NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH

State University of New York at Buffalo

RESPONSE ANALYSIS OF STOCHASTIC STRUCTURES

by

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Technical Report NCEER-88-0029

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NOTICE

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PREFACE

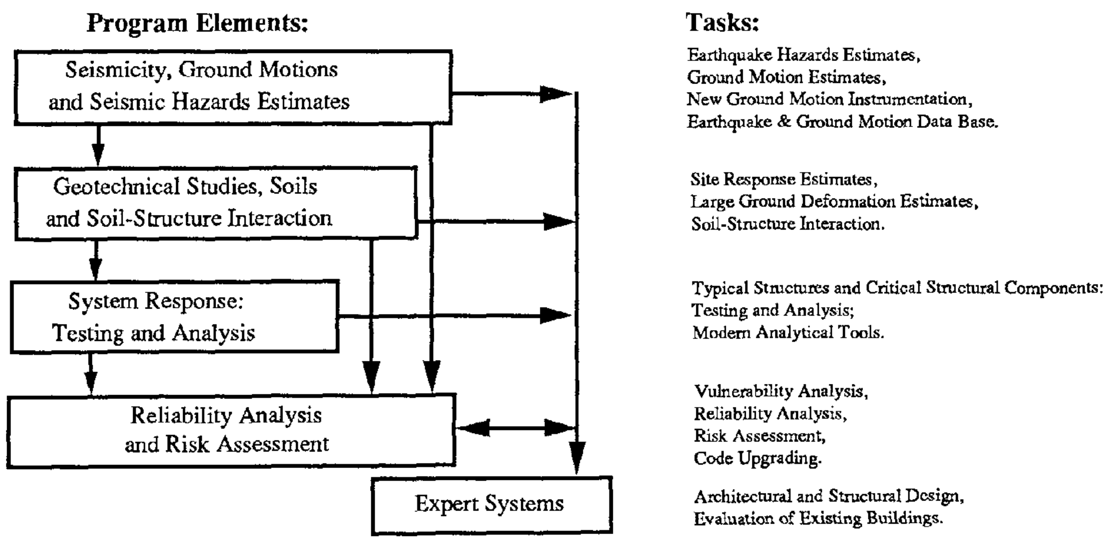
The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 1, Existing and New Structures, and more specifically to Reliability Analysis and Risk Assessment.

The long term goal of research in Existing and New Structures is to develop seismic hazard mitigation procedures through rational probabilistic risk assessment for damage or collapse of structures, mainly existing buildings, in regions of moderate to high seismicity. This work relies on improved definitions of seismicity and site response, experimental and analytical evaluations of systems response, and more accurate assessment of risk factors. This technology will be incorporated in expert systems tools and improved code formats for existing and new structures. Methods of retrofit will also be developed. When this work is completed, it should be possible to characterize and quantify societal impact of seismic risk in various geographical regions and large municipalities. Toward this goal, the program has been divided into five components, as shown in the figure below:



Reliability Analysis and Risk Assessment research constitutes one of the important areas of Existing and New Structures. Current research addresses, among others, the following issues:

1. Code issues - Development of a probabilistic procedure to determine load and resistance factors. Load Resistance Factor Design (LRFD) includes the investigation of wind vs. seismic issues, and of estimating design seismic loads for areas of moderate to high seismicity.
2. Response modification factors - Evaluation of RMFs for buildings and bridges which combine the effect of shear and bending.
3. Seismic damage - Development of damage estimation procedures which include a global and local damage index, and damage control by design; and development of computer codes for identification of the degree of building damage and automated damage-based design procedures.
4. Seismic reliability analysis of building structures - Development of procedures to evaluate the seismic safety of buildings which includes limit states corresponding to serviceability and collapse.
5. Retrofit procedures and restoration strategies.
6. Risk assessment and societal impact.

Research projects concerned with Reliability Analysis and Risk Assessment are carried out to provide practical tools for engineers to assess seismic risk to structures for the ultimate purpose of mitigating societal impact.

Most existing structures are subjected to spatial variations of their material and/or geometrical properties. The extent of response variability of structures arising from such spatial variations may be significant from a structural reliability point of view. In this context, the result of this study has contributed to the reduction of the uncertainty level involved in structural reliability estimation, particularly when the result is combined with other NCEER studies in the reliability area.

ABSTRACT

The response variability of statically indeterminate linear structures due to spatial variation of material and/or geometrical properties, is investigated. Utilizing a Green's function formulation, or the more general flexibility method, the mean square response of statically indeterminate beams and frames (multi-story/multi-bay) is determined without recourse to a finite element analysis. The response variability is expressed in terms of random variables even though the material and/or the geometrical properties (in this case the flexibility) are considered to constitute stochastic fields. This makes it easier to estimate not only the response statistics but also the limit state probability, if the limit state conditions pertain to serviceablity and hence are given in terms of linear structural response. The response variability can be estimated by various methods including the First-Order Second Moment method and Monte Carlo simulation techniques. The safety index for the beam midspan deflection and end moment are evaluated using standard methods. One of the example analyses deals with the effect of such spatial variability on the seismic structural response evaluated under quasi-static conditions. While the loading condition and structure considered in this example analyses are relatively simple, the result provides some insight to the extent of the response variability expected under these circumstances.

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SECTION 1

INTRODUCTION

The response variability of structures is generally caused by random loading or random structural properties. Recently, interest has been generated among engineers and researchers concerning the spatial variation of material properties. In this connection, Shinozuka (1972b) and Shinozuka and Lenoe (1976) presented analytical models for the spatial variability of material strength. Methods for digitally generating sample functions of random fields were developed (Shinozuka, 1974; Shinozuka and Jan, 1972) and applied to the Monte Carlo solution of nonlinear dynamic problems (Shinozuka, 1972a; Shinozuka and Wen, 1972; Vaicaitis, Jan and Shinozuka, 1975; Vaicaitis, Shinozuka and Takeno, 1975).

More recent publications deal with stochastic field theory (Vanmarcke, 1983), digital simulation and related applications (Shinozuka, 1974; Shinozuka, 1985a) as well as discretization of random fields using finite element analysis, perturbation techniques and related reliability analysis (e.g., Der Kiureghian, 1985; Handa and Anderson, 1981; Hisada and Nakagiri, 1980; Vanmarcke and Grigoriu, 1983) or Neumann expansion (Shinozuka, 1985b; Shinozuka and Dasgupta, 1986; Yamazaki and Shinozuka, 1987). Other analytical treatments of the same subject are found elsewhere (Lawrence, Liu and Belytschko, 1986; Liu, Belytschko and Mani, 1985).

In an earlier study (Shinozuka, 1986), the response variability of statically determinate linear structures due to spatial variability of the elastic properties was examined from an analytical point of view. In a more recent work (Bucher and Shinozuka, 1986), (1) the above concept was extended to relatively simple statically indeterminate structures, (2) a Green's function formulation was used to obtain the mean square response quantities in terms of convolution integrals, and (3) it was shown that the response variability can be expressed in terms of random variables only, whose statistics depend on the characteristics of the stochastic fields involved.

The present study extends these concepts even further to more complicated structures, including beams of a higher degree of statical indeterminacy, and multi-bay multi-story frames. The response variability is expressed in terms of random variables, as was mentioned above, which facilitates the estimation of the response statistics. This can be achieved either by analytical methods with the aid, for example, of the First-Order Second-Moment (FOSM) method, or using Monte Carlo simulation techniques. The safety index for the response quantities can be evaluated using the Lagrange multiplier and other methods, if a meaningful limit state can be prescribed. The analysis presented in what follows is based on the assumption that flexibility is the only quantity that varies significantly along the axis of the structure.

One of the example analyses deals with the effect of such spatial variability on the seismic structural response evaluated under quasi-static conditions. While the loading condition and structure considered in this example analyses are relatively simple, the result provides some insight to the extent of the response variability expected under these circumstances.

SECTION 2

DESCRIPTION OF PROBLEM

Consider the problem of a beam loaded with a deterministic load, with flexibility varying randomly along its length. It is assumed that the flexibility of the beam is described by a Gaussian homogeneous stochastic field with autocorrelation function $R(\xi)$. Thus, the randomly fluctuating flexibility is given by :

$$\frac{1}{EI} = \frac{1}{E_0 I_0}\left[1 + f(x)\right] \tag{2.1}$$

where $1/(E_0 I_0)$ is the expected value of the flexibility and $f(x)$ is a zéro mean homogeneous Gaussian random field with autocorrelation function

$$R_{ff}(x-y) = \mathbf{E}[f(x)f(y)] \tag{2.2}$$

in which $\mathbf{E}[\cdot]$ indicates the expectation. In Eq. 2.1, it is implied that both E and I, or one of these alone, can be a stochastic field.

SECTION 3

FORMULATION OF PROBLEM

3.1 Statically Determinate Beams

The governing differential equation for a beam is

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = p(x) \tag{3.1}$$

where w is the deflection, $p(x)$ the distributed load and x the coordinate along the beam axis. For a statically determinate beam, integration of Eq. 3.1 yields

$$\frac{d^2w}{dx^2} = w'' = -\frac{M(x)}{EI} \tag{3.2}$$

where $M(x)$ is the bending moment. Since, in this case, the boundary conditions used to derive Eq. 3.2 do not depend on the elastic properties of the beam, there is only one random quantity in the last member of Eq. 3.2, i.e., the flexibility term $1/(EI)$. Inserting Eq. 2.1 into Eq. 3.2 yields :

$$w'' = -\frac{M(x)}{E_0 I_0}\left[1 + f(x)\right] \tag{3.3}$$

Hence, w'' becomes a Gaussian field and thus $w(x)$ obtained from the above equation through integration is also Gaussian.

In the study by Bucher and Shinozuka (1986), it was shown that statistics of $w(x)$ can be easily obtained by utilizing the Green's function of the bending problem. First of all, Green's function $h(x, \xi)$ must be a solution to the given differential equation with the delta function on the right-hand side:

$$\frac{d^2}{dx^2}h(x,\xi) = \delta(x - \xi) \tag{3.4}$$

Integration yields

$$\begin{aligned} \frac{d}{dx}h(x,\xi) &= a \qquad & x < \xi \\ &= b \qquad & x \geq \xi \end{aligned} \tag{3.5}$$

where, from the properties of the delta function,

$$b - a = 1 \tag{3.6}$$

Further integration results in

$$\begin{aligned} h(x,\xi) &= ax + c & x < \xi \\ &= b(x-\xi) + d & x \geq \xi \end{aligned} \tag{3.7}$$

From the two boundary conditions imposed on $h(x,\xi)$ with respect to x together with Eq. 3.6 and the continuity condition at $x = \xi$, the four constants a, b, c and d can be determined uniquely.

For example, for a cantilever beam, the boundary conditions are $h(0,\xi) = h'(0,\xi) = 0$. From these conditions together with Eq. 3.6 and the continuity requirement, it follows that

$$\begin{aligned} h(x,\xi) &= x - \xi & x \geq \xi \\ &= 0 & x < \xi \end{aligned} \tag{3.8}$$

Utilizing Green's function $h(x,\xi)$, the deflection $w(x)$ of a statically determinate beam may be expressed in terms of a convolution integral as

$$w(x) = -\int_0^L h(x,\xi) \frac{M(\xi)}{E_0 I_0} \left[1 + f(\xi)\right] d\xi \tag{3.9}$$

where L is the beam length. Since $f(x)$ is assumed to be a Gaussian stochastic field, so is $w(x)$. The expected value of $w(x)$ is

$$\mathbf{E}\left[w(x)\right] = -\frac{1}{E_0 I_0} \int_0^L h(x,\xi) M(\xi) d\xi \tag{3.10}$$

and the covariance function $R_{ww}(x,y)$:

$$R_{ww}(x,y) = \frac{1}{E_0^2 I_0^2} \int_0^L \int_0^L h(x,\xi) h(y,\eta) M(\xi) M(\eta) R_{ff}(\xi,\eta) d\xi d\eta \tag{3.11}$$

This form is useful in estimating the autocorrelation-free upper bounds of the response variability. Since

$$R_{ff}(\xi - \eta) \leq R_{ff}(0) = \sigma_{ff}^2 \tag{3.12}$$

the variance $\sigma_{w(x)}^2 = R_{ww}(x, x)$ is bounded by the following inequality:

$$\sigma_{w(x)}^2 \leq \frac{\sigma_{ff}^2}{E_0^2 I_0^2} \int_0^L \int_0^L |h(x,\xi)h(x,\eta)M(\xi)M(\eta)| d\xi d\eta \tag{3.13}$$

If, furthermore, neither the bending moment $M(\xi)$ nor Green's function $h(x, \xi)$ change sign in the interval $[0, L]$, then

$$\sigma_{w(x)}^2 \leq \frac{\sigma_{ff}^2}{E_0^2 I_0^2} \int_0^L \int_0^L h(x,\xi)h(x,\eta)M(\xi)M(\eta) d\xi d\eta = \sigma_{ff}^2 \mathbf{E}^2[w(x)] \tag{3.14}$$

and therefore

$$\frac{\sigma_{w(x)}}{\mathbf{E}[w(x)]} \leq \sigma_{ff} \tag{3.15}$$

This implies that the coefficient of variation of the displacement $w(x)$ is bounded by the coefficient of variation of the flexibility if neither $M(x)$ nor $h(x, \xi)$ change sign in the interval [0,L].

3.2 Statically Indeterminate Beams

In the analysis of statically indeterminate structures, statically indeterminate forces $B_k \quad (k = 1, 2, \ldots, N)$ are introduced in order to satisfy N boundary conditions, which cannot be satisfied from equilibrium only. If the boundary conditions are

$$V(x_i) = V_i \qquad i = 1, 2, \ldots, N \tag{3.16}$$

where $V(x_i)$ represents deflection or its corresponding spatial derivative, then the statically indeterminate forces can be derived from the following set of linear equations:

$$V_0(x_i) + \sum_{k=1}^{N} B_k V_k(x_i) = V(x_i) = V_i \qquad i = 1, 2, \ldots, N \tag{3.17}$$

where the indices k and i refer to locations where statically indeterminate forces are applied and where boundary conditions are satisfied, respectively. In Eq. 3.17, $V_0(x_i)$ represents deflection or its corresponding derivative at x_i of the associated statically determinate structure under external loads only ($B_k = 0$ for all values of k). Also in Eq. 3.17, the quantities $V_k(x_i)$ represent deflection or its corresponding derivative at x_i of the associated statically determinate structure under the statically indeterminate forces $B_k = 1$ and $B_j = 0$ for all $j \neq k$. Solving Eq. 3.17 for B_k, the deflection $w(x)$ at any point can be written as

$$w(x) = w_0(x) + \sum_{k=1}^{N} B_k w_k(x) \tag{3.18}$$

where $w_0(x)$ is the deflection of the statically determinate system under external forces only and $w_k(x)$ the deflection of the same system due to $B_k = 1$ and $B_j = 0 \quad (j \neq k)$. The statically determinate forces B_k obtained from Eq. 3.17 are usually nonlinear combinations of $V_0(x_i), V_k(x_i)$ and $V(x_i)$. Among these, $V_0(x_i)$ and $V_k(x_i)$ are Gaussian random variables with zero mean in the same sense as $w(x)$ in Eq. 3.9 can be so interpreted when x is fixed. Writing $X_1, X_2, \ldots$ for $V_0(x_i)$ and $V_k(x_i) \quad (i, k = 1, 2, \ldots, N)$ for simplicity, $w(x)$ in Eq. 3.18 can be symbolically expressed as $w(x) = g(X_1, X_2, \ldots, X_M)$ where $M = N + N^2$.

The statistics of $w(x)$ can be obtained analytically in principle. However, such an analytical procedure is not very practical particularly as the number of random variables X_k increases. Since X_k are Gaussian random variables with zero mean, a covariance matrix describes their statistical characteristics completely, and can be evaluated numerically as shown later. Upon constructing the covariance matrix **C**, appropriate approximations are used to estimate the statistical characteristics, primarily the second moments, of $w(x)$. A typical technique that can be used for this purpose is the FOSM method where a truncated Taylor expansion of $w(x) =$

$g(X_1, X_2, \cdots X_M)$, about the expected values of X_s', is employed together with the covariance information provided by matrix $\mathbf{C}$. As an alternative to, and/or for the validation of, the FOSM method, Monte Carlo techniques can be used. For this purpose, the covariance matrix $\mathbf{C}$ is diagonalized by means of an eigenvalue analysis:

$$\mathbf{\Phi}^T \mathbf{C} \mathbf{\Phi} = \mathbf{\Lambda} \tag{3.19}$$

where $\mathbf{\Phi}$ is the normalized modal matrix and $\mathbf{\Lambda}$ a diagonal matrix consisting of the eigenvalues of $\mathbf{C}$. Hence, the following transform pair exists for $\mathbf{Z}$ and $\mathbf{X}$:

$$\mathbf{Z} = \mathbf{\Phi}^T \mathbf{X} \qquad\qquad \mathbf{X} = \mathbf{\Phi} \mathbf{Z} \tag{3.20}$$

If $\mathbf{Z}$ is a Gaussian vector consisting of independent components with covariance matrix $\mathbf{\Lambda}$, then it can be shown that $\mathbf{C}$ is the covariance matrix of $\mathbf{X}$:

$$\mathbf{E}[\mathbf{X}\mathbf{X}^T] = \mathbf{\Phi}\mathbf{E}[\mathbf{Z}\mathbf{Z}^T]\mathbf{\Phi}^T = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^T = \mathbf{C} \tag{3.21}$$

A Monte Carlo simulation is carried out beginning with generation of vector $\mathbf{Z}$. Each realization of $\mathbf{Z}$ is transformed into a realization of $\mathbf{X}$ by means of $\mathbf{X} = \mathbf{\Phi}\mathbf{Z}$, and then substituted into Eq. 3.18 recalling that $w(x) = g(X_1, X_2, \ldots, X_M)$. This results in a realization of $w(x)$ from which sample statistics such as moments are estimated.

