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RANDOM VIBRATION AND RELIABILITY ANALYSIS OF PRIMARY-SECONDARY STRUCTURAL SYSTEMS

by

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

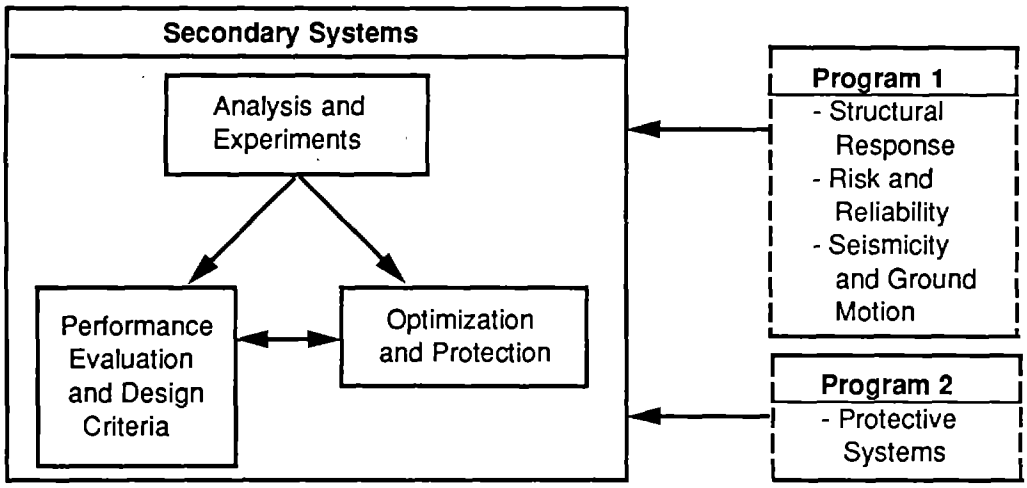
- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself. The research goals in this area are to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom, and nonlinearities in the primary structure.
2. Develop practical criteria and procedures for the analysis and design of secondary systems.
3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.



This report describes a computer code which has been developed for calculating the mean and covariance functions of general linear primary-secondary systems with uncertain parameters subject to non-stationary non-white random excitations. The code can also be applied to determine the probability (reliability) that extreme stresses do not reach critical levels and damage caused by stress cycles is limited provided that the excitation is a Gaussian process. The code is efficient because it is based on closed form expressions for the mean and covariance functions of response conditioned on specified values of uncertain system parameters. Reliability estimates involve concepts of crossing theory of Gaussian vector processes and First- and Second-Order Reliability Methods (FORM/SORM).

ABSTRACT

Primary-secondary structural systems do not generally have classical modes of vibration and are characterized by large differences in masses and stiffnesses associated with various degrees of freedom. Considerable research has been directed at finding efficient and robust techniques for the dynamic analysis of these systems. Methods have also been developed for approximate analysis of primary-secondary system. A variety of approximate methods of analysis, such as cascade analysis, perturbation methods and component mode method, have extensively been applied. In addition, there are several exact methods of analysis that circumvent the associated eigenvalue problem of a primary-secondary system. Most of these methods focus on stationary responses.

In this work, a methodology is proposed for calculating second moment characteristics of response processes and the probability of failure for linear primary-secondary systems with uncertain parameters subject to non-stationary Gaussian excitation. The proposed method is based on methods of linear random vibration, crossing theory of Gaussian processes, and First-and Second-Order Reliability Methods (FORM/SORM). The random vibration analysis follows the state space approach in which excitation is modeled as the output of a linear filter subjected to a uniformly modulated white noise process. Mean crossing rates of responses are used to approximate conditional failure probabilities for a given set of system parameters. The analysis is relatively simple because conditional responses are Gaussian processes. FORM/SORM algorithms are used to approximate unconditional system failure probabilities.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF NOTATIONS

The following notations are used in this report.

1. Random quantities will be uppercase.
2. Deterministic quantities will be lowercase.
3. A vector will be in bold-face.
4. $\mathbf{X}(t)$ or $\mathbf{X}_t$ refers to physical co-ordinates.
5. $\mathbf{Y}(t)$ or $\mathbf{Y}_t$ refers to classical modal co-ordinates.
6. Subscript p refers to primary system.
7. Subscript s refers to secondary system.
8. Subscript f refers to filter system.
9. Supercript $*$ refers to parameters of combined system.
10. ω is the modal frequency.
11. ζ is the modal damping ratio.
12. $\mathbf{M}$ is the mass matrix.
13. $\mathbf{C}$ is the damping matrix
14. $\mathbf{K}$ is the stiffness matrix.
15. $\mathbf{Z}$ is the vector of uncertain parameters.

SECTION 1
Introduction

1.1 Primary-Secondary Systems

There are several features that characterize a combined primary-secondary structural system [31]. These features are :

1. *Large number of degrees of freedom.* The combined degrees of freedom tends generally to be large [31].

2. *Differences in properties.* The secondary system is generally much lighter than the primary system. Also, large differences in the damping and stiffness values are characteristics of these systems. For example, in a nuclear facility the supporting structure, defined as the primary system, is usually a concrete structure while the equipment and piping, which constitute the secondary systems, are made of steel. Thus, differences in the masses, damping and stiffness characteristics of the two systems can be significant. These differences are widely believed to pose numerical difficulties when attempting to solve the combined primary secondary system, i.e., the multidegree of freedom structure consisting of the primary and secondary systems [16, 18].

3. *Non-proportional Damping.* Due to differences in the damping characteristics of the two systems, the overall system is generally characterized by non-proportionally damping. Hence, classical modal analysis techniques cannot be used to analyze the primary-secondary system. One technique of analysis is the state space approach [8].

4. *Tuning.* When a frequency of the secondary system is very close to a modal frequency of the primary system, resonance effects are present. When tuning occurs and the secondary system is light, it has been observed that two closely space modes will occur in the combined system [1]. And the dominant contribution to the response of the secondary system comes from these two modes [1]. For individually classically damped systems, when the frequency of the secondary system matches with one of

the modal frequencies of the primary system, the overall system is called a tuned primary-secondary system.

5. *Attachment Configuration.* The manner in which the secondary system is attached to the primary system may be very complex.

6. *Location of secondary system.* The secondary system can be re-located anywhere in the primary system. Hence, the combined system has to be re-analyzed whenever the secondary system is re-located. Thus, a separate analysis, i.e., the two systems are analyzed separately, is sometimes preferred. Furthermore, in a separate analysis several of the associated problems of a combined analysis such as large number of degrees of freedom, numerical difficulties due to differences in properties, and non-proportional damping are avoided. However, separate analyses are applicable only under certain conditions. This shall be discussed in Chapter 2.

This report examines linear primary-secondary system with uncertain parameters subject to random non-stationary Gaussian input. It evaluates selected response properties, such as mean and variance functions, and the probability that a particular response does not exceed a given level. This probability is related to the failure mode defined as first excursion failures. The combined system is solved directly using the state-space method. A linear filter is used to generate the necessary input and the state space is augmented to incorporate this filter in the analysis. The desired probability is estimated by using mean crossing rates of response processes and First- and Second-Order Reliability Methods (FORM/SORM).

1.2 Combined Equations of Motion

Consider an n_p-degree-of-freedom (dof) primary system and an n_s-degree-of-freedom secondary system. The equations of motion of the combined primary-secondary system subjected to externally applied loads can be written as

$$[\mathbf{M}^*]\ddot{\mathbf{X}}_t^* + [\mathbf{C}^*]\dot{\mathbf{X}}_t^* + [\mathbf{K}^*]\mathbf{X}_t^* = \mathbf{F}_t \tag{1.1}$$

where

$$\mathbf{X}_t^* = \begin{bmatrix} \mathbf{X}_p(t) \\ \mathbf{X}_s(t) \end{bmatrix}, \tag{1.2}$$

$\mathbf{X}_p(t)$ is the n_p-dimensional relative displacement vector of the primary system, $\mathbf{X}_s(t)$ is the n_s-dimensional relative displacement vector of the secondary system, $[\mathbf{M}^*], [\mathbf{C}^*], [\mathbf{K}^*]$ are the combined mass, damping and stiffness matrices respectively, and $\mathbf{F}_t$ is the vector of applied loads. If the system is subjected to a base excitation, then the vector of applied loads become a scalar, F_t, defined as the ground acceleration and Eqn. 1.1 becomes

$$[\mathbf{M}^*]\ddot{\mathbf{X}}_t^* + [\mathbf{C}^*]\dot{\mathbf{X}}_t^* + [\mathbf{K}^*]\mathbf{X}_t^* = -[\mathbf{M}^*]\mathbf{r}F_t \tag{1.3}$$

where $\mathbf{r}$ is the vector of ground displacement influence coefficient of size $n_p + n_s$. If both the primary and secondary systems are modeled such that each floor mass has only one degree of freedom, i.e., translational then Eqn. 1.3 becomes

$$[\mathbf{M}^*]\ddot{\mathbf{X}}_t^* + [\mathbf{C}^*]\dot{\mathbf{X}}_t^* + [\mathbf{K}^*]\mathbf{X}_t^* = -[\mathbf{M}^*]\mathbf{1}F_t \tag{1.4}$$

where $\mathbf{1}$ is a vector of unit components of size $n_p + n_s$. The matrices of the combined system can be shown to take the following forms

$$[\mathbf{M}^*] = \begin{bmatrix} [\mathbf{M}_p] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{M}_s] \end{bmatrix} \tag{1.5}$$

$$[\mathbf{C}^*] = \begin{bmatrix} [\mathbf{C}_p] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{C}_s] \end{bmatrix} + [\mathbf{C}_c] \tag{1.6}$$

$$[\mathbf{K}^*] = \begin{bmatrix} [\mathbf{K}_p] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{K}_s] \end{bmatrix} + [\mathbf{K}_c] \tag{1.7}$$

where $[\mathbf{M}_p]$, $[\mathbf{C}_p]$ and $[\mathbf{K}_p]$ are respectively, mass, damping and stiffness matrices of the primary system, $[\mathbf{M}_s]$, $[\mathbf{C}_s]$ and $[\mathbf{K}_s]$ are corresponding matrices for the secondary system, and $[\mathbf{C}_c]$ and $[\mathbf{K}_c]$ are damping and stiffness coupling matrices respectively. The size of the combined matrices is $n^* = n_p + n_s$. The coupling matrices depend on the point or points at which the secondary system is attached to the primary system. If each floor mass of

the primary and secondary systems has only one degree of freedom, the coupling matrices will have entries only at the dof where the attachments are. For example, consider a 2-dof primary system with a 1-dof secondary system placed on the second floor as shown in Fig. 1.1. The damping and stiffness coupling matrices will then be of the following form:

$$[\mathbf{C}_c] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_s & -c_s \\ 0 & -c_s & 0 \end{bmatrix} \tag{1.8}$$

and

$$[\mathbf{K}_c] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_s & -k_s \\ 0 & -k_s & 0 \end{bmatrix} \tag{1.9}$$

where c_s and k_s are the damping and stiffness of the 1-dof secondary system. Consider now the case where the 1-dof secondary system is attached to both the first and second floor of the primary system as shown in Fig. 1.2. The coupling matrices will now be of the following form :

$$[\mathbf{C}_c] = \begin{bmatrix} c_{s1} & 0 & -c_{s1} \\ 0 & c_{s2} & -c_{s2} \\ -c_{s1} & -c_{s2} & 0 \end{bmatrix} \tag{1.10}$$

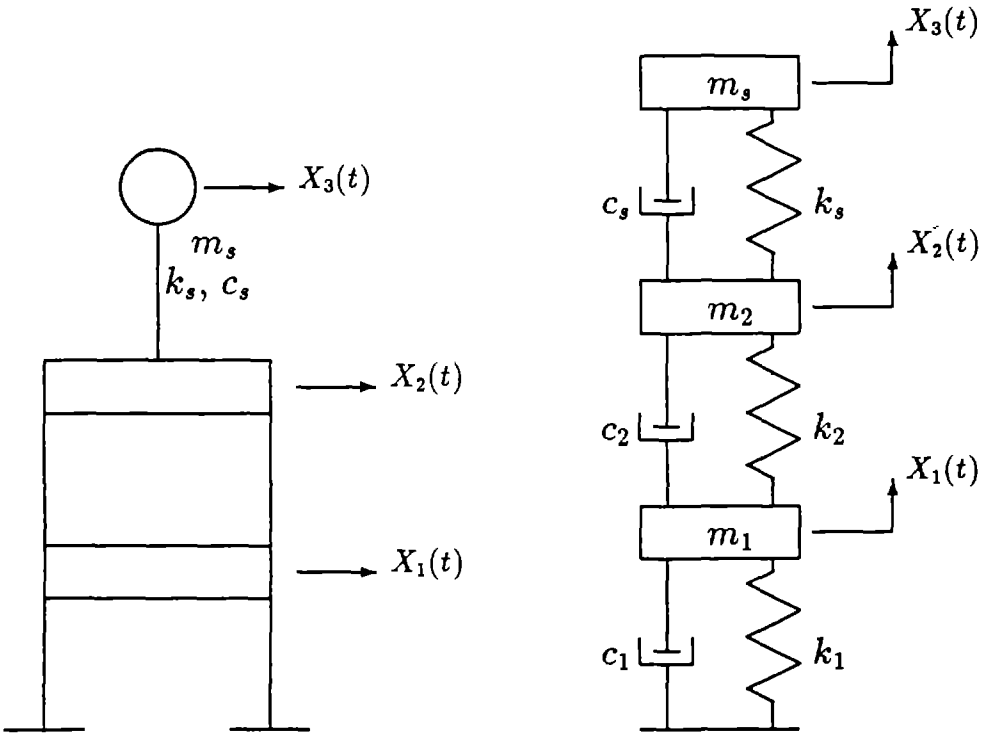
and

$$[\mathbf{K}_c] = \begin{bmatrix} k_{s1} & 0 & -k_{s1} \\ 0 & k_{s2} & -k_{s2} \\ -k_{s1} & -k_{s2} & 0 \end{bmatrix} \tag{1.11}$$

where c_{s1} and c_{s2}, and k_{s1} and k_{s2} are the properties of dashpot and spring systems that characterizes the connection of the secondary system to the first and second floor of the primary system, respectively. The above formulation is valid for any number of secondary systems present. One only has to extend the mass, damping and stiffness matrices of the combined system and the coupling matrices to incorporate other secondary systems. Note that in general the primary system and secondary system can be non-proportionally damped.

