders in the prototype. Thas, a model retaining wall in coarse sand may not represent the behavior of a prototype retaining wall in the same coarse sand, but that of a retaining wall with a backfill composed of boulders.

The use of the centrifuge in geomechanics dates back to the early 1930's when Bucky [4] first used one in the study of some simple mining problems. The use of a soil mechanics centrifuge was also reported in the Soviet Union around the same time [52]. The use of the centrifuge technique, however, has not been extensively practiced since then, although in the past 15 or 20 years it has been gaining in popularity.

At present, a number of centrifuges have been built and used for soil testing. There are three in the United Kingdom, two at Cambridge and one at Manchester, with radii up to 16 ft and acceleration capabilities up to 200 g . It has been reported that "several dozen" centrifuges for soil testing purposes are in use in the Soviet Union [41]. In addition, centrifuges are currently used for geotechnical research in Sweden, Denmark, France, and Japan. Surprisingly, in the United States, where the technique originated, there are only a handful of small centrifuges currently in use. There is one at Princeton, one at Colorado, and one is being developed at the Ames Research Center by the University of California at Davis, in addition to the one at Caltech. The reasons for their limited usage have not been determined.

A compilation of references on centrifugal testing, worldwide, extends to more than 150 papers and a number of books.

With the number of centrifuges built and operational, and the number of tests performed, it might well be thought that the questions above would have been satisfactorily answered by this time; that many comparisons would have been made between models and prototypes. Study of the accessible literature does not show this to be the case in the quantitative sense, although a fair number of studies show qualitatively similar behavior and mechanisms. The particular type of testing involved in this case, the dynamic centrifuge testing of flexible retaining walls, however, has, as far as known, no precedent.

## EQUIPMENT AND INSTRUMENTATION

### 3.1. The Centrifuge

The centrifuge (Figure 3.1) used is a Model A1030 Genisco G-accelerator", which consists of an 80-inch diameter aluminum-alloy arm which rotates in the horizontal plane and is rated at $10,000 \mathrm{~g}$-pounds payload capacity. At each end of the arm is located an $18 \times 22$ inch magnesium mounting frame (Figure 3.2 ) capable of carrying a 200 -pound payload to 50 g or 60 pounds to 175 g . The acceleration range at the approximately 40 -inch radius of the basket is from 1 to 175 g .

The machine is driven by means of a Sabina Electric and Engineering Type RG 2600 Single phase Full Wave Regenerative Static D.C. Drive with a $5 \mathrm{HP}, 1725 \mathrm{rpm}, 230 \mathrm{v}, 3-\mathrm{ph} a \mathrm{se}$, constant torque, doubleended electric drive motor. For accurate determination of the rotational speed, there is located on the main drive shaft a 600 tooth gear wheel, which via a magnetic pickoff produces 600 pulses per revolution. The pulses are read by an electronic counter which converts them to an LED display of RPM accurate to 0.1 rpm . The drift and wow of the system at any given setting is $0.05 \%$. The acceleration arm is housed in an extruded aluminum enclosure, with all the controls and instrumentation, in the interests of safety, located remotely.



FIGURE 3.2 - CENTRIFUGE FRAME

Electrical power and signals to and from the rotating arm or frame are conducted through 44 sliprings of various capacities in the 10 to 30 amp range. Hydraulic pressure is externally generated with a Haskell Engineering and Supply Co. Model DEN. PR51 pump unit with a line capacity of 3000 psi and is transmitted through either two or four lines by means of rotary unions (hydraulic sliprings). Operations on the centrifuge can be observed by means of a television camera mounted on the arm close to the axis; its signal is conveyed either through the rings mentioned above or through coaxial cable and related, separate sliprings to a monitor $T V$ in the instrumentation room.

### 3.2. The "Earthquake Generating" Mechanism

As mentioned previously the centrifuge is rated at $10,000 \mathrm{~g}$-pounds payload capacity. The load ("payload") of model structure, soil, and containment that it can sustain is 1 imited to 200 1bs (taken up to 50g). Consequently, the need for a method of creating an earthquake-like motion in the centrifuge without taking up a substantial amount of the payload was imperative and was developed with the aid of John Lee.

The "earthquake-generating" mechanism (Figures 3.3, 3.4) consists of a $14.6^{\prime \prime} \times 11.6^{\prime \prime} \times 10^{\prime \prime}$ reinforced aluminum container mounted on a bed of ball bearings which lie in horizontal parallel grooves in a steel plate attached to the swinging magnesium centrifage frame. The bearings were separated with a perforated teflon sheet which allowed equal spacing between them and thus an even pressure distribution throughout (Figure 3.5). At one end, between the bucket and the frame is a spring

FIGURE 3.3 - SCHEMATIC OF EXPERIMENT


FIGURE 3.4 - EARTHQUAKE GENERATING MECHANISM


FIGURE 3.5 - BALL BEARINGS SEPARATED BY TEFLON SHEET


FIGURE 3.6 - REACTION SPRING


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(spring constant $=75 \mathrm{kips} / \mathrm{in}$ ) (Figure 3.6). At the other end is a toggle mechanism connected to a hydraulic piston (Figures 3.7,3.8). Under control the piston displaces the center of the toggle, spreading the ends, and thus forcing the bucket to move, deforming the spring at the other end. When the toggle goes over center, it snaps through, driven by the sudden energy release of the spring, and the soil container snaps back until it hits, stops and rebounds. This happens a number of times for one mode1 "earthquake" event. The bucket thus moves back and forth for a couple of tenths of a second in a relatively random motion which resembles that of a short but intense earthquake. The comparison of the model earthquakes with that of one component of a record of the 1966 Parkfield, California earthquake is done in Section 5.2. Because of the simplicity of the "earthquake generating" mechanism, the motion attained resembles that which would occur near a short fault rupture. The production of prolonged earthquake motions typical of sites at intermediate distances from a long fault rupture (a "great" earthquake) would require another (probably more complicated and thus heavier) mechanism.


### 3.3. Mode1 Retaining Wa11s

Ideally, a model retaining wall made of (properly scaled)
reinforced concrete similar to one described in the design example of Section 12.7 of Wang and Salmon's Reinforced Concrete Design [64] would be desirable for centrifuge testing, but as can be seen from the design sketch (Figure 3.9) of a prototype, it would be very difficult to scale


FIGURE 3.7 - PISTON,TOGGLE,AND BUMPER (FRONT VIEW)


FIGURE 3.8 - PISTON,TOGGLE,AND BUMPER (TOP VIEW)


Design sketch fur cantilever retaining wall.

FIGURE 3.9 - FROM (64)
down all the components of the wall to $1 / 50$ th the size shown. Because of the ease of modelling, it was decided to design a retaining wall made of aluminum instead, and then scale it down. The procedure is similar to the procedure used in the design of a regular reinforced concrete cantilever retaining wall.

### 3.3.1. Design of the Retaining Wa11s

It is required to design a prototype, aluminum cantilever retaining wall to support a backfill of earth 16 ft high above the final level of earth at the toe of the wall. The backfill is to be level. A 1ateral earthquake acceleration of 0.25 g is expected for design purposes (in actuality, it doesn't occur though). The following data is given for design:
soil density $\gamma=92$ pcf (Nevada 120 sand medium density)
E1astic Strength of 6061-T6 Aluminum $f_{A}=48,000$ psi
Elastic Modulus $E_{A}=10 \times 10^{6}$ psi
First of all, it is necessary for the wall-soil system to be in a state of equilibrium. A Mononobe-Okabe analysis (see Section 1.1) with $k_{H}=0.25$ will be used.

The Mononobe-Okabe parameters are:

$$
\begin{array}{ll}
\theta=\tan ^{-1}(0.25)=14^{\circ} & \delta=0^{\circ} \\
\gamma=0.092 \mathrm{kcf} & \mathrm{i}=0^{\circ} \\
\gamma=35^{\circ} & \beta=0^{\circ}
\end{array}
$$

## Therefore:

$$
\mathrm{K}_{\mathrm{AE}}=0.43
$$

and the total force $P_{A E}$ is thas

$$
\begin{equation*}
\mathbf{P}_{\mathrm{AE}}=1 / 2 \gamma \mathrm{~h}^{2}\left(1-k_{\mathrm{v}}\right) K_{\mathrm{AE}} \tag{3.1}
\end{equation*}
$$

or

$$
P_{A E}=(1 / 2)(0.092)(18.3)^{2}(0.43)=6.6 \mathrm{kips} / \mathrm{ft}
$$

This is the total lateral force acting on the wall. As recommended by Seed and Whitman [55], the force increment on the wall, ${ }^{\Delta P_{A E}}$, due to the earthquake load should be assumed to act 0.6 h or so above the base. Thus, it is necessary to find the static force $P_{A}$ and place the forces on the wall as shown in Figure 3.10.

From the Rankine static lateral earth pressure theory $P_{A}$ is given by:

$$
\begin{equation*}
P_{A}=1 / 2 \gamma h^{2} K_{A} \tag{3.2}
\end{equation*}
$$

where:

$$
\begin{equation*}
K_{A}=\frac{1-\sin \phi}{1+\sin \phi} \tag{3.3}
\end{equation*}
$$

For the soil involved $K_{A}=0.27$ so:

$$
P_{A}=(1 / 2)(0.092)(18.3)^{2}(0.27)=4.2 \mathrm{kips} / \mathrm{ft}
$$

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FIGURE 3.10 - PRELIMINARY PROPORTIONING OF PROTOTYPE CANTILEVER RETAINING WALL
which acts at $h / 3$ above the base of the wall. Thus:

$$
\Delta P_{\mathrm{AE}}=\mathrm{P}_{\mathrm{AE}}-\mathrm{P}_{\mathrm{A}}=6.6-4.2=2.4 \mathrm{kips} / \mathrm{ft}
$$

which acts at 0.6 h above the base.

The weight of the backfill, W, is:

$$
W=\gamma H x=(0.092)(18) \times=1.6 x \mathrm{kips} / \mathrm{ft} .
$$

Summing moments about point B. $\left(\sum M_{b}=0\right)$

$$
\frac{W x}{2}=\frac{P_{A^{h}}}{3}+0.6 \Delta P_{A E} h=h\left(1 / 3 P_{A}+0.6 \Delta P_{A E}\right)
$$

Consequently:

$$
\frac{1.6 x^{2}}{2}=(18.3)[(1 / 3)(4.2)+(0.6)(2.4)]
$$

Therefore:

$$
x=\left[\frac{(2)(18.3)[(1 / 3)(4.2)+(0.6)(2.4)]}{1.6}\right]^{1 / 2}=8.1 \mathrm{ft}
$$

The entire base length is recommended by Wang and Salmon to be approximately:

$$
\text { Base length } \approx 1.5 x=(1.5)(8.1)=12.2 \mathrm{ft}
$$

The base 1 ength was thus decided upon to be 15.25 feet 1 ong (3.66 in long in the $1 / 50$ scale model) which gives about an extra $25 \%$ or so of
length for safety. A check must now be made for safety against overturning. Recalling that the design base 1 ength is 15.25 ft , the design $x$ (Figure 3.10) is thus $2 / 3$ of this or $10.2 \mathrm{ft}.(10 \mathrm{ft}, 2 \mathrm{in}$ ). Thus the weight $W$ of the backfill is, from above:

$$
W=1.6 x=(1.6)(10.2)=16.3 \mathrm{kips} / \mathrm{ft} .
$$

Taking moments about point $A$ of the base and neglecting the weight of the wall, the resisting moment is:

$$
M_{R}=(10.2)(16.3)=166.3 \mathrm{ft} \mathrm{k} / \mathrm{ft}
$$

The overturning moment is:

$$
M_{0}=h\left(1 / 3 P_{A}+0.6 \Delta P_{A E}\right)
$$

Thus:

$$
M_{0}=(18.3)[(1 / 3)(4.2)+(0.6)(2.4)]=52.0 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

Therefore, the factor of safety against overturning is:

$$
\text { F.S. }=\frac{\mathrm{M}_{R}}{\mathrm{M}_{\mathrm{o}}}=\frac{166.3}{52.0}=3.2
$$

which is greater than the traditional value of 2.0 . This factor of safety does not even include the weight of the wall itself which would provide additional resistance to overturning.

The stem of the wall must now be designed to resist the bending moment $M$ given by:

$$
M=H\left(1 / 3 P_{A}+0.6 \Delta P_{A E}\right)-1 / 3 P_{P E} H_{f}
$$

where $P_{P E}$ is the resultant of the passive force provided by the frost cover of depth $H_{f}$ (Figure 3.10).

The coefficient of passive earth pressure, $K_{P E}$, for a MononobeOkabe analysis is given by

$$
\begin{equation*}
K_{P E}=\frac{\cos ^{2}(\alpha-\theta+\beta)}{\cos \theta \cos ^{2} \beta \cos (\beta-\delta-\theta)\left(1-\sqrt{\frac{\sin (\alpha+\delta) \sin (\alpha-\theta+i)}{\cos (\beta-\delta-\theta) \cos (\beta-i)}}\right)^{2}} \tag{3.4}
\end{equation*}
$$

and:

$$
\begin{equation*}
P_{P E}=1 / 2 \gamma H_{f}^{2}\left(1-k_{v}\right) K_{P E} \tag{3.5}
\end{equation*}
$$

Therefore:

$$
\mathrm{K}_{\mathrm{PE}}=3.18
$$

Therefore:

$$
\mathrm{P}_{\mathrm{PE}}=(1 / 2)(0.092)(4)(3.18)=0.6 \mathrm{kips}
$$

Thus:

$$
M=18[(1 / 3)(4.2)+(0.6)(2.4)]-(1 / 3)(0.6)(2)=50.7 \mathrm{ft} \mathrm{k} / \mathrm{ft}
$$

With a bending factor of safety of 1.7 , the design moment is:

$$
M_{D}=86.1 \mathrm{ft} \mathrm{k} / \mathrm{ft}
$$

The thickness of the stem is determined by the use of the bending formula for a beam:

$$
\begin{equation*}
\sigma=\mathrm{M} / \mathrm{S} \tag{3.6}
\end{equation*}
$$

Where:

$$
\begin{aligned}
& \sigma=\text { stress of the material } \\
& S=\text { unit section modulus of cross section }
\end{aligned}
$$

For a rectangular cross section, the unit section modulus is:

$$
\begin{equation*}
S=\frac{d^{2}}{6} \tag{3.7}
\end{equation*}
$$

Where $d$ is the thickness of the section. Taking the elastic strength of the aluminum $f_{A}$ as $\sigma$, the stem thickness is determined:

This corresponds to a thickness of 0.065 inches in the model wall, at 1/50 scale.

Two models of the 16 ft high cantilever retaining wall were built (Figure 3.11). They were made of two aluminum 6061-T6 plates dip-brazed together by Precision Dipbraze, Inc. of Van Nuys, CA. The base of both walls is made of 0.063 inch plate while the stems are 0.063 inches thick in wall No. 1 and 0.050 in wall No. 2. The thicknesses stated are standard aluminum plate sizes. The 0.063 inch thickness of wall No. 1 is approximately the correct size for the design conditions imposed with the appropriate safety factors. Wall No. 2 has no safety factor (F.S. = 1.0) at all. (Its prototype wall would have a moment capacity of $50.0 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$ versus the calculated acting moment of $52.0 \mathrm{ft} \mathrm{k} / \mathrm{ft})$.

FIGURE 3.11 - MODEL RETAINING WALL

It should be noted that it is generally agreed upon in practice that the Mononabe-Okabe method gives a conservative design (i.e., calls for larger walls than "necessary"), and in most cases is not even used (nor is any other method) when a seismic design is in order.

### 3.3.2. Determination of Actual EI of Walls

In order to determine the true stiffness (EI) of the retaining walls, the Young's Modulus $E$ of the aluminum used had to be measured. To do this a rectangular piece of the same $0.063^{\prime \prime}$ thick plate used to make the walls $6.555^{\prime \prime}$ long and $1.493^{\prime \prime}$ wide was cut. The piece was then clamped and held horizontal so that it formed a cantilever beam $5.026^{\prime \prime}$ long. Weights of $0,0.220,0.441,0.661$ and $0.772 \mathrm{lbs}(0,100,200$, 300 , and 350 grams) were then hung from the free end. The end deflection was measured with a Federal dial gage accurate to 0.0001 inches. Recalling that the end (maximum) deflection $y_{M A X}$ of a cantilever beam with an end point load is:

$$
\begin{equation*}
y_{M A X}=\frac{P 1^{3}}{3 E I} \tag{3.8}
\end{equation*}
$$

where $P$ is the load, 1 the beam length, and $I$ the bending moment of inertia it follows that:

$$
\begin{equation*}
E=\frac{\mathrm{Pl}^{3}}{3 \mathrm{Iy}_{\mathrm{MAX}}} \tag{3.9}
\end{equation*}
$$

The average $E$ then determined from the measurements was found to be $9.699 \times 10^{6} \mathrm{psi}$.

Recalling that the moment of inertia per unit width $I$ of a rectangalar cross section is $\frac{h^{3}}{12}$, where $h$ is the section depth, for retaining wall No. 1 (RWI) the EI was determined as $202.11 b$ in ${ }^{2} /$ in and for (RW2) as $101.0 \mathrm{lb} \mathrm{in}^{2} / \mathrm{in}$.

### 3.3.3. Determination of the Fundamental Frequency of the Wa11-Soil System

The fundamental frequency of the wall-soil system was determined by an examination of the Fourier Amplitude Spectra of the accelerograms recorded at the top and bottom of the wall (in prototype scale) from tests 1CNO001,* 1CN0002, and 1CN1003 for RW1, and from test 2 CNOO11 for RW2 using the FORTRAN program IVMAIN described in Section 4.2 . The accelerograms at the top of the wall indicate the output response of the system while those at the bottom are a measure of the input excitation to the system. Taking the corresponding pairs of Fourier Spectra for each test and finding where the ratio of output (top) to input (bottom) amplitude is a maximum provides an accurate determination of the system's natural frequencies.

Upon examination of the Fourier spectra (Figures 3.12 through 3.19), it was determined that the fundamental frequencies were 2.3 Hz for 1CNO001, 2.7 Hz for 1CNO002, and 2.7 Hz for 1CN1003. There was

```
* The following nomenclature was chosen for test numbering:
Test 吕 b
a = wall number; b = type of wall; c = type of sand; d = backfill
angle (in degrees); e = test number; C = cantilever; N=Nevada 120.
```

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very little relative difference between the frequencies determined from these tests, leading to the conclusion that there is little sensitivity in the system with regard to backfill slope or soil density differences for RW1. The fundamental frequency of the tests where RW1 was used was then taken to be the average of these tests, $2.6 \mathrm{~Hz}(129 \mathrm{~Hz}$ model). Similarly from the spectra for 2 CNOO11, the fundamental frequency of RW2 was taken to be $2.5 \mathrm{~Hz}(123 \mathrm{~Hz}$ model). This is also very close to the frequencies of tests using RW1, so there is little variation of frequency with regard to wall stiffness as well.

From examination of the Fourier spectra it can also be seen that there is only a significant contribution to the response of the systems by only one frequency, the fundamental. This is confirmed upon examination of the displacement curves presented in Chapter 5.

As will be explained in section 5.1 , the fundamental frequencies of the systems are used to create dimensionless time parameters since they are a characteristic of each system.

### 3.4. Soil

The type of soil used was Nevada 120 Silica. This sand is a uniformly-graded, fine grained soil. A grain size distribution is shown in Figure 3.20. The soil was dry in all of the tests. It has a density range of from about 85 pcf in its 100 sest state to 99 pcf in its most dense. For the tests the density ranged from 91 to 99 pcf. For the medium density soil, the angle of internal friction $\phi$ is about $35^{\circ}$.


The soil was chosen because of its fine grained size which is desirable when doing centrifuge work, as already mentioned in Chapter 2 .

### 3.5. Instrumentation

A cross section of the retaining walls indicating the location of all the transducers which will be described below is shown in Figure 3.21 .

### 3.5.1. Strain Gages

Moments on the retaining walls are measured directly by the use of strain gages which in reality measure the curvature, M/EI.

Retaining wall No. 1 (RW1) is instrumented with seven pairs of Micromeasurements Model CEA-13-062 UW-350 strain gages located at distances $1.50^{\prime \prime}, 2.25^{\prime \prime}, 275^{\prime \prime}, 3.15^{\prime \prime}, 3.50^{\prime \prime}, 3.75^{\prime \prime}$ and $4.00^{\prime \prime}$ from the top of the wall, and down the centerline, one strain gage of each pair on the front and one on the back at each location. Retaining wall No. 2 (RW2) is likewise instrumented with four pairs at distances from the top of $1.50^{\prime \prime}, 2.75^{\prime \prime}, 3.50^{\prime \prime}$ and $4.00^{\prime \prime}$ (Figure 3.22 ). The type of strain gage used is a universal general-purpose strain gage. These gages are polymide-encapsulated Constantan ('A' Alloy) gages featuring large, integral, copper-coated terminals. This construction provides optimum capability for direct leadwire attachment. The gage is extremely thin and flexible ( $0.0022^{\prime \prime}$ ). The gage length is $0.062^{\prime \prime}$ and the grid width is $0.062^{\prime \prime}$. The resistance is $350 \pm 0.3 \% \Omega$ with a strain range of $\pm 3 \%$.


- Accelerometer and $\Delta$-beam locations
- Strain gage locations
$x$ Pressure transducer locations
(Parentheses indicate distance of transducer from top of wall in inches)

FIGURE 3.21 - MODEL WALL CROSS SECTION


The gages are bonded to the wall surface according to M-Line Accessories Instruction Bulletin B-130-6 (8/77) with M-Bond 600 epoxy resin adhesive. Soldered to each gage are two lengths of Belden AFG32 magnet wire, The leads were laid on the faces of the wall and coated with a flexible, impermeable protective coating (BLH Barrier J).

The strain gage circuit is arranged as a Chevron Wheatstone bridge circuit as shown in Figure 3.23 . This configaration minimizes the number of balancing resistors used as well as the number of sliprings taken up since all the pairs of strain gages have but one common ground. The excitation voltage is 5V DC.

The location of the Soil Mechanics Centrifage at Caltech is on the roof of Thomas Laboratory in close proximity to air conditioning units and elevator drive motors which make for a very noisy electrical environment. In order to minimize this noise, the signals from the strain gage bridge are amplified with one LF352 amplifier (Figure 3.24) for each pair of strain gages. This is done inside the centrifuge itself as the amplifiers are loaded on the centrifuge arm. The gain is set at 50. The amplified signals then pass through the sliprings to the control room where they are recorded on a Honeywell Model 1858 CRT Visicorder which allows inertialess recording from DC to 5 kHz . The analog signals are recorded on Kodak Type UV 1920-80330Y Visicorder Recording Paper at an amplitude of $200 \mathrm{mV} / \mathrm{division} \mathrm{(1} \mathrm{division}=$ 2.5 cm ). In the dynamic portions of the test, the recording takes


FIGURE 3.23 - STRAIN GAGE CIRCUIT


FIGURE 3.24 - AMPLIFIERS FOR STRAIN GAGES AND ACCELEROMETERS

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FIGURE 3.25 - CENTRIFUGE POWER SUPPLY TO DRIVE BRIDGE
place at a rate of from 50 to 80 inches per second of recording paper depending on the particular test.


### 3.5.2. Accelerometers

At the top and bottom of the centerline of the face of each retaining wall is mounted an Entran Devices Inc. Model EGA-125F-500D miniature accelerometer. In most tests there is an additional one located in the backfill approximately half way between the wall and the wall of the bucket and is buried near the surface.

The accelerometers employ a fully active Wheatstone Bridge consisting of semiconductor strain gages. The strain gages are bonded to a simple cantilever beam which is end-loaded with a mass (Figure 3.26). Dnder acceleration, a "g" force, the force on the cantilever is created by the $g$ effect on the mass ( $F=m a$ ). The accelerated mass creates a force which in turn provides a bending moment to the beam. The moment creates a strain (proportional to the acceleration) which results in a bridge unbalance. With an applied voltage, this unbalance produces a millivolt deviation at the bridge output, which is propor tional to the acceleration vector.

A very attractive feature of this type of accelerometer is its very small size. The entire unit (minus the leads) weighs only 0.02 oz . The accelerometer unit is $0.270^{\prime \prime}$ long by $0.145^{\prime \prime}$ wide by $0.105^{\prime \prime}$ (unit weight of $525 \mathrm{lb} / \mathrm{ft}^{3}$ ) high and is mounted on a $0.270^{\prime \prime} \times 0.370^{\prime \prime} \times 0.040^{\prime \prime}$ flange as shown in Figure 3.27 . The bold-faced arrows indicate the sensitive axis. The accelerometers are attached to the model walls with


FIGURE 3.26 - ACCELEROMETER CUTAWAY (FROM ENTRAN DEVICES)


FIGURE 3.27 - ACCELEROMETER DIMENSIONS (FROM ENTRAN DEVICES)
two 0-80 hex screws. The model of accelerometer used has a range of $\neq 500 \mathrm{~g}$ with a nominal sensitivity of about $0.5 \mathrm{mV} / \mathrm{g}$ (varies slightly from this with each particular unit), an input impedance of about 1150』, an ontput impedance of about $550 \Omega$, and a resonant frequency of $\mathbf{3 0 0 0}$ Hz. In addition, the unit is damped to 0.7 of critical using a viscous fluid medium. This helps to eliminate resonance and allows a nseful frequency range of DC to 1000 Hz . The excitation voltage is 15 V DC.

