SEISMIC ANALYSIS OF ROTATING MECHANICAL SYSTEMS

A REPORT TO

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by

A. H. Soni, Professor

School of Mechanical and Aerospace Engineering

Oklahoma State University

Stillwater, Oklahoma 74078

and

V. Srinivasan*, Research Assistant

School of Mechanical and Aerospace Engineering

Oklahoma State University

Stillwater, Oklahoma 74078

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* Presently with IBM, Watson Research Center

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ABSTRACT

In this report we present the seismic analysis of a rotating mechanical system in the time domain. The earthquake excitation is assumed to be a deterministic function of time. The report is divided into two parts. Part I presents the theoretical developments of the models. Part II presents the corresponding computer programs along with the User's Manuals.

Literature available on seismic analysis of rotating mechanical systems is first reviewed in Part I. A rigid body model is then developed. In the rigid body model the rotating system is modeled as a rigid body spinning in three-dimensional space. Factors such as gyroscopic effects, rotor-bearing interaction effects, base rotation (including Coriolis effects) and base translation are included in the model. A numerical example is solved and the results are presented in graphical form.

Following this, a beam model is presented. The beam model incorporates the flexibility of the rotating system using Timoshenko beam theory. In addition to the factors mentioned in the rigid body model, factors such as rotatory inertia, shear deformation, intermediate disks and flywheels and effects of initial stresses due to axial force and axial torque are included in the beam model. The solution is obtained using finite elements in the spatial domain and finite differences in the time domain. A numerical example is solved and the results are presented in graphical form.

Finally, a three-dimensional elasticity model is proposed. The 3-D elasticity model incorporates the flexibility of the system using the three-dimensional theory of elasticity. The solution is obtained using eight-noded, isoparametric solid of revolution finite elements in the spatial domain and finite differences in the time domain. A numerical example is solved and the results are presented in graphical form. Based on the performance of the

rigid body, beam and 3-D elasticity models, conclusions are drawn at the end of Part I.

In Part II we first present a User's manual and listings of a computer program called GYROT which is based on the rigid body model. This is followed by another User's manual and listings of ROBET, which is based on our beam model. Finally, we present a User's manual and listings of AXIST, which is based on our 3-D elasticity model.

NOMENCLATURE

c_{xxi} etc.	Damping coefficients of the fluid film in i^{th} bearing.
h	Vertical distance between G and b.
$\tilde{i}, \tilde{j}, \tilde{k}$	Unit vectors along X, Y, and Z axes.
k	Timoshenko coefficient = $6(1 + \nu)/(7 + 6\nu)$
k_{xxi} etc.	Stiffness coefficients of the fluid film in i^{th} bearing.
ℓ_i	Distance between G and the i^{th} bearing.
m	Mass of the rotor.
r	Radius of cross section of the rotor
x_G, y_G, z_G	Displacements of G relative to $x_b y_b z_b$ reference system.
A	Area of cross section of the rotor.
E	Young's Modulus of the material.
F_{xi}, F_{yi}	Dynamic reaction forces in i^{th} bearing.
G	Center of the mass of the rotor, also rigidity modulus of the material.
I	Moment of inertia of the rotor about z-axis.
I_o	Moment of inertia of the rotor about x- or y-axis.
X_b, Y_b, Z_b	Absolute displacements of point b.
$\tilde{\alpha}_b$	Angular acceleration of the base.
$\tilde{\varepsilon}_x, \tilde{\varepsilon}_y, \tilde{\varepsilon}_z$	Unit vectors along x, y, and z axes.
$\tilde{\varepsilon}_{xb}, \tilde{\varepsilon}_{yb}, \tilde{\varepsilon}_{zb}$	Unit vectors along x_b, y_b, and z_b axes.
ν	Poisson's ratio of the material.
ρ	Mass density of the material.
ψ, θ, ϕ	Precession, nutation and spin angles.
ω	Rotational speed of the rotor, a constant.
$\tilde{\omega}_b$	Angular velocity of the base.

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PREFACE

This is the final report of an investigation on the seismic behavior of rotating mechanical systems, supported by National Science Foundation. It covers the literature review, a rigid body model, a beam model and a three-dimensional elasticity model.

The present report is divided into two parts. Part I deals with the theoretical developments of various models. Part II presents the corresponding computer programs and User's manuals.

In Part I, Chapter 1 gives a brief introduction to the seismic analysis of rotating mechanical systems. This is followed by a review of the models used and results obtained by various authors. The review covers all available literature and we believe that it presents the current state of the art in this area.

Chapter 2 presents a rigid body model. As a first order of approximation, the rotating system is modeled as a rigid body spinning in three-dimensional space. A rigid body approximation is the first step in our analysis sequence. The reader will notice that much of the kinematic relations developed in this Chapter is carried over to the higher-order models.

Chapter 3 presents a beam model. The flexibility of the rotating system is now taken into account using Timoshenko beam theory. The kinematic relations developed in this Chapter are very similar to their counterparts in Chapter 2. But in developing the kinetic relations, we have departed considerably from the rigid body model. A finite element method is used to obtain the seismic response in the beam model.

Chapter 4 presents a 3-D elasticity model. The flexibility of the rotating system is incorporated in the model using three-dimensional theory of elasticity. The kinematic relations developed in this Chapter are similar to their counterparts in Chapters 2 and 3. In developing the kinetic relations, we have followed a procedure similar to that of beam model. A finite element method is used to obtain the seismic response in the 3-D elasticity model.

Chapter 5 draws conclusions based on the performance of rigid body, beam and 3-D elasticity models. This is followed by references and appendices.

In this report, a deterministic analysis approach is used throughout. This means that the seismic excitation is treated as a known function of time. Historically, a deterministic approach preceeds a non-deterministic approach. A deterministic approach enables us to understand the basic dynamic behavior of the system under investigation. It also helps us to develop the governing equations for a complex system such as a rotating mechanical system. These equations will form the starting point for any non-deterministic analysis to be conducted in the future.

The reader will notice that we have adopted a Newton-Euler approach, rather than a Lagrangian approach, to formulate the governing dynamic equations in this report. A rotating mechanical system is a nonconservative system and as such it does not possess a potential function from which the generalized active forces can be derived. This makes the construction of Lagrangian for this system more difficult, if not impossible. On the other hand, the Newton-Euler approach is more direct and can be applied to a nonconservative system without any difficulty. Hence the Newton-Euler approach has been adopted throughout this report.

In Part II, Chapter 1 presents the GYROT User's manual and the associated listings. GYROT is a computer program based on the rigid body model. Chapter 2 presents the ROBET User's manual and the corresponding listings. ROBET is a computer program based on our beam model. Chapter 3 presents the AXIST User's manual and the corresponding listings. AXIST is a computer program based on our 3-D elasticity model.

PART I

THEORETICAL DEVELOPMENTS

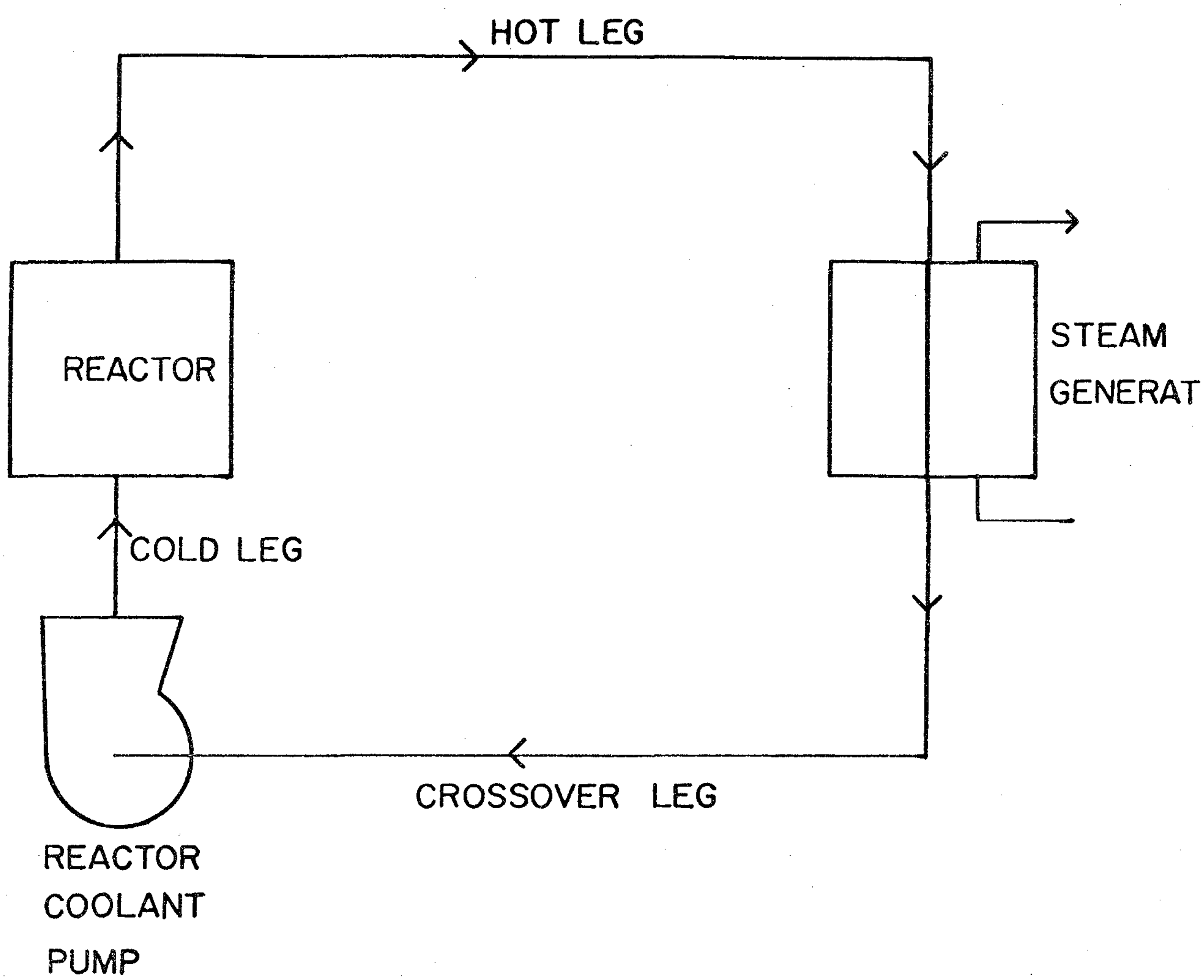

FIG.1.1 TYPICAL NUCLEAR STEAM SUPPLY SYSTEM

1. INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION

The dynamics of rotating machines has been a topic of interest to designers and research engineers for many years. Most studies have focused on the following:

- rotor stability
- balancing of the rotor
- dynamic response of the rotor

Dynamic response studies include response due to mass unbalance and response due to such environmental effects as foundation excitation.

The performance of rotating machines on such moving vehicles as aircraft was a major concern of early designers and led to investigations on the dynamic response of rotating machines to foundation excitation. Research in this area has recently been revitalized because of concern regarding the performance of rotating machines in earthquake environments. In such emergency installations as hospitals and fire stations and in nuclear power plants certain rotating machines must remain functional during and after an earthquake.

Figure 1.1 shows the primary circuit of a typical nuclear steam supply system in a pressurized water reactor. The heat generated in the reactor is carried by a primary fluid that condenses in a steam generator that transfers heat to a secondary fluid. The condensed fluid is then pumped up to the reactor by the reactor coolant pump. This pump is vital to the nuclear steam supply system and is the heart of the power plant. Failure of this pump could lead to catastrophic consequences. It is therefore essential that this pump remain functional in the event of seismic activity.

The seismic analysis of rotating machines basically involves a transient dynamic response computation. The computation is performed after the rotor/bearing system has been suitably modeled and the foundation base has been subjected to a motion that simulates an earthquake. From these computations the designer checks the following, whether

- the lubricant fluid film preserves a minimum thickness at all times so that the rotor and bearing surfaces do not rub against each other

- the dynamic stresses induced in the rotor stay within allowable limits

- the bearing reaction forces can be adequately withstood by the supporting structures

Dynamic response computations in the seismic analysis can be carried out using any of the following methods:

- time history analysis, in which base excitation as well as response are treated in the time domain

- response spectrum analysis, in which excitation and response are considered in the frequency domain

- spectral density analysis, in which excitation and response are analyzed as random vibrations

All of these methods have been employed in the seismic analysis of rotors.

The difference between the seismic analysis of stationary structures and rotating structures must be noted at this point. Seismic analysis of stationary structures is well-developed and has become a routine practice in industry [1, 2]*. Seismic analysis of rotating components is relatively new; it differs from the seismic analysis of stationary structures in that the additional gyroscopic effects and rotor/bearing interactions must be considerred. It is well known that gyroscopic moments are developed whenever the spin axis of a rotating body is rotated. In rotating machines the spin axis rotates for any of the following reasons:

- overall rotation of the structure supporting the rotating machine

- flexibility of the members supporting the rotor

- differential translational motions of the support points on the rotor.

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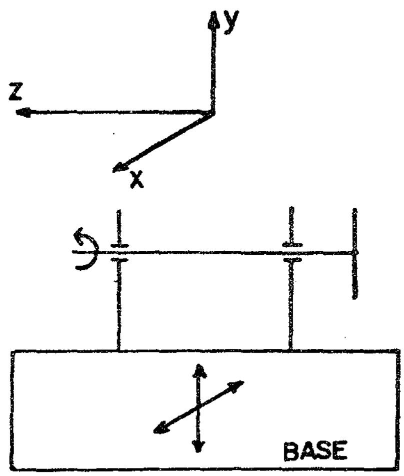
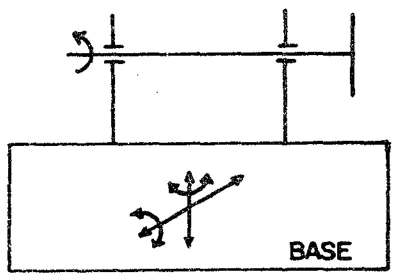
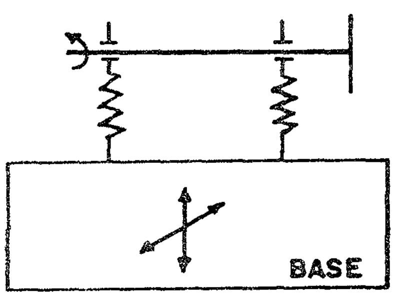
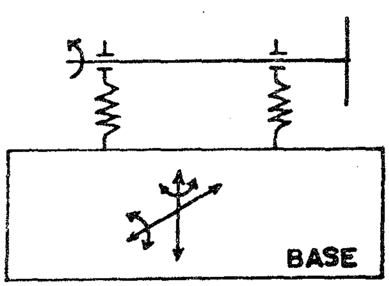
* Numbers in brackets refer to corresponding items under 'References'.

TABLE 1.1 GYROSCOPIC EFFECTS DUE TO BASE MOTION

ROTOR-BEARING SYSTEM	DESCRIPTION	REMARKS
	RIGID ROTOR ON RIGID BEARINGS. BASE TRANSLATION ONLY.	NO GYROSCOPIC EFFECT IS FELT.
	RIGID ROTOR ON RIGID BEARINGS. BASE TRANSLATION AND ROTATION.	GYROSCOPIC EFFECTS ARE PRESENT.
	RIGID ROTOR ON FLEXIBLE BEARINGS. BASE TRANSLATION ONLY.	GYROSCOPIC EFFECTS ARE PRESENT EXCEPT FOR A SYMMETRICAL ROTOR ON IDENTICAL BEARINGS.
	RIGID ROTOR ON FLEXIBLE BEARINGS. BASE TRANSLATION AND ROTATION.	GYROSCOPIC EFFECT ARE PRESENT.

The presence of gyroscopic effects is illustrated in Table 1.1; a simple case of a rigid rotor mounted on two bearings is presented for four cases of base excitation. Three of the four cases involve gyroscopic effects.

A vast literature is available on the general dynamic response of rotors. This review is restricted to models used and results obtained by authors who have specifically addressed the problem of seismic analysis of rotating shafts.

1.2 RIGID BODY MODELS

Useful results have been obtained by modeling the rotor as a spinning rigid body. Some accuracy is sacrificed in ignoring the flexibility of the rotor itself, but such analyses provide certain physical insight into the problem and avoid complex mathematics.

Tessarzik model. The axial dynamic response of a rotating machine supported on a gas thrust bearing and subjected to stationary random environment has been obtained [3]. The rotor/bearing system was modeled by the linear, discrete parameter system shown in Figure 1.2. Only the axial vibration of the rotor was considered; the effect of rotation of the rotor was thus ignored. The film thickness of the gas thrust bearing was the primary concern in the analysis. The theoretical random vibration response compared well with experimental measurements obtained on an actual turbomachine, thereby validating the model.

Nakamura-Asmis model. The dynamic response of a uranium centrifuge subjected to seismic excitation has been obtained [4]. The centrifuge was modeled as a rigid body spinning in three-dimensional space (see Figure 1.3)

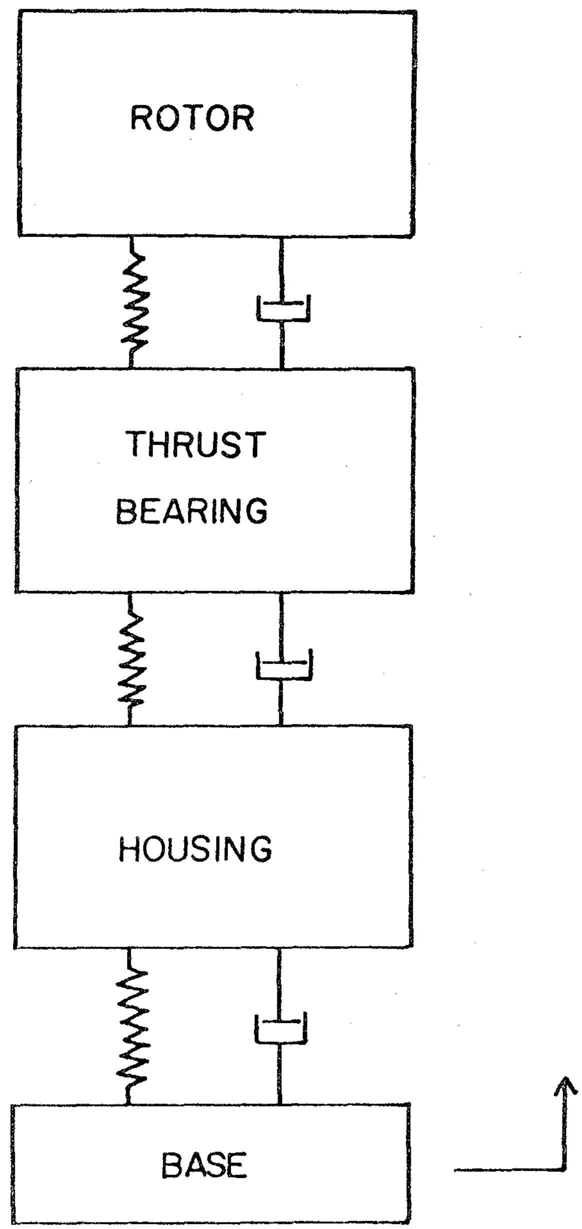


FIG. **1.2** TESSARZIK MODEL

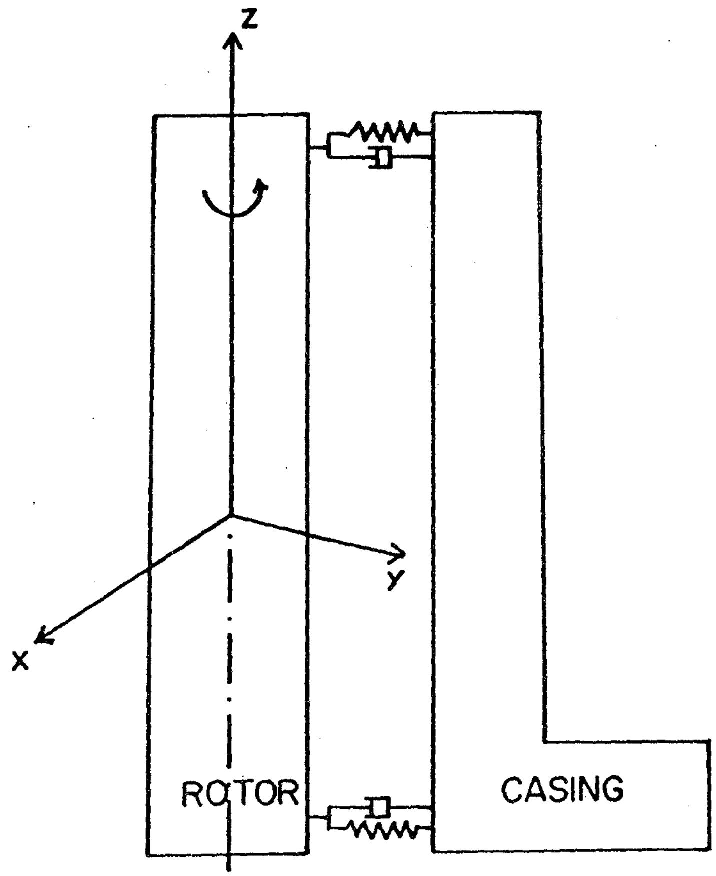


FIG.1.3 NAKAMURA-ASMIS MODEL

Rotor/bearing interactions were modeled by two sets of orthogonal springs and dashpots at each of the two bearing locations. Loci of the journal centers were obtained for Taft and El Centro earthquake excitations. The uranium centrifuge that was analyzed was found to be safely designed; safety was experimentally confirmed.

A similar model was proposed independently [5, 6] to study the dynamic response of a heat transport pump in the CANDU reactor. Response was obtained as a function of time using numerical integration of the governing equations. Responses were obtained for a unit step base excitation and the El Centro earthquake excitation. Gyroscopic effects were found to be of considerable importance. Contrary to common belief, it was found that gyroscopic effects did not necessarily strengthen or reduce motion of the assembly in the direction of excitation. It was also found that the gyroscopically-induced forces could be kept within reasonable values by providing close fitting, stiff supports. It was suggested that gyroscopically-induced forces could be minimized by mounting the equipment so that the external forces excited only the translational modes.

A rigid body model similar to that described above has been proposed [7] to obtain the transient dynamic response to seismic excitation.

Schweitzer-Iwatsubo model. A refined model for a rotating system can be obtained if the lubricant fluid film and the bearing support are modeled separately as springs and dashpots. Such a model facilitates inclusion of the influence of bearing masses in the analysis. One such model has been proposed [8] and used [9] for seismic analysis (see Figure 1.4). Emphasis was on reliability analysis. The earthquake excitation was treated as a nonstationary random process. Such physical parameters as mass, stiffness, and damping coefficients were allowed to vary randomly from their design

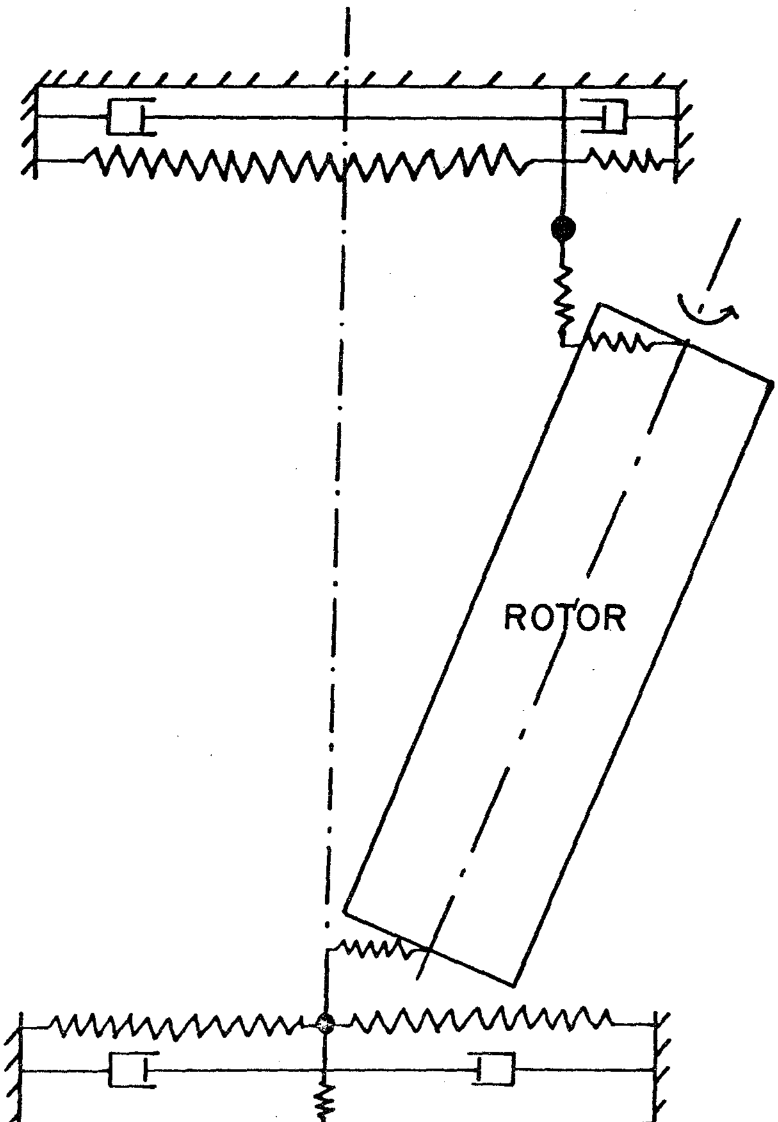


FIG. 1.4 SCHWEITZER-IWATSUBO MODEL

values. The authors used the principles of random vibration to obtain the displacement response, system failure probability, and the period of first collision with the guard.

Dynamic response of gyroscopes to base excitation has been studied [10, 11], but these studies were restricted to gyroscopes. Rotor/bearing interaction effects were not included in these anayses. Such studies are less likely to be of interest to designers of rotating shafts.

1.3 BEAM MODELS

A more realistic model for the rotor is obtained if rotor flexibility is included in the analysis. In a limited number of studies, the rotor has been modeled as a beam for seismic analysis.

Villasor model. The dynamic response of a reactor coolant pump to earthquake excitation has been obtained [12] using the ANSYS finite element computer program. A major feature of this work was the use of beam, spring, and fluid elements to model the rotor and all of its supporting members (see Figure 1.5). The effect of rotation is not included in the analysis. The seismic analysis was performed using the response spectrum method; seismic velocity was the input excitation parameter. Nodal stresses and displacements were obtained. It was concluded that the reactor coolant pump was adequately designed to withstand the imposed seismic loading.

Lund model. An important element in the seismic analysis of rotating systems is the proper inclusion of rotor/bearing interaction effects. The nature of interaction is complicated by the fact that the restoring force

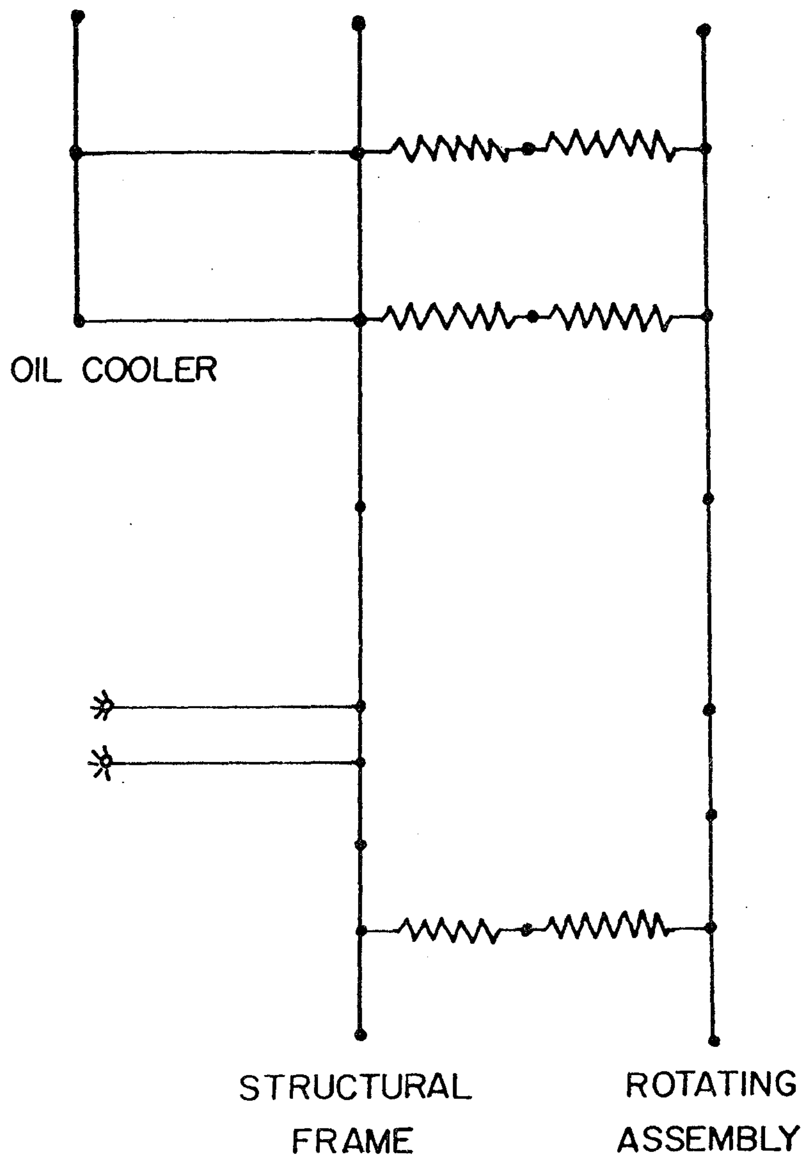


FIG. 1.5 VILLASOR MODEL

acting on the rotor in a fluid film is not collinear with the perturbing force. It is therefore necessary to use at least four stiffness and damping coefficients -- two collinear and two cross-coupled in each case -- to describe the dynamic characteristics of a fluid-film journal bearing [13].

The damping coefficients for the fluid-film bearing are symmetric, but the stiffness coefficients are not symmetric [14, 15]. This important aspect of the problem has been recognized by Lund [16]. He proposed a beam model for the seismic analysis of a rotor that includes shear deformation, rotatory inertia, gyroscopic moments, internal hysteresis damping, and rotor-bearing interaction effects (see Figure 1.6). The vertical amplitude response of the rotor due to foundation shock pulse and to random excitation were obtained using a modal method developed earlier [17].

Shimogo model. The seismic response of a rotor supported on two bearings has been obtained [18] by modeling the rotor as a rigid rotor, a flexible rotor with distributed mass, and a flexible rotor with lumped mass (see Figure 1.7). The seismic excitations acting on the two bearings were assumed to be stationary Gaussian random processes. The rotor-bearing interaction was properly modeled, as done earlier by Lund. The authors concluded that the flexibility of the rotor should be taken into account in the seismic analysis for proper estimation of the bearing reaction forces.

1.4 SUMMARY OF REVIEW

The need to design reliable machines for earthquakes environments has focused attention on the transient dynamic response of rotating machines to

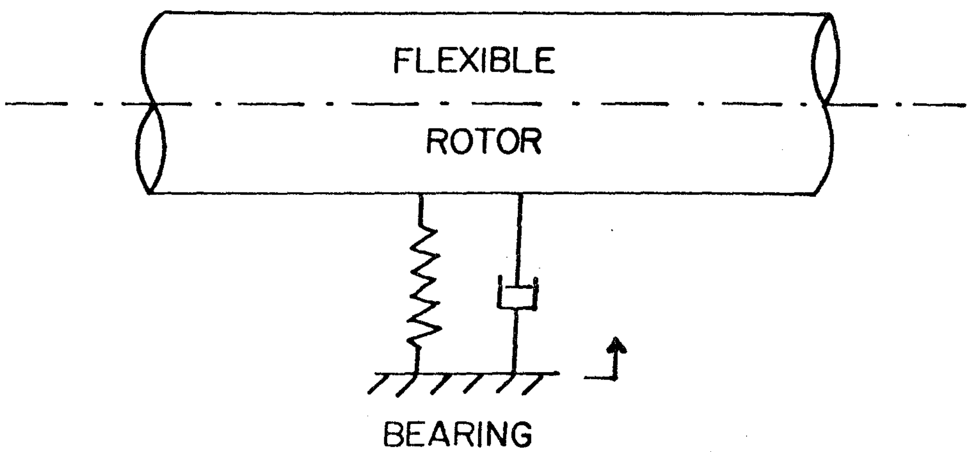


FIG. 1.6 LUND MODEL

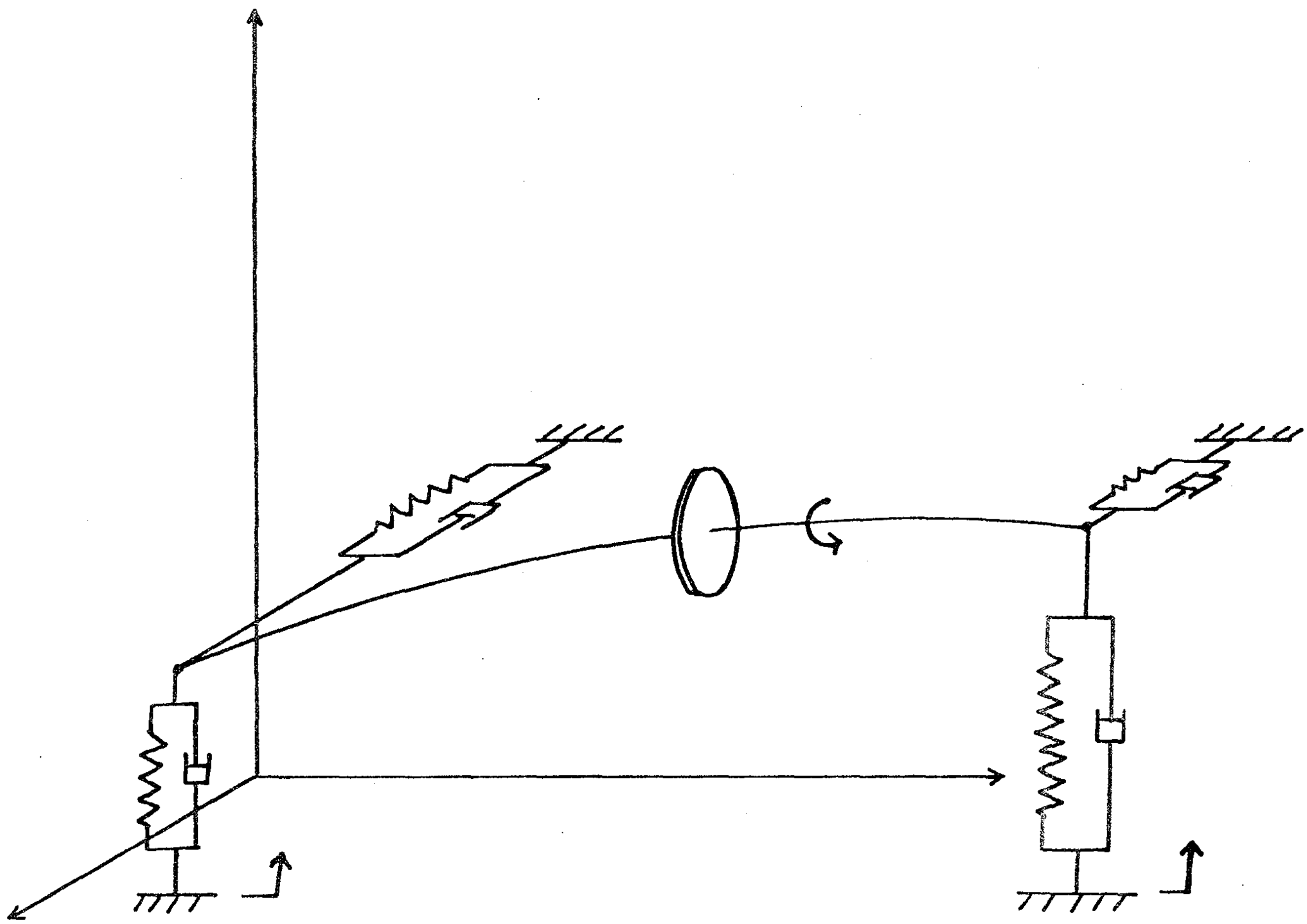


FIG. 1.7 SHIMOGO MODEL

base motions. This type of analysis differs significantly from traditional structural dynamic analysis because of the presence of gyroscopic effects and rotor-bearing interaction effects.

A rigid body model for the rotor spinning in three-dimensional space seems to be satisfactory for predicting lubrication film thickness and bearing reaction forces when the rotor is supported on only two bearings. In all the models reported in this review, the base is subjected only to translational excitations. A rotating machine mounted on a structure would, however, be subjected to base rotations as well as base translations in an earthquake.

When the rotor is supported on more than two bearings or the stresses and deflections in the rotor are to be estimated, the beam model should be used. Existing beam models reported in this review do not include the effects of base rotation.

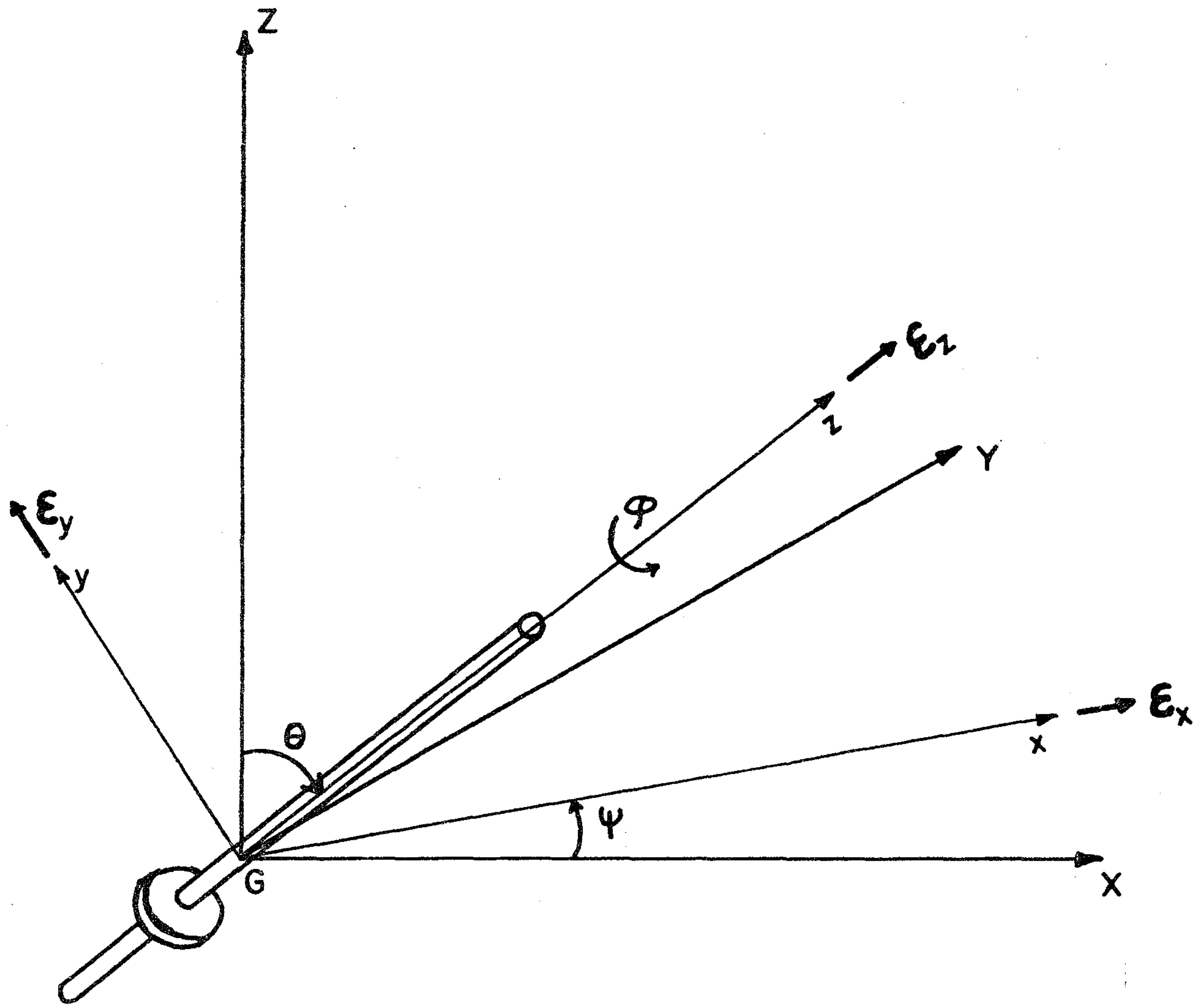


FIG.2.1EULER ANGLES FOR THE GENERAL MOTION OF A RIGID ROTOR

2. RIGID BODY MODEL

2.1 SCOPE OF CHAPTER

In this chapter, we present a seismic analysis in which the rotating system is modeled as a spinning rigid body. A rigid body model represents the first order of approximation in our analysis. It includes such factors as gyroscopic effects, rotor-bearing interaction effects, Coriolis effects due to base rotation and the effects of base translation. The dynamical problem is formulated using Newton-Euler approach. A numerical example is solved for the case of a typical rotating system and the results are presented in graphical form.

2.2 FORMULATION OF THE PROBLEM

Consider an axially symmetric rigid body spinning about its axis of symmetry and executing arbitrary motion in space. Its motion can be conveniently described using Euler angles as shown in Figure 2.1. XYZ is a reference system which preserves fixed orientation in space (i. e. no rotation) with the center of mass of the rotor as its origin. xyz is another, non-spinning, reference system with its origin at the center of mass of the rotor, but xyz can execute precessional (ψ) and nutational (θ) motion. In addition to these precessional and nutational motions, the rigid rotor can possess a spin (ϕ) motion about the z-axis of the xyz reference system.

The Newton's Law of Motion for the rigid body can be written vectorially as

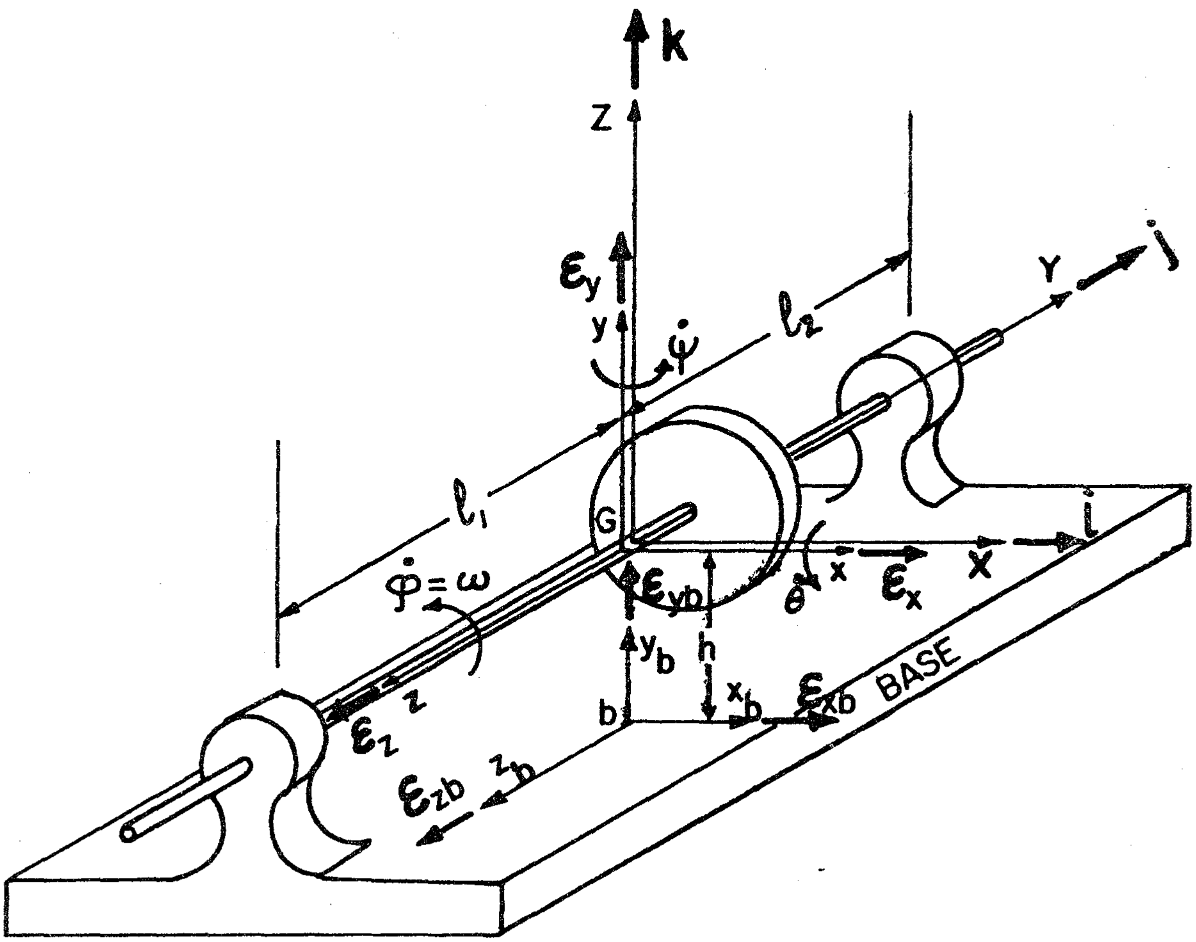


FIG.2.2 ROTOR AND BASE REFERENCE AXES

$$\underset{\sim}{F} = m\underset{\sim}{a}_G$$

and (2.1)

$$\underset{\sim}{M}_G = \dot{\underset{\sim}{H}}_G$$

where $\underset{\sim}{F}$ is the resultant force acting on the rotor, $\underset{\sim}{a}_G$ is the absolute acceleration of the center of mass of the rotor, $\underset{\sim}{M}_G$ is the moment due to exernal forces taken about the center of mass and $\underset{\sim}{H}_G$ is the angular momentum of the rotor computed about its center of mass. A general expression for the time rate of change of the angular momentum can be derived as [19, 20]

$$\begin{aligned}\dot{\underset{\sim}{H}}_G = &\{I_0\ddot{\theta} + I\,\dot{\phi}\dot{\psi}\ \mathrm{Sin}\ \theta + (I - I_0)\,\dot{\psi}^2\ \mathrm{Sin}\ \theta\ \mathrm{Cos}\ \theta\}\ \underset{\sim}{\varepsilon}_x \\ &+ \{I_0\ddot{\psi}\ \mathrm{Sin}\ \theta - I\,\dot{\phi}\dot{\theta} + (2I_0 - I)\,\dot{\psi}\dot{\theta}\ \mathrm{Cos}\ \theta\}\ \underset{\sim}{\varepsilon}_y \\ &+ \{I\ddot{\phi} + I\,\ddot{\psi}\ \mathrm{Cos}\ \theta - I\,\dot{\psi}\dot{\theta}\ \mathrm{Sin}\ \theta\}\ \underset{\sim}{\varepsilon}_z\end{aligned} \qquad (2.2)$$

where $\underset{\sim}{\varepsilon}_x$, $\underset{\sim}{\varepsilon}_y$ and $\underset{\sim}{\varepsilon}_z$ are the unit vectors along the x, y and z axes. I is the moment of inertia of the rotor about the z-axis and I_0 is the moment of inertia about the x- or y-axis. A detailed derivation of (2.2) is given in Appendix A.

Let us consider the case when the xyz reference system assumes an orientation with $\theta \cong \pi/2$ and $\psi \cong 0$ as shown in Figure 2.2. The rotor is supported on two bearings and the bearing-base unit will be considered as another rigid body with a body-fixed reference system $x_b y_b z_b$. The origin b of the $x_b y_b z_b$ coordinate system is so chosen that in equilibrium position point G lies on the y_b axis. The lubricants in the bearings provide stiffness and damping for the relative motion between the rotor and the bearings. In the seismic analysis of such a rotor-bearing system, the base is subjected to known translational and rotational motion. The analyst aims at predicting the translational and rotational response of the rotor.

The base excitation due to earthquake results in small, perturbational rotations and translations of the base about its equilibrium position. The rotor responds with small, perturbational rotations and translations of the xyz reference system from the position of $\theta = \pi/2$ and $\psi = 0$ as shown in Figure 2.2 Let $\underset{\sim}{\omega}_b$ be the known angular velocity and $\underset{\sim}{\alpha}_b$ be the known angular acceleration of the base given by

$$\underset{\sim}{\omega}_b = \dot{\theta}_{xb}\, \underset{\sim}{\varepsilon}_{xb} + \dot{\theta}_{yb}\, \underset{\sim}{\varepsilon}_{yb} + \dot{\theta}_{zb}\, \underset{\sim}{\varepsilon}_{zb}$$

$$\underset{\sim}{\alpha}_b = \ddot{\theta}_{xb}\, \underset{\sim}{\varepsilon}_{xb} + \ddot{\theta}_{yb}\, \underset{\sim}{\varepsilon}_{yb} + \ddot{\theta}_{zb}\, \underset{\sim}{\varepsilon}_{zb} \tag{2.3}$$

The small, perturbational translations of the center of mass of the rotor relative to the $x_b y_b z_b$ reference system can be specified by the displacements x_G, y_G and z_G along the x_b, y_b and z_b axes. Similarly, the small, perturbational rotations of the xyz system relative to the $x_b y_b z_b$ reference system can be specified by the small rotations θ_x, θ_y and θ_z about the x_b, y_b and z_b axes and the sequence in which these rotations take place becomes immaterial. Since the rotations of the base θ_{xb}, θ_{yb} and θ_{zb} about x_b, y_b and z_b axes are also small, perturbational motions, it can be taken that

$$\underset{\sim}{\varepsilon}_{xb} \cong \underset{\sim}{\varepsilon}_{x} \cong \underset{\sim}{i}$$

$$\underset{\sim}{\varepsilon}_{yb} \cong \underset{\sim}{\varepsilon}_{y} \cong \underset{\sim}{k} \qquad (2.4)$$

$$\text{and} \quad \underset{\sim}{\varepsilon}_{zb} \cong \underset{\sim}{\varepsilon}_{z} \cong -\underset{\sim}{j}$$

This leads to the approximate expressions

$$\theta = \pi/2 + \theta_{xb} + \theta_{x} \quad , \psi = \theta_{yb} + \theta_{y}$$

$$\dot{\theta} = \dot{\theta}_{xb} + \dot{\theta}_{x} \quad , \dot{\psi} = \dot{\theta}_{yb} + \dot{\theta}_{y} \qquad (2.5)$$

$$\ddot{\theta} = \ddot{\theta}_{xb} + \ddot{\theta}_{x} \quad , \ddot{\psi} = \ddot{\theta}_{yb} + \ddot{\theta}_{y}$$

In addition, it will be assumed that the spin velocity of the rotor remains constant so that

$$\dot{\phi} = \omega (\text{a constant}) \text{ and } \ddot{\phi} = 0 \qquad (2.6)$$

Substituting (2.5) and (2.6) in equation (2.2) and retaining only the first order terms we get the linearized expression

$$\dot{\underset{\sim}{H}}_{G} = \{I_0 \, (\ddot{\theta}_{xb} + \ddot{\theta}_{x}) + I\omega(\dot{\theta}_{yb} + \dot{\theta}_{y})\} \, \underset{\sim}{\varepsilon}_{xb}$$
$$+ \{I_0 \, (\ddot{\theta}_{yb} + \ddot{\theta}_{y}) - I\omega \, (\dot{\theta}_{xb} + \dot{\theta}_{x})\} \, \underset{\sim}{\varepsilon}_{yb} \qquad (2.7)$$

In the above expression, terms involving $I\omega$ are the familiar gyroscopic moments caused by the rotation of the spin axis.