The safety index for the response quantities can be also evaluated, if the limit state conditions are given, with the aid of the Lagrange multiplier method (Shinozuka, 1983), while other methods (e.g., Rackwitz and Fiessler, 1977) can certainly be utilized for this purpose.

3.3 Statically Indeterminate Frames

The use of Green's function in the response analysis of frames becomes cumbersome and thus in this case it is more appropriate to use a more general flexibility method. For the static analysis of statically indeterminate frames consisting of n components (beams and columns), statically indeterminate forces $B_k \quad (k = 1, 2, \ldots, N)$ are introduced in conjunction with an appropriate associated statically determinate structure in order to satisfy N boundary conditions (N=degree of indeterminacy).

The flexibility of component l is assumed to be

$$\frac{1}{(EI)_l} = \frac{1}{\alpha_l E_0 I_0}[1 + f^{(l)}(x)] \tag{3.22}$$

with

$$\mathbf{E}[\frac{1}{(EI)_l}] = \frac{1}{\alpha_l E_0 I_0} \tag{3.23}$$

The homogeneous stochastic field $f^{(l)}(x)$ has zero mean and represents the spatial variability of the flexibility of component l. The standard deviation $\sigma_{ff}^{(l)}$ of $f^{(l)}(x)$ is then identical with the coefficient of variation of $1/(EI)_l$. In the following discussion, $f^{(l)}(x)$ and $f^{(m)}(x)$ are assumed to have the same autocorrelation function and to be statistically independent if $l \neq m$ for the sake of simplicity.

Coefficients δ_{ik} are then introduced as:

$$\delta_{ik} = \sum_{l=1}^{n} \delta_{ik}^{(l)} \tag{3.24}$$

with

$$\delta_{ik}^{(l)} = \int_0^{L_l} M_i^{(l)}(\xi) M_k^{(l)}(\xi) \frac{1}{\alpha_l E_0 I_0}[1 + f^{(l)}(\xi)] d\xi \tag{3.25}$$

where L_l = length of component l and $M_i^{(l)}(x)$ = bending moment distribution along component l of the associated statically determinate structure under a unit indeterminate force $B_i \quad$ (with $B_j = 0$ if $j \neq i$). The coefficients δ_{ik} represent the deflections or slopes of the associated statically determinate structure at the location of the indeterminate force B_i, under a unit indeterminate force $B_k = 1 \quad$ (with $B_j = 0$ if $j \neq k$).

Upon imposing the boundary conditions at the locations of the indeterminate forces, the unknown indeterminate forces B_k are determined from a set of linear equations:

$$\sum_{k=1}^{N} B_k \delta_{ik} = -\delta_{i0} \qquad i = 1, 2, \cdots, N \tag{3.26}$$

The coefficients δ_{i0} represent the deflection or slope of the associated statically determinate system at the location of the indeterminate force B_i, due to external loads only. In order to obtain the coefficients δ_{ik} and δ_{i0}, the moment diagrams of the statically determinate system under external loads only and also under each of the unit forces B_k, must in principle be constructed. Solution of Eq. 3.26 yields B_k as nonlinear combinations of δ_{ik} and δ_{i0} which depend on the elastic properties of the structure and are Gaussian random variables.

Any desired deflection w_q or moment M_q at point q of the structure may be expressed as:

$$w_q = w_{q0} + \sum_{k=1}^{N} B_k w_{qk} \tag{3.27}$$

$$M_q = M_{q0} + \sum_{k=1}^{N} B_k M_{qk} \tag{3.28}$$

where M_{q0} and M_{qk} are moments at point q of the associated determinate structure due to external loads only and unit indeterminate forces $B_k = 1$ with $B_j = 0$ if $j \neq k$, respectively. These moments as well as any moment calculated from the associated statically determinate structure do not depend on the elastic properties of the structure and thus, they are deterministic. The quantities w_{q0} and w_{qk} can be expressed as:

$$w_{qk} = \sum_{l=1}^{n} w_{qk}^{(l)} \tag{3.29}$$

with

$$w_{qk}^{(l)} = \int_0^{L_l} M_k^{(l)}(\xi) M_q^{(l)}(\xi) \frac{1}{\alpha_l E_0 I_0} [1 + f^{(l)}(\xi)] d\xi \quad k = 0, 1, 2, \ldots, N \tag{3.30}$$

and they represent deflection at point q due to external loads only when $k = 0$ and due to unit indeterminate forces B_k (with $B_j = 0$ if $j \neq k$) when $k \neq 0$. $M_q^{(l)}(x)$ is the moment of component l of the associated statically determinate structure due to a unit concentrated load at point q, and $M_0^{(l)}(x)$ indicates the bending moment distribution along component l of the associated statically determinate structure under external loads only.

As can be seen from Eqs. 3.27 and 3.28, in order to obtain the statistics of moment M_q and deflection w_q, the statistics of the quantities w_{qk} as well as of the indeterminate forces B_k should be obtained first. In this respect, since B_k are obtained as a function of the quantities δ_{ik} and δ_{i0}, the statistics of these quantities are also needed.

The expected values of the quantities δ_{ik} are

$$\mathbf{E}[\delta_{ik}] = \sum_{l=1}^{n} \mathbf{E}[\delta_{ik}^{(l)}] = \sum_{l=1}^{n} \int_0^{L_l} M_i^{(l)}(\xi) M_k^{(l)}(\xi) \frac{1}{\alpha_l E_0 I_0} d\xi \tag{3.31}$$

where $i = 1, 2, \cdots, N$ and $k = 0, 1, 2, \cdots, N$. Assuming independence between the stochastic fields of each and every pair of structural components, then:

$$\sigma^2{}_{\delta_{ik}} = \sum_{l=1}^{n} \sigma^2{}_{\delta_{ik}^{(l)}} \tag{3.32}$$

where

$$\sigma^2{}_{\delta_{ik}^{(l)}} = \frac{1}{\alpha_l^2 E_0^2 I_0^2} \int_0^{L_l} \int_0^{L_l} M_i^{(l)}(\xi) M_k^{(l)}(\xi) M_i^{(l)}(\eta) M_k^{(l)}(\eta) R_{ff}(\xi, \eta) d\xi d\eta \tag{3.33}$$

The covariance matrix of the random variables δ_{ik} becomes

$$Cov(\delta_{ik}, \delta_{mn}) = \sum_{l=1}^{n} Cov(\delta_{ik}^{(l)}, \delta_{mn}^{(l)}) \tag{3.34}$$

with

$$Cov(\delta_{ik}^{(l)}, \delta_{mn}^{(l)}) = \frac{1}{\alpha_l^2 E_0^2 I_0^2} \int_0^{L_l} \int_0^{L_l} M_i^{(l)}(\xi) M_k^{(l)}(\xi) M_m^{(l)}(\eta) M_n^{(l)}(\eta) R_{ff}(\xi, \eta) d\xi d\eta \tag{3.35}$$

Similarly, the expected values of the quantities w_{qk} are

$$\mathbf{E}[w_{qk}] = \sum_{l=1}^{n} \mathbf{E}[w_{qk}^{(l)}] = \sum_{l=1}^{n} \int_0^{L_l} M_k^{(l)}(\xi) M_q^{(l)}(\xi) \frac{1}{\alpha_l E_0 I_0} d\xi \tag{3.36}$$

and the covariance matrix of the quantities w_{qk} becomes

$$Cov(w_{qk}, w_{qm}) = \sum_{l=1}^{n} Cov(w_{qk}^{(l)}, w_{qm}^{(l)}) \tag{3.37}$$

with

$$Cov(w_{qk}^{(l)}, w_{qm}^{(l)}) = \frac{1}{\alpha_l^2 E_0^2 I_0^2} \int_0^{L_l} \int_0^{L_l} M_k^{(l)}(\xi) M_q^{(l)}(\xi) M_m^{(l)}(\eta) M_q^{(l)}(\eta) R_{ff}(\xi, \eta) d\xi d\eta \tag{3.38}$$

Also, the covariance matrix of the quantities δ_{ik} and w_{qm} with $i = 1, 2, \ldots, N$ and $k, m = 0, 1, 2, \ldots, N$, can be expressed in a similar way as:

$$Cov(\delta_{ik}, w_{qm}) = \sum_{l=1}^{n} Cov(\delta_{ik}^{(l)}, w_{qm}^{(l)}) \tag{3.39}$$

with

$$Cov(\delta_{ik}^{(l)}, w_{qm}^{(l)}) = \frac{1}{\alpha_l^2 E_0^2 I_0^2} \int_0^{L_l} \int_0^{L_l} M_i^{(l)}(\xi) M_k^{(l)}(\xi) M_q^{(l)}(\eta) M_m^{(l)}(\eta) R_{ff}(\xi, \eta) d\xi d\eta \tag{3.40}$$

With all the information given above, the statistics of M_q and w_q can be obtained either through Monte Carlo simulation or through the FOSM method. When Monte Carlo techniques are used, covariance matrix $\mathbf{C}$, whose elements are the quantities $Cov(\delta_{ik}, \delta_{mn}), Cov(\delta_{ik}, w_{qn})$ and $Cov(w_{qn}, w_{ql})$ $(i, m = 1, 2, \ldots, N$ and $k, n, l = 0, 1, 2, \ldots, N)$, is diagonalized by means of eigenvalue analysis, as shown previously.

Consider the case of the FOSM method. Expanding the w_q and M_q given by Eqs. 3.27 and 3.28, respectively, in a Taylor series around the mean values and keeping only linear terms:

$$w_q - \bar{w}_q = \sum_{k=1}^{N} (\frac{\partial w_q}{\partial B_k})^* (B_k - \bar{B}_k) + \sum_{m=0}^{N} (\frac{\partial w_q}{\partial w_{qm}})^* (w_{qm} - \bar{w}_{qm}) \tag{3.41}$$

and

$$M_q - \bar{M}_q = \sum_k (\frac{\partial M_q}{\partial B_k})^* (B_k - \bar{B}_k) \tag{3.42}$$

where the quantities with superbars represent mean values and $(\frac{\partial w_q}{\partial B_k})^*$ indicates the partial derivatives evaluated at the mean values of the variables involved, and hence, is equal to $\bar{w}_{qk}$; similar expressions apply to other partial derivatives. The mean values $\bar{B}_k$ of the indeterminate forces B_k can be obtained from Eq. 3.26 with the quantities δ_{ik} and δ_{i0} replaced by their expected values. Also, the mean values $\bar{w}_q$ and $\bar{M}_q$ of the deflection w_q and the moment M_q, respectively, can be obtained from Eqs. 3.27 and 3.28 when the quantities B_k, w_{q0} and w_{qk} are replaced by their expected values. Using Eqs. 3.41 and 3.42, an approximation can be obtained of the variances of w_q and M_q, respectively, as follows:

$$\begin{aligned}
Var(w_q) &= \mathbf{E}[(w_q - \bar{w}_q)^2] \\
&= \sum_{i=1} \sum_{k=1} (\frac{\partial w_q}{\partial B_i})^* (\frac{\partial w_q}{\partial B_k})^* \mathbf{E}[(B_i - \bar{B}_i)(B_k - \bar{B}_k)] \\
&+ \sum_{k=1} \sum_{m=0} (\frac{\partial w_q}{\partial B_k})^* (\frac{\partial w_q}{\partial w_{qm}})^* \mathbf{E}[(B_k - \bar{B}_k)(w_{qm} - \bar{w}_{qm})] \\
&+ \sum_{m=0} \sum_{n=0} (\frac{\partial w_q}{\partial w_{qm}})^* (\frac{\partial w_q}{\partial w_{qn}})^* \mathbf{E}[(w_{qm} - \bar{w}_{qm})(w_{qn} - \bar{w}_{qn})] \\
&= \sum_{i=1} \sum_{k=1} (\frac{\partial w_q}{\partial B_i})^* (\frac{\partial w_q}{\partial B_k})^* Cov(B_i, B_k) \\
&+ \sum_{k=1} \sum_{m=0} (\frac{\partial w_q}{\partial B_k})^* (\frac{\partial w_q}{\partial w_{qm}})^* Cov(B_k, w_{qm}) \\
&+ \sum_{m=0} \sum_{n=0} (\frac{\partial w_q}{\partial w_{qm}})^* (\frac{\partial w_q}{\partial w_{qn}})^* Cov(w_{qm}, w_{qn})
\end{aligned} \tag{3.43}$$

and

$$Var(M_q) = \sum_i \sum_k (\frac{\partial M_q}{\partial B_i})^* (\frac{\partial M_q}{\partial B_k})^* Cov(B_i, B_k) \tag{3.44}$$

where the quantities $Cov(w_{qm}, w_{qn})$ in Eq. 3.43 are obtained from Eq. 3.38, whereas the quantities $Cov(B_i, B_k)$ and $Cov(B_k, w_{qm})$ need to be determined.

In order to obtain the covariance matrix of the indeterminate forces B_k, Eq. 3.26 is written in matrix form:

$$\mathbf{AB} = -\mathbf{W} \tag{3.45}$$

where

$$\mathbf{A} = \begin{pmatrix} \delta_{11} & \dots & \delta_{1N} \\ \vdots & \vdots & \vdots \\ \delta_{N1} & \dots & \delta_{NN} \end{pmatrix} \qquad \mathbf{B} = \begin{Bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{Bmatrix} \qquad \mathbf{W} = \begin{Bmatrix} \delta_{10} \\ \delta_{20} \\ \vdots \\ \delta_{N0} \end{Bmatrix} \tag{3.46}$$

Thus

$$\mathbf{B} = -\mathbf{A}^{-1}\mathbf{W} \tag{3.47}$$

Taking the derivatives of the above expression with respect to the random variables,

$$\frac{\partial \mathbf{B}}{\partial \delta_{ik}} = -\left(\frac{\partial}{\partial \delta_{ik}}\mathbf{A}^{-1}\right)\mathbf{W} \tag{3.48}$$

$$\frac{\partial \mathbf{B}}{\partial \delta_{k0}} = -\mathbf{A}^{-1}\left(\frac{\partial \mathbf{W}}{\partial \delta_{k0}}\right) \tag{3.49}$$

Since $\mathbf{AA}^{-1} = \mathbf{I}$,

$$\frac{\partial}{\partial \delta_{ik}}(\mathbf{AA}^{-1}) = \left(\frac{\partial}{\partial \delta_{ik}}\mathbf{A}\right)\mathbf{A}^{-1} + \mathbf{A}\left(\frac{\partial}{\partial \delta_{ik}}\mathbf{A}^{-1}\right) = 0 \tag{3.50}$$

Hence,

$$\frac{\partial}{\partial \delta_{ik}}\mathbf{A} = \Delta_{ik} \tag{3.51}$$

where Δ_{ik} is a matrix whose components are all zero except for the i-kth component which is equal to unity. With the aid of matrix Δ_{ik},

$$\frac{\partial}{\partial \delta_{ik}}\mathbf{A}^{-1} = -\mathbf{A}^{-1}\Delta_{ik}\mathbf{A}^{-1} \tag{3.52}$$

$$\left(\frac{\partial}{\partial \delta_{ik}}\mathbf{A}^{-1}\right)\mathbf{W} = -\mathbf{A}^{-1}\Delta_{ik}\mathbf{A}^{-1}\mathbf{W} = -\mathbf{A}^{-1}\Delta_{ik}(-\mathbf{B}) = \mathbf{A}^{-1}\Delta_{ik}\mathbf{B} \tag{3.53}$$

Substituting Eq. 3.53 into Eq. 3.48 we obtain

$$\frac{\partial \mathbf{B}}{\partial \delta_{ik}} = -\left(\frac{\partial}{\partial \delta_{ik}} \mathbf{A}^{-1}\right) \mathbf{W} = -\mathbf{A}^{-1} \Delta_{ik} \mathbf{B} \tag{3.54}$$

The element l of the above vector is

$$-\left[\left(\frac{\partial}{\partial \delta_{ik}} \mathbf{A}^{-1}\right) \mathbf{W}\right]_l = -\sum_n \sum_j \beta_{ln} \Delta_{nj} B_j = -\beta_{li} B_k \tag{3.55}$$

where β_{ln} is the $l-n$ component of $\mathbf{A}^{-1}$:

$$(\mathbf{A}^{-1})_{ln} = \beta_{ln} \tag{3.56}$$

Also,

$$\frac{\partial \mathbf{W}}{\partial \delta_{k0}} = \Delta_k \qquad \mathbf{A}^{-1}\left(\frac{\partial \mathbf{W}}{\partial \delta_{k0}}\right) = \mathbf{A}^{-1} \Delta_k \tag{3.57}$$

where Δ_k is a vector whose components are zero except for the kth component which is equal to unity. Substituting Eq. 5.57 into Eq. 3.49,

$$\frac{\partial \mathbf{B}}{\partial \delta_{k0}} = -\mathbf{A}^{-1}\left(\frac{\partial \mathbf{W}}{\partial \delta_{k0}}\right) = -\mathbf{A}^{-1} \Delta_k \tag{3.58}$$