Similarly, as with the strain gages, the output signals were suitably amplified and filtered to minimize the high frequency noise inherent with centrifuge testing. The accelerometer circuit is shown in Figure 3.28. The gain on the amplifiers was set at 10 , and the analog signals recorded on the Honeywell Visicorder at an amplitude of 200 mV/division. The accelerometer signals were recorded directly alongside those of the strain gages on the recording paper.

### 3.5.3. Pressure Transducers

Originally, it was planned to obtain pressure distributions behind the retaining walls by means of differentiating the moment distributions twice with respect to the length coordinate $x$. From elementary relationships it is well known that the shear $Q$ is:

$$
\begin{equation*}
Q=\frac{\partial M}{\partial x} \tag{3.10}
\end{equation*}
$$


[ACCELEROMETER]

FIGURE 3.28
(PRESSURE TRANSDUCER) CIRCUIT
$\{D E L T A-B E A M\}$
where $M$ is the moment distribution. The load (pressure) distribution $P$ is:

$$
\begin{equation*}
P=\frac{\partial Q}{\partial X}=\frac{\partial^{2} M}{\partial x^{2}} \tag{3.11}
\end{equation*}
$$

Unfortunately, becanse of inaccuracies which develop and propagate in namerical differentiation it was found that these simple relations did not give adequate or accurate pressure distributions.

Figure 3.29 (which is fully explained in Section 5.3) shows how inaccurate the use of moment differentiation to arrive at pressure distributions is. It was thus necessary to measure pressure directly by the use of pressure transducers and then integrate the determined pressure distributions (numerical integration is much more stable and accurate than differentiation) to obtain the shear distributions.

Except for test 1CNOOO1, four miniature, low profile pressure transducers were placed at various locations (depending on the particular test) along the centerline of the back of the walls. In tests 1CN0002, 1CN1003, and 1CNO004, the pressure transducers were located $1.68^{\prime \prime}, 2.78^{\prime \prime}, 3.59^{\prime \prime}$, and $4.17^{\prime \prime}$ from the top of the wall; in tests $1 \mathrm{CN} 1505,1 \mathrm{CN} 0006$ at $1.79^{\prime \prime}, 2.75^{\prime \prime}, 3.60^{\prime \prime}, 4.16^{\prime \prime}$, in tests 1CNO007, 1CN0508, 1CN1009, 1CN1510 at 1.86", 2.77", $3.59^{\prime \prime}, 4.21^{\prime \prime}$, and in tests 2CN0011, 2CN0012, 2 CN1013, $2 C N 1514$ at 1.83", $2.92^{\prime \prime}, 3.36^{\prime \prime}, 3.91^{\prime \prime}$.

The pressure transducers used are Entran Devices Inc. Model EPF-200-50 F1at1ine Pressure Transducers. The transducer consists of a semiconductor strain gaged circular diaphragm less than $0.2^{\prime \prime}$ in diameter constructed of $17-4$ PH stainless steel. This is a piezo resistive

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FIGURE 3.29 - PRESSURE DISTRIBUTION OBTAINED FROM MOMENT DOUBLE-DIFFERENTIATON

pressure transducer with a fully active semiconductor bridge. Similarly, as with the accelerometer, a load on the diaphragm will create a strain (proportional to the pressure) which resalts in a bridge unbalance. With an applied voltage, this unbalance produces a millivolt deviation at the bridge output, which is proportional to the pressure.

The transducer is very small (Figure 3.30) and thin being only $0.040^{\prime \prime}$ thick. It has a range of 0 to 50 psis with a nominal sensitivity of about $2.5 \mathrm{mV} / \mathrm{psi}$ (varies slightly from this with each particular anit), an input impedance of about $750 \Omega$, an output impedance of about $250 \Omega$ and a resonant frequency of 50 kHz . The excitation voltage is 6 V DC.

As previously described with the other types of transducers, the output signal is suitably amplified and filtered. The pressure transducer circuit (Figure 3,28 ) is similar to that of the accelerometers with the exception that the signals are amplified with a CA3080 amplifier (Figure 3.31). The amplifier gain was 25, and the signals were recorded alongside those of the other transducers on the Honeywell Visicorder at an amplitude of $200 \mathrm{mV} / \mathrm{div}$.

### 3.5.4. Displacement Transducers ( $\Delta$-beams)

In order to determine the relative displacements of the retaining walls with respect to the centrifuge bucket, the moment distribution

- 100 -

$+15$
(RED)


FIGURE 3.31 - AMPLIFIERS FOR PRESSURE TRANSDUCERS AND DELTA-BEAMS
along the wall must be integrated twice with respect to the length coordinate $x$. Recalling the equation for a the curvature of the deflected shape of a simple beam:

$$
\begin{equation*}
\frac{\partial^{2} y}{\partial x^{2}}=\frac{-M}{E I} \tag{3.12}
\end{equation*}
$$

it follows that the deflected shape $y$ is given by:

$$
\begin{equation*}
y=\frac{1}{E I} \int_{0}^{H} \int_{0}^{H} M d x d x+A x+B \tag{3,13}
\end{equation*}
$$

where $A$ and $B$ are constants of integration dependent on the boundary conditions of the wall. A and B can be determined knowing the displacements at the top and the bottom of the wall. The displacements at these locations can be deduced by integrating the accelerometer records twice with respect to time. This, however, requires the determination of two additional constants of integration dependent on time-imposed conditions. At each location if the initial (static), and final (static, after shaking is over) displacements are known at each of the locations, the pair of time imposed constants of integration can be determined, and thus the relative displacements between the walls and bucket can be determined at the top and bottom of the wall. Knowing this, A, B, and the full displacement carves can thas be determined.

Initial and final displacements at the top and bottom of the walls are measured by means of a pair of cantilever beams (called $\Delta$-beams for simplicity) which are attached to the front of the bucket and connected by means of a very thin wire to the accelerometer locations on the face of the wall. These $\Delta$-beams are very thin ( $0.015^{\prime \prime}$ thick) strips $2.25^{\prime \prime}$
long, $1.00^{\prime \prime}$ wide of spring steel attached to a rigid base and strain gaged, so that, properly calibrated, they can record displacements over a relatively wide range.

The $\Delta$-beam circuit is similar to that of the pressure transducers (Figure 3.28). Since the frequency response is very 1ow, the transducer signals are only recorded on the Visicorder during the static portions of the test. The circuit excitation is 5 V , the gain 25 , and the Visicorder amplitude is $100 \mathrm{mV} / \mathrm{div}$.

### 3.6. Calibration of Transducers

A11 pre-test calibrations were carried out using the entire electronic circuitry, i.e, the calibration signals were routed through those terminals, amplifier channels, filters, sliprings, and Visicorder channels which they would use during the actual testing. The excitations, gains, and recording amplitudes used in calibration were likewise the same as in the tests. The outputs recorded on the Visicorder were converted directly to parameter (moment, displacement, acceleration, etc.) measurements without the use of instrument factors. All transducers are 1 inear and therefore require two calibration factors (slope, intercept) for each. These factors were determined using the 1 inear least-squares function on a Hewlett-Packard 55 pocket calculator.

All calibrations were recorded on the Visicorder and the traces digitized with a Benson-Lehner 099D data reducer anit. The digitizer had a resolution of 790.8 digitizer units (du) per inch of width of recording paper and $792.0 \mathrm{du} /$ inch of length. The calibration slopes
were thus in units of parameter per digitizer unit and the intercepts in units of parameter, Data reduction of the tests will be discussed in Section 4.2.

### 3.6.1. Strain Gages

The strain gages are calibrated to measure moments directly. To accomplish this, the base of the model retaining walls is rigidly secured to the bottom of the centrifuge magnesium frame which was rotated $90^{\circ}$ so that the stem forms as horizontal cantilever beam. Two $1^{\prime \prime}$ thick (each) Plexiglas beams were then clamped in sandwich fashion to the free end of the stem and weights hung from the center. The calibration arrangement is shown in Figure 3.32. The Plexiglas beams distribute the load evenly across the width of the wall. This creates in effect a cantilever beam with a concentrated load at the end, moments of which can be readily determined. Weights of $0,1,2,3,4,5,6$, and 8 pounds were hung and the output recorded at the Visicorder at the other end of the system.

### 3.6.2. Accelerometers

In order to calibrate the accelerometers, they were placed with the sensitive axis facing downward on the upper lip of the centrifage bucket which is at a radins of 30.5 inches from the centrifuge axis. Readings were recorded on the Visicorder with this arrangement, i.e., the accelerometers reading 1 g . The centrifuge was then taken up to accelerations of $10,20,30,40,50,60$, and 70 g respectively. It was


FIGURE 3.33 - DELTA-BEAM CALIBRATION


FIGURE 3.32 - STRAIN GAGE CALIBRATION

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assumed that an amplitude of 0 du on the Visicorder was 1 g . The calibrations were then determined in relation to this.

### 3.6.3. Pressure Transducers

The pressure transducers were calibrated by placing them on the bottom of the centrifnge bucket at a radius of 40.5 inches from the centrifuge axis, and placing $4.90^{\prime \prime}$ of Nevada 120 sand at a density of 93.3 pcf on top of them. Measurements were then taken with the centrifnge stationary (at 1 g ) and spinning at $10,20,30,40$, and 50 g . The increase in g-acceleration to $N g^{\prime} s$ causes an increase in the soil unit weight by $N$ (see Table 2.1 ) and thus an increase in pressure, the pressure simply being the weight density of the soil (at the particular acceleration level) times the depth (4.90"). Thus pressures of 38, 381, 762, 1143, 1524, and 1905 psf corresponded to each g level used in the calibration.

### 3.6.4. $\Delta$-beams

The $\Delta$-beams were calibrated by fixing them to a vice and measuring displacements with the aid of a Federal dial gange accurate to 0.001 in. Displacements of $0,0.01,0.02,0.03,0.04,0.05$, and 0.10 inches were measured (Figure 3.33).

## CLAPTER IV

### 4.1. The Experiment

In every test performed, the following sequence of experimental procedures was carried out.

To begin with, sand was placed on the centrifuge bucket to a depth of about 4 inches (Figare 4.1). If looser conditions were desired, it was just dumped in; if denser, it was tamped and/or vibrated after being placed in one to two inch lifts. Following this, one of the walls, along with all its instrumentation, was placed approximately 6 inches from the front of the bucket (leaving about 8-1/2 inches for backfil) and carefully seated on the sand layer already placed (Figure 4.2). Special care was taken to assure the wall was vertical by following guide lines drawn on the inside of the bucket. Sand was then placed on both sides of the wall following the procedure for looser or denser conditions described above (Figure 4.3). The total depth of sand (for a flat backfill) was 8 inches. For a sloping backfill, it was placed to the desired slope above the 8 inch mark on both sides. The weight of the sand placed was then totalled and, since the bucket dimensions were well known, the unit weight determined.

By placing sand on both sides of the wall and taking the container up to 50 g 's the transducers were thas zeroed. In this manner, the walls were subjected to no moment, lateral acceleration, or displacement and an accurate zero was recorded on the Visicorder at the test centrifuge


FIGURE 4.2


FIGURE 4.3


Figure 4.4
acceleration．The experiment was then returned to one $g$ where the sand on the front of the walls was removed to the design height（Figure 4．4）．

Table 4．1 Soil Densities

| Test | $\underset{(\mathrm{pef})}{\text { Density } ⿴^{(0)} 1 \mathrm{~g}}$ | $\underset{(\mathrm{pcf})}{\text { Density } @} 50 \mathrm{~g}$ | Test | $\underset{(\mathrm{pcf})}{\text { Density } ⿴ 囗 ⿰ 丿 ㇄}$ | $\underset{(\mathrm{pcf})}{\text { Density } @} 50 \mathrm{~g}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 CN 0001 | 92.6 | 4630 | 1CN0 508 | 95.9 | 4797 |
| 1 CNO 002 | 91.2 | 4561 | 1CN1009 | 97.0 | 4849 |
| 1 CN 1003 | 92.0 | 4597 | 1CN1510 | 95.3 | 4764 |
| 1 CNO 004 | 93.9 | 4695 | $2 \mathrm{CN0} 011$ | 98.8 | 4941 |
| 1 CN 1505 | 92.4 | 4621 | $2 \mathrm{CN0012}$ | 95.8 | 4790 |
| 1 CN 0006 | 94.5 | 4726 | $2 \mathrm{CN1} 013$ | 97.3 | 4865 |
| 1 CN 0007 | 98.1 | 4906 | $2 \mathrm{CN1514}$ | 97.7 | 4886 |

The system was next taken back up to 50 g ＇s where all the static outputs were recorded on the Visicorder．The channels which carried the signals of the $\Delta$－beams were then turned off since，due to the poor frequency response of the $\Delta$－beams，they were inadequate for dynamic measurements．After this，the container was subjected to the ＂earthquake＂shaking described in Sections 3.2 and 5．2．The output sig－ nals were recorded on the Visicorder at a recording paper rate of 50 to 80 inches per second depending on the particular test．Usually there were 4 strain gage， 3 accelerometer and 4 pressure transducer outputs （11 traces total）being recorded on paper only 8 inches wide．Needless to say，there was some overlapping of traces，and a high density of ana－ log data，but the recordings were usually clear and easy to follow when digitizing subsequently．

Figure 4.5 is an example of the traces recorded on the Visicorder during part of the dynamic portion of a typical test (2CN0012 in this case).

Following the shaking, the two $\Delta$-beam channels were turned back on, and their outputs taken along with those from the other transducers now static once more. The system was then brought back to rest which concluded the actal experiment itself. Data reduction of the Visicorder output followed.

### 4.2. Data Reduction

The digitizing was performed on a Benson-Lehner 099D data reducer unit and the following procedure used. The cross hairs are manually set to successive $x-y$ coordinates on each record trace. The coordinates are converted to digital position fignres by means of a magnetic readout head, and are stored in a 6-digit accumulator system from which they are automatically read out to an IBM 29 card punch. The resolution of the system is 792.0 du/inch in the $x$ and 790.8 du/inch in the $y$ directions. The Visicorder paper is placed on the $24^{\prime \prime} \times 16^{\prime \prime}$ digitizing table with the horizontal axis lined up by eye to an estimated zero axis. The lining up of the paper need not be too accurate since it will be corrected with respect to a baseline recorded on the paper. All traces are digitized without moving the record on the table.

First of all, a baseline, which will be used to make corrections for deviation from the horizontal, is digitized. Each trace on the Visicorder paper is then digitized individually as follows. The zero

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point of the trace is first digitized. This is the point at 50 g where sand is on both sides of the wall. For the pressure transducers and the $\Delta$-beams the zero point is the reading when the centrifuge is at rest. Next the static point at 50 g (normal experiment, backfill sand only) is digitized followed by the digitization of the dynamic part of the test. The records are digitized on an unequal time basis since this leads to the best definition of the trace for a given number of data points. All significant peaks, points of inflection, etc., are picked, along with as many intermediate points as are needed for an accurate definition of shape.

The digitized data are directly punched on cards which are then read into magnetic disks on a VAX 11 Wordprocessing system. Program P1CHECK (Trifunac, Lee [63]) reads the data and checks whether the time coordinates monotonically increase. It also searches for possible disproportionate jumps of the amplitude data. If any error is found, the program prints out the message. Small errors are corrected immediately. The data are then plotted to the same scale as the digitized record, and the two versions are compared to check the accuracy of digitization. Any portion that is digitized improperly has to be redigitized and replotted until the final plot agrees well with the digitized record.

The corrected digitized data is now fed into the data processing program WALL which will be described below and which is listed in Appendix B. WALL prints out static, maximum dynamic, and final static moment, pressure, shear, and displacement distributions along the wall
to discrete locations; moment, pressure, shear, displacement vs. time distributions at the location of each maximum response at equal time steps; accelerometer, velocity and displacement vs. time records for each of the three accelerometer locations, as well as other data pertaining to the test, namely, centrifuge operation data, material properties, and calibration factors. In addition plots are made of the abovementioned distributions. Contour plots of moment, pressure, shear, and displacement distributions with respect to location and time are also made. This provides a very descriptive and compact representation of the entire test.

It was sometimes desired to obtain characteristics of the motion recorded by the accelerometers in order to have a comparison with actual accelerogram characteristics of real earthquakes. For this purpose, some of the accelerometer records were given the routine computer processing of strong-motion accelerograms developed at Caltech by Trifunac and Lee [63]. Programs P1CHECK, P2 SCALE, and P3TAPE form Volume $I$ of data in which the raw data is converted into uncorrected, scaled, accelerogram data. Program IIMAIN creates Volume II which contains corrected accelerogram, velocity, and displacement data. Volume III, which gives the response spectra of the record, is created using program IIIMAIN. Program IVMAIN creates Volume IV containing the Fourier Spectra. From this volume, the fundamental frequency of the system is determined (see Section 3.3.3). As will be seen in the results, it is the only frequency which contributes significantly to the
response. The standard accelerogram processing is outlined in Figure 4.6 .

The results from the tests are obtained by processing the digitized data with the FORTRAN program WALL. The program is run on an IBM 370/3032 Compater System at the Booth Computing Center at Caltech.

After the raw digitized data is checked by program P1CHECK, the corrected data from the transducers is fed into WALL, along with other experimental data, namely centrifuge speed, distance from centrifuge axis to top of wall, wall/soil properties, order of polynomial desired for least-squares fit (see below), type, number, and location of transducers used, and calibration factors.

All the traces are then corrected with respect to the input baseline to avoid errors due to the slight slope which all the records inherently have becanse of positioning on the digitizer table. This is particularly important in the accelerometer records since double integrations can introduce errors proportional to the square of the running time with just a small initial slope present.

Following this, the data is scaled to model dimensions using the calibration factors.

Since all the separate traces are digitized individually, it is necessary to correlate them to specific, discrete time steps. This is done by smoothing the individual trace data point by point with a cubic spline and then picking off the values from the spline at particular time intervals. For convenience (see Section 5.1), it was decided to use a dimensionless time group $t f_{1}$ to express time. $t$ is the real

## STANDARD ACCELEROGRAM PROCESSING


prototype (or mode1) time and $f_{1}$ is the real prototype (or model) fundamental frequency of the system. $t f_{1}$ is the same for both model and prototype. The discrete time steps are chosen at 150 per $\mathrm{tf}_{1}$ for the first six $t f_{1}$ and 75 per $t f_{1}$ thereafter. Because of the nature of the experimental shaking, most of the critical (maximum and high frequency) response occurs when $0 \leq \mathrm{tf}_{1} \leq 6.0$.

The moments are determined from the scaled strain gage data. It is intended to use a quintic (fifth order) spline fit to the data points at each time step. The spline fitting, however, requires six boundary conditions, the moment and the first and second derivatives of the moment, at the top and base of the wall. At the top of the wall, these are known. The moment and shear (first derivative) are zero since this is the free end of a cantilever beam. The pressure (second derivative) is also zero (no load). Since the bottom-most strain gage is located at some distance from the base of the wall (Section 3.5.1), the boundary conditions at this location are thus not known. In order to estimate these a polynomial least-square fit is made of the data points at each time step. A third or fourth order fit is done and the base boundary conditions are determined from this. Once this is done, the quintic spline is fitted to the data points and the moment distribution determined from this fit at each time step.

If no pressure transducers are used, the moment distributions are numerically differentiated with a fourth order finite difference scheme, once to obtain the shears and once more to obtain pressures. (This is why a quintic-spline was used, since a cubic spline would give straight
line segments in the second derivative.) However, due to the instabilities of numerical differentiation, it was determined that first derivatives were marginally satisfactory and second derivatives very inaccurate (recall Figure 3.29). This spawned the use of pressure transducers in tests.

When pressure transducers were used (all the tests except the first one) at each time step, the pressure transducer data points were polynomial fitted and a cubic spline fitted in a manner similar to the moments. An advantage of the cubic spline is that it requires no boundary conditions to be specified. The pressure distribution at each time step is thus read directly from the spline. The location of the resultants is then determined by finding the centroids of the pressure distributions. The shear distributions are obtained by direct trapezoidal rule integration of the pressure distributions. Numerical integration, as opposed to differentiation, is stable and accurate.

The following step is to determine the displacements at the top and bottom of the wall for every time step. The accelerograms are integrated twice and the $\Delta$-beam readings are used to tie in the initial and final conditions. (In the case of the free-field accelerometer, the initial and final displacements are assumed to be zero). The displacement distributions along the wall are then determined by integrating the moments twice and using the end displacements to find the two constants of integration required (see Section 3.5.4). The velocities at the accelerometer locations are also calculated in this process.

After each parameter distribution was determined, the corresponding printing and plotting described in the previous section was done.

The data processing procedure is outlined in the flow chart of program WALL in Figure 4.7.

## FLOWCHART FOR PROGRAM "WALL"



FIGURE 4.7

## CHAPTER V

## RESULTS

### 5.1. Dimensionless Groups

Henceforth, for convenience, all parameters will be discussed as dimensionless groups. This will make the discussion indifferent as to model or prototype.

The principles of dimensional analysis (reference [3] and Appendix A) are used to determine the dimensionless groups. From the tests, the following parameters are involved in influencing the results:

TABLE 5.1

```
            Parameters Involved in Tests
x - vertical location
H - height of wall
EI - stiffness of wal1*
M - wall moment*
Q - wall shear force*
y - lateral displacement of wall
P - lateral earth pressure
\gamma - density of soil
d - angle of internal friction of soil
e - soil void ratio
g - gravitational acceleration
a - lateral acceleration
v - lateral velocity
t - time
f
*per unit width
```

Parameters like Young's Modulus, Poissons's ratio and wave velocities for the soil were not used since these imply the soil is elastic, and are items that can only be assumed, not measured.

Table 5.1 gives a total number of parameters $n$ of 14. From the Buckingham II theorem, the total number of independent dimensionless groups $k$ that can be derived is $n$ minus the rank $r$ of the dimensional matrix:

$$
\begin{equation*}
\mathbf{k}=\mathbf{n}-\mathbf{r} \tag{5.1}
\end{equation*}
$$

For the parameters listed the dimensional matrix is shown as Table 5.2.

TABLE 5.2

Dimensional Matrix of Test Parameters

| Parameter | Force (F) | Length (L) | Time (T) |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| x | 0 | 1 | 0 |
| H | 0 | 1 | 0 |
| EI | 1 | 1 | 0 |
| M | 1 | 0 | 0 |
| Q | 1 | -1 | 0 |
| y | 0 | 1 | 0 |
| P | 1 | -2 | 0 |
| $\gamma$ | 1 | -3 | 0 |
| $\phi$ | 0 | 0 | 0 |
| e | 0 | 0 | 0 |
| g | 0 | 1 | -2 |
| a | 0 | 1 | -2 |
| $t$ | 0 | 0 | 1 |
| $\mathrm{f}_{1}$ | 0 | 0 | 1 |
| v | 0 | 1 | -1 |

The rank of the above matrix is 3. From equation (5.1), therefore, 12 independent dimensionless groups can be determined. They were chosen as follows:

TABLE 5.3
Dimensionless Parameters

| Parameter | Symbol | Dimensionless Group |
| :---: | :---: | :---: |
| Location | x | x/H |
| Time | t | $\mathrm{tf}_{1}$ |
| Moment (bending) | M | MH/EI |
| Moment (overturning) | M | $6 \mathrm{M} / \gamma \mathrm{H}^{3}$ |
| Shear force | Q | $Q /\left(1 / 2 \gamma H^{2}\right)$ |
| Pressure | P | $\mathrm{P} / \boldsymbol{\gamma} \mathrm{H}$ |
| Displacement | y | y/ ${ }^{\text {\% }}$ |
| Velocity | $\checkmark$ | $\mathrm{v} / \mathrm{f}_{1} \mathrm{H}$ |
| Acceleration | a | a/g |
| Friction angle | $\phi$ | $\phi$ |
| Void ratio | e | e |
| - | - | $\mathrm{V}^{2} / \mathrm{gH}$ |

In addition, the ratio of bending to overturning moment gives the non-independent dimensionless grouping $\gamma \mathrm{H}^{4} / 6 \mathrm{EI}$ which can be used as an indication of the relative stiffness of the wall-soil system.

