STRUCTURAL ENGINEERING

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TWO PAPERS ON APPLICATIONS OF FUZZY SETS

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APPLICATION OF FUZZY MULTI-CRITERIA ANALYSIS TO DAMAGE ASSESSMENT OF STRUCTURES

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ABSTRACT

In this paper, an attempt is made to introduce qualitative information in the damage assessment of structures using fuzzy sets theory. By using the fuzzy quantification theory, the weighting of each damage factor is given through the data obtained from the past observations or inspections. The rank of damages in underlying structures is determined by means of a multi-criteria analysis, which is one of the comprehensive evaluation systems of alternatives and can be used to deal with the rated values of damage factors. The results which are obtained from the multi-criteria analysis are interpreted by using the concept of semi-ordering.

INTRODUCTION

It is widely recognized that a considerable number of structures presently require repairing and alteration. Under this situation, it is a timely and important task to establish a method of evaluating the damage state of structures. However, damage assessment of structures is very difficult because of the lack of available data and the complex mechanism of analysis [1-4].

In this paper, an attempt is made to introduce qualitative information in damage assessment using fuzzy sets theory. By using the fuzzy quantification theory, the weighting of each damage factor is given through the data obtained from the past observations or inspections. Damage factors are chosen as items whose grades of membership to each category are rated by using linguistic variables such as "very small", "small", "medium", "large", "very large". Similarly, the external criteria (e.g., damage state) are given in the verbal form of "no damage", "slight damage", "moderate damage", "severe damage", "destructive damage", through the subjective assessment of engineers.

By using a weighting ratio, the rank of damages in underlying structures is determined by means of the multi-criteria analysis, which is one of the comprehensive evaluation systems of alternatives and can be used to deal with the rated values of damage factors. Moreover, if the structures under consideration increase in number, the new data can be used effectively to lead to a reasonable estimation. Here, the multi-criteria analysis is performed according to the extension principle in fuzzy algebra. To make the calculation easier, L-R type fuzzy numbers are adopted to represent all the linguistic variables. Finally, the results which are obtained from the multi-criteria analysis are interpreted by using the concept of semi-ordering. A numerical example is presented to illustrate the applicability of the method developed herein.

DETERMINATION OF WEIGHTS BASED ON FUZZY QUANTIFICATION THEORY

There are several kinds of information which are available in the damage assessment of structures. For example, we can consider such information as 1) the examination of design documents, 2) the visual examination, 3) the field testing, 4) the laboratory testing, 5) the structural analysis [5]. While the second information (i.e., item 2)) is easier to collect, it is usually obtained in a qualitative manner which is meaningful but not precisely defined. In other words, the results of observation or inspection are generally reported in such evaluation forms as being ranked by A (severely damaged), B (damaged), C (slightly damaged), etc. The boundaries of A and B or B and C are not sharp even according to the existing inspection manuals.

In this paper, an attempt is made to deal with the ambiguity or vagueness associated with the subjective judgement in a quantitative way. To bridge between the subjective judgement and the objective analysis, we use the fuzzy sets theory [6] through which a more meaningful solution may be obtained for complex problems in the state of nature. By using type II fuzzy quantification theory [7], the weighting of each damage factor is determined. The fuzzy quantification theory [8] was proposed for the regression or discrimination analysis of qualitative data. In particular, type II theory is useful in the discrimination of the statistical data.

Here, damage factors are chosen as "items" whose grades of membership to each "category" are rated by using linguistic variables such as "very small (Vs)", "small (S)", "medium (M)", "large (L)", "very large (Vl)". For example, S, M and L are characterized as shown in Fig. 1. Similarly, "external criteria" (e.g., damage states) are given in the verbal form of "no damage (N)", "slight damage (Sl)", "moderate damage (M)", "severe damage (Se)", "destructive damage (D)".

The fuzzy quantification analysis is carried out by using the discrimination function $y(x_\alpha)$ [9]:

$$y(x_\alpha) = \sum_{i=1}^{K} \sum_{k=1}^{l_i} a_{ik}\, \mu_{A_{ik}}(x_\alpha), \quad \alpha = 1, 2, \ldots, n \qquad (1)$$

where $\mu_{A_{ik}}$ is the membership function of fuzzy category A_{ik}, a_{ik} is a category weight, K is the total number of items, l_i is the item number of the i-th category, and n is the sample number. As a measure of discrimination, the ratio of variances, η^2, is employed:

$$\eta^2 = \frac{\sigma_G^2}{\sigma^2} \qquad (2)$$

where σ^2 and σ_G^2 denote the fuzzy total variance and the fuzzy variance between groups, respectively.

$$\sigma^2 = \frac{1}{N} \sum_{r=1}^{M} \sum_{\alpha=1}^{n} \{ \sum_{i=1}^{K} \sum_{k=1}^{l_i} (\mu_{A_{ik}}(x_\alpha) - \bar{\mu}_{A_{ik}}) a_{ik} \}^2 \mu_{B_r}(x_\alpha) \quad (3)$$

$$\sigma_G^2 = \frac{1}{N} \sum_{r=1}^{M} \sum_{\alpha=1}^{n} \{ \sum_{i=1}^{k} \sum_{k=1}^{l_i} (\bar{\mu}^r_{A_{ik}} - \bar{\mu}_{A_{ik}}) a_{ik} \}^2 \mu_{B_r}(x_\alpha) \quad (4)$$

where $\bar{\mu}^r_{A_{ik}}$ is the mean value of A_{ik} with respect to each external criterion B_r, and $\bar{\mu}_{A_{ik}}$ is the mean value of A_{ik} with respect to the total external criterion.

The category weight which provides the best discrimination can be obtained by maximizing the ratio η^2. The maximum value of η^2 is found when its partial derivative with respect to a_{jl} is equal to zero.

$$\sum_{r=1}^{M} \sum_{\alpha=1}^{n} \sum_{i=1}^{K} \sum_{k=1}^{l_i} \mu_{B_r}(x_\alpha) (\bar{\mu}^r_{A_{ik}} - \bar{\mu}_{A_{ik}}) (\bar{\mu}^r_{A_{jl}} - \bar{\mu}_{A_{jl}}) a_{ik}$$
$$= \eta^2 \sum_{r=1}^{M} \sum_{\alpha=1}^{n} \sum_{i=1}^{K} \sum_{k=1}^{l_i} \mu_{B_r}(X_\alpha) (\mu_{A_{ik}}(x_\alpha) - \bar{\mu}_{A_{ik}})$$
$$(\mu_{A_{jl}}(x_\alpha) - \bar{\mu}_{A_{jl}}) a_{jl}$$
$$(j=1,\ldots,K; l=1,\ldots,l_j) \quad (5)$$

Eq. 5 implies that the underlying problem may result in an eigenvalue problem. The eigenvector corresponding to maximum eigenvalue provides the category weights which are sought. Using the category weights obtained, the weights of damage factors are determined as the "range" of items:

$$W_i = \max_k a_{ik} - \min_k a_{ik} \quad (6)$$

The larger the value of W_i is, the greater effect the factor gives to the damage assessment.

APPLICATION OF FUZZY MULTI-CRITERIA ANALYSIS TO DAMAGE ASSESSMENT

Concordance analysis [10], which is a representative multi-criteria analysis, has been developed to provide a comprehensive evaluation of alternatives. While this evaluating method is quite useful for the damage assessment of structures [11], it often suffers from the ambiguity or imprecision of the data obtained from the observations or inspections. To account for this kind of ambiguity or vagueness, linguistic variables defined by fuzzy sets are introduced into the concordance analysis.

In the fuzzy concordance analysis, the impact matrix $\tilde{P}$ and weight W are defined as

$$\tilde{P} = \begin{bmatrix} P_{11} & \cdots & P_{1J} \\ \vdots & P_{ij} & \vdots \\ P_{I1} & \cdots & P_{IJ} \end{bmatrix} \qquad (7)$$

$$W = [W_1, W_2, \ldots, W_i, \ldots, W_J] \qquad (8)$$

where $\tilde{P}_{ij}$ is a fuzzy set representing the damage state of the j-th factor for the i-th structure, W_j is the weight of the j-th factor, and I and J are the numbers of structures and factors. It is noted that W_i is not a fuzzy but crisp number. Using the fuzzy impact matrix $\tilde{P}$ and the weight W, the fuzzy discordance index $\tilde{d}_{ii'}$ is defined as follows:

$$\tilde{d}_{ii'} = \sum_{j \varepsilon D_{ii'}} \{W_j \frac{|\tilde{P}_{ij} - \tilde{P}_{i'j}|}{\max_{1 \leq i, i' \leq I} |\tilde{P}_{ij} - \tilde{P}_{i'j}|}\} \qquad (9)$$

in which $D_{ii'} = \{j \;|\tilde{P}_{ij} > \tilde{P}_{i'j}\}$. The numerator means the difference of the damage states regarding the j-th factor between the i-th and i′-th structures, while the denominator is a normalizing factor.

