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DETECTION AND ASSESSMENT OF SEISMIC STRUCTURAL DAMAGE

by

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16. Abstract (Limit: 200 words)

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A program for the identification of linear structures, based on strong motion records, has been written and implemented by the authors. The theoretical and computational aspects of the identification algorithm are discussed.

Strong motion records from six buildings that experienced different levels of damage during the San Fernando earthquake (1971) are analyzed. The analysis of strong motion records of buildings that have been damaged during the San Fernando earthquake shows a good agreement between the numerical values of the damage indices and the levels of damage observed in the actual structures. If this result is confirmed by further analysis of both strong motion records from actual structures and small scale experiments these damage indices can be used as a measure of structural damage. Areas of future research are outlined.

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ABSTRACT

The problem of earthquake damage assessment is defined. The role of the analysis of strong motion records in damage assessment codes based on expert system is pointed out. A literature survey on damage assessment and structural system identification is presented.

In order to quantify the damage to which a simple structural element is subjected under earthquake or earthquake-like excitation, several indicators (damage indices) have been proposed. Numerical simulations have been used to study the dependence of different damage indices on the paramenters of the structure and of the ground motion. Furthermore, it can be shown that many of the proposed indices are statistically equivalent, and therefore carry the same information about the damage state of the structure.

Both peak deformation and fatigue load contribute to the damage of a structural element. A global damage index for a complex structure should as well consider the two components of damage. Damage indices can be computed from the optimal time variant linear model, that is fitted to recorded strong motion accelerograms. Two of these indices are proposed, and their performance in estimating other response based indices is discussed.

A program for the identification of linear structures, based on strong motion records, has been written and implemented by the authors. The theoretical and computational aspects of the identification algorithm are discussed.

Strong motion records from six buildings that experienced different levels of damage during the San Fernando earthquake (1971) are analyzed using the techniques described in chapter 5. The damage indices proposed in chapter 4 are estimated. The computed values are found to be consistent with the level of damage.

The analysis of strong motion records of buildings that have been damaged during the San Fernando earthquake shows a good agreement between the numerical values of the damage indices and the levels of damage observed in the actual structures. If this result is confirmed by further analysis of both strong motion records from actual structures and small scale experiments, these damage indices can be used as a measure of structural damage. Areas of future research are outlined.

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SECTION 1: INTRODUCTION

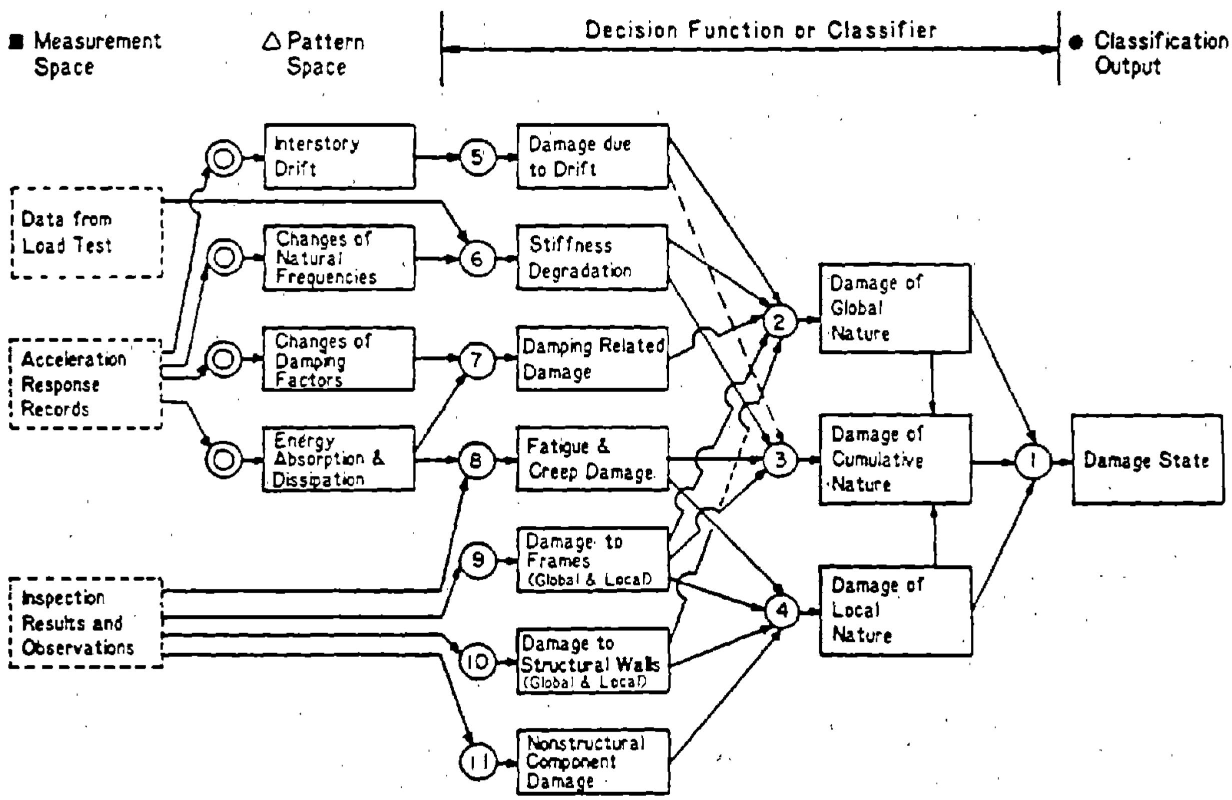
1.1. Statement of the Problem

When a major earthquake strikes an urban area, one of the most compelling problems that engineers face is to evaluate the safety of existing structures. This task is usually accomplished by inspecting the buildings in question. Visual inspection can identify cracks and permanent deformation, and field testing can measure the degradation of the structure. Eventually, expert engineers have to decide what, if any, action should be taken, ranging from cosmetic repairs to the demolition of the building.

The analysis of strong motion accelerograms will play a major role in an automatic damage assessment scheme. In fig. 1.1 (from Yao,1982), a flow chart for a damage assessment procedure based on expert systems is illustrated. From the acceleration response records, informations about both the three damage classifiers (global, cumulative and local damage) can be obtained).

In this study, an attempt is made to define procedures and algorithms, so that informations about global and cumulative damage can be extracted from strong motion records.

When laboratory tests are performed, model structures are extensively instrumented, and local damage can be analyzed in great detail. However, the number of observation points is, in real structures, very limited. One accelerogram array is placed in the basement of the structure, and this record can be used as input to the structural system considered, if soil-structure interactions are neglected (the absence of soil structure interactions is a working assumption throughout this report). Another array would usually be placed at some upper level, so that the response of the building can be observed. These data can still give some information about the global behavior of the structure, when system identification based on modal decomposition is used.



Feature selection and extractor with intermediate diagnostic states

Fig. 1.1 : Conceptual Flow Chart for Damage Assessment Using Expert Systems (from Yao (1982))

1.2. Organization of the work

This report starts with a literature review (chapter 2) of the assessment of damage for simple elements and complex structures, and of system identification techniques applied to structural dynamics.

chapter 3 presents the results of a series of numerical simulations, that have been carried out to test the performances of several damage indices in the damage analysis of SDOF (Single Degree of Freedom) non linear systems to pseudo-earthquake excitation.

Damage indices based on system identification are introduced in chapter 4. The system identification algorithm that has been developed and implemented by the authors is described in chapter 5.

Strong motion records from the San Fernando earthquake (February 9, 1971) have been analyzed. The results are presented in chapter 6.

Several damage indices, both proposed in the literature during the past years or introduced by the authors, are described. A summary is presented in table 1.1.

TABLE 1.1: DAMAGE INDICES		
index	formula	notes
Chapter 2 (literature survey)		
ductility ratio	$\frac{x_M}{x_y}$	Newmark and Rosemblueth (1974)
$\delta_{Bertero}$	$\frac{1}{\sum_{i=1}^{m}\omega_i}\sum_{i=1}^{m}\frac{\omega_i\eta_i s_i}{\gamma_i r_i}$	Bertero and Bresler (1971)
δ_{Banon}	$f\left(\frac{K_f}{K_r}, \frac{\sum_{i=1}^{ncycle}\lvert\theta_i\rvert}{\theta_y}\right)$	Banon and Veneziano (1982)
δ_{Park}	$\frac{x_M}{x_u}+\beta\frac{\int dE}{F_y x_u}$	Park and Ang (1985)
$\delta_{Stephens}$	$\sum_{i=1}^{ncycle}\left(\frac{\Delta\delta_{pt}}{\Delta\delta_{pf}}\right)_i^{\alpha}$	Stephens and Yao (1985)
Chapter 3 (numerical simulations)		
ductility index (μ_x)	$\frac{x_M}{x_u}$	physical component of δ_{Park}
energy index (μ_E)	$\frac{\int dE}{F_y x_u}$	physical component of δ_{Park}
Chapter 4 (system identification)		
ultimate stiffness degradation	$\frac{(T_0)_{final}-(T_0)_{initial}}{(T_0)_{initial}}$	global degradation
maximum softening (δ_M)	$\max_{i=1,nwind}\frac{(\Delta T_0)_i}{(T_0)_{initial}}$	global damage due to peak deformation
cumulative softening (δ_E)	$\sum_{i=1}^{nwind}\frac{(\Delta T_0)_i}{(T_0)_{initial}}\frac{s_i}{(T_0)_i}$	global damage due to fatigue

SECTION 2: DAMAGE ASSESSMENT AND STRUCTURAL SYSTEM IDENTIFICATION: A LITERATURE SURVEY

2.1. Introduction

Structural damage is a complex phenomenon that is very difficult to model analytically or to reproduce in laboratory experiments.

A certain understanding has been achieved when two limit cases for the failure mechanism are considered: static load, when the strain is increased monotonically until the element that is being tested breaks, and low cycle fatigue, when repeated strain cycles, below the level of ultimate strain, although above the yielding level, are applied.

Seismic loading appear to be a random combination of these two limit cases, as the structure experiences several load cycles of different amplitude, many of them beyond its yield limit.

Correspondingly, structural material can present either a fragile behavior, as in the case of concrete, or a ductile behavior, as for steel. The behavior of composite material, such as reinforced concrete, will be a combination of these two limit cases.

The literature regarding these phenomena is very vast. Relevant references for earthquake engineers are Newmark and Rosenblueth (1974), Akiyama (1985), Kranwinkler and Zohrei (1983), Yao et al. (1986).

Blejwas and Bresler (1978) suggested that damage was measured by the relative value (ratio) of a demand (the seismic response) and the capacity of the system. In the case of seismic loads, both these factors depend on different mechanisms, and are stochastic in nature. Furthermore, in complex structures, damage will result in degradation of some characteristics, namely stiffness or yielding limit, introducing a feedback in an apparently simple relationship.

During the past years, the prediction of the maximum response of structural systems to earthquakes has been studied extensively (see Vanmarcke, 1983, for a review). The fatigue load due to earthquakes has not received the same attention, since Crandall and Mark's work (1963) was published.

2.2. Damage of Simple Structural Elements

Most of the methods to quantify damage that have been proposed stem from theoretical and experimental work on simple elements, for which the "displacement" is defined without any ambiguity. In the following, several indices that have been proposed are described. For the reader's convenience, the name of the first author of the paper in which the index is described appears in the symbol of each index. When more complex structures are considered, some authors have concentrated their attention on the interstory displacement (Sozen,1981), or on the displacement of a particular level, namely the roof (Meyer and Roufaiel, 1984), while others have somehow combined the damage that each of the elements has undergone.

Newmark and Rosenblueth (1974) proposed that the ductility ratio, defined as the ratio of the maximum displacement to the yielding displacement $\frac{x_M}{x_y}$ be used as a measure of the structural damage. Other measures or indices, always expressed as a function of the maximum displacement, have been introduced by Oliveria (1975) and by Bertero and Bresler (1971),who took into account the cumulative nature of damage, as well as the complexity of a structure, considered as an assemblage of m elements. The damage index for the global structure was defined as

$$\delta_{Bertero} = \frac{1}{\sum_{i=1}^{m} \omega_i} \sum_{i=1}^{m} \frac{\omega_i \eta_i s_i}{\gamma_i r_i} \qquad (2.2.1)$$

where S_i is the demand and r_i is the capacity, corresponding to the ith element, the ω_i are weights, to account for the relative importance of different elements, and η_i and γ_i are service factors, that model the cumulative nature of the damage.

Banon and Veneziano (1982) pointed out the necessity to consider separately the two components of damage. They defined a damage function

$$\delta_{Banon} = f\ (FDR\ , NCR\ ) \tag{2.2.2}$$

where the flexural damage ratio (FDR) is the ratio between the initial flexural stiffness K_f to the reduced secant stiffness K_r for a reinforced concrete cantilevered element.

$$FDR = \frac{K_f}{K_r}$$

The normalized cumulative rotation NCR is the ratio between the cumulative plastic rotation in n_{cycle} cycles and the yielding rotation of the non linear spring, considered in their model

$$NCR = \frac{\sum_{i=1}^{ncycle} \mid \theta_i \mid}{\theta_y}$$

Park and Ang (1985) suggested the use of a linear combination of a ductility and of an energy factor, defining an index δ

$$\delta_{Park} = \frac{x_M}{x_u} + \beta \frac{\int dE}{F_y x_u} \tag{2.2.3}$$

where x_u is the ultimate displacement, F_y the yielding force, dE the elementary energy dissipated in the system, and β a parameter, estimated from experimental data.

According to Park (1987) this linear relationship must be viewed as a first order approximation to a more complicated, unknown function. This approximation is valid in the region, close to the ultimate displacement of the element.

Stephens (1985) developed a damage function, on the basis of a hypothesis formulated by Yao and Munse (1962). The damage, subsequent to the ith cycle of deformation, is given by:

$$\Delta\delta_{Stephens,i} = \left(\frac{\Delta\delta_{pt}}{\Delta\delta_{pf}}\right)_i^{\alpha} \tag{2.2.4}$$

where

$\Delta\delta_{pt}$ = positive change in plastic deformation

$\Delta\delta_{pf}$ = positive change in plastic deformation to failure

$\alpha = 1 - b\ r_l$, where b is a constant and r_l is relative deformation ratio, $\frac{\Delta\delta_{pc}}{\Delta\delta_{pt}}$, between the negative and the positive change in plastic deformation over a cycle. This index takes into account the dissymetry in the behavior of reinforced concrete elements, as well as the influence of the geometry of the cycle on the accumulation mechanism.

The capacity of the structure is probably the biggest unknown in such a treatment. Park and Ang (1985) studied the behavior of reinforced concrete elements, using nonlinear regression to obtain expressions for the ultimate displacement, depending on the geometric and constructive features of the elements.

Such results represent very useful qualitative guidelines to model the behavior to failure of structural elements. Their use in quantitative analysis is complicated by the wide scatter of experimental results. The coefficient β of eq. (2.2.3), when estimated on the basis of the ultimate displacement, computed using the regression expression, often happen to be negative, and therefore physically meaningless. Stephens and Yao (1985) used arbitrary values for x_u, based on the advice of expert engineers.

In other works, researchers have expressed the contribution of ductility to damage independently of the ultimate displacement. Therefore, indices based on the secant stiffness (Banon and Veneziano, 1982) or on the slope ratio (Toussi and Yao, 1982) have been introduced.

Once a measure of damage has been defined, stochastic methods can be used to predict the future damage on a given structure. Wen (1985) used equivalent linearization to compute the statistics of the index (2.2.3), for a degrading hysteretic Single Degree of Freedom Oscillator, subjected to an earthquake excitation, modeled by a filtered gaussian shot noise.

2.3. Damage indices for Complex Structures

Damage indices for more complex structures have usually been defined as a weighted average of the damage indices, relative to each component that is considered in the modeling process. Park, Ang and Wen (1985) have used a very realistic, although two-dimensional, model to simulate the damage to structures, hit by the San Fernando earthquake (1971), and by the Miyagiken-Oki earthquake (1978). The same authors (1987) extended the study, giving some design recommendation. Stephens and Yao (1987) have studied small scale experiments, performed at the University of Illinois by Sozen and his associates (Aristizabal-Ochoa and Sozen, 1976, Healey and Sozen, 1978) to validate the damage index of equation (2.2.4), and to determine its value for different levels of damage. A damage analysis on the Imperial County Service Building, that was severely damaged during the Imperial Valley earthquake (1979) was also conducted.

2.4. Identification of Structural Systems

In the following chapters, damage indices based on system identification are introduced. Fundamental references on system identification techniques are the works of Sage and Melsa (1971), Eychoff (1974), Box and Jenkins (1976). A more up to date overview of the field can be found in Hajdasinsk et al. (1982) and Ljung (1982). System identification methods have been used in civil engineering as early as 1972 (see Pilkey and Cohen, 1972).

The problem that first received attention was the identification of linear structural models. Gersh et al. (1973) used parametric (Auto Regressive) time series models and Maximum Likelihood techniques for structural identification. Beck (1978) proved that linear models based on modal decomposition are identifiable from earthquake records. Beck and Jennings (1980) and McVerry (1980) studied the behavior of buildings during the San Fernando earthquake, using time variant linear models. A review of the application of system identification to strong motion records has been written by Beck (1983). Shinozuka and Yun (1982) compared several available methods for the identification of linear structures. Recently, the authors (DiPasquale and Cakmak, 1987) studied the estimation of modal parameters from strong motion accelerograms, using Maximum Likelihood techniques.

Linear structural systems are non linear in the parameters (Eychoff, 1974). The identification of such systems is therefore a non linear problem, and the techniques applied are essentially the same as in the case of non linear structures, as long as memoryless (non hysteretic) systems are considered. Yun and Shinozuka (1980) used the extended Kalman filter to study non linear fluid structure interaction. Hysteretic models have been identified by Toussi and Yao (1983). Iwan and Cifuentes (1986,1987) have have introduced a degrading mechanism in their identification scheme. Beck and Jayakumar (1986a,1986b,1986c) applied system identification techniques to the analysis of data, measured during the pseudo-dynamic test of a full-scale, six-story structure.

The review paper by Kozin and Natke (1986) is worth mentioning, as an exhaustive presentation of the applications of system identification to structural dynamics.

2.5. Effect of Structural Damage on the Vibrational Parameters

When time variant linear models are fitted to strong motion records, it is found that the natural frequencies of the structure tend to shift towards lower values (Beck,1983). This can be due to nonlinearities in the mechanical behavior of the structure and to soil-structure interaction, as well as to stiffness degradation, subsequent to structural damage.

The local effect of structural damage will be cracking, buckling, in any case degradation of the resistance properties of structural elements. Very often, especially in the case of reinforced concrete elements or shear walls, the stiffness characteristics of an element or of a joint will degrade. From this local stiffness degradation, a general shift of the natural frequencies towards lower values will result (Rayleigh,1945, Dowell,1979).

Stiffness degradation of both full scale structures and of small scale models, consequent to seismic damage, has been observed by several authors (Chen et al.,1977, Foutch and Housner,1977, Meyer and Roufaiel,1984, Mihai et al.,1980, Vasilescu et al.,1980). Ogawa and Abe (1980) and Carydis and Mouzakis (1986) attempted a correlation between stiffness degradation, as a function of the variation in the fundamental period, and the severity of damage.

2.6. Database on Seismic Structural Damage

For the first part of this study, a very valuable guide in gathering a database on structural damage has been the work of Stephens and Yao (1983). The data acquired so far consist of both strong motion records and of shaking table experiments on small scale models.

The accelerograms, recorded during the San Fernando earthquake (February 9, 1971) contain information about buildings that have experienced very little damage, while in only one case (the Bank of California building) the damage was considered serious enough to repair the building (Jennings et al.,1971). During the Imperial Valley earthquake (November 15, 1979), the Imperial County Service Building was damaged so severely that it was eventually torn down (Saiidi,1981, Wosser et al.,1982, Kreger,1984, Pardoen and Sheperd,1984). The building was very well instrumented (Porter,1983) and an analysis of these records will provide an interesting mean of validing the methodology proposed in the following chapters.

Sozen and his associates (Aristazabal-Ochoa and Sozen,1976, Healey and Sozen,1978) conducted several series of experiments on small scale models of ten story buildings, using the seismic simulator at the University of Illinois at Urbana. Each structure was subjected to earthquakes of increasing intensity, until failure was observed. The reports describing these experiments contain a detailed qualitative description of the damage state, subsequent to each run. This is very important, as a correlation between analytical and qualitative measures of damage is thus possible.

SECTION 3: DAMAGE INDICES AS A MEASURE OF DAMAGE DURING EARTHQUAKES: A STUDY BASED ON NUMERICAL SIMULATIONS

3.1. Introduction

In chapter 2 several indices that have been proposed in literature during the past years have been presented. They can be divided into two categories:

- indices based on the peak displacement,
- indices based on fatigue.

This classification reflects the difference in the behavior of structural materials. Concrete has a fragile response, its force-deformation characteristic degrades very sharply after the first yielding, and its ability to dissipate energy is minimal. Therefore, peak deformation controls the failure process for simple specimens and for more complex elements. The behavior of steel is ductile, the initial stiffness is usually recovered after yielding, and a specimen can go through several cycles of high deformation before a failure occurs, dissipating a considerable amount of energy through hysteretic mechanisms. The mechanisms that generate a failure in either case, although not yet fully understood, are believed to be different.

As it can be expected, reinforced concrete exhibits a mixed behavior. Therefore some combination of these basic indices have been proposed to describe the damage of reinforced concrete elements.

There are therefore physical considerations that suggest that two different aspects of damage, peak deformation and fatigue, be considered. Their contributions may be combined through a damage law, like the ones proposed by Banon and Veneziano (1982), or by Park and Ang (1985).

These physical considerations may be supported by statistical studies, based on numerical simulations. The indices based on peak deformation are found to be uncorrelated to the ones based on fatigue. Although this does not imply statistical independence, it rules out any linear connection between the two phenomena. The relative importance of damage, due to peak deformation or to fatigue, is found to be dependent in a deterministic fashion on the parameters of the ground motion and of the structure considered. Hence the necessity to carry damage indices of both types along the analysis of structural damage.

The numerical study that is described in the following has been devised and conducted in collaboration with Prof. Mircea Grigoriu of Cornell University.

3.2. Description of the Experiment

The first step in the design of the numerical experiment was the selection of the damage indices to be computed.

All the indices, proposed to measure the peak deformation, are functionally dependent on the maximum displacement. Therefore, only the ductility index

$$\mu_x = \frac{x_M}{x_u} \tag{3.2.1}$$

has been computed. The ductility index is the ratio of the maximum displacement x_M to the ultimate displacement x_u, that was conventionally chosen equal to 20 mm. No degradation or failure mechanism was implemented, so that values for μ_x greater than one were not uncommon. $\mu_x = 1$ will therefore imply only that the system developed a ductility equal to five. Reinforced concrete elements, when properly designed and built, can achieve ductility values of 20 and higher (Newmark and Rosenblueth, 1974).

Several indices have been proposed to measure the fatigue load on a structure. In this experiment, four of them have been computed:

- the normalized energy dissipation (energy index) μ_E (eq. 2.2.3)
- the cumulative plastic displacement, equivalent to the normalized cumulative rotation (eq. 2.2.2)

Stephens and Yao's index (eq. 2.2.4), and its simplified version.

