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# **SYSTEM IDENTIFICATION OF STRUCTURES WITH JOINT ROTATION**

by

JERRY S. DIMSDALE

Report to the National Science Foundation



COLLEGE OF ENGINEERING

UNIVERSITY OF CALIFORNIA · Berkeley, California

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#### ABSTRACT

The goal of this research is to investigate the role of joint behavior in the identification of frame models from dynamic response data caused by seismic forcing functions. Including joint rotation and deformation in the mathematical model for even simple structures significantly affects the distribution of stiffness, and the accuracy with which response can be predicted.

An optical method has been devised for accurately measuring joint rotation of a structure during earthquake excitation. This method has been applied to a simple six story frame in which the columns have approximately the same stiffness as the girders. Response data have been collected for a variety of base motion histories. Also studied are data previously collected from a three story frame in which joint rotation information has been inferred from strain measurements.

A number of different mathematical models of these structures are evaluated using system identification. Each mathematical model depends on a number of parameters related to the characteristics of the structure. An iterative method is applied to calculate the values of these parameters which best reproduce the measured response of the structure; The form of the mathematical model has an effect on the degree to which the optimal parameters accurately reflect physical properties of the structure. Further, the form of the model influences not only the number of parameters and degrees of freedom, but also the set of response quantities necessary for calculating an optimal set of parameters.

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### **ACKNOWLEDGEMENT**

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#### CHAPTER 1

#### **INTRODUCTION**

**In** studying the properties of a structure, the analyst typically has a continuum of models from which to choose. For many applications, the simplest and most coarse approximation to actual behavior will suffice. By adding greater refinement to critical portions of the analysis, prediction of response can be improved. This refinement takes its toll both in the analyst's time, and computational effort, usually in a computer. It would therefore be wise to add sophistication to a model in a logical manner.

To predict the response of a structure to a prescribed input using an analytical model, certain parameters reflecting physical properties of the structure must be known or determined. This may not always be trivial, both because the materials used may have some uncertainty, and also because the materials can be used in some unfamiliar way. For example, the properties of concrete subjected to triaxial stress are both variable and imperfectly understood. Further, while there are counter-examples, increasing a model's analytical complexity often increases uncertainty about physical parameters. An analytical model actually represents a continuum of models which, in practice, may be difficult to apply, since its parameters are difficult to estimate.

System identification is a tool which can be used to evaluate a model. By systematically adjusting the parameters to provide the best possible correlation between predicted and measured responses, the form of the analytical model can be appraised. System identification is a generic term for this optimization process, and there are many approaches to applying it to structural engineering. There have been many survey articles written on system identification [4,7,8,11,20,21], so this discussion need not be exhaustive. Evaluating models by adjusting parameters to fit known response data is known as parametric identification. Much of the literature in parametric identification has been devoted to the determination of modal characteristics [13,17]. While it is true that stiffness and damping matrices can be determined from modal properties, little can be inferred about the participation of individual structural elements. An algorithm used by Matzen [18] allows determination of element characteristics but, for economic reasons, is only applicable to a structure with a small number of degrees of freedom. Here, Matzen's algorithm is extended to allow identification using a structure with a relatively large number of degrees of freedom.

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In a frame with even a small number of elements, accurately predicting response can require a large number of degrees of freedom. For instance, in frames commonly approximated by a simple shear model, it is well known that the inclusion of rotational stiffness at its joints significantly affects predictive accuracy [12]. Using data from a three story frame previously built by Clough and Tang [10,24], and a six story frame developed for this study, it is shown that not only the rotational response, but also the deformation of the joints, significantly affects a model's optimal precision.

The optimization algorithm to be used and its implementation in dynamic structural analysis are described in Chapter 2. In Chapter 3 its application to a frame already studied in some detail is discussed. Previously, accurate data concerning the dynamic rotation of frame joints has been unavailable. Chapter 4 describes an optical method of measurement which allows high resolution and accuracy. This method has proved to be extremely useful in the study of the effects of joint behavior on overall dynamic response. In Chapter 5 the application of both the optimization and rotation measurement methods in the identification of parameters of a six story frame are described and conclusions are presented in Chapter 6.

#### CHAPTER 2

#### **ITERATIVE IDENTIFICATION**

#### **General** System

Suppose we have a system subjected to a time dependent input  $p(t)$ , which produces a set of measurable outputs  $y_j(t)$ ,  $j=1,...,n$ . If we have a model which we believe represents the system, this means we have some rule by which, given an input,  $p(t)$ , and some information about the system in terms of a vector of constants,  $\underline{b}$ , we can predict the output of the system,  $x_j(\underline{b},t)$ . Here we include <u>b</u> as an argument to emphasize the dependence of the predicted output of the system on the information supplied to the model.

One measure of how well the predicted response matches the measured response is the squared-error loss function over a time interval  $0 < t < T$ :

$$
J(\underline{b}) = \sum_{j=1}^{n} \int_{0}^{T} g_j [x_j(\underline{b}, t) - y_j(t)]_m^2 dt
$$
 (2.1)

Again,  $\underline{b}$  is included as an argument to emphasize the dependence of J on  $\underline{b}$ . If  $J(\underline{b})=0$ , then the predicted response would exactly match the measured response. We would like to know what value of  $\underline{b}$ , if any, makes J a minimum value.

Unfortunately, very few models permit a closed form solution for *Q* which minimizes J globally. It is, however, often possible to generate an iterative scheme which will produce a *Q* which is a local minimum.

#### **Gauss Newton Method**

Given a set of parameters  $\underline{b}_i$ , we would like a systematic method of discovering a new set  $\underline{b}_{i+1}$  such that  $J(\underline{b}_{i+1}) < J(\underline{b}_{i})$ . Repeated often enough, this will lead to a minimum for J. If the function J is approximately quadratic in a neighborhood of  $\underline{b}_i$ , there will be little error in the approximation

$$
4\!\!
$$

$$
J(\underline{b}_{i+1}) = J(\underline{b}_{i}) + \nabla J^{t}(\underline{b}_{i})(\underline{b}_{i+1} - \underline{b}_{i}) + \frac{1}{2}(\underline{b}_{i+1} - \underline{b}_{i})^{t} \nabla^{2} J(\underline{b}_{i})(\underline{b}_{i+1} - \underline{b}_{i})
$$
\n(2.2a)

where

$$
\nabla J_p = \frac{\partial J}{\partial b_p} \tag{2.2b}
$$

and

$$
\nabla^2 J_{\rho s} = \frac{\partial^2 J}{\partial b_{\rho} \partial b_s}.
$$
\n(2.2c)

To minimize J, its gradient with respect to  $\underline{b}_{i+1}$  is set to the zero vector. If the Hessian matrix is invertible, it follows that

$$
\underline{b}_{i+1} = \underline{b}_i - \left[\nabla^2 J(\underline{b}_i)\right]^{-1} \nabla J(\underline{b}_i)
$$
\n(2.3)

Since J will not, in general, be exactly quadratic, we will want to be able to adjust the size of the correction to  $\underline{b}_i$ . Thus we modify the equation by adding a step size variable,  $\alpha$ :

$$
\underline{b}_{i+1} = \underline{b}_i - \alpha \Big[ \nabla^2 J(\underline{b}_i) \Big]^{-1} \nabla J(\underline{b}_i). \tag{2.4}
$$

The components of  $\nabla J$  and  $\nabla^2 J$  are found by taking the appropriate derivatives of the error function:

$$
\nabla J_p = \frac{\partial J}{\partial b_p} = 2 \sum_{j=1}^n \int_0^T g_j \left[ [x_j(\underline{b}, t) - y_j(t)] \frac{\partial x_j(\underline{b}, t)}{\partial b_p} \right] dt \qquad (2.5a)
$$

$$
\frac{\partial^2 J}{\partial b_p \partial b_s} = 2 \sum_{j=1}^n \left[ \int_0^T g_j \frac{\partial x_j(\underline{b},t)}{\partial b_p} \frac{\partial x_j(\underline{b},t)}{\partial b_s} dt + \int_0^T g_j (x_j(\underline{b},t) - y_j(t)) \frac{\partial^2 x_j(\underline{b},t)}{\partial b_p \partial b_s} dt \right].
$$
\n(2.5b)

J

Experience has shown that the second integral, particularly when  $\underline{b}_i$  is close to a minimum, is negligible when compared to the first. The Gauss-Newton iteration scheme, therefore, is to choose  $\alpha$  and calculate

$$
\underline{b}_{i+1} = \underline{b}_i - \alpha \left[ \underline{AH}(\underline{b}_i) \right]^{-1} \nabla J(\underline{b}_i)
$$
\n(2.6a)

where the approximate Hessian matrix, *AH,* is defined as

$$
\underline{AH}_{ps} = 2 \sum_{j=1}^{n} \left[ \int_{0}^{T} g_{j} \frac{\partial x_{j}(\underline{b},t)}{\partial b_{p}} \frac{\partial x_{j}(\underline{b},t)}{\partial b_{s}} dt \right].
$$
\n(2.6b)

The technique for choosing  $\alpha$  is known as a line search algorithm since the multidimensional

minimization problem has been reduced to a single dimension.

#### Line Search

By establishing a search direction, the error function is reduced to being a function of one variable

$$
J(\alpha) = J[\underline{b}_i - \alpha A H^{-1}(\underline{b}_i) \nabla J(\underline{b}_i)] \qquad (2.7)
$$

whose derivative is

$$
\frac{\partial}{\partial \alpha} J(\alpha) = -\nabla J'(\underline{b}_{i+1}) A H^{-1}(\underline{b}_i) \nabla J(\underline{b}_i).
$$
\n(2.8)

If we are pointed in the right direction,  $J(0) < 0$ . If the error surface were quadratic, then the exact minimum would be at  $\alpha=1$ . If  $J(1) > 0$  then there must be a minimum for  $0 < \alpha < 1$ . In order to find a point closer to the minimum, a cubic polynomial is constructed so that its values and derivatives match J at the end points, and the minimum of the cubic is used as a new trial point. If, on the other hand,  $J(1) < 0$  and  $J(1) < J(0)$ , then a quadratic extrapolation is made. In this way, successive approximations to the functional minimum are made until some stopping criterion is met.

The stopping criterion for the line search will affect the relative amount of time spent on finding search directions and doing line searches. In general, spending too much time on either is not economic. In practice, a good deal of trial and error is necessary to find a reasonable distribution of effort. In this case four or five iterations in the line search is probably a good compromise. It is also desirable to have the line search end fairly soon in the event a poor direction is chosen, since the improvement will be rather slight.

#### Structural Models

The mathematical model associated with dynamic behavior of an n degree of freedom linear elastic structure subjected to rigid base motion is

$$
\underline{m} \frac{d^2 u}{dt^2} + \underline{c} \frac{du}{dt} + \underline{ku} = -\underline{mr} \frac{d^2 \underline{u}_g}{dt^2}
$$
 (2.9a)

$$
\frac{du(0)}{dt} = \underline{u}(0) = 0
$$
\n(2.9b)  
\nhere m is the mass matrix, c is the damping matrix, and k is the stiffness matrix. 
$$
\frac{d^2u}{dt^2}, \frac{du}{dt}
$$

where <u>m</u> is the mass matrix, c is the damping matrix, and k is the stiffness matrix.  $\frac{d^2u}{dt^2}$ ,  $\frac{du}{dt}$ , *d*<sup>2</sup>*u* and  $\mu$  are vectors for relative acceleration, velocity, and displacement.  $\frac{d\mu}{dt^2}$  is the base acceleration.  $r$  is a column vector whose elements are static displacements due to a unit displacement of the base of the structure.

It is possible to find a matrix, P, so that  $M = P'mP$  and  $K = P'kP$  are both diagonal matrices: i.e.,  $M_{ij}=0$  if  $i \neq j$ . If we make the change of variables

$$
\underline{u} = \underline{PY} \tag{2.10}
$$

then the differential equation of motion can be rewritten

$$
\underline{M} \frac{d^2Y}{dt^2} + \underline{C} \frac{dY}{dt} + \underline{KY} = \underline{F}(t) \tag{2.11a}
$$

where

$$
\underline{C} = \underline{P}^t \underline{c} \underline{P} \quad \text{and} \quad \underline{F}(t) = -\underline{P}^t \underline{mr} \frac{d^2 \underline{u_g}}{dt^2}.
$$
 (2.11b)

If the additional assumption of proportional damping is made; that is,

$$
\underline{c} = a_0 \underline{m} + a_1 \underline{k}, \qquad (2.12)
$$

then  $\mathcal{L}, \mathcal{M}$ , and  $\mathcal{K}$  will all be diagonal, and the n coupled differential equations will be decoupled into n equivalent uncoupled single degree of freedom equations. The coefficients *ao* and  $a_1$  can be related to the damping ratios and frequencies by

$$
E_i = \frac{1}{2} \left( \frac{a_0}{w_i} + a_1 w_i \right), \tag{2.13}
$$

where  $E_i$  is the damping ratio, and  $w_i$  is the characteristic frequency of the  $i^h$  mode.