Element l of the above vector is

$$\left[-\mathbf{A}^{-1} \frac{\partial \mathbf{W}}{\partial \delta_{k0}}\right]_l = [-\mathbf{A}^{-1} \Delta_k]_l = -\sum_n \beta_{ln} \Delta_n = -\beta_{lk} \tag{3.59}$$

Expanding B_l in a Taylor series around the mean values and keeping only linear terms

$$B_l - \bar{B}_l = \sum_i \sum_k \left(\frac{\partial \mathbf{B}}{\partial \delta_{ik}}\right)^*_l (\delta_{ik} - \bar{\delta}_{ik}) + \sum_k \left(\frac{\partial \mathbf{B}}{\partial \delta_{k0}}\right)^*_l (\delta_{k0} - \bar{\delta}_{k0}) \tag{3.60}$$

With the aid of Eqs. 3.54, 3.55, 3.58 and 3.59, Eq. 3.60 becomes

$$B_l - \bar{B}_l = -\sum_i \sum_k \bar{\beta}_{li} \bar{B}_k (\delta_{ik} - \bar{\delta}_{ik}) - \sum_k \bar{\beta}_{lk} (\delta_{k0} - \bar{\delta}_{k0}) \tag{3.61}$$

In Eq. 3.61, $\bar{\beta}_{lk}$ is obtained from Eq. 3.56 by replacing all the elements of $\mathbf{A}$ by their respective expected values. Using the above equation, we obtain the covariance matrix of B_l and $B_{l'}$ $\quad (l, l' = 1, 2, \ldots, N)$:

$$
\begin{aligned}
Cov(B_l, B_{l'}) &= \mathbf{E}\big[(B_l - \bar{B}_l)(B_{l'} - \bar{B}_{l'})\big] \\
&= \mathbf{E}\Big\{ \big[-\sum_i \sum_k \bar{\beta}_{li}\bar{B}_k(\delta_{ik} - \bar{\delta}_{ik}) - \sum_k \bar{\beta}_{lk}(\delta_{k0} - \bar{\delta}_{k0})\big] \\
&\qquad \cdot \big[-\sum_m \sum_n \bar{\beta}_{l'm}\bar{B}_n(\delta_{mn} - \bar{\delta}_{mn}) - \sum_m \bar{\beta}_{l'm}(\delta_{m0} - \bar{\delta}_{m0})\big] \Big\} \\
&= \mathbf{E}\Big\{ \sum_i \sum_k \sum_m \sum_n \bar{\beta}_{li}\bar{B}_k\bar{\beta}_{l'm}\bar{B}_n(\delta_{ik} - \bar{\delta}_{ik})(\delta_{mn} - \bar{\delta}_{mn}) \\
&\quad + \sum_i \sum_k \sum_m \bar{\beta}_{li}\bar{B}_k\bar{\beta}_{l'm}(\delta_{ik} - \bar{\delta}_{ik})(\delta_{m0} - \bar{\delta}_{m0}) \\
&\quad + \sum_k \sum_m \sum_n \bar{\beta}_{lk}\bar{\beta}_{l'm}\bar{B}_n(\delta_{mn} - \bar{\delta}_{mn})(\delta_{k0} - \bar{\delta}_{k0}) \\
&\quad + \sum_k \sum_m \bar{\beta}_{lk}\bar{\beta}_{l'm}(\delta_{k0} - \bar{\delta}_{k0})(\delta_{m0} - \bar{\delta}_{m0}) \Big\} \\
&= \sum_i \sum_k \sum_m \sum_n \bar{\beta}_{li}\bar{B}_k\bar{\beta}_{l'm}\bar{B}_n\mathbf{E}\big[(\delta_{ik} - \bar{\delta}_{ik})(\delta_{mn} - \bar{\delta}_{mn})\big] \\
&+ \sum_i \sum_k \sum_m \bar{\beta}_{li}\bar{B}_k\bar{\beta}_{l'm}\mathbf{E}\big[(\delta_{ik} - \bar{\delta}_{ik})(\delta_{m0} - \bar{\delta}_{m0})\big] \\
&+ \sum_i \sum_k \sum_m \bar{\beta}_{l'i}\bar{B}_k\bar{\beta}_{lm}\mathbf{E}\big[(\delta_{ik} - \bar{\delta}_{ik})(\delta_{m0} - \bar{\delta}_{m0})\big] \\
&+ \sum_k \sum_m \bar{\beta}_{lk}\bar{\beta}_{l'm}\mathbf{E}\big[(\delta_{k0} - \bar{\delta}_{k0})(\delta_{m0} - \bar{\delta}_{m0})\big]
\end{aligned} \tag{3.62}
$$

Finally,

$$
\begin{aligned}
Cov(B_l, B_l') &= \sum_i \sum_k \sum_m \sum_n \bar{\beta}_{li}\bar{B}_k\bar{\beta}_{l'm}\bar{B}_n Cov(\delta_{ik}, \delta_{mn}) \\
&+ \sum_i \sum_k \sum_m \bar{\beta}_{li}\bar{B}_k\bar{\beta}_{l'm} Cov(\delta_{ik}, \delta_{m0}) \\
&+ \sum_i \sum_k \sum_m \bar{\beta}_{l'i}\bar{B}_k\bar{\beta}_{lm} Cov(\delta_{ik}, \delta_{m0}) \\
&+ \sum_k \sum_m \bar{\beta}_{lk}\bar{\beta}_{l'm} Cov(\delta_{k0}, \delta_{m0})
\end{aligned} \tag{3.63}
$$

Using Eq. 3.61, the quantities $Cov(B_n, w_{qm})$ are obtained with $n = 1, 2, \ldots N$ and $m = 0, 1, \ldots N$, as below.

$$
\begin{aligned}
\mathbf{E}\left[(B_n - \bar{B}_n)(w_{qm} - \bar{w}_{qm})\right] &= Cov(B_n, w_{qm}) \\
&= \mathbf{E}\left[-\sum_i \sum_k \bar{\beta}_{ni} \bar{B}_k (\delta_{ik} - \bar{\delta}_{ik})(w_{qm} - \bar{w}_{qm}) - \sum_k \bar{\beta}_{nk} (\delta_{k0} - \bar{\delta}_{k0})(w_{qm} - \bar{w}_{qm})\right] \\
&= -\sum_i \sum_k \bar{\beta}_{ni} \bar{B}_k Cov(\delta_{ik}, w_{qm}) - \sum_k \bar{\beta}_{nk} Cov(\delta_{k0}, w_{qm})
\end{aligned}
\tag{3.64}
$$

where $Cov(\delta_{ik}, w_{qm})$ and $Cov(\delta_{k0}, w_{qm})$ in Eq. 3.64 are given by Eq. 3.39.

SECTION 4

NUMERICAL EXAMPLES

In the examples below, the following correlation function is considered for $f^{(l)}(x)$ (for other possible examples of the correlation functions see Shinozuka, 1987):

$$R_{ff}(x-y) = \frac{1-3\left(\frac{x-y}{b}\right)^2}{\left[1+\left(\frac{x-y}{b}\right)^2\right]^3} \cdot \sigma^2{}_{ff} \tag{4.1}$$

The corresponding power spectrum density is

$$S(k) = \frac{1}{2 \cdot 2!} b^3 k^2 e^{-b|k|} \cdot \sigma^2{}_{ff} \tag{4.2}$$

The above functions are plotted in Figs. 4-1 and 4-2 respectively. The parameter b (referred to as correlation distance in this study) in Eq. 4.1 controls the shape of the autocorrelation function. The case where $b \to \infty$ represents a fully correlated stochastic field, whereas the case where $b \to 0$ corresponds to a "finite power white noise" (Shinozuka, 1986). In the example of the fixed beam, another correlation function of the following form is also considered.

$$R_{ff}(\xi) = \sigma^2{}_{ff} \cdot \exp\left(-\frac{\xi^2}{b^2}\right) \tag{4.3}$$

The numerical value for σ_{ff}, which is also the coefficient of variation of the flexibility $1/(EI)$, is chosen to be 0.1 for all examples.

4.1 EXAMPLE 1: Fixed Beam

In this example, a fixed beam as shown in Fig. 4-3a is considered, subjected to a distributed load $p(x)$. The associated statically determinate system is chosen to be a cantilever beam shown in Fig. 4-3b. The bending moments of the cantilever beam due to the distributed load and to unit indeterminate forces B_1 and B_2 are:

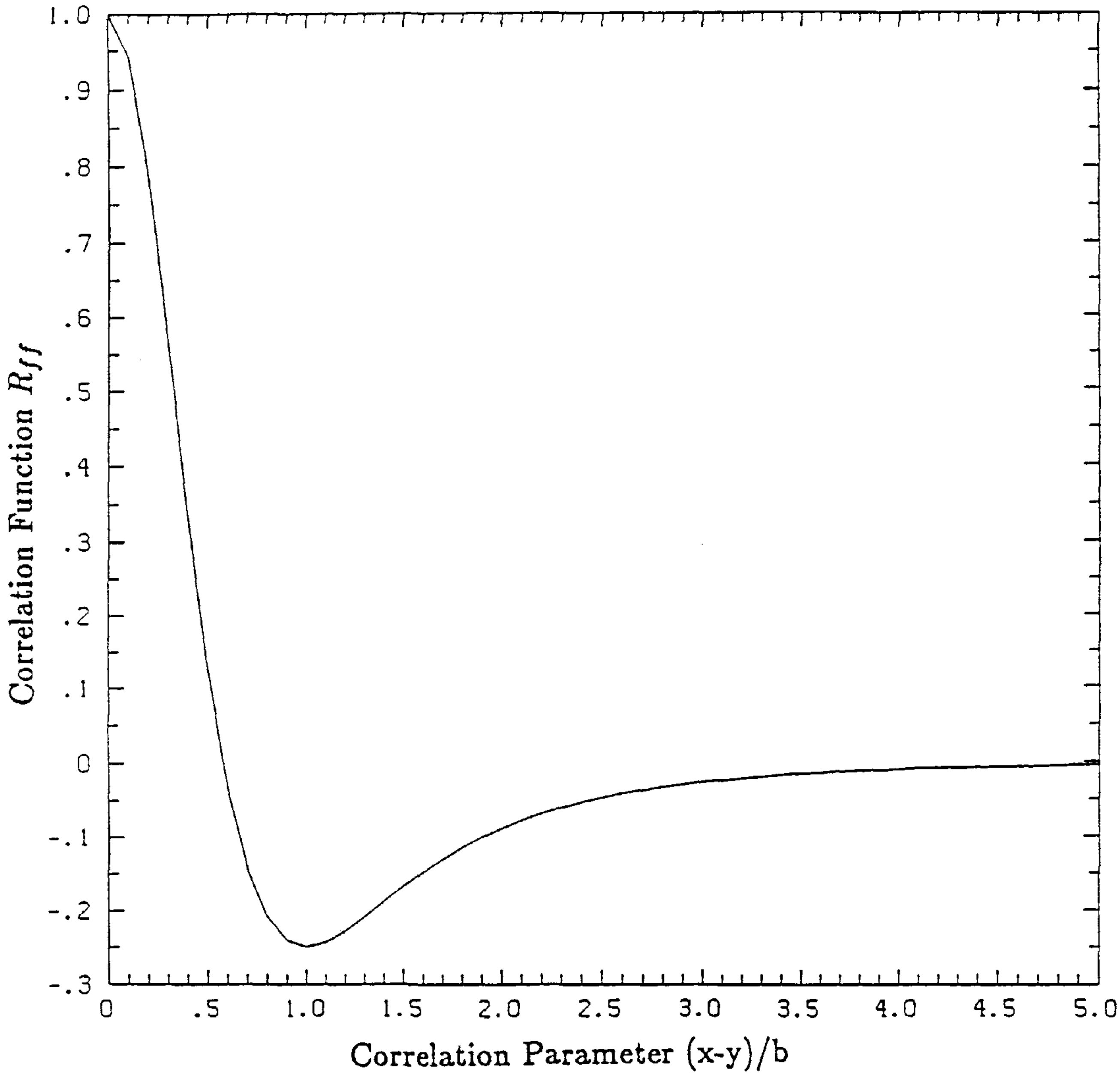


Fig. 4-1 Correlation Function $R_{ff}(x, y)$ (Defined by Eq. 4.1) as Function of Dimensionless Parameter $(x - y)/b$

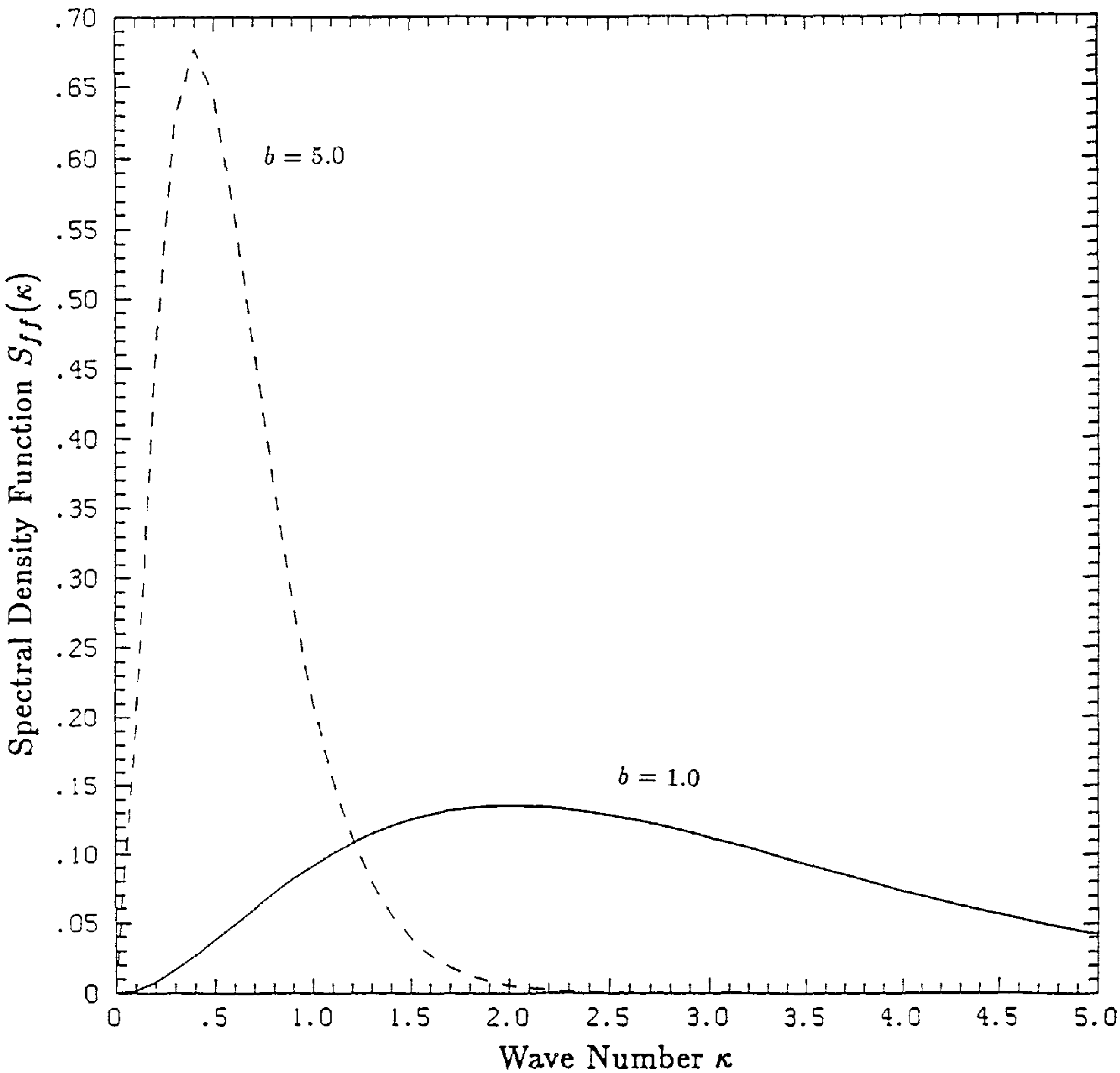


Fig. 4-2 Spectral Density Function $S_{ff}(\kappa)$ (Defined by Eq. 4.2) as Function of Wave Number κ

