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APPROXIMATIONS OF HEAD AND FLOW DISTRIBUTIONS IN LIQUID FILLED PIPING NETWORKS SUBJECT TO SEISMIC EXCITATION USING A STEADY OSCILLATORY FLOW MODEL

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ABSTRACT

The purpose of this study is to develop an economical method of analysis of piping networks subject to a seismic disturbance. A one-dimensional steady oscillatory method was employed and a powerful tool (a computer program for analyzing piping networks subject to steady oscillatory excitations) is developed for piping designers who wish to design pipelines for earthquake zones.

In addition, a model is developed to simulate the geometrical excitation effects of the following piping network junctions: 1) dead-end, 2) 90° elbow, 3) tee, 4) orifice. This model was verified for the dead-end, elbow, and tee connections by comparison with a method of characteristics model. This method of characteristics model as developed by Padron [6], was in turn verified by experimental data obtained by Wood and Chao [8], and energy analysis at resonance.

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CHAPTER I

INTRODUCTION

Earthquakes

An earthquake is a phenomenon of strong vibrations occurring on the ground due to the release of a large amount of strain energy through a sudden slippage in the earth's crust or in the upper part of the mantle [1]*. The span of energy released from the surface of the earth during a major earthquake is of the magnitude of the electricity consumed in the United States over a period of 4 hours to 40 years. Two types of seismic waves propagate from the earthquake, namely: 1) longitudinal compression or P-wave; 2) transverse shear or S-wave. Earthquakes have caused severe effects on human life as well as on structures such as buildings, roads, bridges, railways, dams, pipelines, etc. Over seven million people have lost their lives in earthquakes [2]. Millions of dollars are required every year to repair the damage caused by earthquakes. The study of the causes and methods of preventing this damage involves a wide range of knowledge such as geophysics, geology, seismology, vibration theory, structural dynamics, material dynamics, construction techniques and fluid mechanics. In the study of earthquakes, each of these areas has received considerable attention with the exception of fluid mechanics.

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*Numbers in brackets refer to the references.

Nakagawa [3] as reported by Okamoto [1], estimated the transient overpressures in pipelines subject to a seismic excitation. Young and Hunter [4] used a more rigorous method of analysis and found overpressures of about ten times higher than those estimated by Nakagawa. This result indicated the possibility of pipeline damage due to hydraulic transients induced by earthquakes and therefore prompted further study of the phenomenon. Young [5] employed a one-dimensional method of characteristics and developed a method including a computer program to analyze the piping networks subject to steady oscillatory excitations. Padron [6] modified this program to include the geometrical consideration necessary for the study of hydraulic transients induced in piping networks during earthquakes. He established a geometrical system to define the direction of propagation of the seismic disturbance as related to the orientation of the axes of each pipe segment in the system. He verified his results by comparing them to an available experimental data and energy analysis.

Statement of the Problem

The method employed by Padron [6], consumes a tremendous amount of computation time. The transient response must be calculated before the steady state response can be calculated, and most of the calculations are for the transient state. Often the maximum response is the steady state response and hence a piping designer does not need the transient response; nevertheless, he must pay for them. It would therefore be useful to piping designers if a method were developed to analyze the piping networks only at their steady state response induced during an earthquake. This work sets forth such a method.

Possible Methods of Approach and Method Selected

Streeter and Wylie [7] described a number of methods of analysis of unsteady flow depending upon the restrictive assumptions and also presented an excellent comparison of these methods. These methods all are initiated with the continuity and momentum equations of fluid mechanics and are categorized as arithmetic, graphical, characteristics, algebraic, impedance, and special methods. Young and Hunter [4] applied the impedance (steady oscillatory) method to some simple pipelines (not general piping networks) and found good agreement between their results and the results obtained by the method of characteristics. Their agreement was better for resonant conditions which exhibited the maximum responses. The impedance (steady oscillatory) method was chosen in this study to develop a tool (a method of analyzing piping networks subject to seismic excitation) for piping designers because of the following considerations: 1) piping designers are usually interested in maximum parameter values, 2) a good approximation of maximum response in piping network appears to be a possibility utilizing the impedance method, and 3) since economy is an important consideration. The impedance (steady oscillatory) method is described in detail by Streeter and Wylie [7] and in Chapter II of this work.

Objective

The objective of this thesis is to develop a method of analysis including an efficient computer program for calculation of the head and flow amplitudes in a general piping network subjected to steady oscillatory excitations as an approximation of seismic disturbances. The boundary conditions are chosen to approximate those in a piping system subject to a seismic excitation.

CHAPTER II

STEADY OSCILLATORY FLOW

In this chapter conservation of mass and conservation of momentum are applied to a slightly deformable horizontal pipe to analyze a class of steady oscillatory flow problems. The method of derivation is similar to that used by Streeter and Wylie [7], and it is shown here so that the resulting equations can be used in later chapters.

Conservation of Mass

The continuity equation for the control volume of the pipe shown in Figure 2.1 is written as

$$Q\rho - [Q\rho + \frac{\partial(Q\rho)}{\partial x}\,\delta x] = \frac{\partial(A\rho\delta x)}{\partial t}$$

or

$$\frac{\partial(Q\rho)}{\partial x}\,\delta x + \frac{\partial(A\rho\delta x)}{\partial t} = 0 \qquad (2\text{-}1)$$

Referring to Appendix A, equation 2-1 is condensed to the following form which is applicable to a slightly deformable horizontal pipe.

$$\frac{\partial q'}{\partial x} + \frac{gA}{a^2}\frac{\partial h'}{\partial t} = 0 \qquad (2\text{-}2)$$

Conservation of Momentum

The momentum equation for the slightly deformable horizontal pipe shown in Figure 2.2 is written as

$$PA - [PA + \frac{\partial(PA)}{\partial x}\,\delta x] - \tau_o \pi D \delta x = \rho\delta x(A + \frac{\partial A}{\partial x}\frac{\delta x}{2})\frac{dV}{dt}$$

or

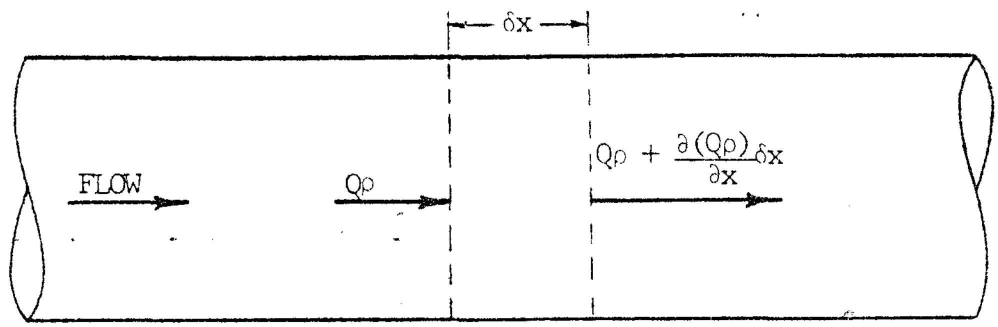

Figure 2.1. Notation for Continuity Equation.

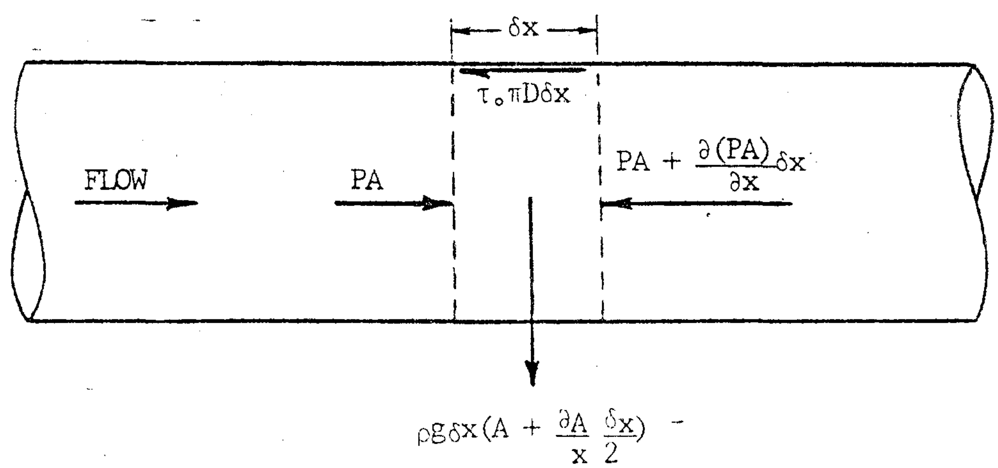

Figure 2.2. Notation for Momentum Equation.

$$-\frac{\partial(PA)}{\partial x}\delta x - \tau_\circ \pi D\delta x = \rho\delta x(A + \frac{\partial A}{\partial x}\frac{\delta x}{2})\frac{dV}{dt} \qquad (2\text{-}3)$$

Referring to Appendix B, equation 2-3 is condensed to the following form.

$$\frac{\partial h'}{\partial x} + \frac{1}{gA}\frac{\partial q'}{\partial t} + Rq' = 0 \qquad (2\text{-}4)$$

Equations 2-2 and 2-4 are used in Appendix C to obtain the steady oscillatory head and flow for a slightly deformable horizontal pipe subjected to steady oscillatory excitation. The following results are obtained for Figure 2.3 in Appendix C after Streeter and Wylie [7] and used in later chapters.

$$Q(x) = -\frac{H_R}{Zc}\text{Sinh}(\gamma x) + Q_R\text{Cosh}(\gamma x) \qquad (2\text{-}5)$$

$$H(x) = H_R\text{Cosh}(\gamma x) - Q_R Zc\text{Sinh}(\gamma x) \qquad (2\text{-}6)$$

$$Q_R = \frac{H_S}{Zc}\text{Sinh}(\gamma L) + Q_S\text{Cosh}(\gamma L) \qquad (2\text{-}7)$$

$$H_R = H_S\text{Cosh}(\gamma L) + ZcQ_S\text{Sinh}(\gamma L) \qquad (2\text{-}8)$$

$$Q_S = -\frac{H_R}{Zc}\text{Sinh}(\gamma L) + Q_R\text{Cosh}(\gamma L) \qquad (2\text{-}9)$$

$$H_S = H_R\text{Cosh}(\gamma L) - Q_R Zc\text{Sinh}(\gamma L) \qquad (2\text{-}10)$$

where

$$Zc = \frac{a^2}{gA\omega}(\beta - i\alpha) \qquad (2\text{-}11)$$

$$\gamma = \alpha + i\beta \qquad (2\text{-}12)$$

$$\alpha = \sqrt{\frac{gA\omega}{a^2}}\,[(\frac{\omega}{gA})^2 + R^2]^{\frac{1}{4}}\text{Sin}(\frac{1}{2}\text{Arctan}\frac{RgA}{\omega}) \qquad (2\text{-}13)$$

$$\beta = \sqrt{\frac{gA\omega}{a^2}}\,[(\frac{\omega}{gA})^2 + R^2]^{\frac{1}{4}}\text{Cos}(\frac{1}{2}\text{Arctan}\frac{RgA}{\omega}) \qquad (2\text{-}14)$$

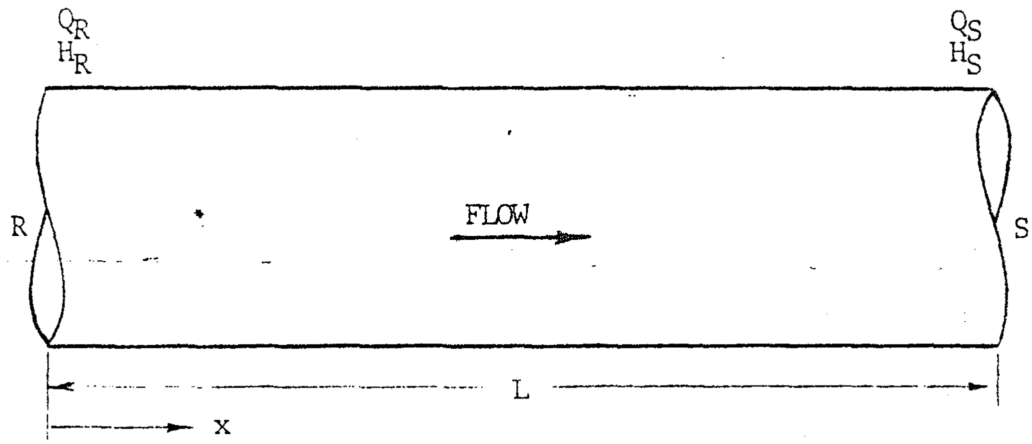

Figure 2.3. Simple Pipeline Showing Receiving and Sending Ends Relative to Flow Direction.

CHAPTER III

FORMULATION

In this chapter the derivation of the governing equations for determining head and flow amplitudes for steady oscillatory flow in a piping network is presented.

Governing Equations

Equations derived in Chapter II are used in this chapter to analyze a network of piping subjected to steady oscillatory excitations.

Flow Direction

The equations derived in the preceding chapter depend on a defined flow direction in a particular pipe. Since this work is not intended for steady flow, but for steady oscillatory flow, the equations will be developed to be independent of the flow direction which will allow the program to deal with a complex piping network without going through a particular system to arbitrarily define the flow direction. However, when flow values are determined, a system must be used to appropriately convey the meaning of the sign of the flow value. Solving equation 2-8 for Q_S, gives:

$$Q_S = \left[\frac{1}{ZcSinh(\gamma L)}\right] H_R + \left[\frac{-Cosh\ (\gamma L)}{ZcSinh(\gamma L)}\right] H_S$$

Head coefficients are defined as

$$X_{ij} = X_{ji} = \frac{1}{Zc_{ij}\mathrm{Sinh}(\gamma_{ij}L_{ij})} \tag{3-1}$$

$$Y_{ij} = Y_{ji} = \frac{-\mathrm{Cosh}(\gamma_{ij}L_{ij})}{Zc_{ij}\mathrm{Sinh}(\gamma_{ij}L_{ij})} \tag{3-2}$$

$$= -X_{ij}\mathrm{Cosh}(\gamma_{ij}L_{ij})$$

where Zc_{ij} is the characteristic impedance of the pipe between nodes i and j, γ_{ij} is the complex constant for the same pipe given by equation 2-12 and L_{ij} is the length of this pipe. Equation 2-8 can then be written as follows.

$$Q_S = X_{ij}\ H_R + Y_{ij}\ H_S \tag{3-3}$$

Rearranging equation 2-10 and solving for Q_R, gives:

$$Q_R = [\frac{-1}{Zc\mathrm{Sinh}(\gamma L)}]\ H_S - [\frac{-\mathrm{Cosh}(\gamma L)}{Zc\mathrm{Sinh}(\gamma L)}]\ H_R$$

Substituting the head coefficients given above, the following equation is obtained.

$$Q_R = -\ X_{ij}\ H_S - Y_{ij}\ H_R \tag{3-4}$$

Assuming oscillatory flow through a segment of pipe shown in Figure 3.1 and applying equations 3-3 and 3-4, the following equation may be written:

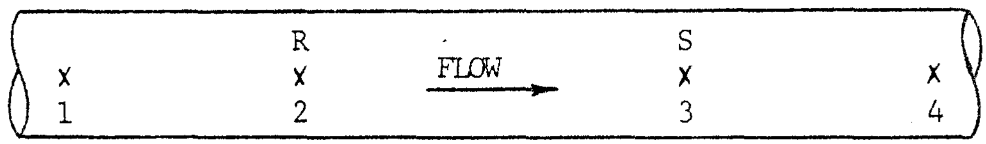

Figure 3.1. Simple Pipeline Showing Receiving and Sending Ends.

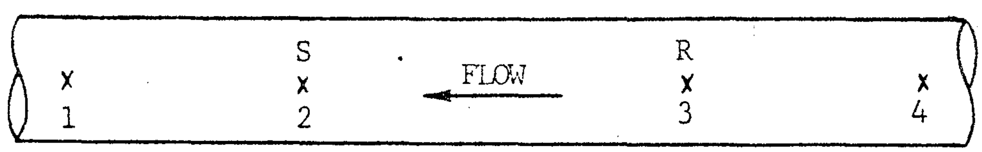

Figure 3.2. Simple Pipeline of Figure 3.1 with Opposite Flow Direction.

$$Q_S = X_{23}\ H_2 + Y_{23}\ H_3$$

$$Q_R = -\ (X_{23}\ H_3 + Y_{23}\ H_2)$$

The following system of nomenclature will be used:

$$Q_S = Q_{3\rightarrow 2}\ @\ 3 = -\ Q_{3\rightarrow 4}\ @\ 3 = -\ Q_{32}\ @_3 = Q_{34}\ @_3$$

$$Q_R = Q_{2\rightarrow 3}\ @\ 2 = Q_{23}\ @_2 = -\ Q_{21}\ @_2$$

Employing these definitions in the above equations, the following results:

$$Q_{32}\ @_3 = -\ (X_{23}\ H_2 + Y_{23}\ H_3) \qquad (3\text{-}3')$$

$$Q_{23}\ @_2 = -\ (X_{23}\ H_3 + Y_{23}\ H_2) \qquad (3\text{-}4')$$

which give the steady oscillatory flow at nodes 2 and 3.

Changing the direction of flow in the same line segment as shown in Figure 3.2 and applying the same equations 3-3 and 3-4, the following may be written:

$$Q_S = X_{23}\ H_3 + Y_{23}\ H_2$$

$$Q_R = -\ (X_{23}\ H_2 + Y_{23}\ H_3)$$

Using the same system of nomenclature as:

$$Q_S = Q_{2\rightarrow 1}\ @\ 2 = -\ Q_{2\rightarrow 3}\ @\ 2 = Q_{21}\ @_2 = -\ Q_{23}\ @_2$$

$$Q_R = Q_{3\rightarrow 2}\ @\ 3 = Q_{32}\ @_3 = -\ Q_{34}\ @_3$$

and substituting into the above equations, the following is obtained:

$$Q_{23}\ @_2 = -\ (X_{23}\ H_3 + Y_{23}\ H_2) \qquad (3\text{-}3'')$$

$$Q_{32}\ @_3 = -\ (X_{23}\ H_2 + Y_{23}\ H_3) \qquad (3\text{-}4'')$$

An analysis of the above equations will show that equations 3-3' and 3-4" are identical, and equations 3-4' and 3-3" are identical. Therefore the following equation can be written for any pipe segment (i,j) independent of the flow direction:

$$Q_{ij} = Q_{ij} @_i = -(X_{ij} H_j + Y_{ij} H_i) \qquad (3\text{-}5)$$

Equations for the Network

In this section, equations will be developed to apply to a network of piping subjected to steady oscillatory flow excitation. Figure 3.3 shows a cross connection with a flow source, Q_i into the center of the cross. Applying conservation of mass to node i of this network

$$Q_{i1} + Q_{i2} + Q_{i3} + Q_{i4} + Q_i = 0 \ ,$$

and substituting equation 3-5 for each Q_{ij}, gives

$$-(X_{i1}H_1+Y_{i1}H_i)-(X_{i2}H_2+Y_{i2}H_i)-(X_{i3}H_3+Y_{i3}H_i)-(X_{i4}H_4+Y_{i4}H_i)+Q_i = 0$$

and by rearranging,

$$X_{i1}H_1+X_{i2}H_2+X_{i3}H_3+X_{i4}H_4+(Y_{i1}+Y_{i2}+Y_{i3}+Y_{i4})-Q_i = 0 \ .$$

In general form, a nodal equation may be written by deduction as

$$\left(\sum_{\substack{j=1 \\ j \neq i}}^{n} X_{ij}H_j\right) + \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij}\right) H_i - Q_i = 0 \qquad (3\text{-}6)$$

where n, is the number of nodes in a piping network.

Real Equations

The paramters in equation 3-6 are complex. In this section, separate equations will be developed for the real and imaginary components of equation 3-6. Expanding the first term of equation 3-6, the following is obtained:

$$X_{ij}H_j = [(X_R)_{ij}+i(X_I)_{ij}]\ [(H_R)_j+i(H_I)_j]$$

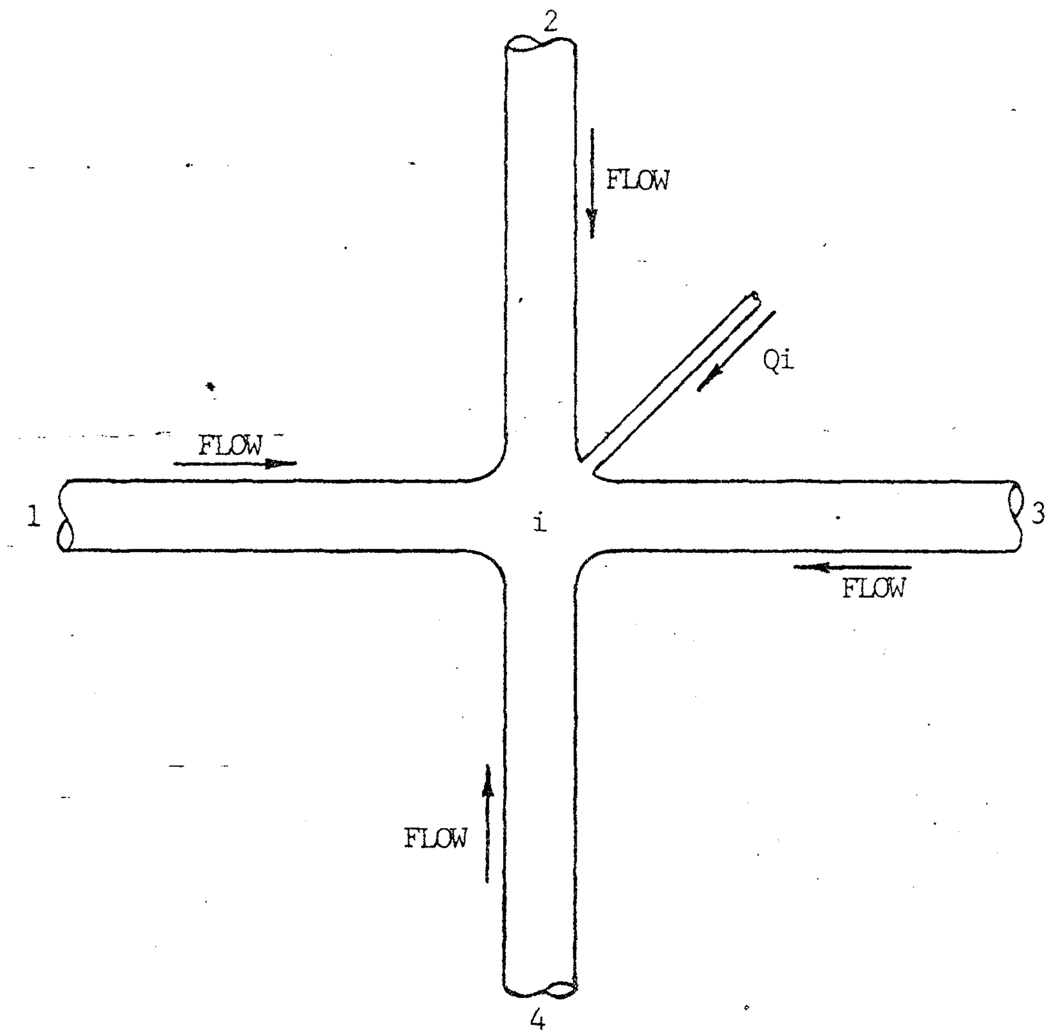

Figure 3.3. Cross Connection with a Flow Source into the Center.

where subscript R, is for the real part, subscript I, is for the imaginary part and the constant i, is $\sqrt{-1}$. The equation above is rearranged as follows:

$$X_{ij}H_j=[(X_R)_{ij}(H_R)_j-(X_I)_{ij}(H_I)_j]+i[(X_R)_{ij}(H_I)_j+(X_I)_{ij}(H_R)_j] \quad (3\text{-}7)$$

Similarly, an expression for the second term of equation 3-6 is

$$Y_{ij}H_i=[(Y_R)_{ij}(H_R)_i-(Y_I)_{ij}(H_I)_i]+i[(Y_R)_{ij}(H_I)_i+(Y_I)_{ij}(H_R)_i] \quad (3\text{-}8)$$

and Q_i can be expressed as

$$Q_i = (Q_R)_i + i(Q_i)_i \quad . \quad (3\text{-}9)$$

Substituting equations 3-7, 3-8 and 3-9 into equation 3-6 and equating the real part and the imaginary part to zero, the two following equations result.

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} [(X_R)_{ij}(H_R)_j-(X_I)_{ij}(H_I)_j + (Y_R)_{ij}(H_R)_i-(Y_I)_{ij}(H_I)_i]-(Q_R)_i = 0 \quad (3\text{-}10)$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} [(X_R)_{ij}(H_I)_j + (X_I)_{ij}(H_R)_j + (Y_R)_{ij}(H_I)_i + (Y_I)_{ij}(H_R)_i]-(Q_I)_i = 0 \quad (3\text{-}11)$$

Solution Matrix

In this section, equations 3-10 and 3-11 are applied to a piping network to find a general matrix representation. The desired flow and head functions can then be found by inverting the matrix. Appendix D shows the creation of this matrix and the following augmented matrix results by deduction for a general piping network subject to steady oscillatory excitation(s).

$$A_{2i,2i} = A_{2i-1,2i-1} = \sum_{j=1}^{n} (Y_R)_{ij} \tag{3-12}$$

$$A_{2i,2i-1} = -A_{2i-1,2i} = \sum_{j=1}^{n} (Y_I)_{ij} \tag{3-13}$$

$$A_{2i,2j} = A_{2i-1,2j-1} = (X_R)_{ij} \tag{3-14}$$

$$A_{2i-1,2j} = -A_{2i,2j-1} = (X_I)_{ij} \tag{3-15}$$

$$A_{2i-1,2n+1} = (Q_R)_i - \sum_{\substack{j=1 \\ j \neq b}}^{n} [(X_R)_{ij}(H_R)_j - (X_I)_{ij}(H_I)_j] \tag{3-16}$$

$$A_{2i,2n+1} = (Q_I)_i - \sum_{\substack{j=1 \\ j \neq b}}^{n} [(X_I)_{ij}(H_R)_j + (X_R)_{ij}(H_I)_j] \tag{3-17}$$

The following conditions are required for the above augmented matrix.

$i = 1,2,\ldots\ldots\ldots,n$

$i \neq m$

$j = 1,2,\ldots\ldots\ldots,n$

$j \neq i$

$j \neq k$

The following are definitions for limiting symbols used in the above augmented matrix:

k, a node number that is not connected to node i

b, a node number at which the head is not given

m, a node number at which the head is given

n, number of nodes in piping network

CHAPTER IV

COMPUTER CONFIGURATION AND PROGRAMMING

This chapter presents the computer program and the method used to obtain the linearized fluid friction term for each pipe for this work. A sample problem is presented at the end of the chapter in order to describe the input and output format.

Computer Program

The equations derived in the preceding chapters are employed to write an efficient computer program to calculate the steady oscillatory head and flow distribution in piping networks subjected to steady oscillatory excitation. The basic procedure of programming is described in Figure 4.1 and the complete listing is presented in Appendix E. The subroutine, "MTINV," was obtained from the master library, tested on several sets of simultaneous equations and after verification was employed in this program. As is shown in Figure 4.1, this program applies a trial solution which assumes a value for average oscillatory flow amplitues at the nodes and the assigned locations*. Using a method that will be described in the next section, the program finds

* See the sample problem at the end of this chapter for these locations.