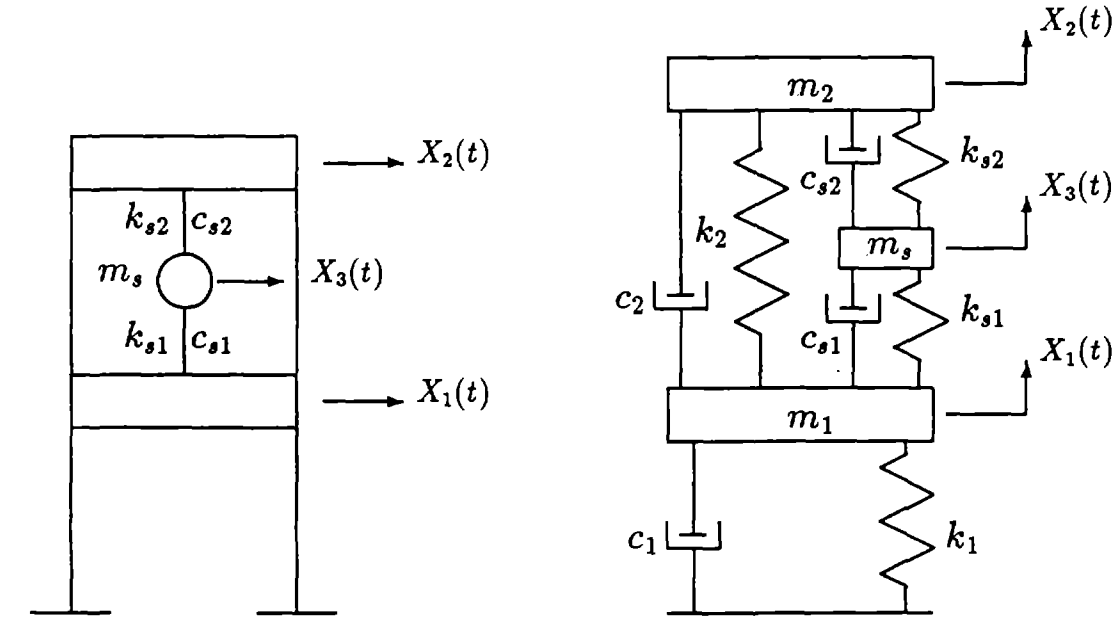
Consider the case in which both the primary system and secondary system are proportionally damped and let

$$\mathbf{X}_t^* = [\Phi^*]\mathbf{Y}_t^* \tag{1.12}$$



Primary-Secondary System **Mathematical Model**

Figure 1.1: 2-DOF Primary System with 1-DOF Secondary System



Primary-Secondary System **Mathematical Model**

Figure 1.2: 2-DOF Primary System with 1-DOF Secondary System on 1st. Floor

where

$$\mathbf{Y}_t^* = \begin{bmatrix} \mathbf{Y}_p(t) \\ \mathbf{Y}_s(t) \end{bmatrix} , \tag{1.13}$$

$$[\Phi^*] = \begin{bmatrix} [\Phi_p] & \mathbf{0} \\ \mathbf{0} & [\Phi_s] \end{bmatrix} , \tag{1.14}$$

$\mathbf{Y}_p(t)$ is the n_p-dimensional generalized displacement vector of the primary system, $\mathbf{Y}_s(t)$ is the n_s-dimensional generalized displacement vector of the secondary system, $[\Phi_p]$ is the modal matrix of the primary system and $[\Phi_s]$ is the modal matrix of the secondary system. The modal matrices are normalized with respect to the appropriate mass matrices. Substituting Eqn. 1.14 into Eqn. 1.4 and pre-multiply the resulting equation by the transpose of the transformation matrix in Eqn. 1.14, Eqn. 1.4 becomes

$$[\mathbf{I}]\ddot{\mathbf{Y}}_t^* + [\tilde{\mathbf{C}}]\dot{\mathbf{Y}}_t^* + [\tilde{\mathbf{K}}]\mathbf{Y}_t^* = -\{\Gamma^*\} F_t \tag{1.15}$$

where

$$[\tilde{\mathbf{C}}] = \begin{bmatrix} 2\zeta_{p1}\omega_{p1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\zeta_{p2}\omega_{p2} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2\zeta_{pn_p}\omega_{pn_p} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 2\zeta_{s1}\omega_{s1} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 2\zeta_{s2}\omega_{s2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 2\zeta_{sn_s}\omega_{sn_s} \end{bmatrix} \tag{1.16}$$

$$+[\Phi^*]^{\mathbf{T}}[\mathbf{C}_c][\Phi^*] ,$$

$$[\bar{\mathbf{K}}] = \begin{bmatrix} \omega_{p1}^2 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_{p2}^2 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \omega_{pn_p}^2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \omega_{s1}^2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \omega_{s2}^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \omega_{sn_s}^2 \end{bmatrix} + [\Phi^*]^{\mathbf{T}}[\mathbf{K}_c][\Phi^*] \,, \tag{1.17}$$

$$\Gamma^* = \begin{bmatrix} \Gamma_p \\ \Gamma_s \end{bmatrix}, \tag{1.18}$$

and ζ and ω are the modal damping ratios and frequencies of the respective systems and Γ is the vector of modal participation factors.

1.2.1 State Space Formulation

Consider Eqn. 1.4 and let

$$\mathbf{V}_t = \begin{bmatrix} \mathbf{X}_t^* \\ \dot{\mathbf{X}}_t^* \end{bmatrix} \tag{1.19}$$

Then,

$$\dot{\mathbf{V}}_t = \begin{bmatrix} [\mathbf{0}] & [\mathbf{I}] \\ -[\mathbf{M}^*]^{-1}[\mathbf{K}^*] & -[\mathbf{M}^*]^{-1}[\mathbf{C}^*] \end{bmatrix} \mathbf{V}_t + \begin{bmatrix} \mathbf{0} \\ -\mathbf{1.0} \end{bmatrix} F_t \tag{1.20}$$

A similar equation of motion can be obtained from Eqn. 1.15 by denoting

$$\mathbf{V}_t = \begin{bmatrix} \mathbf{Y}_t^* \\ \dot{\mathbf{Y}}_t^* \end{bmatrix} \tag{1.21}$$

In this case

$$\dot{\mathbf{V}}_t = \begin{bmatrix} [\mathbf{0}] & [\mathbf{I}] \\ -[\bar{\mathbf{K}}] & -[\bar{\mathbf{C}}] \end{bmatrix} \mathbf{V}_t + \begin{bmatrix} \mathbf{0} \\ -\Gamma \end{bmatrix} F_t \tag{1.22}$$

Appendix A presents methods for finding the mean and covariance function of $\mathbf{V}_t$ when F_t is a white noise process. This approach can be extended to the case of a colored excitation

if the input process is is re-defined as the output of another linear system (filter) driven by a white noise process. Then, Eqns. 1.20 and 1.22 can be augmented to incorporate the state vector of the filter. Subsequently, the final equations of motion in the state-space formulation is

$$\begin{bmatrix} \dot{\mathbf{V}}_t \\ \dot{\mathbf{V}}_{ft} \end{bmatrix} = \begin{bmatrix} [\mathbf{0}] & [\mathbf{I}] & [\mathbf{0}] & [\mathbf{0}] \\ [\mathbf{A}] & [\mathbf{B}] & [\mathbf{E}] & [\mathbf{F}] \\ [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{I}] \\ [\mathbf{0}] & [\mathbf{0}] & [\mathbf{C}] & [\mathbf{D}] \end{bmatrix} \begin{bmatrix} \mathbf{V}_t \\ \mathbf{V}_{ft} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{H}_1 \\ \mathbf{0} \\ \mathbf{H}_2 \end{bmatrix} W_t \tag{1.23}$$

where $\mathbf{V}_{ft}$ is the state vector of the filter system and W_t is a Gaussian white noise process. Matrices [**A**] and [**B**] are related to the combined system, while [**C**] and [**D**] are related to the filter system. Matrices [**E**] and [**F**] are the connectivity matrices relating the output of the filter to the combined system. The vector $\mathbf{H}_1$ and $\mathbf{H}_2$ relates the combined system and the filter to the white noise, respectively. The size of Eqn. 1.23 is $n_{ss} = 2(\, n_p + n_s + n_f)$, where n_f is the number of degree of freedom of the filter system. Non-stationary excitations can be obtained by multiplying the stationary white noise process with a deterministic function of time, defined here as a_t. This function is called the modulating function.

1.3 Summary

In this section, the features of a general primary-secondary system and the objective of this work are presented. The equations of motion of the combined primary-secondary system that are to be used in this work are also presented. This is followed by some simple examples where the equations of motion are shown. For individually classically damped systems, a transformation can be used to transform the combined equations of motion in terms of a set of generalized co-ordinates. Finally, the state space formulation of the equations of motion are presented. In all subsequent discussions, the above formulation of the equations of motion and notation are used.

SECTION 2
Review of Methods of Analyses

2.1 Introduction

In this section a review of the existing methods of analyzing primary-secondary systems is presented. The goal of the review is to present the basic ideas and the advantages and disadvantages of these methods. However, where appropriate a detailed presentation of the method, i.e., formulation and governing equations, is given. Both approximate and exact methods are examined.

2.2 Approximate Methods of Analysis

There are essentially two types of approximate methods :

1. cascade analysis, and

2. approximate combined analysis.

In the first method, the primary system is analyzed separately and responses at the point(s) of attachment are used as inputs to the secondary system. In the second class of methods certain approximations are used in the analysis of the combined system.

2.2.1 Cascade Analysis

The method only considers effects of the primary system. It is usually expressed in the form of floor response spectra [17, 19]. Thus, the possible influences of the secondary system on the primary system and hence on the combined system are ignored [31]. The analysis is satisfactory for small mass ratios and detuned systems [31]. As previously indicated, the method cannot account for the interaction between different support points of a secondary system. Some researchers have introduced the so-called cross floor spectrum to account for this interaction.

2.2.2 Approximate Combined Analysis

Approximate methods attempting to retain the exact behavior of the combined system have also been developed [6, 8, 15, 16, 20, 27]. These methods involve simplified representations of the contributions of the primary or secondary system to the combined properties. Two basic methods have been developed :

1. Perturbation Methods

2. Component Mode Method

Perturbation Methods

Consider the case of light secondary systems. It is assumed that the dynamic properties associated with the degree of freedom of the primary system in a combined analysis are not too different from the original properties of the primary system. These methods have been used to estimate the eigenvalues and eigenvectors of the combined system [1, 15, 16, 20, 27] and develop floor response spectra that accounts for the interaction effects [6].

