SEISMIC BEHAVIOR AND RESPONSE SENSITIVITY OF SECONDARY STRUCTURAL SYSTEMS

by

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October 23, 1989

Technical Report NCEER-89-0030

NCEER Contract Numbers 87-2008 and 88-2005

NSF Master Contract Number ECE 86-07591

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

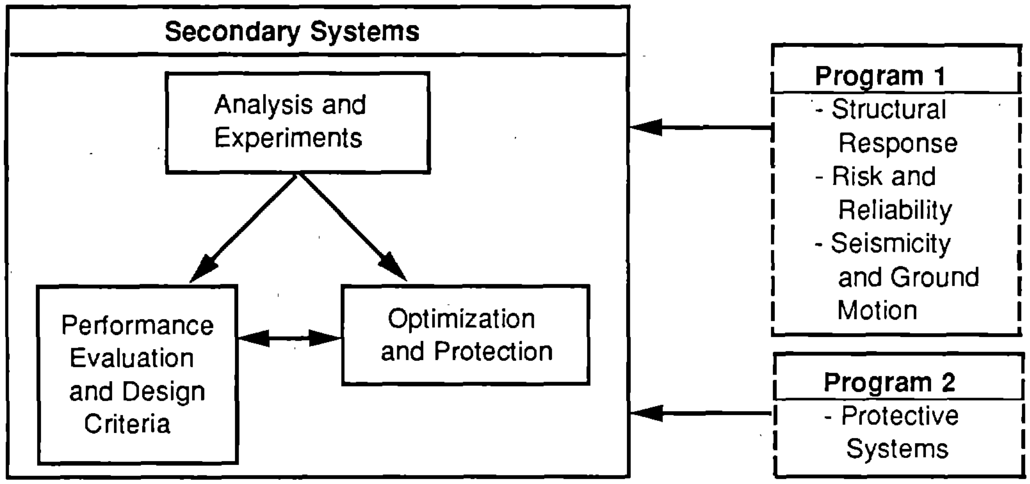
- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself. The research goals in this area are to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom, and nonlinearities in the primary structure.
2. Develop practical criteria and procedures for the analysis and design of secondary systems.
3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.



This report addresses several important issues in seismic design on secondary structural systems. First, some significant effects of primary-secondary system interaction on secondary system response are investigated following a stochastic sensitivity analysis. These effects include nonclassical damping, primary structural yielding, and primary structural parameter uncertainties. These effects are quantified and then translated into necessary modifications of the floor response spectra when they are used in the design procedure.

ABSTRACT

In earthquake engineering research, an area of increasing concern is performance of secondary systems which are anchored or attached to primary structures systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as the failure of the primary structure itself.

The research reported herein is focused on developing a greater understanding of the dynamic behavior of secondary systems under seismic loads. It summarizes: 1) a state-the-art review on seismic response of secondary systems; 2) stochastic response sensitivity analysis of secondary systems to parametric uncertainties in the primary structural systems; 3) the development of an approach for modifying floor response spectra widely used in current design; and 4) response prediction of secondary systems with primary systems exhibiting nonlinear behavior.

This report begins with a comprehensive review and assessment of the state-of-the-art in the area of research on seismic response of secondary systems (structural or non-structural). A sensitivity factor is then proposed based on the concept of spectral moments. It is shown that this factor can be used for a quantitative assessment of secondary system response when uncertainties exist not only in the excitation but also in the parameters of the primary structures, including parameters characterizing inelastic behavior. Furthermore, the analysis leads to a formulation in modifying the floor response spectrum. In this approach, based on the sensitivity analysis, the effect of interaction between the primary and secondary systems and that of nonclassical damping are accounted for. A general modification procedure, not only to determine the effect of structural uncertainties

on structural frequencies, but also on structural response magnitudes, is formulated and illustrated by numerical examples. It lends itself easily as a design tool.

Finally, the problem of inelastic behavior of primary structures and its effect on secondary system response is addressed from a quantitative point of view. Both the approximate analysis method developed here, which is capable of analyzing elasto-plastic shear beam systems, and time history integration method are used to analyze primary-secondary system response with various structural and ground motion parameters. In comparison of the floor response spectrum (FRS) based on yielding primary system with the FRS based on the original elastic system, three cases of amplification impact of nonlinearity in the primary system in terms of frequency shift and multi-support amplification are investigated. The application of these results to design is commented upon.

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SECTION 1

GENERAL INTRODUCTION

Considerable progress has been made over the last two decades in the seismic analysis of structural systems, resulting in substantial improvement in analysis, design and construction of buildings, bridges, dams, etc, under seismic excitations. More recently, an area of increasing concern has been seismic performance of secondary systems which are attached or anchored to primary structural systems. They can be broadly classified into the following categories.

Non-structural secondary systems. Computer systems, control systems, machinery, panels, storage tanks and heavy equipment are examples in this category. Performance integrity of these systems under seismic loads, transmitted through the primary structure system, is important since they serve a vital function and their failure may have far reaching ramifications.

Structural secondary systems. Examples of these systems include stairways, structural partitions, suspended ceilings, piping systems and ducts. For these systems, not only is their seismic behaviour of practical concern, but their interaction with the primary structural systems is also important since their presence is capable of modifying structural behaviour of the primary system to which they are attached. Thus, these primary-secondary interactions cannot be ignored in seismic analysis of either the secondary or the primary system.

While considerable analytical, numerical and experimental work on seismic performance evaluation of secondary systems has been conducted over the last decade, it has been difficult to generate a good understanding of the dynamic behaviour of secondary systems. The fact that their general behaviour is difficult to ascertain is basically due to several inherent dynamic characteristics of the combined primary-secondary systems. They can be summarized as follows:

(a)*Large number of degrees of freedom.* Both primary and secondary systems are multi-degree-of-freedom systems and the number of degrees of freedom of the combined system is in general prohibitively large. Moreover, the large differences in the stiffness, damping and mass terms between the primary and secondary systems pose serious numerical problems.

(b)*Tuning.* Resonance effects must be considered since any number of frequencies of the secondary system may be arbitrarily close to or coincide with the frequencies of the primary system. The presence of other additional secondary systems may cause additional tuning problems.

(c)*Attachment configuration.* Attachment configurations of the secondary systems vary and can be quite complex, causing difficulties in modelling of the combined system.

(d)*Non-classical damping and gyroscopic effects.* Non-classical damping occurs when different damping ratios exist in the primary and secondary systems and its effects are particularly significant at tuning. Moreover, when the secondary system has dynamics of its own, such as rotating machinery, it gives rise to gyroscopic effects.

(e)*Nonlinearity.* Structures are generally designed to dissipate some of the input energy during severe earthquake ground motion by means of inelastic deformation. Hence, seismic analysis of the combined primary-secondary system needs to be extended to the inelastic range.

(f)*Local vibration.* An equipment, for example, can be an oscillator attached at the middle of a floor beam of the primary structure, on which there may be several sets of equipment. The effects of the vibration of the beam and other equipment (Structural and nonstructural elements) may be significant.

The object of this research is to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities described above that exist in the underlying primary-secondary structural systems.

2. Develop practical criteria and procedures for the analysis and design of secondary systems.

In this report, a comprehensive review and assessment of the state-of-the-art in the area of research and practice on seismic response of secondary systems is presented first in **Section 2**. Included are an appraisal of current engineering practice and design, an update of recent advances, and a discussion of possible future research directions.

As indicated above, tuning is an important consideration in the analysis of secondary systems, which is characterized by large peak response values. Thus, for design purposes, sensitivity indices relating peak response of a secondary systems to primary structural parameter variations need to be developed. The stochastic response sensitivity of secondary systems to primary structural uncertainties is introduced in **Section 3**, where the use of spectral moments is suggested as peak response sensitivity indices which provide a quantitative measure of relative importance of parameter uncertainties in the design of secondary systems. The need for a more comprehensive sensitivity analysis is indicated. These results will be particularly useful to designers in order to evaluate relative importance of parameter uncertainties in the primary structure and to determine the desired dynamic characteristics of secondary systems.

One of the applications of the results reported in section 3 is to provide a methodology for modifying the floor response spectrum commonly used in the analysis and design of secondary systems. The proposed procedure, presented in **Section 4**, not only accounts for the effect of structural uncertainties on structural frequencies, but also on the structural response amplitude. In addition, the interaction effect between the primary and secondary systems and nonclassical damping are accounted for.

Under the action of severe earthquakes, engineering structures are expected to dissipate some of the input energy by means of inelastic deformation. The effect on secondary systems of this excursion into the nonlinear range on the part of the primary structure is thus of importance. Some work has been initiated in this area, in which the primary structure is modeled as a single-degree-of-freedom system and the mass ratio is assumed to be small so that a decoupled analysis can be used. It has been shown that the effect of inelastic behavior of the primary structure is a reduction of the secondary system response. While analytical studies of multi-degree-of-freedom systems have already indicated the erroneous nature of this conclusion, better understanding of this nonlinear effect is clearly needed under more general conditions. In **Section 5**, an approximate method of analysis capable of generating floor response spectra for elasto-plastic shear beam primary structures is developed. Both this method and time history integration are used to analyze primary-secondary systems with various structural and ground motion parameters.

SECTION 2

STATE-OF-THE-ART REVIEW

2.1 INTRODUCTION

As seismic safety and integrity of secondary systems become an increasingly important issue, it is instructive to provide first a critical evaluation of current engineering practice in response calculations and design of secondary systems under seismic conditions. This is the objective of this section. A review and appraisal of current practice are given in the next two sections. An assessment of more recent advances is then presented, followed by a discussion of possible future research directions.

2.2 CURRENT PRACTICE IN RESPONSE CALCULATIONS AND DESIGN

Two basic approaches currently exist which provide the basis for engineering analysis and response calculations for secondary systems. They are the conventional floor response spectrum (FRS) approach, on which some design codes are based [1,125,128] and the combined primary-secondary system approach.

2.2.1 Floor Response Spectrum Approach

A conventional method of analysis, in which the primary and secondary systems are decoupled and analysed individually, is the method of floor response spectrum. In this approach, the behaviour of the primary structural system at the support points of a secondary system is first determined while neglecting the effect of the secondary system. The response spectra at the support points, or the *floor response spectra*, are then used as input to the secondary system, from which its response behaviour is determined by using time domain analysis or by using one of several modal combination rules.

To obtain the floor response spectrum, horizontal and vertical time histories at support points of a secondary system are first calculated based upon the time domain analysis of the primary system. These time histories are then used to generate the required floor response spectrum for the secondary system analysis. Spectrum peaks are normally expected to occur at frequencies corresponding to the peaks of the ground motion spectrum and at the natural frequencies of the supporting structure. In cases involving equipment mounted on equipment, the frequencies of all supported structures are normally included [1,128].

Other methods of generating the floor response spectrum are acceptable in current practice when a good agreement can be established when compared with the time-domain analysis [1,125].

Spectrum peak broadening. For practical design purposes, broadening of the peaks of a floor response spectrum is one of the means of accounting for the effect of structural frequency variations resulting from possible uncertainties in the ground motion spectrum and in the material properties of the structure and soil. A suggested method for determining the amount of peak widening associated with the structural frequencies is described in the USNRC code [125], where sensitivities of the structural frequencies to each significant parameter, such as soil modulus or material density, are first performed. The total frequency variation is then determined by taking the square root of the sum of squares (SRSS) of a minimum variation and the individual frequency variations. The amount of spectrum peak broadening is generally based on engineering judgement. However, USNRC

Regulating Guide [128] recommends that the peak be broadened by 15% if the sensitivity study accounting for soil-structure interaction if the calculations are not performed.

Combined spectrum and spectrum envelope. When a two- or three-dimensional analysis is performed, the motion of the primary structure at a given location may have contributions from both vertical and horizontal excitations. The combination from each individual analysis will generate a response spectrum at the same location and in the same direction. In these cases, the ordinates of these individual response spectrum can be combined according to the SRSS criterion to predict the total response spectrum at a given location and for a given direction [1,4,125].

In cases where a secondary system is supported by the primary structure at several locations, an upper-bound envelope of the individual response spectrum at these support points can be developed. It can be used to calculate a conservative maximum response of a multi-supported secondary system [4].

2.2.2 Combined Primary-Secondary System(P-S system) Approach

While the method of floor response spectra provides a relatively simple procedure for response calculations for secondary systems, the use of this approach leads to a number of deficiencies. The most serious is the fact that it ignores the interaction between primary and secondary systems. The floor response spectrum method gives acceptable results for secondary systems with relatively small masses and with frequencies which are not tuned to a frequency of the primary structural system. When the masses of the secondary systems can not be ignored or when the two systems are tuned to each other, however, a gross error in estimation of the secondary system behaviour can result [88,163].

This deficiency can be overcome by performing a coupled analysis in which the secondary systems are considered as an integral part of the primary-secondary structural system. Both modal analysis and time history integration method can be used for this purpose. More recently, a number of improved or more efficient methods of the coupled analysis have been developed. In [76, 79, 148] perturbation methods are used by treating the parameters of secondary systems as small parameters, leading to better accuracies in analysing tuned multi-degree-of-freedom systems. Modal analysis of the P-S system has also been developed [35, 37, 201] in which the mode shapes and frequencies of the combined system are found by using perturbation techniques and the modal responses are subsequently combined using a modal combination rule. A explicit comparison between FRS and combine analysis approaches is given in Table 2.1.

2.2.3 Criteria Pertaining to Decoupled and Coupled Analyses

Both the floor response spectrum (decoupled) and the combined P-S system analysis (coupled) approaches are currently used in response calculations and design practice for secondary systems. Depending on the significance of dynamic interaction between primary and secondary systems, a design criterion is currently employed in engineering practice in deciding whether the decoupled or the coupled approach is required [7,125].

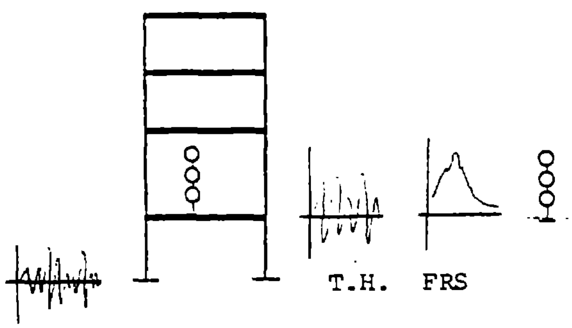
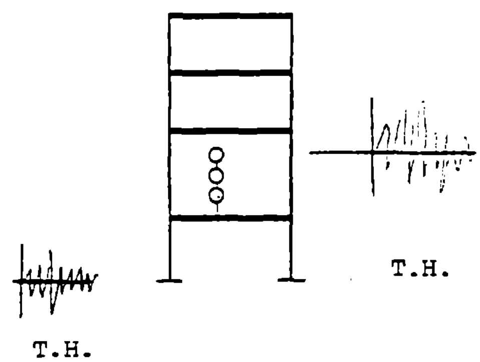
Let the mass ratio R_m be defined by

$$R_m = m_s/m_p \qquad (2-1)$$

where m_s is the total mass of the secondary system and m_p is the total modal mass of the primary structure associated with the dominant frequencies, and let R_ω denote the frequency ratio, i.e.,

$$R_\omega = \omega_s/\omega_p \qquad (2-2)$$

Table 2.1 *Comparison between FRS and combine analysis approaches*

Floor response spectrum (FRS)	Combine analysis
 Time History (T.H.)	 or Modal Analysis
Decoupled analysis	Coupled analysis
Neglect Interaction	Account for interaction
Approximate	Exact
Convenient	Inconvenient
Computationally Simple	Computationally Complex

where ω_s is the fundamental frequency(ies) of the secondary system and ω_p the dominant frequency(ies) of the primary structure. Then, as indicated earlier, greater primary-secondary system interaction is expected as R_m increases and as R_ω approaches one. Thus, regions of validity of decoupled and coupled analysis can be determined according to the values of R_m and R_ω. A typical base for a design rule is indicated in Figure 2-1, which results from a two-degree-of freedom system analysis, i.e., a single-degree-of-freedom secondary system mounted on a single-degree-of-freedom primary structure (SDOF-SDOF P-S system) [51]. In Figure 2-1, R is defined by

$$R = |\omega_c - \omega_p|/\omega_p \qquad (2-3)$$

where ω_c is the natural frequency of the combined system [1,125].

Optional code requirements include torsional and rocking analysis as well as nonlinear analysis of the P-S system[1].

2.3 AN APPRAISAL OF CURRENT ENGINEERING PRACTICE

The floor response spectrum approach is simple and familar to designers, and it allows the analyst to study the secondary system independent of the primary system characteristics. In comparison with the combined P-S system analysis, this approach not only leads to substantial savings in computational costs but also avoids numerical difficulties that could arise in the analysis of the combined system due to large differences that exist between properties of the two systems.

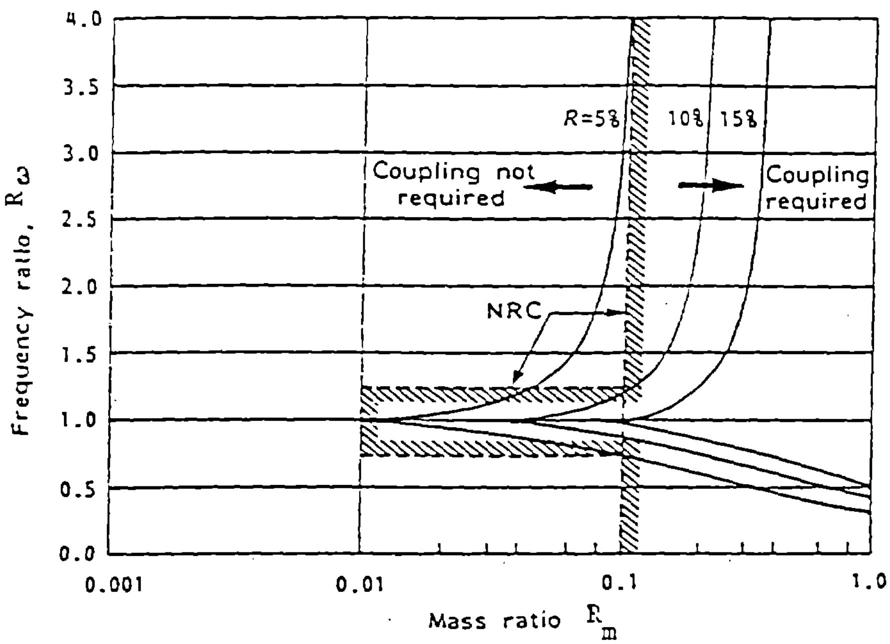


Fig. 2-1 Frequency Error Region

However, as currently applied, the floor response spectrum approach has several serious shortcomings. They include the following:

(a) As indicated earlier, the interaction between the primary and secondary systems is ignored. Important effects such as non-classical damping of the combined system and interaction among closely spaced modes of the P-S system are often neglected or improperly considered.

