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State University of New York at Buffalo

DYNAMIC COMPLIANCE OF VERTICALLY
LOADED STRIP FOUNDATIONS IN
MULTILAYERED VISCOELASTIC SOILS

by

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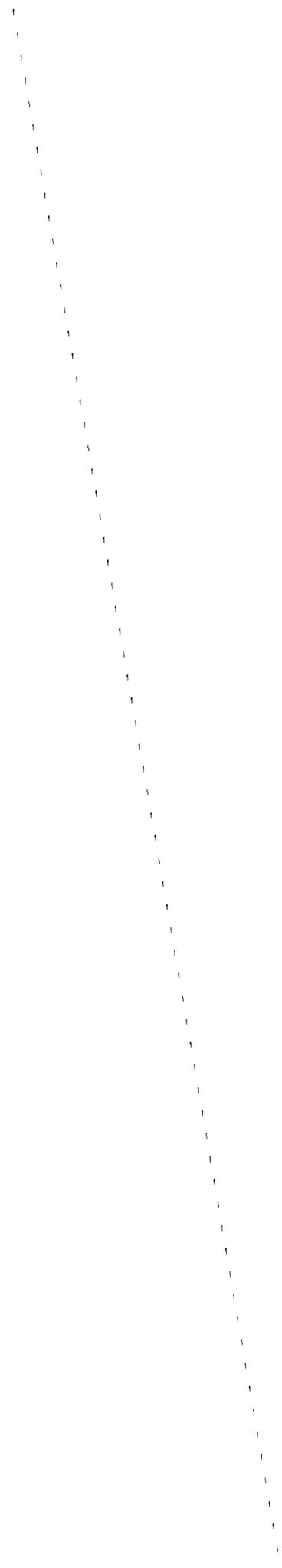
Technical Report NCEER-88-0017

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PREFACE

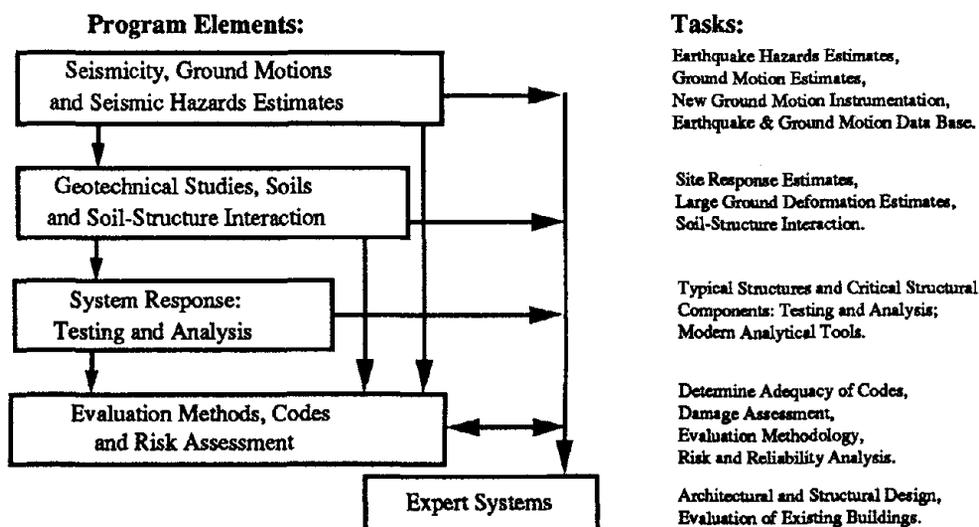
The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. Initially, the emphasis is on structures and lifelines of the types that would be found in zones of moderate seismicity, such as the eastern and central United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 1, Existing and New Structures, and more specifically to Geotechnical Studies.

The long term goal of research in Existing and New Structures is to develop methods for rational probabilistic risk assessment for damage or collapse of structures, mainly existing buildings, especially in regions of moderate seismicity. The work will rely on improved definitions of seismicity and site response, experimental and analytical evaluations of systems response, and more accurate assessment of risk factors. This technology will be incorporated in expert systems tools and improved code formats for existing and new structures. Methods of retrofit will also be developed. When this work is completed, it should be possible to characterize and quantify societal impact of seismic risk in various geographical regions and large municipalities. Toward this goal, the program has been divided into five components, as shown in the figure below:



Geotechnical Studies constitute one of the important areas of research in Existing and New Structures. Current research activities include the following:

1. Development of linear and nonlinear site response estimates.
2. Development of liquefaction and large ground deformation estimates.
3. Investigation of soil-structure interaction phenomena.
4. Development of computational methods.
5. Incorporation of local soil effects and soil-structure interaction into existing codes.

The ultimate goal of projects concerned with Geotechnical Studies is to develop methods of engineering estimation of large soil deformations, soil-structure interaction and site response.

In this report, the dynamic response of a rigid strip foundation under vertical loading is obtained using the boundary element method. The effects of various factors, such as material damping, layering, embedment, and the type of foundation-soil contact, are evaluated. The influence of a soil layer on bedrock or on a half-space are studied. Such layers shift the resonant frequencies and change the amplitudes. Specific results are compared with known solutions.

ABSTRACT

In this report, results of a detailed investigation on the dynamic response of rigid strip foundation, in viscoelastic soils, under vertical excitation are presented. An advanced Boundary Element algorithm developed by incorporating isoparametric quadratic elements and a sophisticated self-adopted numerical integration scheme has been used for this investigation. Foundations supported on three types of soil profiles: half-space, stratum-over-half-space and stratum-over-bedrock are considered. Influence of material properties like Poisson's ratio, material damping as well as the influence of geometrical properties, such as depth of embedment and layer thickness are studied. The effect of the type of contact at the soil-foundation interface are also investigated.

ACKNOWLEDGEMENT

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SECTION 1

INTRODUCTION

The study of the dynamic response of rigid foundations is of significant importance in machine foundation design and soil-foundation interaction problems in general. Observation of earthquake damage indicates that the local soil properties as well as foundation geometries, depth of embedment, soil profile, etc. play an important role in the dynamic behavior of the soil-foundation system.

Considering the importance of dynamic soil-structure interaction, a great deal of research has been carried out in this area in the last few decades. Reissner [24] established the theoretical basis for studying the response of a footing on an elastic half-space based on Lamb's [21] solution of the dynamic point load on an elastic half-space. Quinlan [23], Sung [28] and Bycroft [8] have approached this problem by assuming the dynamic stress distribution under a footing to be similar to the static stress distribution. Awojobi and Grootenhuis [6] solved the problem of a smooth circular rigid disc undergoing vertical oscillation using a set of dual integral equations. Karashudi, Keer and Lee [16] extended this approach to the solution of vertical, horizontal and rocking oscillations of a rigid strip footing on elastic half-space. Luco and Westmann [22] studied the vertical, horizontal and rocking motions of a rigid strip footing bonded to an elastic half-plane using a formulation in terms of Green's functions. Hryniewicz [15] investigated the dynamic behavior of rigid strip foundation under plane strain condition. All of the above work can be classified into broad categories of analytical or semi-analytical approach.

With the advent of the digital computer, numerical methods such as Finite Difference Method (FDM) and Finite Element Method (FEM) have gained considerable popularity. The FDM is not suitable for problems with complex geometries, whereas the FEM can handle problems with non-linearities, layering, complex geometries, and boundary conditions. However, one limitation of FEM is its inability of proper modelling of an unbounded soil media by satisfying the wave radiation condition. Day and Frazier [10] suggested the use of artificial boundaries far away from the region of interest so as to avoid the undesirable wave reflections. Others recommended the use of transmitting or non-reflecting boundaries (Roesset and Ettouney [25], Kausel and Tassoulas [18]) to circumvent this problem.

During the last two decades, the Boundary Element Method (BEM) has emerged as one of the most effective numerical techniques for solving a wide class of engineering problems. In this method, as the name signifies, only the boundary of the domain needs to be modelled, thereby reducing the problem dimensionality by one. Moreover, boundary integral representation is an exact formulation of the problem and the only approximations are those due to the numerical implementation of these integral equations. This method is especially suitable for problems involving infinite or semi-infinite domain because the Green's function which is used in the BEM formulation automatically satisfies the radiation condition at the far-field.

The first numerical implementation of the elastodynamic formulation of BEM was done by Cruse and Rizzo [9]. However, Dominguez [11] was apparently the first to study the dynamic response of rigid surface and embedded rigid foundations by BEM in the frequency domain. Spyrakos and Beskos [26,27] used time domain BEM in the study of dynamic behavior of 2D rigid and flexible foundations. Abascal and Dominguez [1] used BEM to find

the dynamic compliance of rigid strip footing on viscoelastic soil. In all of the above work, the displacements and tractions were assumed to remain constant within an element, i.e., constant elements were used to model the geometry as well as to represent the field variables. However, as suggested by Kobayashi and Nishimura [19], for dynamic problems it is important to use higher order elements so that the variation of the functions are compatible with the wavy nature of the problem. Ahmad [2], and Ahmad and Banerjee [3] used BEM with isoparametric quadratic elements to study some two-dimensional dynamic problems and demonstrated the accuracy of the higher order BEM implementation. Ahmad et al. [4] studied the dynamic stiffness of rectangular foundations using a higher order implementation of the Direct and Indirect BEM for the three-dimensional case.

In this report, the influence of mechanical soil properties like Poisson's ratio, internal material damping and various other geometrical parameters such as depth of embedment, layer on half-space, layer on bed-rock, as well as the effect of the type of contact at the soil foundation interface on the vertical dynamic compliance of rigid strip footing are studied in a comprehensive manner. The soil profiles are considered to be viscoelastic. All of the analyses are carried out by Boundary Element Method incorporating higher order (quadratic) boundary elements. To the best of the authors' knowledge, nowhere in the published literature does such a comprehensive study using BEM exist.



SECTION 2

BOUNDARY INTEGRAL FORMULATION FOR ELASTODYNAMICS

The Navier-Cauchy equation for linear elastodynamics is expressed in terms of displacement, u , as

$$(c_1^2 - c_2^2)u_{i,ij} + c_2^2 u_{j,ii} + b_j = \ddot{u}_j \quad (2.1)$$

where

$u = u(x,t)$, x being the cartesian position vector, and t is the time.

b_j is the body force vector;

c_1 and c_2 are the dilatational and shear wave velocities respectively given in terms of the Lamé constants (λ, μ) by the relations;

$$c_1^2 = (\lambda + 2\mu) / \rho;$$

$$c_2^2 = \mu / \rho;$$

where ρ is the mass density of the medium.

In equation (2.1), summation convention is implied by the repeated indices, commas indicate spatial differentiation and dots represent differentiation with respect to time.

For a well posed problem, equation (2.1) is always accompanied by appropriate initial and boundary conditions.

Application of Fourier Transform on equation (2.1) in conjunction with the assumption of zero initial conditions and zero body force, leads to:

$$(c_1^2 - c_2^2)\bar{u}_{i,ij} + c_2^2 \bar{u}_{j,ii} + \omega^2 \bar{u}_j = 0 \quad (2.2)$$

where ω is the circular natural frequency of excitation and the overbar ($\bar{\quad}$) denotes the functions in the Fourier Transformed domain.

The dynamic equivalent of the Betti-Maxwell reciprocity relation established between the Fourier Transforms of the actual and virtual states (denoted by *), neglecting the body forces, can be written as:

$$\int_S \bar{t}_i \bar{u}_i^* dS = \int_S \bar{t}_i^* \bar{u}_i dS \quad (2.3)$$

where S denotes the boundary surface of the body.

By letting the virtual state be the fundamental solution state such that $\bar{u}_i^* = G_{ij} e_j$ and $\bar{t}_i^* = F_{ij} e_j$, equation (2.3) finally transforms into the following integral equation (Banerjee and Butterfield [7]):

$$c_{ij}(\xi) \bar{u}_i(\xi, \omega) = \int_S [G_{ij}(x, \xi, \omega) \bar{t}_i(x, \omega) - F_{ij}(x, \xi, \omega) \bar{u}_i(x, \omega)] dS(x) \quad (2.4)$$

where x and ξ are the source and field points respectively.

G_{ij} and F_{ij} are the fundamental solution tensors representing the displacements and tractions at the field point in direction i due to a unit harmonic force applied at the source point in the direction j. For two-dimensional case, these fundamental solutions can be found in Ahmad and Banerjee [3].

In equation (2.4), c_{ij} is known as the jump term and it assumes the following values:

- (i) δ_{ij} if ξ is an interior point;
- (ii) 0 if ξ is an exterior point, and
- (iii) if ξ lies on the boundary, it is a function of the geometry of the boundary in the vicinity of ξ ; for smooth surfaces it is 0.5 δ_{ij} .

Using isoparametric boundary elements, the coordinates and the functions (displacements and tractions) at any point on the element can be expressed in terms of the nodal values as:

$$\begin{aligned}
x_i &= N_\alpha(\eta)X_{i\alpha} \\
\bar{u}_i &= N_\alpha(\eta)U_{i\alpha} \\
\bar{t}_i &= N_\alpha(\eta)T_{i\alpha}
\end{aligned}
\tag{2.5}$$

where $i = 1, 2$ (for 2D)

and $\alpha = 1, 2, 3$ (for quadratic elements)

$X_{i\alpha}$, $U_{i\alpha}$ and $T_{i\alpha}$ are the nodal coordinates, nodal displacements and nodal tractions respectively. $N_\alpha(\eta)$ are the shape functions in the intrinsic coordinate (η) of the elements.

The Jacobian of transformation between the cartesian and intrinsic coordinates are expressed as:

$$dS(x) = |J|d\eta$$

After usual discretization using isoparametric boundary elements, equation (2.4), takes the following form:

$$\begin{aligned}
c_{ij}(\xi)\bar{u}_i(\xi, \omega) &= \sum_{m=1}^M \left[\sum_{\alpha=1}^3 T_{i\alpha} \int_0^1 G_{ij}(x(\eta), \xi, \omega) N_\alpha(\eta) |J| d\eta \right. \\
&\quad \left. - \sum_{\alpha=1}^3 U_{i\alpha} \int_0^1 F_{ij}(x(\eta), \xi, \omega) N_\alpha(\eta) |J| d\eta \right]
\end{aligned}
\tag{2.6}$$

where M is the total number of boundary elements.