Since Eq. 9 is an equation of fuzzy quantities, its execution cannot be performed by ordinal arithmetic operations. Therefore, the extension principle [12] is adopted to compute the equation. In general, the extension principle is defined as follows:

$$\mu_{\tilde{y}}(t) = \max_{t=f(s_1,\ldots,s_n)} \min[\mu_{\tilde{X}_1}(s_1),\ldots,\mu_{\tilde{X}_n}(s_n)) \qquad (10)$$

where $\tilde{X}_i$ $(i = 1,2,\ldots,n)$ is a fuzzy set defined on the real number, $\mu_{\tilde{X}i}$ is its membership function, and $\mu_{\tilde{y}}$ is the membership function of $\tilde{y}$ $(= f(\tilde{x}_1,\ldots,\tilde{x}_n))$ which is a function of $\tilde{x}_i$. Because the direct computation of Eq. 10 is usually time-consuming, an approximation is employed herein; all fuzzy numbers are given by L-R type fuzzy numbers [13]. The L-R type fuzzy numbers are defined as

$$\mu_{\tilde{m}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right) & (x < m, \; \alpha > 0) \\ R\left(\frac{x-m}{\beta}\right) & (x > m, \; \beta > 0) \end{cases} \qquad (11)$$

where m is the median, and α and β are the measures of dispersion about left and right sides, respectively. The L function must satisfy the following conditions:

1) $L(-x) = L(x)$

2) $L(0) = 1$

3) $L(x)$ is a decreasing function on $[0, +\infty)$

Also, R function must satisfy the above conditions. Using the expression of L-R functions, any membership function can be specified by only three parameters m, α and β, as shown in Fig. 2.

$$\mu_{\tilde{m}}(x) \stackrel{\Delta}{=} (m, \alpha, \beta)_{LR} \qquad (12)$$

Then, the summation $\oplus$ and product $\odot$ of two fuzzy numbers $\tilde{m}$ and $\tilde{n}$ are easily calculated as follows:

$$(m,\alpha,\beta)_{LR} \oplus (n,\gamma,\delta)_{LR} = (m+n, \alpha+\gamma, \beta+\delta)_{LR} \qquad (13)$$

$$(m,\alpha,\beta)_{LR} \odot (n,\gamma,\delta)_{LR} \simeq (mn, m\gamma+n\alpha, m\delta+n\beta)_{LR} \qquad (14)$$

It should be noted that Eq. 14 is true only when m and n are positive.

For all pairs of the underlying structures, the fuzzy discordance index $\tilde{d}_{ii'}$ is computed and summarized as the fuzzy discordance matrix $\tilde{D}$:

$$\tilde{D} = [\tilde{d}_{ii'}] = \begin{bmatrix} - & \tilde{d}_{12} & \cdot & \tilde{d}_{1I} \\ \tilde{d}_{21} & - & \cdot & \tilde{d}_{2I} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \tilde{d}_{I1} & \tilde{d}_{I2} & \cdot & - \end{bmatrix} \qquad (15)$$

Using the above fuzzy discordance matrix, the fuzzy discordance dominance index $\tilde{D}_i$ is defined as

$$\tilde{D}_i = \sum_{i'=1}^{I} \tilde{d}_{ii'} - \sum_{i'=1}^{I} \tilde{d}_{i'i} \quad (16)$$

$\tilde{D}_i$ is a relative measure of the damage state of the i-th structure. Eq. 16 may be computed according to the extension principle.

As abovementioned, the damage measure $\tilde{D}_i$'s are obtained as fuzzy sets. Now the problem is how to interpret these fuzzy discordance dominance indices. It is often difficult to distinguish which is greater or smaller between two close fuzzy numbers. To determine the more practical ranking for the $\tilde{D}_i$'s obtained, the concept of semi-ordering [14] is used. The semi-ordering, $\lesssim$, between two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as follows [15]:

$$\tilde{A} \lesssim \tilde{B} \Leftrightarrow \tilde{B} = \widetilde{\max}\,(\tilde{A},\tilde{B}) \text{ and } \tilde{A} = \widetilde{\min}\,(\tilde{A},\tilde{B}) \quad (17)$$

where $\widetilde{\max}$ and $\widetilde{\min}$ are specified using the extension principle. Supposing that $\tilde{C} = \widetilde{\max}\,(\tilde{A},\tilde{B})$ and $\tilde{C}' = \widetilde{\min}\,(\tilde{A},\tilde{B})$, the membership functions of $\tilde{C}$ and $\tilde{C}'$ are expressed as

$$\mu_{\tilde{C}}(z) = \max_{z=\max(x,y)} \min(\mu_{\tilde{A}}(X), \mu_{\tilde{B}}(y)) \quad (18)$$

$$\mu_{\tilde{C}'}(z) = \max_{z=\min(x,y)} \min\,(\mu_{\tilde{A}}(X), \mu_{\tilde{B}}(y)) \quad (19)$$

If the semi-ordering $\tilde{D}_i \lesssim \tilde{D}_j$ is satisfied for a pair of $\tilde{D}_i$ and $\tilde{D}_j$, it can be said that the i-th structure is less damaged than the j-th structure. On the other hand, if the semi-ordering is not satisfied, the two structures don't differ in their damage states.

NUMERICAL EXAMPLE

As an illustrative example, consider the diagonosing problems of bridges for earthquake damage. Usually, the following items are considered in the inspection of the damage state of bridges; 1) name, 2) road and river, 3) site, 4) length and width, 5) construction year, 6) design specification, 7) visual examination, 8) documents, 9) ground conditions, etc. For the sake of simplicity, the results of items 5), 6), 7), 9) are used in this example. Assume that those items are evaluated by means of linguistic variables such as very small, small, medium, large, very large and crisp (C) (see Table 1). Here, crisp C means that there is no data about the item. As stated previously, the damage states (e.g., external criteria) are given in the verbal form of no damage, slight damage, moderate damage, severe damage and destructive damage. Using these fuzzy data, the ranges of each damage factor are obtained as shown in Table 2, through the fuzzy quantification analysis. In the calculation, the membership functions shown in Fig. 3 are used for Vs, S, M and L. The results indicate that items 2 and 9 play important roles in the damage assessment of bridges. Note that the results of Table 2 are normalized as the maximum value should be one.

By using the above weights of damage factors, the fuzzy concordance analysis is applied to evaluate the new ten bridges, whose impact matrix is shown in Table 3. In Table

4, the membership functions adopted for Vs, S, M, L, Vl are given, which are specified by the L-R type fuzzy numbers. Performing the fuzzy concordance analysis with these data, the fuzzy discordance dominance indices are obtained as shown in Fig. 4, where the results of five bridges are depicted. Based on the concept of semi-ordering, the results of Fig. 4 can be interpreted as shown in Fig. 5. Fig. 5 implies that the most damaged bridge is No. 6, the least damaged is No. 10, the next less damaged is No. 3, and the damage degrees of No. 2 and No. 9 are between No. 6 and No. 3 but they have no distinct difference, whereas the usual concordance analysis provides the damage order of No. 6, No. 2, No. 9, No. 3, No. 10.

CONCLUSIONS

In this paper, a method of evaluating the damage state is presented, which is important and useful for maintenance of bridge structures. Considering the complex mechanism and the lack of quantitative available data, fuzzy sets theory is introduced to couple the quantification theory and the multi-criteria analysis.

The main conclusions obtained through this investigation are as follows:

1. By using the fuzzy quantification theory, it is possible to account for the ambiguities involved in categorizing items and determining external criteria. This method can

provide rational and reliable weights for damage factors in a quantitative form.

2. The multi-criteria analysis is useful for the damage assessment of structures. Especially, the introduction of linguistic variables makes it possible to enhance the capability of the multi-criteria analysis in the treatment of available information. In other words, qualitative as well as quantitative informations are effectively considered.

3. Since there still exist ambiguities in the final solutions to the problem of damage assessment, the strict ranking of damaged structures may be frequently meaningless. Therefore, the concept of semi-ordering can be used to give more meaningful classification with which an economical program of repair and/or maintenance can be established.

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Table 1. Fuzzy Data Obtained from Fact-Finding Inquiry

	Bridge No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.	Superstructure	M	L	M	L	M	M	S	M	M	L	L	S	S	S	M	M	M	S	S	S
2.	Bearings	V_s	L	V_s	L	S	L	M	S	S	V_s	V_s	M	S	S	M	M	L	M	V_s	V_s
3.	Substructure	M	L	M	L	S	L	L	L	S	L	L	M	S	S	M	L	L	M	S	S
4.	Anchoring	S	S	S	S	S	L	S	S	S	S	L	S	S	S	S	S	S	S	S	S
5.	Ground condition I	M	C	L	C	C	C	C	C	M	C	C	M	S	M	M	C	C	L	S	C
6.	Ground condition II	L	L	M	M	M	S	M	S	S	S	S	S	L	L	L	S	M	S	S	S
7.	Ground condition III	S	L	L	L	S	L	L	S	S	L	L	L	S	S	S	L	S	L	S	S
8.	Constructed year	M	L	L	L	L	M	M	L	M	S	S	S	S	S	S	S	S	S	S	S
9.	Design specification	M	S	S	S	S	M	M	S	M	M	M	M	M	M	M	M	M	S	S	M
	Integrity rank	S_1	D	S_1	D	S_1	S_e	M	S_1	S_e	S_e	N	N	N	N	S_1	M	M	N	N	N

