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HYSTERETIC COLUMNS  
UNDER RANDOM EXCITATIONS

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## PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

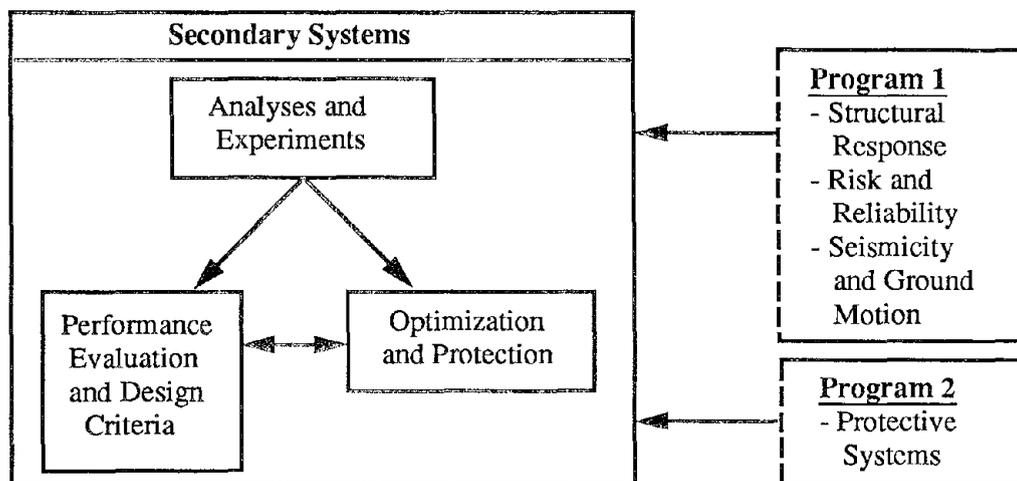
- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself. The research goals in this area are to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom, and nonlinearities in the primary structure.
2. Develop practical criteria and procedures for the analysis and design of secondary systems.
3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.



*Considered in this report is the dynamic response of a hysteretic column under random excitations in both horizontal and vertical directions. Response probability distributions are obtained using the approximate method of dissipation energy balancing. It is shown that the shape of the distribution depends largely on the relative contribution of the hysteretic component.*

*The response distributions provide deeper insight into stochastic response characteristics of the column. The effect of vertical excitation on the column response is also carefully examined.*

## ABSTRACT

In a seismic event, the horizontal ground motion gives rise to an additive random excitation to a column, while the vertical ground motion gives rise to a multiplicative random excitation. The adjectives "additive" and "multiplicative" refer to the fact that these excitations appear in the governing equation as an inhomogeneous term on the right hand side and in a coefficient on the left hand side, respectively. In the present paper, the approximate method of dissipation energy balancing, previously developed for randomly excited nonlinear systems, is applied to obtain the probability density for the response amplitude of a hysteretic column under such excitations. Both types of random excitations are idealized as Gaussian white noises. Three different hysteresis models are used in the formulation: the Hata-Shibata model, a general bi-linear model, and the Bouc-Wen model. Numerical results are obtained for wide ranges of excitation spectral levels and other physical parameters. To verify the accuracy of these results, the root-mean-square responses are also computed and compared with results from Monte-Carlo simulations and from the methods of equivalent linearization and Gaussian closure.



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## SECTION 1

### INTRODUCTION

In recent years there has been considerable interest in the hysteresis behavior of stochastic dynamic systems, which must be taken into account when the deformation of a system becomes large. An excellent review of recent contributions in this area was given by Wen [19].

Various models have been proposed for the analysis of hysteretic systems. Shown in figure 1-1 is the well-known bi-linear model in which the restoring force of the system is idealized as consisting of two components: a purely elastic component, and an elasto-plastic component. Such a restoring force is illustrated in figure 1-2(a), and the part attributable to the elasto-plastic component alone is shown in figure 1-2(b). The Hata-Shibata model shown in figure 1-3 is a special case of the bi-linear model in which the elasto-plastic component becomes purely plastic.

While the bi-linear model is conceptually simple, it is by no means analytically simple as well. The idealization that the slope of the restoring force changes suddenly as the deformation varies in either positive or negative direction is clearly unrealistic, and it may also cause analytical problems.

To remedy the above deficiency, Iwan [12] has proposed a smooth hysteretic model, composed of a distribution of Jenkin's elements in parallel, each of which consists of a linear spring in series with a Coulomb damper. Another smooth hysteresis model proposed initially by Bouc [4] and extended by Wen [17] has gained greater acceptance lately among stochastic dynamicists. It has a deformation and restoring force relationship illustrated in figure 1-4.

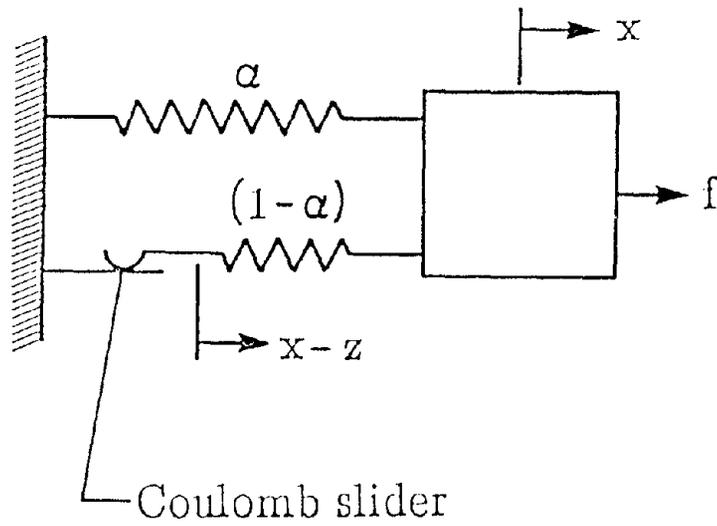


FIGURE 1-1 Schematic Representation of a Bi-Linear Hysteretic System.