The absolute acceleration of the point G can be obtained by considering the motion of the point b and the relative motion of G with respect to the $x_b y_b z_b$ reference system. Even though the unit vectors in various reference systems shown in Figure 2.2 can be approximately equated to their counterparts as shown in equations (2.4), their time derivatives cannot be equated in a similar manner. Hence,

$$\underset{\sim}{a}_G = \underset{\sim}{a}_b + \underset{\sim}{\omega}_b \times (\underset{\sim}{\omega}_b \times \underset{\sim}{r}) + \underset{\sim}{\alpha}_b \times \underset{\sim}{r} + 2\, \underset{\sim}{\omega}_b \times \underset{\sim}{V}_{rel} + \underset{\sim}{a}_{rel} \qquad (2.8)$$

where

$$\underset{\sim}{a}_b = \ddot{X}_b\, \underset{\sim}{\varepsilon}_{xb} + \ddot{Y}_b\, \underset{\sim}{\varepsilon}_{yb} + \ddot{Z}_b\, \underset{\sim}{\varepsilon}_{zb}$$

$$\underset{\sim}{r} = x_G \underset{\sim}{\varepsilon}_{xb} + (h + y_G)\, \underset{\sim}{\varepsilon}_{yb} + z_G \underset{\sim}{\varepsilon}_{zb} \qquad (2.9)$$

$$\underset{\sim}{V}_{rel} = \dot{x}_G \underset{\sim}{\varepsilon}_{xb} + \dot{y}_G \underset{\sim}{\varepsilon}_{yb} + \dot{z}_G \underset{\sim}{\varepsilon}_{zb}$$

and $$\underset{\sim}{a}_{rel} = \ddot{x}_G \underset{\sim}{\varepsilon}_{xb} + \ddot{y}_G \underset{\sim}{\varepsilon}_{yb} + \ddot{z}_G \underset{\sim}{\varepsilon}_{zb}$$

This leads to

$$\underset{\sim}{a}_G = \{\ddot{x}_G - 2\dot{\theta}_{zb}\dot{y}_G + 2\dot{\theta}_{yb}\dot{z}_G - (\dot{\theta}^2_{yb} + \dot{\theta}^2_{zb})x_G + (\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})\, y_G +$$

$$(\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})z_G + \ddot{X}_b + h(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})\}\ \underset{\sim}{\varepsilon}_{xb} +$$

$$\{\ddot{y}_G + 2\dot{\theta}_{zb}\dot{x}_G - 2\dot{\theta}_{xb}\dot{z}_G + (\dot{\theta}_{xb}\dot{\theta}_{yb} + \ddot{\theta}_{zb})x_G -$$

$$(\dot{\theta}^2_{zb} + \dot{\theta}^2_{xb})\, y_G + (\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})z_G + \ddot{y}_b - h(\dot{\theta}^2_{zb} + \dot{\theta}^2_{xb})\}\ \underset{\sim}{\varepsilon}_{yb} +$$

$$\{\ddot{z}_G - 2\dot{\theta}_{yb}\dot{x}_G + 2\dot{\theta}_{xb}\dot{y}_G + (\dot{\theta}_{zb}\dot{\theta}_{xb} - \ddot{\theta}_{yb})\, x_G + (\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb})\, y_G -$$

$$(\dot{\theta}^2_{xb} + \dot{\theta}^2_{yb})\, z_G + \ddot{Z}_b + h(\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb})\}\ \underset{\sim}{\varepsilon}_{zb} \qquad (2.10)$$

The external forces and moments acting on the rigid rotor can be evaluated by considering the rotor-bearing interaction. The nature of the interaction is complicated by the fact that the restoring force acting on the rotor in a fluid film is not collinear with the perturbing force in the $x_b y_b$ plane. Thus a perturbing force in the x_b direction gives rise to restoring forces in both the x_b and y_b directions and vice versa. Therefore, it is necessary to use at least four stiffness and damping coefficients, two collinear and two cross-coupled in each case, to describe the dynamic characteristics of a fluid-film journal bearing [13]. If x_i, y_i and z_i are the displacements of the rotor relative to the i^{th} bearing along the x_b, y_b and z_b axes, then the forces acting on the rotor at the i^{th} station can be written as

$$\underset{\sim}{F}_i = -(k_{xxi}x_i + k_{xyi}y_i + c_{xxi}\dot{x}_i + c_{xyi}\dot{y}_i)\,\underset{\sim}{\varepsilon}_{xb}$$

$$-(k_{yxi}x_i + k_{yyi}y_i + c_{yxi}\dot{x}_i + c_{yyi}\dot{y}_i)\,\underset{\sim}{\varepsilon}_{yb} \qquad (2.11)$$

$$-(k_{zzi}z_i + c_{zzi}\dot{z}_i)\underset{\sim}{\varepsilon}_{zb}$$

Here the damping coefficients may be symmetric ($c_{xyi} = c_{yxi}$) but the stiffness coefficients are not symmetric ($k_{xyi} \neq k_{yxi}$). For oil-lubricated bearings, these coefficients are functions of the rotational speed (they are functions of the bearing Sommerfeld number) [21]. For gas lubricated bearings, they are not only functions of speed but, because of compressibility effects, they also depend on the time history of the rotor motion. Using these coefficients, the external forces and moment acting on the rotor can be written as

$$\underset{\sim}{F} = -\{(k_{xx1} + k_{xx2})\,x_G + (k_{xy1} + k_{xy2})\,y_G + (-\ell_1 k_{xy1} + \ell_2 k_{xy2})\theta_x +$$

$$+ (\ell_1 k_{xx1} - \ell_2 k_{xx2})\theta_y + (c_{xx1} + c_{xx2})\dot{x}_G + (c_{xy1} + c_{xy2})\dot{y}_G +$$

$$(-\ell_1 c_{xy1} + \ell_2 c_{xy2})\dot{\theta}_x + (\ell_1 c_{xx1} - \ell_2 c_{xx2})\dot{\theta}_y\}\,\underset{\sim}{\varepsilon}_{xb}$$

$$-\{(k_{yx1} + k_{yx2})x_G + (k_{yy1} + k_{yy2})y_G + (-\ell_1 k_{yy1} + \ell_2 k_{yy2})\theta_x$$

$$+ (\ell_1 k_{yx1} - \ell_2 k_{yx2})\theta_y + (c_{yx1} + c_{yx2})\dot{x}_G + (c_{yy1} + c_{yy2})\dot{y}_G +$$

$$(-\ell_1 c_{yy1} + \ell_2 c_{yy2})\dot{\theta}_x + (\ell_1 c_{yx1} - \ell_2 c_{yx2})\,\dot{\theta}_y\}\,\underset{\sim}{\varepsilon}_{yb}$$

$$-\{(k_{zz1} + k_{zz2})z_G + (c_{zz1} + c_{zz2})\dot{z}_G\}\,\underset{\sim}{\varepsilon}_{zb} \qquad (2.12)$$

$$
\begin{aligned}
\underset{\sim}{M}_G = - \{ & (-\ell_1 k_{yx1} + \ell_2 k_{yx2}) x_G + (-\ell_1 k_{yy1} + \ell_2 k_{yy2}) y_G \\
& + (\ell_1^2 k_{yy1} + \ell_2^2 k_{yy2})\, \theta_x + (-\ell_1^2 k_{yx1} - \ell_2^2 k_{yx2}) \theta_y \\
& + (-\ell_1 c_{yx1} + \ell_2 c_{yx2})\, \dot{x}_G + (-\ell_1 c_{yy1} + \ell_2 c_{yy2}) \dot{y}_G \\
& + (\ell_1^2 c_{yy1} + \ell_2^2 c_{yy2}) \dot{\theta}_x + (-\ell_1^2 c_{yx1} - \ell_2^2 c_{yx2}) \dot{\theta}_y \} \underset{\sim}{\varepsilon}_{xb} \\
-\{ & (\ell_1 k_{xx1} - \ell_2 k_{xx2}) x_G + (\ell_1 k_{xy1} - \ell_2 k_{xy2})\, y_G \\
& + (-\ell_1^2 k_{xy1} - \ell_2^2 k_{xy2}) \theta_x + (\ell_1^2 k_{xx1} + \ell_2^2 k_{xx2}) \theta_y \\
& + (\ell_1 c_{xx1} - \ell_2 c_{xx2}) \dot{x}_G + (\ell_1 c_{xy1} - \ell_2 c_{xy2}) \dot{y}_G \\
& + (-\ell_1^2 c_{xy1} - \ell_2^2 c_{xy2}) \dot{\theta}_x + (\ell_1^2 c_{xx1} + \ell_2^2 c_{xx2}) \dot{\theta}_y \} \underset{\sim}{\varepsilon}_{yb}
\end{aligned}
\tag{2.13}
$$

Using (2.7), (2.10), (2.12), and (2.13), the governing equations of motion as given by the vector equations (2.1) can be written in convenient matrix form as

$$[M]\,\{\ddot{X}\} + [C]\,\{\dot{X}\} + [K]\,\{X\} = \{F\} \tag{2.14}$$

where

$$\{X\}^T = \lfloor x_G \;\; y_G \;\; z_G \;\; \theta_x \;\; \theta_y \; \rfloor \tag{2.15}$$

[M] is a diagonal mass matrix given by

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & I_o & 0 \\ 0 & 0 & 0 & 0 & I_o \end{bmatrix} \tag{2.16}$$

Matrices [C] and [K] and vector {F} can be further subdivided as

$$[C] = [C_1] + [C_2] + [C_3]$$

$$[K] = [K_1] + [K_2] \tag{2.17}$$

$$\{F\} = \{F_1\} + \{F_2\}$$

$[C_1]$ is a symmetrical damping matrix given by

$$[C_1] = \begin{bmatrix} (c_{xx1} + c_{xx2}) & (c_{xy1} + c_{xy2}) & 0 & (-\ell_1 c_{xy1} + \ell_2 c_{xy2}) & (\ell_1 c_{xx1} - \ell_2 c_{xx2}) \\ (c_{yx1} + c_{yx2}) & (c_{yy1} + c_{yy2}) & 0 & (-\ell_1 c_{yy1} + \ell_2 c_{yy2}) & (\ell_1 c_{yx1} - \ell_2 c_{yx2}) \\ 0 & 0 & (c_{zz1} + c_{zz2}) & 0 & 0 \\ (-\ell_1 c_{yx1} + \ell_2 c_{yx2}) & (-\ell_1 c_{yy1} + \ell_2 c_{yy2}) & 0 & (\ell_1^2 c_{yy1} + \ell_2^2 c_{yy2}) & -(\ell_1^2 c_{yx1} + \ell_2^2 c_{yx2}) \\ (\ell_1 c_{xx1} - \ell_2 c_{xx2}) & (\ell_1 c_{xy1} - \ell_2 c_{xy2}) & 0 & -(\ell_1^2 c_{xy1} + \ell_2^2 c_{xy2}) & (\ell_1^2 c_{xx1} + \ell_2^2 c_{xx2}) \end{bmatrix} \quad (2.18)$$

$[C_2]$ and $[C_3]$ are due to gyroscopic and Coriolis effects, respectively, and both are skew-symmetric. They are given by

$$[C_2] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I\omega \\ 0 & 0 & 0 & -I\omega & 0 \end{bmatrix} \quad (2.19)$$

$$[C_3] = \begin{bmatrix} 0 & -2m\dot{\theta}_{zb} & 2m\dot{\theta}_{yb} & 0 & 0 \\ 2m\dot{\theta}_{zb} & 0 & -2m\dot{\theta}_{xb} & 0 & 0 \\ -2m\dot{\theta}_{yb} & 2m\dot{\theta}_{xb} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (2.20)$$

$[K_1]$ is the stiffness matrix due to the fluid film and is given by

$$[K_1] = \begin{bmatrix} (k_{xx1} + k_{xx2}) & (k_{xy1} + k_{xy2}) & 0 & (-\ell_1 k_{xy1} + \ell_2 k_{xy2}) & (\ell_1 k_{xx1} - \ell_2 k_{xx2}) \\ (k_{yx1} + k_{yx2}) & (k_{yy1} + k_{yy2}) & 0 & (-\ell_1 k_{yy1} + \ell_2 k_{yy2}) & (\ell_1 k_{yx1} - \ell_2 k_{yx2}) \\ 0 & 0 & (k_{zz1} + k_{zz2}) & 0 & 0 \\ (-\ell_1 k_{yx1} + \ell_2 k_{yx2}) & (-\ell_1 k_{yy1} + \ell_2 k_{yy2}) & 0 & (\ell_1^2 k_{yy1} + \ell_2^2 k_{yy2}) & -(\ell_1^2 k_{yx1} + \ell_2^2 k_{yx2}) \\ (\ell_1 k_{xx1} - \ell_2 k_{xx2}) & (\ell_1 k_{xy1} - \ell_2 k_{xy2}) & 0 & -(\ell_1^2 k_{xy1} + \ell_2^2 k_{xy2}) & (\ell_1^2 k_{xx1} + \ell_2^2 k_{xx2}) \end{bmatrix} \tag{2.21}$$

In general $[K_1]$ will be unsymmetrical due to unsymmetry in the fluid film bearing stiffness coefficients ($k_{xyi} \neq k_{yxi}$). $[K_2]$ is the supplementary stiffness matrix due to the base rotation and is given by the unsymmetrical matrix

$$[K_2]= \begin{bmatrix} -m(\dot{\theta}_{yb}^2+\dot{\theta}_{zb}^2) & m(\dot{\theta}_{xb}\dot{\theta}_{yb}-\ddot{\theta}_{zb}) & m(\dot{\theta}_{zb}\dot{\theta}_{xb}+\ddot{\theta}_{yb}) & 0 & 0 \\ m(\dot{\theta}_{xb}\dot{\theta}_{yb}+\ddot{\theta}_{zb}) & -m(\dot{\theta}_{zb}^2+\dot{\theta}_{xb}^2) & m(\dot{\theta}_{yb}\dot{\theta}_{zb}-\ddot{\theta}_{xb}) & 0 & 0 \\ m(\dot{\theta}_{zb}\dot{\theta}_{xb}-\ddot{\theta}_{yb}) & m(\dot{\theta}_{yb}\dot{\theta}_{zb}+\ddot{\theta}_{xb}) & -m(\dot{\theta}_{xb}^2+\dot{\theta}_{yb}^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.22)$$

$\{F_1\}$ is the force vector due to base translation and $\{F_2\}$ is the force vector due to base rotation. They are given by

$$\{F_1\}^T = [-m\ddot{X}_b \quad -m\ddot{Y}_b \quad -m\ddot{Z}_b \quad 0 \quad 0\,] \quad (2.23)$$

$$\{F_2\} = \begin{Bmatrix} -mh(\dot{\theta}_{xb}\dot{\theta}_{yb}-\ddot{\theta}_{zb}) \\ mh\ (\dot{\theta}_{zb}^2+\dot{\theta}_{xb}^2) \\ -mh(\dot{\theta}_{yb}\dot{\theta}_{zb}+\ddot{\theta}_{xb}) \\ -I_0\ddot{\theta}_{xb} - I\omega\,\dot{\theta}_{yb} \\ -I_0\ddot{\theta}_{yb} + I\omega\,\dot{\theta}_{xb} \end{Bmatrix} \quad (2.24)$$

When the excitation is confined only to base translation (i.e. no rotation) $[C_3]$ and $[K_2]$ become null matrices and $\{F_2\}$ becomes a null vector. Then the governing equations reduce to those of Nakamura [4] and Asmis [5, 6] if the cross-coupling terms in the fluid film stiffness and damping matrices are ignored. When both the translation and rotation of base are taken into account, the [C] and [K] matrices become functions of time. It can also be seen that when a symmetrical rotor (i.e. $\ell_1 = \ell_2$) is mounted on identical bearings (i.e. $k_{ij1} = k_{ij2}$ and $C_{ij1} = C_{ij2}$), the translational and rotational motion of the rotor are decoupled and if the base is subjected only to translational excitation, no gyroscopic effect is felt in the rotor motion. This special case has been pointed out in Table 1.1.

In the seismic analysis, solution for $\{X\}$from (2.14) is sought when the rest of the quantities are known.

2.3 NUMERICAL EXAMPLE

The governing equations given by (2.14) can be solved numerically using direct integration approach. Among the many techniques that are available to carry out the numerical integration, the one due to Newmark [22, 23] is highly suitable for seismic analysis. The Newmark's integration scheme is an implicit, unconditionally stable technique and is widely used by seismic engineers. Table 2.1 presents the steps involved in Newmark's integration scheme.

As a numerical examle, the seismic response of a typical rotor-bearing system will now be presented. The parameters of the rotor-bearing system chosen for the analysis are given in Table 2.2. The axial degree of freedom is not considered in the numerical example for the sake of simplicity. The

Table 2.1 Newmark's Integration Scheme

1. Initialize $\{X\}_0$, $\{\dot{X}\}_0$ and $\{\ddot{X}\}_0$ to zero.
2. Set $\delta = 1/2$, $\alpha = 1/4$

$$a_0 = 1/(\alpha \,.\, \Delta t^2) \quad , \quad a_1 = \delta/(\alpha \,.\, \Delta t) \quad , \quad a_2 = 1/(\alpha \,.\, \Delta t)$$

$$a_3 = 1/(2\alpha) - 1 \quad , \quad a_4 = (\delta/\alpha) - 1 \quad , \quad a_5 = (\delta/\alpha - 2) \,.\, \Delta t/2$$

$$a_6 = (1 - \delta) \,.\, \Delta t \quad , \quad a_7 = \delta \,.\, \Delta t$$

3. Calculate

$$\{\hat{F}\}_t = \{F\}_t + [M]_t \,(a_0\{X\}_{t - \Delta t} + a_2\{\dot{X}\}_{t - \Delta t} + a_3 \{\ddot{X}\}_{t - \Delta t})$$
$$+ [C]_t \,(a_1\{X\}_{t - \Delta t} + a_4\{\dot{X}\}_{t-\Delta t} + a_5\{\ddot{X}\}_{t - \Delta t})$$

4. Solve $([K]_t + a_0\, [M]_t + a_1\, [C]_t)\, \{X\}_t = \{\hat{F}\}_t$

5. Compute $\{\ddot{X}\}_t = a_0\, (\{X\}_t - \{X\}_{t - \Delta t}) - a_2\, \{\dot{X}\}_{t - \Delta t} - a_3\, \{\ddot{X}\}_{t - \Delta t}$

$$\{\dot{X}\}_t = \{\dot{X}\}_{t - \Delta t} + a_6\, \{\ddot{X}\}_{t - \Delta t} + a_7\{\ddot{X}\}_t$$

6. Repeat from step 3 for all intervals

Table 2.2 Parameters of the Rotor-Bearing System Analyzed

m	$= 24 \times 10^3$ kg.	ℓ_1	$= 4.52$ m.
I	$= 4.57 \times 10^3$ kg $\cdot$ m^2	ℓ_2	$= 4.74$ m.
I_o	$= 3.60 \times 10^5$ kg$\cdot$ m^2	h	$= 1.00$ m.
		ω	$= 3000$ rpm
k_{xx1}	$= 5.89 \times 10^8$ N/m	k_{xx2}	$= 6.76 \times 10^8$ N/m
k_{xy1}	$= 5.10 \times 10^7$ N/m	k_{xy2}	$= 2.16 \times 10^7$ N/m
k_{yx1}	$= -1.29 \times 10^9$ N/m	k_{yx2}	$= -1.49 \times 10^9$ N/m
k_{yy1}	$= 1.87 \times 10^9$ N/m	k_{yy2}	$= 2.27 \times 10^9$ N/m
c_{xx1}	$= 2.80 \times 10^6$ N $\cdot$ s/m	c_{xx2}	$= 3.10 \times 10^6$ N $\cdot$ s/m
c_{xy1}	$= -4.10 \times 10^6$ N $\cdot$ s/m	c_{xy2}	$= -5.00 \times 10^6$ N $\cdot$ s/m
c_{yx1}	$= -4.10 \times 10^6$ N $\cdot$ s/m	c_{yx2}	$= -5.00 \times 10^6$ N $\cdot$ s/m
c_{yy1}	$= 1.17 \times 10^7$ N $\cdot$ s/m	c_{yy2}	$= 1.37 \times 10^7$ N $\cdot$ s/m

base is subjected to El Centro excitation as shown in Figure 2.3. The data for these translational accelerations are available at intervals of 0.02 seconds [24]. The base is also subjected to simulated angular accelerations as shown in Figure 2.4.

Figure 2.5 presents the displacements of the rotor axis relative to the bearings. Figure 2.6 presents the dynamic forces exerted on the bearings. These forces are in addition to the static weight of the rotor carried by the bearings. The entire computation took only 1.2 seconds of CPU time in IBM System 370/168. Plots such as Figures 2.5 and 2.6 aid the designer in checking whether a minimum lubrication film thickness is maintained and the loads on the bearings are within allowable limits.

A major contribution of the present chapter is the incorporation of the effects of base rotation in the analysis. To assess the importance of the inclusion of base rotation, results were obtained for the case in which the base is subjected only to translational motion given by Figure 2.3. For this case, Figure 2.7 presents the displacements of the rotor in the bearings and Figure 2.8 presents the dynamic forces exerted on the bearings. It can be seen from these figures that the displacements and forces are under-predicted if the effects of base rotation are not included in the response computations.

A computer program called GYROT has been prepared, along with a User's manual, to automate the seismic response computation using the rigid body model presented in this chapter. The User's manual for GYROT and listings of the program can be found in 'Part II: Computer Programs' of this report.

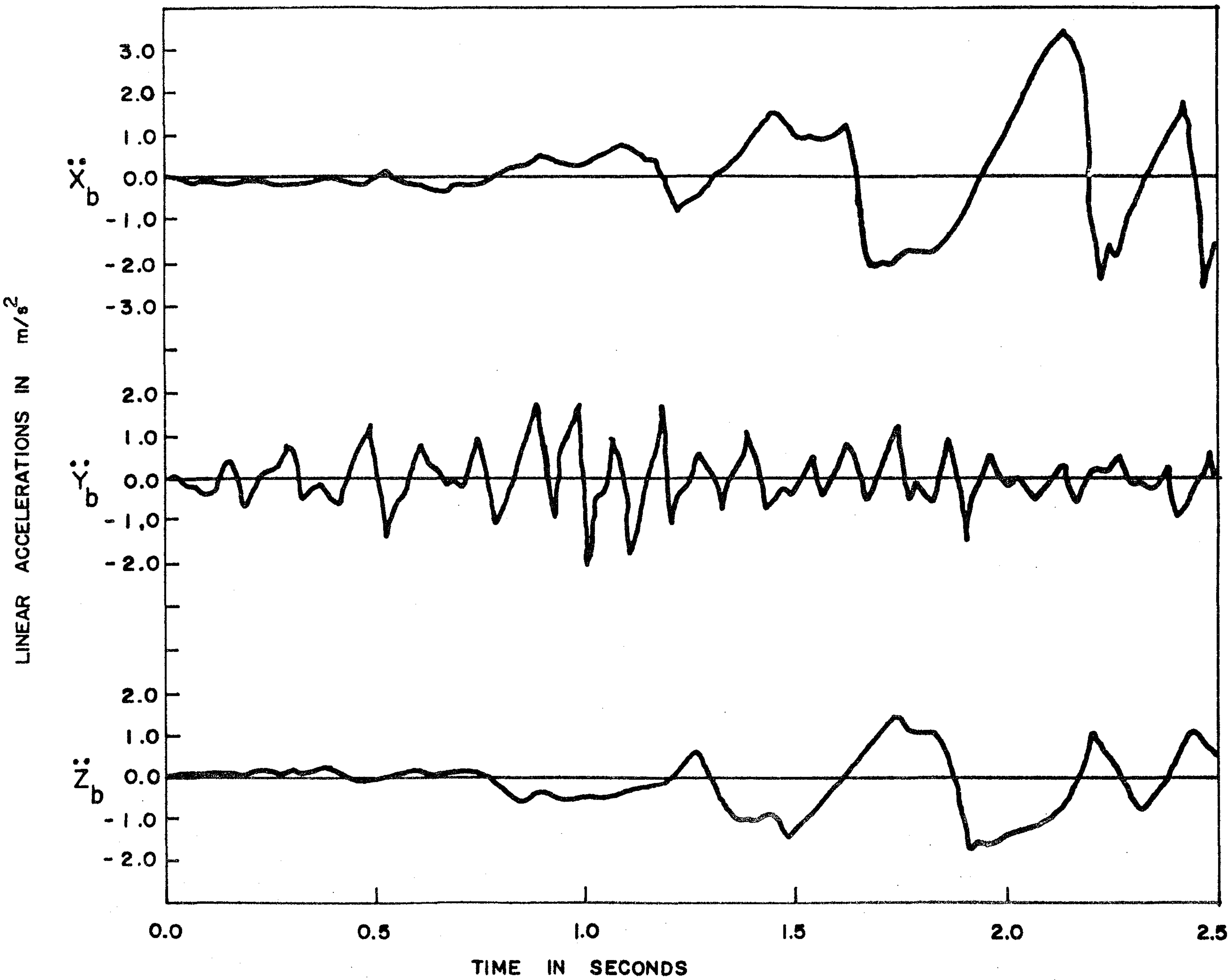


FIG 2.3 LINEAR ACCELERATION OF THE BASE

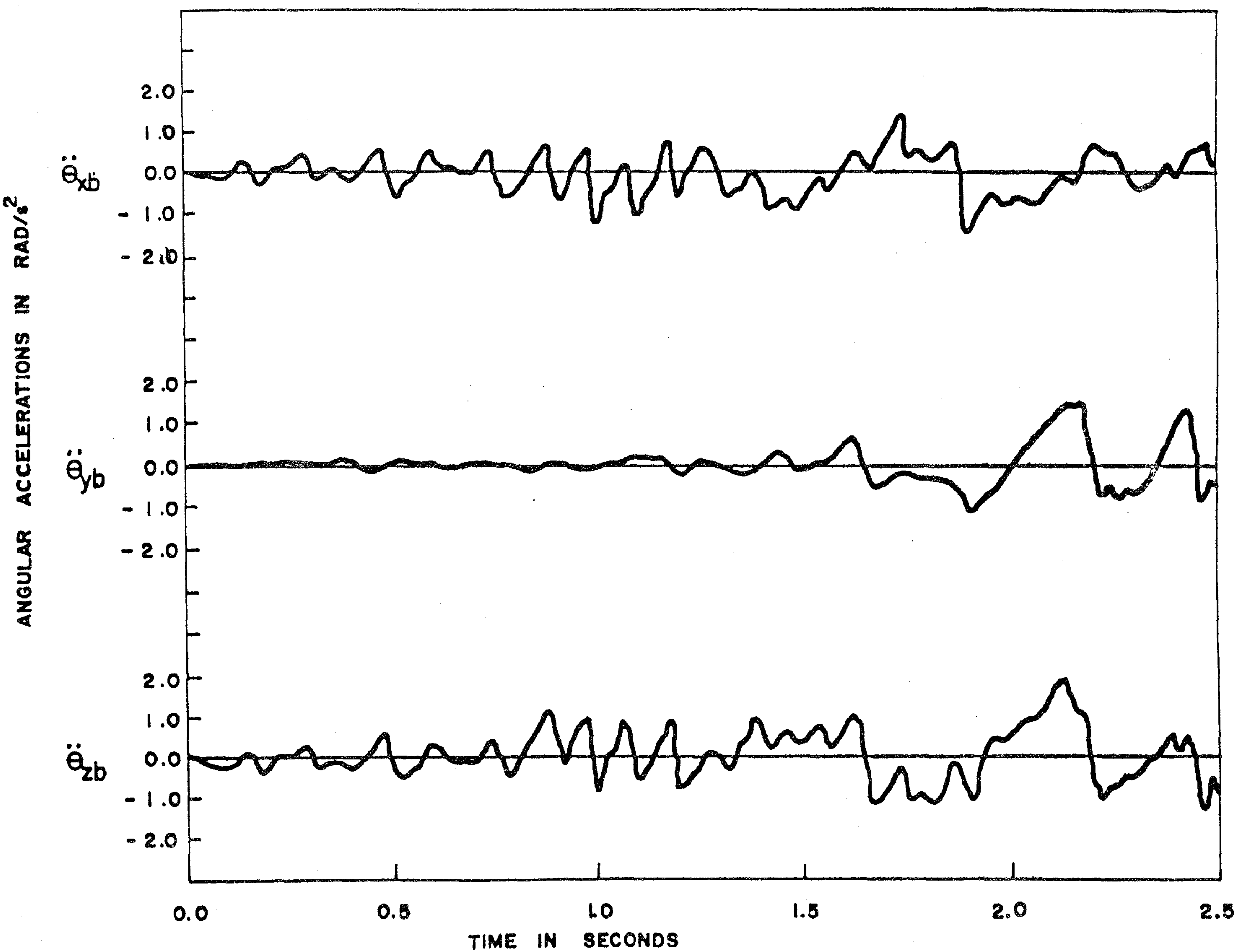


FIG.2.4 ANGULAR ACCELERATION OF THE BASE

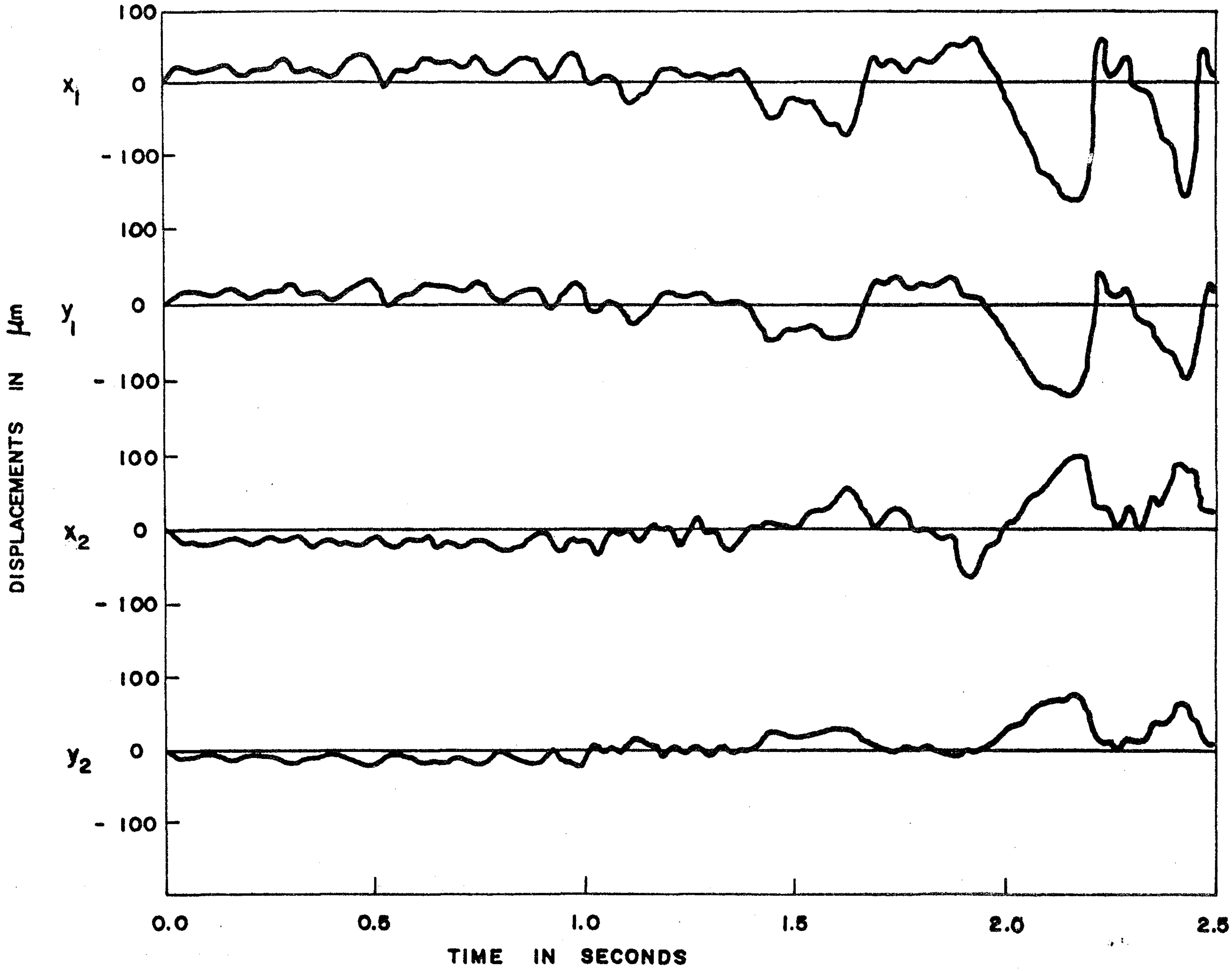


FIG.2.5 DISPLACEMENTS OF ROTOR IN THE BEARINGS

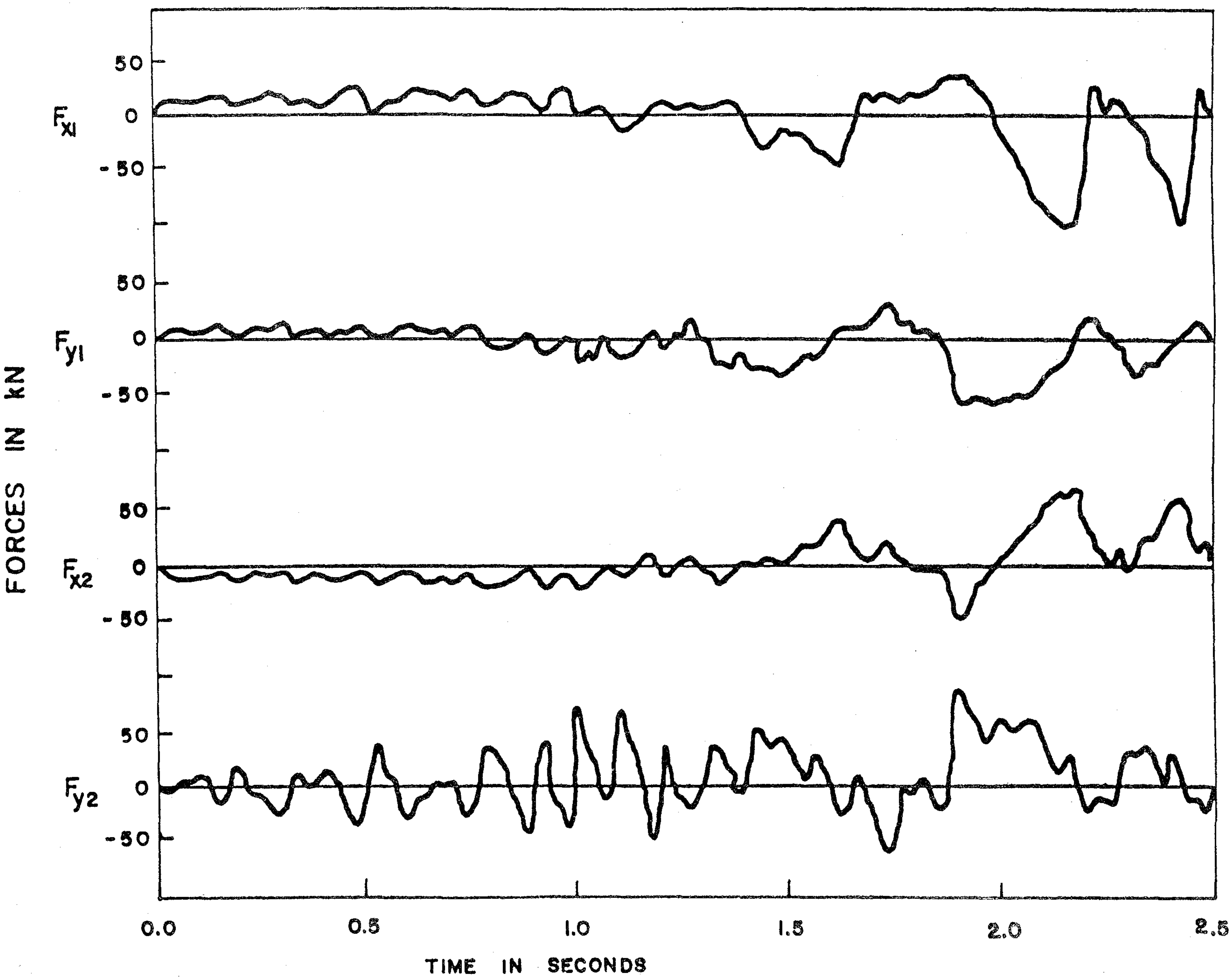


FIG. 2.6 DYNAMIC REACTION FORCES IN THE BEARINGS

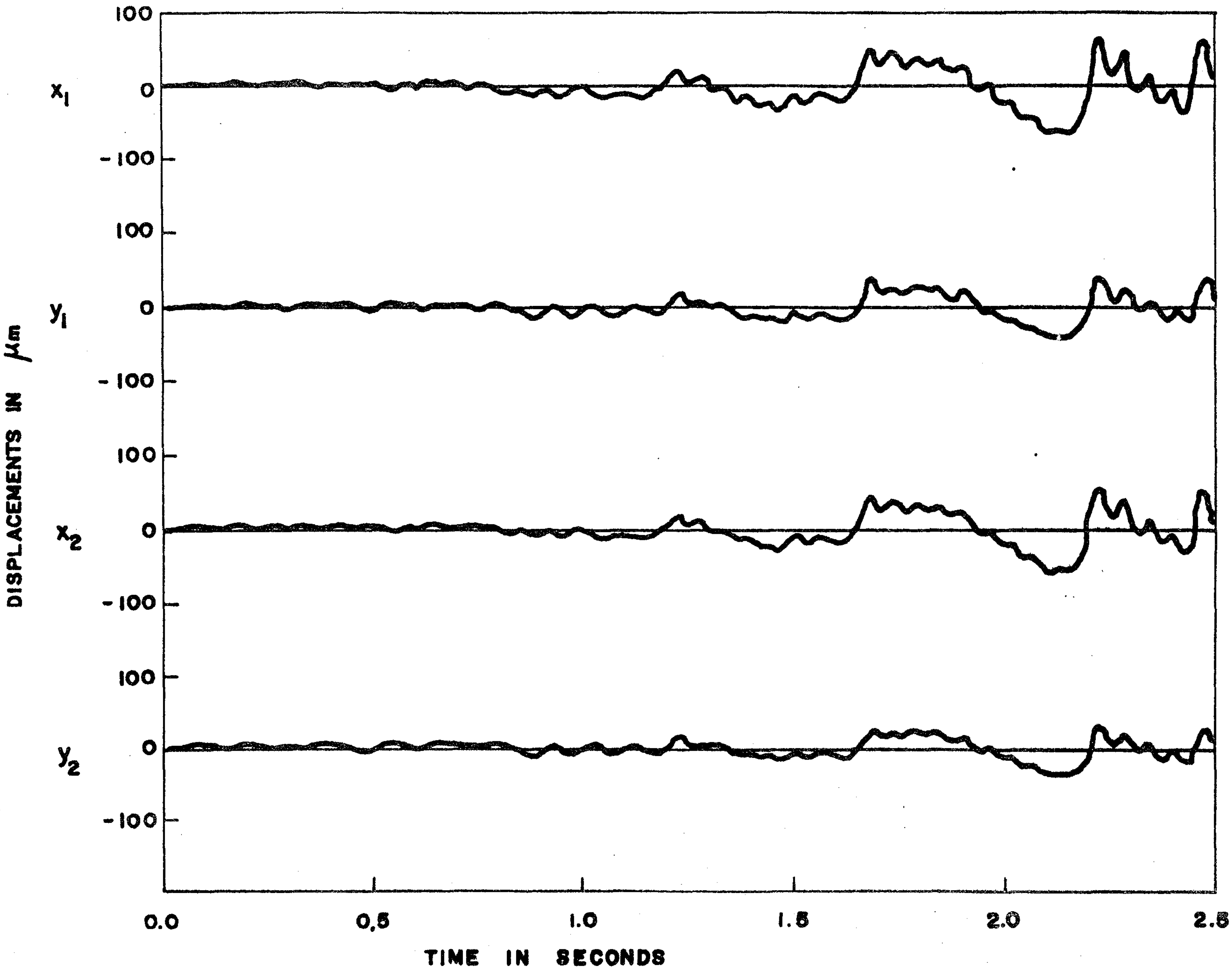


FIG.2.7 DISPLACEMENTS OF ROTOR IN THE BEARINGS

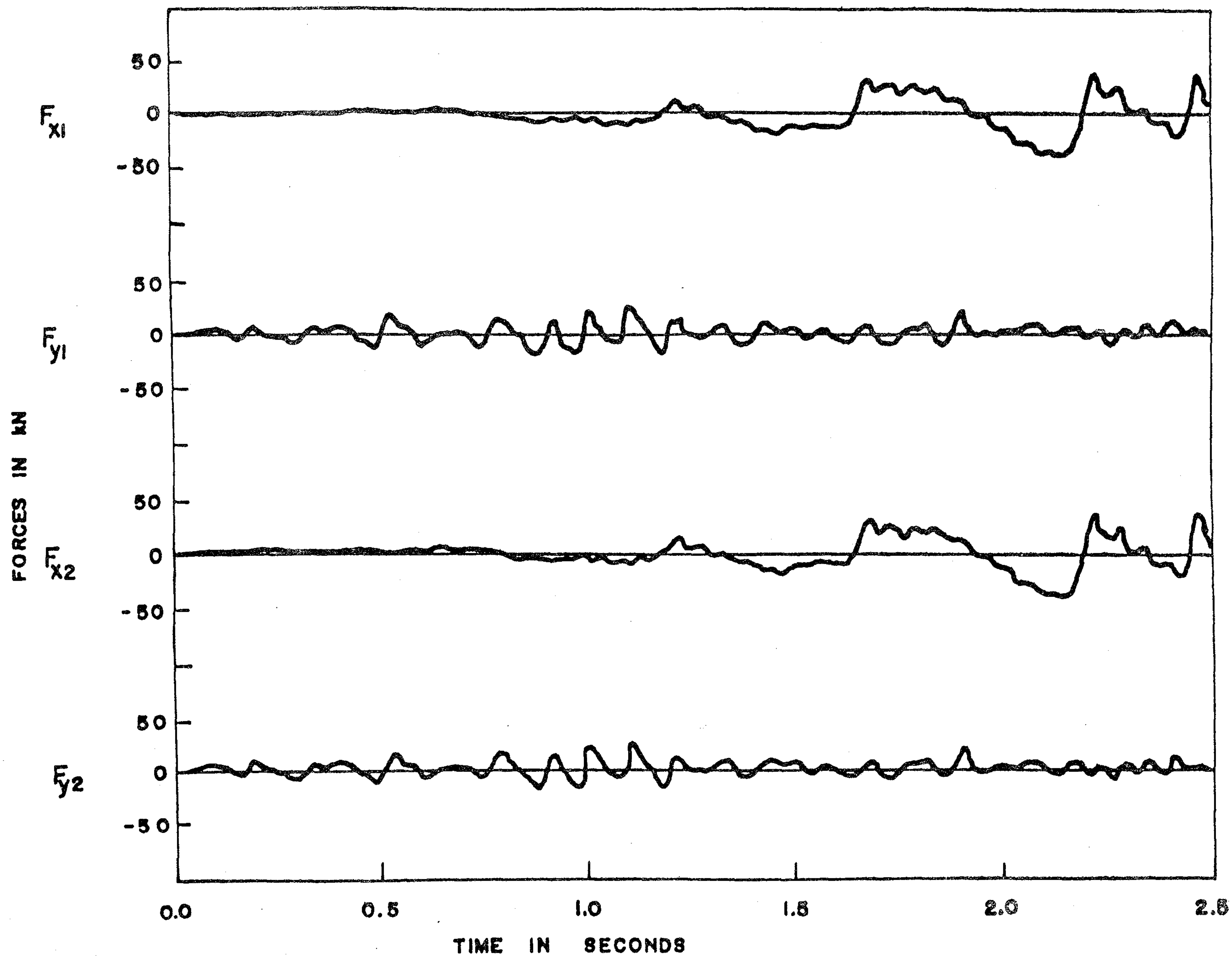


FIG. 2.8 DYNAMIC REACTION FORCES IN THE BEARINGS
(BASE ROTATION EXCLUDED)

2.4 MERITS AND LIMITATIONS OF RIGID BODY MODEL

In this chapter it was shown that factors such as gyroscopic effects, rotor-bearing interaction effects (i.e. stiffness and damping provided by the lubricants in the bearings), and effects of base rotation can be directly and systematically incorporated in the seismic analysis of a rotating mechanical system. Modeling the rotating system as a spinning rigid body enabled us in keeping the mathematical complexities to the minimum and helped us in understanding the role played by the various factors mentioned above. The rigid body model is also computationally economical (i.e. less computational time) and is easy to program.

However, it should be pointed out that in modeling the rotating system as a rigid body we have ignored the flexibility of the body itself. The effects of initial stresses due to axial force, axial torque and spin of the system cannot be included in the rigid body model. Also, if the rotating system is supported on more than two bearings, a rigid body model will not predict the relative motion between the rotor and the bearings correctly.

In the chapters that follow, we will develop models that do not have the above mentioned limitations. This is achieved by including the flexibility of the rotating system in our analysis.

3. BEAM MODEL

3.1 SCOPE OF CHAPTER

Following the development of a rigid body model in the previous chapter, we now present a beam model to predict the dynamic response of a rotating mechanical system. In this beam model, the flexibility of the rotating system is included in the analysis using Timoshenko beam theory. The Timoshenko beam theory is known to be superior to the classical Bernoulli-Euler beam theory in predicting the dynamic response of 'short' as well as 'long' beams. The beam model presented in this chapter includes the following factors:

1. Rotatory inertia
2. Shear deformation
3. Gyroscopic effects
4. Rotor-bearing interaction
5. Intermediate disks and flywheels
6. Axial thrust
7. Axial torque
8. Base translation, and
9. Base rotation.

The dynamical problem is again formulated using Newton-Euler approach.

The governing differential equations are posed in an integral form using Galerkin's method. A numerical solution to the problem is obtained by using finite elements in the spatial domain and finite differences in the temporal domain.

3.2 FORMULATION OF THE PROBLEM

The rotor is considered to be a shaft having circular cross section and is modeled using Timoshenko beam theory. The governing equations of motion of the rotor are derived by isolating an elemental disk of the rotor. This elemental disk will be treated as a rigid body to obtain such kinematic quantities as acceleration and rate of change of angular momentum. The elastic properties of the rotor will be taken into account while evaluating the forces and moments acting on the elemental disk.

3.2.1 KINEMATIC RELATIONS

Consider a rigid, circular elemental disk spinning about its axis and executing arbitrary motion in space. Its motion can be conveniently described using Euler angles as shown in Figure 3.1. XYZ is a reference system which preserves fixed orientation in space (i.e. no rotation) with the center of mass of the elemental disk as its origin. xyz is another, non-spinning reference system with its orign at the center of mass of the rotor, but xyz

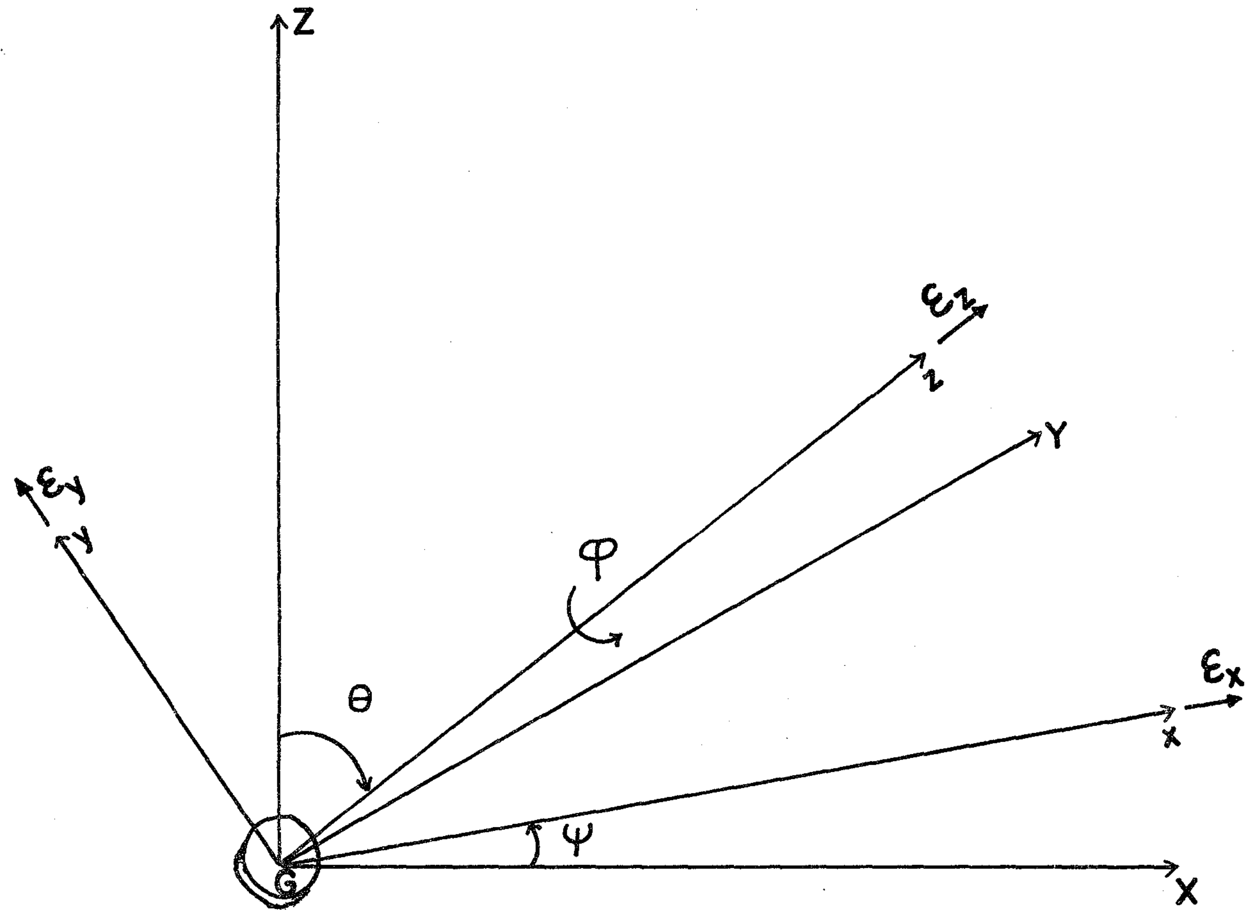


FIG.3.1EULER ANGLES FOR THE GENERAL MOTION OF A RIGID DISK

can execute precessional (ψ) and nutational (θ) motion. In addition to the precessional and nutational motions, the rigid elemental disk can possess a spin (ϕ) motion about the z-axis of the xyz reference system.

The Newton's Law of Motion for the elemental disk can be written vectorially as

$$\underset{\sim}{F} = m\underset{\sim}{a}_G \quad \text{and} \quad \underset{\sim}{M}_G = \dot{\underset{\sim}{H}}_G \tag{3.1}$$

where $\underset{\sim}{F}$ is the resultant force acting on the elemental disk, $\underset{\sim}{a}_G$ is the absolute acceleration of the center of mass of the elemental disk, $\underset{\sim}{M}_G$ is the moment due to external forces taken about the center of mass and $\underset{\sim}{H}_G$ is the angular momentum of the elemental disk computed about its center of mass. A general expression for the time rate of change of the angular momentum can be derived similar to (2.2) as

$$\begin{aligned} \dot{\underset{\sim}{H}}_G = & \rho\{I_T \ddot{\theta} + I_P\dot{\phi}\dot{\psi} \text{ Sin } \theta + I_T \dot{\psi}^2 \text{ Sin } \theta \text{ Cos } \theta\} \text{ ds } \underset{\sim}{\varepsilon}_x \\ & + \rho\{I_T \ddot{\psi}\text{Sin } \theta - I_P \dot{\phi}\dot{\theta}\} \text{ ds } \underset{\sim}{\varepsilon}_y \\ & + \rho\{I_P \ddot{\phi} + I_P\ddot{\psi} \text{ Cos } \theta - I_P \dot{\psi}\dot{\theta} \text{ Sin } \theta\} \text{ ds } \underset{\sim}{\varepsilon}_z \end{aligned} \tag{3.2}$$

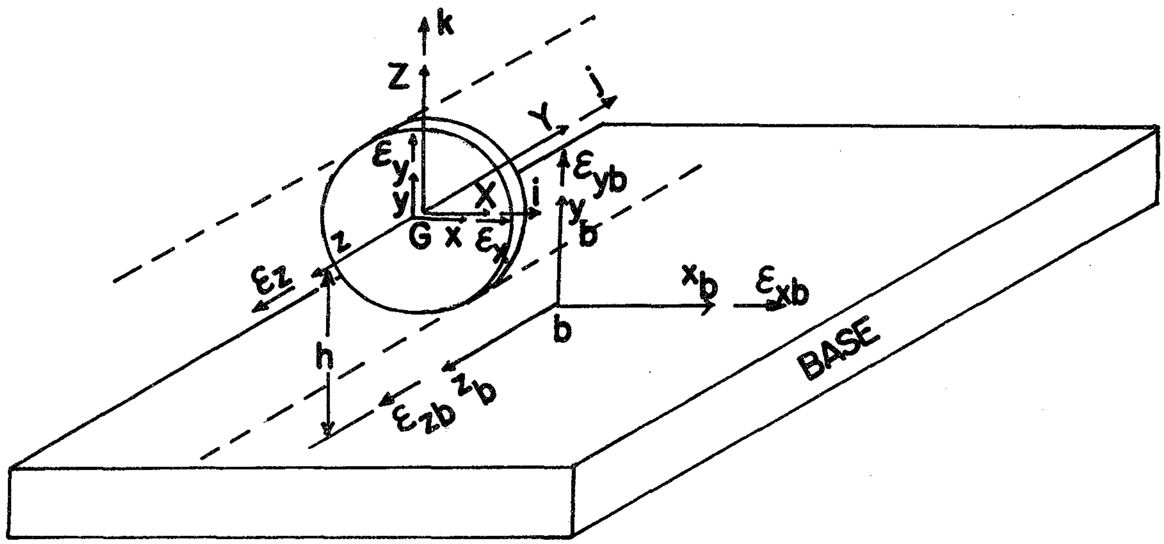


FIG. 3.2 ROTOR AND BASE REFERENCE FRAMES

where $\underset{\sim}{\varepsilon}_x$, $\underset{\sim}{\varepsilon}_y$ and $\underset{\sim}{\varepsilon}_z$ are the unit vectors along the x, y and z axes. I_P is the second moment of the cross sectional area about the z-axis and I_T is the second moment of the cross sectional area about the x- or y-axis. For circular cross sections $I_P = 2I_T$.

Let us consider the case when the xyz reference system assumes an orientation with $\theta \cong \pi/2$ and $\psi \cong 0$ as shown in Figure 3.2 The rotor is supported on bearings and the bearing-base unit will be considered as a rigid body with a body-fixed reference system $x_b y_b z_b$. The origin b of the $x_b y_b z_b$ reference system is so chosen that in equilibrium position the axis of the rotor is parallel to the z_b axis and lies in the $y_b z_b$ plane. The lubricants in the bearings provide stiffness and damping for the relative motion between the rotor and the bearings. In the seismic analysis of such a rotor-bearing system, the base is subjected to known translational and rotational motion. The analyst aims at predicting the transient dynamic response of the rotor.

The base excitation due to earthquake results in small, perturbational rotations and translations of the base about its equilibrium position. The rotor responds with small, perturbational rotations and translations of the xyz reference system from the position of $\theta = \pi/2$ and $\psi = 0$ as shown in Figure 3.2 Let $\underset{\sim}{\omega}_b$ be the known angular velocity and $\underset{\sim}{\alpha}_b$ be the known angular acceleration of the base given by

$$\underset{\sim}{\omega}_b = \dot{\theta}_{xb}\, \underset{\sim}{\varepsilon}_{xb} + \dot{\theta}_{yb}\, \underset{\sim}{\varepsilon}_{yb} + \dot{\theta}_{zb}\, \underset{\sim}{\varepsilon}_{zb}$$

$$\underset{\sim}{\alpha}_b = \ddot{\theta}_{xb}\, \underset{\sim}{\varepsilon}_{xb} + \ddot{\theta}_{yb}\, \underset{\sim}{\varepsilon}_{yb} + \ddot{\theta}_{zb}\, \underset{\sim}{\varepsilon}_{zb} \qquad (3.3)$$

The small, perturbational translations of the center of mass of the elemental disk relative to the $x_b y_b z_b$ reference system can be specified by the displacements u_x, u_y and u_z along the x_b, y_b and z_b axes. Similarly, the small, perturbational rotations of the xyz system relative to the $x_b y_b z_b$ reference system can be specified by the small rotations θ_x, θ_y and θ_z about the x_b, y_b, and z_b axes and the sequence in which these rotations take place become immaterial. Since the rotations of the base θ_{xb}, θ_{yb} and θ_{zb} about the x_b, y_b and z_b axes are also small, perturbational motions, it can be taken that

$$\underset{\sim}{\varepsilon}_{xb} \cong \underset{\sim}{\varepsilon}_{x} \cong \underset{\sim}{i}$$

$$\underset{\sim}{\varepsilon}_{yb} \cong \underset{\sim}{\varepsilon}_{y} \cong \underset{\sim}{k} \qquad (3.4)$$

$$\text{and} \quad \underset{\sim}{\varepsilon}_{zb} \cong \underset{\sim}{\varepsilon}_{z} \cong -\underset{\sim}{j}$$

This leads to the approximate expressions

$$\theta = \pi/2 + \theta_{xb} + \theta_x \quad , \quad \psi = \theta_{yb} + \theta_y$$
$$\dot{\theta} = \dot{\theta}_{xb} + \dot{\theta}_x \quad , \quad \dot{\psi} = \dot{\theta}_{yb} + \dot{\theta}_y \tag{3.5}$$
$$\ddot{\theta} = \ddot{\theta}_{xb} + \ddot{\theta}_x \quad , \quad \ddot{\psi} = \ddot{\theta}_{yb} + \ddot{\theta}_y$$

In addition, it will be assumed that the spin velocity of the rotor remains constant so that

$$\dot{\phi} = \boldsymbol{\omega}(\text{a constant}) \quad \text{and } \phi = 0 \tag{3.6}$$

Substituting (3.5) and (3.6) in equation (3.2) and retaining only the first order terms, we get the linearized expression

$$\begin{aligned} \dot{\underset{\sim}{H}}_G &= \rho\{I_T(\ddot{\theta}_{xb} + \ddot{\theta}_x) + I_p\,\boldsymbol{\omega}(\dot{\theta}_{yb} + \dot{\theta}_y)\}\ ds\ \underset{\sim}{\varepsilon}_{xb} \\ &+ \rho\{I_T(\ddot{\theta}_{yb} + \ddot{\theta}_y) - I_p\,\boldsymbol{\omega}(\dot{\theta}_{xb} + \dot{\theta}_x)\}\ ds\ \underset{\sim}{\varepsilon}_{yb} \end{aligned} \tag{3.7}$$

In the above expression, terms involving $I_p\boldsymbol{\omega}$ are the familiar gyroscopic moments caused by the rotation of the spin axis.

The absolute acceleration of the point G can be obtained by considering the motion of the point b and the relative motion of G with respect to the $x_b y_b z_b$ reference system. Even though the unit vectors in various reference systems shown in Figure 3.2 can be approximately equated to their counterparts as shown in equations (3.4), their time derivatives cannot be equated in a similar manner. Hence,

$$\underset{\sim}{a}_G = \underset{\sim}{a}_b + \underset{\sim}{\omega}_b \times (\underset{\sim}{\omega}_b \times \underset{\sim}{r}) + \underset{\sim}{\alpha}_b \times \underset{\sim}{r} + 2\underset{\sim}{\omega}_b \times \underset{\sim}{V}_{rel} + \underset{\sim}{a}_{rel} \tag{3.8}$$

where

$$\begin{aligned}
\underset{\sim}{a}_b &= \ddot{X}_b \, \underset{\sim}{\varepsilon}_{xb} + \ddot{Y}_b \, \underset{\sim}{\varepsilon}_{yb} + \ddot{Z}_b \, \underset{\sim}{\varepsilon}_{zb} \\
\underset{\sim}{r} &= u_x \underset{\sim}{\varepsilon}_{xb} + (h + u_y) \underset{\sim}{\varepsilon}_{yb} + (s + u_z) \underset{\sim}{\varepsilon}_{zb} \\
\underset{\sim}{V}_{rel} &= \dot{u}_x \underset{\sim}{\varepsilon}_{xb} + \dot{u}_y \underset{\sim}{\varepsilon}_{yb} + \dot{u}_z \underset{\sim}{\varepsilon}_{zb} \\
\underset{\sim}{a}_{rel} &= \ddot{u}_x \underset{\sim}{\varepsilon}_{xb} + \ddot{u}_y \underset{\sim}{\varepsilon}_{yb} + \ddot{u}_z \underset{\sim}{\varepsilon}_{zb}
\end{aligned} \tag{3.9}$$

This leads to

$$a_G = a_x \varepsilon_{xb} + a_y \varepsilon_{yb} + a_z \varepsilon_{zb}$$

where

$$a_x = \ddot{u}_x - 2\dot{\theta}_{zb}\dot{u}_y + 2\dot{\theta}_{yb}\dot{u}_z - (\dot{\theta}^2_{yb} + \dot{\theta}^2_{zb})u_x + (\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})u_y$$
$$+ (\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})u_z + \ddot{X}_b + h(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb}) + s(\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})$$

$$a_y = \ddot{u}_y + 2\dot{\theta}_{zb}\dot{u}_x - 2\dot{\theta}_{xb}\dot{u}_z + (\dot{\theta}_{xb}\dot{\theta}_{yb} + \ddot{\theta}_{zb})u_x - (\dot{\theta}^2_{zb} + \dot{\theta}^2_{xb})u_y$$
$$+ (\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})u_z + \ddot{Y}_b - h(\dot{\theta}^2_{zb} + \dot{\theta}^2_{xb}) + s(\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})$$

and

$$a_z = \ddot{u}_z - 2\dot{\theta}_{yb}\dot{u}_x + 2\dot{\theta}_{xb}\dot{u}_y + (\dot{\theta}_{zb}\dot{\theta}_{xb} - \ddot{\theta}_{yb})u_x + (\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb})u_y$$
$$- (\dot{\theta}^2_{xb} + \dot{\theta}^2_{yb})u_z + \ddot{Z}_b + h(\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb}) - s(\dot{\theta}^2_{xb} + \dot{\theta}^2_{yb}) \quad (3.10)$$

It is worth noting that in the kinematic relations developed in this section, the rotatory inertia, the gyroscopic effects and the base motions (including translation and rotation) have been taken into account.

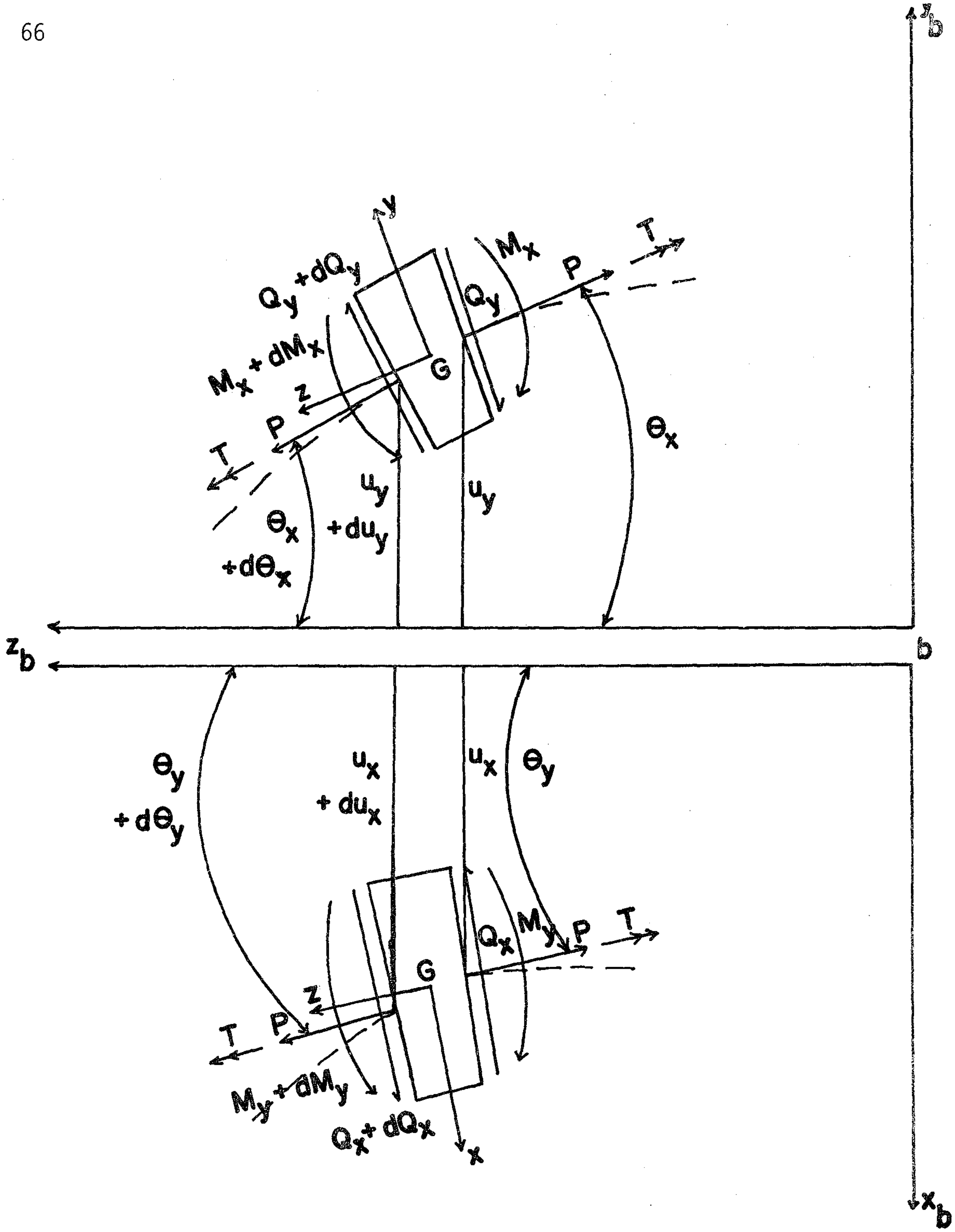


FIG. 3.3 AN ELEMENTAL DISK IN $y_b z_b$ AND $x_b z_b$ PLANES

3.2.2 KINETIC RELATIONS

The free body diagrams of the elemental disk in the $y_b z_b$ and $x_b z_b$ planes are shown in Figure 3.3. It is the flexural motion of the rotor that is of interest to us. Following Timoshenko beam theory [25], the effect of transverse shear can be included in the model by expressing

$$Q_x = kAG\left(\frac{\partial u_x}{\partial s} - \theta_y\right)$$
$$Q_y = kAG\left(\frac{\partial u_y}{\partial s} + \theta_x\right) \qquad (3.11)$$

The moment-curvature relations are given by the classical expressions

$$M_x = EI_T \frac{\partial \theta_x}{\partial s}$$
$$M_y = EI_T \frac{\partial \theta_y}{\partial s} \qquad (3.12)$$

The effects of initial axial force P and initial axial torque T can be included in the analysis by observing from Figure 3.3 that

$$\underset{\sim}{F} = \left(\frac{\partial Q_x}{\partial s} + P \frac{\partial \theta_y}{\partial s} + f_x \right) ds \; \underset{\sim}{\varepsilon}_x + \left(\frac{\partial Q_y}{\partial s} - P \frac{\partial \theta_x}{\partial s} + f_y \right) ds \; \underset{\sim}{\varepsilon}_y$$

$$\underset{\sim}{M}_G = \left\{ \frac{\partial M_x}{\partial s} - Q_y + P\left(\frac{\partial u_y}{\partial s} + \theta_x\right) + T \frac{\partial \theta_y}{\partial s} \right\} ds \; \underset{\sim}{\varepsilon}_x + \frac{\partial M_y}{\partial s} + Q_x - P\left(\frac{\partial u_x}{\partial s} + \theta_y\right) - T \frac{\partial \theta_x}{\partial s} \right\} ds \; \underset{\sim}{\varepsilon}_y \tag{3.13}$$

Here f_x and f_y are the external forces per unit length, distributed along the rotor axis in the x_b and y_b directions. In particular, these forces act at discrete points along the rotor where the bearings are located. If $(u_x)_i$ and $(u_y)_i$ are the displacements of the rotor relative to the i^{th} bearing along the x_b and y_b axes, then we can express

$$f_x = - \sum_{i=1}^{n} \{ (k_{xx})_i (u_x)_i + (k_{xy})_i (u_y)_i + (c_{xx})_i (\dot{u}_x)_i + (c_{xy})_i (\dot{u}_y)_i \} \, \delta(s - s_i)$$

$$f_y = - \sum_{i=1}^{n} \{ (k_{yx})_i (u_x)_i + (k_{yy})_i (u_y)_i + (c_{yx})_i (\dot{u}_x)_i + (c_{yy})_i (\dot{u}_y)_i \} \, \delta(s - s_i) \tag{3.14}$$

where n denotes the total number of bearings and δ stands for Dirac's delta function. s_i's are the z-coordinates of the bearing locations. Here, the damping coefficients may be symmetric ($c_{xyi} = c_{yxi}$) but the stiffness coefficients are not symmetric ($k_{xyi} \neq k_{yxi}$).