Stephens and Yao's index considers the dissymmetry in the behavior of concrete elements, and takes empirically into account the effect of the shape of the cycle in the process of the accumulation of damage. In the simplified version, the exponent α is a constant, equal to 1.77. This takes into account the mixed behavior of reinforced concrete.

Earthquake-like inputs have been generated, following a procedure devised by Ellis, Cakmak and Ledolter (1987). Earthquakes of different magnitudes, and for different soil conditions, have been simulated. These earthquakes were statistically equivalent to earthquakes of the same characteristics, as recorded in Mexico City during the 1985 Michoacan earthquake (Ellis, 1987). The choice of Mexico City was due to the interest in the seismicity of that region, after the disaster of October 1985, and to the stationarity of the frequency content of these records. The analysis of the numerical results was simplified in this way.

Four magnitudes (6.5, 7.0, 7.5 and 8.1) and four types of soil (rock, hard soil, soft soil, very soft soil) have been selected. Figures 3.1, 3.2, 3.3, 3.4 show the simulations corresponding to the four types of soil chosen, for a magnitude 8.1 earthquake. Three statistically independent replicas were generated for each earthquake, for a total of 4*4*3=48 different ground motion simulations.

The simulated earthquakes were applied to a single degree of freedom elasto-plastic oscillator, with a yielding displacement x_y of 4 mm. Three values were considered for the period of the linear oscillations T_0 : 1.0, 1.5 and 2.0 seconds. The model, proposed by Bonc (1967) and made popular by Wen (1976) of a smooth and markovian approximation to an elasto-plastic oscillator, has been implemented for the numerical simulations.

There were, therefore, 48*3=144 records of structural response from which damage indices could be computed. Of these, only 109 showed an appreciable yielding and were considered in the analysis.

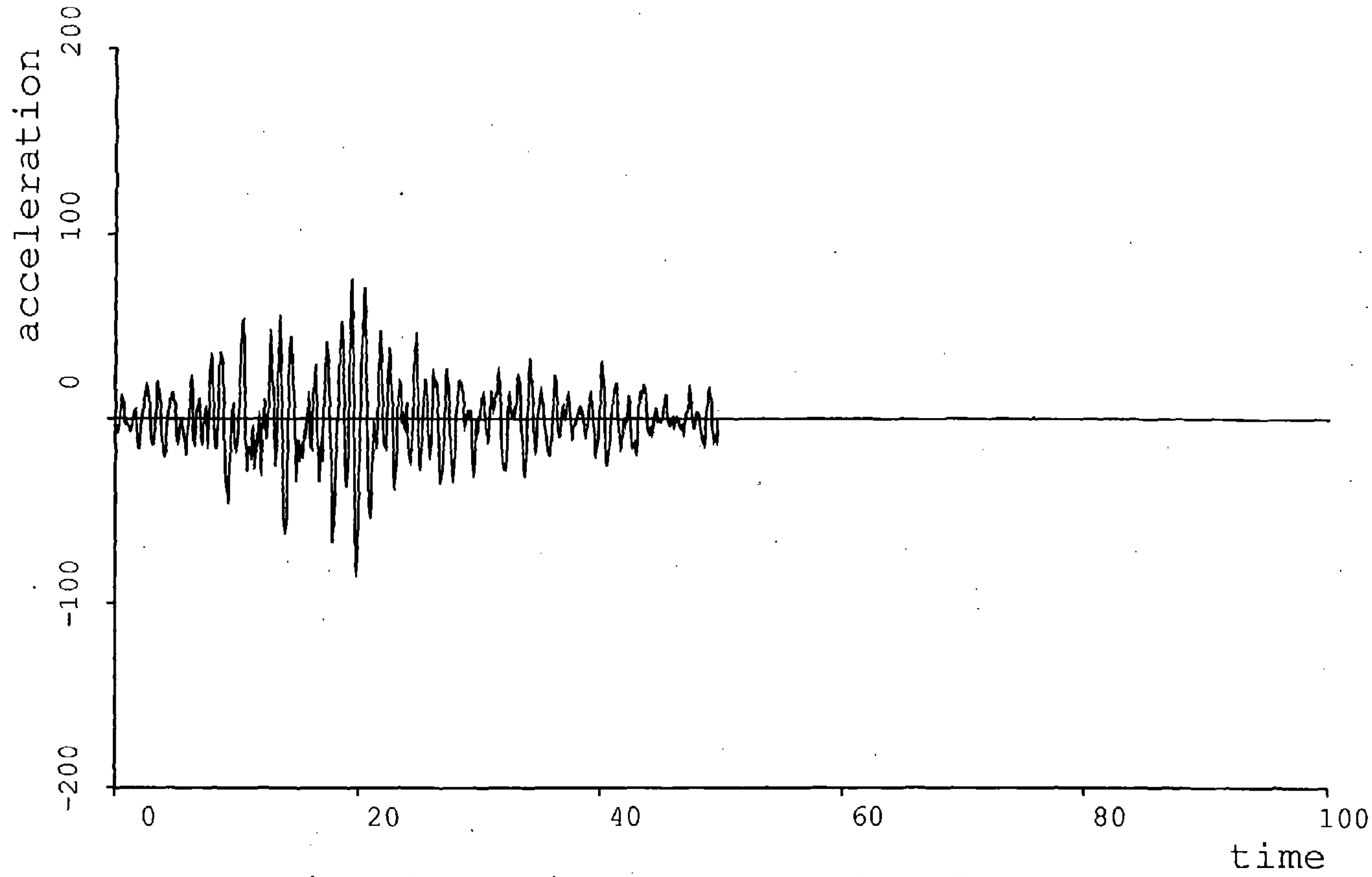


Fig. 3.1: Simulated Earthquake:
Magnitude 8.1, Rock

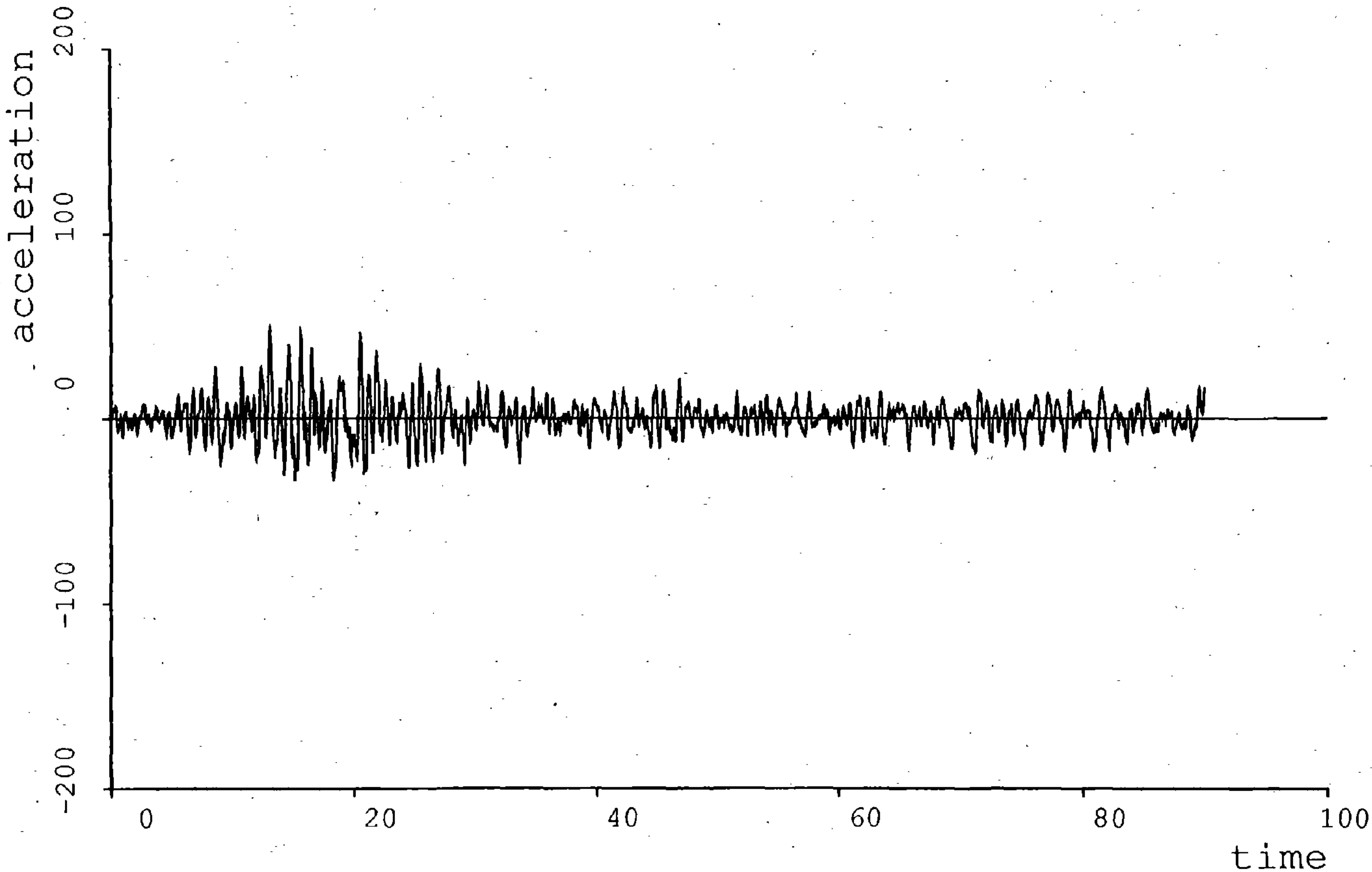


Fig. 3.2: Simulated Earthquake:
Magnitude 8.1, Hard Soil

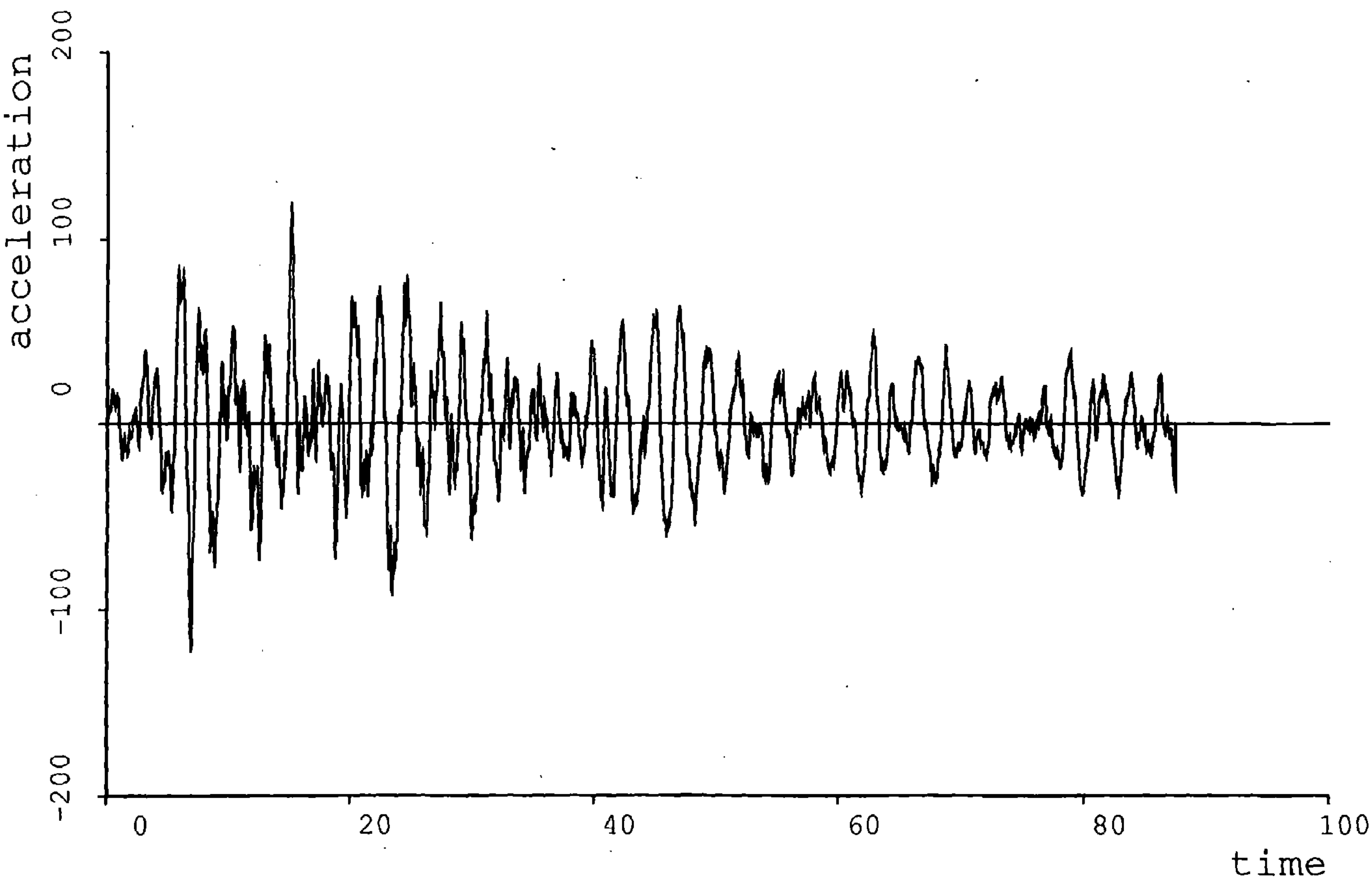


Fig. 3.3: Simulated Earthquake:
Magnitude 8.1, Soft Soil

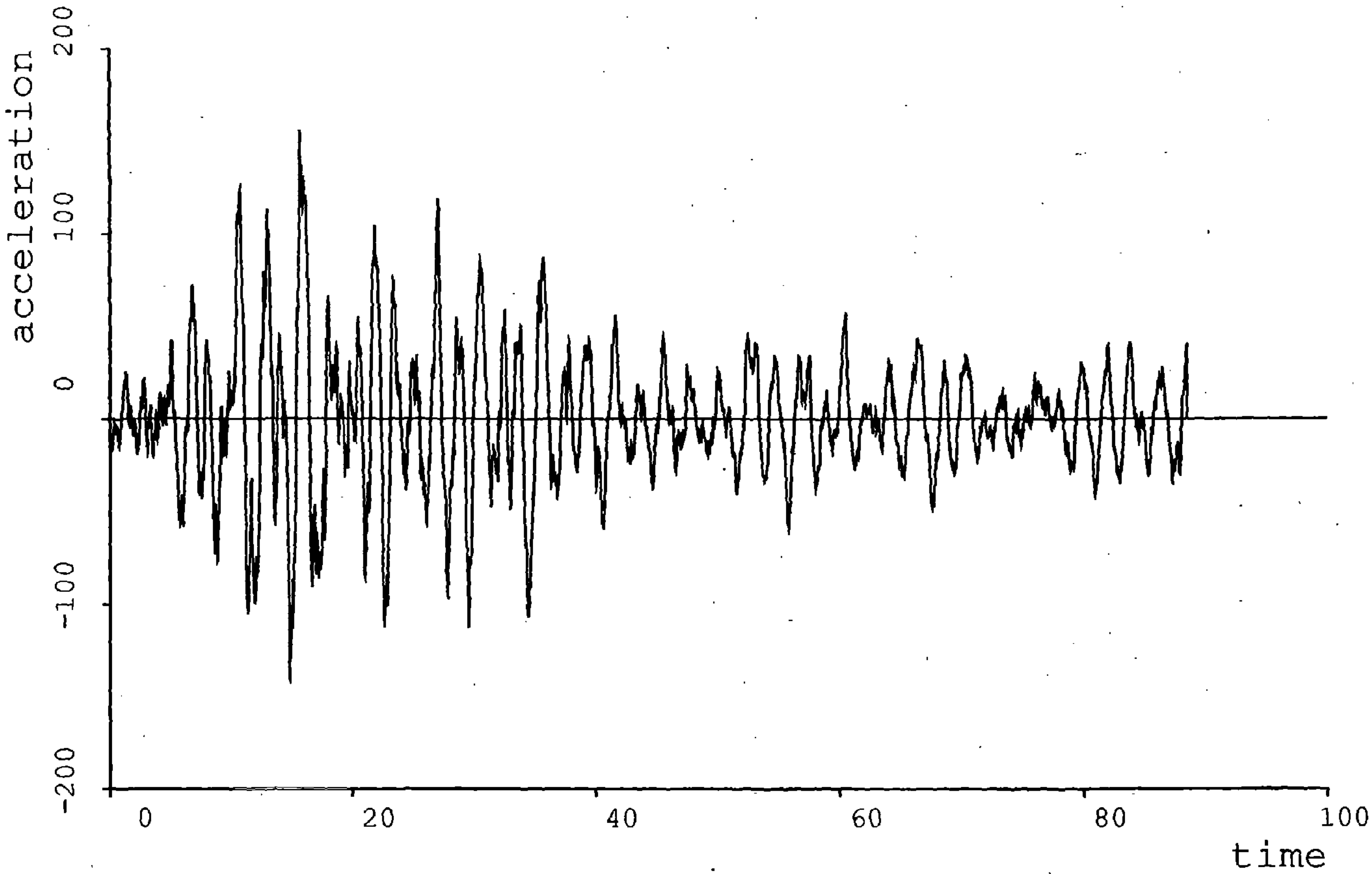


Fig. 3.4: Simulated Earthquake:
Magnitude 8.1, Very Soft Soil

3.3. Correlation between Indices

All the four fatigue indices show very high correlation. In particular, the energy index and the cumulative plastic displacement are virtually undistinguishable (fig 3.5), and the same can be said about Stephens and Yao's index and its simplified version (fig. 3.6). Both these last two indices are very well correlated with the energy index (figures 3.7 and 3.8).

For this limited but interesting database, it can be concluded that all the four fatigue indices considered carry the same information. From now on, therefore, only the energy index μ_E will be carried along.

Fig. 3.9 shows the correlation between the ductility index and the energy index. The coefficient of correlation is still very high, but definitely much lower than the correlation observed between the fatigue indices. Hence, we can conclude that the ductility index carries new information that is not contained in any of the energy indices examined.

3.4. Influence of the Parameters of the Ground Motion and of the Structure on the Damage Indices

An analytical investigation of the relations between numerical values of the damage indices and both ground motion and structural parameters is beyond the scope of this work. Nonetheless, some qualitative analysis is reported in the following.

It may be expected that damage indices should consistently increase when the intensity of the ground motion increases. Figures 3.10 and 3.11 show that this happens. The ductility index μ_x and the energy index μ_E are plotted versus the normalized intensity