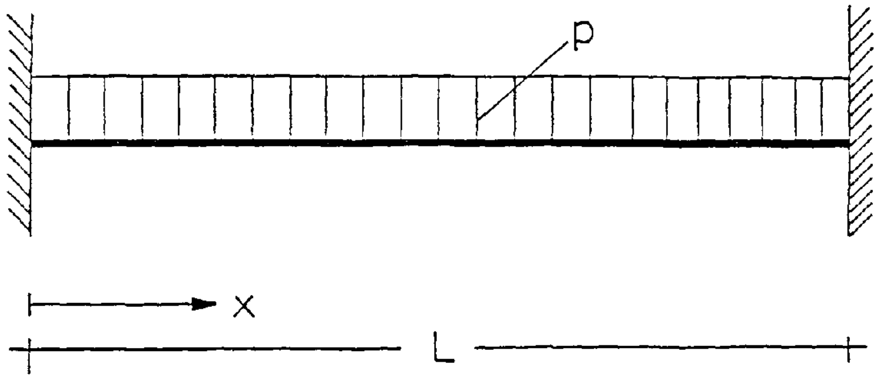


Fig. 4-3a Fixed Beam with Uniform Load

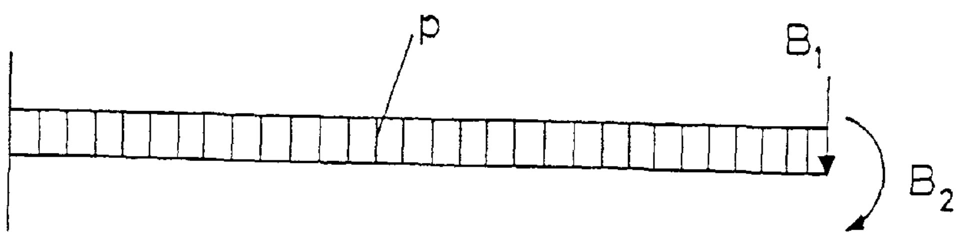


Fig. 4-3b Statically Determinate Structure Associated with Introduced Indeterminate Forces

$$M_0(x) = -\frac{p}{2}(L-x)^2 \tag{4.4}$$

$$M_1(x) = -(L-x) \tag{4.5}$$

$$M_2(x) = -1 \tag{4.6}$$

Utilizing the procedure described before, specifically Eq. 3.8, and Eqs. 4.4-4.6, the deflection $w_0(x)$ due to the applied load, and $w_1(x)$ and $w_2(x)$ due to $B_1 = 1$ and $B_2 = 1$, respectively, can be expressed in terms of the following convolution integrals:

$$w_0(x) = \frac{p}{2E_0I_0}\int_0^x (x-\xi)(L-\xi)^2[1+f(\xi)]d\xi \tag{4.7}$$

$$w_1(x) = \frac{1}{E_0I_0}\int_0^x (x-\xi)(L-\xi)[1+f(\xi)]d\xi \tag{4.8}$$

$$w_2(x) = \frac{1}{E_0I_0}\int_0^x (x-\xi)[1+f(\xi)]d\xi \tag{4.9}$$

Hence, the corresponding end deflections $w_0(L)$, $w_1(L)$ and $w_2(L)$ are

$$w_0(L) = \frac{p}{2E_0I_0}\int_0^L (L-\xi)^3\,[1+f(\xi)]\,d\xi \tag{4.10}$$

$$w_1(L) = \frac{1}{E_0I_0}\int_0^L (L-\xi)^2\,[1+f(\xi)]\,d\xi \tag{4.11}$$

$$w_2(L) = \frac{1}{E_0I_0}\int_0^L (L-\xi)\,[1+f(\xi)]\,d\xi \tag{4.12}$$

Similarly, the corresponding end slopes $w_0'(L), w_1'(L)$ and $w_2'(L)$ are

$$w_0'(L) = \frac{p}{2E_0I_0}\int_0^L (L-\xi)^2\,[1+f(\xi)]\,d\xi \tag{4.13}$$

$$w_1'(L) = \frac{1}{E_0I_0}\int_0^L (L-\xi)\,[1+f(\xi)]\,d\xi \tag{4.14}$$

$$w_2'(L) = \frac{1}{E_0I_0}\int_0^L [1+f(\xi)]\,d\xi \tag{4.15}$$

The deflections and slopes of the original system can be expressed as:

$$w(x) = w_0(x) + B_1 w_1(x) + B_2 w_2(x) \tag{4.16}$$

$$w'(x) = w_0'(x) + B_1 w_1'(x) + B_2 w_2'(x) \tag{4.17}$$

The boundary conditions $w(L) = 0$ and $w'(L) = 0$ should be satisfied:

$$w(L) = w_0(L) + B_1 w_1(L) + B_2 w_2(L) = 0 \tag{4.18}$$

$$w'(L) = w_0'(L) + B_1 w_1'(L) + B_2 w_2'(L) = 0 \tag{4.19}$$

Solution of the above system yields:

$$B_1 = \frac{w_0'(L) w_2(L) - w_0(L) w_2'(L)}{w_1(L) w_2'(L) - w_1'(L) w_2(L)} \tag{4.20}$$

$$B_2 = \frac{w_0(L) w_1'(L) - w_1(L) w_0'(L)}{w_1(L) w_2'(L) - w_1'(L) w_2(L)} \tag{4.21}$$

The deflection $w(x)$ can finally be written as:

$$\begin{aligned} w(x) &= w_0(x) + \frac{w_0'(L) w_2(L) - w_0(L) w_2'(L)}{w_1(L) w_2'(L) - w_1'(L) w_2(L)} w_1(x) + \\ &\quad + \frac{w_0(L) w_1'(L) - w_1(L) w_0'(L)}{w_1(L) w_2'(L) - w_1'(L) w_2(L)} w_2(x) \\ &= X_1 + \frac{X_3 X_6 - X_2 X_7}{X_4 X_7 - X_5 X_6} X_8 + \frac{X_2 X_5 - X_4 X_3}{X_4 X_7 - X_5 X_6} X_9 \end{aligned} \tag{4.22}$$

where X_i $(i = 1, 2, \ldots, 9)$ are written for $w_0(x)$, $w_0(L)$, $w_0'(L)$, $w_1(L)$, $w_1'(L)$, $w_2(L)$, $w_2'(L)$, $w_1(x)$, $w_2(x)$, respectively, partly for simplicity and partly to emphasize that they are Gaussian random variables. The expected values of the random variables appearing on the right-hand side of Eq. 4.22 become

$$\begin{aligned} \mathbf{E}[w_0(x)] &= \frac{p}{24 E_0 I_0} \left[x^4 + 6x^2 L^2 - 4x^3 L \right] \\ \mathbf{E}[w_0(L)] &= \frac{pL^4}{8 E_0 I_0} \end{aligned}$$

$$
\begin{aligned}
\mathbf{E}[w_0'(L)] &= \frac{pL^3}{6E_0I_0} \\
\mathbf{E}[w_1(L)] &= \frac{L^3}{3E_0I_0} \\
\mathbf{E}[w_1'(L)] &= \frac{L^2}{2E_0I_0} \\
\mathbf{E}[w_2(L)] &= \frac{L^2}{2E_0I_0} \\
\mathbf{E}[w_2'(L)] &= \frac{L}{E_0I_0} \\
\mathbf{E}[w_1(x)] &= \frac{1}{6E_0I_0}\left[3x^2L - x^3\right] \\
\mathbf{E}[w_2(x)] &= \frac{x^2}{2E_0I_0}
\end{aligned}
\tag{4.23}
$$

Introducing the following abbreviations

$$
\begin{aligned}
&b_1(x,\xi) = \frac{p}{2E_0I_0}(x-\xi)(L-\xi)^2 \qquad\qquad b_2(x,\xi) = \frac{1}{E_0I_0}(x-\xi)(L-\xi) \\
&b_3(x,\xi) = \frac{1}{E_0I_0}(x-\xi) \qquad b_4(L,\xi) = \frac{p}{2}b_2(L,\xi) \qquad b_5 = \frac{1}{E_0I_0}
\end{aligned}
\tag{4.24}
$$

covariance matrix $\mathbf{C}$ of the nine variables X_k can be expressed in terms of integrals involving these functions and the correlation function $R_{ff}(x,y)$. The following are a few representative elements of the covariance matrix:

$$
\begin{aligned}
\sigma^2{}_{X_1} &= \int_0^x \int_0^x b_1(x,\xi)b_1(x,\eta)R_{ff}(\xi,\eta)d\xi d\eta \\
\varrho_{12}\sigma_{X_1}\sigma_{X_2} &= \int_0^x \int_0^L b_1(x,\xi)b_1(L,\eta)R_{ff}(\xi,\eta)d\xi d\eta \\
\varrho_{13}\sigma_{X_1}\sigma_{X_3} &= \int_0^x \int_0^L b_1(x,\xi)b_4(L,\eta)R_{ff}(\xi,\eta)d\xi d\eta \\
\varrho_{15}\sigma_{X_1}\sigma_{X_5} &= \int_0^x \int_0^L b_1(x,\xi)b_3(L,\eta)R_{ff}(\xi,\eta)d\xi d\eta \\
\sigma^2{}_{X_2} &= \int_0^L \int_0^L b_1(L,\xi)b_1(L,\eta)R_{ff}(\xi,\eta)d\xi d\eta \\
\varrho_{27}\sigma_{X_2}\sigma_{X_7} &= \int_0^L \int_0^L b_1(L,\xi)b_5R_{ff}(\xi,\eta)d\xi d\eta \\
\varrho_{28}\sigma_{X_2}\sigma_{X_8} &= \int_0^L \int_0^x b_1(L,\xi)b_2(x,\eta)R_{ff}(\xi,\eta)d\xi d\eta
\end{aligned}
\tag{4.25}
$$

$$\sigma^2{}_{X_3} = \int_0^L \int_0^L b_4(L,\xi) b_4(L,\eta) R_{ff}(\xi,\eta)\, d\xi d\eta$$
$$\sigma^2{}_{X_4} = \int_0^L \int_0^L b_2(L,\xi) b_2(L,\eta) R_{ff}(\xi,\eta)\, d\xi d\eta$$
$$\sigma^2{}_{X_7} = \int_0^L \int_0^L b_5 b_5 R_{ff}(\xi,\eta)\, d\xi d\eta$$
$$\sigma^2{}_{X_9} = \int_0^x \int_0^x b_3(x,\xi) b_3(x,\eta) R_{ff}(\xi,\eta)\, d\xi d\eta$$

Integration of Eqs. 4.25 is carried out numerically in order to construct the covariance matrix. Then, both the FOSM method and Monte Carlo technique are used for the evaluation of the response statistics.

For the sake of simplicity, the quantities $p(x)$, $(E_0 I_0)$ and also the length of the beam are set equal to one. The sample size used for the Monte Carlo analysis is 200. Using the autocorrelation function defined in Eq. 4.1, the coefficient of variation (C.O.V) of the midspan deflection V_w, based on both the simulation approach and FOSM approximation, is plotted in Fig. 4-4 as a function of the correlation parameter b/L. As can be seen in the figure, the coefficient of variation converges as $b/L \to \infty$ to its limiting value of 0.1 which is the C.O.V of the flexibility.

The same procedure is carried out using the correlation function defined by Eq. 4.3. The resulting coefficient of variation V_w is plotted in Fig. 4-5 for both methods. Use of this correlation function produced, as can be seen in Fig. 4-5, a faster convergence of the coefficient to its limiting value 0.1.

Furthermore, the coefficient of variation of the end moment $M(L) = B_2$ is evaluated for the same correlation functions and the results are plotted in Figs. 4-6 and 4-7. It is also obvious from these figures that the FOSM approximation and the Monte Carlo simulation show good agreement.

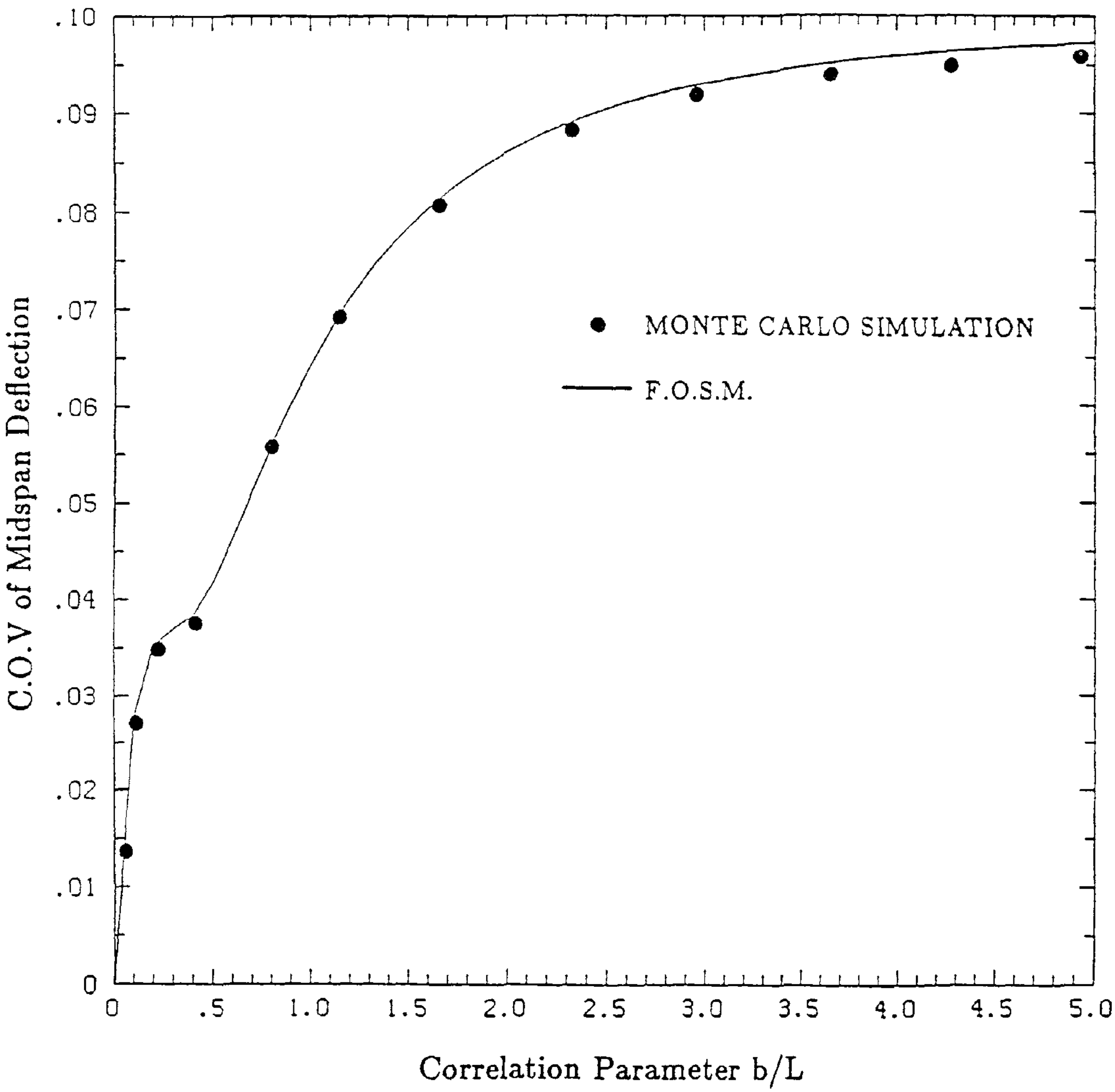


Fig. 4-4 Coefficient of Variation of Midspan Deflection of Fixed Beam as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.1)

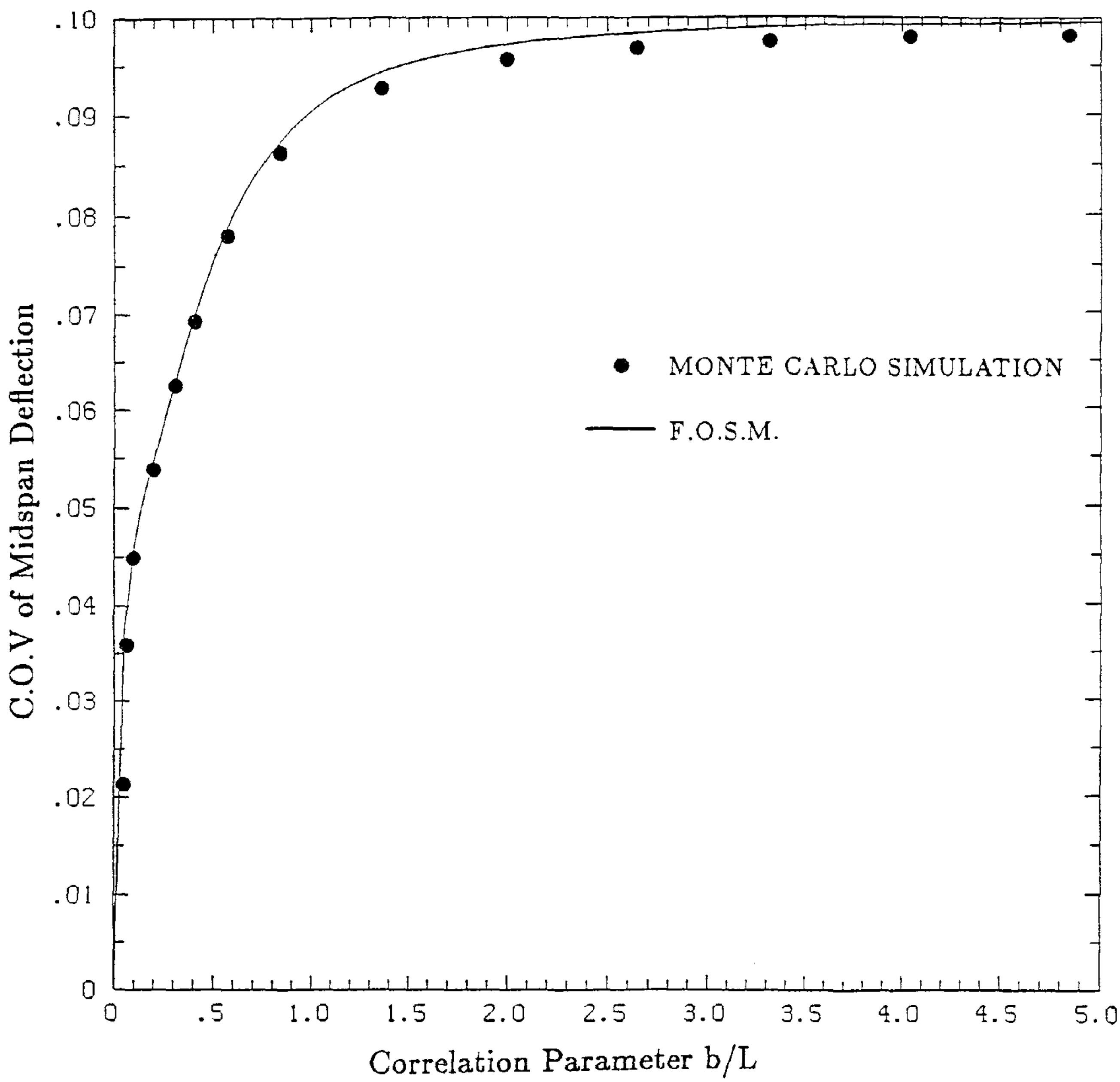


Fig. 4-5 Coefficient of Variation of Midspan Deflection of Fixed Beam as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.3)

