NEW FLOOR RESPONSE SPECTRUM METHOD FOR SEISMIC ANALYSIS OF MULTIPLY SUPPORTED SECONDARY SYSTEMS

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The method defines and utilizes an extension of the conventional floor response spectrum denoted cross-oscillator, cross-floor response spectrum (CCFS). The CCFS is defined to be proportional to the covariance of the responses of two fictitious oscillators subjected to the motions of the primary system at two support points. Through this extended concept, important effects not accounted for in the current floor-spectrum methods, are correctly included in the analysis. These effects include: cross-correlations between motions of the support points, cross-correlations between modal responses of the secondary system, interaction between primary and secondary systems, resonance or tuning between the frequencies of the two subsystems and the non-classical damping effect of the combined primary-secondary system.

The generation of the CCFS requires repeated evaluations of the modal properties of combined oscillator-primary systems. The effects of interaction and non-classical damping are implicitly included in the derived CCFS. The effects of cross-correlation between the support motions and cross-correlation between the modal responses of the secondary system are included in the derived CCFS and in the modal combination rule for the response of the secondary system. Several representative primary-secondary systems are numerically analyzed.

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A New Floor Response Spectrum Method for Seismic Analysis of Multiply Supported Secondary Systems

Abstract

The objective of this study is to present an improved floor-spectrum method for seismic analysis of linear, multi-degree-of-freedom secondary systems multiply supported on linear, multi-degree-of-freedom primary systems. The method defines and utilizes an extension of the conventional floor response spectrum denoted cross-oscillator, cross-floor response spectrum or, in short, cross-cross floor spectrum (CCFS). The CCFS is defined to be proportional to the covariance of the responses of two fictitious oscillators subjected to the motions of the primary system at two support points. Through this extended concept, important effects, which are not accounted for in the current floor-spectrum methods, are correctly included in the analysis. These effects include: cross-correlations between motions of the support points, cross-correlations between modal responses of the secondary system, interaction between primary and secondary systems, resonance or tuning between the frequencies of the two subsystems and the non-classical damping effect of the combined primary-secondary system.

The proposed method consists of two basic steps: (1) Generation of CCFS in terms of the ground response spectrum and the modal properties of the primary system; and (2) determination of the mean peak response of the secondary system by modal combination in terms of the CCFS, the modal properties of the fixed-base secondary system, and the stiffnesses of the elements connecting the secondary to the primary system. The generation of the CCFS requires repeated evaluations of the modal properties of combined oscillator-primary systems. Recent results employing perturbation theory are used to

make this evaluation efficient. The effects of interaction and non-classical damping are implicitly included in the derived CCFS in approximate manners. The effects of cross-correlation between the support motions and cross-correlation between the modal responses of the secondary system are included in the derived CCFS and in the modal combination rule for the response of the secondary system.

Several representative primary-secondary systems are numerically analyzed. Results obtained using the proposed method are compared with results obtained by considering the primary-secondary system as a single structure. Close agreement is found between the two results for all cases.

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CHAPTER 1

Introduction

1.1 General Remarks

During the last three decades, a large amount of effort has been devoted to the development of methods for seismic analysis of light, multiply supported secondary systems which are attached to heavier primary systems. These efforts have been motivated mainly by the use of critically important secondary systems, such as piping networks in nuclear power plants or refining facilities. Although important contributions have been made and a lot has been learned in this period of time, most methods currently used in practice are still heuristic in nature and have important shortcomings. Thus, it is worthwhile to study this problem, once again, with the objective of developing a practical and accurate method.

For the purpose of this study, a secondary system is defined as a linear-elastic, viscously and classically damped system, whose masses and stiffnesses are considerably smaller than the masses and stiffnesses of the system to which it is attached. The system to which the secondary system is attached is defined as the primary system, and it is also assumed to be linear-elastic, viscously and classically damped. The seismic excitation is applied to the primary system as a rigid base excitation. The secondary system is assumed to be attached to the primary system in any arbitrary manner.

Two approaches have been used by researchers to study the seismic response of secondary systems: time history analysis and floor response spectrum analysis. In the former approach, the time histories of motions at the support points of the secondary system are obtained from a separate analysis of the primary system, which might include a crude model of the secondary system to account for the effect of interaction between the two systems. Using these motions as input into the secondary system, a separate analysis of the secondary system is carried out (Kassawara and Peck [1973]). This approach has two important deficiencies; it is very expensive and it is impossible to define the proper input ground motion history which characterizes potential future earthquakes at the site. The floor response spectrum approach involves specification of the motions at the support points of the secondary system in terms of floor response spectra, and modal dynamic analysis of the secondary system in terms of the floor spectra. This approach is more economical than the time history approach and can be based on a response spectrum description of the ground motion. However, as currently applied to multiply supported secondary systems, it does not properly account for important effects such as the cross-correlation between support excitations, the cross-correlation between modal responses, interaction between primary and secondary systems and the effect of non-classical damping in the composite primary-secondary system.

1.2 Current Approaches for Multiple Support Excitations

Amin, Hall, Newmark and Kassawara [1971], Shaw [1975], Vashi [1975] and Thailer [1976], among others, have developed methods based on the floor response spectrum approach to analyze multiply supported secondary systems. All of these studies, however, have used heuristic techniques for combining the responses due to individual support excitations and/or the modal responses of the secondary system. Thus, due to the subjective nature of the combination techniques employed, important effects such as the cross-correlation between modal responses or the cross-correlation between support excitations are not properly accounted for. This leads to large errors in the estimation of the response, as many numerical studies have demonstrated (see, for example, Wang, Subudhi and Bezler [1983]).

Recently, Lee and Penzien [1980] developed a stochastic method to analyze multiply supported secondary systems. This method includes the cross-correlation between support excitations and the cross-correlation between modal responses. Using stationary random vibrations techniques, they proceed in two basic steps: First they determine the cross-power spectral density matrix for the motions at the support points on the primary system. Then, using this matrix as the input for the secondary system, the mean of the extreme value of any response of the secondary system is evaluated. This approach may be very expensive and may not be attractive from practical standpoint, since the design earthquake motion is most conveniently specified in terms of a response spectrum rather than a power spectral density function.

In most of the above mentioned studies the effect of interaction between the primary and secondary systems has not been included. However, Crandall and Mark [1963], Amin, Hall, Newmark and Kassawara [1971], Pickel [1972], Hadjian [1978], Der Kiureghian, Sackman and Nour-Omid [1981], Igusa and Der Kiureghian [1983] have shown that, depending on mass and frequency ratios, there are many practical situations where this effect can be highly significant and must be included in the analysis.

More recently, Igusa and Der Kiureghian [1983], using random vibration techniques, developed an alternative method for the dynamic analysis of multiply supported secondary systems. This method accounts for interaction between primary and secondary systems, cross-correlation between support excitations, cross-correlation between modal responses for stochastic input, resonance or tuning between frequencies of the two systems and non-classical damping effects. In their method, the combined primary-secondary system is considered as a single dynamic assemblage and the ground excitation is utilized as input. The modal characteristics of the combined system are derived, using perturbations methods, from the individual modal characteristics of the fixed base primary and the fixed base secondary systems. They also developed a general modal combination rule for systems with non-classical damping and closely spaced frequencies for stationary inputs or inputs specified in terms of the ground response spectrum. Using this method, they give the response of the secondary system in terms of the derived properties of the combined system and the ground response spectrum.

1.3 The Conventional Floor Spectrum Methods

The current methods used in practice for seismic analysis of multiply supported secondary systems, as described in Appendix N of the ASME Boiler and Pressure Vessel Code [1980], are based on the concept of floor spectrum. This approach has the following important practical advantages:

(a) The approach avoids the dynamic modeling and analysis of the combined primary-secondary system, which can be prohibitively costly if carried out directly.

(b) It avoids numerical difficulties that could arise in the analysis of the combined system due to large differences between the properties of the two systems.

(c) Once the floor response spectra are specified, the method then allows the analyst to work on the secondary system independently of the primary system characteristics.

(d) The floor response spectrum method is inexpensive relative to time-history integration methods.

However, as currently applied, the floor response spectrum approach has several important shortcomings. These include:

(a) The cross-correlation between the excitations at the support points of the secondary system are neglected or improperly considered.

(b) The response is artificially separated into "pseudo-static" and "dynamic" parts, which has the consequence that a proper modal combination rule cannot be developed.

(c) The cross-correlation between responses of closely spaced modes in the primary and secondary systems is often neglected or improperly considered.

(d) The interaction between the primary and secondary systems is neglected. This interaction can be significant when the mass of the secondary system is not negligible in comparison with the mass of the primary system or when the two systems have tuned or nearly tuned natural frequencies.

(e) Finally, the effect of non-classical damping of the combined system, which can be significant even when the two systems are individually modally damped (Warburton and Soni [1977], Singh [1980], Igusa and Der Kiureghian [1983]), is often not considered.

Furthermore, although efficient methods for generating floor response spectra directly in terms of the ground response spectrum have recently become available (Singh [1975], Der Kiureghian, Sackman and Nour-Omid [1981], Igusa and Der Kiureghian [1983]), in most applications the floor response spectra are generated using "spectrum-compatible" ground time histories in conjunction with time-history analysis of the primary system. This approach, besides being expensive, is inappropriate since different time histories compatible with the same ground response spectrum may lead to very different estimates of the peak response (Singh, Singh and Chu [1973]).

1.4 Objectives of Study

The investigation presented herein endeavors to develop a theoretically sound method for modal seismic analysis of linear multi-degree-of-freedom secondary systems with multiple attachment points. The method is developed using elementary concepts from stationary random vibrations. It is based on an extension of the conventional floor response spectrum concept, defined through a cross-oscillator cross-floor response spectrum (CCFS). Thus, the method allows analysis of the secondary system which is separate from the analysis of the primary system. The CCFS include the effects of cross-correlation between modal responses of the secondary system and the cross-correlation between the support excitations. Also, the effects of interaction and non-classical damping are included in these spectra by means of approximate methods. The interaction effect is accounted for, with sufficient accuracy, by defining equivalent modal masses for multiply supported systems. These masses represent the effect of each mode of the secondary system in perturbing the dynamic properties of the primary system. The non-classical damping effect, which is important only in the vicinity of tuning (resonance), is resolved by using matching techniques in conjunction with known solutions for extreme cases of perfect tuning and complete detuning.

The cross-oscillator cross-floor response spectra are evaluated directly in terms of the input ground response spectrum and the modal properties of the primary system. Once the CCFS and the fixed-base modal properties of the secondary system are obtained, the method allows one to carry out a modal analysis of the secondary system independently of the primary system. To combine the modal responses, a combination rule is derived in terms of the modal properties of the secondary system and ordinates of the CCFS. The proposed method accounts for all the important effects mentioned above and, hence, resolves the shortcomings inherent in the conventional floor spectrum methods.

1.5 An Overview

This study is divided in four well defined parts, each constituting a chapter. A brief outline of each chapter follows:

In Chapter 2, an extension of the conventional floor response spectrum is introduced and defined as a cross-oscillator cross-floor response spectrum. Then, a modal combination rule, that accounts for cross-correlations between modal responses and cross-correlations between support excitations, is developed. The formulation of the CCFS and the development of the modal combination rule for multiply supported systems are based on fundamental concepts from stationary random vibration theory. The CCFS are defined to be proportional to the covariance of the response of two fictitious oscillators subjected to base excitations equal to the motions of the support points on the primary system. The dynamic properties of these oscillators correspond to the dynamic properties of the modes of the fixed-base secondary system.

In Chapter 3, equivalent masses are defined for the aforementioned oscillators. These masses represent, in an approximate manner, the effects of the modes of the secondary system in perturbing the dynamic properties of the primary system and they are introduced to account for the effect of interaction between primary and secondary systems.

In Chapter 4, it is shown that the CCFS can be efficiently evaluated by using systems composed of the primary system and attached oscillators. Closed form expressions for the modal characteristics of the combined oscillator-structure systems are presented. These closed form expressions were derived previously by Der Kiureghian, Sackman and Nour-Omid [1981] using perturbations methods and assuming classical damping. These expressions are improved here to approximately account for the effect of non-classical damping, which arises when the primary and secondary systems have unequal modal damping.

In Chapter 5, simple numerical examples having basic features of most important cases encountered in practice are presented. These examples illustrate the application of the method and demonstrate, in each case, close agreement between results obtained using this method and "exact" results obtained by considering the primary-secondary system as a single system.

CHAPTER 2

Modal Combination Rule for Secondary Systems in Terms of Cross-Cross Floor Spectra

2.1 Introduction

The main objective of this chapter is to develop a modal combination rule that accounts for cross-correlations between modal responses and cross-correlations between support motions of a multiply supported secondary system. This is accomplished through the introduction of an extension of the conventional floor spectrum defined as a cross-oscillator, cross-floor response spectrum, or in short, cross-cross floor spectrum.

The analysis in this chapter is based on the assumption of stationary response to stationary input. However, the final results for the secondary system response are given in terms of cross-cross floor spectra. These are in turn derived in terms of the ground response spectrum. Both of these spectra inherently include the non-stationarity of the earthquake excitation. Thus, the stationarity assumption used in this chapter is only an interim assumption.

In the following sections, first the equations of motion of the secondary system are formulated. The power spectral density and the mean square of the response are then obtained as a function of cross terms between floor motions. These cross quantities are closely examined. Their interpretation in terms of the cross-correlation of responses of two oscillators attached to the primary structure leads to the formal definition of cross-cross floor spectra. The final result of this chapter is a modal combination rule for the mean peak response of the secondary system in terms of a four-fold summation involving the cross-cross floor spectra and the modal properties of the fixed base secondary system. Two of these summations are over the fixed base modes of the secondary system, and the other two are over the support points. Thus, cross-correlation between modes and between support motions are included in the modal combination rule.

2.2 Formulation of Equations of Motion

Consider a linear primary system with N degrees of freedom which is subjected to base input $u_g(t)$. Attached to this system at n_a degrees of freedom is an $n_a + n$ degrees of freedom linear secondary system, which may represent an extended equipment item or a piping system. The attached degrees of freedom in the primary and secondary systems are selected to be $I = 1,\ldots,n_a$ and $i = 1,\ldots,n_a$, respectively, so that the unattached degrees of freedom are $I = n_a+1,\ldots,N$ and $i = n_a+1,\ldots,n_a+n$. These definitions are schematically illustrated in Fig. 2.1.

Let $\mathbf{U} = [\mathbf{U}_a \ \mathbf{U}_{\bar{a}}]^T$ denote the vector of total displacements of the primary system, which has been partitioned into attached ($\mathbf{U}_a$) and unattached ($\mathbf{U}_{\bar{a}}$) degrees of freedom. Similarly, let $\mathbf{u} = [\mathbf{u}_a \ \mathbf{u}_{\bar{a}}]^T$ denote the partitioned vector of total displacements of the secondary system. The coupled equations of motion for the primary and secondary systems can be written, respectively, as

$$\mathbf{M}\begin{Bmatrix}\ddot{\mathbf{U}}_a\\ \ddot{\mathbf{U}}_{\bar{a}}\end{Bmatrix} + \mathbf{C}\begin{Bmatrix}\dot{\mathbf{U}}_a\\ \dot{\mathbf{U}}_{\bar{a}}\end{Bmatrix} + \mathbf{K}\begin{Bmatrix}\mathbf{U}_a\\ \mathbf{U}_{\bar{a}}\end{Bmatrix} = \mathbf{CR}\dot{u}_g + \mathbf{KR}u_g + \begin{Bmatrix}\mathbf{f}\\ 0\end{Bmatrix} \tag{2.1}$$

and

$$\begin{bmatrix}\mathbf{m}_a & \mathbf{0}^T\\ \mathbf{0} & \mathbf{m}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{u}}_a\\ \ddot{\mathbf{u}}_{\bar{a}}\end{Bmatrix} + \begin{bmatrix}\mathbf{c}_a & \mathbf{c}_c^T\\ \mathbf{c}_c & \mathbf{c}\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{u}}_a\\ \dot{\mathbf{u}}_{\bar{a}}\end{Bmatrix} + \begin{bmatrix}\mathbf{k}_a & \mathbf{k}_c^T\\ \mathbf{k}_c & \mathbf{k}\end{bmatrix}\begin{Bmatrix}\mathbf{u}_a\\ \mathbf{u}_{\bar{a}}\end{Bmatrix} = -\begin{Bmatrix}\mathbf{f}\\ 0\end{Bmatrix} \tag{2.2}$$

In the above, **M**, **C**, and **K** are the conventional mass, damping, and stiffness matrices of the primary system, respectively; **R** is the influence vector that couples the ground motion to the degrees of freedom of the primary system; $\mathbf{m}$, $\mathbf{c}$, and $\mathbf{k}$ are the mass, damping, and stiffness matrices of the fixed base secondary system; $\mathbf{c}_c$ and $\mathbf{k}_c$ are coupling matrices which include the dampings and stiffnesses of the elements connecting the primary and secondary systems, and $\mathbf{m}_a$, $\mathbf{c}_a$, and $\mathbf{k}_a$ are matrices associated with the attachment points of the secondary system. Note that the full matrices in Eq. (2.2) are the mass, damping, and stiffness matrices of the secondary system when it is considered as a free-free system. The n_a-vector $\mathbf{f}$ represents the interaction forces exerted by the secondary system on the primary system. It should be clear that $\mathbf{U}_a = \mathbf{u}_a$ for compatibility.

The complete solution for the combined system involves a simultaneous solution of Eqs. (2.1) and (2.2). For practical reasons mentioned in the previous chapter, however, a solution of the secondary system which is separate from the solution of the primary system is desired. The coupled equations of motion for the unattached degrees of freedom of the secondary system can be written, using the partitioning in Eq. (2.2) and the identity $\mathbf{u}_a = \mathbf{U}_a$, as

$$\mathbf{m}\ddot{\mathbf{u}}_{\bar{a}} + \mathbf{c}\dot{\mathbf{u}}_{\bar{a}} + \mathbf{k}\mathbf{u}_{\bar{a}} = -\mathbf{c}_c\dot{\mathbf{U}}_a - \mathbf{k}_c\mathbf{U}_a \tag{2.3}$$

Thus, the solution for the secondary system $\mathbf{u}_{\bar{a}}$ is obtained in terms of the motions of the primary system at the attachment degrees of freedom, $\mathbf{U}_a$. These motions are functions of the interaction force $\mathbf{f}$, as shown in Eq. (2.1), and Eq. (2.3) remains coupled. The standard simplifying approximation in order to decouple Eq. (2.3) is to neglect the interaction between primary and secondary systems (ASME Boiler and Pressure Vessel Code, Appendix N, [1980]). In that case $\mathbf{U}_a$ can be obtained from Eq. (2.1) with $\mathbf{f} = 0$, and Eq. (2.3) can be solved directly. It has been shown (Amin, Hall, Newmark and Kassawara [1971], Pickel [1972], Der Kiureghian, Sackman and Nour-Omid [1981], Igusa and Der

Kiureghian [1983]), however, that the effect of interaction is negligible only when the secondary system is sufficiently light in comparison to the primary system and there is no tuning between the frequencies of the secondary system and the dominant frequencies of the primary system. In the present work, interaction will be retained and an approximate method to account for this effect will be described in the next chapter.

For the sake of simplicity, the damping terms on the right-hand side of Eqs. (2.1) and (2.3), which are generally small for structural systems, are neglected. Thus, Eq. (2.3) is rewritten as

$$\mathbf{m}\ddot{\mathbf{u}}_{\sigma} + \mathbf{c}\dot{\mathbf{u}}_{\sigma} + \mathbf{k}\mathbf{u}_{\sigma} = -\mathbf{k}_c \mathbf{U}_a \tag{2.4}$$

Experience has shown that the above approximation has negligible effect for the vast majority of systems encountered in practice.

2.3 Power Spectral Density of Secondary System Response

In order to develop a response spectrum method, it is essential to use a modal approach. Let $\mathbf{\Phi} = [\mathbf{\Phi}_1\ \mathbf{\Phi}_2\ \cdots\ \mathbf{\Phi}_N]$ and $\boldsymbol{\varphi} = [\boldsymbol{\varphi}_1\ \boldsymbol{\varphi}_2\ \cdots\ \boldsymbol{\varphi}_n]$ denote the modal matrices of the primary system and the fixed base secondary system, respectively. Also, let Ω_I, Z_I and ω_i, ζ_i denote the modal frequencies and damping ratios of modes I and i of the primary and secondary systems, respectively. Using Eq. (2.4), and following standard techniques in stationary random vibrations (Clough and Penzien [1975]), the one-sided power spectral density function of the total displacement u_r at degree of freedom r of the secondary system is obtained as

$$G_{u_r u_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} \varphi_{ri}\varphi_{rj}\, h_i(-\omega) h_j(\omega) G_{q_i q_j}(\omega) \tag{2.5}$$

in which φ_{ri} is the $r-th$ element of φ_i, $h_i(\omega) = (\omega_i^2 - \omega^2 + 2i\zeta_i\omega_i\omega)^{-1}$ is the complex frequency response function associated with mode i of the fixed base secondary system where $i = \sqrt{-1}$, and $G_{q_iq_j}(\omega)$ is the cross-power spectral density of the modal loads. These loads are given by

$$q_i = -\varphi_i^T \frac{\mathbf{k}_c}{m_i} \mathbf{U}_a \tag{2.6a}$$

where $m_i = \varphi_i^T \mathbf{m} \varphi_i$ is the modal mass associated with mode i of the fixed base secondary system. Writing the matrix multiplication in Eq. (2.6a) in an expanded form, the modal force is written as

$$q_i = -\frac{1}{m_i} \sum_{K=1}^{n_a} \left(\sum_{l=1}^{n} \varphi_{li} k_{clK} \right) U_K \tag{2.6b}$$

where U_K is the $K-th$ element of $\mathbf{U}_a$ and represents the total displacement of the $K-th$ attachment point, and k_{clK} is the (l,K) element of the coupling stiffness matrix $\mathbf{k}_c$, which is associated with the attached degree of freedom K and unattached degree of freedom l of the secondary system. The cross-power spectral density function $G_{q_iq_j}(\omega)$ is given by the expression

$$G_{q_iq_j}(\omega) = \frac{1}{m_i m_j} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} \left(\sum_{l=1}^{n} \varphi_{li} k_{clK} \right) \left(\sum_{l=1}^{n} \varphi_{lj} k_{clL} \right) G_{U_K U_L}(\omega) \tag{2.7}$$

where $G_{U_K U_L}(\omega)$ is the cross-power spectral density of the total displacement responses of the primary system at the attachment points K and L. Note that the first two sums are over the attachment points, whereas the two sums inside parenthesis are over the unattached degrees of freedom of the secondary system.

Introducing Eq. (2.7) into Eq. (2.5), the power spectral density function $G_{u_ru_r}(\omega)$ takes the form

$$G_{u_r u_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \, \omega_i^2 \omega_j^2 h_i(-\omega) h_j(\omega) G_{\ddot{y}_K \ddot{y}_L}(\omega) \tag{2.8}$$

in which

$$a_{ri} = \frac{\varphi_{ri}}{m_i \omega_i^2} \quad \text{and} \quad b_{iK} = \sum_{l=1}^{n} \varphi_{li} k_{clK}. \tag{2.9}$$

It is noted that a_{ri} is a constant which depends on the $i-th$ mode of the fixed base secondary system, whereas b_{iK} is a constant that depends on the $i-th$ mode shape of the fixed base secondary system and the coupling stiffness matrix $\mathbf{k}_c$. The effect of interaction is implicitly included in the cross-power spectral density function $G_{\ddot{y}_K \ddot{y}_L}(\omega)$.