(b) Cross-correlations between support excitations of the secondary system are improperly considered or neglected

(c) The response is artificially separated into 'pseudostatic' and 'dynamic' parts, leading to difficulties in developing proper modal combination rules.

(d) It is difficult to consider the effect of structural torsion and inelastic deformation problem.

On the other hand, while the combined P-S system analysis should result in an exact response determination, it also gives rise to a number of difficulties as indicated below.

(a) A primary-secondary combination generally results in a system with an excessive number of degrees of freedom.

(b) When large differences exist between properties of primary and secondary systems, such characteristics usually render conventional methods of analysis expensive, inaccurate or inefficient.

(c) In the combined analysis, any secondary system modification necessitates a recalculation involving both primary and secondary systems, a tedious process when only secondary systems are of interest.

In view of the above, it is instructive to address quantitatively various effects that may render the floor response spectrum approach ineffective and indicate areas in which a combined P-S system analysis may become necessary. Clearly, clear-cut answers in this direction are not always possible, but a discussion of general trends can be attempted.

2.3.1 Interaction between Primary and Secondary Systems

Errors in response characteristics and in eigenproperties of secondary systems caused by neglecting primary-secondary system interactions have been studied extensively [55,64, 136]. These investigations clearly indicate that these errors are sensitive to the mass ratio R_m and the frequency ratio R_ω, which can be more than 100% when R_m becomes large and R_ω approaches one. This serious degradation effect can be demonstrated by considering a simple single-degree-of-freedom primary structure subjected to El Centro earthquake excitation. The resulting floor acceleration response spectra obtained from the coupled and decoupled analyses are shown in Figure 2-2, which shows that, when the secondary system is tuned to the primary system, spectrum amplitude error can exceed 100%. This observation is consistent with conclusions reached in studies cited above. In Figure 2-2, T_s and T_p are the periods of secondary system and primary system, respectively.

The effect of cross-correlation between closely-spaced modes of the combined system has also been studied [210], including suggested modification of square root of sum of squares (SRSS) criteria for modal combination [56].

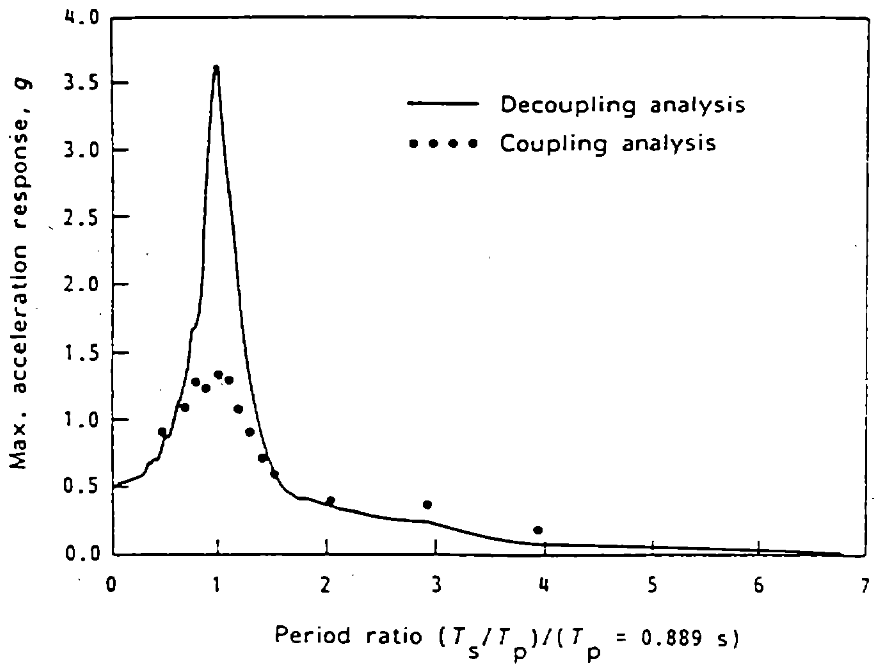


Fig. 2-2 Floor Acceleration Response Spectrum

2.3.2 Non-classical Damping

In general, a primary-secondary structural system is not classically damped due to differences that exist between the primary and secondary damping characteristics. Hence, the damping matrix of the combined system cannot be diagonalized by the eigenvectors of the undamped system. The use of approximate classically damped solution by neglecting the off-diagonal terms of the resulting damping matrix has been shown to lead to significant errors when the damping ratio $R_c = c_s/c_p$ is small, where c_s and c_p denote, respectively, damping constants of the secondary and primary systems. Quantitatively, this error is shown in Table 2.2 for the simple case of a two-degree-of-freedom system, which can be considered as a SDOF-SDOF P-S system with two types of excitations $p_1(t)$ and $p_2(t)$ [207]. The spring constants are taken to be

$$k_s = 0.2382kN/m, \qquad k_p = 104.88kN/m$$

and the masses are

$$m_s = 2.271kg, \qquad m_p = 1000kg$$

It is noted that the resulting errors are significant only when $1/R_c$ is less than 10^{-3} in this case. In [207], conditions are proposed under which the approximate diagonalization procedure can be used without causing serious errors in major response quantities of the secondary system.

It has also been pointed out that the effect of non-classical damping can be significant for tuned secondary systems [59,76]. It is, however, generally difficult to separate the effect of non-classical damping from other primary-secondary system interaction effects. As an example, consider frequency calculations for a SDOF-SDOF P-S system with damping factors $\zeta_p = 0.05$ for the primary system and $\zeta_s = 0.002$ for the secondary system. Both

Table 2.2 *Response of a Two-degree-of-freedom P-S System (from [207], table II)*

c_s (Ns/m)	$1/R_m$ (c_p/c_s)	$p_1(t) = 0.1 sin\omega t(N)$ Max.% error	$p_2(t)$ Step function Max. % error
0.1	0	0.1	0.2
	0.01	0.1	0.1
	0.0025	0.04	0
	0.001	1.1	0.2
	0.00067	3.0	0.4
0.3	0	0.8	0.5
	0.005	1.2	0.2
	0.0025	1.2	0
	0.0017	1.7	0.2
	0.00125	4.5	0.4
1.0	0	8.2	1.7
	0.0033	6.9	0.5
	0.0025	2.1	0.1
	0.002	3.2	0.3
	0.001	26	2.0

undamped frequencies and non-classically damped frequencies are shown in Table 2.3. While the error in ω_s is somewhat greater under tuned conditions, it is not significant when one considers inherent uncertainties that exist in estimating damping characteristics of a structural system.

A more comprehensive parametric study of a single-degree-of-freedom secondary system mounted on a classically damped multi-degree-of-freedom primary structure is made in [213]. Based on both deterministic and stochastic earthquake ground motion inputs, the following conclusions were reached.

(a) The effect of non-classical damping on the secondary system response is significant when the following conditions are satisfied simultaneously: (i) the secondary system frequency is tuned to a primary system frequency, (ii) the mass ratio R_m is small, and (iii)the damping factor ζ_s of the secondary system is smaller than the damping factor ζ_{sc} (unique damping ratio of the equipment) that would result in a classical damped primary-secondary system. Under these conditions, the response solutions using classical damping approximations are usually not conservative.

(b) The effect of non-classical damping on the secondary system response is negligible when it is detuned at low frequency. However, it can be significant under the following conditions:(i) the secondary system is detuned at high frequency, (ii) the mass ratio R_m is small, (iii) the damping factor ζ_{sc} is high, and (iv) the ratio of ζ_s and ζ_{sc} is smaller than unity. Non-conservative results again can be expected under these conditions when the effect of non-classical damping is neglected.

Table 2.3 *Frequency Comparison between Undamped and Nonclassically Damped Systems (R_m =0.1)*

case	R_ω	Freq. No.	Undamped Freq.,ω_1(Hz)	Nonc. Damp. Freq.,ω_2(Hz)	$[\lvert\omega_1 - \omega_2\rvert/\omega_1]\%$
1	0.632	1	0.6900	0.6901	0.014
		2	1.1609	1.1594	0.130
2	0.775	1	0.8207	0.8213	0.076
		2	1.1953	1.1594	0.155
3	0.894	1	0.9080	0.9092	0.134
		2	1.2478	1.2451	0.213
4	1.0	1	0.9615	0.9628	0.132
		2	1.3173	1.3146	0.21
5	1.095	1	0.9929	0.9939	0.1
		2	1.397	1.3949	0.15
6	1.183	1	1.012	1.0125	0.052
		2	1.4808	1.4789	0.125
7	1.265	1	1.0242	1.0244	0.019
		2	1.564	1.5626	0.093

2.3.3 Cross-correlation between Support Excitations

When a secondary system is supported at several locations, support excitations are generally correlated. Response calculations following code procedures in which the effect of this correlation is neglected or the conservative upper-bound envelope response spectrum is used can incur significant errors. A quantitative study is given in [206], showing conditions under which the effect of this cross-correlation between support excitations can become significant.

Other effects, such as those due to torsional motion or nonlinearity in the primary system, can also be significant. However, studies that have been made in these areas do not appear to be adequate for a quantitative assessment.

2.4 RECENT ADVANCES

In view of the preceding discussions, it appears that a desirable approach to secondary response analysis would be one having the accuracy given by the combined P-S system analysis, but the expediency offered by the floor response spectrum approach. Indeed, much of the recent work falls into this category. In addition, some of the P-S system interaction effects on the performance of secondary systems need to be better understood. In what follows, these recent activities are summarized.

2.4.1 Floor Response Spectrum Approach

Recent attempts at improving the floor response spectrum approach include finding more efficient procedures for generating the desired spectrum while including in the spectrum some of the P-S system effects.

In the development of more direct methods of generating the floor response spectrum, a modal analysis approach is used in [19]. By using modal properties of the primary system and the ground response spectrum, floor response spectrum curves can be obtained directly from a prescribed base response spectrum. An alternative approach based on Fourier transforms has also been developed.

A random vibration analysis approach is described in [163,165,199]. For a multi-degree-of-freedom(MDOF) system subjected to a stationary random excitation, it is well known that power spectral density functions of various floors of a structural system can be found directly from that of the ground motion accelerogram and the knowledge of primary system dynamic properties. As an input, these floor power spectral densities can be used to generate the desired floor response spectrum approximately. The results obtained following this approach compare well with the time history results, especially after improvements[164,165]. A similar approach is taken in [198], but utilizing evolutionary power spectra.

Interaction and non-classical damping. Floor response spectrum including the primary-secondary system interaction effect can in principle be extracted from analyses involving a combined system analysis, e.g., using perturbation techniques or simplified modal synthesis [60,80,83,167,184]. If the combined system dynamics has taken into account the effect of non-classical damping, it will also be included in the resulting floor response spectra [59,60,76,166,169,171,185].

Cross-correlation. Currently, a major approach to accounting for the effect of cross-correlation between multiple-supported excitations is to generate cross-correlation spectra through random vibration analysis [6,151]. As a result, a new cross-correlated secondary system response can be found which takes into account the effect of cross-correlation between multiple-supported excitations.

Inelastic response spectrum. Some code provisions for taking into account inelastic primary system behaviour exist in current practice (for example, [1]), but a majority of them are only suitable for small nonlinearities and for specific structural types. In the study of secondary system response behaviour, the determination of inelastic response has been made by using equivalent linearization and time history integration techniques. In [91], for example, inelastic floor response spectra are generated following a time history analysis where the types of hysteresis curves included origin-oriented type, degrading trilinear type and slip type. Other investigations in this area include [201], in which a simplified procedure is introduced, from which corresponding inelastic floor response spectra can be generated.

Response sensitivity to uncertainties. The sensitivity of secondary system response to uncertainties in structural modelling and in parameter values of the primary system has also received some attention [126,179,215]. The uncertain tuning parameter is studied in [81]. In reference [126], the response uncertainty of secondary systems is characterized by a dimensionless K factor, which represents the ratio of actual seismic response to predicted seismic response. The K factor is treated as a random variable in order to define degree of conservatism, separation of random and modelling uncertainty, and nonlinear effects. It may be represented by a product of the form

$$K = K_1 K_2 K_3 \dots K_n \qquad (2-4)$$

where each K_j characterizes a certain type of uncertainty. Included in this consideration can be uncertainties in mass, stiffness and damping magnitudes and distributions, discretization, structural modelling, geometric and material nonlinearities, decoupling and boundary conditions, design errors, and structural degrading effects.

2.4.2 Combined Primary-secondary System Approach

Computational and numerical difficulties associated with the combined P-S system analysis as outlined in the preceding section have spurred the development of improved and more efficient methodology, with major activities in the areas described below.

Complex modal analysis and synthesis. When non-classical damping exists, it has been pointed out that the damping matrix cannot be diagonalized by the eigenvectors of the undamped system. An alternative modal analysis approach is to decouple the equations of motion through the use of eigenvectors of the damped system. Since the damped eigenvectors are complex valued, the resulting decoupled equations contain complex parameters. The application of this procedure to the treatment of non-classically damped P-S systems can be found in [60, 77], where response characteristics of the secondary system can be determined through numerical integration procedure.

More recently, an alternative modal decomposition approach employing canonical transformation is proposed [213]. The resulting decoupled equations following this procedure contain only real parameters, thus avoiding computations to be made in the complex field.

Perturbation analysis. As mentioned earlier, perturbation techniques can be successfully applied to the analysis of the combined P-S system when the mass, stiffness and damping terms of the secondary system can be considered small as compared to those of the primary system [76,79,148]. The procedure allows a direct determination of all dynamic properties of the P-S system in terms of the primary and secondary system characteristics. From these dynamic properties, the secondary system response can be obtained by using modal combination or other dynamic analysis approaches. The effect of cross-correlation for multiple-supported secondary systems can be included as well as multiple effects involving tuning and non-classical damping [79,86].

Substructuring. To overcome numerical difficulties associated with the analysis of large- dimensional P-S systems, substructuring or structural partitioning has received considerable attention in recent years. Substructuring refers to the division of a complete structure into a number of substructures whose boundaries may be arbitrarily specified. It is preferable, however, to make the structural partitioning corresponding to physical partitioning. If the stiffness or flexibility properties of each secondary system can be determined, then each can be treated as a complex structural element. A finite element or other suitable numerical methods can then be written for the partitioned structure. Once the displacements and/or forces on the boundaries of a substructure are found, then each substructure can be analysed separately under known boundary conditions.

There are basically two avenues open for implementation of substructuring in dynamic studies. The first is essentially an extension of static concepts with the addition of inertia and damping terms to the equations of equilibrium, while the second manipulates the eigenvalue-eigenvector representation of a given dynamic system. A more general way is the component mode synthesis. A number of variations on the component mode synthesis technique exist [12,17,33], most of which are modifications of the original method of Hurty [74]. The component mode synthesis can be extended to damped structures [87,95] and to forced vibrations. It is also possible to include rigid-body modes in the formulation [98,129]. If attachment modes are employed in the component mode synthesis method, the question of linear independence arises. This is not a problem when fixed-boundary normal modes or constraint modes are used, for they form a linearly independent vector space. This last problem is alleviated through the use of residual attachment modes when there is no rigid-body motion, and through residual inertia relief attachment modes when rigid body modes are present [34,143].

Recent research in the component mode synthesis method has focused on improving the accuracy [3,102,109,186] as well as efficiency of the methodology, on investigating normal truncation [103,191], and on implementing the technique in general-purpose finite-element programs [15,182]. Furthermore, error analyses have been done [118,134] and attempts have been made to extend the method to systems with non-classical damping and complex eigenvalues and eigenvectors[192].

Other considerations. Other investigations dealing with more specialized problems include the study of response of light equipment mounted on based-isolated structures [94,195,212]. Both analytical and experimental results show that the use of the base isolation not only can attenuate primary system response, but also can reduce the response of secondary systems. Proof tests using full-scale structural models of base-isolated buildings for the purpose of protecting facilities and equipment have been made [18].

Seismic response of equipment located within an asymmetric primary structure subject to lateral and rotational base motion is studied in [85]. In addition, a coupled lateral-torsional floor response spectrum was generated and used to estimate torsional effects on the response of secondary systems. The effect of local vibration of structural and nonstructural elements is discussed in [40]. Optimum design of primary-secondary systems has been presented in [124].

2.4.3 Criteria Pertaining to Decoupled and Coupled Analyses

Coupled with recent advances in the response determination of secondary systems, new criteria have also been proposed in assessing the significance of primary-secondary system interaction. In [55], derivations are given from which P-S system interactions involving multi-degree-of-freedom systems and under other more general conditions can be assessed. A refined criterion based on the perturbation approach is also given in [80].

2.4.4 Experimental Work

Two types of experimental work have been reported. Some tests were performed on the integrated primary-secondary systems and others were performed on the secondary systems themselves.

Full scale testing and model testing of primary-secondary systems in the laboratory have not been as extensive as analytical and numerical work summarized above. A one-half scale piping system model mounted on a three-story steel frame has been tested on a shaking table [151,183]. The performance of specially designed ductile energy absorbers was studied together with conventional hydraulic snubbers to resist the combined thermal and seismic loadings. More than one hundred test runs were made, and time histories of piping response and hysteretic behaviour of energy absorbers were obtained under various two-component excitations (horizontal and vertical). Throughout the tests, the primary structure remained in the elastic range. although both the operating and safe shutdown earthquakes were considered. Light equipment, modelled as a simple cantilever and mounted on the same steel frame was also tested [41,93]. The application of pseudo-dynamic testing techniques to secondary systems was attempted [41,131]. In both studies, only secondary systems were tested physically while the primary structure was replaced by a mathematical model using substructuring methods. The results have shown that the proposed experimental method is viable and economical for testing light secondary systems mounted on an elastic structure. When the interaction between the secondary and primary systems as well as inelastic response of the primary structure is involved, a verification testing may be required. In [109] and [110], a good correlation between experimental evidence and theoretical predictions in using substructuring method is shown.

