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# ARMA MONTE CARLO SIMULATION IN PROBABILISTIC STRUCTURAL ANALYSIS

by

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#### **PREFACE**

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself. The research goals in this area are to:

- 1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom, and nonlinearities in the primary structure.
- 2. Develop practical criteria and procedures for the analysis and design of secondary systems.
- 3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.



*In dealing with nonlinear structural systems, random vibration problems are quite intricate and the technique of Monte Carlo simulation has been widely used. In this report, ARMA systems are reviewed from a perspective of applicability to Monte Carlo simulation in nonlinear random vibration. It is shown that, with the advent of super computers, ARMA systems can be a potent weapon in addressing structural problems of extremely large dimensions and complexities. Their applicability to earthquake engineering is briefly reviewed.* 

# **ABSTRACT**

Autoregressive moving average (ARMA) systems for synthesizing realizations of stochastic processes are discussed in context with the technique of Monte Carlo simulation. Strictly autoregressive CAR) or strictly moving average CMA) systems are considered as special cases of the ARMA systems. Their applicability in wind, ocean, and earthquake engineering is briefly reviewed.

 $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbb{L}$  $\mathbb{E}$  $\mathcal{A}$  $\mathbb T$  $\mathbf{I}$ 

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### **INTRODUCTION**

In designing structural systems reliably, effects of adverse natural environments must often be considered. The flight of an airplane encountering gust, the motion of a ship in a confused sea, and the shaking of a building induced by wind or an earthquake are typical examples. An exact deterministic prediction of the loads generated by these natural phenomena often requires precise global models of the atmosphere, the seas, and the ground motions. In this regard, many concurrent factors must be considered which render the exact physical modeling practically impossible. Thus, the designer is led to use either simplified deterministic representations or realizations of stochastic processes. In the latter case, the natural loads are described by means of stochastic processes whose characteristics such as probability densities and power spectra have been estimated from available data. Then, the response of linear structural models to these natural loads can be determined by means of linear random vibration analysis. However, reliability concerns dictate the use of improved nonlinear models for predicting structural responses to natural loads. The corresponding random vibration problems are quite intricate, and lend themselves to limited analytical treatments. To overcome this difficulty, the technique of Monte Carlo simulation can be used. That is, one can first synthesize many records of the random loads and then proceed to use a code of nonlinear deterministic analysis to compute the structural response to individual load records. Finally, a statistical analysis of the produced population of structural response records can be performed. Clearly, the Monte Carlo technique is quite versatile. However, it can be quite costly computationally when applied to major structural problems. Therefore, it is desirable to discuss pertinent simulation algorithms from the perspective of increased computational efficiency of the Monte Carlo technique.

In this regard, note that the advent of digital instruments and computers has underlined the discrete character of most records of structural loads and of the induced responses. Further, the digital signal processing theory [1,2] has provided a mathematically sound description of the properties of discrete systems. For these reasons, the present article will consider only discrete-time models and simulation algorithms.

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#### **BACKGROUND ON LINEAR TIME-INVARIANT DISCRETE SYSTEMS**

An-input n-output linear time-invariant discrete system is represented by the equation

$$
\underline{Y}_k = \sum_{l=-\infty}^{\infty} H_{k-l} \, \underline{W}_l. \tag{1}
$$

The n-component vectors  $W_k$  and  $Y_k$  are the  $k^{th}$  samples of the input (excitation) and output (response), respectively. Further, the nxn matrix  $H_k$  is the  $k^{th}$  sample of the impulse response sequence of the system.

Of particular importance in digital signal processing is the z transform of a sequence  $P_k$ defined as

$$
P(z) = \sum_{k=-\infty}^{\infty} P_k z^{-k},\tag{2}
$$

wherever the series converges in the complex z plane. The value of this function on the unit circle, I z 1=1, yields the discrete Fourier transform of  $P_k$ 

$$
P(e^{j\omega T}) = \left[ P(z) \right]_{z=e^{j\omega T}} \text{ for } -\pi \le \omega T \le \pi. \tag{3}
$$