The term $\omega_i^2 \omega_j^2 h_i(-\omega) h_j(\omega) G_{\ddot{y}_K \ddot{y}_L}(\omega)$ in Eq. (2.8) may be interpreted as the cross-power spectral density of the total displacement responses of two oscillators having frequencies ω_i and ω_j and damping ratios ζ_i and ζ_j, which are subjected to base inputs U_K and U_L, respectively. Figure 2.2 illustrates this idea schematically. This interpretation is central to the subsequent development of the cross-cross floor spectrum method in this study.

Let X_{iK}^T and X_{jL}^T denote the total displacements of the two fictitious oscillators described above, and let $G_{X_{iK}^T X_{jL}^T}(\omega)$ denote their cross power spectral density function

$$G_{X_{iK}^T X_{jL}^T}(\omega) = \omega_i^2 \omega_j^2 h_i(-\omega) h_j(\omega) G_{\ddot{y}_K \ddot{y}_L}(\omega) \tag{2.10}$$

Introducing this notation in Eq. (2.8), the power spectral density $G_{u_r u_r}(\omega)$ can be rewritten as

$$G_{u_r u_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \, G_{X_{iK}^T X_{jL}^T}(\omega) \tag{2.11}$$

In a similar way, using the identity $G_{XY}(\omega) = \dfrac{G_{\ddot{X}\ddot{Y}}(\omega)}{\omega^4}$, the power spectral density of the absolute acceleration $\ddot{u}_r$ at degree of freedom r of the secondary system is given by

$$G_{\ddot{u}_r \ddot{u}_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} G_{\ddot{X}_{iK}^T \ddot{X}_{jL}^T}(\omega) \tag{2.12}$$

where $G_{\ddot{X}_{iK}^T \ddot{X}_{jL}^T}(\omega)$ is the cross-power spectral density of the total accelerations $\ddot{X}_{iK}^T$ and $\ddot{X}_{jL}^T$ of the two fictitious oscillators, and is given by

$$G_{\ddot{X}_{iK}^T \ddot{X}_{jL}^T}(\omega) = \omega_i^2 \omega_j^2 h_i(-\omega) h_j(\omega) G_{\ddot{y}_K \ddot{y}_L}(\omega) \tag{2.13}$$

in which $G_{\ddot{y}_K \ddot{y}_L}(\omega)$ is the cross-power spectral density of the total acceleration responses of the primary system at the attachment points K and L.

It will be shown later in Chapter 4 that the two formulations in Eqs. (2.11) and (2.12) lead to expressions for the responses of the secondary system given in terms of the total displacement ground response spectrum and the absolute acceleration ground response spectrum, respectively. It is well known that the total displacement ground response spectrum is very sensitive to the base-line correction criterion applied to accelerograms (Trifunac, Udwadia and Brady [1973]), and hence it is unreliable for use in practice. Because of this, it is desirable to find the displacement response of the secondary system as a function of the relative displacement ground response spectrum. Towards that end, it is shown in Appendix A that the power spectral density of the relative displacement of the secondary system with respect to the ground can be expressed as

$$G_{v_r v_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} G_{X_{iK}^R X_{jL}^R}(\omega) \tag{2.14}$$

where $G_{v_r v_r}(\omega)$ is the power spectral density of the relative displacement v_r at degree of freedom r of the secondary system with respect to the base of the primary system, and $G_{X_{iK}^R X_{jL}^R}(\omega)$ is the cross-power spectral density of the relative displacements $X_{iK}^R = X_{iK}^T - u_g$ and $X_{jL}^R = X_{jL}^T - u_g$ of the two fictitious oscillators described above. These terms are schematically defined in Figs. 2.3 and 2.4. It will be shown later in Chapter 4 that Eq. (2.14) will lead to a formulation in terms of the relative displacement ground response spectrum. It must be evident that for oscillators attached to degrees of freedom perpendicular to the direction of the input excitation, the corresponding relative displacements are $X_{iK}^R = X_{iK}^T$ and $X_{jL}^R = X_{jL}^T$.

Since in practice only relative displacements and absolute accelerations of the secondary system are of interest, in the following development only Eqs. (2.12) and (2.14) will be considered. It is worthwhile to note in these equations that due to symmetry, the imaginary parts in the summations on the right-hand sides cancel out so that the left-hand sides are always real-valued.

2.4 Mean Square of the Secondary System Response

The response quantity of engineering interest is the mean of the peak response of the secondary system over the duration of the seismic excitation. It is known (Davenport [1964], Vanmarcke [1976], Der Kiureghian [1980]), that for a stationary process the mean peak response is related to the root-mean square value of the response through a peak factor. Hence, as an intermediate step, the mean square response of the secondary system is formulated in this section. The mean square of a generic response quantity s is related to its one-sided power spectral density function by the relation

$$E[s^2] = \int_0^\infty G_{ss}(\omega)\, d\omega \tag{2.15}$$

Thus, integrating Eqs. (2.12) and (2.14) over the frequency domain, the mean squares of the absolute acceleration and the relative displacement at degree of freedom r are obtained, respectively

$$E[\ddot{u}_r^2] = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \lambda^a_{0,ijKL} \tag{2.16}$$

and

$$E[v_r^2] = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \lambda^v_{0,ijKL} \tag{2.17}$$

where

$$\lambda^a_{0,ijKL} = \mathrm{Re} \int_0^\infty G_{\ddot{X}^T_{iK} \ddot{X}^T_{jL}}(\omega)\, d\omega \tag{2.18}$$

and

$$\lambda^v_{0,ijKL} = \mathrm{Re} \int_0^\infty G_{X^R_{iK} X^R_{jL}}(\omega)\, d\omega \tag{2.19}$$

The above two terms respectively denote cross-correlations between accelerations and displacements of the two fictitious oscillators defined earlier. As indicated earlier, these quantities will be derived in terms of their respective ground response spectra, i.e., $\lambda^v_{0,ijKL}$ will be derived in terms of the relative displacement ground response spectrum and $\lambda^a_{0,ijKL}$ will be derived in terms of the pseudo-acceleration ground response spectrum.

As a final remark for this section, it is noted that the mean square of any general displacement-related quantity, such as the internal force of a member or the relative displacement between two points, can be obtained from Eq. (2.17) by redefining the coefficients a_{ri} and a_{rj}. For this purpose, it is sufficient to replace the terms φ_{ri} and φ_{rj} in the definition of a_{ri} and a_{rj} by the corresponding modal responses when the fixed base secondary system is statically displaced into its $i-th$ and $j-th$ mode shapes, respectively. For a generic response quantity s, the modal response can be expressed as $s_i = \mathbf{g}^T \varphi_i$, where $\mathbf{g}$ is an n-vector of constants. Equation (2.17) may then be used to compute the mean square $E[s^2]$, provided a_{ri} is replaced by $\frac{\mathbf{g}^T \varphi_i}{m_i \omega_i^2}$, i.e.

$$E[s^2] = \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\frac{\mathbf{g}^T \varphi_i}{m_i \omega_i^2} \right] \left[\frac{\mathbf{g}^T \varphi_j}{m_j \omega_j^2} \right] \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \lambda_{0,ijKL}^{v} \tag{2.20}$$

As an example, for relative displacement between degrees of freedom r and s, the vector $\mathbf{g}$ contains 1 and -1 for the $r-th$ and $s-th$ elements and zeros elsewhere.

2.5 Interpretation of Cross-Correlation in Terms of Cross-Cross Floor Spectra

In the previous section, the mean square responses of the secondary system were obtained as functions of the cross-correlation terms $\lambda_{0,ijKL}^{a}$ and $\lambda_{0,ijKL}^{v}$ which were associated with the responses of two fictitious oscillators of frequencies ω_i and ω_j, and with damping ratios ζ_i and ζ_j. These oscillators were assumed to be subjected to base excitations equal to the motions of the primary system at the support points K and L, respectively. In this section, these cross-correlation quantities are interpreted in terms of a generalization of the conventional floor spectra. For notational simplicity, the superscripts on the cross-correlation terms are dropped in the following analysis.

First consider the term $\lambda_{0,iiKK}$. This may be regarded as the mean square of the response of a fictitious oscillator of frequency ω_i and damping ratio ζ_i which is attached to the $K-th$ degree of freedom of the primary system. To clarify the procedure, the interaction between the secondary and primary systems and the interaction between the oscillator and the primary system are ignored at this time. Figure 2.5(a) illustrates this concept schematically. If $\bar{S}_K(\omega_i,\zeta_i)$ denotes the mean floor response spectrum associated with the degree of freedom K, then by definition it is equal to the mean peak response of the oscillator. Thus, using the relation between the mean square and the mean of the peak of a stationary process, the following relation can be written

$$\lambda_{0,iiKK} = \frac{1}{p_{iK}^2}\bar{S}_K^2(\omega_i,\zeta_i) = \frac{1}{p_{iK}^2}\bar{S}_{KK}(\omega_i,\zeta_i;\omega_i,\zeta_i) \qquad (2.21)$$

in which p_{iK} is a peak factor associated with the response of the oscillator (Davenport [1964], Vanmarcke [1976], Der Kiureghian [1980]), and by definition $\bar{S}_{KK}(\omega_i,\zeta_i;\omega_i,\zeta_i) = \bar{S}_K^2(\omega_i,\zeta_i)$. The reason for the latter definition will become clear shortly. Note that $\bar{S}_{KK}(\omega_i,\zeta_i;\omega_i,\zeta_i)$ has the square dimension of the floor spectrum.

Now consider the term $\lambda_{0,ijKK}$. This may be regarded as the cross-correlation of the responses of two fictitious oscillators of frequencies ω_i and ω_j and damping ratios ζ_i and ζ_j , both attached to the $K\text{-}th$ degree of freedom of the primary system. Figure 2.5(b) illustrates this concept. This term may not be interpreted in terms of the conventional floor spectra. However, based on the relation in Eq. (2.21), an extension of the floor spectrum can be defined, which may then be used to interpret $\lambda_{0,ijKK}$. Consider the following relation

$$\lambda_{0,ijKK} = \frac{1}{p_{iK}p_{jK}}\bar{S}_{KK}(\omega_i,\zeta_i;\omega_j,\zeta_j) \qquad (2.22)$$

where p_{iK} and p_{jK} are peak factors associated with motions of the two oscillators. In this relation, $\bar{S}_{KK}(\omega_i,\zeta_i;\omega_j,\zeta_j)$ may be interpreted as a *cross-oscillator* floor response spectrum associated with the $K\text{-}th$ degree of freedom of the primary system.

Next consider the term $\lambda_{0,iiKL}$. This term may be regarded as the cross-correlation of the responses of two identical oscillators of frequencies ω_i and damping ratios ζ_i which are attached to the degrees of freedom K and L of the primary system, as illustrated in Fig. 2.5(c). Following the above idea, the relation

$$\lambda_{0,iiKL} = \frac{1}{p_{iK}p_{iL}}\bar{S}_{KL}(\omega_i,\zeta_i;\omega_i,\zeta_i) \tag{2.23}$$

is defined, where $\bar{S}_{KL}(\omega_i,\zeta_i;\omega_i,\zeta_i)$ may be interpreted as a *cross-floor* response spectrum associated with degrees of freedom K and L of the primary system.

Finally, consider the general term $\lambda_{0,ijKL}$. It is clear now, that this is the cross-correlation of the responses of two fictitious oscillators of frequencies ω_i and ω_j and damping ratios ζ_i and ζ_j , which are attached to degrees of freedom K and L of the primary system, respectively, as illustrated in Fig. 2.5(d). The following relation is defined

$$\lambda_{0,ijKL} = \frac{1}{p_{iK}p_{jL}}\bar{S}_{KL}(\omega_i,\zeta_i;\omega_j,\zeta_j) \tag{2.24}$$

where $\bar{S}_{KL}(\omega_i,\zeta_i;\omega_j,\zeta_j)$ can be interpreted as the *cross-oscillator, cross-floor* response spectrum associated with the degrees of freedom K and L of the primary system. For brevity, this most general term may be called the *cross-cross floor spectrum* or *CCFS*. Note that this term is a cross term not only between two oscillators (modes of the secondary system) but also between two "floors". This is the reason for using two "cross" terms in its definition. It is important to realize that this cross-cross term contains the effect of cross correlation between modal responses of the secondary system as well as the cross correlation between motions of support points K and L. An efficient method for generating the CCFS's directly in terms of the input ground response spectrum is presented in Chapter 4.

Using the relation $E[s_{\max}] = p\, E[s^2]^{1/2}$ between the mean of the peak and the root-mean-square of a stationary process, where p is the peak factor, and substituting Eq. (2.24) in Eq. (2.16), the mean of the peak acceleration at the degree of freedom r of the secondary system is obtained as

$$E[\ddot{u}_{r,max}] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{jL}\frac{p^2}{p_{iK}p_{jL}}\bar{S}^a_{KL}(\omega_i,\zeta_i;\omega_j,\zeta_j)\right]^{1/2} \tag{2.25}$$

In a similar manner, the mean of the peak relative displacement at the degree of freedom r is obtained as

$$E[v_{r,max}] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{jL}\frac{p^2}{p_{iK}p_{jL}}\bar{S}^v_{KL}(\omega_i,\zeta_i;\omega_j,\zeta_j)\right]^{1/2} \tag{2.26}$$

It can be shown (Der Kiureghian [1981]) that the peak factors are relatively insensitive to the characteristics of the response processes and the ratios p/p_{iK} are near unity. Thus, the above expressions can be simplified to

$$E[\ddot{u}_{r,max}] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{jL}\,\bar{S}^a_{KL}(\omega_i,\zeta_i;\omega_j,\zeta_j)\right]^{1/2} \tag{2.27}$$

$$E[v_{r,max}] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{jL}\,\bar{S}^v_{KL}(\omega_i,\zeta_i;\omega_j,\zeta_j)\right]^{1/2} \tag{2.28}$$

It should be noted that in principle there is no need to make this approximation and it is done here for the sake of simplicity and to avoid time consuming calculations. Also, it will be seen in Appendix B, that the peak factors p_{iK} are only intermediate values and need not be evaluated even in cases where the effect of peak factors is to be included in the analysis. Expressions for these factors are given in Appendix B.

Equations (2.27) and (2.28) provide modal combination rules for the mean values of peak responses of the secondary system in terms of the CCFS's. Note that the coefficients a_{ri} and b_{iK}, as given in Eq. (2.9), are functions of the characteristics of the secondary system only. Thus, having the CCFS's, the secondary system can be analyzed independently of the primary system. It is emphasized that the formulation presented here not only includes the effect of correlation between modal responses of the secondary system, but also the correlation that exists between the excitations at the various attachment points. It can be shown that due the narrow-bandedness of the floor motions, strong correlation exists between modal responses even when modal frequencies are well spaced. Also, the floor motions are in general highly correlated due to the filtering of the ground motion through the primary system. Thus, all cross terms in Eqs. (2.27) and (2.28) are important and must be retained in all cases to obtain accurate results. As indicated in Chapter 1, these effects are commonly neglected or improperly handled in the current practice.

In the next chapter, a method will be introduced to approximately account for the effect of interaction between the primary and secondary systems. This will be done through the definition of equivalent masses for the fictitious oscillators introduced above such that the motions of the primary system at the attachment points are properly modified in account of the interaction.

2.6 Properties of the Cross-Cross Floor Spectra

In this section, a short list of the main properties and features of the cross-cross floor spectra is given:

(a) In this chapter, cross-cross floor spectra were defined in terms of the cross-correlation of the responses of two fictitious oscillators associated with two modes of the secondary system. Because the responses are real quantities, the expectation of their product, or cross-correlation, will be real valued. Thus, cross-cross floor spectra are real valued functions. Also, these real functions can be positive or negative depending on how the responses of the two oscillators are correlated.

(b) The cross-cross floor spectra, $\bar{S}_{KL}(\omega_i, \zeta_i; \omega_j, \zeta_j)$, depend on two modes of the fixed base secondary system. Thus, unlike conventional floor spectra which can be represented as two-dimensional curves, the CCFS must be represented by surfaces. Figure 2.6 shows the cross-cross floor spectrum surface $\bar{S}_{13}$ for the five story system shown in Fig. 4.2. In Fig. 2.6(a), only the values around the fundamental frequency of the structure (4.025 rad/s) have been plotted. For this example, it can be observed that $\bar{S}_{13}$ has a large peak when the two oscillators are tuned to the fundamental frequency of the structure, two valleys when only one oscillator is tuned, and practically zero values elsewhere. In Fig. 2.6(b), the peak value was truncated and the figure rotated to see more details on the surface. There, it can be observed that a negative peak begins to appear when the two oscillators are tuned to the second frequency of the structure.

(c) The "diagonal" term $\bar{S}_{KK}(\omega_i, \zeta_i; \omega_i, \zeta_i)$ corresponds to the square of the conventional floor response spectrum for frequency ω_i, damping ratio ζ_i and "floor" K.

(d) The following symmetry property holds

$$\bar{S}_{KL}(\omega_i, \zeta_i; \omega_j, \zeta_j) = \bar{S}_{LK}(\omega_j, \zeta_j; \omega_i, \zeta_i)$$

(e) As mentioned in the previous section, due to the narrow-bandedness of the floor motions, strong correlation exists between the floor motions and between the modal responses of the secondary system. Thus, the "cross" terms $\bar{S}_{KL}(\omega_i, \zeta_i; \omega_j, \zeta_j)$ in general are not negligible in relation to the "diagonal" terms $\bar{S}_{KK}(\omega_i, \zeta_i; \omega_i, \zeta_i)$, and all the cross terms in Eqs. (2.27) and (2.28) must, in general, be retained.

