SEISMIC RESPONSE OF MULTIPLY SUPPORTED SECONDARY SYSTEMS WITH DYNAMIC INTERACTION EFFECTS

by

R. A. Burdisso and M. P. Singh

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16. Abstracts For a proper response analysis of a secondary system with multiple supports, the seismic inputs are required to be defined in terms of the auto and cross floor response spectra. If no feed-back or interaction effect from the secondary system to its supporting primary structure is suspected, these inputs can be developed by a direct analysis of the supporting structure alone. However, sometimes the effect of the interaction on the secondary system response can be quite significant. Herein, a method is developed to incorporate the feed-back effect, through proper modifications of the interaction-free floor spectrum inputs. The interaction coefficients are used to effect such modifications in different floor spectral quantities. A procedure for the calculation of the interaction coefficients is proposed. The modified floor spectra when used as inputs to the secondary system do introduce the interaction effect in the secondary system response. A successful application of this method is demonstrated by numerical examples of the secondary systems with three different secondary-to-primary system mass ratios.

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CHAPTER I

INTRODUCTION

1.1 GENERAL BACKGROUND

Currently for the seismic analysis of the secondary systems with multiple support such as pipings in industrial units, the seismic input are defined in terms of the acceleration floor response spectra. An acceleration floor spectrum is a plot of the maximum (pseudo) acceleration response of an oscillator, subjected to the motion of the floor, versus the oscillator frequency. However, in the analysis of multiple support secondary systems, the characterization of the support motion in terms of the acceleration floor response spectrum alone is not adequate; it is also important to know the correlation between the motions of various support. To characterize this correlation, the concept of the cross floor spectra has recently been introduced [1-6]. Thus, a more complete description of the seismic input for the multiple support secondary system requires the definition of the auto as well as the cross floor response spectra. Recently [3], a response spectrum method has also been developed which can use these floor response spectra directly as the seismic inputs in the calculation of the secondary system response.

The analytical methods have also been developed [4] to generate these floor response spectra directly from the ground response spectra. In generation of the floor spectra, often the primary and secondary systems are assumed to be decoupled [4,7] with no interaction with each other. In particular, the feed-back from the secondary system to the primary system is usually ignored. This is acceptable for most secondary systems which are very light compared to their supporting primary systems. However, if a secondary system is not light relative to the primary system and if one or more of its frequencies are close to the frequencies of the primary system, the effect on the response of the dynamic interaction between the two system can be significant. In such cases it is necessary that this effect be included in the response analysis.

One way to include this dynamic interaction is to prepare a combined analytical model of the two systems for analysis. This approach is, however, not practical because of several reasons. Firstly, it increases the size of the problem. Secondly, because of the large differences in the magnitudes of the mass and stiffness properties of the two systems, the combined analytical model is likely to be ill-conditioned inasmuch as its analysis may introduce numerical erros unless an extended numerical precision is used. Also, a change in the properties or the layout of any of the two systems will necessitate a complete reanalysis of the combined model. As such this approach of combined analysis is rarely used in practice.

Another alternative approach is to synthesize the modal properties of the two systems to obtain the modal properties of the combined system [8,9]. However, if one wants to use the floor response spectra as the inputs in the analysis of the secondary system, the effect of the interaction can also be incorporated in generation of floor response spectra. To do this, the floor spectra obtained with no interaction, or the interaction-free spectra, can be modified to include the interaction effect. Such modified floor spectra when used as the inputs in the analysis of the secondary system will then introduce the effect of the dynamic interaction in the calculated response. Herein this approach has been adopted.

The equations of motion which explicitly show the interaction terms are developed in Chapter II. The interaction terms appear as the interface forces at the supports in the equations of motion of the primary structure. The total response of the secondary system is partitioned into the pseudo-static and dynamic components. The pseudo-static component is due to the displacement of the supports, whereas the dynamic component is due to the inertial forces caused by the acceleration of the supports. The interaction forces appearing in the equations of motion of the primary system are defined in terms of the dynamic component of the response. The coupled equations are then analysed to obtain the floor accelerations and floor spectral density functions. The floor acceleration obtained with interaction effect are expressed in terms of the accelerations without interaction effect. Interaction coefficients are then introduced to incorporate the interaction effect in generation of various auto and cross floor response spectra. A methodology to obtain these coefficients is developed.

Chapter III entitled User Summary of the Proposed Method describes the step-by-step procedure for implementation of the approach. This chapter is provided for a user who is primarily interested in the implementation of the approach rather than in its analytical development. The numerical results showing the application of this procedure are presented in Chapter IV, followed by Chapter V, Summary and Conclusions.

CHAPTER II

INTERACTION EFFECT BETWEEN PRIMARY AND SECONDARY SYSTEMS

2.1 INTRODUCTION

In a previous report [10], it was assumed that the motion propagates from the primary to the secondary system and not backwards. This implies that the secondary system does not interact with the primary system. This assumption is justified for very light secondary systems compared to the supporting structure as well as when there are no secondary system modes tuned to the primary system modes. However, when the secondary system is relatively heavy or when one or more of its modes are tuned to one or more modes of the primary system, the assumption of decoupling could lead to erroneous results.

In this chapter, the effect of the dynamic interaction between the primary and the secondary systems, also called the feed-back effect, is studied. The equations of motion for the primary and secondary subsystems incorporating the dynamic interaction terms are developed and the modifications in the response introduced by the coupling effect are evaluated.

As in References [3,10], the response of the secondary system is split into the dynamic and pseudo-static part. The pseudo-static part is due to the relative displacement of the support with no inertial force effects, whereas the dynamic part is due to the acceleration of the secondary masses caused by the accelerations of its supports. Here, a formulation is developed to evaluate the feed-back effect induced in the primary system by these two parts of the secondary system response.

The feed-back effect caused by the pseudo-static component of the secondary system response manifests itself through the changes in the mass, damping and stiffness matrices of the primary system. This effect, therefore, can be included by proper modification of these matrices of the primary system. The feed-back effect caused by the dynamic component of the response, on the other hand, introduces an additional force input in the equations of motion of the primary system; this force input is, of course, defined in terms of the dynamic response of the secondary system and is applied at the support points where the secondary system is connected to the primary structure. The equations of motion for the secondary system, on the other hand, remain unchanged. These coupled equations of the primary and secondary systems must be solved to include the dynamic interaction effect. Such solutions are difficult to obtain. Herein, a concept of the interaction factor is introduced to bring in the effect of dynamic interaction in generation of floor spectra defined in the previous chapters. The interaction factors are defined for each spectral quantity in terms of the primary and secondary systems properties. These factors are used to modify the interaction-free spectral inputs obtained by the methods developed in References [4,10]. Once the new floor inputs with interaction are obtained, the formulation presented by Burdisso and Singh [3] can be directly used for calculating the maximum or design response for any quantity of interest in the secondary system.

2.2 COUPLED EQUATIONS OF MOTION

To evaluate the interaction effect between the primary and the secondary systems, it is necessary to consider their coupled equations of motion. The combined equations of motion for multiple degree of freedom primary and secondary systems can be written as follows:

$$[M]\{\ddot{V}\} + [C]\{\dot{V}\} + [K]\{V\} = -[M]\{1\}\ddot{X}_g(t) \tag{2.1}$$

in which $\ddot{X}_g(t) =$ the ground acceleration input. The mass, damping and stiffness matrices in Eq.(2.1) have the following form:

$$[M] = \begin{bmatrix} M_{pp} & M_{pa} & 0 \\ M_{ap} & \hat{M}_{aa} & m_{as} \\ 0 & m_{sa} & m_{ss} \end{bmatrix} \;;\; [C] = \begin{bmatrix} C_{pp} & C_{pa} & 0 \\ C_{ap} & \hat{C}_{aa} & c_{as} \\ 0 & c_{sa} & c_{ss} \end{bmatrix} \;;\; [K] = \begin{bmatrix} K_{pp} & K_{pa} & 0 \\ K_{ap} & \hat{K}_{aa} & k_{as} \\ 0 & k_{sa} & k_{ss} \end{bmatrix} \tag{2.2}$$

The relative displacement vector $\{V\}$ and the excitation influence vector $\{1\}$ are given by

$$\{V\}' = \left(\{V_p\}',\{V_a\}',\{V_s\}'\right) \tag{2.3}$$

$$\{1\}' = \left(\{1_p\}',\{1_a\}',\{1_s\}'\right) \tag{2.4}$$

The subscripts p and s are associated with the primary and secondary degrees of freedom, respectively. The susbscript a is related to the attachement points common to both subsystems. The submatrices in Eq.(2.2) related to the primary and the secondary systems are represented by capital and lower case letters, respectively. The submatrices $[\hat{M}_{aa}]$, $[\hat{C}_{aa}]$ and $[\hat{K}_{aa}]$ result from the contribution of both the primary and the secondary substructures and are, thus, defined as

$$\begin{aligned} [\hat{M}_{aa}] &= [M_{aa}] + [m_{aa}] \\ [\hat{C}_{aa}] &= [C_{aa}] + [c_{aa}] \\ [\hat{K}_{aa}] &= [K_{aa}] + [k_{aa}] \end{aligned} \tag{2.5}$$

Considering Eqs.(2.2) through (2.5), Eq.(2.1) can be split into the following equations:

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{V}_p \\ \ddot{V}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa} \end{bmatrix} \begin{Bmatrix} \dot{V}_p \\ \dot{V}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix} \begin{Bmatrix} V_p \\ V_a \end{Bmatrix} = - \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} 1_p \\ 1_a \end{Bmatrix} \ddot{X}_g(t)$$

$$- \begin{Bmatrix} \{0\} \\ [m_{aa}]\{\ddot{U}_a\} + [m_{as}]\{\ddot{U}_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [c_{aa}]\{\dot{V}_a\} + [c_{as}]\{\dot{V}_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [k_{aa}]\{V_a\} + [k_{as}]\{V_s\} \end{Bmatrix} \qquad (2.6)$$

and

$$[m_{ss}]\{\ddot{V}_s\} + [c_{ss}]\{\dot{V}_s\} + [k_{ss}]\{V_s\} = -[m_{ss}]\{1_s\}\ddot{X}_g(t)$$

$$-[m_{sa}]\{\ddot{U}_a\} - [c_{sa}]\{\dot{V}_a\} - [k_{sa}]\{V_a\} \qquad (2.7)$$

where $\{\ddot{U}_a\}$ is the absolute acceleration vector of the attachment points and $\{\ddot{U}_s\}$ is the absolute acceleration vector of the secondary degrees of freedom.

Since a rigid body motion of the secondary system cannot induce either elastic or damping forces, the stiffness and damping matrices of the secondary system satisfy the following relationship:

$$\begin{bmatrix} k_{aa} & k_{as} \\ k_{sa} & k_{ss} \end{bmatrix} \begin{Bmatrix} 1_a \\ 1_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (2.8a)$$

$$\begin{bmatrix} c_{aa} & c_{as} \\ c_{sa} & c_{ss} \end{bmatrix} \begin{Bmatrix} 1_a \\ 1_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (2.8b)$$

In view of Eq.(2.8), Eqs.(2.6) and (2.7) can be written as follows (See Appendix A):

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{V}_p \\ \ddot{V}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa} \end{bmatrix} \begin{Bmatrix} \dot{V}_p \\ \dot{V}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix} \begin{Bmatrix} V_p \\ V_a \end{Bmatrix} = - \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} 1_p \\ 1_a \end{Bmatrix} \ddot{X}_g(t)$$

$$- \begin{Bmatrix} \{0\} \\ [m_{aa}]\{\ddot{U}_a\} + [m_{as}]\{\ddot{U}_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [c_{aa}]\{\dot{U}_a\} + [c_{as}]\{\dot{U}_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [k_{aa}]\{U_a\} + [k_{as}]\{U_s\} \end{Bmatrix} \qquad (2.9)$$

and

$$[m_{ss}]\{\ddot{U}_s\} + [c_{ss}]\{\dot{U}_s\} + [k_{ss}]\{U_s\} = -[m_{sa}]\{\ddot{U}_a\} - [c_{sa}]\{\dot{U}_a\} - [k_{sa}]\{U_a\} \tag{2.10}$$

The secondary response is partitioned into the pseudo-static and dynamic components as follows

$$\{U_s(t)\} = \{U_s^p(t)\} + \{U_s^d(t)\} \tag{2.11}$$

in which the subscripts d and p refer to dynamic and pseudo-static components, respectively.

The pseudo-static part is due to the support points′ motion without any induced dynamic effect. Thus, the pseudo-static displacement vector is linearly related to the displacements of the attachment degrees of freedom through the following transformation

$$\{U_s^p\} = -[k_{ss}]^{-1}[k_{sa}]\{U_a\} \tag{2.12}$$

Replacing Eq.(2.12) into (2.9), and after some algebraic manipulations involving Eq.(2.8), we obtain (See Appendix A)

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & \tilde{M}_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{V}_p \\ \ddot{V}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & \tilde{C}_{aa} \end{bmatrix} \begin{Bmatrix} \dot{V}_p \\ \dot{V}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & \tilde{K}_{aa} \end{bmatrix} \begin{Bmatrix} V_p \\ V_a \end{Bmatrix} = -\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & \tilde{M}_{aa} \end{bmatrix} \begin{Bmatrix} 1_p \\ 1_a \end{Bmatrix} \ddot{X}_g(t) - \{F\} \tag{2.13}$$

where the submatrices $[\tilde{M}_{aa}]$, $[\tilde{C}_{aa}]$ and $[\tilde{K}_{aa}]$ are defined as follows:

$$\begin{aligned} [\tilde{M}_{aa}] &= [M_{aa}] + [m_{aa}] - [m_{as}][k_{ss}]^{-1}[k_{sa}] \\ [\tilde{C}_{aa}] &= [C_{aa}] + [c_{aa}] - [c_{as}][k_{ss}]^{-1}[k_{sa}] \\ [\tilde{K}_{aa}] &= [K_{aa}] + [k_{aa}] - [k_{as}][k_{ss}]^{-1}[k_{sa}] \end{aligned} \tag{2.14}$$

and the vector $\{F(t)\}$ is given by

$$\{F(t)\} = \begin{Bmatrix} \{0\} \\ [m_{as}]\{\ddot{U}_s^d\} + [c_{as}]\{\dot{U}_s^d\} + [k_{as}]\{U_s^d\} \end{Bmatrix} \tag{2.15}$$

Similarly, substituting Eq.(2.12) into (2.10), we obtain

$$[m_{ss}]\{\ddot{U}_s^d\} + [c_{ss}]\{\dot{U}_s^d\} + [k_{ss}]\{U_s^d\} = ([m_{ss}][k_{ss}]^{-1}[k_{sa}] - [m_{sa}])\{\ddot{U}_a\} +$$

$$([c_{ss}][k_{ss}]^{-1}[k_{sa}] - [c_{sa}])\{\dot{U}_a\} \tag{2.16}$$

If the damping matrix of the secondary system is assumed proportional to the stiffness matrix, the term related to $\{\dot{U}_a\}$ in Eq.(2.16) vanishes. Even in a more general case the magnitude of this term associated with damping forces will be small in comparison with the elastic term. Thus, in any case it can be neglected without causing much error. Therefore, Eq.(2.16) can be written as

$$[m_{ss}]\{\ddot{U}_s^d\} + [c_{ss}]\{\dot{U}_s^d\} + [k_{ss}]\{U_s^d\} = [\,r\,]\{\ddot{U}_a\} \tag{2.17}$$

where $[\,r\,] = ([m_{ss}][k_{ss}]^{-1}[k_{sa}] - [m_{sa}])$ is the dynamic influence matrix, the same as defined in References [3,10]

To study the coupling effect, Eqs.(2.13) and (2.17) should be compared with the corresponding equations obtained without considering the dynamic interaction effect. The interaction-free equations of motion for the primary system has the form

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix}\begin{Bmatrix} \ddot{V}_p \\ \ddot{V}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa} \end{bmatrix}\begin{Bmatrix} \dot{V}_p \\ \dot{V}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix}\begin{Bmatrix} V_p \\ V_a \end{Bmatrix} = -\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix}\begin{Bmatrix} 1_p \\ 1_a \end{Bmatrix}\ddot{X}_g(t) \tag{2.18}$$

and the corresponding interaction-free equations of motion for the fixed-base secondary system is given as follows (See References [3,10])

$$[m_{ss}]\{\ddot{U}_s^d\} + [c_{ss}]\{\dot{U}_s^d\} + [k_{ss}]\{U_s^d\} = [\,r\,]\{\ddot{U}_a\} \tag{2.19}$$

Comparison of Eq.(2.19) with (2.17) shows that the equation of motion for the dynamic component of the response of the secondary system is the same in both cases. This suggests that once the floor inputs without interaction effect are modified to include the dynamic interaction, the procedure developed by Burdisso and Singh [3] can be directly used, without modification, to obtain the secondary system response. That is, if the floor inputs have been modified to include the dynamic interaction, then the calculated response will also include the dynamic interaction effect.

The comparison of Eqs.(2.18) and (2.13), on the other hand, shows that the interaction effect manifests itself through (1) the modification of the primary system matrices and (2) an introduction of the forcing function vector in Eq.(2.13). These changes are due to both the pseudo-static and the dynamic components of the secondary response. The effect associated with the pseudo-static part appears through the changes in matrices $[\tilde{M}_{aa}]$, $[\tilde{C}_{aa}]$ and $[\tilde{K}_{aa}]$, as defined in Eq.(2.14), whereas the effect associated with the dynamic part of the secondary system response is reflected in the interaction force vector $\{F(t)\}$ defined by Eq.(2.15).

The interaction force vector represents the induced reaction forces, caused by the dynamic response of the secondary system, to be applied to the primary structure at the attachment points. It consists of elastic, damping and inertial forces. If we assume a lumped mass matrix for the secondary system with matrix $[m_{as}] = [\,0\,]$ the inertial component of this force vector vanishes. The evaluation of the damping force component will require explicit information about the cross damping matrix $[c_{as}]$. Since the energy dissipative characteristic of a system is usually known in terms of the damping ratios , it is hard to define the matrix $[c_{as}]$. However, since the damping force component will be relatively small in comparison with the elastic component in any case, this interaction effect can be neglected from further considerations. With these assumptions, the interaction force vector can be written as

$$\{F(t)\} = \left\{ \begin{matrix} \{0\} \\ [k_{as}]\{U_s^d\} \end{matrix} \right\} \tag{2.20}$$

To investigate the effect of the coupling between the primary and the secondary systems we will now consider the two sets of coupled differential equations, Eqs.(2.13) and (2.17), simultaneously in the following section.

For the sake of clarity of the presentation, the equation of motion, Eq.(2.18), is referred to as the original or unperturbed equation, and Eq.(2.13) which includes the coupling effect is referred to as the modified or perturbed equation. Similarly, any characteristic like frequencies, mode shapes, etc. associated with Eqs.(2.18) and (2.13) are also referred to as unperturbed and perturbed, respectively.

2.3 ACCELERATION OF THE SUPPORTS

Assuming that both primary and secondary systems are classically damped, Eqs.(2.13) and (2.17) can be decoupled, by employing the modal analysis approach, into a set of modal equations as follows:

$$\ddot{Y}_r(t) + 2\,\tilde{\beta}_r\,\tilde{\omega}_r\,\dot{Y}_r(t) + \tilde{\omega}_r^2\,Y_r(t) = -\,\tilde{\gamma}_r\,\ddot{X}_g(t) - \{\tilde{\Psi}_r\}'\{F\} \tag{2.21}$$

$$\ddot{q}_j(t) + 2\,\beta_j\,\omega_j\,\dot{q}_j(t) + \omega_j^2\,q_r(t) = \{P_j\}'\{\ddot{U}_a\} \tag{2.22}$$

where $Y_r(t)$ = r^{th} modified principal coordinate; $\tilde{\omega}_r$ and $\tilde{\beta}_r$ are the r^{th} modified modal frequency and damping ratio, respectively; $\tilde{\gamma}_r = (\{\tilde{\Psi}_r\}'[\tilde{M}]\{1\})/(\{\tilde{\Psi}_r\}'[\tilde{M}]\{\tilde{\Psi}_r\})$ = r^{th} modified participation factor; $[\tilde{M}]$ = modified mass matrix; and $\{\tilde{\Psi}_r\}$ = r^{th} modified mode shape vector, all belonging to the primary structure. Similarly for the secondary system we have, $q_j(t)$ = j^{th} principal coordinate; ω_j and β_j = the j^{th} modal frequency and damping ratio, respectively. $\{P_j\}$ = j^{th} influence vector = $\{\psi_j\}'[r]$; and $\{\psi_j\}$ = j^{th} mode shape vector. A prime (′) over a vector quantity represents its transpose. In general the subscript r will be associated with the quantities belonging to the primary structure and j with the quantities belonging to the secondary structure.

The modal properties of the modified or perturbed primary equation of motion can be obtained by direct eigenvalue analysis of Eq.(2.13) with new mass and stiffness matrices. However, if the original unperturbed eigenproperties of the primary system are known, these can also be used to obtain the new properties by a perturbation approach. In Appendix B a first order perturbation approach is presented based on Reference [11] to obtain these modal properties. That is, to obtain the dynamic characteristics of the modified primary system, a new eigenvalue analysis is not necessary. This is especially helpful if several different configurations of secondary systems are to be analyzed or investigated.

If the absolute accelerations of the support points are known, Eq.(2.22) can be solved to obtain the generalized coordinate $q_j(t)$. Using the expansion theorem, the dynamic part of the displacement vector can be obtained in terms of $q_j(t)$ as follows:

$$\{U_s^d(t)\} = \sum_{j=1}^{n} q_j(t) \{\psi_j\} \tag{2.23}$$

where n = number of modes of the secondary system included in the response. Replacing Eq.(2.23) into Eq.(2.20) and then substituting in Eq.(2.21), we obtain

$$\ddot{Y}_r(t) + 2 \tilde{\beta}_r \tilde{\omega}_r \dot{Y}_r(t) + \tilde{\omega}_r^2 Y_r(t) = - \tilde{\gamma}_r \ddot{X}_g(t) - \sum_{j=1}^{n} C_{rj} q_j(t) \tag{2.24}$$

where C_{rj} is defined as

$$C_{rj} = \{\tilde{\Psi}_r\}' \begin{bmatrix} [0] \\ [k_{as}] \end{bmatrix} \{\psi_j\} \tag{2.25}$$

Eq.(2.24) can be transformed into the frequency domain by applying the Fourier Transform (FT) as

$$(- \omega^2 + i\, 2 \tilde{\beta}_r \tilde{\omega}_r \omega + \tilde{\omega}_r^2) Y_r(\omega) = - \tilde{\gamma}_r \ddot{X}_g(\omega) - \sum_{j=1}^{n} C_{rj} q_j(\omega) \tag{2.26}$$

or

$$Y_r(\omega) = \tilde{Y}_r(\omega) - \tilde{H}_r \sum_{j=1}^{n} C_{rj} q_j(\omega) \tag{2.27}$$

in which $\tilde{H}_r$ = the r^{th} modal frequency response function of the modified primary system, $\tilde{H}_r = 1/(\tilde{\omega}_r^2 - \omega^2 + i\, 2 \tilde{\beta}_r \tilde{\omega}_r \omega)$. In Eq.(2.27), $\tilde{Y}_r(\omega)$ is the FT of the principal coordinate of the perturbed primary system obtained when the interaction force $\{F(t)\}$, in Eq.(2.13), is set equal to zero. Thus, $\tilde{Y}_r(\omega)$ is given by

$$\tilde{Y}_r(\omega) = - \tilde{H}_r \tilde{\gamma}_r \ddot{X}_g(\omega) \tag{2.28}$$

It is noted that Eq.(2.28) includes the interaction effect due to the pseudo-static part of the secondary system response. Henceforth, the superscript ($\sim$) will represent a response quantity obtained by neglecting the interaction forces at the interface but including the interaction effect due to the pseudo-static part of the secondary response.

In the frequency domain, the absolute acceleration of the attachment points at the floor k can be written as the sum of the relative acceleration of the floor and the ground acceleration as follows:

$$\ddot{U}_{ak}(\omega) = \ddot{V}_{ak}(\omega) + 1_k \ddot{X}_g(\omega) \tag{2.29}$$

where $\ddot{V}_{ak}(\omega)$ = FT of the relative acceleration of the attachment point at floor k; $\ddot{X}_g(\omega)$ = FT of the ground acceleration; and 1_k = excitation influence factor at floor k. Substituting in Eq.(2.29) for the relative acceleration, expressed in terms of the generalized coordinates $Y_r(\omega)$ by the expansion theorem, we obtain

$$\ddot{U}_{ak}(\omega) = \sum_{r=1}^{N} \tilde{\Psi}_r(k)\, \ddot{Y}_r(\omega) + 1_k \ddot{X}_g(\omega) \tag{2.30}$$

where N = the number of primary modes included in the response. In the frequency domain, the generalized acceleration $\ddot{Y}_r(\omega)$ is related to the displacement $Y_r(\omega)$ as follows:

$$\ddot{Y}_r(\omega) = - \omega^2 Y_r(\omega) \tag{2.31}$$

By substituting $Y_r(\omega)$ from Eq.(2.27) into (2.31) and then into (2.30), we obtain

$$\ddot{U}_{ak}(\omega) = \tilde{\ddot{U}}_{ak}(\omega) + \sum_{r=1}^{N} \tilde{\Psi}_r(k)\, \omega^2 \tilde{H}_r \sum_{j=1}^{n} C_{rj}\, q_j(\omega) \tag{2.32a}$$

where

$$\tilde{\ddot{U}}_{ak}(\omega) = \sum_{r=1}^{N} \tilde{\Psi}_r(k)\, \omega^2\, \tilde{H}_r\, \tilde{\gamma}_r\, \ddot{X}_g(\omega) + 1_k\, \ddot{X}_g(\omega) \tag{2.32b}$$

is the FT of the absolute acceleration of the floor k when the interaction force vector is neglected.