Consider for illustration, a single degree of freedom (dof) secondary system attached at the $k-$th floor of an n_p-dof proportionally damped primary system [15, 16]. The secondary system is assumed to be light in comparison to the primary system. The formulation of the combined system follows that given in Chapter 1. The n_p+1 dof of the combined system is associated with the dof of the secondary system. The eigenvalue problem of the combined system is set-up as

$$[\mathbf{K}^*]\Phi_i^* = \omega_i^{*2}[\mathbf{M}^*]\Phi_i^*; \quad i = 1, 2, ..., n_p + 1 \tag{2.1}$$

where $[\mathbf{K}^*]$ and $[\mathbf{M}^*]$ are the stiffness and mass matrices of the combined system. Eqn. 2.1

is expanded and separated into two parts shown in Eqns. 2.2 and 2.3 below.

$$[\mathbf{K}_p]\begin{bmatrix}\phi^*_{1i}\\ \phi^*_{2i}\\ \vdots\\ \phi^*_{ki}\\ \vdots\\ \phi^*_{n_pi}\end{bmatrix}+\begin{bmatrix}0\\ \vdots\\ m_s\omega_s^2(\phi^*_{ki}-\phi^*_{n_p+1,i})\\ \vdots\\ 0\end{bmatrix}=\omega_i^{*2}[\mathbf{M}_p]\begin{bmatrix}\phi^*_{1i}\\ \phi^*_{2i}\\ \vdots\\ \phi^*_{ki}\\ \vdots\\ \phi^*_{n_pi}\end{bmatrix} \tag{2.2}$$

and

$$-m_s\omega_s^2(\phi^*_{ki}-\phi^*_{n_p+1,i})=\omega_i^{*2}m_s\phi^*_{n_p+1,i} \tag{2.3}$$

From Eqn. 2.3

$$\phi^*_{n_p+1,i}=-\frac{\omega_s^2}{\omega_i^{*2}-\omega_s^2}\phi^*_{ki}=\alpha_i\phi^*_{ki} \tag{2.4}$$

where

$$\alpha_i=\frac{\phi^*_{n_p+1,i}}{\phi^*_{ki}} \tag{2.5}$$

is the modal amplification factor of the equipment relative to the attachment point. Substitute Eqn. 2.4 into Eqn. 2.2 and we get

$$[\mathbf{K}_p]\begin{bmatrix}\phi^*_{1i}\\ \phi^*_{2i}\\ \vdots\\ \phi^*_{ki}\\ \vdots\\ \phi^*_{n_pi}\end{bmatrix}+\begin{bmatrix}0\\ \vdots\\ m_s\phi^*_{ki}\omega_i^{*2}\omega_s^2/(\omega_i^{*2}-\omega_s^2)\\ \vdots\\ 0\end{bmatrix}=\omega_i^{*2}[\mathbf{M}_p]\begin{bmatrix}\phi^*_{1i}\\ \phi^*_{2i}\\ \vdots\\ \phi^*_{ki}\\ \vdots\\ \phi^*_{n_pi}\end{bmatrix} \tag{2.6}$$

The single non-zero term in the second vector above is small for light equipment such that it will only slightly modify the frequencies and modes shapes of the original primary structure. Thus, as a first approximation, it is assumed that the portions of the modal vectors of the combined system corresponding to the structural degrees of freedom of the primary system retain their shapes after the equipment is attached, i.e., it is assumed that $\phi^*_{mi}=\phi_{pmi}$ for $m=1,2,..,n_p$ and for $i\neq(\mathrm{n}_p+1)$, where ϕ_{pmi} is the $m-$th entry of the $i-$th modal vector of the primary system. Then, pre-multiplying Eqn. 2.6 with Φ_{pi}^T, we get

$$\omega_i^2M_{pi}+m_s\phi_{pki}^2\frac{\omega_i^{*2}\omega_s^2}{(\omega_i^{*2}-\omega_s^2)}=\omega_i^{*2}M_{pi},\ \mathrm{i}=1,2,..,\mathrm{n}_p \tag{2.7}$$

Note that Eqn. 2.7 is valid only for the dof associated with the primary system. All subsequent indices denoted by i runs from 1 to n_p unless otherwise stated. M_{pi} is the $i-$th modal mass of the primary system, given by

$$M_{pi} = \Phi_{pi}^T [\mathrm{M}_p] \Phi_{pi} = \omega_{pi}^{-2} \Phi_{pi}^T [\mathrm{K}_p] \Phi_{pi} \tag{2.8}$$

Let

$$\beta_i = \frac{(\omega_{pi}^2 - \omega_s^2)}{\omega_s^2} \tag{2.9}$$

be a *detuning parameter* and

$$\gamma_i = \frac{m_s}{(M_{pi}/\phi_{pki}^2)} \tag{2.10}$$

is defined as the *effective mass ratio*, respectively for mode i of the primary system. Substituting Eqns. 2.9 and 2.10 into Eqn. 2.7 leads to

$$(1+\beta_i)\left(\frac{\omega_i^*}{\omega_{pi}}\right)^4 - 2\left(1 + \frac{\beta_i + \gamma_i}{2}\right)\left(\frac{\omega_i^*}{\omega_{pi}}\right)^2 + 1 = 0 \tag{2.11}$$

The solution to Eqn. 2.11 is

$$\left(\frac{\omega_i^*}{\omega_{pi}}\right)^2 = \begin{cases} \left[1 + b_i - \left[(1+b_i)^2 - (1+\beta_i)\right]^{\frac{1}{2}}\right]/(1+\beta_i), & \beta_i < 0 \\ \\ \left[1 + b_i + \left[(1+b_i)^2 - (1+\beta_i)\right]^{\frac{1}{2}}\right]/(1+\beta_i), & \beta_i \geq 0 \end{cases} \tag{2.12}$$

where

$$b_i = \frac{\beta_i + \gamma_i}{2} \tag{2.13}$$

Substitute Eqn. 2.12 into Eqn. 2.4 and the modal amplification factor is obtained as

$$\alpha_i = \begin{cases} -\left[b_i - \left[(1+b_i)^2 - (1+\beta_i)\right]^{\frac{1}{2}}\right]^{-1}, & \beta_i < 0 \\ \\ -\left[b_i + \left[(1+b_i)^2 - (1+\beta_i)\right]^{\frac{1}{2}}\right]^{-1}, & \beta_i \geq 0 \end{cases} \tag{2.14}$$

From Eqn. 2.14, notice that for small β_i, i.e., near or perfect tuning, α_i is large, i.e., high modal amplification at the secondary system level. From Eqns. 2.5, 2.12 and 2.14 the eigenvalues and eigenvectors of the dof associated with the primary system can be computed.

To get mode shape $\Phi^*_{n_p+1}$, i.e., the new mode, let

$$\Phi^*_{n_p+1} = \begin{bmatrix} \phi^*_{1,n_p+1} \\ \phi^*_{2,n_p+1} \\ \vdots \\ \phi^*_{n_p,n_p+1} \\ 1 \end{bmatrix} = \begin{bmatrix} \Phi_{n_p+1} \\ 1 \end{bmatrix} \tag{2.15}$$

The above eigenvector must satisfy the orthogonality condition with the other modes, i.e.,

$$\Phi^{*}_{i}[\mathrm{M}^*]\Phi^*_{n_p+1} = 0, \text{for } i \neq (n_p+1) \tag{2.16}$$

Subsequently, it can be shown that

$$\Phi_{n_p+1} = \begin{bmatrix} \sum_{i=1}^{n_p}(\phi_{p1i}/M_{pi})m_s\gamma_i\phi_{pki} \\ \vdots \\ \sum_{i=1}^{n_p}(\phi_{pni}/M_{pi})m_s\gamma_i\phi_{pki} \end{bmatrix} \tag{2.17}$$

Note that all structure modes of the primary system contribute to this mode and that it will be dominated by that structure mode with frequency closest to secondary system frequency. The frequency of the new mode is obtained from Eqn. 2.3, i.e.,

$$-m_s\omega_s^2(\phi^*_{ki} - \phi^*_{n_p+1,i}) = \omega_i^{*2}m_s\phi^*_{n_p+1,i} \tag{2.18}$$

Canceling m_s and substituting for $i = n_p + 1$, the following equation is obtained.

$$\omega^*_{n_p+1} = \left(1 + \sum_{i=1}^{n_p}\gamma_i\alpha_i\right)^{\frac{1}{2}}\omega_s \tag{2.19}$$

Note the contribution from the structure modes. Similar perturbation technique was applied to get the modal damping coefficient of the combined system. Here, it is assumed that the equipment is light and that the combined system very nearly has modal damping [15, 16].

Component Mode Method

Consider a primary-secondary system in which the primary system has classical modes of vibration. The secondary system need not be classically damped. The combined equations of motion for the system subjected to base excitation is given by

$$[\mathrm{M}^*]\ddot{\mathbf{X}}^*_t + [\mathrm{C}^*]\dot{\mathbf{X}}^*_t + [\mathrm{K}^*]\mathbf{X}^*_t = -[\mathrm{M}^*]\mathbf{1}F_t \tag{2.20}$$

where

$$\mathbf{X}_t^* = \begin{bmatrix} \mathbf{X}_p(t) \\ \mathbf{X}_s(t) \end{bmatrix} \tag{2.21}$$

which is of size $(n_p + n_s)$. Introducing the following approximation

$$\mathbf{X}_p(t) = [\Phi_p^1]\mathbf{q}(t) \tag{2.22}$$

where $[\Phi_p^1]$ is a sub-set of the the modal matrix of the primary system of size n_p by $n_p^1 << n_p$ and $\mathbf{q}(t)$ is a vector of generalized displacements of size n_p^1 associated with the modes of the primary system. From Eqns. 2.22 and 2.21,

$$\hat{\mathbf{X}}_t^* = \begin{bmatrix} [\Phi_p^1]\mathbf{q}(t) \\ \mathbf{X}_s(t) \end{bmatrix} \tag{2.23}$$

which is of size $(n_p^1 + n_s) << (n_p + n_s)$. Eqn. 2.23 is then used in the combined equations of motion 2.20. Thus, the analysis of the primary-secondary system involves a smaller number of responses than the exact combined analysis. Eqn. 2.22 represents the approximation used in this method, i.e., the primary system is approximated by only a few of its modes. The major task is the judicious selection of the modes of the primary system to be included in the analysis. Note that if all the modes of the primary system are used then an exact analysis is undertaken.

A comprehensive investigation of this method applied to the analyses of secondary system with classical modes was performed recently [3]. The investigation concluded that accurate results can be obtained by only using the modes of the primary system which are closed to the natural frequency of the secondary system. Responses in the frequency region away from the natural frequency of the secondary system can be obtained using cascade analysis [3].

The component mode method was also used to develop floor response spectra [25]. It has also been applied to the analysis of structures other than primary-secondary systems, e.g., a system with non-classically damped modes [11].

2.3 Exact Methods of Analysis

Any attempt at a combined analysis of a primary-secondary system must contend with the associated problems outlined in Section 1.1. For example, the large differences in masses and other properties of the primary system and the secondary system can create some numerical difficulties in finding the eigenvalues and eigenvectors of the combined system. Furthermore, the combined system is generally non-classically damped. However, with the development of more efficient eigenvalue solvers, a combined analysis is now computationally feasible. Alternative exact methods, i.e., other than solving the eigenvalue problem directly, have been developed [23, 24, 26, 28]. Spanos et al [23] proposed an iterative scheme to determine the response of a primary-secondary system. The proposed scheme circumvents a modal analysis. Suarez and Singh [24, 26, 28] developed a technique of obtaining the eigenvalues of the combined system by solving a non-linear algebraic equation.

2.3.1 Direct Analysis

A direct analysis of the combined equations of motion can be performed using the state space approach [8]. In this approach, the coupled equations of motion is recast into a system of coupled first order differential equation. Let the resulting equation be given by

$$\dot{\mathbf{V}}_t = [\mathbf{A}]\mathbf{V}_t + \mathbf{G}F_t \tag{2.24}$$

where [A] and G are a matrix and vector respectively, of constants, and F_t is a forcing function. The system of first order differential equation is then uncoupled using a transformation involving the complex eigenvalues and eigenvectors of the matrix [A]. Thus, a complex eigenvalue solver is required. The resulting uncoupled equations can then be solved individually. Appendix A outlines one direct method of analysis using the state space approach.