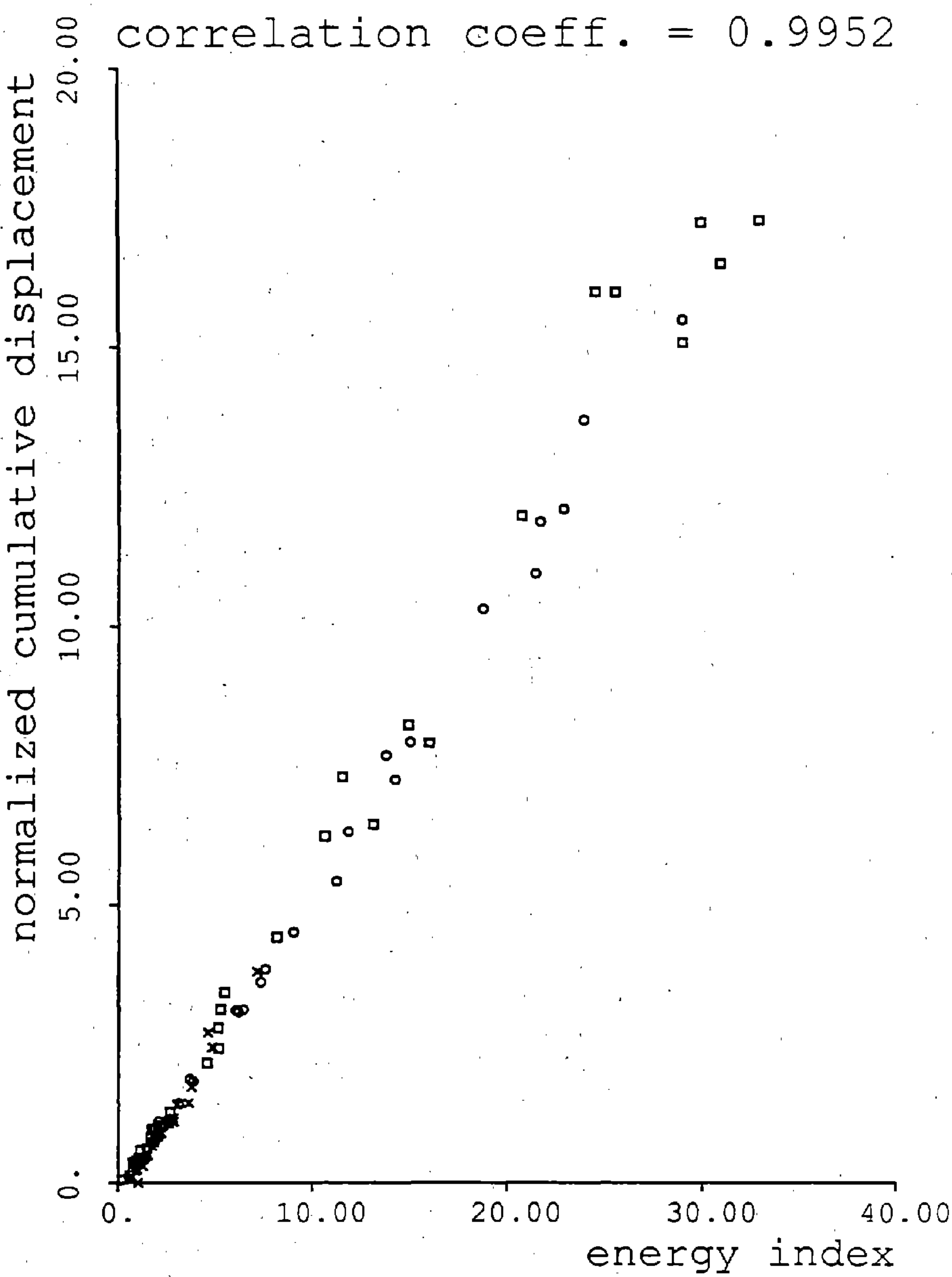


fig.3.5: Energy Index vs. Normalized Cumulative Displacement

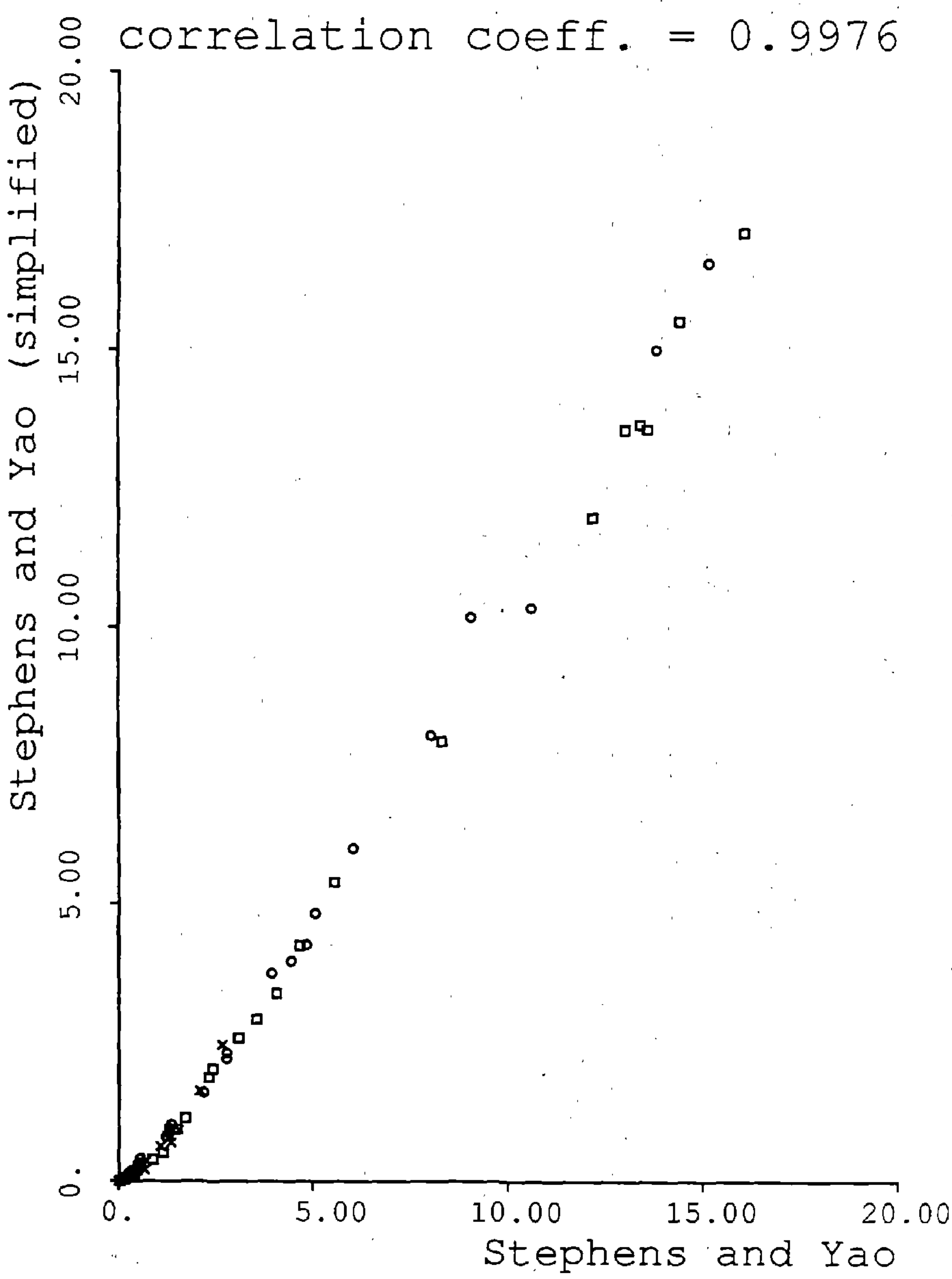


fig.3.6: Stephens' index vs. its simplified version

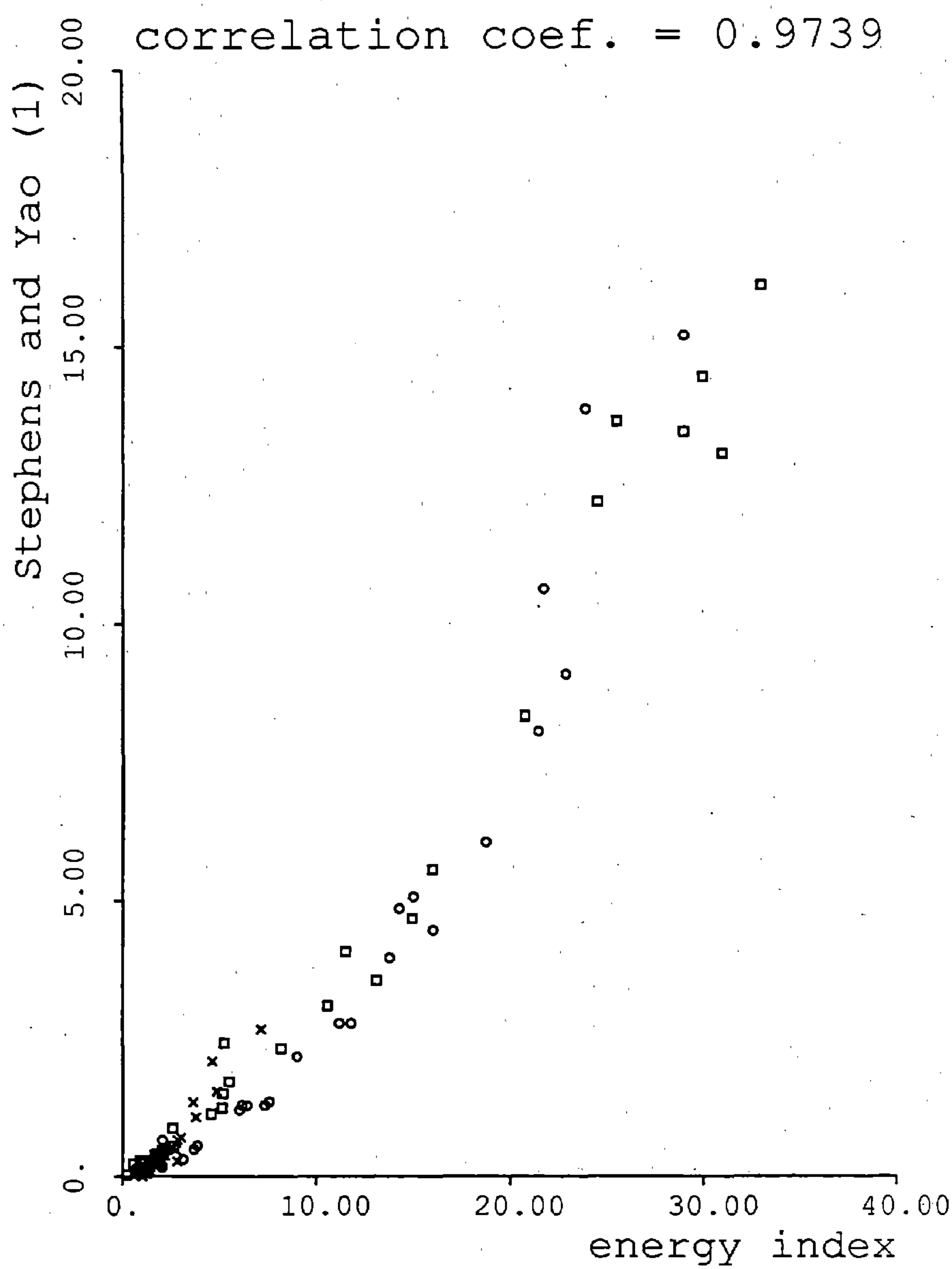


fig.3.7: Energy Index vs. Stephens' Index

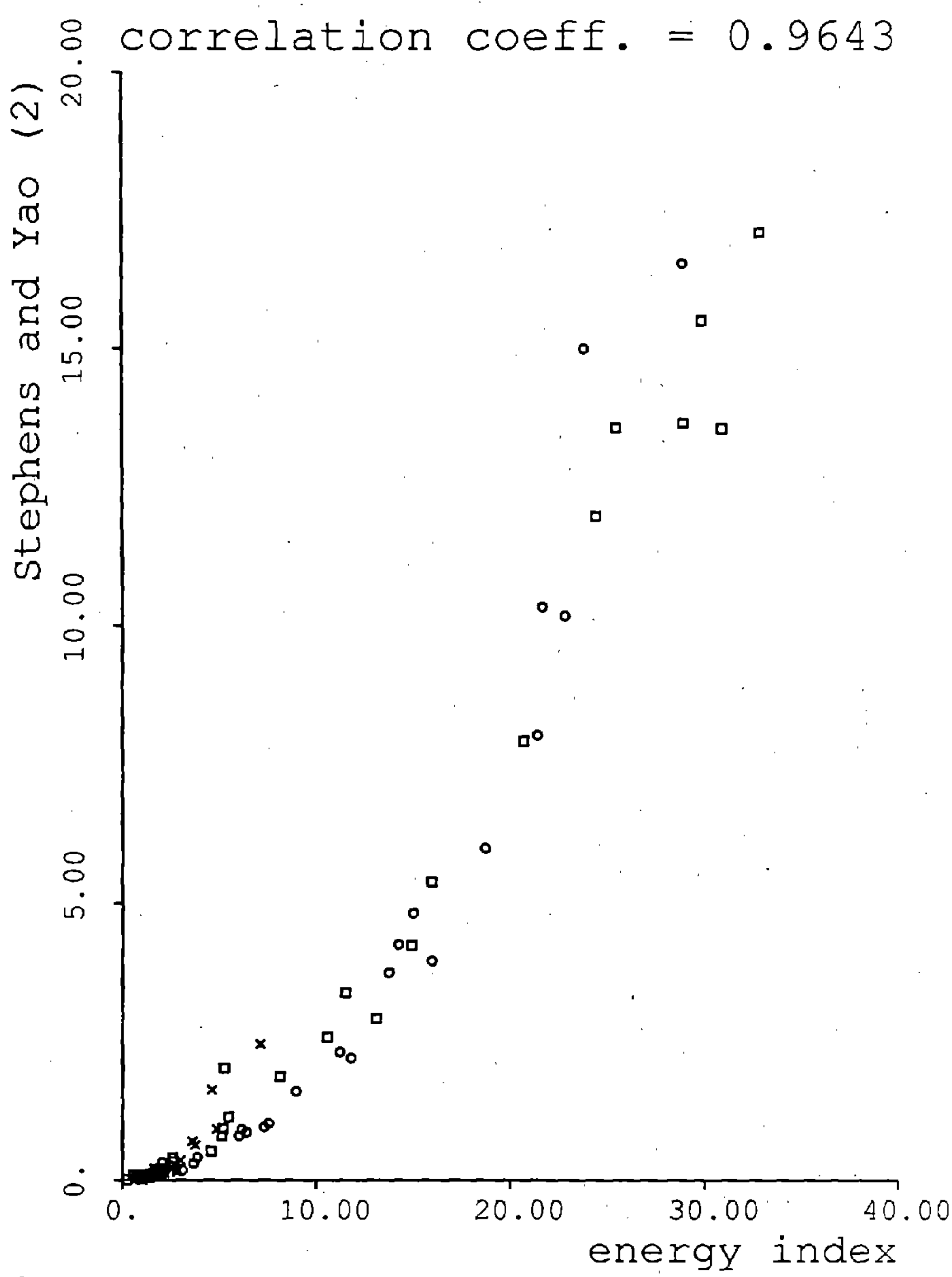


fig.3.8: Energy Index vs. Stephens' Index

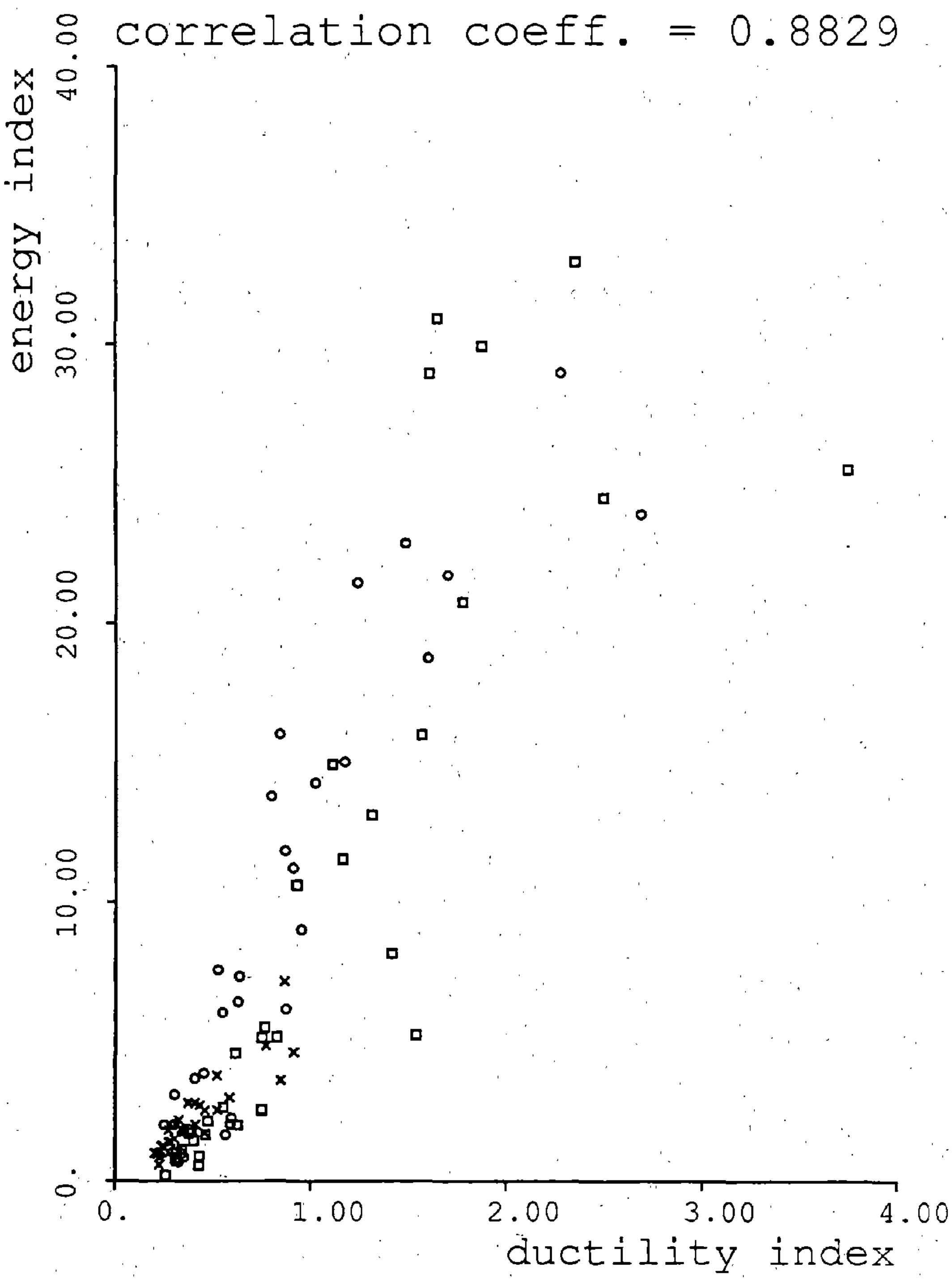


fig.3.9: Ductility Index vs. Energy Index

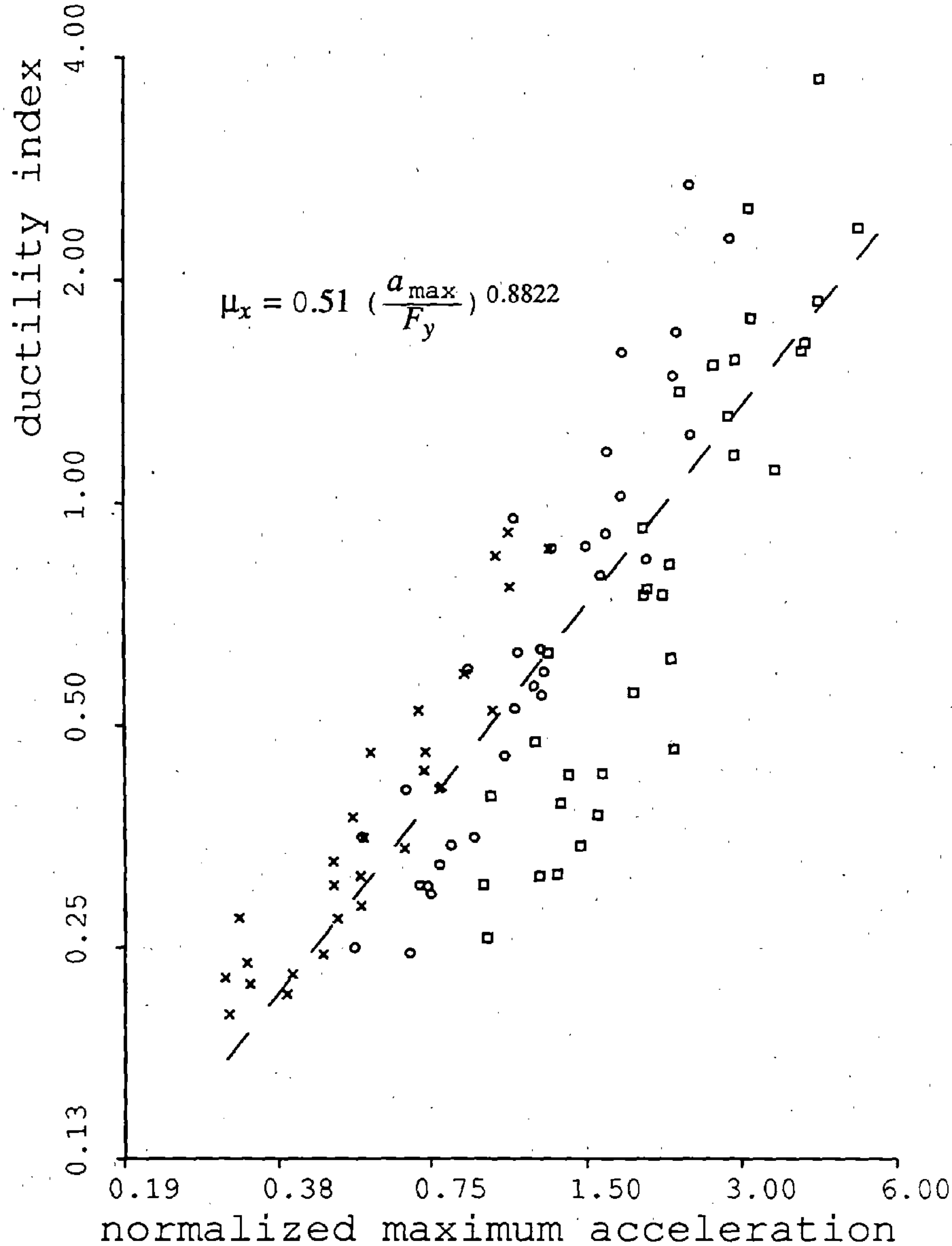


fig.3.10: Ductility Index vs. Normalized Max. Acceleration

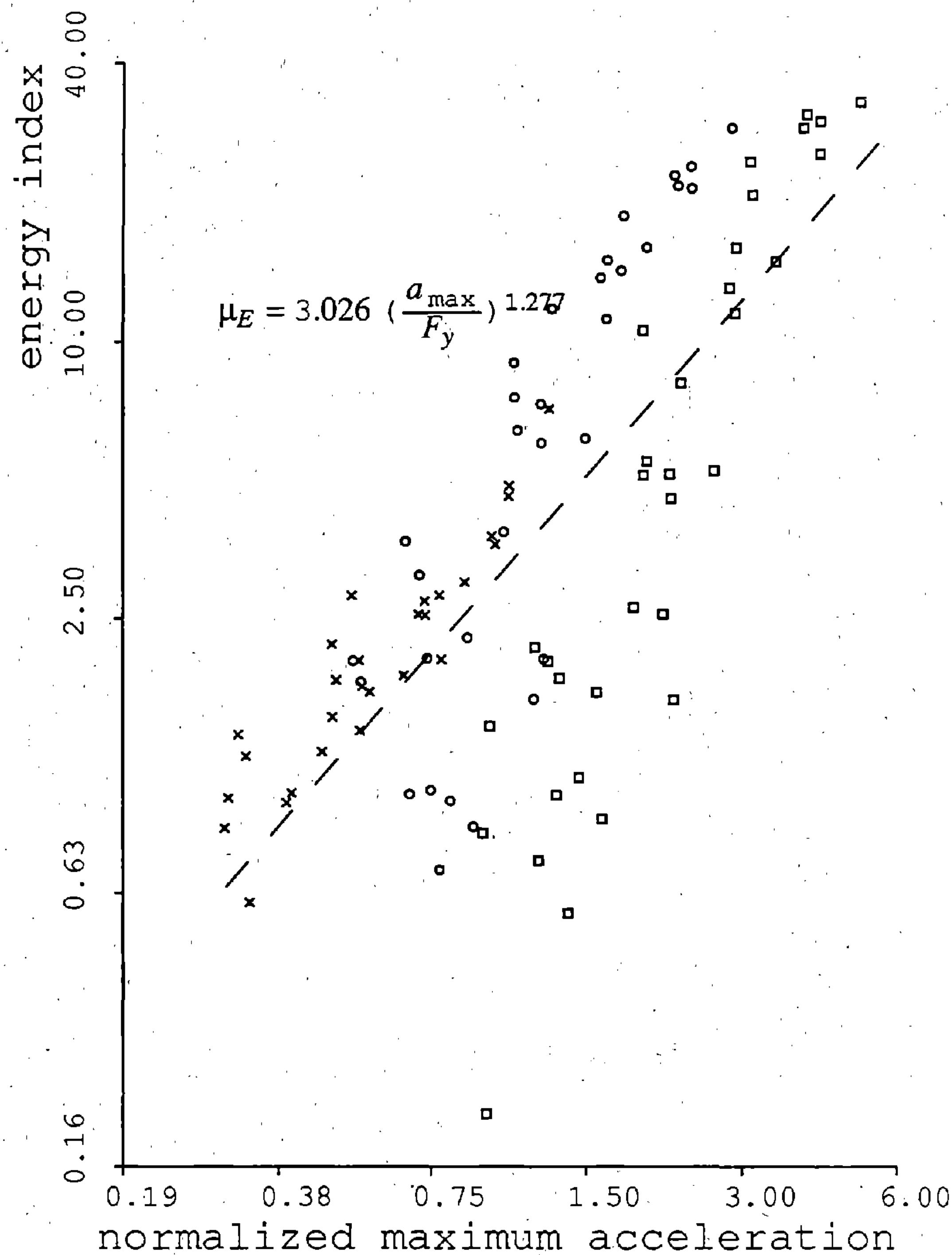


fig.3.11: Energy Index vs. Normalized Max. Acceleration

$$\frac{a_{\max}}{F_y}$$

where $a_{\max}$ is the maximum ground acceleration and F_y the yielding force. The exponential relations indicated have been obtained by fitting an expression of the type

$$\mu = \alpha \left[\frac{a_{\max}}{F_y} \right]^{\beta} \tag{3.3.1}$$

using the least squares method.

In the preceeding section, it was pointed out that the ductility index and the energy index where different from a statistical point of view. The index ratio

$$\gamma_\mu = \frac{\mu_E}{\mu_x} \tag{3.3.2}$$

can be considered a measure of the relative risk of failure by fatigue or by peak deformation.

Physical considerations suggest that fatigue effects should be more important for longer earthquakes. γ_μ has also been found to increase when the amplitude of the response increases, due either to the intensity of the ground motion or to resonance phenomena. As the energy that an elasto-plastic system dissipates in a cycle of deformation is proportional to the amplitude of the cycle, this last behavior is probably to be attributed to large amplitude vibrations that follow an initial strong shaking.