Equation (2.6) can be written for all the nodes on the boundary and then these equations can be put together to form the system matrix of the form:

$$[F]\{u\} = [G]\{t\} \tag{2.7}$$

where $[F]$ and $[G]$ are the coefficient matrices with integrands involving the F_{ij} and G_{ij} kernels, respectively,

{u} and {t} are global vectors of the nodal displacements and tractions, respectively, on the boundary.

The integrands consisting of the kernel-shape-function-Jacobian product could be either singular or non-singular depending on whether the field point ξ lies on the element being integrated or not. The numerical integration of non-singular integrals poses no problem. The singular integrals involving the G_{ij} kernels are weakly singular and can be evaluated numerically as well. However, the integrals involving the F_{ij} kernel are highly singular and exists only in the Cauchy principal value sense. They can be evaluated indirectly by introducing a rigid body type motion as discussed by Lachat and Watson [20] for elastostatics, and by Ahmad and Banerjee [3] for elastodynamics.

Transferring all the known boundary values on the right side and the unknowns on the left, equation (2.7) takes the form:

$$\begin{aligned} & [A]\{X\} = [B]\{Y\} \\ \text{or} & [A]\{X\} = \{b\} \end{aligned} \tag{2.8}$$

Solving equation (2.8), one can obtain all the unknown functions on the boundary. Once all the boundary values are known, the displacement at any interior point, if needed, can be found from equation (2.4). However, for the type of problems discussed in this report, only the functions at the boundary are of interest.

The BEM formulation described here can take into account the material damping (linear hysteretic type) by a simple transformation of the Lamé constants into a complex form given by,

$$\begin{aligned} \lambda^* &= \lambda(1+2i\beta) \\ \mu^* &= \mu(1+2i\beta) \end{aligned}$$

where β is the internal material damping ratio. The Poisson's ratio remains unchanged.

The BEM formulation can also take account of layered media by considering each layer as a separate homogeneous region, forming the BEM equations independently and then assembling together by satisfying equilibrium and capability across common interfaces.

SECTION 3

CONVERGENCE STUDIES AND ACCURACY OF ANALYSIS

3.1 Dynamic Compliance: Definition

Dynamic compliance of a rigid strip foundation indicates the amount of displacement it will undergo upon application of a unit harmonic load. It can be evaluated by inverting the dynamic stiffness which is obtained by summing up the forces developed at the soil-foundation interface due to a prescribed unit displacement of the foundation.

The vertical compliance can be expressed as:

$$f = \frac{u}{P} = \text{Re}[f] + i\text{Im}[f] \quad (3.1)$$

where P denotes the amplitude of vertical load and u , that of vertical displacement.

The compliance given by equation (3.1) can be written in non-dimensionalized form by multiplying it with the shear modulus μ of the soil, i.e., $f\mu$.

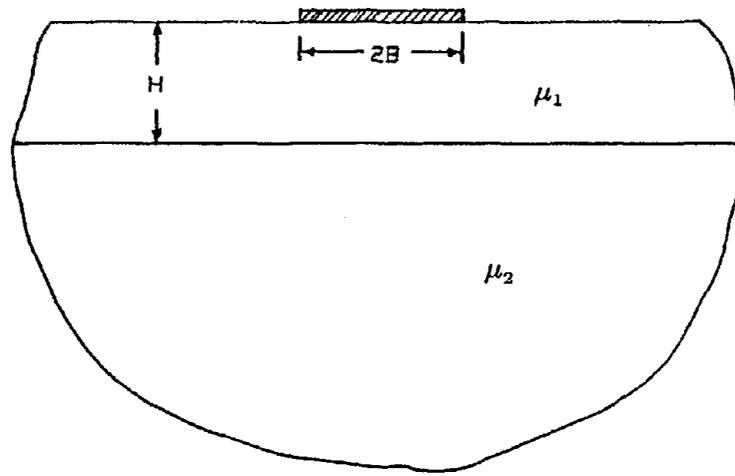
The values of the compliance are obtained for dimensionless frequencies from 0 to 2.5. The dimensionless frequency is defined as (after E. Reissner):

$$a_0 = \frac{\omega B}{c_2}$$

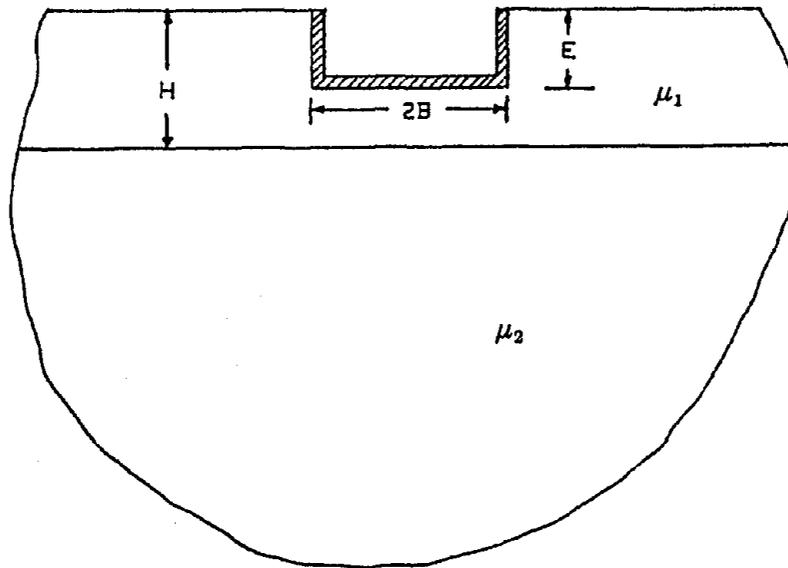
where B is the half-width of the strip foundation and c_2 is the shear wave velocity.

In the case of layered media, the values of μ and c_2 are taken to those corresponding to the top layer.

Figure 3-1 shows a typical surface and an embedded strip foundation on stratum-over-half-space.



(a)



(b)

FIGURE 3-1. Strip foundation on stratum overlaying half-space:

- (a) surface foundation
- (b) embedded foundation

3.2 Convergence Studies and Discretization

A series of convergence studies were conducted to select the optimum meshes for surface and embedded strip footing with bonded and lubricated interface conditions resting on half space as well as on layered soil. Limited convergence studies were carried out on relaxed boundary conditions which exhibited similar behavior as the bonded ones.

For surface foundation on half-space, in addition to the discretization of the soil-foundation contact area, only a small part of the free-surface adjacent to the foundation is needed to be modelled (5B for bonded and 3B for lubricated case). These free field elements were needed to model the deformation of the soil surface in the close neighborhood of the foundation under dynamic loading. They somewhat improved the results but not by a substantial amount.

In the case of embedded foundation on half-space, however, the free-surface discretization had to be extended further (17B for bonded case and 7B for lubricated contact). In bonded contact, the free-surface undergoes high-amplitude vibration than the lubricated case, and they have pronounced influence on the footing response; thus in the former case, extended and refined discretization is needed. However, embedded foundation, in general, needs more free-field modelling to take into account the waves impinging on the surface.

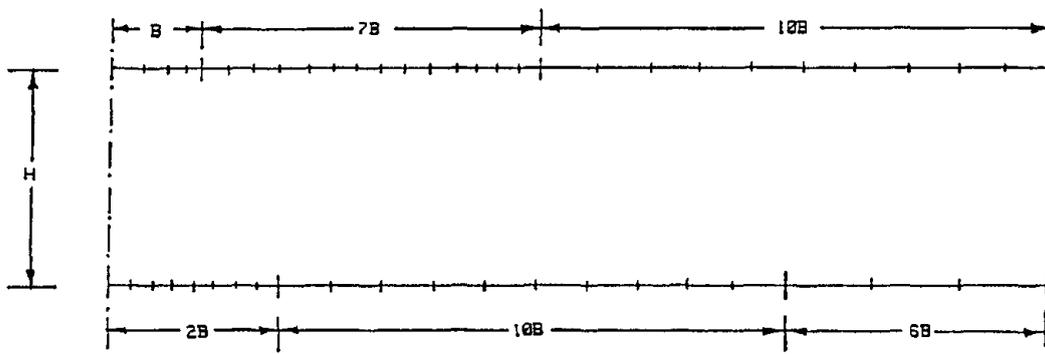
In the case of footings resting on stratum-over-half-space or stratum-over-bed-rock, extended discretization is needed both for the free-surface and the layer-interface (18B from the centerline of the foundation) in all contact cases, since waves are reflected back and forth from the surface and the layer-interface (or bedrock). Finer discretizations were used for parts of the free-surface and layer-interface in close proximity to the foundation (7B for the bonded case, 2B for the lubricated one) with

element length about 1/4 to 1/6 the Raleigh wave-lengths. Coarser elements with lengths about half the wave lengths were found to be adequate for distant locations.

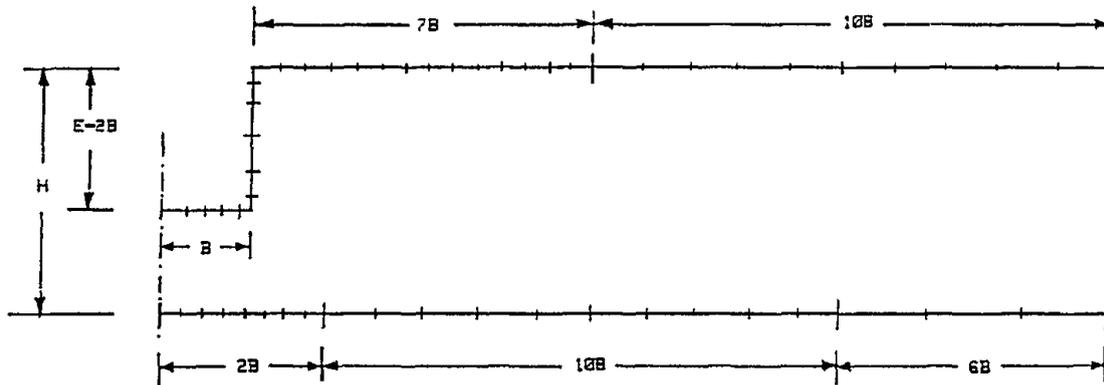
The soil-foundation interface was modelled using four elements for surface foundations and eleven for embedded ones in all cases. Two typical meshes for surface and embedded foundations on stratum-over-half-space are presented in Figure 3-2. It is to be mentioned here that the present BEM formulation can take advantage of the symmetry of loading and geometry so that only one half of the system needs to be modelled and thus reduces the computing time by half.

3.3 Comparison with Published Results

To establish the accuracy of the present analysis, the vertical compliance of a rigid strip footing resting on a viscoelastic soil profile was analyzed. In one case, the soil profile was half-space and in the other case it consisted of a stratum lying over a half-space. In both cases, the soil was assumed to have a Poisson's ratio of 0.40 and 5% material damping. The stratum had a thickness $H/B = 2$ and shear modulus μ_1 , and the underlying half-space had a shear modulus μ_2 . The study was conducted for $\mu_1/\mu_2 = 0.25$. The footing was assumed to be bonded to the soil. The discretization pattern was as mentioned in the previous section. The vertical compliances obtained by the present methodology are compared with those of Gazetas and Roesset [13] (Figure 3-3) who used a semi-analytical method consisted of replacing the dual integrals by a discrete Fast Fourier transform. Reasonable agreement can be noticed between the two sets of results presented in Figure 3-3.



(a)



(b)

FIGURE 3-2. Typical discretization patterns for foundations on stratum-over-half-space:

(a) surface foundation

(b) embedded foundation

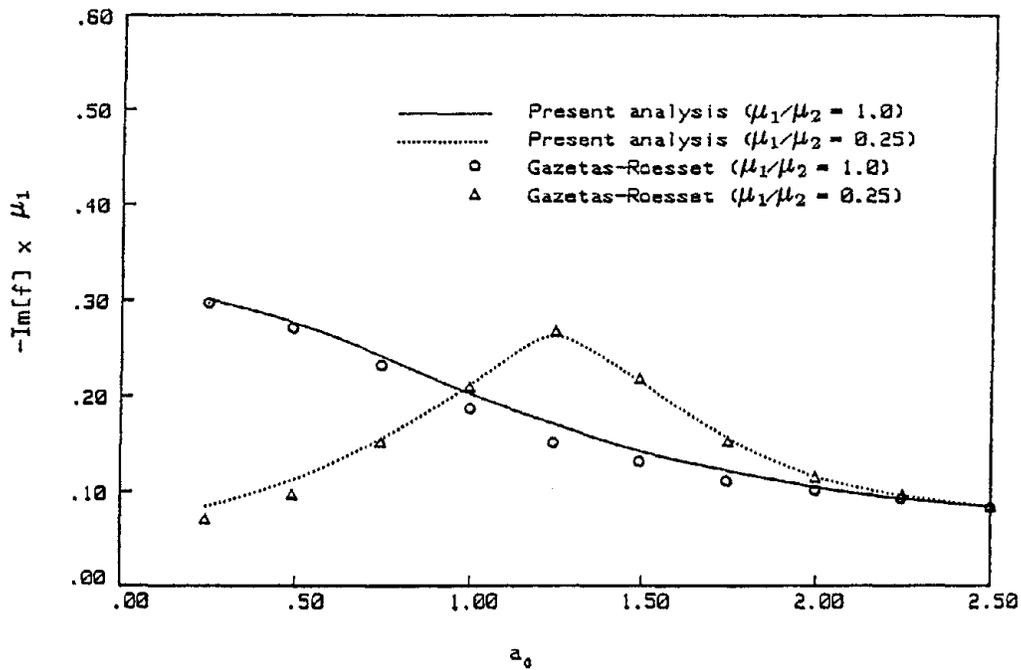
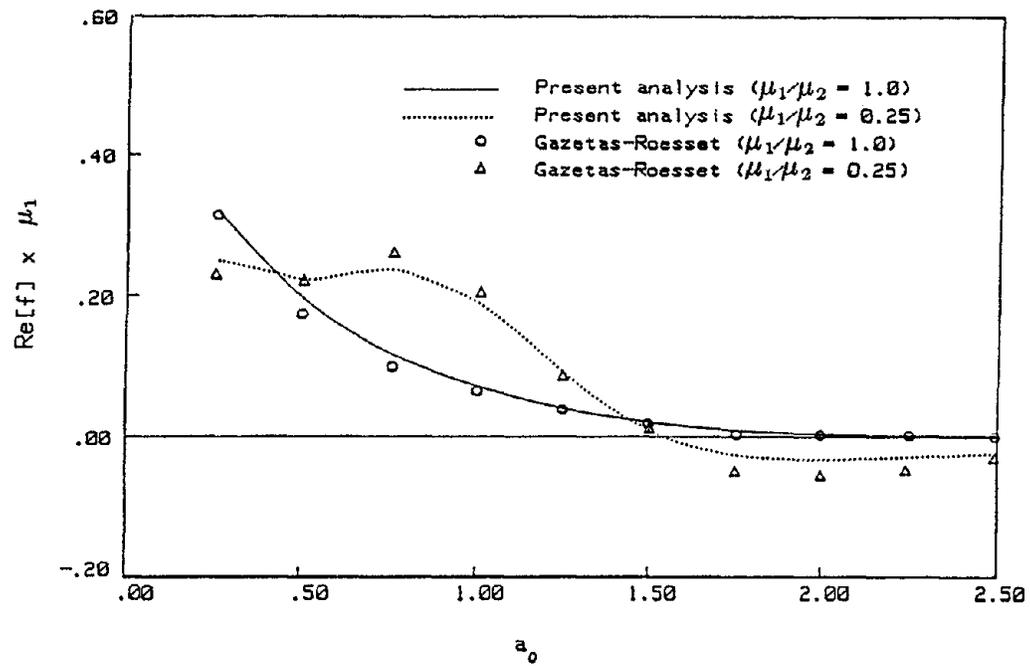


FIGURE 3-3. Comparison of vertical compliances of strip footing on stratum-over-half-space ($\nu = 0.40$; $\beta = .05$; $H/B = 2$)

SECTION 4

PRESENTATION AND ANALYSIS OF RESULTS

4.1 Parameters for the Study

The studies were conducted for foundations on viscoelastic soil-profiles. Poisson's ratio of 0.33 and 5% material damping were used throughout unless otherwise stated. For embedded foundations, embedment depth was chosen to be $E/B = 2$ except for the studies involving effect of embedment. Bonded contact was assumed along the soil-foundation interface.