2.3.2 Analysis Based on Characteristics of Primary and Secondary Systems

Consider a single degree of freedom secondary system placed on the $k-$th floor of a n_p-dof classically damped primary system. The equations of motion of the combined system

is given by Eqn. 1.4. Using the transformation [24]

$$\mathbf{X}_t^* = [\Phi^*]\mathbf{Y}_t^* \tag{2.25}$$

where

$$[\Phi^*] = \begin{bmatrix} [\Phi_p] & \mathbf{0} \\ \mathbf{0} & \phi_s \end{bmatrix} \tag{2.26}$$

and

$$\phi_s = \frac{1}{\sqrt{m_s}} \tag{2.27}$$

the equation of motion for $\mathbf{Y}_t$ becomes

$$[\mathbf{I}]\ddot{\mathbf{Y}}_t^* + [\mathbf{C}^*]\dot{\mathbf{Y}}_t^* + [\mathbf{K}^*]\mathbf{Y}_t^* = -\{\Gamma^*\} F_t \tag{2.28}$$

where

$$[\mathbf{C}^*] = \begin{bmatrix} [\mathbf{C}_p] & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} + 2m_s\omega_s\zeta_s\mathbf{v}\mathbf{v}^T \ , \tag{2.29}$$

$$[\mathbf{K}^*] = \begin{bmatrix} [\mathbf{K}_p] & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} + m_s\omega_s^2\mathbf{v}\mathbf{v}^T \ , \tag{2.30}$$

$$\{\Gamma^*\} = \begin{bmatrix} \Gamma_p \\ \sqrt{m_s} \end{bmatrix} \ , \tag{2.31}$$

$$\mathbf{v}^T = \left\{\phi_{pk1},\ \phi_{pk2}, \cdots, \phi_{pkn_p},\ -\phi_s\right\} \tag{2.32}$$

and ϕ_{pki} is the $k-$th entry (or value) of the $i-$th modal vector of the primary system. The undamped eigenvalue problem associated with Eqn. 2.28 is as follows:

$$\begin{bmatrix} \omega_{p1}^2 & 0 & \cdots & 0 & 0 \\ 0 & \omega_{p2}^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \omega_{pn_p}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Phi_j^* \ + m_s\omega_s^2\mathbf{v}\mathbf{v}^T\Phi_j^* \ = \ \omega_j^*\Phi_j^*; \ \ j = 1, 2, ..., n_p + 1 \tag{2.33}$$

where ω_{pi}, the $i-th$ eigenvalue of the primary system, ω_j^* is the $j-th$ eigenvalue of the combined system, and Φ_j^* is the $j-th$ eigenvector of the combined system. The above equation can be re-written as

$$\begin{bmatrix} \omega_{p1}^2-\omega_j^* & 0 & \cdots & 0 & 0 \\ 0 & \omega_{p2}^2-\omega_j^* & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \omega_{pn_p}^2-\omega_j^* & 0 \\ 0 & 0 & 0 & 0 & -\omega_j^* \end{bmatrix} \Phi_j^* = -m_s\omega_s^2\mathbf{v}\mathbf{v}^T\Phi_j^* \quad j=1,2,...,n_p+1 \tag{2.34}$$

or, in matrix form,

$$[\lambda]\Phi_j^* = -m_s\omega_s^2\mathbf{v}\mathbf{v}^T\,\Phi_j^* \quad j=1,2,...,n_p+1 \tag{2.35}$$

Then

$$\Phi_j^* = -[\lambda]^{-1}m_s\omega_s^2\mathbf{v}\mathbf{v}^T\Phi_j^* \quad j=1,2,...,n_p+1 \tag{2.36}$$

The inverse of matrix $[\lambda]$ is a diagonal matrix. Pre-multiply Eqn. 2.36 with $\mathbf{v}^T$ leads to

$$\mathbf{v}^T\Phi_j^* = -m_s\omega_s^2\mathbf{v}^T[\lambda]^{-1}\mathbf{v}\mathbf{v}^T\Phi_j^* \quad j=1,2,...,n_p+1 \tag{2.37}$$

The product $\mathbf{v}^T\Phi_j^*$ in Eqn. 2.37 is a scalar and therefore can be canceled out. This gives an equation of the form

$$\mathbf{v}^T[\lambda]^{-1}\mathbf{v} + \frac{1}{m_s\omega_s^2} = 0 \tag{2.38}$$

Eqn. 2.38 can be expanded into

$$\sum_{i=1}^{n_p}\frac{\phi_{pki}^2}{(\omega_{pi}^2-\omega^*)} - \frac{1}{m_s\omega^*} + \frac{1}{m_s\omega_s^2} = 0 \tag{2.39}$$

The solution of this equation gives the (n_p+1) eigenvalues of the combined system. The authors proposed the Newton-Raphson method for solving the non-linear equation with the second-order perturbation estimates of the eigenvalues as a starting solution. Numerical results showed that [24]:

1. For light equipment, the final estimate is sensitive to initial estimate.

2. For heavier equipment, the solution converges very fast with the initial estimates.

3. For detuned and tuned cases, the number of iteration increases with mass of the equipment. However, the number increases more rapidly in the tuned case.

The method outlined above entails solving a highly non-linear algebraic equation. First estimates of the roots of the equation based upon a perturbation analysis are suggested. This implies one has to perform a perturbation analysis. Furthermore, since the equation is expressed in terms of the modal properties of the primary system, a modal analysis of the primary system has to be performed. However, this is done only once. The proposed scheme was proven to be superior to a direct solution. A comparison in the efficiency of the proposed scheme to that of a direct solution was performed [26].

The eigenvectors can be evaluated from Eqn. 2.36. First, let

$$m_s\omega_s^2 \mathbf{v}^T \Phi_j^* = 1.0 \tag{2.40}$$

Then, from Eqn. 2.36

$$\Phi_j^* = -[\lambda]^{-1}\mathbf{v} \tag{2.41}$$

or

$$\Phi_j^{*T} = \left(\frac{\phi_{pk1}}{\omega_{p1}^2 - \omega_j^*}, \frac{\phi_{pk2}}{\omega_{p2}^2 - \omega_j^*}, \cdots, \frac{\phi_{pkN_p}}{\omega_{pN_p}^2 - \omega_j^*}, \frac{\phi_s}{\omega_j^*} \right) \tag{2.42}$$

for $j = 1, 2, .., n_p + 1$. This will give the eigenvectors of the combined system.

This method has also been applied to non-classically damped primary systems [26] and multi-degree-of-freedom secondary systems with different attachment configuration [28].

2.3.3 Sequential Analysis

A useful iterative scheme for calculating the time-history of the responses of a primary-secondary system is presented in Ref. [23]. The scheme assumes that each sub-system has classical modes of vibration. The responses are obtained using an iterative approach involving a time-step integration solution of the combined equations of motion. The equations of motion of the combined system are first separated into three different systems of equations. One system describes the degree of freedom at the attachment point (or interface) and the other two describe the equations of motion of the unattached degrees of freedom of the primary and secondary system respectively. The equations of motion for the interface

will invariably be smaller in dimension than the other two. Hence attention is focused on this set of equations first. A predictor-corrector scheme is used to converge interatively to the interface accelerations. At each time point, the subsequent interface accelerations are obtained (or predicted) from a truncated Taylor's expansion involving the values of the third-order derivatives of the dof at the interface at the next time point. The values of the third-order derivatives are obtained from an algebraic equation involving the accelerations and velocities of the unattached dof. Once the accelerations at the interfaces are estimated, the remaining two sets of equations of motion can be integrated independently. Solution at the next time point requires the solution at all the dof at the current time point. The advantages however are that the iteration is performed with the small number of equations of motion involving the dof at the interfaces and a modal analysis of the combined system is not required.

2.4 Summary

In this section, a number of approximate and exact methods of analyses are reviewed. The review attempts to extract the more general ideas of current methods for vibration analysis of primary-secondary systems. More extensive reviews are in References [18] and [31].

Approximate methods of analysis are satisfactory under certain conditions. A cascade analysis can give good results for light and detuned secondary systems. For tuned systems, interaction effects are significant and a cascade analysis can be inaccurate. Perturbation methods can be used effectively for light tuned or detuned secondary systems. The component mode method is also adequate for tuned systems provided the modes of the primary system included in the combined analyses are representative. One distinct advantage of these approximate methods is that the primary system has to be analyzed only once. Despite some limitations and restrictions, approximate methods can be used to obtain a first estimate of the performance of primary-secondary systems.

A direct analysis is currently feasible due to new algorithms for finding eigenvalues. However, the cost of re-analysis due to, e.g., changes in the location of a secondary system

can be expensive. The exact method proposed by Suarez and Singh [24] requires a solution of a highly non-linear equation to find the eigenvalues of the combined system. This is a main disadvantage of their approach. The method proposed by Spanos et al [23] does not require a modal analysis of the combined system. The time-history of the responses of the primary-secondary system are obtained through a numerical integration scheme. The scheme was successfully applied only to deterministic vibrations. The advantage of the scheme for stochastic input has yet to be determined.

SECTION 3
Methods of Analyses of Uncertain Systems and Random Inputs

3.1 Introduction

Considerable research and progress have been made in developing techniques to evaluate the response and performance of a deterministic system subjected to a stochastic excitation. It is usually assumed that uncertainties in the excitation dominate the system response and performance. Hence, it is sufficiently accurate to assume that the system is deterministic. Effects of the uncertainty in system parameters on system responses have only began to be investigated recently [2, 7, 10, 22, 30]. This section presents a brief outline of response and reliability analysis of uncertain systems. The Monte Carlo simulation method and reliability-based techniques are developed for estimating the performance of uncertain primary-secondary system subject to random excitation.

3.2 Response and Reliability Analysis of Uncertain Systems

Consider a dynamic system with uncertain parameters $\mathbf{Z}$ following joint probability density function $f(\mathbf{z})$. The deterministic dynamic system corresponding to a possible value of $\mathbf{Z}$ is referred to as the conditional system. Traditional random vibration techniques can be applied to determine the response statistics and probabilities of failure of the conditional system. Let $p_f(\mathbf{z})$ be the conditional failure probability for $\mathbf{Z} = \mathbf{z}$. Then, the unconditional or simply the probability of failure of uncertain systems can be written as

$$p_f = \int_{\mathbf{Z}} p_f(\mathbf{z}) f(\mathbf{z}) d\mathbf{z} \tag{3.1}$$

Numerical techniques such as integration and simulation can be used to obtain the unconditional failure probability. Integration techniques are not very efficient for large number of

uncertain parameters. A more important disadvantage of numerical integration and simulation schemes is that a significant number of conditional random vibration analyses would have to be performed in order to obtain an accurate estimate. In the next section, Monte Carlo simulation is presented. This is followed by the proposed method which is based upon the so-called First- and Second-Order Reliability Methods (FORM/SORM).

3.2.1 Monte Carlo Simulation

Consider samples $\mathbf{z}_i$ of $\mathbf{Z}$, $i = 1,.., N$. Random vibration analyses can be performed for each of these samples and the conditional failure probability $p_f(\mathbf{z}_i)$ be computed. An unbiased estimator of the system failure probability is then given by

$$\hat{p}_f = \frac{1}{N} \sum_{i=1}^{N} p_f(\mathbf{z}_i) \tag{3.2}$$

The accuracy of the estimator can be measured by its coefficient of variation (COV) and this is given by

$$COV(\hat{p}_f) = \frac{\sqrt{Var(\hat{p}_f)}}{\hat{p}_f} \tag{3.3}$$

where

$$Var(\hat{p}_f) = \frac{1}{N(N-1)} \left[\sum_{i=1}^{N} p_f^2(\mathbf{z}_i) - N\hat{p}_f^2 \right] \tag{3.4}$$

If the conditional failure probability cannot be evaluated easily then an alternative formulation is used. In this case, the load process is also sampled and for each realization the system response is checked for failure in a prescribed time interval. The fraction of samples leading to failure is an unbiased estimator of the system failure probability, that is,

$$\hat{p}_f = \frac{n_f}{N} \tag{3.5}$$

where n_f is the number of samples where system failure was observed. The main disadvantage of Monte Carlo simulation is that a large number of samples are required in order to get an accurate estimate. This is especially the case when the failure probability is very small. Hence, costs considerations become important.

Monte Carlo simulation can also be used to estimate the moments of any response. The formulation is similar to Eqn. 3.2. In this case, for each sample the conditional moments,

i.e., for specified value of the uncertain system parameters, of the desired response are computed. The average of the computed conditional moments is an estimate of the moments of the response.

3.2.2 Proposed Method

Consider a linear primary-secondary system with uncertain parameters subjected to a random non-stationary colored Gaussian input. The goal is to evaluate the probability of failure corresponding to various failure criterion. The proposed method has two phases. The first phase is a random vibration and reliability analysis of a conditional system, i.e., conditional on the system parameters. The second phase is the evaluation of failure probabilities using FORM/SORM. Both these phases are discussed below.

Response Analysis

In this method, the conditional responses of the combined primary-secondary system is solved using the state space approach [8]. This approach can be used to characterize both stationary and non-stationary responses [13]. The state space is augmented to incorporate the filter system for colored inputs. The resulting system of first order differential equations is then solved directly using the method described in Appendix A. Closed-form expressions are developed for the covariance matrix of conditional system responses. Since the input is Gaussian, the mean crossing rates for the conditional responses can be evaluated using results developed by Veneziano et al [32].

