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and
Mohammad Noori

Final Technical Report On

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Submitted To:

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Division of Earthquake Hazards Mitigation
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DEVELOPMENT OF AN APPROXIMATE TECHNIQUE FOR THE NONLINEAR RANDOM VIBRATIONS ANALYSIS OF HYSTERETIC SYSTEMS

A Dissertation
Submitted to the Faculty
of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the
Degree of Doctor of Philosophy
in
Mechanical Engineering

by

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DEDICATION

TO THOSE I AM BEHOLDEN THE MOST

TO MY FATHER AND TO MY GOD;

FOR THE HEART OVERSHADOWING IMPERFECTIONS AND

FOR THE PERFECTIONS OVERCOME BY UNLIMITED COMPASSION

AND TO MY MOTHER, MANSOUREH;

FOR BEING THE BETTER IN BOTH WORLDS.

ABSTRACT

A cumulant-neglect closure scheme is developed for determining the stationary and nonstationary response of structural systems with hysteretic restoring force characteristics. The method is applied for the analysis of a hysteresis model with general strength/stiffness degradation capabilities. This model has been used in the past for stochastic seismic performance evaluation of buildings. Response statistics obtained for the model using this closure technique are compared with equivalent linearization results via Monte Carlo simulation. The study performed shows that the closure results are in better agreement with simulation than those obtained by linearization. This technique also provides information on higher order statistics for hysteresis models. These statistics were not possible to be obtained previously using the existing approximation techniques. These higher order statistics are valuable in the reliability analysis and prediction of the probability of survival of hysteretically yielding structures.

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TIME PASSES BUT TRUE

FRIENDSHIPS ENDURE.

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LIST OF SYMBOLS

A	Hysteretic parameter affecting initial slope and yield level
A_i	dummy variable
A_0	initial value of A
$a_i(Y,t)$	first incremental moment or drift coefficients
$a(t)$	vector relating to forcing function
a_i	initial value of M_i
a_{in}	time independent coefficients
$B(t)$	Brownian motion process
$B_i(t)$	elements of B(t)
B_i	dommy variable
$b_{ij}(Y,t)$	second incremental moment or diffusion coefficients
b	right hand side vector of response covariance matrix equation
$b(t)$	unit Brownian motion process
b_i	initial value of m_i
C	damping coefficients, also Damping matrix
C_{ij}	elements of damping matrix
C_i	coefficients of Gram-Charlier density function
$cof(\Delta)$	cofactor of covariance element
D	a parameter controling W
D_i	dommy variable
dt	differential of time
dU	differential of generalized coordinate

$d\epsilon$	differential of hysteretic energy
E{ }	expected value of
EL	equivalent linearization
e	error
F(t)	forcing function; also forcing function vector
FPK	Fokker-Plank-Kolmgorov
F(Y,t)	n-dimensional vector representing the deterministic part of the model
f(t)	forcing function per unit mass
f_y	yield level force
G	linearized system matrix
G(Y,t)	n x m matrix, whose elements can be functions of system variables and time
g_i	approximates g_i
g_i	a general function of M_i
g( )	a nonlinear function
H_i( )	Hermite polynomial
IG	Ito approach Gaussian
ING	Ito approach Non-Gaussian
K	spring constant; stiffness matrix, and cummulant index $K = K_1 + K_2 + ... + K_n$
K_i	initial stiffness
K_f	post yield stiffness
K_{ij}	elements of stiffness matrix
Ki	power of response variable
Ke	equivalent stiffness
L( )	linear operator

M	mass matrix
M_i	moment of order i
$\dot{M}_i$	time derivative of M
MCS	Monte Carlo simulation
m	mass of system
m_i	approximates M_i; also the mean value of Y_i
m_{ij}	elements of mass metrix
N	# of unknowns in Gram-Charlier density function
NG	Non-Gaussian, using Ito approach
n	parameter controling hysteretic shape; also length of state space vector
$n(\ )$	a nonlinear function
$P(Y,t)$	probability density function
PSD	power spectral density
$P^*(Y)$	Non-Gaussian density function
$P(Y)$	Gaussian density function
Q	square matrix, elements of which consist of dB_i
$q(u,t)$	total restoring force
$R_w(\ )$	auto correlation function for W(t)
S	one time response covariance matrix
S_{YY}	zero time lag covariance matrix
$Sgn(\ )$	sign function
U	generalized coordinate
$\dot{U}$	generalized velocity
$\ddot{U}$	generalized acceleration
U_i	elements of the series approximation for u

W	constant power spectral density; also 1/2 of power spectral density (PSD)
$W(t)$	m dimensional vector of stochastic random process
X^i	dummy variable
Y	dummy variable
$Y_i(t)$	elements of state space vector
Y_0	initial conditions for Y
Z	hysteretic restoring force
Z_{max}	maximum hysteretic force
$(\ )^T$	transpose of
α	ratio of post-yield to pre-yield stiffness
β	energy dissipation parameter of hysteretic restoring force
γ	shape parameter of hysteretic restoring force
δ_A	degradation rate for A
δ_η	degradation rate for η
δ_ν	degradation rate for ν
$\delta(\ )$	delta function
Δt	time increment
$\|\Delta\|$	determinat of covariance matrix
ϵ	total hysteretic energy dissipated; also a small parameter controling the nonlinearity
$\dot{\epsilon}$	time derivative of ϵ
$\epsilon(\ )$	error of linearization
η	stiffness degradation parameter in hysteretic restoring force; a normalizing constant; also a damping factor
η_0	initial value of η

θ_i	frequency component
λ	maximum limiting force
λ_1	parameter controling σ and λ
λ_K	joint cummulant or order K
μ_K	joint central moment of order K
ν	strength degradation parameter in hysteretic restoring force; also mean value of U
ν_0	initial value of ν
ξ	damping ratio
π	constant $\pi = 3.14159$
ρ	a constant parameter
σ	standard deviation
σ^2	variance parameter
${\sigma_i}^2$	element of σ^2
τ	correlation time
$\Phi(\ )$	a function of response coordinate
$\phi(\ )$	charactristic function; also an arbitrary function
Ψ_Y	partial derivative of Ψ with respect to Y_i
Ψ_{YY}	Jacobian of Ψ
$\Psi(Y,t)$	scale-valued real function
ω_0	natural frequency, $(K/m)^{1/2}$
∂	partial differentiation symbol

CHAPTER I
INTRODUCTION

1.1 General

The uncertainties inherent in most loading mechanisms are known to create extremely complicated states of stress which will inevitably challenge the applicability of the basic deterministic techniques. Probability is, therefore, a definite and integral part of any design process. Even so, the current practice in the US overwhelmingly favors the equivalent static response concept. However, major new findings - specifically in regard to random dynamic forces such as earthquake, wind, ocean waves, atmospheric turbalance and jet noise - tend to question the wisdom of such idealogical preferences.

1.2 Background

The concurrent subject of vibration deals with the excitation, the associated behavior, and the ensuing response of the dynamic systems. In the particular area of random vibration, one is confronted with the response of vibratory systems, linear or nonlinear, resulting from random excitation input. When nonlinearities are encountered, they may be through geometric or physical sources. The geometric non-linearities are identified by large deformations, while physical non-linearities arise from the non-linear nature of the material itself. In retrospect, the most studied - and perhaps the most crucial form of nonlinearity - is the one resulted when the restoring force in a system is not proportional to the deformation.

The next logical step after modelling the oscillating system is to solve the governing stochastic differential equations of random motion. Numerous solution techniques for the response of linear systems have been developed and documented in many books, (Robson, 1963; Crandall and Mark, 1963; Lin, 1967; Clough and Penzien, 1975; Nigam, 1983 and Yang, 1986). However, the class of non-linear problems that can be solved in exact forms are limited. If the response is assumed to be Markovian, the Fokker-Plank-Kolmogorov (FPK) equation can be used to derive a partial differential equation for transition probability density function of the response. In many cases, however, it is not possible to solve the FPK equation in closed form. Thus, a variety of approximate techniques for the response analysis of non-linear systems have been developed. These include, Equivalent Linearization (Coughey, 1963; Lin, 1967; Iwan, 1973; and Spanos, 1981), Gaussian and Non-Gaussian Closure schemes (Crandall, 1980, 1985; Wu and Lin, 1984, Ibrahim, 1985, and Ibrahim and Soundararajan, 1985), Perturbation techniques (Crandall, 1963; and Lin, 1967), Functional series representation and Functional Analysis (Ahmadi, 1982; Jahedi and Ahmadi, 1983; and Ibrahim nd Pandy, 1989), Simulation Methods (Shinozuka, 1970), Stochastic Central Difference method (To, 1986; 1988), Stochastic Averaging method (Stratonovich, 1963; and Roberts, 1981a), Finite Element approach (Spencer and Bergman, 1985; Mohammadi and Amin, 1988) and Equivalent Stochastic Systems (Lin and Cai, 1987; Lin, 1988). Of all these techniques, Equivalent Linearization is the most widely used for solving random vibration of Hysteretic systems. This technique can be used for both zero and non-zero mean analysis (Spanos, 1980; Baber, 1984, 1985; and Noori and Baber, 1984).

1.3 Objective of Study

Because the original nonlinear system of equationsin Equivalent Linearization technique is replaced by an equivalent linear system of uncoupled equations, this method is incapable of displaying the effects of non-linearity. In order to improve Equivalent Linearization technique and calculate the response of nonlinear systems more closely, other approximation techniques, such as Non-Gaussian closure method have been developed. Ibrahim and Soundararajan (1985), and Wu and Lin (1984) developed a Non-Gaussian closure scheme for determining the response statistics of non-linear systems under external and/or parametric excitation. This technique can be used for both single or multi-degree of freedom systems, possessing the non-linearity in the restoring force, damping or inertia terms. However, the technique as presented was only applied to systems with polynomial type non-linearities. The objective of this study is to extend the Cumulant-Neglect Closure scheme for the response analysis of nonlinear systems with general hysteretic behavior. This will result in information which is useful in random vibration and reliability analysis of the hysteretically yielding systems.

1.4 Summary of the Contents

The response of a SDOF hysteretic system subjected to zero mean Gaussian white noise excitation is studied in this report. In Chapter 2, first the development of general hysteresis models are briefly discussed, and then the history and the mathematical formulation of the so-called smooth hysteresis model is presented. Approximate solution techniques for nonlinear random vibration problems are reviewed in Chapter 3. The focus of this chapter is on the mathematical development of a Non-Gaussian solution method using Ito-differential formula. Application of the proposed solution technique of Chapter 3 to the smooth hysteresis systems is given in Chapter 4. Chapter 5 contains the numerical studies of Chapter 4. Two studies related to Non-Gaussian random vibration analysis are presented in Chapter 6. The first study is the application of the Non-Gaussin solution technique proposed by Crandall (1980) to a SDOF system having a tangent hyperbolic restoring force. The second study is a proof of the equivalence of Equivalent Linearization and Ito-differential approch by assuming Gaussian response. Conclusion, remarks and suggestions are given in Chapter 7.

CHAPTER II
HYSTERESIS MODELS

2.1 General

Analytical modeling of any structure subject to dynamic loadings requires the compelete knowledge of force-displacement relationships. When subjected to high random excitations, the structure may go through numerous cycles of inelastic response. These responses can be accompained by strength and/or stiffness deteroration in form of hysteresis loops action (Newmark and Rosenblueth, 1971; Vanmarcke and Veneziano, 1973; Bertero, Popov and Wang, 1974; Sozen, 1974; Atalay and Penzieen, 1977; Gosain and Jirsa, 1977; Sues, Wen and Ang, 1983; Keshavarzian and Schnobrich, 1983; and Vielsack, 1987). Because of the practical significace of this type of behavior, specifically in Earthquake engineering and base isolation system design, modelling and analysis of these systems have been the subject of extensive studies over the past two decades. This chapter discusses briefly the development of general hystersis models by focusing, in specific, on the history and the mathematical formulation of the so-called smooth transitional model.

2.2 Existing Models

In recent years, various types of piecewise linear and smoothly varying hysteresis models have been developed by many researches. Ozdemir (1976), Baber and Wen (1981, 1982), Baber and Noori (1985, 1986), Noori and Baber (1984), Noori, Choi and Davoodi (1986), Noori, Davoodi and Choi (1986) and Choi (1986) reviewed much of this work. The two important factors involved in developing analytical models for hysteretic systems are; 1) capability of models in representing the inelastic, hysteretic, degrading and loop pinching behavior, and 2) compatibility of models with the existing approximate solution techniques.

2.2-1 Bilinear Model

The bilinear model of classical plasticity which exhibits a sharp transition from elastic to plastic state is perhaps the simplest and most widely used model for inelastic behavior of structural elements under both random and deterministic cyclic loadings. The model consists of linear spring elements and coulomb slip element (Iwan, 1966). Caughey (1960a, 1960b) studied the behavior of a single degree of freedom system with bilinear hysteresis characteretics under deterministic and random excitation. Using an equivalent linear system, based on Krylov and

Bogoliubov assumption, he showed that bilinear hysteresis are beneficial for small or moderately large inputs. He further concluded that bilinear models are not useful for increasing the effective damping in structures subjected to high random excitation.

For bilinear hysteretic systems subjected to stationary Gaussian white noise, Iwan and Lutes (1967) used electronic-analog techniques to determine response statistics. They showed that the probability distribution of the response is strongly influenced by the excitation level and is, in general, non-Gaussian. Brown (1969) used an analog computer simulation and equivalent linearization technique to analyze the response and failure criteria of a bilinear hysteretic single degree of freedom system. By applying the technique of stochastic averaging, Roberts (1978) obtained the joint distribution of the displacement and velocity of a bilinear single degree of freedom system. Tansirikongkol and Pecknold (1980) used bilinear hysteresis model for studying the earthquake response of multi-degree of freedom systems.

Using an analytical technique based on equivalent linearization, Asano and Iwan (1984) studied a single degree of freedom bilinear system subjected to non-stationary random excitation. Most recently, Mohammad Yar and Hammond (1987) developed a differential equation for true bilinear hysteretic systems. They showed that for properly chosen parameters, their model is capalable of simulating Caughey's bilinear model (1960a, 1960b). Further studies on Bilinear hysteretic models are reported by: Caughey (1963), Takeda, Sozen and Nielsen (1970), Kobori, Minai and Suzuki (1974), Iyengar and Dash (1978), Kelly and Tsai (1985) and Capecchi and Vestroni (1985).

2.2-2 Elasto-Plastic Model

Another popular model used in the nonlinear analysis of structures is Elasto-plastic model which is a limiting case of a bilinear model. The restoring force diagram of this model is characterized by a constant force during its plastic deformation. This model was used by Bycroft (1960) for studying the response of systems subjected to stationary white noise excitation. Penzien and Liu (1969) used recorded and simulated ground motion data and obtained the response statistics of an elasto-plastic model. Ditlevsen (1986) studied the plastic movement process of a SDOF linear elasto-plastic model subjected to stationary Gaussian process excitation. Other studies on elasto-plastic model are reported by Karnopp and Sharton (1966), Kaul and Penzien (1974), Vanmarke (1976), Chopra and Lopez (1979), Yamada and Kawamura (1980) and Grossmayer (1981).

2.2-3 Smooth Hysteresis Model

The bilinear models are not capable of representing the actual behavior of many structures, such as steel or concrete buildings, observed in practice (Newmark and Rosenblueth, 1971; Bertero, Popov and Wang, 1974; Sozen, 1974; Park and Paulay, 1975; Atalay and Penzien, 1977; Higashi, Ohkubo and Ohtsuka, 1977; Matsui and Mitani, 1977; Minai and Wakabayashi, 1977; Mitani, Makino and Matsui, 1977; Sues, Wen and Ang 1983; and Iwan and Cifuentes, 1986). In an effort to simulate a smooth transition from elastic into plastic range, researchers have developed smooth hysteresis models. Ramberg-Osgood model (1943) and Bouc model (1967) are the most popular models of this class of hystereses.

Ramberg-Osgood model is an algebric model based on three parameters: Young's modulus and two secant yield strength. This model when coupled with Masing's rule for unloading and reloading gives a continous transition from elastic to inelastic stage.

The major limitations inheritive to the models based on Ramberg-Osgood formulation are; 1) the difficulty in introducing stiffness and strength degradation, and 2) the problems with adoptability for random vibration analysis. In 1967 Bouc introduced a differential equation for smooth hysteresis models which was later generalized by Wen (1976) and Baber and Wen (1979). A wide range of hysteresis behavior, including bilinear hysteresis, can be obtained by using Wen's model.

Wen (1976) modified Bouc's model and developed an approximate method for general random response analysis of a single degree of freedom system under filtered Gaussian shot noise excitation. Pivovarov and Vinogradov (1987) used Bouc's model to represent the non-linear hysteresis behavior in a vibrating cable. They showed that within some range of load parameters, Bouc model is capable of representing the non-linear effects of a vibrating stranded cable. Hoshiya and Maruyama (1987) developed an identification method, the extended Kalman filter incorporated with a weighted global iteration, identifying parameters on the hysteretic restoring force systems of Bouc's (1967).

Based on the equivalent linearization technique, Mohammadi and Amin (1988) used a stochastic finite element approach to obtain the dynamic response of framed structures supported on hysteretic media. Using Wen's model (1976) they developed a hysteretic beam element with one yield region at each end. Another smooth hysteresis model was presented by Iwan (1977) Iwan's model was based on the behavior of an infinite number of bilinear systems.

2.2-4 Degrading Hysteresis Model

The experimental results show that the stiffness and/or the strength of structures deteriorates when subjected to severe cyclic loads (Celebi and Penzien, 1973; Bertero and Popov, 1977; Sozen, 1974; and Chang and Lee, 1987). Baber and Wen (1981) used hysteretic energy dissipation as an index of response severity and duration to control stiffness and/or strength degradation on dynamical systems using Bouc's model. They used this model to study MDOF shear beam and discrete hinge structures subjected to random excitations (Baber and Wen, 1979, 1982). Ang and Wen (1982) used Baber and Wen's model to study the structural damage under earthquake excitations. Sues, Wen and Ang (1983) modified the hysteresis model of Baber and Wen for sisemic performance evaluation of buildings. They used maximum deformation in each cycle during loading as a measure of degradation for the next cycle.

Constatinou and Tadjabaksh (1985) used rate independent hysteresis model of Wen (1976) to model a hysteresis damper for base isolation systems. Su, Ahmadi and Tadjabaksh (1987) showed that this base isolation system is relatively effective for high amplitude and high frequency earthquakes, but it is not suitable for earthquakes with considerable energy at low frequency. Mostagel and Ahmadi (1988) studied a nonlinear base isolation system based on a friction type isolator in parallel with a hysteresis element.

2.2-5 Pinching Model

Of the available hysteresis models, loop pinching models are the ones best suited to represent more complex forms of yielding behavior encountered in practice (Celebi and Penzien, 1973; Sozen, 1974; Bertero and Popov, 1977; Aoyama, Umeura and Minamino, 1977; Atalay and Penzien, 1977; Matsui and Mitani, 1977; Grossmayer, 1981; and Yamada and Kawamura, 1980). However, many of these models are developed empirically and they require several variables for describing the hysteresis behavior (Banon, Biggs and Irvine, 1981; Stanton and McNiven, 1983; Saatcioglu, Derecho and Corley, 1983; and Filippou, Popov and Bertero, 1983). Recently Baber and Noori (1985) and Noori and Baber (1984) developed several general hysteretic models capable of pinching, stiffness and strength degradation behavior. They also proposed a mathematical approach for constructing such general hysteresis with a wide range of deterioration characteristics. These models incorparate a slip-lock element in series with smooth hysteresis of Baber and Wen's (1979). Choi (1986) studied the zero and non-zero mean random vibration of a SDOF system having the type of degrading hysteresis which was developed by Noori and Baber (1984).

Based on their proposed mathematical technique, Noori and Baber (1984) developed a single element pinching model which can easily be incorporated in stochastic vibration of hysteretic. Noori, Davoodi and Choi (1986) and Noori, Choi and Dvoodi (1986) studied the zero and non-zero mean random vibration analysis of a series model with strength/ stiffness and pinching characteristics. This model was constructed based on the mathematical approach proposed by Noori and Baber (1984) and Baber and Noori (1985). More recently Noori, Saffar, Davoodi and Ghantous (1988) have reported the equivalent linearization of a further generalized hysteresis capable of predicting the stiffness degradation in the unloading cycle. This model will appear in the literature.

The inelastic model which is used in this work is a smooth hysteresis model originaly proposed by Bouc (1967) and generalized by Wen (1976) and Baber and Wen (1979) to include degradation. Application of the proposed approximation technique to other general hysteresis is not considered in this work but it can be easily performed.

2.3 Mathematical Description of Smooth Hysteresis Model

Consider a single degree of freedom system as shown in Figure 2.1. The governing differential equation of motion can be written as

$$M\ddot{U} + C\dot{U} + q(U, t) = F(t) \qquad [2.1]$$

where M is mass of the system, U is the generalized coordinate, q is the total restoring force, and F(t) is the forcing function. The restoring force q consists of two elements which are in parallel, a linear spring force with spring constant K, and a hysteretic restoring force $(1-\alpha)KZ$.

$$q = \alpha KU + (1-\alpha)KZ \qquad [2.3]$$

After subsituting for q and dividing both sides of the equation by M, equation [2.1] can be written as

$$\ddot{U} + 2\zeta\omega_0\dot{U} + \alpha\omega_0^2\, U + (1 - \alpha)\, \omega_0^2\, Z = f(t) \qquad [2.3]$$

where ζ is damping ratio, ω_0 is the natural frequency $(K/m)^{1/2}$, α is the ratio of post yield to pre-yield stiffness, and f(t) is the forcing function per unit mass.

The smooth hysteresis restoring force Z is described by

$$\dot{Z} = A\dot{U} - \beta|\dot{U}||Z|^{n-1}Z - \gamma\dot{U}|Z|^{n} \qquad [2.4]$$

where A, β, γ and n determine the hysteresis shape. Equation [2.4] can approximate a bilinear model when n approaches ∞. The characterstic of equation [2.4] can be better seen if it is divided by $\dot{U}$

$$\frac{dZ}{dU} = A - \beta \frac{|\dot{U}|}{\dot{U}} |Z|^{n-1} Z - \gamma|Z|^{n} \qquad [2.5]$$

The nature of the hysteretic restoring force can be better observed if one looks at equation [2.5] in four different regions. For the case of n=1, the resulting hysteretic shapes, for different combinations of β and γ and when U is a constant amplitude harmonic motion , are shown in Figure 2.2. The choice for A, β and γ are not completely arbitrary; they have to assure that the total energy dissipation for a cycle is positive.

There are three important quantities that define a softening system, maximum value of hysteretic force or ultimate hysteretic strength, initial stiffness and post-yield stiffness. Maximum hystertic force when $\dot{U}$ and Z are positive for an arbitrary n is

$$Z_{max} = \left[\frac{A}{\beta + \gamma} \right]^{1/n} \qquad [2.6]$$

The yield level, f_y, is thus given by

$$f_y = (1-\alpha)\ K \left[\frac{A}{\beta + \gamma} \right]^{1/n} \qquad [2.7]$$

The initial stiffness K_i can be calculated by finding the slope of total restoring force, q, at Z=0. For the case of n=1 K_i is given as:

$$K_i = \alpha K + (1-\alpha)\ K\ A \qquad [2.8]$$

The post-yield stiffness (finial stiffness) is

$$K_f = \alpha K \qquad [2.9]$$

The physical significance of α can be found by dividing equation [2.9] by equation [2.8] after setting A=1.

$$K_f/K_i = \alpha \qquad [2.10]$$

Hysteretic model described by equation [2.4] is not capable of strength and/or stiffness degradation. Baber and Wen (1979, 1981) have considered the total energy dissapation as a measure of system deterioration. Examining a typical hysteretic system, as shown in Figure 2.3, the differential energy of a system can be written as

$$d\epsilon = (1-\alpha)\ K\ Z\ dU = (1-\alpha)\ K\ Z\ \dot{U}\ dt \qquad [2.11]$$

Therefore the rate of energy dissipation is

$$\dot{\epsilon} = (1-\alpha)\ KZ\ \dot{U} \qquad [2.12]$$

and the total energy is

$$\epsilon = (1-\alpha)\ K \int_{t_0}^{t_f} Z\dot{U}\ dt \qquad [2.13]$$

Because of the nature of hysteretic systems ϵ defined by equation [2.13] is always positive. Degradation can then be introduced by modifying equation [2.4] in the following manner;

$$\dot{Z} = (A\dot{U} - \nu[\beta|\dot{U}||Z|^{(n-1)}Z + \gamma\ \dot{U}|Z|^{n}])/\eta \qquad [2.14]$$

where ν and η are considered to be increasing function of ϵ and A is considered to be a decreasing function of ϵ The deterioration parameters were chosen as

$$A = A_0 - \delta_A\ \epsilon \qquad [2.15\text{-a}]$$

$$\eta = \eta_0 + \delta_\eta \epsilon \qquad [2.15\text{-b}]$$

$$\nu = \nu_0 + \delta_\nu \epsilon \qquad [2.15\text{-c}]$$

where A_0, η_0 and ν_0 are the initial values and δ_A, δ_η and δ_ν are parameters controling the rate of degradation. Figure 2.4 illustrates the effect of degradation parameters on a hysteresis restoring force for a cyclic displacement input. ν and η present strength and stiffness degradation respectively and A induces both stiffness and strength degradation. An in depth study of smooth degrading hysteresis model is given by Baber and Wen (1979).

CHAPTER III

SOLUTION TECHNIQUES AND THE PROPOSED GENERAL APPROXIMATION METHOD FOR HYSTERETIC SYSTEMS

3.1 General

Analytic models used for the deterministic response analysis of structures, can be easily adapted for stochastic vibration by replacing the deterministic input excitation with a random process. The theory of random vibration for linear time-invarient systems have been well developed and documented by Crandall and Mark (1963), Robson (1963), Lin (1967), Clough and Penzein (1975), Nigam (1983), Yang (1986), and Schueller and Shinozuka (1988). However, the fundamental solution approach for linear systems under random excitation, such as time domain superposition or frequency domain superposition, are not applicable for non-linear systems, systems with random coefficients. That would leave the use of the FPK equation as still the most powerful tool in obtaining an exact solution to nonlinear random vibration problems. However, the FPK formulation can be applied to only a limited class of nonlinear problems with special forms of nonlinearity. Hence, the lack of a general technique for closed form solution of nonlinear random vibration problems has risen the development of alternative approximate solutions. This chapter briefly reviews some of these techniques, and also focuses on the advancement of a non-Gaussian solution method which is proposed by the author for the random vibration analysis of hysteretic systems.

3.2 Basic Premise

Complete analysis of random vibration problems requires statistical information on the response of the system under study. Random vibration problems can be categorized according to the force-displacement behavior of the system, whether the system behaves linearly or nonlinearly under the excitation; types of the excitations, i.e. external and/or parametric, single random process, finite number of discrete processes or continuously distributed random field, and the behavior of the excitation and the response, i.e. stationary or nonstationary, Gaussian or non-Gaussian nature of the input or the response. The exciting solution techniques can also be classified in a broader sense of exact solutions or approximate solutions.

During the past twenty five years, researchers have developed a number of approximate techniques for estimating the response statistics of non-linear systems to random excitatins. Two of these methods namely Monte Carlo Simulation (MCS) and Equivalent Linearization (EL) are most widley used for highly non-linear problems such as hysteretic systems. Other approximate methods are: Pertubation technique (Crandall, 1963), Stochastic Averaging technique (Stratonovich, 1963), Gaussian and Non-Gaussian techniques (Dash and Iyengar, 1982; Crandall, 1980; Wu and Lin, 1984; Ibrahim and Soundarajan, 1985, and Davoodi, Noori and Saffar, 1988), Winer-Hermite series approximation (Ahmadi, 1982 and Jahedi and Ahmadi, 1983), Finite element approach (Spencer and Bergman, 1985; Mohammadi and Amin, 1988), Stochastic Centeral Difference method (To, 1986, 1988), Equivalent stochastic system (Lin, 1988). A good review of nonlinear random vibration solution technique can be found in articles by Ibrahim and Roberts (1978), Crandall and Zhu (1983), Roberts (1984b), To (1987) and Lin et al (1986).

3.2-1 Fokker-Plank-Kolmogorov Approach

The transition probability density function of a system whose response is a Markov process can be written as the solution of a partial differential equation known as Fokker-Plank-Kolmogorov (FPK) equation. The mathematical derivation of FPK equation has been given in numerous references (Caughey, 1963a; and Lin, 1967). The FPK equation was developed independently by Fokker, Plank and Kolmogorov and has two basic forms, namely the forward and the backward equations. In the forward equation formulation time derivatives are taken with respect to the future time that is:

$$\frac{\partial P(Y,t)}{\partial t} = -\sum_{i=1}^{n} \frac{\partial}{\partial Y_i} [a_i(Y,t)\ P(Y,t)] +$$

$$\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2}{\partial Y_i\ \partial Y_j} [b_{ij}(Y,t)\ P(Y,t)] \qquad [3.1]$$

and in the backward equation time derivatives are taken with respect to the earlier time:

$$\frac{\partial P(Y,t)}{\partial t} = -\sum_{i=1}^{n} \frac{\partial P(Y,t)}{\partial Y_i} a_i(Y,t) +$$

$$\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 P(Y,t)}{\partial Y_i\ \partial Y_j} [b_{ij}(Y,t)\ P(Y,t)] \qquad [3.2]$$

where

$Y(t) = [Y_1,\ Y_2,\ \ldots,\ Y_n]^T$

$P(Y,t) = P[Y(t_{m+1})\,|\,Y(t_n)]$ = transition probability

$a_i(Y,t)$ = first incremental moment or drift coefficients

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} E(Y_i(t+\Delta t) - Y_i(t)) \qquad [3.3]$$

$b_{ij}(Y,t)$ = second incremental moment or diffusion coefficients.

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} E\{[Y_i(t+\Delta t) - Y_i(t)]$$

$$[Y_j(t+\Delta t) - Y_j(t)]\} \qquad [3.4]$$

The initial conditions for these equations are

$$= \lim_{t \to t0} \frac{1}{\Delta t} P(Y|Y_0) = \delta(Y-Y_0) \qquad [3.5]$$

The complete statistical description of the response of a system can be obtained by solving its corresponding FPK equation. However, in many cases it is not possible to derive a closed form solution for the stationary or non-stationary probability density function. The uniquness and the existance of FPK equation and its limitations are discussed by Caughey (1963a, 1971).

In order to overcome the difficulty in solving the FPK equation, this technique is frequently used in conjunction with other methods such as numerical approximation (Spigler, 1985; Kapitaniak, 1985;) and stochastic averaging (Davies, 1983; Roberts, 1984a, 1986). It is also possible to generate the response statistical function of a system, in particular the response moments and correlation functions, without solving FPK equation directly (Ibrahim, 1985, Caughey and Dienes, 1962).

3.2-2 Simulation

Numerical simulation, also known as Monte Carlo simulation, is essentially a deterministic technique based on averaging many response samples. In this technique first a large number, n, of sample excitations according to some probability law are generated. Then the response of the system subjected to each sample excitation is calculated and processed to obtain the desired statistics. Simulation technique is a relatively general and simple technique which is applicable to both single and multi degree of freedom systems of any degree of complication. This technique is often used to inspect the accuracy of the existing approximation techniques when exact solution is not available. The main drawback of this method is the computer cost which increases in proportion to the number of ensembles, n, while the statistical uncertainty in the response statistics decreases in proportion to $n^{\frac{1}{2}}$ (Crandall and Zhu, 1983).

3.2-3 Pertubation

Crandall (1963) extended the classical pertubation method of deterministic vibration for random vibration problems involving small nonlinearity. Consider, a single degree of freedom system whose governing dynamic equation is given by;

$$L(U) + \epsilon\, g(U, \dot{U}) - F(t) = 0 \qquad [3.6]$$

where L is a linear operator and g is a nonlinear function, ϵ is a small parameter and F(t) is a weakly stationary excitation. If ϵ is small, then the solution of Equation [3.6] can be approximated by a power series in terms of ϵ.

$$U(t) = U_0(t) + \epsilon\, U_1(t) + \epsilon^2\, U_2(t) + \ldots \qquad [3.7]$$

Substituting Equation [3.7] into Equation [3.6] and equating the terms which have the same power of ϵ, results

$$L(U_0) = F(t)$$

$$L(U_1) = -g(U_0, \dot{U}_0) \qquad [3.8]$$

Equations [3.8] form a set of linear equations and they can be solved using standard methods of linear random vibration (Lin, 1967). In practice the approximation for the response of a stochastic system is seldom carried out beyond the first term.

3.2-4 Equivalent Linearization

Equivalent linearization technique is perhaps the most widely used approximation technique in random vibration today. This technique was first developed independently by Booton (1954) and Caughey (1963b), and later was generalized by Foster (1968), Iwan and Yang (1972), Atalik and Utku (1976), Spanos (1978, 1979, 1981), Beaman and Hedrick (1981) and Baber and Wen (1982).

The basic idea of equivalent linearization technique is to replace the original nonlinear dynamic system by an equivalent linear system. Then, the solution of the linear system is taken as an approximate solution to the original nonlinear equations. The coeficients of the linear equations are obtained by minimizing the mean square error between the nonlinear and linear equations.

The general problem of defining equivalent linear systems can be described as follows. Given an initial nonlinear set of stochastic differential equations

$$g(\ddot{U},\dot{U},U,t) = F(t) \qquad [3.9]$$

find an equivalent system of linearized equations of the form

$$M\ddot{U} + C\dot{U} + KU + \epsilon(\ddot{U},\dot{U},U,t) = F(t) \qquad [3.10]$$

by minimizing the error of linearization

$$\epsilon(\ddot{U},\dot{U},U,t) = g(\ddot{U},\dot{U},U,t) - [M\ddot{U} + C\dot{U} + KU] \qquad [3.11]$$

keeping in mind that U and F are stochastic processes. The process requires using least square minimization in the mean square sense which results in minimizing $E[\epsilon^T\epsilon]$ with respect to m_{ij}, c_{ij} and k_{ij}, i.e.

$$\partial E[\epsilon^T\epsilon]/\partial m_{ij} = 0 \qquad [3.12a]$$

$$\partial E[\epsilon^T\epsilon]/\partial c_{ij} = 0 \qquad [3.12b]$$

$$\partial E[\epsilon^T\epsilon]/\partial k_{ij} = 0 \qquad [3.12c]$$

It can be shown (Atalik and Utku, 1976) that this is equivalent to

$$m_{ij} = E[\partial g_i(.)/\partial \ddot{U}_j] \qquad [3.13\text{-a}]$$

$$c_{ij} = E[\partial g_i(.)/\partial \dot{U}_j] \qquad [3.13\text{-b}]$$

$$k_{ij} = E[\partial g_i(.)/\partial U_j] \qquad [3.13\text{-c}]$$

provided that u are zero mean Gaussian processes and the partial derivatives exist.

By the transformation of variables the linearized

Equation [3.10] can be written in the vector form as

$$\dot{y} + Gy = a(t) \qquad [3.14]$$

where a(t) is a vector of possibly correlated Gaussian white noise components, and y is the state-space vector of response coordinates. The covariance matrix equation for the linear system of Equation [3.14] can be easily derived by simple matrix manipulations

$$\dot{S}_{yy} + GS_{yy} + S_{yy}G^T = W \qquad [3.15\text{-a}]$$

where $S_{yy} = E[yy^T]$ is the zero time lag covariance matrix and $W = 2\pi D$ with D being the constant power spectral density matrix. Since S_{yy} is symmetric, Equation [3.15-a] can be transformed into a vector form

$$\dot{s} + Ls = b \qquad [3.15\text{-b}]$$

and integrated for s in the transient case, or solved by inverting L in the stationary case.

Because the equivalent linearization method is one of the simplest to implement, it has been used extensively for studying hysteretic systems (Wen, 1976; Baber and Wen, 1979; Baber and Noori, 1986; Noori and Baber, 1984.) Spanos (1980) extended the equivalent linearization technique for the cases involving non-zero mean excitation. Noori and Baber (1984) and Baber and Noori (1986) applied this technique for zero and non-zero mean analysis of several hysteretic systems. Recently Baber (1985) studied the random vibration of hysteretic frames subjected to non-zero mean excitation using equivalent linearization technique. He showed that stationary solution does not always exist. Noori, Davoodi and Choi (1986) applied the equivalent linearization for studying the zero and non-zero mean analysis of a new proposed hysteresis model.

It must be noted that since the linearized coficients of the equivalent linear system are obtained based on mean square minimization of the error, the prediction of the second order moment may not be reliable. An in depth review of Equivalent Linearization technique can be found in references given by: Baber and Wen (1979), Roberts (1981b), Spanos (1981), Noori and Baber (1984) and Beaman and Hedrick (1981).

3.2-5 Wiener Hermite Expansion

The Wiener Hermite series expansion method was first developed by Cameron and Martin (1947) and Wiener (1958). In this method both the forcing function and the response of the system are expanded on the random, statistically Wiener-

Hermite set. The zeroth order term corresponds to the mean value, the first order term corresponds to the Gaussian part and the second and higher order terms correspond to the Non-Gaussian part of the random function. The original dynamic equation of the system are used to derive the unknown kernel functions of the Wiener-Hermite expansion of the response and the corresponding series for the forcing function.

Wiener-Hermite series expansion technique was proposed and developed by Ahmadi (1982) in the analysis of strong electrostatic plasma turbulence. Jahedi and Ahmadi (1983) used a single term expansion to analyze the non-stationary random vibration response of a Duffing oscillator subjected to a modulated white noise excitation. Orabi (1986) and Orabi and Ahmadi (1987b) improved the result of single-term expansion by considereing the first three terms in the Wiener-Hermite series. This technique is very useful for analyzing systems with high nonlinearities. It could also be used for cases of non-stationary input excitation as well as stationary input excitation.

3.2-6 Gaussian and Non-Gaussian Closure

In the area of stochastic dynamics, the term "closure technique" refers to a procedure for truncating the infinite hierarchy of governing statistical moment equations of the system response. The need for such a procedure arises when one tries to obtain, using the avaliable solution techniques, the moment equations of a nonlinear system under the external and/or internal random loading. There are basically two types of closure techniques available; the first type is when the truncation assumption is applied directly to the joint moments of the response, and the second type is when the closure assumption is applied to the joint probability density of the response variables.

It is philosophically convenient to investigate probability density function truncation techniques in terms of Gaussian and non-Gaussian closures. Assaf and Zirkle (1976) proposed a general solution technique for a class of nonlinear problems, discontinuous or non-polunomial type restoring force, by assuming an appropriate density function for the response coordinate. Iyengar and Dash (1978) developed a Gaussian closure technique based on the assumption that the joint probability density function of the response variables and input variables are Gaussian. Beaman (1978) presented an approximation solution technique based on the assumption that the probability density function of the response of a nonlinear system is goverend by a multi-dimensional form of Gram-Charlier series. Crandall (1980) introduced an approximation method by applying a Non-Gaussian closure technique. In outline, the method consists of constructing a Non-Gaussian probability distribution with adjustable parameters for the response and using moment relations derived from the equation of the motion to obtain differential or algebric equations for the unknown parameters. Noori, Saffar and Davoodi (1987) applied Crandall's technique to a system having tangent hyperbolic nonlinearity in the restoring force. The detail study of this work is given in Chapter 6. Davoodi, Noori and Saffar (1988) developed a Gaussian closure technique based on Ito-stochastic approach to study the behavior of a single degree of freedom hysteretic system.

The second type of closure is when the truncation is directly applied to the moments. Wu and Lin (1984) and Ibrahim and Soundarajan (1985) independently developed a cumulant neglect closure method based on Ito stochastic approach. Since the proposed technique in this thesis is an extension and generalization of the method developed by Ibrahim and Soundararajan (1985) and Wu and Lin (1984) which is based on Ito-calculus formulation of the governing differential equation of motion, a review of Ito stochastic differential formulation for nonlinear systems is presented in section 3.2-8.

3.2-7 Other Techniques

There are also other approximation techniques which were not dissussed in detail, these are Stochastic Averaging method, Equivalent Stochastic System and Stochastic Central Difference technique. In this section a brief discussion of tnese methods are presented.

Stochastic Central Difference technique (To, 1986, 1988) for random vibration problem is basically an extension of the deterministic Central Difference method. This technique can be applied to systems having stationary or nonstationary random excitation. Since the formulation of this technique is based on the Finite Element method, therefore it can easily be implemented in the Finite Element packages currently available.

Stochastic Averaging technique is referred to a class of procedures in which rapidly fluctuating terms are averaged out to provide a simpler set of equations for slowly fluctuating response coordinates. This technique involves a procedure for taking into account the effect of a random excitation multiplied by a correlated response. The ground work of Stochastic Averaging method was done by Krylov and Bogoliubov for deterministic excitations, and formulated for stochastic problems by Stratonovich (1963). A good discussion on this solution technique is given by Ibrahim (1985).

Equivalent Stochastic Systems (Lin, 1988) is referred to a class of problems which share the same probabilistic solution. In this technique, the nonlinear system under study is replaced by an equivalent system whose solution is known.

3.3 Ito-Stochastic Differential Approach

The general governing equation of motion of most physical systems can be written as a set of first order differential equations in terms of the state vector Y(t)

$$\frac{dY(t)}{dt} = F(Y,t) + G(Y,t)\ dW(t) \qquad [3.16]$$

where:

Y(t) is an n-dimensional vector of system response variables;

W(t) is an m-dimensional vector of stochastic random process, whose influence on the system is through G(Y,t) matrix;

F(Y,t) is an n-dimensional vector representing the linear or nonlinear deterministic part of the model;

G(Y,t) is an n x m linear or nonlinear matrix, whose elements can be functions of system variables and time.

Here it is assumed that W(t) is a mathematical Gaussian White noise with zero mean and auto-correlation function given as

$$R_w(\Delta\tau) = 2D \quad \delta(\tau) \tag{3.17}$$

where 2D is the power spectral density of the white noise and $\delta(\tau)$ is Dirac delta function. The Gaussian white noise can be formally written as the derivative of Browian motion process, B(t);

$$W(t) = \frac{dB(t)}{d(t)} \tag{3.18}$$

Some authors prefer to use unit Brownian motion which in that case equation [3.18] is written as:

$$W(t) = \sigma^2 \quad \frac{db(t)}{d(t)} \tag{3.19}$$

where σ^2 = 2D and b(t) is a unit Brownian motion process.

By using equation [3.17], Equation [3.16] can be written in the Ito form:

$$dY(t) = F(Y,t)\ dt + G(Y,t)\ dB(t) \tag{3.20}$$

where B(t) is an m-dimensional vector, with the following properties:

$$E\{B_i(t)\} = E\{dB_i(t)\} = 0 \qquad [3.21]$$

$$E\{[dB_i(t)]^2\} = \sigma_i^2\, dt \qquad [3.22]$$

where σ_i^2 is the variance parameter, which is a positive constant. Here and throughout this work E{.} represents expected value.

The Ito state vector differential equation, Equation [3.20], can be used to derive a general moment equation for the system variables. The differential of a scalar function $\Psi(Y,t)$, using Ito differential rule (Jazwinski, 1970; Arnold, 1974; and Ibrahim, 1987) can be written as:

$$d\Psi(Y,t) = (\frac{\partial \Psi}{\partial t} + \tfrac{1}{2}\, \text{Trace}\; G\, Q\, G^T\, \Psi_{YY})dt + \Psi_Y^T\, dY \qquad [3.23]$$

where

$Q\, dt = E\{[dB(t)]\; [dB(t)]^T\}$

$\Psi_Y^T = (\partial\Psi/\partial Y_1,\; \partial\Psi/\partial Y_2,\; \ldots,\; \partial\Psi/\partial Y_n)$

$\Psi(Y,t)$ = Scalar-valued real function, continuously differentiable in time and possessing continous second mix partial derivaties.

$$\Psi_{YY} = \begin{bmatrix} \partial^2\Psi/\partial Y_1^2 & \partial^2\Psi/\partial Y_1 Y_2 & \cdots & \partial^2\Psi/\partial Y_1 Y_n \\ \partial^2\Psi/\partial Y_2 Y_1 & \cdots\cdots & \cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots \\ \partial^2\Psi/\partial Y_n Y_1 & \cdots\cdots & \cdots & \partial^2\Psi/\partial Y_n^2 \end{bmatrix}$$

Replacing the function $\Psi(Y,t)$ of Equation [3.23], by $\Phi(Y) = (Y_1^{K1}, Y_2^{K2}, \ldots, Y_n^{Kn})$, an arbitary scalar function of the response coordintes, yields

$$d\Phi(Y) = [\frac{\partial\Phi(Y)}{\partial Y_i}]^T dY + \frac{1}{2} \text{Trace } [G\ Q\ G^T\ \Phi_{YY}(Y)]\ dt \qquad [3.24]$$

The choice of $\Phi(.)$ depends on the type of the statistical functions to be evaluated. The expression given herein, and suggested by Ito, is used if the joint moments of response are required. Taking the expected value of both sides of Equation [3.24] and dividing it by dt yields the general form of the moment differential equation.

$$E\{d\Phi(Y)\} = E\{[\frac{\partial\Phi(Y)}{\partial Y_i}]^T dY\} + \frac{1}{2} E\{\text{Trace } [G\ Q\ G^T\ \Phi_{YY}(Y)]\ dt\}$$

$$
\Psi_{YY} = \begin{bmatrix} \partial^2\Psi/\partial Y_1{}^2 & \partial^2\Psi/\partial Y_1 Y_2 & \cdots & \partial^2\Psi/\partial Y_1 Y_n \\ \partial^2\Psi/\partial Y_2 Y_1 & \cdots\cdots & \cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots \\ \partial^2\Psi/\partial Y_n Y_1 & \cdots\cdots & \cdots & \partial^2\Psi/\partial Y_n{}^2 \end{bmatrix}
$$

Replacing the function $\Psi(Y,t)$ of Equation [3.23], by $\Phi(Y) = (Y_1^{K1}, Y_2^{K2}, \ldots, Y_n^{Kn})$, an arbitary scalar function of the response coordintes, yields

$$
d\Phi(Y) = [\frac{\partial\Phi(Y)}{\partial Y_i}]^T dY + \frac{1}{2} \text{Trace } [G\ Q\ G^T\ \Phi_{YY}(Y)]\ dt \qquad [3.24]
$$

The choice of $\Phi(.)$ depends on the type of the statistical functions to be evaluated. The expression given herein, and suggested by Ito, is used if the joint moments of response are required. Taking the expected value of both sides of Equation [3.24] and dividing it by dt yields the general form of the moment differential equation.

$$
E(d\Phi(Y)) = E([\frac{\partial\Phi(Y)}{\partial Y_i}]^T dY) + \frac{1}{2} E(\text{Trace } [G\ Q\ G^T\ \Phi_{YY}(Y)]\ dt)
$$

$$= E\{[\frac{\partial\Phi(Y)}{\partial Y_i}]^T [F(Y,t)\ dt + G(Y,t)\ dB(t)]\} +$$

$$1/2\ E\{\text{Trace}\ [G\ Q\ G^T\ \Phi_{YY}(Y)]\ dt\}$$

$$= E\{[\frac{\partial\Phi(Y)}{\partial Y_i}]^T F(Y,t)\}\ dt +$$

$$E\{[\frac{\partial\Phi(Y)}{\partial Y_i}]^T G(Y,t)\ dB(t)]\} +$$

$$1/2\ E\{\text{Trace}\ [G\ Q\ G^T\ \Phi_{YY}(Y)]\ dt\} \qquad [3.25]$$

Because of Equation [3.21] the second term in Equation [3.25] makes no contribution and the moment equation can be written as:

$$\dot{M}_K = E\{[\frac{\partial\Phi(Y)}{\partial Y_i}]^T F(Y,t)\}\ dt +$$

$$\frac{1}{2}\ E\{\text{Trace}\ [G\ Q\ G^T\ \Phi_{YY}(Y)]\ dt\} \qquad [3.26]$$

where $K = K_1 + K_2 + \ldots + K_n$, $F(Y,t)$, and $G(Y,t)$ are as defined in Equation [3.16], and $\dot{M}$ is the time derivative of M. Equation [3.26] can be used to write up to the Kth order moment differential equation of a system which is described by Equation [3.16]. A very good treatment for the development of moment equations, using Ito-Differential approach, can be found in the thesis by Bover (1978).

If the governing dynamic equation of the system, Equation [3.16], were linear then, the moment equations obtained from Equation [3.25] form a closed set of first order differential equations. However, if the governing equations are nonlinear then the resulting moment equations will be coupled with higher order moments. This infinite hierarchy of the moment equations can be solved by using proper closure techniques (Ibrahim ,1981, 1978).

In this work a brief review of the available closure techniques is first presented. Then a new closure technique is proposed by the author for deriving the moment equations of nonlinear systems with general hysteretic form of nonlinearity.

3.4 Closure Techniques

Equation [3.26] can be used to obtain a set of first order differential equation of response moments of a dynamical system subjected to external or parametric random excitation. If the governing equations for the system are nonlinear, then the resulting moment equations are coupled with higher order moments.

The term "Closure Technique", in stochastic dynamics, refers to a procedure for truncating the infinite hierarchy of governing statistical moment equations of the system response. From mathematical point of view, a closure technique is said to be proper if it does not violate the moment properties and other statistical conditions. On the other hand, an appropriate closure technique for an engineer is the one that results into moments which are close to experimental findings.

The problem of infinite hierarchy of moment equations can be described in general form as:

$$\dot{M}_i = g_i(M_1, M_2, \ldots, M_i, M_{i+1}, \ldots) \qquad [3.27]$$

with the initial condition $M_i = a_i$ at time = 0, where M_i is the exact solution of Equation [3.27]. Using an appropriate technique Equation [3.27] can be approximated by:

$$\dot{m}_i = \bar{g}_i(m_1, m_2, \ldots, m_i) \qquad [3.28]$$

with $m_i = b_i$ at time = 0, where m_i is an approximated solution. There are three criteria for choosing a closure technique, accuracy, simpilicity and versatility. Here a number of existing closure techniques are discussed. These techniques have been used in studying the response analysis of randomly excited dynamical systems. A more detail discussion of these closure techniques can be found in an article by Ibrahim (1981, 1978).

3.4-1 Cumulant Neglect Closure

The joint moment of order K of a random variable vector Y can be expressed in terms of derivatives of its characteristic function:

$$M_K = E(Y_1^{K1}\ Y_2^{K2}\ \ldots\ Y_n^{Kn})$$
$$= \frac{1}{iK}\ \frac{\partial^K}{\partial\theta_1^{K1}\ \ldots\ \partial\theta_n^{Kn}}\ \phi_Y(\theta)\ \Big|_{\theta_i=0} \qquad [3.29]$$

where $K = K_1 + K_2 + \ldots + K_n$ and $\phi_Y(\theta) = E\{\exp(i\theta_1 Y_1 + \ldots + i\theta_n Y_n)\}$ is the characterestic function for Y. The joint cumulant of order K, λ_K, is given by:

$$\lambda_K[Y_1^{K1},\ Y_2^{K2},\ \ldots,\ Y_n^{Kn}] =$$
$$\frac{1}{iK}\ \frac{\partial^K}{\partial\theta_1^{K1}\ \ldots\ \partial\theta_n^{Kn}}\ \phi_Y(\theta)\ \Big|_{\theta_i=0} \qquad [3.30]$$

Equation [3.29] and [3.30] can be used to derive a relationship between the joint cumulant of order K and the joint moments of order K and less. The first four cumulants are:

$$\lambda_1[Y_i] = E(Y_i)$$
$$\lambda_2[Y_iY_j] = E(Y_iY_j) - E(Y_i)\ E(Y_j)$$
$$\lambda_3[Y_iY_jY_k] = E(Y_iY_jY_k) - \sum^{3} E(Y_i)\ E(Y_jY_k) +$$
$$2\ E(Y_i)\ E(Y_j)\ E(Y_k)$$

$$\lambda_4[Y_iY_jY_kY_l] = E(Y_iY_jY_kY_l) - \overset{4}{\Sigma}\, E(Y_i)\; E(Y_jY_kY_l) + 2\overset{6}{\Sigma}\, E(Y_i)\; E(Y_j)\; E(Y_kY_l) - \overset{3}{\Sigma}\, E(Y_iY_j)\; E(Y_kY_l) - 6\, E(Y_i)\; E(Y_j)\; E(Y_k)\; E(Y_l) \qquad [3.31]$$

where the numbers above the Σ sign indicate the numbers of possible permutations.

Higher order moments can be written in terms of lower order moments by setting the corresponding cumulant equal to zero (Ibrahim, 1984; and Wu and Lin, 1985). For example in Gaussian closure closure assumption for the random vector Y, cumulants corresponding to the third and higher order moments are set equal to zero.

3.4-2 Central Moments Closure

The joint central moment, μ_K, of a random process Y is given by:

$$\mu_K = E((Y_1-m_1)^{K1}\; (Y_2-m_2)^{K2}\; \ldots\; (Y_n-m_n)^{Kn}) \qquad [3.32]$$

where $m_i = E(Y_i)$. In this method the joint central moments beyound a selected order is set equal to zero. This technique is found to be less accurate than cumulant neglect closure (Bellmanand and Richardson, 1968).

3.4-3 Mean Square Closure Technique

In this technique the higher-order moments are written as functions of lower-order moments:

$$m_{n+1} = \sum_{i=1}^{n} a_{in} M_i \qquad [3.33]$$

where a_{in} are time independent coefficients, $M_n = E(Y^n)$ and m_{n+1} is the approximated (n+1)th order moment. Equation [3.33] is not applicable unless the lower order moments are known.

The resulting error from the above approximation is:

$$e = M_{n+1} - \sum_{i=1}^{n} a_{in} M_i \qquad [3.34]$$

The mean square closure technique procedure requires the minimization of Equation [3.34].

$$e = \int_0^\infty (M_{n+1} - \sum_{i=1}^{n} a_{in} m_i)^2 \, dt \qquad [3.35]$$

This technique becomes very cumbersome for multi-dimensional systems.

3.4-4 Non-Gaussian Closure Technique

In this technique the unknown probability density function of the response is approximated by a truncated non-Gaussian density function; like Gram-Charlier or Edgeworth expansion series. The coefficients of the finite density function are then calculated by using the governing equation of motion for the system. Once the probability density function is known, then any statistical moment of the response can be computed easily. Crandall (1980) introduced an approximate method by applying a Non-Gaussian closure technique. In outline, the method consists of constructing a Non-Gaussian probability distribuion with adjustable parameters for the response and using the moment relations derived from the equation of motion to obtain differential or algebraic equations for the unknowns parameters. Bover (1978) and Beaman (1981) used Ito-differential approach for calculating the response moments of nonlinear systems. They suggested that by assuming a Non-Gaussian density function for the response, the higher order moments can be calculated using the basic moment defination.

To the best of the author's knowledge, none of the proposed Non-Gaussian solution techniques were applied to hysteretic systems. When the nonlinearity in the system is hysteretic, the resulting moment equations are not closed, and they are implicilitly in terms of higher order moments. The purpose of this thesis was to develop a solution technique that can be used for analysis of hysteretic systems.

3.5 The Proposed Closure Technique

As it was mentioned before, many approximate techniques for nonlinear random vibration problems have been developed. Most of these techniques are not applicable for systems having hysteresis nonlinearity, specially when the resulting moment equations are in implicit terms of higher order moments. The technique that is proposed in this thesis is for calculating the implicit higher order moments in terms of lower order moments. This was done by assuming a joint density function for the response variables and calculating the desired moments by using the basic moment defination. The proposed approach was also suggested independently by Assaf and Zirkle (1976), Beaman and Hedrik (1981) and Bover (1978), but was not applied to highly nonlinear systems such as hysteretic systems.

The approximated density function which was used in this work is a multi-dimensional Edgeworth expansion.

$$P^*(Y) = P(Y) - \frac{1}{3!} \sum_{k,l,m} \lambda_{k,l,m} \frac{\partial^3 P(Y)}{\partial Y_k \, \partial Y_l \, \partial Y_m} +$$

$$\frac{1}{4!} \sum_{k,l,m,q} \lambda_{k,l,m,q} \frac{\partial^4 P(Y)}{\partial Y_k \, \partial Y_l \, \partial Y_m \, \partial Y_q} - \ldots \qquad [3.36]$$

where λ is the joint cumulant and P(Y) is a multi-dimensional Gaussian density function;

$$P(Y) = \frac{(2\pi)^{-n/2}}{|\Delta|} \exp\left(- \frac{1}{2|\Delta|} \sum_{i=1}^{n} \sum_{j=1}^{n} \right.$$

$$\left. \mathrm{cof}(\Delta)_{ij} \; (Y_i - m_i) \; (Y_j - m_j) \right) \qquad [3.37]$$

where

$Y(t) = [Y_1(t),\ Y_2(t),\ \ldots,\ Y_n(t)]^T$

$|\Delta|$ = Determinant of the covariance matrix

$\mathrm{cof}(\Delta)_{ij}$ = cofactor of the covariance element, C_{jk}

$m_i = E(Y_i)$

In order to close the moment equations for Gaussian and Non-Gaussian cases, the cumulants of the response behind the second and forth order are set equal to zero. The moments are caculated by using the basic defination:

$$E(g(Y)) = \int_{-\infty}^{\infty} g(Y) \; P^*(Y) \; dY \qquad [3.38]$$

where g(Y) is a nonlinear, non-polynomial function of the response variables and $E(g(Y))$ is the implicit higher order moment. In Chapter 4, the application of the proposd solution technique to a SDOF BBW hysteretic model is demonstrated, and the numerical studies are presented in Chapter 5.

CHAPTER IV

RESPONSE OF A SDOF GENERAL HYSTERETIC SYSTEM

4.1 General

The process of the analysis of most engineering problems consists of three stages: study of the parameters or functions which are input to the system; study of the mathematical model representing the system, and the analysis of the system response to the input. In the particular area of structural dynamics these stages are: 1) Modeling the excitation, 2) Modeling the behavior of the system geometry when subjected to a given excitation, and 3) Obtaining the response of the system. In the previous chapter some of the existing solution techniques for obtaining the response of nonlinear systems were discussed. In this chapter the excitation models adopted in the random vibration of dynamic systems and used in this work are briefly reviewed. However, the main body of this chapter is devoted to presenting the formulation of the proposed approximation technique. This is done by applying this Ito-based method to the analysis of a SDOF smooth hysteresis model and by deriving the corresponding response statistics of the model.

4.2 Excitation Model

Dynamic loads - such as base excitations of a building during an earthquake, wind loads on tall buildings, ocean wave forces on offshore oil drilling platforms and acoustic loads on aircraft studies - are examples of exteremly complex loadings that cannot be described as a definite functions of time. Due to the uncertainty involved, these classes of excitations have to modeled as random time functions, known as random processes.

One of the major elements in random vibration analysis of structures is the modeling of excitations, an extensive review of which is given by Crandall and Zhu (1983). The particuler model used herein is a stationary white noise process with the following properties;

$$E(F(t)) = 0 \qquad [4.1]$$

$$E(F(t)\ F(t+\tau)) = 2\pi D\delta(\tau) \qquad [4.2]$$

where E(.) denotes the expected value; $\delta(\tau)$ is the Dirac delta function and $2\pi D$ represents the constant power spectral density of the input. The important mathematical characteristic of a white noise process is that its energy is uniformly distributed over the entire frequency range. Although the assumption of constant spectral excitation is not realistic and is just a mathematical idealization, when the excitation spectrum varies slowly in the neighborhood of system's natural frequency, this assumption leads to meaningful results. Bycroft (1960) was one of the first who suggested the use of Gaussian white noise process for modeling the eartquake excitations. Excitations modeled as stationary white noise have been used by many other researches (Liu and Davies, 1988; Paola, 1988; Bruckner and Lin, 1987; Sun and Hsu, 1987; Crandall, 1962; Caughey, 1960).

Other types of models for random vibration analysis are non white stationary and temporaly modulated excitations. Non-white stationary excitations can be introduced by adding one or more linear filters between the system and the white Gaussian input (Housner and Jennings, 1964; Lutes and Lilhand, 1979). This type of excitation may result in more complicated system of equations than the original ones. The problem gets even more complicated for non-linear systems and/or parametric excitations. Temporaly modulated excitation can be introduced by either multiplying the white noise input by a deterministic temporaly varying function before passing it through the filters or multiplying the filtered white noise by a temporal factor before passing it through the system. Lin and Yong (1987) used an evolutionary Kanai-Tajimi model, a one dimensional elastic model and a one dimensional Maxwell model to model the ground accelaration during an earthquake,; and they compared their results to 1985 Mexico earthquake. These excitation models can be numericaly generated by using MCS or a more recent technique called Autoregressive-Moving Average (ARMA) modeling (Marple, 1987). For further studies on available models the reader can refer to Levy, Kozin and Moorman (1971), and Spanos (1980). Non white noise and temporaly modulated excitations are not considered in this study. If needed, filtering can easily be incorporated into the model (Ibrahim, 1985).

4.3 Formulation of the Moment Equations for BBW Hysteretic Model

The nonlinear system to be studied here is a single degree of freedom (SDOF) system, with a linear viscous damper and a hysteretic restoring force element as shown in Figure 3.1. The governing equation of motion for the system shown is:

$$\ddot{U} + 2\zeta\omega_0\dot{U} + \alpha\omega_0^2 U + (1-\alpha)\omega_0^2 Z = f(t) \qquad [4.3]$$

where U is the displacement of the mass with respect to a fixed datum; ω_0 is the natural frequency of the system in the linearly elastic range; f(t), a zero mean Gaussian white noise, is the forcing function per unit mass; Z is the BBW hysteretic restoring force model as proposed by Bouc(1967) and extended by Baber and Wen [1981, 1979] to incorporate stiffness and/or strength degradation, and α is the post-yield to pre-yield stiffness ratio. This model can represent a wide variety of hysteretic, deteriorating types of behavior with a considerable range of cyclic energy dissipation (Baber and Wen, 1982, 1981, 1979; Mohammad Yar and Hammond, 1987; and Park, Wen and Ang, 1986). This model has also been utilized for the stochastic seismic performance evaluation of structures (Sues, Wen and Ang, 1981) and for modelling base isolation mechanisms in the response of structures to random excitation (Constantinou and Tadjbaksh, 1985).

The BBW model is written in the form

$$\dot{Z} = (A\dot{U} - \nu[\beta|\dot{U}||Z|^{(n-1)}Z + \gamma\ \dot{U}|Z|^{n}])/\eta \qquad [4.4]$$

in which β, γ and n determine the hysteresis shape; A defines the tangential stiffness, and ν and η are the deterioration control parameters. The parameters A, ν, and η, may be varied as a function of the energy dissipation $\epsilon(t)$, to introduce system degradation. Detailed study of BBW model is reported elsewhere (Baber and Wen, 1979).

Using a coordinate transformation $Y_1 = u$, $Y_2 = \dot{u}$, and $Y_3 = z$, Equation [4.3] can be written as

$$\dot{Y}_1 = Y_2$$
$$\dot{Y}_2 = f(t) - 2\zeta\omega_0 Y_2 - \alpha\omega_0{}^2 Y_1 - (1 - \alpha)\omega_0{}^2 Y_3$$
$$\dot{Y}_3 = (AY_2 - \nu[\beta|Y_2||Y_3|^{n-1}Y_3 - \gamma Y_2|Y_3|^n)/\eta \qquad [4.5]$$

and in the Markov vector form this nonlinear set of state space equations can be expressed as the stochastic differential equation

$$d\ Y(t) = F(Y,t)dt + G(Y,t)\ dB(t) \qquad [4.6]$$

where $Y(t)$ is the state space vector of the system response coordinates, $F(Y,t)$ and $G(Y,t)$ are the matrices of state vector equations defining the nonlinear system of equations governing the system under study, $dB(t)$ is the zero-mean increment of Brownian motion process with mean square differential given as:

$$E\{[dB(t)]^2\} = \sigma^2\ dt \qquad [4.7]$$

for a real constant σ. In this equation $\sigma^2 = 2\pi D$ is the power spectral density of the input Gaussian white noise as described in Equation [4.2].

The general moment equation of a system given by Equations [4.3] and [4.4] can be obtained by using Equation [3.26]. The elements of Equation [3.26] can be defined as:

$$F(Y,t) = \begin{bmatrix} Y_2 \\ -2\zeta\omega_0 Y_2 - \alpha\omega_0^2 Y_1 - (1-\alpha)\omega_0^2 Y_3 \\ \{AY_2 - \nu\,[\,\beta|Y_2||Y_3|^{n-1}Y_3 - \gamma Y_2|Y_3|^n]\}/\eta \end{bmatrix}$$

$$G(Y,t) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$G\;Q\;G^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the general moment equation of the response is:

$$\begin{aligned} \dot{M}_{i,j,k} = {} & \tfrac{1}{2}j(j-1)\sigma^2 M_{i,j-2,k} + iM_{i-1,j+1,k} - \\ & 2j\zeta\omega_0 M_{i,j,k} - \alpha j\omega_0^2 M_{i+1,j-1,k} - \\ & j(1-\alpha)\omega_0^2 M_{i,j-1,k+1} + AkM_{i,j+1,k-1} - \\ & \beta kE\{|Y_2||Y_3|^{n-1}\,Y_1^i Y_2^j Y_3^k\} - \\ & \gamma kE\{|Y_3|^n Y_1^i\,Y_2^{j+1} Y_3^{k-1}\} \end{aligned} \qquad [4.8]$$

where $M_{i,j,k} = E[Y_1^i Y_2^j Y_3^k]$.

In Fquation [4.8], $\sigma^2 = 2\pi D$ is the power spectral density of the input Gaussian white noise as described in Equation

[4.2], and the index summation i+j+k varies from 1 to the order of moments to be evaluated.

As can be seen from Equations [4.8], the right hand sides include expected values in terms of signum functions. These expected values cannot be expressed in terms of $M_{i,j,k}$. Therefore, the approach used by Ibrahim and Soundararajan (1985) and Wu-Lin (1984) for solving these moment equations is not suitable for the case of general hysteresis nonlinearity. In order to solve these moment equations, as proposed in Chapter 3, multi-dimensional Gaussian probability density functions are assumed for the joint probability distribution of the corresponding variables in each expected value. Hence, these non-classic expected functions are evaluated using the proposed technique and then the moment equations are solved.

4.3-1 Gaussian Response

Equation [4.8] can be used to derive a set of simultaneous differential moment equations. First order moments are derived by setting i+j+k = 1. This results in three first order moment equations

$$\dot{M}_{001} = AM_{010} - \beta E(|Y_2|Y_3) - \gamma E(|Y_3|Y_2)$$
$$\dot{M}_{010} = -2\zeta\omega_0 M_{010} - \alpha\omega_0^2 M_{100} - (1-\alpha)\omega_0^2 M_{001}$$

$$\dot{M}_{100} = M_{010} \qquad [4.9]$$

And second order moments are obtained by setting $i+j+k = 2$ as follows

$$\dot{M}_{200} = 2M_{110}$$
$$\dot{M}_{020} = \sigma^2 - 4\zeta\omega_0 M_{020} - 2\alpha\omega_0^2 M_{110} - 2(1-\alpha)\omega_0^2 M_{011}$$
$$\dot{M}_{002} = 2AM_{011} - 2\beta E(|Y_2|Y_3^2) - 2\gamma E(|Y_3|Y_2Y_3)$$
$$\dot{M}_{110} = M_{020} - 2\zeta\omega_0 M_{110} - \alpha\omega_0^2 M_{200} - (1-\alpha)\omega_0^2 M_{101}$$
$$\dot{M}_{101} = M_{011} + AM_{110} - \beta E(|Y_2|Y_1Y_3) - \gamma E(|Y_3|Y_1Y_2)$$
$$\dot{M}_{011} = -2\zeta\omega_0 M_{011} - \alpha\omega_0^2 M_{101} - (1-\alpha)\omega_0^2 M_{002} + AM_{020} - \beta E(|Y_2|Y_2Y_3) - \gamma E(|Y_3|Y_2^2) \qquad [4.10]$$

As it can be seen the set of first order and second order moment equations are not closed and they are coupled with higher order moments. In order to evaluate these higher order moments in term of lower order moments, and close the set of simultaneous differential moment equations, it was assumed that the response of the system is jointly Gaussian with a density function of the general form.

$$P[Y(t)] = \frac{(2\pi)^{-3/2}}{|\Delta|^{\frac{1}{2}}} \exp\left(- \frac{1}{2|\Delta|} \sum_{i=1}^{3} \sum_{j=1}^{3} cof(\Delta)_{ij} \, (Y_i - m_i) \, (Y_j - m_j) \right) \qquad [4.11]$$

where

$Y(t) = [Y_1(t),\ Y_2(t),\ Y_3(t)]^T$

$|\Delta|$ = Determinant of the covariance matrix

$cof(\Delta)_{ij}$ = cofactor of the covariance element, C_{ij}, in the determinant of covariance matrix.

$m_i = E(Y_i)$ = mean value of the response coordinate Y_i - assumed to be zero in this study.

Since the joint probability density function for the response is assumed to be zero mean Gaussian distributed, then Equations [4.9] are eliminated. The expected values in Equations [4.10] can be evaluated in closed form for this case of Gaussian analysis. However since the closed form derivation of the expected values with singular functions is not possible for the case of non-Gaussian analysis, a general numerical and iterative approach is developed for evaluating these expected values. The evaluation process for these expected values is mathematically involved and lengthy and thus cannot be reported herein. A summary of the final forms of these expected values and the integration procedures that have been performed are presented in Appendix A. The closed form derivation of I's can be done by using the approached discussed in Appendix B. The resulting integro-differential moment equations were solved using an appropriate numerical integration routine.

4.3-2 Non-Gaussian Response

It is known that the response of a nonlinear system subjected to a Gaussian process input are not Gaussian distributed. In order to improve the accuracy of the Gaussian results, a Non-Gaussian distribution function for the response is assumed. In this work, the assumed probability density function isin the form of a multi-dimensional Edgeworth expansion:

$$P^*(Y) = P(Y) - \frac{1}{3!} \sum_{k,l,m}^{3} \lambda_{k,l,m} \frac{\partial^3 P(Y)}{\partial Y_k \, \partial Y_l \, \partial Y_m} +$$

$$\frac{1}{4!} \sum_{k,l,m,q}^{3} \lambda_{k,l,m,q} \frac{\partial^4 P(Y)}{\partial Y_k \, \partial Y_l \, \partial Y_m \, \partial Y_q} - \ldots \qquad [4.12]$$

where

$P^*(Y)$ = Non-Gaussian, Multi-dimensional density function

Y = is the response vector (nx1), $Y_1 = U$, $Y_2 = \dot{U}$, $Y_3 = Z$,

$P(Y)$ = is the correspondingmulti-dimensional Gaussian density function

λ = the cumulant

The first term of Equation [4.12] is in terms of the first and the second moments. The second term of that Equation is in terms of third moment, and the third term is in terms of the fourth moments and so on. Herein, it is assumed that only moments up to the fourth order are significant and therefore only the first three terms of Equation [4.12] were considered.

Equation [4.8] can be used to derive 34 simulatenous first order differential equations of first, second, third and fourth order moments. The first and second order moments are as given by Equations [4.9] and [4.10]. The third order moments are:

$$\dot{M}_{300} = M_{210}$$

$$\dot{M}_{030} = 3\sigma^2 M_{010} - 6\zeta\omega_o M_{030} - 3\alpha\omega_o^2 M_{120} - 3(1-\alpha)\omega_o^2 M_{021}$$

$$\dot{M}_{003} = 3AM_{012} - 3\beta E(|Y_2|Y_3^3) - 3\gamma E(|Y_3|Y_2Y_3^2)$$

$$\dot{M}_{210} = 2M_{120} - 2\zeta\omega_o M_{210} - \alpha\omega_o^2 M_{300} - (1-\alpha)\omega_o^2 M_{201}$$

$$\dot{M}_{201} = 2M_{111} + AM_{210} - \beta E(|Y_2|Y_1^2 Y_3) - \gamma E(|Y_3|Y_1^2 Y_2)$$

$$\dot{M}_{120} = \sigma^2 M_{100} + M_{030} - 4\zeta\omega_o M_{120} - 2\alpha\omega_o^2 M_{210} - 2(1-\alpha)\omega_o^2 M_{111}$$

$$\dot{M}_{021} = \sigma^2 M_{001} - 4\zeta\omega_o M_{021} - 2\alpha\omega_o^2 M_{111} - 2(1-\alpha)\omega_o^2 M_{012} + AM_{030} - \beta E(|Y_2|Y_2^2 Y_3) - \gamma E(|Y_3|Y_2^3)$$

$$\dot{M}_{102} = M_{012} + 2AM_{111} - 2\beta E(|Y_2|Y_1Y_3^2) - 2\gamma E(|Y_3|Y_1Y_2Y_3)$$
$$\dot{M}_{012} = -2\zeta\omega_0 M_{012} - \alpha\omega^2 M_{012} - (1-\alpha)\omega_o^2 M_{003} +$$
$$2AM_{021} - 2\beta E(|Y_2|Y_2Y_3^3 - 2\gamma E(|Y_3|Y_2^2Y_3)$$
$$\dot{M}_{111} = M_{021} - 2\zeta\omega_o M_{111} - \alpha\omega_o^2 M_{201} - (1-\alpha)\omega_o^2 M_{012} +$$
$$AM_{120} - \beta E(|Y_2|Y_1Y_2Y_3) - \gamma E(|Y_3|Y_1Y_2^2) \quad [4.13]$$

The fourth order moments are:

$$\dot{M}_{400} = 4M_{310}$$
$$\dot{M}_{040} = 6\sigma^2 M_{020} - 8\zeta\omega_o M_{040} - 4\alpha\omega_o^2 M_{130} -$$
$$4(1-\alpha)\omega_o^2 M_{031}$$
$$\dot{M}_{004} = 4AM_{013} - 4\beta E(|X_2|X_3^4) - 4\gamma E(|X_3|X_2X_3^3)$$
$$\dot{M}_{310} = 3M_{220} - 2\zeta\omega_o M_{310} - \alpha\zeta_o^2 M_{400} - (1-\alpha)\omega_o^2 M_{301}$$
$$\dot{M}_{301} = 3M_{211} + AM_{310} - \beta E(|X_2|X_1^3X_3) - \gamma E(|X_3|X_1^3X_2)$$
$$\dot{M}_{220} = \sigma^2 M_{200} + 2M_{130} - 4\zeta\omega_o M_{220} - 2\alpha\omega_o^2 M_{310} -$$
$$2(1-\alpha)\omega_o^2 M_{211}$$
$$\dot{M}_{211} = 2M_{121} - 2\zeta\omega_o M_{211} - \alpha\omega_o^2 M_{301} - (1-\alpha)\omega_o^2 M_{202} +$$
$$AM_{220} - \beta E(|X_2|X_1^2X_2X_3) - \gamma E(|X_3|X_1^2X_2^2)$$
$$\dot{M}_{202} = 2M_{112} + 2AM_{211} - 2\beta E(|X_2|X_1^2X_3^2) - 2\gamma E(|X_3|X_1^2X_2X_3)$$
$$\dot{M}_{130} = 3\sigma^2 M_{110} + M_{040} - 6\zeta\omega_o M_{130} - 3\alpha\omega_o^2 M_{220} -$$
$$3(1-\alpha)\omega_o^2 M_{121}$$
$$\dot{M}_{121} = \sigma^2 M_{101} + M_{031} - 4\zeta\omega_o M_{121} - 2\alpha\omega^{o2} M^{211} -$$
$$2(1-\alpha)\omega_o^2 M_{112} + AM_{130} - \beta E(|X_2|X_1X_2^2X_3) -$$
$$\gamma E(|X_3|X_1X_2^3)$$
$$\dot{M}_{112} = M_{022} - 2\zeta\omega_o M_{112} - \alpha\omega_o^2 M_{202} - (1-\alpha)\omega_o^2 M_{103} +$$
$$2AM_{121} - 2\beta E(|X_2|X_1X_2X_3^2) - 2\gamma E(|X_3X_1X_2^2X_3)$$
$$\dot{M}_{103} = M_{013} + 3AM_{112} - 3\beta E(|X_2|X_1X_3^3) - 3\gamma E(|X_3|X_1X_2X_3^2)$$

$$\dot{M}_{031} = 3\sigma^2 M_{011} - 6\zeta\omega_o M_{031} - 3\alpha\omega_o^2 M_{121} - 3(1-\alpha)\omega_o^2 M_{022} + AM_{040} - \beta E\{|X_2|X_2^3 X_3\} - \gamma E\{|X_3|X_2^4\}$$

$$\dot{M}_{013} = -2\zeta\omega_o M_{013} - \alpha\omega_o^2 M_{103} - (1-\alpha)\omega_o^2 M_{004} + 3AM_{022} - 3\beta E\{|X_2|X_2 X_3^3\} - 3\gamma E\{|X_3|X_2^2 X_3^2\}$$

$$\dot{M}_{022} = \sigma^2 M_{002} - 4\zeta\omega_o M_{022}\ 2\alpha\omega_o^2 M_{112} - 2(1-\alpha)\omega_o^2 M_{013} + 2AM_{031} - 2\beta E\{|X_2|X_2^2 X_3^2\} - 2\gamma E\{|X_3|X_2^3 X_3\} \qquad [4.14]$$

The right hand sides of Equations [4.13] and [4.14] contain moments which are coupled with moments higher than the third and fourth orders. At the same time these higher order moments cannot be expressed in terms of $M_{i,j,k}$. In order to close the moment equations and derive the higher order moments in terms of the lower order ones, the mathematical definitation of moment was used. This is done in conjunction with Non-Gaussian density function of Equation [4.12] and the technique proposed in Chapter 3 and adapted in the Gaussian analysis. Since the assumed density function has zero mean, therefore only the second, third, and the fourth order moments need to be integrated. The resulting 31 differential equations were solved using a numerical integration subroutine.

CHAPTER V

NUMERICAL STUDIES

5.1 General

As it was discussed in the preceeding chapters, due to highly nonlinear nature of hysteretic systems, exact analysis of these systems under random excitations are difficult. This has resulted in the development of approximate solution techniques such as equivalent linearization and Gaussian and Non-Gaussian closures. Historically, since the development of equivalent linearization technique, this method has been the most extensively used in the field, specifically with regrad to hysteretic systems. However, there are several disadvantages associated with this technique. In particular, the method is not capable of predicting the respone of highly nonlinear systems, and the response predictions are limited to Gaussian. In order to obtain information about Non-Gaussian behavior of general hysteretic systems, an approximate solution technique based on Ito calculus is presented in Chapter 3. In order to investigate the capabilities of the proposed solution technique, a SDOF system with BBW hysteretic restoring force was considered and formulated in Chapter 4. In this chapter numerical studies that have been performed on the

BBW model using the proposed technique are presented .

5.2 The Goals of Numerical Studies

The numerical studies which follow were conducted with several purposes in mind: (1) to investigate the validity of the new Ito-based solution scheme for a nonlinear hysteretic model with a very general form of nonlinearity; (2) to explore any limitation inherent to this technique, and its source, if possible; (3) to compare the results of this anlaysis with equivalent linearization studies via digital Monte Carlo simulation, and to examine possible advantages of this approach. In the studies reported herein, both cases of nondegrading and degrading systems have been presented. Degrading system studies were limited, however, to the consideration of the effects of degradation upon the covariance matrix response and the energy dissipation. Stability of the response has not been studied in this case.

5.3 Classification of Numerical Studies

Figure 2.1 shows a schematic diagram of the system studied in this work. The governing differential equations of motion were previously given by Equations [4.3] and [4.4] and are repeated here:

$$\ddot{U} + 2\xi\omega_0\dot{U} + \alpha\omega_0^2 U + (1 - \alpha)\omega_0^2 Z = f(t) \qquad [5.1]$$

$$\dot{Z} = \{A\dot{U} - \nu[\beta|\dot{U}||Z|^{(n-1)}Z + \gamma \dot{U}|Z|^{n}]\}/\eta \qquad [5.2]$$

where U is the displacement of the sytem; ξ is the damping ratio of the linear viscous damping element; ω_0 is the natural frequency of the system in the linearly elastic range; α is the post-yield to pre-yield stiffness ratio; Z is the BBW hysteretic restoring force model as proposed by Bouc (1967) and modified by Baber and Wen (1981,1979); f(t) is a zero mean Gaussian white noise forcing function per unit mass with a power spectral density of $2\pi D$; β, γ, and n are hysteresis shaped parameters and A, ν, η, are parameters that control the degradation of the model.

The degradation of the system is assumed to be a function of the dissipated hysteretic energy, that is

$$A = A_0 - \delta_A \epsilon \qquad [5.3\text{-a}]$$

$$\eta = \eta_0 + \delta_\eta \epsilon \qquad [5.3\text{-b}]$$

$$\nu = \nu_0 + \delta_\nu \epsilon \qquad [5.3\text{-c}]$$

where A_0, η_0 and ν_0 are the initial values of the corresponding variables; δ_A, δ_η and δ_ν are parameters controling the rate of degradation and ϵ is the dissipated energy which is calculated from the following equation:

$$\dot{\epsilon} = (1-\alpha)\ KZU\ dt \qquad [5.4]$$

The numerical values of the above parameters were chosen as:

$\xi = 0.0,\ 0.1$
$\omega_0 = 1.0$
$\alpha = 1./21.$
$D = 0.05,\ 0.1$
$A_0 = \nu_0 = \eta_0 = 1.0$
$\beta = \gamma = 0.5$
$n = 1$
$\delta_A = 0.0,\ 0.01,\ 0.02$
$\delta_\nu = 0.0,\ 0.01,\ 0.05$
$\delta_\eta = 0.0,\ 0.05,\ 0.1$

where the cases of $(\delta_A = \delta_\nu = \delta_\eta = 0.0)$, $(\delta_A = 0.01,\ \delta_\nu = 0.01,\ \delta_\eta = 0.05)$, and $(\delta_A = 0.02,\ \delta_\nu = 0.05,\ \delta_\eta = 0.1)$ are refered to as non-deteriorating, low-deteriorating and high-deteriorating cases respectively. Also, systems with $\xi = 0.0$ are referred to as undamped systems.

Equation [4.8] is used to derive 31 simultaneous first order differential equation for the second, third and fourth order moments of the response. First order moments are assumed to be zero. The resulting 31 moment equations - Equations [4.10], [4.13] and [4.14] - and the energy equation, Equation [5.4], are integrated simultaneously using an integration subroutine. The results obtained by

Ito's scheme are compared with Monte Carlo Simulation results, using 100 ensembles, and Equivalent Linearization results. In Chapter 6 it will be shown that equivalent linearization and Gaussian analysis using Ito approach result in identical predicted statistics.