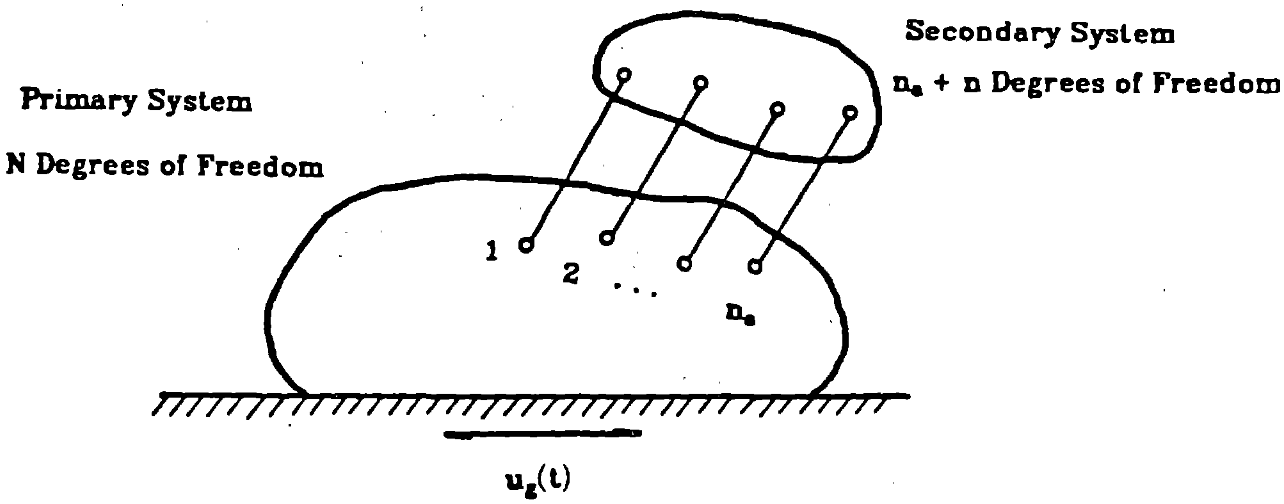


Figure 2.1 Schematic Illustration of Primary-Secondary System

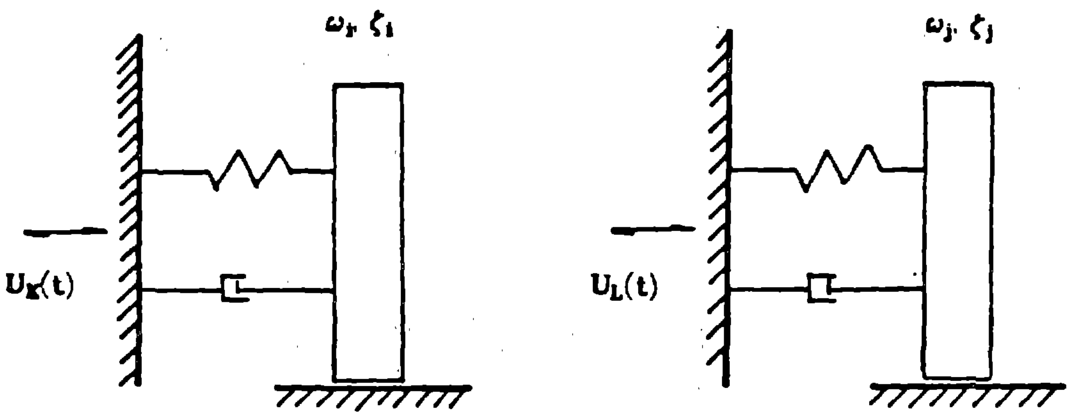


Figure 2.2 Schematic Model for the Cross-Power Spectral Density Function

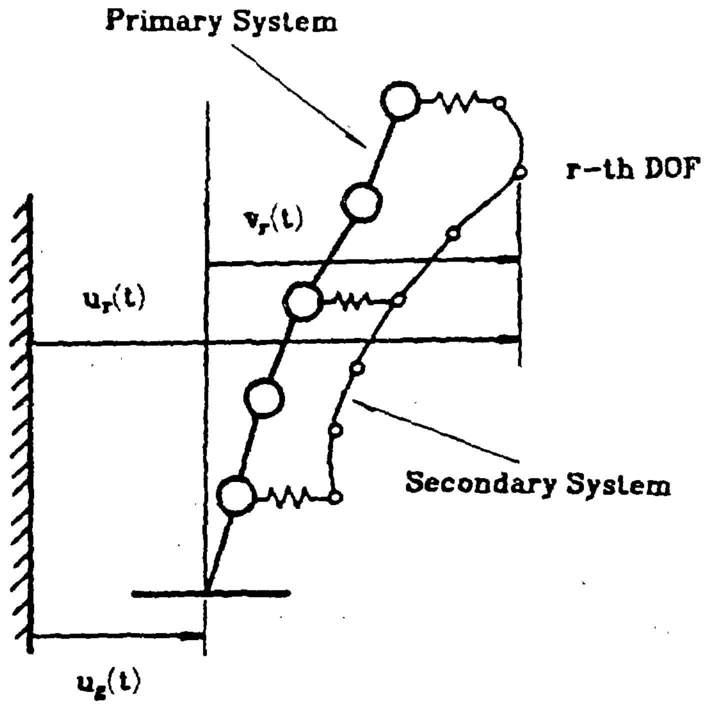


Figure 2.3 Definition of Relative Displacements for the Secondary System

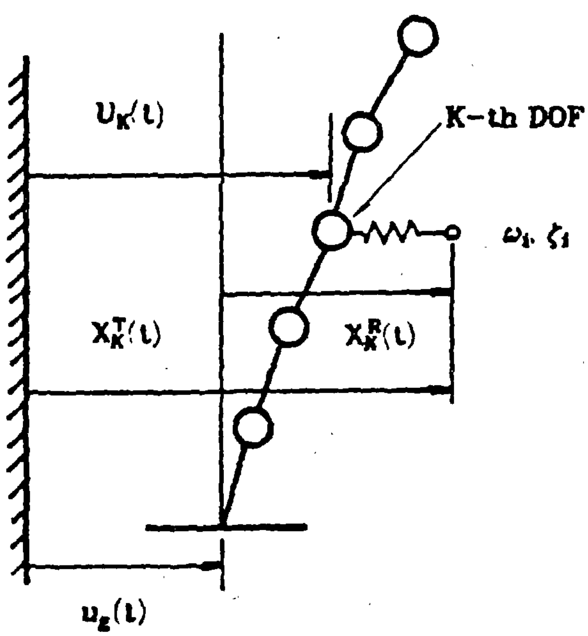


Figure 2.4 Definition of Relative Displacement for the Single Oscillator

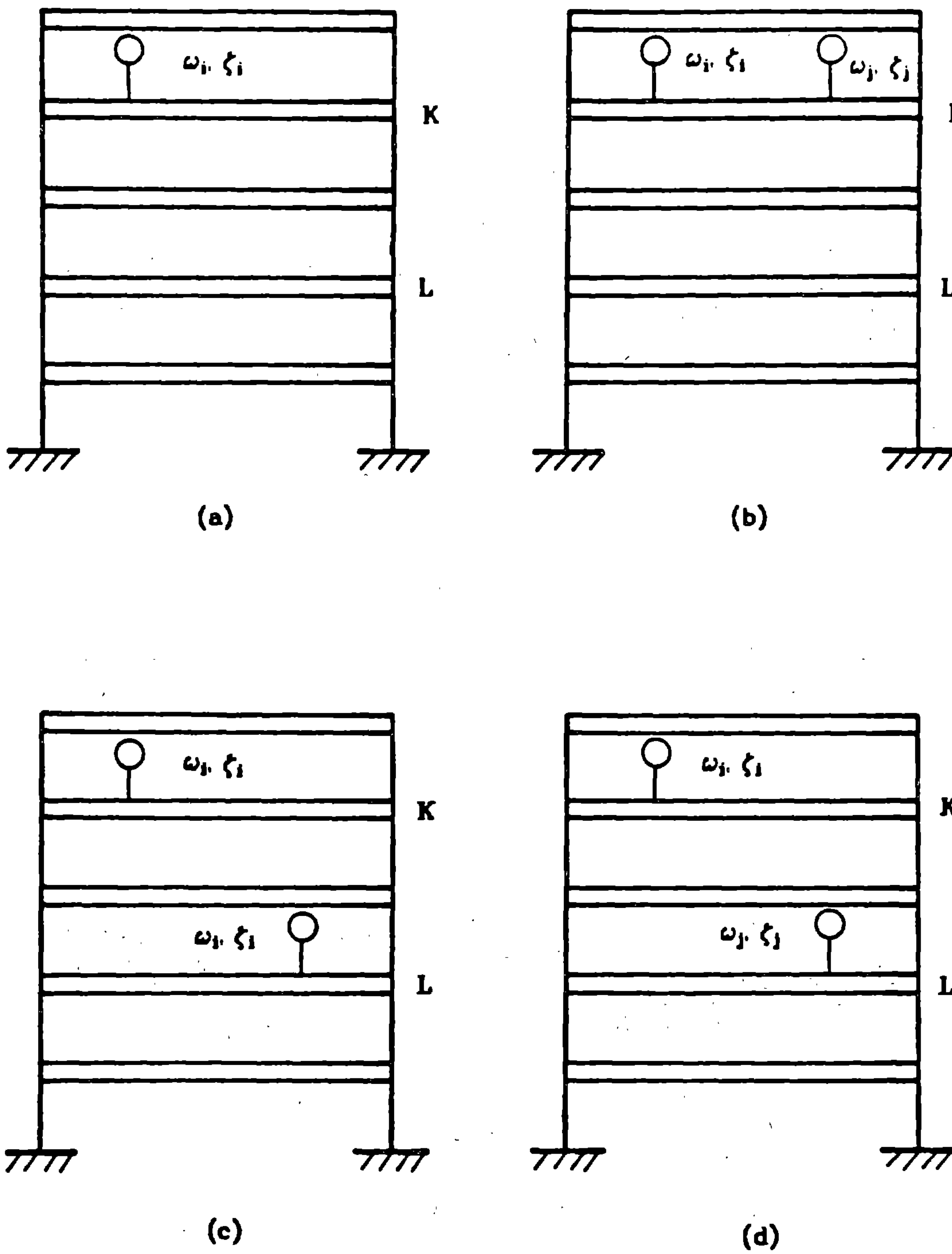
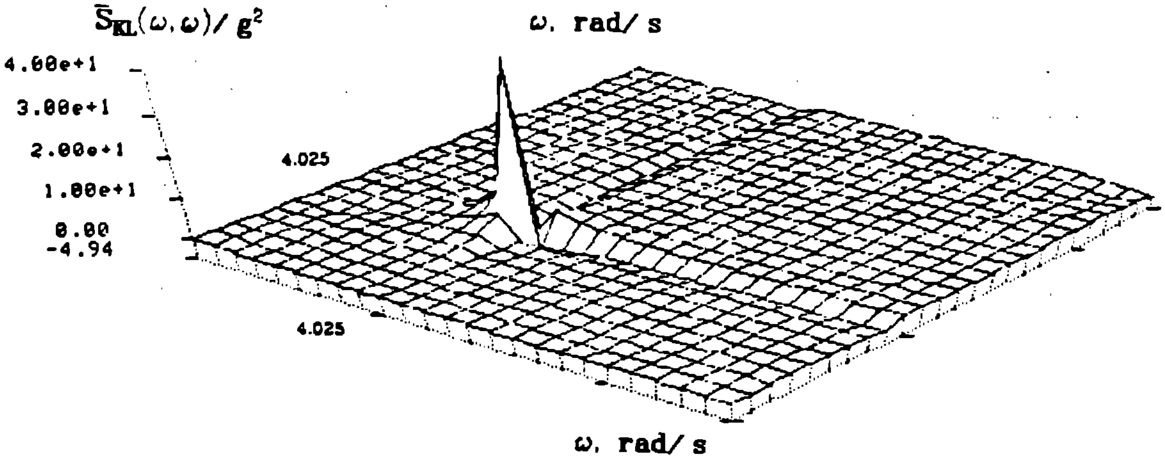
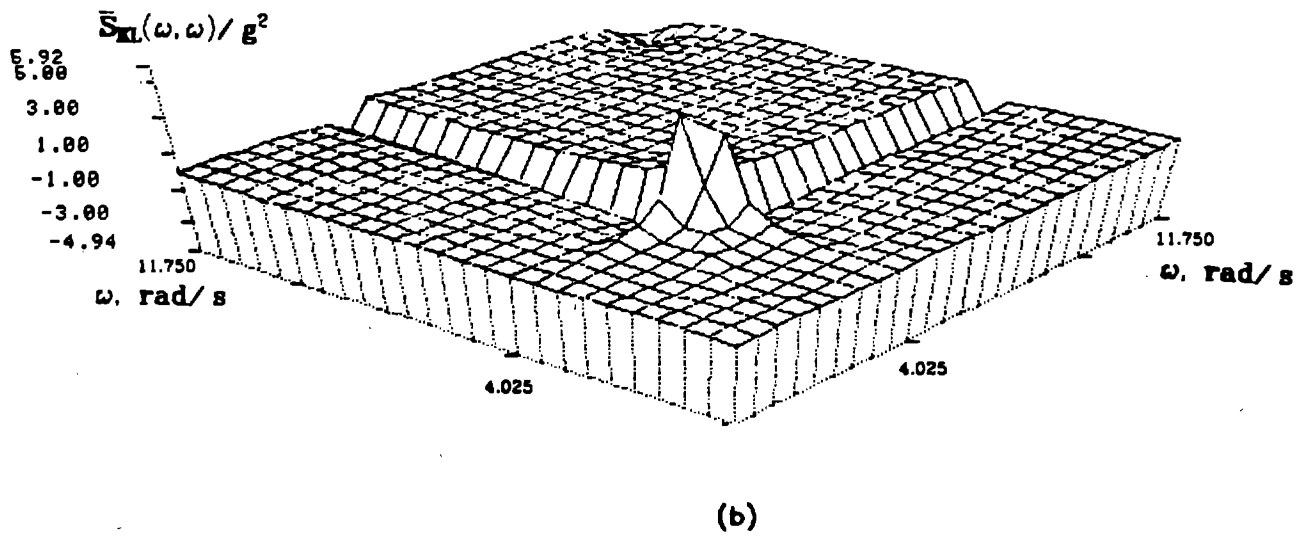


Figure 2.5 Schematic Systems Used in Defining CCFS



(a)



(b)

Figure 2.6 **Cross-Cross Floor Spectrum**

CHAPTER 3

Effect of Interaction Between Primary and Secondary Systems

3.1 Introduction

Research on dynamic behavior of composite primary-secondary systems has shown that the effect of dynamic interaction between the two subsystems can be important in two situations: (a) when the mass of the secondary system is not sufficiently small in relation to the mass of the primary system, and (b) when one or more frequencies of the fixed base secondary system are in resonance (tuning) with one or more of the dominant frequencies of the primary system. Generally, interaction tends to reduce the response of the secondary system. Therefore, neglecting its effect is a conservative measure. However, the conservatism can be very large, i.e., the response can be overestimated by as much as several hundred percent in some cases. Thus, it is important to account for this effect in the floor spectrum method to be developed.

Neglecting the effect of interaction is equivalent to assuming that the motions at the support points on the primary system are the same as those in absence of the secondary system. In reality, the support motions are affected by the presence of the secondary system, and in particular their frequency content is modified. This modification on the frequency content is due to a shift of the modal frequencies of the primary system as the secondary system is attached to it. The shift depends on the ratio of masses of the two systems as well as their respective stiffnesses. To make this point clearer, consider an oscillator with a small mass attached to a primary system and tuned to its fundamental frequency. If the motion of the primary system without interaction is considered as input into the oscillator, i.e., if interaction is neglected, resonance will occur and the oscillator will have a large response. However, if the primary-oscillator system is considered as a single unit, the frequencies of the combined system will shift away from the tuning frequency and resonance will not occur. Thus, the response of the oscillator can be expected to be smaller in the latter case where interaction is included.

Although interaction between primary and secondary systems has been studied before (Crandall and Mark [1963], Amin, Hall, Newmark and Kassawara [1971], Pickel [1972], Newmark [1972], Hadjian [1978]), most previous studies have been limited to single degree of freedom secondary systems. For this case, Der Kiureghian, Sackman and Nour-Omid [1981] recently developed a perturbation technique to evaluate the modal characteristics of the combined equipment-structure system that includes the effect of interaction. Igusa and Der Kiureghian [1983] have more recently extended this method to multi-degree-of-freedom secondary systems with multiple attachment points. In both of these approaches the response of the combined primary-secondary system is given in terms of the ground response spectrum. No methods have yet been developed to account for the effect of interaction in the floor response spectrum approach for multiply-supported secondary systems and this effect is currently neglected in practice (ASME Boiler and Pressure Vessel Code, Appendix N, 1980).

In this chapter, an approximate method for incorporating the effect of interaction in the cross-cross floor spectrum method is developed. For this purpose, the fictitious oscillators representing modes of the secondary system are assigned mass values such that proper shifts in the frequencies of the primary system are achieved. The shifts are essentially equivalent to the actual shifts that occur in the combined primary-secondary system. This results in a modification of the frequency content of the support motion of each oscillator, making it approximately equal to the motion of the support point in the actual composite primary-secondary system. Thus, the cross-cross floor spectra computed with mass oscillators implicitly include the effect of interaction between the primary and secondary systems.

3.2 Equivalent Masses for Fictitious Oscillators

The simple primary-secondary system shown in Fig. 3.1a will be used to first introduce the concept of equivalent masses for fictitious oscillators representing modes of the secondary system.

Since the effect of interaction is most important when tuning exists, consideration will be focused on the special case where the fundamental frequency of the secondary system is tuned to the frequency of the primary system. In the combined 3-degree-of-freedom system, the two tuned frequencies will shift away from one another and two closely spaced modes will result. The third frequency (corresponding to the second mode of the secondary system) will remain approximately constant. A similar effect can be achieved if an oscillator representing the tuned mode of the secondary system, i.e., having the same frequency, is attached to the primary system, provided the oscillator mass is properly adjusted (Fig. 3.1b). This procedure is numerically shown in the following paragraph.

For the system shown in Fig. 3a and described above, the following relation holds for the properties of the individual subsystems:

$$\omega_1^2 = \Omega^2 = 0.3820 \, \frac{k}{m} = \frac{K}{M}$$

where ω_1 is the fundamental frequency of the secondary system and Ω is the frequency of the primary system. If the combined system is analyzed, these two equal frequencies are shifted away from one another by a small amount which depends on the ratio of masses. For example, when $\frac{M}{m} = 100$, the two shifted frequencies of the combined system are:

$$\bar{\Omega}_1^2 = 0.3322 \frac{k}{m}$$

$$\bar{\Omega}_2^2 = 0.4372 \frac{k}{m}$$

The third frequency of the combined system is essentially equal to the second frequency of the secondary system. Now consider the system shown in Fig. 3.1b. It consists of the primary system and an attached oscillator having a frequency equal to the fundamental frequency of the secondary system. If a mass equal to the effective modal mass (Clough and Penzien [1975]) of the first mode of the secondary system, $m_{11} = 1.8944\, m$, is assigned to the oscillator, the two frequencies of this system are found to be:

$$\bar{\Omega}_1^2 = 0.3329 \frac{k}{m}$$

$$\bar{\Omega}_2^2 = 0.4383 \frac{k}{m}$$

As can be observed, these are very close to the actual frequencies of the combined primary-secondary system. It should be clear that with the above equivalent mass, the fictitious oscillator will modify the frequency content of the support motion around ω_1 such that it will be approximately equal to the support motion in the actual system. This concept will now be extended to multiply supported secondary systems such as that shown in Fig. 3.2.

In deriving properties of composite primary-secondary systems, Igusa and Der Kiureghian [1983] have shown that when mode i of a secondary system is perfectly tuned to mode I of a primary system, the resulting shift in the frequency Ω_I of the primary system is

$$\Delta\Omega_I \approx \frac{1}{2}\omega_i \left[\frac{m_i}{M_I} \frac{[\varphi_i^T \mathbf{k}_c \overline{\Phi}_I]^2}{[m_i \omega_i^2]^2} \right]^{1/2} \tag{3.1}$$

where m_i and M_I are the modal masses, $\mathbf{k}_c$ is the coupling stiffness matrix defined in Chapter 2, and $\overline{\Phi}_I$ is a vector containing the elements of the mode shapes of the primary system that are associated with the attachment points. On the other hand, for a single-degree-of-freedom oscillator attached at degree of freedom K of the primary system and having mass m_{iK}, the resulting shift is derived from Eq. (3.1)

$$\Delta\Omega_I \approx \frac{1}{2}\omega_i \left[\frac{m_{iK}}{M_I} \Phi_{KI}^2 \right]^{1/2} \tag{3.2}$$

Imposing the condition that the shift in frequencies given by Eqs. (3.1) and (3.2) be equal, the equivalent mass m_{iK} for the oscillator is obtained

$$m_{iK} = \frac{1}{m_i \omega_i^4} \frac{[\varphi_i^T \mathbf{k}_c \overline{\Phi}_I]^2}{\Phi_{KI}^2} \tag{3.3}$$

In the special case where there is a single attachment point, it can be shown that the above exprnssion is equivalent to the effective mass of mode i of the secondary system. This concept was utilized in the numerical example above.

From Eq. (3.3), it can be seen that the equivalent mass associated with support point K can be very large when the ordinate Φ_{KI} is small. In the limit, m_{iK} tends to infinity when Φ_{KI} tends to zero. This is logical, since when the oscillator is placed near a zero point of a mode shape, it then requires a very large mass in order to cause the proper shift in the frequency of that mode. However use of large masses, which may generate large effective mass ratios, will violate the assumptions made in arriving at Eqs. (3.1) and (3.2). Thus, a limit for the value of the equivalent masses has to be imposed. On the other hand, when the mode shape ordinate Φ_{KI} is small, the I-th modal response associated with support point K makes a small contribution to the total response and the effect of interaction is not important. Thus, in this case, any small mass can be used for the oscillator. For practical use, the following upper limit for the equivalent mass is proposed

$$m_{iK} \leq m_{tot} \tag{3.4}$$

where m_{tot} is the total mass of the secondary system.

3.3 Numerical Examples

The two systems shown in Fig. 3.3 are studied to examine the accuracy of the proposed method to account for the affect of interaction. This is done by comparing the frequency shifts in the actual primary-secondary system with that caused by the fictitious oscillators with equivalent masses. Tuning between primary and secondary systems is assumed in both examples.

3.3.1 Example System A

Consider the system in Fig. 3.3a, where the fundamental frequency of the secondary system is tuned to the fundamental frequency of the primary system. The dimensionless nodal masses of the secondary system are assumed to be $m = 3.203$ and its interstory and connecting dimensionless stiffnesses are assumed to be $k = 100$. The mass ratio between the two systems, $m/M = 0.03203$, is large enough to produce an important effect of interaction. The frequencies of the two systems considered individually and the frequencies of the combined primary-secondary system are given in Table 3.1. In this table, the first five frequencies correspond to the primary system and the remaining five to the secondary system. Table 3.2 shows the equivalent masses m_{iK} for the first mode of the secondary system and for each support point. These are computed using Eq. (3.3). Table 3.3 shows the frequencies of the $N+1$ degree of freedom systems defined by the primary system and oscillators representing the first mode of the secondary system. The three first columns of this table correspond to the oscillator located at support points 1,3 and 5, respectively. The fourth column corresponds to the oscillator located at support 5, but a mass $m_{15} = 16.015$ equal to the total mass of the secondary system is used. The first five frequencies in each column correspond to the primary system and the sixth to the shifted frequency of the oscillator (first mode of the secondary system).

Comparison of the first six frequencies in Tables 3.1 and 3.3, which correspond to the shifted frequencies of the primary system and mode 1 of the secondary system, reveals that the equivalent masses for the fictitious oscillators properly account for the frequency shifts. The biggest discrepancy between frequency shifts occurs when $m_{15} = m_{tot} = 16.015$ is used in place of the computed equivalent mass $m_{15} = 90.616$. As explained earlier, the effect of interaction in this case is not important.

3.3.2 Example System B

This example represents a case where a secondary system is attached between two primary systems. Properties of individual systems are given in Fig. 3.3b, where it is noted that the ratio of mass is $m / M = 0.02$. For simplicity, the foundation is modeled as a very stiff story. The frequencies of the individual systems are given in the first column of Table 3.4. Note that the fundamental frequency of the secondary system is tuned to the fundamental frequency of primary system B1. The second column in Table 3.4 shows computed frequencies of the combined system. A one-to-one correspondence between the frequencies of the combined system and the individual systems can be observed in this table. Table 3.5 shows computed equivalent mass values for the two attachment points and the first mode of the secondary system. Finally, Table 3.6 shows the frequencies of the $N+1$ degree of freedom systems defined by the primary system and the oscillators representing the first mode of the secondary system. In this table, column 1 corresponds to the oscillator attached to support point 1 and column 2 to the oscillator attached to support point 3 of the primary system. Column 3 also corresponds to the oscillator attached to support 3, however, the equivalent mass m_{13} is taken to be equal to the total mass of the secondary system, i.e., $m_{13} = m_{tot} = 6.0$. It is observed in Table 3.5 that the equivalent mass m_{13} is much larger than the total mass of the secondary system. This is because the component of the first primary mode for support 3 is essentially zero. Using this equivalent mass results in shifts in frequencies of the primary system which are not consistent with the actual shifts shown in Table 3.4. (For example, compare the third frequencies in the second columns of Tables 3.4 and 3.6.) On the other hand, the case where $m_{13} = m_{tot} = 6.0$ is used, results in reasonable shifts in the frequencies for all modes. Thus, the upper bound for equivalent masses defined in Eq. (3.4) should be utilized to avoid improper shifts in frequencies of the primary system.

It is worthwhile to analyze this example in more detail. From the first column of Table 3.4, it can be seen that the secondary system is tuned to the primary system B1 and not to primary system B2. Thus, in practice, the secondary system will dynamically interact only with system B1 and not with system B2. This is observed in the second column of Table 3.4, where it is shown that the frequencies of the combined system corresponding to system B2 have practically remained constant and the frequencies of system B1 have been modified due to the presence of the secondary system. From this, it is clear that the motion of system B2 will be essentially the same as the motion in absence of the secondary system. It can be concluded then, that for the evaluation of cross-cross floor spectra, one needs the correct equivalent mass (m'_{11}) attached to system B1 to produce the necessary modification in the motion of support 1, and any small mass attached to system B2, such that the motion on support point 2 is not modified. In that way, the actual motion on the support points is reproduced and the cross-cross floor spectra will account properly for the effect of interaction.

From the results shown in Tables 3.3 and 3.6, it is concluded that equivalent masses defined above can be used to approximately account for the effect of interaction between the primary and secondary systems. The accuracy of this method will be examined further in Chapter 5.