In the following sections, unless otherwise noted, a reference to Pressure (P) will imply its dimensionless group ( $\mathrm{P} / \gamma \mathrm{H}$ ), reference to
time ( $t$ ) will imply $t f_{1}$, and so forth. This will avoid any model/prototype confusion, and will also simplify the discussion.

### 5.2. The Experimental "Earthquake"

Although the "earthquake generating" mechanism employed in the experiment was quite simple, the recorded motions are such that they are within the realm of strong earthquake ground motions which have been recorded in reality.

The accelerograms recorded at the top and bottom of the wall, as well as the free field (i.e., in the backfill some distance behind the wall) during various experiments, are displayed in Figures 5.1 a through 5.40a. Their corresponding velocities and displacements are shown in Figures 5.1 b through 5.40 b and 5.1 c through 5.40 c respectively.

The displacement curves include both the initial static displacements due to the backfill load (assuming that no backfill implies no wall deflection) plus those generated by the shaking. The magnitudes of the displacements prior to the earthquake are greater than $1 / 2 \%$ of the wall height which indicates a state of plastic equilibrium behind the wall, and thus the development of full active pressure.

From the accelerograms it can be seen that the general pattern of shaking is such as one would expect from the motion-generating mechanism involved, namely that of a decaying sinusoid. However, due to the inherent complexity of the experimental system, this basic pattern is enhanced by some extra acceleration noise probably generated from reverberations, collisions, nonlinearities, etc., of the


































FIGURE 5.39

centrifuge-frame-bucket-toggle-spring-bumper-wall-soil system. The accelerograms recorded in the free-field are very similar to the corresponding ones at the base of the wall (which indicate the input excitation into the wall-soil system), although they are not exactly alike. The peak amplitudes range from about 0.25 to 0.70 depending on the test, and the duration of shaking is from about 18 to about 33 (note the dimensionless variables). The accelerograms recorded at the top of the wall indicate that the motion can be amplified by greater than a factor of 2.0. The "earthquakes" can be generally categorized as short but severe.

The shaking exhibited in the experiments is not unlike that which has been recorded very near a ruptured fault. For example, used for comparison is the accelerogram (Figure 5.41) recorded at Station 2 of the Cholame-Shandon array during the Parkfield, California earthquake of June $27,1966\left(M_{L}=5.6\right)$. The strong motion accelerograph was located just a few yards from the San Andreas fault trace. This record also exhibits sharp pulse-like accelerations which decay quite quickly. Although the maximum recorded ground acceleration was $50 \%$ of gravity, there was 1 ittle damage to nearby structures presumably because of the narrowness of the acceleration spikes (low energy content) and because of the short duration of the severe shaking $[8,16]$.

From an engineering standpoint, the response spectrum is very important since it gives an indication of how the response of a structure to an earthquake will be. Comparing the response spectra of the centrifnge accelerograms of tests 1 CNOOO1, 1 CNOOO2, 1CNOO03, and


## RESPONSE ${ }^{-167}$ SPECTRUM

## TEST 1 CNOOO1 CENTRIFUGE EARTHQUAKE 11-12-80

 IIXXO100 80.001 .0 SOIL MECHANICS CENTRIFUSE - TEP OF WPLL PCCELERGNETER CEMP HOA OAMPING VRLUES ARE 0.2.5. 10 AND 20 PERTENT OF CRITICRL

## RESPONS ${ }^{-168}$ E SPECTRUM

## TEST 1 CNOOO1 CENTRIFUGE ERRTHOURKE 11-12-80

 IIXXO100 80.001 .0 SEIL MECHPNICS CENTAIFUSE-BGTTOM OF WALL PCCELERGTETER COMP HLR driming values pre 0.2,5. 10 pand 20 percent of critical

## RESPONSE SPECTRUM

TEST 1CNOOO2 CENTRIFUGE ERRTHQUAKE 3-4-81 IIXX0200 81.002 .0 SOIL MECHRNICS CENTRIFUGE - TOP OF WPLL RCEELERGEETER COMP HER ORMPING VALUES ARE 0, 2. 5. 10 AND 20 PERCENT OF CRITICRL


FIGURE 5.44

## RESPONSE SPECTRUM

TEST 1CNOOO2 CENTRIFUGE EARTHQUAKE 3-4-81
IIXXO200 81.002 .0 SOIL MEOHANICS CENTRIFUCE-BOTTOM OF WMLL RCOELERGETER COMP HER
damping values fre $0,2,5.10$ and 20 percent of chitical


## RESPONSE SPECTRUM

## TEST 1CNOOO2 CENTRIFUGE EARTHQUAKE 3- 4-81

 DAMPING VRLUES RAE 0.2.5. 10 RND 20 PERCENT OF CAITICRL


FIGURE 5.46

# RESPOONSE ${ }^{-172}$ SPECTRUM 

TEST ICNIOO3 CENTRIFUGE ERRTHQUAKE 3- 9-81 IIXX0300 81.009 .0 SOIL MECHWICS CENTRIFUGE - TEP OF HAL RLCELEROETER COMP HOR datping values pre $0,2,5,10$ and 20 percent of chitical


## RESPONSE SPECTRUM

TEST 1CN1003 CENTRIFUGE EARTHQUAKE 3-9-81

## IIXXO300 81.003 .0 SOIL MEDHPNICS CENTRIFUEE-BOTTOH OF HPLL RCCELERONETER COWP HER

damping values fre $0.2,5,10$ mid 20 percent of critical


FIGURE 5.48

# RESPONSE SPECTRUM 

## TEST 2CNOOII CENTRIFUGE EARTHOUAKE <br> 2-25-82

IIXXO100 82.011 .0 SOIL MECHANICS CENTAIFUGE - TEP OF WALL RCCELERGMETER COMP HOR
DAMPING VRLUES RRE 0. 2. 5. 10 FND 20 PERCENT GF CRITICRL


FIGURE 5.49

## RESPONSE SPECTRUM

## TEST 2CNOO11 CENTRIFUGE ERRTHQUAKE 2-25-82

IIXXO100 82.011.0. SUIL MEDHANICS CENTRIFUGE-BUTTOM GF WALL RCCELEROTETER COMP HOR
DAMPING VRLUES PRE 0.2.5. 10 AND 20 PERCENT OF CRITICAL


FIGURE 5.50

## RESPONSE SPECTRUM

```
IIXX0100 82.011.0 SOIL MECHRNICS CENTRIFUGE - FREE FIELD PCCELERONETER CEMP HCR
```

ormping values are 0. 2. 5. 10 and 20 percent of critical


## RESPONSE SPECTRUM

PARKFIELD. CALIFORNIA EARTHGUAKE JUNE 27. 1966 - 2026 PST
IIIBO33 66.001.0 CHOLAME.SHANDON. CALIFORNIA ARRAY NG. 2 COMP N65E
damping values are $0.2,5.10$ and 20 Percent of critical


FIGURE 5.52

2 CNOO11 (Figures 5.42 through 5.51) with that of the stronger horizontal component of Parkfield (Figure 5.52), it can be seen that they are all very similar. They have peaks for periods between 0.4 and 1.5 seconds (prototype) which are at similar levels for similar dampings. The main difference lies in the observation that the centrifuge shaking lacks the longer ( $>2.0$ sec) period components which the Parkfield motion contains. The above would seem to indicate that the prototype structure of the centrifuge model would have behaved very much like the model during an earthquake similar to Parkfield, had it been close to the rupturing fault.

The comparisons clearly show that, although the shaking mechanism employed in the centrifuge is not sophisticated, it does give motions which have realistic characteristics and thus can be used to provide some real insight into the problem at hand. Longer duration shaking would primarily affect walls retaining saturated backfill in which pore pressure effects might be important.

### 5.3. Parameter Diagrams

Figures 5.53 through 5.107 show the moment, pressure, shear force, and 1ateral displacement distributions obtained from the 14 tests performed. As explained in Section 4.2, these figures show the entire response of the system to the particular shaking it was subjected to. Table 5.4 should be used as a key to the interpretation of the figures.

TABLE 5.4
Key To Figures 5.53 Through 5.107

```
Frame a - Contour map of the parameter distribution with respect to location and time.
Frame b - Parameter distribution with respect to time at location where maximum occurs (Section A-A of contour map).
Frame c - Parameter distribution with respect to location-static, maximum dynamic (section B-B of the contour map), and final static after motion ceases.
+ Location of strain gages
\(x\) Location of pressure transducers
O Maximum
\(\Delta\) Data point
```

On Frame c of pressure distribation plots, the following symbols
appear. [Along (P/ $\rho \mathrm{g}$ H) axis]:

0 Location of static resultant
$\square$ Location of maximum resultant
$\rangle$ Location of final static resultant

Rankine/Conlomb (static) and Mononobe-Okabe (maximum dynamic) pressure distributions are also shown in this frame.

Except for tests $1 \mathrm{CN} 0001,1 \mathrm{CNO} 002,1 \mathrm{CN} 1003,1 \mathrm{CN} 0004$, and 1 CN 1505, the time $\left(t f_{1}\right)$ scales (on Frames a and $b$ ) are set up so that the first $20 \%$ of the record is displayed over the first $50 \%$ of the graph and the final $80 \%$ over the other $50 \%$. This was done to enhance the presentation of the more critical part of the tests.

FIGURE 5.54





FIGURE 5.55

TEST ICNOOO1
ALEXANDER ORTIZ
FIGURE 5.56



1.00
FIGURE 5.57

〔 $35 N 3$
$3 d 075$
FIGURE 5.58








FIGURE 5.62



FIGURE 5.64


FIGURE 5.65




FIGURE 5.68


FIGURE 5.69


FIGURE 5.70



FIGURE 5.71


FIGURE 5.72


FIGURE 5.73



FIGURE 5.76




FIGURE 5.77












FIGURE 5.81




FIGURE 5.82


## FIGURE 5.83




FIGURE 5.84







FIGURE 5.88
FIGURE 5.69

FIGURE 5.90




FIGURE 5.91


26*S ヨyก9I」



FIGURE 5.93


FIGURE 5.94







FIGURE 5.96


FIGURE 5.97


TEST 2CNOO12 3-8-82 CANTILEVER WRLL NV. 2 ZERU DEGREE BACKFILL SLOPE
ALEXANDER ORTI2


FIGURE 5.98



$66^{\circ} 5$ 34n9id

TEST 2CNOO12
ALEXANDER OATIZ
FIGURE 5.100






TEST 2CN1013 3-8-82 CANTILEVER WALL NO. ${ }^{2} 10$ DEGAEE BRCKFILL SLOPE
ALEXRNDER ORTIZ
(DRY, DENSE)




## FIGURE 5.103






FIGURE 5.104

FIGURE 5.105






FIGURE 5.107



### 5.4 Static Results

Although the main emphasis of the research project was the study of the dynamic behavior of retaining walls, some interesting results were obtained from a static point of view as well. An important indication that an accurate model has been used is to examine how it behaves statically and compare the results with the accepted Rankine and Coulomb static lateral earth pressure theories.

The Rankine lateral earth pressure theory gives the resultant active force $\mathrm{P}_{\mathrm{A}} /\left(1 / 2 \gamma \mathrm{H}^{2}\right)$ acting on the retaining wall as:

$$
\begin{equation*}
\frac{P_{A}}{1 / 2 \gamma H^{2}}=K_{A}=\frac{(1-\sin \phi)}{(1+\sin \phi)} \tag{5,2}
\end{equation*}
$$

The coefficient $K_{A}$ is referred to as the active earth pressure coefficient. The assumptions under which this theory is formulated are very approximately folfilled by the model tests which have a horizontal backfill, namely:

- The wall is rigid and vertical.
- The backfill is horizontal.
- There is no friction between soil and wall.
- There is active pressure (wall displaces more than $1 / 2 \%$ of its height).

The Coulomb lateral earth pressure theory (of which the Rankine is only a special case) follows the same assumptions as the Mononobe-Okabe theory (Section 5.5), with the exception that there are no lateral or
vertical acceleration coefficients $k_{h}$ or $k_{v}\left(i, e, \theta=0^{\circ}\right.$ ). The
resultant force acting on the wall is expressed as:
$\frac{P_{A}}{1 / 2 \gamma \mathrm{H}^{2}}=K_{A}=\frac{\cos ^{2}(\alpha-\beta)}{\cos ^{2} \beta \cos (\delta+\beta)}\left[1+\left(\frac{\sin (\alpha+\delta) \sin (\alpha-i)}{\cos (\delta+\beta) \cos (i-\beta)}\right)^{1 / 2}\right]^{-2}$
For the previously mentioned assumptions, with the exception that the backfill can be sloping, equation (5.3) can be reduced to:

$$
\begin{equation*}
\frac{P_{A}}{1 / 2 \gamma \mathbb{R}^{2}}=K_{A}=\cos ^{2} \phi\left[1+\left(\frac{\sin \phi \sin (\phi-i)}{\operatorname{cosi}}\right)^{1 / 2-2}\right. \tag{5.4}
\end{equation*}
$$

This equation will be used as a comparison basis for the tests with sloping backfills.

In the Rankine and Coulomb theories, under the assumptions listed, the resultant acts at one third of the height above the wall base since the pressure distribution is assumed triangular. Therefore, the overturning moment $6 \mathrm{M} / \mathrm{\gamma}^{\mathbf{3}}$ from the Rankine/Coulomb theory is:

$$
\begin{equation*}
\frac{6 \mathrm{M}_{\mathrm{A}}}{\gamma \mathrm{H}^{3}}=\mathrm{K}_{\mathrm{A}} \tag{5.5}
\end{equation*}
$$

The maximum bending moment is:

$$
\begin{equation*}
\frac{M_{A} H}{E I}=\frac{\gamma H^{4}}{6 E I} K_{A} \tag{5.6}
\end{equation*}
$$

Table 5.5 gives a comparison of the maximum measured static parameters from the tests with the Rankine/Coulomb theories, recalling that the friction angle of the soil used is $35^{\circ}$.

The 1ateral earth pressure theories (both static and dynamic) unfortunately only estimate the resultant force and its point of application based on the assumption of a triangular pressure distribution.


Therefore, the most accurate comparison that can be made is that of the resultant forces.

Comparing the Rankine/Coulomb resultant forces with the maximum shear forces (which are an integration of the pressure distribution behind the wall) it can be concluded that there is reasonable agreement between theory and experiment in this respect, the maximum difference being of the order of $25 \%$ between the two. The sole exceptions are tests 1 CN1505 and especially 1 CN0006 where the pressure distributions show a small magnitude in the upper $60 \%$ or so of the height and then increase rapidly below that (Figures 5.69c and 5.73c). This then contains a smaller area under the curve, although the maximum pressures (at the bottom of the wall) are comparable to those of similar tests.

From frames $c$ of the pressure distribution figures, it can be observed that the static pressure distributions are not linear, as the Rankine/Coulomb theories assume, although for the most part, the centroid of the distribution (location of the resultant force) is at around $1 / 3$ of the wall height above the base as a triangular distribution would indicate. It should be noted that, for RW2, the more flexible wall, this centroid does generally creep up to about $40 \%$ of the wall height above the base. The maximum pressures (at the bottom of the wall) are much greater in all cases, except 2 CNOO11 (Figure 5.93c), than those predicted by the Rankine/Coulomb assumption. The maximam static pressures recorded are on an average on the order of $60 \%$ higher than those than the Rankine/Coulomb theories would give. From these figures
it can, however, be seen that the traditional theories do seem to predict a correct average pressure distribution.

Since the traditional lateral earth pressure theories are based on the assumption that the wall holding back the soil is rigid, one can only make a qualitative overtarning/ bending moment comparison with the test results which are those of two flexible walls. The Rankine/Coulomb overturning moment is assumed to be the resultant force times the moment arm which is $1 / 3$ of the height above the base. The bending (reaction) moments recorded in the tests are generally greater than the overturning (action) moments given by Rankine/Coulomb. The actual test moments generally vary from just a few percent to about $35 \%$ greater than those predicted. Since stems of cantilever walls are designed as bending beams, the actual factor of safety could thas actually be much less than the usual 1.7. For a $35 \%$ underestimation, the actual safety factor (static) would then only be 1.25 .

Looking at the parameters that do not involve the wall stiffness EI, namely, $6 M / \gamma H^{3}, P / \gamma H$, and $Q /\left(1 / 2 \gamma H^{2}\right)$, it can be seen that there is correspondingly virtually no difference in the values for the two walls. This indicates that, for the range of wall stiffnesses tested, the system stiffness has little or no effect on the static response. The stiffness of RW1 is about twice that of RW2, but its moments MH/EI are about half. Thus the dimensional moments would be correspondingly similar also demonstrating the independence of wall flexibility on the response.

As far as is known, nobody has ever measured actual moments, static or otherwise, in a cantilever retaining wall, or has ever considered it to be a flexible bending beam, which it obviously is. Thus the moments shown in frames $c$ of the moment distribution figures provide a first insight into actual moments in cantilever walls due to lateral earth pressures.

The measurement of lateral displacements seems also to be unprecedented. The static displacements for all the tests indicate that the wall has initially displaced laterally at least $1 / 2 \%$ of its height and thus a state of plastic equilibrium in the traditional sense can be assumed to exist behind the wall, and thas comparisons with the traditional theories (which use this assumption) can be considered valid. The maximum static displacements are of the order of $3 \%$ to $4 \%$ in RW1 and $4 \%$ to $6 \%$ in the less stiff RW2, and, as expected, always occur at the top of the wall. On some of the displacement curves (frames c), one may note a small outward "curl" near the bottom of the wall. This is probably due to slight faults in the measurements of the boundary conditions and should be considered numerical and not physical. This also applies to the maximum dynamic and final static curves.

### 5.5. Dynamic Results

One can compare the maximum dynamic parameters obtained from the tests with those which would be estimated from the Mononobe-Okabe Theory (discussed in detail in Section 1.1) for similar circumstances. The envelopes (upper bounds) of the various parameters with respect to the








(20)


strong-motion characteristics are illustrated in Figures 5.108 through 5.122. How these envelopes were determined will be explained below. In addition, Mononobe-Okabe distributions with respect to the lateral acceleration coefficients $k_{h}$ for an average test soil density are shown in Figures 5.108, 5.111, 5.114 and 5.117.

For a flat backfill under the test assumptions (see Section 5.4), the total resaltant active force $\mathrm{P}_{\mathrm{AE}} /\left(1 / 2 \boldsymbol{\gamma} \mathrm{H}^{2}\right)$, given by Mononobe-Okabe, reduces from equations (1.1) and (1.2) to:

$$
\begin{equation*}
\frac{P_{\mathrm{AE}}}{1 / 2 \gamma \mathrm{H}^{2}}=\cos ^{2} \frac{(\phi-\theta)}{\cos ^{2} \theta}\left[1+\left(\frac{\sin \phi \sin (\phi-\theta)}{\cos \theta}\right)^{1 / 2}\right]^{-2} \tag{5.8}
\end{equation*}
$$

For a sloping backfill of angle $i$, the resultant force is expressed as:

$$
\begin{equation*}
\frac{\mathrm{P}_{\mathrm{AE}}}{1 / 2 \gamma \mathrm{H}^{2}}=\mathrm{K}_{\mathrm{AE}}=\frac{\cos ^{2}(\phi-\theta)}{\cos ^{2} \theta}\left[1+\left(\frac{\sin \phi \sin (\phi-\theta-i)}{\cos \theta \cos i}\right)^{1 / 2}\right]^{-2} \tag{5.9}
\end{equation*}
$$

These equations form the basis for comparison with the maximam dynamic results obtained from the tests.

In the Mononobe-Okabe theory, the resultant force is assumed to act at one third of the height above the base of the wall. Therefore, the overturning moment $6 \mathrm{M}_{\mathrm{AE}} / \gamma \mathrm{H}^{3}$ from the Mononobe-0kabe theory is:

$$
\begin{equation*}
\frac{6 \mathrm{M}_{\mathrm{AE}}}{\gamma \mathrm{H}^{3}}=\mathrm{K}_{\mathrm{AE}} \tag{5.10}
\end{equation*}
$$

The maximum bending moment is:

$$
\begin{equation*}
\frac{\mathrm{M}_{\mathrm{AE}} \mathrm{H}}{\mathrm{EI}}=\frac{\gamma \mathrm{H}^{4}}{6 \mathrm{EI}} \mathrm{~K}_{\mathrm{AE}} \tag{5.11}
\end{equation*}
$$

On the basis of previous studies (up to 1970), Seed and Whitman
suggest that the dynamic portion of the moment acts at 0.6 of the height above the base of the wall. Therefore the overturning moment is:

$$
\begin{equation*}
\frac{6 \mathrm{M}_{\mathrm{AE}}}{\gamma \mathrm{H}^{3}}=\mathrm{K}_{\mathrm{A}}+1.8 \Delta \mathrm{~K}_{\mathrm{AE}}=1.8 \mathrm{~K}_{\mathrm{AE}}-0.8 \mathrm{~K}_{\mathrm{A}} \tag{5.12}
\end{equation*}
$$

where:

$$
\begin{equation*}
\Delta \mathbf{K}_{A E}=\mathrm{K}_{\mathrm{AE}}-\mathrm{K}_{\mathrm{A}} \tag{5.13}
\end{equation*}
$$

Likewise, the bending moment is:

$$
\begin{equation*}
\frac{\mathrm{M}_{\mathrm{AE}} \mathrm{H}}{E I}=\frac{\gamma \mathrm{H}^{4}}{6 \mathrm{EI}}\left(\mathrm{~K}_{\mathrm{A}}+1.8 \Delta \mathrm{~K}_{\mathrm{AE}}\right)=\frac{\gamma \mathrm{H}^{4}}{6 \mathrm{EI}}\left(1.8 \mathrm{~K}_{\mathrm{AE}}-0.8 \mathrm{~K}_{\mathrm{A}}\right) \tag{5.14}
\end{equation*}
$$

This suggestion is also used in the moment comparisons with the experiments, and is shown (for an average test soil density) in Figures 5.108 and 5:111.

The maximum pressure $\mathrm{R}_{\mathrm{AE}} / \gamma H$ at the base of the wall is:

$$
\begin{equation*}
\frac{\mathrm{R}_{\mathrm{AE}}}{\gamma \mathrm{H}}=\mathrm{K}_{\mathrm{AE}} \tag{5.15}
\end{equation*}
$$

One should keep in mind that the Mononobe-Okabe Theory is based on the assumption that the coefficient of lateral earth pressure $k_{h}$ is representative of a constant lateral acceleration which provides a constant lateral body type force to the system. There are no inertia effects. The wall is also assumed to be rigid. In the experiment however, the lateral acceleration was rapidly varying in time, providing for inertia effects, and the retaining walls were flexible.

From equation (5.9) it can be seen that the Mononobe-Okabe equation goes singular when ( $(\alpha-\theta$ - ) is less than zero since the term under the radical goes negative. For $\phi=35^{\circ}$ and a flat backfill $\left(i=0^{\circ}\right)$ this means that $\theta$ has to be less than or equal to $35^{\circ}$ or $k_{h} \leq 0.70$. Likewise, for a $5^{\circ}$ backfill slope $\theta \leq 30^{\circ}$ or $k_{h} \leq 0.58$, for a $10^{\circ}$ backfill slope $\theta \leq 25^{\circ}$ or $k_{h} \leq 0.47$ and for a $15^{\circ}$ backfill slope, $\theta \leq 20^{\circ}$ or $k_{h} \leq 0.36$. From the lateral acceleration values (comparable to $k_{h}$ ) 1isted in Table 5.6, it can be seen that the upper limits just mentioned are exceeded, or very nearly exceeded, in some of the tests, especially in those where sloping backfills were used. From a Mononobe-0kabe analysis one would then have expected a complete collapse of the walls. In fact, there was never a complete collapse in any of the tests although lateral displacements were in some cases quite high (about $10 \%$ of the wall height in tests $1 \mathrm{CNO} 508,2 \mathrm{CN} 1013$, and 2 CN 1514 ). Complete collapse would probably have occurred if the lateral acceleration was constant and inertialess as assumed by Mononobe-0kabe. The level of maximam acceleration was only achieved momentarily, however, being followed by changes in acceleration which in time would lead to a restoring force holding the wall back. There might have been a momentary collapse of the system in some cases, which was quickly arrested. It should be noted that in most tests the maximum accelerations recorded (especially at the top of the wall) occur while the wall is being
"pushed" back into the backfill (i.e., while the system is being restrained from collapse).

The envelopes of the various parameters with respect to the strong motion characteristics were arrived at in the following manner:

As mentioned in Section 5.2 , the ground motion of the centrifuge earthquakes has the shape mainly of a decaying sinusoid with some additional noise added (see the bottom-of-wall accelerograms). In most of the tests there is an initial acceleration spike (positive acceleration) followed by a trough (negative acceleration), then a smaller spike, then a smaller trough, and thereafter low amplitude accelerations. The corresponding velocity diagrams, which, as one would expect, have their extreme values when the acceleration curves cross the zero axis, give the total area under the acceleration curves. The velocity changes from one extreme velocity to the other thas give the area under their respective acceleration spikes. The velocity and velocity changes are important in that they can be used as an indication of the energy content of the acceleration spikes, which is thus an indication of the energy put into the system by the earthquake. Recall that there was little damage from the Parkfield earthquake (Section 5.2), although there were high accelerations, because of the low energy content of the acceleration spikes.