On the basis of these considerations, two parameters have been chosen to characterize the behavior of γ_μ: the normalized cumulative energy

$$E_n = \frac{\int_0^s a^2(t)\, dt}{F_y x_u} \tag{3.3.3}$$

and the normalized ground frequency

$$f_n = \frac{f_g}{f_0} \tag{3.3.4}$$

Above, $a(t)$ is the ground acceleration, with average frequency f_g, as computed by Ellis (1987), F_y, x_u, f_0 are respectively the yielding force, the ultimate displacement and the frequency of the linear oscillation of the system considered.

Because of the randomness of the numerical simulation, only scattered data points were available. The range of values of E_n has been divide, using a trial and error procedure, in three non overlapping bands. In fig. 3.12, a scattered plot of the data points on the E_n/f_n plane is shown. When the values of the index ratio are plotted versus the normalized frequency for eah of these intervals (figures 3.13 through 3.15), two features are evident:

- the values of γ_μ present a peak for certain values of the normalized frequency,
- the value of f_n for which γ_μ is maximum decreases as E_n increases.

The three interpolating curves drawn in figures 3.13, 3.14 and 3.15 have been fitted to the experimental data using the least square method. The analytical model has been adapted from Caughey (1963). From fig. 3.16, where the three curves are compared, it can be observed that the index ratio exhibits a softenig resonance (Caughey, 1963, Iwan,1974).

Although this analysis is purely qualitative, it clearly shows a deterministic dependence exists between γ_μ and the paramenters of the ground motion and of the structure. Therefore, the mechanism that is more likely to provoke failure will depend on both these parameters.

The designer can exploit this determinism to identify which failure mode might be more dangerous. On the other hand, when "a priori" informations are not available, the variability of γ_μ implies that both mechanisms of failure should always be considered in a seismic damage assessment analysis.

The softening resonance is probably enphasized by the absence of a deterioration mechanism in the model considered. Simulation with more realistic models are necessary to assert definitely that such a phenomenon will take place in real structures.

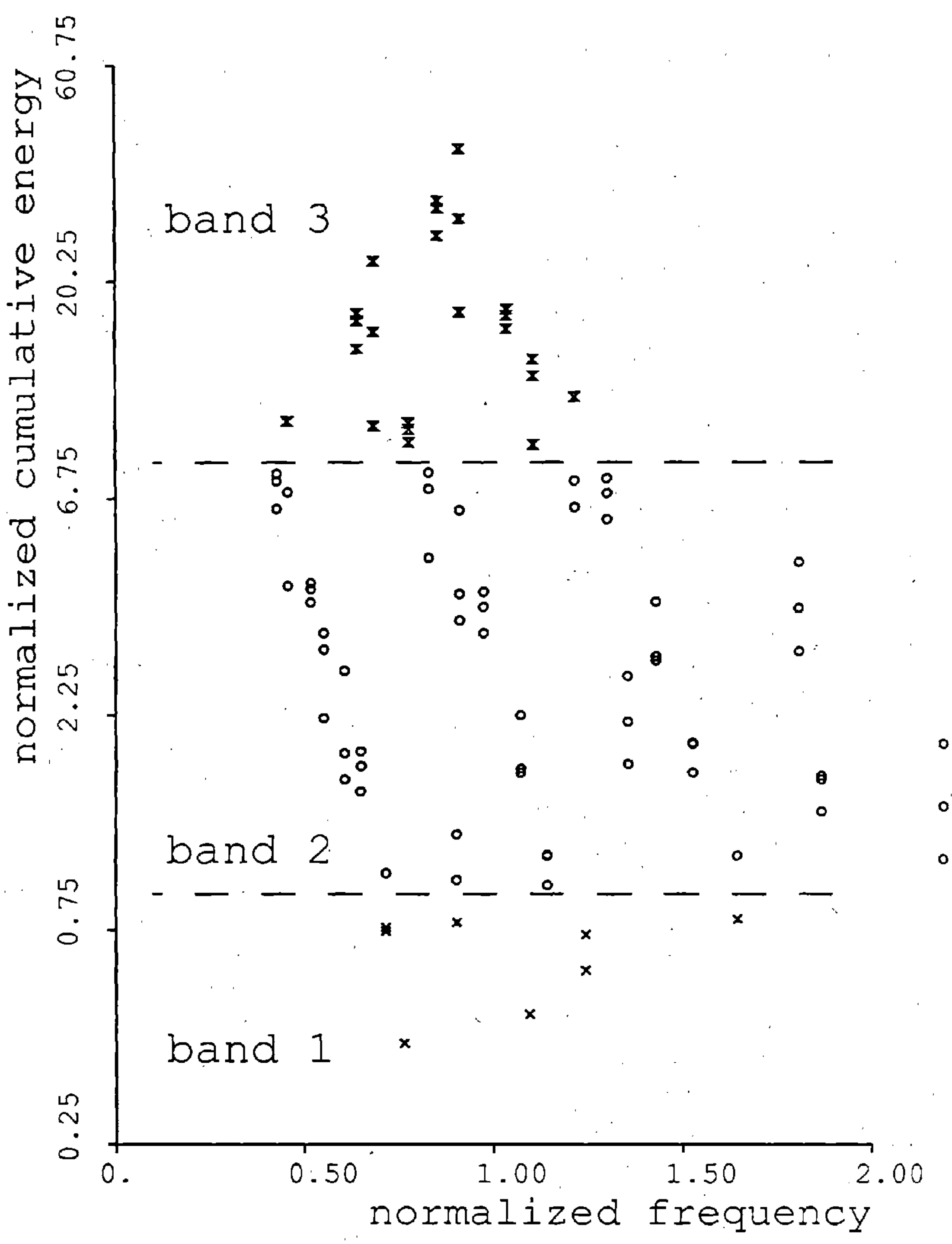


Fig. 3.12: Normalized Frequency and Cumulative Energy for the Earthquakes Simulated

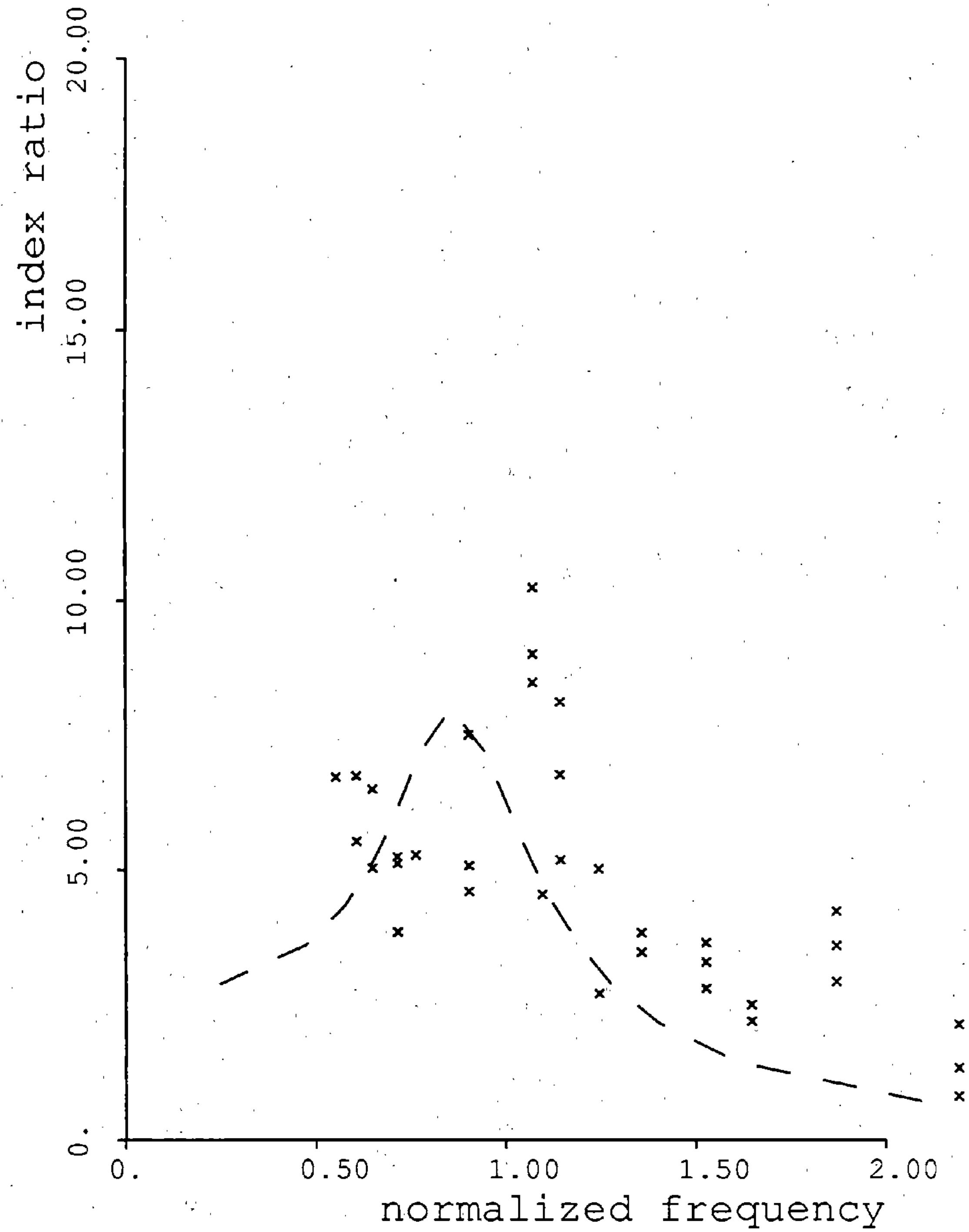


Fig. 3.13: Index Ratio vs. Normalized Frequency:
low cumulative energy

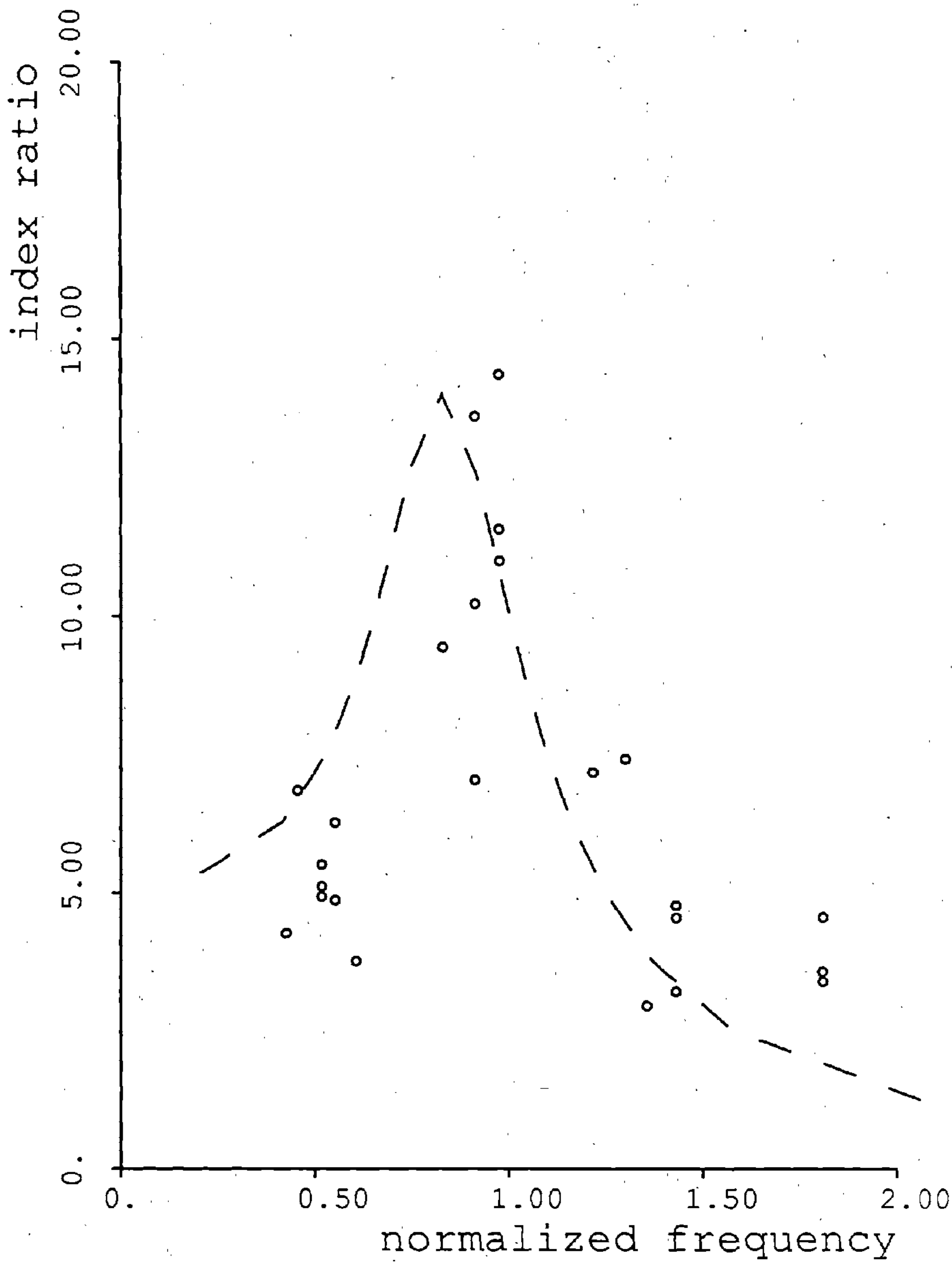


Fig. 3.14: Index Ratio vs. Normalized Frequency:
Intermediate Cumulative Energy

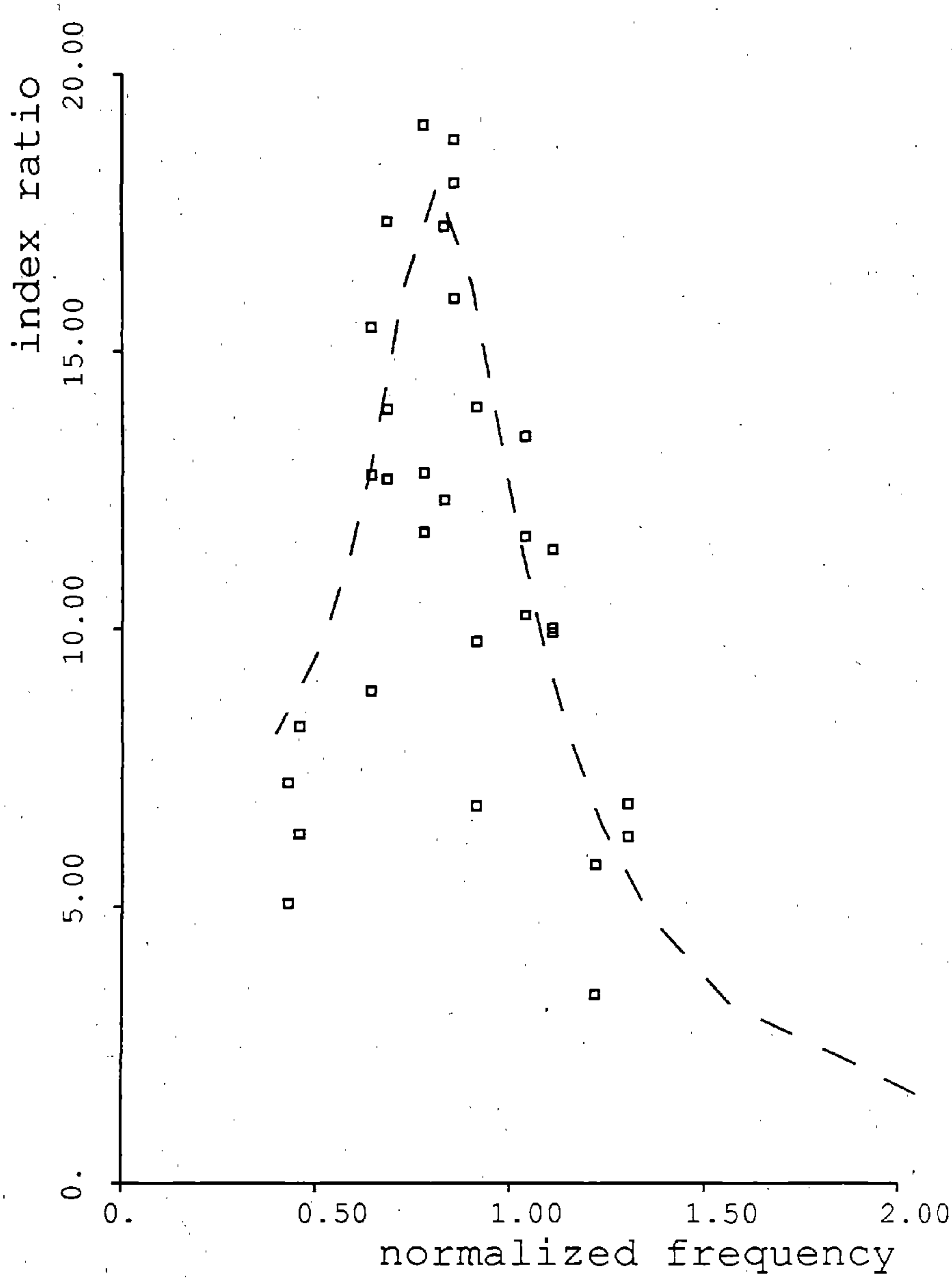


Fig. 3.15: Index Ratio vs. Normalized Frequency:
High Cumulative Energy

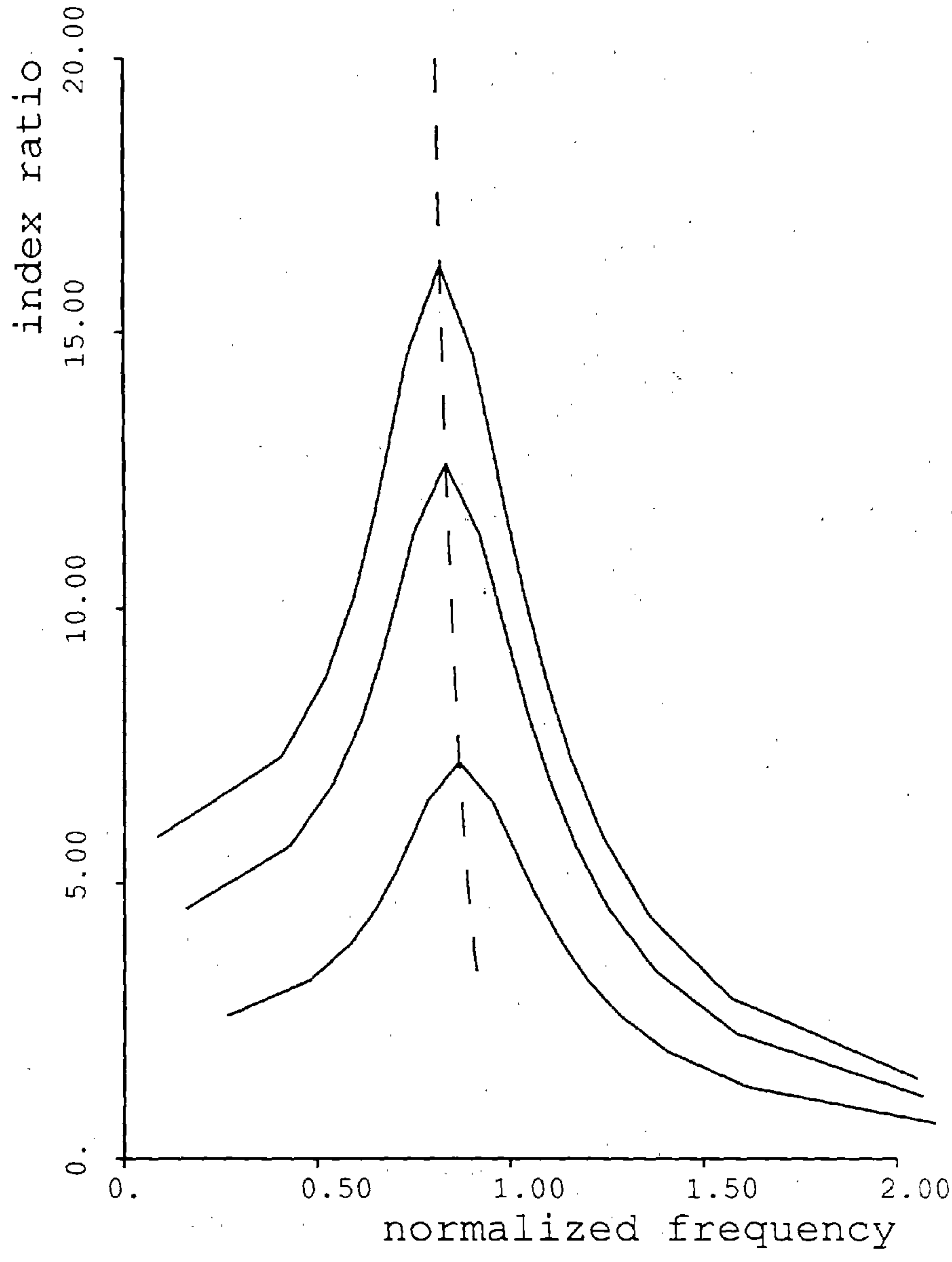


Fig. 3.16: Comparison of the Resonance Curves

SECTION 4: DAMAGE INDICES BASED ON SYSTEM IDENTIFICATION USING LINEAR MODELS

4.1. Introduction

In order to assess seismic structural damage, a damage state for the structure in question must be defined. The first approach can be to consider the damage state at each point, and define the damage state of the global structure as a combination thereof. In theory, a damage state can be defined for each point.

The definition of "point" depends on the structural model that is chosen for the analysis. If the structure is modeled as an assemblage of beams in two or three dimensions, there will be ∞^1 points for which a damage state must be defined. If a complete 3-dimensional model is used, there will be ∞^3 points to be considered.

The damage state can also be defined in several ways. One can think of a binary damage state (failure/no failure), or of a discrete valued damage state, using qualitative indicators such as none, minor, relevant, major, failure, or of continuous damage indices, as proposed by Ju (1987). The damage state may or may not be dependent on the history of loads, depending on the model that has been chosen to describe the structural behavior.

In general, therefore, a damage event that the structure undergoes can be described as a function

$$f : \mathbf{R}^n \rightarrow \mathbf{R}^n$$

The damage state can thus be defined as a functional of f.

$$D = \int_{\mathbf{R}^n} w(\mathbf{x}) \, f(\mathbf{x}) \, d\mathbf{x} \tag{4.1.1}$$

where $w(\mathbf{x})$ is an appropriate weighting function.

The formulation of the damage problem, based on an infinite dimensional damage state, is theoretically correct but not very practical. The damage state must be reduced to a finite number of dimensions in order to solve the problem of damage assessment for a real structure. The need for this reduction has been first pointed out by Yao (1982).

Once the number of dimensions has been reduced, the damage state of the structure can be inferred from the history of a finite number of structural parameters. The analysis of each of these histories yields a numerical value for the corresponding damage index.

This reduction can be obtained by lumping procedures. The structure is modeled as an assemblage of elements and joints, for each of which a damage index is computed from the history of load during the earthquake. The global damage index for the structure is then defined as a weighted average of the damage indices for the single elements (Stephens and Yao, 1987, Park, Ang and Wen, 1985). If n_{el} elements and joints are considered, for each of them a damage index δ_i and a weight β_i can be defined, so that the global damage would be measured as:

$$\delta = \frac{1}{\sum_{i=1}^{nel} \beta_i} \sum_{i=1}^{nel} \beta_i \delta_i \tag{4.1.2}$$

Reduction by lumping requires that generalized displacements and restoring forces are available for a large number of nodes in order to be significant. This is possible in simulation studies, in the analysis of shaking table experiments, or for full scale structures that are extensively instrumented. In the practical case of a structure, where only two accelerometer arrays are installed, reduction by lumping is not possible directly. The nodal displacements that are not observed can be estimated using non linear Kalman filter techniques, but these would fail if the behavior of the structure is of the hysteretic type.