Table 3.1. Shift in Frequencies for Example A

Frequencies, rad/s		
Individual systems		Combined system
Primary	4.025	3.711
	11.750	11.795
	18.520	18.548
	23.790	23.804
	27.140	27.148
Secondary	4.025	4.359
	5.588	5.589
	8.494	8.489
	9.678	9.669
	11.410	11.405

Table 3.2. Equivalent Masses m_{iK} for Example A

Mode	Support 1	Support 3	Support 5
	m_{11}	m_{13}	m_{15}
1	7.348	12.594	90.616

Table 3.3. Frequencies of $N+1$ Systems for Example A

Frequencies, rad/s			
supp 1, m_{11}	supp 3, m_{13}	supp 5, m_{15}	supp 5, m_{tot}
3.709	3.709	3.668	3.882
11.787	11.760	11.890	11.776
18.529	18.540	18.668	18.548
23.797	23.796	23.892	23.811
27.139	27.152	27.169	27.144
4.359	4.358	4.309	4.155

Table 3.4. Shift in Frequencies for Example B

Frequencies, rad/s		
Individual systems		Combined system
Primary B1	8.740 22.883	9.047 22.778
Primary B2	10.705 28.025	10.589 27.803
Foundation	70.711	74.338
Secondary	8.603 15.897 20.770	8.116 15.936 20.770

Table 3.5. Equivalent Masses m_{iK} for Example B

Mode	Support 1	Support 3
	m_{11}	m_{13}
1	1.710	247.928

Table 3.6. Frequencies of $N+1$ Systems for Example B

Frequencies, rad/s		
supp 1, m_{11}	supp 3,m_{13}	supp 3, m_{tot}
9.088 22.760	8.618 22.755	8.617 22.751
10.495 27.789	16.364 29.048	10.884 27.812
74.338	74.338	74.338
8.141	5.268	8.274

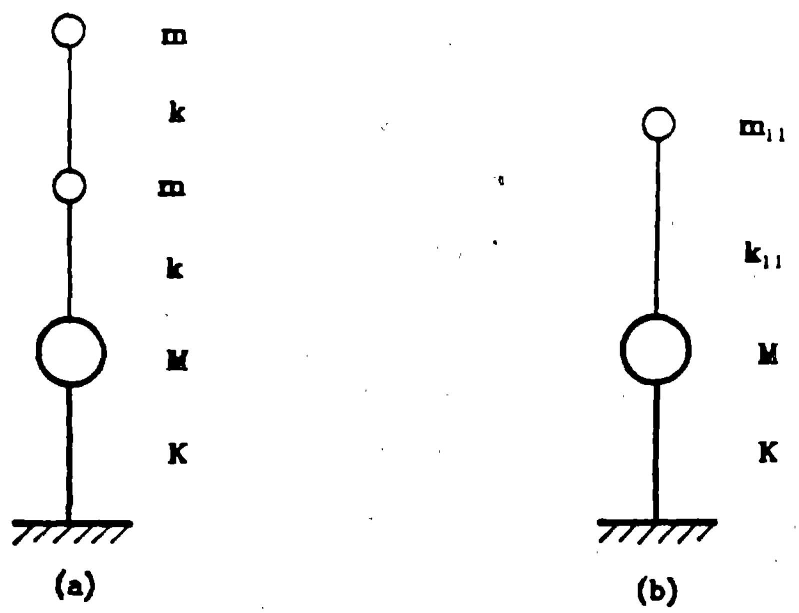


Figure 3.1 Schematic Illustration of Combined Systems

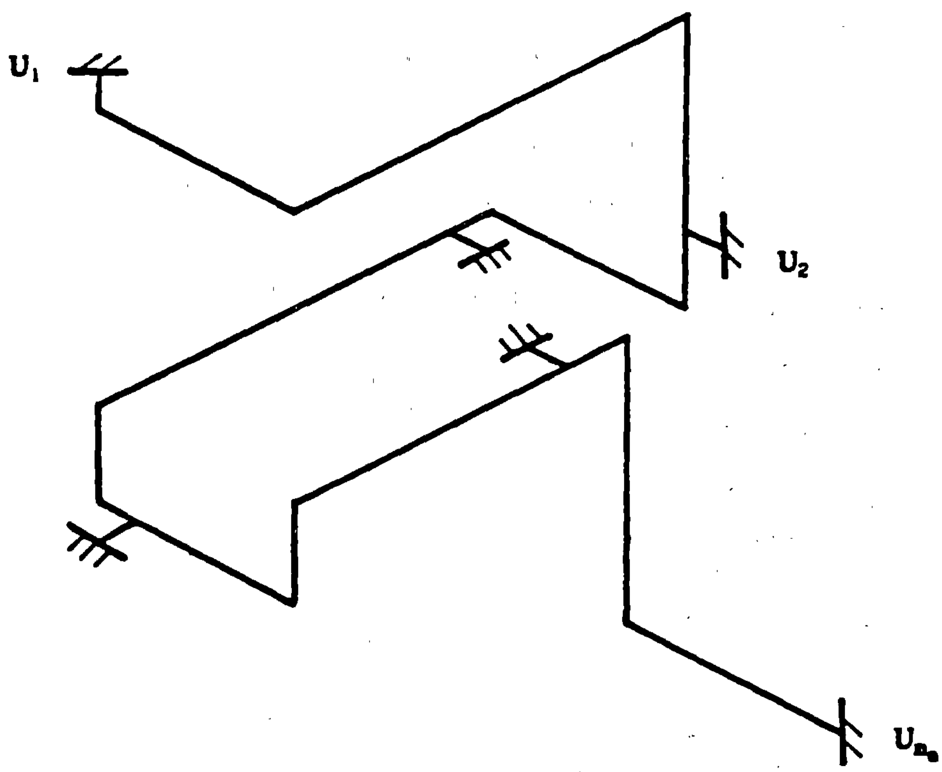
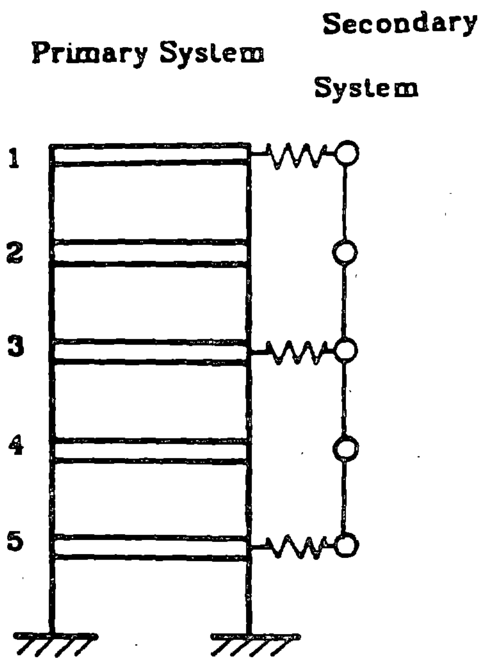


Figure 3.2 Schematic Illustration of Multiply Supported Structure

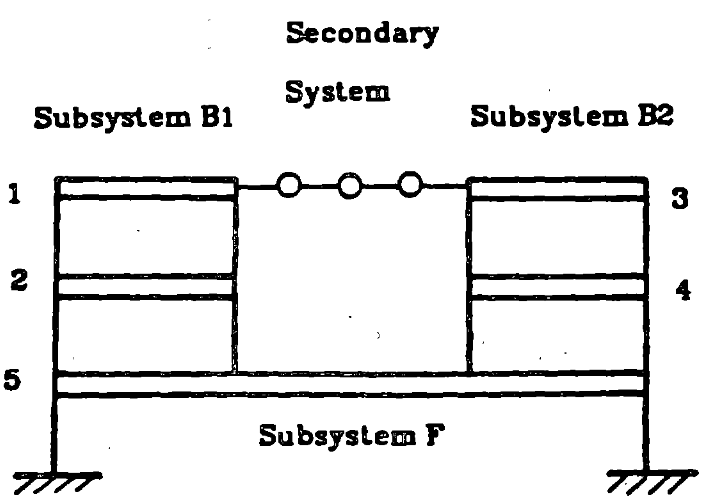


(a)

DIMENSIONLESS PROPERTIES

SECONDARY SYSTEM:
- Nodal Mass ; $m = 3.203$
- Internodal Stiffness ; $k = 100$
- Connection Stiffness ; $k = 100$

PRIMARY SYSTEM:
- Floor Mass ; $M = 100$
- Interstory Stiffness ; $K = 20,000$



(b)

DIMENSIONLESS PROPERTIES

SECONDARY SYSTEM:
- Nodal Mass ; $m = 2.0$
- Internodal Stiffness ; $k = 252.713$
- Connection Stiffness ; $k = 252.713$

PRIMARY SUBSYSTEM B1:
- Floor Mass ; $M = 100$
- Interstory Stiffness ; $K = 20,000$

PRIMARY SUBSYSTEM B2:
- Floor Mass ; $M = 100$
- Interstory Stiffness ; $K = 30,000$

PRIMARY SUBSYSTEM F:
- Floor Mass ; $M = 100$
- Interstory Stiffness ; $K = 500,000$

Figure 3.3 Schematic Illustration of Combined Systems

CHAPTER 4

Evaluation of Cross-Cross Floor Spectra

4.1 Introduction

In Chapter 2, a modal combination rule was developed giving the response of a secondary system in terms of cross-cross floor spectra (CCFS). For practical implementation of the method, it is essential that an efficient procedure for generating the cross-cross floor spectra is developed. This will be the main objective of this chapter.

The key for the generation of the CCFS's is the $N+2$ degree of freedom system shown in Fig. 2.5. This system is composed of the primary system to which are attached two fictitious oscillators at degrees of freedom ("floors") K and L. The cross-cross floor spectrum associated with floors K and L was defined to be proportional to the cross-correlation of the responses of the two oscillators as the composite system is subjected to the base input (see Section 2.5). The use of this system permits expressing the response of the two oscillators in terms of the known ground input excitation rather than in terms of the unknown motions of the two support points.

In Section 4.2, it is shown that the $N+2$ degree of freedom system can be replaced by two $N+1$ degree of freedom systems. In Sections 4.3 and 4.4, the modal properties of a typical $N+1$ DOF system are obtained by use of perturbation techniques. Using the modal properties of the two $N+1$ DOF systems, an expression for the cross-cross floor spectrum in terms of the ground design spectrum is derived in Section 4.5.

4.2 Approach for Evaluating the Cross-Cross Floor Spectra

The cross-cross floor spectrum associated with floors K and L was defined to be proportional to the cross-correlation of the responses of two oscillators attached to these floors. If interaction between primary and secondary systems is not considered, the motion of the support points will not be affected by the presence of the secondary system. Thus, the motion of these points can be considered to be the same as those generated in absence of the secondary system. It is obvious then, that in this case the response of each oscillator in Fig. 4.1a can be obtained from the corresponding $N+1$ degree of freedom system shown in Fig. 4.1b. Furthermore, the cross-correlation of their responses can be evaluated using these two $N+1$ DOF systems.

When interaction is considered, the real motions of the support points are different from those in absence of the secondary system. To introduce this effect in an approximate way, equivalent masses for oscillators attached to support points were defined in Chapter 3. These masses were obtained using $N+1$ degree of freedom systems such that proper shifts in the frequencies of the primary system were achieved. It should be clear that the $N+1$ system with equivalent masses can also be used in the manner shown in Fig. 4.1b to obtain cross-cross floor spectra which approximately include the effect of interaction. The first $N+1$ degree system is composed of the primary system and the oscillator representing mode i of the secondary system. The oscillator has mass m_{iK} and is attached to degree of freedom K of the primary system. For convenience, this system will be called the Ki system. The second $N+1$ degree system, the Lj system, is composed of the primary system and the oscillator representing mode j of the secondary system. Its mass is m_{jL} and it is attached to degree of freedom L of the primary system.

Der Kiureghian, Sackman and Nour-Omid [1981] used perturbation methods to derive the dynamic properties of an $N+1$ degree of freedom system composed of an N degree of freedom primary structure and a light appendage modeled as a single-degree-of-freedom oscillator. This approach will be used here to avoid the numerical solution of the eigenvalue problem for the many $N+1$ degrees of freedom systems that are needed, thus providing an efficient and practical method for generation of the CCFS's. Following this scheme, the evaluation of the cross-cross floor spectra involves two basic steps:

(a) Synthesis of the dynamic modal properties of the $N+1$ degrees of freedom systems using perturbation methods which exploit the relative lightness of the oscillators. This process involves the modal characteristics of the primary system and the properties of the two oscillators.

(b) Determination of the cross-cross floor spectra by combining the product of the modal responses of two $N+1$ degrees of freedom systems defined by the primary system and the oscillators with dynamic characteristics $(\omega_i, \zeta_i, m_{iK})$ and $(\omega_j, \zeta_j, m_{jL})$, respectively.

4.3 Modal Characteristics of an Oscillator-Structure System

In this section, for the sake of completeness of this study, the closed form expressions obtained by Der Kiureghian, Sackman and Nour-Omid [1981] for the modal characteristics of an $N+1$-degree of freedom combined oscillator-structure system are presented. These are functions of the modal properties of the N-degree of freedom primary structure and the dynamic properties of the light oscillator attached to it. The study considered cases of gross detuning between the oscillator and frequencies of the primary system and near or perfect tuning between the oscillator and one of the frequencies of the primary system. Well spaced modes in the primary system were assumed in both cases.

Their results will be used in the evaluation of the cross-cross floor spectra and they will be listed in this section without further explanation.

For notational purposes, the modal properties of the primary system alone are denoted by capital letters and the dynamic properties of the oscillator by lower case letters. The properties of the combined $N+1$-degree system are denoted by capital letters superposed by a capital letter indicating the degree of freedom of the primary system where the oscillator is attached. The $N+1-th$ degree of freedom of the combined system corresponds to the oscillator which is assumed to be attached to the $K-th$ degree of freedom of the primary system. Also, the first N modes of the combined system are assumed to correspond to the modified modes of the primary system and the $N+1$-st mode corresponds to the new mode generated by the oscillator. With this notation the mode shapes for the first N modes of the combined system are:

$$\Phi_I^K = \begin{Bmatrix} \Phi_{1I}^K \\ \cdot \\ \cdot \\ \Phi_{KI}^K \\ \cdot \\ \cdot \\ \Phi_{NI}^K \\ \Phi_{N+1,I}^K \end{Bmatrix} \qquad I = 1,\ldots,N \tag{4.1}$$

in which $\Phi_{N+1,I}^K = \alpha_{IJ}\,\Phi_{KI}^K$, where α_{IJ} is a modal amplification factor. As a first approximation, it is assumed that the portion of modal vectors corresponding to the degrees of freedom of the primary system retain their shapes; i.e. $\Phi_{JI}^K = \Phi_{JI}$ for $J = 1,\ldots,N$; $I = 1,\ldots,N$. The frequencies for these modes are given by

$$\left[\frac{\Omega_I^K}{\Omega_I}\right]^2 = \begin{cases} \dfrac{1 + \dfrac{\beta_{iI} + \gamma_{iIK}}{2} - \left[\left(1 + \dfrac{\beta_{iI} + \gamma_{iIK}}{2}\right)^2 - (1 + \beta_{iI})\right]^{1/2}}{1 + \beta iI} & \beta_{iI} < 0 \\ \dfrac{1 + \dfrac{\beta_{iI} + \gamma_{iIK}}{2} + \left[\left(1 + \dfrac{\beta_{iI} + \gamma_{iIK}}{2}\right)^2 - (1 + \beta_{iI})\right]^{1/2}}{1 + \beta iI} & \beta_{iI} \geq 0 \end{cases} \quad (4.2)$$

and the modal amplification factors, for $I = 1,...,N$, are given by

$$\alpha_{iI} = \begin{cases} -\left[\dfrac{\beta_{iI} + \gamma_{iIK}}{2} - \left[\left(1 + \dfrac{\beta_{iI} + \gamma_{iIK}}{2}\right)^2 - (1 + \beta_{iI})\right]^{1/2}\right]^{-1} & \beta_{iI} < 0 \\ -\left[\dfrac{\beta_{iI} + \gamma_{iIK}}{2} + \left[\left(1 + \dfrac{\beta_{iI} + \gamma_{iIK}}{2}\right)^2 - (1 + \beta_{iI})\right]^{1/2}\right]^{-1} & \beta_{iI} \geq 0 \end{cases} \quad (4.3)$$

where β_{iI} and γ_{iIK} are the detuning parameter and the effective mass ratio for mode I, respectively, defined by

$$\beta_{iI} = \frac{\Omega_I^2 - \omega_i^2}{\omega_i^2} \qquad \gamma_{iIK} = \frac{m_{iK}}{M_I}\Phi_{KI}^2 \quad (4.4)$$

where m_{iK} is the mass and ω_i is the frequency of the oscillator. M_I is the modal mass corresponding to mode I of the primary system.

The mode shape of the $N+1$-st mode is given by

$$\Phi_{N+1}^K = \begin{Bmatrix} \Phi_{1,N+1}^K \\ \cdot \\ \cdot \\ \Phi_{K,N+1}^K \\ \cdot \\ \cdot \\ \Phi_{N,N+1}^K \\ 1 \end{Bmatrix} = \begin{Bmatrix} \Phi_0^K \\ 1 \end{Bmatrix} \quad (4.5)$$

where

$$\Phi_0^K = -\begin{Bmatrix} \sum_{J=1}^{N} \alpha_{iJ}\gamma_{iJK} \frac{\Phi_{1J}}{\Phi_{KJ}} \\ \vdots \\ \sum_{J=1}^{N} \alpha_{iJ}\gamma_{iJK} \frac{\Phi_{NJ}}{\Phi_{KJ}} \end{Bmatrix} \tag{4.6}$$

The frequency of this mode is given by

$$\Omega_{N+1}^K = \left[1 + \sum_{J=1}^{N} \alpha_{iJ}\gamma_{iJK}\right]^{1/2} \omega_i \tag{4.7}$$

When the frequency of the oscillator is near or perfectly tuned to the frequency Ω_T of the primary system but well spaced from the other modes, it is necessary to improve the results for the $T-th$ mode shape of the combined system. The refined mode shape for this mode is given by

$$\Phi_T^K = \begin{Bmatrix} \sum_{J \neq T}^{N} \alpha_{iJ}\gamma_{iJK} \frac{\Phi_{1J}}{\Phi_{KJ}} - \frac{1}{\alpha_{iT}} \frac{\Phi_{1T}}{\Phi_{KT}} \\ \vdots \\ \sum_{J \neq T}^{N} \alpha_{iJ}\gamma_{iJK} \frac{\Phi_{NJ}}{\Phi_{KJ}} - \frac{1}{\alpha_{iT}} \frac{\Phi_{NT}}{\Phi_{KT}} \\ -1 \end{Bmatrix} \tag{4.8}$$

A theoretically sound criterion to define when the frequency of the oscillator can be considered detuned from a frequency of the primary system can be found in Igusa and Der Kiureghian [1983].

In the evaluation of the damping ratios for the combined system, when the primary system is proportionally damped, Der Kiureghian, Sackman and Nour-Omid [1981] assumed that, for light damping, the combined system also very nearly has proportional damping. With this assumption, the following expressions for damping ratios were derived

$$Z_I^K = \begin{cases} \dfrac{\sqrt{1+\beta_{iJ}}\, Z_I + (1-\alpha_{iJ})^2 \gamma_{iJK} \zeta_i}{\sqrt{1+\beta_{iJ}}\,(1+\alpha_{iJ}^2 \gamma_{iJK})} \dfrac{\omega_i}{\Omega_I^K} & I = 1,\ldots,N \\[2ex] \dfrac{\sum_{J=1}^{N} \sqrt{1+\beta_{iJ}}\, \alpha_{iJ}^2 \gamma_{iJK} Z_J + (1+\sum_{J=1}^{N} \alpha_{iJ}\gamma_{iJK})^2 \zeta_i}{1+\sum_{J=1}^{N} \alpha_{iJ}^2 \gamma_{iJK}} \dfrac{\omega_i}{\Omega_I^K} & I = N+1 \end{cases} \tag{4.9}$$

where Z_I is the damping ratio of the primary system and ζ_i is the damping ratio of the oscillator. For perfect tuning between modes i and T, the damping ratios for the combined system become

$$Z_I^K \approx \begin{cases} Z_I & I \neq T, N+1 \\ \dfrac{Z_T + \zeta_i}{2} & I = T, N+1 \end{cases} \tag{4.10}$$

In the expressions in Eq. (4.10), small effective mass ratios γ_{iJK} were assumed.

As it will be seen in the next section, the validity of the above damping ratios, for the case of near or perfect tuning and very small effective mass ratio, is questionable. In the next section it will be shown that, for very small effective mass ratios, the use of the damping ratios in Eqs. (4.9) and (4.10) produces unacceptable results for the responses evaluated in the vicinity of tuning.

4.4 Modal Damping Ratios

A system is said to be classically or proportionally damped, if its equations of motion can be uncoupled using the modal shapes resulting from the undamped eigen-problem. For practical reasons it is desirable to treat structures as classically damped. Unfortunately, for primary-secondary systems, it is known that even if both systems are individually classically damped, the combined system will not be necessarily classically damped. When the mass of the secondary system is small with respect to the mass of the primary system, the effect of non-classical damping becomes crucial in the response of the secondary system for cases of near or perfect tuning. This effect is also present in the $N+1$ degree systems used in evaluating the cross-cross floor spectra. Thus, it is necessary to consider this effect to properly evaluate the spectra in the cases of near of perfect tuning between the frequency of the oscillator and one or more frequencies of the primary system. An extensive discussion of this effect can be found in Igusa and Der Kiureghian [1983].

Figures 4.3 to 4.6 show the effect of considering classical damping in the evaluation of a typical cross-cross floor spectrum for the structure shown in Fig. 4.2. The dotted line in these figures corresponds to the exact CCFS obtained through random vibration theory. The solid line corresponds to the solution obtained making use of the classical damping assumption. A very small mass ratio ($m/M = 10^{-5}$) was assumed to dramatize this effect. As it may be seen from the figures, for very small mass ratios, the assumption of classical damping is unacceptable in the vicinity of tuning. The error due to the assumption of classical damping decreases rapidly for larger effective mass ratios. This is shown in Fig. 4.7, where a mass ratio $m/M = 10^{-3}$ has been assumed. In this figure, because the mass ratio is still small, the exact solution was obtained as before, without considering interaction.

The mathematically exact method to analyze non-classical damping is solving the eigenproblem in the complex domain (Hurty and Rubinstein [1964]). To avoid dealing with complex modes, in this section an approximate procedure is presented to consider the effect of non-classical damping in the $N+1$ degree of freedom systems. To make the procedure clear, three frequency regions are considered for the oscillator:

Complete detuning

In this case, the frequency of the oscillator is far from all frequencies of the primary system. From Figs. 4.3 to 4.6, it may be seen that the assumption of proportionality in the damping matrix gives very good results in this region. Thus, the expressions in Eq. (4.9) can be used. When the effective mass ratios γ_{IJK} are small, the damping ratios can be approximated by

$$Z_I^K = \begin{cases} Z_I & I = 1,\dots,N \\ \zeta_i & I = N+1 \end{cases} \tag{4.11}$$

Perfect tuning

When ω_i is identical to the $T-th$ frequency of the primary system, Ω_T, the use of proportional damping tends to underestimate the true value of the response (Igusa and Der Kiureghian [1983]). Based on comparisons between exact solutions and solutions considering proportional damping, Der Kiureghian, Sackman and Nour-Omid [1981] have shown that the use of the following damping ratios will produce better results:

$$Z_I^K = \begin{cases} Z_I & I \neq T,N+1 \\ \sqrt{Z_T \zeta_i} & I = T,N+1 \end{cases} \tag{4.12}$$

The use of these values for the damping ratios will tend to slightly overestimate the response.

Near tuning

The behavior of Eq. (4.10) in the region of near tuning, i.e., small but nonzero β_{iT}, is shown in Fig. 4.8. It is seen that the assumption of classical damping results in damping ratios for modes T and $N+1$ which are almost constant up to a frequency very close to the perfect tuning, and then abruptly change to the values given by Eq. (4.10).

This abrupt change is not realistic and gives rise to erroneous estimates of the response in the vicinity of tuning (see Figs. 4.3 to 4.6). To improve the results, the following method is proposed. Once the solution is known for the two extremes, i.e., complete detuning and perfect tuning, the solution for the intermediate cases is determinated through a matching process. In the present case, a smooth variation in the values of the modal damping ratios between their extreme values is assumed. This is schematically shown in Fig. 4.9. In this figure the dotted line represents the variation of the damping ratios assuming classical damping and the solid line represents the assumed smooth variation.

The parameter β_0 in Fig. 4.9 is based on a detuning criterion defined by Igusa and Der Kiureghian [1983], and it is given by the relation:

$$\frac{4\omega_i^4}{(\Omega_T + \omega_i)^4}\beta_0^2 < \frac{1}{e}\left[\frac{Z_T + \zeta_i}{2}\right]^2\left[4 + \frac{\gamma_{iTK}}{Z_T\zeta_i}\right] \tag{4.13}$$

where e is the relative error tolerance for evaluation of the mean square response to white noise input excitation.

Figures 4.10 to 4.13 show the improved results for the acceleration cross-cross floor spectra shown in Figs. 4.3 to 4.6. The solid lines represent the approximate solution and the dotted lines represent the exact solution. To produce a smooth variation of the damping ratios in the near tuning region, the following interpolation expressions were used:

$$Z_I^K = \begin{cases} \dfrac{Z_g + Z_I}{2} + \dfrac{Z_g - Z_I}{2}\cos(\pi\,|\dfrac{\beta_{iI}}{\beta_0}|) & I = T \\ \dfrac{Z_g + \zeta_i}{2} + \dfrac{Z_g - \zeta_i}{2}\cos(\pi\,|\dfrac{\beta_{iT}}{\beta_0}|) & I = N+1 \end{cases} \tag{4.14}$$

From the figures, it may be seen that the proposed method to account for the effect of non-classical damping gives good results for reasonable differences in damping between secondary and primary systems. The detuning parameter β_c was evaluated assuming $e = 0.5$.