Full-scale testings of components and equipment of nuclear power plants were conducted by using a large shaking table [129]. The test included the primary loop recirculating system (pipeline, pumps, valves and motor), reactor core internals (reactor vessel, fuel assemblies, core barrel and support structure), and primary coolant systems (piping, pump and hydraulic snubbers). All the test results were compared with computer simulations and good correlations were reported. These results provide invaluable information regarding the dynamic behaviour of secondary systems as they are the only experimental work using full-scale models. Several shaking table tests of piping systems have also been reported (see, for example, [130, 156]).

Electric equipment, in particular that used in the nuclear industry, requires a thorough investigation under severe vibratory conditions. Usually the equipment is isolated from its environment and tested under severe conditions on shaking tables [75]. A 500 kv gas circuit breaker has been tested on an earthquake simulator [44] without evaluating the interaction of the breaker with the foundation.

The results of a study of seismic fragility of nuclear power plant equipment are reported in [13]. In the first phase of this study, existing test data were collected and evaluated, followed by the development of a methodology for establishing seismic fragility levels in terms of test response spectra. In the second phase, additional test data were collected and analysed for a group of electrical equipment.

Owing to scarce availability of large shaking tables, only a limited number of experiments have been performed on secondary systems exclusively. Field testing of secondary systems [159] and damage inspections after major earthquakes [189] have provided some information in this regard.

The dynamics of various cladding and connection has been investigated experimentally [36,50,133]. The influence of windows on structures, and of structural vibrations on the glass has been investigated, where flexible framing techniques were suggested. The influence of cladding , in particular precast panels, has been investigated experimentally on a 25-story building under construction [49,131]. As a result, an elastic-plastic connection possessing stable hysteretic response was proposed to reduce the overall response of the structure

2.5 CONCLUDING REMARKS AND POSSIBLE FUTURE RESEARCH DIRECTIONS

2.5.1 Analysis

As summarized in the preceding section, significant progress has been made over the last few years, leading to a better understanding of the dynamic behaviour of secondary systems. A major thrust has been to develop more rigorous methods that can account for more realistically the dynamic environment in which secondary systems operate. As work progresses, it appears that future research in this area having greatest impact potential falls into the following areas.

From the point of view of engineering practice and design, improvements on the recently developed methodologies are needed in several problem areas. They include the following:

Effect of nonlinear primary structural behaviour. Under the action of severe earthquakes, engineering structures are expected to dissipate some of the input energy by means of inelastic deformation. The effect on secondary systems of this excursion into the nonlinear range on the part of the primary structure is thus of importance. As indicated earlier, some work has been initiated in this area. In [105], for example, the primary structure is modelled as a SDOF system and the mass ratio R_m is assumed to be small so that the decoupled analysis can be used. It is shown that the effect of inelastic behaviour of the primary structure is a reduction of the secondary system response in most cases.

Better understanding of this nonlinear effect is clearly needed under more general conditions. For example, similar studies must be performed on MDOF P-S systems and on MDOF secondary systems.

Effect of uncertainties in primary structural parameters. As stressed in the preceding sections, tuning is an important consideration in the analysis of secondary systems, which is characterized by large peak response values. Thus, for design purposes, sensitivity indices relating peak response of a secondary system to primary structural parameter variations need to be developed. Work is progressing in this area [179,215]. In [179], for example , the use of spectral moments are suggested as peak response sensitivity indices, which provide a quantitative measure of relative importance of parameter uncertainties in the design of secondary systems.

The need for a more comprehensive sensitivity analysis is indicated. These results will be particularly useful to designers in order to evaluate relative importance of parameter uncertainties in the primary structure and to determine the desired dynamic characteristics of secondary systems.

2.5.2 Optimization and Protection

Mitigation of potential seismic damage to secondary systems can be achieved in several ways. The most direct route is to consider enhancing their performance through optimization in their placement within a primary structure or in innovative design of their supports. Preliminary results show that, for example, judicious placement of a secondary system not only can enhance its own response characteristics, but also benefit the overall P-S structural system. A systematic study of these possible optimization schemes does not exist now and is clearly needed.

As pointed out earlier, base isolation of the primary structure has been considered as a means of protecting secondary systems. More work is warranted in the study of potential applicability of active as well as passive control devices. While considerations of passive and active control have been mainly directed to primary structures, the protection of secondary systems using similar devices at the substructure level also merits serious consideration.

2.5.3 Code and Standard

The ultimate impact of new methodologies and approaches rests with their usage by the design industry. While considerable advances in secondary system analysis have been made, crude guidelines such as the use of amplification factors are still being practised in various building codes. Similar crude guidelines are being proposed for the revised ASME code related to nuclear power plants. A challenging task for researchers in this area is thus the development of accurate yet simple response calculation procedures, which can be incorporated into codes and standards.

2.5.4 Experimental Work

As indicated in the preceding section, experimental work on secondary systems has been fragmentary and not as extensive as analytical work. Better understanding of various factors entering the design and analysis of secondary systems must be gained through experimental investigation in the laboratory and in the field. Hence, systematic experimental work focusing on the dynamics of secondary systems together with various methods of optimization and protection must be considered as one of the important research tasks in this important area of investigation.

SECTION 3

STOCHASTIC RESPONSE SENSITIVITY OF SECONDARY SYSTEMS TO PRIMARY STRUCTURAL UNCERTAINTIES

3.1 INTRODUCTION

Response and eigenvalue sensitivities of a dynamic system to variations of its parameter values are one of the basic problems in dynamic analysis of structural systems. The problem is important because, on the one hand, the parameters of a real structure cannot be identified exactly and the degree of inaccuracy in the response calculation caused by uncertainties in parameter values is of practical importance. On the other hand, deterioration and degradation take place as structures age, resulting in changes in parameter values. This sensitivity problem is particularly important when the response of secondary systems is considered. For example, parameter uncertainties in the primary structural system can cause tuning in the primary-secondary system, which is characterized by large peak values in the secondary system response [27,72,215]. Response and eigenvalue sensitivities are also useful in determining the degree of broadening of the floor response spectrum.

Sensitivity analysis for deterministic systems has been well developed (see, for example, [45]). More recently, attention has been given to systems under random excitations or with random parameter variations. In [175], for example, an integral measure and a supremum measure are proposed as output sensitivity factors for stochastic systems. The analyses of deterministic and stochastic structures are generally carried out following the flow charts given in Fig. 3-1.

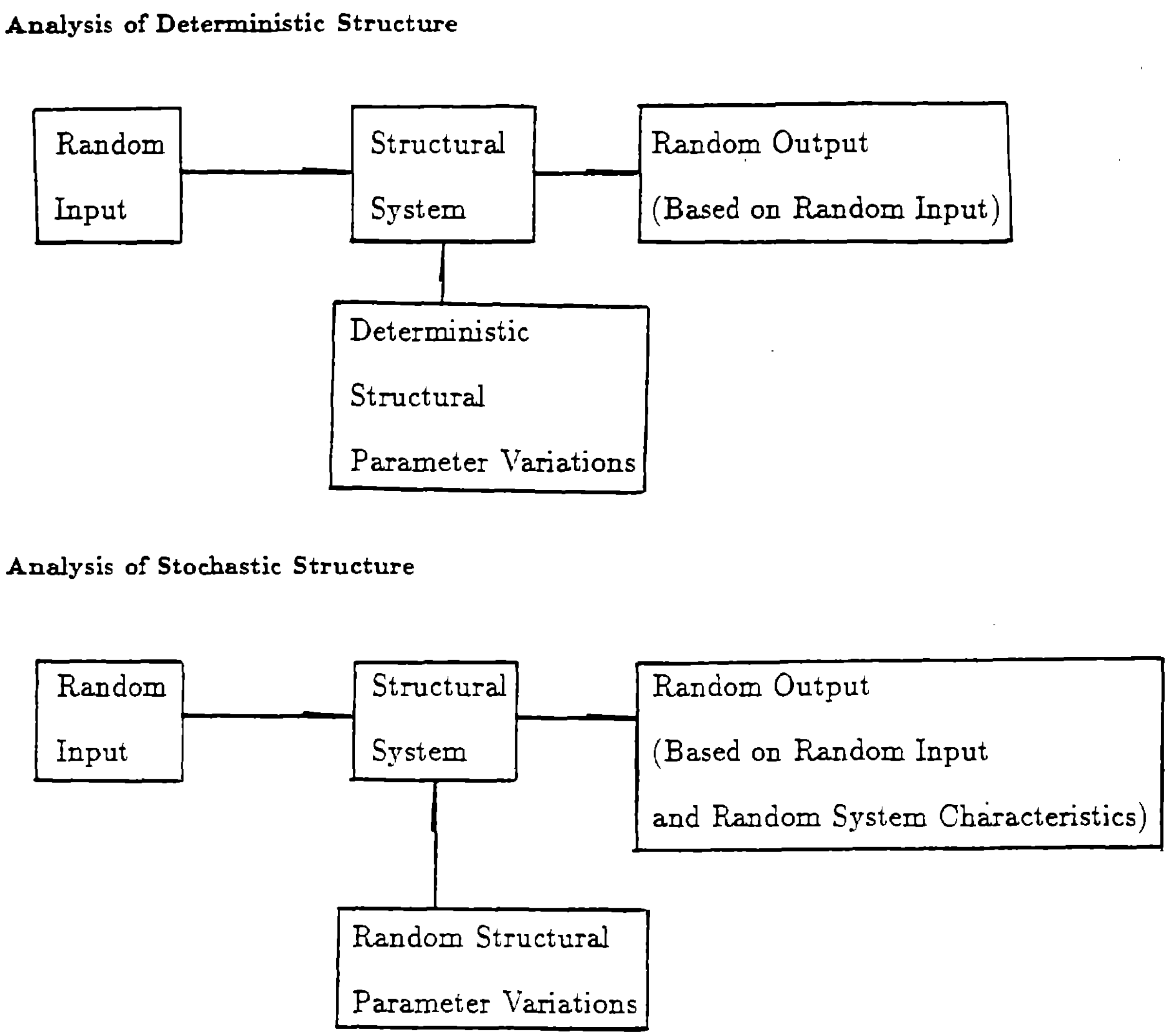


Fig. 3-1 Flow Diagrams for Sensitivity Analysis

In this section, response and eigenvalue sensitivities of secondary systems to primary structural uncertainties are considered. The emphasis is the peak response sensitivity under seismic load. Assuming that peak response is strongly dependent only on the strong-motion portion of the excitation, the system input can be assumed to be weakly stationary and, for linear systems, the power spectral density of the response can be obtained. Under these conditions, it is proposed that the response spectral moments be used as a response sensitivity measure and that second order derivative be used to determine the eigenvalue sensitivity. Calculation procedures and numerical results are examined by means of numerical examples.

3.2 SENSITIVITY FACTOR

3.2.1 Response Sensitivity

Consider an n-degree-of-freedom primary-secondary structural system subjected to a weakly stationary input. The equation of motion has the form

$$M\ddot{\underline{X}}(t) + C\dot{\underline{X}}(t) + K\underline{X}(t) = \underline{F}(t) \qquad (3-1)$$

where M, C and K are respectively, $n \times n$ mass, damping and stiffness matrices. The n-vectors $\underline{X}(t)$ is the displacement vector and $\underline{F}(t)$ is the input vector. In steady state, the input-output spectral density relationship is [177]

$$S_{XX}(\omega) = H^{*}(j\omega)S_{FF}(\omega)H^{T}(j\omega) \qquad (3-2)$$

where $S_{FF}(\omega)$ and $S_{XX}(\omega)$ are the spectral density functions of the excitations and the structural responses, respectively; $H(j\omega)$ is the system frequency response and the superscripts * and ^{T} denote complex conjugate and matrix transpose, respectively.

Let the components of an m-vector $\underline{\beta}$ represent system parameters with uncertainties. They can be written in the form

$$\underline{\beta} = diag(1+\varepsilon_1, 1+\varepsilon_2, \ldots, 1+\varepsilon_m)\underline{\beta}_0 \qquad (3-3)$$

where $\underline{\beta}_0$ represents nominal system parameter values. It is assumed that the random uncertain vector $\underline{\varepsilon}^T = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m]$ has zero mean and covariance matrix $E[\underline{\varepsilon}\underline{\varepsilon}^T] = [\mu_{ij}]$. With $\underline{\beta}$ being stochastic, $H(j\omega)$ and $S_{XX}(\omega)$ are stochastic and Eq.(3-2) can be written as

$$S_{XX}(\omega) = H^*(j\omega, \underline{\beta}) S_{FF}(\omega) H^T(j\omega, \underline{\beta}) \qquad (3-4)$$

The expectation of the ij^{th} elements of $S_{XX}(\omega, \underline{\beta})$ can be represented by [158]

$$E[S_{XX}(\omega, \beta)]_{ij} = (vecI)^T [S_{FF}(\omega) \otimes E\{\underline{h}_i^*(j\omega, \underline{\beta}) \underline{h}_j^T(j\omega, \underline{\beta})\}] vecI \qquad (3-5)$$

In the above, $\underline{h}_j$ is the j^{th} column of H, the symbol $\otimes$ denotes the Kronecker product of two matrices[157], and "vec" of a matrix is a vector formed by stacking the columns of the matrix. For example, let $\underline{w}_j$ be the j^{th} column of a matrix W. Then

$$vecW = \begin{pmatrix} \underline{w}_1 \\ \underline{w}_2 \\ \vdots \\ \underline{w}_s \end{pmatrix} \qquad (3-6)$$

Expanding $H(j\omega, \underline{\beta})$ about $\underline{\beta}_o$ and keeping terms of second order in $\underline{\varepsilon}$, Eq.(3-5) leads to

$$E[S_{XX}(\omega, \underline{\beta})] = S_{XX}(\omega, \underline{\beta}_o) + S_1(\omega, \underline{\beta}_o) \qquad (3-7)$$

where

$$[S_1(\omega, \underline{\beta}_0)]_{ij} = (vec\underline{I})^T [S_{FF}(\omega) \otimes \sum_{r=1}^{m} \sum_{s=1}^{m} \frac{\beta_{or}\beta_{os}}{2} R_{rs}(\omega, \underline{\beta}_o) \mu_{rs}] vec\underline{I} \qquad (3-8)$$

with

$$R_{r,s}(\omega,\underline{\beta}_o) = \underline{h}_i^*(\omega,\underline{\beta}_o)\frac{\partial^2 \underline{h}_j^T(\omega,\underline{\beta}_o)}{\partial\beta_{or}\partial\beta_{os}} + \frac{\partial^2 \underline{h}_i^*(\omega,\underline{\beta}_o)}{\partial\beta_{or}\partial\beta_{os}}\underline{h}_j^T(\omega,\underline{\beta}_o) + \frac{\partial \underline{h}_i^*(\omega,\underline{\beta}_o)}{\partial\beta_{or}}\frac{\partial \underline{h}_j^T(\omega,\underline{\beta}_o)}{\partial\beta_{os}} \qquad (3-9)$$

As seen from Eq.(3-7), the average of the change in the response spectral density is $S_1(\omega,\underline{\beta}_0)$ and the corresponding changes in the response spectral moments are

$$\Delta\Lambda_k = \int_0^\infty \omega^k S_1(\omega,\underline{\beta}_o)d\omega, \qquad k = 0,1,2\ldots \qquad (3-10)$$

Let

$$\Lambda_{ok} = \int_0^\infty \omega^k S_{XX}(\omega,\underline{\beta}_o)d\omega \qquad (3-11)$$

The ratios

$$[\Gamma_k]_{ij} = \frac{[\Delta\Lambda_k]_{ij}}{[\Lambda_{ok}]_{ij}}, \qquad k = 0,1,2,\ldots \qquad (3-12)$$

can be considered as rational sensitivity factors since spectral moments are related to peak response characteristics. In particular, Γ_o gives a sensitivity measure of the squared average maximum response with respect to parameter uncertainties [197].

3.2.2 Eigenvalue Sensitivity [13,32,119,138,160,176]

Similar sensitivity factors can be defined for the eigenvalues. Let $\Omega_i(\underline{\beta})$ be the i^{th} natural frequency of the structure. Expanding $\Omega(\underline{\beta})$ about $\underline{\beta}_0$ and taking expectations, we have

$$E[\Omega_i(\underline{\beta})] = \Omega_i(\underline{\beta}_0) + \Omega_{1i}(\underline{\beta}_0) \qquad (3-13)$$

where

$$\Omega_{1i}(\underline{\beta}_0) = \sum_{r=1}^{m}\sum_{s=1}^{m}[\frac{\beta_{or}\beta_{os}}{2}\frac{\partial^2\Omega_i(\underline{\beta}_o)}{\partial\beta_{or}\partial\beta_{os}}]\mu_{rs} \qquad (3-14)$$

The rational sensitivity factor for $\Omega_i(\underline{\beta}_o)$ can be defined as

$$\gamma_{i\Omega} = \frac{\Omega_{1i}(\underline{\beta}_0)}{\Omega_i(\underline{\beta}_0)} \qquad (3-15)$$

The second order derivative in Eq.(3-14) can be derived in a straight forward manner and the details are given in Appendix C.