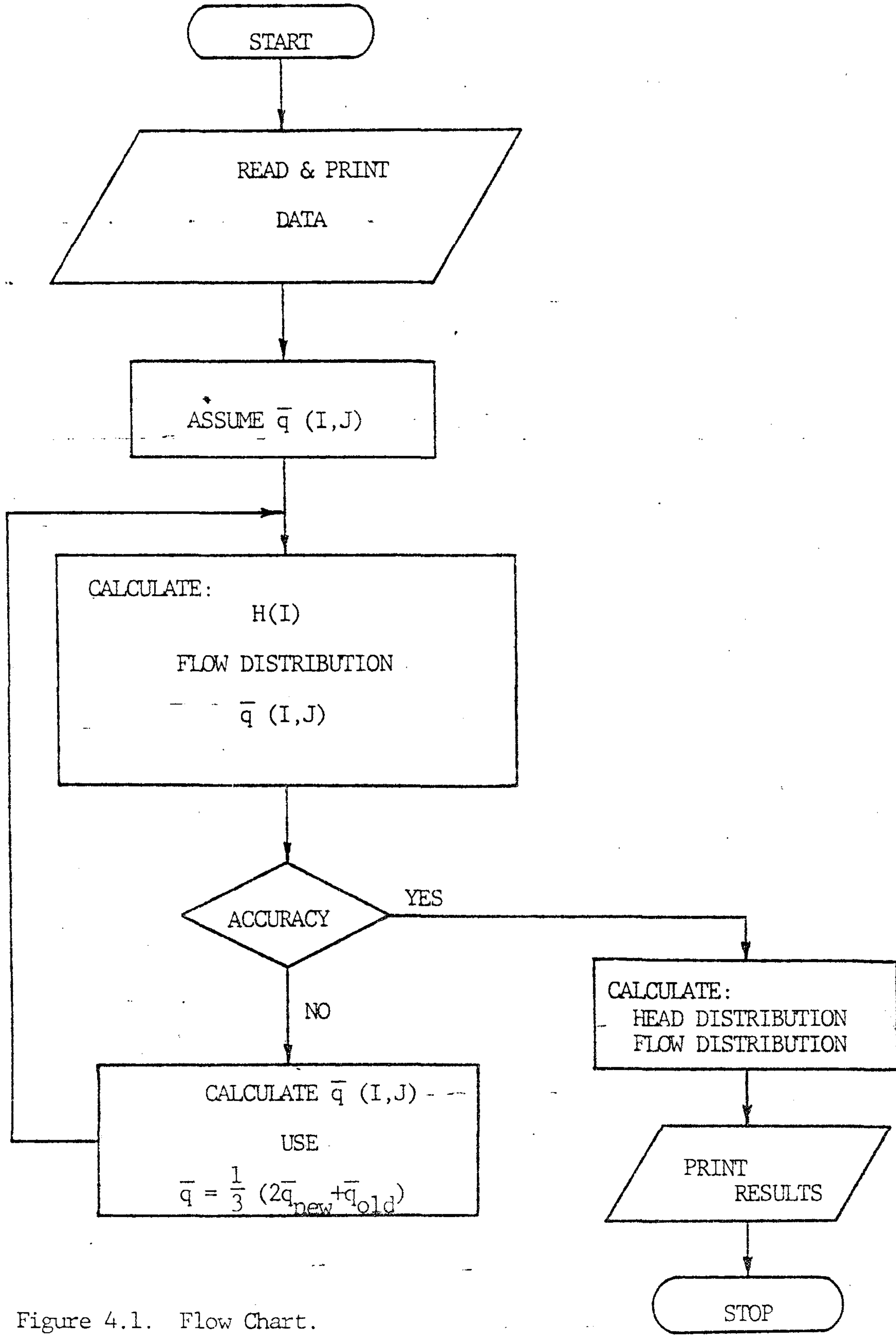

Figure 4.1. Flow Chart.

the average oscillatory flow through each line segment, compares these values with the preceding one, and uses an average oscillatory flow through each line segment as

$$q = \frac{1}{3}(2\bar{q}_{new} + \bar{q}_{old})$$

to calculate the linearized fluid friction term for the next iteration. With these values, computations are initiated for the determination of head and flow amplitudes at the nodes and at the assigned locations. If the difference between the new and old average flows are within the desired accuracy, the results are printed. Otherwise, the program calculates the flow amplitudes and the new average flow for each line segment for the next comparison. The limits of the accuracy assigned to this program are a maximum change in average flow in any pipe segment of 0.001 ft^3/s. and to an average change of 0.0005 ft^3/s. for all pipe segments in the network.

Average Steady Oscillatory Flow and Correction Factor

Average Steady Oscillatory Flow

In this section, a method is described to determine the average steady oscillatory flow, '$\bar{Q}$', that appears in equation B-8, the linearized fluid friction term. This method is employed in the computer program to obtain the results of this work; however, an alternate method is developed and is described in Appendix F. Streeter and Wylie [7] neglected the effect of oscillatory flow in their linearized fluid friction term. They probably assumed that the oscillatory flow is very small in comparison to the steady state flow. Since this program may deal with the systems or parts of systems with low steady state flow, the average flow through each pipe segment is defined as

$$\overline{Q} = \overline{Q}_{steady} + \overline{q}_{osc}. \qquad (4\text{-}1)$$

where the steady state component must be defined as an input condition. The oscillatory component is assumed to be varying linearly through the pipe and is defined as

$$\overline{q} = \frac{1}{2} (q_{max} + q_{min}) \frac{2\omega}{\pi} \int_0^{\frac{\pi}{2\omega}} \text{Sin}(\omega t) \; d(\omega t)$$

where q_{max} and q_{min} are the maximum and minimum flow amplitudes along the length of the pipe segment, respectively. After simplifying the average oscillatory flow is given by

$$\overline{q} = \frac{1}{\pi} (q_{max} + q_{min})$$

and equation 4-1, becomes

$$\overline{Q} = \overline{Q}_{steady} + \frac{1}{\pi} (q_{max} + q_{min}). \qquad (4\text{-}2)$$

Since the flow does not vary linearly along the length of the pipe, equation 4-2 is a rough estimate, unless each line is divided into enough sections and the averaging process is applied for each section separately. To find the number of sections into which each straight pipe must be divided, the following tests were performed on a straight pipe, 5000 feet long, 30 inches in diameter, having a friction factor of 0.1, connected to a constant pressure tank at one end and a steady oscillatory flow excitation of 4.909 ft^3/s. amplitude at the other end. The speed of sound in the liquid is assumed to be 3000 ft/s. For the non-dimensional excitation frequencies* of 1.0 to 3.0 with intervals of 0.1, the length of the pipe is divided into 1,2, 3,.....,10,19 sections and the system is defined as 2,3,4,.....,11,20-node piping network. Table-4.1 shows the resulting non-dimensional pressure* for these tests. An inspection of Table 4.1, shows that, for the frequency range of 1.3 to 2.7,, results are independent of the number of sections into which the pipe is divided. Beyond this range, the results are functions of the number of segments into which the pipe is divided as the frequency approaches the resonant frequency. At resonance the results are highly dependent of the number of segments. This is because at resonance the energy input into the system is dissipated by friction only. At resonance the result of a 19-section line differs by a maximum of 1.2% from the result of a 10-section line. The result of a 5-section line, however, differs by a maximum of 5.8% from the result of a 19-section line. For the purpose of this study and since computation time must be considered, each straight pipe is divided into five or ten sections. This should be a good approximation of the average of the steady oscillatory flow for the first few frequency harmonics.

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*Non-dimensional frequency and non-dimensional pressure are defined on page 24 of Chapter V.

Frequency	NUMBER OF SECTIONS INTO WHICH THE PIPE IS DIVIDED										
	1	2	3	4	5	6	7	8	9	10	19
1.0	9.318	7.871	7.670	7.600	7.567	7.549	7.538	7.530	7.525	7.522	7.508
1.1	5.871	5.549	5.489	5.465	5.454	5.448	5.444	5.441	5.439	5.438	5.433
1.2	3.215	3.195	3.191	3.186	3.188	3.187	3.186	3.186	3.186	3.186	3.185
1.3	2.200	2.197	2.197	2.196	2.196	2.196	2.196	2.196	2.196	2.196	2.196
1.4	1.701	1.700	1.700	1.700	1.700	1.700	1.700	1.700	1.700	1.700	1.700
1.5	1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.414
1.6	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236
1.7	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122
1.8	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051
1.9	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012
2.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.1	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.012
2.2	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051
2.3	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122
2.4	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236
2.5	1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.413	1.414	1.414	1.414
2.6	1.701	1.700	1.700	1.700	1.700	1.700	1.699	1.699	1.700	1.700	1.700
2.7	2.200	2.198	2.199	2.197	2.197	2.196	2.196	2.196	2.196	2.196	2.196
2.8	3.215	3.309	3.211	3.199	3.194	3.192	3.189	3.190	3.187	3.187	3.185
2.9	5.920	5.860	5.883	5.641	5.561	5.542	5.512	5.491	5.483	5.474	5.446
3.0	9.581	9.392	9.480	8.180	7.948	7.855	7.728	7.671	7.641	7.602	7.509

Table 4.1. Non-Dimensional Head Response as a Function of Non-Dimensional Frequency for a Pipe Segment Divided into Different Numbers of Sections.

Correction Factor

Since the purpose of this work is to compare the accuracy of the method of characteristics with the steady oscillatory method, a more computationally economical method, the system that is defined in the preceding sub-section, is tested against the method of characteristics at the first resonant frequency. The maximum non-dimensional head obtained by the method of characteristics was 9.64. To obtain this value by the steady oscillatory method, a correction factor of 0.605 must be applied to the average steady oscillatory flow. Then the corrected form of equation 4-2 becomes

$$\overline{Q} = \overline{Q}_{steady} + 0.1926(q_{max} + q_{min}) . \qquad (4\text{-}3)$$

This equation is used in the computer program.

Sample Problem

The length of the pipe of the system defined in the preceding section is divided into eight sections to form a 9-node piping network. This network is excited with a steady oscillatory flow of 4.909 ft^3/s. amplitude and a frequency equal to the resonant frequency of the pipe. Input and output formats are shown and described in Appendix G.

CHAPTER V

RESULTS AND CONCLUSIONS

The computer program described in the preceding chapter will be used to analyze the same systems used by Padron [6] in investigating piping networks subjected to a seismic excitation. Results obtained by the method of characteristics for corresponding boundary conditions will be compared with the steady oscillatory method presented here.

Cases of Study and Results

A piping network can be described as an orderly combination of nodes such as dead-ends, elbows, tees, crosses, valves, reducers, orifices, etc. interconnected by line segments. The local effects of these connections on steady flow through piping networks are limited to frictional effects, while in steady oscillatory flow through the piping networks, the geometric effects may be much more important than the frictional effects and must be considered. Further study is required to model these geometric effects as steady oscillatory flow sources, steady oscillatory head sources, etc.

In this work, a simple piping network consisting of a constant head tank and connected by a pipe segment to a dead-end was selected for initial study. The system was excited by a compression seismic wave with a velocity amplitude of 1 ft/s. and various excitation frequencies. Liquid in the network was chosen to be water at 80^{o}F.

The piping network was assumed to be at one elevation. The velocity of wave propagation in the liquid was assumed to be 3000 ft/s. with no steady flow through the piping network. For each case, tests were performed and the results compared with the results obtained by the method of characteristics. Since the results are conveniently presented as non-dimensional parameters, a system of reducing the parameters is defined. Non-dimensional frequency is the ratio of excitation frequency to the resonance frequency in the liquid of the particular line segment involved, where the resonance frequency of the line segment is defined as the inverse of the time required for the completion of one cycle of wave propagation in the line. Non-dimensional head is the ratio of the actual head increase to the head rise which would occur if the liquid velocity were instantaneously changed by the amplitude of the excitation velocity.

Dead-End

A simple system consisting of a pipe 5000 feet long and 30 inches in diameter connected to a constant head tank at one end and a dead-end at the other end was selected for initial study. The system was excited at the dead-end by a longitudinal compression seismic wave with a velocity amplitude of 1 ft/s. and an angle of propagation, θ, with respect to the longitudinal axis of the pipe. Figure 5.1 shows the sketch for this piping network. The pipe was assumed to be buried in the ground and to have no slippage between the pipe wall and the ground. The dead-end connection was assumed to have the same

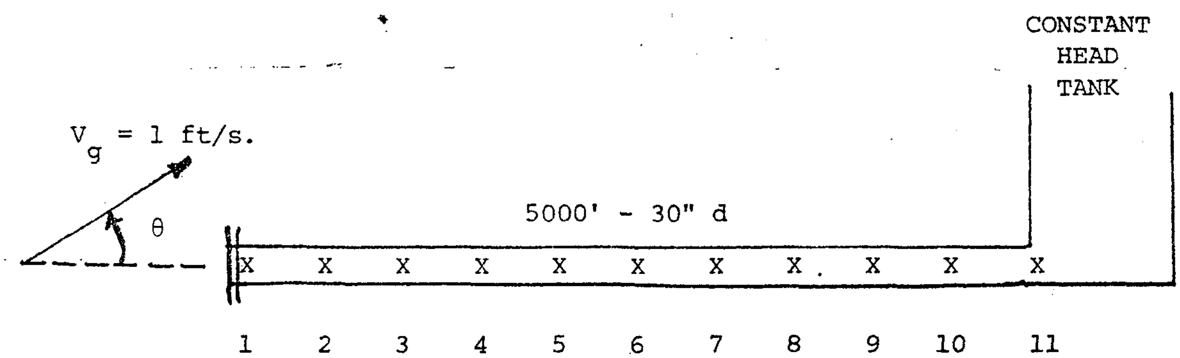


Figure 5.1 Schematic Diagram for a Dead-End Connection Showing the Direction of Seismic Excitation

velocity component as the ground motion parallel to the longitudinal axis of the pipe. Therefore, the motion of the liquid particles in contact with the dead-end was the same as the motion of the dead-end connection. As an approximation the transient seismic ground motion was replaced with a steady oscillatory flow through the cross-sectional area of the pipe at the dead-end with a velocity amplitude equal to the component of the ground motion velocity parallel to the longitudinal axis of the pipe as shown below.

$$Q = Vg\ A\text{Cos}(\theta)e^{i\omega t} \tag{5-1}$$

In order to obtain a good approximation of the effects of the linearized friction term, the 5000-foot length was divided into ten equal sections (as discussed in Chapter IV) and the system defined as an 11-node piping network. The amplitude of the steady oscillatory head at node 11 was zero due to the constant head tank, and at node 1, the steady oscillatory flow amplitude calculated by equation 5-1, was 4.909Cos(θ) ft/s. The resonance frequency of the system was $\frac{2\pi a}{4L}$ = .9425 rad/s. For these boundary conditions the following tests were performed: a) for a constant angle of wave propagation, $\theta = 0$, and pipe friction factors of, 0.02, 0.05, 0.10, 0.20, and non-dimensional frequencies of 0.5 to 3.5 with intervals of 0.1; b) for $\theta = 0$ to 90° in intervals of 15°, pipe friction factors of, 0.02, 0.05, 0.10, 0.20, and non-dimensional frequencies of, 0.5 to 3.0 with intervals of 0.5. The results for tests (a) are presented in Figures 5.2 to 5.5 using open circles while the results obtained by the method of

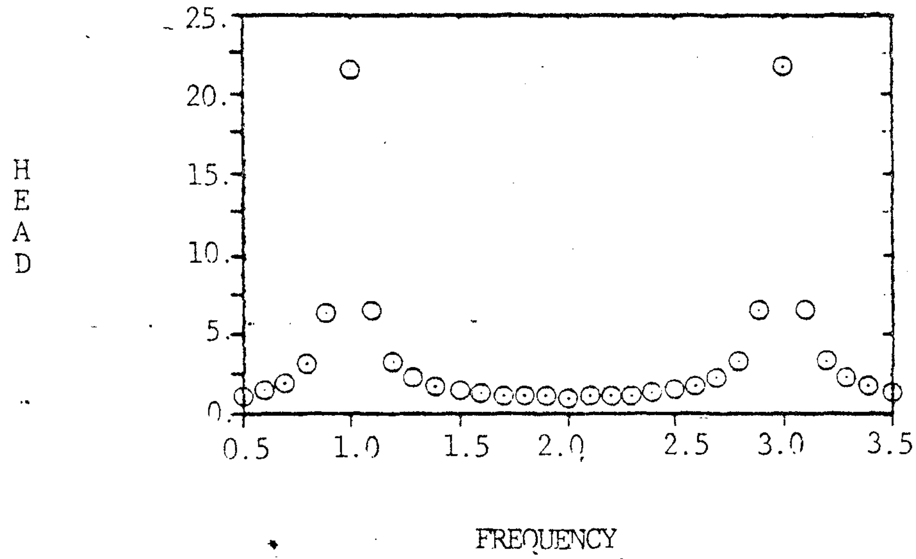


Figure 5.2. Non-Dimensional Head Response as a Function of Non-Dimensional Frequency for Tests (a) of the Dead-End Connection (Friction Factor = 0.02).

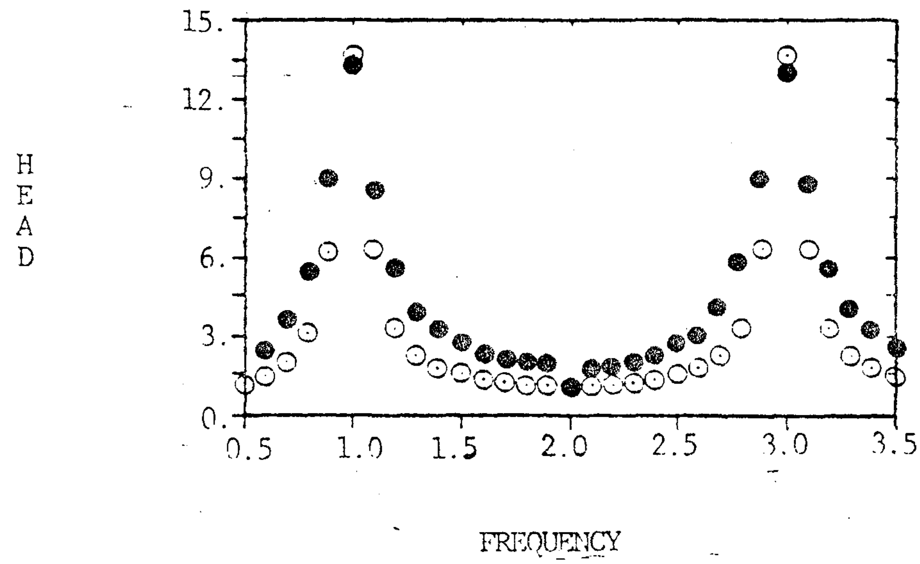


Figure 5.3. Non-Dimensional Head Response as a Function of Non-Dimensional Frequency for Tests (a) of the Dead-End Connection (Friction Factor = 0.05).

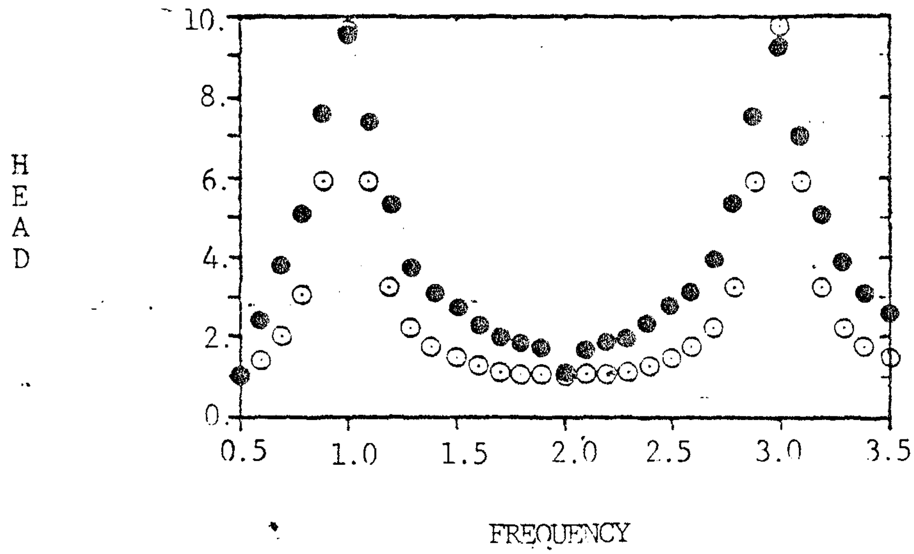


Figure 5.4. Non-Dimensional Head Response as a Function of Non-Dimensional Frequency for Tests (a) of the Dead-End Connection (Friction Factor = 0.10).

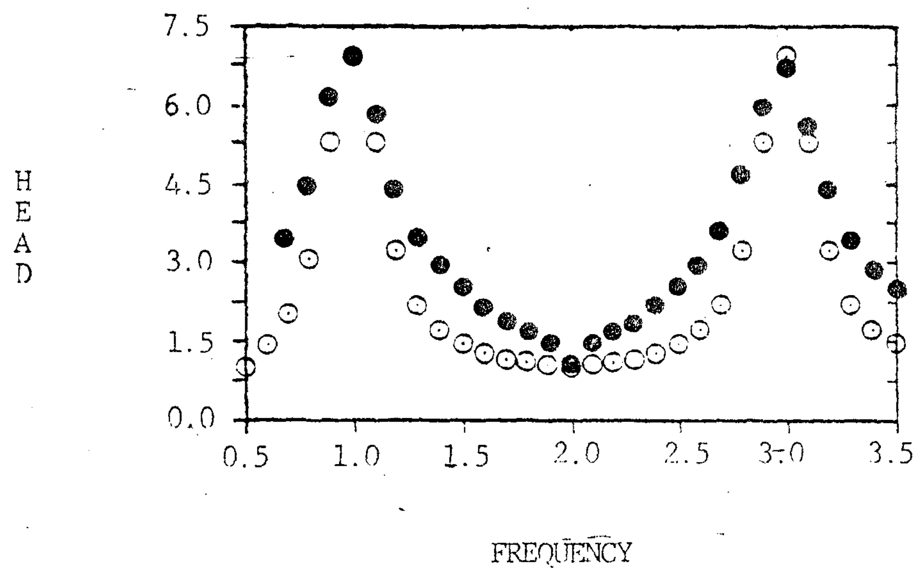
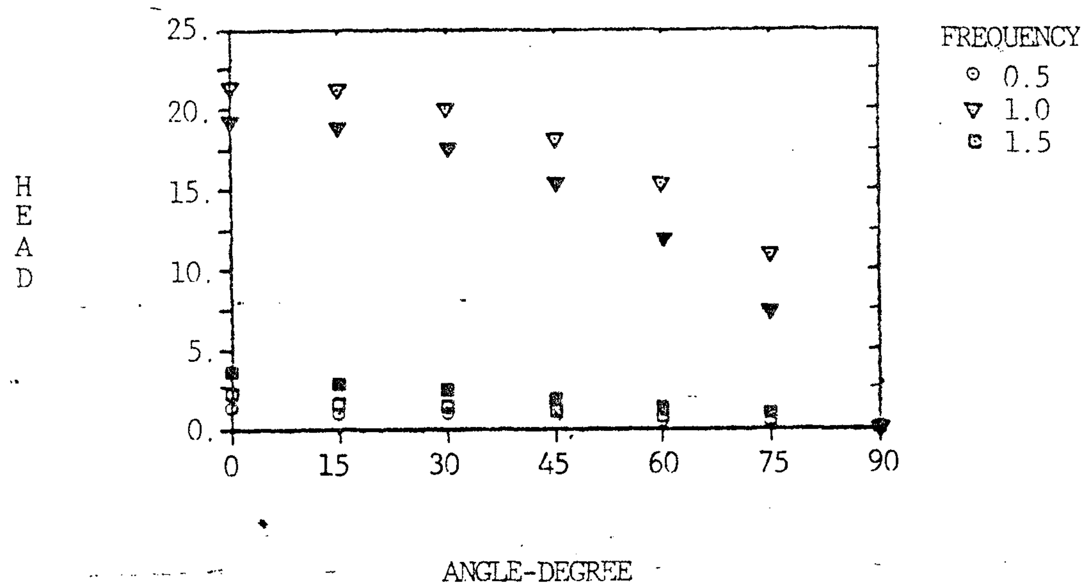


Figure 5.5. Non-Dimensional Head Response as a Function of Non-Dimensional Frequency for Tests (a) of the Dead-End Connection (Friction Factor = 0.20).

characteristics for the same tests are shown in the same figures using solid circles. Comparing the results, good agreement for the resonance and anti-resonance frequencies are exhibited while there is considerable differences at values between these two frequencies, and these differences are larger as the peaks get sharper, i.e. as the pipe friction factor gets smaller. This is expected since the method of characteristics gives the maximum overpressure in either steady oscillatory flow or for the transient case. The results for tests (b) are presented in Figures 5.6 to 5.9 employing open symbols, while solid symbols show the results obtained by the method of characteristics for the same boundary conditions. The results obtained by both methods for the frequencies of 0.5 and 2.0 are very similar and are shown with open circles. Comparing the results of tests (b), considerable differences can be observed between the results of the two methods for friction factors other than 0.1. That is because the method of calculating the linearized friction term contained a correction factor which was chosen to make the methods agree at a friction factor of 0.1 (as discussed in Chapter IV). To minimize these differences, the method described in Appendix F was developed. Also slight differences can be observed between the results of the two methods as the angle of wave propagation becomes larger.

Elbow Connection

The elbow connection was modeled by a 90° elbow connecting two



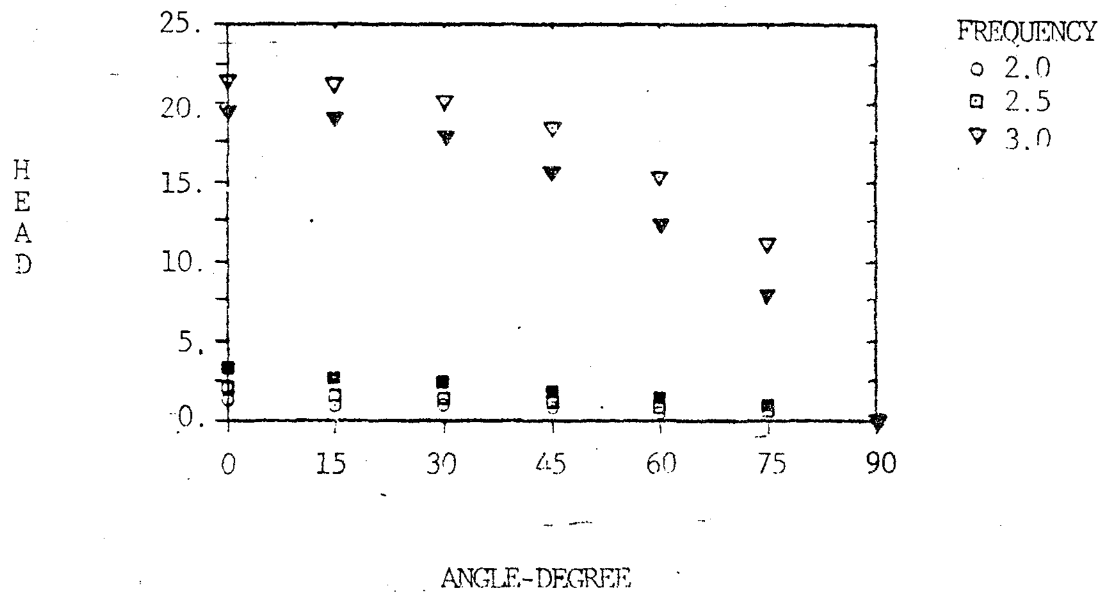
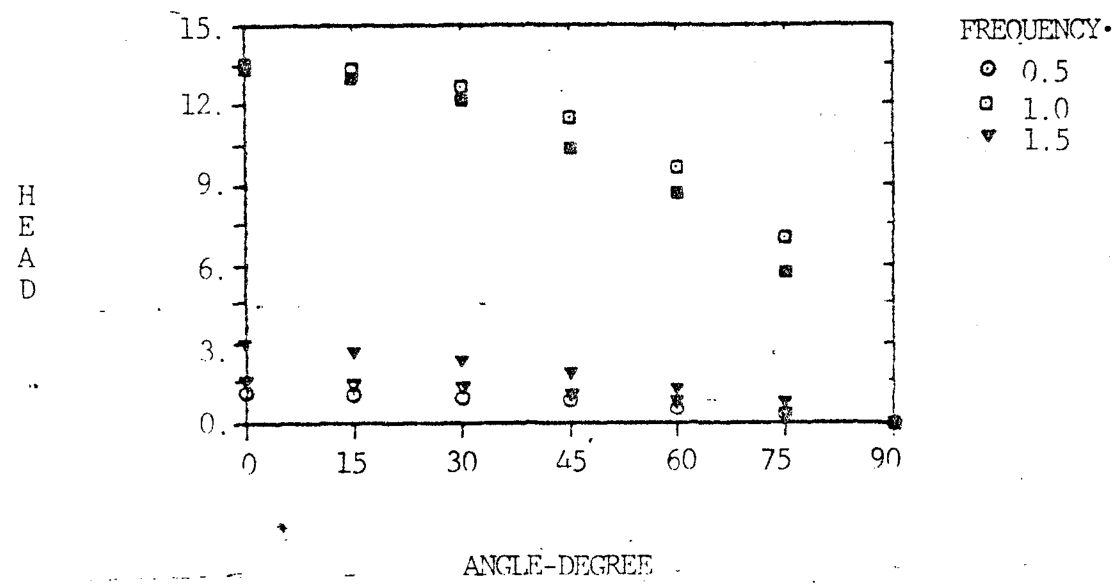


Figure 5.6. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for Tests (b) of the Dead-End Connection (Friction Factor = 0.02).