4.5 Modal Combination Rule for Evaluating Cross-Cross Floor Spectra

Once the modal properties of the combined $N+1$ degree of freedom systems are known, the evaluation of the cross-cross floor spectra is straightforward. Using stationary random vibrations techniques, the cross power spectral density function of typical responses (i.e., accelerations or displacements) of the two oscillators is given by

$$G_{ijKL}(\omega) = \sum_{I=1}^{N+1} \sum_{J=1}^{N+1} \Psi_{N+1,I}^{Ki} \Psi_{N+1,J}^{Lj} H_I^{Ki}(\omega)\, H_J^{Lj}(-\omega)\, G_{\ddot{u}_g \ddot{u}_g}(\omega) \tag{4.15}$$

where $\Psi_{N+1,I}^{Ki}$ and $H_I^{Ki}(\omega)$ are, respectively, the effective participation factor and the complex frequency response function associated with mode I of the $N+1$ DOF Ki system. $G_{\ddot{u}_g \ddot{u}_g}(\omega)$ is the power spectral density function of the input ground acceleration. Integrating $G_{ijKL}(\omega)$ over the frequency range and following the procedure used by Der Kiureghian [1981], the cross-correlation $\lambda_{0,ijKL}$ of the responses of the two oscillators is given by

$$\lambda_{0,ijKL} = \sum_{I=1}^{N+1} \sum_{J=1}^{N+1} \Psi_{N+1,I}^{Ki} \Psi_{N+1,J}^{Lj} \rho_{0,IJ}^{KiLj} \frac{1}{p_I^{Ki} p_J^{Lj}} \bar{S}(\Omega_I^{Ki}, Z_I^{Ki}) \bar{S}(\Omega_J^{Lj}, Z_J^{Lj}) \tag{4.16}$$

where p_I^{Ki} and $\bar{S}(\Omega_I^{Ki}, Z_I^{Ki})$ are the peak factor and the ground response spectrum ordinate associated with mode I of the $N+1$ degree of freedom Ki system. The term $\rho_{0,IJ}^{KiLj}$ is the cross- modal correlation coefficient for the two $N+1$ DOF systems, Ki and Lj, and is given in terms of the modal frequencies and damping ratios of these systems by the expressions previously derived by Der Kiureghian [1980]. This coefficient, in case of white noise input, is given by:

$$\rho_{0,IJ}^{KL} = \frac{2\sqrt{Z_I^K Z_J^L}\left[(\Omega_I^K+\Omega_J^L)^2(Z_I^K+Z_J^L)+([\Omega_I^K]^2-[\Omega_J^L]^2)(Z_I^K-Z_J^L)\right]}{4(\Omega_I^K-\Omega_J^L)^2+(Z_I^K+Z_J^L)^2(\Omega_I^K+\Omega_J^L)^2} \tag{4.17}$$

Using Eq. (4.16) together with Eq. (2.24), and approximating the ratios $\frac{p_{iK}p_{jL}}{p_I^K p_J^L}$ by unity, the cross-cross floor spectrum ordinate is given by

$$\bar{S}_{KL}(\omega_i,\zeta_i;\omega_j,\zeta_j) = \sum_{I=1}^{N+1}\sum_{J=1}^{N+1} \Psi_{N+1,I}^K \Psi_{N+1,J}^L \rho_{0,IJ}^{KL}\, \bar{S}(\Omega_I^K,Z_I^K)\bar{S}(\Omega_J^L,Z_J^L) \tag{4.18}$$

Although the effect of the peak factors is seldom significant in the evaluation of $\bar{S}_{KL}$, for completeness the necessary expressions to consider them are given in Appendix B.

From Eq. (4.18), the cross-cross floor spectrum of pseudo-acceleration and the cross-cross floor spectrum of relative displacement can be written as

$$\bar{S}_{KL}^a(\omega_i,\zeta_i;\omega_j,\zeta_j) = \sum_{I=1}^{N+1}\sum_{J=1}^{N+1} \Phi_{N+1,I}^K\Gamma_I^K\,\Phi_{N+1,J}^L\Gamma_J^L\,\rho_{0,IJ}^{KL}\,\bar{S}_a(\Omega_I^K,Z_I^K)\,\bar{S}_a(\Omega_J^L,Z_J^L) \tag{4.19}$$

$$\bar{S}_{KL}^v(\omega_i,\zeta_i;\omega_j,\zeta_j) = \sum_{I=1}^{N+1}\sum_{J=1}^{N+1} \frac{\Phi_{N+1,I}^K\Gamma_I^K}{(\Omega_I^K)^2}\,\frac{\Phi_{N+1,J}^L\Gamma_J^L}{(\Omega_J^L)^2}\,\rho_{0,IJ}^{KL}\,\bar{S}_a(\Omega_I^K,Z_I^K)\,\bar{S}_a(\Omega_J^L,Z_J^L) \tag{4.20}$$

where

$$\Gamma_I^K = -\frac{(\Phi_I^K)^T\,\mathbf{M}^K\,\mathbf{R}^K}{M_I^K} \tag{4.21}$$

is the modal participation factor, and

$$M_I^K = (\Phi_I^K)^T\,\mathbf{M}^K\,\Phi_I^K \tag{4.22}$$

is the modal mass associated with mode I of the combined system. In the above expressions, $\mathbf{M}^K$ is the mass matrix and $\mathbf{R}^K$ is the influence vector associated with the $N+1$ degree of freedom Ki system. $\bar{S}_a(\Omega_I^K,Z_I^K)$ is the corresponding ground acceleration response spectrum ordinate.

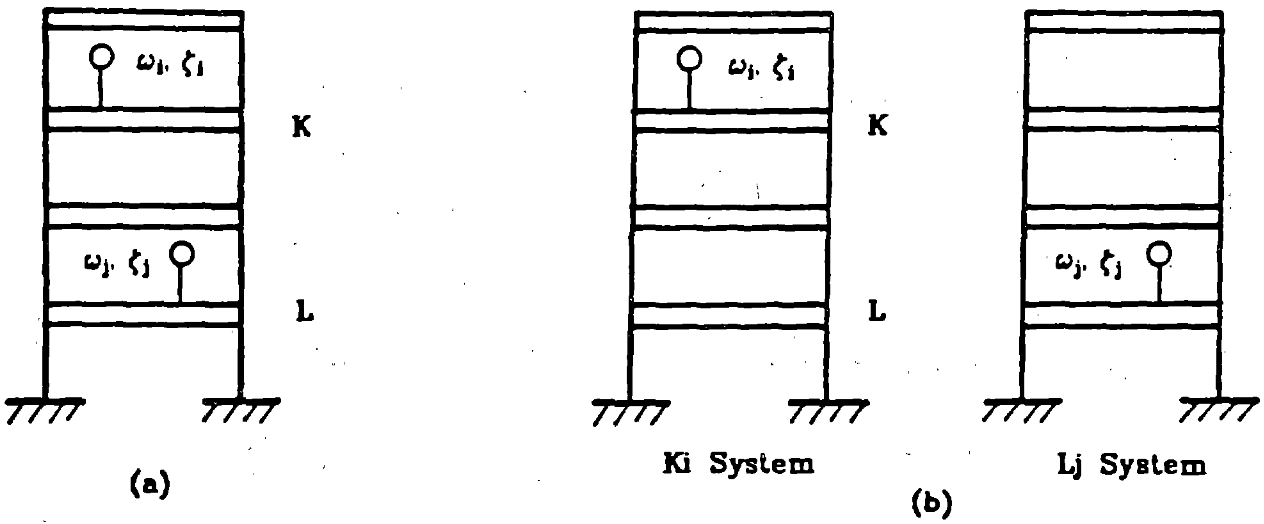
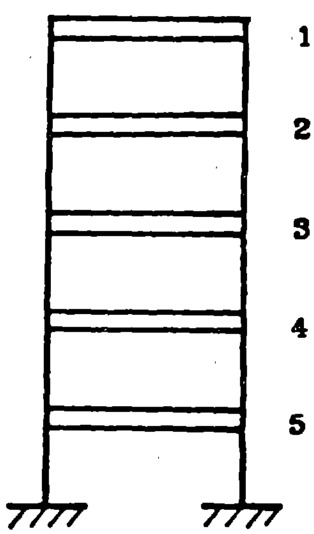


Figure 4.1 Schematic Illustration of Oscillator-Structure Systems



PROPERTIES:
Floor Mass = M
Interstory Stiffness = K
$K/M = 200 \quad s^{-2}$
$m_{osc}/M = 10^{-5}$

GROUND ACCELERATION SPECTRUM:
$$S_a(\omega,\zeta) = g\left(\frac{\pi\omega}{2000\zeta}\right)^{1/2}$$