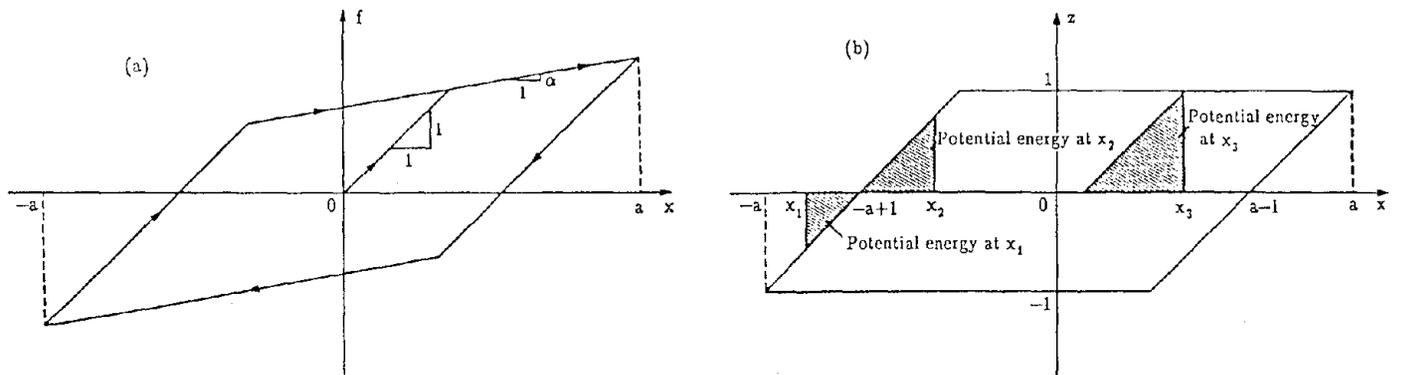


FIGURE 1-2 A Bi-Linear Hysteretic System: (a) Restoring Force-Deformation Relationship; (b) Elasto-Plastic Component.

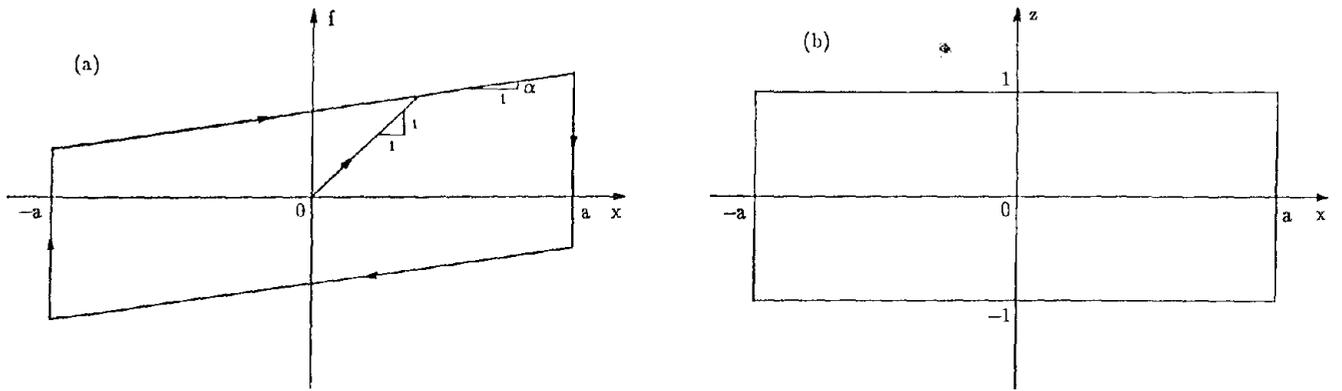


FIGURE 1-3 A Hata-Shibata Hysteretic System: (a) Restoring Force-Deformation Relationship; (b) Hysteretic Component.

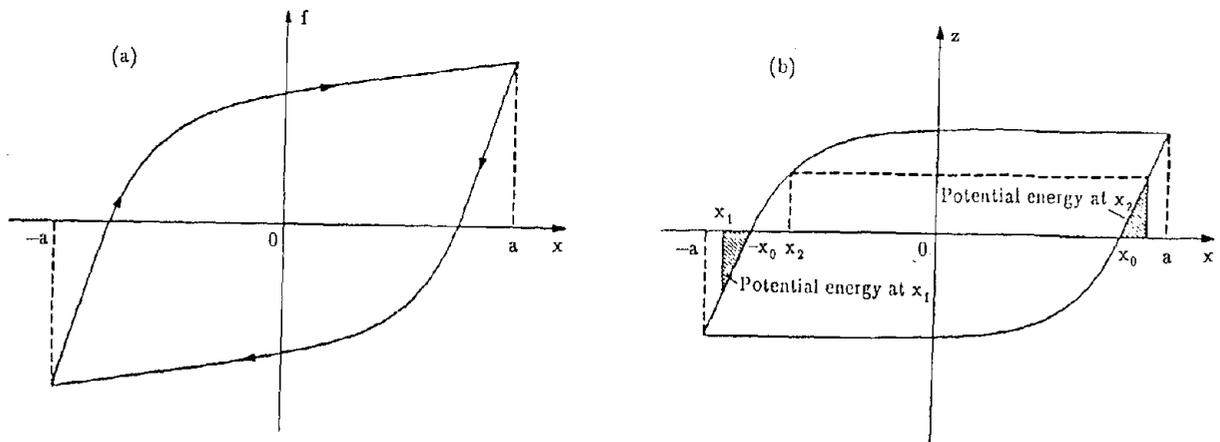


FIGURE 1-4 A Smooth Hysteretic System: (a) Restoring Force-Deformation Relationship; (b) Hysteretic Component.

Further refinements of the Bouc-Wen model have also appeared in the literature, e.g. [1,2]. For other even more complicated models, see [3,8].

Exact solutions for randomly excited hysteretic systems are not obtainable at the present time, and a number of approximation techniques have been developed to compute either the statistical moments or probability distributions of the response. Perhaps the most well-known procedures are the methods of equivalent linearization and Gaussian closure. These methods can be applied easily to most nonlinear systems. It has been applied, for example, to the Hata-Shibata model [10,13], although the results so obtained may not be very accurate. For the more general type of bi-linear systems, a combination of equivalent linearization and the slowly varying parameter method [9,11] led to good results for systems with weak or moderate hysteresis. However, if a system is nearly elasto-plastic, the accuracy is generally poor. For smooth hysteretic systems, Wen [18] has devised a special form of equivalent linearization and obtained the mean-square values of the system response which are in very good agreement with the simulation results for wide ranges of excitation intensities and hysteresis properties. One possible reason for his success is that the linearized system obtained from this special scheme is capable of exhibiting some hysteretic properties of its own.

A common shortcoming of linearization and Gaussian closure techniques is that the results are restricted only to the statistical properties of the system response, in particular the first and second order statistical moments. In order to obtain the more useful information of probability distribution of the response, another well-known approximate method, called stochastic averaging, has been applied to both bi-linear and smooth hysteretic systems [20,15]. Under the assumptions that energy dissipation is small and that the random

excitations are weak, the total energy, namely the sum of the kinetic and potential energies, in the system is a slowly varying function of time. It is then possible to derive an approximate Ito type stochastic differential equation for the total energy, from which the probability density of the total energy may be obtained as the solution to the associated Fokker-Planck equation.

The assumption of small energy dissipation appears to be too restrictive for most practical hysteretic systems. More recently, a new procedure called energy dissipation balancing, was developed by the authors [6] without such a restriction. This procedure has been applied to hysteretic systems of both the bi-linear type and the smooth type [7] to obtain the mean-square response and the probability densities of the total energy and the amplitude of the response. The results were found to be in very good agreement with simulations.