#### **Structural Identification**

Geometric and material information describing a structure can be organized into a vector  $\underline{b}$ , so that using the finite element method mass, damping, and stiffness matrices can be constructed which depend on  $b$ :

$$
\epsilon
$$

$$
\underline{m} = \underline{m} \, (\underline{b}) \quad \underline{c} = \underline{c} \, (\underline{b}) \quad \underline{k} = \underline{k} \, (\underline{b}). \tag{2.14}
$$

By solving the differential equation

$$
\underline{m}(\underline{b})\frac{d^2\underline{u}}{dt^2} + \underline{c}(\underline{b})\frac{d\underline{u}}{dt} + \underline{k}(\underline{b})\underline{u} = -\underline{m}(\underline{b})\underline{r}\frac{d^2\underline{u}_g}{dt^2}
$$
\n(2.15)

with boundary value

$$
\frac{du(0)}{dt} = \underline{u}(0) = 0,\tag{2.16}
$$

we arrive at a solution  $\underline{x}(\underline{b})$  which is the predicted response of the structure, given  $\underline{b}$ , subjected to a ground motion  $\underline{u}_g(t)$ .

Given measured response histories at a number of locations,  $y_i(t)$ , the error of the model which the differential equation represents, over some time interval  $0 < t < T$  is defined as:

$$
J(\underline{b}) = \sum_{j=1}^{n} \int_{0}^{T} g_j [x_j(\underline{b},t) - y_j(t)]_m^2 dt
$$
 (2.17)

Applying the iterative scheme previously described requires calculating the sensitivities  $\frac{\partial x_j(\underline{b})}{\partial b_p}$ , and the responses  $x_j(\underline{b})$ . Given the structural matrices <u>m</u>, <u>c</u>, and <u>k</u>, the differential equations could be solved directly. However, for even moderately small problems this can be quite time-consuming since there must be n+1 integrations each time  $\nabla J$  and AH are calculated. Using modal decomposition this effort can be significantly reduced.

Using the modal equations

$$
\underline{MY} + \underline{CY} + \underline{KY} = \underline{F}(t) = -\underline{P}^t \underline{m} \underline{r} \underline{i}_g \tag{2.18}
$$

 $Y$  can be calculated directly. The specific single degree of freedom integration algorithm will be described subsequently. Differentiating (2.18) we get

$$
\underline{M} \frac{\partial \dot{Y}}{\partial b_p} + \underline{C} \frac{\partial \dot{Y}}{\partial b_p} + \underline{K} \frac{\partial Y}{\partial b_p} = -\frac{\partial C}{\partial b_p} \dot{Y} - \frac{\partial K}{\partial b_p} \underline{Y} - \frac{\partial P'}{\partial b_p} \text{ mriig}
$$
(2.19)

where it is assumed that

$$
\frac{\partial M}{\partial b_p} = 0. \tag{2.20}
$$

The time histories of the modal sensitivities are obtained from (2.19). This equation has the

same form as (2.18), and the pseudo-forcing function on the right hand side of (2.19) is well known after (2.18) has been solved. The terms  $\frac{\partial C}{\partial b_p}$ ,  $\frac{\partial K}{\partial b_p}$ , and  $\frac{\partial P'}{\partial b_p}$  can all be calculated using finite differences. This may require great precision since  $P$  can be insensitive to changes in parameters. It may prove desirable to calculate these sensitivities concurrently with calculating the structural property matrices.

Once the modal sensitivities  $\frac{\partial Y}{\partial b_p}$  are known, the sensitivities  $\frac{\partial x_j}{\partial b_p}$  can be calculated from

$$
\frac{\partial x_j}{\partial b_p} = \frac{\partial}{\partial b_p} [PY] = \frac{\partial P}{\partial b_p} + P \frac{\partial Y}{\partial b_p}
$$
 (2.21)

These can then be used to calculate a search direction for the Gauss-Newton iteration.

#### Numerical Integration Algorithm

The predicted dynamic response of a structure which has estimated structural property matrices  $\underline{k}$ ,  $\underline{m}$ , and  $\underline{c}$ , depends to some extent on the algorithm by which the equations of motion are integrated. This can affect the values of identified parameters, particularly those which affect response at higher frequencies. In this work, we have chosen to use the Newmark-Wilson algorithm [5] for linear stepwise integration, primarily since it is unconditionally stable.

#### Model Specification

A number of physical constants are incorporated into a finite element model for a structure. These constants specify the geometric dimensions and material properties of the structure.

In general, it is neither practical nor possible to optimize the finite element model with respect to all the parameters describing a structure. As will be shown, some sets of parameters are not independent. Additionally, The computational effort required for a large number of parameters can be tremendous. Some parameters will be known with great precision, and may have a relatively small effect on response. Little time need be spent optimizing these. Other

parameters, that are known less exactly, may have a pronounced effect on overall response. It would be wise to spend the bulk of the computational effort on these.

In structures with a repeated set of elements, the parameters associated with these elements may be known somewhat imprecisely. However, the analyst may be quite certain that all the elements of a certain type behave identically. With this in mind it is desirable to group all these parameters together. Thus, if one is studying the effective lengths of identical girders of a three story frame, it may be more appropriate to optimize a single parameter representing all these lengths. Thus, if the parameters describing a structure form a set  $p_k$ , the parameters to be identified will form a new set  $b_j$ . In general

$$
p_k = b_j \text{ [initial estimate for } p_k \text{]}
$$
 (2.22)

for some j.

#### Redundant Degrees of Freedom

Suppose we model a structure that appears as though it had only two degrees of freedom;



Figure 2.1 Two degree of freedom frame

using symmetry, we use the reduced model



Figure 2.2 Symmetric simplification

Let 
$$
\alpha_i = \left[\frac{2EI}{L}\right]_i
$$
, then the stiffness matrix for this model is  

$$
\underline{k} = \begin{bmatrix} 6\alpha_1 & 3\alpha_1 \\ 3\alpha_1 & 2\alpha_1 + \frac{3}{2}\alpha_2 \end{bmatrix}.
$$

Noting that we are going to try to identify  $\alpha_1$  and  $\alpha_2$  using only lateral force F, we can apply static condensation:

$$
\begin{bmatrix} F \\ 0 \end{bmatrix} = \underline{k} = \begin{bmatrix} 6\alpha_1 & 3\alpha_1 \\ 3\alpha_1 & 2\alpha_1 + \frac{3}{2}\alpha_2 \end{bmatrix} \begin{bmatrix} v \\ \theta \end{bmatrix}
$$
 (2.24)

(2.23)

so that

$$
F = \left[\frac{3\alpha_1^2 + 9\alpha_1\alpha_2}{2\alpha_1 + \frac{3}{2}\alpha_2}\right] \nu
$$
\n
$$
\theta = \frac{-3\alpha_1}{2\alpha_1 + \frac{3}{2}\alpha_2} \nu.
$$
\n(2.25b)

Now given any response pair  $(v_m, F_m)$ , it is possible to identify

$$
\frac{3\alpha_1^2+9\alpha_1\alpha_2}{2\alpha_1+\frac{3}{2}\alpha_2}
$$

but not  $\alpha_1$  and  $\alpha_2$  separately. Given any  $\theta$  response data which satisfies the assumptions of static condensation, this cannot further reduce the problem.

However, suppose the structure we are studying is, in fact, accurately represented by a three degree of freedom model



Figure 2.3 Three degree of freedom frame

where  $\phi$  represents the distortion of a joint element, with no physical dimension. If the constitutive law of the joint is  $M=k_j \phi$ , the stiffness of this model is

$$
\underline{k} = \begin{bmatrix} 6\alpha_1 & 3\alpha_1 & \frac{3}{2}\alpha_1 \\ 3\alpha_1 & 2\alpha_1 + \frac{3}{2}\alpha_2 & \alpha_1 \\ \frac{3}{2}\alpha_1 & \alpha_1 & \frac{1}{2}\alpha_1 + k_j \end{bmatrix}
$$
 (2.26)

Applying static condensation:

$$
\begin{bmatrix} F \\ 0 \\ 0 \end{bmatrix} = \underline{k} \begin{bmatrix} v \\ \theta \\ \phi \end{bmatrix}
$$
 (2.27)

we find that

$$
\theta = \frac{-3\alpha_1}{2\alpha_1 + \frac{3}{2}\alpha_2} v - \frac{\alpha_1}{2\alpha_1 + \frac{3}{2}\alpha_2} \phi.
$$
 (2.28)

So, response data from a structure correctly modelled by the three degree of freedom model cannot satisfy the assumptions we used in identifying the two degree of freedom model (see Eqn. 2.23b). This problem will be amplified if, instead of measuring the joint rotation, we measure the rotation of the top of the column. Then, the measurement will be  $\theta_m=+\phi$ . Thus, while using two response quantities from the three degree of freedom model, the basic assumptions made in this identification will be violated.

It appears that if identification fails to converge for a set of response data, but does converge when we include more data that we assumed was linearly related, this implies an underlying problem with the mathematical model, rather than anything about the parameters of the model that is being identified!

#### Statistical Interpretation **of** the Error Function

A dynamic structural model can be defined as a functional G which predicts a vector response  $Y(t)$  from an input function  $v_g(t)$ , and a vector of parameters  $\underline{b}$ :

$$
\underline{Y} = G(v_{\varrho}, \underline{b}) + \underline{\epsilon} \tag{2.29}
$$

where  $\epsilon$  is a vector of error terms, and is often supposed to be normally distributed with zero mean.

A question which often arises is whether the complete set of parameters is needed in the model. In particular, if  $\underline{b}$  is partitioned so that  $\underline{b}^t=(\underline{b}_1,\underline{b}_2)^t$ , an hypothesis which could be tested is whether  $\underline{b}_2 = \underline{b}_2^*$ , where  $\underline{b}_2^*$  could be a zero vector. The test is based upon the relative reduction in the error function when all the parameters are estimated as compared with the error when  $\underline{b}_1$  is estimated with  $\underline{b}_2=\underline{b}_2^*$ . If the reduction is large, the hypothesis that  $\underline{b}_2=\underline{b}_2^*$  is untenable.

Suppose  $\underline{b}_1$  has p parameters and  $\underline{b}_2$  has q. Let  $\underline{b}^*$  be the optimal set of parameters with  $\underline{b}_2 = \underline{b}_2^*$ . With appropriate statistical assumptions, the statistic

$$
F = \frac{[J(b^*) - J(b)]/q}{J(b)/(n-p)} \tag{2.30}
$$

will approximately have what is called an  $F(q, n-p)$  distribution. This distribution is tabled in any of a number of sources [7]. The hypothesis that  $\underline{b}_2=\underline{b}_2^*$  is tested at the  $\alpha$  level of significance by comparing F with the critical value  $F_{1-\alpha}(q, n-p)$ . If this critical value is exceeded, the hypothesis that  $\underline{b}_2 = \underline{b}_2^*$  is rejected.

The use of a squared error loss function, therefore, is not only a good subjective measure of goodness-of-fit, but can be utilized as a quantitative tool for comparing alternative models.

#### CHAPTER 3

#### THREE STORY FRAME

#### Model Description

The test structure consisted of two parallel single-bay, three-story, moment resistant steel frames. The frames were fabricated from standard rolled shapes of ASTM A-36 grade steel. Detailed in Figure 3.1, the two frames designated A and B are separated by a distance of 6 ft. They are connected at floor levels by removable cross beams and bracing angles producing the effect of a floor diaphragm rigid in its own plane. The total height of the structure is 17 ft. 4 in. The story heights are 6 ft. 8 in., 5 ft. 4 in., and 5 ft. 4 in. The bar width is 12 ft. 0 in. Sections W5-16 and W6-12 are used for columns and girders, respectively.

Fully penetrated welded girder to column connections are used in this structure. Figure 3.2 depicts the details of these connections. The panel zone thickness is 1/4 in. (i.e. the column web thickness) for phase I of the experiments, and 1 in. (column web reinforced by 3/8 in. doubler plates on both sides) for phase II. Figure 3.3 lists the nominal section properties, and Figure 3.4 summarizes the estimated weights of the structure.



Figure 3.1 Structure plan and elevations of the test structure





	Girder W6x12	Column W5x16
	Nominal	Nominal
b(in)	4.00	5.00
d(in)	6.00	5.00
$t_w$ (in)	0.23	0.24
$t_f$ (in)	0.28	0.36
$\tilde{A}$ (in <sup>2</sup> )	3.54	4.70
$I_{\rm x}$ (in <sup>4</sup> )	21.7	21.3
$S_{\rm r}$ (in <sup>3</sup> )	7.25	8.53
$Z_{r}(in^3)$	8.23	9.61

Figure 3.3 Section properties

Weight
9300
9288
9290

Figure 3.4 Floor weights (lb)

#### **Instrumentation**

The frames were instrumented with linear potentiometers at each floor to measure floor translation. The frames had strain gauges attached to both flanges at the top and bottom of each column. Assuming a linear variation of bending strain along the length of the column, the relative rotation of the ends will be given by

$$
\theta = \int_0^L \frac{1}{\rho} dx = \frac{1}{h} \int_0^L \left[ \left( 1 - \frac{x}{L} \right) \epsilon_a + \frac{x}{L} \epsilon_b \right] dx
$$
\n
$$
= \frac{L}{h} \frac{\epsilon_a + \epsilon_b}{2}
$$
\n(3.1)

where  $\epsilon_a$  and  $\epsilon_b$  are the bending strains at either end, L is the length of the column, h is its height, and  $\rho$  is its curvature.

Additionally, in the Phase I experiments, LVDT's were attached to the first floor column bases to permit measurement of the column end rotation. **In** Phase II, the column bases were stiffened so that it was felt that the base would remain essentially rigid. Utilizing this information, the rotation of each joint relative to the base could be calculated.

