

<b>REPORT DOCUMENTATION PAGE</b>	<b>1. REPORT NO.</b> NCEER-88-0021	<b>2.</b>	<b>Recipient's Report No.</b> N89-1221961A5
<b>4. Title and Subtitle</b> Seismic Interaction of Structures and Soils: Stochastic Approach		<b>5. Report Date</b> July 21, 1988	
<b>7. Author(s)</b> A.S. Veletsos and A.M. Prasad		<b>6.</b>	
<b>9. Performing Organization Name and Address</b> National Center for Earthquake Engineering Research SUNY/Buffalo Red Jacket Quadrangle Buffalo, NY 14261		<b>8. Performing Organization Rept. No.</b>	
<b>12. Sponsoring Organization Name and Address</b>		<b>10. Project/Task/Work Unit No.</b>	
		<b>11. Contract(C) or Grant(G) No.</b> ECE-86-07591 86-2034 & 87-1314	
		<b>13. Type of Report &amp; Period Covered</b> Technical Report	
<b>15. Supplementary Notes</b> This research was conducted at Rice University and was partially supported by the National Science Foundation under Grant No. ECE 86-07591.		<b>14.</b>	
<b>16. Abstract (Limit: 200 words)</b> A study of soil-structure interaction for seismically excited simple structures is made considering both kinematic and inertial interaction effects. The information and concepts presented elucidate the nature and relative importance of the two effects and make it possible to assess readily the influences of the more important parameters. The response quantities examined are the ensemble means of the peak values of the lateral and torsional components of the foundation input motion and of the associated structural deformations. The results are evaluated over wide ranges of the parameters involved and compared with those obtained for no soil-structure interaction and for kinematic interaction only. Simple, physically motivated interpretations are given for the observed differences. For the important special case of vertically incident incoherent waves, simple closed-form approximate expressions are presented for the transfer functions of circular massless foundations.			
<b>17. Document Analysis</b>			
<b>a. Descriptors</b>			
<b>b. Identifiers/Open-Ended Terms</b> SOIL-STRUCTURE INTERACTION. EARTHQUAKE ENGINEERING. SEISMIC EXCITATION. STRUCTURE-FOUNDATION-SOIL SYSTEM.			
<b>c. COSATI Field/Group</b>			
<b>18. Availability Statement</b> Release unlimited	<b>19. Security Class (This Report)</b> unclassified	<b>21. No. of Pages</b> 60	
	<b>20. Security Class (This Page)</b> unclassified	<b>22. Price</b> NOL	

PB89-12219o



**NATIONAL CENTER FOR EARTHQUAKE  
ENGINEERING RESEARCH**

State University of New York at Buffalo

---

---

**SEISMIC INTERACTION OF  
STRUCTURES AND SOILS:  
STOCHASTIC APPROACH**

by

**A.S. Veletsos and A.M. Prasad**

Department of Civil Engineering  
Rice University  
Houston, Texas 77251

Technical Report NCEER-88-0021

July 21, 1988

This research was conducted at Rice University and was partially supported by the National Science Foundation under Grant No. ECE 86-07591.

REPRODUCED BY  
U.S. DEPARTMENT OF COMMERCE  
NATIONAL TECHNICAL INFORMATION SERVICE

## NOTICE

This report was prepared by Rice University as a result of research sponsored by the National Center for Earthquake Engineering Research (NCEER) and the National Science Foundation. Neither NCEER, associates of NCEER, its sponsors, Rice University, nor any person acting on their behalf:

- a. makes any warranty, express or implied, with respect to the use of any information, apparatus, method, or process disclosed in this report or that such use may not infringe upon privately owned rights; or
- b. assumes any liabilities of whatsoever kind with respect to the use of, or for damages resulting from the use of, any information, apparatus, method, or process disclosed in this report.



---

**SEISMIC INTERACTION OF STRUCTURES AND SOILS:  
STOCHASTIC APPROACH**

by

A.S. Veletsos<sup>1</sup> and A.M. Prasad<sup>2</sup>

July 21, 1988

Technical Report NCEER-88-0021

NCEER Contract Numbers 86-2034 and 87-1314

NSF Master Contract Number ECE 86-07591

1 Brown and Root Professor, Dept. of Civil Engineering, Rice University

2 Graduate Student, Dept. of Civil Engineering, Rice University

**NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH**

State University of New York at Buffalo

Red Jacket Quadrangle, Buffalo, NY 14261

---

## PREFACE

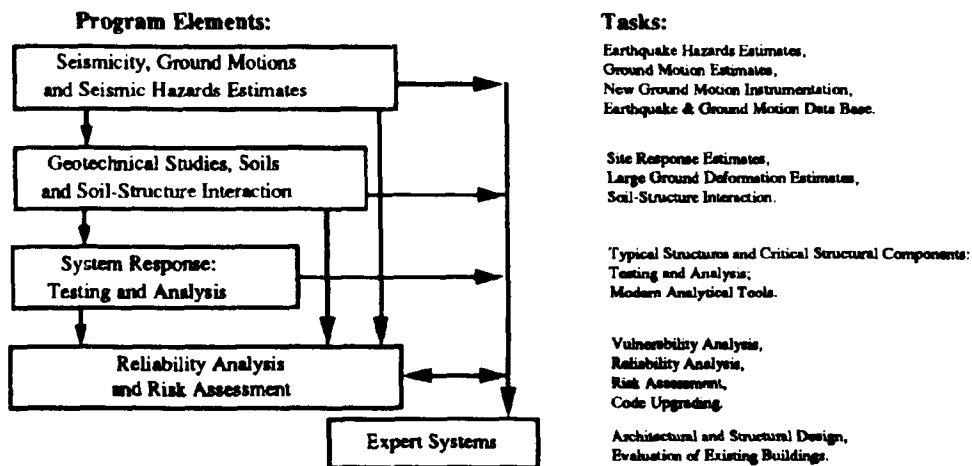
The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 1, Existing and New Structures, and more specifically to Geotechnical Studies.

The long term goal of research in Existing and New Structures is to develop seismic hazard mitigation procedures through rational probabilistic risk assessment for damage or collapse of structures, mainly existing buildings, in regions of moderate to high seismicity. The work relies on improved definitions of seismicity and site response, experimental and analytical evaluations of systems response, and more accurate assessment of risk factors. This technology will be incorporated in expert systems tools and improved code formats for existing and new structures. Methods of retrofit will also be developed. When this work is completed, it should be possible to characterize and quantify societal impact of seismic risk in various geographical regions and large municipalities. Toward this goal, the program has been divided into five components, as shown in the figure below:



Geotechnical Studies constitute one of the important areas of research in Existing and New Structures. Current research activities include the following:

1. Development of linear and nonlinear site response estimates.
2. Development of liquefaction and large ground deformation estimates.
3. Investigation of soil-structure interaction phenomena.
4. Development of computational methods.
5. Incorporation of local soil effects and soil-structure interaction into existing codes.

The ultimate goal of projects concerned with Geotechnical Studies is to develop methods of engineering estimation of large soil deformations, soil-structure interaction and site response.

*This report presents the result of a study of soil-structure interaction for seismically excited simple structures considering both kinematic and inertial interaction effects. The information and concepts presented elucidate the nature and relative importance of the two effects and make it possible to assess readily the influences of the more important parameters. The response quantities examined are the ensemble means of the peak values of the lateral and torsional components of the foundation input motion and of the associated structural deformations. The results are evaluated over wide ranges of the parameters involved and compared with those obtained for no soil-structure interaction and for kinematic interaction only. Simple, physically motivated interpretations are given for the observed differences. For the important special case of vertically incident incoherent waves, simple closed-form approximate expressions are presented for the transfer functions of circular massless foundations.*

## ABSTRACT

A study of soil-structure interaction for seismically excited simple structures is made considering both kinematic and inertial interaction effects. The information and concepts presented elucidate the nature and relative importance of the two effects and make it possible to assess readily the influences of the more important parameters. The response quantities examined are the ensemble means of the peak values of the lateral and torsional components of the foundation input motion and of the associated structural deformations. The results are evaluated over wide ranges of the parameters involved and compared with those obtained for no soil-structure interaction and for kinematic interaction only. Simple, physically motivated interpretations are given for the observed differences. For the important special case of vertically incident incoherent waves, simple closed-form approximate expressions are presented for the transfer functions of circular massless foundations.

#### **ACKNOWLEDGEMENT**

This study was supported by Grants 86-2034 and 87-1314 awarded to Rice University by the National Center for Earthquake Engineering Research, State University of New York at Buffalo. This support is appreciated greatly. Appreciation also is expressed to Yu Tang, Post-Doctoral Research Associate at Rice, and K. Dotson and P. Malhotra, graduate students at Rice, for reading an earlier draft of the manuscript and offering valuable comments.



## TABLE OF CONTENTS

TITLE	PAGE
1. INTRODUCTION . . . . .	.1-1
2. SYSTEM CONSIDERED . . . . .	.2-1
3. KINEMATIC INTERACTION EFFECTS . . . . .	.3-1
3.1 Spectral Characterization of Foundation Input Motion . . . . .	.3-1
3.1.1 Integration of Equations . . . . .	.3-3
3.1.2 Presentation of Results. . . . .	.3-4
3.1.3 Accuracy of Solutions. . . . .	.3-4
3.1.4 Other Meanings for Results . . . . .	.3-7
3.2 Spectral Characterization of Structural Response . . . . .	.3-7
3.3 Characterization of Free-Field Earthquake Ground Motions . . . . .	.3-9
3.4 Foundation Input Motion. . . . .	3-11
3.5 Effects of Ground-Motion Incoherence on Structural Response. . . . .	3-13
3.5.1 Comparison of Incoherence and Wave Passage Effects . . . . .	3-15
4. INERTIAL INTERACTION EFFECTS . . . . .	.4-1
4.1 Results for Vertically Propagating Incoherent Waves. . . . .	.4-2
5. CONCLUSIONS. . . . .	.5-1
6. REFERENCES . . . . .	.6-1
APPENDIX A. DERIVATION OF EQUATIONS 9 . . . . .	.A-1
APPENDIX B. EVALUATION OF PEAK VALUES OF INPUT AND RESPONSE . . . . .	.B-1
APPENDIX C. HARMONIC RESPONSE OF SYSTEMS WITH INERTIAL INTERACTION. . . . .	.C-1
C.1 Torsionally Excited Systems. . . . .	.C-1
C.2 Laterally Excited Systems. . . . .	.C-3
APPENDIX D. NOTATION. . . . .	.D-1

## LIST OF FIGURES

FIGURE	TITLE	PAGE
2-1	System Considered . . . . .	2-2
3-1	Magnitudes of Transfer Functions between Free-Field Ground Motion and Foundation Input Motions. . . . .	3-5
3-2	Normalized Cross PSD Function for Horizontal and Torsional Components of Foundation Input Motion . . . . .	3-6
3-3	Comparison of Approximate and Exact Magnitudes of Foundation Transfer Functions . . . . .	3-8
3-4	Normalized PSD Functions for Free-Field Ground Motions Considered. . . . .	3-10
3-5	Normalized Mean Peak Values of Lateral and Torsional Components of Foundation Input Accelerations, Velocities and Displacements . . . . .	3-12
3-6	Effects of Ground Motion Incoherence on Maximum Deformations of Structures with $\zeta_x = \zeta_\theta = 0.02$ for Vertically Propagating Waves. . . . .	3-14
3-7	Effects of Ground Motion Incoherence and Wave Passage on Maximum Deformations of Structures with $\zeta_x = \zeta_\theta = 0.02$ . . . . .	3-16
4-1	Comparison of Effects of Kinematic and Inertial Interaction on Maximum Deformations of Structures with $\zeta_x = \zeta_\theta = 0.02$ . . . . .	4-3
4-2	Natural Frequencies and Damping of Modified Systems in Approximate Analysis of Inertial Interaction Effects. . . . .	4-4

## SECTION 1 INTRODUCTION

In evaluating the response of structures to earthquakes, it is normally assumed that all points of the ground surface beneath the foundation are excited synchronously and experience the same free-field motion [3,6,26]; the latter term refers to the motion which would be induced at the foundation-soil interface if no structure were present. The assumption of synchronous interface free-field ground motions is strictly valid only for vertically propagating coherent wave fields; in reality, the motions may vary from one point to the next [1,8,13,34]. Even when the wave front is plane and propagates in a perfectly homogeneous medium, it may impinge the foundation at a finite angle, leading to motions at neighboring points which in the words of Kausel and Pais [11] are "delayed replicas" of each other. Known as the **wave passage effect**, the consequences of such action have been the subject of numerous previous studies [4,14,19,20,23,24,32,33] and are reasonably well understood.

Several additional factors contribute to the spatial variability of the free-field ground motion. The individual wave trains may emanate from different points of an extended source and may impinge the foundation at different instants and with different angles of incidence, or they may propagate through paths of different physical properties and may be affected differently in both amplitude and phase by the characteristics of the travel paths and by reflections from, and diffractions around, the foundation. The spatial variability of the ground motion due to these factors will be referred to as the **ground motion incoherence effect**. This effect, which would exist even for horizontally polarized vertically propagating shear waves, has been the subject of only exploratory recent studies [9,15,16,17,18,21,22].

The motion experienced by a rigid foundation is clearly different from the free-field ground motion. The actual motion may conveniently be evaluated in two steps. First, the so-called **foundation input motion** is computed; this is defined as the motion which would be experienced by the foundation if both it and the superimposed structure were massless.

Computed with due provision for the rigidity of the foundation, the foundation input motion includes both horizontal and torsional components even for a purely horizontal free-field ground shaking. The difference in the responses of the structure computed for the foundation input motion and the free-field motion at some reference or control point of the ground surface is known as the **kinematic interaction effect**. The greater the degree of ground motion incoherence or the plan dimensions of the foundation in comparison to the length of the dominant seismic waves, the more important this effect is likely to be.

The actual motion of the foundation is also influenced by its own inertia and the inertia of the structure, and by the interaction or coupling between them and the supporting soils. For a structure subjected to a purely horizontal free-field ground shaking, not only are the horizontal and torsional components of the actual foundation motion different from those of the corresponding input motion, but the actual motion may also include rocking components about horizontal axes. Contributed by the overturning tendency of the superstructure, the latter components may be particularly prominent for tall slender structures and for soft soils. These factors are provided for in the second step of the evaluation process.

The term **inertial interaction effect** refers to the difference in structural responses computed for the actual motion of the foundation and the foundation input motion. The total **soil structure interaction** is clearly the sum of the kinematic and inertial interaction effects.

Although the inertial interaction effects have been the subject of numerous studies [6,23,26,27,28], they have generally been examined at the exclusion of the kinematic interaction effects, and the interrelationship of the two effects has not been adequately assessed. The objectives of this paper are: to elucidate the nature of both types of interaction for seismically excited simple structures; to assess the effects and relative importance of the numerous parameters involved; and to present information and concepts with which the effects of the

principal parameters may be evaluated readily. Primary emphasis is placed on the kinematic interaction effects.

The structures investigated are presumed to have one lateral and one torsional degree of freedom in their fixed-base condition and to be excited by obliquely incident, horizontally polarized, incoherent shear waves. The temporal variation of the free-field ground motion is expressed stochastically by a local power spectral density (psd) function, and its spatial variability is specified by a cross psd function. The response quantities examined include the ensemble means of the peak values of the lateral and torsional components of the foundation input motion and of the corresponding structural deformations. These deformations are displayed in the form of pseudo-velocity response spectra and compared, over wide ranges of the parameters involved, with those obtained for no soil-structure interaction and for kinematic interaction only. Simple, physically motivated interpretations are given for the observed differences.