Nearly all the analyses of stochastic hysteretic systems, including those cited above, are restricted to additive random excitations, namely, random excitations which appears as inhomogeneous terms in the governing differential equations. One exception is a paper by Shih and Lin [16] on a hysteretic column subjected to both horizontal and vertical seismic excitations. While the horizontal seismic excitation is additive, it can be shown that the vertical excitation is multiplicative which appears on the left hand side of the governing equation in a coefficient for the unknown. The hysteretic behavior of the column considered in [16] was described by the Hata-Shibata model, and statistical moments were computed for the response using a Gaussian closure procedure. However, since the Hata-Shibata model represents a very strong hysteresis behavior, results obtained from either Gaussian closure or

equivalent linearization may not be very accurate. They can at best be used to infer to some general trends.

In the present paper, the method of dissipation energy balancing is applied to hysteretic systems under both additive and multiplicative random excitations. Specifically, we use again one of the simplest examples of such systems, namely, a hysteretic column. The Hata-Shibata model, the general bi-linear model, and the Bouc-Wen model are used in the formulations. Both horizontal and vertical random excitations are replaced by Gaussian white noises, and the probability densities of the response amplitude at the stationary state are computed for wide ranges of excitation spectral levels and other physical parameters. To verify the accuracy of the present method the root-mean-square responses are also computed and compared with the results obtained from the Monte-Carlo digital simulations and from other approximate procedures, such as equivalent linearization and Gaussian closure.

SECTION 2  
FORMULATION

Consider a massless column, supporting a concentrated mass at the top and rigidly clamped in the ground at the lower end shown in figure 2-1. If the column deformation remains within a linear elastic range, the equation of motion for the concentrated mass is given approximately by [14]

$$\ddot{\delta} + 2\zeta \omega_0 \dot{\delta} + \omega_0^2 \left(1 - \frac{mg}{P_{cr}} - \frac{m\ddot{V}}{P_{cr}}\right) \delta = -\ddot{U} \quad (2.1)$$

in which  $P_{cr}$  = the static buckling load of the column;  $U$  and  $V$  = the horizontal and vertical ground displacements; each over-dot denotes one differentiation with respect to time; and  $\omega_0$  and  $\zeta$  = the natural frequency and the modal damping if the effects of gravitation and vertical ground acceleration are not taken into account. Equation (2.1) may be nondimensionalized in terms of new variables  $\tau = \omega_0 t$  and  $x = \delta / \delta_e$  as follows:

$$x'' + 2\zeta x' + [1 - \kappa_1 - \kappa_2 \eta(\tau)]x = \xi(\tau) \quad (2.2)$$

in which  $\delta_e$  = the elastic limit of deflection (chosen to be the normalization length) beyond which (2.1) is no longer valid, each prime signifies one differentiation with respect to  $\tau$ ,  $\xi = U'' / \delta_e$ ,  $\eta = V'' / \delta_e$  and  $\kappa_1$  and  $\kappa_2$  are constants given by  $\kappa_1 = mg / P_{cr}$  and  $\kappa_2 = m\omega_0^2 / P_{cr} = (12/\pi^2)(\delta_e/l)$ .

For hysteretic columns, (2.2) is replaced by

$$x'' + 2\zeta x' + [\alpha - \kappa_1 - \kappa_2 \eta(\tau)]x + (1 - \alpha) z(x, x') = \xi(\tau) \quad (2.3)$$

where  $z$  represents the hysteretic component of the restoring force. In the cases of the bi-linear model, the Hata-Shibata model and the Bouc-Wen model, the hysteretic components are given, respectively, by

$$z' = \begin{cases} 0, & |z| = 1 \\ x', & |z| < 1 \end{cases} \quad (2.4a)$$

$$|z| < 1 \quad (2.4b)$$

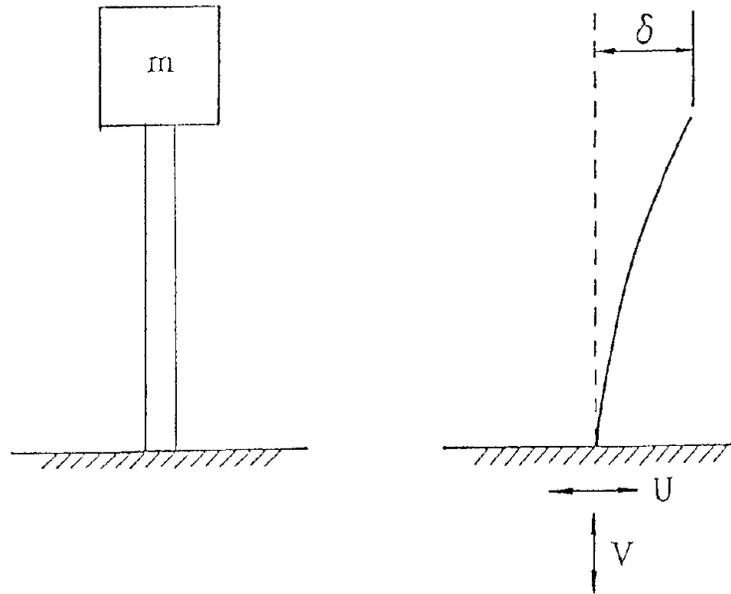


FIGURE 2-1 Column Excited by Vertical and Horizontal Ground Motions.

$$z = \text{sgn } x' \quad (2.5)$$

and

$$z' = -\gamma |x'| |z| |z|^{n-1} - \beta x' |z|^n + Ax' \quad (2.6)$$

These models have been illustrated in figures 1-2, 1-3 and 1-4, respectively.

In what follows, the random excitations  $\eta(\tau)$  and  $\xi(\tau)$  will be replaced by white noises; namely,

$$E[\eta(\tau)\eta(\tau + u)] = 2\pi K_{\eta\eta}\delta(u) \quad (2.7a)$$

$$E[\xi(\tau)\xi(\tau + u)] = 2\pi K_{\xi\xi}\delta(u) \quad (2.7b)$$

$$E[\eta(\tau)\xi(\tau + u)] = 2\pi K_{\eta\xi}\delta(u) \quad (2.7c)$$

These substitutions are permissible provided that the correlation times of the excitations are short compared with the relaxation time of the dynamic system, in which case  $K_{\eta\eta}$ ,  $K_{\xi\xi}$  and  $K_{\eta\xi}$  may be taken as their spectral densities and the cross spectral density at the zero frequency.



### SECTION 3

#### DISSIPATION ENERGY BALANCING

When random excitations are Gaussian white noises, the response state vector of a dynamic system is a Markov vector, whose probability density is governed by the well-known Fokker-Planck equation. In certain cases, exact stationary solutions to the associated Fokker-Planck equation are obtainable. To the knowledge of the authors, all the known exact solutions to date belong to a general class, called the class of generalized stationary potential [5]. This solvable class of stochastic systems is quite large. Therefore, even if a given stochastic system does not belong to this class, it may be substituted by one within this class which is closest to the original system using a suitable statistical criterion. The exact solution of the substituting system may be considered as an approximate solution of the substituted system. The criterion for substitution is that the ensemble average of the dissipated energy remains the same for the substituting and the substituted systems; thus the procedure is called dissipation energy balancing.