#### Finite Element Model - Three Story Frame

In a previous study [15], Kaya and McNiven were able to show that by constructing a mathematical model of this frame, using system identification, they were able to gain physical insight into the seismic response. However, they found that when the effective column and girder lengths were adjusted to minimize the difference between the predicted and actual response, these lengths were substantially different from those in the real structure, particularly in the frame whose joints were not reinforced, suggesting that joint behavior was important in predicting total response. Their model did not include a joint element. They used static condensation to reduce the number of degrees of freedom to a size manageable for their identification method, but as was discussed in the previous chapter, this can lead to errors.

Developed here is a finite element model where joint panel zones are assumed rigid for flexural and axial deformations, but shear distortions are allowed. The column element stiffnesses will be given by

$$
\underline{k} = k' \begin{bmatrix} 2+\beta & 1-\beta & 0 \\ 1-\beta & 2+\beta & 0 \\ 0 & 0 & \frac{A}{2I}(1+2\beta) \end{bmatrix} .
$$
 (3.2)

The girder stiffnesses will be given by

$$
k = \frac{3EI}{L(1+2\beta)}\tag{3.3}
$$

The joint stiffnesses will be given by

$$
k = Gbht \tag{3.4}
$$

where

$$
k' = \frac{2EI}{L(1+2\beta)} \quad \beta = \frac{6EI}{L^2 A'G} \tag{3.5}
$$

and E, I, A, and *A'* denote Young's modulus, moment of inertia, section area, and effective shear area, respectively.

The displacement transformation matrices, *Ai,* for each element are given by (see Figure 35 for global coordinates)





The global stiffness matrix will be

$$
\underline{K} = \sum \underline{A_i} \underline{k_i} \underline{A_i} \tag{3.6}
$$

 $\bar{z}$ 





#### Identification

The primary goal of this study of the three story frame was to investigate the role of joint behavior in the overall response. Therefore, a sequence of models of increasing complexity is proposed. The nature and predictive power of each of these models is compared. In this way, a number of interesting facts about the behavior of the joints is revealed. Additionally, the identification algorithm is shown to be a valuable analytical tool.

In all cases the data being used are from the results of a test in which the El Centro earthquake record was used as a seismic forcing function. These earthquake records were scaled to produce elastic response in the test structure. In the phase II tests, the test earthquake was 40% of that recorded. In the phase I tests, the test earthquake was only 10% of that recorded.

#### Identification Using Displacement

The first four models were analyzed using only 6 seconds of displacement response in the identification. It is shown that displacement, while providing some useful information about stiffness distribution, offers limited capability for identifying many structural properties.

#### *Model* 1

In the first model, 4 parameters were used. The first three parameters were associated with the effective column lengths of the Phase I structure. All the columns on each floor were taken to have effective lengths that were their clear span times one of these parameters. One parameter was associated with mass proportional damping. The table stiffness was set very high, simulating a rigid base. All the other physical constants were set at their measured values.

The computer program converged from an error of 20.2 to an error of 0.35 in five steps. The rapid convergence can be seen in Figure 3.6.



Figure 3.6 Algorithm convergence

The resulting effective column length factors were 1.05, 1.04, and 1.12 from the top of the structure down. The displacement time histories, both before and after identification can be seen in Figure 3.7. The resulting match can be seen to be quite close. The closeness of the first two column factors suggests that one parameter could be used for both floors.




 $\bar{z}$ 



Figure 3.7b Third story displacement after identification (inches vs. seconds)



Figure 3.7c Second story displacement before<br>identification (inches vs. seconds)

 $\sim$ 



Figure 3.7d Second story displacement after identification (inches vs. seconds)

 $22$ 

÷.







Figure 3.7f First story displacement after identification (inches vs. seconds)

23

 $\bar{z}$ 

## *Model* 2

To determine the necessity of incorporating three parameters for effective column lengths, a new model was entered with only three parameters. One parameter was associated with the effective column lengths of the top two floors, one parameter with the first floor, and one parameter with mass proportional damping. Otherwise this model is identical to model 1.

Again, convergence of the algorithm is quite rapid, reducing the error from 20.4 to 0.35 in seven steps. The column length factors were 1.05 for the top two floors and 1.11 for the first floor. While the number of parameters has decreased, the error associated with the optimized parameters remains unchanged.

#### *Model* 3

It appears that the accuracy of the model is relatively insensitive to changes in distribution of parameters among the columns. To emphasize this, a new model was entered with 5 parameters. Only one of these was associated with the columns. One parameter was associated with the effective girder lengths at each of the three floors. Finally, one parameter was associated with mass proportional damping. Convergence was even more rapid, resulting in a final error of only 0.08 in 4 steps. More importantly, the resulting column effective length factor was 1.009. This indicates the variations in girder length are more critical than column lengths. However, the resulting girder length factors are far different than 1.0 - ranging from 0.32 to 52.3! The most reasonable explanation for this is that the girder lengths are not independent with respect to translation. That is, while the algorithm converges using only displacements, the identified parameters do not form an independent set. Thus one could expect another set of girder lengths to form a model with the same error. One could reasonably expect a whole class of models with the same error. In fact, investigations of the error surface in the vicinity of this minimum have shown the possibility of wide variations in parameter values, without significant change in the error function.

# **Identification Using Displacement and** Rotation.

It appears that the girder and column length factors do not form an independent set of parameters with respect to displacement response data. However, the rotation data, inferred from strain measurements, are not of the same magnitude as the displacement data. If used directly, identifying the parameters with the use of the rotation data could be expected to have little or no effect. The rotation data, therefore, are scaled by the modulus of elasticity of the steel,  $E=29.6x10^6$  psi. While somewhat arbitrary, this constant causes the two sets of data to be of the same order of magnitude. It should be pointed out that the relative weighting of the response variables will undoubtedly have appreciable effect on the parameter values, as it influences the response that the identification procedure will attempt to accommodate. Subsequent identifications were performed utilizing approximately 12 seconds of data.

#### *Mode/4*

This model was the same as Model 3, but all the response data was used to identify the phase II structure. After five iterations, the model had converged to an error of 3.84. Note that this error is summed over twice as many integrals as without the inclusion of the rotational response. The resulting effective column length factor was 1.12. The resulting effective girder length factors were 0.958, 0.721, and 0.694, listed from the top of the structure down. The apparent reduction in girder lengths and increase in column lengths was also noted in [15]. The important characteristic is that these factors are roughly equal.

#### *Mode/5*

In Kaya and Tang [15,24] it was noted that the change in girder stiffness could be attributed to the pitching motion of the table. In the previous models, we used the effective table pitch stiffness as was suggested in Tang. Model 5 is the same as Model 4, with the addition of a table stiffness parameter. In the phase II structure this reduced the error to only 3.78. However, in the phase I structure, this model reduced the error to 0.53. In the phase I model the resulting column length factor was 1.15 and the resulting girder length factors were 0.53, 0.63, and 0.55. In both cases, the resulting table stiffness factor was about 0.6. It appears that the

algorithm tends to soften up the system by increasing the base stiffness, and in order to compensate, decreases the effective girder lengths. Thus, it appears that the girder and base factors do not form an independent set.

### *Model* 6

3.9.

In the previous models, the parameter adjustment primarily took place in the effective girder lengths. In contrast, Model 6 is an attempt to permit the joints to accommodate the response. Thus, a four parameter model was entered, with one parameter associated with the columns, one parameter with the base stiffness, and one parameter with the effective joint panel thickness. After identification the errors in the phase I and phase II models were 0.632 and 4.08, respectively. This is only slightly larger than in Model 5, but the identification was done with two fewer parameters. The resulting parameter values are also interesting

parameter	Phase I value	Phase II value
column	1.09	1.07
base	0.96	0.97
joint	2.16	5.27
damping	1.35	1.54

Figure 3.8 Model 6 parameters

in that while the column and base parameters are much closer to the estimated values, the identified joint parameters accurately reflect the fact that the frame in the phase II experiments has reinforced joints. Thus, while it is possible to adjust the effective girder lengths to accommodate the behavior of the frame, it appears more sensible to attribute this behavior to the joints, particularly in light of the crude approximation inherent in the joint modelling. The results of the identification using this model utilizing the phase II data can be seen in Figure



Figure 3.9a Displacement at floor 3, before identification (inches vs. seconds)



Figure 3.9b Displacement at floor 3, after identification (inches vs. seconds)

 $\hat{\mathcal{A}}$ 

 $\hat{\mathcal{A}}$ 



Figure 3.9c Displacement at floor 2, before identification (inches vs. seconds)

 $\sim$ 





 $\bar{\gamma}$ 



Figure 3.9e Displacement at floor 1, before identification (inches vs. seconds)





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Figure 3.9g Rotation at floor 3, before identification (milliradians vs. seconds)







Figure 3.9i Rotation at floor 2, before identification (milliradians vs. seconds)



Figure 3.9j Rotation at floor 2, after identification (milliradians vs. seconds)

31



Figure 3.9k Rotation at floor 1, before identification (milliradians vs. seconds)



Figure 3.91 Rotation at floor 1, after identification (milliradians vs. seconds)

 $\bar{z}$ 

 $\hat{\mathcal{L}}$ 

### **CHAPTER 4**

### DYNAMIC MEASUREMENT OF SMALL ANGLES

### Design Objectives

Typically, structural models have fairly low frequency response - the highest frequency of interest being on the order of 100hz. Therefore, an angle measurement device capable of response up to 2 khz would be adequate. The maximum amplitude of rotational response is often in the milliradian range. This poses two problems:

- $(1)$  The instrument must be very sensitive.
- (2) The instrument must be very insensitive to motions other than those which are to be measured.

It is very desirable, additionally, that the instrument be both economic, easy to fabricate, and easy to use.

Existing Method§

### RVDT - Rotational Variable Differential Transformer

This is a transformer with variable coupling produced by moving a ferromagnetic core within the coils. In order to effect zero output for zero displacement, two secondary windings connected in electrical opposition are used. A mathematical analysis of the performance of this type of instrument was first described in detail by Atkinson and Hynes [3].

A number of manufacturers produce this type of device. A Schaevitz model RVDT was acquired to test its suitability. In static tests, the RVDT was capable of resolving  $10^{-4}$  radians. However, when the device was subjected to vibration, such as tapping the case lightly with a pencil, the induced signal noise reduced the resolution by at least an order of magnitude. Thus, while an RVDT might prove useful in static small angle measurement, its dynamic behavior is not adequate.

### **LVDT With Stationary Arm**

A balanced lever arm is attached to the structure on a pivot. A perfectly balanced arm will not rotate if its pivot is rotated or translated. This should permit measuring a distance change at the end of the arm with an LVDT. It is, however, difficult to balance the arm "perfectly". Further, the measurement apparatus at the end of the moment arm is likely to have significant effect. Clough and others have reported attempting this method without appreciable success.

### Strain Gauge

If strain gauges are placed on opposite sides of the neutral axis of a beam, strains at the extreme fibers can be measured. Using a linear elastic analysis, one can calculate the relative rotation between the ends. Theoretically, this can be extended throughout a structure. Indeed, this method is quite straight-forward to implement. It relies heavily on two assumptions:

(1) The entire structure has linear elastic response.

(2) The method of connecting the beams does not contribute to relative rotation.



Figure 4.1 Strain gauge placement

Obviously, in a variety of situations, these assumptions will not be appropriate. Kaya [15] has done some analysis work with this type of measurement using data collected by Clough and Tang [10].

# **Rotational** Accelerometers

There are a number of highly accurate rotational accelerometers built commercially, primarily for aerospace and military applications. Servo accelerometers are among the most accurate and stable.

Typically, the servo accelerometer is a closed loop, torque balance system. In the ilIustration below, relative motion of a balanced mass is detected by a position sensor whose output signal is applied to an electronic amplifier. The output current from the servo amplifier is applied to the torque motor. Thus mass is held in the same relative position. The current through the torque motor is accurately proportional to input acceleration.



Figure 4.2 Rotational servo accelerometer

Specifications for a typical range of servo accelerometers are as follows [22]:



There are two main drawbacks in the use of these devices for structural testing. Most important, in a typical case, rotational acceleration will be far smaller than linear acceleration at a given point. This makes the cross-axis sensitivity an important factor. Second, these devices are only reliable at approximately 60% of their natural frequency. Generally, the higher the natural frequency, the greater the cross-axis sensitivity. These devices could be useful in a number of tests, particularly if it is possible to order special designs aimed at minimizing the shortcomings.

### Ring Lasers

A ring laser is one which supports circulating light beams: independent oscillations for two counter-rotating beams. For instance, Figure 4.3 shows a resonator where three mirrors define a triangular path.



Figure 4.3 Ring laser

There are two mode conditions, one for each route around the ring:

$$
P = q \lambda \tag{4.1}
$$

where

- $P =$  optical perimeter of ring
- $q = an integer$

 $\lambda$  = resonant wavelength.

If there is a region in the resonator where the velocities for the two directions are unequal, let them be  $(u + du)$  and  $(u - du)$ . Then there are two oscillating frequencies:

$$
f_i = f_0 \left[ 1 + (-1)^i \frac{\delta u}{u} \frac{l}{P} \right] \quad i = 1, 2 \tag{4.2}
$$

where

$$
f_0 = \frac{qc}{P} \ . \tag{4.3}
$$

If samples of both beams are fed to a common detector, a beat frequency will be generated, given by

$$
f_b = f_2 - f_1 = 2f_0 \frac{\delta u}{u} \frac{l}{P}.
$$
\n(4.4)

If the resonator is rotated about an axis perpendicular to its plane, the resolved component of translational velocity, at a given position in the path, is then added to the light velocity. An integration must be performed around the ring to find the net unbalance between the two beams [23].

