Seismic Response Analysis of Multiply Connected Secondary Systems

by

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TECHNICAL REPORT OF RESEARCH SUPPORTED BY
THE NATIONAL SCIENCE FOUNDATION
Under Grant No. CEE-8208897

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August 1985

ACKNOWLEDGEMENTS

The work on this report was funded by the National Science Foundation, Grant No. CEE-8208897, with Dr. M. P. Gaus, Dr. J. E. Goldberg and Dr. S. C. Liu as its Program Directors. This support is gratefully acknowledged. Any opinion, findings and conclusions or recommendations expressed in this report are those of the writers and do not necessarily reflect the views of the National Science Foundation.

BIBLIOGRAPHIC DATA SHEET	1. Report No. VPI-E-84-30	2.	3. Recipient's Accession No. PB87 129334/AS

4. Title and Subtitle Seismic Response Analysis of Multiply Connected Secondary Systems	5. Report Date August 1985
	6.
7. Author(s) R. A. Burdisso and M. P. Singh	8. Performing Organization Rept. No.
9. Performing Organization Name and Address Department of Engineering Science & Mechanics Virginia Polytechnic Institute & State University 227 Norris Hall, Blacksburg, Virginia 24061	10. Project/Task/Work Unit No.
	11. Contract/Grant No. CEE-8208897
12. Sponsoring Organization Name and Address National Science Foundation Washington, D.C. 20550	13. Type of Report & Period Covered
	14.

15. Supplementary Notes

16. Abstracts

An analytical formulation for seismic analysis of multiply supported secondary systems is developed. The formulation is based on the random vibration theory of structural systems subjected to correlated inputs at several points. The response of the secondary systems is expressed as a combination of the dynamic, pseudo-static and cross response components. The dynamic part is associated with the inertial effect induce by the support accelerations. The pseudo-static part is due to the relative displacement between supports, and the cross part takes into account the correlation between these two parts of the response. The seismic input in this approach is defined in terms of auto and cross pseudo-acceleration and relative velocity floor spectra. Also the information about floor displacements and velocities as well as their correlations is required. By the analysis of the supporting primary structure, these inputs can be directly obtained from the ground response spectra. The application of the proposed method is demonstrated by numerical examples.

17. Key Words and Document Analysis. 17a. Descriptors

Secondary Systems, Primary Systems, Miltiple support systems, Piping Systems, Power plant facilities, Seismic analysis, Correlated inputs, Vibrations, Structural dynamics, Response spectra, Spectrum analysis, floor response spectra, auto floor spectra, Cross floor spectra.

17b. Identifiers/Open-Ended Terms

17c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
Release Unlimited	UNCLASSIFIED	159
	20. Security Class (This Page) UNCLASSIFIED	22. Price

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CHAPTER I

INTRODUCTION

I.1 GENERAL BACKGROUND

For seismic design of important industrial facilities, the earthquake input loading is often prescribed in terms of the ground response spectra [8,14,15]. For such loadings, a proper seismic design of the primary systems as well as the subsystems is of vital importance for safe operation of the facility. For the analysis of the primary systems, rational analytical method which can effectively use the ground spectra as inputs have already been developed. However, for the analysis of the secondary systems, especially the systems with multiple supports (such as piping attached at several points of a main structure), the methods are still in the development stage, and research efforts [1,2,9-12,16,18,27,30] are being continually reported in the literature.

In the current practice, the time history and the single response spectrum methods are commonly used [1] to analyze such multiply supported subsystem. The time history method, though analytically most accurate for a given earthquake motion time history, does not provide unique response results suitable for a design. To obtain the design response, i.e. the response which can be used for the design of these subsystems, one must consider a set of time histories as inputs in the analysis. This, however, requires a large computational effort and thus is not be economically feasible.

The response spectrum method, on the other hand, is computationally inexpensive. Currently, it is a common practice to use the envelope of all the support point floor spectra as input in this approach. To account for the effect of the relative displacements between the supports, some approximate and conservative methods are employed [1]. These methods, however, do not account for the effect of the correlation between the support motions and may, sometimes, give overly conservative results.

In this report, the analysis of the multiply connected secondary systems subjected to correlated random excitations at the supports is examined in details. The random vibration approach is employed. This analysis leads to the development of a rational response spectrum approach. The different types of floor response spectra required as inputs in this spectrum approach are identified. The procedure to obtain these floor spectra directly from the prescribed ground response spectra are developed. Use of these spectra as inputs in the calculation of response is, then, demostrated on several examples of the multiply connected secondary systems.

I.2 ORGANIZATION OF REPORT

The report consists of several chapters in which the theoretical development are presented. However, for a user who is primarily interested in the implementation of the approach, Chapter VI entitled USER SUMMARY OF THE PROPOSED METHOD is provided. This chapter gives a step-by-step procedure for the implementation of the approach.

In Chapter II, the equations of motions for a free-free primary-secondary system are developed, and the strategy of partitioning the total response displacement into the so-called pseudo-static and dynamic components in elaborated upon. The basic equations of equilibrium necessary to define these two components are developed. As an immediate extension, other response quantities such as member forces are also partitioned into their pseudo-static and dynamic components. The equations for the mean square values of these response components and their correlation, defined in terms of the so-called cross terms, are developed in this Chapter. Since the main aim is to develop a response spectrum approach employing response spectra as the design inputs, the modal analysis approach has been used in the formulation.

Finally, the expressions are developed for calculating the dynamic, pseudo-static and cross responses directly from the design inputs defined in terms of the floor response spectra. Here, the need for defining the inputs for the secondary system in terms of different types floor response spectra is identified. It is shown that the conventionally employed pseudo-acceleration floor response spectra is just one of the several other types of floor spectra which must be defined for a proper seismic analysis of multiply connected secondary systems. The other types of floor response spectra are: (1) auto relative velocity floor spectra; (2) coincident velocity and displacement floor spectra and; (3) quadrature velocity and displacement floor spectra. The procedure for the calculation of the pseudo-static and cross response components in terms of the support displacement, velocities and floor spectral quantities is also developed in this Chapter.

In Chapter III, the procedures are developed to define various floor response spectral quantities, which were identified in Chapter II as the necessary inputs for the calculation of the dynamic as well as the cross response contributions. The procedures are also developed to calculate other floor inputs, such as the maximum displacements, velocities, the correlation between the displacements of various supports etc., which are required in the calculation of the pseudo-static and cross response terms. These methods employ the dynamic characteristic of the supporting primary structure and directly use the ground response spectra as the base input.

The special case of a secondary system with one or more of its supports on the ground is examined in Chapter IV. The development of the cross floor spectra, correlations between support displacements as well as support velocities in this case is somewhat different from that described earlier in Chapter II; these are thus covered in this chapter.

The numerical results obtained for various floor response spectral quantities and the response of two secondary systems are presented in Chapter V. The relative contributions of the dynamic, pseudo-static and cross response terms to the total response are evaluated. Also some of the currently used procedures of combining the various response contributions, specially the dynamic response and the response due to relative displacement between the supports, are evaluated vis-a-vis the results obtained by this proposed method. Chapter VII summarizes the report and presents the general conclusions.

CHAPTER II

RESPONSE ANALYSIS

II.1 INTRODUCTION

In this Chapter, the development of the equations of motion and their solution technique are presented. A response quantity of interest is devided into the dynamic and pseudo-static part. The dynamic part is associated with the inertial effects induced by the support acceleration. The pseudo-static part is due to the relative displacement between the supports. The methods to obtain the contributions of these two types of the responses to the design response as well as the effect of their correlation are developed. Various types of the support (or floor) inputs, required in these methods, are identified.

II.2 EQUATIONS OF MOTION

The equations of motion of a secondary system attached at several points of a primary structure, considered as a free-free system, can be written as

$$\begin{bmatrix} M_{ss} & M_{sa} \\ M_{as} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{U}_s \\ \ddot{U}_a \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sa} \\ C_{as} & C_{aa} \end{bmatrix} \begin{Bmatrix} \dot{U}_s \\ \dot{U}_a \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sa} \\ K_{as} & K_{aa} \end{bmatrix} \begin{Bmatrix} U_s \\ U_a \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (2.1)$$

where the subscript a is associated with the degrees-of-freedom of the support points and s with the degrees-of-freedom of the active or unattached mass points of the secondary system. The displacement vectors U_s and U_a, respectively, denote the absolute displacements of the unattached points and the support points of the structure, measured in the Newtonian frame of reference. The vector U_s is of dimension n, the degrees-of-freedom of the unattached masses, and U_a is of dimension m, the degrees-of-freedom of the support points or the masses on the primary structure. The dot over a time dependent vector quantity denotes its time derivative. M_{ss}, C_{ss} and K_{ss}, respectively, are the mass, damping, and stiffness matrices associated with the active degree of freedom, and thus are of dimension n x n; similarly M_{aa}, C_{aa} and K_{aa} which are of dimension m x m, are the respective matrices associated with the support points. The other matrices in Eq.(2.1) introduce the existing coupling effects between the support and active degree of freedom through the inertial, damping and elastic forces.

By taking the right hand side of Eq.(2.1) to be zero, we imply that the support points are connected to the rest of the primary structure by springs of zero stiffness. Thus, no force is transmitted between the rest of the primary structure and the support points to which the secondary system is attached. However, as we will see later the support point motion U_a constitute the input to the rest of the secondary system and to define these motions we will consider the entire primary structure. Thus, the motion is assumed to propagate only in one direction, i.e. from the primary to the secondary structure and not backwards. Such systems are also called as the systems in cascade.

In this formulation, the total response is partitioned into the pseudo-static and dynamic components. Earlier, a similar partitioning of the response was also utilized by Lee and Penzien [12]. The pseudo-static component is due to the relative displacement between the supports, without any dynamic influence. The dynamic component comes from the inertial forces induced in the unattached masses due to the support inputs. Obviously, the dynamic response of the attached degrees of freedom is zero. Thus, we write

$$U_s(t) = U_s^p(t) + U_s^d(t) \tag{2.2a}$$

$$U_a(t) = U_a^p(t) \tag{2.2b}$$

where U_s^d and U_s^p, respectively, are the dynamic and the pseudo-static components of the active degree of freedom and U_a are the support motion time histories.

Substituting Eq.(2.2) into (2.1) , the first set of equations associated with the dynamic component of response can be written as

$$[M_{ss}]\{\ddot{U}_s^d + \ddot{U}_s^p\} + [C_{ss}]\{\dot{U}_s^d + \dot{U}_s^p\} + [K_{ss}]\{U_s^d + U_s^p\} = - [M_{sa}]\{\ddot{U}_a\} - [C_{sa}]\{\dot{U}_a\} - [K_{sa}]\{U_a\} \tag{2.3}$$

Since the pseudo-static response does not include any dynamic effect , it can be obtained by making the forces associated with the mass and damping matrix as zero in Eq.(2.1). That is,

$$\begin{bmatrix} K_{ss} & K_{sa} \\ K_{as} & K_{aa} \end{bmatrix} \begin{Bmatrix} U_s^p \\ U_a \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2.4}$$

Thus, the pseudo-static response of the n degrees-of-freedom can be written in terms of the prescribed support inputs as

$$[K_{ss}]\{U_s^p\} + [K_{sa}]\{U_a\} = \{0\} \tag{2.5}$$

or

$$\{U_s^p\} = (-[K_{ss}]^{-1}[K_{sa}])\{U_a\} \tag{2.6}$$

Substituting Eq.(2.6) into Eq.(2.3) and rearranging terms, we obtain

$$[M_{ss}]\{\ddot{U}_s^d\} + [C_{ss}]\{\dot{U}_s^d\} + [K_{ss}]\{U_s^d\} = ([M_{ss}][K_{ss}]^{-1}[K_{sa}] - [M_{sa}]\{\ddot{U}_a\}$$

$$+ ([C_{ss}][K_{ss}]^{-1}[K_{sa}] - [C_{sa}])\{\dot{U}_a\} \tag{2.7}$$

If the complete damping matrix is assumed proportional to the complete stiffness matrix, i.e.

$$\begin{bmatrix} C_{ss} & C_{sa} \\ C_{as} & C_{aa} \end{bmatrix} = \alpha \begin{bmatrix} K_{ss} & K_{sa} \\ K_{as} & K_{aa} \end{bmatrix} \tag{2.8}$$

then the terms dependent on $\dot{U}_a$ in Eq.(2.7) vanish. In a more general case these terms will not be zero. Here we assume such proportionality to simplify the analysis, although more general case can also be treated analytically. Furthermore, since these terms are associated with damping terms, their magnitude compared to the other terms will be relatively small [5], and thus they can be neglected. With these assumptions, the equations of motions associated with the dynamic response can, then, be expresed as,

$$[M_{ss}]\{\ddot{U}_s^d\} + [C_{ss}]\{\dot{U}_s^d\} + [K_{ss}]\{U_s^d\} = [r]\{\ddot{U}_a\} \qquad (2.9)$$

where $[r]=([M_{ss}][K_{ss}]^{-1}[K_{sa}]-[M_{sa}])$ is dynamic influence matrix in which each column represents the distribution of force in the unattached degrees-of-freedom due to the acceleration of each support.

II.3 DESIGN RESPONSE

A response quantity linearly related to the displacement response, can also be expresed as a sum of the dynamic and pseudo-static component as

$$S(t) = S^d(t) + S^p(t) \qquad (2.10)$$

where $S^d(t)$ corresponds to the diplacement vector $U_s^d(t)$ and $S^p(t)$ corresponds to the vector $U_s^p(t)$.

We are interested in calculating the maximum response induced by the ground motions which are likely to occur at a site. We assume these site motions to be the sample functions of a random process. The maximum response or the design response for such random motions can be obtained in terms of the root mean square response and its peak factor, as

$$R_d = C_d \sigma_s \qquad (2.11)$$

where R_d= maximum response, C_d= the peak factor and σ_s= root mean square value of response S(t).

To obtain the root mean square response we first develop the covariance function of the response. From Eq.(2.10) this can be obtained as,

$$R_s(t_1,t_2) = E[S^d(t_1)S^d(t_2)] + E[S^p(t_1)S^p(t_2)]$$

$$+ E[S^d(t_1)S^p(t_2)] + E[S^p(t_1)S^d(t_2)] \qquad (2.12)$$

where $R_s(t_1,t_2)$= covariance function of the response. The first two terms represent the contribution of the dynamic and pseudo-static components, respectively, to the total response, while the last two terms take into account the cross correlation between them. The variance of the response is obtained by setting $t_1=t_2$ in Eq.(2.12), and the design response can be written as follows

$$R_d^2 = C_d^2(\sigma_{dd}^2+\sigma_{pp}^2+2C_{dp}) \qquad (2.13)$$

where σ_{dd}= variance due to the dynamic component, σ_{pp}= variance due to the pseudo-static component and and C_{dp}= the cross-covariance between the dynamic and pseudo-static components. The three terms in Eq.(2.13) are referred to as the dynamic, pseudo-static and cross response components.

For design purposes, the earthquake motions are usually precribed in terms of the ground response spectra for the primary structures and in terms of the floor response spectra for the secondary system. It is, thus, desired to evaluate R_d in terms of such response spectra. In the following, therefore, the response spectrum methods employing the ground and floor response spectra as inputs, are developed for the calculation of the contributions of the dynamic, pesudo-static and cross response components.

II.4 DYNAMIC RESPONSE CONTRIBUTION

In Eq.(2.13) the contribution of the dynamic response to the total response, here denoted by R_{dd}, is

$$R_{dd}^2 = C_d^2 \sigma_{dd}^2 \tag{2.14}$$

To obtain σ_{dd}, the solution of Eq.(2.9) is required. Employing the modal analysis approach and standard manipulations involving orthogonal properties of the normal modes, a decoupled modal equation (for a classically damped system) can be written as

$$\ddot{q}_j(t) + 2\beta_j\omega_j\dot{q}_j(t) + \omega_j^2 q_j(t) = \{P_j\}'\{\ddot{U}_a\} \tag{2.15}$$

where q_j= j^{th} principal coordinates; ω_j= j^{th} modal frequency, β_j= the modal damping ratio, and $\{\Psi_j\}$= j^{th} modal shape vector. $\{P_j\}$= the influence vector= $\{\Psi_j\}'[r]$; each element of this vector represents the contribution of a support motion to the response in the j^{th} mode. A prime (') over a vector quantity represents its transpose. Here we have assumed that the secondary system is classically damped. However, analysis can also be made for nonclassically damped system, as indicated by Singh [23].

For given support motion time histories, Eq.(2.15) can be solved to define q_j. In terms of q_j any component of the displacement vector or the response quantity of interest which is linearly related to the displacement can be obtained by the expansion theorem as

$$S^d(t) = \sum_{j=1}^{n} \rho_j q_j(t) \qquad (2.16)$$

where ρ_j is the so called modal response in the j^{th} mode which can be evaluated from the displacement mode shape by a simple linear transformation.

For random site motions, the motions of the support points of the secondary system will also be random processes. To simplify the analysis we assume that the ground motion, the motions of the support points defined by U_a as well as the induced dynamic response of the secondary systems are stationary random processes. Although, these assumption are not strictly valid for earthquake type of ground motions and responses, they have been found to be acceptable in the calculation of primary system response and in generation of floor response spectra, required as inputs for the secondary system response [17]. With these assumptions, the stationary value of the variance of the dynamic response ,can be shown to be as follows (See Appendix I):

$$\sigma_{dd}^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_i \rho_j \sum_{k=1}^{m} \sum_{\ell=1}^{m} P_{ik} P_{j\ell} \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega) H_i^* H_j \, d\omega \qquad (2.17)$$

in which P_{ik} is the k^{th} component of the modal force influence vector P_i; H_j is the frequency response function which is defined as

$$H_j = 1/(\omega_j^2 - \omega^2 + 2i\beta_j \omega_j \omega) \qquad (2.18)$$

The asterisk over the frequency response function denotes its complex conjugate and i is the imaginary number=$\sqrt{-1}$.

$\Phi_{ak\ell}(\omega)$ is the cross spectral density function of the absolute accelerations at supports k and ℓ. It can be obtained by a dynamic analysis of the supporting primary structure as shown in Chapter III.

In Eq.(2.17), the first double summation represents the contribution of the modes of the secondary system to the dynamic response. The second double summation gives the auto and cross correlation effects of the excitation at various support points. Often the cross correlation effects of the support motions are neglected. However, this could lead to erroneous results.

Substituting Eq.(2.17) into Eq.(2.14), the contribution of the dynamic response to the total response is obtained as follows,

$$R_{dd}^2 = C_d^2 \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_i \rho_j \sum_{k=1}^{m} \sum_{\ell=1}^{m} P_{ik} P_{j\ell} I_{ak\ell ij} \tag{2.19}$$

where the frequency integral, $I_{ak\ell ij}$, is defined as,

$$I_{ak\ell ij} = \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega) H_i^* H_j d\omega \tag{2.20}$$

If the auto and cross spectral density functions of the support accelerations are known, R_{dd} can be obtained from Eq.(2.19). However, our aim is to develop a response spectrum approach wherein the support inputs are defined in terms of floor spectra, and not the spectral density functions, for the calculation of design response.

In the current practice, the seismic floor input is most commonly defined in terms of pseudo-acceleration floor spectra. This input is, however, adequate only for the single-degree-of-freedom secondary systems. For the multi-support secondary systems, it becomes necessary to define floor inputs in other forms of floor spectra to obtain the frequency integral in Eq.(2.20). Here, these floor spectra are classified as the auto and cross floor spectra, and are presented in the followings sections.

II.4.1 AUTO-FLOOR RESPONSE SPECTRA

For the evaluation of the terms with $k=\ell$ in Eqs.(2.19) and (2.20), we need the (auto) spectral density function for the motion of the K^{th} support. That is, in such a case we are concerned with the motion of a single support. However, we need to obtain such terms for $i=j$ and $i\neq j$ separately. Evaluation of the frequency integral in Eq.(2.20) for these two cases is now described.

CASE 1: $k=\ell$ and $i=j$

The frequency integral Eq.(2.20) for this case, defined as

$$I_{akkjj} = \int_{-\infty}^{\infty} \Phi_{akk}(\omega)\ |H_j|^2 d\omega \qquad (2.21)$$

represents the mean square displacement response of an oscillator with parameters ω_j (frequency) and β_j (damping ratio) subjected to the base acceleration of k^{th} floor. This can be defined in terms of the conventionally used pseudo-acceleration or the relative displacement floor response spectra and the peak factor of the oscillator response. Here, we will call these floor spectra as auto pseudo-acceleration and auto displacement floor spectra and denote them by $R_{pk}(\omega_j)$ and $R_{dk}(\omega_j)$, respectively. In terms of the frequency integral they are defined as

$$R_{dk}^2(\omega_j) = [R_{pk}^2(\omega_j)/\omega_j^4] = C_{dj}^2 \int_{-\infty}^{\infty} \Phi_{akk}(\omega)|H_j|^2 d\omega \qquad (2.22)$$

In Eq.(2.22) and hereafter, the suffix p is associated with term pseudo, d with displacement, k with the floor number and j with the oscillator of parameters ω_j and β_j. C_{dj} is the peak factor associated with the displacement response.

In term of the displacement spectrum, the frequency integral in Eq.(2.21) is obtained as,

$$I_{akkjj} = R_{dk}^2(\omega_j)/C_{dj}^2 \qquad (2.23)$$

The evaluation of the terms with $k=\ell$ and $i=j$,thus, requires only the auto displacement (or pseudo-acceleration) floor spectra of the support point accelerations. The procedures to obtain these spectra, directly from ground response spectra, have been developed in Reference [9,17,20,22]. The expressions for these spectra are also given in Chapter III.

CASE 2: $k=\ell$ and $i\neq j$

For the case of $i\neq j$ but with $k=\ell$ the frequency integral of Eq.(2.20) can be written as,

$$I_{akkij} = \int_{-\infty}^{\infty} \Phi_{akk}(\omega)\ H_i^* H_j d\omega \tag{2.24}$$

The complex part of $H_i^* H_j$ will finally cancel out when summed up in Eq.(2.19) or when integrated over the frequency range due to it being an odd function of ω. Thus, in Eq.(2.24) we will consider only the real part of $H_i^* H_j$ which can be written as,

$$\text{Real}\ (H_i^* H_j) = N(\omega)\ |H_i|^2 |H_j|^2 \tag{2.25}$$

where $N(\omega)$ is defined as

$$N(\omega) = \omega^4 + \omega^2(4\beta_i\beta_j\omega_i\omega_j - \omega_i^2 - \omega_j^2) + (\omega_i\omega_j)^2 \tag{2.26}$$

The right hand side of Eq.(2.25) can be resolved into partial fractions as

$$N(\omega)\ |H_i|^2 |H_j|^2 = (A+\omega^2 B)|H_i|^2 + (C+\omega^2 D)|H_j|^2 \tag{2.27}$$

where the coefficients of the partial fraction A, B, C and D are obtained from the solution of the following simultaneous equations

$$[Y_{ij}]\{V_1\} = \{W_1\} \tag{2.28}$$

where the matrix $[Y_{ij}]$, and vectors V_1 and W_1 are defined as,

$$[Y_{ij}] = \begin{bmatrix} \omega_j^4 & 0 & \omega_i^4 & 0 \\ 2\omega_j^2(2\beta_j^2-1) & \omega_j^4 & 2\omega_i^2(2\beta_i^2-1) & \omega_i^4 \\ 1 & 2\omega_j^2(2\beta_j^2-1) & 1 & 2\omega_i^2(2\beta_i^2-1) \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{2.29}$$

$$\{V_1\}' = \{A, B, C, D\} \tag{2.30a}$$

$$\{W_1\}' = \{(\omega_i\omega_j)^2, (4\beta_i\beta_j\omega_i\omega_j - \omega_i^2 - \omega_j^2), 1, 0\} \tag{2.30b}$$

Substituting Eq.(2.27) into Eq.(2.24), and noting that the imaginary part is equal to zero, we obtain,

$$I_{akkij} = \int_{-\infty}^{\infty} \Phi_{akk}(\omega)\{(A+\omega^2 B)|H_i|^2 + (C+\omega^2 D)|H_j|^2\}d\omega \tag{2.31}$$

The terms associated with A, and C in Eq.(2.31) can be obtained in terms of the auto displacement (or pseudo-acceleration) floor response spectra, as explained above. However, the terms associated with B and D are obtained in terms of different spectra called the auto velocity floor response spectra. The velocity response spectrum value for floor k and the oscillator parameters of ω_j and β_j, denoted by $R_{vk}(\omega_j)$, is defined as,

$$R_{vk}^2(\omega_j) = C_{vj}^2 \int_{-\infty}^{\infty} \Phi_{akk}(\omega)\omega^2|H_j|^2 d\omega \tag{2.32}$$

C_{vj} is the peak factor of the relative velocity response of the oscillator.

In terms of these floor response spectra, the frequency integral of Eq.(2.24) or Eq.(4.31) can now be written as

$$I_{akkij} = A\, I_{1ki} + B\, I_{2ki} + C\, I_{1kj} + D\, I_{2kj} \tag{2.33}$$

where I_{1ki} and I_{2ki} are defined as follows

$$I_{1ki} = R_{dk}^2(\omega_i)/C_{di}^2 \tag{2.34a}$$

$$I_{2ki} = R_{vk}^2(\omega_i)/C_{vi}^2 \tag{2.34b}$$

From Eqs.(2.23) and (2.34a), it is noted that $I_{1ki}=I_{akkii}$. We also note that for the calculation of terms with $k=\ell$ for any combination of the indices i and j, the support point motions need to be defined in terms of the auto displacement (or pseudo-acceleration) and (relative) velocity floor response spectra.

II.4.2 CROSS FLOOR SPECTRA

In a most general case with $k\neq\ell$ and $i\neq j$, the evaluation of the frequency integral in Eq.(2.20) involves the cross spectral density function, $\Phi_{ak\ell}(\omega)$. This functions defines the correlation between the (absolute) accelerations of two differents supports. It consists of the real and imaginary parts and can be written as

$$\Phi_{ak\ell}(\omega) = \Phi_{ak\ell}^R(\omega) + i\ \Phi_{ak\ell}^I(\omega) \tag{2.35}$$

where the superscripts R and I denote the real and imaginary parts, respectively. It is noted that the real and imaginary components , respectively, are even and odd functions of ω. The expressions for these components of $\Phi_{ak\ell}(\omega)$ are developed in terms of the ground motion spectral density function and the properties of the primary structure in Chapter III.

To evaluate Eq.(2.20) for the most general case, we rewrite the term $H_i^*H_j$ in the integrand as,

$$H_i^* H_j = \{N(\omega) + i\omega M(\omega)\}|H_i|^2|H_j|^2 \qquad (2.36)$$

where $N(\omega)$ is defined by Eq.(2.26). $M(\omega)$ is an even function of ω, and is defined as

$$M(\omega) = 2\{\omega_i\omega_j(\beta_i\omega_j-\beta_j\omega_i) + \omega^2(\beta_j\omega_j-\beta_i\omega_i)\} \qquad (2.37)$$

Substituting Eqs.(2.35) and (2.36) into Eq.(2.20), we obtain

$$I_{ak\ell ij} = \int_{-\infty}^{\infty} [\{\Phi^R_{ak\ell}(\omega)N(\omega) - \omega\Phi^I_{ak\ell}(\omega)M(\omega)\} + i\{\Phi^I_{ak\ell}(\omega)\ N(\omega) + \omega\Phi^R_{ak\ell}(\omega)\ M(\omega)\}|H_i|^2|H_j|^2 d\omega \qquad (2.38)$$

It is noted that the imaginary part in Eq.(2.38) is zero as its integrand is an odd function of ω. As in Eq.(2.27), we decompose the term $M(\omega)|H_i|^2|H_j|^2|$ of Eq.(2.37) into its partial fraction as follows

$$M(\omega)|H_i|^2|H_j|^2 = (E+\omega^2F)|H_i|^2+(G+\omega^2H)|H_j|^2 \qquad (2.39)$$

The coefficients of the partial fraction in Eq.(2.39) are obtained from the solution of the following simultaneous equations

$$[Y_{ij}]\{V_2\} = \{W_2\} \qquad (2.40)$$

where the matrix $[Y_{ij}]$ is defined in Eq.(2.29) and the vectors V_2 and W_2 are given by

$$\{V_2\}' = \{E,\ F,\ G,\ H\} \qquad (2.41a)$$

$$\{W_2\}' = \{2\omega_i\omega_j(\beta_i\omega_j-\beta_j\omega_i),\ 2(\beta_j\omega_j-\beta_i\omega_i),\ 0,\ 0\} \qquad (2.41b)$$

Substituting Eqs.(2.27) and (2.39) into Eq.(2.38), we obtain

$$I_{ak\ell ij} = \{A \int_{-\infty}^{\infty} \Phi^R_{ak\ell}(\omega)|H_i|^2 d\omega + B \int_{-\infty}^{\infty} \omega^2 \Phi^R_{ak\ell}(\omega)|H_i|^2 d\omega$$

$$+ C \int_{-\infty}^{\infty} \Phi^R_{ak\ell}(\omega)|H_j|^2 d\omega + D \int_{-\infty}^{\infty} \omega^2 \Phi^R_{ak\ell}(\omega)|H_j|^2 d\omega\}$$

$$- \{E \int_{-\infty}^{\infty} \omega \Phi^I_{ak\ell}(\omega)|H_i|^2 d\omega + F \int_{-\infty}^{\infty} \omega^3 \Phi^I_{ak\ell}(\omega)|H_i|^2 d\omega$$

$$+ G \int_{-\infty}^{\infty} \omega \Phi^I_{ak\ell}(\omega)|H_j|^2 d\omega + H \int_{-\infty}^{\infty} \omega^3 \Phi^I_{ak\ell}(\omega)|H_j|^2 d\omega\} \qquad (2.42)$$

To evaluate the frequency integrals in Eq.(2.42) in terms of floor spectra, we introduce the following floor response spectral quantities:

Coincident displacement spectra :

$$C^2_{dk\ell}(\omega_i) = P^2_{di} \int_{-\infty}^{\infty} \Phi^R_{ak\ell}(\omega)|H_i|^2 d\omega \qquad (2.43)$$

Coincident velocity spectra :

$$C^2_{vk\ell}(\omega_i) = P^2_{vi} \int_{-\infty}^{\infty} \omega^2 \Phi^R_{ak\ell}(\omega)|H_i|^2 d\omega \qquad (2.44)$$

Quadrature displacement spectra :

$$Q^2_{dk\ell}(\omega_i) = Q^2_{di} \int_{-\infty}^{\infty} \omega \, \Phi^I_{ak\ell}(\omega)|H_i|^2 d\omega \qquad (2.45)$$

Quadrature velocity spectra :

$$Q^2_{vk\ell}(\omega_i) = Q^2_{vi} \int_{-\infty}^{\infty} \omega^3 \Phi^I_{ak\ell}(\omega)|H_i|^2 d\omega \qquad (2.46)$$

where P_{di}, P_{vi}, Q_{di} and Q_{vi} are the peak factors associated with various response quantities of concern in Eqs.(2.43) through (2.46).