Consider a general single-degree-of-freedom nonlinear system

$$x'' + h(x, x') = f_j(x, x') W_j(\tau) \quad (3-1)$$

where  $W_j(\tau)$  are Gaussian white noises with spectral densities  $K_{ij}$ . Express the stationary probability density of the system response in the form of

$$p_S(x, x') = C \exp(-\phi) \quad (3-2)$$

where  $C$  is a positive normalization constant. If the system belongs to the class of generalized stationary potential, then  $\phi$  is a non-negative function of the total energy

$$\lambda = \frac{1}{2} (x')^2 + G(x) \quad (3-3)$$

where  $G(x)$  is the potential energy if multiplicative random excitations do not appear in the coefficient of  $x'$ ; otherwise  $G(x)$  is the "effective" potential energy [5]. For a system not belonging to the class of generalized stationary potential, an approximate  $\phi$  based on the criterion of dissipation energy balancing may be computed from [6]:

$$\phi'(\lambda) = \frac{\int_{a_1(\lambda)}^{a_2(\lambda)} \{ [h + \pi K_{ij} f_i \frac{\partial f_j}{\partial x'}]_{x'=\sqrt{2\lambda-2G(x)}} - [h + \pi K_{ij} f_i \frac{\partial f_j}{\partial x'}]_{x'=-\sqrt{2\lambda-2G(x)}} \} dx}{\pi \int_{a_1(\lambda)}^{a_2(\lambda)} \{ x' K_{ij} f_i f_j \}_{x'=\sqrt{2\lambda-2G(x)}} - \{ x' K_{ij} f_i f_j \}_{x'=-\sqrt{2\lambda-2G(x)}} \} dx} \quad (3-4)$$

The integration limits,  $a_1(\lambda)$  and  $a_2(\lambda)$ , in the above expression are the two roots of the equation  $G(x) = \lambda$ .

In many cases equations (3-4) and (3-2) can be reduced to closed forms. Even when this is not possible they can still be calculated numerically, once the effective potential energy  $G(x)$  is determined, which is not difficult for a specific system. With the knowledge of the probability density  $p(x, x') = \rho(\lambda)$ , the statistical properties of the response can be computed easily. For example, the mean square value of the displacement is given by

$$E[x^2] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x^2 p_S(x, x') dx dx' = \int_0^{+\infty} \rho(\lambda) \left[ \int_{a_1(\lambda)}^{a_2(\lambda)} \frac{2x^2}{\sqrt{2\lambda-2G(x)}} dx \right] d\lambda \quad (3-5)$$

The stationary probability densities for the total energy  $\lambda$  and the amplitude  $a$ , respectively, can also be readily determined. Specifically,

$$p_S(\lambda) = 2\rho(\lambda) \int_{a_1(\lambda)}^{a_2(\lambda)} \frac{dx}{\sqrt{2\lambda-2G(x)}} \quad (3-6)$$

$$p_S(a) = 2 |G'(a)| \rho[\lambda(a)] \int_{a_1(\lambda)}^{a_2(\lambda)} \frac{dx}{\sqrt{2\lambda-2G(x)}} \quad (3-7)$$

The principle of energy dissipation balancing can now be applied to the hysteretic column governed by equation (2-3). It leads to

$$\phi'(\lambda) = \frac{2\zeta \int_{-a(\lambda)}^{a(\lambda)} \sqrt{2\lambda - 2G(x)} dx + \frac{1}{2} A_r(\lambda)}{\pi \int_{-a(\lambda)}^{a(\lambda)} (K_{\xi\xi} + K_{\eta\eta} \kappa_2^2 x^2) \sqrt{2\lambda - 2G(x)} dx} \quad (3-8)$$

where  $A_r(\lambda)$  is the area of the hysteresis loop corresponding to a given  $\lambda$ , and where use is made of the fact that  $a_2(\lambda) = -a_1(\lambda) = a(\lambda)$  for a symmetric hysteresis constitutive law such as equations (2-4), (2-5) and (2-6). The area  $A_r(\lambda)$  represents the dissipated energy per cycle due to hysteresis at an energy level  $\lambda$ . Once  $A_r(\lambda)$  and the potential energy  $G(x)$  are determined, the probability densities, equations (3-2), (3-6) and (3-7), and the mean square displacement, equation (3-5), can be computed numerically.

The response amplitude  $a$  is usually of greater interest in practical applications. The area of the hysteresis loop  $A_r$  for the Hata-Shibata model in terms of  $a$  can be obtained easily by referring to figure 1-3,

$$A_r = 4a(1 - \alpha) \quad (3-9)$$

Furthermore, it is obvious that the hysteretic component  $z$  of this model is purely dissipative. Therefore, all the potential energy of the system is stored in the linear spring alone, and is given by

$$G(x) = \frac{1}{2} (\alpha - \kappa_1)x^2 \quad (3-10)$$

This potential energy represents the ability of the system to return to a local equilibrium upon removal of the external force.

In the case of a bi-linear column with a hysteretic component  $z$  governed by equation (2-4), the hysteresis loop area may be expressed as

$$A_r = \begin{cases} 0 & a \leq 1 \\ 4(1-\alpha)(a-1) & a \geq 1 \end{cases} \quad (3-11)$$

The potential energy of the system is stored in the two spring elements shown in figure 1-1. Its values for different ranges of  $x$  can be computed by referring to the shaded area in figure 1-2(b). Specifically,

$$G(x) = \begin{cases} \frac{1}{2}(1-\kappa_1)x^2 & a \leq 1 \\ \frac{1}{2}(\alpha - \kappa_1)x^2 + \frac{1}{2}(1-\alpha)(x + a - 1)^2 & a \geq 1, -a \leq x \leq -a + 2 \\ \frac{1}{2}(\alpha - \kappa_1)x^2 + \frac{1}{2}(1-\alpha) & a \geq 1, -a + 2 \leq x \leq a \end{cases} \quad (3-12)$$

Following the same principle, the hysteresis loop area and the potential energy for the Bouc-Wen model can be evaluated. The loop area  $A_r$  is obtained from the integral

$$A_r = 2(1-\alpha) \int_{-a}^a z(x) dx \quad (3-13)$$

The potential energy  $G(x)$  consists of two parts: one part is stored in the linear element, another in the hysteretic element. The latter may be computed by referring to the shaded area shown in figure 1-4(b) for different values of  $x$ . For a specific system with known parameters  $\gamma$ ,  $\beta$ ,  $A$  and  $n$ , the functional relationship between  $z$  and  $x$  can be derived by integrating equation (2-6), and the area  $A_r$  and the potential energy  $G(x)$  can be obtained in closed forms. In what follows, the case of  $n = 1$ ,  $A = 1$  and  $\gamma = \beta = 0.5$  will be considered in more detail. For this case

$$A_r = (1 - \alpha)[4x_0 - (a - x_0)^2] \quad (3-14)$$

$$G(x) = \begin{cases} \frac{1}{2}(\alpha - \kappa_1)x^2 + \frac{1}{2}(1 - \alpha)(x + x_0)^2 & -a \leq x \leq -x_0 \\ \frac{1}{2}(\alpha - \kappa_1)x^2 + \frac{1}{2}(1-\alpha)[1 - e^{-(x+x_0)}]^2 & -x_0 \leq x \leq a \end{cases} \quad (3-15)$$

where  $x_0$ , shown in figure 1-4(b), is uniquely determined for a given amplitude  $a$  by solving  $z(\pm x_0) = 0$ .