5.4 The Response Statistics for the SDOF BBW System

Numerical studies for the response analysis of a SDOF BBW model were conducted for nine cases of degradation, two levels of damping, and two levels of excitations. The nine cases of degradation are:

Case #1	$\delta_A = 0.0$,	$\delta_\nu = 0.0$,	$\delta_\eta = 0.0$
Case #2	$\delta_A = 0.01$,	$\delta_\nu = 0.01$,	$\delta_\eta = 0.05$
Case #3	$\delta_A = 0.02$,	$\delta_\nu = 0.05$,	$\delta_\eta = 0.1$
Case #4	$\delta_A = 0.01$,	$\delta_\nu = 0.0$,	$\delta_\eta = 0.0$
Case #5	$\delta_A = 0.02$,	$\delta_\nu = 0.0$,	$\delta_\eta = 0.0$
Case #6	$\delta_A = 0.0$,	$\delta_\nu = 0.01$,	$\delta_\eta = 0.0$
Case #7	$\delta_A = 0.0$,	$\delta_\nu = 0.05$,	$\delta_\eta = 0.0$
Case #8	$\delta_A = 0.0$,	$\delta_\nu = 0.0$,	$\delta_\eta = 0.05$
Case #9	$\delta_A = 0.0$,	$\delta_\nu = 0.0$,	$\delta_\eta = 0.1$

The numerical studies were performed utilizing Equivalent Linearization (EL) and Ito approach (IA). The results of each technique were compared against Monte Carlo Simulation (MCS). Ito approach was used for both Gaussian (IG) and Non-Gaussian (ING) analysis. As it was observed in chapter 6, the results obtained by EL were identical to the results obtained by IG. This finding verified the theoretical argument on the identity of two approaches as postulated by

Wu and Lin (1984) and Orabi and Ahamidi (1987a). The contents of this section are the results of numerical studies for the mean dissipated hysteretic energy, and the second, third and fourth order moments of displacement, velocity, and the restoring force. Study on the third order moment has been limited to only one case.

5.4-1 RMS Displacement Response, Figures 5.1-5.18

As can be seen from displacement response figures, the response statistics predicted by the ING method are generally in good agreement with the simulated responses generated by MCS. The following observations are worth to be pointed out.

1) For high-deterioration rates, i.e. any of δ's>0.02 - see cases #3, #5 and #7 - when D = 0.1, ING slightly underestimates MCS. In case of undamped systems the results of ING and EL crossover each other at about 40 seconds.
2) In case of low and high degradation for η, cases #8 and #9, when D = 0.05 and ξ = 0.0 ING slightly overestimates the simulated response. However, the response predictions by ING are in better agreement with MCS than those obtained by EL.

In general, as can be seen from Figures 5.1 through 5.18 in all the studies for the RMS displacement analysis, EL underestimates the response while the proposed ING results are in very good agreement with simulation. As ξ increases the agreement between ING and simulation is improved. The best agreement between ING and MCS, for all cases of deteroration is observed when $D = 0.05$ and $\xi = 0.1$, except for case #8 and case #9; that is for $\xi = 0.0$ where the results for $D = 0.1$ show a better agreement with MCS.

5.4-2 RMS Velocity Response, Figures 5.19-5.36

The results for RMS velocity response for all levels of degradation and for undamped and damped systems show that while ING and MCS are in good agreement, EL always underestimates the MCS. Noting the following observations:

1) As the damping ratio increases the underestimation by the EL is slightly improved, however in general there is still a better agreement between ING and MCS.
2) For cases #3 and #7, when $\xi = 0.0$ and $D = 0.1$, the ING slightly underestimates the MCS results.

In all cases an increase in damping, ξ, or a decrease in excitation level, D, results in a decrease in the response level.

5.4-3 RMS of the Restoring Force, Figures 5.37-5.54

As can be seen from the restoring force figures, at all levels of excitation, for any degradation rate and for $\xi = 0$ or $\xi = 0.1$, the ING predictions slightly overestimate the simulated response while the linearization underestimates the MCS largley. The following exceptions for high degradation cases - cases #3, #5, #7, and #9 - can be noticed.

1) When $D = 0.05$ and $\xi = 0.1$, the ING and EL results are very close to each other while EL results seems to be closer to MCS results.
2) For case #9, when $\xi = 0.0$ and $D = 0.05$ or $D = 0.1$, the ING results agree well with the MCS while EL underestimates the MCS.

In all cases an increase in damping, ξ, results in a decrease in the response level, and also an increase in the excitation level, D, results in an increase in the response level.

5.4-4 Fourth Order Moment of the Displacement, Figures 5.55-5.72

The following observations were made concerning the fourth order moment of the displacement:

1) For constant damping if D increases $E(U^4)$ increases severly.
2) For constant excitation level if ξ increases $E(U^4)$ decreases severaly.

3) For non-deterorating systems, case #1, ING results are in good agreement with MCS except when ξ = 0.1 and D = 0.05 for which ING underestimates MCS.

4) For low and high deterioration rates systems, cases #2 through #9, ING slightly underestimates MCS results.

5) The increase of $E(U^4)$ for high deterioration when ξ = 0.0, is quite severe as compared to systems with low levels of degradation.

5.4-5 Fourth Order Moment of the Velocity, Figures 5.73-5.90

The ING method are shown to produce results overestimating the MCS with the following exceptions:

1) For non-deteriorating and low deterorating systems - cases #1, #2, #4, #6, and #8 - when D = 0.1 and ξ = 0.0, the ING results are in good agreement with MCS.

2) The overestimation of ING in case #9 for D = 0.05 is very small if ξ = 0.0 and ξ = 0.1.

3) For high deterioration -cases #3, #5, #7 and #9 - when ξ = 0.1 and D = 0.1, ING results are in good agreement with MCS.

Moreover an increase in damping value, or decrease in excitation level, results in a decrease in the response level.

5.4-6 Fourth Order Moment of the Restoring Force, Figures 5.91-5.108

The following points for the fourth order moment of the restoring force were observed:

1) The ING results always overestimate MCS.
2) The response increases if damping, ξ, decreases or excitation increases.
3) Discrepancy between ING and MCS decreases by increasing ξ or decreasing D, except for highly deteriorating systems - cases #3, #5 and #7. Noticeable exceptions are when D = 0.1 and ξ changes from zero to 0.1, and also when ξ = 0.0 and D changes from 0.1 to 0.05.

5.4-7 Mean of Dissipated Hysteretic Energy, Figures 5.109-5.126

Generaly, there is a good agrements between the mean of energy obtained by ING, EL and MCS for all cases of degradation, damping coefficents and excitation levels. There are two points that can be seen from Figures 5.109 to 5.126:

1) For $\xi = 0.0$, ING and EL results have almost coincided, slightly underestimating the MCS.

2) For $\xi = 0.1$, EL can predict the MCS results very closely while ING slightly underestimstes MCS.

It can also be observed that increasing damping, ξ, decreases the dissipated energy level through hysteretic action, and increasing the excitation level increases the dissipated hysteretic energy.

CHAPTER VI

SOME EXAMPLES

6.1 General

The materials presented in this chapter may not seem to be related to the chronology of the topics discussed in this thesis. However, there are two objectives for presenting these materials here. First, this discussion helps the reader in a better understanding of other approximation techniques for nonlinear random vibration in general, and other closure methods in particular. Second, this work represents the initial effort and investigation process that led to the formulation and development of the proposed approximation technique.

This chapter is divided into two parts. In the first part, the Non-Gaussian closure technique, developed by Crandall (1980), is applied to a single degree of freedom system with hyperbolic tangent restoring force. The probability density functions predicted by this technique are then compared with density functions constructed by exact solution via the FPK equation and statistical linearization. This part of the work was published at the early stage of this research in form of a journal paper (Noori, Saffar and Davoodi 1987) and several conference papers and proceedings (Davoodi and Noori, 1988a, 1988b, 1988c). In the second part, a comparative study is performed between the Equivalent Linearization and the closure method of Ibrahim and Soundararajan (1985) and Wu and Lin (1984). Gaussian response statistics are obtained for three nolinear systems: a Duffing oscillator; a system with a set-up spring and a general hysteretic system. It is shown that the moment equations obtained by the Itô-based closure technique of Ibrahim and Soundararajan and Wu-Lin are identical to the covariance equations derived from equivalent linearization. This latter part of the work has also been submitted for journal publication (Noori and Davoodi, 1989a) and has appeared in several conference proceedings and presentations (Noori and Davoodi, 1988a, 1988b, 1989b).

6.2 Application of Crandall's Non-Gaussian Closure Technique to a System with Tangent Hyperbolic Restoring Force

The nonlinear system to be studied herein is a single degree of freedom (SDOF) oscillator with a "softening spring" restoring force characteristics. Figure 6.1 is a schematic representation of this system. This type of nonlinear system has applications in the dynamics of package cushioning (Mindlin, 1945).

The governing differential equation of motion for the system shown in Figure 6.1 can be written as

$$M\ddot{U} + C\dot{U} + h(U) = F(t) \qquad [6.1]$$

in which M is considered here as unit mass, and F(t) is an ideal white noise random process with auto-correlation function

$$R_F(\tau) = 2W\ \delta(\tau) \qquad [6.2]$$

where 2W is the power spectral density. The nonlinear restoring force, h(U) has the form

$$h(U) = \lambda \tanh\ (KU/\lambda) \qquad [6.3]$$

where K and λ are parameteres controlling the rate of stiffness softening, and maximum limiting force respectively.

6.2-1 Non-Gaussian Approach

The Non-Gaussian closure approach, as proposed by Crandall (1980), is applied to the nonlinear system defined by Equations [6.1] and [6.3]. In order to evaluate response statistics the fol'owing procedure is considered. Both sides of Equation [6.1] are multiplied by a set of arbitary continuously differentiable functions $\phi(U)$. Taking the expected values of both sides of the resulting expression yields

$$E(\phi\ddot{U}) + (C/M)\ E(\phi\dot{U}) + (1/M)\ E(\phi h(U)) = (1/M)\ E(\phi F(t)) \qquad [6.4]$$

Because of the stationary response characterestic and following Crandall's physical argument, Equation [6.4] can be written as

$$E(\partial\phi/\partial U)\ E(\dot{U}^2) = (1/M)\ E(\phi h(U)) \qquad [6.5]$$

The expected value, $E(\dot{U}^2)$ can be evaluated by multiplying both sides of Equation [6.1] by $\dot{U}$ and taking expected values of both sides,

$$E(\dot{U}^2) = (1/C)\ E(UF(t)) \qquad [6.6]$$

where zero mean velocity is assumed. It can be further shown that (Crandall, 1980)

$$E(\dot{U}F(t)) = W/M \qquad [6.7]$$

thus

$$E(\dot{U}^2) = W/(CM) \qquad [6.8]$$

Substitution of $E(\dot{U}^2)$ from Equation [6.8] into Equation [6.5] results in

$$(W/C)\ E(\partial\phi/\partial U) = E(\phi h(U)) \qquad [6.9]$$

By using Equation [6.9] and selecting appropriate functions for $\phi(U)$ a wide range of relations among response statistics can be generated.

Evaluation of the expected values in Equation [6.9] requires a knowledge of the probability density function for the response. On the other hand, the same equation can be used in producing the relations between the moments of the probability density function. The assumed density function is approximated by a truncated Gram-Charlier expansion in the form

$$P(U) = \frac{1}{\sqrt{2\pi}\ \sigma} \exp[- \frac{(U-\nu)^2}{2\sigma^2}]$$

$$(1 + \sum_{n=3}^{N} (C_n/n!)\ H_n[(U-\nu)/\sigma]) \qquad [6.10]$$

where ν and σ are the mean and the standard deviation of the response, respectively, and $H_n(.)$ are Hermite polynomials defined as

$$H_n(Z) = (-1)^n \exp(\frac{Z^2}{2}) \frac{d^n}{dZ^n} \exp(- \frac{Z^2}{2}) \qquad [6.11]$$

The differentiation and recurrence formulas for the Hermite polynomial are

$$\frac{d}{dZ} H_n(Z) = n\, H_{n-1}(Z) \qquad [6.12]$$

$$H_{n+1}(Z) = ZH_n(Z) - nH_{n-1}(Z) \qquad [6.13]$$

The coefficients C_n in Equation [6.10] can be evaluated as

$$C_n = E(H_n[(U-\nu)/\sigma]) \quad n = 3, \ldots, N$$

where the following truncation is used

$$C_n = E(H_n[(U-\nu)/\sigma]) = 0 \quad n = N+1, N+2, \ldots$$

The first six coefficients are

$$C_0 = 1$$
$$C_1 = C_2 = 0$$
$$C_3 = \mu_3/\sigma^3$$
$$C_4 = (\mu_4/\sigma^4) - 3$$
$$C_5 = (\mu_5/\sigma^5) - 10\ (\mu_3/\sigma^3)$$
$$C_6 = (\mu_6/\sigma^6) - 15\ (\mu_4/\sigma^4) + 30$$

where

$$\mu_n = E((U-\nu)^n)$$

The density function of Equation [6.10] has N unknowns (ν, σ, C_3, C_4, ..., C_N). In order to find these unknowns, N constraints are needed. Through the selection of appropriate ϕ functions, Equation [6.9] is used to generate a set of simultaneous nonlinear equations. The calculations are somewhat simpler if Hermite polynomials are selected as $\phi(U)$ functions.

Since the nonlinear restoring force is an odd function and since the density function, as given by Equation [6.10], is an even function, all odd ordered moments are equal to zero. Substituting $\phi(U) = H_n(U/\sigma)$ into Equation [6.9] will result in

$$\frac{W}{C} \cdot \frac{n}{\sigma} \; E(H_{n-1}(U/\sigma)) = E(\phi h(U)) \qquad [6.14]$$

Setting N = 6 in Equation [6.10], and combining the outcome with that of Equation [6.14] results

$$H_n(X) = \frac{W\, n\, C_{n-1}}{C\, \sigma} \qquad [6.15]$$

where

$$X^j = [\lambda/(\sqrt{2\pi}\ \sigma^{j+1})] \sum_{i=1}^{4} A_{2(i-1)+j}\ D_i \qquad [6.16]$$

$$A_i = \int_{-\infty}^{+\infty} U^i \tanh(\frac{KU}{\lambda}) \exp(-\frac{U^2}{2\sigma^2})\ dU \qquad [6.17\text{-}a]$$

where

$$D_1 = 1 + 3 \frac{C_4}{4!} - 15 \frac{C_6}{6!} \qquad [6.18\text{-}a]$$

$$D_2 = 45 \frac{C_6}{6!\sigma^2} - 6 \frac{C_4}{4!\sigma^2} \qquad [6.18\text{-}b]$$

$$D_3 = \frac{C_4}{4!\sigma^4} - 15 \frac{C_6}{6!\sigma^4} \qquad [6.18\text{-}c]$$

$$D_4 = \frac{C_6}{6!\sigma^6} \qquad [6.18\text{-}d]$$

Denoting the fact that for an even i, A_i has a value of zero, one can construct a set of three independent simultaneous equations by setting n = 1, 3, 5 in Equation [6.15]. The enusing system can then be used to solve for the three unknowns σ, C_4 and C_6. In doing so, the complicated form of A_i requires the employment of a numerical technique. Since σ is one of the unknowns, one cannot evaluate A_i directly; a change of variables has to be performed.

The integrand in Equation [6.17-a] is an even function, thus allowing one to rewrite Equation [6.17-a] as

$$A_i = 2\int_0^{+\infty} U^i \tanh(KU/\lambda)\ \exp(-\frac{U^2}{2\sigma^2})\ dU \qquad [6.17\text{-b}]$$

In order to take σ out of this integral, Variable Y is defined as

$$Y = U/\sigma \qquad [6.19]$$

Since the argument of the hyperbolic tangent does not have a U/σ factor λ is defined as

$$\lambda = \lambda_1\sigma \qquad [6.20]$$

where σ is the unknown standard deviation and λ_1 is a parameter which controls σ and the maximum limiting force λ. With this change, Equation [6.17-b] is written as

$$A_i = 2\sigma^{n+1}\int_0^{+\infty} Y^i \tanh(KY/\lambda_1)\ \cdot$$

$$\exp(-\frac{Y^2}{2})\ dY \qquad [6.21\text{-a}]$$

or

$$A_i = 2\sigma^{n+1} B_i \qquad [6.21\text{-}b]$$

where

$$B_i = \int_0^{+\infty} Y^i \tanh(KY/\lambda_1) \exp\left(-\frac{Y^2}{2}\right) dY \qquad [6.21\text{-}c]$$

Equation [6.21-c] is evaluated numerically using Gauss-Laguerre quadrature (Stroud and Secrest, 1966).

Evaluation of λ_1 which is needed for calculation of B_i is performed through a fixed point iteration. Initially, a small positive value is assumed for λ_1. Using Equation [6.15], σ, C_4, and C_6 are evaluated based on this current value of λ_1. A new value for λ_1 is obtained by substituting the calculated value of σ into Equation [6.20]. This approach continues until the difference between the new value of λ_1 and the previous one is negligible. Calculation of σ, C_4, and C_6 completes the analysis by this technique and defines the probability density function given by Equation [6.10].

6.2-2 Statistical Linearization

In order to make a comparison between response statistics provided by Crandall's Non-Gaussian closure approach and those recorded from statistical linearization, the following analysis is performed (Lin, 1967). The nonlinear system of Equation [6.1] is replaced by an equivalent linear system

$$M\ddot{U} + C\dot{U} + K_e U = F(t) \qquad [6.22]$$

The coefficient, K_e, of the linear system is found by minimizing the error between Equations [6.1] and [6.22] in the mean square sense. The excitatioin F(t) is assumed to be zero mean stationary Gaussian white noise with autocorrelation function as given by Equation [6.3]. The standard deviation of the response is given as (Clough and Penzien, 1975)

$$\sigma_U{}^2 = W/(CK_e) \qquad [6.23]$$

Detailed derivation of K_e is given in Appendix C. Since the response is Gaussian, Equation [6.23] is sufficient for obtaining the probabilty density function of the response.

6.2-3 Fokker-Plank-Kolmogorov Equation

The response statistics by Non-Gaussian closure and statistical linearization are both compared with the exact solution via the FPK equation. The probability density function, using FPK formulatioin, can be derived following Caughey's approach (Caughey, 1963a). For the nonlinear system of Equation [6.1], the density function of the response is given by

$$P(U) = \rho \exp \left[- \frac{C}{W} \int_0^U h(y) \right] dy \qquad [6.24]$$

where in this case $h(y) = \lambda \tanh(Ky/\lambda)$, and ρ is a constant parameter. Substitution of h(y) into Equation [6.24] results in

$$P(U) = \eta[\cosh(KU/\lambda)]^{-\alpha} \qquad [6.25]$$

in which

$$\alpha = \frac{C\lambda^2}{WK} \qquad [6.26]$$

and η is a normalizing constant.

6.2-4 Numerical Studies

Two cases of low and high nonlinearity were considered for these studies. For both cases the maximum restoring force was limited to 1.0. For the case of low nonlinearity a spring constant parameter of K = 0.1, input power spectral density of PSD = 0.5, and a damping coefficient of C = 0.5 were chosen. The resulting density functions presented in Figure 6.2 show a very close agreement between both NGC and SL techniques and the FPK solution. The SL results show, however, a slightly closer agreement with the FPK at P(0). Construction of the density function for the SL case has been based on Gaussian distribution assumption. In order to make a more thorough comparison between the two approximation techniques for this case two additional studies were performed. In the first study, the effect of the change in the spring constant was considered. Figure 6.3 shows the results of this investigation. For the value of spring constant parameter in the range of K = 0.1-10.0, and the damping kept at a constant value of 0.5, the two approximate techniques show the same trend in predicting the RMS of system displacement response. The SL results however, underestimate the response whereas the NGC is in good agreement with the FPK. Beyond K = 10.0, where the nonlinearity increases sharply, the FPK solution shows an increase in σ_U. For this case both NGC and SL results underestimate the FPK solution. Figure 6.4 demonstrates the predicted response vs the variation of damping in the system. Both NGC and SL are in good agreement with FPK solution.

For the case of high nonlinearity, spring constant parameter was K = 10.0, with input PSD of 0.1, and damping coefficient of 0.05. The corresponding density functions, as shown in Figure 6.5, exhibit deviations of up to 31%, with NGC technique providing a more precise solution than the one obtained from SL. An interesting phenomenon observed in this investigation is the presence of oscillation in the NGC solution. This behavior is not present in the FPK or SL solution. Figure 6.6 shows the effect of variation of the spring constant on the RMS response predicted by the approximate techniques. In this study a constant damping of 0.05 was considered. For the spring constant changing from K =0.1 to 7, the two techniques show close agreement with FPK solution. SL results slightly underestimate the response. When the spring constant increases beyond K = 7, there is a sharp disagreement between FPK and the two approximate results. A similar study was done for the RMS response vs variation of system damping. When the damping is very low, between 0.0 and 0.03, there is no good agreement between the results. This may stem from the fact that since the system is slightly damped, the stationarity is unlikely. However, for damping value greater than 0.03 both NGC and SL agree very well with FPK results. These results are shown in Figure 6.7.

6.3 A Comparative Study Between Equivalent Linearization and Ito-Gaussian Closure Technique

In this section, method of Equivalent Linearization as extended by Atalik and Utku, Spanos and others (Section 3.2-4) and the Cumulant-Neglect closure scheme independently developed by Ibrahim and soundararajan, and Wu and Lin (Section 3.2-6) are utilized for the random vibration analysis of three nonlinear systems: A Duffing oscillator; A system with a set-up spring, and the general hysteretic system of BBW. It is shown that the differntial equations for the moments derived by this closure scheme are identical to the covariance matrix equations obtained from Equivalent Linearization. This comparison is made both analytically and through numerical studies. Comparison is also made with Monte Carlo simulation, available exact solutions, and other available closure techniques (Noori and Davoodi, 1989a).

6.3-1 Duffing Oscillator

The first case for illustrating the proposed work is the following Duffing oscillator considered by Crandall (1980)

$$\ddot{U} + \eta\dot{U} + (U + \epsilon U^3) = \sqrt{\eta}\, f(t) \qquad [6.27]$$

where f(t) is a Gaussain white noise with autocorrelation function of $R_{ff}(\tau) = 2\delta(t)$ and η is a damping factor. By the coordinate transformation $Y_1 = U$ and $Y_2 = \dot{U}$, Equation [6.27] can be written as

$$\dot{Y}_1 = Y_2 \qquad [6.28\text{-a}]$$

$$\dot{Y}_2 = -\eta Y_2 - Y_1 - \epsilon Y_1{}^3 + \sqrt{\eta}\, f(t) \qquad [6.28\text{-b}]$$

By choosing a function $\Phi(Y) = (Y_1{}^i\ Y_2{}^j)$ the moment equation, Equation [3.26], can be derived as

$$\dot{M}_{i,j} = iM_{i-1,j+1} - \eta jM_{i,j} - jM_{i+1,j-1} - j\epsilon M_{i+3,j-1} + \eta j(j-1)M_{i,j-2} \qquad [6.29]$$

where $M_{i,j} = E(Y_1{}^i\ Y_2{}^j)$; $i+j=2$. Neglecting the fourth order cumulant, Equation [6.29] results in the following set of differential equations for the second order moments

$$\dot{M}_{20} = 2M_{11}$$
$$\dot{M}_{11} = -M_{20} - 3\epsilon M_{20}{}^2 - \eta M_{11} + M_{02}$$
$$\dot{M}_{02} = -6\epsilon M_{20}M_{11} - 2M_{11} - 2\eta M_{02} + 2\eta M_{01} \qquad [6.30]$$

In order to derive the corresponding covariance matrix equation for the linearized system, Equation [3.13-c] is utilized where $g(.) = U + \epsilon U_3$. The state vector equation for the system, Equation [3.14], can be written in the form

$$\begin{aligned} \dot{Y}_1 &= Y_2 \\ \dot{Y}_2 &= -\eta Y_2 - K_e Y_1 + \sqrt{\eta}\, f(t) \end{aligned} \qquad [6.31]$$

where $K_e = (\partial/\partial Y_1)\ E(Y_1 + \epsilon Y_1^3) = 1 + 3\epsilon E(Y_1^2)$. The covaraince matrix equation can be easily derived as

$$\begin{aligned} \dot{S}_1 &= 2S_2 \\ \dot{S}_2 &= S_1 + 3\epsilon S_1^2 - \eta S_2 + S_3 \\ \dot{S}_3 &= -2S_2 - 6\epsilon S_1 S_2 - 2\eta S_3 + 2\sqrt{\eta} \end{aligned} \qquad [6.32]$$

where $S_n = M_{i,j} = E(Y_1^i Y_2^j)$. As can be seen Equations [6.30] and [6.32] are identical and this completes the formulation. Figures 6.8 and 6.9 present the results for the RMS displacement of this oscillator as obtained by the numerical solution of Equations [6.30] (Itô Gaussian), and by Equations [6.32] (linearization) for two cases of low and high nonlinearities. These results have also been compared with the exact solution and Non-Gaussian results derived by Crandall (1980). As expected, the Itô-Gaussian and linearization results are identical.

6.3-2 A SYSTEM WITH A SET-UP SPRING

The second example studied is a nonlinear system with a set-up spring. This system has been studied earlier by Crandall (1962) using a statistical linearization approach.

The equation of motion for the system is given as

$$\ddot{U} + 2\xi\omega_0\dot{U} + \omega_0{}^2(U + \epsilon\,\mathrm{sgn}\,U) = W(t) \qquad [6.33]$$

where W(t) is a Gaussian white noise. A coordinate transformation $Y_1 = U$ and $Y_2 = \dot{U}$ results

$$\begin{aligned} \dot{Y}_1 &= Y_2 \\ \dot{Y}_2 &= -K_e Y_1 - 2\xi\omega_0 Y_2 + W(t) \end{aligned} \qquad [6.34]$$

where K_e is the equivalent linearized coefficient given by

$$\begin{aligned} K_e &= (\partial/\partial Y_1)\ E(\omega_0{}^2(Y_1 + \epsilon\ \mathrm{sgn}\ Y_1)) \\ &= \omega_0{}^2[1+2\epsilon/(\sqrt{2\pi}\ \sigma_{Y1})] \end{aligned} \qquad [6.35]$$

As obtained by applying Equation [3.13-c]. The corresponding covariance matrix can be easily derived yielding

$$\begin{aligned} \dot{S}_1 &= 2S_2 \\ \dot{S}_2 &= -K_e S_1 - 2\xi\omega_0 S_2 + S_3 \end{aligned} \qquad [6.36]$$

$$\dot{S}_3 = -2K_e S_2 - 4\xi\omega_0 S_3$$

where $S_n = E(Y_1^{\,i}\ Y_2^{\,j})$; $i+j = 2$.

By choosing similar $\Phi(.)$ functions as used for the first case, the moment differential equations can be derived. This is done by applying Equation [3.26] to the state-space vector form of the equation of motion resulting

$$\begin{aligned}
\dot{M}_{20} &= 2M_{11} \\
\dot{M}_{11} &= M_{02} - 2\xi\omega_0 M_{11} - \omega_0^2 M_{20} - \omega_0^2 \epsilon E(Y_1 \mathrm{sgn}(Y_1)) \qquad [6.37] \\
\dot{M}_{02} &= \sigma^2 - 4\xi\omega_0 M_{02} - 2\omega_0^2 M_{11} - 2\omega_0^2 \epsilon E(\mathrm{sgn}(Y_1 Y_2))
\end{aligned}$$

where $M_{ij} = S_n$ and σ^2 is the power spectral density of the input Gaussian white noise. The two expected values in these equations are evaluated as $E(Y_1 \mathrm{sgn}(Y_1)) = 2M_{20}/(2\pi)^{\frac{1}{2}}$ and $E(\mathrm{sgn}(Y_1 Y_2)) = 2M_{11}/[(2\pi)^{\frac{1}{2}}\ \sigma_{Y1}]$. As can be seen the two Equations [6.36] and [6.37] are identical in form. Corresponding numerical results for the RMS of displacement are presented in Figures 6.10 and 6.11 for two cases of low and high nonlinearities and are compared with the available exact solution and the statistical linearization (Crandall, 1962).

6.3-3 A General Hysteretic System

The third example studied is a nonlinear SDOF system with BBW hysteretic restoring force element. This is the system that was studied in this thesis. The equation of motion for the system is given by Equation [4.3] and is given here again

$$\ddot{U} + 2\xi\omega_0\dot{U} + \alpha\omega_0^2 U + (1 - \alpha)\omega_0^2 Z = f(t) \qquad [6.38]$$

where the hysteretic restoring force z is described by Equation [4.4] and here as

$$\dot{Z} = (A\dot{U} - \nu[\beta|\dot{U}||z|^{n-1}z + \gamma\dot{U}|z|^{n}])/\eta \qquad [6.39]$$

in which β, γ, and n are shape parameters; A is the tangent stiffness and ν and η are deterioration parameters. Detailed study of this model is reported elsewhere (Baber and Wen, 1981). The equivalent linearization of this model has been studied before (Baber and Wen, 1981). The moment equations for the Gaussian case are derived as given by Equation [4.10] and are given here again

$$\dot{M}_{200} = 2M_{110}$$
$$\dot{M}_{110} = M_{020} - 2\xi\omega_0 M_{110} - \alpha\omega_0^2 M_{200} - (1-\alpha)\omega_0^2 M_{101}$$
$$\dot{M}_{101} = M_{011} + AM_{110} - \beta E(|Y_2|Y_1Y_3) - \gamma E(|Y_3|Y_1Y_2)$$
$$\dot{M}_{020} = \sigma^2 - 4\xi\omega_0 M_{020} - 2\alpha\omega_0^2 M_{110} - 2(1-\alpha)\omega_0^2 M_{011}$$

$$\dot{M}_{011} = -2\xi\omega_0 M_{011} - \alpha\omega_0^2 M_{101} - (1-\alpha)\omega_0^2 M_{002} + AM_{020}$$
$$-\beta E(|Y_2|Y_2Y_3) - \gamma E(|Y_3|Y_2^2)$$
$$\dot{M}_{002} = 2AM_{011} - 2\beta E(|Y_2|Y_3^2) - 2\gamma E(|Y_3|Y_2Y_3) \qquad [6.40]$$

where $M_{i,j,k} = E(Y_1^i\ Y_2^j\ Y_3^k)$; $i+j+k=2$ and σ^2 is the PSD of the input Gaussian white noise. The corresponding covariance matrix for the Linearized system is

$$\dot{S}_1 = -2S_2$$
$$\dot{S}_2 = \alpha.\omega_0^2 S_1 + 2\xi.\omega_0 S_2 + (1-\alpha)\omega_0^2 S_3 - S_4$$
$$\dot{S}_3 = -C_e S_2 - K_e S_3 - S_5$$
$$\dot{S}_4 = 2\alpha\omega_0^2 S_2 + 4\xi\omega_0 S_4 + 2(1-\alpha)\omega_0^2 S_5 + \sigma^2$$
$$\dot{S}_5 = \alpha\omega^2 S_3 - C_e S_4 + (2\xi\omega - K_e)S_5 + (1-\alpha)\omega^2 S_6$$
$$\dot{S}_6 = -2C_e S_5 - 2K_e S_6 \qquad [6.41]$$

where $S_i = M_{i,j,k} = E(y_1^i\ y_2^j\ y_3^k)$; $i+j+k=2$, and

$$K_e = -\nu[\beta E(|U|(\partial/\partial Z)(|Z|^{n-1}Z)) + \gamma E(U(\partial/\partial Z)|Z|^n)]/\eta$$
$$C_e = [A-\nu(\beta E(|Z|^{n-1}Z(\partial|U|/\partial U)) + \gamma E(|Z|^n))]/\eta \qquad [6.42]$$

it can be shown that Equations [6.41] and [6.40] are identical.

Figure 6.12 and 6.13 represent the RMS displacement and velocity respectively. These statistics have been obtained by solving the moment equations, Equation [6.40], and covariance equations, Equation. [6.41]. The system studied is a nondeteriorating smooth hysteresis given by A=1, $\beta=\gamma=0.5$, $\alpha=1/21$, n=1, $\nu=1$, $\xi=0$, and $\omega_0=1.0$. This model has been previously studied via Fokker Plank equation (Wen, 1976) and by other Gaussian closure techniques (Iyengar and Dash, 1978). A comparison is made between the Itô Gaussian and equivalent linearization results and the digitized Monte Carlo simulation for a wide range of input power spectral densities. This study verifies that the two techniques lead to identical results.

CHAPTER VII

SUMMARY, CONCLUSION AND RECOMMENDATIONS

7.1 General

The main objective of this research was to develop an approximation technique for random vibration analysis of nonlinear systems with general hysteretic behavior. This solution technique had to be capable of representing the Non-Gaussian effect of the response. This goal was accomplished by extending the Cumulant-Neglect closure method of Wu-Lin (1984) and Ibrahim and Soundarajan (1985) to include this form of nonlinearities. In the proposed approach, first a joint density function for the response of the system is assumed, and then using Ito-differential approach a set of first order differential equations for the system moments are derived. Because of the nonlinearity in the BBW hysteresis model and other general hysteresis, the resulting moment equations are coupled with higher order moments. In order to close the moment equations, the basic mathematical defination of moments was used and the higher order moments were obtained in terms of the lower order ones. The approach presented in this thesis can be used for both Gaussian and Non-Gaussian response analysis of nonlinear systems. This chapter contains a summary of previous chapters, and also the conclusions and suggestions that are resulted from this study.

7.2 Summary

Chapter 2 contains a brief discussion on the developement of general hysteresis models, and in specific the history and the mathematical formulation of the so-called smooth transitional model. The models that are discussed include: Bilinear model, Elasto-Plastic model, Smooth hysteresis models, degrading hysteresis models and pinching models. The mathematical formulation of the smooth hysteresis model is based on the model originally introduced by Bouc (1967) and later was generalized by Wen (1976) and Baber and Wen (1979). The degradation model for the smooth hysteresis is based on hysteretic energy dissipation developed by Baber and Wen (1981).

In Chapter 3 a brief review of approximate solution techniques and different closure methods for nonlinear random vibration problems are presented. The solution techniques discussed are: Fokker-Plank-Kolmogorov formulation, Monte Carlo Simulation, Pertubation, Equivalent Linearization, Wiener-Hermite Expansion, Gaussian and Non-Gaussian closure, Central Difference, Stochastic Equivalent Systems, and Stochastic Averaging method. Chapter 3 also contains the development of a Non-Gaussian solution method which was proposed by the author for the random vibration analysis of hysteretic systems.

In Chapter 4 the application of the proposed solution technique to a BBW hysteresis model is demonstrated. The moment equations for the response of a SDOF system subjected to zero mean stationary white noise excitation are derived by assuming a 3 dimensional joint density function for the response.

Chapter 5 contains the numerical studies for the random vibration of the BBW model using this proposed technique. A wide variaty of parameters were considered. The Non-Gaussian results obtained by the proposed technique are compared with Equivalent Lineraization via Monte Carlo Simulation. Higher order moment of response coordinate for this system are for the firt time evaluated by the new approach and are presented.

In Chapter 6 a comparative study was performed for different approximate solution techniques. First, the Non-Gaussian technique developed by Crandall (1980) is used to analyze a system with tangent hyperbolic restoring force. Then, through studing three different nonlinear systems, it is shown that the Equivalent Linearization results are identical to the results obtained by Ito-differential approach assuming Gaussian response.

7.3 Conclusion

This study has resulted in a number of conclusions regrading to the Non-Gaussian Ito technique, and they are as follows:

1) The results from this technique are in good agreement with Monte Carlo simulation.
2) In general the Non-Gaussian Ito approach predicts the second order moments of the response better than Equivalent Linearization.
3) This technique can be applied to any nonlinear system, but the major advantage is when the nonlinearity in the system is of non-polynomial type or when it has discontinuty.
4) Due to the highly nonlinear nature of the smooth hysteresis model the exact solution of systems having this type of nonlinearity is very difficult, if not impossible. Equivalent Linearization has been the most widely used approximation technique for analyzing systems with hysteretic nonlinearity. The advandage of this Ito Non-Gaussain approach over Equivalent Linearization is that it not only does provide more accurate results for second order moments of the response but it can also provide the

information on higher order moments. Therefore, the density function obtained by Ito Non-Gaussian is more accurate than the density function obtained by Equivalent Linearization.

5) When the number of unknown moments are greater than the number of avialable simultaneous equations, it is easy to implement this approach with Equivalent Linearization.

6) The computer run time for this solution technique, assuming Non-Gaussian response, is much higher than Equivalent Linearization method, while it is the same if the response is assumed to be Gaussian.

7) The solution technique presented in this work is for systems subjected to stationary zero mean excitation, while Equivalent Linearization can be used for both zero and non-zero mean excitations.

8) This technique becomes very cumbersome as the number of independent system variables increase. This is the case when more complicated hysteresis such as Baber-Noori or Noori-Baber slip-lock models are considered.

7.4 Suggestions and Recommendations

The numerical studies, as reported in Chapter 5, clearly indicate that the proposed solution technique of Chapter 3 is capable of accurately estimating the response moments of nonlinear hysteretic systems. Although good agreements were found for moments of up to the fourth order, there still exist further areas to be investigated:

1) the applicability of the proposed solution technique to multi degree of freedom systems,
2) extension of the present solution technique to the cases of non-zero mean input excitation,
3) studying the effect of higher order moments on the response level of hysteretic systems,
4) using a different index for degradation of the system instead of hysteretic energy dissipation. An alternative is the maximum deformation in each cycle during loading as a measure of degradation for the next cycle (Sues, Wen and Ang, 1983.)
5) Since this technique can provide better information on the probability density function for the response coordinates, it is strongly adviseable that the first passage problem for hysteretic systems to be restudied using this approach.
6) The Non-Gaussian statistics obtained by the proposed technique also provide a ground for reliability and risk analysis of hysteretic structures.
7) In a number of cases the numerical studies indicate that the dissipated energy prediction, by the Gaussian analysis, is in a better agreement with simulation than the Non-Gaussian results. In the

same studies, a cross-over behavior is also observed between Gaussian and Non-Gaussian response with the Gaussian results being in better agreement with MCS after the cross-over. One source of this problem is suspected to be the fact that the Non-Gausian density function in the vicinity of the cross-over becomes unrealizeable, i.e. becomes negative. Therefore, it is suggested that other possible forms of expansions to be considered for the assumed density function such that the density function stays realizeable at all times.

8) The numerical values for the parameters that were used for describing the hysteretic behavior were chosen arbitarily. In order to model the response behavior of real structures, it is necessary to obtain the appropriate values for the hysteresis shape parameters. There are many system identification procedures avialable (Distefano and Rath, 1974; Distefano and Pena-Pardo, 1976; Yar and Hammond, 1987; and Hoshiya and Maruyama, 1987.) This is also another tak that can be investigated.

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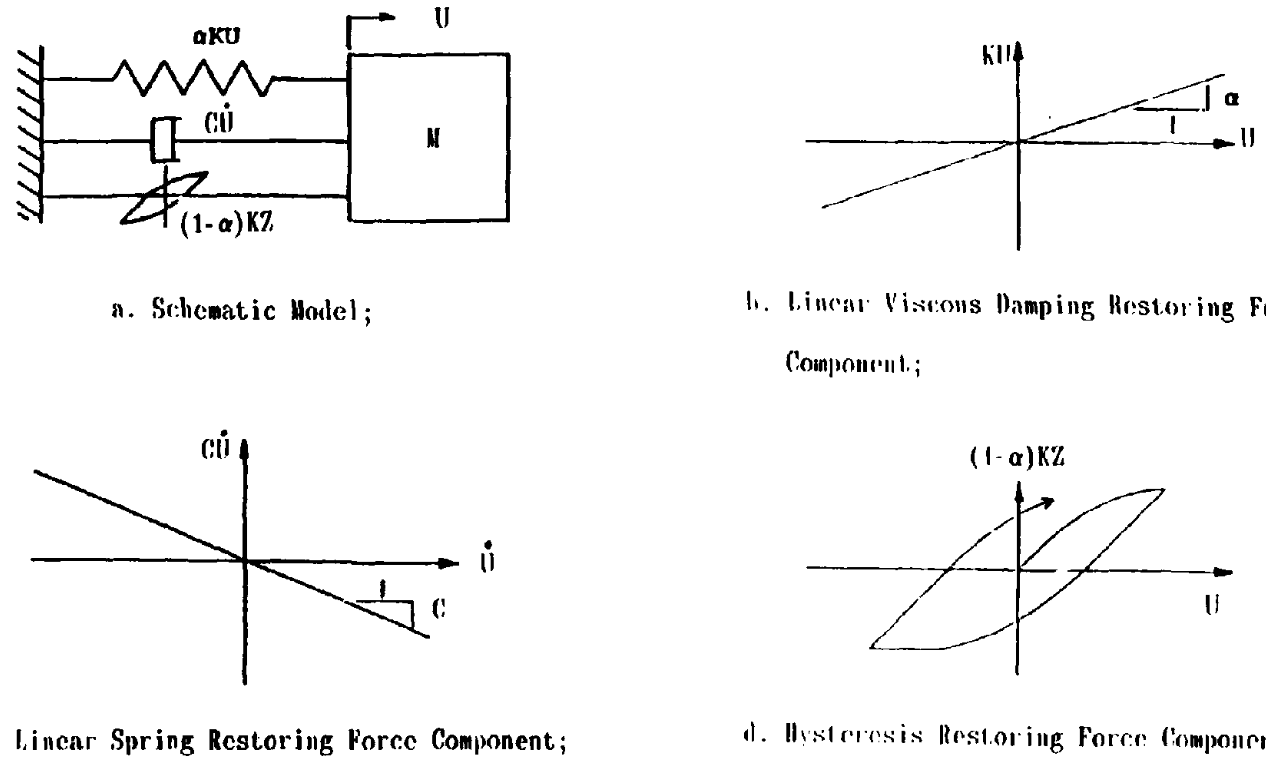
FIGURES



a. Schematic Model;

b. Linear Viscous Damping Restoring Force Component;

c. Linear Spring Restoring Force Component;

d. Hysteresis Restoring Force Component;

Fig. 2.1- The Nonlinear SDOF Model (After Baber & Wen; 1979).

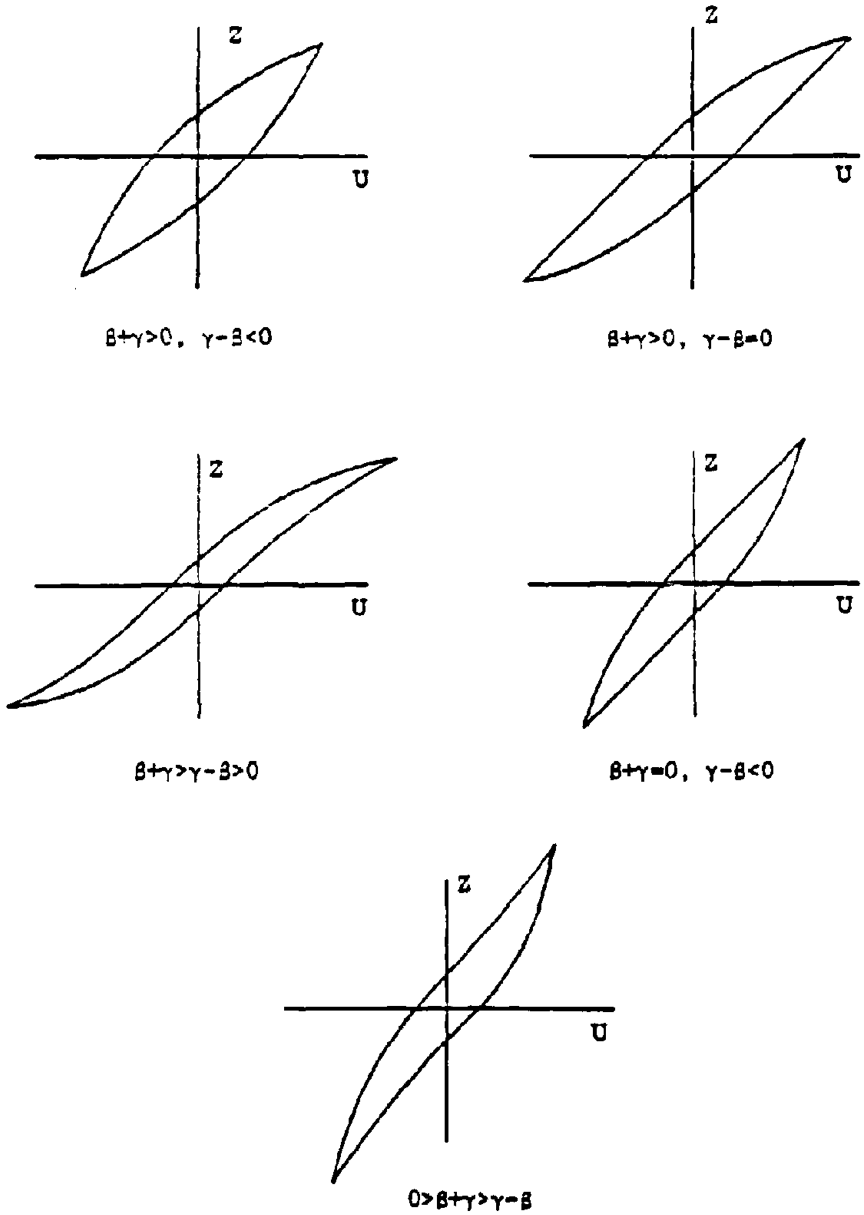


Fig. 2.2- Hysteretic Restoring Force for $n=1$ & different Combinations of β and γ (After Baber & Wen; 1979).

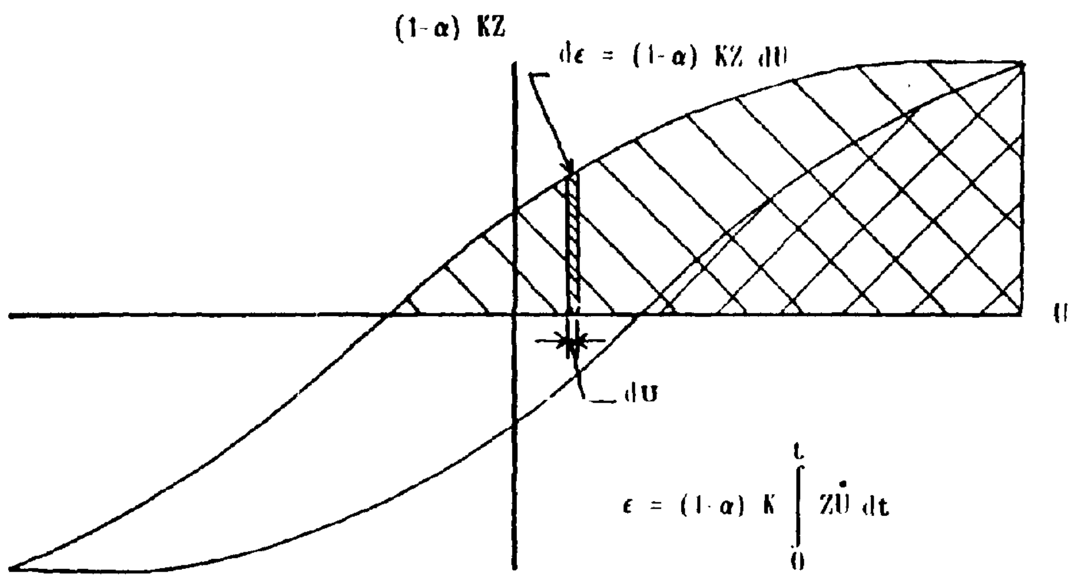


Fig. 2.3- Energy Dissipation Through Hysteresis Loop
(After Baber & Wen; 1979).

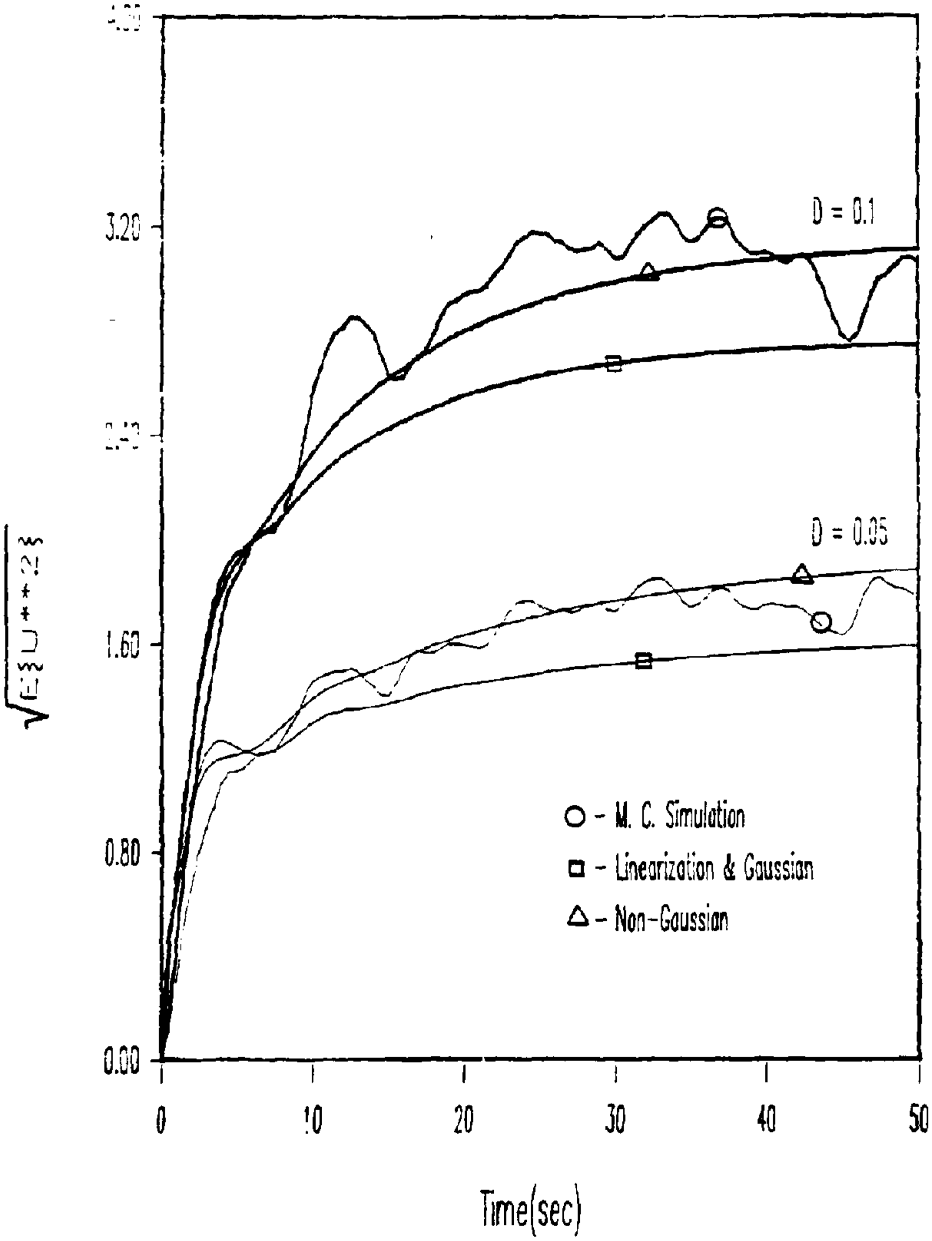


Fig. 5.1- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

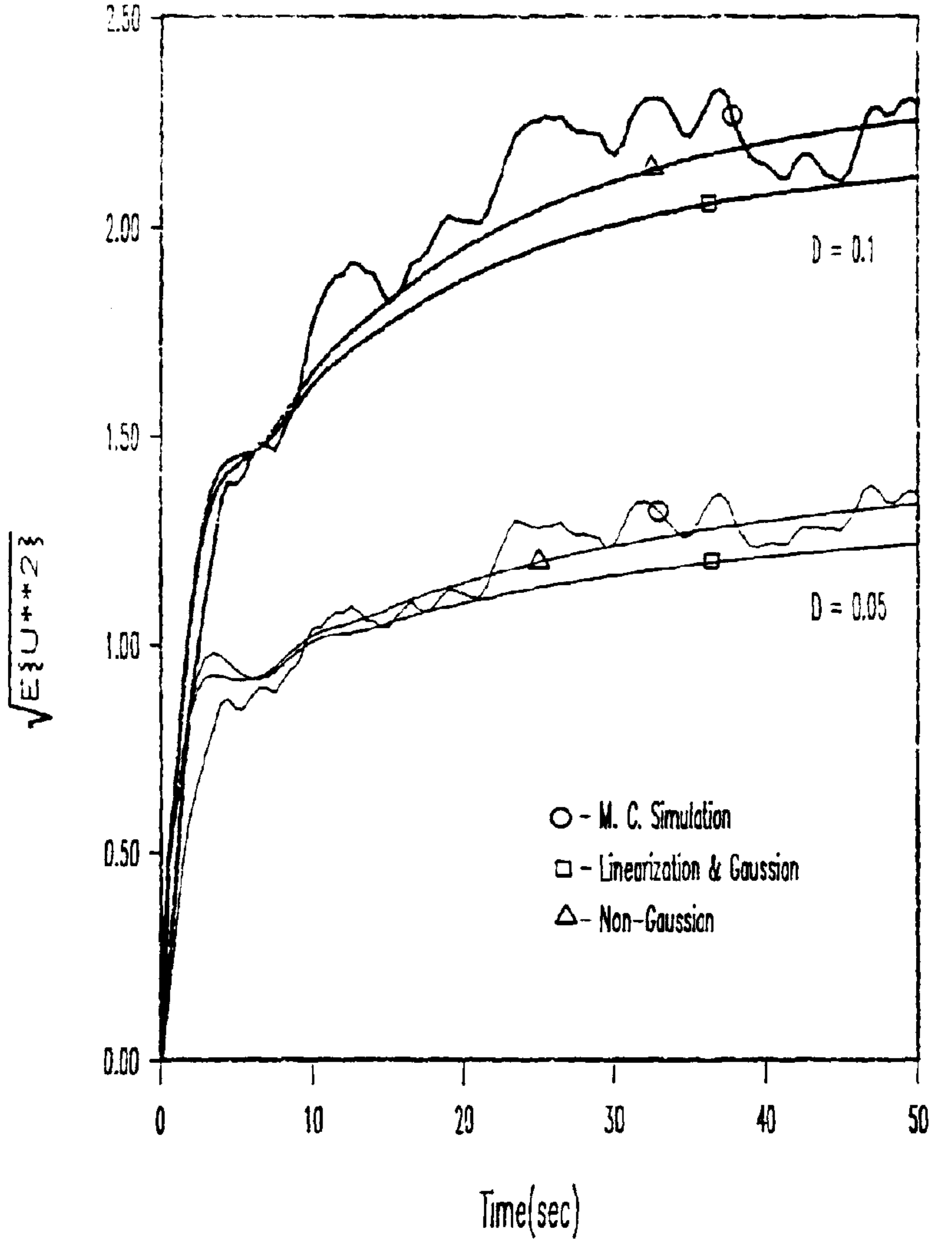


Fig. 5.2- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

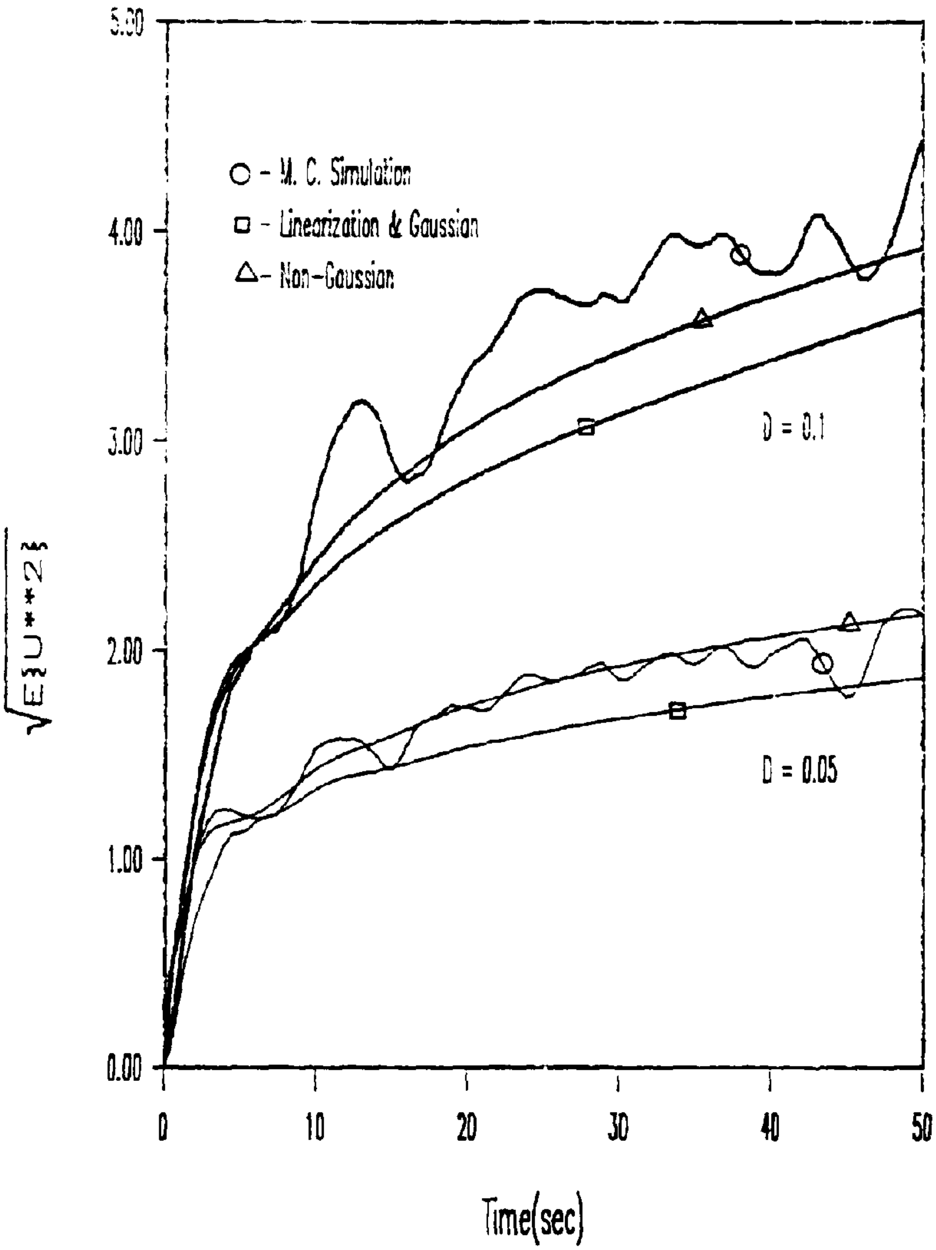


Fig. 5.3- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.0$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

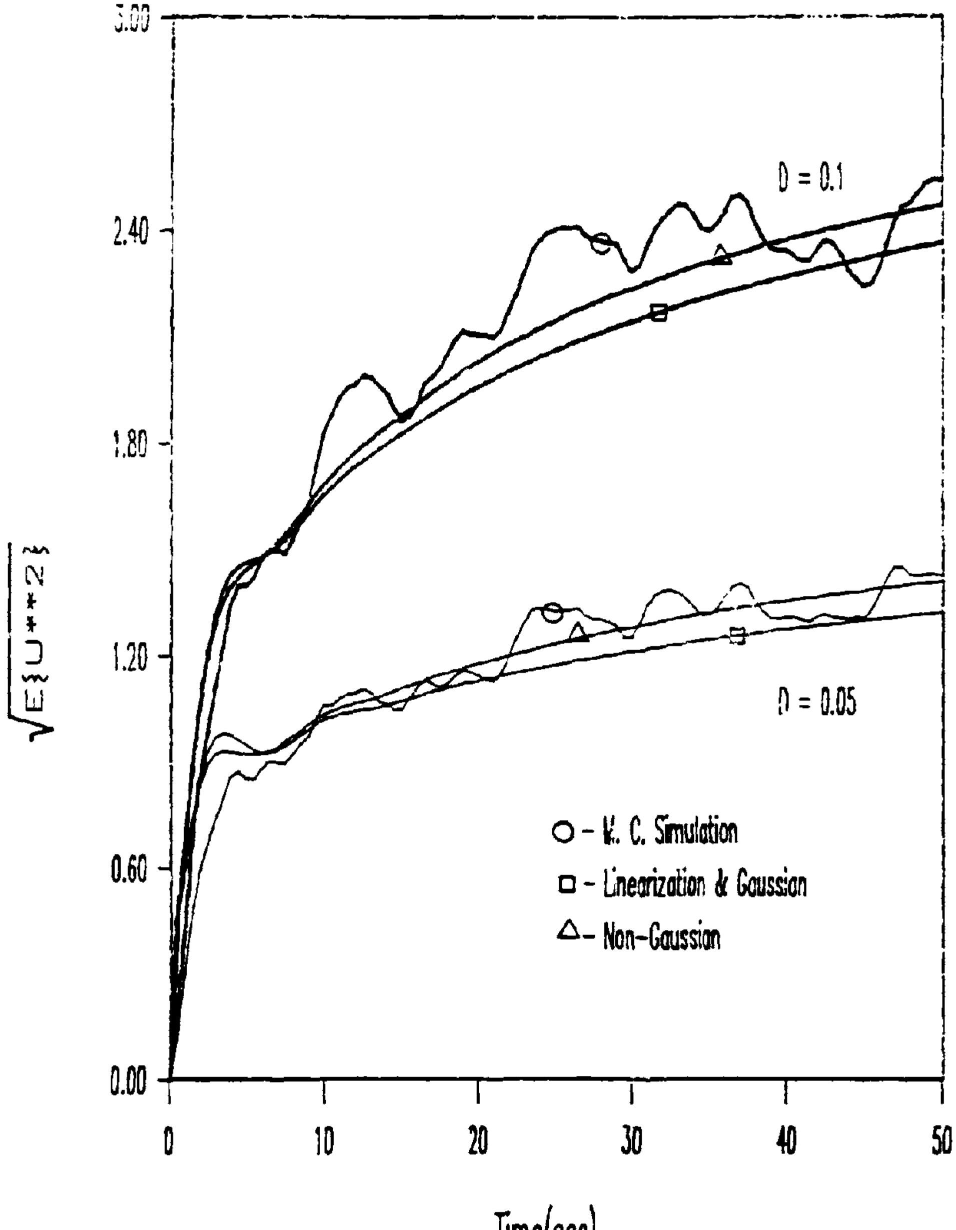


Fig. 5.4- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

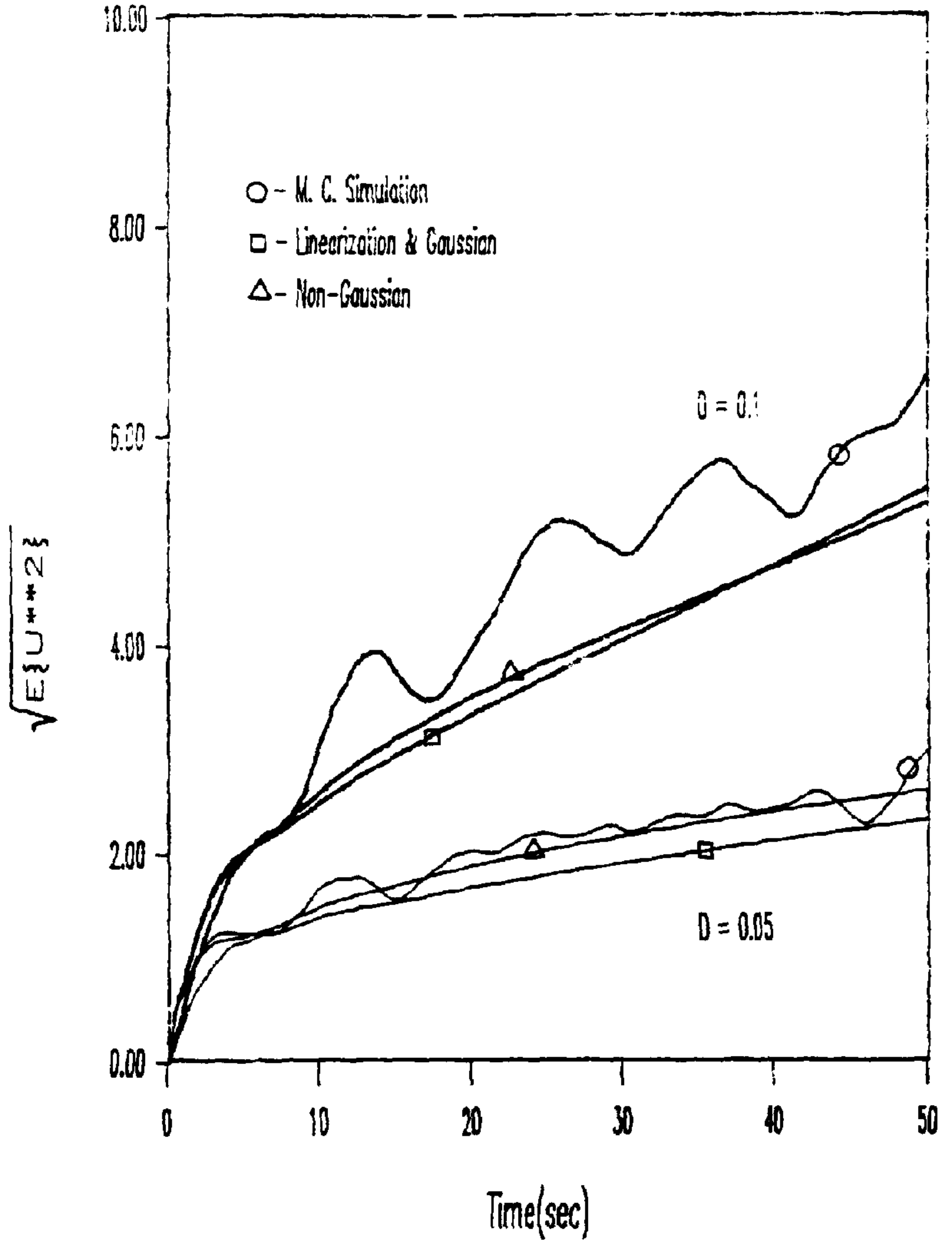


Fig. 5.5- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.0,\ \delta_A = 0.02,\ \delta_\nu = 0.05,\ \delta_\eta = 0.1.$

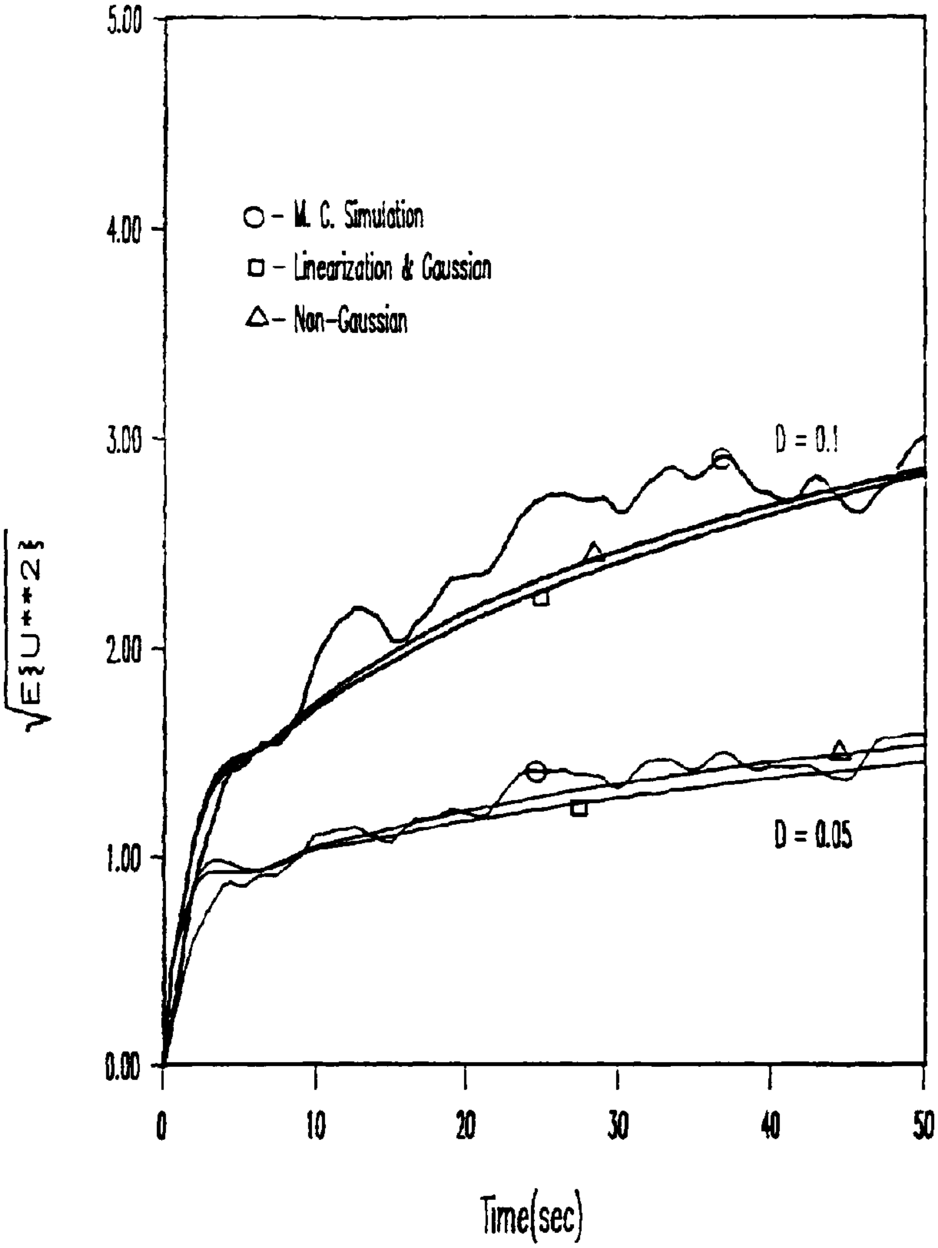


Fig. 5.6- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.1,\ \delta_A = 0.02,\ \delta_\nu = 0.05,\ \delta_\eta = 0.1.$

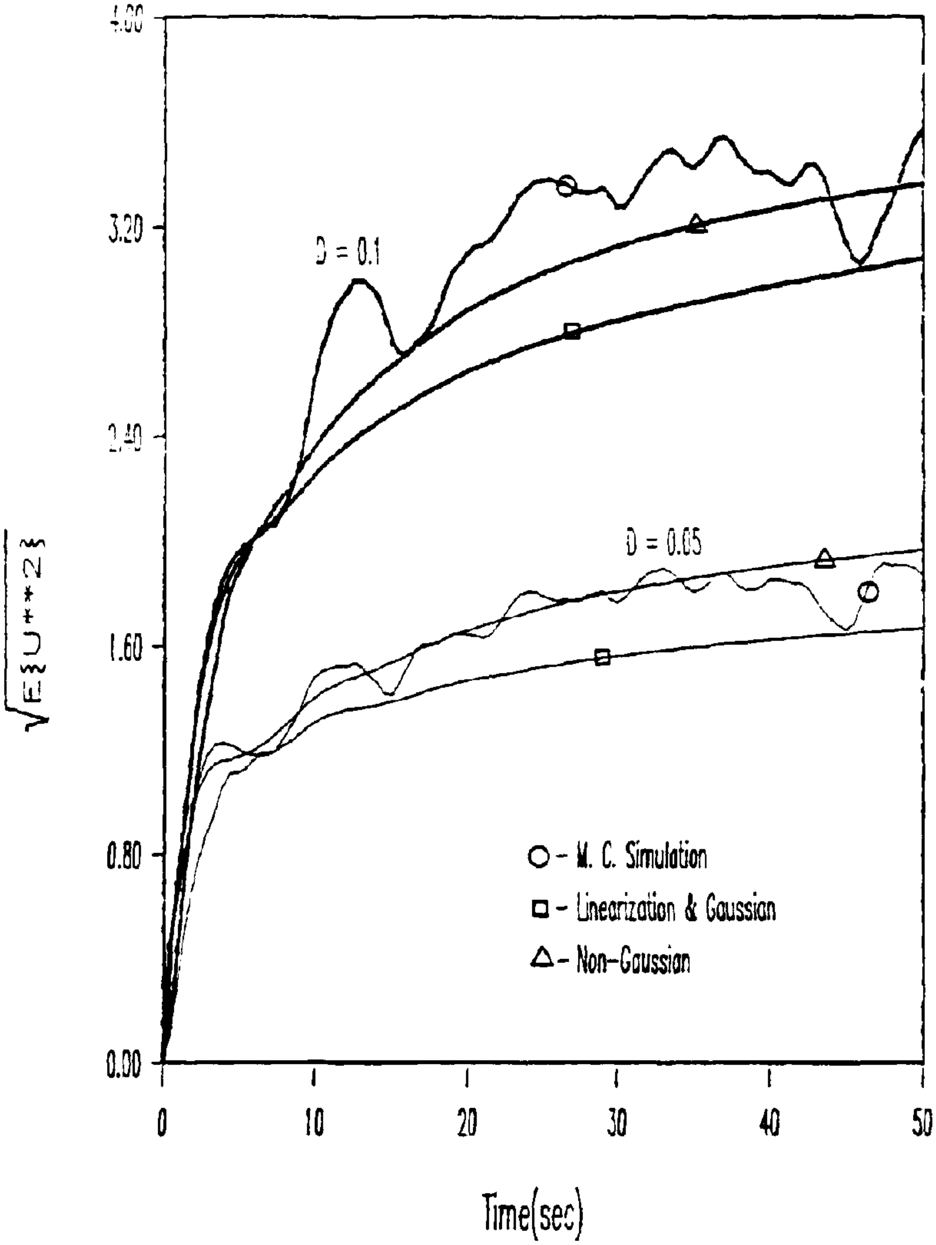


Fig. 5.7- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.0$, $\bar{\sigma}_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

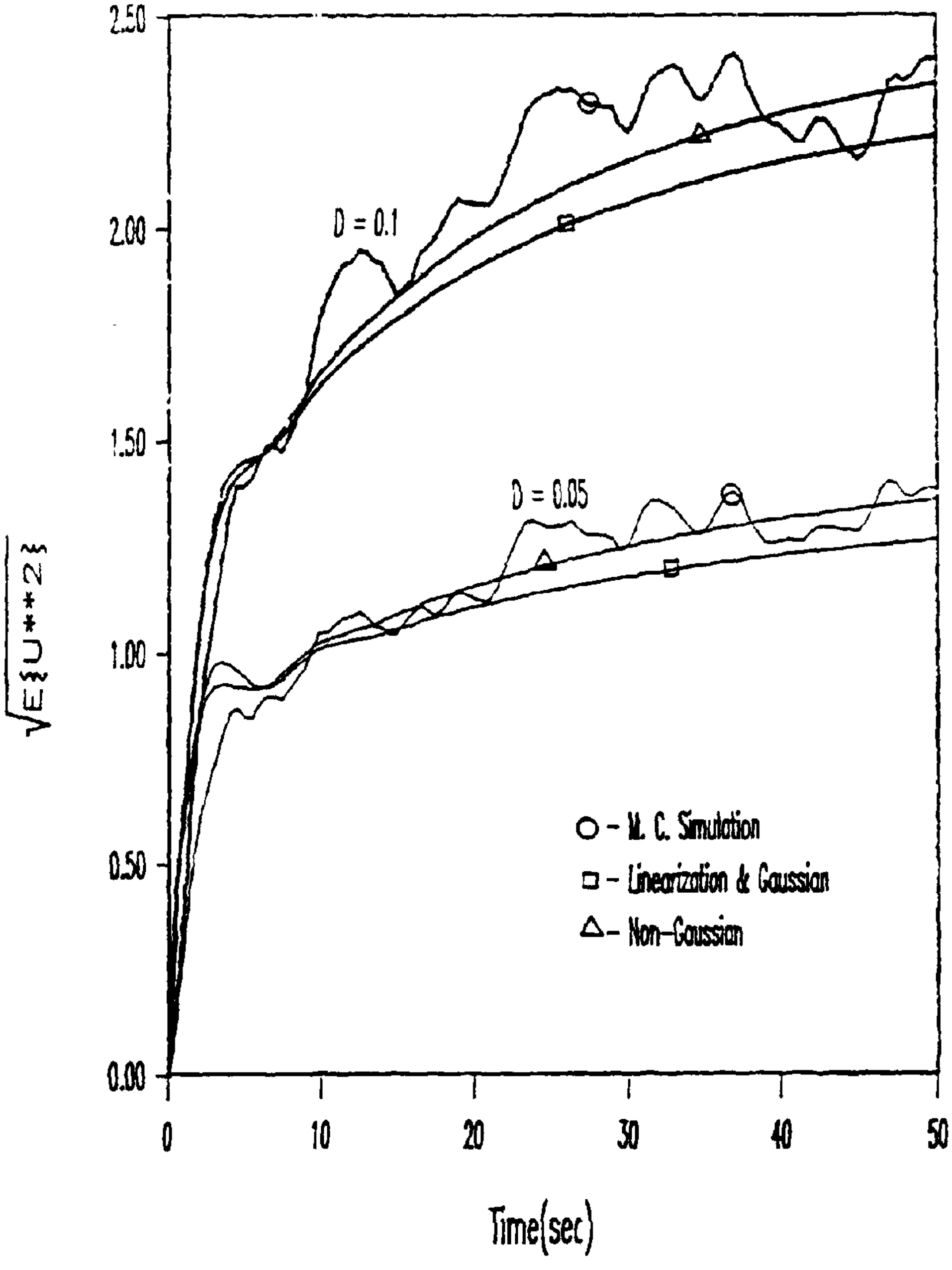


Fig. 5.8- RMS Displacement Response of a BBW SDOF System. ξ = 0.1, δ_A = 0.01, δ_ν = 0.0, δ_η = 0.0.

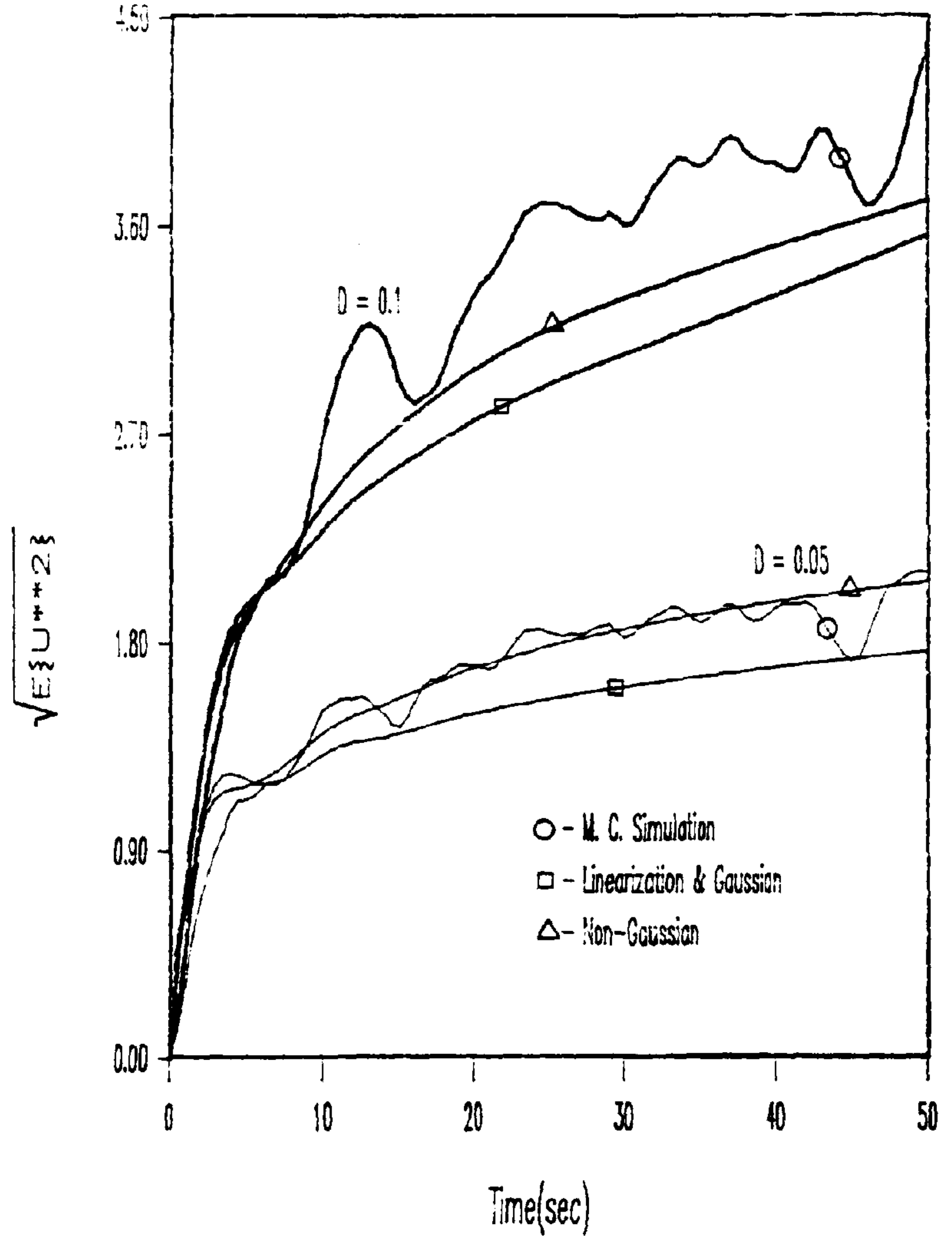


Fig. 5.9- RMS Displacement Response of a BBW SDOF System.
ξ = 0.0, δ_A = 0.02, δ_ν = 0.0, δ_η = 0.0.