N: No S_1: Slight M: Moderate S_e: Severe D: Destructive

Table 2. Weights of Damage Factors

	J
1	0.001
2	0.858
3	0.119
4	0.001
5	0.184
6	0.015
7	0.001
8	0.465
9	1.000

Table 3. Impact Matrix

	i								
j	1	2	3	4	5	6	7	8	9
1	L	S	L	V_s	S	S	S	S	S
2	V_l	M	L	V_s	M	M	M	M	M
3	V_s	M	V_s	L	M	S	M	S	M
4	M	L	L	S	V_s	V_s	V_s	V_s	V_s
5	V_s	V_s	V_l	S	S	V_s	S	V_s	S
6	V_l	S	V_s	M	V_l	V_l	V_l	V_l	V_l
7	S	L	V_s	V_s	M	S	V_s	S	V_l
8	V_l	S	M	V_s	L	V_l	L	M	L
9	L	V_l	L	V_s	S	S	S	S	S
10	M	V_s	V_l	V_s	S	M	M	S	V_s

Table 4. Membership Functions Defined by L-R Fuzzy Numbers

M.F. / L.V.	Left Side	Right Side	Parameters
Very small	0	$y = A(\frac{x-B}{C})^2 + 1$	A = -25, B = 0, C = 0.1
Small	$y = A(\frac{x-B}{C})^2 + 1$	$y = A(\frac{x-B}{C})^2 + 1$	A = -25, B = 0.2, C = 0.1
Medium	$y = A(\frac{x-B}{C})^2 + 1$	$y = A(\frac{x-B}{C})^2 + 1$	A = -25, B = 0.5, C = 0.4
Large	$y = A(\frac{x-B}{C})^2 + 1$	$y = A(\frac{x-B}{C})^2 + 1$	A = -25, B = 0.8, C = 0.1
Very large	$y = A(\frac{x-B}{C})^2 + 1$	0	A = -25, B = 1, C = 0.1

L.V.: Linguistic Variable M.F.: Membership Function

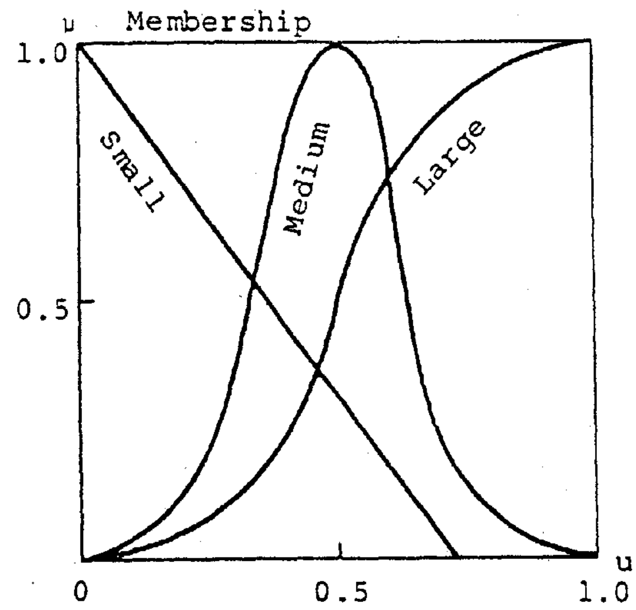

Fig. 1   Membership functions of linguistic variables

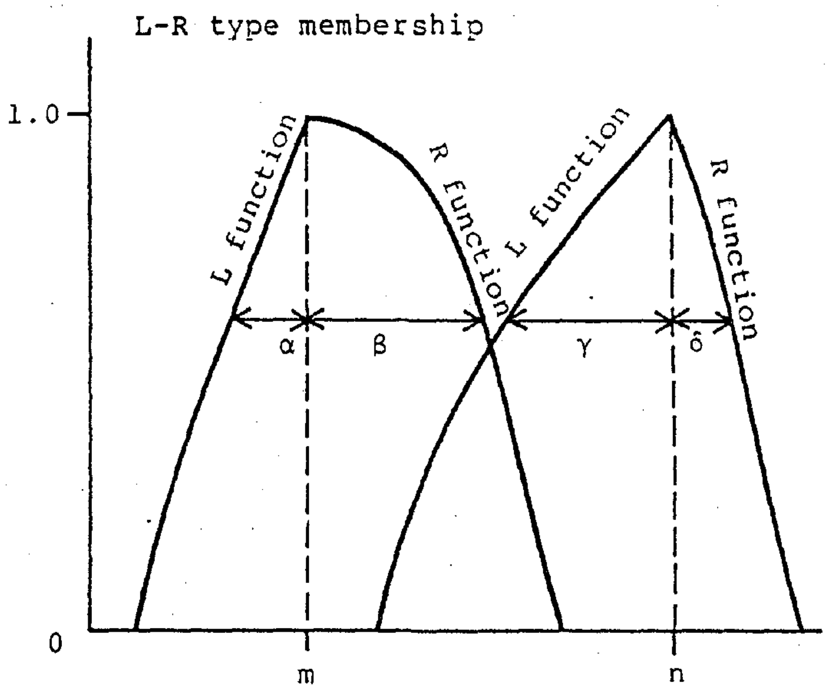

Fig. 2   L-R type fuzzy numbers

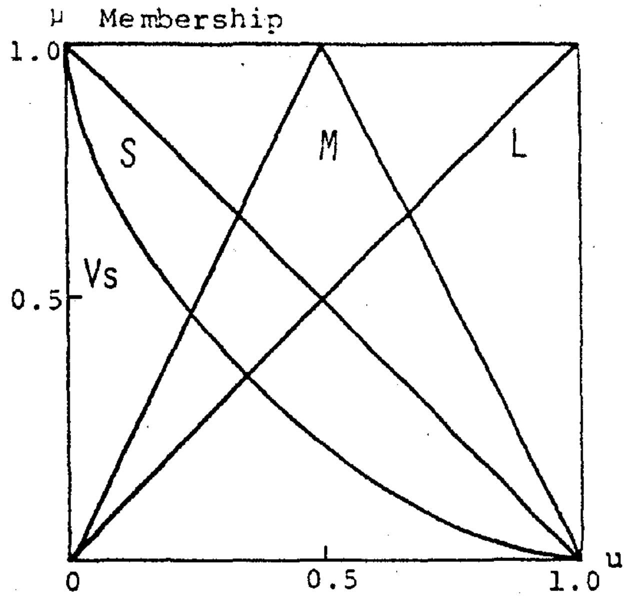

Fig. 3 Membership functions used for Vs, S, M, L

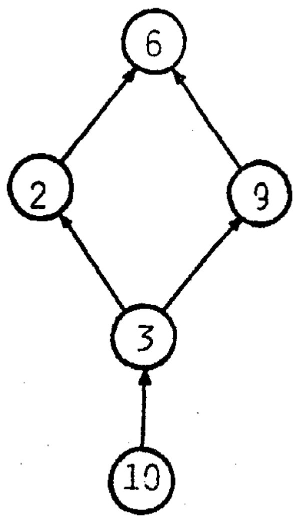

Fig. 5 Total evaluation of damage state

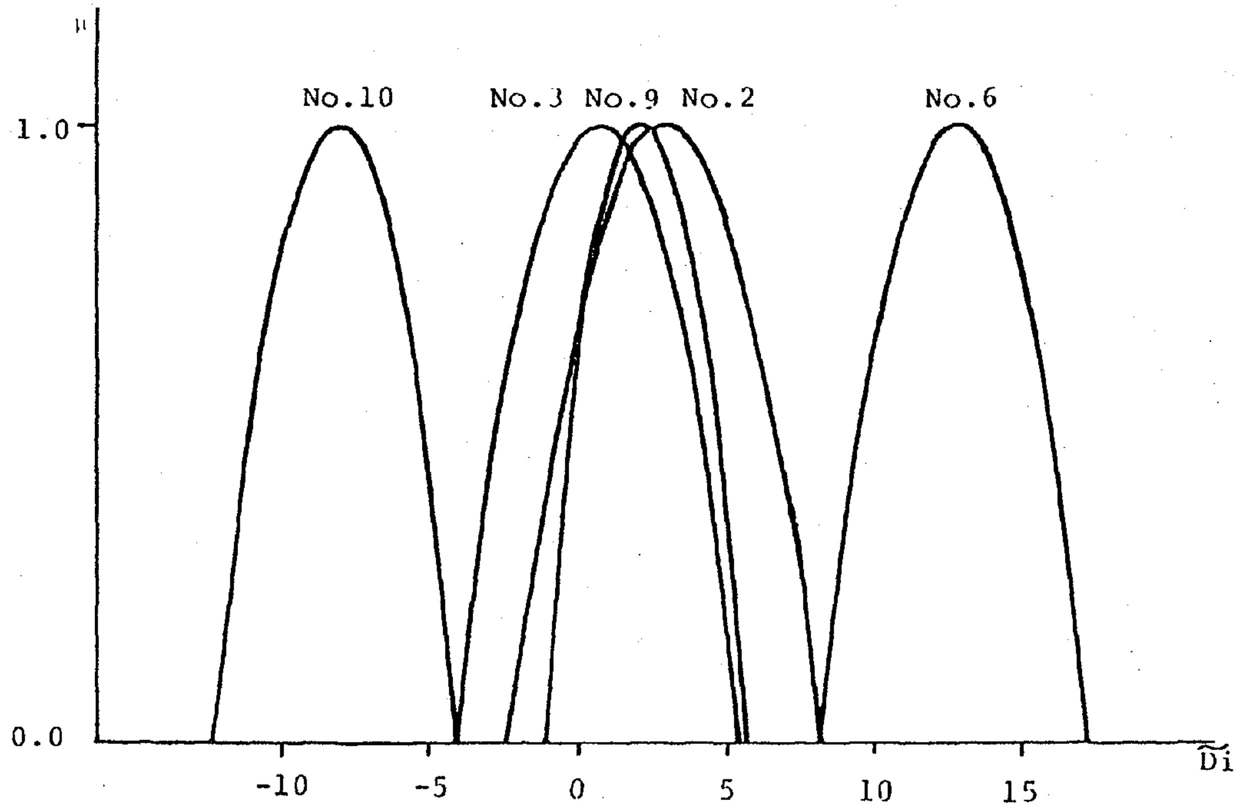

Fig. 4 Fuzzy discordance indices

PROBABILISTIC AND FUZZY REPRESENTATION OF REDUNDANCY IN STRUCTURAL SYSTEMS

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ABSTRACT

It is desirable to design structures with redundant load paths in order to minimize the potential of catastrophic failures. In mathematical analyses of relatively simple and idealized structures, the degree of redundancy is usually clearly defined. However, for more complex and real-world structures, it is difficult to define or quantify the amount and effect of redundancy. In this paper, several available definitions of redundancy are reviewed and examined. To account for the often unclear definition of structural redundancy, probabilistic and fuzzy interpretations of several alternative measures are given and discussed using several illustrative examples.