It was observed, from the frames b of the parameter diagrams (Figures 5.53 b through 5.107 b ) that, in almost every experiment, peaks in the maximum moment, pressure and shear distributions at the base of the walls with respect to time were obtained in between the time when


#### Abstract

the acceleration spikes reached their peaks and the time when they crossed the zero axis (where the corresponding velocities reached their peaks). It was likewise observed that troughs in the maximum moment, pressure and shear distributions were obtained in between the times when accelerations reached troughs (negative maxima) and the times when they recrossed the zero axis (where the corresponding velocities reached their negative maxima). The opposite correlation between accelexation/velocity extremes and the maximum displacements at the top of the walls was also observed.


The peaks and troughs of the parameter distributions were then plotted with respect to their corresponding accelerations, velocities, and velocity changes (which are the areas under individual acceleration spikes) in Figures 5.108 through 5.122. These values are also tabulated in Table 5.6. It should be noted that static values were not included as peaks or troughs in the analysis, as they are probably neither. These values would have been plotted along the axis where acceleration and velocity are zero. However, in dynamic motion, when the acceleration is zero, the velocity might not be, and vice-versa, so the inclusion of static values in the envelope analysis would not have been appropriate to the other dynamic points. Only dynamic values were included.

It should also be noted that the Mononobe-Okabe analysis reduces to the static Rankine/Coulomb analysis for no lateral acceleration which does not seem accurate from a dynamic motion point of view.

| TABLE 5.6Maximum Extrome Dynamio Valnos (Poaka and Tronen |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tost 1CN0001 | $\begin{gathered} \text { Test } \\ \text { 1CNO002 } \end{gathered}$ |  |  |  | $\begin{gathered} \text { Test } \\ 1 \mathrm{CN} 1003 \end{gathered}$ |  |  |  | Tost1CN0004 |  |  | $\begin{gathered} \text { Tost } \\ \text { 1CN1505 } \end{gathered}$ |  |  |
| Ma/EI | 0.387 | 0.383 | $0.271^{T}$ | 0.382 | $0.354^{\text {T }}$ | 0.450 | $0.343^{\text {T }}$ | 0.433 | $0.401{ }^{\text {T }}$ | 0.323 | 0.292 | $0.305^{\text {T }}$ | 0.406 | $0.346^{\text {T }}$ | 0.314 |
| $6 \mathrm{M} / \mathrm{YH}^{3}$ | 0.503 | 0.505 | $0.358^{\text {T }}$ | 0.504 | 0.467 T | 0.589 | $0.489{ }^{\text {T }}$ | 0.566 | $0.525^{T}$ | 0.414 | $0.374^{\text {T }}$ | 0.391 | 0.529 | $0.451{ }^{\text {T }}$ | 0.487 |
| $\mathrm{P} / \mathrm{HH}$ | - | 0.813 | $0.318^{T}$ | 0.622 | $0.430^{\text {T }}$ | 0.929 | $0.324^{\text {T }}$ | 0.560 | $0.378{ }^{T}$ | 0.605 | $0.391^{\text {T }}$ | 0.489 | 0.661 | $0.389{ }^{\text {T }}$ | 0.532 |
| $a /\left(1 / 2 \gamma h^{2}\right)$ | 0.538 | 0.518 | - | 0.452 | - | 0.558 | $0.293{ }^{\text {T }}$ | - | - | 0.415 | - | 0.362 | 0.420 | $0.370^{\text {T }}$ | 0.488 |
| y/ H | $0.0181^{T}$ | $0.0238{ }^{\text {T }}$ | 0.0534 | $0.0149^{\text {T }}$ | 0.0365 | 0.0238 | 0.0462 | 0.0274 | 0.0596 | $0.0305^{\text {T }}$ | 0.0607 | $0.0383{ }^{\text {T }}$ | $0.0300^{\text {T }}$ | 0.0572 | $0.0331^{\text {T }}$ |
| 2/8 | 0.430 | 0.401 | -0.538 | 0.181 | -0.162 | 0.483 | -0.529 | 0.234 | -0.165 | 0.279 | -0.203 | 0.090 | 0.344 | -0.371 | 0.072 |
| $v / f_{1} \mathrm{E}$ | 0.0305 | 0.0369 | -0.0249 | 0.0101 | -0.0109 | 0.0364 | -0.0249 | 0.0094 | -0.0117 | 0.0249 | -0.0114 | -0.0001 | 0.0335 | -0.0132 | 0.0041 |
| $\mathrm{Av} / \mathrm{f}_{1} \mathrm{H}$ | 0.0305 | 0.0369 | -0.0618 | 0.0350 | -0.0210 | 0.0364 | -0.0613 | 0.0343 | -0.0211 | 0.0249 | -0.0363 | 0.0113 | 0.0335 | -0.0467 | 0.0173 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | Tost1CN0007 |  |  |  |  |  |  |  |  |  |  |  | Tost CN1510 |
| 20H/EI | $0.229{ }^{\text {T }}$ | 0.393 | 0.423 | $0.108{ }^{\text {T }}$ | 0.296 | $0.249^{\text {T }}$ | 0.327 | $0.199^{\text {T }}$ | 0.327 | $0.264^{\text {T }}$ | 0,491 | $0.301{ }^{\text {T }}$ | 0.493 | $0.351{ }^{\text {T }}$ | 0.554 |
| $6 \mathrm{M} / \mathrm{YH}^{3}$ | $0.292{ }^{\text {T }}$ | 0.500 | 0.519 | $0.133^{T}$ | 0.363 | $0.305^{\text {T }}$ | 0.410 | $0.250^{\text {T }}$ | 0.410 | $0.331{ }^{\text {T }}$ | 0.609 | $0.373^{\text {T }}$ | 0.611 | $0.436^{T}$ | 0.700 |
| P/r ${ }^{\text {B }}$ | $0.250^{\text {T }}$ | 0.366 | 0.712 | $0.179{ }^{\text {T }}$ | 0.455 | $0.235^{\text {T }}$ | 0.723 | $0.300^{\text {T }}$ | 0.549 | $0.306^{\mathrm{T}}$ | 0.557 | $0.233^{T}$ | 0.506 | $0.322^{T}$ | 0.673 |
| $a /\left(1 / 2 \gamma H^{2}\right)$ | $0.110^{\text {T }}$ | 0.169 | 0.458 | $0.185^{T}$ | 0.307 | $0.276{ }^{\text {T }}$ | 0.479 | $0.254^{\text {T }}$ | 0.397 | $0.312{ }^{\text {T }}$ | 0.473 | $0.272{ }^{\text {T }}$ | 0.404 | $0.291{ }^{\text {T }}$ | 0.611 |
| y/ H |  | - | $0.0210^{\text {T }}$ | 0.0638 | $-0.0029{ }^{\text {T }}$ | $0.0182^{\text {T }}$ | $0.0236^{\text {T }}$ | 0.0647 | $0.0612^{\text {T }}$ | - $0.0412^{\text {T }}$ | T | - | - | - | $0.0200^{\text {T }}$ |
| 1/8 | -0.472 | 0.157 | 0.549 | -0.652 | 0.332 | -0.151 | 0.629 | -0.614 | 0.302 | -0.161 | 0.634 | -0.593 | 0.379 | -0.185 | 0.573 |
| v/fi ${ }_{1}$ | -0.0285 | 0.0075 | 0.0601 | -0.0413 | 0.0104 | -0.0113 | 0.0525 | -0.0412 | 20.0087 | -0.0119 | 0.0452 | -0.0517 | 0.0036 | -0.0217 | 0.0445 |
| $\Delta v / \mathrm{P}_{1} \mathrm{H}$ | -0.0285 | 0.0360 | 0.0601 | -0.1014 | 0.0517 | -0.0217 | 0.0525 | -0.0937 | 70.0499 | -0.0206 | 0.0452 | -0.0969 | 0.0553 | -0.0253 | 0.0445 |


|  | Tost1CN1510(Cont.) |  |  | Test2CN0011 |  |  |  | $\begin{gathered} \text { Tost } \\ 2 \mathrm{CN} 0012 \end{gathered}$ |  |  |  | Tost2 CN1013 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mu/EI | $0.239^{\text {r }}$ | 0.560 | $0.409^{\text {T }}$ | 0.736 | $0.622^{\text {T }}$ | 0.826 | $0.752^{\text {T }}$ | 0.698 | $0.492{ }^{\text {T }}$ | 0.762 | $0.680^{\text {T }}$ | 0.746 | $0.570^{\text {T }}$ | 0.804 | $0.738{ }^{\text {T }}$ |
| $6 \mathrm{M} / \mathrm{\gamma}^{4} \mathrm{H}^{3}$ | $0.302{ }^{\text {T }}$ | 0.707 | $0.517^{\text {T }}$ | 0.448 | $0.379{ }^{\text {P }}$ | 0.503 | $0.458{ }^{\text {T }}$ | 0.438 | $0.309{ }^{\text {T }}$ | 0.478 | $0.427^{\text {T }}$ | 0.461 | $0.352^{T}$ | 0.497 | $0.456{ }^{\text {r }}$ |
| P/ $/ \mathbf{H}$ | $0.374^{\text {P }}$ | 0.578 | $0.426^{\text {T }}$ | 0.703 | $0.142^{\text {T }}$ | 0.414 | - | 0.615 | $0.090{ }^{\text {T }}$ | 0.423 | - | 0.814 | $0.239^{\text {T }}$ | 0.456 | $0.261^{T}$ |
| $a /\left(1 / 2 \gamma^{\prime} \mathrm{H}^{2}\right)$ | $0.351^{\text {T }}$ | 0.541 | $0.452^{\text {T }}$ | 0.338 | $0.277^{T}$ | 0.377 | $0.240^{\text {T }}$ | 0.390 | $0.209^{\text {T }}$ | 0.400 | - | 0.408 | $0.203{ }^{\text {T }}$ | 0.375 | $0.331{ }^{\text {T }}$ |
| $\mathrm{FH}^{\text {H }}$ | 0.0640 | $0.0024^{\text {T }}$ | 0.0210 | $0.0229^{\text {T }}$ | 0.0488 | -0.0376 | -0.0222 | $0.0279{ }^{\text {T }}$ | 0.0666 | $-0.0124^{\text {T }}$ | 0.0135 | $0.0368^{\text {T }}$ | 0.0706 | $0.0048^{\text {T }}$ | 0.0296 |
| 2/8 | -0.615 | 0.367 | -0.190 | 0.629 | -0.703 | 0.390 | -0.198 | 0.723 | -0.612 | 0.408 | -0.151 | 0.729 | -0.596 | 0.400 | -0.193 |
| $\mathrm{V} / \mathrm{f}_{1} \mathrm{H}$ | -0.494 | 0.0095 | -0.0142 | 0.0569 | -0.0376 | 0.0137 | -0.0114 | 0.0595 | -0.0373 | 0.0115 | -0.0071 | -0.0593 | -0.0398 | 0.0138 | -0.0098 |
| $\chi_{v / f}{ }_{1} \mathrm{~B}$ | -0.0939 | 0.0589 | -0.0237 | 0.0369 | -0.0945 | 0.0513 | -0.0251 | 0.0595 | -0.0968 | 0.0488 | -0.0186 | 0.0593 | -0.0944 | 0.0536 | -0.0236 |


|  | $\begin{gathered} \text { Tost } \\ 2 \mathrm{CN} 514 \end{gathered}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| M $\mathrm{H}^{\text {/ }}$ RI | 0.798 | $0.616^{\text {T }}$ | 0.902 | 0.830 |
| $6 \mathrm{M} / \mathrm{r}^{\mathbf{3}}{ }^{3}$ | 0.491 | $0.379^{\text {T }}$ | 0.555 | $0.511^{\text {T }}$ |
| $\mathbf{P} / \mathbf{\gamma} \mathbf{H}$ | 0.730 | $0.202{ }^{\text {T }}$ | 0.659 | $0.267{ }^{\text {T }}$ |
| $a /\left(1 / 2 \gamma H^{2}\right)$ | 0.394 | $0.263^{\text {T }}$ | 0.323 | $0.203{ }^{\text {T }}$ |
| 9/8 | $0.0282{ }^{\text {T }}$ | 0.0472 | -0.0351 ${ }^{\text {T }}$ | -0.0288 |
| a/8 | 0.701 | -0.624 | 0.422 | -0.202 |
| $v / f_{1} \mathrm{H}$ | 0.0526 | -0.0474 | 0.0074 | -0.0200 |
| $\Delta_{v / 1} 1_{1} H$ | 0.0526 | -0.1000 | 0.0548 | -0.0274 |

The extreme points seem to follow, with the exception of the displacement, a general trend; that is, the higher the lateral acceleration, velocity or change in velocity, the higher the extreme. It was decided to fit least-squares straight lines through each of the sets of points one for each backfill slope; $0^{\circ}, 10^{\circ}, 15^{\circ}$. The maximum slope from each of the three sets of data was taken as the slope for the envelopes. The envelopes were drawn with these slopes as tangents to the individual sets of points. From the linear correlation coefficients of the least-squares fits, it was determined that the best fits were the maximum moment vs. change in velocity (Figures 5.110 and 5.113), the maximum pressure vs. velocity (Figure 5.115 ) and, the maximum shear vs. acceleration (Figure 5.117). No conclusions could be drawn from the displacement curves (Figures 5.120 through 5.122).

The best fits would indicate that moment and pressure are more momentum- or energy-governed parameters since they are better related to velocity effects. Similarly, the shear force is more a force-governed parameter (which is logical) since it is better related to acceleration.

The envelopes presented thas provide an upper bound for the various parameters with respect to actual dynamic strong motion characteristics for at least a range of system stiffnesses ( $\gamma \mathrm{H}^{4} / 6 \mathrm{EI}$ ) between about 0.75 and 1.75 which the experiments encompassed (for $\phi \approx 35^{\circ}$ ).

As in the static case, the various parameters are indicated to be independent of the stiffness of the walls at least for the range of stiffnesses tested. It would be difficult to say if this would hold for
rigid walls, or very flexible walls, since the actual walls tested appeared quite flexible as indicated by the deflection shapes.

The only logical comparisons that can be made are those between the envelopes obtained and the corresponding Mononobe-Okabe predictions (which have been simplified for an average of the soil densities encountered). These can be seen in Figures 5.108 and 5.111 for moments, 5.114 for pressures, and 5.117 for shears. In addition, Figures 5.108 and 5.111 show the values for moments suggested by Seed and Whitman which were previously discussed. No comparisons with previous investigators can be made in terms of the envelopes involving the velocity parameters since this does not sem to have been examined before. Having the envelopes with respect to accelerations, velocity and change in velocity should, however, help in better understanding the problem at hand.

Since the Mononobe-0kabe curves and Seed and Whitman curves (in the moment diagrams) generally intersect at one point and at relatively steep angles to each other, it appears that the traditional methods anderestimate the actual values of maximum moments below the point of intersection and overestimate them above. It appears that going even a small distance above or below the intersection points leads to large differences between the actual experimental maximum values and those predicted by the theory. For example, from Figure 5.111 , for a flat backfill and a lateral acceleration of 0.25 g the Mononobe-Okabe method gives a maximum moment around $60 \%$ as large as that determined from the envelope. Seed and Whitman indicate one about $80 \%$ as large. For 0.50 g ,
however, Mononobe-Okabe predicts a maximum moment about as large as the envelope while Seed and Whitman shows one 1.5 times larger. Similar comparisons can readily be made for the other parameters as well by examining the respective diagrams. The designer would observe, therefore, that the envelopes obtained from the experiments generally give what appear to be conservative values for lateral accelerations less than about 0.50 g (which is probably the practical extreme for the use of the Mononobe-Okabe theory in any case). It shoald also be noted that the envelopes do not seem to be as sensitive to backfill slope as the Mononobe-0kabe theory is.

From the parameter diagrams (Figures 5.53 through 5.107) it can be observed that the maximum moments recorded ranged from about $40 \%$ to about $100 \%$ higher than the maxima recorded statically (with the exception of test 1 CN 0007 which had a relatively very low static maximum moment). As mentioned previously, this ratio is more dependent on the energy input into the system (represented by the velocity) than on the peak accelerations. The moment distributions with respect to the location (frames c and vertical cuts of frames a) seem to be smooth curves which could possibly be approximated using low order polynomials, for example, quadratic functions.

The maximum dynamic pressures ranged anywhere from 1 to 2-1/2 times the maximum static ones and like the moments this ratio was more dependent on the velocities recorded. Although the pressure distributions are by no means 1 inear (as assumed by the Mononobe-Okabe theory), their centroids (locations of resultants) generally appear to be at or
very near the location of the static centroids, that is, somewhere between $30 \%$ to $40 \%$ above the wall base. As with the static pressure distributions, this indicates that the distributions are like an "average" of a linear pressure distribution although they are generally difficult to relate to a Mononobe-Okabe distribution. In any case, the dynamic centroid appears to hold steady at around $1 / 3$ the height above the base in contradiction to Seed and Whitman [55] and Prakash and Basavanna [42] (see Section 1.3).

The maximum shear forces recorded in the tests are generally $50 \%$ to $100 \%$ higher than the maximum recorded statically for the range of maximum test accelerations. It appears that the percentage is more closely associated with the acceleration than the velocity levels. One should keep in mind that shear requirements are usually amply met if a bending design is used unless the beam is short with respect to its thickness (behaving like a shear beam). For reinforced concrete beams, shear is important, however, and some shear reinforcement is usually required by design.

As can be seen from Figures 5.120 through 5.122 no clear trend could be determined between the maximum displacements (at the top of the wall) and the strong motion characteristics.

Richards and Elms [43] performed some tests on a gravity retaining wall on a ( 1 g ) shaking table which was subjected to a scaled El Centro, California (1940) earthquake record. They measured the displacements on the wall and noted that the wall always moved outward away from the backfill and continued to move outward until the shaking ceased. By
contrast, barring the author's prejudice toward 1 g shaking table tests (Section 1.2), in the cantilever retaining wall tests of this investigation, the walls were observed to displace both outwardly and inwardly with respect to the backfill. The maximum displacements were observed to be not necessarily the final ones although in some tests they were. This is as it should be. At 1 g , the soil grains are undex low stresses and are rigid, so the only displacements axe due to grain slipping which is all irreversible. In the centrifage, the soil behavior is properly elastic/plastic so dynamic to and fro movements are observed. In addition to the sliding and rotation of the base, there is also the flexing of the stem (and base) so the elastic wall can rebound somewhat as well. The maximum deflections ranged from $5 \%$ to $9 \%$ of the wall height for RW1 and from 7\% to $11 \%$ for the more flexible RW2. These magnitudes of deflections conld lead to some severe cracking in reinforced concrete walls although it should be remembered that part of the deflection is due to a rotation of the base.

From the shape of the deflection curves (frames and vertical sections through frames a of the parameter diagrams) it can be seen that the wall motion is basically in the first mode with apparently little or no contribution from other modes. This is also confirmed by the Fourier Spectra discussed in Section 3.3.3.

### 5.6. Final Static (Residual) Results

A visual idea of the results of the earthquake on the retaining walls can be observed from Figures 5.123 (Test 1CNO007), 5.124 (Test
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FIGURE 5.123 - TEST ICNOOO7.POST TEST VIEW (AT IG)


FIGURE 5.124-TEST ICNI009,POST TEST VIEW (AT IG)


FIGURE 5.125 - TEST 2 CNOO11,POST TEST VIEW (AT |G)

1CN1009), and 5.125 (Test 2CN0011). Although the photographs were taken after the centrifuge was brought back down to rest, one can see that there was a large amount of motion of the backfill and wall. There was, of course, an amount of "rebounding" of the system as the artificial gravitational field decreased. One can observe that the backfill, which was originally flush with the 1 ip of the wall, has displaced downard $1 / 4$ to $1 / 2$ of an inch. These kinds of displacements are quite sizeable and it can be safe to speculate that, if colored sand, or slightly moistened sand (with some apparent cohesion) had been used, some cracks in the backfill would have been observed.

Not apparent from the photographs is a "mounding" of the sand observed at the base of the wall. This was obviously produced by the outward movement of the wall during the tests.

An important observation related to the downward sliding of the backfill and the "mounding" at the base is that these featares were uniform across the width of the wall and there was no apparent change near the edges. This can be taken as a good indication that the system behaved in a plane strain fashion (as assumed) and that the edge effects (if any) were minimal.

Seed and Whitman [55] mention the fact that after a retaining structure with granular backfill has been subjected to a base excitation, a residual pressure acts on it which is substantially greater than the initial pressure before base excitation. This pressure is also a

| Maximum Final (Rosidual) Statio Values From Tests |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tost 1 CN0001 | Test 1CN0002 | $\begin{gathered} \text { Toat } \\ \text { 1CN1003 } \end{gathered}$ | $\begin{gathered} \text { Test } \\ \text { 1CN0004 } \end{gathered}$ | Test 1CN1505 | $\begin{gathered} \text { Tost } \\ 1 \mathrm{CNO006} \end{gathered}$ | $\begin{gathered} \text { Test } \\ \text { 1CN0007 } \end{gathered}$ | $\begin{gathered} \text { Tost } \\ \text { 1CN0s08 } \end{gathered}$ | Tost $1 \mathrm{CN1009}$ | Tost 1 CNI 510 | Tast $2 \mathrm{CN0011}$ | Tost $2 \mathrm{CN0012}$ | Tost $2 \mathrm{CN1013}$ | Tost 2CN1S14 |
| Momont MH/EI | 0.338 | 0.358 | 0.409 | 0.303 | 0.371 | 0.381 | 0.302 | 0.270 | 0.423 | 0.456 | 0.790 | 0.708 | 0.760 | 0.856 |
| Moment 6M/ $\mathrm{Y}^{\mathbf{3}}{ }^{3}$ | 0.439 | 0.472 | 0.535 | 0.388 | 0.483 | 0.485 | 0.370 | 0.379 | 0.525 | 0.576 | 0.481 | 0.445 | 0.470 | 0.527 |
| Prosanco P/ $\boldsymbol{\gamma}$ E | - | 0.478 | 0.408 | 0.461 | 0.501 | 0.314 | 0.318 | 0.308 | 0.354 | 0.444 | $0.268{ }^{\text {c }}$ | 0.275** | 0.297 | 0.276 |
| Shoar $\mathbf{Q} /\left(1 / 2 \boldsymbol{\gamma} \mathrm{~B}^{2}\right.$ ) | 0.413 | 0.428 | 0.423 | 0.375 | 0.419 | 0.177 | 0.320 | 0.317 | 0.381 | 0.482 | 0.359 | 0.398 | 0.365 | 0.282 |
| Displacoment f/EI | 0.0474 | 0.0536 | 0.0599 | 0.0433 | 0.0548 | 0.0328 | 0.0481 | 0.0552 | 0.0580 | 0.0561 | 0.0653 | 0.0731 | 0.0856 | 0.0947 |

[^0]| ```TABLE 5.8 \\ 1 Rosidual to Maximum Static and Marimum Dynamic Values (Test only).``` |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tost 1CN0001 |  | Test 1CN0002 |  | Tost 1 CN 1003 |  | Test 1CN0004 |  |
|  | Statio | Dynamic | Static | Dynamic | Static | Dynamic | Statio | Dynamia |
| Momont MH/EId6M/ $\mathrm{rlH}^{3}$ | 1.40 | 0.87 | 1.32 | 0.93 | 1.37 | 0.91 | 1.23 | 0.94 |
| Pressure P/ $\boldsymbol{\gamma} \mathbf{H}$ | - | - | 0.96 | 0.59 | 0.92 | 0.44 | 0.87 | 0.76 |
| Shear Force $\mathrm{Q} /\left(1 / 2 \boldsymbol{\gamma} \mathbf{H}^{\mathbf{2}}\right.$ | 1.55 | 0.77 | 1.44 | 0.84 | 1.48 | 0.76 | 1.36 | 0.90 |
| Displacement y/H | 1.70 | $1.00 *$ | 1.68 | 1.00* | 1.59 | 1.00* | 1.37 | 0.71 |



| Tost 1CN1510 | Tost 2CN0011 |  | Tost 2CN0012 |  | Tost 2CN1013 |  | Test 2CN1514 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Statio | Dynamic | Statio | Dynamic | Static | Dynamic | Static | Dynamic | Static |
| 1.57 | 0.81 | 1.56 | 0.97 | 1.58 | 0.93 | 1.53 | 0.95 | 1.61 |
| 0.84 | 0.66 | 0.95 | 0.38 | 0.87 | 0.45 | 0.62 | 0.36 | 0.66 |
| 1.44 | 0.79 | 1.07 | 0.95 | 1.58 | 0.97 | 0.38 |  |  |
| 1.72 | 0.88 | 1.49 | 1.00 | 1.58 | 1.00 | 1.54 | 0.89 | 1.11 |
| 1.72 | 0.76 | 1.73 | 1.00 |  |  |  |  |  |

- Final = Maximum
substantial portion of the maximum pressure developed during the excitation. This statement is quantitatively demonstrated by the experiments. The maximam residual parameters are 1 isted in Table 5.7 and their ratios to maximum static and maximum dynamic values in Table 5.8.