A fundamental step in the analysis of the a structure-foundation-soil system is the evaluation of the transfer functions of the foundation. Defined for harmonically excited massless foundations, these functions relate the amplitudes of the horizontal and torsional components of foundation input motion to the amplitude of the free-field ground motion. The relevant functions are evaluated herein by a relatively simple, approximate procedure, and their accuracy is assessed through comparisons with available exact solutions for special cases. In addition, simple closed-form expressions are presented for these functions for the important special case of vertically incident, incoherent waves.

## SECTION 2 SYSTEM CONSIDERED

The system investigated is shown in Fig. 2-1. It is a linear structure of mass  $m$  and height  $h$ , which is supported through a foundation of mass  $m_f$  at the surface of a homogeneous elastic halfspace. The circular natural frequencies of lateral and torsional modes of vibration for the structure when fixed at its base are denoted by  $p_x = 2\pi f_x$  and  $p_\theta = 2\pi f_\theta$ , respectively, in which  $f_x$  and  $f_\theta$  are the associated frequencies in cycles per unit of time; and the corresponding percentages of critical damping are denoted by  $\zeta_x$  and  $\zeta_\theta$ , respectively. The foundation mat is idealized as a rigid circular plate of negligible thickness and radius  $R$  which is bonded to the halfspace so that no uplifting or sliding can occur, and the columns of the structure are presumed to be massless and axially inextensible. Both  $m$  and  $m_f$  are assumed to be uniformly distributed over identical circular areas. The supporting medium is characterized by its mass density,  $\rho$ , shear wave velocity,  $v_s$ , and Poisson's ratio,  $\nu$ . This structure may be viewed either as the direct model of a single-story building frame or, more generally, as the model of a multistory, multimode structure that responds as a system with one lateral and one torsional degrees of freedom in its fixed-base condition.

The free-field ground motion for all points of the foundation-soil interface is considered to be a uni-directional excitation directed parallel to the horizontal  $x_1$ -axis, as shown in Fig. 2-1, with the detailed histories of the motions varying from point to point. Such motions may be induced by horizontally polarized, incoherent shear waves propagating either vertically or at an arbitrary angle with the vertical,  $\alpha_v$ . The intense portions of the motions are represented by a stationary random process of limited duration,  $t_0$ , and a space-invariant, local psd function,  $S_g = S_g(\omega)$ , in which  $\omega$  = the circular frequency of the motions. The spatial variability of the motions is defined by a cross psd function,  $S(\vec{r}_1, \vec{r}_2, \omega)$ , in which  $\vec{r}_1$  and  $\vec{r}_2$  are the position vectors for two arbitrary points.

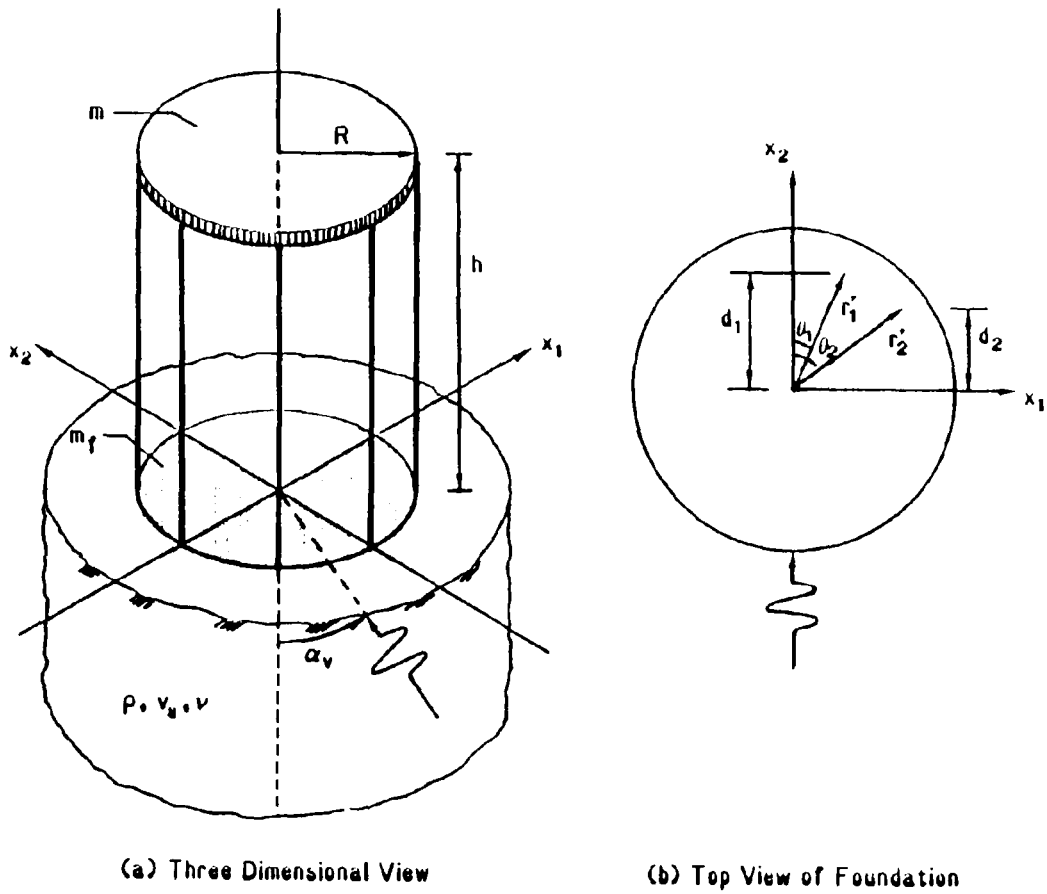


FIG. 2-1 System Considered

A decreasing function of the frequency  $\omega$  and of the distance between the two points,  $|\vec{r}_1 - \vec{r}_2|$ , the function  $S(\vec{r}_1, \vec{r}_2, \omega)$  is taken in the form suggested by Harichandran and Vanmarcke [7] as

$$S(\vec{r}_1, \vec{r}_2, \omega) = \Gamma(|\vec{r}_1 - \vec{r}_2|, \omega) \exp[-i\omega \frac{d_1 - d_2}{c}] S_g(\omega) \quad (1)$$

in which  $\Gamma$ , referred to as the incoherence function, is a dimensionless, decreasing function of  $|\vec{r}_1 - \vec{r}_2|$ ;  $i = \sqrt{-1}$ ;  $d_1$  and  $d_2$  = the components of  $\vec{r}_1$  and  $\vec{r}_2$  in the direction of propagation of the wave front (see Fig. 2-1b); and  $c$  = the apparent horizontal velocity of the front. The latter quantity is related to the angle of incidence of the waves,  $\alpha_v$ , by

$$c = \frac{v_s}{\sin \alpha_v} \quad (2)$$

The product of the exponential term in Eq. 1 and  $S_g$  represents the wave passage effect, whereas the product  $\Gamma S_g$  represents the effect of ground motion incoherence. The peak value of  $\Gamma$  is unity and occurs at  $\vec{r}_1 = \vec{r}_2$ .

Several different expressions have been suggested for the incoherence function (e.g., Refs. 8,9,13,16,18), and there is no general agreement at this time on the form that may be the most appropriate for realistic earthquakes. In this study, the single-parameter, second order function recommended by Mita and Luco [18] is used,

$$\Gamma(|\vec{r}_1 - \vec{r}_2|, \omega) = \exp \left[ - \left( \frac{\gamma \omega |\vec{r}_1 - \vec{r}_2|}{v_s} \right)^2 \right] \quad (3)$$

in which  $\gamma$  is a dimensionless factor, taken between zero and 0.5.

A different approach to the study of this problem has been taken by Pais and Kausel [22]. They have attributed the ground motion incoherence to arrays of uncorrelated, obliquely incident waves arriving from different directions within a sector of the supporting medium. The kinematic interaction effects in this approach are represented by weighted averages of the component wave passage effects.



**SECTION 3**  
**KINEMATIC INTERACTION EFFECTS**

**3.1 Spectral Characterization of Foundation Input Motion**

Let  $S_x$  be the psd function of the horizontal component of the foundation input displacement, and  $S_y$  be the corresponding function for the circumferential or tangential displacement component along the periphery of the foundation. Further, let  $S_{xy}$  for the cross spectral density function for the component displacements. Whereas  $S_x$  and  $S_y$  are real-valued,  $S_{xy}$  is generally complex-valued.

These functions were evaluated from the cross spectral density function,  $S(\vec{r}_1, \vec{r}_2, \omega)$ , by application of the averaging technique employed by Iguchi [10] and Scanlan [24] in their studies of wave propagation effects. This approach leads to

$$S_x = \frac{1}{A^2} \int_A \int_A S(\vec{r}_1, \vec{r}_2, \omega) dA_1 dA_2 \quad (4a)$$

$$S_y = \frac{R^2}{I_\theta^2} \int_A \int_A d_1 d_2 S(\vec{r}_1, \vec{r}_2, \omega) dA_1 dA_2 \quad (4b)$$

$$S_{xy} = \frac{R}{I_\theta A} \int_A \int_A d_2 S(\vec{r}_1, \vec{r}_2, \omega) dA_1 dA_2 \quad (4c)$$

in which  $dA_1$  and  $dA_2$  are elemental areas of the foundation;  $A = \pi R^2$  = the area of the foundation; and  $I_\theta = AR^2/2$  = its polar moment of inertia about a vertical centroidal axis.

As is true of the corresponding exact expressions presented by Luco and Mita [15], Eqs. 4 represent weighted averages of  $S(\vec{r}_1, \vec{r}_2, \omega)$ . However, whereas the weighting functions in the exact formulation are the complex distributions of the actual tractions at the foundation-soil interface, in the procedure employed herein they are taken as linear functions. This is tantamount to representing the restraining action of the supporting medium by a series of mutually independent springs of the Winkler type [24]. There are two main advantages to the use of the approximate formulation over the exact formulation: (a) it reduces the number of indepen-

dent parameters that must be considered, thereby simplifying the interpretation of the results; and (b) for important special cases, it leads to simple, closed-form expressions for the desired quantities. Additionally, the results are generally of good accuracy.

For the circular foundations examined herein, it is convenient to express  $\vec{r}_1$  and  $\vec{r}_2$  in Eqs. 1 and 3 in terms of polar coordinates. On substituting Eq. 1 into Eqs. 4, and making use of the appropriate coordinate transformation, one obtains

$$\frac{S_x}{S_g} = \frac{1}{\pi^2} \int_0^1 \int_0^1 \int_0^{2\pi} \int_0^{2\pi} \epsilon_1 \epsilon_2 \exp(-b_0^2 \Delta_1) \cos(c_0 \Delta_2) d\theta_1 d\theta_2 d\epsilon_1 d\epsilon_2 \quad (5a)$$

$$\frac{S_y}{S_g} = \frac{4}{\pi^2} \int_0^1 \int_0^1 \int_0^{2\pi} \int_0^{2\pi} \epsilon_1^2 \epsilon_2^2 \exp(-b_0^2 \Delta_1) \cos(c_0 \Delta_2) \cos \theta_1 \cos \theta_2 d\theta_1 d\theta_2 d\epsilon_1 d\epsilon_2 \quad (5b)$$

$$\frac{S_{xy}}{S_g} = -\frac{2}{\pi} i \int_0^1 \int_0^1 \int_0^{2\pi} \int_0^{2\pi} \epsilon_1 \epsilon_2^2 \exp(-b_0^2 \Delta_1) \sin(c_0 \Delta_2) \cos \theta_2 d\theta_1 d\theta_2 d\epsilon_1 d\epsilon_2 \quad (5c)$$

in which

$$\Delta_1 = \epsilon_1^2 + \epsilon_2^2 - 2\epsilon_1 \epsilon_2 \cos(\theta_1 - \theta_2) \quad (6a)$$

$$\Delta_2 = \epsilon_1 \cos \theta_1 - \epsilon_2 \cos \theta_2 \quad (6b)$$

$\epsilon_1$  and  $\epsilon_2$  are the radial distances of the two points normalized with respect to the radius,  $R$ ;  $\theta_1$  and  $\theta_2$  are the corresponding angular coordinates measured from the direction of wave propagation, as shown in Fig. 2-1(b); and  $b_0$  and  $c_0$  are dimensionless parameters related to the well known frequency parameter,  $a_0 = \omega R/v_s$ , as follows:

$$b_0 = \gamma a_0 \quad (7)$$

$$\text{and } c_0 = (v_s/c) a_0 = (\sin \alpha_v) a_0 \quad (8)$$

In the exact formulation of the problems presented in Refs. 14 and 15, the quantities  $\gamma$ ,  $a_0$  and  $v_s/c$  appear independently.

**3.1.1 Integration of Equations.** For vertically incident incoherent waves,  $c_0 = 0$  and the interrelationship of the free-field ground motion and the foundation input motion is defined by the single parameter  $b_0$ . Equations 5 in this case can be integrated exactly to yield

$$S_x = \frac{1}{b_0^2} \left\{ 1 - \exp(-2b_0^2) [I_0(2b_0^2) + I_1(2b_0^2)] \right\} S_g \quad (9a)$$

$$S_y = \frac{1}{b_0^2} \left\{ 1 - \exp(-2b_0^2) [I_0(2b_0^2) + 2I_1(2b_0^2) + I_2(2b_0^2)] \right\} S_g \quad (9b)$$

$$S_{xy} = 0 \quad (9c)$$

in which  $I_0$ ,  $I_1$  and  $I_2$  are modified Bessel functions of the first kind of the order indicated by the subscript. Eq. 9c indicates that the horizontal and torsional components of the foundation input motion are statistically uncorrelated. The derivation of these expressions is given in Appendix A.

For obliquely incident coherent waves, for which  $\gamma = b_0 = 0$ , the interrelationship of the two motions is defined completely by  $c_0$ , and Eqs. 5 can again be integrated exactly to yield

$$S_x = \left[ 2 \frac{J_1(c_0)}{c_0} \right]^2 S_g \quad (10a)$$

$$S_y = \left[ 4 \frac{J_2(c_0)}{c_0} \right]^2 S_g \quad (10b)$$

$$S_{xy} = i \left[ 8 \frac{J_1(c_0)J_2(c_0)}{c_0^2} \right] S_g \quad (10c)$$

in which  $J_1$  and  $J_2$  are Bessel functions of the first kind of order one and two, respectively. The latter expressions have been presented previously in Ref. 22. Note that  $S_{xy}$  is purely imaginary, indicating that there is a 90° phase angle in this case between the horizontal and torsional components of foundation input motion.

For the more general case involving combinations of wave passage and incoherence effects, formal integration of Eqs. 5 has not proved possible, and the relevant expressions were integrated numerically.

**3.1.2 Presentation of Results.** The quantities  $\sqrt{S_x/S_g}$  and  $\sqrt{S_y/S_g}$  define the transfer functions for the amplitudes of the horizontal and rotational components of the foundation input motion, and the magnitude of  $S_{xy}/\sqrt{S_x S_y}$  define the degree of correlation or coherence of the components of the motion. A numerical value of unity for the latter quantity indicates that the component motions are fully correlated, while a zero value indicates that they are uncorrelated. These quantities are plotted conveniently in Figs. 3-1 and 3-2 as functions of the modified frequency parameter,

$$\bar{a}_0 = \sqrt{b_0^2 + c_0^2} = \sqrt{\gamma^2 + \sin^2 \alpha_v} a_0 \quad (11)$$

and the modified incoherence parameter,

$$\bar{\gamma} = b_0/c_0 = \gamma/\sin \alpha_v \quad (12)$$

For incoherence effects only,  $\alpha_v = 0$ ,  $\bar{\gamma} = \infty$  and  $\bar{a}_0$  reduces to  $\gamma a_0 = b_0$ . Similarly, for wave passage effects only,  $\gamma = \bar{\gamma} = 0$  and  $\bar{a}_0$  reduces to  $a_0 \sin \alpha_v = c_0$ .