4.2 Effect of Poisson's Ratio

Poisson's ratio significantly influences the dilatational wave velocity. In the vertical vibration, the relative contribution of the dilatational wave is higher than the other waves. So variation of Poisson's ratio affects the vertical response to a considerable extent. Figure 4-2 depicts the effect of Poisson's ratio. It can be seen that as the Poisson's ratio increases, decrease in the compliance is noticed, implying that foundation becomes stiffer. When soil is loaded vertically, with increased Poisson's ratio, there will be more lateral deformation. Thus, the lateral confinement of soil will offer more resistance, thereby stiffening the system. For embedded foundation, as shown in Figure 4-2b, decrease in compliance due to the increase in Poisson's ratio is comparatively small to that of surface foundation.

4.3 Effect of Material Damping

The internal friction between soil particles and hysteresis behavior cause some energy dissipation during vibration of foundations. Experimental evidence indicates that the dissipation of energy per cycle is almost independent of frequency for small amplitude vibrations. This

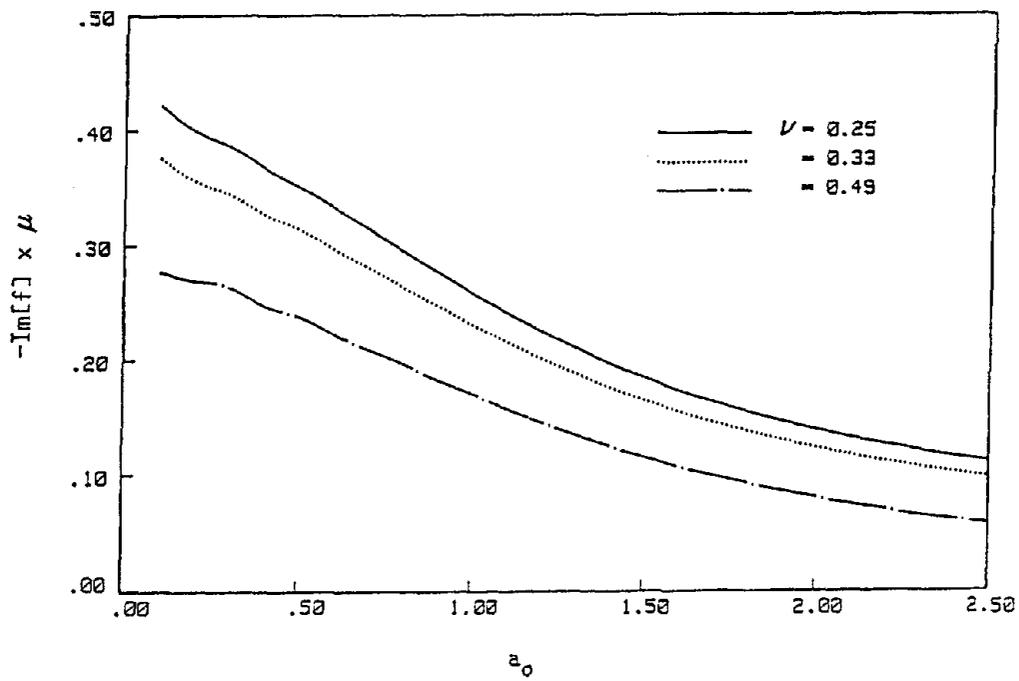
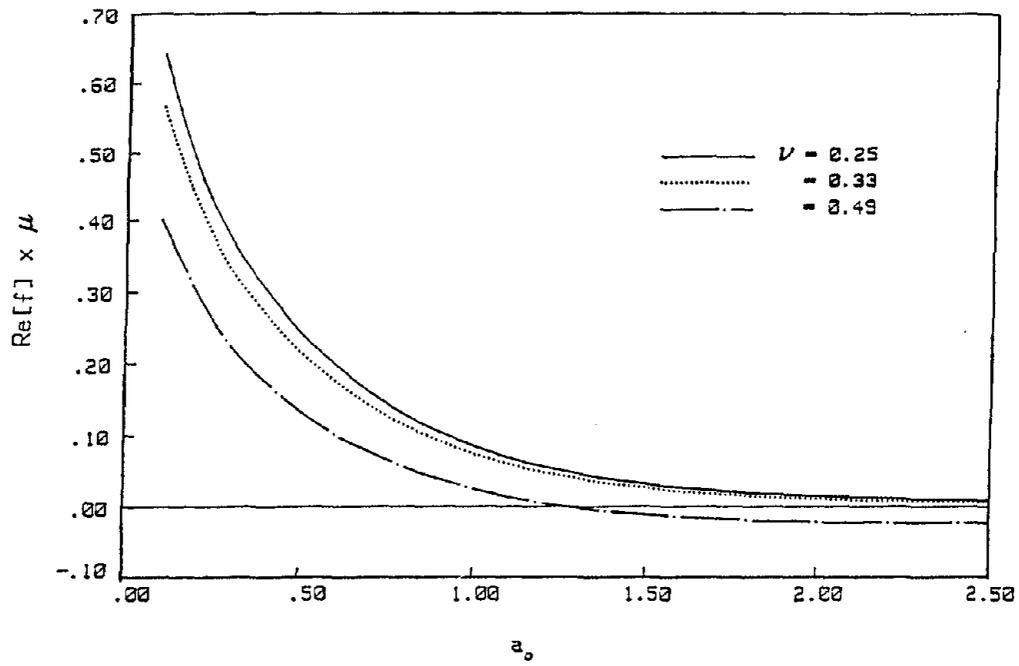


FIGURE 4-2a. Effects of Poisson's ratio: Surface foundation

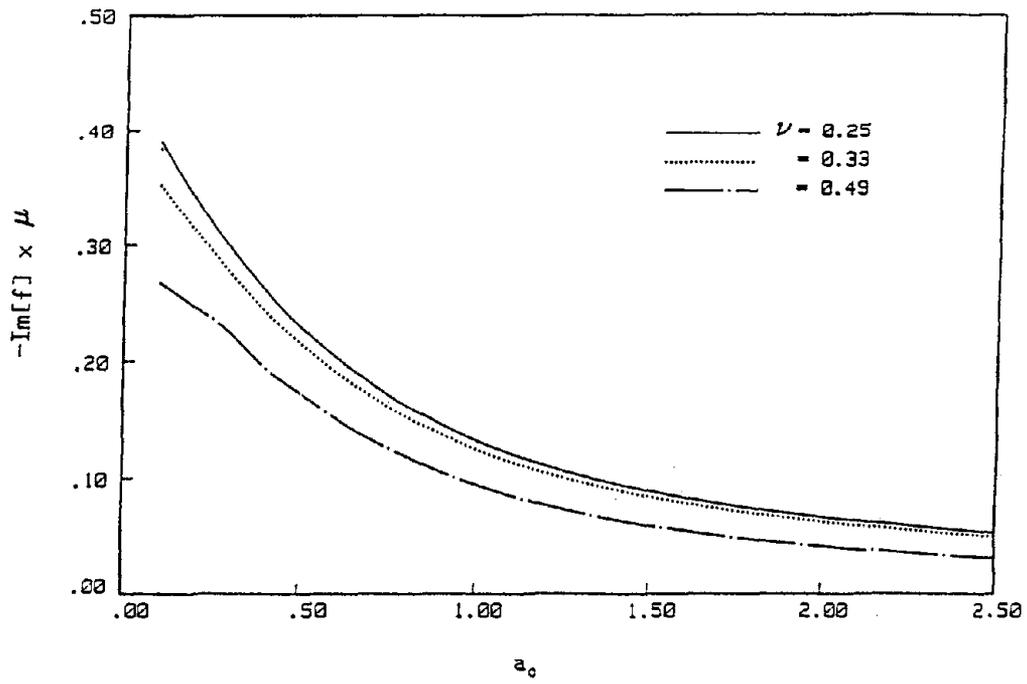
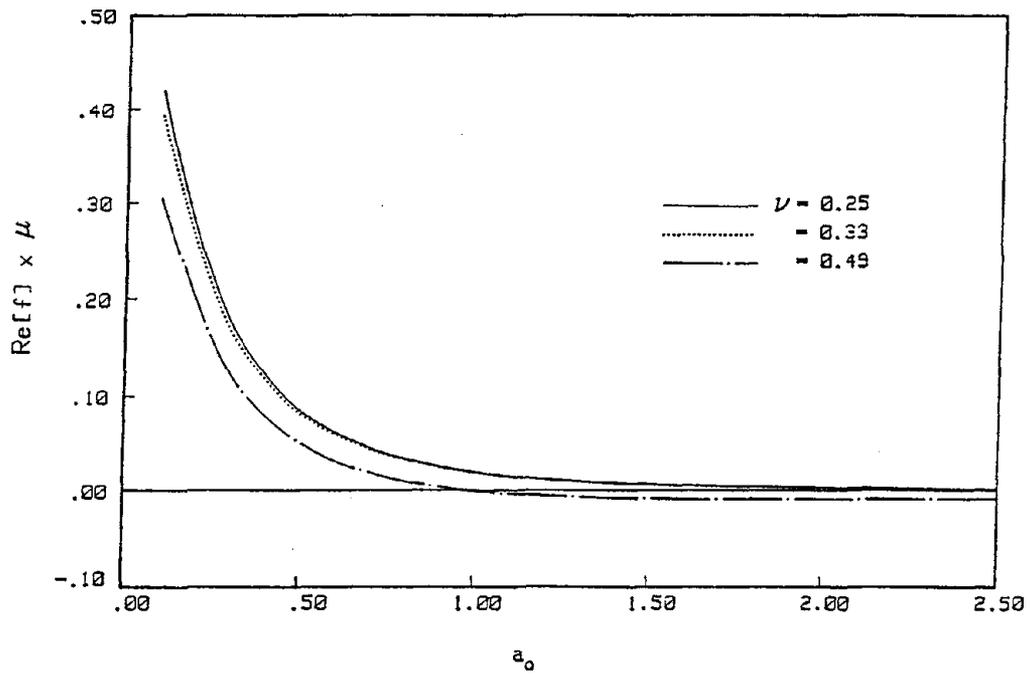


FIGURE 4-2b. Effect of Poisson's ratio; Embedded foundation

justifies the use of a linear hysteretic damping model to account for the internal energy losses. This model can be accommodated in theory by replacing the elastic moduli with the complex ones.

Figure 4.3 shows that increased material damping decreases the values of the compliance. For foundation on half-space, the difference is noticeable at lower frequencies, but not so at higher frequencies. The real part of the compliance is affected most, whereas the complex part remains almost unchanged except at very low frequencies. Influence of damping on embedded foundation in half-space follows a similar pattern as that of surface foundation, but to a diminutive scale. Material damping has a pronounced effect on foundations resting on layered soil. As seen in Figure 4-3c for a surface foundation lying on a stratum-over-bedrock, inclusion of material damping attenuates the resonance amplitudes significantly.

4.4 Effect of μ_1/μ_2

This study covers the influence of layering characteristics of soils on the vertical compliance. A stratum with a shear modulus of μ_1 overlaying a half-space with shear modulus μ_2 represents a general soil profile. When μ_1/μ_2 is 1, it represents the half-space case, and when $\mu_1/\mu_2 = 0$, the model reduces to a stratum-over-bed-rock. Five different values of μ_1/μ_2 (1, 0.5, 0.25, 0.05, 0.0) were used in this study covering the entire range and the results are presented in Figure 4-4. As expected, the effect of a layering is noticeable as μ_1/μ_2 changes; with extreme influence noticed for the case of bed-rock ($\mu_1/\mu_2 = 0$) and no affect for the half-space case ($\mu_1/\mu_2 = 1$).