Figure 4.2 Structure used for Figures 4.3 to 4.6 and 4.10 to 4.13

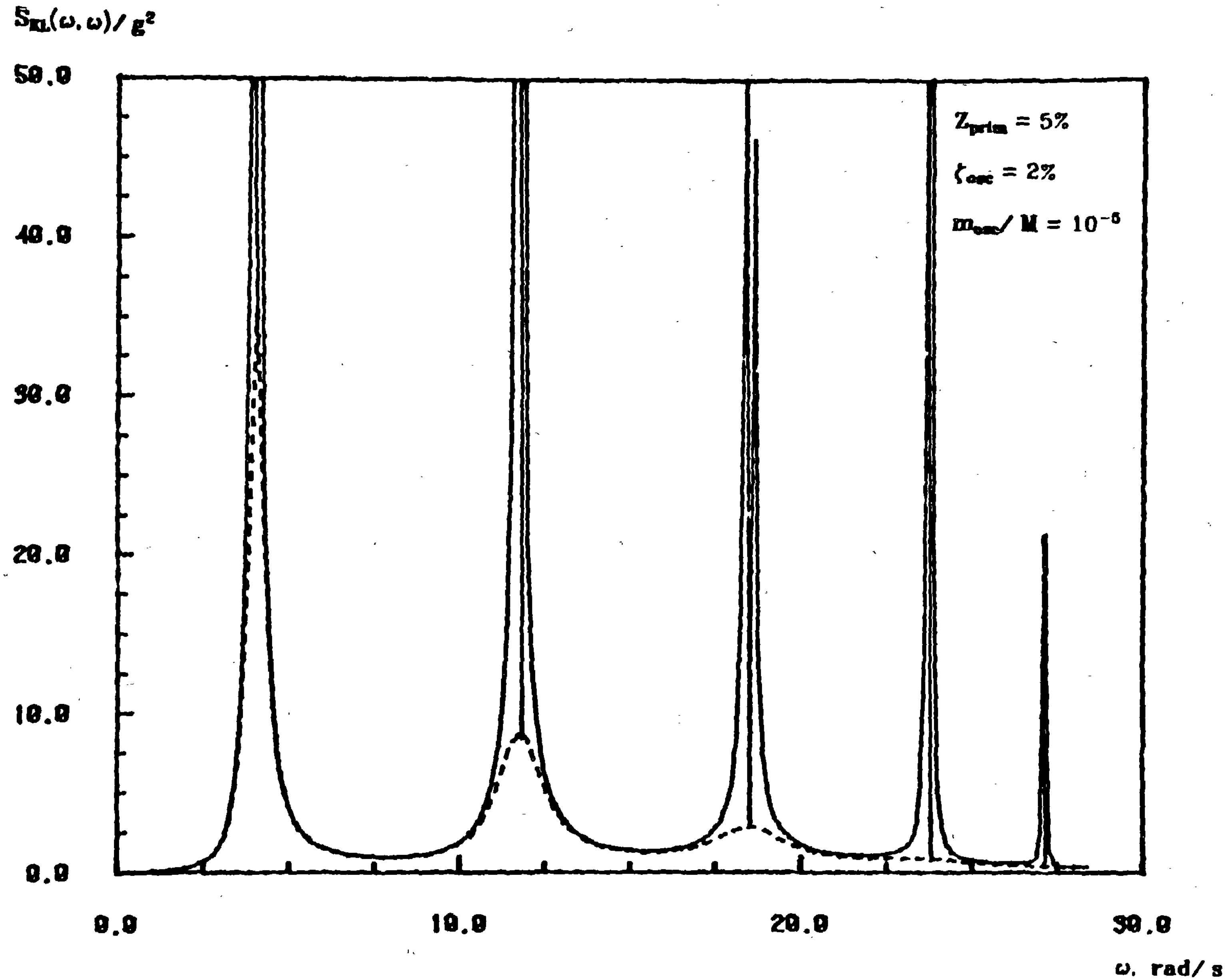


Figure 4.3 $\bar{S}_{11}$, Conventional Floor Spectrum

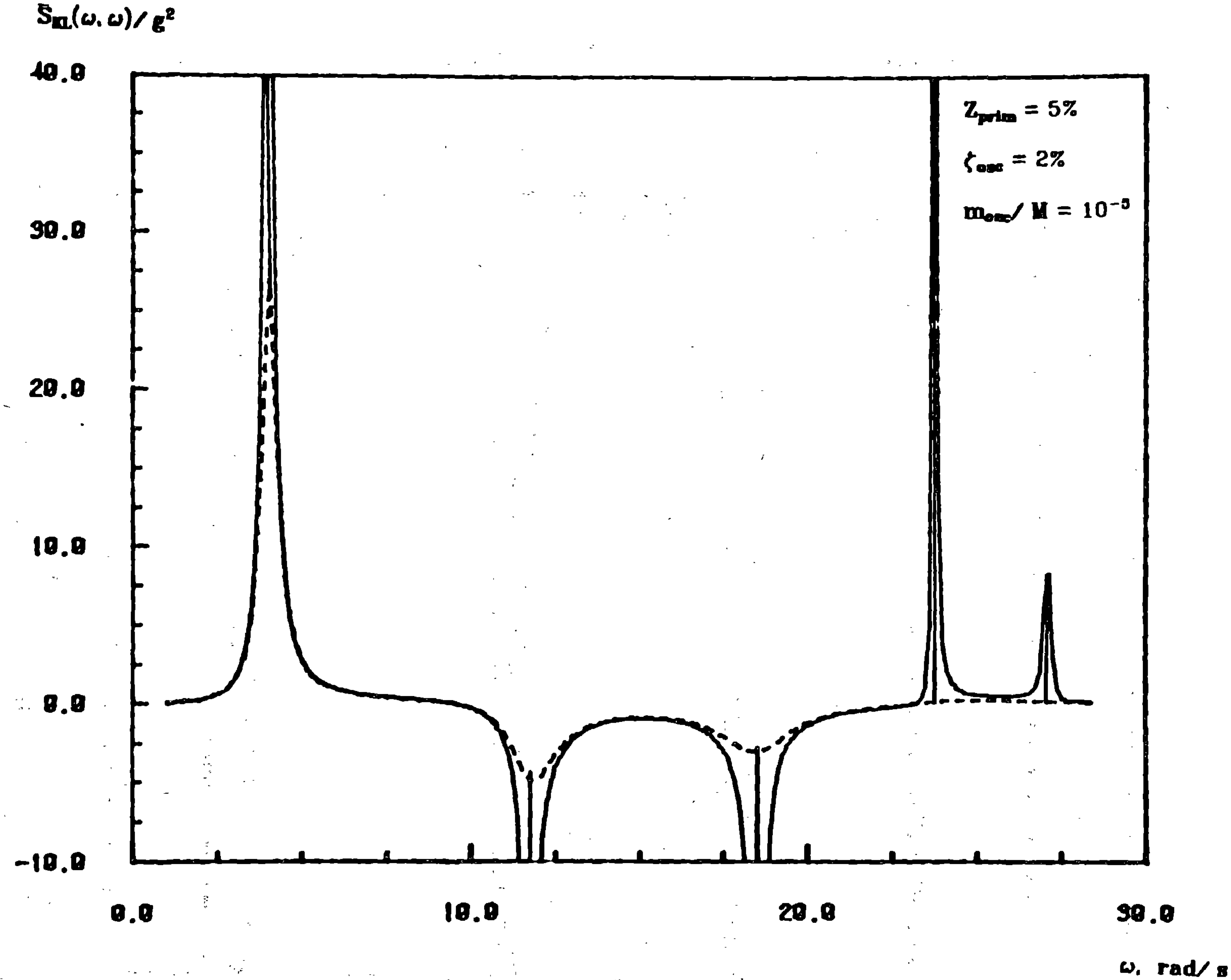


Figure 4.4 $\bar{S}_{13}$, Cross-Cross Floor Spectrum

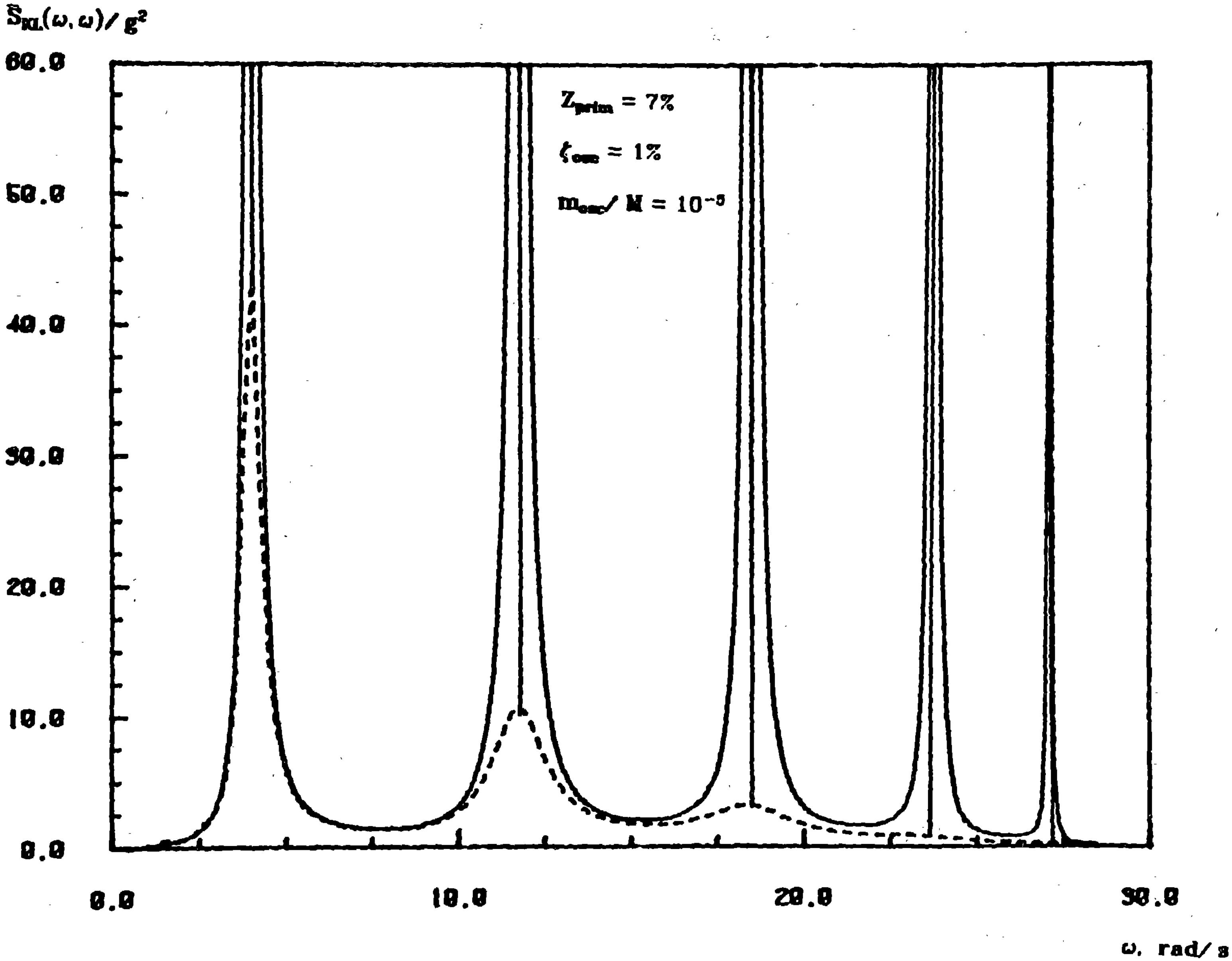


Figure 4.5 $\tilde{S}_{11}$, Conventional Floor Spectrum

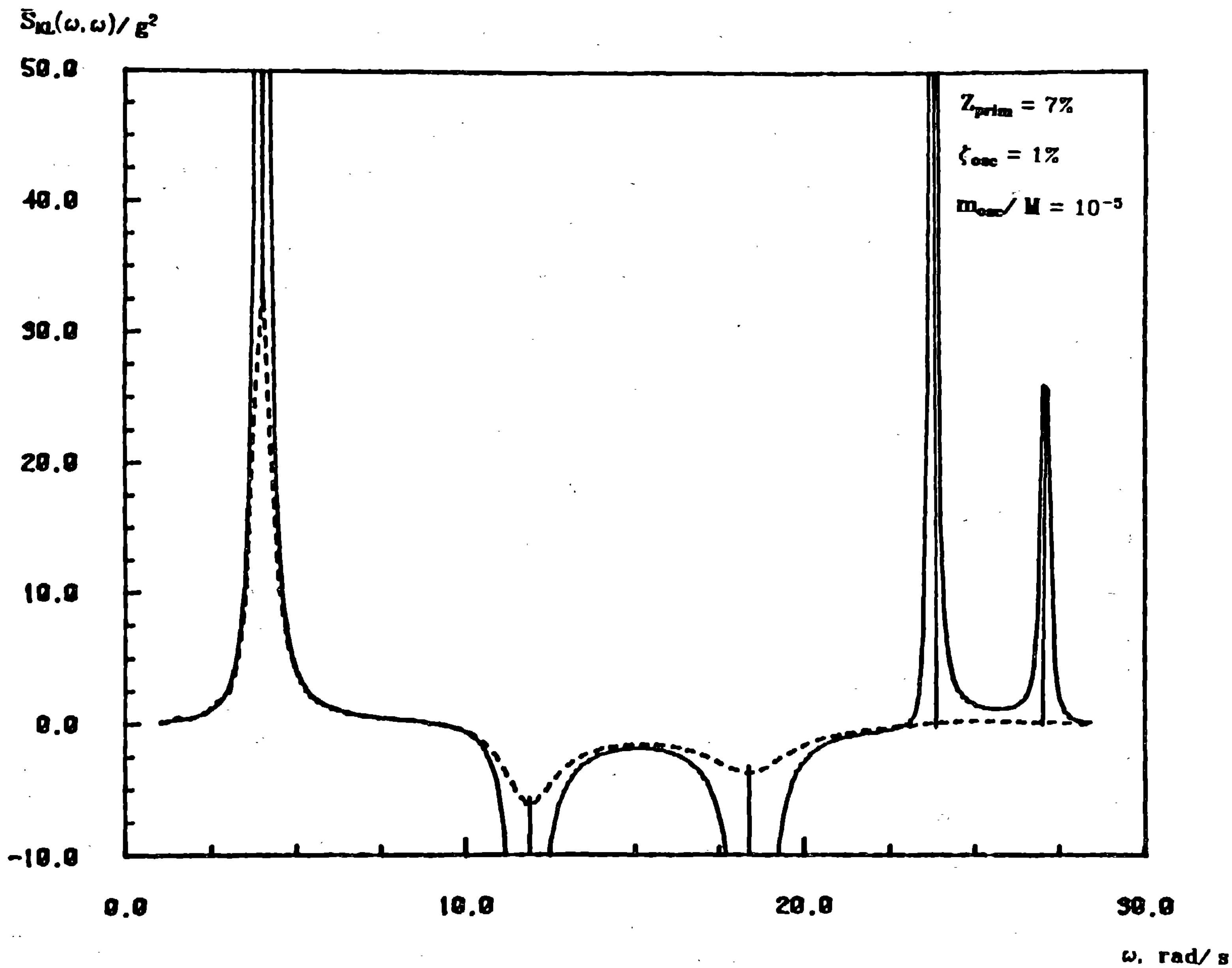


Figure 4.6 $\bar{S}_{13}$. Cross-Cross Floor Spectrum

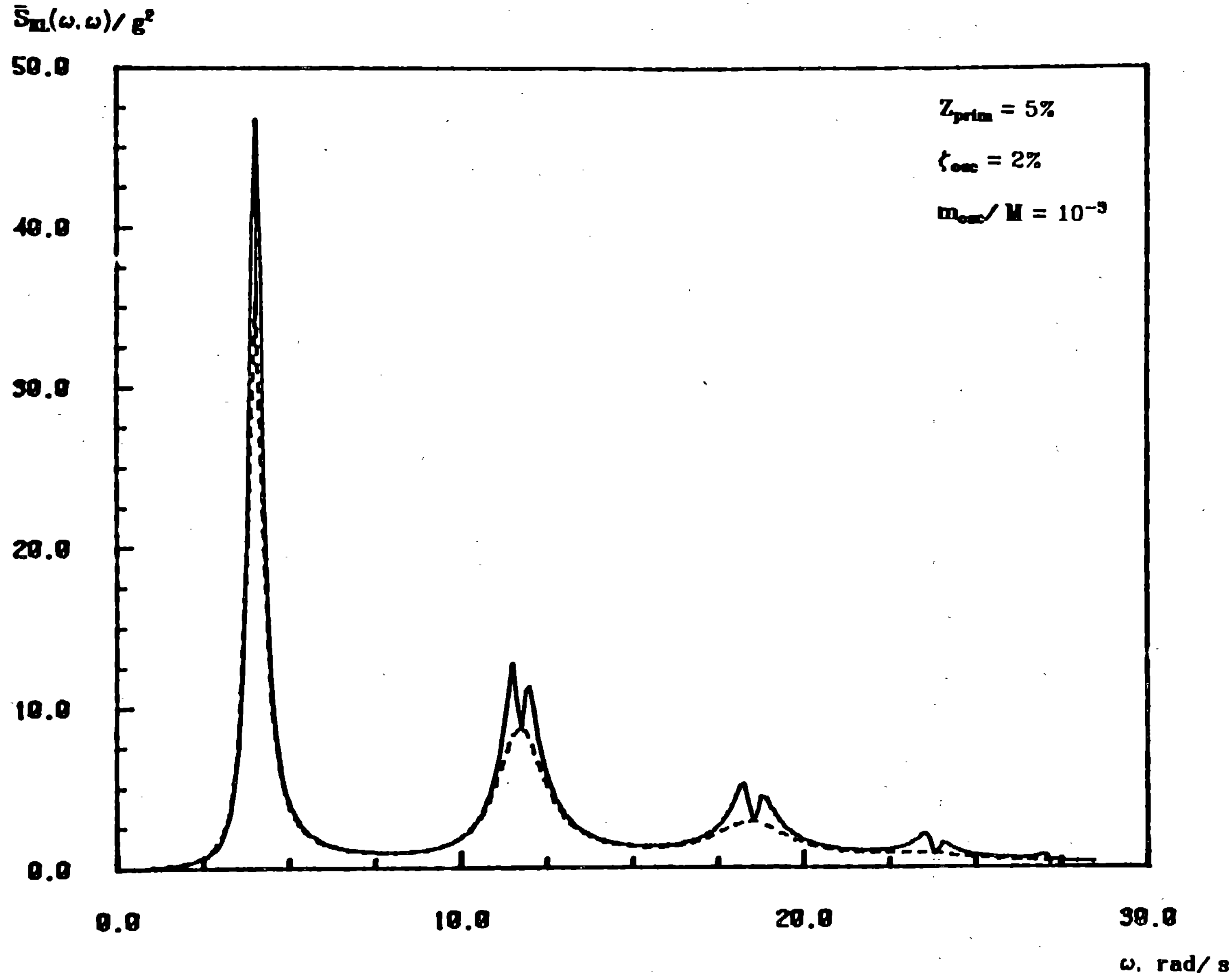


Figure 4.7 $\bar{S}_{11}$. **Conventional Floor Spectrum**

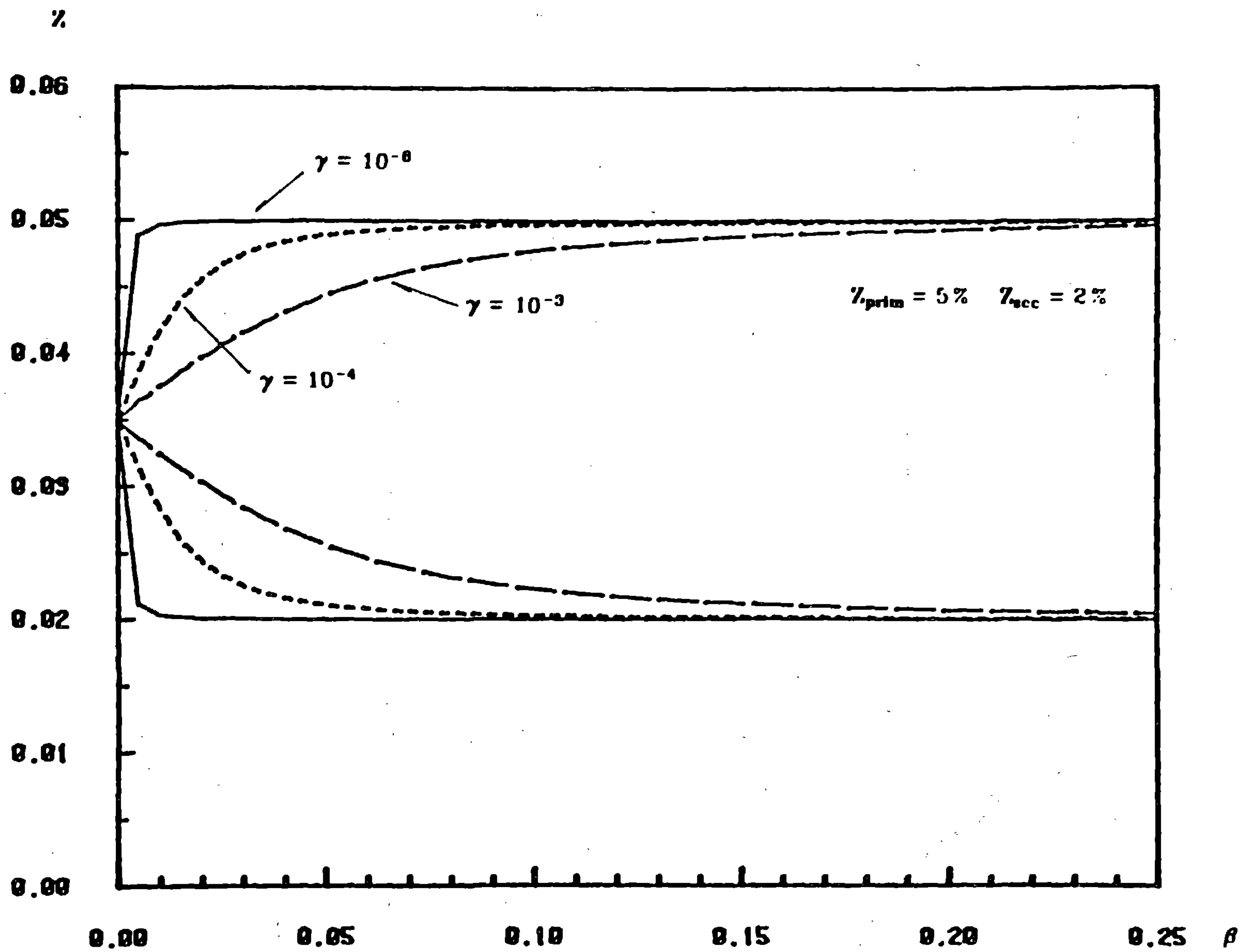


Figure 4.8 Schematic Variation of Modal Damping Ratios

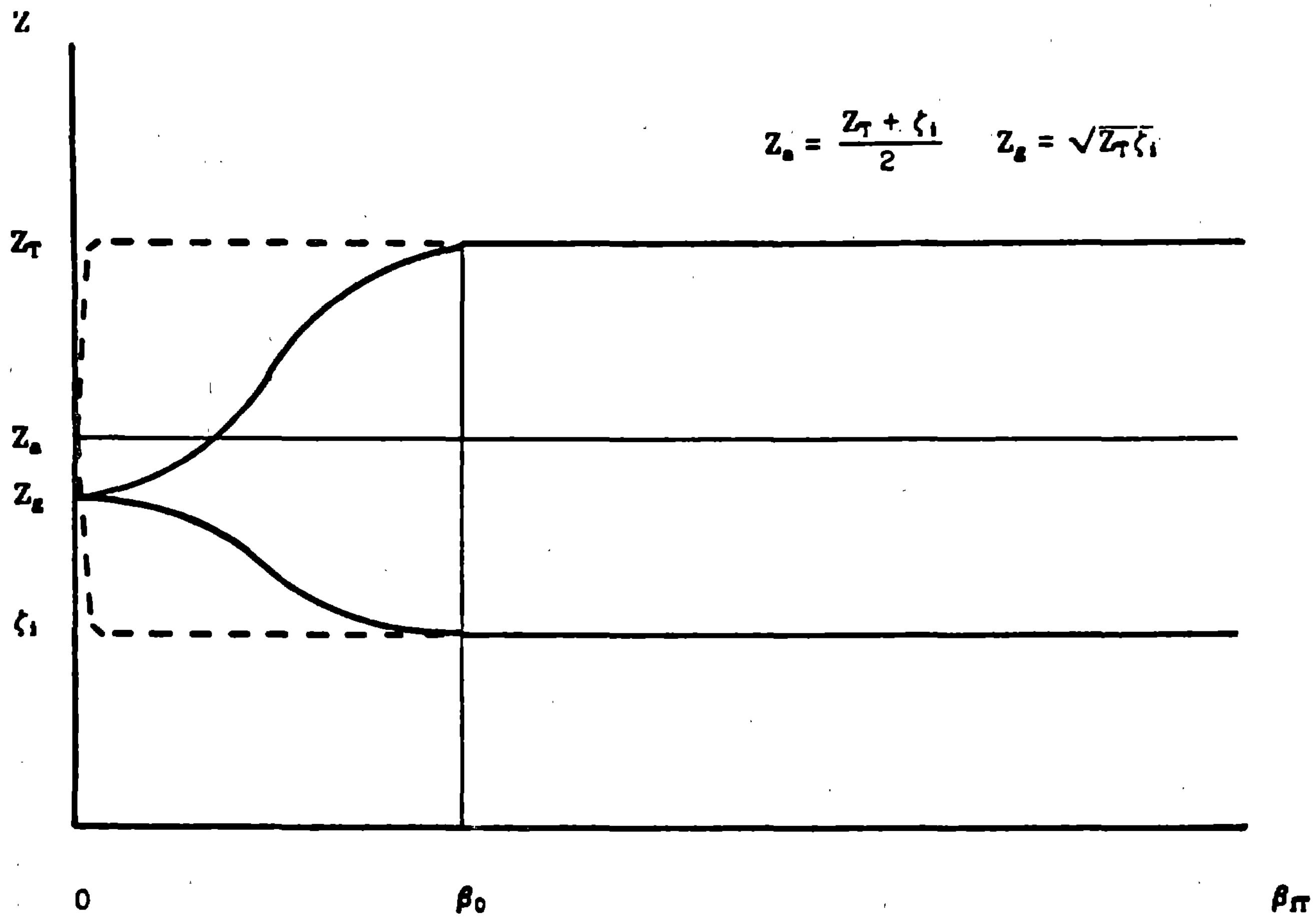


Figure 4.9 Smooth Variation of Modal Damping Ratios

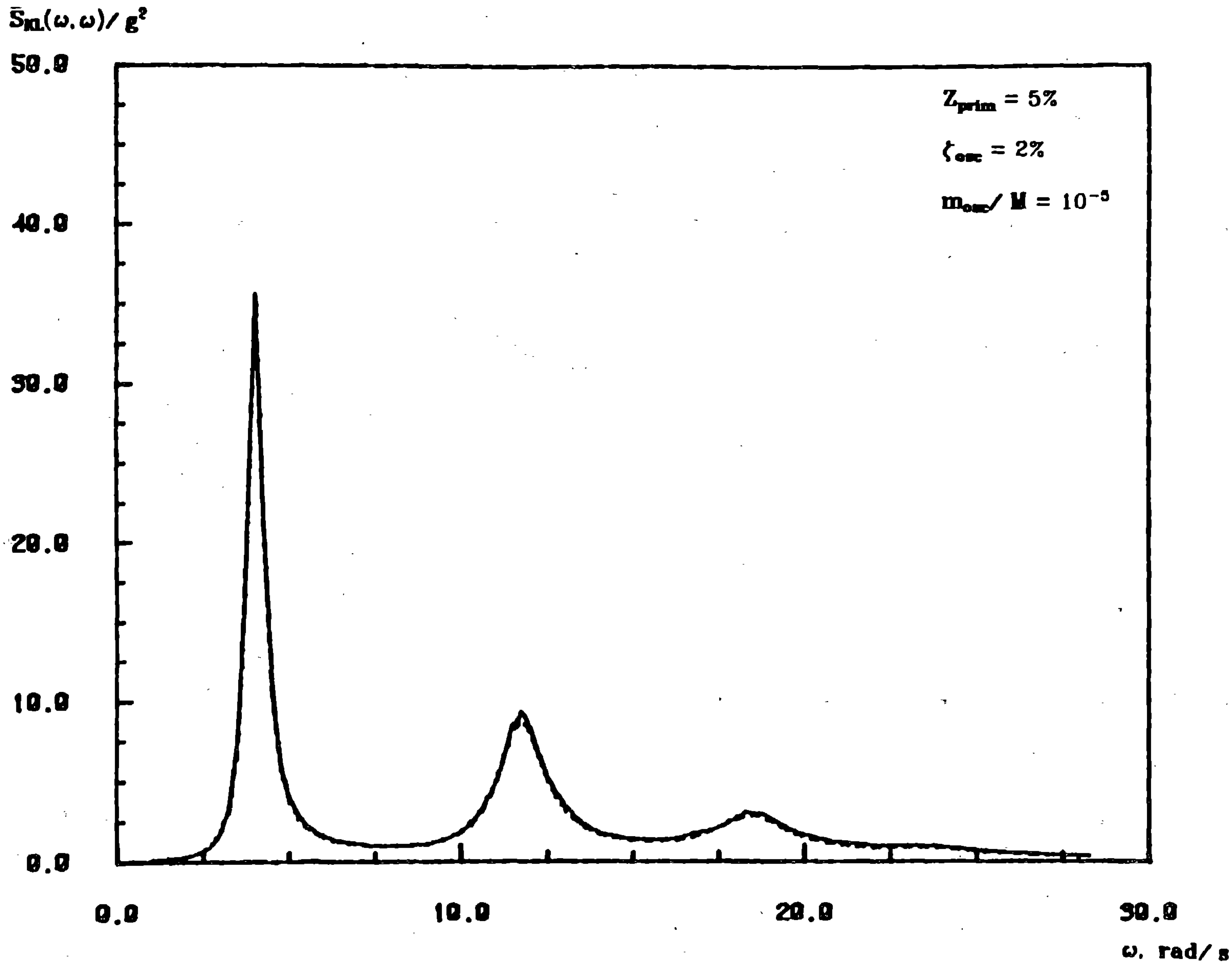


Figure 4.10 $\bar{S}_{11}$. Conventional Floor Spectrum

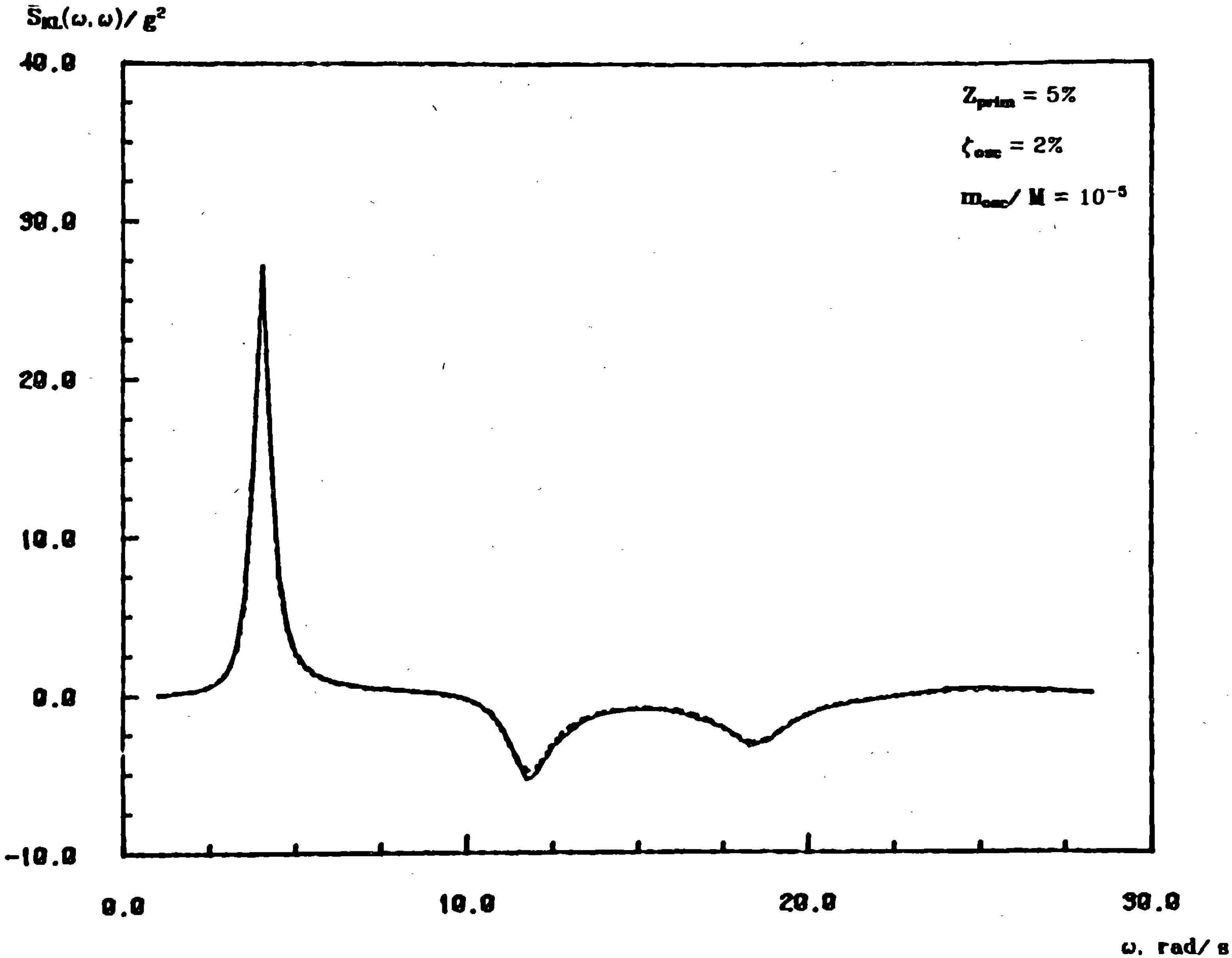


Figure 4.11 $\bar{S}_{13}$, Cross-Cross Floor Spectrum

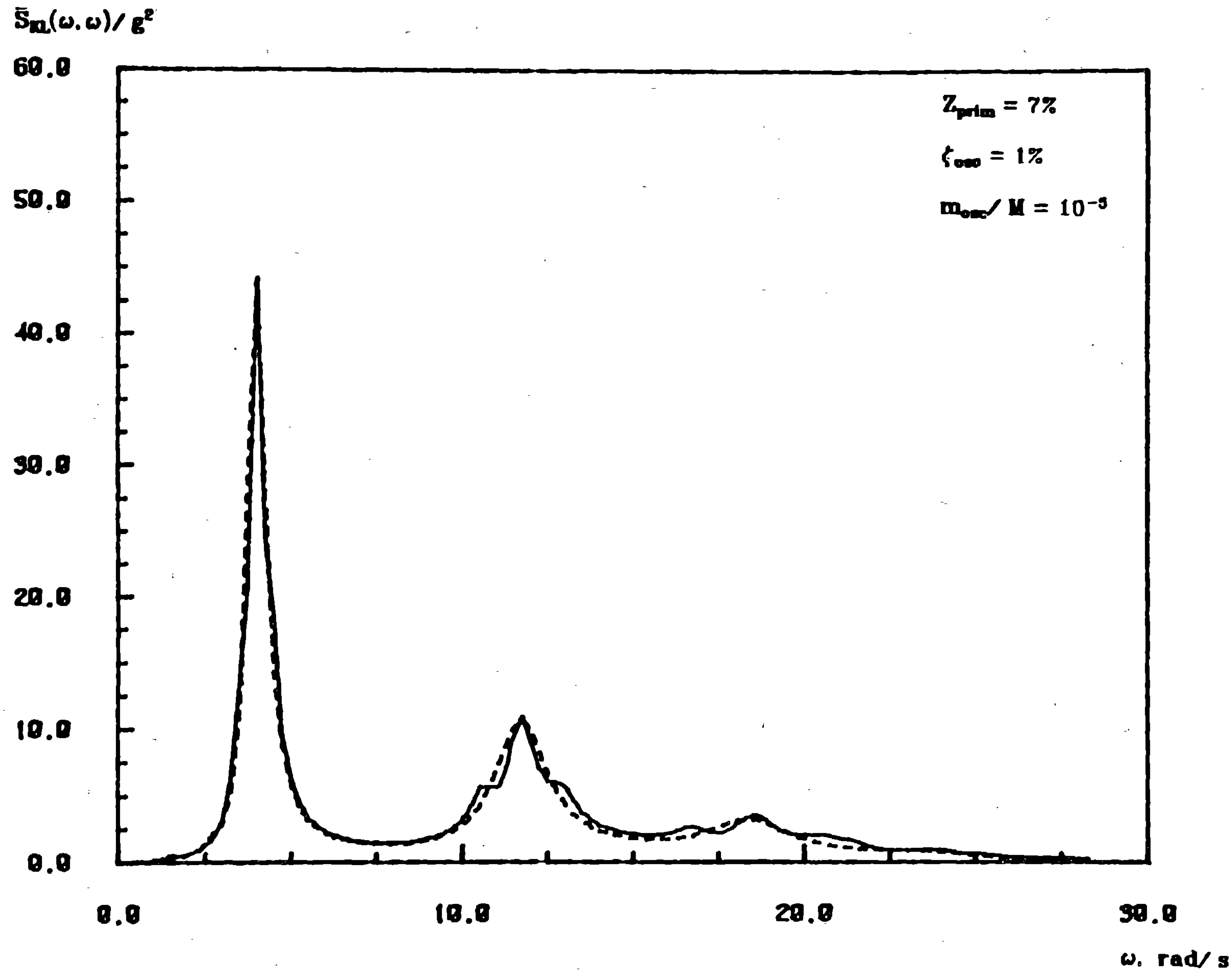


Figure 4.12 $\bar{S}_{11}$, Conventional Floor Spectrum

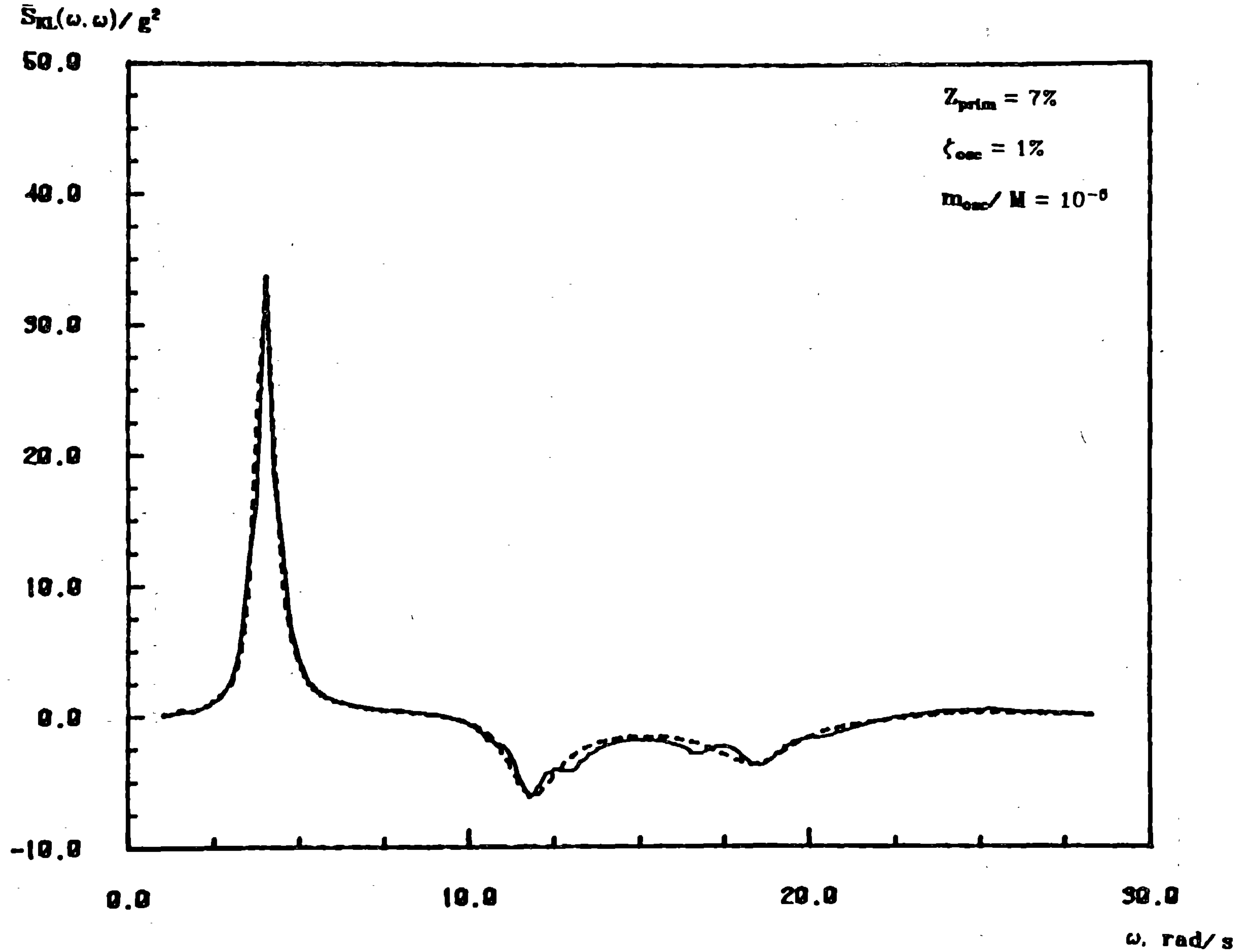


Figure 4.13 $\bar{S}_{13}$, Cross-Cross Floor Spectrum

CHAPTER 5

Numerical Examples

5.1 Introduction

In the previous chapters, the theory for a method to analyze secondary systems was presented. In this chapter, two simple systems are studied in detail, to examine the accuracy of the cross-cross floor spectrum approach in different situations. These two systems represent two extreme cases encountered in practice; namely, a secondary system attached to an ordinary structure, representing a situation where support motions are strongly correlated, and a secondary system attached between two structures, representing situations where support motions are weakly correlated. Each system is studied for : (a) different ratios of masses between the secondary and primary systems to examine how the method accounts for the effect of interaction, (b) tuning between frequencies of the primary and secondary systems and (c) different damping ratios for the primary and secondary systems to examine the accuracy of the method in cases of non-classical damping.

5.2 Example A

The first example primary-secondary system is shown in Fig. 5.1. This combined system represents a regular shear building supporting a secondary system, which is also modeled as a shear structure. The properties of the primary system and the acceleration ground response spectrum used in the modal analysis are listed in the aforementioned figure. Twelve different cases are solved using the cross-cross floor spectrum approach. The results are compared with results obtained by a modal spectrum analysis of the combined primary-secondary system using the CQC modal combination rule (Der Kiureghian [1981]). The latter results will be considered "exact". The twelve cases are separated into four groups which are subsequently presented in the following subsections. The results of the analysis of each case are presented in tables at the end of this chapter.

5.2.1 Examples A1 - Detuned, Classically Damped Systems.

This group of examples are intented to examine the proposed method for detuned and classically damped systems. Thus, frequencies of the secondary system are selected such that there is no tuning between the primary and secondary system frequencies. Also, the modal damping ratios of the two systems are assumed to be $\zeta_{secondary} = Z_{primary} = 0.05$, which will give rise to classical damping in the combined system. Different ratios of masses are considered to examine the effect of interaction under these conditions. The ratios of masses and stiffnesses between secondary and primary systems are shown in Table 5.1, where m is the nodal mass of the secondary system, M is the nodal mass of the primary system, k is the interstory and connecting stiffnesses of the secondary system and K is the interstory stiffness of the primary system. The natural frequencies of the secondary system are listed in Table 5.2. These can be compared with the frequencies of the primary system in Fig. 5.1.

Dimensionless total accelerations and relative (to the ground) displacements are obtained using the CCFS approach and are compared with the "exact" values in Table 5.3. A remarkable agreement between "exact" and CCFS results can be observed in this table. Also, comparing the results for the three mass ratios, it is concluded that interaction effects have not importance when the frequencies of the two systems are not tuned.

5.2.2 Examples A2 - Detuned, Non-Classically Damped Systems.

This group of example systems have the same properties as the previous group, except that the modal damping ratios of the primary and secondary systems are now assumed to be different, i.e., $\zeta_{secondary}$ = 0.02 and Z_{prim} = 0.05 are assumed. Results for accelerations and displacements are presented in Table 5.4. Note the good agreement between the approximate and the "exact" solution, especially for displacements which are the necessary quantities for evaluation of stresses. These results show the adequacy of the approximate method employed to account for the effect of non-classical damping, at least for detuned systems.

5.2.3 Examples A3 - Tuned, Classically Damped Systems.

In this group of examples systems, the damping ratios are assumed to be equal, i.e., $\zeta_{secondary} = Z_{primary} = 0.05$ are assumed. However, the masses and stiffnesses of the secondary system are selected such that perfect tuning occurs between the fundamental modes of the primary and secondary systems. Table 5.5 gives the mass and stiffness ratios for each case. Table 5.6 shows the frequencies of the secondary system which can be compared with the frequencies of the primary system in Fig. 5.1. Note that not only are the fundamental frequencies tuned, but the 5-th frequency of the secondary system is very close to the 2-nd frequency of the primary system.

Results for the three examples systems are shown in Table 5.7. Again very good agreement is found between the solution obtained employing the CCFS method and the "exact" solution. In this table, the effect of interaction between the primary and secondary systems can be observed by comparing the results for the increasing mass ratio. In particular, example system A31, which corresponds to a mass ratio of 0.00032, represents a case where there is practically no interaction. Example system A33, on the other hand, has a mass ratio of 0.032 and exhibits approximately 40 percent reduction in the response due to interaction. By comparing the results in Table 5.7, it is concluded that the CCFS method accurately accounts for the effect of interaction, at least for classically damped systems.

5.2.4 Examples A4 - Tuned,Non-Classically Damped Systems.

This group of examples have the same properties as the previous group, except that the modal damping values are assumed to be unequal, i.e., $\zeta_{secondary}$ = 0.02 and $Z_{primary}$ = 0.05.

The results of the analysis are shown in Table 5.8. The larger errors observed for cases A41 and A42 are due to the increased importance of non-classical damping effect for light secondary systems. The results are generally good, indicating the ability of the proposed method to account for effects such as interaction, tuning and non-classical damping. Again, it is interesting to compare cases A41 and A43. The former represents a case where the interaction is negligible and the latter represents a case where the interaction is important. In example A43, the effect of interaction is found to be a reduction of more than 50 percent in the response quantities.

5.3 Example B

The combined primary-secondary system shown in Fig. 5.2 is analyzed in this section. It represents two independent shear buildings, B1 and B2, which for simplicity have been connected to a common foundation, F. The foundation is modeled as a very rigid story. Connecting the two primary buildings is a secondary system which is modeled as a three-degree-of-freedom system. The properties of buildings B1, B2 and foundation F, and the acceleration ground response spectrum employed in the analysis are listed in Fig. 5.2. The modal frequencies of the two primary structures are given in Table 3.4 in Chapter 3. The same twelve analyses performed for Example A are carried out for the system defined as Example B. Since these cases were already described in Section 5.2, in this section only a brief description is presented for each group of examples.

5.3.1 Examples B1 - Detuned, Classically Damped Systems.

The characteristics of three example secondary systems and their frequencies are shown in Tables 5.9 and 5.10, respectively. These examples are intented to examine the effect of variation of the ratio of masses on the response of the secondary system. No tuning or non-classical damping are considered. The results for these three cases are shown in Table 5.11. Again, a good agreement between the "exact" results and CCFS results is observed. As before, it is noted that in cases of detuning, no important dynamic interaction occurs. This indicates that the response of the secondary system is insensitive to the mass ratio, as long as the frequencies remain the same.

5.3.2 Examples B2 - Detuned, Non-Classically Damped Systems.

Results for this group of example systems, which include the effect of non-classical damping due to different damping ratios for the secondary and primary systems ($\zeta_{secondary} = 0.02; Z_{primary} = 0.05$), are shown in Table 5.12. Although unconservative results are observed, the errors are negligible for all practical purposes.

5.3.3 Examples B3 - Tuned, Classically Damped Systems.