Using (3.4), (3.7), (3.10) and (3.13), the governing equations of motion (3.1) can be written as

$$\frac{\partial Q_x}{\partial s} + P\frac{\partial \theta_y}{\partial s} + f_x = \rho A a_x$$

$$\frac{\partial Q_y}{\partial s} - P\frac{\partial \theta_x}{\partial s} + f_y = \rho A a_y \tag{3.15}$$

$$\frac{\partial M_x}{\partial s} - Q_y + P\left(\frac{\partial u_y}{\partial s} + \theta_x\right) + T\frac{\partial \theta_y}{\partial s} = \rho\{I_T(\ddot{\theta}_{xb} + \ddot{\theta}_x) + I_p\omega(\dot{\theta}_{yb} + \dot{\theta}_y)\}$$

$$\frac{\partial M_y}{\partial s} + Q_x - P\left(\frac{\partial u_x}{\partial s} - \theta_y\right) - T\frac{\partial \theta_x}{\partial s} = \rho\{I_T(\ddot{\theta}_{yb} + \ddot{\theta}_y) - I_p\,\omega(\dot{\theta}_{xb} + \dot{\theta}_x)\}$$

The above four equations and the four equations given by (3.11) and (3.12) form a total of eight equations. The eight unknowns to be solved from these equations are: two shear forces, Q_x and Q_y; two bending moments M_x and M_y; two displacements, u_x and u_y; and two rotations, θ_x and θ_y.

3.3 METHOD OF SOLUTION

The equations of motion as given by (3.15) are in the form of partial differential equations involving spatial variable s and temporal variable t. A numerical solution to the problem will be attempted by employing finite elements in the spatial domain and finite differences in the time domain.

It has been recognized that the inclusion of the effects of initial axial torque and the effects of rotor-bearing interaction renders the problem nonconservative [26, 27]. A finite element approach to solve the rotor dynamic problem has been attempted in the past [28] using Hamilton's extended principle. In this chapter we have used a more direct approach by deriving the governing equations from Newton's laws of motion. This approach has the advantage of giving the designer a better physical insight into the problem. The governing differential equations as given by (3.11), (3.12) and (3.15) must be rendered in an integral form before they can be solved using finite element method. This is achieved by the application of Galerkin's technique.

3.3.1 GALERKIN'S TECHNIQUE

In the Galerkin's technique, the displacements u_x, u_y and rotations θ_x, θ_y will be treated as the primary unknowns. Let δu_x, δu_y, $\delta\theta_x$ and $\delta\theta_y$ be arbitrary variations from their actual values. Then, according to Galerkin's technique, the equations of motion given by (3.15) can be written in an integral form as

$$\int_{s_1}^{s_2} [(\rho\ Aa_x - \frac{\partial Q_x}{\partial s} - P\frac{\partial \theta_y}{\partial s} - f_x)\ \delta u_x + (\rho Aa_y - \frac{\partial Q_y}{\partial s} + P\frac{\partial \theta_x}{\partial s} - f_y)\ \delta u_y$$

$$+ \{\rho I_T(\ddot{\theta}_{xb} + \ddot{\theta}_x) + \rho\ I_p\omega(\dot{\theta}_{yb} + \dot{\theta}_y) - \frac{\partial M_x}{\partial s} + Q_y - P(\frac{\partial u_y}{\partial s} + \theta_x) - T\frac{\partial \theta_y}{\partial s}\}\ \delta\theta_x$$

$$+ \{\rho I_T\ (\ddot{\theta}_{yb} + \ddot{\theta}_y) - \rho\ I_p\omega(\dot{\theta}_{xb} + \dot{\theta}_x) - \frac{\partial M_y}{\partial s} - Q_x + P(\frac{\partial u_x}{\partial s} - \theta_y)$$

$$+ T\frac{\partial \theta_x}{\partial s}\}\ \delta\theta_y]\ ds = 0 \tag{3.16}$$

subject to the constraints (3.11) and (3.12). It is to be noted that equation (3.16) is also a statement of the principle of virtual work. The equivalence between the principle of virtual work and the Galerkin's technique is well known to finite element analysts [29]. Using partial integration, equation (3.16) can be written in a more convenient form as

$$\int_{s_1}^{s_2} [\rho\ Aa_x\delta u_x + \rho Aa_y\ \delta u_y + \rho I_T(\ddot{\theta}_{xb} + \ddot{\theta}_x)\ \delta\theta_x + \rho I_p\omega\ (\dot{\theta}_{yb} + \dot{\theta}_y)\ \delta\theta_x$$

$$+ \rho I_T\ (\ddot{\theta}_{yb} + \ddot{\theta}_y)\ \delta\theta_y - \rho I_p\ \omega(\dot{\theta}_{xb} + \dot{\theta}_x)\delta\theta_y - f_x\delta u_x - f_y\ \delta u_y$$

$$+ Q_x \delta(\frac{\partial u_x}{\partial s} - \theta_y) + Q_y \delta(\frac{\partial u_y}{\partial s} + \theta_x) + M_x \delta(\frac{\partial \theta_x}{\partial s}) + M_y \delta(\frac{\partial \theta_y}{\partial s})$$

$$+ P\theta_y \delta(\frac{\partial u_x}{\partial s}) - P\theta_x \delta(\frac{\partial u_y}{\partial s}) - P(\frac{\partial u_y}{\partial s} + \theta_x)\delta\theta_x + P(\frac{\partial u_x}{\partial s} - \theta_y) \delta\theta_y$$

$$+ T\theta_y\delta(\frac{\partial \theta_x}{\partial s}) - T\theta_x (\frac{\partial \theta_y}{\partial s})] ds$$

$$= [(Q_x + P\theta_y) \delta u_x + (Q_y - P\theta_x) \delta u_y + (M_x + T\theta_y)\delta\theta_x + (M_y - T\theta_x) \delta\theta_y]_{s_1}^{s_2} \tag{3.17}$$

The shear forces Q_x, Q_y and bending moments M_x, M_y appearing on the left hand side of equation (3.17) can be replaced by the unknown displacements, rotations and their derivatives using equations (3.11) and (3.12).

3.3.2 FINITE ELEMENTS

Consider a typical rotor element with two nodes (see Figure 3.4). The unknown displacements and rotations will be expressed in terms of unknown nodal values and known shape functions as

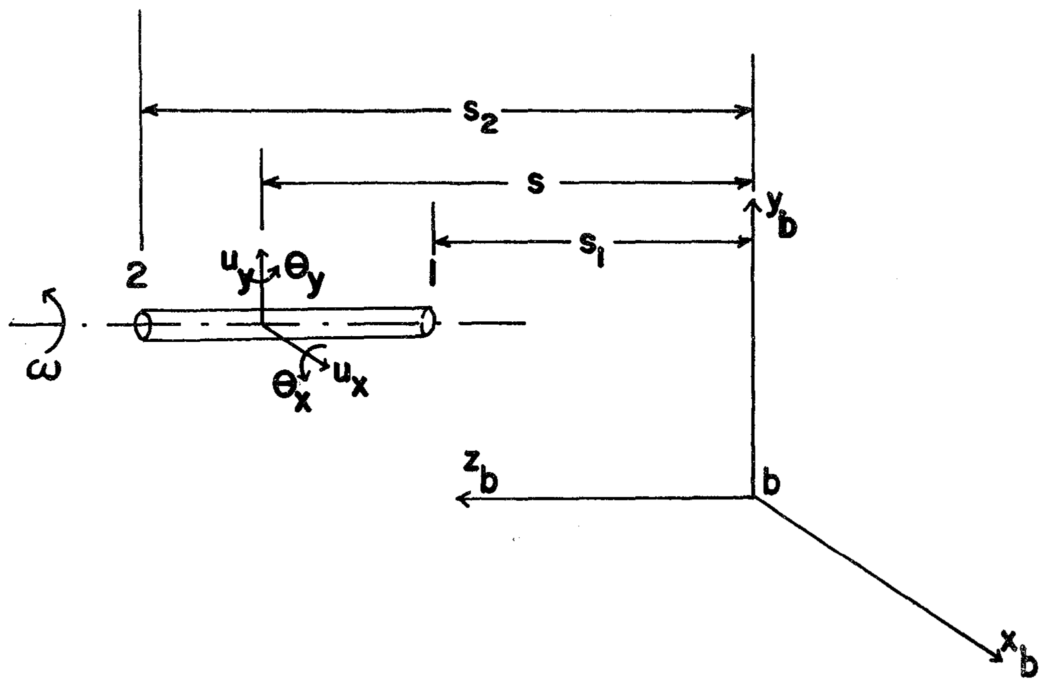


FIG. 3.4 A FINITE ROTOR ELEMENT

$$
\begin{aligned}
u_x &= (U_x)_1 \, N_1(s) + (U_x)_2 \, N_2(s) \\
u_y &= (U_y)_1 \, N_1(s) + (U_y)_2 \, N_2(s) \\
\theta_x &= (\Theta_x)_1 \, N_1 \, (s) + (\Theta_x)_2 N_2(s) \\
\theta_y &= (\Theta_y)_1 \, N_1(s) + (\Theta_y)_2 \, N_2(s)
\end{aligned} \tag{3.18}
$$

where

$$
\begin{aligned}
N_1(s) &= (s_2 - s)/(s_2 - s_1) \\
N_2(s) &= (s - s_1)/(s_2 - s_1)
\end{aligned} \tag{3.19}
$$

It is worth noting that a simple linear interpolation for the displacements and rotations has been used. Equations (3.18) can be expressed more conveniently in a matrix form as

$$
\{u\}_e = [N]_e \; \{q\}_e \tag{3.20}
$$

where $[N]_e$ is a matrix of shape functions and $\{q\}_e$ is a vector of nodal displacements and rotations given by

$$
\{q\}_e^T = [(U_x)_1 \; (U_y)_1 \; (\Theta_x)_1 \; (\Theta_y)_1 \; (U_x)_2 \; (U_y)_2 \; (\Theta_x)_2 \; (\Theta_y)_2] \tag{3.21}
$$

We also note that

$$\delta \{u\}_e = [N]_e \, \delta\{q\}_e \tag{3.22}$$

Substituting (3.20) and (3.22) in (3.17) and carrying out the necessary differentiations and integrations, we can express (3.17) in a matrix form as

$$\delta \{q\}_e^T [\,[M]_e \{\ddot{q}\}_e + [C]_e \{\dot{q}\}_e + [K]_e \{q\}_e] = \delta\{q\}_e^T \{Q\}_e \tag{3.23}$$

Here, $[M]_e$ is the elemental inertia matrix. $[C]_e$ is an elemental matrix that can be written as

$$[C]_e = [C_G]_e + [C_C]_e + [C_D]_e \tag{3.24}$$

where

$[C_G]_e$ - Gyroscopic matrix,

$[C_C]_e$ - Coriolis matrix due to base rotation,

$[C_D]_e$ - Damping matrix due to bearing(s) located at the node(s).

$[K]_e$ is an elemental matrix that can be written as

$$[K]_e = [K_C]_e + [K_P]_e + [K_T]_e + [K_R]_e + [K_B]_e \tag{3.25}$$

where

$[K_C]_e$ - Conventional stiffness matrix for the beam element,
$[K_P]_e$ - Geometric stiffness matrix due to initial axial force,
$[K_T]_e$ - Geometric stiffness matrix due to initial axial torque,
$[K_R]_e$ - Supplementary stiffness matrix due to base rotation,
$[K_B]_e$ - Stiffness matrix due to bearing(s) located at the node(s).

$\{Q\}_e$ is a vector of nodal forces and moments due to base translation and rotation. The elemental matrices and the nodal force vector are given explicitly in Appendix B. It can be seen that $[M]_e$, $[C_D]_e$, $[K_C]_e$ and $[K_P]_e$ are symmetric matrices; $[C_G]_e$ and $[C_C]_e$ are skew-symmetric matrices; $[K_T]_e$, $[K_R]_e$ and $[K_B]_e$ are nonsymmetric matrices. These elemental matrices are to be properly assembled to obtain the global matrices.

3.3.3 INTERMEDIATE DISKS AND FLYWHEELS

The effect of intermediate disks and flywheels can be included in the analysis by considering them as spinning rigid disks that execute motion in three-dimensional space. Consider the node i where a rigid disk has been mounted on the rotor. The equations of motion for such a rigid disk have been derived in considerable detail in Chapter 2. It suffices to point out that these equations of motion can be written in a form similar to equation (3.23) as

$$\delta\{q\}_i^T \quad [[M]_d \; \{\ddot{q}\}_i \; + [C]_d \, \{\dot{q}\}_i + [K]_d \, \{q\}_i] \; = \delta\{q\}_i^T \; \{Q\}_i \qquad (3.26)$$

Here, $\{q\}_i$ is the vector of displacements and rotations at the i^{th} node. $[M]_d$ is the inertia matrix for the disk. $[C]_d$ is a matrix that can be written as

$$[C]_d \; = \; [C_G]_d \; + \; [C_C]_d \qquad (3.27)$$

where

$[C_G]_d$ - Gyroscopic matrix for the disk,

$[C_C]_d$ - Coriolis matrix due to base rotation

$[K]_d$ is a supplementary stiffness matrix due to the base rotation. $\{Q\}_i$ is a vector of nodal forces and moments due to base translation and rotation. These matrices and vector for the disk are given explicitly in Appendix C.

3.3.4 CHECK PROBLEMS

Before solving for the seismic response, which involves transient dynamic response computation, the performance of the finite elements formulated above must be tested against some known, closed form dynamic solutions available in literature. Three such check problems were solved and they are given below.

3.3.4.1 FREE VIBRATION OF A TIMOSHENKO BEAM

Consider a beam of uniform cross section, simply supported at both ends. When both shear deformation and rotatory inertia are taken into account, the frequencies of free vibration ω_n of such a beam are given by the roots of the equation

$$\frac{\rho^2 \omega_n^4}{kEG} - \left[\frac{\rho}{E} \left(1 + \frac{E}{kG}\right)(n\pi/\ell)^2 + \frac{\rho A}{EI_T} \right]\omega_n^2 + (n\pi/\ell)^4 = 0 \quad (3.28)$$

Using the finite elements developed in this chapter, the eigenproblem is posed as

$$[M]\{X\} = \frac{1}{\omega_n^2} [K_C]\{X\} \quad (3.29)$$

It is well known in finite element literature that when simple linear interpolations as given by equations (3.19) are used, difficulties arise due to 'shear locking' phenomenon, particularly at low aspect ratios. This problem can be overcome by resorting to reduced, single point integration [29, 30, 31] of the element stiffness matrix. Table 3.1 shows the comparison of the finite element and exact natural frequencies for various aspect ratios of

TABLE 3.1 COMPARISON OF NON-DIMENSIONAL FREQUENCY PARAMETER $\ell^2\omega_n\sqrt{\frac{\rho A}{EI_T}}$

FOR A SIMPLY SUPPORTED TIMOSHENKO BEAM (ν = 0.3)

Mode n	Aspect Ratio r/2ℓ	10 ELEMENTS		15 ELEMENTS		EXACT
		Exact Integration	Reduced Integration	Exact Integration	Reduced Integration	
1	0.02	12.92	9.918	11.28	9.855	9.794
	0.04	10.48	9.696	9.988	9.632	9.580
	0.06	9.693	9.364	9.452	9.305	9.258
	0.08	9.131	8.963	8.985	8.910	8.867
	0.10	8.623	8.528	8.522	8.479	8.440
2	0.02	51.75	40.21	44.59	39.14	38.32
	0.04	39.68	37.05	37.34	36.16	35.47
	0.06	34.22	33.29	32.99	32.58	32.02
	0.08	30.10	29.70	29.31	29.13	28.68
	0.10	26.73	26.53	26.16	26.07	25.70

the beam. It is seen that reduced integration provides very good results at both low and high aspect ratios.

3.3.4.2 BUCKLING OF A TIMOSHENKO BEAM

Consider a simply supported beam of uniform cross section, acted on by an axial compressive load P_c. When shear deformation is taken into account, the buckling loads are given by the roots of the following equation.

$$P_c^2 + kAGP_c - kEI_TAG \ (n\pi/\ell)^2 = 0 \qquad (3.30)$$

Using the finite elements developed in this chapter, the corresponding eigenproblem is posed as

$$[K_C]\{X\} = P_c [K_P]\{X\} \qquad (3.31)$$

Table 3.2 compares the finite element and exact results. The reduced, single point integrations were performed on both the conventional and geometric stiffness matrices. It is again seen that reduced integration leads to better results at low as well as high aspect ratios.

TABLE 3.2 COMPARISON OF NON-DIMENTIONAL BUCKLING LOAD $\ell^2 P_n/EI_T$

FOR A SIMPLY SUPPORTED TIMOSHENKO BEAM (ν = 0.3)

Mode n	Aspect Ratio $r/2\ell$	10 ELEMENTS		15 ELEMENTS		EXACT
		Exact Integration	Reduced Integration	Exact Integration	Reduced Integration	
1	0.02	16.97	9.924	12.94	9.845	9.758
	0.04	11.31	9.604	10.27	9.520	9.450
	0.06	9.879	9.151	9.395	9.074	9.012
	0.08	9.026	8.635	8.738	8.566	8.510
	0.10	8.346	8.107	8.150	8.045	7.995
2	0.02	69.06	40.32	51.20	38.89	37.80
	0.04	42.68	36.11	37.75	34.94	34.04
	0.06	34.33	31.65	31.85	30.71	29.98
	0.08	29.16	27.76	27.59	26.99	26.40
	0.10	25.40	24.55	24.27	23.90	23.41

3.3.4.3 FREE VIBRATION OF A ROTATING TIMOSHENKO BEAM

Consider a rotating beam of uniform cross section, simply supported by bearings at both ends. When shear deformation, rotatory inertia and gyroscopic effects are included in the analysis, the frequencies of free vibration ω_n are given by the roots of the equation

$$\frac{\rho^2}{kEG}\,\omega_n^4 - \frac{2\rho^2\omega}{kEG}\,\omega_n^3 - \left\{ \frac{\rho}{E}\left(1 + \frac{E}{kG}\right)(n\pi/\ell)^2 + \frac{\rho A}{EI_T} \right\}\omega_n^2 + \frac{2\rho\omega}{E}(n\pi/\ell)^2\omega_n + (n\pi/\ell)^4 = 0 \tag{3.32}$$

Using the finite elements of this chapter, the corresponding eigenproblem is posed as

$$-\,\omega_n^2\,[M]\,\{X\} + i\,\omega_n\,[C_G]\,\{X\} + [K_C]\,\{X\} = 0 \tag{3.33}$$

The above form of matrix eigenproblem was solved using an algorithm due to Gupta [32]. The forward and backward travelling frequencies thus obtained are compared in Table 3.3 against the exact values for various rotational speeds. Reduced integration was used in the evaluation of the stiffness matrix. The agreement is found to be satisfactory.

TABLE 3.3 COMPARISON OF NON-DIMENSIONAL FREQUENCY PARAMETERS $\ell^2\omega_n\sqrt{\frac{\rho A}{EI_T}}$ FOR A SIMPLY SUPPORTED, ROTATING TIMOSHENKO BEAM($r/2\ell = 0.06$, $\nu = 0.3$)

Mode n	Rotational Speed $\ell^2\omega\sqrt{\frac{\rho A}{EI_T}}$	FORWARD			BACKWARD		
		10 Elements	15 Elements	Exact	10 Elements	15 Elements	Exact
1	4.0	9.473	9.418	9.372	9.243	9.187	9.145
	8.0	9.592	9.529	9.488	9.131	9.076	9.032
	12.0	9.712	9.648	9.603	9.020	8.965	8.920
	16.0	9.831	9.767	9.720	8.909	8.845	8.810
	20.0	9.950	9.886	9.837	8.798	8.742	8.700
2	4.0	33.57	32.84	32.30	33.00	32.27	31.74
	8.0	33.86	33.12	32.58	32.71	31.98	31.46
	12.0	34.14	33.41	32.85	32.39	31.73	31.18
	16.0	34.43	33.66	33.13	32.11	31.44	30.90
	20.0	34.68	33.95	33.40	31.82	31.15	30.62

3.3.5 NUMERICAL INTEGRATION

The governing equations for the rotor can be obtained by properly assembling the elemental and disk matrices and vectors. The final set of equations can be written in a matrix form as

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \qquad (3.34)$$

The above matrix equations can be solved numerically using direct integration approach. We have used the Newmark's integration technique outlined in Table 2.1.

3.4 EXAMPLE PROBLEM

As an example problem, the seismic analysis of a rotor-bearing system shown schematically in Figure 3.5 was carried out using the beam model. Additional parameters for the rotor-bearing system needed for the calculation are given in Table 3.4. The base of the rotor-bearing system was subjected to El Centro excitation shown in Figure 3.6. In some cases, the base was also subjected to simulated angular accelerations shown in Fig. 3.7. The rotor was divided into 19 finite elements.

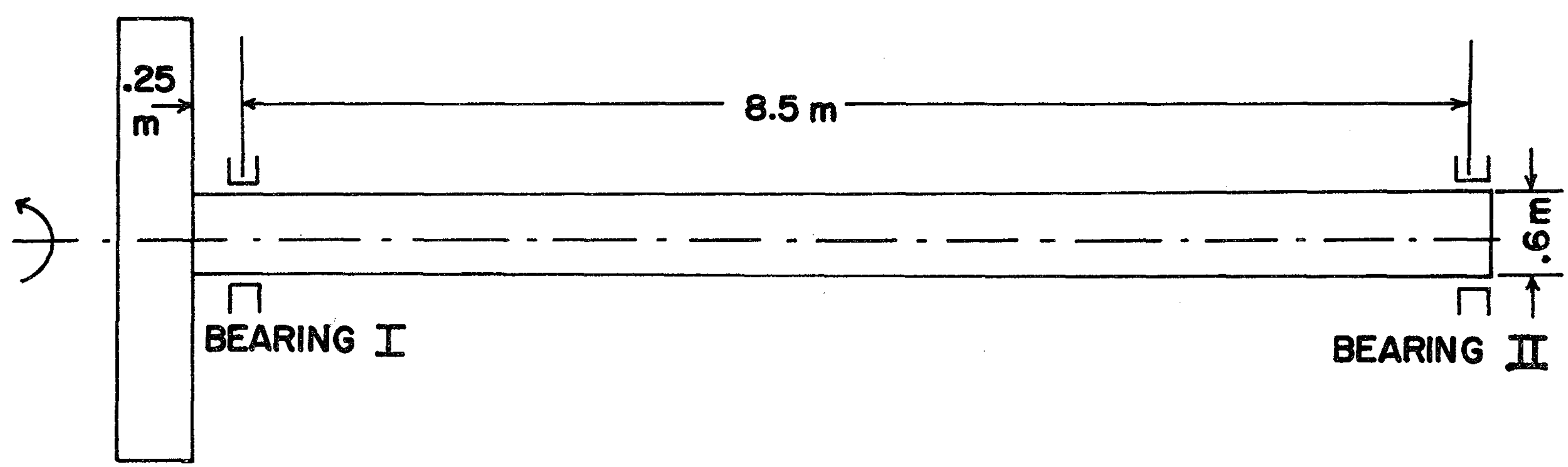


FIG. 3.5 ROTOR - BEARING SYSTEM FOR EXAMPLE PROBLEM

TABLE 3.4 PARAMETERS FOR THE ROTOR-BEARING SYSTEM

$E = 20 \times 10^{10}$ N/m^2	$\nu = 0.3$
$\rho = 7800$ kg/m^3	$\omega = 3000$ rpm
FOR THE FLYWHEEL	
$m = 5000$ kg	
$I = 2500$ kg.m^2	$I_0 = 1267$ kg.m^2
FOR THE BEARINGS	
$(k_{xx})_I = 0.5890 \times 10^9$ N/m	$(k_{xy})_I = 0.5100 \times 10^8$ N/m
$(k_{yx})_I = -0.1290 \times 10^{10}$ N/m	$(k_{yy})_I = 0.1870 \times 10^{10}$ N/m
$(c_{xx})_I = 0.2800 \times 10^7$ N.s/m	$(c_{xy})_I = -0.4100 \times 10^7$ N.s/m
$(c_{yx})_I = -0.4100 \times 10^7$ N.s/m	$(c_{yy})_I = 0.1170 \times 10^8$ N.s/m
$(k_{xx})_{II} = 0.6760 \times 10^9$ N/m	$(k_{xy})_{II} = 0.2160 \times 10^8$ N/m
$(k_{yx})_{II} = -0.1490 \times 10^{10}$ N/m	$(k_{yy})_{II} = 0.2270 \times 10^{10}$ N/m
$(c_{xx})_{II} = 0.3100 \times 10^7$ N.s/m	$(c_{xy})_{II} = -0.5000 \times 10^7$ N.s/m
$(c_{yx})_{II} = -0.5000 \times 10^7$ N.s/m	$(c_{yy})_{II} = 0.1370 \times 10^8$ N.s/m

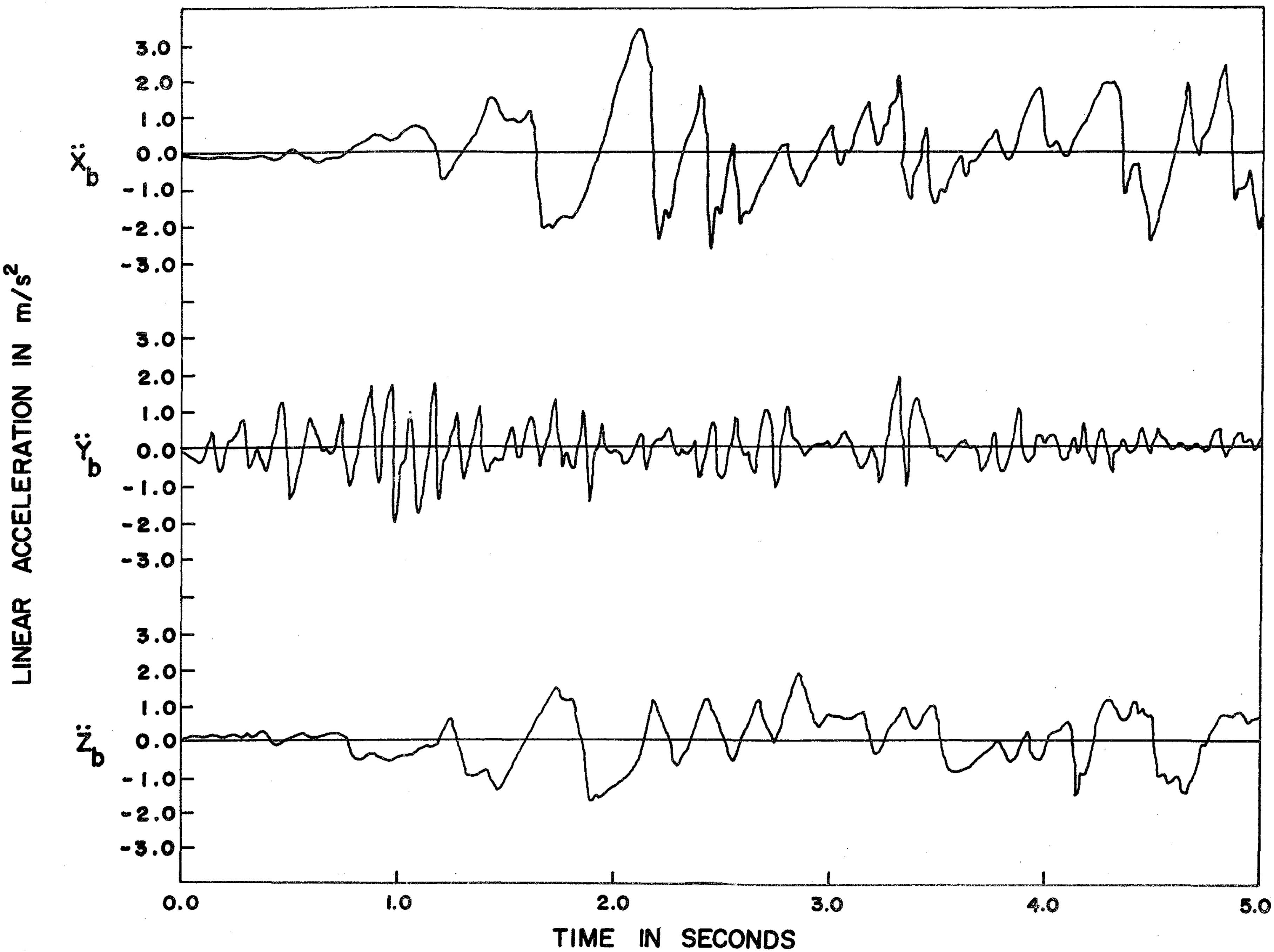


FIG.3.6 LINEAR ACCELERATION OF THE BASE

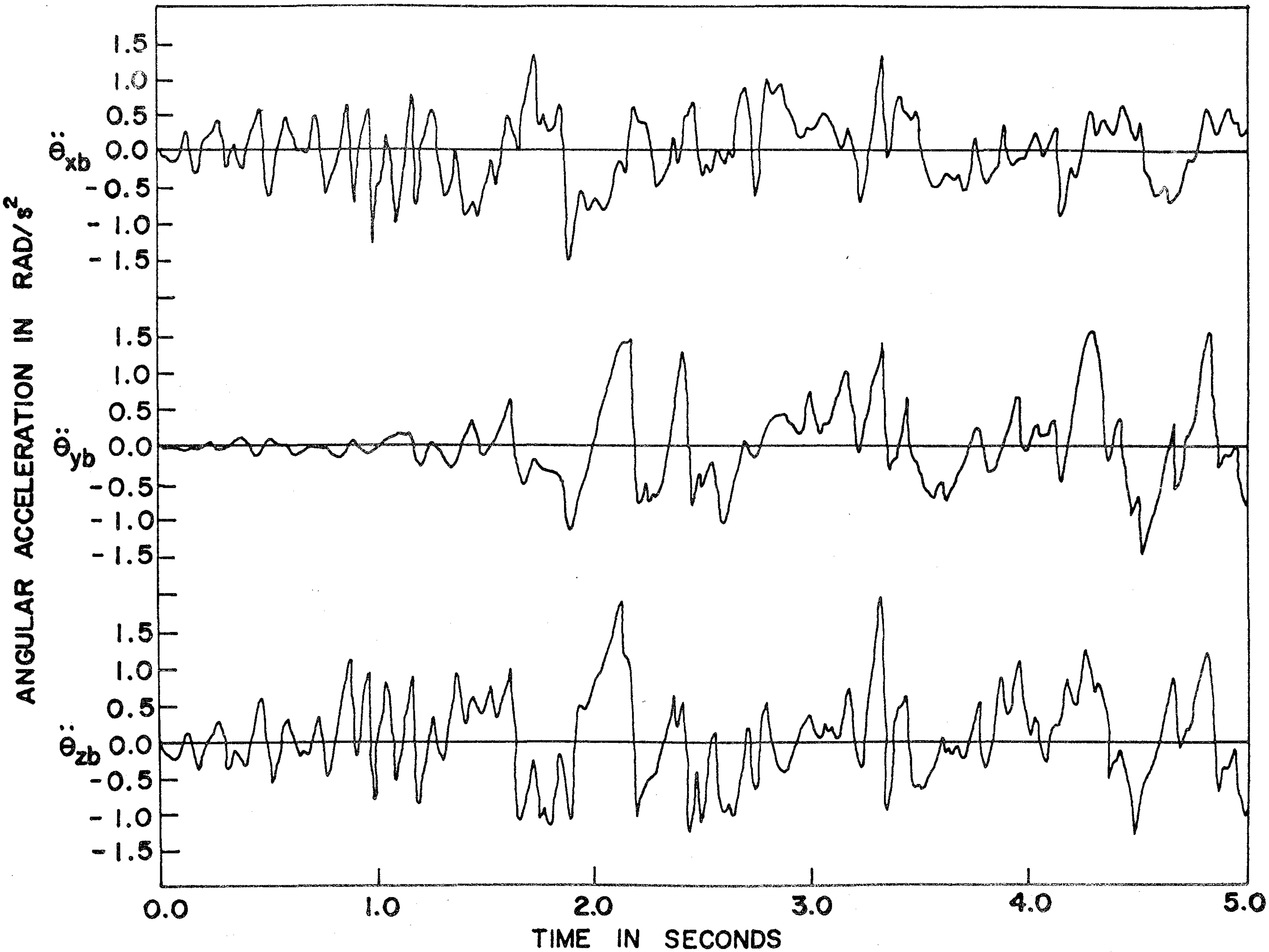


FIG. 3.7 ANGULAR ACCELERATION OF THE BASE

The results can be better presented and discussed under the following four cases.

Case 1. Spin vs. No Spin

An important aspect of the seismic analysis of rotating system that distinguishes it from the seismic analysis of a stationary system is the presence of gyroscopic effects.

Figures 3.8, 3.9 and 3.10 present the displacements of rotor in the bearings, dynamic reaction forces in the bearings and shear forces and bending moments at midspan of the rotor as functions of time when the rotor is spinning at 3000 rpm and the base is subjected to only translational excitations shown in Figure 3.6. The x and y components of displacements and forces in bearings I and II are denoted with proper subscripts in Figures 3.8 and 3.9, and in the subsequent figures to follow.

Similar results are presented in Figures 3.11, 3.12 and 3.13 when the gyroscopic effects are not taken into account. This means that the effect of spin speed is ignored, but the stiffness and damping provided by the fluid film lubricants in the two bearings are retained. This is analogous to the analysis procedure of Villasor [12] and is similar to the various existing structural dynamics computer codes.

It can be clearly seen by comparing the two sets of figures that the

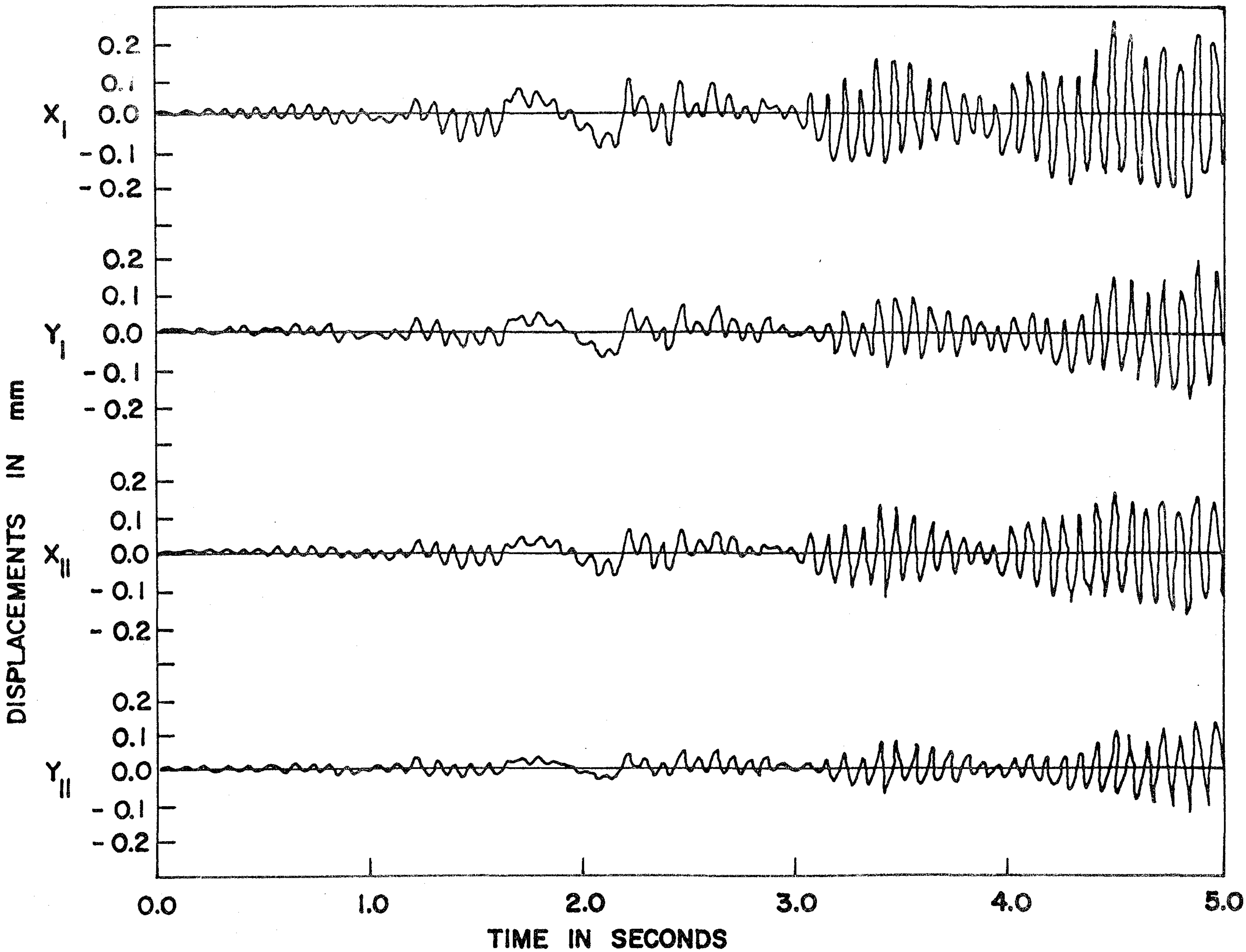


FIG. 3.8 DISPLACEMENTS OF ROTOR IN THE BEARINGS

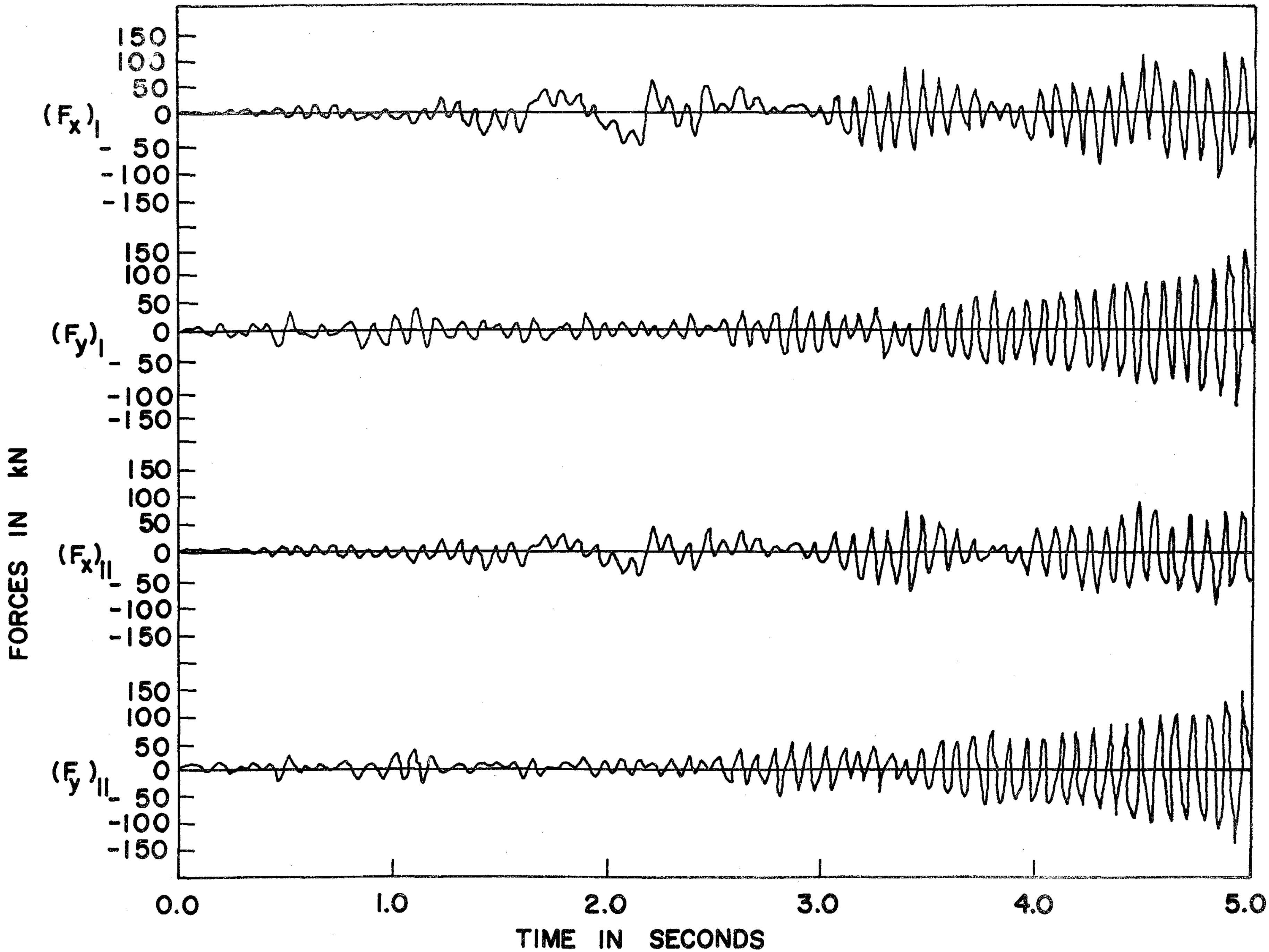


FIG.3.9 DYNAMIC REACTION FORCES IN THE BEARINGS
(BASE ROTATION EXCLUDED)

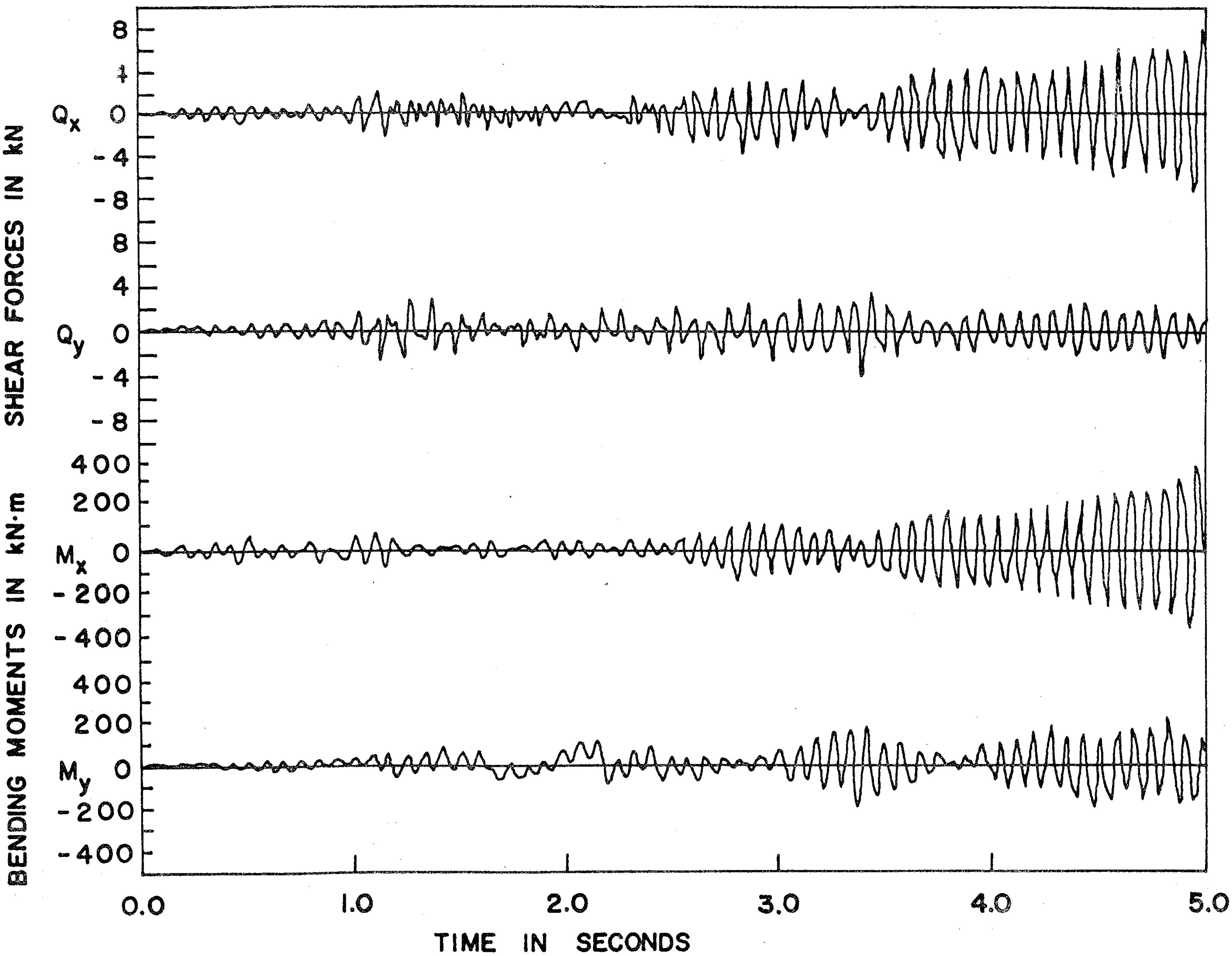


FIG. 3.10 SHEAR FORCES AND BENDING MOMENTS AT MIDSPAN

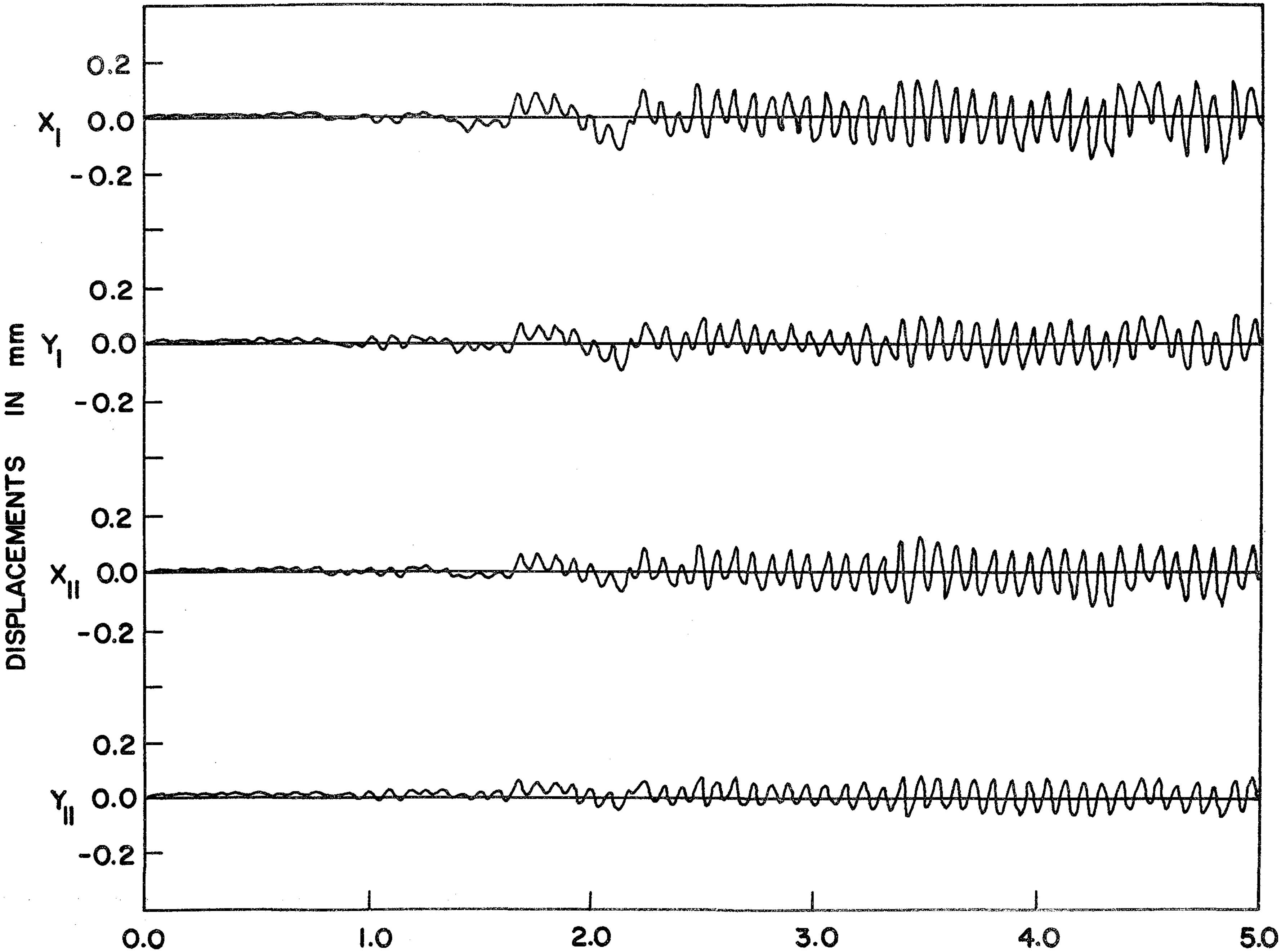


FIG.3.11 DISPLACEMENTS OF ROTOR IN THE BEARINGS
(BASE ROTATION EXCLUDED. RPM = 0)

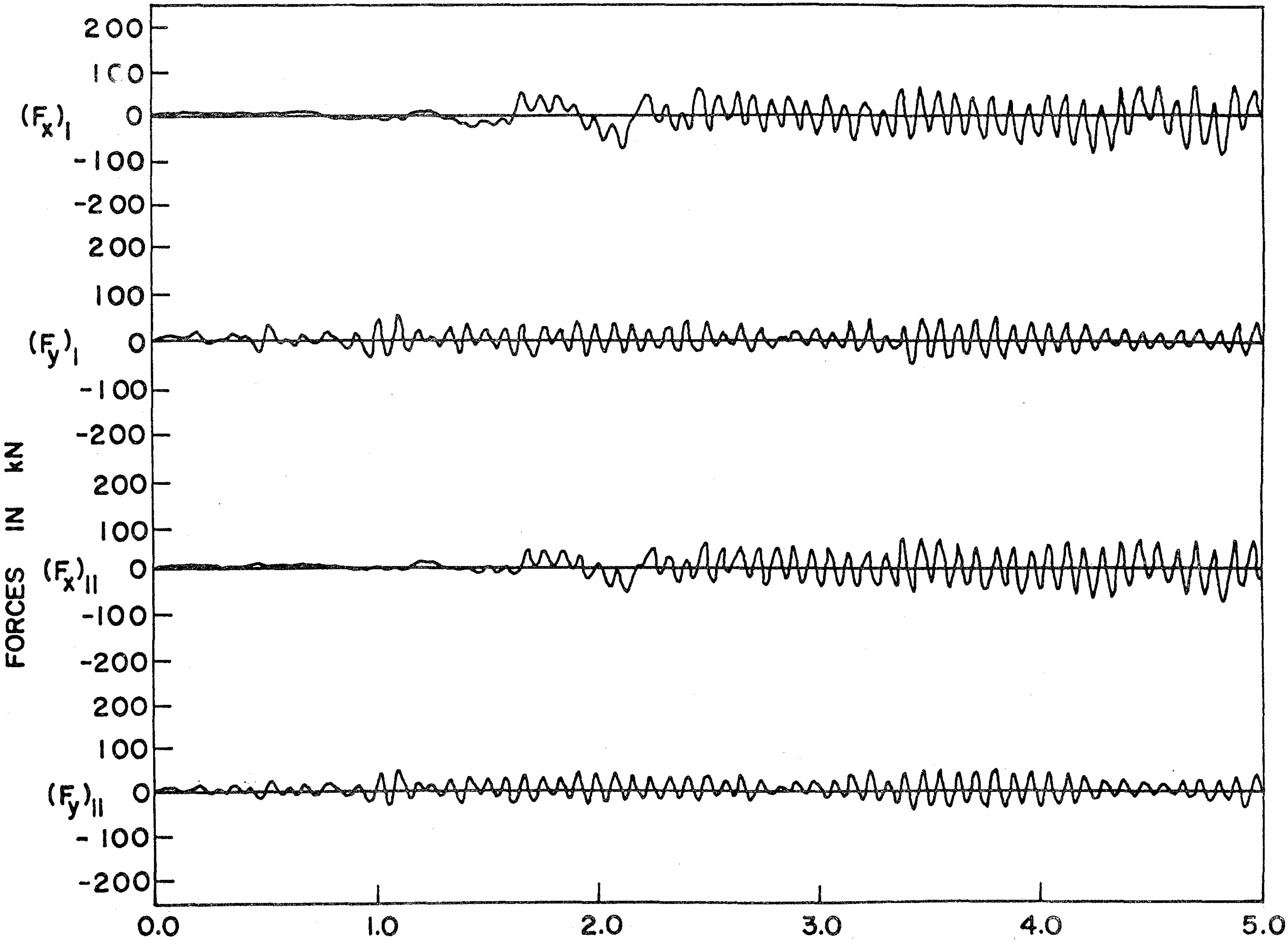


FIG.3.12 DYNAMIC REACTION FORCES IN THE BEARINGS (BASE ROTATION EXCLUDED, RPM = 0)

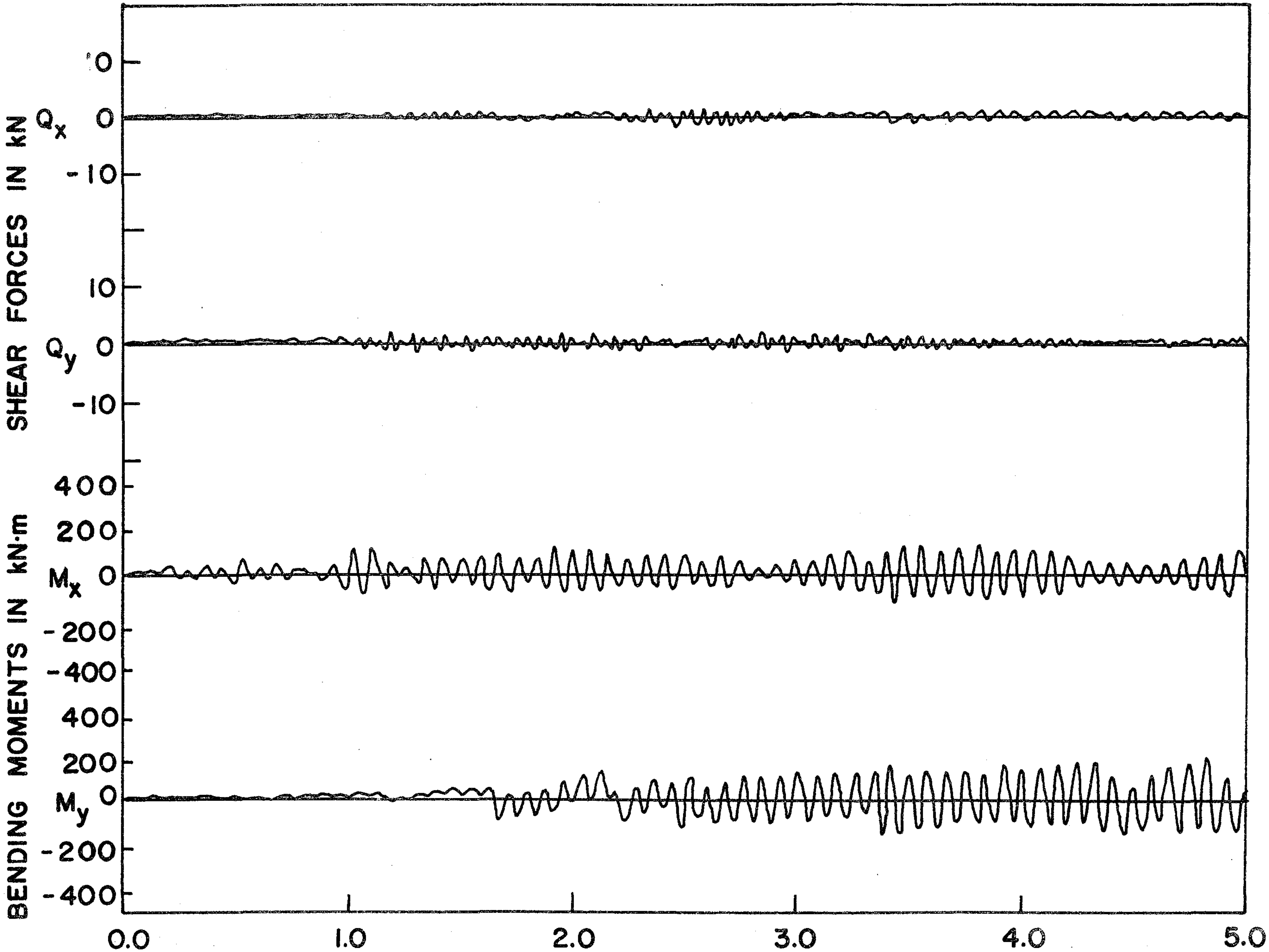


FIG. 3.13 SHEAR FORCES AND BENDING MOMENTS AT MIDSPAN
(BASE ROTATION EXCLUDED, RPM = 0)

gyroscopic effects tend to magnify the response of the rotating system. Ignoring the effects of spin speed will underpredict the response and lead to unreliable estimates.