As the relative stiffness between the top and bottom layer decreases (decrease in μ_1/μ_2), more and more waves are reflected from the interface and cause resonance in the top layer at its natural frequencies. The

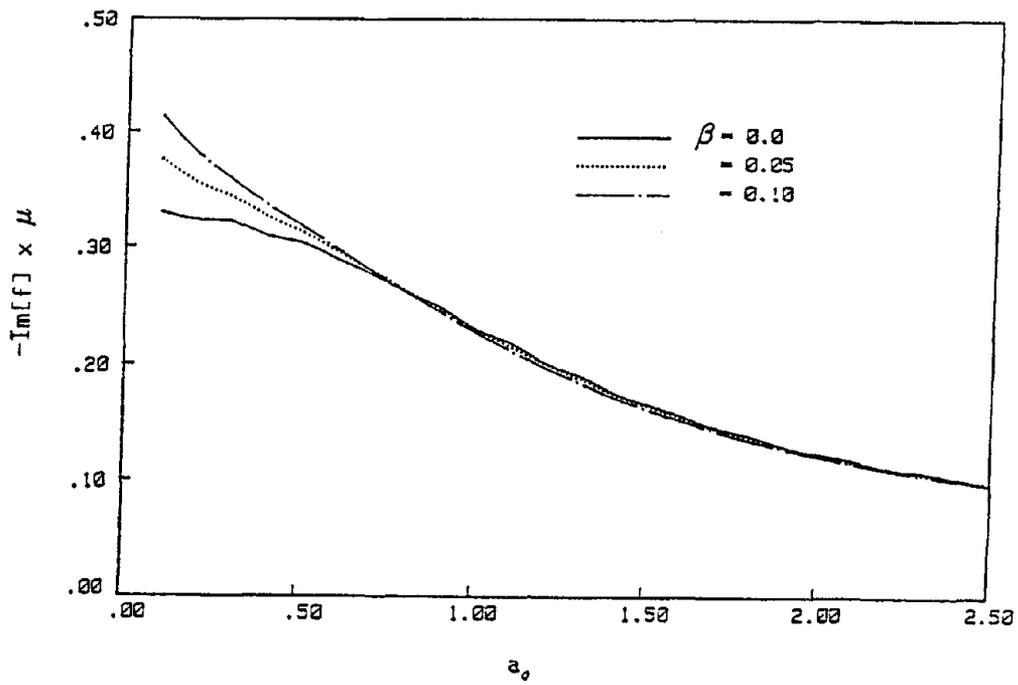
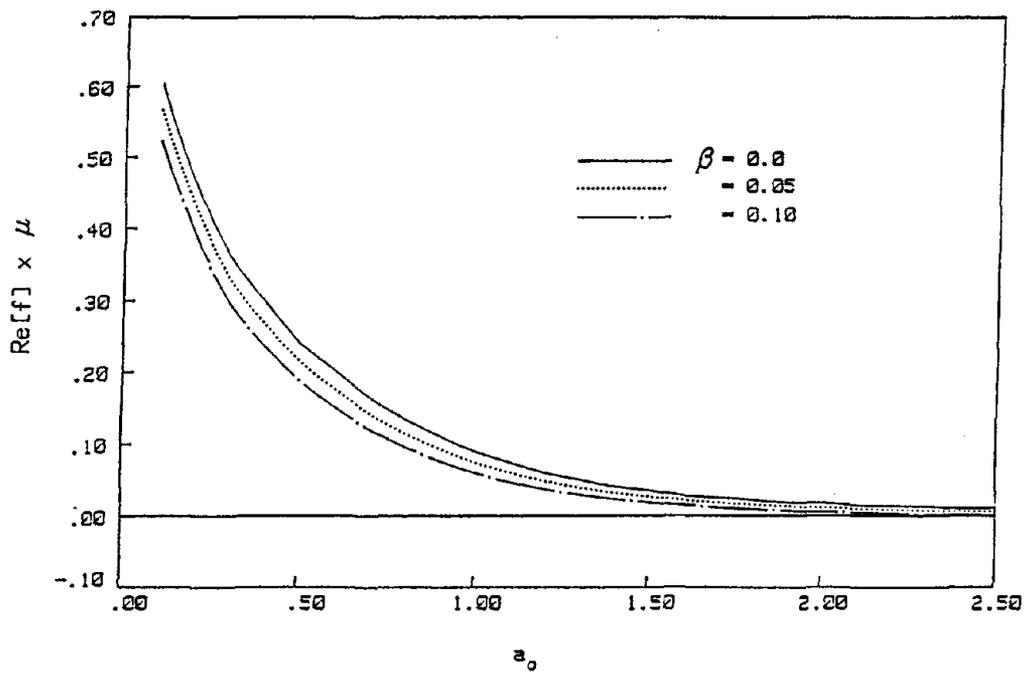


FIGURE 4-3a. Effect of material damping: Surface foundation on half-space

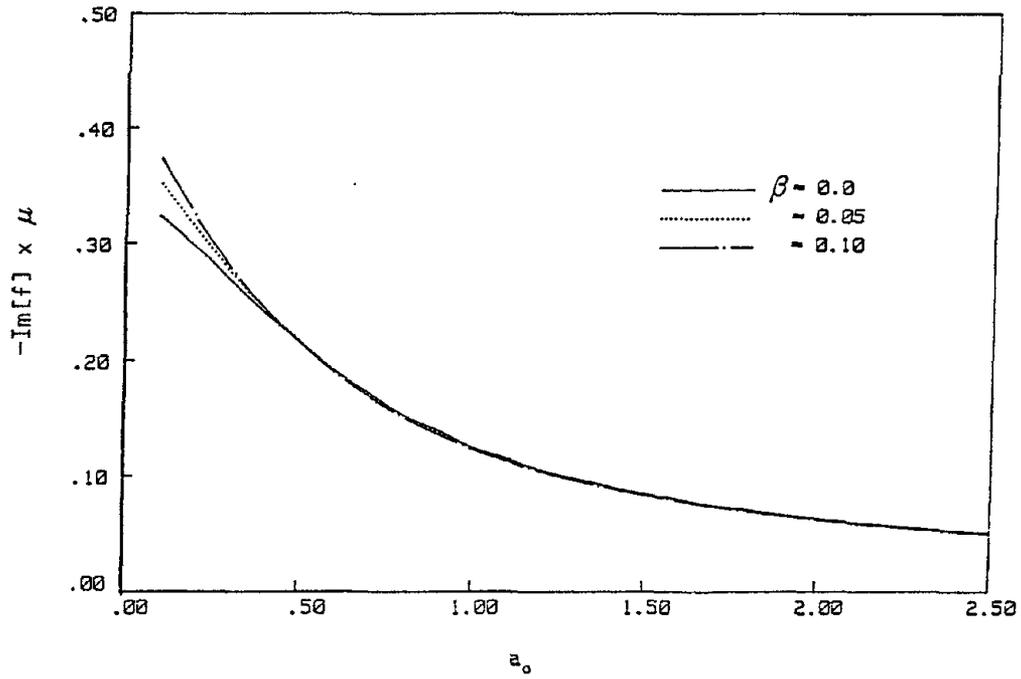
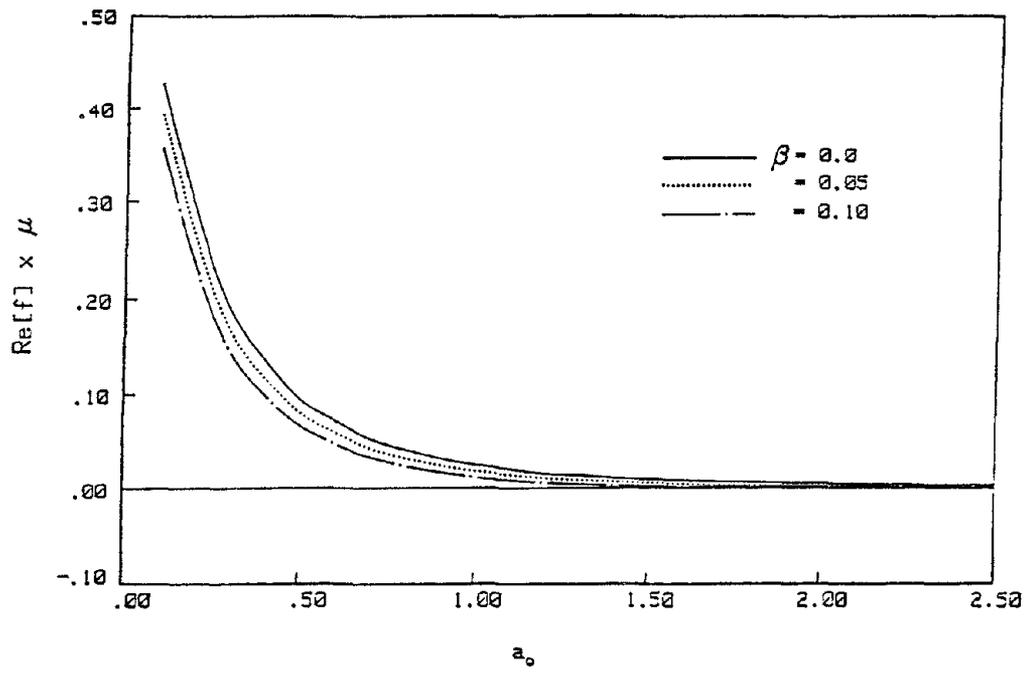


FIGURE 4-3b. Effect of material damping: Embedded foundation on half-space

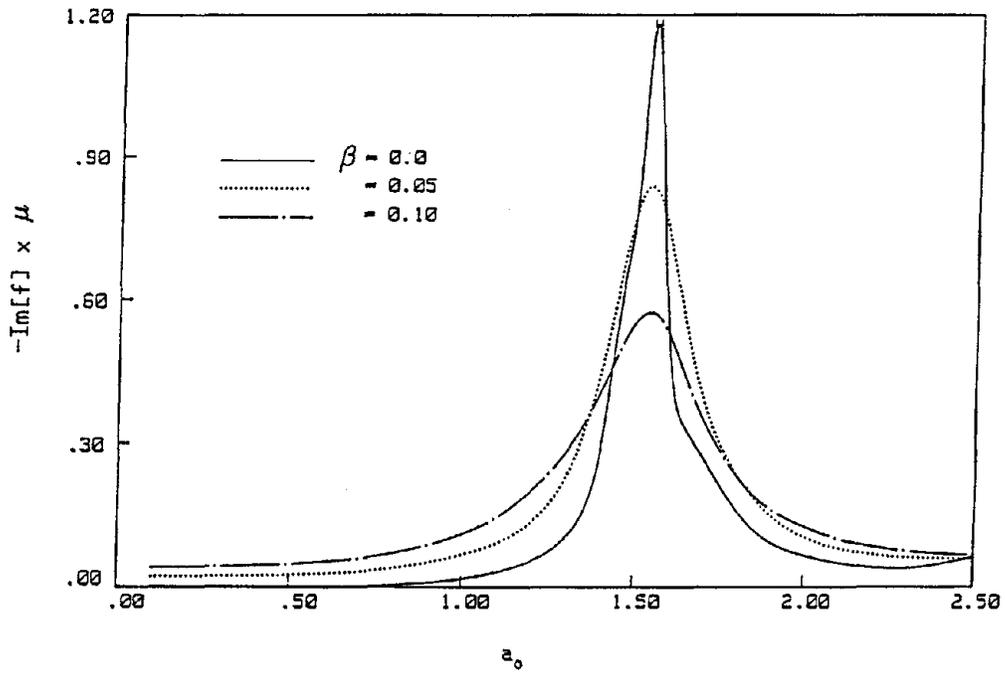
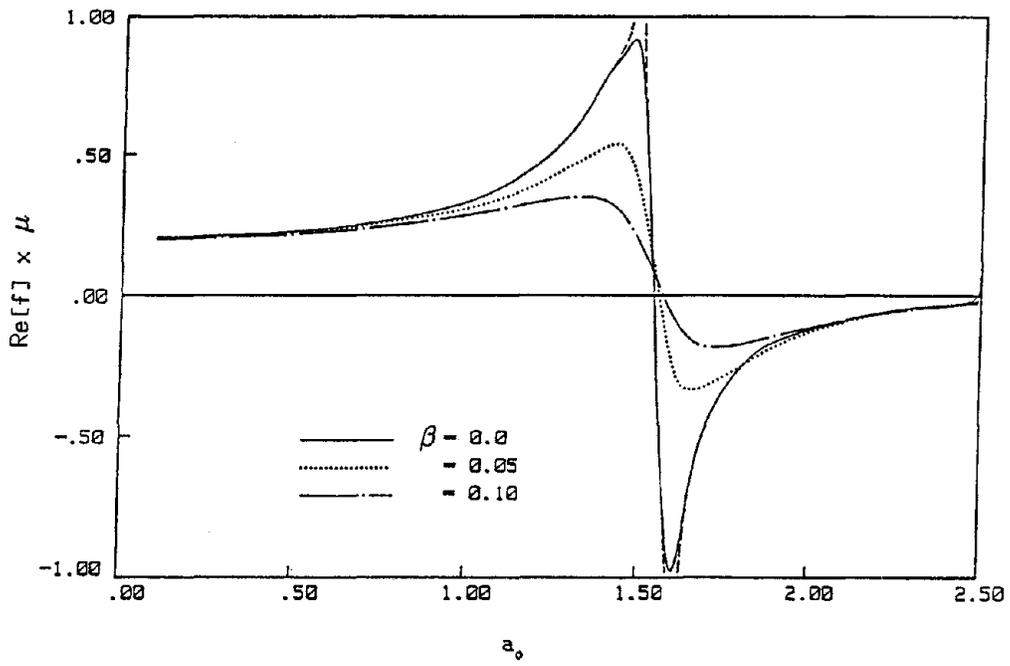


FIGURE 4-3c. Effect of material damping: Surface foundation on stratum-over-bedrock ($H/B = 2$)