The sampling period T denotes the time interval between two consecutive samples of the sequence  $P_k$ . Considering equations (1) and (2), it is readily shown that the z transforms of  $Y_k$ ,  $W_k$  and  $H_k$ satisfy the relation

$$
\underline{Y}(z) = H(z) \underline{W}(z). \tag{4}
$$

Note that the transfer function,  $H(z)$ , completely describes the system. Next, assume that the excitation  $W_l$  is a stationary random sequence with autocorrelations  $R_{WW}(l)$  and spectral matrix  $S_{WW}(\omega)$ related by the equations

$$
R_{\underline{WW}}(l) = E\left[\underline{W}_k \underline{W}_{k+l}^{\dagger}\right] = \int_{-\infty_b}^{\infty} S_{\underline{WW}}(\omega) e^{jl\omega T} d\omega \tag{5}
$$

and

$$
S_{WW}(\omega) = \frac{1}{2\omega_b} \sum_{l=-\infty}^{\infty} R_{WW}(l) e^{-jl\omega T},
$$
 (6)

where <sup>t</sup> denotes the operator of transposition. Further,  $\omega_b$  is the cut-off frequency which satisfies the Nyquist relation

$$
\omega_b = \frac{\pi}{T}.\tag{7}
$$

The steady-state response of the system,  $Y_k$ , is also a stationary random sequence whose spectral matrix is

$$
S_{\underline{YY}}(\omega) = H^*(e^{j\omega T}) S_{\underline{WW}}(\omega) H^{\dagger}(e^{j\omega T}). \tag{8}
$$

The symbol \* denotes the operator of complex conjugation. Note in equations (6) and (8) that both the spectral matrices of the excitation and the response are periodic functions of period  $2 \frac{\pi}{T}$  or  $2\omega_b$ . T Let  $\underline{Y}(t)$  and  $\underline{Y}_k$  be the continuous-time target random vector process and its sampled discrete-time equivalent. Further, assume that the eigenvalues of  $S_{\underline{Y}(t)\underline{Y}(t)}(\omega)$  are negligible for  $|\omega| \ge \omega_b$  so that no aliasing occurs. That is, the autocorrelations  $R_{Y(t)Y(t)}(\tau)$  can be exactly recovered, for any lag  $\tau$ , from the values  $R_{YY}(k)$ , k =0,±1,±2,... Then, it can be shown that the periodic spectral matrix  $S_{YY}(\omega)$  of the discrete process satisfies the equation

$$
S_{\underline{Y}\underline{Y}}(\omega) = S_{\underline{Y}(t)\underline{Y}(t)}(\omega) \quad \text{for } -\frac{\pi}{T} \le \omega \le \frac{\pi}{T}.
$$

Briefly stated, digital Monte Carlo simulation requires that the excitation sequence  $W_k$  and the transfer function  $H(e^{j\omega T})$  be chosen so that the spectral matrix  $S_{YY}(\omega)$  of equation (8) represents a close approximation of a given (target) expression,  $S_{Y(t)Y(t)}(\omega)$ , in the frequency band  $\omega \in [-\omega_b, \omega_b].$ 

A computationally simple excitation process which can be generated is the bandlimited discrete white noise  $W_k$ . Its components involve sequences of independent and identically distributed random numbers. The corresponding spectral and autocorrelation matrices are

$$
R_{WW}(l) = 2\omega_b I_n \,\delta_{l0} \tag{10}
$$

and

$$
S_{WW}(\omega) = I_n \tag{11}
$$

where  $\delta_{ij}$  and  $I_n$  are the Kronecker symbol and the nxn identity matrix, respectively.

Given the target matrix and the white noise input, an appropriate transfer function  $H(e^{j\omega T})$  can be determined by several procedures. This is the subject of the ensuing sections.