3.3 STRUCTURES UNDER GROUND EXCITATION

When a primary-secondary structure is subjected to ground acceleration $\ddot{X}_o(t)$, the input term in Eq.(3-1) becomes

$$\underline{F}(t) = -M\underline{r}\ddot{X}_o(t) \qquad (3-16)$$

where

$$\underline{r}^T = [1 \quad 1 \quad \ldots \quad 1]$$

In this case, it is convenient to define

$$\underline{g}(j\omega, \underline{\beta}) = -H(j\omega, \underline{\beta})M\underline{r} \qquad (3-17)$$

and

$$T(\omega, \underline{\beta}) = |\underline{g}(j\omega, \underline{\beta})|^2 = \underline{g}^*(j\omega, \underline{\beta})\underline{g}^T(j\omega, \underline{\beta}) \qquad (3-18)$$

Equations (3-4) and (3-8) then take the forms

$$S_{XX}(\omega, \underline{\beta}) = T(\omega, \underline{\beta})S_{\ddot{x}_o\ddot{x}_o} \qquad (3-19)$$

and

$$S_1(\omega, \underline{\beta}_o) = \sum_{r=1}^{m}\sum_{s=1}^{m}[\frac{\beta_{or}\beta_{os}}{2}\frac{\partial^2 T(\omega, \underline{\beta})}{\partial\beta_{or}\partial\beta_{os}}]\mu_{rs}S_{\ddot{x}_o\ddot{x}_o} \qquad (3-20)$$

The method of determining the second order derivative will be shown in Appendix B. The sensitivity factors Γ_k defined by Eq.(3-12) pertain to displacement response. They can be modified to correspond to other response quantities by premultiplying or postmutiplying $\underline{g}(j\omega,\underline{\beta})$ by appropriate matrices. For example, with C_1 given by

$$C_1 = (\overset{\text{P-system}}{0 \quad 0 \quad \ldots \quad 0 \quad \underset{i}{-1} \quad 0 \quad \ldots \quad 0} \mid \overset{\text{S-system}}{0 \quad \ldots \quad 0 \quad \underset{j}{1} \quad 0 \quad \ldots \quad 0})$$

a modification from $\underline{g}(j\omega,\underline{\beta})$ to $C_1\underline{g}(j\omega,\underline{\beta})$ produces a response which gives the relative displacements $X_{sj} - X_{pi}$, where the subscripts s and p are used to identify displacements associated with the secondary system and those associated with the primary system, respectively.

If the absolute acceleration at position X_{sj} is of interest, the required transformation is $-C_2[\omega^2\underline{g}(j\omega,\underline{\beta}) + \underline{r}]$, where

$$C_2 = (\overset{\text{P-system}}{0 \quad \ldots \quad 0 \quad \ldots \quad 0} \mid \overset{\text{S-system}}{0 \quad \ldots \quad 0 \quad \underset{j}{1} \quad 0 \quad \ldots \quad 0})$$

In practice, the maximum displacement responses of structures are more significant. They can be represented by the standard deviation of the response[197]

$$(X_{max})_{s;p} = \Upsilon_{s;p}\sigma_x(s) \tag{3-21}$$

where the subscript s is strong-motion duration, p is exceedance probability corresponding to X_{max}, and $\Upsilon_{s;p}$ is the peak factor. When p=0.5, a representation of the average maximum displacement response ($\bar{X}_{max}$) can be obtained. Assuming that the peak factor $\Upsilon_{s;p}$ remains constant under parametric variations, it is seen from Eq.(3-12) that

$$\gamma_0 = \frac{\int_0^\infty S_1(\omega, \underline{\beta}_o) d\omega}{\int_0^\infty S_{XX}(\omega, \underline{\beta}_o) d\omega} \tag{3-22}$$

which gives

$$\gamma_0 = \frac{\sigma_1^2}{\sigma_{X0}^2} \tag{3-23}$$

or, based on Eq.(3-21),

$$\gamma_0 = \frac{X_{1,max}^2}{X_{0,max}^2} \tag{3-24}$$

Thus, γ_0 is also associated with the sensitivity of squared maximum displacement response. It thus takes an added significance in sensitivity studies.

3.4 SDOF-SDOF P-S SYSTEM

3.4.1 Formulation

Consider first a simple single-degree-of-freedom(SDOF) secondary system anchored to a SDOF primary structure as shown in Fig. 3-2. Let m_p, ω_p and ζ_P be the mass, frequency and damping ratio of the primary system and m_s, ω_s *and* ζ_s be the corresponding quantities for the secondary system. We are primarily interested in the sensitivity of the relative displacement, $X_s - X_p$, to random damping variations in the primary structure. Thus, $\beta = \zeta_p = \zeta_{op}(1 + \varepsilon)$ where ε has mean zero and variance σ^2.

For this case, $T(\omega, \beta)$ as defined in Eq.(3-18) is a scalar and has the form

$$T(\omega, \beta) = |H(j\omega, \beta)|^2 = \frac{D_1}{D_2 D_3} \tag{3-25}$$

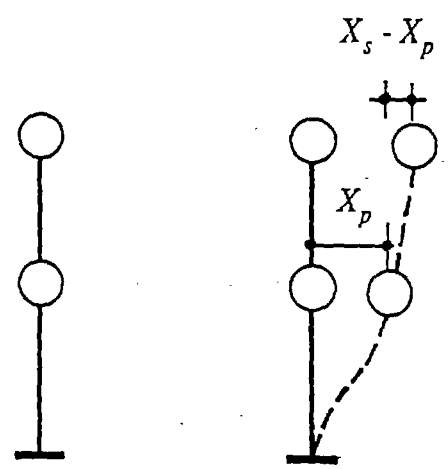


Fig. 3-2 SDOF-SDOF P-S System.

where ω is the frequency of the combined primary-secondary two-degree-of-freedom system and

$$D_1 = 4\zeta_p^2\omega_p^2\omega^2 + \omega_p^4 \tag{3-26}$$

$$D_2 = \omega^4 - 2i\omega^3(\omega_p\zeta_p + \omega_s\zeta_s + R_m\zeta_s\omega_s) - \omega^2(\omega_p^2 + \omega_s^2 + R_m\omega_s^2 + 4\zeta_p\zeta_s\omega_s\omega_p)$$

$$+2i\omega(\zeta_s\omega_s\omega_p^2 + \zeta_p\omega_s^2\omega_p) + \omega_p^2\omega_s^2 \tag{3-27}$$

$$D_3 = D_2^* \tag{3-28}$$

We have

$$\frac{\partial T(\omega)}{\partial\zeta_p} = T(\omega)(\frac{1}{D_1}\frac{\partial D_1}{\partial\zeta_p} - \frac{1}{D_2}\frac{\partial D_2}{\partial\zeta_p} - \frac{1}{D_3}\frac{\partial D_3}{\partial\zeta_p}) \tag{3-29}$$

where

$$\frac{\partial D_1}{\partial\zeta_p} = 8\zeta_p\omega_p^2\omega^2 \tag{3-30}$$

$$\frac{\partial^2 D_1}{\partial\zeta_P^2} = 8\omega_p^2\omega^2 \tag{3-31}$$

$$\frac{\partial D_2}{\partial\zeta_p} = -2i\omega_p\omega^3 - 4\zeta_s\omega_p\omega_s\omega^2 + 2i\omega_p\omega_s^2\omega \tag{3-32}$$

$$\frac{\partial^2 D_2}{\partial\zeta_p^2} = 0$$

$$\frac{\partial D_3}{\partial\zeta_p} = \frac{\partial D_2}{\partial\zeta_p}^* \quad and \quad \frac{\partial^2 D_3}{\partial\zeta_p^2} = \frac{\partial^2 D_2}{\partial\zeta_p^2}^*$$

and

$$\frac{\partial^2 T(\omega)}{\partial\zeta_p^2} = T(\omega)[\frac{1}{D_1}\frac{\partial^2 D_1}{\partial\zeta_p^2} + \frac{2}{D_2^2}(\frac{\partial D_1}{\partial\zeta_p})^2 + \frac{2}{D_3^2}(\frac{\partial D_3}{\partial\zeta_p})^2$$

$$-\frac{2}{D_1 D_2}\frac{\partial D_1}{\partial \zeta_p}\frac{\partial D_2}{\partial \zeta_p}+\frac{2}{D_2 D_3}\frac{\partial D_2}{\partial \zeta_p}\frac{\partial D_3}{\partial \zeta_p}-\frac{2}{D_1 D_3}\frac{\partial D_1}{\partial \zeta_p}\frac{\partial D_3}{\partial \zeta_p}] \qquad (3-33)$$

3.4.2 Example 3.1 SDOF-SDOF P-S System.

It is of interest to consider first the tuned case. Using parameter values listed in Table 3.1, $T(\omega,\beta)$ and its second derivative are plotted in Fig. 3-3 and Fig. 3-4, respectively. It is seen that the combined P-S system exhibits two resonant frequencies straddling ω_p and ω_s.

The sensitivity factors γ_o, γ_1 and γ_2 are shown in Fig. 3-5 as functions of $R_\omega = \omega_s/\omega_p$ under white noise excitation with spectral density S_o. It is shown that damping uncertainties in the primary structure have a significant effect on the peak secondary system response at tuning, but their effect diminishes rapidly as the P-S system moves away from the tuned case.

It is also seen from Fig. 3-5 that γ_o, γ_1 and γ_2 provide similar and consistent sensitivity information, and this property has also been verified in other numerical calculations. Hence, it is sufficient to consider only γ_o in what follows.

Figures 3-6 and 3-7 show the sensitivity factor γ_o as a function of the mass ratio $R_m = m_s/m_p$ and the damping ratio $R_\zeta = \zeta_{os}/\zeta_p$. They show that the value of γ_o stays relatively constant as the mass of the secondary system increases although a heavier secondary mass will cause an increase in the interaction effect between the primary and the secondary system. On the other hand, the response sensitivity is the greatest when the damping ratio is small, decreasing as ζ_{os} approaches ζ_p.

Table 3.1 *Properties of P-S System in Example 3.1*

	Mass	Stiffness	Damping Ratio	Frequency (Hz)
S-system	m_s	k_s	0.02	1.1254
P-system	$10m_s$	$10k_s$	0.05	1.1254
Combined				0.9614, 1.3173

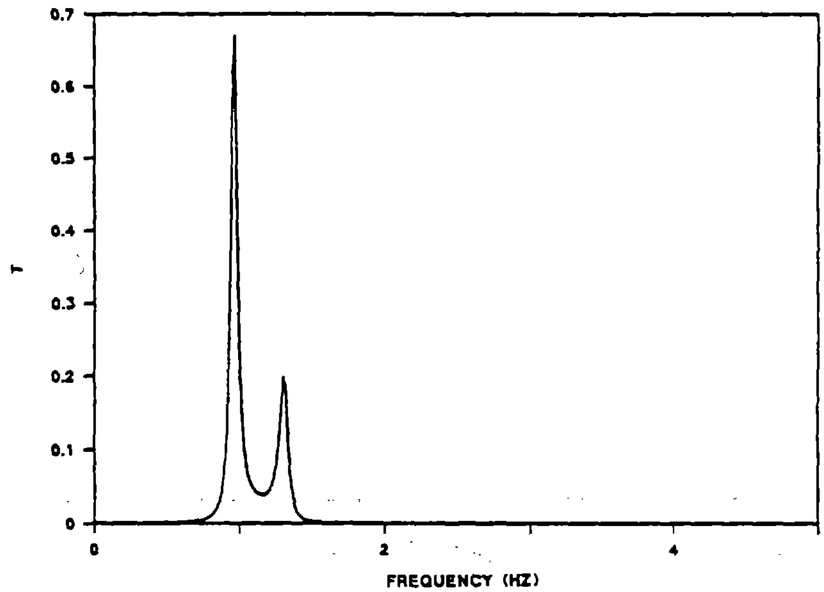


Fig. 3-3 $T(\omega,\beta_0)$ in Example 3.1.

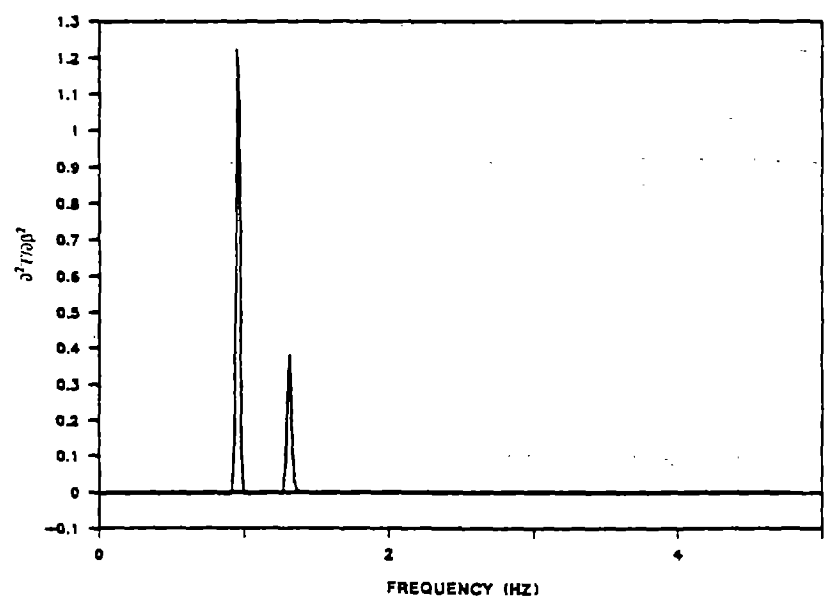


Fig. 3-4 $\partial^2 T/\partial\beta_0^2$ in Example 3.1.

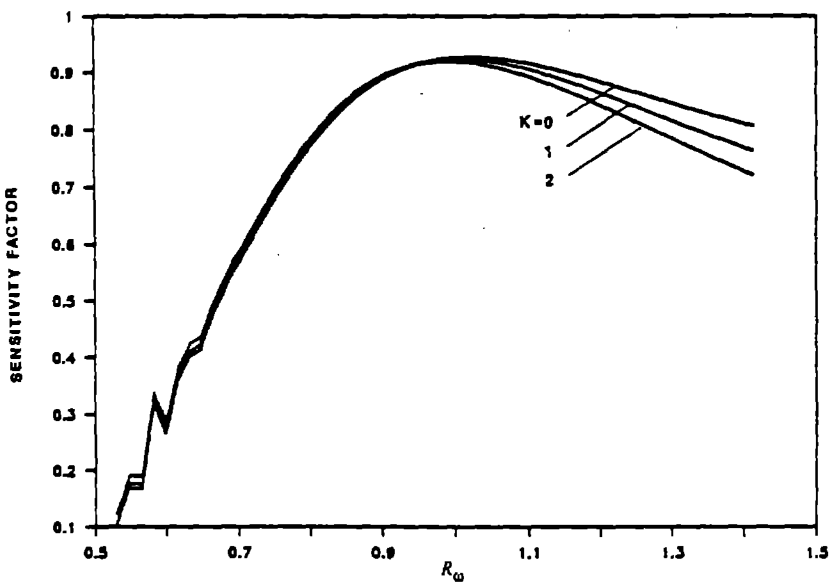


Fig. 3-5 Sensitivity Factor vs. Frequency Ratio.

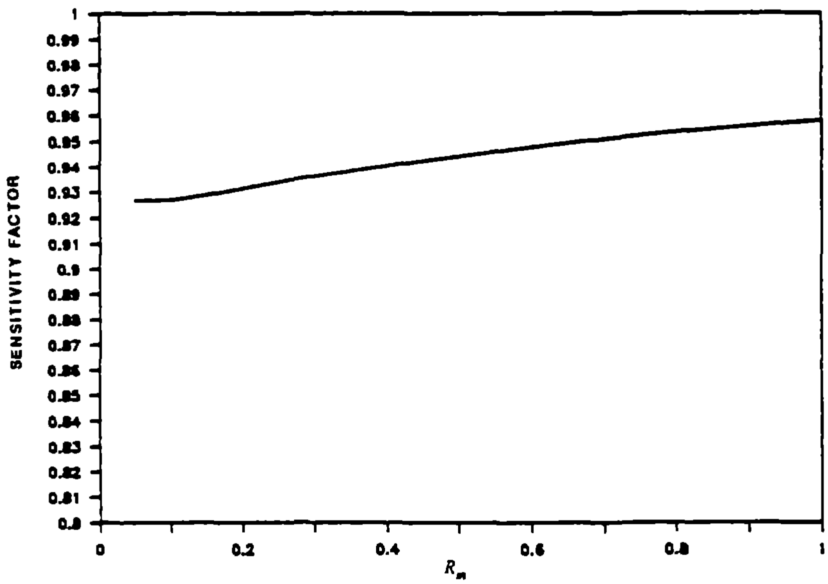


Fig. 3-6 Sensitivity Factor vs. Mass Ratio.

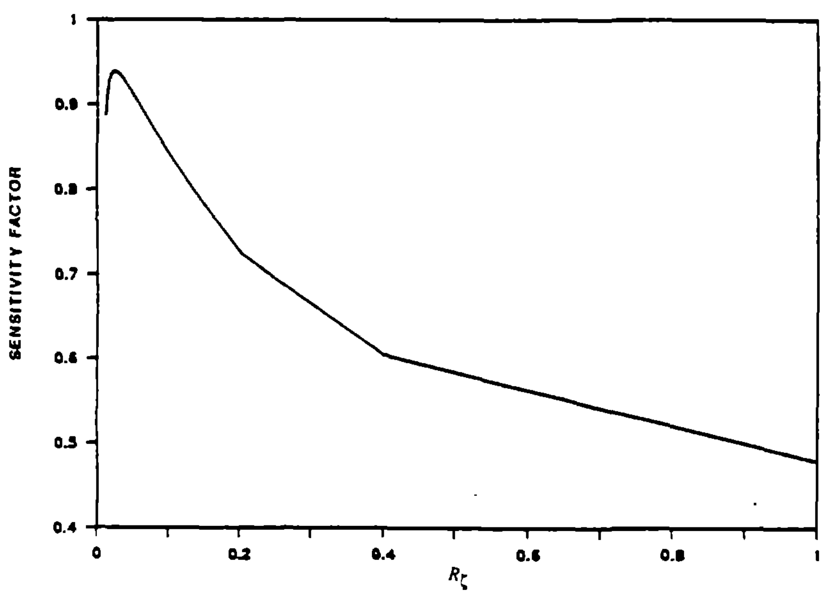


Fig. 3-7 Sensitivity Factor vs. Damping Ratio.