For a beam length dl

$$
\delta u = \Omega r \cos \theta \tag{4.5}
$$

where

 $\Omega$  = angular velocity

 $r =$  distance from rotation axis to dl

 $\theta$  = angle between light path and translation velocity.

But cos  $\theta = r \frac{d\phi}{dl}$  where  $\phi$  is angle about the axis, so

$$
\int \delta u \, dl = \int \Omega r^2 d\phi = 2\Omega A \tag{4.6}
$$

where  $A =$  the area enclosed by the resonating beams.

The beat frequency from rotation is then

$$
f_b = \frac{4\Omega A}{\lambda P}.\tag{4.7}
$$

For a square resonator with sides of length L, and a ReNe laser oscillating at 632.8 nm, this is

$$
f_h = L \, \Omega \, (1.58 \times 10^6). \tag{4.8}
$$

Thus, a practical instrument can be constructed with very high resolution, digital output, and virtually no cross-axis sensitivity. In fact, a square resonator with  $L = 1m$  has been used to accurately measure the rotation of the earth! Since these devices are presently being constructed out of a single crystalline block, they should be quite durable and economic, if constructed in any quantity.

There are two principal disadvantages to using ring lasers to instrument a structure. First, they tend to drift, though some researchers have been able to achieve drift rates as low as 0.1 deg/hr [l6l. Second, the size of the apparatus makes it unsuitable for small models. It would, however, be easily applied in instrumenting a real structure.

### Optical Lever

A lever arm permits amplifying an angle change, therefore increasing the ease with which it may be measured. In several of the previous instruments, a mechanical lever is employed. The fundamental problem with most of these methods, in dynamic application, is the mass of the lever arm. A light beam has no mass and, from this point of view, is most desirable.

If a mirror is attached to a structure, and a beam of light directed at it, rotations of the mirror will cause the reflection of the beam to move. Statically, an observer can merely measure the deflection and, depending on his distance from the mirror, obtain any desired degree of accuracy.



Figure 4.4 Mirror attached to frame

For small mirror angle change,  $\theta$ , the deflection d will be given by  $d=2r\theta$ , where r is the distance from the observer to the mirror.



Figure 4.5 Measuring spot deflection

For example, from a 1 milliradian structural rotation, at a distance of 2 meters we could expect a 4 mm movement.

One of the simplest methods of electronically measuring the beam deflection is to direct the beam at a pair of adjacent photocells.



Figure 4.6 Adjacent photocells

As the spot moves onto either photocell, the output from that photocell will increase, and the output from the other photocell will decrease. The output from the differential amplifier will reflect the spot motion.

Since the detector will be some distance from the structure, and presently available photocells are fairly small, it is desirable to use a source with small dispersion. A typical helium neon laser has a dispersion of only about 1 milliradian. A simple two lense system can be employed to optimize the size of the spot.



Figure 4.7 Beam dispersion

The response characteristics of the system depend, to a large extent, on the light source used. Lasers tend to have an approximately Gaussian distribution of intensity.



Figure 4.8 Gaussian intensity distribution

The differential amplifier output will therefore be approximately proportional to a cumulative normal distribution:

$$
o(x) = C_0^x e^{-bt^2} dt. \tag{4.9}
$$

### Secondary Effects

If the spot is centered on two photocells with the same conversion characteristics, variations in ambient light will have little effect on differential photocell output. However, if the photocells are unbalanced, or the beam is not centered, ambient light can produce noise. This is primarily due to nonlinear photocell response at high intensity - particularly at the center of the laser spot. The simplest method of reducing this effect is to conduct tests in the dark. However, filters are available which can remove virtually all ambient light, leaving virtually all the laser beam intact. This application makes the monochromaticity of lasers extremely important.

It is difficult to place the laser, mirror, and detector collinear. If the detector is placed to the side, linear motion of the mirror will produce a deflection of the spot:



Figure 4.9 Induced secondary output

This deflection will be at right angles to the measurement we are interested in if the plane of the laser beam contains the motion of the mirror. Properly aligning the detector will minimize this effect.

Use of a pentagonal prism in place of the mirror will remove the effect of rotations orthogonal to those being measured:



Figure 4.10 Pentagonal prism

Rotations of the prism about an axis out of the paper will not produce a beam deflection.

As was previously discussed, the output from the split photocells will not be a linear function of spot displacement. The instrument could be made linear by moving the photocells to keep the spot centered and measuring the movement of the photocells directly. The technique used for this linear measurement would depend on the maximum displacement and frequency response expected. The success of this approach would depend on the accuracy and rapidity with which the photocells can be positioned. A possible alternative is to keep the spot centered by rotating the mirror. The principal advantage of this system is that the mirror need only move a minor amount. It is conceivable that the mirror could be constructed out of a single piezoelectric crystal so that the motion of the mirror could be controlled by applying a voltage to the crystal. While providing a linear output, all these techniques require greater expense and development time than merely measuring photocell output. Further, the photocell output is close to linear over a large segment of its response.

The desired output is the difference of two relatively strong signals. It is necessary to minimize the effect of the magnitude of each photocell output on the measurement of the difference. Electronic devices designed to do this are known as instrumentation amplifiers. Fortunately, it is now possible to construct a high quality instrumentation amplifier incorporating only one integrated circuit. The circuit diagram and printed circuit layout for this device is shown in Figures 4.11 and 4.12.

While designed to measure joint rotation, it is interesting to note that this device has since proved valuable in measuring the physical properties of the materials used in a model structure. A rod of material was clamped at one end and set vibrating. The beam was aimed directly at the photocells with the material interposed. As less of the beam was interrupted, the photocell . output increased. Thus, the characteristic frequency of the vibrating rod was measured by the photocell output. From this, the modulus of elasticity could be derived.

43



Figure 4.11 Printed circuit (2x) - Instrumentation Amplifier



Figure 4.12 Parts layout (2x)-Instrumentation Amplifier

#### CHAPTER 5

### SIX STORY FRAME

Studies of the three story frame proved quite fruitful - insight has been gained into both the behavior of the structure and the performance of the identification procedure. A number of questions were left open. To answer some of these, a new test model was developed.

The three story frame was a simple structure with relatively few degrees of freedom. We wished to determine the effect of increasing the number of degrees of freedom on the optimization procedure without greatly increasing the structure's complexity. In the hope of eventually being able to generalize previous speculations made by Kaya and McNiven [15] on data necessary for identification, the new structure was again a simple moment resistant frame, but with six stories.

The methods outlined in the previous chapter for making dynamic measurements of joint rotation, have not been applied in a structural test environment. While there are several advantages to being able to make kinematic measurements of rotation, testing the methods on a simple frame was essential. This also allows an assessment of their value in the identification of structural properties.

### A **Model** Design Note

Data acquisition equipment is constrained to measuring a limited frequency bandwidth. Therefore, even when studying relatively simple frames, it is important to keep the important frequency content of response within the limits of the measuring apparatus.

In a model which has repetitive elements, it is desirable to be able to relate the modal frequencies of the whole structure to the modal frequencies of a single element. To illustrate, consider a shear structure model where each story has the same stiffness and mass.



Figure 5.1 Repetitive shear model

The stiffness and mass matrices for this structure are given by

$$
\underline{K}_{N} = \begin{bmatrix} k & -k & & & \\ -k & 2k & -k & & \\ & -k & 2k & . & \\ & & \ddots & \ddots & \ddots & \\ & & & -k & 2k \end{bmatrix}
$$
 (5.1a)  

$$
\underline{M}_{N} = mI_{NxN}.
$$
 (5.1b)

The natural frequencies,  $w^2$ , of the structure will be given by the solutions of

$$
P_N(w^2) = \det(K_N - w^2 M_N) = \det(K_N - m w^2 I_{N \times N}) = 0.
$$
 (5.2)

By expanding about the last column, the frequencies can be found as the roots of the recursively defined polynomials:

$$
P_1(w^2) = k - w^2 m \tag{5.3a}
$$

$$
P_2(w^2) = m^2 w^4 - 3mkw^2 + k^2
$$
\n(5.3b)

 $\sqrt{2}$ 

$$
P_n = (2k - w^2m)P_{n-1} - k^2P_{n-2}.
$$
\n(5.3c)

If we use the change of variables  $w^2 = tk/m$ , we get

$$
P_1(t) = k(1-t) \tag{5.4a}
$$

$$
P_2(t) = k^2(t^2 - 3t + 1) \tag{5.4b}
$$

$$
P_n(t) = k[(2-t)P_{n-1} - kP_{n-2}]. \tag{5.4c}
$$

The roots, t, of these polynomials will be the same as the roots of

$$
P_1(t) = 1-t \tag{5.5a}
$$

$$
P_2(t) = t^2 - 3t + 1 \tag{5.5b}
$$

$$
P_n(t) = (2-t)p_{n-1} - P_{n-2}.
$$
\n(5.5c)

An alternative formulation can be obtained by expanding the determinant about the first column:

$$
P_n = (k - w^2 m) Q_{n-1} + k^2 Q_{n-2} \tag{5.6}
$$

where

$$
Q_n = \det_{n \times n} \begin{bmatrix} 2k - m w^2 & -k & & & & \\ -k & 2k - m w^2 & -k & & & \\ & -k & 2k - m w^2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \ddots & 2k - m w^2 & -k \\ & & & & -k & 2k - m w^2 \end{bmatrix} .
$$
 (5.7)

However, the  $Q_n$  have the same recursion formula as the  $P_n$ :

$$
Q_n = (2k - mw^2)Q_{n-1} + k^2 Q_{n-2}
$$
\n(5.8)

with

$$
Q_1 = 2k - w^2m
$$
  
 
$$
Q_2 = m^2w^4 - 4mkw^2 + 3k^2.
$$

Hence,

$$
P_n = (k - w^2 m) Q_{n-1} + k^2 Q_{n-2}
$$
  
= 
$$
[(2k - mw^2)Q_{n-1} + k^2 Q_{n-2}] - kQ_{n-1}
$$
  
= 
$$
Q_n - kQ_{n-1}.
$$
 (5.9)

Applying the same change of variables,  $w^2 = tk/m$ , to the Q's gives

$$
Q_1 = k(2-t) \tag{5.10a}
$$

$$
Q_2 = k^2(t^2 - 4t + 3) \tag{5.10b}
$$

$$
Q_n = k \left[ (2-t) Q_{n-1} + k Q_{n-2} \right]. \tag{5.10c}
$$

So the zeros of the  $P_n'$  may be discovered by using the alternative set of formulas:

$$
P_n' = Q_n' - Q_{n-1}' \tag{5.11}
$$

where the  $Q'_n$  are defined by

$$
Q_n' = (2-t)Q_{n-1}' + Q_{n-2}'
$$
\n(5.12a)  
\n
$$
Q_1' = 2-t
$$
\n(5.12b)  
\n
$$
Q_2' = t^2 - 4t + 3;
$$
\n(5.12c)

hence,

$$
P'_n = Q'_n - Q'_{n-1}
$$
  
= (2-t)Q'\_{n-1} + Q'\_{n-2} - Q'\_{n-1}  
= (1-t)Q'\_{n-1} + Q'\_{n-2}. (5.13)

It is interesting to note that

$$
P'_1(1) = 0
$$
  
\n
$$
P'_2(1) = -1
$$
  
\n
$$
P'_3(1) = -1
$$
  
\n
$$
P'_4(1) = 0
$$
  
\n
$$
P'_5(1) = 1
$$
  
\n
$$
P'_6(1) = 1
$$
  
\n
$$
P'_7(1) = 0
$$
  
\n
$$
P'_8(1) = -1
$$

Since, by definition,  $P'_n(1) = P'_{n-1}(1) - P'_{n-2}(1)$ , this pattern evidently repeats. Hence,

$$
P'_{3n+1}(1) = 0 \quad n=0,1,2,... \tag{5.15}
$$

Thus,

*Proposition:* A repetitive shear structure with  $3n+1$  stories has, as one of its modal frequencies, the same frequency as that of a single bay.

# Asymptotic **Behavior**

Consider story i of an N story repetitive shear structure, with story shears  $V_i$  and  $V_{i-1}$ .



Figure 5.2 Single story of shear structure

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Statics suggests that

$$
V_i - V_{i-1} = f_I = m \frac{d^2 v_i}{dt^2}.
$$
 (5.16)

This and the constitutive relation

$$
k(v_i - v_{i-1}) = v_{i-1} \tag{5.17}
$$

combine to give

$$
m\frac{d^2v_i}{dt^2} = k(v_{i+1}-2v_i+v_{i-1}).
$$
\n(5.18)

If the time and distance scales of  $v_i(x) = v(i,x)$  are changed by setting

$$
u_N(s,t) = v(Ns, Nt) \quad 0 < s < 1 \tag{5.19}
$$

where  $\nu$  is here extended differentiably between integer values of  $i$ , we have

$$
m\frac{d^2u}{dt^2} = N^2m \frac{d^2v}{dt^2}
$$
\n
$$
= N^2k (v_{i+1} - 2v_i + v_{i+1})
$$
\n
$$
= k \frac{u (s + \frac{1}{N}) - 2u (s) + u (s - \frac{1}{N})}{\frac{1}{N^2}}.
$$
\n(5.20)

The effect of the distance translation is to make the domain of u independent from N. The last expression contains the second central difference of u so, as N approaches infinity, this becomes

$$
\frac{\partial^2 u}{\partial t^2} = \frac{k}{m} \frac{\partial^2 u}{\partial s^2}.
$$
\n(5.21)

Applying the technique of separation of variables, and applying the boundary conditions for a cantilever

 $u(0) = 0$  $u'(0) = 0$  $u''(1) = 0$  $u'''(1) = 0,$ (5.22)

lead to

$$
w^2 = n^2 \frac{k}{m} \quad \text{where } n = 1, 3, 5, 7, \dots \tag{5.23}
$$

Thus, as N gets large, the roots of the  $P_n$  will have the approximate ratios  $1^2$ ,  $3^2$ ,  $5^2$ ,  $7^2$ ,...