The floor spectral inputs are customarily defined in terms of the pseudo-acceleration rather than the displacement response, specially for auto floor spectra. Here thus, we define coincident and quadrature pseudo-acceleration spectra as follows

$$C_{pk\ell}(\omega_i) = \omega_i^2 C_{dk\ell}(\omega_i) \qquad (2.47a)$$

$$Q_{pk\ell}(\omega_i) = \omega_i^2 Q_{dk\ell}(\omega_i) \qquad (2.47b)$$

The procedure to obtain these coincident and quadrature floor response spectral quantities directly in terms of ground response spectra and the primary system properties are given in Chapter III.

It is seen that for $k=\ell$, the quadrature spectra, Eqs.(2.45) and (2.46), are zero, whereas the coincident displacement and velocity spectra as defined by Eqs.(2.43) and (2.44), revert back to the previously mentioned auto displacement and velocity floor response spectra.

The influence of the correlation between the support accelerations is reflected through the imaginary component of the cross spectral density function. If the signals are strongly correlated, the quadrature terms become less important in comparison with the coincident terms. In fact, for two perfectly correlated floor motions the contribution of the quadrature terms is zero.

Substitution of Eqs.(2.43) through (2.46) into Eq.(2.42), the frequency integrals can now be written in terms of these spectral quantities as

$$I_{ak\ell ij} = \{A\ I_{3k\ell i} + B\ I_{4k\ell i} + C\ I_{3k\ell j} + D\ I_{4k\ell j}\}$$
$$- \{E\ I_{5k\ell i} + F\ I_{6k\ell i} + G\ I_{5k\ell j} + H\ I_{6k\ell j}\} \qquad (2.48)$$

where $I_{3k\ell i}$, $I_{4k\ell i}$, $I_{5k\ell i}$ and $I_{6k\ell i}$ are defined in terms of the coincident and quadrature spectra as follows

$$I_{3k\ell i} = C^2_{dk\ell}(\omega_i)/P^2_{di} \qquad (2.49a)$$

$$I_{4k\ell i} = C^2_{vk\ell}(\omega_i)/P^2_{vi} \qquad (2.49b)$$

$$I_{5k\ell i} = Q^2_{dk\ell}(\omega_i)/Q^2_{di} \qquad (2.49c)$$

$$I_{6k\ell i} = Q^2_{vk\ell}(\omega_i)/Q^2_{vi} \qquad (2.49d)$$

Evaluation of terms with i=j, but with $k \neq \ell$, is a special case of Eq.(2.20). To define this, we substitute for the cross spectral density function in term of its real and imaginary parts into Eq.(2.20) to obtain

$$I_{ak\ell ij} = \int_{-\infty}^{\infty} \{\Phi^R_{ak\ell}(\omega) + i\ \Phi^I_{ak\ell}(\omega)\}|H_i|^2 d\omega \qquad (2.50)$$

Since the imaginary part of the spectral density function is an odd function of ω, its integral is zero. Thus,

$$I_{ak\ell ii} = \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega_i)|H_i|^2 d\omega \qquad (2.51)$$

which from Eq.(2.43) can be obtained in terms of the coincident displacement spectrum as,

$$I_{ak\ell ii} = C^2_{dk\ell}(\omega_i)/P^2_{di} \qquad (2.52)$$

Thus, although two different floor motion are involved in the evaluation of this term, only one type of the cross floor spectra are required.

This presentation clearly identifies the types of floor spectra we need for the calculation of response of the secondary system with multiple supports. The methods for the generation of these spectra are described in Chapter III.

II.5 PSEUDO-STATIC RESPONSE CONTRIBUTION

In Eq.(2.13), the pseudo-static part of the total design response is written as

$$R_{pp}^2 = C_d^2 \, \sigma_{pp}^2 \tag{2.53}$$

In Eq.(2.53), σ_{pp} is the root mean square value of the response due to the relative displacement between supports. The displacement response due to the relative support displacement is defined by Eq.(2.6), which is rewritten as,

$$\{U_s^p\} = [A]\{U_a\} \tag{2.54}$$

where $[A]=(-[Kss]^{-1}[Ksa])$ is called the pseudo-static influence matrix. A generic term A_{rs} of this matrix represents the displacement of the active degree of freedom r due to a unit displacement of support s. Therefore, each column of matrix [A] defines the constrained displacement configuration of the secondary system associated to each support motion.

Assuming the linear behavior of the structure, displacement component, or any other response linearly related to the displacement, can be obtained as a linear sum of the contribution of each support displacement. That is,

$$S^p(t) = \sum_{k=1}^{m} \eta_k U_{ak}(t) \tag{2.55}$$

where η_k is the constrained response associated with support k, or the response due to a unit displacement of support k; and $U_{ak}(t)$ is the absolute displacement time history of support k.

The covariance function of the response in Eq.(2.55) can now be written as

$$R_{pp}(t_1,t_2) = \sum_{k=1}^{m} \sum_{\ell=1}^{m} \eta_k \eta_\ell \; E[U_{ak}(t_1) U_{a\ell}(t_2)] \tag{2.56}$$

Here also, we assume that the support displacements and the induced response are stationary random processes. With this, the covariance functions of the support displacements in Eq.(2.56) can be expresed in terms of their auto and cross spectral density functions.

By substituting $t_1=t_2$, the variance of the pseudo-static response component can be shown to be as

$$\sigma_{pp}^2 = \sum_{k=1}^{m} \sum_{\ell=1}^{m} \eta_k \eta_\ell \int_{-\infty}^{\infty} \Phi_{dk\ell}(\omega) d\omega \tag{2.57}$$

where $\Phi_{dk\ell}(\omega)$ is the cross spectral density function of the displacement at supports k and ℓ. Substituting Eq.(2.57) into Eq.(2.53), the contribution of the pseudo-static response to the design response can be written as

$$R_{pp}^2 = C_d^2 \sum_{k=1}^{m} \sum_{\ell=1}^{m} \eta_k \eta_\ell \int_{-\infty}^{\infty} \Phi_{dk\ell}(\omega) d\omega \tag{2.58}$$

in terms of the spectral density function of the absolute displacement of the supports.

Separating the terms with $k=\ell$ and $k\neq\ell$, we obtain

$$R_{pp}^2 = C_d^2 \{\sum_{k=1}^{m} \eta_k^2 \int_{-\infty}^{\infty} \Phi_{dkk}(\omega)d\omega + \sum_{k=1}^{m} \sum_{\substack{\ell=1 \\ k\neq\ell}}^{m} \eta_k\eta_\ell \int_{-\infty}^{\infty} \Phi_{dk\ell}(\omega)d\omega\} \quad (2.59)$$

The first frequency integral in this equation is the mean square value of the absolute displacement of support k. It can be obtained in terms of the maximum support displacement $\bar{U}_{ak}$, as

$$\int_{-\infty}^{\infty} \Phi_{dkk}(\omega)d\omega = (\frac{\bar{U}_{ak}}{C_{uk}})^2 \quad (2.60)$$

where C_{uk}= the peak factor of the absolute displacement response of floor k. The integral in the double summation term can be expressed in terms of the correlation coefficient, $\delta_{k\ell}$, of the absolute displacement response, defined as

$$\delta_{k\ell} = \frac{\text{Real } \{\int_{-\infty}^{\infty} \Phi_{dk\ell}(\omega)d\omega\}}{\{\int_{-\infty}^{\infty} \Phi_{dkk}(\omega)d\omega \int_{-\infty}^{\infty} \Phi_{d\ell\ell}(\omega)d\omega\}^{1/2}} \quad (2.61)$$

In terms of these quantities, the pseudo-static response contribution can be expressed as

$$R_{pp}^2 = C_d^2 \{\eta_\ell \bar{U}_{a\ell}/C_{u\ell}\}'[\delta]\{\eta_k \bar{U}_{ak}/C_{uk}\} \quad (2.62)$$

where $[\delta]$ is the matrix of correlation coefficients of the absolute displacement, with diagonal terms being equal to 1.

It will be shown that the maximum absolute displacement $\bar{U}_{ak}$ and correlation coefficient $\delta_{k\ell}$, required in Eq.(2.62) are related to the auto and cross coincident floor spectra. Therefore the approach to obtain these quantities, as described in the next Chapter , can be directly used for the evaluation of $\bar{U}_{ak}$ and $\delta_{k\ell}$.

In the preceding formulation we expressed the pseudo-static response in terms of the absolute displacement of the support or floors. However, the pseudo-static response can also be obtained in terms of the relative displacement of the supports. For this, the absolute displacement in Eq.(2.55) can be written as a sum of the ground displacement and displacement of the supports with respect to ground as,

$$S^p(t) = \sum_{k=1}^{m} \eta_k \{V_{ak}(t) + r_k X_g(t)\} \qquad (2.63)$$

where V_{ak}= relative displacement of the support k and r_k=1 if the displacement of support k is in the direction of the ground displacement otherwise it is zero.

The second part of Eq.(2.63) is the response due to the rigid body displacement of X_g, of the supports of the secondary system applied in the direction of ground motion. For force quantities, this term should be equal to zero. Thus if $S^p(t)$ represents the force response, and not the displacement response, of the secondary system then

$$S^p(t) = \sum_{k=1}^{m} \eta_k \, V_{ak}(t) \qquad (2.64)$$

where now $V_{ak}(t)$ represents the relative displacement of support k with respect to ground.

The mean square response can now be written in terms of the spectral density functions of the relative displacements as,

$$\sigma_{pp}^2 = \sum_{k=1}^{m} \sum_{\ell=1}^{m} n_k n_\ell \int_{-\infty}^{\infty} \Phi_{vk\ell}(\omega)d\omega \qquad (2.65)$$

where $\Phi_{vk\ell}(\omega)$= cross spectral density function of the relative displacements of supports k and ℓ.

By substituting Eq.(2.65) into Eq.(2.53), we obtain the design response in terms of the spectral density function $\Phi_{vk\ell}(\omega)$. To express this in terms of the relative floor displacement response, we separate the terms with $k=\ell$, and $k\neq\ell$ as follows

$$R_{pp}^2 = C_d^2\{\sum_{k=1}^{m} n_k^2 \int_{-\infty}^{\infty} \Phi_{vkk}(\omega)d\omega + \sum_{\substack{k=1 \\ k\neq\ell}}^{m} \sum_{\ell=1}^{m} n_k n_\ell \int_{-\infty}^{\infty} \Phi_{vk\ell}(\omega)d\omega\} \qquad (2.66)$$

in which the first frequency integral represents the mean square value of the displacement of floor k. This can be obtained in terms of the maximum relative floor displacement and its peak factor as,

$$\int_{-\infty}^{\infty} \Phi_{vkk}(\omega)d\omega = (\frac{\bar{V}_{ak}}{C_{vk}})^2 \qquad (2.67)$$

where $\bar{V}_{ak}$ is the maximum relative floor displacement and C_{vk} is its peak factor obtained by a straight forward response spectrum analysis of the primary structure. The expression to obtain this is given in Chapter III.

The term related to the cross spectral density term in Eq.(2.66) can be defined in terms of the correlation coefficient between the support displacements, expressed as

$$\delta'_{k\ell} = \frac{\text{Real } \{\int_{-\infty}^{\infty} \Phi_{vk\ell}(\omega)d\omega\}}{\{\int_{-\infty}^{\infty} \Phi_{vkk}(\omega)d\omega \int_{-\infty}^{\infty} \Phi_{v\ell\ell}(\omega)d\omega\}^{1/2}} \qquad (2.68)$$

The method to obtain this coefficient in terms of the ground response spectra and properties of the primary structure is also given in Chapter III.

In terms of the maximum relative floor displacement and the correlation coefficient, the pseudo-static response contribution can now be expressed as,

$$R_{pp}^2 = C_d^2 \{\eta_\ell V_\ell / C_{v\ell}\}'[\delta']\{\eta_k V_{ak}/C_{vk}\} \tag{2.69}$$

where $[\delta']$ is the matrix of correlation coefficients with its diagonal terms being equal to 1.

It is noted that the single summation terms in Eqs.(2.59) and (2.66) represent the response due to each support displacement added up as the square-root-of-the-sum-of-the-squares, whereas the double summation terms give the contribution of the cross correlation between the support displacements.

II.6 CROSS RESPONSE CONTRIBUTION

This contribution is given by the last term of Eq.(2.13),

$$R_{dp}^2 = 2C_d^2 C_{dp} \tag{2.70}$$

where C_{dp} is the real part of the cross covariance between the dynamic and pseudo-static parts. To obtain this cross covariance, Eqs.(2.16) and (2.55) or (2.64) are used. This covariance can be expressed in terms of the cross spectral density function, $\Phi_{adk\ell}(\omega)$, of the absolute acceleration of support k and absolute displacement of support ℓ. At $t_1=t_2$, the third term of Eq.(2.12) can be shown (See Appendix I) to be as follows,

$$\sigma_{dp}^2 = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i n_\ell \sum_{k=1}^{m} P_{ik} \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) H_i^* d\omega \tag{2.71}$$

Similarly , the fourth term in Eq.(2.12) can be shown to be

$$\sigma_{pd}^2 = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i n_\ell \sum_{k=1}^{m} P_{ik} \int_{-\infty}^{\infty} \Phi_{da\ell k}(\omega) H_i d\omega \tag{2.72}$$

The integrands of Eqs.(2.71) and (2.72) are complex conjugate of each other. Also since the imaginary part of the integrand is an odd function of ω, its integral is zero. Thus, the third term in the parenthesis of Eq.(2.13) which is sum of Eqs.(2.71) and (2.72) can be written as,

$$2C_{dp} = \sigma_{dp}^2 + \sigma_{pd}^2 = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i n_\ell \sum_{k=1}^{m} 2P_{ik} \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) H_i^* d\omega \tag{2.73}$$

Substituting Eq.(2.73) into Eq.(2.70), the cross term contribution is obtained as,

$$R_{pd}^2 = C_d^2 \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i n_\ell \sum_{k=1}^{m} 2P_{ik} \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) H_i^* d\omega \tag{2.74}$$

Here Eq.(2.74) is expressed in terms of the cross spectral density function of the absolute displacement and absolute acceleration of the supports. One could also use the cross spectral density function between the absolute acceleration and relative displacement of the supports. The relative displacement formulation is, however, more involved as it requires additional types of cross floor spectra. The absolute displacement formulation has been found to be more convenient and will be persued further here.

From the stationary random vibration analysis, it is known that the cross spectral density function between absolute acceleration and absolute displacement, $\Phi_{adk\ell}(\omega)$, is related to the power spectral density of the absolute acceleration, $\Phi_{ak\ell}(\omega)$, by the following expression

$$\Phi_{ak\ell}(\omega) = -\omega^2 \Phi_{adk\ell}(\omega) \tag{2.75}$$

Thus , a generic frequency integral in Eq.(2.74), denoted as

$$I_{adk\ell i} = \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) H_i^* d\omega \tag{2.76}$$

can be written in terms of the power spectral density function of the absolute acceleration as

$$I_{adk\ell i} = \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega) \left(\frac{1}{-\omega^2}\right) H_i^* d\omega \tag{2.77}$$

The right hand side of Eq.(2.77) is, however, the same as the integral in Eq.(2.20) when ω_j is taken equal to zero. That is,

$$I_{adk\ell i} = (I_{ak\ell ij})_{\omega_j=0} \tag{2.78}$$

Thus, this frequency integral can be obtained by simply substituting $\omega_j=0$ in Eq.(2.42). For this case it can be shown that the coefficients C and G of the partial fraction in Eq.(2.42) are identically zero. Thus, from Eq.(2.42),

$$I_{adk\ell i} = \{A \int_{-\infty}^{\infty} \Phi^R_{ak\ell}(\omega)|H_i|^2 d\omega + B \int_{-\infty}^{\infty} \omega^2 \Phi^R_{ak\ell}(\omega)|H_i|^2 d\omega$$

$$+ D \int_{-\infty}^{\infty} \omega^2 \Phi^R_{ak\ell}(\omega)(\frac{1}{\omega^4})\ d\omega\} - \{E \int_{-\infty}^{\infty} \omega \Phi^I_{ak\ell}(\omega)|H_i|^2 d\omega$$

$$+ F \int_{-\infty}^{\infty} \omega^3 \Phi^I_{ak\ell}(\omega)|H_i|^2 d\omega + H \int_{-\infty}^{\infty} \omega^3 \Phi^I_{ak\ell}(\omega)(\frac{1}{\omega^4})\ d\omega\} \quad (2.79)$$

in which the coefficients A, B, D, etc can be obtained from the solution of Eqs.(2.28) and (2.40) for $\omega_j=0$, or they can be explicitly defined as follows

$$A = (4\beta_i^2 - 1) \qquad E = 4\beta_i/\omega_i(2\beta_i^2-1)$$

$$B = 1/\omega_i^2 \qquad F = 2\beta_i/\omega_i^3 \qquad (2.80)$$

$$D = -1/\omega_i^2 \qquad H = -2\beta_i/\omega_i^3$$

In Eq.(2.79), the frequency integrals associated with coefficients A, B, E and F can be defined in terms of cross floor spectra and the associated peak factors, as in Eqs.(2.43) through (2.46). The integrals associated with coefficients D and H are, however, additional floor response quantities which are required to be defined to obtain these terms. The physical characteristics of these terms are examined in the following.

It is noted that these terms are associated with absolute floor velocities. They can be defined in terms of cross floor spectral quantities. For example, the term associated with D is,

$$I_{4k\ell} = \int_{-\infty}^{\infty} \omega^2 \Phi^R_{ak\ell}(\omega)\ (\frac{1}{\omega^4})\ d\omega = C^2_{vk\ell}(\omega_i=0)/P^2_{vi} \qquad (2.81)$$

For $k=\ell$, this term is the mean square value of the absolute floor velocity. For $k\neq\ell$, this term represents the correlation between the coincident components of the velocities of the two floors, and it is the same as the coincident relative velocity spectrum, defined by Eq.(2.44), for $\omega_i=0$. In fact, this frequency integral for all values of k and ℓ can be defined in terms of ground spectra and primary structure properties by using the coincident velocity spectrum generation algorithm. This algorithm is developed in the next chapter.

The frequency integral associated with H in Eq.(2.79) can also be obtained similarly. For $k=\ell$, this term is zero. For $k\neq\ell$, this represents the correlation between the quadrature components of the absolute velocities of the two floors. It is same as the quadrature velocity spectrum at $\omega_i=0$ and is defined by Eq.(2.46). That is

$$I_{6k\ell} = \int_{-\infty}^{\infty} \omega^3 \Phi^I_{ak\ell}(\omega) \left(\frac{1}{\omega^4}\right) d\omega = Q^2_{vk\ell}(\omega_i=0)/Q^2_{vi} \tag{2.82}$$

This can be easily obtained in terms of ground response spectra by using the quadrature velocity spectrum generation algorithm. This algorithm is also developed in the next Chapter.

We can now rewrite Eq.(2.79) in terms of various floor response spectra quantities, defined by Eqs.(2.49a) through (2.49d) as follows

$$\begin{aligned} I_{adk\ell i} = {} & A\, I_{3k\ell i} + B\, I_{4k\ell i} + D\, I_{4k\ell} \\ & - E\, I_{5k\ell i} - F\, I_{6k\ell i} - H\, I_{6k\ell} \end{aligned} \tag{2.83}$$

Again, it is noted that Eq.(2.83) is the same as Eq.(2.48) with $\omega_i=0$.

II.7 PEAK FACTOR AND RESPONSE

For the calculation of R_{dd}, R_{pp} and R_{dp} terms as defined in the previous section, we require the peak factor of the response C_d as well as the peak factors associated with several other floor spectral response quantities. The calculation of these peak factors accurately is rather a sensitive task. Such calculations require that ground motion spectral density function be defined explicitly. This information will usually not be available in practice. An approximate evaluation of these factors can, however, be made for a band-limited white noise spectral density function, and the use of these peak factors may provide a better estimate of design response. Such an approach was used for generation of floor spectra[17]. Implementation of this approach to the current problem is under further study. However, in the mean time the analysis can be simplified, without zeopardizing the accuracy of the results as observed in Reference[17], by assuming that all the peak factors are the same. If this assumption is made, then the response becomes independent of the peak factors. That is any value can be assumed for the peak factors for the calculation of response. In the numerical results presented here, all the peak factors were assumed equal to 1.

II.8 SUMMARY

The response expressions developed in the previous sections for the dynamic, pseudo-static and cross term responses have been utilized to obtain numerical results. For convenience these expressions are summarized in the following

DYNAMIC RESPONSE TERMS:

$$R_{dd}^2 = C_d^2 \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_i \rho_j \sum_{k=1}^{m} \sum_{\ell=1}^{m} P_{ik} P_{j\ell} I_{ak\ell ij} \qquad (2.84)$$

where $I_{ak\ell ij}$, for various combination of the suffixes, is defined as

i) $k=\ell$ and $i=j$.

$$I_{akkii} = R_{dk}^2(\omega_j)/C_{di}^2 \qquad (2.85)$$

ii) $k=\ell$ and $i\neq j$.

$$I_{akkij} = A\, I_{1ki} + B\, I_{2ki} + C\, I_{1kj} + D\, I_{2kj} \qquad (2.86)$$

iii) $k\neq\ell$ and $i=j$.

$$I_{ak\ell ii} = C_{dk\ell}^2(\omega_i)/P_{di}^2 \qquad (2.87)$$

iv) $k\neq\ell$ and $i\neq j$.

$$I_{ak\ell ij} = \{A\, I_{3k\ell i} + B\, I_{4k\ell i} + C\, I_{3k\ell j} + D\, I_{4k\ell j}\}$$
$$- \{E\, I_{5k\ell i} + F\, I_{6k\ell i} + G\, I_{5k\ell j} + H\, I_{6k\ell j}\} \qquad (2.88)$$

where A, B, etc are defined by Eqs.(2.28) and (2.40).

The method to obtain various auto and cross response spectra quantities are developed in the next Chapter.

PSEUDO-STATIC RESPONSE TERM:

This term can be obtained easily using the relative displacement or absolute acceleration formulation. The final expression obtained in the two formulation are as follows

Relative Displacement Formulation

$$R_{pp}^2 = C_d^2 \{\eta_\ell \bar{V}_\ell / C_{v\ell}\}' [\delta'] \{\eta_k \bar{V}_{ak} / C_{vk}\} \tag{2.89}$$

Absolute Acceleration Formulation

$$R_{pp}^2 = C_d^2 \{\eta_\ell \bar{U}_{a\ell} / C_{u\ell}\}' [\delta\,] \{\eta_k \bar{U}_{ak} / C_{uk}\} \tag{2.90}$$

CROSS RESPONSE TERMS:

$$R_{pd}^2 = C_d^2 \sum_{i=1}^{n} \rho_i \sum_{\ell=1}^{m} \sum_{k=1}^{m} 2\eta_\ell P_{ik} I_{adk\ell i} \tag{2.91}$$

where $I_{adk\ell i}$, for various combinations of k and ℓ, is defined as

i) $k=\ell$.

$$I_{adkki} = A\, I_{3kki} + B\, I_{4kki} + D\, I_{4kk} \tag{2.92}$$

ii) $k \neq \ell$.

$$I_{adk\ell i} = A\, I_{3k\ell i} + B\, I_{4k\ell i} + D\, I_{4k\ell}$$

$$- E\, I_{5k\ell i} - F\, I_{6k\ell i} - H\, I_{6k\ell} \tag{2.93}$$

The coefficients A, B, etc in these expressions are again obtained from Eq.(2.28) and (2.40) but with ω_j=0. However, their explicit values are also given by Eq.(2.80)

CHAPTER III

SEISMIC INPUTS FOR MULTIPLE SUPPORT SECONDARY SYSTEMS

III.1 INTRODUCTION

The previous Chapter has identified several types of seismic inputs which must be prescribed for a proper seismic evaluation of multiple support secondary systems. To define these inputs, the primary supporting structure is analyzed for the base seismic input. In this Chapter, the methods are developed to obtain such inputs in terms of the properties of the primary structure and the prescribed ground response spectra.

To define the frequency characteristics of the support or the floor inputs, auto displacement (or pseudo-acceleration) and relative velocity spectra for the floor motion are used. The methods to obtain these floor spectra have been developed earlier [17,22]. However, for the completeness of the treatment and also because of the association of these inputs with other inputs, the formulation used to develop these inputs is described. To characterize the correlation between various floor inputs, the concept of cross floor spectra for displacement and velocity response of an oscillator is used. The methods to obtain these cross spectra directly from ground spectra are developed. These floor spectral inputs are primarily used to calculate the dynamic component of the total response. For the calculation of the pseudo-static component, the floor or support displacements and their correlation are required. Also required are the floor velocities and their correlations for the calculation of the cross response terms. The methods to obtain these floor inputs directly from prescribed ground input and primary structure properties are developed.

III.2 FLOOR SPECTRAL INPUTS FOR DYNAMIC RESPONSE

To define these inputs, we are required to solve the equations of motion of the supporting primary structure, subjected to a base excitation, viz:

$$[M]\{\ddot{V}\} + [C]\{\dot{V}\} + [K]\{V\} = -[M]\{1\}\ddot{X}_g(t) \qquad (3.1)$$

in which [M], [C] and [K] are the mass, damping and stiffness matrices of the structure, respectively; $\{V\}$= the relative displacement vector; $\{1\}$= the excitation influence vector; and $\ddot{X}_g(t)$= the ground acceleration input.

We are interested in expressing the response quantities of this structure in terms of the ground response spectra. Thus, we must use the modal analysis approach. If the system is classically (proportionally) damped, the normal mode approach can be used. However, if the system is nonclassically damped, the complex mode approach is required. Here we will develop the solution only for the proportionally damped system. The formulation involving the nonclassically damped system is also possible (see Reference 17) but will not be given here.

For the classically damped system, Eq.(3.1) can be decoupled into the equations of the principal coordinates as

$$\ddot{Y}_r(t) + 2\beta_r\omega_r\dot{Y}_r(t) + \omega_r^2 Y_r(t) = -\gamma_r \ddot{X}_g(t) \tag{3.2}$$

where Y_r= the r^{th} principal coordinate or modal displacement; ω_r and β_r are the natural frequency and damping ratio of the r^{th} mode, and γ_r= the mode participation factor defined as

$$\gamma_r = \{\psi_r\}'[M]\{1\} \;/\; \{\psi_r\}'[M]\{\psi_r\} \tag{3.3}$$

In Eq.(3.3), $\{\psi_r\}$= the r^{th} displacement mode shape of the system. The absolute acceleration of the floor k can be obtained as a sum of the ground acceleration and the relative acceleration of the floor as

$$\ddot{U}_k(t) = 1_k\ddot{X}_g(t) + \ddot{V}_k(t) \tag{3.4}$$

where 1_k is the k^{th} element of the vector {1} corresponding to the displacement of the floor k.

In Eq.(3.4), the relative floor acceleration can be expressed in terms of the generalized coordinates Y_r by expansion theorem. Thus,

$$\ddot{U}_k(t) = 1_k\ddot{X}_g(t) + \sum_{r=1}^{N} \psi_r(k)\ddot{Y}_r(t) \tag{3.5}$$

Eq.(3.5) can be directly used to define the auto and cross spectral density function for the floor acceleration. This will lead to the mode acceleration formulation [17]. This formulation has some specific computational advantage. However, here only the mode displacement formulation will be given.

In the mode displacement formulation, Eq.(3.5) can be further manipulated to give the following [20]

$$\ddot{U}_k(t) = -\sum_{r=1}^{N} \psi_r(k)[2\beta_r\omega_r\dot{Y}_r(t) + \omega_r^2 Y_r(t)] \tag{3.6}$$

Eq.(3.6) will be used to define the auto and cross spectral density functions of the floor accelerations which are required to define various floor spectral inputs.

CROSS SPECTRAL DENSITY FUNCTION OF FLOOR ACCELERATIONS

The cross correlation function of the accelerations of two floors k and ℓ can be expressed as

$$R_{ak\ell}(t_1,t_2) = E[\ddot{U}_k(t_1)\ddot{U}_\ell(t_2)] \tag{3.7}$$

Substituting from Eq.(3.6), we obtain

$$R_{ak\ell}(t_1,t_2) = \sum_{r=1}^{N}\sum_{s=1}^{N} \psi_r(k)\psi_s(\ell)\, E[\{2\beta_r\omega_r\dot{Y}_r(t_1) + \omega_r^2 Y_r(t_1)\}$$

$$\{2\beta_s\omega_s\dot{Y}_s(t_2) + \omega_s^2 Y_s(t_2)\}] \tag{3.8}$$

Assuming that the ground input is a stationary random process with the spectral density function $\Phi_g(\omega)$, and the response is also stationary, the correlation in Eq.(3.8) can be shown to be given by

$$R_{ak\ell}(t_1,t_2) = \sum_{r=1}^{N}\sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell) \int_{-\infty}^{\infty}(-2i\beta_r\omega_r\omega + \omega_r^2)$$

$$(\,2i\beta_s\omega_s\omega + \omega_s^2)H_r^*H_s\Phi_g(\omega)e^{i\omega(t_2-t_1)}d\omega \tag{3.9}$$

where H_r is the frequency response function of the primary structure defined as

$$H_r = 1/(\omega_r^2 - \omega^2 + 2i\beta_r\omega_r) \tag{3.10}$$

From Eq.(3.9), the cross spectral density function can be identify as

$$\Phi_{ak\ell}(\omega) = \sum_{r=1}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k)\psi_s(\ell)\ (\omega_r^2 - 2i\beta_r\omega_r\omega)$$

$$(\omega_s^2 + 2i\beta_s\omega_s\omega)\ H_r^* H_s \Phi_g(\omega) \qquad (3.11)$$

For $k=\ell$, Eq.(3.11) is a real quantity, whereas for $k\neq\ell$ it will have the real and imaginary parts.