To solve Eq.(2.32a), the generalized coordinates of the secondary system, $q_j(\omega)$, need to be known. This can be obtained from the solution of Eq.(2.22). Taking the FT of Eq.(2.22), $q_j(\omega)$ can be expressed as a linear combination of the absolute acceleration of the support points where the secondary system is attached. That is,

$$q_j(\omega) = H_j \sum_{j=1}^{m} P_{jl}\, \ddot{U}_{al}(\omega) \tag{2.33}$$

where H_j = the frequency response function of the j^{th} secondary mode; P_{jl} = l^{th} element of vector $\{P_j\}$; and $\ddot{U}_{al}(\omega)$ = FT of the absolute acceleration of the attachment point at the floor l; and m = number of support points. Substituting Eq.(2.33) into (2.32a) and rearranging the terms, we obtain

$$\tilde{\ddot{U}}_{ak}(\omega) = \ddot{U}_{ak}(\omega) - \sum_{r=1}^{N} \tilde{\Psi}_r(k)\, \omega^2\, \tilde{H}_r \sum_{j=1}^{n} C_{rj}\, H_j \sum_{l=1}^{m} P_{jl}\, \ddot{U}_{al}(\omega) \tag{2.34}$$

Similar equations, obtained for each floor where the support points are located, can be assembled into the following matrix equation:

$$\{\tilde{\ddot{U}}_a(\omega)\} = [E(\omega)]\, \{\ddot{U}_a(\omega)\} \tag{2.35}$$

in which $[E(\omega)]$ is a complex frequency-dependent matrix of dimension equal to the number of supports m. The elements of this matrix are defined as follows:

$$E_{kk} = 1 - \sum_{r=1}^{N} \tilde{\Psi}_r(k)\, \omega^2\, \tilde{H}_r \sum_{j=1}^{n} C_{rj}\, H_j\, P_{jk} \tag{2.36a}$$

$$E_{kl} = - \sum_{r=1}^{N} \tilde{\Psi}_r(k)\, \omega^2\, \tilde{H}_r \sum_{j=1}^{n} C_{rj}\, H_j\, P_{jl} \qquad , \quad k \neq l \tag{2.36b}$$

For a harmonic ground input, matrix $[E(\omega)]$ relates the phases and magnitudes of the support accelerations obtained with and without the interaction effect due to the dynamic part of the secondary system response.

For the special case of a secondary system supported on ground, we assume that the absolute acceleration of this support is not changed by the dynamic interaction. Thus, the diagonal element in matrix $[E(\omega)]$ associated with this support point is set equal to one and its corresponding row is set equal to zero.

Inverting matrix $[E(\omega)]$ in Eq.(2.35), we obtain

$$\{\ddot{U}_a(\omega)\} = [E(\omega)]^{-1}\{\tilde{\ddot{U}}_a(\omega)\} = [G(\omega)]\{\tilde{\ddot{U}}_a(\omega)\} \tag{2.37}$$

From Eq.(2.37), we note that the absolute acceleration of a support point with interaction, $\ddot{U}_{ak}(\omega)$, is given as a linear combination of the absolute acceleration of all the supports without dynamic interaction. That is,

$$\ddot{U}_{ak}(\omega) = \sum_{u=1}^{m} G_{ku}(\omega)\, \tilde{\ddot{U}}_{au}(\omega) \tag{2.38}$$

where coefficients $G_{ku}(\omega)$ are the elements of matrix $[G(\omega)]$.

2.4 SPECTRAL DENSITY FUNCTIONS OF FLOOR ACCELERATIONS

Eq.(2.38) can be used to obtain the auto and cross power spectral density function (PSDF) of the absolute acceleration with interaction for floor k in terms of the auto and cross PSDF of the absolute floor accelerations without interaction as follows:

$$\Phi_{akl}(\omega) = \sum_{u=1}^{m} \sum_{v=1}^{m} G_{ku}^{*}(\omega)\, G_{lv}(\omega)\, \tilde{\Phi}_{auv}(\omega) \tag{2.39}$$

where $\tilde{\Phi}_{auv}(\omega)$ represents the cross PSDF of the absolute accelerations for floors u and v obtained by setting the interaction force vector, $\{F(t)\}$, to zero in Eq.(2.13). An asterisk over a quantity denotes its complex conjugate. Splitting Eq.(2.39) into its real and imaginary parts, we obtain

$$\Phi_{akl}^{R}(\omega) = \sum_{u=1}^{m} \sum_{v=1}^{m} \left[T_1\, \tilde{\Phi}_{auv}^{R}(\omega) - T_2\, \tilde{\Phi}_{auv}^{I}(\omega) \right] \tag{2.40}$$

$$\Phi_{akl}^{I}(\omega) = \sum_{u=1}^{m} \sum_{v=1}^{m} \left[T_2\, \tilde{\Phi}_{auv}^{R}(\omega) + T_1\, \tilde{\Phi}_{auv}^{I}(\omega) \right] \tag{2.41}$$

where superscripts R and I refer to the real and imaginary parts. Coefficients T_1 and T_2 are defined as follows:

$$\begin{aligned} T_1 &= G_{ku}^{R}(\omega)\, G_{lv}^{R}(\omega) + G_{ku}^{I}(\omega)\, G_{lv}^{I}(\omega) \\ T_2 &= G_{ku}^{R}(\omega)\, G_{lv}^{I}(\omega) - G_{ku}^{I}(\omega)\, G_{lv}^{R}(\omega) \end{aligned} \tag{2.42}$$

The auto PSDF of the absolute acceleration for floor k is obtained from Eq.(2.40) by setting $k = l$. Because the imaginary part of the cross PSDF is an odd function, the auto PSDF takes the special form

$$\Phi_{akk}(\omega) = \sum_{u=1}^{m} \sum_{v=1}^{m} \left[G_{ku}^{R}(\omega)\, G_{kv}^{R}(\omega) + G_{ku}^{I}(\omega)\, G_{kv}^{I}(\omega) \right] \tilde{\Phi}_{auv}^{R}(\omega) \tag{2.43}$$

Eqs.(2.40), (2.41) and (2.43) can now be used to obtain the auto and cross floor spectra which include the effect of dynamic interaction between the primary and the secondary systems.

2.5 INTERACTION COEFFICIENTS FOR FLOOR SPECTRA

In principle, knowing the spectral density function of the floor acceleration from Eq.(2.43), we can now define the relative displacement auto floor spectra with interaction effect for an oscillator parameters of ω_i (frequency) and β_i (damping ratio) as follows:

$$R_{dk}^2(\omega_i) = C_{di}^2 \int_{-\infty}^{\infty} \Phi_{akk}(\omega)\, |H_i|^2\, d\omega \tag{2.44}$$

where in Eq.(2.44), H_i is the frequency response function of the oscillator.

Eq.(2.44) cannot be simplified any further to express this in terms of the modal responses. That is, it is not possible to obtain the floor spectrum which incorporates the effect of dynamic interaction in terms of the ground response spectrum values directly, as it is possible when the interaction-free spectrum is obtained [4,10]. Even if the ground motion spectral density function were available, the integral in Eq.(2.44) will still have to be calculated numerically. It is because the frequency dependent terms $G_{ku}^o(\omega)$ appearing in Eq.(2.42) can only be obtained numerically at discrete frequencies by inversion of the complex frequency-dependent matrix $[E(\omega)]$.

We can, however, still bring in the effect of dynamic interaction in generation of the floor spectra through some correction terms, called the interaction coefficients. These coefficients are obtained for each floor spectrum quantity of interest as follows:

The relative displacement auto floor spectra for an oscillator with parameters ω_i and β_i when the interface force vector ,$\{F\}$, is neglected in Eq.(2.13) can be written as follows

$$\tilde{R}_{dk}^2(\omega_i) = \tilde{C}_{di}^2 \int_{-\infty}^{\infty} \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega \tag{2.45}$$

This floor spectrum can be obtained using the method of References [4,10] except that here we do include the effect of the pseudo-static response of the secondary system in modifying the primary system matrices as in Eq.(2.13). The only difference in Eq.(2.44) and (2.45) is that the former includes the effect of the interaction force $\{F(t)\}$ induced at the support by the secondary system, whereas the latter does not. Using Eqs.(2.44) and (2.45), we now define the interaction coefficient for the relative displacement spectrum as

$$\mu_{dk}(\omega_i) = \frac{R_{dk}(\omega_i)}{\tilde{R}_{dk}(\omega_i)} \tag{2.46}$$

Substituting Eqs.(2.44) and (2.45) into (2.46) and assuming that the peak factors are equal, we obtain

$$\mu_{dk}^2(\omega_i) = \frac{\int_{-\infty}^{\infty} \Phi_{akk}(\omega)\, |H_i|^2\, d\omega}{\int_{-\infty}^{\infty} \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega} \tag{2.47}$$

To obtain this interaction coefficient, we still need to know the PSDF of the ground acceleration. For the ground input given in terms of ground response spectra, a spectrum consistent PSDF can be obtained and used in this equation. However, a good estimate of these factors can also be obtained by simply using a band-limited white noise spectral density function as the input. The numerator of Eq.(2.46), however, must still be calculated numerically. Appendix C presents the required details for the numerical integration for a white-noise type ground input.

We can similarly define the interaction coefficients for the other floor spectrum quantities as follows:

Relative velocity spectrum interaction coefficient:

$$\mu_{vk}^2(\omega_i) = \frac{\int_{-\infty}^{\infty} \omega^2\, \Phi_{akk}(\omega)\, |H_i|^2\, d\omega}{\int_{-\infty}^{\infty} \omega^2\, \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega} \tag{2.48}$$

Coincident displacement spectrum interaction coefficient:

$$\tau_{dkl}^2(\omega_i) = \frac{\int_{-\infty}^{\infty} \Phi_{akl}^R(\omega)\, |H_i|^2\, d\omega}{\int_{-\infty}^{\infty} \tilde{\Phi}_{akl}^R(\omega)\, |H_i|^2\, d\omega} \tag{2.49}$$

Coincident velocity spectrum interaction coefficient:

$$\tau_{vkl}^2(\omega_i) = \frac{\int_{-\infty}^{\infty} \omega^2\, \Phi_{akl}^R(\omega)\, |H_i|^2\, d\omega}{\int_{-\infty}^{\infty} \omega^2\, \tilde{\Phi}_{akl}^R(\omega)\, |H_i|^2\, d\omega} \tag{2.50}$$

Quadrature displacement spectrum interaction coefficient:

$$\zeta_{dkl}^2(\omega_i) = \frac{\int_{-\infty}^{\infty} \omega\, \Phi_{akl}^I(\omega)\, |H_i|^2\, d\omega}{\int_{-\infty}^{\infty} \omega\, \tilde{\Phi}_{akl}^I(\omega)\, |H_i|^2\, d\omega} \tag{2.51}$$

Quadrature velocity spectrum interaction coefficient:

$$\zeta_{vkl}^2(\omega_i) = \frac{\int_{-\infty}^{\infty} \omega^3\, \Phi_{akl}^I(\omega)\, |H_i|^2\, d\omega}{\int_{-\infty}^{\infty} \omega^3\, \tilde{\Phi}_{akl}^I(\omega)\, |H_i|^2\, d\omega} \tag{2.52}$$

The interaction coefficients when multiplied by the respective floor spectrum quantities, calculated without interaction, will give the floor spectrum quantity with interaction. These modified floor spectra (with interaction) when used in the analysis of secondary systems will provide the design response which will now incorporate the effect of dynamic interaction.

The interaction-free floor spectrum quantities to be used in these computations can be calculated in terms of the ground spectra directly as explained in References [4,10]. Of course in the computation of the modal properties of the primary structure to be used in the calculation of these interaction-free floor spectra and the interaction coefficient, the modified matrices in Eq.(2.13) are to be used.

2.6 DAMPING FOR TUNED MODES

In this work, we are dealing with only classically damped primary and secondary systems where the energy dissipative characteristic is defined in terms of the modal damping ratios of the two subsystems. To obtain the auto spectral density function as well as the components of the cross spectral density functions required in Eqs.(2.47) through (2.52), we need to know the modal damping ratios of the primary structure which include the dynamic interaction effect. We also need to use the correct modal damping ratios of the secondary system when we calculate its response from the floor inputs. In an interaction free analysis, these damping characteristics remain unaffected, but they may be significantly altered when the interaction effect between the two systems is considered. Because of the interaction, a combined primary-secondary system, in general, is likely to be nonclassically damped even if the individual subsystems are classically damped. It is especially so when the modes of the two systems are in tune with each other. A proper analysis of such a system will necessitate the use of the complex mode approach [13]. However, it turns out that this nonclassical damping effect can still be incorporated with reasonable accuracy if the modal damping ratios of the primary and the secondary systems are appropriately changed in the normal mode approach. Here, this simple approach has been found to be quite effective. In the sequel, we describe an approach to obtain the modified modal damping ratios to be used in the preceding formulation to incorporate the nonclassicality of the combined system damping.

Since we are only interested in calculating the modified modal damping ratios, we will consider only the homogeneous part of Eq.(2.1). We now introduce the following coordinate transformation in this equation:

$$\{V\} = [\Psi_c]\{Y\} = \begin{bmatrix} [\Psi] & [\,0\,] \\ [\,0\,] & [\psi] \end{bmatrix}\{Y\} \tag{2.53}$$

where $\{Y\}$ is the vector of new coordinates. The matrices $[\Psi]$ and $[\psi]$, respectively, are the modal matrices of the primary and the secondary systems. These matrices can be obtained by separate eigenvalue analyses of the two fixed-base systems. The matrix $[\Psi_c]$ here will be called the combined modal matrix. Substituting Eq.(2.53) in the homogeneous part of Eq.(2.1), and then premultiplying by $[\Psi_c]'$, we obtain the following transformed equation:

$$[M_t]\{\ddot{Y}\} + [C_t]\{\dot{Y}\} + [K_t]\{Y\} = \{0\} \tag{2.54}$$

where

$$[M_t] = [\Psi_c]'[M][\Psi_c] \quad , \quad [C_t] = [\Psi_c]'[C][\Psi_c] \quad , \quad [K_t] = [\Psi_c]'[K][\Psi_c] \tag{2.55}$$

are the transformed matrices of size $(n + N) \, x \, (n + N)$.

It is noted that for lumped mass matrices of the two systems, $[M_t]$ will be a diagonal matrix, and if the modal matrices of the two systems are orthonormalized, this matrix will also be an identity matrix. For a consistent mass matrix formulation, there will be off diagonal coupling terms, as they would appear in $[K_t]$. Likewise, if the two systems are classically damped and also if the cross coupling matrices $[c_{as}]$ and $[c_{sa}]$, which represent the damping coupling elements connected to the primary system masses at the support to the secondary system masses, are zero, then the transformed matrix $[C_t]$ will be also a diagonal one. The first N diagonal elements of this matrix will be defined in terms of the modal damping ratios and frequencies of the primary system, whereas the remaining n diagonal elements will be in terms of the secondary system modal damping ratios and frequencies.

The transformed stiffness matrix, on the other hand, will have off diagonal terms. These off diagonal terms, however, will only be of significance when there are some closely spaced frequencies in the combined system. Such frequencies will occur as a result of the tuning of the modes of the two systems. It is this coupling which primarily changes the effective damping ratios of the interactive modes; the damping characteristics of the other modes essentially remain unchanged.

To evaluate the effect of this coupling on damping ratios of the primary and secondary system modes, we will examine a simple yet important case in which a single mode of the secondary system is tuned to a single mode of the primary system. We assume that the r^{th} primary system mode is in tune with the j^{th} secondary system mode. The coupled equations corresponding to these modes now can be extracted out of Eq.(2.54) and written as follows:

$$\begin{bmatrix} m_{rr} & m_{rj} \\ m_{jr} & m_{jj} \end{bmatrix} \begin{Bmatrix} \ddot{Y}_r \\ \ddot{Y}_j \end{Bmatrix} + \begin{bmatrix} c_{rr} & c_{rj} \\ c_{jr} & c_{jj} \end{bmatrix} \begin{Bmatrix} \dot{Y}_r \\ \dot{Y}_j \end{Bmatrix} + \begin{bmatrix} k_{rr} & k_{rj} \\ k_{jr} & k_{jj} \end{bmatrix} \begin{Bmatrix} Y_r \\ Y_j \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2.56}$$

where, in general, the elements of the matrices in Eq.(2.56) are given as follows:

$$m_{rr} = \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}^{\prime} [M] \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}; \quad m_{jj} = \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}^{\prime} [M] \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}; \quad m_{rj} = \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}^{\prime} [M] \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}; \quad m_{jr} = m_{rj} \tag{2.57a}$$

$$c_{rr} = \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}^{\prime} [C] \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}; \quad c_{jj} = \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}^{\prime} [C] \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}; \quad c_{rj} = \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}^{\prime} [C] \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}; \quad c_{jr} = c_{rj} \tag{2.57b}$$

$$k_{rr} = \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}^{\prime} [K] \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}; \quad k_{jj} = \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}^{\prime} [K] \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}; \quad k_{rj} = \begin{Bmatrix} \Psi_r \\ 0 \end{Bmatrix}^{\prime} [K] \begin{Bmatrix} 0 \\ \psi_j \end{Bmatrix}; \quad k_{jr} = k_{rj} \tag{2.57c}$$

Here $\{\Psi_r\}$ = r^{th} primary mode shape; and $\{\psi_j\}$ = j^{th} secondary mode shape. If we assume that the cross coupling damping terms $[c_{sa}]$ and $[c_{as}]$ in Eq.(2.1) are zero, then the combined damping matrix can be written as follows:

$$[C] = \begin{bmatrix} [C_p] & [\,0\,] \\ [\,0\,] & [C_s] \end{bmatrix} \tag{2.58}$$

where $[C_p]$ and $[C_s]$ are the primary and secondary system damping matrices, respectively. Here it is mentioned that if the damping characteristics of the secondary system are defined only in terms of the modal damping ratios, then these cross damping terms $[c_{sa}]$ and $[c_{as}]$ are undefined in any case. Furthermore, if there are no dampers connecting the primary and secondary masses, then the cross damping matrices indeed can be taken zero identically. With Eq.(2.58), the damping matrix in Eq.(2.56) will now be of the following form:

$$\begin{bmatrix} c_{rr} & c_{rj} \\ c_{jr} & c_{jj} \end{bmatrix} = \begin{bmatrix} \{\Psi_r\}'[C_p]\{\Psi_r\} & 0 \\ 0 & \{\psi_j\}'[C_s]\{\psi_j\} \end{bmatrix} = \begin{bmatrix} 2\,\beta_r\,\omega_r\,m_{rr} & 0 \\ 0 & 2\,\beta_j\,\omega_j\,m_{jj} \end{bmatrix} \tag{2.59}$$

where ω_r and β_r are the natural frequency and damping ratio of the r^{th} primary system mode; and ω_j and β_j are the same parameters belonging to the j^{th} secondary system mode.

To obtain the effective damping ratios and frequencies associated with the two tuned modes, we need to solve the following complex-value eigenvalue problem associated with Eq.(2.56):

$$\begin{bmatrix} m_{rr} & m_{rj} & 0 & 0 \\ m_{jr} & m_{jj} & 0 & 0 \\ 0 & 0 & -k_{rr} & -k_{rj} \\ 0 & 0 & -k_{jr} & -k_{jj} \end{bmatrix} \{\chi_i\} + \begin{bmatrix} 0 & 0 & m_{rr} & m_{rj} \\ 0 & 0 & m_{jr} & m_{jj} \\ m_{rr} & m_{jr} & c_{rr} & c_{rj} \\ m_{jr} & m_{jj} & c_{jr} & c_{jj} \end{bmatrix} \{\chi_i\}\,p_i = \{\,0\,\} \tag{2.60}$$

where $\{\chi_i\}$ and p_i are the complex valued eigenvector and eigenvalues. These eigenvalues will occur in of complex conjugate pairs. The real and imaginary parts of the eigenvalues can be used to obtain the effective frequencies ω_{ie} and damping ratios β_{ie} for the two modes as follows:

$$\omega_{ie} = \text{Abs}\,|p_i| \quad ; \quad i = 1,2 \tag{2.61a}$$

$$\beta_{ie} = \frac{-\,\text{Real}\,(p_i)}{\omega_{ie}} \quad ; \quad i = 1,2 \tag{2.61b}$$

It is pointed out that these equivalent damping ratios will be different from β_r and β_j appearing in Eq.(2.59) only if the two modes are in tune. The stronger the tuning, the larger the difference between these two damping ratios.

These two damping ratios are associated with the modes of Eq.(2.56). We, however, need the equivalent values of the damping ratios which are associated with Y_r and Y_j, which, respectively, are the primary and secondary system modes. To obtain these equivalent damping ratios, the following scheme can be used:

For the damping ratios defined by Eq.(2.61b), we can define a classically damping matrix which is proportional to mass and stiffness terms of Eq.(2.56) as follows:

$$\begin{bmatrix} c_{rr} & c_{rj} \\ c_{jr} & c_{jj} \end{bmatrix}_p = a_1 \begin{bmatrix} m_{rr} & m_{rj} \\ m_{jr} & m_{jj} \end{bmatrix} + a_2 \begin{bmatrix} k_{rr} & k_{rj} \\ k_{jr} & k_{jj} \end{bmatrix} \tag{2.62}$$

in which the coefficients a_1 and a_2 are defined as

$$a_1 = \frac{2\,\omega_{1e}\,\omega_{2e}\,(\beta_{1e}\,\omega_{2e} - \beta_{2e}\,\omega_{1e})}{\omega_{2e}^2 - \omega_{1e}^2} \tag{2.63a}$$

$$a_2 = \frac{2\,(\beta_{1e}\,\omega_{1e} - \beta_{2e}\,\omega_{2e})}{\omega_{1e}^2 - \omega_{2e}^2} \tag{2.63b}$$

and subscript p with the damping matrix in Eq.(2.62) denotes that it is a proportional or classical damping matrix. It is noted that a modal matrix obtained from an undamped classical eigenvalue analysis of the system in Eq.(2.56) will decouple the damping matrix defined in Eq.(2.62). Moreover, the damping ratios associated with the undamped modes will also be equal to the equivalent damping ratios obtained from Eq.(2.61b) through the complex eigenvalue analysis of this system.

A good estimate of the damping ratios associated with Y_r and Y_j can now be obtained if the off-diagonal elements of the proportional damping matrix in Eq.(2.62) are neglected. With this assumption we now obtain the following equivalent damping ratios for the tuned primary and secondary modes:

$$\beta_{re} = \frac{c_{rr}}{2\,\omega_r\,m_{rr}} = \frac{a_1\,m_{rr} + a_2\,k_{rr}}{2\,\omega_r\,m_{rr}} \tag{2.64a}$$

$$\beta_{je} = \frac{c_{jj}}{2\,\omega_j\,m_{jj}} = \frac{a_1\,m_{jj} + a_2\,k_{jj}}{2\,\omega_j\,m_{jj}} \tag{2.64b}$$

These equivalent damping ratios will take into account the non-classical damping effect in the calculation of the floor inputs as well as in the computation of the secondary systems' design response.

For the case of more than two modes in resonance, the procedure developed here can still be applied. We define n_r to be the number of modes (primary and secondary) that are in resonance ($n_r \geq 2$). The steps to follow for the damping analysis would be to select the n_r degrees of freedom that are in resonance from Eq.(2.54) and perform a complex analysis of this reduced system to obtain its complex eigenvalues. These values are then used to obtain the equivalent natural frequencies and damping ratios which in turn are needed to define the classical damping matrix, $[C]_p$. To compute this matrix for $n_c > 2$ the following general equation can be used [14]:

$$[C]_p = [M] \sum_{i=0}^{n_r - 1} a_{i+1}([M]^{-1}[K])^i \tag{2.65}$$

where $[M]$ and $[K]$ are the mass and stiffness matrices of the reduced system. It is noted that for $n_c = 2$ Eq.(2.65) gives Eq.(2.62). The coefficients a_i in Eq.(2.65) are obtained from the solution of the following system of equations:

$$\begin{Bmatrix} \beta_{1e} \\ \cdot \\ \beta_{ie} \\ \cdot \\ \beta_{n_r e} \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} \omega_{1e} & \omega_{1e}^3 & \omega_{1e}^5 & \cdot\,\cdot & \omega_{1e}^{2n_r - 1} \\ \cdot & \cdot & \cdot & \cdot\,\cdot & \cdot \\ \omega_{ie} & \omega_{ie}^3 & \omega_{ie}^5 & \cdot\,\cdot & \omega_{ie}^{2n_r - 1} \\ \cdot & \cdot & \cdot & \cdot\,\cdot & \cdot \\ \omega_{n_r e} & \omega_{n_r e}^3 & \omega_{n_r e}^5 & \cdot\,\cdot & \omega_{n_r e}^{2n_r - 1} \end{bmatrix} \begin{Bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_{n_r} \end{Bmatrix} \tag{2.66}$$

where β_{ie} and ω_{ie} are the equivalent damping ratios and natural frequencies obtained from the complex eigenvalue analysis. Neglecting the off diagonal elements of the proportional damping matrix $[C]_p$, the damping ratios for the u^{th} tuned mode are given as follows:

$$\beta_{ue} = \frac{c_{uu}}{2\,\omega_u\, m_{uu}} \tag{2.67}$$

CHAPTER III

USER SUMMARY OF THE PROPOSED METHOD

3.1 INTRODUCTION

This chapter provides the step-by-step procedure for implementation of the approach developed in the previous chapter. This is for the benefit of a user who is primarily interested in its implementation rather than in its analytical development.