1 1.00 2 0.382.62 0.20 1.55 3.25 0.12 1.00 2.35 3.53 0.080.69 1.72 2.83 3.68 0.060.50 1.29 2.24 3.14 3.77 0.040.38 1.00 1.792.623.343.83 0.03 0.300.79 1.452.18 2.893.483.86 0.03 0.24 0.65 1.20 1.84 2.49 3.10 3.58 3.89 0.020.200.53 1.00 1.562.152.743.243.663.91

Figure 5.3 Roots of polynomials  $P'_n$ 

# Design of the Six Story Frame

 $\underline{n}$ 

Utilizing the design considerations of the previous section, a frame was designed to be tested on a small shaking table test facility at the University of California at Berkeley (see Figure 5.4). The frame was designed to have a highest modal frequency of about 20hz, since that is the approximate limit of the response of this table.



Figure 5.4 Six story frame on the table

To make the joints as simple and as continuous as possible, the columns were constructed out a single piece of material. The girders, also made of the same material were then set into the columns (see Figure 5.5).



Figure 5.5a Geometry of six story frame

After testing the properties of several different materials, lexan was chosen since it was easy to machine, had material properties which were apparently independent of frequency, and could be glued with a solvent to produce very homogeneous joints.



Figure 5.5b Joint detail

# **Instrumentation**

Schaevitz LVDT accelerometers were attached to the joints on one side of the structure. These were chosen since they respond to frequencies up to about 100 hz, and also down to and including 0 hz. Mirrors were attached to the joints on the other side of the structure to become a part of an optical rotation measurement apparatus described in the previous chapter. The photocell targets were mounted on a stage which could be translated a controlled amount and measured with a micrometer.



Figure 5.6 Photocell targets

Lasers and optical benches were mounted in a stable rack. This rack had adjustable shelves, permitting precise alignment and adjustment of the laser systems:



Figure 5.7 Lasers and optical benches

The lasers were carefully aligned to minimize parallax errors. As was noted in the previous chapter, the output of each target will only be linear over a moderate range. However, each photocell target was calibrated over a wide range. In this way, by fitting a polynomial to the calibration curves for the targets, the output of the apparatus could be linearized over a much larger range.

### Finite Element Model - Six Story Frame

Developed here is a finite element model where joint panel zones and beams are allowed to distort both in shear and in flexure, but are assumed rigid for axial deformations. Using Figure 5.8 as a reference for global coordinates, the transformations from the local element coordinates to global coordinates will be given.



Figure 5.8 Six story frame global coordinates

### Horizontal member i:

 $\underline{A}^h = [\underline{0}_{3x3} \cdots \underline{0}_{3x6} \underline{a}_{3x6}^h \underline{0}_{3x6} \cdots \underline{0}_{3x6}]$ 

(5.24)

where

$$
\underline{a}^{h} = \begin{bmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{b}{2} & \frac{b}{4} & 0 & 0 & 0 \end{bmatrix}
$$
(5.25)

Vertical member i:

$$
\underline{A}^{\nu} = [\underline{0}_{4x3} \cdots \underline{0}_{4x6} \underline{a}_{4x6}^{\nu} \underline{b}_{4x6}^{\nu} \underline{0}_{4x6} \cdots \underline{0}_{4x6}] \quad 2 \leq i \leq 6
$$
\n(5.26a)  
\n
$$
\underline{A}^{\nu} = [\underline{a}_{4x3}^{\nu} \ b_{4x6} \underline{0}_{4x6} \cdots \underline{0}_{4x6}] \quad i=1
$$
\n(5.26b)

where

$$
\underline{a}^{\nu} = \begin{bmatrix} 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{h}{2} & -\frac{h}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{h}{2} & \frac{h}{4} & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 & 0 & 0 \end{bmatrix}
$$
(5.27a)  

$$
\underline{a}^{\prime} = \begin{bmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ -\frac{h}{4} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$
(5.27b)

**Joint i:**

$$
\underline{A}^j = [\underline{0} \cdots \underline{0} I_{3x3} \underline{0} \cdots \underline{0}] \quad 1 \leq i \leq 6
$$
\n
$$
\underline{A} = [\underline{I}_{3x3} \underline{0} \cdots \underline{0}] \quad i = \text{base}
$$
\n(5.28a)\n(5.28b)

The beam stiffness matrices will be given by

$$
k = k_e - k_g
$$
 where (5.29)

$$
\underline{k}_e = \frac{2EI}{L^3} \begin{bmatrix} 2L^2 & L^2 & -3L & 3L \\ L^2 & 2L^2 & -3L & 3L \\ -3L & -3L & 6 & -6 \\ 3L & 3L & -6 & 6 \end{bmatrix}
$$
 (5.30)

 $\hat{\boldsymbol{\beta}}$ 

 $\bar{z}$ 

 $\overline{\phantom{a}}$ 

 $\tilde{\gamma}$ 

$$
k_g = \frac{N}{30L} \begin{vmatrix} 4L^2 - L^2 - 3L & 3L \\ -L^2 & 4L^2 - 3L & 3L \\ -3L & -3L & 36 & -36 \\ 3L & 3L & -36 & 36 \end{vmatrix} .
$$
 (5.31)

The beam mass matrices will be given by

$$
\underline{m} = \frac{\overline{m}L}{420} \begin{bmatrix} 4L^2 & -3L^2 & -22L & -13L \\ -3L^2 & 4L^2 & 13L & 22L \\ -22L & 13L & 156 & 54 \\ -13L & 22L & 54 & 156 \end{bmatrix}
$$
(5.32)

L is the length,  $\overline{m}$  = mass/length, and N = normal force. The joint stiffness matrices will be given by

$$
\underline{k}_J = \begin{bmatrix} Gbht & & & \\ & \underline{EI}_x & & \\ & & \underline{E}I_z & \\ & & & h \end{bmatrix} . \tag{5.33}
$$

### **Material Properties**

The total frame weighed 18.4kg. The consistent mass matrix was calculated using this number. In addition, the accelerometers and mirrors contributed to the frame's translational mass. The accelerometers and associated hardware averaged 65.5g. The mirrors with their hardware averaged 20.1g.

The modulus of elasticity of the lexan is listed as 1-300,000 by the manufacturer. To estimate this more exactly, a piece of material was clamped at one end, and a number of loads were applied to the end. By measuring the deflections at midspan, the modulus of elasticity in flexure was estimated at 237,000.

Modal damping ratios were determined by exciting the model in the *nth* mode, and recording the exponential decay. For any two positive peaks m cycles apart,  $v_0$ and  $v_m$ , the damping ratio  $\xi$  can be determined from

$$
\ln\left[\frac{v_0}{v_m}\right] = 2m\pi \frac{\xi}{(1-\xi^2)^{\frac{1}{2}}} \tag{5.34}
$$
which can be simplified for low damping to

$$
\xi = \frac{\nu_0 - \nu_m}{2\pi m \nu_m}.\tag{5.35}
$$

For the six story model, modal ratios and frequencies were determined to be



The damping coefficients  $a_k$  and  $a_m$  can be derived from two sets of modal damping ratios  $\xi_i$  and natural periods  $T_i$ , as follows:

$$
a_m = \frac{4\pi (T_2 \xi_2 - T_1 \xi_2)}{T_2^2 - T_1^2}
$$
(5.36)  

$$
a_k = \frac{T_1 T_2 (T_2 \xi_1 - T_1 \xi_2)}{(T_2^2 - T_1^2)}
$$
(5.37)

**In** the six story model, using the first two modes we have

$$
a_m = 0.13 \tag{5.38}
$$
  

$$
a_k = 0.00012 \tag{5.39}
$$

## **Identification**

The six story frame was subjected to a reproduction of the same **El** Centro earthquake record as the three story frame. As in the identification of the three story frame, the rotation measurements were scaled by the modulus of elasticity of the material to produce quantities of the same order of magnitude. **In** all of these identifications, both the acceleration and rotation time histories were used.

## **Model** 1

The first model was developed using only two parameters - one for the columns, and one for mass proportional damping. **In** the six story frame, the column and girder parameters were not associated with the effective lengths, but rather with the moment of inertia. After three steps, the error had decreased from 23,400 to 7,199. The column parameter changed from l.0 to 0.93, while the damping factor changed from 1.0 to 6.6. Thus while the overall response is approximated using close to the actual column geometry, it is apparent that some further investigation is required.

#### Model 2

The second model included one more parameter associated with the shear modulus of the joint. The estimate of shear modulus was derived from the measured elastic modulus by assuming a Poisson's ratio of 0.25. This produced still more improvement in response matching, producing an error of 5090 in five steps, with resulting parameters of 1.03 for the columns, 0.86 for the joint modulus, and 1.16 for the damping parameter. It is apparent that the inclusion of the joint parameter significantly improves response matching. It would seem that, for this model at least, the joint behaves more flexibly than would be expected from measurements of material behavior.

#### Model 3

Response history matching using the joint shear modulus parameter reflects the degree to which joint shear deformation accounts for the total response of the frame. **In** order to assess the relative effect of joint flexure, a new model was applied using four parameters - one again associated with the column moment of inertia, one with mass proportional damping, and one associated with each joint moment of inertia. In four steps, the error function arrived at 3149, with the column parameter reaching 1.04 and the damping parameter reaching 1.2. The joint moment of inertia parameters, however, became 0.18 and 1.4. Interestingly, while the flexural deformations seem to account for more error than the shear deformations, it is unclear from this analysis whether the two flexures were in fact different, or just impossible to identify separately. Since the construction of the frame was designed to keep the joints as homogeneous as possible, it seemed reasonable to assume that the joint parameters should be lumped together.

## Model 4

In this model the joint moments of inertia were associated with a single parameter, and an additional parameter was included to represent the shear modulus. With the addition of the column and damping parameters, this model has four parameters. In 5 steps the error was reduced to 3077, which is not a very substantial improvement over Model 3. The column parameter was 1.04, and the damping factor was 0.75. The inclusion of the joint shear parameter is therefore relatively unimportant. Further, the shear factor became 0.34 while the flexural factor became 1.48, indicating that the two are not independent.

## Model 5

In Model 5 the shear parameter of Model 4 was replaced by a parameter associated with the moment of inertia of the girders. If the joint flexural parameters are the logical set, the inclusion of the girder parameter should not decrease the error significantly, and also should not be independent. In fact, the error became 2700. This is only slightly smaller than in Model 4. The parameter associated with the girder became 0.86 while the parameter associated with the joint became 2.37. Figure 5.9 depicts the measured and predicted response histories of this model.



Figure 5.9a Sixth floor acceleration before<br>identification  $(in/sec<sup>2</sup>$  vs. seconds)















Figure 5.9e Fourth floor acceleration before<br>identification  $(in/sec<sup>2</sup>$  vs. seconds)







Figure 5.9g Third floor acceleration before<br>identification  $(in/sec<sup>2</sup>$  vs. seconds)







Figure 5.9i Second floor acceleration before identification  $(in/sec<sup>2</sup>$  vs. seconds)







Figure 5.9k First floor acceleration before<br>identification  $(in/sec<sup>2</sup>$  vs. seconds)

 $\mathcal{L}_{\mathcal{A}}$ 







Figure 5.9m Sixth floor rotation before identification (milliradians vs. seconds)



Figure 5.9n Sixth floor rotation after identification (milliradians vs. seconds)



Figure 5.90 Fifth floor rotation before identification (milliradians vs. seconds)



Figure 5.9p Fifth floor rotation after identification (milliradians vs. seconds)



Figure 5.9q Fourth floor rotation before identification (milliradians vs. seconds)







**Figure** 5.9s **Third floor rotation before** identification (milliradians vs. seconds)



**Figure 5.9t Third floor rotation after**  $i$ dentification (milliradians vs. seconds)



Figure 5.9u Second floor rotation before identification (milliradians vs. seconds)



Figure 5.9v Second floor rotation after identification (milliradians vs. seconds)

 $\sim$ 



Figure 5.9w First floor rotation before identification (milliradians vs. seconds)



Figure 5.9x First floor rotation after identification (milliradians vs. seconds)

#### CHAPTER 6

#### CONCLUSIONS

It has been shown that the use of modal decomposition in the Gauss-Newton iterative identification procedure greatly reduces the computational effort required. At the same time, the technique appears to be quite stable, and converges rapidly.

This technique was applied to data resulting from seismic tests of a pair of three story frames previously investigated in some detail. It was discovered that even though the frames were very similar the identification procedure was able to determine accurately which of these frames had reinforced joints, but only when the identification included information gathered about the rotation of the joints in addition to their translations.