Reliability Analysis

The safety condition can be expressed in terms of the response vector $\mathbf{X}_t$. For example, the system is considered safe when an interstory displacement does not exceeds a certain threshold in a certain time interval. That is, the system is safe when

$$\Delta_i(t) \leq S \ , \quad 0 \leq \mathrm{t} \leq \tau \tag{3.6}$$

where $\Delta_i(t) = X_i(t) - X_{i-1}(t)$ is the i-th interstory displacement, S is the allowable threshold and τ is the time interval. The above relation can also be written as

$$\Delta_i(t) - S \leq 0 \ , \quad 0 \leq t \leq \tau \tag{3.7}$$

Since the inter-story displacement is a function of the response vector and the uncertain parameters, the safety condition can be expressed using a function called the g-function, i.e., the system survives when

$$g(\mathbf{X}_t, S, \mathbf{Z}) \leq 0 \ , \quad 0 \leq t \leq \tau \tag{3.8}$$

The reliability is given by

$$p_r(\tau) \ = P\left[g(\mathbf{X}_t, S, \mathbf{Z}) \leq 0\right] \quad 0 \leq t \leq \tau \tag{3.9}$$

Note that if other inter-story displacements are considered then several g-functions must be evaluated.

The calculation of the reliability is performed in two steps. First, the conditional reliability is obtained using random vibration methods. The conditional reliability is defined as $p_r(\tau, \mathbf{z})$. This quantity can be evaluated using the conditional mean crossing rates. In the second step, the unconditional reliability is obtained as

$$p_r(\tau) = E_{\mathbf{Z}}\left[p_r(\tau, \mathbf{Z})\right] \tag{3.10}$$

where $E_{\mathbf{Z}}$ is the expectation with respect to $\mathbf{Z}$. Eqn. 3.10 is computationally difficult when the dimension of $\mathbf{Z}$ is very large. However, it can be approximated using First- and Second-Order Reliability Methods (FORM/SORM) [9]. The advantage of using FORM/SORM over the numerical techniques is that the number of conditional random vibration analyses is generally much smaller.

The FORM/SORM algorithms can be used to approximate the probability content of a set define by a g-function (Appendix B). The random variables $\mathbf{Z}$ are first transformed into independent unit normal random variables. The β-point is defined as the point on the limit state surface $g = 0$ that is closest to the origin in the space of unit normal random variables. The limit state surface is approximated by either a hyperplane or a second-order surface which is tangential at the β-point. Reliability is then approximated by the probability content of these approximate surfaces.

FORM/SORM algorithms have previously been used to evaluate the reliability of uncertain dynamic systems [4, 10, 30]. The approach proposed by Wen and Chen [30] is used in this work. Let

$$\mathbf{U} = \mathbf{T}(\mathbf{Z}) \tag{3.11}$$

be a vector of standard independent normal random variables and $\mathbf{T}$ a transformation from $\mathbf{Z}$ to $\mathbf{U}$. Then Eqn. 3.1 can be written as

$$p_f = \int_{\mathbf{u}} p_f \left[\mathbf{T}^{-1}(\mathbf{u})\right] f(\mathbf{u}) d\mathbf{u} \tag{3.12}$$

It can be shown that the failure probability p_f is given by

$$p_f = P[g(\mathbf{U}) \leq 0] \tag{3.13}$$

where

$$g(\mathbf{u}) = u_{n+1} - \Phi^{-1}\left[p_f(\mathbf{T}^{-1}(\mathbf{u}))\right] \tag{3.14}$$

The above g-function was proposed by Wen and Chen [30]. U_{n+1} is a standard independent normal auxiliary random variable and Φ is the cumulative distribution function of a unit standard normal. The first order approximation of the event $P[g(\mathbf{U}) \leq 0]$ is $\Phi(-\beta_F)$ where β_F is the first-order β found using FORM analysis. Hence the failure probability is approximated by

$$p_f \approx \Phi(-\beta_F) \tag{3.15}$$

If a second-order approximation is used, then

$$p_f \approx \Phi(-\beta_S) \tag{3.16}$$

where β_S is from a SORM analysis.

Failure Due to First Crossing

Suppose a system fails when an inter-story displacement exceeds a certain allowable value or threshold. The probability of failure is given by

$$p_f(\tau) = 1.0 - p_r(\tau) = 1.0 - P\left[\Delta_i(t) \geq S\right],\ 0 \leq t \leq \tau \tag{3.17}$$

where $p_r(\tau)$ is the reliability, $\Delta_i(t)$ is the i-th interstory displacement and S is the threshold which can be deterministic or random. Note that the interstory displacement is given by

$$\Delta_i(t) = X_i(t) - X_{i-1}(t) \tag{3.18}$$

where $X_i(t)$ is the relative displacement of the i-th floor. The conditional reliability can be evaluated easily using the mean crossing rate of a Gaussian process. The conditional reliability is approximated by

$$p_r(s, \mathbf{z}, \tau) = \exp\left[-\int_0^\tau \nu_\Delta(s, \mathbf{z}, t)dt\right] \tag{3.19}$$

where $\nu_\Delta(s, \mathbf{z}, t)$ is the time-dependent s-crossing rate of the conditional random process $\Delta(t) \mid \mathbf{Z}$. The conditional reliability is a function of the given threshold s and uncertain parameters $\mathbf{z}$. Note that

$$p_f(s, \mathbf{z}, \tau) = 1. - p_r(s, \mathbf{z}, \tau) \tag{3.20}$$

It can be shown that for a zero mean non-stationary Gaussian input

$$\nu_\Delta(s, \mathbf{z}, t) = \frac{1}{\sigma_{\Delta(t)}}\phi(\frac{s}{\sigma_{\Delta(t)}})\left[\sigma^{'}(t)\phi\left\{\frac{\mu^{'}(t)}{\sigma^{'}(t)}\right\} + \mu^{'}(t)\Phi\left\{\frac{\mu^{'}(t)}{\sigma^{'}(t)}\right\}\right] \tag{3.21}$$

where

$$\mu^{'}(t) = \frac{sE\left[\Delta(t)\dot{\Delta}(t)\right]}{\sigma^2_{\Delta(t)}}\,, \tag{3.22}$$

$$\sigma^{'2}(t) = \sigma^2_{\dot{\Delta}(t)} - \frac{\left\{E\left[\Delta(t)\dot{\Delta}(t)\right]\right\}^2}{\sigma^2_{\Delta(t)}}\,, \tag{3.23}$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})\,, \tag{3.24}$$

$$\Phi(y) = \int_{-\infty}^{y}\phi(x)dx\,, \tag{3.25}$$

$$\sigma^2_{\Delta(t)} = E\left[X_i^2(t)\right] + E\left[X_{i-1}^2(t)\right] - 2E\left[X_i(t)X_{i-1}(t)\right]\,, \tag{3.26}$$

$$\sigma^2_{\dot{\Delta}(t)} = E\left[\dot{X}_i^2(t)\right] + E\left[\dot{X}_{i-1}^2(t)\right] - 2E\left[\dot{X}_i(t)\dot{X}_{i-1}(t)\right]\,, \tag{3.27}$$

and

$$\begin{aligned} E\left[\Delta(t)\dot{\Delta}(t)\right] = \; & E\left[X_i(t)\dot{X}_i(t)\right] + E\left[X_{i-1}(t)\dot{X}_{i-1}(t)\right] - \\ & E\left[X_i(t)\dot{X}_{i-1}(t)\right] - E\left[X_{i-1}(t)\dot{X}_i(t)\right] \end{aligned} \tag{3.28}$$

From Eqns. 3.26, 3.27, and 3.28, clearly the second moments of the responses are required for the above analysis. Note that similar results can be obtained for other responses due to the linearity of the problem.

3.2.3 Example

Consider the 2-dof system shown in Fig. 3.1. The first floor is a 1-dof primary system and the second floor constitutes a 1-dof secondary system. The system is subjected to a non-stationary zero-mean Gaussian white noise process, i.e.,

$$F(t) = a(t)W(t) \tag{3.29}$$

where $a(t)$ is a modulating (deterministic) function and $W(t)$ is the stationary zero-mean Gaussian white noise process. The one-sided power spectral density (PSD) function of $W(t)$, i.e., defined here as G_0, is set to 1.0. The modulating function used was

$$a(t) = 2.32[\exp(-0.09t) - \exp(-1.49t)] \tag{3.30}$$

The forcing function was applied for 20 seconds. A mass ratio (i.e., ratio of the mass of the

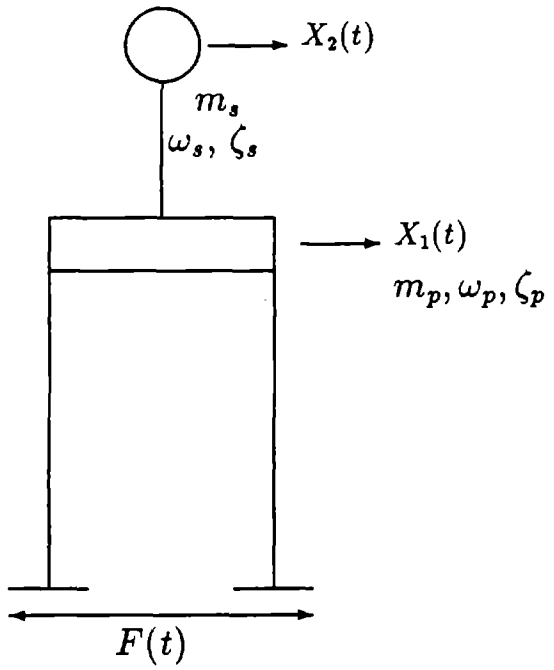


Figure 3.1: 2-DOF Uncertain Primary-Secondary System

secondary system to that of the primary system) of 0.01 was used. The uncertain parameters are the damping ratios and frequencies of the two systems and the respective distributions are shown in Table 3.1. Note that similar distributions are used for the frequencies of the two systems. Hence, the overall system is tuned. The probability of failure is defined as the

Parameters	Distribution
ω_p & ω_s	$U[0.65\pi\text{rads/s}, 3.35\pi\text{rads/s}]$
ζ_p & ζ_s	$U[0.01, 0.07]$

Table 3.1: Table of Distribution of Parameters

probability that the interstory displacement of the second floor, i.e., the secondary system, exceeds the given deterministic threshold. The probability of failure is estimated using the proposed method.

The purpose of this example is to demonstrate the proposed method. This is done by investigating nine different cases shown in Table 3.2. Each case is associated with none or some uncertainties present. In cases where one or some parameters are fixed, these parameters are fixed at their mean values. A secondary objective of the cases investigated is

Case No.	ω_p(rads/sec)	ζ_p	ω_s(rads/sec)	ζ_s
1	2π	0.01	2π	0.01
2	2π	0.04	2π	0.04
3	2π	0.07	2π	0.07
4	2π	0.04	$U[0.65\pi, 3.35\pi]$	0.04
5	$U[0.65\pi, 3.35\pi]$	0.04	2π	0.04
6	$U[0.65\pi, 3.35\pi]$	0.04	$U[0.65\pi, 3.35\pi]$	0.04
7	2π	0.04	$U[0.65\pi, 3.35\pi]$	$U[0.01, 0.07]$
8	$U[0.65\pi, 3.35\pi]$	$U[0.01, 0.07]$	2π	0.04
9	$U[0.65\pi, 3.35\pi]$	$U[0.01, 0.07]$	$U[0.65\pi, 3.35\pi]$	$U[0.01, 0.07]$

Table 3.2: Table of Cases Investigated

also to study the effects of uncertainties in damping ratios and frequencies on the probability of failure for a tuned primary-secondary-system. Clearly, the cases investigated are not exhaustive.

Four different thresholds were used for each of the 9 cases. In evaluating the conditional failure probability for a given set of deterministic parameters, a simple numerical integration scheme was used. The non-stationary mean crossing rates were evaluated at 80 time points and the reliability evaluated using Simpson's rule. For cases where there are some uncertainties in the system parameters, the probability of failure were evaluated using the method proposed. However, for high thresholds, a convergence problem was encountered in the program. That is, the program took a long time to converge to the design point or in some cases, it did not converge at all. As an example of the convergence problem, consider the case where only ω_p is uncertain. For a threshold of 10.0, a β_F of 3.118 was obtained. However, the algorithm took 198 gradient evaluations and 618 random vibration analyses to search for the design point. A plot of the g-function in u-space for different thresholds is shown in Fig. 3.2. Note the rapid changes in the gradients around the β-point.

Due to the preceding reasons, for this example, other numerical schemes were used instead. For Cases 4 to 8, the probability of failure were evaluated using a simple numerical integration scheme. Each random variable was discretized into 40 intervals. Simple Monte Carlo simulation was used for case 9. The results are given in Tables 3.3, 3.4 and 3.5.

From Table 3.3, when the damping ratios are very small, the probability of failure is the highest, i.e., Case 1. Recall that for a 1-dof system subjected to Gaussian white noise, the variance of the response is inversely related to the damping ratio. Hence there tend to be larger displacements for lower damping ratios. This fact is reflected in the results obtained.