3.2 STEP-BY-STEP PROCEDURE

The following steps procedure can be used to include in the dynamic interaction effect between the primary and secondary systems in generation of auto and cross floor response spectral quantities.

1. Define the elements of the mass, damping and stiffness matrices of the secondary equations of motion as follows:

$$\begin{bmatrix} m_{aa} & m_{as} \\ m_{sa} & m_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{U}_a \\ \ddot{U}_s \end{Bmatrix} + \begin{bmatrix} c_{aa} & c_{as} \\ c_{sa} & c_{ss} \end{bmatrix} \begin{Bmatrix} \dot{U}_a \\ \dot{U}_s \end{Bmatrix} + \begin{bmatrix} k_{aa} & k_{as} \\ k_{sa} & k_{ss} \end{bmatrix} \begin{Bmatrix} U_a \\ U_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.1}$$

where matrices $[m_{ss}]$, $[c_{ss}]$ and $[k_{ss}]$, associated to the unattached secondary mass points, are of dimension $n \times n$. Matrices $[m_{aa}]$, $[c_{aa}]$ and $[k_{aa}]$, associated to the support points, are of dimension $m \times m$. Matrices $[m_{as}]$, $[c_{as}]$ and $[k_{as}]$ are of dimension $m \times n$ where $n =$ unconstrained degree-of-freedom (DOF) of the secondary system and $m =$ number of support on the primary system.

2. Solve the following real eigenvalue problem for the secondary system:

$$[k_{ss}]\{\psi_j\} = \omega_j^2 [m_{ss}]\{\psi_j\} \tag{3.2}$$

where $\omega_j = j^{th}$ natural frequency and $\{\psi_j\} = j^{th}$ mode shape belonging to the secondary system. Normalize the mode shape with respect to the mass matrix such that

$$\{\psi_j\}'[m_{ss}]\{\psi_j\} = 1 \tag{3.3}$$

A prime (′) over a vector quantity denotes its transpose.

3. Define the elements of the mass, damping and stiffness matrices of the primary equations of motions

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{V}_p \\ \ddot{V}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa} \end{bmatrix} \begin{Bmatrix} \dot{V}_p \\ \dot{V}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix} \begin{Bmatrix} V_p \\ V_a \end{Bmatrix} = - \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} 1_p \\ 1_a \end{Bmatrix} \ddot{X}_g(t) \quad (3.4)$$

where subscripts p and a both refer to the primary DOF. The subscript a refers to the primary masses where the secondary system is attached and subscript p refers to the primary DOF where there are not support points. The dimension of the complete matrices in Eq.(3.4) are $N \times N$, where $N =$ total number of primary DOF.

4. Solve the following real eigenvalue problem for the primary structure:

$$[K]\{\Psi_r\} = \omega_r^2 [M]\{\Psi_r\} \quad (3.5)$$

where $\omega_r = r^{th}$ natural frequency and $\{\Psi_r\} = r^{th}$ mode shape of the primary system. Normilize the mode shape with respect to the mass matrix such that

$$\{\Psi_r\}'[M]\{\Psi_r\} = 1 \quad (3.6)$$

Matrices $[K]$ and $[M]$ in Eqs.(3.5) and (3.6) are given as follows:

$$[K] = \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix} \quad ; \quad [M] = \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \quad (3.7)$$

5. Compare the natural frequencies of the secondary system with the natural frequencies of the priamry system and identify the modes of both system that are tuned or nearly tuned.

6. If there is no tuning and the secondary system is light, the decoupled analysis as presented in Reference [4,10] is acceptable. On the other hand, if the secondary system is heavy or tuned or both compute the modified or perturbed primary mass and stiffness matrices as follows:

$$[\tilde{K}] = \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & \tilde{K}_{aa} \end{bmatrix} \quad ; \quad [\tilde{M}] = \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & \tilde{M}_{aa} \end{bmatrix} \tag{3.8}$$

where

$$[\tilde{M}_{aa}] = [M_{aa}] + [m_{aa}] - [m_{as}][k_{ss}]^{-1}[k_{sa}] \tag{3.9a}$$

$$[\tilde{K}_{aa}] = [K_{aa}] + [k_{aa}] - [k_{as}][k_{ss}]^{-1}[k_{sa}] \tag{3.9b}$$

7. Solve the following real eigenvalue problem associated with the perturbed primary system

$$[\tilde{K}]\{\tilde{\Psi}_r\} = \tilde{\omega}_r^2 [\tilde{M}]\{\tilde{\Psi}_r\} \tag{3.10}$$

where $\tilde{\omega}_r$ = r^{th} natural frequency and $\{\tilde{\Psi}_r\}$ = r^{th} mode shape of the modified primary system. Normilize the mode shape with respect to the mass matrix such that

$$\{\tilde{\Psi}_r\}'[\tilde{M}]\{\tilde{\Psi}_r\} = 1 \tag{3.11}$$

To avoid solution of a second eigenvalue problem, the eigenproperties of Eq.(3.10) can also be obtained by a perturbation approach if the eigenvalues and eigenvectors of Eq.(3.2) are known. The steps for a first order perturbation approach are as follows:

a. Define the matrices perturbation as

$$[K_1] = [\tilde{K}] - [K] \quad ; \quad [M_1] = [\tilde{M}] - [M] \tag{3.12}$$

b. Compute the natural frequencies perturbation as

$$\omega_{1r}^2 = \{\Psi_r\}'([K_1] - \omega_r^2 [M_1])\{\Psi_r\} \tag{3.13}$$

c. Compute the perturbed natural frequencies and mode shapes as follows

$$\tilde{\omega}_r^2 = \omega_r^2 + \omega_{1r}^2 \tag{3.14a}$$

$$\{\tilde{\Psi}_r\} = \{\Psi_r\} + \sum_{k=1}^{N} \varepsilon_{rk} \{\Psi_k\} \tag{3.14b}$$

where the coefficients ε_{rs} are given as follows

$$\varepsilon_{rs} = \frac{\{\Psi_s\}'([K_1] - \omega_r^2 [M_1])\{\Psi_r\}}{\omega_s^2 - \omega_r^2} \qquad r \neq s \quad ; \qquad \varepsilon_{rr} = 0 \tag{3.15}$$

8. Compute the equivalent damping ratio for the tuned modes to take into account the non-classical damping effect. First, we assume that the j^{th} secondary system mode is tuned to the r^{th} primary mode. Then, the procedure is as follows:

   a. Obtain the following reduced homogeneous two DOF system of equations

$$\begin{bmatrix} m_{rr} & m_{rj} \\ m_{jr} & 1 \end{bmatrix} \begin{Bmatrix} \ddot{Y}_r \\ \ddot{Y}_j \end{Bmatrix} + \begin{bmatrix} 2\beta_r\omega_r & 0 \\ 0 & 2\beta_j\&wj \end{bmatrix} \begin{Bmatrix} \dot{Y}_r \\ \dot{Y}_j \end{Bmatrix} + \begin{bmatrix} k_{rr} & k_{rj} \\ k_{jr} & \omega_j^2 \end{bmatrix} \begin{Bmatrix} Y_r \\ Y_j \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.16}$$

   where Y_r and Y_j are, respectively, the generalized coordinates associated to the tuned primary and secondary system modes. β_r and β_j are their damping ratios defined when the interaction effect is ignored. The elements in the matrices are obtained as follows

$$m_{rr} = \{\Psi_r\}' \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & (M_{aa} + m_{aa}) \end{bmatrix} \{\Psi_r\} \quad ; \quad k_{rr} = \{\Psi_r\}' \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & (K_{aa} + k_{aa}) \end{bmatrix} \{\Psi_r\} \tag{3.17a}$$

$$m_{rj} = \{\Psi_r\}' \begin{bmatrix} 0 \\ m_{as} \end{bmatrix} \{\psi_j\} \quad ; \quad k_{rj} = \{\Psi_r\}' \begin{bmatrix} 0 \\ K_{as} \end{bmatrix} \{\psi_j\} \tag{3.17b}$$

   b. Solve the complex-value eigenvalue problem associated with the system of Eq.(3.16) to obtain the complex eigenvalues p_i

$$\begin{bmatrix} m_{rr} & m_{rj} & 0 & 0 \\ m_{jr} & 1 & 0 & 0 \\ 0 & 0 & -k_{rr} & -k_{rj} \\ 0 & 0 & -k_{jr} & -\omega_j^2 \end{bmatrix} \{\chi_i\} + \begin{bmatrix} 0 & 0 & m_{rr} & m_{rj} \\ 0 & 0 & m_{jr} & m_{jj} \\ m_{rr} & m_{jr} & 2\beta_r\omega_r & 0 \\ m_{jr} & 1 & 0 & 2\beta_j\omega_j \end{bmatrix} \{\chi_i\}\, p_i = \{\,0\,\} \qquad (3.18)$$

c. Compute the equivalent natural frequencies and damping ratios as follows

$$\omega_{ie} = |p_i| \quad ; \quad i = 1,2 \qquad (3.19a)$$

$$\beta_{ie} = \frac{-\,\mathrm{Real}\,(p_i)}{\omega_{ie}} \quad ; \quad i = 1,2 \qquad (3.19b)$$

d. Compute the new damping ratios for the tuned modes of the primary and secondary systems as follows:

$$\beta_{re} = \frac{a_1\, m_{rr} + a_2\, k_{rr}}{2\,\omega_r} \quad ; \quad \beta_{je} = \frac{a_1 + a_2\, k_{jj}}{2\,\omega_j} \qquad (3.20)$$

where a_1 and a_2 are obtained as

$$a_1 = \frac{2\,\omega_{1e}\,\omega_{2e}\,(\beta_{1e}\,\omega_{2e} - \beta_{2e}\,\omega_{1e})}{\omega_{2e}^2 - \omega_{1e}^2} \quad ; \quad a_2 = \frac{2\,(\beta_{1e}\,\omega_{1e} - \beta_{2e}\,\omega_{2e})}{\omega_{1e}^2 - \omega_{2e}^2} \qquad (3.21)$$

The damping ratios of the modes of both systems that are not in resonance remain unchanged.

For the case of more than two modes in resonance, a similar procedure developed in Section 2.6 can be used.

9. To calculate the interaction coefficients assume a band-limited white noise ground input PSDF with ω_c as the cut-off frequency. Divide the interval $[0\,,\ \omega_c]$ into subintervals and compute the

complex frequency dependant matrix $[E(\omega)]$ at the Gauss points of each subinterval. The elements of the matrix $[E(\omega)]$ are given as follows

$$E_{kk} = 1 - \sum_{r=1}^{N} \{\tilde{\Psi}_r\}(k)\, \omega^2\, \tilde{H}_r \sum_{j=1}^{n} C_{rj}\, H_j\, P_{jk} \tag{3.22a}$$

$$E_{kl} = - \sum_{r=1}^{N} \{\tilde{\Psi}_r\}(k)\, \omega^2\, \tilde{H}_r \sum_{j=1}^{n} C_{rj}\, H_j\, P_{jl} \qquad , \quad k \neq l \tag{3.22b}$$

where the frequency response functions $\tilde{H}_r$ and H_j are defined as

$$\tilde{H}_r = \frac{1}{(\tilde{\omega}_r^2 - \omega^2 + i\, 2\, \beta_{re}\, \tilde{\omega}_r\, \omega)} \quad ; \quad H_j = \frac{1}{(\omega_j^2 - \omega^2 + i\, 2\, \beta_{je}\, \omega_j\, \omega)} \tag{3.23}$$

and the coefficient C_{rj} is given by

$$C_{rj} = \{\tilde{\Psi}_r\} \begin{bmatrix} 0 \\ k_{as} \end{bmatrix} \{\psi_j\} \tag{3.24}$$

P_{jk} = k^{th} element of the modal influence vector $\{P_j\}$ defined as

$$\{P_j\}' = \{\psi_j\}\, ([m_{ss}]\, [k_{ss}]^{-1}\, [k_{sa}] - [m_{sa}]) \tag{3.25}$$

Matrix $[E(\omega)]$ is of dimension $m \times m$. The subscripts k and l refer to the floors where the attachment points are located.

10. Obtain the inverse of matrix $[E(\omega)]$ as $[G(\omega)] = [E(\omega)]^{-1}$ and identify its complex coefficient $G_{uv}(\omega)$.

11. Define the following floor interaction coefficients:

a. Auto displacement spectrum interaction coefficient:

$$\mu_{dk}^2(\omega_i) = \frac{\int_0^{\omega_c} \Phi_{akk}(\omega)\, |H_i|^2\, d\omega}{\int_0^{\omega_c} \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega} \tag{3.26}$$

b. Auto velocity spectrum interaction coefficient:

$$\mu_{vk}^2(\omega_i) = \frac{\int_0^{\omega_c} \omega^2\, \Phi_{akk}(\omega)\, |H_i|^2\, d\omega}{\int_0^{\omega_c} \omega^2 \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega} \tag{3.27}$$

c. Coincident displacement spectrum interaction coefficient:

$$\tau_{dkl}^2(\omega_i) = \frac{\int_0^{\omega_c} \Phi_{akl}^R(\omega)\, |H_i|^2\, d\omega}{\int_0^{\omega_c} \tilde{\Phi}_{akl}^R(\omega)\, |H_i|^2\, d\omega} \tag{3.28}$$

d. Coincident velocity spectrum interaction coefficient:

$$\tau_{vkl}^2(\omega_i) = \frac{\int_0^{\omega_c} \omega^2\, \Phi_{akl}^R(\omega)\, |H_i|^2\, d\omega}{\int_0^{\omega_c} \omega^2\, \tilde{\Phi}_{akl}^R(\omega)\, |H_i|^2\, d\omega} \tag{3.29}$$

e. Quadrature displacement spectrum interaction coefficient:

$$\xi_{dkl}^2(\omega_i) = \frac{\int_0^{\omega_c} \omega\, \Phi_{akl}^I(\omega)\, |H_i|^2\, d\omega}{\int_0^{\omega_c} \omega\, \tilde{\Phi}_{akl}^I(\omega)\, |H_i|^2\, d\omega} \tag{3.30}$$

f. Quadrature velocity spectrum interaction coefficient:

$$\xi_{vkl}^2(\omega_i) = \frac{\int_0^{\omega_c} \omega^3\, \Phi_{akl}^I(\omega)\, |H_i|^2\, d\omega}{\int_0^{\omega_c} \omega^3\, \tilde{\Phi}_{akl}^I(\omega)\, |H_i|^2\, d\omega} \tag{3.31}$$

The supscripts R and I refer to the real and imaginary part of the cross PSDF. The auto and cross PSDF required in Eqs.(3.26) through (3.31) are given as follows

$$\Psi^R_{akl}(\omega) + i\,\Psi^I_{akl}(\omega) = \sum_{u=1}^{m} \sum_{v=1}^{m} G^*_{ku}(\omega)\, G_{lv}(\omega) \left[\tilde{\Psi}^R_{auv}(\omega) + i\,\tilde{\Psi}^I_{auv}(\omega)\right] \tag{3.32}$$

The auto PSDF is obtained setting $k = l$ in Eq.(3.32). The cross PSDF $\tilde{\Psi}^R_{auv}(\omega) + i\,\tilde{\Psi}^I_{auv}(\omega)$ is obtained in terms of the properties of the modified or perturbed primary system as follows

$$\tilde{\Psi}^R_{auv}(\omega) + i\,\tilde{\Psi}^I_{auv}(\omega) = \sum_{r=1}^{N} \sum_{s=1}^{N} \tilde{\gamma}_r\, \tilde{\gamma}_s\, \tilde{\Psi}_r(u)\, \tilde{\Psi}_s(v)\, (\tilde{\omega}_r^2 - i\,2\,\tilde{\beta}_{re}\,\tilde{\omega}_r\,\omega)$$

$$(\tilde{\omega}_s^2 - i\,2\,\tilde{\beta}_{se}\,\tilde{\omega}_s\,\omega)\, \tilde{H}_r^{\times}\, \tilde{H}_s \tag{3.33}$$

where $\tilde{\gamma}_r$ = r^{th} modified participation factor given as

$$\tilde{\gamma}_r = \frac{\{\tilde{\Psi}_r\}'\,[\tilde{M}]\,\{1\}}{\{\tilde{\Psi}_r\}'\,[\tilde{M}]\,\{\tilde{\Psi}_r\}} \tag{3.34}$$

An asterisk (*) over a complex quantity denotes its complex conjugate.

12. Perform the integration in Eqs.(3.26) through (3.31) as a weighted sum by Gauss Quadrature scheme.

13. Compute the auto, coincident and quadrature floor spectra as given in References [4,10] using the modified primary system properties. All these spectral quantities pertain to an oscillator of frequency ω_i and damping ratio β_i. Let the floor spectral inputs for the modified primary system excluding the dynamic interaction effect be defined as:

    a. $\tilde{R}_{dk}(\omega_i)$ = Auto displacement spectrum for the k^{th} floor.
    b. $\tilde{R}_{vk}(\omega_i)$ = Auto velocity spectrum for the k^{th} floor.
    c. $\tilde{C}_{dkl}(\omega_i)$ = Coincident displacement spectrum for floors k and l.
    d. $\tilde{C}_{vkl}(\omega_i)$ = Coincident velocity spectrum for floors k and l.
    e. $\tilde{Q}_{dkl}(\omega_i)$ = Quadrature displacement spectrum for floors k and l.

f. $\tilde{Q}_{vkl}(\omega_i)$ = Quadrature velocity spectrum for floors k and l.

14. Compute the floor spectral inputs including the dynamic interaction effect as the product of the interaction coefficient and corresponding floor spectrum. That is,

a. $R_{dk}(\omega_i) = \mu_{dk}(\omega_i)\ \tilde{R}_{dk}(\omega_i)$

b. $R_{vk}(\omega_i) = \mu_{vk}(\omega_i)\ \tilde{R}_{vk}(\omega_i)$

c. $C_{dkl}(\omega_i) = \tau_{dkl}(\omega_i)\ \tilde{C}_{dk}(\omega_i)$

d. $C_{vkl}(\omega_i) = \tau_{vkl}(\omega_i)\ \tilde{C}_{vk}(\omega_i)$

e. $Q_{dkl}(\omega_i) = \xi_{dkl}(\omega_i)\ \tilde{Q}_{dk}(\omega_i)$

f. $Q_{vkl}(\omega_i) = \xi_{vkl}(\omega_i)\ \tilde{Q}_{vk}(\omega_i)$

The modified floor spectra when used as inputs in the approach developed in References [3,10] will include the effect of interaction in the calculation of the secondary system response.

CHAPTER IV

NUMERICAL RESULTS

4.1 INTRODUCTION

In this chapter, the numerical results which demonstrate the applicability of the approach developed in the previous chapters are presented for two different structural configurations, shown in Figs.1 and 2. The secondary system in configuration A has all the supports on the primary structure, whereas the system in configuration B has one of the supports on ground. The primary structure in both these problems is the same. It consists of five floors connected by columns which primarily deform in the shearing mode. This system has five degrees of freedom. The mass and stiffness properties of the primary system are shown in Figs.1 and 2, with K = 10,075 Kips/ft and M = 35.5 Kips- Sec ^{2}/ft. The natural frequencies, participation factors and modal displacements are given in Tables 1 and 2. The primary structure is assumed to have 5% damping in each mode.

To study the interaction effect between the primary and secondary systems, the mass and stiffness properties of the secondary system are chosen such that the fundamental frequencies of the primary and the secondary systems are tuned. In such cases, it becomes essential to include the effect of dynamic interaction to calculate accurate response. The mass and stiffness values for the secondary system in configuration A are given by m = 3.35 Kips-Sec^2/ ft, k = 550 Kips/ft and k_c = 300 Kips/ft. These properties for the system in configuration B are m = 3.35 Kips-Sec^2/ ft, k = 570 Kips/ft and k_c = 160 Kips/ft. The natural frequencies and mode shapes of these two systems, assumed fixed at all the supports, are given in Tables 3 and 4. The dynamic influence coefficient defining P_{ik} are given in Table 5. The modal damping ratio of 2% is assumed in each mode of both the secondary systems. By proportionally scaling the mass and stiffness parameters of the secondary system, their natural frequencies are kept unchanged for different mass ratios of the two systems. The structures shown in Figs.1 and 2 were analyzed for three different ratios of the nodal mass of the secondary system , m, to the nodal mass of the primary system, M. These mass ratios m/M are 0.1, 0.05 and 0.01. The mode shapes and dynamic influence coefficients of the secondary system for the different mass ratios can be obtained from Tables 3, 4 and 5 by a proportional scaling. For example, the properties of the secondary system for a mass ratio of 0.05 is $\frac{1}{\sqrt{2}}$ times the values tabulated in Tables 4 and 5. Tables 6 and 7 show the frequencies of the primary, secondary and the combined systems for configurations A and B, respectively. Columns (4), (5) and (6) in these tables give the natural frequencies of the combined system, obtained by complex eigenvalue analysis of the combined system for the three different mass ratios. It can be seen that the two tuned frequencies shift away from each other in the combined analysis. This effect becomes more important for large mass ratios.

4.2 FLOOR SPECTRAL INPUTS

The seismic ground input to the entire system is defined in the form of pseudo-acceleration and relative velocity ground response spectra. These spectrum curves are shown in Figs. 3 and 4 for three different oscillator damping ratios.

The various floor spectral quantities which are required as inputs for the calculation of the secondary system member forces response are shown in Figs. 5 through 38. Figs.5 through 22 pertain to the secondary System A and the remaining figures are for System B. These floor inputs are obtained according to the procedure described in Chapter II for the mass ratio $m/M = 0.1$ and , thus, include the effect of dynamic interaction. Also, the floor spectra without interaction are shown by the discontinuous curves in these figures. These are shown here to demonstrate the effect of the interaction through coupling between the primary and the secondary systems.

Figs.5 through 10 show the auto pseudo-acceleration and relative velocity for floors 1, 3 and 5. The cross floor spectra (coincident and quadrature components) for the interconnected floors 1, 3 and 5 are given in Figs. 11 through 22. The auto floor spectra for system B are given in Figs. 3 and 4 for the support at the ground, and Figs. 23 through 26 show the corresponding spectra for the attachment points at the floors 2 and 4. The cross floor spectra required in this case are given in Figs. 26 through 38. It is noted that even though the ground motion is assumed not to be changed by the response of the secondary system, the cross floor spectra between the support points on the primary structure and the ground are modified because of the dynamic interaction.

It is also noted that the effect of interaction is seen to be very important in the vicinity of the tuned frequency. At the resonance frequency, the interaction free floor spectra show a peak whereas the floor spectra with interaction show a valley. The peaks in the floor spectrum now appear on either side of the tuned mode.

These spectral figures will change if the mass ratio of the two system is changed. To provide the information about the spectral input in a compact form for various mass ratios, here Tables 8 through 19 are used. Tables 8,9 and 10 show the auto displacement (AD) and velocity (AV) floor spectra for floors 1, 3 and 5 for configuration A. The associated coincident displacement (CD), coincident velocity (CV), quadrature displacement (QD) and quadrature velocity (QV) cross floor spectra are tabulated in Tables 11, 12 and 13. The same information for the system B is tabulated in Tables 14 through 19. It is noted that these tables include the case of zero oscillator frequency; these values are required for calculating the pseudo-static and cross response components. As the floor spectrum figures presented earlier were obtained for a mass ratio of 0.1, the spectrum values in the tables for this mass ratio are the same as those in the figures, except for the unit difference in the displacement spectrum values. In these tables the displacement spectrum units are in ft. whereas in the figures, the displacement spectrum values having been plotted in terms of pseudo-acceleration, are in units of ft-/sec^2. These tables, 8 through 19, by themself define the inputs to the two secondary systems of Figs. 1 and 2 completely for all mass ratios.

4.3 RESPONSE OF SECONDARY SYSTEMS

The force response is obtained for the two secondary systems for three different mass ratios. The results are shown in Tables 20 and 21. The results in Columns (3) of these tables are obtained by a direct analysis of the combined primary-secondary system models. These are the bench-marck values against which the values obtained by the proposed method with and without the interaction effect are compared. The values in Columns (4) are obtained from a decoupled interaction free analysis; that is, in calculating these values the interaction-free floor spectra were used as the inputs to the secondary systems. The ratio of the interaction free response to the combined analysis response is given in Column (5). It is noted that as the secondary system becomes heavier, the error in the interaction free response increases considerably. It is also noted that the decoupled analysis does not necessarily give conservative results in all cases; see the results in Table 21 obtained for the System B for the mass ratio m/M = .01. Columns (6) and (7) give similar results for the response obtained with the dynamic interaction effect. As the values in Column (7) are close to 1.0 for all cases, the proposed method does provide an accurate estimate of the design response which includes the dynamic interaction effect. This validates the applicability of the approach presented here to incorporate the coupling effect in calculating the secondary systems response.