This data, however, was not kinematic in nature. Rather, it was inferred from strain measurements using the assumption that the whole frame remained linear and elastic. An optical technique was developed for the dynamic measurement of joint rotation.

A new six story frame was built, and instrumented with these optical instruments. This frame was built with joints that were as homogeneous as possible. With the use of the rotational data, it was found that the joint behavior still plays an important role in the structure's behavior, and in the identification.

Therefore, a tool has been developed for the identification of elements in structures with a relatively large number of degrees of freedom. This tool, when applied to the data made available through the use of the optical instrumentation, is valuable in describing structural behavior.

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# **APPENDIX**

## **COMPUTER PROGRAMS**

# **Computer Program Input**

I. Integration parameters

 $\delta, \alpha, \theta$  for integration

) Rank parameters

NP - number of parameters

NDOF - # of total degrees of freedom

NINDF - # of recorded degrees of freedom

NXNT - # of modes for integration

NGP - # of global coordinates

3. Parameter specification - for each range of same values

 $E_i$  - fixed value associated with global parameter i

 $I E_i$  - code for computing global coordinates:

-i same as global parameter i

0 fixed

i  $x\beta$ ,

IGPS - start of range

IGPE - end of range

4. NFN - # of normal forces

 $f_i^N$  - normal forces, if any

5. NDEL -  $#$  of dof to delete from stiffness matrix

degrees of freedom, if any

6. NCNDNS - # of dof to condense

degrees of freedom, if any

7.  $R_i$  - static displacements for  $i=1,...,N$ 

8. Measurement identification

For i=l,...,NINDF *IR*<sup>j</sup> <sup>=</sup>

1 displacement

2 velocity

3 acceleration

 $IDF_i =$  dof for this measurement

9. End tolerance

SLMIN - minimum slope for line search termination

IT - maximum iterations

ENDTOL - necessary improvement for continuing

DDF - factor for finite differences

10. Initial estimates.  $\beta_{i,j} = 1, ..., NP$ 

If absent, use data file TEMP.

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ROUTINES FOR THREE STORY FRAME

common/#rf1g/iwr<br>common/gp/esteel,ri(6),area(6),areap(6),rl(6),ra(3),rb(3),gj(4),<br>tablek,tw,ak,am,rgp)<br>common /feduce/phi(39,39),dphi(39,39,10)<br>common /feduce/phi(39,39),dphinr(39,10)<br>dimension phitmr(39),dphtmr(39,10)<br>dim common /gpl/ e(69),ieval(69),fn(6)<br>common/gp/esteel,ri(6),area(6),areap(6),rl(6),ra(3),rb(3),g(4),<br>† tablek,tw,ak,am.ngp<br>common/transf/ag(14,3),ac(3,14,3) nindf,nxnt,ndel,nondns,idel(39),iondns(39) subroutine dir(b,k,m,level,dk,ks,err,grad,ah,u,dudb,f,ncall common /gpl/ e(69),ie(65),fri(6)"<br>dimension b(1),k(noall,1),m(noall,1),dk(noall,10),<br>\* grad(1),ah(noall,noall),<br>dudb(6,noall,1),u(6,1),f(1) subroutine addk(tm,sm,n,ncall)<br>dimension tm(ncall,ncall),sm(ncall,ncall)<br>do 100 i=i,n<br>tm(i,j)=i,n<br>tm(i,j)=i,1,j)+sm(i,j) ,tdudb(10),teu(39)<br>,db(1),bt(20),temtu(39)  $\arg(\frac{7}{3}, 1) = 1, \zeta(1)$ <br>  $\arg(8, 1) = -1, \zeta(1)$ <br>  $\arg(9, 1) = -1, \zeta(1)$ <br>  $\arg(9, 1) = -1, \zeta(1)$  $12 - 11$ subroutine afix  $ac(1, 1, 1) = -d1$  $= -d1$ -<br>-<br>"  $3 = d1$ ac(1,3,3)=-d1<br>ac(2,3,3)=-d1 ,db,alpha,dj)<br>real k,m,ks  $3 = d1$  $\frac{1}{2}$  $\overline{a}$  $\widehat{\mathcal{S}}$  $1/r1(1)$  $d1 = 1.7r1(3)$ ,ks(ncall)  $ac(2, 3, 2)$  $ac(1, 3, 2)$  $ac(2, 2, 2)$ return  $ac(1,3)$  $ac(1,2$  $ac(2, 2)$  $ac(2, 1)$  $d = 1$ return י<br>ד end<br>end end  $100$ 

dphtmr(i,ii)=0.0<br>do 95 j=1,ndof<br>phitmr(i)=phitmr(i)+phi(j,i)\*xmr(j)<br>do 93 ii=1,np<br>dphtmr(i,ii)=dphtmr(i,ii) + dphi(j,i,ii)\*xmr(j) common /fdiff/ df:(39),tdi(39),r<br>common /fdiff/ df:(39),tdi(39),r<br>ifd=0<br>da=alpha\*ddf<br>da=alpha\*ddf<br>da=alpha\*ddf<br>grad(1)=1,p<br>do 60 l=1,3<br>do 6011nue 1,1)=0.0<br>continue 1,1)=0.0 common /static/ ir(39),idf(39),r(39) read (1) npts,delt<br>read (1) vgc(1),(v(111),111=1,nindf)<br>if (1wr.eq.0) go to 101<br>write (3) npts,delt ks(j)=1./(k(j,j)\*a1k+m(j,j)\*a01m)<br>do 102 i=1,3 70 contrine<br>90 call mfind  $(k, m, dk, \text{nea11}, b)$ <br>80 call mfind  $(k, m, dk, \text{nea11}, b)$ <br>20 phitmr(1) = 0.0<br>20 91 sines  $\frac{1}{2}$  (3) (z, 111=1, nindf+1) if (ic.ge.npts) go to 205<br>k1=1  $f (k2, eq. 1) k1 = 2$ do 105 j=1, nxnt<br>perr(j)=0.  $k2 = mod(10, 2) + 1$ a01m=a0+a1<sup>\*</sup>am do 70  $j=1, np$ <br>ah(i,j)=0.0  $(k-1)$  $a$   $k = 1, a$   $a$ <sup>\*</sup> $a$   $k$  $\begin{array}{c} 121 \text{ s}1111 \\ 131 \text{ s}2111 \\ 141 \text{ s}111 \\ 112 \text{ s}1111 \\ 112$ continue continue  $11 = 1 + 1$ tk ្នុ  $u(1,j)=0.$ rewind 1 continue  $0.0 - r$  $tk1=3*$  $tk2=3*$  $i$ c= $i$ c+  $2 = 0.0$  $rac{0}{2}$  $252$  $\frac{1}{2}$ 

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aiphab= 1.0<br>call check(b,bn,db,alphab,alphab,okb)<br>call dir(bn,k,m,level,dk,ks,errb,grad,ah,u,dudb,f,ncall,db,alphab<br>call dir(bn,k,m,level,dk,ks,errb,grad,ah,u,dudb,f,ncall,db,alphab if  $(djb)$  300,450,227<br>227 alphan=cubic(erra,dja,errb,djb,alphaa,alphab)<br>1f ((okb.lt.1.0).and.(intr.eq.1)) bound=yes<br>print 6,alphaa,erra,dja,alphab,errb,djb,alphan,bound<br>6 format (11x,7e10.3,7x,a3)<br>6 format (11x,7e10.3,7x,a call dir (bn,k,m,level,dk,ks,errn,grad,ah,u,dudb,f,ncall,db, alphan=sq(erra,dja,errb,djb,alphaa,alphab)<br>call check(b,bn,db,alphan,alphab,okn)<br>if (okn.lt.1.0) bound=yes<br>bound=no ,alphaa,erra,dja,alphab,errb,djb,alphan,bound<br>bound=no ,k,m,level,dk,ks,errn,grad,ah,u,dudb,f,ncall<br>call d call dir(bn,k,m,level,dk,ks,errn,grad,ah,u,dudb,f,ncall print 6 ,alphaa,erra,dja,alphab,errb,djb,alphan,bound print 6,alphaa,erra,dja,alphab,errb,djb,alphan,bound 227 alphan=cubic(erra,dja,errb,djb,alphaa,alphab) aifuan-corpicteria, paralelista (intr.eq.1)) bound=yes<br>if ((okb.1t.1.0).and.(intr.eq.1)) bound=yes alphan=sq(erra,dja,errb,djb,alphaa,alphab) intr=0<br>if(err.gt.preerr) go to 9006<br>if (abs(err-preerr).le.endtol) go to 600 if (absolution).le.endtol) go to 600<br>erra=err<br>erra=err call check(b,bn,db,alphab,alphab,okb) call check(b,bn,db,alphan,alphab,okn) erraerr<br>
200 do 210 1=1, np<br>
200 do 210 1=1, np<br>
210 db(1)=grad(1)<br>
call mprint(ah, np, np, ncall, 'ah')<br>
call mprint(ah, np, ncall, 'ah')<br>
print 94ms.lda.<br>
do 215 i=1, np. 210 db(l)=grad(l)<br>call mprint(ah,np,np,ncall,'ah') call symsol(ah,db,np , 1,0,ncall) alphan, djn<br>if (abs(djn). lt. slmin) go to 455 226 if (abs(djb).lt.slmin) go to 450 if (abs(djn).lt.slmin) go to 455 uv rij it.)<br>print 946,i,b(i),grad(i),db(i)<br>continue print 946,i,b(1),grad(i),db(i) c if(err.gt.preerr) go to 9006 go to 400<br>300 if (errb.gt.erra) go to 227<br>300 if (errb.gt.erra) go to 227 300 if (errb.gt.erra) go to 227 if(numit.gt.it) go to 700<br>print 1005 if(numit.gt.it) go to 700 if (okn.1t.1.0) bound=yes 6 format (11x,7el0.3,7x,a3) 230 bn(l)=b(l)-alphan\*db(l) \* alphan,djn ) dja=dja-db(l)\*grad(l) if (djn) 240,455,245 if (djb) 300,450,227 if (djn) 240,455,245 print 945,numit 240 erra=errn<br>dja=djn<br>alphaa=alphan djb=djn<br>alphab=alphan do 215 1=l,np do 220 1=1,np do 230 1=1,np 200 do 210 1=1,np alphaa=alphan alphab=alphan \* ,djb)<br>if (abs(djb<br>intr=intr+l<br>intr=intr+l go to 400 alphaa=O.O dja=0.0<br>do 220 l=1<br>bn(l)=b(l) 240 erra=errn 245 errb=errn 215 continue 220 continue 245  $215$  $\ddot{\mathbf{o}}$ 

end<br>subroutine init<br>: common /kdyn/ a0,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint<br>: rintt,ndof,np,delt2 \_ subroutine init<br>common /kdyn/ aO,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint<br>common /kdyn/ aO,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint common/transf/ag(14,3),ac(3,14,3)<br>read (\*\*\*) delta,alpha,thc(3,14,3)<br>read (\*\*\*) delta,alpha,thc(3,14,3)<br>bormat (4f10.0)<br>tautheta+delt<br>al=a2\*delta<br>al=a2\*delta<br>al=a2\*delta \* ,rintt,ndof,np,delt2 \* ,rint,ncndns,idel(39),icndns(39)<br>\* ,rintt,ndof,np,nindf,nxnt,ndel,ncndns,idel(39),icndns(39) print 3000,dja,erra<br>print 3000,dja,erra<br>format (1hO,"final slope:",g12.6/" final error:",g12.6) print 6,alphaa,erra,dja,alphab,errb,djb,alphan,bound write (\*,"('f1nal parameters: ')") common/transf/ag(14,3),ac(3,14,3)  $*$ , db, alphan, djn)<br>
if (abs(djn).lt.slmin) go to 455<br>
1f (djn) 320, 455, 245<br>
320 if (errn.le.errb) go to 245<br>
alphaa=alphab if (abs(djn).lt.slmin) go to 455 rint=theta=float(ifix(theta))<br>rintt=1.0-rint<br>do 100 i=1,3 rint=theta-float(ifix(theta» commonic and is alpha,theta<br>read (#,\*) delta,alpha,theta format (" increasing error") write (2,3001) (b(i),i=1,np) continue<br>if (errb.gt.erra) go to 227 write (\*,\*) (b(t) ,i=l ,np) 320 if (errn.le.errb) go to 245 400 if (intr,lt.S) go to 226 if (djn) 320,455,245 a4=delta/alpha-1:0<br>a5=.5#tau#(a4-1.0)<br>a6=delt#(1.0-delta) a6=delt\*(1.0-delta) a4:delta/alpha-l;0 a5=.5\*tau\*(a4-1.0) format (1x, 8e15.7) a2=1.0/(alpha\*tau) • ,db,alphan,djn) a3=.5/alpha-1.0 a3=.5/alpha-l.0 1000 format (4fl0.0)  $d = 1 t 2 = 2.04 d = 1 t$ tau=theta\*delt delt2=2.0 <sup>#delt</sup> rintt=1.0-rint a7=delt#delta dtt=delt\*delt a7=deltlldelta dtt=delt<sup>#</sup>delt alphan=alphab 455 err=errn 460 do 465 l=l,np a9=alpha<sup>sdtt</sup>  $a8 = .5*dtt - a9$ alphaa=alphab a9=alpha <sup>s</sup>dtt a8=.5\*dtt-a9 do 100 i=1,3 al=a2\*delta  $465 b(1)=bn(1)$ go to 190<br>print 2007<br>print 2007<br>format (11)<br>format (11)<br>printe (\*<br>wintite (\*<br>stop ite (\* go to 245<br>if (intr.<br>go to 455 erreerrb<br>alphan=al<br>print 6,a<br>go to 460 bound=no 450 continue dja=djb