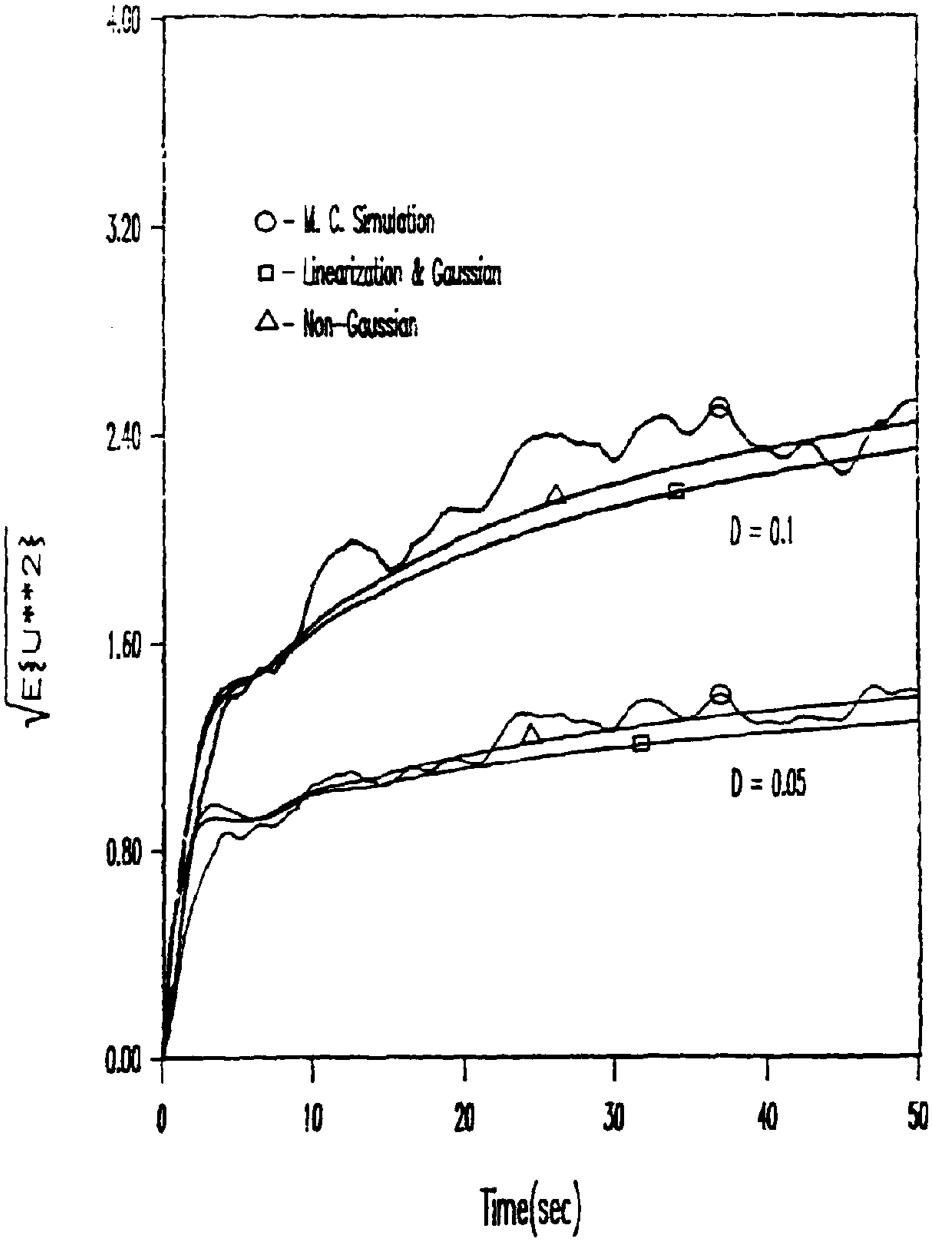


Fig. 5.10- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.1,\ \delta_A = 0.02,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

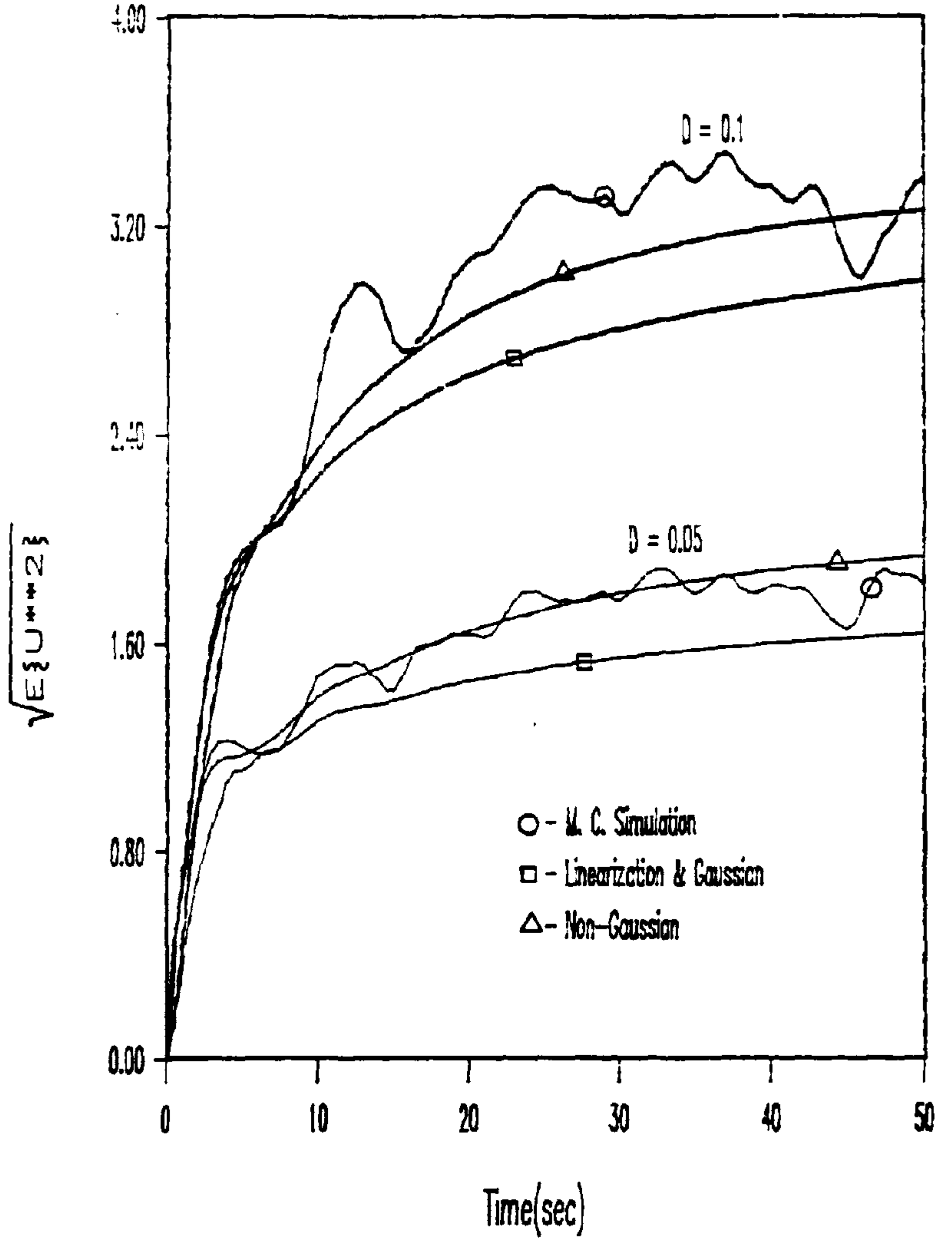


Fig. 5.11- RMS Displacement Response of a BDW SDOF System. $\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.01$, $\delta_\eta = 0.0$.

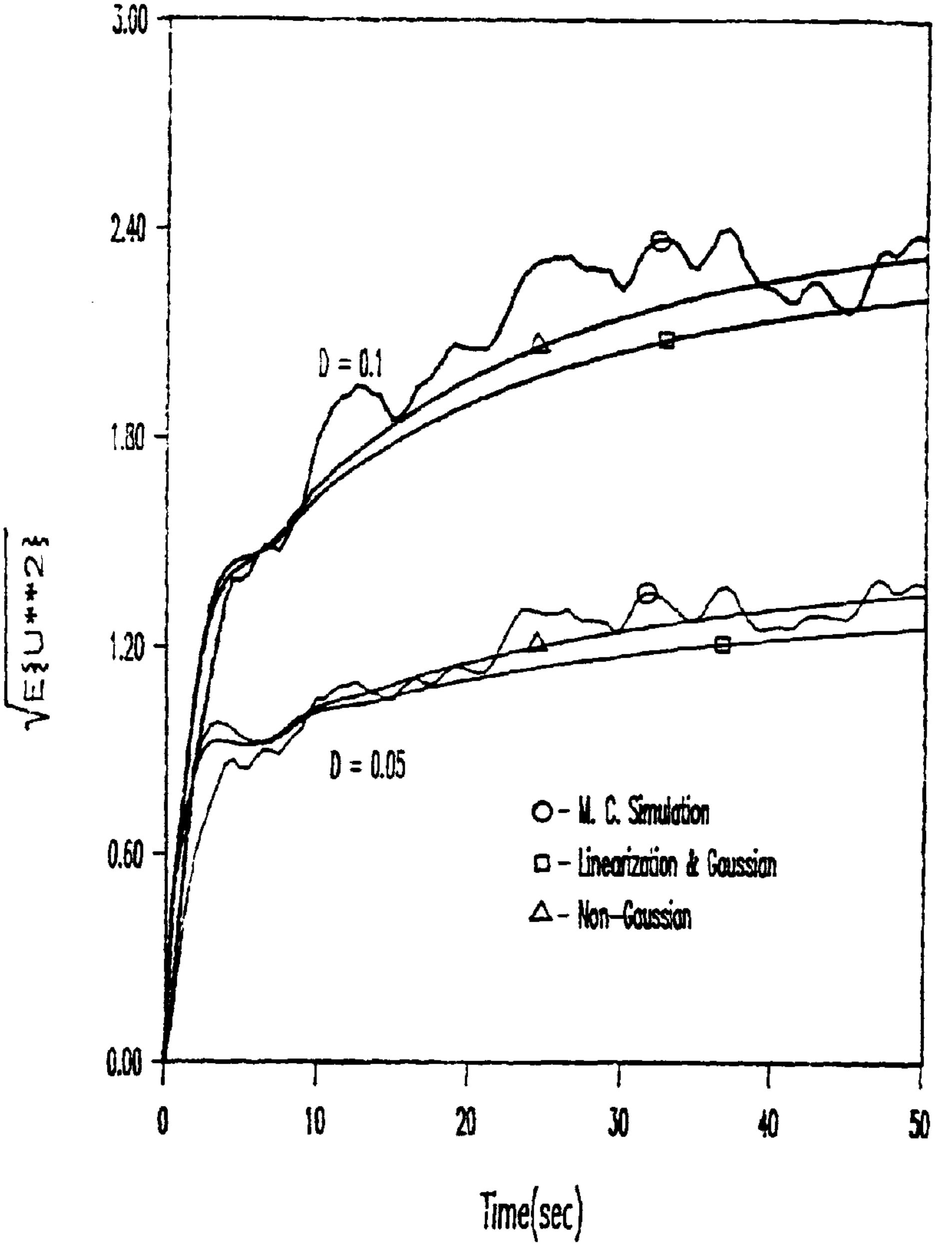


Fig. 5.12- RMS Displacement Response of a BBW SDOF System. $\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.01$, $\delta_\eta = 0.0$.

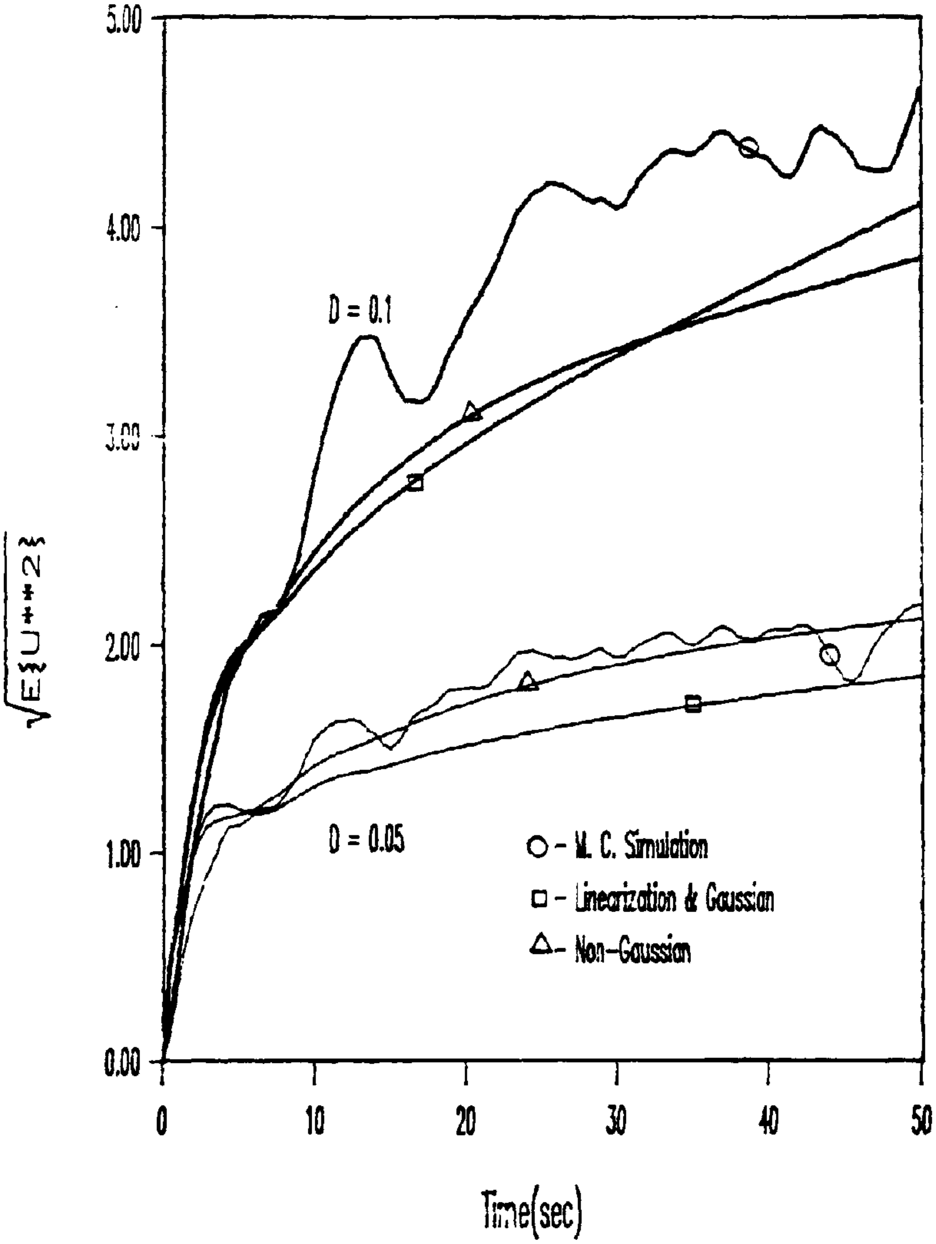


Fig. 5.13- RMS Displacement Response of a BBW SDOF System.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.05, δ_η = 0.0.

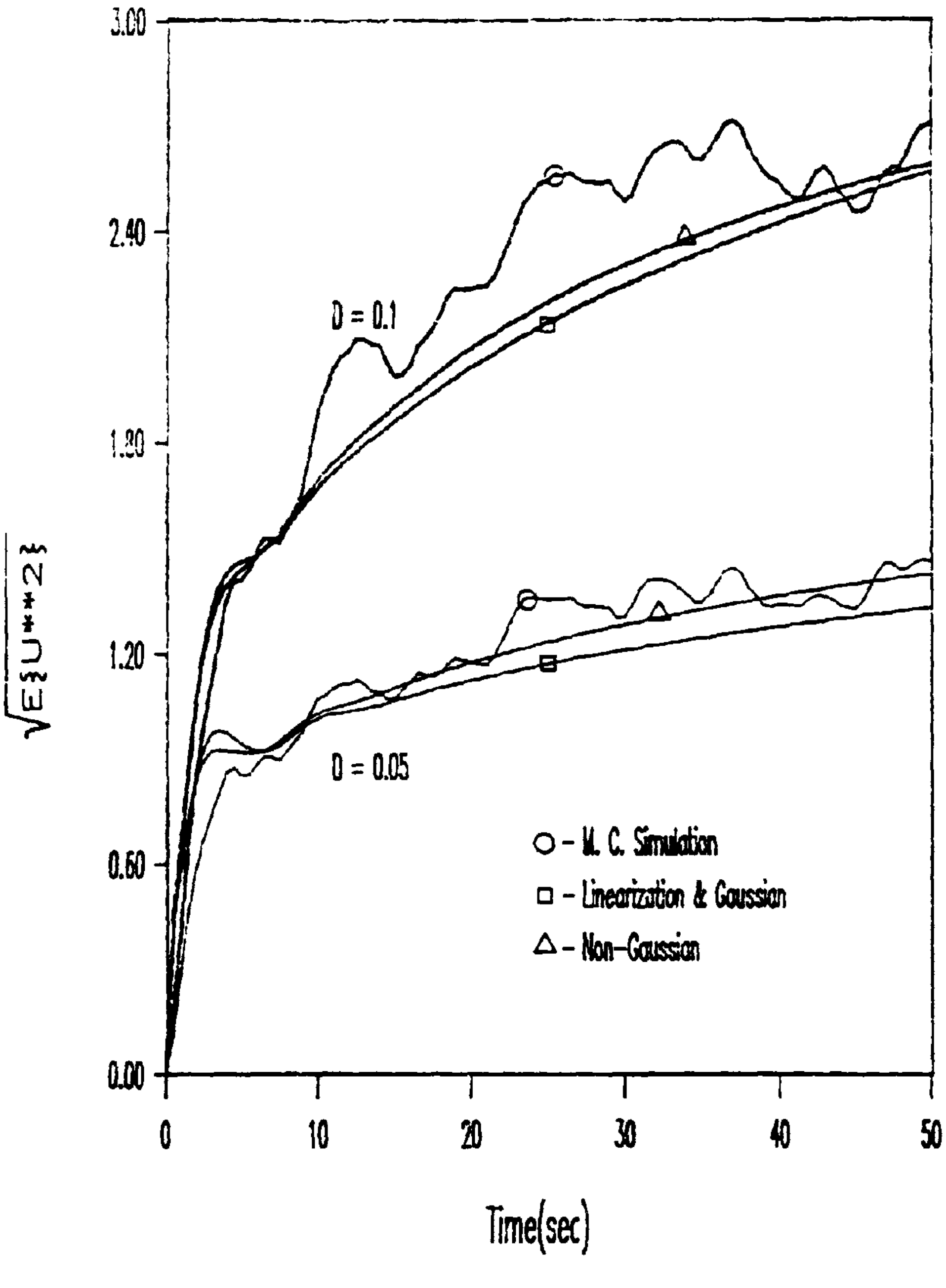


Fig. 5.14- RMS Displacement Response of a BBW SDOF System. $\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.05$, $\delta_\eta = 0.0$.

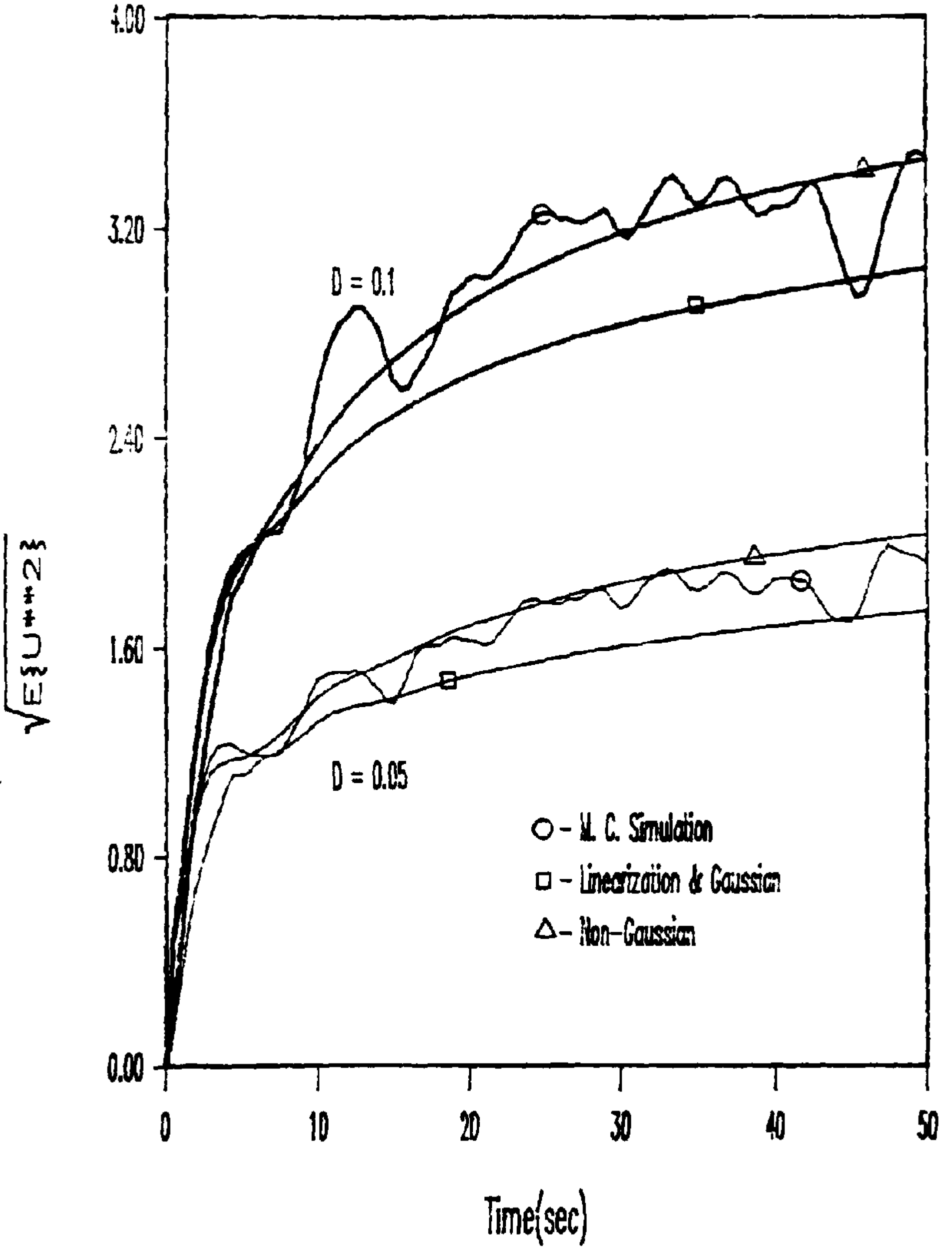


Fig. 5.15- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.05.$

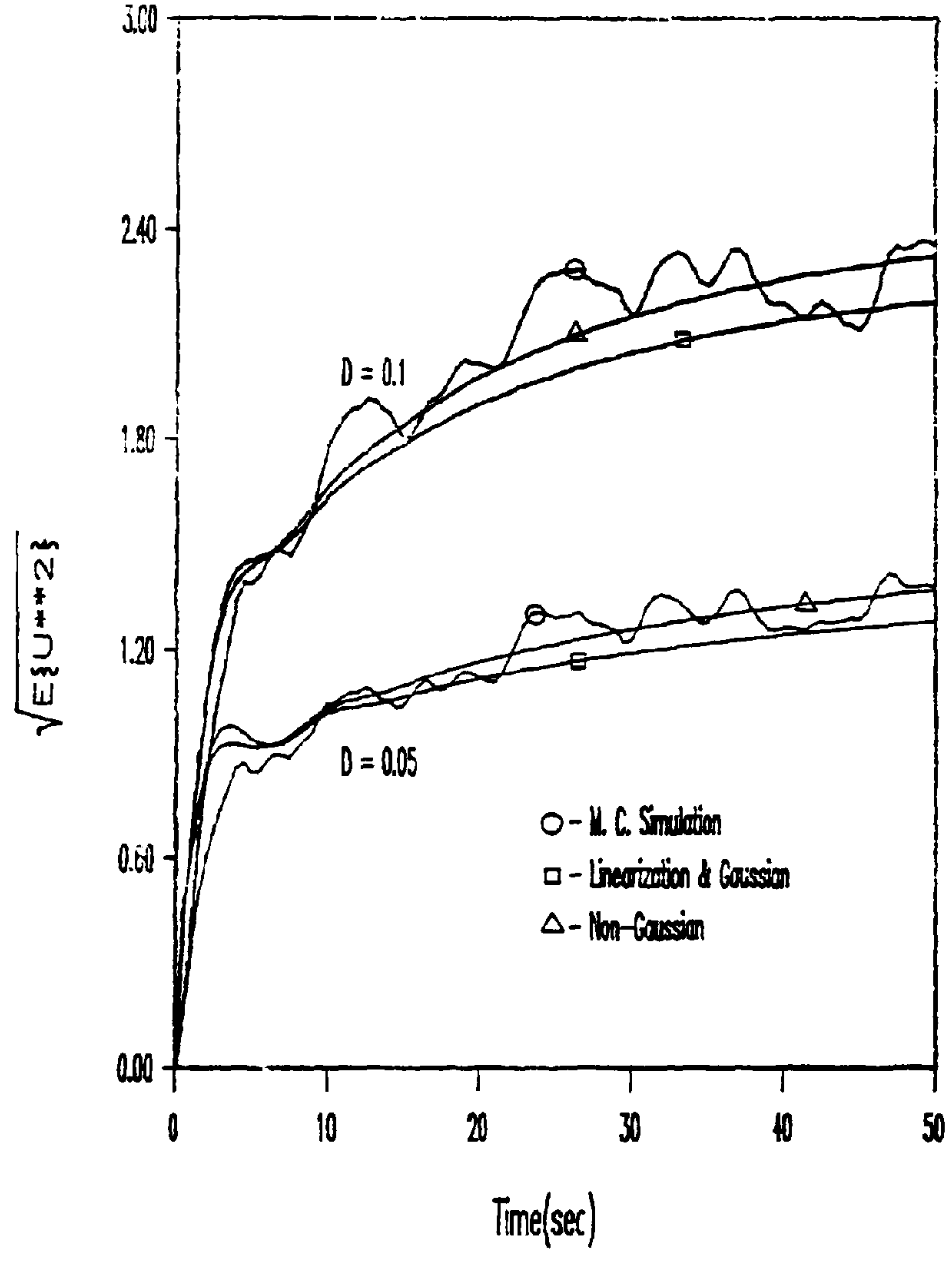


Fig. 5.16- RMS Displacement Response of a BBW SDOF System.
ξ = 0.1, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.05.

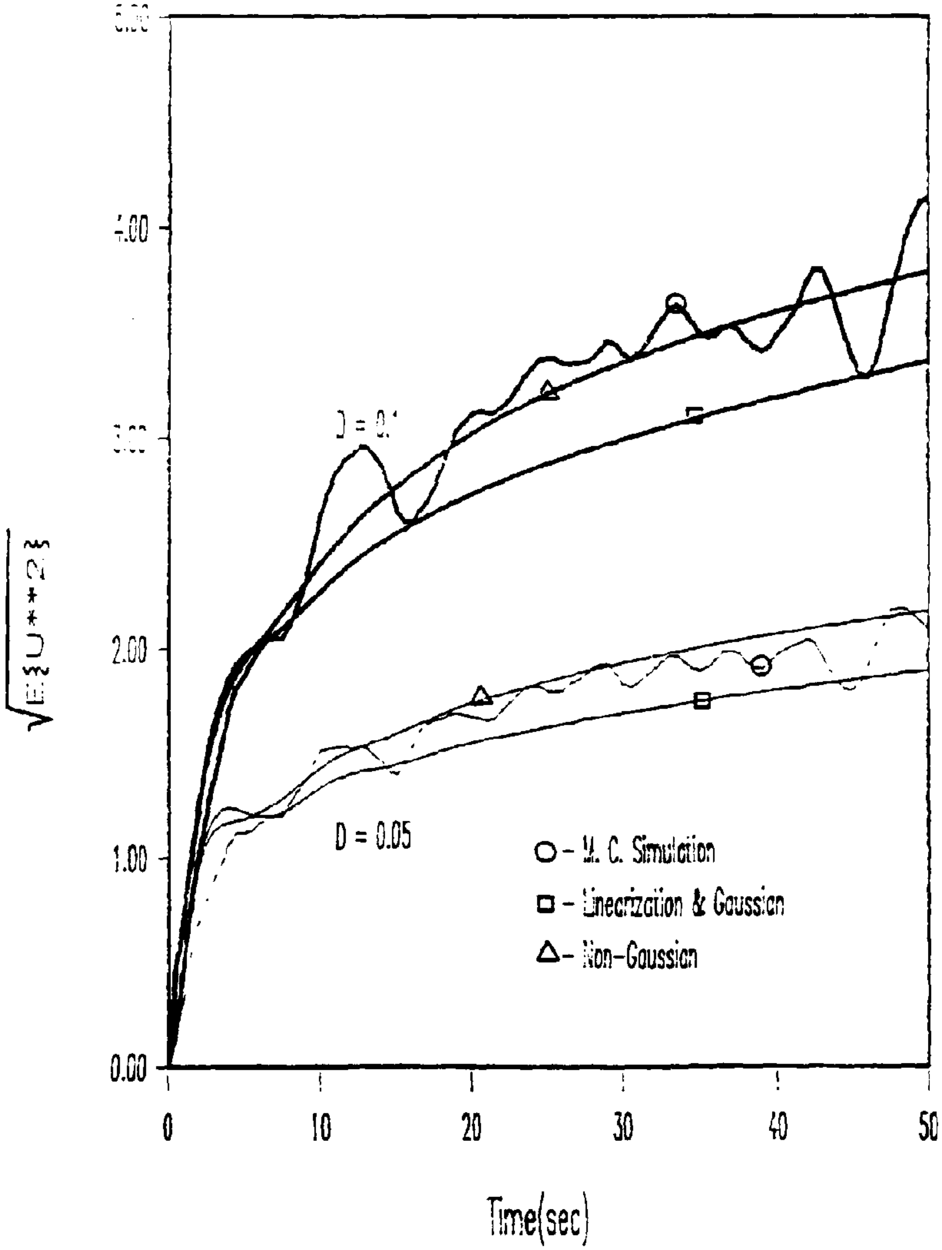


Fig. 5.17- RMS Displacement Response of a BBW SDOF System.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.1.

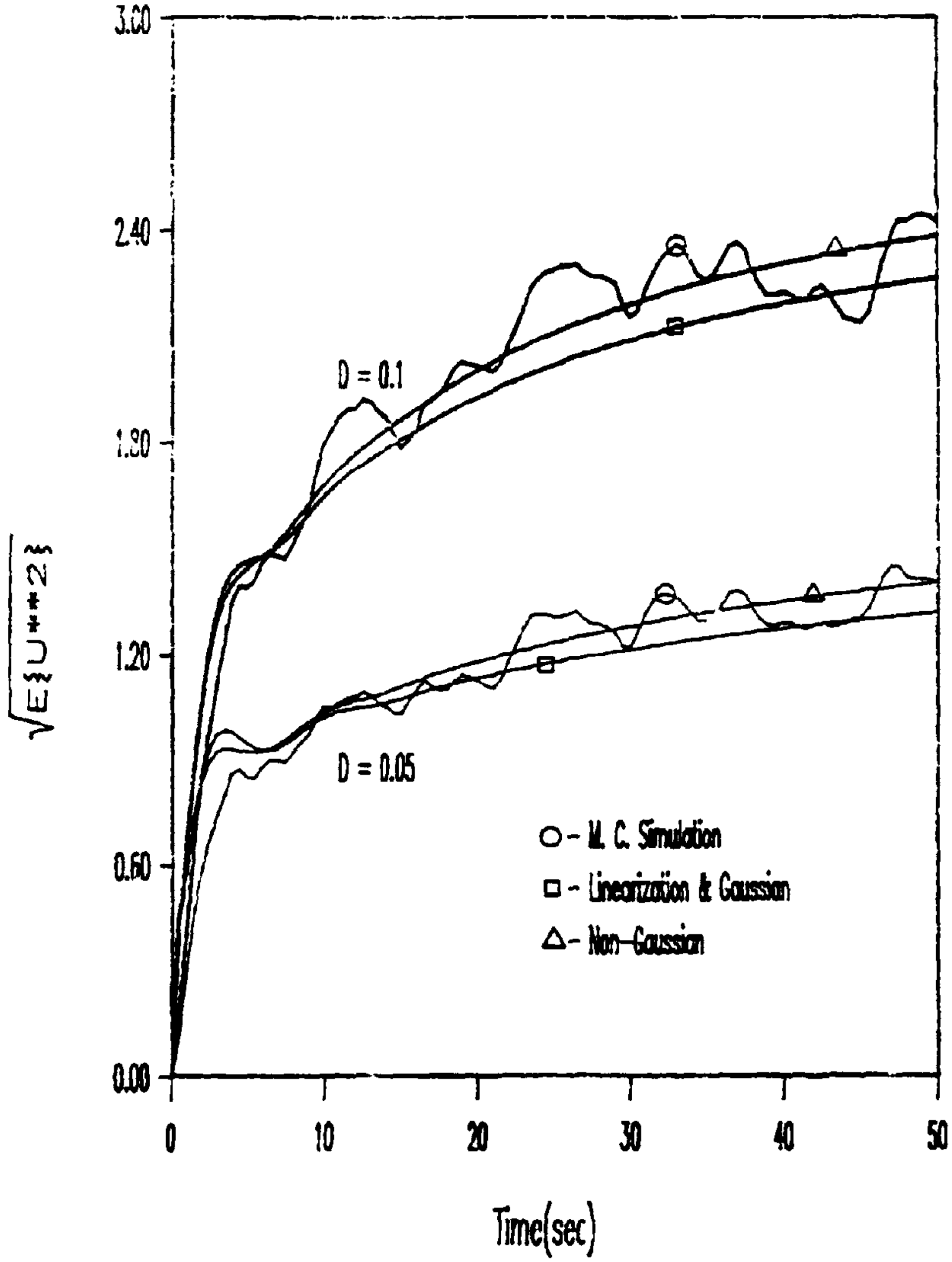


Fig. 5.18- RMS Displacement Response of a BBW SDOF System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.1$.

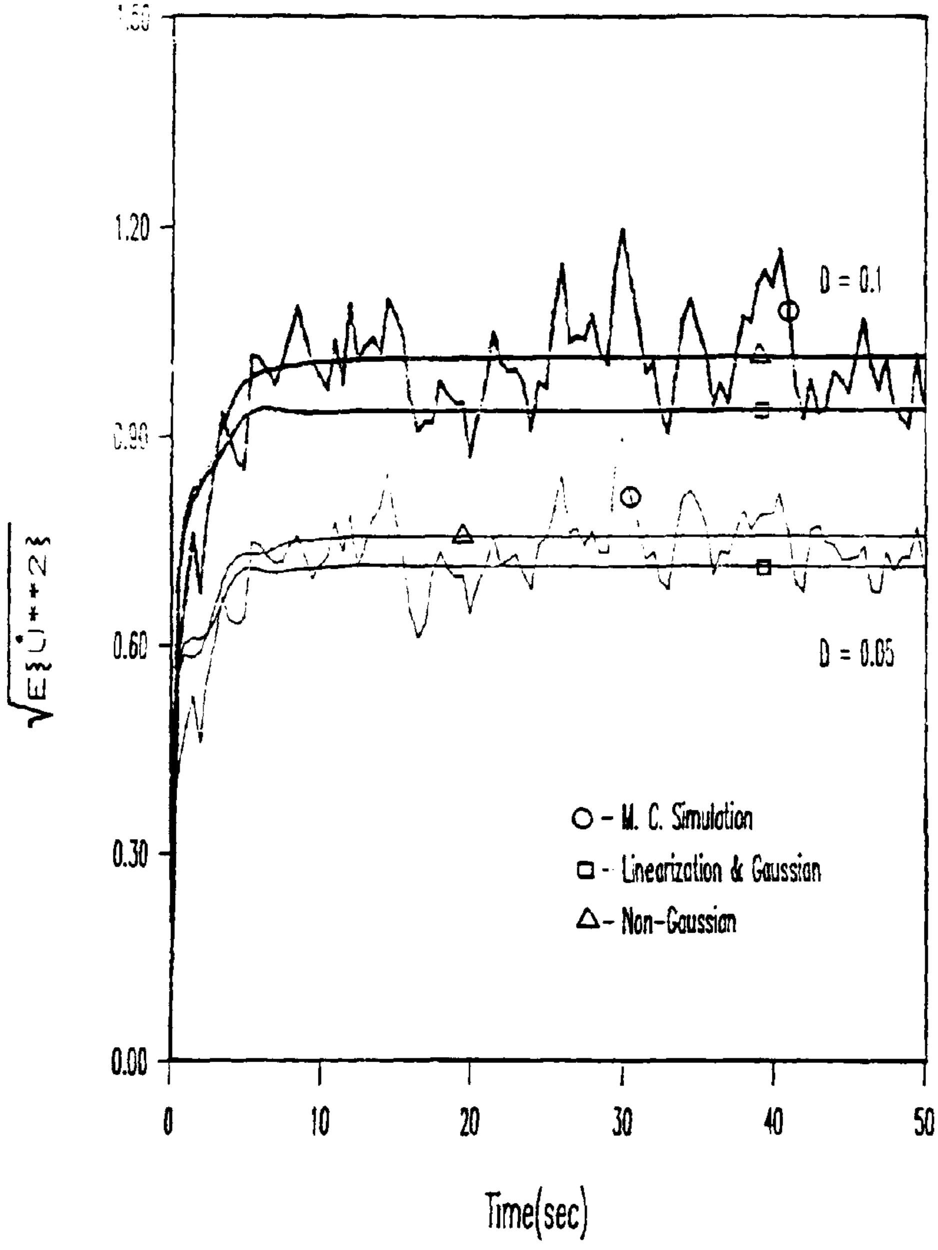


Fig. 5.19- RMS Velocity Response of a BBW SDOF System.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.0.

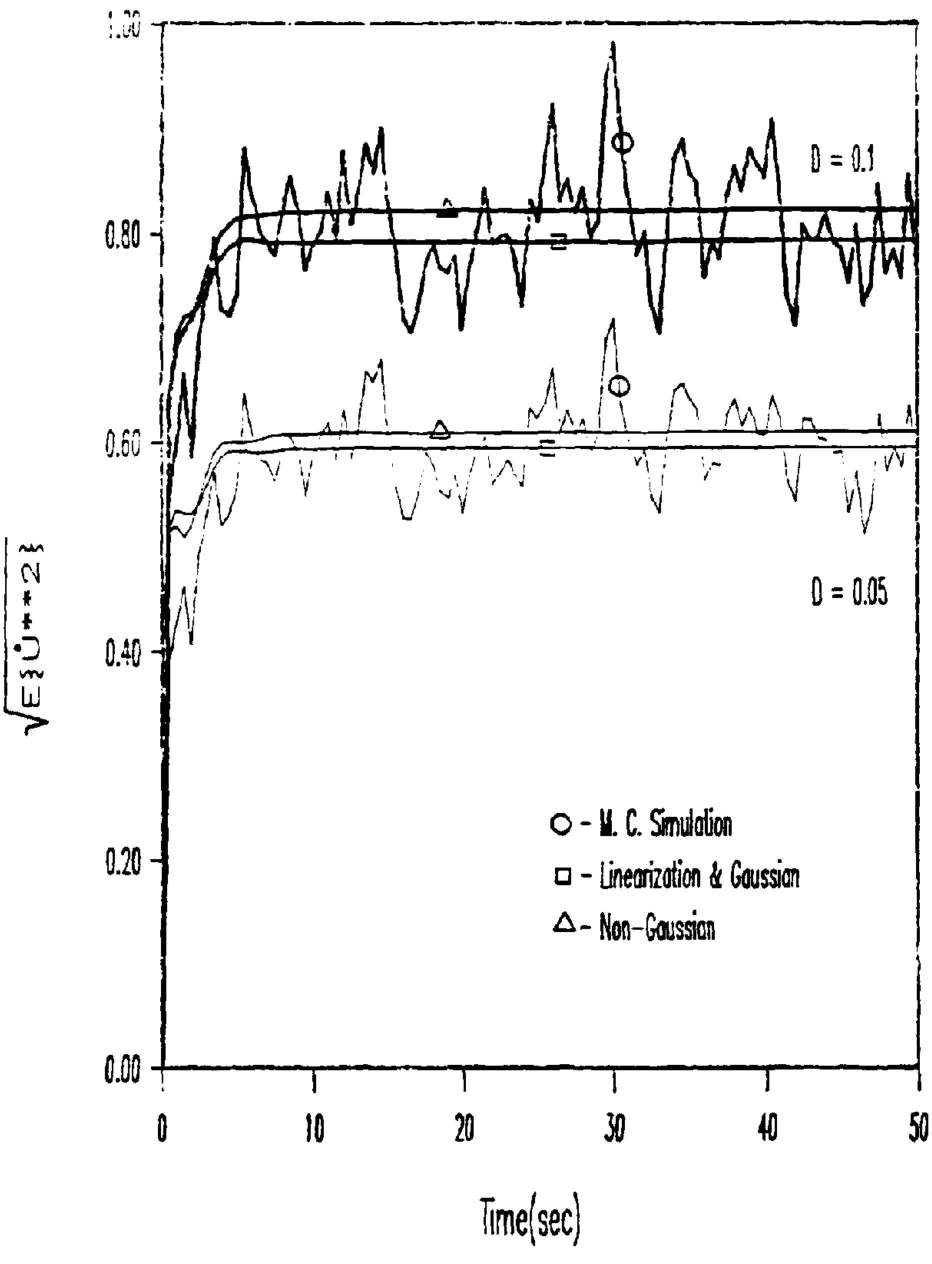


Fig. 5.20- RMS Velocity Response of a BBW SDOF System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

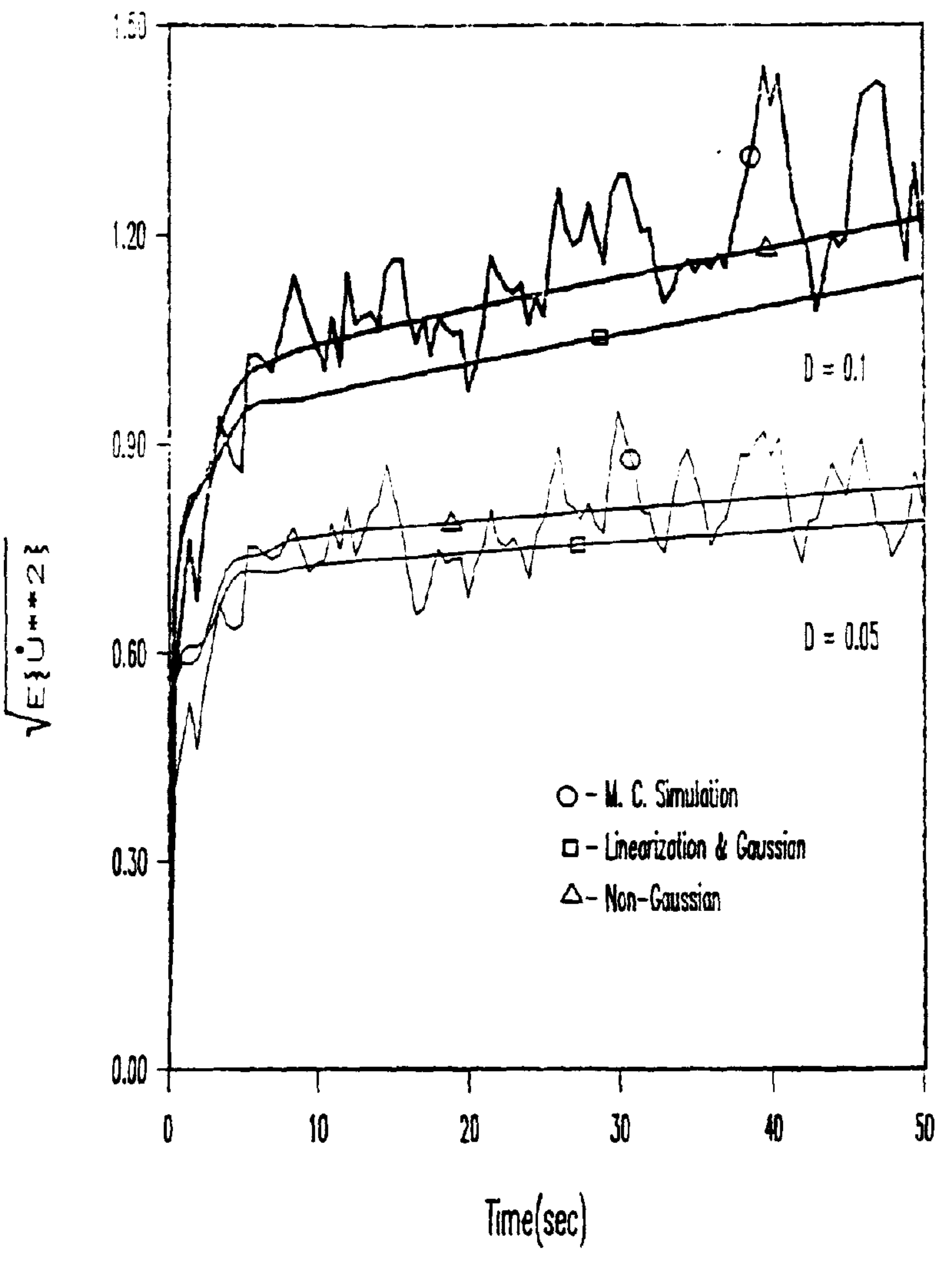


Fig. 5.21- RMS Velocity Response of a BBW SDOF System.
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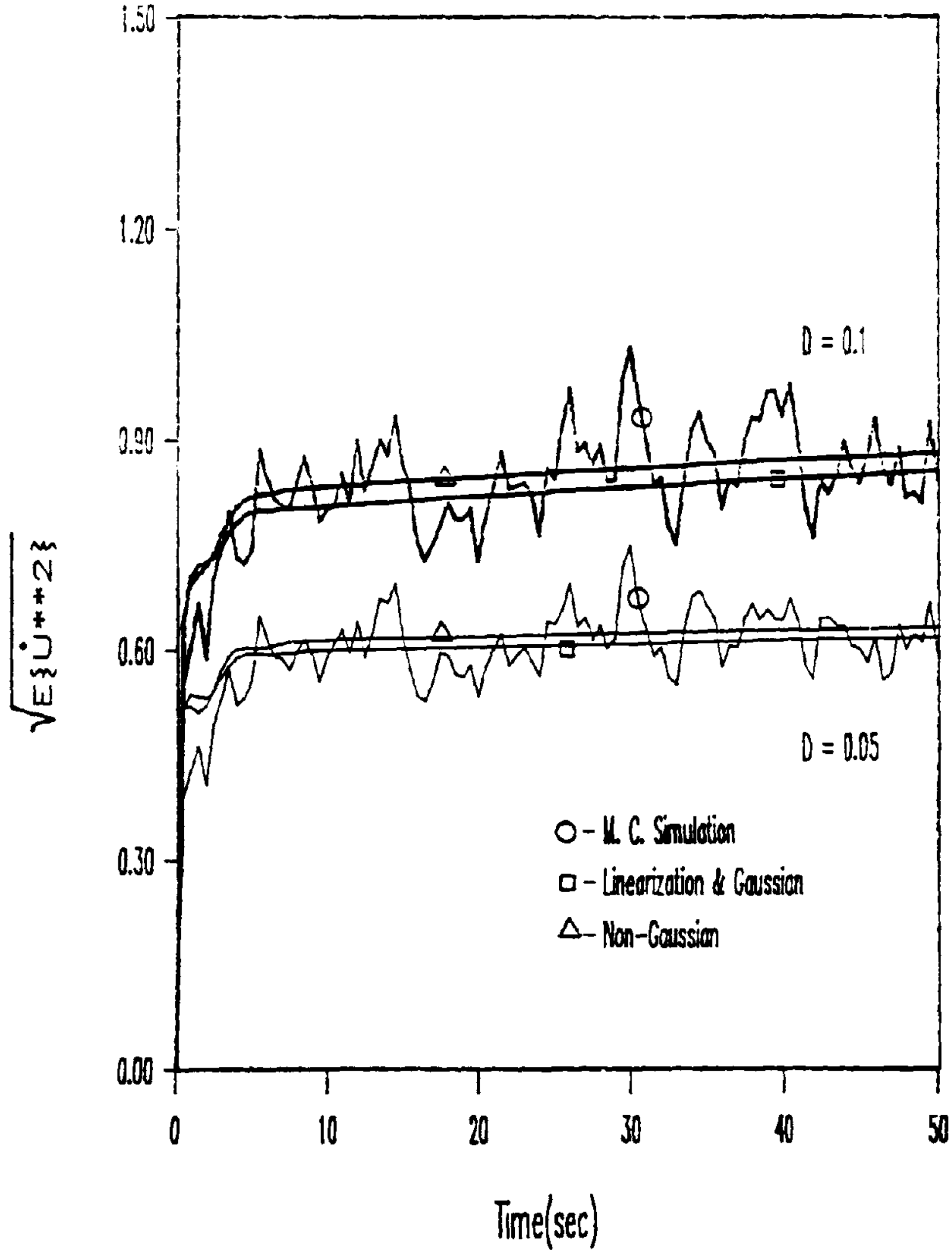

Fig. 5.22- RMS Velocity Response of a BBW SDOF System.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

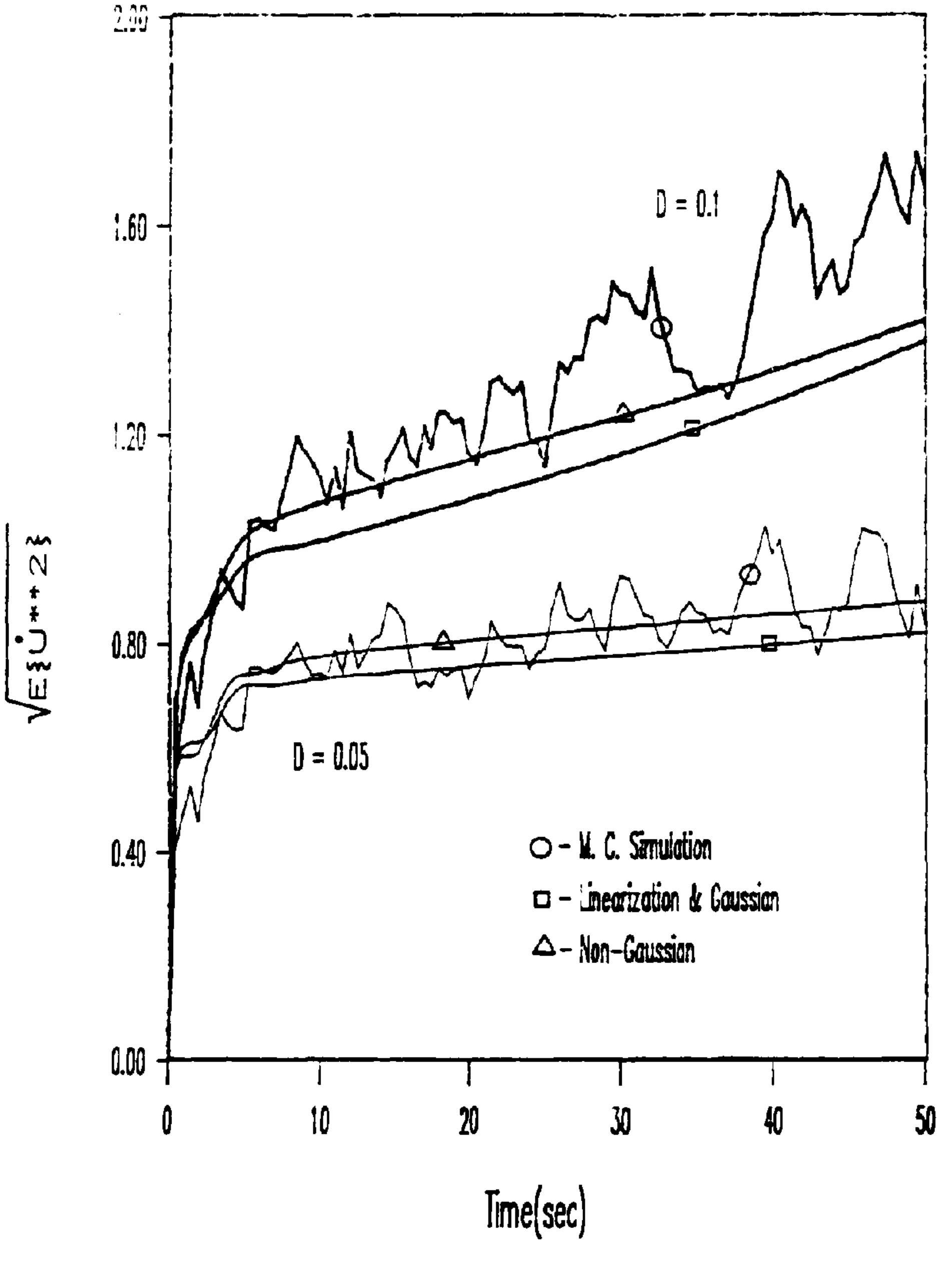

Fig. 5.23- **RMS Velocity Response of a BBW SDOF System.**
$\xi = 0.0$, $\delta_A = 0.02$, $\delta_\nu = 0.05$, $\delta_\eta = 0.1$.

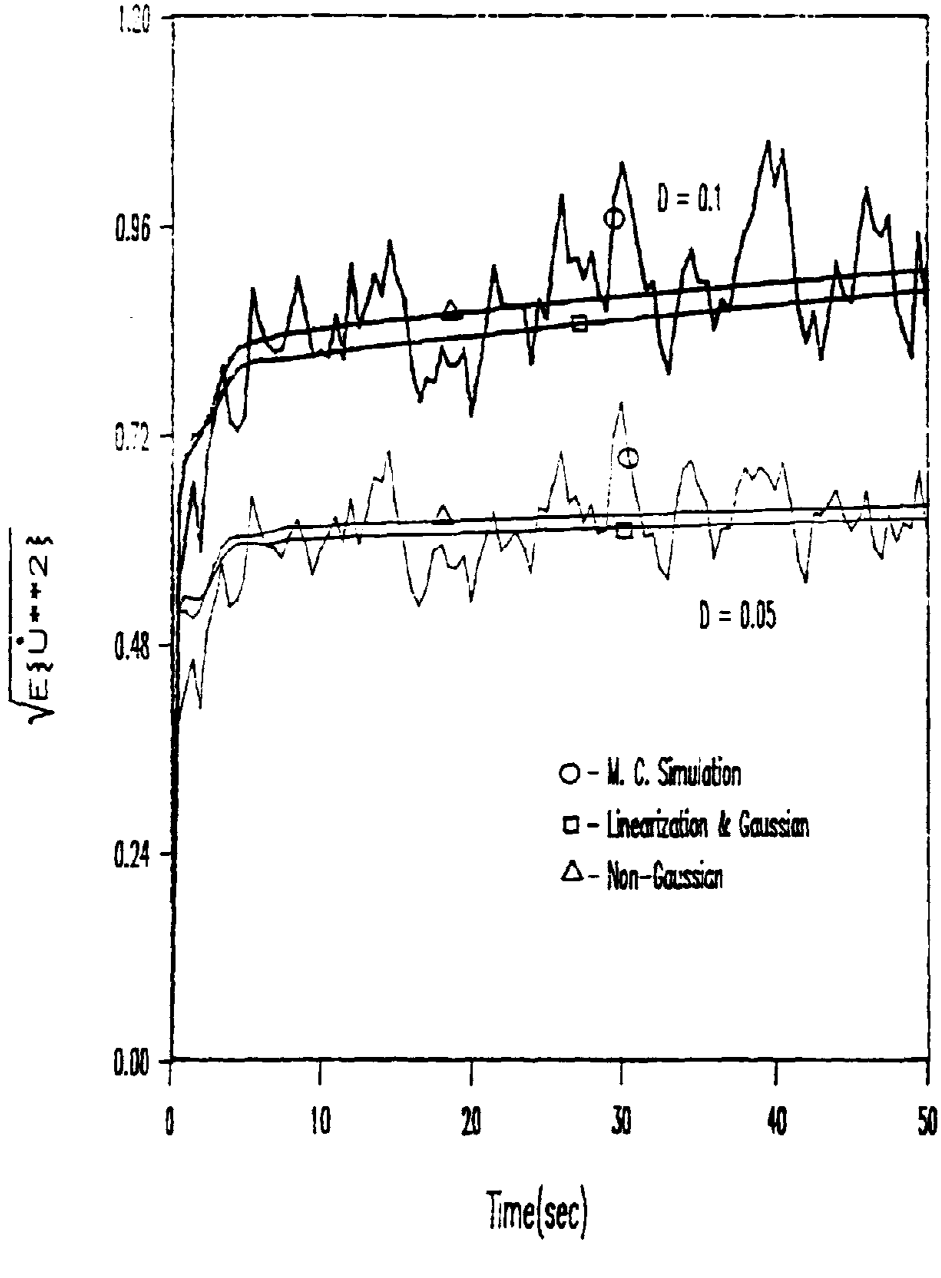

Fig. 5.24- RMS Velocity Response of a BBW SDOF System.
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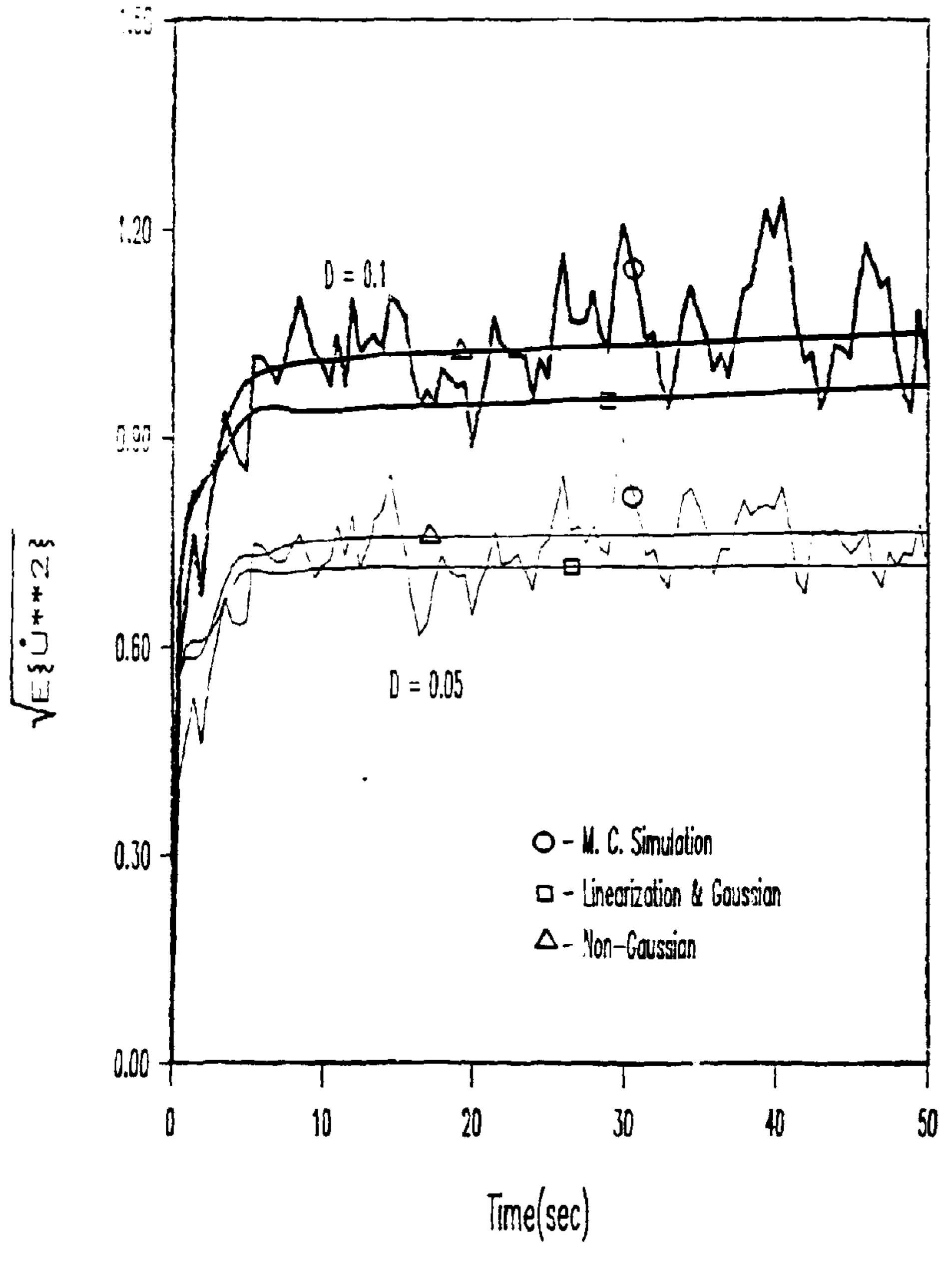

Fig. 5.25- RMS Velocity Response of a BBW SDOF System. $\xi = 0.0$, $\delta_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

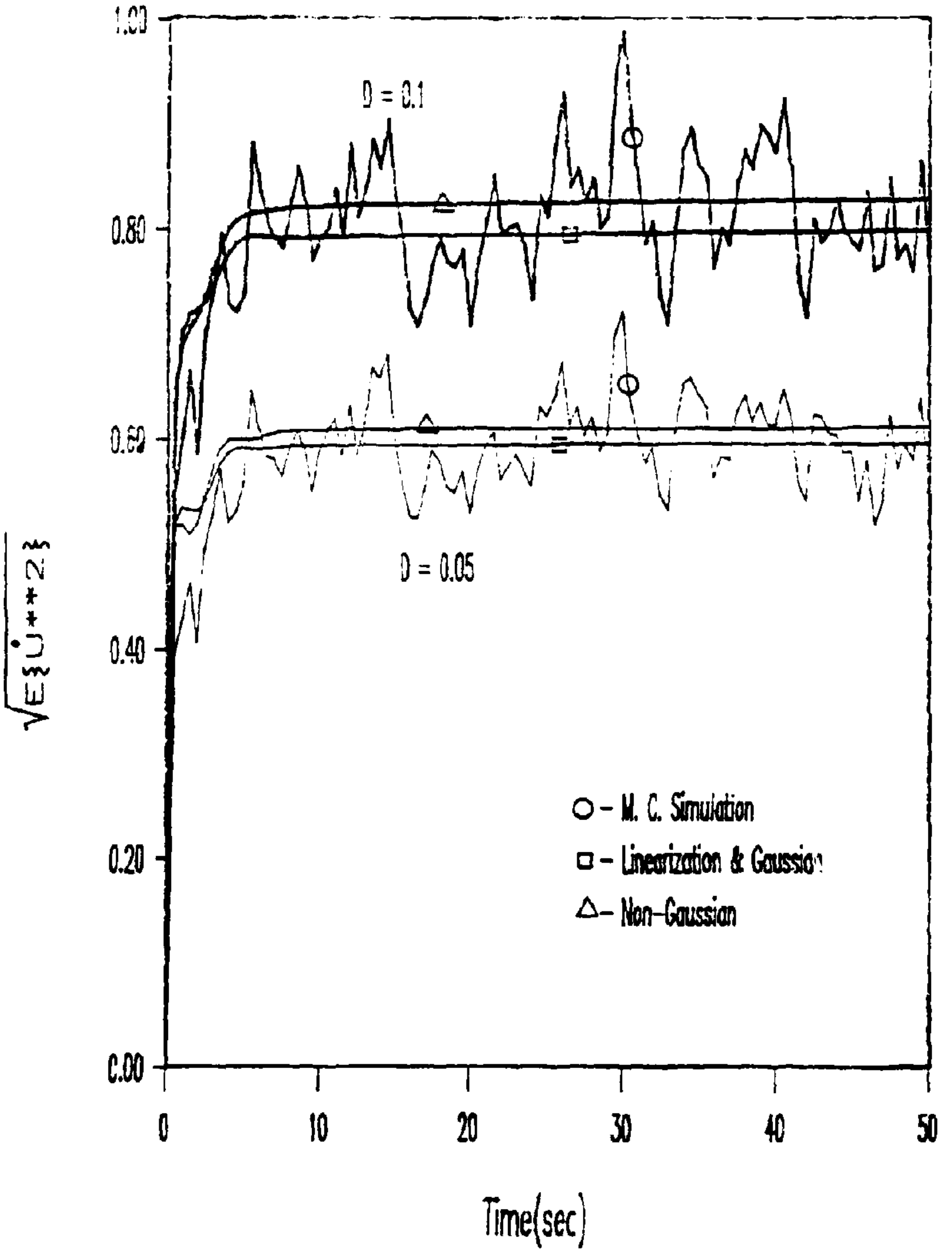

Fig. 5.26- RMS Velocity Response of a BBW SDOF System.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

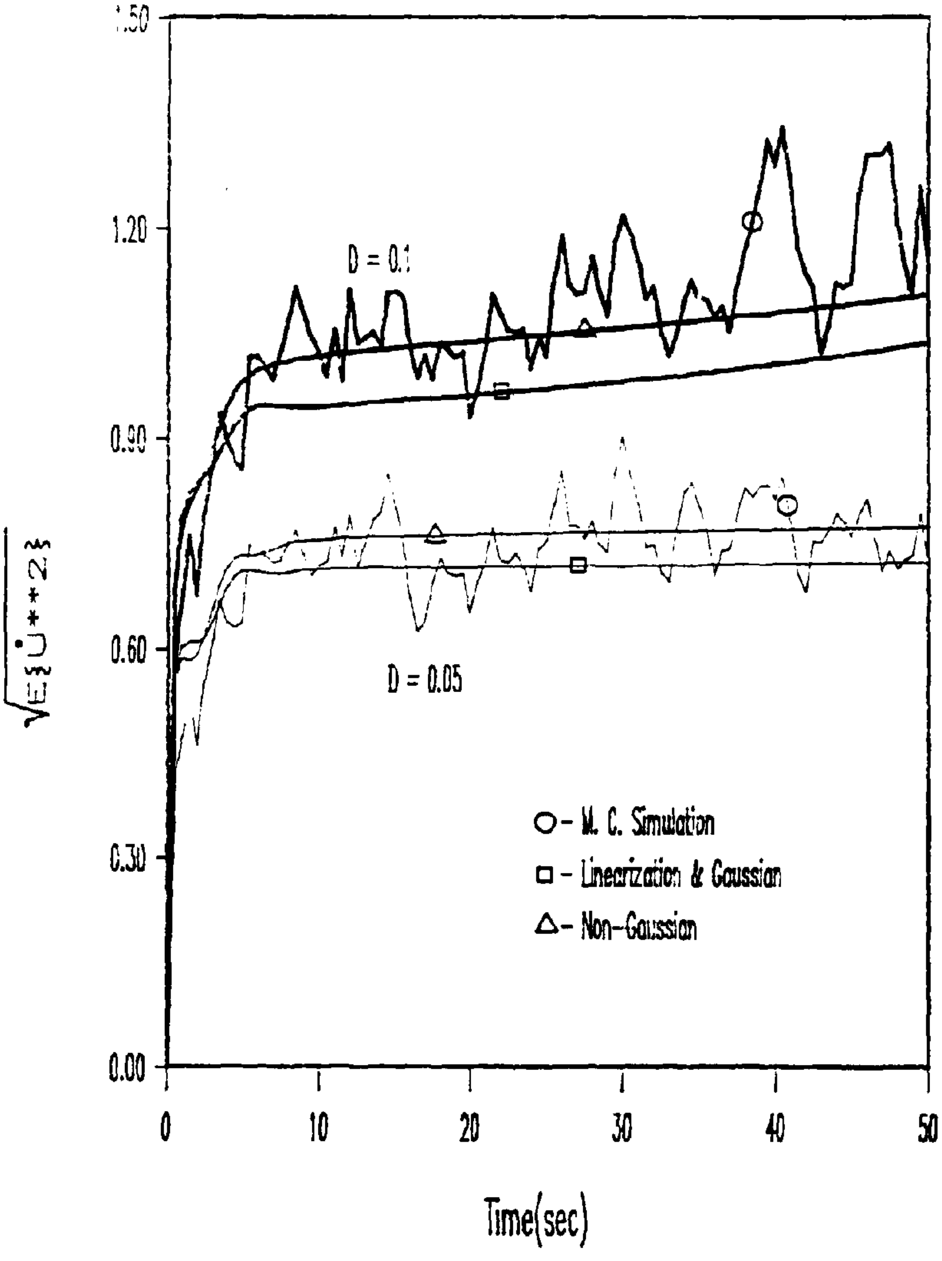

Fig. 5.27- RMS Velocity Response of a BBW SDOF System.
$\xi = 0.0$, $\delta_A = 0.02$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

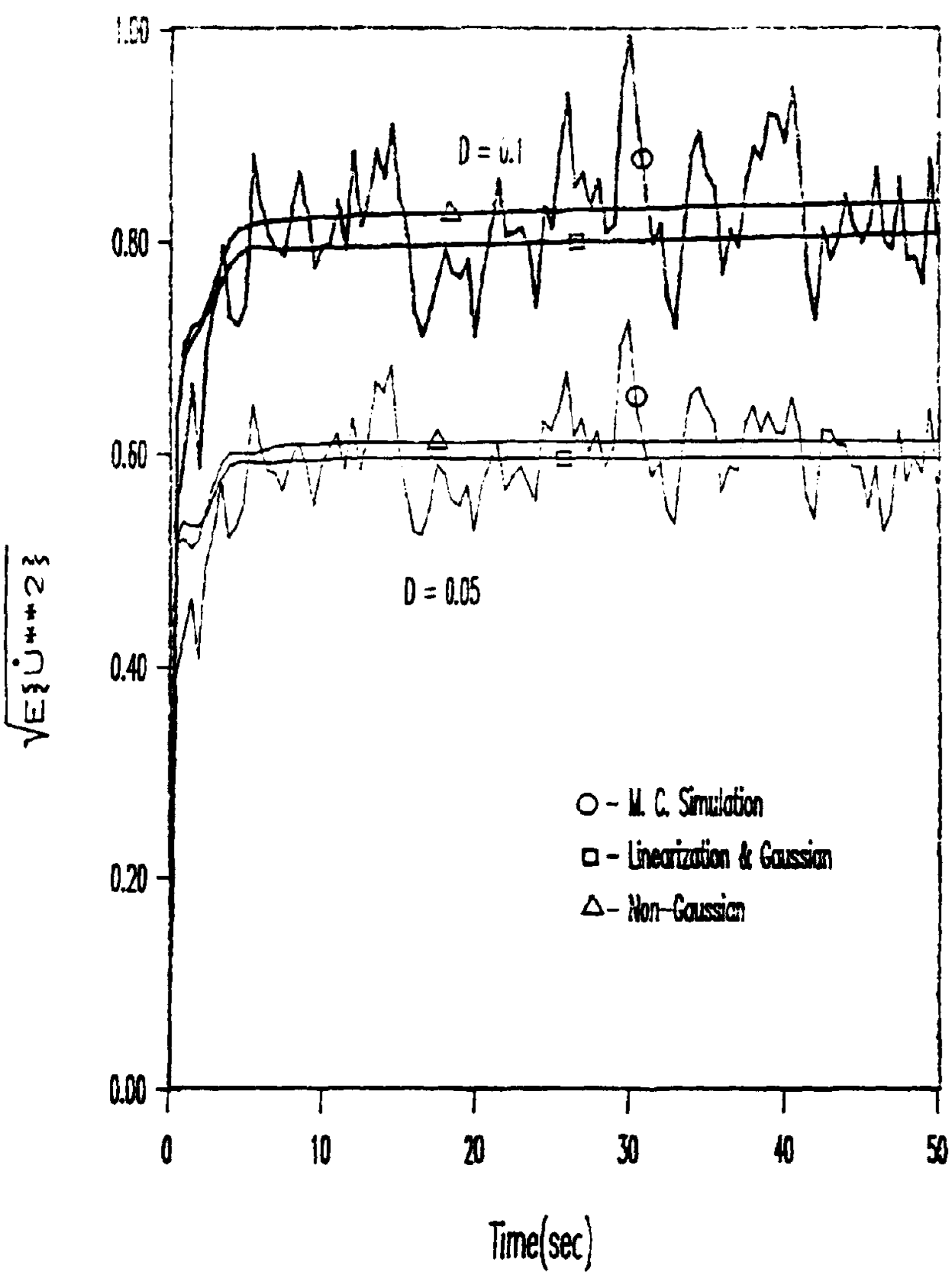

Fig. 5.28- RMS Velocity Response of a BBW SDOF System. $\xi = 0.1$, $\delta_A = 0.02$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

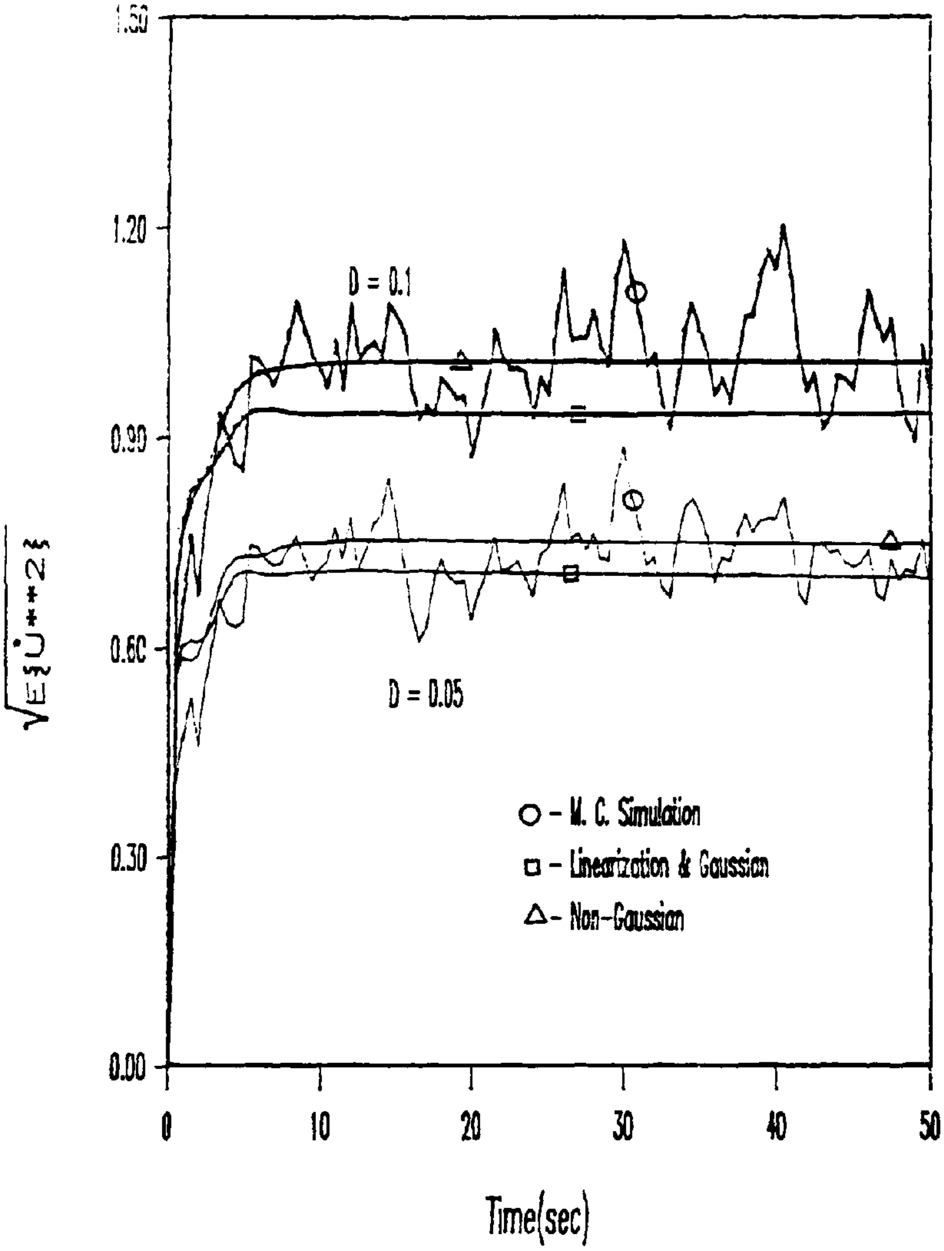

Fig. 5.29- RMS Velocity Response of a BBW SDOF System.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.01, δ_η = 0.0.

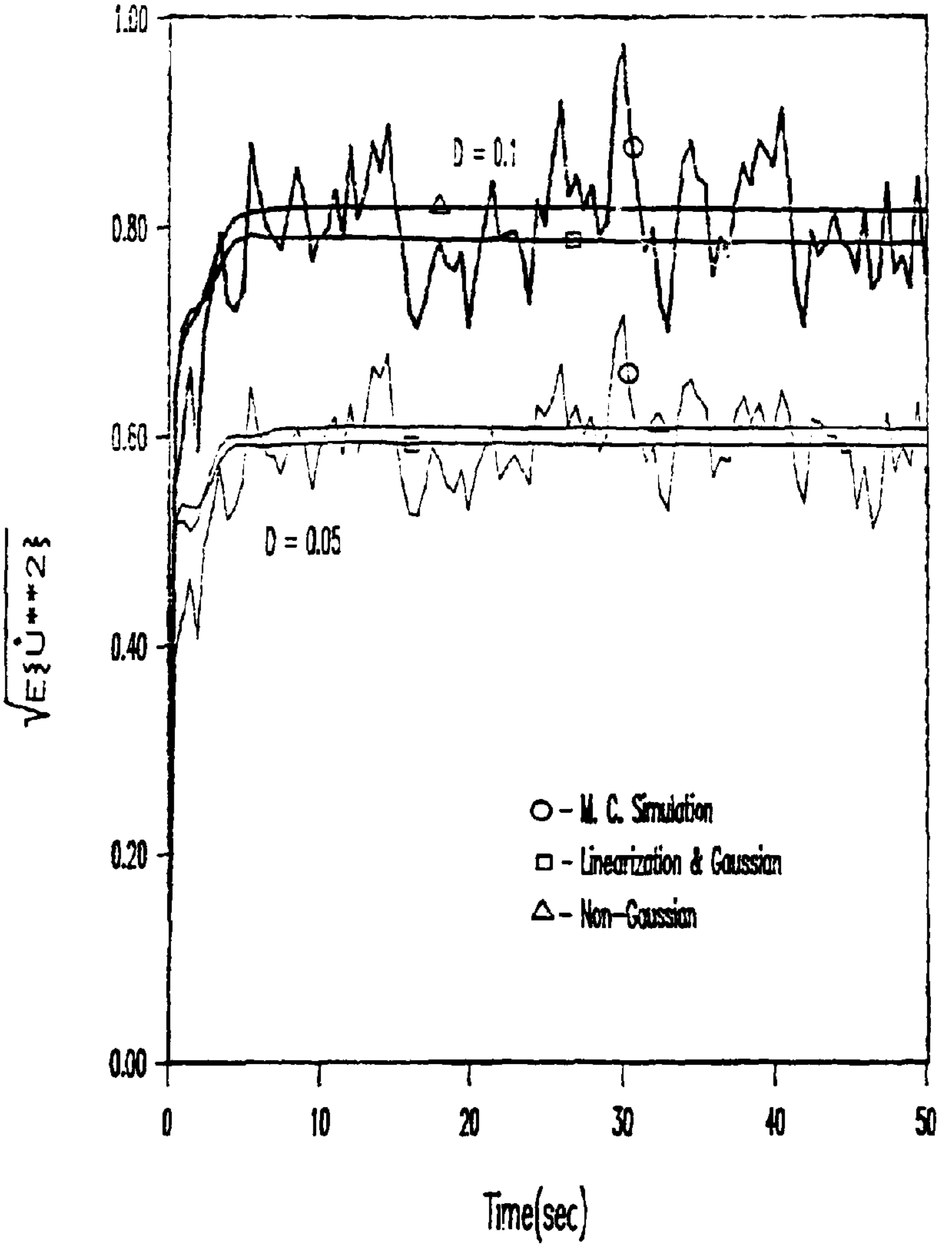

Fig. 5.30- RMS Velocity Response of a BBW SDOF System.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.01,\ \delta_\eta = 0.0.$

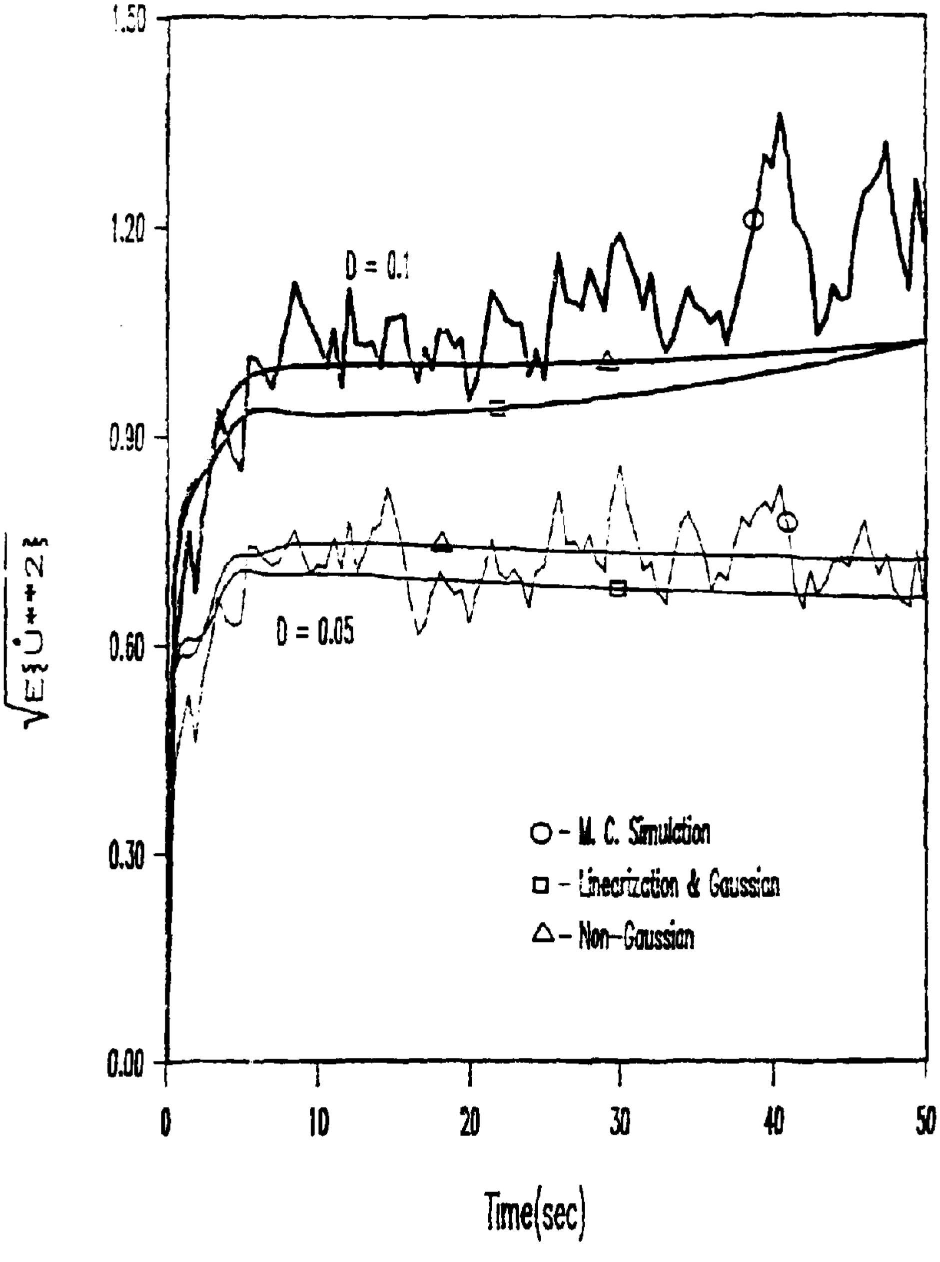

Fig. 5.31- RMS Velocity Response of a BBW SDOF System.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.05, δ_η = 0.0.