As an accuracy check of the results presented above, a Monte-Carlo simulation is performed. For the tuned case considered above, assume the stiffness of the primary system, $10K_s$, is normally distributed with mean $10k_s$ and standard deviation k_s. The combined P-S system is subjected to sixty sets of artificial white-noise ground motions. Maximum displacement responses and response standard deviations are obtained by means of step-by-step integration together with statistical averaging. The values of the sensitivity factor γ_0 using Eq.(3-23) and eq(3-24) are calculated and compared with the analytical results. These results are tabulated in Table 3.2, showing good agreement between Monte-Carlo simulation and analytical results.

3.5 MDOF-MDOF P-S SYSTEM EXAMPLES

Some typical primary-secondary (P-S) structural system configurations are shown in Fig. 3-8. In the following examples, we are primarily interested in the eigenvalue sensitivity of the system and the response sensitivity of the secondary system to structural parameter variations in the primary system.

Two examples are discussed in what follows. As shown in Fig. 3.8, system S3-1 represents a three-degree-of freedom secondary system attached to a six-degree-of-freedom primary system and system S3-2 replaces the secondary system by a two-degree-of-freedom system. Some of their dynamic properties are given in Tables 3.3 and Table 3.4. White noise base excitation is again assumed with spectral density S_o.

3.5.1 Random Uncertainty at Each Floor for Fixed Secondary System

Let us first consider sensitivities of both eigenvalues and response of the secondary system if there exists a random uncertainty at a given floor of the primary system.

Table 3.2 *Comparison of γ_0 Value Using Monte-Carlo Simulation and Analytical Results*

	Max. Shift	Monte-carlo	Simulation	Analytical Results
		$\gamma_0 = \frac{\sigma_1^2}{\sigma_{X0}^2}$	$\gamma_0 = \frac{X_{1.max}^2}{X_{0.max}^2}$	γ_0
γ_0/σ^2	$X_s - X_p$	1.77	1.694	1.611
	X_p	1.9	1.728	1.732

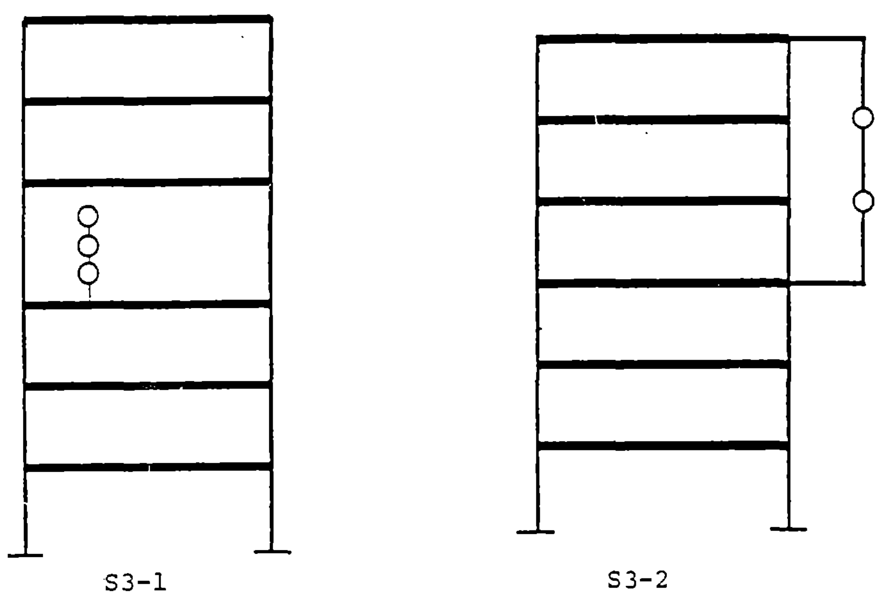


Fig. 3-8 . Examples of P-S System

Table 3.3 *Properties of P-S System S3-1*

	P-system						S- system		
Mode	1	2	3	4	5	6	1	2	3
ω(Hz)	2.714	7.981	12.788	16.847	19.92	21.85	2.716	7.609	10.996
			Combined		system				
Mode	1	2	3	4	5	6	7	8	9
ω(Hz)	1.839	2.873	5.556	7.878	8.287	12.809	16.913	19.933	21.899

Table 3.4 *Properties of P-S System S3-2*

	P-system						S-system	
	1	2	3	4	5	6	1	2
ω(Hz)	2.714	7.981	12.788	16.835	19.92	21.85	2.715	4.691
			Combined		system			
Mode	1	2	3	4	5	6	7	8
ω(Hz)	1.973	3.719	4.638	8.228	12.872	16.937	19.947	21.909

Eigenvalue sensitivity. Assuming that there exists mass or stiffness uncertainty on one of the floors (in system S3-1), the eigenvalue sensitivity factors for the first three modes are listed in Table 3.5. It is seen that, in most cases, eigenvalue sensitivities decrease as mass uncertainty moves from the top floor to the first floor of the primary system, i.e., mass variability at the top floors has greater effect than that at the bottom. On the other hand, the impact of stiffness uncertainty is the greatest when it occurs at the first floor, diminishing as stiffness uncertainty moves to the top.

Response Sensitivity. For system S3-1, the response quantities of interest are the displacement of the top secondary mass relative to the third floor connecting point and the absolute acceleration of the top secondary mass. The quantity $T(\omega, \beta)$ for the displacement and its second-order derivative are shown in Figs. 3-9 and 3-10. Their sensitivity to either mass uncertainty or stiffness uncertainty at each floor of the primary system is of interest.

Table 3.6 shows the sensitivity factors as the mass uncertainty or the stiffness uncertainty moves from the first floor to the sixth floor of the primary system. Similar to the trend seen in the eigenvalue sensitivity, the response quantities become less sensitive to mass uncertainty as it moves from the top floor to the first floor of the primary system, whereas just the opposite is true in the case of stiffness uncertainty. The response sensitivity are the greatest when the stiffness uncertainty is at the first floor and the smallest when it is at the top floor.

The conclusions in eigenvalue sensitivity study and in response sensitivity study are in agreement.

Table 3.5 γ_Ω/σ^2 for P-S System S3-2

		Floor		of	Uncertainty		
β	γ_Ω/σ^2	6	5	4	3	2	1
Mass	Ω_1	0.0125	0.0105	0.0060	0.0019	0.0030	0.0002
	Ω_2	0.0041	0.0046	0.0049	0.0017	0.0014	0.0004
	Ω_3	0.0119	0.0049	0.0023	0.0109	0.0028	0.0088
Stiff.	Ω_1	0.0096	0.0273	0.0489	0.0891	0.1169	0.1153
	Ω_2	0.0019	0.0021	0.0014	0.0027	0.0122	0.0727
	Ω_3	0.0384	0.0772	0.0435	0.0021	0.0473	0.0105

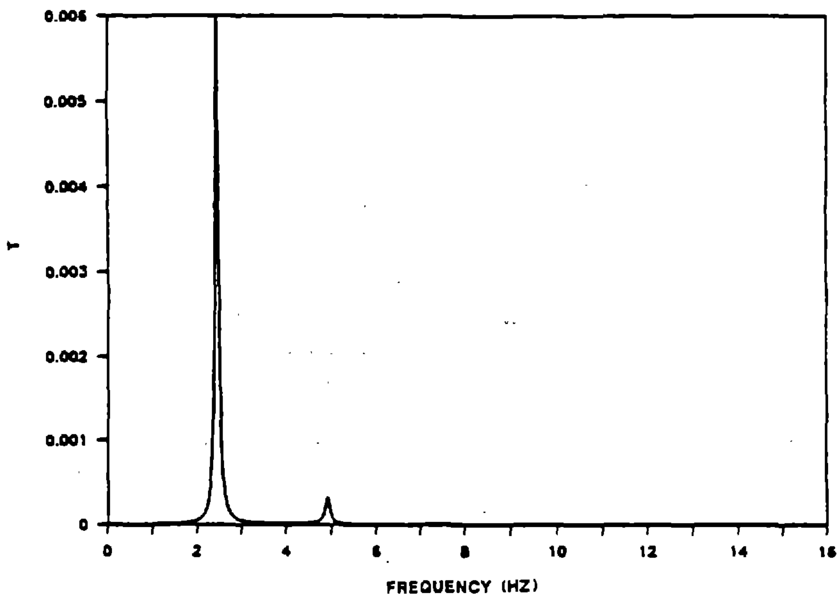


Fig. 3-9 $T(\omega,\beta_0)$ in P-S System S3-2.

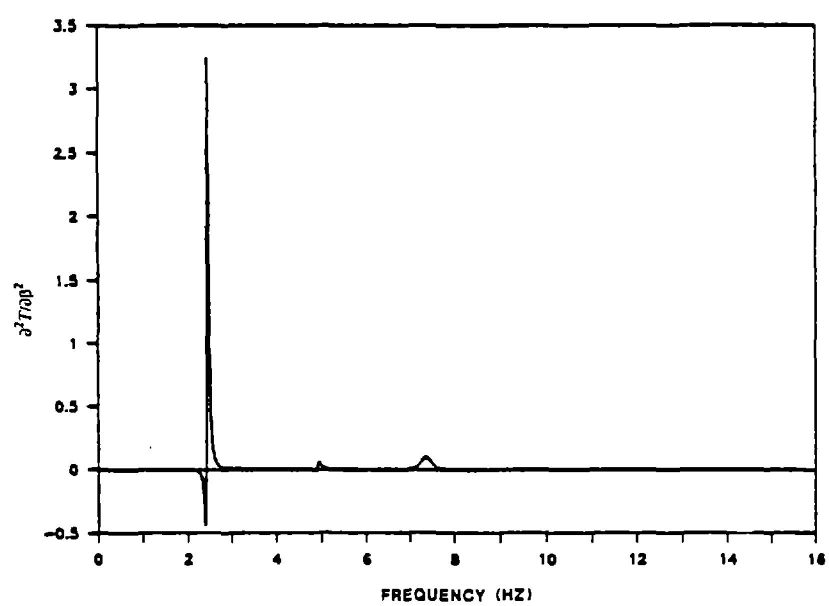


Fig. 3-10 $\partial^2 T/\partial\beta_0^2$ in P-S System S3-2.

Table 3.6 γ_0/σ^2 *for P-S System S3-2*

		Floor	of	Uncertainty			
β	γ_0/σ^2	6	5	4	3	2	1
Mass	Disp.	0.0265	0.0218	0.0160	0.0074	0.0045	0.0028
	Acc.	0.0414	0.0248	0.0141	0.0082	0.0035	0.0036
Stiff.	Disp.	0.0203	0.0830	0.1563	0.3688	0.4161	0.4346
	Acc.	0.0657	0.1426	0.2432	0.3716	0.4223	0.4452

3.5.2 System Sensitivity at Each Floor for Fixed Random Uncertainty

Let us now consider sensitivity results in terms of x_{pk} and x_{sk}, where x_pk is the relative displacement of the k^{th} floor of the primary system with respective to the third floor, $k = 4, 5, 6$, and x_{sk} is the corresponding quantities, $k = 1, 2, 3$, for the secondary system.

It is shown (in Table 3.7) that the sensitivities to both mass and stiffness variations, of the secondary system are more significant than those of the primary system.

3.6 CONCLUDING REMARKS

Spectral moments are proposed in this section as sensitivity factors in the study of response sensitivity of structural systems to parameter uncertainties under random excitations. Their application to the study of response sensitivity of primary-secondary systems has shown that spectral moments provide quantitative and consistent response sensitivity measures.

In design and performance evaluation of secondary systems, spectral moments can thus provide important information concerning relative importance of parameter uncertainties in the primary system. Although not explored in detail here, numerical experimentations show that they can also serve as useful design tools in the determination of optimal locations and arrangements of secondary systems within a primary structure.

Table 3.7 γ_0/σ^2 *for P-S System S3-1*

		Position of response					
β	γ_0/σ^2	x_{p6}	x_{p5}	x_{p4}	x_{s3}	x_{s2}	x_{s1}
Mass	Disp.	0.0124	0.0115	0.0105	0.0204	0.0203	0.0207
	Acc.	0.0212	0.0129	0.0194	0.0283	0.0249	0.0447
Stiff.	Disp.	0.2665	0.2655	0.2639	0.4648	0.4625	0.4558
	Acc.	0.2786	0.2698	0.2683	0.4571	0.4472	0.3510

The formulas derived for determining sensitivities of eigenvalues and eigenvectors are also useful in examining the total response sensitivity of secondary systems subjected to primary system parameters uncertainties.

SECTION 4

AN APPROACH TO FLOOR RESPONSE SPECTRUM MODIFICATION

4.1 INTRODUCTION

As presented in the preceding section, the sensitivity factors based on spectral moments provide a quantitative and consistent response sensitivity measure. They can be used to obtain important information on the response of secondary systems. As one of their applications, a methodology for modifying floor response spectrum is studied in this section.

The seismic design or evaluation of secondary systems placed in structures is frequently based on the method of floor response spectrum (FRS) [27]. Current procedures are to first calculate a raw floor response spectrum. To account for the effect on structural frequency variations of possible uncertainties in the material properties of the structure and soil, and approximations in the modeling techniques used in seismic analysis, the initially computed floor response spectra are usually smoothed, and peaks associated with the structural frequencies are broadened. A recommended method of determining the amount of peak widening associated with the structural frequency is described below[1,128].

Let ω_j be the j^{th} mode structural frequency which is determined from the structural model. The variation in the structural frequency is determined by evaluating individual frequency variations due to the variations in each parameter having a significant effect, such as the soil modulus, material density, etc. The total frequency variation $\Delta\omega_j$ is then determined by taking the SRSS of a minimum variation of $0.05\omega_j$ plus the individual frequency variation $(\Delta\omega_j)_n$, that is,

$$\Delta\omega_j = \sqrt{(0.05\omega_j)^2 + \sum(\Delta\omega_j)_n^2}$$

This is widely used as an acceptable method. However, it is not actually shown how to determine the frequency variation $\Delta\omega_j$ from the uncertainties. In practice, the recommended procedures for smoothing and peak broadening of FRS are usually empirical in nature. In [69], a more rigorous procedure is presented which is based on probability distributions of the structural and soil parameters and how they jointly determine the structural frequency distributions. Both the conventional approach and the approach described above are based on the following idea: The amplitude of the floor response spectra is determined by the dynamic characteristics of the structure, and the location of the peak of the floor response spectra is a function of the structural frequency.

In these analyses, the following problems may exist: The shift of the primary system frequency may cause both a horizontal and a vertical shift of the floor response spectrum, which need to be accounted for. In addition, while the method of floor response spectra is attractive because of its simplicity, it ignores dynamic interaction between primary and secondary systems. This interaction can be significant for certain P-S structural systems.

In this section, a more rational quantitative procedure based on the sensitivity study is proposed, which determines the amount of variation of the floor response spectrum due to structural parameter variations. The interaction between secondary and primary systems and nonclassical damping are taken into account as well. Moreover, the effect of nonlinear primary structural behavior is also considered using the method of equivalent linearization. Numerical examples are presented.

4.2 DEVELOPMENT OF MODIFICATION PROCEDURE

4.2.1 Response Amplitude Modification

Consider an n-degree-of-freedom primary-secondary structural system subjected to a weakly stationary input. Under ground motions, for a particular response following Eq.(3-7,3-8 and 3-9), the spectral moments can be expanded in the form

$$\lambda_k(\underline{\beta}) = \lambda_{ok}(\underline{\beta}_0) + \sum_{r=1}^{N}[\beta_{or}\frac{\partial \lambda_{ok}(\underline{\beta}_0)}{\partial \beta_{or}}]\varepsilon_{or} + \sum_{r=1}^{m}\sum_{s=1}^{m}[\frac{\beta_{or}\beta_{os}}{2}\frac{\partial^2 \lambda_{ok}(\underline{\beta}_0)}{\partial \beta_{or}\partial \beta_{os}}]\varepsilon_r\varepsilon_s + \ldots \qquad (4-1)$$

where

$$\frac{\partial \lambda_{ok}(\underline{\beta}_0)}{\partial \beta_{or}} = \int_0^\infty \omega^k \frac{\partial T(\omega, \underline{\beta}_o)}{\partial \beta_{or}} S_{\ddot{x}_o\ddot{x}_o}(\omega)d\omega \qquad (4-2)$$

$$\frac{\partial^2 \lambda_{ok}(\underline{\beta}_0)}{\partial \beta_{or}\partial \beta_{os}} = \int_0^\infty \omega^k [\frac{\partial^2 T(\omega, \underline{\beta})}{\partial \beta_{or}\partial \beta_{os}}] S_{\ddot{x}_o\ddot{x}_o} d\omega \qquad (4-3)$$

$$E[\lambda_k(\underline{\beta})] = \lambda_{ok}(\underline{\beta}_0) + \sum_{r=1}^{m}\sum_{s=1}^{m}[\frac{\beta_{or}\beta_{os}}{2}\frac{\partial^2 \lambda_{ok}(\underline{\beta}_0)}{\partial \beta_{or}\partial \beta_{os}}]\mu_{rs} + \ldots \qquad (4-4)$$

In the above, the m-vector $\underline{\beta}$ represents uncertain structural parameters. The ratios

$$\begin{aligned}\gamma_k &= \frac{\Delta \lambda_k(\underline{\beta}_0)}{\lambda_{ok}(\underline{\beta}_0)} \\ &= \frac{1}{\lambda_{ok}(\underline{\beta}_0)}\sum_{r=1}^{m}\sum_{s=1}^{m}[\frac{\beta_{or}\beta_{os}}{2}\frac{\partial^2 \lambda_{ok}(\underline{\beta}_0)}{\partial \beta_{or}\partial \beta_{os}}]\mu_{rs} \quad k = 0, 1, 2\end{aligned} \qquad (4-5)$$

can be considered as sensitivity factors since the zeroth-order spectral moment is related to peak response characteristics (Eq. 3-24).