For the cases where the frequency of the secondary system is uncertain, the secondary system is safer for the case where the frequency of the primary system is deterministic, i.e., compare results of Case 4 and 6 in Table 3.4. In Case 4, tuning occurs at one point while in Case 6, tuning occurs over the whole range of frequencies. However, the probability of tuning in both cases is zero. The failure probability increases when the damping ratio of the secondary system is also uncertain, i.e., cases 4 and 7. The least safe case is when all the parameters are uncertain, i.e., case 9, and the safest case is when only ω_s is uncertain.

Relative to the deterministic system, i.e., case 2, the following observations can be made regarding the effects of uncertainties on the failure probability. Uncertainties in either of the natural frequencies of the two systems resulted in smaller failure probabilities. This is

because tuning has zero probability of occurrence. Also, the input energy would be concentrated about ω_p or ω_s if the excitation were stationary. When both natural frequencies are uncertain, the failure probability either increases or decreases depending upon the threshold. Although tuning still has zero probability of occurrence, the amplification of response of secondary system can be much higher than for deterministic system when ω_s and/or ω_p is less than their mean values. When all system parameters are uncertain in either or both systems, the failure probability could either increase or decrease. This may be due to the effects of damping and tuning. Further study is required in order to understand fully the effects of uncertainties on the failure probability of the secondary system. However, the utility of the method in providing a tool to examine the exact behavior of primary-secondary system is clearly demonstrated in this example.

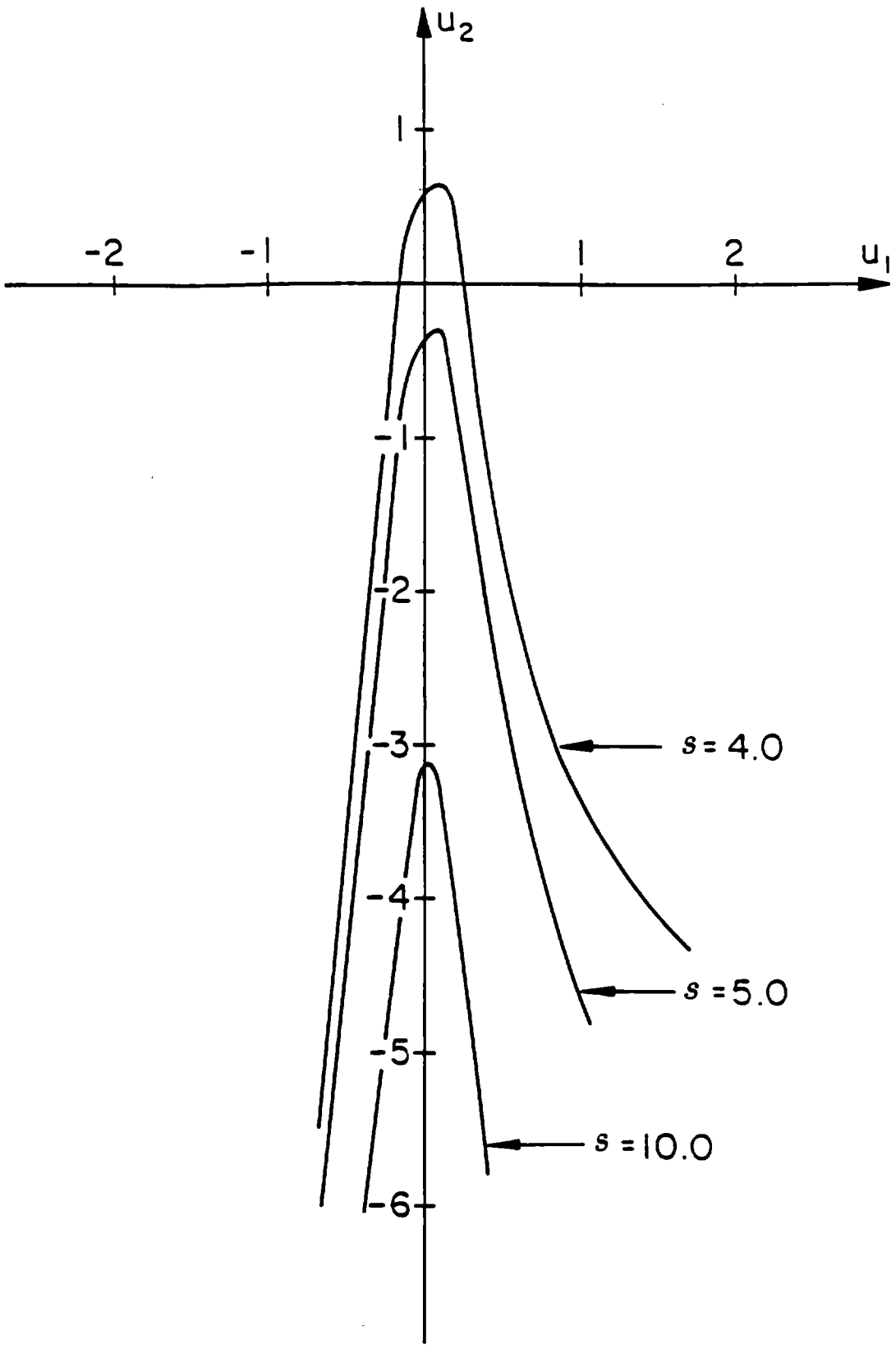


Figure 3.2 : Plots of the g-function for ω_p uncertain (u_1 corresponds to ω_p).

Threshold	Case 1	Case 2	Case 3
6.0	9.9637E-01	2.1658E-01	5.58E-04
8.0	9.2473E-01	1.9258E-02	1.0E-06
10.0	6.4193E-01	8.6E-04	0.0
12.0	3.0275E-01	2.1E-05	0.0

Table 3.3: Prob. of Failure of Tuned Systems : Deterministic Cases

Threshold	Case 4	Case 5	Case 6	Case 7	Case 8
6.0	3.1401E-02	2.2941E-02	6.2229E-02	4.9154E-02	3.5367E-02
8.0	2.3907E-03	1.5666E-03	2.9356E-02	8.0833E-03	5.8289E-03
10.0	1.0666E-04	5.8266E-05	1.5354E-02	1.0875E-03	7.2928E-04
12.0	2.8084E-06	1.2149E-06	8.3648E-03	1.1915E-04	7.1771E-05

Table 3.4: Prob. of Failure of Tuned Systems : Random Cases

Threshold	Prob. of Failure	Estimated COV	# of Samples
6.0	7.36568E-02	0.396816E-01	5000
8.0	3.84944E-02	0.565171E-01	5000
10.0	1.90848E-02	0.759416E-01	5000
12.0	9.59821E-03	0.990306E-01	5000

Table 3.5: Prob. of Failure of Tuned Systems : Case 9

SECTION 4
Conclusion

An exact and general method was proposed for calculating second moment characteristics of response processes and the probability of failure for linear primary-secondary systems with uncertain parameters subject to non-stationary Gaussian excitation. The proposed method was based on methods of linear random vibration, crossing theory of Gaussian processes, and First-and Second-Order Reliability Methods (FORM/SORM). The state space approach was used in the random vibration analysis. The excitation was modeled as the output of a linear filter subjected to a uniformly modulated white noise process. Mean crossing rates of responses were used to approximate conditional failure probabilities for set of system parameters. The FORM/SORM algorithms were then used to approx. unconditional system failure probabilities.

The main disadvantage of the proposed method is that the combined system has to be re-analyzed whenever the location or the parameters of the secondary system are changed. However, re-analysis is not a major task due to the availability of stable and efficient eigenvalue solvers.

The method was demonstrated on an example. The example consisted of an uncertain single-degree-of-freedom (dof) secondary system attached to an uncertain single-dof primary system subjected to non-stationary zero-mean Gaussian white noise process. Uncertainties were present in both the damping ratios and natural frequencies of both systems. The probability that the relative displacement of the secondary system to the primary system exceeds a given threshold was computed for different cases of uncertainties present. The responses were obtained efficiently and without too much difficulty. The following observations can be made regarding the effects of uncertainties on the failure probability relative to a deterministic system. Uncertainties in either of the natural frequencies of the two systems resulted in smaller failure probabilities. When both natural frequencies are uncertain, the failure probability either increases or decreases depending upon the threshold. When all system parameters are uncertain in either or both systems, the failure probability could either increase or decrease. This may be due to the effects of damping and tuning.

SECTION 5

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APPENDIX A
Mathematical Formulations

A.1 Solution of System of First Order Differential Equations

Consider the following system of size n of first order differential equations (ode)

$$\dot{\mathbf{V}} = [\mathbf{A}]\mathbf{V} + \mathbf{G}F_t \tag{A.1}$$

where $[\mathbf{A}]$ is a matrix of constants which is generally unsymmetrical, $\mathbf{G}$ is a vector of constants and F_t is a forcing function. Eqn. A.1 is generally coupled. The above equation can be uncoupled using the modal matrix of $[\mathbf{A}]$. Let λ_a, and $[\Phi_a]$ be the eigenvalues and matrix of eigenvectors of $[\mathbf{A}]$, respectively. These quantities are generally complex-valued. Since $[\mathbf{A}]$ is generally unsymmetrical, it can be shown that the eigenvectors are not orthogonal with respect to $[\mathbf{A}]$. Hence, Eqn. A.1 cannot be diagonalized by the usual modal transformation used in classically damped systems. However, the modal transformation can still be used if one is willing to work with the inverses of complex matrices. That is, let

$$\mathbf{V} = [\Phi_a]\mathbf{U} \tag{A.2}$$

then

$$[\Phi_a]\dot{\mathbf{U}} = [\mathbf{A}][\Phi_a]\mathbf{U} + \mathbf{G}F_t \tag{A.3}$$

Pre-multiply Eqn. A.3 with $[\Phi_a]^{-1}$ leads to

$$\dot{\mathbf{U}} = [\Lambda]\mathbf{U} + [\Phi_a]^{-1}\mathbf{G}F_t \tag{A.4}$$

where $[\Lambda]$ is a diagonal matrix whose entries are the eigenvalues of $[\mathbf{A}]$. Eqn. A.4 is uncoupled, however, the inverse of the modal matrix is required.

Alternatively, a canonical transformation can be used where partial decoupling takes place. An advantage of such a transformation is that one has only to invert a real matrix. The transformation matrix $[\mathbf{T}]$ is define, for example, as

$$[\mathbf{T}] = \left[\; Re(\mathbf{v}_1) \;\; Im(\mathbf{v}_1) \;\; Re(\mathbf{v}_3) \;\; Im(\mathbf{v}_3) \;\; \cdots \;\; \mathbf{v}_k \;\; \cdots \;\; Re(\mathbf{v}_{n-1}) \;\; Im(\mathbf{v}_{n-1}) \; \right] \tag{A.5}$$

where $Re()$ is the real part, $Im()$ is the imaginary part and $\mathbf{v}_i$ is the $i-th$ eigenvector of [**A**]. That is, if the first, third,..., and n-1 eigenvectors are complex, then the real and imaginary parts of the eigenvectors are used in the transformation matrix. Thus, if the k-th eigenvector is real, it is used in the transformation matrix. Note that in the above example of the transformation matrix [**T**], the second, fourth,..., and n-th eigenvectors are just the conjugate of the first, third,.., and n-1 eigenvectors respectively. The basic idea behind the above transformation is that a complex eigenvalue will invariably generate a pair of conjugate eigenvectors. Hence, one of these pair is used in the transformation matrix. If the above transformation is used on Eqn. A.1 and the subsequent equation is de-coupled by pre-multiplying with $[\mathbf{T}]^{-1}$, then the following equation will result

$$\dot{\mathbf{U}} = [\Lambda']\mathbf{U} + [\mathbf{T}]^{-1}\mathbf{G}F_t \tag{A.6}$$

where $[\Lambda']$ is generally block diagonal, i.e.,

$$[\Lambda'] = \begin{bmatrix}
Re(\lambda_1) & Im(\lambda_1) & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\
-Im(\lambda_1) & Re(\lambda_1) & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\
0 & 0 & Re(\lambda_3) & Im(\lambda_3) & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & -Im(\lambda_3) & Re(\lambda_3) & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \lambda_k & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & \cdots & \cdots & 0 & 0 & Re(\lambda_{n-1}) & Im(\lambda_{n-1}) \\
0 & 0 & \cdots & \cdots & \cdots & 0 & 0 & -Im(\lambda_{n-1}) & Re(\lambda_{n-1})
\end{bmatrix} \tag{A.7}$$