We further expand Eq.(3.11) by separating terms with $r=s$ and $r\neq s$ as follows

$$\Phi_{ak\ell}(\omega) = \sum_{r=1}^{N} \gamma_r^2 \psi_r(k)\psi_s(\ell) X(\omega) |H_r|^2 \Phi_g(\omega)$$

$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[\omega_r^2\omega_s^2 + 4\beta_r\beta_s\omega_r\omega_s\omega^2$$

$$- 2i\omega\omega_r\omega_s(\omega_s\beta_r - \omega_r\beta_s)]\ H_r^* H_s \Phi_g(\omega) \qquad (3.12)$$

where

$$X(\omega) = (\omega_r^4 + 4\beta_r^2\omega_r^2\omega^2) \qquad (3.13)$$

Separating $H_r^* H_s$ into real and imaginary parts as

$$H_r^* H_s = \{N(\omega) + i\omega M(\omega)\}|H_r|^2|H_s|^2 \qquad (3.14)$$

where $N(\omega)$ and $M(\omega)$ are defined by Eqs.(2.26) and (2.37), which for the primary structure parameters are as follows

$$N(\omega) = \omega^4 + \omega^2(4\beta_r\beta_s\omega_r\omega_s - \omega_r^2 - \omega_s^2) + \omega_r^2\omega_s^2 \qquad (3.15)$$

$$M(\omega) = -2\{\omega_r\omega_s(\beta_s\omega_r - \beta_r\omega_s) - \omega^2(\beta_r\omega_r - \beta_s\omega_s)\} \tag{3.16}$$

Substituting Eq.(3.14) into (3.12) and separating the real and imaginary parts, we obtain

$$\begin{aligned}\Phi_{ak\ell}(\omega) = \Big(&\sum_{r=1}^{N} \gamma_r^2\psi_r(k)\psi_r(\ell)X(\omega)|H_r|^2 + \\ &\sum_{\substack{r=1 \\ r\neq s}}^{N}\sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[T(\omega) + i\omega Z(\omega)]|H_r|^2|H_s|^2\Big)\Phi_g(\omega)\end{aligned} \tag{3.17}$$

where

$$T(\omega) = (\omega_r^2\omega_s^2 + 4\beta_r\beta_s\omega_r\omega_s\omega^2)N(\omega) + 2\omega_r\omega_s(\beta_r\omega_s - \beta_s\omega_r)\omega^2M(\omega) \tag{3.18}$$

$$Z(\omega) = -2\omega_r\omega_s(\beta_r\omega_s - \beta_s\omega_r)N(\omega) + (\omega_r^2\omega_s^2 + 4\beta_r\beta_s\omega_r\omega_s\omega^2)M(\omega) \tag{3.19}$$

Eq.(3.17) defines the real and imaginary parts of the cross spectral density functions as

$$\begin{aligned}\Phi^R_{ak\ell}(\omega) = &\sum_{r=1}^{N} \gamma_r^2\psi_r(k)\psi_s(\ell)X(\omega)|H_r|^2\Phi_g(\omega) \\ &+ \sum_{\substack{r=1 \\ r\neq s}}^{N}\sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)\, T(\omega)\, |H_r|^2|H_s|^2\Phi_g(\omega)\end{aligned} \tag{3.20}$$

and

$$\Phi^I_{ak\ell}(\omega) = \sum_{\substack{r=1 \\ r\neq s}}^{N}\sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)\omega Z(\omega)|H_r|^2|H_s|^2\Phi_g(\omega) \tag{3.21}$$

For $k=\ell$, Eq.(3.20) provides the auto spectral density function of the k^{th} floor.

We further reform Eqs.(3.20) and (3.21) by splitting the double summation terms into partial fractions.

$$\Phi^R_{ak\ell}(\omega) = \sum_{r=1}^{N} \gamma_r^2 \psi_r(k)\psi_s(\ell)X(\omega)|H_r|^2\Phi_g(\omega)$$

$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)\{(A_3+\omega^2B_3)|H_r|^2 + (C_3+\omega^2D_3)|H_s|^2\}\Phi_g(\omega) \quad (3.22)$$

$$\omega\Phi^I_{ak\ell}(\omega) = \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[(A_4 + \omega^2B_4)|H_r|^2 + (C_4 + \omega^2D_4)|H_s|^2]\Phi_g(\omega) \quad (3.23)$$

where the coefficients of the partial fraction in Eqs.(3.22) and (2.23) are obtained as the solution of the following simultaneous equations

$$[Y_{rs}]\{V_3\} = \{W_3\} \quad (3.24a)$$

$$[Y_{rs}]\{V_4\} = \{W_4\} \quad (3.24b)$$

Matrix $[Y_{rs}]$ is the same as Eq.(2.29) except that the subscripts i and j, respectively, are now replaced by the subscripts r and s which pertain to the primary structure and is defined as,

$$[Y_{rs}] = \begin{bmatrix} \omega_s^4 & 0 & \omega_r^4 & 0 \\ 2\omega_s^2(2\beta_s^2-1) & \omega_s^4 & 2\omega_r^2(2\beta_r^2-1) & \omega_r^4 \\ 1 & 2\omega_s^2(2\beta_s^2-1) & 1 & 2\omega_r^2(2\beta_r^2-1) \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad (3.25)$$

The vectors $\{V_3\}$, $\{W_3\}$, etc are defined as follows

$$\{V_3\}' = \{A_3, B_3, C_3, D_3\} \quad (3.26a)$$

$$\{V_4\}' = \{A_4, B_4, C_4, D_4\} \quad (3.26b)$$

$$W_3(1) = \omega_r^4\omega_s^4 \quad (3.26c)$$

$$W_3(2) = \omega_r^2\omega_s^2[-\ \omega_r^2(1-4\beta_s^2)\ -\ \omega_s^2(1-4\beta_r^2)] \tag{3.26d}$$

$$W_3(3) = \omega_r^2\omega_s^2(1-4\beta_s^2)\ (1-4\beta_r^2) \tag{3.26e}$$

$$W_3(4) = 4\beta_r\beta_s\omega_r\omega_s \tag{3.26f}$$

$$W_4(1) = 0 \tag{3.26g}$$

$$W_4(2) = 0 \tag{3.26h}$$

$$W_4(3) = -2\omega_r\omega_s(\beta_s\omega_r^3 - \beta_r\omega_s^3) \tag{3.26i}$$

$$W_4(4) = -2[\omega_r\omega_s^2\beta_r(1-4\beta_s^2) - \omega_r^2\omega_s\beta_s(1-4\beta_r^2)] \tag{3.26j}$$

Eqs.(3.22) and (3.23) will now be used to define the auto and cross floor spectra.

III.2.1 AUTO FLOOR SPECTRA

A method to obtain auto floor response spectra was developed by Singh[20,22] where the expression for the absolute floor acceleration spectra was provided. In the response analysis of the secondary system, however, we need the displacement (or pseudo-acceleration) and relative velocity floor spectra. Here, using Singh's approach, the explicit expression for these two types of spectra are developed.

AUTO DISPLACEMENT FLOOR SPECTRA

Here we will obtain the expression for the displacement floor spectrum. The pseudo-acceleration floor spectrum can, then, be obtained by merely multiplying the displacement spectrum by the square of the oscillator frequency.

The displacement floor spectrum at the oscillator frequency ω_i and damping ratio β_i is defined as

$$R_{dk}^2(\omega_i) = C_{di}^2 \int_{-\infty}^{\infty} \Phi_{akk}(\omega)|H_i|^2 d\omega \tag{3.27}$$

where C_{di} is the peak factor for the displacement response. Substituting for the spectral density function $\Phi_{akk}(\omega)$ from Eq.(3.22),

$$R_{dk}^2(\omega_i) = C_{di}^2 \int_{-\infty}^{\infty} \Big(\sum_{r=1}^{N} \gamma_r^2 \psi_r^2(k) X(\omega)|H_r|^2|H_i|^2 + \sum_{\substack{r=1 \\ r \neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell)$$

$$[(A_3 + \omega^2 B_3)|H_r|^2 + (C_3 + \omega^2 D_3)|H_s|^2]|H_i|^2 \Phi_g(\omega)\Big) d\omega \tag{3.28}$$

To express the frequency integrals in Eq.(3.28) in terms of the ground response spectra, we must further resolve the products involving $|H_r|^2$, $|H_s|^2$ and $|H_i|^2$ into their linear sum by partial fractions. Such partial fractions can be obtained if H_r or H_s are not equal to H_i. Assuming such a case, we obtain

$$X(\omega)|H_r|^2|H_i|^2 = (A_5 + \omega^2 B_5)|H_r|^2 + (C_5 + \omega^2 D_5)|H_i|^2 \tag{3.29}$$

$$(A_3 + \omega^2 B_3)|H_r|^2|H_i|^2 = (A_6 + \omega^2 B_6)|H_r|^2 + (C_6 + \omega^2 D_6)|H_i|^2 \tag{3.30}$$

$$(C_3 + \omega^2 D_3)|H_s|^2|H_i|^2 = (A_7 + \omega^2 B_7)|H_s|^2 + (C_7 + \omega^2 D_7)|H_i|^2 \tag{3.31}$$

The coefficients of the partial fractions A_5, B_5, ..., A_6, B_6,... and A_7, B_7, etc are obtained from the solution of the following set of linear simultaneous equations

$$[Y_{ri}]\{V_5\} = \{W_5\} \tag{3.32a}$$

$$[Y_{ri}]\{V_6\} = \{W_6\} \tag{3.32b}$$

$$[Y_{si}]\{V_7\} = \{W_7\} \tag{3.32c}$$

where the elements of Y_{ri}, Y_{si} are the same as Eq.(3.25) except for the change in subscripts. The vectors $\{V_5\}$, $\{V_6\}$, etc are defined as

$$\{V_5\}' = \{A_5, B_5, C_5, D_5\} \tag{3.33a}$$

$$\{V_6\}' = \{A_6, B_6, C_6, D_6\} \tag{3.33b}$$

$$\{V_7\}' = \{A_7, B_7, C_7, D_7\} \tag{3.33c}$$

$$\{W_5\}' = \{\omega_r^4, 4\beta_r^2\omega_r^2, 0, 0\} \tag{3.33d}$$

$$\{W_6\}' = \{A_3, B_3, 0, 0\} \tag{3.33e}$$

$$\{W_7\}' = \{C_3, D_3, 0, 0\} \tag{3.33f}$$

Now the frequency integrals in Eq.(3.28) can be expressed in terms of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and their peak factors, obtained at appropriate frequency and damping ratio values as follows

$$R_{dg}^2(\omega_r) = R_{pg}^2(\omega_r)/\omega_r^4 = C_{dgr}^2 \int_{-\infty}^{\infty} \Phi_g(\omega)|H_r|^2 d\omega \tag{3.34}$$

$$R_{vg}^2(\omega_r) = C_{vgr}^2 \int_{-\infty}^{\infty} \omega^2 \Phi_g(\omega)|H_r|^2 d\omega \tag{3.35}$$

where $R_{dg}(\omega_r)$, $R_{pg}(\omega_r)$ and $R_{vg}(\omega_r)$ are the relative displacement, pseudo-accelration and relative velocity ground response spectrum values obtained at parameters ω_r and β_r, respectively. C_{dgr} and C_{vgr} are the peak factors values associated with these response quantities.

Substituting for the frequency integrals in Eq.(3.28) in terms of the ground spectra, we obtain the following expression for the displacement response spectrum

$$R_{dk}^2(\omega_i) = C_{di}^2 \{\sum_{r=1}^{N} \gamma_r^2\psi_r^2(k)[A_5 I_{1gr} + B_5 I_{2gr} + C_5 I_{1gi} + D_5 I_{2gi}]$$
$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(k) ([A_6 I_{1gr} + B_6 I_{2gr} + C_6 I_{1gi} + D_6 I_{2gi}]$$
$$+ [A_7 I_{1gs} + B_7 I_{2gs} + C_7 I_{1gi} + D_7 I_{2gi}])\} \quad (3.36)$$

where

$$I_{1gr} = [R_{pg}(\omega_r)/(C_{pgr}\omega_r^2)]^2 \quad (3.37a)$$

$$I_{2gr} = [R_{vg}(\omega_r)/C_{vgr}]^2 \quad (3.37b)$$

The case when H_r or H_s are identically equal to H_i is referred to as the resonance case. In such a case it is not possible to define the coefficients of partial fraction A_5, B_5, etc. in Eq.(3.29) through (3.31). This case, however, can be treated as described by Singh[22]. The frequency integrals for the resonance case required in Eq.(3.28) are obtained as a special case of the following integral

$$I_R(a_1,a_2) = \int_{-\infty}^{\infty} (a_1\omega_i^8 + a_2\omega_i^6\omega^2)|H_i|^4\Phi_g(\omega)d\omega \quad (3.38a)$$

Following Singh[22], it can be shown that

$$I_R(a_1,a_2) = I_{1gi}\,\omega_i^4[F(\omega_i)C_m'+a_1\{1-F(\omega_i)\}] + a_2\omega_i^2 D_m' \quad (3.38b)$$

where $F(\omega_i)$, C_m' and D_m' are defined in Appendix II.

For $H_r = H_i$, the frequency integral associated with H_r and H_i in Eq.(3.28) can be defined in term of I_R as follows

$$I_{R1} = \int_{-\infty}^{\infty} X(\omega)|H_r|^4 \Phi_g(\omega)d\omega = \frac{1}{\omega_r^4} I_R(1,4\beta_r^2) \qquad (3.39a)$$

$$I_{R2} = \int_{-\infty}^{\infty} (A_3+\omega^2 B_3)|H_r|^4 \Phi_g(\omega)d\omega = \frac{1}{\omega_r^6} I_R(A_3/\omega_r^2, B_3) \qquad (3.39b)$$

Substituting these in Eq.(3.28), the displacement floor response spectrum expression for the resonance case becomes

$$R_{dk}^2(\omega_i) = C_{di}^2 \{\gamma_i^2 \psi_i^2(k) I_{R1} + \sum_{\substack{r=1 \\ r\neq i}}^{N} \gamma_r^2 \psi_r^2(k)[A_5 I_{1gr} + B_5 I_{2gr} + C_5 I_{1gi} + D_5 I_{2gi}]$$
$$+ \sum_{\substack{s=1 \\ s\neq i}}^{N} \gamma_i \gamma_s \psi_i(k)\psi_s(k)[I_{R2} + A_7 I_{1gs} + B_7 I_{2gs} + C_2 I_{1gi} + D_7 I_{2gi}]$$
$$+ \sum_{\substack{r=1 \\ r\neq i}}^{N} \sum_{\substack{s=1 \\ r\neq s}}^{N} \gamma_r \gamma_s \psi_r(k)\psi_s(k)[A_6 I_{1gr} + B_6 I_{2gr} + C_6 I_{1gi} + D_6 I_{2gi}$$
$$+ A_7 I_{1gs} + B_7 I_{7gs} + C_7 I_{1gi} + D_7 I_{2gi}]\} \qquad (3.40)$$

AUTO VELOCITY FLOOR SPECTRA

The velocity spectrum for floor k at the oscillator frequency ω_i and damping ratio β_i is defined as

$$R_{vk}^2(\omega_i) = C_{vi}^2 \int_{-\infty}^{\infty} \omega^2 \Phi_{akk}(\omega)|H_i|^2 d\omega \qquad (3.41)$$

where C_{vi} is the peak factor for the velocity response. Substituting for $\Phi_{akk}(\omega)$ from Eq.(3.22), we obtain

$$R_{vk}^2(\omega_i) = C_{vi}^2 \int_{-\infty}^{\infty} (\sum_{r=1}^{N} \gamma_r^2 \psi_r^2(k)\ \omega^2 X(\omega)|H_r|^2 |H_i|^2$$
$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k)\psi_s(k)\omega^2 [(A_3 + \omega^2 B_3)|H_r|^2$$
$$+ (C_3 + \omega^2 D_3)|H_s|^2]|H_i|^2 \Phi_g(\omega))d\omega \qquad (3.42)$$

To express the frequency integrals in Eq.(3.42) in terms of the ground response spectra, we again resolve the products involving $|H_r|^2$, $|H_s|^2$ and $|H_i|^2$ into their linear sum by partial fractions. If H_r or H_s are not equal to H_i, we obtain

$$\omega^2 X(\omega)|H_r|^2|H_i|^2 = (A_8 + \omega^2 B_8)|H_r|^2 + (C_8 + \omega^2 D_8)|H_i|^2 \quad (3.43)$$

$$\omega^2(A_3 + \omega^2 B_3)|H_r|^2|H_i|^2 = (A_9 + \omega^2 B_9)|H_r|^2 + (C_9 + \omega^2 D_9)|H_i|^2 \quad (3.44)$$

$$\omega^2(C_3 + \omega^2 D_3)|H_s|^2|H_i|^2 = (A_{10} + \omega^2 B_{10})|H_s|^2 + (C_{10} + \omega^2 D_{10})|H_i|^2 \quad (3.45)$$

where the coefficients of the partial fractions A_8, B_8,..., A_9, B_9,... and A_{10}, B_{10}, etc are obtained from the solution of the following set of linear simultaneous equations

$$[Y_{ri}]\{V_8\} = \{W_8\} \quad (3.46a)$$

$$[Y_{ri}]\{V_9\} = \{W_9\} \quad (3.46b)$$

$$[Y_{si}]\{V_{10}\} = \{W_{10}\} \quad (3.46c)$$

where $[Y_{ri}]$ is defined by Eq.(3.25) and the vectors $\{V_8\}$, $\{V_9\}$, etc are defined as

$$\{V_8\}' = \{A_8, B_8, C_8, D_8\} \quad (3.47a)$$

$$\{V_9\}' = \{A_9, B_9, C_9, D_9\} \quad (3.47b)$$

$$\{V_{10}\}' = \{A_{10}, B_{10}, C_{10}, D_{10}\} \quad (3.47c)$$

$$\{W_8\}' = \{0, \omega_r^4, 4\beta_r^2\omega_r^2, 0\} \quad (3.47d)$$

$$\{W_9\}' = \{0, A_3, B_3, 0\} \quad (3.47e)$$

$$\{W_{10}\}' = \{0,\ C_3,\ D_3,\ 0\} \tag{3.47f}$$

Again each frequency integral in Eq.(3.42) can be expressed in terms of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and their peak factors, obtained at appropriate frequency and damping ratio. That is

$$\begin{aligned} R_{vk}^2(\omega_i) = C_{vi}^2 \Big(\sum_{r=1}^{N} \gamma_r^2 \psi_r^2(k) [A_8 I_{1gr} + B_8 I_{2gr} + C_8 I_{1gi} + D_8 I_{2gi}] \\ + \sum_{\substack{r=1 \\ r \neq s}}^{N} \sum_{s=2}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(k) [A_9 I_{1gr} + B_9 I_{2gr} + C_9 I_{1gi} + D_9 I_{2gi} \\ + A_{10} I_{1gs} + B_{10} I_{2gs} + C_{10} I_{1gi} + D_{10} I_{2gi}] \Big) \end{aligned} \tag{3.48}$$

The case where H_r or H_s is equal to H_i can again be treated as described by Singh [22]. For this case the required frequency integrals can be obtained as a special case of the following integral

$$I_S(a_2,a_3) = \int_{-\infty}^{\infty} (a_2\omega_i^6\omega^2 + a_3\omega_i^4\omega^4)|H_i|^4 \Phi_g(\omega)d\omega \tag{3.49a}$$

which can be defined in terms of ground spectra as follows

$$I_S(a_2,a_3) = I_{1gi}\ \omega_i^4\ F(\omega_i)F_m' + a_2\omega_i^2 I_{2gi} D_m' \tag{3.49b}$$

where $F(\omega_i)$, F_m and D_m are defined in Appendix II. For H_r=H_i, the frequency integral in Eq.(3.42), which are associated with H_r and H_i can be defined in terms of I_S

$$I_{S1} = \int_{-\infty}^{\infty} \omega^2 X(\omega)|H_i|^4 \Phi_g(\omega)d\omega = \frac{1}{\omega_i^2}\ I_S(1,4\beta_i^2) \tag{3.50a}$$

$$I_{S2} = \int_{-\infty}^{\infty} \omega^2 (A_3+\omega^2 B_3)|H_i|^4 \Phi_g(\omega)d\omega = \frac{1}{\omega_i^4}\ I_S(\omega_i^2 A_3, B_3) \tag{3.50b}$$

Substituting these in Eq.(3.42), the velocity floor response spectrum expression for the resonance case can be written as

$$R_{vk}^2(\omega_i) = C_{vi}^2\{\gamma_i^2\psi_i^2(k)I_{S1} + \sum_{\substack{r=1\\r\neq i}}^{N} \gamma_r^2\psi_r^2(k)[A_8I_{1gr} + B_8I_{2gr} + C_8I_{1gi} + D_8I_{2gi}]$$
$$+ \sum_{\substack{s=1\\s\neq i}}^{N} \gamma_i\gamma_s\psi_i(k)\psi_s(k)[I_{S2} + A_{10}I_{1gs} + B_{10}I_{2gs} + C_{10}I_{1gi} + D_{10}I_{2gi}]$$
$$+ \sum_{\substack{r=1\\r\neq i}}^{N} \sum_{\substack{s=1\\r\neq s}}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(k)[A_9I_{1gr} + B_9I_{2gr} + C_9I_{1gi} + D_9I_{2gi}$$
$$+ A_{10}I_{1gs} + B_{10}I_{2gs} + C_{10}I_{1gi} + D_{10}I_{2gi}]\} \tag{3.51}$$

III.2.2 CROSS FLOOR SPECTRA

In the response analysis of the secondary systems with multiply supports, the coincident and quadrature cross floor spectra for the displacement and velocity responses are required. Here, using similar approach as in the case of the auto floor spectra, expressions for these two type of spectra are developed.

COINCIDENT DISPLACEMENT SPECTRA

The coincident displacement spectrum at the frequency ω_i and damping ratio β_i is defined as

$$C_{dk\ell}^2(\omega_i) = P_{di}^2 \int_{-\infty}^{\infty} \Phi_{ak\ell}^R(\omega)|H_i|^2 d\omega \tag{3.52}$$

where P_{di} is the peak factor for the displacement response. The coincident pseudo-acceleration spectrum is obtained from the displacement spectrum as

$$C_{pk\ell}(\omega_i) = \omega_i^2 \, C_{dk\ell}(\omega_i) \tag{3.53}$$

Substituting for $\Phi^R_{ak\ell}(\omega)$ from Eq.(3.22) into (3.52), we obtain

$$C^2_{dk\ell}(\omega_i) = P^2_{di} \Big(\sum_{r=1}^{N} \gamma_r^2 \psi_r(k)\psi_s(\ell) \int_{-\infty}^{\infty} X(\omega)|H_r|^2|H_i|^2\Phi_g(\omega)d\omega$$
$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell) \int_{-\infty}^{\infty} [(A_3 + \omega^2 B_3)|H_r|^2$$
$$+ (C_3 + \omega^2 D_3)|H_s|^2]|H_i|^2\Phi_g(\omega)d\omega) \tag{3.54}$$

The frequency integrals in Eq.(3.54) have already been defined in connection with the development of the auto displacement floor spectra. Thus, by substituting Eq.(3.29), (3.30) and (3.31) into Eq.(3.54), each frequency integral can be expressed in term of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and peak factors, obtained at appropriate frequency and damping ratio values. This leads to

$$C^2_{dk\ell}(\omega_i) = P^2_{di}\Big(\sum_{r=1}^{N} \gamma_r^2\psi_r(k)\psi_r(\ell)[A_5 I_{1gr} + B_5 I_{2gr} + C_5 I_{1gi} + D_5 I_{2gi}]$$
$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[A_6 I_{1gr} + B_6 I_{2gr} + C_6 I_{1gi} + D_6 I_{2gi}]$$
$$+ A_7 I_{1gs} + B_7 I_{2gs} + C_7 I_{1gi} + D_7 I_{2gi}] \tag{3.55}$$

For the resonance case when $H_r = H_i$, the frequency integrals I_{R1} and I_{R2} of Eq.(3.39) can be directly used. In terms of these integrals, the expression of the coincident displacement spectra can be written as

$$C^2_{dk\ell}(\omega_i) = P^2_{di}\{\gamma_i^2\psi_i(k)\psi_i(\ell)I_{R1} + \sum_{\substack{r=1\\ r\neq 1}}^{N} \gamma_r^2\psi_r^2(k)[A_5I_{1gr} + B_5I_{2gr} + C_5I_{1gi}$$
$$+ D_5I_{2gi}] + \sum_{\substack{s=1\\ s\neq i}}^{N} \gamma_i\gamma_s\psi_i(k)\psi_s(\ell)[I_{R2} + A_7I_{1gs} + B_7I_{2gs} + C_7I_{1gi}$$
$$+ D_7I_{2gi}] + \sum_{\substack{r=1\\ r\neq i}}^{N}\sum_{\substack{s=1\\ r\neq s}}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[A_6I_{1gr} + B_6I_{2gr} + C_6I_{1gi}$$
$$+ D_6I_{2gi} + A_7I_{1gs} + B_7I_{2gs} + C_7I_{1gi} + D_7I_{2gi}]\} \qquad (3.56)$$

COINCIDENT VELOCITY SPECTRA

The coincident velocity spectrum at frequency ω_i and damping ratio β_i is defined as

$$C^2_{vk\ell}(\omega_i) = P^2_{vi}\int_{-\infty}^{\infty} \omega^2\Phi^R_{ak\ell}(\omega)|H_i|^2d\omega \qquad (3.57)$$

where P_{vi} is the peak factor of the velocity response. Substituting for $\Phi^R_{ak\ell}(\omega)$ from Eq.(3.22), we obtain an equation exactly similar to Eq.(3.54), except that the integrands of each integral will be now be multiply by ω^2. We further break this integrand into partial fractions. The frequency integrals and participation factors involved in this expressions are the same those involved in the calculation of the auto velocity floor spectra. Thus, the expression for this spectrum is essentially the same as Eq.(3.48) except for the fact that here we are concerned with two floors. With a proper substituting for the modal displacements of the two floors, we obtain:

$$C^2_{vk\ell}(\omega_i) = P^2_{vi}\ (\sum_{r=1}^{N} \gamma_r^2\psi_r(k)\psi_s(\ell)[A_8I_{1gr} + B_8I_{2gr} + C_8I_{1gi} + D_8I_{2gi}]$$

$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[A_9I_{1gr} + B_9I_{2gr} + C_9I_{1gi} + D_9I_{2gi}$$

$$+ A_{10}I_{1gs} + B_{10}I_{2gs} + C_{10}I_{1gi} + D_{10}I_{2gi}] \qquad (3.58)$$

For the resonance case when H_r is equal to H_i, the frequency integral can be written in terms of integrals I_{S1} and I_{S2}. Thus, we obtain

$$C^2_{vk\ell}(\omega_i) = P^2_{vi}\{\gamma_i^2\psi_i(k)\psi_r(\ell)I_{S1} + \sum_{\substack{r=1 \\ r\neq i}}^{N} \gamma_r^2\psi_r(k)\psi_r(\ell)[A_8I_{1gr} + B_8I_{2gr} + C_8I_{1gi}$$

$$+ D_8I_{2gi}] + \sum_{\substack{s=1 \\ s\neq i}}^{N} \gamma_i\gamma_s\psi_i(k)\psi_s(\ell)[I_{s2} + A_{10}I_{1gs} + B_{10}I_{2gs} + C_{10}I_{1gi}$$

$$+ D_{10}I_{2gi}] + \sum_{\substack{r=1 \\ r\neq i}}^{N} \sum_{\substack{s=1 \\ r\neq s}}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[A_9I_{1gr} + B_9I_{2gr} + C_9I_{1gi}$$

$$+ D_9I_{2gi} + A_{10}I_{1gs} + B_{10}I_{2gs} + C_{10}I_{1gi} + D_{10}I_{2gi}]\} \qquad (3.59)$$

QUADRATURE DISPLACEMENT SPECTRA

The quadrature displacement spectrum at the frequency ω_i and damping ratio β_i is defined as

$$Q^2_{dk\ell}(\omega_i) = Q^2_{di} \int_{-\infty}^{\infty} \omega\ \Phi^I_{ak\ell}(\omega)|H_i|^2 d\omega \qquad (3.60)$$

where Q_{di} is the peak factor for the displacement response. The quadrature pseudo-acceleration spectrum is obtained from displacement spectrum as

$$Q_{pk\ell}(\omega_i) = \omega_i^2 Q_{dk\ell}(\omega_i) \qquad (3.61)$$

Substituting Eq.(3.23) into (3.60), we obtain

$$Q^2_{dk\ell}(\omega_i) = Q^2_{di} \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k)\psi_s(\ell) \int_{-\infty}^{\infty} [(A_4 + \omega^2 B_4)|H_r|^2 + (C_4 + \omega^2 D_4)|H_s|^2]|H_i|^2 \Phi_g(\omega)d\omega \quad (3.62)$$

To express the frequency integrals in Eq.(3.61) in terms of ground response spectra, we resolve the products involving $|H_r|^2$, $|H_s|^2$ and $|H_i|^2$ into their partial fractions. If H_r or H_s are not identically equal to H_i, we can write

$$(A_4 + \omega^2 B_4)|H_r|^2|H_i|^2 = (A_{11} + \omega^2 B_{11})|H_r|^2 + (C_{11} + \omega^2 D_{11})|H_i|^2 \quad (3.63)$$

$$(C_4 + \omega^2 D_4)|H_s|^2|H_i|^2 = (A_{12} + \omega^2 B_{12})|H_s|^2 + (C_{12} + \omega^2 D_{12})|H_i|^2 \quad (3.64)$$

The coefficients of the partial fraction A_{11}, B_{11}, ..., A_{12}, B_{12}, etc are obtained from the solution of the following set of simultaneous equations

$$[Y_{ri}]\{V_{11}\} = \{W_{11}\} \quad (3.65a)$$

$$[Y_{si}]\{V_{12}\} = \{W_{12}\} \quad (3.65b)$$

The vectors $\{V_{11}\}$, $\{V_{12}\}$, etc. are defined as

$$\{V_{11}\}' = \{A_{11}, B_{11}, C_{11}, D_{11}\} \quad (3.66a)$$

$$\{V_{12}\}' = \{A_{12}, B_{12}, C_{12}, D_{12}\} \quad (3.66b)$$

$$\{W_{11}\}' = \{A_4, B_4, 0, 0\} \quad (3.66c)$$

$$\{W_{12}\}' = \{C_4, D_4, 0, 0\} \quad (3.66d)$$

The frequency integrals in Eq.(3.62) can now be expressed in term of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and peak factors, obtained at appropriate frequency and damping ratio values. This leads to

$$Q^2_{dk\ell}(\omega_i) = Q^2_{di} \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell) [A_{11'} I_{1gr} + B_{11} I_{2gr} + C_{11} I_{1gi}$$

$$+ D_{11} I_{2gi} + A_{12} I_{1gs} + B_{12} I_{2gs} + C_{12} I_{1gi} + D_{12} I_{2gi}] \quad (3.67)$$