As seen from Eq.(3-24),

$$\gamma_0 = \frac{X_{1,max}^2}{X_{0,max}^2}$$

Eq.(4-4) leads to

$$\frac{E[X_{max}^2]}{X_{o,max}^2} = 1 + \gamma_o \tag{4-6}$$

In addition, expanding $\sqrt{\lambda_k(\underline{\beta})}$ about $\underline{\beta}_o$ we have

$$\sqrt{\lambda_k(\underline{\beta})} = \sqrt{\lambda_{ok}(\underline{\beta}_0)} + \sum_{r=1}^{N}[\beta_{or}\frac{\partial\sqrt{\lambda_{ok}(\underline{\beta}_0)}}{\partial\beta_{or}}]\varepsilon_{or} + \sum_{r=1}^{m}\sum_{s=1}^{m}[\frac{\beta_{or}\beta_{os}}{2}\frac{\partial^2\sqrt{\lambda_{ok}(\underline{\beta}_0)}}{\partial\beta_{or}\partial\beta_{os}}]\varepsilon_r\varepsilon_s + \ldots \tag{4-7}$$

where

$$\frac{\partial\sqrt{\lambda_{ok}(\underline{\beta}_0)}}{\partial\beta_{or}} = \frac{1}{2}\lambda_{ok}^{-\frac{1}{2}}(\underline{\beta}_0)\frac{\partial\lambda_{ok}(\underline{\beta}_0)}{\partial\beta_{or}} \tag{4-8}$$

$$\frac{\partial^2\sqrt{\lambda_{ok}(\underline{\beta}_0)}}{\partial\beta_{or}\partial\beta_{os}} = \frac{1}{2}\lambda_{ok}^{-\frac{1}{2}}(\underline{\beta}_0)\frac{\partial^2\lambda_{ok}(\underline{\beta}_0)}{\partial\beta_{or}\beta_{os}} - \frac{1}{4}\lambda_{ok}^{-\frac{3}{2}}(\underline{\beta}_0)(\frac{\partial\lambda_{ok}(\underline{\beta}_0)}{\partial\beta_{or}}\frac{\partial\lambda_{ok}(\underline{\beta}_0)}{\partial\beta_{os}}) \tag{4-9}$$

$$\frac{E[\sqrt{\lambda_k(\underline{\beta})}]}{\sqrt{\lambda_k(\underline{\beta}_0)}} = 1 + \frac{1}{\sqrt{\lambda_{ok}(\underline{\beta}_0)}}\sum_{r=1}^{m}\sum_{s=1}^{m}[\frac{\beta_{or}\beta_{os}}{2}\frac{\partial^2\sqrt{\lambda_{ok}(\underline{\beta}_0)}}{\partial\beta_{or}\partial\beta_{os}}]\mu_{rs} \tag{4-10}$$

Hence,

$$\frac{E[X_{max}]}{X_{o,max}} = 1 + \nu_o \qquad (4-11)$$

$$\nu_0 = \frac{1}{\sqrt{\lambda_{ok}(\underline{\beta}_0)}} \sum_{r=1}^{m} \sum_{s=1}^{m} [\frac{\beta_{or}\beta_{os}}{2} \frac{\partial^2 \sqrt{\lambda_{ok}(\underline{\beta}_0)}}{\partial\beta_{or}\partial\beta_{os}}] \mu_{rs} \qquad (4-12)$$

The difference between the maximum responses with and without parameter uncertainties is

$$\begin{aligned} \frac{E[(\Delta X(\underline{\beta}))^2]}{X_{o,max}^2} &= \frac{E[X_{max}^2] - 2E[X_{max}X_{o,max}]}{X_{o,max}^2} + 1 \\ &= \gamma_o - 2\nu_0 \end{aligned} \qquad (4-13)$$

The variance of the difference is

$$\begin{aligned} \frac{Var[\Delta X(\underline{\beta})]}{X_{0,max}^2} &= \frac{\sigma^2_{\Delta X(\underline{\beta})}}{X_{o,max}^2} \\ &= \frac{E[(\Delta X(\underline{\beta}))^2] - [E(\Delta X(\underline{\beta}))]^2}{X_{o,max}^2} \\ &= \gamma_o - 2\nu_0 - \nu_0^2 \end{aligned} \qquad (4-14)$$

Using the mean plus and minus one standard deviation as the modification criterion, one obtains

$$\frac{E[\Delta X(\underline{\beta})] \pm \sigma_{\Delta X(\underline{\beta})}}{X_{o,max}} = \nu_o \pm \sqrt{\gamma_o - 2\nu_0 - \nu_0^2} \qquad (4-15)$$

Assuming the uncertainties are independent random variables, μ_{rs} in Eqs. (4-5) and (4-12) are non-zero only when $r = s$.

In this procedure of determining the sensitivity factor, the effect of the interaction between primary and secondary systems and nonclassical damping have been accounted for. This is superior to a totally decoupled analysis.

4.2.2 Frequency Shift

Consider random structural parameters, it is seen from Eq.(3-13) that

$$\frac{E[\Omega_i(\underline{\beta}) - \Omega_i(\underline{\beta}_0)]}{\Omega_i(\underline{\beta}_0)} = \frac{\Omega_{i1}(\underline{\beta}_0)}{\Omega_i(\underline{\beta}_0)} \tag{4-16}$$

or

$$\frac{E[\Delta\Omega_i]}{\Omega_i(\underline{\beta}_0)} = \frac{1}{\Omega_i(\underline{\beta}_0)} \sum_{r=1}^{m} \sum_{s=1}^{m} [\frac{\beta_{or}\beta_{os}}{2} \frac{\partial^2 \Omega_i(\underline{\beta}_0)}{\partial \beta_{or} \partial \beta_{os}}] \mu_{rs} = \gamma_{\Omega_i} \tag{4-17}$$

This can be used as the sensitivity factor for the average eigenvalue. The second-order expansion of the structural frequency about $\underline{\beta}_o$ gives

$$\Omega_i(\underline{\beta}) = \Omega_{i0}(\underline{\beta}_0) + \sum_{r=1}^{m} [\beta_{or} \frac{\partial \Omega_i(\underline{\beta}_0)}{\partial \beta_{or}}] \varepsilon_{or} + \sum_{r=1}^{m} \sum_{s=1}^{m} [\frac{\beta_{or}\beta_{os}}{2} \frac{\partial^2 \Omega_i(\underline{\beta}_0)}{\partial \beta_{or} \partial \beta_{os}}] \varepsilon_r \varepsilon_s \cdots \tag{4-18}$$

Neglecting terms of orders higher than two, the variance of the difference is

$$\begin{aligned} \frac{Var[\Delta\Omega_i)]}{\Omega_i^2(\underline{\beta}_0)} &= \frac{\sigma^2_{\Delta\Omega_i}}{\Omega_i^2(\underline{\beta}_0)} \\ &= \frac{1}{\Omega_i^2(\underline{\beta}_0)} \sum_{r=1}^{m} \sum_{s=1}^{m} \beta_{or}\beta_{os} \frac{\partial \Omega_i(\underline{\beta}_0)}{\partial \beta_{or}} \frac{\partial \Omega_i(\underline{\beta}_0)}{\partial \beta_{os}} \mu_{rs} - \gamma^2_{\Omega_i} \end{aligned} \tag{4-19}$$

and

$$\frac{\sigma_{(\Delta\Omega_i)}}{\Omega_i(\underline{\beta}_0)} = \sqrt{\frac{1}{\Omega_i^2(\underline{\beta}_0)} \sum_{r=1}^{m} \sum_{s=1}^{m} \beta_{or}\beta_{os} \frac{\partial\Omega_i(\underline{\beta}_0)}{\partial\beta_{or}} \frac{\partial\Omega_i(\underline{\beta}_0)}{\partial\beta_{os}} \mu_{rs} - \gamma_{\Omega_i}^2} \tag{4-20}$$

The mean plus and minus one standard deviation of the frequency difference is given by

$$\frac{E[\Delta\Omega_i] \pm \sigma_{\Delta\Omega_i}}{\Omega_i(\underline{\beta}_0)} = \gamma_{\Omega_i} \pm \sqrt{\alpha_{\Omega_i}^2 - \gamma_{\Omega_i}^2} \tag{4-21}$$

where

$$\alpha_{\Omega i} = \frac{1}{\Omega_i(\underline{\beta}_0)} \sqrt{\sum_{r=1}^{m} \sum_{s=1}^{m} \beta_{or}\beta_{os} \frac{\partial\Omega_i(\underline{\beta}_0)}{\partial\beta_{or}} \frac{\partial\Omega_i(\underline{\beta}_0)}{\partial\beta_{os}} \mu_{rs}} \tag{4-22}$$

Assuming again that the random uncertainties are independent, $\mu_{rs} \neq 0$ only when $r = s$.

The first- and second-order derivatives contained in Eqs.(4-17) to (4-22) are derived in Appendix C.

4.3 NONLINEAR PRIMARY SYSTEM

4.3.1 Sensitivity of Substitute Structure

The above discussion is limited to linear primary structures. When the primary structures undergo inelastic deformation, it is also of interest to consider necessary modifications of FRS.

For nonlinear primary systems, the determination of the sensitivity factor is much more difficult than that for the linear case. Several approximate methods are being used to predict the nonlinear structural response. One of the more popular methods is that of equivalent linearization. If an equivalent linearized primary system is obtained, it is easy to combine this with the secondary system to form the primary-secondary system. The linearized primary-secondary system can be represented by

$$M\ddot{\underline{X}}(t) + C_E\dot{\underline{X}}(t) + K_E\underline{X}(t) = \underline{F}(t) \tag{4-23}$$

where K_E and C_E consist of the primary system's equivalent linear stiffness and damping coefficients, K_e and C_e, and elastic secondary parameters K_s and C_s, respectively. Therefore,

$$H_E(\omega) = [-\omega^2 M + i\omega C_E + K_E]^{-1} \tag{4-24}$$

If we are interested in obtaining the response and frequency sensitivity factors corresponding to the yielding level of a primary system, we need to find the following partial derivatives:

$$\frac{\partial C_E}{\partial Y} \quad and \quad \frac{\partial K_E}{\partial Y}; \qquad \frac{\partial^2 C_E}{\partial Y^2} \quad and \quad \frac{\partial^2 K_E}{\partial Y^2}$$

With them we can derive

$$\frac{\partial H_E}{\partial Y} \quad and \quad \frac{\partial^2 H_E}{\partial Y^2} \quad and\ then \quad \frac{\partial T_E}{\partial Y} \quad and \quad \frac{\partial^2 T_E}{\partial Y^2}$$

where Y can be either the yielding strength ratio R or ductility ratio μ. The yielding strength ratio is defined by

$$R = F_y/F_e \leq 1 \tag{4-25}$$

where F_y is the inelastic yielding force and F_e is the elastic force and the ductility ratio is

$$\mu = X_m/X_y \tag{4-26}$$

where X_m is the maximum displacement and X_y is the yielding point displacement as shown in Fig. 4-1.

If we can successfully obtain an equivalent linearized system. The above derivative values can be found by numerical means (computer programs RSPS and ESPS). These derivatives can then be used to obtain the sensitivity of substitute systems and necessary modifications to the FRS.

4.3.2 Substitute Structure

A simple practical analytical-empirical method for a MDOF yielding system is used to perform equivalent linearization. For non-deteriorating systems such as steel frames, it has been shown[208] that the effective post-yielding stiffness K_e can be reasonably predicted by the average stiffness method. For example, if a steel frame is modeled as a bilinear hysteretic system, K_e can be written as

$$K_e(\mu) = K_o[\frac{1-\alpha}{\mu}(1 + ln\mu) + \alpha] \tag{4-27}$$

where K_o is the pre-yielding stiffness and α the ratio of post to pre-yielding stiffness. It has also been shown that, compared with empirical data, the effective damping based on an average stiffness and using an energy method is

$$\beta_m = \frac{2K_o(1-\alpha)\frac{1}{\mu}(\mu-1)^2 + (\pi\beta_o K_o/\mu)[(1-\alpha)(\mu^2-\frac{1}{3}) + \frac{2}{3}\alpha\mu]}{\frac{2}{3}\pi K_e(\mu)\mu^2} \tag{4-28}$$

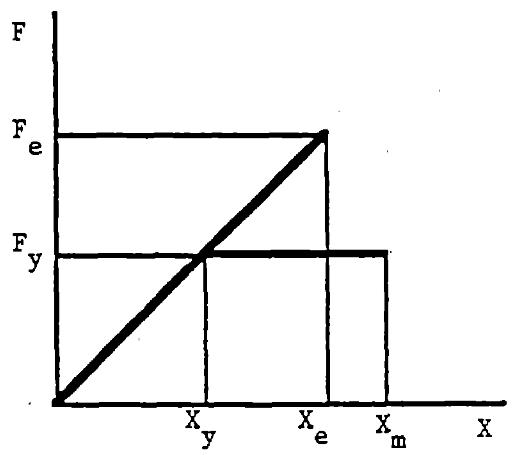


Fig. 4-1 Force-Displacement Relationship

For simplicity, consider an N-story single-bay frame structure with rigid beams and negligible column axial deformation (shear-beam type) with μ, α and K_o as interstory parameters; K_o and β_o being interstory stiffness and damping ratio, respectively. The damping ratio of the j^{th} mode of the substitute structure, β_j, can be obtained from the interstory damping ratio β_m as

$$\beta_j = \sum_{i=1}^{N}(\phi_{ij} - \phi_{i-1,j})^2 \beta_{mi} / \sum_{i=1}^{N}(\phi_{i,j} - \phi_{i-1,j})^2 \qquad (4-29)$$

where $\phi_{i,j}$ is the modal displacement at the i^{th} floor in mode j.

In order to obtain the derivatives $\frac{\partial C_E}{\partial Y}$ and $\frac{\partial K_E}{\partial Y}$ using numerical method, we need to obtain $\Delta\mu$ from ΔR. One simple energy relationship equating the energy increment due to ΔR with that due to $\Delta\mu$ is used (the two shaded areas are equal to each other in Fig. 4-2). Other approximate equivalent linearization methods, such as that presented in [209], can also be used to obtain the equivalent linearization results.

4.5 EXAMPLES

4.5.1 Elastic Case

The equipment in this example is mounted on the third floor of a six-story shear type structure (Fig. 4-3). The primary structure is the same as that of S3-1 whose dynamic properties are listed in Table 3.3.

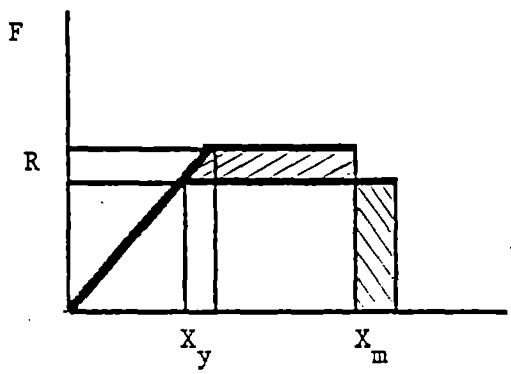


Fig. 4-2 Force-Displacement Relationship

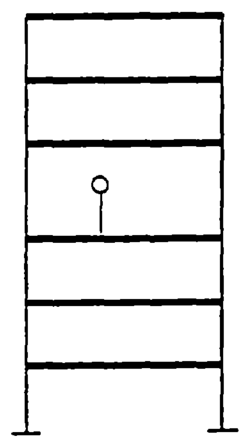


Fig. 4-3 P-S System (S4-1)

Spectral Amplitude Modification. The original FRS in this example is the average floor response spectrum when the structure is subjected to 20 artificial ground motions, SAS filtered white noise, compatible with ATC-3 acceleration peak A=400 gal, soil type S=2 response spectrum (Appendix D).

For this original FRS the following three cases are analyzed:

(1) The secondary system is tuned to the structural fundamental frequency indicated as the first peak in Table 4.1. The maximum sensitivity will occur in this case[180].

(2) The secondary system is tuned to the structural second frequency (second peak).

(3) The frequency of the secondary system is equal to the frequency corresponding to the lowest point of the floor response spectrum.

Based on design code requirements[5], let us assume that the coefficients of variance (CoV) for stiffness and mass are 0.3 and for damping ratio is 0.5. From the calculated sensitivity factor, we can obtain the means and standard deviations of modified values for the spectral peaks as shown in Table 4.1. Using Eq.(4-15) we can obtain the modified values. For example, considering first peak with only the mass uncertainties

$$\frac{E[\Delta X_1(\beta)] \pm \sigma_{\Delta X_1}(\beta)]}{X_{1o,max}} = -0.1668 \pm 0.001$$

in the design, it is more conservative to consider only $+\sigma$. Therefore, in the following figures we just show these results.

Frequency Shift Modification. Similarly, the calculated modification factors for the first through the sixth mode frequencies are listed in Table 4.2 for masses, stiffnesses,

Table 4.1 *Amplitude Modification Values for Example S4-1*

No.	Uncert.	Mass	CoV(0.3)	Stiff.	CoV(0.3)	Damp.	CoV(0.5)	Total	
		$E[\Delta X]$	$\sigma_{\Delta X}$	$E[\Delta X]$	$\sigma_{\Delta X}$	$E[\Delta X]$	$\sigma_{\Delta X}$	$E[\Delta X]$	$\sigma_{\Delta X}$
1	1st Peak	-0.1668	0.001	-0.2617	0.001	0.0457	0.1844	-0.3828	0.2091
2	2nd Peak	-0.0667	0.1033	-0.1185	0.2913	0.1129	0.2834	-0.0722	0.449
3	lowst	0.0267	0.0811	-0.0832	0.0870	0.1437	0.2659	0.0871	0.3249

Table 4.2 *Frequency Modification Values for Example S4-1*

Mode	Mass	CoV(0.3)	Stiff.	CoV(0.3)	Damping	CoV(0.5)	Total	
	$E[\Delta\Omega]$	$\sigma_{\Delta\Omega}$	$E[\Delta\Omega]$	$\sigma_{\Delta\Omega}$	$E[\Delta\Omega]$	$\sigma_{\Delta\Omega}$	$E[\Delta\Omega]$	$\sigma_{\Delta\Omega}$
1	0.0061	0.0720	-0.0389	0.0607	-0.0001	0.0003	-0.0329	0.0964
2	0.0068	0.0715	-0.0381	0.0614	-0.0001	0.0022	-0.0314	0.0969
3	0.0102	0.0706	-0.0362	0.0630	-0.0018	0.0056	-0.0278	0.0982
4	0.0123	0.0700	-0.0317	0.0661	-0.0033	0.0100	-0.0227	0.0999
5	0.0241	0.0660	-0.0198	0.0711	-0.0047	0.0148	-0.0004	0.0010
6	0.0886	0.0010	0.0450	0.0589	-0.0057	0.0178	0.1279	0.0001

damping ratios and total uncertainties. Assume the CoV for stiffnesses, masses and damping ratios are the same as above. From Eq.(4-21), the first mode frequency shift values for the mass uncertainty case are

$$\frac{E[\Delta\Omega_1(\beta)] \pm \sigma_{\Delta\Omega_1(\beta)}}{\Omega_1(\beta_0)} = 0.0061 \pm 0.072$$

Figures 4-4 and 4-5 show the original and modified FRS with mass and stiffness uncertainties, respectively. Considering the first peak point, this is a tuned case, and any change of the mass or the stiffness will cause the frequency shift of the primary structure from the tuned case, leading to a reduction of the secondary system response. Obviously, this is due to the interaction between the primary system and secondary system.

