NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo

NONSTATIONARY MODELS OF SEISMIC GROUND ACCELERATION

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PREFACE

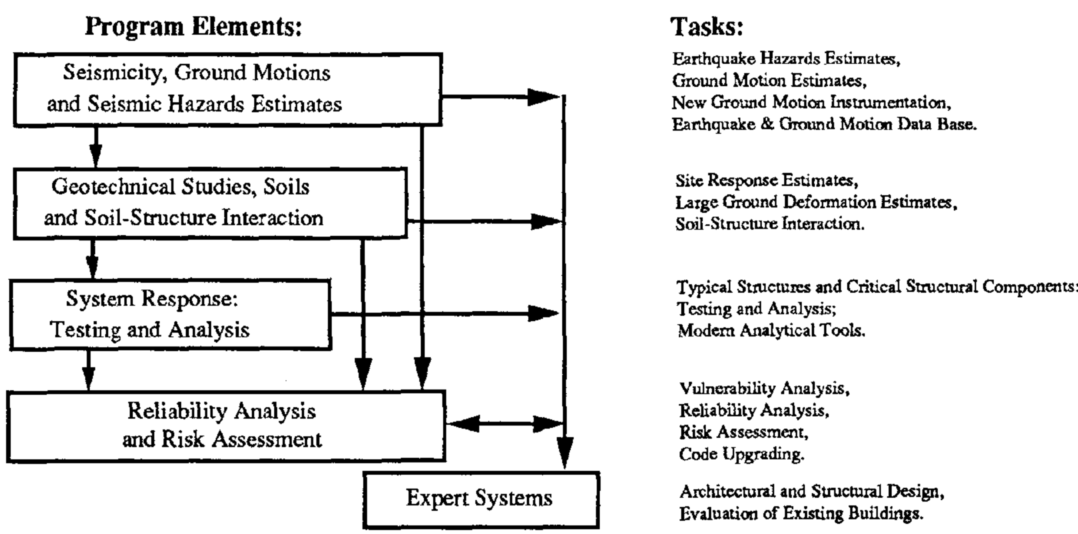
The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 1, Existing and New Structures, and more specifically to geotechnical studies, soils and soil-structure interaction.

The long term goal of research in Existing and New Structures is to develop seismic hazard mitigation procedures through rational probabilistic risk assessment for damage or collapse of structures, mainly existing buildings, in regions of moderate to high seismicity. The work relies on improved definitions of seismicity and site response, experimental and analytical evaluations of systems response, and more accurate assessment of risk factors. This technology will be incorporated in expert systems tools and improved code formats for existing and new structures. Methods of retrofit will also be developed. When this work is completed, it should be possible to characterize and quantify societal impact of seismic risk in various geographical regions and large municipalities. Toward this goal, the program has been divided into five components, as shown in the figure below:



Geotechnical studies, soils and soil-structure interaction constitute one of the important areas of research in Existing and New Structures. Current research activities include the following:

1. Development of linear and nonlinear site response estimates.
2. Development of liquefaction and large ground deformation estimates.
3. Investigation of soil-structure interaction phenomena.
4. Development of computational methods.
5. Incorporation of local soil effects and soil-structure interaction into existing codes.

The ultimate goal of projects in this area is to develop methods of engineering estimation of large soil deformations, site response, and the effects that the interaction of structures and soils have on the resistance of structures against earthquakes.

The authors of this report have developed a nonstationary process for modeling ground acceleration during earthquakes. The model was calibrated and evaluated using accelerograms from the 1985 Mexico City earthquake. The results showed good agreement between the actual and model-generated accelerograms.

ABSTRACT

A new nonstationary model is applied to represent seismic ground acceleration records. The model can be obtained by modulating both the amplitude and the frequency of a stationary process. It is not included in the class of oscillatory processes. Accelerograms recorded during the 1985 Michoacan earthquake in Mexico City are used to calibrate and evaluate the usefulness of the proposed and other nonstationary models of ground acceleration. Results suggest that the proposed nonstationary model is more adequate for reliability studies. System responses to acclerograms generated by this model have coefficients of variation that are consistent with those corresponding to actual accelerograms.

ACKNOWLEDGEMENTS

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

SECTION 1
INTRODUCTION

The generation of artificial ground acceleration records usually resorts to stationary processes modulated by deterministic functions that specify the temporal variation of seismic intensity [1]. These nonstationary models, referred to as uniformly modulated processes, are characterized by spectra with time-invariant shapes [2,3]. They have been generalized in several ways. For example, it has been proposed to generate accelerograms by passing a uniformly modulated stationary white noise through a shaping filter [4] or to represent consecutive nonoverlapping segments of a ground acceleration record by different uniformly modulated stationary processes [5]. Accelerograms generated by the latter model have sample discontinuities at the instants of changes from one random process to another; they are accordingly not considered in this study. Another type of representation proposed for the ground acceleration process is based on the response of a linear system to a modulated train of impulses with random magnitudes [6].

This paper considers alternative nonstationary models for the ground acceleration process. The models are divided in two classes: oscillatory processes characterized by evolutionary spectra and processes obtained by modulating the amplitude and the frequency of stationary processes. The relationship between these classes is also examined. Accelerograms recorded during the 1985 Michoacán earthquake in Mexico City are used to calibrate the nonstationary models of ground accelerations considered in this paper and to evaluate their potential usefulness in earthquake engineering.

SECTION 2
OSCILLATORY PROCESSES

The class of oscillatory processes has the form

$$X(t) = \int_{-\infty}^{\infty} a(t;\ \omega)e^{i\omega t}\ dZ(\omega) \tag{2.1}$$

in which $Z(\omega)$ = a process with orthogonal increments of variance $E[|dZ\ (\omega)|^2] = dF(\omega)$ and $\{a(t;\ \omega)\}$ = a family of slowly varying functions of time for all values of ω. The condition that the functions $a(t;\ \omega)$ vary slowing in time is essential to retain the frequency interpretation of the parameter ω. When this condition is satisfied $a(t;\ \omega)e^{i\omega t}$ can be interpreted as an amplitude modulated wave. From Eq. 1, $X(t)$ has mean zero, covariance function

$$B(t,\ t + \tau) = E[X(t)X(t + \tau)^*] = \int_{-\infty}^{\infty} a(t;\ \omega)a(t + \tau;\ \omega)^* e^{-i\omega\tau} dF(\omega) \tag{2.2}$$

where the asterisk denotes complex conjugate, and variance

$$\sigma^2(t) = B(t,\ t) = \int_{-\infty}^{\infty} a(t;\ \omega)|^2 \tag{2.3}$$

Let F be a differentiable function and $dF(\omega) = S(\omega)\ d\omega$. The evolutionary power spectral density of $X(t)$ at time t is [7]

$$S_t(\omega) = |a(t;\ \omega)|^2\ S(\omega) \tag{2.4}$$

and represents the frequency content of the process in a small vicinity of t. Process $X(t)$ becomes stationary in the wide sense when functions $a(t;\ \omega)$ are time-invariant.

Consider the special case in which F is a piecewise constant function with jumps of magnitude $\sigma_q{}^2/2$ at a finite number of frequencies ω_q, $q = -Q$, ... -1, 1, ..., Q. In this case the integral in Eq. 2.1 is replaced with a finite sum that can be given in terms of trigonometric functions in the form

$$X(t) = \sum_{q=1}^{Q} a_q(t)\, \sigma_q (V_q \cos \omega_q t + W_q \sin \omega_q t) \tag{2.5}$$

in which a_q = slowly varying functions of time, and V_q and W_q = uncorrelated zero-mean, unit-variance random variables. The one-sided density of the process is

$$G_t(\omega) = \sum_{q=1}^{Q} |a_q(\omega)|^2 \, \sigma_q^2 \, \delta(\omega - \omega_q) \tag{2.6}$$

where δ = Dirac's delta function.

From Eqs. 2.5 and 2.6, the energy of an oscillatory process is associated with a fixed set of frequencies and fluctuates in time according to the modulating functions $a(t; \omega)$ or $a_q(t)$. For example, the oscillatory process in Eq. 2.6 has energy at frequencies ω_q and the energy associated with these frequencies is $|a_q(t)|^2\sigma_q{}^2$ at any time t. Similarly, when X(t) has a continuous spectrum the energy is distributed at any time within the range of frequencies where $G_t(\omega) > 0$. Therefore, the characteristics of the process $Z(\omega)$ have to be so chosen that the spectral density of X(t) be nonzero in the frequency range relevant to a particular task.

An elementary example of an oscillatory process is [1]

$$X(t) = c(t)Y(t) \tag{2.7}$$

in which c(t) = a slowly varying real-valued deterministic function modulating the variance of X(t), and Y(t) = a real-valued zero-mean wide sense stationary process with variance $\sigma_Y{}^2$, covariance function B_Y, spectral density S_Y, and one-sided spectral density G_Y. The family of oscillatory functions of the process is $\{c(t)e^{i\omega t}\}$ so that X(t) has the following one-sided evolutionary spectral density

$$G_t(\omega) = [c(t)]^2 G_Y(\omega) \tag{2.8}$$

of time-invariant shape. Nevertheless, the process in Eq. 2.7 has been applied extensively to model and generate seismic accelerograms, although these functions have time-dependent spectral shape. Another criticism of the model is that its samples generally have finite power at $\omega = 0$ even when $G_Y(0) = 0$ [8]. Thus, the model predicts the existence of a static load that is meaningless in seismic analysis.