Note that whereas the transfer function for the lateral component of the foundation input motion,  $\sqrt{S_x/S_g}$ , decreases monotonically in Fig. 3-1 with increasing  $\bar{a}_0$ , the corresponding function for the torsional component,  $\sqrt{S_y/S_g}$ , increases from zero to a peak and then decreases monotonically.

**3.1.3 Accuracy of Solutions.** As a measure of the accuracy of the reported data, the results computed for incoherence effects only and for wave passage effects only are compared in Fig. 3-3 with the corresponding exact solutions of Luco and Mita [14,15]. Since the factors  $\gamma$  and  $\sin \alpha_v = v_s/c$  appear independently in the exact solutions, several different values are considered for these parameters. No comparisons are made for combinations of incoherence and wave passage as the exact solutions are not available in this case.

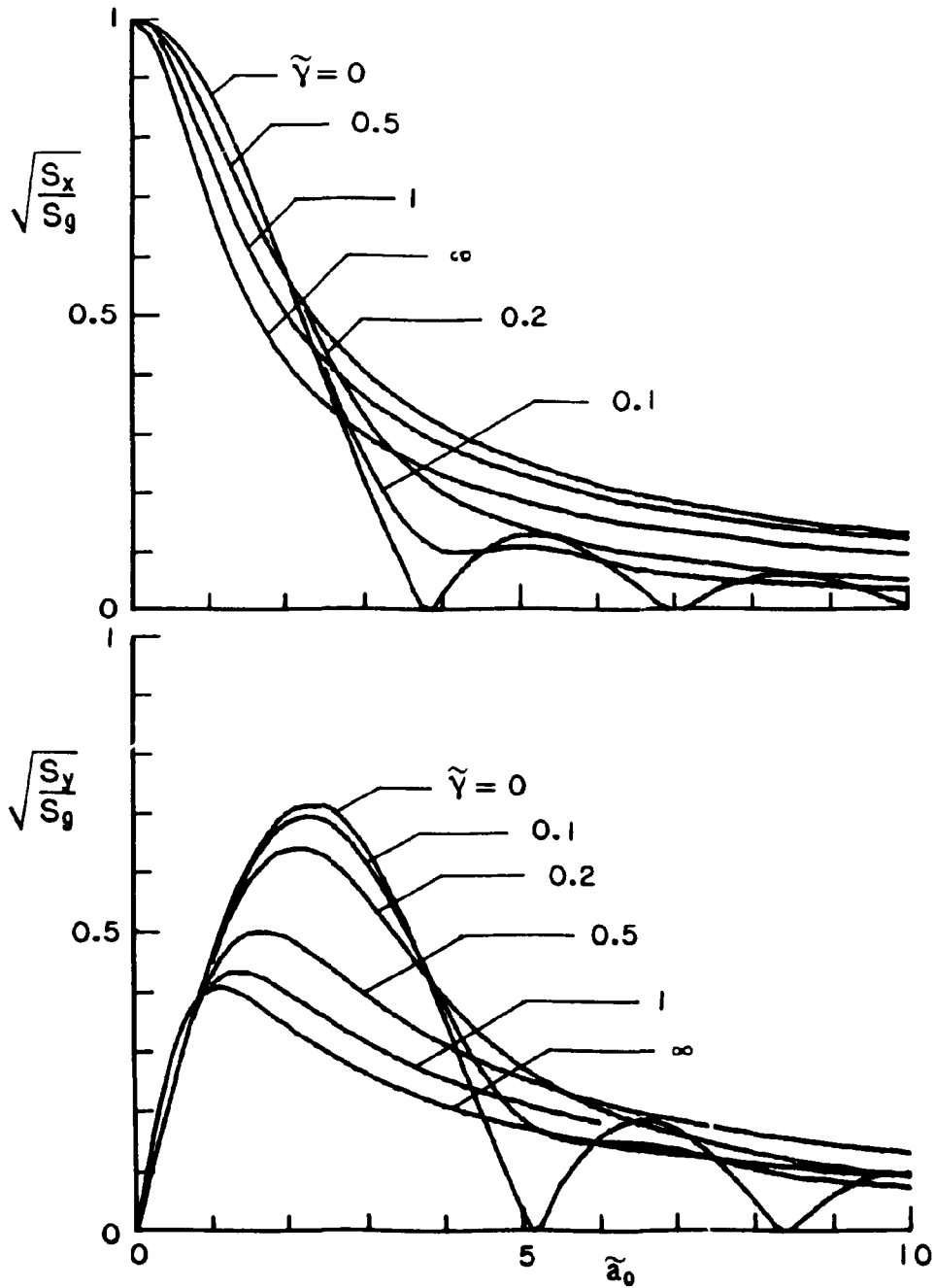


FIG. 3-1 Magnitudes of Transfer Functions between Free-Field Ground Motion and Foundation Input Motions

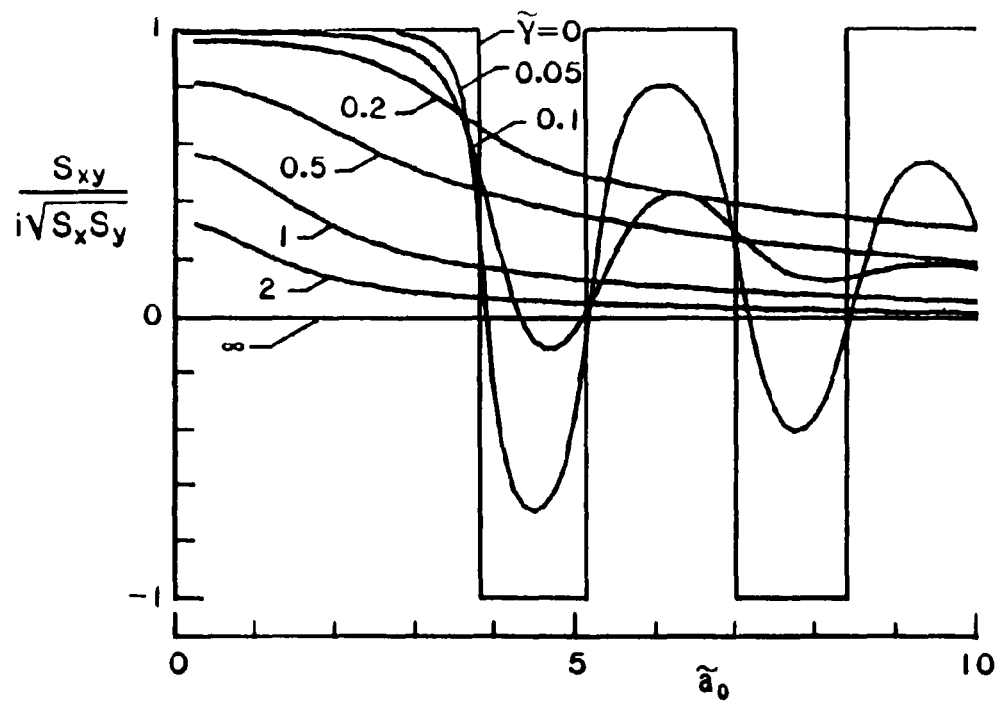


FIG. 3-2 Normalized Cross PSD Function for Horizontal and Torsional Components of Foundation Input Motion

Considering the uncertainties that are inherent in the definition of the incoherence function and in the choice of the parameter  $\gamma$ , the degree of agreement in the two sets of results displayed in Fig. 3-3 is deemed to be quite satisfactory. Note should also be taken of the fact that, excepting the narrow frequency ranges where the curves for wave passage only exhibit notch-like trends, the approximate solutions overestimate the amplitudes of foundation input motions.

**3.1.4 Other Meanings for Results.** Although defined specifically for the displacement histories of the foundation input motion, the spectral density ratios  $S_x/S_g$ ,  $S_y/S_g$  and  $S_{xy}/S_g$  also define the ratios  $S_{\dot{x}}/S_{\dot{g}}$ ,  $S_{\dot{y}}/S_{\dot{g}}$ ,  $S_{\ddot{x}y}/S_{\ddot{g}}$  and  $S_{\ddot{x}}/S_{\ddot{g}}$ ,  $S_{\ddot{y}}/S_{\ddot{g}}$ ,  $S_{\ddot{x}\ddot{y}}/S_{\ddot{g}}$  of the corresponding velocity and acceleration histories. Recall that the psd function for the first derivative of a process is given by the product of  $(2\pi f)^2$  and the psd function of the original process.

### 3.2 Spectral Characterization of Structural Response

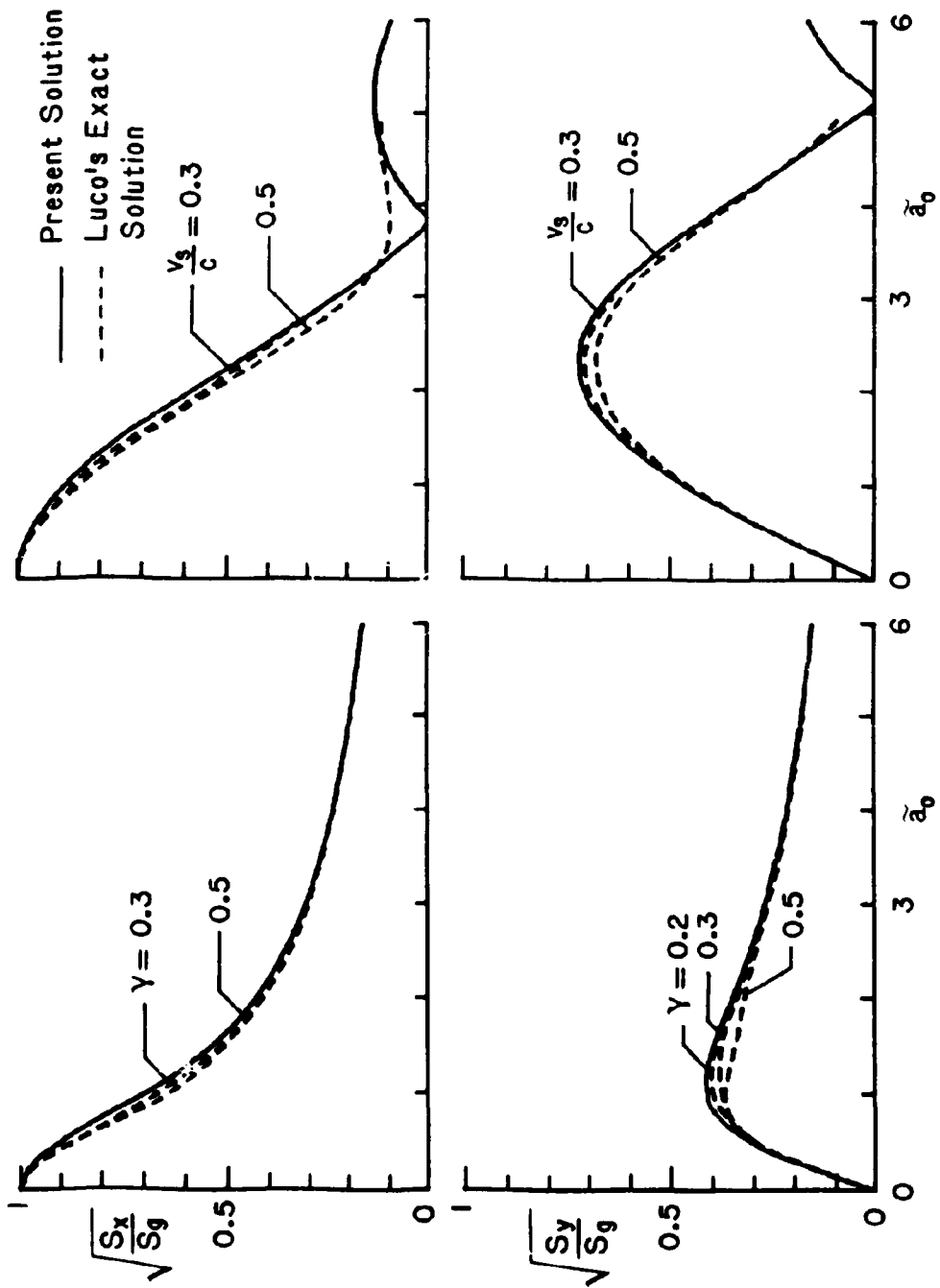
With the psd functions of the foundation input motion established, the corresponding functions of the structural response can be obtained by well-established procedures (e.g., Ref. 12). Let  $S_u$  be the psd function of the structural deformation,  $u$ , induced by the lateral component of the foundation input motion; and let  $S_v$  be the corresponding function of the deformation,  $v = \psi R$ , induced along the perimeter of the structure by the torsional component of response. The quantity  $\psi$  represents the angular deformation of the structure. These functions are related to the psd functions of the foundation input accelerations,  $S_{\ddot{x}}$  and  $S_{\ddot{y}}$ , by

$$S_u = |H_u|^2 S_{\ddot{x}} \quad (13)$$

$$\text{and } S_v = |H_v|^2 S_{\ddot{y}} \quad (14)$$

in which  $H_u$  = the transfer function for lateral response, given by

$$H_u = -\frac{1}{p_x^2} \frac{1}{1 - (\omega/p_x)^2 + i2\zeta_x(\omega/p_x)} \quad (15)$$



(a) Incoherence Only,  $\frac{v_s}{c} = 0$  (b) Wave Passage Only,  $\gamma = 0$

FIG. 3-3 Comparison of Approximate and Exact Magnitudes of Foundation Transfer Functions



$H_v$  = the corresponding function for torsional response, obtained from Eq. 15 by replacing  $p_x$  by  $p_\theta$  and  $\zeta_x$  by  $\zeta_\theta$ ; and vertical bars indicate the modulus of the enclosed quantity. Similarly, the psd function  $S_w$  for the total deformation at the most highly stressed point on the periphery of the structure,  $w = u + v$ , is given by [12]

$$S_w = S_u + S_v + 2 | \operatorname{Re}(H_u H_v^* S_{xy}^{\dots}) | \quad (16)$$

in which  $S_{xy}^{\dots}$  = the cross psd function of the lateral and circumferential components of the foundation input accelerations; Re denotes the real part of the indicated quantity; and a star superscript denotes the complex conjugate of the quantity to which it is attached.

### 3.3 Characterization of Free-Field Earthquake Ground Motions

The local psd function for the set of acceleration traces considered in the remainder of this paper is taken in the form

$$S_{\ddot{g}} = \begin{cases} \frac{f^4}{0.5 + f^4} \left(1 - \frac{f^2}{f_0^2}\right) S_0 & \text{for } f \leq f_0 \\ 0 & \text{for } f \geq f_0 \end{cases} \quad (17)$$

in which  $S_0$  = a constant;  $f = \omega/2\pi$  = the exciting frequency in cps; and  $f_0$  = the cut-off frequency, taken as 15 cps. Of the same general form as that employed in a related study by Pais and Kausel [22], the function  $S_{\ddot{g}}$ , along with the associated functions for ground velocity and ground displacement, are plotted in Fig. 3-4, with all peaks normalized to a unit value. As would be expected, the psd function for velocity decays much more rapidly with frequency than that for acceleration, and the corresponding displacement function decays even faster.