One can observe that, although the maximum residual pressure is always somewhat lower than the maximum static pressure ( $5 \%$ to $25 \%$ ), and considerably lower than the maximam dynamic ( $25 \%$ to $60 \%$ lower), the resultant (shear) forces (i.e., the areas under the pressure distributions) are in accordance with the Seed and Whitman observation. The residual resultants can be up to $60 \%$ higher than the static! This appears to be random with respect to the slope. From frames $c$ of the pressure distribution diagrams, it can also be observed that the final residual resultant is usually located some $20 \%$ to $40 \%$ above the static and dynamic resultants indicating that a triangular (or "average triangular") pressure distribution no longer exists.

The residual moments are also substantially higher than the static and are only a few percent lower than the maximum. This, again, develops regardless of the magnitude of the shaking the wall was subjected to. This could indicate that a retaining wall which has survived an earthquake intact could be pre-stressed for the following earthquake or aftershock to the point where there is virtually no safety factor and thus fail under an even mild event. It should be noted that in Test 1 CN1505 the centrifuge was left running for 3 hours after the shaking occurred. This is the equivalent of 150 hours (over 6 days) in prototype time, and in this period, no rebounding or relaxation was
observed in either the strain gage (moment) or pressure transducer readings.
As mentioned in Section 5.5 the walls displaced out and in with respect to the backfill and then generally crept out toward some final displacement which in some tests was the maximum observed. The final displacements were found to be much greater than the static ones in any case. This then gives rise to the question of whether or not such large displacements can be tolerated from a safety or aesthetic point of view although the retaining wall survived the earthquake.

## CHAPTER VI

## CONCLUSIONS AND RECOMMENDATTONS


#### Abstract

The purpose of this investigation was to observe the natural behavior of an 18 ft high cantilever retaining wall when subjected to only a gravity body force with a dynamic lateral earthquake excitation. The retaining walls were properly modelled and were subjected to some earthquake-1ike motions which were considered to be in a realistic range. Moment, pressure, shear, and displacement distributions (static, dynamic, and residual) were obtained. It was also novel that the retaining walls were considered flexible (as they are in real life) as opposed to rigid, which seems to be the norm in 1 g model retaining wall studies and in theoretical analyses. A large amount of data was obtained directly from transducers and indirectly from simple mathematical manipulations of transducer data and was presented in as concise a manner as possible. Some empirical curves for relating the upper bound responses of the retaining walls to strong motion characteristics were also obtained.


From the information acquired from the tests, the following conclusions and recommendations can be made.

### 6.1. Conclusions

1. The simple "earthquake generating" mechanism employed was found to give realistic characteristics and could thus be employed in the
studies of other earthquake-related problems in geotechnics in the centrifuge.
2. The static earth pressure distributions obtained were not triangular as the Rankine/Coulomb lateral earth pressures assume. The experimental centroids were generally located at about $1 / 3$ the height above the base of the wall. The resultant forces (areas under the pressure curves) were in reasonable agreement with the Rankine/Conlomb theory. This indicates that the Rankine/Coulomb theory estimates an "average" pressure distribution which is taken as triangular.
3. The static moments measured were generally higher than those which would be obtained using a Rankine/Coulomb resultant force with a $1 / 3$ of the height moment arm (by as mach as $35 \%$ ), indicating that the properly designed wall might have a safety factor lower than estimated.
4. Static displacements were sufficiently large to create a state of full active pressure behind the wall.
5. Static and dynamic reaction parameters (moments, pressares, etc.) appear to be independent of wall stiffness, at least for the range of experimental system stiffnesses ( $0.75 \leq \gamma \mathrm{H}^{4} / 6 \mathrm{EI} \leq 1.75, \phi \approx 35^{\circ}$ )

6a. The only significant dynamic response of the system is in the fundamental mode.

6 b . The two walls had fundamental frequencies of 2.6 Hz and 2.5 Hz with the soils employed.
7. The dynamic response of the system is not only dependent on lateral accelerations, as the Mononobe-Okabe theory assumes, but also on the energy content of the earthquake indicated by the velocities. Maximum moments were found to be more closely associated with the areas under the individual acceleration spikes (changes in velocity), maximum pressures with the velocities, and maximum shear (resultant) forces with the accelerations, although there is a general dependence on all the strong motion characteristics. There is a strong correlation between maximum and minimum (maximum negative) accelerations, velocities, and changes in velocities; and peaks and troughs in the maximum response curves.
8. The experimental envelopes presented in Chapter $V$ provide an upper bound for the various parameters with respect to actual dynamic strong motion characteristics for at least the system stiffness range $\left(0.75 \leq \gamma H^{4} / 6 \mathrm{EI} \leq 1.75, \phi \approx 35^{\circ}\right)$ which was studied. These envelopes can be used as a design aid (Section 6.2).
9. The Mononobe-Okabe theory underestimates responses (in some cases severely) below certain lateral acceleration levels for each individual case (Figures $5.108,5.111,5.114$, and 5.117 ) and overestimates them above that acceleration when compared to the experimental envelopes. This is due to the steep slope of intersection (at only one point) between the recorded parameter envelopes and the Mononobe-Okabe curves. This makes the envelopes appear conservative for $k_{h}$ values less than 0.5 g , but they are not, because they came from tests.
10. The experimental envelopes are not as sensitive to backfill slope as the Mononobe-Okabe theory is.
11. Dynamic moment distributions with respect to wall location are generally smooth, monotonic curves which resemble some low order polynomial, possibly quadratic.
12. As in the static cases, the dynamic pressures were not triangular as the Mononobe-Okabe theory assumes, although the centroids did remain at about $1 / 3$ the height above the base, contradicting other investigators which state that it rises to between $1 / 2$ and $2 / 3$ of the height. The dynamic pressure distributions could thas be considered an "average" of a linear distribution, although they could not generally be related to Mononobe-Okabe.
13. The walls displaced both outwardly and inwardly with respect to the backfills during the severe parts of the shaking and crept outwardly during the milder shaking towards the end. Maximum deflections could be considered excessive in some cases even though the structure survived the event intact. Deflected shapes gave an indication of first mode (only) flexible bending beam behavior.
14. The fact was confirmed that, after a retaining structure with a granular backfill undergoes severe dynamic excitation, a residual pressure acts on it which is substantially greater than the initial pressure before excitation, and is a substantial portion of the maximum pressure developed during the excitation. This also applies to moments, shears, and displacements.
15. No noticeable experimental "edge effects" were observed, and a plane strain condition for the tests could be assumed to hold.
16. Elastic solutions for retaining wall problems should be avoided. This includes the use of elastic finite elements (Appendix D).

### 6.2. Recommendations

Based on the concluded investigation, it is highly recommended that some type of dynamic analysis in the design of large retaining structures be employed, as the dynamic responses generated can be considerably greater than the static ones. There should be extreme caution in accepting the following quote from Seed and Whitman [55]:
"Thas many walls adequately designed for static earth pressures will automatically have the capacity to withstand earthquake ground motions of substantial magnitudes and in many cases, special seismic earth pressure provisions need not be required".

As an example of how the experimental data from this investigation might be used as a design aid consider the following practical problem:

It is required to design a 20 ft high cantilever retaining wall with a flat, granular backfill with $\phi=35^{\circ}$. The wall is to be subjected to a scaled down Parkfield Earthquake (Figure 5.41a) to one half the magnitude shown.

Since the wall/soil description is similar to that of the experiments, the fundamental frequency can be assumed to be about 2.5 Hz . From Figare 5.41, based on test experience, the second acceleration spike (that whose peak is at about 4.1 seconds) should probably generate
the critical response. The peak design acceleration is then 215 cri/sec ${ }^{2}$, the corresponding velocity $39 \mathrm{~cm} / \mathrm{sec}$ (which occurs at about 4.6 sec) and the area under the acceleration spike is $49 \mathrm{~cm} / \mathrm{sec}$ (which is the peak-trough difference on the velocity curve). Based on test experience, the peak response of the wall should then occur sometime between the 4 and 5 second mark.

For $a=215 \mathrm{~cm} / \mathrm{sec}^{2}, a / g=0.22$. Therefore, from Figure 5.111

$$
\frac{6 M}{\gamma H^{3}}=0.58
$$

For $v=39 \mathrm{~cm} / \mathrm{sec},\left(f_{1}=2.5 \mathrm{~Hz}\right)$.

$$
\frac{\nabla}{f_{1} H}=\frac{39}{(2.5)(20)(30.48)}=0.026
$$

Therefore, from Figure 5.112,

$$
\frac{6 M}{\gamma \mathrm{H}^{3}}=0.55
$$

For $\Delta v=49 \mathrm{~cm} / \mathrm{sec},\left(f_{1}=2.5 \mathrm{~Hz}\right)$.

$$
\frac{\Delta v}{f_{1} H}=0.032
$$

Therefore, from Figure 5.113,

$$
\frac{6 \mathrm{M}}{\gamma \mathrm{H}^{3}}=0.57
$$

The maximum moment could then be taken as the average of the three values obtained from the envelopes, therefore

$$
\left(\frac{6 \mathrm{M}}{\gamma \mathrm{H}^{3}}\right)_{\mathrm{MAX}}=0.57
$$

Having this value, the stem could then be designed as a regular bending beam using, for example, a quadratic moment distribution for simplicity and having all the design requirements (as was done in Section 3.3.1).

It should be noted that had a Mononobe-Okabe analysis been performed, using the maximum scaled Parkfield acceleration of $240 \mathrm{~cm} / \mathrm{sec}^{2}$ and equation (5.10), the maximum moment would have been:

$$
\left(\frac{6 \mathrm{M}}{\gamma \mathrm{H}^{3}}\right)_{\mathrm{MAX}}=0.42
$$

which is $35 \%$ below the one obtained from the other analysis. It was based on one dynamic parameter (the peak acceleration) whereas, the other was based on three. If a standard factor of safety of 1.7 is used, it would in actuality only be 1.25 when compared to the previous analysis.

One could also use a similar analysis to investigate the pressures and shears and perhaps refine the design.

Future research could be done using identical types of tests with different wall heights, stiffnesses, different soils and longer earthquake durations.

The data analysis should concentrate more on the high1ights (peaks, troughs, etc.) of the dynamic characteristics related to the system responses instead of the detailed, time-consuming, expensive, and tedious data analysis which was performed in this investigation.

Sheetpile walls, channel sections, and other types of bending beam retaining walls should also be studied.

Retaining wall problems with wet or saturated soils should also be examined with the centrifuge, although there could be some problems with retaining the water in the backfill as well as having two time scales (dynamic and consolidation - see Appendix A).

The centrifuge would also be an ideal tool for studying static and dynamic retaining wall behavior with clays.

It wonld be desirable to develop a better shaker which could be implemented into a centrifuge. There is also a need for some fall-scale testing of bending beam retaining structures. Sinusoidal shakers could be used on actual retaining structures to determine some natural frequencies and modes of vibration and perhaps test some to failure.

An actual retaining wall should also be instrumented with two strong motion accelerographs (one at the base and one at the top) and with at least some kind of pressure transducers which could record pres sures during an actual earthquake. The recording devices could be triggered by the accelerographs.

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## APPENDIX A

SCALING RELATIONS (Hoek [15])

Every quantity of physics and mechanics has a dimension which can be expressed as a function of the fundamental dimensions:

$$
\begin{array}{lll}
M-\text { mass } & F-\text { force }\left(F=M L T^{-2}\right) \\
L-\text { length } & \text { or } & L-\text { length } \\
T-\text { time } & & T-\text { time }
\end{array}
$$

If a formula is dimensionally correct, it is valid in all systems of units.

By the method of dimensional analysis ([3],[15]) relations between the equations governing the states of the model and prototype can be derived.

The stress and displacement at a point in the structure will depend upon the following factors:
1.) The geometry of the structure. The behavior of a point defined by the coordinates $x, y, z$ can be described by a typical length dimen-
sion $L$ and set of dimensionless ratios $L_{R}$ relating all other
lengths to L.
2.) Material properties: For example, for a linearly elastic isotropic material.
$\rho=$ mass density of the material.
$E=$ Young's modulus of the material.
$V=$ Poisson's ratio of the material (dimensionless).
Other material properties can be related to $\rho$ and $E$ by sets of dimensionless ratios $\rho_{R}, E_{R}$.
3.) Applied stress conditions:
$\mathbf{P}=$ externally applied load.
$Q=$ externally applied stress.
$u_{0}=$ externally induced displacement.
$\sigma_{0}=$ internal stress.
$g$ = acceleration of gravity.
$a \quad=$ externally applied acceleration.
Other stress conditions are related to $P, Q, u_{0}, \sigma_{0}$, a by sets of
dimensionless ratios $P_{R}, Q_{R}, U_{O R}, \sigma_{O R},{ }^{a_{R}}$.
The behavior of a point $x, y, z$ in the structure at time $t$ is defined by a resulting stress $\sigma$ and a resulting displacement $u$ and depend upon the abovementioned parameters and dimensionless ratios.

The quantities $\sigma, u, x, y, z, t, L, \rho, E, V, P, Q, \sigma_{0}, u_{0}, g$, a are all derived from the three fundamental units of force $F$, length $L$, and time $T$. The Poisson's ratio $V$ is already dimensionless.

The dimensions of the 1 isted parameters are given in Table A. 1.

TABLE A. 1

|  | $\sigma$ | $\square$ | x | y | $z$ | t | L | p | E | V | P | Q | $\sigma_{0}$ | $\mathrm{u}_{0}$ | g | a |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| L | -2 | 1 | 1 | 1 | 1 | 0 | 1 | -4 | -2 | 0 | 0 | -2 | -2 | 1 | 1 | 1 |
| T | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | -2 | -2 |

The table consists of a matrix of rank 3. According to
Buckingham's first theorem, one may obtain 16-3 = 13 dimensionless independent groups of parameters from those 1isted. Hoek chooses the following:

$$
\frac{\sigma L^{2}}{P}, \frac{u}{L}, \frac{x}{L}, \frac{Y}{L}, \frac{z}{L}, \frac{t^{2} a}{L}, \frac{E L^{2}}{P}, \frac{\rho a L^{3}}{P}, V, \frac{a}{g}, \frac{Q L^{2}}{P}, \frac{u_{0}}{L}, \frac{\sigma_{0} L^{2}}{P}
$$

It should be noted that other combinations than those listed above are possible. For this particular set, however, all other groups would be combinations of those listed.

Buckingham's second theorem (Buckingham's $\Pi$ Theorem) states that a dimensionally homogeneous equation (one which does not depend on the units of measurement) can be reduced to a relationship between a complete set of dimensionless products.

From Buckingham's $I I$ Theorem then the displacement $u$, and the stress $\sigma$ at a point ( $x, y, z$ ) can be expressed by the following dimensionless equations

$$
\begin{gather*}
\frac{u}{L}=F\left(\frac{X}{L}, \frac{Y}{L}, \frac{Z}{L}, \frac{t^{2} a}{L}, \frac{E L^{2}}{P}, \frac{\rho \cdot L^{3}}{P}, V, \frac{a}{g}, \frac{O L^{2}}{P},\right. \\
\left.\frac{u_{0}}{L}, \frac{\sigma_{0} L^{2}}{P}, L_{R}, E_{R}, P_{R}, Q_{R},{ }^{u_{O R}}, \rho_{R}, \sigma_{O R}, a_{R}\right)  \tag{A.1}\\
\frac{\sigma L^{2}}{P}=G\left(\frac{x}{L}, \frac{Y}{L}, \ldots, a_{R}\right) \tag{A.2}
\end{gather*}
$$

in which $F$ and $G$ are undetermined functions. The parameter $t$ is the dynamic time scale.

For the two systems, model and prototype to by physically similar, the functions $F$ and $G$ must be the same for each. Therefore, the following conditions of similitude are established.

The subscripts m and $p$ will refer to model and prototype parameters respectively.
1.) Model similitude related to natural properties: Since Poisson's ratio is dimensionless, the model and prototype must have the same Poisson's ratio:

$$
\begin{equation*}
V_{\mathrm{m}}=V_{\mathrm{p}} \tag{A.3}
\end{equation*}
$$

Combining the remaining nataral properties $E$ and $\rho$ by dimensionless grouping:

$$
\begin{equation*}
\frac{\rho_{a L^{3}}}{P} \cdot \frac{g}{a} \cdot \frac{P}{\mathrm{EL}^{2}}=\frac{\rho \mathrm{g} L}{\mathrm{E}} \tag{A.4}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
\frac{\rho_{m} g_{m} L_{m}}{E_{m}}=\frac{\rho_{p} g_{p} L_{p}}{E_{p}} \tag{A.5}
\end{equation*}
$$

Or

$$
\begin{equation*}
\frac{L_{p}}{L_{m}}=\frac{E_{p}}{E_{m}} \frac{\rho_{p}}{\rho_{m}} \frac{g_{m}}{g_{p}} \tag{A.6}
\end{equation*}
$$

If the model material is identical to the prototype material $\left(E_{m}=E_{p} ; \rho_{m}=\rho_{p} ; V_{m}=V_{p}\right)$ and the model is subjected to an artificial acceleration $N$ - $g$ ( $N$ is the scale factor) then:

$$
\begin{equation*}
\frac{L_{p}}{L_{m}}=\frac{g_{m}}{g_{p}}=\frac{N g}{g}=N \tag{A.7}
\end{equation*}
$$

It can be thus seen that by use of the centrifuge, scale models manufactured of the prototype material are suitable.
2.) Model similitude in relation to applied stresses: Applied stresses are defined by the parameters $P, Q, \sigma_{0}, \dot{u}_{0}$ and a and appear in the dimensionless groups:

$$
\frac{t^{2} a}{L}, \frac{E L^{2}}{P}, \frac{\rho a L^{3}}{P}, \frac{a}{g}, \frac{Q L^{2}}{P}, \frac{\mathrm{a}_{0}}{L}, \frac{\sigma_{0} L^{2}}{P}
$$

Taking the grouping:

$$
\begin{equation*}
\frac{Q}{E}=\frac{Q L^{2}}{P} \cdot \frac{P}{E L^{2}} \tag{A.8}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
\frac{Q_{m}}{Q_{p}}=\frac{E_{m}}{E_{p}} \tag{A.9}
\end{equation*}
$$

also:

$$
\begin{equation*}
\frac{P_{m}}{E_{m} L_{m}^{2}}=\frac{P_{p}}{E_{p} L_{p}^{2}} \quad \text { or } \quad \frac{P_{m}}{P_{p}}=\frac{E_{m} L_{m}^{2}}{E_{p} L_{p}^{2}} \tag{A.10}
\end{equation*}
$$

From the grouping:

$$
\begin{equation*}
\frac{\sigma_{0}}{E}=\frac{\sigma_{0} L^{2}}{P} \cdot \frac{P}{E L^{2}} \tag{A.11}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
\frac{\sigma_{o m}}{\sigma_{o p}}=\frac{E_{m}}{E_{p}} \tag{A.12}
\end{equation*}
$$

Displacements are scaled directly by:

$$
\begin{equation*}
\frac{u_{o m}}{u_{o p}}=\frac{L_{m}}{L_{p}} \tag{A.13}
\end{equation*}
$$

Inertia and gravity forces in the model and the prototype are characterized by the dimensionless groups $\frac{\rho a L^{3}}{p}$ and $\frac{a}{g}$ which were already used in deriving expression (A.4).

Finally, dynamic or inertial forces involve a time scale which can be derived from the grouping:

$$
\begin{equation*}
\frac{t^{2} E}{L^{2} \rho}=\frac{t^{2} a}{L} \cdot \frac{P}{\rho a L^{3}} \cdot \frac{E L^{2}}{P} \tag{A.14}
\end{equation*}
$$

## Therefore:

$$
\begin{equation*}
\frac{t_{m}}{t_{p}}=\left(\frac{\rho_{m}}{\rho_{p}} \frac{E_{p}}{E_{m}}\right)^{1 / 2} \frac{L_{m}}{L_{p}} \tag{A.15}
\end{equation*}
$$

Using a centrifuge model made of the same material as the prototype $\left(E_{m}=E_{p} ; \rho_{m}=\rho_{p} ; V_{m}=V_{p}\right)$ and subjecting it to the centrifuge artificial gravitational acceleration $N$ • (A.7).
(A,9) reduces to:

$$
\begin{equation*}
Q_{m}=Q_{p} \tag{A.16}
\end{equation*}
$$

(A,10) reduces to: $\quad \frac{P_{m}}{P_{p}}=\frac{1}{N^{2}}$
(A.12) reduces to: $\quad \sigma_{o m}=\sigma_{o p}$
(A.13) reduces to: $\quad \frac{\mathfrak{u}_{o p}}{u_{o m}}=N$
(A.15) reduces to: $\quad \frac{t_{p}}{t_{m}}=N$

One can clearly see the convenience of centrifuge modeliing. From (A.16), (A.18) and the fact that $E_{p}=E_{m}$ can also note that the strains in the model and prototype are identical. In the event that the soil behavior exhibits its usual nonlinearity, the same considerations hold, if prototype and model soils are the same.

In the experiments, it was necessary to model reinforced concrete walls by means of aluminum. The stiffness of the wall EI is modelled as follows. The dimensions of EI are FL (actually $\mathrm{FL}^{2} \mathrm{~L}^{-1}$ ). It has been shown, by equation (A.17) that force scales as $N^{2}$, and length of course, scales as $N$, so that the $E I$ of the
model must be equal to $1 / N^{3}$ the $E I$ of the prototype. For a given, but arbitrary design of a prototype reinforced concrete wall, the EI can be calculated. In the model, the $E$ of the aluminum is known, and the wall thickness can therefore be selected to produce the appropriate, scaled value of EI.

The yield characteristics of the wall itself were not modelled. In the prototype, yield would be indicated by the creation of a plastic hinge at the point of maximum moment, i.e., at the base of the stem. In order to model this, a notch or groove would have to be cut along the base of the model to a point so that the stem would fail easily at that point and thus simulate the plastic hinge.

Consolidation time scale (Rowe [46]):
In the study of 1 iquefaction, the time rate of flow of water from the soil is considered in comparison with the rate at which pore pressures are generated. The consolidation process thas requires consolidation time scaling.

The time factor $T$ of consolidation is defined by:

$$
\begin{equation*}
T=\frac{c_{v} t_{c}}{(n H)^{2}} \tag{A.21}
\end{equation*}
$$

where
$c_{v}$ is the coefficient of consolidation
$t_{c}$ is consolidation time
$H$ is the height of the stratum to be drained
$n$ is the number of drainage boundaries (1 or 2)

It is required that $T_{m}=T_{p}$. If the soil materials
are identical then:

$$
\begin{equation*}
\frac{C_{v} t_{c}{ }^{m}}{n^{2} H_{m}^{2}}=\frac{C_{\nabla} t_{c} p}{n^{2} H_{p}^{2}} \tag{A.22}
\end{equation*}
$$

since

$$
\frac{H_{m}}{H_{p}}=\frac{1}{N}
$$

then

$$
\begin{equation*}
\frac{t_{c} m}{t_{c} p}=\frac{1}{N^{2}} \tag{A.23}
\end{equation*}
$$

Which establishes the consolidation time scale.

APPENDIX B

WALL PROGRAM LISTING

Following is a listing of the data processing program WALL described in Section 4.2. The following subroutines were developed:

| MAIN(program) | DIGIT | PAPRNT |
| :--- | :--- | :--- |
| ALGEQN | INTEG | PRESS |
| APLOT | MAP | QUINT |
| BASCOR | MAXARR | SHEAR |
| BIGMAX | MOMENT | SPLINE |
| CRUNCH | PAGE | SUBU |
| DERIV | PAPLOT | IDISP |

The following called subroutines are system subroutines of the IBM 370/3032 system at the Booth Computing Center of Caltech. EQSOV - System of equations solving routine.

LSQUAR - Polynomial least-squares fitting routine.
SYSSYM* - Symbol plotting routine.
VLABEL* - Axis/axis label plotting routine.
XYPLOT* - Line plotting routine。
XYPLT* - Point Plotting routine.

* Calcomp plotter.