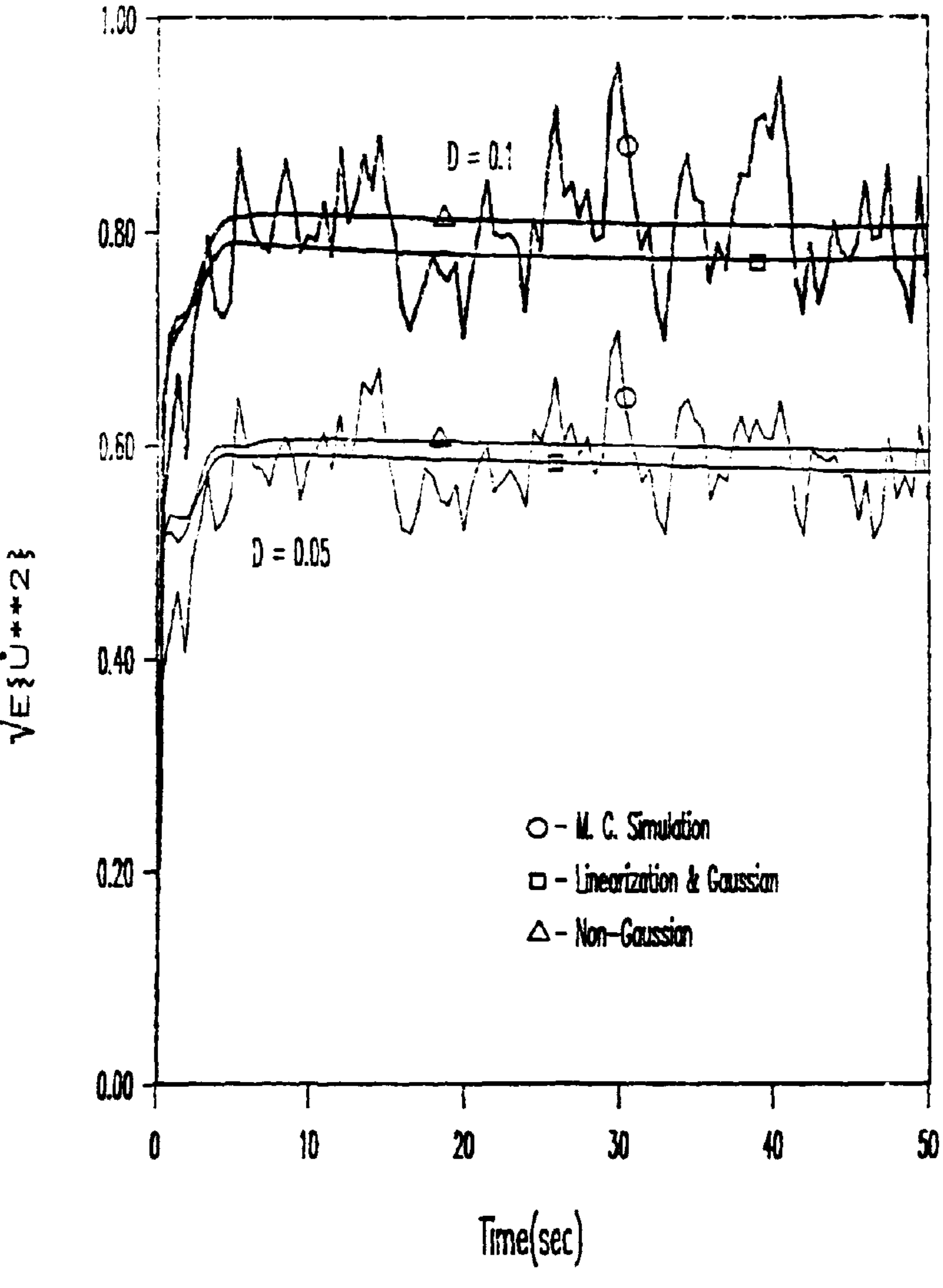

Fig. 5.32- RMS Velocity Response of a BBW SDOF System. $\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.05$, $\delta_\eta = 0.0$.

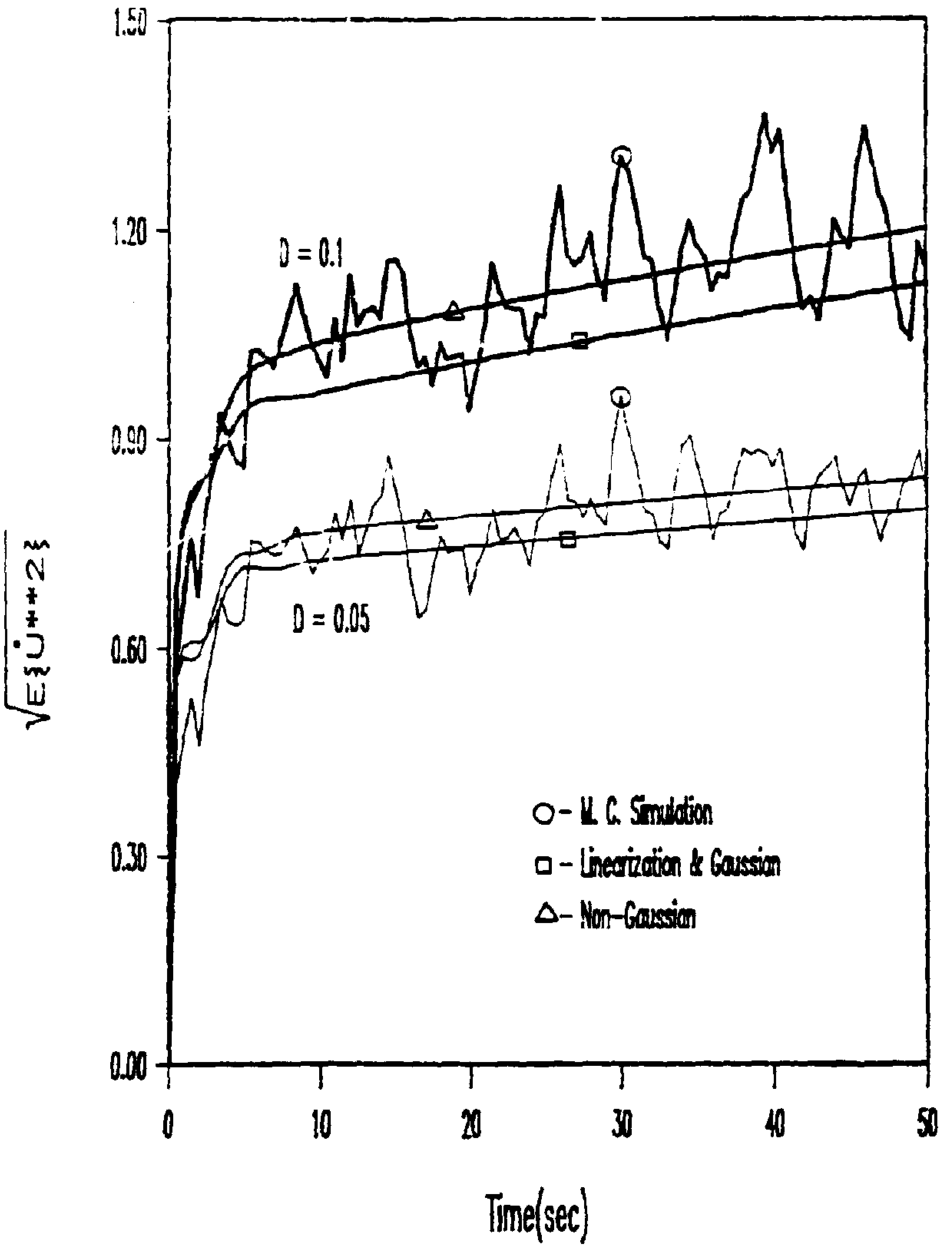

Fig. 5.33- RMS Velocity Response of a BBW SDOF System.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.05.

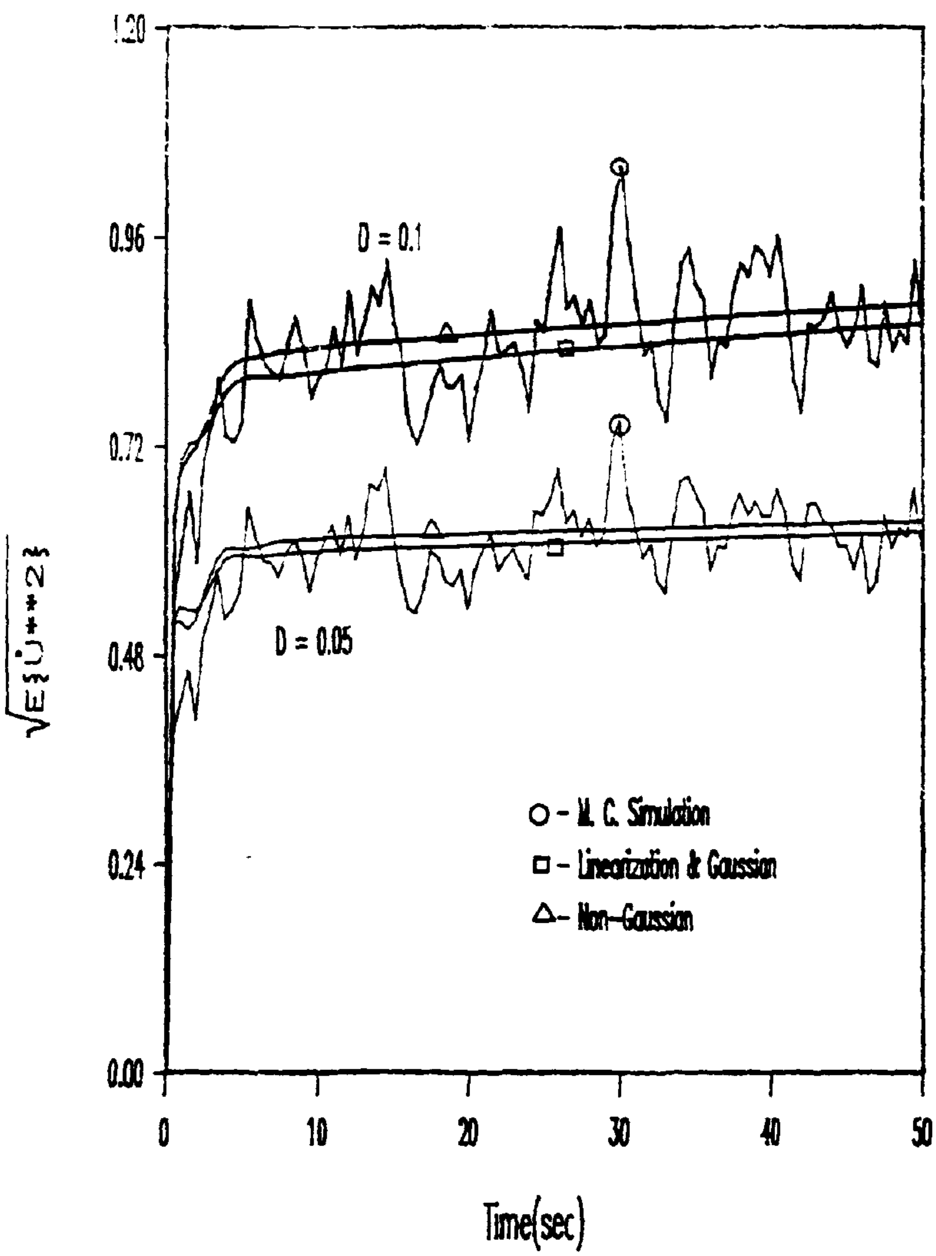

Fig. 5.34- RMS Velocity Response of a BBW SDOF System. $\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.05$.

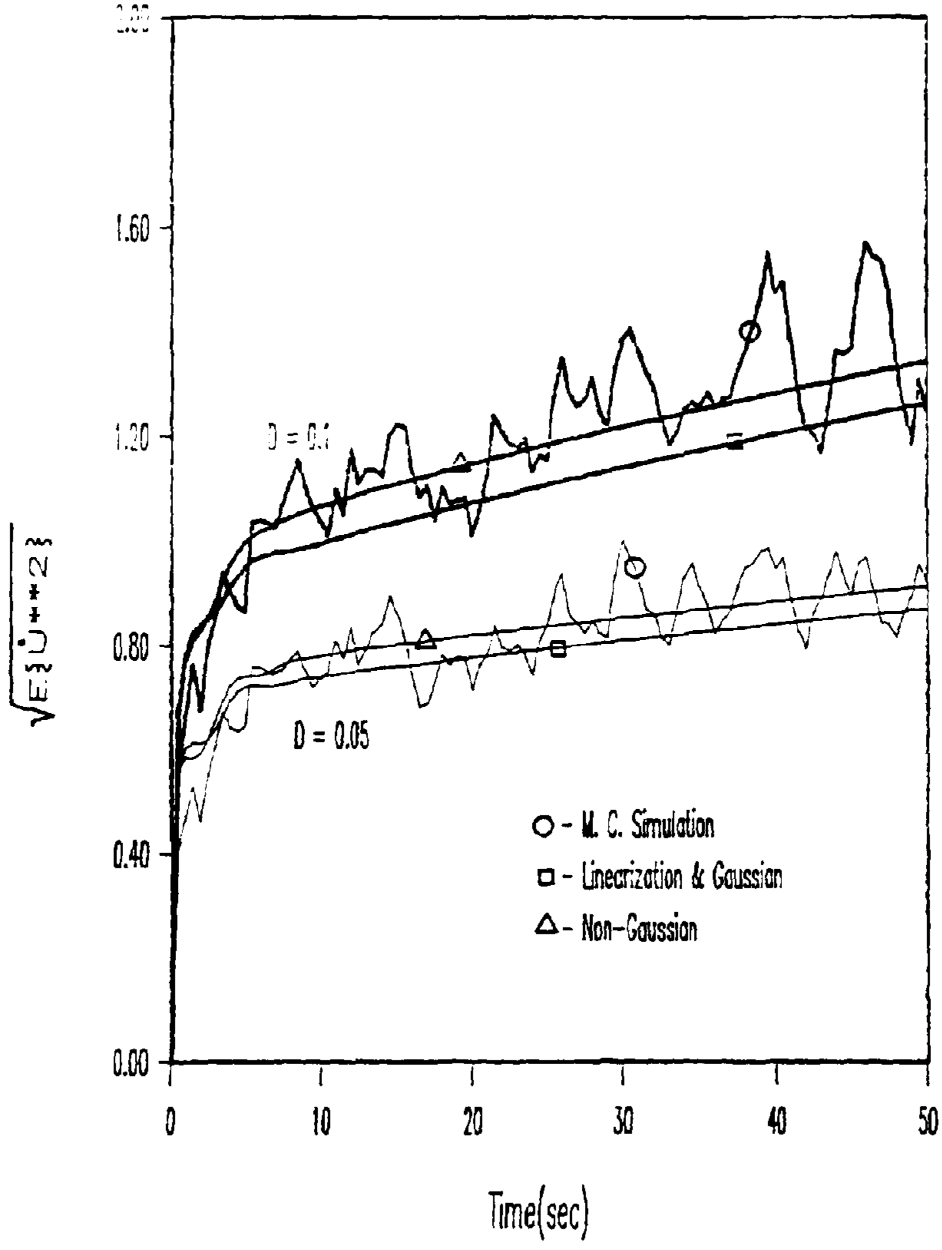

Fig. 5.35- RMS Velocity Response of a BBW SDOF System.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.1.$

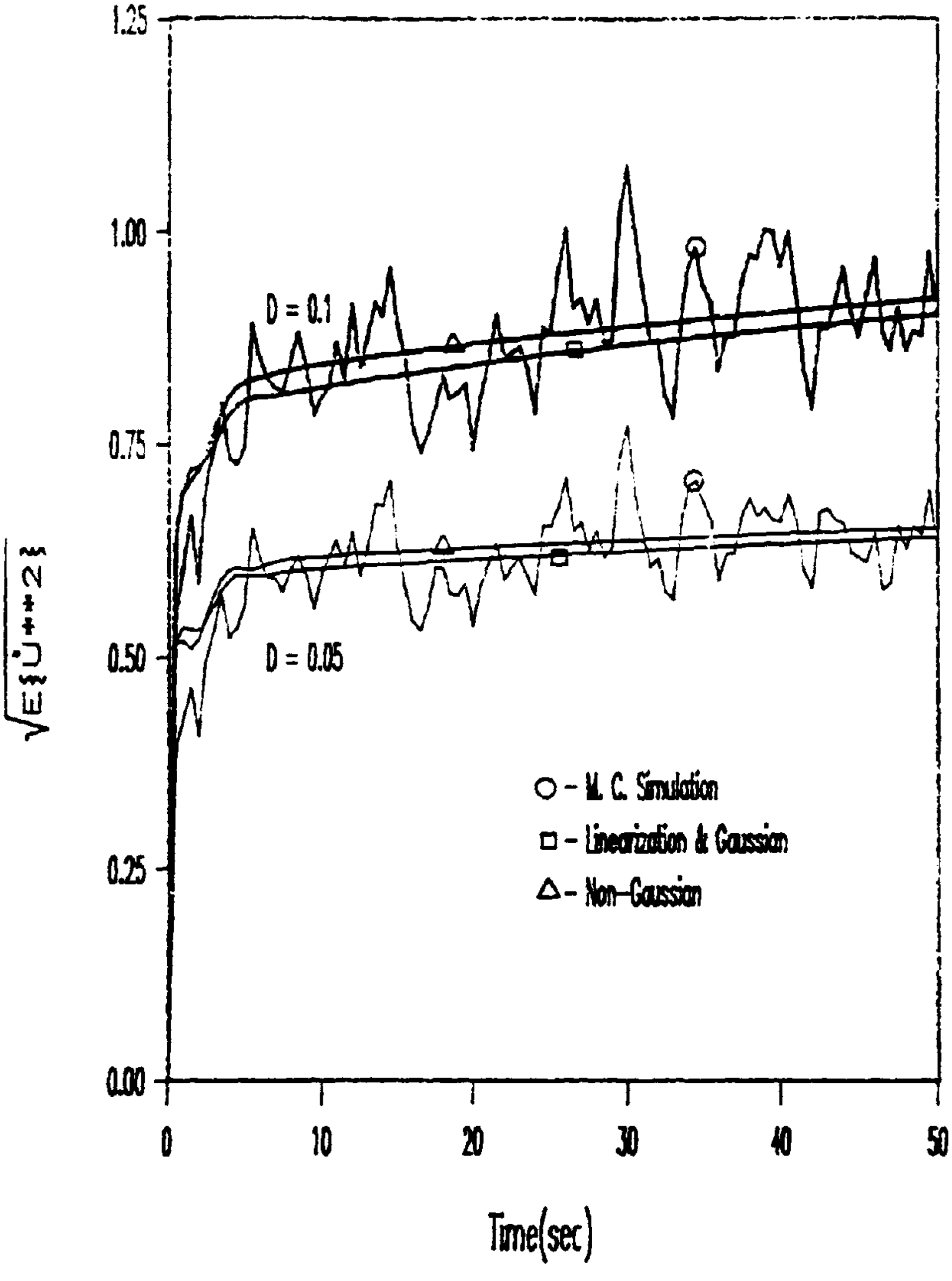

Fig. 5.36- RMS Velocity Response of a BBW SDOF System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.1$.

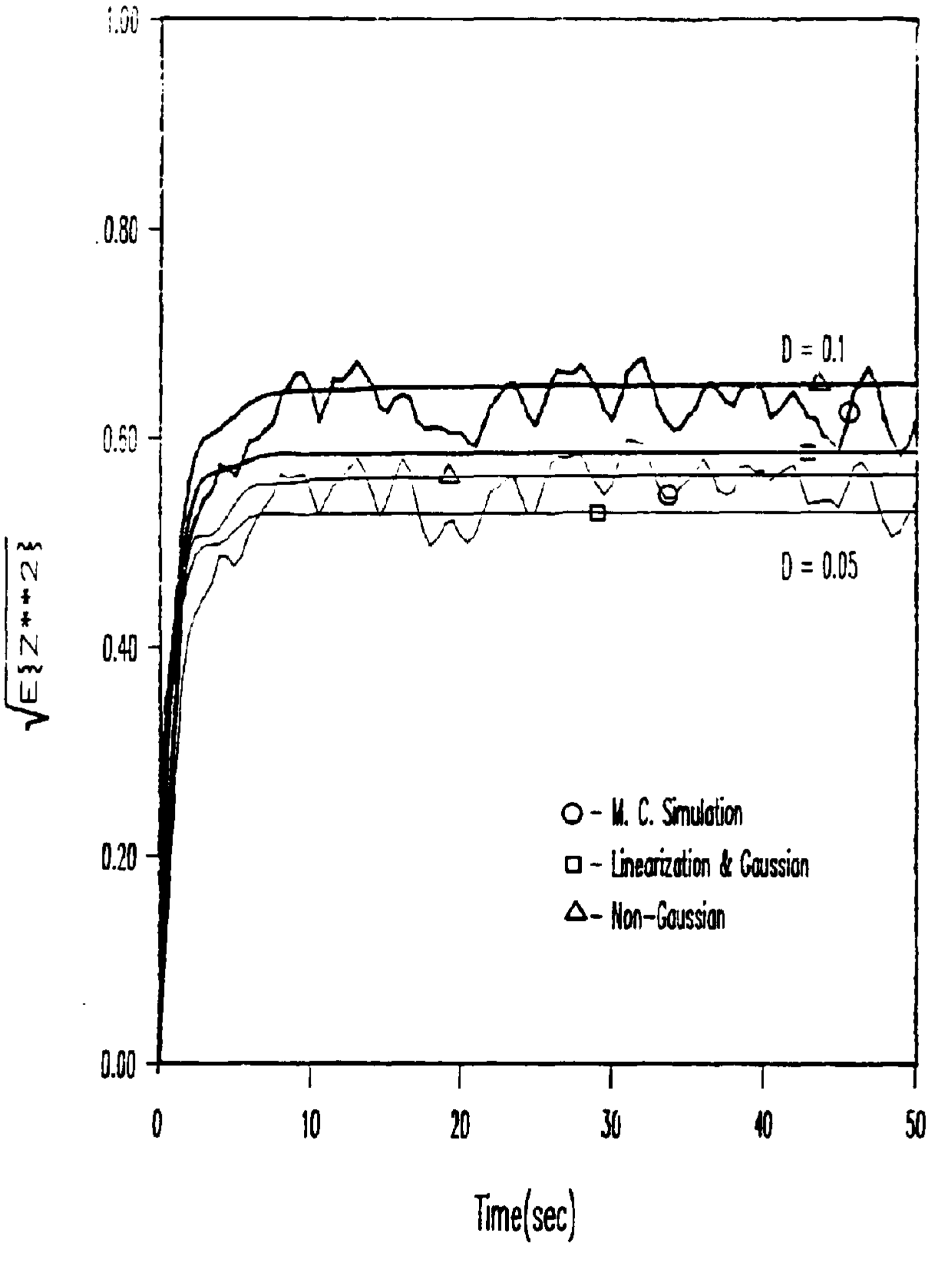

Fig. 5.37- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

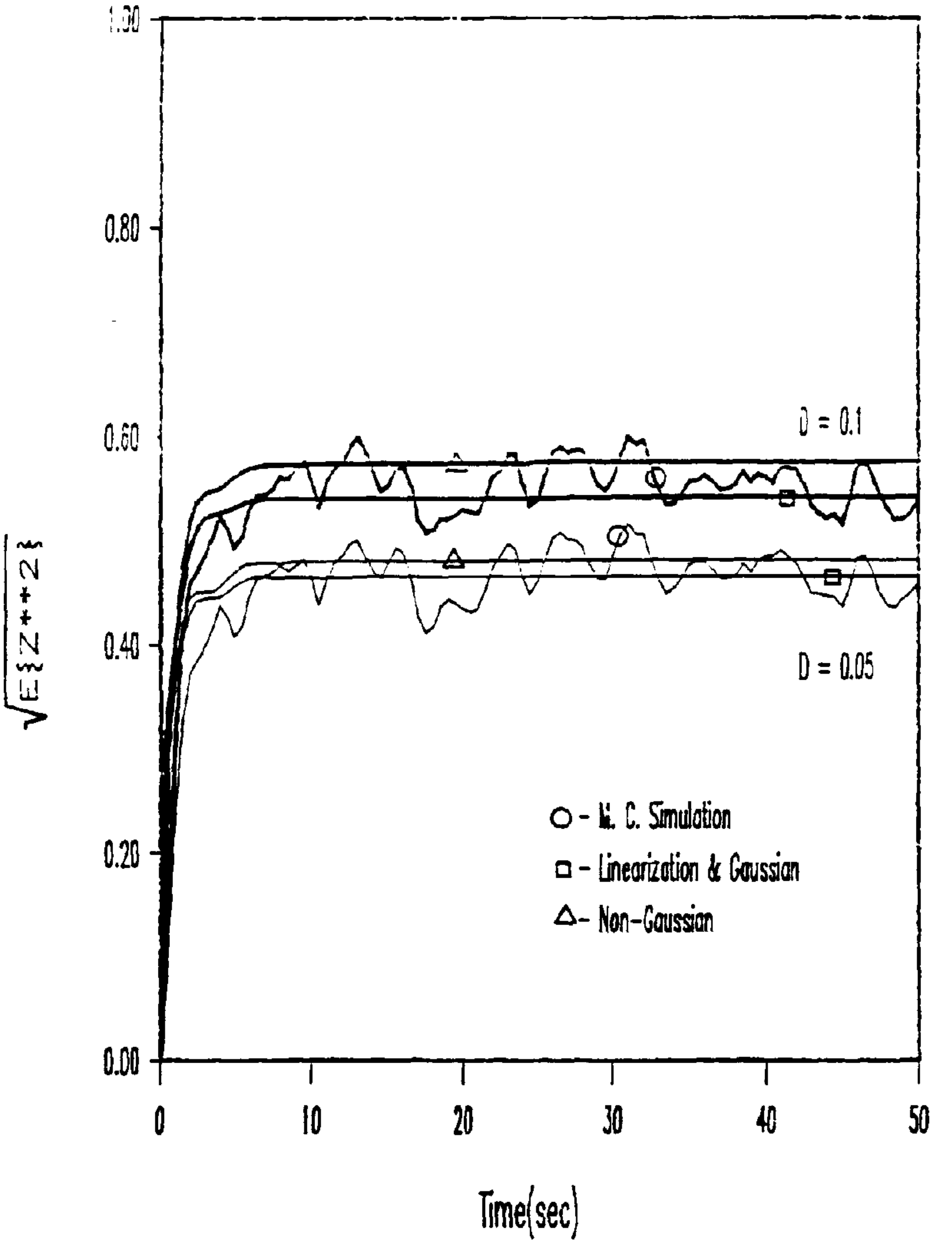

Fig. 5.38- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

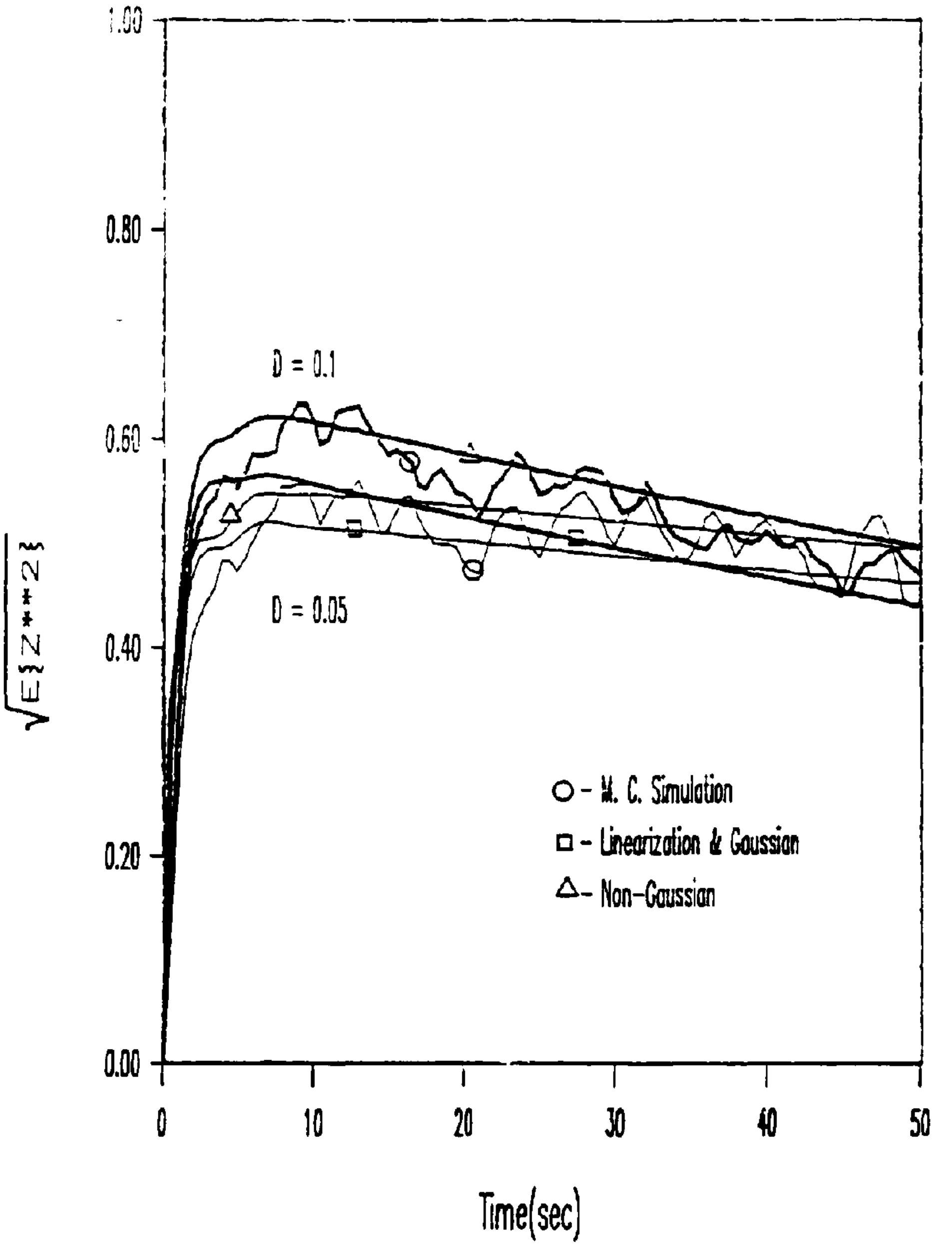

Fig. 5.39- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.01$. $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

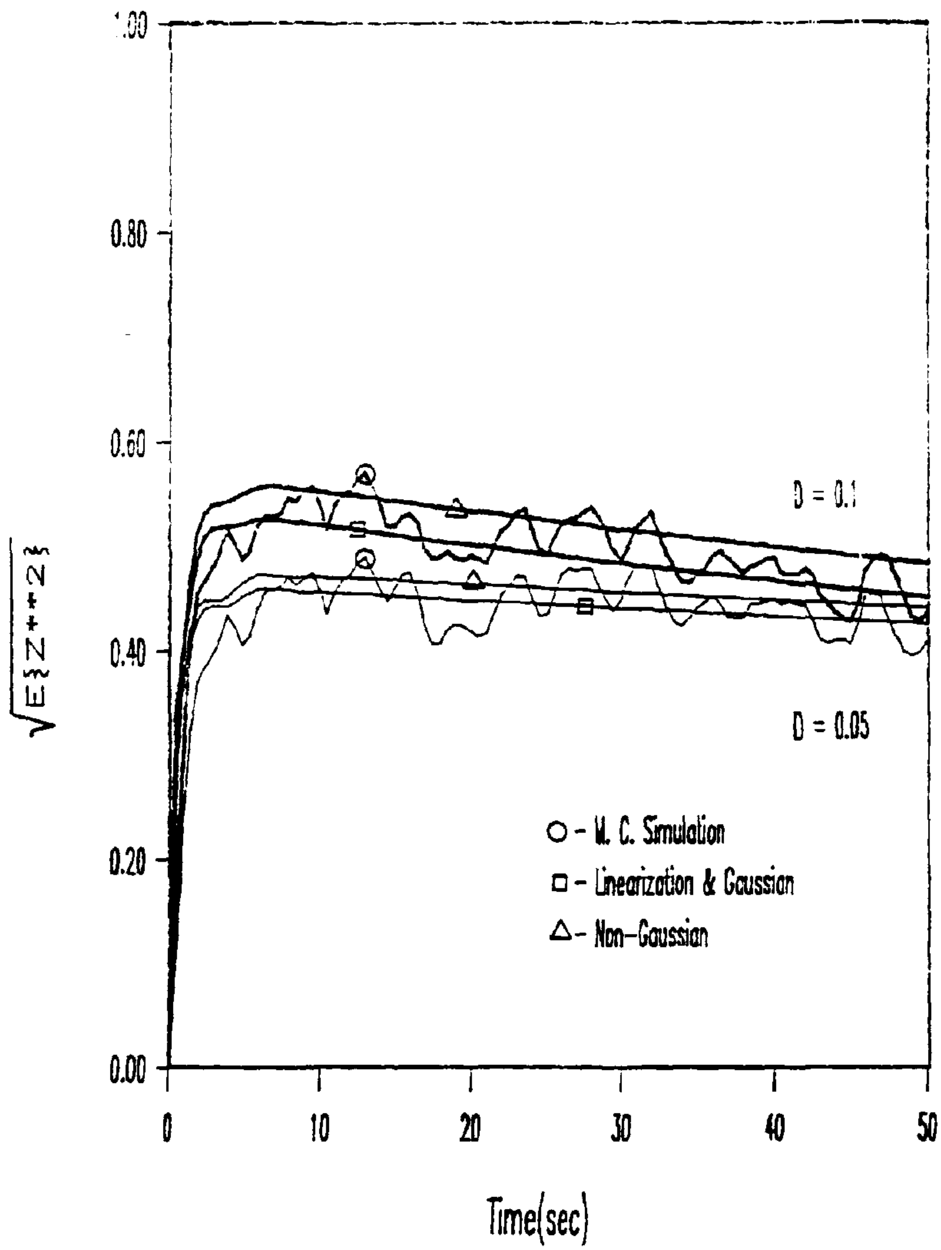

Fig. 5.40- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

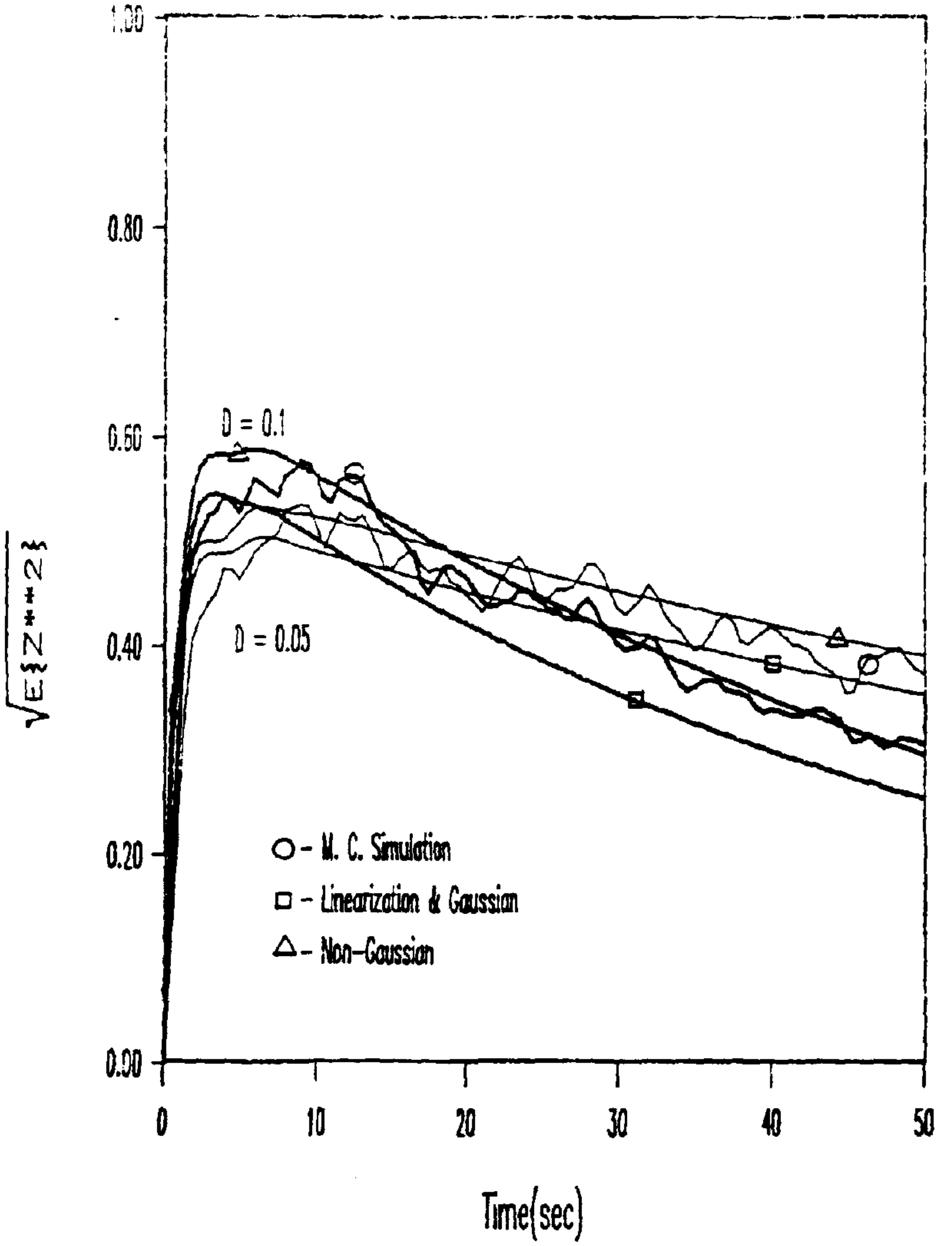

Fig. 5.41- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.0,\ \delta_A = 0.02,\ \delta_\nu = 0.05,\ \delta_\eta = 0.1.$

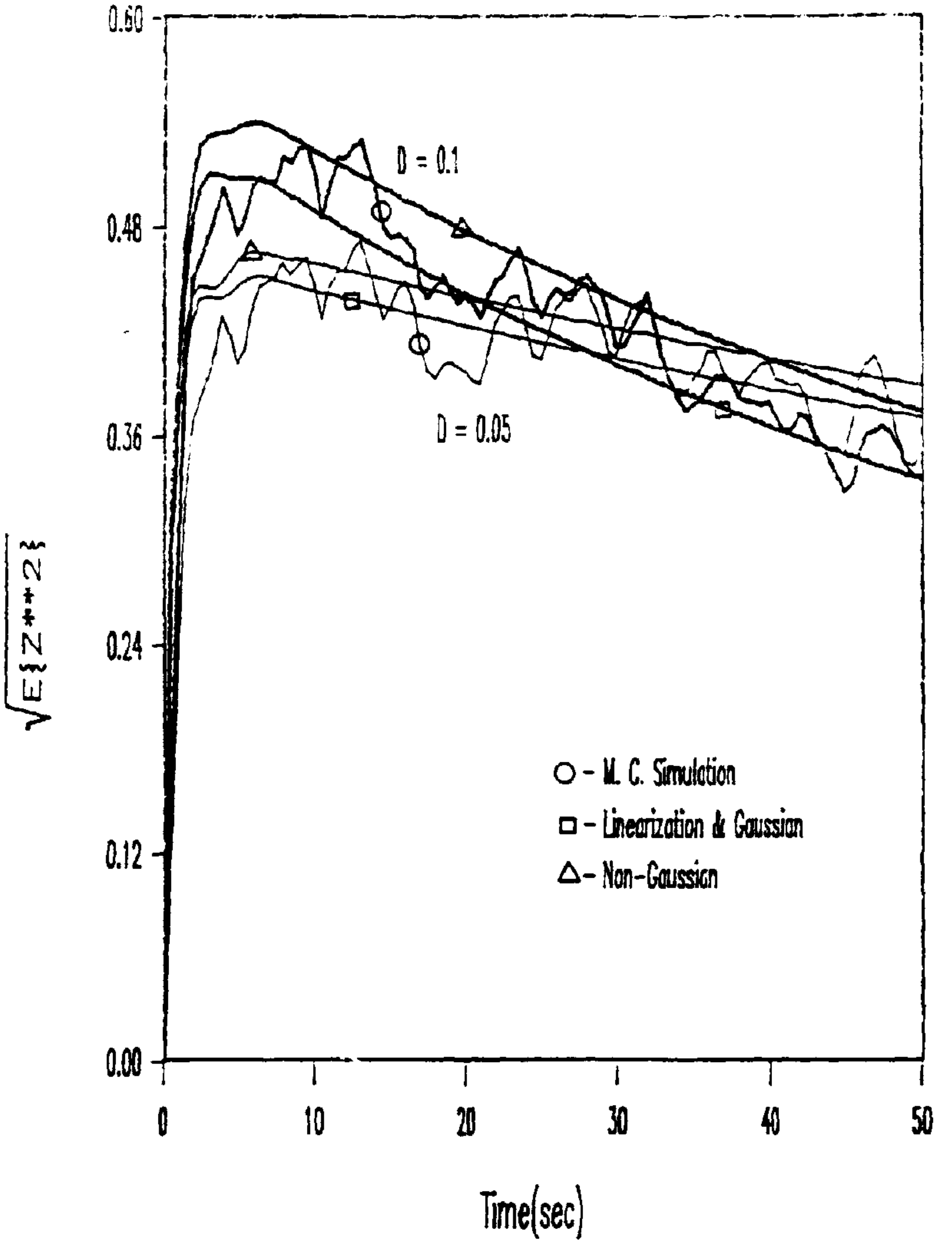

Fig. 5.42- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.02$, $\delta_\nu = 0.05$, $\delta_\eta = 0.1$.

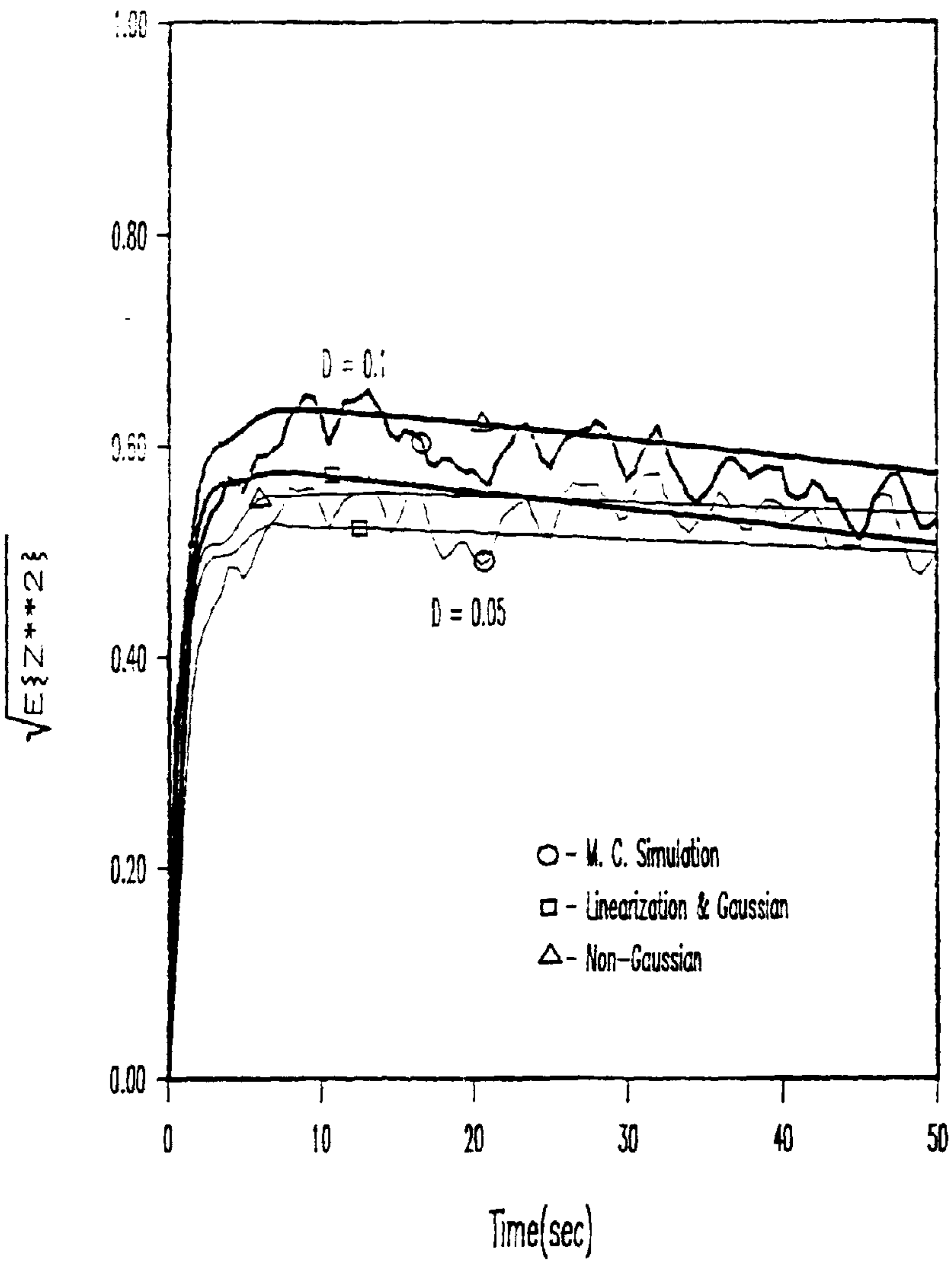

Fig. 5.43- RMS Response of the Restoring Force for a SDOF BDW System.
$\xi = 0.0$, $\delta_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

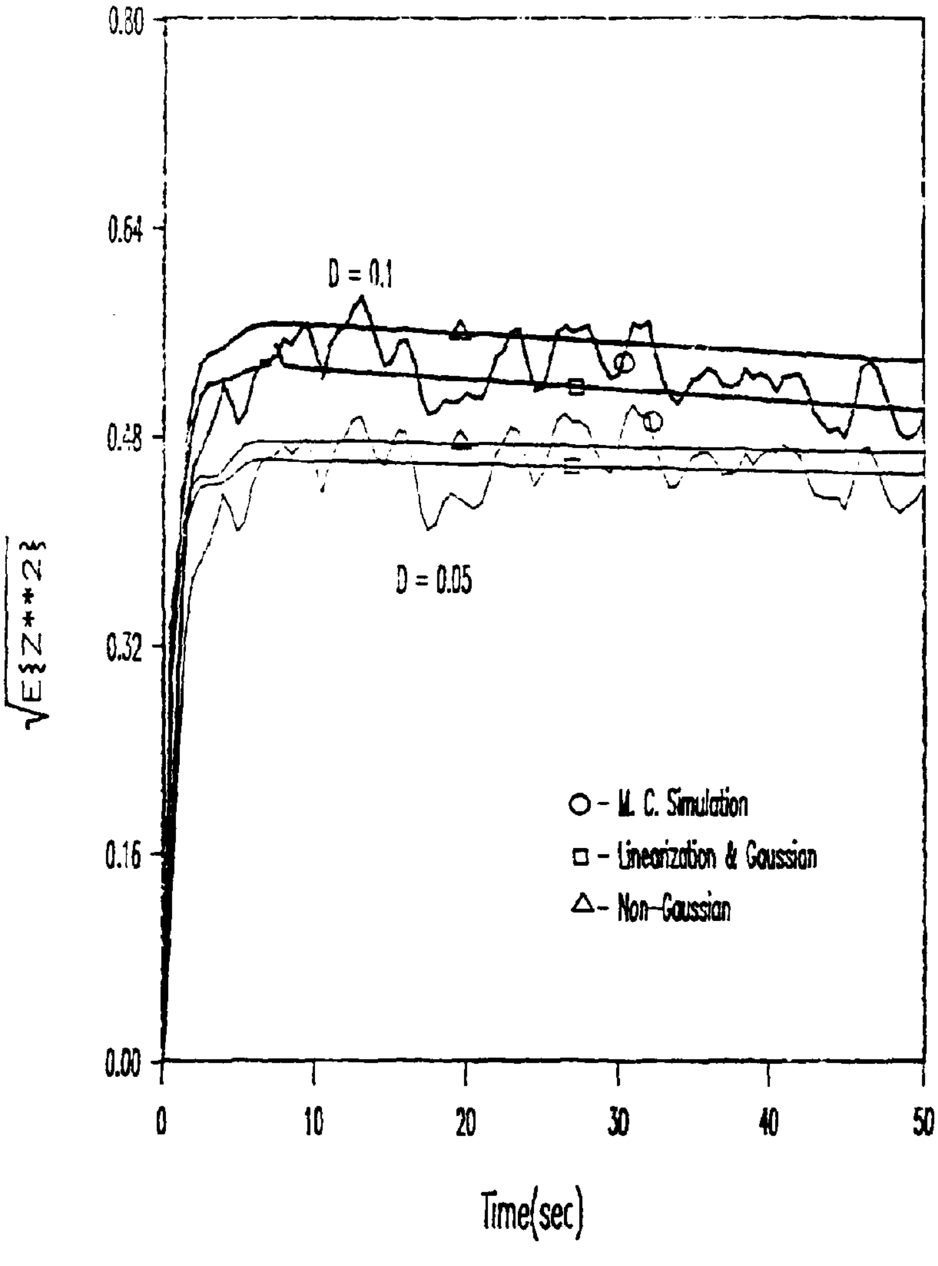

Fig. 5.44- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

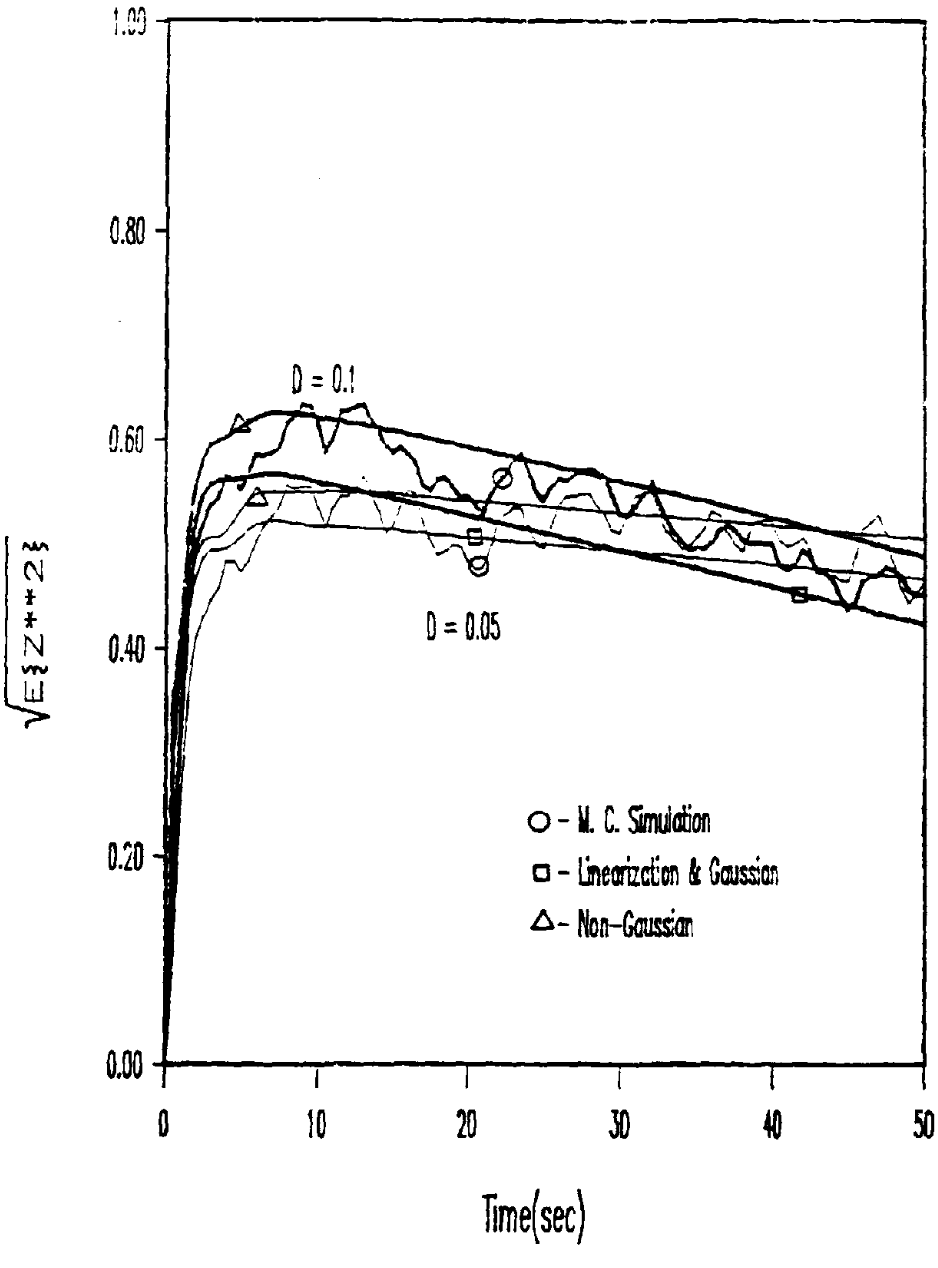

Fig. 5.45- RMS Response of the Restoring Force for a SDOF BBW System.
ξ = 0.0, δ_A = 0.02, δ_ν = 0.0, δ_η = 0.0.

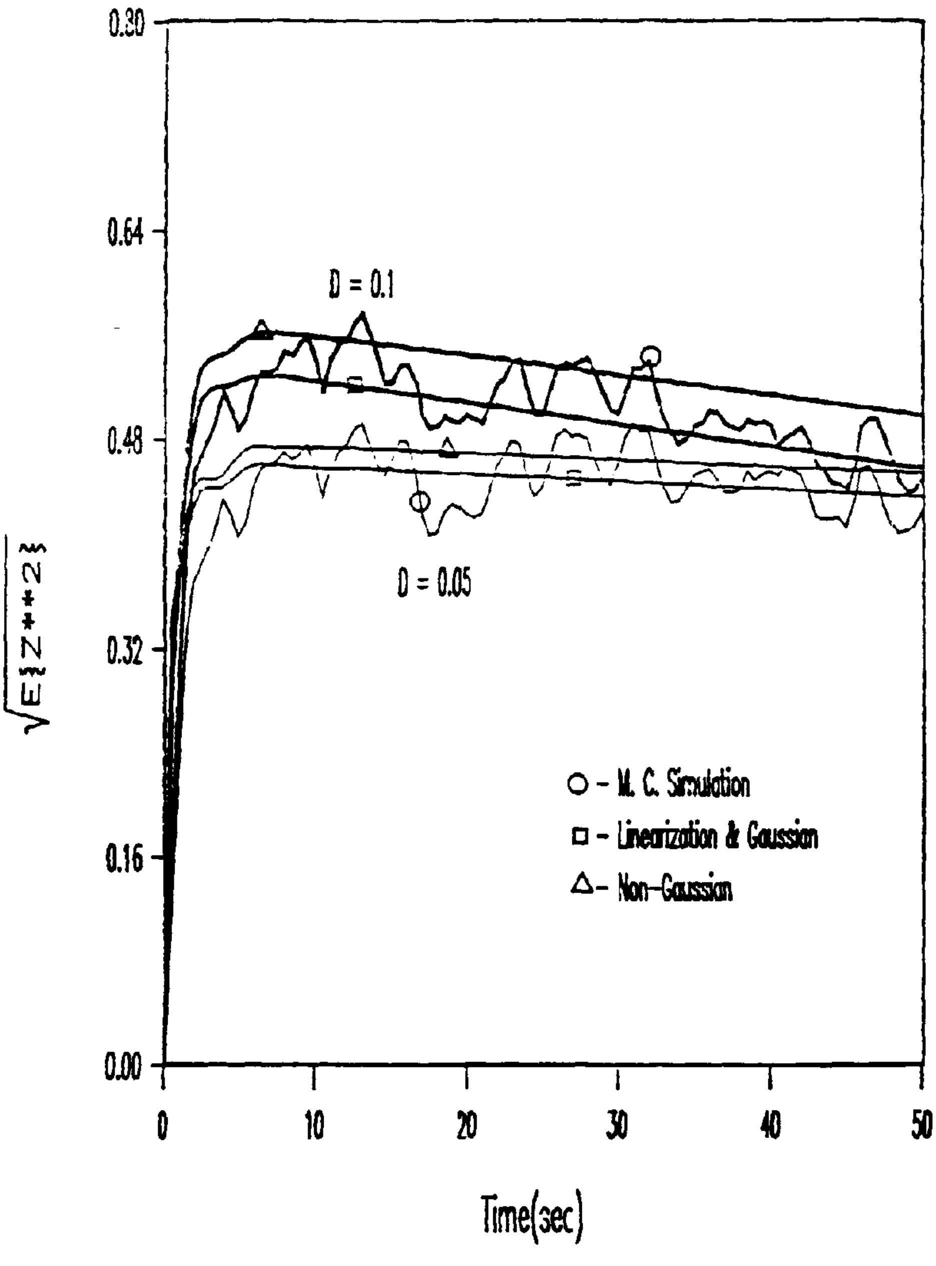

Fig. 5.46- RMS Response of the Restoring Force for a SDOF BBW System.
ξ = 0.1, δ_A = 0.02, δ_ν = 0.0, δ_η = 0.0.

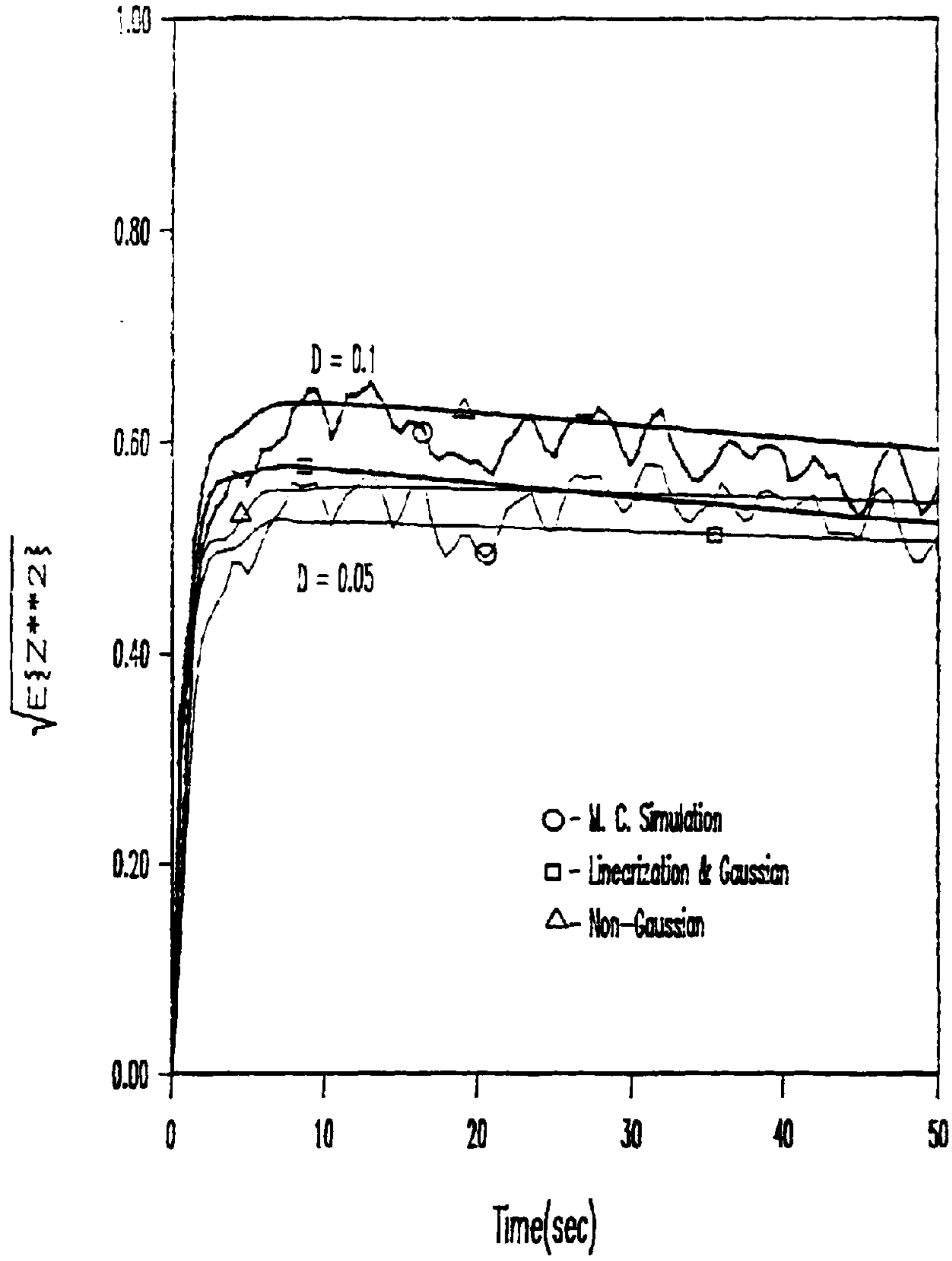

Fig. 5.47- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.01$, $\delta_\eta = 0.0$.

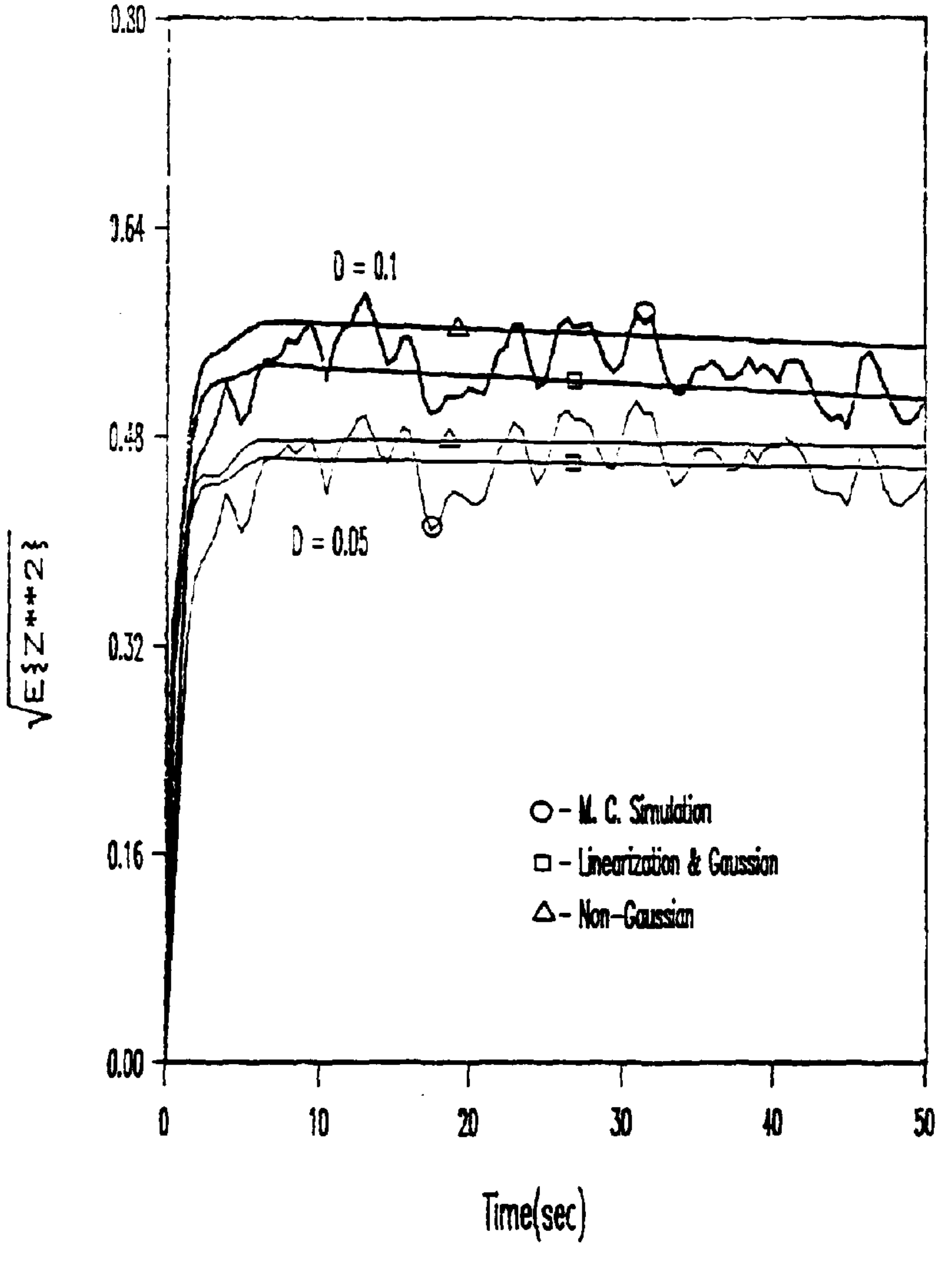

Fig. 5.48- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.01,\ \delta_\eta = 0.0.$

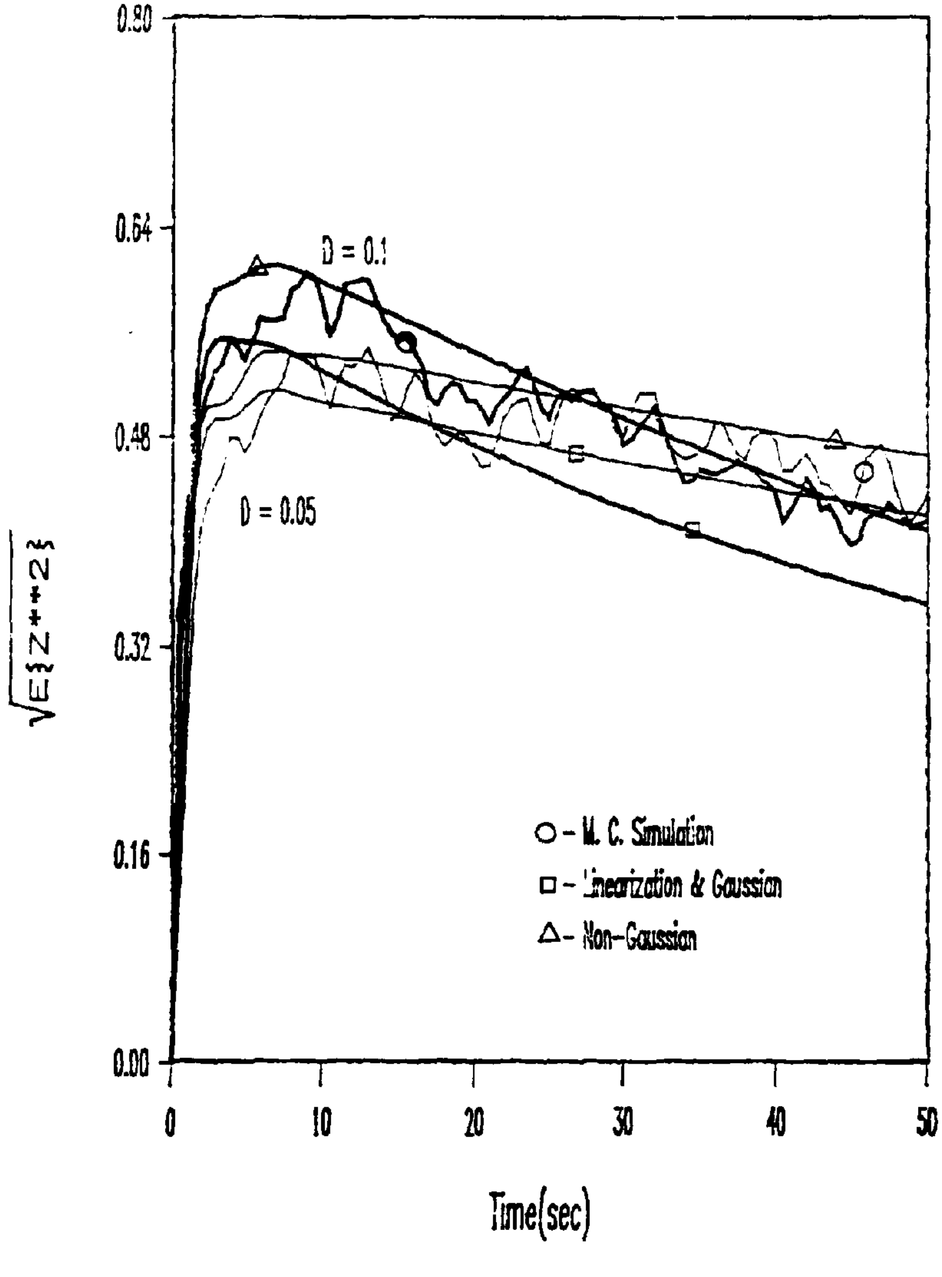

Fig. 5.49- RMS Response of the Restoring Force for a SDOF BBW System.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.05, δ_η = 0.0.

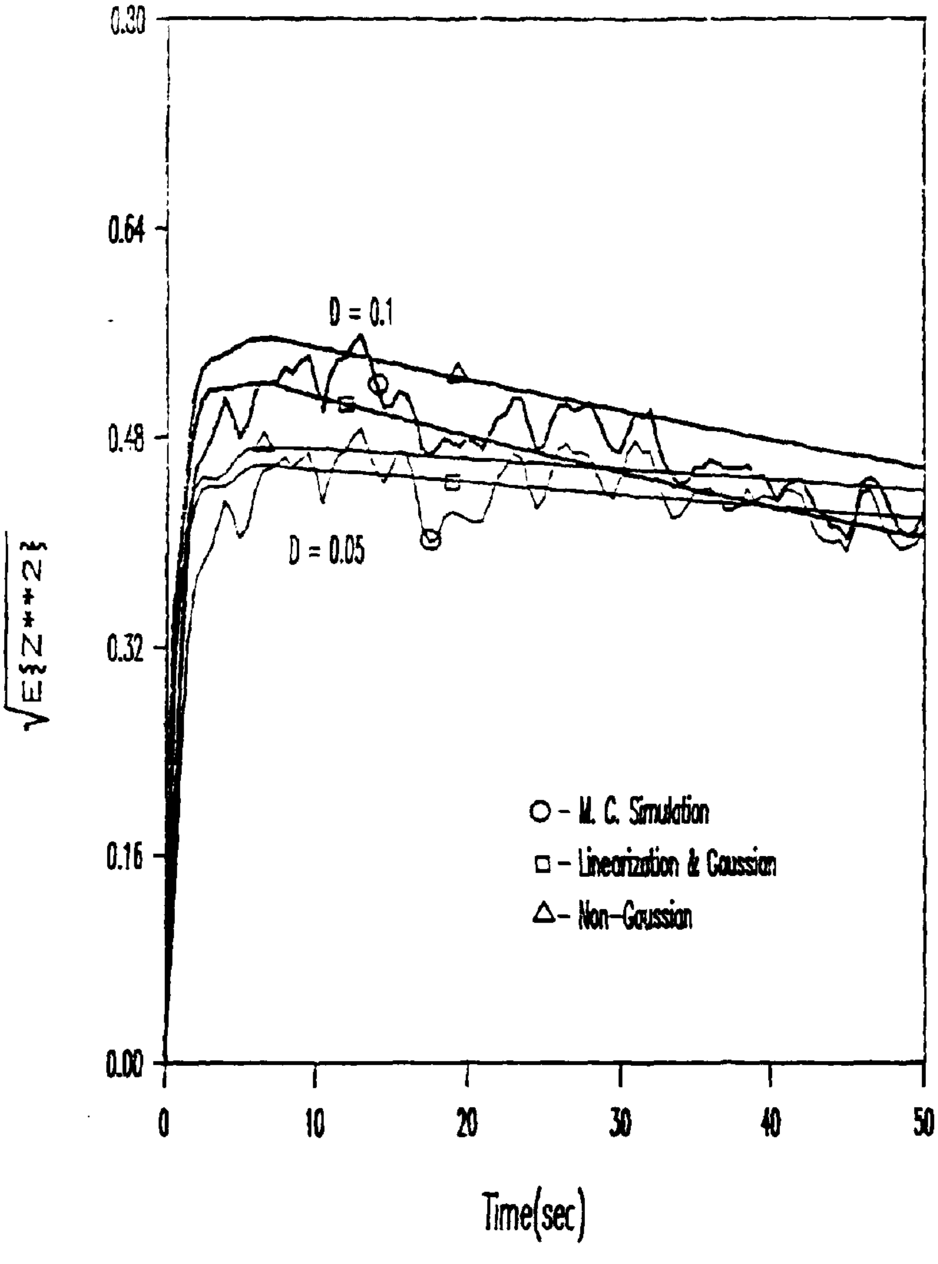

Fig. 5.50- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1$, $\delta_\lambda = 0.0$, $\delta_\nu = 0.05$, $\delta_\eta = 0.0$.

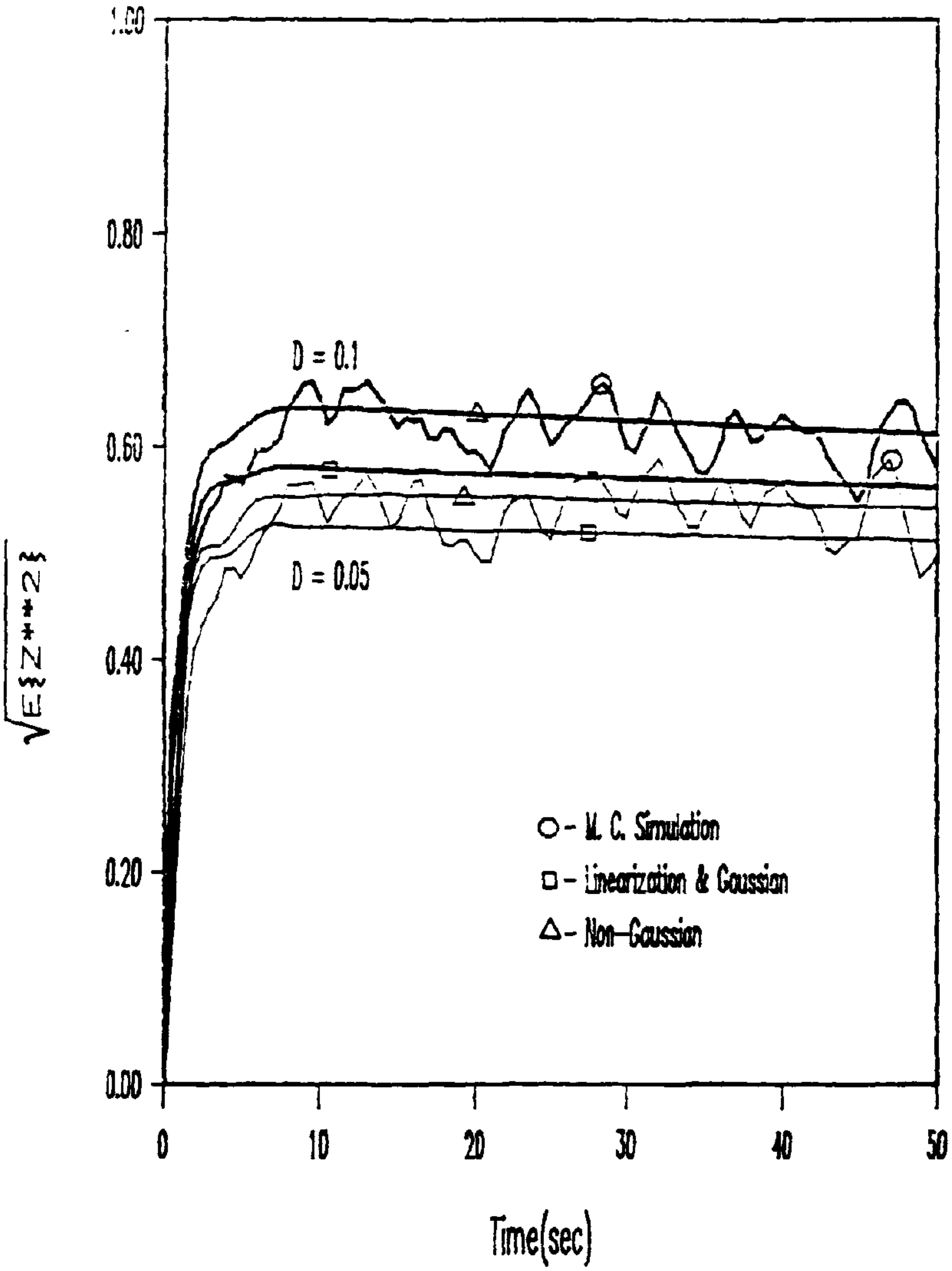

Fig. 5.51- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.05.$

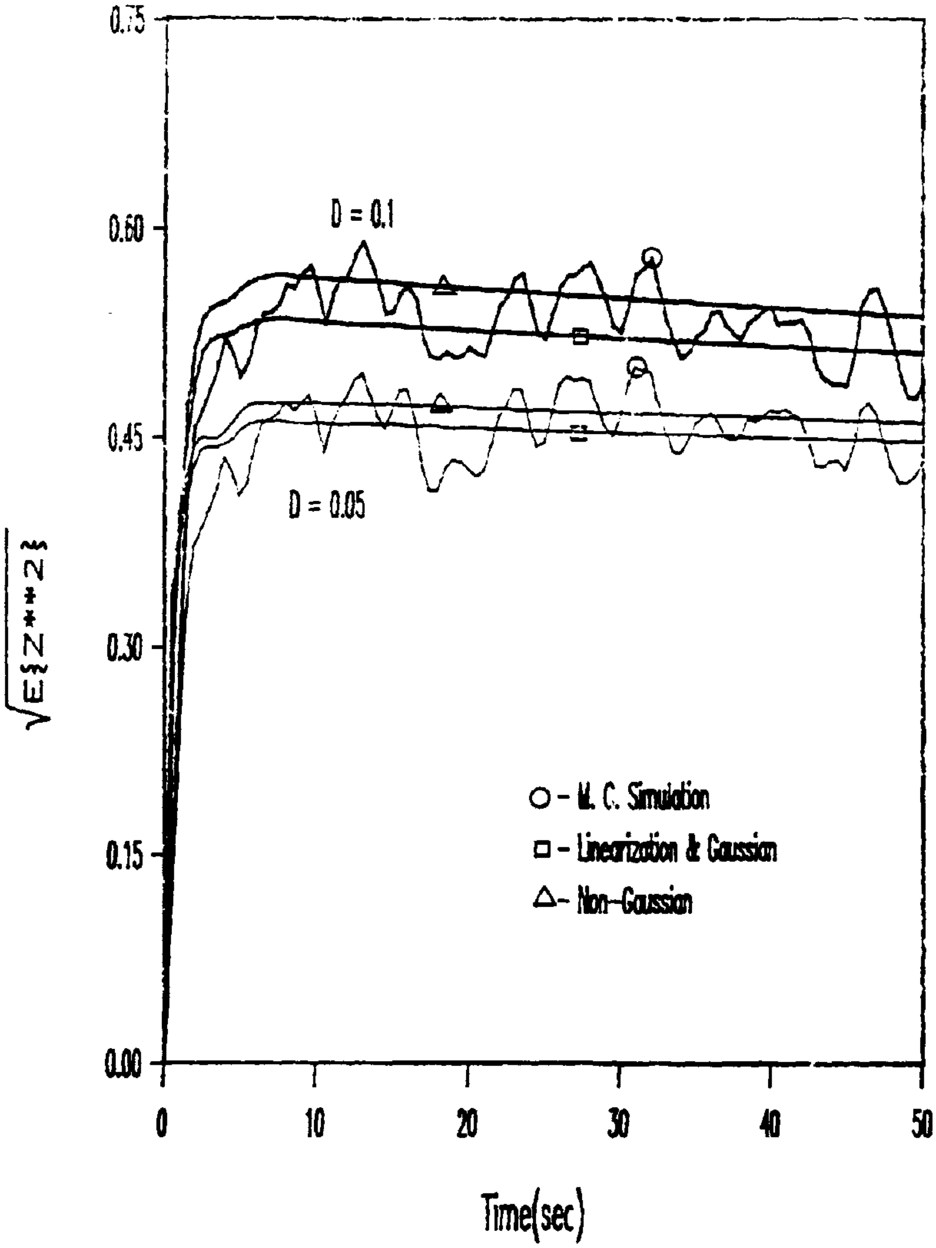

Fig. 5.52- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.05$.