It is known that the recommended modification value in the design code is 15%[1]. From our calculations, this value is close to the mean plus 2σ for most modes following the probabilistic approach.

The damping uncertainties may cause either an increase or a decrease of the amplitude. One safe design rule is to increase the amplitude (Fig. 4-6). In general, smaller frequency shift is caused by damping uncertainties as compared to those caused by mass or stiffness uncertainties.

If all masses, stiffnesses and damping ratios are uncertain, the corresponding modification values are -17.38% at first peak, +37.62% at second peak and +41.20% at the lowest peak in amplitude, noticing that the first peak is reduced. The resulting frequency shifts are +6.35% and -12.93% (first mode) and +6.55% and -12.83% (second mode).

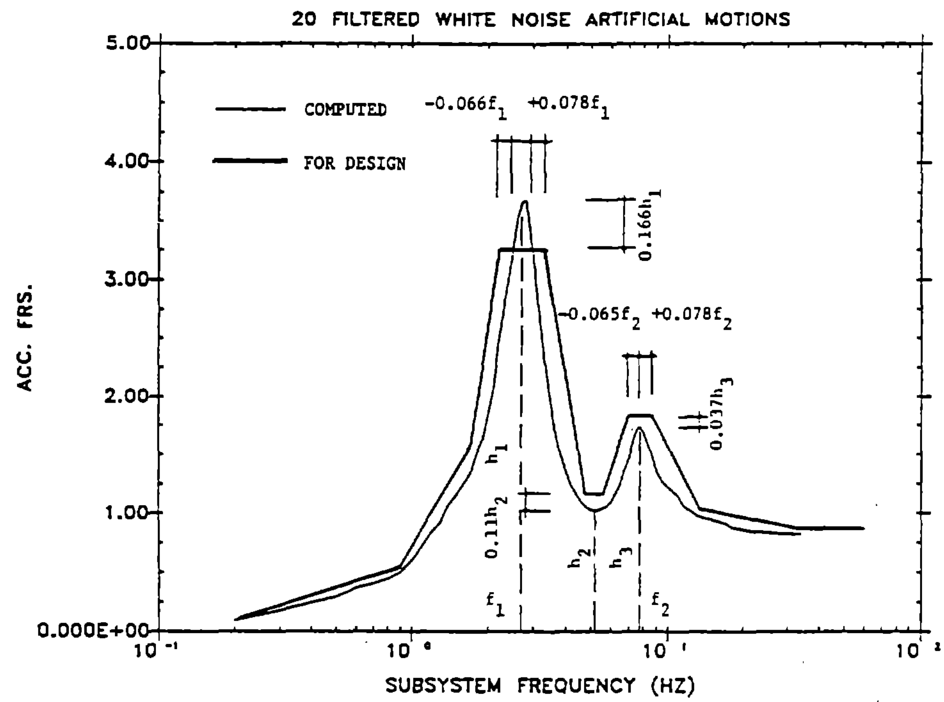


Fig. 4-4 Computed and Design FRS, Uncertain Masses

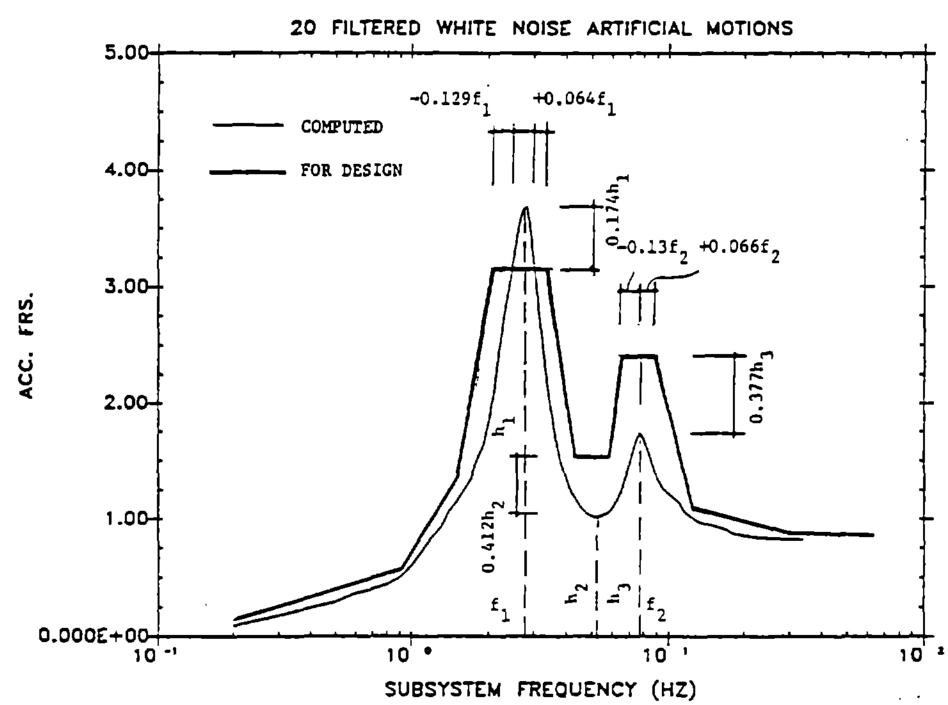


Fig. 4-5 Computed and Design FRS, Uncertain Stiffnesses

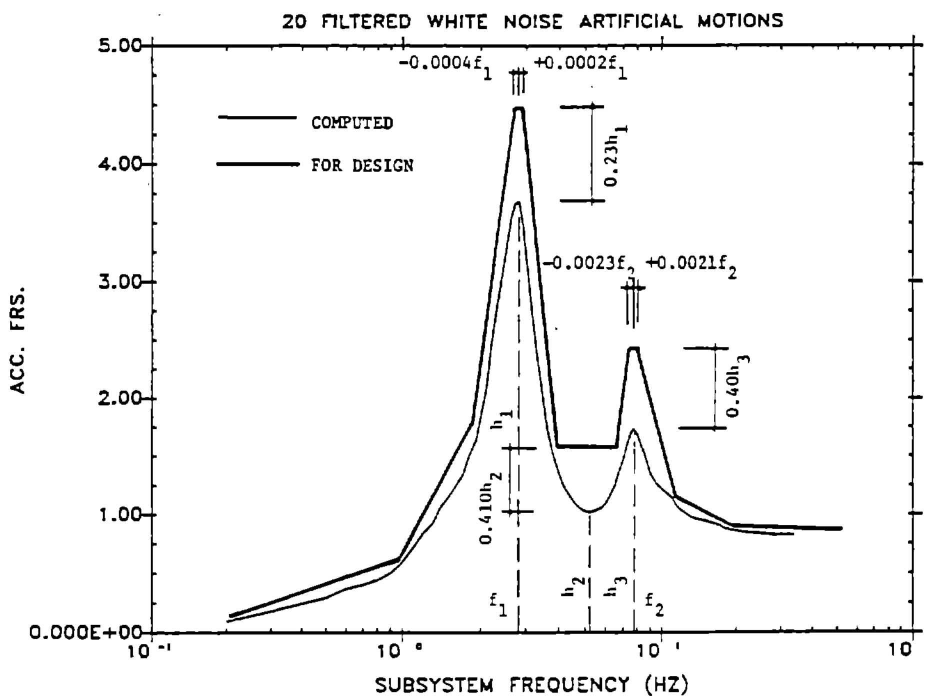


Fig. 4-6 Computed and Design FRS, Uncertain Damping Ratios

In accordance with these amplitude modification values of the three peaks and the frequency shift values, the final design FRS curves are plotted in Fig. 4-7.

4.5.2 Inelastic Case

We now investigate the same primary system subjected to the same artificial ground motion (filtered white noise) in which first story columns undergo inelastic deformation (assuming ductility ratio $\mu = 4$). The tri-linear restoring force model with deteriorating stiffness is used (Fig. 4-8).

The resulting FRS curves for the inelastic primary system and the elastic secondary system are shown in Fig. 4-9. If yielding strength level of the first story is uncertain, the FRS curve should be modified not only in spectral amplitude but also in frequency as shown in Table 4.3 and Fig. 4-10.

4.6 SUMMARY

An FRS modification method based on the response and frequency sensitivity analysis of secondary systems to primary structural uncertainties is suggested. General conclusions are:

1) Instead of relying upon an engineer's experience judgment or upon a simple determination analysis, this approach gives a quantitative computational procedure by means of stochastic analysis.

2) Instead of considering only the frequency shift in the current code [1], both frequency shift and amplitude modification are accounted for. This provides a more complete FRS modification if the structure has uncertainties.

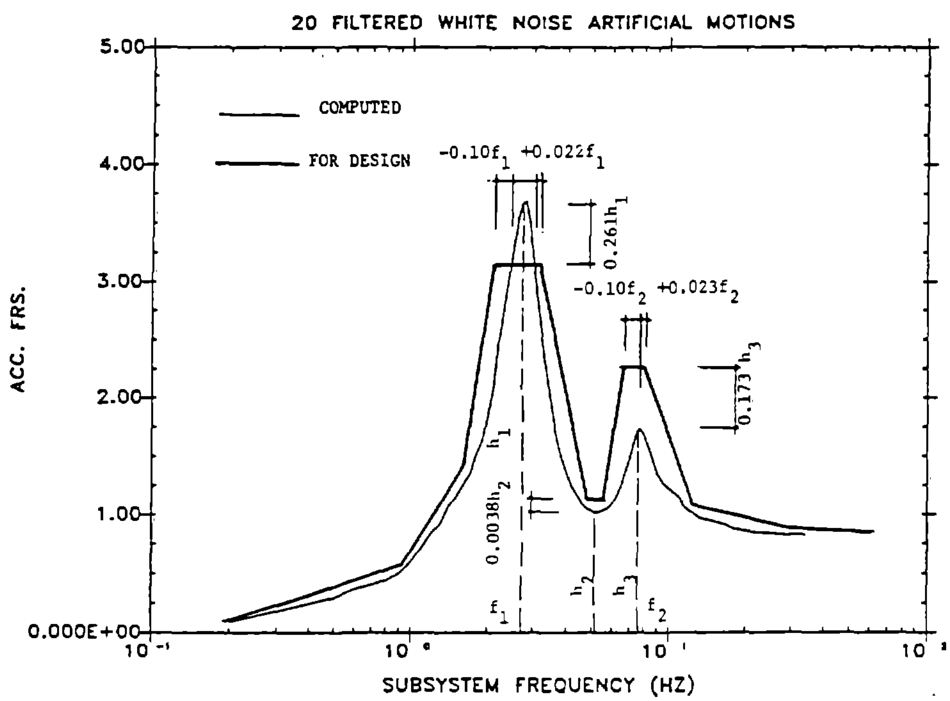


Fig. 4-7 Computed and Design FRS,

Uncertain Masses, Stiffnesses, and Damping Ratios

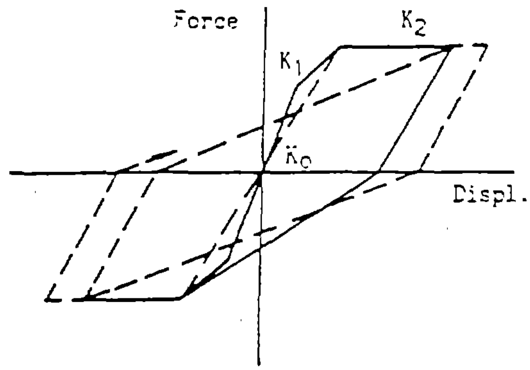


Fig. 4-8 Restoring Force Model

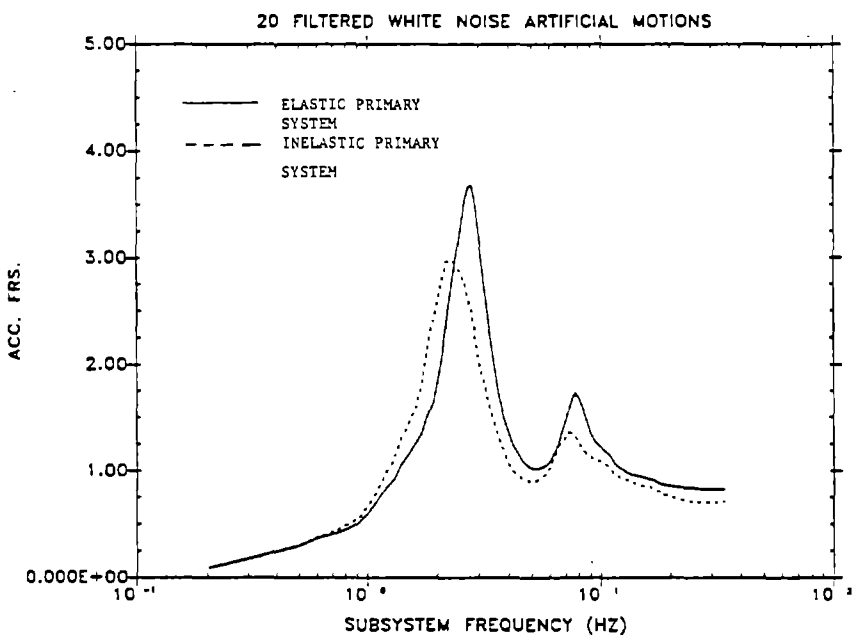


Fig. 4-9 FRS of Linear and Nonlinear Primary Systems

Table 4.3 *Modification Values of Response and Frequency for First-Story Yielding Primary System*

	ΔX			$\Delta\Omega$	
	1st Peak	2nd Peak	Lowest Peak	1st Freq.	2nd Freq.
Mean	-0.01154	0.6390	0.6003	0.0855	0.0708
σ	0.3604	0.2110	0.4885	0.2048	0.2051

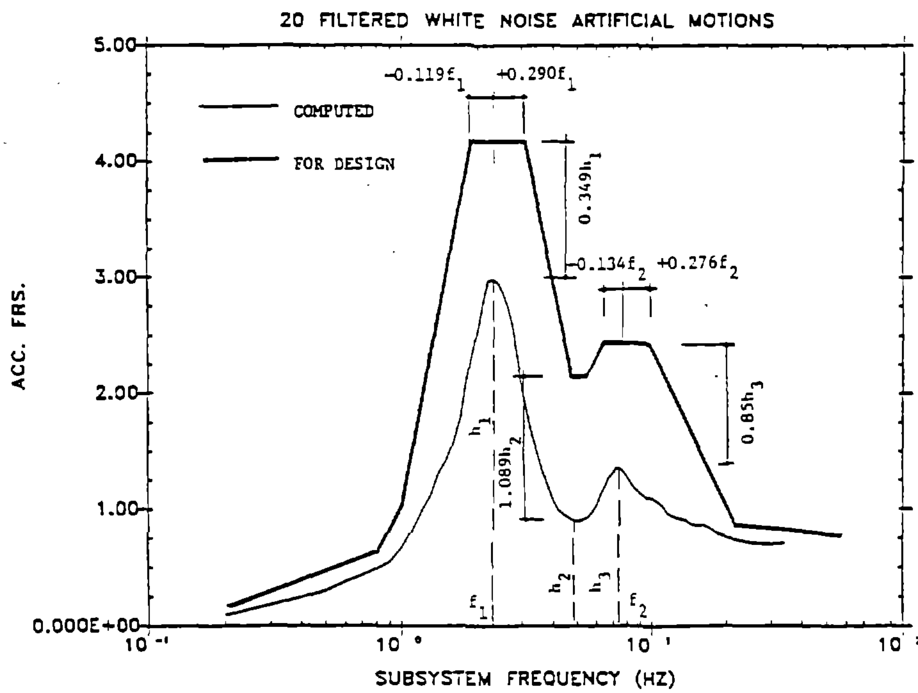


Fig. 4-10 Computed and Design FRS for Uncertain Yielding Level

3) Major deficiencies of the decoupled analysis, which is used by the current code in most cases, such as neglecting the interaction effect between the primary and secondary systems and neglecting nonclassical damping, are rectified through modification of the decoupled FRS.

4) In this analysis, both uncertainties of dynamic properties in elastic structures and uncertainties in yielding level in inelastic primary structures can be considered.

5) In the numerical examples, probabilistic significance of the modification values in the current design code is given.

6) Upon simplifying the system to a SDOF-SDOF Primary-Secondary system, some simple results and a suggested approach of FRS modification is advanced.

Table 4.4 *Comparison between Conventional and New FRS Broadening Approaches*

Conventional	Suggested
Experience judgement	Quantitative computation
Deterministic analysis	Stochastic analysis
Frequency shift	Frequency shift and amplitude modification
Neglect Interaction	Account for interaction
Neglect nonclassical damping	Account for nonclassical damping

SECTION 5

RESPONSE OF SECONDARY SYSTEM TO NONLINEAR PRIMARY SYSTEMS

5.1 INTRODUCTION

The nonlinear floor response spectrum together with a method of spectrum modification has been introduced in the preceding section. This is sufficient for design of secondary systems in the case of a yielding primary system. However, it is necessary for designers to better understand response behaviour of secondary systems, especially in relation to known differences between elastic and inelastic primary systems. This information can lead to response evaluations of secondary systems placed on a nonlinear primary structure based on their response on a corresponding linear primary structure.