For the case when H_r is equal to H_i, the frequency integral I_{R2} of Eq.(3.39) can be directly used. In terms of these integrals, the expression of the quadrature spectra can be written as

$$Q^2_{dk\ell}(\omega_i) = Q^2_{di} \{ \sum_{\substack{s=1 \\ s\neq i}}^{N} \gamma_i \gamma_s \psi_i(k) \psi_s(\ell) [I_{R2} + A_{12} I_{1gs} + B_{12} I_{2gs} + C_{12} I_{1gi}$$

$$+ D_{12} I_{2gi}] + \sum_{\substack{r=1 \\ r\neq i}}^{N} \sum_{\substack{s=1 \\ r\neq s}}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell) [A_{11} I_{1gr} + B_{11} I_{2gr}$$

$$+ C_{11} I_{1gi} + D_{11} I_{2gi} + A_{12} I_{1gs} + B_{12} I_{2gs} + C_{12} I_{1gi} + D_{12} I_{2gi}]\} \quad (3.68)$$

<u>QUADRATURE VELOCITY SPECTRA</u>

The quadrature velocity spectrum at frequency ω_i and damping ratio β_i is defined as

$$Q^2_{vk\ell}(\omega_i) = Q^2_{vi} \int_{-\infty}^{\infty} \omega^3 \Phi^I_{ak\ell}(\omega) |H_i|^2 d\omega \quad (3.69)$$

where Q_{vi} is the peak factor of the velocity response. Substituting for $\Phi^I_{ak\ell}(\omega)$ from Eq.(3.23) into (3.69) will give an equation exactly similar to Eq.(3.62), except that the integrand will be multiplied by ω^2. Here again we resolve the product terms involving $|H_r|^2$, $|H_s|^2$ and $|H_i|^2$ into their partial fractions. If H_r or H_s are not identically equal to H_i, we can write

$$\omega^2(A_4 + \omega^2 B_4)|H_i|^2|H_i|^2 = (A_{13} + \omega^2 B_{13})|H_r|^2 + (C_{13} + \omega^2 D_{13})|H_i|^2 \quad (3.70)$$

$$\omega^2(C_4 + \omega^2 D_4)|H_s|^2|H_i|^2 = (A_{14} + \omega^2 B_{14})|H_s|^2 + (C_{14} + \omega^2 D_{14})|H_i|^2 \quad (3.71)$$

where the coefficients of the partial fractions A_{13}, B_{13},... and A_{14}, B_{14}, etc are obtained from the solutions of the following simultaneous equations

$$[Y_{ri}]\{V_{13}\} = \{W_{13}\} \quad (3.72a)$$

$$[Y_{si}]\{V_{14}\} = \{W_{14}\} \quad (3.72b)$$

The vectors $\{V_{13}\}$, $\{V_{14}\}$, etc are defined as

$$\{V_{13}\}' = \{A_{13}, B_{13}, C_{13}, D_{13}\} \quad (3.73a)$$

$$\{V_{14}\}' = \{A_{14}, B_{14}, C_{14}, D_{14}\} \quad (3.73b)$$

$$\{W_{13}\}' = \{0, A_4, B_4, 0\} \quad (3.73c)$$

$$\{W_{14}\}' = \{0, C_4, D_4, 0\} \quad (3.73d)$$

The frequency integrals in Eq.(3.69) can be expressed in terms the of pseudo-acceleration (or relative displacement) and relative velocity ground spectra and associated peak factors, to give us the following

$$Q^2_{vk\ell}(\omega_i) = Q^2_{vi} \sum_{r=1}^{N} \sum_{\substack{s=1 \\ r\neq s}}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell) [A_{13} I_{1gr} + B_{13} I_{2gr} + C_{13} I_{1gi} + D_{13} I_{1gi} + A_{14} I_{1gs} + B_{14} I_{2gs} + C_{14} I_{1gi} + D_{14} I_{2gi}] \quad (3.74)$$

For the resonance case when Hr is equal to Hi, the frequency integral can be written in terms of integrals I_{S2}. In such a case, Eq.(3.69) can be written as

$$Q^2_{vk\ell}(\omega_i) = Q^2_{vi}\{\sum_{\substack{s=1\\s\neq i}}^{N} \gamma_i\gamma_s\psi_i(k)\psi_s(\ell)[I_{S2} + A_{14}I_{1gs} + B_{14}I_{2gs} + C_{14}I_{1gi}$$

$$+ D_{14}I_{2gi}] + \sum_{\substack{r=1\\r\neq i}}^{N} \sum_{\substack{s=1\\r\neq s}}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(\ell)[A_{13}I_{1gr} + B_{14}I_{2gr} + C_{14}I_{1gi}$$

$$+ D_{14}I_{2gi} + A_{14}I_{1gs} + B_{14}I_{2gs} + C_{14}I_{1gi} + D_{14}I_{2gi}]\} \qquad (3.75)$$

III.3 FLOOR INPUTS FOR PSEUDO-STATIC RESPONSE

For the calculation of the pseudo-static response term, we require the maximum values of either the relative or absolute displacement of each supporting floor. We also require the correlation between these response quantities. Again, these quantities are obtained from the dynamic analysis of the primary structure. The expressions to obtain these are developed in the following sections.

MAXIMUM FLOOR RESPONSE-RELATIVE DISPLACEMENT

The relative floor displacement response for floor k can be written in terms of modal quantities as follows

$$V_k(t) = \sum_{r=1}^{N} \psi_r(k)Y_r(t) \qquad (3.76)$$

The maximum value of $V_k(t)$, here denoted as $\bar{V}_k$, can be defined in terms of its mean square response and peak factor value, C_{vk}, as

$$\bar{V}^2_k = C^2_{vk}\ E[V^2_k(t)] \qquad (3.77)$$

Substituting for $V_k(t)$ from Eq.(3.76), we obtain

$$\bar{V}_k^2 = C_{vk}^2 \sum_{r=1}^{N} \sum_{s=1}^{N} \psi_r(k)\psi_s(k)E[Y_rY_s] \qquad (3.78)$$

Substituting for the expected value of $E[Y_rY_s]$ in terms of the ground spectral density function and considering only the stationary response, we obtain

$$\bar{V}_k^2 = C_{vk}^2 \sum_{r=1}^{N} \sum_{s=1}^{N} \psi_r(k)\psi_s(k)\gamma_r\gamma_s \int_{-\infty}^{\infty} \Phi_g(\omega)H_r^*H_s d\omega \qquad (3.79)$$

To express the frequency integral in terms of ground spectra, we separate terms with r=s and r≠s, to obtain

$$\bar{V}_k^2 = C_{vk}^2 \Big(\sum_{r=1}^{N} \gamma_r^2\psi_r^2(k) \int_{-\infty}^{\infty} \Phi_g(\omega)|H_r|^2 d\omega$$

$$+ 2 \sum_{r=1}^{N-1} \sum_{s=r+1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(k) \int_{-\infty}^{\infty} N(\omega)|H_r|^2|H_s|^2 \Phi_g(\omega)d\omega \Big) \qquad (3.80)$$

where $N(\omega)$ is given in Eq.(3.15).

The first frequency integral in Eq.(3.80) can be directly expressed in terms of the relative displacement or pseudo-acceleration spectrum values. We resolve the second integral into its partial fractions as in Eq.(2.27), so that each term can then be expressed in terms of the pseudo-acceleration and relative velocity spectra. The final expression for the relative displacement response can, then, be shown to be as follows

$$\bar{V}_k^2 = C_{vk}^2 \Big(\sum_{r=1}^{N} \gamma_r^2\psi_r^2(k)\ I_{1gr} + 2 \sum_{r=1}^{N-1} \sum_{s=r+1}^{N} \gamma_r\gamma_s\psi_r(k)\psi_s(k)$$

$$[AI_{1gr} + BI_{2gr} + CI_{1gs} + DI_{2gs}]\Big) \qquad (3.81)$$

where I_{1gr}, I_{2gr} are defined in terms of the ground response spectra by Eqs.(3.37a) and (3.37b). This equation is similar to the one obtained by Singh and Chu[24].

RELATIVE DISPLACEMENT CORRELATION COEFFICIENT

The expression for the correlation between the relative displacement of two floors can be developed similarly. For two floors k and ℓ, it can be shown that

$$E[V_k V_\ell] = \sum_{r=1}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell) \int_{-\infty}^{\infty} \Phi_g(\omega) H_r^* H_s d\omega \qquad (3.82)$$

Separating Eq.(3.82) into terms with r=s and r≠s, we obtain

$$E[V_k V_\ell] = \sum_{r=1}^{N} \gamma_r^2 \psi_r(k) \psi_s(\ell) \int_{-\infty}^{\infty} \Phi_g(\omega) |H_r|^2 d\omega + \sum_{r=1}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell) \int_{-\infty}^{\infty} \Phi_g(\omega) H_r^* H_s d\omega \qquad (3.83)$$

The first frequency integral can be directly expressed in terms of the ground spectra. The second integral will have the real and imaginary part. However, the imaginary part, being an odd function of ω, will give zero when integrated out over its range. Thus, considering only the real term, we can write for Eq.(3.83) as,

$$E[V_k V_\ell] = \sum_{r=1}^{N} \gamma_r^2 \psi_r(k) \psi_s(\ell) \int_{-\infty}^{\infty} \Phi_g(\omega) |H_r|^2 d\omega + \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell) \int_{-\infty}^{\infty} N(\omega) |H_r|^2 |H_s|^2 \Phi_g(\omega) d\omega \qquad (3.84)$$

The integrand in the second frequency integral can now be split into its partial fraction, to give us the expression for this correlation similar to Eq.(3.81) as follows

$$E[V_k V_\ell] = \sum_{r=1}^{N} \gamma_r^2 \psi_r(k)\psi_s(\ell) I_{1gr} + \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k)\psi_s(\ell)$$

$$[A I_{1gr} + B I_{2gr} + C I_{1gs} + D I_{2gs}]) \qquad (3.85)$$

The correlation coefficient can now be defined as

$$\delta'_{k\ell} = \frac{E[V_k V_\ell]}{(\bar{V}_k/C_{vk})(\bar{V}_\ell/C_{v\ell})} \qquad (3.86)$$

MAXIMUM FLOOR RESPONSE-ABSOLUTE DISPLACEMENT

As mentioned in the previous chapter, the pseudo-static response can also be obtained in terms of the absolute floor displacement of the supports. Here the methods to obtain the absolute displacement response and correlation between the responses of two different floors are presented.

The maximum absolute displacement response can also be obtained using the same approach as employed for the calculation of the relative displacement response. That is

$$U_k(t) = 1_k X_g(t) + \sum_{r=1}^{N} \psi_r(k) Y_r(t) \qquad (3.87)$$

where $X_g(t)$= ground displacement time history. Eq.(3.87) can be used to obtain the mean square value of $U_k(t)$ which when multiplied by the peak factor will give the maximum displacement. Using this approach, an expression similar to the expression for the maximum relative displacement can be developed for the maximum absolute displacement and the correlation coefficient.

Here, however, a different approach is taken. For this, the displacement and correlation between the displacements is expressed as the limiting cases of the auto floor and coincident cross floor spectra.

The maximum value of the absolute displacement response can be written in terms of the spectral density function of displacement as follows

$$\bar{U}_k^2 = C_{uk}^2 \int_{-\infty}^{\infty} \Phi_{dkk}(\omega)d\omega \tag{3.88}$$

where $\Phi_{dkk}(\omega)$= spectral density function of the absolute floor displacement and C_{uk}= the peak factor.

From the stationary random vibration analysis, it is known that the cross or auto spectral density function of the absolute displacement, $\Phi_{dk\ell}(\omega)$, and absolute acceleration, $\Phi_{ak\ell}(\omega)$ are related by the following expression

$$\Phi_{ak\ell}(\omega) = \omega^4 \, \Phi_{dk\ell}(\omega) \tag{3.89}$$

Thus, the maximum displacement in terms of the acceleration spectral density function can be written as,

$$\bar{U}_k^2 = C_{uk}^2 \int_{-\infty}^{\infty} \frac{1}{\omega^4} \Phi_{akk}(\omega)d\omega \tag{3.90}$$

The right hand side of Eq.(3.90) is, however, the same as the following expression with ω_i=0

$$\bar{U}_k^2 = C_{uk}^2 \int_{-\infty}^{\infty} \Phi_{akk}(\omega)|H_i|^2 d\omega \tag{3.91}$$

Comparing the right hand sides of Eqs.(3.27) and (3.91) we notice that $\bar{U}_k$ is nothing but the relative displacement floor response spectrum value obtained from Eq.(3.36) for ω_i=0.

That is,

$$\bar{U}_k^2 = R_{dk}^2(\omega_i=0) \qquad (3.92)$$

Using the expression for R_{dk}^2, as in Eq.(3.36), we obtain

$$\bar{U}_k^2 = C_{di}^2 \Big(\sum_{r=1}^{N} \gamma_r^2 \psi_r^2(k)[A_5 I_{1gr} + B_5 I_{2gr} + C_5 I_{1g} + D_5 I_{2g}]$$

$$+ \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k)\psi_x(k)[A_6 I_{1gr} + B_6 I_{2gr} + C_6 I_{1g} + D_6 I_{2g}$$

$$+ A_7 I_{1gs} + B_7 I_{2gs} + C_7 I_{2g} + D_7 I_{2g}]\Big) \qquad (3.93)$$

in which

$$I_{1g} = [R_{dg}(\omega_i=0)/C_{dg}]^2 = (D_g/C_{dg})^2 \qquad (3.94)$$

$$I_{2g} = [R_{vg}(\omega_i=0)/C_{vg}]^2 = (V_g/C_{vg})^2 \qquad (3.95)$$

where D_g= maximun ground displacement and C_{dg}= peak factor for the ground displacement random process; V_g= maximun ground velocity and C_{vg}= peak factor for the ground velocity random process. It is noted that C_{di} obtained for ω_i=0 is the same as C_{uk}.

It is mentioned that no numerical problem is encountered in evaluation of the coefficients A_5, B_5,..., A_6, B_6,..., A_7, B_7, etc for ω_i=0 by Eqs.(3.29) through (3.31). Thus, to obtain the maximum floor displacement, the algorithm for the evaluation of the displacement auto floor spectrum can be directly used.

ABSOLUTE DISPLACEMENT CORRELATION COEFFICIENT

The cross correlation between the absolute displacements of two floors can be written in terms of the cross spectral density function as,

$$E[U_k U_\ell] = \int_{-\infty}^{\infty} \Phi_{dk\ell}^{R}(\omega) d\omega \tag{3.96}$$

Using Eq.(3.90), this can also be written in terms of the cross spectral density function of the absolute acceleration as

$$E[U_k U_\ell] = \int_{-\infty}^{\infty} \Phi_{ak\ell}^{R}(\omega) \frac{1}{\omega^4} d\omega \tag{3.97}$$

Comparing the right hand sides of Eq.(3.52) and (3.97) we notice that this correlation is the same as the coincident displacement cross spectrum obtained at ω_i=0. That is

$$E[U_k U_\ell] = [C_{dk\ell}(\omega_i = 0)/P_{di}]^2 \tag{3.98}$$

The right hand side of Eq.(3.98) can, thus, be directly obtained by using Eq.(3.55) for ω_i=0, and is written as

$$\begin{aligned} E[U_k U_\ell] = & \sum_{r=1}^{N} \gamma_r^2 \psi_r(k)\psi_s(\ell)[A_5 I_{1gr} + B_5 I_{2gr} + C_5 I_{1g} + D_5 I_{2g}] \\ & + \sum_{\substack{r=1 \\ r\neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k)\psi_s(\ell)[A_6 I_{1gr} + B_6 I_{2gr} + C_6 I_{1g} + D_6 I_{2g} \\ & + A_7 I_{1gs} + B_7 I_{2gs} + C_7 I_{1g} + D_7 I_{2g}] \end{aligned} \tag{3.99}$$

Here again, the terms like I_{1g} and I_{2g} are expressed in terms of maximun ground displacement and maximun ground velocity, as in Eqs.(3.94) and (3.95).

Thus, to evaluate the correlation, the algorithm used for the development of coincident displacement cross floor spectra can be directly used. Here again, no problem is encountered in the calculation of the partial fraction coefficients for $\omega_i=0$.

Using Eqs.(3.93) and (3.99), the correlation coefficient $\delta_{k\ell}$ can be easily obtained as

$$\delta_{k\ell} = \frac{E[U_k U_\ell]}{(\bar{U}_k/C_{uk})/(\bar{U}_\ell/C_{u\ell})} \tag{3.100}$$

III.4 FLOOR INPUTS FOR CROSS RESPONSE

We observed in Chapter II that we require the cross floor response spectrum values as well as their limiting values for $\omega_i=0$ to calculate the contribution of the cross term. The evaluation of the cross spectra was presented in the earlier section. Here the expression for the limiting cases are developed.

MAXIMUM FLOOR VELOCITY RESPONSE

One of the terms required to define Eq.(2.79) was the mean square value of the absolute floor velocity response. It is defined as

$$E[\dot{U}_k^2] = \int_{-\infty}^{\infty} \Phi_{akk}(\omega) \frac{1}{\omega^2} d\omega \tag{3.101}$$

which, as we saw in Eq.(3.41) is the auto velocity spectrum value obtained for $\omega_i=0$. This can be written from Eq.(3.41) as

$$E[\dot{U}_k^2] = R_{vk}^2(\omega_i = 0)/C_{vi}^2 \tag{3.102}$$

This value can be directly obtained from Eq.(3.48) as follows

$$E[\dot{U}_k^2] = \sum_{r=1}^{N} \gamma_r^2 \psi_r^2(k)[A_8 I_{1gr} + B_8 I_{2gr} + C_8 I_{1g} + D_8 I_{2g}]$$

$$+ \sum_{\substack{r=1 \\ r \neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(k) [A_9 I_{1gr} + B_9 I_{2gr} + C_9 I_{1g} + D_9 I_{2g}$$

$$+ A_{10} I_{1gs} + B_{10} I_{2gs} + C_{10} I_{1g} + D_{10} I_{2g}] \quad (3.103)$$

It is again mentioned that no numerical problem is encountered in evaluation of the coefficients A_8, B_8, ..., A_9, B_9, ..., and A_{10}, B_{10}, etc for $\omega_i=0$ by Eqs.(3.46). Thus, to obtain the maximum floor velocity, the algorithm for the evaluation of the velocity auto floor spectrum can be directly used.

COINCIDENT CROSS FLOOR VELOCITY RESPONSE

In Eq.(2.79) we also required a term associated with the cross correlation of the absolute velocities of two floors. This was defined in Eq.(2.81) as

$$I_{4k\ell} = \int_{-\infty}^{\infty} \Phi_{\ddot{a}k\ell}^{R}(\omega) \frac{1}{\omega^2} d\omega \quad (3.104)$$

Comparing Eq.(3.104) with Eq.(3.57), we notice that this term is nothing but the coincident cross velocity spectrum obtained at $\omega_i=0$. Thus, this can be written as

$$I_{4k\ell} = [C_{vk\ell}(\omega_i=0)/P_{vi}]^2 \quad (3.105)$$

This can be directly obtained by using Eq.(3.58). That is

$$I_{4k\ell} = \sum_{r=1}^{N} \gamma_r^2 \psi_r(k)\psi_r(\ell)[A_8 I_{1gr} + B_8 I_{2gr} + C_8 I_{1g} + D_8 I_{2g}]$$

$$+ \sum_{\substack{r=1 \\ r \neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k)\psi_s(\ell)[A_9 I_{1gr} + B_9 I_{2gr} + C_9 I_{1g} + D_9 I_{2g}$$

$$+ A_{10} I_{1gs} + B_{10} I_{2gs} + C_{10} I_{1g} + D_{10} I_{2g}] \quad (3.106)$$

Thus, the algorithm developed for the coincident relative velocity cross floor spectra can be directly used without any numerical problem. This term can also be expressed as a correlation coefficient, defined as follows

$$\delta''_{k\ell} = \frac{I_{4k\ell}}{\{E[\dot{U}_k^2]E[\dot{U}_\ell^2]\}^{1/2}} \quad (3.107)$$

A complete description of this velocity related input , as expressed by Eqs.(3.103) and (3.107), can now be given in terms of a matrix of correlation coefficients and the maximum floor velocity.

QUADRATURE CROSS FLOOR VELOCITY RESPONSE

The term associated with coefficient H in Eq.(2.79) represents the correlation between the quadrature components of absolute velocities between two floors. This was defined in Eq.(2.82) as

$$I_{4k\ell} = \int_{-\infty}^{\infty} \omega^3 \phi^I_{ak\ell}(\omega) \frac{1}{\omega^4} d\omega \quad (3.108)$$

This term is nothing but the quadrature cross velocity spectrum obtained at ω_i=0. Thus, this can be written as follows

$$I_{4k\ell} = Q^2_{vk\ell}(\omega_i{=}0)/Q^2_{vi} \quad (3.109)$$

For the evaluation of this term, also the algorithm used for the quadrature velocity spectra can be directly used. That is,

$$I_{6k\ell} = \sum_{\substack{r=1 \\ r \neq s}}^{N} \sum_{s=1}^{N} \gamma_r \gamma_s \psi_r(k) \psi_s(\ell) [A_{13} I_{1gr} + B_{13} I_{2gr} + C_{13} I_{1g} + D_{13} I_{2g}$$

$$+ A_{14} I_{1gs} + B_{14} I_{2gs} + C_{14} I_{1g} + D_{14} I_{2g}] \qquad (3.110)$$

For $k=\ell$, this term is zero. Again, we can also express this term as a correlation coefficient, defined as follows

$$\delta'''_{k\ell} = \frac{I_{6k\ell}}{\{E[\dot{U}_k^2]\ E[\dot{U}_\ell^2]\}^{1/2}} \qquad (3.111)$$

This part of the input can also be defined as a matrix. The diagonal term of this matrix will be zero and the matrix is skew symmetric.

CHAPTER IV

SUPPORT INPUTS FOR SECONDARY SYSTEMS ATTACHED TO GROUND AND PRIMARY STRUCTURE

IV.1 INTRODUCTION

In industrial facilities, a multiply supported piping system could be directly attached to the ground. In this case the ground is like a floor. The seismic inputs required to be defined for this case are the special cases of the inputs described in the previous chapters. These inputs will now be explicitly developed in this chapter.

Auto floor spectra for the support on the ground are simply the ground response spectra. The cross floor spectra for the motions between a floor and ground support are developed in section IV.2.

In addition to floor spectra, we also need to define the cross correlation between the displacement of a floor and ground support for the calculation of pseudo-static response. These are developed in Section IV.3.

The quantities required in the calculation of the cross terms are developed in Section IV.4.

IV.2 CROSS FLOOR SPECTRA

The cross correlation function of the absolute accelerations of the ground and floor k can be expressed as

$$R_{agk}(t_1,t_2) = E[\ddot{X}_g(t_1)\ddot{U}_k(t_2)] \tag{4.1}$$

Substituting from Eq.(3.6), we obtain

$$R_{agk}(t_1,t_2) = -\sum_{r=1}^{N} \psi_r(k)\{2\beta_r\omega_r E[\ddot{X}_g(t_1)\dot{Y}_r(t_2)] + \omega_r^2 E[\ddot{X}_g(t_1)Y_r(t_2)]\} \qquad (4.2)$$

For stationary ground motion and the floor response ,this cross correlation can be shown to be given by

$$R_{agk}(t_1,t_2) = \sum_{r=1}^{N} \gamma_r\psi_r(k) \int_{-\infty}^{\infty} (\omega_r^2+i2\beta_r\omega_r\omega)H_r\Phi_g(\omega)e^{i\omega(t_2-t_1)}d\omega \qquad (4.3)$$

From Eq.(4.3), the cross spectral density function of the absolute acceleration of the ground and floor k can be identified as

$$\Phi_{agk}(\omega) = \sum_{r=1}^{N} \gamma_r\psi_r(k)(\omega_r^2 + i2\beta_r\omega_r\omega)H_r\Phi_g(\omega) \qquad (4.4)$$

This cross spectral density function has the real and imaginary parts. These parts can be separated as

$$\Phi_{agk}(\omega) = \sum_{r=1}^{N} \gamma_r\psi_r(k)[\omega_r^4-\omega_r^2\omega^2(1-4\beta_r^2)-i2\beta_r\omega_r\omega^3]|H_r|^2\Phi_g(\omega) \qquad (4.5)$$

in which the real part is as follows

$$\Phi_{agk}^R(\omega) = \sum_{r=1}^{N} \gamma_r\psi_r(k)[\omega_r^4-\omega_r^2\omega^2(1-4\beta_r^2)]|H_r|^2\Phi_g(\omega) \qquad (4.6)$$

and the imaginary part as follows

$$\Phi_{agk}^I(\omega) = \sum_{r=1}^{N} \gamma_r\psi_r(k) \ (-2\beta_r\omega_r\omega^3)|H_r|^2\Phi_g(\omega) \qquad (4.7)$$

The cross spectral density function associated with cross correlation R_{akg}, here denoted by $\Phi_{akg}(\omega)$, is merely the complex conjugate of $\Phi_{agk}(\omega)$. That is,

$$\Phi_{akg}(\omega) = \Phi^R_{agk}(\omega) - i\Phi^I_{agk}(\omega) \qquad (4.8)$$

Eqs.(4.6) and (4.7) will now be used to developed various types of cross spectra, defined by Eq.(2.42) through (2.46).