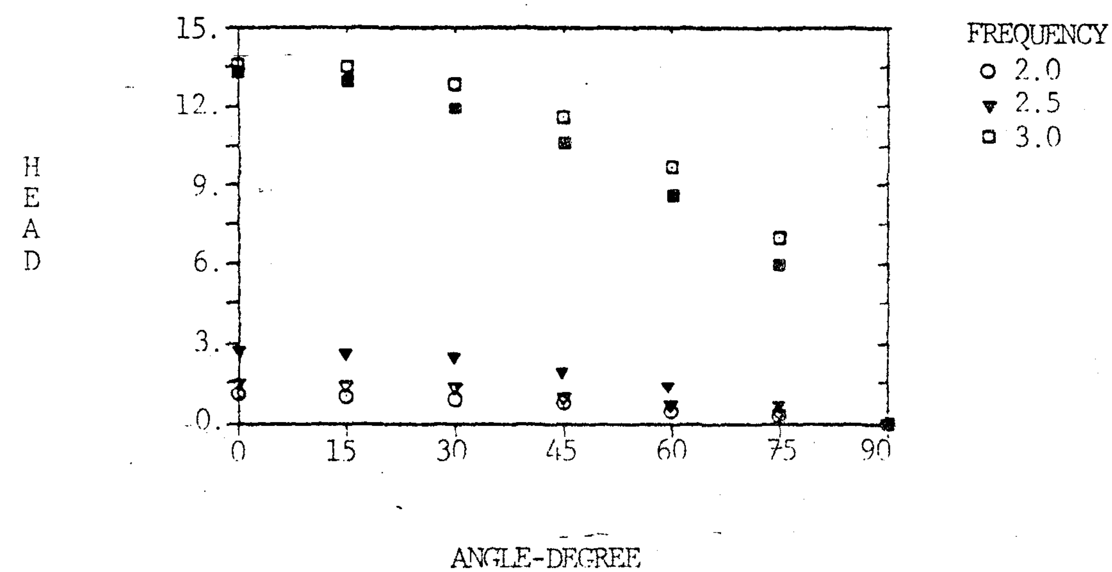
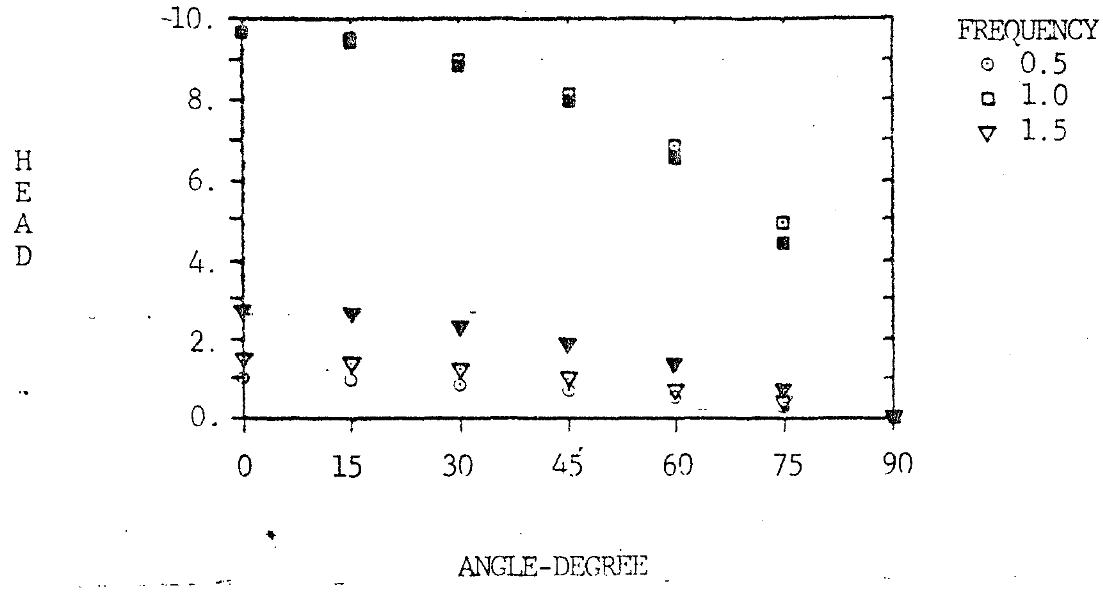


Figure 5.7. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for Tests (b) of the Dead-End Connection (Friction Factor = 0.05).



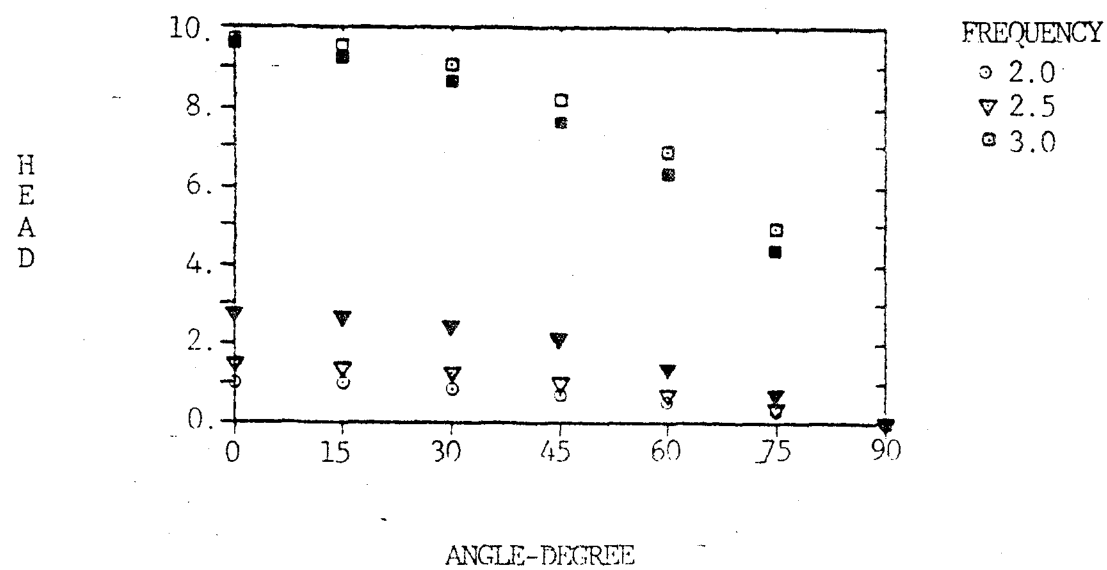
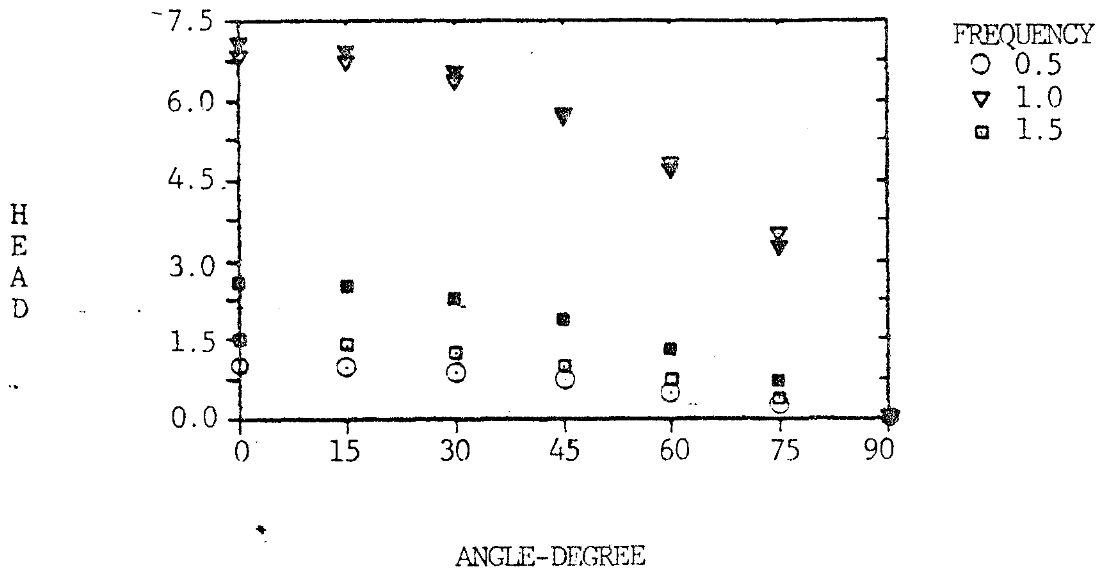


Figure 5.8. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for Tests (b) of the Dead-End Connection (Friction Factor = 0.10).



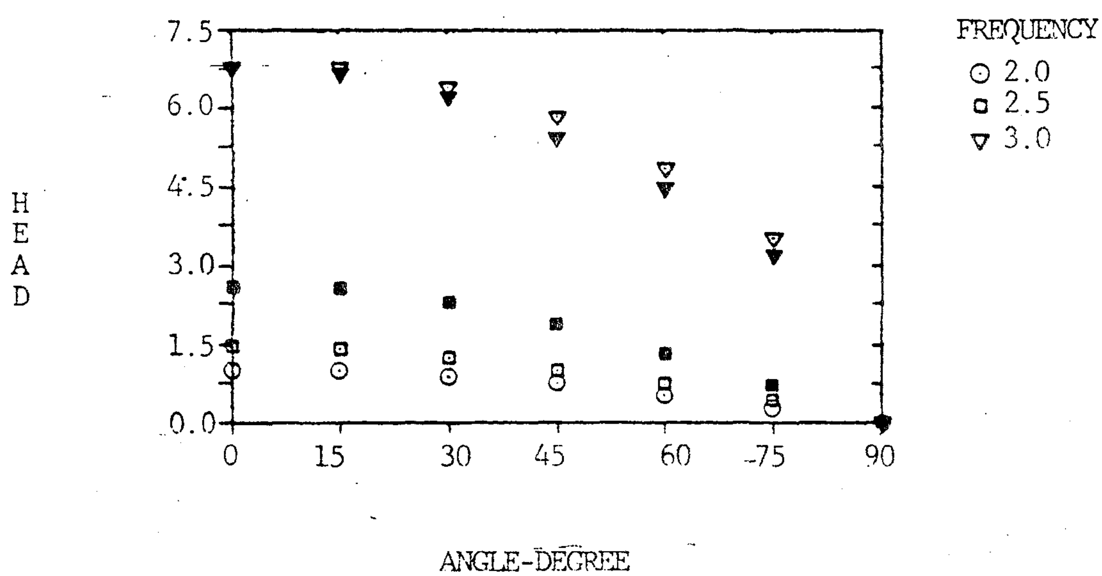


Figure 5.9. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for Tests (b) of the Dead-End Connection (Friction Factor = 0.20).

pipes which were terminated at the other ends by two constant head tanks. The length and the diameter of one pipe was kept fixed at 5000 feet and 30 inches, respectively. The length and the diameter of the second pipe was varied as shown by the four different piping networks in Figures 5.10 to 5.13. The elbow connection was assumed to have the same effect as the dead-end connection in each of the pipes except that flow could occur between the two pipes. With this assumption, equation 5-1 was modified for an elbow connection as follows:

$$Q = Vg\ [A_1\mathrm{Cos}(\theta)+A_2\mathrm{Sin}(\theta)]e^{i\omega t}\ . \qquad (5\text{-}2)$$

Each pipe length was divided into 5 equal sections to define an 11-node piping network. For a pipe friction factor of 0.1, four groups of tests were performed for the frequency ranges of 0.5 to 3.0, with intervals of 0.5, and angles of wave propagation ranging from 0^o to 180^o, with intervals of 15^o as follows: a) pipe (2) was chosen to be the same length and diameter as pipe (1); b) the length of pipe (2) was maintained at 5000 feet and its diameter was chosen to be 15 inches; c) pipe (2) was chosen to be 4000 feet long and 30 inches in diameter; d) the diameter of pipe (2) was maintained at 30 inches and its length changed to 2500 feet. Figures 5.14 to 5.17 show the results of tests (a) through tests (d), respectively using open symbols. Solid symbols show the results obtained by the method of characteristics. Open triangles in Figures 5.14 and 5.15 show the results obtained by both methods for frequencies of 0.5 and 2.0. The two methods exhibit the same characteristics as they did with the dead-end connection. That

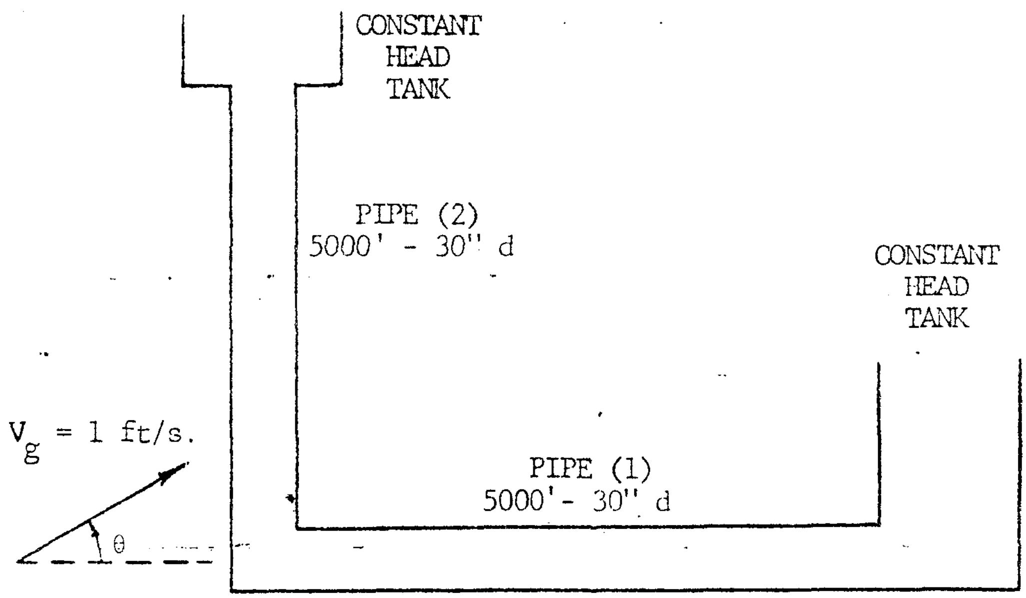


Figure 5.10. Schematic Diagram Showing the Piping Network used for Tests (a) of the Elbow Connection.

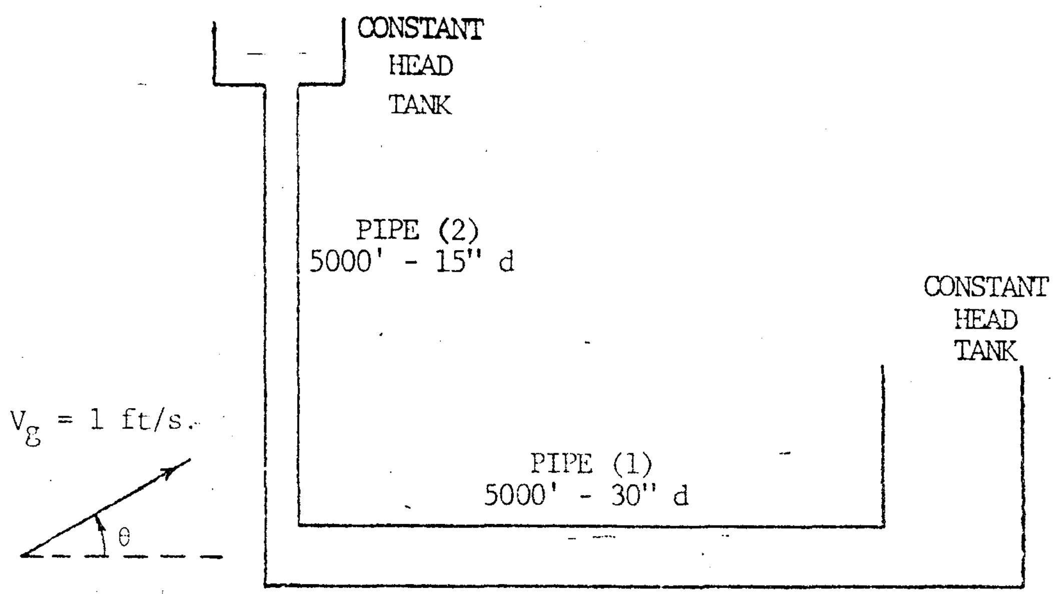


Figure 5.11. Schematic Diagram Showing the Piping Network used for Tests (b) of the Elbow Connection.

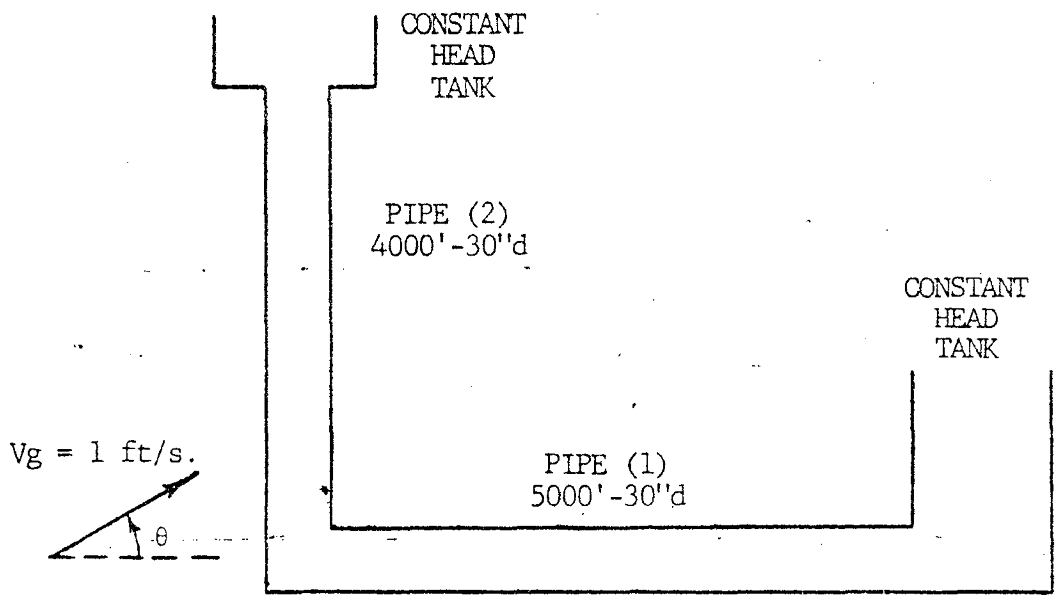


Figure 5.12. Schematic Diagram Showing the Piping Network Used for Tests (c) of the Elbow Connection.

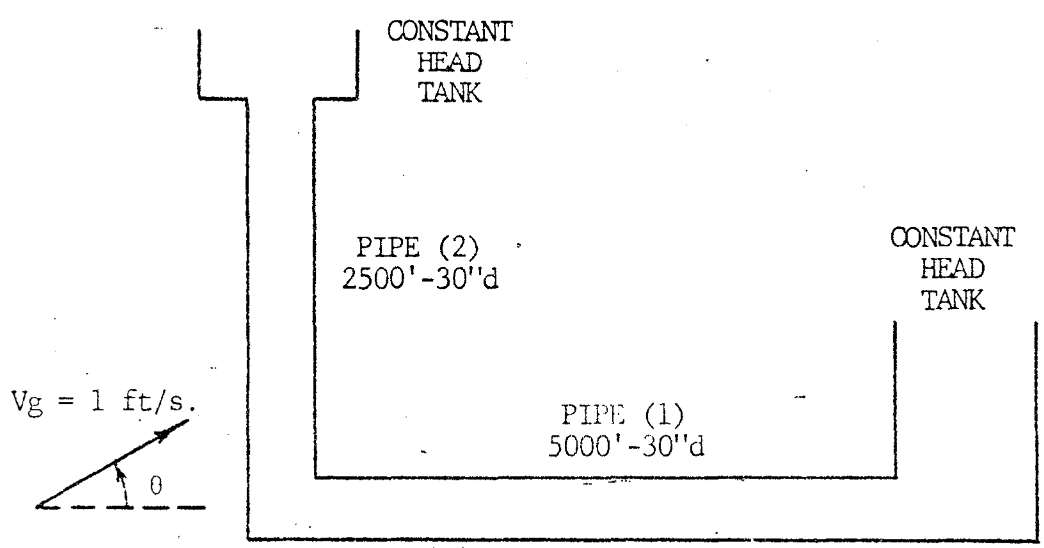
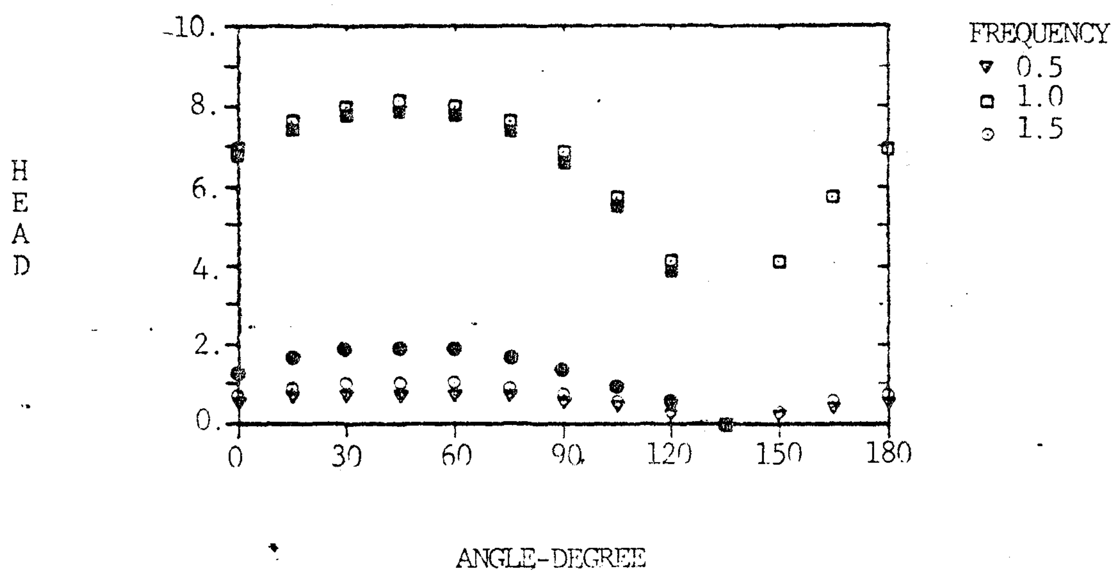


Figure 5.13. Schematic Diagram Showing the Piping Network Used for Tests (d) of the Elbow Connection.



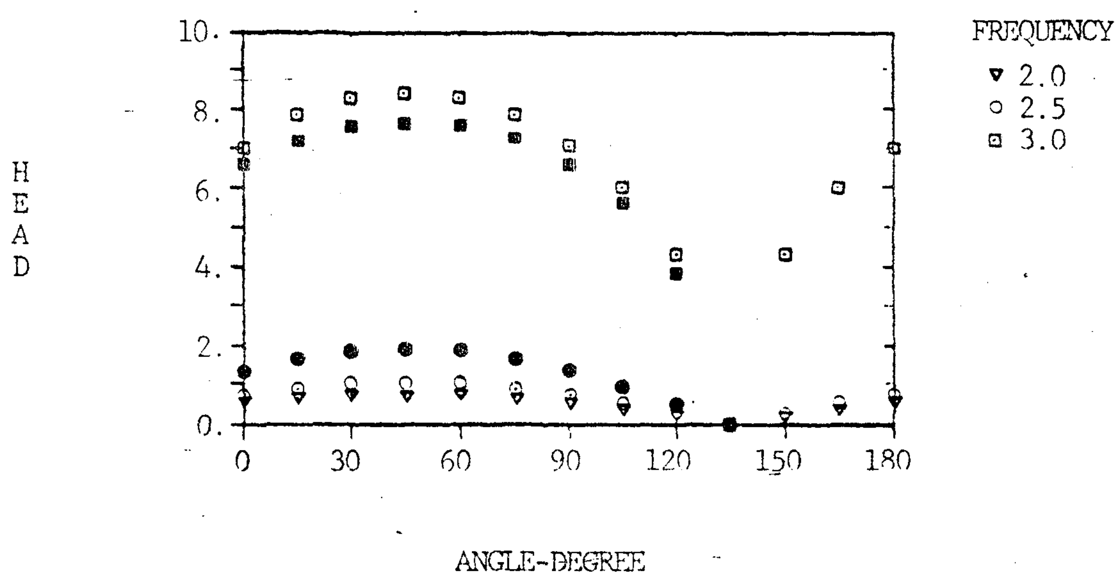
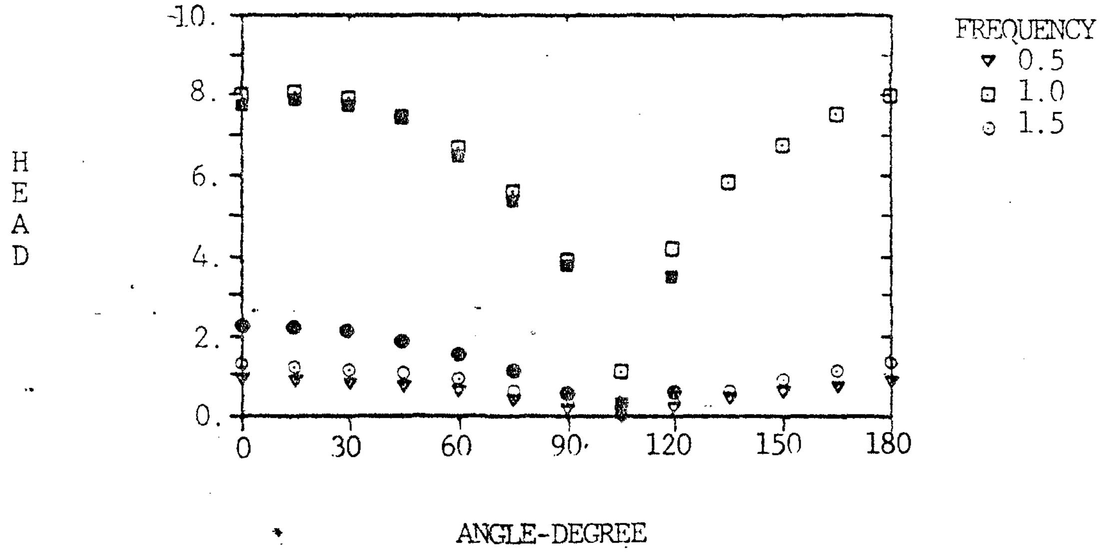


Figure 5.14. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for the Elbow Connection Shown in Figure 5.10.



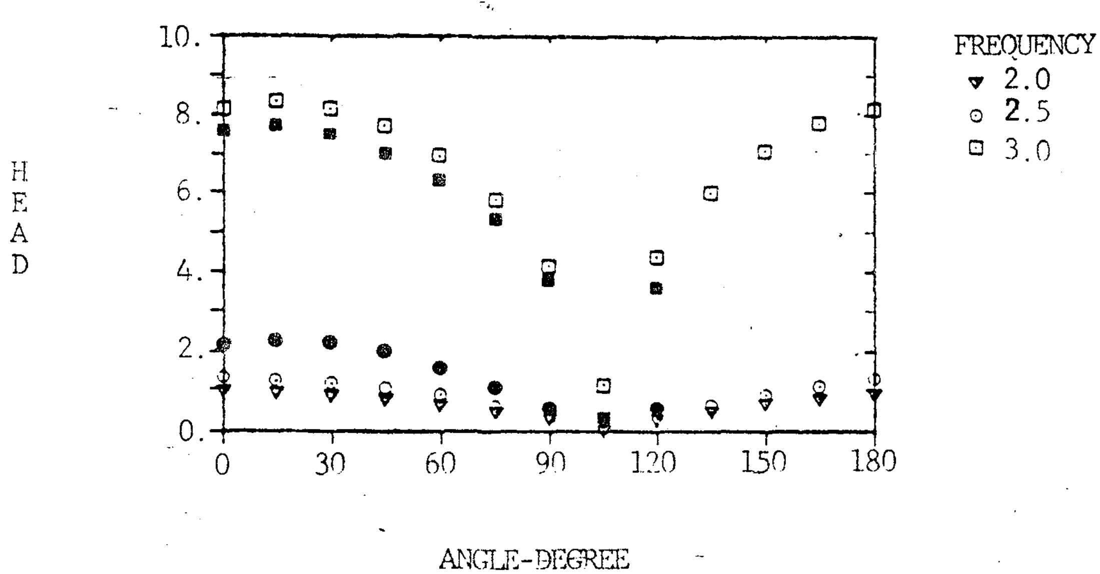
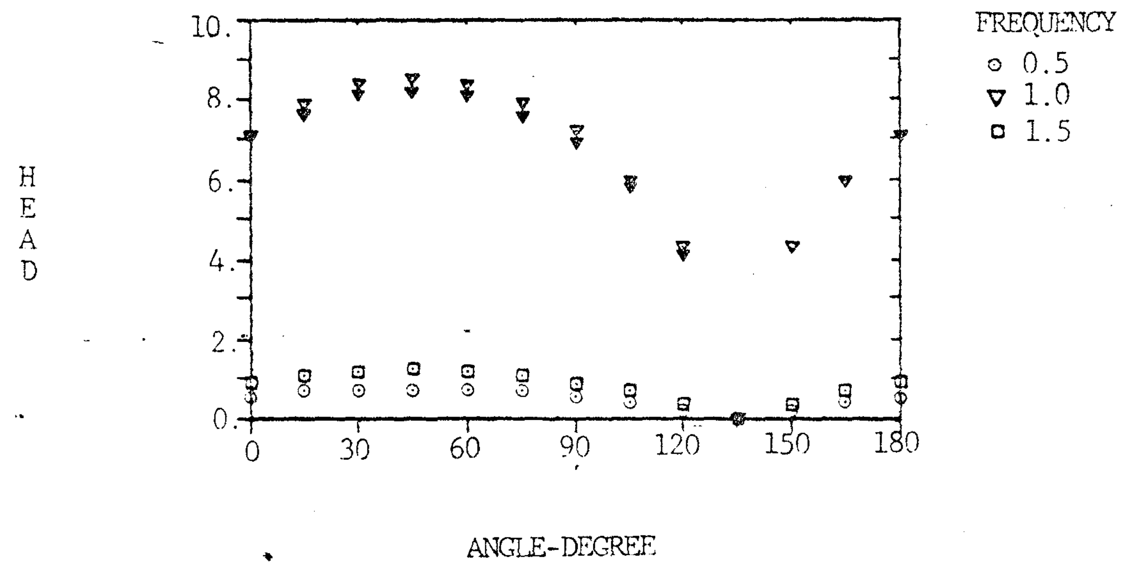


Figure 5.15. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for the Elbow Connection Shown in Figure 5.11.