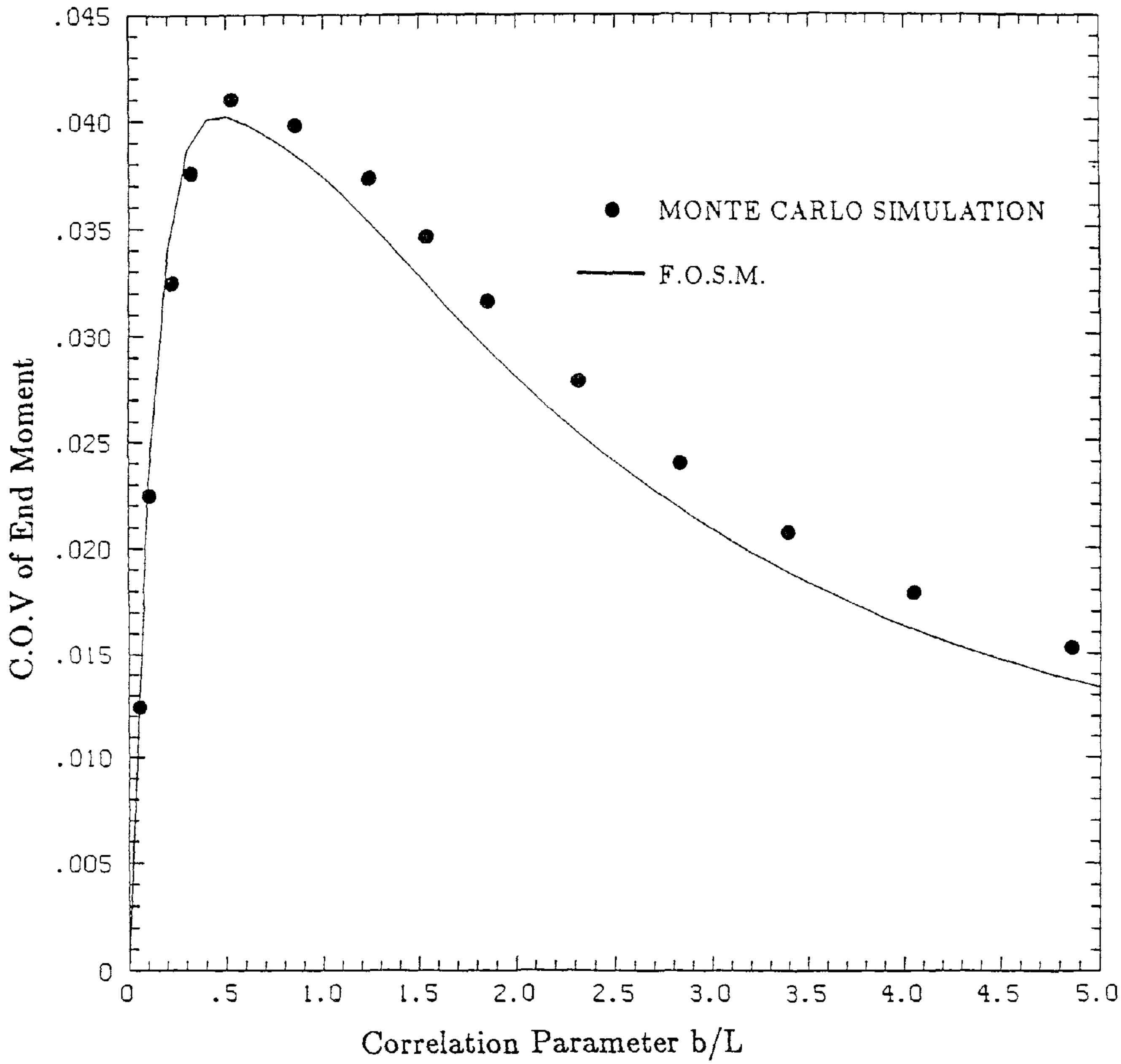


Fig. 4-6 Coefficient of Variation of End Moment of Fixed Beam as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.1)

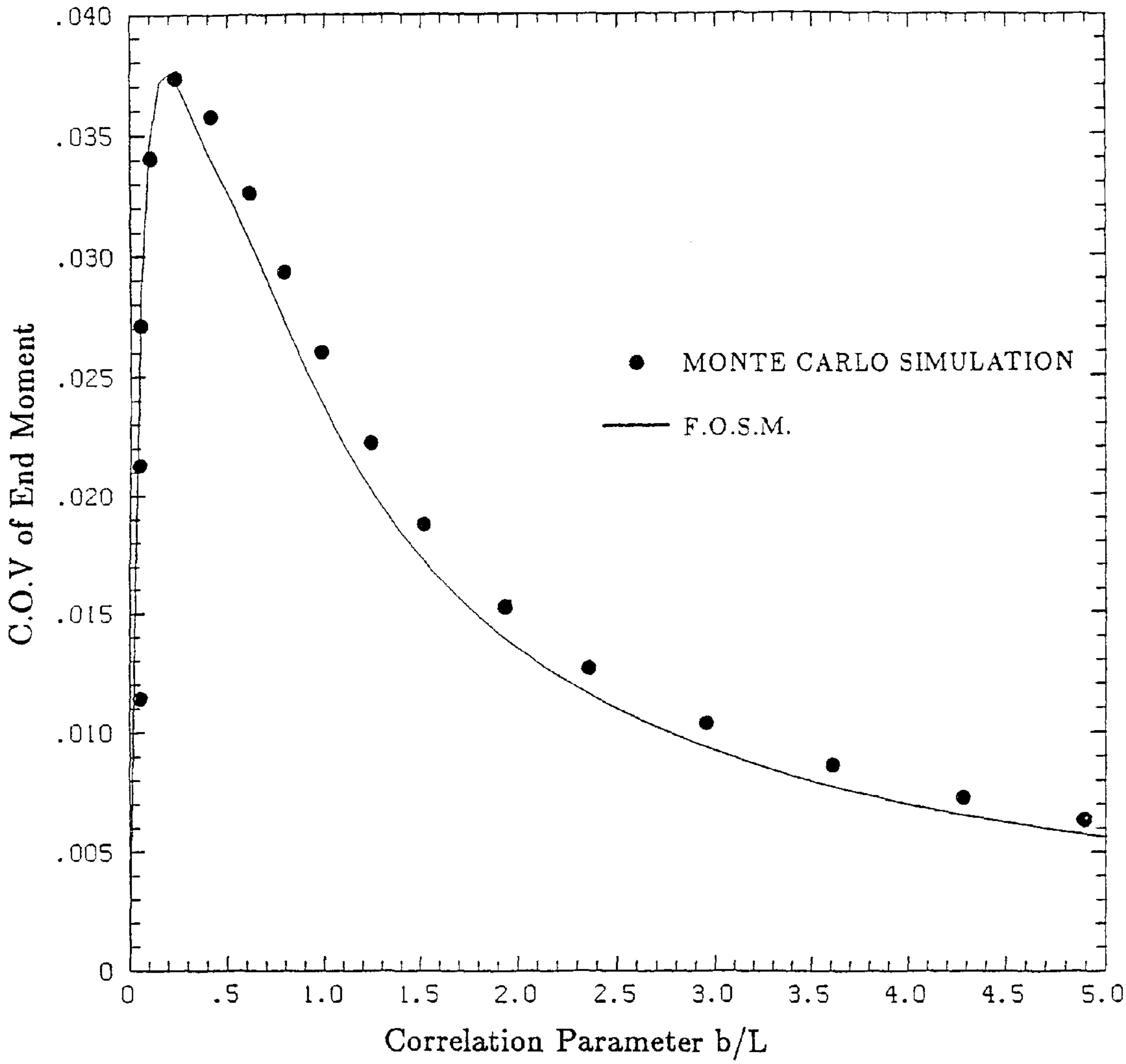


Fig. 4-7 **Coefficient of Variation of End Moment of Fixed Beam as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.3)**

Finally, using the Lagrange multiplier method, the safety index of the end moment B_2 is evaluated, assuming that the deterministic resisting moment B_R is given by the form:

$$B_R = \mathbf{E}[B_2] + k\sigma_{B_2} \tag{4.26}$$

If the parameter k was chosen to be 3, the limit state condition becomes

$$\begin{aligned} g(\mathbf{X}) = B_R - B_2 = B_R - \frac{X_2X_5 - X_4X_3}{X_4X_7 - X_5X_6} \\ = g(X_2, X_3, X_4, X_5, X_6, X_7) \end{aligned} \tag{4.27}$$

As can be seen in Eq. 4.27, $g(\mathbf{X})$ is a function of six random variables. The Lagrange multiplier method yielded a safety index $\beta_{B_2} = 2.53$.

The safety index of the midspan deflection $w(L/2)$ was also evaluated assuming that the maximum deflection w_R allowed is given by:

$$w_R = \mathbf{E}[w(L/2)] + k\sigma_{w(L/2)} \tag{4.28}$$

The limit state condition in terms of the deflection $w(L/2)$ is

$$g(\mathbf{X}) = w_R - w(L/2) = w_R - \left[X_1 + \frac{X_3X_6 - X_2X_7}{X_4X_7 - X_5X_6}X_8 + \frac{X_2X_5 - X_4X_3}{X_4X_7 - X_5X_6}X_9\right] \tag{4.29}$$

The safety index of the midspan deflection β_w is then found to be 2.83, again by means of the Lagrange miltiplier method.

4.2 Example 2: Two-Story Two-Bay Frame

A two-story two-bay frame as shown in Fig. 4-8a is considered, subjected to a distributed load $p(x)$ on the beams. The degree of indeterminacy for this frame is 12 and thus 12 redundant forces should be selected. The associated statically determinate system together with the introduced indeterminate forces B_k $(k = 1, 2, \ldots, 12)$ are shown in Fig. 4-8b.

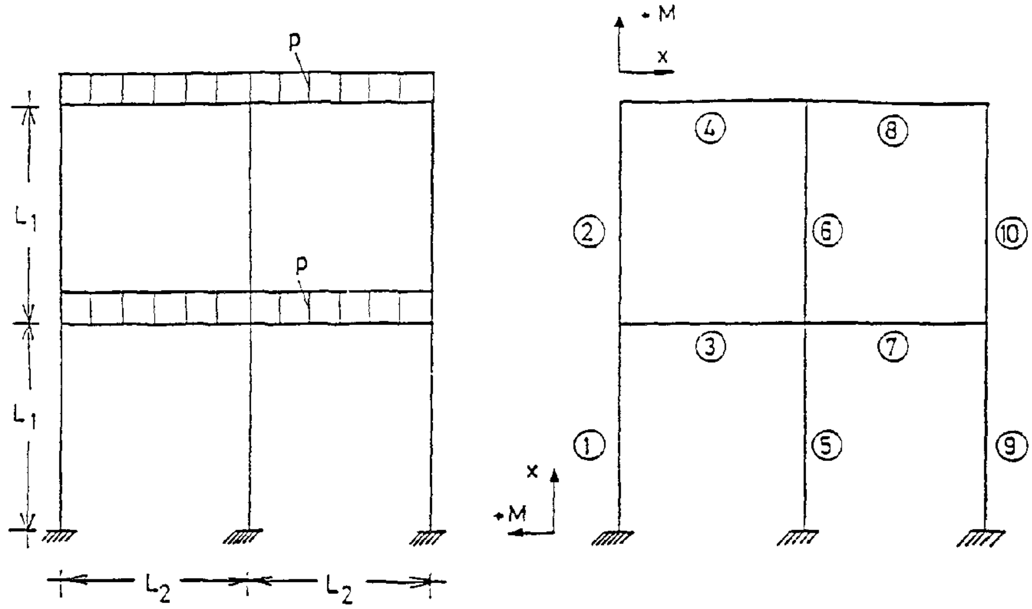


Fig. 4-8a Two-Story Two-Bay Frame with Uniform Load on Beams

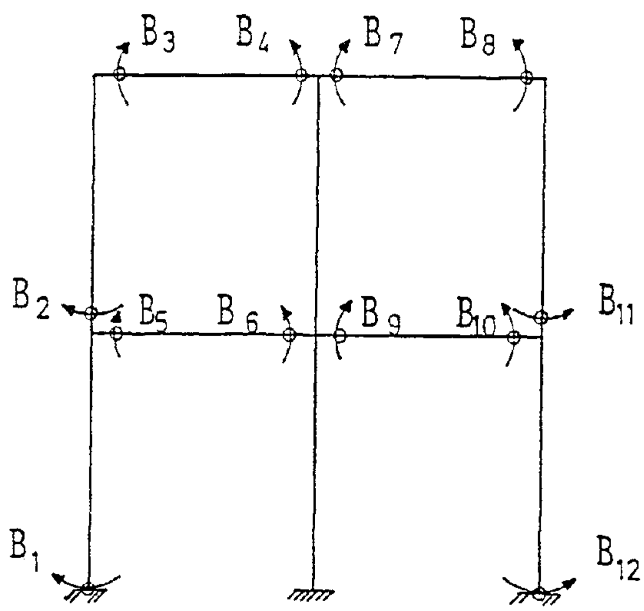


Fig. 4-8b Statically Determinate System of Two-Story Two-Bay Frame Associated with Introduced Statically Indeterminate Forces

In this case, since the indeterminate forces introduced are all moments, the coefficients δ_{ik} represent slopes at the locations of the indeterminate moments B_i, due to unit indeterminate moments B_k. The moment diagrams of the statically determinate system due to external load and unit indeterminate forces B_k are plotted in Fig. 4-9.

Upon imposing the 12 boundary conditions at the locations of the selected indeterminate forces, forces B_k follow from Eq. 3.26. The distributed load $p(x)$, lengths $L1$ and $L2$ and mean value of the flexibility $1/(\alpha_l E_0 I_0)$, are assumed to be one.

Following the procedure described in the section dealing with indeterminate frames, covariance matrix **C** is constructed and then the statistics of the indeterminate forces as well as of the deflections and moments are evaluated using both the FOSM and Monte Carlo methods. Monte Carlo analysis is again performed with 200 simulations.

The coefficients of variation of the indeterminate forces B_3, B_4, B_5 are plotted in Figs. 4-10, 4-11, and 4-12 respectively, as a function of the correlation distance b/L_2.