# **AUTOREGRESSIVE** (AR) **SYSTEMS**

The  $r^{th}$  sample,  $\hat{Y}_r$ , of the response of a n-input n-output autoregressive system of order m, AR(m), can be computed from the m previous ones and from the corresponding value of the excitation in the following manner

$$
\hat{\underline{Y}}_{r} = -\sum_{k=1}^{m} \hat{A}_{k} \hat{\underline{Y}}_{r-k} + \hat{B}_{0} \underline{W}_{r}.
$$
\n(12)

The symbols  $A_k$  and  $B_0$  denote nxn matrix coefficients. The transfer function of this system is

matrix coefficients. The transfer function of this system is  
\n
$$
H(z) = \left[ I_n + \sum_{k=1}^{m} \hat{A}_k z^{-k} \right]^{-1} \hat{B}_0.
$$
\n(13)

The unknown parameters  $\hat{A}_k$  and  $\hat{B}_0$  should be selected in such a manner that the output spectral matrix  $S_{\hat{Y}\hat{Y}}(\omega)$  as defined by equations (8) and (11) represents a good approximation of the target expression  $S_{YY}(\omega)$ . It has been shown [3] that a meaningful measure of the error involved in this modeling is

$$
\varepsilon = \frac{1}{2\omega_b} \int_{-\omega_b}^{\omega_b} tr \left[ H^{-*}(e^{j\omega T}) S_{\underline{Y} \underline{Y}}(\omega) H^{-\dagger}(e^{j\omega T}) \right] d\omega \tag{14}
$$

where  $tr(A)$  and  $A^{-1}$  denotes the trace and inverse of an arbitrary matrix A. The minimization of  $\varepsilon$ with respect to the elements of the matrix  $\hat{A}_k$  yields the following linear system of equations (Yule-Walker equations) [4-7]

$$
R_{\underline{Y}Y}^{\dagger}(l) + \sum_{k=1}^{m} \hat{A}_k R_{\underline{Y}Y}(k-l) = 0 \quad \text{for } l=1,...,m.
$$
 (15)

The parameter  $\hat{B}_0$  is obtained by specifying that the total energy of the target and the AR processes are equal. Specifically, it is found that

$$
\hat{B}_0 \,\hat{B}_0^{\,\dagger} = \frac{1}{2\omega_b} \left[ R_{\underline{YY}}(0) + \sum_{k=1}^{m} \hat{A}_k \, R_{\underline{YY}}(k) \right]. \tag{16}
$$

Note that this equation is nonlinear in the unknown coefficient  $\hat{B}_0$ . Further, it can be shown [4] that there exists an infinite number of solutions  $\hat{B}_0$ . The response of the corresponding AR systems has identical response spectral and autocorrelation matrices [4]. Thus, any solution of equation (16) is a bonafide parameter  $\hat{B}_0$ . For the sake of simplicity, the matrix  $\hat{B}_0$  can be assumed to be lower triangular. Thus, it can be conveniently computed through the Cholesky decomposition of equation (16) [4,8,9].

Note that equations (15) and (16) could also be obtained by postmultiplying the input-output relation (12) by  $Y_{r-1}^{\dagger}$  and  $W_r^{\dagger}$ , taking mathematical expectations, and postulating the equality of the

autocorrelations of the AR and target processes [8-13], A posterior analysis [4,14] can be performed to show that the assumed autocorrelation matching is indeed achieved.

Further, it can be proved [14] that the AR system whose parameters are computed from equations (15) and (16) is stable. That is, its response to a bounded excitation is bounded.

The quality of the AR approximation can be assessed by a visual comparison of the behavior of the components of the target and AR spectral matrices. A quantitative measure of the agreement between the target curves and their AR counterparts has also been introduced [4] as

$$
\varepsilon_m = \det \left[ \hat{B}_0 \, \hat{B}_0^\dagger \right] - \exp \left\{ \frac{1}{2\omega_b} \int_{-\omega_b}^{\omega_b} tr \left[ \log S_{\underline{Y} \underline{Y}}(\omega) \right] d\omega \right\}.
$$
 (17)

Finally, it should be noted that obtaining a reliable AR approximation of a target process can become a quite delicate task if the trace of the logarithm of the target spectral matrix is not integrable in the domain  $[-\omega_b, \omega_b]$  [4,11].