The properties of the secondary systems analyzed in this subsection and their frequencies are shown in Tables 5.13 and 5.14, respectively. Note that the fundamental frequency of the secondary system is tuned to the fundamental frequency of the primary system, which is also to the fundamental frequency of building B1. Due to this tuning, interaction effects are expected to be important. It is interesting to note that in this case interaction will occur only between the secondary system and building B1. As in the equivalent Examples A3, the damping ratios for primary and secondary systems are considered to be equal, i.e., $\zeta_{secondary} = Z_{primary} = 0.05$ are assumed. Results for these cases are shown in Table 5.15. Again, for all practical purposes, the errors are found to be acceptably small. Also, by comparing cases B31 and B33, observe that the effect of interaction is significant and it is closely predicted by the CCFS method.

5.3.4 Examples B4 - Tuned, Non-Classically Damped Systems.

Finally, three cases, which include all the effects considered above, are studied. The damping ratios are considered to be $\zeta_{secondary} = 0.02$ and $Z_{primary} = 0.05$, and the tuned frequencies in Table 5.14 are assumed. Results of the analyses are presented in Table 5.16. These indicate errors of 10-15 percent, all on the conservative side. From the examples in the preceding sections, it should be clear that for small mass ratios the main source of errors is the non-classical damping effect, and for large mass ratios the main source of the error is the approximation in accounting for the interaction effect. For all practical purposes, these errors are believed to be acceptably small.

5.4 Concluding Remarks for Numerical Examples

Two example systems of very different nature have been studied in detail for different and extreme situations and results for 24 cases are presented. From these results, it can be concluded that the proposed method is able to properly account for effects such as the correlation between support excitations, the correlation between modal responses, the effect of dynamic interaction between primary and secondary systems, the effect of tuning and the effect of non-classical damping. The errors listed in the Tables at the end of this chapter are considered to be acceptable for engineering purposes and negligible in comparison with the expected errors resulting from methods currently employed. In this regard, it is worth noting that in the existing methods errors exceeding several hundred or thousand percent are often encountered (Wang, Subudhi and Bezler [1983]). Thus, it is concluded that the proposed cross-cross floor spectrum method is a powerful and accurate tool for seismic analysis of multiply supported secondary systems.

Table 5.1. Properties of Example Systems A1 and A2

Case	m / M (%)	k / K (%)
A11, A21	0.02	0.05
A12, A22	0.20	0.50
A13, A23	2.00	5.00

Table 5.2. Frequencies of Example Systems A1 and A2

Mode	Freq. (rad/s)
1	16.11
2	22.36
3	33.99
4	38.73
5	45.66

Table 5.3. Comparison of Results for Example System A1

Case	DOF	Accelerations			Displacements x 10^{-2}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
A11	1	0.487	0.487	0.0	2.609	2.610	0.0
	2	0.468	0.468	0.0	2.385	2.386	0.0
	3	0.399	0.399	0.0	2.086	2.087	0.0
	4	0.419	0.419	0.0	1.707	1.709	-0.1
	5	0.372	0.372	0.0	1.273	1.275	-0.2
A12	1	0.486	0.486	0.0	2.610	2.609	0.0
	2	0.466	0.467	-0.2	2.385	2.385	0.0
	3	0.399	0.398	0.3	2.086	2.087	0.0
	4	0.418	0.418	0.0	1.707	1.709	-0.1
	5	0.370	0.371	-0.3	1.273	1.276	-0.2
A13	1	0.474	0.478	-0.8	2.615	2.608	0.3
	2	0.452	0.457	-1.1	2.390	2.388	0.1
	3	0.398	0.396	0.5	2.088	2.094	-0.3
	4	0.405	0.407	-0.5	1.706	1.720	-0.8
	5	0.359	0.361	-0.6	1.271	1.290	-1.5

Table 5.4. Comparison of Results for Example System A2

Case	DOF	Accelerations			Displacements x 10^{-2}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
A21	1	0.552	0.541	2.0	2.610	2.610	0.0
	2	0.538	0.532	1.1	2.386	2.386	0.0
	3	0.411	0.411	0.0	2.086	2.087	0.0
	4	0.498	0.490	1.6	1.708	1.710	-0.1
	5	0.450	0.431	4.4	1.274	1.276	-0.2
A22	1	0.547	0.538	1.7	2.611	2.609	0.1
	2	0.532	0.528	0.8	2.386	2.386	0.0
	3	0.411	0.411	0.0	2.086	2.087	0.0
	4	0.492	0.487	1.0	1.708	1.710	-0.1
	5	0.444	0.428	3.7	1.274	1.276	-0.2
A23	1	0.513	0.517	-0.8	2.615	2.608	0.3
	2	0.494	0.504	-2.0	2.391	2.389	0.1
	3	0.407	0.406	0.2	2.088	2.094	-0.3
	4	0.453	0.458	-1.1	1.707	1.720	-0.8
	5	0.406	0.403	0.7	1.272	1.291	-1.5

Table 5.5. Properties of Example Systems A3 and A4

Case	m / M (%)	k / K (%)
A31, A41	0.03203	0.005
A32, A42	0.32030	0.050
A33, A43	3.20300	0.500

Table 5.6. Frequencies of Example Systems A3 and A4

Mode	Freq. (rad/s)
1	4.025
2	5.588
3	8.494
4	9.678
5	11.410

Table 5.7. Comparison of Results for Example System A3

Case	DOF	Accelerations			Displacements x 10^{-1}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
A31	1	1.777	1.773	0.2	1.136	1.134	0.2
	2	2.547	2.541	0.2	1.614	1.611	0.2
	3	2.039	2.034	0.2	1.294	1.292	0.2
	4	2.509	2.504	0.2	1.573	1.570	0.2
	5	1.703	1.700	0.2	1.063	1.061	0.2
A32	1	1.635	1.608	1.7	1.051	1.037	1.4
	2	2.331	2.296	1.5	1.485	1.464	1.4
	3	1.862	1.837	1.4	1.189	1.174	1.3
	4	2.289	2.261	1.2	1.441	1.422	1.3
	5	1.555	1.537	1.2	0.974	0.962	1.2
A33	1	1.071	1.008	6.3	0.715	0.693	3.2
	2	1.467	1.384	6.0	0.976	0.932	4.7
	3	1.154	1.103	4.6	0.775	0.748	3.6
	4	1.408	1.353	4.1	0.924	0.882	4.8
	5	0.962	0.935	2.9	0.623	0.600	3.8

Table 5.8. Comparison of Results for Example System A4

Case	DOF	Accelerations			Displacements x 10^{-1}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
A41	1	3.205	2.924	9.6	2.002	1.834	9.2
	2	4.684	4.251	10.2	2.920	2.656	9.9
	3	3.762	3.414	10.2	2.349	2.434	10.1
	4	4.661	4.224	10.3	2.893	2.622	10.3
	5	3.159	2.865	10.3	1.955	1.772	10.3
A42	1	2.651	2.443	8.5	1.665	1.543	7.9
	2	3.853	3.532	9.1	2.414	2.219	8.8
	3	3.089	2.835	9.0	1.939	1.783	8.7
	4	3.826	3.509	9.0	2.382	2.184	9.1
	5	2.598	2.386	8.9	1.610	1.477	9.0
A43	1	1.359	1.298	4.7	0.882	0.869	1.5
	2	1.894	1.790	5.8	1.236	1.182	4.6
	3	1.492	1.426	4.6	0.983	0.949	3.6
	4	1.848	1.772	4.3	1.192	1.132	5.3
	5	1.273	1.235	3.1	0.806	0.773	4.3

Table 5.9. Properties of Example Systems B1 and B2

Case	m/M (%)	k/K (%)
B11, B21	0.02	0.04
B12, B22	0.20	0.40
B13, B23	2.00	4.00

Table 5.10. Frequencies of Example Systems B1 and B2

Mode	Freq. (rad/s)
1	15.31
2	28.28
3	36.96

Table 5.11. Comparison of Results for Example System B1

Case	DOF	Accelerations			Displacements x 10^{-2}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
B11	1	1.029	1.028	0.1	1.025	1.025	0.0
	2	1.208	1.207	0.1	1.047	1.047	0.0
	3	1.052	1.051	0.1	0.874	0.873	0.1
B12	1	1.022	1.025	-0.3	1.025	1.027	-0.2
	2	1.201	1.201	0.0	1.045	1.046	-0.1
	3	1.046	1.043	0.3	0.871	0.870	0.1
B13	1	0.962	0.998	-3.6	1.021	1.047	-2.5
	2	1.123	1.142	-1.7	1.020	1.042	-2.1
	3	0.988	0.974	1.4	0.849	0.846	0.4

Table 5.12. Comparison of Results for Example System B2

Case	DOF	Accelerations			Displacements x 10^{-2}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
B21	1	1.361	1.348	1.0	1.080	1.080	0.0
	2	1.630	1.631	-0.1	1.150	1.150	0.0
	3	1.381	1.367	1.0	0.939	0.939	0.0
B22	1	1.347	1.341	0.4	1.078	1.081	-0.3
	2	1.613	1.620	-0.4	1.144	1.147	-0.3
	3	1.367	1.356	0.8	0.935	0.934	0.1
B23	1	1.222	1.281	-4.6	1.059	1.092	-3.0
	2	1.447	1.531	-5.5	1.092	1.128	-3.2
	3	1.244	1.259	-1.2	0.895	0.901	-0.7

Table 5.13. Properties of Example Systems B3 and B4

Case	m/M (%)	k/K (%)
B31, B41	0.02	0.0126
B32, B42	0.20	0.1260
B33, B43	2.00	1.2600

Table 5.14. Frequencies of Example Systems B3 and B4

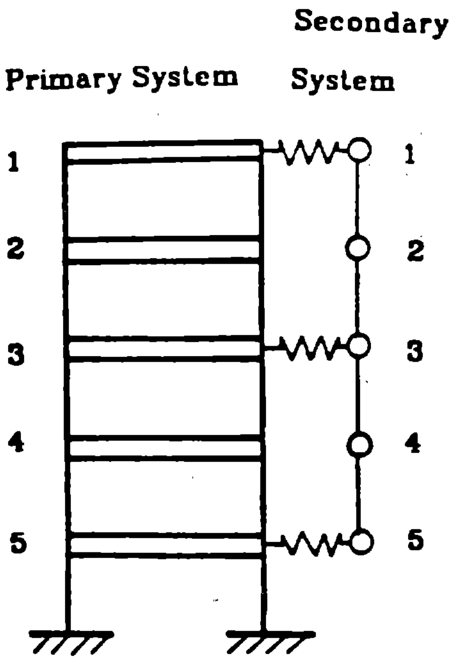
Mode	Freq. (rad/s)
1	8.603
2	15.900
3	20.770

Table 5.15. Comparison of Results for Example System B3

Case	DOF	Accelerations			Displacements x 10^{-2}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
B31	1	2.543	2.535	0.3	3.533	3.523	0.3
	2	3.364	3.354	0.3	4.661	4.648	0.3
	3	2.277	2.271	0.3	3.204	3.197	0.2
B32	1	2.477	2.422	2.3	3.448	3.376	2.1
	2	3.259	3.193	2.1	4.526	4.439	2.0
	3	2.194	2.157	1.7	3.098	3.053	1.5
B33	1	2.024	1.821	11.1	2.897	2.613	10.9
	2	2.559	2.333	9.7	3.664	3.385	8.2
	3	1.651	1.549	6.6	2.413	2.293	5.2

Table 5.16. Comparison of Results for Example System B4

Case	DOF	Accelerations			Displacements x 10^{-2}		
		CCFS	Exact	Error (%)	CCFS	Exact	Error (%)
B41	1	4.320	3.967	8.9	5.934	5.472	8.4
	2	5.956	5.414	10.0	8.164	7.440	9.7
	3	4.152	3.761	10.4	5.714	5.193	10.0
B42	1	3.974	3.615	9.9	5.475	5.011	9.8
	2	5.447	4.913	10.9	7.488	6.780	10.4
	3	3.781	3.409	10.9	5.222	4.731	10.4
B43	1	2.662	2.296	15.9	3.786	3.313	14.1
	2	3.495	3.010	16.1	4.975	4.311	15.4
	3	2.340	2.064	13.4	3.354	3.007	11.5



DIMENSIONLESS PROPERTIES

PRIMARY SYSTEM:
Floor Mass ; $M = 100$
Interstory Stiffness ; $K = 20{,}000$

GROUND ACCELERATION SPECTRUM:

$$S_a(\omega,\zeta) = g\left(\frac{\pi\omega}{2000\zeta}\right)^{1/8}$$

Frequencies of Primary System

Mode	Freq. (rad/s)
1	4.025
2	11.750
3	18.520
4	23.790
5	27.140

Figure 5.1 Combined System. Examples A

DIMENSIONLESS PROPERTIES

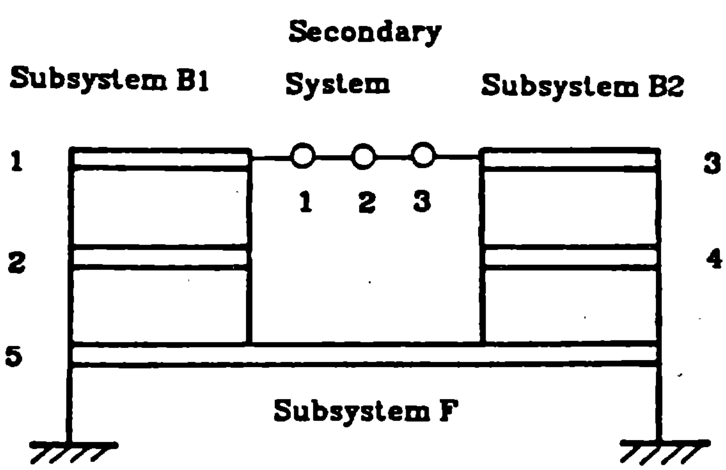
PRIMARY SUBSYSTEM B1:
Floor Mass ; $M = 100$
Interstory Stiffness ; $K = 20{,}000$

PRIMARY SUBSYSTEM B2:
Floor Mass ; $M = 100$
Interstory Stiffness ; $K = 30{,}000$

PRIMARY SUBSYSTEM F:
Floor Mass ; $M = 100$
Interstory Stiffness ; $K = 500{,}000$

GROUND ACCELERATION SPECTRUM:

$$S_a(\omega,\zeta) = g\left(\frac{\pi\omega}{2000\zeta}\right)^{1/8}$$



Frequencies of Primary System

Mode	Freq. (rad/s)
1	8.803
2	10.490
3	22.750
4	27.790
5	74.340

Figure 5.2 Combined System. Examples B

CHAPTER 6

Summary and Conclusions

6.1 Summary

A method to evaluate the seismic response of multiply supported secondary systems was developed within the framework of a stationary random vibration theory and the response spectrum method of describing the ground motion. This new method can be seen as an extension of the conventional floor response spectrum method. It allows one to analyze the secondary system separately of the primary system.