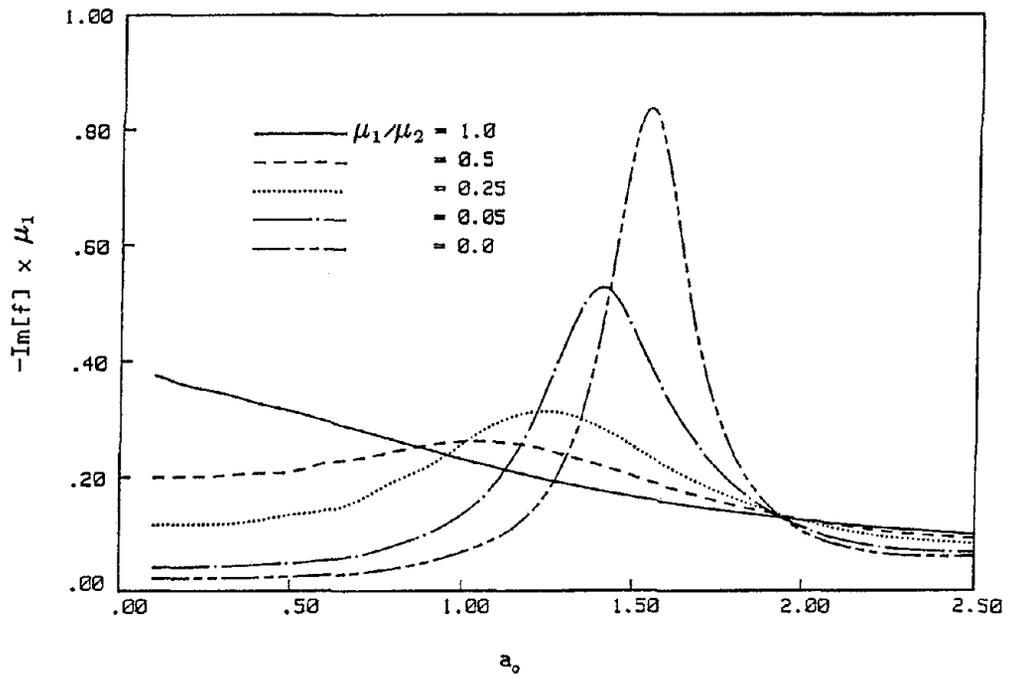
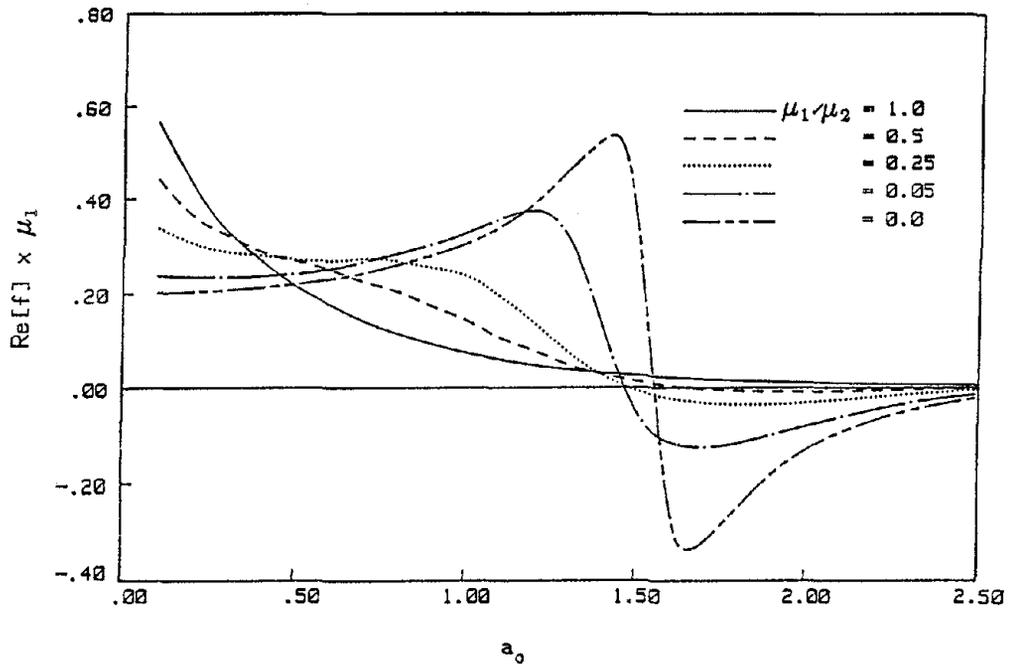


FIGURE 4-4a. Effect of μ_1/μ_2 : Surface foundation ($H/B = 2$)

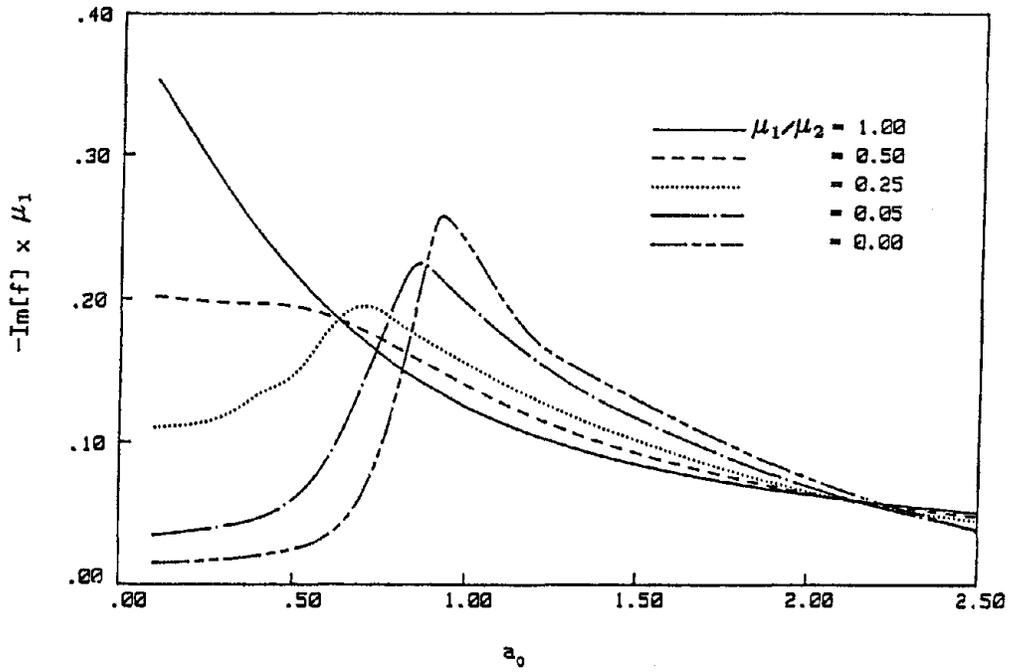
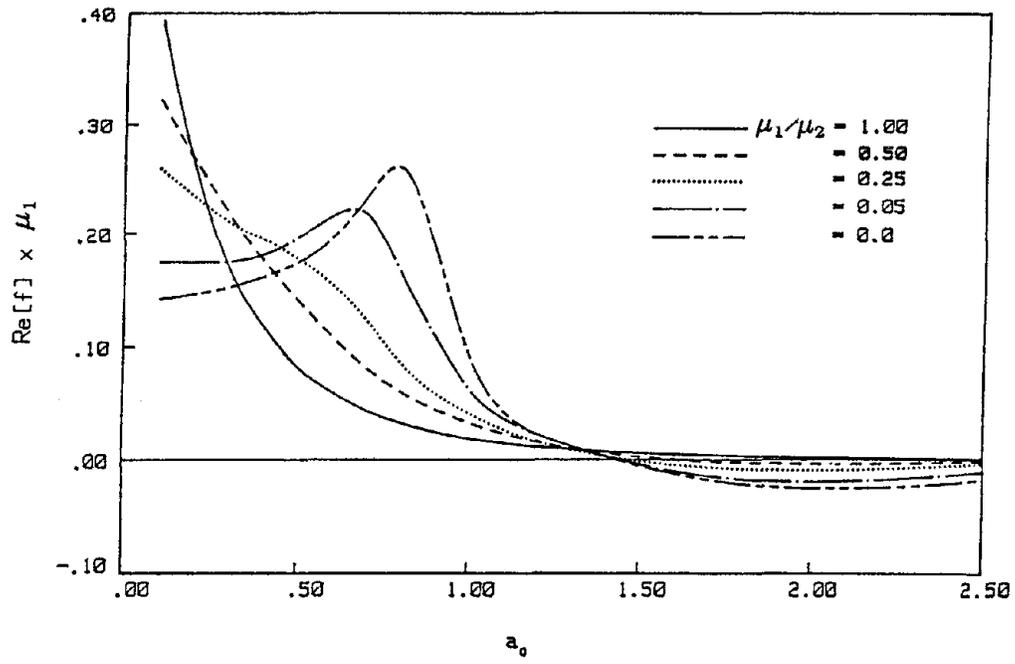


FIGURE 4-4b. Effect of μ_1/μ_2 : Embedded foundation ($H/B = 4$)

resonance peaks become narrower and steeper and the resonant frequencies shift to higher values as μ_1/μ_2 decreases. Since embedded foundation offers more damping, the resonant amplitudes are lower than surface foundations.

4.5 Effect of H/B for Stratum Over Bedrock

If bedrock, located below the soil layer, significantly affects the dynamic response of a foundation resting on or embedded in the top layer. Waves are reflected back and forth between the bedrock and the free-surface, modifying the response. In this study, for surface foundation, the layer depths were chosen to be $H/B = 1, 3, 5$ and ∞ , and in the case of embedded ($E/B = 2$) foundation, layer depths were taken as $H/B = 3, 4, 6$ and ∞ .

It can be seen in Figure 4-5 that the presence of bedrock modifies the compliance terms which exhibit peaks and valleys at the natural frequencies of the stratum. As the layer depth increases, the resonant frequency decreases, and the pattern shows gradual merging towards the half-space case. One interesting point to note is that in the case of a surface foundation on a very shallow soil deposit ($H/B = 1$), only a single flat resonance takes place, which represents the behavior of a highly damped system. The possible explanation for such a behavior is that at frequencies below the natural frequency of the top layer, some leakage of energy occurs in the form of laterally propagating P, S and R waves.

4.6 Effect of H/B for Stratum Over Half-space

The vertical oscillations have large 'zones of influence' in the downward direction. Thus, depth of the top layer will influence the response. Moreover, waves have different propagational velocities in the top and bottom profile and some wave reflection and refraction will occur

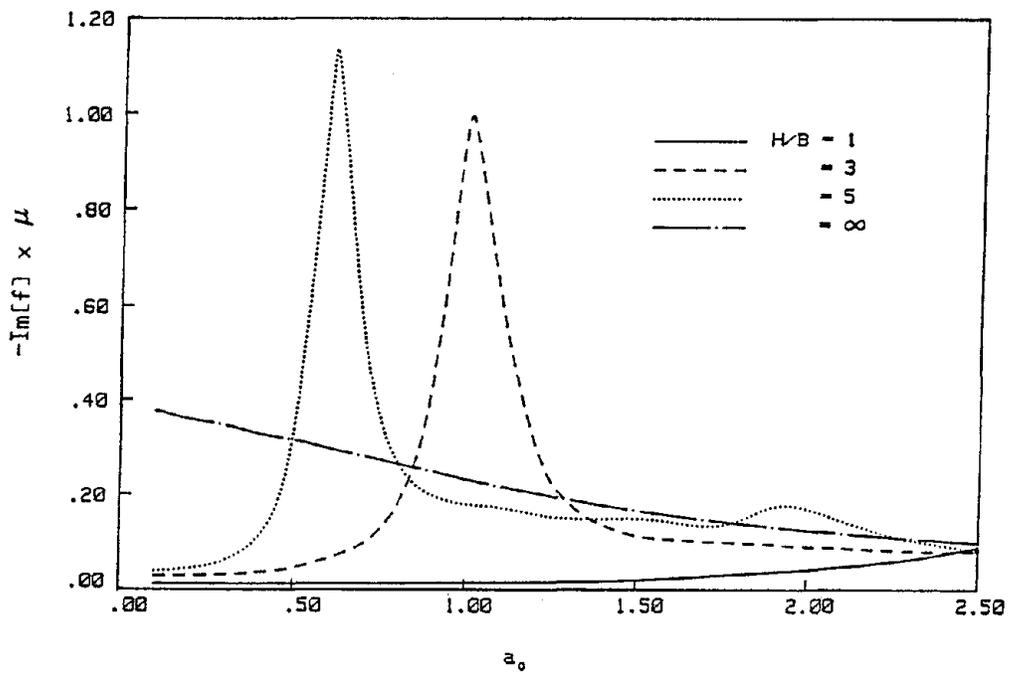
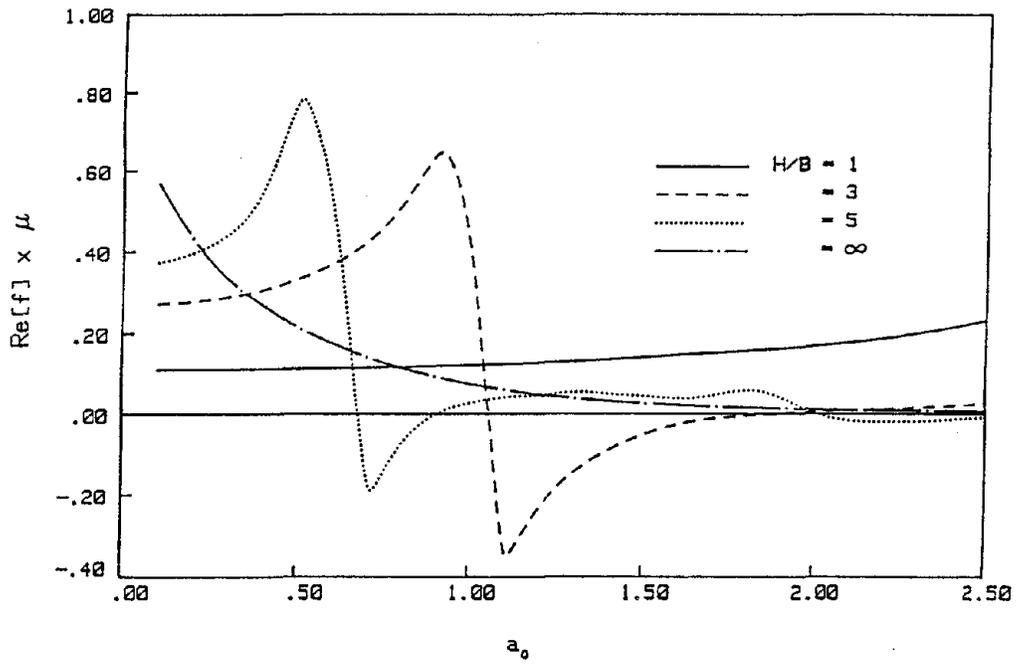