CHAPTER V

SUMMARY AND CONCLUSIONS

An analytically rational procedure is presented for including the effect of dynamic interaction between the primary and secondary systems in generation of auto and cross floor response spectra which are required as the inputs in the seismic analysis of multiply supported secondary systems. The equations of motion of the two systems are developed which clearly show the interaction terms which couple the two equations. The interaction effects menifests itself through the modification of the primary system matrices as well as through the introduction of the interface forces applied to the primary structure at the support points. These interface forces are seen to depend only on the dynamic component of the secondary system response.

To incorporate these effects in the calculation of the floor spectral quantities, here the interaction coefficients are introduced. These interaction coefficients are required to be obtained for each of the floor response spectra required in the analysis of the secondary systems. A method to obtain these interaction coefficients is developed. As the combined system can be nonclassical in damping, a method to include this nonclassicality in generation of floor spectra as well as in the calculation of the secondary system response is proposed.

The numerical results are presented to demenstrate the applicability of the proposed approach. The effect of the interaction on the floor response spectra and secondary system response is shown. The effect is observed to be significant when the secondary system is relatively heavy and tuned to the primary structure.

Table 1. Natural Frequencies (rad/sec) and Participation Factors of the Original Primary System.

Mode	Natural Frequency	Participation factor
1	6.98	383.8
2	20.38	-120.8
3	32.12	-63.7
4	41.26	35.4
5	47.06	-16.2

Table 2. Mode Shapes of the Original Primary System - [ft/100].

Node	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	0.093	-.249	-.326	0.299	-.178
2	0.178	-.326	-.093	-.249	0.299
3	0.249	-.178	0.299	-.093	-.326
4	0.299	0.093	0.178	0.326	0.249
5	0.326	0.299	-.249	-.178	-.093

Table 3. Natural Frequencies of the Secondary System (Rad/sec).

Mode	Secondary System	
	Configuration "A"	Configuration "B"
1	7.054	6.980
2	11.076	11.459
3	17.602	15.522
4	21.434	19.082
5	25.306	--

Table 4. Mode Shapes of the Secondary System - [ft/10].

	Secondary System								
	Configuration "A"					Configuration "B"			
Mode	dof 6	dof 7	dof 8	dof 9	dof 10	dof 6	dof 7	dof 8	dof 9
1	0.068	0.085	0.076	0.085	0.068	-.050	-.074	-.114	-.093
2	-.095	-.076	0.0	0.076	0.095	-.125	-.072	0.028	0.089
3	-.095	0.032	0.098	0.032	-.095	-.085	0.051	0.092	-.107
4	0.076	-.095	0.0	0.095	-.076	-.066	0.128	-.086	0.039
5	0.034	-.081	0.119	-.081	0.034	--	--	--	--

Table 5. Dynamic Influence Coefficient , P_{ik}

	Configuration "A"			Configuration "B"		
Mode	dof 1	dof 3	dof 5	dof 0	dof 2	dof 4
1	-41.446	-45.925	-41.446	32.043	35.030	44.292
2	23.349	0.0	-23.349	29.571	12.750	-15.696
3	9.191	- 9.545	9.191	10.953	- 4.904	10.219
4	- 4.977	0.0	4.977	5.635	- 8.090	-2.494
5	- 1.613	- 5.607	- 1.613	--	--	--

Table 6. Natural Frequencies for Configuration "A".

Mode	Primary System	Secondary System	Combined System		
			m/M = .01	m/M = .05	m/M = 0.1
1	6.98	--	6.71	6.34	6.08
2	--	7.05	7.33	7.76	8.09
3	--	11.07	11.07	11.06	11.06
4	--	17.60	17.59	17.58	17.56
5	20.37	--	20.38	20.42	20.46
6	--	21.43	21.44	21.46	21.50
7	--	25.30	25.30	25.30	25.31
8	32.12	--	32.13	32.18	32.25
9	41.26	--	41.26	41.28	41.31
10	47.06	--	47.06	47.08	47.11

Table 7. Natural Frequencies for Configuration "B".

Mode	Primary System	Secondary System	Combined System		
			m/M = .01	m/M = .05	m/M = 0.1
1	--	6.98	6.79	6.53	6.35
2	6.98	--	7.17	7.47	7.69
3	--	11.46	11.45	11.45	11.44
4	--	15.52	15.52	15.51	15.51
5	--	19.08	19.08	19.06	19.05
6	20.37	--	20.38	20.42	20.47
7	32.12	--	32.12	32.12	32.13
8	41.26	--	41.26	41.28	41.31
9	47.06	--	47.06	47.08	47.10

Table 8. **Auto Floor Spectra for Floor 1 for Structural Configuration "A".**

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra	
			AD	AV
.01	7.05	.0347	0.1759	1.2130
	11.07	.02	0.0372	0.4364
	17.60	.02	0.0461	0.8447
	21.43	.02	0.0451	0.9453
	25.30	.02	0.0118	0.2697
	0.0	---	0.3042	0.4414
.05	7.05	.0347	0.1056	0.7150
	11.07	.02	0.0364	0.4319
	17.60	.02	0.0451	0.8298
	21.43	.02	0.0456	0.9352
	25.30	.02	0.0119	0.2667
	0.0	---	0.3041	0.4412
0.1	7.05	.0344	0.0856	0.5698
	11.07	.02	0.0377	0.4377
	17.60	.02	0.0443	0.8119
	21.43	.02	0.0451	0.9431
	25.30	.02	0.0117	0.2650
	0.0	---	0.3041	0.4418

Table 9. **Auto Floor Spectra for Floor 3 for Structural Configuration "A".**

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra	
			AD	AV
.01	7.05	.0347	0.4498	3.1355
	11.07	.02	0.0523	0.4740
	17.60	.02	0.0216	0.3542
	21.43	.02	0.0379	0.7775
	25.30	.02	0.0167	0.4071
	0.0	---	0.3162	0.6274
.05	7.05	.0347	0.2445	1.6695
	11.07	.02	0.0496	0.4651
	17.60	.02	0.0195	0.3389
	21.43	.02	0.0365	0.7625
	25.30	.02	0.0161	0.4036
	0.0	---	0.3164	0.6271
0.1	7.05	.0344	0.1819	1.2049
	11.07	.02	0.0517	0.5007
	17.60	.02	0.0181	0.3236
	21.43	.02	0.0353	0.7515
	25.30	.02	0.0165	0.4011
	0.0	---	0.3161	0.6274

Table 10. Auto Floor Spectra for Floor 5 for Structural Configuration "A".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra	
			AD	AV
.01	7.05	.0347	0.5899	4.1225
	11.07	.02	0.0912	0.9094
	17.60	.02	0.0471	0.8354
	21.43	.02	0.0567	1.1667
	25.30	.02	0.0186	0.4197
	0.0	---	0.3246	0.7737
.05	7.05	.0347	0.3204	2.1999
	11.07	.02	0.0879	0.8995
	17.60	.02	0.0469	0.8178
	21.43	.02	0.0556	1.1472
	25.30	.02	0.0176	0.4173
	0.0	---	0.3246	0.7717
0.1	7.05	.0344	0.2731	1.5924
	11.07	.02	0.0891	0.9160
	17.60	.02	0.0458	0.7973
	21.43	.02	0.0555	1.1451
	25.30	.02	0.0186	0.4191
	0.0	---	0.3246	0.7707

Table 11. Cross Floor Spectra for Floors 1 and 3 for Structural Configuration "A".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra			
			CD	CV	QD	QV
.01	7.05	.0347	0.2756	1.8993	0.3574	2.5354
	11.07	.02	0.0068	-.2911	0.0756	0.8430
	17.60	.02	0.0264	0.4816	0.0712	1.3119
	21.43	.02	0.0389	0.8005	0.1139	2.3721
	25.30	.02	0.0029	-.1221	0.0467	1.1225
	0.0	---	0.3091	0.4778	0.0455	0.3956
.05	7.05	.0347	0.1506	1.0144	0.2387	1.6987
	11.07	.02	0.0095	-.3029	0.0901	1.0156
	17.60	.02	0.0252	0.4645	0.0703	1.2911
	21.43	.02	0.0378	0.7875	0.1120	2.3587
	25.30	.02	0.0027	-.1156	0.0464	1.1235
	0.0	---	0.3090	0.4772	0.0455	0.3939
0.1	7.05	.0344	0.1124	0.7324	0.1815	1.2935
	11.07	.02	0.0072	-.2733	0.1136	1.2821
	17.60	.02	0.0245	0.4447	0.0689	1.2678
	21.43	.02	0.0377	0.7842	0.1148	2.3919
	25.30	.02	0.0037	-.1067	0.0465	1.1308
	0.0	---	0.3092	0.4782	0.0455	0.3952

Table 12. Cross Floor Spectra for Floors 1 and 5 for Structural Configuration "A".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra CD	CV	QD	QV
.01	7.05	.0347	0.3129	2.1556	0.4400	3.1012
	11.07	.02	-.0465	-.5467	0.0889	0.9034
	17.60	.02	-.0455	-.8276	-.0334	-.7278
	21.43	.02	-.0490	-1.0259	-.1023	-2.1634
	25.30	.02	-.0086	-.4088	-.0438	-1.0788
	0.0	---	0.3102	0.4812	0.0535	0.3523
.05	7.05	.0347	0.1686	1.1267	0.2936	2.0658
	11.07	.02	-.0512	-.5487	0.1056	1.1289
	17.60	.02	-.0445	-.8117	-.0312	-.7008
	21.43	.02	-.0491	-1.0123	-.1019	-2.1348
	25.30	.02	-.0092	-.1413	-.0435	-1.0792
	0.0	---	0.3118	0.4822	0.0538	0.3521
0.1	7.05	.0344	0.1238	0.7913	0.2223	1.5631
	11.07	.02	-.0489	-.5318	0.1335	1.4748
	17.60	.02	-.0432	-.7939	-.0298	-.6673
	21.43	.02	-.0486	-1.0167	-.1017	-2.1432
	25.30	.02	-.0083	-.1189	-.0445	-1.0902
	0.0	---	0.3112	0.4827	0.0531	0.3581

Table 13. Cross Floor Spectra for Floors 3 and 5 for Structural Configuration "A".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra CD	CV	QD	QV
.01	7.05	.0347	0.5147	3.5908	0.2661	1.8892
	11.07	.02	0.0677	0.6125	0.0646	0.7243
	17.60	.02	-.0249	-.4903	0.0649	1.1678
	21.43	.02	-.0446	-.9374	0.0856	1.7764
	25.30	.02	-.0165	-.4032	0.0251	0.5632
	0.0	---	0.3206	0.6779	0.0344	0.2998
.05	7.05	.0347	0.2793	1.9087	0.1778	1.2645
	11.07	.02	0.0643	0.6023	0.0798	0.8956
	17.60	.02	-.0243	-.4756	0.0637	1.1532
	21.43	.02	-.0438	-.9309	0.0856	1.7815
	25.30	.02	-.0173	-.4035	0.0252	0.5690
	0.0	---	0.3205	0.6767	0.0341	0.2988
0.1	7.05	.0344	0.2077	1.3745	0.1345	0.9677
	11.07	.02	0.0656	0.6298	0.1024	1.1476
	17.60	.02	-.0234	-.4534	0.0629	1.1378
	21.43	.02	-.0428	-.9129	0.0882	1.8412
	25.30	.02	-.0168	-.4012	0.0251	0.5727
	0.0	---	0.3203	0.6769	0.0345	0.3023

Table 14. Auto Floor Spectra for Ground for Structural Configuration "B".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra AD	Floor Spectra AV
.01	6.98	.0350	0.0781	0.5763
	11.46	.02	0.0566	0.6498
	15.52	.02	0.0365	0.5584
	19.08	.02	0.0260	0.4806
	0.0	---	0.3000	0.4000
.05	6.98	.0349	0.0782	0.5771
	11.46	.02	0.0566	0.6498
	15.52	.02	0.0365	0.5584
	19.08	.02	0.0260	0.4806
	0.0	---	0.3000	0.4000
0.1	6.98	.0347	0.0783	0.5771
	11.46	.02	0.0566	0.6498
	15.52	.02	0.0365	0.5584
	19.08	.02	0.0260	0.4806
	0.0	---	0.3000	0.4000

Table 15. Auto Floor Spectra for Floor 2 for Structural Configuration "B".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra AD	Floor Spectra AV
.01	6.98	.0350	0.4159	2.8867
	11.46	.02	0.0316	0.3155
	15.52	.02	0.0312	0.5081
	19.08	.02	0.0601	1.1679
	0.0	---	0.3108	0.5286
.05	6.98	.0349	0.2559	1.7570
	11.46	.02	0.0317	0.3442
	15.52	.02	0.0300	0.4989
	19.08	.02	0.0588	1.1460
	0.0	---	0.3108	0.5238
0.1	6.98	.0347	0.1961	1.3245
	11.46	.02	0.0368	0.4237
	15.52	.02	0.0294	0.4905
	19.08	.02	0.0574	1.1201
	0.0	---	0.3107	0.5230

Table 16. Auto Floor Spectra for Floor 4 for Structural Configuration "B".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra AD	Floor Spectra AV
.01	6.98	.0350	0.6940	4.8346
	11.46	.02	0.0736	0.7041
	15.52	.02	0.0308	0.3900
	19.08	.02	0.0256	0.4358
	0.0	---	0.3223	0.7040
.05	6.98	.0349	0.4264	2.9458
	11.46	.02	0.0717	0.6970
	15.52	.02	0.0287	0.3797
	19.08	.02	0.0244	0.4267
	0.0	---	0.3221	0.6880
0.1	6.98	.0347	0.3266	2.2225
	11.46	.02	0.0715	0.6983
	15.52	.02	0.0281	0.3755
	19.08	.02	0.0239	0.4178
	0.0	---	0.3220	0.6855

Table 17. Cross Floor Spectra for Ground and Floor 2 for Structural Configuration "B".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra CD	Floor Spectra CV	Floor Spectra QD	Floor Spectra QV
.01	6.98	.0350	0.0664	0.3809	-.4245	-2.9901
	11.46	.02	0.0140	0.2102	-.0764	-.9147
	15.52	.02	0.0289	0.4573	-.0642	-1.0820
	19.08	.02	0.0291	0.5375	-.1308	-2.5597
	0.0	---	0.3031	0.3749	-.0477	-.4417
.05	6.98	.0349	0.0428	0.2215	-.3339	-2.3572
	11.46	.02	0.0135	0.2058	-.1017	-1.2157
	15.52	.02	0.0287	0.4536	-.0639	-1.0749
	19.08	.02	0.0289	0.5338	-.1297	-2.5397
	0.0	---	0.3032	0.3750	-.0479	-.4447
0.1	6.98	.0347	0.0380	0.2033	-.2744	-1.9369
	11.46	.02	0.0128	0.1979	-.1267	-1.5120
	15.52	.02	0.0284	0.4492	-.0634	-1.0660
	19.08	.02	0.0286	0.5291	-.1284	-2.5144
	0.0	---	0.3031	0.3751	-.0480	-.4482

Table 18. Cross Floor Spectra for Ground and Floor 4 for Structural Configuration "B".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra CD	CV	QD	QV
.01	6.98	.0350	0.0480	0.2674	-.5498	-3.8488
	11.46	.02	-.0553	-.6201	-.0819	-.8404
	15.52	.02	-.0295	-.4447	-.0224	-.1228
	19.08	.02	-.0218	-.3967	0.0650	1.3033
	0.0	---	0.3047	0.3442	-.0579	-.3921
.05	6.98	.0349	-.0157	0.3282	-.4314	-3.0220
	11.46	.02	-.0553	-.6198	-.1035	-1.1039
	15.52	.02	-.0295	-.4437	-.0226	0.0984
	19.08	.02	-.0217	-.3944	0.0646	1.2952
	0.0	---	0.3047	0.3443	-.4909	-.3938
0.1	6.98	.0347	-.0365	-.2835	-.3536	-2.4705
	11.46	.02	-.0553	-.6197	-.1267	-1.3770
	15.52	.02	-.0294	-.4425	-.0229	0.0641
	19.08	.02	-.0215	-.3914	0.0640	1.2847
	0.0	---	0.3047	0.3445	-.0583	-.3955

Table 19. Cross Floor Spectra for Floors 2 and 4 for Structural Configuration "B".

Mass Ratio	Natural Frequency	Damping Ratio	Floor Spectra CD	CV	QD	QV
.01	6.98	.0350	0.5361	3.7245	0.3684	2.5913
	11.46	.02	0.0308	0.3055	0.0722	0.8246
	15.52	.02	-.0225	-.3809	0.0501	0.8046
	19.08	.02	-.0343	-.0681	0.0765	1.4817
	0.0	---	0.3154	0.5799	0.0414	0.3482
.05	6.98	.0349	0.3287	2.2592	0.2846	2.0067
	11.46	.02	0.0248	0.3525	0.0965	1.1052
	15.52	.02	-.0258	-.3819	0.0499	0.7993
	19.08	.02	-.0340	-.6694	0.0750	1.4543
	0.0	---	0.3153	0.5714	0.0415	0.3479
0.1	6.98	.0347	0.2510	1.6956	0.2331	1.6432
	11.46	.02	0.0286	0.2519	0.1202	1.3784
	15.52	.02	-.0262	-.3779	-.0497	-.7924
	19.08	.02	-.0333	-.6548	0.0733	1.4207
	0.0	---	0.3153	0.5701	0.0417	0.3510

Table 20. Force Response Configuration "A" - [Kips].

Mass Ratio m/M (1)	Elem. (2)	Combined System (3)	Interaction Excluded		Interaction Included	
			Response (4)	Ratio (5)	Response (6)	Ratio (7)
.01	10-9	4.859	5.552	1.142	4.780	0.983
	9-8	3.093	3.319	1.073	3.062	0.989
	8-7	2.928	3.403	1.162	2.982	1.018
	7-6	5.080	5.597	1.101	4.925	0.969
	10-5	10.014	12.370	1.122	10.696	0.970
	8-3	12.200	13.737	1.125	11.869	0.972
	6-1	11.151	12.502	1.120	10.853	0.972
.05	10-9	13.213	27.741	2.092	12.863	0.970
	9-8	10.095	16.548	1.635	10.799	1.070
	8-7	9.510	16.969	1.784	9.586	1.007
	7-6	14.569	27.970	1.919	14.661	1.006
	10-5	29.690	61.942	2.086	28.786	0.969
	8-3	32.961	68.828	2.088	32.116	0.974
	6-1	30.848	62.601	2.029	30.535	0.989
0.1	10-9	19.088	55.457	2.905	18.618	0.975
	9-8	17.264	33.117	1.918	19.359	1.120
	8-7	16.261	33.959	2.088	16.265	1.000
	7-6	22.287	55.916	2.508	23.903	1.072
	10-5	42.294	123.598	2.922	41.941	0.991
	8-3	47.058	137.592	2.916	47.188	1.000
	6-1	45.387	124.915	2.748	46.957	1.033

Table 21. Force Response Configuration "B" - [Kips].

Mass Ratio m/M (1)	Elem. (2)	Combined System (3)	Interaction Excluded Response (4)	Interaction Excluded Ratio (5)	Interaction Included Response (6)	Interaction Included Ratio (7)
.01	9-8	3.010	2.804	0.931	2.930	0.973
	8-7	6.034	5.310	0.880	5.747	0.952
	7-6	3.707	3.281	0.885	3.622	0.977
	6-0	7.609	6.845	0.899	7.342	0.965
	9-4	10.288	9.187	0.892	9.776	0.950
	7-2	8.142	7.295	0.895	7.706	0.946
.05	9-8	8.930	14.030	1.571	9.324	1.044
	8-7	18.338	26.579	1.449	18.542	1.011
	7-6	11.695	16.403	1.372	12.653	1.058
	6-0	23.072	34.087	1.477	23.672	1.026
	9-4	29.933	46.089	1.539	29.965	1.001
	7-2	23.742	36.553	1.538	23.017	0.969
0.1	9-8	13.531	28.081	2.075	14.045	1.037
	8-7	28.380	52.964	1.866	29.617	1.043
	7-6	18.714	32.781	1.751	21.126	1.128
	6-0	35.582	68.354	1.921	38.196	1.073
	9-4	44.176	92.020	2.083	45.041	1.019
	7-2	35.115	73.056	2.081	35.137	1.001

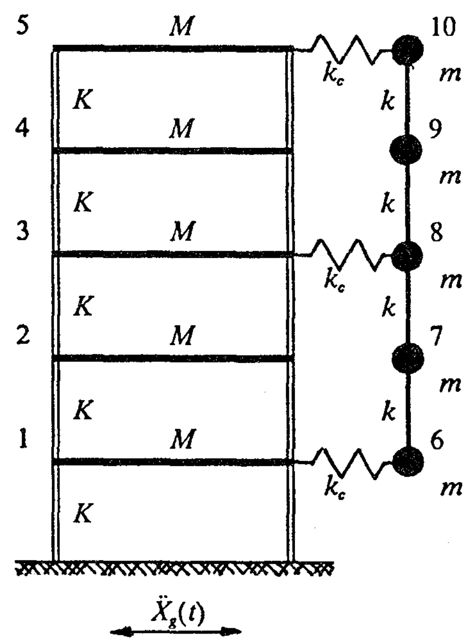

Figure 1. Structural Configuration "A".

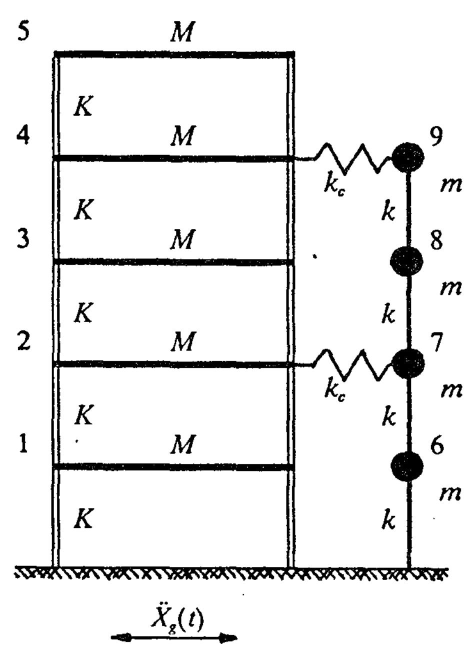

Figure 2. Structural Configuration "B".

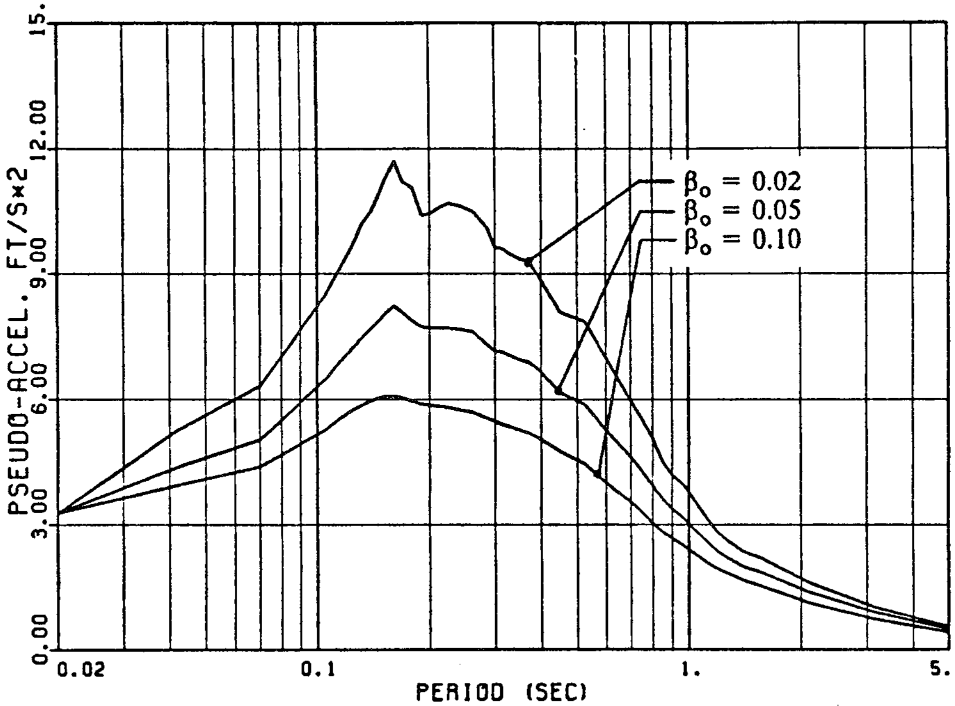

Figure 3. Auto Pseudo-acceleration Ground Spectrum.