Some extensions of traditional AR approximations are advantageous. First, by allowing parameters  $\hat{A}_k$  and  $\hat{B}_0$  to vary with time, an autoregressive modeling approach to the simulation of nonstationary processes can be formulated [15-17]. Second, by allowing the summation involved in equation (12) to include samples  $Y_{r-k,s-l...v-p}$  lagged in all directions [18,19], records of values of a multidimensional field  $Y_{r,s,\ldots,v}$  can be synthesized. Recently, a multivariate AR modeling technique was suggested [20-22] that considers each of the correlated components of the vector process  $Y$  as a time-invariant linear combination of a series of independent processes. This representation leads to computational savings in the determination of the AR model of the process  $Y$  which involves only a series of univariate approximations as opposed to a multivariate one. Note however that this improvement is achieved at the expense of the matching between the AR and target autocorrelations at lags  $1, 2, \ldots$ 

## **MOVING AVERAGE (MA) SYSTEMS**

A moving average system is characterized by a finite length impulse response sequence. Specifically, a MA(m) approximation,  $Y'_r$ , of the target process is expressed by means of the equation

$$
Y_{r}' = \sum_{l=-m}^{m} B_{l}' \, W_{r-l} \tag{18}
$$

where  $W_r$ , designates a bandlimited white noise sequence characterized by Eq. (10) and (11). The transfer function of the MA system is readily computed as

$$
H(e^{j\omega T}) = \sum_{l=-m}^{m} B'_{l} e^{-jl\omega T}
$$
 (19)

The corresponding response spectral matrix is

$$
S_{\underline{Y'}\underline{Y}}(\omega) = H^*(e^{j\omega T}) H^{\dagger}(e^{j\omega T}).
$$
\n(20)

In the context of Monte Carlo simulation,  $S_{YY'}(\omega)$  must be a close approximation of the target expression  $S_{Y(t)Y(t)}(\omega)$ . Assume that the latter matrix can be decomposed in the form

$$
S_{\underline{Y}(t)\underline{Y}(t)}(\omega) = Q^*(\omega) Q^{\dagger}(\omega). \tag{21}
$$

Then, a reliable MA approximation is obtained by selecting  $H(e^{j\omega T})$  as the limited Fourier series of  $Q(\omega)$ . The corresponding model parameters are readily computed as

$$
B'_{l} = \frac{1}{2\omega_b} \int_{-\omega_b}^{\omega_b} Q(\omega) e^{jl\omega T} d\omega.
$$
 (22)

Consider first the solution of equation (21) in the scalar, single input single output case. Seek a function  $Q(\omega)$  such that

$$
S_{\underline{Y}(t)\underline{Y}(t)}(\omega) = \left| Q(\omega) \right|^{2}.
$$
 (23)

The everywhere nonnegative character of  $S_{Y(t)Y(t)}(\omega)$  ensures the existence of  $Q(\omega)$ . Further, only the modulus of this function is specified by equation (23). The phase can be arbitrarily chosen. In particular, a zero phase, real decomposition

$$
Q(\omega) = \sqrt{S_{Y(t)Y(t)}(\omega)}\tag{24}
$$

can be selected [7,23-25]. The corresponding MA parameters are real as required and satisfy the equation

$$
B'_l = B_{-l} \tag{25}
$$

Note that the zero phase decomposition has also been used in connection with other simulation techniques [26-30].  $4-1$ 

A simple solution to the general, n-input n-output, decomposition problem of equation (21) can be obtained by selecting  $Q(\omega)$  to be lower triangular. It is readily found as the Cholesky decomposition of the spectral matrix  $S_{Y(t),Y(t)}(\omega)$  [31]. The corresponding MA matrix parameters  $B_t$  are real and lower triangular.

One-sided MA models,  $B'_1 = 0$  for 1<0, have also been suggested in the scalar case [24,32-34]. However, the design equations for the remaining non-zero MA parameters are nonlinear and can only be solved by some iterative scheme. A generalization of the MA approximation technique to the case of multidimensional and/or nonstationary processes has been investigated [35-37].