To correct this undesirable sample properly of the process in Eq. 2.7, it has been proposed to generate artificial accelerograms from a different process obtained by passing a uniformly modulated white noise through a linear filter [8]. This model belongs to the class of oscillatory processes since it can be regarded as the output of a time-variant linear filter to stationary white noise. It has samples with no power at $\omega = 0$ but still has a time-invariant spectral shape. Generation of artificial accelerograms based on this model involves some numerical difficulties. In addition, the design of the filters for shaping the white noise input is not straightforward when, e.g., $X(t)$ has a multimodal spectrum. This is a common situation with seismic ground acceleration records.

SECTION 3
PROCESSES WITH MODULATED AMPLITUDE AND FREQUENCY

Consider the process $Y(t)$ and the amplitude modulating function $c(t)$ in Eq. 2.7. Let $\phi(t)$ be a (frequency) modulation function that is positive, real valued, and satisfies the conditions $\phi(0) = 0$ and $\phi'(t) > 0$ for $t \geq 0$. The process

$$X(t) = c(t)Y(\phi(t)) \tag{3.1}$$

is derived from $Y(t)$ by modulating its amplitude and frequency. It can also be given in the form

$$X(t) = c(t) \int_{-\infty}^{\infty} e^{i\omega\phi(t)}\, dZ(\omega) \tag{3.2}$$

if $Y(t)$ has the spectral representation $\int_{-\infty}^{\infty} e^{i\omega t}\, dZ(\omega)$ in which $Z(\omega)$ has orthogonal increments of variance $E[|dZ(\omega)|^2] = S_Y(\omega)d\omega$. The covariance function of $X(t)$ is obtained directly from Eq. 3.1 or from Eq. 3.2. It is

$$\begin{aligned} B(t,\ t + \tau) &= E[X(t)X(t + \tau)^*] \\ &= c(t)c(t + \tau)\ B_Y[\phi(t + \tau) - \phi(t)] \\ &= c(t)c(t + \tau) \int_{-\infty}^{\infty} e^{i\omega[\phi(t+\tau)-\phi(t)]}\ S_Y(\omega)\ d\omega \end{aligned} \tag{3.3}$$

so that the variance of $X(t)$ at time t has the expression

$$\sigma^2(t) = B(t,\ t) = c(t)^2\ \sigma_Y^2 \tag{3.4}$$

From Eqs. 2.1 and 3.2, one might conclude that $X(t)$ in Eq. 3.2 is an oscillatory process relative to the family of functions $\{a(t;\ \omega) = c(t)e^{i\omega[\phi(t)-t]}\}$. According to this interpretation $X(t)$ has, from Eq. 2.4, the evolutionary spectral density $c(t)^2 S_Y(\omega)$. However, this interpretation is incorrect because functions $a(t;\ \omega)$ do not vary slowly in time as required by developments in Ref. 7 so that ω loses the meaning of frequency. Examples are presented in subsequent paragraphs to illustrate differences between the nonstationary processes defined in Eqs. 2.1 and

3.2. These differences can be better appreciated if the concept of time-dependent spectrum is introduced for the process in Eq. 3.1.

According to Eq. 3.2, process X(t) is obtained from Y(t) by (i) scaling the amplitude of the random waves $dZ(\omega)e^{i\omega t}$ in the spectral decomposition of Y(t) from $dZ(\omega)$ to $c(t)\ dZ(\omega)$, and (ii) associating the scaled amplitudes with the time-dependent frequencies

$$\omega(t) = \omega \frac{\phi(t)}{t} \tag{3.5}$$

This observation suggests that the time-dependent one-sided spectral density of X(t) can be defined as

$$G_t(\omega) = c^2(t)\ G_Y\ (\omega \frac{t}{\phi(t)}) \tag{3.6}$$

As expected, G_t coincides with the spectrum in Eq. 2.8 when $\phi(t) = t$ because in this case X(t) in Eq. 3.1 is a uniformly modulated process. The modulating function $\phi(t)$ determines the rate at which the spectrum of X(t) changes with time. In contrast with oscillatory process, that are defined for slowly varying spectra, $G_t(\omega)$ in Eq. 3.7 is valid for both slow and rapid changes in the frequency content.

According to Eqs. 2.1, 3.5, and 3.6, processes with modulated amplitude and frequency belong to the class of oscillatory processes when their spectral characteristics do not change rapidly with time, for a slowly varying family of functions $\{a(t;\ \omega)\}$ can be found. The representation of these processes as oscillatory ones requires that the increments of process $Z(\omega)$ in Eq. 2.1 have nonzero variance for all frequencies ω where $G_t(\omega)$ in Eq. 3.6 is not zero for at least one value of t.

<u>Example 1</u>. Assume that $\phi(t) = \alpha t$ with $\alpha > 0$, $c(t) \equiv 1$, and Y(t) has power at a single frequency ω_0, i.e., $G_Y(\omega) = \sigma_Y^2\ \delta(\omega - \omega_0)$. Then,

$$X(t) = \sigma_Y(V \cos \omega_0\ \alpha t + W \sin \omega_0\ \alpha t) \tag{3.7}$$

in which V and W = uncorrelated random variables with zero mean and unit variance. In this case, X(t) is a wide-sense stationary process having power only at frequency $\omega_0\alpha$. The one-sided spectral density of this stationary process is $\sigma_Y{}^2(\omega - \omega_0\alpha)$ and coincides with the spectrum $G_t(\omega) = \sigma^2\delta(\omega - \omega_0\alpha)$ in Eq. 3.6.

If the frequency modulation function $\phi(t)$ has a more general form, X(t) is still a monochromatic process with time-invariant total power $\sigma_Y{}^2$. However, the power of X(t) is associated at any time t with the time dependent frequency $\omega_0(t) = \omega_0\phi(t)/t$ in Eq. 3.5. Therefore, X(t) is a nonstationary process. This is not an oscillatory process because there is no slowly varying family of functions $a(t; \omega)$ that can describe the evolution of the energy content of the process in time. Thus the class of processes with modulated amplitude and frequency is not included in the class of oscillatory processes.

Example 2. Let Y(t) in Eq. 3.1 have the discrete one-sided spectrum $G_Y(\omega) = \sum_{q=1}^{Q} \sigma_Y{}^2\delta(\omega - \omega_q)$ so that

$$X(t) = c(t) \sum_{q=1}^{Q} \sigma_q(V_q \cos \omega_q(t)t + \sin \omega_q \sin \omega_q(t)t) \tag{3.8}$$

in which $\omega(t) = \omega_q\phi(t)/t$ and V_q and W_q = uncorrelated random variables with zero mean and unit variance. From Eq. 3.8, X(t) has energy of magnitude $c^2(t)\ \sigma_q{}^2$ at the time dependent frequencies $\omega_q(t)$, q = 1 ..., Q. This observation is consistent with the definition in Eq. 3.6 showing that $G_t(\omega) = c^2(t) \sum_{q=1}^{Q} \sigma_q^2\ \delta(\omega t/\phi(t) - \omega_q) = c^2(t) \sum_{q=1}^{Q} \sigma_q^2\ \delta(\omega - \omega_q(t))$. Note also that the ratio of any two time-dependent frequencies $\omega_{q2}(t)/\omega_{q1}(t) = \omega_{q2}(t) - \omega_{q1}(t) = (\phi(t)/t)(\omega_{q2} - \omega_{q1})$ varies with time. Thus, the mapping from the spectrum of Y(t) to the time-dependent spectrum of X(t) involves, in addition to a scaling and a translation of the spectral ordinates, a distortion of the spectral shape.

In accordance with Eqs. 2.5 and 3.8, oscillatory processes are characterized by a fixed set of frequencies with nonzero power of time-dependent intensity. On the other hand, the modulated processes in Eq. 3.1 have time invariant spectral coordinates when c(t) is constant but these coordinates are associated with time dependent frequencies.

The generalization to processes Y(t) with continuous spectra follows directly from Eq. 3.7. The summation in this equation is replaced with an integral over all frequencies and the random variables V_q and W_q become increments of uncorrelated processes with orthogonal increments. As previously mentioned, these processes are oscillatory ones if their spectral shape changes slowly with time.