INTRODUCTION

It is desirable to design structures with redundant load paths to minimize the potential of catastrophic failures which could cause death and injury to people. For example, the accident of Alexander Keeland gave us a valuable lesson to re-recognize the importance of structural reliability. Meanwhile, several symposiums or workshops have been held with the emphasis on the necessity of redundancy [1].

Structures with multiple load paths (i.e., redundant structures) are said to be fail-safe, because these structures remain safe even with the failure of certain elements. However, the meaning of fail-safe in structural systems is somewhat different from that in electrical and mechanical systems. While electrical circuits or mechanical systems have redundant elements to compensate the function of the elements which are broken or failed, there are no such completely redundant elements in structural systems. Usually, some bracing or horizontal members of space trusses are said to be redundant members, but they may not always resist applied loads when certain primary members fail. This implies that the problem of structural redundancy should be discussed in conjunction with damage assessment. It is timely and important to evaluate how redundant members or elements of structures can be used to avoid a catastrophic failure in the presence of severe damage. Then, the structural redundancy has significant implications in terms of (a) the tolerance of the structure for flaws and overload, and (b) the strategy for inspection and repair throughout the life of the structure.

For relatively simple structures with no damage, their failure modes may be clearly defined. In addition, their probabilities of failure can be calculated using the theory of structural reliability. On the other hand, for complex structures with damage resulting from corrosion, cracks and other types of deterioration, their failure modes can no longer be defined in a clear manner. Moreover, the alternative load paths in actual structures and future environmental conditions are frequently unknown. When we wish to assess the effect of redundancy, it is difficult to evaluate the importance of various structural members using conventional analyses.

In this paper, available definitions of redundancy (such as redundant factor [2], reserve resistance factor, and residual resistance factor [3]) are reviewed and examined. To account for the often unclear definition of structural redundancy, probabilistic and fuzzy interpretations of several alternative measures are given and discussed. In addition, potential applications are illustrated using numerical examples.

SEVERAL DEFINITIONS OF STRUCTURAL REDUNDANCY

In mathematical analyses of relatively simple structures (*e.g., idealized trusses*), the degree of redundancy is clearly defined. However, the degree of redundancy itself does not necessarily express the safety level of the truss. It is, for instance, not always true that structures with a high degree of redundancy are safer than structures with a low degree of redundancy. The advantage of highly redundant trusses may be attained when the members are appropriately designed and carefully constructed. In order to illustrate the relationship between structural reliability and redundancy, consider a ten-bar truss as shown in Fig. 1. Because the truss has one degree of redundancy, the failure of any one member among members number 3 through 8 does not necessarily lead to the collapse. On the other hand, the failure of any one member among members number 1,2,9 and 10 immediately causes the collapse. Moreover, if the redundancy is created by very weak members, the degree of redundancy can be deceiving. Therefore, the structural redundancy should be considered from the standpoints of both member strength and system strength.

To account for the significant uncertainty in applied loads and resistances, reserve strength is needed in structural design. As a measure of reserve strength, reserve resistance factor (REF) is defined by the ratio of the ultimate strength to the design load [3].

$$REF = \frac{\text{environmental load at collapse}}{\text{design environmental load}} \quad (1)$$

Residual resistance factor (RIF) is also given by the ratio of the residual strength to the collapse strength [3].

$$RIF = \frac{\text{environmental load at collapse (damaged)}}{\text{environmental load at collapse (undamaged)}} \quad (2)$$

Product of REF and RIF indicates whether the damaged structure will survive the design load without suffering collapse. As an alternative measure of redundancy, redundant factor (RF) is defined as follows [2]:

$$RF = \frac{\text{intact strength}}{\text{intact strength-damaged strength}} \quad (3)$$

These factors are useful to evaluate the structural redundancy, and at least better than the degree of redundancy. There are, however, difficulties in calculating these factors, i.e., the estimation of the strength of damaged structures. The property and degree of damage can vary widely in individual structures depending on their environmental and other conditions. It is therefore doubtful that sufficient data will be collected to establish a precise analytical model. Moreover, there exists a continuum of damage states instead of the binary situation (failure or survival) as treated in the usual theory. Several investigators [4,5] established descriptive terms such as "slight", "moderate", and "severe" damage states, which are useful in practice but cannot be clearly defined for complex existing structures. In order to assess the effect of structural redundancy on the safety of damaged structures, it is desirable and useful to blend the structural analysis with the subjective feelings of inspectors or designers.

PROBABILISTIC AND FUZZY REPRESENTATION OF REDUNDANCY

If a structure is very simple, e.g., a uni-axial tensile bar, the collapse load is proportional to the limit strength of the material. Then, the structural redundancy is supplied by its member reserve strength. The reserve strength is a result of the conservative design procedure. As implicit reserves, the following sources are considered: (a) code safety factor, (b) material reserve strength, (c) member reserve strength, (d) corrosion allowance, (e) conservative hypothesis in analysis, and (f) nonstructural elements. In the above simple structure, the reserve strength can be calculated without much difficulty. Meanwhile, the complexity of a structure may add load-carrying capacity to the structure. This added capacity is due to the system redundancy, which is closely related to the degree of redundancy. As an example, consider a single bent frame shown in Fig. 2. In this case, the collapse load is no longer directly proportional to the ultimate strength of material or member, but depends on its topology and geometry and the applied load as well. The collapse load can be estimated by using the plastic theorem.

As mentioned previously, difficulty lies in the calculation of the residual resistance factor. Since the boundaries between several damage categories are unclear and imprecise, the damage state is represented in terms of linguistic variables which are defined by fuzzy sets. Thus, the residual resistance factor is also defined by a fuzzy set. To explain how to obtain the fuzzified residual strength, consider a single bent frame again. It is considered that the overall reserve strength can be represented by the overall collapse load. Based on the lower bound in the plastic theorem, the overall collapse load of the frame can be found by solving the following linear programming problem [6].

$$\begin{aligned} &\text{maximize} && \lambda \\ &\text{Subject to} && CM = \lambda F \\ &&& |M_i| \leq M_{p_i} \end{aligned} \qquad (4)$$

where

λ = amplification factor providing the collapse load λ F

F = applied load

M_i = bending moment at the i-th portion of the frame

M = bending moment vector consisting of M_i

M_{p_i} = plastic moment capacity corresponding to M_i

C = connection matrix

The above constraints represent the equilibrium and yield conditions.

Now assume that the plastic moment capacity M_{pi} changes to $\tilde{M}^*_{pi}$ due to some damage. As abovementioned, $\tilde{M}^*_{pi}$ is specified by a fuzzy set. Then, the collapse load of the frame with damage can be obtained as a solution of the following fuzzy linear programming problem [7]:

$$\text{maximize} \quad \tilde{\lambda}$$
$$\text{Subject to} \quad CM = \tilde{\lambda}F \tag{5}$$
$$|M_i| \leq \tilde{M}^{*}_{p_i}$$

where the symbol $\tilde{}$ denotes a fuzzy quantity. Using λ and $\tilde{\lambda}$, the fuzzy redundant factor $\tilde{r}$ can be defined as

$$\tilde{r} = \frac{\lambda}{\lambda - \tilde{\lambda}} \tag{6}$$

It is usually difficult to estimate the collapse load because of the uncertainties associated with the applied load, the structural strength, and other environmental conditions. To consider the variations of the load strength, REF is probabilistically defined as

$$REF = \frac{S_c}{S_D} \tag{7}$$

where S_c and S_D are random variables representing load and design load, respectively. In this paper, we define the reserve strength in terms of the probability of reserve strength, based on the definition of REF.

$$p = P[\frac{S_c}{S_D} < \nu] \tag{8}$$

where P[*] is the probability of event * and ν is a constant being greater than 1. On the other hand, we define the residual strength in terms of a fuzzy set. RIF is defined here as a fuzzy reduction factor $\tilde{\phi}$, which is evaluated by using such linguistic variables as "severe", "more or less", "slight", and "no" damages.

$$RIF = \frac{\tilde{S}_c^*}{S_c} = \tilde{\phi} \qquad (9)$$

where $\tilde{S}_c^*$ denotes the collapse load of damaged structures. Similarly to Eq. 8, the probability of residual strength can be defined as follows, using the ratio of $\tilde{S}_c^*$ to the design load S_D.

$$\tilde{p}^* = P[\frac{\tilde{S}_c^*}{S_D} < \nu] \qquad (10)$$

It should be noted that $\tilde{p}^*$ is a fuzzy probability. By using the reduction factor $\tilde{\phi}$, Eq. 10 is rewritten as

$$\tilde{p}^* = P[\frac{\tilde{\phi} S_c}{S_D} < \nu]$$

$$= P[\frac{S_c}{S_D} < \frac{\nu}{\tilde{\phi}}] \qquad (11)$$

From Eqs. 8 and 11, we can define another redundancy factor as a new measure of structural ability to sustain damage without collapse.

$$\tilde{r}_f = \frac{\text{probability of reserve strength}}{\text{probability of residual strength}} \qquad (12)$$

where $\tilde{r}_f$ varies within the range between 0 and 1 with $\tilde{r}_f = 0$ indicating a nonredundant structure and $\tilde{r}_f = 1$ indicating a "completely" redundant structure.