#### **AUTOREGRESSIVE MOVING AVERAGE CARMA) SYSTEMS**

The excitation-response relation of a n-input n-output  $ARMA(p,q)$  system can be written as

$$
\underline{\tilde{Y}}_{r} = -\sum_{k=1}^{p} A_{k} \underline{\tilde{Y}}_{r-k} + \sum_{l=1}^{q} B_{l} \underline{W}_{r-l}
$$
\n(26)

where  $A_k$  and  $B_l$  are nxn real matrices. Clearly, the ARMA system represents a generalization of both the AR and MA systems and thus, exhibits enhanced versatility in terms of the lowest number of parameters which must be used in a Monte Carlo scheme associated with a target spectral matrix. One approach of ARMA modeling relies on postmultiplying equation (26) by  $\tilde{Y}_{r-u}^{\dagger}$  and taking mathematical expectations. It can be shown [10,38-40] that the autocorrelations of the ARMA process satisfy the equation

$$
R_{\underline{\tilde{Y}}\tilde{\underline{Y}}}^{\dagger}(u) + \sum_{k=1}^{p} A_k R_{\underline{\tilde{Y}}\tilde{\underline{Y}}}^{\dagger}(k-u) = 0 \quad \text{for } u > q.
$$
 (27)

The next step of the modeling procedure involves the assumption that the ARMA autocorrelations are a close approximation of the target ones. Thus the parameters  $A_k$  are computed to satisfy equation (27) for  $u = q+1, \ldots, q+p$  and with  $R_{\tilde{\gamma}\tilde{\gamma}}(u) = R_{YY}(u)$  [38-40]. Another approach involves a least squares solution of these equations for  $u = q+1, \ldots$ , N with N>q+p [10]. The computation of the remaining coefficients,  $B_l$  for 1=0,..,q, represents the last and arduous step of this ARMA modeling procedure. In the scalar case,  $n = 1$ , it has been suggested [40] to select these coefficients to ensure the autocorrelation matching property,  $R_{\tilde{Y}\tilde{Y}}(u) = R_{YY}(u)$ , discussed earlier. However, the corresponding equations are coupled and nonlinear so that an iterative solution scheme must be used.

An alternative ARMA modeling approach is based on the assumption that an initial reliable AR approximation of the target spectral matrix is available. Then, the parameters of the ARMA system are selected so that its transfer function

$$
H(e^{j\omega T}) = D^{-1}(e^{j\omega T}) N(e^{j\omega T})
$$
\n(28)

where

$$
D(e^{j\omega T}) = I_n + \sum_{k=1}^{p} A_k e^{-jk\omega T}
$$
 (29)

and

$$
N(e^{j\omega T}) = \sum_{l=0}^{q} B_l e^{-jl\omega T}
$$
 (30)

represents a close approximation of  $H_{AR}(e^{j\omega T})$ , Eq. (13). The two-stage least squares 2SLS or ACM procedure is based on the minimization of the error [4,41]

ture is based on the minimumization of the error [4,41]  
\n
$$
\varepsilon = \frac{1}{2\omega_b} \int_{-\omega_b}^{\omega_b} tr \left\{ \left[ D(e^{j\omega T}) H(e^{j\omega T}) - N(e^{j\omega T}) \right]^* \left[ D(e^{j\omega T}) H(e^{j\omega T}) - N(e^{j\omega T}) \right]^T \right\} d\omega. \quad (31)
$$

It is found that the ARMA parameters satisfy the following system of linear equations [4,41]

$$
\sum_{k=0}^{p} A_k R_{\hat{\underline{Y}} \hat{\underline{Y}}} (k-i) - \sum_{l=0}^{q} B_l R_{\hat{\underline{Y}} \hat{\underline{W}}} (i-l) = 0 \quad \text{for } i=1,..,p
$$
\n(32)

and

$$
2\omega_b B_l = \sum_{k=0}^{\min(l,p)} A_k R_{\underline{\hat{Y}W}}(k-l) \text{ for } l=0,..,q
$$
 (33)

where  $A_0 = I_n$ . The symbol  $R_{\hat{Y}W}(l)$  denotes the output-input crosscorrelation matrix of lag 1 of the AR process. These matrices can be recursively computed from the AR model as follows [4]

$$
R_{\hat{Y}W}(l) = 0 \quad \text{for } l > 0 \tag{34}
$$

$$
R_{\hat{Y}W}(0) = 2\omega_b \,\hat{B}_0\tag{35}
$$

and

$$
R_{\underline{\hat{Y}W}}(-l) = -\sum_{k=1}^{\min(m,l)} \hat{A}_k R_{\underline{\hat{Y}W}}(k-l) \text{ for } l > 0.
$$
 (36)