<u>Example 3</u>. Consider a superposition of processes with modulated amplitude and frequency,

$$X(t) = \sum_k X_k(t) = \sum_k c_k(t)\ Y_k(\phi_k(t)) \tag{3.9}$$

in which Y_k, c_k, and ϕ_k have the same properties as in Eq. 3.1 and the processes $Y_k(t)$ are uncorrelated. The covariance function of X(t) is

$$\begin{aligned} B(t,\ t+\tau) &= \sum_k c_k(t)\ c_k(t+\tau)\ B_k(\phi_k(t+\tau) - \phi_k(t)) \\ &= \sum_k\ c_k(t)\ c_k(t+\tau) \int_{-\infty}^{\infty} e^{i\omega[\phi_k(t+\tau)]-\phi_k(t)]}\ S_k(\omega)\ d\omega \end{aligned} \tag{3.10}$$

in which B_k and S_k = covariance and spectral density functions of $Y_k(t)$, respectively. Since the processes $Y_k(t)$ are uncorrelated, the concept of time dependent spectrum in Eq. 3.6 can be extended with the same interpretation to the process in Eq. 3.9. The corresponding time-dependent spectrum is then

$$G_t(\omega) = \sum_k\ c_k^2(t)\ G_k(\omega t/\phi_k(t)) = \sum_k c_k^2\ G_{t,k}(\omega) \tag{3.11}$$

where G_k = one-sided spectrum of $Y_k(t)$, as illustrated in Fig. 3-1. Note that ordinates of spectra of different components $X_k(t)$ of X(t) that do not overlap at a given time may overlap at a later time and visa versa depending on the frequency content of the processes $Y_k(t)$ and the modulation functions $\phi_k(t)$.

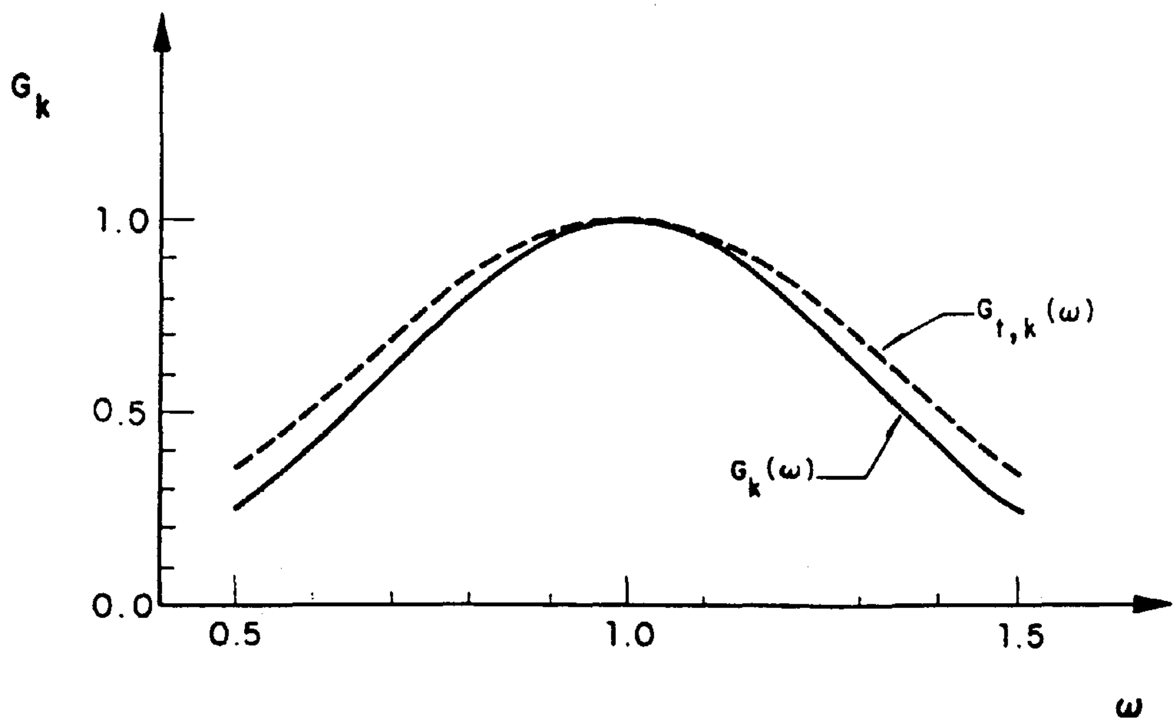


Figure 3-1 Spectral Densities of Example 3

Use of process X(t) in modelling is straightforward. Assume, e.g., that the ground acceleration spectrum at a site has K bell-shaped modes of variable height and width that are centered at the time-dependent frequencies $\bar{\omega}_k(t)$. From Eq. 3.12, this accelerogram can be modeled by a process as in Eq. 3.9 with K components and time dependent spectrum

$$G_t(\omega) = \sum_k c_k^2(t) \exp\left[-\frac{1}{2}\left[\frac{\omega - \bar{\omega}_k(t)}{\bar{\sigma}_k(t)}\right]^2\right] \tag{3.12}$$

in which $G_k(\omega) = \exp\left[-\frac{1}{2}\left[\frac{\omega - \bar{\omega}_k}{\bar{\sigma}_k}\right]^2\right]$, $\bar{\omega}_k(t) = \bar{\omega}_k\phi(t)/t$, and $\bar{\sigma}_k(t) = \bar{\sigma}_k\phi_k(t)/t$. The height and width of the modes are controlled by the modulation functions c_k and ϕ_k and by the parameters $\bar{\sigma}_k$ of the spectral density of $Y_k(t)$. A restriction of the model is that the frequency modulating functions control the variation in time of both the location and width of the spectral models. Once the functions $\omega_k(t)$ are specified, so is the variation in time of the mode widths.

This limitation of the model can be overcome if the representation in Eqs. 3.9 and 3.12 is modified so that any spectral mode k be modeled by a process.

$$X_k(t) = \sum_{j=0}^{2} c_{kj}(t)\, Y_{kj}(\phi_{kj}(t)) \tag{3.13}$$

in which c_{kj} and ϕ_{kj} = modulation functions $\phi_{k0}(t) = t$, and $\{Y_{kj}\}$ = zero-mean uncorrelated stationary processes. The one-sided spectral density $G_{k0} = G_k$ of Y_{k0} corresponds to the narrowest frequency content of mode k. Process $c_{kj}(t)\, Y_{kj}(\phi_{kj}(t))$, j = 1, 2, are used as "correction terms" on the uniformly modulated process $c_{k0}(t)\, Y_{k0}(\phi_{k0}(t)) = c_{k0}(t)\, Y_{k0}(t)$ that account for changes in the frequency content of the mode relative to G_k. For example, the time dependent spectral density of $X_k(t)$ extends to higher or lower frequencies if $\phi_{k1}(t) = t$ and $\phi_{k2}(t) > t$ or $\phi_{k1}(t) < t$ and $\phi_{k2}(t) = t$, respectively. Figure 3-1 shows the spectral densities G_{k0} and $G_{t,k}$ of $Y(_{k0})$ and $X_k(t)$ in Eq. 3.13 for $G_{k0}(t)$ =

$\exp\left[-\frac{1}{2}\left[\frac{\omega - 1}{0.3}\right]^2\right]$ and $G_{kj}(t) = \exp\left[-\frac{1}{2}\left[\frac{\omega - \bar{\omega}_{kj}(t)}{\sigma_{kj}(t)}\right]^2\right]$, $j = 1, 2$ at a time when $\bar{\sigma}_{kj}(t) = 0.3$, $\bar{\omega}_{k1}(t) = 0.8$, and $\bar{\omega}_{k2}(t) = 1.2$. For this choice of the correction factors, the time dependent spectral density of $X_k(t)$ has the peak at the same frequency as $Y_{k0}(t)$.

When spectral modes have practically constant location and shape, the frequency modulation is not necessary and the process in Eq. 3.9 can be simplified to

$$X(t) = \sum_k c_k(t)\, Y_k(t) \tag{3.14}$$

From Eqs. 2.1, 2.7, 2.8, and 3.6, it follows that X(t) is an oscillatory process relative to the family of slowly varying functions $a(t; \omega) = \sum_k c_k(t)\, S_k^{1/2}(\omega)$ and a process $Z(\omega)$ with orthogonal increments $dZ(\omega)$ of variance $E[|\, dZ(\omega)^2|d\omega$. The process can be used for modeling accelerograms with multimodal spectra whose modes preserve shape and location. This is a useful feature because spectra of some accelerograms may have several peaks whose magnitudes only change with time, as shown in the next section. The process has the same undesirable sample property as the one in Eq. 2.7. There is a direct generalization of the process in Ref. 8 that can be obtained by filtering uniformly modulated independent white noise process $c_k(t)\, W_k(t)$ through linear systems of unit impulse response functions $h_k(t)$. The generalized model is

$$X(t) = \sum_k \int_0^t h_k(t - \tau)\, c_k(\tau)\, W_k(\tau)\, d\tau \tag{3.15}$$

A useful feature of the processes in Eqs. 3.14 and 3.15 is that they can model the arrival of various seismic waves if the amplitude modulation functions c_k are selected adequately. These processes can also be applied to model spectral modes of fixed location but time dependent width following the principle outlined in Eq. 3.13. However, the use of these processes for this purpose can be less satisfactory due to their limited number of parameters.

SECTION 4
MODEL CALIBRATION AND SIMULATION

Two stochastic families of the East-West component of the motion recorded in the neighborhood of the Ministry of Communications and Transportation's main building (SCT-EW) in Mexico City on September 19, 1985 were simulated. The first one corresponds to an oscillatory process (Model I), and the second to a process with modulated amplitude and frequency (Model II).