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call afix<br>do 500 if=1,6<br>calco 500 if=1,6<br>calcolical member stiffness vm(1,j,1)<br>pl=rlv(if) c calculate vertical member stiffness vm(1,j,1)<br>pl=rlv(1f)  $\begin{array}{l} \mathsf{v}\mathfrak{m}(3,2,1):=1\\ \mathsf{v}\mathfrak{m}(1,2,1):=1\\ \mathsf{tr}(\mathfrak{u},2,1):=k\\ \mathsf{v}\mathfrak{m}(1,2,1):=k\\ \mathsf{v}\mathfrak{m}(1,3,1):=1\\ \mathsf{v}\mathfrak{m}(1,3,1):=1\\ \mathsf{v}\mathfrak{m}(1,1,1):=k\\ \mathsf{c}\mathfrak{m}(1,1):=k\\ \mathsf{c}\mathfrak{m}(1,1):=k\\ \mathsf{c}\mathfrak{m}(1,1):=k\\ \mathsf{c}\mathfrak{$ c calculate horizontal stiffness hm(i,j,1) ppl=pl\*pl<br>pppl=pl\*ppl<br>avk=2.\*rev(if)\*riv(if)/pppl<br>avkg=fn(if)(30.\*ripl)<br>tkg=fn(if)=tk<br>vm(i,1,1)=tk ppl=pl\*pl<br>pppl=pl\*ppl<br>avk=2.\*reh(if)\*rih(if)/pppl aVk=2. llrev(if)\*riv(if)/pppl avk=2.\*reh(if)\*rih(if)/pppl<br>tk=avk\*2.\*ppl do 20 i=1,ncall<br>do 20 j=1,ncall<br>k(i,j)=O.O<br>if (icm.eq.2) m(i,j)=O.O  $a_0$  ioo 11 + 69<br>ie=iabs(leval(1))<br>if (ieval(1)) 40,50,60<br>40 gpe(1)=gpe(ie)<br>50 gpe(1)=e(1) if (icm.eq.2) m(i,j)=O.O vm(2,2,1)=tk<br>vm(2,1,1)=(avk+avkg)\*ppl<br>tk=(avk-avkg)\*3.\*pl vm(2,1,1)=(avk+avkg)\*ppl  $\begin{array}{l} \text{Im}(3,1,1)=\text{tk} \\ \text{Im}(3,2,1)=\text{tk} \\ \text{Im}(3,2,1)=\text{tk} \\ \text{Im}(3,3,1)=\text{tk} \\ \text{Im}(3,3,1)=\text{tk} \end{array}$ <br> $\begin{array}{l} \text{Im}(3,2,1)=\text{tk} \\ \text{Im}(3,3,1)=\text{tk} \end{array}$ to 102  $if (1cm+eq, 1) go to 102$ tk=(avk#2.-avkg#4.)#ppl<br>vm(1,1,1)=tk if (ieval(l» 40,50,60 c calculate vertical mass<br>tbm=barm\*8.0<br>tbm=barm\*8.0 if  $(mfix_eq.0)$  ion=2<br> $mfix = 1$ if (mfix.eq.O) icm=2  $avkg=fn(1f)/(30. *p1)$  $t$ k=(avk-avkg)#3.\*pl tk=avk<sup>#6</sup>.-avkg<sup>#</sup>36. go to 100<br>60 gpe(1)=e(1)\*b(ie)<br>100 continue 60  $gpe(1) = e(1) * b(ie)$ ie=iabs(ieval(l» hm(3,3,l)=6.\*avk  $\begin{array}{l} \texttt{hm}(2,2,1) \pm \texttt{tk} \\ \texttt{hm}(2,1,1) \pm \texttt{tk}/2, \\ \texttt{tk} = \texttt{avk}^3 \cdot \texttt{N} \\ \end{array}$ do 20 1=1,ncall  $h$ m $(2,1,1)$ =tk/2. 40 gpe(l)=gpe(ie) do 100 1=1,69 do 500 if=l,6 vm(3, 1, 1)=-tk  $vm(3,1,1)=-tk$ vm(3,2.1)=-tk  $van(4,3,1) = -tx$ tk=avk\*2.\*ppl  $hm(3,1,1)=-tk$ hm(3,2,l )=-tk bmv=tbm\*rb hm(1,1,1)=tk tk=avk<sup>#</sup>3.<sup>#</sup>pl bmv=tbm\*rb  $\nu$ m(2,2,1)=tk  $van(4,1,1)=tk$ i=tk  $\sqrt{n}$  $(4, 2, 1)$ =tk vm(3,3,1)=tk  $\nu$ m $(4, 4, 1)$ =tk hm (  $1, 1, 1$  ) = tk  $h m(2,2,1)=tk$ ppl=pl\*ppl 50 gpe(l)=e(l) pppl=ppl<sup>#</sup>ppl  $\widehat{\mathbf{r}}$ continue 20 continue 100 continue  $\frac{1}{\sqrt{2}}$  $\frac{1}{2}$ ನ್ನ  $\ddot{\mathbf{c}}$ d  $\ddot{\mathbf{c}}$ end<br>sumnon /static/ ir(39),idf(39),r(39)<br>common /static/ ir(39),idf(39),r(39)<br>emmon /kdyn/ a0,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint<br>f ,rintt,ndof,np,delt2 subroutine mck(k,m,ncall,b)<br>common /static/ ir(39),idf(39),r(39)<br>common /kdyn/ aO,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint II ,rintt,ndof,np,delt2 II . ,nindf,nxnt,ndel,ncndns,idel(39),icndns(39) real k(ncall,l),m(ncall,l) common *Igpl* rev(6),reh(6),rej(7),riv(6),rih(6),rijx(7),rijz(7), end<br>fumeion kpt(j,id,ist)<br>kpc=ist+idisp(j)<br>kpc=ist+idisp(j) II rlv(6),rlh(6),rgj(7),rb,rh,barm,ak,am common Igpll e(69),ieval(69),fn(6) common *Itransfl* av(4,B),ah(3,6),nv,nh,istv,isth,idisp(B) function kpt(j,id,ist)<br>common /transf/ av(4,8),ah(3,6),nv,nh,istv,isth,idisp(8)<br>kpt=ist+idisp(j) common /first/ mf1x,xm(39,39),xmr(39)<br>common /reduce/ phi(39,39),dphi(39,39,10)<br>equivalence (gpe(1),rev(1))<br>icm=1 dimension gpe(69)<br>common /first/ mfix,xm(39,39),xmr(39)<br>common /first/ mfix,xm(39,39),xmr(39)<br>equivalence (gpe(l),rev(l)) dimension b(10),vm(4,4,2),hm(3,3,2) end<br>subroutine mass (bm,rl,n,hm) subroutine mass (bm,rl,n,hm) if (id.ne.1) return<br>if (j.lt.5) kpt=kpt+1<br>return if (j.lt.5) kpt=kpt+l if (id.ne.l) return dimension tt(39,39) if (n.eq.3) return dimension gpe(69)<br>common /first/ mf<br>common /reduce/ pl  $dimension$   $lm(n,n)$ <br> $rrl=rl*rl$ dimension hm(n,n)  $\text{Im}(2,1)=-3.$   $\text{Per1}$  $\text{Im}(3, 1) = -22.$  \*brl hm(4,1)=-hm(3,2) hm(4,2)=-hm(3,1)  $\text{hm}(3,2)$ =13.  $\text{Br1}$  $h$ m $(3,3)$ =156.  $\frac{1}{2}$ hm(4,4)=156. \*b b=(bm/420.)\*rl  $h$ m $(4, 3)$ =54.  $\ast b$ idisp(B)=10  $disp(3)=2$ tm=4. \*brrl<br>nm(1,1)=tm idisp(l)=O idisp(4)=4 idisp(5)=6 idisp(6)=7 idisp(7)=B brrl=b\*rrl<br>brl=b\*rl hm(l,1)=tm hm(2,2):11<br>hm(2,3):11<br>hm(2,3):11<br>hm(3,3):11<br>hm(2,2):11<br>hm(2,11):12<br>hm(2,11):12<br>hm(2,11):12<br>hm(2,11):12  $disp(2)=1$ return

 $m($ ist,ist)= $m($ ist,ist)+8.\*rh\*(rlh(if)+rb)<sup>&</sup>barm call mconj (hm(l,l,kk),ah,tt,3,6,ncall,3,3) call mconj(vm(i,i,kk),av,tt,4,8,ncall,4,4) ist=ist+1<br>k(ist,ist)=k(ist,ist)+rej(i1)\*rijx(i1)/rb ist=ist+1<br>k(ist,ist)=k(ist,ist)+rej(i1)\*rijz(i1)/rh<br>continue k(ist,ist)=k(ist,ist)+rej(il)\*rijx(il)/rb k(ist,ist)=k(ist,ist)+rej(il)\*rijz(il)/rh ist=ist+2<br>k(ist,ist)=k(ist,ist)+rgj(i1)\*rb\*rh\*8. k(ist,ist)=k(ist,ist)+rgj(il)\*rb\*rh\*8. call mass (bmv,rlv(if),4,vm(1,1,2» call mass (bmh,rlh(if),3,hm(1,1,2» 200 bitatiravieniravieniravieniravieniravieniravieniravieniravieniravieniravieniravieniravieniravieniravienira<br>500 continue<br>k(1,1)=k(1,1)+rgj(1)\*rb\*rb\*8. k(1,1)=k(1,l)+rgj(1)\*rb\*rh\*8. 103 vm(jj,kk,ii)=vm(kk,jj,ii) hm(jj,kk,ii)=hm(kk,jj,ii) m(ii,jj)=m(ii,jj)+tt(i,j) 120 k(ii,jj)=k(ii,jj)+tt(i,j)  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{3}$ )=m(ii, j))+tt(i,j) 135 k(ii,jj)=k(ii,jj)+tt(i,j)  $if (kk.eq.2) go to 200$ <br>il=if+1 if (kk.eq.2) go to 200 calculate horizontal mass  $if (ii, 1t, 1) go to 130"$ if (jj.lt.1) go to 130 if (kk.eq.l) go to 120 add *in* translational mass in translational mass<br>if (kk.lt.2) go to 132 if (kk.eq.l) go to 135 do 200 kk=1.icm ii=kpt(i,if,ist) jj=kpt(j,if,ist) do 103 j j = 1 ,kk 1 102 do 106 *ii=l,icm* do  $104 \text{ Jj=1,kk}$ ist=4+6~(if-2) ist=4+6\*( if-l) do 103 kk=2,4 do 104 kk=2,3 bmh=tbm<sup>#</sup>rh bmh=tbm\*rh do 130 *i=* 1 ,8 do  $130 \text{ } j=1,8$ 132 do 140  $i = 1, 6$ do 140 j=l,6 )<br>jj=j+ist-l<br>if (kk.eq.<br>m(11,j)=n<br>go to 140 140 continue kkl=xkl=xkl=xkl<br>
do 103<br>
continue<br>
continue<br>
continue<br>
continue<br>
continue<br>
continue<br>
continue<br>
continue<br>
continue<br>
go 11-1+125+1<br>
continue<br>
go 21<br>
continue<br>
go 21<br>
continue<br>
continue<br>
continue<br>
continue<br>
continue<br>
continue 130 continue 200 continue 500 continue 1<br>106<br>110 115 c c

k ( 2 , 2) =k (2,2)+ re j ( 1 ) • r i j x ( 1 ) *Ir* b  $k(3,3) = k(3,3) + rej(1) *r1jz(1)/rh$ kt.3,37=Kt.3,37+redti7-r<br>if (icm.eq.1) goto6 call delet(m,ncall) do 550 *i=* 1, ndof  $xmr(i) = 0.0$ do 550 j=l,ndof xmr(i)=xmr(i)+m(i,j)"r(j)  $x$ m(i,j)=m(i,j) continue call delet(k,ncall) return end