The concept of cross-oscillator cross-floor response spectrum was introduced. This can be seen as an extension of the conventional floor response spectrum which accounts for effects such as correlation between support excitations, correlation between modal responses, interaction between primary and secondary systems, tuning between frequencies of primary and secondary systems, and non-classical damping due to difference in damping ratios of the primary and secondary systems.

The method consists of two main steps: (a) evaluation of CCFS in terms of ground response spectrum, and (b) evaluation of secondary system response by modal combination in terms of the CCFS.

The CCFS is evaluated by employing a set of two $N+1$-degree-of-freedom systems, each composed of the N-DOF primary system and an oscillator attached to a selected floor of the primary system. The modal properties of the $N+1$-DOF systems are obtained, using perturbation techniques, in closed form in terms of the modal properties of the primary system and the properties of the two oscillators. An equivalent mass is assigned to each oscillator to account for the effect of interaction between the primary and secondary systems. Finally, the CCFS are expressed in terms of the modal properties of the two $N+1$-DOF systems and the ordinates of the ground response spectrum.

Using methods from stationary random vibration theory, a modal combination rule for systems subjected to multiple support excitations was developed. This combination rule expresses the mean maximum response of the secondary system in terms of the CCFS and the fixed-base modal properties of the secondary system. All the effects mentioned above are included in this combination rule.

6.2 General Conclusions

The main contribution of this work is the development of a practical and accurate method for the modal analysis of multiply supported secondary systems. This method is based on a floor spectrum approach which allows one to analyze the secondary system separately of the primary system. Although the method, in its final form, was presented in terms of response spectra, it also can be used in the framework of random vibrations with the input motions described by their power spectral density functions.

From the basic assumption in the theoretical development and from the numerical results presented in Chapter 5, it can be concluded that the method is adequate for seismic analysis of general secondary systems used in structural engineering practice. Also, it can be concluded that the approximations employed to account for the effect of interaction and non-classical damping are sufficiently accurate for all practical purposes. These effects, which normally result in significant reduction in the response, are entirely ignored in the current practice.

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Appendix A

Cross Power Spectral Density Function for Relative Displacements

In this appendix, it will be shown that the power spectral density of the relative displacement of the secondary system with respect to the ground, v_r, is given by

$$G_{v_r v_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} G_{X_{iK}^R X_{jL}^R}(\omega) \tag{A.1}$$

where a_{ri} and b_{iK} are defined in Eq. (2.9) and $G_{X_{iK}^R X_{jL}^R}(\omega)$ is the cross-power spectral density of the relative displacements X_{iK}^R and X_{jL}^R of the two oscillators shown in Fig. 2.4. For simplicity, Eq. (A.1) will be proven first for systems having all their dynamic degrees of freedom in the direction of the input excitation. Then, the proof will be extended for general tridimensional systems. The dynamic degrees of freedom in the direction of the input are called "parallel" degrees of freedom and the dynamic degrees of freedom orthogonal to these are called "normal" degrees of freedom.

Considering a system with only parallel degrees of freedom, the relative displacements are given by:

$$v_r = u_r - u_g \qquad X_{iK}^R = X_{iK}^T - u_g \qquad X_{jL}^R = X_{jL}^T - u_g \tag{A.2}$$

Substituting these expressions into Eq. (A.1), the following relation is obtained

$$G_{u_r u_r}(\omega) - G_{u_r u_g}(\omega) - G_{u_g u_r}(\omega) + G_{u_g u_g}(\omega) =$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \left(G_{X_{iK}^T X_{jL}^T}(\omega) - G_{X_{iK}^T u_g}(\omega) - G_{u_g X_{jL}^T}(\omega) + G_{u_g u_g}(\omega) \right) \tag{A.3}$$

Changing the order of summation for the last three terms, the right hand side of the above expression can be written as

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK} b_{jL}\, G_{X_{iK}^T X_{jL}^T}(\omega) - \sum_{j=1}^{n}\sum_{L=1}^{n_a} a_{rj} b_{jL} \sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK}\, G_{X_{iK}^T u_g}(\omega)$$

$$-\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK} \sum_{j=1}^{n}\sum_{L=1}^{n_a} a_{rj} b_{jL}\, G_{u_g X_{jL}^T}(\omega) + \sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK} \sum_{j=1}^{n}\sum_{L=1}^{n_a} a_{rj} b_{jL}\, G_{u_g u_g}(\omega)$$

Using the expressions for the coefficients a_{ri} and b_{iK} given in Eq. (2.9), the double sums inside parenthesis in the above expression, which are independent of ω, can be evaluated. These can be written, in general, as

$$\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK} = \sum_{i=1}^{n} \frac{\varphi_{ri}}{m_i \omega_i^2} \varphi_i^T \mathbf{k}_c \{1\}_{n_a}$$

where $\{1\}_{n_a}$ is an n_a-vector of ones and $\mathbf{k}_c$ is the coupling stiffness matrix. Assuming that all the dynamic degrees of freedom are translational, the matrix $\mathbf{k}_c$ is related to the stiffness matrix of the fixed base secondary system, $\mathbf{k}$, through

$$\mathbf{k}\{1\}_n + \mathbf{k}_c\{1\}_{n_a} = \mathbf{0}$$

The above relation is obtained by observing that a rigid body translation of the secondary system should not cause any internal forces. Thus,

$$\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK} = -\sum_{i=1}^{n} \frac{\varphi_{ri}}{m_i \omega_i^2} \varphi_i^T \mathbf{k}\{1\}_n \tag{A.4}$$

Now express the vector $\{1\}_n$ as a linear combination of the eigenvectors of the fixed base secondary system, i.e.,

$$\{1\}_n = \sum_{j=1}^{n} \alpha_j \varphi_j$$

Introducing this expression in Eq. (A.4), and using the orthogonality of the modal shapes with respect to the stiffness matrix,

$$\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri}\, b_{iK} = -\sum_{i=1}^{n} \varphi_{ri}\,\alpha_i$$

The sum on the right hand side of the above expression corresponds to the $r-th$ element of the vector $\{1\}_n$. Thus

$$\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri}\, b_{iK} = -1 \qquad \text{(A.5)}$$

Using this result, Eq. (A.3) is written

$$G_{u_r u_r}(\omega) - G_{u_r u_g}(\omega) - G_{u_g u_r}(\omega) + G_{u_g u_g}(\omega) =$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}\, a_{rj} \sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK} b_{jL}\, G_{\chi_{iK}^T \chi_{jL}^T}(\omega) + \sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri}\, b_{iK}\, G_{\chi_{iK}^T u_g}(\omega) + \qquad \text{(A.6)}$$

$$\sum_{j=1}^{n}\sum_{L=1}^{n_a} a_{rj}\, b_{jL}\, G_{u_g \chi_{jL}^T}(\omega) + G_{u_g u_g}(\omega)$$

From Eqs. (A.6) and (2.11) it is clear that in order to prove Eq. (A.1), it is sufficient to show that

$$G_{u_r u_g}(\omega) = -\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri}\, b_{iK}\, G_{\chi_{iK}^T u_g}(\omega) \qquad \text{(A.7)}$$

Using a standard techniques of random vibration theory, the following relation can be obtained for the cross-power spectral density function, $G_{u_r u_g}(\omega)$:

$$G_{u_r u_g}(\omega) = -\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri}\, b_{iK}\, \omega_i^2 h_i(-\omega)\, G_{\ddot{y}_K u_g}(\omega) \qquad \text{(A.8)}$$

In this expression, the term $\omega_i^2 h_i(-\omega) G_{\ddot{y}_K u_g}(\omega)$ may be interpreted as the cross-power spectral density of the ground displacement u_g and the total displacement response of an oscillator of frequency ω_i and damping ratio ζ_i which is subjected to base input U_K. This is the same as $G_{X^T_{iK} u_g}(\omega)$ and, thus, Eq. (A.1) is proven.

Now, let us consider a general tridimensional system. For parallel degrees of freedom, the relative displacements are give by Eq. (A.2). For normal degrees of freedom, the relative displacements are equal to the total displacements, i.e.,

$$v_r = u_r \quad X^R_{iK} = X^T_{iK} \quad X^R_{jL} = X^T_{jL} \tag{A.9}$$

In order to prove Eq. (A.1), the sums over the support points is split into sums over the parallel support points (subindices $K1$ and $L1$) and sums over the normal support points (subindices $K2$ and $L2$). Then, Eq. (A.1) is rewritten as is given by

$$G_{v_r v_r}(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri} a_{rj} \left\{ \sum_{K1}^{n_{a1}}\sum_{L1}^{n_{a1}} b_{iK1} b_{jL1} G_{X^R_{iK1} X^R_{jL1}}(\omega) + \right.$$

$$\sum_{K2}^{n_{a2}}\sum_{L1}^{n_{a1}} b_{iK2} b_{jL1} G_{X^R_{iK2} X^R_{jL1}}(\omega) + \sum_{K1}^{n_{a1}}\sum_{L2}^{n_{a2}} b_{iK1} b_{jL2} G_{X^R_{iK1} X^R_{jL2}}(\omega) + \tag{A.10}$$

$$\left. \sum_{K2}^{n_{a2}}\sum_{L2}^{n_{a2}} b_{iK2} b_{jL2} G_{X^R_{iK2} X^R_{jL2}}(\omega) \right\}$$

where n_{a1} is the number of parallel support points and n_{a2} is the number of normal support points. Introducing the relations given by Eqs. (A.2) and (A.9) into Eq. (A.10), the following expression is obtained

$$G_{v_r v_r}(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri} a_{rj} \left\{ \right.$$

$$\sum_{K1}^{n_{a1}}\sum_{L1}^{n_{a1}} b_{iK1} b_{jL1} [\, G_{X^T_{iK1} X^T_{jL1}}(\omega) - G_{X^T_{iK1} u_g}(\omega) - G_{u_g X^T_{jL1}}(\omega) + G_{u_g u_g}(\omega) \,] +$$

$$\sum_{K2}^{n_{a2}} \sum_{L1}^{n_{a1}} b_{iK2} b_{jL1} \left[G_{X_{iK2}^T X_{jL1}^T}(\omega) - G_{X_{iK2}^T u_g}(\omega) \right] + \tag{A.11}$$

$$\sum_{K1}^{n_{a1}} \sum_{L2}^{n_{a2}} b_{iK1} b_{jL2} \left[G_{X_{iK1}^T X_{jL2}^T}(\omega) - G_{u_g X_{jL2}^T}(\omega) \right] +$$

$$\left. \sum_{K2}^{n_{a2}} \sum_{L2}^{n_{a2}} b_{iK2} b_{jL2} G_{X_{iK2}^T X_{jL2}^T}(\omega) \right\}$$

grouping and combining terms, this equation is written as

$$G_{v_r v_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} G_{X_{iK}^T X_{jL}^T}(\omega) -$$

$$\sum_{j=1}^{n} \sum_{L1}^{n_{a1}} a_{rj} b_{jL1} \sum_{i=1}^{n} \sum_{K=1}^{n_a} a_{ri} b_{iK} G_{X_{iK}^T u_g}(\omega) -$$

$$\sum_{i=1}^{n} \sum_{K1}^{n_{a1}} a_{ri} b_{iK1} \sum_{j=1}^{n} \sum_{L=1}^{n_a} a_{rj} b_{jL} G_{u_g X_{jL}^T}(\omega) + \tag{A.12}$$

$$\sum_{i=1}^{n} \sum_{K1}^{n_{a1}} a_{ri} b_{iK1} \sum_{j=1}^{n} \sum_{L1}^{n_{a1}} a_{rj} b_{jL1} G_{u_g u_g}(\omega)$$

As before, let us evaluate the generic term

$$\sum_{i=1}^{n} \sum_{K=1}^{n_a} a_{ri} b_{iK}$$

This double summation can be written in terms of the parallel and normal support points as

$$\sum_{i=1}^{n} \sum_{K=1}^{n_a} a_{ri} b_{iK} = \sum_{i=1}^{n} \sum_{K1}^{n_{a1}} a_{ri} b_{iK1} + \sum_{i=1}^{n} \sum_{K2}^{n_{a2}} a_{ri} b_{iK2} \tag{A.13}$$

or, as in the previous proof,

$$\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK} = \sum_{i=1}^{n} a_{ri}\varphi_i^T \mathbf{k}_c \begin{Bmatrix} \{1\}_{n_{a1}} \\ \{0\}_{n_{a2}} \end{Bmatrix} + \sum_{i=1}^{n} a_{ri}\varphi_i^T \mathbf{k}_c \begin{Bmatrix} \{0\}_{n_{a1}} \\ \{1\}_{n_{a2}} \end{Bmatrix} \tag{A.14}$$

If a rigid body translation is given in the parallel direction, the following relation is obtained

$$\mathbf{k}_c \begin{Bmatrix} \{1\}_{n_{a1}} \\ \{0\}_{n_{a2}} \end{Bmatrix} + \mathbf{k} \begin{Bmatrix} \bar{1} \\ \bar{0} \end{Bmatrix} = 0 \tag{A.15}$$

where $\bar{1}$ is a vector of ones associated with the parallel dynamic degrees of freedom and $\bar{0}$ is a vector of zeros associated with the normal degrees of freedom. Again, it is considered that only translational dynamic degrees of freedom exist. If the translation is given in the normal direction, a similar expression is obtained

$$\mathbf{k}_c \begin{Bmatrix} \{0\}_{n_{a1}} \\ \{1\}_{n_{a2}} \end{Bmatrix} + \mathbf{k} \begin{Bmatrix} \bar{0} \\ \bar{1} \end{Bmatrix} = 0 \tag{A.16}$$

Introducing Eqs. (A.15) and (A.16) into Eq. (A.14), this becomes

$$\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK} = -\sum_{i=1}^{n} a_{ri}\varphi_i^T \mathbf{k} \begin{Bmatrix} \bar{1} \\ \bar{0} \end{Bmatrix} - \sum_{i=1}^{n} a_{ri}\varphi_i^T \mathbf{k} \begin{Bmatrix} \bar{0} \\ \bar{1} \end{Bmatrix} \tag{A.17}$$

The vectors of zeros and ones can be expressed as a linear combination of the eigenvectors of the fixed base secondary system, i.e.,

$$\begin{Bmatrix} \bar{1} \\ \bar{0} \end{Bmatrix} = \sum_{j=1}^{n} \alpha_j \varphi_j \quad ; \quad \begin{Bmatrix} \bar{0} \\ \bar{1} \end{Bmatrix} = \sum_{j=1}^{n} \beta_j \varphi_j \tag{A.18}$$

Introducing Eq. (A.18) into Eq. (A.17) and using the orthogonality of the mode shapes with respect to the stiffness matrix, Eq. (A.17) becomes

$$\sum_{i=1}^{n}\sum_{K=1}^{n_a} a_{ri} b_{iK} = -\sum_{i=1}^{n} \varphi_{ri}\alpha_i - \sum_{i=1}^{n} \varphi_{ri}\beta_i \tag{A.19}$$

From Eq. (A.18), it can be seen that

$$\sum_{i=1}^{n} \varphi_{ri}\alpha_i = 1 \quad ; \quad \sum_{i=1}^{n} \varphi_{ri}\beta_i = 0 \qquad \text{for } r \textit{ parallel}$$

$$\sum_{i=1}^{n} \varphi_{ri}\alpha_i = 0 \quad ; \quad \sum_{i=1}^{n} \varphi_{ri}\beta_i = 1 \qquad \text{for } r \textit{ normal}$$

and

$$\sum_{i=1}^{n} \sum_{K=1}^{n_a} a_{ri} b_{iK} = -1$$

Then, for a response in the direction of the input the following relations hold:

$$v_r = u_r - u_g$$

$$\sum_{i=1}^{n} \alpha_i \varphi_{ri} = 1 \quad \rightarrow \quad \sum_{i=1}^{n} \sum_{K1}^{n_{a1}} a_{ri} b_{iK1} = -1$$

$$\sum_{i=1}^{n} \beta_i \varphi_{ri} = 0 \quad \rightarrow \quad \sum_{i=1}^{n} \sum_{K2}^{n_{a2}} a_{ri} b_{iK2} = 0$$

and Eq. (A.12) becomes

$$G_{v_r v_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL}\, G_{X_{iK}^T X_{jL}^T}(\omega) +$$

$$\sum_{i=1}^{n} \sum_{K=1}^{n_a} a_{ri} b_{iK}\, G_{X_{iK}^T u_g}(\omega) + \sum_{j=1}^{n} \sum_{L=1}^{n_a} a_{rj} b_{jL}\, G_{u_g X_{jL}^T}(\omega) + G_{u_g u_g}(\omega) \qquad \text{(A.20)}$$

This equation is identical to Eq. (A.6) which has been already proven. Thus, Eq. (A.1) or Eq. (A.10) are valid relations for the response of parallel degrees of freedom.

For a response normal to the direction of the input excitation, the following relations hold:

$$v_r = u_r$$

$$\sum_{i=1}^{n} \alpha_i \varphi_{ri} = 0 \quad \rightarrow \quad \sum_{i=1}^{n} \sum_{K1}^{n_{a1}} a_{ri} b_{iK1} = 0$$

$$\sum_{i=1}^{n} \beta_i \varphi_{ri} = 1 \quad \rightarrow \quad \sum_{i=1}^{n} \sum_{K2}^{n_{a2}} a_{ri} b_{iK2} = -1$$

and Eq. (A.12) becomes

$$G_{v_r v_r}(\omega) = G_{u_r u_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} G_{X_{iK}^T X_{jL}^T}(\omega) \tag{A.21}$$

which, obviously, is true.

Thus, it is concluded that Eq. (A.1) is valid for any general tridimensional system when only translational dynamic degrees of freedom are considered, which is common practice for most systems.

Appendix B

Response Including Peak Factors

In Section 4.5, the ratio between the peak factors was approximated by unity in order to have simple and efficient expressions for evaluating the cross-cross floor spectra and the final response of the secondary system. This is not a condition for the method, and in this appendix, the necessary expressions to consider those peak factors will be presented. In order to avoid to define again every term, the same notation used in Chapter 4 of the main text will be used here. It was shown, in the main text, that the mean square of a general response s of a secondary multiply supported structure can be written as

$$E[s^2] = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \lambda_{0,ijKL} \tag{B.1}$$

where σ_i can be determinated by static analysis (Eq. 2.20). From the relation $E[s_{max}] = pE[s^2]^{1/2}$ and the expression for $\lambda_{0,ijKL}$ given by Eq. (4.16), it is clear that $E[s_{max}]$ depends only on the ratio $\dfrac{p^2}{p_i^K p_j^L}$ and not on the peak factors $p_{iK} \cdot p_{jL}$ used to define the cross-cross floor spectrum (see Chapter 2). Expressions for evaluating p, p_i^K and p_j^L can be found in Der Kiureghian [1980], [1981]. These expressions, which are based on improvements of expressions given by Davenport [1964], are written below without further explanations.

The general expression for the peak factor over duration τ is

$$p = \sqrt{2 \ln \nu_e(0)\tau} + \frac{0.5772}{\sqrt{2 \ln \nu_e(0)\tau}} \tag{B.2}$$

where $\nu_e(0)\tau$ is an equivalent-statistically-independent mean zero-crossing rate given by

$$\nu_e(0)\tau = \begin{cases} \max[2.1, \delta\nu(0)\tau] & 0.00 < \delta \le 0.10 \\ (1.63\delta^{0.45} - 0.38)\nu(0)\tau & 0.10 < \delta < 0.69 \\ \nu(0)\tau & 0.69 \le \delta < 1.00 \end{cases} \tag{B.3}$$

Assuming wide band input, $\nu(0)$ and δ for the modal peak factor p_I^K are given by:

$$\nu(0) = \frac{\Omega_I^K}{\pi} \;; \qquad \delta = 2\left[\frac{Z_I^K}{\pi}\right]^{1/2}$$

and for the global peak factor p by:

$$\nu(0) = \frac{1}{\pi}\left[\frac{\lambda_2}{\lambda_0}\right]^{1/2} \;; \qquad \delta = \left[1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}\right]^{1/2}$$

For the problem presented in this work, λ_m, $m = 0,1,2$, are given by

$$\lambda_m = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_i \sigma_j \sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK} b_{jL} \lambda_{m,ijKL} \tag{B.4}$$

where

$$\lambda_{m,ijKL} = \sum_{I=1}^{N+1}\sum_{J=1}^{N+1} \Psi_{N+1,I}^K \Psi_{N+1,J}^L \rho_{m,IJ}^{KL} \frac{w_{m,IJ}^{KL}}{p_I^K p_J^L} \bar{S}(\Omega_I^K, Z_I^K)\bar{S}(\Omega_J^L, Z_J^L) \tag{B.5}$$

where

$$\rho_{m,IJ}^{KL} = \frac{\mathrm{Re}\int_0^\infty \omega^m H_I^K(\omega) H_J^L(-\omega) G_{\ddot{u}_g\ddot{u}_g}(\omega)\, d\omega}{\left[\int_0^\infty \omega^m \left|H_I^K(\omega)\right|^2 G_{\ddot{u}_g\ddot{u}_g}(\omega)\, d\omega \int_0^\infty \omega^m \left|H_J^L(\omega)\right|^2 G_{\ddot{u}_g\ddot{u}_g}(\omega)\, d\omega\right]^{1/2}} \tag{B.6}$$

and

$$w_{m,IJ}^{KL} = \frac{\left[\int_0^\infty \omega^m \left|H_I^K(\omega)\right|^2 G_{\ddot{u}_g\ddot{u}_g}(\omega)\, d\omega \int_0^\infty \omega^m \left|H_J^L(\omega)\right|^2 G_{\ddot{u}_g\ddot{u}_g}(\omega)\, d\omega\right]^{1/2}}{\left[\int_0^\infty \left|H_I^K(\omega)\right|^2 G_{\ddot{u}_g\ddot{u}_g}(\omega)\, d\omega \int_0^\infty \left|H_J^L(\omega)\right|^2 G_{\ddot{u}_g\ddot{u}_g}(\omega)\, d\omega\right]^{1/2}} \tag{B.7}$$

Expressions for approximating these coefficients, for wide-band inputs, can be found in Der Kiureghian [1981] and Der Kiureghian and Smeby [1983].

In the evaluation of the peak factors, the spectral moments λ_0, λ_1, and λ_2 are needed. This implies that the numerical effort for incorporating the peak factors is equivalent to evaluating the response three times. Since the improvement in the final results is usually small, the use of peak factors is not deemed to be necessary in the practical implementation of the cross-cross floor spectrum method.

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