Let  $\ddot{\bar{x}}_g$  be the mean of the absolute maximum peaks of the acceleration traces, and  $\dot{\bar{x}}_g$  and  $\bar{x}_g$  be the corresponding means of the velocity and displacement traces. These values were computed from Der Kiureghian's empirical expressions [5] summarized in Appendix B, considering the duration of the intense portion of the excitation to be  $t_0 = 20$  sec. The

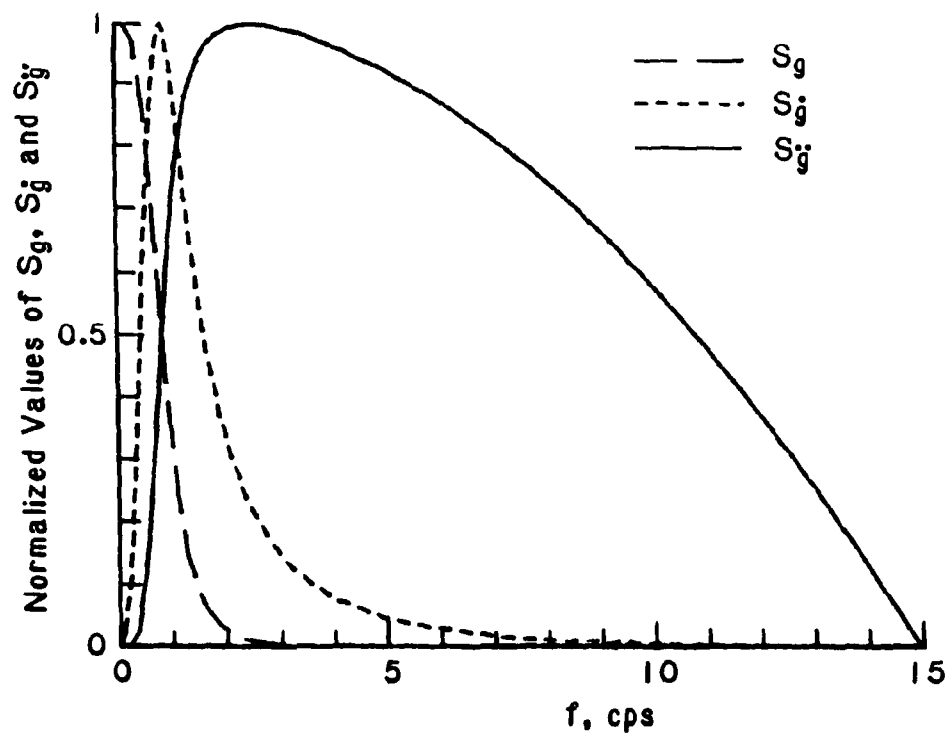


FIG. 3-4 Normalized PSD Functions for Free-Field Ground Motions Considered

resulting values are  $\ddot{X}_g = 26.17 \sqrt{S_0}$ ,  $\dot{X}_g = 1.417 \sqrt{S_0}$  and  $X_g = 0.2468 \sqrt{S_0}$ .

### 3.4 Foundation Input Motion

Before examining the response of the structure, it is desirable to compute the mean peak values of the acceleration, velocity and displacement traces of the horizontal and circumferential components of the foundation input motion. The relevant values for the horizontal component of motion are denoted by  $\ddot{X}$ ,  $\dot{X}$  and  $X$ , and those for the circumferential component along the periphery of the foundation are denoted by  $\ddot{Y}$ ,  $\dot{Y}$  and  $Y$ . Computed by Der Kiureghian's approximation from the appropriate psd functions, these values are plotted in Fig. 3-5 normalized with respect to the mean peak values of the corresponding histories of the free-field ground motion.

For the multifrequency, transient excitation considered in this section, the solution is controlled by the effective transit time,

$$\bar{\tau} = \sqrt{\gamma^2 + \sin^2 \alpha_v} \tau \quad (18)$$

in which  $\tau = R/v_s$  = the time required for the shear wave to traverse the radius of the foundation; and by the modified incoherence parameter,  $\bar{\gamma}$ , defined by Eq. 12.

The following observations may be made and inferences drawn from the data presented in Fig. 3-5:

1. The reduction in the horizontal component of the foundation input motion and the corresponding increase in the rotational component are greatest for acceleration, much smaller for velocity, and almost negligible for displacement. Since the foundation filters the high-frequency wave components more effectively than the low-frequency wave components, the acceleration traces of the ground motion, which are richer in high-frequency content than the velocity and displacement traces, are influenced more than the latter traces.

2. Considering that the response of high-frequency systems is acceleration-sensitive whereas that of low-frequency systems is

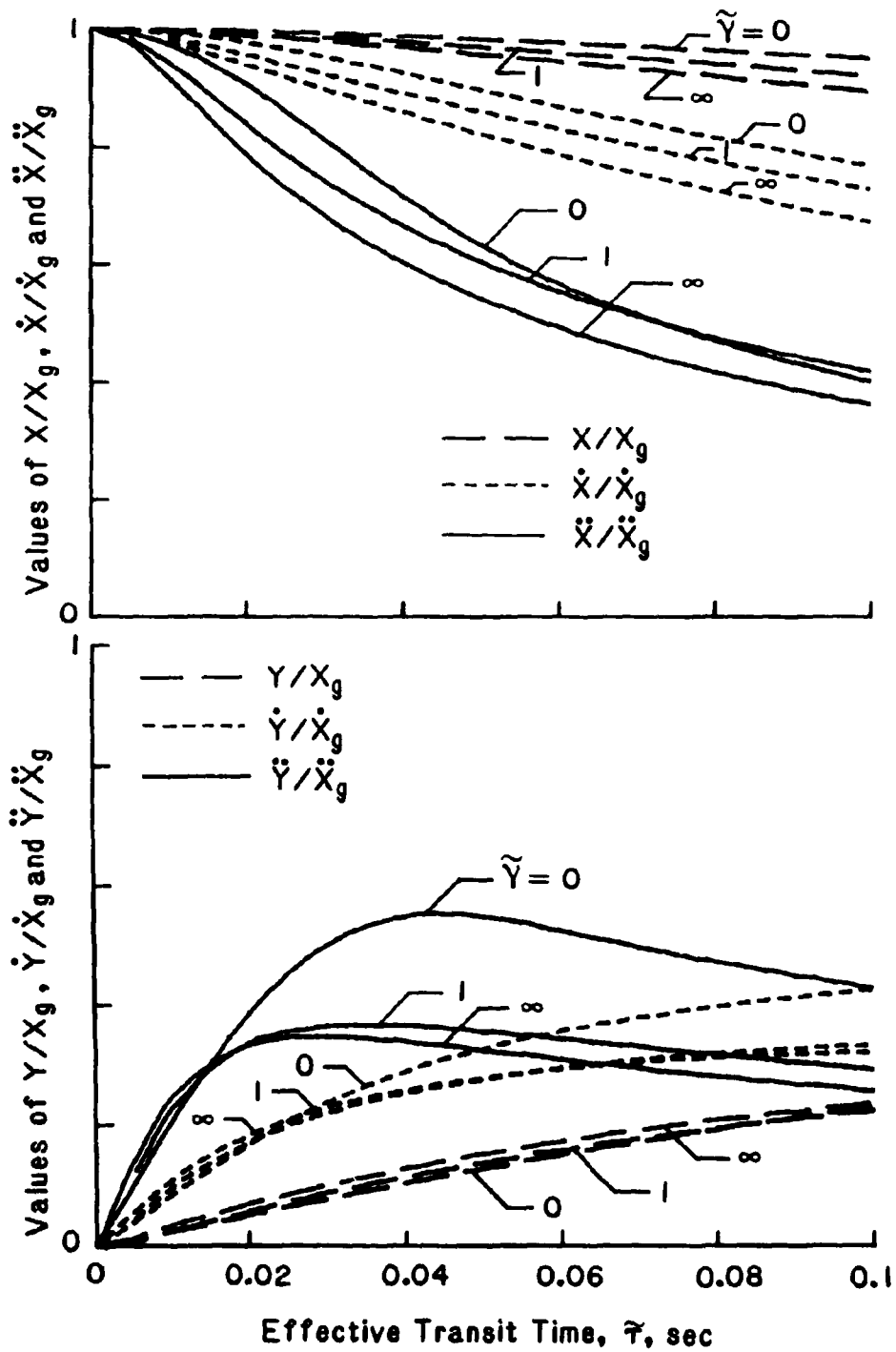


FIG. 3-5 Normalized Mean Peak Values of Lateral and Torsional Components of Foundation Input Accelerations, Velocities and Displacements

displacement-sensitive, it should be clear that the effects of kinematic interaction would be important for high-frequency systems and inconsequential for low-frequency systems. Furthermore, medium-frequency systems which are velocity-sensitive would be expected to be affected moderately. That this is indeed the case is confirmed by the data presented in the following sections.

### 3.5 Effects of Ground-Motion Incoherence on Structural Response

Let  $U_x$  = the mean of the maximum values of the structural deformations induced by the ensemble of lateral components of the foundation input motions, and  $U_y$  = the corresponding mean of the deformations induced at the periphery of the deck by the torsional components. These quantities have been evaluated for vertically propagating incoherent shear waves ( $\tilde{\gamma} = \infty$ ), and the results are displayed in Fig. 3-6 in the form of tripartite response spectra. The solid curves in the upper part of the figure refer to lateral response, and the lower curves refer to torsional response. Several values of the effective transit time parameter,  $\tilde{\tau}$ , are considered, including the limiting value of  $\tilde{\tau} = 0$  for which there is no kinematic interaction. The damping factors for both modes of response are taken as  $\zeta_x = \zeta_\theta = 0.02$ .

The left-hand diagonal scale in the upper part of Fig. 3-6 represents  $U_x$  normalized with respect to the mean peak value of the free-field displacement,  $X_g$ ; the vertical scale represents the corresponding pseudo-velocity,  $V_x = p_x U_x$ , normalized with respect to  $\dot{X}_g$ ; and the right-hand diagonal scale represents the corresponding pseudo-acceleration,  $A_x = p_x V_x$ , normalized with respect to  $\ddot{X}_g$ . In an analogous manner, the three scales in the lower part of the figure represent the deformation ratio,  $U_y/X_g$ ; the pseudo-velocity ratio,  $V_y/\dot{X}_g$ , in which  $V_y = p_\theta U_y$ ; and the pseudo-acceleration ratio,  $A_y/\ddot{X}_g$ , in which  $A_y = p_\theta V_y = p_\theta^2 U_y$ .

As anticipated from examination of the peak values of the foundation motions, the lateral component of the response of high-frequency systems in Fig. 3-6 is affected materially by ground incoherence, and this effect is particularly large in the practically important region of the response

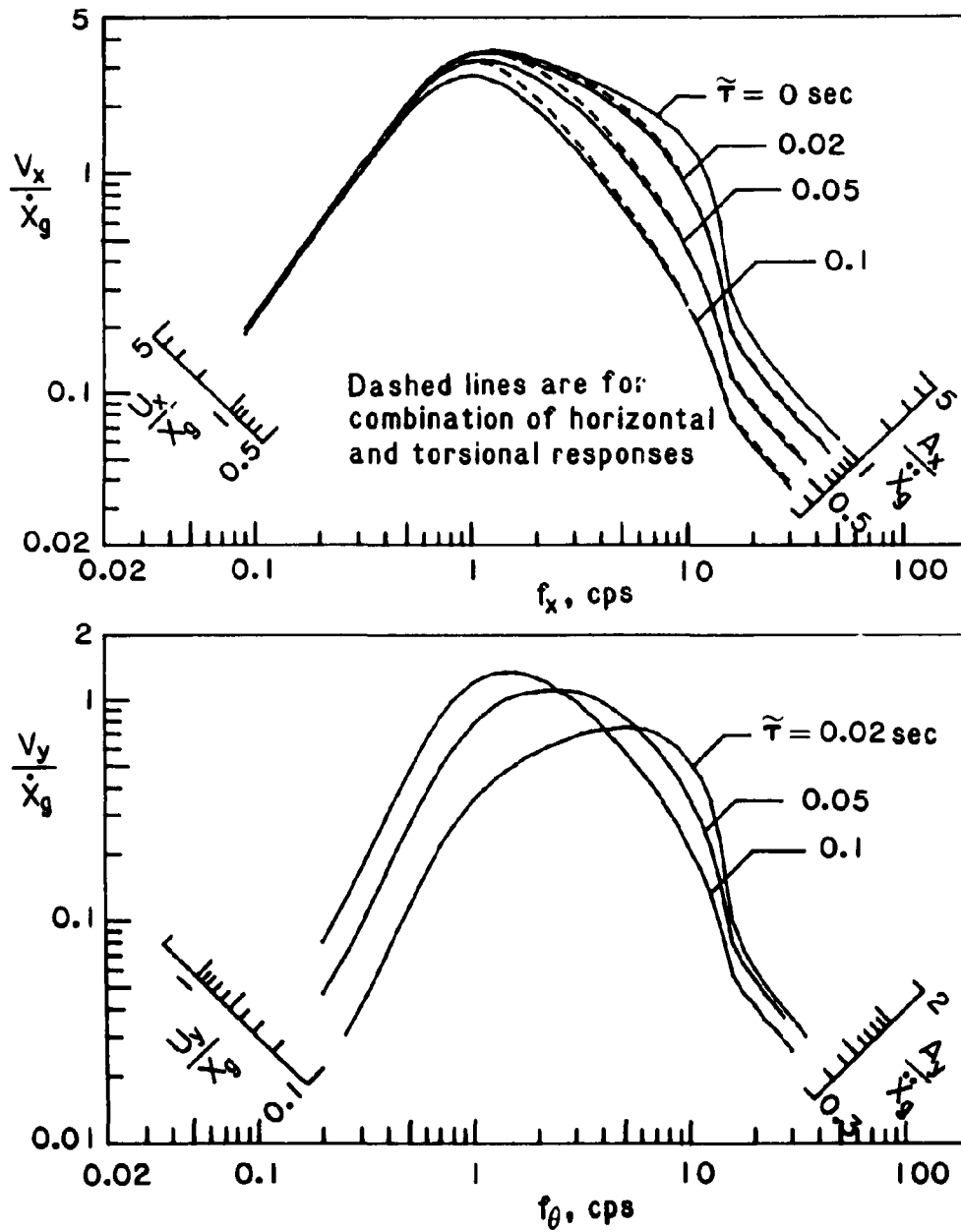


FIG. 3-6 Effects of Ground Motion Incoherence on Maximum Deformations of Structures with  $\tau_x = \tau_\theta = 0.02$  for Vertically Propagating Waves

spectrum within which the pseudo-acceleration attains its maximum value. For  $\gamma = 0.2$  and  $\bar{\tau} = 0.02$  sec. (a value corresponding to, say,  $R = 100$  ft. and  $v_s = 1000$  ft/sec), the maximum value of  $A_x$  is 78 percent of that obtained for a fully coherent, uniform free-field ground motion; for  $\bar{\tau} = 0.05$  sec., the corresponding ratio is 55 percent. The reductions are significantly less pronounced for medium-frequency systems and practically negligible for low-frequency systems. For systems of very high-frequency, for which  $A_x$  may be considered to be equal to the mean peak value of the foundation input acceleration, the percentage reductions are, of course, identical to those indicated in Fig. 3-5 for the foundation input acceleration.

The general trends of the response spectra for the torsional deformation in Fig. 3-6 are consistent with those of the corresponding curves for the foundation input motion presented in Fig. 3-5. Specifically, in the low-frequency, displacement-sensitive region, the response increases with increasing values of the effective transit time,  $\bar{\tau}$ , whereas in the high-frequency, acceleration-sensitive region, the response values for  $\bar{\tau} = 0.02$  sec. are higher than those for the higher values of  $\bar{\tau}$  considered. Furthermore, the percentage changes in response are comparable to those for the controlling values of the foundation input motion.

The component of the response contributed by the rotation of the foundation is generally small, and the combined effect of lateral and torsional responses is generally only slightly greater than that due solely to lateral response. The mean maximum values of the total deformation for the most highly stressed column along the periphery of the structure were evaluated considering  $p_\theta/p_x = 1.5$ , and the results are shown by the dashed lines in Fig. 3-6. These results were computed by Der Kieureghian's approximation making use of Eq. 16 for the psd function of the combined motion.