FIGURE 4-5a. Effect of stratum-depth (overlying bedrock):
Surface foundation

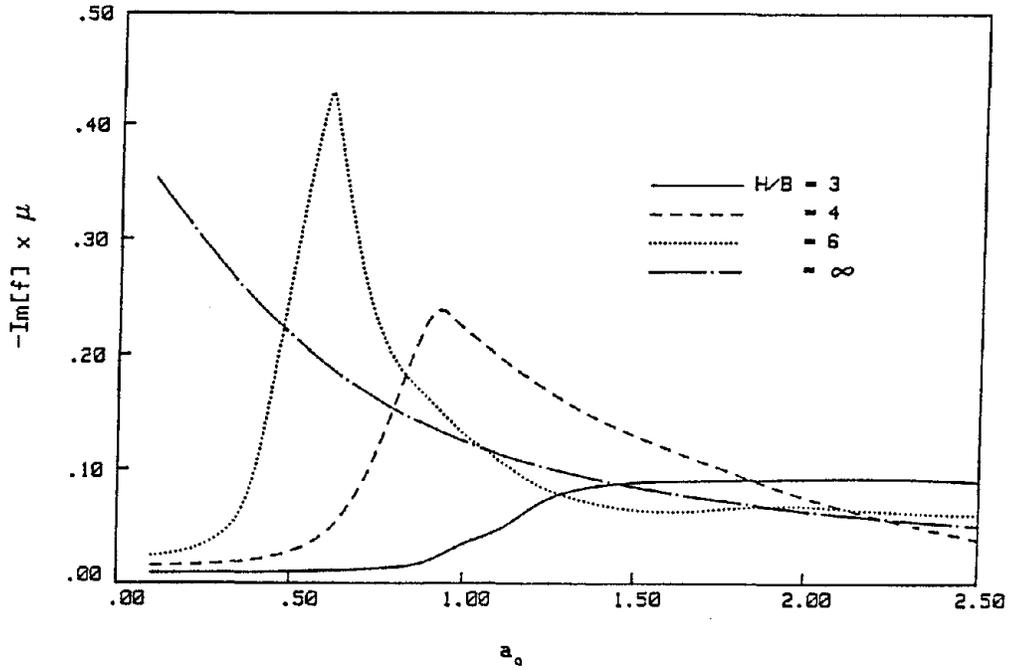
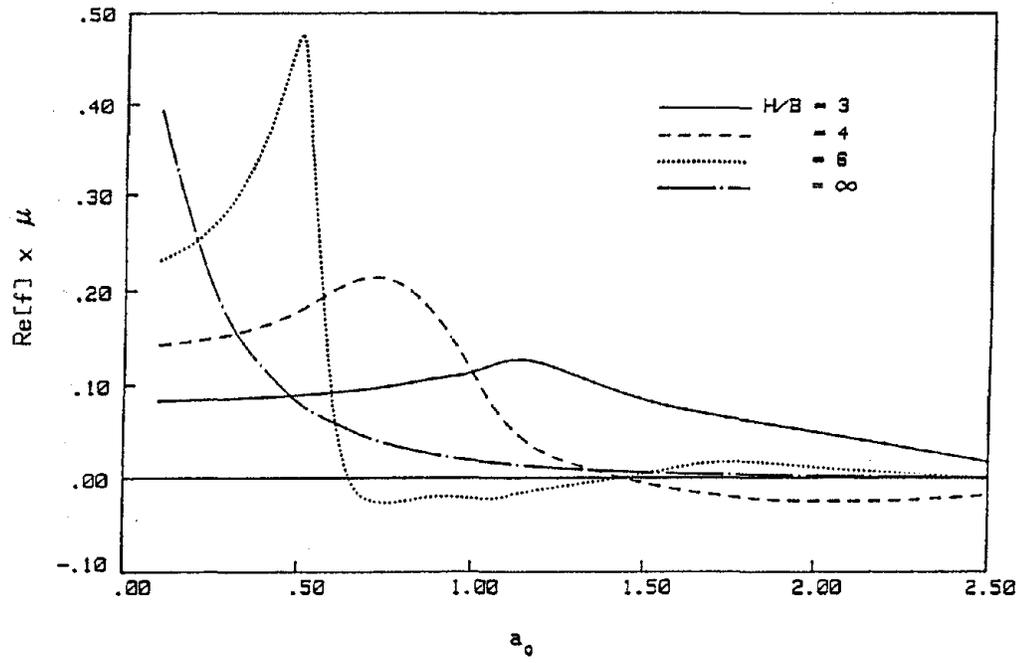


FIGURE 4-5b. Effect of stratum-depth (overlying bedrock):
Embedded foundation

at the interface between the layers. In this study, the depth of the top layer was varied using $H/B = 1, 2, 3$ for surface foundation and $H/B = 3, 4, 6$ for embedded foundation ($E/B = 2$). Ratio between the shear moduli of top and bottom layer (μ_1/μ_2) was kept at 0.5. Figure 4-6 shows that the resonance peaks are more pronounced in the complex part of the compliance than the real part. For a very shallow top layer, no resonance is noticed due to possible energy leakage in the form of laterally propagating waves. With increased layer-depth, the resonant peaks become more noticeable and exhibit low resonant frequency and the pattern shows gradual transition to the half-space curves.

4.7 Effect of Type of Contact

Contact condition at the soil-foundation interface may vary. Perfect contact between the foundation and soil exists in the bonded case and the adjacent soil moves with the foundation. All degrees of freedom are coupled with each other. In the lubricated contact, a roller-like boundary condition is assumed along the interface. A third kind of contact condition termed as 'relaxed' is quite often assumed in which the degrees of freedom are uncoupled, i.e., response in one direction is not affected by the force and displacements in the other directions. For example, if the response in the vertical direction is sought, it is assumed that no force is developed in the horizontal direction.

Figure 4-7 shows the influence of the type of contact on vertical compliance. For surface foundations, both relaxed and lubricated conditions refer to the same type of contact, which are very different from a bonded contact. However, negligible difference is noticed between the bonded and the lubricated (or relaxed) contact as seen in Figure 4-7a. In the case of an embedded foundation, a lubricated boundary condition

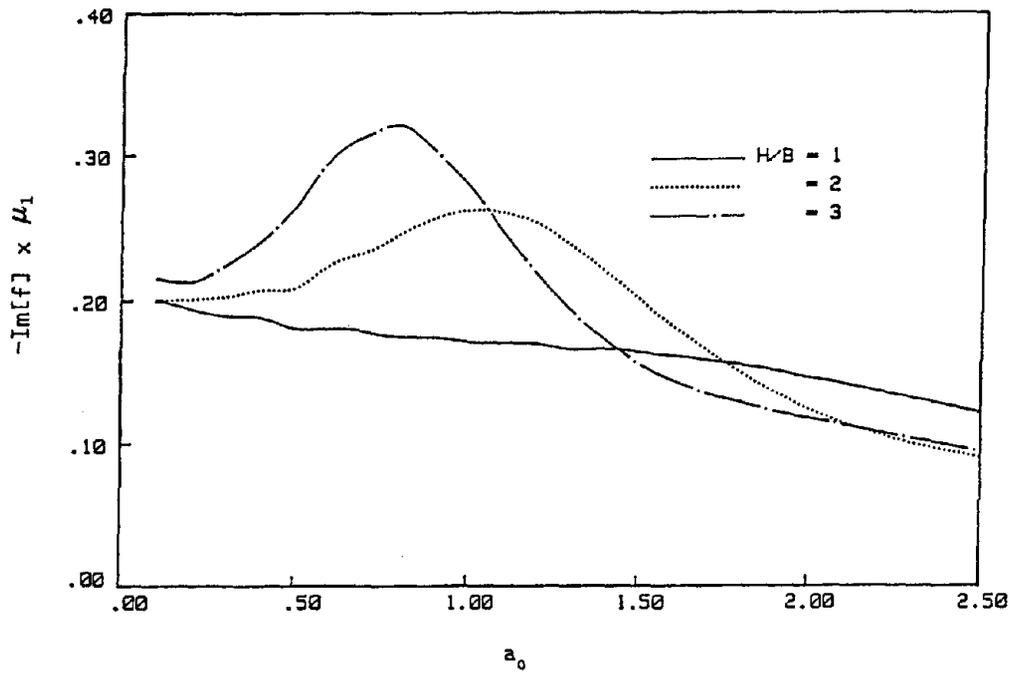
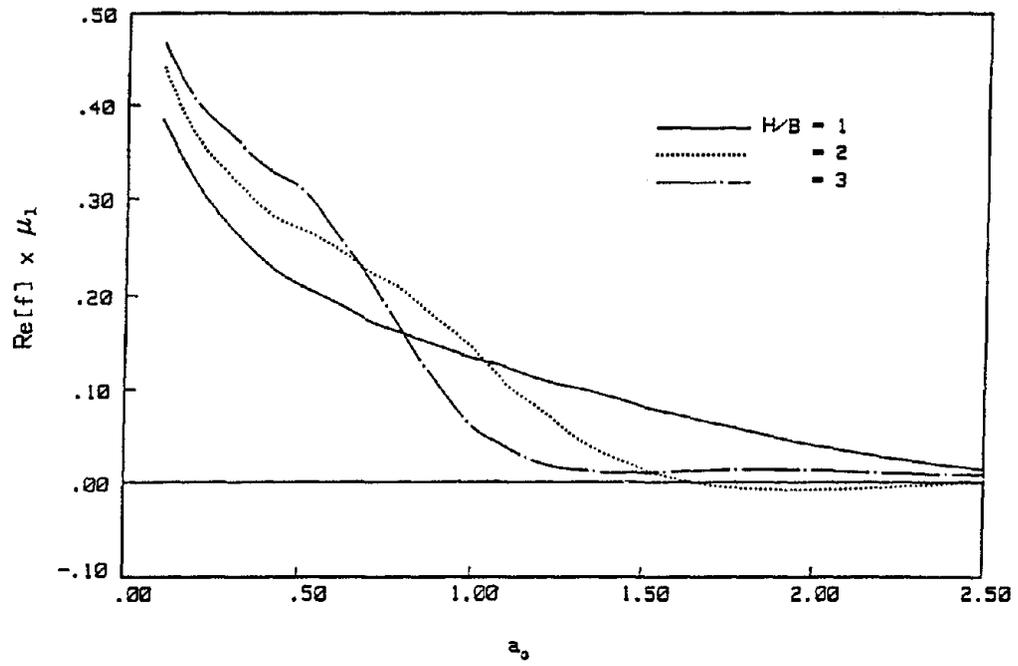


FIGURE 4-6a. Effect of depth of top layer (overlying half-space):
Surface foundation

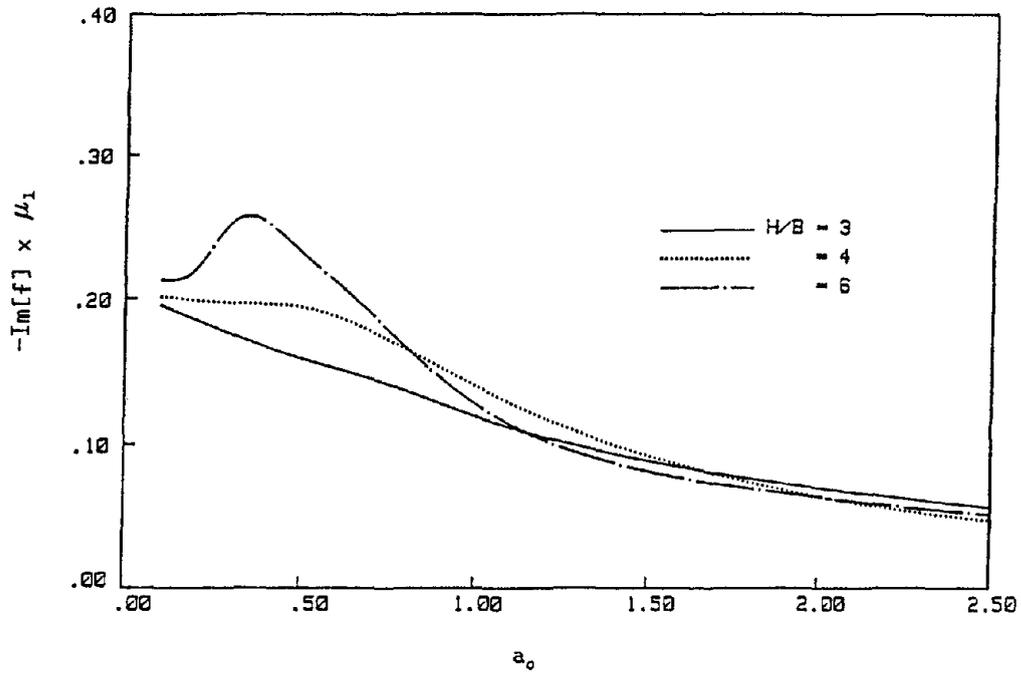
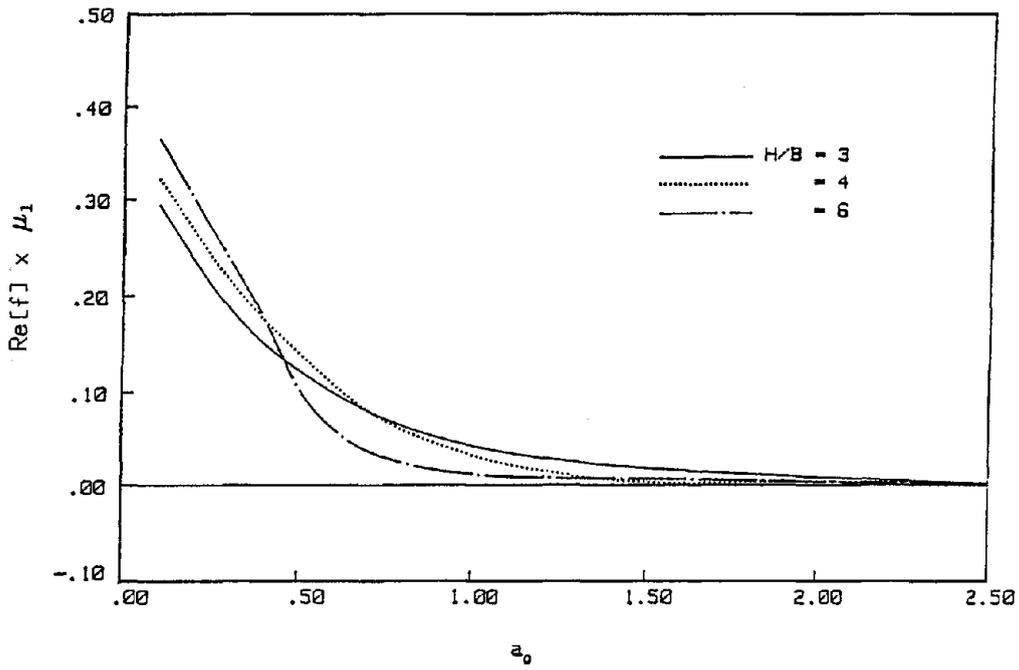


FIGURE 4-6b. Effect of depth of top layer (overlying half-space);
 Embedded foundation
 ($\mu_1/\mu_2 = 0.5$)