Also, the statistics of the midspan deflection and moment (point q) of element 4 are obtained, using both Monte Carlo simulation and FOSM approximation. The midspan deflection and moment of component 4, given in general by Eqs. 3.27 and 3.28, can be written in this particular case, respectively, as:

$$w_q = w_{q0} + B_3 w_{q3} + B_4 w_{q4} \tag{4.30}$$

$$M_q = M_{q0} + B_3 M_{q3} + B_4 M_{q4} \tag{4.31}$$

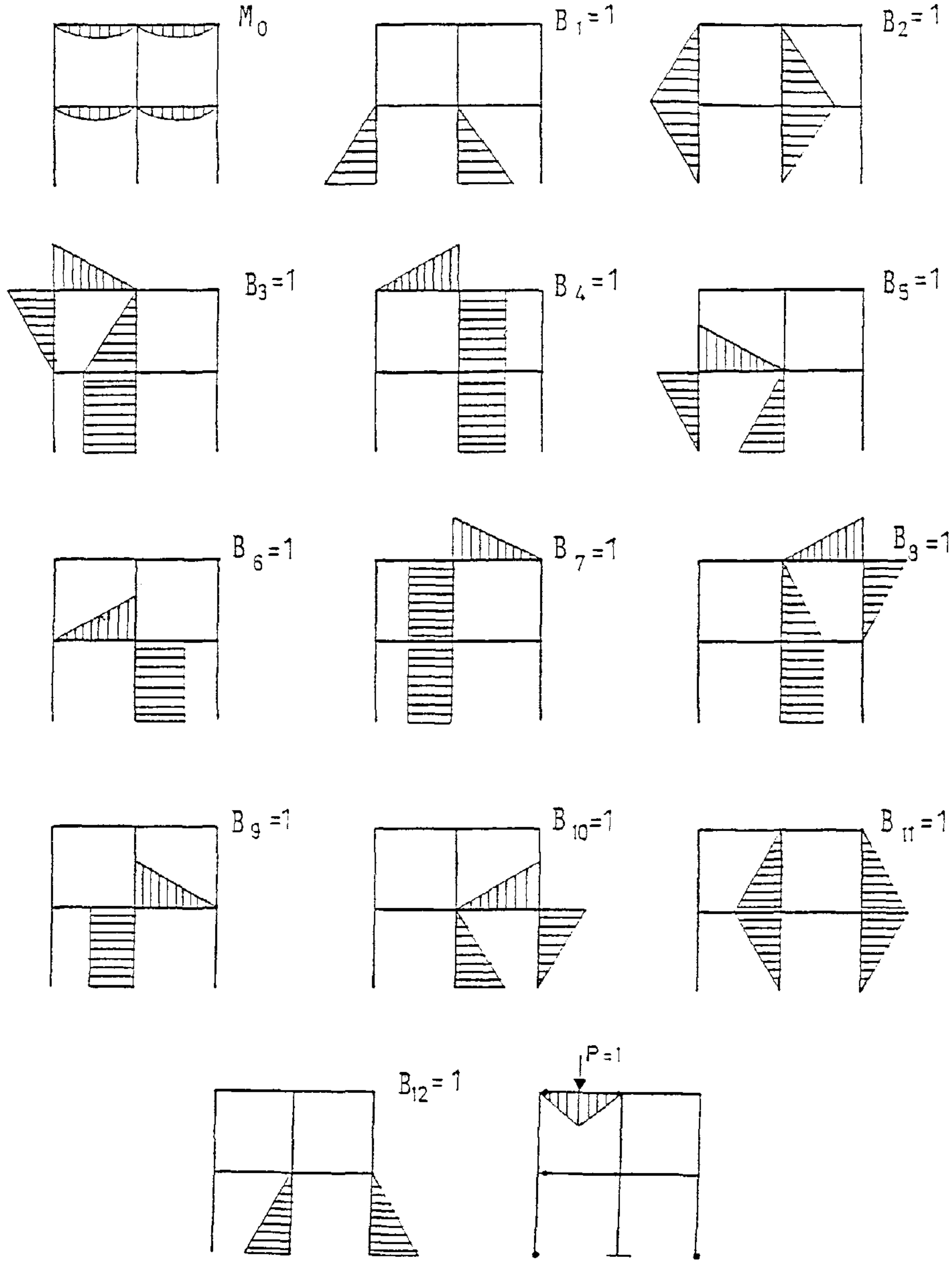


Fig. 4-9 Moment Diagrams for the Applied Load $p(x)$, Unit Indeterminate Forces B_k $(k = 1, 2, \ldots, 12)$ and Unit Concentrated Load at Midspan of Component 4

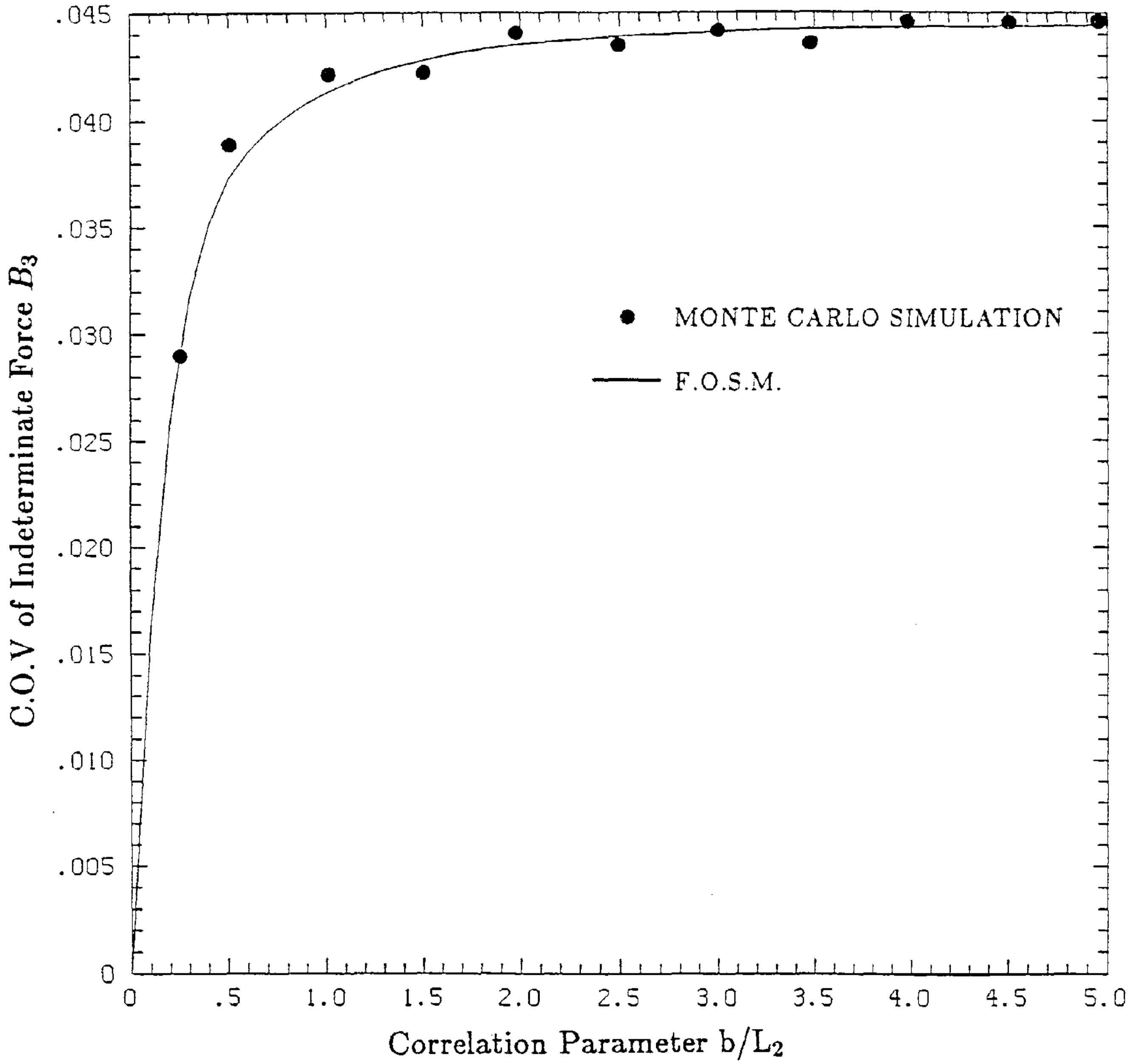


Fig. 4-10 **Coefficient of Variation of Indeterminate Force B_3 as Function of Dimensionless Parameter b/L_2 (Correlation Function Defined by Eq. 4.1**

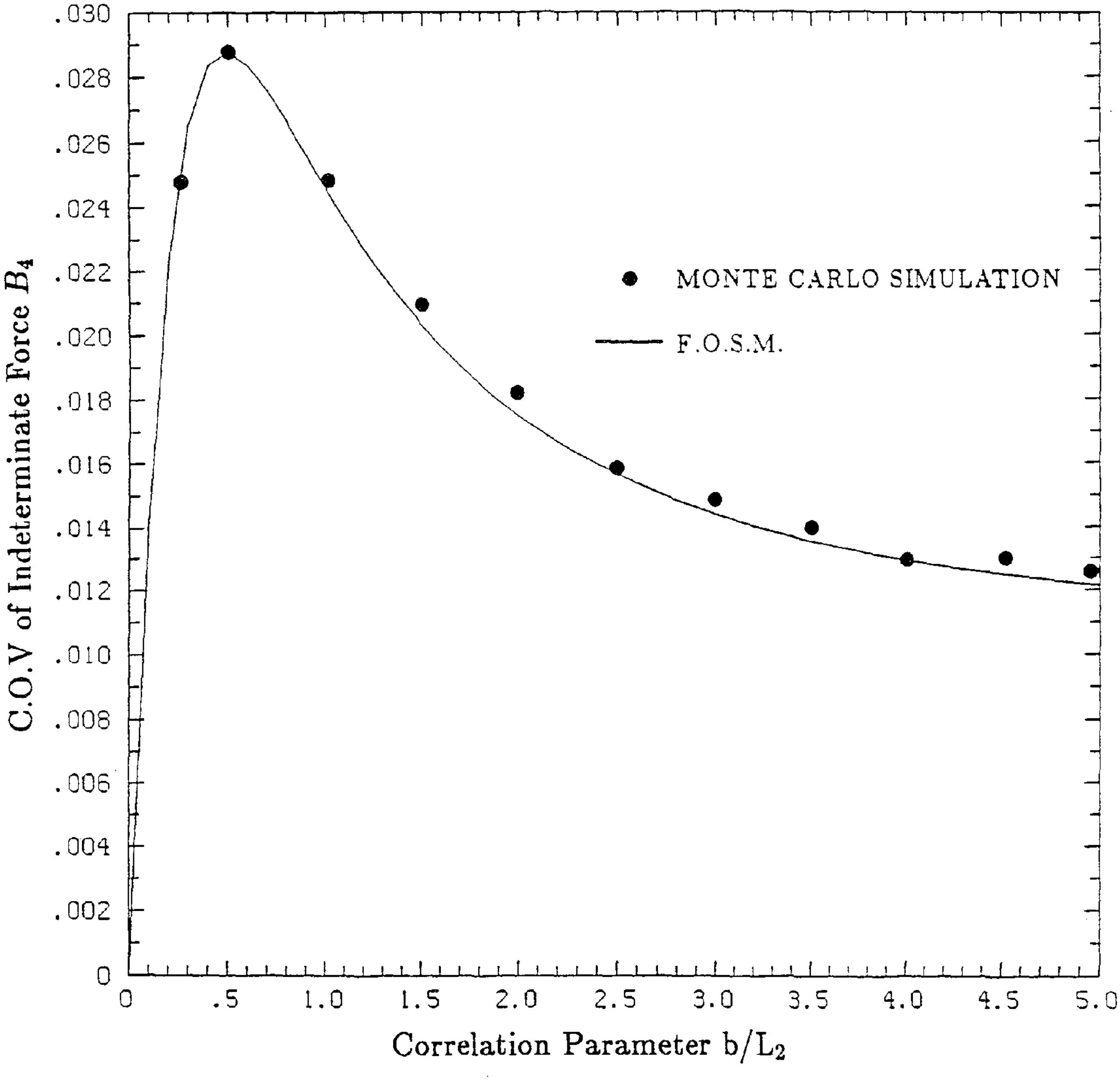


Fig. 4-11 Coefficient of Variation of Indeterminate Force B_4 as Function of Dimensionless Parameter b/L_2 (Correlation Function Defined by Eq. 4.1

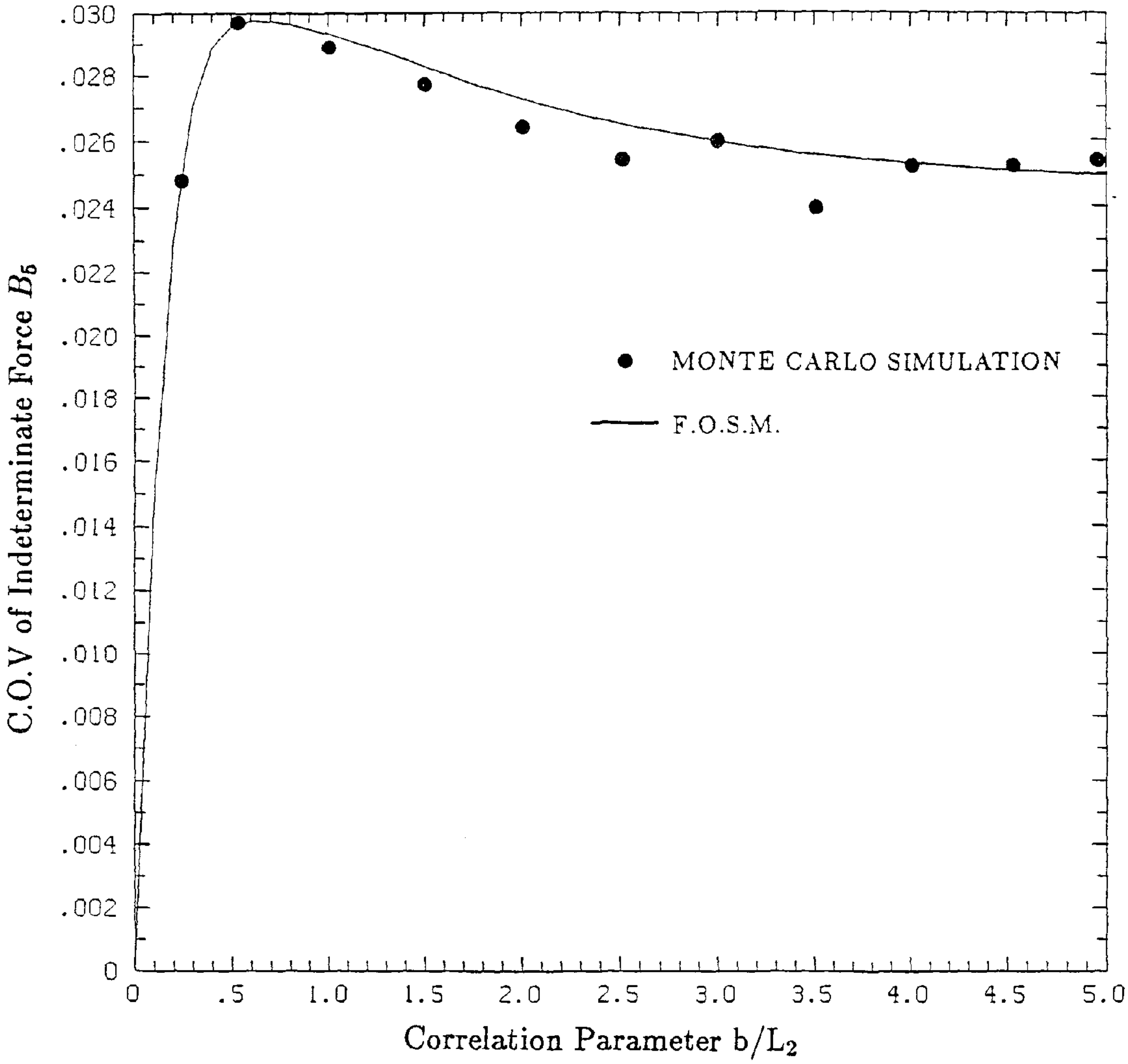


Fig. 4-12 Coefficient of Variation of Indeterminate Force B_5 as Function of Dimensionless Parameter b/L_2 (Correlation Function Defined by Eq. 4.1

where w_{q0} and M_{q0} are the midspan deflection and moment due to the external loads only, w_{q3} and M_{q3} are the midspan deflection and moment due to unit indeterminate force B_3 (with $B_k = 0$ if $k \neq 3$) and w_{q4} and M_{q4} are the midspan deflection and moment, respectively, due to the unit indeterminate force B_4 (with $B_k = 0$ if $k \neq 4$).

As can be seen from the above equations, only the indeterminate forces B_3 and B_4 are involved in the expressions for the midspan deflection and moment of component 4, since, in this case, the quantities w_{qk} and M_{qk} with $k = 1, 2, 5, \ldots, 12$ are zero. Deflection w in Eq. 4.30 is a function of five random variables. In order to apply the FOSM method, the quantities $\sigma^2_{w_{q0}}$, $\sigma^2_{w_{q3}}$, $\sigma^2_{w_{q4}}$, $Cov(w_{q0}, w_{q3})$, $Cov(w_{q0}, w_{q4})$ and $Cov(w_{q3}, w_{q4})$ must be determined from Eq. 3.37, the quantities $\sigma^2_{B_3}$, $\sigma^2_{B_4}$, $Cov(B_3, B_4)$ from Eq. 3.63 and finally, the quantities $Cov(B_l, w_{qn})$ $(l = 3, 4 \quad n = 0, 3, 4)$ from Eq. 3.64.

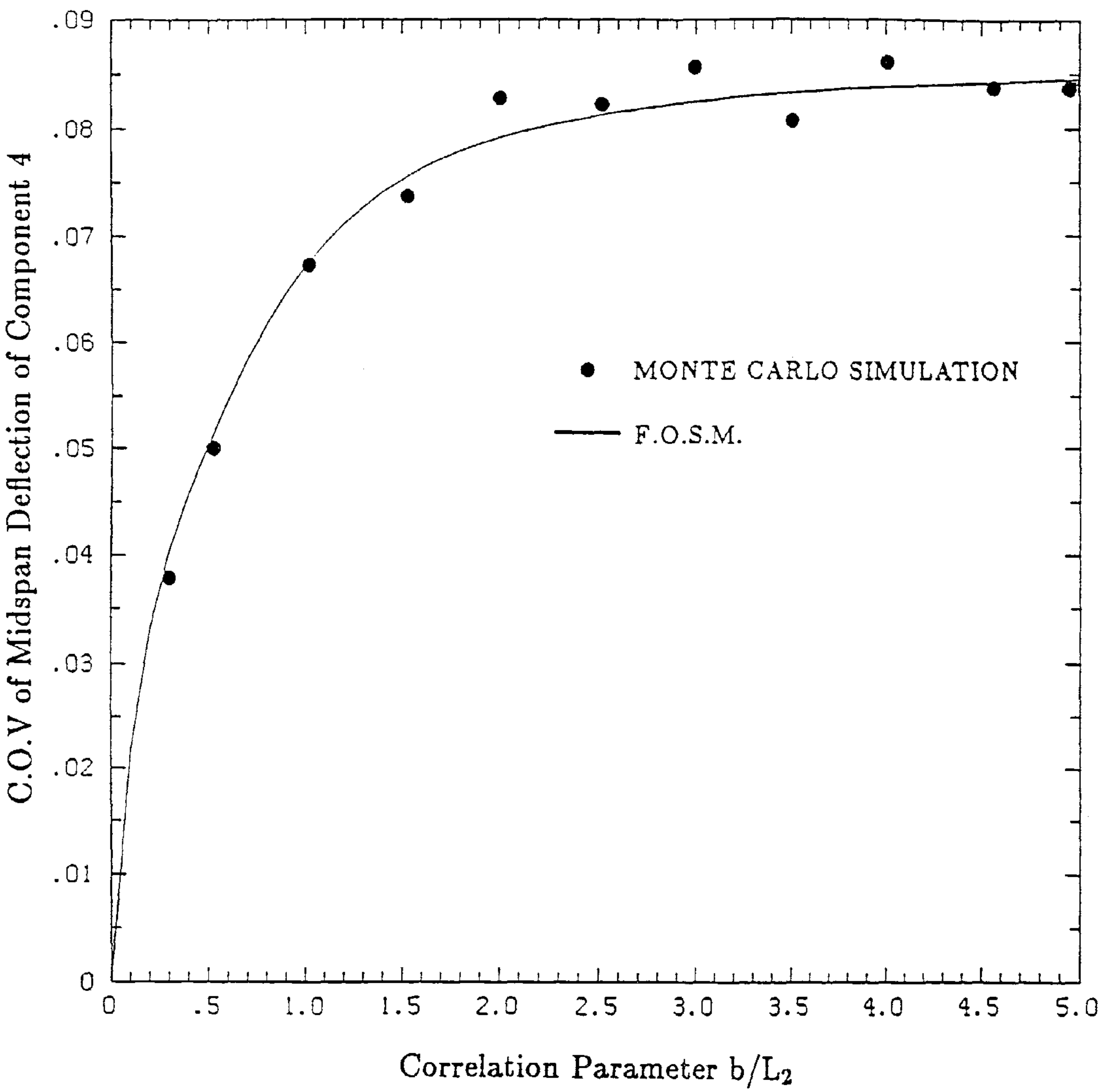
The coefficient of variation of the midspan deflection of element 4 is plotted in Fig. 4-13 as a function of the correlation parameter b/L_2. Also the coefficient of variation of the midspan moment is plotted in Fig. 4-14.