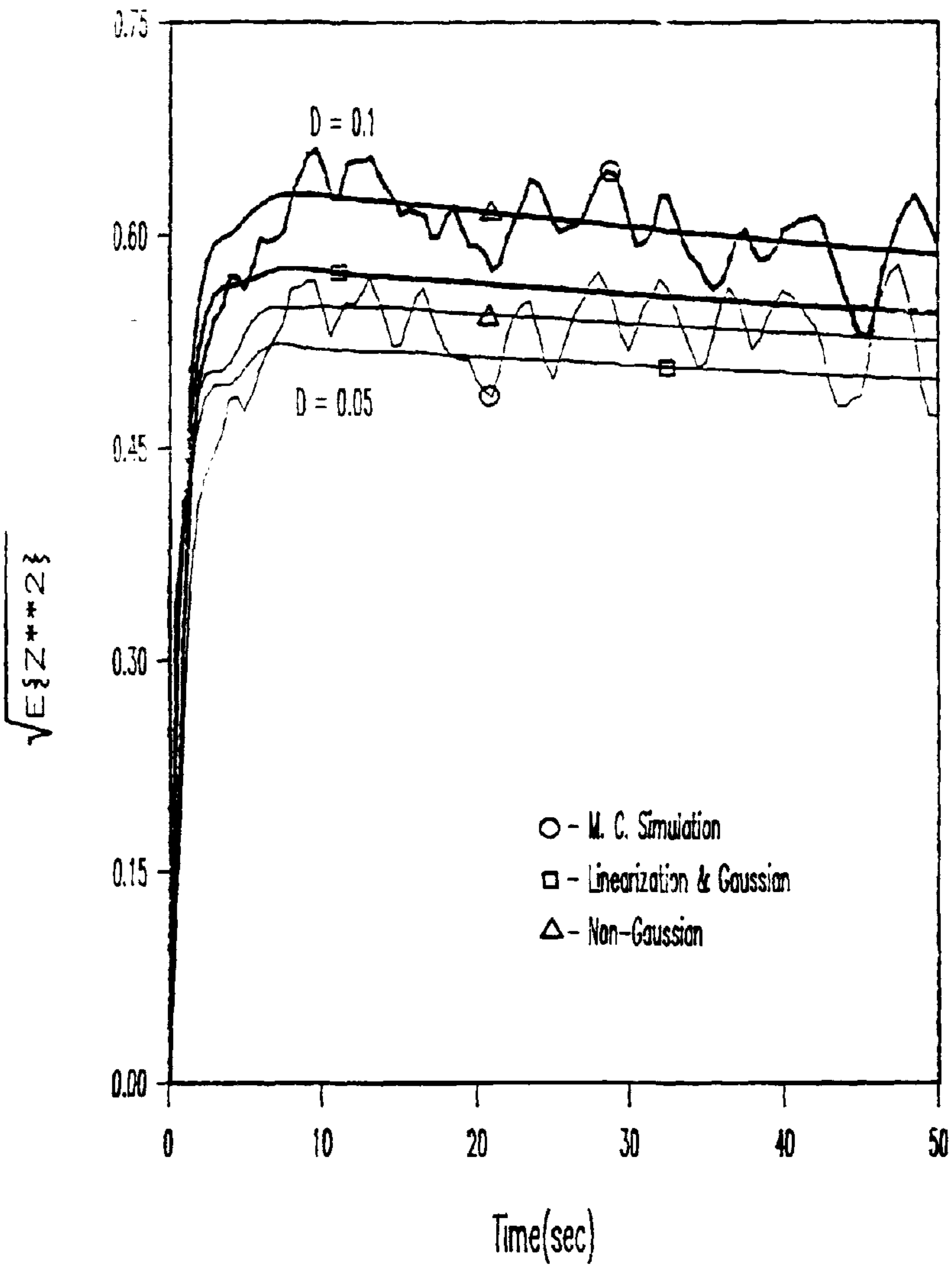

Fig. 5.53- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.1.$

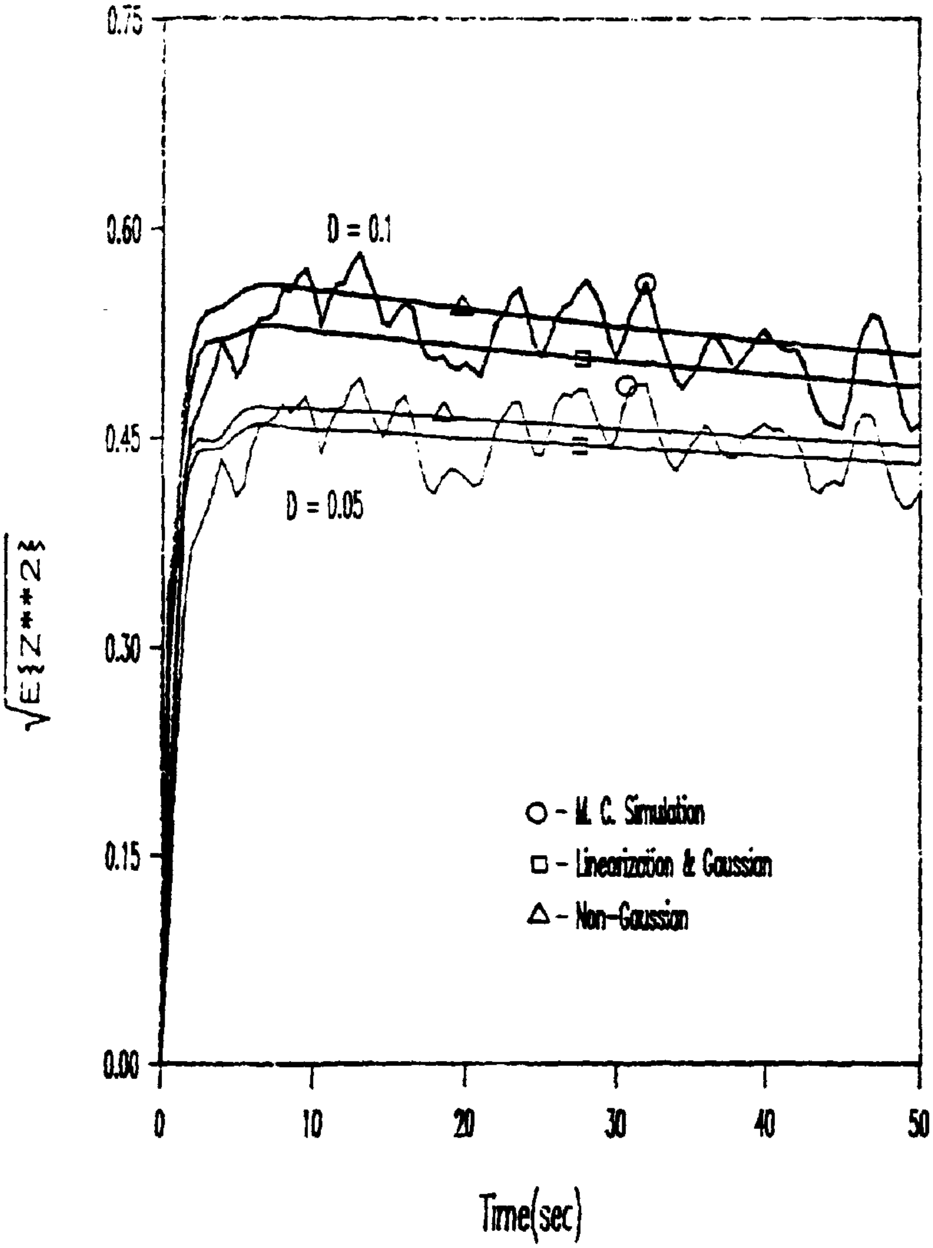

Fig. 5.54- RMS Response of the Restoring Force for a SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.1$.

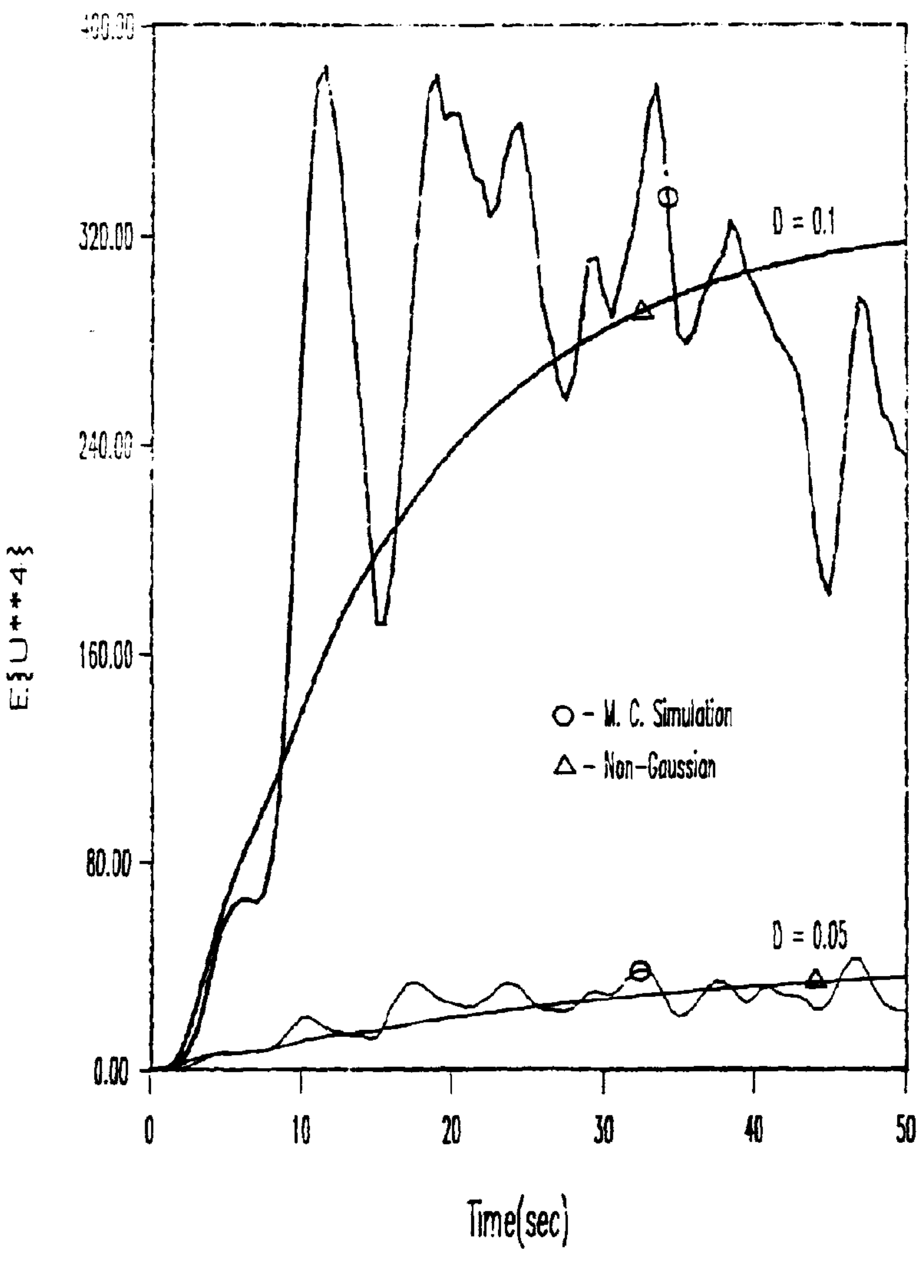

Fig. 5.55- **Fourth Order Moment of the Displacement Response.**
ξ = 0.0, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.0.

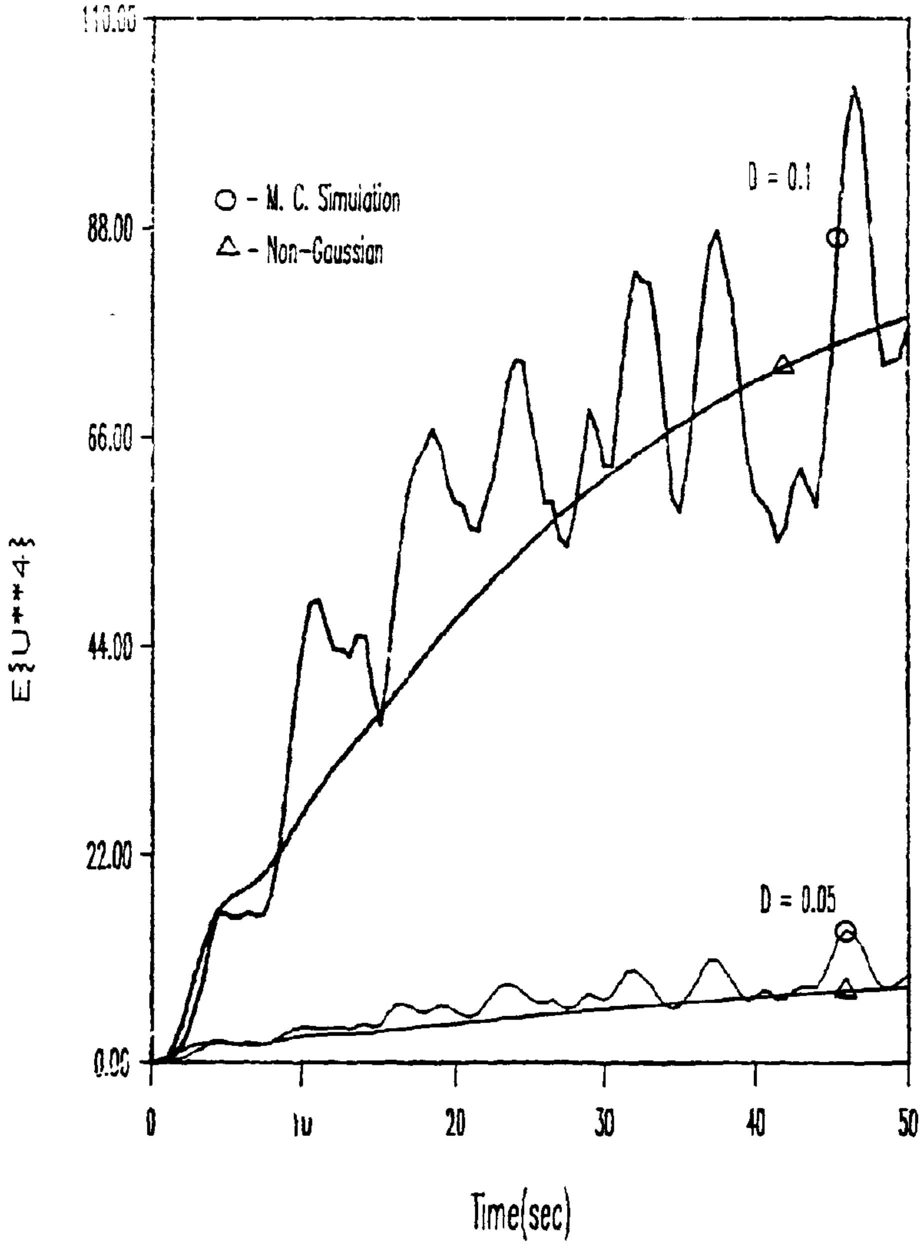

Fig. 5.56- Fourth Order Moment of the Displacement Response.
$\xi = 0.1,\ \delta_{\lambda} = 0.0,\ \delta_{\nu} = 0.0,\ \delta_{\eta} = 0.0.$

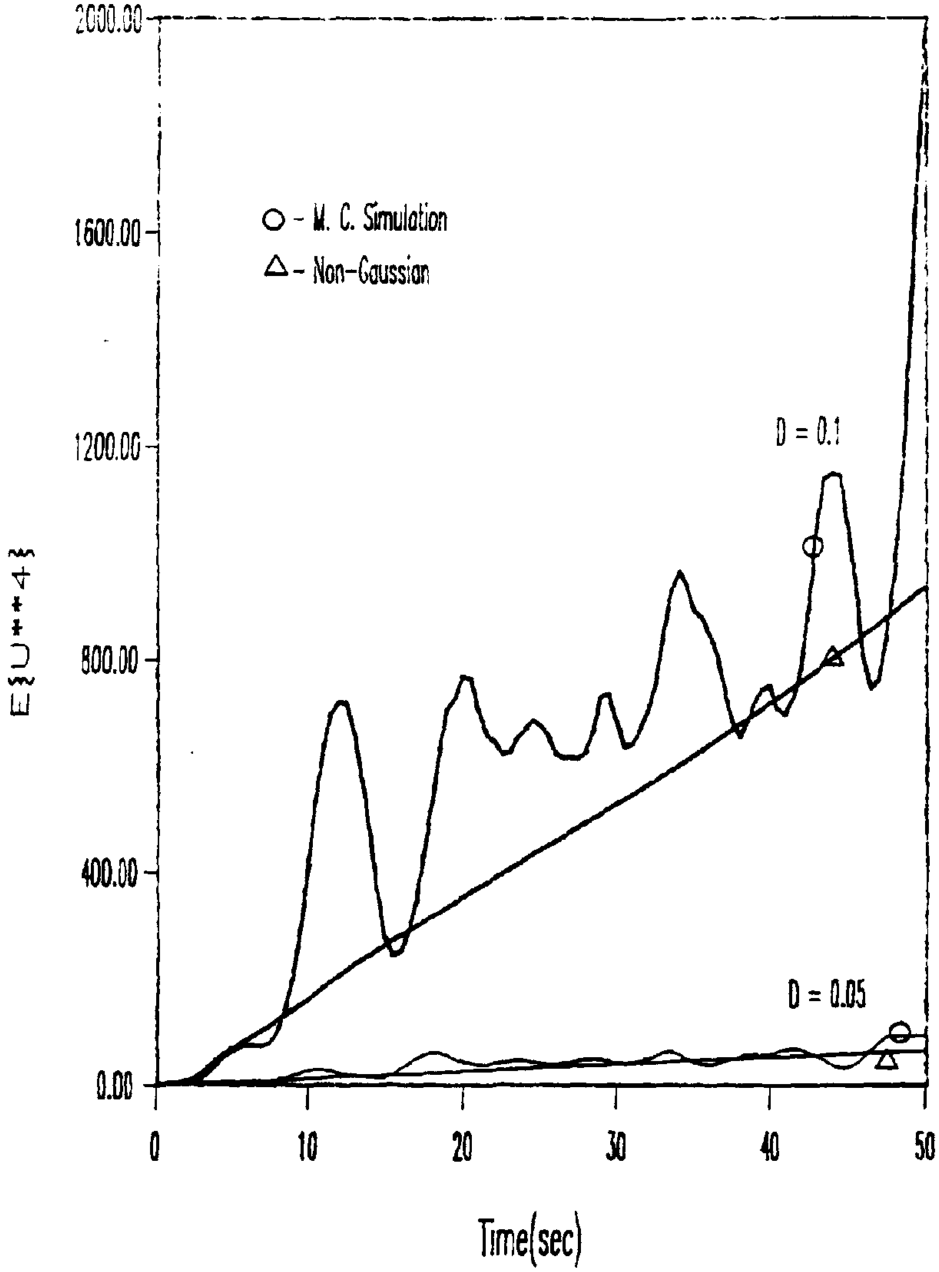

Fig. 5.57- Fourth Order Moment of the Displacement Response. $\xi = 0.0$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

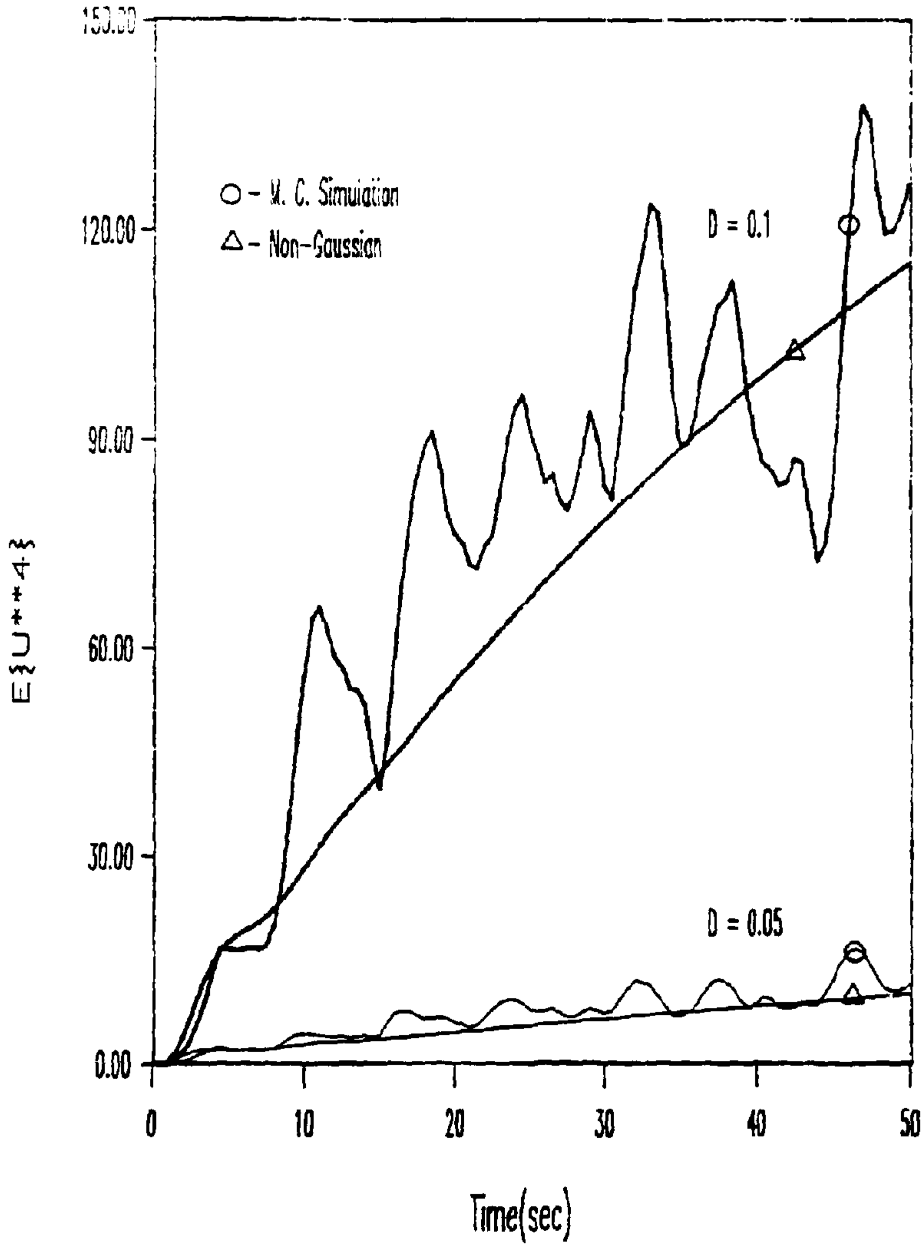

Fig. 5.58- Fourth Order Moment of the Displacement Response.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

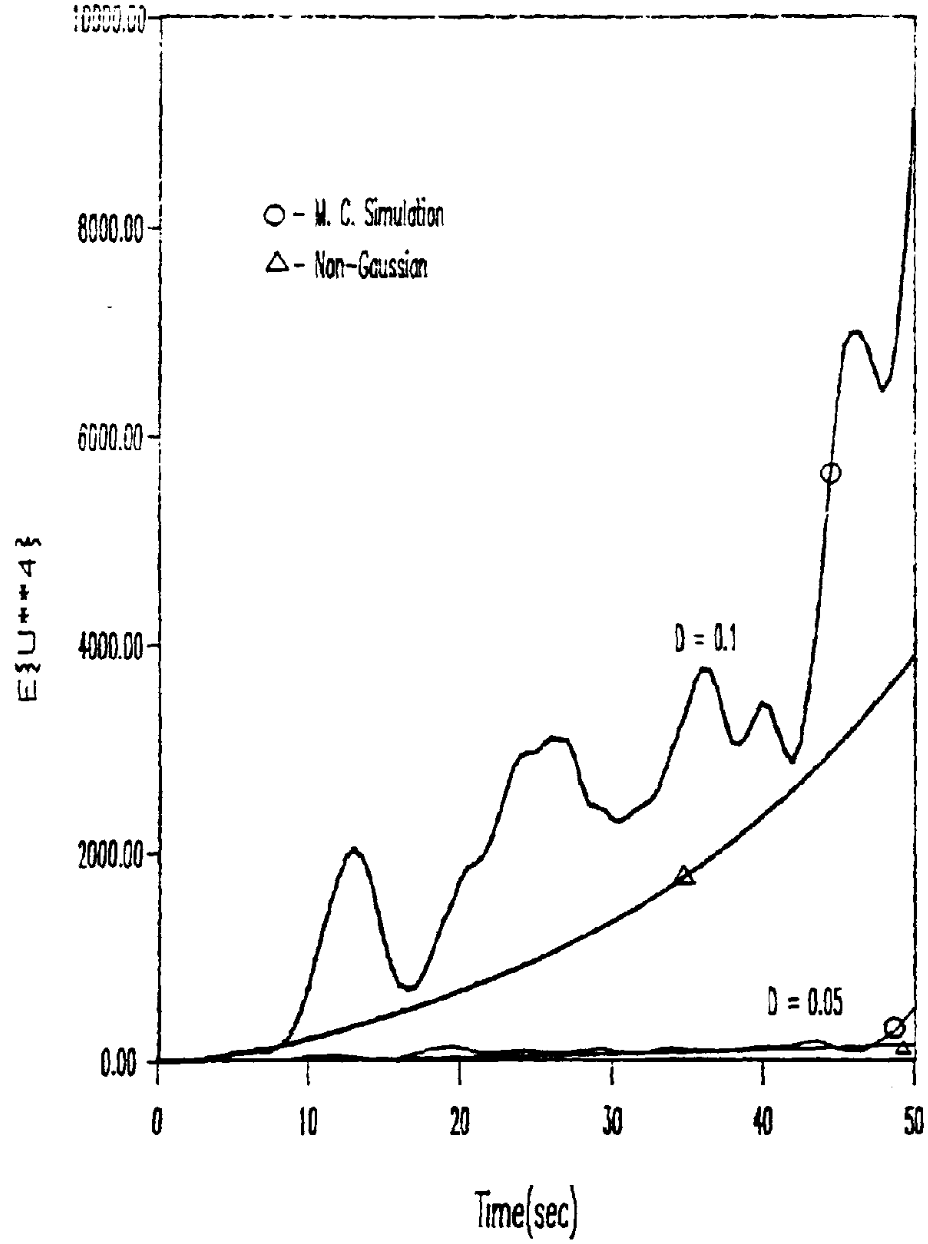

Fig. 5.59- Fourth Order Moment of the Displacement Response.
$\xi = 0.0,\ \delta_A = 0.02,\ \delta_\nu = 0.05,\ \delta_\eta = 0.1.$

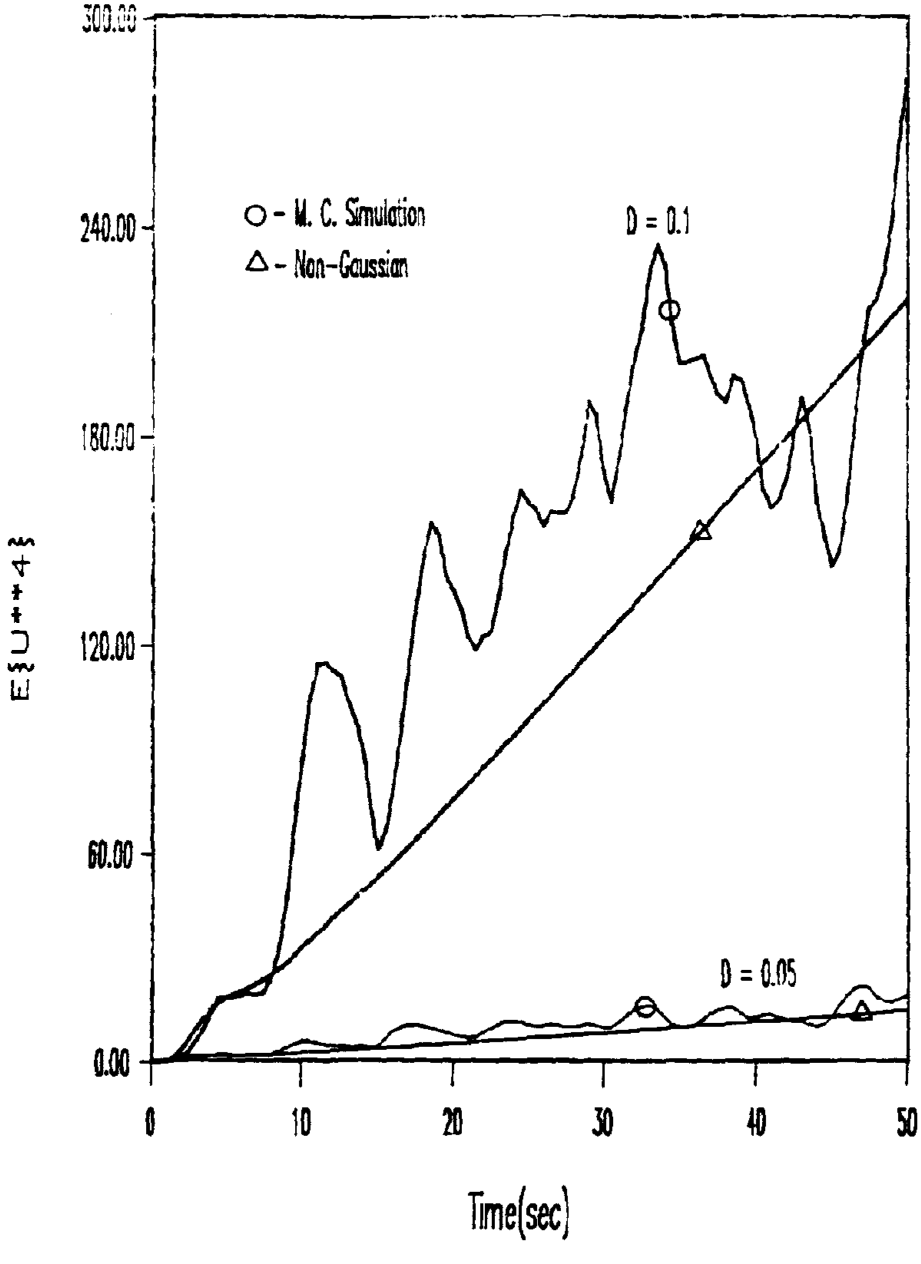

Fig. 5.60- Fourth Order Moment of the Displacement Response.
ξ = 0.1, δ_A = 0.02, δ_ν = 0.05, δ_η = 0.1.

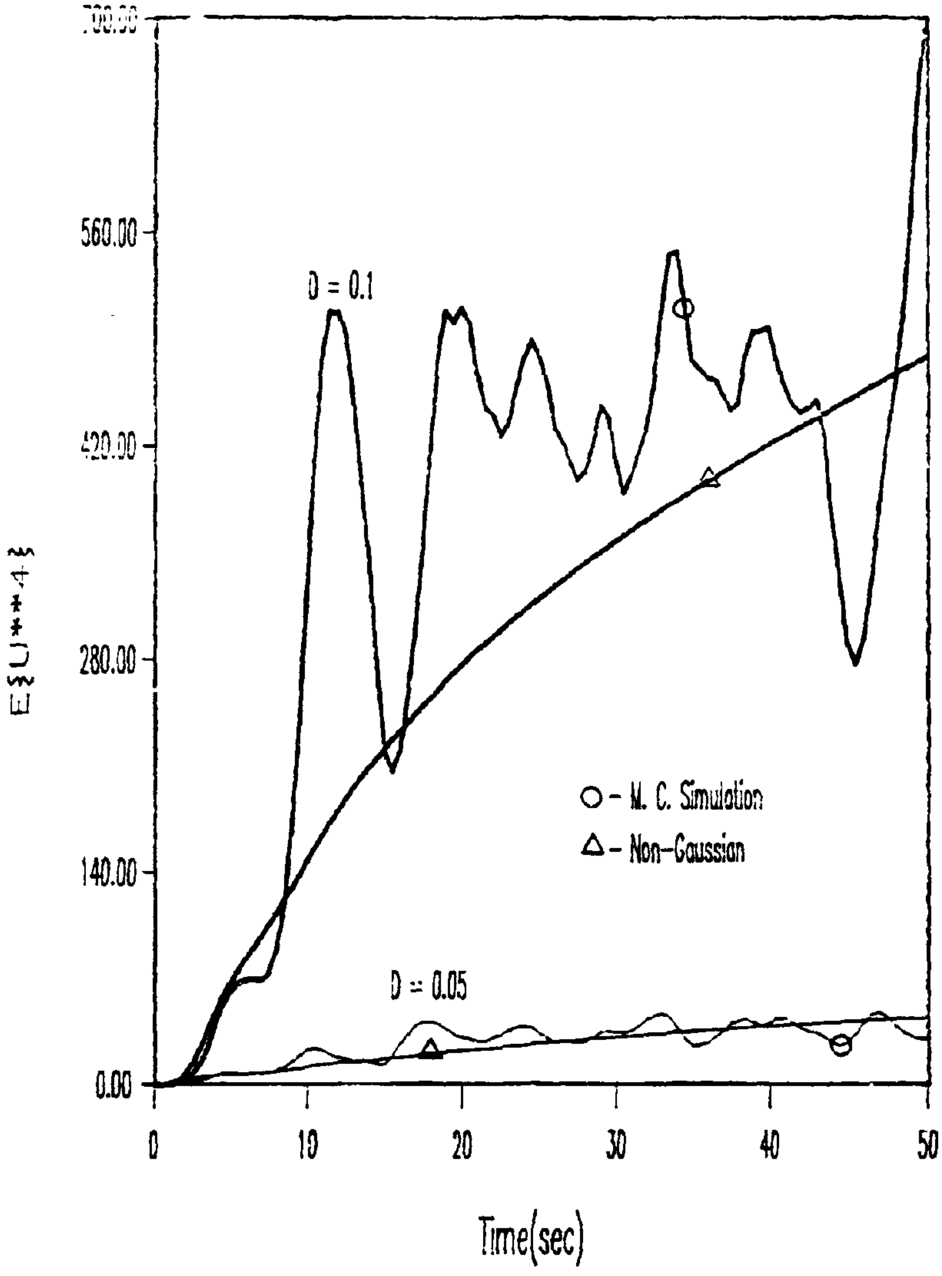

Fig. 5.61- Fourth Order Moment of the Displacement Response. ξ = 0.0, δ_{a} = 0.01, δ_{ν} = 0.0, δ_{η} = 0.0.

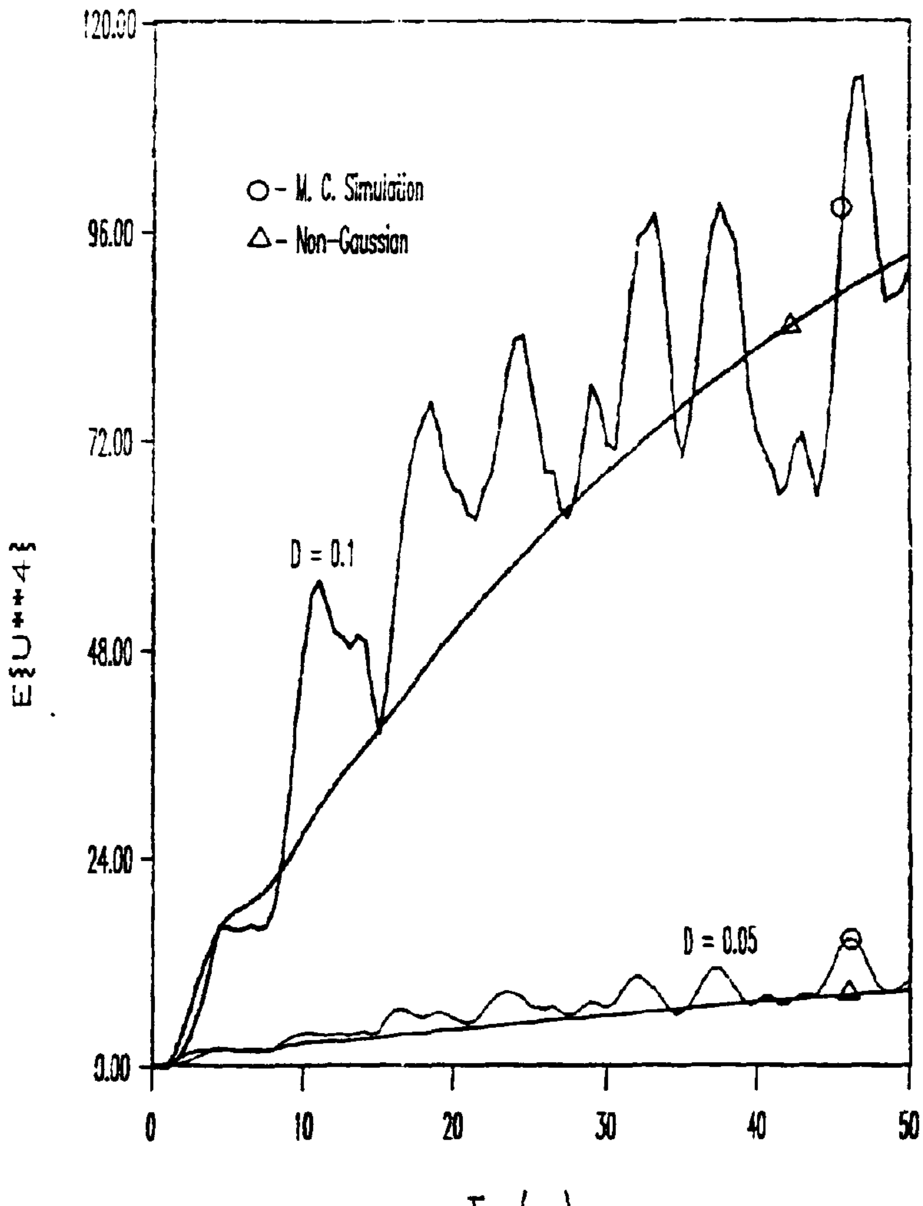

Fig. 5.62- Fourth Order Moment of the Displacement Response.
$\xi = 0.1,\ \delta_{\lambda} = 0.01,\ \delta_{\nu} = 0.0,\ \delta_{\eta} = 0.0.$

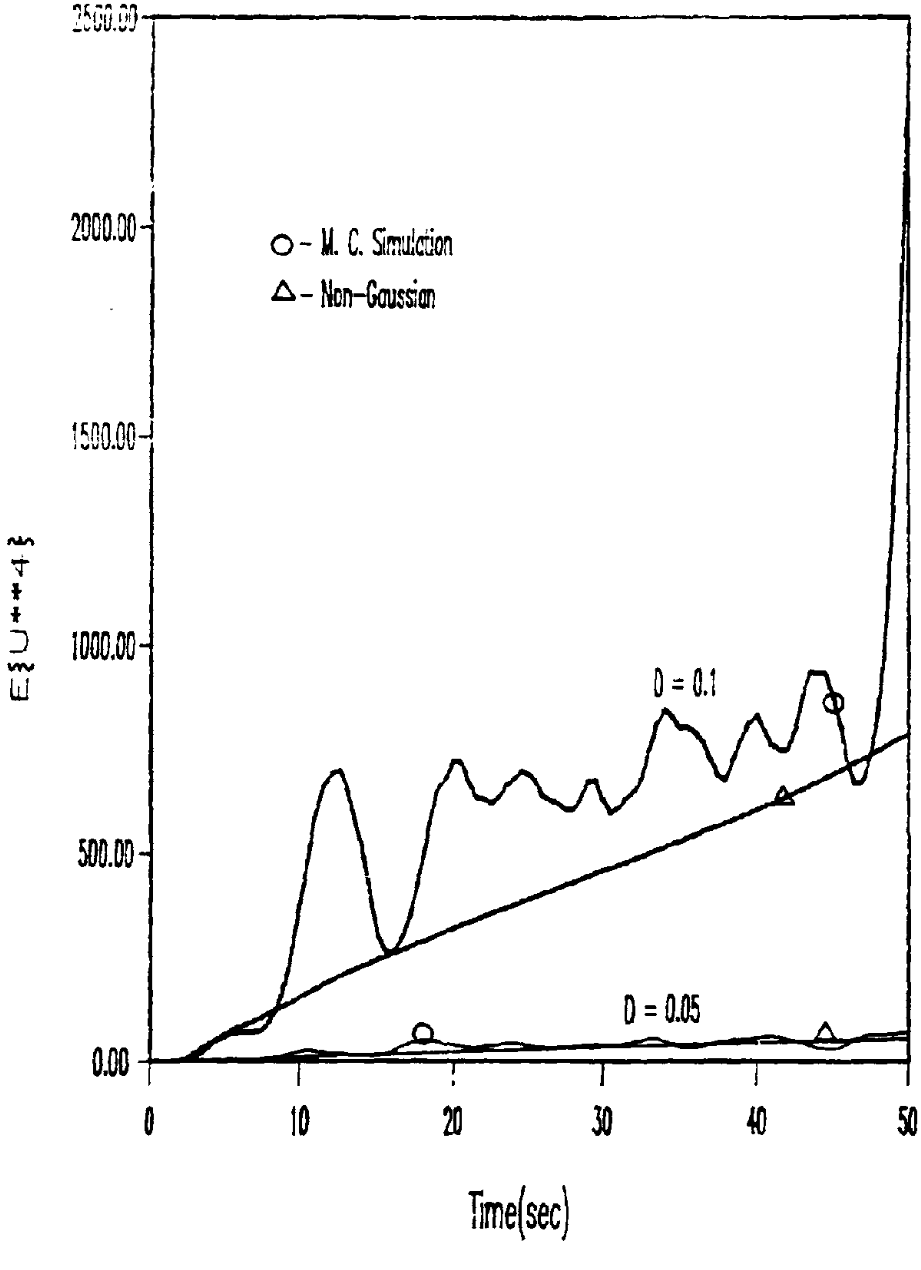

Fig. 5.63- Fourth Order Moment of the Displacement Response.
$\xi = 0.0,\ \delta_A = 0.02,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

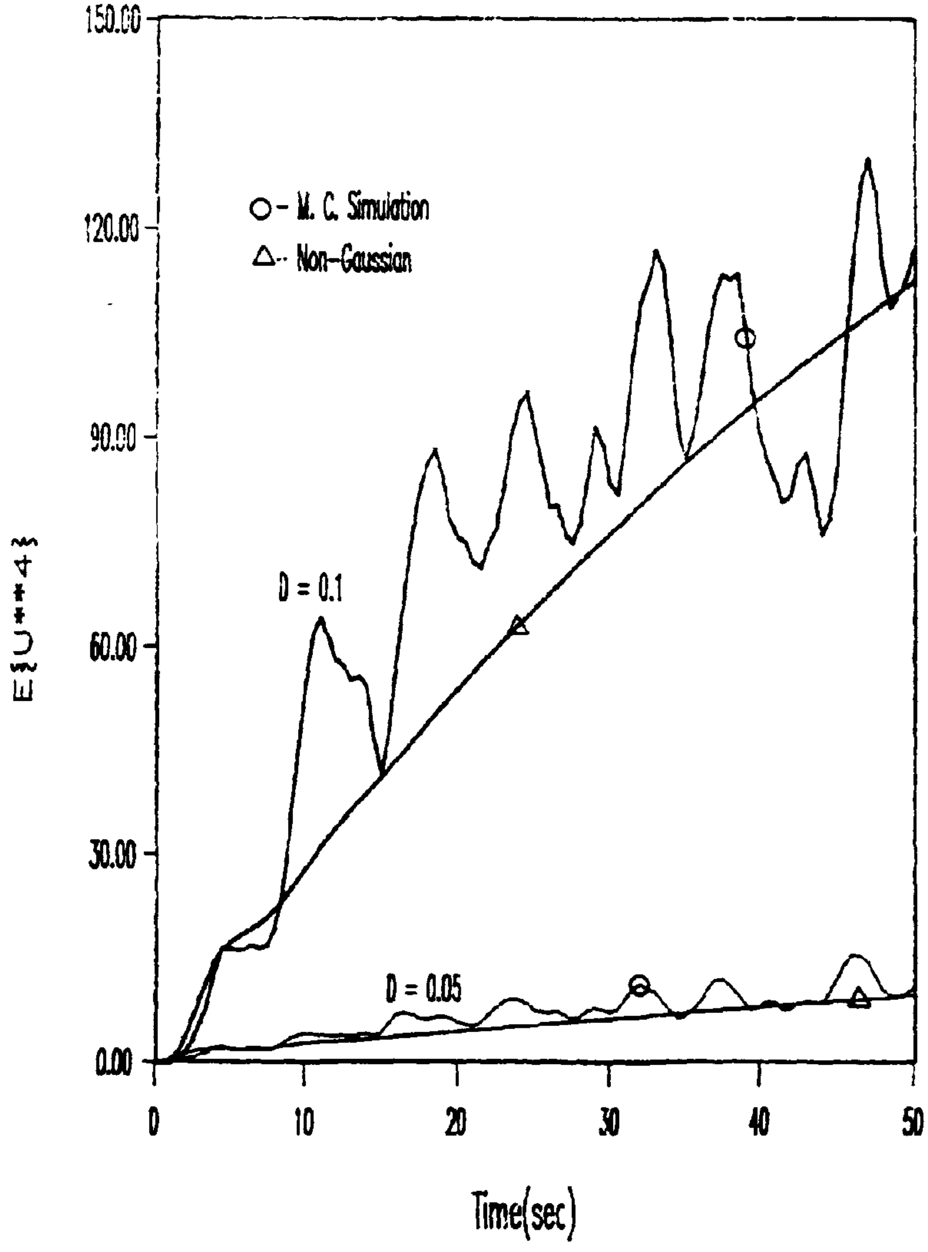

Fig. 5.64- Fourth Order Moment of the Displacement Response. $\xi = 0.1$, $\delta_A = 0.02$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

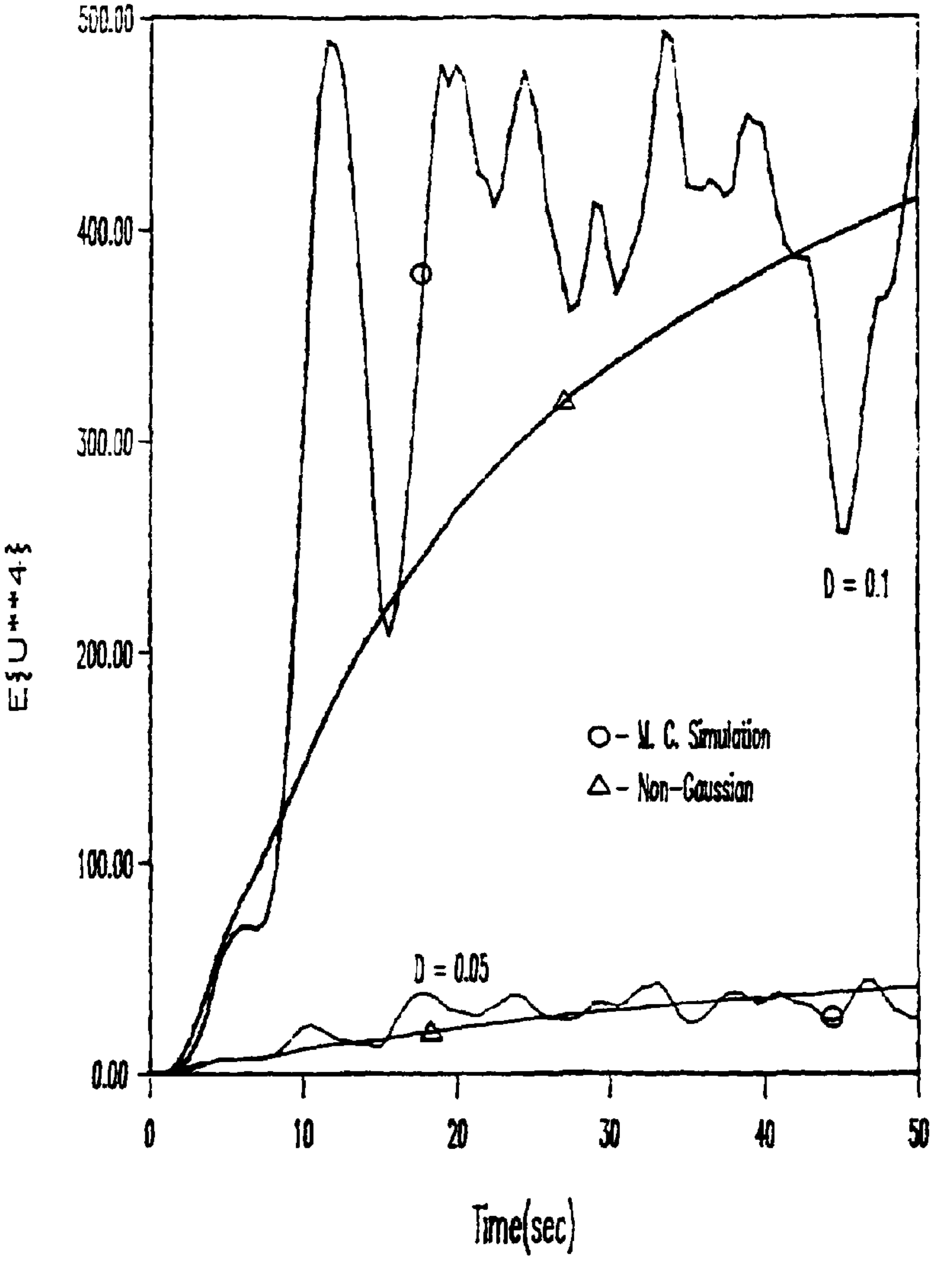

Fig. 5.65- Fourth Order Moment of the Displacement Response.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.01,\ \delta_\eta = 0.0.$

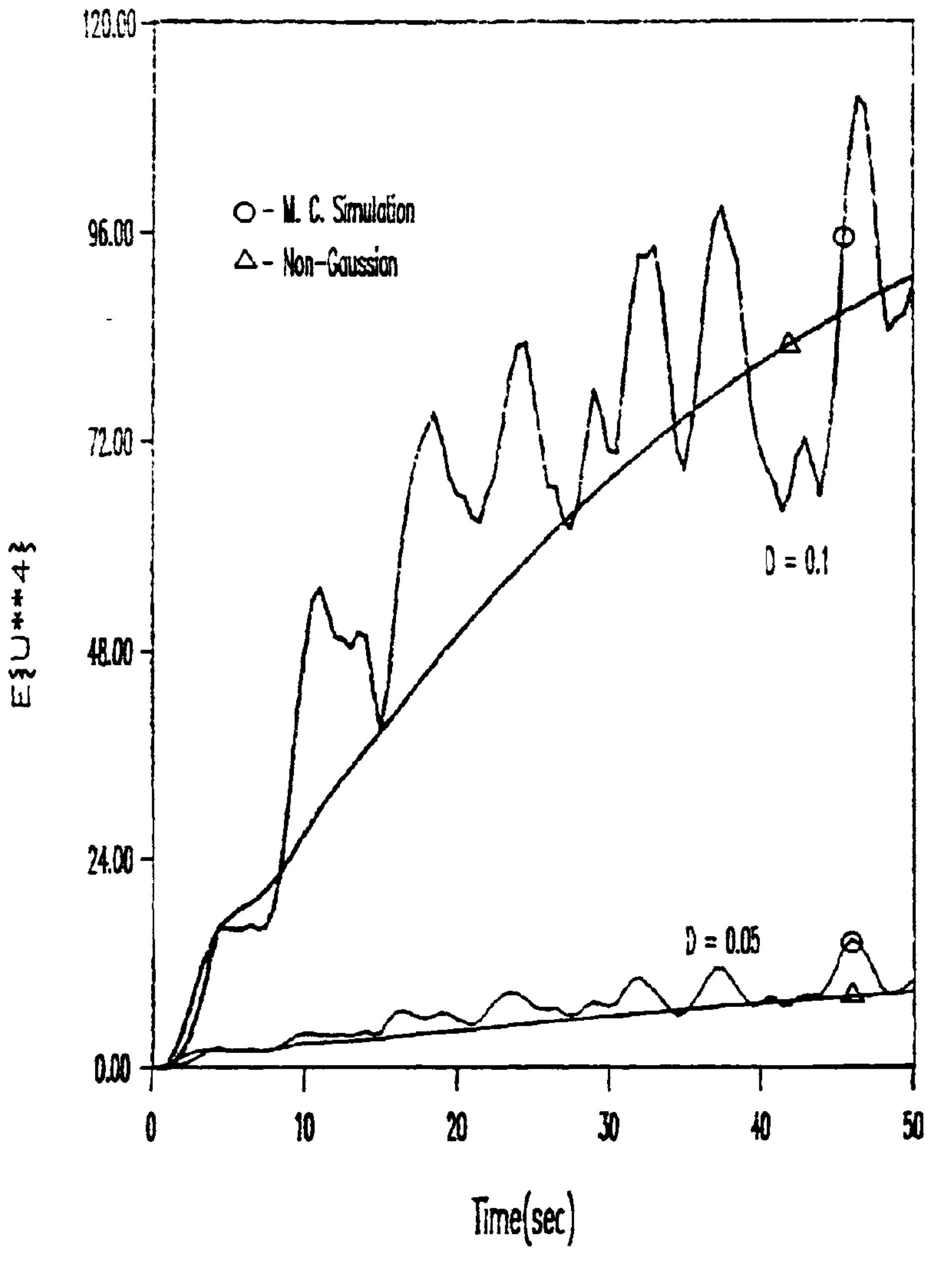

Fig. 5.66- Fourth Order Moment of the Displacement Response. $\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.01$, $\delta_\eta = 0.0$.

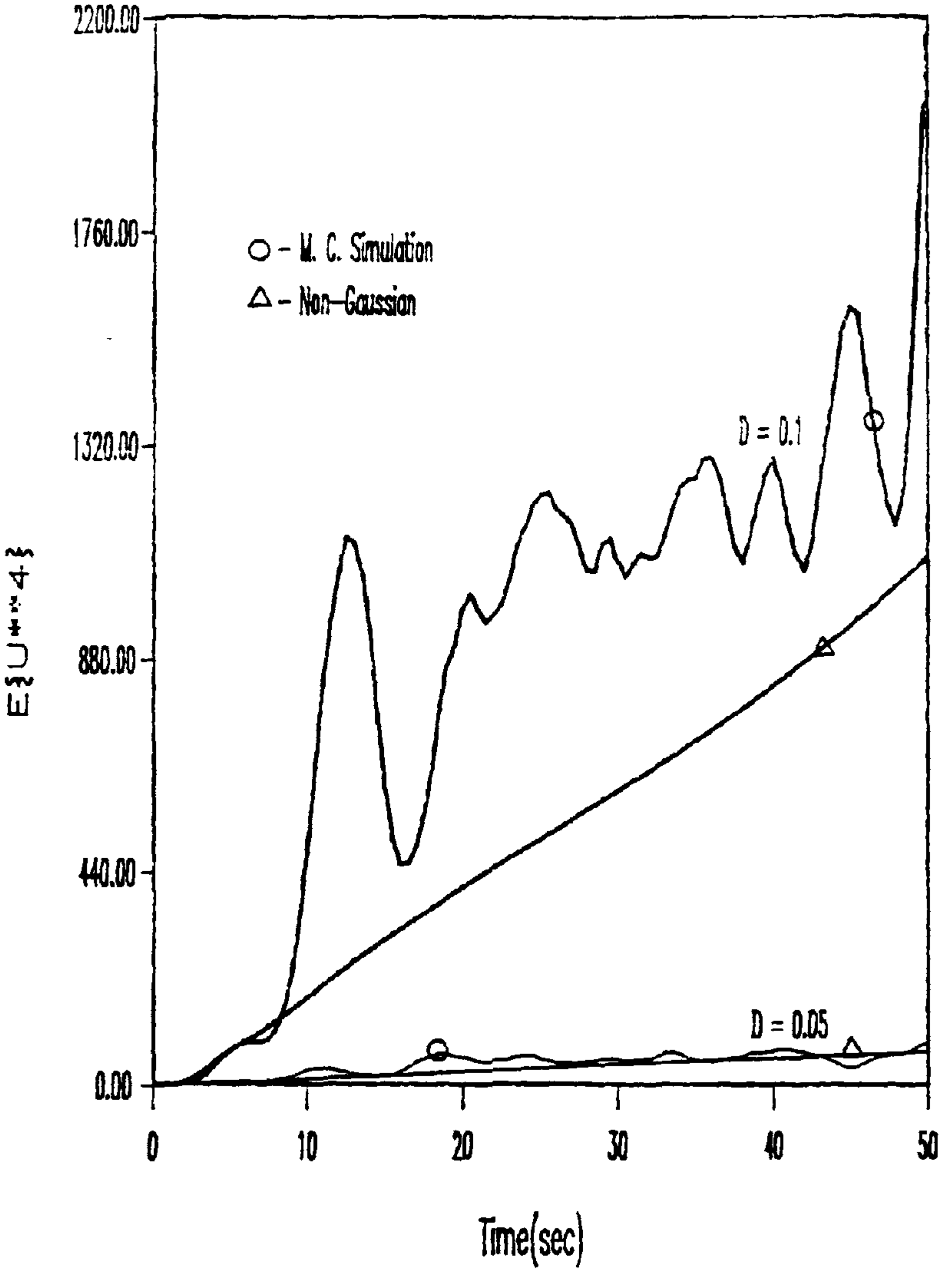

Fig. 5.67- Fourth Order Moment of the Displacement Response.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.05$, $\delta_\eta = 0.0$.

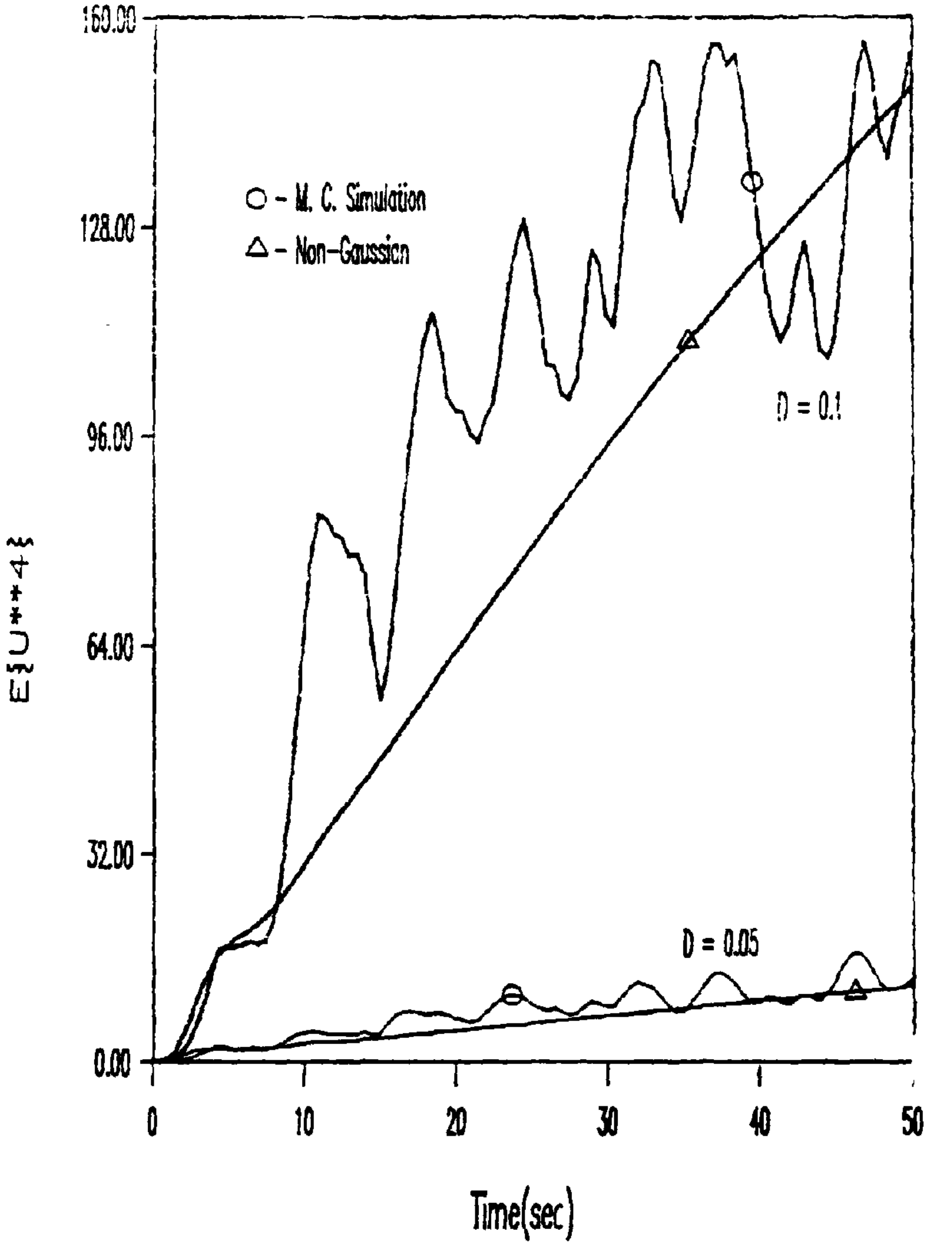

Fig. 5.68- Fourth Order Moment of the Displacement Response.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.05$, $\delta_\eta = 0.0$.

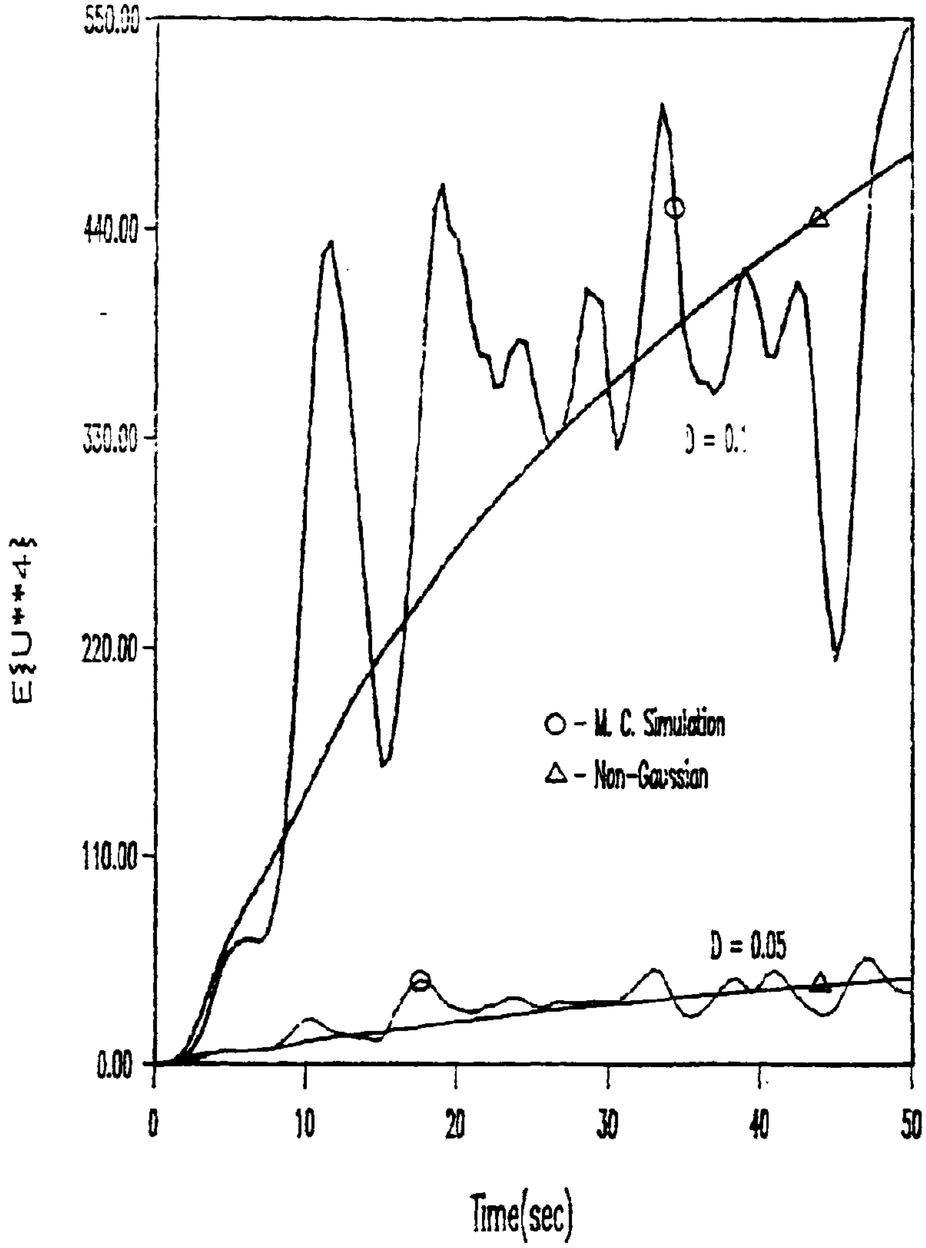

Fig. 5.69- Fourth Order Moment of the Displacement Response.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.05$.

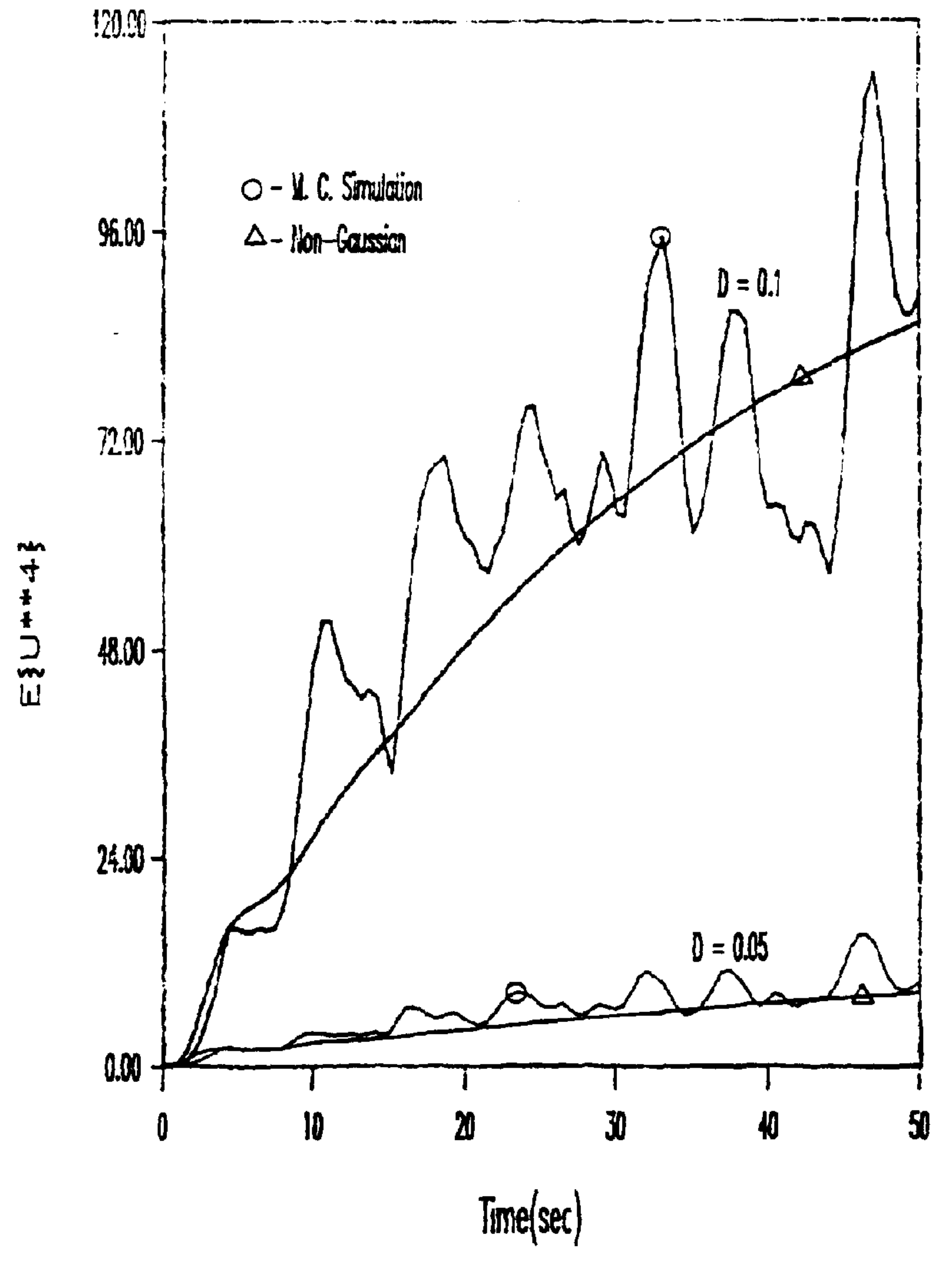

Fig. 5.70- Fourth Order Moment of the Displacement Response.
ξ = 0.1, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.05.

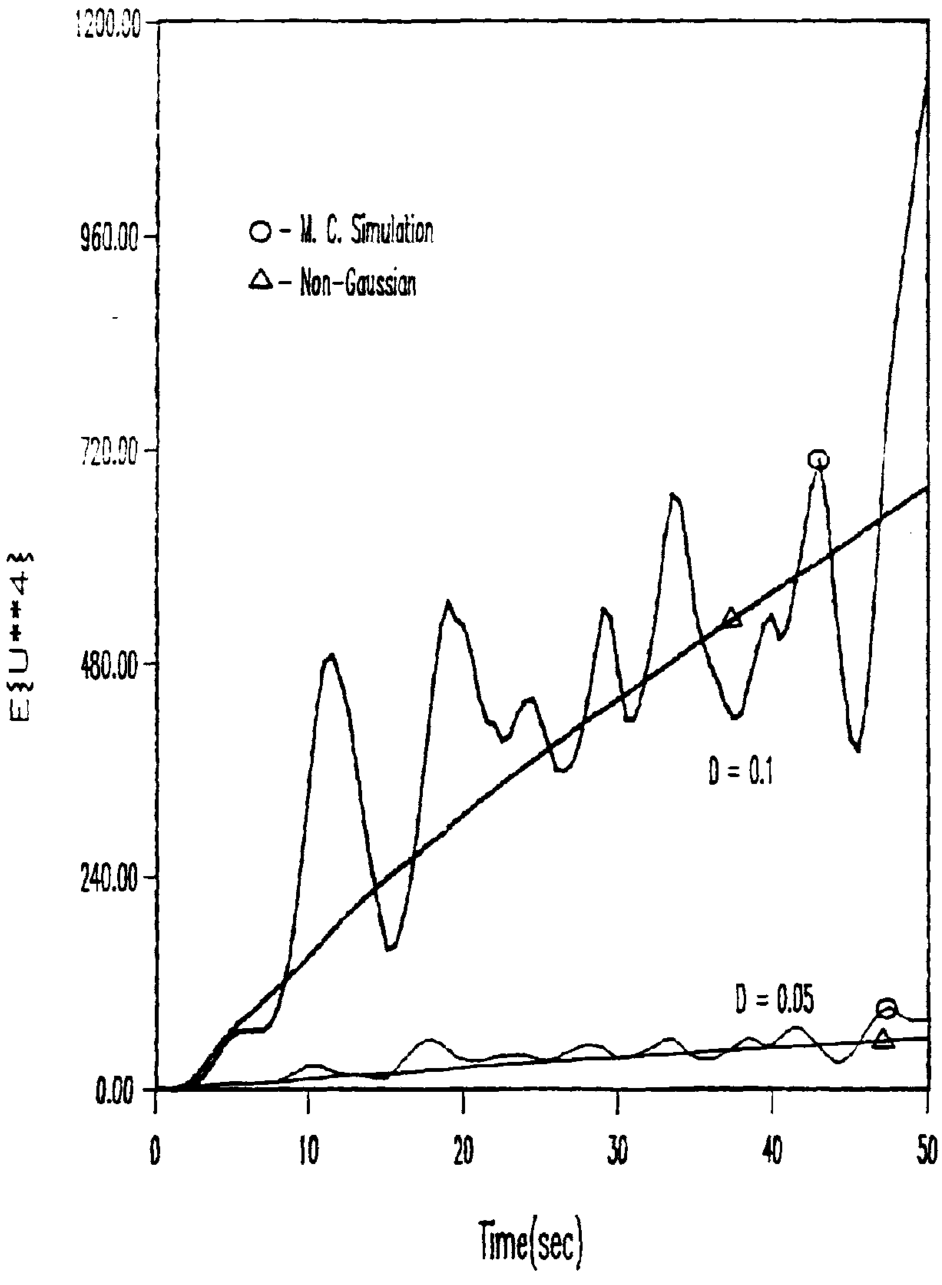

Fig. 5.71- Fourth Order Moment of the Displacement Response.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.1$.

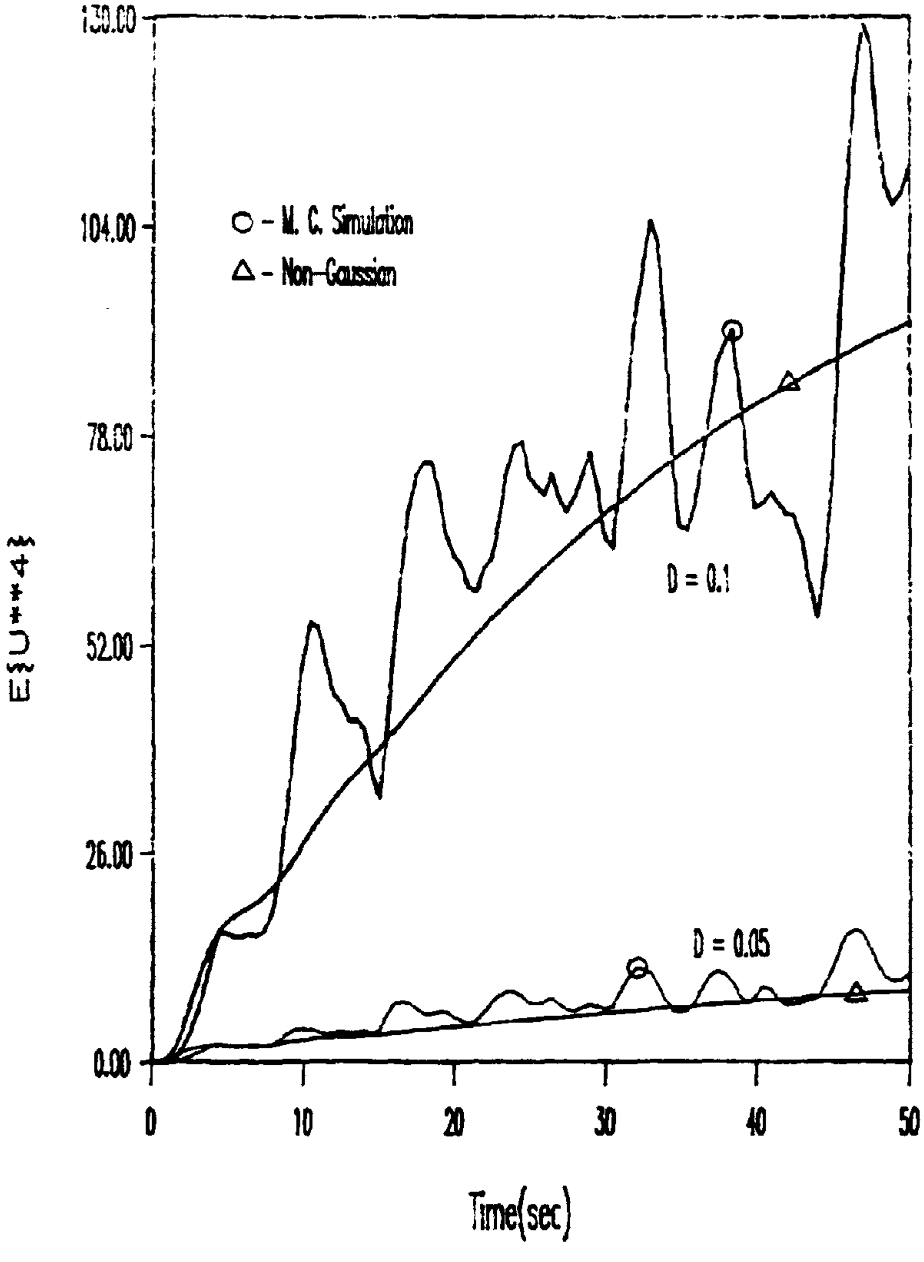

Fig. 5.72- Fourth Order Moment of the Displacement Response.
ξ = 0.1, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.1.

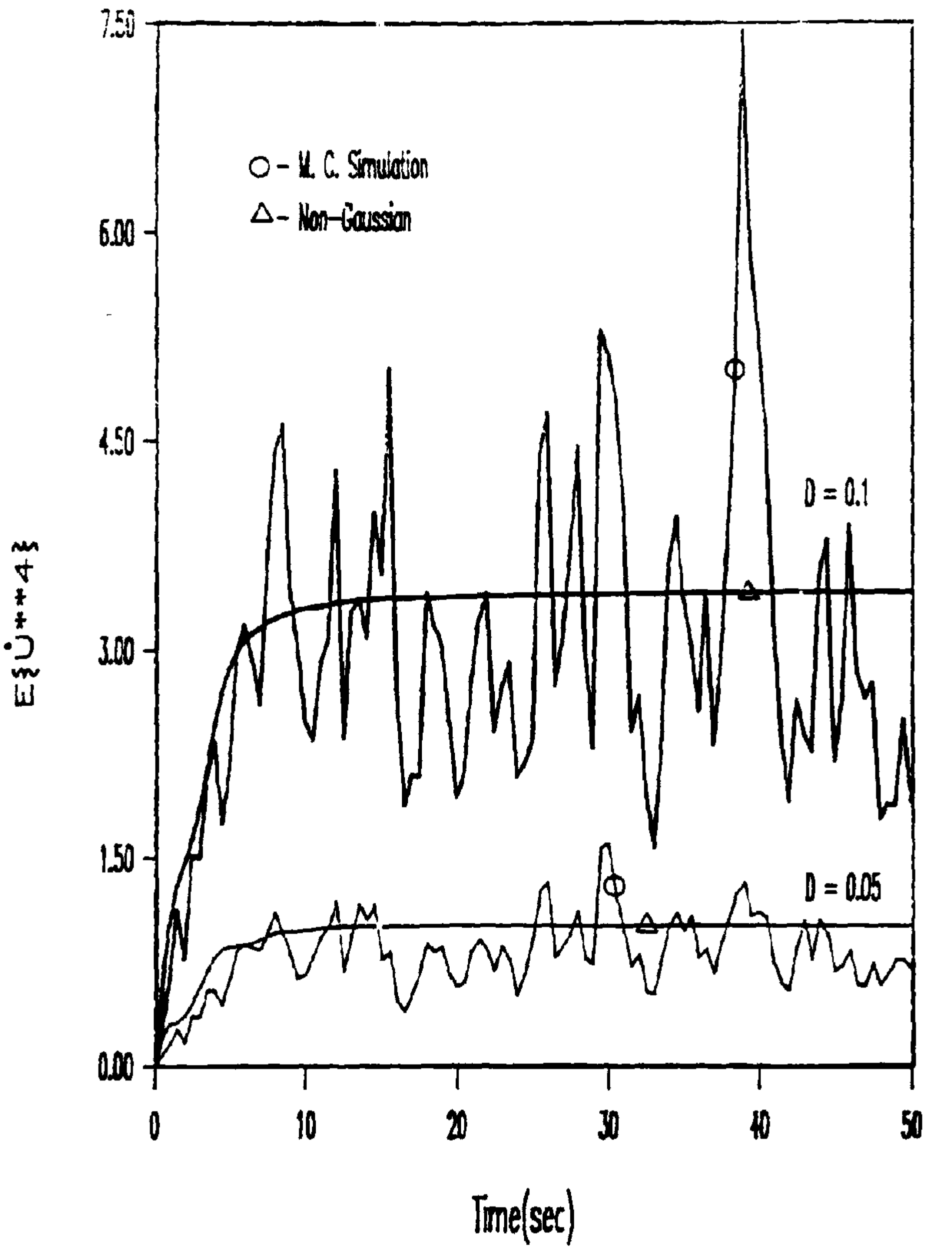

Fig. 5.73- Fourth Order Moment of the Velocity Response. ξ = 0.0, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.0.