Case 2. Base Rotation vs. No Base Rotation

It is a common practice in seismic analysis to consider only the translational excitations of the base. Figures 3.14, 3.15, and 3.16 present the response of the rotating system when the base is simultaneously subjected to base translational excitations shown in Figure 3.6 and base rotational excitations shown in Figure 3.7. Comparison of Figures 3.8, 3.9 and 3.10 (base translation only) and Figures 3.14, 3.15 and 3.16 (base translation and rotation) shows the relative importance of inclusion of base rotation in the analysis. The base rotational components amplify the dynamic response and must be included in the analysis for reliable response computations.

Case 3. Rigid Body Model vs. Beam Model

In the previous chapter, we developed a rigid body model for the rotor that takes the effects of base rotation into account. Figures 3.17 and 3.18 present the displacements of rotor in the bearings and dynamic reaction forces in the bearings using the rigid body model. Comparison of these figures with

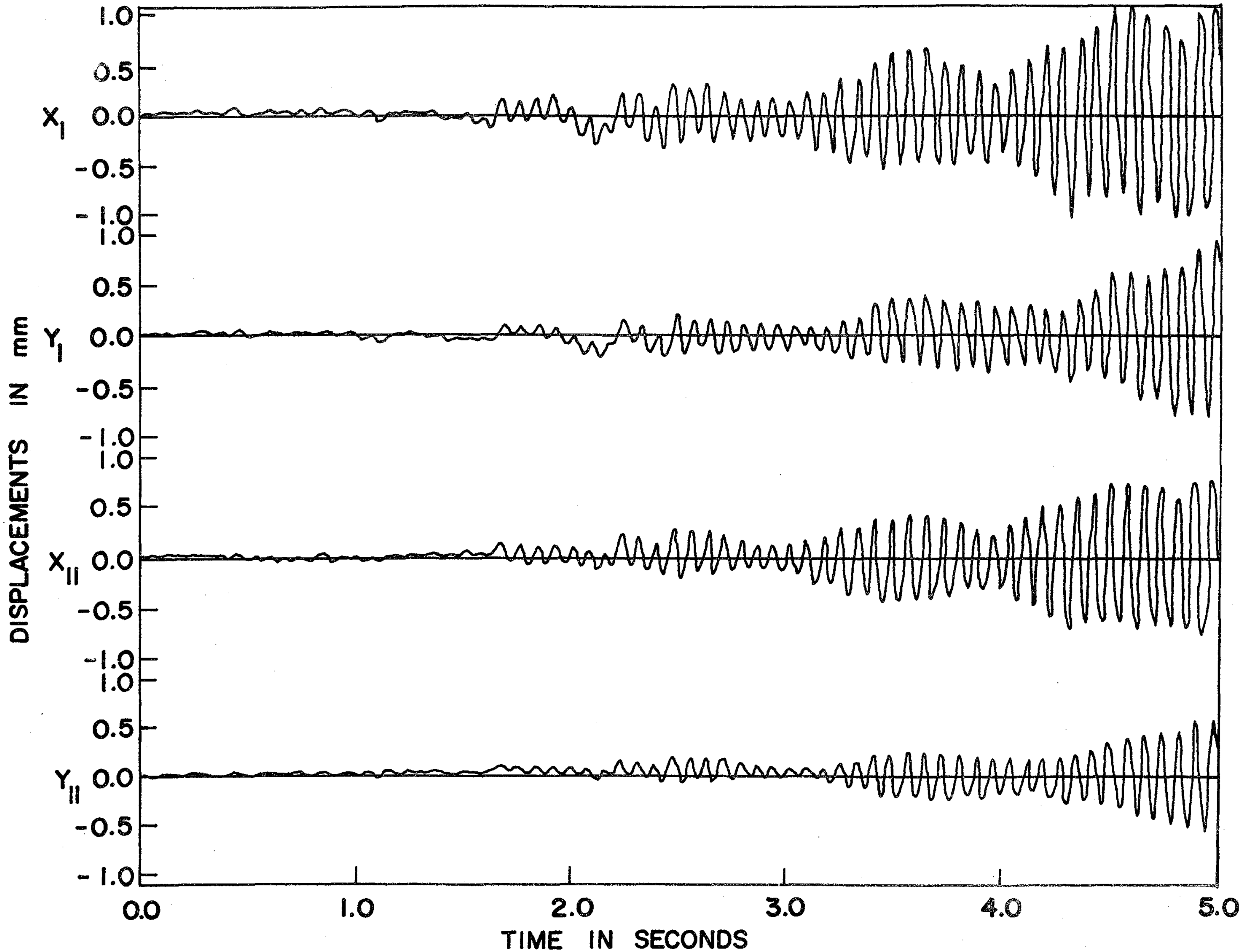


FIG. 3.14 DISPLACEMENTS OF ROTOR IN THE BEARINGS
(BASE ROTATION INCLUDED)

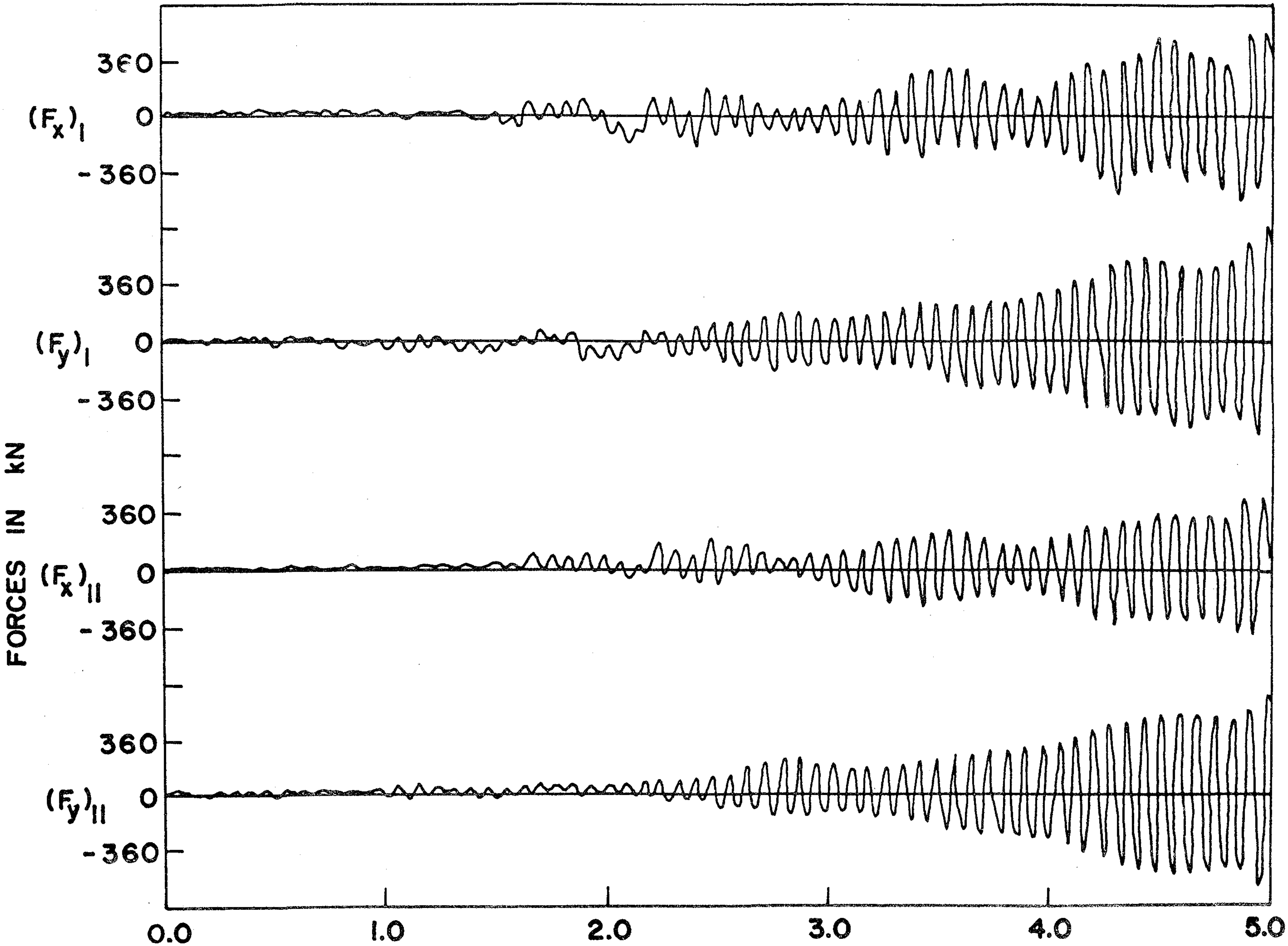


FIG. 3.15 DYNAMIC REACTION FORCES IN THE BEARINGS
(BASE ROTATION INCLUDED)

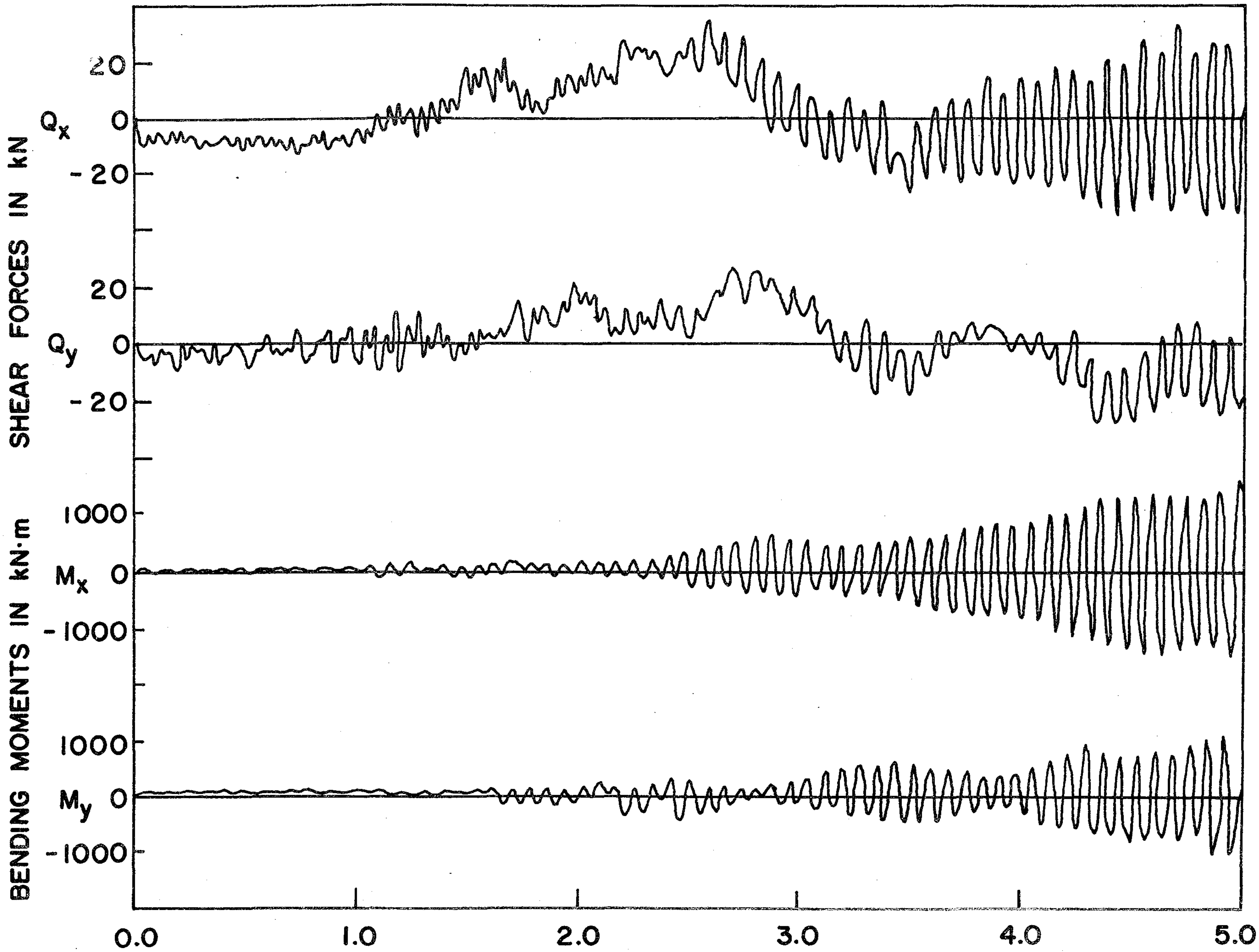


FIG. 3.16 SHEAR FORCES AND BENDING MOMENTS AT MIDSPAN
(BASE ROTATION INCLUDED)

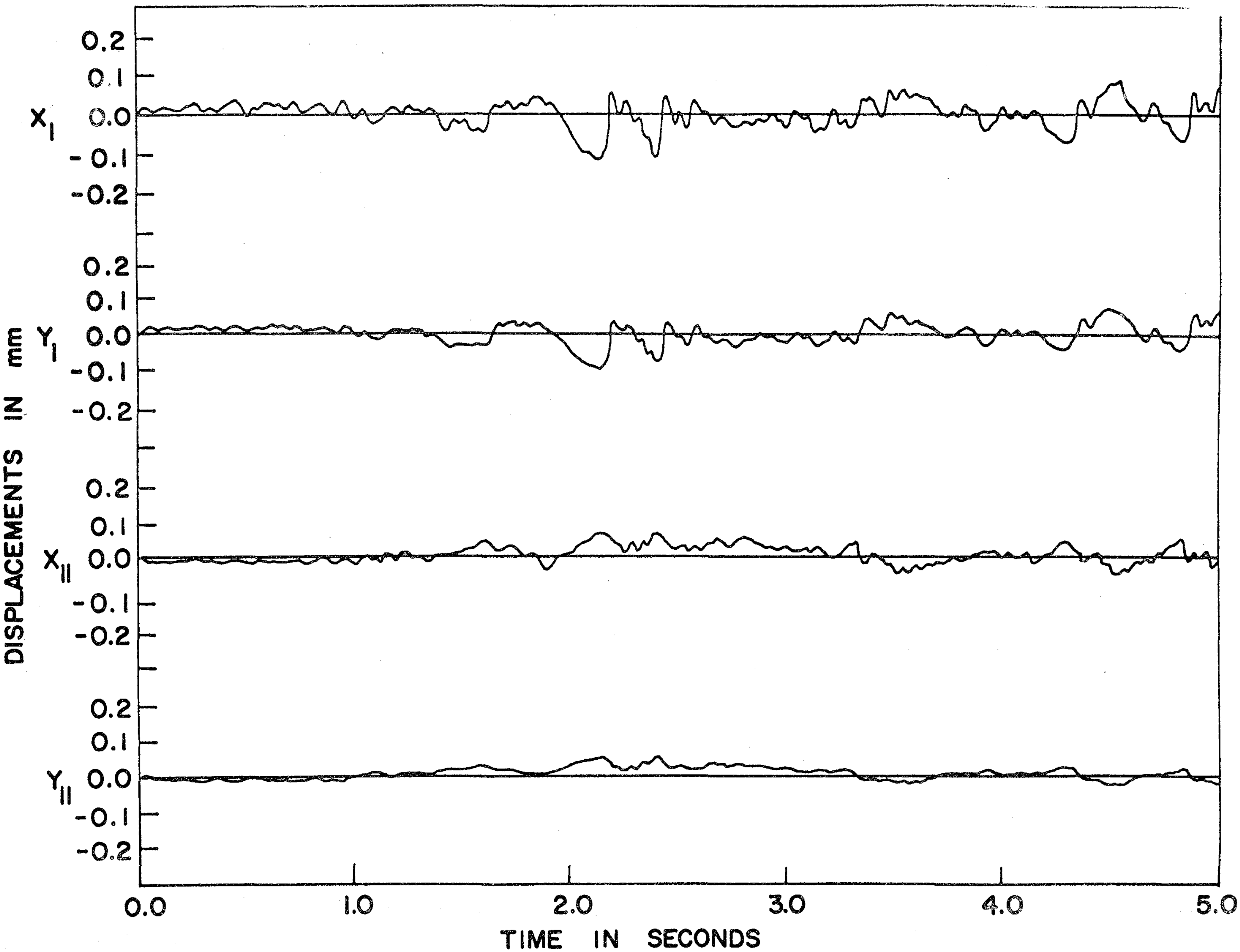


FIG.3.17 DISPLACEMENTS OF ROTOR IN THE BEARINGS
(RIGID BODY MODEL)

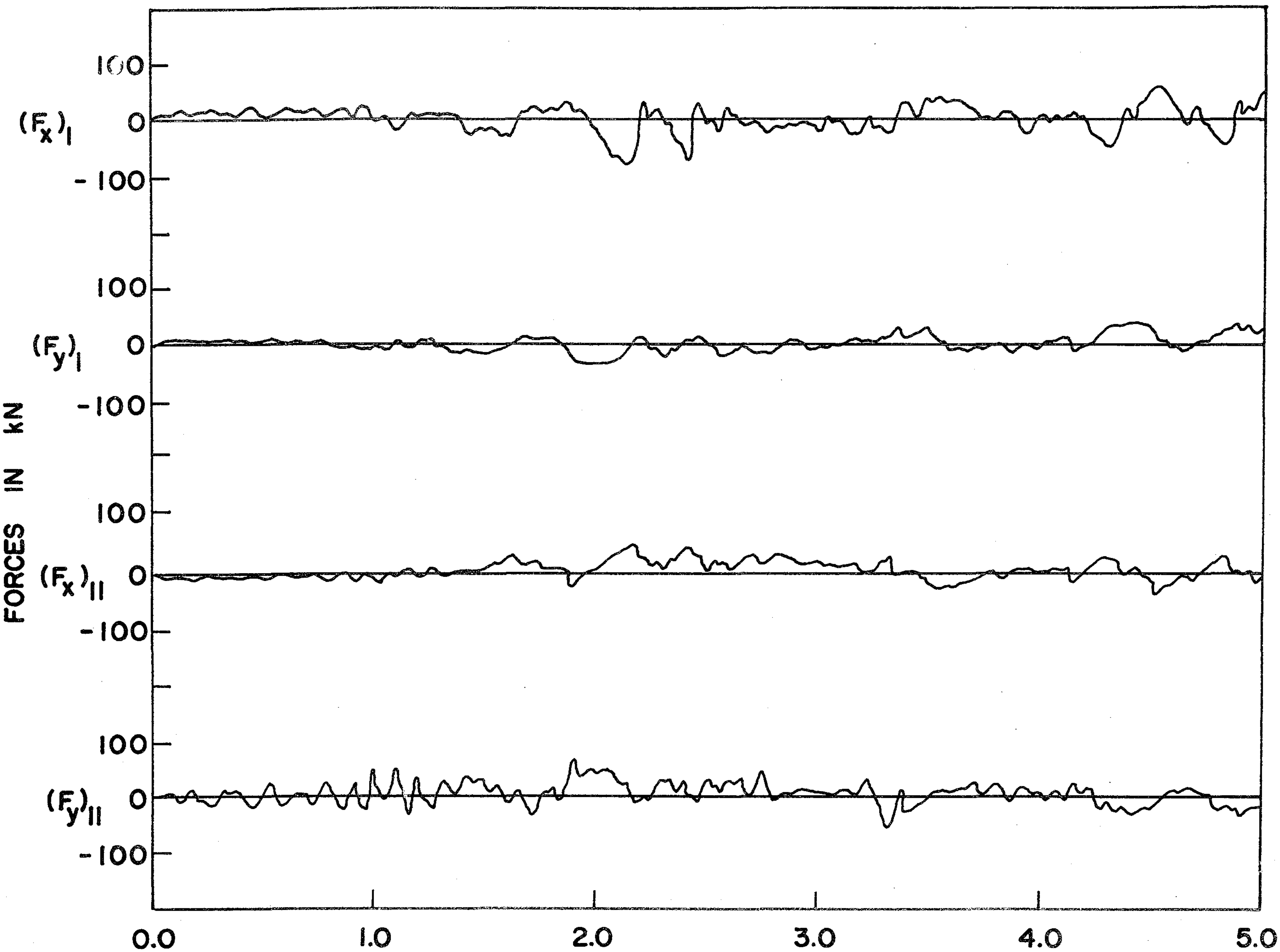


FIG.3.18 DYNAMIC REACTION FORCES IN THE BEARINGS
(RIGID BODY MODEL)

Figures 3.14 and 3.15 shows that the rigid body model overly underpredicts the response of the rotating sytem.

A single run for the beam model took about 36 seconds of CPU time in IBM System 370/165 for execution. A single run for the rigid body model took about 2 seconds of CPU time for execution.

Case 4. Effects of Axial Force and Axial Torque

The results presented so far do not include the effects of any axial force or axial torque. Figures 3.19, 3.20 and 3.21 present the responses when an axial force of 2500 kN is applied and an axial torque of 500 kN.m is transmitted. Comparison of these figures with Figures 3.14, 3.15, and 3.16 reveals the magnification in the response due to initial axial force and torque and demonstrates the necessity to include them in the analysis.

A computer program called ROBET has been prepared, along with a User's manual, to automate the seismic response computation using the beam model presented in this chapter. The User's manual for ROBET and a listing of the program can be found in "Part II: Computer Programs' of this report.

3.5 MERITS AND LIMITATIONS OF BEAM MODEL

In this chapter it was shown that the flexibility of the rotating system

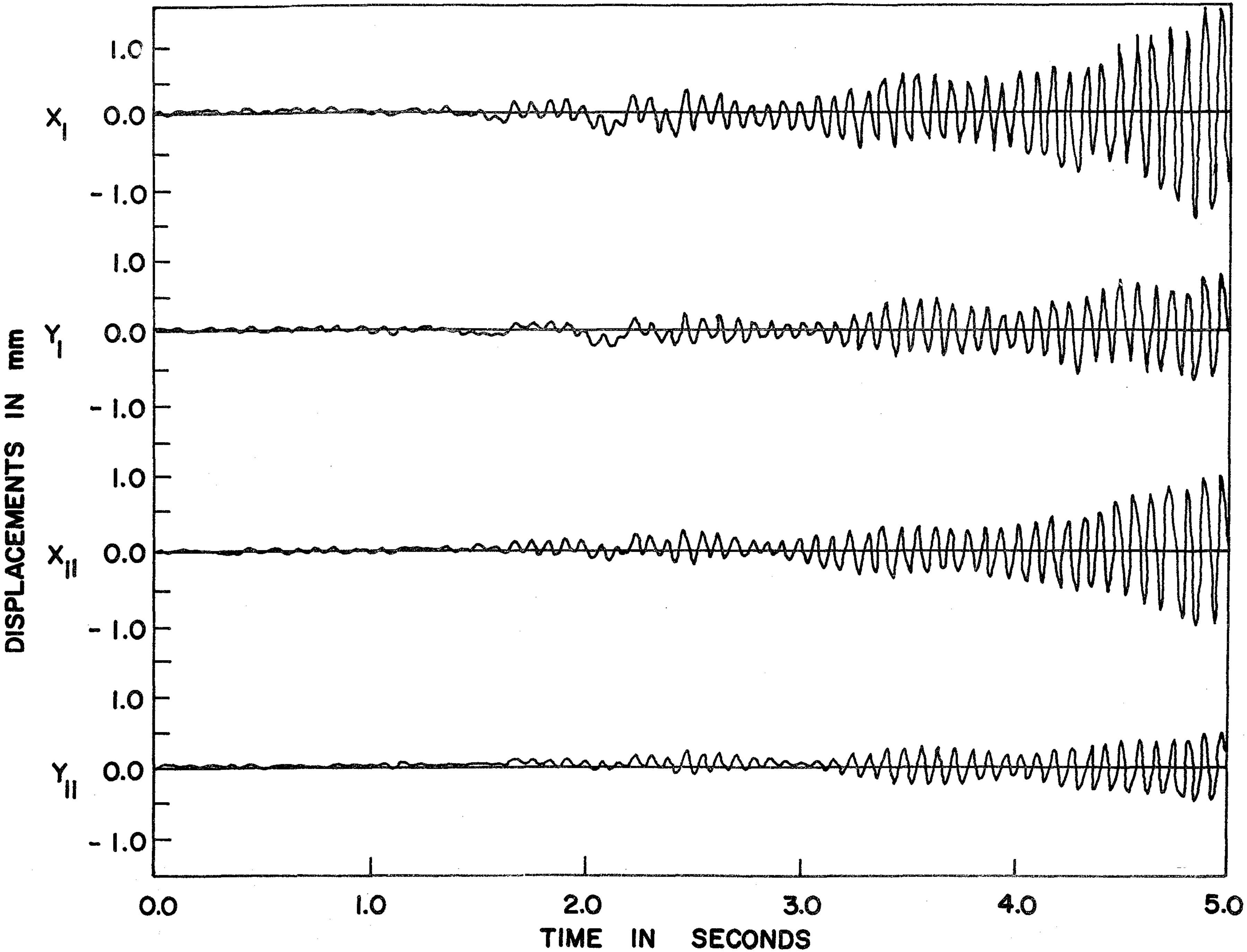


FIG. 3.19 DISPLACEMENTS OF ROTOR IN THE BEARINGS
(BASE ROTATION, AXIAL FORCE AND AXIAL TORQUE INCLUDED)

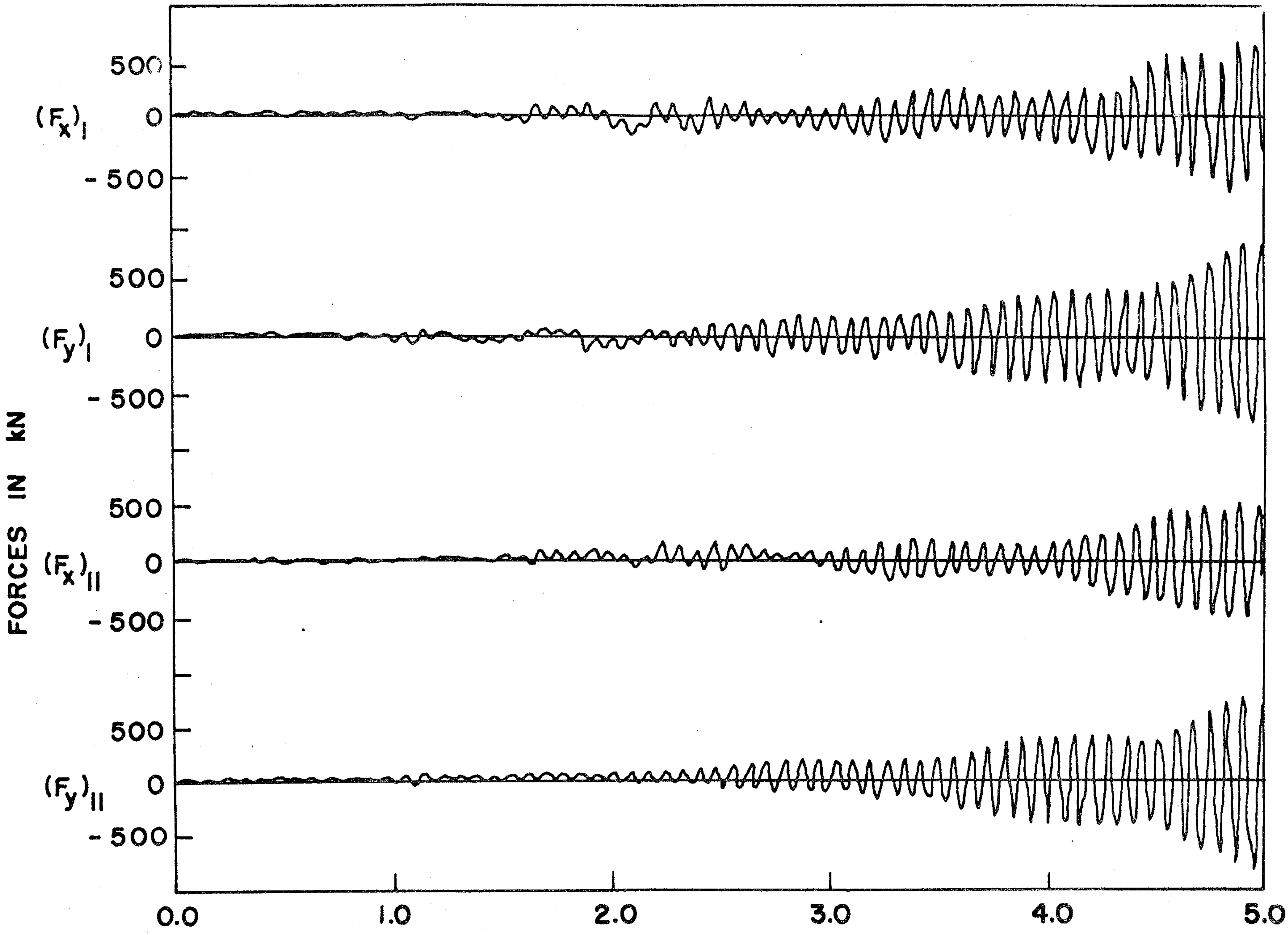


FIG. 3.20 DYNAMIC REACTION FORCES IN THE BEARINGS
(BASE ROTATION, AXIAL FORCE AND AXIAL TORQUE INCLUDED)

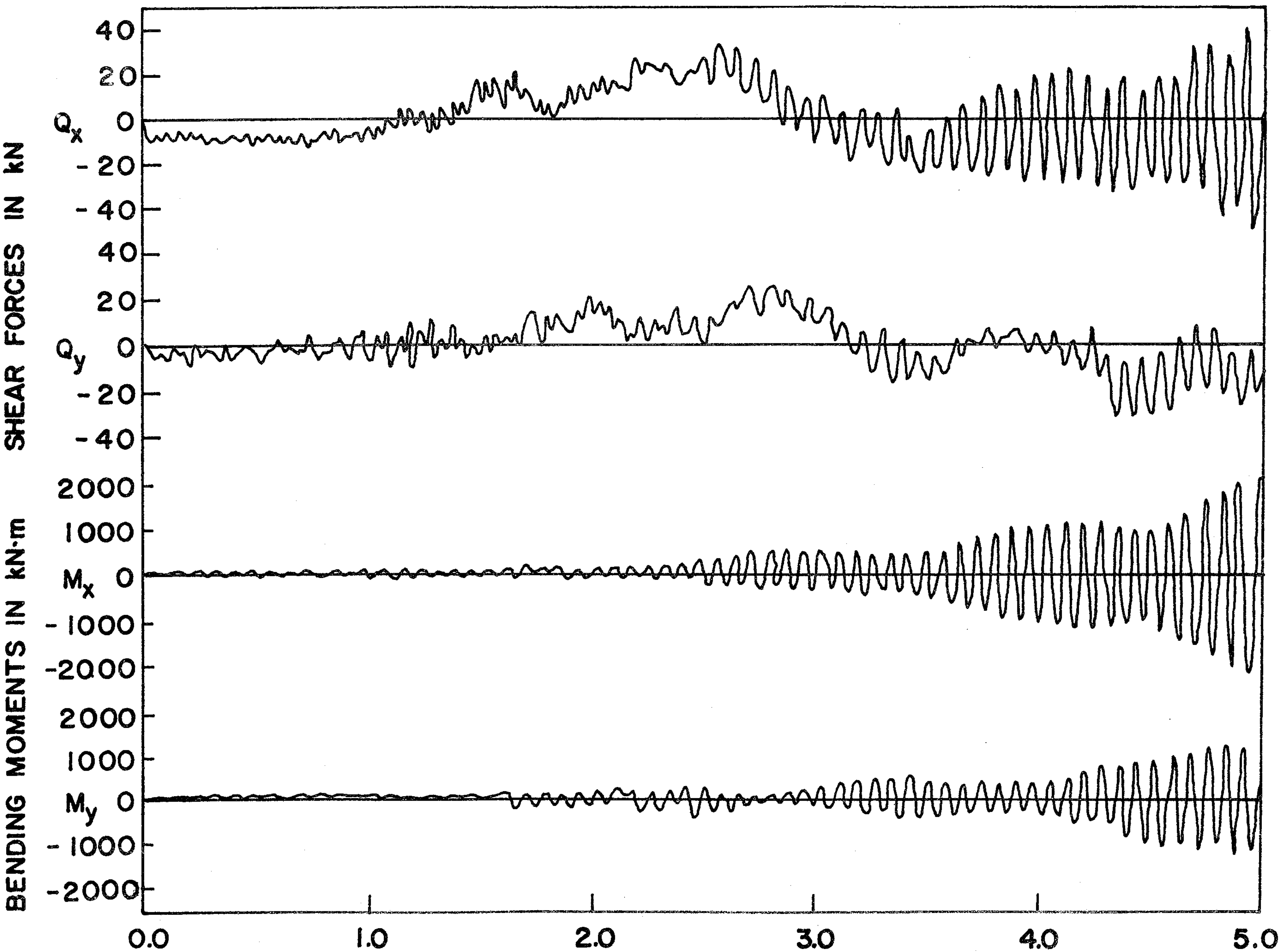


FIG.3.21 SHEAR FORCES AND BENDING MOMENTS AT MIDSPAN
(BASE ROTATION, AXIAL FORCE AND AXIAL TORQUE INCLUDED)

can be included in the seismic analysis using Timoshenko beam theory. The beam model includes such factors as rotatory inertia, shear deformation, gyroscopic effects, rotor-bearing interaction (i.e. stiffness and damping provided by the lubricants in the bearings), intermediate disks and flywheels, initial stresses due to axial thrust and axial torque, base translation and base rotation (including Coriolis effects). The beam model is clearly superior to the rigid body model developed earlier. Since the beam model developed in this chapter uses a finite element solution procedure, the corresponding computer program has been written along the familiar, widely used finite element codes such as SAP IV and NASTRAN. Hence the user will find it easier to use the beam model and may also choose to include it along with other general purpose seismic analysis codes available in his organization. For the beam model, the computing time and the cost of computing are very reasonable.

One limitation of the beam model is that it can be used only for shaft-like rotating systems. A uranium centrifuge, for example, is a cylindrical shell-type structure rotating about its axis at a high speed and a beam model cannot be used to obtain the seismic response of such a centrifuge. Another, though minor, limitation of the beam model is that the initial stresses due to spin of the system cannot be included in the analysis. A three-dimensional model of the rotating system will avoid these limitations and is likely to enlarge the range of application.

4. 3-D ELASTICITY MODEL

4.1 SCOPE OF CHAPTER

In the last two chapters we presented a rigid body model and a beam model to obtain the seismic response of a rotating mechanical system. We now present a three dimensional elasticity model in which the rotating system will be modeled as a spinning elastic solid of revolution.

The dynamical problem is formulated using Newton-Euler approach. The governing differential equations are derived from the three dimensional theory of elasticity. A numerical solution to the problem is obtained using eight-noded isoparametric, ring-type elements in the spatial domain and finite differences in the temporal domain.

4.2 FORMULATION OF THE PROBLEM

The rotating body is a body of revolution, with the spin axis as the axis of revolution. The governing equations of motion of the rotating body are derived by isolating an elemental ring of the body. This elemental ring will be treated as a rigid body to obtain such kinematic quantities as acceleration and rate of change of angular momentum. The elastic properties of the rotating body will be taken into account while evaluating the forces and moments acting on the elemental ring.

4.2.1 KINEMATIC RELATIONS

Consider a rigid, circular elemental ring spinning about its axis and executing arbitrary motion in space. Its motion can be conveniently described using Euler angles as shown in Figure 4.1. XYZ is a reference system which

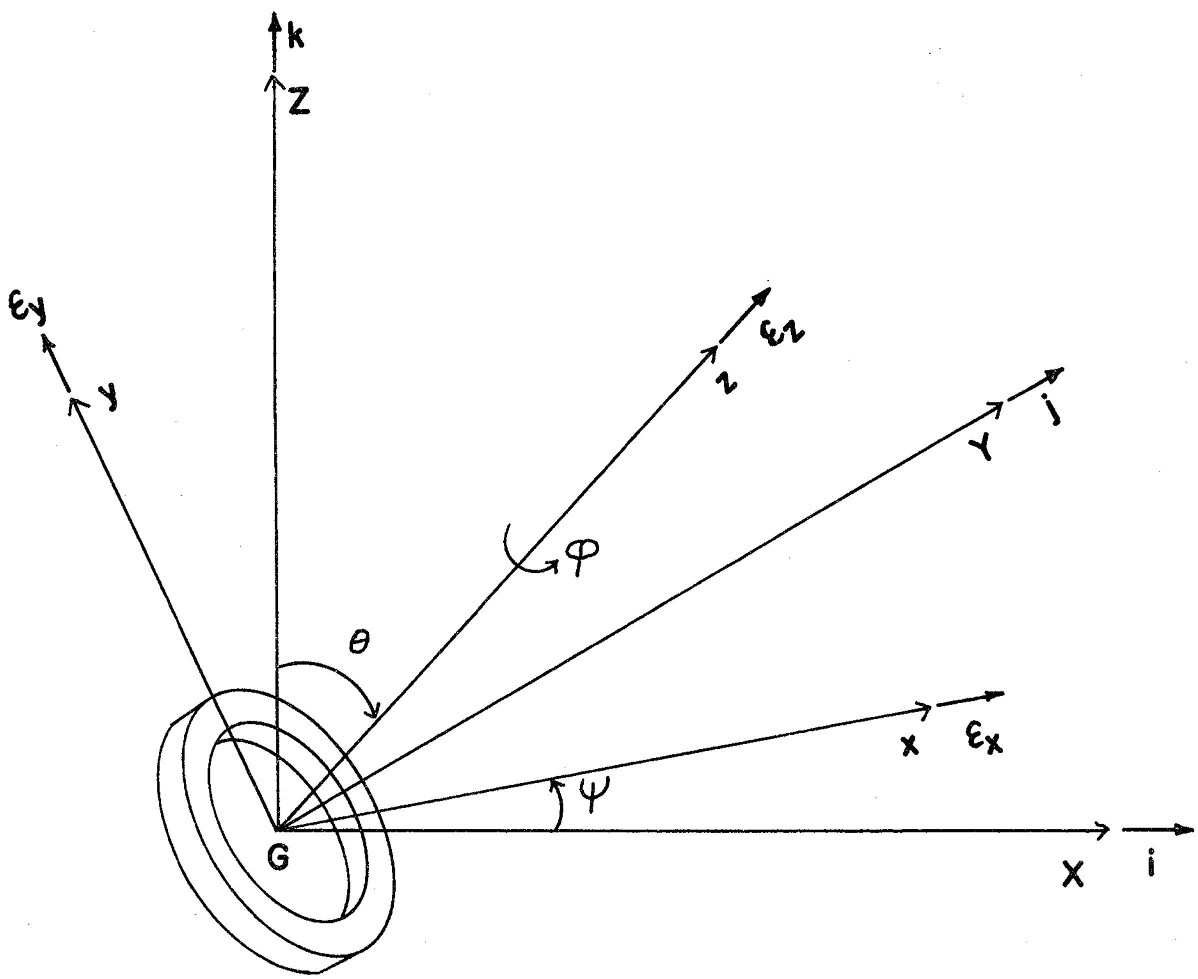


FIG. 4.1 EULER ANGLES FOR THE GENERAL MOTION OF A RIGID RING

preserves fixed orientation in space (i.e. no rotation) with the center of mass of the elemental ring as its origin. xyz is another, nonspinning reference system with its origin at the center of mass of the ring, but xyz can execute precessional (ψ) and nutational (θ) motion. In addition to the precessional and nutational motion, the rigid elemental ring can possess a spin (ϕ) motion about z-axis of the xyz reference system.

The Newton's Law of Motion for the elemental ring can be written vectorially as

$$\underset{\sim}{F} = (2\rho\pi r dr dz)\underset{\sim}{a}_G$$

and (4.1)

$$\underset{\sim}{M}_G = \dot{\underset{\sim}{H}}_G$$

where $\underset{\sim}{F}$ is the resultant force acting on the elemental ring, $\underset{\sim}{a}_G$ is the absolute acceleration of the center of mass of the elemental ring, $\underset{\sim}{M}_G$ is the moment due to external forces taken about the center of mass and $\underset{\sim}{H}_G$ is the angular momentum of the elemental ring computed about its center of mass. A general expression for the time rate of change of the angular momentum can be derived as

$$\begin{aligned} \dot{\underset{\sim}{H}}_G &= \rho\pi r^3\{\ddot{\theta} + 2\dot{\phi}\dot{\psi}\text{Sin}\theta + \dot{\psi}^2\text{Sin}\theta\ \text{Cos}\theta\}\ drdz\ \underset{\sim}{\varepsilon}_x \\ &+ \rho\pi r^3\{\ddot{\psi}\text{Sin}\theta - 2\dot{\phi}\dot{\theta}\}\ drdz\ \underset{\sim}{\varepsilon}_y \\ &+ 2\rho\pi r^3\{\ddot{\phi} + \ddot{\psi}\text{Cos}\theta - \dot{\psi}\dot{\theta}\text{Sin}\theta\}\ drdz\ \underset{\sim}{\varepsilon}_z \end{aligned} \quad (4.2)$$

where $\underset{\sim}{\varepsilon}_x$, $\underset{\sim}{\varepsilon}_y$ and $\underset{\sim}{\varepsilon}_z$ are the unit vectors along the x, y and z axes. ρ is the mass density of the material and r is the radius of the elemental ring. The cross sectional area of the elemental ring is given by drdz.

Let us consider the case when the xyz reference system assumes an orientation with $\theta \cong \pi/2$ and $\psi \cong 0$ as shown in Figure 4.2. The rotating system is supported on bearings and the bearing-base unit will be considered as a rigid body with a body fixed reference system $x_b y_b z_b$. The origin b of the $x_b y_b z_b$ reference system is so chosen that in equilibrium position the axis of the rotating body is parallel to the z_b axis and lies in the $y_b z_b$ plane. The lubricants in the bearings provide stiffness and damping for the relative motion between the rotating body and the bearings. In the seismic analysis of such a rotating system, the base is subjected to known translational and rotational motion. The analyst aims at predicting the transient dynamic response of the rotating body.

The base excitation due to earthquake results in small, perturbational rotations and translations of the base about its equilibrium position. The rotating body responds with small, perturbational rotations and translations of the xyz reference system from the position of $\theta = \pi/2$ and $\psi = 0$ as shown in Figure 4.2. Let $\underset{\sim}{\omega}_b$ be the known angular velocity and $\underset{\sim}{\alpha}_b$ be the known angular acceleration of the base given by

$$\begin{aligned} \underset{\sim}{\omega}_b &= \dot{\theta}_{xb}\,\underset{\sim}{\varepsilon}_{xb} + \dot{\theta}_{yb}\underset{\sim}{\varepsilon}_{yb} + \dot{\theta}_{zb}\underset{\sim}{\varepsilon}_{zb} \\ \underset{\sim}{\alpha}_b &= \ddot{\theta}_{xb}\underset{\sim}{\varepsilon}_{xb} + \ddot{\theta}_{yb}\underset{\sim}{\varepsilon}_{yb} + \ddot{\theta}_{zb}\underset{\sim}{\varepsilon}_{zb} \end{aligned} \tag{4.3}$$

The small, perturbational translations of the center of mass of the elemental ring relative to the $x_b y_b z_b$ reference system can be specified by the displacements u_x, u_y and u_z along the x_b, y_b and z_b axes. Similarly, the small perturbational rotations of the xyz reference system relative to the $x_b y_b z_b$ reference system can be specified by the small rotations θ_x, θ_y and θ_z about

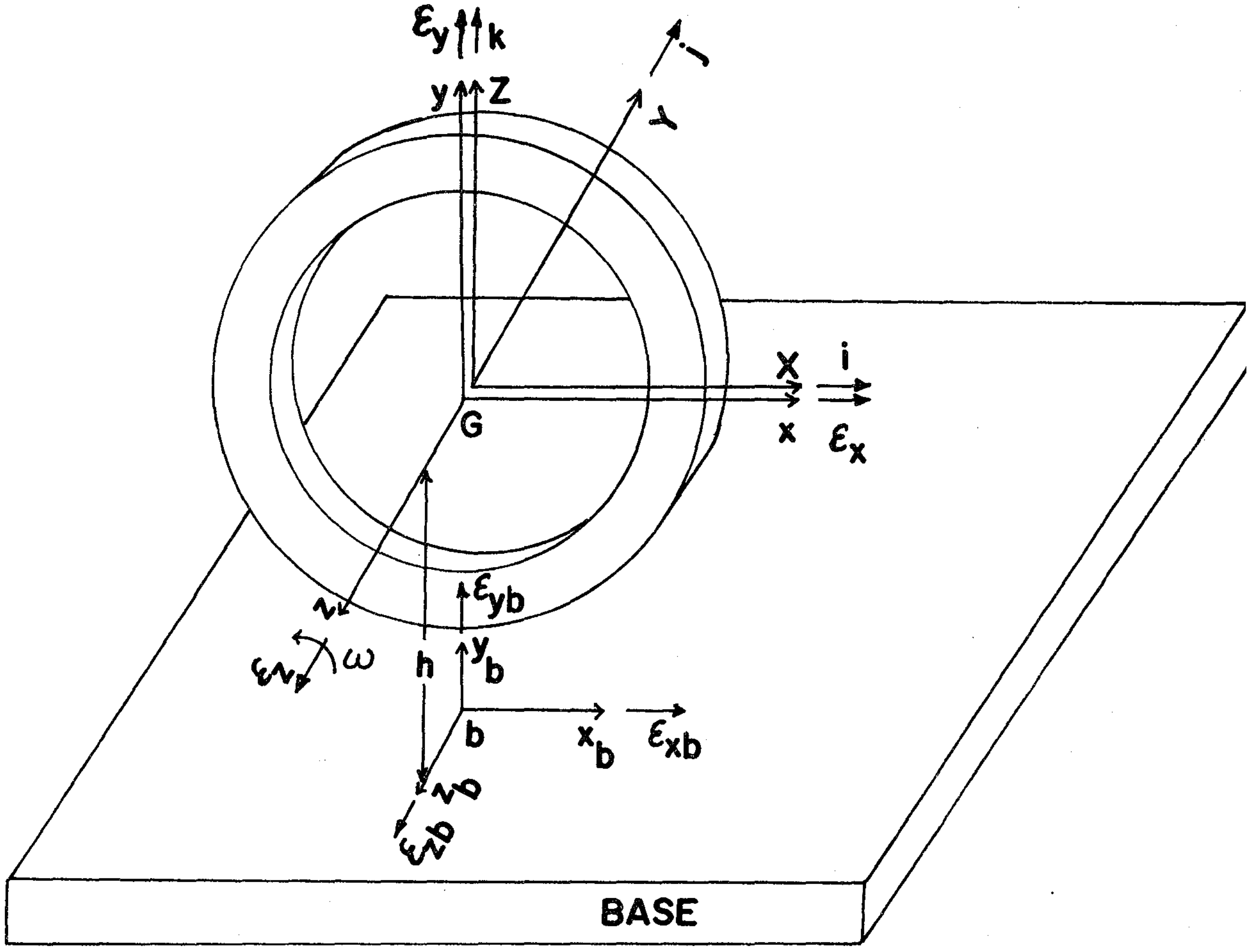


FIG.4.2 RING AND BASE REFERENCE FRAMES

the x_b, y_b, and z_b axes and the sequence in which these rotations take place becomes immaterial. Since the rotations of the base θ_{xb}, θ_{yb} and θ_{zb} about the x_b, y_b and z_b axes are also small, perturbational motions, it can be taken that

$$\begin{aligned} \underset{\sim}{\varepsilon}_{xb} &\cong \underset{\sim}{\varepsilon}_{x} \cong \underset{\sim}{i} \\ \underset{\sim}{\varepsilon}_{yb} &\cong \underset{\sim}{\varepsilon}_{y} \cong \underset{\sim}{k} \\ \text{and} \quad \underset{\sim}{\varepsilon}_{zb} &\cong \underset{\sim}{\varepsilon}_{z} \cong -\underset{\sim}{j} \end{aligned} \tag{4.4}$$

This leads to the approximate expressions

$$\begin{aligned} \theta &= \pi/2 + \theta_{xb} + \theta_{x} \quad , \quad & \psi &= \theta_{yb} + \theta_{y} \\ \dot{\theta} &= \dot{\theta}_{xb} + \dot{\theta}_{x} \quad , \quad & \dot{\psi} &= \dot{\theta}_{yb} + \dot{\theta}_{y} \\ \ddot{\theta} &= \ddot{\theta}_{xb} + \ddot{\theta}_{x} \quad , \quad & \ddot{\psi} &= \ddot{\theta}_{yb} + \ddot{\theta}_{y} \end{aligned} \tag{4.5}$$

In addition, it will be assumed that the spin velocity of the rotor remains constant so that

$$\dot{\phi} = \omega \text{ (a constant) and } \ddot{\phi} = 0 \tag{4.6}$$

Substituting (4.5) and (4.6) in equation (4.2) and retaining only the first order terms, we get the linearized expression

$$\dot{\underset{\sim}{H}}_G = \rho\pi r^3 \{(\ddot{\theta}_{xb} + \ddot{\theta}_x) + 2\omega(\dot{\theta}_{yb} + \dot{\theta}_y)\} \text{ drdz } \underset{\sim}{\varepsilon}_{xb}$$
$$+ \rho\pi r^3\{(\ddot{\theta}_{yb} + \ddot{\theta}_y) - 2\omega(\dot{\theta}_{xb} + \dot{\theta}_x)\} \text{ drdz } \underset{\sim}{\varepsilon}_{yb} \tag{4.7}$$

In the above expression, terms involving ω are the familiar gyroscopic moments caused by the rotation of the spin axis.

The absolute acceleration of the point G can be obtained by considering the motion of the point b and the relative motion of G with respect to the $x_b y_b z_b$ reference system. Even though the unit vectors in various references systems shown in Figure 4.2 can be approximately equated to their counterparts as shown in equation (4.4), their time derivatives cannot be equated in a similar manner. Hence,

$$\underset{\sim}{a}_G = \underset{\sim}{a}_b + \omega_b x(\underset{\sim}{\omega}_b \times \underset{\sim}{r}) + \underset{\sim}{\alpha}_b \times \underset{\sim}{r} + 2\underset{\sim}{\omega}_b \times \underset{\sim}{V}_{rel} + \underset{\sim}{a}_{rel} \tag{4.8}$$

where

$$\underset{\sim}{a}_b = \ddot{X}_b \underset{\sim}{\varepsilon}_{xb} + \ddot{Y}_b \underset{\sim}{\varepsilon}_{yb} + \ddot{Z}_b \underset{\sim}{\varepsilon}_{zb}$$

$$\underset{\sim}{r} = u_x \underset{\sim}{\varepsilon}_{xb} + (h + u_y)\, \underset{\sim}{\varepsilon}_{yb} + (z + u_z) \underset{\sim}{\varepsilon}_{zb}$$

$$\underset{\sim}{V}_{rel} = \dot{u}_x \underset{\sim}{\varepsilon}_{xb} + \dot{u}_y \underset{\sim}{\varepsilon}_{yb} + \dot{u}_z \underset{\sim}{\varepsilon}_{zb} \tag{4.9}$$

$$\underset{\sim}{a}_{rel} = \ddot{u}_x \underset{\sim}{\varepsilon}_{xb} + \ddot{u}_y \underset{\sim}{\varepsilon}_{yb} + \ddot{u}_z \underset{\sim}{\varepsilon}_{zb}$$

This leads to

$$\underset{\sim}{a}_G = a_x \underset{\sim}{\varepsilon}_{xb} + a_y \underset{\sim}{\varepsilon}_{yb} + a_z \underset{\sim}{\varepsilon}_{zb}$$

where

$$a_x = \ddot{u}_x - 2\dot{\theta}_{zb}\dot{u}_y + 2\dot{\theta}_{yb}\dot{u}_z - (\dot{\theta}_{yb}^2 + \dot{\theta}_{zb}^2)u_x + (\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})u_y$$

$$+ (\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})u_z + \ddot{X}_b + h(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb}) + z(\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})$$

$$a_y = \ddot{u}_y + 2\dot{\theta}_{zb}\dot{u}_x - 2\dot{\theta}_{xb}\dot{u}_z + (\dot{\theta}_{xb}\dot{\theta}_{yb} + \ddot{\theta}_{zb})u_x - (\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2)u_y$$

$$+ (\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})u_z + \ddot{Y}_b - h(\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2) + z(\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})$$

and

$$a_z = \ddot{u}_z - 2\dot{\theta}_{yb}\dot{u}_x + 2\dot{\theta}_{xb}\dot{u}_y + (\dot{\theta}_{zb}\dot{\theta}_{xb} - \ddot{\theta}_{yb})u_x + (\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb})u_y$$

$$-(\dot{\theta}_{xb}^2 + \dot{\theta}_{yb}^2)u_z + \ddot{Z}_b + h(\dot{\theta}_{yb}\dot{\theta}_{zb} + \ddot{\theta}_{xb}) - z(\dot{\theta}_{xb}^2 + \dot{\theta}_{yb}^2) \qquad (4.10)$$

It should be pointed out that in the kinematic relations developed in this section, the gyroscopic effects and the base motions (including translation and rotation) have been taken into account.

4.2.2 KINETIC RELATIONS

We now turn to the evaluation of forces and moments acting on the elemental ring. Since the rotating system is a solid revolution, the stresses acting on the elemental ring, and hence the forces and moments, can be evaluated more conveniently in cylindrical polar coordinates.

4.2.2.1 RELATIONS IN CYLINDRICAL POLAR COORDINATES

The rotating solid of revolution is under a state of initial stress. The initial stresses can result from:

(1) Steady spin of the body about the axis of revolution. This will result in initial stresses $\sigma_{rr}^{(0)}$, $\sigma_{\phi\phi}^{(0)}$, $\sigma_{zz}^{(0)}$ and $\tau_{zr}^{(0)}$, forming an axi-symmetric distribution of initial stresses.

(2) Axial thrust on the rotating system. This will result in initial streses $\sigma_{rr}^{(0)}$, $\sigma_{\phi\phi}^{(0)}$, $\sigma_{zz}^{(0)}$ and $\tau_{zr}^{(0)}$, again forming an axi-symmetric distribution of initial stresses.

(3) Torque transmitted by the rotating system. This will result in initial stresses $\tau_{r\phi}^{(0)}$ and $\tau_{\phi z}^{(0)}$, forming an anti-symmetric distribution of initial stresses.

All the six components of initial stresses mentioned above are independent of ϕ.

Let σ_{rr}, $\sigma_{\phi\phi}$, σ_{zz}, $\tau_{r\phi}$, $\tau_{\phi z}$ and τ_{zr} be the incremental stress components and u, v and w be the incremental displacement components along the r, ϕ and z directions, respectively. The components of incremental strain, within the framework of linear theory, are given by

$$e_{rr} = \frac{\partial u}{\partial r}$$

$$e_{\phi\phi} = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \phi}$$

$$e_{zz} = \frac{\partial w}{\partial z}$$

$$e_{r\phi} = \frac{1}{r}\frac{\partial u}{\partial \phi} + \frac{\partial v}{\partial r} - \frac{v}{r} \tag{4.11}$$

$$e_{\phi z} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \phi}$$

$$e_{zr} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}$$

The components of incremental rotation are given by

$$\omega_r = 1/2\,(\frac{1}{r}\frac{\partial w}{\partial \phi} - \frac{\partial v}{\partial z})$$

$$\omega_\phi = 1/2\,(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}) \tag{4.12}$$

$$\omega_z = 1/2\,(\frac{\partial v}{\partial r} + \frac{v}{r} - \frac{1}{r}\frac{\partial u}{\partial \phi})$$

The incremental stress and strain components are related to each other by Hooke's law

$$\sigma_{rr} = \frac{E}{(1 + \nu)}\{e_{rr} + \frac{\nu}{(1 - 2\nu)}(e_{rr} + e_{\phi\phi} + e_{zz})\}$$

$$\sigma_{\phi\phi} = \frac{E}{(1 + \nu)}\{e_{\phi\phi} + \frac{\nu}{(1 - 2\nu)}(e_{rr} + e_{\phi\phi} + e_{zz})\}$$

$$\sigma_{zz} = \frac{E}{(1+\nu)} \quad e_{zz} + \frac{\nu}{(1-2\nu)} (e_{rr} + e_{\phi\phi} + e_{zz})$$

$$\tau_{r\phi} = \frac{E}{2(1+\nu)} \; e_{r\phi} \tag{4.13}$$

$$\tau_{\phi z} = \frac{E}{2(1+\nu)} \; e_{\phi z}$$

$$\tau_{zr} = \frac{E}{2(1+\nu)} \; e_{zr}$$

where E is the Young's modulus of the material and ν is the Poisson's ratio.