The reduction procedure and the damage indices that are proposed in the following are based on the modal analysis of the structure in the linear phase, and of an equivalent linear structure in the non linear phase. The functional form of the damage indices is derived from the theoretical and experimental analysis of damage of simple structural elements.

4.2. Damage Indices Based on Equivalent Modal Parameters

A linear structural system described in terms of the modal parameters has been proved to be identifiable by Beck (1979). The modal parameters are the damping factor, the natural frequency and the effective participation factor for each natural mode, considered in an approximate description of the structural motion based on modal decomposition. When the structural behavior is non linear, a system identification algorithm based on linear models will yield estimates of equivalent linear models. The nature of such an equivalence will depend on the criteria that the analyst has chosen for the purpose of identification. Traditional equivalent linearization techniques (Caughey, 1963, Valdimarsson et al., 1981) seek equivalent linear models in the "error in the equation" sense, Beck and Jennings (1980) introduced "error in the output" criteria in structural analysis, and DiPasquale and Cakmak (1987) treated "maximum likelihood" criteria. These concepts are discussed in detail in chapter 5.

The equivalent linear model that fits strong motion records is unique as long as the behavior of the structure is linear. When the structure enters a nonlinear phase, such a uniqueness is lost. In particular, the structure will have an apparent softening as the amplitude of the oscillation increases, and the equivalent natural frequencies will decrease. By fitting a time variant linear model to the records, a history of the equivalent linear parameters is obtained.

The goal is now to extract information about damage from the history of the modal parameters. It is clear from the start that only the natural frequencies will provide valuable information. Damping factors are entities of uncertain physical meaning, and their estimation, when the structure is in the non linear phase, yields results of questionable reliability. Furthermore, as it has been noted by Beck (1978), the estimates of damping factors and of effective participation factors are statistically correlated. This correlation is reduced but not eliminated when constraints are imposed on the effective participation factors, as it is described in chapter 5.

In this phase of the research, only the first (fundamental) natural frequency is considered. All the computations are actually carried out on its inverse, the fundamental period of vibration, because the fundamental period is the quantity most commonly considered in the engineering practice.

The interval $(0,s)$ of duration of the earthquake is divided into n_{wind} non overlapping windows of width s_i sec. For each of these windows, an equivalent fundamental period $(T_0)_i$ is computed. The first window can be made small enough so that $(T_0)_1$ can be assumed to be equal to the fundamental period of the linear oscillation of the building before the earthquake, $(T_0)_{initial}$. When the record is long enough, so that the vibrations due to the strong motion have abated at the end of the record, and the behavior of the structure can be considered linear, $(T_0)_{nwind}$ can be assumed to be equal to the fundamental period of the linear oscillation after the earthquake $(T_0)_{final}$. Otherwise, when $(T_0)_{final}$ was available from post-earthquake tests, a fake window of length zero has been added, so that $(T_0)_{nwind} = (T_0)_{final}$.

The first attempt to characterize seismic damage using the evolution of the fundamental period was based on the assumption that damage in a structural element would result in a degradation of its stiffness. This has been found in laboratory experiments on reinforced concrete elements and shear walls (Newmark and Rosenblueth, 1974), but it is not always true for steel elements.

A decrease in the stiffness of some elements will result in a decrease of the global stiffness of the structure (Rayleigh, 1945, Dowell, 1979), henceforth in an increase of its fundamental period. A damage index can therefore be defined as the ultimate stiffness degradation

$$\delta_{st} = \frac{(T_0)_{final} - (T_0)_{initial}}{(T_0)_{initial}} \tag{4.2.1}$$

Small amplitude vibration tests on both full scale structure and small scale models indicate that δ_{st} would be always positive and consistently increasing with the severity of the damage. (Chen et al.,1977, Foutch and Housner,1977, Meyer and Roufaiel,1984, Mihai et al.,1980, Vasilescu et al.,1980, Ogawa and Abe,1980, Carydis and Mouzakis,1986)

The choice of ultimate stiffness degradation as a damage index, although intuitively appealing, lends itself to some major criticisms. The digitized records available are rarely long enough, so that $(T_0)_{final}$ can be extracted, and the need for post-earthquake vibration testing would make the method unpractical. Furthermore, there is no basis to predict that stiffness degradation would take place in steel structures.

However, the most important shortcoming of damage measurements based on stiffness degradation is that the information contained in the actual strong motion records is neglected, and only quantities that could be measured independently of the earthquake are considered.

Hence, damage indices have been sought, that could be computed from the history of the acceleration during the strong motion.

It has been pointed out in chapters 2 and 3 that peak deformation and fatigue contribute independently to the damage of a simple structural element. Even though their effect on the performance of the element may be the same, the mechanisms through which such an effect is obtained are different. When numerical simulations are carried out, the indices that are computed to quantify the effects of these two components are uncorrelated.

The reduced global damage space for a structure should therefore have at least dimension two. One damage index should reflect the peak response, and another the fatigue load that the structure experiences.

The maximum softening δ_M can be used to measure the peak deformation.

$$\delta_M = \max_{i=1,nwind} \frac{(\Delta T_0)_i}{(T_0)_{initial}} \tag{4.2.2}$$

For a simple (SDOF) non linear system, δ_M is a good measure of the peak deformation, as it will be seen later. For complex (MDOF) systems, some authors have suggested that the maximum displacement of a node, namely the roof, be used to measure the maximum strain on a structure (Meyer and Roufaiel, 1984). The maximum softening is, in the authors' opinion, a more meaningful measure of the maximum strain. Furthermore, the computation of the displacement through a double integration of the digitized accelerograms is by no means an easy task (Stephens et al., 1985).

The cumulative softening δ_E can be used to measure the energy dissipated.

$$\delta_E = \sum_{i=1}^{nwind} \frac{(\Delta T_0)_i}{(T_0)_{initial}} \frac{s_i}{(T_0)_i} \tag{4.2.3}$$

This form of δ_E is suggested by a qualitative analysis of the behavior of hysteretic systems. As the amplitude of the cycle and therefore the energy dissipated increases, the structure appears to soften (Beck, 1979, McVerry, 1979).

Notice that no knowledge of the ultimate characteristics of a structure is needed for the computation of the indices (4.2.2) and (4.2.3).This greatly simplifies the procedure and reduces the uncertainties.

These indices, being based on the results of parameter estimation procedures, will be called parameter based indices, in contrast to those described in chapter 3, that can be referred to as response based indices.

With respect to the response based indices, the parameter based indices present some conceptual and practical advantages. They characterize the global state of the structure using as little as two observation records. The modal parameters of a structure are non linear functions of the local force-deformation characteristics, so that the softening phenomena that take place at a local level are averaged. This is similar to using reduction by lumping. By using indices of the kind (4.1.2), the analyst controls the averaging process, but much more data are needed. Furthermore, the indices (4.2.2) and (4.2.3) can be computed without any prior knowledge of the structure in question. In particular, no ultimate capacity is needed, and the normalization factor $(T_0)_{initial}$ is computed from the strong motion records.

In order to test the ability of δ_M and of δ_E to measure response based indices, the numerical simulations described in chapter 3 have been studied.

In fig. 4.1, the maximum softening δ_M is compared to the ductility index. The two indices correlate very well up to very high ductilities (a ductility index of two indicates that the maximum displacement was greater than the yielding displacement by a factor of ten)

The numerical value of the coefficient of correlation is lowered by a few outliers that appear at very high ductility, mostly for the weakest oscillator ($T_0 = 2.5$sec , $F_y = 39.5\ mm/sec^2$). These outliers are due to large plastic drifts in one direction, as in fig. 4.2, that make the cycle dissymmetric and cannot be detected by an equivalent linear analysis. As long as the cycle is symmetrical, the maximum softening performs very well as a measure of the peak deformation. Hysteretic cycles that have been measured for real structures, subjected to earthquake loads, are approximately symmetric. This is true even for severe damage, such as the Imperial County Service Building during the Imperial Valley earthquake in 1979 (Cifuentes, 1984).

In fig. 4.3, the energy index and the cumulative softening are compared. The correlation index is very large, (ca. 0.95) and the correspondence between the two quantities is very good. Further study is necessary to state with certainty that the cumulative softening is a reliable measure of the energy dissipated in simple structures.

It can be concluded that, for the numerical simulation (a sample of 109 damaging earthquakes) that has been considered, the parameter based indices carry the same information as the response based indices. They are, therefore, physically meaningful. In order to use them for the analysis of damage, records from structures that have been actually damaged must be analyzed. This analysis is presented in chapter 6.

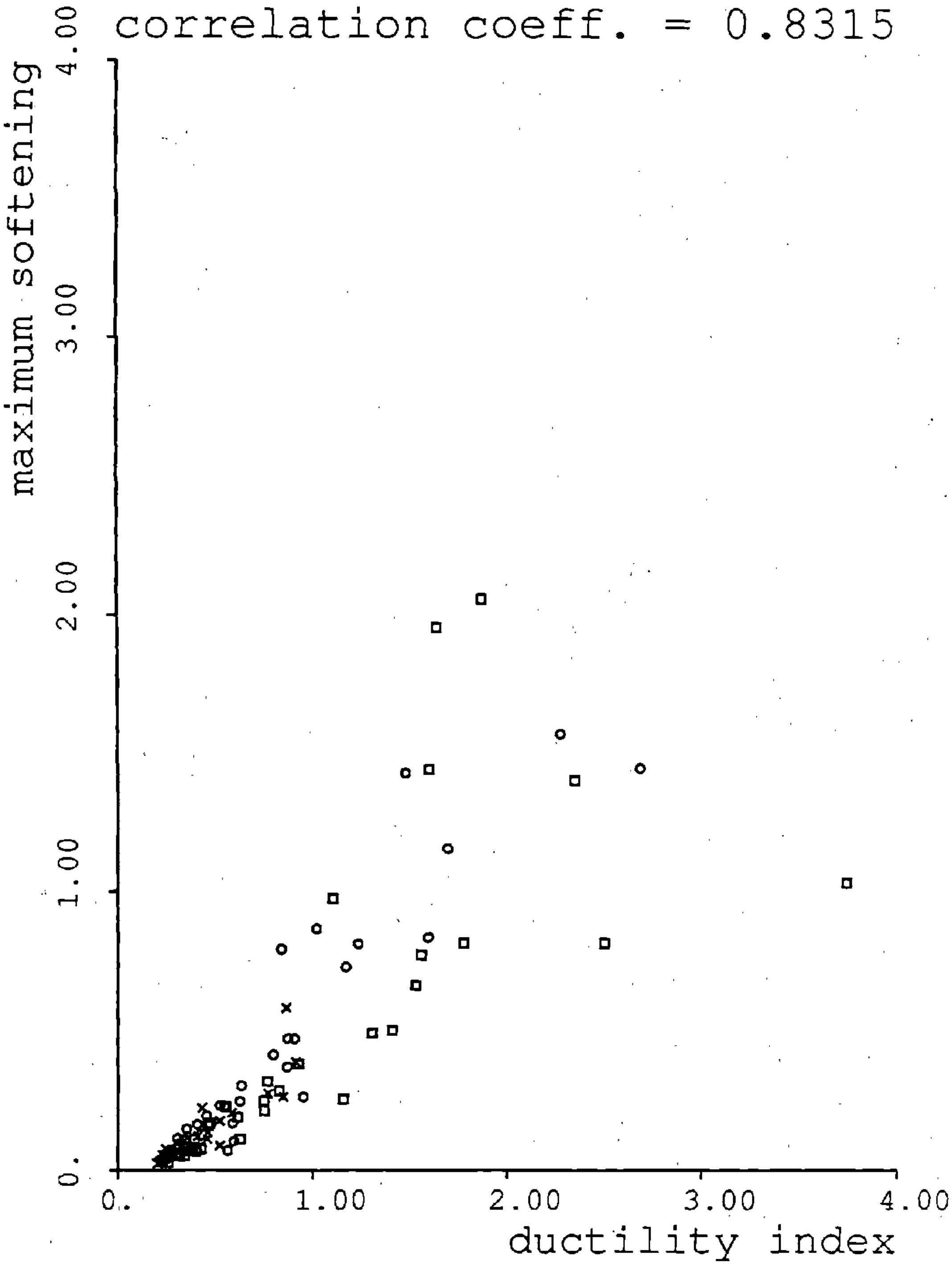


fig.4.1: Ductility Index vs. Maximum Softening

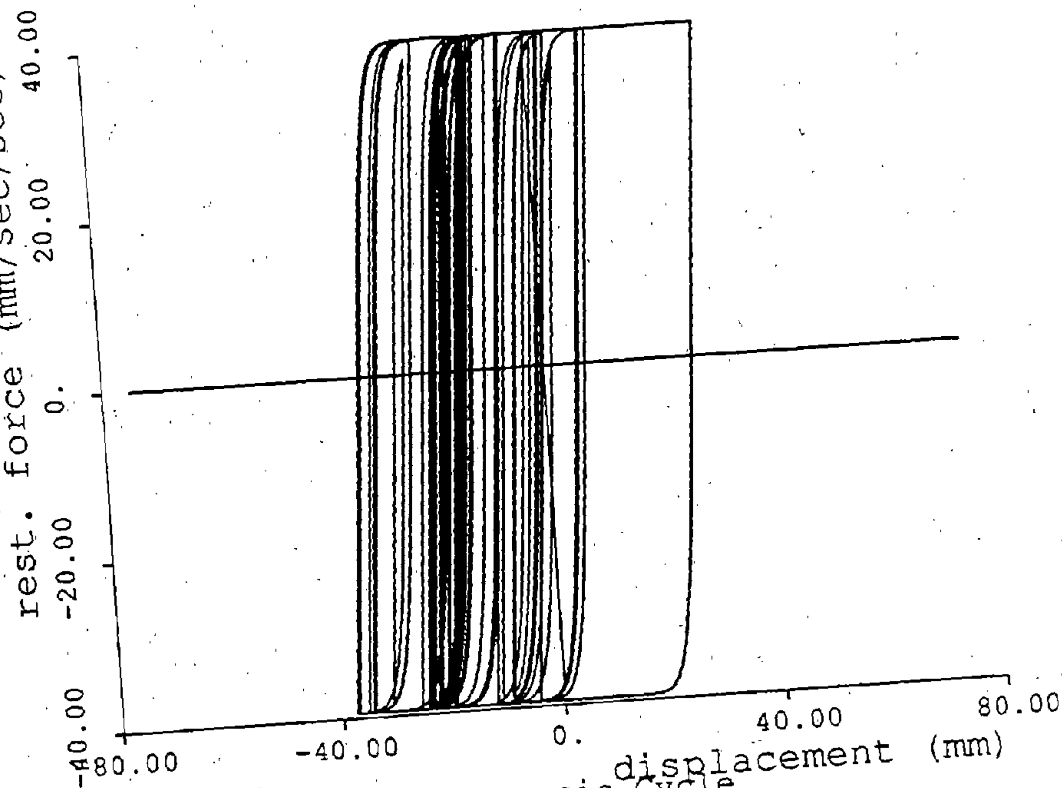


Fig. 4.2b: Nonsymmetric Hysteresis Cycle

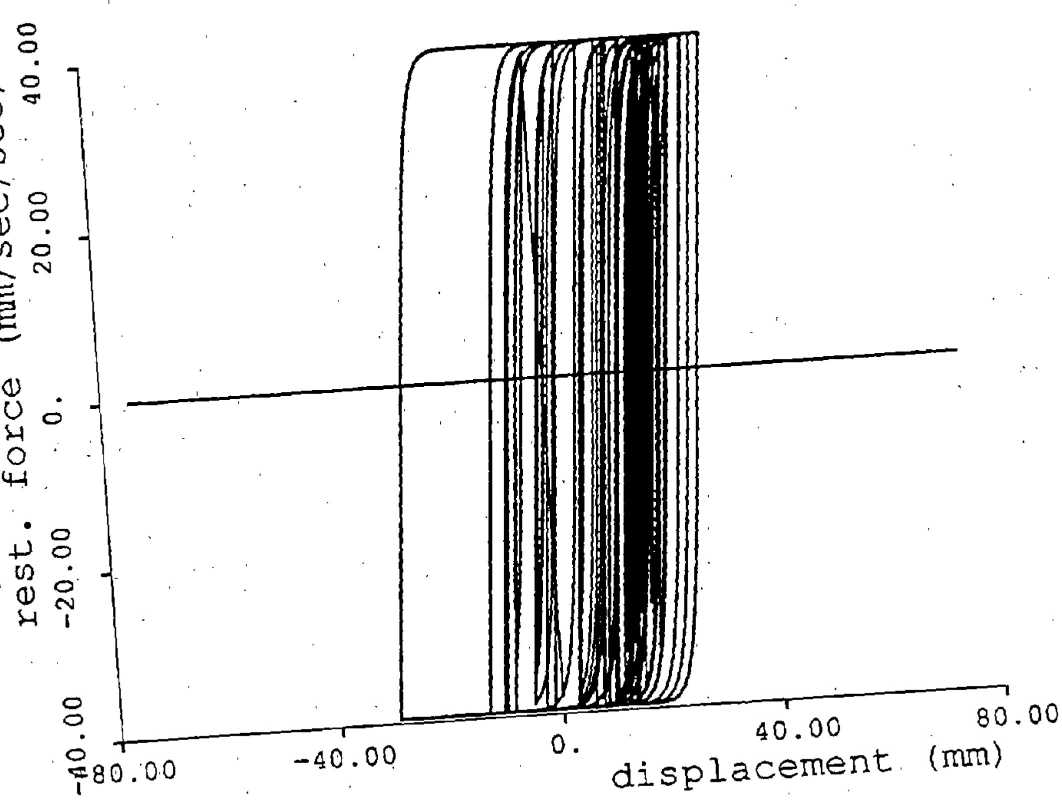


Fig. 4.2a: Symmetric Hysteresis Cycle

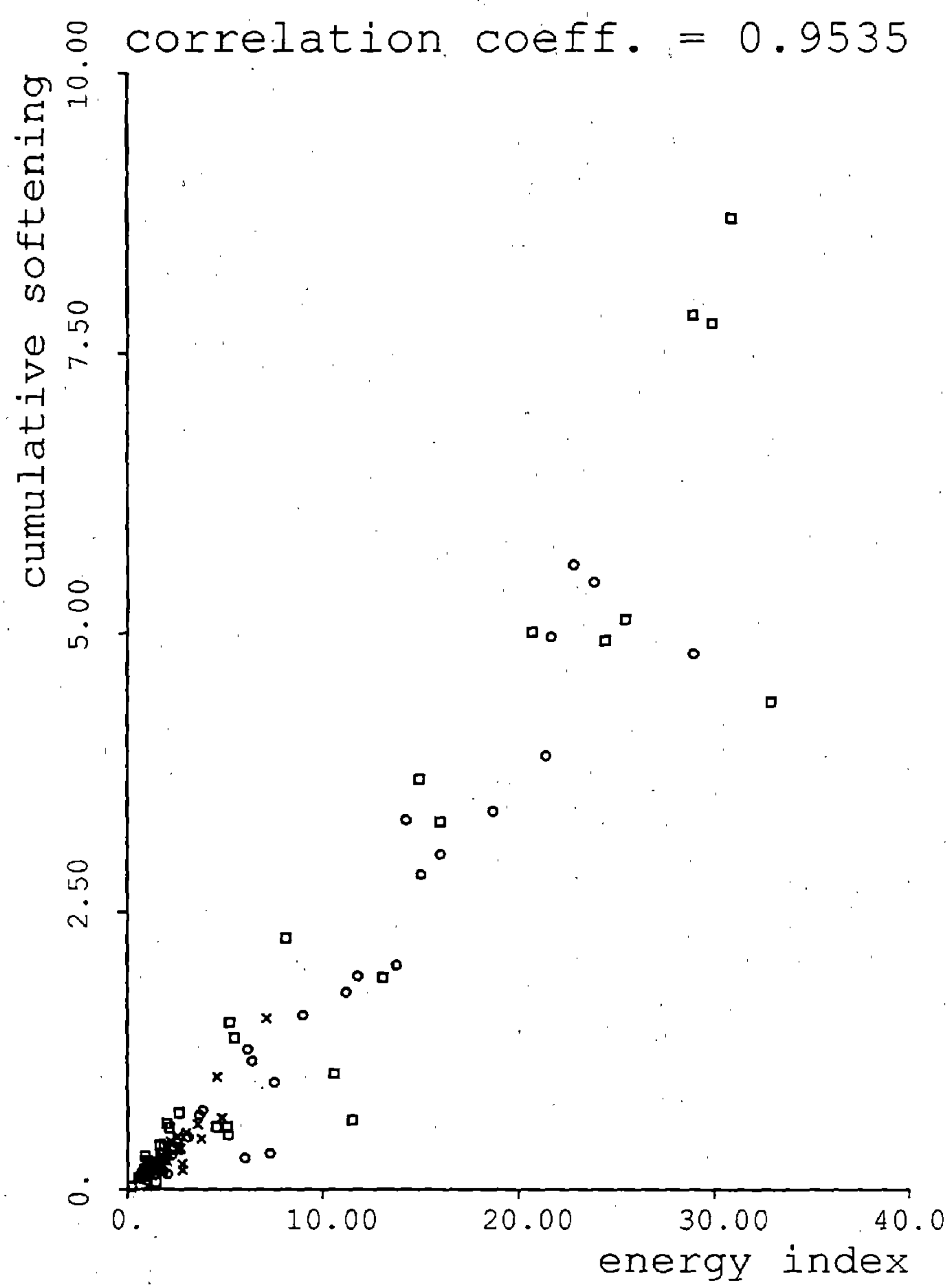


fig.4.3: Energy Index vs. Cumulative Softening

SECTION 5: MUMOID: A PROGRAM FOR THE IDENTIFICATION OF LINEAR STRUCTURAL SYSTEMS

5.1. Introduction

In this chapter, a procedure for Maximum Likelihood Estimation of the modal parameters is presented. It yields statistically optimal estimates of the time varying modal parameters (natural frequencies, damping ratios, participation factors, mode shapes) of a structure using earthquake records. The estimation procedure leads to a computationally efficient algorithm and still yields direct estimates of the modal parameters.

5.2. Model of the Structure

The structure is modeled as an n degree of freedom (ndof) linear system, with measured (earthquake) and unmeasured (wind, traffic, etc.) excitation, the latter modeled as white noise. The observations are noisy samples of the accelerations at one point of the building. The sources of uncertainty are assumed to be the unmeasured excitation and an observation error (sequence of uncorrelated random variables). The equations of motion are

$$\underset{\sim}{\mathbf{M}}\ddot{\mathbf{x}} + \underset{\sim}{\mathbf{C}}\dot{\mathbf{x}} + \underset{\sim}{\mathbf{K}}\mathbf{x} = -\mathbf{m}u + \mathbf{f}v \tag{5.2.1a}$$

In the following, the mass matrix M is assumed to be diagonal, u is the ground acceleration and v is a white noise process that represents the unmeasured excitation. The absolute acceleration of a node, say the nth, is measured with an error $\hat{w}_k$ at intervals Δt ,

$$y_k = \ddot{x}^{(n)}(k\Delta t) + u(k\Delta t) + \hat{w}_k \tag{5.2.1b}$$

The first step in dealing with identification problems is to build a model that is consistent with physical reality and contains as few parameters as possible. In the case of structural systems, classical dynamics indicates modal analysis as the most suitable approach.