For such simple cases as the uni-axial tensile bar, $\tilde{p}^*$ can be easily calculated as follows provided that the joint probability density function of S_c and S_D, $f_{S_c S_D}(s_c, s_D)$, and $\tilde{\phi}$ are given [8].

$$\tilde{p}^* = \iint_\Omega f_{S_c S_D}(s_c, s_D)\, ds_c ds_D \tag{13}$$

where Ω is the domain of $S_c/S_D < \nu/\tilde{\phi}$. It is, however, noted that Eq. 13 must be executed by means of the extension principle [9] because $\tilde{\phi}$ is a fuzzy quantity. Supposing that S_c and S_D are statistically independent, Eq. 13 is expressed as

$$\tilde{p}^* = \int_0^\infty \int_0^{\frac{s_D}{\tilde{\phi}}} f_{S_c}(s_c)\, f_{S_D}(s_D)\, ds_c ds_D$$

$$= \int_0^\infty F_{S_c}\left(\frac{s_D}{\tilde{\phi}}\right) f_{S_D}(s_D)\, ds_D \tag{14}$$

where F_{S_c} is the cumulative distribution function of S_c and f_{Sc} and f_{SD} are the probability density functions of S_c and S_D, respectively.

For structures with multiple load paths, it is necessary to consider the effect of system redundancy. As it is well known, system redundancy of a structure is influenced by its topology and geometry, the nature of component failure, and the relationship between member failure modes. As an illustrative example, consider truss as shown in Fig. 3. The mechanism of its system failure is ideally described by use of the fault tree diagram, as shown in Fig. 4. In this case, p is calculated as [10].

$$p = (1 - p_1)(1 - p_2 p_3) \tag{15}$$

where p_1, p_2 and p_3 are the probabilities of reserve strength with respect to members 1, 2 and 3, respectively. Similarly, $\tilde{p}^*$ is calculated as follows:

$$\tilde{p}^* = (1 \ominus \tilde{p}_1^*) \odot (1 \ominus \tilde{p}_2^* \odot \tilde{p}_3^*) \qquad (16)$$

where $\ominus$ and $\odot$ denote the subtraction and multiplication obeying the extension principle, respectively. In addition to the calculation of p and $\tilde{p}^*$, the fault tree analysis is useful for rating member importance. For structures without damage, member importance can be determined by use of the importance analysis in the fault tree strategy. The redundancy factor $\tilde{r}_f$ developed herein can be used as an alternative measure of member importance for structures with damage.

$$I_i = \tilde{r}_{fi} = \frac{p}{\tilde{p}_i^*} \qquad (17)$$

where $\tilde{p}_i^*$ is the probability of residual strength which is calculated for the structure with damage in only the i-th member. The smaller $\tilde{r}_f$, the greater the member force. This is why the damage on that member provides a greater influence on the system carrying capacity.

ILLUSTRATIVE EXAMPLES

Example 1. Single-Bent Frame

Assume that the beam is slightly damaged and the columns are very slightly damaged. In this case, the moment capacities of the beam and column, M_{pB} and M_{pc}, are changed as

$$M_{pB}^* = \tilde{\phi}_B \cdot M_{pB} \qquad (18)$$

$$M_{pc}^* = \tilde{\phi}_c \cdot M_{pc} \qquad (19)$$

where the reduction factors Φ_B and Φ_c are given by "slight" and "very slight", whose membership functions are shown in Fig. 5. In Fig. 5, the abscissa u is the support of Φ such that u = o means collapse and u = 1 means no damage. Solving the fuzzy linear programming problem given by Eq. 5, the fuzzy redundancy factor $\tilde{r}$ is calculated as shown in Fig. 6. In Fig. 6, the solid line means the redundancy factor using the overall collapse load, whereas the broken line means the redundancy factor using the weakest member strength. These results imply that the influence of structural damage on the overall strength is greater than that on the member strength.

Example 2. Uni-axial Tensile Bar

This example is used to explain how to calculate the probability of residual strength. Assume that S_c and S_D are statistically independent and follow the lognormal distributions whose mean values and coefficients of variation are (240 MPa, 0.1) and (140 MPa, 0.2), respectively. It is also postulated that ν is one and Φ is evaluated as "slight" whose membership function is specified on Fig. 5. Using Eq. 14, the probability of residual strength is expressed as

$$\tilde{p}^* = \Phi\left(- \frac{\ln \tilde{\Phi}\, \mu_{S_c}/\mu_{S_D}}{\sqrt{v_{S_c}^2 + v_{S_D}^2}}\right)$$

$$= 0.2/0.604 + 0.4/0.45 + 0.6/0.315 + 0.8/0.208 + 1/0.131$$

$$+ 0.8/0.079 + 0.6/0.046 + 0.4/0.026 + 0.2/0.015 \qquad (20)$$

where V_{S_c} and V_{S_D} are the coefficients of variation of S_c and S_D, respectively. It is noted that the effect of the reduction factor is neglected in the calculation of V_{S_c}. The above results indicate that 0.131 is the most dependable, but other values have some possibility of residual strength. From the definition of the redundancy factor (Eq. 12), $\tilde{r}_f$ is calculated as Fig. 7. Comparing these diagrams of $\tilde{r}_f$, it is possible to rate the degree of residual strength in the more informative manner.

Example 3. A Lifeline System

Consider an example of lifeline system to demonstrate the efficiency of the redundancy factor $\tilde{r}_f$ proposed herein. The problem is what degree of decreasing in serviceability will be estimated when lifeline systems suffer from certain kinds of damage. For the sake of simplicity, consider a simple water supply network shown in Fig. 8 [11]. In this case, serviceability is measured by using the probability that the flow at node B is less than a required amount.

Considering the event that no water is supplied at node B as the top event (failure event), we can obtain the fault tree diagram shown in Fig. 9. It is, however, not easy to calculate the occurrence probability of the top event, because there exist several basic events which appear twice in the fault tree. To solve the complex fault tree problem, the concept of cut sets are adopted. If the flow capacities of individual links are deterministic, the flow capacity of the network system is represented by the capacity of the minimum cut set [12]. However, since there are uncertainties in the capacity of the individual link, the network capacity will be well-defined in terms of probability. The mean capacity and corresponding coefficients of variation of the links are summarized in Table 1. In this case, the capacity of a cut set is the sum of the link capacities:

$$C_i = \sum_{j=1}^{n_i} C_{ij} \tag{21}$$

where C_i is the capacity of the i-th cut set and C_{ij} is the capacity of the j-th link, and n_i is the number of the links. For the eight possible cut sets shown in Table 2, the fuzzy probabilities of less than 200 cfs flow are given in Fig. 10. These results are obtained on the assumptions that link 1 is very slightly damaged and link 6 is slightly damaged, but other links have no damage. It is also postulated that the individual link capacities C_{ij} are uncorrelated and the effects of the damage on the coefficients of variation of C_{ij} can be neglected. In Fig. 10 it can be seen that this damage provides no influence for cut sets No. 2, No. 4, No. 5 and No. 6, but causes the reduction of the functional performance of cut sets No. 1, No. 3, No. 7 and No. 8. While the usual redundant factor (e.g., Eq. 3) cannot distinguish the damage state of cut set 1 from that of cut set 3, the present factor $\tilde{r}_f$ enables us to elucidate their difference by means of the shapes of their membership grades. This is also true in the comparison of the damage states of cut set 7 and cut set 8.

Using the PNET method [13], the probability that the network flow is less than 200cfs can be easily calculated, which accounts for the effect of the correlations between the cut sets. Analogously to the PNET method, the fuzzy probability for the damage state can be derived, that is, the capacity of the individual link is multiplied by the reduction factor $\tilde{\phi}$ and all the arithmetic operations are executed according to the extension principle. Fig. 11 indicates the calculated result of $\tilde{r}_f$ that the damage causes no serious deterioration in the capability of water supply. However, maintenance should be carefully carried out because the possibility of great influence still remains. Using Eq. 17, the importance of the individual links is calculated, as shown in Fig. 12. In this case, $\tilde{r}^*_{fi}$ is defined as the fuzzy probability that the flow capacity at node B is less than 200 cfs when only the i-th link is slightly damaged. Fig. 12 implies that the order of link importance is No. 4, No. 1, No. 7, No. 5, No. 2, No. 3, and No. 6. The link No. 6 can be considered to be a redundant link the failure of which will not result in a functional failure of the water supply network.