Consider an isolated element of the ring as shown in Figure 4.3. In evaluating the resultant force acting on this element of the ring due to seismic excitation, the effects of initial stresses must be taken into account. Within the framework of linear theory, the forces acting on the element of the ring along the r, ϕ and z directions are given by [33] as

$$F_r = \frac{1}{r} [\frac{\partial}{\partial r} \{r\sigma_{rr} + r(1 + e_{rr})\sigma_{rr}^{(0)} + r(\tfrac{1}{2} e_{r\phi} - \omega_z)\tau_{r\phi}^{(0)} + r(\tfrac{1}{2} e_{zr} + \omega_\phi)\tau_{zr}^{(0)}\}$$

$$+ \frac{\partial}{\partial \phi} \{\tau_{r\phi} + (1 + e_{rr})\tau_{r\phi}^{(0)} + (\tfrac{1}{2} e_{r\phi} - \omega_z)\sigma_{\phi\phi}^{(0)} + (\tfrac{1}{2} e_{zr} \; \omega_\phi)\tau_{\phi z}^{(0)}\}$$

$$+ \frac{\partial}{\partial z} \{r\tau_{zr} + r(1 + e_{rr})\tau_{zr}^{(0)} + (\tfrac{1}{2} e_{r\phi} - \omega_z)T_{\phi z}^{(0)} + (\tfrac{1}{2} e_{zr} + \omega_\phi)\sigma_{zz}^{(0)}\}$$

$$-\{\sigma_{\phi\phi} + (\tfrac{1}{2} e_{r\phi} + \omega_z)\tau_{r\phi}^{(0)} + (1 + e_{\phi\phi})\sigma_{\phi\phi}^{(0)} + (\tfrac{1}{2} e_{\phi z} - \omega_r)\tau_{\phi z}^{(0)}\}] \, rdrd\phi dz$$

$$F_\phi = \frac{1}{r} [\frac{\partial}{\partial r} \{r\tau_{r\phi} + r(\tfrac{1}{2} e_{r\phi} + \omega_z)\sigma_{rr}^{(0)} + r(1 + e_{\phi\phi})\tau_{r\phi}^{(0)} + r(\tfrac{1}{2} e_{\phi z} - \omega_r)\tau_{zr}^{(0)}\}$$

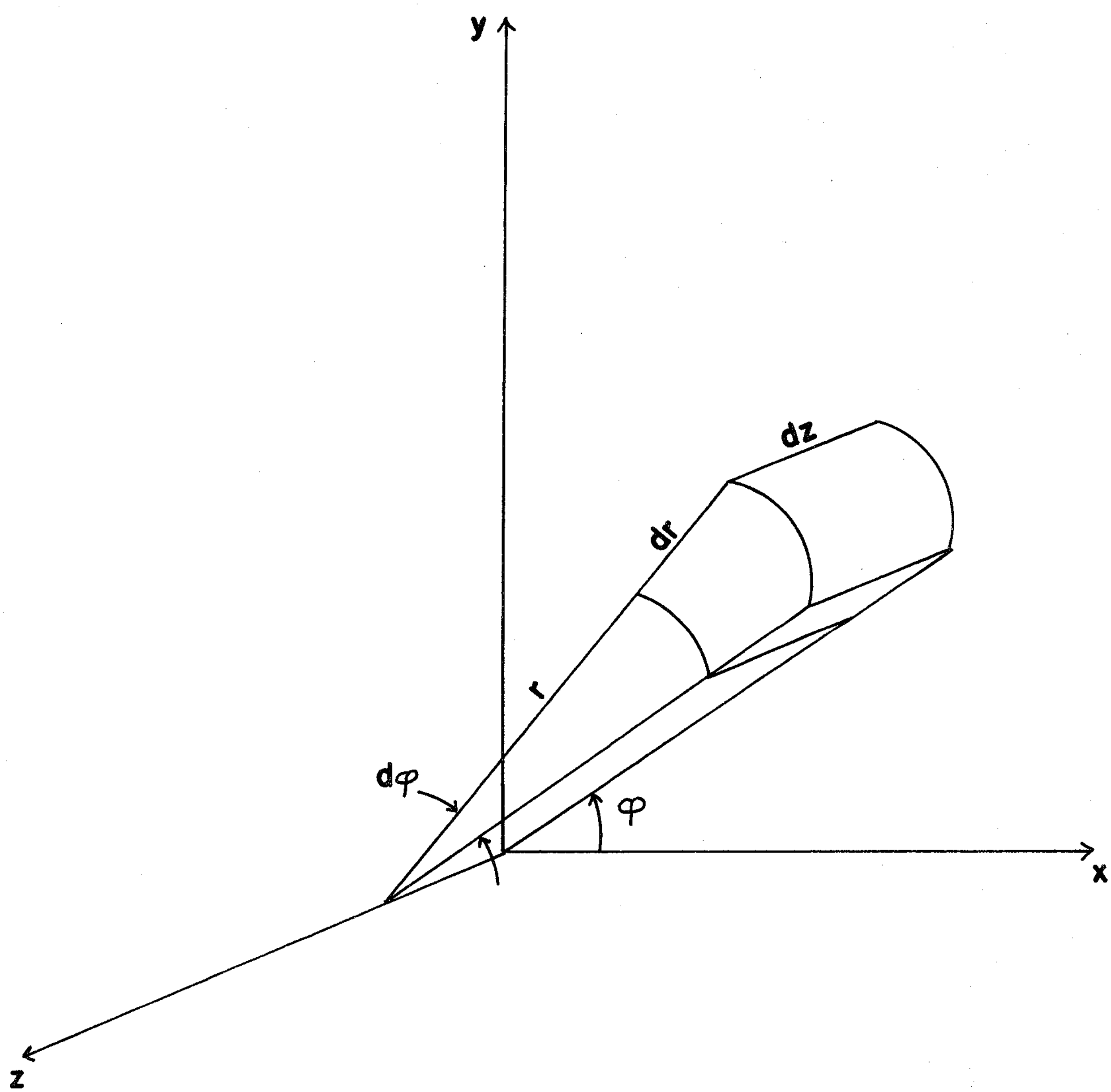


FIG.4.3 DIFFERENTIAL ELEMENT OF A RING

$$+ \frac{\partial}{\partial\phi} \{\sigma_{\phi\phi} + (\tfrac{1}{2} e_{r\phi} + \omega_z)\tau_{r\phi}^{(0)} + (1 + e_{\phi\phi})\sigma_{\phi\phi}^{(0)} + (\tfrac{1}{2} e_{\phi z} - \omega_r)\tau_{\phi z}^{(0)}\}$$

$$+ \frac{\partial}{\partial z} \{r\tau_{\phi z} + r(\tfrac{1}{2} e_{r\phi} + \omega_z)\tau_{zr}^{(0)} + r(1 + e_{\phi\phi})\tau_{\phi z}^{(0)} + r(\tfrac{1}{2} e_{\phi z} - \omega_r)\sigma_{zz}^{(0)}\}$$

$$+ \{\tau_{r\phi} + (1 + e_{rr})\tau_{r\phi}^{(0)} + (\tfrac{1}{2} e_{r\phi} - \omega_z)\sigma_{\phi\phi}^{(0)} + (\tfrac{1}{2} e_{zr} + \omega_{\phi}^{(0)}\} \; rdrd\phi dz$$

$$F_z = \frac{1}{r} [\frac{\partial}{\partial r} \{r\tau_{zr} + r(\tfrac{1}{2} e_{zr} - \omega_\phi)\sigma_{rr}^{(0)} + r(\tfrac{1}{2} e_{\phi z} + \omega_r)\tau_{r\phi}^{(0)} + r(1 + e_{zz})\tau_{zr}^{(0)}\}$$

$$+ \frac{\partial}{\partial\phi} \{\tau_{\phi z} + (\tfrac{1}{2} e_{zr} - \omega_\phi)\tau_{r\phi}^{(0)} + (\tfrac{1}{2} e_{\phi z} + \omega_r)\sigma_{\phi\phi}^{(0)} + (1 + e_{zz})\tau_{\phi z}^{(0)}\}$$

$$+ \frac{\partial}{\partial z} \{r\sigma_{zz} + r(\tfrac{1}{2} e_{zr} - \omega_\phi)\tau_{zr}^{(0)} + r(\tfrac{1}{2} e_{\phi z} + \omega_r)\tau_{\phi z}^{(0)} + (1 + e_{zz})\sigma_{zz}^{(0)}\}] \; rdrd\phi dz$$

(4.14)

4.2.2.2 RELATIONS IN CARTESIAN COORDINATES

Since the incremental displacements, stresses and strains are periodic in ϕ, one can seek the solution to the problem using harmonic decomposition. In this, the unknown displacements, stresses and strains will be expanded as fourier series in ϕ. The displacements, for example, will be expanded as

$$u = U_0 + \sum_{n=1}^{\infty} U_{nc} \; \text{Cos} n\phi + U_{ns} \; \text{Sin} n\phi$$

$$v = V_0 + \sum_{n=1}^{\infty} V_{nc} \; \text{Cos} n\phi + V_{ns} \text{Sin} n\phi \qquad (4.15)$$

$$w = W_0 + \sum_{n=1}^{\infty} W_{nc} \text{Cos} n\phi + W_{ns} \text{Sin} n\phi$$

Here, U_0, V_0, and W_0 correspond to the zero harmonic, and U_{nc}, U_{ns}, . . . , W_{ns} correspond to the n^{th} harmonic. Solutions for the various harmonics can be found independently and then superposed to obtain the overall solution.

The zero harmonic modes, U_0 and W_0, can be excited by a base motion along the axial direction of the rotating system. However, the more important bending modes due to base excitation are represented by the first harmonic terms [34]. So we shall restrict our attention to the expansion

$$u = U_{1c} \cos\phi + U_{1s} \sin\phi$$

$$v = V_{1s} \sin\phi + V_{1c} \cos\phi \tag{4.16}$$

$$w = W_{1c} \cos\phi + W_{1s} \sin\phi$$

It should be noted that both the symmetric and anti-symmetric harmonics have been retained in (4.16). This is because of the presence of gyroscopic effects that couple the flexural motion in two mutually perpendicular planes.

The expansion in (4.16) leaves us with six independent, unknown Fourier coefficients of displacements; namely U_{1c}, U_{1s}, V_{1c}, V_{1s}, W_{1c} and W_{1s}. These coefficients, however, have the disadvantage that they cannot be interpreted as the displacements of a physical point on the rotating system. This makes it difficult to relate these coefficients to the bearing reaction forces which are specified in literature as functions of the displacements and velocities of the rotor axis relative to the bearing. So we shall adopt an expansion, which is similar to (4.16) but morre in line with our kinematic relations, as

$$u = u_x \text{Cos}\phi + u_y \text{Sin}\phi$$

$$v = -u_x \text{Sin}\phi + u_y \text{Cos}\phi \qquad (4.17)$$

$$w = -r\theta_y \text{Cos}\phi + r\theta_x \text{Sin}\phi$$

The above expansion involves only four unknown coefficients, namely u_x, u_y, θ_x , and θ_y, and this time these coefficients can be interpreted as the displacements of the center of mass of the elemental ring and the rotations of the elemental ring.

In a similar fashion, the incremental stresses and strains can be expanded as

$$\sigma_{rr} = \sigma_{rrc} \text{Cos}\phi + \sigma_{rrs} \text{Sin}\phi$$

$$\sigma_{\phi\phi} = \sigma_{\phi\phi c} \text{Cos}\phi + \sigma_{\phi\phi s} \text{Sin}\phi$$

$$\sigma_{zz} = \sigma_{zzc} \text{Cos}\phi + \sigma_{zzs} \text{Sin}\phi \qquad (4.18)$$

$$\tau_{r\phi} = \tau_{r\phi c} \text{Cos}\phi + \tau_{r\phi s} \text{Sin}\phi$$

$$\tau_{\phi z} = \tau_{\phi zc} \text{Cos}\phi + \tau_{\phi zs} \text{Sin}\phi$$

$$\tau_{zr} = \tau_{zrc} \text{Cos}\phi + \tau_{zrs} \text{Sin}\phi$$

and

$$e_{rr} = e_{rrc}\text{Cos}\phi + e_{rrs}\text{Sin}\phi$$

$$e_{\phi\phi} = e_{\phi\phi c}\text{Cos}\phi + e_{\phi\phi s}\text{Sin}\phi$$

$$e_{zz} = e_{zzc}\text{Cos}\phi + e_{zzs}\text{Sin}\phi$$

$$e_{r\phi} = e_{r\phi c}\text{Cos}\phi + e_{r\phi s}\text{Sin}\phi \qquad (4.19)$$

$$e_{\phi z} = e_{\phi zc}\text{Cos}\phi + e_{\phi zs}\text{Sin}\phi$$

$$e_{zr} = e_{zrc}\text{Cos}\phi + e_{zrs}\text{Sin}\phi$$

Equations (4.11), (4.12), (4.17) and (4.18) can now be substituted in (4.14) to find the forces acting on an element of the ring along the r, ϕ and z directions. Resolving these components of forces along the x and y directions and taking their moments about the center of mass of the ring, and integrating these forces and moments along ϕ, we finally obtain the forces and moments acting on the elemental ring shown in Figure 4.2 as

$$\underset{\sim}{F} = \pi[\frac{\partial}{\partial r}(r\sigma_{rrc} - r\tau_{r\phi s}) + \frac{\partial}{\partial z}(r\tau_{zrc} - r\tau_{\phi zs}) + 2\frac{\partial}{\partial r}(r\frac{\partial u_x}{\partial r}\sigma_{rr}^{(0)} + r\frac{\partial u_x}{\partial z}\tau_{zr}^{(0)})$$

$$+ 2\frac{\partial}{\partial z}(r\frac{\partial u_x}{\partial r}\tau_{zr}^{(0)} + r\frac{\partial u_x}{\partial z}\sigma_{zz}^{(0)})]\, drdz\, \underset{\sim}{\varepsilon}_x$$

$$+ \pi[\frac{\partial}{\partial r}(r\sigma_{rrs} + r\tau_{r\phi c}) + \frac{\partial}{\partial z}(r\tau_{zrs} + r\tau_{\phi zc}) + 2\frac{\partial}{\partial r}(r\frac{\partial u_y}{\partial r}\sigma_{rr}^{(0)} + r\frac{\partial u_y}{\partial z}\tau_{zr}^{(0)})$$

$$+ 2\frac{\partial}{\partial z}(r\frac{\partial u_y}{\partial r}\tau_{zr}^{(0)} + r\frac{\partial u_y}{\partial z}\sigma_{zz}^{(0)})]\, drdz\, \underset{\sim}{\varepsilon}_y$$

$$\underset{\sim}{M}_G = \pi[\frac{\partial}{\partial r}(r\tau_{zrs}) + \frac{\partial}{\partial z}(r\sigma_{zzs}) - \tau_{\phi zc} + \frac{\partial}{\partial r}\{r\frac{\partial(r\theta_x)}{\partial r}\sigma_{rr}^{(0)} + r\theta_y\tau_{r\phi}^{(0)} + r\frac{\partial(r\theta_x)}{\partial z}\tau_{zr}^{(0}$$

$$+ \frac{\partial}{\partial z}\{r\frac{\partial(r\theta_x)}{\partial r}\tau_{zr}^{(0)} + r\theta_y\tau_{\phi z}^{(0)} + r\frac{\partial(r\theta_x)}{\partial z}\sigma_{zz}^{(0)}\}$$

$$+ \frac{\partial(r\theta_y)}{\partial r}\tau_{r\phi}^{(0)} - \theta_x\sigma_{\phi\phi}^{(0)} + \frac{\partial(r\theta_y)}{\partial z}\tau_{\phi z}^{(0)}]\ rdrdz\ \underset{\sim}{\varepsilon}_x$$

$$-\pi[\frac{\partial}{\partial r}(r\tau_{zrc}) + \frac{\partial}{\partial z}(r\sigma_{zzc}) + \tau_{\phi zs} + \frac{\partial}{\partial r}\{-r\frac{\partial(r\theta_y)}{\partial r}\sigma_{rr}^{(0)} + r\theta_x\tau_{r\phi}^{(0)} - r\frac{\partial(r\theta_y)}{\partial z}\tau_{zr}^{(0)}\}$$

$$+ \frac{\partial}{\partial z}\{-r\frac{\partial(r\theta_y)}{\partial r}\tau_{zr}^{(0)} + r\theta_x\tau_{\phi}^{(0)} - r\frac{\partial(r\theta_y)}{\partial z}\sigma_{zz}^{(0)}\}$$

$$+ \frac{\partial(r\theta_x)}{\partial r}\tau_{r\phi}^{(0)} + \theta_y\sigma_{\phi\phi}^{(0)} + \frac{\partial(r\theta_x)}{\partial z}\tau_{\phi z}^{(0)}]\ rdrdz\ \underset{\sim}{\varepsilon}_y$$

(4.20)

Using (4.4), (4.7), (4.10) and (4.20), the governing equations of motion for the spinning elemental ring can be written as

$$\pi[\frac{\partial}{\partial r}(r\sigma_{rrc} - r\tau_{r\phi s}) + \frac{\partial}{\partial z}(r\tau_{zrc} - r\tau_{\phi zs}) + 2\frac{\partial}{\partial r}(r\frac{\partial u_x}{\partial r}\sigma_{rr}^{(0)} + r\frac{\partial u_x}{\partial_z}\tau_{zr}^{(0)})$$

$$+ 2\frac{\partial}{\partial z}(r\frac{\partial u_x}{\partial r}\tau_{zr}^{(0)} + r\frac{\partial u_x}{\partial z}\sigma_{zz}^{(0)})] = 2\pi\rho\ ra_x$$

$$\pi[\frac{\partial}{\partial r}(r\sigma_{rrs} + r\tau_{r\phi c}) + \frac{\partial}{\partial z}(r\tau_{zrs} + r\tau_{\phi zc}) + 2\frac{\partial}{\partial r}(r\frac{\partial u_y}{\partial r}\sigma_{rr}^{(0)} + r\frac{\partial u_y}{\partial z}\tau_{zr}^{(0)})$$

$$+ 2\frac{\partial}{\partial z}(r\frac{\partial u_y}{\partial r}\tau_{zr}^{(0)} + r\frac{\partial u_y}{\partial z}\sigma_{zz}^{(0)})] = 2\pi\rho ra_y$$

$$\pi r[\frac{\partial}{\partial r}(r\tau_{zrs}) + \frac{\partial}{\partial z}(r\sigma_{zzs}) - \tau_{\phi zc} + \frac{\partial}{\partial r}\{r\frac{\partial(r\theta_x)}{\partial r}\sigma_{rr}^{(0)} + r\theta_y\tau_{r\phi}^{(0)} + r\frac{\partial(r\theta_x)}{\partial z}\tau_{zr}^{(0)}\}$$

$$+ \frac{\partial}{\partial z} \{r \frac{\partial (r\theta_x)}{\partial r} \tau_{zr}^{(0)} + r\theta_y \tau_{\phi z}^{(0)} + r \frac{\partial (r\theta_x)}{\partial x} \sigma_{zz}^{(0)}\}$$

$$+ \frac{\partial (r\theta_y)}{\partial r} \tau_{r\phi}^{(0)} - \theta_x \sigma_{\phi\phi}^{(0)} - \frac{\partial (r\theta_y)}{\partial z} \tau_{\phi z}^{(0)}] = \pi\rho r^3 \{(\ddot{\theta}_{xb} + \ddot{\theta}_x) + 2\omega(\dot{\theta}_{yb} + \dot{\theta}_y)\}$$

$$-\pi r[\frac{\partial}{\partial r} (r\tau_{zrc}) + \frac{\partial}{\partial z} (r\sigma_{zzc}) + \tau_{\phi zs} + \frac{\partial}{\partial r} \{-r \frac{\partial (r\theta_y)}{\partial r} \sigma_{rr}^{(0)} + r\theta_x \tau_{r\phi}^{(0)} - r \frac{\partial (r\dot{\theta}_y)}{\partial z} \tau_{zr}^{(0)}\}$$

$$+ \frac{\partial}{\partial z} \{- r \frac{\partial (r\theta_y)}{\partial r} \tau_{zr}^{(0)} + r\theta_x \tau_{\phi z}^{(0)} - r \frac{\partial (r\theta_y)}{\partial z} \sigma_{zz}^{(0)}\}$$

$$+ \frac{\partial (r\theta_x)}{\partial r} \tau_{r\phi}^{(0)} + \theta_y \sigma_{\phi\phi}^{(0)} + \frac{\partial (r\theta_x)}{\partial z} \tau_{\phi z}^{(0)}] = \pi\rho r^3 \{(\ddot{\theta}_{yb} + \ddot{\theta}_y) - 2\omega(\dot{\theta}_{xb} + \dot{\theta}_x)\}$$

(4.21)

4.3 METHOD OF SOLUTION

The equations of motion given by (4.21) are in the form of partial differential equations involving spatial variables r, z, and temporal variable t. A numerical solution to the problem can be obtained by employing finite elements in the spatial domain and finite differences in the time domain.

Solid of revolution elements have been used in the past to study the seismic behaviour of axisymmetric tower structures [34]. The solid of revolution elements developed in this paper differ from those of Liaw and Chopra [34] because of the inclusion of both the symmetric and anti-symmetric harmonics in the displacement functions given by (4.16) and the special form of Fourier coefficients given by (4.17). The governing differential equations must be rendered in an integral form before they can be solved using finite element method. This is achieved by the applicationi of Galerkin's technique.

4.3.1 GALERKIN'S TECHNIQUE

In the Galerkin's technique, the displacements u_x, u_y, and rotations θ_x, θ_y will be treated as the primary unknowns. Let δu_x, δu_y, $\delta\theta_x$ and $\delta\theta_y$ be arbitrary variations from their actual values. Then, according to Galerkin's technique, the equations of motion given by (4.21) can be written in an integral form as

$$\pi\iint[\{2\rho r a_x - \frac{\partial}{\partial r}(r\sigma_{rrc} - r\tau_{r\phi s}) - \frac{\partial}{\partial z}(r\tau_{zrc} - r\tau_{\phi zs})$$

$$- 2\frac{\partial}{\partial r}(r\frac{\partial u_x}{\partial r}\sigma_{rr}^{(0)} + r\frac{\partial u_x}{\partial z}\tau_{zr}^{(0)}) - 2\frac{\partial}{\partial z}(r\frac{\partial u_x}{\partial r}\tau_{zr}^{(0)} + r\frac{\partial u_x}{\partial z}\sigma_{zz}^{(0)})\}\delta u_x$$

$$+ \{2\rho r a_y - \frac{\partial}{\partial r}(r\sigma_{rrs} + r\tau_{r\phi c}) - \frac{\partial}{\partial z}(r\tau_{zrs} + r\tau_{\phi zc})$$

$$-2\frac{\partial}{\partial r}(r\frac{\partial u_y}{\partial r}\sigma_{rr}^{(0)} + r\frac{\partial u_y}{\partial z}\tau_{zr}^{(0)}) - 2\frac{\partial}{\partial z}(r\frac{\partial u_y}{\partial r}\tau_{zr}^{(0)} + r\frac{\partial u_y}{\partial z}\sigma_{zz}^{(0)})\}\ \delta u_y$$

$$+ r\{\rho r^2((\ddot{\theta}_{xb} + \ddot{\theta}_x) + 2\omega(\dot{\theta}_{yb} + \dot{\theta}_y)) - \frac{\partial}{\partial r}(r\tau_{zrs}) - \frac{\partial}{\partial z}(r\sigma_{zzs}) + \tau_{\phi zc}$$

$$- \frac{\partial}{\partial r}(r\frac{\partial(r\theta_x)}{\partial r}\sigma_{rr}^{(0)} + r\theta_y\tau_{r\phi}^{(0)} + r\frac{\partial(r\theta_x)}{\partial z}\tau_{zr}^{(0)})$$

$$- \frac{\partial}{\partial z}(r\frac{\partial(r\theta_x)}{\partial r}\tau_{zr}^{(0)} + r\theta_y\tau_{\phi z}^{(0)} + r\frac{\partial(r\theta_x)}{\partial z}\sigma_{zz}^{(0)})$$

$$- \frac{\partial(r\theta_y)}{\partial r}\tau_{r\phi}^{(0)} + \theta_x\sigma_{\phi\phi}^{(0)} - \frac{\partial(r\theta_y)}{\partial z}\tau_{\phi z}^{(0)}\}\ \delta\theta_x$$

$$+ r\{\rho r^2((\ddot{\theta}_{yb} + \ddot{\theta}_y) - 2\omega(\dot{\theta}_{xb} + \dot{\theta}_x)) + \frac{\partial}{\partial r}(r\tau_{zrc}) + \frac{\partial}{\partial z}(r\sigma_{zzc}) + \tau_{\phi zs}$$

$$+ \frac{\partial}{\partial r} \left(- r \frac{\partial (r\theta_y)}{\partial r} \sigma_{rr}^{(0)} + r\theta_x \tau_{r\phi}^{(0)} - r \frac{\partial (r\theta_y)}{\partial z} \tau_{zr}^{(0)}\right)$$

$$+ \frac{\partial}{\partial z} \left(-r \frac{\partial (r\theta_y)}{\partial r} \tau_{zr}^{(0)} + r\theta_x \tau_{\phi z}^{(0)} - r \frac{\partial (r\theta_y)}{\partial z} \sigma_{zz}^{(0)}\right)$$

$$+ \frac{\partial (r\theta_x)}{\partial r} \tau_{r\phi}^{(0)} + \theta_y \sigma_{\phi\phi}^{(0)} + \frac{\partial (r\theta_x)}{\partial z} \tau_{\phi z}^{(0)}\} \, \delta\theta_y] \, drdz = 0$$

(4.22)

The above equation (4.22) is also a statement of the principle of virtual work. When (4.22) is partially integrated, certain boundary terms appear as a result. These boundary terms correspond to the boundary conditions at the locations of the bearings. If $(u_x)_i$ and $(u_y)_i$ are the displacements of the rotor axis relative to the ith bearing along the x_b and y_b axes, then the forces due to bearing lubricants can be expressed as

$$f_x = - \sum_{i=1}^{n} \{(k_{xx})_i (u_x)_i + (k_{xy})_i (u_y)_i + (c_{xx})_i (\dot{u}_x)_i + (c_{xy})_i (\dot{u}_y)_i\} \, \delta(z - z_i)$$

$$f_y = - \sum_{i=1}^{n} \{(k_{yx})_i (u_x)_i + (k_{yy})_i (u_y)_i + (c_{yx})_i (\dot{u}_x)_i + (c_{yy})_i (\dot{u}_y)_i\} \, \delta(z - z_i)$$

(4.23)

where n denotes the total number of bearings and δ stands for Dirac's delta function, z_i's are the z-coordinates of the bearing locations. The damping coefficients may be symmetric ($c_{xyi} = c_{yxi}$) but stiffness coefficients are not symmetric ($k_{xyi} \neq k_{yxi}$).

4.3.2 FINITE ELEMENTS

Consider a typical solid of revolution element with eight nodes (see Figure 4.4). It is an eight-noded isoparametric element where the displacements are assumed to vary parabolically within each element. The unknown displacements and rotations will be expressed in terms of unknown nodal values and known shape functions as

$$u_x = \sum_{i=1}^{8} N_i(\xi_1, \xi_2)\ (U_x)_i$$

$$u_y = \sum_{i=1}^{8} N_i(\xi_1, \xi_2)\ (U_y)_i$$

$$\theta_x = \sum_{i=1}^{8} N_i(\xi_1, \xi_2)\ (\Theta_x)_i \qquad (4.24)$$

$$\Theta_y = \sum_{i=1}^{8} N_i(\xi_1, \xi_2)\ (\Theta_y)_i$$

where ξ_1 and ξ_2 are the natural coordinates for the element. Expressions for the shape functions $N_i(\xi_1, \xi_2)$ can be found in [ ] and, in fact, it is possible to formulate a variable node element in which the number of nodes can be chosen between 4 and 8.

Equations (4.24) can be expressed more conveniently in a matrix form as

$$\{u\}_e = [N]_e\{q\}_e \qquad (4.25)$$

where $[N]_e$ is a matrix of shape functions and $\{q\}_e$ is a vector of nodal displacements and rotations given by

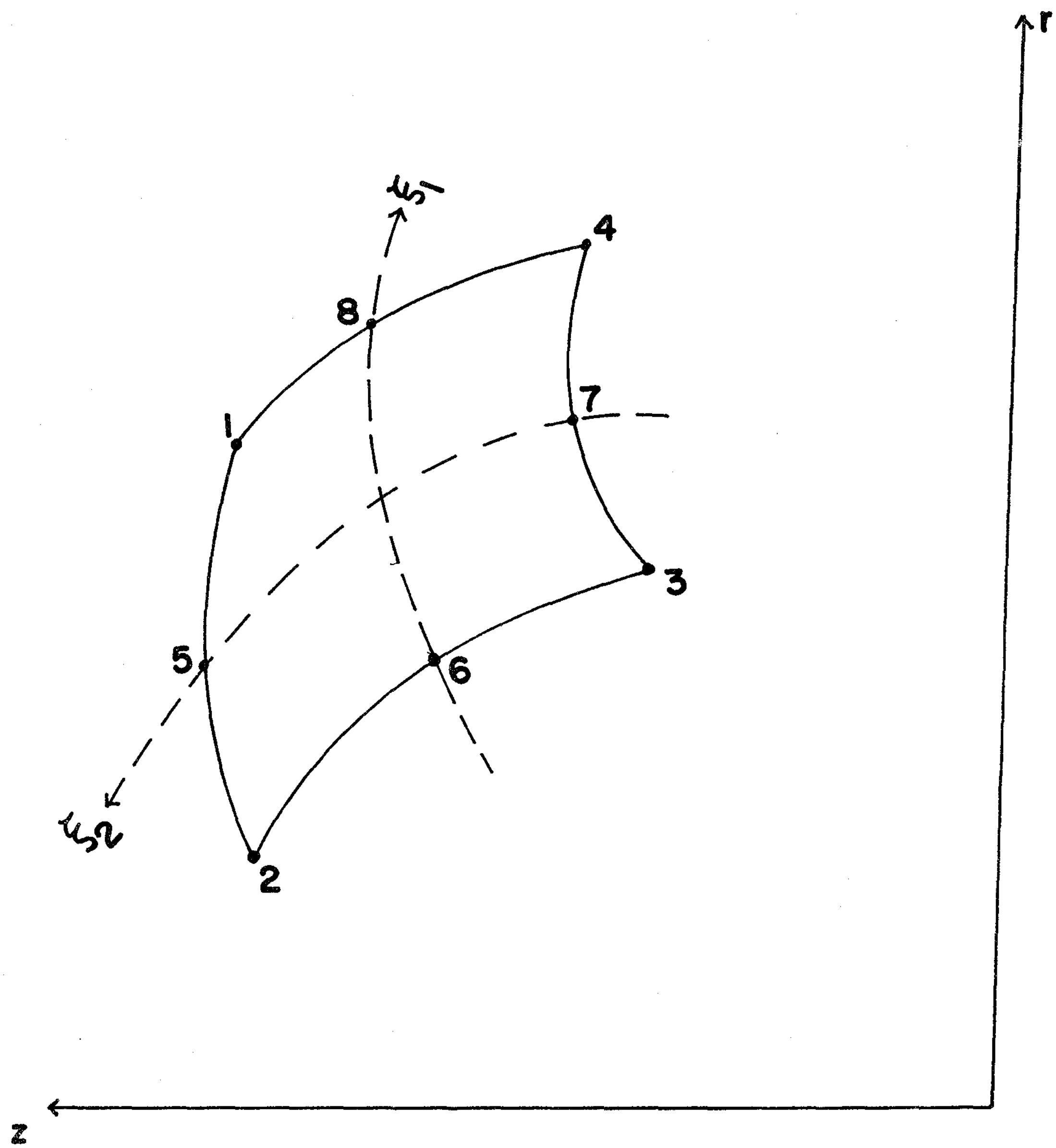


FIG. 4.4 ISOPARAMETRIC, SOLID OF REVOLUTION ELEMENT

$$\{q\}_e^T = [(U_x)_1(U_y)_1(\Theta_x)(\Theta_y)_1 \cdot \cdot \cdot (U_x)_8(U_y)_8(\Theta_x)_8(\Theta_y)_8] \qquad (4.26)$$

It should be pointed out that $(U_x)_i$ and $(U_y)_i$ are not the displacements of the i^{th} node itself; they are the displacements of the center of mass of the elemental ring passing through the i^{th} node. $(\Theta_x)_i$ and $(\Theta_y)_i$ are the rotations of this elemental ring.

We note that

$$\delta\{u\}_e = [N]_e\delta\{q\}_e \qquad (4.27)$$

Substituting (4.25) and (4.27) in the partially integrated form of (4.22) and carrying out the differentiations and integrations we get

$$\delta\{q\}_e^T\ [[M]_e\{\ddot{q}\}_e + [C]_e\{\dot{q}\}_e + [K]_e\{q\}_e] = \delta\{q\}_e^T\ \{Q\}_e \qquad (4.28)$$

Here, $[M]_e$ is the elemental intertia matrix. $[C]_e$ is an elemental matrix that can be written as

$$[C]_e = [C_G]_e + [C_C]_e + [C_D]_e \qquad (4.29)$$

where

$[C_G]_e$ - Gyroscopic matrix,

$[C_C]_e$ - Coriolis matrix due to base rotation,

$[C_D]_e$ - Damping matrix due to bearing(s) located at the node(s).

$[K]_e$ is an elemental matrix that can be written as

$$[K]_e = [K_C]_e + [K_G]_e + [K_R]_e + [K_B]_e \tag{4.30}$$

where

$[K_C]_e$ - Conventional stiffness matrix,

$[K_G]_e$ - Geometric stiffness matrix due to initial stresses,

$[K_R]_e$ - Supplementary stiffness matrix due to base rotation,

$[K_B]_e$ - Stiffness matrix due to bearing(s) located at the node(s).

$\{Q\}_e$ is a vector of nodal forces and moments due to base translation and rotation. $[M]_e$, $[C_D]_e$, $[K_C]_e$ and $[K_G]_e$ are symmetric matrices; $[C_G]_e$ and $[C_C]_e$ are skew-symmetric matrices; $[K_R]_e$ and $[K_B]_e$ are non-symmetric matrices. These elemental matrices are to be properly assembled to obtain the global matrices.

The elemental matrices mentioned above are obtained after performing a numerical integration over the element using a Gaussian scheme. It may be mentioned that the conventional and geometric stiffness matrices can be derived by minimizing the potential

$$\pi = \tfrac{1}{2}\iiint[(\sigma_{rr}e_{rr} + \sigma_{\phi\phi}e_{\phi\phi} + \sigma_{zz}e_{zz} + \tau_{r\phi}e_{r\phi} + \tau_{\phi z}e_{\phi z} + \tau_{zr}e_{zr})$$

$$+ 2\{\sigma_{rr}^{(0)}e'_{rr} + \sigma_{\phi\phi}^{(0)}e'_{\phi\phi} + \sigma_{zz}^{(0)}e'_{zz} + \tau_{r\phi}^{(0)}e'_{r\phi} + \tau_{\phi z}^{(0)}e'_{\phi z} + \tau_{zr}^{(0)}e'_{zr}\}] \, rdrd\phi dz \tag{4.31}$$

where

$$e'_{rr} = ½\{(\frac{\partial u}{\partial r})^2 + (\frac{\partial v}{\partial r})^2 + (\frac{\partial w}{\partial r})^2\}$$

$$e'_{\phi\phi} = ½\{(\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \phi})^2 + (\frac{1}{r}\frac{\partial u}{\partial \phi} - \frac{v}{r})^2 + (\frac{1}{r}\frac{\partial w}{\partial \phi})^2\}$$

$$e'_{zz} = ½\{(\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2 + (\frac{\partial w}{\partial z})^2\}$$

$$e'_{r\phi} = (\frac{\partial u}{\partial r})(\frac{1}{r}\frac{\partial u}{\partial \phi} - \frac{v}{r}) + (\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \phi})(\frac{\partial v}{\partial r}) + (\frac{\partial w}{\partial r})\ (\frac{1}{r}\frac{\partial w}{\partial \phi})$$

$$e'_{\phi z} = (\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \phi})(\frac{\partial v}{\partial z}) + (\frac{\partial u}{\partial z})\ (\frac{1}{r}\frac{\partial u}{\partial \phi} - \frac{v}{z}) + (\frac{\partial w}{\partial z})\ (\frac{1}{r}\frac{\partial w}{\partial \phi})$$

$$e'_{zr} = (\frac{\partial u}{\partial r})\ (\frac{\partial u}{\partial z}) + (\frac{\partial v}{\partial r})(\frac{\partial v}{\partial z}) + (\frac{\partial w}{\partial r})(\frac{\partial w}{\partial z}) \tag{4.32}$$

We have adopted a more direct Newtonian approach to the problem because it seems to be more appropriate to the rotating system under consideration.

4.3.3 CHECK PROBLEMS

The performance of finite elements formulated above must be tested against some known, closed form dynamic solutions available in literature, before we use them in our seismic analysis. Two such check problems are given below.

4.3.3.1 FREE VIBRATION OF A BEAM

The frequencies of free vibration of a simply supported Timoshenko beam are given by the roots of equation (3.28). Using the finite elements developed in this chapter, the eigenproblem can be posed as

$$[M]\{X\} = \frac{1}{\omega_n^2}[K_c]\{X\} \tag{4.33}$$

Table 4.1 shows the comparison between the finite element and Timoshenko beam natural frequencies for various aspect ratios. It can be seen from Table 4.1 that the eight-noded element gives better results than the four-noded element. It is also observed that reduced, 2 x 2 integration gives better results for low aspect ratio beams. But for higher aspect ratios, 3 x 3 integration gives better results.

4.3.3.2 BUCKLING OF A BEAM

The buckling loads for a simply supported, Timoshenko beam are given by the roots of equation (3.30). Using the finite elements developed in this chapter, the eigenproblem can be posed as

$$[K_c]\{X\} = P_c[K_G]\{X\} \tag{4.34}$$

Table 4.2 compares the finite element and Timoshenko beam buckling loads for various aspect ratios. Here again we see that the eight-noded element has a superior performance over the four-noded element. For low aspect-ratios, the reduced 2 x 2 integration gives better results. But for higher aspect ratios, the 3 x 3 integration gives better results.

TABLE 4.1 FREE VIBRATION OF A SIMPLY SUPPORTED BEAM: $\ell^2\omega\sqrt{\rho A/EI}$, FIVE ELEMENTS

Mode m	Aspect Ratio r/2ℓ	4 - noded elements		8 - noded elements		Timoshenko beam theory
		3x3 inegration	2x2 integration	3x3 integration	2x2 integration	
1	0.02	21.20	21.21	11.67	11.35	9.794
	0.04	13.88	13.87	10.29	10.00	9.580
	0.06	11.69	11.68	9.628	9.123	9.258
	0.08	10.42	10.40	8.767	8.084	8.867
	0.10	9.388	9.368	7.737	6.996	8.440
2	0.02	90.98	90.96	42.29	40.58	38.32
	0.04	55.69	55.61	37.30	34.29	35.47
	0.06	42.78	42.66	31.20	27.37	32.02
	0.08	34.36	34.24	24.52	21.22	28.68
	0.10	27.61	27.54	18.52	16.28	25.70

TABLE 4.2 BUCKLING OF A SIMPLY SUPPORTED BEAM, $\ell^2 P/EI$

Mode m	Aspect Ratio $r/2\ell$	Five 4 Noded Elements		Five 8 Noded Elements		Timoshenko Beam Theory
		3x3 integration	2x2 integration	3x3 integration	2x2 integration	
1	0.02	44.07	44.10	13.80	13.11	9.758
	0.04	18.91	18.89	10.85	10.42	9.450
	0.06	13.57	13.53	9.921	9.238	9.012
	0.08	11.07	11.01	9.007	8.064	8.510
	0.10	9.395	9.324	8.075	6.964	7.995
2	0.02	184.3	184.2	45.21	41.89	37.80
	0.04	69.62	69.40	36.61	32.40	34.04
	0.06	42.62	42.30	29.09	24.11	29.98
	0.08	29.80	29.43	22.88	18.01	26.40
	0.10	22.11	21.72	18.27	14.26	23.41

4.4 EXAMPLE PROBLEM

As an example problem, the seismic analysis of a rotor bearing system was obtained using the 3-D elasticity model. The geometry of the rotor is shown in Figure 4.5. The bearings are located at nodes 3 and 18. The stiffness and damping coefficients for the lubricants in the bearings were taken from Table 3.4

The base was first subjected to purely translational excitations given by the El Centro earthquake history in Figure 3.6. The displacements of the rotor in the bearings are given in Figure 4.6 with proper subscripts. The dynamic reaction forces on the bearings are shown in Figure 4.7. Figure 4.8 shows the bending stress at midspan.

The base was then subjected to translational as well as simulated rotational excitations as given in Figure 3.7. The displacements of the rotor in the bearings are given in Figure 4.9. The dynamic reaction forces on the bearings are shown in Figure 4.10. Figure 4.11 shows the bending stress at midspan.

A typical run for the example problem took about 2 minutes of CPU time in IBM System 3081.

4.5 MERITS AND LIMITATIONS OF 3-D MODEL

In this chapter, we have shown that the flexibility of the rotating system can be included in the seisimic analysis using three-dimensional theory of elasticity. This represents the most general treatment of the problem thus far. The three-dimensional elasticity model is superior in formulation to the beam and rigid body models. Since the method of solution is based on a finite

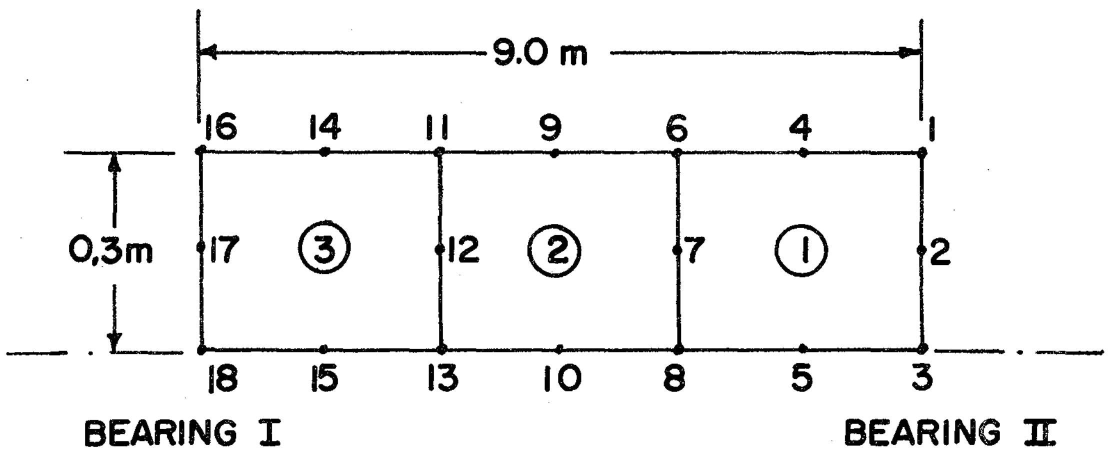


FIG. 4.5 GEOMETRY OF EXAMPLE PROBLEM

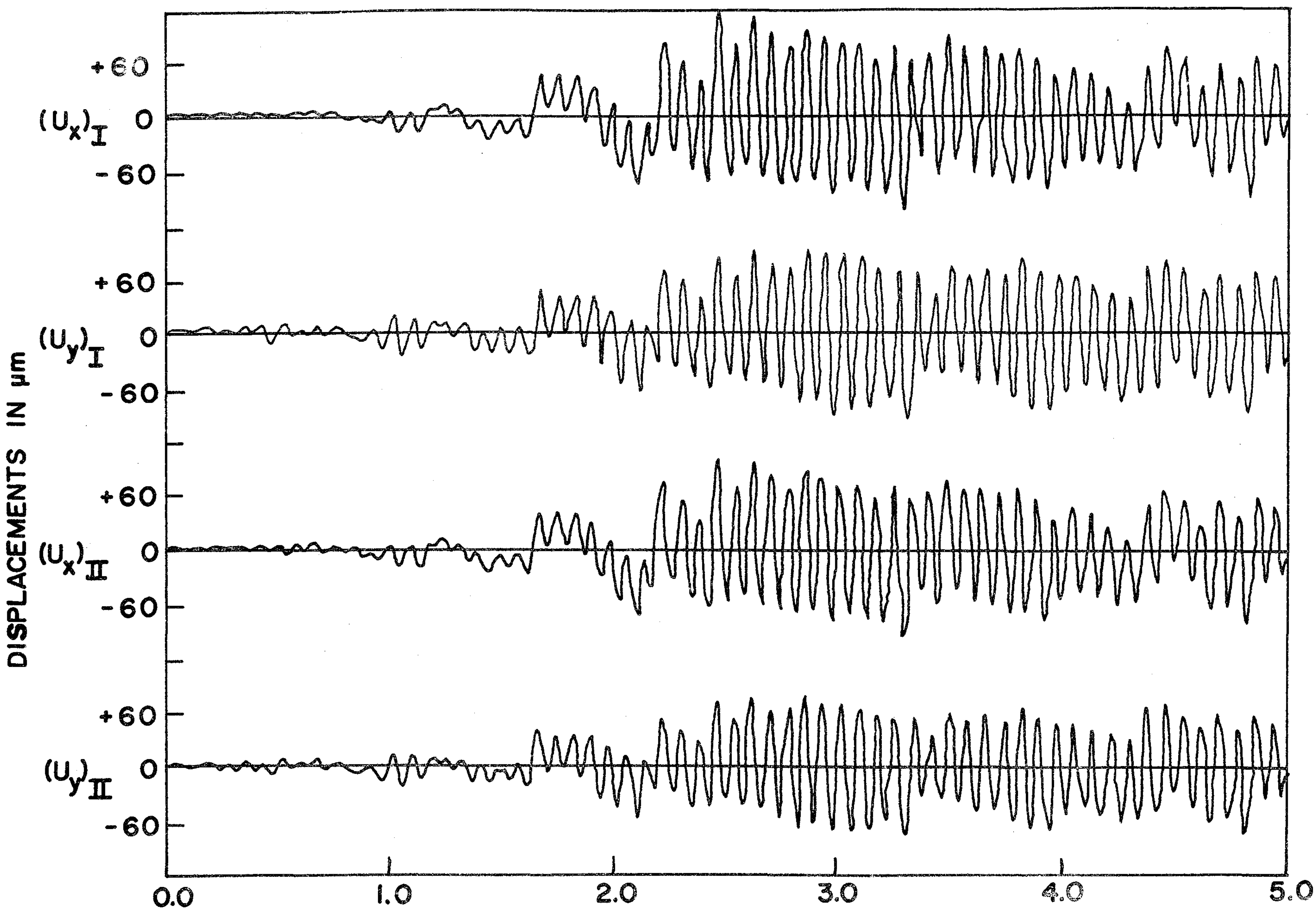


FIG. 4.6 DISPLACEMENTS OF ROTOR IN BEARINGS (NO BASE ROTATION)

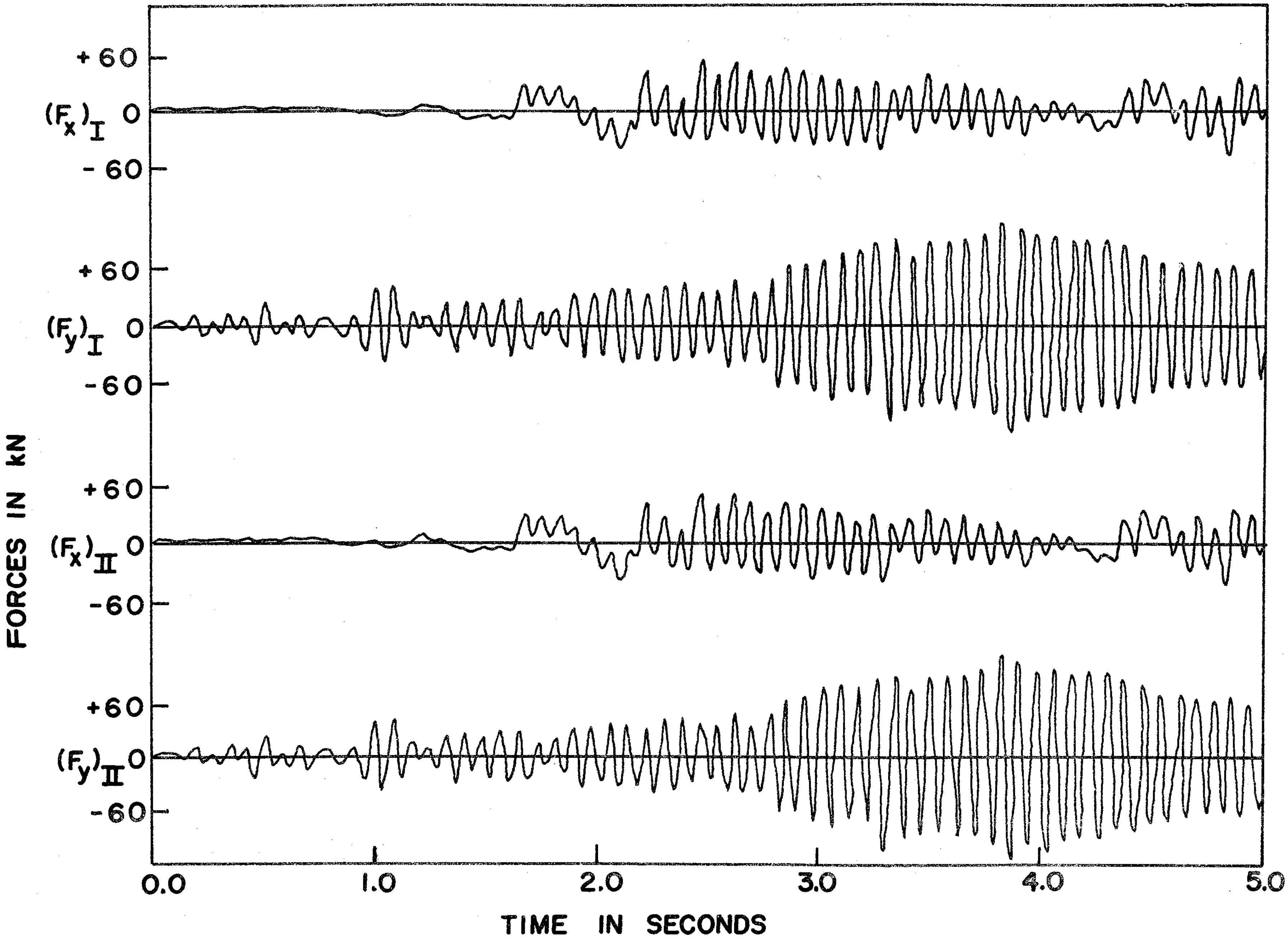


FIG.4.7 DYNAMIC REACTION FORCES IN BEARINGS (NO BASE ROTATION)

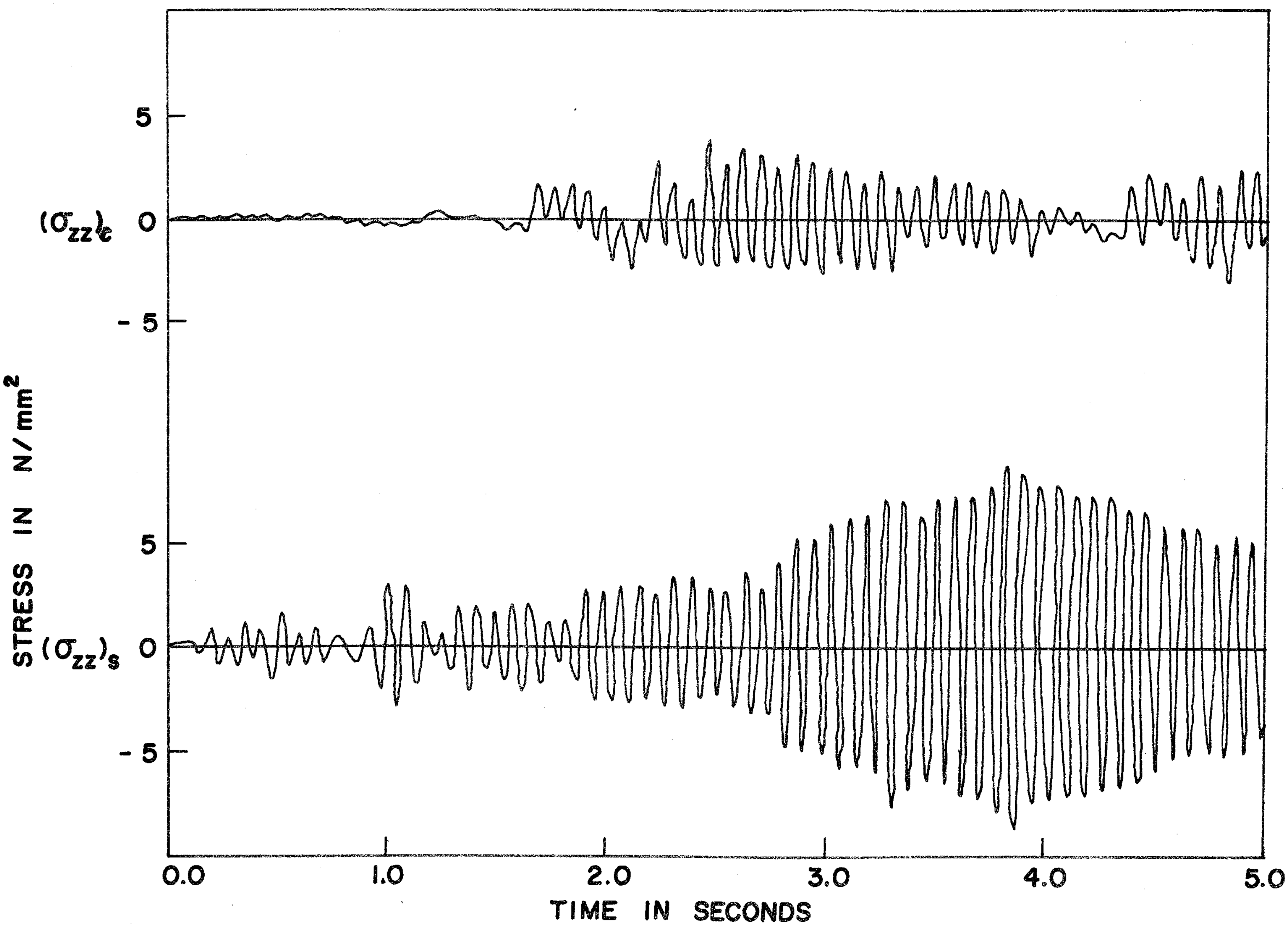


FIG.4.8 BENDING STRESS AT MIDSPAN (NO BASE ROTATION)

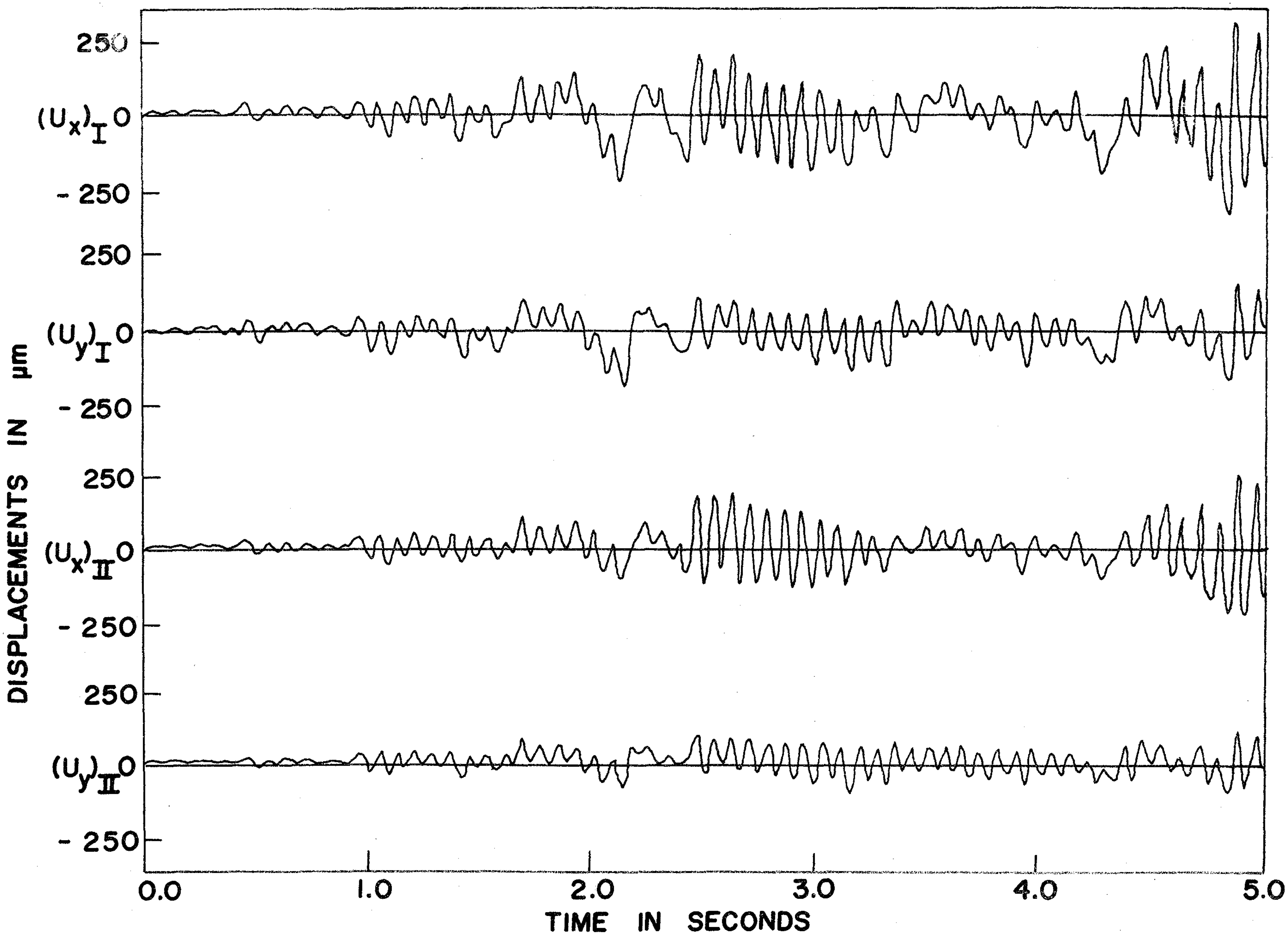


FIG.4.9 DISPLACEMENTS OF ROTOR IN BEARINGS

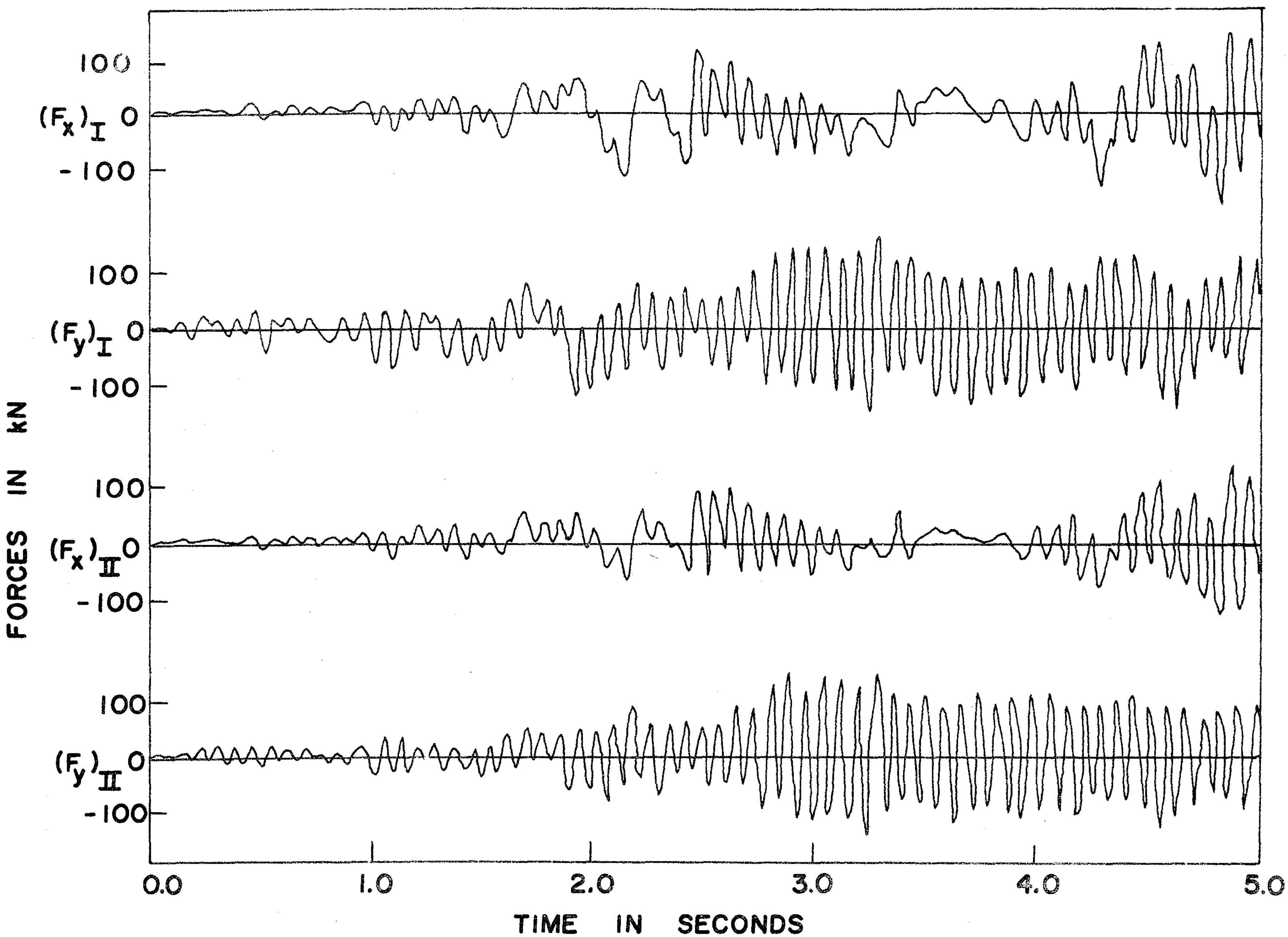


FIG. 4.10 DYNAMIC REACTION FORCES IN BEARINGS

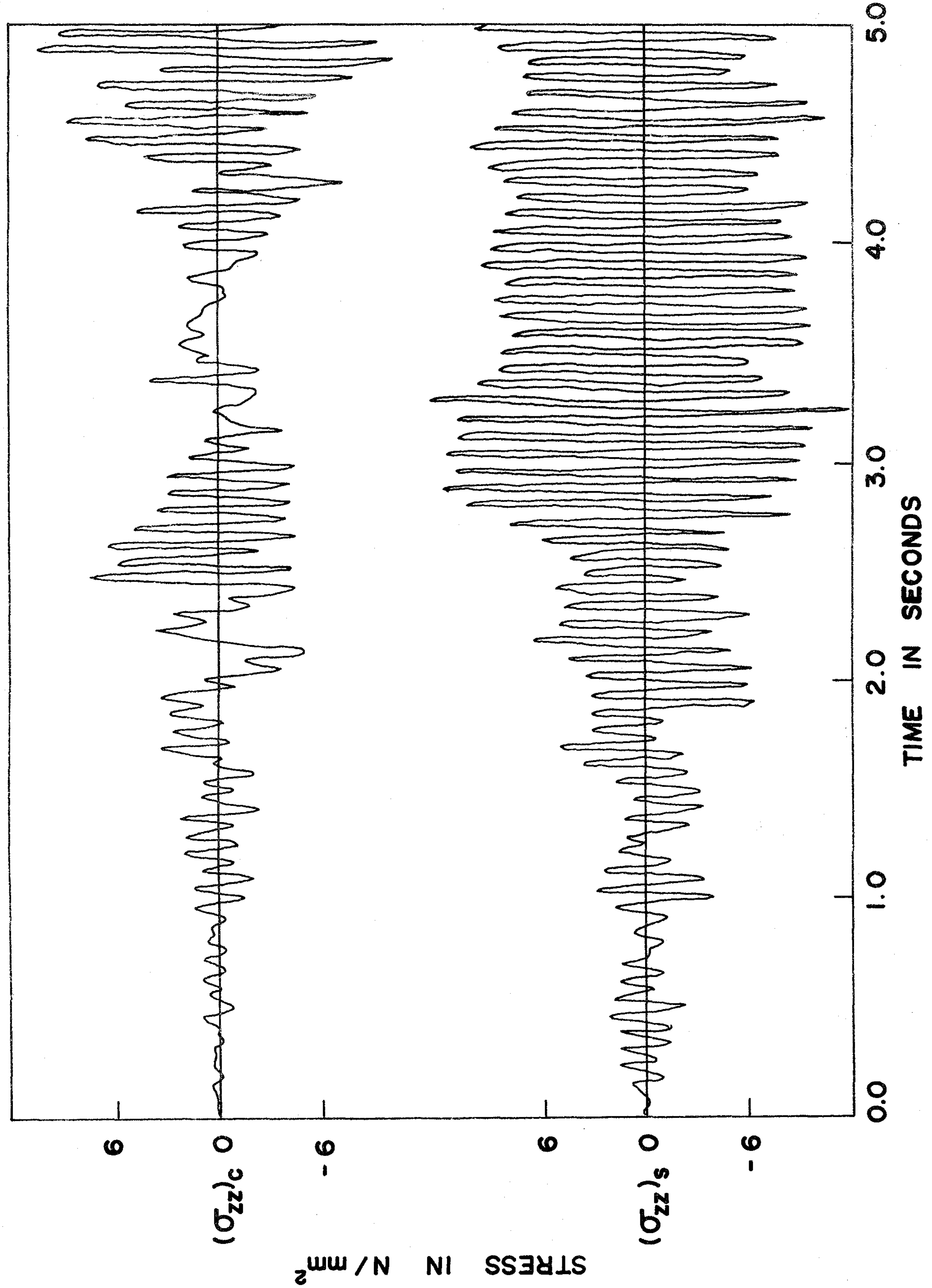


FIG. 4.11 BENDING STRESS AT MIDSPAN

element procedure, it can be easily implemented along with other finite element codes in the user's organization.