## SECTION 4

### NUMERICAL EXAMPLES

Numerical calculations were carried out for hysteretic columns with moderate hysteresis ( $\alpha = 0.5$ ) and strong hysteresis ( $\alpha = 0.1$ ) and with  $\kappa_1 = 0.04$ ,  $\kappa_2 = 0.1$ ,  $\zeta = 0.025$ . The horizontal and vertical excitations were assumed to be white noises. Two nondimensional quantities  $D_\eta = \sqrt{2K_\eta \eta}$  and  $D_\xi = \sqrt{2K_\xi \xi}$  are introduced to represent the strength of the vertical and horizontal ground excitations. The root-mean-square response would be proportional to  $D_\xi$ , if the system were linear and if the vertical excitation were absent.

Some computed results of the stationary probability density of the response amplitude are shown in figures 4-1 and 4-2 for the Bouc-Wen type hysteretic columns. The strengths of the horizontal and vertical excitations used in the computation were  $D_\xi = 0.1, 0.5, 1, 5$  and  $D_\eta = 0, 0.3, 0.34, 0.4, 0.8, 1.0, 1.2$ . The low  $D_\xi = 0.1$  represents a very weak horizontal excitation and the high  $D_\xi = 5$  a very strong horizontal excitation. The case of  $D_\eta = 0$  was included so that the error incurred by neglecting the vertical (multiplicative) excitation could be determined.

Several general trends are revealed in figures 4-1 and 4-2. The presence of the vertical excitation shifts the peak of the amplitude probability density toward right, decreases the distribution for lower amplitudes, and increases the distribution for higher amplitudes. However, these effects are significant only when the horizontal excitation is sufficiently strong (see figures 4-1d and 4-2d). The general shape of the probability density curve depends primarily on the relative contribution of the hysteresis components and to a lesser degree on the horizontal excitation intensity. In the case of

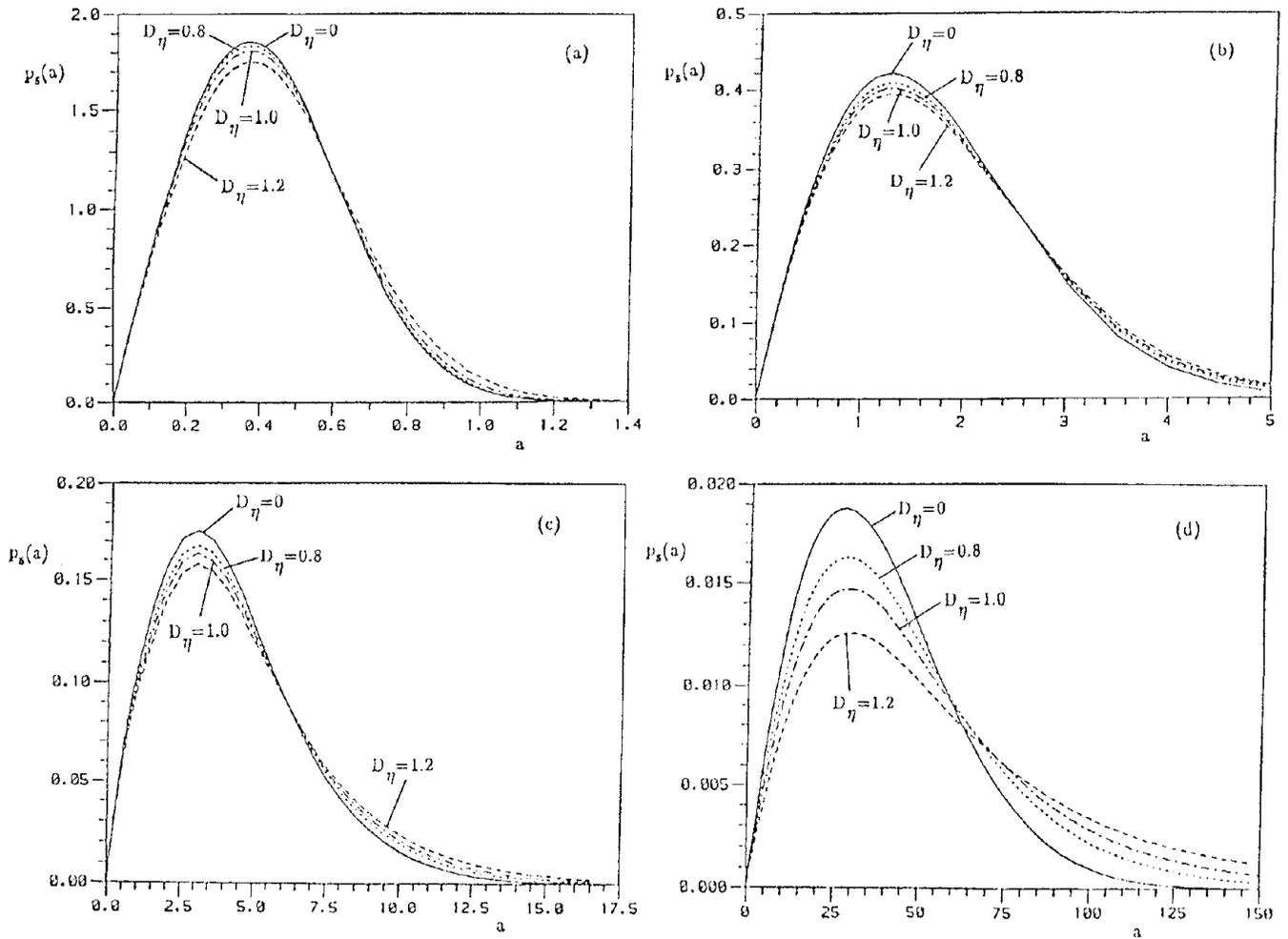


FIGURE 4-1 Stationary Probability Density of the Response Amplitude of a Smooth Hysteretic Column with  $\zeta = 0.025$ ,  $\kappa_1 = 0.04$ ,  $\kappa_2 = 0.1$ ,  $\gamma = \beta = 0.5$  and  $\alpha = 0.05$ :

- (a) Horizontal Excitation Spectral level  $D_\eta = 0.1$ ;
- (b) Horizontal Excitation Spectral Level  $D_\eta = 0.5$ ;
- (c) Horizontal Excitation Spectral Level  $D_\eta = 1$ ;
- (d) Horizontal Excitation Spectral Level  $D_\eta = 5$ .