**3.5.1 Comparison of Incoherence and Wave Passage Effects.** Some of the response spectra for the incoherent ground motions presented in Fig. 3-6 are compared in Fig. 3-7 with those computed considering only wave passage effects, and combinations of wave passage and incoherence

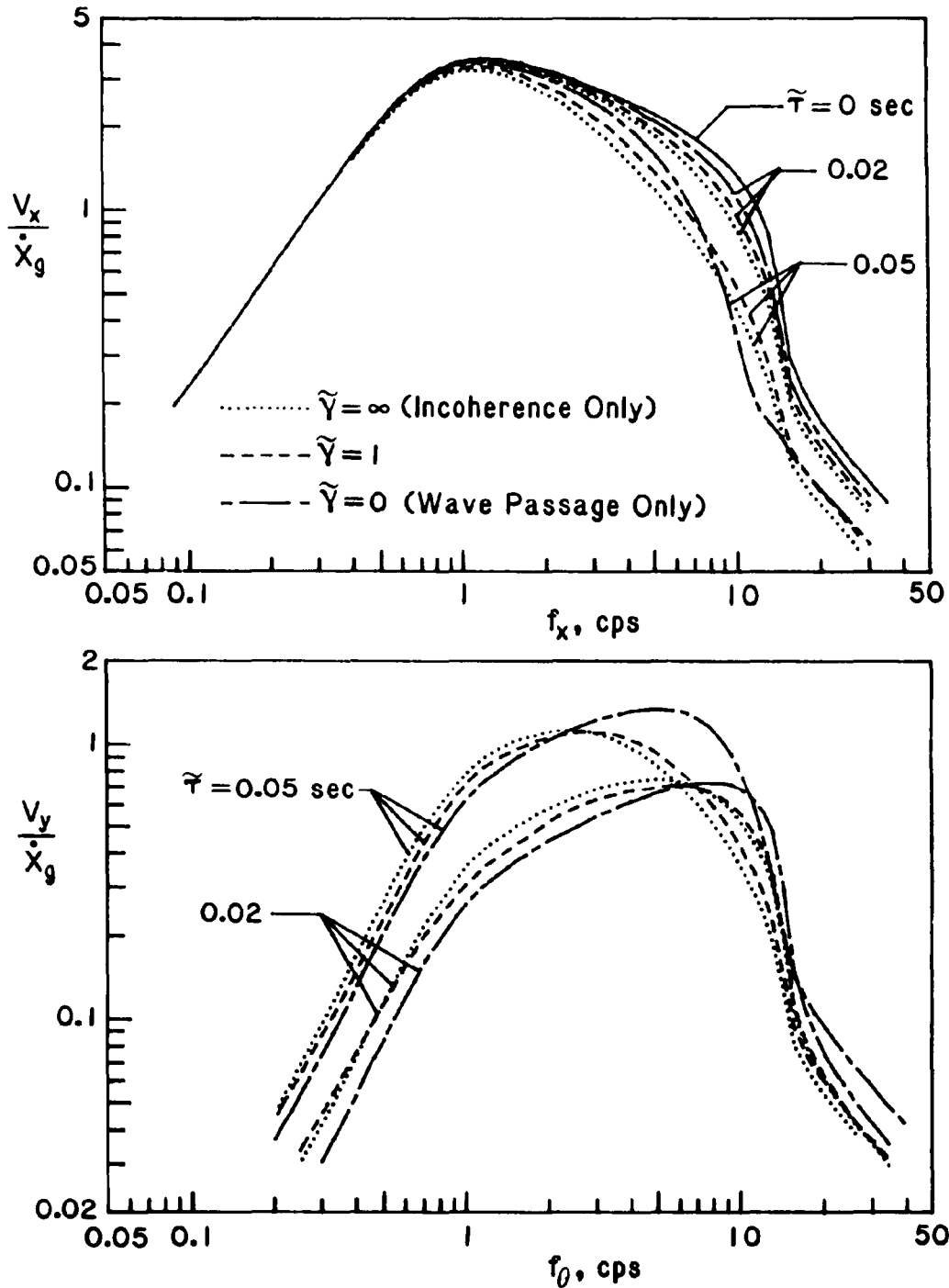


FIG. 3-7 Effects of Ground Motion Incoherence and Wave Passage on Maximum Deformations of Structures with  $\zeta_x = \zeta_\theta = 0.02$



represented by a value of  $\bar{\gamma} = 1$ . It should be clear that the results are not particularly sensitive to the choice of the parameter  $\bar{\gamma}$ , and that this insensitivity is fully compatible with that observed in Fig. 3-5 for the peak values of the foundation input motions. Indeed, the ratio of the low-frequency limiting values of  $U_x$  for  $\bar{\gamma} = 0$  and  $\bar{\gamma} = \infty$  in Fig. 3-7 is almost identical to that of the peak values of the lateral component of the foundation input displacements in Fig. 3-5, and the ratio of the corresponding values of  $U_y$  is almost identical to the displacement ratio of the torsional component of the foundation input motion. Similarly, the ratios of the high-frequency limits of  $A_x$  and  $A_y$  in Fig. 3-7 are identical to those obtained from Fig. 3-5 for the mean peak values of the lateral and torsional components of the foundation input accelerations. It follows that, to the degree of approximation represented by the differences in the results displayed in Fig. 3-7, the effects of ground motion incoherence may be replaced by those of wave passage and vice versa. This possibility has also been suggested by Luco and Wong [16] from examination of the relevant foundation transfer functions. In implementing this replacement, it is important that the value of  $\bar{\tau}$  be the same in the two cases.

## SECTION 4 INERTIAL INTERACTION EFFECTS

The inertial interaction effects are now evaluated by a simple modification of the procedure used in previous studies in which the effects of kinematic interaction were neglected (e.g., Refs. 26, 28). For each mode of excitation, it is only necessary to replace the free-field motion by the appropriate component of the foundation input motion.

The following steps are involved in the analysis: First, the harmonic response of the system is evaluated making use of the appropriate complex-valued foundation impedance functions. Next, the psd functions of the torsional and lateral components of structural response are determined. The desired mean peak values of the responses are finally computed from Der Kiureghian's approximation. Additional details are given in Appendix C.

The foundation impedances for the torsional mode of vibration were computed from the approximate closed-form expressions of Veletsos and Nair [29], and those for the horizontal and rocking motions were computed from the corresponding expressions of Veletsos and Verbic [30]. The cross coupling terms between horizontal and rocking motions were presumed to be negligible.

The principal parameters that influence the response of the system are the characteristics of the free-field ground motion; the fixed-base natural frequencies of the structure,  $f_x$  and  $f_\theta$ , and the associated damping factors,  $\zeta_x$  and  $\zeta_\theta$ ; the height to base radius ratio,  $h/R$ ; the mass density ratio for the structure, defined conveniently as  $\delta = m/(\pi\rho R^2h)$ , in which the denominator represents the total mass of the structure when filled with the supporting soil; and the wave transit times,  $\tau$  and  $\bar{\tau}$ . It is important to note that whereas the kinematic interaction effects are defined completely by  $\bar{\tau}$ , the evaluation of the inertial interaction effects requires the separate specification of the parameters  $\gamma$  and  $\tau$ . Other parameters affecting the response of the system are Poisson's ratio for the supporting medium,  $\nu$ ; the mass ratio of the foundation and super-

structure,  $m_f/m$ ; the ratio  $I_f/I$  of the mass moments of inertia of the foundation and structure about horizontal centroidal axes; and the ratio  $J_f/J$  of the corresponding polar moments of inertia. For the solutions presented herein,  $\zeta_x = \zeta_\phi = 0.02$ ;  $\delta = 0.15$ ;  $\nu = 1/3$ ; and  $m_f$  (and hence  $I_f$  and  $J_f$ ) are considered to be negligible.

#### 4.1 Results for Vertically Propagating Incoherent Waves

Fig. 4-1 shows response spectra for lateral and torsional response obtained for vertically propagating incoherent waves, taking  $\gamma = 0.4$  and  $\tau = R/v_s = 0.05$  sec. Three sets of solutions are presented: (a) making no provision for soil-structure interaction, i.e., considering the foundation motion to be equal to the free-field ground motion; (b) providing only for the kinematic interaction effects, i.e., using as base excitation the foundation input motions; and (c) providing for both kinematic and inertial interaction effects; i.e., analyzing the structure-foundation-soil system exactly as a coupled system. In the analysis of the inertial interaction effects, two values of  $h/R$  are used: a unit value, corresponding to short stubby structures, and a value of 3, corresponding to taller, more slender structures. Solutions (a) and (b) are independent of  $h/R$ , whereas solutions (c) are valid for all combinations of  $\gamma$  and  $\tau$  for which  $\gamma\tau = \bar{\tau} = 0.02$  sec.

Previous studies of soil-structure interaction involving only inertial interaction effects [3,6,26,27] have shown that these effects may be evaluated to a high degree of approximation using the free-field ground motion as the foundation input motion and merely modifying the relevant natural frequency and damping of the structure. The modified frequency and damping are taken such that, for each mode of vibration, the magnitude and location of the resonant peak of the relevant harmonic response are identical for the actual and replacement systems. For structures for which the kinematic interaction effects are important, this approach would require that the response of the structure be evaluated for the horizontal and torsional components of the foundation input motion rather than for the free-field ground motion.

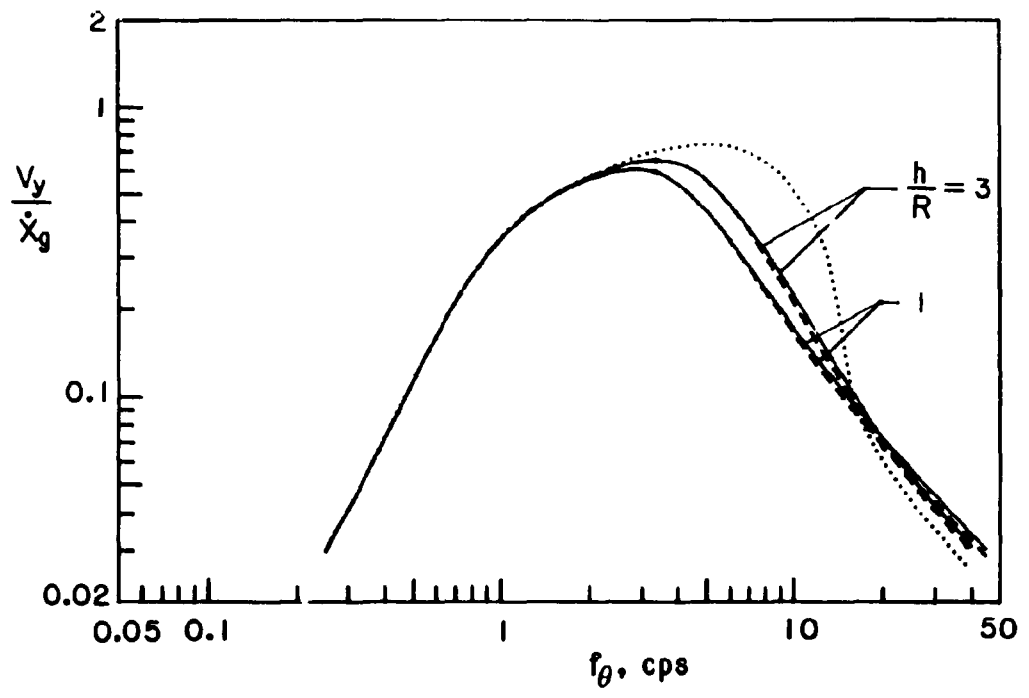
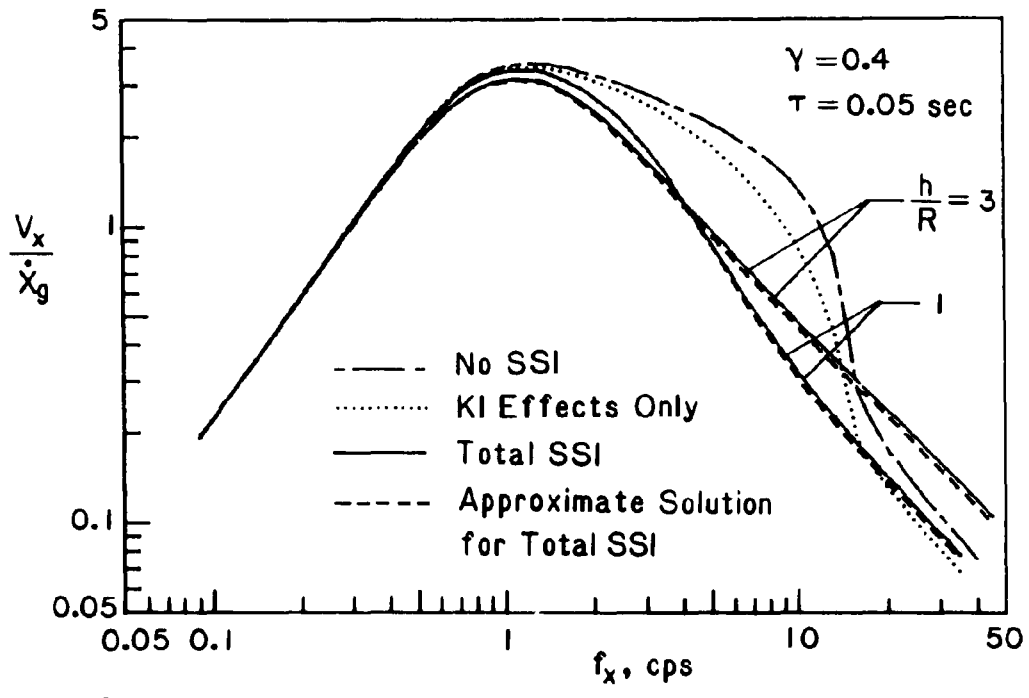


FIG. 4-1 Comparison of Effects of Kinematic and Inertial Interaction on Maximum Deformations of Structures with  $\zeta_x = \zeta_\theta = 0.02$