COINCIDENT DISPLACEMENT SPECTRA

The coincident displacement spectrum for the ground and floor k at frequency ω_i and damping ratio β_i is defined as

$$C^2_{dgk}(\omega_i) = P^2_{di} \int_{-\infty}^{\infty} \Phi^R_{agk}(\omega)|H_i|^2 d\omega \qquad (4.9)$$

The pseudo-acceleration spectrum is obtained from the displacement spectrum as

$$C_{pgk}(\omega_i) = \omega_i^2 C_{dgk}(\omega_i) \qquad (4.10)$$

Substituting Eq.(4.6) into (4.9), we obtain

$$C^2_{dgk}(\omega_i) = P^2_{di} \sum_{r=1}^{N} \gamma_r\psi_r(k) \int_{-\infty}^{\infty} [\omega_r^4-\omega_r^2\omega^2(1-4\beta_r^2)]|H_r|^2|H_i|^2\Phi_g(\omega)d\omega \qquad (4.11)$$

To express the frequency integral in Eq.(4.11) in terms of the ground response spectra, we resolve the product involving $|H_r|^2$ and $|H_i|^2$ into their partial fractions. If H_r is not identically equal to H_i, we obtain

$$[\omega_r^4-\omega_r^2\omega^2(1-4\beta_r^2)]|H_r|^2|H_i|^2 = (A_{15} + \omega^2 B_{15})|H_r|^2 + (C_{15} + \omega^2 D_{15})|H_i|^2 \qquad (4.12)$$

The coefficients of the partial fraction A_{15}, B_{15}, etc are obtained from the solution of the following set of linear simultaneous equations

$$[Y_{ri}]\{V_{15}\} = \{W_{15}\} \tag{4.13}$$

where the vectors $\{V_{15}\}$ and $\{W_{15}\}$ are defined as

$$\{V_{15}\}' = \{A_{15}, B_{15}, C_{15}, D_{15}\} \tag{4.14a}$$

$$\{W_{15}\}' = \{\omega_r^4, -\omega_r^2(1-\beta_r^2 4), 0, 0\} \tag{4.14b}$$

The frequency integral in Eq.(4.11) can now be expressed in terms of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and their peak factors. That is

$$C_{dgk}^2(\omega_i) = P_{di}^2 \sum_{r=1}^{N} \gamma_r \psi_r(k)[A_{15}I_{1gr} + B_{15}I_{2gr} + C_{15}I_{1gi} + D_{15}I_{2gi}] \tag{4.15}$$

For the case H_r is equal to H_i (resonance case) the approach developed in Section III.2.1 for the auto floor displacement spectra is directly applicable. In terms of the integral I_R of Eq.(3.38a), the expression for Eq.(4.11) becomes

$$C_{dgk}^2(\omega_i) = P_{di}^2 \; \gamma_i \psi_i(k) I_R(1, 4\beta_i^2 - 1)/\omega_i^4$$
$$+ P_{di}^2 \sum_{\substack{r=1 \\ r \neq i}}^{N} \gamma_r \psi_r(k)[A_{15}I_{1gr} + B_{15}I_{2gr} + C_{15}I_{1gi} + D_{15}I_{2gi}] \tag{4.16}$$

COINCIDENT VELOCITY SPECTRA

The coincident velocity spectrum for the ground and floor k at frequency ω_i and damping ratio β_i is defined as

$$C_{vgk}^2(\omega_i) = P_{vi}^2 \int_{-\infty}^{\infty} \omega^2 \Phi_{agk}^R(\omega)|H_i|^2 d\omega \tag{4.17}$$

Substituting Eq.(4.6) into (4.17), we obtain

$$C^2_{vgk}(\omega_i) = P^2_{vi} \sum_{r=1}^{N} \gamma_r \psi_r(k) \int_{-\infty}^{\infty} [\omega^2\omega_r^4 - \omega_r^2\omega^4(1-4\beta_r^2)]|H_r|^2|H_i|^2\Phi_g(\omega)d\omega \quad (4.18)$$

To express the frequency integral in Eq.(4.18) in terms of the ground response spectra, we again resolve the product involving $|H_r|^2$ and $|H_i|^2$ into their partial fractions. If H_r is not identically equal to H_i, we obtain

$$[\omega^2\omega_r^4 - \omega_r^2\omega^4(1-4\beta_r^2)]|H_r|^2|H_i|^2 = (A_{16} + \omega^2 B_{16})|H_r|^2 + (C_{16} + \omega^2 D_{16})|H_i|^2 \quad (4.19)$$

The coefficients of the partial fraction A_{16}, B_{16}, etc are obtained from the solution of the following set of linear simultaneous equations

$$[Y_{ri}]\{V_{16}\} = \{W_{16}\} \quad (4.20)$$

where the vectors $\{V_{16}\}$ and $\{W_{16}\}$ are defined as

$$\{V_{16}\}' = \{A_{16}, B_{16}, C_{16}, D_{16}\} \quad (4.21a)$$

$$\{W_{16}\}' = \{0, \omega_r^4, -\omega_r^2(1-4\beta_r^2), 0\} \quad (4.21b)$$

The frequency integral in Eq.(4.18) can now be expressed in terms of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and their peak factors. That is

$$C^2_{vgk}(\omega_i) = P^2_{vi} \sum_{r=1}^{N} \gamma_r \psi_r(k)[A_{16}I_{1gr} + B_{16}I_{2gr} + C_{16}I_{1gi} + D_{16}I_{2gi}] \quad (4.22)$$

For the case H_r is equal to H_i (resonance case) the approach developed in Section III.2.2 for the auto floor velocity spectra is directly applicable. In terms of the integral I_S of

Eq.(3.49a), the expression for Eq.(4.18) becomes

$$C^2_{vgk}(\omega_i) = P^2_{vi}\ \gamma_i \psi_i(k) I_S(1, 4\beta_i^2 - 1)/\omega_i^2$$

$$+ P^2_{vi} \sum_{\substack{r=1 \\ r \neq i}}^{N} \gamma_r \psi_r(k) [A_{16} I_{1gr} + B_{16} I_{2gr} + C_{16} I_{1gi} + D_{16} I_{2gi}] \quad (4.23)$$

QUADRATURE DISPLACEMENT SPECTRA

The quadrature displacement spectrum for the ground and floor k at frequency ω_i and damping ratio β_i is defined in terms of the imaginary part of the cross spectral density function as

$$Q^2_{dgk}(\omega_i) = Q^2_{di} \int_{-\infty}^{\infty} \omega \Phi^I_{agk}(\omega) |H_i|^2 d\omega \quad (4.24)$$

The pseudo-acceleration spectrum is obtained from the displacement spectrum as

$$Q^2_{pgk}(\omega_i) = \omega_i^2\ Q^2_{dgk}(\omega_i) \quad (4.25)$$

Substituting Eq.(4.7) into (4.24), we obtain

$$Q^2_{dgk}(\omega_i) = Q^2_{di} \sum_{r=1}^{N} \gamma_r \psi_r(k) \int_{-\infty}^{\infty} (-2\beta_r \omega_r \omega^4) |H_r|^2 |H_i|^2 \Phi_g(\omega) d\omega \quad (4.26)$$

To express the frequency integral in Eq.(4.26) in terms of ground response spectra, we resolve the product involving $|H_r|^2$ and $|H_i|^2$ into their partial fractions. If H_r is not identically equal to H_i, we obtain

$$-2\beta_r \omega_r \omega^4 |H_r|^2 |H_i|^2 = (A_{17} + \omega^2 B_{17}) |H_r|^2 + (C_{17} + \omega^2 D_{17}) |H_i|^2 \quad (8.27)$$

The coefficients of the partial fraction A_{17}, B_{17}, etc are obtained from the solution of the following set of linear simultaneous equations

$$[Y_{ri}]\{V_{17}\} = \{W_{17}\} \tag{4.28}$$

where the vectors $\{V_{17}\}$ and $\{W_{17}\}$ are defined as

$$\{V_{17}\}' = \{A_{17}, B_{17}, C_{17}, D_{17}\} \tag{4.29a}$$

$$\{W_{17}\}' = \{0, 0, -2\beta_r\omega_r, 0\} \tag{4.29b}$$

The frequency integral in Eq.(4.26) can now be expressed in terms of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and their peak factors. That is

$$Q^2_{dgk}(\omega_i) = Q^2_{di} \sum_{r=1}^{N} \gamma_r\psi_r(k)[A_{17}I_{1gr} + B_{17}I_{2gr} + C_{17}I_{1gi} + D_{17}I_{2gi}] \tag{4.30}$$

For the case H_r is equal to H_i (resonance case) the approach developed in Section II.2.1 for the auto floor velocity spectra is directly applicable. In terms of the integral I_S of Eq.(3.49a), the expression for Eq.(4.26) becomes

$$Q^2_{dgk}(\omega_i) = Q^2_{di}\, \gamma_i\psi_i(k)\; I_S\,(0,-2\beta_i/\omega_i^3)$$

$$+\, Q^2_{di} \sum_{\substack{r=1 \\ r\neq i}}^{N} \gamma_r\psi_r(k)[A_{17}I_{1gr} + B_{17}I_{2gr} + C_{17}I_{1gi} + D_{17}I_{2gi}] \tag{4.31}$$

QUADRATURE VELOCITY SPECTRA

The quadrature velocity spectrum for the ground and floor k at frequency ω_i and damping ratio β_i is defined as

$$Q^2_{vgk}(\omega_i) = Q^2_{vi} \int_{-\infty}^{\infty} \omega^3 \Phi^I_{agk}(\omega) |H_i|^2 d\omega \qquad (4.32)$$

Substituting Eq.(4.7) into (4.32), we obtain

$$Q^2_{vgk}(\omega_i) = Q^2_{vi} \sum_{r=1}^{N} \gamma_r \psi_r(k) \int_{-\infty}^{\infty} (-2\beta_r \omega_r \omega^6) |H_r|^2 |H_i|^2 \Phi_g(\omega) d\omega \qquad (4.33)$$

To express the frequency integral in Eq.(4.33) in terms of ground response spectra, we resolve the product involving $|H_r|^2$ and $|H_i|^2$ into their partial fractions. If H_r is not identically equal to H_i, we obtain

$$-2\beta_r \omega_r \omega^6 |H_r|^2 |H_i|^2 = (A_{18} + \omega^2 B_{18}) |H_r|^2 + (C_{18} + \omega^2 D_{18}) |H_i|^2 \qquad (4.34)$$

The coefficients of the partial fraction A_{18}, B_{18}, etc are obtained from the solution of the following set of linear simultaneous equations

$$[Y_{ri}]\{V_{18}\} = \{W_{18}\} \qquad (4.35)$$

where the vectors $\{V_{18}\}$ and $\{W_{18}\}$ are defined as

$$\{V_{18}\}' = \{A_{18}, B_{18}, C_{18}, D_{18}\} \qquad (4.36a)$$

$$\{W_{18}\}' = \{0, 0, 0, -2\beta_r \omega_r\} \qquad (4.36b)$$

The frequency integral in Eq.(4.33) can now be expressed in terms of the pseudo-acceleration (or relative displacement) and relative velocity ground spectra and their peak factors. That is

$$Q^2_{vgk}(\omega_i) = Q^2_{vi} \sum_{r=1}^{N} \gamma_r \psi_r(k) [A_{18} I_{1gr} + B_{18} I_{2gr} + C_{18} I_{1gi} + D_{18} I_{2gi}] \qquad (4.37)$$

For the case H_r is equal to H_i (resonance case) is treated as described by Singh[22]. Following this approach, the required frequency integral is obtained for the generic case

$$I_E(a_4) = \int_{-\infty}^{\infty} a_4\omega_i^2\omega^6|H_i|^4\Phi_g(\omega)d\omega \qquad (4.38a)$$

Thus,

$$I_E(a_4) = a_4 I_{1gr}\,\omega_r^4 F(\omega_r)(G_m-r/4)/(E_m-2r) \qquad (4.38b)$$

where $F(\omega_r)$, G_m, E_m and r are defined in Appendix II.

Thus, for $H_r=H_i$ the frequency integral in Eq.(4.33) can be written in terms of I_E as follows

$$Q_{vgk}^2(\omega_i) = Q_{vi}^2\,\gamma_i\psi_i(k)I_E(-2\beta_i/\omega_i)$$

$$+ Q_{vi}^2 \sum_{\substack{r=1\\ r\neq i}}^{N} \gamma_r\psi_r(k)[A_{18}I_{1gr} + B_{18}I_{2gr} + C_{18}I_{1gi} + D_{18}I_{2gi}] \qquad (4.39)$$

IV.3 INPUTS FOR PSEUDO-STATIC RESPONSE

As mentioned in Chapter III, we could use either the relative or absolute displacement of the supports points and their respective cross correlation coefficients for the calculation of the pseudo-static response.

The maximum relative displacement of the ground is obviously zero. Therefore, the correlation coefficient between the relative displacement of any floor and the ground support vanish automatically. That is

$$\delta'_{gk} = 0 \qquad (4.40)$$

However, while working with the absolute displacements, the maximum ground displacement as well as its correlation with the displacements of each support points are required. The maximum ground displacement is an input parameter now, which if not explicitly provide, can be estimated as suggested in Reference [8]. However, as observed later the force response quantity is not affected by this input parameter. That is, any value can be assumed for the maximum ground displacement.

The correlation coefficient of the absolute displacements of the ground and floor k evaluated at the same time instant is defined as follows

$$\delta_{gk} = \frac{E[X_g U_k]}{(D_g/C_{dg})(\bar{U}_k/C_{uk})} \tag{4.41}$$

The cross correlation between the ground and floor displacements measured at the same time instant, and as required in Eq.(4.41), can be expressed in terms of the cross spectral density function as,

$$E[X_g U_k] = \int_{-\infty}^{\infty} \Phi_{dgk}^{R}(\omega) d\omega \tag{4.42}$$

Using Eq.(3.89), this can also be written in terms of the cross spectral density function of the absolute acceleration as

$$E[X_g U_k] = \int_{-\infty}^{\infty} \Phi_{agk}^{R}(\omega) \frac{1}{\omega^4} d\omega \tag{4.43}$$

Comparing the right hand sides of Eq.(4.9) and (4.43) we notice that

$$E[X_g U_k] = [C_{dgk}(\omega_i{=}0)/P_{di}]^2 \tag{4.44}$$

Thus, Eq.(4.44) can be directly obtained by using Eq.(4.15) for

$\omega_i=0$. That is

$$E[X_g U_k] = \sum_{r=1}^{N} \gamma_r \psi_r(k)[A_{15} I_{1gr} + B_{15} I_{2gr} + C_{15} I_{1g} + D_{15} I_{2g}] \quad (4.45)$$

The terms I_{1g} and I_{2g} are expressed in terms of the maximum ground displacement and velocity, and are given by Eqs.(3.94) and (3.95), respectively.

IV.4 INPUTS FOR CROSS RESPONSE

As shown in Eq.(2.79), various kind of cross floor spectra as well as their limiting values for $\omega_i=0$ are required to calculate the cross response component. The cross floor spectra for the ground and a floor were developed in Chapter III. In addition, we need to obtain the maximum ground velocity response and the cross velocity response for the ground and each floor. The maximum ground velocity parameter can be prescribed or estimated from the maximum acceleration as suggested in Reference [14]. The cross velocity response, for the coincident and quadrature components, are presented in the following sections.

COINCIDENT CROSS VELOCITY RESPONSE

The term associated with coefficient D in Eq.(2.79) represents the cross correlation between the coincident component of the velocities of two different supports. If one of the supports is on the ground, then we can write

$$I_{4gk} = \int_{-\infty}^{\infty} \Phi_{agk}^{R}(\omega) \frac{1}{\omega^2} d\omega \quad (4.46)$$

Comparing the right hand sides of Eqs.(4.16) and (4.46), we notice that

$$I_{4gk} = [C_{vgk}(\omega_i=0)/P_{vi}]^2 \qquad (4.47)$$

Therefore, the coincident cross velocity response can be directly obtained by using Eq.(4.22) as follows

$$I_{4gk} = \sum_{r=1}^{N} \gamma_r\psi_r(k)[A_{16}I_{1gr} + B_{16}I_{2gr} + C_{16}I_{1g} + D_{16}I_{2g}] \qquad (4.48)$$

Thus, the algorithm developed in Section III.2.2. can be used without any numerical problems.

We can also define Eq.(4.48) in terms of a correlation coefficient as follows

$$\delta''_{gk} = \frac{I_{4gk}}{(V_g/C_{vg})\{E[\dot{U}_k^2]\}^{1/2}} \qquad (4.49)$$

QUADRATURE CROSS VELOCITY RESPONSE

The term associated with coefficient H in Eq.(2.79) represents the correlation between the quadrature components of the absolute velocities of any two supports. If one of the supports is on the ground, then we can write for this term as

$$I_{6gk} = \int_{-\infty}^{\infty} \omega^3\phi^I_{agk}(\omega)\,\frac{1}{\omega^4}\,d\omega \qquad (4.50)$$

Comparing the right hand sides of Eq.(4.32) and (4.50), we notice that

$$I_{6gk} = Q^2_{vgk}(\omega_i=0)/Q^2_{vi} \qquad (4.51)$$

Thus, the algorithm used for the quadrature velocity spectra can be directly used. That is, from Eq.(4.37)

$$I_{6gk} = \sum_{r=1}^{N} \gamma_r \psi_r(k)[A_{18} I_{1gr} + B_{18} I_{1gr} + C_{18} I_{1g} + D_{18} I_{2g}] \quad (4.52)$$

Again we can express this term as a correlation coefficient, defined as follows

$$\delta'''_{gk} = \frac{I_{6gk}}{(V_g/C_{vg})\{E[\dot{U}_k^2]\}^{1/2}} \quad (4.53)$$

This part of the input can again be defined as a skew symmetric matrix.

CHAPTER V

NUMERICAL RESULTS

V.1 INTRODUCTION

In the preceding chapters, a response spectrum approach is developed for the calculation of seismic design response of multiply connected secondary systems. The approach requires an analysis of the supporting primary structure to define various types of floor spectra and other inputs. These inputs are then used in the analysis of the supported secondary system to obtain its response. The response spectrum method to define various inputs are developed in Chapter III and IV, and utilization of these inputs for the calculation of the secondary system response, again through a generalized response spectrum approach, is described in Chapter II.

In this Chapter, the numerical results demostrating the applicability of the approach are presented for two different structural configurations, shown in Fig.1 and 2. The primary structure in both these problems is the same. It consists of five floors connected by columns which primarily deform in the shearing mode. The system has five degrees of freedom. The mass and stiffness properties of the system are shown in Fig.1 and 2, with K=10.075 Kips/ft and M=35.5 Kips-Sec^2/ft. The natural frequencies, participation factors and modal displacements are given in Tables I and II. The primary structure is assumed to have 5% damping in each mode.

The secondary systems shown in Fig.1 and 2 are almost similar except that the system in Fig.2 has a support attached to the ground. The mass and stiffness properties of the secondary system are also shown in Fig.1 and 2, with k=150 kips/ft and m=.1 kips-Sec^2/ft. The natural frequencies and mode shapes of these two systems, assumed fixed at all the supports are given in Tables III and IV. The dynamic influence coefficients matrix defining P_{ik} in Eq.(2.17) are given in Table V. Even though the two systems have different configurations, their dynamic properties, as listed in Tables I to V, are identical as their mass and stiffness characteristic are exactly the same. These systems are assumed to possess 2% damping ratio in each mode.

V.2 FLOOR SPECTRAL INPUTS

The seismic ground input to the entire system is defined in the form of pseudo-acceleration and relative velocity ground response spectra and these are shown in Figs.3 and 4, respectively. These curves represent the average spectra obtained for an ensemble of 75 synthetically generated acceleragrams. They have also been used in earlier studies [6,17].

Various floor spectral inputs developed for the analysis of the two secondary systems are shown in Figs.3 through 30. Figs.5, 6 and 7 show the auto pseudo-acceleration floor spectra for floors 2, 3 and 4, obtained by employing Eqs.(3.36) and (3.40). The auto velocity floor response spectra were obtained from Eqs.(3.48) and (3.51), and are shown in Figs.8, 9 and 10 for these floors. The cross floor spectra, both for pseudo-acceleration and velocity responses, have coincident and quadrature components. These spectra are to be defined for all the floors interconnected through the secondary system. Figs.11 through 13 show the coincident pseudo-acceleration spectra, obtained by employing Eqs.(3.55) and (3.56). The coincident velocity spectra are shown in Figs.14, 15 and 16, and these were obtained from Eqs.(3.58) and (3.59). The quadrature spectra for the pseudo-acceleration and velocity responses, respectively obtained from Eqs.(3.67), (3.68), (3.74) and (3.75), are shown in Figs.17 through 22. For the analysis of system in Fig.2, the cross floor spectra between the ground and various connected floors are required. These are shown in Figs.23 through 30 and were obtained from the equations developed in Chapter IV.

Here all the floor spectra have been developed for the oscillator damping ratio of 2%, because the secondary systems being examined in this work are assumed to have 2% damping ratio in all the modes. However, if different damping ratios are assumed in different modes, or if the secondary system is assumed to be nonproportional, then floor spectra for all possible modal damping ratios must be developed.

It is seen that for the development of all these floor spectral inputs no time history analysis is required. The prescribed ground response spectra can be directly used. In addition to the ground spectra, the dynamic characteristic for the primary system defined in terms of the natural frequencies, mode shapes, participation factors and modal damping ratios are also required.

It is noted that although the auto floor spectra will always have positive values, the cross floor spectra can assume negative values.

For design purposes these spectra should incorporate the effect of the uncertanties in the parameters of the primary structure. This can possibly be incorporated as described by Singh[21] and Ghafory-Ashtiany and Singh[6]. The methods to do this for various types of floor inputs defined here are under developement at this moment.

V.3 RESPONSE OF SECONDARY SYSTEMS

Various floor spectra inputs developed in Chapter III are utilized here to obtain the displacement and the force response of the secondary systems shown in Figs.1 and 2. In addition to the floor spectra, it is also necessary to define for various floors: (1) the maximum absolute displacement and their correlation coefficients and (2) the maximum absolute velocity and their correlation coefficient, as described in Chapter III. For the primary system being examined here, these values are given in Table VI. The matrices of the correlation coefficients are given in Tables VII, VIII and the matrix of the quadrature velocity coefficient, as defined by Eq.(3.111), is defined in Table IX. These represent the correlation between the quadrature velocity components of two floors normalized by the maximum velocity of the corresponding floors. Unlike correlation coefficient, these values could be greater than 1.0, as indeed they are in Table IX. These tables cover the input requirements of both the systems of Figs.1 and 2.

For given ground response spectra, these inputs quantities are also obtained by a direct analysis of the primary structure by employing Eqs.(3.93) and (3.103) for the displacement and velocity; Eqs.(3.100) and (3.107) for the correlation coefficients and Eq.(3.111) for the quadrature coefficient.

In Chapter IV, we noted that the maximum ground displacement and velocity values are also required if the absolute displacement formulation is used. It, however, turns out that any value can be prescribed for these two parameters without affecting the force response values. This has been verified by numerical results obtained for two widely different values of the ground parameters which still provide identical values within the numerical accuracy of the computations performed. Probably, it can also be analytically shown that these two parameters contribute only to the rigid body response of the entire system. At this stage, however, it is not immediately apparent.

The force response results obtained for the two systems are shown in Table X. The results in columns (3) and (6) were obtained by the approach developed in Chapter II employing the floor spectra and other inputs presented earlier. For comparison the parallel results were also obtained by a straight forward response spectrum analysis of the combined systems with ground response spectra as inputs. In this combined analysis, the primary and secondary system were considered jointly as one single system. Thus, these results for the combined system, shown in columns (2) and (5) of Table X, do also incorporate the possible dynamic interaction between the two systems. However, because the secondary systems considered here are relatively very light, as well as their frequencies are well separated, the interaction effect in the results presented here are believed to be very small.

It is seen that, the results obtained by the two approaches compare very well. This verifies the analytical development presented here. Also the verification of the results against the well established response spectrum approach, commonly used for seismic response evaluation of primary structures, clearly demostrates the applicability of the response spectrum approach developed here for the analysis of multiply connected secondary systems as well. It also ratifies the concept of cross floor spectra as a valid form of the seismic input; such spectra also must be prescribed along with auto floor response spectra for a proper seismic analysis of the secondary systems with multiple supports.

V.4 RELATIVE CONTRIBUTION OF DYNAMIC, PSEUDO-STATIC AND CROSS TERMS TO TOTAL RESPONSE

For the examples considered here, the relative importance of the dynamic, pseudo-static and cross response terms in the calculation of the total force response is evaluated. For system "A", Table XI shows the total response variance in Col.(2) for various structural elements. In Col.(3), (4) and (5) are shown the contributions of various components as a fraction of the total variance. For system "B", similar results are reported in Table XII. It is seen that in same cases the pseudo-static and cross components are seen to contribute negligibly to the total response, yet in other cases their contributions can be relatively large. In particular, it is noted that correlation between the dynamic and pseudo-static part, as measured by the contribution of the cross terms can be very significant and must be properly considered in the analysis. Thus, no particular component can be disregarded as trivial with respect to the other components in all situations and as a rule all components should be properly calculated and combined to obtain the total response.

V.5 EVALUATION OF SOME CURRENT RESPONSE EVALUATION PROCEDURES

Several different seismic analysis procedures are used in the industry to calculate the design response of such systems. As mentioned before, some analysts employ only time history approach, as it provides most accurate response, at least for that time history as well as the phase relationship between the motions of various floors are correctly accounted. This procedure is, however, only acceptable if an ensemble of time histories representing the prescribed design input are used. The currently employed response spectrum methods, on the other hand, use the envelop spectra to obtain the dynamic response or a combination of time history and response spectrum methods for the calculation of pseudo-static response due to differential support movement. Here some of these approaches are evaluated vis-a-vis the approach presented in this report.

To obtain the dynamic component of response, the envelop response spectra approach is used. In this approach, the seismic inputs are defined as the spectra which envelop the spectra of all the floors at which the secondary system is supported. Such inputs, defined in terms of the pseudo-acceleration and relative velocity spectra are shown in Figs.31 though 34 for system "A" and "B". These inputs are then used with the fixed base model of the secondary system to obtain response by the response spectrum approach [24].

To obtain the pseudo-static response due to it support displacement, first the system was analyzed for each support displacement applied individually, keeping the other supports fixed. The responses obtained for such individual support displacements were then combined by two, supposely, conservative rules: (1) Square-root-of-the-sum-of-the-squares procedure which assumes that support displacement are uncorrelated and (2) the absolute sum procedure. These values are refered to as SS and SA. These values were, in turn, combined with the dynamic component obtained above, again by the (1) square-root-sum procedure and (2) absolute sum procedures. The response combination procedure where the dynamic and SS are combined by square-root-sum procedure is designated as procedure SS1. Likewise SS2 means combination of the dynamic and SS as an absolute sum. The parallel combinations for SA with the dynamic response are designated as SA1 and SA2, respectively. The ratio of the values obtained by these procedures to the value obtained by the proposed approach are shown in Tables XIII and XIV for systems "A" and "B", for comparison purposes. If a particular ratio values is greater than 1.0, it indicates that this particular combination procedure gives a more conservative estimate of the response than the proposed approach. It is seen that for system "A", all these procedures give rather overly conservative estimates of response. This, however, is not true for system "B". In this case the approach SA2, in which the pseudo-static response is obtained by absolute sum procedure which in turn is combined with the dynamic response also as an absolute sum, only gives the conservative response in all the terms.

This evaluation indicates that these ad hoc and approximate rules of response combinations are not reliable and should be used with restraints.

CHAPTER VI

USER SUMMARY OF THE PROPOSED METHOD

VI.1 INTRODUCTION

Although, the detailed analytical development of the proposed method are given in Chapters II and III, all the necessary steps required to obtain the response of multiple support secondary systems are given in this chapter for the benefit of a user not interested in the mathematical details but interested in the application of the method.

In the development of this method, it is assumed that the secondary system is light so that the dynamic interaction between the system and its supporting primary structure can be neglected. With this assumption, the primary system can be analyzed independently of the secondary system to define the characteristics of the input motions at the supports of the latter system. Chapter III is exclusively devoted to the development of these inputs. Herein, the steps of the procedure which employs these inputs in the calculation of the secondary system response are described.

VI.2 STEP-BY-STEP PROCEDURE

The following step-by-step procedure can be used for the calculation of the secondary system response.

1. Define the elements of the mass, damping and stiffness matrices required in the following equations-of-motion:

$$\begin{bmatrix} M_{ss} & M_{sa} \\ M_{as} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{U}_s \\ \ddot{U}_a \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sa} \\ C_{as} & C_{aa} \end{bmatrix} \begin{Bmatrix} \dot{U}_s \\ \dot{U}_a \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sa} \\ K_{as} & K_{aa} \end{bmatrix} \begin{Bmatrix} U_s \\ U_a \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (6.1)$$

More specifically, only matrices M_{ss}, C_{ss}, K_{ss}, K_{sa} and M_{sa} are required. For a lumped mass system M_{sa} is zero. The matrices M_{ss}, C_{ss} and K_{ss} are n x n and the matrices K_{sa} and M_{sa} are m x n, where n=unconstrained degree-of-freedom of the secondary system and m=number of support on the primary system.

2. Define the static influence matrix $[A]_{nxm}$ as follows:

$$[A]=(-[Kss]^{-1}[Ksa]) \quad (6.2)$$

3. Define the dynamic influence matrix $[r]_{nxm}$ as follows:

$$[r]=([M_{ss}][K_{ss}]^{-1}[K_{sa}]-[M_{sa}]) \quad (6.3)$$

4. Obtain the natural frequencies and mode shapes of the secondary system assumed fixed at the supports, as a solution of the following eigenvalue problem:

$$[K_{ss}]\{\Psi_j\} = \omega_j^2[M_{ss}]\{\Psi_j\} \quad (6.4)$$

where ω_j=jth natural frequency and Ψ_j=jth mode shape. Normalize the mode shape with respect to mass matrix such that

$$\{\Psi_j\}'[M_{ss}]\{\Psi_j\} = 1 \quad (6.5)$$

5. Obtain the modal influence vector $\{P_j\}_{nx1}$ for each mode:

$$\{P_j\} = [r]'\{\Psi_j\} \tag{6.6}$$

6. Obtain the mode shape for the response quantity of interest by simple linear transformation as

$$\{\rho_j\} = \{T\}'\{\Psi_j\} \tag{6.7}$$

where {T} is the transformation vector which transform Ψ_j into the response quantity of interest ρ_j. If only the displacement response is required, then $\rho_j=\Psi_j(u)$.

7. Obtain the static response influence coefficients η_k for the response quantity of interest. η_k represents the response of interest induced by a unit displacement of support k, and can be obtained by a simple static solution of the secondary system.

8. The total response consists of (a) dynamic, (b) pseudo-static and (c) cross components of the response. These individual components are calculated as follows:

a. CALCULATION OF DYNAMIC RESPONSE

The seismic inputs for the calculation of dynamic response are defined in terms of auto, coincident and quadrature floor spectra for the displacement and velocity responses. The procedures for the development of these spectra are given in Chapter III. The following notations have been used to designate various spectral quantities earlier in the report. All these spectral quantities pertain to an oscillator of frequency ω_j and damping ratio β_j.

FLOOR SPECTRAL INPUTS

$R_{dk}(\omega_j)$ = auto displacement spectrum for the k^{th} floor (See section III.2.1)

$R_{vk}(\omega_j)$ = auto velocity spectrum for the k^{th} floor (See section III.2.1)

$C_{dk\ell}(\omega_j)$ = cross coincident displacement spectrum for floors k and ℓ (See section III.2.2)

$C_{vk\ell}(\omega_j)$ = cross coincident velocity spectrum for floors k and ℓ (See section III.2.2)

$Q_{dk\ell}(\omega_j)$ = cross quadrature displacement spectrum for floors k and ℓ (See section III.2.2)

$C_{vk\ell}(\omega_j)$ = cross quadrature velocity spectrum for floors k and ℓ (See section III.2.2)

It is assumed that all peak factors are equal. Thus, all the peak factors, required for generation of these floor spectral inputs according to Chapter III, should be taken equal to 1.

The dynamic response component is now defined as

$$R_{dd}^2 = C_d^2 \sum_{i=1}^{n'} \sum_{j=1}^{n'} \rho_i \rho_j \sum_{k=1}^{m} \sum_{\ell=1}^{m} P_{ik} P_{j\ell} I_{ak\ell ij} \tag{6.8}$$

where P_{ik} = the k^{th} component of the influence vector $\{P_i\}$,

ρ_i = i^{th} modal response quantity,

n' = the number of modes desired to be included in the analysis $\leqslant$ n.

$I_{ak\ell ij}$ is defined in terms of various floor response spectral quantities for various combinations of i, j, k and ℓ as follows:

i) $k=\ell$ and $i=j$

$$I_{akkjj} = R^2_{dk}(\omega_j) \tag{6.9}$$

ii) $k=\ell$ and $i \neq j$

$$I_{akkij} = A\, R^2_{dk}(\omega_i) + B\, R^2_{vk}(\omega_i) + C\, R^2_{dk}(\omega_j) + D\, R^2_{vk}(\omega_j) \tag{6.10}$$

Coefficients A, B, C and D are obtained as a solution of

$$[Y_{ij}]\{V_1\} = \{W_1\} \tag{6.11}$$

where

$$\{V_1\}' = \{A,\ B,\ C,\ D\} \tag{6.12}$$

$$\{W_1\}' = \{(\omega_i\omega_j)^2,\ (4\beta_i\beta_j\omega_i\omega_j - \omega_i^2 - \omega_j^2),\ 1,\ 0\}' \tag{6.13}$$

$$[Y_{ij}] = \begin{bmatrix} \omega_j^4 & 0 & \omega_i^4 & 0 \\ 2\omega_j^2(2\beta_j^2-1) & \omega_j^4 & 2\omega_i^2(2\beta_i^2-1) & \omega_i^4 \\ 1 & 2\omega_j^2(2\beta_j^2-1) & 1 & 2\omega_i^2(2\beta_i^2-1) \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{6.14}$$

iii) $k \neq \ell$ and $i=j$

$$I_{ak\ell ii} = C^2_{dk\ell}(\omega_i) \tag{6.15}$$

iv) $k \neq \ell$ and $i \neq j$

$$I_{ak\ell ij} = \{A\ C^2_{dk\ell}(\omega_i) + B\ C^2_{vk\ell}(\omega_i) + C\ C^2_{dk\ell}(\omega_j) + D\ C^2_{vk\ell}(\omega_j)\}$$

$$- \{E\ Q^2_{dk\ell}(\omega_i) + F\ Q^2_{vk\ell}(\omega_i) + G\ Q^2_{dk\ell}(\omega_j) + H\ Q^2_{vk\ell}(\omega_j) \quad (6.16)$$

Coefficients A, B, C and D are the same as obtained in Eq.(6.11). Coefficients E, F, G and H are ontained as a solution of the following equation:

$$[Y_{ij}]\{V_2\} = \{W_2\} \quad (6.17)$$

where

$$\{V_2\}' = \{E, F, G, H\} \quad (6.18)$$

$$\{W_2\}' = \{2\omega_i\omega_j(\beta_i\omega_j - \beta_j\omega_i),\ 2(\beta_j\omega_j - \beta_i\omega_i),\ 0,\ 0\} \quad (6.19)$$

and the matrix $[Y_{ij}]$ is same as in Eq.(6.14).

b. CALCULATION OF PSEUDO-STATIC RESPONSE

The pseudo-static component can be obtained from the relative or absolute displacement formulations. Here, however, only the steps of the approach employing absolute displacement formulation are given.