The Fourier transform of the 1985 STC-EW accelerogram is characterized by three modes with time-dependent location, bandwidth, and magnitude.

Model I. Consider the elementary oscillatory process in Eq. 2.7 with one-sided evolutionary spectral density in Eq. 2.8 and variance $\sigma(t)^2 = c(t)^2 \, \sigma_Y^2$. The spectrum of the stationary process Y(t) in Eq. 2.8 has, consistent with the SCT-EW record, three modes and can be represented by

$$G_Y(\omega) = \sum_{i=1}^{3} d_i \exp\left[- A_i \, (\omega - \Omega_i)^2\right] \tag{4.1}$$

in which estimates of d_i, A_i, and Ω_i, $i = 1, 2, 3$, are given in Table 4-I. The modulation function follows the evolution in time of the variance $\sigma(t)^2 = c(t)^2 \sigma_Y^2$ of the process. It has been found by curve fitting that

$$c(t)^2 = 0.033 \exp\left[- (t-58)^2/25\right] + 0.007 \exp\left[- (t-54)^2/400\right] \qquad t < 100 \text{ sec.} \tag{4.2}$$

Model II. According to Eq. 3.11 and the observation that the spectral modes of the SCT-EW record have time-dependent central frequency, bandwidth, and magnitude, a process X(t) with spectral density

$$G_t(\omega) = \sum_{k=1}^{3} c_k^2(t) \exp\left[- A_k(t) \, (\omega - \Omega_k(t))^2\right] \tag{4.3}$$

is selected. The modulation functions $c_k(t)$, $A_k(t)$, and $\Omega_k(t)$ have been calibrated to the STC-EW accelerogram and are shown in Figs. 4-1 and 4-2.

TABLE 4-1 PARAMETERS OF MODEL I

Mode	d_i	A_i	Ω_i
1	0.1802	10	2.482
2	0.6608	20	3.142
3	0.00878	4.75	8.87

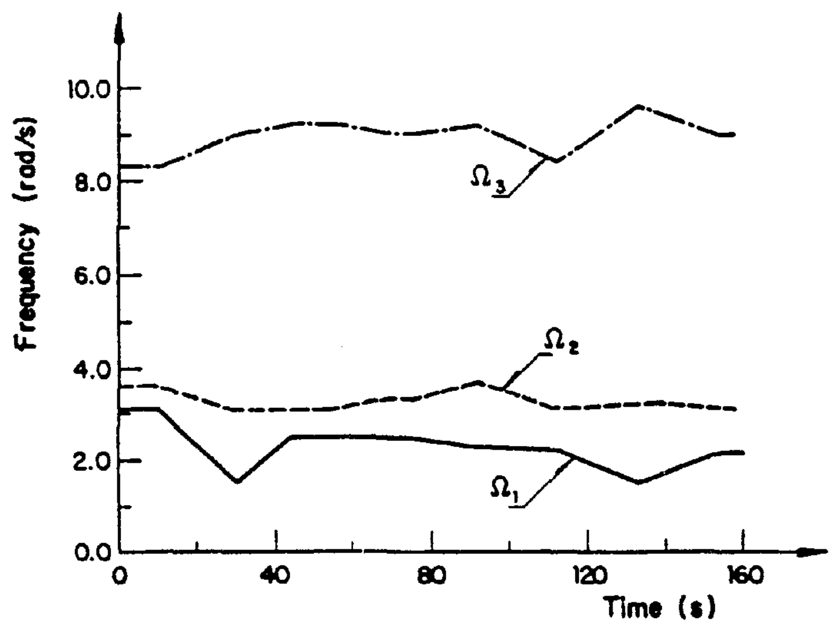


Figure 4-1a Modulating Functions of Frequency

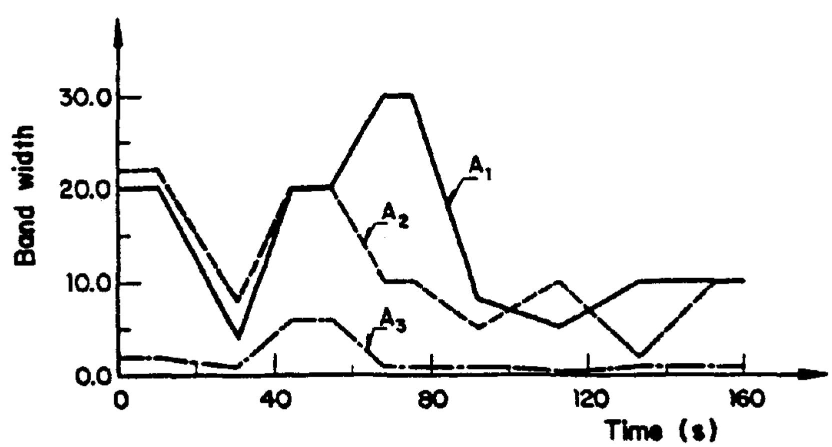


Figure 4-1b Modulating Functions of Band Width

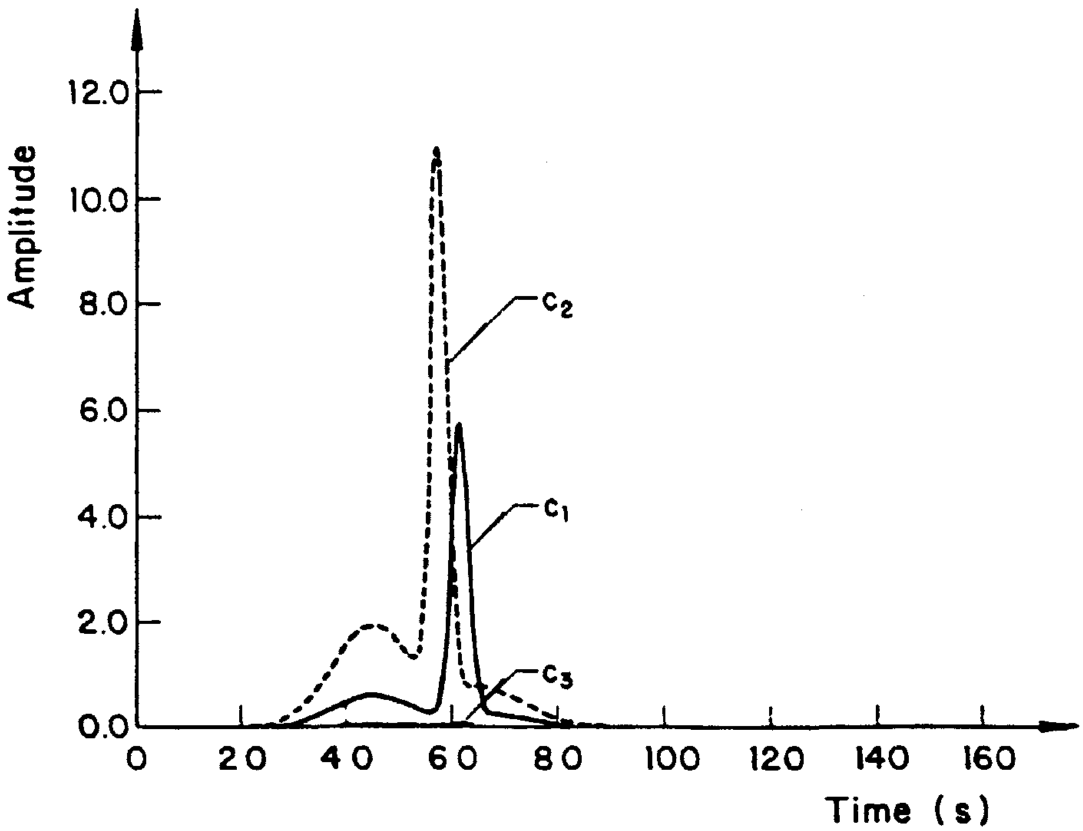


Figure 4-2 Modulating Functions of Amplitude

Response spectra for a 5% damping ratio and spectral density functions were computed for twelve simulated accelerograms corresponding to Models I and II, respectively. Figures 4-3 to 4-6 show second-moment characteristics of these response spectra and of Fourier transforms for simulated accelerograms by both models. Results for the SCT-EW record are also shown. The mean elastic responses of single-degree systems having 5 percent damping computed using the two models are close to each other (Figs. 4-3 and 4-5). However, the coefficient of variation of the peak structural acceleration is substantially greater for systems subjected to earthquakes generated with Model II than for those obtained using Model I.

To evaluate the potential usefulness of the Models I and II in earthquake engineering, ductility demands have been evaluated for single degree-of-freedom hysteretic bilinear systems subject to simulated and actual accelerograms. The systems are characterized by restoring forces with and without stiffness degradation, initial natural periods T = 0.2, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 5.0 sec., damping ratio $\xi = 5\%$, and ratios of elastic to yield force equal to four.