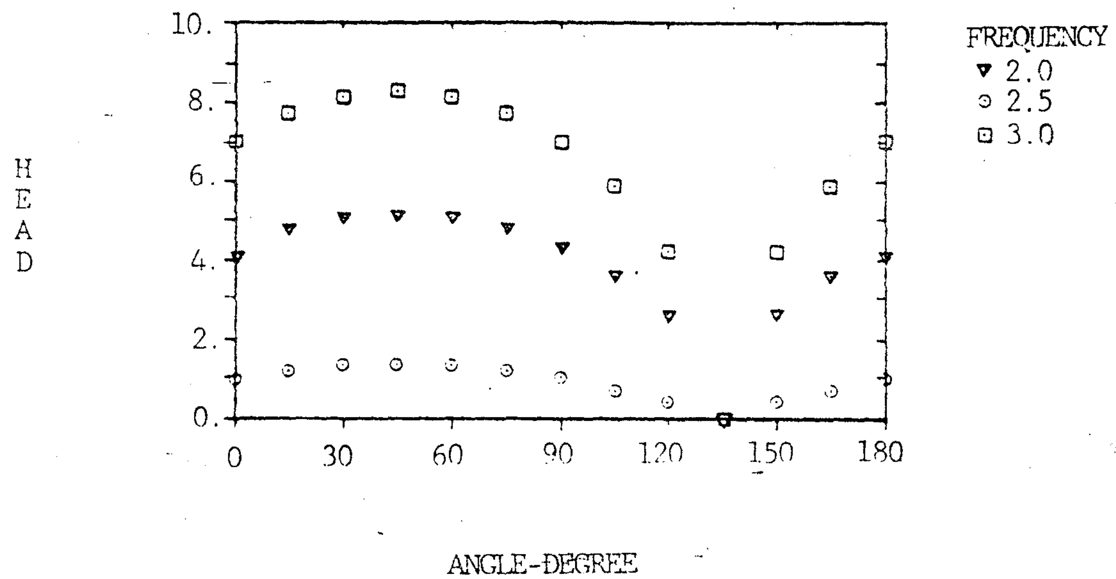
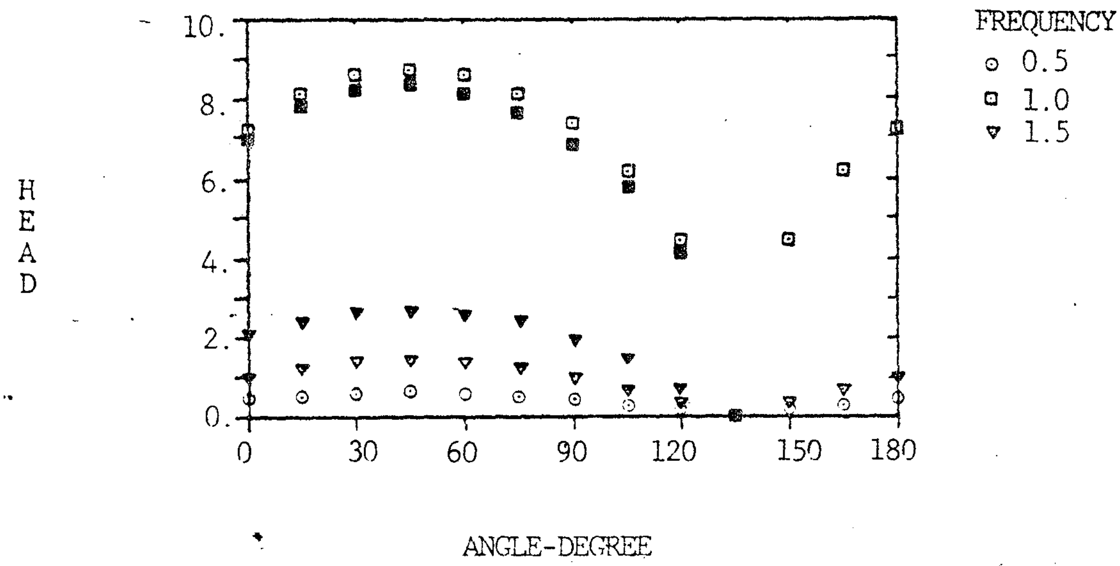


Figure 5.16. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for the Elbow Connection Shown in Figure 5.12.



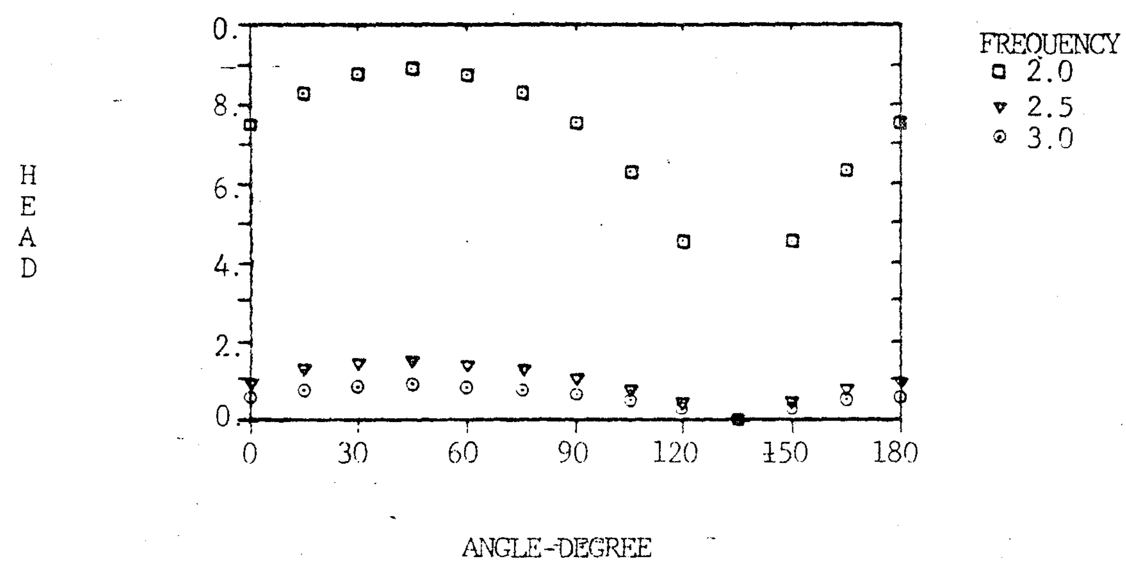


Figure 5.17. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for the Elbow Connection Shown in Figure 5.13.

is, the results exhibit good agreement for resonance and anti-resonance frequencies and differ at other frequencies for which the steady response would be less than the transient response.

Tee Connection

A tee connection was modeled by three pipes 5000 feet long each connected to constant head tanks at one end and to a tee connection at the other end. The diameter of the pipes were varied, as shown in Figures 5.18 to 5.20, to define three piping networks for the study of this connection. The procedure was employed that was used for the modeling of the elbow. Equation 5-2 was modified as

$$Q = Vg\ [(A_3-A_1)\text{Cos}(\theta)+A_2\text{Sin}(\theta)]\ e^{i\omega t} \qquad (5\text{-}3)$$

for this study. Each pipe length was divided into 5 equal sections to define a 16-node piping network. For a pipe friction factor of 0.1, three groups of tests were performed in the frequency ranges of 0.5 to 3.0, with intervals of 0.5, and angles of wave propagation from 0^o to 90^o, with intervals of 15^o, as follows: a) the diameter of all pipes were chosen to be 30 inches; b) the diameter of pipe (2) was chosen to be 20 inches, while the diameters of the other two pipes were maintained at 30 inches; c) pipe (1) was chosen to be 20 inches in diameter and the diameters of pipes (2) and (3) were 30 inches. Figures 5.21 to 5.23 show the results of tests (a) through tests (c), respectively, using open symbols. Solid symbols show the results obtained by the method of characteristics. Open circles

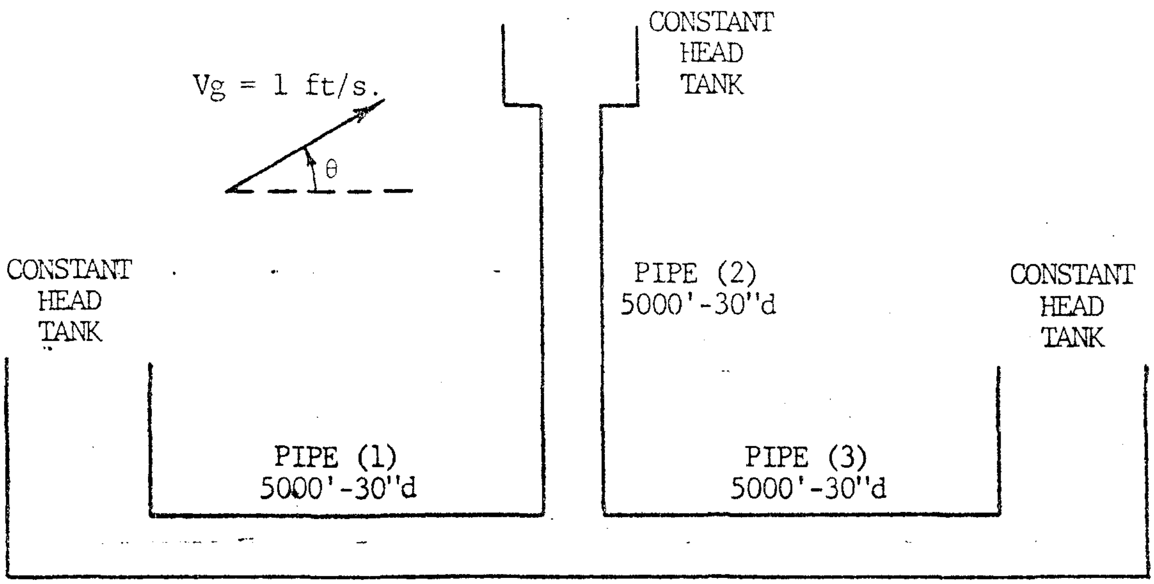


Figure 5.18. Schematic Diagram Showing the Piping Network Used for Tests (a) of the Tee Connection.

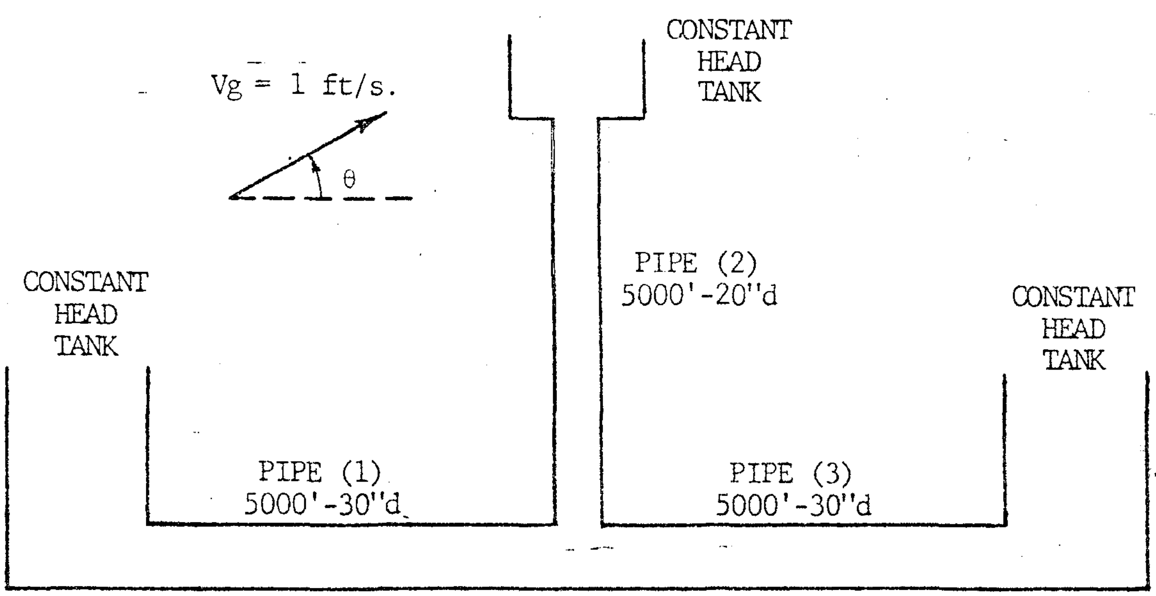


Figure 5.19. Schematic Diagram Showing the Piping Network Used for Tests (b) of the Tee Connection.

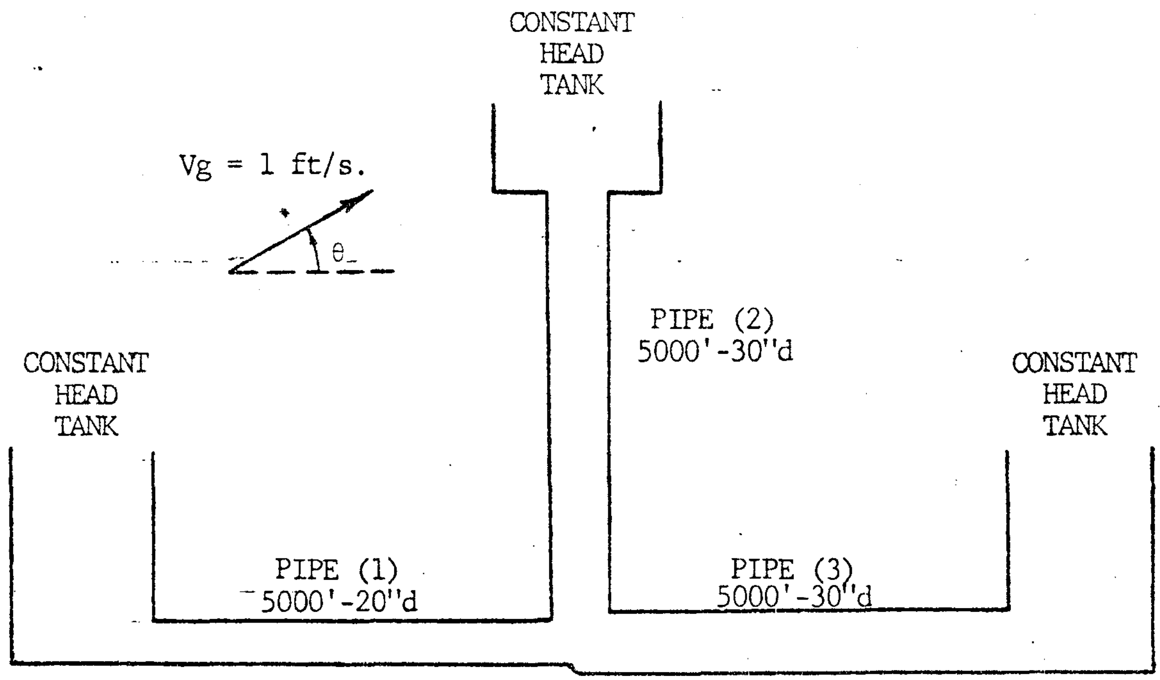
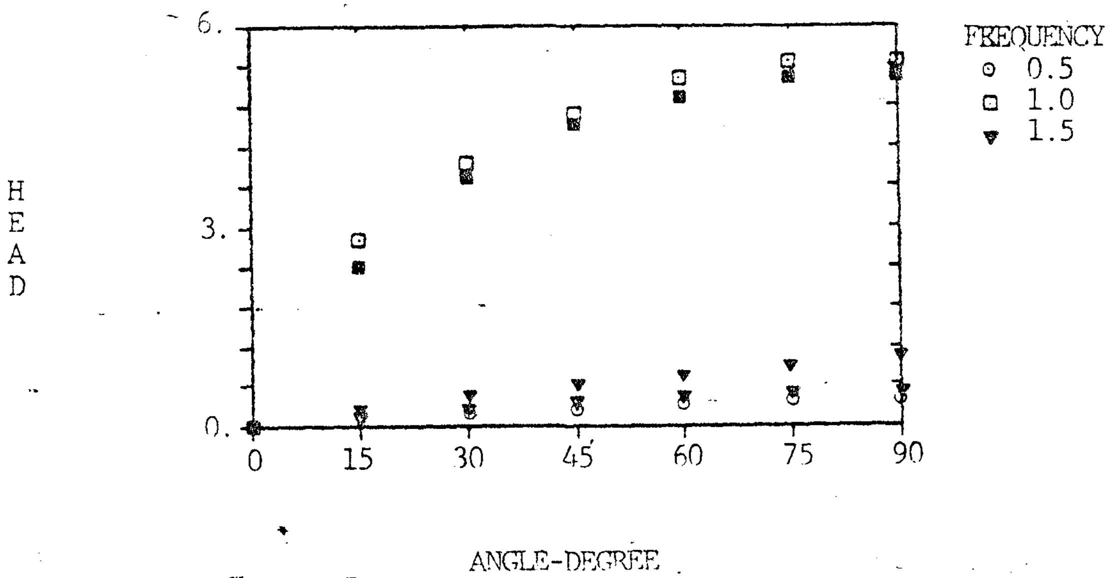


Figure 5.20. Schematic Diagram Showing the Piping Network Used for Tests (c) of the Tee Connection.



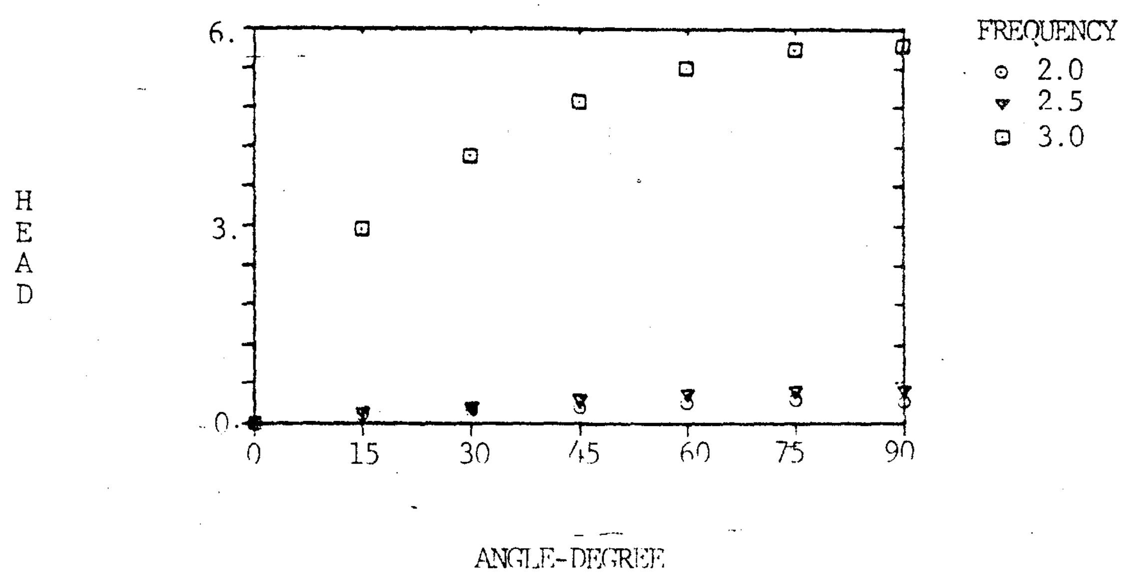
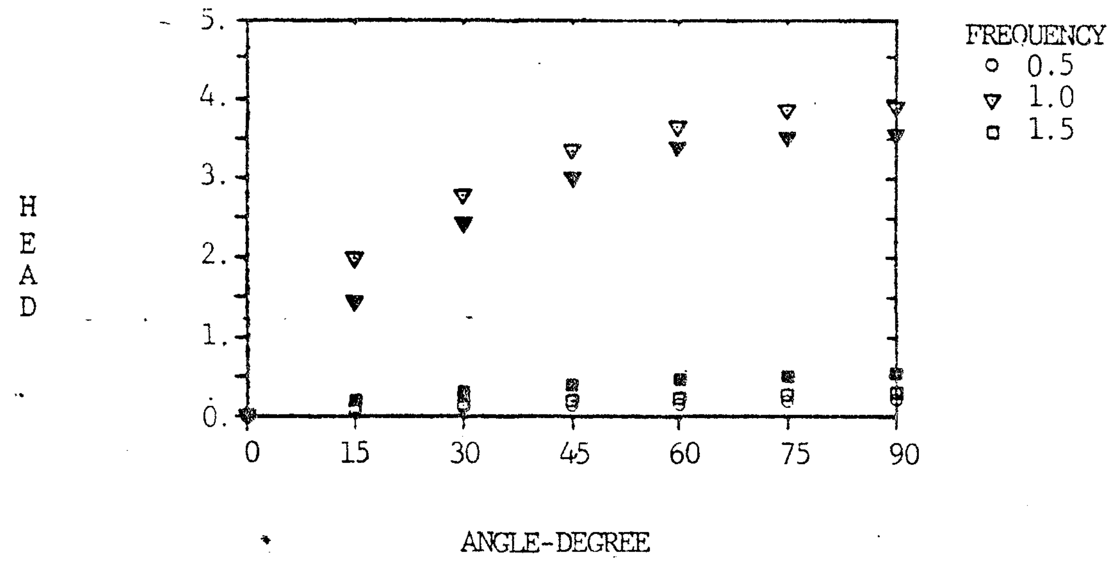


Figure 5.21. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for the Tee Connection Shown in Figure 5.18.



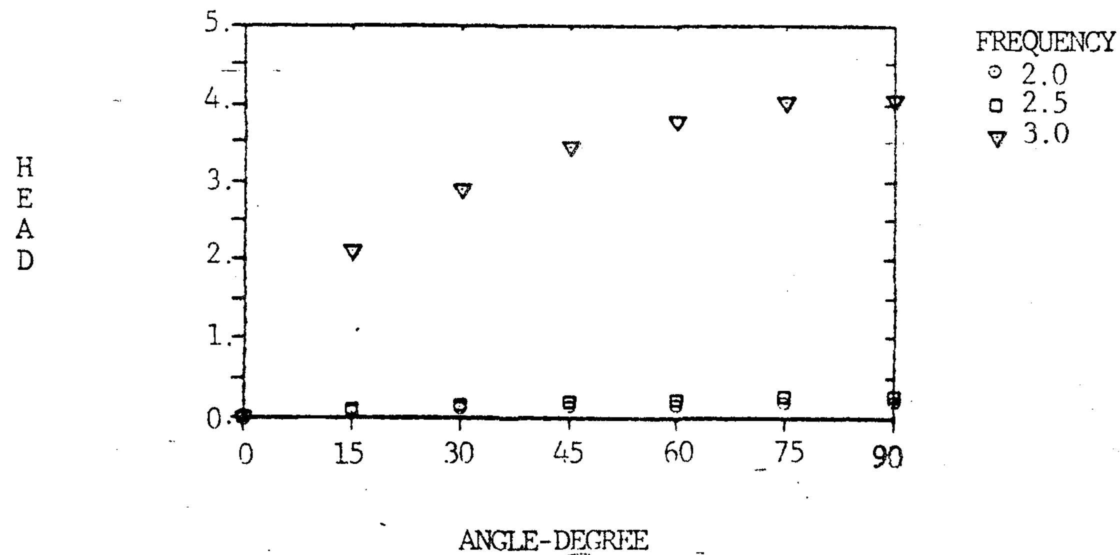
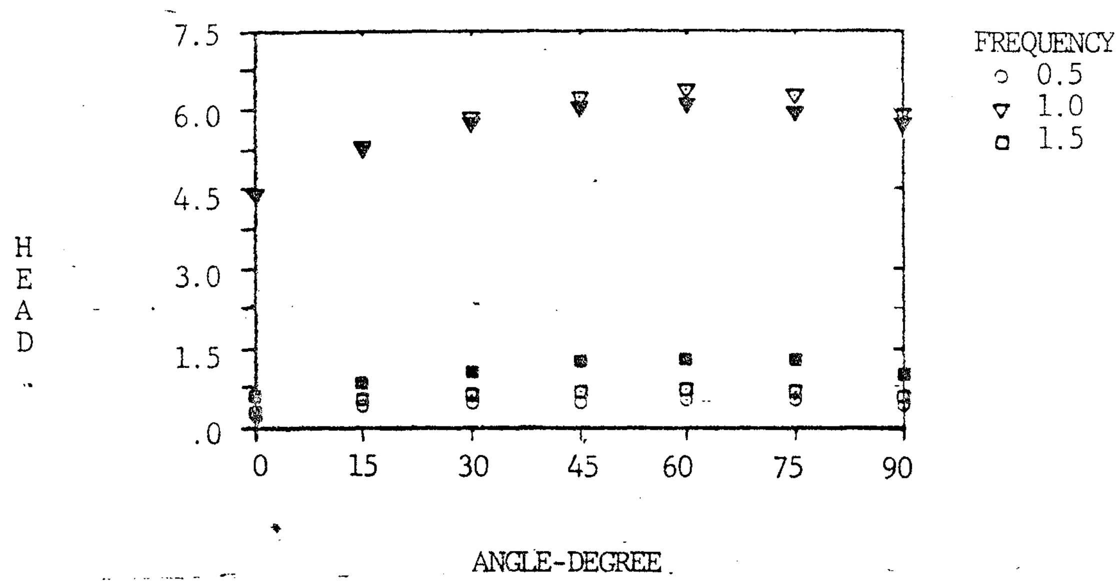


Figure 5.22. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for the Tee Connection Shown in Figure 5.19.



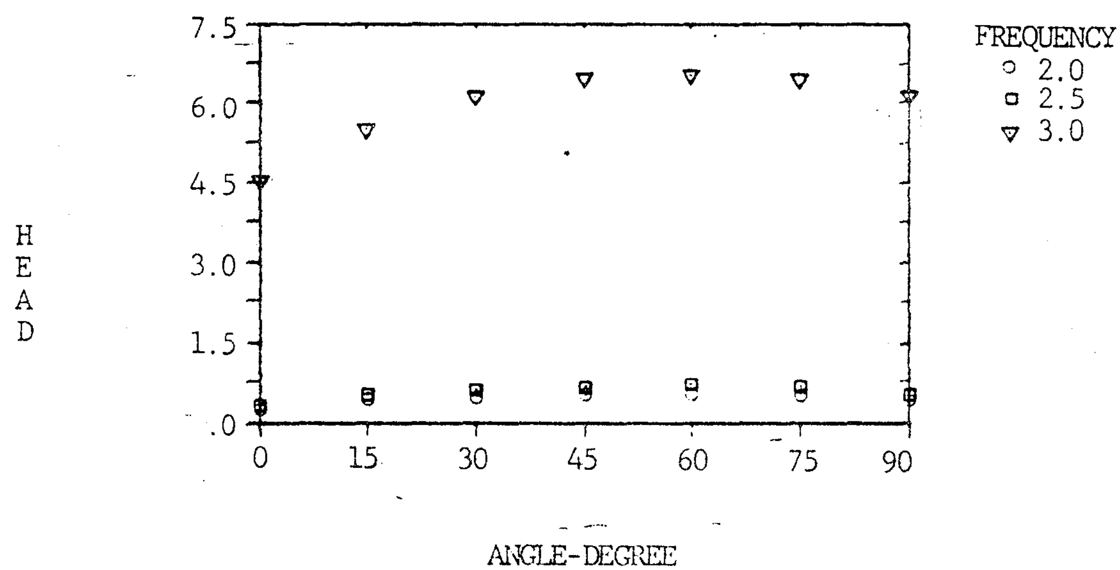


Figure 5.23. Non-Dimensional Head Response as a Function of the Angle of Seismic Wave Propagation for the Tee Connection Shown in Figure 5.20.

show the results obtained by both methods for frequencies of 0.5 and 2.0. The two methods agree about as well as the results agreed for the elbow connection.