A major limitation of the 3-D model is the cost. A price has been paid for the general treatment of the rotor-bearing system in the form of large memeory requirement and relatively large computing time. The 3-D model is recommended for those problems where accurate modeling is of greater concern over the cost of computing.

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5. CONCLUSIONS

In this report we have presented a rigid body model, a beam model and a 3-D elasticity model to predict the seismic response of a rotating mechanical system.

In the rigid body model, the rotating system is modeled as a rigid body spinning about its axis of symmetry. It is shown that factors such as gyroscopic effects, rotor-bearing interaction effects (i.e. stiffness and damping provided by the lubricants in the bearings), effects of base rotation (including Coriolis effects) and base translation can be directly and systematically incorporated in the seismic analysis. The rigid body model keeps mathematics to the minimum and is easy to program. It is computationally economical.

The beam model incorporates the flexibility of the rotating system using Timoshenko beam theory. In addition to the factors mentioned in the rigid body model, factors such as rotatory inertia, shear deformation, intermediate disks and flywheels and effects of initial stresses due to axial force and axial torque are included in the beam model. The beam model uses a finite element approach and the solution can be obtained within reasonable computer time and cost. We strongly recommended it for all shaft-like systems.

The 3-D elasticity model incorporates the flexibility of the rotating system using the three-dimensional theory of elasticity. This enables the

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model to take into account such factors as effect of initial stresses due to spin and systems that do not have the appearance of a shaft, in addition to the factors mentioned in the beam model. The 3-D elasticity model is the most-rigorous of the three model and also the most expensive. We recommend the 3-D model for those systems that do not look like a shaft and where cost of computation is not of great concern.

A more general three-dimensional model is under development. The three-dimensional model uses eight-noded, isoparametric solid-of-revolution finite elements. It is expected that the three-dimensional model will increase the range of problems that can be solved under seismic analysis of rotating mechanical systems.

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APPENDIX A

EXPRESSION FOR THE RATE OF CHANGE OF ANGULAR MOMENTUM OF A RIGID BODY USING EULER ANGLES

Consider the axially symmetric rigid body shown in Figure 2.1. We have an xyz coordinate system with its origin at the center of mass G. The xyz-system undergoes precessional (ψ) and nutational (θ) motions. In addition, the rigid body undergoes a spin (ϕ) motion about the z-axis.

Because of the rotational symmetry of the body about the z-axis, the x, y, and z axes become the principal axes of inertia with the corresponding principal moments of inertia being I_0, I_0 and I respectively. The rate of change of angular momentum for such a body is given by

$$\begin{aligned} \dot{\underset{\sim}{H}}_G &= \{I_0\alpha_x + (I - I_0)\,\omega_y\omega_z\}\,\underset{\sim}{\varepsilon}_x \\ &\quad + \{I_0\alpha_y + (I_0 - I)\,\omega_z\,\omega_x\}\,\underset{\sim}{\varepsilon}_y \\ &\quad + I\alpha_z\underset{\sim}{\varepsilon}_z \end{aligned} \tag{A.1}$$

where the angular velocity and angular acceleration of the rigid body are given by

$$\begin{aligned} \underset{\sim}{\omega} &= \omega_x\underset{\sim}{\varepsilon}_x + \omega_y\underset{\sim}{\varepsilon}_y + \omega_z\underset{\sim}{\varepsilon}_z \\ &= \dot{\theta}\,\underset{\sim}{\varepsilon}_x + \dot{\psi}\sin\theta\,\underset{\sim}{\varepsilon}_y + (\dot{\theta} + \dot{\psi}\cos\theta)\,\underset{\sim}{\varepsilon}_z \end{aligned} \tag{A.2}$$

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$$\underset{\sim}{\alpha} = \alpha_x \underset{\sim}{\varepsilon}_x + \alpha_y \underset{\sim}{\varepsilon}_y + \alpha_z \underset{\sim}{\varepsilon}_z$$

$$= (\ddot{\theta} + \dot{\psi}\dot{\phi} \sin \theta) \underset{\sim}{\varepsilon}_x$$

$$+ (\ddot{\psi} \sin \theta + \dot{\psi}\, \dot{\theta} \cos \theta - \dot{\phi}\dot{\theta}) \underset{\sim}{\varepsilon}_y$$

$$+ (\ddot{\psi} \cos \theta - \dot{\psi}\dot{\theta} \sin \theta + \ddot{\phi}) \underset{\sim}{\varepsilon}_z \qquad \text{(A.3)}$$

Substituting (A.2) and (A.3) in (A.1) we get

$$\dot{\underset{\sim}{H}}_G = \{ I_o\ddot{\theta} + I\dot{\psi}\dot{\phi} \text{ Sin } \theta + (I - I_o)\dot{\psi}^2 \text{ Sin } \theta \cos \theta\} \underset{\sim}{\varepsilon}_x$$

$$+ \{I_o\ddot{\psi} \text{ Sin } \theta - I\dot{\phi}\dot{\theta} + (2I_o - I)\, \dot{\psi}\dot{\theta} \cos \theta\} \underset{\sim}{\varepsilon}_y \qquad \text{(A.4)}$$

$$+ \{I\ddot{\phi} + I\, \ddot{\psi} \cos \theta - I\, \dot{\psi}\dot{\theta} \sin \theta\} \underset{\sim}{\varepsilon}_z$$

APPENDIX B

BEAM ELEMENT MATRICES

Let $\ell = s_2 - s_1$

(1) Inertia Matrix

$$[M]_e = \begin{bmatrix} \rho A\ell/3 & 0 & 0 & 0 & \rho A\ell/6 & 0 & 0 & 0 \\ & \rho A\ell/3 & 0 & 0 & 0 & \rho A\ell/6 & 0 & 0 \\ & & \rho I_T\ell/3 & 0 & 0 & 0 & \rho I_T\ell/6 & 0 \\ & & & \rho I_T\ell/3 & 0 & 0 & 0 & \rho I_T\ell/6 \\ & & & & \rho A\ell/3 & 0 & 0 & 0 \\ & (SYM) & & & & \rho A\ell/3 & 0 & 0 \\ & & & & & & \rho I_T\ell/3 & 0 \\ & & & & & & & \rho I_T\ell/3 \end{bmatrix}$$

(2) $[C]_e = [C_G]_e + [C_C]_e + [C_D]_e$

(2a) Gyroscopic Matrix

$$[C_G]_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & \rho I_p\omega\ell/3 & 0 & 0 & 0 & \rho I_p\omega\ell/6 \\ & & & 0 & 0 & 0 & -\rho I_p\omega\ell/6 & 0 \\ & (SKEW\text{-}SYM) & & & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 \\ & & & & & & 0 & \rho I_p\omega\ell/3 \\ & & & & & & & 0 \end{bmatrix}$$

(2b) Coriolis Matrix

$$[C_C]_e = \begin{bmatrix} 0 & -2\rho A\dot{\theta}_{zb}\ell/3 & 0 & 0 & 0 & -2\rho A\dot{\theta}_{zb}\ell/6 & 0 & 0 \\ & 0 & 0 & 0 & 2\rho A\dot{\theta}_{zb}\ell/6 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 \\ & \text{(SKEW - SYM)} & & & 0 & -2\rho A\dot{\theta}_{zb}\ell/3 & 0 & 0 \\ & & & & & 0 & 0 & 0 \\ & & & & & & 0 & 0 \\ & & & & & & & 0 \end{bmatrix}$$

(2c) Bearing Damping Matrix

$$[C_D]_e = \begin{bmatrix} (C_{xx})_1 & (C_{xy})_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ (C_{yx})_1 & (C_{yy})_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (C_{xx})_2 & (C_{xy})_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (C_{yx})_2 & (C_{yy})_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above damping matrix is applicable only to those elements whose node(s) are supported on bearing(s).

(3) $[K]_e = [K_C]_e + [K_P]_e + [K_T]_e + [K_R]_e + [K_B]_e$

(3a) Conventional Stiffness Matrix

$$[K_C]_e = \begin{bmatrix} kAG/\ell & 0 & 0 & kAG/2 & -kAG/\ell & 0 & 0 & kAG/2 \\ & kAG/\ell & -kAG/2 & 0 & 0 & -kAG/\ell & -kAG/2 & 0 \\ & & (kAG\ell/3 + EI_T/\ell) & 0 & 0 & kAG/2 & (kAG\ell/6 - EI_T/\ell) & 0 \\ & & & (kAG\ell/3 + EI_T/\ell) & -kAG/2 & 0 & 0 & (kAG\ell/6 - EI_T/\ell) \\ & \text{(SYM)} & & & kAG/\ell & 0 & 0 & -kAG/2 \\ & & & & & kAG/\ell & kAG/2 & 0 \\ & & & & & & (kAG\ell/3 + EI_T/\ell) & 0 \\ & & & & & & & (kAG\ell/3 + EI_T/\ell) \end{bmatrix}$$

Upon reduced (single point) integration the above conventional stiffness matrix reduces to

$$[K_C]_e = \begin{bmatrix} kAG/\ell & 0 & 0 & kAG/2 & -kAG/\ell & 0 & 0 & kAG/2 \\ & kAG/\ell & -kAG/2 & 0 & 0 & -kAG/\ell & -kAG/2 & 0 \\ & & (kAG\ell/4 + EI_T/\ell) & 0 & 0 & kAG/2 & (kAG\ell/4 - EI_T/\ell) & 0 \\ & & & (kAG\ell/4 + EI_T/\ell) & -kAG/2 & 0 & 0 & (kAG\ell/4 - EI_T/\ell) \\ & \text{(SYM)} & & & kAG/\ell & 0 & 0 & -kAG/2 \\ & & & & & kAG/\ell & kAG/2 & 0 \\ & & & & & & (kAG\ell/4 + EI_T/\ell) & 0 \\ & & & & & & & (kAG\ell/4 + EI_T/\ell) \end{bmatrix}$$

(3b) Geometric Stiffness Matrix Due to Axial Force

$$[K_p]_e = P \begin{bmatrix} 0 & 0 & 0 & -1/2 & 0 & 0 & 0 & -1/2 \\ & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ & & -\ell/3 & 0 & 0 & -1/2 & -\ell/6 & 0 \\ & & & -\ell/3 & 1/2 & 0 & 0 & -\ell/6 \\ & \text{(SYM)} & & & 0 & 0 & 0 & 1/2 \\ & & & & & 0 & -1/2 & 0 \\ & & & & & & -\ell/3 & 0 \\ & & & & & & & -\ell/3 \end{bmatrix}$$

Upon reduced (single point) integration, the above geometric stiffness matrix reduces to

$$[K_p]_e = P \begin{bmatrix} 0 & 0 & 0 & -1/2 & 0 & 0 & 0 & -1/2 \\ & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ & & -\ell/4 & 0 & 0 & -1/2 & -\ell/4 & 0 \\ & & & -\ell/4 & 1/2 & 0 & 0 & -\ell/4 \\ & \text{(SYM)} & & & 0 & 0 & 0 & 1/2 \\ & & & & & 0 & -1/2 & 0 \\ & & & & & & -\ell/4 & 0 \\ & & & & & & & -\ell/4 \end{bmatrix}$$

(3c) Geometric Stiffness Matrix Due to Axial Torque

$$
[K_T]_e = T \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1/2 & 0 & 0 & 0 & -1/2 \\
0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 \\
0 & 0 & -1/2 & 0 & 0 & 0 & -1/2 & 0
\end{bmatrix}
$$

(3d) Supplementary Stiffness Matrix Due to Base Rotation

$$[K_R]_e = \rho A \begin{bmatrix} -(\dot\theta_{yb}^2 + \dot\theta_{zb}^2)\ell/3 & (\dot\theta_{xb}\dot\theta_{yb} - \ddot\theta_{zb})\ell/3 & 0 & 0 & -(\dot\theta_{yb}^2 + \dot\theta_{zb}^2)\ell/6 & (\dot\theta_{xb}\dot\theta_{yb} - \ddot\theta_{zb})\ell/6 & 0 & 0 \\ (\dot\theta_{xb}\dot\theta_{yb} + \ddot\theta_{zb})\ell/3 & -(\dot\theta_{zb}^2 + \dot\theta_{xb}^2)\ell/3 & 0 & 0 & (\dot\theta_{xb}\dot\theta_{yb} + \ddot\theta_{zb})\ell/6 & -(\dot\theta_{zb}^2 + \dot\theta_{xb}^2)\ell/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(\dot\theta_{yb}^2 + \dot\theta_{zb}^2)\ell/6 & (\dot\theta_{xb}\dot\theta_{yb} - \ddot\theta_{zb})\ell/6 & 0 & 0 & -(\dot\theta_{yb}^2 + \dot\theta_{zb}^2)\ell/3 & (\dot\theta_{xb}\dot\theta_{yb} - \ddot\theta_{zb})\ell/3 & 0 & 0 \\ (\dot\theta_{xb}\dot\theta_{yb} + \ddot\theta_{zb})\ell/6 & -(\dot\theta_{zb}^2 + \dot\theta_{xb}^2)\ell/6 & 0 & 0 & (\dot\theta_{xb}\dot\theta_{yb} + \ddot\theta_{zb})\ell/3 & -(\dot\theta_{zb}^2 + \dot\theta_{xb}^2)\ell/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(3e) Bearing Stiffness Matrix

$$[K_B]_e = \begin{bmatrix} (k_{xx})_1 & (k_{xy})_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ (k_{yx})_1 & (k_{yy})_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (k_{xx})_2 & (k_{xy})_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (k_{yx})_2 & (k_{yy})_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above stiffness matrix is applicable only to those elements whose node(s) are supported on bearing(s)

(4) Force Vector Due to Base Motion

$$\{Q\}_e = \begin{Bmatrix} -\rho A\ddot{X}_b \ell/2 - \rho Ah\,\ell(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})/2 & -\rho A\ell(\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})(2s_1 + s_2)/6 \\ -\rho A\ddot{Y}_b \ell/2 + \rho Ah\ell(\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2)/2 & -\rho A\ell(\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})(2s_1 + s_2)/6 \\ -\rho\ell(I_T\ddot{\theta}_{xb} + I_p\omega\,\dot{\theta}_{yb})/2 & \\ -\rho\ell(I_T\ddot{\theta}_{yb} - I_p\omega\,\dot{\theta}_{xb})/2 & \\ -\rho A\ddot{X}_b \ell/2 - \rho Ah\ell(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb})/2 & -\rho A\ell(\dot{\theta}_{zb}\dot{\theta}_{xb} + \ddot{\theta}_{yb})(s_1 + 2s_2)/6 \\ -\rho A\ddot{Y}_b \ell/2 + \rho Ah\ell(\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2)/2 & -\rho A\ell(\dot{\theta}_{yb}\dot{\theta}_{zb} - \ddot{\theta}_{xb})(s_1 + 2s_2)/6 \\ -\rho\ell(I_T\ddot{\theta}_{xb} + I_p\,\omega\dot{\theta}_{yb})/2 & \\ -\rho\ell(I_T\ddot{\theta}_{yb} - I_p\,\omega\dot{\theta}_{xb})/2 & \end{Bmatrix}$$

INTENTIONALLY BLANK

APPENDIX C

DISK MATRICES

(1) Inertia Matrix

$$[M]_d = \begin{bmatrix} m_i & 0 & 0 & 0 \\ 0 & m_i & 0 & 0 \\ 0 & 0 & (I_0)_i & 0 \\ 0 & 0 & 0 & (I_0)_i \end{bmatrix}$$

(2) $[C]_d = [C_G]_d + [C_C]_d$

(2a) Gyroscopic Matrix

$$[C_G]_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (I)_i\omega \\ 0 & 0 & -(I)_i\omega & 0 \end{bmatrix}$$

(2b) Coriolis Matrix

$$[C_C]_d = \begin{bmatrix} 0 & -2m_i\dot{\theta}_{zb} & 0 & 0 \\ 2m_i\dot{\theta}_{zb} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(3) Supplementary Stiffness Matrix

$$[K]_d = \begin{bmatrix} -m_i(\dot{\theta}_{yb}^2 + \dot{\theta}_{zb}^2) & m_i(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb}) & 0 & 0 \\ m_i(\dot{\theta}_{xb}\dot{\theta}_{yb} + \ddot{\theta}_{zb}) & -m_i(\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(4) Force Vector Due to Base Motion

$$\{Q\}_i = \begin{Bmatrix} -m_i\ddot{X}_b - m_i h(\dot{\theta}_{xb}\dot{\theta}_{yb} - \ddot{\theta}_{zb}) \\ -m_i\ddot{Y}_b + m_i h(\dot{\theta}_{zb}^2 + \dot{\theta}_{xb}^2) \\ -(I_0)_i\,\ddot{\theta}_{xb} - (I)_i\,\omega\dot{\theta}_{yb} \\ -(I_0)_i\,\ddot{\theta}_{yb} + (I)_i\,\omega\dot{\theta}_{xb} \end{Bmatrix}$$

where

m_i = Mass of the i^{th} disk

$(I)_i$ = Moment of inertia of the i^{th} disk about the spin axis

$(I_o)_i$ = Moment of inertia of the i^{th} disk about an axis perpendicular to the spin axis and passing through the center of mass of the disk.

PART II

COMPUTER PROGRAMS

1. GYROT USER'S MANUAL

1.1 PURPOSE

GYROT is a computer program written in Fortran to carry out the seismic analysis of a rigid rotor in time domain. GYROT is a part of a series of computer program packages that are developed at the School of Mechanical and Aerospace Engineering, Oklahoma State University under the sponsorship of National Science Foundation. These computer programs are intended for the use of designers who want to carry out seismic calculations for rotating mechanical systems.

This user's manual describes the way in which the data is supplied to the program. It also includes a listing of the program and a sample output.

1.2 BACKGROUND THEORY

GYROT is based on the rigid body model developed in Part I, Chapter 2 of this report.

GYROT is a self-contained program. The only external subroutine used is a commonly available IMSL routine LEQT2F to solve a set of linear simultaneous equations.

1.3 INPUT DATA

Card #	Data and Description
1	AMASS,AI,AIO,AL1,AL2,H,RPM (7F10.5)

AMASS - Mass of the rigid rotor, in kgs.

AI - Moment of inertia of the rotor about the axis of rotation, in kg.m^2.

AIO - Moment of inertia of the rotor about an axis perpendicular to the axis of rotation and passing through the center of mass, in kg.m^2.

AL1 - Distance between the location of the first bearing and the center of mass of the rotor, in m.

AL2 - Distance between the location of the second bearing and the center of mass of the rotor, in m.

H - Vertical height of the center of mass of the rotor from the base reference point b, in m.

RPM - Rotational speed of the rotor in revolutions per minute.

2 NFREE (I5)

NFREE - Number of degrees of freedom for the rotor.

Note: - NFREE is either 5 or 4. If NFREE = 5, then the stiffness and damping coefficients of the bearings in the axial direction (i.e. of the thrust bearing) must be supplied in the following cards. If the axial degree of freedom is to be deleted from the analysis, then set NFREE = 4. If NFREE is set to 4, the computer prints out a message that "THE Z DEGREE OF FREEDOM IS DELETED".

3 AKXX1,AKXY1,AKYX1,AKYY1,AKZZ1 (5E11.4)

These are the stiffness coefficients of bearing #1. If NFREE = 4, then leave AKZZ1 blank.

4 CXX1,CXY1,CYX1,CYY1,CZZ1 (5E11.4)

These are the damping coefficients of bearing #1. If NFREE = 4, then leave CZZ1 blank.

5 AKXX2,AKXY2,AKYX2,AKYY2,AKZZ2 (5E11.4)

These are the stiffness coefficients of bearing #2. If NFREE = 4, then leave AKZZ2 blank.

6 CXX2,CXY2,CYX2,CYY2,CZZ2 (5E11.4)

These are the damping coefficients of bearing #2. If NFREE = 4, then leave CZZ2 blank.

7 TIME,ACCX,ACCY,ACCZ,VELX,VELY,VELZ (7F10.5)

These are the initial conditions for the base translation. Set TIME = 0.0

ACCX,ACCY,ACCZ are the initial accelerations of ponit b in the x_b, y_b and z_b directions, respectively, in m/s^2.

VELX,VELY,VELZ are the initial velocities of point b in the x_b, y_b, and z_b directions, repsectively, in m/s.

8 ACCTX,ACCTY,ACCTZ,VELTX,VELTY,VELTZ (6F10.5)

These are the initial conditions for the base rotation. ACCTX,ACCTY,ACCTZ are the initial angular acceleration of the base about the x_b, y_b and z_b axes, respectively, in rad/s^2.

VELTX,VELTY,VELTZ are the initial angular velocity of the base about the x_b, y_b, and z_b axes, respectively, in rad/s.

9 TIME,AX,AY,AZ,TX,TY,TZ (7F10.5)

TIME - Time at which the acceleration data is specified, in seconds.

AX,AY,AX are the linear acceleration of the base reference point b in the x_b, y_b, and z_b directions, respectively, in m/s^2.

TX,TY,TZ are the angular acceleration of the base about the x_b, y_b, and z_b axes, respectively, in rad/s^2.

<u>Note:</u> Card #9 must be repeated for all the time values at which the base acceleration data is given. The program is terminated by supplying a blank card in this place. The value of TIME must be supplied in the increasing sequence and the program stops whenever it reads the value of TIME as zero.

1.4 LISTING OF GYROT

```
C****
C**** GYROT - PROGRAM TO COMPUTE THE SEISMIC RESPONSE OF A
C**** RIGID ROTOR IN THE TIME DOMAIN.
C**** WRITTEN BY DR.V.SRINIVASAN, MARCH 1982.
C****
      DIMENSION AM(5,5),AK(5,5),C(5,5),F(5),AK1(5,5),AK2(5,5)
      DIMENSION C1(5,5),C2(5,5),C3(5,5),F1(5),F2(5)
      DIMENSION XOLD(5),VXOLD(5),AXOLD(5),XNEW(5),VXNEW(5),AXNEW(5)
      DIMENSION A(5,5),B(5,1),WKAREA(50),DIS(6),FOR(6)
      PI=4.0*ATAN(1.0)
C****
C**** READ AND PRINT MASS,MOMENTS OF INERTIA,L1,L2,H,RPM AND NFREE
C****
      READ(5,100) AMASS,AI,AIO,AL1,AL2,H,RPM
  100 FORMAT(7F10.5)
      READ(5,101) NFREE
  101 FORMAT(I5)
      WRITE(6,105) AMASS,AI,AIO,AL1,AL2,H,RPM,NFREE
  105 FORMAT(//5X,'MASS=',E11.4,5X,'I=',E11.4,5X,'IO=',E11.4//
     *5X,'L1=',E11.4,5X,'L2=',E11.4,5X,'H=',E11.4,5X,'RPM=',E11.4,
     *5X,'NFREE=',I5//)
      SPIN=2.0*RPM*PI/60.0
      IF (NFREE.EQ.4) WRITE(6,102)
  102 FORMAT(//5X,'THE Z DEGREE OF FREEDOM IS DELETED'//)
C****
C**** READ AND PRINT COEFFICIENTS FOR BEARING # 1
C****
      READ(5,110) AKXX1,AKXY1,AKYX1,AKYY1,AKZZ1
      READ(5,110) CXX1,CXY1,CYX1,CYY1,CZZ1
  110 FORMAT(5E11.4)
      WRITE(6,120) AKXX1,AKXY1,AKYX1,AKYY1,AKZZ1
  120 FORMAT(/5X,'KXX1=',E11.4,5X,'KXY1=',E11.4,5X,'KYX1=',E11.4,5X,
     *'KYY1=',E11.4,5X,'KZZ1=',E11.4/)
      WRITE(6,125) CXX1,CXY1,CYX1,CYY1,CZZ1
  125 FORMAT(/5X,'CXX1=',E11.4,5X,'CXY1=',E11.4,5X,'CYX1=',E11.4,
     *5X,'CYY1=',E11.4,5X,'CZZ1=',E11.4/)
C****
C**** READ AND PRINT COEFFICIENTS FOR BEARING # 2
C****
      READ(5,110) AKXX2,AKXY2,AKYX2,AKYY2,AKZZ2
      READ(5,110) CXX2,CXY2,CYX2,CYY2,CZZ2
      WRITE(6,130) AKXX2,AKXY2,AKYX2,AKYY2,AKZZ2
  130 FORMAT(/5X,'KXX2=',E11.4,5X,'KXY2=',E11.4,5X,'KYX2=',E11.4,5X,
     *'KYY2=',E11.4,5X,'KZZ2=',E11.4/)
      WRITE(6,135) CXX2,CXY2,CYX2,CYY2,CZZ2
  135 FORMAT(/5X,'CXX2=',E11.4,5X,'CXY2=',E11.4,5X,'CYX2=',E11.4,5X,
     *'CYY2=',E11.4,5X,'CZZ2=',E11.4/)
C****
C**** FORM TIME-INDEPENDENT MATRICES
C****
      CALL MASS(AMASS,AIO,AM)
```

```
      CALL CMAT1(CXX1,CXY1,CYX1,CYY1,CZZ1,CXX2,CXY2,CYX2,CYY2,CZZ2,
     *AL1,AL2,C1)
      CALL CMAT2(AI,SPIN,C2)
      CALL KMAT1(AKXX1,AKXY1,AKYX1,AKYY1,AKZZ1,AKXX2,AKXY2,AKYX2,AKYY2,
     *AKZZ2,AL1,AL2,AK1)
C****
C**** SET INITIAL CONDITIONS FOR THE ROTOR AND BASE
C****
      TIME=0.0
      DO 200 I=1,5
      XNEW(I)=0.0
      VXNEW(I)=0.0
  200 AXNEW(I)=0.0
      READ(5,230) (TIME,ACCX,ACCY,ACCZ,VELX,VELY,VELZ)
      READ(5,225) (ACCTX,ACCTY,ACCTZ,VELTX,VELTY,VELTZ)
  225 FORMAT(6F10.5)
  230 FORMAT(7F10.5)
      WRITE(6,210)
  210 FORMAT(//5X,'**** INITIAL CONDITIONS OF THE BASE AND ROTOR ****')
      WRITE(6,215) TIME,ACCX,ACCY,ACCZ,ACCTX,ACCTY,ACCTZ,VELX,VELY,VELZ,
     *VELTX,VELTY,VELTZ
  215 FORMAT(//5X,'TIME=',F10.5/5X,'ACCX=',E11.4,5X,'ACCY=',E11.4,5X,
     *'ACCZ=',E11.4,5X,'ACCTX=',E11.4,5X,'ACCTY=',E11.4,5X,'ACCTZ=',
     *E11.4/5X,'VELX=',E11.4,5X,'VELY=',E11.4,5X,'VELZ=',E11.4,5X,
     *'VELTX=',E11.4,5X,'VELTY=',E11.4,5X,'VELTZ=',E11.4)
      GO TO 2000
C****
C**** READ BASE ACCELERATIONS
C****
 1000 READ(5,235) (TIME,AX,AY,AZ,TX,TY,TZ)
  235 FORMAT(7F10.5)
      IF (TIME.LE.0.00001) STOP
      WRITE(6,220) TIME,AX,AY,AZ,TX,TY,TZ
  220 FORMAT(//5X,'TIME=',F10.5/5X,'ACCX=',E11.4,5X,'ACCY=',E11.4,5X,
     *'ACCZ=',E11.4,5X,'ACCTX=',E11.4,5X,'ACCTY=',E11.4,5X,'ACCTZ=',
     *E11.4)
      DT=TIME-TOLD
      VELX=VELX + 0.5*DT*(AX+ACCX)
      VELY=VELY + 0.5*DT*(AY+ACCY)
      VELZ=VELZ + 0.5*DT*(AZ+ACCZ)
      ACCX=AX
      ACCY=AY
      ACCZ=AZ
      VELTX=VELTX + 0.5*DT*(TX+ACCTX)
      VELTY=VELTY + 0.5*DT*(TY+ACCTY)
      VELTZ=VELTZ + 0.5*DT*(TZ+ACCTZ)
      ACCTX=TX
      ACCTY=TY
      ACCTZ=TZ
C****
C**** FORM TIME-DEPENDENT MATRICES AND VECTORS
```

```
C****
      CALL CMAT3(AMASS,VELTX,VELTY,VELTZ,C3)
      CALL KMAT2(AMASS,VELTX,VELTY,VELTZ,ACCTX,ACCTY,ACCTZ,AK2)
      CALL FVEC1(AMASS,ACCX,ACCY,ACCZ,F1)
      CALL FVEC2(AMASS,AI,AIO,SPIN,H,VELTX,VELTY,VELTZ,ACCTX,ACCTY,
     *ACCTZ,F2)
      DO 240 I=1,5
      F(I)=F1(I)+F2(I)
      DO 240 J=1,5
      AK(I,J)=AK1(I,J)+AK2(I,J)
  240 C(I,J)=C1(I,J)+C2(I,J)+C3(I,J)
      IF(NFREE.EQ.4) CALL CHANG(AM,C,AK,F)
C****
C**** USE NEWMARK'S ALGORITHM
C****
      DELTA=0.5
      ALFA=0.25
      AO=1.0/(ALFA*DT*DT)
      A1=DELTA/(ALFA*DT)
      A2=1.0/(ALFA*DT)
      A3=(0.5/ALFA)-1.0
      A4=(DELTA/ALFA)-1.0
      A5=DT*((DELTA/ALFA)-2.0)*0.5
      A6=DT*(1.0-DELTA)
      A7=DELTA*DT
      DO 250 I=1,5
      B(I,1)=F(I)
      DO 250 J=1,5
      B(I,1)=B(I,1)+AM(I,J)*(AO*XOLD(J)+A2*VXOLD(J)+A3*AXOLD(J))
     *+C(I,J)*(A1*XOLD(J)+A4*VXOLD(J)+A5*AXOLD(J))
  250 A(I,J)=AK(I,J)+AO*AM(I,J)+A1*C(I,J)
      CALL LEQT2F(A,1,NFREE,5,B,4,WKAREA,IER)
      DO 260 I=1,5
      XNEW(I)=B(I,1)
      AXNEW(I)=AO*(XNEW(I)-XOLD(I))-A2*VXOLD(I)-A3*AXOLD(I)
  260 VXNEW(I)=VXOLD(I)+A6*AXOLD(I)+A7*AXNEW(I)
 2000 CALL RESULT(NFREE,AKXX1,AKXY1,AKYX1,AKYY1,AKZZ1,AKXX2,AKXY2,AKYX2,
     *AKYY2,AKZZ2,CXX1,CXY1,CYX1,CYY1,CZZ1,CXX2,CXY2,CYX2,CYY2,CZZ2,AL1,
     *AL2,XNEW,VXNEW,DIS,FOR)
      WRITE(6,280) (DIS(I),I=1,6)
  280 FORMAT(7X,'X1=',E11.4,7X,'Y1=',E11.4,7X,'Z1=',E11.4,8X,'X2=',E11.4
     *,8X,'Y2=',E11.4,8X,'Z2=',E11.4)
      WRITE(6,290) (FOR(I),I=1,6)
  290 FORMAT(6X,'FX1=',E11.4,6X,'FY1=',E11.4,6X,'FZ1=',E11.4,7X,'FX2=',
     *E11.4,7X,'FY2=',E11.4,7X,'FZ2=',E11.4//)
      DO 270 I=1,5
      XOLD(I)=XNEW(I)
      VXOLD(I)=VXNEW(I)
  270 AXOLD(I)=AXNEW(I)
      TOLD=TIME
      GO TO 1000
```

```
      END

      SUBROUTINE MASS(AMASS,AIO,AM)
      DIMENSION AM(5,5)
      DO 100 I=1,5
      DO 100 J=1,5
  100 AM(I,J)=0.0
      AM(1,1)=AMASS
      AM(2,2)=AMASS
      AM(3,3)=AMASS
      AM(4,4)=AIO
      AM(5,5)=AIO
      RETURN
      END

      SUBROUTINE CMAT1(CXX1,CXY1,CYX1,CYY1,CZZ1,CXX2,CXY2,CYX2,CYY2,
     *CZZ2,AL1,AL2,C1)
      DIMENSION C1(5,5)
      DO 100 I=1,5
      DO 100 J=1,5
  100 C1(I,J)=0.0
      C1(1,1)=CXX1 + CXX2
      C1(1,2)=CXY1 + CXY2
      C1(1,4)=-AL1*CXY1 + AL2*CXY2
      C1(1,5)=AL1*CXX1 - AL2*CXX2
      C1(2,1)=CYX1 + CYX2
      C1(2,2)=CYY1 + CYY2
      C1(2,4)=-AL1*CYY1 + AL2*CYY2
      C1(2,5)=AL1*CYX1 - AL2*CYX2
      C1(3,3)=CZZ1 + CZZ2
      C1(4,1)=-AL1*CYX1 + AL2*CYX2
      C1(4,2)=-AL1*CYY1 + AL2*CYY2
      C1(4,4)=AL1*AL1*CYY1 + AL2*AL2*CYY2
      C1(4,5)=-AL1*AL1*CYX1 - AL2*AL2*CYX2
      C1(5,1)=AL1*CXX1 - AL2*CXX2
      C1(5,2)=AL1*CXY1 - AL2*CXY2
      C1(5,4)=-AL1*AL1*CXY1 - AL2*AL2*CXY2
      C1(5,5)=AL1*AL1*CXX1 + AL2*AL2*CXX2
      RETURN
      END

      SUBROUTINE CMAT2(AI,SPIN,C2)
      DIMENSION C2(5,5)
      DO 100 I=1,5
      DO 100 J=1,5
```

```
100 C2(I,J)=0.0
    C2(4,5)=AI*SPIN
    C2(5,4)=-AI*SPIN
    RETURN
    END


    SUBROUTINE CMAT3(AMASS,VELTX,VELTY,VELTZ,C3)
    DIMENSION C3(5,5)
    DO 100 I=1,5
    DO 100 J=1,5
100 C3(I,J)=0.0
    C3(1,2)=-2.0*AMASS*VELTZ
    C3(1,3)=2.0*AMASS*VELTY
    C3(2,1)=2.0*AMASS*VELTZ
    C3(2,3)=-2.0*AMASS*VELTX
    C3(3,1)=-2.0*AMASS*VELTY
    C3(3,2)=2.0*AMASS*VELTX
    RETURN
    END


    SUBROUTINE KMAT1(AKXX1,AKXY1,AKYX1,AKYY1,AKZZ1,AKXX2,AKXY2,
   *AKYX2,AKYY2,AKZZ2,AL1,AL2,AK1)
    DIMENSION AK1(5,5)
    DO 100 I=1,5
    DO 100 J=1,5
100 AK1(I,J)=0.0
    AK1(1,1)=AKXX1 + AKXX2
    AK1(1,2)=AKXY1 + AKXY2
    AK1(1,4)=-AL1*AKXY1 + AL2*AKXY2
    AK1(1,5)=AL1*AKXX1 - AL2*AKXX2
    AK1(2,1)=AKYX1 + AKYX2
    AK1(2,2)=AKYY1 + AKYY2
    AK1(2,4)=-AL1*AKYY1 + AL2*AKYY2
    AK1(2,5)=AL1*AKYX1 - AL2*AKYX2
    AK1(3,3)=AKZZ1 + AKZZ2
    AK1(4,1)=-AL1*AKYX1 + AL2*AKYX2
    AK1(4,2)=-AL1*AKYY1 + AL2*AKYY2
    AK1(4,4)=AL1*AL1*AKYY1 + AL2*AL2*AKYY2
    AK1(4,5)=-AL1*AL1*AKYX1 - AL2*AL2*AKYX2
    AK1(5,1)=AL1*AKXX1 - AL2*AKXX2
    AK1(5,2)=AL1*AKXY1 - AL2*AKXY2
    AK1(5,4)=-AL1*AL1*AKXY1 - AL2*AL2*AKXY2
    AK1(5,5)=AL1*AL1*AKXX1 + AL2*AL2*AKXX2
    RETURN
    END
```

```
      SUBROUTINE KMAT2(AMASS,VELTX,VELTY,VELTZ,ACCTX,ACCTY,ACCTZ,AK2)
      DIMENSION AK2(5,5)
      DO 100 I=1,5
      DO 100 J=1,5
  100 AK2(I,J)=0.0
      AK2(1,1)=-AMASS*(VELTY*VELTY + VELTZ*VELTZ)
      AK2(1,2)=AMASS*(VELTX*VELTY - ACCTZ)
      AK2(1,3)=AMASS*(VELTZ*VELTX + ACCTY)
      AK2(2,1)=AMASS*(VELTX*VELTY + ACCTZ)
      AK2(2,2)=-AMASS*(VELTZ*VELTZ + VELTX*VELTX)
      AK2(2,3)=AMASS*(VELTY*VELTZ - ACCTX)
      AK2(3,1)=AMASS*(VELTZ*VELTX - ACCTY)
      AK2(3,2)=AMASS*(VELTY*VELTZ + ACCTX)
      AK2(3,3)=-AMASS*(VELTX*VELTX + VELTY*VELTY)
      RETURN
      END


      SUBROUTINE FVEC1(AMASS,ACCX,ACCY,ACCZ,F1)
      DIMENSION F1(5)
      F1(1)=-AMASS*ACCX
      F1(2)=-AMASS*ACCY
      F1(3)=-AMASS*ACCZ
      F1(4)=0.0
      F1(5)=0.0
      RETURN
      END


      SUBROUTINE FVEC2(AMASS,AI,AIO,SPIN,H,VELTX,VELTY,VELTZ,ACCTX,
     *ACCTY,ACCTZ,F2)
      DIMENSION F2(5)
      F2(1)=-AMASS*H*(VELTX*VELTY - ACCTZ)
      F2(2)=AMASS*H*(VELTZ*VELTZ + VELTX*VELTX)
      F2(3)=-AMASS*H*(VELTY*VELTZ + ACCTX)
      F2(4)=-AIO*ACCTX - AI*SPIN*VELTY
      F2(5)=-AIO*ACCTY + AI*SPIN*VELTX
      RETURN
      END


      SUBROUTINE CHANG(AM,C,AK,F)
      DIMENSION AM(5,5),C(5,5),AK(5,5),F(5)
      DO 100 I=1,4
      IF (I.LE.2) IN=I
      IF (I.GT.2) IN=I+1
      F(I)=F(IN)
```

```
      DO 100 J=1,4
      IF (J.LE.2) JN=J
      IF (J.GT.2) JN=J+1
      AM(I,J)=AM(IN,JN)
      C(I,J)=C(IN,JN)
  100 AK(I,J)=AK(IN,JN)
      F(5)=0.0
      DO 110 I=1,5
      AM(I,5)=0.0
      AM(5,I)=0.0
      C(I,5)=0.0
      C(5,I)=0.0
      AK(I,5)=0.0
  110 AK(5,I)=0.0
      RETURN
      END


      SUBROUTINE RESULT(NFREE,AKXX1,AKXY1,AKYX1,AKYY1,AKZZ1,AKXX2,AKXY2,
     *AKYX2,AKYY2,AKZZ2,CXX1,CXY1,CYX1,CYY1,CZZ1,CXX2,CXY2,CYX2,CYY2,
     *CZZ2,AL1,AL2,XNEW,VXNEW,DIS,FOR)
      DIMENSION XNEW(5),VXNEW(5),DIS(6),FOR(6)
      IF (NFREE.EQ.5) GO TO 100
      DIS(1)=XNEW(1) + AL1*XNEW(4)
      DIS(2)=XNEW(2) - AL1*XNEW(3)
      DIS(3)=0.0
      DIS(4)=XNEW(1) - AL2*XNEW(4)
      DIS(5)=XNEW(2) + AL2*XNEW(3)
      DIS(6)=0.0
      VEL1=VXNEW(1) + AL1*VXNEW(4)
      VEL2=VXNEW(2) - AL1*VXNEW(3)
      VEL3=0.0
      VEL4=VXNEW(1) - AL2*VXNEW(4)
      VEL5=VXNEW(2) + AL2*VXNEW(3)
      VEL6=0.0
      GO TO 200
  100 DIS(1)=XNEW(1) + AL1*XNEW(5)
      DIS(2)=XNEW(2) - AL1*XNEW(4)
      DIS(3)=XNEW(3)
      DIS(4)=XNEW(1) - AL2*XNEW(5)
      DIS(5)=XNEW(2) + AL2*XNEW(4)
      DIS(6)=XNEW(3)
      VEL1=VXNEW(1) + AL1*VXNEW(5)
      VEL2=VXNEW(2) - AL1*VXNEW(4)
      VEL3=VXNEW(3)
      VEL4=VXNEW(1) - AL2*VXNEW(5)
      VEL5=VXNEW(2) + AL2*VXNEW(4)
      VEL6=VXNEW(3)
  200 FOR(1)=AKXX1*DIS(1) + AKXY1*DIS(2) + CXX1*VEL1 + CXY1*VEL2
      FOR(2)=AKYX1*DIS(1) + AKYY1*DIS(2) + CYX1*VEL1 + CYY1*VEL2
```

```
FOR(3)=AKZZ1*DIS(3) + CZZ1*VEL3
FOR(4)=AKXX2*DIS(4) + AKXY2*DIS(5) + CXX2*VEL4 + CXY2*VEL5
FOR(5)=AKYX2*DIS(4) + AKYY2*DIS(5) + CYX2*VEL4 + CYY2*VEL5
FOR(6)=AKZZ2*DIS(6) + CZZ2*VEL6
RETURN
END
```

1.5 SAMPLE INPUT DATA

```
24297.0    3368.0    200440.0    3.22    5.28    1.0    3000.0
   4
+0.5890E+09+0.5100E+08-0.1290E+10+0.1870E+10
+0.2800E+07-0.4100E+07-0.4100E+07+0.1170E+08
+0.6760E+09+0.2160E+08-0.1490E+10+0.2270E+10
+0.3100E+07-0.5000E+07-0.5000E+07+0.1370E+08
   0.0                                   -0.0466    0.0297    0.1183

   0.02    -0.014     0.024    0.003
   0.04    -0.108    -0.230    0.019
   0.06    -0.101    -0.275    0.068
   0.08    -0.088    -0.397    0.029
   0.10    -0.095    -0.390    0.029
   0.12    -0.120    -0.060    0.054
```

1.6 SAMPLE RESULTS

```
MASS= 0.2430E+05      I= 0.3368E+04      IO= 0.2004E+06

L1= 0.3220E+01      L2= 0.5280E+01      H= 0.1000E+01      RPM= 0.3000E+04      NFREE=    4


THE Z DEGREE OF FREEDOM IS DELETED


KXX1= 0.5890E+09      KXY1= 0.5100E+08      KYX1=-0.1290E+10      KYY1= 0.1870E+10      KZZ1= 0.0

CXX1= 0.2800E+07      CXY1=-0.4100E+07      CYX1=-0.4100E+07      CYY1= 0.1170E+08      CZZ1= 0.0

KXX2= 0.6760E+09      KXY2= 0.2160E+08      KYX2=-0.1490E+10      KYY2= 0.2270E+10      KZZ2= 0.0

CXX2= 0.3100E+07      CXY2=-0.5000E+07      CYX2=-0.5000E+07      CYY2= 0.1370E+08      CZZ2= 0.0


**** INITIAL CONDITIONS OF THE BASE AND ROTOR ****

TIME=   0.0
ACCX= 0.0             ACCY= 0.0             ACCZ= 0.0             ACCTX= 0.0            ACCTY= 0.0            ACCTZ= 0.0
VELX=-0.4660E-01      VELY= 0.2970E-01      VELZ= 0.1183E+00      VELTX= 0.0            VELTY= 0.0            VELTZ= 0.0
  X1= 0.0               Y1= 0.0               Z1= 0.0                X2= 0.0               Y2= 0.0               Z2= 0.0
 FX1= 0.0              FY1= 0.0              FZ1= 0.0               FX2= 0.0              FY2= 0.0              FZ2= 0.0


TIME=   0.02000
ACCX=-0.1400E-01      ACCY= 0.2400E-01      ACCZ= 0.3000E-02      ACCTX= 0.0            ACCTY= 0.0            ACCTZ= 0.0
  X1= 0.2090E-06        Y1=-0.2183E-08        Z1= 0.0                X2= 0.1190E-06        Y2= 0.4251E-08        Z2= 0.0
 FX1= 0.1824E+03       FY1=-0.3620E+03       FZ1= 0.0               FX2= 0.1153E+03       FY2=-0.2212E+03       FZ2= 0.0


TIME=   0.04000
ACCX=-0.1080E+00      ACCY=-0.2300E+00      ACCZ= 0.1900E-01      ACCTX= 0.0            ACCTY= 0.0            ACCTZ= 0.0
  X1= 0.2768E-05        Y1= 0.2518E-05        Z1= 0.0                X2= 0.1599E-05        Y2= 0.1382E-05        Z2= 0.0
 FX1= 0.1383E+04       FY1= 0.3126E+04       FZ1= 0.0               FX2= 0.8461E+03       FY2= 0.1956E+04       FZ2= 0.0


TIME=   0.06000
ACCX=-0.1010E+00      ACCY=-0.2750E+00      ACCZ= 0.6800E-01      ACCTX= 0.0            ACCTY= 0.0            ACCTZ= 0.0
```

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```
  X1= 0.3901E-05      Y1= 0.5007E-05      Z1= 0.0             X2= 0.2186E-05      Y2= 0.2628E-05      Z2= 0.0
 FX1= 0.2226E+04     FY1= 0.4790E+04     FZ1= 0.0            FX2= 0.1358E+04     FY2= 0.2922E+04     FZ2= 0.0

TIME=    0.08000
ACCX=-0.8800E-01    ACCY=-0.3970E+00    ACCZ= 0.2900E-01    ACCTX= 0.0          ACCTY= 0.0          ACCTZ= 0.0
  X1= 0.8995E-06      Y1= 0.3856E-05      Z1= 0.0             X2= 0.5521E-06      Y2= 0.1967E-05      Z2= 0.0
 FX1= 0.6850E+03     FY1= 0.5475E+04     FZ1= 0.0            FX2= 0.4164E+03     FY2= 0.3341E+04     FZ2= 0.0

TIME=    0.10000
ACCX=-0.9500E-01    ACCY=-0.3900E+00    ACCZ= 0.2900E-01    ACCTX= 0.0          ACCTY= 0.0          ACCTZ= 0.0
  X1= 0.1915E-05      Y1= 0.4244E-05      Z1= 0.0             X2= 0.1210E-05      Y2= 0.2261E-05      Z2= 0.0
 FX1= 0.1511E+04     FY1= 0.6080E+04     FZ1= 0.0            FX2= 0.9232E+03     FY2= 0.3705E+04     FZ2= 0.0

TIME=    0.12000
ACCX=-0.1200E+00    ACCY=-0.6000E-01    ACCZ= 0.5400E-01    ACCTX= 0.0          ACCTY= 0.0          ACCTZ= 0.0
  X1= 0.2960E-05      Y1= 0.3617E-05      Z1= 0.0             X2= 0.1627E-05      Y2= 0.1874E-05      Z2= 0.0
 FX1= 0.2311E+04     FY1= 0.1171E+04     FZ1= 0.0            FX2= 0.1407E+04     FY2= 0.7154E+03     FZ2= 0.0
```

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2. ROBET USER'S MANUAL

2.1 PURPOSE

ROBET is a computer program written in Fortran to carry out the seismic analysis of a flexible rotor in time domain. It is the second of a series of computer program packages that are developed at the School of Mechanical and Aerospace Engineering, Oklahoma State University under the sponsorship of National Science Foundation.

This user's manual describes the way in which the data is supplied to the program. It also includes a listing of the program and a sample output.

2.2 BACKGROUND THEORY

ROBET is based on the beam model developed in Part I, Chapter 3 of this report. ROBET uses two-noded finite rotor elements, such as the one shown in Figure 3.4.

ROBET is a self-contained program. The only external subroutine used is a commonly available IMSL routine LEQT1B to solve a set of linear, banded simultaneous equations lacking symmetry.

2.3 INPUT DATA

Card #		Data and Description
1		NNOD,NELEM,NNOEL,NFREE,NEQ,NLC,NUC,IKC,IKP (9I5)
	NNOD -	Number of nodes in the model
	NELEM -	Number of elements in the model
	NNOEL -	Number of nodes per element = 2 in our case
	NFREE -	Number of degrees of feedom per node = 4 in our case
	NEQ -	Number of final set of equations
	NLC -	Number of lower codiagonals (excluding diagonal)
	NUC -	Number of upper codiagonals (excluding diagonal)
	IKC -	Index for conventional stiffness matrix = 0 for reduced integration, ≠ 0 for exact integration.
	IKP -	Index for geometric stiffness matrix due to axial force = 0 for reduced integration, ≠ 0 for exact integration.
2		RPM,P,T,H,NBEAR,NDISK,NPINT (4E11.4,3I5)
	RPM -	Spin speed of the rotor in revolutions per minute
	P -	Axial tension on the rotor, in N.
	T -	axial torque on the rotor in the +z direction, in N-m.
	H -	Height of the rotor axis from the base, in m.
	NBEAR -	Number of bearings in the system.
	NDISK -	Number of disks (and flywheels) in the system.

NPINT - Number of points at which internal stresses are to be evaluated.

3 ZC(I),ID(I,1),ID(I,2),ID(I,3),ID(I,4) (E11.4,4I5)

ZC(I) - z coordinate of the i^{th} node, in m.

ID(I,J) - Index for the j^{th} degree of freedom at the i^{th} node.

Note: This card must be repeated for each of the NNOD nodes.

J=1 corresponds to $(U_x)_i$
J=2 corresponds to $(U_y)_i$
J=3 corresponds to $(\Theta_x)_i$
J=4 corresponds to $(\Theta_y)_i$
ID(I,J) ≠ 0 to delete the j^{th} degree of freedom at i^{th} node.

ID(I,J) = 0 to keep the j^{th} degree of freedom at i^{th} node. (Leave it blank).

4 NOD(LK,1),NOD(LK,2),EYM(LK),EPR(LK),ERHO(LK), EAREA(LK),EAIT(LK) (2I5,5E11.4)

NOD(LK,J) - J^{TH} NODE OF THE $(LK)^{TH}$ element.

EYM(LK) - Young's modulus for the $(LK)^{th}$ element, in N/m^2.

EPR(LK) - Poisson's ratio for the $(LK)^{th}$ element

ERHO(LK) - Mass density of the $(LK)^{th}$ element, in kg/m^3.

EAREA(LK) - Area of cross-section of the $(LK)^{th}$ element, in m^2.

EAIT(LK) - Transverse second moment of area of the $(LK)^{th}$ element, in m^4.

Note: This card must be repeated for each of the NELEM elements.

5 NODIS(I),DMASS(I),DIO(I),DI(I) (I5,3E11.4)

NODIS(I) - Node number at which the i^{th} disk (or flywheel) is located.

DIMASS(I) - Mass of the i^{th} disk (or flywheel), in Kg.

DIO(I) - Transverse moment of inertia of the i^{th} disk (or flywheel), in $Kg.m^2$.

DI(I) - Polar moment of inertia of the i^{th} disk (or flywheel), in $Kg.m^2$.

Note: This card must be repeated for each of the NDISK disks (or flywheels) in the model. If NDISK=0, skip this card.

6 NOBER(I),BK(I,1,1),BK(I,1,2),BK(I,2,1), BK(I,2,2) (I5,4E11.4)

NOBER(I) - Node number at which the i^{th} bearing is located.

BK(I,J,K) - The $(j,k)^{th}$ coefficient in the stiffness matrix for the lubricants in i^{th} bearing.

7 BC(I,1,1),BC(I,1,2),BC(I,2,1),BC(I,2,2) (4E11.4)

BC(I,J,K) - The $(j,k)^{th}$ coefficient in the damping matrix for the lubricants in i^{th} bearing.

Note: The 6th and 7th cards must be repeated for each of the NBEAR bearings. If NBEAR=0, skip these cards.

8 NELP(I) (I5)

NELP(I) - Element number in which the i^{th} internal stress point is located.

Note: This card must be repeated for each of the NPINT points. If NPINT=0, skip this card.

9 TIME,ACCX,ACCY,ACCZ,VELX,VELY,VELZ (7F10.5)
TIME=0.0
ACCX,ACCY,ACCZ are the initial acceleration of point b in the x_b, y_b, and z_b directions, in m/s^2.
VELX,VELY,VELZ are the initial velocity of point b in the x_b, y_b, and z_b directions, in m/s.

10 ACCTX,ACCTY,ACCTZ,VELTX,VELTY,VELTZ (6F10.5)
ACCTX,ACCTY,ACCTZ are the initial angular accleration of the base along x_b, y_b, and z_b axes, in rad/s^2.
VELTX, VELTY, VELTZ are the initial angular velocity of the base along x_b, y_b and z_b axes, in rad/s.

11 TIME,AX,AY,AZ,TX,TY,TZ

TIME	-	Time at which the accleration data is specified, in s.
AX,AY,AZ		are the linear acceleration of the point b in the x_b, y_b and z_b directions, in m/s^2.
TX,TY,TZ		are the angular acceleration of the base along the x_b, y_b and z_b directions, in rad/s^2.

Note: This card must be repeated for all the time values at which the base acceleration data is given. The program is terminated by supplying a blank card in this place. The value of TIME must be supplied in the increasing sequence, and the program stops whenever it reads the value of TIME as zero.

2.4 LISTING OF ROBET