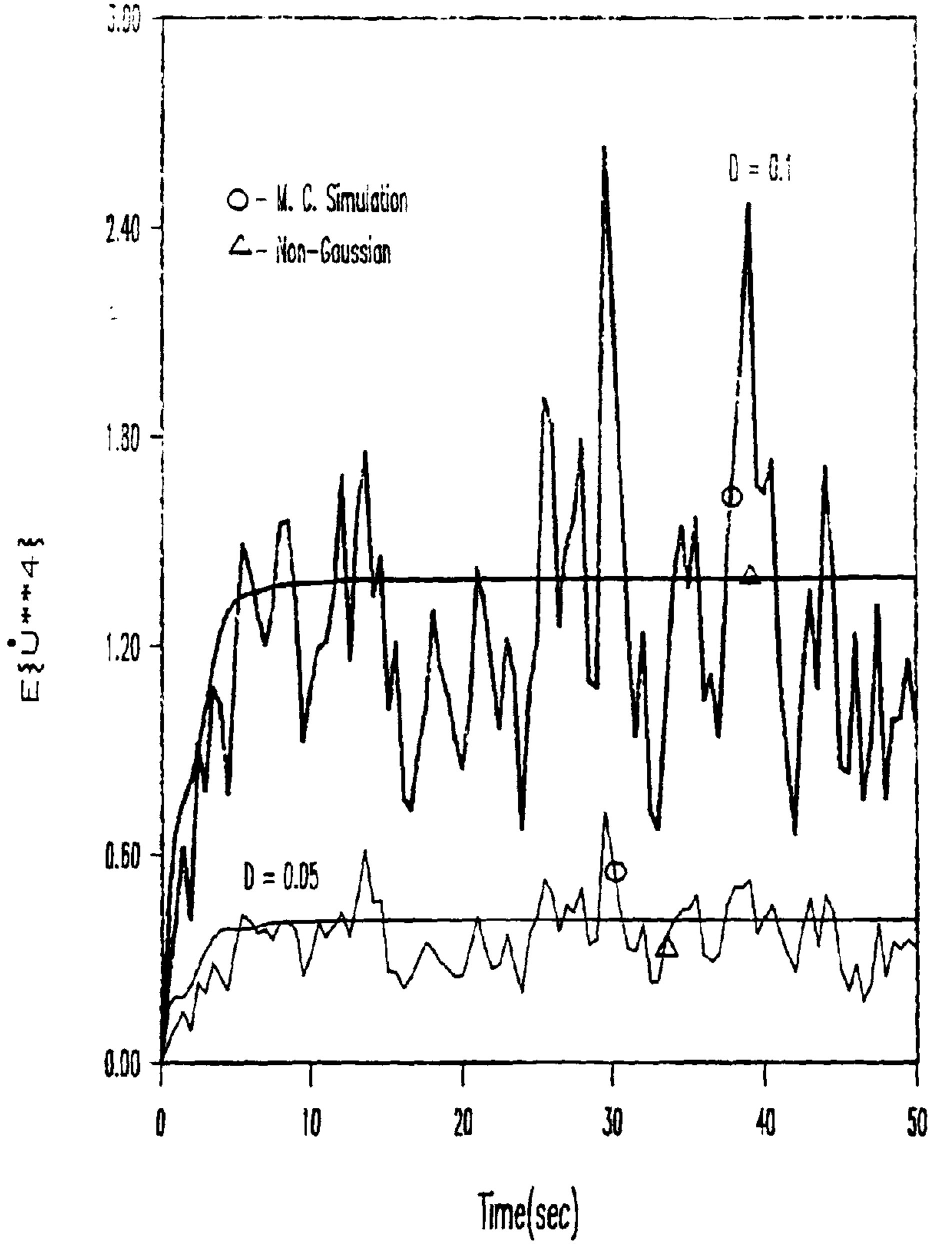

Fig. 5.74- Fourth Order Moment of the Velocity Response. $\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

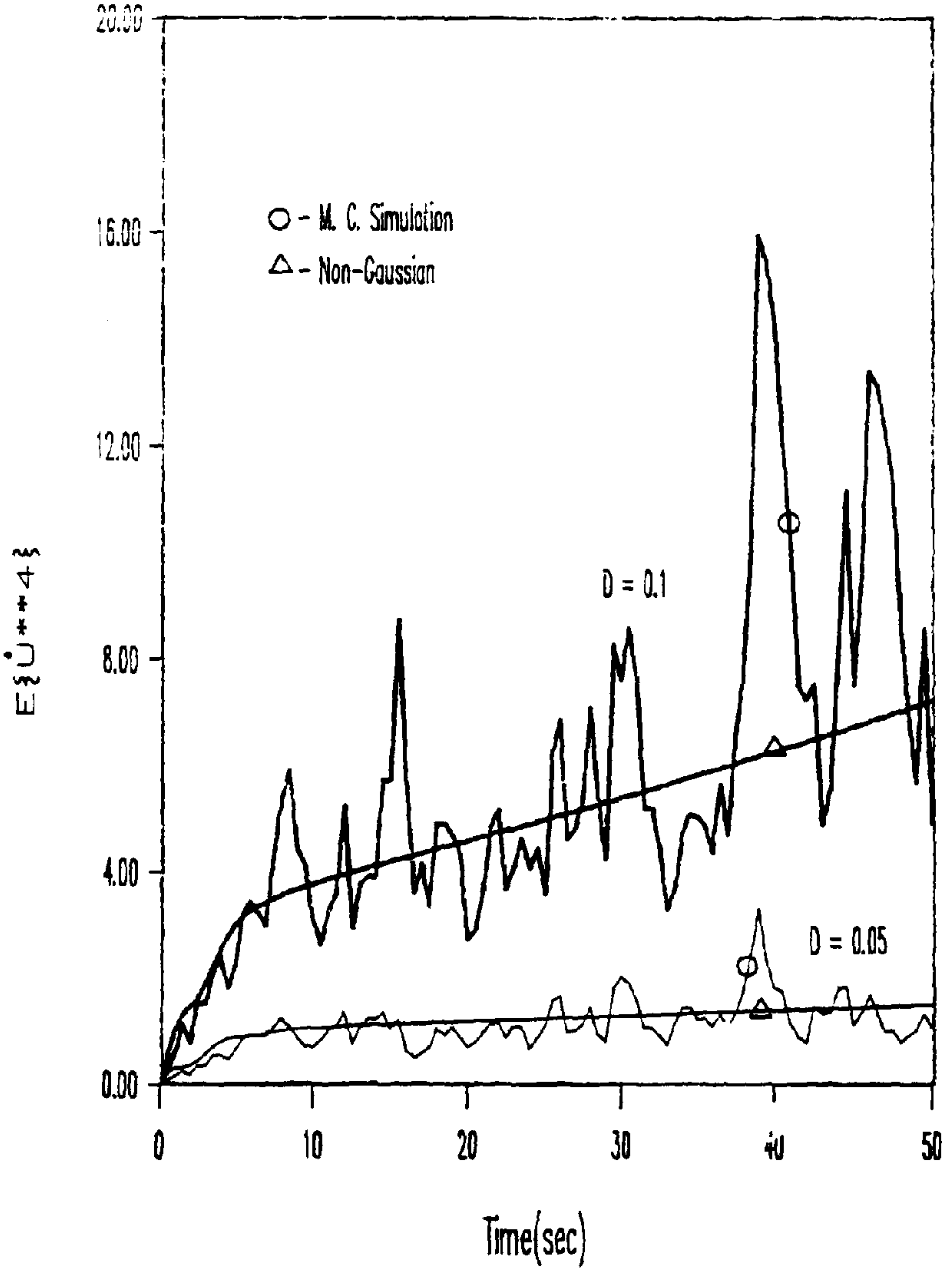

Fig. 5.75- Fourth Order Moment of the Velocity Response.
$\xi = 0.0$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

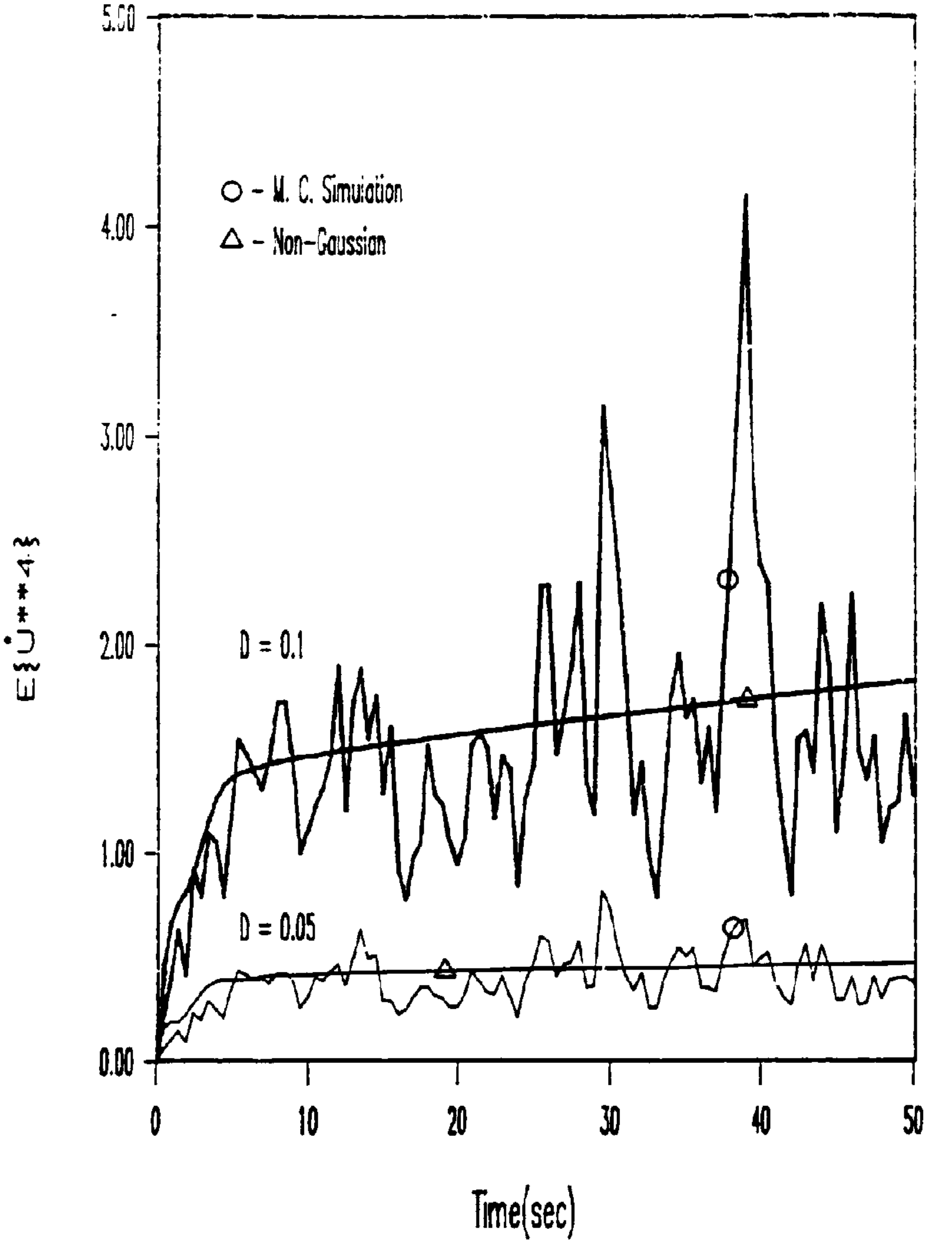

Fig. 5.76- Fourth Order Moment of the Velocity Response.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

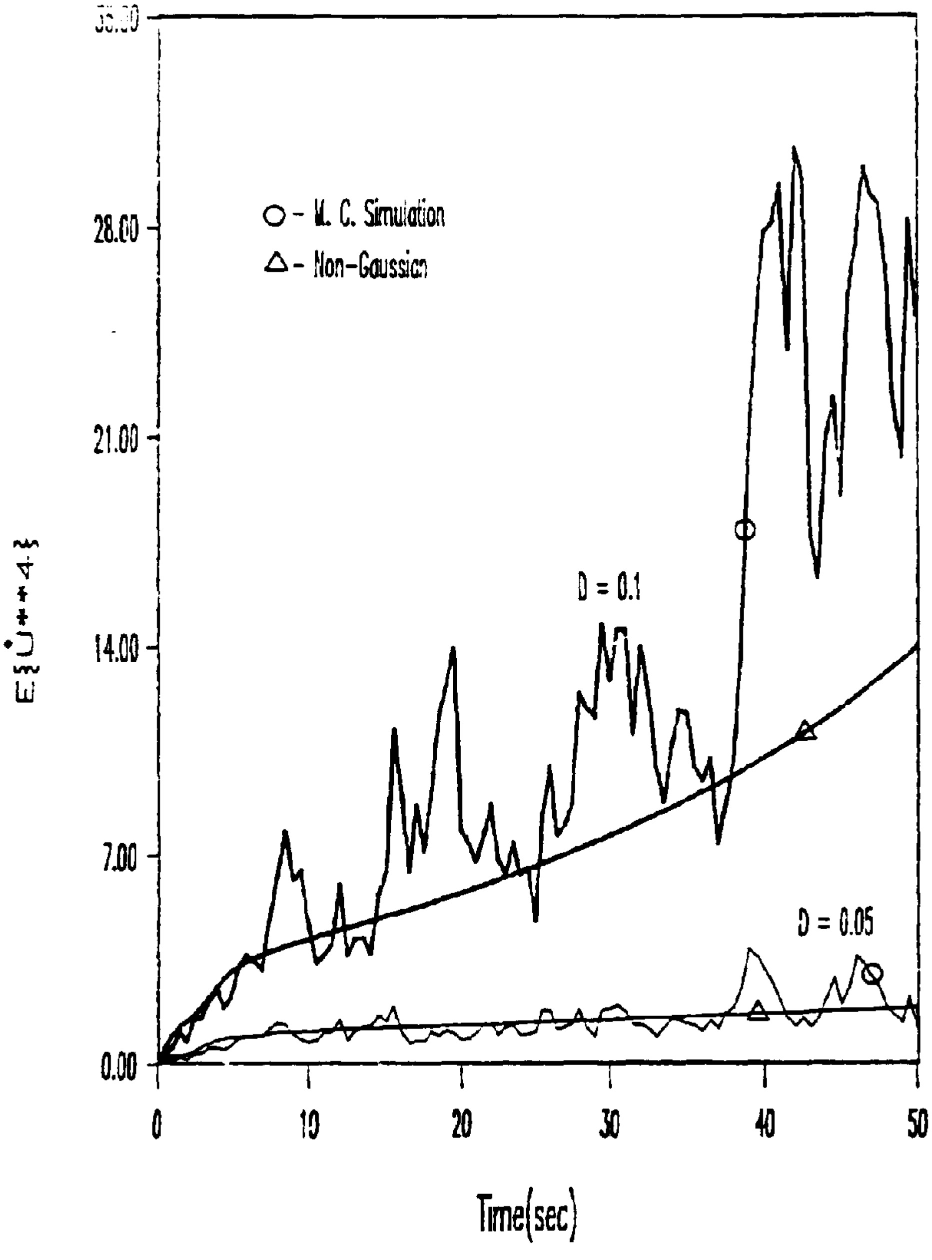

Fig. 5.77- Fourth Order Moment of the Velocity Response.
ξ = 0.0, δ_A = 0.02, δ_ν = 0.05, δ_η = 0.1.

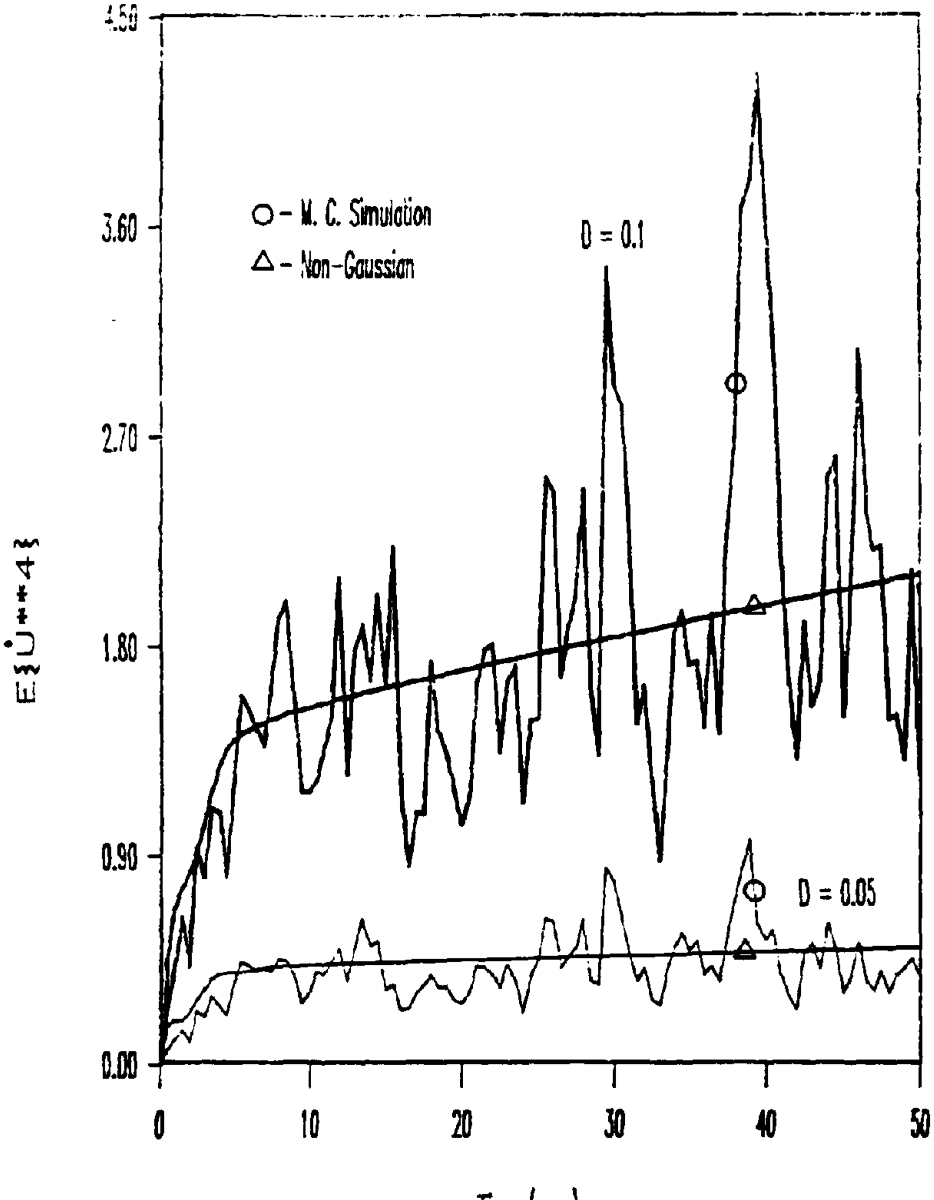

Fig. 5.78- Fourth Order Moment of the Velocity Response.
$\xi = 0.1,\ \delta_A = 0.02,\ \delta_\nu = 0.05,\ \delta_\eta = 0.1.$

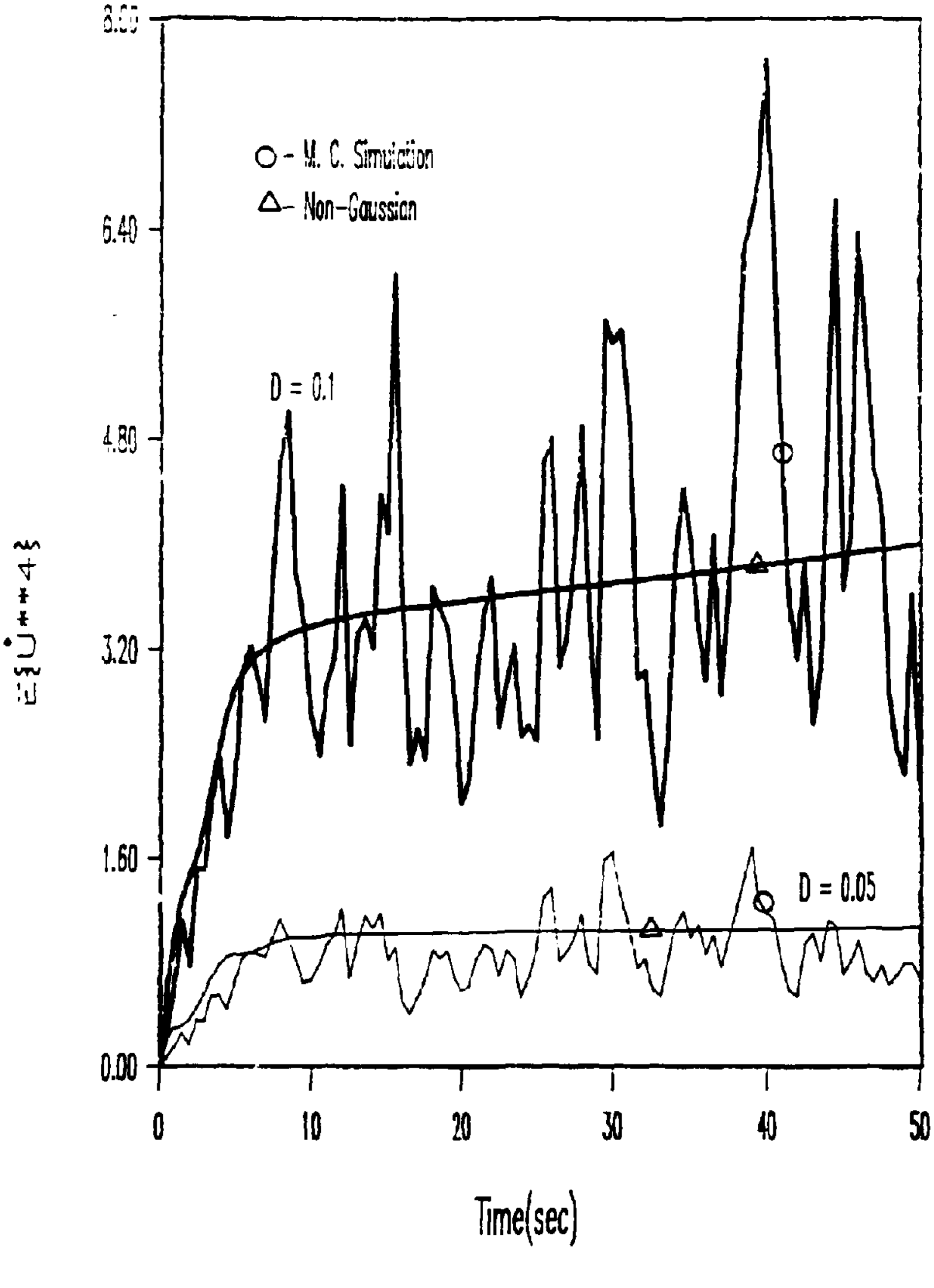

Fig. 5.79- Fourth Order Moment of the Velocity Response.
$\xi = 0.0,\ \delta_A = 0.01,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

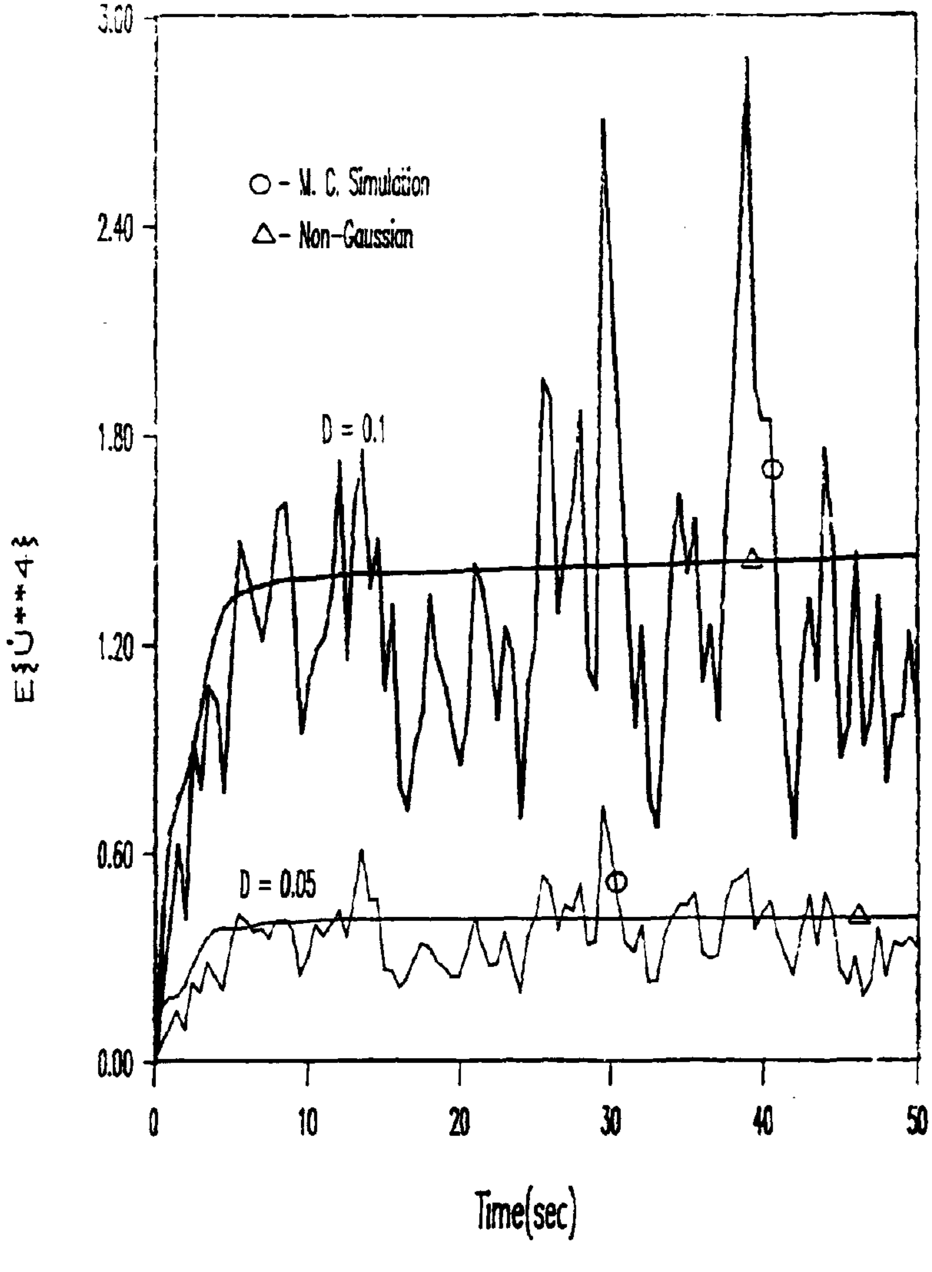

Fig. 5.80- Fourth Order Moment of the Velocity Response.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

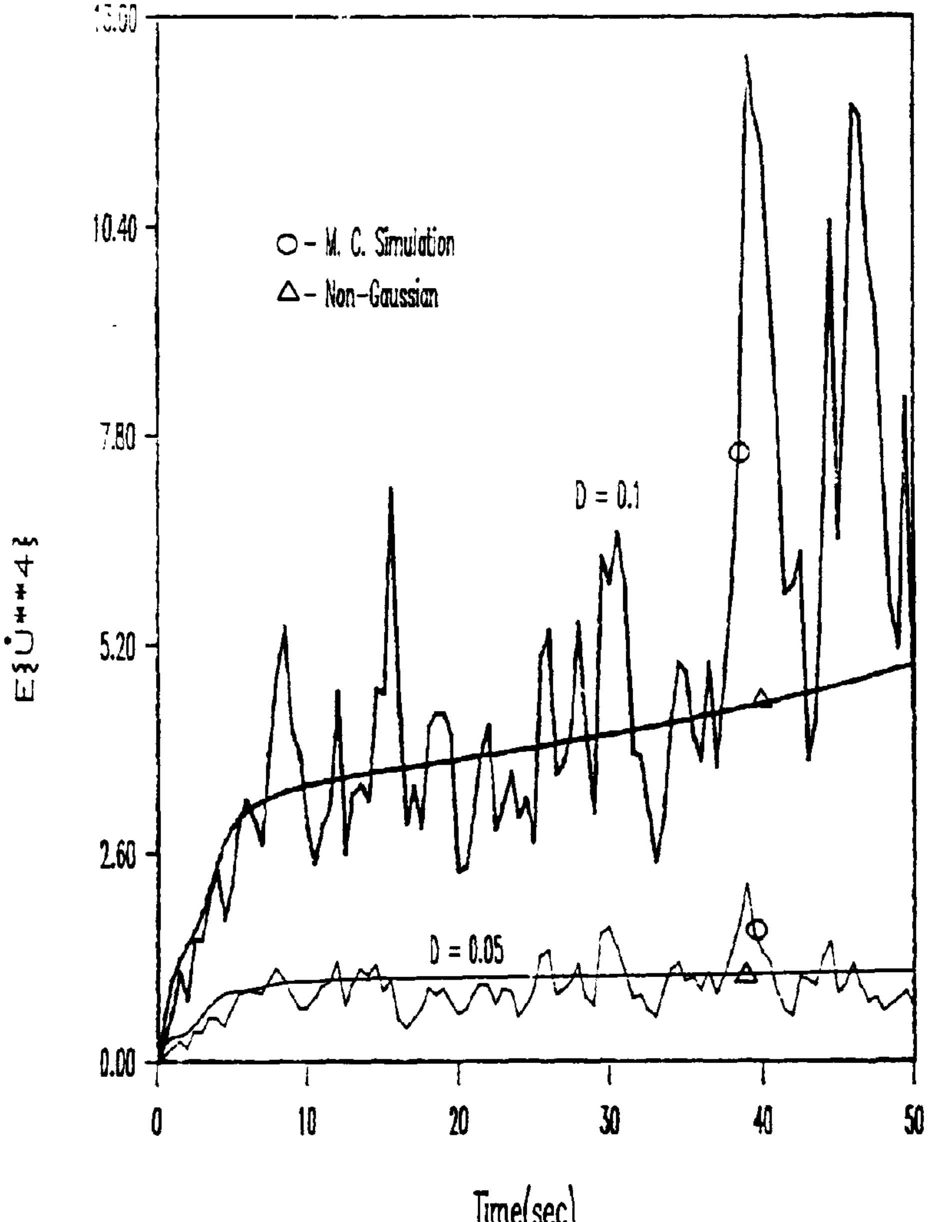

Fig. 5.81- Fourth Order Moment of the Velocity Response.
$\xi = 0.0$, $\delta_A = 0.02$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

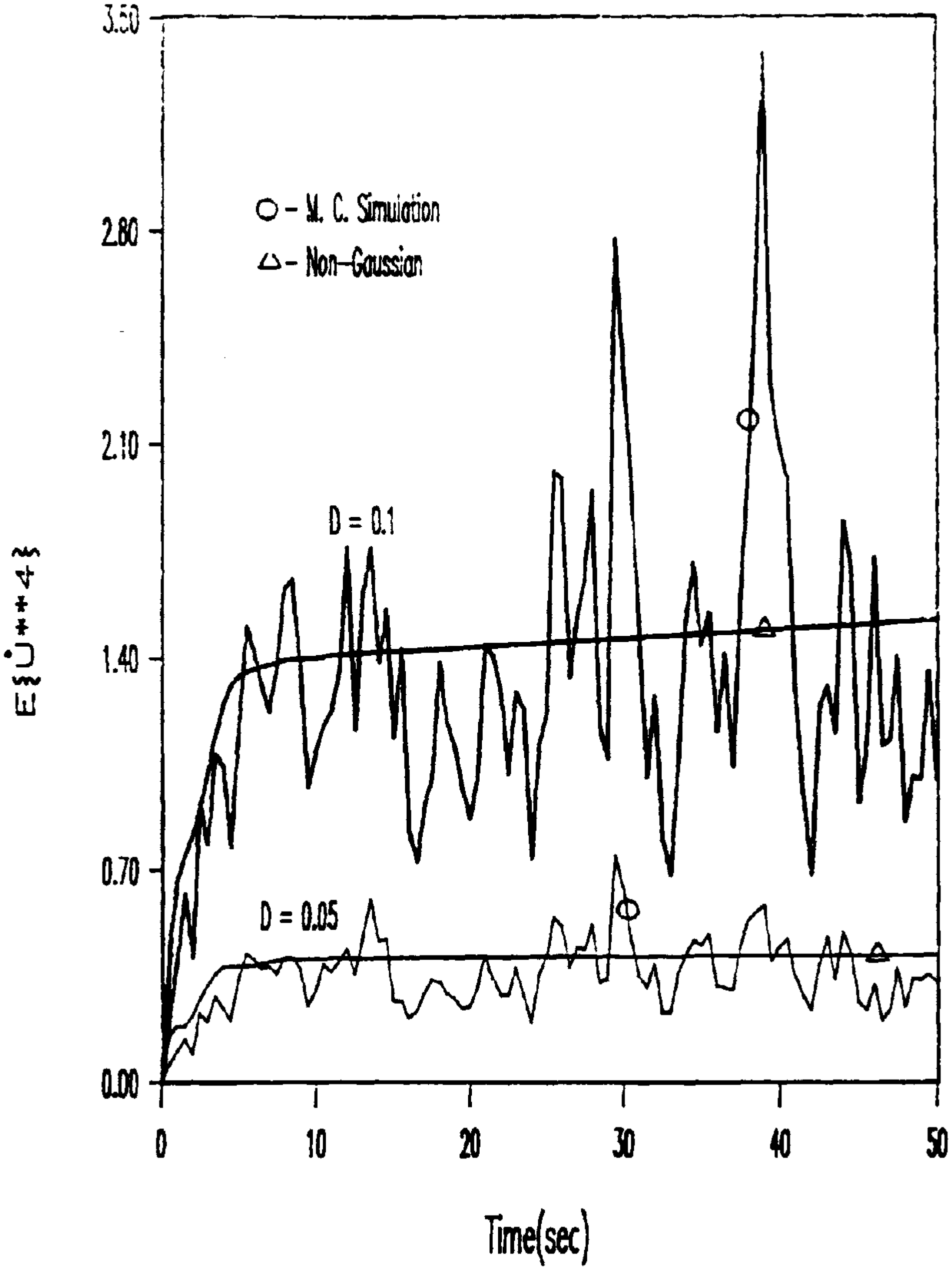

Fig. 5.82- Fourth Order Moment of the Velocity Response. $\xi = 0.1,\ \delta_A = 0.02,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0$.

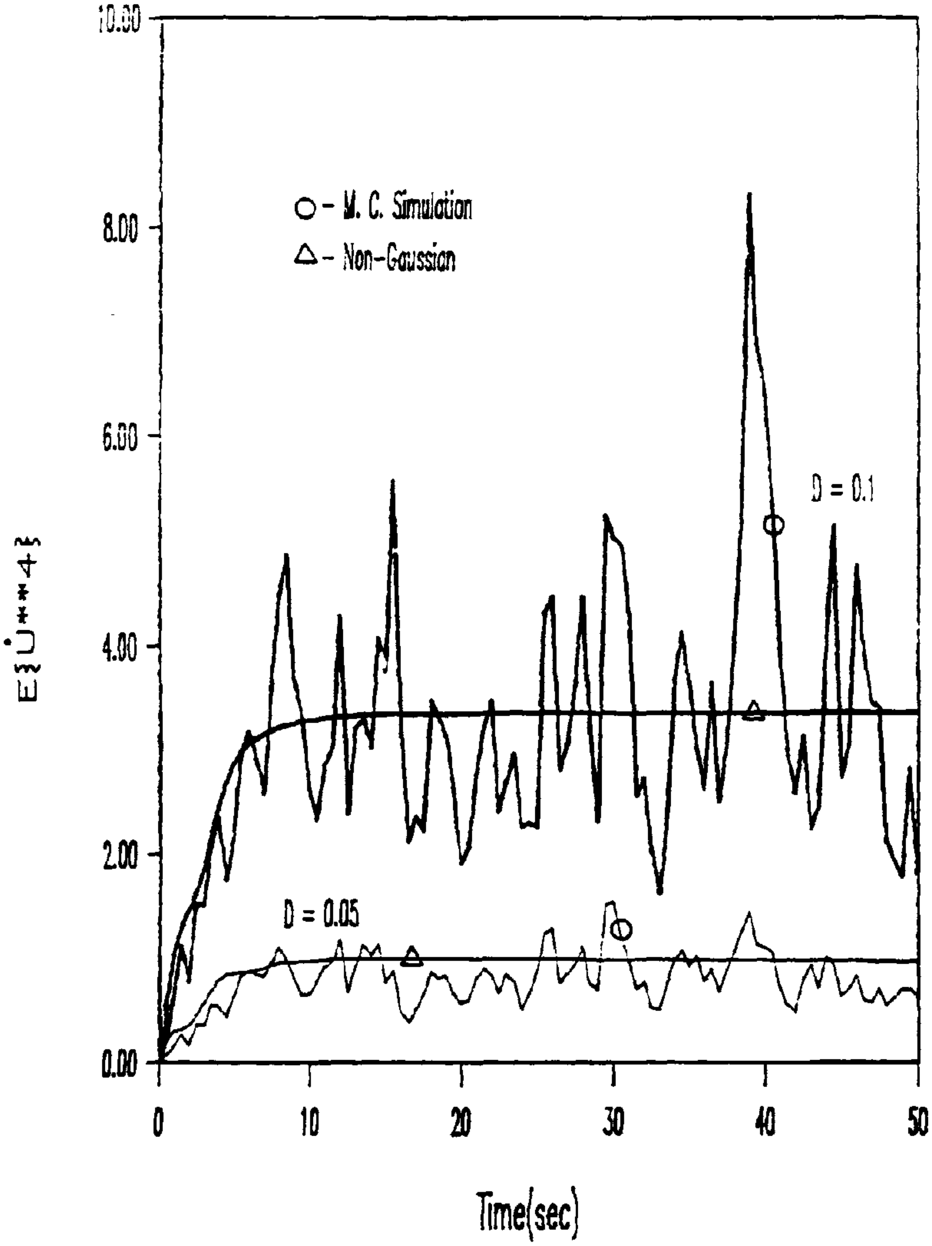

Fig. 5.83- Fourth Order Moment of the Velocity Response.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.01$, $\delta_\eta = 0.0$.

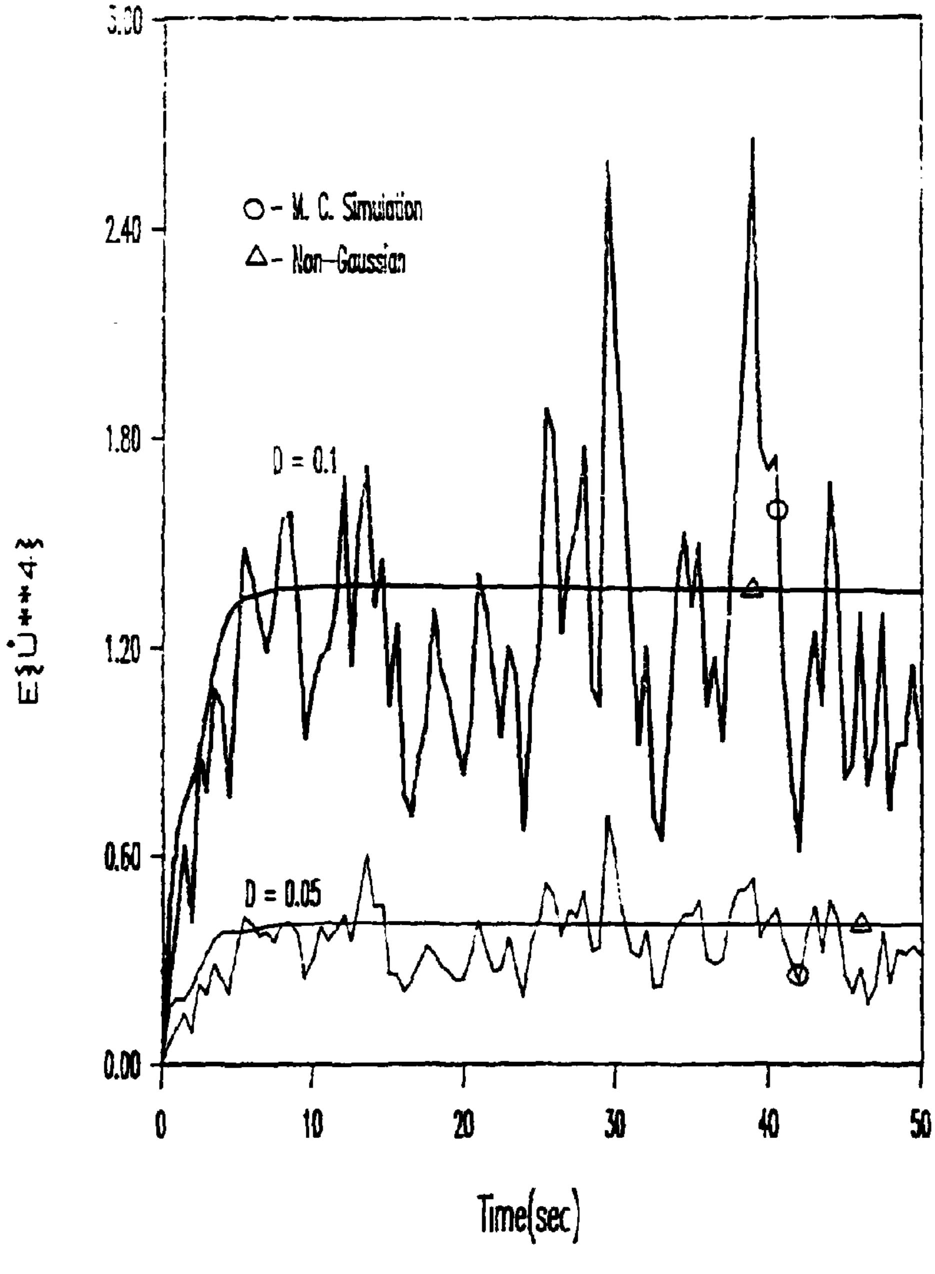

Fig. 5.84- Fourth Order Moment of the Velocity Response.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.01,\ \delta_\eta = 0.0.$

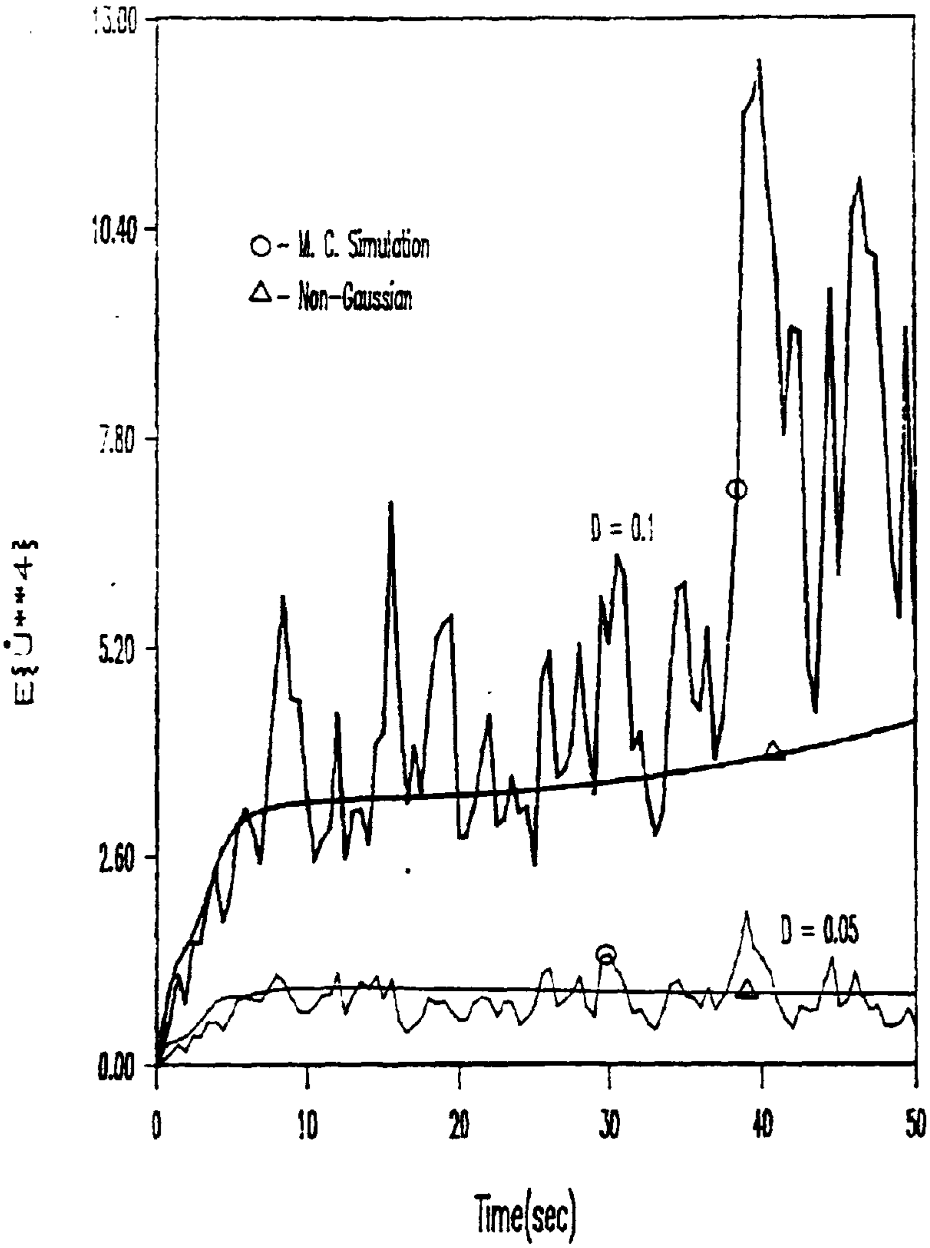

Fig. 5.85- Fourth Order Moment of the Velocity Response.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.05,\ \delta_\eta = 0.0.$

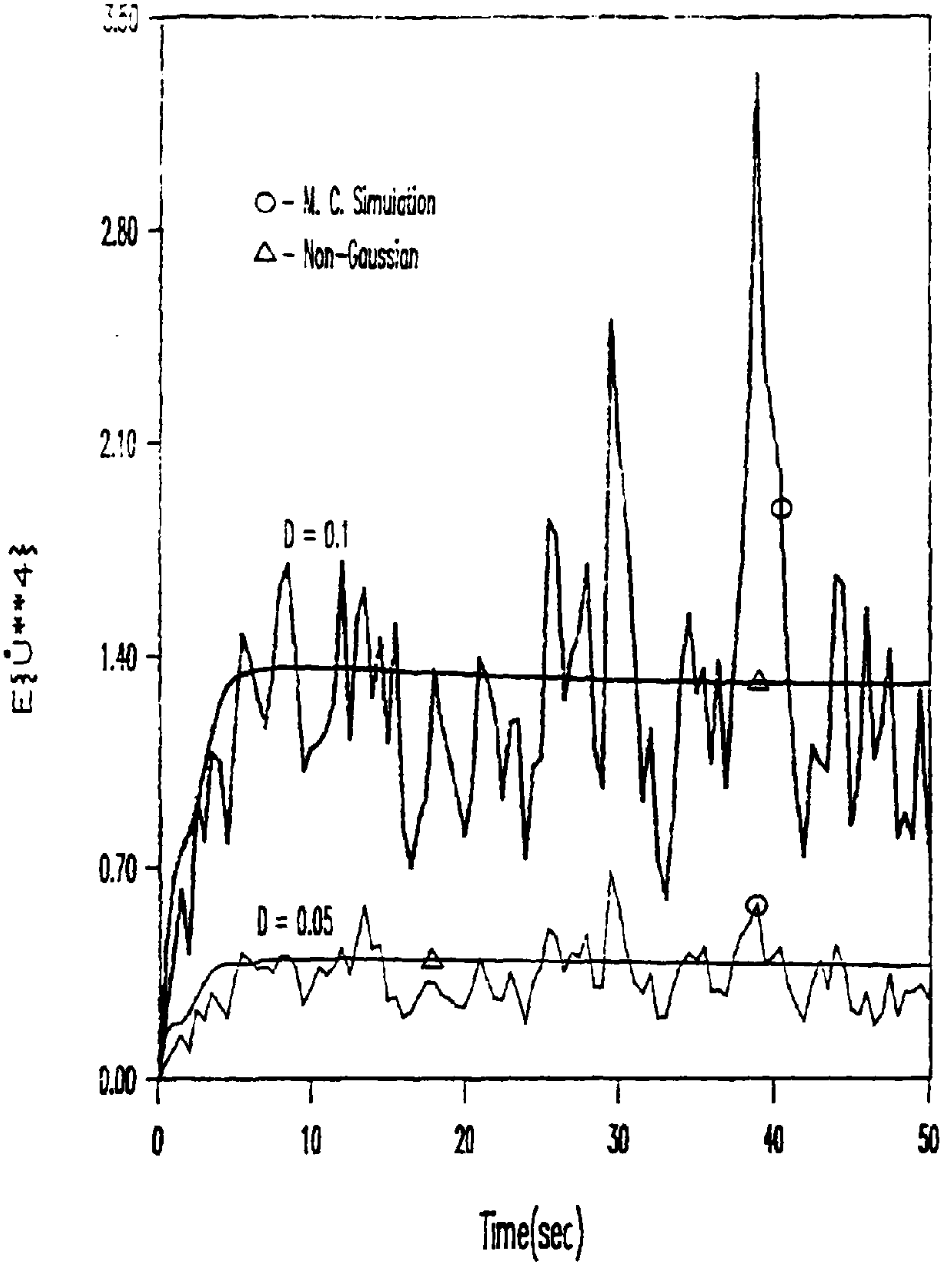

Fig. 5.86- Fourth Order Moment of the Velocity Response.
$\xi = 0.1,\ \delta_{A} = 0.0,\ \delta_{\nu} = 0.05,\ \delta_{\eta} = 0.0.$

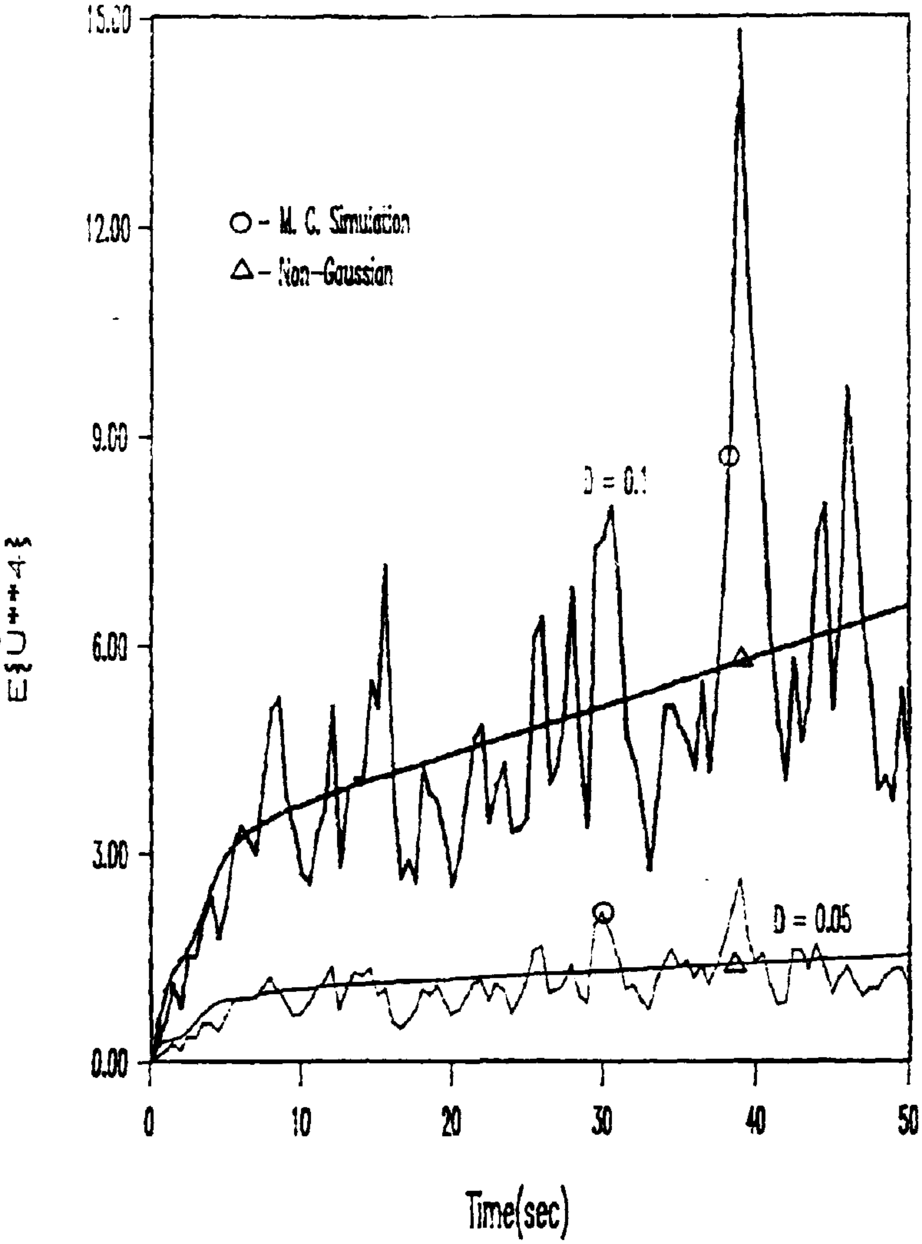

Fig. 5.87- Fourth Order Moment of the Velocity Response.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.05.$

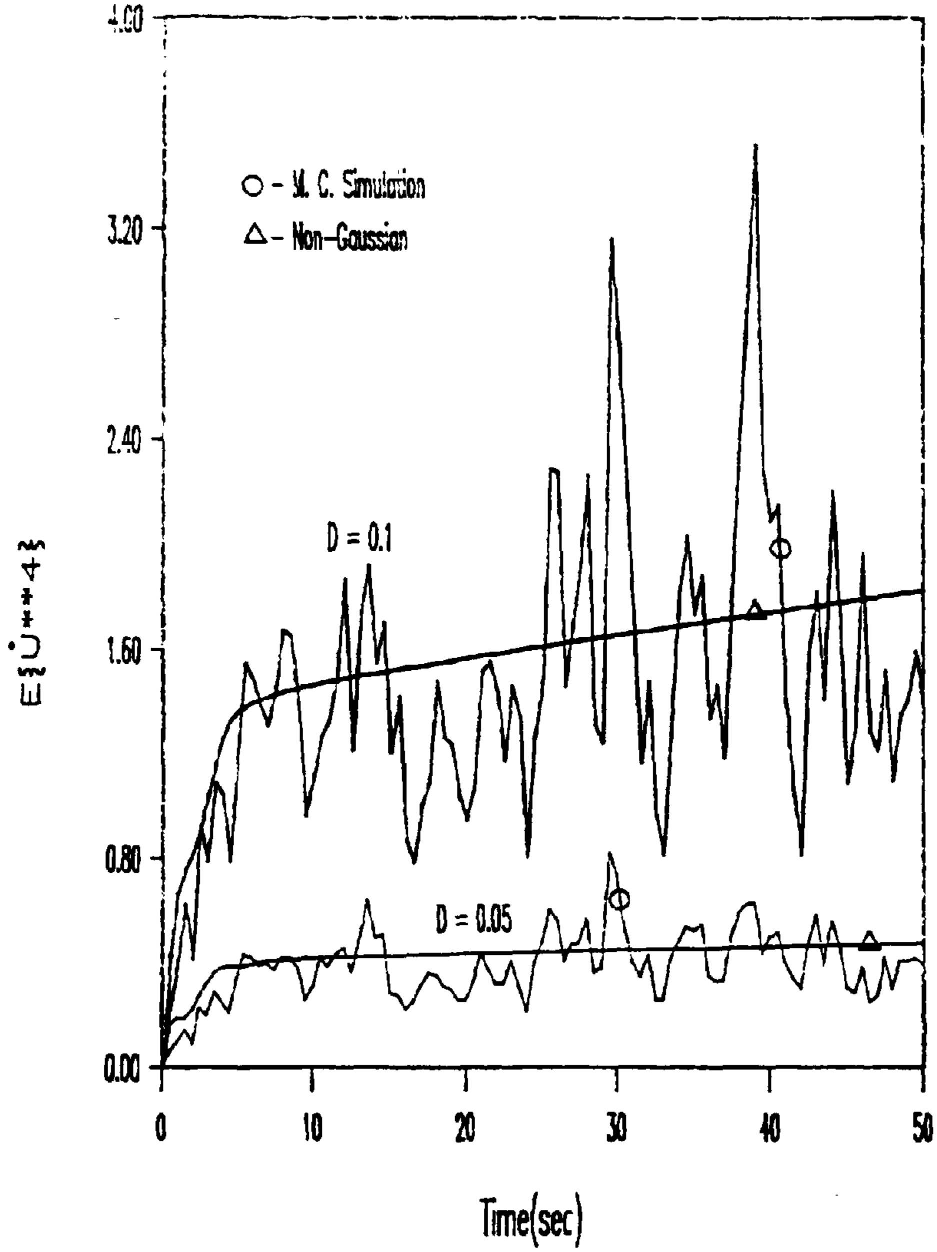

Fig. 5.88- Fourth Order Moment of the Velocity Response.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.05.$

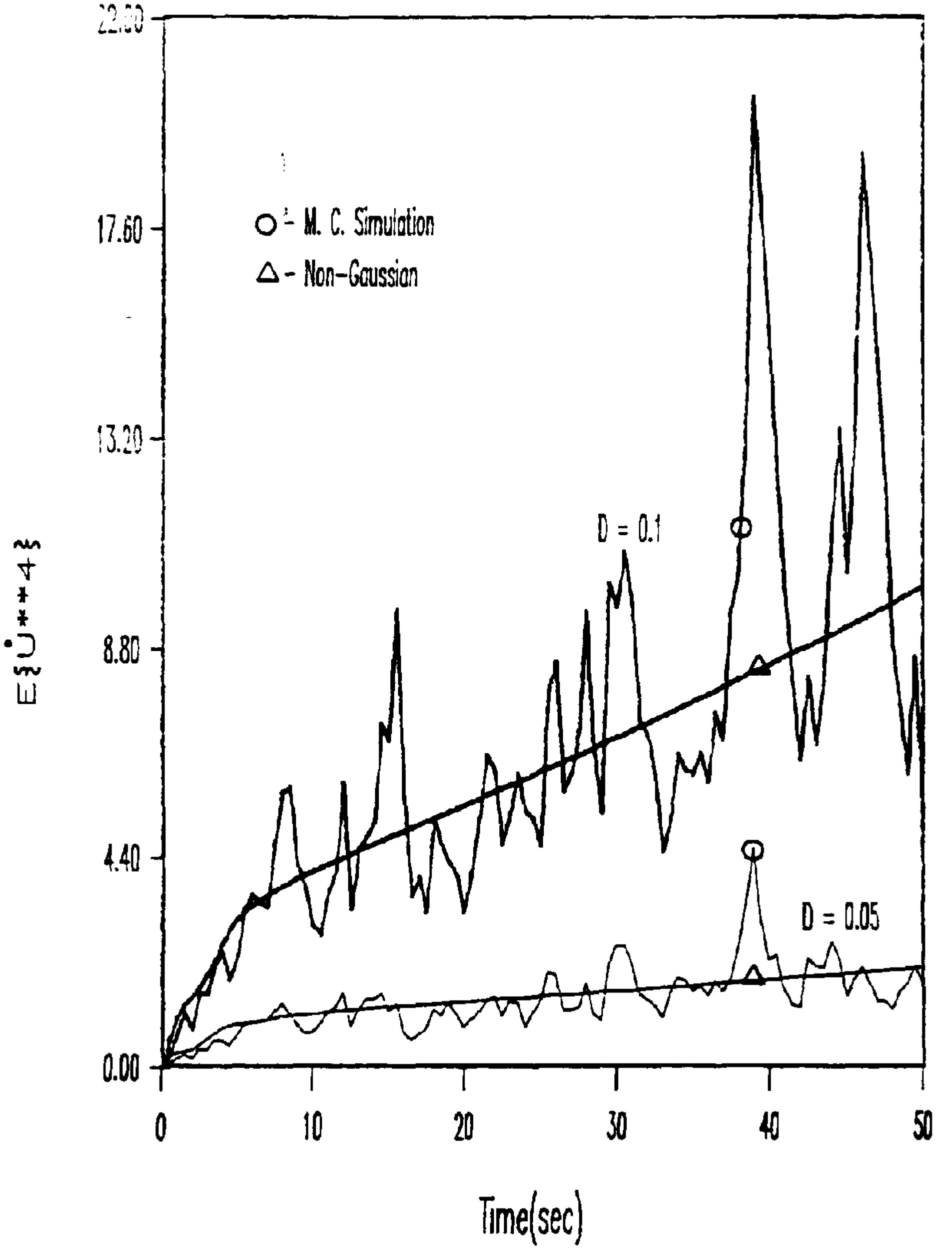

Fig. 5.89- Fourth Order Moment of the Velocity Response.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.1.$

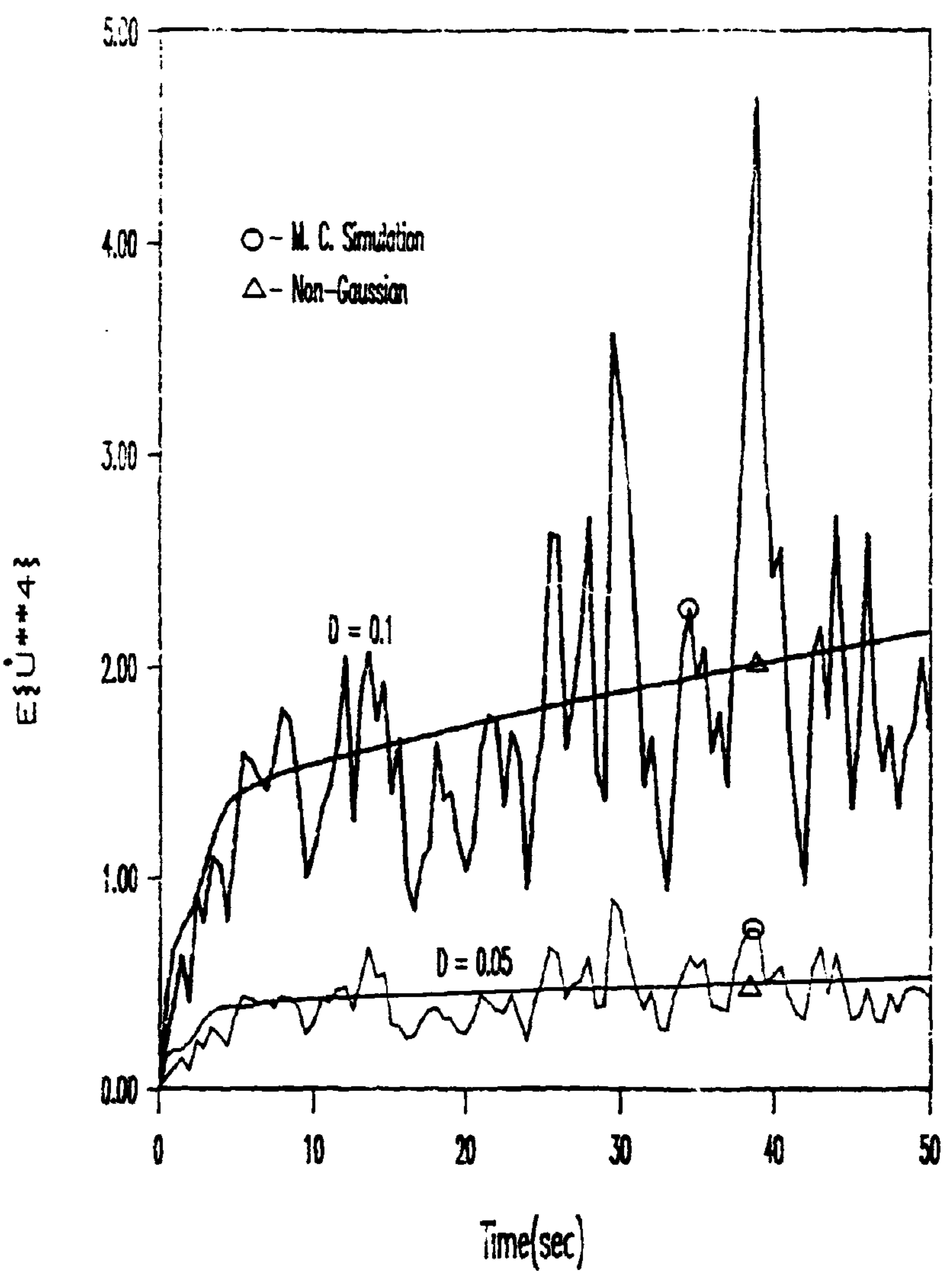

Fig. 5.90- Fourth Order Moment of the Velocity Response.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.1$.

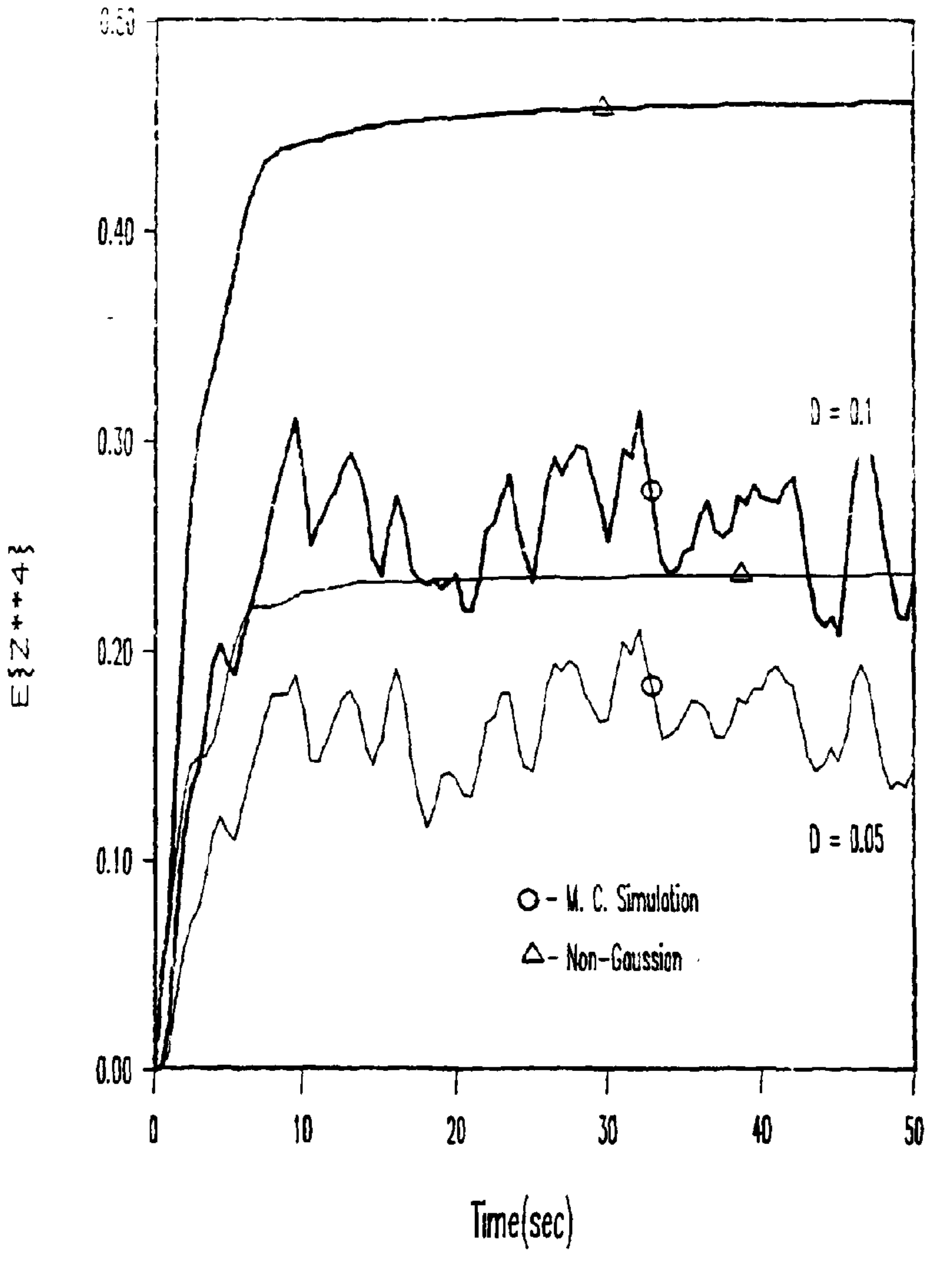

Fig. 5.91- Fourth Order Moment of the Restoring Force.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

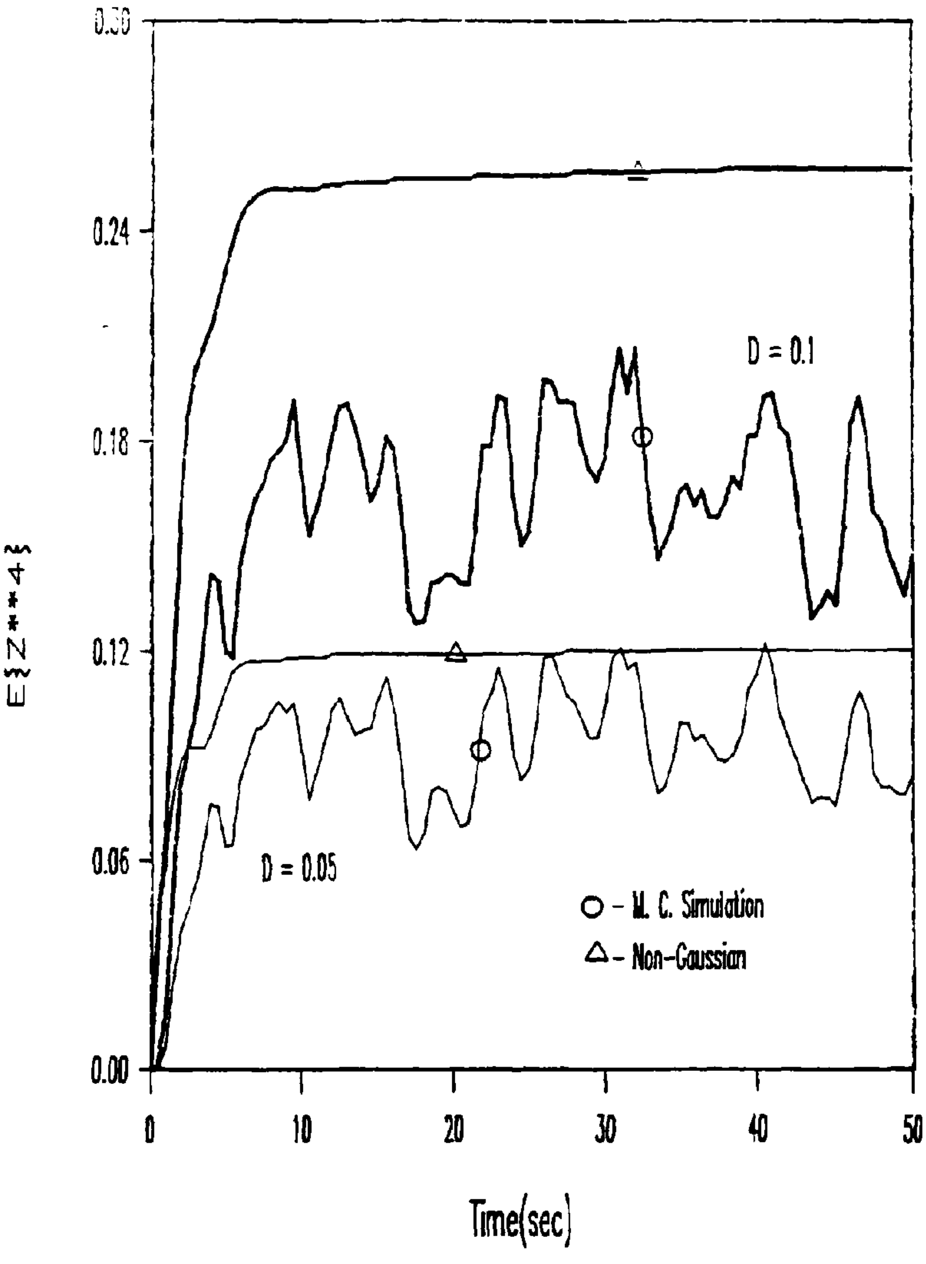

Fig. 5.92- Fourth Order Moment of the Restoring Force.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

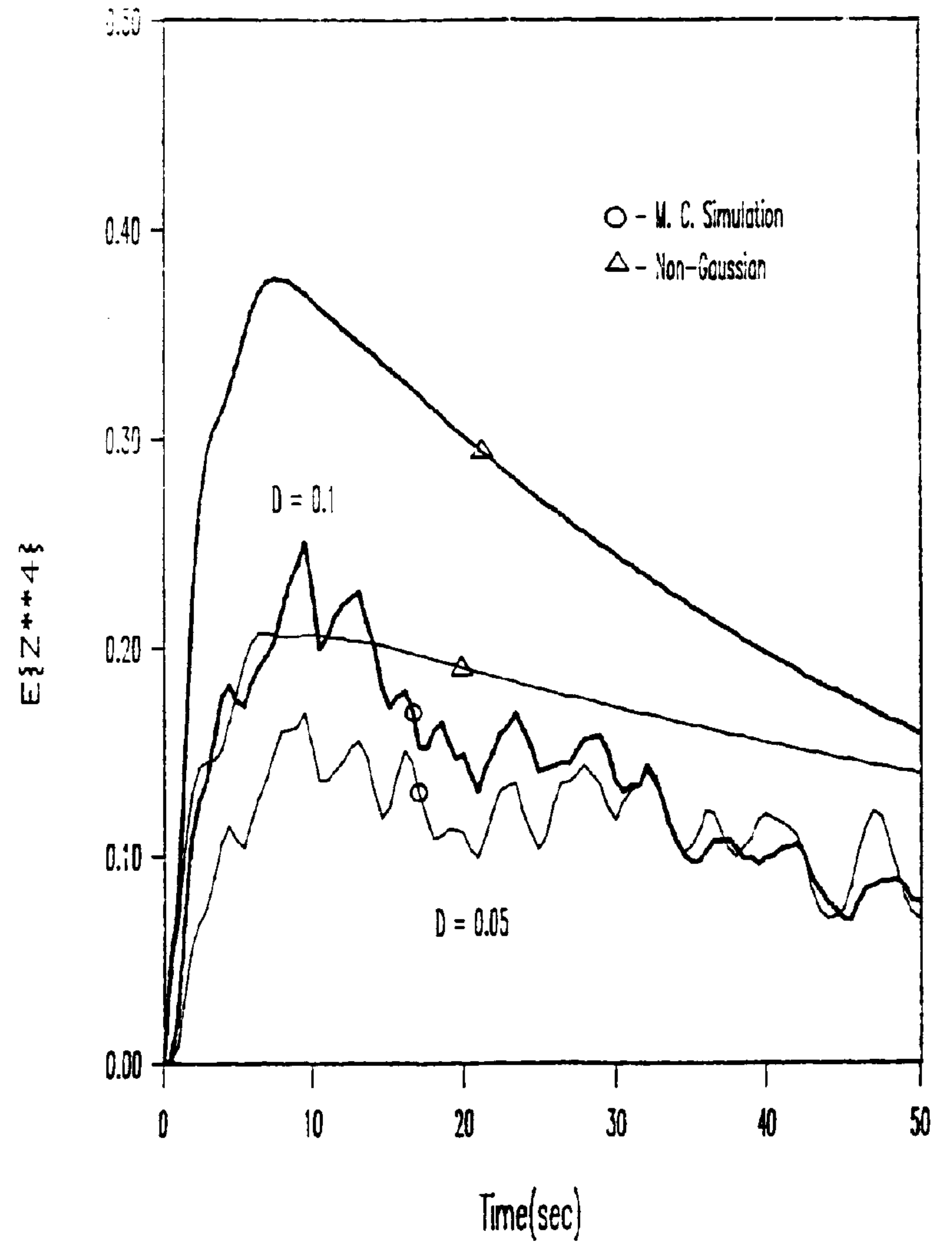

Fig. 5.93- Fourth Order Moment of the Restoring Force.
$\xi = 0.0,\ \delta_{\Lambda} = 0.01,\ \delta_{\nu} = 0.01,\ \delta_{\eta} = 0.05.$

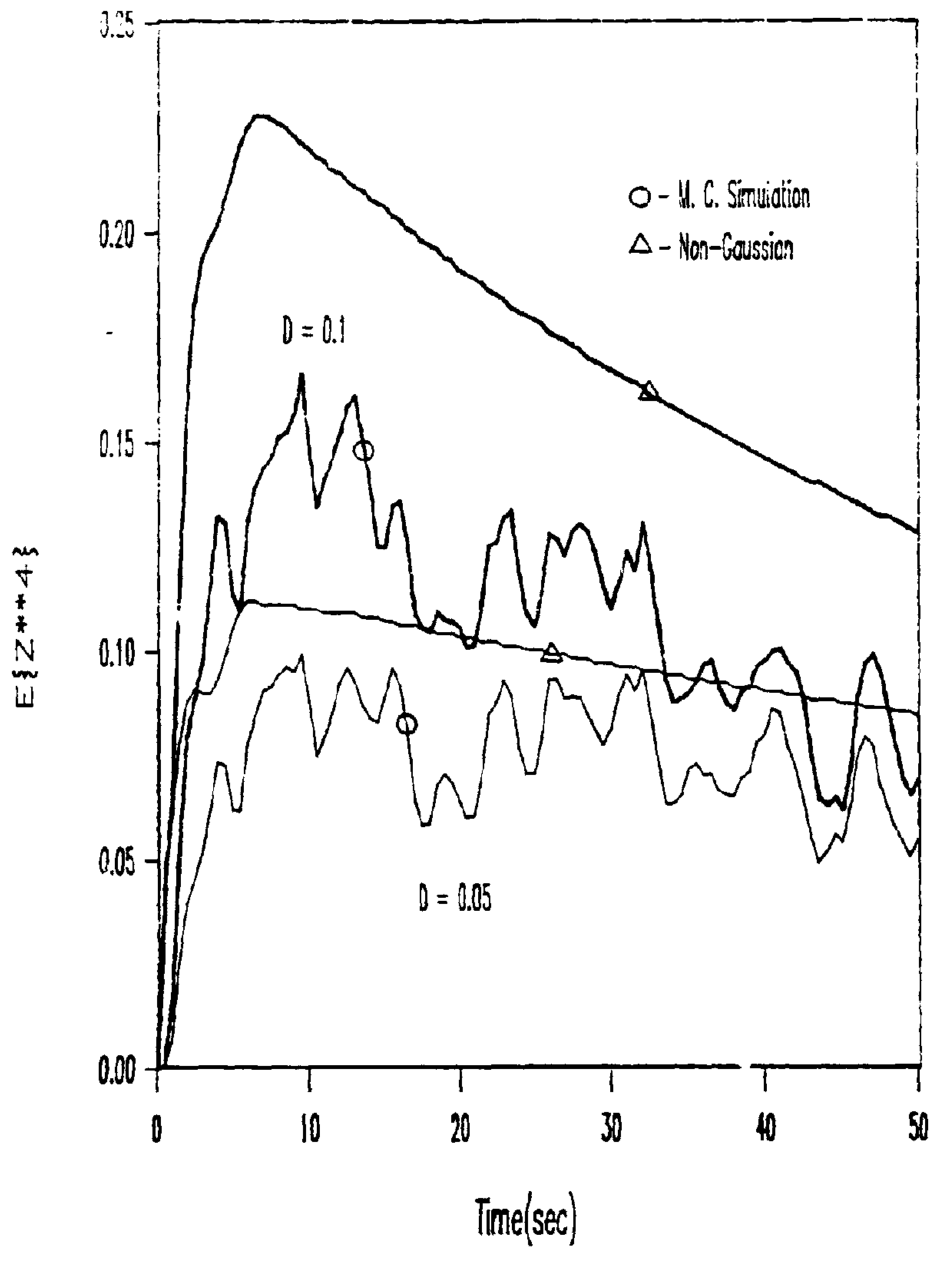

Fig. 5.94- Fourth Order Moment of the Restoring Force.
$\xi = 0.1,\ \delta_A = 0.01,\ \delta_\nu = 0.01,\ \delta_\eta = 0.05.$

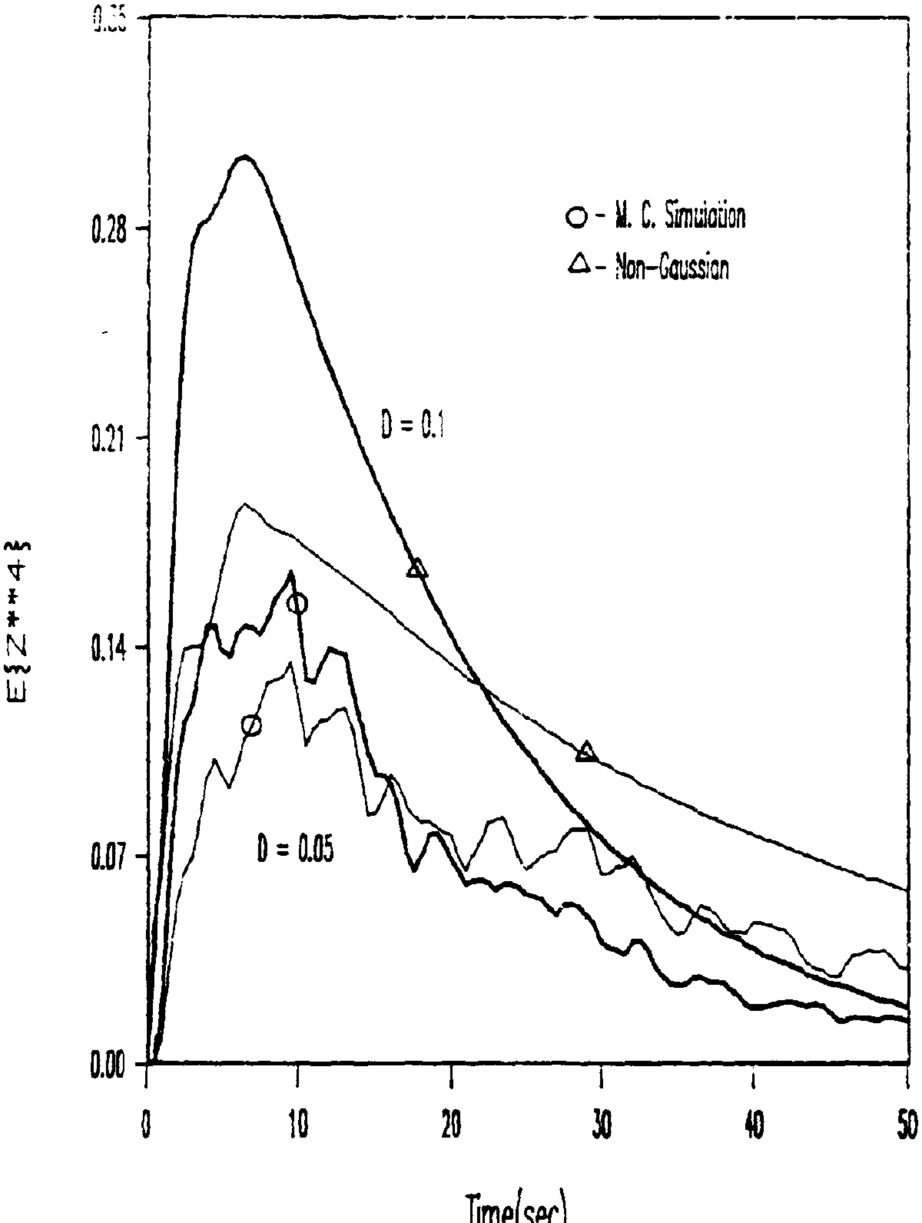

Fig. 5.95- Fourth Order Moment of the Restoring Force.
$\xi = 0.0,\ \delta_A = 0.02,\ \delta_\nu = 0.05,\ \delta_\eta = 0.1.$

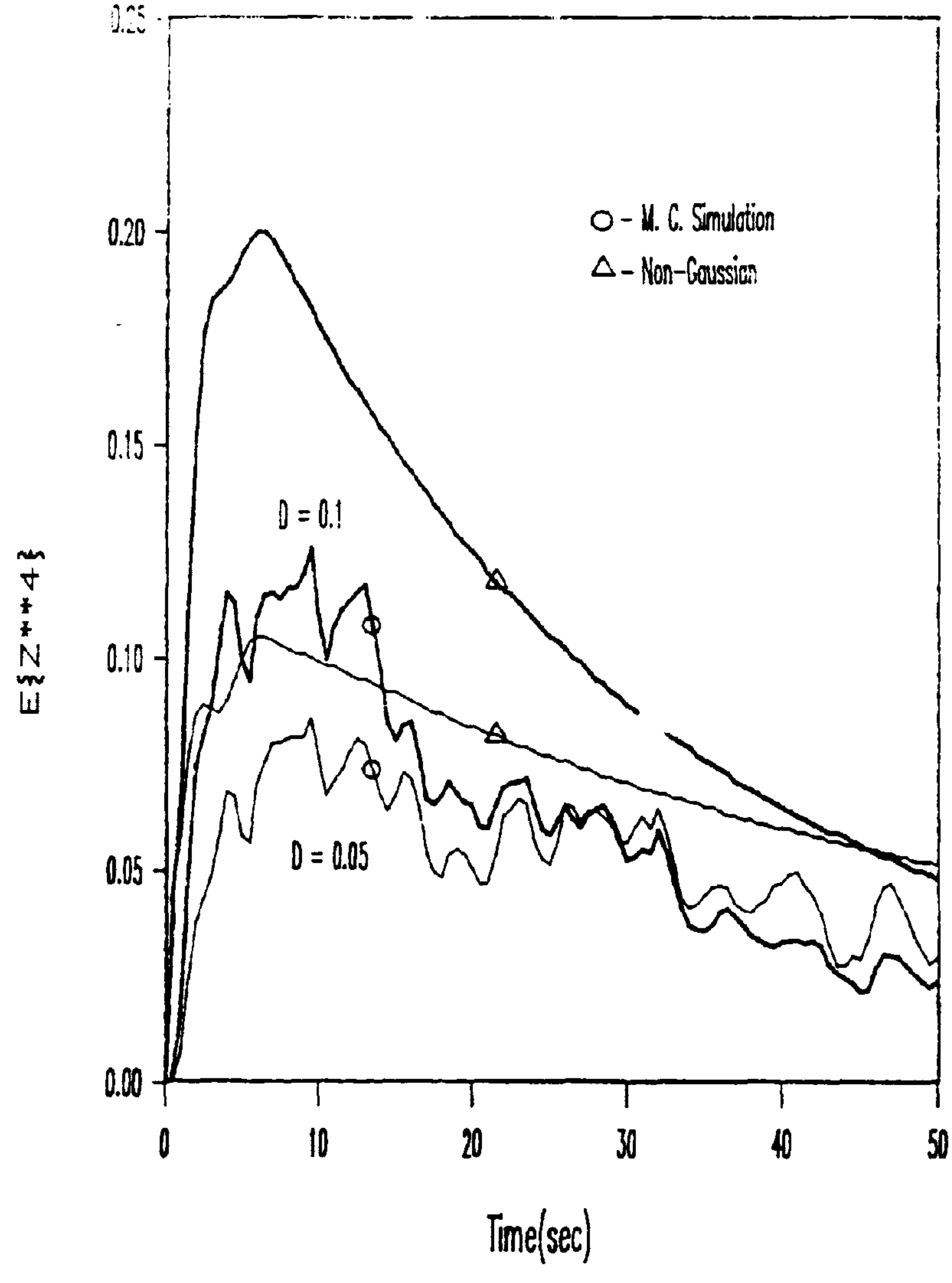

Fig. 5.96- Fourth Order Moment of the Restoring Force. $\xi = 0.1$, $\delta_A = 0.02$, $\delta_\nu = 0.05$, $\delta_\eta = 0.1$.