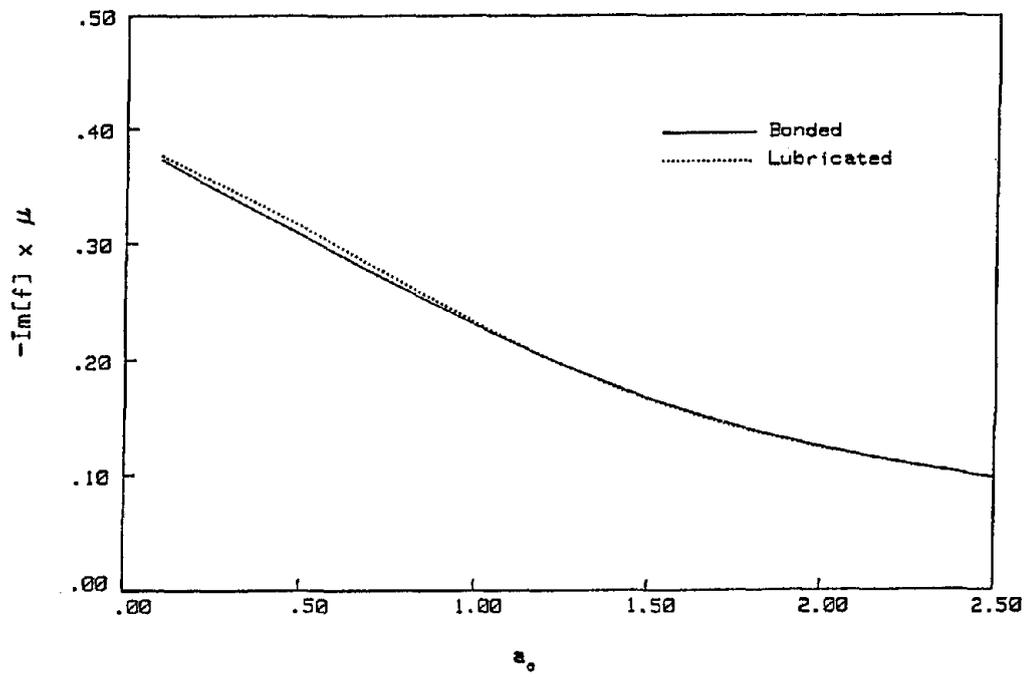
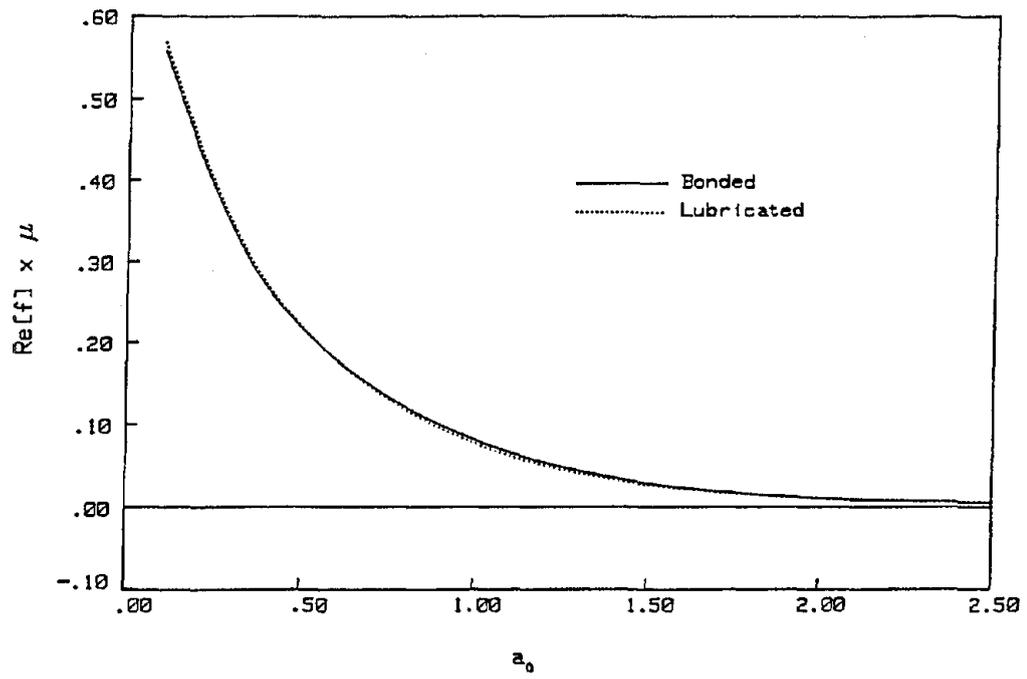


FIGURE 4-7a. Effect of type of contact at the soil-foundation interface: Surface foundation

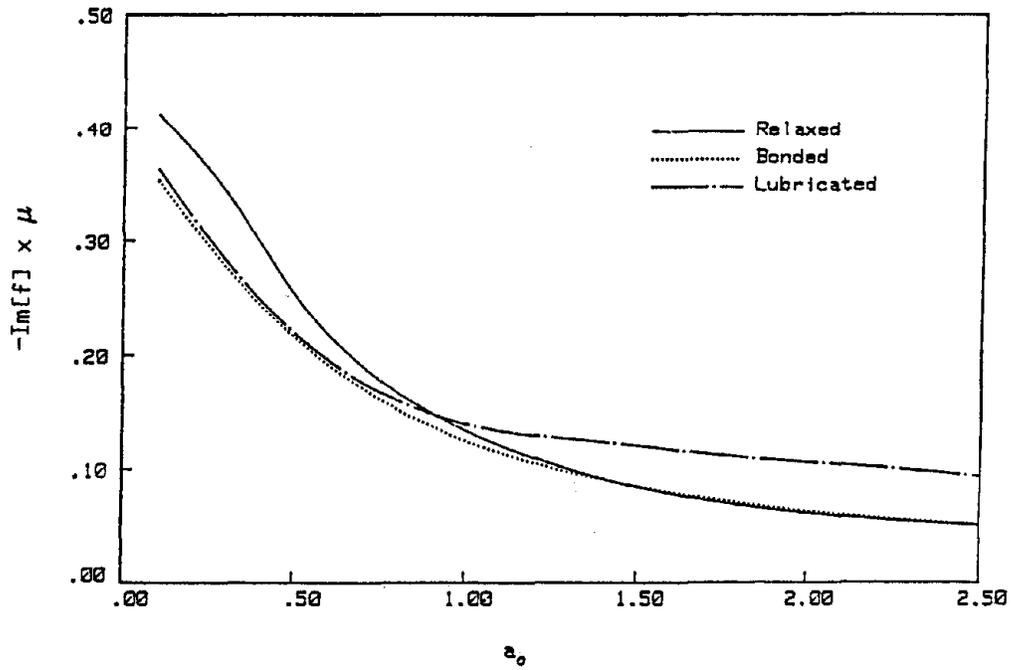
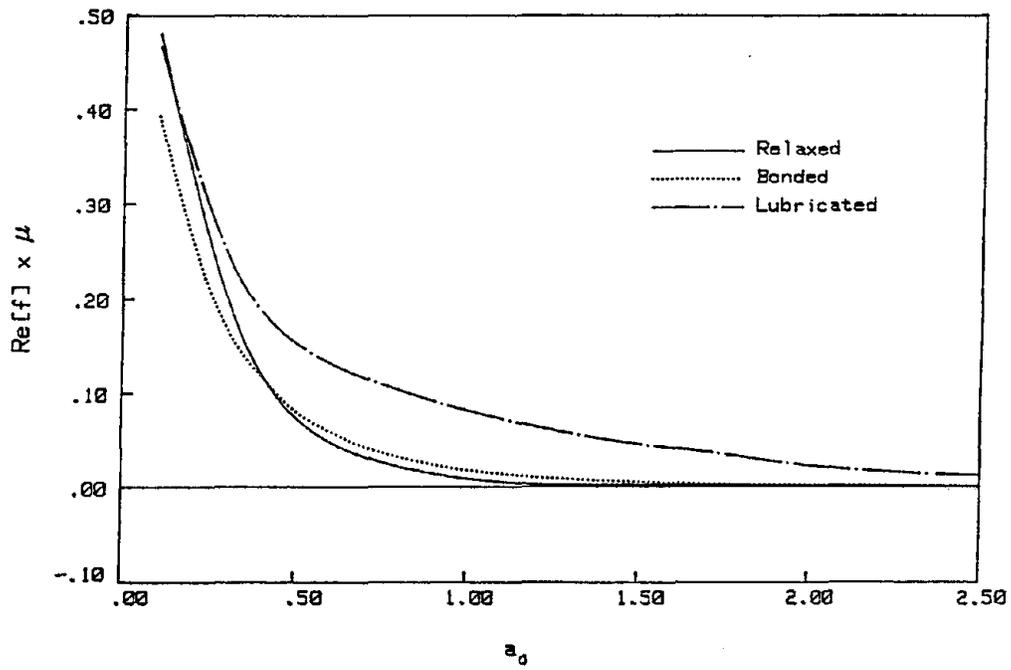


FIGURE 4-7b. Effect of type of contact at the soil-foundation interface: Embedded foundation

represents significantly different contact than the other two, since in this case there is no vertical contact along the side-wall interface. As a result, the foundation-compliance is higher in the lubricated contact, meaning the foundation will undergo greater vertical oscillation compared to the other contact conditions. This phenomenon is depicted in Figure 4-7b. Relaxed and bonded contacts exhibit virtually identical response at higher frequencies.

4.8 Effect of Embedment

Most foundations in the real world are embedded in the soil to a certain extent. Embedment significantly increases the stiffness of the foundation mainly because the contact area between the foundation and the soil is increased. That increase, however, depends on the type and degree of contact with the surrounding soil. For example, if there is no contact between the side-wall and backfill, the stiffness of an embedded foundation will be very close to that of a surface foundation. Wave interference due to embedment may also attenuate the amplitude of the response, thus showing increased damping.

Studies were done to find the effect of embedment for foundations resting on half-space, on layer-over-half-space and on stratum-over-bed-rock. Three types of embedment were chosen with $E/B = 0, 1$ and 2 . For a layered profile, the depth of the top layer was taken $H/B = 3$. Bonded contact was assumed along the foundation-soil interface. Figure 4-8 depicts the previously stated fact that with increased embedment, the foundation becomes stiffer, i.e., compliance decreases. When foundation lies over a layer-over-half-space or layer-over-bed-rock, embedment reduces the resonant peaks (Figures 4-9,4-10). Significant attenuation is noticed in the case of bedrock. Wave interference due to embedment might be the reason for increase in damping and reduction in the amplitude of response.

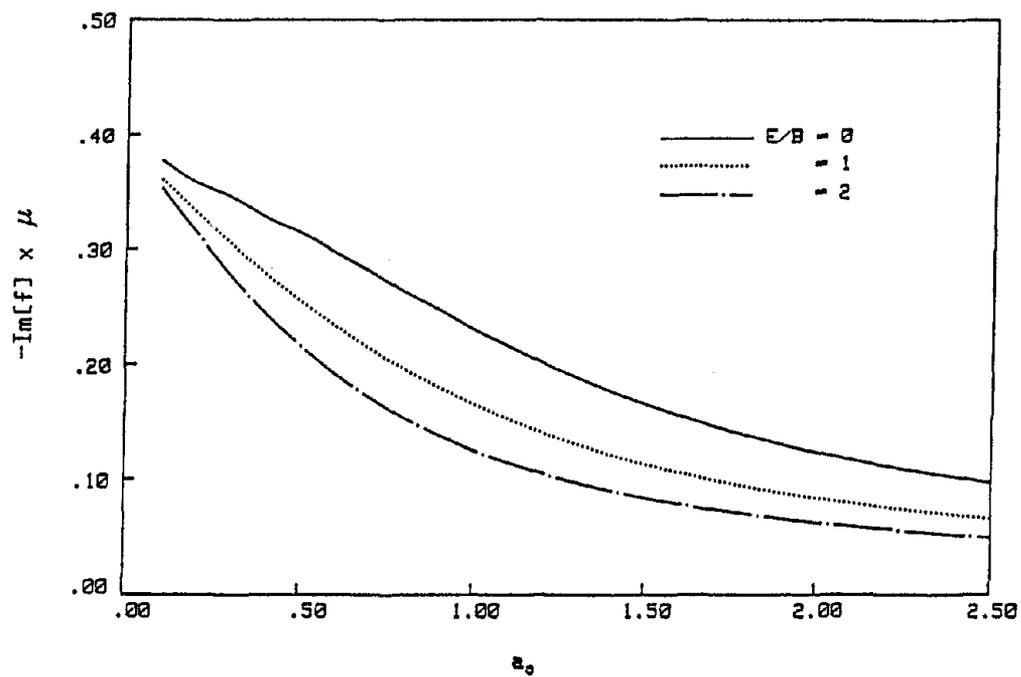
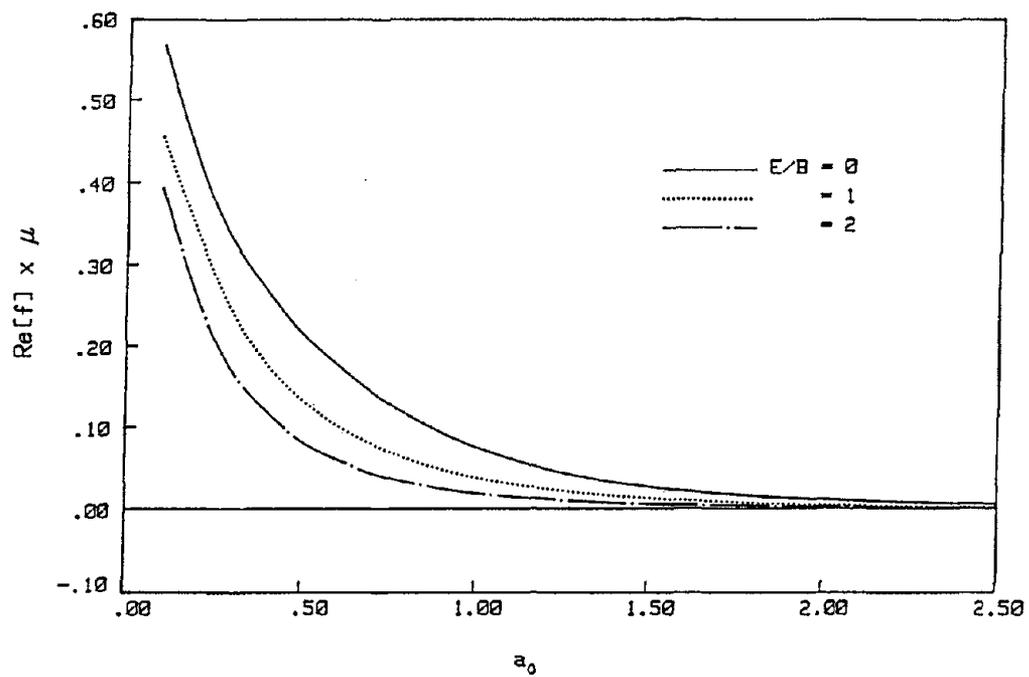