The pseudo-static response is given by

$$R^2_{dp} = \sum_{k=1}^{m} U^2_{ak} + \sum_{k=1}^{m} \sum_{\substack{\ell=1 \\ k\neq\ell}}^{m} \eta_k \eta_\ell \delta_{k\ell} \bar{U}_{ak} \bar{U}_{a\ell} \quad (6.20)$$

where $\bar{U}_{ak}$= the maximum displacement of floor k,(See Section III.3)

$\delta_{k\ell}$= correlation coefficient between the displacement of floors k and ℓ. (See Section III.3 for its calculation)

c. CALCULATION OF CROSS RESPONSE

The cross response is obtained from the following equation as

$$R_{pd}^2 = C_d^2 \sum_{i=1}^{n'} \rho_i \sum_{\ell=1}^{m} \sum_{k=1}^{m} 2\eta_\ell P_{ik} I_{adk\ell i} \tag{6.21}$$

where the term $I_{adk\ell i}$ is defined as follows:

i) $k=\ell$

$$I_{adkki} = A\, R_{dk}^2(\omega_i) + B\, R_{vk}^2(\omega_i) + D\, \bar{\dot{U}}_k \tag{6.22}$$

where $\bar{\dot{U}}_k$= maximum velocity of floor k (See Section III.4 for its calculation). Coefficients A, B and D can be obtained from Eq.(6.11) by setting $\omega_j=0$ or they are defined as

$$A = (4\beta_i^2-1)$$
$$B = 1/\omega_i^2 \tag{6.23}$$
$$D = -1/\omega_i^2$$

ii) $k\neq\ell$

$$I_{adkki} = \{A\, C_{dk\ell}^2(\omega_i) + B\, C_{vk\ell}^2(\omega_i) + D\, \delta''_{k\ell}\bar{\dot{U}}_k\bar{\dot{U}}_\ell\}$$
$$- \{E\, Q_{dk\ell}^2(\omega_i) + F\, Q_{vk\ell}^2(\omega_i) + H\, \delta'''_{k\ell}\, \bar{\dot{U}}_k\bar{\dot{U}}_\ell\} \tag{6.24}$$

Coefficients A, B and D are given in Eq.(6.23). Coefficients E, F and H can be obtained from Eq.(6.17) by setting $\omega_j=0$ or they are defined as:

$$E = 4\beta_i/\omega_i(2\beta_i^2-1)$$

$$F = 2\beta_i/\omega_i^3 \qquad (6.25)$$

$$H = -2\beta_i/\omega_i^3$$

$\delta''_{k\ell}$= correlation coefficient between the velocity of floor k and ℓ (See Section III.4) $\delta'''_{k\ell}$= quadrature coefficient between the velocity of floor k and ℓ (See Section III.4)

Therefore, the total response is calculated as

$$R_d^2 = R_{dd}^2 + R_{pd}^2 + R_{pd}^2 \qquad (6.26)$$

The procedure outlined above can be used for the calculation of force or absolute displacement response of the secondary system.

CHAPTER VII

SUMMARY AND CONCLUTIONS

VII.1 GENERAL SUMMARY

A rational response spectrum procedure for seismic analysis of multiply supported secondary systems is developed. The development of the procedure is based on the random vibration analysis of structural systems subjected to several correlated inputs applied at several supports. The support inputs are defined in the spectral form like floor spectra, and herein the methods are developed to characterize the correlated support motions in this form. The information about floor displacements and velocities as well as correlation among these quantities is also required as input.

The total response is expressed as a combination of the dynamic and pseudo-static parts. The dynamic part is associated with the inertial effects of the support accelerations, whereas the pseudo-static part is due to the displacement of the supports relative to each other. Since these two components of the response are correlated, this correlation must be properly reflected in the analysis. The procedure developed here include this correlation through the terms, herein being referred as the cross response terms.

The development of the floor spectral inputs, of course, requires the dynamic analysis of the supporting primary structure. The correlated support motions are characterized in terms of the auto and cross floor response spectra for the displacement (or pseudo-acceleration) and velocity response of the oscillators on the floors. The cross spectra consists of the two components: (1) the coincident and (2) the quadrature floor spectra. These two spectra must also be defined for the displacement and velocity response of an oscillator. The methods are developed to obtain such cross floor ,spectra even for the special case of a secondary system with one of its supports being on the ground.

The methods for development of these various floor spectral inputs also employ response spectrum approaches. Thus, the ground response spectra can be directly used in these methods for generation of floor spectra.

Various numerical results, showing the development of various floor spectra are presented. These floor spectra are then used as inputs in the analysis of the secondary systems for the calculation of the force responses. The numerical results for these response quantities are also presented.

For the benefit of a user not interested in the analytical developments presented in various chapters, a step-by-step procedure for the implementation of the method is provided in Chapter VI: USER SUMMARY OF THE PROPOSED METHOD.

VII.2 DISCUSSION AND CONCLUSIONS

The analysis presented in this report clearly shows that for a proper seismic evaluation of the multiply connected secondary system, it is necessary to define the seismic inputs not only in terms of the (conventional) auto floor displacement spectra, but also the (1) auto relative velocity spectra and (2) the cross floor spectra for displacement and velocity responses. A complete description of the cross floor spectra, characterizing the correlation between any two floor accelerations, requires the definition of the coincident and quadrature floor response spectra. In addition, the displacement and velocity responses of various interconnected floors and their correlation must also be defined as a part of the input for seismic analysis of such systems.

The analytical feasibility of the methods to obtain various types of floor response spectra, and also the effective use of these spectral inputs in the calculation of the force and displacement responses of the secondary systems, are clearly demonstrated by the numerical examples.

The comparison of the numerical results obtained by the proposed approach with the results obtained with the help of some currently used procedures, shows that the latter procedures may not always provide a conservative estimate of the response. Since some of the currently used procedures, such as an enveloping of the support point floor spectra and the rules for combination of the dynamic and pseudo-static responses, lack analytical rationality, their use will not give analytically consistent results.

Often in the designs of such secondary systems,the pseudo-static component of the response is considered secondary of self limiting type in nature because it is induced by the differential displacements of the supports (anchor movement stresses). See ASME code, section III, Appendix N 1. For such components of stresses, higher allowable stresses can be used. Although, there are some interpretational differences in considering these stresses as secondary self limiting stresses, their separate rational evaluation is possible in the proposed approach. In case one intends to treat these stresses separately from the dynamic component of stresses, their correlation, which can sometime be very significant as represented by the cross term , must not be ignored. It is writers personal opinion that no distinction be made between the dynamic and pseudo-static components of stresses with regard to the allowable stresses and that all these stresses be considered as primary stresses in design evaluation of the secondary systems.

The approach is valid for linearly behaving light secondary systems for which their dynamic interaction with the supporting primary structure can be ignored. In practice many secondary systems are usually light enough, even for the tuned case, such that this interaction is insignificant and thus a decouple analysis of the two systems can be quite justified. Such decouple analysis also facilitate the design of such systems. The combined analysis to incorporate dynamic interaction, on the other hand, will require the information about the two systems simultaneously which may be difficult, and sometimes practically impossible, to obtain. The methods to incorporate the dynamic interaction are presented in a separate report [26]. However, the proposed approach can be used with confidence to obtain the improved response results whenever dynamic coupling is considered unimportant.

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TABLE I: Natural Frequncies (rad/S) and Participation Factors of the Primary System.

Mode	Natural Frequency	Participation factor
1	6.98	383.8
2	20.38	-120.8
3	32.12	-63.7
4	41.26	35.4
5	47.06	-16.2

TABLE II: Mode Shapes of the Primary System - [ftx10^{-2}].

Node	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	.093	-.249	-.326	.299	-.178
2	.178	-.326	-.093	-.249	.299
3	.249	-.178	.299	-.093	-.326
4	.299	.093	.178	.326	.249
5	.326	.299	-.249	-.178	-.093

TABLE III: Natural Frequencies of the Secondary System (rad/S).

Mode	Natural Frequency
1	17.32
2	24.49
3	34.64

TABLE IV: Mode Shapes of the Secondary System - [ftx10^{-1}].

Node	Mode 1	Mode 2	Mode 3
1	.577	.577	.577
2	-.707	.0	.707
3	.408	-.816	.408

TABLE V: Dynamic Influence Coefficient P_{jk}.

Support	Mode 1	Mode 2	Mode 3
1	-5.773	-5.773	-5.773
2	3.535	.0	-3.535
3	-1.020	2.041	1.020

TABLE VI: Maximum Absolute Support Displacements and Velocities.

Support	Maximun Disp. [ft]	Maximun Vel. [ft/s]
Ground	.300	.400
2	.309	.505
3	.315	.588
4	.319	.665

TABLE VII: Displacement Correlation Coefficients.

Support	Ground	2	3	4
Ground	1.	.991	.978	.970
2	.991	1.	.998	.994
3	.978	.998	1.	.997
4	.970	.994	.997	1.

TABLE VIII: Coincident Velocity Correlation Coefficients.

Support	Ground	2	3	4
Ground	1.	.701	.549	.451
2	.701	1.	.962	.890
3	.549	.962	1.	.975
4	.451	.890	.975	1.

TABLE IX: Quadrature Velocity Coefficients.

Support	Ground	2	3	4
Ground	0.	-1.038	-.837	-.684
2	1.038	0.	.366	.353
3	.837	-.366	0.	.203
4	.684	-.353	-.203	0.

TABLE X: Force Response - [Lb].

Configuration "A"			Configuration "B"		
Elem. (1)	Combined system (2)	Proposed method (3)	Elem. (4)	Combined system (5)	Proposed method (6)
2-6	713.2	708.6	G-6	1120.3	1114.6
6-7	378.8	426.2	6-7	728.4	738.4
3-7	513.9	532.1	2-7	836.5	822.3
7-8	367.2	367.5	7-8	529.0	496.6
4-8	475.5	476.9	3-8	858.9	847.1

TABLE XI: Fractional Contribution of the Dynamic, Pseudo-static and Cross Components to the Total Force Response Variance of Various Members in Structure "A".

Elem. (1)	Total variance (2)	Dynamic component (3)	Pseudo-static component (4)	Cross component (5)
2-6	502149.	.455	.156	.388
6-7	181702.	.307	.431	.261
3-7	283203.	1.070	.008	-.078
7-8	135124.	.408	.442	.148
4-8	227473.	1.794	.262	-1.057

TABLE XII: Fractional Contribution of the Dynamic, Pseudo-static and Cross Components to the Total Force Response Variance of Various Members in Structure "B".

Elem. (1)	Total variance (2)	Dynamic component (3)	Pseudo-static component (4)	Cross component (5)
G-6	1242430.	.623	.287	.089
6-7	545329.	.152	.654	.193
2-7	676232.	1.010	.067	-.078
7-8	246666.	.286	.631	.082
3-8	717683.	1.039	.216	-.256

TABLE XIII: Comparison of the Results Obtained by Various Response Combination Rules and Proposed Approach for Structure "A".

Elem.	Proposed method	SS1 ratio	SS2 ratio	SA1 ratio	SA2 ratio
2-6	708.6	1.544	2.186	2.160	2.949
6-7	426.2	1.948	2.475	3.176	3.743
3-7	532.1	2.315	2.703	3.678	4.080
7-8	367.5	2.918	3.519	4.330	4.961
4-8	476.9	2.245	2.693	3.338	3.805

TABLE XIV: Comparison of the Results Obtained by Various Response Combination Rules and Proposed Approach for Structure "B".

Elem.	Proposed method	SS1 ratio	SS2 ratio	SA1 ratio	SA2 ratio
G-6	1114.6	.786	1.072	.871	1.223
6-7	738.4	.689	.949	.892	1.178
2-7	822.3	1.086	1.327	1.489	1.739
7-8	496.6	1.733	2.164	2.268	2.715
3-8	847.1	1.011	1.255	1.325	1.578

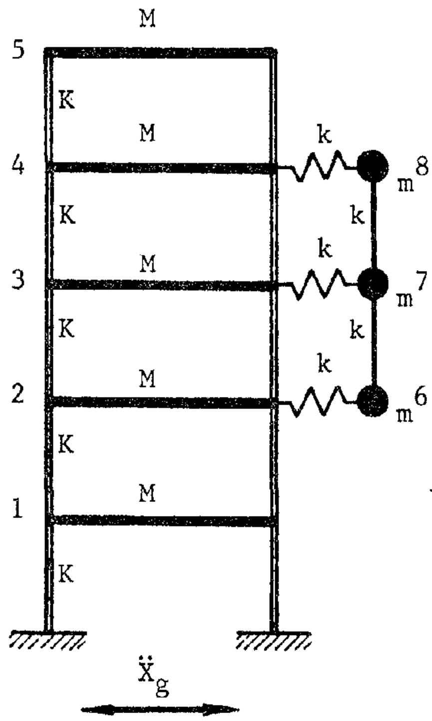

FIG.1: Structural Configuration "A".

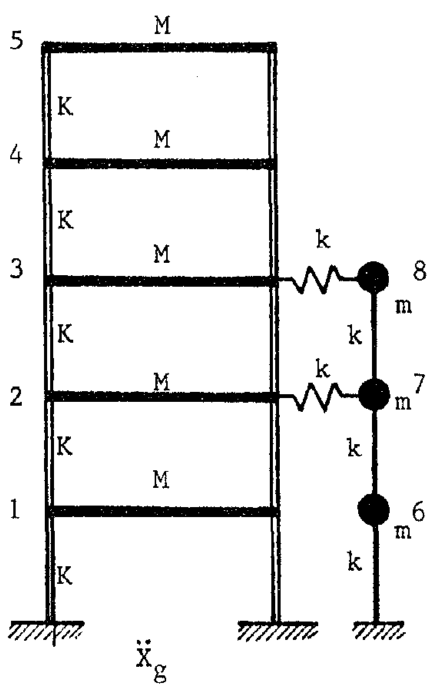

FIG.2: Structural Configuration "B".

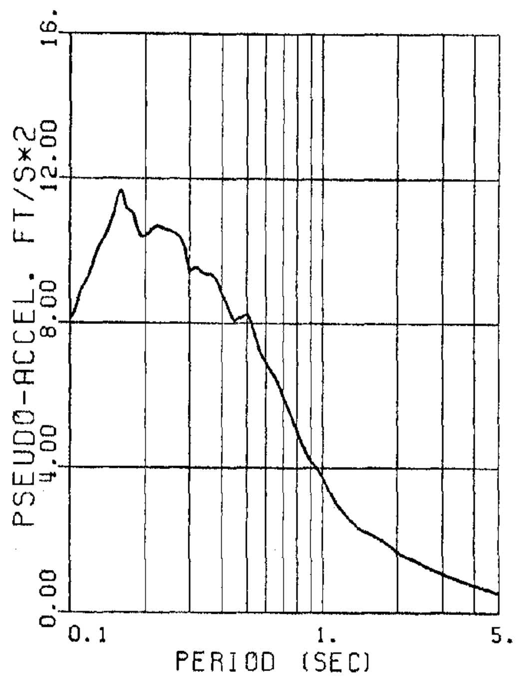

FIG.3: Auto Pseudo-accelerarion Ground Spectrum.

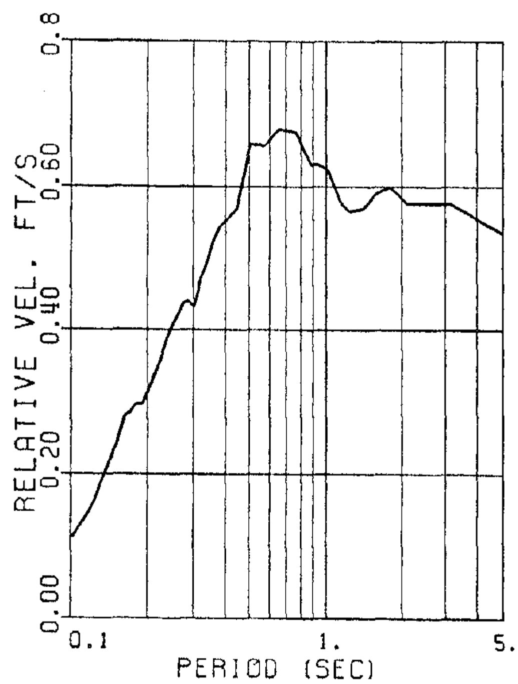

FIG.4: Auto Relative Velocity Ground Spectrum.

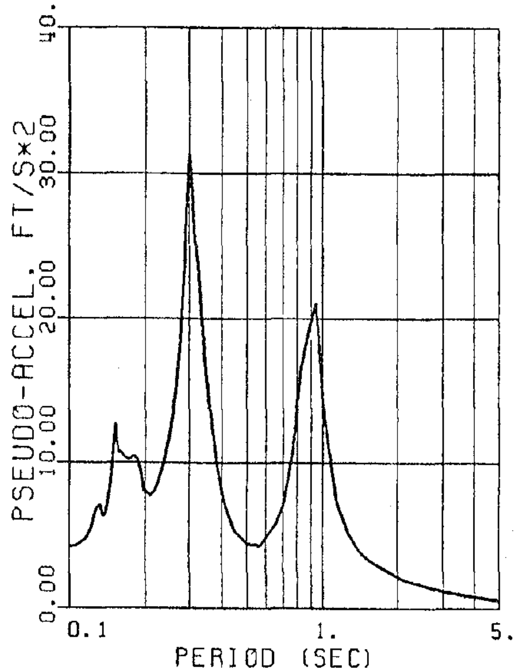

FIG.5: Auto Pseudo-accelerarion Spectrum for Floor 2.

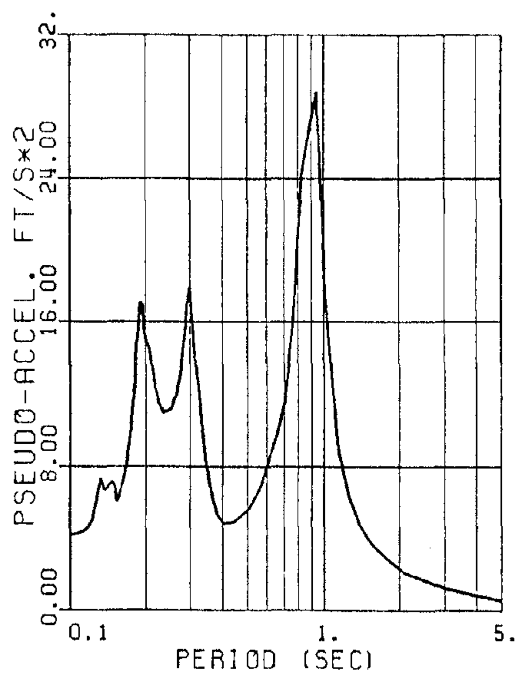

FIG.6: Auto Pseudo-acceleration Spectrum for Floor 3.

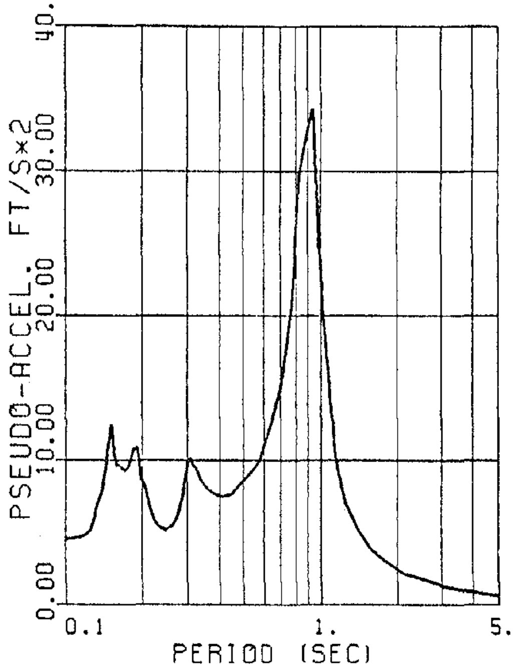

FIG.7: Auto Pseudo-accelerarion Spectrum for Floor 4.

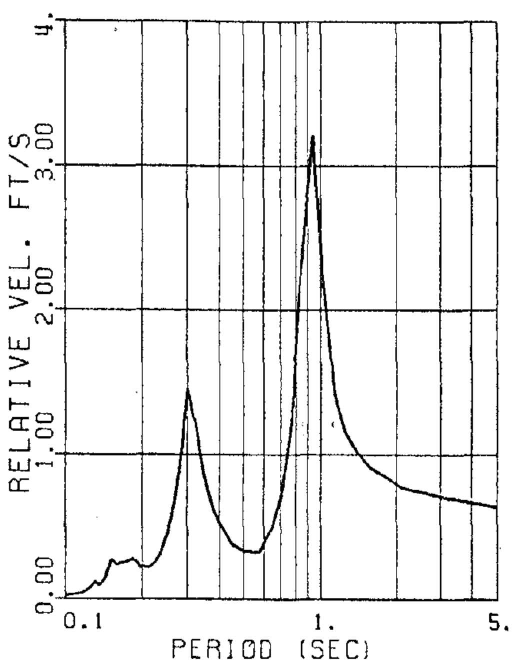

FIG.8: Auto Velocity Spectrum for Floor 2.

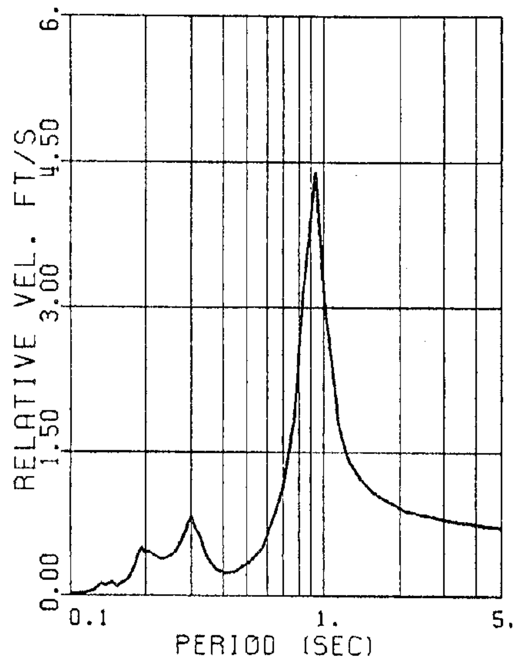

FIG.9: Auto Velocity Spectrum for Floor 3.

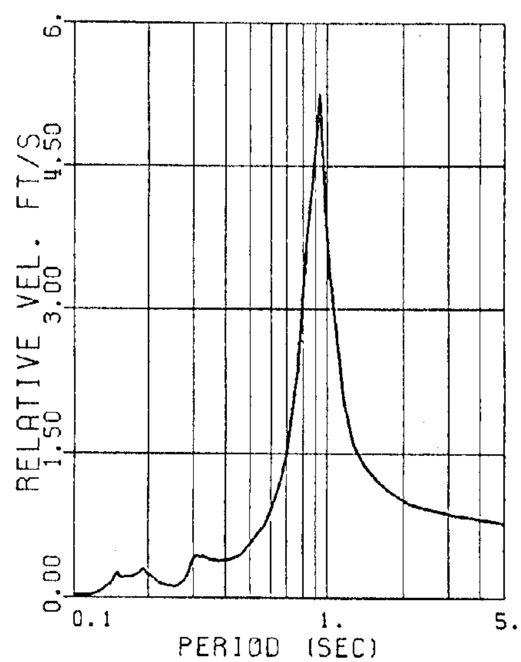

FIG.10: Auto Velocity Spectrum for Floor 4.

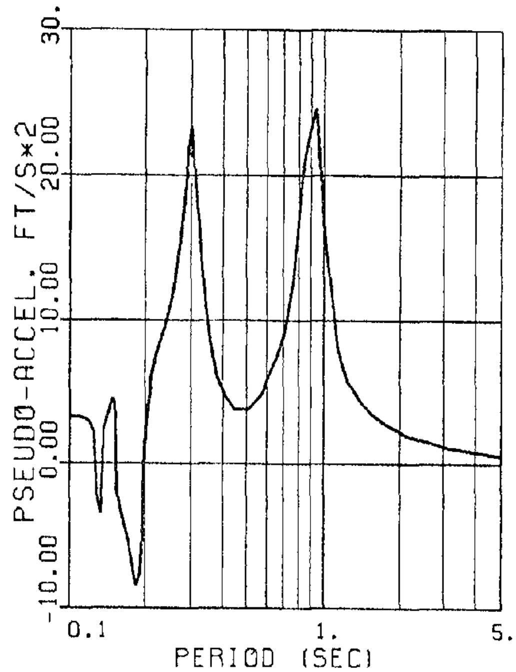

FIG.11: Coincident Pseudo-acceleration

Spectrum for Floors 2 and 3.

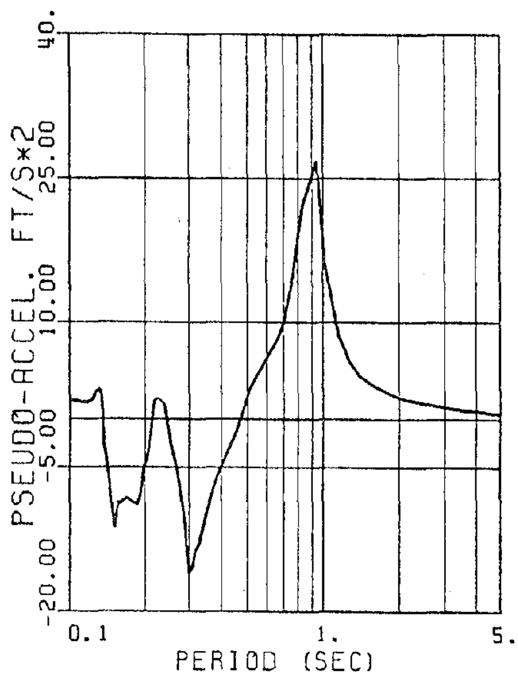

FIG.12: Coincident Pseudo-acceleration

Spectrum for Floors 2 and 4.

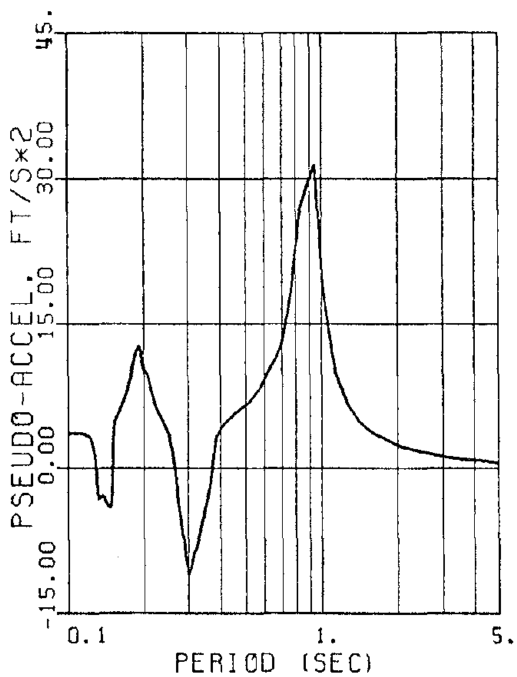

FIG.13: Coincident Pseudo-acceleration

Spectrum for Floors 3 and 4.

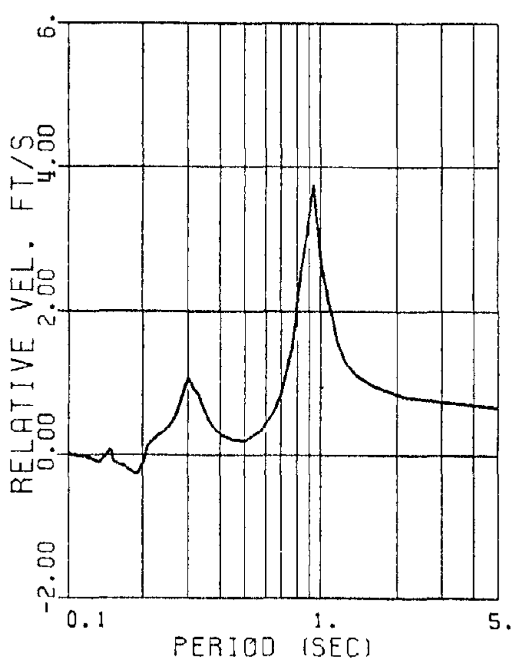

FIG.14: Coincident Velocity

Spectrum for Floors 2 and 3.

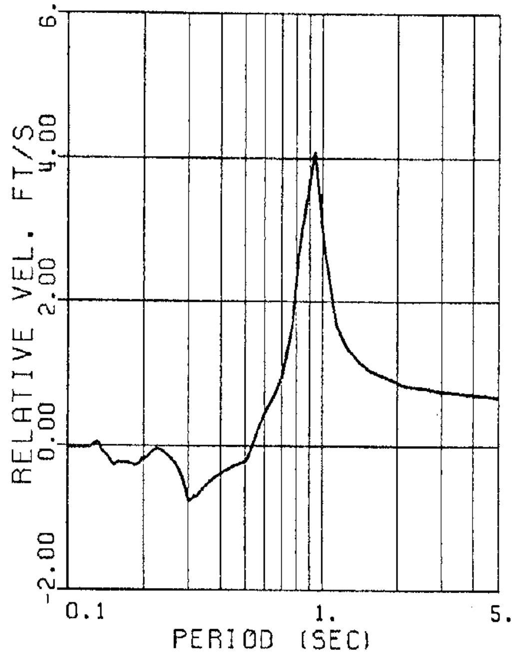

FIG.15: Coincident Velocity Spectrum for Floors 2 and 4.