Figures 4-7 and 4-8 show the mean values of ductility demand of the systems normalized with respect to the ductility demand of the same systems excited with the SCT-EW,1985 accerologram $(\mu/\bar{\mu}_{SCT})$, as well as the coefficients of variation of the ductility demand, CV_{μ}, for different natural periods. The ratio $\bar{\mu}/\mu_{SCT}$ is near one for both models and for the different kinds of behavior. However, the coefficients of variation of the ductility demands are systematically lower for systems excited with Model I than for those subjected to Model II earthquakes. In addition, values of CV_{μ} for Model II follow closer corresponding values for the actual accelerograms with response spectra in Fig. 4-11. It seems, therefore, that the latter model is preferable, at least when the simulated motions are to be used in reliability analyses, since failure probabilities can be underestimated when based on Model I.

The normalized ductility demands of the systems excited with Model II accelerograms are compared in Figs. 4-9 and 4-10 with those obtained by exciting the structures with five scaled motions recorded on soft soil in

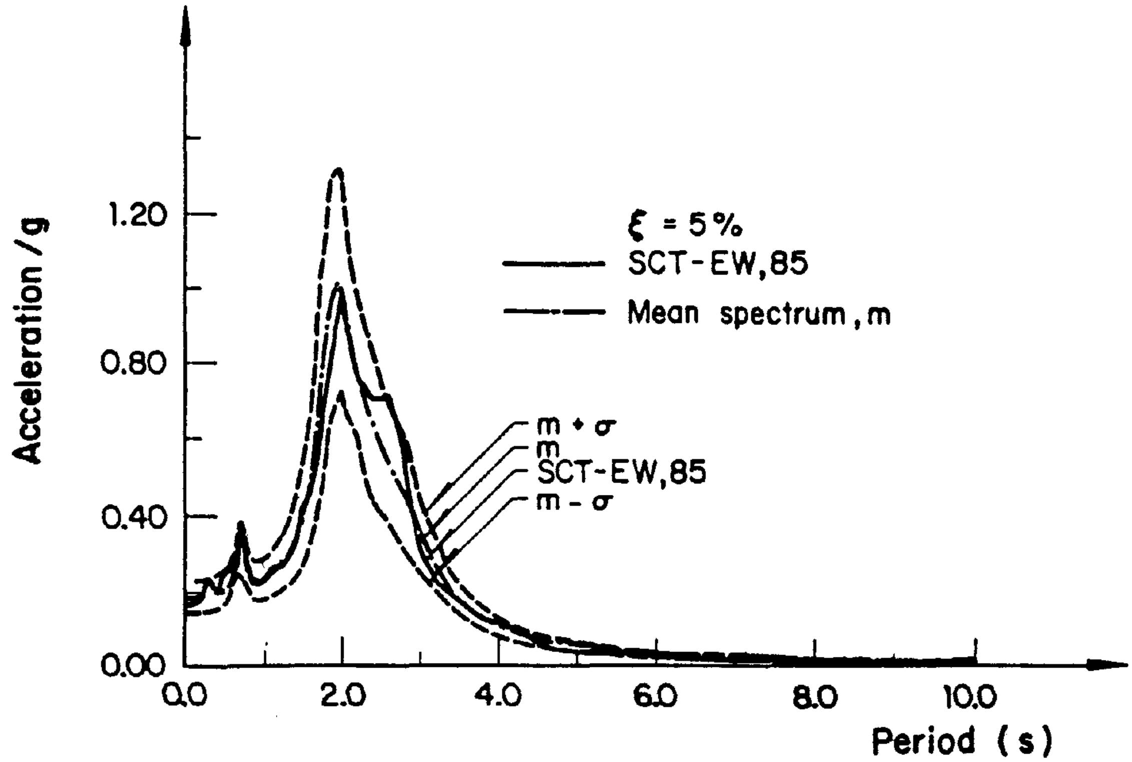


Figure 4-3 Response Spectra. Model I

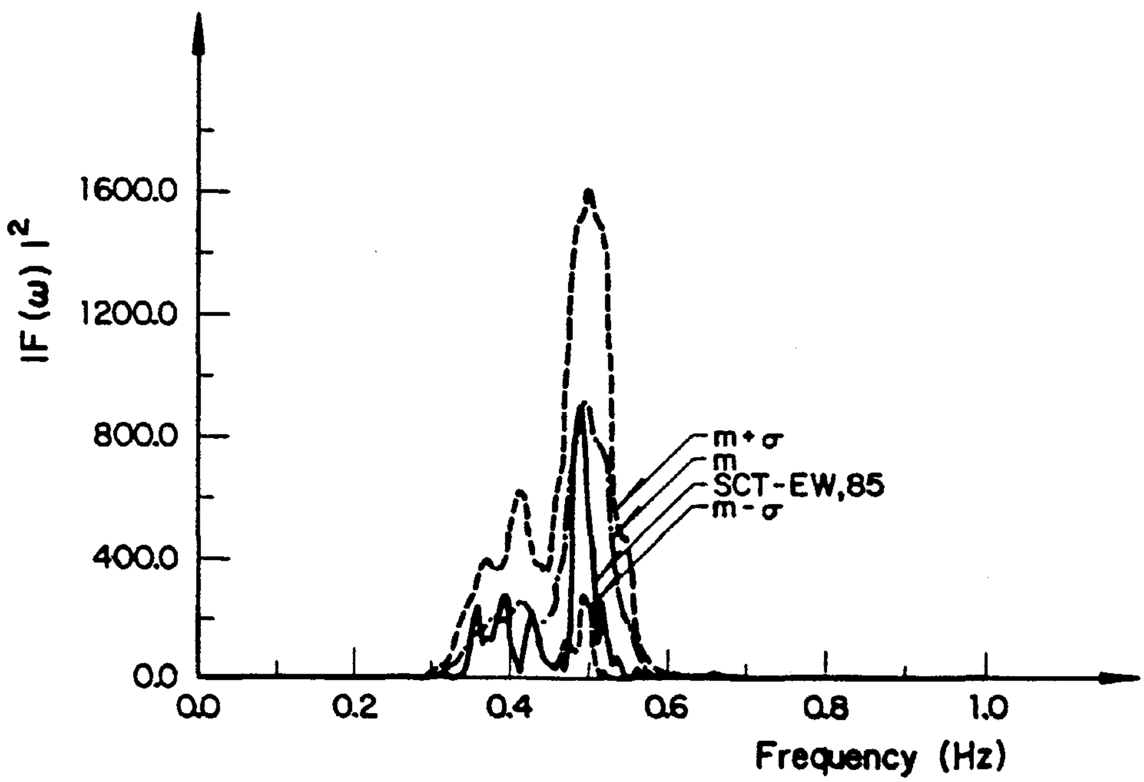


Figure 4-4 Power Spectra. Model I

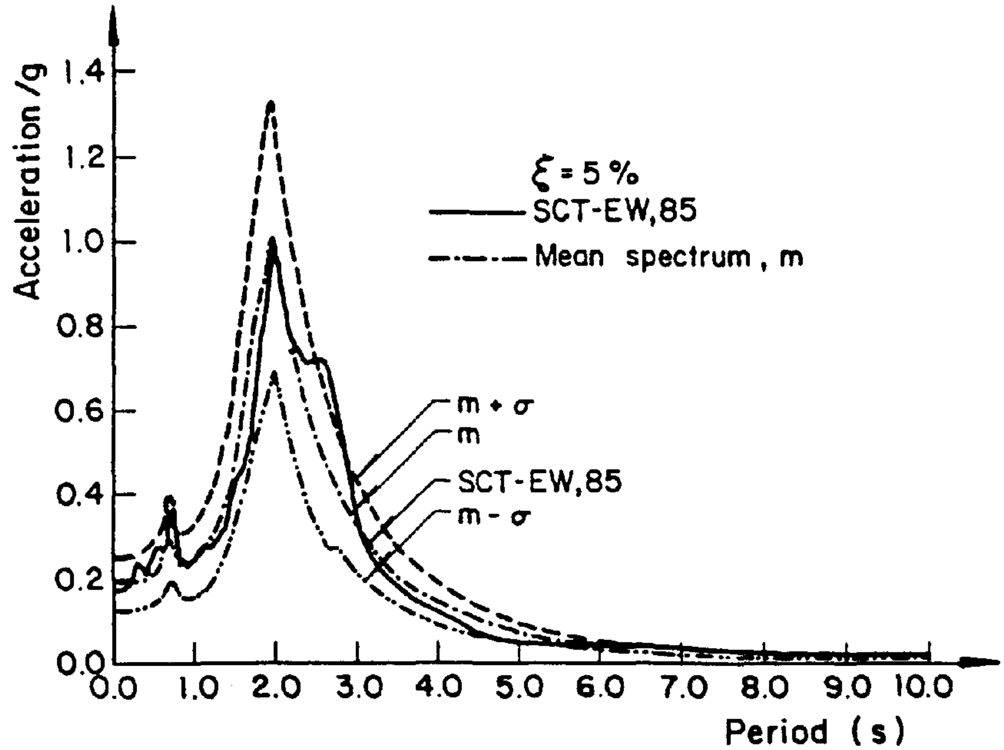


Figure 4-5 Response Spectra. Model II

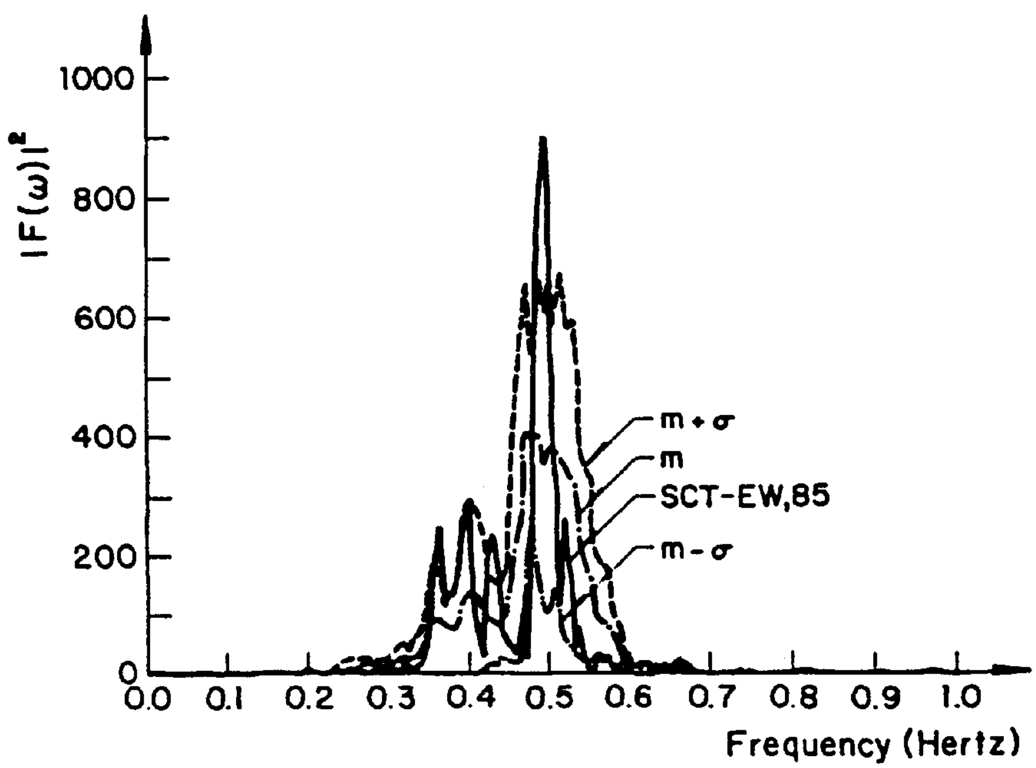


Figure 4-6 Power Spectra. Model II

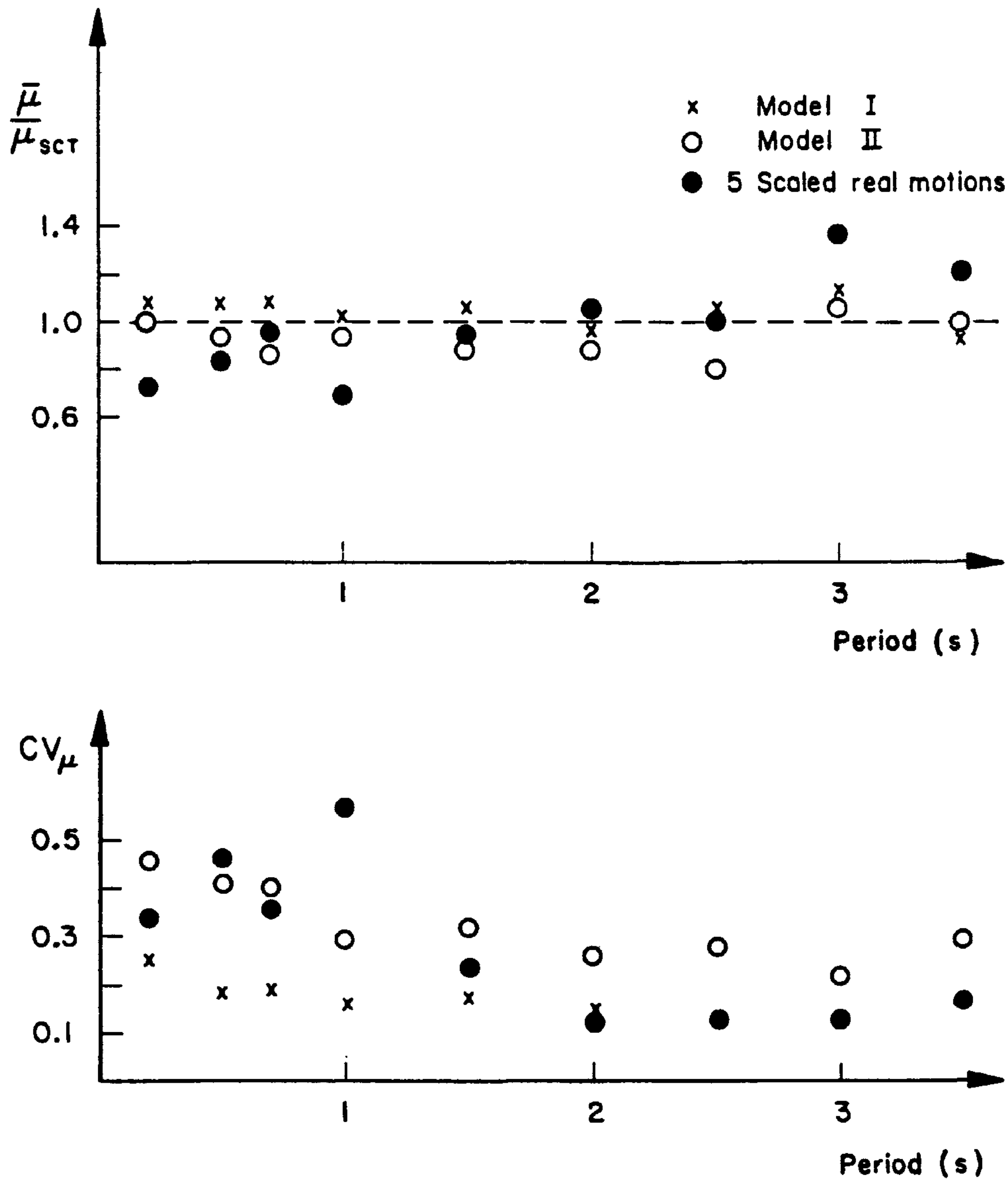
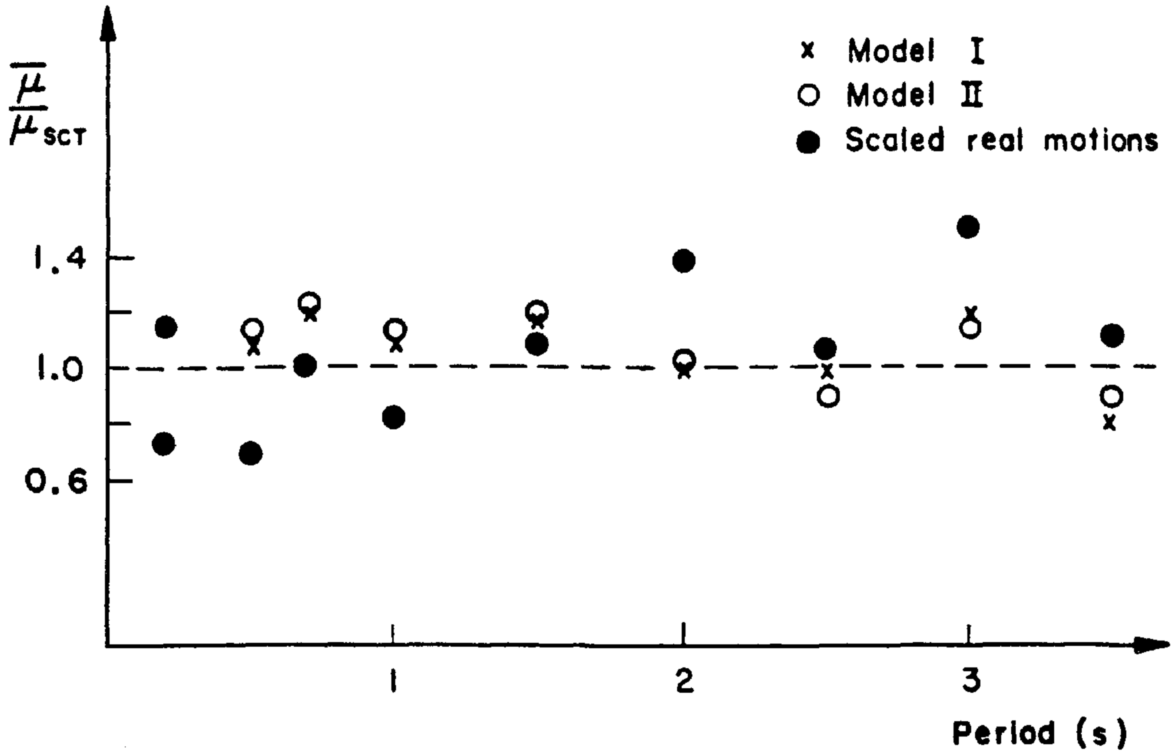