```
C****
C**** ROBET - PROGRAM TO COMPUTE THE SEISMIC RESPONSE OF A ROTOR/
C**** BEARING SYSTEM IN THE TIME DOMAIN.
C**** WRITTEN BY DR.V.SRINIVASAN, JULY 1982.
C****
      DIMENSION ZC(25),NOD(25,2),ID(25,4),ZE(2)
      DIMENSION CI(100,15),C(100,15),A(100,15),B(100,1),F(100),XL(800)
      DIMENSION AM(8,8),ACG(8,8),ACC(8,8),AKC(8,8),AKP(8,8),AKT(8,8)
      DIMENSION AKR(8,8),FV(8),DM(4,4),DCG(4,4),DCC(4,4),DKR(4,4),FD(4)
      DIMENSION BK(20,4,4),BC(20,4,4),ERHO(25),EAREA(25),EAIT(25)
      DIMENSION EAIP(25),DMASS(20),NODIS(20),NOBER(20),DIO(20),DI(20)
      DIMENSION XOLD(100),VXOLD(100),AXOLD(100),XNEW(100)
      DIMENSION VXNEW(100),AXNEW(100),EYM(25),EPR(25),NELP(25)
      REAL M(100,15),KI(100,15),K(100,15)
      PI=4.0*ATAN(1.0)
C****
C**** READ AND PRINT CONTROL DATA
C****
      READ(5,100) NNOD,NELEM,NNOEL,NFREE,NEQ,NLC,NUC,IKC,IKP
  100 FORMAT(9I5)
      READ(5,105) RPM,P,T,H,NBEAR,NDISK,NPINT
  105 FORMAT(4E11.4,3I5)
      WRITE(6,110) NNOD,NELEM,NNOEL,NFREE
  110 FORMAT(//
     *5X,'NUMBER OF NODES ********************* =',I5//
     *5X,'NUMBER OF ELEMENTS ****************** =',I5//
     *5X,'NUMBER OF NODES PER ELEMENT ********* =',I5//
     *5X,'NUMBER OF DEGREES OF FREEDOM PER NODE =',I5)
      WRITE(6,111) NEQ,NLC,NUC,IKC
  111 FORMAT(/
     *5X,'NUMBER OF EQUATIONS ***************** =',I5//
     *5X,'NUMBER OF LOWER CODIAGONALS ********* =',I5//
     *5X,'NUMBER OF UPPER CODIAGONALS ********* =',I5//
     *5X,'INDEX FOR KC MATRIX ***************** =',I5)
      WRITE(6,112) IKP,RPM,P,T
  112 FORMAT(/
     *5X,'INDEX FOR KP MATRIX ***************** =',I5//
     *5X,'SPIN SPEED IN RPM ******************* =',E11.4//
     *5X,'AXIAL FORCE ************************* =',E11.4//
     *5X,'AXIAL TORQUE ************************ =',E11.4)
      WRITE(6,113) H,NBEAR,NDISK,NPINT
  113 FORMAT(/
     *5X,'HEIGHT OF THE ROTOR FROM BASE ******* =',E11.4//
     *5X,'NUMBER OF BEARINGS ****************** =',I5//
     *5X,'NUMBER OF DISKS (AND FLYWHEELS) ***** =',I5//
     *5X,'NUMBER OF STRESS POINTS ************* =',I5)
      SP=2.0*RPM*PI/60.0
C****
C**** READ AND PRINT NODAL DATA
C****
      WRITE(6,121)
```


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```
  121 FORMAT(//7X,'NODE #',7X,'Z',25X,'ID MATRIX'/)
      DO 10 I=1,NNOD
      READ(5,120) ZC(I),ID(I,1),ID(I,2),ID(I,3),ID(I,4)
   10 WRITE(6,125) I,ZC(I),ID(I,1),ID(I,2),ID(I,3),ID(I,4)
  120 FORMAT(E11.4,4I5)
  125 FORMAT(5X,I5,5X,E11.4,4(5X,I5))
C****
C**** FORM AND PRINT CONNECTIVITY MATRIX
C****
      WRITE(6,131)
  131 FORMAT(//7X,'NODE #',12X,'CONNECTIVITY MATRIX'/)
      ISUM=0
      DO 20 I=1,NNOD
      DO 25 J=1,NFREE
      IF (ID(I,J)) 35,30,35
   30 ISUM=ISUM + 1
      ID(I,J)=ISUM
      GO TO 25
   35 ID(I,J)=0
   25 CONTINUE
      WRITE(6,130) I,ID(I,1),ID(I,2),ID(I,3),ID(I,4)
   20 CONTINUE
  130 FORMAT(5X,5(I5,5X))
      NEQ=ISUM
      WRITE(6,135) NEQ
  135 FORMAT(//5X,'COMPUTED NUMBER OF EQUATIONS =',I5//)
      NBAND=NLC+NUC+1
C****
C**** INITIALIZE THE MATRICES
C****
      DO 40 I=1,NEQ
      DO 40 J=1,NBAND
      M(I,J)=0.0
      CI(I,J)=0.0
   40 KI(I,J)=0.0
C****
C**** FORM TIME-INDEPENDENT MATRICES M,CI,AND KI
C****
C**** ASSEMBLE ELEMENT MATRICES
C****
      WRITE(6,141)
  141 FORMAT(5X,'ELEMENT #',3X,'NODE 1',4X,'NODE 2',4X,'YOUNG*S',8X,
     *'POISSON*S',8X,'DENSITY',9X,'AREA OF',8X,'TRANSVERSE',8X,'POLAR'/
     *37X,'MODULUS',10X,'RATIO',23X,'CROSS-SECTION',3X,'SECOND MOMENT',
     *3X,'SECOND MOMENT'/101X,'OF AREA',9X,'OF AREA'/)
      DO 50 LK=1,NELEM
      READ(5,140) (NOD(LK,J),J=1,NNOEL),EYM(LK),EPR(LK),ERHO(LK),EAREA(L
     *K),EAIT(LK)
  140 FORMAT(2I5,5E11.4)
      YM=EYM(LK)
      PR=EPR(LK)
```

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```
      RHO=ERHO(LK)
      AREA=EAREA(LK)
      AIT=EAIT(LK)
      AIP=2.0*AIT
      WRITE(6,145) LK,(NOD(LK,J),J=1,NNOEL),YM,PR,RHO,AREA,AIT,AIP
  145 FORMAT(3(5X,I5),6(5X,E11.4))
      DO 55 IP=1,NNOEL
      II=NOD(LK,IP)
   55 ZE(IP)=ZC(II)
      AL=ABS(ZE(2)-ZE(1))
      RM=0.5*YM/(1.0+PR)
      TIMC=6.0*(1.0+PR)/(7.0+6.0*PR)
      CALL MASS(RHO,AREA,AL,AIT,AM)
      CALL GYRO(RHO,AL,AIP,SP,ACG)
      CALL KCMAT(AREA,RM,YM,AIT,AL,TIMC,IKC,AKC)
      CALL KPMAT(P,AL,IKP,AKP)
      CALL KTMAT(T,AKT)
      DO 60 IT=1,NNOEL
      II=NOD(LK,IT)
      IM=NFREE*(IT-1)
      DO 60 JT=1,NNOEL
      JJ=NOD(LK,JT)
      JN=NFREE*(JT-1)
      DO 65 I=1,NFREE
      MMI=ID(II,I)
      IF (MMI.EQ.0) GO TO 65
      IMI=IM+I
      DO 70 J=1,NFREE
      NJJ=ID(JJ,J)
      IF (NJJ.EQ.0) GO TO 70
      NNJ=NJJ-MMI+NLC+1
      JNJ=JN+J
      M(MMI,NNJ)=M(MMI,NNJ) + AM(IMI,JNJ)
      CI(MMI,NNJ)=CI(MMI,NNJ) + ACG(IMI,JNJ)
      KI(MMI,NNJ)=KI(MMI,NNJ) + AKC(IMI,JNJ)+AKP(IMI,JNJ)+AKT(IMI,JNJ)
   70 CONTINUE
   65 CONTINUE
   60 CONTINUE
   50 CONTINUE
C****
C**** ASSEMBLE DISK MATRICES
C****
      IF (NDISK.EQ.0) GO TO 90
      DO 75 I=1,NDISK
      READ(5,150) NODIS(I),DMASS(I),DIO(I),DI(I)
  150 FORMAT(I5,3E11.4)
      AMASS=DMASS(I)
      AIO=DIO(I)
      AI=DI(I)
      NDUM=NODIS(I)
      WRITE(6,155) I,NDUM,AMASS,AIO,AI
```

```
155 FORMAT(/5X,'DISK #=',I5,5X,'AT NODE #=',I5,5X,'MASS=',E11.4,5X,
   *'IO=',E11.4,5X,'I=',E11.4/)
    CALL DISKI(AMASS,AIO,AI,SP,DM,DCG)
    DO 80 J=1,NFREE
    JJ=ID(NDUM,J)
    IF (JJ.EQ.O) GO TO 80
    DO 85 KA=1,NFREE
    KK=ID(NDUM,KA)
    IF (KK.EQ.O) GO TO 85
    KKI=KK-JJ+NLC+1
    M(JJ,KKI)=M(JJ,KKI) + DM(J,KA)
    CI(JJ,KKI)=CI(JJ,KKI) + DCG(J,KA)
 85 CONTINUE
 80 CONTINUE
 75 CONTINUE
 90 CONTINUE
C****
C**** ASSEMBLE BEARING MATRICES
C****
    IF (NBEAR.EQ.O) GO TO 300
    DO 305 I=1,NBEAR
    READ(5,160) NOBER(I),BK(I,1,1),BK(I,1,2),BK(I,2,1),BK(I,2,2)
160 FORMAT(I5,4E11.4)
    READ(5,165) BC(I,1,1),BC(I,1,2),BC(I,2,1),BC(I,2,2)
165 FORMAT(4E11.4)
    NDUM=NOBER(I)
    WRITE(6,170) I,NDUM,BK(I,1,1),BK(I,1,2),BK(I,2,1),BK(I,2,2),BC(I,1
   *,1),BC(I,1,2),BC(I,2,1),BC(I,2,2)
170 FORMAT(//5X,'BEARING #=',I5,5X,'AT NODE #=',I5,5X/5X,'BK(1,1)=',
   *E11.4,5X,'BK(1,2)=',E11.4,5X,'BK(2,1)=',E11.4,5X,'BK(2,2)=',E11.4,
   *5X/5X,'BC(1,1)=',E11.4,5X,'BC(1,2)=',E11.4,5X,'BC(2,1)=',E11.4,5X,
   *'BC(2,2)=',E11.4/)
    DO 310 J=1,2
    JJ=ID(NDUM,J)
    IF (JJ.EQ.O) GO TO 310
    DO 315 KA=1,2
    KK=ID(NDUM,KA)
    IF (KK.EQ.O) GO TO 315
    KKI=KK-JJ+NLC+1
    CI(JJ,KKI)=CI(JJ,KKI) + BC(I,J,KA)
    KI(JJ,KKI)=KI(JJ,KKI) + BK(I,J,KA)
315 CONTINUE
310 CONTINUE
305 CONTINUE
300 CONTINUE
    IF (NPINT.EQ.O) GO TO 210
    DO 220 I=1,NPINT
    READ(5,225) NELP(I)
225 FORMAT(I5)
220 CONTINUE
210 CONTINUE
```

```
C****
C**** SET INITIAL CONDITIONS FOR THE ROTOR AND BASE
C****
      DO 320 I=1,NEQ
      XNEW(I)=0.0
      VXNEW(I)=0.0
  320 AXNEW(I)=0.0
      WRITE(6,175)
  175 FORMAT('1')
      READ(5,180) (TIME,ACCX,ACCY,ACCZ,VELX,VELY,VELZ)
      READ(5,185) (ACCTX,ACCTY,ACCTZ,VELTX,VELTY,VELTZ)
  180 FORMAT(7F10.5)
  185 FORMAT(6F10.5)
      WRITE(6,190)
  190 FORMAT(//5X,'**** INITIAL CONDITIONS OF THE BASE AND ROTOR ****')
      WRITE(6,195) TIME,ACCX,ACCY,ACCZ,ACCTX,ACCTY,ACCTZ,VELX,VELY,VELZ,
     *VELTX,VELTY,VELTZ
  195 FORMAT(//5X,'TIME=',F10.5/5X,'ACCX=',E11.4,5X,'ACCY=',E11.4,5X,
     *'ACCZ=',E11.4,5X,'ACCTX=',E11.4,5X,'ACCTY=',E11.4,5X,'ACCTZ=',E11.
     *4/5X,'VELX=',E11.4,5X,'VELY=',E11.4,5X,'VELZ=',E11.4,5X,'VELTX=',
     *E11.4,5X,'VELTY=',E11.4,5X,'VELTZ=',E11.4)
      GO TO 3000
C****
C**** READ BASE ACCELERATIONS
C****
 1000 READ(5,200) (TIME,AX,AY,AZ,TX,TY,TZ)
  200 FORMAT(7F10.5)
      IF (TIME.LE.0.00001) STOP
      WRITE(6,175)
      WRITE(6,215) TIME,AX,AY,AZ,TX,TY,TZ
  215 FORMAT(//5X,'TIME=',F10.5//5X,'ACCX=',E11.4,5X,'ACCY=',E11.4,5X,
     *'ACCZ=',E11.4,5X,'ACCTX=',E11.4,5X,'ACCTY=',E11.4,5X,'ACCTZ=',E11.
     *4)
      DT=TIME-TOLD
      VELX=VELX + 0.5*DT*(AX+ACCX)
      VELY=VELY + 0.5*DT*(AY+ACCY)
      VELZ=VELZ + 0.5*DT*(AZ+ACCZ)
      ACCX=AX
      ACCY=AY
      ACCZ=AZ
      VELTX=VELTX + 0.5*DT*(TX+ACCTX)
      VELTY=VELTY + 0.5*DT*(TY+ACCTY)
      VELTZ=VELTZ + 0.5*DT*(TZ+ACCTZ)
      ACCTX=TX
      ACCTY=TY
      ACCTZ=TZ
C****
C**** FORM TIME-DEPENDENT MATRICES AND VECTORS
C****
      DO 340 I=1,NEQ
      F(I)=0.0
```

```
      DO 340 J=1,NBAND
      C(I,J)=CI(I,J)
  340 K(I,J)=KI(I,J)
C****
C**** ASSEMBLE ELEMENT MATRICES AND VECTORS
C****
      DO 350 LK=1,NELEM
      RHO=ERHO(LK)
      AREA=EAREA(LK)
      AIT=EAIT(LK)
      AIP=2.0*AIT
      DO 355 IP=1,NNOEL
      II=NOD(LK,IP)
  355 ZE(IP)=ZC(II)
      AL=ABS(ZE(2)-ZE(1))
      CALL CORI(RHO,AL,AREA,VELTZ,ACC)
      CALL KRMAT(RHO,AL,AREA,VELTX,VELTY,VELTZ,ACCTX,ACCTY,ACCTZ,AKR)
      CALL FVEC(RHO,AL,AREA,H,ZE(1),ZE(2),SP,AIT,AIP,ACCX,ACCY,VELTX,VEL
     *TY,VELTZ,ACCTX,ACCTY,ACCTZ,FV)
      DO 360 IT=1,NNOEL
      II=NOD(LK,IT)
      IM=NFREE*(IT-1)
      DO 360 JT=1,NNOEL
      JJ=NOD(LK,JT)
      JN=NFREE*(JT-1)
      DO 365 I=1,NFREE
      MMI=ID(II,I)
      IF (MMI.EQ.0) GO TO 365
      IMI=IM+I
      DO 370 J=1,NFREE
      NJJ=ID(JJ,J)
      IF (NJJ.EQ.0) GO TO 370
      NNJ=NJJ-MMI+NLC+1
      JNJ=JN+J
      C(MMI,NNJ)=C(MMI,NNJ) + ACC(IMI,JNJ)
      K(MMI,NNJ)=K(MMI,NNJ) + AKR(IMI,JNJ)
  370 CONTINUE
  365 CONTINUE
  360 CONTINUE
      DO 380 IT=1,NNOEL
      II=NOD(LK,IT)
      IM=NFREE*(IT-1)
      DO 385 I=1,NFREE
      MMI=ID(II,I)
      IF (MMI.EQ.0) GO TO 385
      IMI=IM+I
      F(MMI)=F(MMI) + FV(IMI)
  385 CONTINUE
  380 CONTINUE
  350 CONTINUE
C****
```

```
C**** ASSEMBLE DISK MATRICES
C****
      IF (NDISK.EQ.0) GO TO 390
      DO 395 I=1,NDISK
      AMASS=DMASS(I)
      NDUM=NODIS(I)
      CALL DISKD(AMASS,VELTX,VELTY,VELTZ,ACCTZ,DCC,DKR)
      DO 400 J=1,NFREE
      JJ=ID(NDUM,J)
      IF (JJ.EQ.0) GO TO 400
      DO 405 KA=1,NFREE
      KK=ID(NDUM,KA)
      IF (KK.EQ.0) GO TO 405
      KKI=KK-JJ+NLC+1
      C(JJ,KKI)=C(JJ,KKI) + DCC(J,KA)
      K(JJ,KKI)=K(JJ,KKI) + DKR(J,KA)
  405 CONTINUE
  400 CONTINUE
      AIO=DIO(I)
      AI=DI(I)
      CALL FDIS(AMASS,AIO,AI,H,SP,ACCX,ACCY,VELTX,VELTY,VELTZ,ACCTX,ACCT
     *Y,ACCTZ,FD)
      DO 410 J=1,NFREE
      JJ=ID(NDUM,J)
      IF (JJ.EQ.0) GO TO 410
      F(JJ)=F(JJ) + FD(J)
  410 CONTINUE
  395 CONTINUE
  390 CONTINUE
C****
C**** USE NEWMARK'S ALGORITHM
C****
      DELTA=0.5
      ALFA=0.25
      A0=1.0/(ALFA*DT*DT)
      A1=DELTA/(ALFA*DT)
      A2=1.0/(ALFA*DT)
      A3=(0.5/ALFA) - 1.0
      A4=(DELTA/ALFA) - 1.0
      A5=DT*((DELTA/ALFA)-2.0)*0.5
      A6=DT*(1.0-DELTA)
      A7=DELTA*DT
      DO 420 I=1,NEQ
      B(I,1)=F(I)
      DO 420 J=1,NBAND
      JN=J+I-NLC-1
      IF (JN.LE.0.OR.JN.GT.NEQ) GO TO 425
      B(I,1)=B(I,1) + M(I,J)*(A0*XOLD(JN) + A2*VXOLD(JN) + A3*AXOLD(JN))
     *+ C(I,J)*(A1*XOLD(JN) + A4*VXOLD(JN) + A5*AXOLD(JN))
  425 CONTINUE
  420 A(I,J)=K(I,J) + A0*M(I,J) + A1*C(I,J)
```

```
      CALL LEQT1B(A,NEQ,NLC,NUC,100,B,1,100,0,XL,IER)
      DO 430 I=1,NEQ
      XNEW(I)=B(I,1)
      AXNEW(I)=A0*(XNEW(I)-XOLD(I)) - A2*VXOLD(I) - A3*AXOLD(I)
  430 VXNEW(I)=VXOLD(I) + A6*AXOLD(I) + A7*AXNEW(I)
 3000 CALL RESULT(NNOD,NFREE,ZC,NOD,ZE,XNEW,VXNEW,ID,NBEAR,NOBER,BK,BC,
     *NPINT,NELP,EYM,EPR,EAIT,EAREA)
      DO 440 I=1,NEQ
      XOLD(I)=XNEW(I)
      VXOLD(I)=VXNEW(I)
  440 AXOLD(I)=AXNEW(I)
      TOLD=TIME
      GO TO 1000
      END


      SUBROUTINE MASS(RHO,AREA,AL,AIT,AM)
      DIMENSION AM(8,8)
      DO 100 I=1,8
      DO 100 J=1,8
  100 AM(I,J)=0.0
      CM1=RHO*AREA*AL/3.0
      CM2=RHO*AREA*AL/6.0
      CI1=RHO*AIT*AL/3.0
      CI2=RHO*AIT*AL/6.0
      AM(1,1)=CM1
      AM(1,5)=CM2
      AM(2,2)=CM1
      AM(2,6)=CM2
      AM(3,3)=CI1
      AM(3,7)=CI2
      AM(4,4)=CI1
      AM(4,8)=CI2
      AM(5,5)=CM1
      AM(6,6)=CM1
      AM(7,7)=CI1
      AM(8,8)=CI1
      DO 110 I=1,8
      DO 110 J=1,I
  110 AM(I,J)=AM(J,I)
      RETURN
      END


      SUBROUTINE GYRO(RHO,AL,AIP,SP,ACG)
      DIMENSION ACG(8,8)
      DO 100 I=1,8
      DO 100 J=1,8
  100 ACG(I,J)=0.0
```

```
      C3=RHO*AIP*SP*AL/3.0
      C6=RHO*AIP*SP*AL/6.0
      ACG(3,4)=C3
      ACG(3,8)=C6
      ACG(4,7)=-C6
      ACG(7,8)=C3
      DO 110 I=1,8
      DO 110 J=1,I
  110 ACG(I,J)=-ACG(J,I)
      RETURN
      END


      SUBROUTINE CORI(RHO,AL,AREA,VELTZ,ACC)
      DIMENSION ACC(8,8)
      DO 100 I=1,8
      DO 100 J=1,8
  100 ACC(I,J)=0.0
      C3=2.0*RHO*AREA*VELTZ*AL/3.0
      C6=2.0*RHO*AREA*VELTZ*AL/6.0
      ACC(1,2)=-C3
      ACC(1,6)=-C6
      ACC(2,5)=C6
      ACC(5,6)=-C3
      DO 110 I=1,8
      DO 110 J=1,I
  110 ACC(I,J)=-ACC(J,I)
      RETURN
      END


      SUBROUTINE KCMAT(AREA,RM,YM,AIT,AL,TIMC,IKC,AKC)
      DIMENSION AKC(8,8)
      DO 100 I=1,8
      DO 100 J=1,8
  100 AKC(I,J)=0.0
      CS1=TIMC*AREA*RM/AL
      CS2=TIMC*AREA*RM/2.0
      IF (IKC.NE.0) CS3=TIMC*AREA*RM*AL/3.0
      IF (IKC.NE.0) CS6=TIMC*AREA*RM*AL/6.0
      IF (IKC.EQ.0) CS3=TIMC*AREA*RM*AL/4.0
      IF (IKC.EQ.0) CS6=TIMC*AREA*RM*AL/4.0
      CF1=YM*AIT/AL
      AKC(1,1)=CS1
      AKC(1,4)=CS2
      AKC(1,5)=-CS1
      AKC(1,8)=CS2
      AKC(2,2)=CS1
      AKC(2,3)=-CS2
```


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```
      AKC(2,6)=-CS1
      AKC(2,7)=-CS2
      AKC(3,3)=CS3 + CF1
      AKC(3,6)=CS2
      AKC(3,7)=CS6 - CF1
      AKC(4,4)=CS3 + CF1
      AKC(4,5)=-CS2
      AKC(4,8)=CS6 - CF1
      AKC(5,5)=CS1
      AKC(5,8)=-CS2
      AKC(6,6)=CS1
      AKC(6,7)=CS2
      AKC(7,7)=CS3 + CF1
      AKC(8,8)=CS3 + CF1
      DO 110 I=1,8
      DO 110 J=1,I
  110 AKC(I,J)=AKC(J,I)
      RETURN
      END


      SUBROUTINE KPMAT(P,AL,IKP,AKP)
      DIMENSION AKP(8,8)
      DO 100 I=1,8
      DO 100 J=1,8
  100 AKP(I,J)=0.0
      C2=P/2.0
      IF (IKP.NE.0) C3=P*AL/3.0
      IF (IKP.NE.0) C6=P*AL/6.0
      IF (IKP.EQ.0) C3=P*AL/4.0
      IF (IKP.EQ.0) C6=P*AL/4.0
      AKP(1,4)=-C2
      AKP(1,8)=-C2
      AKP(2,3)=C2
      AKP(2,7)=C2
      AKP(3,3)=-C3
      AKP(3,6)=-C2
      AKP(3,7)=-C6
      AKP(4,4)=-C3
      AKP(4,5)=C2
      AKP(4,8)=-C6
      AKP(5,8)=C2
      AKP(6,7)=-C2
      AKP(7,7)=-C3
      AKP(8,8)=-C3
      DO 110 I=1,8
      DO 110 J=1,I
  110 AKP(I,J)=AKP(J,I)
      RETURN
      END
```

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```
      SUBROUTINE KTMAT(T,AKT)
      DIMENSION AKT(8,8)
      DO 100 I=1,8
      DO 100 J=1,8
  100 AKT(I,J)=0.0
      C2=T/2.0
      AKT(3,4)=-C2
      AKT(3,8)=-C2
      AKT(4,3)=C2
      AKT(4,7)=C2
      AKT(7,4)=C2
      AKT(7,8)=C2
      AKT(8,3)=-C2
      AKT(8,7)=-C2
      RETURN
      END


      SUBROUTINE KRMAT(RHO,AL,AREA,VELTX,VELTY,VELTZ,ACCTX,ACCTY,ACCTZ,A
     *KR)
      DIMENSION AKR(8,8)
      DO 100 I=1,8
      DO 100 J=1,8
  100 AKR(I,J)=0.0
      C113=-RHO*AREA*(VELTY*VELTY + VELTZ*VELTZ)*AL/3.0
      C123=RHO*AREA*(VELTX*VELTY - ACCTZ)*AL/3.0
      C213=RHO*AREA*(VELTX*VELTY + ACCTZ)*AL/3.0
      C223=-RHO*AREA*(VELTZ*VELTZ + VELTX*VELTX)*AL/3.0
      C116=C113/2.0
      C126=C123/2.0
      C216=C213/2.0
      C226=C223/2.0
      AKR(1,1)=C113
      AKR(1,2)=C123
      AKR(1,5)=C116
      AKR(1,6)=C126
      AKR(2,1)=C213
      AKR(2,2)=C223
      AKR(2,5)=C216
      AKR(2,6)=C226
      AKR(5,1)=C116
      AKR(5,2)=C126
      AKR(5,5)=C113
      AKR(5,6)=C123
      AKR(6,1)=C216
      AKR(6,2)=C226
      AKR(6,5)=C213
```

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```
      AKR(6,6)=C223
      RETURN
      END


      SUBROUTINE FVEC(RHO,AL,AREA,H,S1,S2,SP,AIT,AIP,ACCX,ACCY,VELTX,VEL
     *TY,VELTZ,ACCTX,ACCTY,ACCTZ,FV)
      DIMENSION FV(8)
      C2=RHO*AL*AREA/2.0
      C6=RHO*AL*AREA/6.0
      FV(1)=-C2*ACCX - C2*H*(VELTX*VELTY - ACCTZ) - C6*(VELTZ*VELTX + AC
     *CTY)*(2.0*S1+S2)
      FV(2)=-C2*ACCY + C2*H*(VELTZ*VELTZ + VELTX*VELTX) - C6*(VELTY*VELT
     *Z - ACCTX)*(2.0*S1+S2)
      FV(3)=-RHO*AL*(AIT*ACCTX + AIP*SP*VELTY)/2.0
      FV(4)=-RHO*AL*(AIT*ACCTY - AIP*SP*VELTX)/2.0
      FV(5)=-C2*ACCX - C2*H*(VELTX*VELTY - ACCTZ) - C6*(VELTZ*VELTX + AC
     *CTY)*(S1+2.0*S2)
      FV(6)=-C2*ACCY + C2*H*(VELTZ*VELTZ + VELTX*VELTX) - C6*(VELTY*VELT
     *Z - ACCTX)*(S1+2.0*S2)
      FV(7)=-RHO*AL*(AIT*ACCTX + AIP*SP*VELTY)/2.0
      FV(8)=-RHO*AL*(AIT*ACCTY - AIP*SP*VELTX)/2.0
      RETURN
      END


      SUBROUTINE DISKI(AMASS,AIO,AI,SP,DM,DCG)
      DIMENSION DM(4,4),DCG(4,4)
      DO 100 I=1,4
      DO 100 J=1,4
      DM(I,J)=0.0
  100 DCG(I,J)=0.0
      DM(1,1)=AMASS
      DM(2,2)=AMASS
      DM(3,3)=AIO
      DM(4,4)=AIO
      DCG(3,4)=AI*SP
      DCG(4,3)=-AI*SP
      RETURN
      END


      SUBROUTINE DISKD(AMASS,VELTX,VELTY,VELTZ,ACCTZ,DCC,DKR)
      DIMENSION DCC(4,4),DKR(4,4)
      DO 100 I=1,4
      DO 100 J=1,4
      DCC(I,J)=0.0
  100 DKR(I,J)=0.0
```

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```
      DCC(1,2)=-2.0*AMASS*VELTZ
      DCC(2,1)=2.0*AMASS*VELTZ
      DKR(1,1)=-AMASS*(VELTY*VELTY + VELTZ*VELTZ)
      DKR(1,2)=AMASS*(VELTX*VELTY - ACCTZ)
      DKR(2,1)=AMASS*(VELTX*VELTY + ACCTZ)
      DKR(2,2)=-AMASS*(VELTZ*VELTZ + VELTX*VELTX)
      RETURN
      END


      SUBROUTINE FDIS(AMASS,AIO,AI,H,SP,ACCX,ACCY,VELTX,VELTY,VELTZ,ACCT
     *X,ACCTY,ACCTZ,FD)
      DIMENSION FD(4)
      FD(1)=-AMASS*ACCX - AMASS*H*(VELTX*VELTY - ACCTZ)
      FD(2)=-AMASS*ACCY + AMASS*H*(VELTZ*VELTZ + VELTX*VELTX)
      FD(3)=-AIO*ACCTX - AI*SP*VELTY
      FD(4)=-AIO*ACCTY + AI*SP*VELTX
      RETURN
      END


      SUBROUTINE RESULT(NNOD,NFREE,ZC,NOD,ZE,XNEW,VXNEW,ID,NBEAR,NOBER,
     *BK,BC,NPINT,NELP,EYM,EPR,EAIT,EAREA)
      DIMENSION XNEW(100),VXNEW(100),ID(25,4),NOBER(20),BK(20,4,4)
      DIMENSION BC(20,4,4),NELP(25),EYM(25),EPR(25),EAIT(25),EAREA(25)
      DIMENSION DIS(4),VEL(4),ZC(25),NOD(25,2),ZE(2),DUM(4,2)
      WRITE(6,100)
  100 FORMAT(//5X,'**** NODAL DISPLACEMENTS ****'//7X,'NODE #',6X,'UX',
     *14X,'UY',12X,'THETAX',10X,'THETAY'/)
      DO 200 I=1,NNOD
      DO 210 J=1,NFREE
      II=ID(I,J)
      IF (II.EQ.0) GO TO 220
      DIS(J)=XNEW(II)
      GO TO 210
  220 DIS(J)=0.0
  210 CONTINUE
      WRITE(6,110) I,(DIS(J),J=1,NFREE)
  110 FORMAT(5X,I5,4(5X,E11.4))
  200 CONTINUE
      IF (NBEAR.EQ.0) GO TO 250
      WRITE(6,120)
  120 FORMAT(/5X,'**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEAR
     *INGS ****'/)
      DO 230 I=1,NBEAR
      IN=NOBER(I)
      DO 240 J=1,NFREE
      II=ID(IN,J)
      IF (II.EQ.0) GO TO 235
```

```
      DIS(J)=XNEW(II)
      VEL(J)=VXNEW(II)
      GO TO 240
  235 DIS(J)=0.0
      VEL(J)=0.0
  240 CONTINUE
      FX=BK(I,1,1)*DIS(1) + BK(I,1,2)*DIS(2) + BC(I,1,1)*VEL(1)
     *+ BC(I,1,2)*VEL(2)
      FY=BK(I,2,1)*DIS(1) + BK(I,2,2)*DIS(2) + BC(I,2,1)*VEL(1)
     *+ BC(I,2,2)*VEL(2)
      WRITE(6,130) I,IN,DIS(1),DIS(2),FX,FY
  130 FORMAT(5X,'BEARING #',I5,5X,'AT NODE #',I5,5X,'UX=',E11.4,5X,'UY='
     *,E11.4,5X,'FX=',E11.4,5X,'FY=',E11.4)
  230 CONTINUE
  250 CONTINUE
      IF (NPINT.EQ.0) GO TO 300
      WRITE(6,140)
  140 FORMAT(/5X,'**** SHEAR FORCES AND BENDING MOMENTS ****'/)
      DO 310 I=1,NPINT
      IN=NELP(I)
      YM=EYM(IN)
      PR=EPR(IN)
      AREA=EAREA(IN)
      AIT=EAIT(IN)
      TIMC=6.0*(1.0+PR)/(7.0+6.0*PR)
      RM=0.5*YM/(1.0+PR)
      DO 320 J=1,2
      NUM=NOD(IN,J)
      ZE(J)=ZC(NUM)
      DO 320 IA=1,NFREE
      II=ID(NUM,IA)
      IF (II.EQ.0) DUM(IA,J)=0.0
      IF (II.NE.0) DUM(IA,J)=XNEW(II)
  320 CONTINUE
      AL=ZE(2)-ZE(1)
      QX=TIMC*AREA*RM*(((DUM(1,2)-DUM(1,1))/AL)-0.5*(DUM(4,1)+DUM(4,2)))
      QY=TIMC*AREA*RM*(((DUM(2,2)-DUM(2,1))/AL)+0.5*(DUM(3,1)+DUM(3,2)))
      AMX=YM*AIT*(DUM(3,2)-DUM(3,1))/AL
      AMY=YM*AIT*(DUM(4,2)-DUM(4,1))/AL
      WRITE(6,150) IN,QX,QY,AMX,AMY
  150 FORMAT(5X,'AT THE MIDPOINT OF ELEMENT #',I5,5X,'QX=',E11.4,5X,'QY=
     *',E11.4,5X,'MX=',E11.4,5X,'MY=',E11.4)
  310 CONTINUE
  300 CONTINUE
      RETURN
      END
```

2.5 SAMPLE INPUT DATA

```
   19   18     2     4    76     7     7
+0.3000E+04                          +0.1000E+01     2     1     1
+0.0000E+00
+0.5000E+00
+0.1000E+01
+0.1500E+01
+0.2000E+01
+0.2500E+01
+0.3000E+01
+0.3500E+01
+0.4000E+01
+0.4500E+01
+0.5000E+01
+0.5500E+01
+0.6000E+01
+0.6500E+01
+0.7000E+01
+0.7500E+01
+0.8000E+01
+0.8500E+01
+0.8750E+01
    1    2+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    2    3+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    3    4+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    4    5+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    5    6+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    6    7+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    7    8+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    8    9+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
    9   10+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   10   11+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   11   12+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   12   13+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   13   14+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   14   15+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   15   16+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   16   17+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   17   18+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   18   19+0.2000E+12+0.3000E+00+0.7800E+04+0.2827E+00+0.6362E-02
   19+0.5000E+04+0.1267E+04+0.2500E+04
    1+0.6760E+09+0.2160E+08-0.1490E+10+0.2270E+10
+0.3100E+07-0.5000E+07-0.5000E+07+0.1370E+08
   18+0.5890E+09+0.5100E+08-0.1290E+10+0.1870E+10
+0.2800E+07-0.4100E+07-0.4100E+07+0.1170E+08
    9
                                             -0.0466     0.0297     0.1183

   0.02      -0.014      0.024      0.003
   0.04      -0.108     -0.230      0.019
   0.06      -0.101     -0.275      0.068
   0.08      -0.088     -0.397      0.029
   0.10      -0.095     -0.390      0.029
   0.12      -0.120     -0.060      0.054
```

2.6 SAMPLE RESULTS

```
NUMBER OF NODES ******************** =   19
NUMBER OF ELEMENTS ***************** =   18
NUMBER OF NODES PER ELEMENT ********* =    2
NUMBER OF DEGREES OF FREEDOM PER NODE =    4
NUMBER OF EQUATIONS **************** =   76
NUMBER OF LOWER CODIAGONALS ********* =    7
NUMBER OF UPPER CODIAGONALS ********* =    7
INDEX FOR KC MATRIX **************** =    0
INDEX FOR KP MATRIX **************** =    0
SPIN SPEED IN RPM ****************** = 0.3000E+04
AXIAL FORCE ************************ = 0.0
AXIAL TORQUE *********************** = 0.0
HEIGHT OF THE ROTOR FROM BASE ******* = 0.1000E+01
NUMBER OF BEARINGS ***************** =    2
NUMBER OF DISKS (AND FLYWHEELS) ***** =    1
NUMBER OF STRESS POINTS ************* =    1
```

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NODE #	Z	ID MATRIX			
1	0.0	0	0	0	0
2	0.5000E+00	0	0	0	0
3	0.1000E+01	0	0	0	0
4	0.1500E+01	0	0	0	0
5	0.2000E+01	0	0	0	0
6	0.2500E+01	0	0	0	0
7	0.3000E+01	0	0	0	0
8	0.3500E+01	0	0	0	0
9	0.4000E+01	0	0	0	0
10	0.4500E+01	0	0	0	0
11	0.5000E+01	0	0	0	0
12	0.5500E+01	0	0	0	0
13	0.6000E+01	0	0	0	0
14	0.6500E+01	0	0	0	0
15	0.7000E+01	0	0	0	0
16	0.7500E+01	0	0	0	0
17	0.8000E+01	0	0	0	0
18	0.8500E+01	0	0	0	0
19	0.8750E+01	0	0	0	0

NODE #	CONNECTIVITY MATRIX			
1	1	2	3	4
2	5	6	7	8
3	9	10	11	12
4	13	14	15	16
5	17	18	19	20
6	21	22	23	24
7	25	26	27	28
8	29	30	31	32
9	33	34	35	36
10	37	38	39	40
11	41	42	43	44
12	45	46	47	48
13	49	50	51	52
14	53	54	55	56
15	57	58	59	60
16	61	62	63	64
17	65	66	67	68
18	69	70	71	72
19	73	74	75	76

COMPUTED NUMBER OF EQUATIONS = 76

ELEMENT #	NODE 1	NODE 2	YOUNG*S MODULUS	POISSON*S RATIO	DENSITY	AREA OF CROSS-SECTION	TRANSVERSE SECOND MOMENT OF AREA	POLAR SECOND MOMENT OF AREA
1	1	2	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
2	2	3	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
3	3	4	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
4	4	5	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
5	5	6	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
6	6	7	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
7	7	8	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
8	8	9	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
9	9	10	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
10	10	11	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
11	11	12	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
12	12	13	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
13	13	14	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
14	14	15	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
15	15	16	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
16	16	17	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
17	17	18	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01
18	18	19	0.2000E+12	0.3000E+00	0.7800E+04	0.2827E+00	0.6362E-02	0.1272E-01

DISK #= 1 AT NODE #= 19 MASS= 0.5000E+04 IO= 0.1267E+04 I= 0.2500E+04

BEARING #= 1 AT NODE #= 1
BK(1,1)= 0.6760E+09 BK(1,2)= 0.2160E+08 BK(2,1)=-0.1490E+10 BK(2,2)= 0.2270E+10
BC(1,1)= 0.3100E+07 BC(1,2)=-0.5000E+07 BC(2,1)=-0.5000E+07 BC(2,2)= 0.1370E+08

BEARING #= 2 AT NODE #= 18
BK(1,1)= 0.5890E+09 BK(1,2)= 0.5100E+08 BK(2,1)=-0.1290E+10 BK(2,2)= 0.1870E+10
BC(1,1)= 0.2800E+07 BC(1,2)=-0.4100E+07 BC(2,1)=-0.4100E+07 BC(2,2)= 0.1170E+08

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```
**** INITIAL CONDITIONS OF THE BASE AND ROTOR ****

TIME=   0.0
ACCX= 0.0              ACCY= 0.0              ACCZ= 0.0              ACCTX= 0.0              ACCTY= 0.0              ACCTZ= 0.0
VELX=-0.4660E-01       VELY= 0.2970E-01       VELZ= 0.1183E+00       VELTX= 0.0              VELTY= 0.0              VELTZ= 0.0

**** NODAL DISPLACEMENTS ****

  NODE #      UX            UY            THETAX         THETAY

     1       0.0           0.0           0.0            0.0
     2       0.0           0.0           0.0            0.0
     3       0.0           0.0           0.0            0.0
     4       0.0           0.0           0.0            0.0
     5       0.0           0.0           0.0            0.0
     6       0.0           0.0           0.0            0.0
     7       0.0           0.0           0.0            0.0
     8       0.0           0.0           0.0            0.0
     9       0.0           0.0           0.0            0.0
    10       0.0           0.0           0.0            0.0
    11       0.0           0.0           0.0            0.0
    12       0.0           0.0           0.0            0.0
    13       0.0           0.0           0.0            0.0
    14       0.0           0.0           0.0            0.0
    15       0.0           0.0           0.0            0.0
    16       0.0           0.0           0.0            0.0
    17       0.0           0.0           0.0            0.0
    18       0.0           0.0           0.0            0.0
    19       0.0           0.0           0.0            0.0

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING #   1   AT NODE #   1   UX= 0.0        UY= 0.0        FX= 0.0        FY= 0.0
BEARING #   2   AT NODE #  18   UX= 0.0        UY= 0.0        FX= 0.0        FY= 0.0

**** SHEAR FORCES AND BENDING MOMENTS ****

AT THE MIDPOINT OF ELEMENT #    9   QX= 0.0        QY= 0.0        MX= 0.0        MY= 0.0
```

TIME= 0.02000

ACCX=-0.1400E-01 ACCY= 0.2400E-01 ACCZ= 0.3000E-02 ACCTX= 0.0 ACCTY= 0.0 ACCTZ= 0.0

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.7735E-07	0.4696E-08	0.5244E-06	0.3192E-06
2	0.2370E-06	-0.2577E-06	0.5123E-06	0.3125E-06
3	0.3900E-06	-0.5080E-06	0.4785E-06	0.2936E-06
4	0.5307E-06	-0.7365E-06	0.4272E-06	0.2649E-06
5	0.6549E-06	-0.9354E-06	0.3623E-06	0.2285E-06
6	0.7593E-06	-0.1099E-05	0.2873E-06	0.1862E-06
7	0.8411E-06	-0.1223E-05	0.2051E-06	0.1395E-06
8	0.8987E-06	-0.1304E-05	0.1183E-06	0.8972E-07
9	0.9309E-06	-0.1341E-05	0.2938E-07	0.3817E-07
10	0.9369E-06	-0.1334E-05	-0.5931E-07	-0.1408E-07
11	0.9168E-06	-0.1282E-05	-0.1455E-06	-0.6594E-07
12	0.8710E-06	-0.1188E-05	-0.2269E-06	-0.1163E-06
13	0.8004E-06	-0.1055E-05	-0.3010E-06	-0.1641E-06
14	0.7068E-06	-0.8868E-06	-0.3649E-06	-0.2080E-06
15	0.5924E-06	-0.6896E-06	-0.4157E-06	-0.2466E-06
16	0.4602E-06	-0.4708E-06	-0.4499E-06	-0.2782E-06
17	0.3142E-06	-0.2394E-06	-0.4635E-06	-0.3008E-06
18	0.1593E-06	-0.6878E-08	-0.4521E-06	-0.3124E-06
19	0.8153E-07	0.1033E-06	-0.4428E-06	-0.3167E-06

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING # 1 AT NODE # 1 UX= 0.7735E-07 UY= 0.4696E-08 FX= 0.7402E+02 FY=-0.1368E+03
BEARING # 2 AT NODE # 18 UX= 0.1593E-06 UY=-0.6878E-08 FX= 0.1409E+03 FY=-0.2917E+03

**** SHEAR FORCES AND BENDING MOMENTS ****

AT THE MIDPOINT OF ELEMENT # 9 QX= 0.8219E+00 QY= 0.6779E+01 MX=-0.2257E+03 MY=-0.1330E+03

TIME= 0.04000

ACCX=-0.1080E+00 ACCY=-0.2300E+00 ACCZ= 0.1900E-01 ACCTX= 0.0 ACCTY= 0.0 ACCTZ= 0.0

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.1153E-05	0.9065E-06	-0.4069E-05	0.2678E-05
2	0.2493E-05	0.2942E-05	-0.3975E-05	0.2620E-05
3	0.3775E-05	0.4884E-05	-0.3713E-05	0.2457E-05
4	0.4952E-05	0.6657E-05	-0.3323E-05	0.2207E-05
5	0.5984E-05	0.8208E-05	-0.2835E-05	0.1888E-05
6	0.6842E-05	0.9493E-05	-0.2276E-05	0.1517E-05
7	0.7502E-05	0.1048E-04	-0.1670E-05	0.1109E-05
8	0.7951E-05	0.1116E-04	-0.1034E-05	0.6777E-06
9	0.8180E-05	0.1152E-04	-0.3834E-06	0.2375E-06
10	0.8189E-05	0.1155E-04	0.2669E-06	-0.1985E-06
11	0.7982E-05	0.1125E-04	0.9036E-06	-0.6168E-06
12	0.7571E-05	0.1064E-04	0.1513E-05	-0.1004E-05
13	0.6977E-05	0.9738E-05	0.2078E-05	-0.1346E-05
14	0.6224E-05	0.8564E-05	0.2582E-05	-0.1629E-05
15	0.5346E-05	0.7155E-05	0.3003E-05	-0.1837E-05
16	0.4384E-05	0.5560E-05	0.3313E-05	-0.1955E-05
17	0.3389E-05	0.3840E-05	0.3484E-05	-0.1964E-05
18	0.2418E-05	0.2074E-05	0.3476E-05	-0.1846E-05
19	0.1973E-05	0.1223E-05	0.3452E-05	-0.1766E-05

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING # 1 AT NODE # 1 UX= 0.1153E-05 UY= 0.9065E-06 FX= 0.6595E+03 FY= 0.1071E+04
BEARING # 2 AT NODE # 18 UX= 0.2418E-05 UY= 0.2074E-05 FX= 0.1262E+04 FY= 0.2340E+04

**** SHEAR FORCES AND BENDING MOMENTS ****

AT THE MIDPOINT OF ELEMENT # 9 QX=-0.5545E+02 QY=-0.3439E+02 MX= 0.1655E+04 MY=-0.1109E+04

TIME= 0.06000

ACCX=-0.1010E+00 ACCY=-0.2750E+00 ACCZ= 0.6800E-01 ACCTX= 0.0 ACCTY= 0.0 ACCTZ= 0.0

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.2338E-05	0.2561E-05	-0.1619E-04	0.6178E-05
2	0.5430E-05	0.1066E-04	-0.1589E-04	0.6057E-05
3	0.8400E-05	0.1846E-04	-0.1500E-04	0.5709E-05
4	0.1114E-04	0.2567E-04	-0.1361E-04	0.5162E-05
5	0.1357E-04	0.3207E-04	-0.1178E-04	0.4446E-05
6	0.1559E-04	0.3745E-04	-0.9584E-05	0.3594E-05
7	0.1716E-04	0.4166E-04	-0.7110E-05	0.2637E-05
8	0.1823E-04	0.4456E-04	-0.4436E-05	0.1611E-05
9	0.1877E-04	0.4609E-04	-0.1643E-05	0.5493E-06
10	0.1878E-04	0.4621E-04	0.1185E-05	-0.5119E-06
11	0.1826E-04	0.4491E-04	0.3967E-05	-0.1538E-05
12	0.1724E-04	0.4224E-04	0.6620E-05	-0.2493E-05
13	0.1577E-04	0.3829E-04	0.9061E-05	-0.3344E-05
14	0.1389E-04	0.3317E-04	0.1121E-04	-0.4056E-05
15	0.1171E-04	0.2707E-04	0.1299E-04	-0.4596E-05
16	0.9294E-05	0.2018E-04	0.1432E-04	-0.4932E-05
17	0.6770E-05	0.1274E-04	0.1514E-04	-0.5030E-05
18	0.4259E-05	0.5031E-05	0.1536E-04	-0.4861E-05
19	0.3071E-05	0.1212E-05	0.1536E-04	-0.4725E-05

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING # 1 AT NODE # 1 UX= 0.2338E-05 UY= 0.2561E-05 FX= 0.1316E+04 FY= 0.3271E+04
BEARING # 2 AT NODE # 18 UX= 0.4259E-05 UY= 0.5031E-05 FX= 0.2336E+04 FY= 0.5038E+04

**** SHEAR FORCES AND BENDING MOMENTS ****

AT THE MIDPOINT OF ELEMENT # 9 QX=-0.9121E+02 QY=-0.2689E+02 MX= 0.7198E+04 MY=-0.2701E+04

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TIME= 0.08000

ACCX=-0.8800E-01 ACCY=-0.3970E+00 ACCZ= 0.2900E-01 ACCTX= 0.0 ACCTY= 0.0 ACCTZ= 0.0

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.1672E-05	0.3044E-05	-0.2199E-04	0.5471E-05
2	0.4409E-05	0.1404E-04	-0.2157E-04	0.5370E-05
3	0.7044E-05	0.2462E-04	-0.2036E-04	0.5077E-05
4	0.9488E-05	0.3442E-04	-0.1846E-04	0.4614E-05
5	0.1166E-04	0.4310E-04	-0.1596E-04	0.4001E-05
6	0.1349E-04	0.5039E-04	-0.1297E-04	0.3264E-05
7	0.1493E-04	0.5607E-04	-0.9582E-05	0.2427E-05
8	0.1592E-04	0.5997E-04	-0.5917E-05	0.1517E-05
9	0.1644E-04	0.6199E-04	-0.2084E-05	0.5618E-06
10	0.1648E-04	0.6206E-04	0.1800E-05	-0.4094E-06
11	0.1603E-04	0.6019E-04	0.5619E-05	-0.1368E-05
12	0.1511E-04	0.5644E-04	0.9256E-05	-0.2286E-05
13	0.1375E-04	0.5093E-04	0.1260E-04	-0.3134E-05
14	0.1198E-04	0.4384E-04	0.1553E-04	-0.3888E-05
15	0.9857E-05	0.3539E-04	0.1794E-04	-0.4521E-05
16	0.7454E-05	0.2589E-04	0.1972E-04	-0.5010E-05
17	0.4845E-05	0.1566E-04	0.2079E-04	-0.5335E-05
18	0.2117E-05	0.5090E-05	0.2104E-04	-0.5477E-05
19	0.7478E-06	-0.1391E-06	0.2099E-04	-0.5508E-05

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING # 1 AT NODE # 1 UX= 0.1672E-05 UY= 0.3044E-05 FX= 0.1068E+04 FY= 0.4472E+04
BEARING # 2 AT NODE # 18 UX= 0.2117E-05 UY= 0.5090E-05 FX= 0.1312E+04 FY= 0.6610E+04

**** SHEAR FORCES AND BENDING MOMENTS ****

AT THE MIDPOINT OF ELEMENT # 9 QX= 0.8895E+01 QY=-0.3406E+02 MX= 0.9885E+04 MY=-0.2471E+04

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TIME= 0.10000

ACCX=-0.9500E-01 ACCY=-0.3900E+00 ACCZ= 0.2900E-01 ACCTX= 0.0 ACCTY= 0.0 ACCTZ= 0.0

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.3552E-06	0.2063E-05	-0.1544E-04	0.2164E-05
2	0.1437E-05	0.9787E-05	-0.1513E-04	0.2118E-05
3	0.2474E-05	0.1720E-04	-0.1423E-04	0.1992E-05
4	0.3430E-05	0.2402E-04	-0.1283E-04	0.1802E-05
5	0.4276E-05	0.3003E-04	-0.1102E-04	0.1561E-05
6	0.4991E-05	0.3505E-04	-0.8882E-05	0.1283E-05
7	0.5559E-05	0.3892E-04	-0.6495E-05	0.9760E-06
8	0.5967E-05	0.4155E-04	-0.3940E-05	0.6481E-06
9	0.6207E-05	0.4286E-04	-0.1296E-05	0.3057E-06
10	0.6273E-05	0.4284E-04	0.1361E-05	-0.4556E-07
11	0.6162E-05	0.4150E-04	0.3956E-05	-0.4005E-06
12	0.5873E-05	0.3888E-04	0.6414E-05	-0.7533E-06
13	0.5409E-05	0.3508E-04	0.8660E-05	-0.1097E-05
14	0.4776E-05	0.3022E-04	0.1062E-04	-0.1425E-05
15	0.3984E-05	0.2446E-04	0.1221E-04	-0.1725E-05
16	0.3050E-05	0.1800E-04	0.1336E-04	-0.1987E-05
17	0.1997E-05	0.1109E-04	0.1399E-04	-0.2195E-05
18	0.8548E-06	0.4002E-05	0.1401E-04	-0.2333E-05
19	0.2706E-06	0.5384E-06	0.1392E-04	-0.2395E-05

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING # 1 AT NODE # 1 UX= 0.3552E-06 UY= 0.2063E-05 FX= 0.4953E+03 FY= 0.3415E+04
BEARING # 2 AT NODE # 18 UX= 0.8548E-06 UY= 0.4002E-05 FX= 0.9949E+03 FY= 0.5802E+04

**** SHEAR FORCES AND BENDING MOMENTS ****

AT THE MIDPOINT OF ELEMENT # 9 QX= 0.3206E+02 QY=-0.1258E+03 MX= 0.6762E+04 MY=-0.8940E+03

TIME= 0.12000

ACCX=-0.1200E+00 ACCY=-0.6000E-01 ACCZ= 0.5400E-01 ACCTX= 0.0 ACCTY= 0.0 ACCTZ= 0.0

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.6177E-06	0.1071E-05	-0.4398E-05	0.3868E-05
2	0.2552E-05	0.3271E-05	-0.4325E-05	0.3793E-05
3	0.4412E-05	0.5398E-05	-0.4113E-05	0.3578E-05
4	0.6131E-05	0.7385E-05	-0.3772E-05	0.3244E-05
5	0.7657E-05	0.9171E-05	-0.3317E-05	0.2812E-05
6	0.8944E-05	0.1070E-04	-0.2761E-05	0.2300E-05
7	0.9957E-05	0.1193E-04	-0.2121E-05	0.1727E-05
8	0.1067E-04	0.1282E-04	-0.1416E-05	0.1110E-05
9	0.1107E-04	0.1335E-04	-0.6651E-06	0.4652E-06
10	0.1114E-04	0.1349E-04	0.1088E-06	-0.1901E-06
11	0.1088E-04	0.1324E-04	0.8834E-06	-0.8398E-06
12	0.1030E-04	0.1261E-04	0.1636E-05	-0.1468E-05
13	0.9410E-05	0.1160E-04	0.2343E-05	-0.2057E-05
14	0.8239E-05	0.1026E-04	0.2983E-05	-0.2591E-05
15	0.6818E-05	0.8621E-05	0.3535E-05	-0.3052E-05
16	0.5186E-05	0.6727E-05	0.3978E-05	-0.3421E-05
17	0.3396E-05	0.4641E-05	0.4296E-05	-0.3676E-05
18	0.1509E-05	0.2430E-05	0.4472E-05	-0.3795E-05
19	0.5643E-06	0.1308E-05	0.4528E-05	-0.3835E-05

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING # 1 AT NODE # 1 UX= 0.6177E-06 UY= 0.1071E-05 FX= 0.8072E+03 FY= 0.7606E+03
BEARING # 2 AT NODE # 18 UX= 0.1509E-05 UY= 0.2430E-05 FX= 0.1553E+04 FY= 0.1068E+04

**** SHEAR FORCES AND BENDING MOMENTS ****

AT THE MIDPOINT OF ELEMENT # 9 QX= 0.1342E+02 QY= 0.6215E+02 MX= 0.1969E+04 MY=-0.1668E+04

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3. AXIST USER'S MANUAL

3.1 PURPOSE

AXIST is a computer program written in Fortran to carry out the seismic analysis of an elastic rotor in the time domain. It is the third of a series of computer program packages that are developed at the School of Mechanical and Aerospace Engineering, Oklahoma State University under the sponsorship of the National Science Foundation.

This user's manual describes the way in which the data is supplied to the program. It also includes a listing of the program and a sample output.

3.2 BACKGROUND THEORY

AXIST is based on the 3-D elasticity model developed in Part I, Chapter 4 of this report. AXIST uses eight-noded isoparametric, solid of revolution elements, such as the one shown in Figure 4.4.

AXIST is a self-contained program. An external subroutine used is a commonly available IMSL routine LEQT1B to solve a set of linear, banded simultaneous equations lacking symmetry.

The user is also required to supply a subroutine called INSTR which gives the initial stresses in the rotating system at any (r, z) location.

3.3 INPUT DATA

Card #	Data and Description
1	NNOD, NELEM, NNOEL, NFREE, NEQ, NLC, NUC, NGAS1, NGAS2 (9I5)
	NNOD - Number of nodes in the model NELEM - Number of elements in the model

NNOEL - Number of nodes per element = 4 or 8
NFREE - Number of degrees of freedom per node = 4 in our case
NEQ - Number of final set of equations
NLC - Number of lower codiagonals (excluding diagonal)
NUC - Number of upper codiagonals (excluding diagonal)
NGAS1 - Number of Gaussian points in the ξ_1 direction
NGAS2 - Number of Gaussian points in the ξ_2 direction

2 RPM, P, T, H, NBEAR, NPINT (4E11.4, 2I5)

RPM - Spin speed of the rotor in revolutions per minute
P - Axial tension on the rotor, in N.
T - Axial torque on the rotor in the +z direction, in N-m.
H - Height of the rotor axis from the base, in m.
NBEAR - Number of bearings in the system.
NPINT - Number of points at which internal stresses are to be evaluated.

3 RC(I), ZC(I), ID(I,1), ID(I,2), ID(I,3), ID(I,4) (2E11.4, 4I5)

RC(I) - r coordinate of the i^{th} node, in m.
ZC(I) - z coordinate of the i^{th} node, in m.
ID(I,J) - Index for the j^{th} degree of freedom at the i^{th} node.

Note: This card must be repeated for each of the NNOD nodes.

J=1 corresponds to $(U_x)_i$
J=2 corresponds to $(U_y)_i$
J=3 corresponds to $(\Theta_x)_i$
J=4 corresponds to $(\Theta_y)_i$

ID(I,J) $\neq$ 0 to delete the j^{th} degree of freedom at i^{th} node.

ID(I,J) = 0 to keep the jth degree of freedom at i^{th} node. (Leave it blank)

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4 (NOD(LK,J), J=1, NNOEL), EYM(LK), EPR(LK), ERHO(LK). (8I3, 3E11.4)

NOD(LK,J) - jth node of the (LK)th element
EYM(LK) - Young's modulus for the (LK)th element, N/m^2
EPR(LK) - Poisson's ratio for the (LK)th element.
ERHO(LK) - Mass density of the (LK)th element, in kg/m^3

Note: This card must be repeated for each of the NELEM elements.

5 NOBER(I), BK(I,1,1), BK(I,1,2), BK(I,2,1), BK(I,2,2) (I5,4E11.4)

NOBER(I) - Node number at which the i^{th} bearing is located.
BK(I,J,K) - The (j,k)th coefficient in the stiffness matrix for the lubricants in i^{th} bearing.

6 BC(I,J,K) - The (j,k)th coefficient in the damping matrix for the lubricants in ith bearing.

Note: The 5th and 6th cards must be repeated for each of the NBEAR bearings. If NBEAR = 0, skip these cards.

7 NELP(I), XI1(I), XI2(I) (I5,2F10.5)

NELP(I) - Element number in which the i^{th} internal stress point is located.
XI1(I) - Gaussian coordinate along ξ_1 direction for the i^{th} stress point.
XI2(I) - Gaussian coordinate along ξ_2 direction for the i^{th} stress point.

8 TIME, ACCX, ACCY, ACCZ, VELX, VELY, VELZ (7F10.5)

These are the initial conditions for the base translation. Set TIME= 0.0. ACCX, ACCY, ACCZ are the initial accelerations of point b in the x_b, y_b and z_b directions, respectively, m/s^2. VELX, VELY, VELZ are the initial velocitiees of point b in the x_b, y_b, and z_b directions, respectively, in m/s.

9 ACCTX, ACCTY, ACCTZ, VELTX, VELTY, VELTZ (6F10.5)

These are the initial conditions for the base rotation.
ACCTX, ACCTY, ACCTZ are the initial angular accelerations of the base about the x_b, y_b and z_b axes, respectiely, in rad/s^2.
VELTX, VELTY, VELTX are the initial angular velocities of the base about the x_b, y_b and z_b axes, respectively, in rad/s.

10 TIME, AX, AY, AZ, TX, TY, TZ (7F10.5)

TIME	-	Time at which the acceleration data is specified, in seconds.
AX, AY, AZ		are the linear accelerations of the base reference point b in the x_b, y_b, and z_b directions, respectiely, in m/s^2
TX, TY, TZ		are the angular accelerations of the base about the x_b, y_b and z_b axes, respectively, in rad/s^2.

Note: Card #10 must be repeated for all the time values at which the base acceleration data is given. The program is terminated by supplying a blank card in this place. The value of TIME must be supplied in the increasing sequence and the program stops whenever it reads the vaue of TIME as zero.