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If INA．EG．JJGC IC 44
If INA．EG．2JGC TC 43
```



```
EC TC \(4 E\)
```



```
CE TC 45
```



```
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4E CCNTINLE
47 CCNTINLE
CE TC 77
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ti CC \(\epsilon 4 \quad 1=1, N F F\)
（4）PF（I）＝XF（I）／トTN
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LC tet \(1=1, A G\)
\(k=I+\Lambda T\)
\(l=1+1\)
te ffint zut，l，f（CALI（K，w），J＝1，2），XP（L）
［5 67 1＝1，NF：
ET \(\lambda F(1)=X f(1) *+T N\)
GC IC 27
```

6
TC CALL FFESS
CALL SHEAH
call minent
GC TC 20
$c$
71 （CATINLE
$C$
101 FCFMAT（18A4／18A4）
102 fCFYAT（EF10．e．415）
103 fCFNAT（EFIC．C）
LU．FCFMAT（IE）
LCE FCFNAT（4F1C．0．15）
$c$
20］f［rwat（1F1）
201 FLFMAT（ $5 x, 18 A 4,1,5 x, 1 \varepsilon \Delta 4,1,5 x, 1 * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$

202 FCFNET（／／，IX，＇CENTFIFLGE RFN＝＇，31X，FE．2，
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-AIN


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    CIPENSICN A(150),3),L(15DC,3),C(1500.3),E(1).Y(1)
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    {(1.2}=&{1,2 }
    L(1,Z)=A(1, : )
    i(1)=E(1)
    EL 22 1=20, ^
    II= [-1
    {(1,2)=101)
        6(i, 2i=4(I, Z)
        {(1,1)=\Delta(1,1)/U(11., 己)
        L(1,z)=A(I,2)-C{1,1)*し(11, 3)
        If{AES(L(I,<)).LT.L.LE-CE)GC IC ミ5
        2) CRNTINLE
        <z Y(I)=E(1)-[(1, 1)#Y(11)
        Y(N)=Y(N)/L(N,2)
        l=A
        <5 11= 1-1
        Y(II}={Y(11)-6(11, ミ)*Y(I||/G(II,E)
        I=I-I
        If(I.EG.1)GC TC El
        CC 1C <5
        E1.hEILFA
        シE fFINT 167
        C
```



```
    C
        FETLFA
    EAC
```

APLCI


AFLCT

```
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C
G4G GALL HLAELLIL.75.5.O.ANINOANAX.2.5.4.OACCEL (AAC)O,D2.LO
        x'(f4.11*04)
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C,C45
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CCe7
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CC73
C674
CC7%
C,C7E
6.77
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```



```
**(5s.2)*.5)
call face
LAE=0
AA|1)=1.75
AA(2)= 5.7E
EE(1)=2.75
EE(2)=2.i5
CALL XYFLC\IZ,AA,EE,C.C,15.C,O.1).10.O.CLC.LAE!
Et{1}=4.C
EE\2|=4:C
(ALL XYFLCTIZ,AA,EE,G.C,15.O.O.I),10.C,EL(,LAE)
tE(1)=t.25
EE(2)=&.己z
(ALL XYFLCT(Z̈,AA,EE,G.C.1E.C,O.).10.O.CC(,LAE)
EE|I|=7.E
tE(2)= %.0
(ALL XYFLCT(Z,AA,EE,O,C,15,N),Q.'),1.1.O.CLC,LAE)
A&(1)=C.JE
&t(1)=105
Et(2)=4.C
CALL XYFLCICZ,AA,EE,C.C,15.O.C.O.IC.C.CLC,LAES
EE(1)=5..2
Et(2)=7.5
(ALL XYPLCT(Z,AA,EE,U.C,15.0.1).j.,10.O,C[C,LAE)
C
EC 13t J=1.11:4
A1(J)=A(J,|)
```



```
    CAA(1)=Am**(I)
    EEE{1)=T(NAXX{I!)
    CALL XYFOLT(L,LEE,AAL,TPN,THX,ANN,ANX,ECC.LAE,IJ
    AAA(l)=UMXX(I)
    EEE(1)=T(MCx又(1))
    CALL XYFLT(I,EEE,SAA,TPN,TNX,YNN,YNX,ECC,LAB,I)
    CALL XYFLC]IITM,Y,AL,TMN,INX,AMN,AMX,CCC,LAE)
        lAt=-1
        (ALL XYFLGTIITN,T,AZ,THA,THX,YNA,YNX,ECC,LAR)
    10.)CCATINLE
        C
    Z7U FCFNATIEFIC.CI
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    GETUFA
    EAC
```

```
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atceax

```
CGCA SLEFCLTINE EIGNAX(A,F,ANAX,KNAX)
```



```
    cipEAsIGAm(1)
CCIZ ANAX=C.C
        CL 770 I= 1,K
        IF(AES(A(II).GT.AES(AMAX))CO TC 7E5
        GC IC 77C
    7EE ARAX=A(1)
    kNAX=1
    77C CCNtIAle
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        fe ILFA
    ENC
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## CRUACT

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    C SGLRCliJNE IC SUET ClT parameters frgm time-space af.ray xX fem
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        X TCALI,ITH,NT,NA,NFCLY,NSZ,NINT,F,EIF,FTH,AGS,GAMFAF,
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        find maximlr paranetea ane cer̃esfencing time ar.e icgaticn
        MIPTT=AINT-2
1゙GE
    CALL *AXfFF(xX,ITM,NINIT,XXNAX,ITMAX,IX,MAX)
    6
        [C 14.) I=1,N1NT
        >1(1)=x\times(1,1)
        x(1)=xx(11NAx,I)
        (4.) )E(1)=xx(1TM,1)
        x+(1)=xxpa>
        Y(1)=-Ex(1XNAX)/rTN
        1%(1)=|(||\Deltay|#f1N
        [C 141 i=1, iTM
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|  | c $C$ $C$ $C$ $C$ | slerclitne ic scet cli parameters frin timespace afray xx for flctilag anc cliflt <br> pafanetefi chcsen afe static，maximg cyaamic．anc final siatir． |
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| 1015 |  |  |
| cers |  | $x 1(1)=x \times(1,1)$ |
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| 1165 | 14．） |  |
| cict |  | $x+(1)=x \times p A x$ |
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\hline C614 & & \(11=(5 *+)^{-4}\) \\
\hline C615 & & \(12=11+4\) \\
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\hline 1617 & & ［C 30．2 \(k=11,12\) \\
\hline CCle & & Lf（IXEASE（K）．EG．esssssicc ic 3Ce \\
\hline ubls & & XEASESK）＝FLCAT（IXEASEIK） \\
\hline C6ad & EC2 & YteSE（K）\(\times\) FLCAI（IYEASE（K） \\
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\hline C6：2 & Evt & \(\mu k=k-1\) \\
\hline へいご & & CC 3C7 L＝1，kk \\
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\hline LC 25 & & \([A] \Delta(2,1)=Y \in \Delta S E(1)\) \\
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luz} & E＇i & \([A] A(3,1)=1 . C\) \\
\hline & & Cr \(156=0.6\) \\
\hline & c & \\
\hline \multirow[t]{2}{*}{cize} & & CALL LSGLAF（CATA，Kk，2，CHISG，STCRI \\
\hline & \(c\) & \\
\hline ctis & & LC＝XEESE（1） \\
\hline C6 \(=0\) & & \(E C=C(1)+(X E A S E(1) * C(2))\) \\
\hline CCJ 1 & & CC＝XEASE（KK） \\
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\hline C6i5 & 355 & \(A(1, d)=A(1, d)-A(1, j)\) \\
\hline ccit & & Al \(1 . J 1=C \cdot 0\) \\
\hline C6i7 & 357 & clatinue \\
\hline & \(c\) & SCALE RECORDS \\
\hline ccie & & DE 3E0 \＝1，ITM \\
\hline （cis & & \(\times x(1, d)=T C A L I * x \times(1, J)\) \\
\hline cosc & 360 & A（ \(1, J)=A(1, J) * C A L I(J, 1)+C A L I(J, 2)\) \\
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6
6 & mgeel parameters tave nom been scaled \\
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\hline Ci¢ 4 & & \(\times \times(1, J)=x \times(1+2, J)\) \\
\hline 6しと & 365 & A（1，J）\(=4(1+2, J)\) \\
\hline しくとも & & 1TM＝1TM1 \\
\hline CCE7 & & GC 10375 \\
\hline & \(c\) & \\
\hline Ccte & 370 & \(17 p=1\) \\
\hline CCs & & \(A(1, J)=A(2, J)\) \\
\hline LCS¢ & 375 & CCATINUE \\
\hline しiく1 & &  \\
\hline \(10^{\text {c }}\) & & ITFM（J）\(=1\) TM \\
\hline OCs． & 280 & cratinue \\
\hline & c & \\
\hline Cしs4 & & CC \(3811=1, \mathrm{NT}\) \\
\hline ccss & & IFIICRECK（1）．NE．O）STCP \\
\hline 6uso & ミ81 & ccatinle \\
\hline & C & \\
\hline cis 7 & & CALL EICPAX（TMAX，NT，TMX，ICUM） \\
\hline cces & &  \\
\hline LCsts & & 1F（TMX．CE．3．0）GO T0 383 \\
\hline いいい & & \(11 \mu=(1 \mu x * 150.0)+1.0\) \\
\hline CiCl & & \(17 \mathrm{~F}=1 \mathrm{TM+1}\) \\
\hline －1） \(\mathrm{C}^{\text {a }}\) & & T11）\(=0.0\) \\
\hline 1143 & & CC \(382 \mathrm{I}=2.1 \mathrm{TM}\) \\
\hline c1：4 & 382 & 1（1）＝（FLCAT（1－1）／15C．0）／FIN \\
\hline cics & & EC TC 388 \\
\hline ulit & 363 & \(17 \mu=(17 \mu x-3.0) * 75.0)+451.0\) \\
\hline 6.15 & & \(1 T \mu=1 T M+1\) \\
\hline CICE & & 1（1）＝0．0 \\
\hline ulis & & CC 3 E4 I＝2，450 \\
\hline C110 & 384 & 1（1）＝（FLCAT（1－1）／150．0）／FIN \\
\hline 0111 & & CC 385 I 451 l 1TM \\
\hline \％112 & Esf & 1（1）＝（FLCAT（1－226）／7S．0）／FIN \\
\hline （1） 12 & & IFIITM．LE．1500）GC IC 388 \\
\hline 0114 & & PRINT 601．ITM \\
\hline C115 & & ITM＝1500 \\
\hline & \(C\)
\(c\) & SMCCTH Clt data bith cleic spline \\
\hline
\end{tabular}

\footnotetext{
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}

\section*{CIEIT}
cile
C117
u 118
C 115
\(01<0\)
\(01: 1\)
（1）22
4123
C124

U125
016 e
C1ぐ 7
©lzé
ciss
\(01 \equiv 0\)
C 121
C132
－1 1 2
（： 134
612
1） 32
U1こ7
C 125
1439
0140
C141
C142
\(1 \cdot 14=\)
014,4

614．5
CI4e

3EE［（ \(357 \mathrm{~J}=1, \mathrm{H}]\)
11s＝17ヶค（よ）
CC \(395 \mathrm{~J}=1 \mathrm{I}\) ITS
（1）（1）\(x \times 11, j)\)
三95 \(\mathrm{A}_{\mathrm{c}}(\mathrm{f})=\Delta(I, \mathrm{~J})\)
CELL SPLINEIITS．AI，A2，17M，T．III
Et \(35 t \quad 1=1,11 \mathrm{M}\)
39E \(f(1, d)=1111)\)
357 CGATINUE
\(C\)
\(C\)
\(C\)
\(C\)
feac in initial anc final cigitized cisplacerent valuts
C FEACIN INITIAL ANC FINAL
\(C\)
GYTCF ANE ECTTCM CF WALL
HEAD EGz，ITLFI，ITCPF，IECII，IRETF
1（FI＝FICAT（IICPI）
ICFF＝FLCAIITOPF）
ECTI＝FLCAT（IEOTI）
ECTF＝FLCET（IECTF）
－ \(1=\mathrm{AT}+1\)
－ \(2=\mathrm{NT}+2\)
If（NC．EG．0IGL TC 359
TCFI＝TCFI＊CALI（N1，1）＋CALI（M1，2）
TCFF＝TCFF＊（ALICNI，L）＋CALI（NI，Z）
IFIACOEG．1）CL TC 355
ECTI＝PCTI＊CALI（NE，1）＋CALIfN2，z）
ECTF＝ECTF＊CALI（ME，L）＋CALI（M2，2）
シsc CCMIINLE
\(\Lambda T=\Lambda \Delta+\Lambda S\)
C
Eul F（FNat（4IE）
sot FCfuftigfic．C）
sez FCFNAT（lCIE）
c
 \(x: \times x(I-1, j)=0, F I C .2,1 \times(I, j)=1, F 10,2)\)
 x＇ghly fifst 1500 tire siefs cculi bf lsec iue tic stcrace limitsil

6
fetuhn
Ef．C
```

INIEG

```


C SLEHCLTIAE TC FAKE CCATCLR FLCTS CF REGTANGLLER GRIC CGATAIAIAG 6 CECRCINATES \((X, Y)\) ANE CCRRESFCAUING FLACTICN \(Z(\lambda, y)\)

C
\(C 6 i 3\)
6.64

Cibe
iguc
（6． 7
coce
－ccs
－6d
－i． 11
vide
1012
C6．4 CCis vide 1617 Cle icis （ 60.6 CCS 1．iく2 tizs 664 Clis cict 4． \(6<7\) （CZ8 16くら
vis．
Clal
182
CCI
しくミ4
OCE
Cました
Cミ7
しごも
cris C（4C CC4 しくなく CC42

CL 2 1＝1，
2．［cc \(11=0.5\)
6
\(\stackrel{6}{6}\)
sef lf CEKICLKS
RCIRSACIR－1
THSaflCATINCTESI
（2＝（2MAX－2MIN）／TFS
C7F11）\(=2 \mathrm{MiN}\)

\(\geq \mathrm{CTF}(1)=\mathrm{CTH}(1-1)+C Z\)
LAt＝1
\(1 \mathrm{Crk=-1}\)
\(A x=\Lambda \times x-6\)
\(A Y=A Y Y-2\)
C（5） \(1=1, A x, c\)
\(x x(1)=x(18\)
\(x \times(2)=x(I+\epsilon)\)
）\(x(3)=x(t+t)\)
）\(x(4)=x(1)\)
CC 5 n \(j=1, n \mathrm{y}, 2\)
H（1）＝Y（J）
YY（2）\(=Y(J)\)
YY（2）\(=Y(J+z)\)
\(r(4)=r(j+z)\)
（2）（1）＝2（1．j）
\(z 2(2)=2(1+6, j)\)
\(22(2)=2(1+t, j+2)\)
\(22(4)=2(1,+2)\)
CC \(50 \mathrm{~K}=1\) ， ACT R
\(c\)
\(c\)
\(c\)
inteffllate
\(51=0\)
CC \(17 \quad 11=1,4\)
\(\omega \sim N(C(11,4)+1\)
IF（CYR（x）．ET．LZ（IX））GE TC 7
IF（CTR（K）．GT．ZZ（JJ）IGC T．C 10
GC TC 17
7 If（CIR（K）．LE．（Z（JJ））GO TC 10
CL TC 17
\(20 \quad L=t+1\)
If（Z2（It）．E6．2Z（jd）） 6 TC 12
ZSL＝（CTA（K）－2Z（II））／（Z2（JJ）－21（11））
\(x(L)=x \times(11)+(x \times(6 d)-x \times(11))=2 S L\)
\(Y C(L)=Y Y(I I)+(Y Y(d J)-Y Y(I!)\}=2 S L\)
GE TC 17

CCCl SLEFCLTIAE FAXAKF(A,KX,KY,ANAX,KXMAX,KYMAX)
    C SLERCLTINE TC pick clit tre lafeest aescllte value cfa
، 6.2
U61:3
Cl6.
CCC
clet
LCC
いたと
- (t) 5
1.616
vell
Cくな
icla
    \({ }_{6}^{C}\)
    TwL-EINEASICAAL EREAY A(KX.KY)
    EIPENSICA 411502.11
    \(c\)
    . \(\operatorname{An} A X=0.0\)
        -Ci \(7701=1, \mathrm{kx}\)
        [C 770 =1,ky
        If(AES(A)H,V)).CT.AES(AMAXIICC TC TES
        GC TC i7c
        its \(A N(x=A(1, \mathrm{~J})\)
        \(k x * \Delta x=1\)
        -YA \(A X=J\)
    975 C[ATINLE
    c
        kE TuんA
        EAC
moment
```

```
CCSL
```

```
CCSL
ilC2
ilC2
ひいこ
ひいこ
CCO4
CCO4
ccs:
ccs:
C6Co
C6Co
CG6
CG6
G6ie
G6ie
LiG
LiG
1.C10
1.C10
(ぶ)
(ぶ)
心!く
心!く
いごミ
いごミ
CCl4
CCl4
CCls
CCls
cicle
cicle
6017
6017
401%
401%
CG1c
CG1c
Cca%
Cca%
##%)
##%)
66%
66%
0023
0023
CC4
CC4
Cく5
Cく5
CC2t
CC2t
LC%
LC%
62t
62t
ccis
ccis
COC
COC
10:1
10:1
COE2
COE2
じミZ
じミZ
LCミ4
LCミ4
COE
COE
lCze
lCze
CC37
CC37
CO2&
```

CO2\&

```

\section*{preat}
```

UES
C4\
1C41
C442
C42
Uい44
(C45
C(4)
IC47
CC<E
(C45
CET
CCE1
C6%%
CC:3
C454
16こ5
C6%
6.65
し6\&
(<<)
C\&G
C\&1
uc\&z
C(C)
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ule?
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C(E)
Cじ心
CC71
CC72
C(:3
CCi4
C(i5
CC7t
U677

```
```

    CC.617 J=1,NS2
    ```
    CC.617 J=1,NS2
    617 &F(j)=0.0
    617 &F(j)=0.0
        CC 6l& J=ぐ,NS2
        CC 6l& J=ぐ,NS2
        [c ti& k={,NF
        [c ti& k={,NF
    6lE AP(J)=AM(J)+C(K)*x(J)**(K-1)
    6lE AP(J)=AM(J)+C(K)*x(J)**(K-1)
        If(NTYFE.EG.1)AN(AS2)=C.C
        If(NTYFE.EG.1)AN(AS2)=C.C
    C
    calCllate eclacagy ccacitillas at ectic% cF tre hall
    calCllate eclacagy ccacitillas at ectic% cF tre hall
        CC E2C :=1,NF
        CC E2C :=1,NF
        EC(4)=EC(4)+C(J)*XfNS2)**(J-1)
        EC(4)=EC(4)+C(J)*XfNS2)**(J-1)
        EC(5)=EC(E)*FLCDT(J-1)*C(J)*x(NSE)**(d-2)
        EC(5)=EC(E)*FLCDT(J-1)*C(J)*x(NSE)**(d-2)
    (2) EC(O)=EC(t)+FLCAT(J-2):FLCAT(J-1)*C(J)*x(NS2)**(J-3)
    (2) EC(O)=EC(t)+FLCAT(J-2):FLCAT(J-1)*C(J)*x(NS2)**(J-3)
        If(ATYFE.NE.l)GC TC EZ厶
        If(ATYFE.NE.l)GC TC EZ厶
        EC(4)=C.0
        EC(4)=C.0
c
```

c

```


```

    C
    ```
    C
        [C E2E ==1,NINT
```

        [C E2E ==1,NINT
    ```


```

        tg) CCPTINLE
    ```
        tg) CCPTINLE
CESuL [C ENES 1=1,ITN CCCCCOCOCCCC
CESuL [C ENES 1=1,ITN CCCCCOCOCCCC
C [C EGGS j=1,NINT
C [C EGGS j=1,NINT
cocci If(J)=xx(1,d)
cocci If(J)=xx(1,d)
C CALL UHTEGIN|!T,AX,TH,TS,OIC
C CALL UHTEGIN|!T,AX,TH,TS,OIC
T TH
T TH
CoO2C xx(l.w)=TS(J) C
CoO2C xx(l.w)=TS(J) C
cegzs lCMtIAlE CGCCGCCCGCGC6CCGCCC
cegzs lCMtIAlE CGCCGCCCGCGC6CCGCCC
C
C
        wFITE(2l)x*
        wFITE(2l)x*
        ENCFILE zl
        ENCFILE zl
        FEbINC ご1
        FEbINC ご1
        C&bl&blact
        C&bl&blact
C
C
            C[ EES !=1,ITN
            C[ EES !=1,ITN
            EC EEZ J=1,N|NT
            EC EEZ J=1,N|NT
        t=c xx(!,J)=>x([!,J)#FTM/E!ん
        t=c xx(!,J)=>x([!,J)#FTM/E!ん
        [C E55 I=1,NINI
        [C E55 I=1,NINI
        *l(l)=xl(l)*FT*/ElN
        *l(l)=xl(l)*FT*/ElN
        \(I)=x二(I)*FTM/EIN
        \(I)=x二(I)*FTM/EIN
    OE5 x2(I)=x3(I)*FTN/EIN
    OE5 x2(I)=x3(I)*FTN/EIN
            P&(1)=xN(1)*FTM/EIM
            P&(1)=xN(1)*FTM/EIM
            [C E56 I= L,ITM
            [C E56 I= L,ITM
        C5E 11(1)=11(1)*FTM/EIN
        C5E 11(1)=11(1)*FTM/EIN
C
        NSC=NS1-1
        NSC=NS1-1
        LC ese I=L,NSn
        LC ese I=L,NSn
    t5\varepsilon EM(I)=-x(I+1)/H7M
    t5\varepsilon EM(I)=-x(I+1)/H7M
            [c 060 1=1,2
            [c 060 1=1,2
        ctC [C(1)I=C.0
        ctC [C(1)I=C.0
            LAE=0
            LAE=0
            CIATX=1AESICMAX(1)-CMIN(1)/)/2.5
            CIATX=1AESICMAX(1)-CMIN(1)/)/2.5
            CINTC=(AES(CNAX-LNIN)//2.5
            CINTC=(AES(CNAX-LNIN)//2.5
            vM1A=-4.C*CINTX
```

            vM1A=-4.C*CINTX
    ```
```

\i7t
C675
ymAX= C.0*CINIX
\#1A=-7.5*C1NTL+CN1N
XHAX= 7.5*CIATC+CMIN
C [C 670 J=1,3
C GC TC (t\inE , t\in3,t65),J
C teI CL tez I=1,nsu
C Et2 C*(I)=Z.O*C(1,I+NA)*FTH/EIN
GC IC Et7
CEE [C Et4 l=1,N:\
\& Et4 fr(I)=2.O*\&(ITMAX,I+NA)*FTN/EIN
C CL I[ te]
C EtS [C EtE I=1,N\leq]
C CEE \&N(II=2.C\#A(ITN,I+NA)*FTN/EIN
C EET CAlL XYFLI(NSO,AP,EN,XRIN,XRAX,YMIN,YRAX,CCC,LAE,Z)
C 67] CCNTINLE
E
CC E7C }\downarrow=1,
CLTL (=氏1,t\in3,(E5),J
tc) :P(1)=x1(2t)
AN(2)=>1.447)
CM(3)=x1(55)
AN(4)=X|(\epsilonE)
GC TC EET
tt3 fr(1)=x2(2t)
AN(2)=x我(47)
AN(3)=x二(ES)
Ar(4)=x2(tE)
GC TC te7
ttE fM(1)=x3(2t)
AM(2)=>2(47)
AN(2)=xZ(5c)
CN(4)=>3(tE)
CET (ALL XYFLIINSN,AN,EN,XPIN,XNAX,YMIN,YNAX,[CC,LAE,Z)
e7J CCNTINLE
C
CAlL fapfat(l)
C
CC EEC l=1,[1M
[C 6e0 J=1,NINT
\epsilon\&C >>(I,J)=>>(I,J)*EIM/HTM
C
701 FCFNAT(EF17.0)
C
FEILFA
EN[

```
\begin{tabular}{|c|c|c|}
\hline 0001 & & slibroutine page \\
\hline & C
6
\(C\) & SUPROUTINE TO SET UP PLOTting on an 8-1/2 11 INCH AREA \\
\hline 0002 & & OIMENSICN A(2), \(8(2), C C C(3)\) \\
\hline & c & \\
\hline 0003 & & OC \(120 \quad 1=1,3\) \\
\hline 0004 & 120 & CCCIII=0.0 \\
\hline 0005 & \(\therefore\) & \(L A B=0\) \\
\hline 0006 & & \(\Delta(1)=0.0\) \\
\hline 0007 & & \(A(2)=11.0\) \\
\hline 0008 & & \(\mathrm{B}(1)=0.0\) \\
\hline 0009 & & \(\mathrm{B}(2)=0.0\) \\
\hline 0010 & & CALL XYPLCTI2,A, \(8,0.0,15,0,0.0,10.0, D O C, L 481\) \\
\hline 1011 & & A(1)=11.0 \\
\hline 0012 & & \(\mathrm{E}(2)=8.5\) \\
\hline 0013 & & CALL XYPLCTI \(2, A, 3,0.0,15.0,0.0,10.0\), DCC. \(14 B 1\) \\
\hline 0014 & & \(A(1)=0.0\) \\
\hline 0015 & & \(\mathrm{A}(2)=0.0\) \\
\hline 0016 & & CALL XYPLCTI2, A, Q, 0.0,15.0,0.0.10.0,0CC,LAB1 \\
\hline 0017 & & \(A(2)=11.0\) \\
\hline 0018 & & 8(1)=8.5 \\
\hline 0019 & &  \\
\hline & 6 & \\
\hline 0020 & & RETURN \\
\hline 0021 & & ERO \\
\hline
\end{tabular}