Figure 4-7 Stiffness Degrading Systems



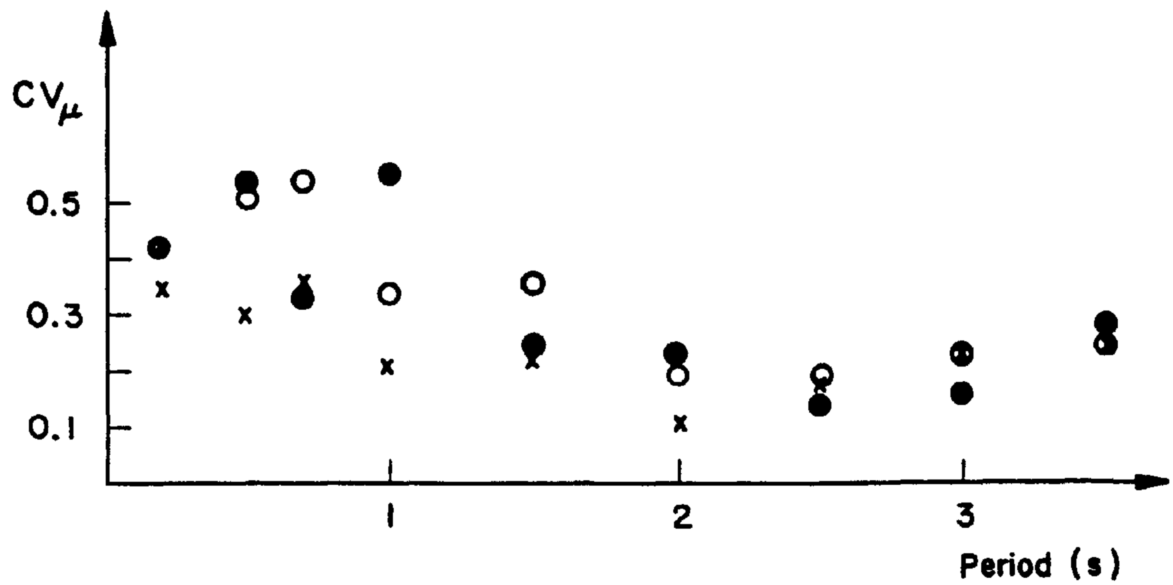


Figure 4-8 Systems Without Stiffness Degradation

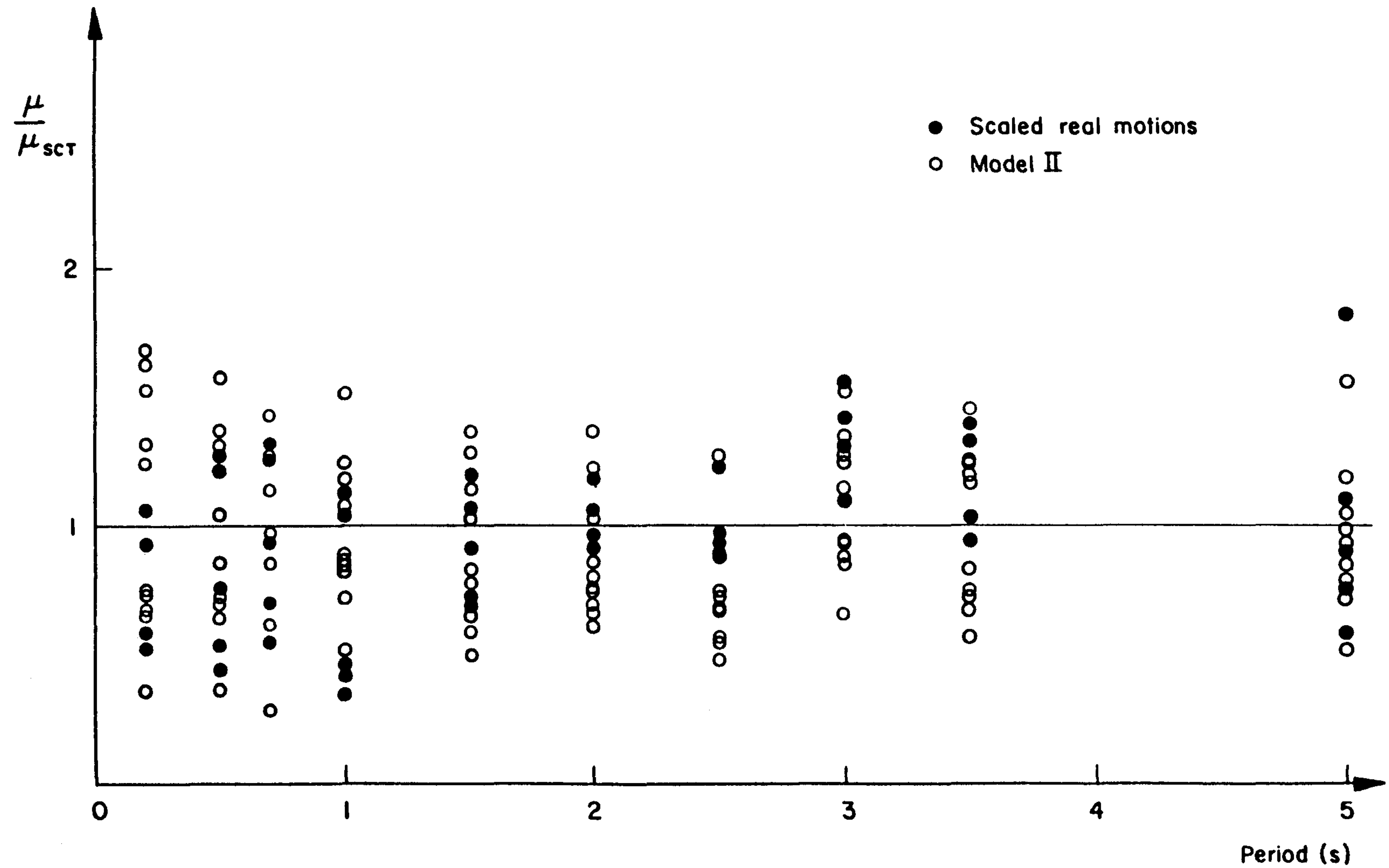


Figure 4-9 Stiffness Degrading Systems

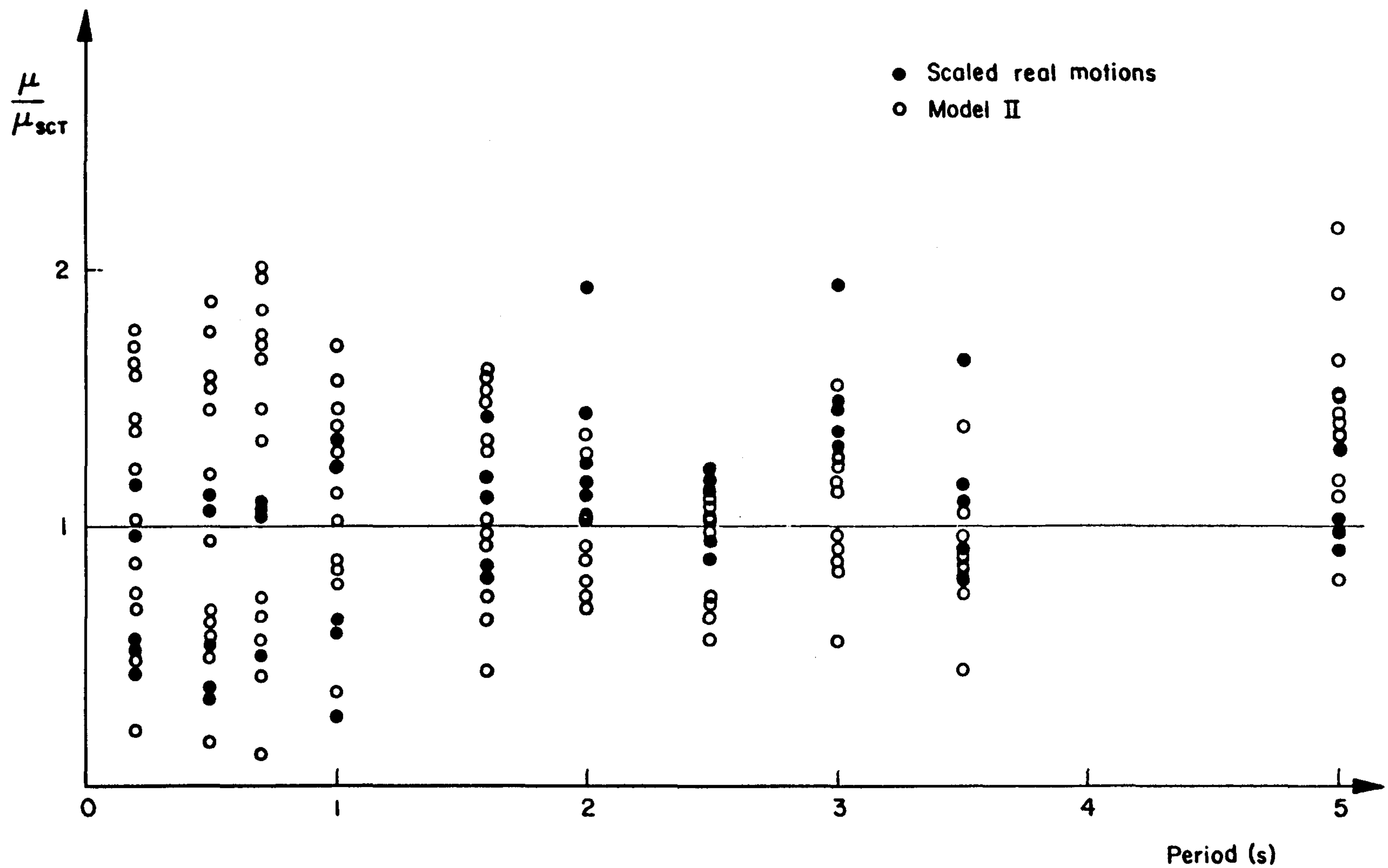


Figure 4-10 Systems Without Stiffness Degradation

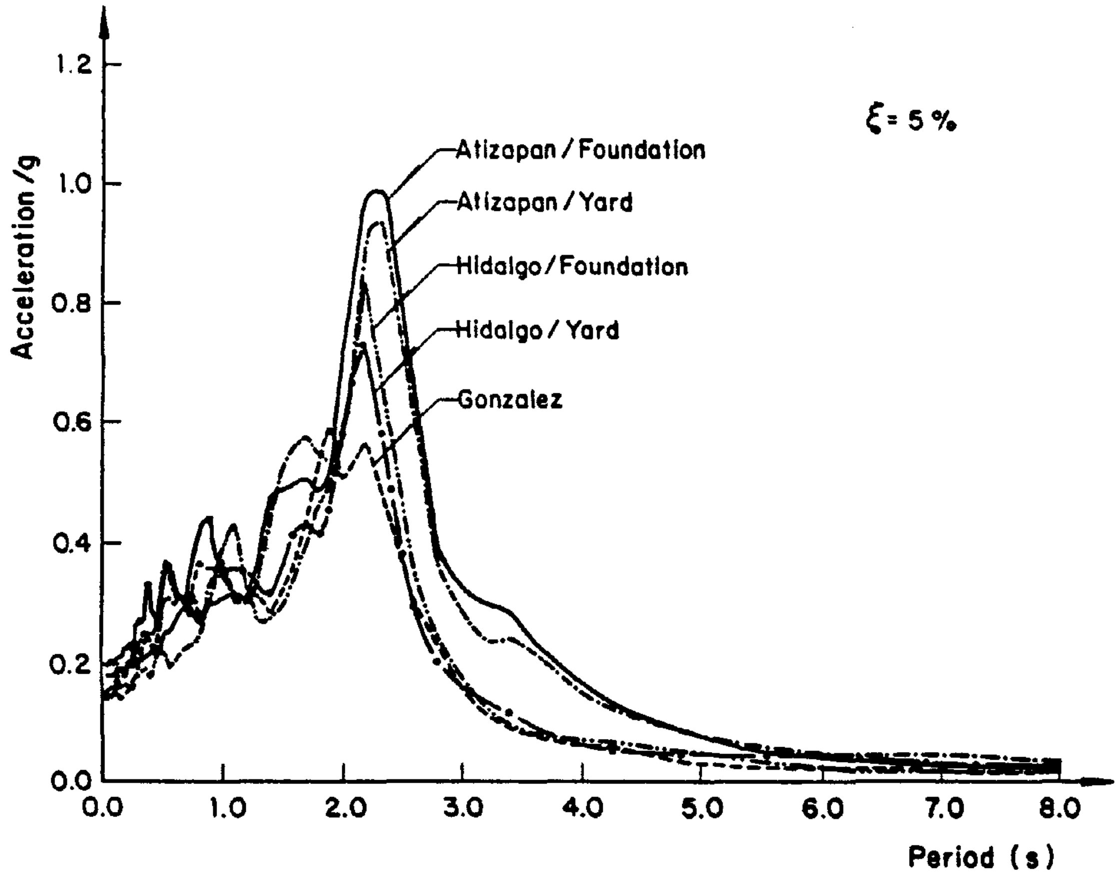


Figure 4-11 Response Spectra of Five Real Accelerograms

Mexico City. Response spectra for 5% damping ratio of scaled real accelerograms are shown in Fig. 4-11. Cumulative square accelerations as functions of time for each motion are compared with that of SCT-EW,85 record in Fig. 4-12. Results in Figs. 4-9 and 4-10 suggest again that Model II can provide a useful representation of seismic motions that can be used in the analysis of structural systems having a broad range of dynamic characterics.

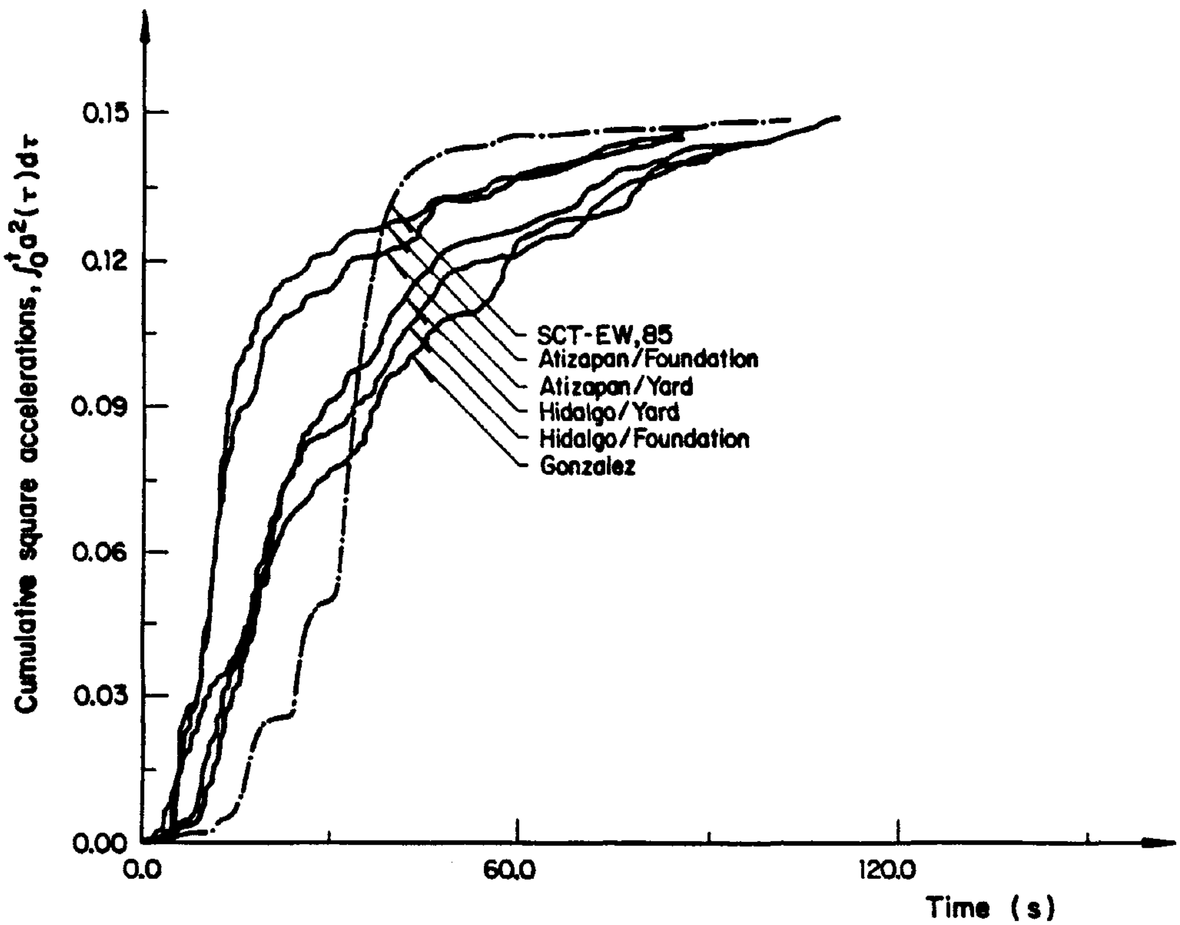


Figure 4-12 Cumulative Square Accelerations of Real Accelerograms as Functions of Time

SECTION 5
CONCLUSIONS

A nonstationary process has been proposed for modeling ground acceleration during seismic events. The process can be obtained by modulating both the amplitude and the frequency of a stationary process. It has been shown that the class of processes with modulated amplitude and frequency is not included in that of oscillatory process.

Accelerograms recorded during the 1985 Michocán Earthquake in Mexico have been used to calibrate the proposed process and an elementary oscillatory process. Samples of these processes and actual accelerograms have been used to calculate extreme responses of linear and nonlinear single-degree-of-freedom systems. Results suggest that the proposed model is more adequate for reliability studies. System responses to accelerograms generated by this model have coefficients of variation consistent with those corresponding to actual accelerograms.

SECTION 6
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