Various approaches have been followed in deriving equations (32) and (33). Early applications of these relations were concerned with the estimation, from a recorded time series, of the parameters of econometric models [42-44], of natural frequencies and damping ratios of MDOF vibrating systems [45-49], and of soil characteristics [50]. Another formulation of this method, based on postmultiplications by  $\tilde{Y}_{r-1}^{\dagger}$  and  $W_r^{\dagger}$  and mathematical expectations has also been suggested [8,9,11,51]. This approach has been extended for the simulation of multidimensional random fields [18,19].

The error based derivation of equations (32) and (33) has the advantage of providing a simple measure of the quality of the approximation, namely the minimum value of the error

$$
\varepsilon_{\min} = \frac{1}{2\omega_b} \text{ tr } \left[ \sum_{k=0}^{p} A_k R_{\hat{\underline{Y}} \hat{\underline{Y}}} (k) - \sum_{l=0}^{q} B_l R_{\hat{\underline{Y}} \hat{\underline{W}}}^{+} (-l) \right]. \tag{37}
$$

It has been shown [14] that the system designed by this two-stage procedure is stable in the bounded-input bounded-output sense. Further, some matching properties between the response autocorrelations and excitation-response crosscorrelations of the AR and ARMA systems have been proved [14].

Another two-stage modeling approach, based on a Pade-type approximation of the AR transfer function and called the POM procedure, has also been suggested [4,11,12,52]. The ARMA parameters are selected to ensure the matching of the first  $p+q+1$  samples of the impulse response sequences of the AR and ARMA systems. Specifically, it is found that the ARMA coefficients must satisfy the equations

$$
\sum_{k=0}^{\min(l,p)} A_k R_{\hat{Y}W}(k-l) = 2\omega_b B_l \text{ for } l=0,..,q
$$
\n(38)

and

$$
\sum_{k=0}^{\min(l,p)} A_k R_{\hat{Y}W}(k-l) = 0 \text{ for } l=q+1, \dots, q+p
$$
 (39)

Note that equations (33) and (38) are identical. This fact has led to the introduction of a general two-stage modeling procedure which incorporates as special cases both the ACM and POM techniques [4]. The matching properties and stability of the resulting ARMA systems have been analyzed in some detail [14].

The inverse of an ARMA transfer function is also a rational expression in  $e^{-j\omega T}$ . Therefore, the previous methods can be applied with  $H_{AR}^{-1}(e^{j\omega T})$  in place of  $H_{AR}(e^{j\omega T})$ . The equations for the ARMA parameters are still linear and comparisons of the respective merits of the  $H_{AR}(e^{j\omega T})$  and  $H_{AB}^{-1}(e^{j\omega T})$  based methods is possible [4,14].

A two-stage MA to ARMA modeling technique has also been recently proposed [53] which relies on the minimization of a frequency error similar to  $\varepsilon$ , Eq. (31).

Finally, it should be noted that some of the previous techniques have been applied to problems of spectrum estimation and modeling. The pertinent literature can be consulted for additional insight [54-58].

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$ 

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 $\mathbb{F}_2$  $\mathbb{F}^2$ 

 $\pm$  $\pm$ 

 $\mathbf{1}$ 

# **STRUCTURAL ENGINEERING APPLICATIONS**

In this section the applicability of the ARMA models for the description and simulation of natural processes, such as turbulent wind velocities, ocean waves kinematics, and earthquake ground motions, is discussed.

#### **6.1 Turbulence.**

The generation of time histories of turbulent wind velocities represents an important application of the modeling techniques described in the previous sections. Both MA [25,32-34] and AR [20-22] simulation algorithms have been successfully applied in conjunction with a variety of target spectra. The two-stage AR to ARMA procedures [9,12] have also been found to produce realizations of turbulent wind velocities which are highly compatible with the von Karman spectrum. In particular, a 9-coefficient ARMA model of the trivariate velocity process whose spectral matrix is indistinguishable from the target expression has been obtained [9]. This method has also been successfully applied to another spectral matrix encountered in wind engineering [14].