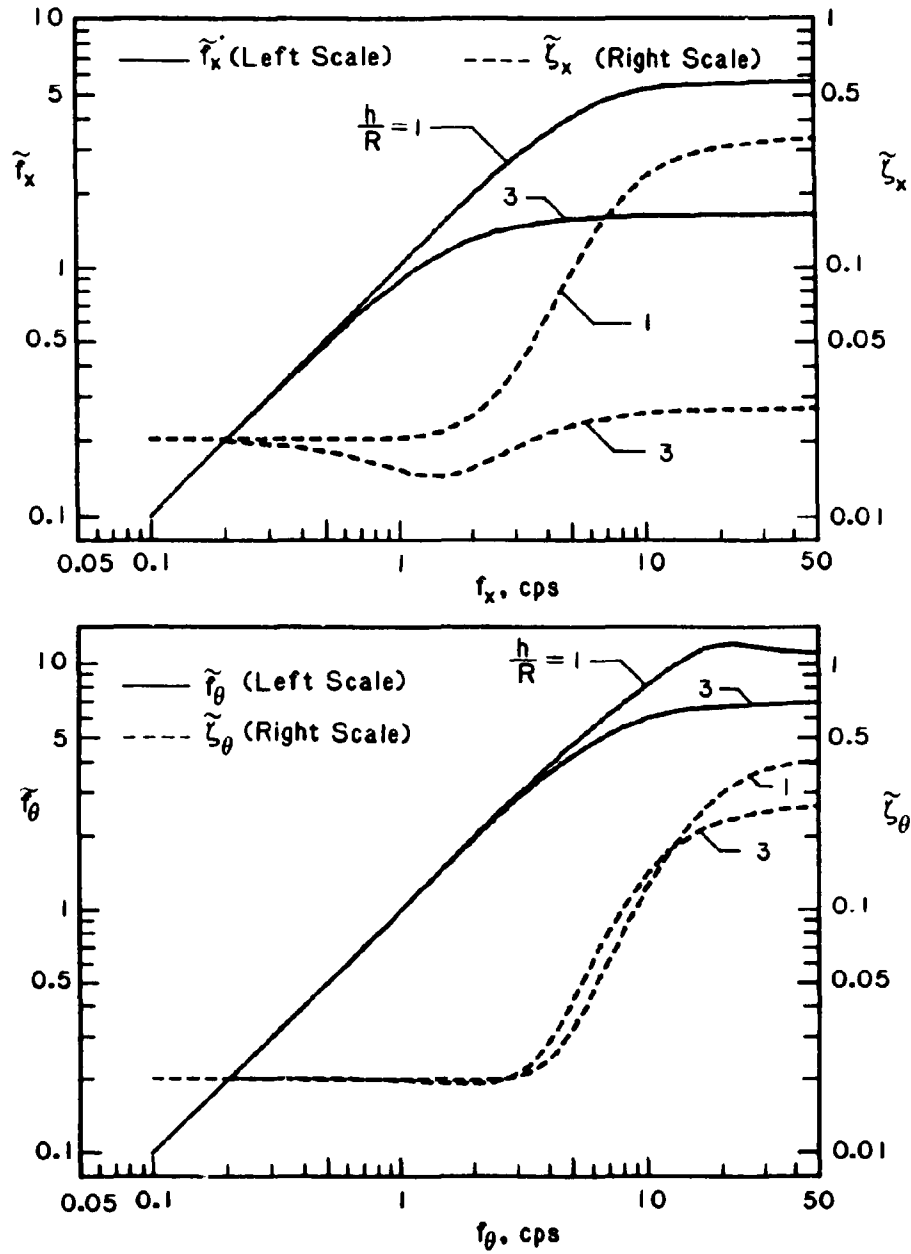


FIG. 4-2 Natural Frequencies and Damping of Modified Systems in Approximate Analysis of Inertial Interaction Effects

The mean maximum values of the responses obtained by this approximate procedure are shown by the dashed lines in Fig. 4-1, and the values of the modified natural frequencies and damping factors are identified in Fig. 4-2. Denoted with a tilde superscript, the modified frequencies are, of course, lower than the corresponding fixed-base frequencies, and the modified damping factors are higher than the value of  $\zeta_x = \zeta_\theta = 0.02$  assumed for the fixed-base structure.

The following trends should be observed in these figures:

1. Like kinematic interaction (KI), inertial interaction (II) may affect significantly the responses of systems in the medium- and high-frequency spectral regions.
2. The II effects are generally more important than the KI effects.
3. Unlike kinematic interaction which generally reduces the lateral response, inertial interaction may increase the corresponding response of tall, slender structures in the high frequency region of the response spectrum. Such structures, however, typically fall in the middle-frequency region of the spectrum, for which the interaction effects are relatively small.
4. The II effects for low-frequency, highly compliant structures are negligible because such systems "see" the halfspace as a very stiff, effectively rigid medium.
5. Provided the base excitation for the structure is taken equal to the foundation input motion rather than the free-field ground motion, the concept of modifying the fixed-base natural frequencies and associated damping values of the system provides a simple and highly reliable practical means for assessing the II effects.

It may be surprising that the values of  $\tilde{f}_\theta$  and  $\tilde{\zeta}_\theta$  in Fig. 4-2 are functions of the ratio  $h/R$ . This is due to the fact that with the value of the mass ratio,  $\delta$ , fixed in these solutions, the polar mass moment of inertia of the system,  $J$ , is different for different values of  $h/R$ .

## SECTION 5 CONCLUSIONS

1. The information and concepts presented herein provide valuable insight into the nature of kinematic and inertial interaction effects for simple structures subjected to earthquakes, and into the effects and relative importance of the numerous parameters involved.
2. In the approximate method of analysis employed, the kinematic interaction effects are defined completely by the effective transit time,  $\bar{\tau}$ , and the modified incoherence parameter,  $\bar{\gamma}$ .
3. Even for vertically propagating waves, kinematic interaction may reduce significantly the critical responses of high-frequency systems. These reductions are generally smaller than, but of approximately the same order of magnitude as, those due to inertial interaction.
4. Reliable estimates of the effects of kinematic interaction on the peak values of structural response may be obtained from knowledge of the corresponding values of the acceleration, velocity and displacement traces of the foundation input motion. The latter quantities may be computed from analyses of the response of the massless foundation to the free-field ground motion.
5. Insofar as the mean maximum values of the responses are concerned, the kinematic interaction effects due to ground motion incoherence are similar to those due to wave passage, and the two effects may be interrelated.
6. An excellent approximation to the inertial interaction effects may be obtained by a previously recommended simple procedure [3,6,26,27] using as base excitation the foundation input motion rather than the free-field motion. The inertial interaction effects in this approach are expressed by changes in the natural frequency of vibration and the associated damping of the structure for the mode of vibration considered.

**SECTION 6**  
**REFERENCES**

1. Abrahamson, N. A., and Bolt, B. A. (1985). "The Spatial Variation of the Phasing of Seismic Strong Ground Motion," Bulletin of the Seismological Society of America, 75(5), 1247-1264.
2. Abramowitz, M., and Stegun, I. A. (1970). Handbook of Mathematical Functions, Dover Publications, Inc., New York, NY.
3. Applied Technology Council (ATC) (1978). Tentative Provisions for the Development of Seismic Regulations for Buildings, ATC-3-06, Palo Alto, California.
4. Bogdanoff, J. L., Goldberg, J. E. and Schiff, A. J. (1965). "The Effect of Ground Transmission Time on the Response of Long Structures," Bulletin of Seismological Society of America, 55, 627-640.
5. Der Kiureghian, A. (1980). "Structural Response to Stationary Excitation," Journal of Engineering Mechanics Division, ASCE, 106(6), 1195-1213.
6. Federal Emergency Management Agency (FEMA) (1986). NEHRP Recommended Provisions for the Development of Seismic Regulations for New Buildings, Building Seismic Safety Council, 1015 15th Street, N.W., Suite 700, Washington, D.C. 20005.
7. Gradshteyn, I. S., and Ryzhik, I. M. (1980). Table of Integrals, Series and Products, Academic Press, Inc., Orlando, Florida.
8. Harichandran, R. S. and Vanmarcke, E. H. (1986). "Stochastic Variation of Earthquake Ground Motion in Space and Time," Journal of Engineering Mechanics Division, ASCE, 112(2), 154-174.
9. Hoshiya, M., and Ishii, K. (1983). "Evaluation of Kinematic Interaction of Soil Foundation Systems by a Stochastic Model," Journal of Soil Dynamics and Earthquake Engineering, 2(3), 128-134.
10. Iguchi, M. (1984). "Earthquake Response of Embedded Cylindrical Foundation to SH and SV Waves," Proceedings of the Eighth World Conference on Earthquake Engineering, San Francisco, 1081-1088.
11. Kausel, E., and Pais, A. (1987). "Stochastic Deconvolution of Earthquake Motions," Journal of Engineering Mechanics Division, ASCE, 113(2), 266-277.
12. Lin, Y. K. (1976). Probabilistic Theory of Structural Dynamics, Robert E. Krieger Publishing Co., Huntington, NY, 155-202.
13. Loh, C-H. (1985). "Analysis of the Spatial Variation of Seismic Waves and Ground Movements from Smart-1 Array Data," Journal of Earthquake Engineering and Structural Dynamics, 13, 561-581.



14. Luco, J. E., and Mita, A. (1987). "Response of a Circular Foundation on a Uniform Half-Space to Elastic Waves," Journal of Earthquake Engineering and Structural Dynamics, 15, 105-118.
15. Luco, J. E., and Mita, A. (1987). "Response of a Circular Foundation to Spatially Random Ground Motion," Journal of Engineering Mechanics Division, ASCE, 113(1), 1-15.
16. Luco, J. E., and Wong, H. L. (1986). "Response of a Rigid Foundation to a Spatially Random Ground Motion," Journal of Earthquake Engineering and Structural Dynamics, 14, 891-908.
17. Matsushima, Y. (1977). "Stochastic Response of Structure Due to Spatial Variant Earthquake Excitations," Sixth World Conference on Earthquake Engineering, New Delhi, 1077-1082.
18. Mita, A., and Luco, J. E. (1987). "Response of Structures to Spatially Random Ground Motion," Proceedings of Third U.S. National Conference on Earthquake Engineering, Charleston, S.C., 907-918.
19. Morgan, J. R., Hall, W. J., and Newmark, N. M. (1983). "Seismic Response Arising from Traveling Waves," Journal of Structural Engineering, ASCE, 109(4), 1010-1027.
20. Newmark, N. (1969). "Torsion of Symmetrical Buildings," Proceedings of 4th World Conference on Earthquake Engineering, Santiago, Chile, A-3, 19-32.
21. Novak, M., and Suen, E. (1987). "Dam-Foundation Interaction Under Spatially Correlated Random Ground Motion," Proceedings of 3rd International Conference on Soil Dynamics and Earthquake Engineering, Princeton University, Princeton, NJ.
22. Pais, A., and Kausel, E. (1985). "Stochastic Response of Foundations," Report No. R85-6, MIT Department of Civil Engineering, Cambridge, Massachusetts.
23. Roesset, J. M. (1980). A Review of Soil-Structure Interaction, Lawrence Livermore National Laboratory, Livermore, Calif., UCRL-15262.
24. Scanlan, R. H. (1976). "Seismic Wave Effects on Soil-Structure Interaction," Earthquake Engineering and Structural Dynamics, 4, 379-388.
25. Vanmarcke, E. H. (1975). "On Distribution of the First-Passage Time for Normal Stationary Random Processes," Journal of Applied Mechanics, 42, 215-220.
26. Veletsos, A. S. (1977). "Dynamics of Structure-Foundation Systems," Structural and Geotechnical Mechanics, W. J. Hall, editor, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 333-361.

27. Veletsos, A. S. (1978). "Soil-Structure Interaction for Buildings During Earthquakes," Proceedings of Second International Conference on Microzonation, San Francisco, California, 1, 111-133.
28. Veletsos, A. S., and Meek, J. W. (1976). "Dynamic Behavior of Building-Foundation Systems," Journal of Earthquake and Structural Dynamics, 3, 121-138.
29. Veletsos, A. S., and Nair, V. V. D. (1974). "Torsional Vibration of Viscoelastic Foundations," Journal of Geotechnical Engineering Division, 100(3), 225-246.
30. Veletsos, A. S., and Verbic, B. (1973). "Vibration of Viscoelastic Foundations," Journal of Earthquake Engineering and Structural Dynamics, 2, 87-102.
31. Veletsos, A. S., and Wei, Y. T. (1971). "Lateral and Rocking Vibration of Footings," Journal of Soil Mechanics and Foundation Division, ASCE, 97(9), 1227-1248.
32. Veletsos, A. S., Erdik, M. O., and Kuo, P. T. (1976). "Structural Response to Traveling Seismic Motions," Proceedings of 5th European Conference on Earthquake Engineering, Istanbul, Turkey, Chapter 4, 63-1 to 63-14.
33. Werner, S. D., Lee, L. C., Wong, H. L., and Trifunac, M. D. (1979). "Structural Response to Traveling Seismic Waves," Journal of Structural Division, ASCE, 105(12), 2547-2564.
34. Yamahara, H. (1970). "Ground Motions during Earthquakes and the Input Loss of Earthquake Power to an Excitation of Building," Soils and Foundations, Japan Society of Soil-Mechanics and Foundation Engineering, 10(2), 145-161.

**APPENDIX A**  
**DERIVATION OF EQUATIONS 9**

For incoherence effects only, the integrands in Eqs. 5a and 5b are symmetric about  $\epsilon_1 = \epsilon_2$ . This symmetry may be provided for by multiplying these expressions by 2 and changing the upper limit of integration of  $\epsilon_2$  from unity to  $\epsilon_1$ . On using the identity

$$I_n(z) = \frac{1}{\pi} \int_0^\pi e^{z \cos \theta} (\cos n\theta) d\theta \quad (19)$$

given as Eq. 9.6.19 in Abramowitz and Stegun [2], the specialized form of Eqs. 5a and 5b are integrated with respect to the circumferential co-ordinates to yield

$$\frac{S_x}{S_g} = 8 \int_0^1 \int_0^1 \epsilon_1 \epsilon_2 \exp[-b_0^2 (\epsilon_1^2 + \epsilon_2^2)] I_0(2b_0^2 \epsilon_1 \epsilon_2) d\epsilon_1 d\epsilon_2 \quad (20a)$$

$$\frac{S_y}{S_g} = 16 \int_0^1 \int_0^1 (\epsilon_1 \epsilon_2)^2 \exp[-b_0^2 (\epsilon_1^2 + \epsilon_2^2)] I_1(2b_0^2 \epsilon_1 \epsilon_2) d\epsilon_1 d\epsilon_2 \quad (20b)$$

The dummy variable  $\epsilon_2$  in these equations is then expressed as  $\epsilon_2 = s\epsilon_1$ , and the resulting expressions are integrated with respect to  $s$  by making use of the identity (See Eq. 6.631.8 in Ref. 7)

$$\int_0^1 s^{n+1} \exp^{-\alpha s^2} I_n(2\alpha s) ds = \frac{1}{4\alpha} [e^\alpha - e^{-\alpha} \sum_{y=-n}^n I_y(2\alpha)] \quad (21)$$

to yield

$$\frac{S_x}{S_g} = \frac{1}{b_0^2} \int_0^1 2\epsilon_1 [1 - \exp(-2b_0^2 \epsilon_1^2)] I_0(2b_0^2 \epsilon_1^2) d\epsilon_1 \quad (22a)$$

$$\frac{S_y}{S_g} = \frac{4}{b_0^2} \int_0^1 \epsilon_1^3 \{1 - \exp(-2b_0^2 \epsilon_1^2) [I_0(2b_0^2 \epsilon_1^2) + 2I_1(2b_0^2 \epsilon_1^2)]\} d\epsilon_1 \quad (22b)$$

Finally, on letting  $a = 2b_0^2 \epsilon_1^2$  and making use of the identities

$$\int_0^z e^{-a} a^n I_n(a) da = \frac{e^{-z} z^{n+2}}{2n+1} [I_n(z) + I_{n+1}(z)] \quad (23)$$

$$\int_0^z a^n I_{n-1}(a) da = z^n I_n(z) \quad (24)$$

$$I_0(z) - I_2(z) = \frac{2}{z} I_1(z) \quad (25)$$

given as Eqs. 11.3.12, 11.3.25 and 9.6.26 in Abramowitz and Stegun [2], Eqs. 22a and 22b are integrated to yield Eqs. 9a and 9b. Equation 9c follows from the fact that the integrand of Eq. 5c is antisymmetric in this case.