Orifice Connection

A pipe 7500 feet long and 30 inches in diameter was connected to a constant head tank at each end with a one inch thick orifice located 2500 feet from one tank, and excited at the location of the orifice by a longitudinal compression seismic wave parallel to the axis of the pipe with a velocity amplitude of 1 ft/sec. Figure 5.24 shows the sketch for this piping network. A model similar to that for the dead-end was used, except that the inside area of the orifice was subtracted from the total area. The orifice connection was also modeled to have compression effect on one side and an expansion effect on the other side. Incorporating these considerations into those made for the dead-end connection, equation 5-1 was modified for an orifice connection as

$$Q = Vg(A-A_o)\mathrm{Cos}(\theta)e^{i\omega t} \tag{5-4}$$

for the compression side of the orifice and the same equation with a negative sign for the expansion side. Figure 5.25 shows a sketch of the model for the orifice where the length l_o is one inch. The piping network shown in Figure 5.25 was analyzed for different ranges of frequency and orifice diameter (d_o). The results showed zero head amplitude along the entire network for all cases, which means that the orifice has no effect on the piping network during a

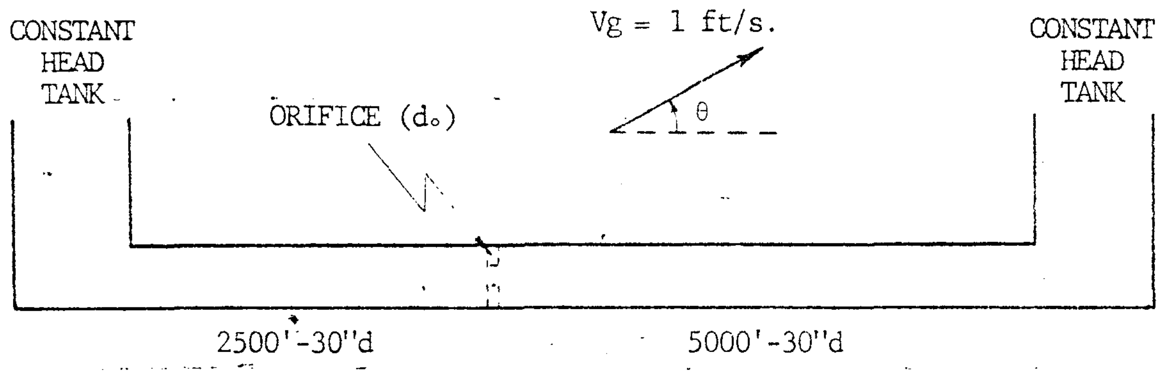


Figure 5.24. Schematic Diagram for the Orifice Connection Showing the Direction of the Seismic Excitation.

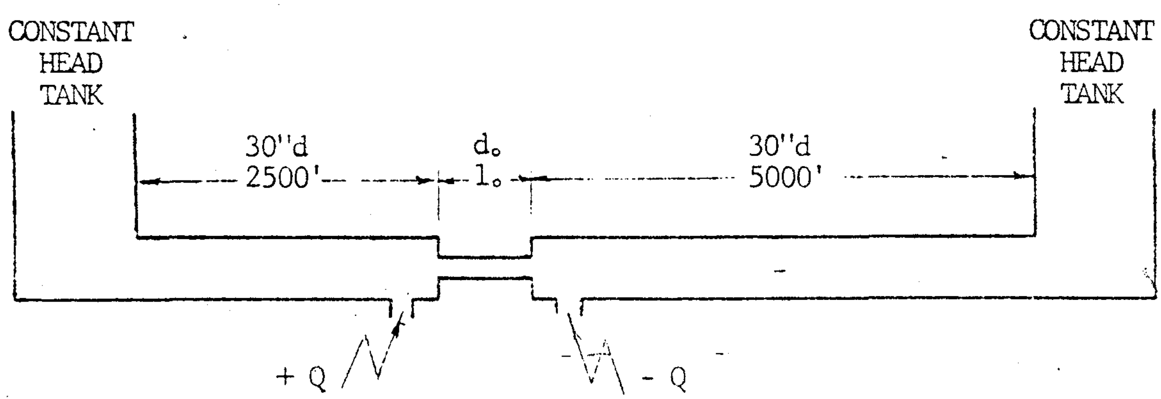


Figure 5.25. Schematic Diagram Showing the Model for the Orifice Connection Shown in Figure 5.24.

seismic disturbance. This method did not include pressure drop across the orifice; therefore the inflow at one side was cancelled by the outflow at the other side of the orifice. Then a new model was created which would account for the pressure drop across the orifice. This modeling was to replace the orifice with a pipe segment of the same diameter and a length selected to have the same frictional effect as the orifice. Referring to Appendix H, the lengths of these pipe segments, l_o, for the 5", 10", 15", and 20" diameter orifices are 11.2', 22.1', 31.5', and 35.9', respectively, for a pipe friction factor of 0.1. With this assumption, the above piping network was analyzed for the orifice diameters of 5", 10", 15", and 20" with excitation non-dimensional frequency ranges of 0.3 to 3.0, with intervals of 0.1. Table 5.1 shows the results of these tests including results of the method of characteristics for the same boundary conditions. Comparing the results, the following can be observed: 1) results of the steady oscillatory method are much less than the results of the method of characteristics; 2) the maximum head responses for the method of characteristics are at a non-dimensional excitation frequency of 1.0, while for the steady oscillatory method they are at frequencies between 1.4 and 1.5; 3) for the method of characteristics, head increases as the orifice diameter increased, while for the steady oscillatory method it is in reverse order. These differences may result because maximum values are obtained throughout as transients rather than at longer times which would correspond to the steady oscillatory flow model.

S.O.M. = STEADY OSCILLATORY METHOD

M.O.C. = METHOD OF CHARACTERISTICS

d_o / FREQUENCY	5"		10"		15"		20"	
	S.O.M.	M.O.C.	S.O.M.	M.O.C.	S.O.M.	M.O.C.	S.O.M.	M.O.C.
0.3	0.18		0.02		0.01		0.00	
0.4	0.19	1.20	0.02	0.70	0.01	0.70	0.00	0.70
0.5	0.20	1.30	0.02	0.85	0.01		0.00	0.85
0.6	0.21		0.02	1.05	0.01	1.10	0.01	1.05
0.7	0.22	1.75	0.03	1.10	0.01	1.15	0.01	1.15
0.8	0.23	2.55	0.03	1.60	0.02	1.70	0.01	1.65
0.9	0.25	3.35	0.04	2.60	0.02	2.65	0.01	2.65
1.0	0.28	3.85	0.05	4.30	0.02	4.60	0.01	4.65
1.1	0.35	2.70	0.06	3.00	0.03	2.60	0.01	2.60
1.2	0.50	2.40	0.08	1.80	0.04	1.80	0.02	1.80
1.3	0.67		0.14	1.50	0.07	1.40	0.03	1.50
1.4	1.00	2.00	0.36	1.40	0.16	1.40	0.07	1.45
1.5	0.96		0.87		0.73		0.54	
1.6	0.49		0.22		0.13		0.06	
1.7	0.37		0.14		0.08		0.04	
1.8	0.31		0.10		0.06		0.03	
1.9	0.28		0.09		0.05		0.02	
2.0	0.26		0.08		0.04		0.02	
2.1	0.26		0.08		0.04		0.02	
2.2	0.27		0.08		0.04		0.02	
2.3	0.29		0.09		0.05		0.02	
2.4	0.34		0.10		0.05		0.02	
2.5	0.40		0.11		0.06		0.03	
2.6	0.53		0.14		0.07		0.03	
2.7	0.75		0.20		0.10		0.05	
2.8	1.09		0.35		0.16		0.07	
2.9	1.43		1.14		0.43		0.17	
3.0	0.97		0.88		0.75		0.55	

Table 5.1. Non-Dimensional Head Response as a Function of Non-Dimenionsal Frequency for the Tests of Orifice Connection by Steady Oscillatory Method and Method of Characteristics.

Conclusions

A one-dimensional steady oscillatory method of analysis of hydraulic transients was employed to develop an economical, powerful tool (a computer program for analyzing the piping networks subject to steady oscillatory excitations) for piping designers who wish to design pipelines for earthquake zones. The use of this tool requires consideration of earthquake characteristics such as ground motion velocity, direction of wave propagation, frequency of the ground motion, etc. as well as an understanding of the model which converts the geometrical effects of junctions in the piping networks into excitation sources.

In this work, a model is developed to convert geometrical effects into excitation flow sources for the following junctions of piping networks: 1) dead-end, 2) 90° elbow, 3) tee, 4) orifice. This model has been verified for dead-end, elbow, and tee connections at resonant frequencies by comparison with the method of characteristics model, developed by Padron [6], which was in turn verified by experimental data obtained by Wood and Chao [8], and an energy analysis at resonance. Results of this model were considerably lower than the results obtained by the method of characteristics model at off-resonant frequencies. At these frequencies the maximum response occurs during the transient state. Since the frequency of the ground motion is a spectrum instead of a single frequency, the piping designer would always consider the worst case which is resonance.

The head response amplitudes calculated by this method are, for certain conditions, higher than the difference between the liquid steady head and the evaporation head. Column separation can result, therefore extra damping in the system and lower head response would be expected. Since larger head responses would be calculated than would be experienced, designers could use this method and expect an additional unknown factor of safety.

The method developed in this study is capable of handling single frequency excitations only. Conventional methods of linear superposition would apply when the system could be treated as approximately linear.

The effects of the steady oscillatory flow component on the linearized fluid friction term were neglected by Streeter and Wylie [7]. A model of this effect was included in calculations of this study by two different methods presented in Chapter IV and Appendix F. The results of the method described in Chapter IV depend on the pipe friction factor and frequency harmonic number. This method is a good approximation of steady oscillatory flow affects for the first harmonic frequency and a pipe friction factor of 0.1. The result of the method described in Appendix F is approximately independent of the frequency harmonics number and the pipe friction factor. The method described in Appendix F has been verified by the method of characteristics for friction factors ranging from 0.02 to 0.20 and is expected to give good approximations for any fruction factor.

An excitation velocity amplitude of 1 ft/s. was chosen for all cases of this study. Slight differences were noted between the results of the steady oscillatory method and the method of characteristics as the angle of wave propagation was increased, which corresponds to lower excitation velocities. This result may indicate that there may be considerable difference between the results of the two methods for higher excitation velocities.

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APPENDIX A

CONTINUITY EQUATION

In this Appendix, the continuity equation 2-1 is expanded after Streeter and Wylie [7]. Small terms are neglected to form a suitable equation for slightly deformable horizontal pipes.

$$\frac{\partial (Q\rho)}{\partial x}\delta x + \frac{\partial (\rho A \delta x)}{\partial t} = 0 \qquad (2\text{-}1)$$

Expanding this equation and dividing by $\rho A \delta x$, gives

$$\frac{Q}{\rho A}\frac{\partial P}{\partial x} + \frac{1}{A}\frac{\partial Q}{\partial x} + \frac{1}{A}\frac{\partial A}{\partial t} + \frac{1}{\delta x}\frac{\partial \delta x}{\partial t} + \frac{1}{\rho}\frac{\partial \rho}{\partial t} = 0 \ .$$

The term δx is a function of time only. Replacing the partial derivative of δx with its total derivative, and rearranging, the following results.

$$\frac{1}{\rho}\left(V\frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial t}\right) + \frac{1}{A}\left[\frac{\partial A}{\partial t} + \frac{\partial (VA)}{\partial x}\right] + \frac{1}{\delta x}\frac{d\delta x}{dt} = 0$$

Expanding the term $\frac{\partial VA}{\partial x}$ and using $V = \frac{dx}{dt}$, the following equation results.

$$\frac{1}{\rho}\left(\frac{dx}{dt}\frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial t}\right) + \frac{1}{A}\left(\frac{dx}{dt}\frac{\partial A}{\partial x} + \frac{\partial A}{\partial t}\right) + \frac{\partial V}{\partial t} + \frac{1}{\delta x}\frac{d\delta x}{dt} = 0 \qquad (A\text{-}1)$$

Using the definition of the total derivative, that is $\frac{\partial \rho}{\partial t} + \frac{dx}{dt}\frac{\partial \rho}{\partial x} = \frac{d\rho}{dt}$, for the first term and $\frac{\partial A}{\partial t} + \frac{dx}{dt}\frac{\partial A}{\partial x} = \frac{dA}{dt}$, for the second term of the equation A-1, the following equation results.

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{A}\frac{dA}{dt} + \frac{\partial V}{\partial t} + \frac{1}{\delta x}\frac{d\delta x}{dt} = 0 \qquad (A\text{-}2)$$

The bulk compressibility modulus K is defined as

$$K = \frac{dP/dt}{d\rho/\rho dt}.$$

Using this definition, the first term of equation A-2 becomes

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{K}\frac{dP}{dt}\ . \tag{A-3}$$

Referring to Figure A.1, the second term of equation A-2 is determined.

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{\pi D^2/4}\frac{d(\pi D\frac{D}{2}\xi_2)}{dt} = 2(\frac{d\xi_2}{dt}) \tag{A-4}$$

ξ_2, is the lateral strain for the pipe and defined as

$$\xi_2 = \frac{1}{E}(\sigma_2-\mu\sigma_1) \tag{A-5}$$

where E is the bulk modulus of elasticity; σ_2 is the lateral stress; σ_1 is the axial stress; and μ is the Poisson ratio of the pipe. The third term of equation A-2 may be expressed as

$$\frac{1}{\delta x}\frac{d\delta x}{dt} = \frac{1}{\delta x}\frac{d}{dt}\xi_1\delta x = \frac{d\xi_1}{dt} \tag{A-6}$$

where ξ_1 is the axial strain of the pipe and is defined as

$$\xi_1 = \frac{1}{E}(\sigma_1-\mu\sigma_2). \tag{A-7}$$

Substituting definitions A-5 and A-7 into expressions A-4 and A-6, respectively, then substituting the resulting equations and equation A-3 into equation A-2 and rearranging, the following equation results:

$$\frac{1}{K}\frac{dP}{dt} + \frac{1}{E}[(2-\mu)\ \frac{d\sigma_2}{dt} + (1-2\mu)]\frac{d\sigma_1}{dt}\ + \frac{\partial V}{\partial x} = 0 \tag{A-8}$$

Referring to Figure A.2, the axial and lateral stresses are written as

$$\sigma_1 = \frac{F_1}{\pi De}\quad \text{and}$$

$$\sigma_2 = \frac{F_2}{2e\delta x}$$

respectively, where F_1 is $\frac{\pi}{4}D^2P$ and F_2 is $PD\delta x$. Substituting for F_1 and F_2 into the above equations and taking their time derivative, the following relations result:

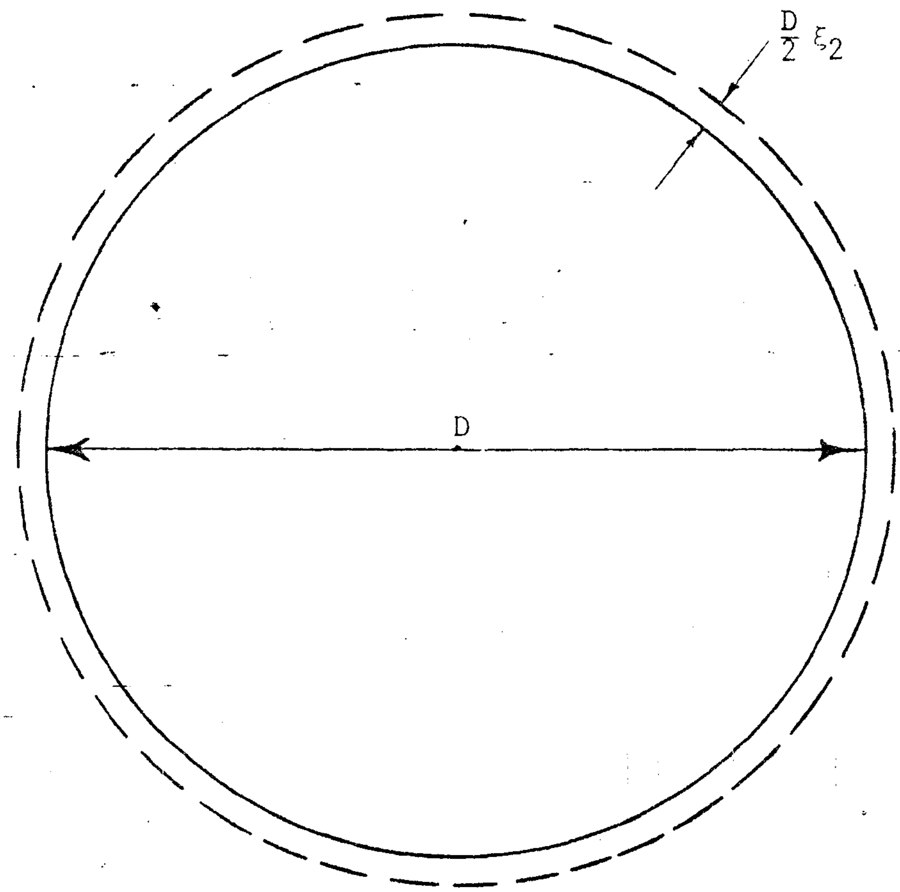

Figure A.1. Cross Sectional View of a Simple Pipe Showing the Lateral Expansion.

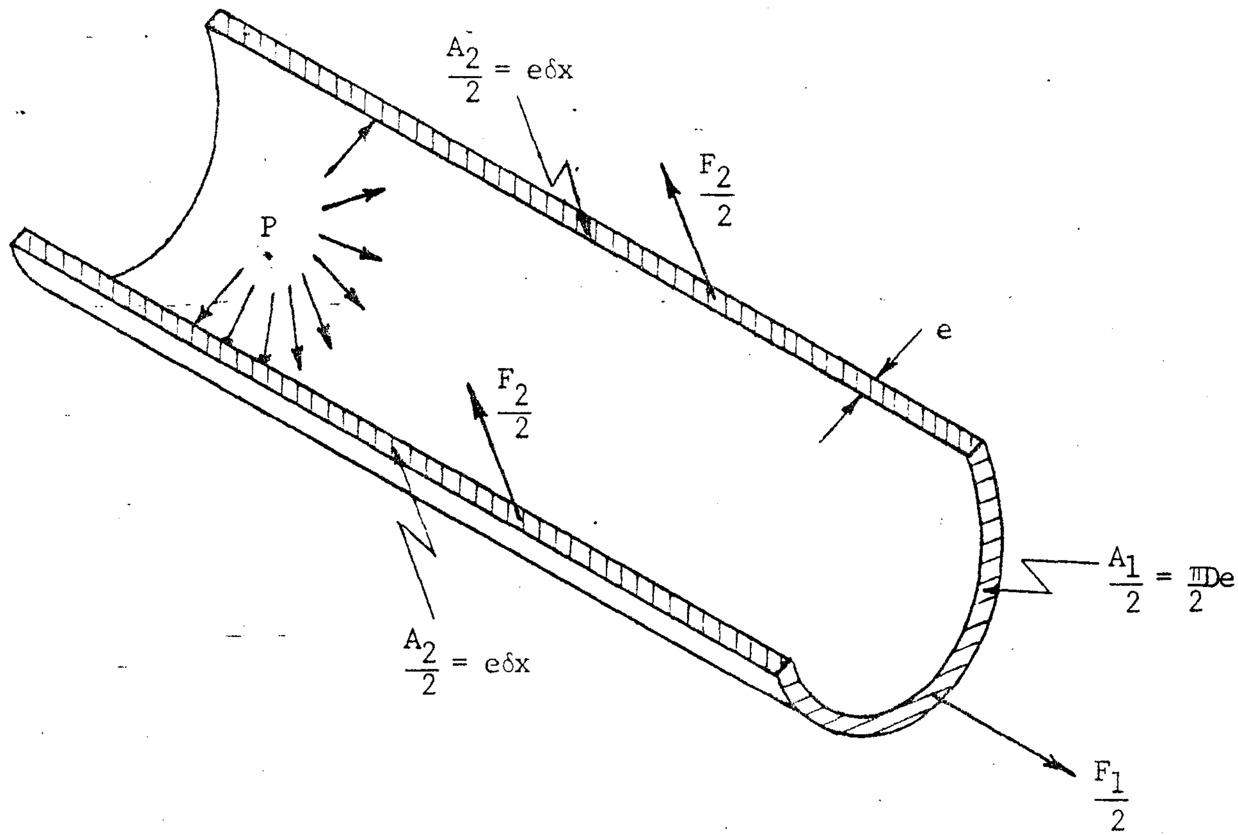

Figure A.2. Sectional View of a Simple Pipe.

$$\frac{d\sigma_1}{dt} = \frac{D}{4e}\frac{dP}{dt}$$

$$\frac{d\sigma_2}{dt} = \frac{D}{2e}\frac{dP}{dt}$$

Substituting these relations into equation A-8, rearranging and collecting like terms, the following equation results:

$$\frac{\partial V}{\partial x} + \frac{1}{K}\frac{dP}{dt}[1 + \frac{KD}{eE}(\frac{5}{4} - \mu)] = 0 \qquad \text{(A-9)}$$

Letting $(\frac{5}{4} - \mu)$ be equal to C_1, which depends only on the type of support of the pipe, and by defining the acoustic velocity, a, as

$$a = \sqrt{\frac{\frac{K}{\rho}}{1 + \frac{K}{E}\frac{D}{e}C_1}} \quad \text{or} \quad \frac{KD}{Ee}C_1 = \frac{K}{\rho a^2} - 1 \ ,$$

and rearranging, equation A-9 can be written as

$$\frac{\partial V}{\partial x} + \frac{1}{\rho a^2}\frac{dp}{dt} = 0 \ .$$

Using $V = \frac{Q}{A}$ and writing the total derivative $\frac{dP}{dt}$ in terms of its partial derivatives, multiplying by A and rearranging,.the following equation is obtained:

$$\frac{\partial Q}{\partial x} + \frac{A}{\rho a^2}\frac{\partial P}{\partial t} + V(\frac{A}{\rho a^2}\frac{\partial P}{\partial x} - \frac{\partial A}{\partial x}) = 0$$

The rate of change of pressure and area with respect to position is very small in comparison with the time rate of change of pressure and position rate of change of flow. Neglecting these small terms, the following equation can be written:

$$\frac{\partial Q}{\partial x} + \frac{A}{\rho a^2}\frac{\partial P}{\partial t} = 0$$

The time rate of change of ρ is very small and can be treated as a constant. Employing this constant, and using $H = \frac{P}{\rho g}$ the following equation results:

$$\frac{\partial Q}{\partial x} + \frac{gA}{a^2}\frac{\partial H}{\partial t} = 0 \qquad \text{(A-10)}$$

Q and H in the above equation are the instantaneous value of the flow and pressure-head. They can be expressed in terms of an average component and a fluctuating component by

$$Q = \overline{Q} + q' \text{ and } H = \overline{H} + h'.$$

Substituting these definitions into equation A-10, the following equation results:

$$\frac{\partial \overline{Q}}{\partial x} + \frac{\partial q'}{\partial x} + \frac{gA}{a^2}\frac{\partial \overline{H}}{\partial t} + \frac{gA}{a^2}\frac{\partial h'}{\partial t} = 0$$

The rate of change of the average values, "$\frac{\partial \overline{Q}}{\partial x}$ and $\frac{\partial \overline{H}}{\partial t}$," are small in comparison to the change in the fluctuating components. Neglecting these small terms, the following equation results,

$$\frac{\partial q'}{\partial x} + \frac{gA}{a^2}\frac{\partial h'}{\partial t} = 0 \qquad \text{(A-11)}$$

which is the continuity equation for a slightly deformable horizontal pipe.

APPENDIX B

MOMENTUM EQUATION

In this Appendix, after the Streeter and Wylie [7], the momentum equation 2-3 is applied to a slightly deformable horizontal pipe, and after neglecting small terms, an approximation is obtained.

$$-\frac{\partial(PA)}{\partial x}\delta x-\tau_o\pi D\delta x = \rho\delta x\left(A + \frac{\partial A}{\partial x}\frac{\delta x}{2}\right)\frac{dV}{dt} \tag{2-3}$$

Expanding the partial derivative, $\frac{\partial(PA)}{\partial x}$, and dividing the equation by δx, the following equation is obtained:

$$A\frac{\partial P}{\partial x} + P\frac{\partial A}{\partial x} + \tau_o\pi D + \rho A\frac{dV}{dt} + \rho\frac{\delta x}{2}\frac{\partial A}{\partial x}\frac{dV}{dt} = 0$$

The position rate of change of area is small in comparison to the other terms. Neglecting these small terms, the following equation is obtained:

$$A\frac{\partial P}{\partial x} + \tau_o\pi D + \rho A\frac{dV}{dt} = 0 \tag{B-1}$$

The shear stress τ_o at the wall of the pipe can be expressed as $\tau_o = \frac{\rho f V^2}{8}$ where f is the friction factor of the pipe. Substituting this expression into equation B-1 and dividing equation by $\rho g A$, the following equation is obtained:

$$\frac{1}{gA}\frac{\partial P}{\partial x} + \frac{fV^2}{2gD} + \frac{1}{g}\frac{dV}{dt} = 0$$

Density changes very little compared with pressure and therefore is treated as a constant. Taking density into the differential and using $H = \frac{P}{\rho g}$, the following equation is obtained:

$$\frac{\partial H}{\partial x} + \frac{fV^2}{2gD} + \frac{1}{g}\frac{dV}{dt} = 0$$

The time rate of change of area is small in comparison to the change in head or velocity, and can be treated as a constant. Using $V = \frac{Q}{A}$, the following equation is obtained:

$$\frac{\partial H}{\partial x} + \frac{fQ^2}{2gDA^2} + \frac{1}{gA}\frac{dQ}{dt} = 0 \qquad \text{(B-2)}$$

If the special case for laminar flow is desired, the friction factor f can be expressed as

$$f = \frac{64}{Re} = \frac{64A\nu}{QD}$$

and by substituting this expression into the equation B-2, the following momentum equation for laminar flow through a slightly deformable horizontal pipe results:

$$\frac{\partial H}{\partial x} + \frac{32\nu Q}{gAD^2} + \frac{1}{gA}\frac{\partial Q}{\partial t} = 0 \qquad \text{(B-3)}$$

The instantaneous values of head H, and flow Q, can be expressed in terms of an average component and a fluctuating component as

$$H = \bar{H} + h' \text{ and } Q = \bar{Q} + q'.$$

Substituting these definitions into equations B-2 and B-3, the equation for laminar flow is

$$\frac{\partial \bar{H}}{\partial x} + \frac{\partial h'}{\partial x} + \frac{32\nu(\bar{Q}+q')}{gAD^2} + \frac{1}{gA}\left(\frac{\partial \bar{Q}}{\partial t} + \frac{\partial q'}{\partial t}\right) = 0 \qquad \text{(B-4)}$$

and the equation for turbulent flow is similarly

$$\frac{\partial \bar{H}}{\partial x} + \frac{\partial h'}{\partial x} + \frac{f(\bar{Q}+q')^2}{2gDA^2} + \frac{1}{gA}\left(\frac{\partial \bar{Q}}{\partial t} + \frac{\partial q'}{\partial t}\right) = 0. \qquad \text{(B-5)}$$

The time rate of change of average flow is small in comparison to the other terms and can be neglected. Assuming the values of the position rate of change of average heads to be $\frac{\partial \bar{H}}{\partial x} = -\frac{f\bar{Q}^2}{2gDA^2}$ for turbulent flow and $\frac{\partial \bar{H}}{\partial x} = -\frac{32\nu\bar{Q}}{gAD^2}$ for laminar flow, equations B-4 and B-5 can be written in the form of B-6 and B-7 respectively.

$$\frac{\partial h'}{\partial x} + \frac{32\nu}{gAD^2}q' + \frac{1}{gA}\frac{\partial q'}{\partial t} = 0 \qquad \text{(B-6)}$$

$$\frac{\partial h'}{\partial x} + \frac{fQ}{gDA^2}q' + \frac{fq'^2}{2gDA^2} + \frac{1}{gA}\frac{\partial q'}{\partial t} = 0 \qquad \text{(B-7)}$$

To have a linear differential equation, the term $\frac{fq'^2}{2gDA^2}$, must be linearized in equation B-7. By defining the friction term for turbulent flow as

$$R = \frac{fQ}{gDA^2} \qquad \text{(B-8)}$$

and the friction term for laminar flow as

$$R = \frac{32\nu}{gAD^2} \qquad \text{(B-9)}$$

both equations B-6 and B-7 can then be written in a linear form

$$\frac{\partial h'}{\partial x} + \frac{1}{gA}\frac{\partial q'}{\partial t} + Rq' = 0 \qquad \text{(B-10)}$$

which is the momentum equation for flow through a slightly deformable horizontal pipe.