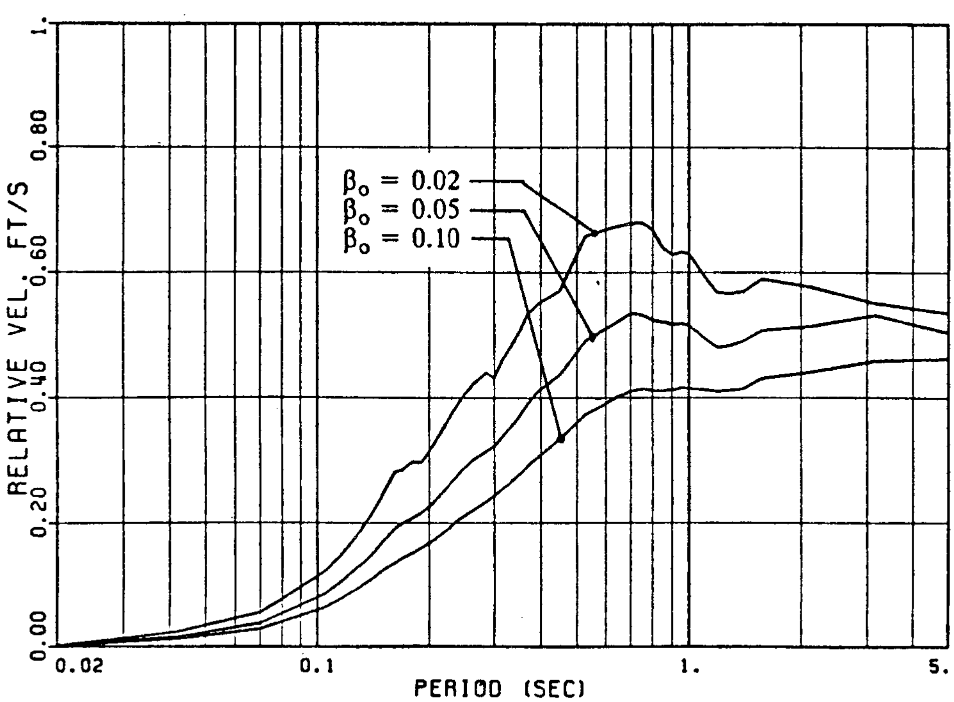

Figure 4. Auto Relative Velocity Ground Spectrum.

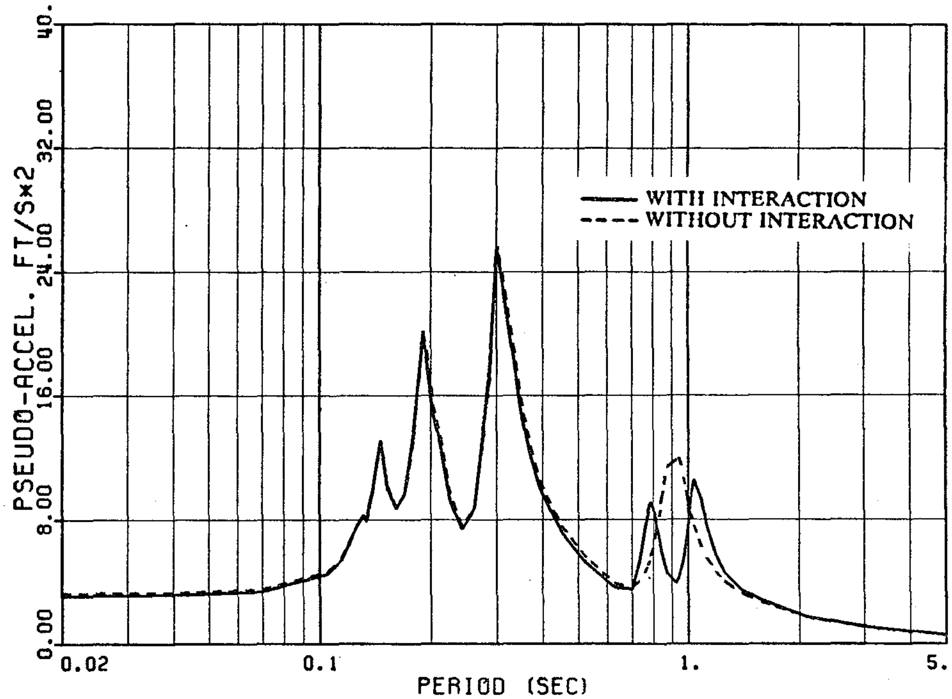

Figure 5. Pseudo-acceleration Spectrum for Floor 1 With and Without Interaction for Configuration "A".

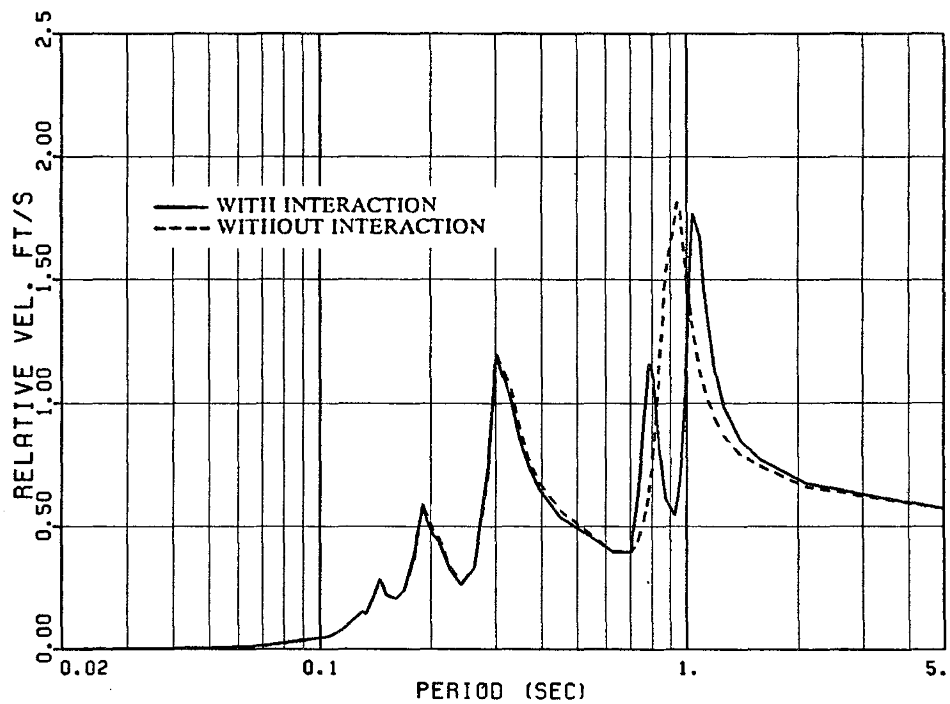

Figure 6. Relative Velocity Spectrum for Floor 1 With and Without Interaction for Configuration "A".

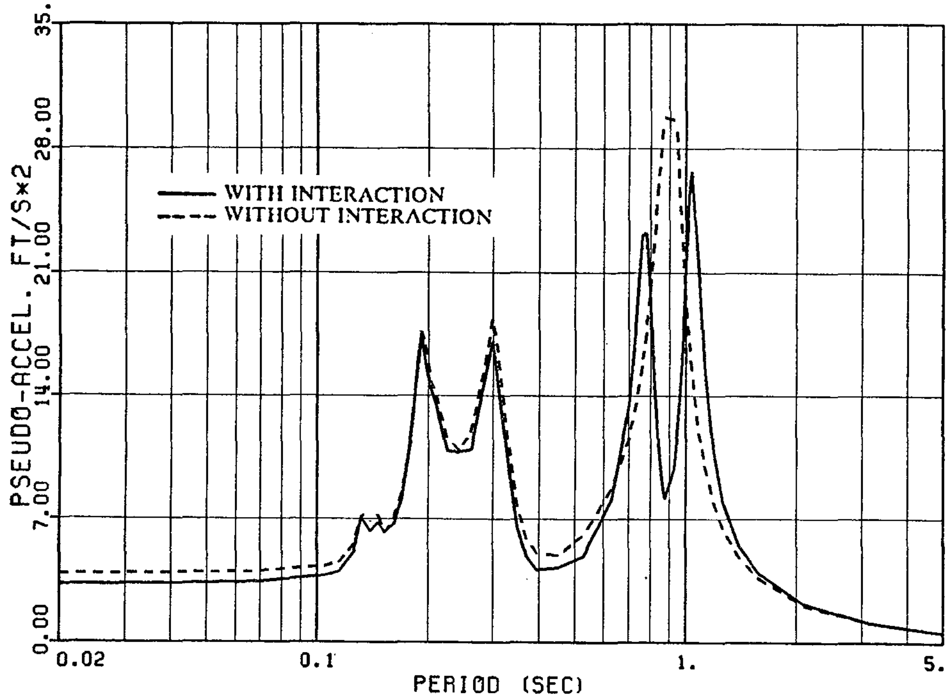

Figure 7. Pseudo-acceleration Spectrum for Floor 3 With and Without Interaction for Configuration "A".

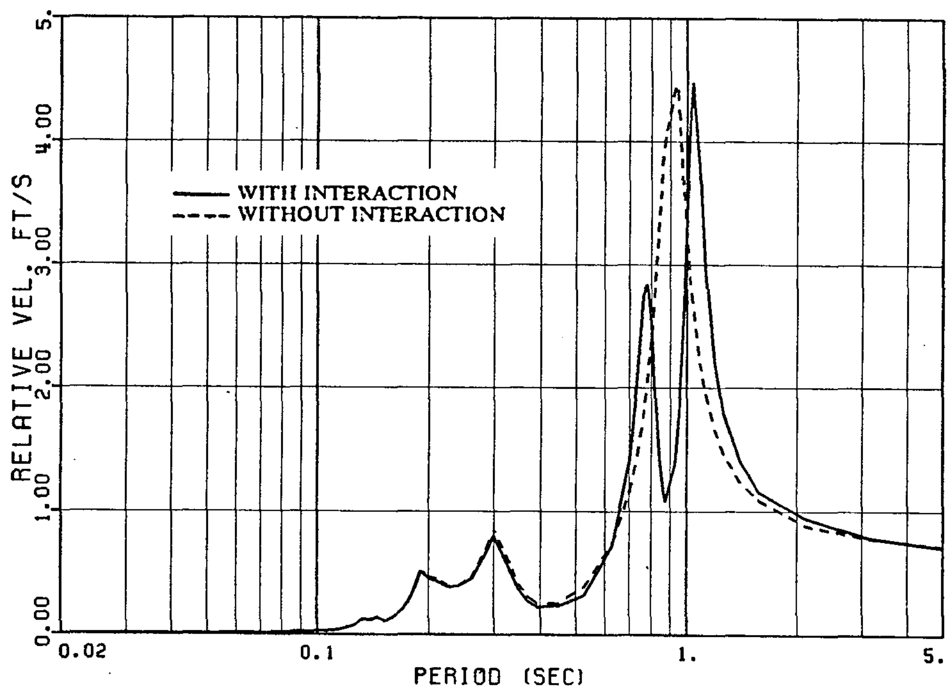

Figure 8. Relative Velocity Spectrum for Floor 3 With and Without Interaction for Configuration "A".

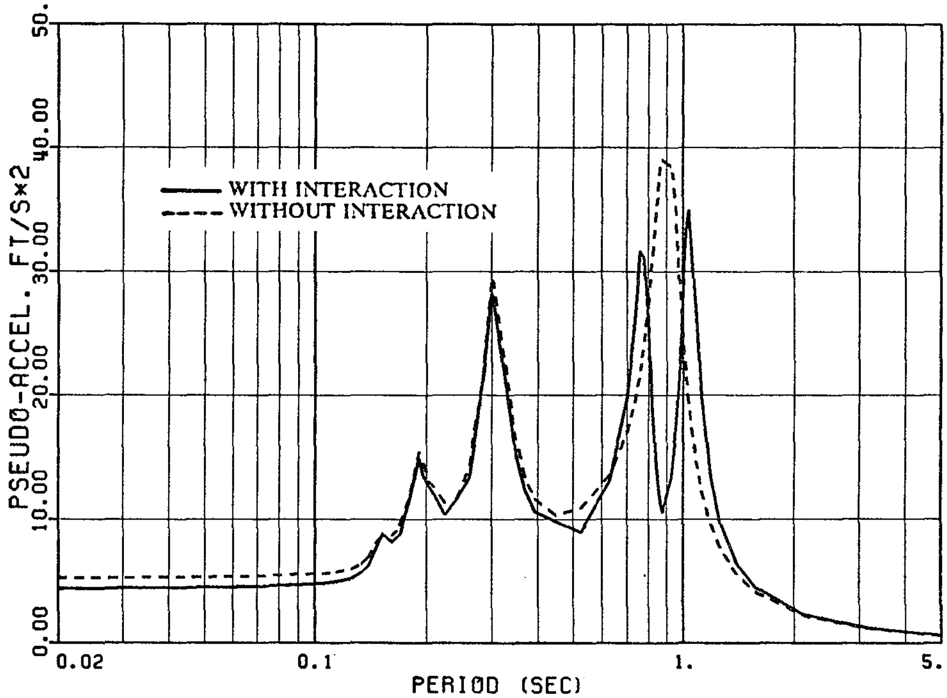

Figure 9. Pseudo-acceleration Spectrum for Floor 5 With and Without Interaction for Configuration "A".

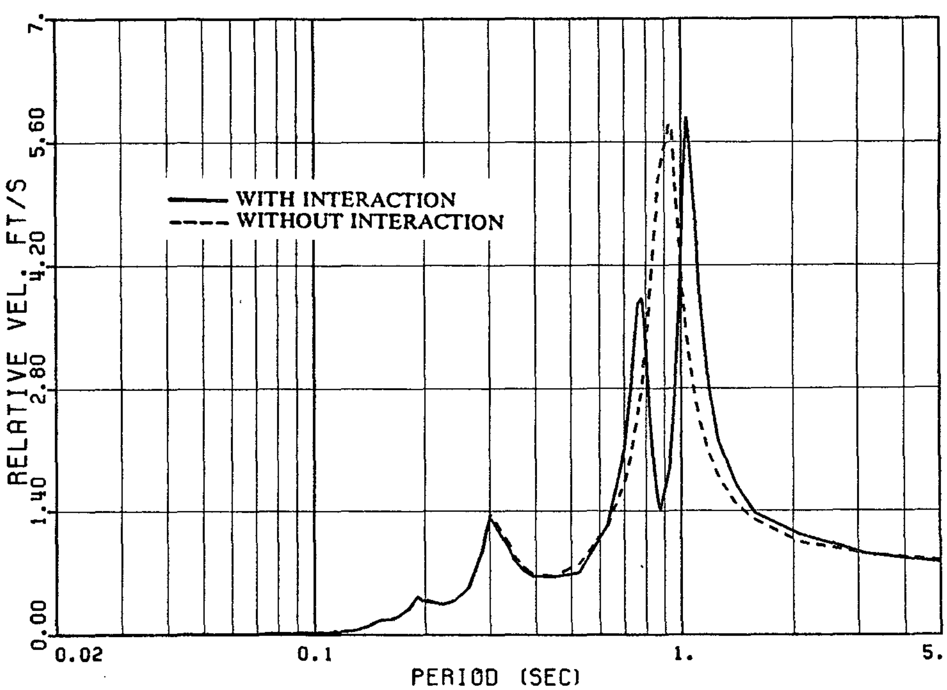

Figure 10. Relative Velocity Spectrum for Floor 5 With and Without Interaction for Configuration "A".

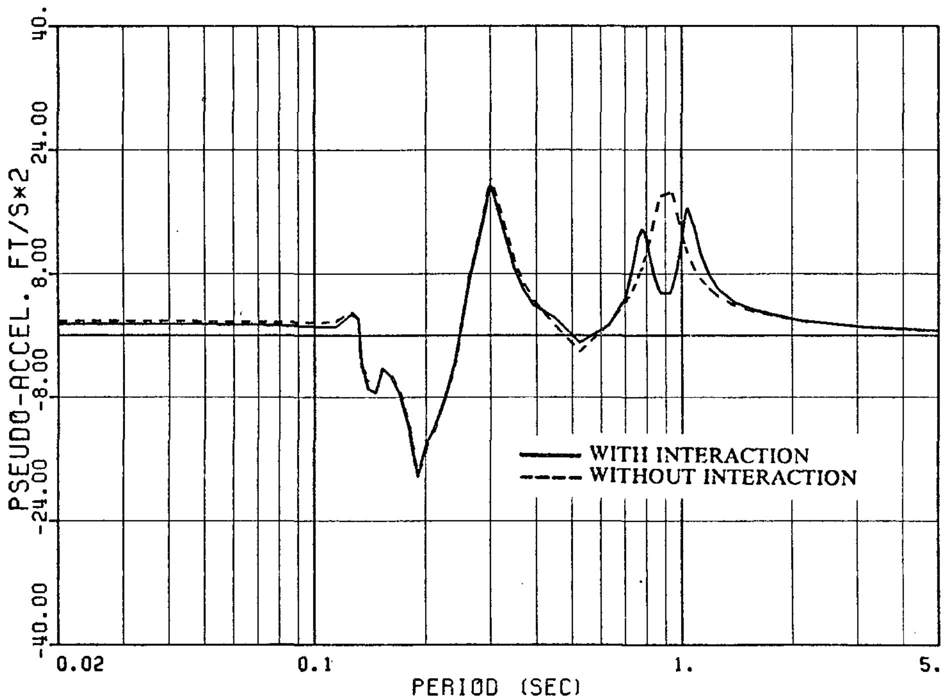

Figure 11. Coincident Pseudo-acceleration Spectrum for Floors 1 and 3 With and Without Interaction for Configuration "A"

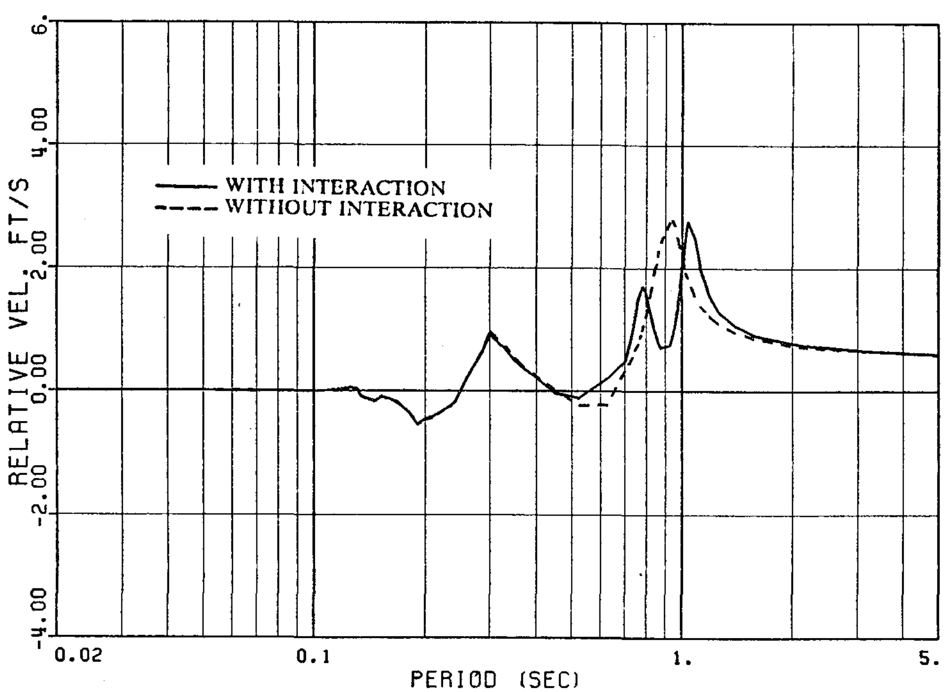

Figure 12. Coincident Relative Velocity Spectrum for Floors 1 and 3 With and Without Interaction for Configuration "A".

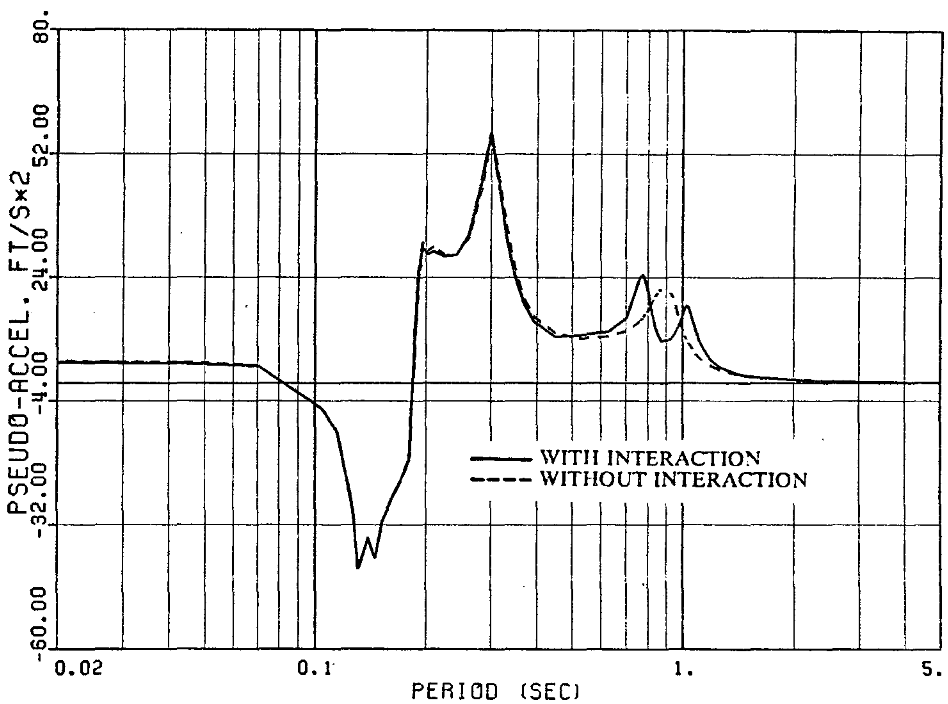

Figure 13. Quadrature Pseudo-acceleration Spectrum for Floor 1 and 3 With and Without Interaction for Configuration "A".

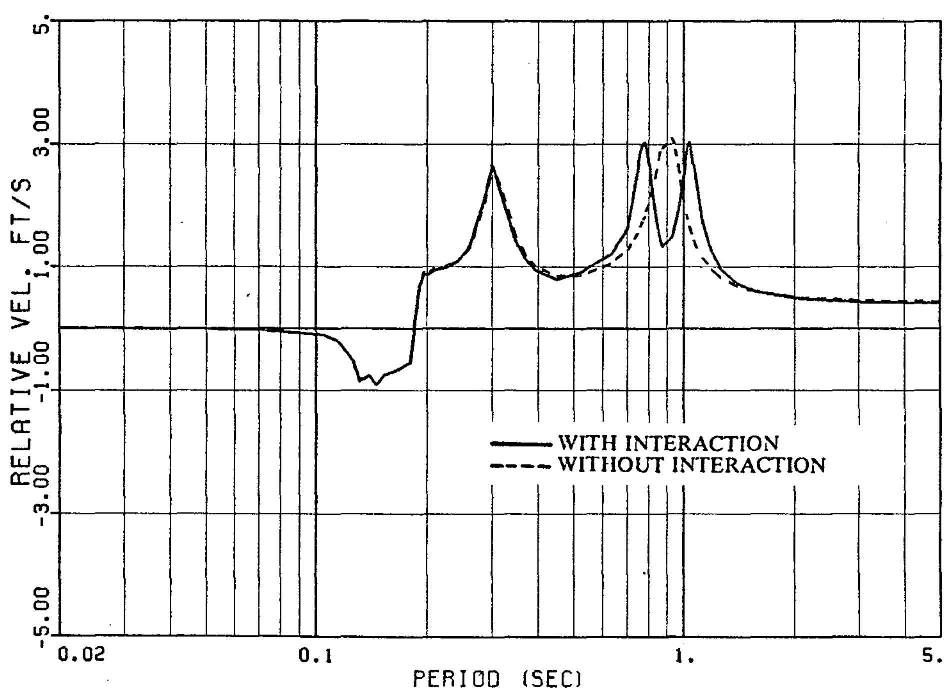

Figure 14. Quadrature Relative Velocity Spectrum for Floor 1 and 3 With and Without Interaction for Configuration "A".