For small values of  $b_0$ , application of Taylor's series expansion to Eqs. 9a and 9b yields

$$S_x = [1 - b_0^2 + \frac{5}{6} b_0^4 + \dots] S_g \quad (26a)$$

$$S_y = [\frac{1}{2} b_0^2 - \frac{2}{3} b_0^4 + \dots] S_g \quad (26b)$$

**APPENDIX B**  
**EVALUATION OF PEAK VALUES OF INPUT AND RESPONSE**

Let  $z(t)$  be a stationary, ergodic random Gaussian process with zero mean and limited duration,  $t_0$ , and let  $Z$  be the ensemble mean of its peak values. Further, let  $G(\omega)$  be the one-sided power spectral density of the process, and  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$  be its first three moments, defined by

$$\lambda_n = \int_0^{\infty} \omega^n G(\omega) d\omega \quad n = 0, 1, 2 \quad (27)$$

The value of  $Z$  in Der Kiureghian's approach is evaluated conservatively from

$$Z = \left[ \sqrt{2 \ln(\mu_e t_0)} + \frac{0.5772}{\sqrt{2 \ln(\mu_e t_0)}} \right] \sqrt{\lambda_0} \quad (28)$$

in which

$$\mu_e t_0 = \begin{cases} 2.1 \text{ or } 2q\mu t_0 \text{ if greater than } 2.1 & \text{for } q \leq 0.1 \\ (1.63 q^{0.45} - 0.38) \mu t_0 & \text{for } 0.1 \leq q \leq 0.69 \\ \mu t_0 & \text{for } q \geq 0.69 \end{cases} \quad (29)$$

$\mu = \sqrt{(\lambda_2/\lambda_0)}/\pi$  = the mean zero-crossing rate of the process; and

$q = \sqrt{1 - \lambda_1^2/(\lambda_0 \lambda_2)}$  = Vanmarcke's bandwidth parameter [25].

**APPENDIX C**  
**HARMONIC RESPONSE OF SYSTEMS WITH INERTIAL INTERACTION**

**C.1 Torsionally Excited System**

Let  $x = x(t)$  and  $x_f = x_f(t)$  be the torsional components of the foundation input displacement and the actual foundation displacement, respectively, and  $\psi = \psi(t)$  be the resulting torsional deformation of the structure. The equations of motion for the system may then be written as

$$\ddot{\psi} + 2c_\theta p_\theta \dot{\psi} + p_\theta^2 \psi = -\ddot{x}_f \quad (30)$$

$$\text{and } J(\ddot{\psi} + \ddot{x}_f) + J_f \ddot{x}_f + Q_\theta(t) = 0 \quad (31)$$

in which a dot superscript denotes differentiation with respect to time;  $J$  and  $J_f$  are the polar mass moments of inertia of the structure and foundation, respectively; and  $Q_\theta(t)$  = the instantaneous value of the torque at the foundation-soil interface. Eq. 30 expresses the dynamic equilibrium of the forces acting on the structural mass, whereas Eq. 31 expresses the fact that the sum of the torsional moments due to the inertia of the structure and the foundation equals the torque acting at the foundation-soil interface.

For the harmonic response considered

$$x(t) = X e^{i\omega t} \quad (32a)$$

$$x_f(t) = X_f e^{i\omega t} \quad (32b)$$

$$\psi(t) = \Psi e^{i\omega t} \quad (32c)$$

$$\text{and } Q_\theta(t) = K_\theta (X_f - X) e^{i\omega t} \quad (33)$$

in which  $X$ ,  $X_f$  and  $\Psi$  are the complex-valued amplitudes of  $x$ ,  $x_f$  and  $\psi$ , respectively; and  $K_\theta$  = the complex-valued torsional impedance of the massless foundation.

Equations 30 and 31 are solved in three steps as follows: First, on substituting Eqs. 32b and 32c and its derivatives into Eq. 30, the deformation amplitude of the structure,  $\Psi$ , is expressed in terms of the amplitude of the torsional component of the foundation acceleration,  $\ddot{x}_f$ , as

$$\Psi = H_V \ddot{x}_f \quad (34)$$

in which  $\ddot{x}_f = -\omega^2 x_f$ , and  $H_V =$  the transfer function for torsional response, given by

$$H_V = -\frac{1}{p_\theta^2} \frac{1}{1 - (\omega/p_\theta)^2 + i2\zeta_\theta(\omega/p_\theta)} \quad (35)$$

Next, Eq. 34, along with Eqs. 32b, 32c and 33, are substituted into Eq. 31, and the resulting expression is solved for  $\ddot{x}_f$ . This step yields

$$\ddot{x}_f = T_V \ddot{x} \quad (36)$$

in which

$$T_V = \frac{\kappa_\theta}{\kappa_\theta - [(TR)_\theta + (J_f/J)a_0^2]} \quad (37)$$

$\kappa_\theta = K_\theta R^2 / (J_V S^2)$ ; and  $(TR)_\theta =$  the torsional transmissibility of the system, given by

$$(TR)_\theta = -(p_\theta^2 + i2\zeta_\theta \omega p_\theta) H_V = \frac{1 + i2\zeta_\theta(\omega/p_\theta)}{1 - (\omega/p_\theta)^2 + i2\zeta_\theta(\omega/p_\theta)} \quad (38)$$

The expression for  $\ddot{x}_f$  defined by Eq. 36 is finally substituted into Eq. 34 to yield

$$\Psi = H_V T_V \ddot{x} \quad (39)$$

from which it follows that the psd function of the deformation of the structure along its periphery,  $S_V$ , is given by

$$S_V = |H_V|^2 |T_V|^2 S_y \quad (40)$$

The factor  $|\tau_v|^2$  in the latter expression represents the effect of inertial interaction.

For the circular foundation considered,  $K_\theta$  is defined by [29]

$$K_\theta = \frac{16}{3} GR^3 (\alpha_\theta + ia_0 \beta_\theta) \quad (41)$$

in which  $\alpha_\theta$  and  $\beta_\theta$  are dimensionless functions of the frequency parameter  $a_0 = \omega R/v_s$ . On making use of the expression  $v_s = \sqrt{G/\rho}$ , the dimensionless stiffness factor,  $\kappa_\theta$ , in Eq. 37 may also be written as

$$\kappa_\theta = \frac{16}{3} \frac{\rho R^5}{J} (\alpha_\theta + ia_0 \beta_\theta) \quad (42)$$

### C.2 Laterally Excited System

Let  $x = x(t)$  and  $x_f = x_f(t)$  be the lateral components of the foundation input displacement and actual foundation displacement, respectively, and  $\phi(t)$  be the angular rocking displacement of the foundation. Further, let  $u = u(t)$  be the resulting lateral deformation of the structure. The equations of motion of the system may then be expressed as

$$\ddot{u} + 2\zeta_x p_x \dot{u} + p_x^2 u = -(\ddot{x}_f + h\ddot{\phi}) \quad (43)$$

$$m_f \ddot{x}_f + m(\ddot{x}_f + h\ddot{\phi} + \ddot{u}) + Q_x(t) = 0 \quad (44)$$

$$I_T \ddot{\phi} + mh(\ddot{x}_f + h\ddot{\phi} + \ddot{u}) + Q_\phi(t) = 0 \quad (45)$$

in which  $I_T$  = the total mass moment of inertia of the structure and its foundation about a horizontal axis through the centroid of the foundation, and  $Q_x(t)$  and  $Q_\phi(t)$  are the horizontal shear and overturning moment at the foundation-soil interface. Equation 43 expresses the dynamic equilibrium of the forces acting on the structural mass, whereas Eqs. 44 and 45 express the equality of the interface forces to the total horizontal force and the base moment induced by the inertia forces acting on the structure and its foundation.



For the harmonic response considered,

$$x(t) = X_0 e^{i\omega t} \quad (46a)$$

$$x_f(t) = X_f e^{i\omega t} \quad (46b)$$

$$\phi(t) = \phi e^{i\omega t} \quad (46c)$$

$$u(t) = U e^{i\omega t} \quad (46d)$$

$$\text{and } \begin{Bmatrix} Q_x(t) \\ Q_\phi(t) \end{Bmatrix} = \begin{bmatrix} & \\ K_f & \end{bmatrix} \begin{Bmatrix} X_f - X_0 \\ \phi \end{Bmatrix} e^{i\omega t} \quad (47)$$

in which  $[K_f]$  = a 2x2 complex-valued impedance matrix for the massless foundation.

The solution of Eqs. 43 through 45 may be obtained in three steps in a manner analogous to that described for the torsionally excited system. First, Eq. 43 is solved for  $U$  in terms of  $\ddot{X}_f + h\ddot{\phi}$ , in which  $\ddot{X}_f = -\omega^2 X_f$  and  $\ddot{\phi} = -\omega^2 \phi$ . Second, the quantity  $U$  is eliminated from Eqs. 44 and 45 utilizing the result of the first step, and the resulting equations are solved for  $\ddot{X}_f$  and  $\ddot{\phi}$  by making use of Eq. 47. Finally, the expressions for  $\ddot{X}_f$  and  $\ddot{\phi}$  are back substituted in the expression for  $U$  obtained in the first step.

Implementation of the first step leads to

$$U = H_U (\ddot{X}_f + h \ddot{\phi}) \quad (48)$$

in which  $H_U$  is defined by Eq. 15; and implementation of the second step leads to the following system of algebraic equations in  $\ddot{X}_f$  and  $\ddot{\phi}$

$$\begin{bmatrix} m(\text{TR})_x + m_f & mh(\text{TR})_x \\ mh(\text{TR})_x & I_T + mh^2(\text{TR})_x \end{bmatrix} \begin{Bmatrix} \ddot{X}_f \\ \ddot{\phi} \end{Bmatrix} - \frac{1}{\omega^2} \begin{bmatrix} & \\ K_f & \end{bmatrix} \begin{Bmatrix} \ddot{X}_f \\ \ddot{\phi} \end{Bmatrix} = \frac{1}{\omega^2} \begin{bmatrix} & \\ K_f & \end{bmatrix} \begin{Bmatrix} \ddot{X}_0 \\ 0 \end{Bmatrix} \quad (49)$$

in which  $(TR)_x$  = the transmissibility factor for lateral response, defined by

$$(TR)_x = -(p_x^2 + i2\zeta_x \omega p_x) H_u = \frac{1 + i2\zeta_x (\omega/p_x)}{1 - (\omega/p_x)^2 + i2\zeta_x (\omega/p_x)} \quad (50)$$

On solving Eqs. 49 and substituting the resulting values of  $\ddot{x}_f$  and  $\ddot{\phi}$  into Eq. 48, one obtains the desired U.

For the solutions presented in this report, the off-diagonal terms of  $[K_f]$  are presumed to be negligible, and the diagonal terms are denoted by  $K_x$  and  $K_\phi$ . On letting  $\epsilon_i = I_T/mh^2$ ,  $\epsilon_m = m_f/m$ ,  $\kappa_x = K_x R^2/(mv_s^2)$  and  $\kappa_\phi = K_\phi R^2/(mh^2 v_s^2)$ , the solution for U may be expressed in the form

$$U = H_u T_u \ddot{x}_0 \quad (51)$$

in which  $\ddot{x}_0 = -\omega^2 x_0$ ; and  $T_u$  = the dimensionless factor that provides for the inertial interaction effects. The latter factor is given by

$$T_u = \frac{B_3}{B_1 a_0^4 - B_2 a_0^2 + B_3} \quad (52)$$

in which

$$B_1 = (\epsilon_i + \epsilon_m)(TR)_x + \epsilon_i \epsilon_m \quad (53a)$$

$$B_2 = (\kappa_x + \kappa_\phi)(TR)_x + \epsilon_m \kappa_\phi \quad (53b)$$

$$B_3 = \kappa_x (\kappa_\phi - \epsilon_i a_0^2) \quad (53c)$$

For the circular foundations considered, the expressions for  $K_x$  and  $K_\phi$  are given by [31]

$$K_x = \frac{8GR}{(2-\nu)} (\alpha_x + i a_0 \beta_x) \quad (54)$$

$$K_\phi = \frac{8GR^3}{3(1-\nu)} (\alpha_\phi + i a_0 \beta_\phi) \quad (55)$$

in which  $\alpha_x$ ,  $\beta_x$ ,  $\alpha_\phi$  and  $\beta_\phi$  are dimensionless factors that depend on Poisson's ratio for the halfspace material,  $\nu$ , and the dimensionless frequency parameter,  $a_0$ . On making use of the expression  $v_s = \sqrt{G/\rho}$ , the stiffness factors  $\kappa_x$  and  $\kappa_\phi$  in Eqs. 53 may be written as

$$\kappa_x = \frac{8}{2-\nu} \frac{\rho R^3}{m} (\alpha_x + ia_0 \beta_x) \quad (56)$$

$$\kappa_\phi = \frac{8}{3(1-\nu)} \frac{\rho R^5}{mh^2} (\alpha_\phi + ia_0 \beta_\phi) \quad (57)$$

The psd function for the deformation of the interacting system,  $S_u$ , is then given by

$$S_u = |H_u|^2 |T_u|^2 S_{\ddot{x}} \quad (58)$$

**APPENDIX D**  
**NOTATION**

The following symbols are used:

- $a_0$  =  $\omega R/v_s$  = frequency parameter;
- $\bar{a}_0$  = modified frequency parameter for combined wave passage and incoherence effects, defined by Eq. (11);
- $A$  = foundation contact area;
- $A_x$  =  $p_x^2 U_x$  = pseudo-acceleration value of the mean maximum deformation induced by the lateral component of the foundation input motion;
- $A_y$  =  $p_\theta^2 U_y$  = pseudo-acceleration value of the mean maximum deformation induced along the perimeter of the structure by the torsional component of the foundation input motion;
- $B_1, B_2, B_3$  = dimensionless parameters in expression for  $T_u$ , defined by Eqs. 53;
- $b_0$  =  $\gamma a_0$  = modified frequency parameter for incoherence only;
- $c$  =  $v_s/(\sin \alpha_v)$  = apparent horizontal velocity of wave front;
- $c_0$  =  $(v_s/c)a_0 = (\sin \alpha_v)a_0$  = modified frequency parameter for wave passage effect only;
- $d_1, d_2$  = components of  $\vec{r}_1$  and  $\vec{r}_2$  in the direction of propagation of the seismic wave front;
- $f_0$  = cut-off frequency of excitation;
- $f_x, \bar{f}_x$  = natural frequencies of rigidly and elastically supported structures in lateral mode of vibration, in cps;
- $f_\theta, \bar{f}_\theta$  = natural frequencies of rigidly and elastically supported structures in torsional mode of vibration, in cps;
- $G(\omega)$  = one side-power spectral density function for a stationary Gaussian random process;
- $h$  = height of structure;
- $H_u, H_v$  = transfer functions relating the lateral and torsional deformations of the structure to the corresponding components of the foundation input acceleration;

$i$	= $\sqrt{-1}$ ;
$I_0, I_1, I_2$	= modified Bessel functions of the first kind of order zero, one and two, respectively;
$I, I_f$	= mass moments of inertia of structure and foundation about a horizontal centroidal axis;
$I_\theta$	= polar area moment of inertia of foundation about a vertical centroidal axis;
$J_1, J_2$	= Bessel functions of first kind of order one and two, respectively;
$J, J_f$	= polar mass moments of inertia of structure and foundation about a vertical centroidal axis;
$K_x, K_\phi, K_\theta$	= complex-valued foundation impedances of massless foundations in lateral, rocking and torsional modes of vibration, respectively;
$m, m_f$	= mass of structure and foundation, respectively;
psd	= power spectral density;
$p_x$	= $2\pi f_x$ = fixed-base circular natural frequency of structure in lateral mode of vibration;
$p_\theta$	= $2\pi f_\theta$ = fixed-base circular natural frequency of structure in torsional mode of vibration;
$q$	= Vanmarcke's bandwidth parameter;
$Q_x, Q_\phi, Q_\theta$	= lateral force, overturning moment and torsional moment at foundation-soil interface;
$\vec{r}_1, \vec{r}_2$	= position vectors for two arbitrary points on foundation-soil interface;
$R$	= radius of foundation;
$S(\vec{r}_1, \vec{r}_2, \omega)$	= cross psd function for motions at points $\vec{r}_1$ and $\vec{r}_2$ ;
$S_0$	= constant in expression for psd function of the free-field ground acceleration;
$S_g, S_{\dot{g}}, S_{\ddot{g}}$	= local psd functions for the displacement, velocity and acceleration histories of the free-field ground motion;