\section*{PAPLCT}
\begin{tabular}{|c|c|c|}
\hline \multirow[t]{2}{*}{0001} & & SUBROUTINE PAPLOTISI, S2.LS.DMIN, DMAXI \\
\hline & \[
\begin{aligned}
& c \\
& c \\
& c \\
& c \\
& c
\end{aligned}
\] & SLbroutine to plct cut, Ef a single b-1/2 x 11 page a cantojr map cf a parameter alcng with plots of static initial ano final values as WELL AS maximum dynamic values \\
\hline 0002 & - &  \\
\hline 0003 & & ```
COMMON/BLUE/X1(112),X2(112),x3(112),TT(1502),XX(1502,112),XM(1).
X YMIll,TM(II,ITMAX,IXM4X
``` \\
\hline 000: & & CCMMCN/GREEN/CMAX(2).CMIN(2),IPLTCD \\
\hline 0005 & & CCMMCN/SRAY/TITLEI(18).TITLE2(18) \\
\hline 0006 & &  \(\times \quad\) LT(4),CX(10),CY(10),CZ(10),PX(1),PY(1),TO(1) \\
\hline \multirow[t]{2}{*}{0007} & & \begin{tabular}{l}
 \\

\end{tabular} \\
\hline & c & \\
\hline 0009 & & OC \(701 \quad 1=1,3\) \\
\hline 0009 & 701 & CCC(1)=0.0 \\
\hline 0010 & & OC \(702 \mathrm{I}=1.15 \mathrm{M}\) \\
\hline 0011 & 702 & T(I) \(=\) T(I) \(\mathrm{F}_{\text {F1M }}\) \\
\hline \multirow[t]{2}{*}{0012} & & NINTININT-3 \\
\hline & 6 & \\
\hline 0013 & & Call page \\
\hline 0014 & & LAB=0 \\
\hline 0015 & & CALL VLABEL(1.5.2.0,CMAX(1),CMIN(1),5.0,5,51,LT(1),1, (FF3.1)',3) \\
\hline 0016 & &  \\
\hline 0017 & & CALL VLABEL(7.5.1.5.CMIN,C1AX, 2.5,2,S2, (S (2),0, (F5.2)',5) \\
\hline 0018 & & CALL VLABEL(7.5.5.0.LMIN.OMAX,2.5,2,S2,LS(2).1.0'(F5.2)',5) \\
\hline 0019 & & START \(=1.5+(15.0-(F L D A T(L S S(1) \mid\) (5.0/12.0) \() / 2.0)\) \\
\hline 0020 & & CALL SYSSYM(START,7.25,0.5.51.LS(1),0.0) \\
\hline 0021 & & CALL SYSSYM(1.0.1.15,0.1. TITLE1,72,0.0) \\
\hline 0022 & & CALL SYSSYM(1.0.1.0,0.1,TITLE2,72,0.01 \\
\hline 0023 & & IFIPLTCD.EE.OIGE 10 703 \\
\hline 0024 & & CALL VLAEEL (1.5.2.0,CMIN(2),CMAX (2),5.0,5,T2,LT(2),0, (F4.1), 4) \\
\hline 0025 & &  \\
\hline 0026 & & GC 10704 \\
\hline 0027 & 703 & CALL VLAREL (7.5,5.0,CMIN(2),CNAX(2),2.5,1,T4,LT(4),0, (F4.1) , 4) \\
\hline 0028 & & CALL VLAEFL(1,5,2.0,CMIN(2),CMAX (2),5.0,1,T2,LT(2),0, (F4.1)*,4) \\
\hline 0075 & & \(C C=C N A X(2)-C M I N(2) ~\) \\
\hline 0030 & & - \(4=0.2 * C C+C M 1 N(2)\) \\
\hline 0031 & & \(P 8=0.8 * C C+C M I N(2)\) \\
\hline 0037 & & \(P C=0.05\) *CC+CMIN(2) \\
\hline 0033 & & \(P D=0.15 * C C+C, 14(2)\) \\
\hline 003. & & PE \(=0.1\) * \(C C+C M I N(2)\) \\
\hline 0035 & & DFE0.6*CC+CMIN(2) \\
\hline 0036 & &  \\
\hline 0027 & & CALL VLABEL (2.125,2.0, PC, PD, 1.25,2,10,4,0, (F4.1) , 4 4 \\
\hline 0038 & & CALL VLABEL (8,125,5.0,PE,PF,1.25,1,T0,4,0, (F4.1) \({ }^{\text {, }}\), 4) \\
\hline \multirow[t]{2}{*}{0039} & &  \\
\hline & \(c\) & \\
\hline 0047 & 704 & \(A 1(1)=1.5\) \\
\hline 0041 & & \(A A(2)=6.5\) \\
\hline 0042 & & \(B B(1)=7.0\) \\
\hline 0043 & & \(B E(2)=7.0\) \\
\hline
\end{tabular}

\section*{PAPLCT}

0044 \(00 \div 5\) 0046 0047 \(0 C 48\) 0049 0050 0051 0052 0053 0054 0055 0056 0057 005 . 0059 0060 0061 0 ce 2 0063 0064 0065 0066 0067 0068 0 C69 0070 0071 0072 0072 0074

0075 0076 0077 0079 0079 0080 0081

0082 0083 0084 0085 0026 0087

0088
0089 0090 0091
    CALL XYPLCT(2,A1,BB,0.0,15.0,0.0,10.0.DOC,LAB)
        \(A(1)=6.5\)
        B8(1) 2.0
        CALS XYPLCT( \(2, A A, B B, 0,0,15.0,0.0,10.0, D U C, L A B)\)
        AA(1)=7.5
        \(B(1)=4.0\)
        \(A A(2)=10.0\)
        Be(2)=4.0
\(\therefore \quad\) CALL XYPLCT(2,AA, QB, 0.0.15,0,0.0.10.0.0DC. LAB)
        \(A A(1)=10.0\)
        \(B B(1)=1.5\)
        CALL XYPLOT(2.44, EB, 0.0, 15.0.0.0,10.0.00C.24E)
        \(A A(1)=7.5\)
        \(B B(1)=7.5\)
        \(11(2)=10.0\)
        \(8 P(2)=7.5\)
        CALL XYPLCT(2.AA, BB, 0.0.15.0,0.0.10.0.DOC.LAE)
        \(A A(1)=10.0\)
        Be(1)=5.0
        CALL XYPLCTI2,AA.BB,0.0.15.0.0.0.10.0.DOC,LAB8
    C
        IFIIPLTCD.NE.OIGC TC 112
        \(C A=C L / 12 . C *(P A-C M I N(2))\}\)
        \(C B=(1.0-C A) * C M I N(2)\)
        DA=CC/(2. \(0 *(C \operatorname{MAX}(2)-P A))\)
        CE=(CMAX(2) CMIN(2))/2.01-(DA*Pq)
        DE 710 1=1.17M
        IF(TII).LT.PAIGOTC TOB
        \(T(I)=(T(1) * D A)+D B\)
        GC TC 710
    70 T (1)=(T11)(\#A)+CB
    710 CCNTINUE
    C
    712 NG \(=\mathrm{NT}-\mathrm{NA}\)
        DC \(714 \mathrm{~T}=1, \mathrm{NB}\)
        CX(I) \(=-X(1+1) / H T M\)
    114 CY(I) \(=0.0\)
        IF(NG.EG.OIGE TO 718
        OC 717 \{=1,N6
        717 CZ(1) \(=-X P(1+1) /\) HTN
        \(c\)
        c
    718 CINTx(ABSICMAXII)-CMINILJH//5.0
        XMIN=-7. D*CINT
        \(X N A X=3.0 * C I N T\)
        CINT=(ABSICMAX(2)-CMIN(2)))/5.0
        TMIN=-1.5*CINT
        TMAX=13.5*CINT
    \(c\)
\(c\)
\(c\)
    plet cantours
        IFINC.EO.OIGC TO 720
        CALL XYPLTINC,CY,CZ,TMIN,TMAX,XMIN,XMAX,CCC,LAB,41
    720 CALL XYPLTINB,CY,CX,TYIN,TYAX,XMIN,XMAX, DOC,LAB,3)
    AA(1) =CMIN(2)

\section*{PAPLOT}

0092
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0055
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0142 0143

BE(I)=8X(IXMAX)
\(\triangle A\{2\}=C M A X\{2\}\)
\(B E(2)=B X(X X A X)\)
CALL XYPLCTIZ,AA, BB,TMIN,TMAX,XMIN,XMAX, COC,LABJ
\(A \operatorname{A}(1)=T(1 T M A X)\)
BB\{1:n-CmIN\{1\}
AA(2) =TIITMAXI
BB(2) =-CHAX(1)
\({ }^{-}\)CALL XYPLCT(2,AA,BE,TMIN,TMAX,XMIN,XMAX, DOC,LABI
-PX(1)=TITTMAX)
PY(I)= \(8 \times(I X M A X)\)
CALL XYPLTII,PX,PY,TMIN,TMAX,XMIN,XMAX,DOC,LAB,II
CALL MAP\{T, BX,XX,TMAX,TMIN, XMAX,XMIN, OMAX, DMIN, ITM,NINT, 21)
PLOT INITIAL AND FINAL STATIC PARARETERS AS WELL AS MAXIMUA OYNAMIG CNES
CINTX=(ABS(CMAX(1)-CMIN(1)))/2.5
CINTT=(ABSICMAX(2)-CMIN(2)))/2.5
CINTC=(ABSIDMAX-CMIN) )/2.5
\(Y M I N=-4.0\) \# CINTX
\(Y\) MAX \(=6.0\) © CIATX
XMIN \(=-7.5 *\) CINTO + OMIN
XMAX \(=7.5\) © CINTCHCMIN
CALL XYPLTII,XM,YM,XMIN, XMAX,YMIN,YMAX, DOC, LAR, I)
IFING.EQ.OIGC TO 722
CALL XYDLTINC, CY, CZ,XMIN,XMAX,YMIN,YMAX,DOC,LAB, 4 I
722 CALL XYPLT(NA,CY,CX,XMIN,XMAX,YMIN,YMAX,OCC,LAR,3)
\(\Delta \Delta(1)=0.0\)
\(\Delta \Delta(2)=0.0\)
BR(1) \(=-\) CMAX(I)
BE(2)=-CMIN11)
CALL XYPLCTIZ, AA.E日, XNIN,XMAX,YMIN,YMAX, DOC, LABI
CALL XYPLCTININT, XI, BX, XMIN, XMAX,YMIN,YMIX,DCL,LABI
CALL XYPLCT \{NINT, X \(2, ~ Q X, X M I N, X M A X, Y M I N, Y M A X, D O C, L A B\}\)
CALL XYPLCT(NINT, XZ, EX, XMIN, XMAX,YMIN,YMAX,DCC,LAB)
YMIN=-5.0*CINTOHOMIN
\(Y \because A X=5.0 * C\) INTD \(\because C H I N\)
XMIN \(=-7.5 * C I N T T\)
XNAX= T.5*CINTT
IFIIPLTCD.NE.OIGE TO 726
IFITMII).LT.PASGC TO 725
\(T M(1)=(T M(1) \neq C A)+C B\)
GC TC 726
\(725 \mathrm{TH}(1)=(\mathrm{TM}(1) \neq(4)+C 8\)
726 CALL XYPLTII,TM,XM,XMIN, XNAX,YMIN,YMAX, OCC,LAB, I)
\(B E(1)=0.0\)
Be(2)=0.0
\(A A(1)=C M I N(2)\)
\(\Delta \Delta(2)=C M \Delta X(2)\)
CALL XYPLOT\{2,AA, EB, XMIN,XMAX,YYIN,YYAX, OUC,LAB)
\(L A B=-1\)
CALL XYPLCTIITN,T,TT,XNIN,XNAX,YNIN,YMAX,COC,LABI
C
IFIIPLTCD.NE,OIGC TC 734
\(P G=(C M A X(2)+C M I N(2)) / 2.0\)
กC \(729 \quad 1=1.17 M\)
- 326 -
paplet


\section*{PAPRNT}

0001

0003

0004 0005 0006 0007 0009 0009 0010 0011 0012 0012 0014 2015 0016 0017 0013 0019 0020 0021 0022 0023 0024 0025 0026 0027 \(00 ? 8\) 0029 0030 0031 0032 0033 0037 0035 0036 0037 0038 0039 0040 0041 0042 004 ? 0044 \(00+5\) 0046 0047 \(00+8\)

SUBROUTINE PAPRNTIIPARMI
sugroutine to print cut parameters
CCYYON/RED/A(1502,12),T(1502), AX(112),BX(1121,CAL1(15,2), X(108, \(X\) TCALI,ITM,NT,NA,NPOLY,NSZ,NINT,H,EIM,HTM,AGS,GAYMAM, \(X \quad\) NTYPE,FIM,NG, XP(9)
 x YM(1).TM(I),ITMAX,IXMAX
\(c\)
NINT=NINT-Z
\(T A=T(1) * F 1 M\)
TEFTIITMAXIFFIM
TCST(ITM)AFIM
万C \(61 \Lambda=1,3\)
PQINT 200
GC TC(51.52,53.54),1PARM
51. PRINT 201

GC TC 57
52 PDINT 202
GC TC 57
53 PRINT 203
GO TC 57
54 PRINT 204
57 PRINT 220.TA.TB.TC
\(L=N * 50\)
DC \(60 \mathrm{~J}=1,50\)
\(I=(L-50)+J\)
IFII.GT.NINTIGC TO 62
TX \(\mathrm{X}=-\mathrm{BX}(1)\)
60 PRINT 221.I. X1III, X2(II, X3(1), TX
61 CENTINUE
62 CCNTINUE
\(c\)
กC \(91 \mathrm{~N}=1.15\)
PRINT 200
GC TC(71,72,73,74),IPARM
71 PRINT \(2 \pi 1\)
GC TC 71
72 PRINT 202
GE TC 77
73 PRINT 203
GC TC 77
74 PRINT 204
77 TX=-BX\{1XMAX|
PRINT 222,TX.TX
\(L=N * 50\)
00 80 J=1.50
\([=(L-50)+J\)
\(K=(+(N-1) \geqslant 50\)
\(K K=K+50\)
IF(KK.GT.ITM)GO TO 82
TY=T(K)*F1M
\(T Z=T(K K) * F I M\)
80 PRINT 223,K,TT(K),TY,KK,TT(KK),TZ
91 CCNTINUE

\section*{PAPRNT}

82 CENTINUE NS=(2*N-1)*50
TFIK.EQ.NSIGC TO 86
DO 85 =K。NS
TY=TIIB\#FIM
If(I.GT.ITA)GOTC 86
85 PRINT 224-I.TTIIJ.IY
86. CCNTINUE
\(-N I N T=N 1 N T+3\)
E
200 FCRMAT (IHI)
201 FCRMATア3IX************ \(x\) /. \(31 x\). \({ }^{\circ}\) MCMENT (M*H/EI) *







20́ FCRMATI29x************



 XE \(10.3,5 X_{*} *(X / H)^{*} / .12 X_{*}\)


221 FQRMATI6X.I4.E15.3.2E20.2.F10.31

 X'IT*FII', /. 10 X. \(\qquad\)
223 FCRNAT \(6 X, 14,2 E 15.3,7 X, 14,2 E 15.31\)
224 FCRYAT \(6 X, 14,2 E 25.3\) )
C
RETURN
Eñ

\section*{PRESS}

0001

SLGRCUTINE PRESS
```

C SUBROUTINF TO DERIVE EARTH PRFSSURFS BY SHEAR DIFFERENTIATIGR
C OR bY CUINTIC SPLINE FITS CF PRESSURE TRANSDUCER DATA AS DINE
IN SUBRCUTINE MOMENT
CCMMON/KED/A(1502,121,T(1502),AX(112),BX{112),CAL1(15,2), X(10),
X TCALI,ITM,NT,NA,NPCLY,NSZ,NINT,H,EIM,HTM,IGS,GAMMAM,
X NTYPE,FlM,NG;XP(9)
-CCMMON/BLUE/X1(112),X2(112),X3(1112),TT11502),XX(1502,112),XM11).
X YM(1),TM(1),ITM4X,IXMAX
C[MMCN/GREEN/CMAX(2),CWIN(2),IPLTCD
CEMMCN/YELLOL/TR(112),TS(112)
RE\&L*\& STCR{11,25)
DIMENSICN S1(2),S2(3),LS(2),RE{1502),DOC(3),AA(1),8B(1), AMP10%.
X BY(10),CATA{2.10),C(11),XD11101,AM1(10)
DATA SI/PPRES'.'SURF'/,S2/'P/(R'.'O\#G*','H)'/.LS/B,1O/
c
C
READ 991.DMIN,DMAX
NI=NINT-4
IF(NG.NE.OIGE TO 900
OC 825 I=1,ITM
DC 309 J=I,NINT
809 TR(J)=x\{1.J)
CALL DERIV(NINT,H,TR.TS)
c
FIND LCGATION CF PRESSUFE RESLLTTANT (REIII)
AR=0.0
YA=0.0
OC 815 J=1,NI
DA=0.5*(AX(J+l)-\DeltaX(J))*(TS{J+1)+TS(J))
Y=(AX(J+1)+AX(J) 1/2.0
AR=AR+DA
815 YA=YA+Y\#DA
RE(I)=YA/AR
DC 820 J=1,NINT
820 XX(I.Jl=TS(J)
925 CCNTINUE
C
827 CALL CRUNCH
C
nC 830 !=1,5TM
OO 830 J=1,NMNT
830 XX(I,J)=XXII,J)/(GAMMGM*FTM)
DC 835 I=1,NINT
x(1I)=xI(I)/(GANNAM\#HTM)
X2(I)=X2(I)/{GAMMAN*HTM)
835 X3(I)=X3(I)/{GAMNAM\#HTN)
XN(1)=XM(1)/(GAMMAM*NTM)
nC 836 I=1,ITM
RE(I)=-(RE(I)/HTM)
836 TT(I)=FT(I)/(GAMMAN*HTM)
C
DC 840 1=1.3

```
\begin{tabular}{|c|c|c|}
\hline \(00 \div 0\) & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\(840 \begin{gathered}\text { DCC } \\ L A B=0\end{gathered}\)}} \\
\hline 0041 & & \\
\hline 0042 & & CINTX= (ABSICMAXIl)-CMIN(1)1/12.5 \\
\hline 0043 & & CINTC=(ABS(OMAX-DFIN)]/2.5 \\
\hline 0044 & & YMIN \(=-4.0 * C\) INTX \\
\hline 0045 & & YNaX=6.0\#CINTX \\
\hline 0046 & & XMIN=-7.5FCINTOLOMIN \\
\hline 0047 & & - XMAX= 7.5*CINTD+CMIN \\
\hline 0048 & & AAPII \(=0.0\) \\
\hline 0045 & & - \(\mathrm{B}(1)=R \mathrm{E}\) (1) \\
\hline 0050 & & CALL XYPLT(I,AA, B , XMIN, XMAX,YMIN,YMAX, DOC, LAB, 1 I \\
\hline 0051 & & BBII) =REITMAXI \\
\hline 0052 & & CALL XYPLTII, AA, BR,XMIA, XPAX,YMIN,YMAX, DCC,LAE, 03 \\
\hline \multirow[t]{3}{*}{\[
\begin{aligned}
& 0053 \\
& 0054
\end{aligned}
\]} & & BB(1):ERF(ITM) \\
\hline & & CALL XYPLTII,AA, ER, XMIN, X4AX,YMIV,YMAX,DOC,LAB, 51 \\
\hline & \multicolumn{2}{|l|}{\(e\)} \\
\hline 0055 & & IFING.EG. 1 ) 5010940 \\
\hline 0056 & & NSO=NW-2 \\
\hline 0057 & & DO \(8581=1 . \mathrm{NSO}\) \\
\hline 0059 & 859 & BNII)=-XP(1+1)/HTN \\
\hline 0059 & & DC. \(8601=1.3\) \\
\hline 0060 & 860 & CCCIt)=0.0 \\
\hline 0061 & & \(1 \Delta \theta=0\) \\
\hline 0 Cb 2 & & CINTX=(AESICMAXII)-CMIN(1)) \(/ 2.5\) \\
\hline 0063 & & CINTD=(ABS (DMAX-DMIN) \(/ 2.5\) \\
\hline 0064 & & YNIN=-4.n*CINTX \\
\hline 0065 & & YMAX \(=6.0 * C\) INYX \\
\hline 0066 & & XMIN=-7.5*CINTD+ CM M IN \\
\hline 0067 & & XMAX \(=7.5 * C I N T D+C M I N\) \\
\hline 0068 & & OC \(870 \mathrm{~J}=1.2\) \\
\hline 0060 & & GC TC (961,963,865).J \\
\hline 0070 & 861 & OE \(862 \quad 1=1, N S 0\) \\
\hline 0071 & 962 & AM(I)=A(l, I+NT)/(GAMNAM*HTM) \\
\hline 0072 & & GC TC 867 \\
\hline 0073 & 863 & DC 864 I=1,NSO \\
\hline 0074 & 964 & AM(I) \(\times 4\) (ITMAX,I+NT)/(GAMMAM*HTM) \\
\hline 0075 & & GC TC 867 \\
\hline 0076 & 865 & O2 \(866 \mathrm{I}=1\), N 50 \\
\hline 0077 & 866 & AN(I) \(=\) A (ITM, I + NT)/(GAMMAN*HTM) \\
\hline 0078 & 367 & CALL XYOLTINSO, AM, BM, XYIN, XYAX,YMIN,YMAX, CDC, LAB, 21 \\
\hline 0079 & 970 & CCNTINLE \\
\hline \multirow[t]{2}{*}{0090} & & GC PC 940 \\
\hline & c & \\
\hline 0081 & 900 & \(N P P=N P C L Y+1\) \\
\hline 0082 & & \(N P T=N C+1\) \\
\hline 0093 & & \(\mathrm{Nh}=\mathrm{NPT}+1\) \\
\hline 0084 & & AM(1) \(=0.0\) \\
\hline 0085 & & DC \(905 \mathrm{I}=1 . \mathrm{NPT}\) \\
\hline 0086 & 905 & DATA(3.1) \(=1.0\) \\
\hline \multirow[t]{2}{*}{0087} & & CHISC=0.0 \\
\hline & \(c\) & \\
\hline 0088 & & DC \(930 \mathrm{I}=1.174\) \\
\hline 0089 & & \(00907 \mathrm{~J}=2 . \mathrm{NPT}\) \\
\hline 0090 & 907 & \(\Delta M(J)=A(1, J+N T-1)\) \\
\hline 0091 & & OC 910 J=2.NPT \\
\hline 0092 & & OATA(1, \(5-1)=\mathrm{XP}(\mathrm{J})\) \\
\hline
\end{tabular}
```

0093 910 DATA(2,J-l)=AM(J)
0094
0 0 9 5
0056
0097
0098
0 0 9 9
0 1 0 0
0101
0102
0103
0104
0 1 0 5
0 1 0 6
0107
0102
0 1 0 9
0110
0111
0 1 1 2
0113
0114
0115
0116
0117
0 1 1 8
0119
0120
0121
0122
0123
910 DATA(2.J-1)=AM(J)

```NPT \(=\) NPT-1
```

C

C
CALL LSCUARIDATA,APT,NPP,C,CHISO,STORI
$N F T=N P T+1$
DC $917 \mathrm{~J}=1 . \mathrm{NH}$
917. AM (J) $=0.0$

- 00 918 $\mathrm{J}=2 . \mathrm{NH}$

OC $918 \mathrm{~K}=1$, NPP
918 AY(J)=AM(J)+C(K)*XP(J)**(K-1)
$\mathrm{NHI}=\mathrm{NH}-1$
DC 922 JFI INH1
$X P I(J)=X P(J)$
$922 \Delta M 1(J)=A M(J)$
$c$
$c$ CALCLLATE PRESSURES WITH CUBIC SDLINE
CALL SPLINE(NWL,XPI,AMI,MIAT,AX,TS)
$C$
$C$ FINO LECATICN OF PRESSURE RESULTANT (REIII)
$A R=0.0$
$Y A=0.0$
DC $925 \mathrm{~J}=1, \mathrm{NI}$
$0 a=0.5 *(\Delta \times(J+1)-\Delta \times(J)) *(T S(J+1)+T S(J))$ $r=(\Delta x(J+1)+\Delta x(J)) / 2.0$
$A R=A P+C A$
$925 \mathrm{YA}=\mathrm{YA+Y*DA}$
RE(I)=YA/AR
DC $927 \mathrm{~J}=1$ ININT
$927 \times(1, J)=T S(J)$
930 CONTINUF
GC TC 827
$c$
940 (ALL DAPRNT(3)
CALL PAPLDT(SL.S2,LS.OMIN,DMAX)
c
DC 943 I=1.1TM
DC $943 \mathrm{~J}=1, \mathrm{NINT}$
$9+3 \times x([, J)=X X(I, J) *(G A \sim v A M * F T Y)$
C
991 FERMAT(2F10.01
$c$
RETURN
ENO