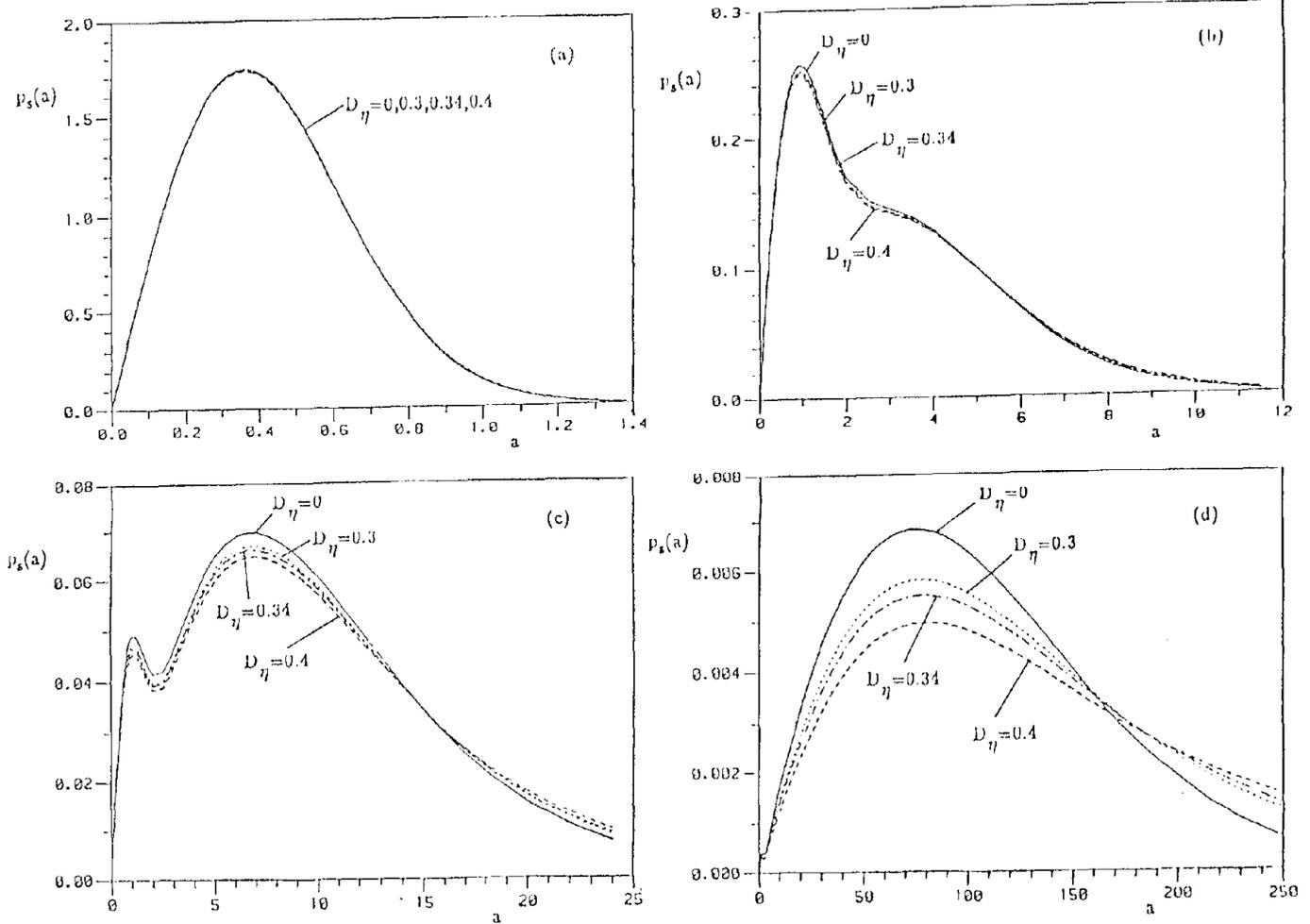


FIGURE 4-2 Stationary Probability Density of the Response Amplitude of a Smooth Hysteretic Column with  $\zeta = 0.025$ ,  $\kappa_1 = 0.04$ ,  $\kappa_2 = 0.1$ ,  $\gamma = \beta = 0.5$  and  $\alpha = 0.1$ :

- (a) Horizontal Excitation Spectral level  $D_\eta = 0.1$ ;
- (b) Horizontal Excitation Spectral Level  $D_\eta = 0.5$ ;
- (c) Horizontal Excitation Spectral Level  $D_\eta = 1$ ;
- (d) Horizontal Excitation Spectral Level  $D_\eta = 5$ .

moderate hysteresis (figure 4-1,  $\alpha = 0.5$ ) neither the vertical excitation nor the horizontal excitation has much influence on the shape, implying that the amplitude is approximately Rayleigh distributed. If the system hysteresis is strong ( $\alpha = 0.1$ ), the response amplitude is close to being Rayleigh distributed only if the horizontal excitation is either very weak (figure 4-2a,  $D_{\xi} = 0.1$ ) or very strong (figure 4-2d,  $D_{\xi} = 5$ ). If the strength of the horizontal excitation is in the intermediate range (figure 4-2b,  $D_{\xi} = 0.5$  and figure 4-2c,  $D_{\xi} = 1$ ) the amplitude distribution is far from Rayleigh, and in some cases it can have two peaks.

The same characteristics are found in the computed probability distributions for the response of bi-linear hysteretic columns.

The root-mean-square displacements  $\sigma_x$  normalized with respect to  $D_{\xi}$  and computed for the Hata-Shibata model, the bilinear model and the smooth hysteretic model are shown in figures 4-3 through 4-5, respectively, in solid lines for different values of  $D_{\eta}$ . The results confirm that the presence of the vertical ground motion increases the displacement response of the column, and that the effect of the vertical ground motion is significant only when the horizontal ground acceleration is sufficiently large.

It is expected that an actual physical system should exhibit a linear behavior when the excitation levels are very low. Therefore, the  $\sigma_x/D_{\xi}$  vs  $D_{\xi}$  curve should approach a horizontal line in the low  $D_{\xi}$  region. This expectation is borne out with bi-linear model, figure 4-4, but not so with the Hata-Shibata model, figure 4-5. This suggests that the Hata-Shibata model is not a suitable model for systems subjected to weak excitations.