CONCLUSIONS

In this paper, several definitions of structural redundancy are reviewed and examined from the viewpoint of its importance in a damaged condition. While a structure is highly redundant, some elements or members may not be effective in carrying the applied load, their presence will contribute significantly to the system safety in a damaged state. It is, therefore, important to evaluate how redundant elements or members can contribute to avoid a catastrophic failure when the structure suffers from damage. However, damage assessment intrinsically includes unclear and imprecise features due to the individuality of structures or the variety of environmental conditions.

To consider the often unclear definition of structural redundancy, probabilistic and fuzzy interpretations of several alternative measures are given and discussed by using several illustrative examples. By representing damage states in terms of fuzzy sets, the redundancy factor can be fuzzified, which may provide a more informative basis for the design of damage-tolerant structures. To calculate the collapse load (or system reserve strength), it is convenient and desirable to use the fuzzy mathematical programming technique.

Both the reserve resistance factor and the residual resistance factor are probabilistically represented to consider the uncertainties associated with applied loads and structural resistances. In addition, the residual resistance factor is defined by introducing a fuzzy reduction factor which is given in terms of linguistic variables such as severe, more or less, and slight. Based on probabilistic and fuzzy considerations, a new redundancy factor is proposed herein. This factor may be used as a measure of the system redundancy and the importance of individual elements or members to the system strength in a damaged condition. Using this the fault tree analysis and the system reliability theory are useful for evaluating the redundancy factor numerically.

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Table 1. Statistics of Link Capacities

Link No.	Mean Capacity	C.O.V. of Capacity
1	70 cfs	0.15
2	80	0.15
3	90	0.15
4	100	0.15
5	100	0.15
6	40	0.15
7	70	0.15

(After Ang and Tang [11]).

Table 2. Possible Cut Sets

Cut Set No.	Links in Cut Set
1	1,4,7
2	2,4,7
3	1,3,5
4	1,4,-6,5
5	2,3,5
6	2,4,5,-6
7	1,3,6,7
8	2,3,6,7

"-6" denotes the fact that water in link 6 flow in an opposite direction. Hence its capacity is not included when calculating the total capacity of the cut set. (After Ang and Tang [11])

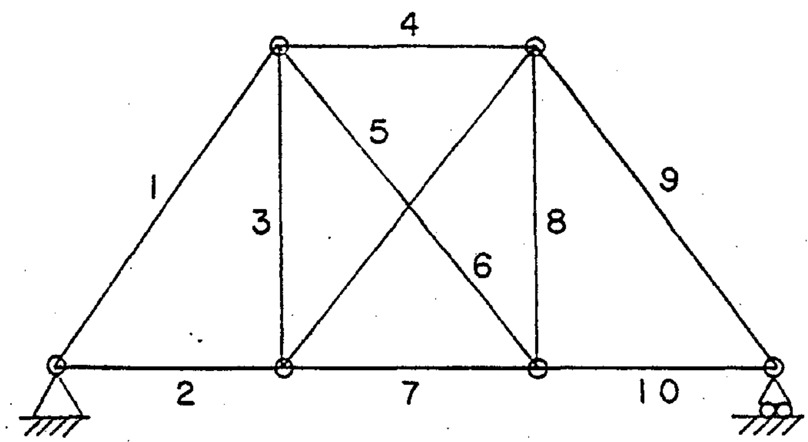


Fig. 1 A Ten-Bar Truss

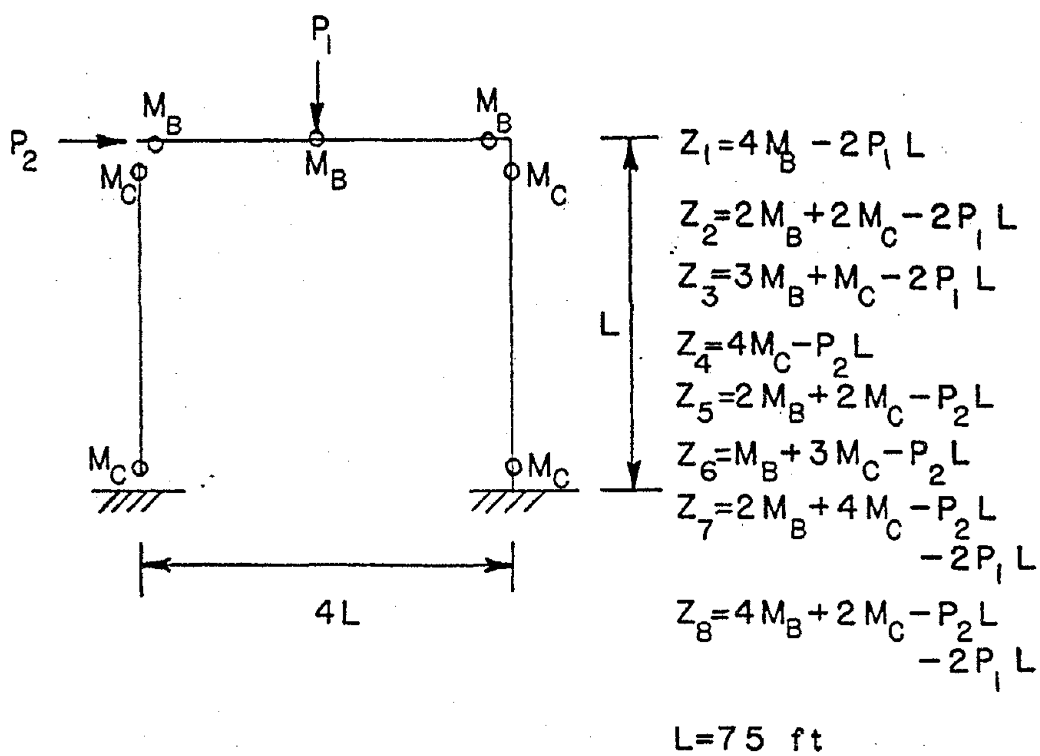


Fig. 2 A Single-Bent Frame

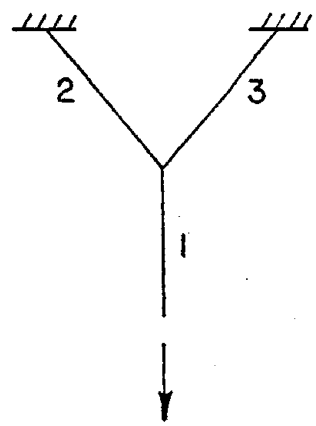
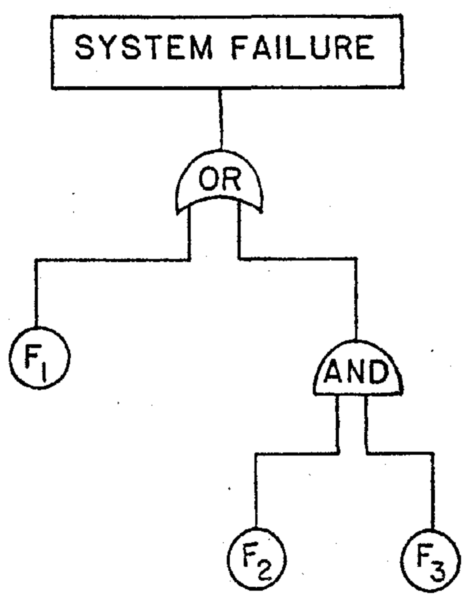