QUINT


0047
0048
0049
0050
0051
0052
0053 0054 0055 0056 0057

0058 0059 0060 0061
$006 ?$
0063
0064

0065 00e6
CUINT
C .e..eESSOV IS A SYSTEM SUBROUTINE
$\stackrel{c}{c}$
CALL EGSCVIN6,A,8,10,1.OE-4,C,1T,0)
DC 60 Ini.M
IF(S(1).LT. $\mathrm{X}(1)$ GC TO 53
DC $52 \mathrm{~J}=1, \mathrm{~N}$
IFiS(I).LE.X(J+1)) GO TO 55
CONTINUE
ccccceccccccccccecec
GC TC 55
CCCECCRCCCECCCCCECCE
53 PRINT 106.I
GE IC }6
55 CCNTINUE
SX=S<br>|-X{J)

```

```

            x(C:5,J)+(SX/5.)*C(6,J)|))|
                IF(JMANT.EG.OI GC TC 60
            SEC(I)=C(3.J)+SX*{C{4,J)+{SX/2.)*{C(5.J)+{SX/3.)*C{6.J)|)
        6) CCNTINUE
            RETURN
    C
        99 PFINT 107
    C
        106 FCRMATI/:" THE',IS."TH ELEMENT OF THE ARRAY S IS CUT OF RANGEQ*/O
            X* ERPCR MESSAGE FROM SUINT',/1
        107 FGRMATI/.' N IS LARGER THAN 501',/%
            X' ERROR MESSAGE FRCM SUINT*./'
        C
            RETURN
            END
    ```

SHEAR
```

0001
0 0 0 2

```

```

    X YM{1},TM{1},ITMAX,IXMAX
        CCMMCN/YELLCH/TR(112),TS(112)
        OIMENSICN SI(2),S2(3),LS(2)
        DATA SI/'SHEA','R'/,SZ/'G/(P*,'AE/K',*AE)'/.LS/5.11/
    0005
0006
0 0 0 7
00C8
0009
0 0 1 0
0 0 1 1
0 0 1 2
0 0 1 3
0014
0 0 1 5
0 0 1 6
0nl?
0018
0019
0025
0 0 2 1
0 0 2 2
0 0 2 3
ON2%
025
0 0 2 6
0027
0 0 2 8
0c79
003n
0 0 3 1
0 0 3 2
0 0 3 3
0034
0025
0036

```

0001

\section*{C SUBRCUTINE SHER}
```SUBRCUTINE TO DERIVE SHEAFS BY MCMENT EIFFERENTIATION ORc PRESSURE DISTRIBUTION INTEGRATIONcCCMMON/PED/A(1502.121,T(15021,AX(1121,BX11121,CAL1115.2), X110).
```

$X$ TCALI,ITM,NT,NA,NPOLY,NS2,NINT,H,EIM,HTH,AGS,GAYYAME

```NTYPE,FIM,NG,XP(S)
CCMMON/BLUE/X111121, X2(1121, X3(112), 1911502), XX(1502,112), XM(1).
\(x \quad\) YY(1),TM(1),ITMAX,IXMAX
DIMENSICN SI(2).S2131.LS(2)
DATA SI/'SHEA'.'R'/,SZ/'G/(P', 'AE/K', 'AE)'/.LS/5.11ر
\(c\)
\(c\)
REAC 8OL.CMIN.DMAX
6
DC \(725 \mathrm{~J}=1.1 \mathrm{TN}\)
DC \(709 \mathrm{~J}=1 . \mathrm{NINT}\)
709 TR(JIEXX(1, J)
IFING.AE.OIGO TO 712
CALL OERIVININT,H,TR,TSI
GC TC 714
112 CALL INTEGININT,AX,TR,TS,OI
714 ⿹丁 \(720 \mathrm{~J}=1 . \mathrm{NINT}\)
\(720 \times \times(1.1)=\mathrm{TS}(\mathrm{J})\)
725 ECNTINUE
C
\(C\)
\(C\) PAE/KAE=1).S*RC*G*(H**2) - FROM \(Y\) - 0 ANALYSXS
DC \(727 \mathrm{I}=1.1 \mathrm{IM}\)
DC \(727 \mathrm{~J}=1, \mathrm{NINT}\)
\(727 \times \times(1, J)=\times(1, J) /(0.5 * G A M M A N *(H M * * 2))\)
DC \(735 \mathrm{I}=1\), NINT
```



```
X2(1) = \(\times 2(1) /(0.5 * G A M N A M *(H T M * * 2))\)
\(735 \times 3(\{ )=\times 3(1) /(0.5 *\) GAMNGN*(HTN**2))
XM(1) =x*(1)/(0.5*GAYMAN* (HTM**2)) DC \(7361=1,1 \mathrm{TN}\)
736 TT(I) 1 TT(I)/(0.5*GAMMAM*(HTM**2))
\(c\)
C
CALL PAPRAT(2)
CILL PAPLOT(SI,S2,LS.DMIN,OMAX)
OC \(744 \mathrm{I}=1.17 \mu\)
DC. \(744 \mathrm{~J}=1 . \mathrm{NINT}\)
\(744 \times \times(1, J)=x \times(I, J) *(0.5 * G A M 4 A M *(H T 4 * * 2))\)
c
801 FCRMAT(2F10.0)
C
RETURN
ENO
```

:
SUBRCUTINE SHEAR

## SPLINE

0001

0002
0003
0004
0005 0006 0007 0008 0009 0010 0011 0012 0013 0014 0015 0016 0017 0018 0 O19 0020 0021

0022
0023
0024
0025
0026
0027

0028 0029

0030 0031

0032
0033 0034 0035 0036 0037 0038 0039 0040 0041 0042 0043

SLBROUTINE SPLINE(NN,X,Y,M,S.TI

- IFINN.GT.1501IGC TC 90
$\mathrm{N}=\mathrm{NN}-1$
NM $1=\mathrm{N}-1$
DC $5 i=1, N$
$5 H(1)=x(1+1)-x(1)$
$0015\{=1, \mathrm{NM}\}$
$A(1,1)=H(1) / H(1+1)$
$\Delta(I, 2)=2.0 *(H(I+1)+H(I)) / H(I+1)$
$A([, 3)=1,0$

$\Delta(1,1)=0$
$\Delta(A M 1.3)=C$
CALL ALGEGN(NMI,A,B,DI
DC $45 I=1, M$
IFIS(1).aT. $\times(11) G C$ TC 26
OC $25 \mathrm{~J}=1 \mathrm{~N}$
[FiSiIf-LE.X(J+1))GO TC 29
25 CENTINUE
ccccecce
GC TO 28
cccccccc
26 PRINT 106.1
GC TO 45
29 IFiJ.EO.lIGO TC 30
IF(J.EG.NICO TC 40
T(II) (P) $J-1) *(x(J+1)-511)) * * 3+$

X(6.0*Y(J)-H(J)**Z*P(J-1))*(X(J+1)-SII) )/16.0*H(J)
GL TC 45
30 T(1) $=(P(J) *(S(I)-X(J)) * * 3+(6.0 * Y(J+1)-H(J) * * 2 * P(J)) *(S(1)-X(J)) *$

GC TC 45
$40 T(1)=(P(J-1) *(x(J+1)-5(1)) * * 3+6.0 * Y(J+1) *(5(1)-x(J))+$

45 CCNTINUE
IFIIKANT.EG.OIRETLRN
DC 80 I=1.M
IFIS(l).LT.X(11)GC TO 52
00 $50 \mathrm{~J}=1 \mathrm{~N}$
IFis(ll.LE.x(J+1))GO TC 54
50 CCNTINUE
52 . PqINT 106, 1
GC TO 80
54 IFIJ.EQ.IIGI TC 60
IFIJ.EO.NIGO TC 70
DER(I)=(3.0*(P\{d)*(S(1)-x(J))**2-P(J-1)*(x(J+1)-S(I))**2)*
X6. 0 (Y(J+1)-Y(J))-H(J)**2*(P(J)-P(J-11))/(6.0*H(J))
GC 7080
60 DER(I)=(3.0*P\{J)*\{SII)-X(J))**2*6.0*(Y(J+1)-Y(J))-H(J)**2*O(J)//

0046
0047
0048
0049 01350

0051
0052
0053
0054

```
```

                X(6.0*H(J))
    ```
```

```
                X(6.0*H(J))
```

```
                GC TO 80
```

                GC TO 80
    70 OER(I)={-3.0*P(J-1)*(X(J+1)-S(I):*** 2+6.0*(Y(J+1)-Y{J))+
    70 OER(I)={-3.0*P(J-1)*(X(J+1)-S(I):*** 2+6.0*(Y(J+1)-Y{J))+
        XH(J)**2*P(J-1)//16.0*H(J))
        XH(J)**2*P(J-1)//16.0*H(J))
        80 CENTINUE
        80 CENTINUE
        RETURN
        RETURN
        90 PQINT 107 :
        90 PQINT 107 :
    C
C
10G FCRMAT('0 THE ©.15.'TH ELEMENT OF ARRAY S IS OUT OF RANGE
10G FCRMAT('0 THE ©.15.'TH ELEMENT OF ARRAY S IS OUT OF RANGE
XERROR MESSAGE FROM SPLINE'I
XERROR MESSAGE FROM SPLINE'I
107 FCRMAT ('O N IS LARGER THAN 1501, SCRRY'I
107 FCRMAT ('O N IS LARGER THAN 1501, SCRRY'I
C
C
RETURN
RETURN
ENO

```
        ENO
```

```
ONO1. SCBROUTINE SUBU{X.US
C
0002
0003
0 0 0 4
0005
0006
0 0 0 7
0008
0009
0 0 1 0
0 0 1 1
0012
0013
0 0 1 4
0015
0016
0017
0 0 1 8
0 0 1 9
0 0 2 0
0 0 2 1
```



0001
slbroutine yoisp

CCMMON/RED/A(1502,12), T(1502), AX(112),BX(112),CAL1(15,2), X(10),
$X$ TCALI,ITM,NT,NA,NPCLY,NSZ,NINT,H,EIM,HTM,AGS,GAMMAM, NTYPE,FIM,NG,XP(S)
CCMMCN/BLUE/X1(112), X2(112), X31112), TT(1502), XX(1502,112), XM(1),

- $X \quad$ YMill,TM(1),ITMAX,IXMAX
- CCMMCN/YFLLCH/TR(112),TS(112)

CCMMCN/BLACK/D1511502,31
OINEASICN S1(3).S2(1).LS(2)
DATA SI/'DISP', LLACE', MENT'/,S2/'Y/H'/,LS/12,3/
$c$
REAC(21)XX
REAC 381, DMIN, DYAX
DC 201 I=1.ITM
DC $201 \mathrm{~J}=1,2$
201 DISII.J)=CIS(I,J)*EIM
$c$
C DETERMINE OISPLACEMENTS
$c$
OC 250 1=1.17N
ne $212 \mathrm{~J}=1$ NiNT
212 TR(J) $=x \times(1 . J)$
CALL INTEGININT,AX,TR,TS,I)
CALL INTFGININT, AX,TS,TR, I)
EE=(TRII)-TR(NINT)+DIS(I,2)-DIS(I,I))/(AX(NINTI-AX(1)
FF=D(S\{I-1)-(EE*AX(1)\}-TR\{1)
DC $237 \mathrm{~J}=1$,NINT
$237 \times \times(I, J)=(T R(J)+(E E * \Delta X(J))+F F) / E I M$
250 CCNTINUE

CALL CRUNCH
DC 274 I=1.1TM
DC $274 \mathrm{~J}=1, \mathrm{NINT}$
$274 \times \times(1, J)=\times \times(1, J) / \mu T M$
DC $277 \mathrm{I}=1$,NINT
x1(1)=x1(1)/HTM
$\times 2(1)=\times 211) /$ TM
$277 \times 3(1)=\times 3(1) / H^{M}$
XM(1) $=X M(1) / H T M$
nC $2731=1,1 \mathrm{TM}$
278 TTII $=$ TTHII/HTM
c
CALL PAPRNT(4)
CALL PAPLCT(S1,S2,LS,DMIN,DMAX)
c
DC 280 $1=1.1 T M$
DO 280 J=1,NINT
$280 \times \times(I, J)=X \times(I, J) * H T M$
DC $288 \mathrm{I}=1 \mathrm{ITM}$
DC $288 \mathrm{~J}=1.2$
283 OSSII.J)=CIS(I.J)/EIM

## voisp



0043 0044
381 FCRMATI2F10.0)

C
FCRMATI2F10.0)
RETURA
END

## blk data

| 0001 | block data |
| :---: | :---: |
| 0002 | CCMMCN/WHITE/IWANT, LER(1500) |
| 000? | CEMMEN/PURPLE/JWANT, FIR(500), SEC (500) |
| 0004 | DATA [HANT/O/.JWANT/O/ |
| 00 |  |

## APPENDIX C

## LIST OF SYMBOLS

Symbols are defined where they first appear in the text. A summary of the symbols employed and their dimensions is given in this appendix.

## LOWER CASE SYMBOLS

## Symbol

Definition
Dimensions
a externally applied acceleration ..... $L_{T}^{-2}$$a_{R}$ set of dimensionless external acceleration ratios
d thickness ..... L
du digitizer unit ..... -
e void ratio ..... -
$f_{a} \quad$ elastic strength of aluminum ..... $\mathrm{FL}^{-2}$
$f_{m} f_{p}$ frequency of vibration of model, prototype ..... $\mathrm{T}^{-1}$
$f_{1}$ fundamental frequency ..... $\mathrm{T}^{-1}$
g gravitational acceleration ..... $L_{T}^{-2}$
$g_{m} g_{p}$ gravitational acceleration of model, prototype ..... $L T^{-2}$
h height ..... L
i angle of backfill slope ..... 0
$k$ number of dimensionless groups ..... -
1 length of beam ..... L

| n | number of drainage boundaries | - |
| :---: | :---: | :---: |
| II | number of parameters | - |
| t | time | T |
| $t_{c}$ | consolidation time | T |
| $t_{c m}{ }^{\text {cp }}$ | consolidation time of model, prototype | T |
| $t_{m} t_{p}$ |  | T |
| ${ }_{0}$ | externally induced displacement | L |
| $\mathbf{u}_{\text {om }}{ }^{\text {rap }}$ | externally induced displacement of model, prototype | L |
| $u_{o R}$ | set of dimensionless externally induced displacement ratios | L |
| v | lateral velocity | $\mathrm{LT}^{-1}$ |
| $\mathrm{x}, \mathrm{y}, \mathrm{z}$ | length and distance in coordinate directions | L |
| y | wa11 displacement | L |

## UPPER CASE SYMBOLS

| Symbol | Definition | Dimensions |
| :---: | :---: | :---: |
| A | constant of integration | - |
| B | constant of integration | L |
| $\mathrm{C}_{\mathrm{a}}$ | expression dependent on Mononobe-Okabe parameters | - |
| $\mathrm{C}_{\mathrm{v}}$ | coefficient of consolidation | $L^{2} \mathrm{~T}^{-1}$ |
| E | Young's modulus | FL ${ }^{-2}$ |
| $\mathrm{E}_{\mathrm{A}}$ | Young's modulus of aluminum | $F L^{-2}$ |
| $\mathrm{E}_{\mathrm{m}} \mathrm{E}_{\mathrm{p}}$ | Young's modulus of model, prototype | $\mathrm{FL}^{-2}$ |
| $\mathrm{E}_{\mathrm{R}}$ | set of dimensionless Young's modulus ratios | - |
| EI | stiffness pex nnit width of wall | $\mathrm{FL}^{2} \mathrm{~L}^{-1}$ |
| F | typical force dimension | F |
| F () | function of | - |
| F.S. | factor of safety | - |
| G | shear modulus | $\mathrm{FL}^{-2}$ |
| $\mathbf{G}_{\mathbf{m}} \mathrm{G}_{\mathrm{p}}$ | shear modulus of model, prototype | $E L^{-2}$ |
| G() | function of | - |
| G.S. | Ground surface | - |
| H | height | $L$ |
| $\mathrm{H}_{\mathbf{A}}$ | height at which resultant force acts | L |
| $\mathbf{H}_{\mathbf{f}}$ | depth of frost cover in front of wall | L |
| $H_{m} H_{p}$ | height of model, prototype | L |
| I | moment of inertia per unit width of wall | $L^{4} L^{-1}$ |


| $\mathrm{K}_{\mathrm{A}}$ | coefficient of static active lateral earth pressure | - |
| :---: | :---: | :---: |
| $\mathrm{K}_{\mathrm{AE}}$ | coefficient of total active lateral earth pressure | - |
| $\mathrm{K}_{\mathrm{PE}}$ | coefficient of total passive lateral earth pressure | - |
| L | typical length dimension | L |
| $L_{m} L_{p}$ | length scale of model, prototype | L |
| $L_{\text {L }}$ | set of dimensionless length ratios | - |
| M | typical mass dimension | M |
| M | moment | FLL ${ }^{-1}$ |
| $M_{\text {A }}$ | active static moment | FLL ${ }^{-1}$ |
| $M_{\text {AE }}$ | active total (static + dynamic) moment | FLL ${ }^{-1}$ |
| $M_{\text {D }}$ | design moment | FLL ${ }^{-1}$ |
| $M_{0}$ | overturning moment | FLL ${ }^{-1}$ |
| $M_{R}$ | resisting moment | FLL ${ }^{-1}$ |
| MMI | Modified Mercalli Intensity | - |
| N | centrifuge gravitational acceleration scale factor | - |
| N | ratio of prototype to model length scales | - |
| P | pressure | $\mathrm{FL}^{-2}$ |
| P | externally applied load | F |
| $\mathrm{P}_{\mathrm{A}}$ | active static resultant wall force | $\mathrm{FL}^{-1}$ |
| $\mathrm{P}_{\mathrm{m}} \mathrm{P}_{\mathrm{p}}$ | externally applied load of model, prototype | F |
| $\mathrm{P}_{\text {AE }}$ | total (static + dynamic) active wall force | $\mathrm{FL}^{-1}$ |


| $\mathbf{P}_{\text {PE }}$ | total (static + dynamic) passive wall force | $\mathrm{KL}^{-1}$ |
| :---: | :---: | :---: |
| $\mathrm{P}_{\mathrm{R}}$ | set of dimensionless external load ratios | - |
| Q | shear force | FL ${ }^{-1}$ |
| Q | externally applied stress | FL ${ }^{-2}$ |
| $Q_{m} Q_{P}$ | externally applied stress of model, prototype | $E L^{-2}$ |
| ${ }^{\text {Q }}$ R | set of dimensionless externally applied stress ratios | - |
| $\mathrm{R}_{\mathrm{A}}$ | maximum static active pressure | $F L^{-2}$ |
| $\mathrm{R}_{\text {AE }}$ | maximum total (static + dynamic) active pressure | $F L^{-2}$ |
| RW1 | Retaining Wall \#1 - | - |
| RW2 | Retaining Fall \#2 | - |
| S | unit section modulus of cross section | $L^{3} L^{-1}$ |
| T | typical time dimension | T |
| T | time factor of consolidation | - |
| $T_{m} T_{p}$ | time factor of consolidation of model, prototype | - |
| W | weight of soil wedge behind wall | $\mathrm{FL}^{-1}$ |
| W | weight of backfill | $\mathrm{FL}^{-1}$ |

## GREEK STMBOLS

| $\beta$ | angle of wall back slope | 0 |
| :---: | :---: | :---: |
| $\gamma$ | unit weight of soil | $\mathrm{FL}^{-3}$ |
| $\delta$ | angle of wall-soil friction | 0 |
| $\theta$ | $\tan ^{-1}\left[k_{h} /\left(1-k_{v}\right)\right]$ |  |
| v | Poisson's ratio | - |
| $\nu_{m} \nu_{p}$ | Poisson's ratio of model, prototype | - |
| $\rho$ | mass density | $M_{L}^{-3}$ |
| $\rho^{\prime m}{ }^{\rho}{ }_{p}$ | mass density of model, prototype | $M^{-3}$ |
| $\sigma_{0}$ | internal stress | $\mathrm{FL}^{-2}$ |
| $\sigma_{o m}{ }^{\text {opp }}$ | internal stress of model, prototype | $\mathrm{FL}^{-2}$ |
| $\sigma_{\text {oR }}$ | set of dimensionless internal stress ratios | - |
| d | angle of internal friction of soil | 0 |
| $\Delta^{\text {P }}$ AE | active wall force increment due to earthquake load | $\mathrm{FL}^{-1}$ |

## APPENDIX D

FINITE ELEMENT COMPARISON


#### Abstract

For an analytical comparison, it was decided to perform a finite element analysis on the wall-soil system of test 1 CNOOO2 using the linearly elastic structural analysis program SAPIV (Bathe, et. al. [1]).

The finite element grid was first drawn up as shown in Figure D. 1 with the retaining wall (shown with speckles) embedded in the soil. Prototype dimensions were used (i.e., wall height was 18 ft ) and the boundaries were determined to be those existing in a postulated prototype centrifuge bucket (i.e., 50 times larger than their actaal size). The wall illustrated is much thicker than that which would be the prototype (1 ft thick vs. $3.15^{\prime \prime}$ thick if it were aluminum), but its Young's Modulus was chosen much less so that the stiffnesses EI would be the same. This was done in order to get a more suitable aspect ratio for the elements which form the wall and base. Incompatible modes were used in the wall and base quads in order to have better bending behavior in these elements, especially since the wall was modelled with only one layer of elements.

Unfortunately, the soil elements had to be attached to the beam (wall) elements as there was no provision in the code to have sliding between elements. This would have been more desirable.



FIGURE D.I - FINITE ELEMENT GRID (SCALE 7.8 FT. PER IN.)

The soil shear moduli were determined from the relationship given by Seed and Idriss [54] between the shear modulus and the confining pressure:

$$
\begin{equation*}
G=1000 K_{2}\left(\sigma_{m}^{\prime}\right)^{1 / 2} \tag{D,1}
\end{equation*}
$$

in which
$G=$ shear modulus of soil
$\sigma^{\prime}{ }_{m}=$ mean principal effective stress.
$K_{2}=$ a parameter which is primarily a function of void ratio and strain amplitude

Because of the high strain range involved in a retaining wall problem, $K_{2}$ was chosen from the extreme right of Figure D. 2 to be 4. The soil moduli were then calculated from equation (D.1) for the varions depths, making some adjustments for the soil in the vicinity of the toe of the wall for the fact that the soil level in front of the wall is lower than that in back.

First of all, the problem was run for a static gravity body load in the negative vertical direction. The problem was then run dynamically as a forced response problem using modal superposition and the free-field acceleration record (prototype) of test 1 CNO002 (Figure $5.5 a$ ) in the horizontal direction. The damping used was assumed $10 \%$ of critical. The total dynamic response was then obtained by superposition of the static response and the lateral dynamic one.


SHEAR MODULI OF SANDS AT DIFFERENT RELATIVE DENSITIES.
FIGURE D. 2 - FROM (54)

The first six natural frequencies of the finite element system were found to be $1.188 \mathrm{~Hz}, 1.388 \mathrm{~Hz}, 1.45 \mathrm{~Hz}, 1.987 \mathrm{~Hz}, 2.449 \mathrm{~Hz}$, and 2.536 Hz . Only the 6 th frequency of 2.536 Hz even resembled the actual fundamental frequency of 2.57 Hz and its mode shape is most likely very different.

Figures D.3, D.4, and D.5 illustrate the static and maximum dynamic displacement, pressure, and moment distributions along the wall for both the centrifuge model test and the finite element problem. As can be seen from these figares there is virtually no correlation between the two in any of the cases.

From this illustration one can see the perils in using elastic theories (which are the basis for the finite element program used) in trying to model the retaining wall problem which after all is the classic most simple plasticity example. Elastic solutions for retaining wall problems should be avoided.





[^0]:    Maximum Momont alwaya at bottom of wall unlos: othorwiac spoolfiod.
    Maximun Shear Forice always at bottom of wall unloss othorwiso apocified.
    Maximum Displacozent alway at top of wall.
    *. Oocur: at $x / H=0.625$
    **. Ooours at x/H $=0.431$