FIGURE 4-8. Effect of embedment for foundation over half-space

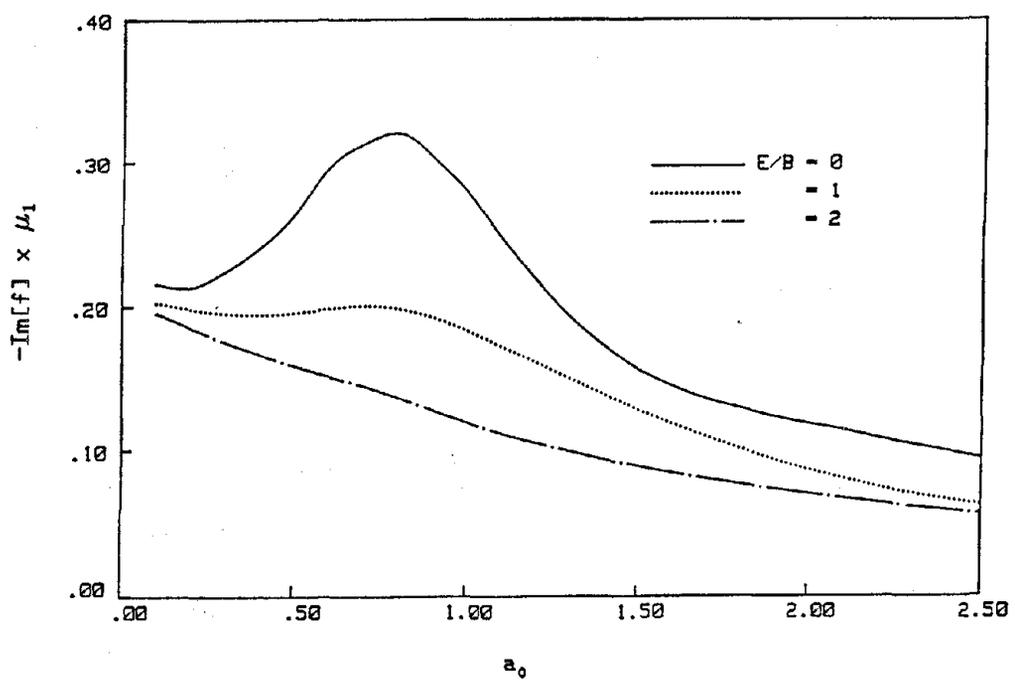
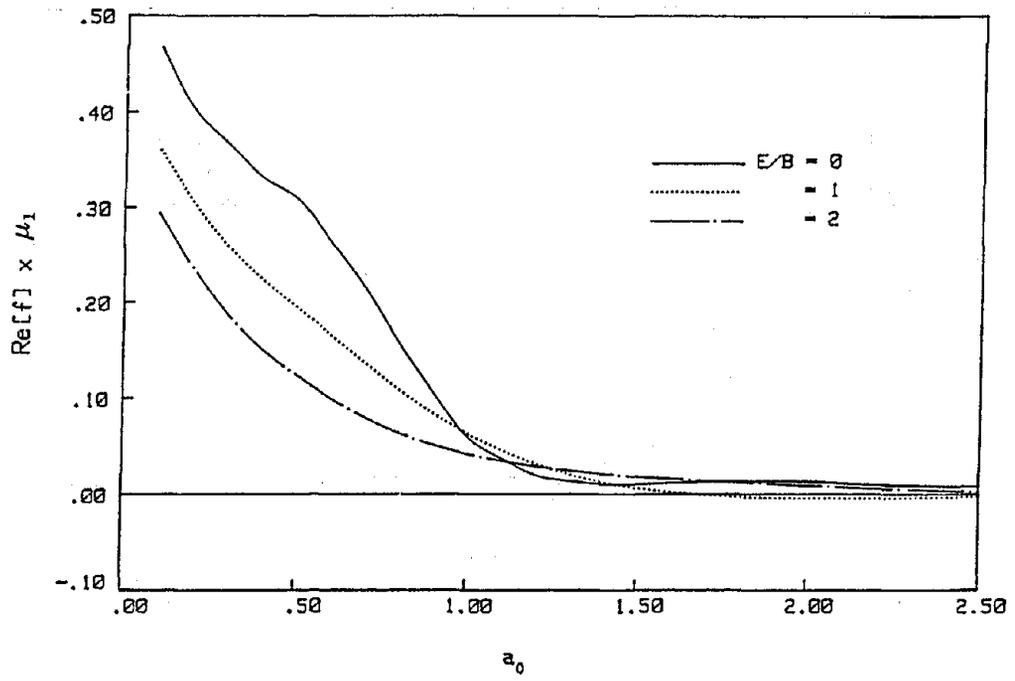


FIGURE 4-9. Effect of embedment for foundation on soil layer overlaying half-space ($H/B = 3$; $\mu_1/\mu_2 = 0.5$)

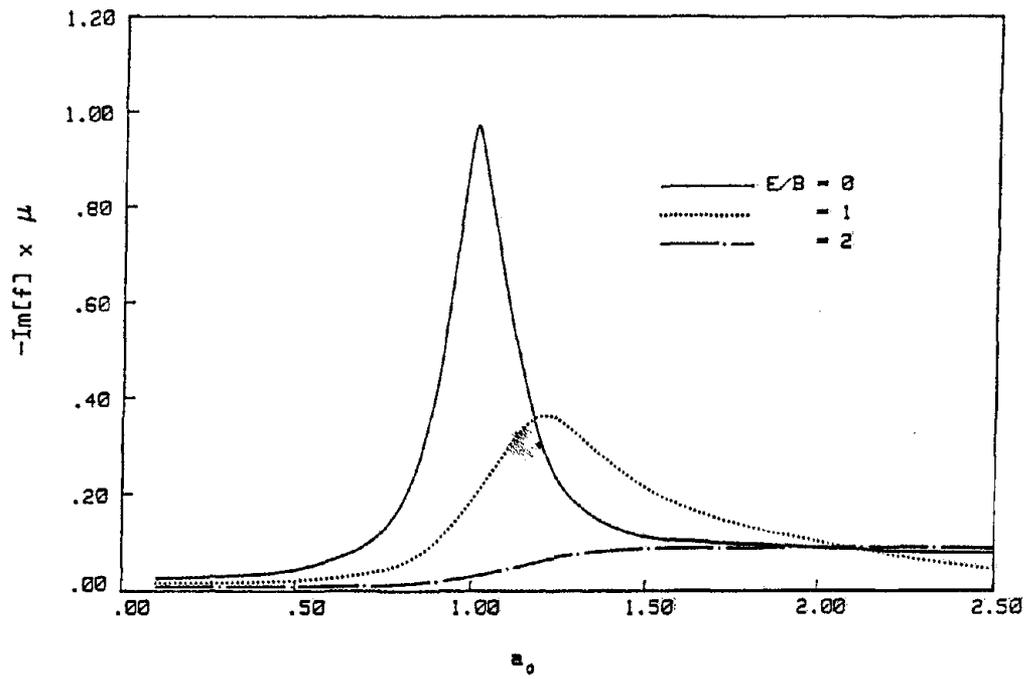
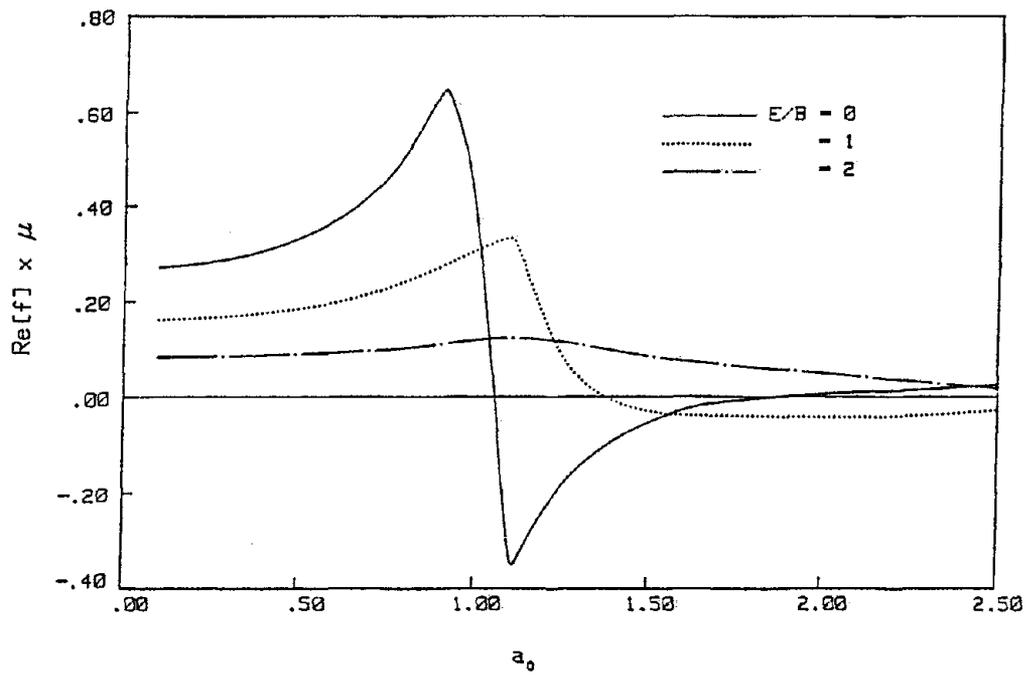
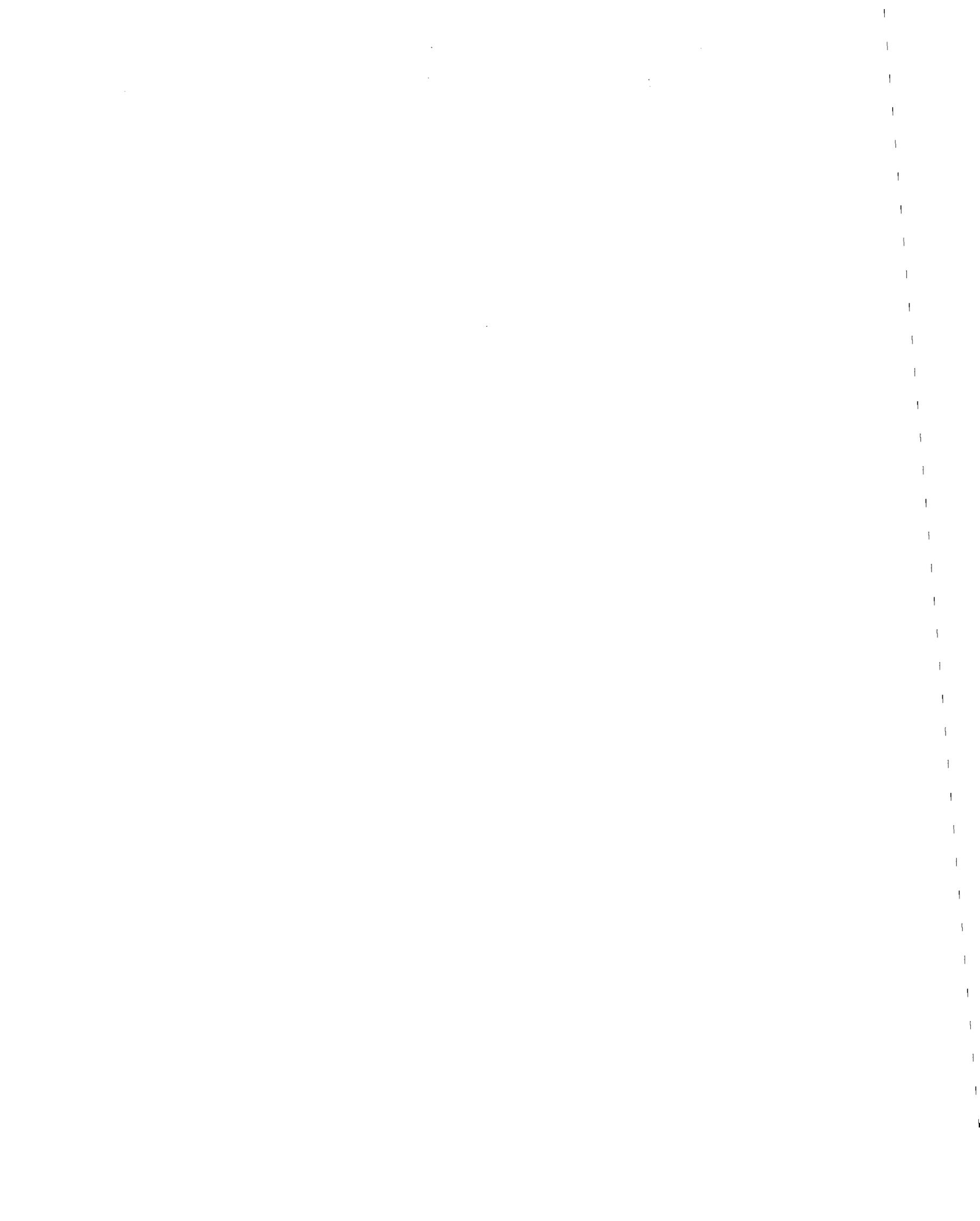


FIGURE 4-10. Effect of embedment for foundation on stratum-over-bedrock ($H/B = 3$)

SECTION 5

CONCLUSION

A comprehensive study on the dynamic response of rigid strip foundations under vertical loading using an advanced BEM algorithm is presented. Investigations were carried out on the effects of Poisson's ratio, internal material damping, layering, layer-depth, embedment and type of contact at the foundation-soil interface. As a result of this study, it was found that the influence of the presence of a soil layer on bedrock or on half-space is to introduce resonant amplitudes in the compliance at frequencies close to the fundamental frequency of the layer. As the depth of the top layer increases, the resonant peaks become sharper and narrower, the resonant frequency decreases, and the compliance curves approach towards their half-space components. Presence of material damping attenuates the peak responses for foundations on a layered stratum. Embedment introduces additional damping in the system. Increase in the soil's Poisson's ratio makes the system stiffer. Lubricated contact at the soil-foundation interface increases the compliance for embedded foundations.



SECTION 6

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