Let ω_i and ϕ_i be defined as the natural frequencies and mode shapes for the undamped eigenproblem:

$$(- \omega^2 \underset{\sim}{\mathbf{M}} + \underset{\sim}{\mathbf{K}})\, \phi = 0$$

If these are known, then it is possible to express the nodal displacements in a new set of coordinates d_i, i=1,...,n (modal coordinates)

$$\mathbf{x} = \sum_{i=1}^{n} \phi_i d_i(t) \tag{5.2.2}$$

In this new set of coordinates, assuming that the damping matrix can be diagonalized by the transformation above, the equations of motion are:

$$\ddot{d}_i + 2\xi_i \omega_i \dot{d}_i + \omega_i^2 d_i = -\hat{\Gamma}_i u + \hat{\Psi}_i v \qquad i = 1,n \tag{5.2.3a}$$

with observations

$$y_k = \sum_{i=1}^{n} \left[-2\xi_i \omega_i \dot{d}_i(k\Delta t) - \omega_i^2 d_i(k\Delta t) \right] \phi_i^{(n)} + \tag{5.2.3b}$$

$$+ \left[1 - \sum_{i=1}^{n} \hat{\Gamma}_i \phi_i^{(n)}) \right] u(k\Delta t) + \sum_{i=1}^{n} \hat{\Psi}_i \phi_i^{(n)} v(k\Delta t) + \hat{w}_k$$

where, for the ith mode:

ξ_i is the damping factor
$\hat{\Gamma}_i = \phi_i^T \mathbf{m}$ is the participation factor for the ground motion
$\hat{\Psi}_i = \phi_i^T \mathbf{f}$ is the participation factor for the unmeasured excitation

From eqs. (5.2.3a) and (5.2.3b) it is evident that only the products $\hat{\Gamma}_i \phi_i^{(n)}$ or $\hat{\Psi}_i \phi_i^{(n)}$ and not the values of the modal amplitude or of the participation factor are relevant for the response. Therefore it is customary (see Beck and Jennings, 1980) to introduce the effective participation factors

$$\Gamma_i = \hat{\Gamma}_i \phi_i^{(n)} \tag{5.2.4a}$$

$$\Psi_i = \hat{\Psi}_i \phi_i^{(n)} \tag{5.2.4b}$$

where $\phi_i^{(n)}$ is the nth component of the mode shape.

It can be shown (see appendix A) that

$$\sum_{i=1}^{n} \Gamma_i = 1 \tag{5.2.5a}$$

$$\sum_{i=1}^{n} \Psi_i = \frac{f_n}{m_n} \tag{5.2.5b}$$

In most engineering situations, the participation factors of the higher modes are negligible, so that the response of the structure can be approximated considering only the first m modes, with m relatively small. There are now only m equations of motion (5.2.3a). Also, (5.2.5a) and (5.2.5b) are assumed to hold when the summations are truncated at the mth element.

Equations (5.2.5a) and (5.2.5b) represent constraints for the identification problem to be solved. The physical meaning is that the participation factors are not allowed to vary freely, but must be consistent with the description of the motion chosen. Using the approximate (5.2.5a) and (5.2.5b), equation (5.2.3b) is rewritten:

$$y_k = \sum_{i=1}^{m} \left[-2\xi_i \omega_i \dot{d}_i(k\Delta t) - \omega_i^2 d_i(k\Delta t) \right] + \frac{f_n}{m_n} v(k\Delta t) + w_k \tag{5.2.6}$$

where the new observation errors w_k model the contribution of the neglected modes, and therefore are only approximately uncorrelated.

The modal approach is the most parsimonious possible. The problem involves 4m parameters, natural frequencies, damping factors, effective participation factors for the earthquake input, and effective participation factors for the unmeasured excitation for each of the m modes that describe the motion. The two constraints of eqs. (5a) and (5b) permit the reduction of the 4m parameters, so that only 4m-2 are left. This set of parameters will be called "modal parameters", with θ representing ξ,ω,Γ,Ψ collectively.

The identification problem for the model introduced in the preceeding section can now be stated. Given the model (5.2.3a),(5.2.6), and sequences of samples of the input $u_k = u(k\Delta t)$ and of the output y_k, the problem consists of estimating the modal parameters and the variance ratio $\frac{\sigma_v^2}{\sigma_w^2}$.

5.3. Estimation of the Parameters

Among the multitude of estimation procedures, Maximum Likelihood Estimators (MLEs) (Eychoff,1974, Box and Jenkins,1976) have been chosen. Before the measurement, the probability density function (pdf) of observing certain values of the response can be thought of as being a function of the values themselves and of the input as well as the parameters:

$$p\,(y_1,....,y_N;u_1,\ldots,u_N;\theta;\frac{\sigma_v^2}{\sigma_w^2})$$

After the observations have been made, u_i and y_i can be held constant and the function above can be thought of as depending on the parameters alone. The MLEs are the set of parameters that maximize this pdf for the time series actually observed. The actual maximization is usually carried out for the logarithm of the pdf, which is called the Likelihood function:

$$L\,(\mathbf{y};\mathbf{u};\theta;\frac{\sigma_v^2}{\sigma_w^2}) = \log p\,(\mathbf{y};\mathbf{u};\theta;\frac{\sigma_v^2}{\sigma_w^2})$$

For this class of problems, the MLEs are preferred not only for their intuitive appeal, but also for their asymptotic properties of consistency, efficiency, normality (Cramer, 1946).

The likelihood function is computed from a difference equation model. As shown in appendix B, eqs (3a) and (6) lead to a difference equation relating the observation to the samples values of the ground motion.

$$y_t+\sum_{i=1}^{2m}a_i y_{t-i}=\sum_{i=1}^{2m}b_i u_{t-i}+\lambda(e_t+\sum_{i=1}^{2m}c_i e_{t-i}) \qquad (5..3.1)$$

Above, the sequence of uncorrelated random variables e_t is the innovation process, which is gaussian, with zero mean and unit variance,and the coefficients a_i,b_i,c_i,are known functions of the parameters.

The final step consists in writing the likelihood function in terms of the coefficients a_i, b_i, c_i and maximizing it (Astrom and Bohlin, 1966, Kashyapp, 1977).

The optimal predictor $\hat{y}_t$ for (7) in the least square sense is given by:

$$\hat{y}_t + \sum_{i=1}^{2m} c_i \hat{y}_{t-i} = \sum_{i=1}^{2m} b_i u_{t-i} + \sum_{i=1}^{2m} (c_i - a_i) y_{t-i} \tag{5.3.2}$$

The prediction error ε_t is defined as:

$$\varepsilon_t = y_t - \hat{y}_t \tag{5.3.3}$$

It can be shown (Schwappe, 1965) that the likelihood function is given by:

$$L(\mathbf{y};\mathbf{u};\theta;\frac{\sigma_v^2}{\sigma_w^2}) = -\frac{1}{2\lambda^2} \sum_{t=1}^{N} \varepsilon_t^2 - N \ln\lambda + const. \tag{5.3.4}$$

where N is the number of observations. Furthemore, if the sum of squares of the prediction error is defined:

$$V(\boldsymbol{\theta}) = \frac{1}{2} \sum_{t=1}^{N} \varepsilon_t^2 \tag{5.3.5}$$

it is found that the MLEs $\hat{\boldsymbol{\theta}}$ minimize $V(\boldsymbol{\theta})$:

$$\hat{\boldsymbol{\theta}} = \min_{\boldsymbol{\theta}}! \, V(\boldsymbol{\theta}) \tag{5.3.6}$$

$$\hat{\lambda} = \frac{2}{N} V(\hat{\boldsymbol{\theta}}) \tag{5.3.7}$$

An approximate expression for the covariance of $\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0$ can also be obtained:

$$cov(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \approx 2\hat{\lambda}^2 (V_{\boldsymbol{\theta\theta}})^{-1}_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \tag{5.3.8}$$

where $V_{\boldsymbol{\theta\theta}}$ is the Hessian matrix of V and $\boldsymbol{\theta}_0$ is the actual set of structural parameters.

It should be noted that the likelihood function is computed and maximized as a function of the modal parameters, instead of seeking estimates of the coefficients of (7). This procedure not only yields direct estimates of physically meaningful parameters, but also avoids the constrained optimization problem that would arise. Indeed, only $4m-2$ independent parameters are present in the physical model, whereas the difference equation contains $6m$ parameters.

The likelihood function as defined in (5.3.5) will depend in general on the initial unknown values of the prediction error, $\varepsilon_0, \cdots, \varepsilon_{1-n}$, as well as on the parameters. For a lightly damped system, the transient induced by neglecting this dependence will be significant. Therefore, a procedure has been developed to estimate these initial values. In appendix C it is shown that the unconditional likelihood function is $L(\theta)$ is to be maximized:

$$L\ (\theta) = L\ (\theta \mid \hat{\varepsilon}_*) \qquad (5.3.9)$$

where $\hat{\varepsilon}_*$ is the least square estimate of the initial conditions for ε_t.

5.4. Implementation of the Procedure

The procedure described above has been implemented on the MicroVax "TREMOR" of the Department of Civil Engineering at Princeton University. for the case in which no input noise is present. The general architecture and the computational details of the program MUMOID are described in appendix D.

Actual records of acceleration of the basement and of an upper level of buildings are usually not synchronized. This is due both to malfunctioning of the triggering devices and to digitization errors. This was first noticed by Beck(1978). Therefore, the difference equation model (5.3.1) contains an unknown time lag τ:

$$y_t{+}\tau + \sum_{i=1}^{2m} a_i y_t{+}tau{-}i = \sum_{i=1}^{2m} b_i u_{t-i} + \lambda\ (\ e_t + \sum_{i=1}^{2m} c_i e_{t-i}\ )$$

Box and Jenkins (1976) suggest that τ be selected so that the sum of squares (5.3.10) be minimum for the MLE values of θ estimated at different delays.

SECTION 6: APPLICATION TO STRONG MOTION RECORDS FROM THE SAN FERNANDO EARTHQUAKE

The methods illustrated in chapters 4 and 5 must be validated by the analysis of earthquake records. This analysis must make use of:

- small scale experiments on seismic simulators (shaking tables)
- full scale experiments
- strong motion records from instrumented buildings.

In this report, preliminary results regarding the San Fernando earthquake (February 9, 1971) are discussed. Although this is a very small database, the damage indices previously defined appear to be consistently increasing with the severity of the damage, and very sensitive to minor damage.

Records from six buildings have been analyzed:

1. Building with significant (repairable) damage

    Bank of California

2. Buildings with minor or nonstructural damage

    Holiday Inn Orion
    Union Bank

3. Buildings with no structural damage

    Millikan Library
    Sheraton Hotel
    611 6th St.

The three undamaged buildings have been examined to provide a reference.

figures 6.1-6.5 show the acceleration records from the basement and from the upper level of the structures considered. In the case of the Union Bank building, no records of acceleration were available, and the equivalent fundamental periods have been taken from Beck (1979). Although there are some differences in the identification

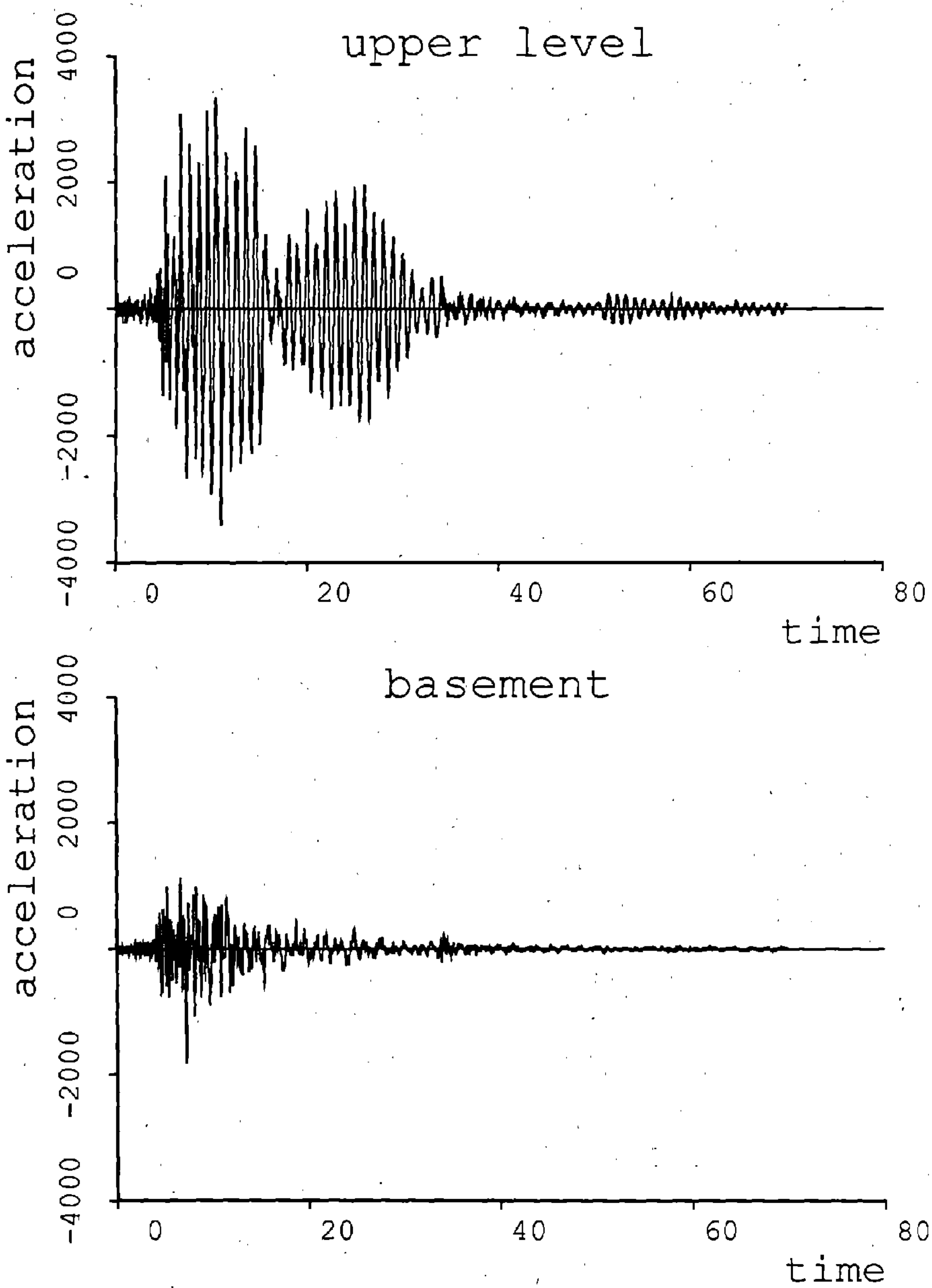


Fig. 6.1: Accelerograms from Millikan Library (EW)

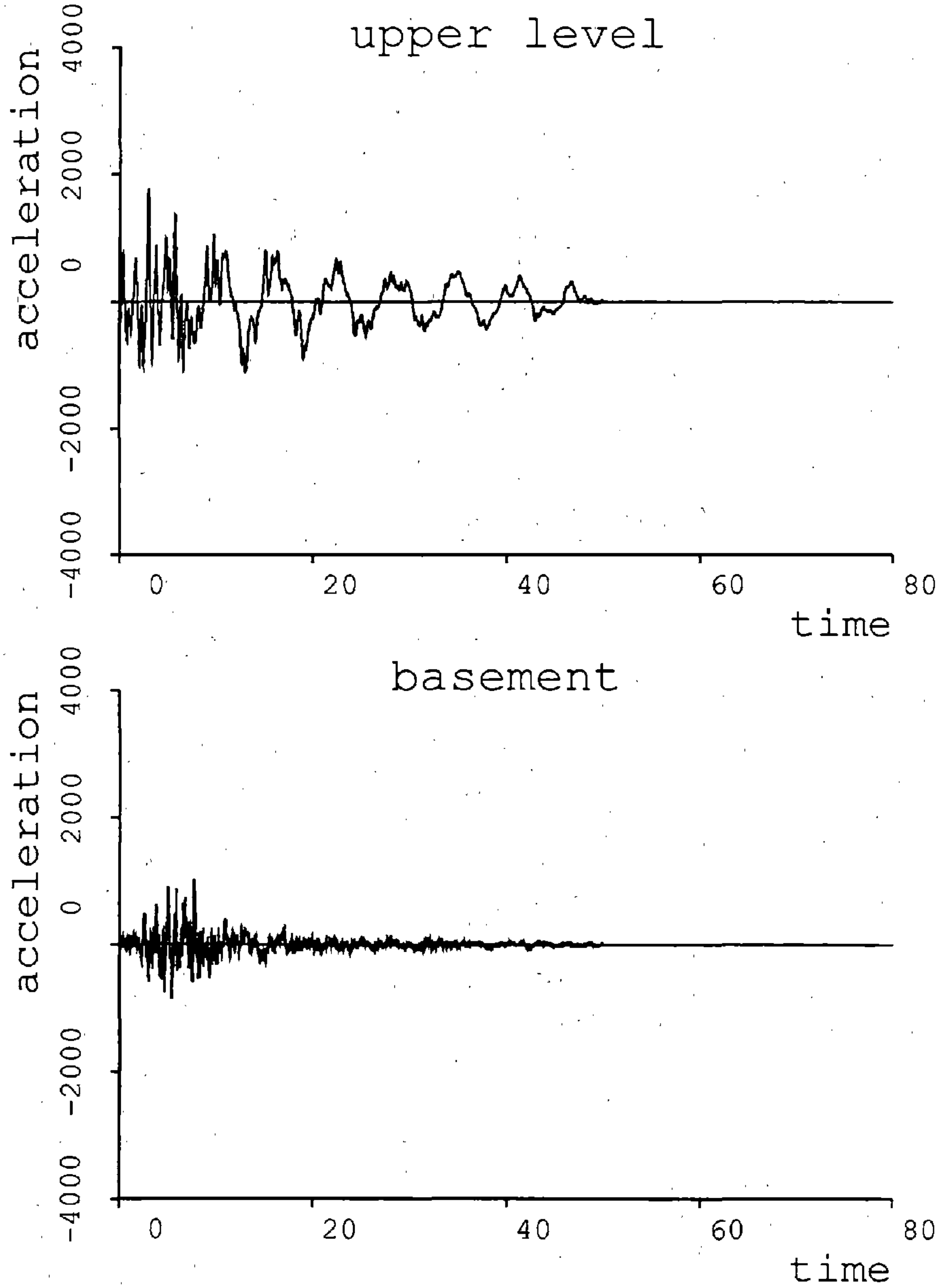


Fig. 6.2: Accelerograms from

611 West 6th St. (N38E)

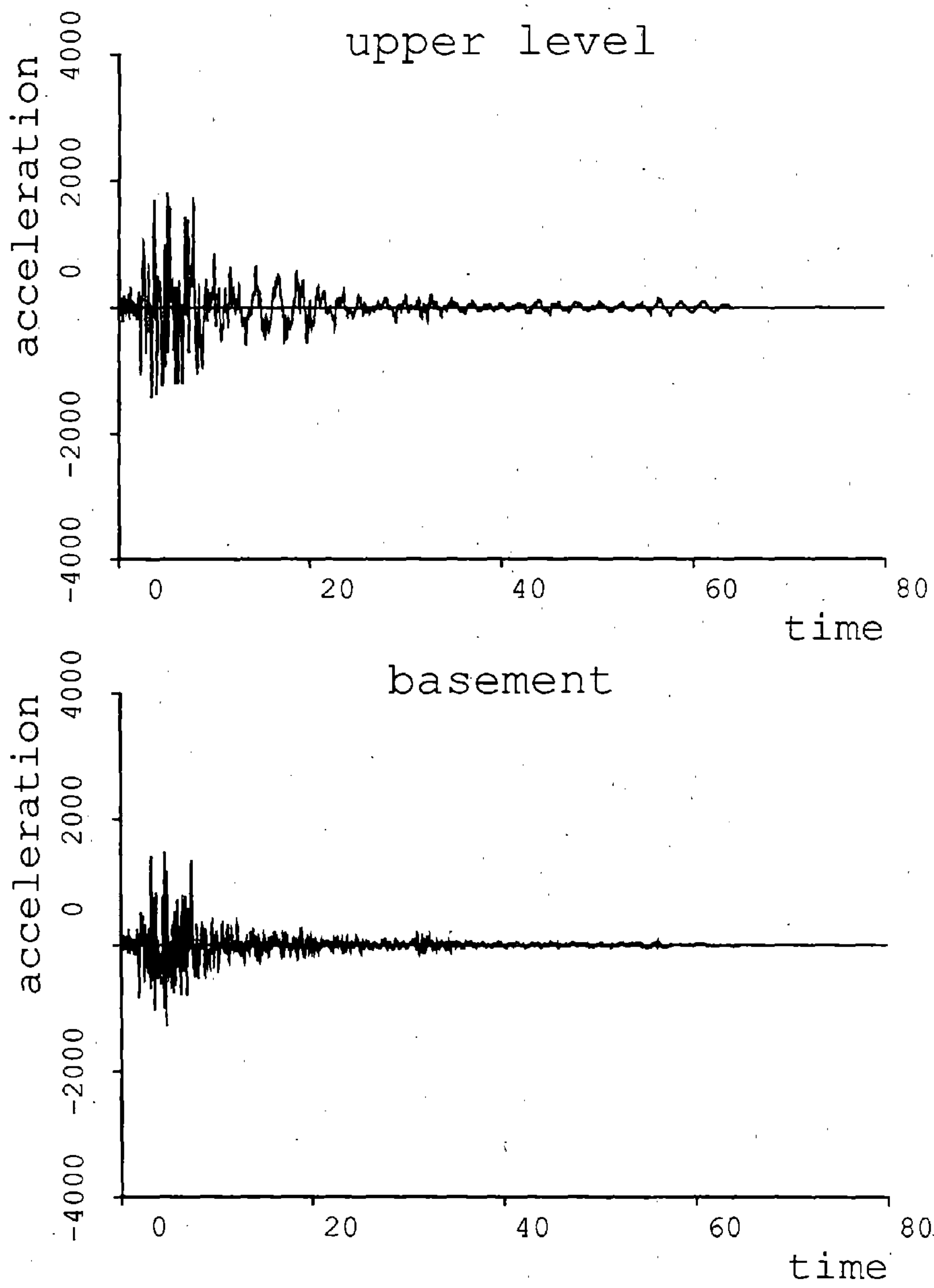


Fig. 6.3: Accelerograms from

Sheraton Hotel (S90W)