dimension rm(ncall,ncall)<br>common /static/ir(39),idf(39),r(39)<br>common /kdyn/ a0,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint<br>\* \* .rintt,ndof,np,delt2 common /static/ir(39),idf(39),r(39)<br>common /kdyn/ a0,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint<br>, rintt,ndof,np,delt2 common *Ikdynl* aO,al,a2,a3,a4,aS.a6,a7.a8,a9,delta.theta,delt,rint common *Ikdynl* aO,al,a2,a3,a4,aS,a6.a7,a8,a9,delta,theta,delt,rint 1=1 cxxxxxxx do not use input for deleted dof •••• "" ••".\*•• \*.\*.\*\*\*\*&&" •• G& , nindf, nxnt, ndel, ncndns, idel(39), icndns(39) • ,nindf,nxnt,ndel,ncndns.idel(39),icndns(39) • ,rintt,ndof,np,delt2 • ,nindf,nxnt,nde1 ,ncndns,ide1(39) ,icndns(39) idil=idi-l c xxxxxxx delete matrix elements xxxxxxxxxxxxxxxxxxxxxx go to 150<br>1050 print 5<br>5 format (\* cubic interpolation appears constant\*)<br>1060 cubic=x2 S format **(.** cubic interpolation appears constant.) common Istatic/ir(39),idf{39).r(39) common Istatic/ir(39),idf(39),r(39)  $\frac{1}{2}$ 1000 if (abs(b).lt.l.0e-l0) go to 10S0 2S if (idf(1).ne.ide1(i» go to 40 150 if(test.gt.0.0) cubic=amin2<br>return lS0 if(test.gt.O.O) cubic=amin2 end<br>subroutine delet(rm,ncall) incolarized.ndof) go to 400<br>1f (idi.eq.ndof) go to 400  $20$  if  $(1,gt.$ nindf) go to 60 subroutine delet(rm,ncall) dimension rm(ncall,ncall) • ,rintt.ndof,np,delt2 test=6.0\*a\*amin2+2.0\*b test=6.0\*a\*amin2+2.0\*b if (ndel.eq.0) return<br>ii=ndel+1 if (ndel.eq.O) return  $290$  rm(j,j)=rm(j,j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm(j)=rm() do 500 j=1,ndof<br>c fill in lower triangle<br>do 500 jj=1,j<br>500 rm(j,jj)=rm(jj,j) 280 rm{jj,j)=rm(jj,j+l) c fill in lower triangle do 300 j=idi,ndof 500 rm(jj)=rm(jj,j) do 290 jj=j,ndof common /ib/ibdof common /ib/ibdof do 280 jj=l,idil  $anin2=-c/(2.0")$ do 400 i= 1, ndel<br>ii= ii-1<br>idi= idel(ii)<br>ndof= ndof-1 do SOO j=l,ndof do 400 i=l,ndel nindf=nindf-l subroutine did do 60 i=l,ndel idi=idel<ii) ndof=ndof-l ndof= ibdof cubic=x2<br>test=2.0\*b  $1060$  cubic= $x2$ 300 continue 400 continue 10S0 print S return return 20<br>20 COMMON SUBROUTINE LIBRARY COMMON SUBROUTINE LIBRARY subroutine check(b,bn,db,alpha,alphab,ok)<br>common /kdyn/ aO,a1,a2,a3,a4,a5,a6,a7,a8,a9,delta,theta,delt,rint<br>,rintt,ndof,np,delt2 common *Ikdynl* aO,al,a2,a3,a4,aS.a6.a7,a8,a9,delta,theta,delt,rint dimension b(1),db(1),hi(1)<br>ok=1.0<br>almax=alpha • ,nindf.nxnt,ndel,ncndns,idel(39),icndns{39) end<br>funtion cubic(y1,d1,y2,d2,x1,x2)<br>dimension t(4,4),coeff(1)),(b,coeff(2)),(c,coeff(3))<br>dimension t(4,4),coeff(4),id4(4) equivalence (a,coeff(1)),(b,coeff(2)),(c,coeff(3) if (alphab.le.almax) alpha = .5\*(almax+alphab) aif (alphab.le.almax) alpha = .5\*(almax+alphab)<br>if (alphab.le.almax) alpha = .5\*(almax+alphab) subroutine check{b,bn,db,alpha,alphab,ok) if((almax.gt.a).and.(a.gt.0.0)) almax=a if«almax.gt.a).and.(a.gt.O.O» almax=a t(4,1)=3.0\*x2\*x2<br>t(4,2)=2.0\*x2\*x2<br>call solveq(t,coeff,4,1,4,1d4)<br>disc=bt=3.0\*a\*1.0e-10) go to 1000<br>disc=bt=3.0\*a\*<br>if (disc.1t.0.0) go to 1060 caii solveq(t, goeii, 4, 1, 1, 1, 1, 000<br>if (abs(a).lt.l.0e-10) go to 1000 function cUbic(yl,dl,y2,d2,xl,x2) dimension t(4,4),coeff(4),id4(4) if (alpha.le.almax) go to 60<br>ok=0.0<br>alpha=.5\*almax -alpha\*db(1) if (alpha.le.almax) go to 60  $100$  bn(1)=b(1)  $-$ alpha<sup>\*db(1)</sup> call solveq(t,coeff,4,1,4,id4) **c ••• \*\* •••••••••••••••••••••••••\*•• 家庭收集事实事故收集审核事业事业审查收集审核事业检察事务生事业检察室** if (disc.lt.O.O) go to 1060 dimension b(1),db(1),bn(1) • ,rintt,ndof,np,delt2  $k$ m=5-k<br>t t (1,km)=x1<sup>4</sup>t (1,km+1)<br>t (3,km)=x2<sup>4</sup>t (3,km+1) t $(1, km) = x1*t(1, km+1)$ t(3,km)=x2\*t(3,km+l) disc=sqrt(disc)<br>ta=3.0\*a<br>amin1=(disc-b)/ta<br>amin2=(-disc-b)/ta amin2={-disc-b)/ta aminl=(disc-b)/ta  $t(2,1)=3.0*x1*x1$ t(4,1)=3.0\*x2\*x2 alpha=.S·almax  $t(2,1)=3.0*x1*x1$ disc:b\*b-3.0\*a\*c disc=sqrt{disc)  $d_0$  50 l=1, np<br>a = b(1)/db(1)  $a = b(1)/db(1)$ do 100 l=1,np<br>bn(l)=b(l)<br>return  $t(2, 2) = 2.0$ <sup>\*</sup>\*1 do 100 l=l.np  $t(4,2)=2.0*x2$ t(2,2)=2.0\*xl  $coeff(2)=d1$ <br> $coeff(3)=y2$ <br> $coeff(4)=d2$ do 50  $l=1, np$ do 100 k=2,4  $t(1, 4)=1.0$ <br> $t(2, 3)=1.0$  $c_0erf(1)=y1$ cubic=amin 1 cubic=amin1 almax=alpha  $\c{o}$ eff $(1)=y$ <sup>1</sup>  $\csc f(3)=y2$ coeff(4)=d2 continue  $coeff(2)=d1$ continue SO continue 60 continue t(2,3)=1.0<br>t{2,4)=0.0<br>t{3,4)=1.0<br>t{4,3)=1.0<br>t{4,4)=0.0 continue 100 continue  $100$  $100$ **S** 50 c c



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1f (n-nn) 200,500,200 200 do 450 j=nl,nn 500 nl=n n=n-1<br>if (n) 700,700,550 1f (n) 700,700,550 550 do 600 1=1,11 do 600 j=nl,nn 600 b(n,l)=b(n,l)-a(n,j)\*b(j,l) go to 500 ccc end on the contract of th function sq  $(y1, d1, y2, d2, x1, x2)$ equivalence (a,coeff(1»,(b,coeff(2» dimension t(3,3),coeff(3),1d3(3) function sq  $(y_1, d_1, y_2, d_2, x_1, x_2)$ <br>equivalence  $(a, coeff(1)), (b, coeff(2))$ <br>dimension t $(3, 3), coeff(3), id3(3)$ <br>t $(1, 3)=1.0$ c modify  $a(1,j)$ 350 do 400 1=1,11 400 b(j,l)=b(j,l)-a(j,n)\*b(n,l) 450 continue 475 continue 200 do 450 j=n1, nn<br>
c 200 do 450 j=n1, nn<br>
c form h(n,j) 250, 350, 250<br>
c 250 a(n,j) 250, 350, 250<br>
c modify a(i,j)<br>
c 300 a(i,j)=a(i,j)-a(i,n)\*a(n,j)<br>
c 300 a(i)=a(i,j)-a(i,n)\*a(n,j)<br>
c modify b(i,l)<br>
c 475 continue<br>
c do 300 1=nl,nn do 300 i=n1, nn<br>300 a(i,j)=a(i,j)-a(i,n)\*a(n,j) 1f  $(a(n, j))$  250, 350, 250<br>250  $a(n, j) = a(n, j) / a(n, n)$ a(1,j)=a(i,j)-a(i,n)\*a(n,j) (a(n,j» 250,350,250 250 a(n,j)=a(n,j)/a(n,n) return modify b(i,l) back-substitution  $550$  do 600 1=1,11<br>600 do 600 j=n1,m<br>600 b(n,1)=b(n,1)-a(n,j)\*b(j,1)<br>8 do 500<br>8 reorder unknowns 700 do 950 n=l,nn do 900 1=n,nn if (1d(1)-n) 900,750,900 750 do 800 1=1,11 700 do 950 n=1,nn<br>do 900 i=n,nn<br>do 900 i=n,nn<br>d=b(n,1)<br>750 d=b(n,1)  $b(n,1)=b(1,1)$  $800 b(1,1)=d$ go to 950 900 continue 950 10(1)=id(n)

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 $\begin{array}{ccccccc}\n\textcolor{red}{\mathbf{0}} & \textcolor{red}{\mathbf{0}} & \textcolor{red}{\mathbf{0}}\n\end{array}$ 

subroutine solveq(a,b,nn,ll,max,id) dimension a(max, max), b(max, 1), id(1) subrout1ne s01veq(a,b,nn,11,max,1d) d1mens10n a(max,max),b(max,I),1d(l) ac ioo j=n;in<br>if (abs(a(1,j))-d) 100,90,90 oc reoord oolumn interchange c cc looste largest element<br>  $\begin{array}{c}\n 0.0 \text{ } \text{if } 1.00, 0.0$ locate largest element 150 b(n,1)=b(n,1)/a(n,n) cc interchange rows c do 120 j=n,nn<br>d=a(n,j)<br>a(n,j)=a(i1,j)<br>120 a(ii,j)=d do 130 1=1,11<br>d=b(n,1)<br>b(n,1)=b(ii,1)<br>130 b(ii,1)=d  $a(n,j)$ = $a(11,j)$  $b(n,1)=b(11,1)$ set i.d. array  $a(1,n)=a(1, j)$ ) set 1.d. array do  $475$  n=1, nn<br>n1=n+1 do 130 1=1,11 do 150 1=1,11 do 110 i=l,nn do 120 j=n,nn do 475 n=l,nn 100 i=n,nn do 100 j=n,nn 90 d=abs(a(1,j»  $id(n)=id(jj)$ 50 n=l,nn 120 a(ii,j)=d  $130$  b(ii, 1)=d a(i,jj)=d  $j=1$ <br>100 continue  $i = id(n)$ <br> $id(n) = id$ <br> $id(j) = 1$ 100 continue  $d = a(i, n)$ 50 id(n)=n d=O.O do 11=i do  $\frac{0}{11}$ c cccc ccc



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end<br>subroutine symsol (a,b,nn,ll,m,ncall)<br>m=1 triangulaze and solve<br>m=2 forward r+duction and back substitution only<br>dimension a(ncall,ncall),b(ncall,ll) c m=2 forward r+duction and back substitution only Forward reduction and backsubstitution<br>
soo if (m.eq.1) return<br>
do 700 n=1,m<br>
do 700 n=1,m<br>
600 to 100 l=1,11<br>
600 to 100 l=1,11<br>
600 to 100 l=1,11<br>
do 700 l=1,1<br>
700 b(1,1)=b(i,1)-a(i,n)\*b(n,1) c forward reduction and backsubstltution subroutine symsol (a,b,nn,ll,m,ncall) dimension a(ncall,ncall),b(ncall,ll) call solved (t, coeff, 3, 1, 3, 1d3)<br>
if (a.lt. 1, 0e-10) go to 200<br>
sq=-b/(2.0\*a)<br>
if(sq., 1t. sqmax) return<br>
200 sq=x2-y2/d2<br>
return (sq, sqmax) call solveq(t,coeff,3,1,3,id3) if (a(n,j).eq.O.O) go to 300 c m=O triangularize and solve if (a.lt.1.0e-10) go to 200<br>11 (a.lt.1.0e-10) go to 200  $a(1,j)=a(1,j)-a(1,n)^{8}a(n,j)$ 700 b(i,l)=b(i,l)-a(i,n)\*b(n,l) if (d.eq.O.O) print 2000,n if (n.eq.nn) go to 500 b(n.i)=b(n.i)/a(n.u)<br>if (n.eq.nn) go to 800 if(sq.lt.sqmax) return if (rn.eq.2) go to 500 t ( 1 ,km) =x1\*t ( 1 , km+ 1 ) t(2,km)=x2\*t(2,km+1) c m=1 triangulize only 600 b(n,l)=b(n,l)/a(n,n) 500 if (m.eq.1) return sq=amin1(sq,sqmax) a(n,jl=a(n,j)/d  $t(3,1)=2,0*x2$ <br>  $\csc f(1)=y1$ <br>  $\csc f(2)=y2$ <br>  $\csc f(3)=y2$ <br>  $\csc f(3)=dz$ do 300 j=n1,nn do 700 1=n1,nn do 600 1=1,11 do 700 1=1,11 t<3,1)=2.0\*x2 sq=-b/(2.0\*a) do 400 n=1,nn do 200 i=j,nn 200 a(j,i)=a(i,j) do 700 n=1,nn sqmax=5.0\*x2 do 150 k=2,3  $c$ coeff $(1)=y1$  $coerf(2)=y2$  $c$ coeff $(3)$ =d2 200 sq=x2-y2/d2 t(2,3)=1.0<br>t(3,2)=1.0<br>t(3,3)=0.0<br>sqmax=5.0\*<br>do 150 k=2<br>km=4-k 150 continue d=a(n,n) n1=n.1 300 cont1nue 400 continue 800 n1=n

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n=n-1

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if (n.eq.0) return
## EARTHOUAKE ENGINEERING RESEARCH CENTER REPORTS

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