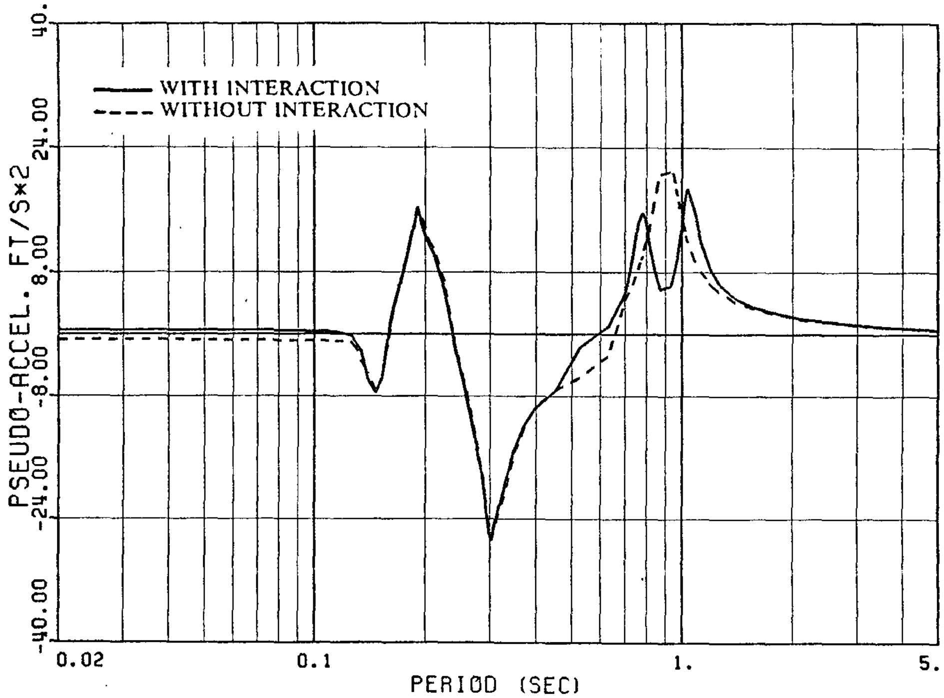

Figure 15. Coincident Pseudo-acceleration Spectrum for Floors 1 and 5 With and Without Interaction for Configuration "A"

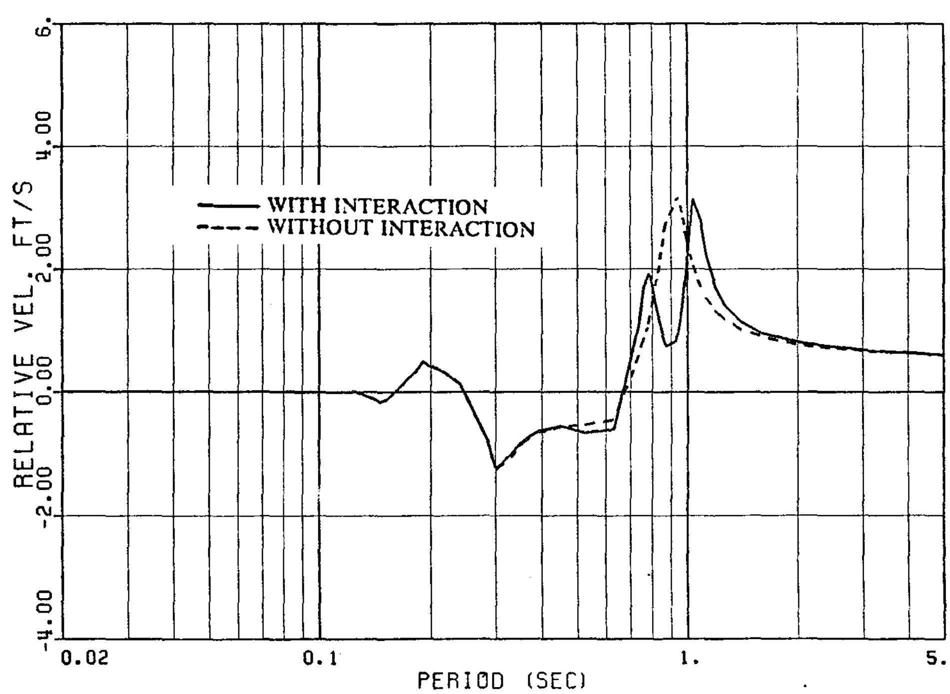

Figure 16. Coincident Relative Velocity Spectrum for Floors 1 and 5 With and Without Interaction for Configuration "A".

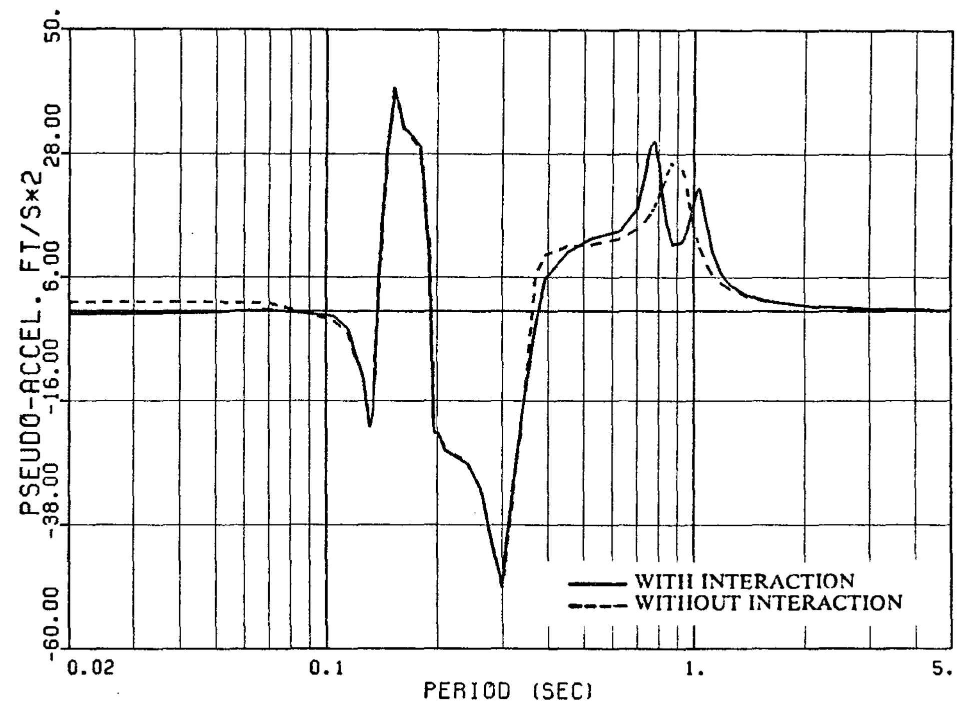

Figure 17. Quadrature Pseudo-acceleration Spectrum for Floor 1 and 5 With and Without Interaction for Configuration "A".

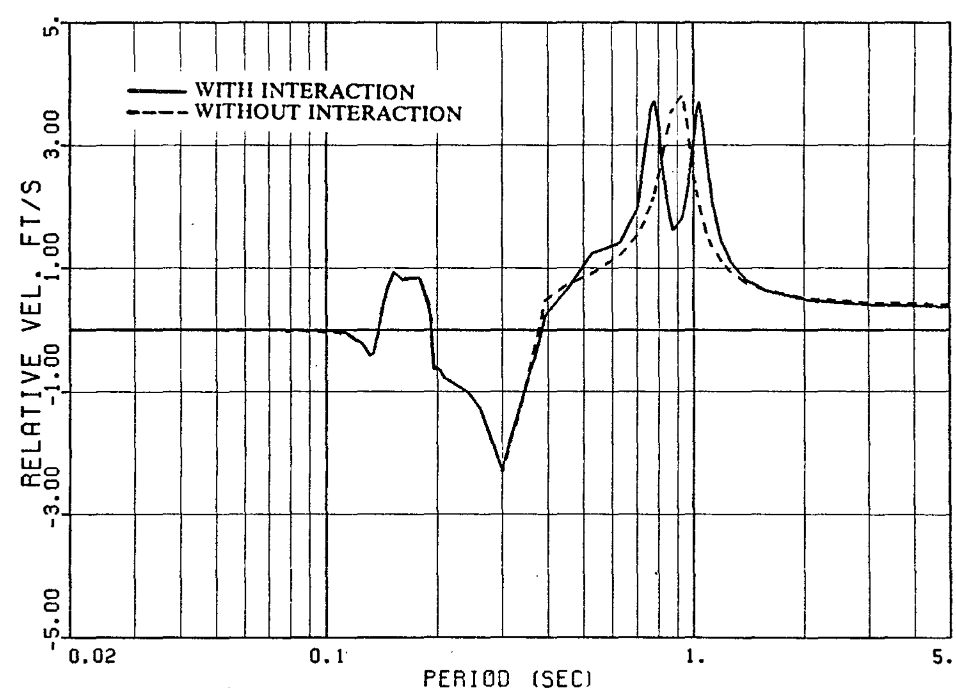

Figure 18. Quadrature Relative Velocity Spectrum for Floor 1 and 5 With and Without Interaction for Configuration "A".

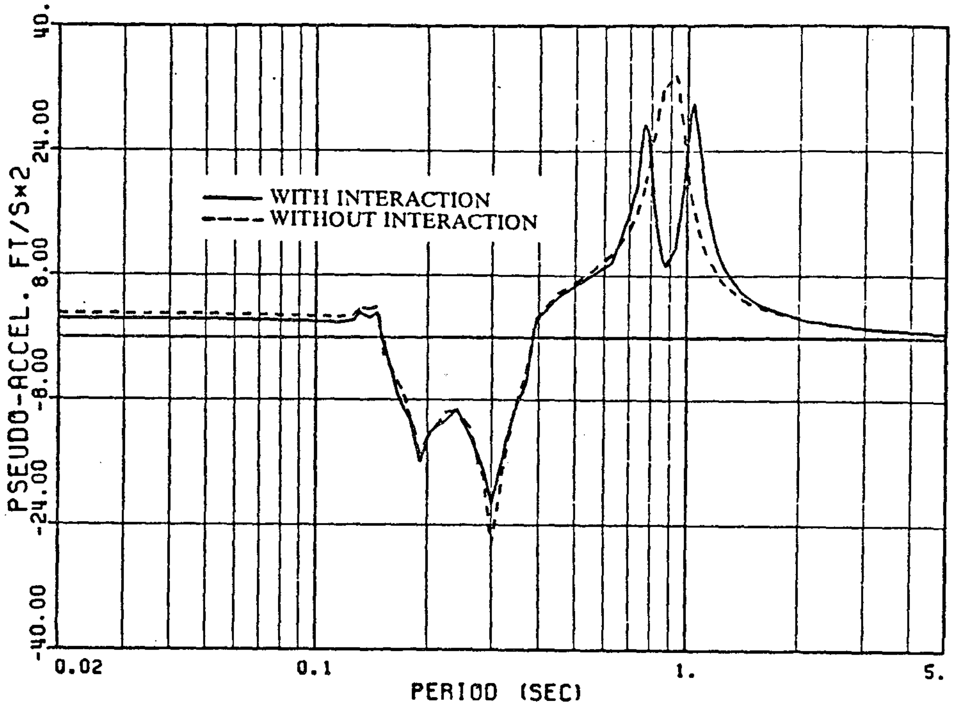

Figure 19. Coincident Pseudo-acceleration Spectrum for Floors 3 and 5 With and Without Interaction for Configuration "A"

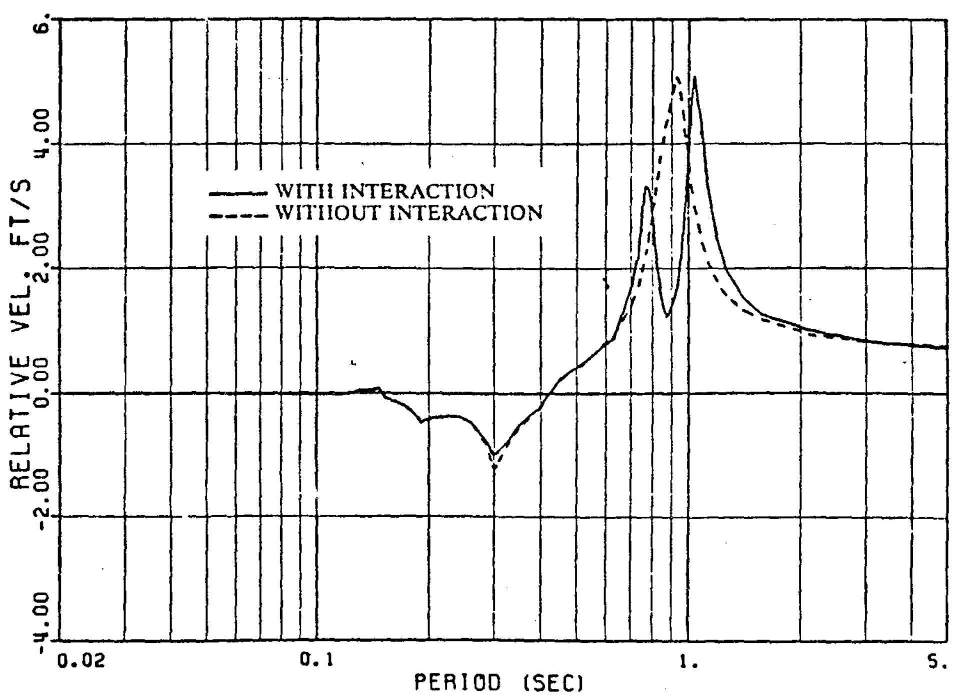

Figure 20. Coincident Relative Velocity Spectrum for Floors 3 and 5 With and Without Interaction for Configuration "A".

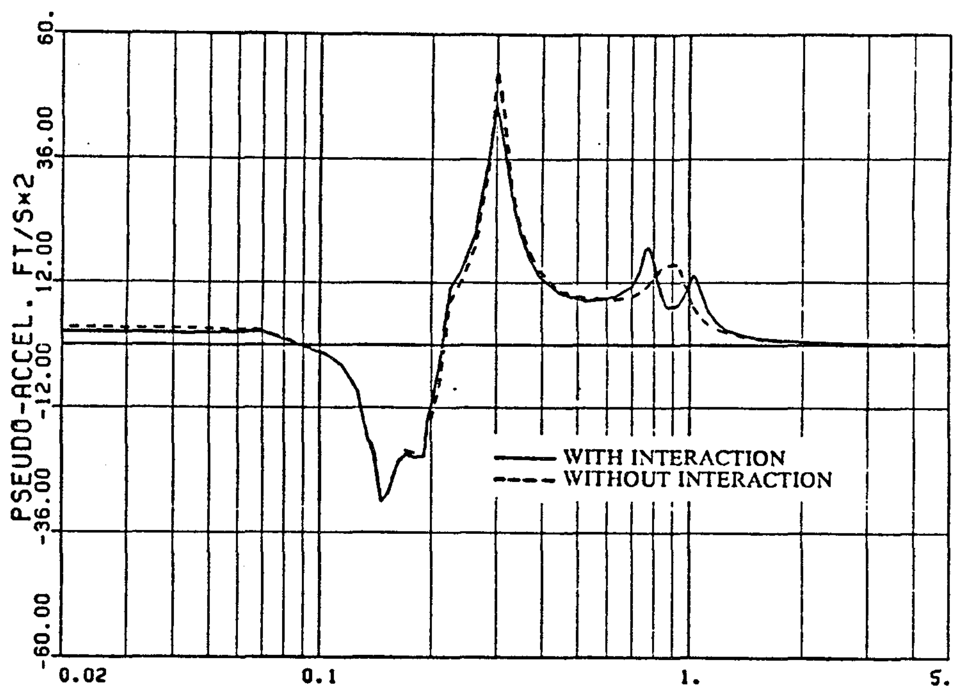

Figure 21. Quadrature Pseudo-acceleration Spectrum for Floor 3 and 5 With and Without Interaction for Configuration "A".

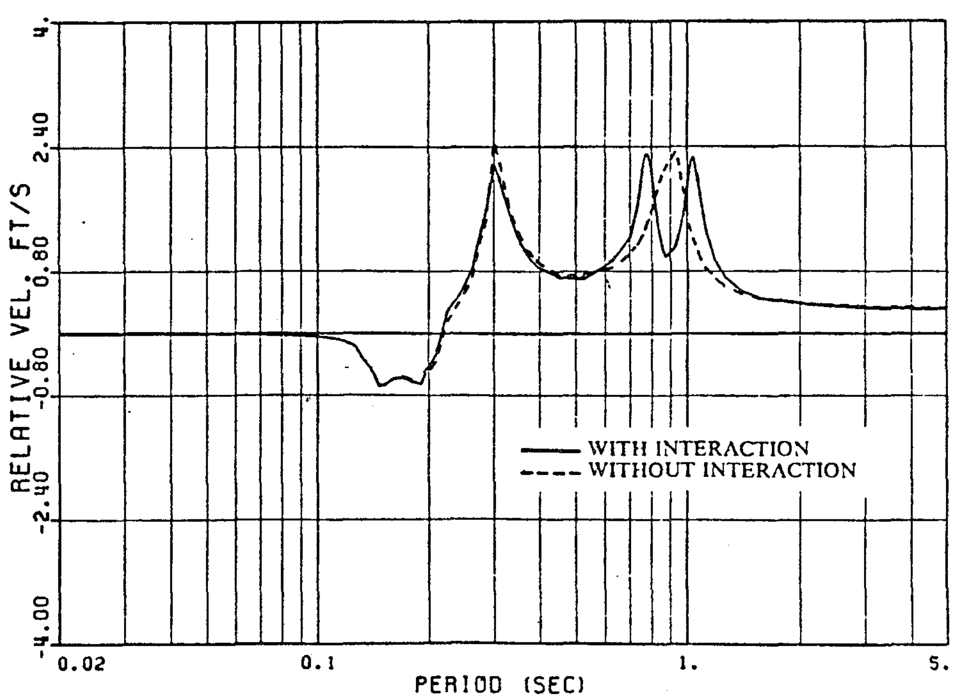

Figure 22. Quadrature Relative Velocity Spectrum for Floor3 and 5 With and Without Interaction for Configuration "A".

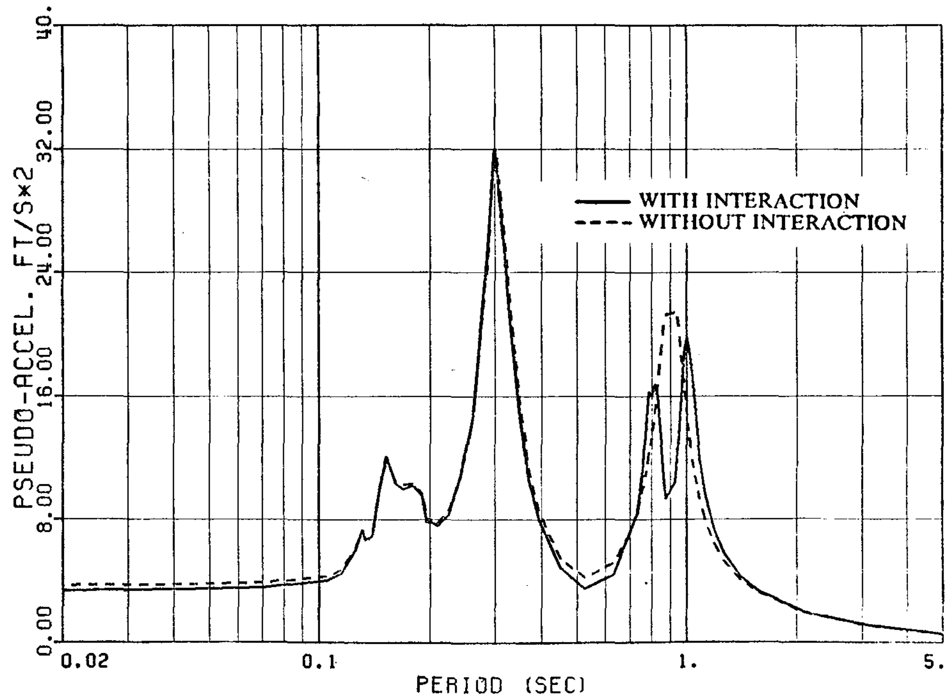

Figure 23. Pseudo-acceleration Spectrum for Floor 2 With and Without Interaction for Configuration "B".

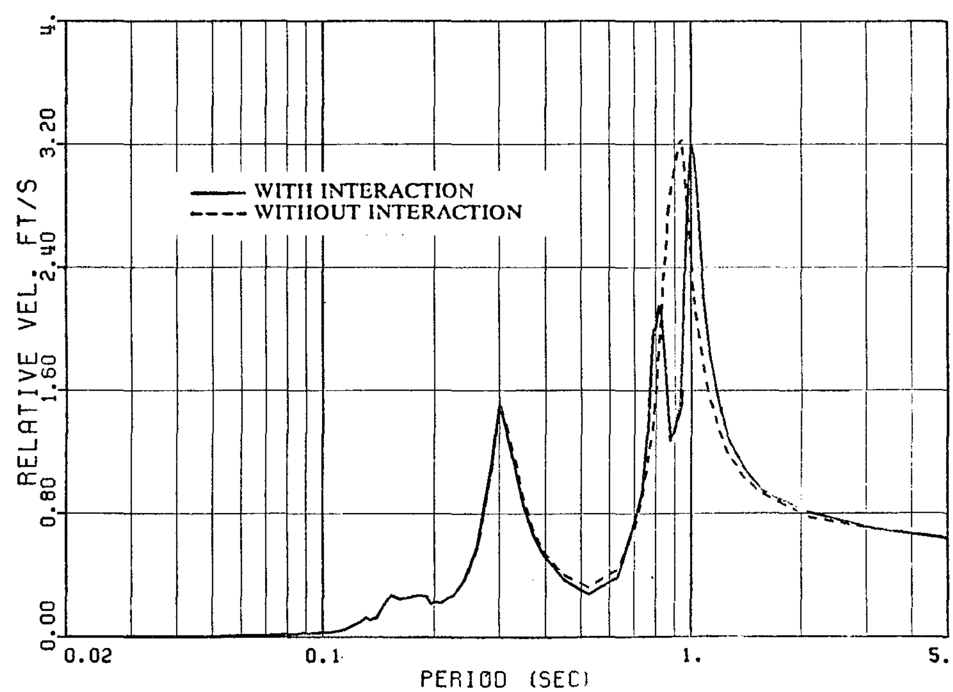

Figure 24. Relative Velocity Spectrum for Floor 2 With and Without Interaction for Configuration "B".

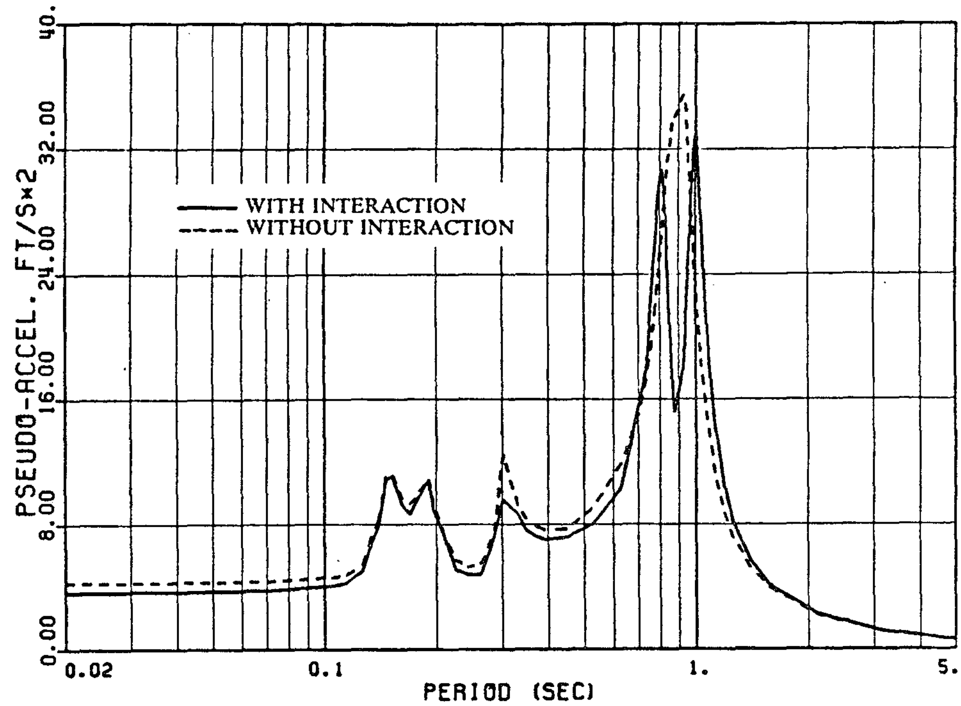

Figure 25. Pseudo-acceleration Spectrum for Floor 4 With and Without Interaction for Configuration "B".

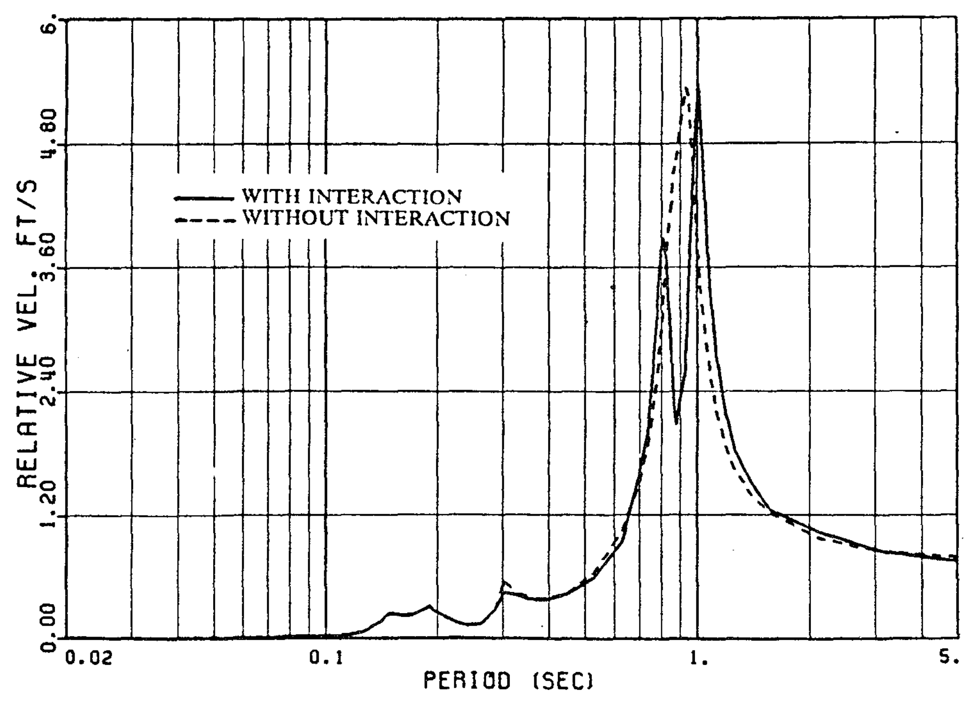

Figure 26. Relative Velocity Spectrum for Floor 4 With and Without Interaction for Configuration "B".