APPENDIX C

STEADY OSCILLATORY FLOW EQUATIONS

In this Appendix, the momentum and continuity equations derived in Appendixes A and B for a slightly deformable horizontal pipe are used to find solutions for steady oscillatory flow after Streeter and Wylie [7].

$$\frac{\partial q'}{\partial x} + \frac{gA}{a^2}\frac{\partial h'}{\partial t} = 0 \qquad (2\text{-}2)$$

$$\frac{\partial h'}{\partial x} + \frac{1}{gA}\frac{\partial q'}{\partial t} + Rq' = 0 \qquad (2\text{-}4)$$

Taking partial derivatives of equation 2-2 and 2-4 with respect to the position x, and with respect to time t, the following equations are obtained:

$$\frac{\partial^2 q'}{\partial x^2} + \frac{gA}{a^2}\frac{\partial^2 h'}{\partial x \partial t} = 0 \qquad (C\text{-}1)$$

$$\frac{\partial^2 q'}{\partial t \partial x} + \frac{gA}{a^2}\frac{\partial^2 h'}{\partial t^2} = 0 \qquad (C\text{-}2)$$

$$\frac{\partial^2 h'}{\partial x^2} + \frac{1}{gA}\frac{\partial^2 q'}{\partial x \partial t} + R\frac{\partial q'}{\partial x} = 0 \qquad (C\text{-}3)$$

$$\frac{\partial^2 h'}{\partial t \partial x} + \frac{1}{gA}\frac{\partial^2 q'}{\partial t^2} + R\frac{\partial q'}{\partial t} = 0 \qquad (C\text{-}4)$$

Substituting equations 2-2 and C-2 into equation C-3 and rearranging, the following equation is obtained:

$$\frac{\partial^2 h'}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 h'}{\partial t^2} + \frac{gAR}{a^2}\frac{\partial h'}{\partial t} \qquad (C\text{-}5)$$

Substituting equation C-1 into equation C-4, multiplying by $\frac{gA}{a^2}$, and rearranging, the following equation is obtained:

$$\frac{\partial^2 q'}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 q'}{\partial t^2} + \frac{gAR}{a^2}\frac{\partial q'}{\partial t} \qquad (C\text{-}6)$$

which is identical in form with equation C-5. A separation of variables technique can be used to solve differential equation C-5 or C-6, which assumes

$$h' = X(x)T(t)$$

for equation C-5, where X is a function of position only and T is a function of time only. Taking the first and second derivatives of this assumed solution with respect to t and with respect to x, the following equations are obtained:

$$\frac{\partial h'}{\partial t} = X\frac{dT}{dt} \qquad \text{(C-8)}$$

$$\frac{\partial^2 h'}{\partial t^2} = X\frac{d^2 T}{dt^2} \qquad \text{(C-9)}$$

$$\frac{\partial h'}{\partial x} = T\frac{dX}{dx} \qquad \text{(C-10)}$$

$$\frac{\partial^2 h'}{\partial x^2} = T\frac{d^2 X}{dx^2} \qquad \text{(C-11)}$$

Substituting equations C-8, C-9 and C-11 into equation C-5 and dividing the equation by TX, the following equation results:

$$\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{a^2T}\frac{d^2T}{dt^2} + \frac{gAR}{a^2T}\frac{dT}{dt} = \gamma^2 \qquad \text{(C-12)}$$

Equation C-12 is equated to a constant because each side of this equation can vary independently of the other side. The constant γ, is the propagation constant and is equal to $(\alpha + i\beta)$ which will be defined later. To find this constant γ, the solution for T can be restricted to the steady oscillatory case, by assuming a particular solution for T as a harmonic oscillation. The solution can then be expressed as

$$T = C\,e^{i\omega t}, \qquad \text{(C-13)}$$

where ω is the angular frequency. Taking the first and second time derivatives of this expression, the following equations are obtained:

$$\frac{dT}{dt} = Ci\omega e^{i\omega t} \tag{C-14}$$

$$\frac{d^2T}{dt^2} = -C\omega^2 e^{i\omega t} \tag{C-15}$$

Substituting equations C-14 and C-15 into equation C-12 and solving for γ^2, the following equation is obtained:

$$\gamma^2 = \frac{Ag\omega}{a^2}\left(-\frac{\omega}{gA} + i\ R\right)$$

Referring to the Figure C.1, γ^2 can be expressed as follows:

$$\gamma = \frac{gA}{a^2}\sqrt{\left(\frac{\omega}{gA}\right)^2 + R^2}\ \ e^{i\theta_1}$$

Taking the square root of this equation, the following equation is obtained:

$$\gamma = (\alpha + i\beta) = \sqrt{\frac{gA\omega}{a^2}}\left[\left(\frac{\omega}{gA}\right)^2 + R^2\right]^{\frac{1}{4}}\ e^{i\frac{\theta_1}{2}}$$

Using the definition of exponential functions, $e^{i\frac{\theta_1}{2}}$ can be defined as, $\text{Cos}(\frac{\theta_1}{2}) + i\text{Sin}(\frac{\theta_1}{2})$, or by writing in terms of θ_2, Figure C.1,

$$e^{i\frac{\theta_1}{2}} = \text{Cos}\left(\frac{\pi}{2} - \frac{\theta_2}{2}\right) + i\text{Sin}\left(\frac{\pi}{2} - \frac{\theta_2}{2}\right)$$

or

$$e^{i\frac{\theta_1}{2}} = \text{Sin}\frac{\theta_2}{2} + i\text{Cos}\frac{\theta_2}{2}. \tag{C-17}$$

Referring to the Figure C.1, θ_2 can be defined as follows:

$$\theta_2 = \tan^{-1}\left(\frac{R}{-\omega/Ag}\right) = \tan^{-1}\frac{gAR}{\omega} \tag{C-18}$$

Substituting equation C-18 into equation C-17 and then the result obtained equation into equation C-16, separating the real and the imaginary parts of the resulting equation, the value of α and β can be defined as

$$\alpha = \sqrt{\frac{\omega gA}{a^2}}\left[\left(\frac{\omega}{gA}\right)^2 + R^2\right]^{\frac{1}{4}}\ \text{Sin}\left(\tfrac{1}{2}\tan^{-1}\frac{RgA}{\omega}\right) \tag{C-19}$$

$$\beta = \sqrt{\frac{\omega gA}{a^2}}\left[\left(\frac{\omega}{gA}\right)^2 + R^2\right]^{\frac{1}{4}}\ \text{Cos}\left(\tfrac{1}{2}\tan^{-1}\frac{RgA}{\omega}\right) \tag{C-20}$$

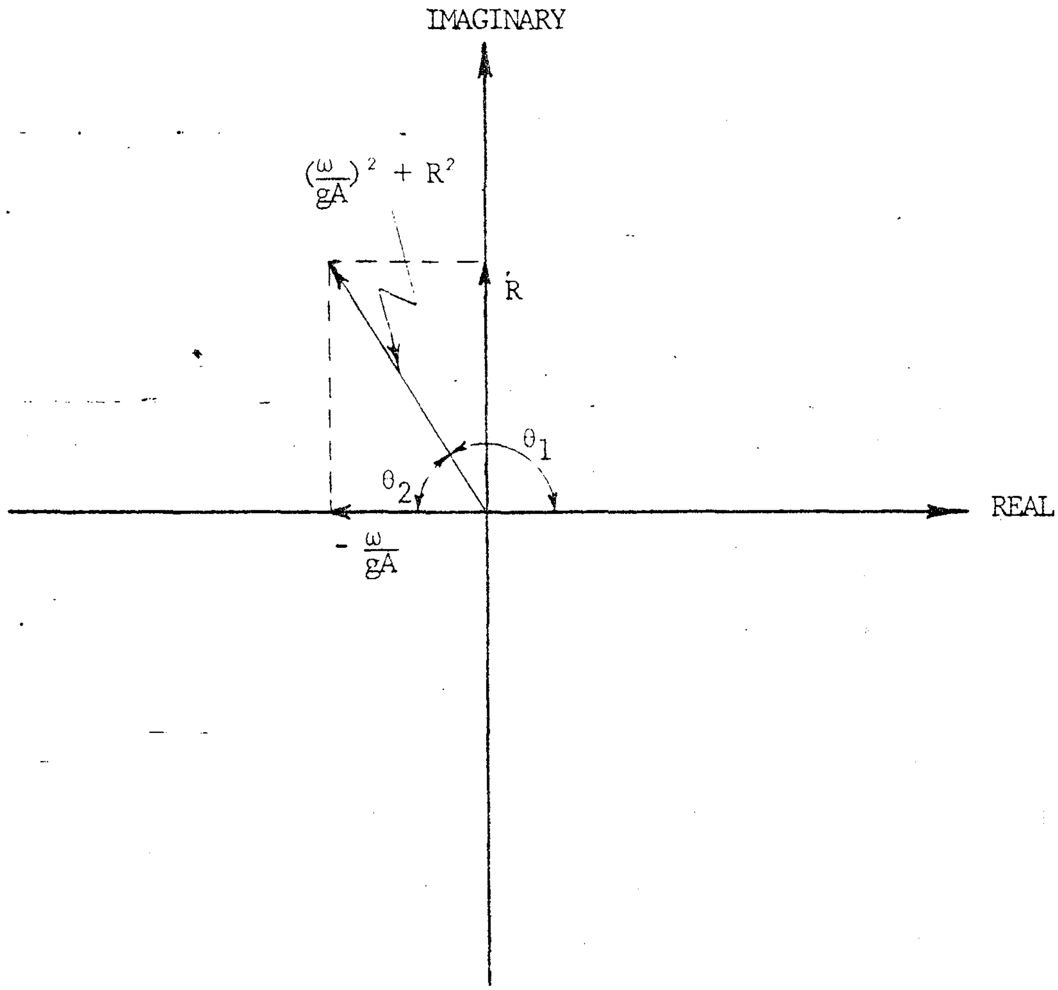

Figure C.1. Axis of Complex Variables.

where α and β are always real, positive numbers. To find the solutions for the oscillatory head and flow, the left side of equation C-12 can be developed as

$$\frac{d^2X}{dx^2} - X\gamma^2 = 0$$

for which the solution for X, is

$$X = C_1 e^{\gamma x} + C_2 e^{-\gamma x} \qquad \text{(C-21)}$$

where C_1 and C_2 are the constants of integration. Substituting equations C-21 and C-13 into equation C-7 and combining the constants, the following equation is obtained:

$$h' = e^{i\omega t}(C_1 e^{\gamma x} + C_2 e^{-\gamma x}) \qquad \text{(C-22)}$$

Taking the position derivative of the above equation and substituting into equations 2-2 and 2-4, then integrating and solving for q', the following equation is obtained:

$$q' = \frac{gA\omega}{ia^2\gamma} e^{i\omega t}(C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \qquad \text{(C-23)}$$

The fluctuating head, h', and the fluctuating flow, q', are functions of t and x, and can be expressed as

$$h'(x,t) = H(x)\ e^{i\omega t}$$

and

$$q'(x,t) = Q(x)\ e^{i\omega t} .$$

Substituting these expressions into equations C-22 and C-23, the following equations are obtained:

$$H(x) = C_1 e^{\gamma x} + C_2 e^{-\gamma x} \qquad \text{(C-24)}$$

$$Q(x) = \frac{gA\omega}{ia^2\gamma}(C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \qquad \text{(C-25)}$$

The ratio of the fluctuating head, h', over the fluctuating flow, q', is defined to be hydraulic impedance, Z(x),

$$Z(x) = \frac{h'}{q'} = -\frac{\gamma a^2}{igA\omega}\,\frac{C_1 e^{\gamma x} + C_2 e^{-\gamma x}}{C_1 e^{\gamma x} - C_2 e^{-\gamma x}} \qquad \text{(C-26)}$$

where the term $\frac{\gamma a^2}{igA\omega}$, depends upon the physical properties of the pipe and is defined to be characteristic impedance, Zc.

$$Zc = \frac{\gamma a^2}{igA\omega} = \frac{a^2}{gA\omega}(\beta - i\alpha) \qquad \text{(C-27)}$$

Using definition C-27 in equation C-25, the following equation results.

$$Q(x) = -\frac{1}{Zc}(C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \qquad \text{(C-28)}$$

Equations C-24 and C-28 are applied to a segment of pipe shown in Figure 2.3 in order to evaluate the integration constants. The boundary conditions at X = 0, are

$$H(0)e^{i\omega t} = H_R \text{ and } Q(0)e^{i\omega t} = Q_R$$

where subscripts R stands for receiving end of the pipe and subscript S refers to the sending end. Applying these boundary conditions to equations C-24 and C-28, the constants can be determined as follows:

$$C_1 = \frac{1}{2}(H_R - ZcQ_R)$$

$$C_2 = \frac{1}{2}(H_R + ZcQ_R)$$

Substituting the values of the above constants into the equations C-24 and C-28 and rearranging, the following equations are obtained:

$$H(x) = H_R \text{Cosh}\gamma x - Q_R Zc \text{Sinh}\gamma x \qquad \text{(C-29)}$$

$$Q(x) = -\frac{H_R}{Zc}\text{Sinh}\gamma x + Q_R \text{Cosh}\gamma x \qquad \text{(C-30)}$$

Using the other boundary conditions at x = L, where

$$H(L)e^{i\omega t} = H_S \text{ and } Q(L)e^{i\omega t} = Q_S,$$

then the following equations are obtained for Figure 2.3.

$$H_S = H_R \text{Cosh}\gamma L - Q_R Zc \text{Sinh}\gamma L \qquad \text{(C-31)}$$

$$Q_S = -\frac{H_R}{Zc}\text{Sinh}\gamma L + Q_R \text{Cosh}\gamma L \qquad \text{(C-32)}$$

If the solutions for H_R and Q_R are desired, then equations C-31 and C-32 can be combined and rearranged to give the following equations:

$$H_R = H_S \text{Cosh}\gamma L + Zc Q_S \text{Sinh}\gamma L \qquad \text{(C-33)}$$

$$Q_R = \frac{H_S}{Zc}\text{Sinh}\gamma L + Q_S \text{Cosh}\gamma L \qquad \text{(C-34)}$$

APPENDIX D

MATRIX GENERATION

In this Appendix, equations 3-10 and 3-11, obtained in Chapter III, are applied to each non-boundary node of a piping network to derive the head-flow solutions for this piping network in the form of simultaneous equations. This solution is then presented in a matrix representation for general piping networks. The piping network adopted for this derivation is shown in Figure D.1. Two flow excitation sources are applied to the node numbers 1 and 7, and two head excitation sources are applied to the node numbers 3 and 5 of this piping network.

Simultaneous Equations

Equations 3-10 and 3-11 are applied to the non-boundary node numbers 1, 2, 4, 6 and 7 of the piping network shown in Figure D.1 and after rearranging, the following ten simultaneous equations are obtained:

$$[(Y_R)_{12}+(Y_R)_{13}+(Y_R)_{14}](H_R)_1-[(Y_I)_{12}+(Y_I)_{13}+(Y_I)_{14}](H_I)_1+ (X_R)_{12}(H_R)_2-(X_I)_{12}(H_I)_2+(X_R)_{14}(H_R)_4-(X_I)_{14}(H_I)_4 = -[(X_R)_{13}(H_R)_3-(X_I)_{13}(H_I)_3-(Q_R)_1] \quad \text{(D-1)}$$

$$[(Y_I)_{12}+(Y_I)_{13}+(Y_I)_{14}](H_R)_1+[(Y_R)_{12}+(Y_R)_{13}+(Y_R)_{14}](H_I)_1+ (X_I)_{12}(H_R)_2+(X_R)_{12}(H_I)_2+(X_I)_{14}(H_R)_4+(X_R)_{14}(H_I)_4 = -[(X_R)_{13}(H_I)_3+(X_I)_{13}(H_R)_3+(Q_I)_1] \quad \text{(D-2)}$$

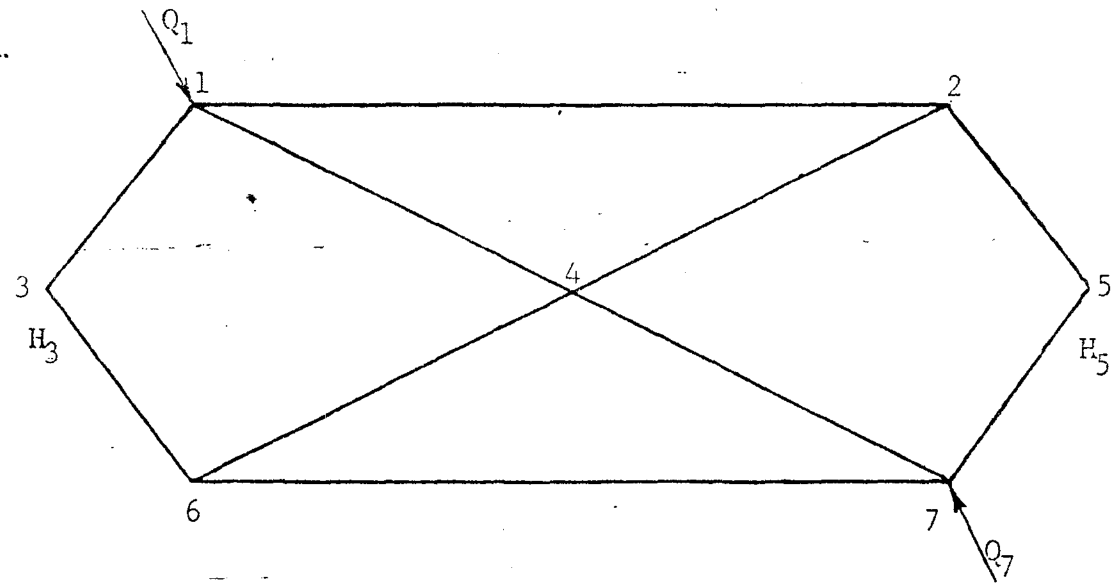

Figure D.1. Schematic Diagram of a Piping Network used to obtain a General Matrix Solution to Piping Networks.

$$(X_R)_{12}(H_R)_1-(X_I)_{12}(H_I)_1+[(Y_R)_{12}+(Y_R)_{24}+(Y_R)_{25}](H_R)_2-$$
$$[(Y_I)_{12}+(Y_I)_{24}+(Y_I)_{25}](H_I)_2+(X_R)_{24}(H_R)_4-(X_I)_{24}(H_I)_4=$$
$$-[(X_R)_{25}(H_R)_5-(X_I)_{25}(H_I)_5] \quad \text{(D-3)}$$

$$(X_I)_{12}(H_R)_1+(X_R)_{12}(H_I)_1+[(Y_I)_{12}+(Y_I)_{24}+(Y_I)_{25}](H_R)_2+$$
$$[(Y_R)_{12}+(Y_R)_{24}+(Y_R)_{25}](H_I)_2+(X_I)_{24}(H_R)_4+(X_R)_{24}(H_I)_4=$$
$$-[(X_R)_{25}(H_I)_5+(X_I)_{25}(H_R)_5] \quad \text{(D-4)}$$

$$(X_R)_{14}(H_R)_1-(X_I)_{14}(H_I)_1+(X_R)_{24}(H_R)_2-(X_I)_{24}(H_I)_2 +$$
$$[(Y_R)_{14}+(Y_R)_{24}+(Y_R)_{46}+(Y_R)_{47}](H_R)_4-[(Y_I)_{14}+(Y_I)_{24} +$$
$$(Y_I)_{46}+(Y_I)_{47}](H_I)_4+(X_R)_{46}(H_R)_6-(X_I)_{46}(H_I)_6+(X_R)_{47}(H_R)_7 -$$
$$(X_I)_{47}(H_I)_7 = 0 \quad \text{(D-5)}$$

$$(X_I)_{14}(H_R)_1+(X_R)_{14}(H_I)_1+(X_I)_{24}(H_R)_2+(X_R)_{24}(H_I)_2 +$$
$$[(Y_I)_{14}+(Y_I)_{24}+(Y_I)_{46}+(Y_I)_{47}](H_R)_4+[(Y_R)_{14}+(Y_R)_{24}+(Y_R)_{46} +$$
$$(Y_R)_{47}](H_I)_4+(X_I)_{46}(H_R)_6+(X_R)_{46}(H_I)_6+(X_I)_{47}(H_R)_7 +$$
$$(X_I)_{47}(H_I)_7 = 0 \quad \text{(D-6)}$$

$$(X_R)_{46}(H_R)_4-(X_I)_{46}(H_I)_4+[(Y_R)_{36}+(Y_R)_{46}+(Y_R)_{67}](H_R)_6 -$$
$$[(Y_I)_{36}+(Y_I)_{46}+(Y_I)_{67}](H_I)_6+(X_R)_{67}(H_R)_7-(X_I)_{67}(H_I)_7 =$$
$$-[(X_R)_{36}(H_R)_3-(X_I)_{36}(H_I)_3] \quad \text{(D-7)}$$

$$(X_I)_{46}(H_R)_4+(X_R)_{46}(H_I)_4+[(Y_I)_{36}+(Y_I)_{46}+(Y_I)_{67}](H_R)_6 +$$
$$[(Y_R)_{36}+(Y_R)_{46}+(Y_R)_{67}](H_I)_6+(X_I)_{67}(H_R)_7+(X_R)_{67}(H_I)_7 =$$
$$-[(X_R)_{36}(H_I)_3+(X_I)_{36}(H_R)_3] \quad \text{(D-8)}$$

$$(X_R)_{47}(H_R)_4-(X_I)_{47}(H_I)_4+(X_R)_{67}(H_R)_6-(X_I)_{67}(H_I)_6 +$$
$$[(Y_R)_{47}+(Y_R)_{57}+(Y_R)_{67}](H_R)_7-[(Y_I)_{47}+(Y_I)_{57}+(Y_I)_{67}](H_I)_7 =$$
$$-[(X_R)_{57}(H_R)_5-(X_I)_{57}(H_I)_5-(Q_R)_7] \quad \text{(D-9)}$$

$$(X_I)_{47}(H_R)_4+(X_R)_{47}(H_I)_4+(X_1)_{67}(H_R)_6+(X_R)_{67}(H_I)_6 +$$
$$[(Y_I)_{47}+(Y_I)_{57}+(Y_I)_{67}](H_R)_7+[(Y_R)_{47}+(Y_R)_{57}+(Y_R)_{67}](H_I)_7 =$$
$$-[(X_R)_{57}(H_I)_5+(X_I)_{57}(H_R)_5-(Q_I)_7] \qquad \text{(D-10)}$$

Matrix Formation

The ten simultaneous equations, D-1 to D-10, indicated in the preceding section, are written in matrix form as shown in Figure D.2. By inspecting this matrix, a general method can be deduced for the construction of a particular matrix for any piping network. These equations are shown in Chapter III.

$$
\begin{pmatrix}
(Y_R)_{12}+(Y_R)_{13}+(Y_R)_{14} & -(Y_I)_{12}-(Y_I)_{13}-(Y_I)_{14} & (X_R)_{12} & -(X_I)_{12} & (X_R)_{14} & -(X_I)_{14} & 0 & 0 & 0 & 0 \\
(Y_I)_{12}+(Y_I)_{13}+(Y_I)_{14} & (Y_R)_{12}+(Y_R)_{13}+(Y_R)_{14} & (X_I)_{12} & (X_R)_{12} & (X_I)_{14} & (X_R)_{14} & 0 & 0 & 0 & 0 \\
(X_R)_{21} & -(X_I)_{21} & (Y_R)_{21}+(Y_R)_{24}+(Y_R)_{25} & -(Y_I)_{21}-(Y_I)_{24}-(Y_I)_{25} & (X_R)_{24} & -(X_I)_{24} & 0 & 0 & 0 & 0 \\
(X_I)_{21} & (X_R)_{21} & (Y_I)_{21}+(Y_I)_{24}+(Y_I)_{25} & (Y_R)_{21}+(Y_R)_{24}+(Y_R)_{25} & (X_I)_{24} & (X_R)_{24} & 0 & 0 & 0 & 0 \\
(X_R)_{41} & -(X_I)_{41} & (X_R)_{42} & -(X_I)_{42} & (Y_R)_{41}+(Y_R)_{42}+(Y_R)_{46}+(Y_R)_{47} & -(Y_I)_{41}-(Y_I)_{42}-(Y_I)_{46}-(Y_I)_{47} & (X_R)_{46} & -(X_I)_{46} & (X_R)_{47} & -(X_I)_{47} \\
(X_I)_{41} & (X_R)_{41} & (X_I)_{42} & (X_R)_{42} & (Y_I)_{41}+(Y_I)_{42}+(Y_I)_{46}+(Y_I)_{47} & (Y_R)_{41}+(Y_R)_{42}+(Y_R)_{46}+(Y_R)_{47} & (X_I)_{46} & (X_R)_{46} & (X_I)_{47} & (X_R)_{47} \\
0 & 0 & 0 & 0 & (X_R)_{64} & -(X_I)_{64} & (Y_R)_{63}+(Y_R)_{64}+(Y_R)_{67} & -(Y_I)_{63}-(Y_I)_{64}-(Y_I)_{67} & (X_R)_{67} & -(X_I)_{67} \\
0 & 0 & 0 & 0 & (X_I)_{64} & (X_R)_{64} & (Y_I)_{63}+(Y_I)_{64}+(Y_I)_{67} & (Y_R)_{63}+(Y_R)_{64}+(Y_R)_{67} & (X_I)_{67} & (X_R)_{67} \\
0 & 0 & 0 & 0 & (X_R)_{74} & -(X_I)_{74} & (X_R)_{76} & -(X_I)_{76} & (Y_R)_{74}+(Y_R)_{75}+(Y_R)_{76} & -(Y_I)_{74}-(Y_I)_{75}-(Y_I)_{76} \\
0 & 0 & 0 & 0 & (X_I)_{74} & (X_R)_{74} & (X_I)_{76} & (X_R)_{76} & (Y_I)_{74}+(Y_I)_{75}+(Y_I)_{76} & (Y_R)_{74}+(Y_R)_{75}+(Y_R)_{76}
\end{pmatrix}
\begin{pmatrix}
(H_R)_1 \\ (H_I)_1 \\ (H_R)_2 \\ (H_I)_2 \\ (H_R)_4 \\ (H_I)_4 \\ (H_R)_6 \\ (H_I)_6 \\ (H_R)_7 \\ (H_I)_7
\end{pmatrix}
=
\begin{pmatrix}
-(X_R)_{13}(H_R)_3-(X_I)_{13}(H_I)_3+(Q_R)_1 \\
-(X_I)_{13}(H_R)_3-(X_R)_{13}(H_I)_3+(Q_I)_1 \\
-(X_R)_{25}(H_R)_5-(X_I)_{25}(H_I)_5 \\
-(X_I)_{25}(H_R)_5-(X_R)_{25}(H_I)_5 \\
0 \\
0 \\
-(X_R)_{63}(H_R)_3-(X_I)_{63}(H_I)_3 \\
-(X_I)_{63}(H_R)_3-(X_R)_{63}(H_I)_3 \\
-(X_R)_{75}(H_R)_5-(X_I)_{75}(H_I)_5+(Q_R)_7 \\
-(X_I)_{75}(H_R)_5-(X_R)_{75}(H_I)_5+(Q_I)_7
\end{pmatrix}
$$

Figure D.2. Solution Matrix for the Piping Network Shown in Figure D.1.