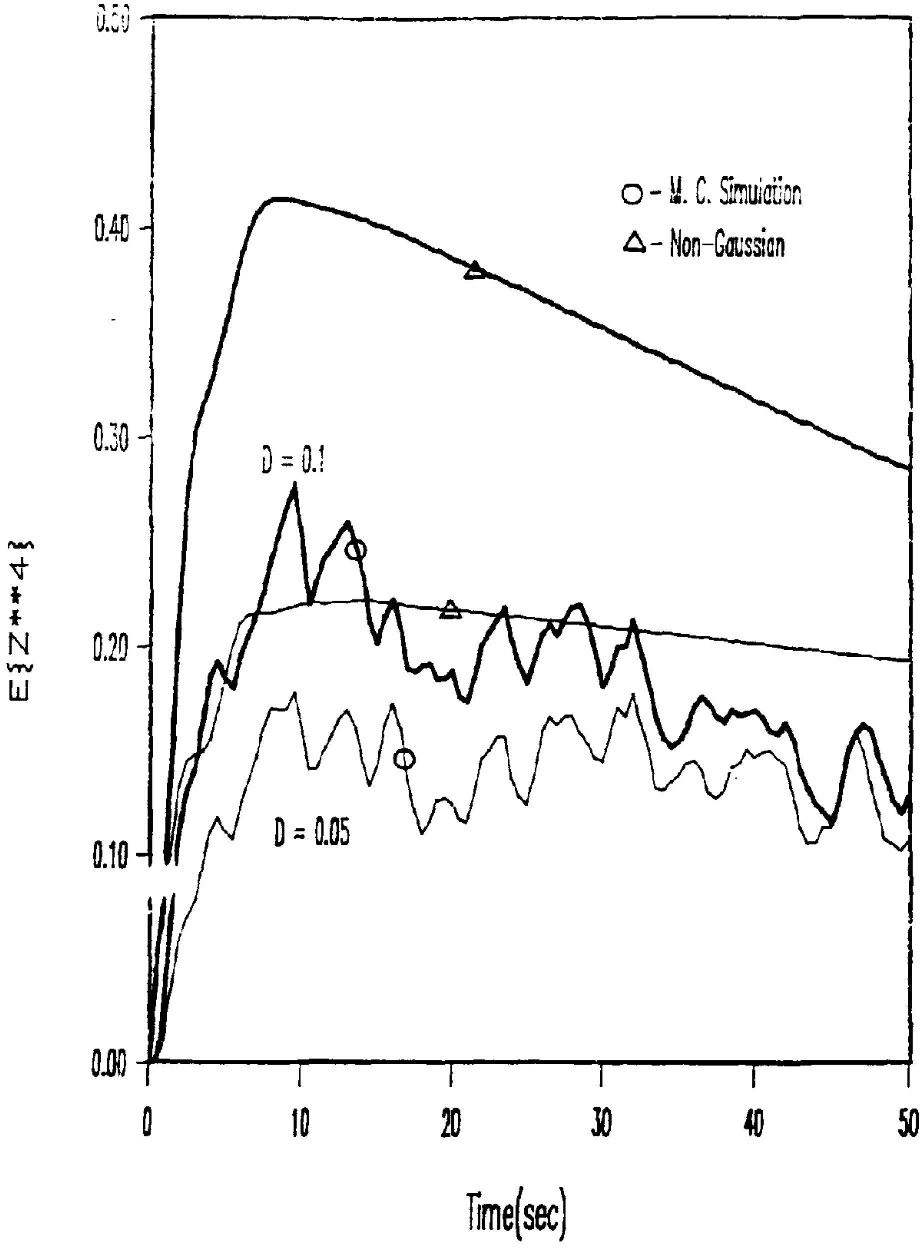

Fig. 5.97- Fourth Order Moment of the Restoring Force.
$\xi = 0.0,\ \delta_A = 0.01,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

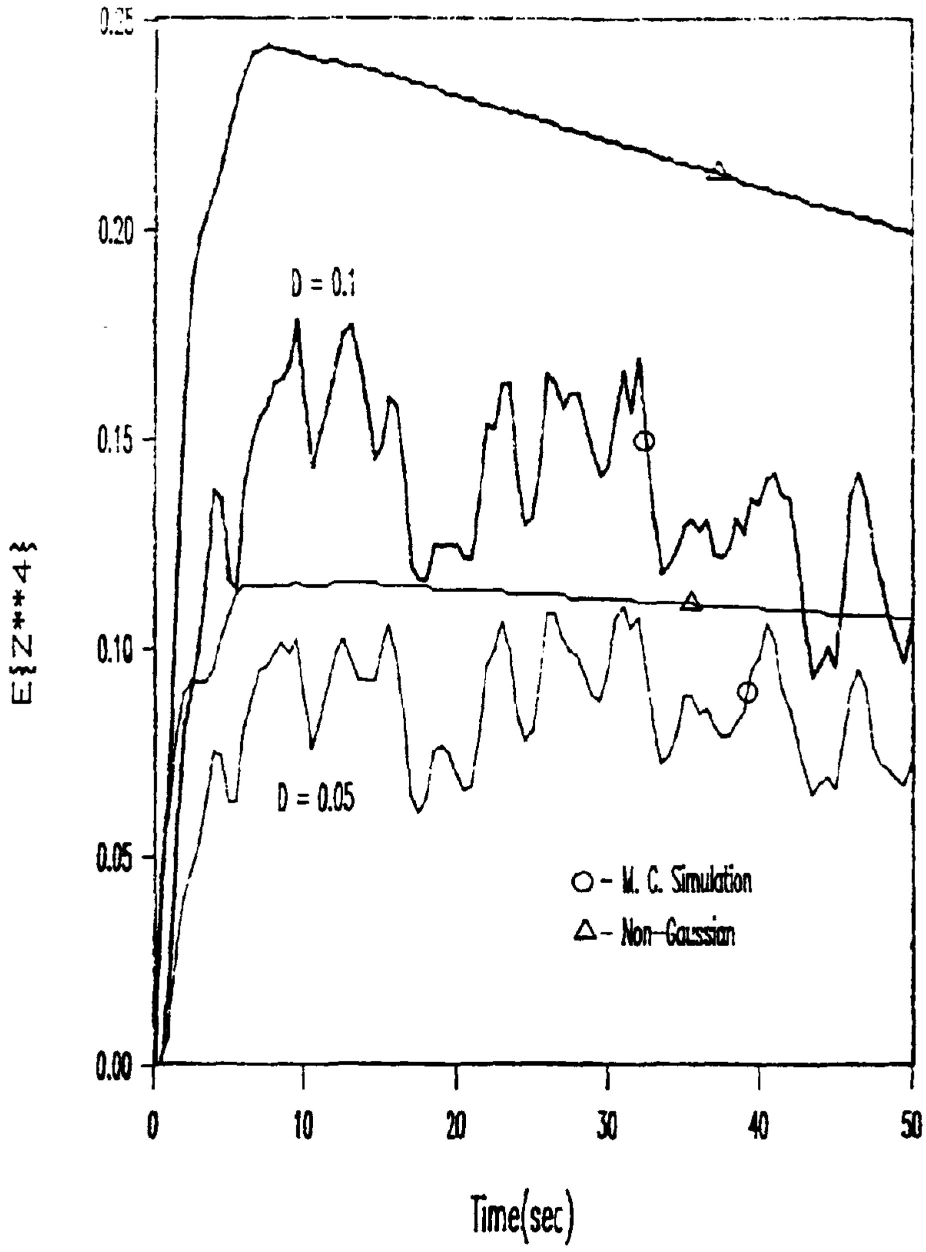

Fig. 5.98- Fourth Order Moment of the Restoring Force. $\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

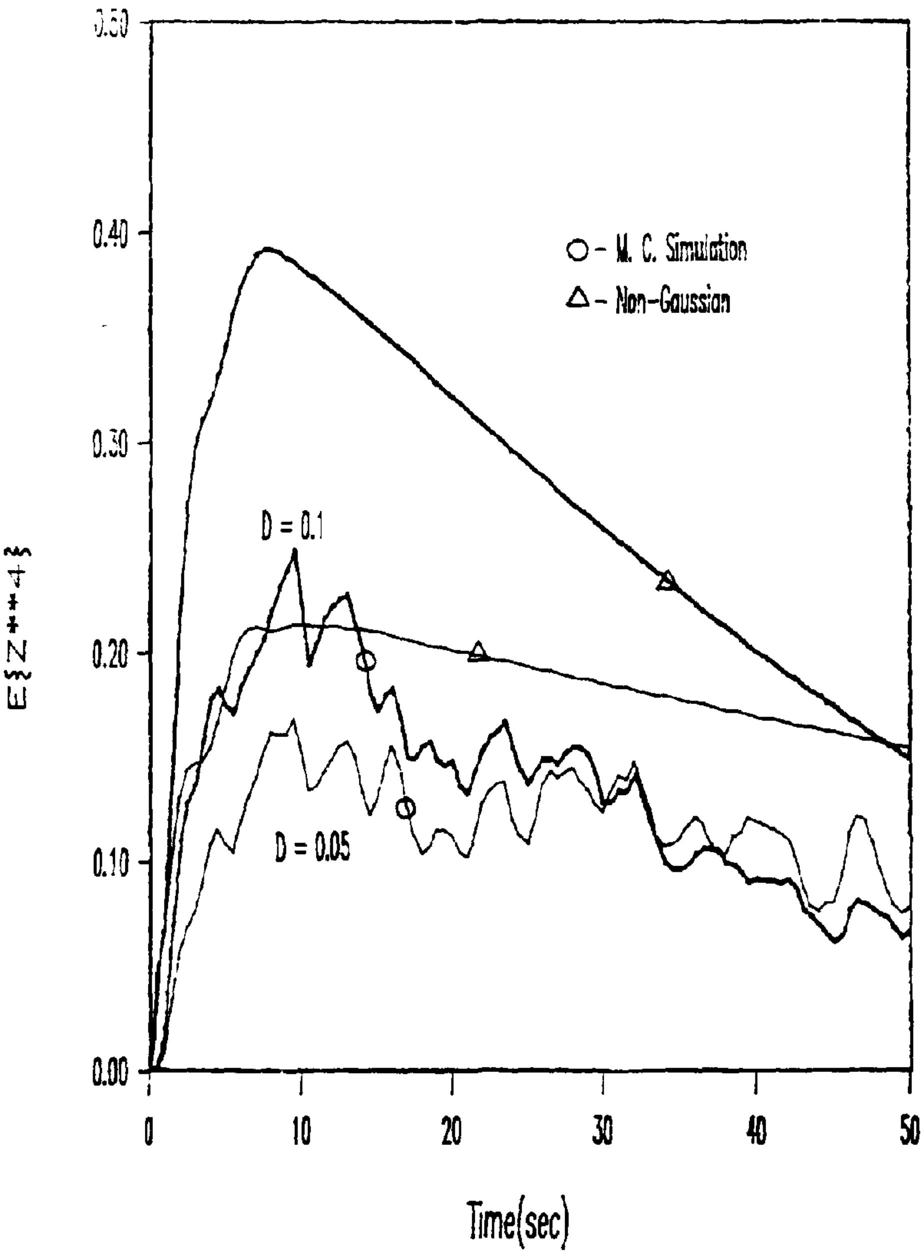

Fig. 5.99- Fourth Order Moment of the Restoring Force.
$\xi = 0.0$, $\delta_A = 0.02$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

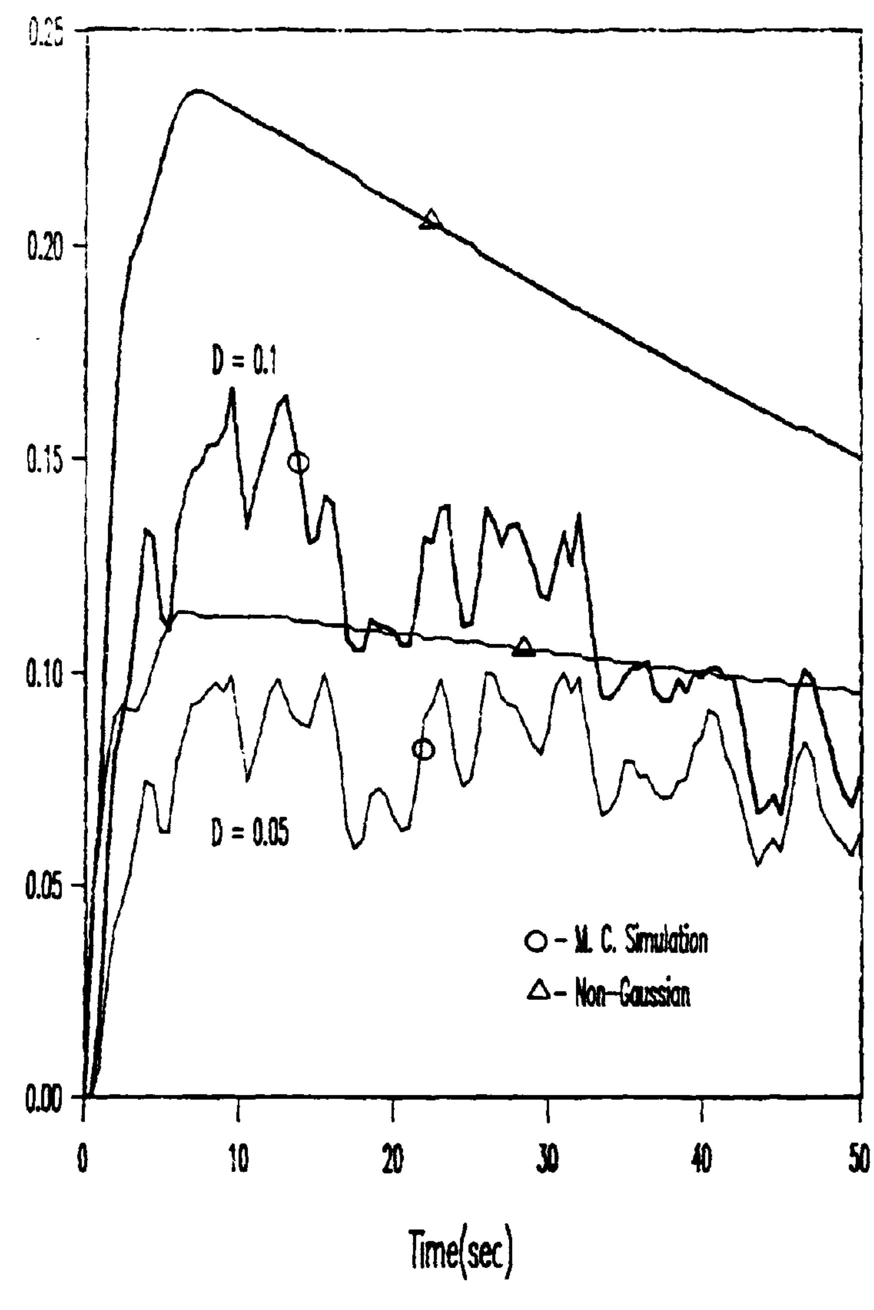

Fig. 5.100- Fourth Order Moment of the Restoring Force.
$\xi = 0.1,\ \delta_A = 0.02,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

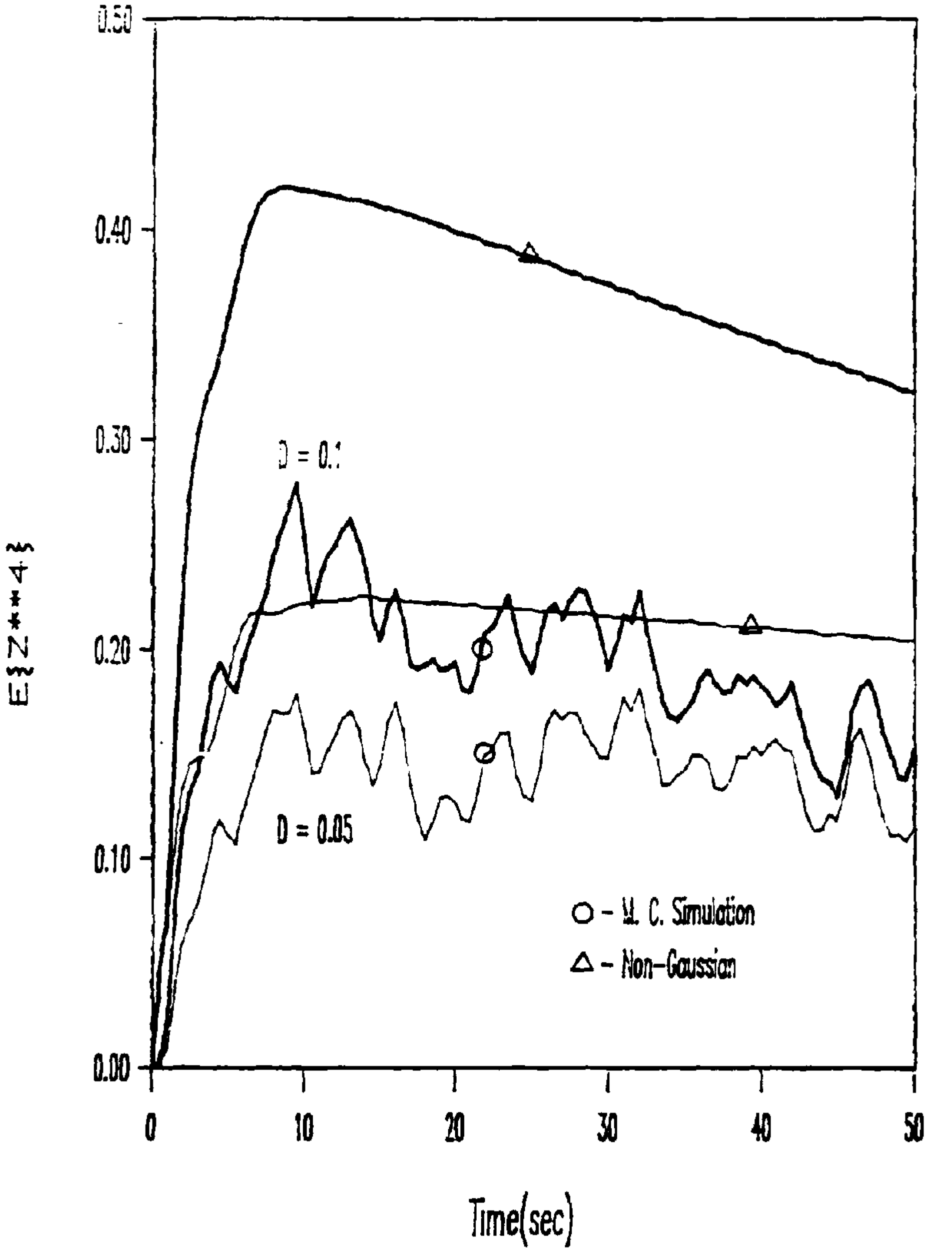

Fig. 5.101- Fourth Order Moment of the Restoring Force.
ξ = 0.0, δ_A = 0.0, δ_ν = 0.01, δ_η = 0.0.

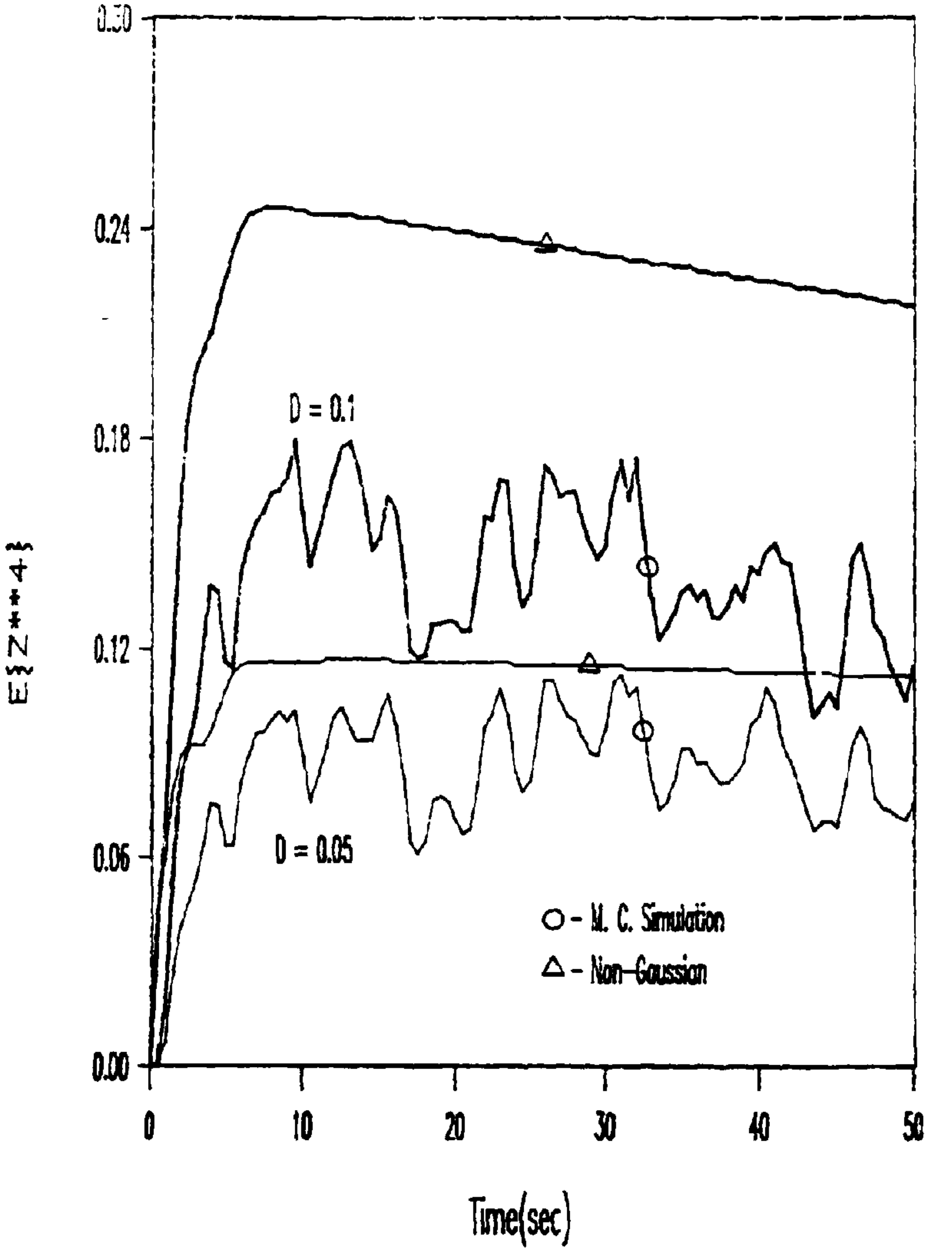

Fig. 5.102- Fourth Order Moment of the Restoring Force.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.01$, $\delta_\eta = 0.0$.

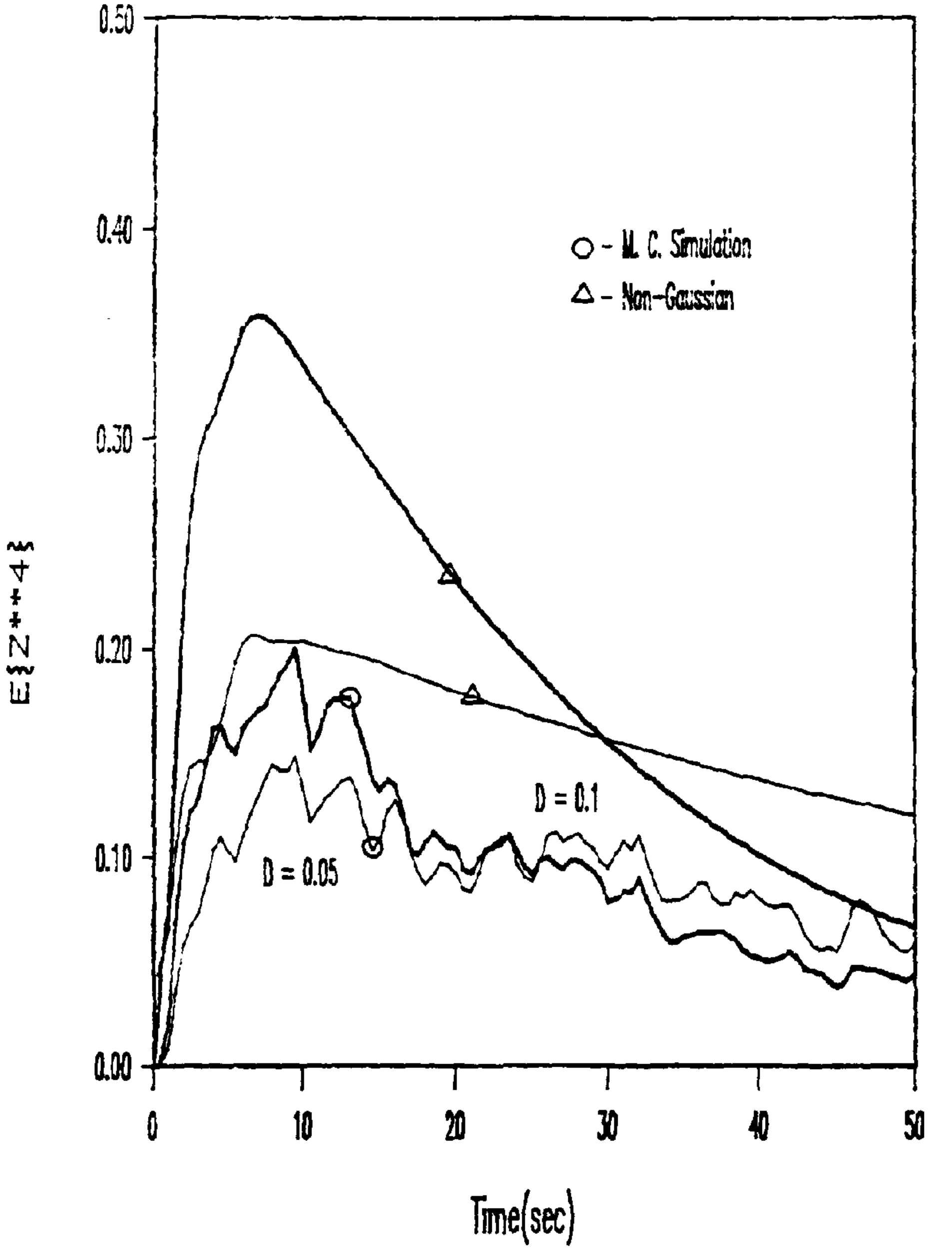

Fig. 5.103- Fourth Order Moment of the Restoring Force
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.05,\ \delta_\eta = 0.0.$

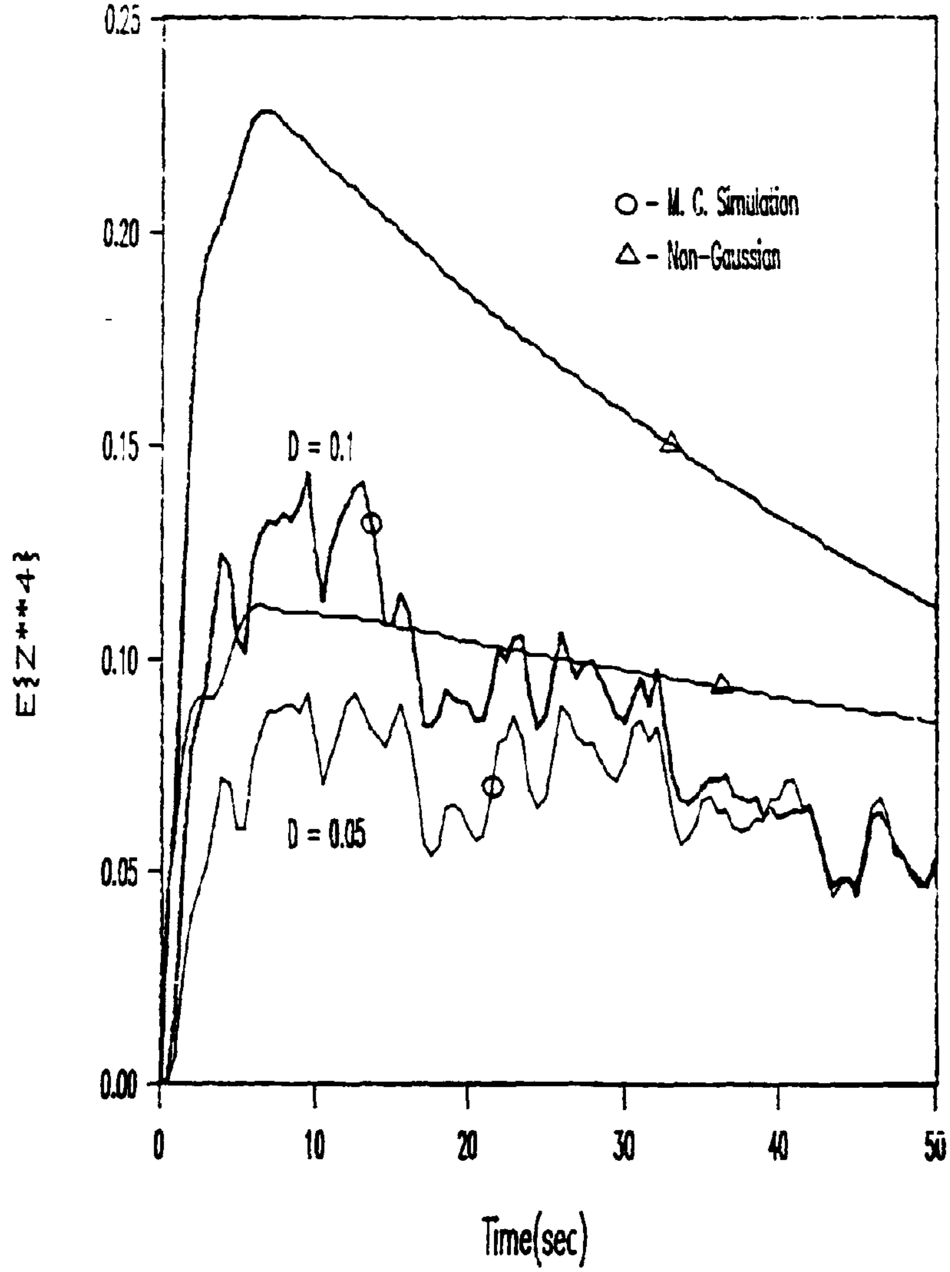

Fig. 5.104- Fourth Order Moment of the Restoring Force.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.05,\ \delta_\eta = 0.0.$

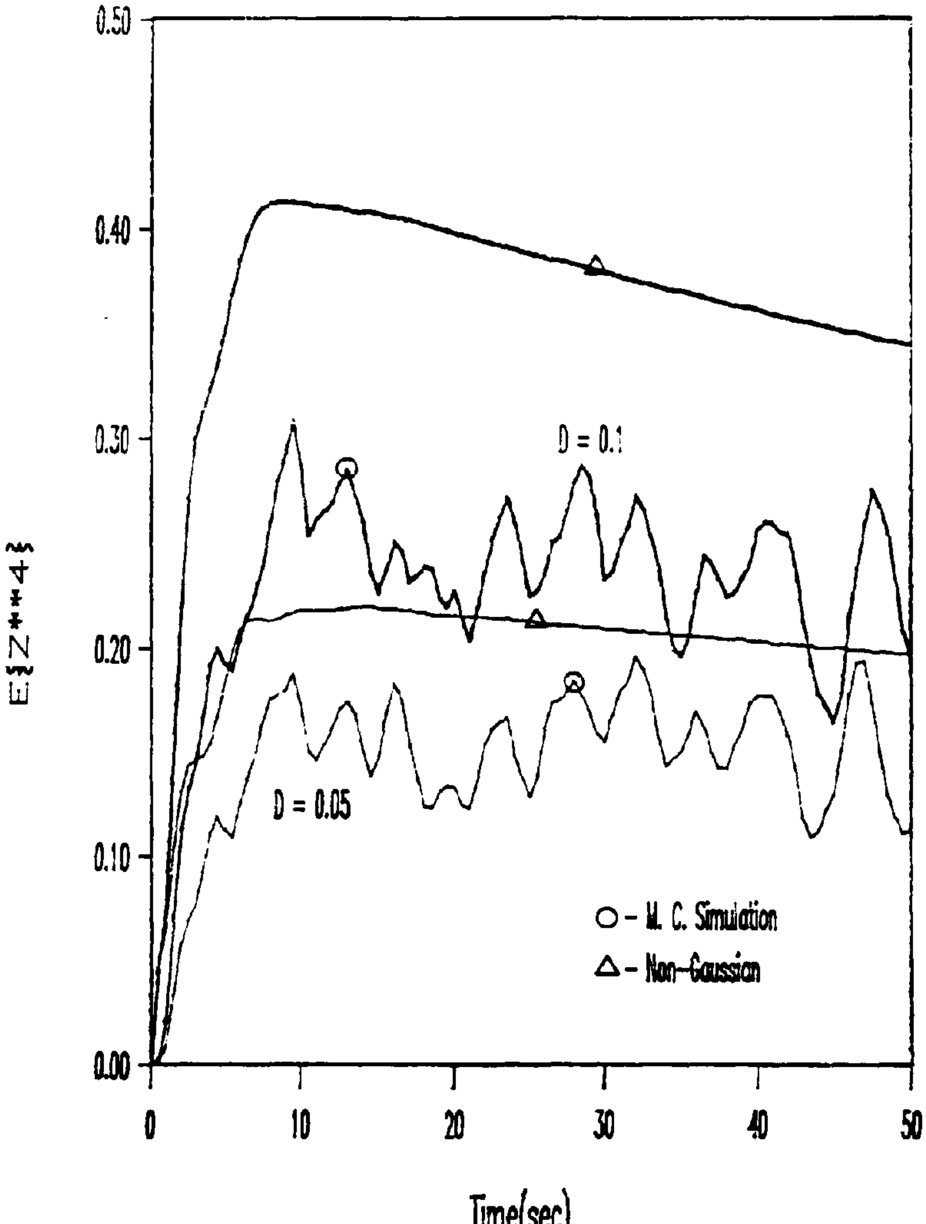

Fig. 5.105- Fourth Order Moment of the Restoring Force.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.05$.

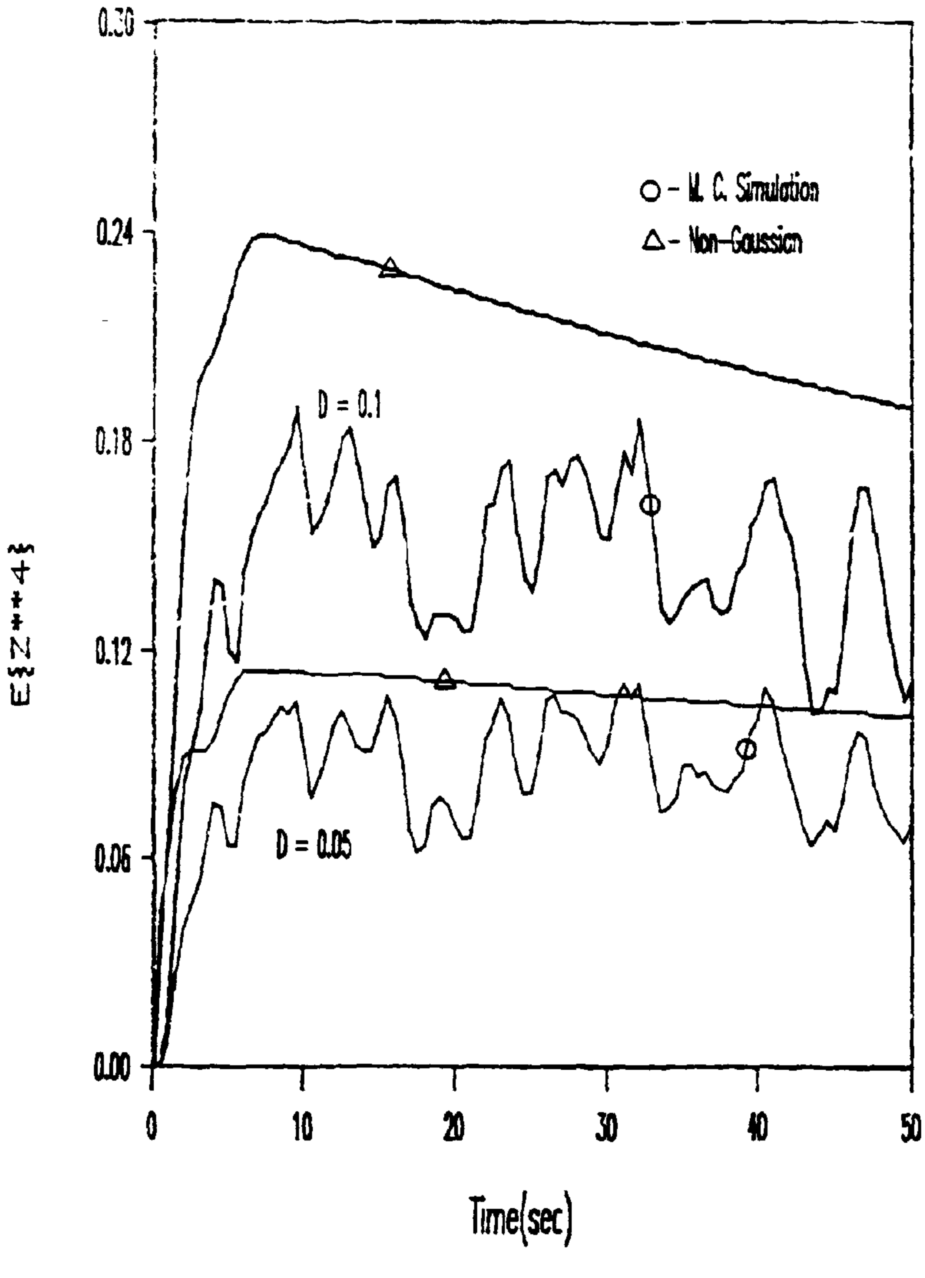

Fig. 5.106- Fourth Order Moment of the Restoring Force.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.05.$

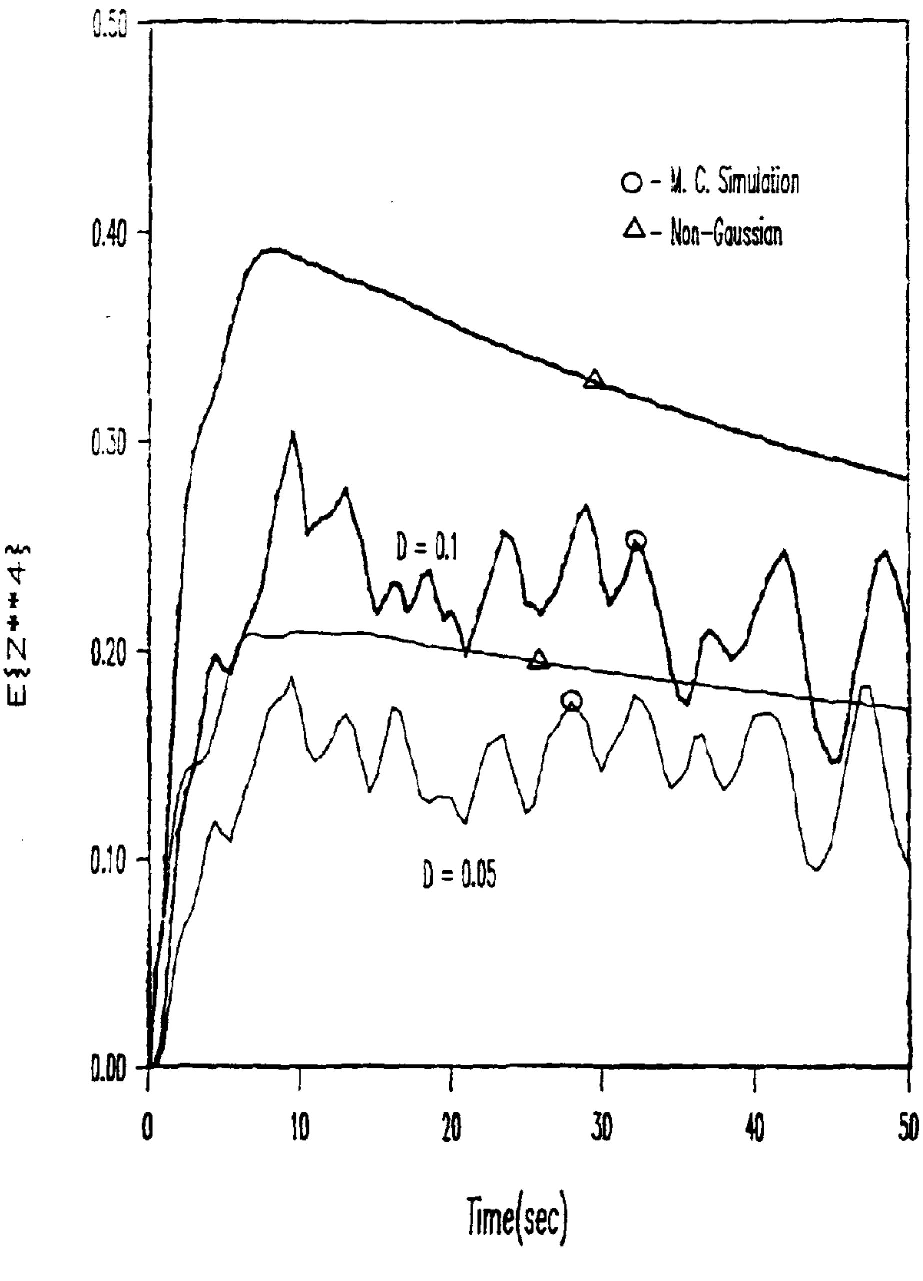

Fig. 5.107- **Fourth Order Moment of the Restoring Force.**
ξ = 0.0, δ_A = 0.0, δ_ν = 0.0, δ_η = 0.1.

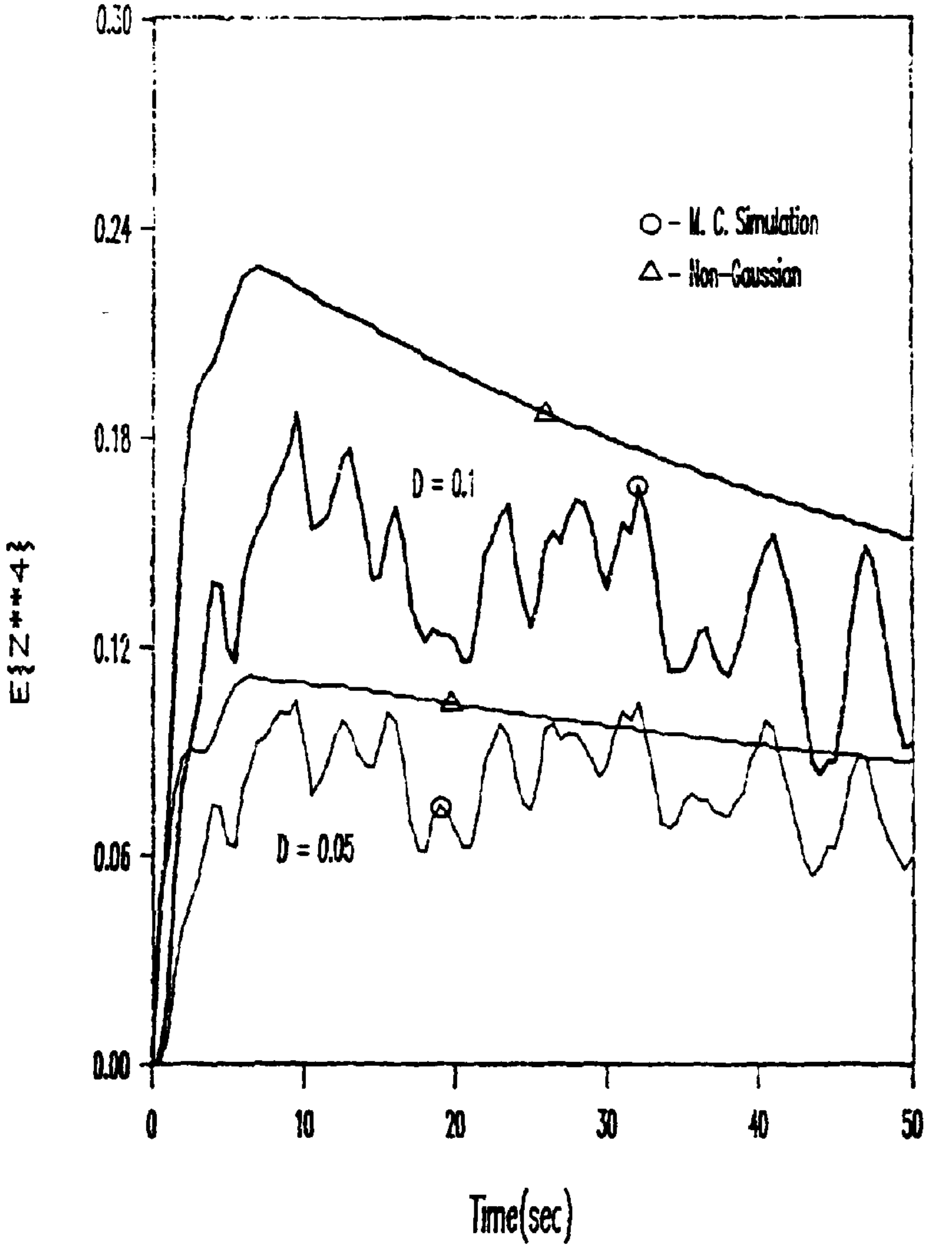

Fig. 5.108- Fourth Order Moment of the Restoring Force.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.1$.

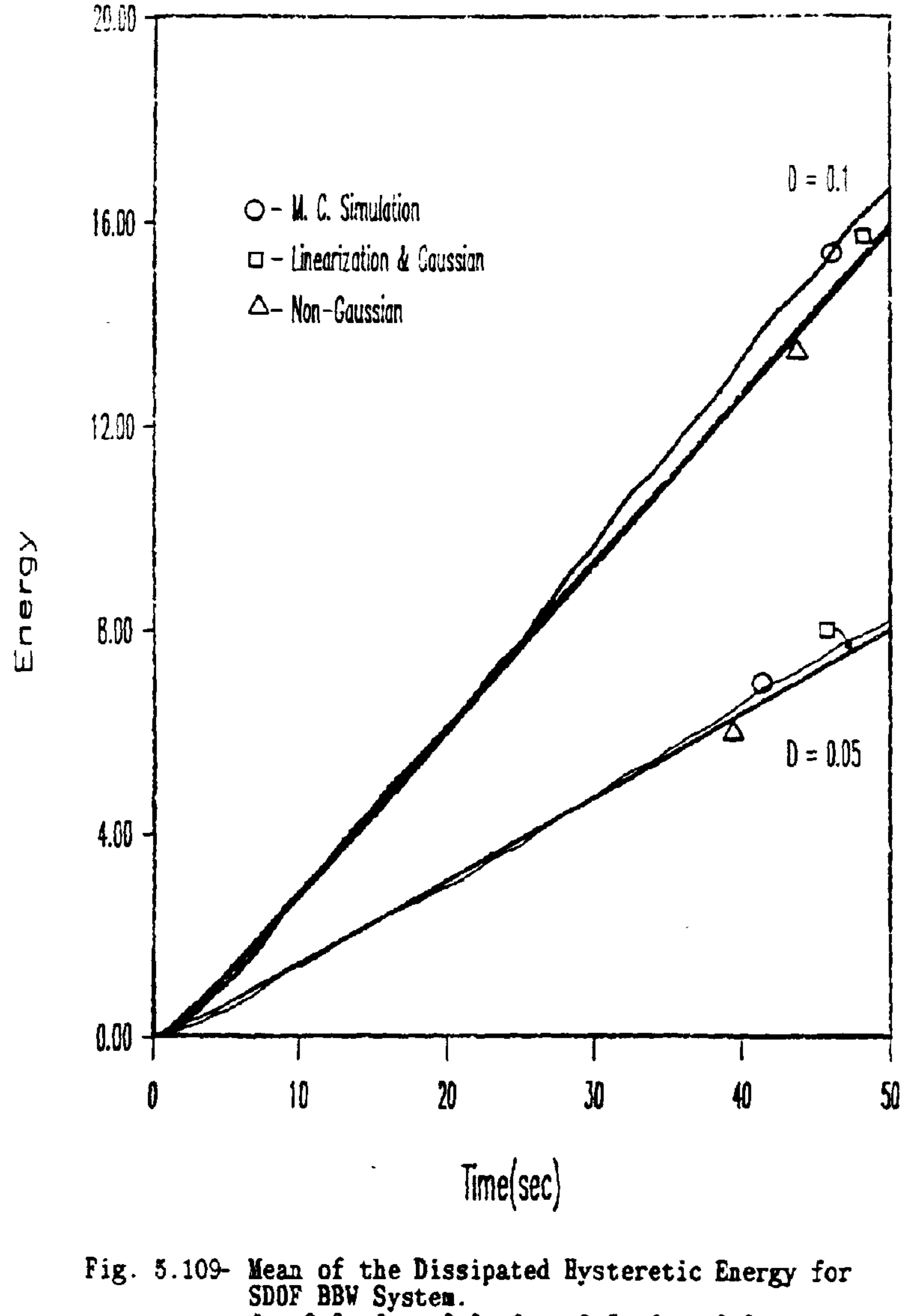

Fig. 5.109- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.0,\ \delta_A = 0.0,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

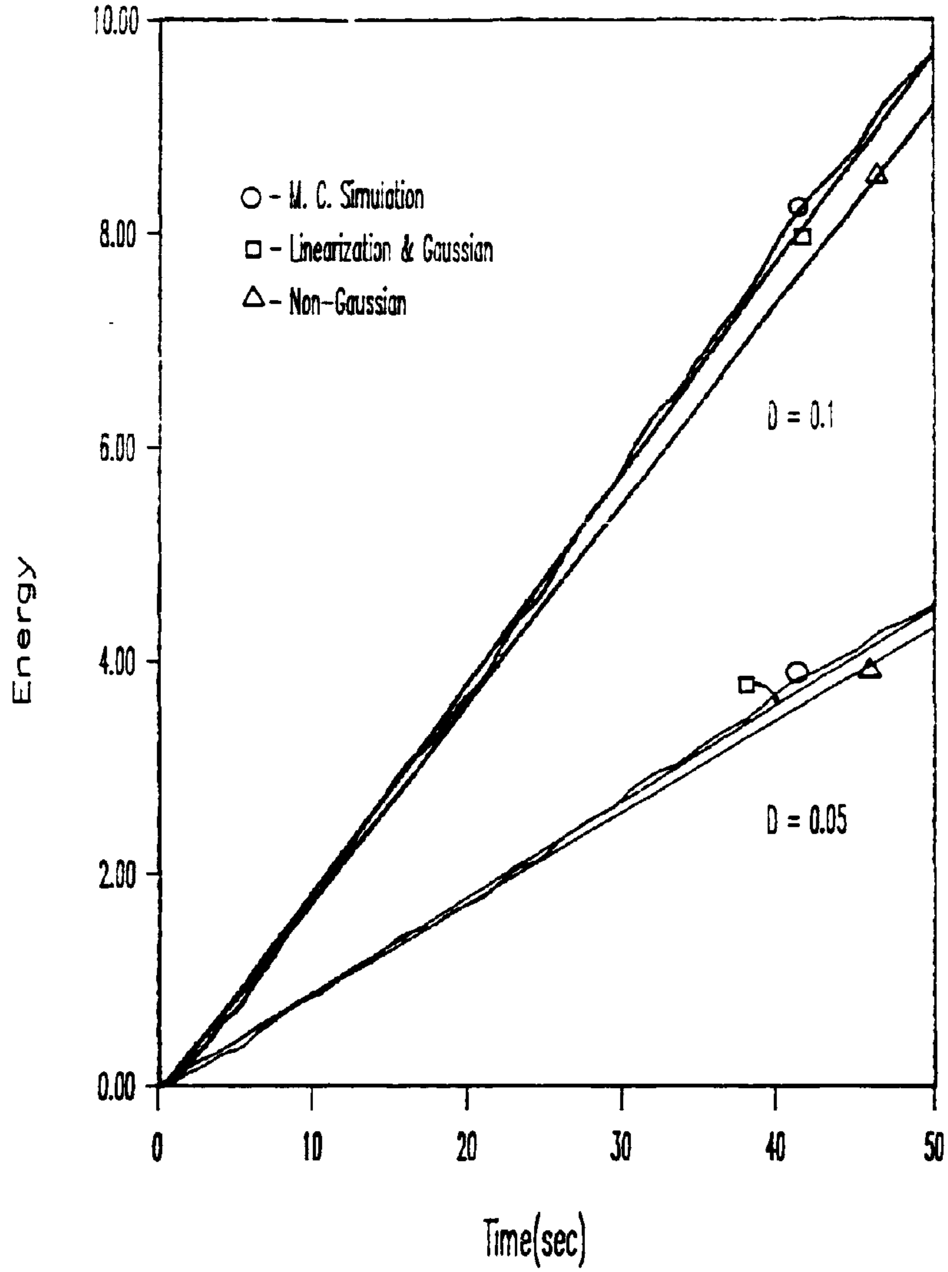

Fig. 5.110- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

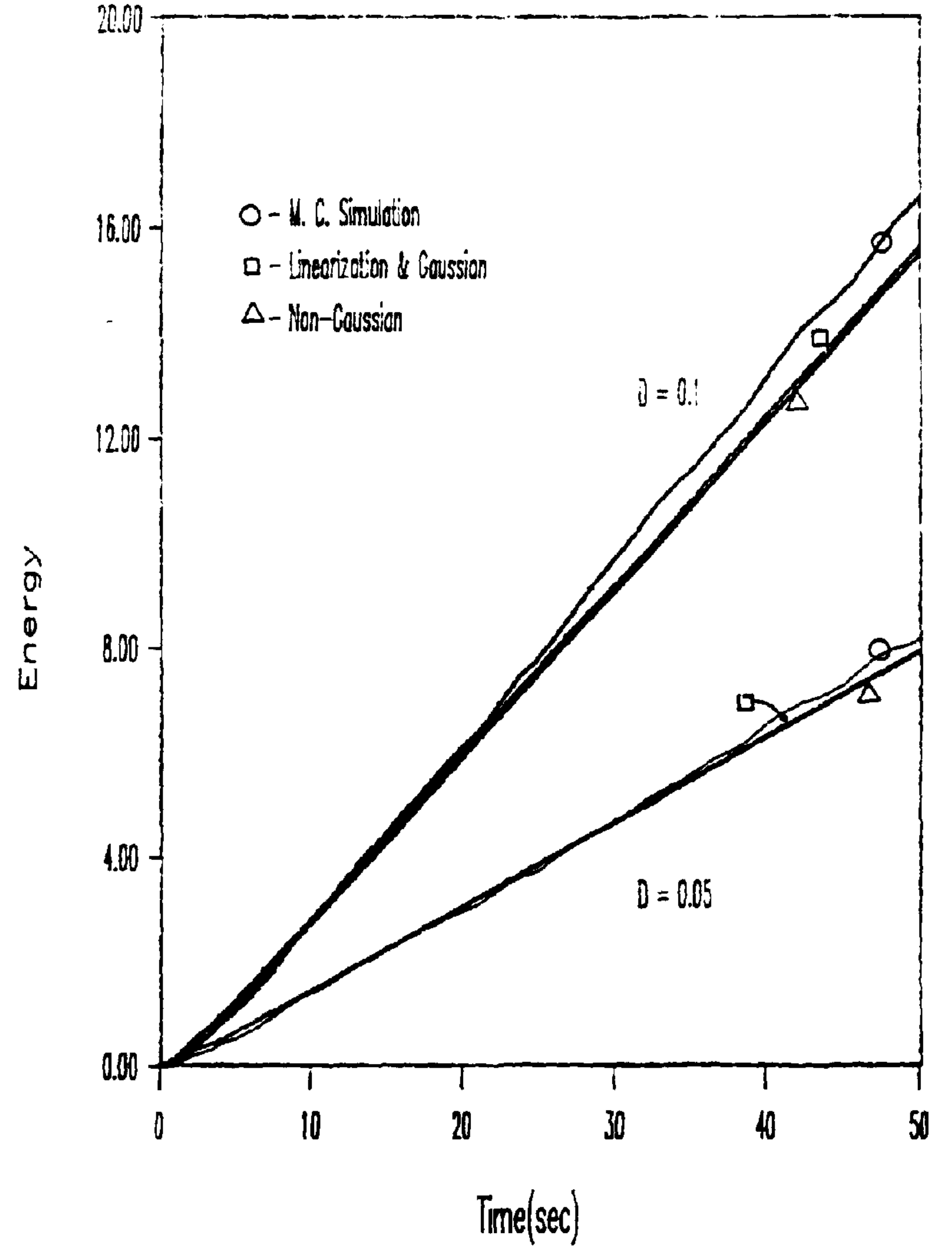

Fig. 5.111- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

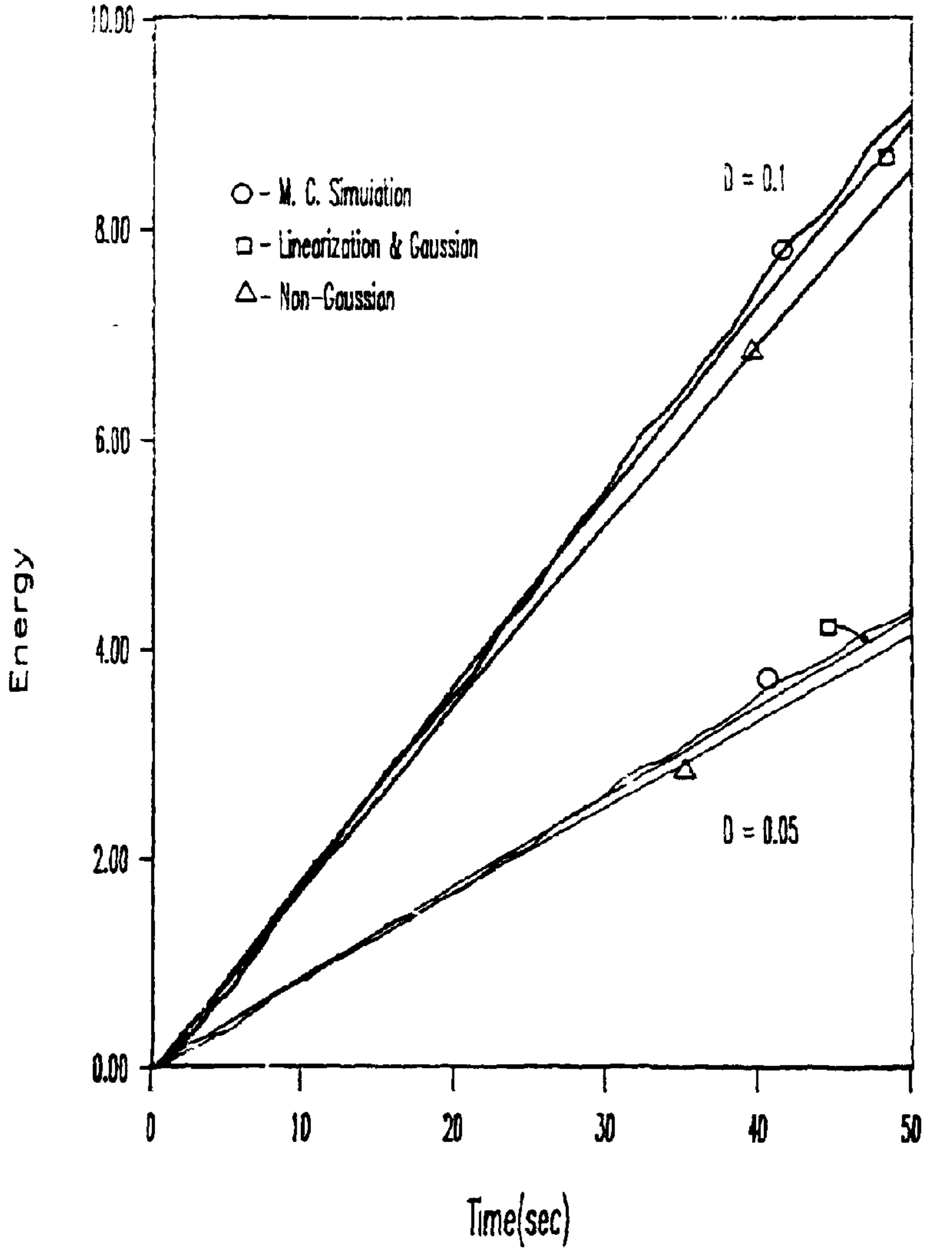

Fig. 5.112- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.01$, $\delta_\nu = 0.01$, $\delta_\eta = 0.05$.

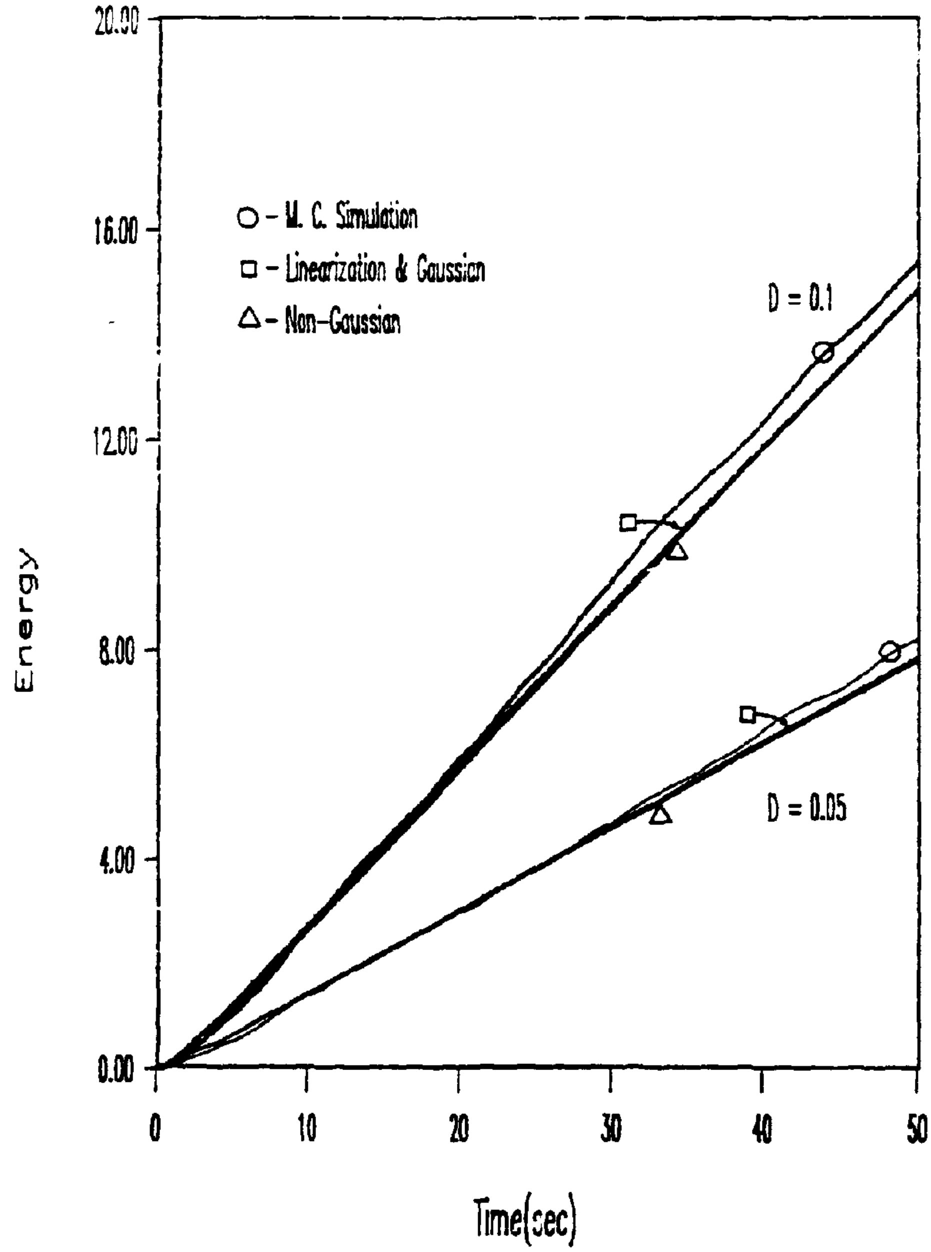

Fig. 5.113- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.02$, $\delta_\nu = 0.05$, $\delta_\eta = 0.1$.

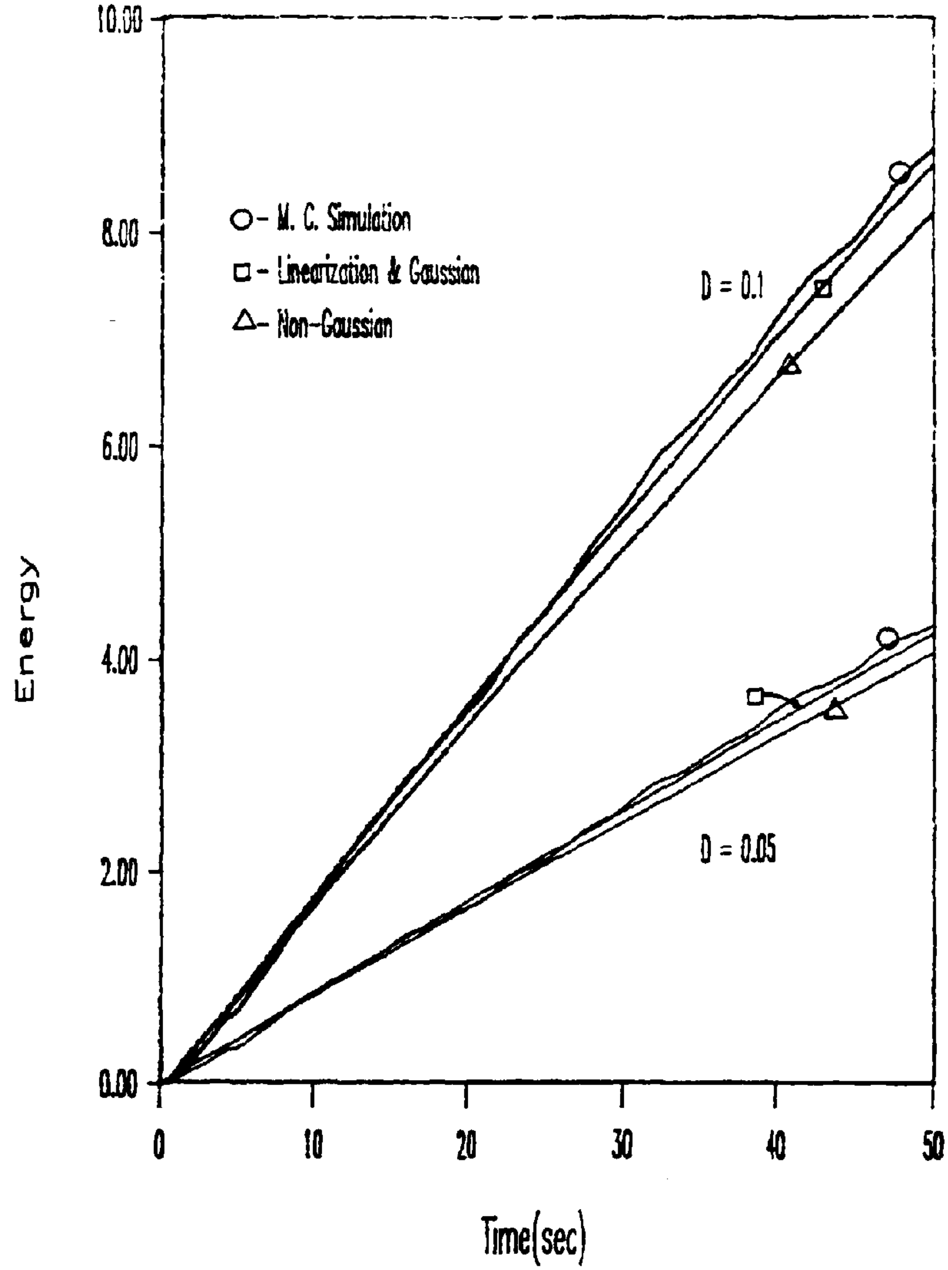

Fig. 5.114- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.02$, $\delta_\nu = 0.05$, $\delta_\eta = 0.1$.

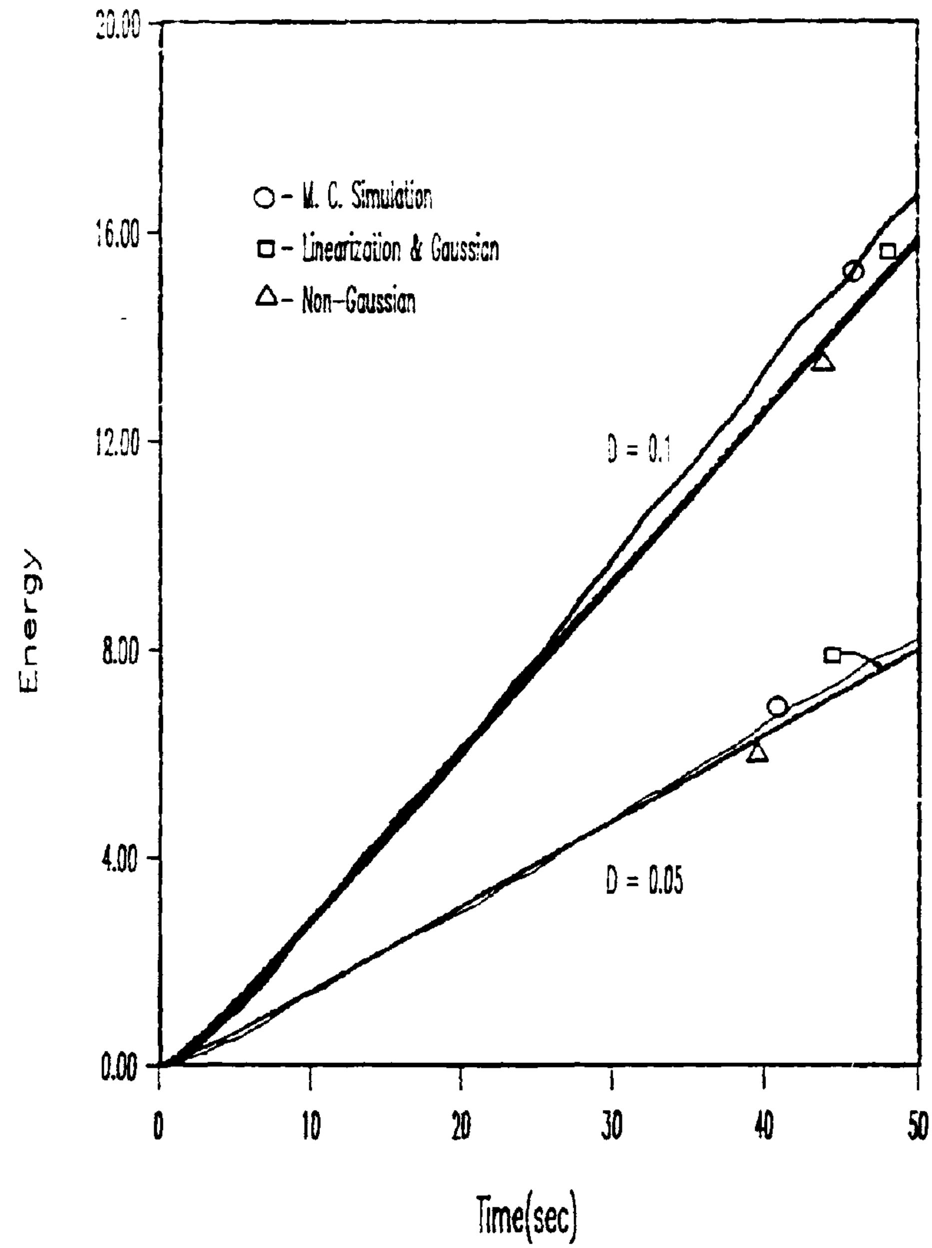

Fig. 5.115- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.01$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

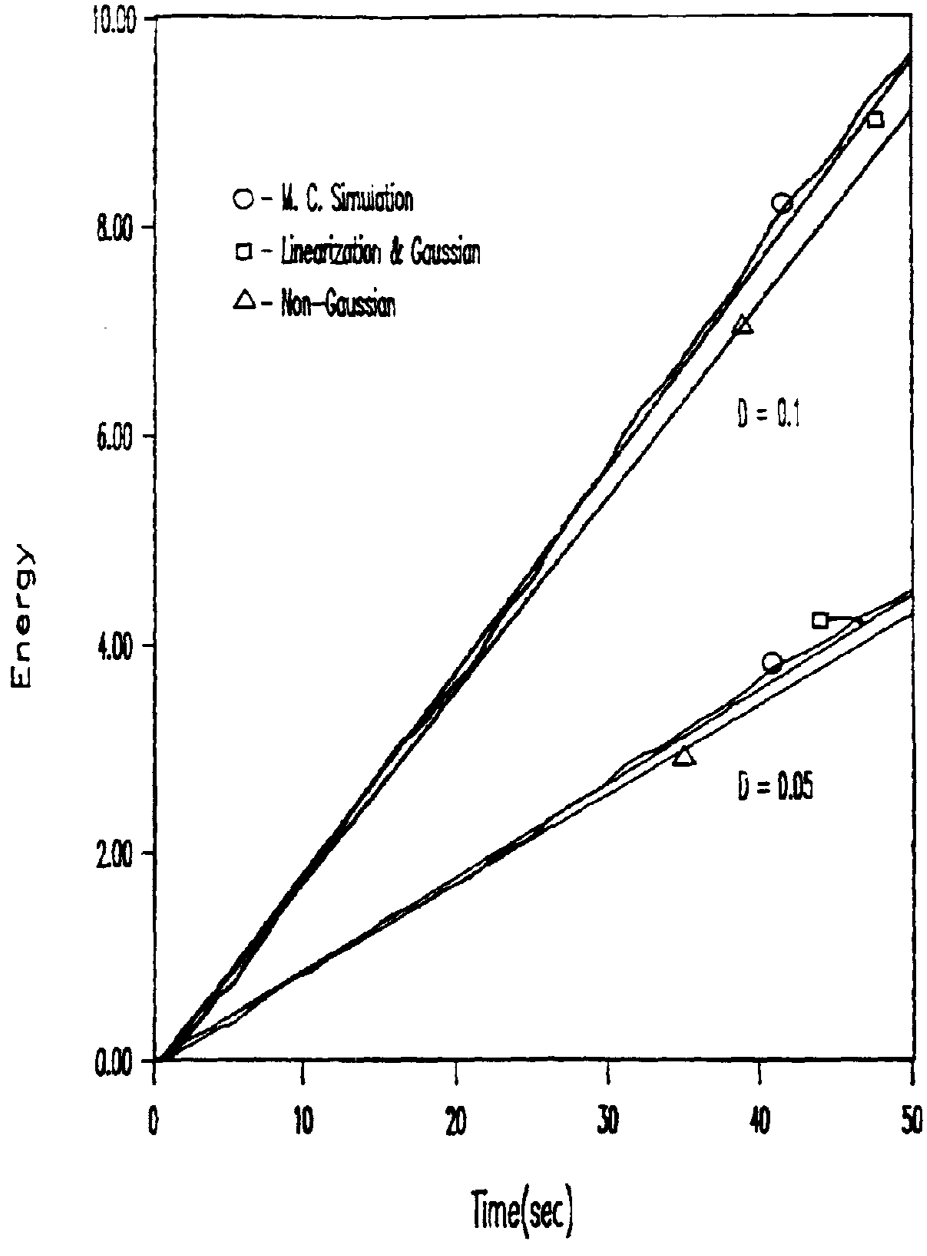

Fig. 5.116- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1,\ \delta_A = 0.01,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

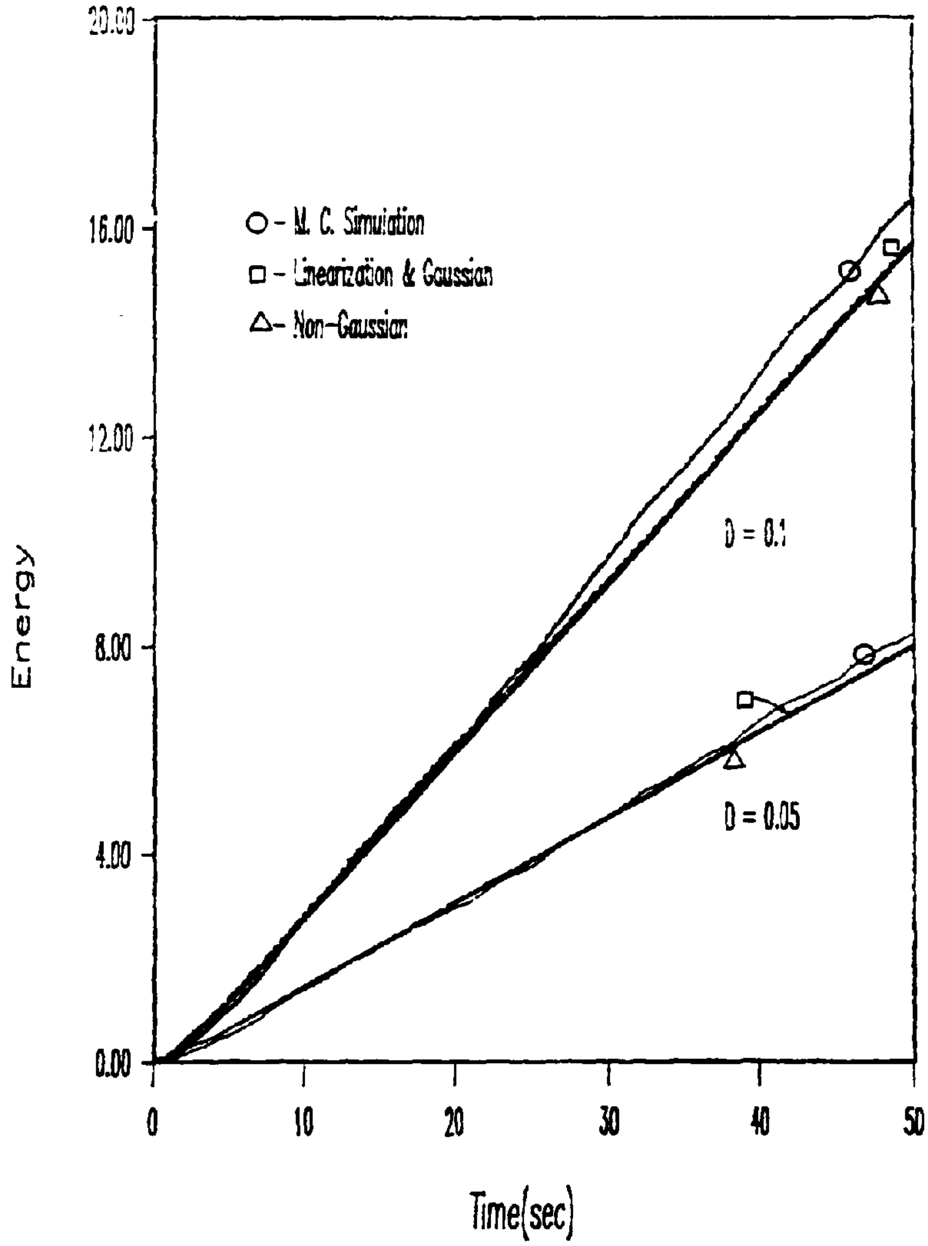

Fig. 5.117- **Mean of the Dissipated Hysteretic Energy for SDOF BBW System.**
$\xi = 0.0,\ \delta_A = 0.02,\ \delta_\nu = 0.0,\ \delta_\eta = 0.0.$

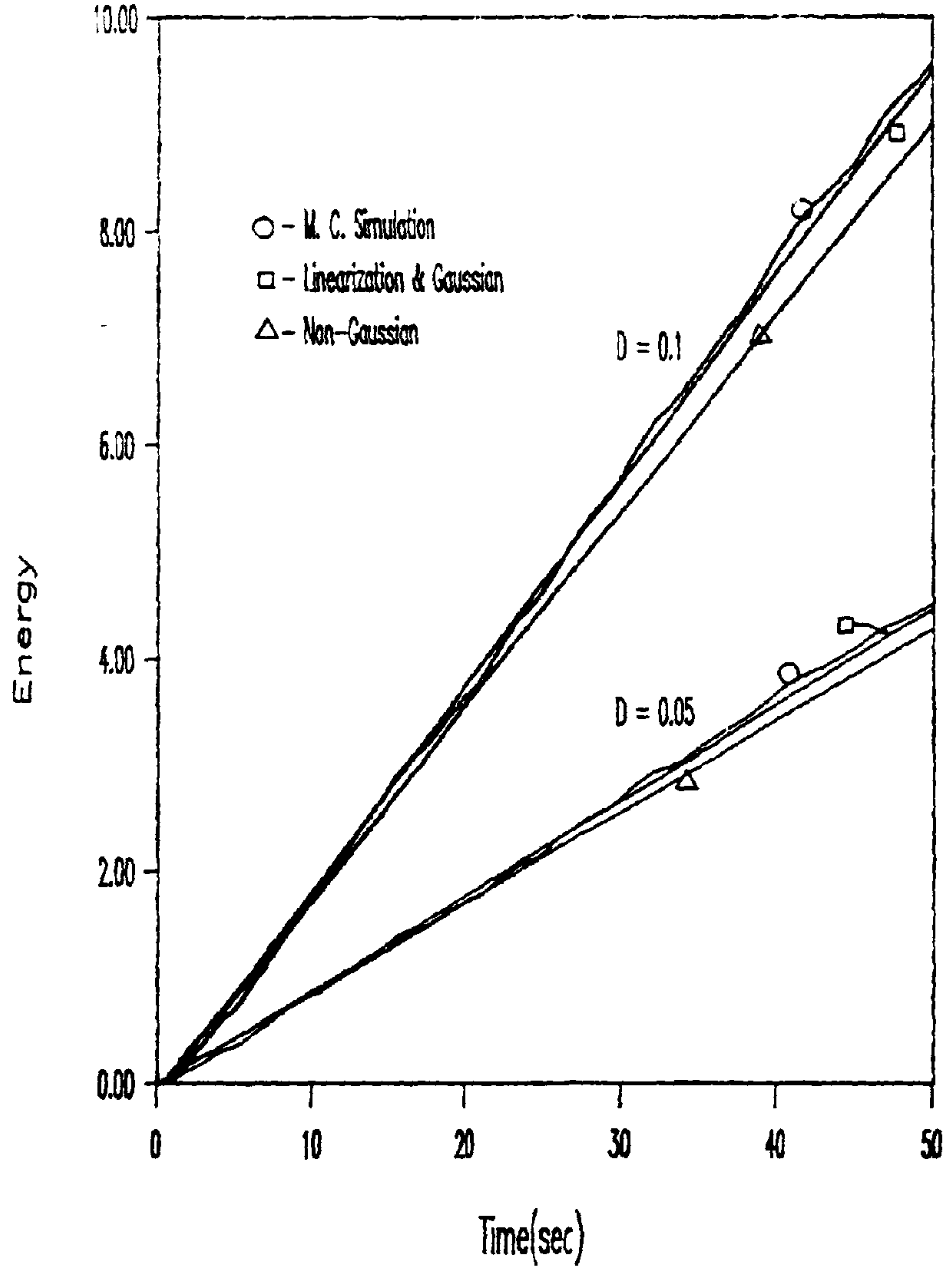

Fig. 5.118- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.02$, $\delta_\nu = 0.0$, $\delta_\eta = 0.0$.