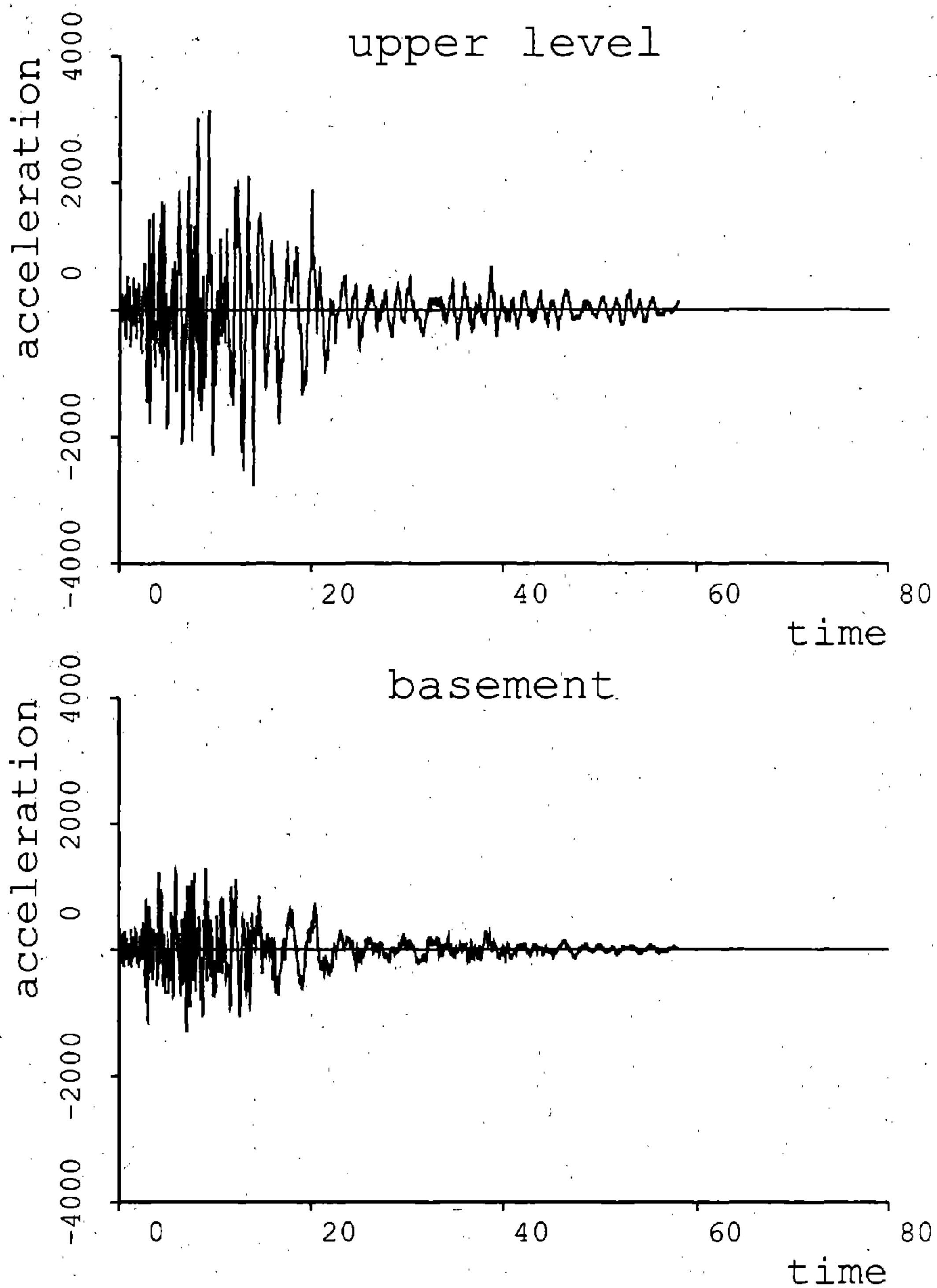


Fig. 6.4: Accelerograms from Holiday Inn Orion (S90W)

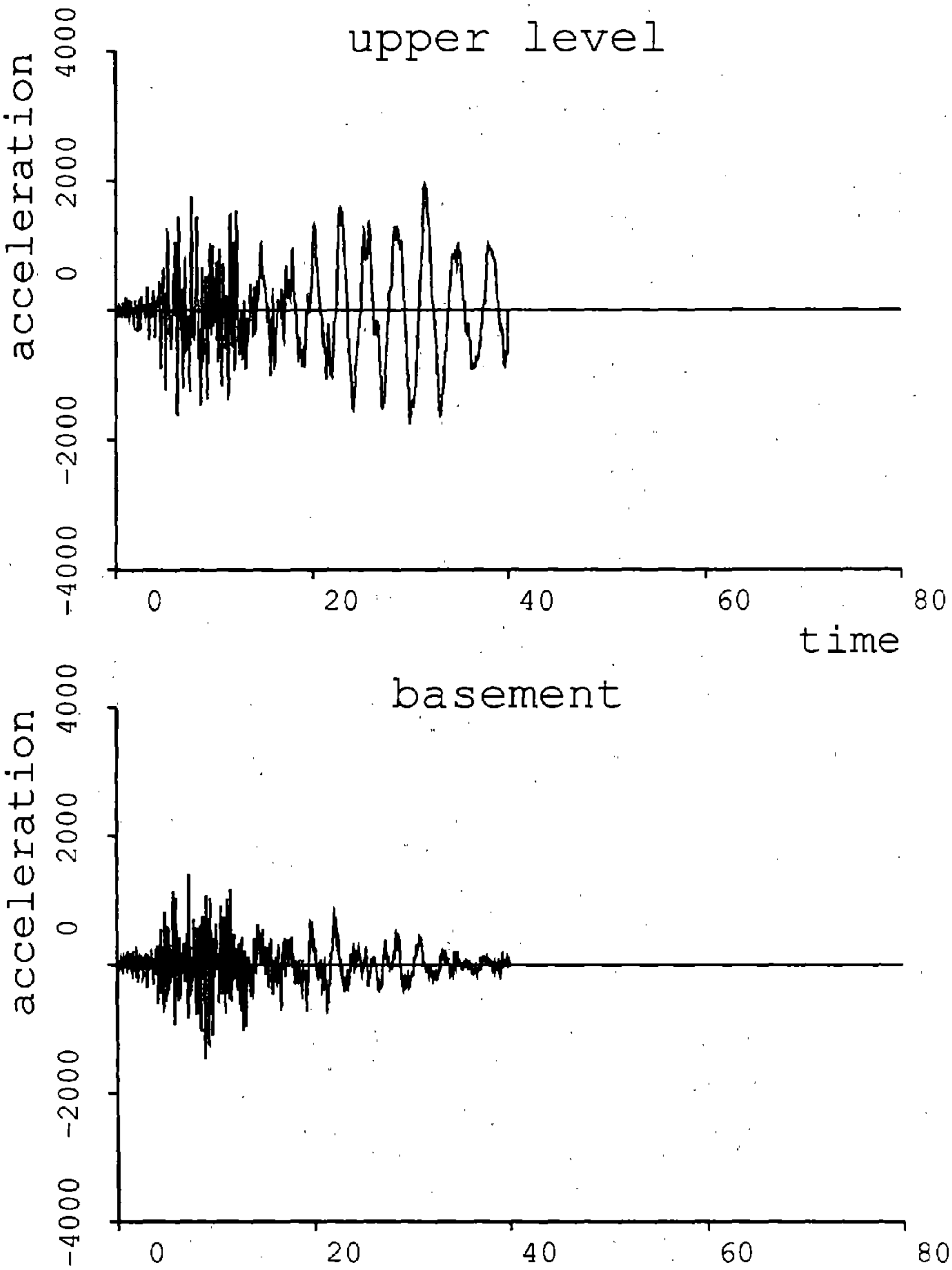


Fig. 6.5: Accelerograms from Bank of California (N79E)

procedure used, the values of the natural frequencies estimated by the authors and by Prof. Beck show perfect agreement for all the other structures considered.

Fig. 6.6 shows the evolution of the equivalent fundamental period during the earthquake, for each of the six buildings considered. The values of $(T_0)_i$ have been normalized with respect to $(T_0)_{initial}$, so that a comparison is possible between buildings with different structural and dynamical characteristics. The increase in the numerical value of the equivalent fundamental period as the damage increases is evident, and also some difference in the pattern may be noted. In fact, while for none or minor damage the equivalent fundamental period reaches a maximum during the strong shaking, for more significant damage (as it is the case of the Bank of California), the structure does not seem to "settle" for some perhaps degraded state, but keeps on yielding until the earthquake lasts. Due to the dependence of the estimates on the characteristics of the response, some plots show deviations from the trend, increasing during the first part of the record, decreasing during the last part of the record. This happens mostly when the structures experience cycles of lower amplitude.

In figures 6.7 and 6.8, the maximum softening δ_M and the cumulative softening δ_E are plotted, again versus the qualitative level of damage (none, minor, repairable). It can be observed that the numerical values of the damage indices are consistently increasing with the level of damage, and they are sensitive to minor levels of damage.

The maximum and the cumulative softening reflect two different aspects of the history of loading that the structure experiences, as discussed in chapter 4. The damage level should therefore depend on a function

$$f\ (\delta_M, \delta_E)$$

of the two indices. The simplest form possible for f is a linear combination:

$$f = \delta_M + \beta \delta_E$$

as Ang and Park (1985) proposed for similar indices, in the case of reinforced concrete elements. A more complicated expression has been proposed by Banon and Veneziano (1982).

Ideally, one expects to identify regions in the δ_M, δ_E plane, to which different levels of damage can be ascribed. In order to address this issue, a much larger database is needed, as the problem is 2-dimensional.

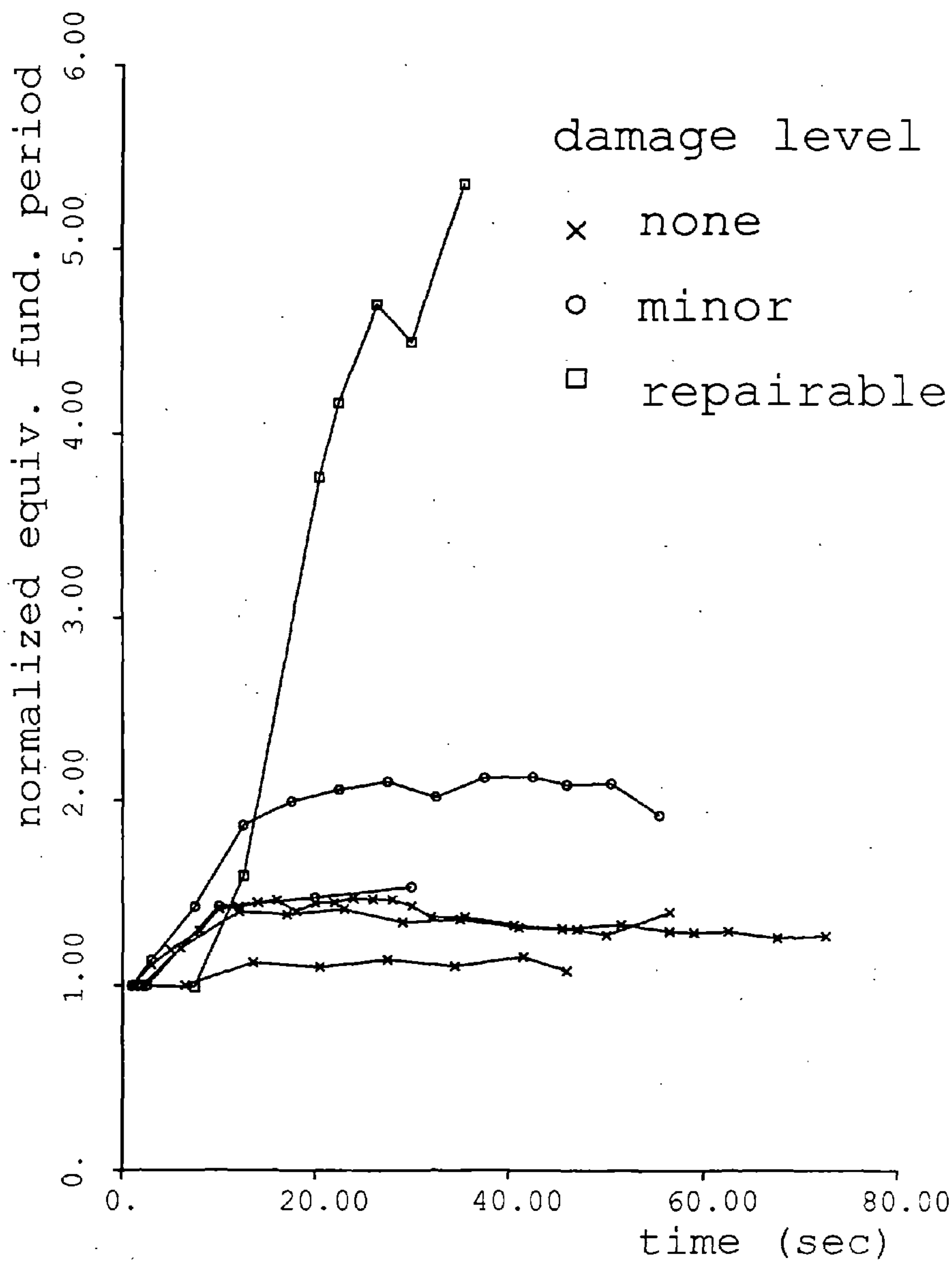


Fig. 6.6: Evolution of the Equivalent Fundamental Period

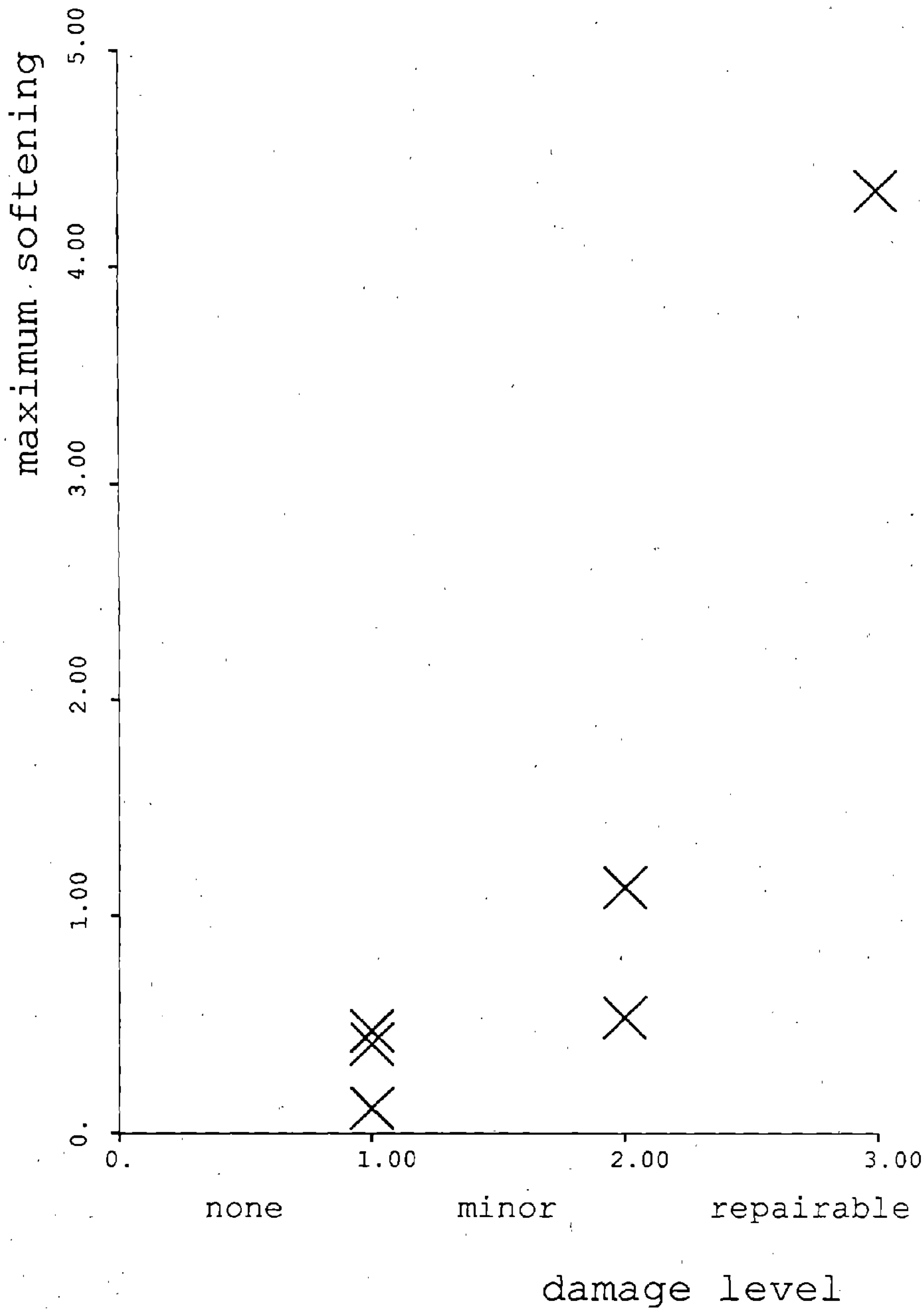


Fig. 6.7: Maximum Softening vs. Damage Level

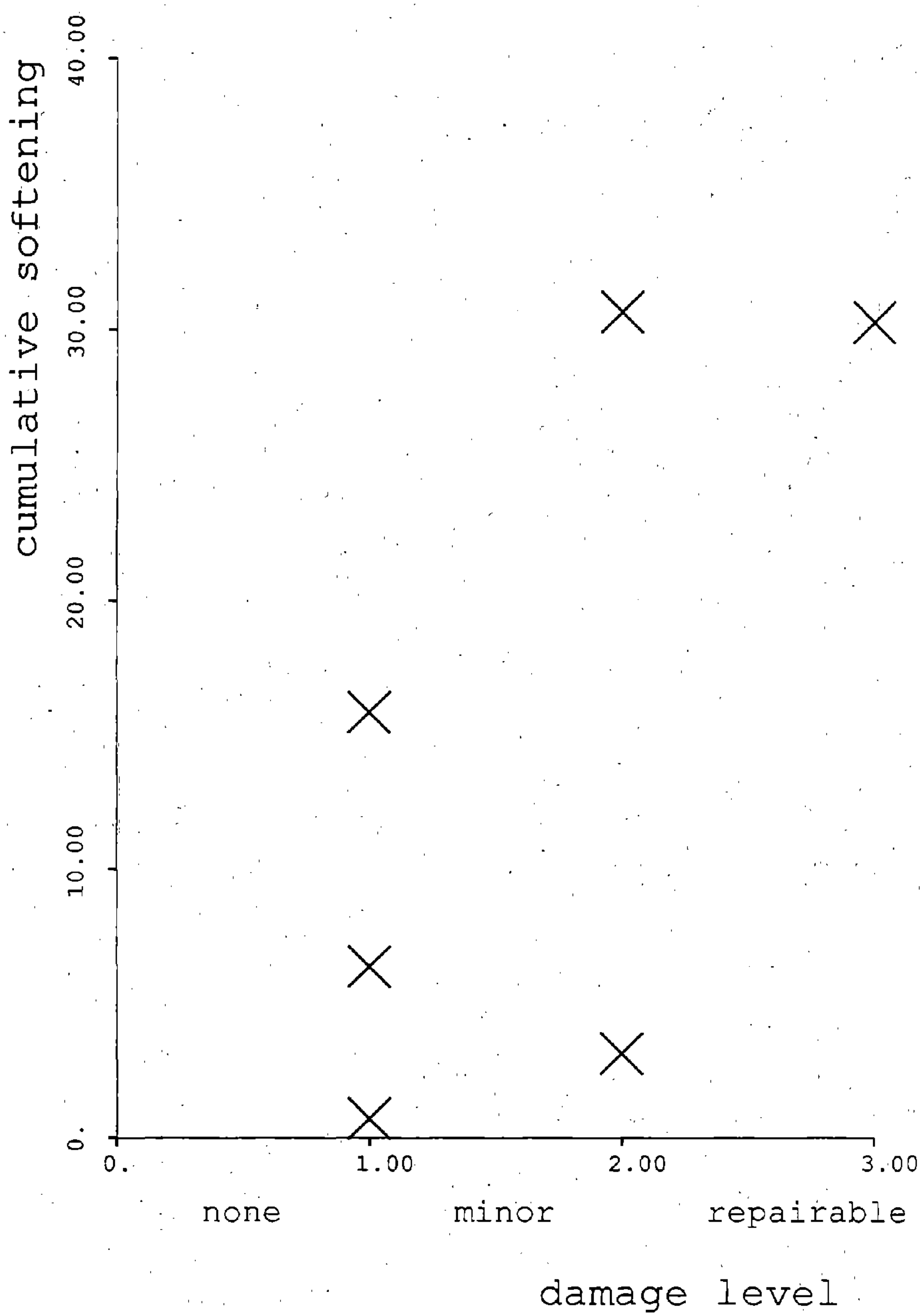


Fig. 6.8: Cumulative Softening vs. Damage Level

SECTION 7: CONCLUSIONS

The analysis of strong motion records of buildings that have been damaged during the San Fernando earthquake shows a good agreement between the numerical values of the damage indices and the levels of damage observed in the actual structures.

Hence, the first application of parameter based indices to damage analysis of strong motion records seems to yield encouraging results. Nonetheless, much is still to be done before such indices can be used in engineering practice.

Further validation is needed. A much larger database must be examined, consisting both of shaking table and full scale experiments, and strong motion records. The relation between the numerical values of damage indices and the level of damage must be obtained.

The application of the system identification techniques described in this report must be further studied. In chapter 4, it has been stated that when the structure is in a non linear phase, the equivalent linear model is not unique, but it depends on the input acceleration. The damage indices introduced are an attempt to exploit empirically this lack of uniqueness to obtain information about seismic damage. There is certainly much work to be done in this area.

Throughout this work, it has been assumed, as a working hypotesis, that the acceleration, recorded at the basement of the structure, was in fact the input motion. Any interaction between foundation and soil was thus neglected. This has been so far assumed, to the author's knowledge, by all the researchers in the field, but it is in general not true. Unfortunately, the importance of soil-structure interaction increases with the amplitude of the motion, and therefore with the expected severity of the damage. This is expecially true for low-rise building, as well as nuclear facilities, whereas soil-structure interactions would definitely be less important in the analysis of tall, slender structures. The study of a more realistic model that takes into account soil-structure interactions definitely deserves attention.

A system identification approach to soil-structure interaction phenomena involves more complicated problems than the ones analyzed in chapter 5 and in general than the ones considered by researchers in earthquake engineering, because the input to the system (i.e. the motion of the bedrock underlying the soil-structure system) is unknown. The problem can be approached using standard techniques of soil dynamics (Clough and Penzien, 1975), when free field motion can be measured at a location close to the structure analyzed, or when stationarity of the ground motion is assumed (Simionan, 1981). Engineering approaches may also be considered, for instance an analysis of the variations of the equivalent second mode that is less sensitive to soil-structure interactions (Newmark and Rosenblueth, 1974).

A linear approximation to the structural behavior, although useful, fails to capture important information about damage, such as unidirectional plastic drifts. Therefore, a more realistic, non linear, Multi-Degree Of Freedom model should be used in the analysis. For the numerical simulations studied in chapter 4, a damage analysis based on equivalent linear models could capture all the features of damage to SDOF non linear oscillators. If these results can be shown to have general validity, damage analysis using SDOF non linear system will not be any more informative than its linear counterpart. Nonetheless, a Multi-Degree of Freedom non-linear system is the only model that can capture complex structural behavior. A first approach may be to consider structures with localized nonlinearities, in the limit with one localized nonlinearity, to model situations in which damage is concentrated in particular points or areas of the building. This model would be realistic and simpler than the general case.