4.3 Example 3: Portal Frame

In this example, a portal frame shown in Fig. 4-15a, is analyzed. The frame is subjected to gravity as well as horizontal loading. The horizontal concentrated load may represent the effect of wind or earthquake. In this example, the horizontal load is assumed to be a quasi-static force representing the effect of an earthquake, and it is taken to be 0.25 of the total gravity load. The dimensions of the frame are shown in Fig. 4-15a and the parameters used in the analysis are

$$\mathrm{p(x)} = 1 \text{ kip/ft}$$

$$\mathrm{I_0} = 200 \text{ in}^2$$



4-13 Coefficient of Variation of Midspan Deflection of Component 4 as Function of Dimensionless Parameter b/L_2 (Correlation Function Defined by Eq. **4.1**)

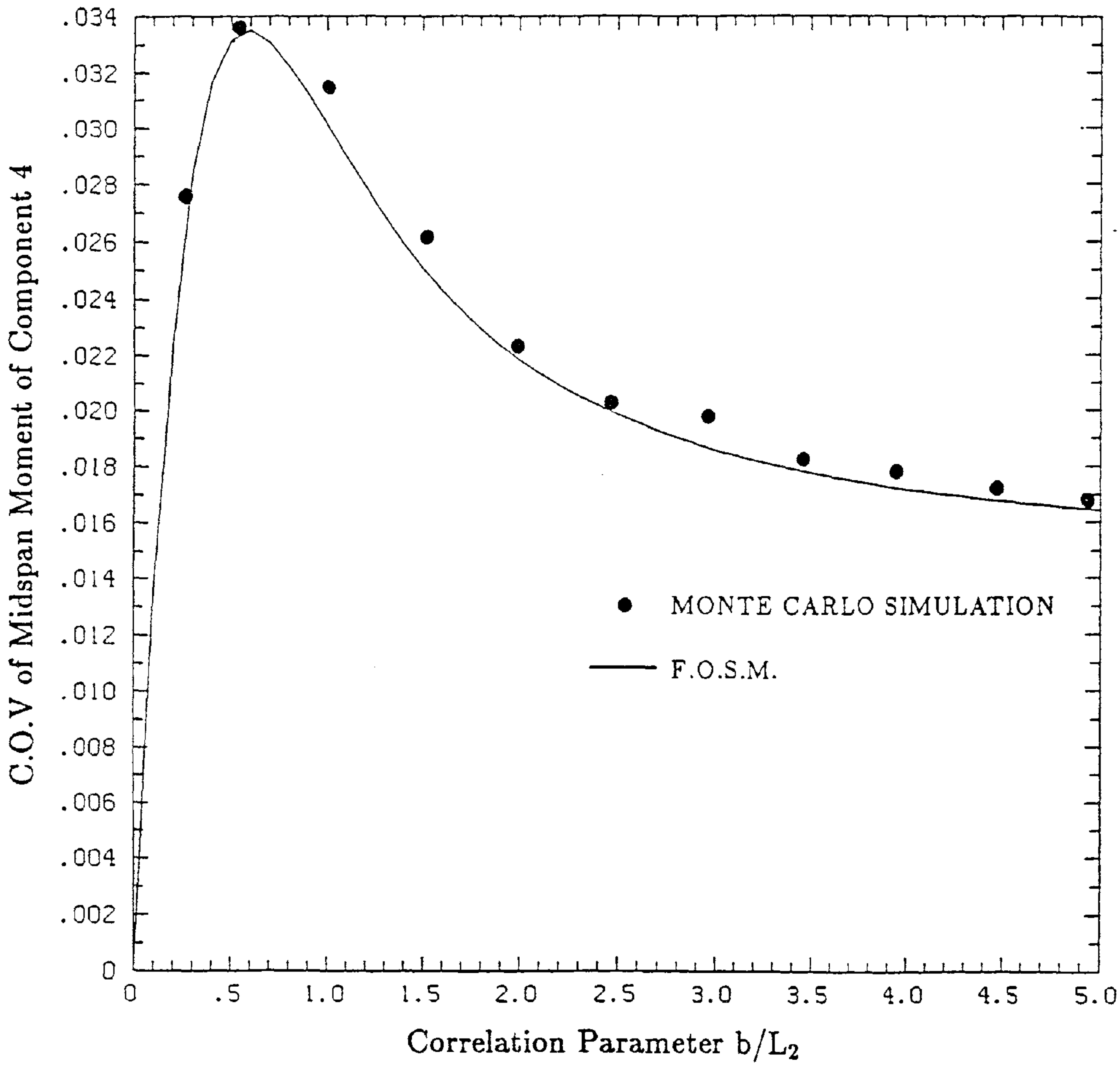


Fig. 4-14 Coefficient of Variation of Midspan Moment of Component 4 as Function of Dimensionless Parameter b/L_2 (Correlation Function Defined by Eq. 4.1

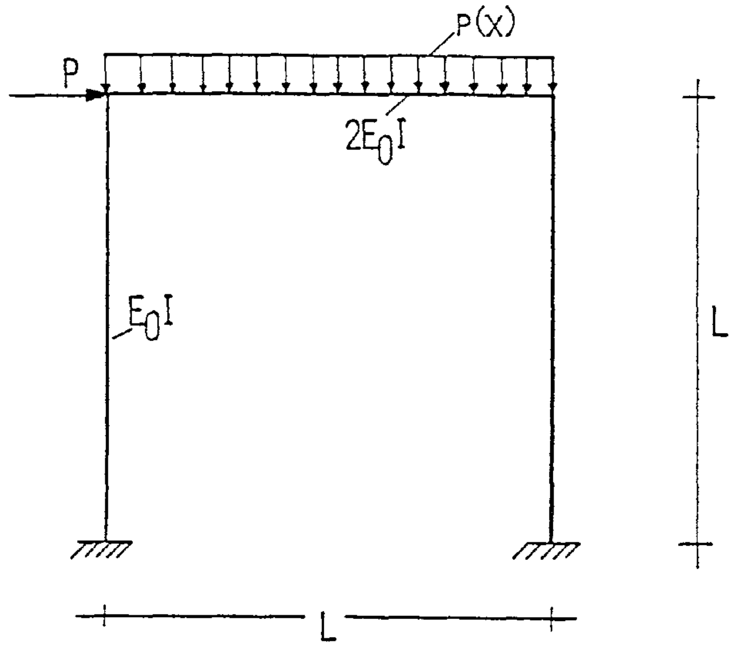


Fig. 4-15a Portal Frame with Uniform Load on Beam and Horizontal Concentrated Force

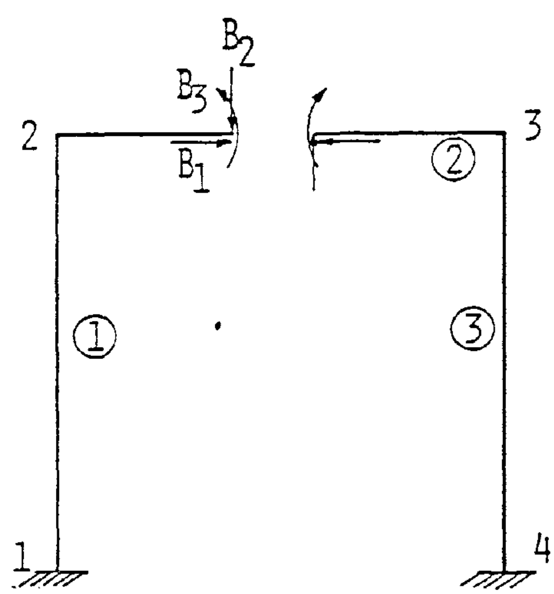


Fig. 4-15b Statically Determinate Structure Associated with Introduced Indeterminate Forces

$$E_0 = 30,000 \text{ ksi}$$

$$L = 10 \text{ ft}$$

$$P = 2.5 \text{ kips}$$

$$\alpha_1 = \alpha_3 = 1$$

$$\alpha_2 = 2$$

The associated statically indeterminate system with selected indeterminate forces is shown in Fig. 4-15b. The moment diagrams corresponding to external loads, unit indeterminate forces B_1, B_2, B_3, and unit concentrated horizontal force at node 2 and vertical at midspan, are shown in Fig. 4-16.

The coefficient of variation of the indeterminate forces B_1 and B_3 is plotted in Figs. 4-17 and 4-18 respectively, as a function of the dimensionless correlation parameter b/L. Also, the coefficient of variation of moment at node 1 is plotted in Fig. 4-19. as a function of the same parameter. Finally, the coefficient of variation of the midspan deflection as well as the horizontal displacement are plotted in Figs. 4-20 and 4-21 respectively.

The number of simulations used for the Monte Carlo solution is 500 in this example, and thus, the agreement of the two methods is very good, as can be seen from Figs. 4-17 $\sim$ 4-21.

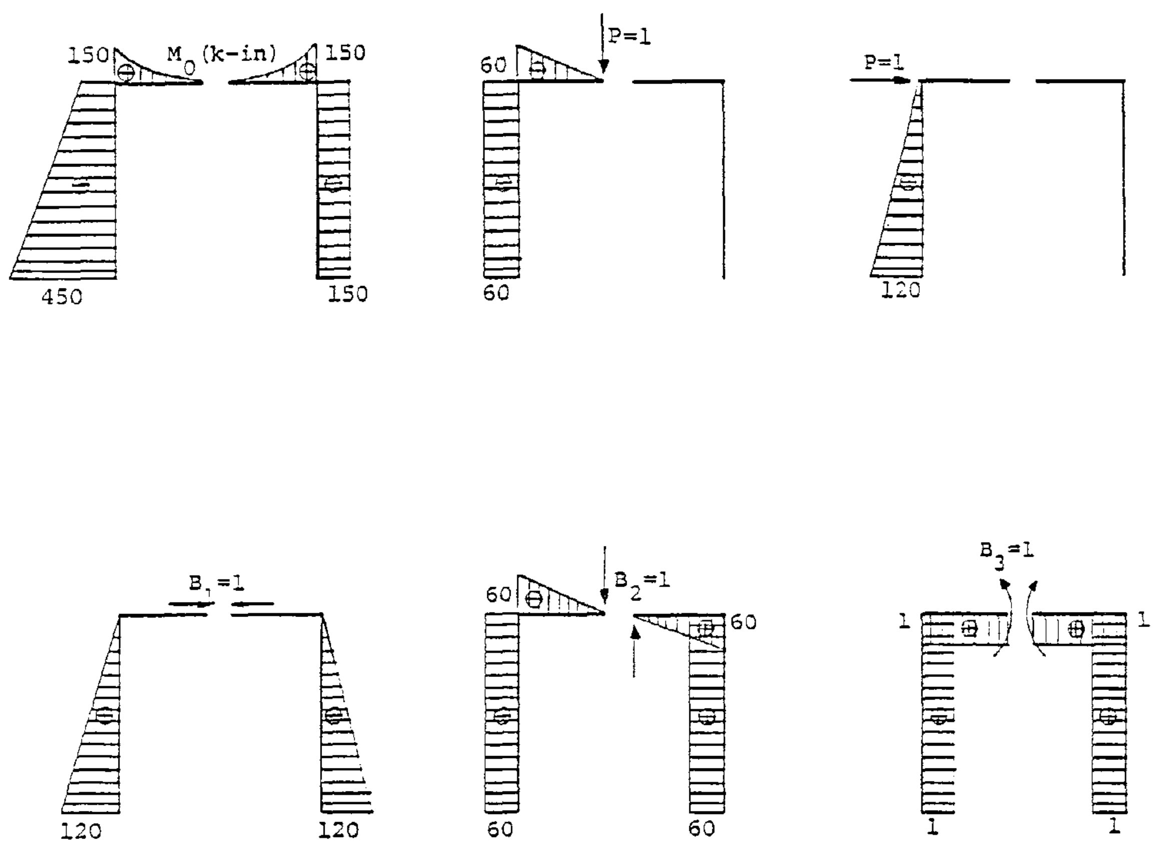


Fig. 4-16 Moment Diagrams for External Loads, Unit Forces B_k $(k = 1,2,3)$ and Unit Horizontal and Vertical Concentrated Forces

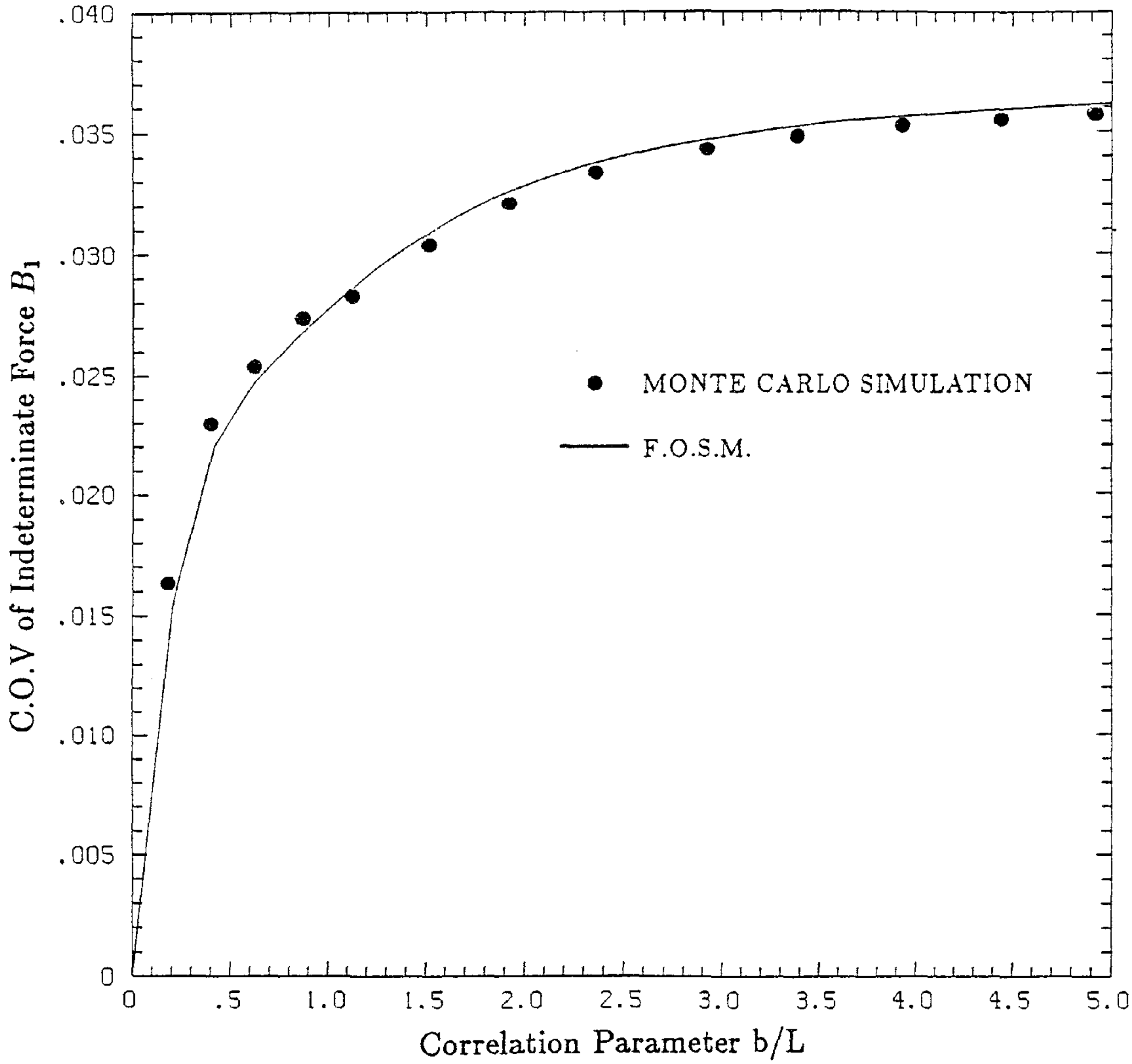


Fig. 4-17 Coefficient of Variation of Indeterminate Force B_1 as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.1