Hence, the modes which form conjugate pairs are separated from the other real or conjugate pairs. The solution of each of the separated pair is very easy. Consider the case where the mode is real, then

$$\dot{U}_k(t) = \lambda_k U_k(t) + c_k F_t \tag{A.8}$$

where c_k is the $k-$th constant of the vector $[\mathbf{T}]^{-1}\mathbf{G}$ of Eqn. A.6. Eqn. A.8 has the following solution

$$U_k(t) = U_k(0)\exp(\lambda_k t) + c_k \exp(\lambda_k t)\int_0^t \exp(-\lambda_k u)F_u du \tag{A.9}$$

where $U_k(0)$ is the initial condition. Consider now the conjugate pair, i.e., the $r-$th and $(r+1)-$th equations of Eqn. A.6, or the $l-$th pair of Eqn. A.6

$$\left\{ \begin{array}{c} \dot{U}_r \\ \dot{U}_{r+1} \end{array} \right\} = \left[\begin{array}{cc} a_j & b_j \\ -b_j & a_j \end{array} \right] \left\{ \begin{array}{c} U_r \\ U_{r+1} \end{array} \right\} + \left\{ \begin{array}{c} c_r \\ c_{r+1} \end{array} \right\} F_t \tag{A.10}$$

where Eqn. A.10 is related to the $j-$th mode the matrix [**A**], i.e.,

$$\lambda_j = a_j \pm i \mid b_j \mid \tag{A.11}$$

where λ_j is the $j-$th eigenvalue of [**A**]. Eqn. A.10 can be solved easily, using the modal decomposition. Note this will result in complex quantities, but it is only a 2-dimensional problem and can be solved easily in closed-form. The eigenvalues and eigenvectors of the coefficient matrix of Eqn. A.10 are

$$\tilde{\lambda}_m = \; a_j \; \pm \; i \mid b_j \mid \quad \text{for } m = r, r+1 \tag{A.12}$$

and

$$[\tilde{\Phi}] = \left[\begin{array}{cc} 1 & 1 \\ i & -i \end{array} \right] \tag{A.13}$$

Notice that

$$\tilde{\lambda}_m = \lambda_j \tag{A.14}$$

The inverse of the modal matrix is

$$[\tilde{\Phi}]^{-1} = \left[\begin{array}{cc} 0.5 & -0.5i \\ 0.5 & 0.5i \end{array} \right] \tag{A.15}$$

Hence, if the following transformation is used for Eqn. A.10

$$\left\{ \begin{array}{c} U_r(t) \\ U_{r+1}(t) \end{array} \right\} = [\tilde{\Phi}] \left\{ \begin{array}{c} Z_r(t) \\ Z_{r+1}(t) \end{array} \right\} \tag{A.16}$$

and performing the usual method of diagonalizing the coefficient matrix leads to

$$\dot{Z}_m(t) = \tilde{\lambda}_l Z_m(t) + D_l F_t \tag{A.17}$$

where $m = r, r+1$, and

$$\left\{ \begin{array}{c} D_r \\ D_{r+1} \end{array} \right\} = [\tilde{\Phi}]^{-1} \left\{ \begin{array}{c} c_r \\ c_{r+1} \end{array} \right\} \tag{A.18}$$

The solution to Eqn. A.17 is

$$Z_m(t) = Z_m(0)\exp(\tilde{\lambda}_m t) + D_m \exp(\tilde{\lambda}_m t)\int_0^t \exp(-\tilde{\lambda}_m u)F_u du \tag{A.19}$$

for $m = r, r+1$. In Eqn. A.19, the appropriate eigenvalues of [**A**] can be used instead, i.e., see Eqn. A.14. The solution to the real mode and the complex conjugate modes can be combined in a single transformation matrix. Consider the following matrix

$$[\mathbf{T_0}] = \begin{bmatrix} [\tilde{\Phi}_1] & \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\tilde{\Phi}_2] & \cdots & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 1.0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} & [\tilde{\Phi}_{n-1}] \end{bmatrix} \tag{A.20}$$

Let

$$\mathbf{U} = [\mathbf{T_0}]\mathbf{Z} \tag{A.21}$$

Substitute the above transformation into Eqn. A.6 and diagonalize the coefficient matrix with the inverse of [$\mathbf{T_0}$]. This will result in a set of decoupled first order differential equations, i.e.,

$$\dot{Z}_i(t) = \lambda_i Z_i(t) + D_i F_t \quad i = 1,2,..,n \tag{A.22}$$

where λ_i are the original eigenvalues of [**A**]. The inverse of [$\mathbf{T_0}$] is simply

$$[\mathbf{T_0}]^{-1} = \begin{bmatrix} [\tilde{\Phi}_1]^{-1} & \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\tilde{\Phi}_2]^{-1} & \cdots & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 1.0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} & [\tilde{\Phi}_n]^{-1} \end{bmatrix} \tag{A.23}$$

The system of equations given by Eqn. A.22 is simply those equations given by either Eqns. A.8 and A.17 for the real or complex case respectively. Hence, the transformation of Eqn. A.1 to a de-coupled set of equations is

$$\mathbf{V} = [\mathbf{T}][\mathbf{T_0}]\mathbf{Z} \tag{A.24}$$

A.2 Covariance Function

Consider the case where the forcing function is a modulated white noise process, i.e.,

$$F_t = a_t W_t \tag{A.25}$$

where a_t is a deterministic function of time, and W_t is a zero-mean white noise process. Consider two solutions of Eqn. A.22 with zero-initial conditions, i.e.,

$$Z_p(t) = D_p \exp(\lambda_p t) \int_0^t \exp(-\lambda_p u) a_u W_u du \tag{A.26}$$

and

$$Z_q(s) = D_q \exp(\lambda_q s) \int_0^s \exp(-\lambda_q u) a_u W_u du \tag{A.27}$$

In general λ and D is complex. Hence, complex algebra is required. The expected value of the product of $Z_{tp} Z_{sq}$ is given by

$$E[Z_p(t)\overline{Z_q(s)}] = D_p \overline{D_q} \exp(\lambda_p t + \overline{\lambda_q} s) \int_0^{min(t,s)} \exp\left[-(\lambda_p + \overline{\lambda_q})u\right] a_u^2 E(W_u^2) du \tag{A.28}$$

where the overbar means the conjugate. The conjugate of Eqn. A.27 is simply Eqn. A.27 replaced with the conjugate terms. The expectation on the right hand side is simply πG_0, where G_0 is the one-sided power spectral density (PSD) of the white noise process. Hence, Eqn. A.28 becomes

$$E[Z_p(t)\overline{Z_q(s)}] = \pi G_0 D_p \overline{D_q} \exp(\lambda_p t + \overline{\lambda_q} s) \int_0^{min(t,s)} \exp\left[-(\lambda_p + \overline{\lambda_q})u\right] a_u^2 du \tag{A.29}$$

The term outside the integral will simply be a complex number. Let it be B. Then, for $t < s$, Eqn. A.29 becomes

$$E[Z_p(t)\overline{Z_q(s)}] = B \int_0^t \exp\left[-(\lambda_p + \overline{\lambda_q})u\right] a_u^2 du \tag{A.30}$$

The solution of Eqn. A.30 depends only on the form of a_t. The exponent in the integral is generally complex. However, it can be decomposed into a real and imaginary part. This permits the use of arbitrary forms of a_t which in turn requires numerical integration schemes to evaluate the above integral.

A.2.1 Solution of the Integral

Consider the following form for a_t

$$a_t = a_1 t^k \exp(-a_2 t) \tag{A.31}$$

where $2k$ is zero or an integer and a and b are constants. Substitute Eqn. A.31 into Eqn. A.30 and considering only the integral, the following is obtained

$$I(t) = \int_{t_0}^{t} \exp\left[-(\lambda_p + \overline{\lambda_q})u\right] a_1^2 u^{2k} \exp(-2a_2 u) du \tag{A.32}$$

where the integral is generalized for an arbitrary lower limit. Simplifying,

$$I(t) = a_1^2 \int_{t_0}^{t} u^m \exp\left[-(\lambda_p + \overline{\lambda_q} + 2a_2)u\right] du \tag{A.33}$$

where $m = 2k$. Consider the following standard integral

$$I_1(t) = \int_{t_0}^{t} u^m \exp(-cu) du \tag{A.34}$$

The solution to Eqn. A.34 is

$$I_1(t) = \left[\frac{u^m \exp(-cu)}{-c}\right]_{t_0}^{t} - \left[\frac{m}{-c}\right] \int_{t_0}^{t} u^{m-1} \exp(-cu) du \tag{A.35}$$

or

$$I_1(t) = \left[\frac{u^m \exp(-cu)}{-c}\right]_{t_0}^{t} - \left[\frac{m}{-c}\right] \left\{ \left[\frac{u^{m-1} \exp(-cu)}{-c}\right]_{t_0}^{t} - \left[\frac{m-1}{-c}\right] \int_{t_0}^{t} u^{m-2} \exp(-cu) du \right\} \tag{A.36}$$

Hence, the integral is recursive and each succeeding integral has the same form as the first term. Evaluating the first term leads to

$$\frac{1}{c}\left[t_0^m \exp(-ct_0) - t^m \exp(-ct)\right] \tag{A.37}$$

Hence, for each succeeding integral, the form is as in Eqn. A.37 with the multiplier change and the powers of t and t_0 change.

For different values of a_1, k and a_2, a_t simplifies to other standard form often used as modulating functions, i.e.,

1. For $a_1 \neq 0$ and k and a_2 equal to 0, a step function is obtained.

2. For a_1 and a_2 not equal to zero and $k = 0$, the exponential function is obtained.

Modulating functions which are sums of exponentials, i.e.,

$$a_t = \sum_{i=1}^{j} a_{1i} \exp(-a_{2i} t) \tag{A.38}$$

can also be set up with the general a_t.

A.3 Moments of the Variables

From Eqn. A.24, the moments of the original variables can be written as follows

$$E[\mathbf{V}_t \overline{\mathbf{V}_s}^T] = [\mathbf{T}] \left\{ [\mathbf{T}_0] E[\mathbf{Z}_t \overline{\mathbf{Z}_s}^T] \overline{[\mathbf{T}_0]}^T \right\} [\mathbf{T}]^T$$

where the superscript T means transpose. Note that on right hand side of Eqn. A.25, other than $[\mathbf{T}]$, all other quantities are complex, hence the conjugate transpose must be used, i.e., the overbar in all the appropriate quantities.

APPENDIX B
First- and Second-Order Reliability Methods (FORM/SORM)

In these methods, the failure probability is estimated by approximating the failure surface either by a hyperplane or by a second-order surface. These approximate surfaces are such that one point on these surfaces lies on the true failure surface. The probability content of the approximate surface is then computed and taken as an estimate of the probability content of the true failure surface. A function describing the failure surface is also needed for these methods. This function is normally called the g-function. There are three basic steps involved in FORM/SORM [9].

1. Transformation of the basic random variables into unit independent normal random variables. The original space of the basic variables is usually called x-space and the space of unit independent normal variables is called u-space.

2. The failure surface is next approximated by a simple surface such as a hyperplane or a paraboloid. The expansion point for this approximation is the point on the failure surface which is nearest to the origin in u-space. The expansion point is also called the design point (or the beta point). This is shown in Fig. B.1

3. The probability content in the failure set defined by the approximate failure surface is computed and taken as an approximation of the true probability content. If the hyperplane is used then

$$P(g(\mathbf{X}) \leq 0) \approx \Phi(-\beta) \tag{B.1}$$

where $\Phi()$ is the cumulative distribution function of the unit standard normal distribution and β is the distance from the origin to the design point in u-space. (See Fig. B.1). This estimate is normally called the FORM estimate. If instead a second-order surface is used, then the estimate is called the SORM estimate. For a second-order surface the probability content can be approximated by the equations developed by Tvedt [9]. The true curvatures of the failure surface at the design point will have to be computed for the second order estimate.

In the above steps, a search for the design point must be conducted. A gradient-based algorithm is commonly used [11]. The design point is searched by successively linearizing the g-function at a sequence of points where each point represents a deterministic set of the random variables. Each linearization point is obtained from the previous point. And these points are computed using the slope (and hence gradients) of the g-function. Eventually, the sequence of points will converge to the design point.