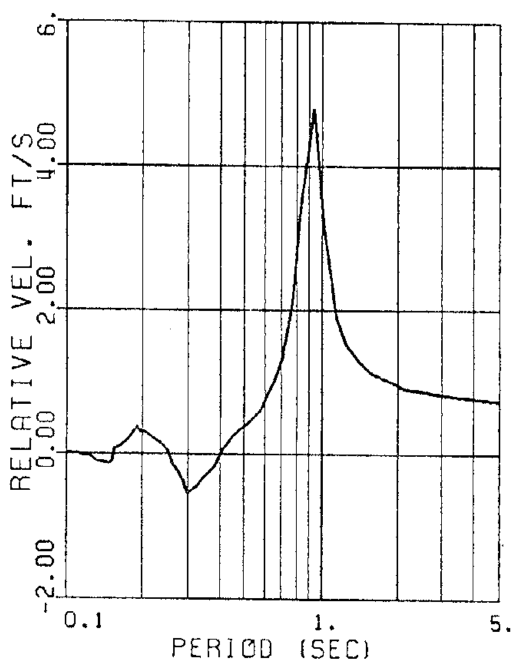

FIG.16: Coincident Velocity Spectrum for Floors 3 and 4.

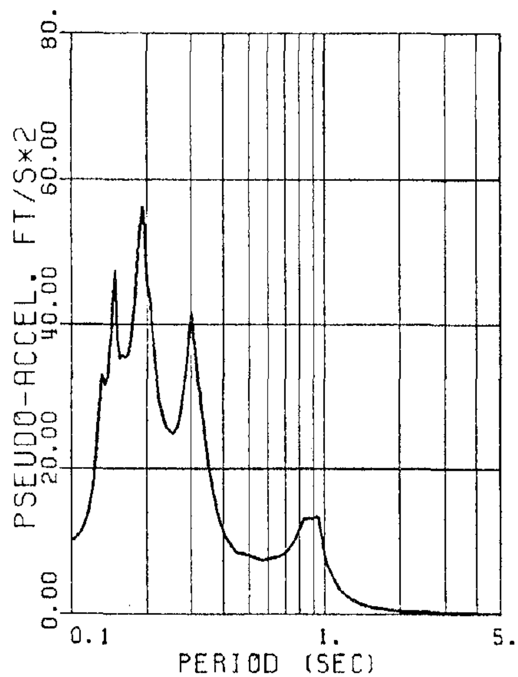

FIG.17: Quadrature Pseudo-acceleration

Spectrum for Floors 2 and 3.

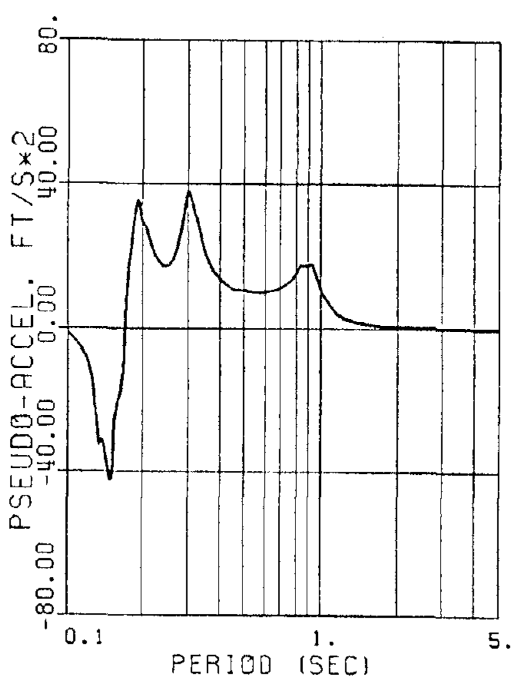

FIG.18: Quadrature Pseudo-acceleration

Spectrum for Floors 2 and 4.

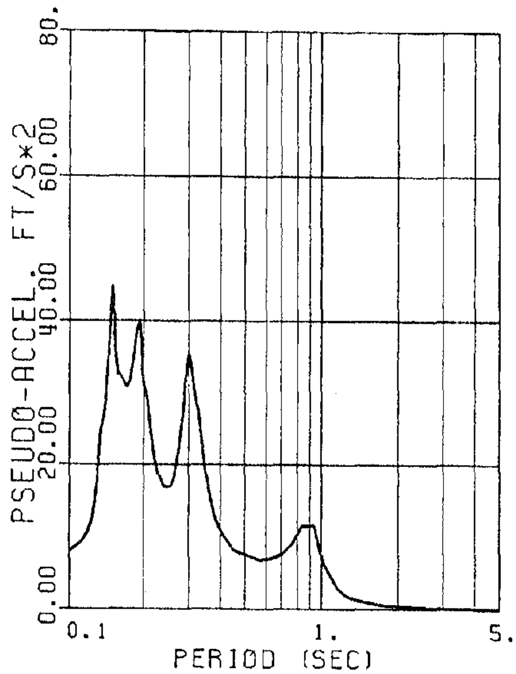

FIG.19: Quadrature Pseudo-acceleration

Spectrum for Floors 3 and 4.

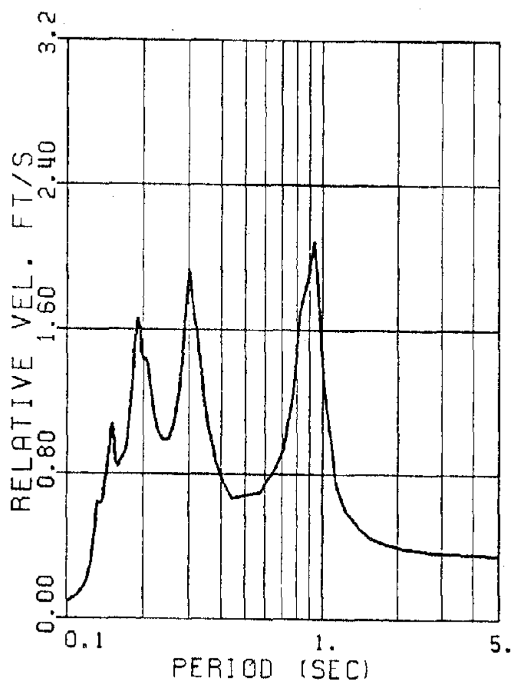

FIG.20: Quadrature Velocity

Spectrum for Floors 2 and 3.

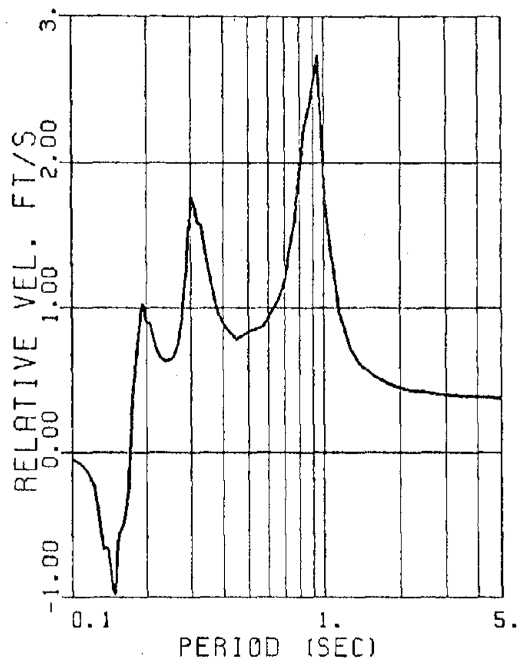

FIG.21: Quadrature Velocity

Spectrum for Floors 2 and 4.

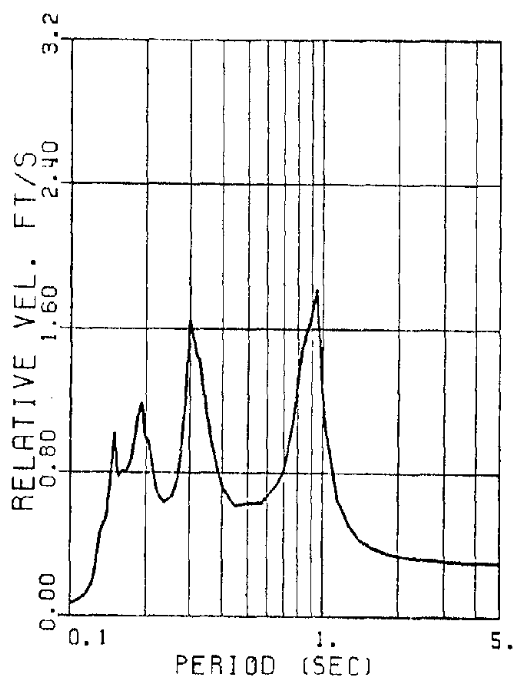

FIG.22: Quadrature Velocity

Spectrum for Floors 3 and 4.

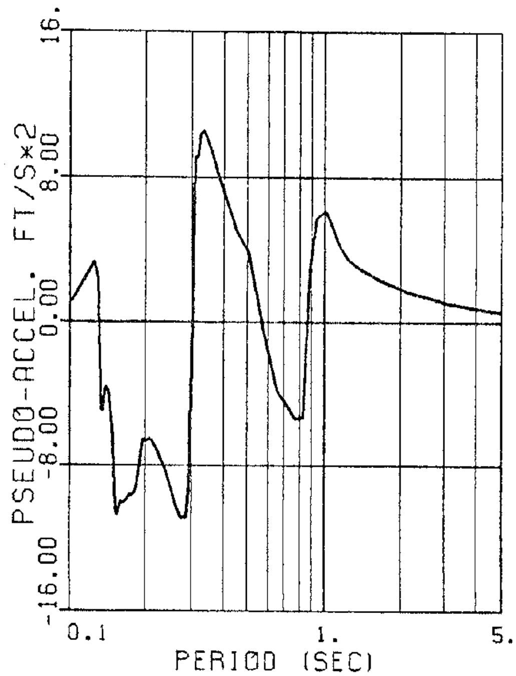

FIG.23: Coincident Pseudo-acceleration

Spectrum for Ground and Floor 2.

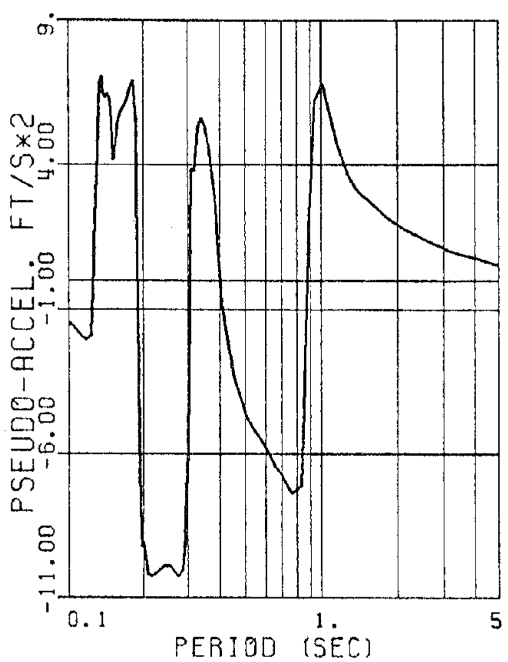

FIG.24: Coincident Pseudo-acceleration

Spectrum for Ground and Floor 3.

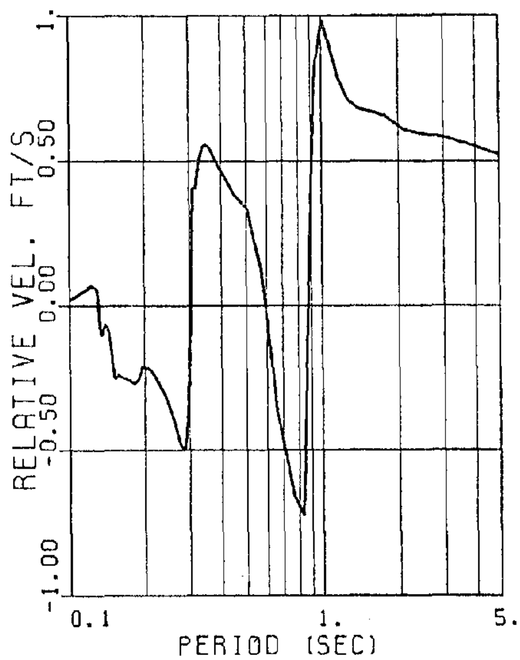

FIG.25: Coincident Velocity Spectrum

for Ground and Floor 2.

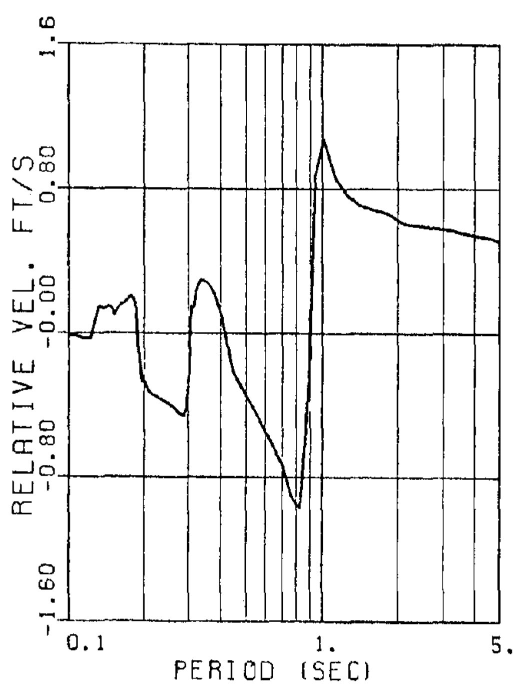

FIG.26: Coincident Velocity Spectrum

for Ground and Floor 3.

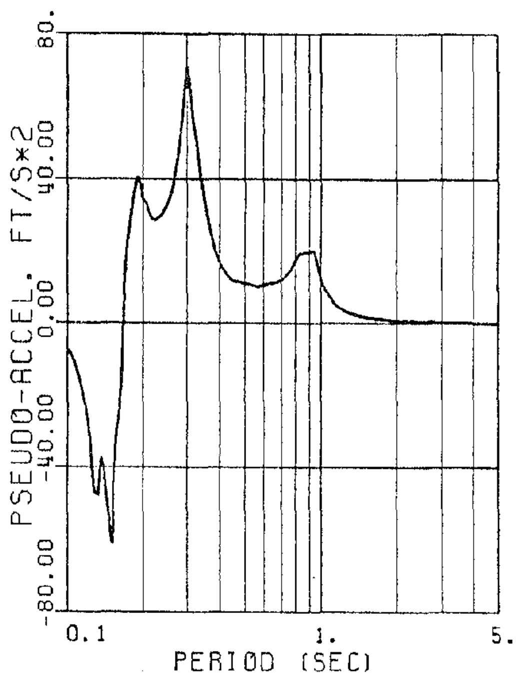

FIG.27: Quadrature Pseudo-acceleration

Spectrum for Ground and Floor 2.

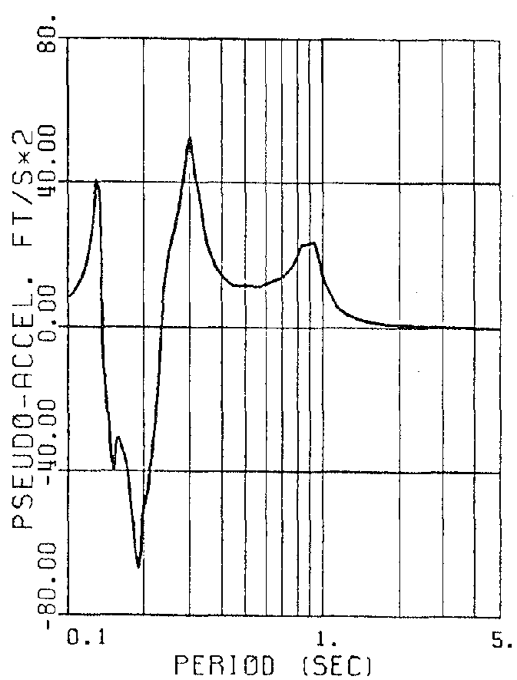

FIG.28: Quadrature Pseudo-acceleration

Spectrum for Ground and Floor 3.

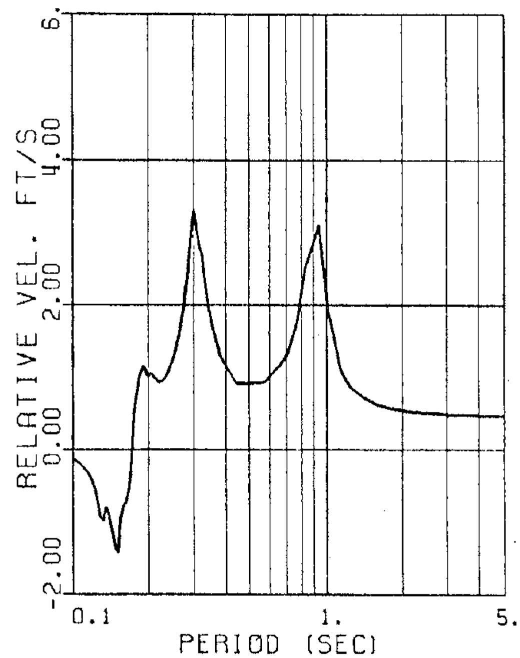

FIG.29: Quadrature Velocity Spectrum

for Ground and Floor 2.

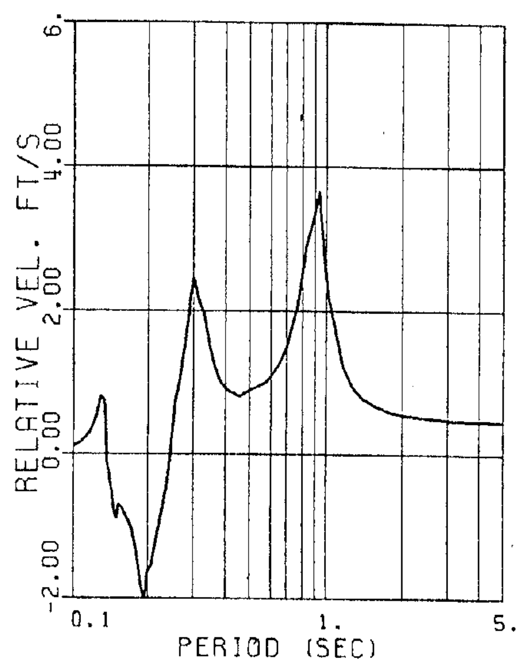

FIG.30: Quadrature Velocity Spectrum

for Ground and Floor 3.

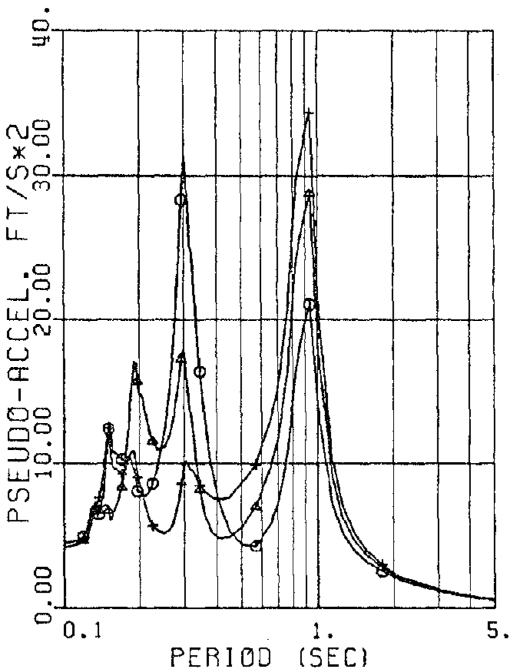

FIG.31: Pseudo-acceleration Envelope

Spectrum for Structural Configuration "A".

(o=Floor 2 | Δ=Floor 3 | +=Floor 4)

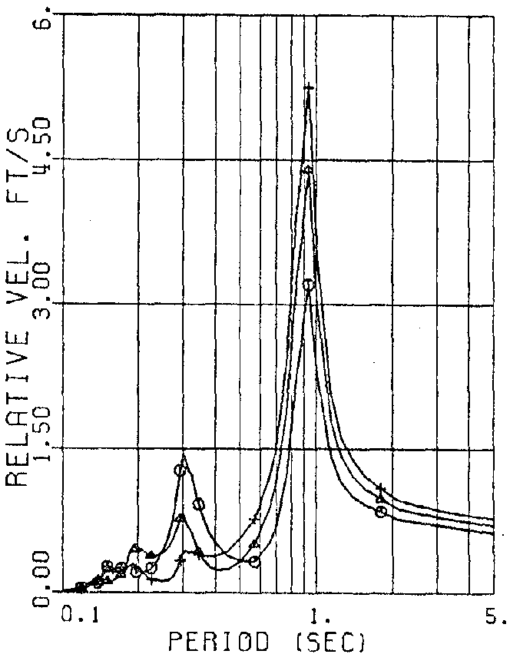

FIG.30: Relative Velocity Envelope

Spectrum for Structural Configuration "A".

(o=Floor 2 | Δ=Floor 3 | +=Floor 4)

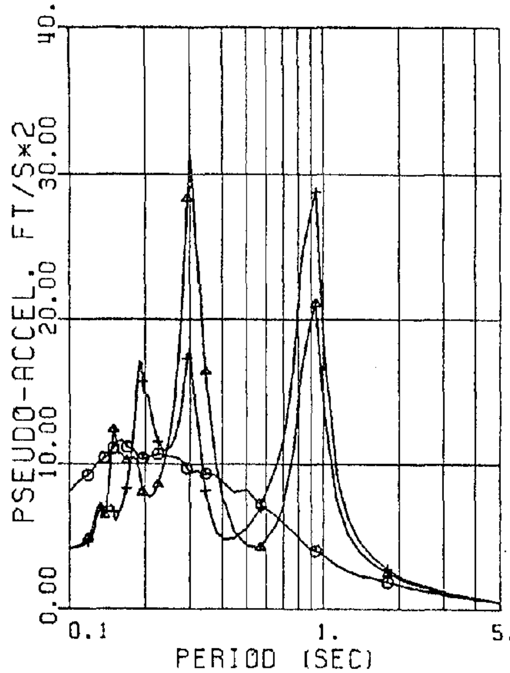

FIG.33: Pseudo-acceleration Envelope

Spectrum for Structural Configuration "B".

(o=Ground | Δ=Floor 2 | +=Floor 3)

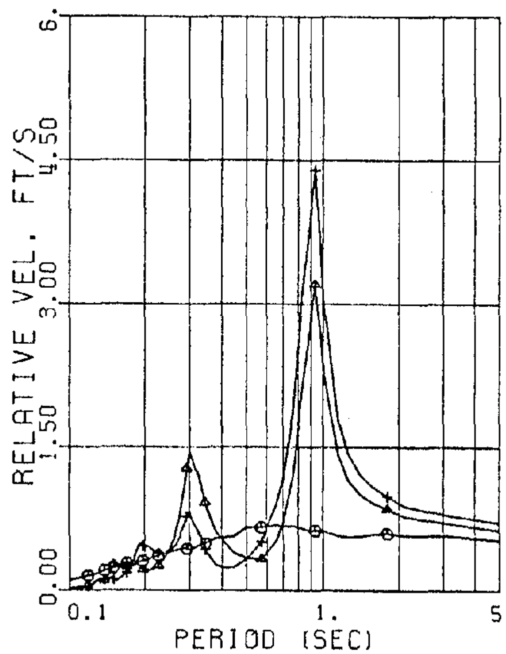

FIG.34: Relative Velocity Envelope

Spectrum for Structural Configuration "B".

(o=Ground | Δ=Floor 2 | +=Floor 3)

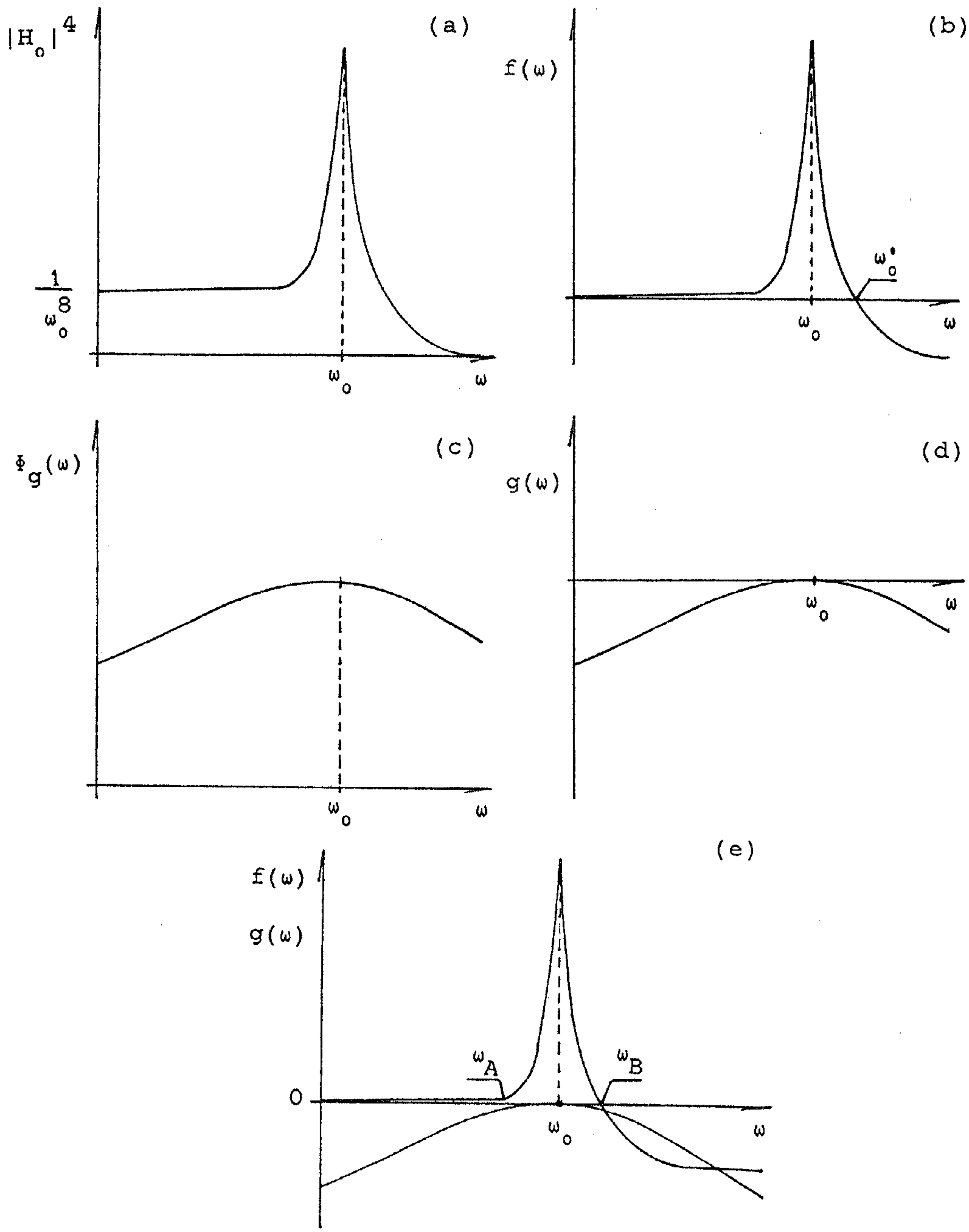

FIG.35: Analysis of Resonance Case.

APPENDIX I. VARIANCE OF THE DYNAMIC AND CROSS RESPONSES

In this Appendix the variances of the dynamic and cross responses, given in Eqs.(2.17), (2.71) and (2.72), are developed.