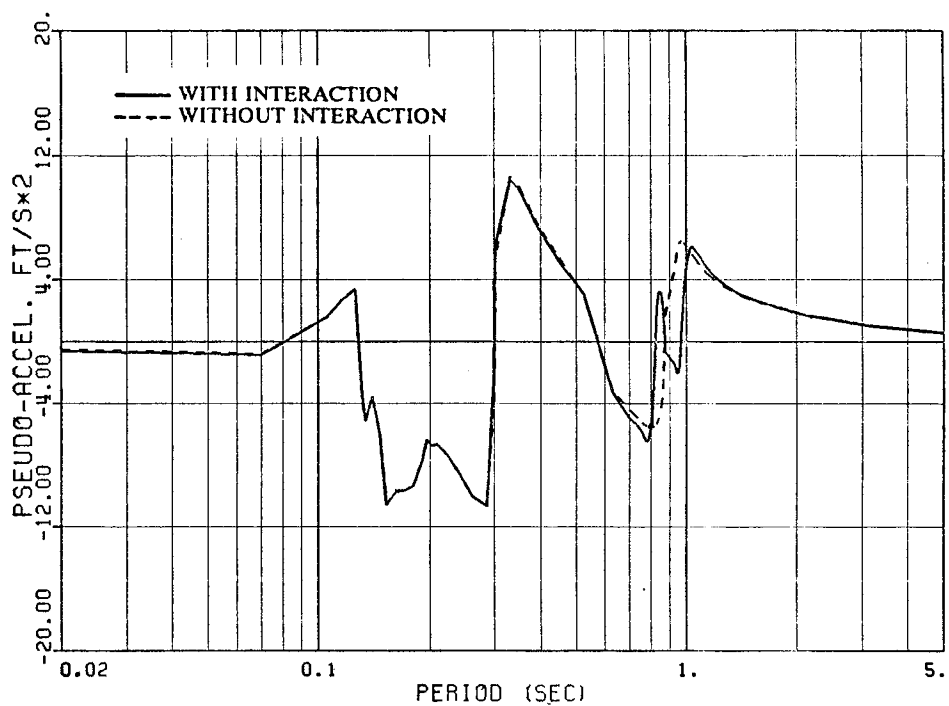

Figure 27. Coincident Pseudo-acceleration Spectrum for Ground and Floor 2 With and Without Interaction for Configuration "B".

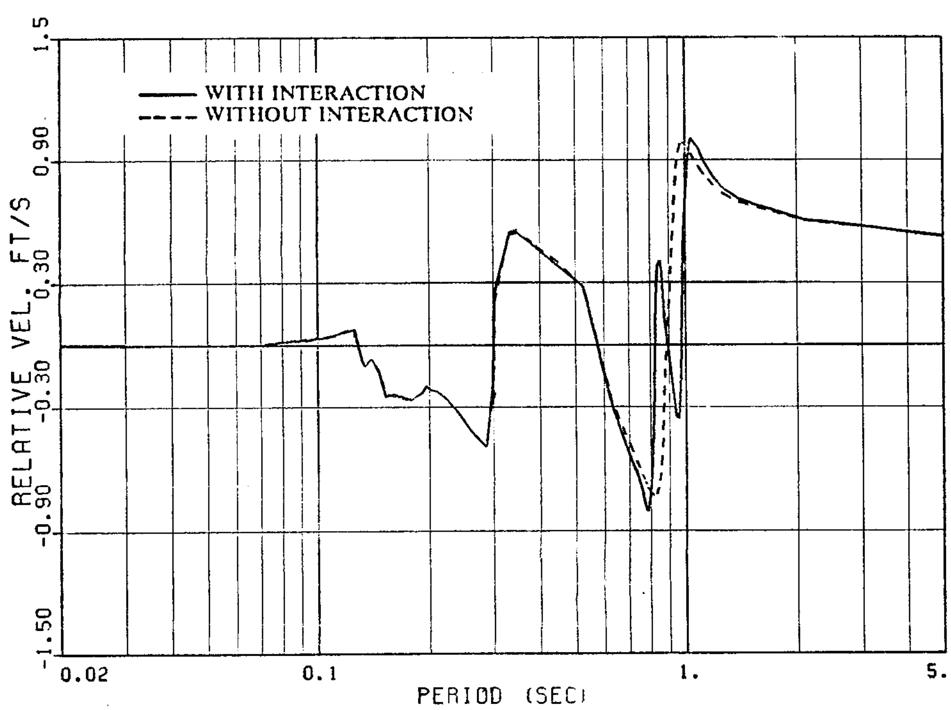

Figure 28. Coincident Relative Velocity Spectrum for Ground and Floor 2 With and Without Interaction for Configuration "B".

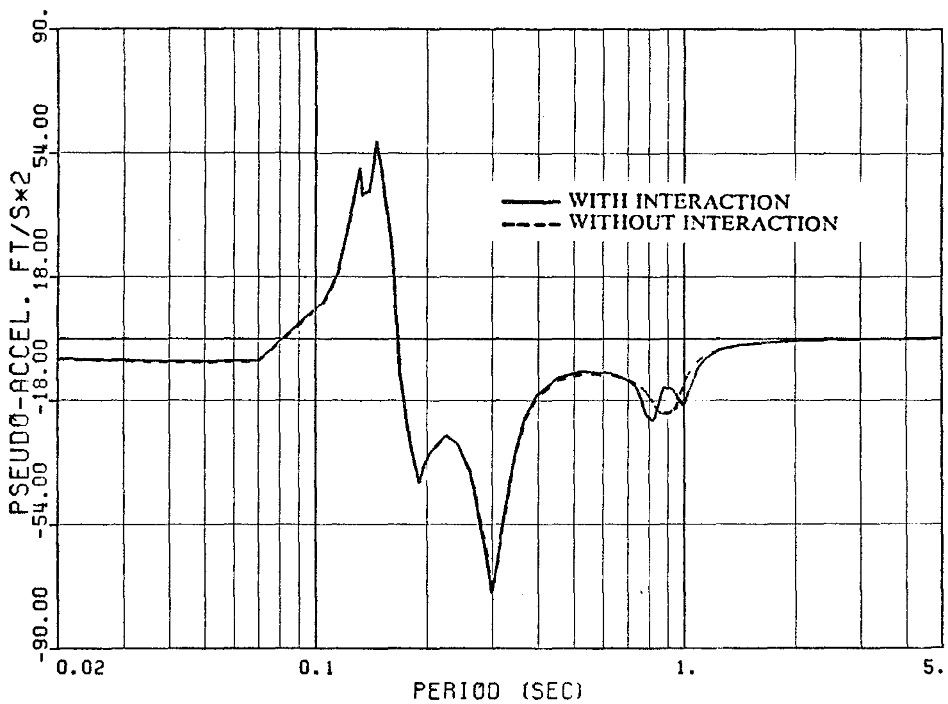

Figure 29. Quadrature Pseudo-acceleration Spectrum for Ground and Floor 2 With and Without Interaction for Configuration "B".

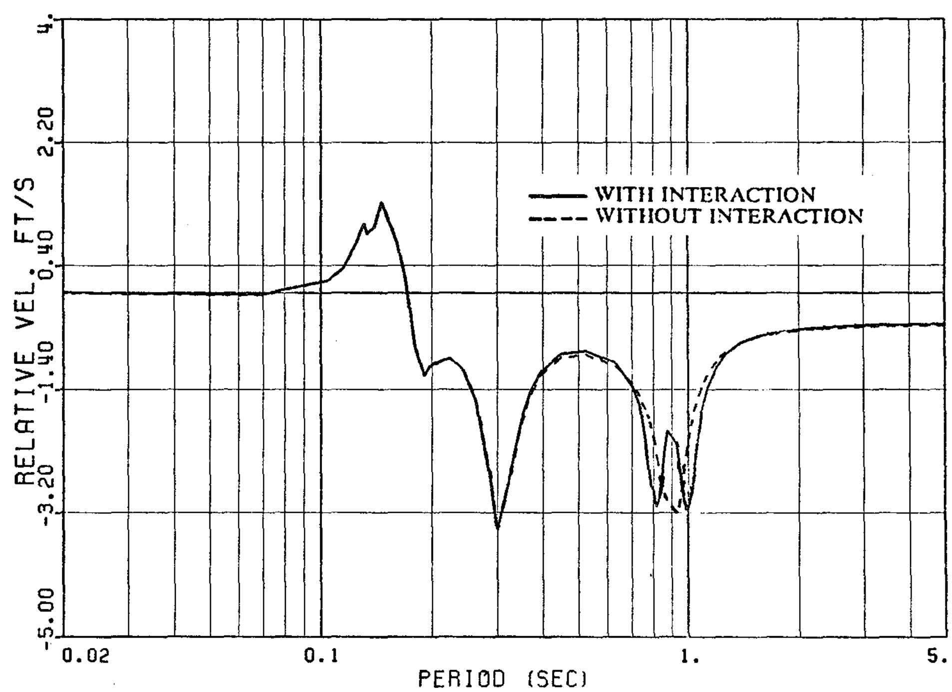

Figure 30. Quadrature Relative Velocity Spectrum for Ground and Floor 2 With and Without Interaction for Configuration "B".

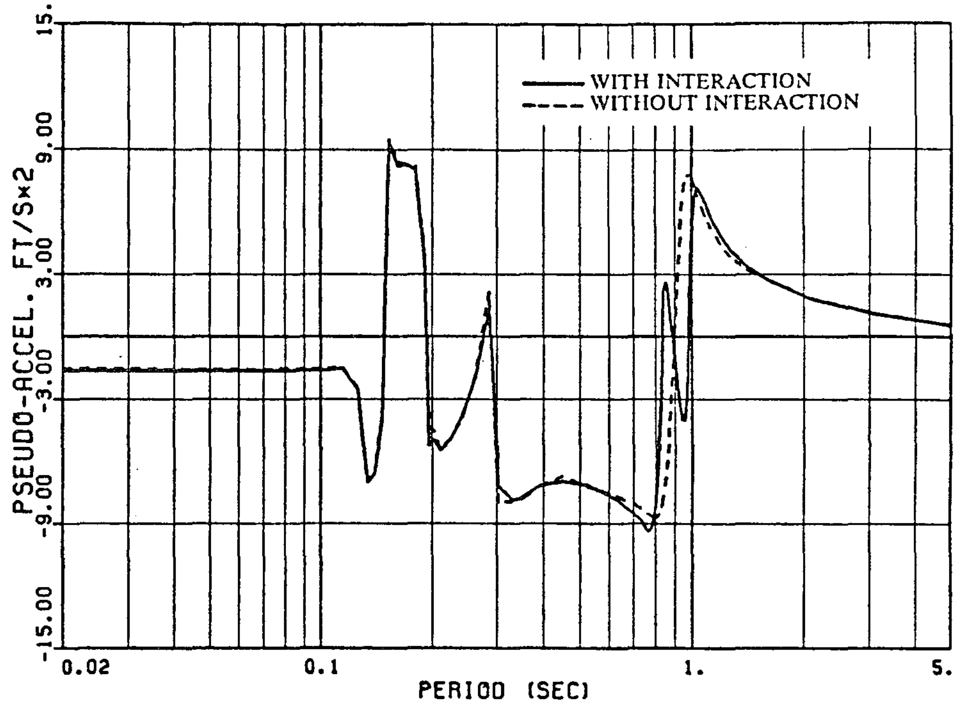

Figure 31. Coincident Pseudo-acceleration Spectrum for Ground and Floor 4 With and Without Interaction for Configuration "B".

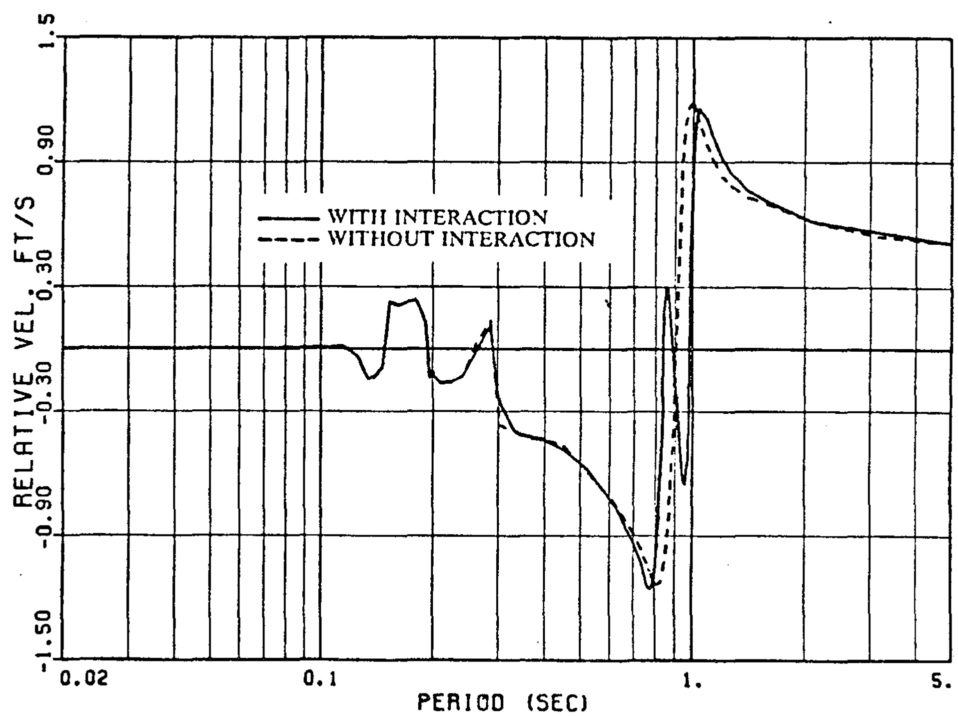

Figure 32. Coincident Relative Velocity Spectrum for Ground and Floor 4 With and Without Interaction for Configuration "B".

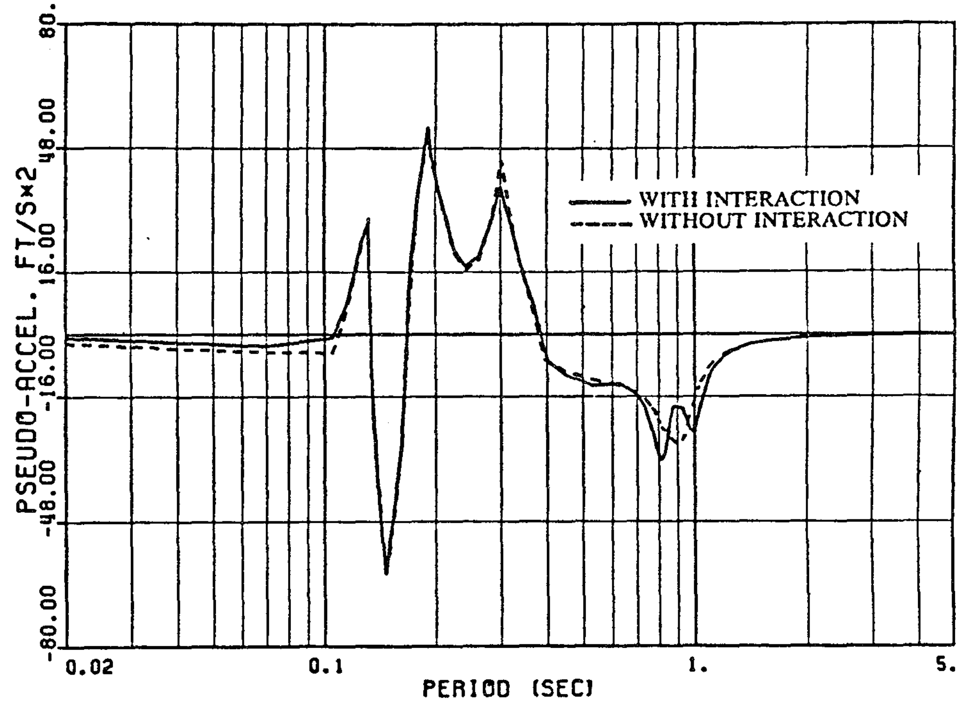

Figure 33. Quadrature Pseudo-acceleration Spectrum for Ground and Floor 4 With and Without Interaction for Configuration "B".

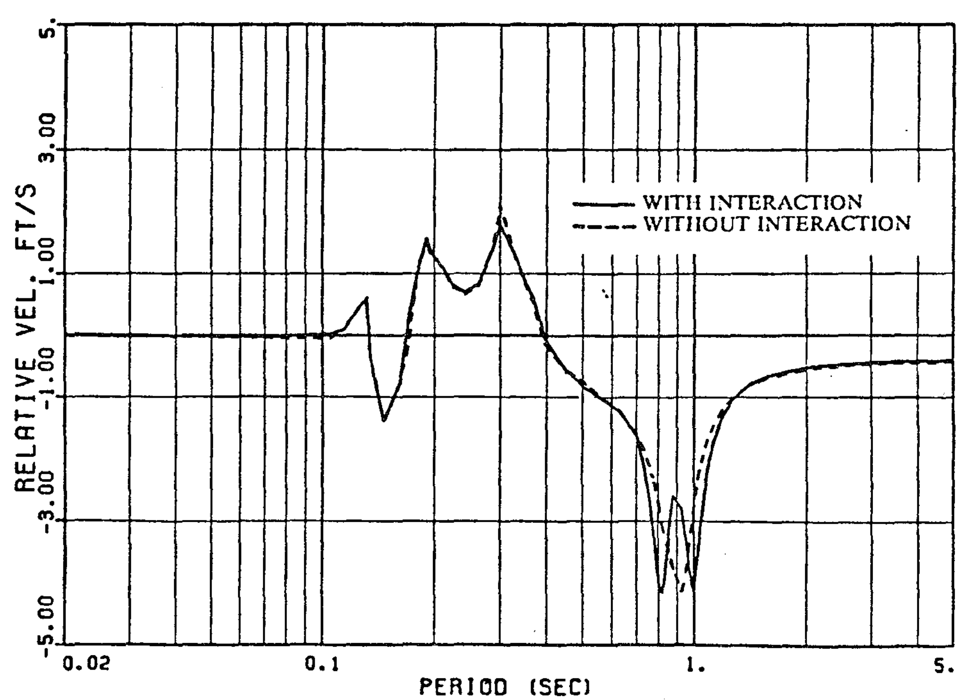

Figure 34. Quadrature Relative Velocity Spectrum for Ground and Floor 4 With and Without Interaction for Configuration "B".

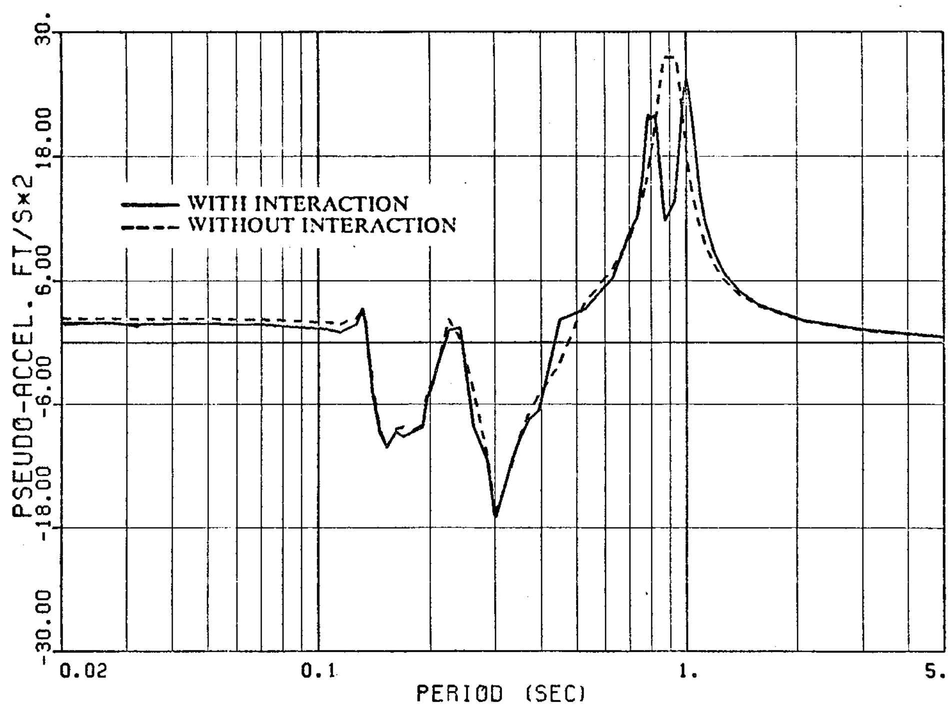

Figure 35. Coincident Pseudo-acceleration Spectrum for Floors 2 and 4 With and Without Interaction for Configuration "B".

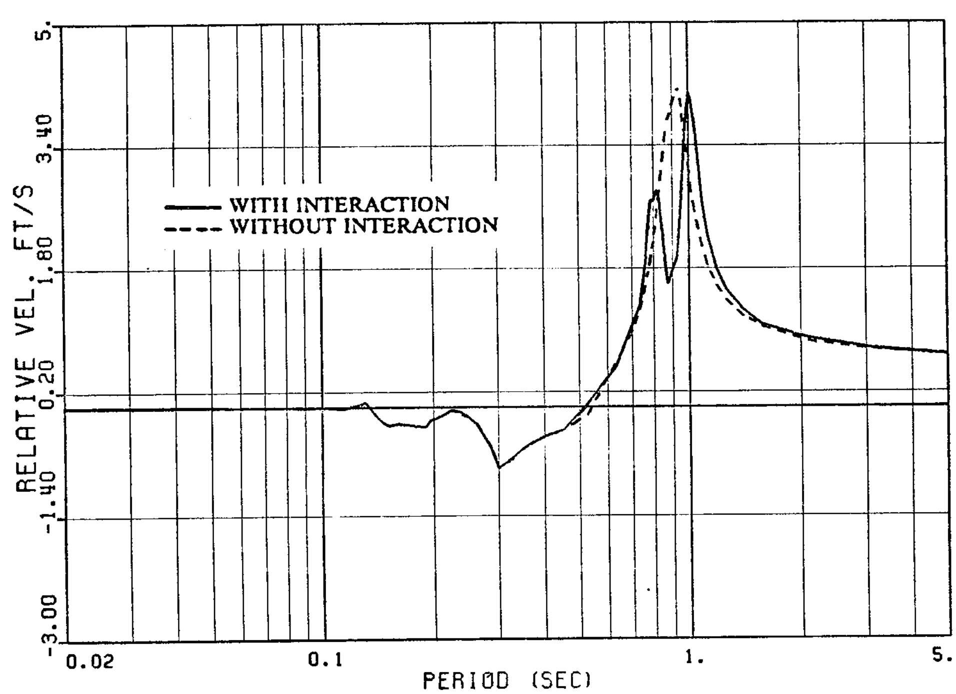

Figure 36. Coincident Relative Velocity Spectrum for Floors 2 and 4 With and Without Interaction for Configuration "B".

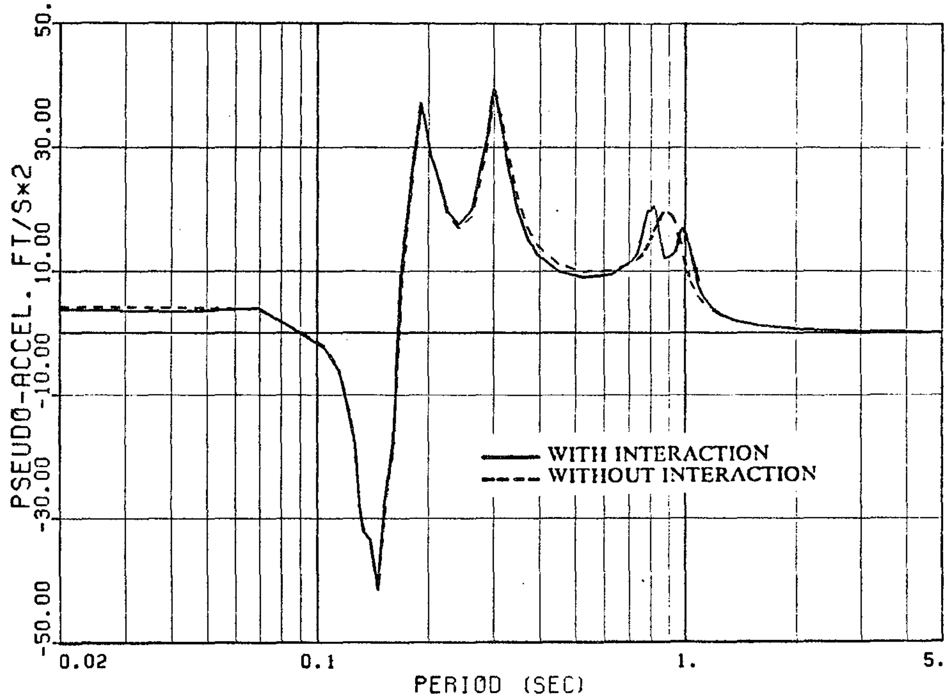

Figure 37. Quadrature Pseudo-acceleration Spectrum for Floors 2 and 4 With and Without Interaction for Configuration "B".