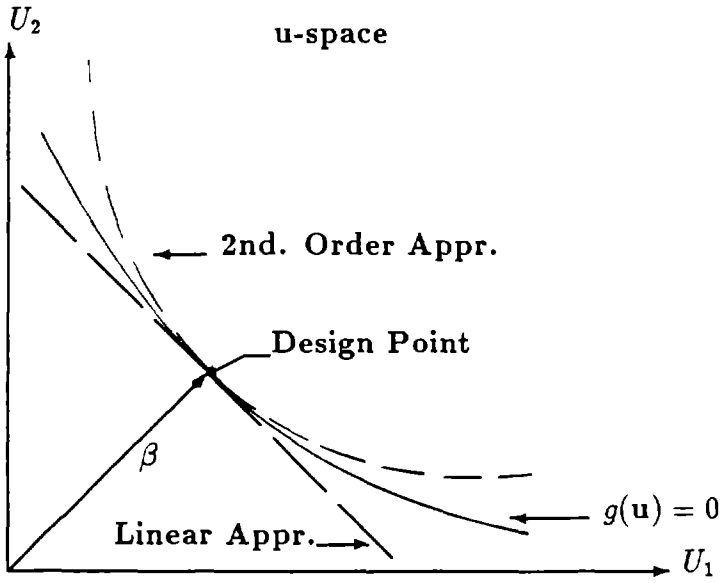

Figure B.1: FORM/SORM

APPENDIX C
Computer Code

C.1 Description of the Code

A computer code has been developed based on the proposed method. The program can be divided into 2 parts :

1. Primary-secondary system.

2. Input (or excitation) system comprised of the filter system and parameters that define the non-stationary Gaussian white noise process.

The items above constitute the input to the program. The code allows uncertainty in the system parameters. Currently, it handles only independent variables. The output of the code for uncertain systems is some desired failure probabilities. If the system is deterministic, the code will generate the desired statistics of responses of the primary-secondary system. A flow chart of the code is shown in Fig. C.1.

C.1.1 Input

The input to the program consists of the primary-secondary system and the excitation.

Primary System

The mass, damping and stiffness matrices of the primary system are required. If the system is proportionally damped, then the modal damping ratios are needed. For uncertain parameters, the mean, standard deviation and distribution type are required.

Secondary Systems

For each secondary system, the properties, i.e., mass, damping and stiffness characteristics, are required. Also, the attachment configuration is needed, i.e., the location of the masses of the secondary system to the primary system.

Excitation

There are two parts to the excitation. The first part is the filter system. The mass, damping and stiffness characteristics of the filter system are required. Also, the response of the filter system which is to be the input to the primary-secondary system must be specified.

The second part is the non-stationary zero-mean Gaussian white noise process. For this, the intensity of the one-sided power spectral density function of a stationary Gaussian white noise, G_0, and the modulation function, a_t, are required. Several standard modulating functions are available in the code. These include

1. A step function, $a_t = a_0$; $t > 0$.
2. Sums of exponentials, i.e., $a_t = \sum_{i=1}^{n} a_{1i} \exp(-a_{2i} t)$.
3. Of the form $a_t = a_1 t^k \exp(-a_2 t)$, where $2k$ is even.

C.1.2 Solution Technique

The solution technique adopted is shown in Fig. C.2. This technique is for a deterministic system. For uncertain systems, samples of the uncertain parameters will have to be generated first before attempting the following steps. The technique makes use of the solution of system of first order differential equation outlined in Appendix A. The technique is as follows :

1. The combined equations of motion for the primary-secondary system is obtained. The numbering system for the responses starts with the degrees of freedom associated the primary system. That is the response $X_1(t)$ to $X_{n_p}(t)$ are for the primary system. Responses from $X_{n_p+1}(t)$ onwards are for the secondary system(s). Examples of these can be found in Chapter 1.
2. The equations of motion are recast into the state space formulation. If a filter system exists, then the equations of motion of the filter system are also recast correspondingly. The resulting equation of motion is a system of first order differential equation such as

$$\dot{\mathbf{V}} = [\mathbf{A}]\mathbf{V} + \mathbf{G} a_t W_t \tag{C.1}$$

where [A] is a matrix of constants comprised of the mass, damping and stiffness matrices of the primary, secondary and filter system, G is a vector of constants and $a_t W_t$ is the non-stationary Gaussian input.

3. The above equation is decoupled according to the technique outlined in Appendix A. The eigenvalues and eigenvectors of the matrix [A] are first obtained. The transformation matrix [T] and $[\mathbf{T_0}]$ are set-up (Eqns. A.5 and A.20 respectively) and their respective inverses obtained. Note that the inverse of $[\mathbf{T_0}]$ can be obtained easily in closed-form, i.e., Eqns. A.15 and A.23. The product $[\mathbf{T_0}]^{-1}[\mathbf{T}]^{-1}\mathbf{G}$ is computed and the system of uncoupled equations are obtained, i.e., Eqn. A.22.

4. The covariance function for the uncoupled system of equations are then evaluated. For purposes of reliability estimations, only the expectation $E[\mathbf{Z}_t\mathbf{Z}_t^T]$ is required, where the superscript T means transpose. (See Eqn. A.30). Since the input is non-stationary the expected product is evaluated at discrete and equal intervals of time.

5. For each time point, the expected product in the original space, i.e., $E[\mathbf{V}_t\mathbf{V}_t^T]$ is obtained from $E[\mathbf{Z}_t\mathbf{Z}_t^T]$ by matrix multiplication, i.e., Eqn. A.39.

6. The second moment statistics obtained can then be used to evaluate the mean-crossing rate or the statistics of any other response process. The mean crossing rate is evaluated using Eqn. 3.21.

For reliability purposes, the code makes use of FORM/SORM. (See Appendix B). The search for the expansion point is essentially an optimization problem. That is,

$$\beta = \min\sqrt{\mathbf{u}^*\mathbf{u}^{*T}} \tag{C.2}$$

subject to the condition that $g(\mathbf{u}^*) = 0$, where $\mathbf{u}^*$ is the expansion point. The g-function used in the code is

$$g(\mathbf{u}, u_{n+1}) = u_{n+1} - \Phi^{-1}\left[p_f(\mathbf{T}^{-1}(\mathbf{u}))\right] \tag{C.3}$$

(See Section 3.2.2). In order to use the above g-function, the conditional failure probability must be computed. This can be done using the solution technique outlined above. Since the program is general any optimization routine can be used in conjunction with the part of the code that evaluates the conditional responses, i.e., the solution given above.

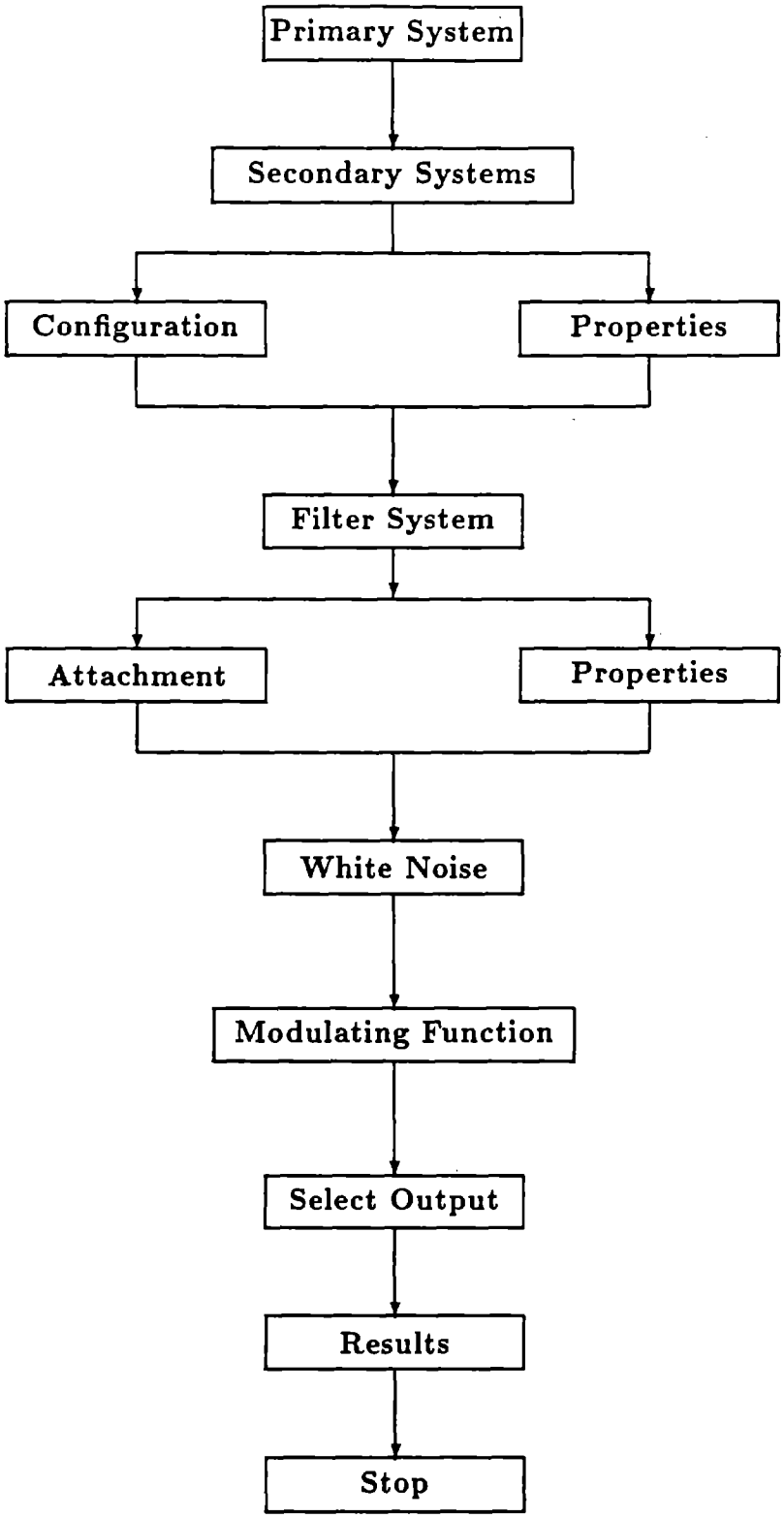

Fig. C.1 : Flow Chart of Computer Code

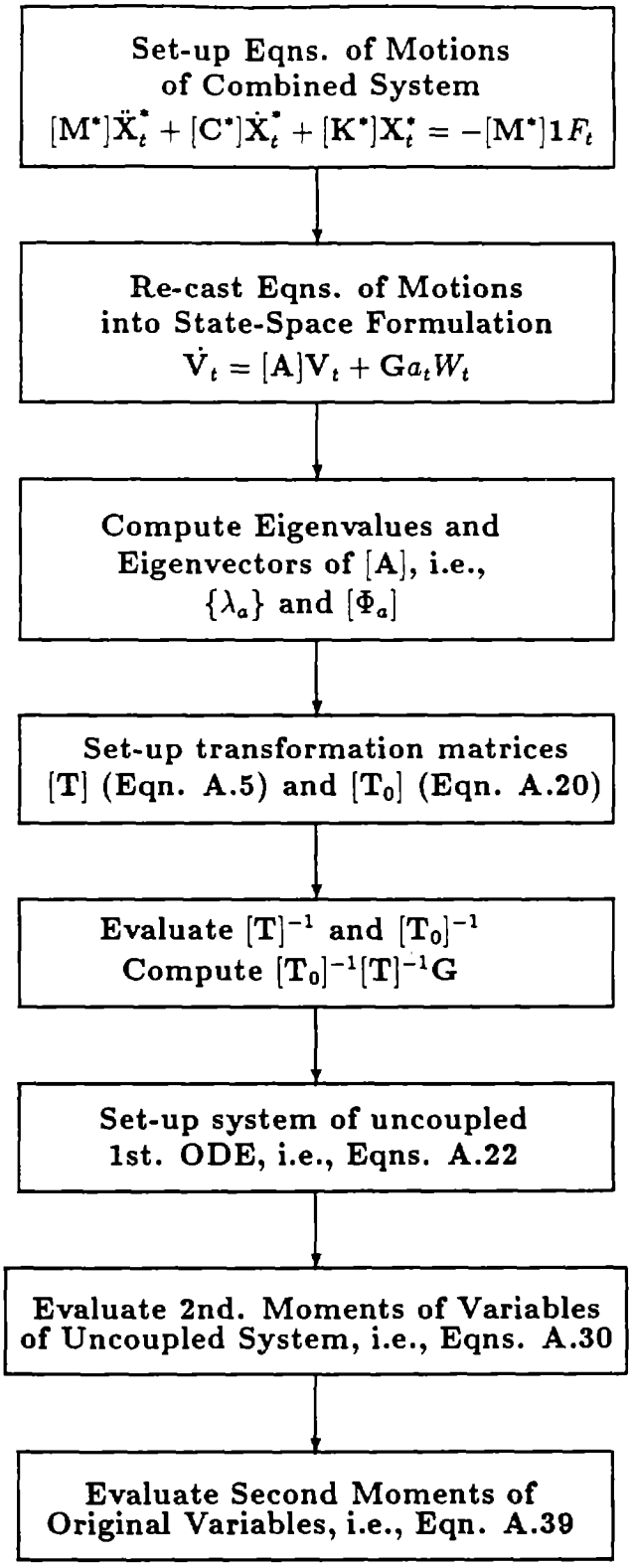

Fig. C.2 : Flow Chart of Solution Technique

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