Fig. 3 A Three-Bar Truss



F_1, F_2, F_3, FAILURE EVENTS OF MEMBERS 1, 2, AND 3

Fig. 4 A Fault-Tree Diagram

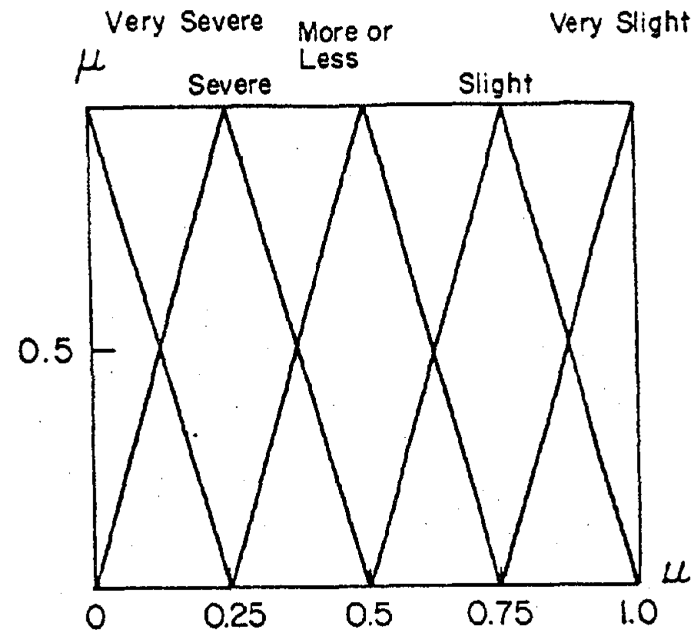


Fig. 5 Membership Functions for Linguisitc Variables

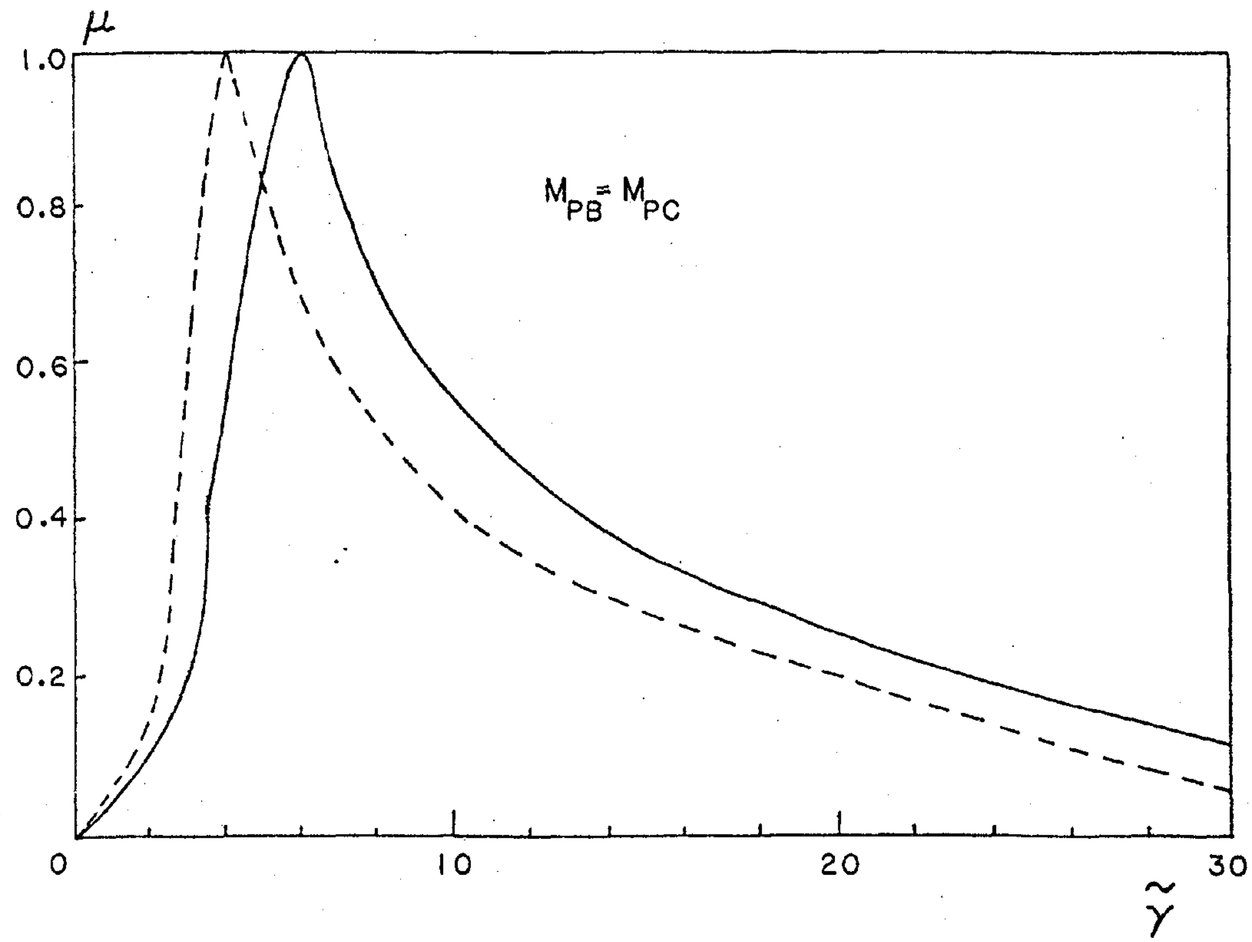


Fig. 6 Membership Functions for Redundancy Factor

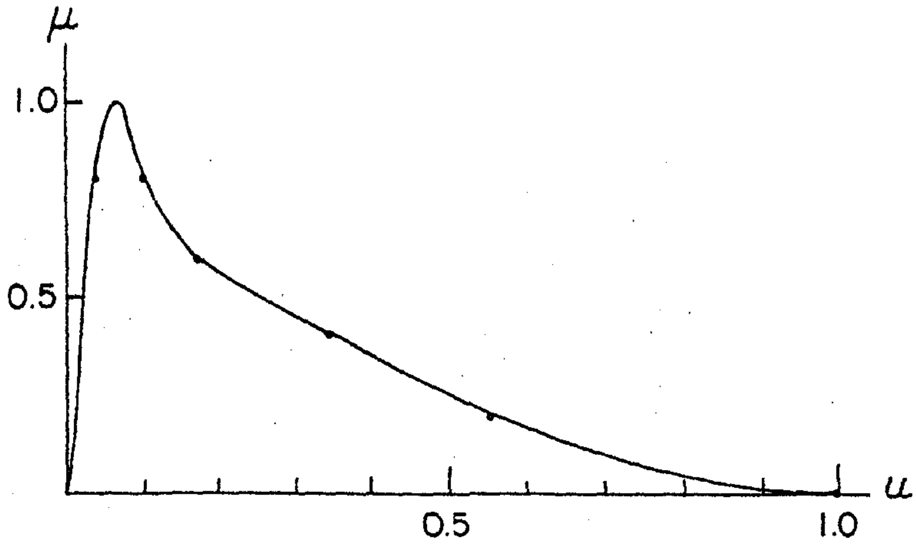


Fig. 7 A Membership Function for Redundancy Factor

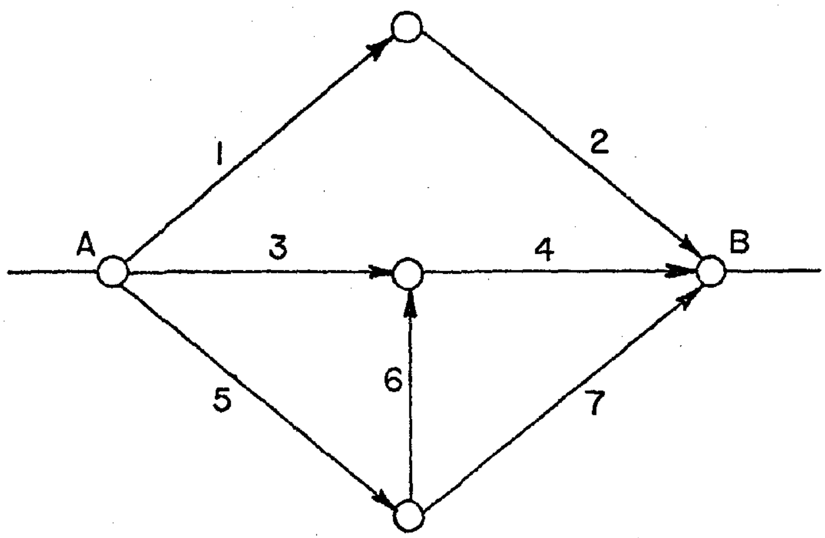