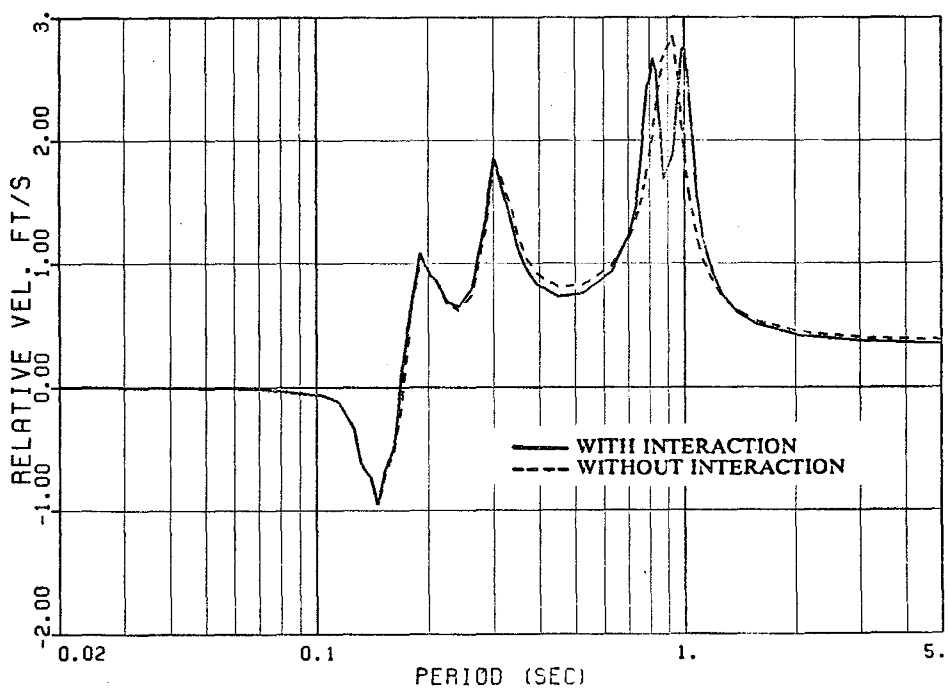

Figure 38. Quadrature Relative Velocity Spectrum for Floors 2 and 4 With and Without Interaction for Configuration "B".

BIBLIOGRAPHY

1. R. A. Burdisso and M. P. Singh, "Seismic analysis of secondary systems with multiple supports", Proceeding of the Conference on Structural Analysis and Design of Nuclear Power Plants, Porto Alegre, Brasil, October 1984.

2. M. P. Singh, "Response spectrum methods for secondary systems", Symposium on Earthquake Effects on Plant and Equipment, Hyderabad, India, Dec. 1984.

3. R. A. Burdisso and M. P. Singh, "Multiply supported secondary systems part I: Response spectrum analysis", Earthquake Engineering & Structural Dynamics, 1986 (To be published).

4. M. P. Singh and R. A. Burdisso, "Multiply supported secondary systems part II: Seismic inputs", Earthquake Engineering & Structural Dynamics, 1986 (To be published).

5. A. Asfura and A. Der Kiureghian, "Floor response spectrum method for seismic analysis of multiply supported secondary systems", J. of Earthquake Engr. & Str. Dyn., Vol. 14, 245-265 (1986).

6. A. Asfura and A. Der Kiureghian, "A new floor response spectrum method for seismic analysis of multiply supported secondary systems", Report No. UCB/EERC-84/04, Earthquake Engineering Research Center, Univ. of Cal., Berkeley, CA, 1984.

7. A. M. Sharma and M. P. Singh, "Direct generation of seismic floor response spectra for classically and nonclassically damped structures", Report No. VPI-E-83-44, Virginia Polytechnic Institute and State University, Nov. 1983.

8. T. Igusa and A. Der Kiureghian, "Dynamic analysis of multiply tuned and arbitrarily supported secondary systems", Report No. EERC-83/07, Earthquake Engineering Research Center, University of California, Berkeley, July 1983.

9. L. E. Suarez and M. P. Singh, "Mode synthesis of multiply connected secondary systems", Report No. VPI-E-86, Virginia Polytechnic Institute & State University, Blacksburg, VA, May 1986.

10. R. A. Burdisso and M. P. Singh, "Seismic analysis of multiply connected secondary systems:, Report No. VPI-E-84-30, Virginia Polytechnic Institute & State University, Blacksburg, VA, March 1986.

11. L. Meirovitch, *Computational Methods in Structural Dynamics*, Sijthoff & Noordhoff, Maryland, 1980.

12. D. Gasparini and E. H. Vanmarcke, "Simulated earthquake motions compatible with prescribed response spectra", Publication No. R76-4, Dept. of Civil Eng., MIT, Jan. 1975.

13. M. P. Singh, "Seismic response by SRSS for nonproportional damping", Journal of the Engineering Mechanics Division, ASCE, Vol. 106, No. EM6, Proc. paper 15948, December 1980, pp. 1405-1419.

14. R. W. Clough and J. Penzien, *Dynamic of Structures*, McGraw Hill, 1975.

Appendix A. MODIFIED PRIMARY EQUATIONS

In this Appendix the derivation of Eqs.(2.9), (2.10) and (2.13) are presented.

Case 1: Derivation of Eq.(2.9)

: The absolute displacement of the attachement points and the secondary degree of freedom can be written as the sum of the relative and ground displacement. That is,

$$\{U_a\} = \{V_a\} + \{1_a\}\, X_g(t) \tag{A.1}$$

$$\{U_s\} = \{V_s\} + \{1_s\}\, X_g(t) \tag{A.2}$$

Replacing the vectors $\{V_a\}$ and $\{V_s\}$ into the last two terms of the right hand side of Eq.(2.6), we can write

$$[c_{aa}]\{\dot{V}_a\} + [c_{as}]\{\dot{V}_s\} = [c_{aa}](\{\dot{U}_a\} - \{1_a\}\, \dot{X}_g(t)\,) + [c_{as}](\{\dot{U}_s\} - \{1_s\}\, \dot{X}_g(t)\,) \tag{A.3}$$

$$[k_{aa}]\{V_a\} + [k_{as}]\{V_s\} = [k_{aa}](\{U_a\} - \{1_a\}\, X_g(t)\,) + [k_{as}](\{U_s\} - \{1_s\}\, X_g(t)\,) \tag{A.4}$$

Rewritting Eqs.(A.3) and (A.4) as follows

$$[c_{aa}]\{\dot{V}_a\} + [c_{as}]\{\dot{V}_s\} = [c_{aa}]\{\dot{U}_a\} + [c_{as}]\{\dot{U}_s\} - ([c_{aa}]\{1_a\} + [c_{as}]\{1_s\})\,\dot{X}_g(t) \qquad (A.5)$$

$$[k_{aa}]\{V_a\} + [k_{as}]\{V_s\} = [k_{aa}]\{U_a\} + [k_{as}]\{U_s\} - ([k_{aa}]\{1_a\} + [k_{as}]\{1_s\})\,X_g(t) \qquad (A.6)$$

From the first set of equations in Eqs.(2.8a) and (2.8b), we obtain

$$[c_{aa}]\{1_a\} + [c_{as}]\{1_s\} = \{\,0\,\} \qquad (A.7)$$

$$[k_{aa}]\{1_a\} + [k_{as}]\{1_s\} = \{\,0\,\} \qquad (A.8)$$

Thus, replacing Eqs.(A.5) and (A.6) into Eq.(2.6) and considering Eqs.(A.7) and (A.8) gives Eq.(2.9)

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix}\begin{Bmatrix} \ddot{V}_p \\ \ddot{V}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa} \end{bmatrix}\begin{Bmatrix} \dot{V}_p \\ \dot{V}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix}\begin{Bmatrix} V_p \\ V_a \end{Bmatrix} = -\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix}\begin{Bmatrix} 1_p \\ 1_a \end{Bmatrix}\ddot{X}_g(t)$$

$$-\begin{Bmatrix} \{0\} \\ [m_{aa}]\{\ddot{U}_a\} + [m_{as}]\{\ddot{U}_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [c_{aa}]\{\dot{U}_a\} + [c_{as}]\{\dot{U}_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [k_{aa}]\{U_a\} + [k_{as}]\{U_s\} \end{Bmatrix} \qquad (2.9)$$

Case 2: Derivation of Eq.(2.10)

Replacing Eqs.(A.1) and (A.2) into Eq.(2.7) and after some rearranging, gives

$$[m_{ss}]\{\ddot{U}_s\} + [c_{ss}]\{\dot{U}_s\} + [k_{ss}]\{U_s\} = -[m_{sa}]\{\ddot{U}_a\} - [c_{sa}](\{\dot{U}_a\} - \{1_a\}\,\dot{X}_g(t))$$

$$- [k_{sa}](\{U_a\} - \{1_a\}\,X_g(t)) + [c_{ss}]\{1_s\}\,\dot{X}_g(t) + [k_{ss}]\{1_s\}\,X_g(t) \qquad (A.9)$$

From the second set of equations in Eqs.(2.8a) and (2.8b), we obtain

$$[c_{sa}]\{1_a\} + [c_{ss}]\{1_s\} = \{\,0\,\} \qquad (A.10)$$

$$[k_{sa}]\{1_a\} + [k_{ss}]\{1_s\} = \{\,0\,\} \qquad (A.11)$$

Replacing Eqs.(A.10) and (A.11) into (A.9) gives Eq.(5.10)

$$[m_{ss}]\{\ddot{U}_s\} + [c_{ss}]\{\dot{U}_s\} + [k_{ss}]\{U_s\} = -[m_{sa}]\{\ddot{U}_a\} - [c_{sa}]\{\dot{U}_a\} - [k_{sa}]\{U_a\} \tag{2.10}$$

Case 2: Derivation of Eq.(2.13)

: Replacing Eq.(2.11) into the last three terms of the right hand side of Eq.(2.9), we can write

$$[m_{aa}]\{\ddot{U}_a\} + [m_{as}]\{\ddot{U}_s\} = [m_{aa}]\{\ddot{U}_a\} + [m_{as}](\{\ddot{U}_s^d\} + \{\ddot{U}_s^p\}) \tag{A.12}$$

$$[c_{aa}]\{\dot{U}_a\} + [c_{as}]\{\dot{U}_s\} = [c_{aa}]\{\dot{U}_a\} + [c_{as}](\{\dot{U}_s^d\} + \{\dot{U}_s^p\}) \tag{A.13}$$

$$[k_{aa}]\{U_a\} + [k_{as}]\{U_s\} = [k_{aa}]\{U_a\} + [k_{as}](\{U_s^d\} + \{U_s^p\}) \tag{A.14}$$

Replacing Eq.(2.12) into Eqs.(A.12) through (A.14), we obtain

$$[m_{aa}]\{\ddot{U}_a\} + [m_{as}]\{\ddot{U}_s\} = ([m_{aa}] - [m_{as}][k_{ss}]^{-1}[k_{sa}])\{\ddot{U}_a\} + [m_{as}]\{\ddot{U}_s^d\} \tag{A.15}$$

$$[c_{aa}]\{\dot{U}_a\} + [c_{as}]\{\dot{U}_s\} = ([c_{aa}] - [c_{as}][k_{ss}]^{-1}[k_{sa}])\{\dot{U}_a\} + [c_{as}]\{\dot{U}_s^d\} \tag{A.16}$$

$$[k_{aa}]\{U_a\} + [k_{as}]\{U_s\} = ([k_{aa}] - [k_{as}][k_{ss}]^{-1}[k_{sa}])\{U_a\} + [k_{as}]\{U_s^d\} \tag{A.17}$$

Substituting Eq.(A.1) into Eqs.(A.15), (A.16) and (A.17) gives

$$[m_{aa}]\{\ddot{U}_a\} + [m_{as}]\{\ddot{U}_s\} = ([m_{aa}] - [m_{as}][k_{ss}]^{-1}[k_{sa}])(\{\ddot{V}_a\} + \{1_a\}\ddot{X}_g(t)) + [m_{as}]\{\ddot{U}_s^d\} \tag{A.18}$$

$$[c_{aa}]\{\dot{U}_a\} + [c_{as}]\{\dot{U}_s\} = ([c_{aa}] - [c_{as}][k_{ss}]^{-1}[k_{sa}])(\{\dot{V}_a\} + \{1_a\}\dot{X}_g(t)) + [c_{as}]\{\dot{U}_s^d\} \tag{A.19}$$

$$[k_{aa}]\{U_a\} + [k_{as}]\{U_s\} = ([k_{aa}] - [k_{as}][k_{ss}]^{-1}[k_{sa}])(\{V_a\} + \{1_a\}X_g(t)) + [k_{as}]\{U_s^d\} \tag{A.20}$$

From Eq.(A.11) we can write

$$\{1_s\} = -[k_{ss}]^{-1}[k_{sa}]\{1_a\} \tag{A.21}$$

Substituting Eq.(A.21) into Eqs.(A.19) and (A.20) and taking into account Eqs.(A.7) and (A.8), we obtain

$$([c_{aa}] - [c_{as}][k_{ss}]^{-1}[k_{sa}])\{1_a\} = [c_{aa}]\{1_a\} + [c_{as}]\{1_s\} = \{0\} \tag{A.22}$$

$$([k_{aa}] - [k_{as}][k_{ss}]^{-1}[k_{sa}])\{1_a\} = [k_{aa}]\{1_a\} + [k_{as}]\{1_s\} = \{0\} \tag{A.23}$$

Replacement of Eqs.(A.18) through (A.20) into Eq.(2.9) and by combining appropiated terms gives Eq.(2.13)

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & \tilde{M}_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{V}_p \\ \ddot{V}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & \tilde{C}_{aa} \end{bmatrix} \begin{Bmatrix} \dot{V}_p \\ \dot{V}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & \tilde{K}_{aa} \end{bmatrix} \begin{Bmatrix} V_p \\ V_a \end{Bmatrix} = - \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & \tilde{M}_{aa} \end{bmatrix} \begin{Bmatrix} 1_p \\ 1_a \end{Bmatrix} \ddot{X}_g(t) - \{F\} \tag{2.13}$$

where matrices $[\tilde{M}_{aa}]$, $[\tilde{C}_{aa}]$ and $[\tilde{K}_{aa}]$ and vector $\{F\}$ are defined in Eqs.(2.14) and (2.15), respectively.

Appendix B. PERTURBATION ANALYSIS OF EIGENVALUE PROBLEM

In this Appendix the firts-order perturbation approach is presented to obtain the frequencies and mode shapes of the modified primary system in Eq.(2.14), in terms of the frequencies and modes shapes of the original primary system of Eq.(2.19).

The eigenvalues λ_{0i} and eigenvectors $\{\Psi_{oi}\}$ of the original or unperturbed system satisfy the following relationship

$$[K_0]\{\Psi_{0i}\} = \lambda_{0i}[M_0]\{\Psi_{0i}\} \qquad ; \quad i = 1,2,...,N \tag{B.1}$$

where $[K_0]$ and $[K_0]$ represent the stiffness and mass matrices of Eq.(2.19). That is

$$[K_0] = \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix} \qquad ; \qquad [M_0] = \begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \tag{B.2}$$

The eigenvectors are assumed to be normilized so that they satisfy

$$\{\Psi_{0i}\}'[K_0]\{\Psi_{0j}\} = \lambda_{0i}\,\delta_{ij} \qquad ; \qquad \{\Psi_{0i}\}'[M_0]\{\Psi_{0j}\} = \delta_{ij} \tag{B.3}$$

in which δ_{ij} is the delta Dirac function. The perturbed eigenvalue problem can be written as follows

$$[K]\{\Psi_i\} = \lambda_i [M]\{\Psi_i\} \qquad ; \quad i = 1,2,...,N \tag{B.4}$$

where λ_i and $\{\Psi_i\}$ are the eigenvalues and eigenvectors of the perturbed system. The stiffness and mass matrices in Eq.(B.4) are given from Eq.(2.14) as follows

$$[K] = \begin{bmatrix} K_{pp} & K_{pa} \\ \sim & \\ K_{ap} & K_{aa} \end{bmatrix} \qquad ; \qquad [M] = \begin{bmatrix} M_{pp} & M_{pa} \\ \sim & \\ M_{ap} & M_{aa} \end{bmatrix} \tag{B.5}$$

We next express matrices $[K]$ and $[M]$ in the form

$$[K] = [K_0] + [K_1] \qquad ; \qquad [M] = [M_0] + [M_1] \tag{B.6}$$

in which $[K_1]$ and $[M_1]$ represent small changes from the original matrices. Thus, we also assume that the perturbed eigenvalues and eigenvectors can be written as follows

$$\lambda_i = \lambda_{0i} + \lambda_{1i} \quad ; \quad \{\Psi_i\} = \{\Psi_{0i}\} + \{\Psi_{1i}\} \quad ; \quad i = 1,2,...,N \tag{B.7}$$

Replacing Eqs.(B.6) and (B.7) into (B.4), we can write

$$([K_0] + [K_1])(\{\Psi_{0i}\} + \{\Psi_{1i}\}) = (\lambda_{0i} + \lambda_{1i})([M_0] + [M_1])(\{\Psi_{0i}\} + \{\Psi_{1i}\}) \tag{B.8}$$

Arranging terms of the same order of magnitude in Eq.(B.8) and considering only the first order term, gives

$$[K_0]\{\Psi_{1i}\} + [K_1]\{\Psi_{0i}\} = \lambda_{0i}[M_0]\{\Psi_{1i}\} + \lambda_{0i}[M_1]\{\Psi_{0i}\} + \lambda_{1i}[M_0]\{\Psi_{0i}\} \tag{B.9}$$

Premultiplying Eq.(B.9) by $\{\Psi_{0j}\}'$, we obtain

$$\{\Psi_{0j}\}'[K_0]\{\Psi_{1i}\} + \{\Psi_{0j}\}'[K_1]\{\Psi_{0i}\} = \lambda_{0i}\{\Psi_{0j}\}'[M_0]\{\Psi_{1i}\} + \lambda_{0i}\{\Psi_{0j}\}'[M_1]\{\Psi_{0i}\} + \lambda_{1i}\,\delta_{ij} \tag{B.10}$$

Assuming that the eigenvector perturbation takes the form

$$\{\Psi_{1i}\} = \sum_{k=1}^{N} \varepsilon_{ik}\{\Psi_{0k}\} \qquad \varepsilon_{ii} = 0 \quad ; \quad i = 1,2,...,N \tag{B.11}$$

We can write using Eq.(B.3),

$$\{\Psi_{0j}\}'[K_0]\{\Psi_{1i}\} = \{\Psi_{0j}\}'[K_0] \sum_{k=1}^{N} \varepsilon_{ik} \{\Psi_{0k}\} = \sum_{k=1}^{N} \varepsilon_{ik} \lambda_{0j} \delta_{jk} = \lambda_{0j} \varepsilon_{ij} \qquad (B.12)$$

$$\{\Psi_{0j}\}'[M_0]\{\Psi_{1i}\} = \{\Psi_{0j}\}'[M_0] \sum_{k=1}^{N} \varepsilon_{ik} \{\Psi_{0k}\} = \sum_{k=1}^{N} \varepsilon_{ik} \delta_{jk} = \varepsilon_{ij} \qquad (B.13)$$

Thus, Eq.(B.10) becomes

$$\varepsilon_{ij} (\lambda_{0j} - \lambda_{0i}) + \{\Psi_{0j}\}'([K_1] - \lambda_{0i} [M_1])\{\Psi_{0i}\} = \lambda_{1i} \delta_{ij} \qquad (B.14)$$

But when $i = j$, $\varepsilon_{ii} = 0$ and $\delta_{ii} = 1$, so that Eq.(B.14) yields the eigenvalue perturbation as

$$\lambda_{1i} = \{\Psi_{0i}\}'([K_1] - \lambda_{0i} [M_1])\{\Psi_{0i}\} \qquad (B.15)$$

On the other hand, when $i \neq j$, $\delta_{ij} = 0$ and Eq.(B.14) gives the coefficients ε_{ij} as

$$\varepsilon_{ij} = \frac{\{\Psi_{0j}\}'([K_1] - \lambda_{0i} [M_1])\{\Psi_{0i}\}}{\lambda_{0j} - \lambda_{0i}} \quad i \neq j \quad ; \quad \varepsilon_{ii} = 0 \qquad (B.16)$$

Thus, the coefficients ε_{ij} are replaced into Eq.(B.11) to obtain $\{\Psi_{1i}\}$ and this toghether with λ_{1i}, from Eq.(B.15) substitute into Eq.(B.7) to obtain the modified or perturbed eigenvalue and eigenvectors.

Appendix C. NUMERICAL INTEGRATION OF FREQUENCY INTEGRALS

In this Appendix, the details for the numerical integration required in the calculation of the interaction coefficients are presented.

The auto relative displacement interaction coefficient was defined in Eq.(2.47) as

$$\mu_{dk}^2(\omega_i) = \frac{\int_{-\infty}^{\infty} \Phi_{akk}(\omega)\, |H_i|^2\, d\omega}{\int_{-\infty}^{\infty} \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega} \tag{2.47}$$

where $\Phi_{akk}(\omega)$ = the absolute acceleration PSDF for floor k including the interaction effect between primary and secondary systems; $\tilde{\Phi}_{akk}(\omega)$ = the absolute acceleration PSDF for floor k when the interaction force in the interface is set equal to zero. The integrants in Eq.(2.47) are even functions. Thus, the limits in Eq.(2.47) can be replaced as

$$\mu_{dk}^2(\omega_i) = \frac{\int_{0}^{\infty} \Phi_{akk}(\omega)\, |H_i|^2\, d\omega}{\int_{0}^{\infty} \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega} \tag{C.1}$$

The PSDF $\Phi_{akk}(\omega)$ required in Eq.(C.1) is obtained from Eq.(2.44). That is,

$$\Phi_{akk}(\omega) = \sum_{u=1}^{m} \sum_{v=1}^{m} \left[G_{ku}^{R}(\omega)\, G_{kv}^{R}(\omega) + G_{ku}^{I}(\omega)\, G_{kv}^{I}(\omega) \right] \tilde{\Phi}_{auv}^{R}(\omega) \qquad (2.44)$$

where $\tilde{\Phi}_{auv}^{R}(\omega)$ = real part of the absolute acceleration cross PSDF for floor u and v obtained from the perturbed primary system. It can be shown that [..]

$$\tilde{\Phi}_{auv}(\omega) = \sum_{r=1}^{N} \sum_{s=1}^{N} \tilde{\gamma}_r \tilde{\gamma}_s \tilde{\Psi}_r(u)\, \tilde{\Psi}_s(v)\, (\tilde{\omega}_r^2 - 2\,i\, \tilde{\beta}_r \tilde{\omega}_r \omega)(\tilde{\omega}_s^2 + 2\,i\, \tilde{\beta}_r \tilde{\omega}_r \omega)\, \tilde{H}_r^* \tilde{H}_s \Phi_g(\omega) \quad (C.2)$$

The absolute acceleration PSDF from the modified primary system for floor k, $\tilde{\Phi}_{akk}(\omega)$, required in the denominator of Eq.(C.1) is obtained from Eq.(C.2) replacing u and v by k.

In Eqs.(C.2), $\Phi_g(\omega)$ = the PSDF of the ground input. Here, a band-limited white noise ground input is assumed. That is,

$$\Phi_g(\omega) = \begin{cases} S_0 & |\omega| \le \omega_c \\ 0 & |\omega| > \omega_c \end{cases} \qquad (C.3)$$

where ω_c = the cut-off ground frequency. With this assumtion, Eq.(C.1) can be written as follows

$$\mu_{dk}^2(\omega_i) \cong \frac{\int_0^{\omega_c} \Phi_{akk}(\omega)\, |H_i|^2\, d\omega}{\int_0^{\omega_c} \tilde{\Phi}_{akk}(\omega)\, |H_i|^2\, d\omega} \qquad (C.4)$$

Here, the integrations required in Eq.(C.5) are performed by the Gauss Quadrature scheme. The given interval of integration is divided into suitably chosen subintervals. In each subintervals the value of the integrand is calculated at the Gauss points and a estimate of the integral is obtained as a weighted sum. To obtain the value of the integrand in the numerator of Eq.(C.5), the real part of Eq.(C.2) is replaced into Eq.(2.44). The coefficients G_{ku}^{R}, G_{kv}^{R}, etc. needed in Eq.(2.44) are obtained from inverting matrix $[E(\omega)]$ (See Eq.(2.37)) evaluated at the Gauss points.