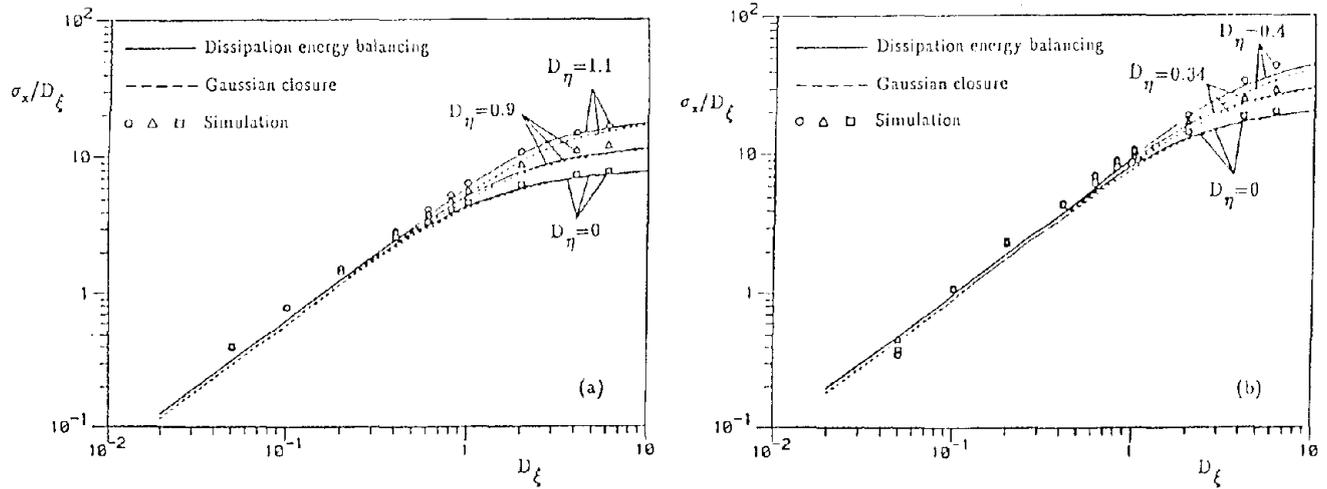


FIGURE 4-3 RMS Response of a Hata-Shibata Hysteretic Column with  $\zeta = 0.025$ ,  $\kappa_1 = 0.04$  and  $\kappa_2 = 0.1$ : (a) Moderate Hysteresis  $\alpha = 0.5$ ; (b) Strong Hysteresis  $\alpha = 0.1$ .

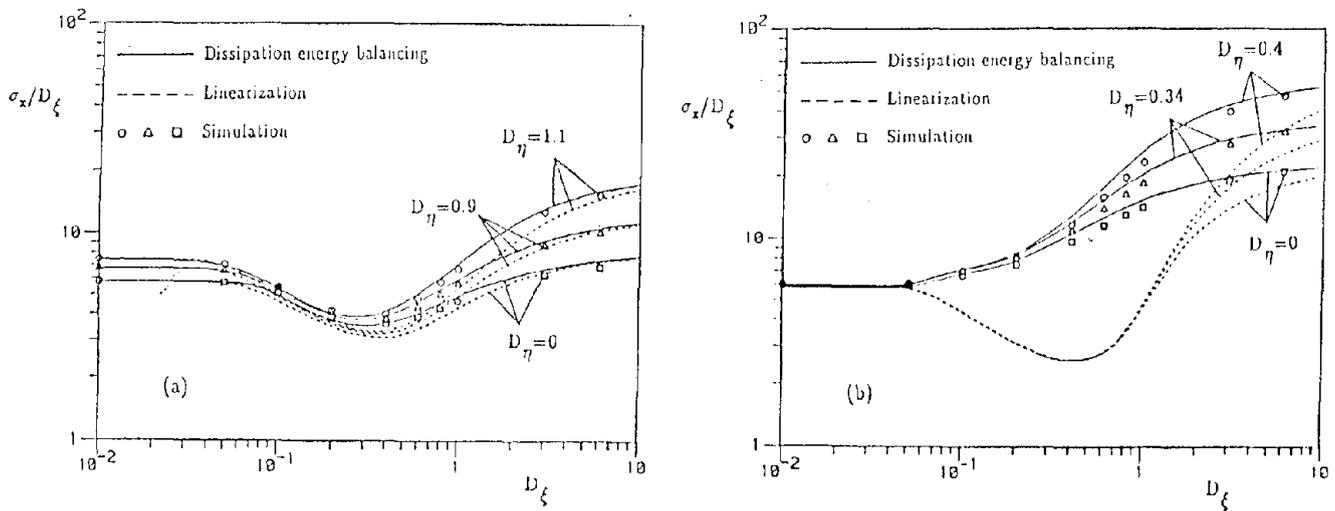


FIGURE 4-4 RMS Response of a Bi-Linear Column  $\zeta = 0.025$ ,  $\kappa_1 = 0.04$  and  $\kappa_2 = 0.1$ : (a) Moderate Hysteresis  $\alpha = 0.5$ ; (b) Strong Hysteresis  $\alpha = 0.1$ .

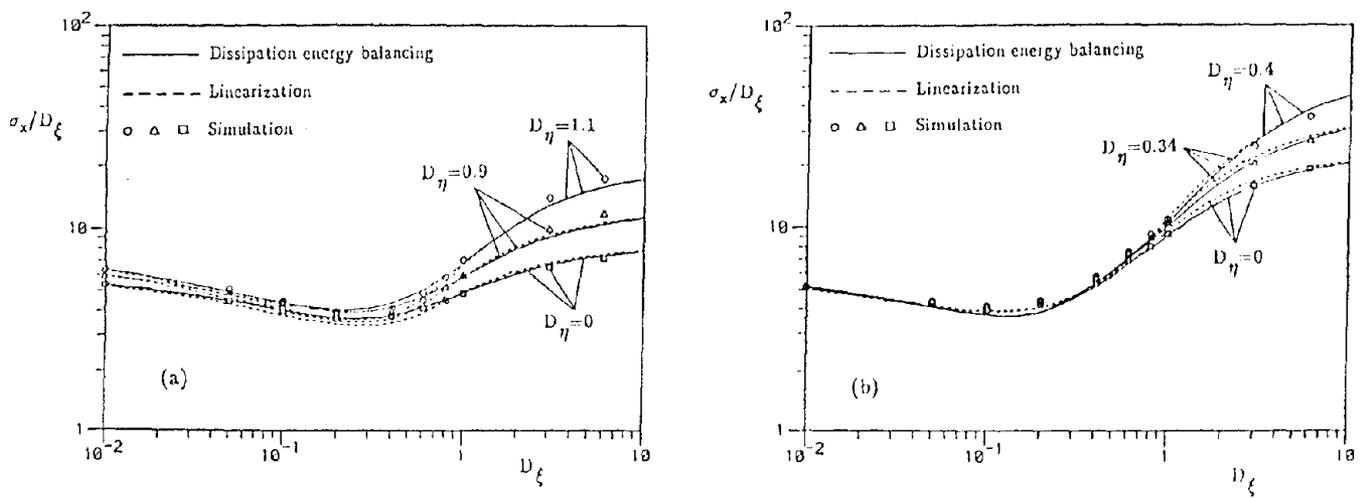


FIGURE 4-5 RMS Response of a Smooth Hysteretic  $\zeta = 0.025$ ,  $\kappa_1 = 0.04$  and  $\kappa_2 = 0.1$ : (a) Moderate Hysteresis  $\alpha = 0.5$ ; (b) Strong Hysteresis  $\alpha = 0.1$ .