The parameter that is referred to as fundamental period is the period of vibration of the first translational mode of the structure in the direction considered. In this work, torsional effect have been neglected, but they should be included if a complete and realistic analysis of damage is sought.

Therefore, the goals of the project for the year 1987-88 are:

(1) to extend damage analysis using linear models to a larger database,

(2) to include soil-structure interaction effects,

(3) to devise a procedure for damage analysis using non linear models.

(4) to include the torsional effects in the damage analysis.

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APPENDIX A: Derivation of Eqs. 5.2.5a-5.2.5b

If $\underset{\sim}{\Phi} = [\phi_i^{(j)}]$ is the matrix of mode shapes, then the orthonormality requirement is

$$\underset{\sim}{\Phi}^T \underset{\sim}{M} \underset{\sim}{\Phi} = \underset{\sim}{I} \tag{A.1}$$

therefore

$$\underset{\sim}{\Phi}^T \underset{\sim}{\Phi} = \underset{\sim}{M}^{-1} = \begin{bmatrix} \frac{1}{m_1} & & \\ & \ddots & \\ & & \frac{1}{m_n} \end{bmatrix} \tag{A.2}$$

Let n auxiliary variables $r_1, \cdots, r_n$ be defined as follows:

$$r_p = \sum_{i=1}^{n} \sum_{k=1}^{n} \phi_i^{(k)} f_k \, \phi_i^{(p)} \qquad p = 1, n \tag{A.3}$$

Then, writing (A.3) in matrix form and recalling (A.2), the expression below is obtained:

$$\begin{bmatrix} r_1 \\ \cdot \\ \cdot \\ r_n \end{bmatrix} = \underset{\sim}{\Phi}\underset{\sim}{\Phi}^T \mathbf{f} = \underset{\sim}{\Phi}^T \underset{\sim}{\Phi} \mathbf{f} = \underset{\sim}{M}^{-1} \mathbf{f} = \begin{bmatrix} \frac{f_1}{m_1} \\ \cdot \\ \cdot \\ \frac{f_n}{m_n} \end{bmatrix} \tag{A.4}$$

Then, eq. (5.2.5b) follows for a general vector **f**, recalling that

$$r_n = \sum_{i=1}^{n} \Psi_i .sp 4 \tag{A.5}$$

If $f_k = m_k$, eq. (5a) is derived as a special case of (5b).

APPENDIX B: Derivation of the Difference Equation (5.3.1) from the Modal Equations of Motion (5.2.3a) and (5.2.6)

Towards the proposed goal, equations (5.2.3a) and (5.2.6) are first written in a state space form. The state vector is defined:

$$\mathbf{s} = \begin{bmatrix} \mathbf{d} \\ \cdots \\ \dot{\mathbf{d}} \end{bmatrix} \tag{B.1}$$

and the equations of motion can be written as:

$$\dot{\mathbf{s}} = \begin{bmatrix} 0 & \mathbf{I} \\ diag\,(-\omega_i^2) & diag\,(-2\xi_i\,\omega_i) \end{bmatrix} \mathbf{s} + \begin{bmatrix} 0 \\ -\Gamma \end{bmatrix} u + \begin{bmatrix} 0 \\ \Psi \end{bmatrix} v \tag{B.2a}$$

$$= \underset{\sim}{\mathbf{A}}\mathbf{s} + \mathbf{b}'u + \mathbf{g}'v$$

Introducing the state vector $\mathbf{s}$, eq. (6) reads:

$$y_k = \mathbf{h}^T \mathbf{s}(k\,\Delta t) + \kappa v\,(k\,\Delta t\,) + w_k \tag{B.2b}$$

where

$$\mathbf{h}^T = [-\omega_1^2,...,-\omega_m^2,-2\xi_1\omega_1, \cdots ,-2\xi_m\,\omega_m]$$

and

$$\kappa = \sum_{i=1}^{m} \Psi_i \approx \sum_{i=1}^{n} \Psi_i = \frac{f_n}{m_n}$$

Defining (Rosko,1972) the state transition matrix:

$$\underset{\sim}{\Xi} = \exp(\underset{\sim}{A}\Delta t\,) = \sum_{n=0}^{\infty} \frac{(\underset{\sim}{A}\Delta t\,)^n}{n\,!} \tag{B.3}$$

the discrete .PLroximation of (B.2) becomes:

$$\mathbf{s}_{t+1} = \underset{\sim}{\Xi}\mathbf{s}_t + \mathbf{b}u_t + \mathbf{g}v_t \tag{B.4a}$$

$$y_t = \mathbf{h}^T \mathbf{s}_t + \kappa v_t + w_t \tag{B.4b}$$

where the subscript t is the discretized time, representing the sampling operation and

(B.4b)

$$\mathbf{b} = [\int_0^{\Delta t} \exp(\underset{\sim}{A}\Delta\tau) d\tau]\ \mathbf{b}'$$

$$\mathbf{g} = [\int_0^{\Delta t} \exp(\underset{\sim}{A}\Delta\tau) d\tau]\ \mathbf{g}'$$

The scheme above, being explicit, is particularly suitable for the manipulations that follow.

The observability matrix for (B.4) is defined as

$$\underset{\sim}{O} = \begin{bmatrix} \mathbf{h}^T \\ \mathbf{h}^T \underset{\sim}{\Xi} \\ \cdot \\ \cdot \\ \mathbf{h}^T \underset{\sim}{\Xi}^{2m-1} \end{bmatrix} \tag{B.5}$$

If rank $\underset{\sim}{O} = 2m$, the system is said to be observable (see Kaylath, 1976) and there exists an invertible matrix $\underset{\sim}{T}$ such that, with the transformation:

$$\mathbf{s} = \underset{\sim}{T}\ \eta \tag{B.6}$$

the canonical form of (B.4) is obtained:

$$\eta_{t+1} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & -\alpha_{2m} \\ 1 & 0 & & & & \cdot \\ 0 & 1 & \cdot & & & \\ \cdot & 0 & \ddots & & & \\ & & & \ddots & & \cdot \\ \cdot & & & & 0 & -\alpha_2 \\ 0 & & & & 1 & -\alpha_1 \end{bmatrix} \eta_t + \beta\, u_t + \gamma\, v_t \tag{B.7a}$$

$$y_t = \eta_t^{(2m)} + \kappa v_t \tag{B7.b}$$

It is interesting to note that the elements of the last column of the state transition matrix in (B.7a) are nothing but the coefficients of the characteristic polynomial of $\underset{\sim}{\Xi}$:

$$p(z) = z^{2m} + \alpha_1 z^{2m-1} + \cdots + \alpha_{2m} \tag{B.8}$$

The roots of (B.8) come in complex conjugate pairs, with

$$\mu_i = \exp[-\xi_i \omega_i \Delta t + j\,\omega_i \Delta t \sqrt{1-\xi_i^2}] \tag{B.9}$$

$$\overline{\mu_i} = \exp[-\xi_i \omega_i \Delta t - j\,\omega_i \Delta t \sqrt{1-\xi_i^2}]$$

The state transition matrix in the canonical form is thus computable from the modal parameters, and so is the transformation matrix :

$$T = \underset{\sim}{\mathbf{O}}_{old}^{-1}\ \underset{\sim}{\mathbf{O}}_{new} \tag{B.10}$$

Above, $\underset{\sim}{\mathbf{O}}_{old}$ is the observability matrix of (B.4) and $\underset{\sim}{\mathbf{O}}_{new}$ of (B.7), which has a particularly simple form in terms of the α_i. Finally, substituting beetween (B.7a) and (B.7b) a difference equation is obtained:

$$y_t + \sum_{i=1}^{2m} a_i y_{t-i} = \sum_{i=1}^{2m} b_i u_{t-i} + \xi_t \tag{B.11}$$

where

$$a_i = \alpha_i$$

$$b_i = \beta_i$$

$$\xi_t = w_t + \sum_{i=1}^{2m} \alpha_i w_{t-i} + \kappa v_t + \sum_{i=1}^{2m} (\gamma_i + \alpha_i \kappa)\, v_{t-i}$$

In order to compute the likelihood function, an alternative form of (B.11) is obtained by rewriting the noise term ξ_t, so that eq. (7) is obtained:

$$y_t + \sum_{i=1}^{2m} a_i y_{t-i} = \sum_{i=1}^{2m} b_i u_{t-i} + \lambda \left(e_t + \sum_{i=1}^{2m} c_i e_{t-i}\right) \tag{B.12}$$

The noise process $\lambda \left(e_t + \sum_{i=1}^{2m} c_i e_{t-i} \right)$ must have the same variance and the same correlation structure of ξ_t. Therefore λ and $c_i s$ are solution of the nonlinear system:

$$\lambda^2 \sum_{i=0}^{2m-k} c_i c_{i+k} = \sigma_w^2 \sum_{i=0}^{2m-k} \alpha_i \alpha_{i+k} + \sigma_v^2 \sum_{i=0}^{2m-k} \delta_i \delta_{i+k} \tag{B.13}$$

$$k = 0,\ldots,2m$$

with

$$\alpha_0 = c_0 = 1$$

$$\delta_0 = \kappa$$

$$\delta_i = \gamma_i + \kappa \alpha_i$$

APPENDIX C: Unconditional Likelihood Function for a Transfer Function Model

The model

$$y_t + \sum_{i=1}^{n} a_i y_{t-i} = \sum_{i=1}^{n} b_i u_{t-i} + \lambda \left(e_t + \sum_{i=1}^{n} c_i e_{t-i}\right) \qquad \text{(C.1)}$$

is considered, where u_t is the input and y_t the output of the system, and the a_i,b_i,c_i are some complicated functions of the original set of parameters $\boldsymbol{\theta}$, and e_t is a gaussian white noise with unit variance.

As proven earlier, the Maximum Likelihood estimators $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$ are the set of parameters that minimize the sum of squares

$$V\,(\,\boldsymbol{\theta};\mathbf{y};\mathbf{u}\,) = \frac{1}{2} \sum_{t=1}^{N} \varepsilon_t^2 \qquad \text{(C.2)}$$

where the prediction errors $\varepsilon_t s$ are defined by the difference equation

$$\varepsilon_t + \sum_{i=1}^{n} c_i \varepsilon_{t-i} = y_t + \sum_{i=1}^{n} a_i y_{t-i} - \sum_{i=1}^{n} b_i u_{t-i} \qquad \text{(C.3)}$$

The ε_t will depend on the initial values

$$\varepsilon_* = (\varepsilon_0, \ldots, \varepsilon_{1-n})^T$$

so that, in general

$$V = V(\boldsymbol{\theta},\boldsymbol{\varepsilon}_*;\mathbf{y};\mathbf{u}) = V(\boldsymbol{\theta},\boldsymbol{\varepsilon}_*)$$

and the ε_* must be estimated.

A common option is to minimize the "conditional" sum of squares of the prediction error:

$$V(\boldsymbol{\theta};\mathbf{y};\mathbf{u}\,|\,\boldsymbol{\varepsilon}_*\!=\!0)$$

However, the conditional estimation, with ε_*=0, induces a transient effect in the series ε_t, that can become dominant for short samples (N<500), and lightly damped systems.

Another possibility is to inbed the $\boldsymbol{\varepsilon}_*$ in the non linear minimization problem that is solved for the parameters $\boldsymbol{\theta}$. This has been found an excessive burden for the algorithm (Gauss-Newton) that was being used.

The procedure that is presented below has given good results and is also correct from the theoretical point of view.

<u>Proposition</u>

The Maximum Likelihood Estimate of $\boldsymbol{\varepsilon}_*$ given $\boldsymbol{\theta}$ is the least square estimate.

<u>Proof</u>

The least square estimates $\hat{\boldsymbol{\varepsilon}}_*$ of the initial values ($\varepsilon_{1-n}, \cdots, \varepsilon_0$) are the set of initial values such that the sum of squares of the ε_t , $t>0$ is minimized. When $\boldsymbol{\varepsilon}_*$ is known, ε_t , $t>0$ will be determined using (C.3). Defining

$$\boldsymbol{\varepsilon} = (\varepsilon_{1-n}, \cdots, \varepsilon_0, \varepsilon_1, \cdots, \varepsilon_N)^T$$

and writing

$$\varepsilon_{1-n} = \varepsilon_{1-n}$$

$$\vdots$$

$$\varepsilon_0 = \varepsilon_0$$

$$\varepsilon_1 = -c_1\varepsilon_0 - \cdots - c_n\varepsilon_{1-n} + y_1 + a_1 y_0 + \cdots + a_n y_{1-n} - b_1 u_0 - \cdots - b_n u_{1-n}$$

$$\varepsilon_N = -c_1\varepsilon_{N-1} - \cdots c_n\varepsilon_{N-n} + y_N + a_1 y_{N-1} + \cdots + a_n y_{N-n} - b_1 u_{N-1} - \cdots - b_n u_{N-n}$$

$\boldsymbol{\varepsilon}$ is seen to be a linear function of $\mathbf{y}$ and $\boldsymbol{\varepsilon}_*$:

$$\boldsymbol{\varepsilon} = \underset{\sim}{\mathbf{A}}\mathbf{y} + \mathbf{b} + \underset{\sim}{\mathbf{X}}\boldsymbol{\varepsilon}_* \qquad \text{(C.4)}$$

where $\mathbf{b}$ takes into account the contribution of the initial condition $y_0, \cdots, y_{1-n}$ and of the input as well.

Since

$$p\ (\varepsilon \mid \sigma_\varepsilon) = \frac{1}{(2\pi)^{\frac{n}{2}}\sigma_\varepsilon^n} \exp(-\frac{\varepsilon^T \cdot \varepsilon}{2\sigma_\varepsilon^2}) \tag{C.5}$$

noting that the transformation (C.4) has a unit Jacobian, the joint distribution of **y** and ε is

$$p\ (\mathbf{y},\varepsilon_* \mid \theta,\sigma_\varepsilon,\mathbf{u}) = \frac{1}{(2\pi)^{\frac{n}{2}}\sigma_\varepsilon^n} \exp(-\frac{V(\theta,\varepsilon_*)}{2\sigma_\varepsilon^2}) \tag{C.6}$$

with

$$V\ (\theta,\varepsilon_*) = (\underset{\sim}{\mathbf{A}}\mathbf{y}+\mathbf{b}+\underset{\sim}{\mathbf{X}}\varepsilon_*)^T(\underset{\sim}{\mathbf{A}}\mathbf{y}+\mathbf{b}+\underset{\sim}{\mathbf{X}}\varepsilon_*) \tag{C.7}$$

Adding and subtracting $\underset{\sim}{\mathbf{X}}\hat{\varepsilon}_*$ in the above expression, and using the orthogonality properties of the least square estimates, the following decomposition of $V\ (\theta,\varepsilon_*)$ is obtained:

$$V\ (\theta,\varepsilon_*) = V\ (\theta) + (\varepsilon_* - \hat{\varepsilon}_*)^T \underset{\sim}{\mathbf{X}}^T \underset{\sim}{\mathbf{X}}(\varepsilon_* - \hat{\varepsilon}_*) \tag{C.8}$$

with

$$V\ (\theta) = (\underset{\sim}{\mathbf{A}}\mathbf{y} + \underset{\sim}{\mathbf{X}}\hat{\varepsilon}_* + \mathbf{b})^T\ (\underset{\sim}{\mathbf{A}}\mathbf{y} + \underset{\sim}{\mathbf{X}}\hat{\varepsilon}_* + \mathbf{b}\ ) \tag{C.9}$$

$V\ (\theta)$ is the unconditional likelihood function. Once $\hat{\theta}$ that minimizes $V\ (\theta)$ is found, then $V\ (\theta,\varepsilon_*)$ is minimum for $\varepsilon = \hat{\varepsilon}_*(\hat{\theta})$, therefore $\hat{\theta}$ and ε_* are maximum likelihood estimators for the model considered.

<u>COMPUTATION OF THE $\hat{\varepsilon}_*$</u>

In the actual implementation, the $\hat{\varepsilon}_*$ are computed following the procedure described below, and $L\ (\theta)$ is computed using (C.2) and (C.3).

$V(\theta,\varepsilon_*)$ can be differentiated with respect to the components of $\varepsilon_* = (\varepsilon_0,\ldots,\varepsilon_{1-n})$:

$$\frac{\partial V}{\partial \varepsilon_{-j}} = \frac{\partial}{\partial \varepsilon_{-j}} \sum_{t=1}^{N} \varepsilon_t^2 = 2 \sum_{t=1}^{N} \varepsilon_t \frac{\partial \varepsilon_t}{\partial \varepsilon_{-j}}$$

The least square equations thus are

$$\sum_{t=1}^{N} \varepsilon_t \frac{\partial \varepsilon_t}{\partial \varepsilon_{-j}} = 0 \tag{C.10}$$

$$j = 0,\ldots,n-1$$

Differentiating (C.3), an expression for $\frac{\partial \varepsilon_t}{\partial \varepsilon_{-j}}$ is obtained:

$$\frac{\partial \varepsilon_t}{\partial \varepsilon_{-j}} + \sum_{i=1}^{n} c_i \frac{\partial \varepsilon_{t-i}}{\partial \varepsilon_{-j}} = 0 \tag{C.11a}$$

$$j = 0, \cdots, n-1$$

with

$$\frac{\partial \varepsilon_{-k}}{\partial \varepsilon_{-j}} = \delta_{jk} \tag{C11.b}$$

because the $\varepsilon_t s$ are independent random variables.

Since ε_t is linear in the initial values, it can be solved from (C.3) as

$$\varepsilon_t = \varepsilon_{(0),t} + \sum_{k=0}^{n-1} \varepsilon_{(-k),t} \varepsilon_{-k} \tag{C.12}$$

Above, $\varepsilon_{(0),t}$ is the particular solution of (C.3), with homogeneous initial conditions:

$$\varepsilon_{(0),t} + \sum_{i=1}^{n} c_i \varepsilon_{(0),t-i} = y_i + \sum_{i=1}^{n} a_i y_{t-i} - \sum_{i=1}^{n} b_i u_{t-i}$$

$$\varepsilon_{(0),-j} = 0 \qquad j=0,\ldots,n-1$$

Similarly, $\varepsilon_{(-k),t}$ are the fundamental homogeneous solutions:

$$\varepsilon_{(-k),t} + \sum_{i=1}^{n} c_i \varepsilon_{(-k),t-i} = 0 \qquad k=0,\ldots,n-1$$

$$\varepsilon_{(-k),-j} = \delta_{jk} \qquad j=0,\ldots,n-1$$

Substituting the solution in (C.10), we obtain:

$$\sum_{k=0}^{n-1} \varepsilon_{-k} \left[\sum_{t=1}^{N} \varepsilon_{(-k),t} \frac{\partial \varepsilon_t}{\partial \varepsilon_{-j}}\right] = -\sum_{t=1}^{N} \varepsilon_{(0),t} \frac{\partial \varepsilon_t}{\partial \varepsilon_{-j}} \qquad \text{(C.12)}$$

This is a system of n linear equations in the n unknown initial values ε_* .

APPENDIX D: Description of MUMOID

MUMOID is a program for the identification of the modal parameter of a linear structural system, based on the accelerograms recorded at the basement and at some upper level of the structure.

Besides its I/O procedures, MUMOID computes the sum of the squares of the prediction errors $L(\theta)$ (eq. 5.3.9) for certain values of the modal parameters. This function is actually minimized by the IMSL routine ZXMIN, that makes use of a modified Gauss-Newton algorithm.

The input file for MUMOID must be named MUMO.PAR. MUMO.PAR must contain the following ceds (records):

card 1 (A19) job title

card 2 (A19) filename for the input acceleration (GROUND)

card 3 (A19) filename for the output acceleration (ROOF)

both GROUND and ROOF must contain the following cards:

card 1 (I5)

NT (total number of data points)

other cards (4E13.4)

data points

card 4 (I4) NM (number of modes in the model)
ISTART (initial sample of the time window)
IFIN (final sample of the time window)
NLAG (time shift between the two records)

card 5 (F10.4) DT (sampling interval)

card 6...5+NM (3F10.4)

CSI(I),OM(I),PART(I) (initial values for the modal parameters; the last card contains only CSI(NM) and OM(NM)).

The function L (θ) is computed by the routine FUNLIN0, that, in sequence, does the following:

(1) Initialization: the vector PAR is splitted into CSI, OM, PART, and the constraint $\sum_{i=1}^{nm} PART(i) = 1$ is enforced.

(2) Generation of the system vectors (subroutine SYSVEC): the vectors $\mathbf{b}'$ (B) and $\mathbf{h}$ (C) of eq. B.2 are computed.

(3) Computation of the discrete approximation (subroutine DISAPP). The matrix $\underset{\sim}{\Xi}$ (ADI) and the vector $\mathbf{b}$ (BDI) of equation B.4 are computed. $\underset{\sim}{\Xi} = \exp(\underset{\sim}{\mathbf{A}}\Delta t)$ is computed from the similarity decomposition:

$$\underset{\sim}{\mathbf{A}} = \underset{\sim}{\mathbf{P}}^{-1} \; diag\,(\lambda_i)\, \underset{\sim}{\mathbf{P}} \tag{D.1}$$

so that

$$\underset{\sim}{\Xi} = \underset{\sim}{\mathbf{P}}^{-1} \; diag\,(e^{\lambda_i}) \underset{\sim}{\mathbf{P}} \tag{D.2}$$

The eigenvalues λ_i and the eigenvectors of $\underset{\sim}{\Xi}$ are computed by the routine EIGMOD. The particularily simple structure of $\underset{\sim}{\Xi}$ is exploited. Infact, the eigenvalues are given by (B.9), and the eigenvectors can be computed from

$$\underset{\sim}{\mathbf{P}}^{(i)} = \begin{array}{cl} \begin{bmatrix} 0 \\ \dot{0} \\ 1 \\ 0 \\ \dot{\lambda_i} \\ \dot{0} \end{bmatrix} & \begin{array}{l} \\ \\ \text{ithrow} \\ \\ \text{NM+ithrow} \\ \\ \end{array} \end{array}$$

(4) Computation of the coefficients of the difference equation (subroutine DIFCOF). The coefficients a_i=c_i (ALFA) and b_i (BETA) of equation 5.8 are computed.

The computation of the transformation matrix $\underset{\sim}{T}$ is complicated, because the observability matrix $obmat_{old}$ (eq. B.5), although nonzero, is ill conditioned for low damping factors. In fact, the coefficients a_i are computed directly by imposing that the polynomial (B.8) has roots as in (B.9). The coefficients a_i are solution of a linear system of algebraic equations, that has a Vandermonde determinant and is well behaved.

(5) Computation of the initial values of the prediction errors (subrutine LOURD), following the procedure of appendix C.

(6) Computation of the prediction errors and of the sum of heir squares (subroutine LIKFUN), from the difference equation

$$\varepsilon_t + \sum_{i=1}^{2nm} c_i \varepsilon_{t-i} = y_t + \sum_{i=1}^{2nm} a_i y_{t-i} - \sum_{i=1}^{2nm} b_i u_{t-i} \qquad \text{(D.3)}$$