I.1. VARIANCE OF THE DYNAMIC RESPONSE

The covariance function of the dynamic component is obtained as follows

$$R_{dd}(t_1,t_2) = E[S^d(t_1)S^d(t_2)] \tag{I.1}$$

Replacing Eq.(2.16) into (I.1) gives

$$R_{dd}(t_1,t_2) = \sum_{i=1}^{n} \sum_{j=1}^{m} \rho_i \rho_j E[q_i(t_1)q_j(t_2)] \tag{I.2}$$

where the generalized coordinate can be expressed as a solution of Eq.(2.15)

$$q_i(t_1) = \int_0^{t_1} \{P_i\}'\{\ddot{U}_a(\theta_1)\}h_i(t_1-\theta_1)d\theta_1 \tag{I.3}$$

in which $h_i(t)$ is the impulse response function of Eq.(2.15). Replacing Eq.(I.3) into (I.2), and after some standard manipulations, we obtain

$$R_{dd}(t_1,t_2) = \sum_{i=1}^{n} \sum_{j=1}^{m} \rho_i \rho_j \int_0^{t_2} \int_0^{t_1} E[\{P_i\}'\{\ddot{U}_a(\theta_1)\}\{\ddot{U}_a(\theta_2)\}'\{P_j\}]$$

$$h_i(t_1-\theta_1)h_j(t_2-\theta_2)d\theta_1 d\theta_2 \tag{I.4}$$

in which the expected value can be replaced by

$$E[\{P_i\}'\{\ddot{U}_a(\theta_1)\}\{\ddot{U}_a(\theta_2)\}'\{P_j\}] = \sum_{k=1}^{m} \sum_{\ell=1}^{m} P_{ik}P_{j\ell}E[\ddot{U}_{ak}(\theta_1)\ddot{U}_{a\ell}(\theta_2)] \tag{I.5}$$

Assuming stationary response, the ecpected value of the absolute acceleration between floors k and ℓ can be obtained from the inverse Fourier transform of the cross spectral density function of the absolute acceleration as follows

$$E[\ddot{U}_{ak}(\theta_1)\ddot{U}_{a\ell}(\theta_2)] = \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega)e^{i\omega(\theta_2-\theta_1)}d\omega \qquad (I.6)$$

Substituting Eqs.(I.6) and (I.5) into (I.4) and introducing the change of variables $u_1=t_1-\theta_1$ and $u_2=t_2-\theta_2$, gives

$$R_{dd}(t_1,t_2) = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_i\rho_j \sum_{k=1}^{m}\sum_{\ell=1}^{m} P_{ik}P_{j\ell} \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega) \left(\int_0^{t_1} h_i(u_1)e^{i\omega u_1}du_1\right) \left(\int_0^{t_2} h_j(u_2)e^{-i\omega u_2}du_2\right) e^{i\omega(t_2-t_1)}d\omega \qquad (I.7)$$

The stationary value of the covariance function is

$$R_{dd}(t_1,t_2) = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_i\rho_j \sum_{k=1}^{m}\sum_{\ell=1}^{m} P_{ik}P_{j\ell} \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega)H_i^*H_j e^{i\omega(t_2-t_1)}d\omega \qquad (I.8)$$

Finally, the variance of the dynamic response is obtained setting $t_1=t_2$ in the covariance function. That is,

$$\sigma_{dd}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_i\rho_j \sum_{k=1}^{m}\sum_{\ell=1}^{m} P_{ik}P_{j\ell} \int_{-\infty}^{\infty} \Phi_{ak\ell}(\omega)H_i^*H_j d\omega \qquad (I.9)$$

I.2. VARIANCE OF CROSS RESPONSE

The covariance function of the cross component is obtained as follows

$$R_{dp}(t_1,t_2) = E[S^d(t_1)\ S^P(t_2)] \qquad (I.10)$$

Replacing Eqs.(2.16) and (2.55) into (I.10) gives

$$R_{dp}(t_1,t_2) = \sum_{i=1}^{n} \sum_{j=1}^{m} \rho_i \eta_\ell \, E[q_i(t_1) U_{a\ell}(t_2)] \tag{I.11}$$

Substituting Eq.(I.3) into the expected value in Eq.(I.11)

$$R_{dp}(t_1,t_2) = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i \eta_\ell \int_0^{t_1} E[\{P_i\}'\{\ddot{U}_a(\theta_1)\} U_{a\ell}(t_2)] h_i(t_1-\theta_1) d\theta_1 \tag{I.12}$$

or

$$R_{dp}(t_1,t_2) = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i \eta_\ell \sum_{k=1}^{m} P_{ik} \int_0^{t_1} E[\ddot{U}_{ak}(\theta_1) U_{a\ell}(t_2)] h_i(t_1-\theta_1) d\theta_1 \tag{I.13}$$

The expected values in Eqs.(I.11) and (I.12) can be substituted in terms of the cross spectral density function as follows

$$E[\ddot{U}_{ak}(\theta_1) U_{a\ell}(t_2)] = \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) \, e^{i\omega(t_2-\theta_1)} d\omega \tag{I.14}$$

Employing Eq.(I.14) and introducing the change of variable $u_1=t_1-\theta_1$, Eq.(I.13) becomes

$$R_{dp}(t_1,t_2) = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i \eta_\ell \sum_{k=1}^{m} P_{ik} \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) \left(\int_0^{t_1} h_i(u_1) e^{i\omega u_1} du_1\right) e^{i\omega(t_2-t_1)} d\omega \tag{I.15}$$

As t_1 approaches infinity, the correlation function in Eq.(I.15) becomes stationary as follows,

$$R_{dp}(t_1,t_2) = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i \eta_\ell \sum_{k=1}^{m} P_{ik} \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) \, H_i^* \, e^{i\omega(t_2-t_1)} d\omega \tag{I.16}$$

Following the same steps, it can be shown that

$$R_{pd}(t_1,t_2) = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i \eta_\ell \sum_{k=1}^{m} P_{ik} \int_{-\infty}^{\infty} \Phi_{dak\ell}(\omega) H_i e^{i\omega(t_2-t_1)} d\omega \tag{I.17}$$

Finally, the variance of the cross response is obtained by setting $t_1=t_2$ in the covariance functions in Eqs.(I.16) and (I.17) as follows

$$\sigma_{dp}^2 = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i \eta_\ell \sum_{k=1}^{m} P_{ik} \int_{-\infty}^{\infty} \Phi_{adk\ell}(\omega) H_i^* d\omega \qquad (I.18)$$

$$\sigma_{pd}^2 = \sum_{i=1}^{n} \sum_{\ell=1}^{m} \rho_i \eta_\ell \sum_{k=1}^{m} P_{ik} \int_{-\infty}^{\infty} \Phi_{da\ell k}(\omega) H_i d\omega \qquad (I.19)$$

APPENDIX II. FREQUENCY INTEGRALS - RESONANCE CASE

In this Appendix the functions required for computing the integrals $I_R(a_1,a_2)$, $I_S(a_2,a_3)$ and $I_E(a_4)$ in Eqs.(3.38), (3.49) and (4.38) are obtained.

To define the frequency integrals for the resonance case an approximation for the following general integral

$$I_i = \int_0^\infty \omega^i |H_o|^4 \Phi_g(\omega) d\omega \qquad \text{(II.1)}$$

is required. To evaluate this approximately, the PSDF of the ground acceleration, $\Phi_g(\omega)$, and the frequency response function can be written as follows (see Fig. 35),

$$\Phi_g(\omega) = \Phi_g(\omega_o) + g(\omega) \qquad \text{(II.2a)}$$

$$|H_o|^4 = \frac{1}{\omega_o^8} + f(\omega) \qquad \text{(II.2b)}$$

Replacing Eq.(II.2a) into (II.1) gives

$$I_i = \Phi_g(\omega_o) \int_0^\infty \omega^i |H_o|^4 d\omega + \int_0^\infty \omega^i g(\omega) |H_o|^4 d\omega \qquad \text{(II.3)}$$

Substituting Eq.(II.2b) into the second integral in Eq.(II.3)

$$I_i = \Phi_g(\omega_o) \int_0^\infty \omega^i |H_o|^4 d\omega + \frac{1}{\omega_o^8} \int_0^\infty \omega^i g(\omega) d\omega + \int_0^\infty \omega^i g(\omega) f(\omega) d\omega \qquad \text{(II.4)}$$

The last integral in Eq.(II.4) can be split as following

$$\int_0^\infty \omega^i g(\omega) f(\omega) d\omega = \int_0^{\omega_o'} \omega^i g(\omega) f(\omega) d\omega + \int_{\omega_o'}^\infty \omega^i g(\omega) f(\omega) d\omega \qquad \text{(II.5)}$$

in which ω_o' is a frequency slightly higher than ω_o as shown in Fig. 35(b). From Fig. 35(e) we observe that the first integral in Eq.(II.5) is approximately zero as between 0 and

ω_A, $f(\omega)$ is zero and between ω_A and ω_B, $g(\omega)$ is nearly equal to zero. Furthermore, we assume that $f(\omega) = -1/\omega_o^8$ for $\omega > \omega_o'$ (which is indeed true for large ω). Thus,

$$\int_0^{\infty} \omega^i g(\omega) f(\omega) d\omega = - \frac{1}{\omega_o^8} \int_{\omega_o'}^{\infty} g(\omega) d\omega \qquad \text{(II.6)}$$

substituting Eq.(II.6) into (II.4), we obtain

$$I_i = \Phi_g(\omega_o) \int_0^{\infty} \omega^i |H_o|^4 d\omega + \frac{1}{\omega_o^8} \int_0^{\omega_o'} \omega^i g(\omega) d\omega \qquad \text{(II.7)}$$

Using Eq.(II.2a) to define $g(\omega)$ in terms of $\Phi_g(\omega_o)$ and $\Phi_g(\omega)$, we obtain

$$I_i = \Phi_g(\omega_o) \int_0^{\infty} \omega^i |H_o|^4 d\omega - \Phi_g(\omega_o) \frac{(\omega_o')^{i+1}}{(i+1)\omega_o^8} + \frac{1}{\omega_o^8} \int_0^{\omega_o'} \omega^i \Phi_g(\omega) d\omega \qquad \text{(II.8)}$$

This integral is also given by Vanmarcke (28). However, the authors were unable to find the proof of this in any reference. Hence the above proof was developed and is given here for ready reference. This approximation will now be used to obtain $I_R(a_1, a_2)$, $I_S(a_2, a_3)$ and $I_E(a_4)$ required in Chapters III and IV.

CASE 1: Integral $I_R(a_1, a_2)$

$$I_R(a_1, a_2) = \int_{-\infty}^{\infty} (a_1 \omega_o^8 + a_2 \omega_o^6 \omega^2) |H_o|^4 \Phi_g(\omega) d\omega \qquad \text{(II.9)}$$

Using Eq.(II.8), $I_R(a_1, a_2)$ can be approximated as follows

$$I_R(a_1, a_2) = \Phi_g(\omega_o) \int_{-\infty}^{\infty} (a_1 \omega_o^8 + a_2 \omega_o^6 \omega^2) |H_o|^4 d\omega - \Phi_g(\omega_o) \omega_o 2\{a_1 + a_2/3\}$$

$$+ a_1 \int_{-\omega_o}^{\omega_o} \Phi_g(\omega) d\omega + \frac{a_2}{\omega_o^2} \int_{-\omega_o}^{\omega_o} \omega^2 \Phi_g(\omega) d\omega \qquad \text{(II.10)}$$

Replacing the limits in the first integral by the cut-off frequency ω_c, it can be obtained inclosed form as follows

$$\int_{-\omega_c}^{\omega_c} (a_1\omega_o^8 + a_2\omega_o^6\omega^2)|H_o|^4 d\omega = \omega_c C_m \qquad (II.11)$$

Expression for C_m is given later in Eq.(II.31).

The second frequency integral in Eq.(II.10) which represents the partial area under the PSDF is denoted by

$$I_b = \int_{-\omega_o}^{\omega_o} \Phi_g(\omega)d\omega \qquad (II.12)$$

To express I_b in spectral terms, the following relationship between the mean square values of the pseudo-acceleration and relative velocity is used [22],

$$\omega_o^4 I_{1go} \cong \omega_o^2 I_{2go} + I_b \qquad (II.13)$$

in which I_{1go} and I_{2go} are defined in Eq.(3.37). Thus,

$$I_b = I_{1go}\omega_o^4\{1 - F(\omega_o)\} \qquad (II.14)$$

where

$$F(\omega_o) = I_{2go}/(\omega_o^2 I_{1go}) \qquad (II.15)$$

The third integral in Eq.(II.10) which represents the second moment of the ground acceleration is denoted by

$$I_{b2} = \int_{-\omega_o}^{\omega_o} \omega^2\Phi_g(\omega)d\omega \qquad (II.16)$$

To express I_{b2} in spectral terms, the relative velocity ground spectra, I_{2go}, can be approximated as follows

$$I_{2go} = \int_{-\omega_c}^{\omega_c} \omega^2\Phi_g(\omega)|H_o|^2 d\omega$$

$$\cong \Phi_g(\omega_o)\int_{-\omega_c}^{\omega_c} \omega^2|H_o|^2 d\omega - \Phi_g(\omega)\,\frac{2}{3\omega_o} + \frac{1}{\omega_o^4}\, I_{b2} \qquad \text{(II.17)}$$

where

$$\int_{-\omega_c}^{\omega_c} \omega_o^2\omega^2|H_o|^2 d\omega = \omega_c D_m \qquad \text{(II.18)}$$

The expression for D_m is given later by Eq.(II.31). Solving Eq.(II.17) for I_{b2}, we obtain

$$I_{b2} = \omega_o^4 I_{2go} - \omega_o^2\Phi_g(\omega_o)\omega_c\{D_m - 2/3\ r\} \qquad \text{(II.19)}$$

Finally, to obtain $\Phi_g(\omega_o)$ in terms of response spectrum values, the approximation for I_{1go} is used:

$$I_{1go} = \int_{-\omega_c}^{\omega_c} \Phi_g(\omega)|H_o|^2 d\omega$$

$$\simeq \Phi_g(\omega_o)\int_{-\omega_c}^{\omega_c} |H_o|^2 d\omega - \Phi_g(\omega_o)\,\frac{2}{\omega_o^3} + \frac{1}{\omega_o^4}\, I_b \qquad \text{(II.20)}$$

where

$$\omega_o^4 \int_{-\omega_c}^{\omega_c} |H_o|^2 d\omega = \omega_c E_m \qquad \text{(II.21)}$$

The expression for E_m is provided in Eq.(II.31). Replacing Eq.(II.14) into (II.19) and rearranging terms gives

$$\Phi_g(\omega_o) = \omega_o^4 I_{1go} F(\omega_o)/\{E_m - 2r\}\omega_c \qquad \text{(II.22)}$$

in which $r = \omega_o/\omega_c$.

Substituting Eqs.(II.11), (II.14), (II.19) and (II.22) into (II.10) and after some algebraic manipulation, we obtain

$$I_R(a_1,a_2) = \omega_o^4 I_{1go}[F[(\omega_o)C_m' + a_1\{1 - F(\omega_o)\}] + a_2\omega_o^2 I_{2go} D_m' \qquad \text{(3.38b)}$$

where C'_m and D'_m are as follows:

$$C'_m = [C_m - 2r(a_1 + a_2/3]/(E_m - 2r) \qquad \text{(II.23a)}$$

$$D'_m = 1 - (D_m - 2r/3)/(E_m - 2r) \qquad \text{(II.23b)}$$

CASE 2: Integral $I_S(a_2, a_3)$

$$I_3(a_2,a_3) = \int_{-\infty}^{\infty} (a_2\omega_o^6\omega^2 + a_3\omega_o^4\omega^4)|H_o|^4\Phi_g(\omega)d\omega \qquad \text{(II.24)}$$

Using Eq.(II.8), $I_S(a_2, a_3)$ can be approximated as follows

$$I_S(a_2,a_3) \simeq \Phi_g(\omega_o)\int_{-\infty}^{\infty} (a_2\omega_o^6\omega^2 + a_3\omega_o^4\omega^4)|H_o|^4d\omega - \Phi_g(\omega_o)2\omega_o(\frac{a_2}{3} + \frac{a_3}{5})$$

$$+ \frac{a_2}{\omega_o^2}\int_{-\omega_o}^{\omega_o} \omega^2\Phi_g(\omega)d\omega + \frac{a_3}{\omega_o^4}\int_{-\omega_o}^{\omega_o} \omega^4\Phi_g(\omega)d\omega \qquad \text{(II.25)}$$

It can be shown that the last integral in Eq.(II.25) is small in comparison with the other terms and can be neglected. Replacing the symbolic limits in the first integral in terms of the realistic limit with the cut-off frequency ω_c and evaluating it in closed form we obtain

$$\int_{-\omega_c}^{\omega_c} (a_2\omega_o^6\omega^2 + a_3\omega_o^4\omega^4)|H_o|^4d\omega = \omega_cF_m \qquad \text{(II.26)}$$

where F_m is defined by Eq.(II.31) later.

Replacing Eqs.(II.19), (II.22) and (II.26) into (II.25) gives Eq.(3.49b)

$$I_S(a_2,a_3) = \omega_o^4I_{1go}F(\omega_o)F'_m + a_2\omega_o^2I_{2go}D'_m \qquad \text{(3.49b)}$$

where F'_m is

$$F'_m = [F_m - 2r(a_2/3 + a_3/5)]/(E_m - 2r) \qquad (II.27)$$

CASE 3: Integral $I_E(a_4)$

$$I_E(a_4) = \int_{-\infty}^{\infty} a_4\omega_o^2\omega^6|H_o|^4 d\omega \qquad (II.28)$$

Using Eq.(II.8), $I_E(a_4)$ can be approximated as follows

$$I_E(a_4) \simeq \Phi_g(\omega_o)\int_{-\infty}^{\infty} a_4\omega_o^2\omega^6|H_o|^4 d\omega - \Phi_g(\omega_o)2\omega_o\frac{a_4}{7} + \frac{a_7}{\omega_o^6}\int_{-\omega_o}^{\omega_o}\omega^6\Phi_g(\omega)d\omega \qquad (II.29)$$

It can be shown that the last integral in Eq.(II.29) is small in comparison with the other terms and can be neglected. Replacing the limits in the first integral by the cut-off frequency ω_c and evaluating in closed form we obtain

$$\int_{-\omega_c}^{\omega_c} a_4\omega_o^2\omega^6|H_o|^4 d\omega = \omega_c G_m \qquad (II.30)$$

where G_m is defined by Eq.(II.31) later. Replacing Eqs.(II.22) and (II.30) into (II.29) gives Eq.(4.38a)

$$I_E(a_4) = I_{1go}\omega_o^4 F(\omega_o)(G_m - a_4\ 2r/7)/(E_m - 2r) \qquad (4.38b)$$

The functions C_m, D_m, E_m, F_m and G_m used earlier are defined as

$$C_m = A_m(r,\beta_o,a_1,a_2,0,0)$$

$$D_m = B_m(r,\beta_o,1,0)$$

$$E_m = B_m(r,\beta_o,0,1) \qquad (II.31)$$

$$F_m = A_m(r,\beta_o,0,a_2,a_3,0)$$

$$G_m = A_m(r,\beta_0,0,0,0,a_4)$$

in which the functions A_m and B_m are related to the following integrals

$$\omega_c A_m(r,\beta_0,a_1,a_2,a_3,a_4) =$$

$$= \int_{-\omega_c}^{\omega_c} (a_1\omega_0^8 + a_2\omega_0^6\omega^2 + a_3\omega_0^4\omega^4 + a_4\omega_0^2\omega^6)|H_0|^4 d\omega \quad \text{(II.32)}$$

$$\omega_c B_m(r,\beta_0,a,b) = \int_{-\omega_c}^{\omega_c} (a\omega_0^2\omega^2 + b\omega_0^4)|H_0|^2 d\omega \quad \text{(II.33)}$$

Equations (II.32) and (II.33) can be obtained in closed form to define A_m and B_m as follows:

$$A_m(r,\beta_0,a_1,a_2,a_3,a_4) =$$

$$= \frac{m_1}{2\beta_0 r}\,\theta_r + \frac{2N_1 + m_1}{2r\sqrt{1-\beta_0^2}}\,\ln\left\{\frac{(1+r^2) - 2r\sqrt{1-\beta_0^2}}{(1+r^2) + 2r\sqrt{1-\beta_0^2}}\right\}$$

$$- \frac{2N_2\{(1-r^2) + 2\beta_0^2 r^2\} - m_2/2(1+r^2)}{r^2(1-\beta_0^2)\,\{(1-r^2)^2 + 4\beta_0^2 r^2\}}$$

$$- \frac{2N_2 + M_2/2}{4r^3(1-\beta_0^2)\,\sqrt{1-\beta_0^2}}\,\ln\left\{\frac{1+r^2 - 2r\sqrt{1-\beta_0^2}}{1+r^2 + 2r\sqrt{1-\beta_0^2}}\right\} \quad \text{(II.34)}$$

with

$$N_1 = -(r^2/16\beta_0^2)[a_4(1-4\beta_0^2)+a_3+a_2+a_1(1+4\beta_0^2)]$$

$$N_2 = -(r^4/16\beta_0^2)[(a_4+a_1)(1-4\beta_0^2)+a_3+a_2]$$

$$m_1 = (r^2/16\beta_0^2)[(a_4+a_1)(1+4\beta_0^2)+a_3+a_2]$$

$$m_2 = r^4/2[2(1-2\beta_0^2)a_4+a_3-a_1] \quad \text{(II.35)}$$

$$M_1 = m_1/2i\beta_0 r$$

$$M_2 = m_2/4i\beta_0 r$$

and

$$Q_r = 2(\pi-\theta) \qquad r < 1$$
$$= \pi \qquad r = 1 \qquad \text{(II.36)}$$
$$= 2\theta \qquad r > 1$$

in which $\theta = \tan^{-1}(2\beta_0/|1-r|^2)$. And,

$B_m(r,\beta_0,a,b) =$

$$= \frac{m_3}{2\beta_0 r}\,\theta_r + \frac{2N_3+m_3}{2r\sqrt{1-\beta_0^2}}\,\ell n\left\{\frac{1+r^2 - 2r\sqrt{1-\beta_0^2}}{1+r^2 + 2r\sqrt{1-\beta_0^2}}\right\} \qquad \text{(II.37)}$$

where

$$m_3 = (a+b)r^2/r$$

$$N_3 = -br^2/2 \qquad \text{(II.38)}$$

NOTATION

$[A]$= Pseudo-static influence matrix defined by Eq.(2.54).

A,B,C,D= Coefficients of partial fraction.

C_d= Peak factor of the design response.

C_{dp}= Cross covariance between dynamic and pseudo-static parts of the response.

C_{dj}= Peak factor associated with auto displacement spectra.

C_{vj}= Peak factor associated with auto relative velocity spectra.

C_{vk}= Peak factor associated with the relative displacement of support k.

C_{uk}= Peak factor associated with the absolute displacement of support k.

C_{dg}= Peak factor for the ground displacement.

C_{vg}= Peak factor for the ground velocity.

$C_{dk\ell}(\omega_i)$= Coincident displacement spectra between floors k and ℓ defined by Eq.(2.43).

$C_{vk\ell}(\omega_i)$= Coincident velocity spectra between floors k and ℓ defined by Eq.(2.44).

$C_{pk\ell}(\omega_i)$= Coincident pseudo-acceleration spectra between floors k and ℓ defined by Eq.(2.47a).

$C_{dgk}(\omega_i)$= Coincident displacement spectra between ground and floor k defined by Eq.(4.9).

$C_{pgk}(\omega_i)$= Coincident pseudo-acceleration spectra between ground and floor k defined by Eq.(4.10).

$C_{vgk}(\omega_i)$= Coincident relative velocity spectra between ground and floor k defined Eq.(4.32).

C_m= Expression defined in Appendix II.

D_g= Maximum ground displacement.

D_m= Expression defined in Appendix II.

E[.]= Expected value of [.].

E_m= Expression defined in Appendix II.

F_m= Expression defined in Appendix II.

$F(\omega_i)$= Expression defined in Appendix II.

G_m= Expression defined in Appendix II.

H_i= Complex frequency response function defined by Eq.(2.18).

$h_i(t)$= Impulse response function.

$I_{ak\ell ij}$= Frequency integral defined in Eq.(2.20).

$I_{adk\ell i}$= Expression defined by Eq.(2.78) and (2.79).

I_{1ki}, I_{2ki}= Expressions defined by Eq.(2.34).

$I_{3k\ell i}, I_{4k\ell i}, I_{5k\ell i}, I_{6k\ell i}$= Expressions defined by Eq.(2.53).

$I_{4k\ell}, I_{6k\ell}$= Frequency integrals defined by Eqs.(2.81) and (2.82), respectively.

I_{1gr}, I_{2gr}= Expressions defined by Eq.(3.37).

$I_R(a_1, a_2)$= Frequency integral defined by Eq.(3.38).

I_{R1}, I_{R2}= Frequency integrals defined in Eq.(3.39).

$I_S(a_1, a_2)$= Frequency integral defined by Eq.(3.49).

I_{S1}, I_{S2}= Frequency integrals defined in Eq.(3.50).

I_{1g}, I_{2g}= Expressions defined by Eqs.(3.94) and (3.95), repectively.

$I_E(a_4)$= Frequency integral defined by Eq.(4.38).

$[M_{ss}], [C_{ss}], [K_{ss}]$= Mass, damping and stiffness matrices of the active degree-of-freedom of the secondary system.

$[M_{aa}]$, $[C_{aa}]$, $[K_{aa}]$= Mass, damping and stiffness matrices associated to support points of the secondary system.

$[M_{sa}]$, $[C_{sa}]$, $[K_{sa}]$= Mass, damping and stiffness cross matrices between active an support points.

$[M]$, $[C]$, $[K]$= Mass, damping and stiffness matrices of the supporting primary system.

$M(\omega)$= Expression defined by Eq.(2.37).

m= Number of support of the secondary system.

$N(\omega)$= Expression defined by Eq.(2.26).

N= Number of modes of the primary system.

n= Number of active degree-of-freedom of the secondary system.

n'= Number of modes of the secondary system included in the analysis.

$\{P_j\}$= j^{th} influence vetor defined in Eq.(2.15).

P_{ik}= k^{th} component of influence vector $\{P_j\}$.

P_{di}= Peak factor associated with coincident displacement spectra.

P_{vi}= Peak factor associated with coincident velocity spectra.

Q_{di}= Peak factor associated with quadrature displacement spectra.

Q_{vi}= Peak factor associated with quadrature velocity spectra.

$Q_{dk\ell}(\omega_i)$= Quadrature displacement spectra between floors k and ℓ defined by Eq.(2.45).

$Q_{vk\ell}(\omega_i)$= Quadrature velocity spectra between floors k and ℓ defined by Eq.(2.46).

$Q_{pk\ell}(\omega_i)$= Quadrature pseudo-acceleration spectra between floors k and ℓ defined by Eq.(2.47b).

$Q_{pgk}(\omega_i)$= Quadarture pseudo-acceleration spectra between ground and floor k defined by Eq.(4.25).

$Q_{vgk}(\omega_i)$= Quadarture relative velocity spectra between ground and floor k defined by Eq.(4.32).

$Q_{dgk}(\omega_i)$= Quadarture displacement spectra between ground and floor k defined by Eq.(4.24).

$q_j(t)$= j^{th} principal coordinates of the secondary system.

R_d= Maximum or design response defined by Eq.(2.11).

$R_s(t_1,t_2)$= Covariance function of the response S(t) defined by Eq.(2.12).

R_{dd}= Dynamic component of maximum or design response defined by Eq.(2.14).

R_{pp}= Pseudo-static component of maximum or design response defined by Eq.(2.53).

R_{dp}= Cross response of the maximum or design response defined in Eq.(2.70).

$R_{pk}(\omega_j)$= Auto pseudo-acceleration floor spectra defined by Eq.(2.22).

$R_{dk}(\omega_j)$= Auto displacement floor spectra defined by Eq.(2.22)

$R_{vk}(\ell_j)$= Auto relative velocity floor spectra defined by Eq.(2.32).

$R_{ak\ell}(t_1,t_2)$= Cross correlation function of the absolute acceleration between floors k and ℓ.

$R_{agk}(t_1,t_2)$= Cross correlation function of the absolute acceleration of ground and floor k.

$R_{dg}(\omega_r)$= Auto displacement ground spectra defined by

Eq.(3.34).

$R_{pg}(\omega_r)$= Auto pseudo-acceleration ground spectra defined by Eq.(3.34).

$R_{vg}(\omega_r)$= Auto relative velocity ground spectra defined by Eq.(3.41).

[r]= Dynamic influence matrix defined in Eq.(2.9).

S(t)= Response quantity.

$S^d(t)$= Dynamic part of the response S(t).

$S^p(t)$= Pseudo-static part of the response S(t).

{T}= Transformation vector defined in Eq.(6.7).

$T(\omega)$= Expression defined by Eq.(3.18).

$\{U_s(t)\}$= Absolute displacement vector of active degree-of-freedom of the secondary system.

$\{U_a(t)\}$= Absolute displacement vector of support points.

$\{U_s^d(t)\}$= Dynamic component of displacement of the active degree-of-freedom.

$U_{ak}(t)$= Absolute displacement of support k.

$\bar{U}_{ak}$= Maximum displacement of support k.

$V_{ak}(t)$= Relative (to ground) displacement of support k.

$\bar{V}_{ak}$= Maximum relative (to ground) displacement of support k.

V_g= Maximum ground velocity.

$X(\omega)$= Expression defined by Eq.(3.13).

$[Y_{ij}]$= Matrix of coefficients defined by Eq.(2.29).

$Y_r(t)$= r^{th} principal coordinate of the primary system.

$Z(\omega)$= Expression defined by Eq.(3.19).

$\Phi_{ak\ell}(\omega)$= Cross power spectral density function (PSDF) of the absolute acceleration at supports (or floors) k and ℓ.

$\Phi^{R}_{ak\ell}(\omega)$= Real part of the cross PSDF $\Phi_{ak\ell}(\omega)$.

$\Phi^{I}_{ak\ell}(\omega)$= Imaginary part of the cross PSDF $\Phi_{ak\ell}(\omega)$.

$\Phi_{dk\ell}(\omega)$= Cross PSDF of the displacement at support k and ℓ.

$\Phi_{vk\ell}(\omega)$= Cross PSDF of the relative displacement of supports k and ℓ.

$\Phi_{adk\ell}(\omega)$= Cross PSDF of the absolute acceleration of floor k and absolute displacement of floor ℓ.

$\Phi_{dak\ell}(\omega)$= Cross PSDF of the absolute displacement of floor k and absolute acceleration of floor ℓ.

$\Phi_{g}(\omega)$= PSDF of the ground input.

$\Phi_{agk}(\omega)$= Cross PSDF of the absolute acceleration of ground and floor k.

$\Phi_{akg}(\omega)$= Cross PSDF of the absolute acceleration of floor k and ground.

$\Phi_{dgk}(\omega)$= Cross PSDF of the absolute displacement of ground and floor k.

$\Phi_{dkg}(\omega)$= Cross PSDF of the absolute displacement of floor k and ground.

$\Phi^{R}_{agk}(\omega)$= Real part of $\Phi_{agk}(\omega)$.

$\Phi^{I}_{agk}(\omega)$= Imaginary part of $\Phi_{agk}(\omega)$.

$\Phi^{R}_{dgk}(\omega)$= Real part of $\Phi_{dgk}(\omega)$.

$\Phi^{I}_{dgk}(\omega)$= Imaginary part of $\Phi_{dgk}(\omega)$.

σ_{s}= Variance of the response S(t).

σ_{dd}= Variance of the dynamic part of the response $S^{d}(t)$.

σ_{pp}= Variance of the pseudo-static part of the response $S^{p}(t)$.

σ_{pd}, σ_{dp}= Cross variance of the dynamic and pseudo-static response.

$[\delta]$= Correlation coefficient matrix of the absolute displacement of the supports.

$\delta_{k\ell}$= Element of matrix $[\delta]$.

$[\delta']$= Correlation coefficient matrix of the relative displacement of the supports.

$\delta'_{k\ell}$= Element of matrix $[\delta']$.

$\delta''_{k\ell}$= Correlation coefficient associated with the coincident velocity cross floor spectra defined by Eq.(3.107).

$\delta'''_{k\ell}$= Correlation coefficient associated with the quadrature velocity cross floor spectra defined by Eq.(3.111).

δ'_{gk}= Correlation coefficient between relative displacement of floor k and ground displacement.

δ_{gk}= Correlation coefficient between absolute displacement of floor k and ground displacement defined by Eq.(4.41).

δ''_{gk}= Correlation coefficient associated with the coincident velocity cross floor spectra defined by Eq.(4.49).

δ'''_{gk}= Correlation coefficient associated with the quadrature velocity cross floor spectra defined by Eq.(4.53).

β_j, ω_j= j^{th} damping ratio and natural frequency of the secondary system, respectively.

β_r, ω_r= r^{th} damping ratio and natural frequency of the primary system, respectively.

ρ_j= Modal response in the j^{th} mode.

η_k= Constraint response associated with support k.

γ_r= r^{th} mode participation factor of the primary system defined by Eq.(3.3).

$\{\psi_r\}$= r^{th} mode shape vector of primary system.

$\{\Psi_j\}$= j^{th} modal shape vector of the secondary system.

$\{1\}$= Excitation influence vector.

1_k= k^{th} component of vector $\{1\}$.