A different approach, based on a continued fraction expansion of the longitudinal and lateral von Karman spectra, has been suggested [59]. The model parameters are computed by approximating the solution of the corresponding ordinary differential equation by an ARMA transfer function. A spatial and temporal simulation technique of the velocity field has also been formulated [35-37] which involves a MA approximation of the coherence function.

The class of ARMA systems has been found to be quite useful also in the modeling of experimental wind data. A series of studies [60-63] have shown that the wind velocities and pressure on a cooling tower can be accurately represented by low order stationary AR and ARMA processes. A similar conclusion has recently been drawn in connection with low buildings [64]. Nonstationary or nonhomogeneous ARMA models have also been suggested to represent wind profiles [17], wind gusts in aircraft flight simulation [65] and payload acceleration during Space Shuttle lift-off [51]. Finally, an AR model of wind velocity has been used to study the problem of glass cladding [66].

#### **6.2 Ocean** Waves

The simulation of wave kinematics in a fully developed sea is a classical problem of offshore engineering. Early attempts to use digital systems for the generation of such time histories have focused on MA models [7,23,31,67]. Low order AR approximations have also been suggested [13]. However, the mathematical peculiarities of the wave elevation spectrum [11] render the computation of an accurate AR approximation a quite delicate task [6,7]. Specifically, the sampling period must be carefully selected to avoid unwanted oscillations of the AR spectrum [7,11,14]. Recently, a reliable AR model obtained with the appropriate selection of T was used as the first step of the two-stage procedures [11,14]. It was found that the spectrum of an ARMA(7,7) model was almost indistinguishable from the target, Pierson-Moskowitz wave elevation expression. An ARMA(11,10) system yielded a similar matching of the velocity spectrum. An earlier, less conclusive, application of the two-stage ACM procedure to this spectrum had been reported [68]. A two-stage MA to

ARMA modeling technique recently proposed [53] has shown great promises from the standpoint of ARMA simulation of ocean wave elevations; it combines the robustness of the MA approximations and the computational efficiency of the ARMA algorithms. In connection with the Pierson-Moskowitz expression, this new method has led to an ARMA(7,7) model whose spectrum is almost identical to the target one.

Experimental data have also shown that low order ARMA models can be used to accurately represent the wave elevation process [69-71]. Finally, a MA approach for the simulation of a random sea according to a directional spectrum has been suggested [72].

#### 6.3 Earthquake Ground Motions

The safe design of structures in an earthquake prone region must incorporate the effects of possible ground motions. Various descriptions of the nonstationary acceleration process have been suggested and in each case, the usefulness of the ARMA systems has been demonstrated on the basis of available seismic data. A simple model involves a stationary process modulated by a deterministic envelope [73-76]. Excellent AR [77] and ARMA [14] approximations of the Kanai-Tajimi spectrum, describing the stationary component, have been produced. Moreover, the spatial propagation of the ground motions has recently been studied on the basis of a multivariate AR model [78]. Finally, a first order autoregression has also been suggested [79] to characterize the amplitudes of a random pulse train model of earthquake ground motions.

Another analysis of experimentally measured ground accelerations has been performed by decomposing the records into a finite number of time segments [80,81]. It has been recognized that low orders stationary ARMA systems can accurately represent the ground motions in almost all intervals.

Finally, AR [82-90], MA [85-88] and ARMA [15,16,91,92] models with time varying coefficients have been suggested to describe the nonstationary character of the ground acceleration process. It appears from all these studies that low orders ARMA models can be used to generate reliable time histories of the ground acceleration during an earthquake.

# **CONCLUDING REMARKS**

ARMA systems have been reviewed from a perspective of applicability to Monte Carlo simulation of engineering problems. Procedures have been discussed for determining efficient simulation algorithms for a given spectral matrix of structural loads. It is believed that with the advent of digital computing, and the availability of mini and micro computers to ordinary engineering offices, ARMA simulation procedures can become routine analysis and design tools for complex structural engineering problems. Furthermore, with the gradual availability of super-computers, ARMA systems offer the potential of addressing extremely large and intricate problems which were considered intractable just a few years ago.

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$ 

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