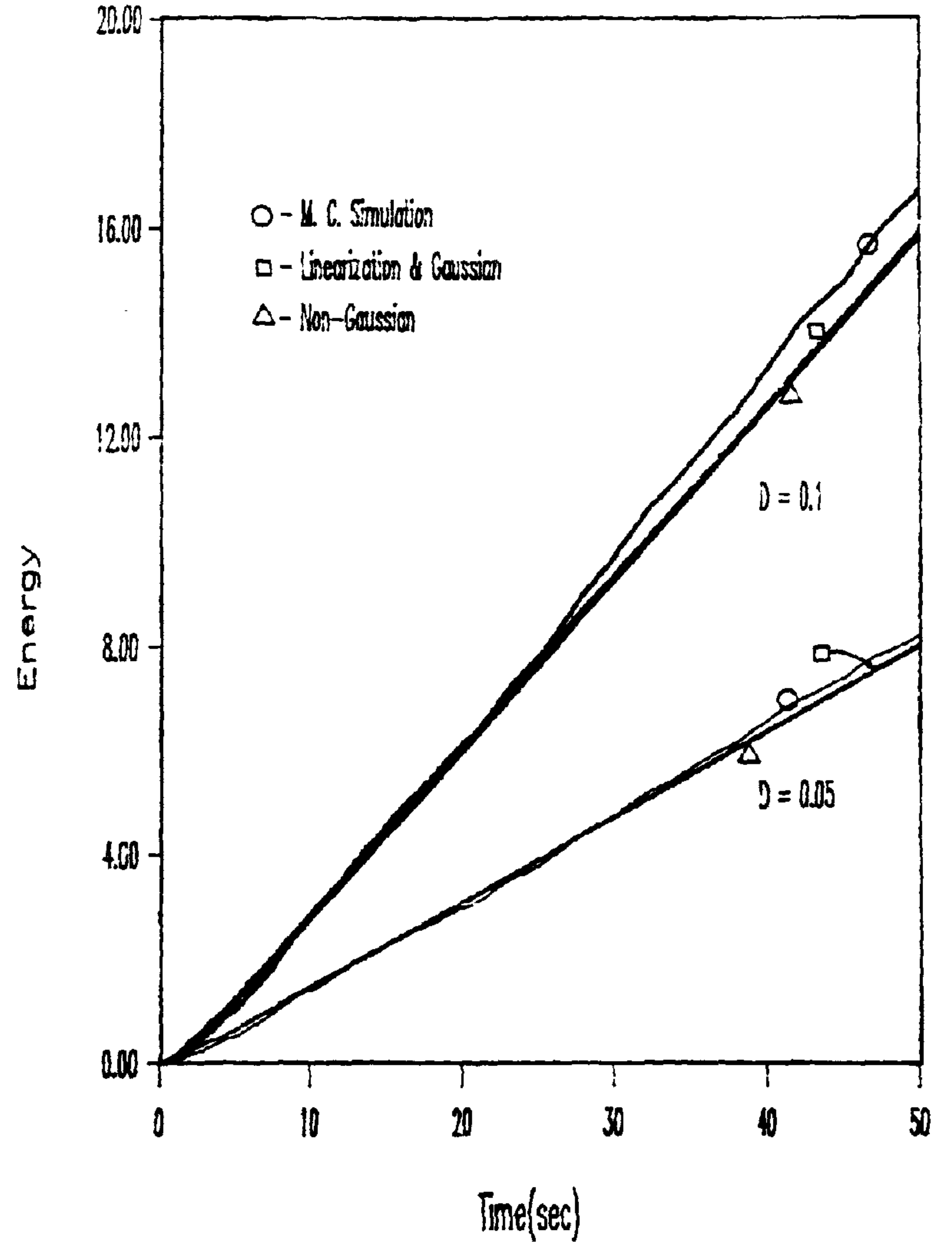

Fig. 5.119- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.01$, $\delta_\eta = 0.0$.

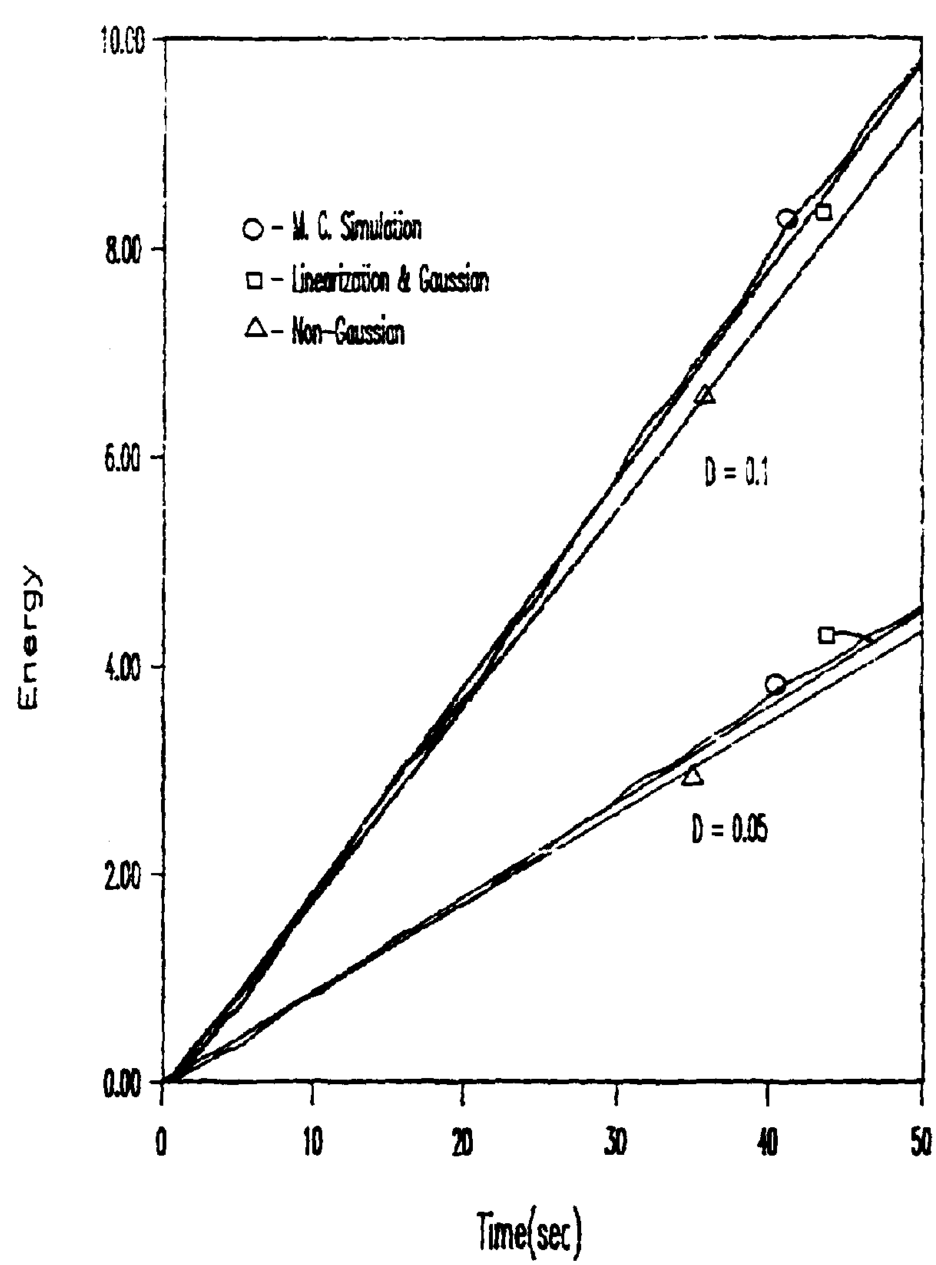

Fig. 5.120- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.01,\ \delta_\eta = 0.0.$

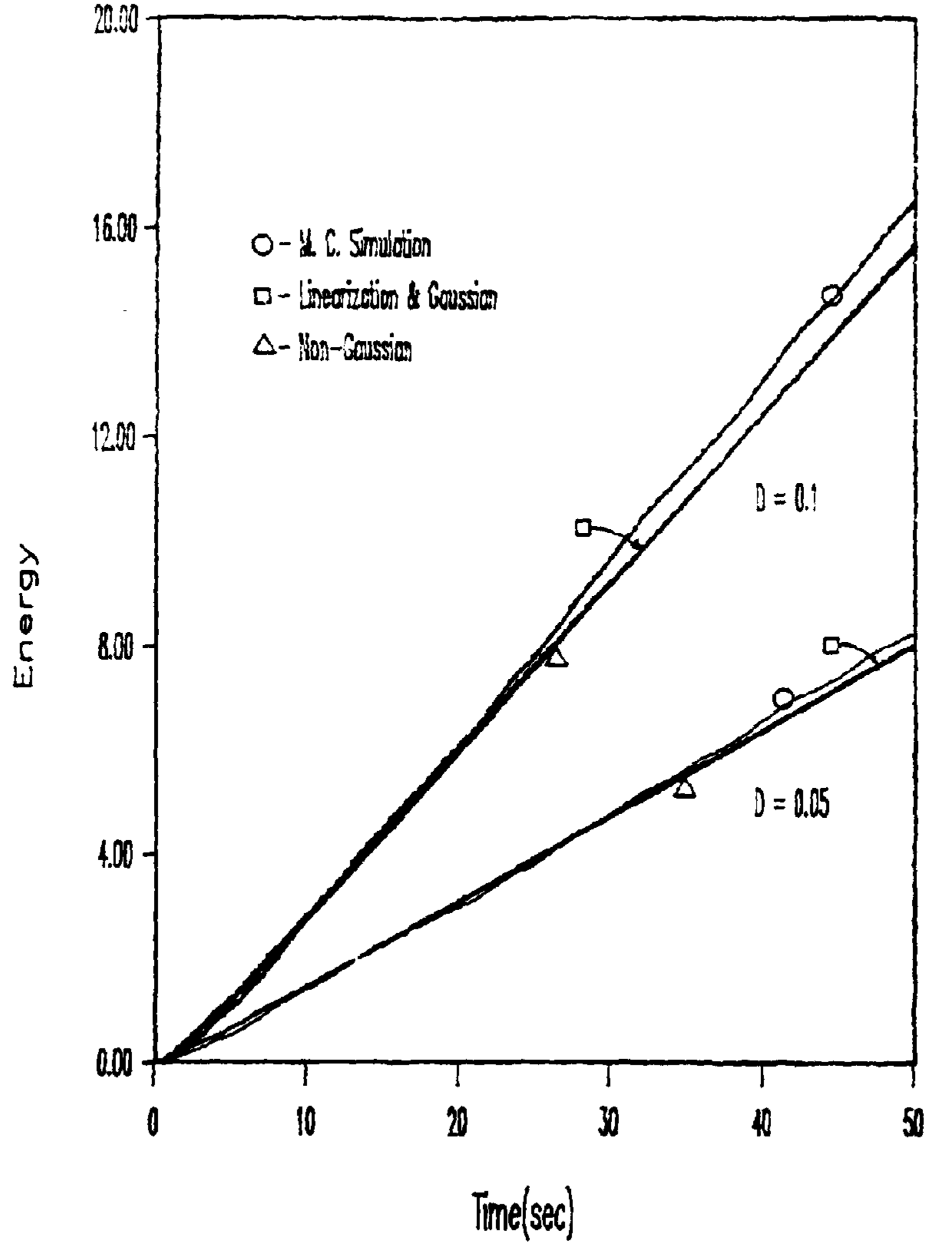

Fig. 5.121- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\epsilon = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.05$, $\delta_\eta = 0.0$.

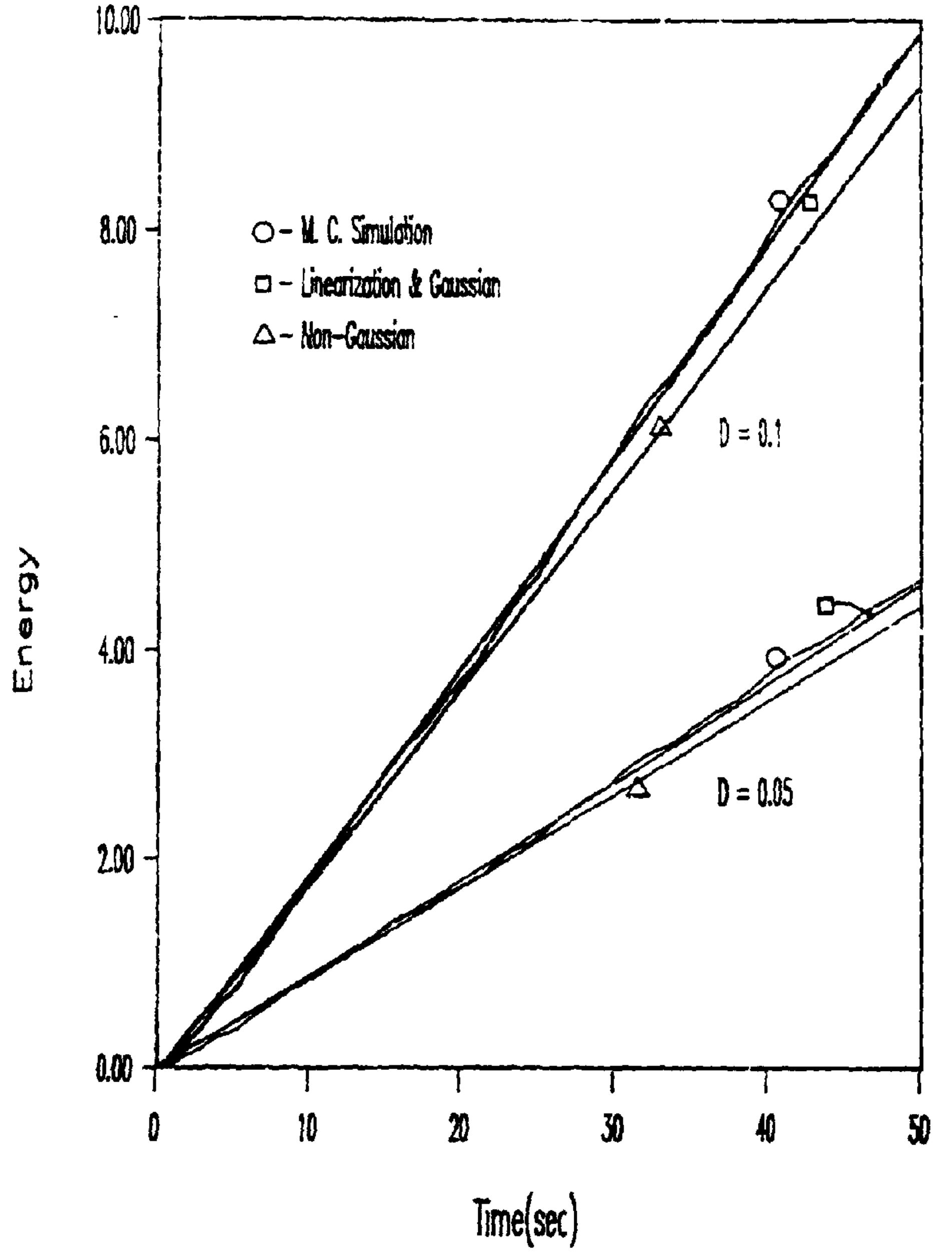

Fig. 5.122- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1,\ \delta_A = 0.0,\ \delta_\nu = 0.05,\ \epsilon_\eta = 0.0.$

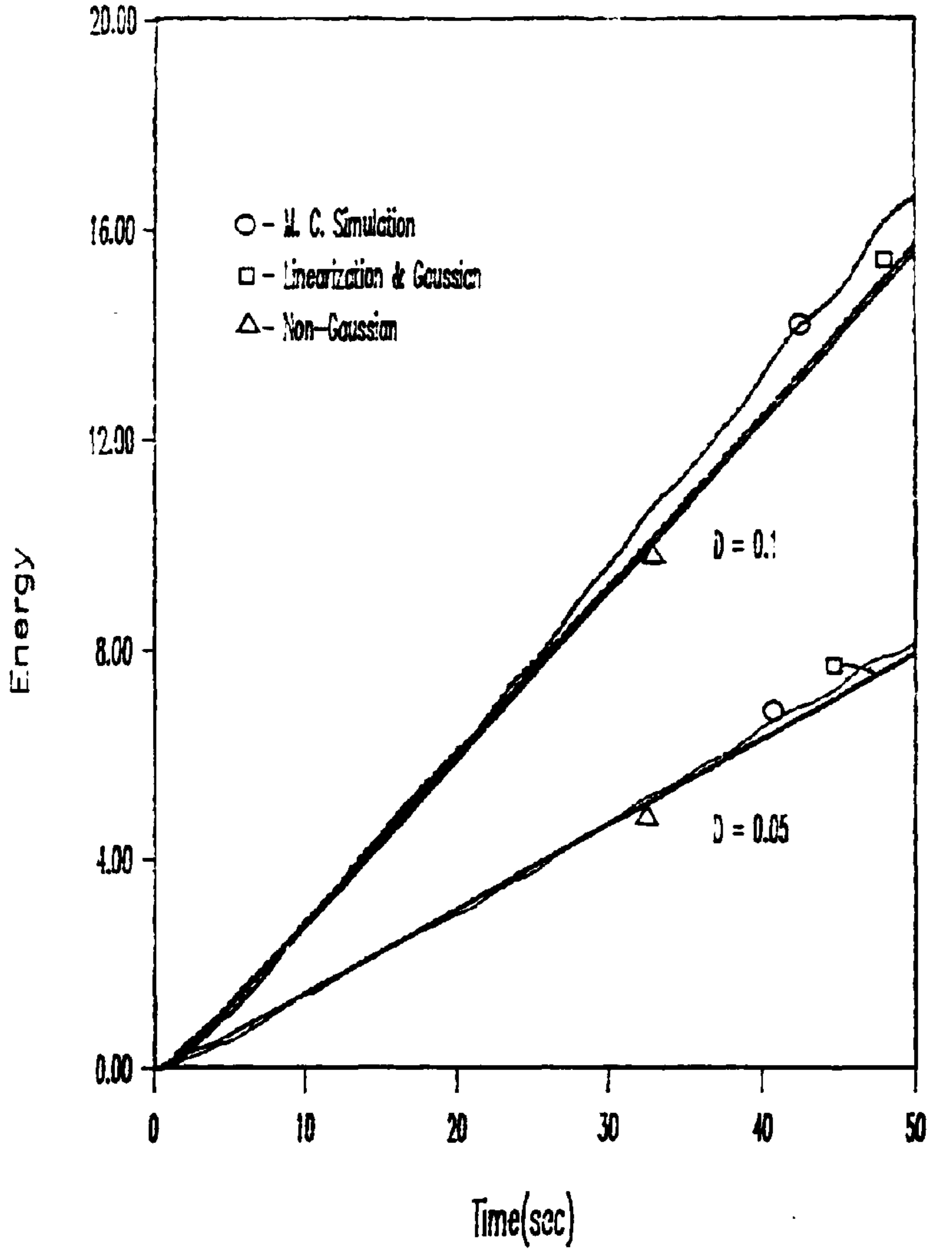

Fig. 5.123- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.05$.

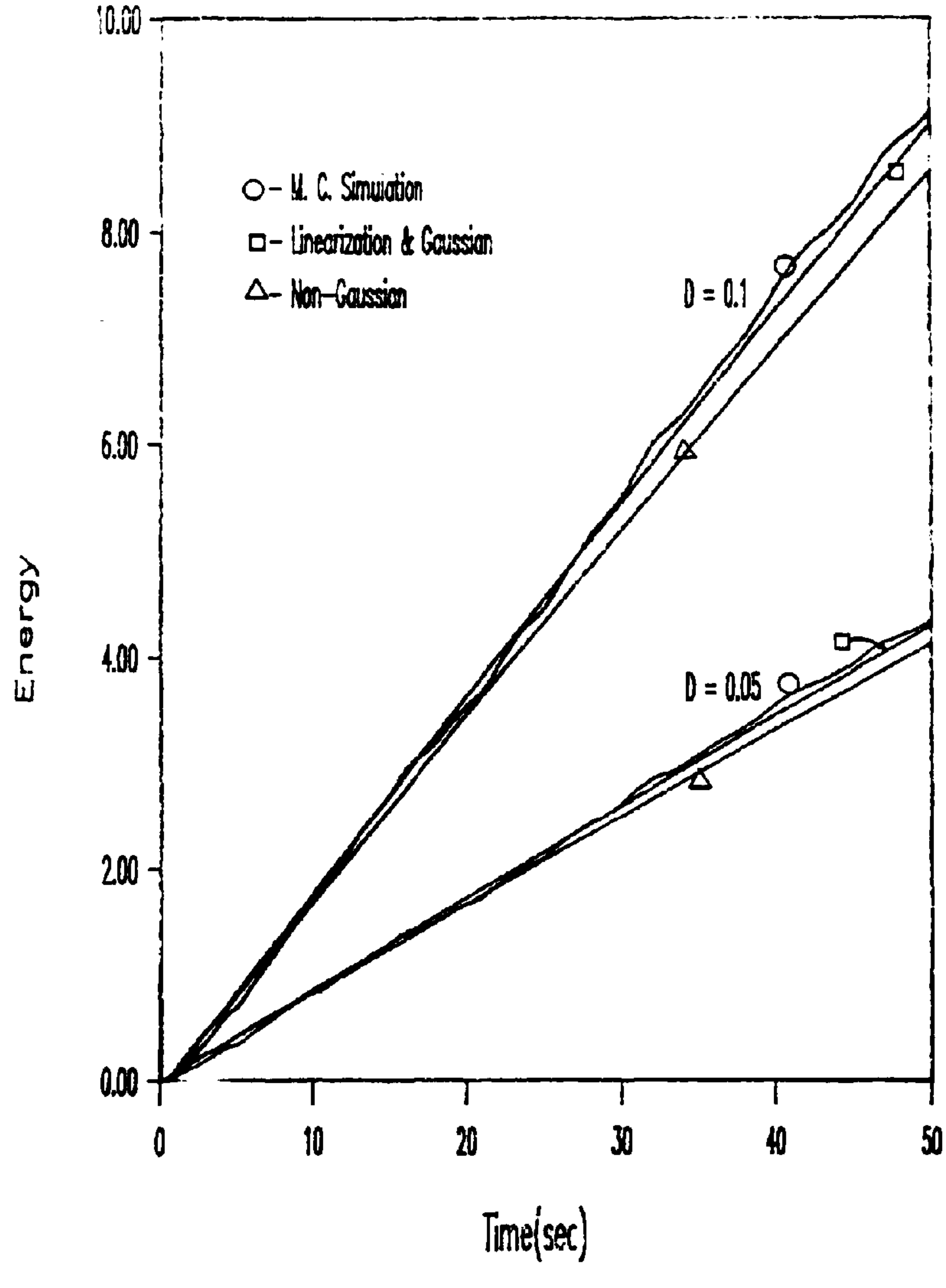

Fig. 5.124- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.05$.

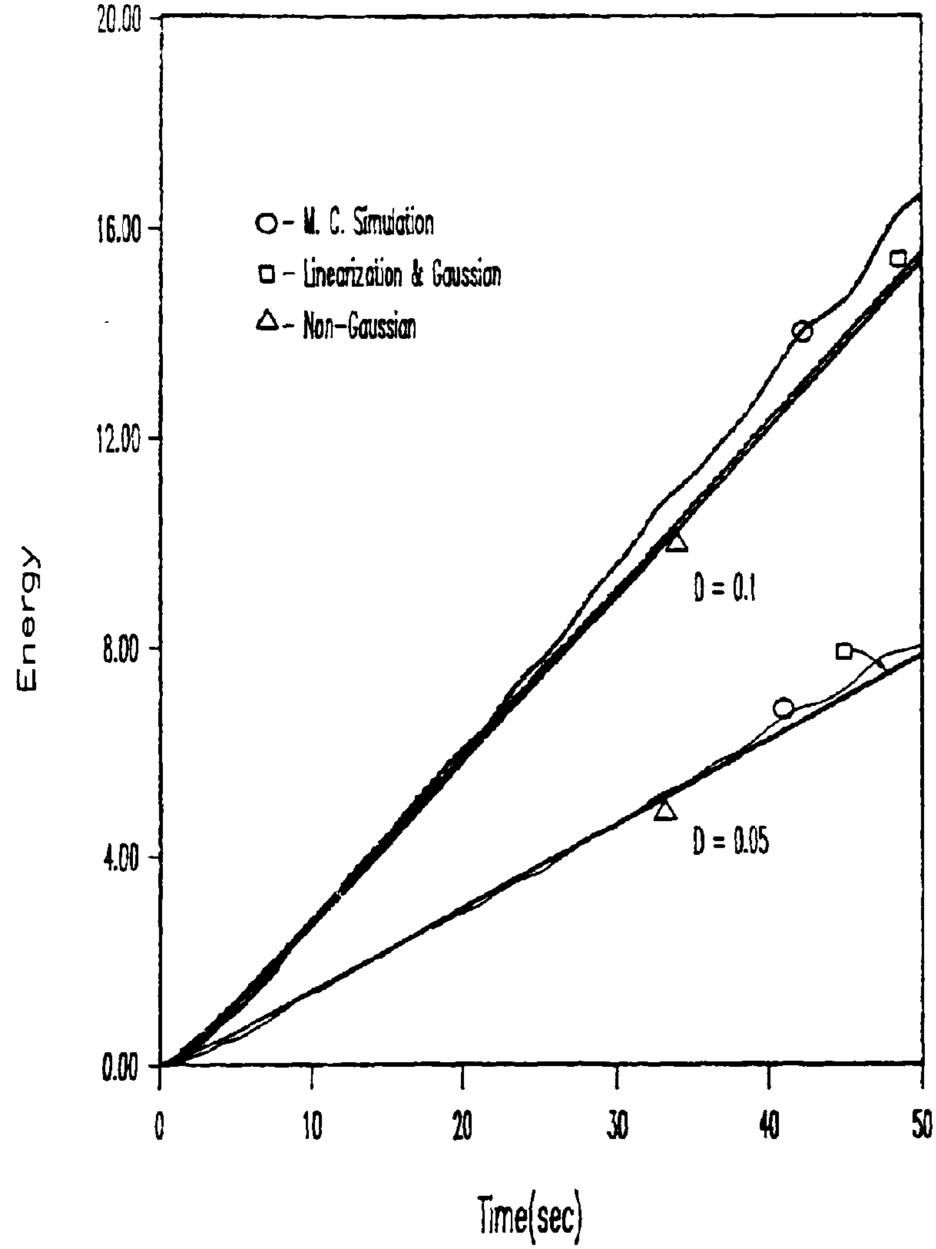

Fig. 5.125- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.0$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.1$.

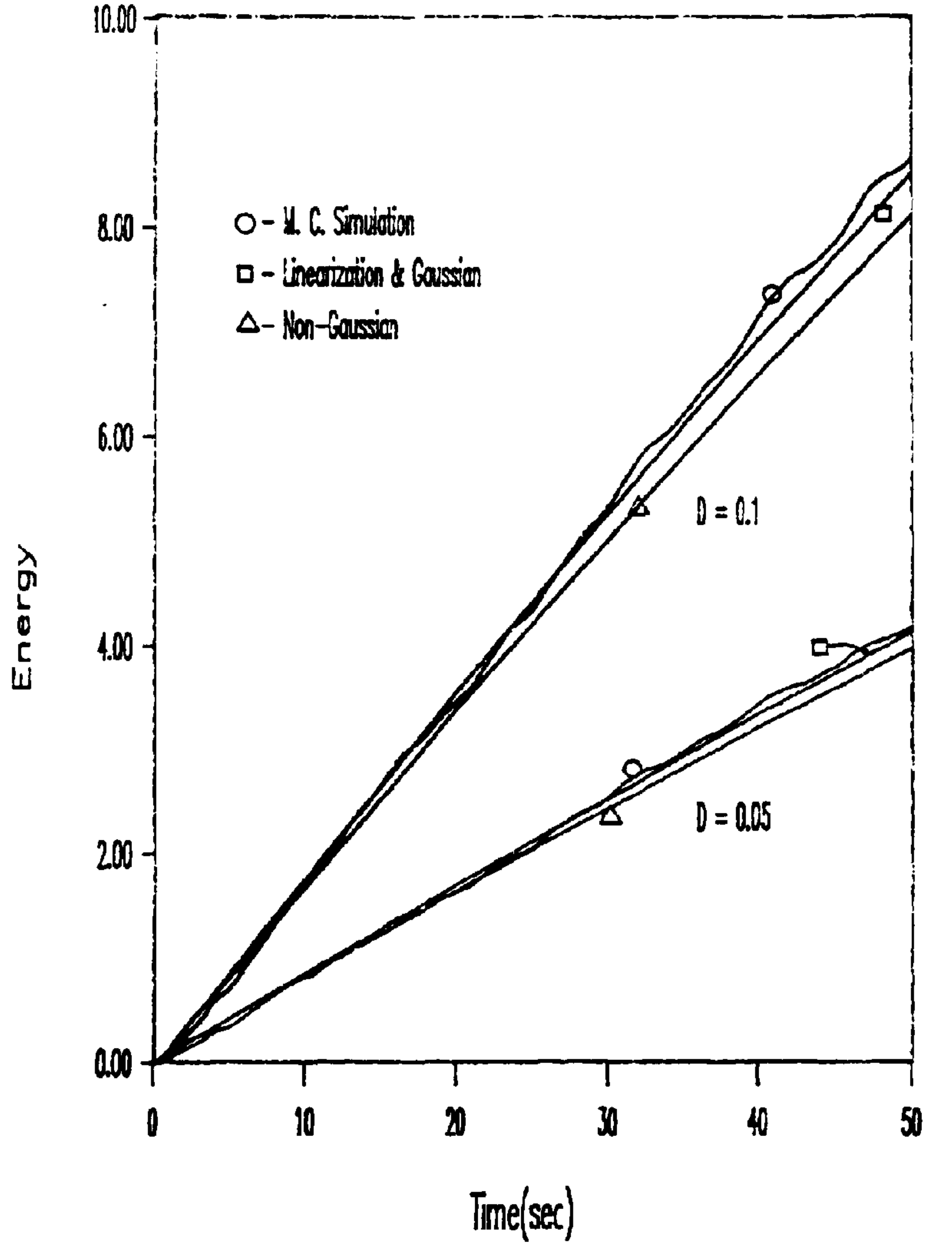

Fig. 5.126- Mean of the Dissipated Hysteretic Energy for SDOF BBW System.
$\xi = 0.1$, $\delta_A = 0.0$, $\delta_\nu = 0.0$, $\delta_\eta = 0.1$.

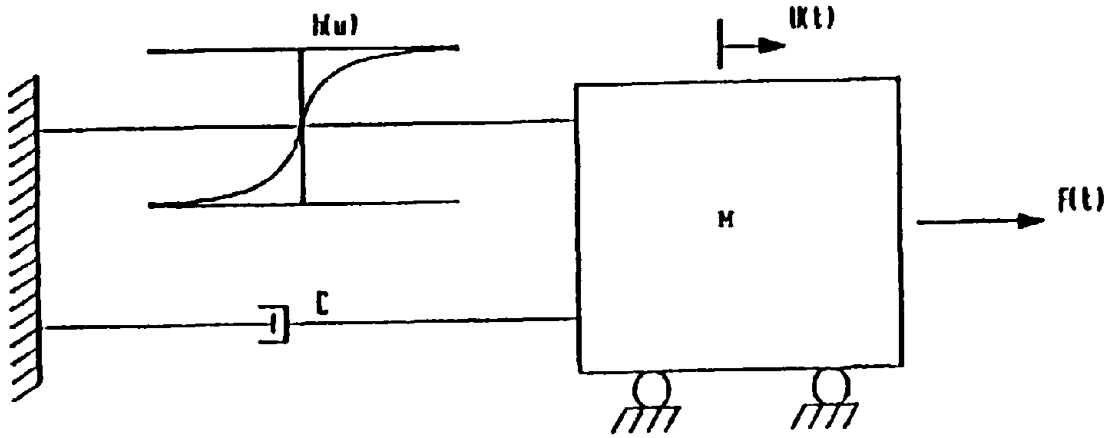

Fig. 6.1- SDOF System with Hyperbolic Tangent Restoring Force.

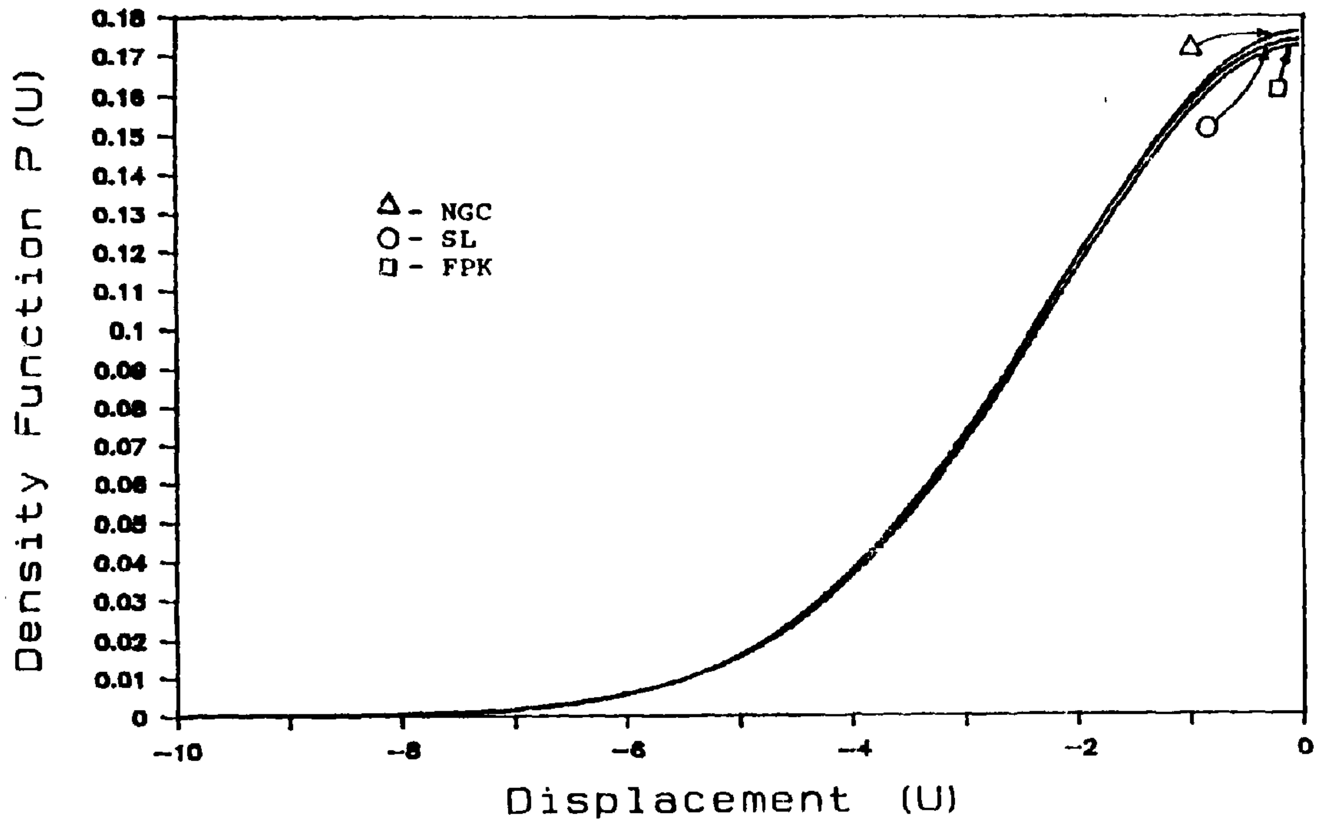

Fig. 6.2- Density Function of the Response for the Hyperbolic Tangent Spring System (Low Nonlinearity) .

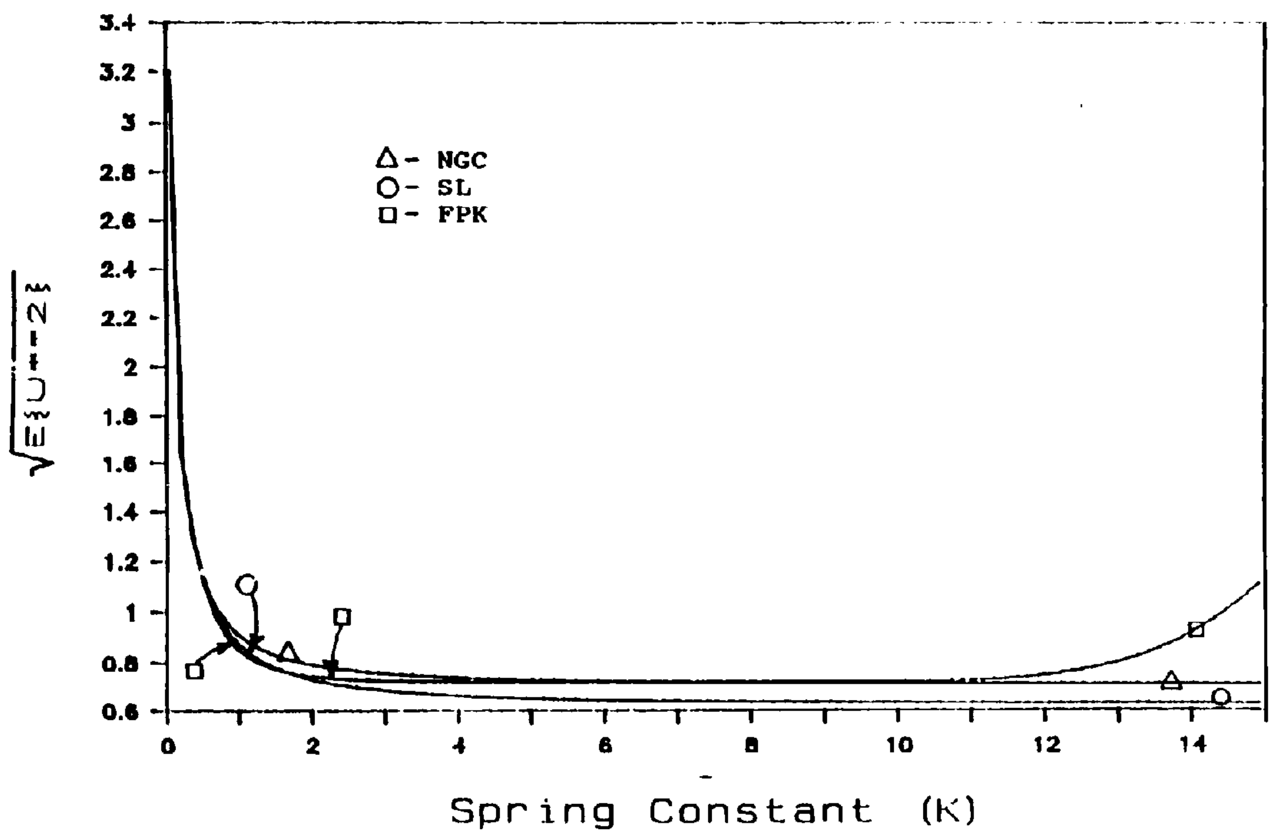

Fig. 6.3- RMS Displacement Response vs Spring Constant (Low Nonlinearity).

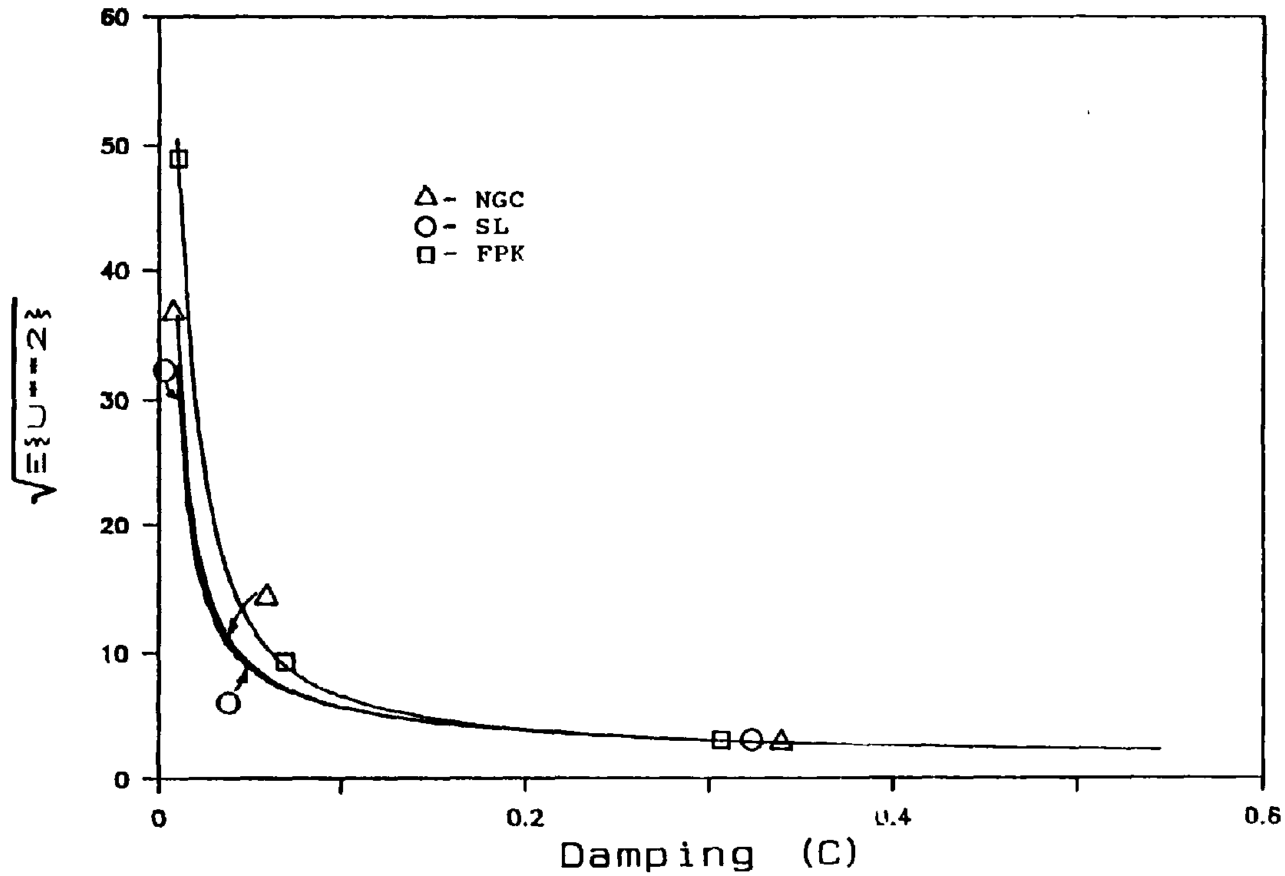

Fig. 6.4- RMS Displacement Response Response vs Damping (Low Nonlinearity).

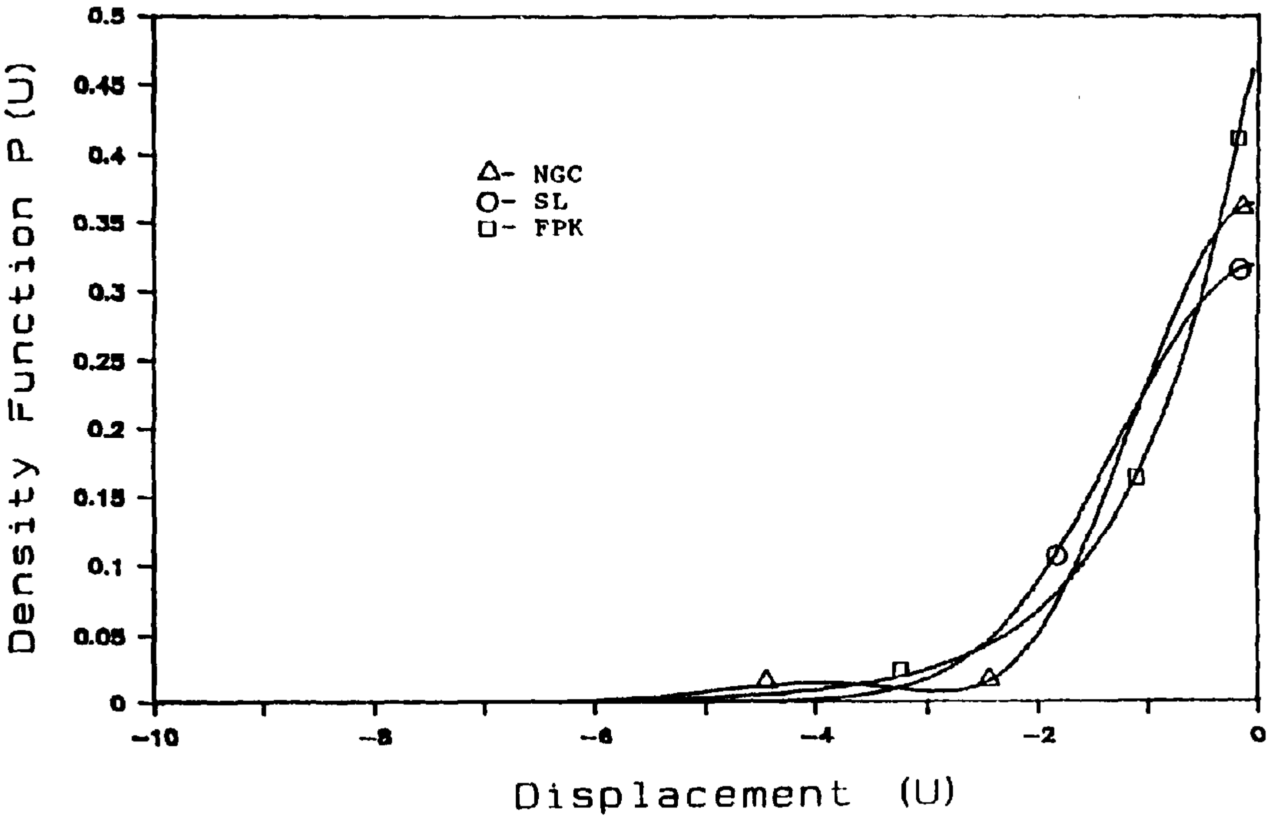

Fig. 6.5- Density Function of the Response for the Hyperbolic Tangent Spring System (High Nonlinearity).

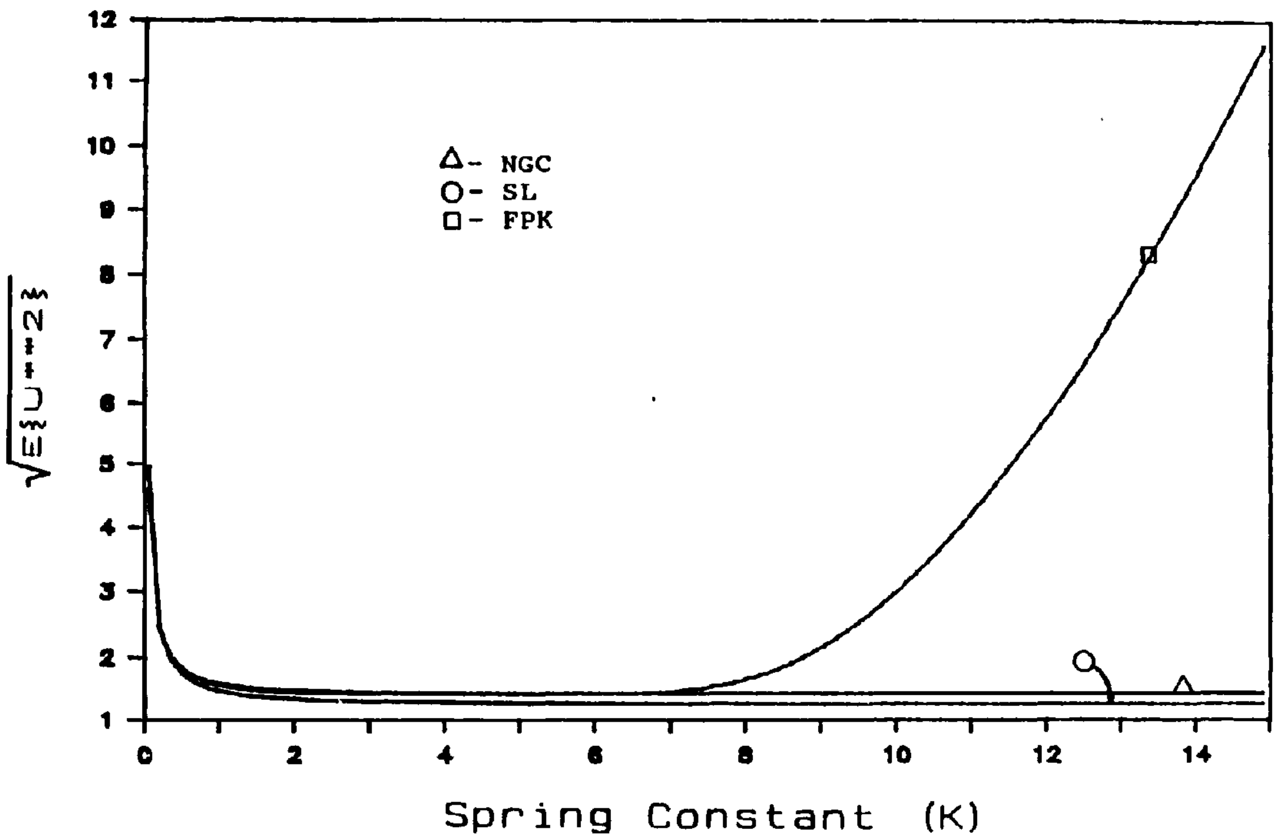

Fig. 6.6- RMS Displacement Response vs Spring Constant (High Nonlinearity).

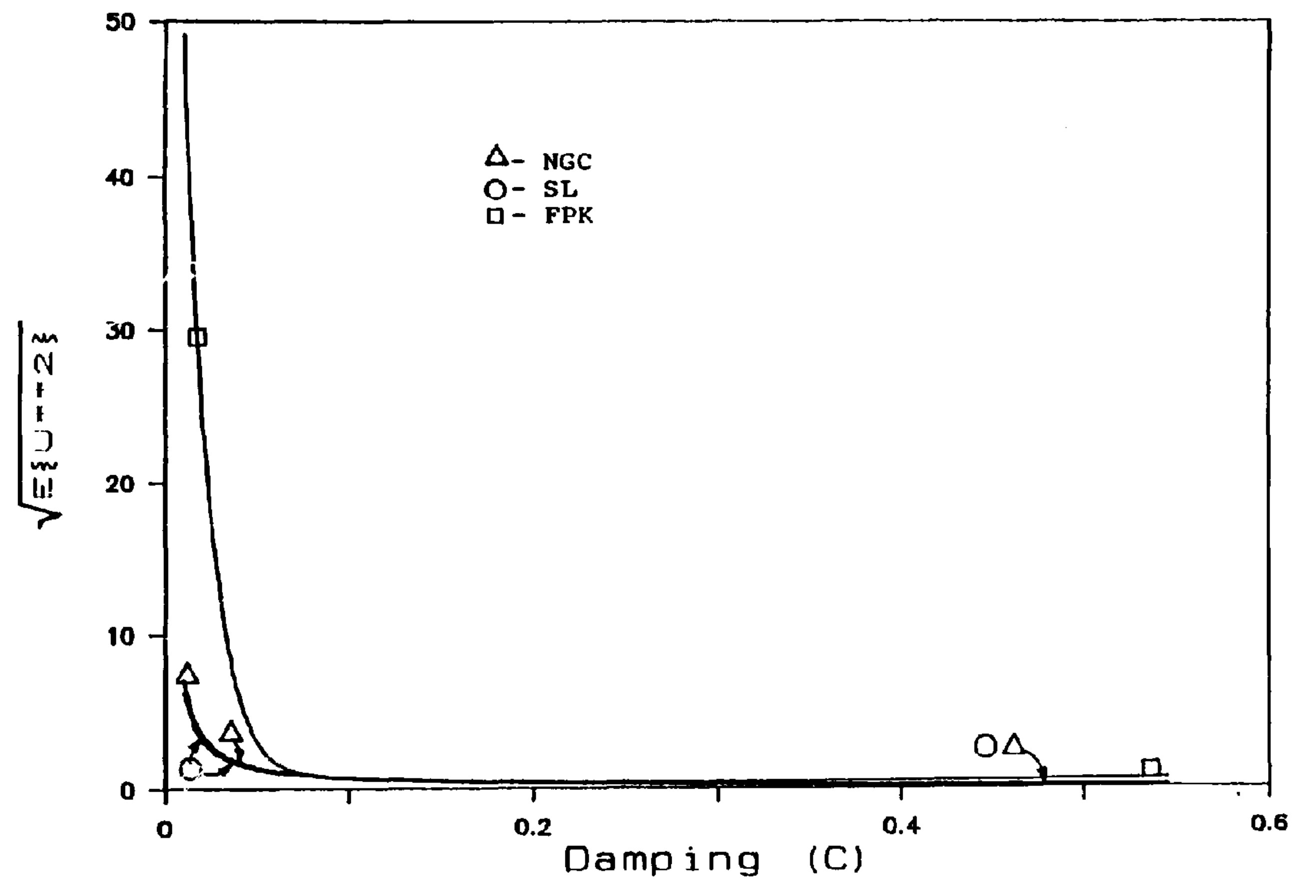

Fig. 6.7- RMS Displacement Response vs Damping (High Nonlinearity).

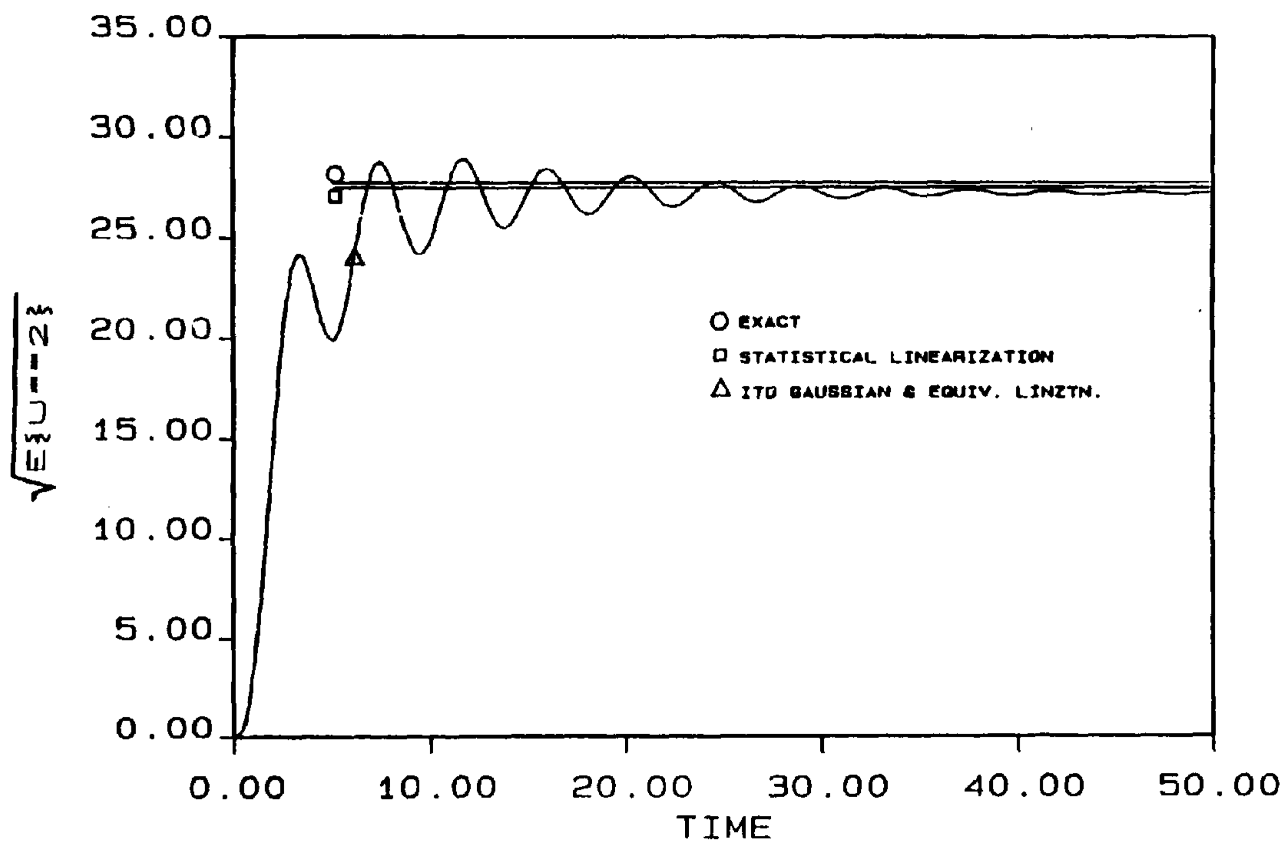

Fig. 6.8- RMS Displacement Response of Duffing Oscillator to White Noise (Low Nonlinearity).

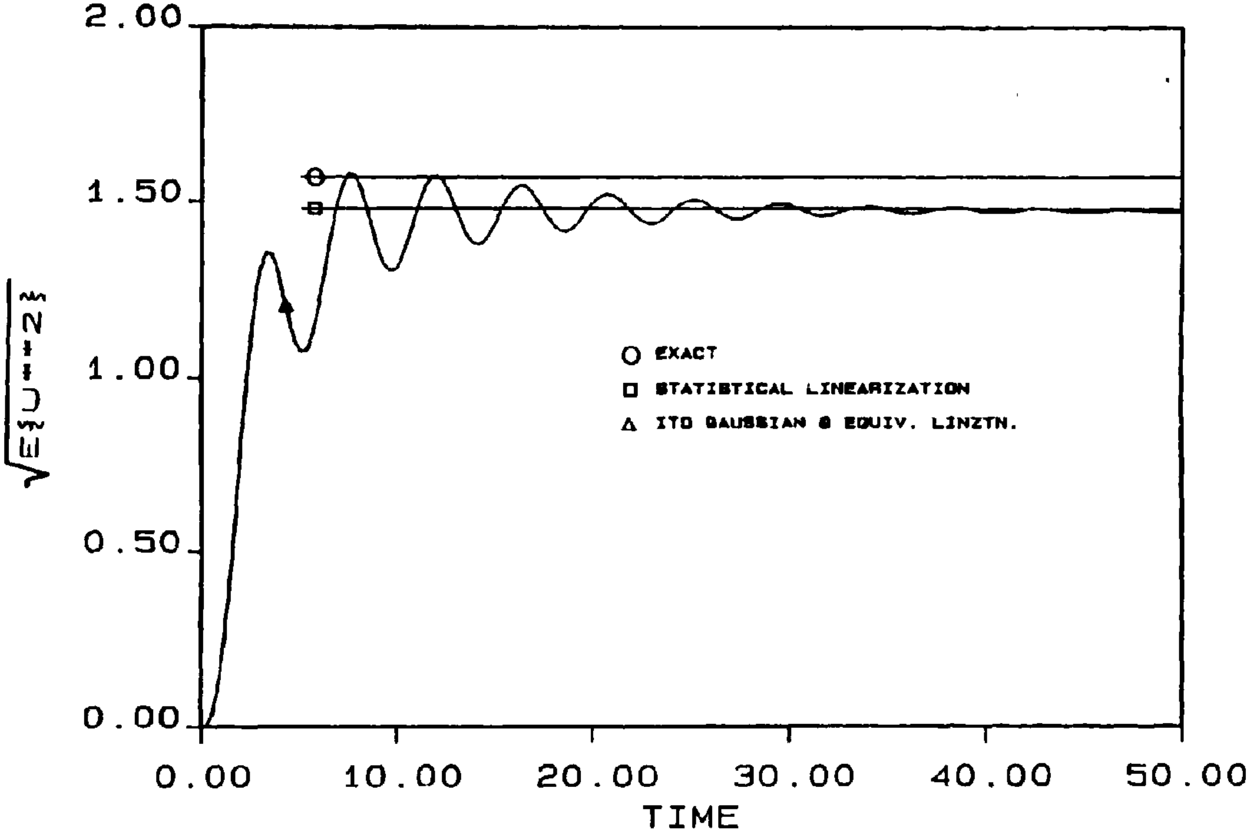

Fig. 6.9- RMS Displacement Response of Duffing Oscillator to White Noise (High Nonlinearity).

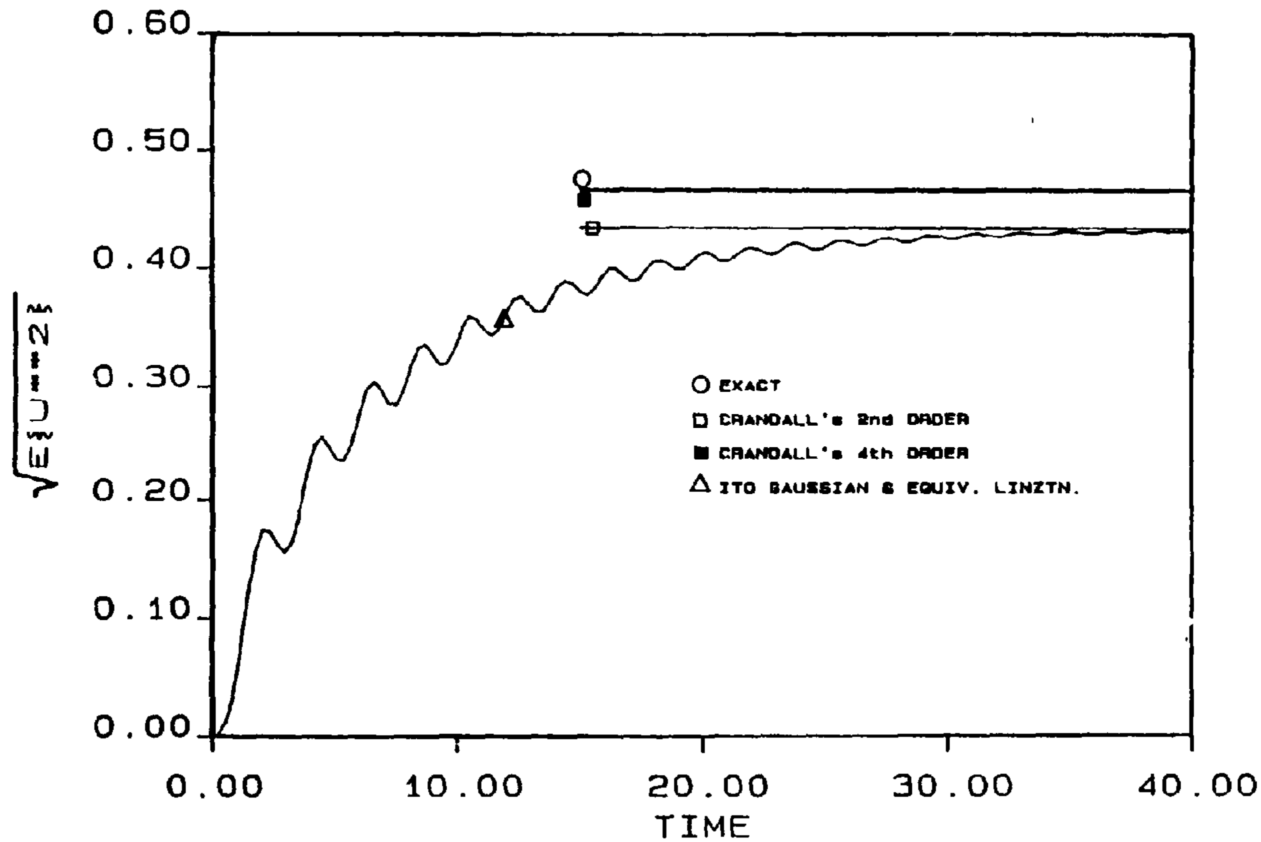

Fig. 6.10- RMS Displacement Response of Set-Up Spring System to White Noise (Low Nonlinearity).

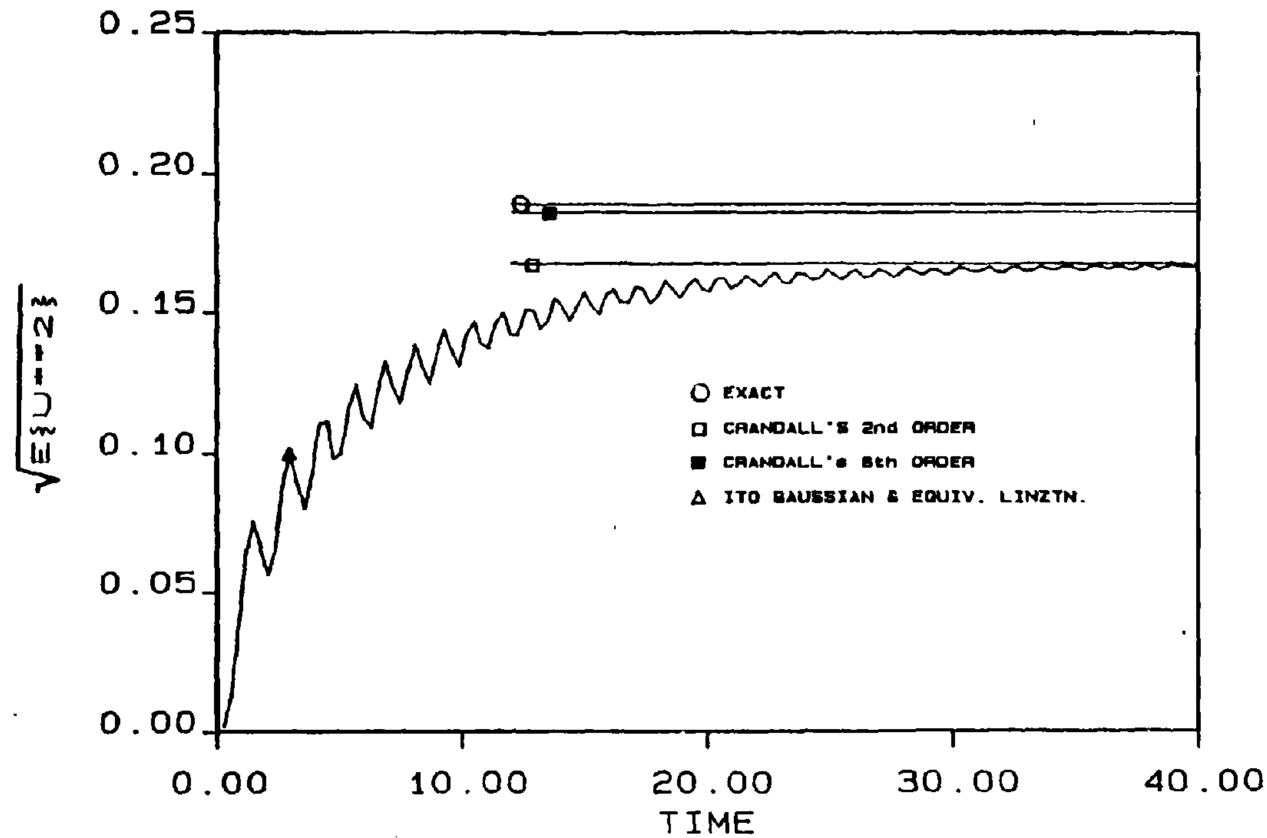

Fig. 6.11- RMS Displacement Response of Set-Up Spring System to White Noise (High Nonlinearity).

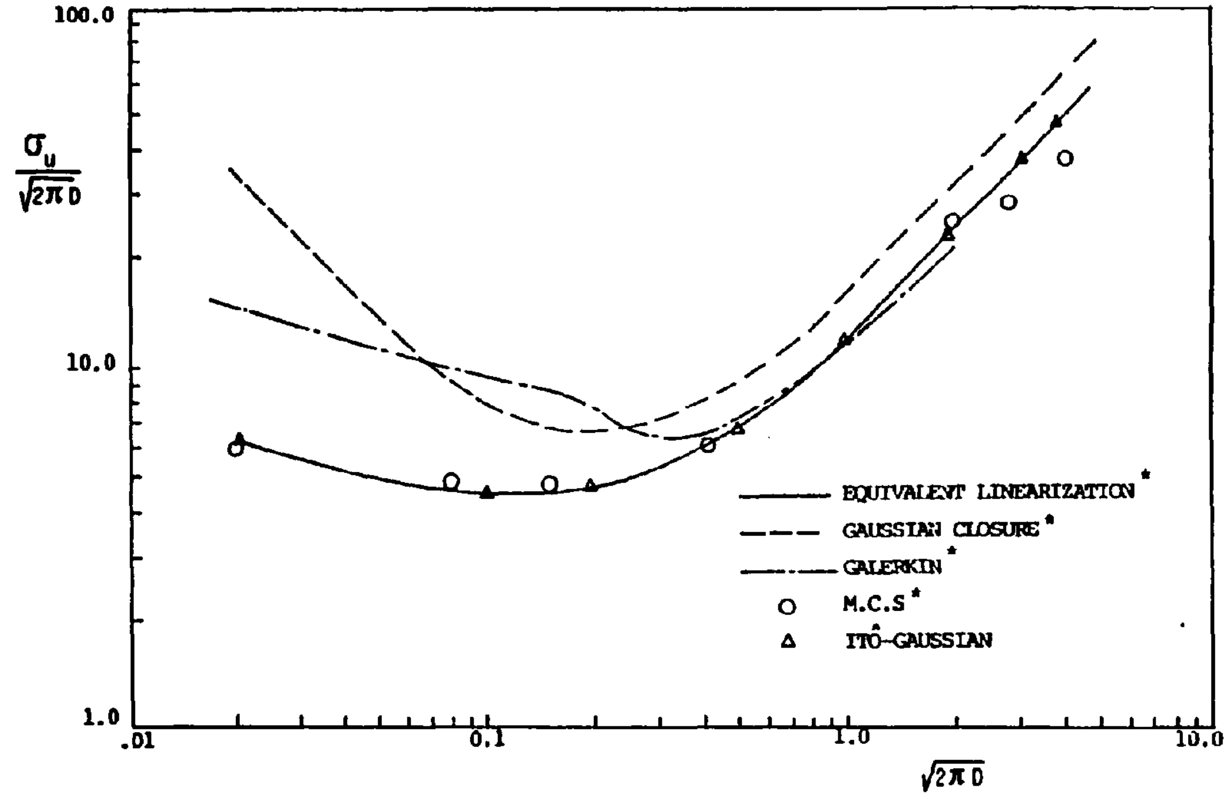

Fig. 6.12- RMS Displacement Response of BBW model to White Noise (*After Baber and Wen; 1979).

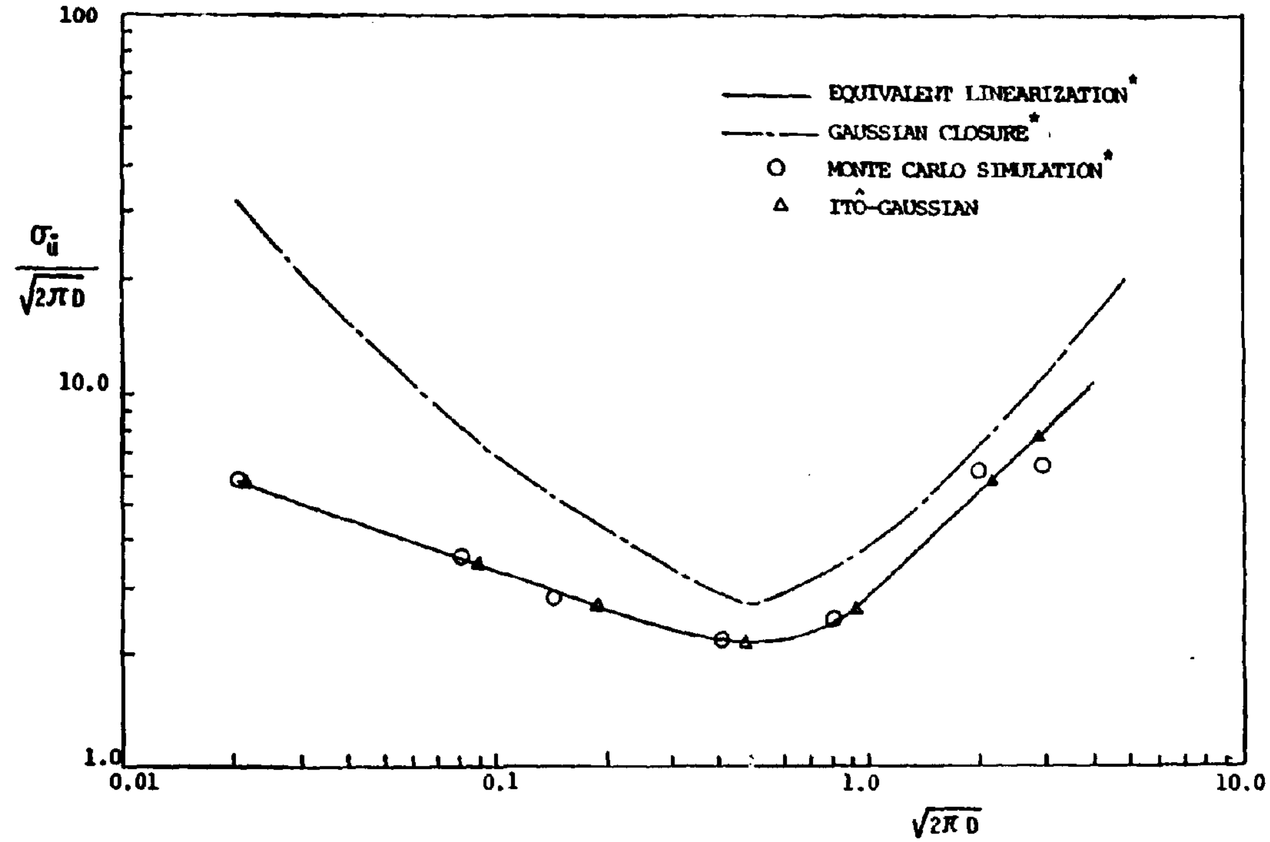

Fig. 6.13- RMS Velocity Response of BRW model to White Noise (*After Baber and Wen; 1979).

APPENDICES

APPENDIX A

The expected values in Equations [4.9] and [4.10] are tabulated as follows

$$I_1 = E(|Y_2|Y_3) \quad [A1]$$

$$I_2 = E(|Y_3|Y_2) \quad [A2]$$

$$I_3 = E(|Y_2|Y_3^2) \quad [A3]$$

$$I_4 = E(|Y_3|Y_2Y_3) \quad [A4]$$

$$I_5 = E(|Y_2|Y_1Y_3) \quad [A5]$$

$$I_6 = E(|Y_3|Y_1Y_2) \quad [A6]$$

$$I_7 = E(|Y_2|Y_2Y_3) \quad [A7]$$

$$I_8 = E(|Y_3|Y_2^2) \quad [A8]$$

In order to evaluate these I_i's in terms of the response moments the following three dimensional Gaussian density function is assumed

$$p[Y(t)] = \frac{(2\pi)^{-(3/2)}}{|\Delta|^{\frac{1}{2}}} \exp(-\frac{1}{2|\Delta|} \sum_{i=1}^{3} \sum_{j=1}^{3} cof(\Delta)_{ij}(Y_i-m_i)(Y_j-m_j)) \quad [A9]$$

where

$$Y(t) = [Y_1(t),\ Y_2(t),\ Y_3(t)]^T$$

$|\Delta|$ = Determinant of the covariance matrix.

$cof(\Delta)_{ij}$ = cofactor of the covariance element c_{ij} in the determinant of covariance matrix.

m_i = $E[Y_i]$

Using the density function in [A9] the expected values given by Equations [A1] through [A8] are evaluated to be

$$I_1 = A_1\,\phi(2;1.5;a) + A_2\,\phi(1;0.5;a) \quad [A10]$$

$$I_2 = A_3\,\phi(2;1.5;b) + A_4\,\phi(1;0.5;b) \quad [A11]$$

$$I_3 = A_5\,\phi(2;0.5;a) + A_6\,\phi(2;1.5;a) + A_7\,\phi(1;0.5;a) \quad [A12]$$

$$I_4 = A_8\,\phi(2;0.5;b) + A_9\,\phi(2;1.5;b) \quad [A13]$$

$$I_5 = A_{10}\,\phi(2;0.5;c) + A_{11}\,\phi(2;1.5;c) + A_{12}\,\phi(1;0.5;c) \quad [A14]$$

$$I_6 = A_{13}\,\phi(2;0.5;d) + A_{14}\,\phi(2;1.5;d) + A_{15}\,\phi(1;0.5;d) \quad [A15]$$

$$I_7 = A_{16}\,\phi(2;0.5;a) + A_{17}\,\phi(2;1.5;a) \quad [A16]$$

$$I_8 = A_{18}\,\phi(2;0.5;b) + A_{19}\,\phi(2;1.5;b) + A_{20}\,\phi(1;0.5;b) \quad [A17]$$

where A_i, a, b, c and d are functions of the response moments and $\phi(.)$ is the Kummer function as given by:

$$\phi(\alpha,\beta,\gamma) = \frac{\Gamma(\beta)}{\Gamma(\alpha)}\,\gamma^{1-\beta}\,\frac{d^{\alpha-\beta}}{d\gamma^{\alpha-\beta}}[\gamma^{\alpha-1}\exp(\gamma)] \quad [A18]$$

$$\Gamma(\alpha) = \text{Gamma function} = \int_0^\infty t^{\alpha-1} \exp(-t)\, dt \qquad \text{for } \alpha>0 \qquad [A19]$$

These parameters are lengthy expressions and their detailed derivations which are not reported in this paper have been obtained using the MAC's Symbolic Manipulation (MACSYMA) available through Massachusetts Institute of Technology.

APPENDIX B

The expected values of Equations [4.9], [4.10], [4.13], and [4.14] can be categorized into two groups. The first group contains the expected values of only two variables, Y_2 and Y_3, and the second group contains the expected values of three variables Y_1, Y_2 and Y_3. In general, these expected values are expressed as:

$$E\left\{|Y_i|\ Y_i^m\ Y_j^n\right\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |Y_i|\ Y_i^m\ Y_j^n\ P(Y_i, Y_j)\ dY_i\ dY_j \qquad [B1]$$

$$E\left\{|Y_i|\ Y_i^l\ Y_j^m\ Y_k^n\right\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |Y_i|\ Y_i^l\ Y_j^m\ Y_k^n\ P(Y_i, Y_j, Y_k)\ dY_i\ dY_j\ dY_k \qquad [B2]$$

where $P(Y_i, Y_j)$ and $P(Y_i, Y_j, Y_k)$ are either a Gaussian distributed density function as given by Equation [4.11] or a Non-Gaussian density function as given by Equation [4.12]. The evaluation of the expected values in Equations [B1] and [B2], after subsituting for the appropriate density functions, requires the calculation of the following integrals:

$$I_{mn} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |Y_i| \; Y_i^m \; Y_j^n$$

$$\exp[a_1 Y_j^2 + a_2 Y_i Y_j + a_3 Y_i^2] \; dY_i \; dY_j \qquad [B3]$$

$$I_{lmn} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |Y_i| \; Y_i^l \; Y_j^m \; Y_k^n \exp[s_1 Y_j Y_i +$$

$$s_2 Y_j Y_k + s_3 Y_i Y_k + s_4 Y_j^2 +$$

$$s_5 Y_i^2 + s_6 Y_k^2] \; dY_i \; dY_j \; dY_k \qquad [B4]$$

a_1 through a_3 and s_1 through s_6 are constants. These constants are in terms of the response moments. The mathematical steps leading to these integrals are not presented here. Closed form solution of Equations [B3] and [B4] are obtained as follows:

$$I_{mn} = \frac{n! \; (-\pi/a_1)^{\frac{1}{2}}}{(-2a_1)^n} \sum_{Q=0}^{E(n/2)} \frac{(-a_1)^Q \; a_2^{(n-2Q)} \; A1}{(n-2Q)! \; Q! \; A2} \qquad [B5]$$

$$I_{lmn} = \frac{m! \; \pi}{(a_4 \; b_2)^{\frac{1}{2}} \; 2^m} \sum_{Q=0}^{E(m/2)} \sum_{J=0}^{m-2Q} \sum_{I=0}^{A3} \frac{A4 \; A6}{A5 \; A7} \qquad [B6]$$

where E( ) is the lowest integer number. The detailed derivations leading to these results are lenghty and cannot be reported here. The variables in Equations [B5] and B[6] are given as:

$$\mathbf{A1} = \left[\frac{m + n - 2Q}{2} \right]!$$

$$\mathbf{A2} = (-B1)^{[(m+n-2Q+2)/2]}$$

$$\mathbf{A3} = E\left(\frac{n+J}{2}\right)$$

$$\mathbf{A4} = a_1^{m-2Q-J}\ a_2^{J}\ (-a_4)^{Q-m}\ (n+J)!\ [-b_3/(2b_2)]^{n+J}\ [-b_2/(b_3^2)]^{I}$$

$$\mathbf{A5} = I!\ J!\ Q!\ (m - 2Q - J)!\ (n + J - 2I)!$$

$$\mathbf{A6} = \left[\frac{1 + m + n - 2Q - 2I}{2} \right]!$$

$$\mathbf{A7} = (-B2)^{[(1+m+n-2Q-2I+2)/2]}$$

$$B1 = a_3 - \frac{a_2^2}{4a_1}$$

$$B2 = b_1 - \frac{b_3^2}{4b_2}$$

$$b_1 = a_5 - \frac{a_1^2}{4a_4}$$

$$b_2 = a_6 - \frac{a_2^2}{4a_4}$$

$$b_3 = a_3 - \frac{2a_1 a_2}{4a_4}$$

Equation [B5] and [B6] are valid if $a_1<0$, B1<0, $s_4<0$, B2<0, and $b_2<0$.

$$I_{mn} \begin{cases} =0 & \text{if } m+n = \text{odd} \\ =[B5] & \text{if } m+n = \text{even} \end{cases}$$

$$I_{lmn} \begin{cases} =0 & \text{if } l+m+n = \text{odd} \\ =[B6] & \text{if } l+m+n = \text{even} \end{cases}$$

APPENDIX C

The differential equation of motion is given by

$$M\ddot{U} + C\dot{U} + h(U) = F(t) \quad [C1]$$

where

$$h(U) = \lambda \tanh(KU/\lambda) \quad [C2]$$

Equation [C1] can be written in a linear form as

$$M\ddot{U} + C\dot{U} + K_e(U) = F(t) \quad [C3]$$

The coefficient, K_e was found by minimizing the error between Equation [C1] and [C3]. The error of linearization is

$$\Delta = K_e U - h(U) \quad [C4]$$

The minimization can be accomplished by

$$\frac{\partial E(\Delta^2)}{\partial K_e} = 0 \quad [C5]$$

which results in

$$K_e = \frac{E(Uh(U))}{E(U^2)} \qquad [C6]$$

The numerator of Equation [C6] can be calculated numerically by a technique that was explained in Section 6.2.

$$E(U \tanh(KU/\lambda)) = \frac{2\sigma B_1}{\sqrt{2\pi}} \qquad [C7]$$

where

$$B_1 = \int_0^\infty y \tanh\left(\frac{Ky}{\lambda_1} \right) \exp\left(- \frac{y^2}{2} \right) dy \qquad [C8]$$

and

$$\lambda = \lambda_1 \sigma \qquad [C9]$$