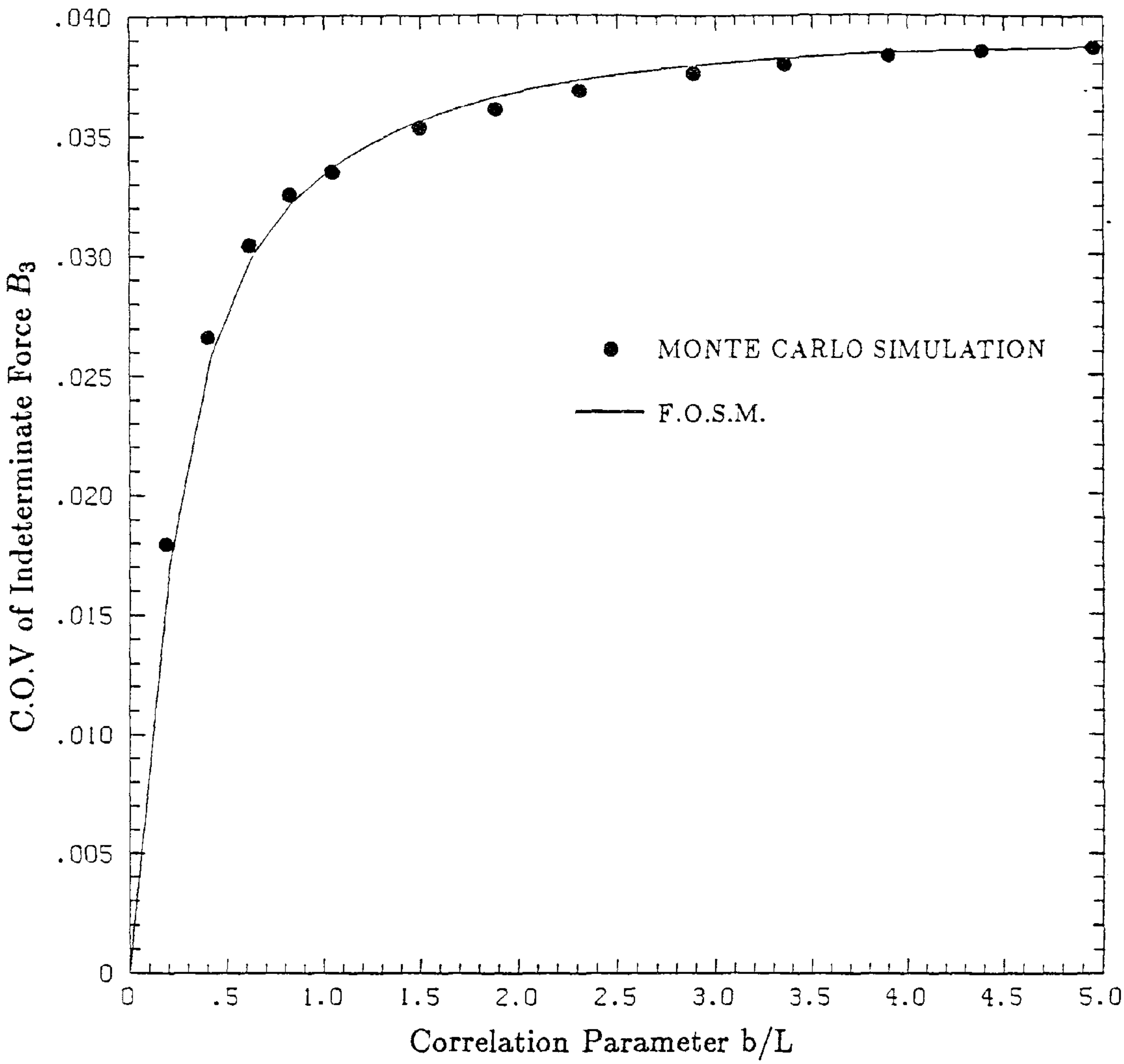


Fig. 4-18 Coefficient of Variation of Indeterminate Force B_3 as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.1

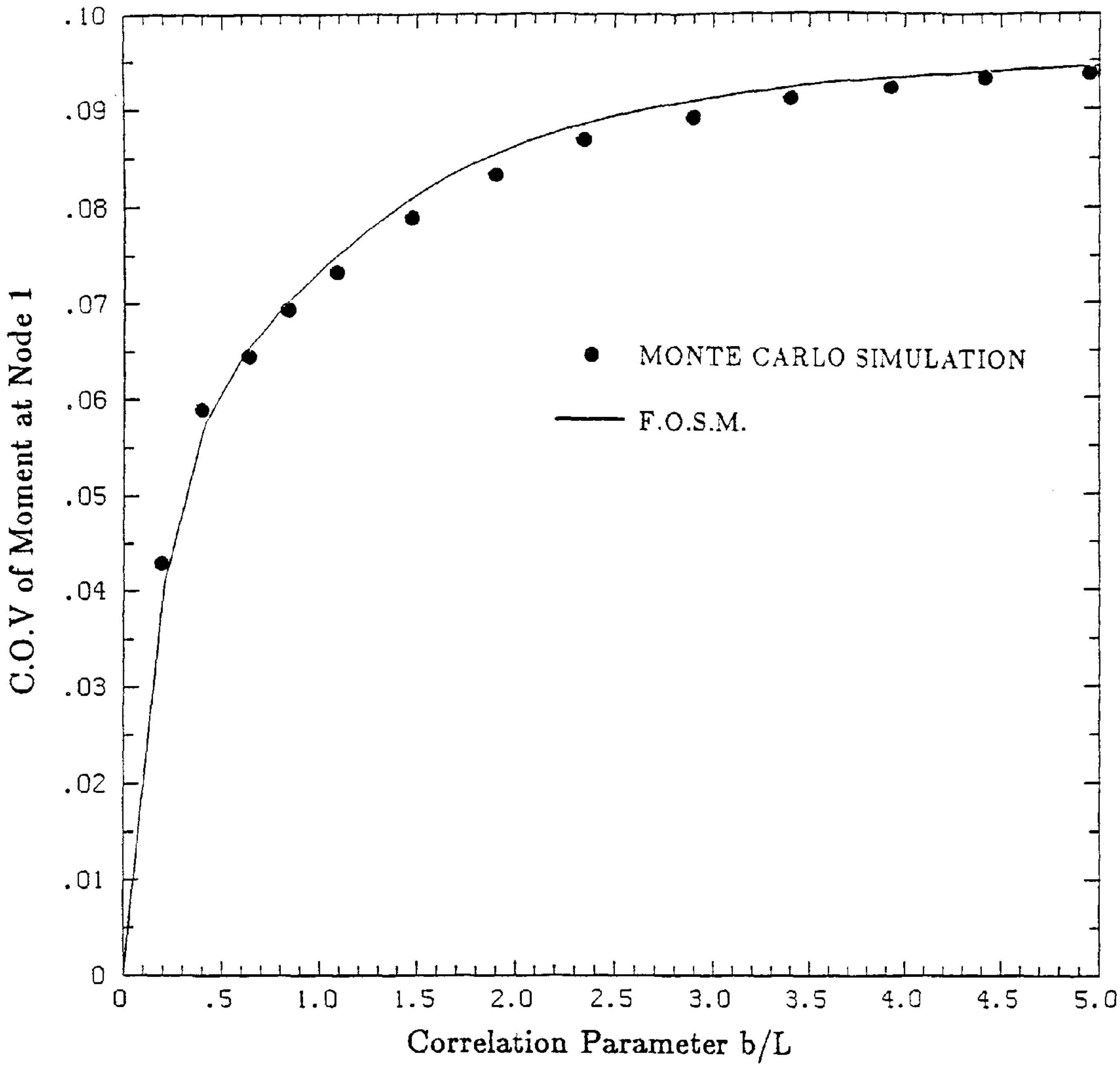


Fig. 4-19 Coefficient of Variation of Moment at Node 1 as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.1

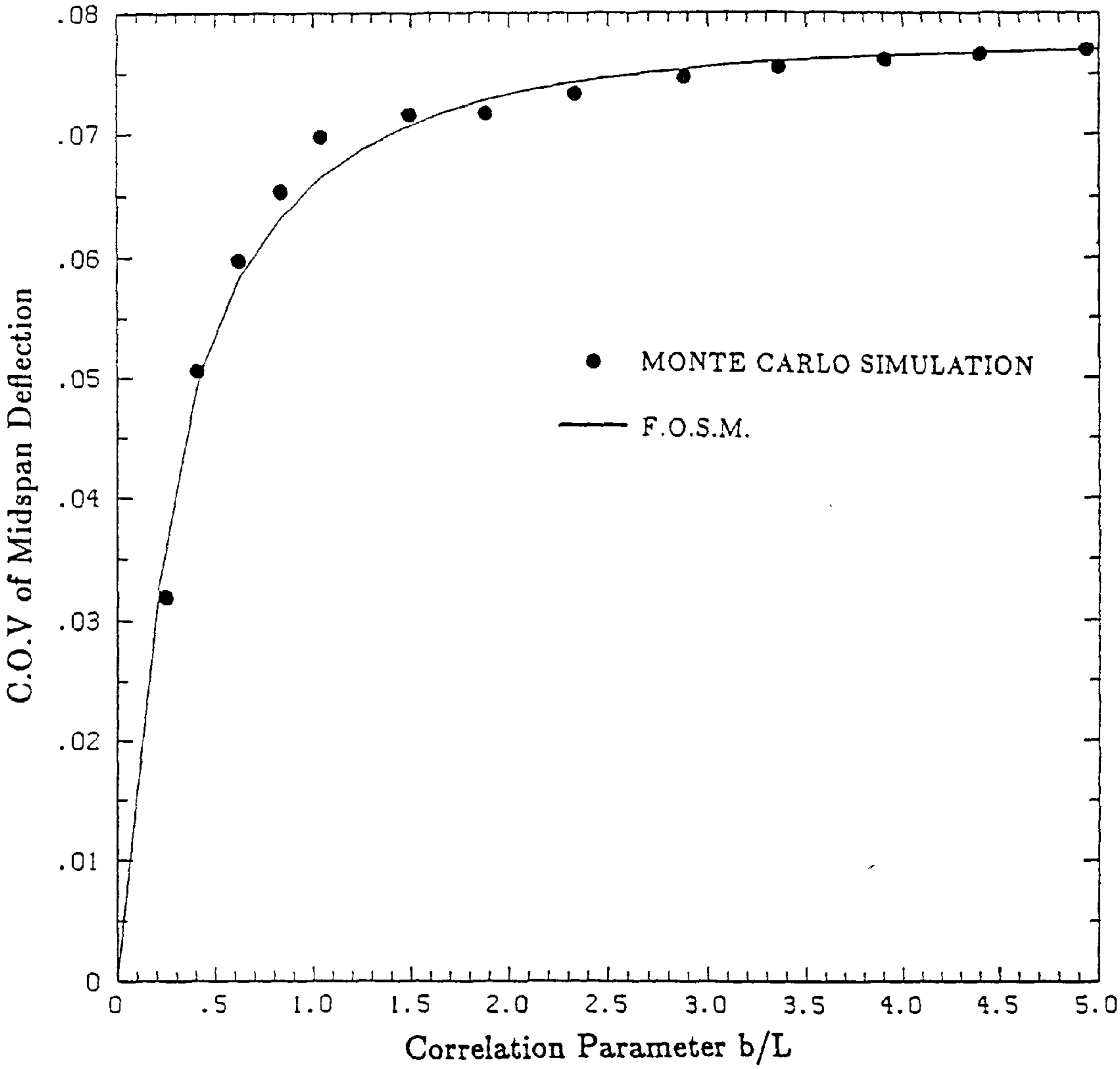


Fig. 4-20 Coefficient of Variation of Midspan Deflection as Function of Dimensionless Parameter b/L (Correlation Function Defined by Eq. 4.1

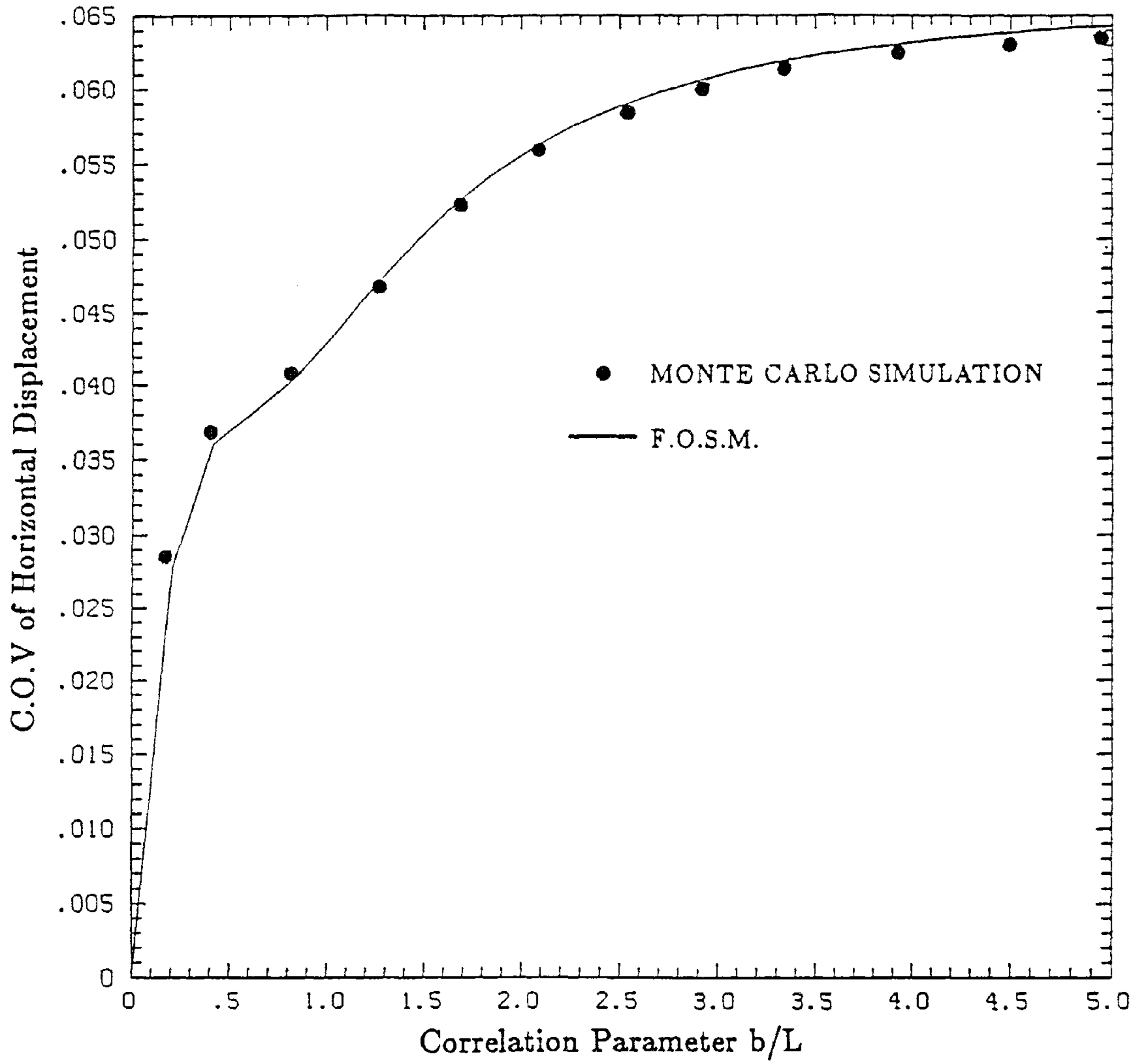


Fig. 4-21 Coefficient of Variation of Horizontal Displacement as Function of Dimensionless Parameter b/L Correlation Function Defined by Eq. 4.1

SECTION 5

CONCLUSIONS

The present study shows that it is possible to evaluate the response variability of statically indeterminate structures due to spatial variation of their material properties and, in principle, their geometries, without recourse to finite element analysis.

To achieve this, a Green's function formulation was utilized in the case of simple structures such as a fixed beam and the more general flexibility method in the case of more complex structures such as a two-story two-bay frame and a portal frame. Using the general approach presented herein, structures with a large degree of static indeterminacy (e.g, multi-bay multi-story frames) can also be treated. The amount of numerical effort involved depends on the number of degrees of statical indeterminacy. The mean square statistics of the desired response quantities as well as statistics of the indeterminate forces are obtained in simple integral form which is evaluated by numerical methods.

Most importantly, it was shown that the response variability problem is reduced to a problem involving only random variables, even if the material property is considered to constitute stochastic fields. The response variability was estimated using the First-Order Second-Moment method and the Monte Carlo simulation technique. The results based on these methods show good agreement. The safety indices of the midspan deflection and end moment of the fixed beam were also evaluated with the aid of the Lagrange multiplier method under certain limit state conditions.

Of more relevance to earthquake engineering is the result of the portal frame analysis under quasi-static conditions. The result indicates that the variabilities of various response quantities measured in terms of coefficient of variation are of the same order of magnitude as that of the flexibility variability.

SECTION 6
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