## SECTION 5

### COMPARISON WITH RESULTS FROM MONTE-CARLO SIMULATIONS AND OTHER APPROXIMATE METHODS

To assess the accuracy of results obtained from the energy dissipation balancing approach, Monte-Carlo simulations were carried out for the above hysteretic columns. The root-mean-square displacements were obtained by numerically integrating equations (2.3) and (2.4), or (2.3) and (2.5), or (2.3) and (2.6) using a fourth order Runge-Kutta algorithm. Each sample function of a Gaussian white-noise excitation, either  $\eta(\tau)$  or  $\xi(\tau)$ , is digitally simulated as a sequence of pulses with random heights but with identical durations  $\Delta\tau$ ; namely,

$$w_s(\tau) = \sum_{k=1}^N Y_k \{U[\tau - (k - 1)\Delta\tau] - U(\tau - k\Delta\tau)\} \quad (5-1)$$

where  $Y_k$  are independent identically distributed Gaussian random variables with a zero mean and a variance equal to  $2\pi K/\Delta\tau$ , and where  $U(\tau)$  denotes a unit step function; i.e.

$$U(\tau) = \begin{cases} 1 & \tau > 0 \\ 0 & \tau < 0 \end{cases} \quad (5-2)$$

Such a sample function is illustrated in figure 5-1. It can be shown that the spectral density of the simulated random process tends to constant  $K$  over the entire frequency range as  $\Delta\tau$  approaches zero. In practice, however,  $\Delta\tau$  is chosen to be sufficiently short so that the spectral density of the simulated process is essentially flat over the frequency range in which the response of the system is expected to be significant. By numerical experimentation, we found that  $\Delta\tau$  should be less than 0.05 for the purpose of present comparison.

The results obtained from digital simulations are shown also in figures 4-3 through 4-5. These results are represented by either circles, triangles or

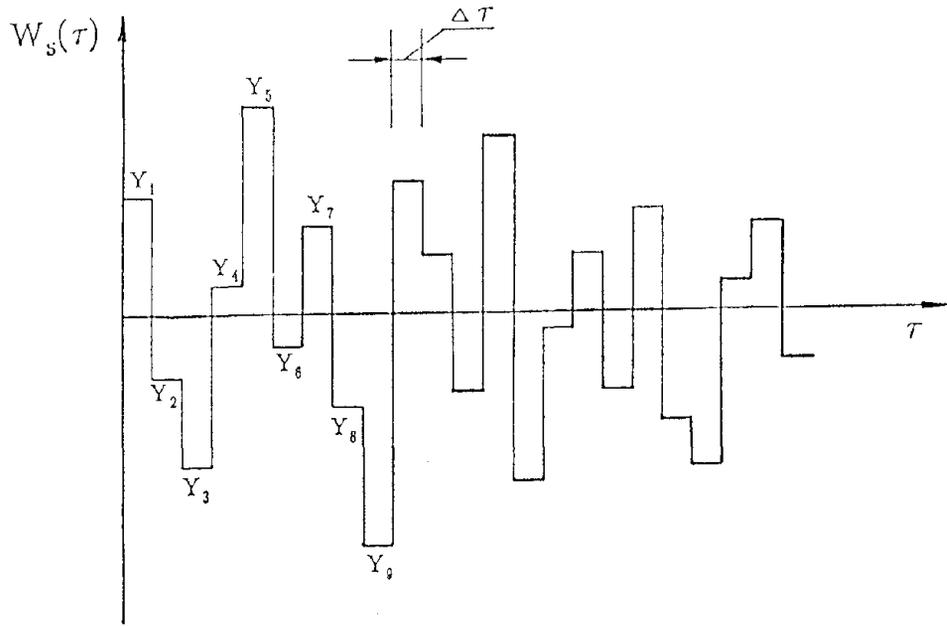


FIGURE 5-1 Typical Sample Function of a Excitation Process for Simulation Study.

squares, and they appear to agree very well with the analytical results obtained from the dissipation energy balancing procedure in all cases investigated, including (i) the Hata-Shibata model, the bi-linear model and the smooth model; (ii) a moderate hysteresis ( $\alpha = 0.5$ ) and a strong hysteresis ( $\alpha = 0.1$ ); (iii) a wide range of horizontal ground acceleration spectra and (iv) when the vertical ground excitation is either strong, or weak, or entirely absent.

The dashed lines in figures 4-3 through 4-5 show the analytical results obtained from other approximate procedures. In figure 4-3 they depict the Gaussian closure results for the Hata-Shibata hysteretic columns, and in figure 4-5 they represent the results obtained by Wen [18] for smooth hysteretic columns using a special form of equivalent linearization. In each of these two cases, they are in very good agreement with the results from dissipation energy balancing and from simulations.

The dashed lines in figure 4-4 are results obtained from employing the linearization technique proposed originally by Caughey [9] and applied by Iwan and Lutes [11] to bi-linear columns. In the case of moderate hysteresis ( $\alpha=0.5$ ) the linearization results appear to be in good agreement with those from simulation and from dissipation energy balancing, but in the case of nearly elasto-plastic systems ( $\alpha=0.1$ ) they deviate far from the simulation results in the range of intermediate horizontal ground excitations. However, it is in this range that the hysteretic component in the restoring force plays a dominant role. The large discrepancy is attributable to the assumptions made in the linearization method that the response is of a narrow frequency band, and that the response amplitude is approximately Rayleigh distributed.



## SECTION 6

### CONCLUSION

The approximation procedure of dissipation energy balancing has been applied to obtain the probability distribution for the response amplitude of a hysteretic column under horizontal and vertical random excitations at the ground support. Similar characteristics are found in the calculated results for both the bi-linear and Bouc-Wen hysteresis models. The presence of the vertical excitation increases the distribution for higher amplitudes especially when the horizontal excitation is sufficiently strong. The shape of the distribution, however, depends largely on the relative contribution of the hysteresis component. If the system is strongly hysteretic, and if the spectral level of the horizontal excitation is neither very high nor very low, the amplitude distribution may show two peaks, very much different from a Rayleigh type distribution.

Comparison of the calculated mean-square response and digital simulation results show that the proposed approximation procedure is accurate for wide ranges of excitation levels and system parameters.

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