APPENDIX E

LISTING OF COMPUTER PROGRAM

```
10**RUN *06; /10804/D.1"05"
20C
30C          THIS PROGRAM CALCULATES THE HEAD AND FLOW
40C       AMPLITUDES AND THEIR PHASE ANGLES WITH THE FIRST
50C       ENTERED EXCITING SOURCE AT ANY DESIRED LOCATION
60C       IN A LARGE STEADY OSCILLATORY NETWORK OF PIPING.
70C
80        REAL PPX(501)/501*0./,ZTX(501)/501*0./,P(20)/20*0./
90        REAL ZQX(501)/501*0./,XI(20,20)/400*0./,QGUS(20,20)
100       REAL TET(20)/20*0./,YI(20,20)/400*0./,PI(20)/20*0./
110       REAL QFI(20,20)/400*0./,QR(20)/20*0./,GI(20)/20*0./
120       REAL QFR(20,20)/400*0./,HH(40)/40*0./,PR(20)/20*0./
130       REAL XR(20,20)/400*0./,YR(20,20)/400*0./,CC1(20,20)
140       REAL AF(20,20)/400*0./,GA(20,20)/400*0./,CC2(20,20)
150       REAL QF(20,20)/400*0./,RP(20,20)/400*0./,CC3(20,20)
160       REAL ZET(20,20)/400*0./,GAA(20,20)/400*0./,SCR(40)
170       REAL AA(40,41)/1640*0./,QQX(501)/501*0./,CC4(20,20)
180       REAL L(20,20)/400*0./,R(20,20)/400*0./
190       DIMENSION NP(20,20)/400*0/,QBAR(20,20)/400*0./
200       LOGICAL HEAD(20)/20*.F./,CONN(20,20)/400*.F./
210C
220C         READS AND PRINTS THE INPUT DATA, AND MAKES SOME
230C      PRE-CALCULATIONS. (UPTO LINE NO. 1000)
240C
250       READ(5,470)AMU,RHO,GG
260       WRITE(6,760)AMU,RHO,GG
270       ANU=32.174*AMU/RHO
280       EPS=1.E-15
290       PY=3.1415927
300       READ(5,480)N,NCH,KLM
310       WRITE(6,770)N,NCH,KLM
320       IF(NCH.LT.1) GO TO 30
330       DO 20 I=1,NCH
340       READ(5,480)J
350       WRITE(6,670)J
360    20 HEAD(J)=.T.
370    30 READ(5,480)NFX,NSP
380       WRITE(6,780)NFX,NSP
390       NREF=-1
400       IF(NFX.LE.0) GO TO 70
410       DO 60 I=1,NFX
420       READ(5,490)J,P(J)
430       HEAD(J)=.T.
440       IF(I.GT.1) GO TO 40
450       NREF=J
460       KD=5
470       PR(J)=P(J)
480       GO TO 50
490    40 READ(5,500)TET(J)
500       PR(J)=P(J)*COS(TET(J))
510       PI(J)=P(J)*SIN(TET(J))
520    50 WRITE(6,790)J,P(J),TET(J)
530    60 CONTINUE
```

```
   70 NFX=NFX+NCH
      IF(NSP.LE.0) GO TO 110
      DO 100 I=1,NSP
      IF(NREF.GT.0) GO TO 80
      READ(5,510)J,Q
      NREF=J
      RAD=0.
      KD=0
      GO TO 90
   80 READ(5,520)J,Q,RAD
   90 QR(J)=Q*COS(RAD)
      QI(J)=Q*SIN(RAD)
      WRITE(6,800)J,Q,RAD
  100 CONTINUE
      WRITE(6,660)
  110 MYT=0
      DO 140 MLK=1,KLM
      READ(5,530)I,J,L(I,J),DDDD,A,AF(I,J),NP(I,J),DOM
      WRITE(6,810)I,J,L(I,J),DDDD,A,AF(I,J),NP(I,J),DOM
      QBAR(I,J)=DOM
      CONN(I,J)=.T.
      CONN(J,I)=CONN(I,J)
      L(J,I)=L(I,J)
      NP(J,I)=NP(I,J)
      QBAR(J,I)=QBAR(I,J)
      D=DDDD/12.
      QGUS(I,J)=QBAR(I,J)+20.
      QGUS(J,I)=QGUS(I,J)
      QQ=QGUS(I,J)
      RP(I,J)=16./(PY*GG*D**4)
      RP(J,I)=RP(I,J)
      AF(J,I)=AF(I,J)
      RE=4.*QQ/(PY*D*ANU)
      IF(RE.GT.2200.) GO TO 120
      R(I,J)=8.*RP(I,J)*ANU
      GO TO 130
  120 R(I,J)=RP(I,J)*AF(I,J)*QQ/(PY*D)
  130 GA(I,J)=PY*GG*D*D/4.
      GAA(I,J)=GA(I,J)/(A*A)
      GA(J,I)=GA(I,J)
      GAA(J,I)=GAA(I,J)
      R(J,I)=R(I,J)
      MYT=MYT+NP(I,J)
  140 CONTINUE
      WRITE(6,660)
      READ(5,540)LLLL,OMG,DOMG
      WRITE(6,320)LLLL,OMG,DOMG
      IF(KD.LT.2) GO TO 150
      WRITE(6,680)NREF
      GO TO 160
  150 WRITE(6,690)NREF
  160 DIV=REAL(KLM)
C
```

```
C         THIS DO-LOOP "460", MAKES COMPLETE CALCULATION
C     AND PRINTS THE RESULTS FOR EACH FREQUENCY.
C
      DO 460 MOG=1,LLLL
      HUM=0.
      ICC=0
      ICONT=0
      IF(MYT.GT.0) GO TO 170
      WRITE(6,700) OMG
      GO TO 180
  170 WRITE(6,710) OMG
      WRITE(6,720)
C
C         THIS DO-LOOP "190", CALCULATES THE REAL AND THE
C     IMAGINARY PARTS OF X AND Y FOR THE LINE SEGMENTS IN
C     THE NETWORK.
C
  180 DO 190 I=2,N
      DO 190 J=1,I-1
      IF(.NOT.CONN(I,J)) GO TO 190
      AL=L(I,J)
      AR=R(I,J)
      AGA=GA(I,J)
      AGB=GAA(I,J)
      CALL X(AL,AR,AGA,AGB,OMG,C1,C2,C3,C4,C5,C6,ZCR,ZCI)
      CC1(I,J)=C1
      CC2(I,J)=C2
      CC3(I,J)=C3
      CC4(I,J)=C4
      CC1(J,I)=C1
      CC2(J,I)=C2
      CC3(J,I)=C3
      CC4(J,I)=C4
      DENOM=(ZCR**2+ZCI**2)*(C1**2+C2**2)
      XR(I,J)=(ZCR*C1-ZCI*C2)/DENOM
      XI(I,J)=(ZCR*C2+ZCI*C1)/(-DENOM)
      YR(I,J)=(ZCI*C6-ZCR*C5)/DENOM
      YI(I,J)=(ZCI*C5+ZCR*C6)/DENOM
      XR(J,I)=XR(I,J)
      XI(J,I)=XI(I,J)
      YR(J,I)=YR(I,J)
      YI(J,I)=YI(I,J)
  190 CONTINUE
C
C         THIS DO-LOOP "240" AND DO-LOOP "250" USE THE
C     RESULTING EQUATIONS FROM CHAPTER III AND BUILD THE
C     AUGMENTED MATRIX.
C
      M=0
      DO 240 I=1,N
      IF(HEAD(I)) GO TO 240
      M=M+2
      SUMR=0.
```

```
      SUMI=0.
      MM=0
      HH(M)=-QI(I)
      HH(M-1)=-QR(I)
      DO 230 J=1,N
      IF(I.EQ.J) MM=MM+2
      IF(I.EQ.J) GO TO 230
      IF(HEAD(J)) GO TO 200
      MM=MM+2
      AA(M,MM)=XR(I,J)
      AA(M-1,MM-1)=XR(I,J)
      AA(M,MM-1)=XI(I,J)
      AA(M-1,MM)=-XI(I,J)
  200 IF(.NOT.CONN(I,J)) GO TO 210
      SUMR=SUMR+YR(I,J)
      SUMI=SUMI+YI(I,J)
  210 IF(HEAD(J).AND.CONN(I,J)) GO TO 220
      GO TO 230
  220 HH(M)=HH(M)+(XR(I,J)*PI(J)+XI(I,J)*PR(J))
      HH(M-1)=HH(M-1)+(XR(I,J)*PR(J)-XI(I,J)*PI(J))
  230 CONTINUE
      AA(M,M)=SUMR
      AA(M-1,M-1)=SUMR
      AA(M-1,M)=-SUMI
      AA(M,M-1)=SUMI
  240 CONTINUE
      NN=2*(N-NFX)
      NNN=NN+1
      DO 250 I=1,NN
      AA(I,NNN)=-HH(I)
  250 CONTINUE
      CALL MTINV(AA,NN,NNN,40,SCR)
C
C          AFTER MATRIX ARE SOLVED. THE DO-LOOP "260"
C     PLACES THE RESULTS INTO THEIR LOCATIONS.
C
      M=0
      DO 260 I=1,N
      IF(HEAD(I)) GO TO 260
      M=M+2
      PR(I)=AA(M-1,NNN)
      PI(I)=AA(M,NNN)
  260 CONTINUE
C
C          DO-LOOP "300" CALCULATES THE HEAD AND FLOW
C     AMPLITUDES AND THEIR PHASE ANGLES FOR EACH NODE.
C
      DO 300 I=1,N
      IF(ICC.LE.0) GO TO 280
      P(I)=SQRT(PR(I)**2+PI(I)**2)
      IF(ABS(PR(I)).LE.EPS) GO TO 270
      TET(I)=ATAN(PI(I)/PR(I))
      IF(PR(I).LT.0..AND.PI(I).LT.0.) TET(I)=TET(I)-PY
```

```
      IF(PR(I).LT.0..AND.PI(I).GT.0.) TET(I)=TET(I)+PY
      GO TO 280
  270 IF(PI(I).GT.0.) TET(I)=PY/2.
      IF(PI(I).LT.0.) TET(I)=-0.5*PY
  280 DO 300 J=1,N
      IF(.NOT.CONN(I,J)) GO TO 300
      AR=PR(J)*XR(I,J)+PR(I)*YR(I,J)
      AI=PI(I)*YR(I,J)+PR(I)*YI(I,J)
      QFR(I,J)=PI(J)*XI(I,J)+PI(I)*YI(I,J)-AR
      QFI(I,J)=-XR(I,J)*PI(J)-XI(I,J)*PR(J)-AI
      QF(I,J)=SQRT(QFR(I,J)**2+QFI(I,J)**2)
      IF(ICC.LE.0) GO TO 300
      IF(ABS(QFR(I,J)).LE.EPS) GO TO 290
      ZET(I,J)=ATAN(QFI(I,J)/QFR(I,J))
      DOM=ZET(I,J)
      IF(QFR(I,J).LT.0..AND.QFI(I,J).LT.0.)DOM=DOM-PY
      IF(QFR(I,J).LT.0..AND.QFI(I,J).GT.0.)DOM=DOM+PY
      ZET(I,J)=DOM
      GO TO 300
  290 IF(QFI(I,J).GT.0.) ZET(I,J)=PY/2.
      IF(QFI(I,J).LT.0.) ZET(I,J)=-0.5*PY
  300 CONTINUE
      IF(ICC.LE.0) GO TO 330
C
C          DO-LOOP "320" PRINTS THE TABULATED RESULTS FOR
C     THE NETWORK AT ONE FREQUENCY.
C
      DO 320 I=1,N
      NJ=0
      DO 320 J=1,N
      IF(.NOT.CONN(I,J)) GO TO 320
      IF(NJ.GT.0) GO TO 310
      WRITE(6,730)I,P(I),TET(I),J,QF(I,J),ZET(I,J)
      NJ=NJ+1
      GO TO 320
  310 WRITE(6,740)J,QF(I,J),ZET(I,J)
  320 CONTINUE
      WRITE(6,550)
  330 NCH=0
C
C     DO-LOOP "440"
C          CALCULATES THE FLOW AMPLITUDES AT THE DESIRED
C     LOCATIONS IN THE NETWORK INORDER TO FIND NEW
C     ABSOLUTE FLOWS, IF ICC=0 , OR
C          CALCULATES THE FLOW AND HEAD AMPLITUDES, THEIR
C     PHASE ANGLES, AND PRINTS THE RESULTS ON THE LINE
C     SEGMENTS, IF ICC > 0 .
C
      SCOR=0.
      DO 440 J=2,N
      DO 440 I=1,J-1
      IF(.NOT.CONN(I,J)) GO TO 440
      IF(AF(I,J).LE.0..AND.ICC.LE.0) GO TO 440
```

```
      IF(NP(I,J).LE.0) GO TO 390
      AD=REAL(NP(I,J)+1)
      DL=L(I,J)/AD
      AL=0.
      MR=NP(I,J)+2
      DO 350 NNP=1,MR
      AR=R(I,J)
      AGA=GA(I,J)
      AGB=GAA(I,J)
      O=OMG
      CALL X(AL,AR,AGA,AGB,O,C1,C2,C3,C4,DM1,DM2,DM3,DM4)
      C11=CC1(I,J)
      C12=CC2(I,J)
      C13=CC3(I,J)
      C14=CC4(I,J)
      P1=PR(J)
      P2=PI(J)
      P3=PR(I)
      P4=PI(I)
      IF(ICC.LE.0) GO TO 340
      CALL YYYY(P1,P2,P3,P4,C1,C2,C3,C4,C11,C12,C13,C14,
     6DM3,DM4,P5,Z5,5)
C
      PPX(NNP)=P5
      ZTX(NNP)=Z5
      IF(P5.LE.HQM) GO TO 340
      HQM=P5
      IHQ=I
      JHQ=J
      AHQ=AL
  340 CALL YYYY(P1,P2,P3,P4,C4,C3,C2,C1,C11,C12,C13,C14,
     6DM3,DM4,Q5,Z6,ICC)
C
      QQX(NNP)=Q5
      ZQX(NNP)=Z6
      AL=AL+DL
  350 CONTINUE
      IF(ICC.LE.0) GO TO 390
      II1=I
      JJ1=J
      NAA=NP(I,J)+1
      IF(NAA.GT.11) GO TO 360
      WRITE(6,610)
      WRITE(6,560) (PPX(NNP),NNP=1,MR)
      WRITE(6,570) (ZTX(NNP),NNP=1,MR)
      CALL PIPE(NAA,II1,JJ1,0.)
      WRITE(6,580) (QQX(NNP),NNP=1,MR)
      WRITE(6,570) (ZQX(NNP),NNP=1,MR)
      GO TO 440
  360 WRITE(6,610)
      WRITE(6,560) (PPX(NNP),NNP=1,11)
      WRITE(6,570) (ZTX(NNP),NNP=1,11)
      WRITE(6,590) I
```

```
      WRITE(6,580) (QQX(NNP),NNP=1,11)
      WRITE(6,570) (ZQX(NNP),NNP=1,11)
      MMA=22
  370 MAA=MMA-10
      IF(NAA.LE.MMA) GO TO 380
      WRITE(6,600)
      WRITE(6,620)(PPX(K),K=MAA,MMA),(ZTX(K),K=MAA,MMA)
      WRITE(6,630)
      WRITE(6,620)(QQX(K),K=MAA,MMA),(ZQX(K),K=MAA,MMA)
      MMA=MMA+11
      GO TO 370
  380 WRITE(6,600)
      WRITE(6,640)(PPX(NNP),NNP=MAA,MR)
      WRITE(6,650)(ZTX(NNP),NNP=MAA,MR)
      NOPT=NAA-MAA+1
      CALL PIPE(NOPT,II1,JJ1,1.)
      WRITE(6,640)(QQX(NNP),NNP=MAA,MR)
      WRITE(6,650)(ZQX(NNP),NNP=MAA,MR)
      GO TO 440
  390 IF(QF(J,I).GT.QF(I,J)) GO TO 391
      QMAX=QF(I,J)
      QMIN=QF(J,I)
      GO TO 392
  391 QMAX=QF(J,I)
      QMIN=QF(I,J)
  392 IF(NP(I,J).LE.0) GO TO 410
      DO 400 NDO=1,MR
      IF(QQX(NDO).GT.QMAX)QMAX=QQX(NDO)
      IF(QQX(NDO).LT.QMIN)QMIN=QQX(NDO)
  400 CONTINUE
  410 QQ=QBAR(I,J)+.1926*(QMAX+QMIN)
      D=2./(PY*GG*RP(I,J))**.25
      RE=4.*QQ/(PY*D*ANU)
      IF(RE.GT.2200.) GO TO 420
      RG=8.*RP(I,J)*ANU
      GO TO 430
  420 RG=RP(I,J)*AF(I,J)*QQ/(PY*D)
  430 QGS=ABS(QGUS(I,J)-QQ)
      IF(QGS.GT..001) NCH=NCH+1
      SCOR=SCOR+QGS
      QGUS(I,J)=(QGUS(I,J)+2.*QQ)/3.
      QGUS(J,I)=QGUS(I,J)
      R(I,J)=(R(I,J)+2.*RG)/3.
      R(J,I)=R(I,J)
  440 CONTINUE
C
C         THIS PART OF THE PROGRAM CHECKS FOR THE
C     ACCURACIES.
C
      ICONT=ICONT+1
      IF(ICC.GT.0) GO TO 450
      SCOR=SCOR/DIV
      IF(NCH.LT.1.AND.SCOR.LT..0005) ICC=1
```

```
      GO TO 180
  450 WRITE(6,750)ICONT,HOM,AHQ,IHQ,IHQ,JHG
      OMG=OMG+DOMG
  460 CONTINUE
C
  470 FORMAT(5X,F10.7,F6.2,F6.3)
  480 FORMAT(5X,3I2)
  490 FORMAT(5X,I2,F10.3)
  500 FORMAT(5X,F6.2)
  510 FORMAT(5X,I2,F10.6)
  520 FORMAT(5X,I2,F10.6,F6.2)
  530 FORMAT(5X,2I2,F8.2,F6.2,F5.0,F6.4,I3,F10.6)
  540 FORMAT(5X,I3,2F10.0)
  550 FORMAT(2("!---------!------------!---------"),"!"/)
  560 FORMAT("HD AMP    ",12(F8.3,1X))
  570 FORMAT("PH ANG",5X,12(F5.2,4X))
  580 FORMAT("FW AMP    ",12(F8.3,1X))
  590 FORMAT(3X,"NODE(",I2,")  ",11("O========"))
  600 FORMAT(//)
  610 FORMAT(////)
  620 FORMAT(9X,11(F8.3,1X)/11X,11(F5.2,4X))
  630 FORMAT(13X,11("O========"))
  640 FORMAT(9X,12(F8.3,1X))
  650 FORMAT(11X,12(F5.2,4X))
  660 FORMAT(2(20X,"*",90X,"*"/))
  670 FORMAT(20X,"*",5X,"CONSTANT HEAD NODE NO. =",I3,58X
     6,"*")
C
  680 FORMAT(///10X,"ALL THE PHASE ANGLES ARE COMPARED"/
     6 10X,"TO THE HEAD AT NODE NO. ",I2)
C
  690 FORMAT(///10X,"ALL THE PHASE ANGLES ARE COMPARED"/
     6 10X,"TO THE FLOW AT NODE NO. ",I2)
C
  700 FORMAT(1H1///21X,"FREQUENCY=",F6.3,"  RAD/SEC"/67(
     6"=")/"!   NODE   !     HEAD     !  PHASE  ! FLOW TO !"
     6,"    FLOW     !  PHASE  !"/"!     NO    !  AMPLITUDE"
     6," !  ANGLE  ! NODE NO !  AMPLITUDE !  ANGLE  !"/
     6"!          !     FEET     !   RAD   !         ! CUBI"
     6,"C FT/S !   RAD   !"/2("!---------!------------!-"
     6,"--------"),"!")
C
  710 FORMAT(1H1///21X,"FREQUENCY=",F6.3,"  RAD/SEC",30X,
     6"NOTE:"/67("="),9X,5("-")/"!   NODE   !     HEAD     "
     6,"!  PHASE  ! FLOW TO !    FLOW     !  PHASE  !",12X
     6,"DEFINITIONS OF FLOW DIRECTIONS"/"!     NO    !   AM"
     6,"PLITUDE !  ANGLE  ! NODE NO !  AMPLITUDE !  ANGL"
     6,"E  !",12X,"MAY BE DIFFERENT BY ONE PI IN")
C
  720 FORMAT("!          !     FEET     !   RAD   !         ",
     6"  ! CUBIC FT/S !   RAD   !",12X,"THE TABLE AT THE"
     6," LEFT AND THE"/2("!---------!------------!------"
     6,"---"),"!",12X,"DISTRIBUTIONS PRINTED BELOW.")
```

```
 730 FORMAT(2("!              !              !              "),"!"/
&    "!   ",I4,"    ! ",F10.3," ! ",F6.3,"   !    ",I2,3X,
&    "! ",E10.4," ! ",F6.3,"  !"/2("!",9X,"!",12X,"!",9X
&    ),"!")
C
 740 FORMAT("!              !              !              !    ",I2,
&    3X,"! ",E10.4," ! ",F6.3,"  !"/2("!              !    "
&    ,"      !   ",7X),"!")
C
 750 FORMAT(//9X,"NO. OF TRIAL = ",I3/9X,"FLOW DIRECTIO"
&    ,"N  ------>"/9X,"MAXIMUM HEAD AMPLITUDE RECORDED "
&    ,"= ",F9.3,"  FEET"/9X,"LOCATION OF MAX. HEAD: ",
&    F7.0,"  FEET FROM NODE(",I2," )  ON LINE(",I2,"-",
&    I2," )"///)
C
 760 FORMAT(58X,"(( INPUT DATA ))"/20X,92("*")/20X,"*",
&    17X,"ABS. VISC. = ",F10.7,50X,"*"/20X,"*",17X,"SPE"
&    ,"C. MASS = ",F6.2,54X,"*"/20X,"*",26X,"G = ",F6.3,
&    54X,"*")
C
 770 FORMAT(20X,"*",15X,"NO. OF NODES = ",I2,58X,"*"/20X
&    ,"* NO. OF CONSTANT HEAD NODES = ",I2,58X,"*"/20X,
&    "*",7X,"NO. OF LINE SEGMENTS = ",I2,58X,"*")
C
 780 FORMAT(20X,"*",5X,"NO. OF HEAD OSC. NODES = ",I2,
&    58X,"*"/20X,"*",5X,"NO. OF FLOW OSC. NODES = ",I2,
&    58X,"*")
C
 790 FORMAT(20X,"*",9X,"HEAD OSC. NODE NO. = ",I2,6X,
&    "HEAD AMP. = ",F10.3,6X,"PH. ANGLE = ",F6.2,6X,"*")
C
 800 FORMAT(20X,"*",9X,"FLOW OSC. NODE NO. = ",I2,6X,
&    "FLOW AMP. = ",F13.8,5X,"PH. ANGLE = ",F6.2,4X,"*")
C
 810 FORMAT(20X,"* LINE(",I2,"-",I2,"):     L=",F6.0,4X,
&    "D=",F6.2,4X,"A=",F5.0,4X,"F=",F6.4,4X,"NP=",I3,4X,
&    "QBAR=",F10.6," *")
C
 820 FORMAT(20X,"*",5X,"RUNNING FOR ",I3,"  DIFFERENT ",
&    "FREQUENCIES,      1ST. W= ",F11.7,4X,"DW= ",F11.7,
&    3X,"*"/2(20X,"*",90X,"*"/),20X,92("*"))
C
     STOP
     END
```

```
C     - THIS SUBROUTINE INVERTS AND SOLVES THE MATRIX.
C
      SUBROUTINE MTINV(A,NRARG,NCARG,IDIM,LABEL)
      DIMENSION A(IDIM,NCARG),LABEL(NRARG)
      NR=NRARG
      NC=NCARG
      DO 10 J1=1,NR
   10 LABEL(J1)=J1
      DO 80 J1=1,NR
      TEMP=0.0
      DO 20 J2=J1,NR
      IF(ABS(A(J2,J1)).LT.TEMP) GO TO 20
      TEMP=ABS(A(J2,J1))
      IBIG=J2
   20 CONTINUE
      IF(IBIG.EQ.J1)GO TO 40
      DO 30 J2=1,NC
      TEMP=A(J1,J2)
      A(J1,J2)=A(IBIG,J2)
   30 A(IBIG,J2)=TEMP
      I=LABEL(J1)
      LABEL(J1)=LABEL(IBIG)
      LABEL(IBIG)=I
   40 TEMP=A(J1,J1)
      A(J1,J1)=1.0
      DO 50 J2=1,NC
   50 A(J1,J2)=A(J1,J2)/TEMP
      DO 70 J2=1,NR
      IF(J2.EQ.J1) GO TO 70
      TEMP=A(J2,J1)
      A(J2,J1)=0.0
      DO 60 J3=1,NC
   60 A(J2,J3)=A(J2,J3)-TEMP*A(J1,J3)
   70 CONTINUE
   80 CONTINUE
      N1=NR-1
      DO 120 J1=1,N1
      DO 90 J2=J1,NR
      IF(LABEL(J2).NE.J1) GO TO 90
      IF(J2.EQ.J1) GO TO 120
      GO TO 100
   90 CONTINUE
  100 DO 110 J3=1,NR
      TEMP=A(J3,J1)
      A(J3,J1)=A(J3,J2)
  110 A(J3,J2)=TEMP
      LABEL(J2)=LABEL(J1)
  120 CONTINUE
      RETURN
      END
```

```
C     THIS SUBROUTINE CALCULATES THE CHARACTERISTIC
C     IMPEDANCE, TRIGONOMETRIC AND HYPERBOLIC FUNCTIONS.
C
      SUBROUTINE X(AL,AR,A,B,OMG,C1,C2,C3,C4,C5,C6,ZR,ZI)
      B1=.5*ATAN(AR*A/OMG)
      B2=SQRT(B*OMG)*((OMG/A)**2+AR*AR)**.25
      B3=SIN(B1)
      B4=COS(B1)
      ALP=B2*B3
      BET=B2*B4
      ZR=BET/(B*OMG)
      ZI=-ALP/(B*OMG)
      YY=EXP(ALP*AL)
      SINH=.5*(YY-1./YY)
      COSH=.5*(YY+1./YY)
      SI=SIN(BET*AL)
      CO=COS(BET*AL)
      C1=SINH*CO
      C2=COSH*SI
      C3=SINH*SI
      C4=COSH*CO
      C5=COSH*SINH
      C6=CO*SI
      RETURN
      END
```

```
C     THIS SUBROUTINE DRAWS THE PROPER SIZE LINE SEGMENTS
C     FOR SOME OF THE PRINT OUTS IN DO-LOOP "440".
      SUBROUTINE PIPE(N,I,J,D)
C
      CHARACTER*2 M(56),L
      DO 5 K=2,49
    5 M(K)="=="
      M(1)="O="
      M(5)="=O"
      M(52)=" N"
      M(53)="OD"
      M(54)="E("
      M(56)=")  "
      M(10)=M(1)
      M(19)=M(1)
      M(28)=M(1)
      M(37)=M(1)
      M(46)=M(1)
      M(14)=M(5)
      M(23)=M(5)
      M(32)=M(5)
      M(41)=M(5)
      M(50)=M(5)
      M(51)="  "
      ENCODE(L,20)J
      M(55)=L
      NX=(N*9+1)/2
      NC=N/2
      XN=REAL(NC)
      XM=REAL(N)
      XM=XM/2.
      X=XM-XN
      IF(X.LT..1)M(51)="O "
      IF(D.GT.0.) GO TO 10
      WRITE(6,30)I,(M(K),K=1,NX),(M(KK),KK=51,56)
      RETURN
   10 WRITE(6,40)(M(K),K=1,NX),(M(KK),KK=51,56)
   20 FORMAT(I2)
   30 FORMAT(3X,"NODE(",I2,")   ",56A2)
   40 FORMAT(13X,56A2)
      RETURN
      END
```

```
C     THIS SUBROUTINE CALCULATES THE HEAD OR FLOW
C     AMPLITUDE WITH ITS PHASE ANGLE AT THE DESIRED
C     LOCATION IN THE NETWORK.
C
      SUBROUTINE YYYY(P1,P2,P3,P4,C1,C2,C3,C4,C5,C6,C7,C8
     ,D3,D4,X,Y,I)
C
      V=3.1415927
      EPS=1.E-15
      A=C5*C5+C6*C6
      B=(C1*C5+C2*C6)/A
      C=(C2*C5-C1*C6)/A
      D=C8*B-C7*C
      E=C8*C+C7*B
      F=C4-D
      O=C3-E
      G=P1*B-P2*C
      P=P1*C+P2*B
      H=P3*F-P4*O
      U=P4*F+P3*O
      R=H+G
      S=U+P
      IF(I.GT.2) GO TO 10
      DM=D3*D3+D4*D4
      T=-(R*D3+S*D4)/DM
      S=(R*D4-S*D3)/DM
      R=T
   10 X=SQRT(R*R+S*S)
      Y=0.
      IF(I.LE.0) RETURN
      IF(ABS(R).LE.EPS) GO TO 20
      Y=ATAN(S/R)
      IF(R.LT.0..AND.S.LT.0.) Y=Y-V
      IF(R.LT.0..AND.S.GT.0.) Y=Y+V
      RETURN
   20 IF(S.GT.0.) Y=V/2.
      IF(S.LT.0.) Y=-V/2.
      RETURN
      END
```

APPENDIX F

METHOD OF CALCULATING THE LINEARIZED FRICTION TERM FOR STEADY OSCILLATORY FLOW

A method to calculate the linearized friction term for steady oscillatory flow is presented in Chapter IV and was used to obtain the results of this study. This Appendix presents another method of approach which was derived during the last days of this study.