Fig. 8 A Lifeline System

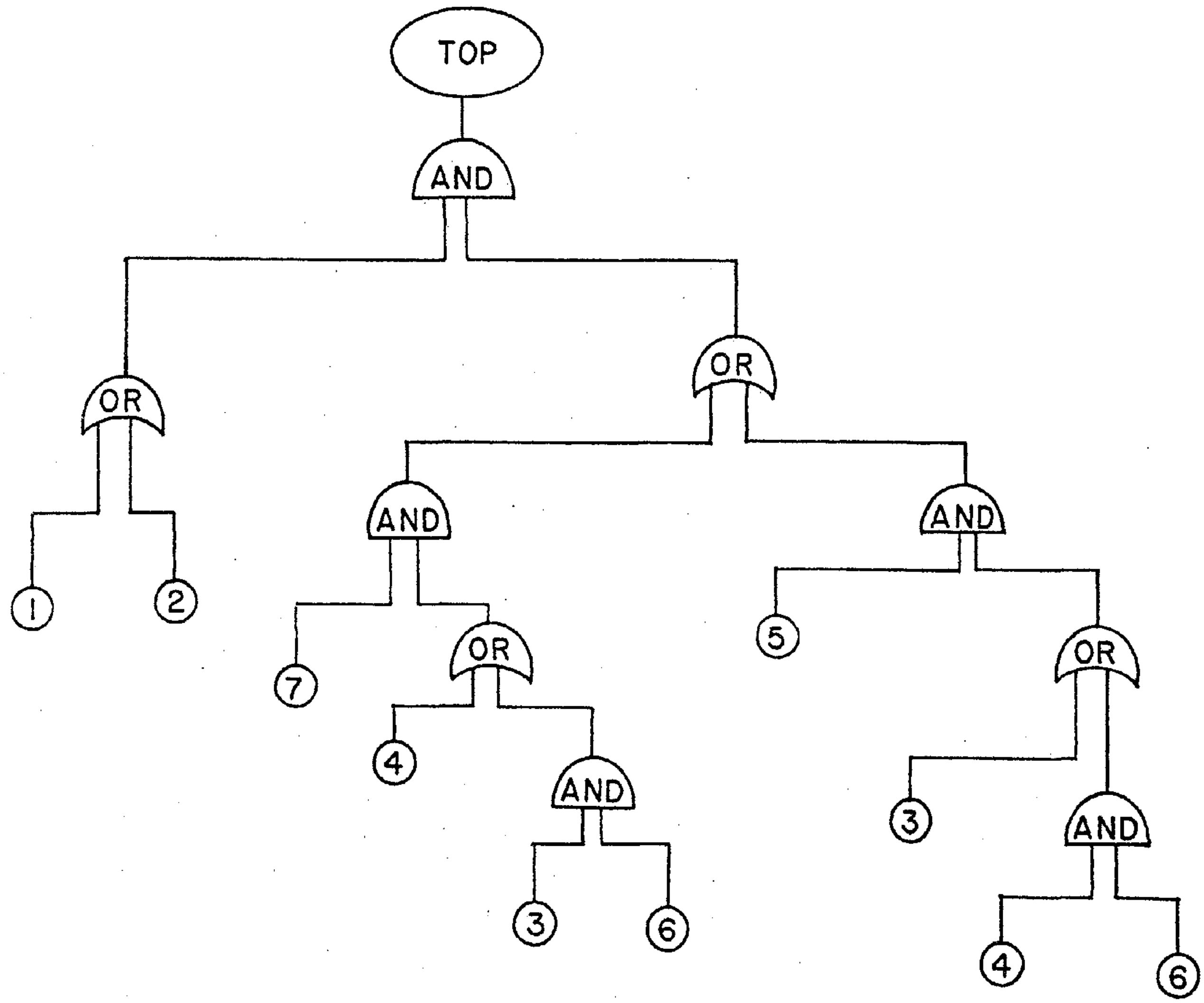


Fig. 9 A Fault-Tree Diagram

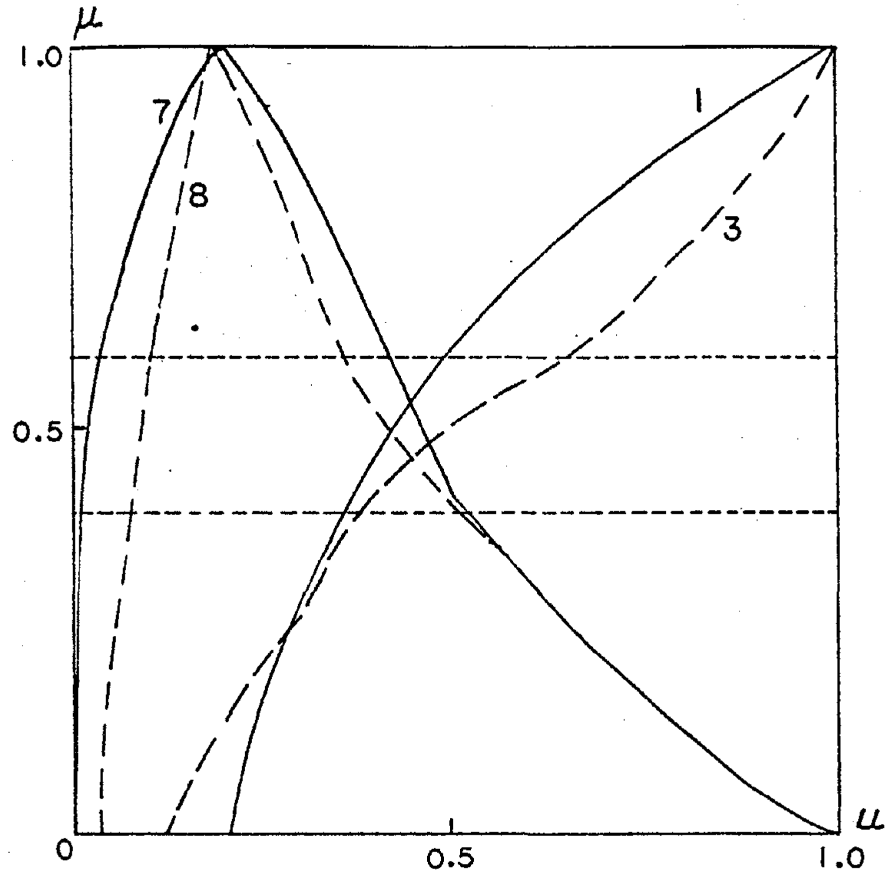


Fig. 10 Membership Functions for Redundancy Factors of Each Cut Set

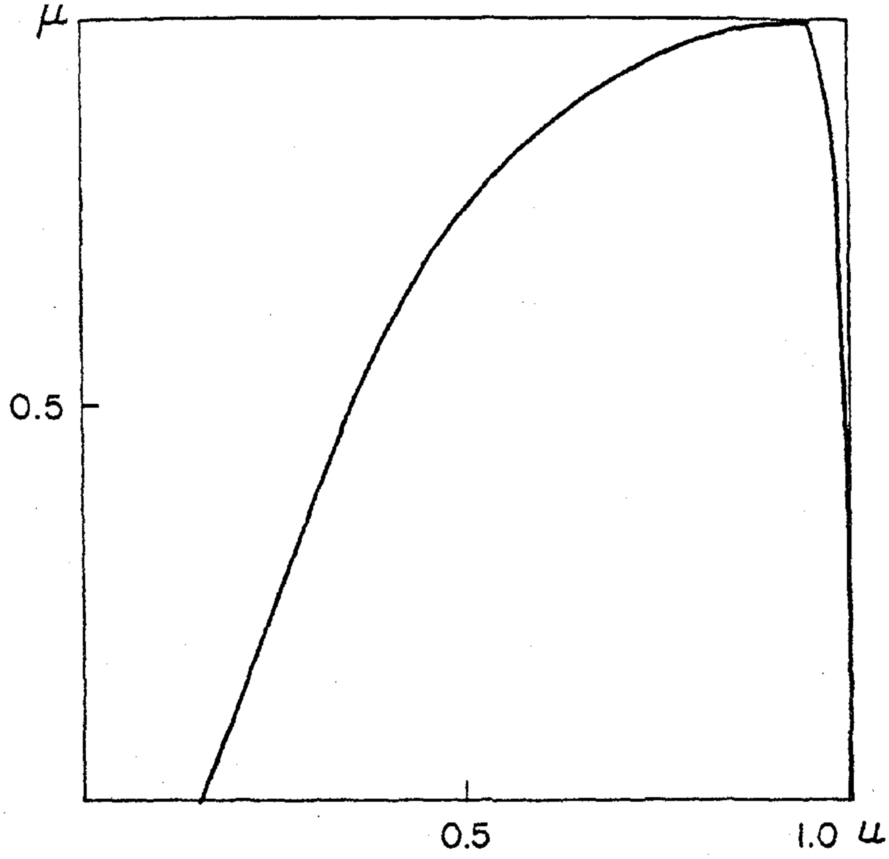


Fig. 11 Membership Function for System Redundancy

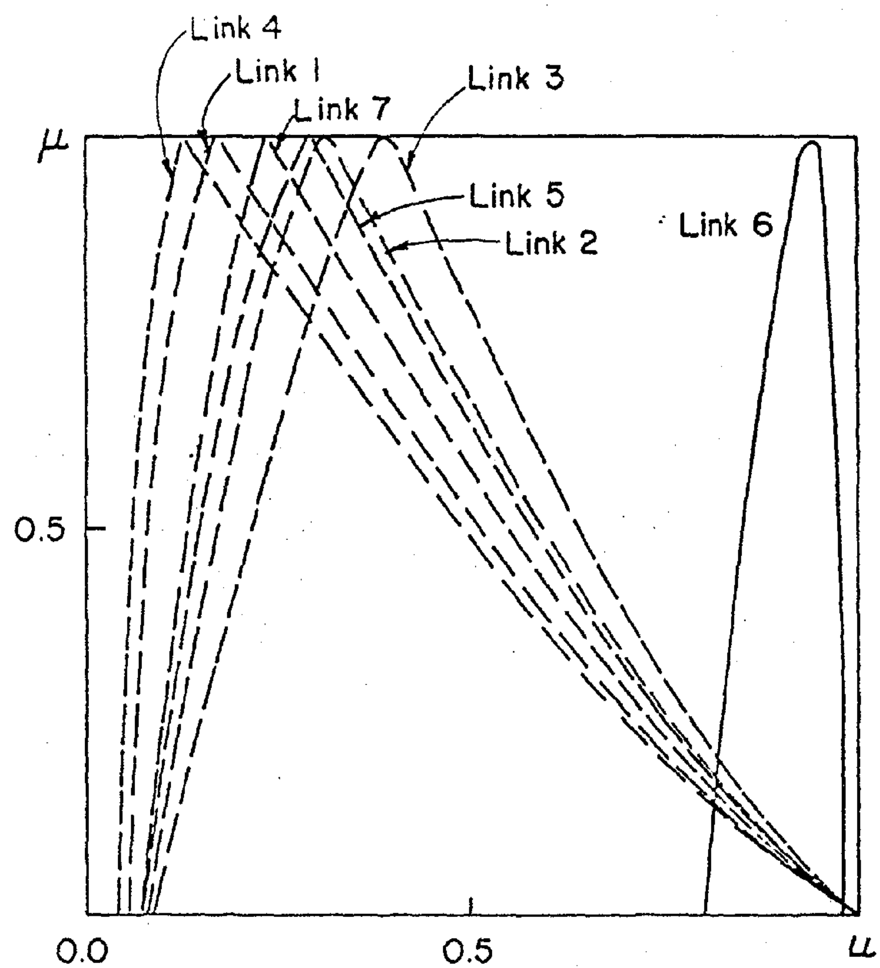


Fig. 12 Membership Functions for the Importance of Each Link