3.4 LISTING OF AXIST

```
C****
C**** AXIST - PROGRAM TO COMPUTE THE SEISMIC RESPONSE OF A
C**** ROTATING AXISYMMETRIC SYSTEM IN THE TIME DOMAIN.
C**** WRITTEN BY DR.V.SRINIVASAN, MARCH 1983.
C****
      DIMENSION RC(60),ZC(60),NOD(25,8),ID(60,4),RE(8),ZE(8)
      DIMENSION CI(75,63),C(75,63),A(75,63),B(75,1),F(75),XL(2400)
      DIMENSION EKC(32,32),EKG(32,32),EM(32,32),ECG(32,32)
      DIMENSION ECC(32,32),EKR(32,32),EF(32)
      DIMENSION BK(20,4,4),BC(20,4,4),NOBER(20)
      DIMENSION XOLD(75),VXOLD(75),AXOLD(75),XNEW(75)
      DIMENSION VXNEW(75),AXNEW(75),EYM(25),EPR(25),ERHO(25),NELP(50)
      DIMENSION XI1(50),XI2(50)
      REAL M(75,63),KI(75,63),K(75,63)
      PI=4.0*ATAN(1.0)
C****
C**** READ AND PRINT CONTROL DATA
C****
      READ(5,100) NNOD,NELEM,NNOEL,NFREE,NEQ,NLC,NUC,NGAS1,NGAS2
  100 FORMAT(9I5)
      READ(5,105) RPM,P,T,H,NBEAR,NPINT
  105 FORMAT(4E11.4,2I5)
      WRITE(6,110) NNOD,NELEM,NNOEL,NFREE
  110 FORMAT(//
     *5X,'NUMBER OF NODES ********************* =',I5//
     *5X,'NUMBER OF ELEMENTS ****************** =',I5//
     *5X,'NUMBER OF NODES PER ELEMENT ********* =',I5//
     *5X,'NUMBER OF DEGREES OF FREEDOM PER NODE =',I5)
      WRITE(6,111) NEQ,NLC,NUC,NGAS1
  111 FORMAT(/
     *5X,'NUMBER OF EQUATIONS ***************** =',I5//
     *5X,'NUMBER OF LOWER CODIAGONALS ********* =',I5//
     *5X,'NUMBER OF UPPER CODIAGONALS ********* =',I5//
     *5X,'NUMBER OF GAUSSIAN POINTS ALONG XI1  =',I5)
      WRITE(6,112) NGAS2,RPM,P,T
  112 FORMAT(/
     *5X,'NUMBER OF GAUSSIAN POINTS ALONG XI2  =',I5//
     *5X,'SPIN SPEED IN RPM ******************* =',E11.4//
     *5X,'AXIAL FORCE ************************* =',E11.4//
     *5X,'AXIAL TORQUE ************************ =',E11.4)
      WRITE(6,113) H,NBEAR,NPINT
  113 FORMAT(/
     *5X,'HEIGHT OF THE ROTOR FROM BASE ******* =',E11.4//
     *5X,'NUMBER OF BEARINGS ****************** =',I5//
     *5X,'NUMBER OF STRESS POINTS ************* =',I5)
      SP=2.0*RPM*PI/60.0
C****
C**** READ AND PRINT NODAL DATA
C****
      WRITE(6,121)
  121 FORMAT(//7X,'NODE #',7X,'R',16X,'Z',25X,'ID MATRIX'/)
```

```
      DO 10 I=1,NNOD
      READ(5,120) RC(I),ZC(I),ID(I,1),ID(I,2),ID(I,3),ID(I,4)
   10 WRITE(6,125) I,RC(I),ZC(I),ID(I,1),ID(I,2),ID(I,3),ID(I,4)
  120 FORMAT(2E11.4,4I5)
  125 FORMAT(5X,I5,2(5X,E11.4),4(5X,I5))
C****
C**** FORM AND PRINT CONNECTIVITY MATRIX
C****
      WRITE(6,131)
  131 FORMAT(//7X,'NODE #',12X,'CONNECTIVITY MATRIX'/)
      ISUM=0
      DO 20 I=1,NNOD
      DO 25 J=1,NFREE
      IF (ID(I,J)) 35,30,35
   30 ISUM=ISUM + 1
      ID(I,J)=ISUM
      GO TO 25
   35 ID(I,J)=0
   25 CONTINUE
      WRITE(6,130) I,ID(I,1),ID(I,2),ID(I,3),ID(I,4)
   20 CONTINUE
  130 FORMAT(5X,5(I5,5X))
      NEQ=ISUM
      WRITE(6,135) NEQ
  135 FORMAT(//5X,'COMPUTED NUMBER OF EQUATIONS =',I5//)
      NBAND=NLC+NUC+1
C****
C**** INITIALIZE THE MATRICES
C****
      DO 40 I=1,NEQ
      DO 40 J=1,NBAND
      M(I,J)=0.0
      CI(I,J)=0.0
   40 KI(I,J)=0.0
C****
C**** FORM TIME-INDEPENDENT MATRICES M,CI,AND KI
C****
C**** ASSEMBLE ELEMENT MATRICES
C****
      WRITE(6,141)
  141 FORMAT(1X,'ELEMENT #',1X,'NODE 1',1X,'NODE 2',1X,'NODE 3',1X,
     *'NODE 4',1X,'NODE 5',1X,'NODE 6',1X,'NODE 7',1X,'NODE 8',1X,
     *'YOUNG*S',5X,'POISSON*S',5X,'DENSITY'/67X,'MODULUS',7X,
     *'RATIO'/)
      DO 50 LK=1,NELEM
      READ(5,140) (NOD(LK,J),J=1,NNOEL),EYM(LK),EPR(LK),ERHO(LK)
  140 FORMAT(8I3,3E11.4)
      YM=EYM(LK)
      PR=EPR(LK)
      RHO=ERHO(LK)
      WRITE(6,145) LK,(NOD(LK,J),J=1,NNOEL),YM,PR,RHO
```

```
  145 FORMAT(9(2X,I5),3(2X,E11.4))
      DO 55 IP=1,NNOEL
      II=NOD(LK,IP)
      RE(IP)=RC(II)
   55 ZE(IP)=ZC(II)
      CALL CONMAT(SP,P,T,LK,NNOEL,RE,ZE,YM,PR,RHO,NGAS1,NGAS2.
     *EKC,EKG,EM,ECG)
      DO 60 IT=1,NNOEL
      II=NOD(LK,IT)
      IM=NFREE*(IT-1)
      DO 60 JT=1,NNOEL
      JJ=NOD(LK,JT)
      JN=NFREE*(JT-1)
      DO 65 I=1,NFREE
      MMI=ID(II,I)
      IF (MMI.EQ.O) GO TO 65
      IMI=IM+I
      DO 70 J=1,NFREE
      NJJ=ID(JJ,J)
      IF (NJJ.EQ.O) GO TO 70
      NNJ=NJJ-MMI+NLC+1
      JNJ=JN+J
      M(MMI,NNJ)=M(MMI,NNJ) + EM(IMI,JNJ)
      CI(MMI,NNJ)=CI(MMI,NNJ) + ECG(IMI,JNJ)
      KI(MMI,NNJ)=KI(MMI,NNJ) + EKC(IMI,JNJ)+EKG(IMI,JNJ)
   70 CONTINUE
   65 CONTINUE
   60 CONTINUE
   50 CONTINUE
C****
C**** ASSEMBLE BEARING MATRICES
C****
      IF (NBEAR.EQ.O) GO TO 300
      DO 305 I=1,NBEAR
      READ(5,160) NOBER(I),BK(I,1,1),BK(I,1,2),BK(I.2,1),BK(I,2,2)
  160 FORMAT(I5,4E11.4)
      READ(5,165) BC(I,1,1),BC(I,1,2),BC(I,2,1),BC(I,2,2)
  165 FORMAT(4E11.4)
      NDUM=NOBER(I)
      WRITE(6,170) I,NDUM,BK(I,1,1),BK(I,1,2),BK(I,2,1),BK(I,2,2),BC(I,1
     *,1),BC(I,1,2),BC(I,2,1),BC(I,2,2)
  170 FORMAT(//5X,'BEARING #=',I5,5X,'AT NODE #=',I5,5X/5X,'BK(1,1)=',
     *E11.4,5X,'BK(1,2)=',E11.4,5X,'BK(2,1)=',E11.4,5X,'BK(2,2)=',E11.4,
     *5X/5X,'BC(1,1)=',E11.4,5X,'BC(1,2)=',E11.4,5X,'BC(2,1)=',E11.4,5X,
     *'BC(2,2)=',E11.4/)
      DO 310 J=1,2
      JJ=ID(NDUM,J)
      IF (JJ.EQ.O) GO TO 310
      DO 315 KA=1,2
      KK=ID(NDUM,KA)
      IF (KK.EQ.O) GO TO 315
```

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```
      KKI=KK-JJ+NLC+1
      CI(JJ,KKI)=CI(JJ,KKI) + BC(I,J,KA)
      KI(JJ,KKI)=KI(JJ,KKI) + BK(I,J,KA)
  315 CONTINUE
  310 CONTINUE
  305 CONTINUE
  300 CONTINUE
      IF (NPINT.EQ.0) GO TO 210
      DO 220 I=1,NPINT
      READ(5,225) NELP(I),XI1(I),XI2(I)
  225 FORMAT(I5,2F10.5)
  220 CONTINUE
  210 CONTINUE
C****
C**** SET INITIAL CONDITIONS FOR THE ROTOR AND BASE
C****
      DO 320 I=1,NEQ
      XNEW(I)=0.0
      VXNEW(I)=0.0
  320 AXNEW(I)=0.0
      WRITE(6,175)
  175 FORMAT('1')
      READ(5,180) TIME,ACCX,ACCY,ACCZ,VELX,VELY,VELZ
      READ(5,185) ACCTX,ACCTY,ACCTZ,VELTX,VELTY,VELTZ
  180 FORMAT(7F10.5)
  185 FORMAT(6F10.5)
      WRITE(6,190)
  190 FORMAT(//5X,'**** INITIAL CONDITIONS OF THE BASE AND ROTOR ****')
      WRITE(6,195) TIME,ACCX,ACCY,ACCZ,ACCTX,ACCTY,ACCTZ,VELX,VELY,VELZ,
     *VELTX,VELTY,VELTZ
  195 FORMAT(//5X,'TIME=',F10.5/5X,'ACCX=',E11.4,5X,'ACCY=',E11.4,5X,
     *'ACCZ=',E11.4,5X,'ACCTX=',E11.4,5X,'ACCTY=',E11.4,5X,'ACCTZ=',E11.
     *4/5X,'VELX=',E11.4,5X,'VELY=',E11.4,5X,'VELZ=',E11.4,5X,'VELTX=',
     *E11.4,5X,'VELTY=',E11.4,5X,'VELTZ=',E11.4)
      GO TO 3000
C****
C**** READ BASE ACCELERATIONS
C****
 1000 READ(5,200) TIME,AX,AY,AZ,TX,TY,TZ
  200 FORMAT(7F10.5)
      IF (TIME.LE.0.00001) STOP
      WRITE(6,175)
      WRITE(6,215) TIME,AX,AY,AZ,TX,TY,TZ
  215 FORMAT(//5X,'TIME=',F10.5//5X,'ACCX=',E11.4,5X,'ACCY=',E11.4,5X,
     *'ACCZ=',E11.4,5X,'ACCTX=',E11.4,5X,'ACCTY=',E11.4,5X,'ACCTZ=',E11.
     *4)
      DT=TIME-TOLD
      VELX=VELX + 0.5*DT*(AX+ACCX)
      VELY=VELY + 0.5*DT*(AY+ACCY)
      VELZ=VELZ + 0.5*DT*(AZ+ACCZ)
      ACCX=AX
```

```
      ACCY=AY
      ACCZ=AZ
      VELTX=VELTX + 0.5*DT*(TX+ACCTX)
      VELTY=VELTY + 0.5*DT*(TY+ACCTY)
      VELTZ=VELTZ + 0.5*DT*(TZ+ACCTZ)
      ACCTX=TX
      ACCTY=TY
      ACCTZ=TZ
C****
C**** FORM TIME-DEPENDENT MATRICES AND VECTORS
C****
      DO 340 I=1,NEQ
      F(I)=0.0
      DO 340 J=1,NBAND
      C(I,J)=CI(I,J)
  340 K(I,J)=KI(I,J)
C****
C**** ASSEMBLE ELEMENT MATRICES AND VECTORS
C****
      DO 350 LK=1,NELEM
      RHO=ERHO(LK)
      DO 355 IP=1,NNOEL
      II=NOD(LK,IP)
      RE(IP)=RC(II)
  355 ZE(IP)=ZC(II)
      CALL VARMAT(LK,NNOEL,H,RE,ZE,RHO,ACCX,ACCY,ACCZ,VELTX,VELTY,
     *VELTZ,ACCTX,ACCTY,ACCTZ,NGAS1,NGAS2,SP,ECC,EKR,EF)
      DO 360 IT=1,NNOEL
      II=NOD(LK,IT)
      IM=NFREE*(IT-1)
      DO 360 JT=1,NNOEL
      JJ=NOD(LK,JT)
      JN=NFREE*(JT-1)
      DO 365 I=1,NFREE
      MMI=ID(II,I)
      IF (MMI.EQ.0) GO TO 365
      IMI=IM+I
      DO 370 J=1,NFREE
      NJJ=ID(JJ,J)
      IF (NJJ.EQ.0) GO TO 370
      NNJ=NJJ-MMI+NLC+1
      JNJ=JN+J
      C(MMI,NNJ)=C(MMI,NNJ) + ECC(IMI,JNJ)
      K(MMI,NNJ)=K(MMI,NNJ) + EKR(IMI,JNJ)
  370 CONTINUE
  365 CONTINUE
  360 CONTINUE
      DO 380 IT=1,NNOEL
      II=NOD(LK,IT)
      IM=NFREE*(IT-1)
      DO 385 I=1,NFREE
```

```
      MMI=ID(II,I)
      IF (MMI.EQ.O) GO TO 385
      IMI=IM+I
      F(MMI)=F(MMI) + EF(IMI)
  385 CONTINUE
  380 CONTINUE
  350 CONTINUE
C****
C**** USE NEWMARK'S ALGORITHM
C****
      DELTA=0.5
      ALFA=0.25
      A0=1.0/(ALFA*DT*DT)
      A1=DELTA/(ALFA*DT)
      A2=1.0/(ALFA*DT)
      A3=(0.5/ALFA) - 1.0
      A4=(DELTA/ALFA) - 1.0
      A5=DT*((DELTA/ALFA)-2.0)*0.5
      A6=DT*(1.0-DELTA)
      A7=DELTA*DT
      DO 420 I=1,NEQ
      B(I,1)=F(I)
      DO 420 J=1,NBAND
      JN=J+I-NLC-1
      IF (JN.LE.O.OR.JN.GT.NEQ) GO TO 425
      B(I,1)=B(I,1) + M(I,J)*(A0*XOLD(JN) + A2*VXOLD(JN) + A3*AXOLD(JN))
     *+ C(I,J)*(A1*XOLD(JN) + A4*VXOLD(JN) + A5*AXOLD(JN))
  425 CONTINUE
  420 A(I,J)=K(I,J) + A0*M(I,J) + A1*C(I,J)
      CALL LEQT1B(A,NEQ,NLC,NUC,75,B,1,75,O.XL,IER)
      DO 430 I=1,NEQ
      XNEW(I)=B(I,1)
      AXNEW(I)=A0*(XNEW(I)-XOLD(I)) - A2*VXOLD(I) - A3*AXOLD(I)
  430 VXNEW(I)=VXOLD(I) + A6*AXOLD(I) + A7*AXNEW(I)
 3000 CALL RESULT(NNOD,NFREE,NNOEL,NOD,ID,NBEAR,NOBER,NPINT,NELP,
     *XI1,XI2,EYM,EPR,RC,ZC,BK,BC,XNEW,VXNEW)
      DO 440 I=1,NEQ
      XOLD(I)=XNEW(I)
      VXOLD(I)=VXNEW(I)
  440 AXOLD(I)=AXNEW(I)
      TOLD=TIME
      GO TO 1000
      END


      SUBROUTINE DMAT(YM,PR,D)
      DIMENSION D(5,5)
      AM=YM/(1.0+PR)
      AL=(PR*YM)/((1.0+PR)*(1.0-2.0*PR))
      DO 100 I=1,5
```

```
      DO 100 J=1,5
  100 D(I,J)=0 0
      D(1,1)=AM+AL
      D(1,2)=AL
      D(2,1)=AL
      D(2,2)=AM+AL
      D(3,3)=0.5*AM
      D(4,4)=0.5*AM
      D(5,5)=0.5*AM
      RETURN
      END


      SUBROUTINE SHFMAT(NNOEL,XII1,XII2,SHF,DSHF)
      DIMENSION SHF(8),DSHF(8,2)
      UPX1=1.0 + XII1
      UMX1=1.0 - XII1
      UPX2=1.0 + XII2
      UMX2=1.0 - XII2
      SHF(1)=0.25*UPX1*UPX2
      SHF(2)=0.25*UMX1*UPX2
      SHF(3)=0.25*UMX1*UMX2
      SHF(4)=0.25*UPX1*UMX2
      DSHF(1,1)=0.25*UPX2
      DSHF(2,1)=-0.25*UPX2
      DSHF(3,1)=-0.25*UMX2
      DSHF(4,1)=0.25*UMX2
      DSHF(1,2)=0.25*UPX1
      DSHF(2,2)=0.25*UMX1
      DSHF(3,2)=-0.25*UMX1
      DSHF(4,2)=-0.25*UPX1
      IF (NNOEL.EQ.4) RETURN
      SHF(5)=0.5*UPX1*UMX1*UPX2
      SHF(6)=0.5*UMX1*UPX2*UMX2
      SHF(7)=0 5*UPX1*UMX1*UMX2
      SHF(8)=0.5*UPX1*UPX2*UMX2
      SHF(1)=SHF(1)-0.5*SHF(5)-0.5*SHF(8)
      SHF(2)=SHF(2)-0.5*SHF(5)-0.5*SHF(6)
      SHF(3)=SHF(3)-0.5*SHF(6)-0.5*SHF(7)
      SHF(4)=SHF(4)-0.5*SHF(7)-0.5*SHF(8)
      DSHF(5,1)=-XII1*UPX2
      DSHF(6,1)=-0.5*UPX2*UMX2
      DSHF(7,1)=-XII1*UMX2
      DSHF(8,1)=0.5*UPX2*UMX2
      DSHF(1,1)=DSHF(1,1)-0.5*DSHF(5,1)-0.5*DSHF(8,1)
      DSHF(2,1)=DSHF(2,1)-0.5*DSHF(5,1)-0.5*DSHF(6,1)
      DSHF(3,1)=DSHF(3,1)-0.5*DSHF(6,1)-0.5*DSHF(7,1)
      DSHF(4,1)=DSHF(4,1)-0.5*DSHF(7,1)-0.5*DSHF(8,1)
      DSHF(5,2)=0.5*UPX1*UMX1
      DSHF(6,2)=-UMX1*XII2
```

```
      DSHF(7,2)=-0.5*UPX1*UMX1
      DSHF(8,2)=-UPX1*XII2
      DSHF(1,2)=DSHF(1,2)-0.5*DSHF(5,2)-0.5*DSHF(8,2)
      DSHF(2,2)=DSHF(2,2)-0.5*DSHF(5,2)-0.5*DSHF(6,2)
      DSHF(3,2)=DSHF(3,2)-0.5*DSHF(6,2)-0.5*DSHF(7,2)
      DSHF(4,2)=DSHF(4,2)-0.5*DSHF(7,2)-0.5*DSHF(8,2)
      RETURN
      END


      SUBROUTINE JACMAT(LK,NNOEL,RE,ZE,DSHF,AJ,DET)
      DIMENSION RE(8),ZE(8),DSHF(8,2),AI(2,2),AJ(2,2)
      DO 100 I=1,2
      DO 100 J=1,2
  100 AI(I,J)=0.0
      DO 110 I=1,2
      DO 110 K=1,NNOEL
      AI(I,1)=AI(I,1) + RE(K)*DSHF(K,I)
  110 AI(I,2)=AI(I,2) + ZE(K)*DSHF(K,I)
      DET=AI(1,1)*AI(2,2) - AI(1,2)*AI(2,1)
      IF (DET.GT.0.00000001) GO TO 120
      WRITE(6,1000) LK
      STOP
  120 DUM=1.0/DET
      AJ(1,1)=AI(2,2)*DUM
      AJ(1,2)=-AI(1,2)*DUM
      AJ(2,1)=-AI(2,1)*DUM
      AJ(2,2)=AI(1,1)*DUM
      RETURN
 1000 FORMAT(' **** ERROR, ZERO OR NEGATIVE JACOBIAN FOR ELEMENT #',I5)
      END


      SUBROUTINE BCMAT(NNOEL,R,SHF,DSHF,AJ,B)
      DIMENSION SHF(8),DSHF(8,2),AJ(2,2),B(5,32)
      DO 100 I=1,5
      DO 100 J=1,32
  100 B(I,J)=0.0
      DO 110 J=1,NNOEL
      J1=4*(J-1) + 1
      J2=J1+1
      J3=J2+1
      J4=J3+1
      DR=AJ(1,1)*DSHF(J,1) + AJ(1,2)*DSHF(J,2)
      DZ=AJ(2,1)*DSHF(J,1) + AJ(2,2)*DSHF(J,2)
      B(1,J1)=DR
      B(2,J4)=-R*DZ
      B(3,J1)=-DR
      B(4,J4)=SHF(J)
```

```
      B(4,J1)=-DZ
      B(5,J1)=DZ
      B(5,J4)=-SHF(J) - R*DR
  110 CONTINUE
      RETURN
      END


      SUBROUTINE BSMAT(NNOEL,R,SHF,DSHF,AJ,B)
      DIMENSION SHF(8),DSHF(8,2),AJ(2,2),B(5,32)
      DO 100 I=1,5
      DO 100 J=1,32
  100 B(I,J)=0.0
      DO 110 J=1,NNOEL
      J1=4*(J-1) + 1
      J2=J1+1
      J3=J2+1
      J4=J3+1
      DR=AJ(1,1)*DSHF(J,1) + AJ(1,2)*DSHF(J,2)
      DZ=AJ(2,1)*DSHF(J,1) + AJ(2,2)*DSHF(J,2)
      B(1,J2)=DR
      B(2,J3)=R*DZ
      B(3,J2)=DR
      B(4,J3)=SHF(J)
      B(4,J2)=DZ
      B(5,J2)=DZ
      B(5,J3)=SHF(J) + R*DR
  110 CONTINUE
      RETURN
      END


      SUBROUTINE SMAT(SRR,SPP,SZZ,TRP,TPZ,TZR,S)
      DIMENSION S(10,10)
      DO 100 I=1,10
      DO 100 J=1,10
  100 S(I,J)=0.0
      S(1,1)=2.0*SRR
      S(1,2)=2.0*TZR
      S(2,2)=2.0*SZZ
      S(3,3)=2.0*SRR
      S(3,4)=2.0*TZR
      S(4,4)=2.0*SZZ
      S(5,5)=SPP
      S(5,9)=-TRP
      S(5,10)=-TPZ
      S(6,6)=SPP
      S(6,7)=TRP
      S(6,8)=TPZ
```

```
      S(7,7)=SRR
      S(7,8)=TZR
      S(8,8)=SZZ
      S(9,9)=SRR
      S(9,10)=TZR
      S(10,10)=SZZ
      DO 110 I=1,10
      DO 110 J=1,I
  110 S(I,J)=S(J,I)
      RETURN
      END


      SUBROUTINE GMAT(NNOEL,R,SHF,DSHF,AJ,G)
      DIMENSION SHF(8),DSHF(8,2),AJ(2,2),G(10,32)
      DO 100 I=1,10
      DO 100 J=1,32
  100 G(I,J)=0.0
      DO 110 J=1,NNOEL
      J1=4*(J-1) + 1
      J2=J1+1
      J3=J2+1
      J4=J3+1
      DR=AJ(1,1)*DSHF(J,1) + AJ(1,2)*DSHF(J,2)
      DZ=AJ(2,1)*DSHF(J,1) + AJ(2,2)*DSHF(J,2)
      G(1,J1)=DR
      G(2,J1)=DZ
      G(3,J2)=DR
      G(4,J2)=DZ
      G(5,J3)=SHF(J)
      G(6,J4)=SHF(J)
      G(7,J3)=SHF(J) + R*DR
      G(8,J3)=R*DZ
      G(9,J4)=SHF(J) + R*DR
      G(10,J4)=R*DZ
  110 CONTINUE
      RETURN
      END


      SUBROUTINE ENMAT(NNOEL,SHF,EN)
      DIMENSION SHF(8),EN(4,32)
      DO 100 I=1,4
      DO 100 J=1,32
  100 EN(I,J)=0.0
      DO 110 J=1,NNOEL
      DO 120 I=1,4
      JI=4*(J-1) + I
  120 EN(I,JI)=SHF(J)
```

```
  110 CONTINUE
      RETURN
      END


      SUBROUTINE M1MAT(RHO,R,EM1)
      DIMENSION EM1(4,4)
      DO 100 I=1,4
      DO 100 J=1,4
  100 EM1(I,J)=0.0
      EM1(1,1)=2.0*RHO
      EM1(2,2)=2.0*RHO
      EM1(3,3)=RHO*R*R
      EM1(4,4)=RHO*R*R
      RETURN
      END


      SUBROUTINE G1MAT(RHO,SP,R,G1)
      DIMENSION G1(4,4)
      DO 100 I=1,4
      DO 100 J=1,4
  100 G1(I,J)=0.0
      G1(3,4)=2.0*RHO*SP*R*R
      G1(4,3)=-G1(3,4)
      RETURN
      END


      SUBROUTINE CONMAT(SP,P,T,LK,NNOEL,RE,ZE,YM,PR,RHO,NGAS1,NGAS2,
     *EKC,EKG,EM,ECG)
      DIMENSION XG(4,4),WGT(4,4),D(5,5),EKC(32,32),EKG(32,32)
      DIMENSION EM(32,32),ECG(32,32),SHF(8),DSHF(8,2),AJ(2,2)
      DIMENSION B(5,32),DUM(10,32),S(10,10),G(10,32),EN(4,32)
      DIMENSION EM1(4,4),G1(4,4),RE(8),ZE(8)
      DATA XG/0.0E0,0.0E0,0.0E0,0.0E0,
     *-0.57735027E0,0.57735027E0,0.0E0,0.0E0,
     *-0.77459667E0,0.0E0,0.77459667E0,0.0E0,
     *-0.86113631E0,-0.33998104E0,0.33998104E0,0.86113631E0/
      DATA WGT/2.0E0,0.0E0,0.0E0,0.0E0,
     *1.0E0,1.0E0,0.0E0,0.0E0,
     *0.55555556E0,0.88888889E0,0.55555556E0,0.0E0,
     *0.34785485E0,0.65214515E0,0.65214515E0,0.34785485E0/
      PI=4.0*ATAN(1.0)
      JN=4*NNOEL
      CALL DMAT(YM,PR,D)
C****
C**** INITIALIZE THE MATRICES
```

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```
C****
      DO 100 I=1,JN
      DO 100 J=1,JN
      EKC(I,J)=0.0
      EKG(I,J)=0.0
      EM(I,J)=0.0
      ECG(I,J)=0.0
  100 CONTINUE
C****
C**** START THE GAUSSIAN LOOP
C****
      DO 110 L1=1,NGAS1
      XII1=XG(L1,NGAS1)
      DO 110 L2=1,NGAS2
      XII2=XG(L2,NGAS2)
      CALL SHFMAT(NNOEL,XII1,XII2,SHF,DSHF)
      R=0.0
      Z=0.0
      DO 120 I=1,NNOEL
      R=R + SHF(I)*RE(I)
  120 Z=Z + SHF(I)*ZE(I)
      CALL JACMAT(LK,NNOEL,RE,ZE,DSHF,AJ,DET)
      WT=WGT(L1,NGAS1)*WGT(L2,NGAS2)*DET*PI*R
      CALL BCMAT(NNOEL,R,SHF,DSHF,AJ,B)
      DO 130 I=1,5
      DO 130 J=1,JN
      DUM(I,J)=0.0
      DO 130 K=1,5
  130 DUM(I,J)=DUM(I,J) + D(I,K)*B(K,J)
      DO 140 I=1,JN
      DO 140 J=1,JN
      DUM1=0.0
      DO 150 K=1,5
  150 DUM1=DUM1 + B(K,I)*DUM(K,J)
  140 EKC(I,J)=EKC(I,J) + DUM1*WT
      CALL BSMAT(NNOEL,R,SHF,DSHF,AJ,B)
      DO 330 I=1,5
      DO 330 J=1,JN
      DUM(I,J)=0.0
      DO 330 K=1,5
  330 DUM(I,J)=DUM(I,J) + D(I,K)*B(K,J)
      DO 340 I=1,JN
      DO 340 J=1,JN
      DUM1=0.0
      DO 350 K=1,5
  350 DUM1=DUM1 + B(K,I)*DUM(K,J)
  340 EKC(I,J)=EKC(I,J) + DUM1*WT
      CALL INSTR(R,Z,SP,P,T,SRR,SPP,SZZ,TRP,TPZ,TZR)
      CALL SMAT(SRR,SPP,SZZ,TRP,TPZ,TZR,S)
      CALL GMAT(NNOEL,R,SHF,DSHF,AJ,G)
      DO 160 I=1,10
```

```
      DO 160 J=1,JN
      DUM(I,J)=0.0
      DO 160 K=1,10
  160 DUM(I,J)=DUM(I,J) + S(I,K)*G(K,J)
      DO 170 I=1,JN
      DO 170 J=1,JN
      DUM1=0.0
      DO 180 K=1,10
  180 DUM1=DUM1 + G(K,I)*DUM(K,J)
  170 EKG(I,J)=EKG(I,J) + DUM1*WT
      CALL ENMAT(NNOEL,SHF,EN)
      CALL M1MAT(RHO,R,EM1)
      DO 190 I=1,4
      DO 190 J=1,JN
      DUM(I,J)=0.0
      DO 190 K=1,4
  190 DUM(I,J)=DUM(I,J) + EM1(I,K)*EN(K,J)
      DO 200 I=1,JN
      DO 200 J=1,JN
      DUM1=0.0
      DO 210 K=1,4
  210 DUM1=DUM1 + EN(K,I)*DUM(K,J)
  200 EM(I,J)=EM(I,J) + DUM1*WT
      CALL G1MAT(RHO,SP,R,G1)
      DO 220 I=1,4
      DO 220 J=1,JN
      DUM(I,J)=0.0
      DO 220 K=1,4
  220 DUM(I,J)=DUM(I,J) + G1(I,K)*EN(K,J)
      DO 230 I=1,JN
      DO 230 J=1,JN
      DUM1=0.0
      DO 240 K=1,4
  240 DUM1=DUM1 + EN(K,I)*DUM(K,J)
  230 ECG(I,J)=ECG(I,J) + DUM1*WT
  110 CONTINUE
      RETURN
      END


      SUBROUTINE C1MAT(RHO,ACCTZ,C1)
      DIMENSION C1(4,4)
      DO 100 I=1,4
      DO 100 J=1,4
  100 C1(I,J)=0.0
      C1(1,2)=-4.0*RHO*ACCTZ
      C1(2,1)=-C1(1,2)
      RETURN
      END
```

```
      SUBROUTINE R1MAT(RHO,VELTX,VELTY,VELTZ,ACCTZ,R1)
      DIMENSION R1(4,4)
      DO 100 I=1,4
      DO 100 J=1,4
  100 R1(I,J)=0.0
      R1(1,1)=-2.0*RHO*(VELTY*VELTY + VELTZ*VELTZ)
      R1(1,2)=2.0*RHO*(VELTX*VELTY - ACCTZ)
      R1(2,1)=2.0*RHO*(VELTX*VELTY + ACCTZ)
      R1(2,2)=-2.0*RHO*(VELTZ*VELTZ + VELTX*VELTX)
      RETURN
      END


      SUBROUTINE FVEC(RHO,H,R,Z,ACCX,ACCY,VELTX,VELTY,VELTZ,ACCTX,
     *ACCTY,ACCTZ,SP,F1)
      DIMENSION F1(4)
      F1(1)=-2.0*RHO*(ACCX + H*(VELTX*VELTY - ACCTZ)
     * + Z*(VELTZ*VELTX + ACCTY))
      F1(2)=-2.0*RHO*(ACCY - H*(VELTZ*VELTZ + VELTX*VELTX)
     * + Z*(VELTY*VELTZ - ACCTX))
      F1(3)=-RHO*R*R*(ACCTX + 2.0*SP*VELTY)
      F1(4)=-RHO*R*R*(ACCTY - 2.0*SP*VELTX)
      RETURN
      END


      SUBROUTINE VARMAT(LK,NNOEL,H,RE,ZE,RHO,ACCX,ACCY,ACCZ,
     *VELTX,VELTY,VELTZ,ACCTX,ACCTY,ACCTZ,NGAS1,NGAS2,SP,ECC,EKR,EF)
      DIMENSION XG(4,4),WGT(4,4),DUM(4,32),C1(4,4),R1(4,4),EN(4,32),
     *ECC(32,32),EKR(32,32),EF(32),SHF(8),DSHF(8,2),AJ(2,2),F1(4)
      DIMENSION RE(8),ZE(8)
      PI=4.0*ATAN(1.0)
      DATA XG/0.0E0,0.0E0,0.0E0,0.0E0,
     *-0.57735027E0,0.57735027E0,0.0E0,0.0E0,
     *-0.77459667E0,0.0E0,0.77459667E0,0.0E0,
     *-0.86113631E0,-0.33998104E0,0.33998104E0,0.86113631E0/
      DATA WGT/2.0E0,0.0E0,0.0E0,0.0E0,
     *1.0E0,1.0E0,0.0E0,0.0E0,
     *0.55555556E0,0.88888889E0,0.55555556E0,0.0E0,
     *0.34785485E0,0.65214515E0,0.65214515E0,0.34785485E0/
      JN=4*NNOEL
      CALL C1MAT(RHO,ACCTZ,C1)
      CALL R1MAT(RHO,VELTX,VELTY,VELTZ,ACCTZ,R1)
C****
C**** INITIALIZE THE MATRICES
C****
      DO 100 I=1,JN
```

```
      EF(I)=0.0
      DO 100 J=1,JN
      ECC(I,J)=0.0
  100 EKR(I,J)=0.0
C****
C**** START THE GAUSSIAN LOOP
C****
      DO 110 L1=1,NGAS1
      XII1=XG(L1,NGAS1)
      DO 110 L2=1,NGAS2
      XII2=XG(L2,NGAS2)
      CALL SHFMAT(NNOEL,XII1,XII2,SHF,DSHF)
      R=0.0
      Z=0.0
      DO 120 I=1,NNOEL
      R=R + SHF(I)*RE(I)
  120 Z=Z + SHF(I)*ZE(I)
      CALL JACMAT(LK,NNOEL,RE,ZE,DSHF,AJ,DET)
      WT=WGT(L1,NGAS1)*WGT(L2,NGAS2)*DET*PI*R
      CALL ENMAT(NNOEL,SHF,EN)
      DO 130 I=1,4
      DO 130 J=1,JN
      DUM(I,J)=0.0
      DO 130 K=1,4
  130 DUM(I,J)=DUM(I,J) + C1(I,K)*EN(K,J)
      DO 140 I=1,JN
      DO 140 J=1,JN
      DUM1=0.0
      DO 150 K=1,4
  150 DUM1=DUM1 + EN(K,I)*DUM(K,J)
  140 ECC(I,J)=ECC(I,J) + DUM1*WT
      DO 160 I=1,4
      DO 160 J=1,JN
      DUM(I,J)=0.0
      DO 160 K=1,4
  160 DUM(I,J)=DUM(I,J) + R1(I,K)*EN(K,J)
      DO 170 I=1,JN
      DO 170 J=1,JN
      DUM1=0.0
      DO 180 K=1,4
  180 DUM1=DUM1 + EN(K,I)*DUM(K,J)
  170 EKR(I,J)=EKR(I,J) + DUM1*WT
      CALL FVEC(RHO,H,R,Z,ACCX,ACCY,VELTX,VELTY,VELTZ,ACCTX,ACCTY,
     *ACCTZ,SP,F1)
      DO 190 I=1,JN
      DUM1=0.0
      DO 200 K=1,4
  200 DUM1=DUM1 + EN(K,I)*F1(K)
  190 EF(I)=EF(I) + DUM1*WT
  110 CONTINUE
      RETURN
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```
      END

      SUBROUTINE RESULT(NNOD,NFREE,NNOEL,NOD,ID,NBEAR,NOBER,NPINT,
     *NELP,XI1,XI2,EYM,EPR,RC.ZC,BK,BC,XNEW,VXNEW)
      DIMENSION XNEW(75),VXNEW(75),ID(60,4),NOBER(20),BK(20,4,4)
      DIMENSION BC(20,4,4),DIS(32),VEL(4),NOD(25,8)
      DIMENSION NELP(50),XI1(50),XI2(50),D(5,5),B(5,32)
      DIMENSION EYM(25),EPR(25),RC(60),ZC(60),RE(8),ZE(8)
      DIMENSION SHF(8),DSHF(8,2),AJ(2,2),SN(5),SC(5),SS(5)
      WRITE(6,100)
  100 FORMAT(//5X,'**** NODAL DISPLACEMENTS ****'//7X,'NODE #',6X,'UX',
     *14X,'UY',12X,'THETAX',10X,'THETAY'/)
      DO 200 I=1,NNOD
      DO 210 J=1,NFREE
      II=ID(I,J)
      IF (II.EQ.0) GO TO 220
      DIS(J)=XNEW(II)
      GO TO 210
  220 DIS(J)=0.0
  210 CONTINUE
      WRITE(6,110) I,(DIS(J),J=1,NFREE)
  110 FORMAT(5X,I5,4(5X,E11.4))
  200 CONTINUE
      IF (NBEAR.EQ.0) GO TO 250
      WRITE(6,120)
  120 FORMAT(/5X,'**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEAR
     *INGS ****'/)
      DO 230 I=1,NBEAR
      IN=NOBER(I)
      DO 240 J=1,NFREE
      II=ID(IN,J)
      IF (II.EQ.0) GO TO 235
      DIS(J)=XNEW(II)
      VEL(J)=VXNEW(II)
      GO TO 240
  235 DIS(J)=0.0
      VEL(J)=0.0
  240 CONTINUE
      FX=BK(I,1,1)*DIS(1) + BK(I,1,2)*DIS(2) + BC(I,1,1)*VEL(1)
     *+ BC(I,1,2)*VEL(2)
      FY=BK(I,2,1)*DIS(1) + BK(I,2,2)*DIS(2) + BC(I,2,1)*VEL(1)
     *+ BC(I,2,2)*VEL(2)
      WRITE(6,130) I,IN,DIS(1),DIS(2),FX,FY
  130 FORMAT(5X,'BEARING #',I5,5X,'AT NODE #',I5,5X,'UX=',E11.4,5X,'UY='
     *,E11.4,5X,'FX=',E11.4,5X,'FY=',E11.4)
  230 CONTINUE
  250 CONTINUE
      IF (NPINT.EQ.0) GO TO 300
      WRITE(6,140)
```

```
140 FORMAT(/5X,'**** DYNAMIC STRESSES IN THE ROTOR ****'/)
    WRITE(6,150)
150 FORMAT(/1X,'EL #',6X,'R',7X,'PHI',7X,'Z',10X,'SRR',10X,'SPP',
   *10X,'SZZ',10X,'TRP',10X,'TPZ',10X.'TZR'/)
    NF=NNOEL*NFREE
    DO 310 I=1,NPINT
    IN=NELP(I)
    YM=EYM(IN)
    PR=EPR(IN)
    XII1=XI1(I)
    XII2=XI2(I)
    CALL DMAT(YM,PR,D)
    DO 320 J=1,NNOEL
    NUM=NOD(IN,J)
    RE(J)=RC(NUM)
    ZE(J)=ZC(NUM)
    DO 320 IA=1,NFREE
    JA=(J-1)*NFREE + IA
    II=ID(NUM,IA)
    IF (II.EQ.0) DIS(JA)=0.0
    IF (II.NE.0) DIS(JA)=XNEW(II)
320 CONTINUE
    CALL SHFMAT(NNOEL,XII1,XII2,SHF,DSHF)
    R=0.0
    Z=0.0
    DO 330 J=1,NNOEL
    R=R + SHF(J)*RE(J)
330 Z=Z + SHF(J)*ZE(J)
    CALL JACMAT(IN,NNOEL,RE,ZE,DSHF,AJ,DET)
    CALL BCMAT(NNOEL,R,SHF.DSHF,AJ,B)
    DO 340 J=1,5
    SN(J)=0.0
    DO 340 K=1,NF
340 SN(J)=SN(J) + B(J,K)*DIS(K)
    DO 350 J=1,5
    SC(J)=0.0
    DO 350 K=1,5
350 SC(J)=SC(J) + D(J,K)*SN(K)
    SPPC=D(1,2)*SN(1) + D(2,1)*SN(2)
    CALL BSMAT(NNOEL.R,SHF,DSHF,AJ,B)
    DO 360 J=1,5
    SN(J)=0.0
    DO 360 K=1,NF
360 SN(J)=SN(J) + B(J,K)*DIS(K)
    DO 370 J=1,5
    SS(J)=0.0
    DO 370 K=1,5
370 SS(J)=SS(J) + D(J,K)*SN(K)
    SPPS=D(1,2)*SN(1) + D(2,1)*SN(2)
    PHI=0.0
    WRITE(6,160) IN,R,PHI,Z,SC(1),SPPC,SC(2),SS(3),SS(4),SC(5)
```

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237

```
      PHI=90.0
      WRITE(6,160) IN,R,PHI,Z,SS(1),SPPS,SS(2),SC(3),SC(4),SS(5)
  160 FORMAT(2X,I2,2X,E10.3.2X.F4.1,2X.E10.3,6(2X,E11.4))
  310 CONTINUE
  300 CONTINUE
      RETURN
      END


      SUBROUTINE INSTR(R,Z,SP,P,T,SRR,SPP,SZZ,TRP,TPZ,TZR)
C****
C**** THIS SUBROUTINE MUST BE SUPPLIED BY THE USER
C****
      SRR=0.0
      SPP=0.0
      SZZ=0.0
      TRP=0.0
      TPZ=0.0
      TZR=0.0
      RETURN
      END
```

3.5 SAMPLE INPUT DATA

```
   18     3     8     4    72    31    31     2     2
+0.3000E+04+0.0000E+00+0.0000E+00+0.1000E+01     2     3
+0.3000E+00+0.0000E+00
+0.1500E+00+0.0000E+00
+0.0000E+00+0.0000E+00
+0.3000E+00+0.1500E+01
+0.0000E+00+0.1500E+01
+0.3000E+00+0.3000E+01
+0.1500E+00+0.3000E+01
+0.0000E+00+0.3000E+01
+0.3000E+00+0.4500E+01
+0.0000E+00+0.4500E+01
+0.3000E+00+0.6000E+01
+0.1500E+00+0.6000E+01
+0.0000E+00+0.6000E+01
+0.3000E+00+0.7500E+01
+0.0000E+00+0.7500E+01
+0.3000E+00+0.9000E+01
+0.1500E+00+0.9000E+01
+0.0000E+00+0.9000E+01
  6  8  3  1  7  5  2  4+0.2000E+12+0.3000E+00+0.7800E+04
 11 13  8  6 12 10  7  9+0.2000E+12+0.3000E+00+0.7800E+04
 16 18 13 11 17 15 12 14+0.2000E+12+0.3000E+00+0.7800E+04
    3+0.6760E+09+0.2160E+08-0.1490E+10+0.2270E+10
+0.3100E+07-0.5000E+07-0.5000E+07+0.1370E+08
   18+0.5890E+09+0.5100E+08-0.1290E+10+0.1870E+10
+0.2800E+07-0.4100E+07-0.4100E+07+0.1170E+08
    1
    2
    3
   0.0       -0.014      0.024      0.003     -0.04664    0.02972    0.1183

   0.02      -0.108     -0.23       0.019
   0.04      -0.101     -0.275      0.068
   0.06      -0.088     -0.397      0.029
```

3.6 SAMPLE RESULTS

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```
NUMBER OF NODES ********************* =   18
NUMBER OF ELEMENTS ****************** =    3
NUMBER OF NODES PER ELEMENT ********* =    8
NUMBER OF DEGREES OF FREEDOM PER NODE =    4
NUMBER OF EQUATIONS ***************** =   72
NUMBER OF LOWER CODIAGONALS ********* =   31
NUMBER OF UPPER CODIAGONALS ********* =   31
NUMBER OF GAUSSIAN POINTS ALONG XII1  =    2
NUMBER OF GAUSSIAN POINTS ALONG XII2  =    2
SPIN SPEED IN RPM ******************* = 0.3000E 04
AXIAL FORCE ************************* = 0.0000E 00
AXIAL TORQUE ************************ = 0.0000E 00
HEIGHT OF THE ROTOR FROM BASE ******* = 0.1000E 01
NUMBER OF BEARINGS ****************** =    2
NUMBER OF STRESS POINTS ************* =    3
```

NODE #	R	Z	ID MATRIX			
1	0.3000E 00	0.0000E 00	0	0	0	0
2	0.1500E 00	0.0000E 00	0	0	0	0
3	0.0000E 00	0.0000E 00	0	0	0	0
4	0.3000E 00	0.1500E 01	0	0	0	0
5	0.0000E 00	0.1500E 01	0	0	0	0
6	0.3000E 00	0.3000E 01	0	0	0	0
7	0.1500E 00	0.3000E 01	0	0	0	0
8	0.0000E 00	0.3000E 01	0	0	0	0
9	0.3000E 00	0.4500E 01	0	0	0	0
10	0.0000E 00	0.4500E 01	0	0	0	0
11	0.3000E 00	0.6000E 01	0	0	0	0
12	0.1500E 00	0.6000E 01	0	0	0	0
13	0.0000E 00	0.6000E 01	0	0	0	0
14	0.3000E 00	0.7500E 01	0	0	0	0
15	0.0000E 00	0.7500E 01	0	0	0	0
16	0.3000E 00	0.9000E 01	0	0	0	0
17	0.1500E 00	0.9000E 01	0	0	0	0
18	0.0000E 00	0.9000E 01	0	0	0	0

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NODE #	CONNECTIVITY MATRIX			
1	1	2	3	4
2	5	6	7	8
3	9	10	11	12
4	13	14	15	16
5	17	18	19	20
6	21	22	23	24
7	25	26	27	28
8	29	30	31	32
9	33	34	35	36
10	37	38	39	40
11	41	42	43	44
12	45	46	47	48
13	49	50	51	52
14	53	54	55	56
15	57	58	59	60
16	61	62	63	64
17	65	66	67	68
18	69	70	71	72

COMPUTED NUMBER OF EQUATIONS = 72

ELEMENT #	NODE 1	NODE 2	NODE 3	NODE 4	NODE 5	NODE 6	NODE 7	NODE 8	YOUNG*S MODULUS	POISSON*S RATIO	DENSITY
1	6	8	3	1	7	5	2	4	0.2000E 12	0.3000E 00	0.7800E 04
2	11	13	8	6	12	10	7	9	0.2000E 12	0.3000E 00	0.7800E 04
3	16	18	13	11	17	15	12	14	0.2000E 12	0.3000E 00	0.7800E 04

BEARING #= 1 AT NODE #= 3

BK(1,1)= 0.6760E 09 BK(1,2)= 0.2160E 08 BK(2,1)=-0.1490E 10 BK(2,2)= 0.2270E 10

BC(1,1)= 0.3100E 07 BC(1,2)=-0.5000E 07 BC(2,1)=-0.5000E 07 BC(2,2)= 0.1370E 08

BEARING #= 2 AT NODE #= 18

BK(1,1)= 0.5890E 09 BK(1,2)= 0.5100E 08 BK(2,1)=-0.1290E 10 BK(2,2)= 0.1870E 10

BC(1,1)= 0.2800E 07 BC(1,2)=-0.4100E 07 BC(2,1)=-0.4100E 07 BC(2,2)= 0.1170E 08

**** INITIAL CONDITIONS OF THE BASE AND ROTOR ****

TIME= 0.00000

ACCX=-0.1400E-01 ACCY= 0.2400E-01 ACCZ= 0.3000E-02 ACCTX= 0.0000E 00 ACCTY= 0.0000E 00

VELX=-0.4664E-01 VELY= 0.2972E-01 VELZ= 0.1183E 00 VELTX= 0.0000E 00 VELTY= 0.0000E 00

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
2	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
3	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
4	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
5	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
6	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
7	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
8	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
9	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
10	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
11	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
12	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
13	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
14	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
15	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
16	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
17	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
18	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING # 1 AT NODE # 3 UX= 0.0000E 00 UY= 0.0000E 00 FX= 0.0000E 00 FY= 0.0000E

BEARING # 2 AT NODE # 18 UX= 0.0000E 00 UY= 0.0000E 00 FX= 0.0000E 00 FY= 0.0000E

**** DYNAMIC STRESSES IN THE ROTOR ****

EL #	R	PHI	Z	SRR	SPP	SZZ	TRP	TPZ	TZR
1	0.150E 00	0.0	0.150E 01	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
1	0.150E 00	90.0	0.150E 01	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
2	0.150E 00	0.0	0.450E 01	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
2	0.150E 00	90.0	0.450E 01	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
3	0.150E 00	0.0	0.750E 01	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
3	0.150E 00	90.0	0.750E 01	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00

TIME= 0.02000

ACCX=-0.1080E 00 ACCY=-0.2300E 00 ACCZ= 0.1900E-01 ACCTX= 0.0000E 00 ACCTY= 0.0000E 00

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.1125E-05	0.1178E-05	-0.4767E-05	0.1966E-05
2	0.1062E-05	0.1038E-05	-0.4983E-05	0.2063E-05
3	0.9985E-06	0.8945E-06	-0.5746E-05	0.2398E-05
4	0.3901E-05	0.7919E-05	-0.4119E-05	0.1698E-05
5	0.3969E-05	0.8073E-05	-0.4151E-05	0.1711E-05
6	0.5988E-05	0.1297E-04	-0.2185E-05	0.8995E-06
7	0.5932E-05	0.1285E-04	-0.2270E-05	0.9370E-06
8	0.5872E-05	0.1272E-04	-0.2332E-05	0.9633E-06
9	0.6615E-05	0.1449E-04	-0.1434E-07	0.1164E-07
10	0.6686E-05	0.1465E-04	-0.1486E-07	0.1190E-07
11	0.6021E-05	0.1302E-04	0.2157E-05	-0.8800E-06
12	0.5966E-05	0.1289E-04	0.2243E-05	-0.9177E-06
13	0.5906E-05	0.1276E-04	0.2305E-05	-0.9432E-06
14	0.3958E-05	0.8003E-05	0.4091E-05	-0.1685E-05
15	0.4026E-05	0.8157E-05	0.4123E-05	-0.1698E-05
16	0.1199E-05	0.1303E-05	0.4739E-05	-0.1955E-05
17	0.1137E-05	0.1163E-05	0.4956E-05	-0.2051E-05
18	0.1073E-05	0.1020E-05	0.5720E-05	-0.2388E-05

**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING #	1	AT NODE #	3	UX= 0.9985E-06	UY= 0.8945E-06	FX= 0.5566E 03	FY= 0.1269E
BEARING #	2	AT NODE #	18	UX= 0.1073E-05	UY= 0.1020E-05	FX= 0.5664E 03	FY= 0.1276E

**** DYNAMIC STRESSES IN THE ROTOR ****

EL #	R	PHI	Z	SRR	SPP	SZZ	TRP	TPZ	TZR
1	0.150E 00	0.0	0.150E 01	-0.5468E 05	-0.1973E 05	-0.1107E 05	-0.3953E 05	-0.5143E 04	-0.1342E 04
1	0.150E 00	90.0	0.150E 01	-0.1227E 06	-0.4365E 05	-0.2278E 05	0.1748E 05	0.1844E 04	-0.3896E 04
2	0.150E 00	0.0	0.450E 01	-0.5310E 05	-0.1664E 05	-0.2376E 04	-0.4139E 05	-0.2610E 02	-0.2459E 02
2	0.150E 00	90.0	0.450E 01	-0.1188E 06	-0.3606E 05	-0.1346E 04	0.1823E 05	0.3449E 02	-0.5964E 01
3	0.150E 00	0.0	0.750E 01	-0.5468E 05	-0.1970E 05	-0.1098E 05	-0.3949E 05	0.5133E 04	0.1381E 04
3	0.150E 00	90.0	0.750E 01	-0.1226E 06	-0.4358E 05	-0.2270E 05	0.1749E 05	-0.1882E 04	0.3915E 04

TIME= 0.04000

ACCX=-0.1010E 00 ACCY=-0.2750E 00 ACCZ= 0.6800E-01 ACCTX= 0.0000E 00 ACCTY= 0.0000E 00

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.2726E-05	0.3415E-05	-0.1526E-04	0.5938E-05
2	0.2568E-05	0.3032E-05	-0.1585E-04	0.6180E-05
3	0.2409E-05	0.2644E-05	-0.1796E-04	0.7036E-05
4	0.1119E-04	0.2517E-04	-0.1336E-04	0.5199E-05
5	0.1136E-04	0.2560E-04	-0.1351E-04	0.5256E-05
6	0.1764E-04	0.4172E-04	-0.7379E-05	0.2877E-05
7	0.1750E-04	0.4139E-04	-0.7593E-05	0.2965E-05
8	0.1735E-04	0.4103E-04	-0.7894E-05	0.3085E-05
9	0.1969E-04	0.4695E-04	-0.5465E-07	0.3829E-07
10	0.1987E-04	0.4741E-04	-0.5687E-07	0.3955E-07
11	0.1774E-04	0.4188E-04	0.7274E-05	-0.2821E-05
12	0.1761E-04	0.4155E-04	0.7490E-05	-0.2909E-05
13	0.1746E-04	0.4119E-04	0.7791E-05	-0.3025E-05
14	0.1135E-04	0.2548E-04	0.1328E-04	-0.5176E-05
15	0.1152E-04	0.2590E-04	0.1343E-04	-0.5235E-05
16	0.2906E-05	0.3830E-05	0.1519E-04	-0.5931E-05
17	0.2749E-05	0.3447E-05	0.1578E-04	-0.6172E-05
18	0.2590E-05	0.3059E-05	0.1790E-04	-0.7033E-05

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**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING #	1	AT NODE #	3	UX= 0.2409E-05	UY= 0.2644E-05	FX= 0.1386E 04	FY= 0.3377E
BEARING #	2	AT NODE #	18	UX= 0.2590E-05	UY= 0.3059E-05	FX= 0.1388E 04	FY= 0.3390E

**** DYNAMIC STRESSES IN THE ROTOR ****

EL #	R	PHI	Z	SRR	SPP	SZZ	TRP	TPZ	TZR
1	0.150E 00	0.0	0.150E 01	-0.1368E 06	-0.4801E 05	-0.2328E 05	-0.1085E 06	-0.1898E 05	-0.4663E 04
1	0.150E 00	90.0	0.150E 01	-0.3320E 06	-0.1151E 06	-0.5160E 05	0.4437E 05	0.6876E 04	-0.1313E 05
2	0.150E 00	0.0	0.450E 01	-0.1328E 06	-0.3753E 05	0.7654E 04	-0.1169E 06	-0.7669E 02	-0.1431E 03
2	0.150E 00	90.0	0.450E 01	-0.3220E 06	-0.8829E 05	0.2773E 05	0.4761E 05	0.1916E 03	0.8643E 01
3	0.150E 00	0.0	0.750E 01	-0.1364E 06	-0.4769E 05	-0.2259E 05	-0.1085E 06	0.1898E 05	0.4784E 04
3	0.150E 00	90.0	0.750E 01	-0.3319E 06	-0.1149E 06	-0.5118E 05	0.4434E 05	-0.7055E 04	0.1316E 05

250

TIME= 0.06000

ACCX=-0.8800E-01 ACCY=-0.3970E 00 ACCZ= 0.2900E-01 ACCTX= 0.0000E 00 ACCTY= 0.0000E 00

**** NODAL DISPLACEMENTS ****

NODE #	UX	UY	THETAX	THETAY
1	0.2166E-05	0.4182E-05	-0.1986E-04	0.5561E-05
2	0.2027E-05	0.3679E-05	-0.2064E-04	0.5776E-05
3	0.1885E-05	0.3170E-05	-0.2341E-04	0.6538E-05
4	0.1010E-04	0.3251E-04	-0.1740E-04	0.4880E-05
5	0.1026E-04	0.3307E-04	-0.1759E-04	0.4936E-05
6	0.1616E-04	0.5405E-04	-0.9604E-05	0.2721E-05
7	0.1604E-04	0.5363E-04	-0.9884E-05	0.2798E-05
8	0.1591E-04	0.5315E-04	-0.1028E-04	0.2915E-05
9	0.1811E-04	0.6087E-04	-0.7455E-07	0.2938E-07
10	0.1827E-04	0.6147E-04	-0.7781E-07	0.3051E-07
11	0.1624E-04	0.5428E-04	0.9459E-05	-0.2679E-05
12	0.1612E-04	0.5385E-04	0.9742E-05	-0.2756E-05
13	0.1599E-04	0.5337E-04	0.1013E-04	-0.2870E-05
14	0.1022E-04	0.3293E-04	0.1728E-04	-0.4867E-05
15	0.1038E-04	0.3348E-04	0.1747E-04	-0.4924E-05
16	0.2292E-05	0.4762E-05	0.1976E-04	-0.5562E-05
17	0.2152E-05	0.4259E-05	0.2054E-04	-0.5776E-05
18	0.2011E-05	0.3750E-05	0.2332E-04	-0.6543E-05

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**** DISPLACEMENTS AND DYNAMIC REACTION FORCES ON BEARINGS ****

BEARING #	1	AT NODE #	3	UX=	0.1885E-05	UY=	0.3170E-05	FX=	0.1217E 04	FY=	0.4405E
BEARING #	2	AT NODE #	18	UX=	0.2011E-05	UY=	0.3750E-05	FX=	0.1224E 04	FY=	0.4455E

**** DYNAMIC STRESSES IN THE ROTOR ****

EL #	R	PHI	Z	SRR	SPP	SZZ	TRP	TPZ	TZR
1	0.150E 00	0.0	0.150E 01	-0.1211E 06	-0.4210E 05	-0.1918E 05	-0.1422E 06	-0.2450E 05	-0.4666E 04
1	0.150E 00	90.0	0.150E 01	-0.4355E 06	-0.1512E 06	-0.6846E 05	0.3952E 05	0.6831E 04	-0.1690E 05
2	0.150E 00	0.0	0.450E 01	-0.1176E 06	-0.3208E 05	0.1063E 05	-0.1531E 06	-0.9193E 02	-0.1166E 03
2	0.150E 00	90.0	0.450E 01	-0.4225E 06	-0.1164E 06	0.3461E 05	0.4275E 05	0.1602E 03	0.3394E 02
3	0.150E 00	0.0	0.750E 01	-0.1208E 06	-0.4181E 05	-0.1858E 05	-0.1422E 06	0.2449E 05	0.4759E 04
3	0.150E 00	90.0	0.750E 01	-0.4354E 06	-0.1510E 06	-0.6794E 05	0.3949E 05	-0.6981E 04	0.1693E 05