Average Steady Oscillatory Flow

Steady oscillatory flow through a pipe is a function of position and time. Assuming sinusoidal variation of the flow with respect to time, the following equation may be written:

$$Q(x,t) = Q(x)\ \mathrm{Sin}(\omega t) \qquad \text{(F-1)}$$

The time average of the flow may be found if equation F-1 is integrated over a range of 0 to $\frac{\pi}{2}$ and divided by $\frac{\pi}{2}$ as shown below.

$$\overline{Q}(x) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} Q(x)\ \mathrm{Sin}(\omega t)\ d(\omega t) = \frac{2}{\pi}\, Q(x) \qquad \text{(F-2)}$$

The average flow over the length of the pipe may be found if the flow distributions along the length of the pipe are integrated over the length. Since the flow distributions are different for different frequencies, and it is not convenient to have a separate averaging routine for each frequency, a method is developed to give an approximation of the average flow for any frequency. Tests presented in Table 4.1, showed that the linearized friction term is important only for excitation near the resonant frequency. Assuming the flow distributions for the frequencies close to the resonances are the same as they are for resonance, then a method to calculate the average flow can be developed for the resonant frequencies (which are the critical frequencies), and subsequently used for any frequency. Since the flow distributions for resonance of different harmonics are different, the sample problem defined in Chapter IV was tested at excitation frequencies of $(2n-1)\ \omega_R$, for $n = 1, 2, 3, 4$, where $\omega_R = \frac{2\pi a}{4L}$, is the natural frequency of the system. Figure F.1 shows the flow distributions along the non-dimensional length of the pipe for different values of n. The excitation source is placed at $x = 0$, and the tank is connected at $x = 1$. This family of curves has the same value of $\frac{Q(x)}{(Qmax)n}$ at any location of $\frac{x}{(2n-1)}$. If these curves passed through the origin of the figure, they would all repeat with periods of $\frac{4}{(2n-1)}$, and would have the same average value as a function of their maximum values. It is assumed that these smooth parts of the curves are straight lines connecting the location $\frac{1}{5(2n-1)}$ of the curves to the origin. The areas between these straight lines and the corresponding curves were neglected and will be considered later in this section. Flow distributions between $x = 0$ and $x = \frac{1}{(2n-1)}$, may be defined as

$$Q(x) = (Qmax)_n\ (aY+bY^2+\ldots\ldots\ldots+jY^{10}) \qquad \text{(F-3)}$$

where $Y = \frac{x}{(2n-1)}$. Substituting equation F-3 into equation F-2 and integrating over the range of 0 to $\frac{1}{(2n-1)}$, the following equation results:

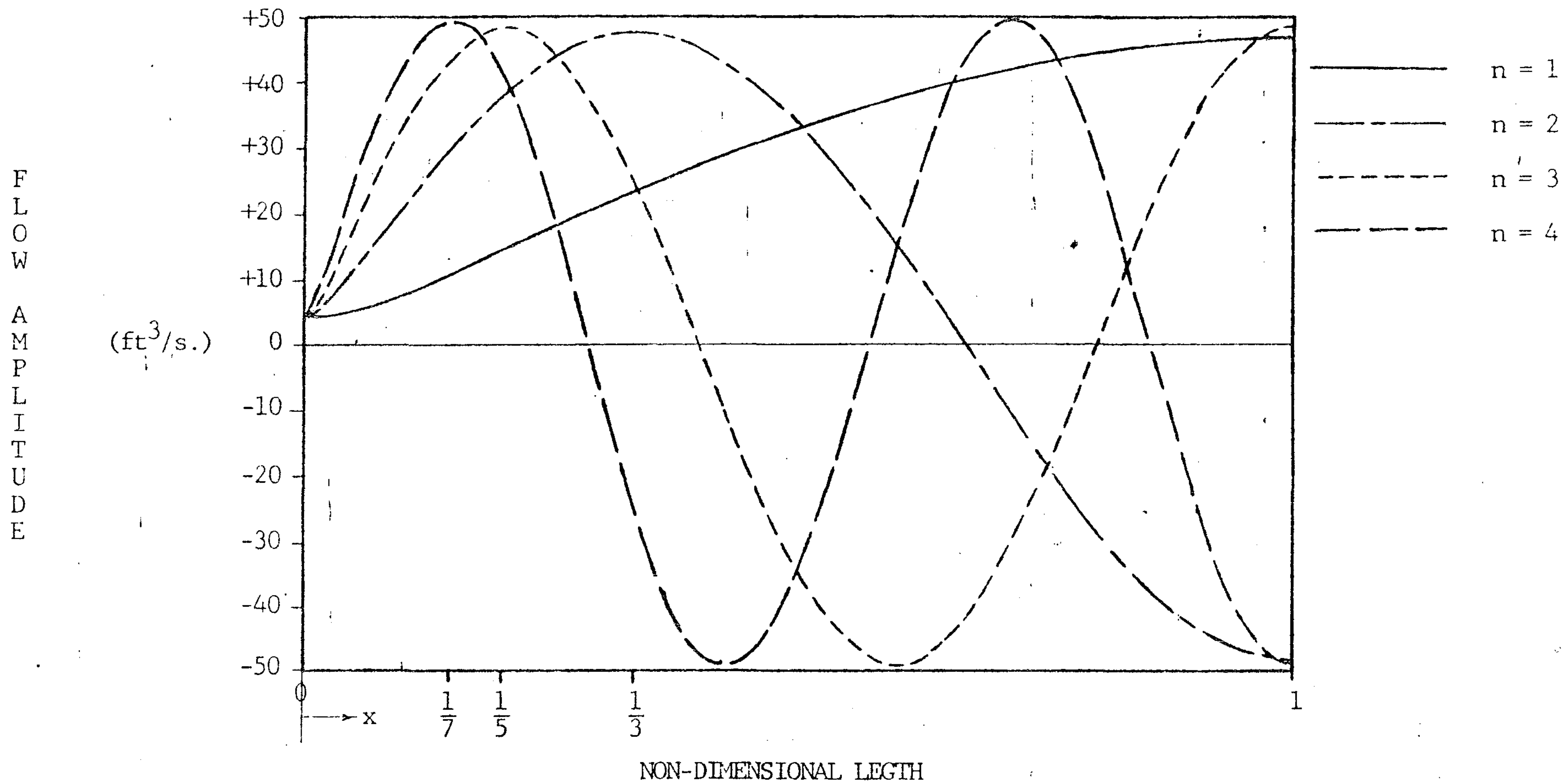

Figure F.1. Flow Amplitude Distributions along the Non-Dimensional Length of Pipe of the System defined in this Section for Resonant Frequencies of Different Harmonics.

$$\overline{Q} = \frac{2}{\pi} (Qmax)_n \left(\frac{a}{2} + \frac{b}{3} + \ldots\ldots + \frac{j}{11}\right) (2n-1) \qquad \text{(F-4)}$$

Choosing the first harmonic curve and applying ten boundary conditions at Y = 0.1, 0.2,........,1.0 to the equation F-3, ten simultaneous equations result which are presented in a matrix form shown in Figure F.2. The solution to this matrix is as follows:

a=0.7594 , b=19.806 , c=-182.31 , d=892.26

e=-2661.08 , f=5064.33 , g=-6195.23 , h=4720.14

i=-2039.36 , j=381.69

Substituting these values into equation F-4, the following equation results:

$$\overline{Q} = \frac{2}{\pi} (0.6375Qmax) \qquad \text{(F-5)}$$

This equation gives a good approximation of the average steady oscillatory flow through the pipe. However, consideration of the neglected area between each curve and the corresponding line further improves the average. In this case, the average of the steady oscillatory flow through a pipe is written as

$$\overline{Q} = \frac{2}{\pi} (0.6375Qmax + Qc). \qquad \text{(F-6)}$$

In order to find the value of Qc the shaded area shown in Figure F.3, which is a portion of the first harmonic curve shown in Figure F.1, must be calculated and divided by 5(2n-1). Using the same method employed earlier in this Appendix, the value of Qc is evaluated by the following equation:

$$Qc = \frac{1}{(2n-1)} (0.1049\ Qmin - 0.00493\ Qmax) \qquad \text{(F-7)}$$

The term, $\frac{1}{(2n-1)}$, can be expanded as

$$\frac{1}{(2n-1)} = \frac{\omega_R}{\omega} = \frac{2\pi a/4L}{\omega} = \frac{\pi a}{2\omega L}.$$

$$\begin{pmatrix} 0.1 & 0.1^2 & 0.1^3 & 0.1^4 & 0.1^5 & 0.1^6 & 0.1^7 & 0.1^8 & 0.1^9 & 0.1^{10} \\ 0.2 & 0.2^2 & 0.2^3 & 0.2^4 & 0.2^5 & 0.2^6 & 0.2^7 & 0.2^8 & 0.2^9 & 0.2^{10} \\ 0.3 & 0.3^2 & 0.3^3 & 0.3^4 & 0.3^5 & 0.3^6 & 0.3^7 & 0.3^8 & 0.3^9 & 0.3^{10} \\ 0.4 & 0.4^2 & 0.4^3 & 0.4^4 & 0.4^5 & 0.4^6 & 0.4^7 & 0.4^8 & 0.4^9 & 0.4^{10} \\ 0.5 & 0.5^2 & 0.5^3 & 0.5^4 & 0.5^5 & 0.5^6 & 0.5^7 & 0.5^8 & 0.5^9 & 0.5^{10} \\ 0.6 & 0.6^2 & 0.6^3 & 0.6^4 & 0.6^5 & 0.6^6 & 0.6^7 & 0.6^8 & 0.6^9 & 0.6^{10} \\ 0.7 & 0.7^2 & 0.7^3 & 0.7^4 & 0.7^5 & 0.7^6 & 0.7^7 & 0.7^8 & 0.7^9 & 0.7^{10} \\ 0.8 & 0.8^2 & 0.8^3 & 0.8^4 & 0.8^5 & 0.8^6 & 0.8^7 & 0.8^8 & 0.8^9 & 0.8^{10} \\ 0.9 & 0.9^2 & 0.9^3 & 0.9^4 & 0.9^5 & 0.9^6 & 0.9^7 & 0.9^8 & 0.9^9 & 0.9^{10} \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{pmatrix} \begin{pmatrix} j \\ i \\ h \\ g \\ f \\ e \\ d \\ c \\ b \\ a \end{pmatrix} = \begin{pmatrix} 0.1588 \\ 0.3176 \\ 0.4577 \\ 0.5895 \\ 0.7078 \\ 0.8093 \\ 0.8911 \\ 0.9511 \\ 0.9877 \\ 1.0 \end{pmatrix}$$

Figure F.2. Matrix Representation of Simultaneous Equations obtained by Equation F-4.

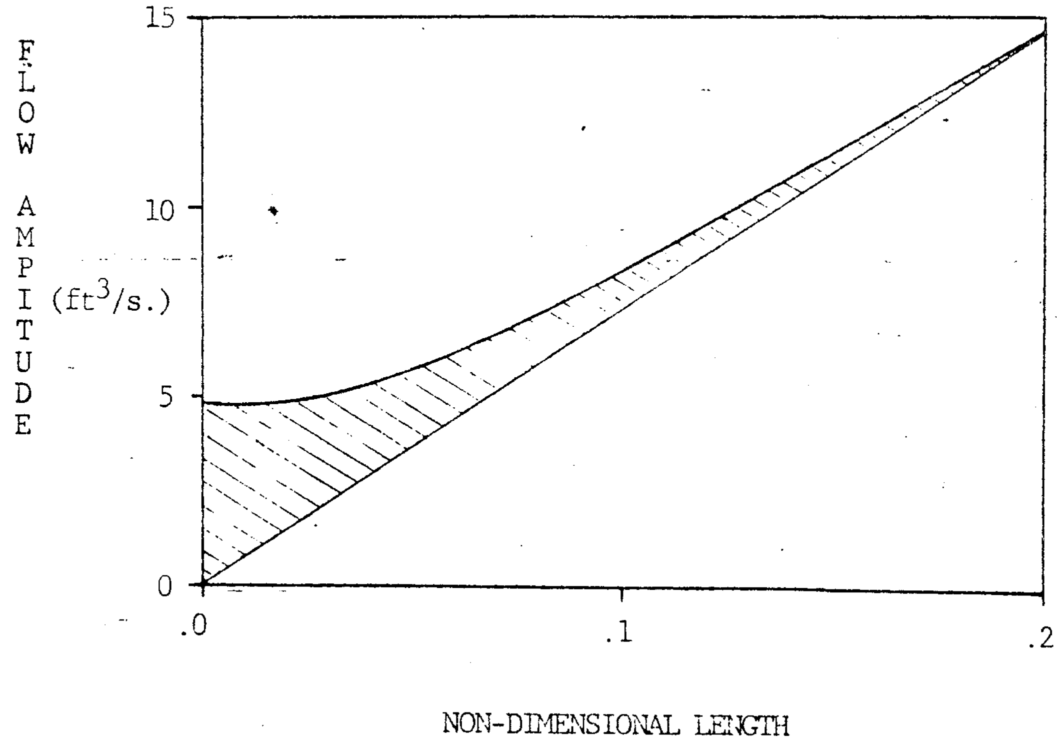

Figure F.3. Portion of the First Harmonic Curve Shown in Figure F.1.

Substituting this value into equation F-7, the following equation results:

$$Qc = \frac{\pi a}{2L\omega} (0.1049\ Qmin - 0.00493\ Qmax) \tag{F-8}$$

Substituting equation F-8 into equation F-6 and rearranging, the following equation results:

$$\overline{Q} = 0.4058Qmax + \frac{a}{L\omega} (0.1049Qmin - 0.00493Qmax) \tag{F-9}$$

Correction Factor

Equation F-8 was used in the main program and applied for the system defined in the Sample Problem for pipe friction factors from 0.02 to 0.20 and an excitation frequency equal to the natural frequency of the pipe. In these tests the system was defined as a 2-node piping network. Results of these tests including the results obtained by the method of characteristics for the same system are shown in the Table F.1. To obtain the same results by this method as it was obtained by the method of characteristics, equation F-9 must be modified as

$$\overline{Q} = (CF)\ [0.4058Qmax + \frac{a}{L\omega} (0.1049Qmin - 0.00493Qmax)] \tag{F-10}$$

where CF is the correction factor and its value varies with friction factor as shown in the fourth column of Table F.1. Interpolating between friction factors of 0.02 and 0.05, shows that the two methods coincide for the friction factor of 0.02072. To generalize the correction factor, one may define the following equation:

$$CF = \left(\frac{0.02072}{f}\right)^{(A + Bf + Cf^2 + Df^3)} \tag{F-11}$$

M.O.C. = METHOD OF CHARACTERISTICS

S.O.M. = STEADY OSCILLATORY METHOD

FRICTION FACTOR	NON-DIMENSIONAL HEAD		CORRECTION FACTOR
	M.O.C.	S.O.M.	
0.02	19.24	19.27	1.0026
0.05	13.22	12.20	0.8508
0.075	11.00	9.98	0.8204
0.076	10.94	9.91	0.8197
0.08	10.68	9.66	0.8171
0.10	9.64	8.65	0.8036
0.15	8.01	7.09	0.7812
0.16	7.78	6.37	0.7772
0.20	7.05	6.17	0.7595

Table F.1. Non-Dimensional Head Response and Correction Factor needed to make Methods Agree as the Functions of Friction Factor for the System Defined in this Section.

Applying four data points from Table F.1, for friction factors of 0.05, 0.10, 0.15, 0.20, four simultaneous equations are obtained with four unknowns A, B, C and D. Solving these sets of equations for A, B, C, D, and substituting into the equation F-11, the following equation results:

$$CF = \left(\frac{0.02072}{f}\right)^{(0.2778-2.518f+13.895f^2-26.092f^3)} \qquad \text{(F-12)}$$

This equation and the equation F-10 were used in the main program and applied for the same system for the friction factors from 0.02 to 0.20. A maximum difference of 0.26% was noted between the results of this method and the results of the method of characteristics. For use of this method, Figure F.4 shows the required modification to the main program.

```
171        REAL AC(20,20)
721        AC(I,J)=A
722        AC(J,I)=A
3470C
3490 410   U=AF(I,J)
3491       POWER=.2778-2.518*U+13.895*U*U-26.092*U*U*U
3492       CFAC=(.02072/U)**POWER
3493       QQQ=AC(I,J)*(.1049*QMIN-.00493*QMAX)/(L(I,J)*OMG)
3494       QQ=QBAR(I,J)+CFAC*(.4058*QMAX+QQQ)
```

Figure F.4. Modification of Main Program for the use of Method Presented in this Appendix.

APPENDIX G

SAMPLE PROBLEM DESCRIPTION

This Appendix presents the input and output format of the sample problem defined in Chapter IV.

Input Format

Referring to the data listed on page 101, the two-digit numbers on the left are the line numbers and each line contains the following information:

Line No. 10: Absolute viscosity of the liquid in $\frac{\text{lb-Sec}}{\text{ft}^2}$, mass density of the liquid in $\frac{\text{lb}_m}{\text{ft}^3}$, and the gravitational acceleration in $\frac{\text{ft}}{\text{Sec}^2}$.

Line No. 11: Number of nodes, number of constant head nodes (tanks), and number of pipe segments in the piping network.

Line No. 12: Constant head node number. Note: If there are more than one tank in the system, each tank's node number must be entered in separate line following this line.

Line No. 13: Number of head excitation and number of flow excitation sources. Note: The program is able to analyze a piping network with several excitation sources, but they must all have the same excitation frequency. In this case, line numbers 370 to 670 of the main program must be followed.

Line No. 14: Flow excitation node number and its flow amplitude in $\frac{\text{ft}^3}{\text{Sec}}$.

```
10     0.000018   62.4   32.2
11     9 1 8
12     9
13     0 1
14     1 4.9087385
15     1 2    625.   30.   3000.    .1    3   0.
16     2 3    625.   30.   3000.    .1    3   0.
17     3 4    625.   30.   3000.    .1    3   0.
18     4 5    625.   30.   3000.    .1    3   0.
19     5 6    625.   30.   3000.    .1    3   0.
20     6 7    625.   30.   3000.    .1    3   0.
21     7 8    625.   30.   3000.    .1    3   0.
22     8 9    625.   30.   3000.    .1    3   0.
23      1   .9424778   0.
```

Line Nos. 15 to 22: Each line contains the information about a pipe segment. This information is: terminating node numbers, length in feet, diameter in inches, speed of sound through the liquid in ft/Sec, friction factor, number of extra locations for calculation of head and flow amplitudes, and steady state flow in ft^3/Sec.

Line No. 23: Number of frequencies, first frequency, and frequency intervals.

Output Format

Page numbers 103 to 106 in this Appendix, show the computer output for this sample problem as follows:

Page No. 103: Echo format of input data.

Page No. 104: Head and flow amplitudes and their phase angles at each node of the piping network.

Page Nos. 105 and 106: Head and flow amplitudes and their phase angles along the length of each pipe segment in the piping network. The maximum head and its location in the piping network.

```
                                    (( INPUT DATA ))
*******************************************************************************************
*                    ABS. VISC. =  0.0000180                                               *
*                   SPEC. MASS =  62.40                                                    *
*                            G = 32.200                                                    *
*                 NO. OF NODES =  9                                                        *
* NO. OF CONSTANT HEAD NODES =  1                                                          *
*        NO. OF LINE SEGMENTS =  8                                                         *
*     CONSTANT HEAD NODE NO. =  9                                                          *
*     NO. OF HEAD OSC. NODES =  0                                                          *
*     NO. OF FLOW OSC. NODES =  1                                                          *
*          FLOW OSC. NODE NO. =  1      FLOW AMP. =    4.90873849    PH. ANGLE =   0.      *
*                                                                                          *
*                                                                                          *
* LINE( 1- 2):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
* LINE( 2- 3):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
* LINE( 3- 4):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
* LINE( 4- 5):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
* LINE( 5- 6):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
* LINE( 6- 7):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
* LINE( 7- 8):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
* LINE( 8- 9):   L=  625.    D= 30.00    A=3000.    F=0.1000    NP=  3    QBAR=  0.        *
*                                                                                          *
*                                                                                          *
*     RUNNING FOR   1  DIFFERENT FREQUENCIES.     1ST. W=   0.9424778    DW=   0.          *
*                                                                                          *
*                                                                                          *
*******************************************************************************************
```

ALL THE PHASE ANGLES ARE COMPARED
TO THE FLOW AT NODE NO. 1

FREQUENCY= 0.942 RAD/SEC

NODE NO	HEAD AMPLITUDE FEET	PHASE ANGLE RAD	FLOW TO NODE NO	FLOW AMPLITUDE CUBIC FT/S	PHASE ANGLE RAD
1	898.724	3.064	2	0.4909E 01	3.142
2	882.567	3.044	1	0.1008E 02	-1.152
			3	0.1008E 02	1.990
3	832.404	3.025	2	0.1833E 02	-1.398
			4	0.1833E 02	1.743
4	750.016	3.007	3	0.2629E 02	-1.490
			5	0.2629E 02	1.651
5	638.500	2.992	4	0.3335E 02	-1.538
			6	0.3335E 02	1.604
6	502.105	2.979	5	0.3918E 02	-1.566
			7	0.3918E 02	1.575
7	346.083	2.970	6	0.4353E 02	-1.583
			8	0.4353E 02	1.558
8	176.502	2.964	7	0.4620E 02	-1.593
			9	0.4620E 02	1.549
9	0.	0.	8	0.4711E 02	-1.596

```
HD AMP      898.724   897.887   894.910   889.799   882.567
PH ANG        3.06      3.06      3.05      3.05      3.04
   NODE( 1)  0=========0=========0=========0=========0  NODE( 2)
FW AMP        4.909     5.260     6.472     8.161    10.079
PH ANG       -3.14      2.69      2.35      2.13      1.99


HD AMP      882.567   873.123   861.595   848.011   832.404
PH ANG        3.04      3.04      3.03      3.03      3.02
   NODE( 2)  0=========0=========0=========0=========0  NODE( 3)
FW AMP       10.079    12.103    14.173    16.255    18.327
PH ANG        1.99      1.90      1.83      1.78      1.74


HD AMP      832.404   814.679   795.007   773.436   750.016
PH ANG        3.02      3.02      3.02      3.01      3.01
   NODE( 3)  0=========0=========0=========0=========0  NODE( 4)
FW AMP       18.327    20.377    22.392    24.365    26.287
PH ANG        1.74      1.71      1.69      1.67      1.65


HD AMP      750.016   724.674   697.597   668.849   638.500
PH ANG        3.01      3.00      3.00      3.00      2.99
   NODE( 4)  0=========0=========0=========0=========0  NODE( 5)
FW AMP       26.287    28.153    29.956    31.691    33.352
PH ANG        1.65      1.64      1.62      1.61      1.60


HD AMP      638.500   606.504   573.051   538.224   502.105
PH ANG        2.99      2.99      2.99      2.98      2.98
   NODE( 5)  0=========0=========0=========0=========0  NODE( 6)
FW AMP       33.352    34.937    36.439    37.855    39.182
PH ANG        1.60      1.60      1.59      1.58      1.58


HD AMP      502.105   464.682   426.142   386.577   346.083
PH ANG        2.98      2.98      2.97      2.97      2.97
   NODE( 6)  0=========0=========0=========0=========0  NODE( 7)
FW AMP       39.182    40.415    41.552    42.590    43.525
PH ANG        1.58      1.57      1.57      1.56      1.56
```

```
HD AMP       346.083   304.686   262.556   219.793   176.502
PH ANG         2.97      2.97      2.97      2.97      2.96
    NODE( 7)  0=========0=========0=========0=========0  NODE( 8)
FW AMP        43.525    44.356    45.081    45.697    46.204
PH ANG         1.56      1.56      1.55      1.55      1.55


HD AMP       176.502   132.750    88.678    44.392     0.000
PH ANG         2.96      2.96      2.96      2.96      1.39
    NODE( 8)  0=========0=========0=========0=========0  NODE( 9)
FW AMP        46.204    46.599    46.882    47.052    47.109
PH ANG         1.55      1.55      1.55      1.55      1.55


NO. OF TRIAL =  11
FLOW DIRECTION   ------>
MAXIMUM HEAD AMPLITUDE RECORDED =    898.724  FEET
LOCATION OF MAX. HEAD:      0. FEET FROM NODE(1) ON LINE(1-2)
```

APPENDIX H

FRICTIONAL EFFECT OF ORIFICE

In this Appendix, the frictional effect of the orifice connection is calculated, and the length of a pipe segment with the same diameter and frictional effect as the orifice is determined in order to model the orifice connection for the tests referred to in Chapter V.

The head loss across the orifice may be calculated by the following equation:

$$h_L = \frac{V_o^2}{2g} \frac{C}{K^2} \tag{H-1}$$

where C = 1, if $\frac{d_o}{D} < 0.3$, and $C = 1 - (\frac{d_o}{D})$ if $\frac{d_o}{D} > 0.3$; K is the orifice coefficient which varies from 0.52 to 0.98 depending on the type of orifice, and K = 0.61 for a sharp edged orifice. V_o is the fluid velocity at the orifice, and g is gravitational acceleration.

The head loss across the length of a pipe segment is defined as

$$h_L = f(\frac{L}{D}) \frac{V^2}{2g} \tag{H-2}$$

where f is the friction factor of the pipe, L is the length of the pipe, D is the diameter of the pipe, which in this case is equal to d_o. V is the bulk fluid velocity inside the pipe and in this case is equal to V_o, and g is gravitational acceleration. Combining equations H-1 and H-2, the length of a pipe segment with the same diameter and frictional effect as the orifice, is calculated by the following equation:

$$L = \frac{Cd_o}{fK^2} \tag{H-3}$$

with f = 0.1, D = 30 inches and K = 0.61, the length of the pipe segment with the same diameter and frictional effect as the orifice, for orifice diameters of 5, 10, 15 and 20 inches, and calculated by equation H-3, are 11.2, 22.1, 31.5 and 35.9 feet, respectively.

NOMENCLATURE

Parameter	Definition	Unit(M,L,T)
A	Constant; pipe area	L^2
A_o	Orifice area	L^2
$A_{i,j}$	Two dimensional array	-
a	Acoustic velocity	L/T
B,b,C	Constant	-
CF	Correction factor	-
c	Constant; subscript for correction	-
D	Pipe diameter; constant	L;-
d	Constant	-
d_o	Orifice diameter	L
E	Modulus of elastisity	M/LT^2
e	Pipe wall thickness; constant	L;-
F	Force	ML/T^2
f	Pipe friction factor; constant	-
g	Gravitational acceleration; constant; subscript for ground	L/T^2;-
H	Instantaneous total head; head amplitude	L
$\bar{H}$	Average head	L
h	Constant	-
h'	Instantaneous oscillatory head	L
i	Subscript for node number; constant; $\sqrt{-1}$	-

Parameter	Definition	Unit(M,L,T)
j	Subscript for node number; constant	-
K	Bulk compressibility modulus	M/LT^2
k	Constant	-
L	Length	L
l	Length	L
m	Constant	-
n	Number of nodes	-
P	Pressure	M/LT^2
Q	Instantaneous total flow; flow amplitude	L^3/T
$\bar{Q}$	Average flow	L^3/T
q	Flow amplitude	L^3/T
$\bar{q}$	Average of oscillatory flow	L^3/T
q'	Instantaneous oscillatory flow	L^3/T
R	Linearized resistant per unit length; subscript for real; subscript for receiving end; subscript for resonant	T/L;-;-;-
Re	Reynold number	-
S	Subscript for sending end	-
V	Velocity	L/T
X	Constant	-
x	Position; non-dimensional position	L;-
Y	Constant	-
Zc	Characteristics impedance	-
Z(x)	Ratio of h' to q'	-
α	Real part of γ	-
β	Imaginary part of γ	-

Parameter	Definition	Unit(M,L,T)
γ	Propagation constant	-
θ	Angle	-
μ	Poisson's ratio	-
ν	Kinematic viscosity	L^2/T
ξ_1	Pipe axial strain	-
ξ_2	Pipe lateral strain	-
ρ	Mass density	M/L^3
σ_1	Pipe axial stress	M/LT^2
σ_2	Pipe lateral stress	M/LT^2
τ_o	Wall shear stress	M/LT^2
ω	Angular frequency	$1/T$
$_o$	Subscript for orifice	-