- $S_x, S_x^{\dot{}} , S_x^{\ddot{}}$  = psd functions for the displacement, velocity and acceleration histories of the lateral component of foundation input motion;
- $S_y, S_y^{\dot{}} , S_y^{\ddot{}}$  = psd functions for the displacement, velocity and acceleration histories of the motion along the perimeter of the foundation induced by the torsional component of foundation input motion;
- $S_{xy}, S_{xy}^{\dots}$  = cross psd functions for the horizontal and torsional components of the foundation input displacement and foundation input acceleration, respectively;
- $S_u$  = psd function for the structural deformation induced by the lateral component of foundation input motion;
- $S_v$  = psd function for the deformation  $v = \psi R$  induced at the periphery of the structure by the torsional component of foundation input motion;
- $S_w$  = psd function for the total deformation at the most highly stressed point on the periphery of the structure;
- $t_0$  = duration of strong motion portion of earthquake;
- $T_u, T_v$  = dimensionless transfer factors that provide for the effects of inertial interaction for laterally and torsionally excited systems;
- $(TR)_x$  = transmissibility of laterally excited system defined by Eq. 50;
- $(TR)_\theta$  = transmissibility of torsionally excited system defined by Eq. 37;
- $U_x$  = mean value of maximum structural deformations induced by the lateral component of foundation input motion;
- $U_y$  = mean value of maximum deformations induced along the perimeter of the structure by the torsional component of foundation input motion;
- $v$  =  $\psi R$  = structural deformation induced along the perimeter of the structure by the torsional component of foundation input motion;
- $v_s$  = shear wave velocity for soil medium;
- $V_x$  =  $p_x U_x$  = pseudo-velocity value corresponding to  $U_x$ ;
- $V_y$  =  $p_y U_y$  = pseudo-velocity value corresponding to  $U_y$ ;

$w$	= $u + v$ = total deformation at the most highly stressed point at the periphery of the structure;
$x$	= lateral component of foundation input displacement;
$x_f$	= lateral component of actual foundation displacement;
$X, \dot{X}, \ddot{X}$	= mean maximum values of the horizontal components of the displacement, velocity and acceleration histories of the foundation input motion;
$X_g, \dot{X}_g, \ddot{X}_g$	= mean maximum values of the displacement, velocity and acceleration histories of the free-field, control-point ground motion;
$X_0$	= amplitude of $x$ for harmonic motion;
$Y, \dot{Y}, \ddot{Y}$	= mean maximum values of the displacement, velocity and acceleration at the periphery of the foundation induced by the torsional component of foundation input motion;
$\alpha_v$	= angle of incidence of seismic waves, measured from vertical axis;
$\alpha_x, \alpha_\phi, \alpha_\theta$	= dimensionless stiffness coefficients in expressions for foundation impedances $K_x, K_\phi$ and $K_\theta$ ;
$\beta_x, \beta_\phi, \beta_\theta$	= dimensionless damping coefficients in expressions for foundation impedances $K_x, K_\phi$ and $K_\theta$ ;
$\gamma$	= dimensionless incoherence parameter;
$\tilde{\gamma}$	= $\gamma c/v_s$ = modified incoherence parameter;
$\Gamma$	= spatial coherence function for free-field ground motion;
$\delta$	= mass density ratio for the structure;
$\Delta_1, \Delta_2$	= dimensionless distance parameters, defined by Eqs. 6;
$\epsilon_i$	= $I_T/mh^2$ = dimensionless measure of mass moment of inertia of structure-foundation system about a horizontal centroidal axis;
$\epsilon_m$	= $m_f/m$ = mass ratio of foundation and structure;
$\zeta_x, \zeta_\theta$	= percentages of critical structural damping for fixed-base structure in lateral and torsional modes of vibration, respectively;
$\bar{\zeta}_x, \bar{\zeta}_\theta$	= effective structural damping factors for elastically supported system in lateral and torsional modes of vibration, respectively;

- $\theta_1, \theta_2$  = circumferential co-ordinates of points  $\vec{r}_1$  and  $\vec{r}_2$ , respectively;
- $\kappa_x$  =  $K_x R^2 / (m v_s^2)$  = dimensionless measure of lateral foundation impedance;
- $\kappa_\phi$  =  $K_\theta R^2 / (J v_s^2)$  = dimensionless measure of torsional foundation impedance;
- $\kappa_\phi$  =  $K_\phi R^2 / (m h^2 v_s^2)$  = dimensionless measure of rocking foundation impedance;
- $\lambda_n$  = nth moment of one-sided power spectral density, given by Eq. 27;
- $\mu$  = mean rate of zero crossings for stationary process;
- $\mu_e$  = effective mean rate of zero crossings;
- $\nu$  = Poisson's ratio of soil medium;
- $\xi_1, \xi_2$  = normalized radial coordinates of points  $\vec{r}_1$  and  $\vec{r}_2$ , respectively;
- $\rho$  = mass density of soil medium;
- $\tau$  =  $R/v_s$  = transit time;
- $\tilde{\tau}$  =  $\sqrt{\gamma^2 + \sin^2 \alpha_v} \tau$  = effective transit time.
- $\phi$  = actual rocking displacement of foundation;
- $\Phi$  = complex-valued amplitude of  $\phi$  for harmonic motion;
- $\chi$  = torsional component of foundation input motion;
- $\chi$  = complex-valued amplitude of  $\chi$  for harmonic motion;
- $\chi_f$  = torsional component of actual foundation  $\phi$  displacement;
- $\chi_f$  = complex-valued amplitude of  $\chi_f$  for harmonic motion;
- $\psi$  = torsional deformation of structure;
- $\Psi$  = complex-valued amplitude of  $\psi$  for harmonic motion;
- $\omega$  = circular frequency of excitation and resulting motion.



**NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH  
LIST OF PUBLISHED TECHNICAL REPORTS**

The National Center for Earthquake Engineering Research (NCEER) publishes technical reports on a variety of subjects related to earthquake engineering written by authors funded through NCEER. These reports are available from both NCEER's Publications Department and the National Technical Information Service (NTIS). Requests for reports should be directed to the Publications Department, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Red Jacket Quadrangle, Buffalo, New York 14261. Reports can also be requested through NTIS, 5285 Port Royal Road, Springfield, Virginia 22161. NTIS accession numbers are shown in parenthesis, if available.

- NCEER-87-0001 "First-Year Program in Research, Education and Technology Transfer," 3/5/87, (PB88-134275/AS).
- NCEER-87-0002 "Experimental Evaluation of Instantaneous Optimal Algorithms for Structural Control," by R.C. Lin, T.T. Soong and A.M. Reinhorn, 4/20/87, (PB88-134341/AS).
- NCEER-87-0003 "Experimentation Using the Earthquake Simulation Facilities at University at Buffalo," by A.M. Reinhorn and R.L. Ketter, to be published.
- NCEER-87-0004 "The System Characteristics and Performance of a Shaking Table," by J.S. Hwang, K.C. Chang and G.C. Lee, 6/1/87, (PB88-134259/AS).
- NCEER-87-0005 "A Finite Element Formulation for Nonlinear Viscoplastic Material Using a Q Model," by O. Gyebe and G. Dasgupta, 11/2/87, (PB88-213764/AS).
- NCEER-87-0006 "Symbolic Manipulation Program (SMP) - Algebraic Codes for Two and Three Dimensional Finite Element Formulations," by X. Lee and G. Dasgupta, 11/9/87, (PB88-219522/AS).
- NCEER-87-0007 "Instantaneous Optimal Control Laws for Tall Buildings Under Seismic Excitations," by J.N. Yang, A. Akbarpour and P. Ghaemmaghami, 6/10/87, (PB88-134333/AS).
- NCEER-87-0008 "IDARC: Inelastic Damage Analysis of Reinforced Concrete-Frame Shear-Wall Structures," by Y.J. Park, A.M. Reinhorn and S.K. Kunnath, 7/20/87, (PB88-134325/AS).
- NCEER-87-0009 "Liquefaction Potential for New York State: A Preliminary Report on Sites in Manhattan and Buffalo," by M. Budhu, V. Vijayakumar, R.F. Giese and L. Baumgras, 8/31/87, (PB88-163704/AS).
- NCEER-87-0010 "Vertical and Torsional Vibration of Foundations in Inhomogeneous Media," by A.S. Veletsos and K.W. Dotson, 6/1/87, (PB88-134291).
- NCEER-87-0011 "Seismic Probabilistic Risk Assessment and Seismic Margins Studies for Nuclear Power Plants," by Howard H.M. Hwang, 6/15/87, (PB88-134267/AS).
- NCEER-87-0012 "Parametric Studies of Frequency Response of Secondary Systems Under Ground-Acceleration Excitations," by Y. Yong and Y.K. Lin, 6/10/87, (PB88-134309/AS).
- NCEER-87-0013 "Frequency Response of Secondary Systems Under Seismic Excitation," by J.A. HoLung, J. Cai and Y.K. Lin, 7/31/87, (PB88-134317/AS).
- NCEER-87-0014 "Modelling Earthquake Ground Motions in Seismically Active Regions Using Parametric Time Series Methods," G.W. Ellis and A.S. Cakmak, 8/25/87, (PB88-134283/AS).
- NCEER-87-0015 "Detection and Assessment of Seismic Structural Damage," by E. DiPasquale and A.S. Cakmak, 8/25/87, (PB88-163712/AS).
- NCEER-87-0016 "Pipeline Experiment at Parkfield, California," by J. Isenberg and E. Richardson, 9/15/87, (PB88-163720/AS).
- NCEER-87-0017 "Digital Simulation of Seismic Ground Motion," by M. Shinozuka, G. Deodatis and T. Harada, 8/31/87, (PB88-155197/AS).

- NCEER-87-0018 "Practical Considerations for Structural Control: System Uncertainty, System Time Delay and Truncation of Small Control Forces," J. Yang and A. Akbarpour, 8/10/87, (PB88-163738/AS).
- NCEER-87-0019 "Modal Analysis of Nonclassically Damped Structural Systems Using Canonical Transformation," by J.N. Yang, S. Sarkani and F.X. Long, 9/27/87, (PB88-187851/AS).
- NCEER-87-0020 "A Nonstationary Solution in Random Vibration Theory," by J.R. Red-Horse and P.D. Spanos, 11/3/87, (PB88-163746/AS).
- NCEER-87-0021 "Horizontal Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by A.S. Veletsos and K.W. Dotson, 10/15/87, (PB88-150859/AS).
- NCEER-87-0022 "Seismic Damage Assessment of Reinforced Concrete Members," by Y.S. Chung, C. Meyer and M. Shinozuka, 10/9/87, (PB88-150867/AS).
- NCEER-87-0023 "Active Structural Control in Civil Engineering," by T.T. Soong, 11/11/87, (PB88-187778/AS).
- NCEER-87-0024 "Vertical and Torsional Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by K.W. Dotson and A.S. Veletsos, 12/87, (PB88-187786/AS).
- NCEER-87-0025 "Proceedings from the Symposium on Seismic Hazards, Ground Motions, Soil-Liquefaction and Engineering Practice in Eastern North America, October 20-22, 1987, edited by K.H. Jacob, 12/87, (PB88-188115/AS).
- NCEER-87-0026 "Report on the Whittier-Narrows, California, Earthquake of October 1, 1987," by J. Pantelic and A. Reinhorn, 11/87, (PB88-187752/AS).
- NCEER-87-0027 "Design of a Modular Program for Transient Nonlinear Analysis of Large 3-D Building Structures," by S. Srivastav and J.F. Abel, 12/30/87, (PB88-187950/AS).
- NCEER-87-0028 "Second-Year Program in Research, Education and Technology Transfer," 3/8/88, (PB88-219480/AS).
- NCEER-88-0001 "Workshop on Seismic Computer Analysis and Design of Buildings With Interactive Graphics," by J.F. Abel and C.H. Conley, 1/18/88, (PB88-187760/AS).
- NCEER-88-0002 "Optimal Control of Nonlinear Flexible Structures," J.N. Yang, F.X. Long and D. Wong, 1/22/88, (PB88-213772/AS).
- NCEER-88-0003 "Substructuring Techniques in the Time Domain for Primary-Secondary Structural Systems," by G. D. Manolis and G. Juhn, 2/10/88, (PB88-213780/AS).
- NCEER-88-0004 "Iterative Seismic Analysis of Primary-Secondary Systems," by A. Singhal, L.D. Lutes and P. Spanos, 2/23/88, (PB88-213798/AS).
- NCEER-88-0005 "Stochastic Finite Element Expansion for Random Media," P. D. Spanos and R. Ghanem, 3/14/88, (PB88-213806/AS).
- NCEER-88-0006 "Combining Structural Optimization and Structural Control," F. Y. Cheng and C. P. Pantelides, 1/10/88, (PB88-213814/AS).
- NCEER-88-0007 "Seismic Performance Assessment of Code-Designed Structures," H.H.M. Hwang, J-W. Jaw and H-J. Shau, 3/20/88, (PB88-219423/AS).
- NCEER-88-0008 "Reliability Analysis of Code-Designed Structures Under Natural Hazards," H.H.M. Hwang, H. Ushiba and M. Shinozuka, 2/29/88.

- NCEER-88-0009 "Seismic Fragility Analysis of Shear Wall Structures," J-W Jaw and H.H-M. Hwang, 4/30/88.
- NCEER-88-0010 "Base Isolation of a Multi-Story Building Under a Harmonic Ground Motion - A Comparison of Performances of Various Systems," F-G Fan, G. Ahmadi and I.G. Tadjbakhsh, 5/18/88.
- NCEER-88-0011 "Seismic Floor Response Spectra for a Combined System by Green's Functions," F.M. Lavelle, L.A. Bergman and P.D. Spanos, 5/1/88.
- NCEER-88-0012 "A New Solution Technique for Randomly Excited Hysteretic Structures," G.Q. Cai and Y.K. Lin, 5/16/88.
- NCEER-88-0013 "A Study of Radiation Damping and Soil-Structure Interaction Effects in the Centrifuge," K. Weissman, supervised by J.H. Prevost, 5/24/88, to be published.
- NCEER-88-0014 "Parameter Identification and Implementation of a Kinematic Plasticity Model for Frictional Soils," J.H. Prevost and D.V. Griffiths, to be published.
- NCEER-88-0015 "Two- and Three-Dimensional Dynamic Finite Element Analyses of the Long Valley Dam," D.V. Griffiths and J.H. Prevost, 6/17/88, to be published.
- NCEER-88-0016 "Damage Assessment of Reinforced Concrete Structures in Eastern United States," A.M. Reinhorn, M.J. Seidel, S.K. Kunnath and Y.J. Park, 6/15/88.
- NCEER-88-0017 "Dynamic Compliance of Vertically Loaded Strip Foundations in Multilayered Viscoelastic Soils," S. Ahmad and A.S.M. Israil, 6/17/88.
- NCEER-88-0018 "An Experimental Study of Seismic Structural Response With Added Viscoelastic Dampers," R.C. Lin, Z. Liang, T.T. Soong and R.H. Zhang, 6/30/88.
- NCEER-88-0019 "Experimental Investigation of Primary - Secondary System Interaction," G.D. Manolis, G. Juhn and A.M. Reinhorn, 5/27/88, to be published.
- NCEER-88-0020 "A Response Spectrum Approach For Analysis of Nonclassically Damped Structures," J.N. Yang, S. Sarkani and F.X. Long, 4/22/88.
- NCEER-88-0021 "Seismic Interaction of Structures and Soils: Stochastic Approach," A.S. Veletsos and A.M. Prasad, 7/21/88.