Some results in this direction have been reported in the literature, most of them utilizing time history integration to obtain numerical solutions. Lin and Mahin[107] simplified the MDOF primary-secondary system to a SDOF-SDOF system. Their analytical results suggest that the effect of inelastic deformations of the structure is to lower the spectra. However, some MDOF analyses [21,123,153] have found that equivalent SDOF models of nonlinear MDOF structures often lead to inaccurate representations of secondary system response. The calculated results based on MDOF systems directly without reduction of the number of degrees of freedom showed that the spectra at high frequencies (i.e., frequencies greater than the structure's fundamental mode frequency) are occasionally increased and sometimes quite substantially, where maximum floor response spectra values of two to three times the corresponding linear-structure values can be observed. Reference [21] presents the results for a simple 10-story shear beam model subjected to 45 different input time histories. A clear observation is that, for some input records, nonlinear behavior causes an increases in high frequency secondary system response.

Sewell and Cornell[153] made a somewhat more systematic effort in examining the influence of ground motion and MDOF primary structural nonlinearity on the response of secondary systems. They discussed the effect of various structural parameters and ground motions in detail and confirmed the high frequency amplification phenomenon which can be understood as being "internally induced high frequency motion". Obviously, their conclusion is easier to understand. When we analyze and test base isolation systems, with their associated reduced frequency and increased damping, increased high-mode response can be observed. This phenomenon is somewhat similar to the case of inelastic primary systems.

However, for this very difficult and important engineering problem, it is not enough that we only know that the high frequency amplification phenomenon "occasionally" occurs, but also know clearly cases in which the values of the floor response may be increased when yielding occurs in the primary structure. Thus, a more generalized approach to determining the response amplification phenomenon is necessary. For this purpose, an approximate formulation for determining the floor response spectrum in both linear and nonlinear primary structures is first derived, which is based on stochastic analysis and the substitute elastic structure method. Then, using this approach and a numerical time history integration method, the response of the primary-secondary system with a yielding primary structure is investigated. The emphasis is placed on the relationship between responses of secondary system before and after yielding in the primary structure. In order to establish a safe design criterion, focus is placed on the analysis of response amplification after primary structural yielding. Finally, some comments on design are advanced.

5.2 AN APPROACH TO DETERMINATION OF INELASTIC FLOOR RESPONSE SPECTRUM

The attempt here is to establish an approximate analytical approach which can be utilized to obtain the floor response spectrum (FRS) directly from ground motion characteristics in the case of both linear and nonlinear primary systems. Several methods in this direction have been presented in the linear case[163,198]. A direct and simplified approach for the nonlinear case based on stochastic analysis and the substitute linear system method is developed in this section. The procedure is as follows:

(1) In accordance with frequency domain analysis, the power spectral density function (PSDF) of the acceleration response process is obtained from PSDF of the associated ground motion.

(2) The acceleration floor response spectrum of interest is derived from PSDF of the acceleration response process.

(3) The acceleration floor response spectrum is used to solve the nonlinear primary system problem by finding an equivalent elastic system as a substitute for the target nonlinear system.

5.2.1 Floor Response Power Spectrum

As mentioned in Section 2.2, the power spectrum density function of the floor response of interest is obtained by the following equation:

$$S_0 = |H(\omega)|^2 S_{FF} \tag{5-1}$$

where

$$H = -(\omega^2 M + j\omega C + K)^{-1} M \underline{r} \tag{5-2}$$

Obviously, the resulting floor PSDF reflects the characteristics of the primary system. As an example, Fig. 5-1 shows the third-floor PSDF of a six-story shear beam system.

5.2.2 Floor Power Spectrum and Response Spectrum

Earthquake-induced ground motions are generally characterized as stochastic processes. A rational approach to seismic response analysis must therefore be based on a stochastic model of ground motion. When a set of ground motions is modeled as a stochastic process, the relationship between PSDF of the ground motion process and the associated response spectrum with a probability exceedance level (for example, a mean response spectrum or a 40% probability of exceedance) can be established. Rosenblueth and Bustamante[142] established a relationship between the maximum energy of a secondary system and the constant power spectrum assumed for the earthquake excitation. Housner and Jennings[73] later utilized this relationship to obtain the PSDF of the earthquake acceleration process from the associated average undamped velocity spectrum. Two significant methods linking ground motion PSDF to structural response are given by Vanmarcke[198] and Kaul[89]. Here, these methods are used for linking floor motion PSDF to floor response spectrum.

Vanmarcke's Approach. The frequency content of the support point motion on the primary system can be described by a (time-dependent) spectral density function $S_0(\omega, t)$.

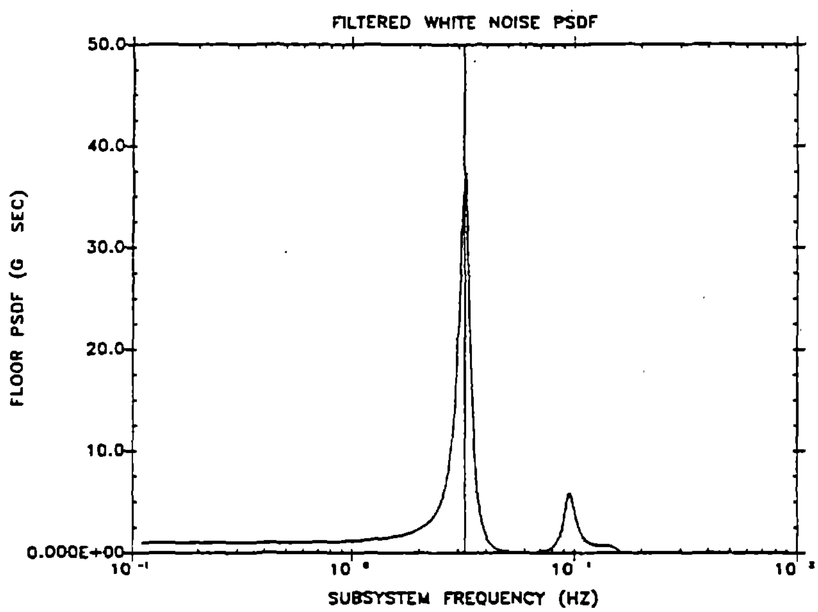


Fig. 5-1 Floor Power Spectral Density Function

In the random vibration analysis, the time-dependent spectral density function of secondary system response is expressed by

$$S_s(\omega,t) = |H_s(\omega,t)|^2 S_0(\omega,T) \tag{5-3}$$

where T is the duration of ground motion. The transient squared amplification function for the secondary system can be approximated by

$$|H_s(\omega,t)|^2 = [\omega_s^2 - \omega^2)^2 + 4\zeta_{st}^2\omega_s^2\omega^2]^{-1} \tag{5-4}$$

where

$$\zeta_{st} = \zeta_s(1 - e^{-2\zeta_s\omega_s t})^{-1} \tag{5-5}$$

and ω_s and ζ_s are, respectively, the frequency and damping ratio of the secondary system. Our attention is focused on the response variance at its peak value when $t = T$. Working in terms of the pseudo-acceleration response variance, it can be approximated by

$$\begin{aligned}\sigma_s^2(T) &= \omega_s^4 \int_0^\infty S_s(\omega,T)d\omega \\ &\simeq \int_0^{\omega_s} S_0(\omega,T)d\omega + S_0(\omega_s,T)\omega_s[\frac{\pi}{4\zeta_{sT}} - 1]\end{aligned} \tag{5-6}$$

As mentioned in Section 3.2, the pseudo-acceleration response spectrum and response variance have a proportional relationship as given in Eq.(3-21). Equations (5-6) and (3-21) can be used to obtained the floor response spectrum from the corresponding ground motion PSDF.

Kaul's Approach. A Gaussian stationary acceleration process is assumed here. The relationship between PDSF of the secondary system response and PSDF at the support point becomes

$$S_s(\omega, \omega_s) = |H_s(\omega, \omega_s)|^2 S_0(\omega) \qquad (5-7)$$

where

$$|H_s(\omega, \omega_s)|^2 = \frac{1 + 4\zeta_s^2(\frac{\omega}{\omega_s})^2}{[1 - (\frac{\omega}{\omega_s})^2]^2 + 4\zeta_s^2(\frac{\omega}{\omega_s})^2} S_o(\omega) \qquad (5-8)$$

Instead of obtaining the relationship between the response PSDF and response spectrum through response variance (Eq. 3-21), a probability density function for the value $X_{max}(\omega)$ of a local maximum of the secondary system time history and spectral moments are used here [23]. The probabily density function has the form

$$P(X_{max}) = \frac{1}{\sqrt{2\pi\lambda_0}}[\varepsilon E(\eta/\varepsilon) + a\varepsilon E(\eta)\int_{-\infty}^{a} E(u)du] \qquad (5-9)$$

in which

$$E(u) = exp(-\frac{1}{2}u^2) \qquad (5-10)$$

$$\eta = X_{max}/\sqrt{\lambda_0} \qquad (5-11)$$

$$\varepsilon = \sqrt{1 - \lambda_2^2/\lambda_0\lambda_4)} \qquad (5-12)$$

$$a = \eta\sqrt{1 - \varepsilon^2}/\varepsilon \qquad (5-13)$$

and the spectral moments are

$$\lambda_n = \int_{-\infty}^{\infty} \omega^n S(\omega, \omega_s)d\omega \qquad n = 0, 2, 4 \qquad (5-14)$$

Note that $0 \leq \varepsilon \leq 1$ and that λ_n, ε and η are functions of the secondary frequency ω.

The acceleration response spectrum value $X_{max,p}(\omega)$ at the secondary system frequency ω can be obtained from the spectral moment as

$$X_{max,p}(\omega) = \{-2\lambda_0 ln[\frac{-\pi}{T}(\lambda_0/\lambda_2)^{\frac{1}{2}} ln(1-r)]\}^{\frac{1}{2}} \quad (5-15)$$

where r is the probability of exceedance of response spectrum.

An approximate procedure results in a direct inverse relation between the floor response spectrum and the associated PSDF at the support point, which is

$$S_0(\omega) = \frac{2\zeta_e}{\pi\omega} X^2_{max,p}(\omega)/\{-2ln[\frac{-\pi}{\omega T} ln(1-r)]\} \quad (5-16)$$

Both Vanmarcke's and kaul's methods are pretested to obtain the floor response spectrum of the P-S system from a ground motion PSDF. Roughly, the approximate results match the numerical results, although errors exist between analytical and numerical results. This is shown in Fig. 5-2 using Vanmarcke's approach. Since qualitative analysis is more important in this section, we will use Vanmarcke's method of analysis.

5.2.3 Substitute Structure

The method of substitute structure is indicated in Sec. 4.3. Here, the same method is used.

5.3. NUMERICAL COMPUTER ANALYSIS

The errors associated with approximate methods in solving nonlinear structural systems impose various limitations on their applications. At the present, the only generally applicable method for the analysis of arbitrary nonlinear systems is the numerical step-by-step integration of the coupled equations of motion[31]. Thus, in the research on primary-

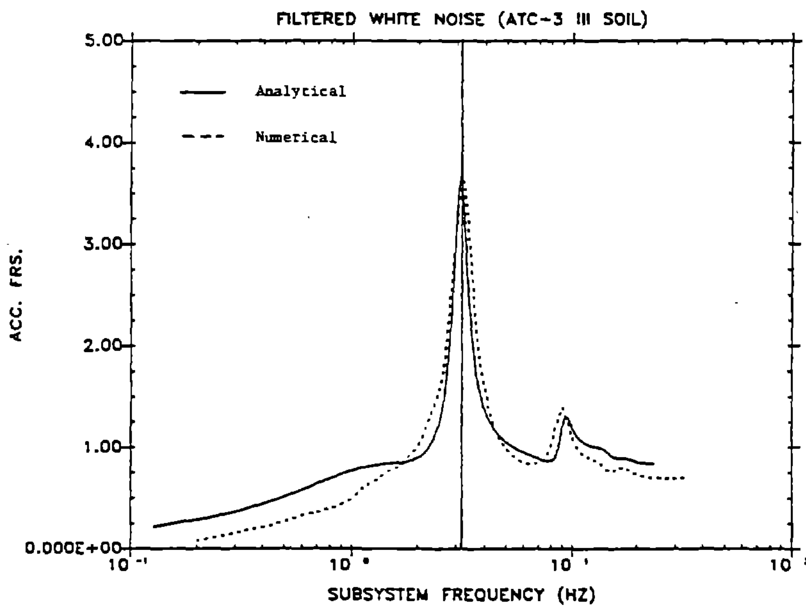


Fig. 5-2 Acceleration Floor Response Spectrum

secondary systems, numerical analysis plays an important role, although it does not often provide a general result.

A modified time history integration program THPS, which is capable of analyzing P-S systems as two connected shear type beams, is used to investigate the response behavior of inelastic primary systems.

5.3.1 Input Accelerogram

The input acceleration waves include natural earthquake records shown in Table 5.1, white noise or SAS-filtered white noise artificial ground motion generated by the ASEW computer program. The filtered white noise is formed by the Kanai-Tajimi model (see Appendix D). This is Eq.(5-8) with frequency ω_s and damping ratio ζ_s of secondary system replaced by those of soil filter ω_g and ζ_g.

5.3.2 Structural Systems

The different structural systems which are analyzed for different purposes in this section are shown in Table 5.2 and Fig. 5-3 (S5-1, S5-2).

5.3.3 Structural Parameters

The following parameters are discussed.

a) Location of the secondary system, i.e., the floor response spectra of interest.

b) Story in which yielding occurs.

c) Degree of yielding.

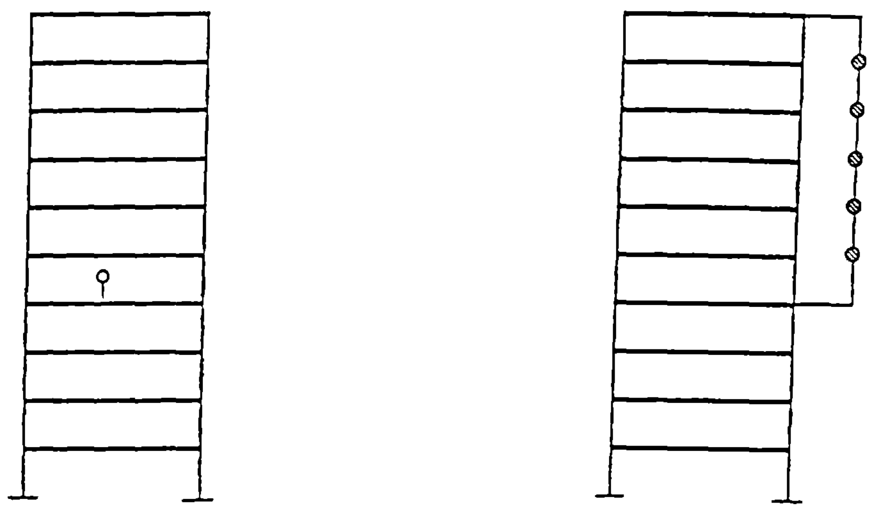
d) One-story or many-story yielding.

Table 5.1 *Earthquake Records*

No.	Earthquake	Date	Station Site	Comp.	Peak Acc. (gal)
1	Imperial Valley	5/18/1941	El Centro	S00N	341.7
2	Mexico City	9/19/1985	Secretariat	N90W	167.92
3	Kern County	7/11/1952	Taft	S69E	175.9

Table 5.2 *Structural Systems*

N0.	N.S.(P)	N.S.(S)	No. of Supports	First Three Freq.(Hz)		
S4-1	6	1	1	2.7	7.21	12.5
S5-1	10	1	1	0.75	5	10
S3-2	6	2	2	2.7	8	13
S5-2	10	5	2	0.8	8	13



S5-1 S5-2

Fig. 5-3 Examples of P-S Structural Systems

e) Type of restoring force model.

f) Type of structure.

g) Type of seismic records.

h) Simply or multiply supported secondary systems.

While these eight cases have been analyzed, their results will not be presented individually and only major conclusions are discussed in the next section.

Two sets of information are used in the analysis. One is the acceleration floor response spectra (FRS), and the other is the acceleration amplification factor in the modification of elastic floor response spectra to the inelastic case.

5.4 FACTORS INFLUENCING RESPONSE CHARACTERISTICS

As mentioned earlier, the analysis of a SDOF-SDOF P-S system has suggested that an effect of inelastic deformation in the primary structure is a response reduction in the secondary system. The present analyses show that, while inelastic response spectrum depends only on the SDOF systems including their frequency and yielding level (for example, ductilities), the floor response spectrum depends on both the SDOF systems and the inelastic primary structure. The complicated inelastic primary structural properties result in very complicated behavior of floor response spectra. As mentioned before, the problem of interest is the amplification case, i.e., the case in which the secondary system response associated with an inelastic primary system is greater than that associated with an elastic primary system. As a result of extensive numerical computation and analyses, the following three factors responsible for secondary system response amplification associated with a yielding primary structure are commented upon.

5.4.1 Lower Primary Structural Frequency

As is well known, structural yielding can cause a reduction of structural frequency and an increase in structural damping, the so-call "base isolation effect"[153]. However, there is a significant difference between the base isolation effect and the structural yielding effect. For base isolated structures, the increase in the period of isolated structures can be of the order of two to three seconds, which is far greater than the peak range of the response spectrum. In this case, even if in the tuned case, the floor secondary system response may not be pronounced. However, primary system yielding may cause only a little change in the frequency of the primary system. The floor response spectrum curve may shift only slightly from the frequency peak. In this case, the frequency of the secondary system may locate within a frequency range around a new peak. Thus, the tuning phenomenon will have great effect on the secondary system response.

The results using the approximate method developed show the above conclusion clearly. After yielding of the primary system, the peaks of both response PSDF and FRS at the third floor, the supported point, move to a lower frequency range (Figs. 5-4 and 5-5). Fig. 5-6 shows the corresponding amplification factor. As an example, structure S4-1 subjected to 20 sets of filtered white noise artificial ground motion[26] is analyzed. The third floor response spectrum (FRS) in both linear and nonlinear cases (first floor yielding displacement ratio $Ry/Re = 0.5$) and the amplification factor of FRS are shown in Figs. 5-7 and 5-8, respectively. For this system, it is clear from analytical and numerical results that the lower frequency response may increase when the lower story (lower than the floor of interest) of the primary system yields.

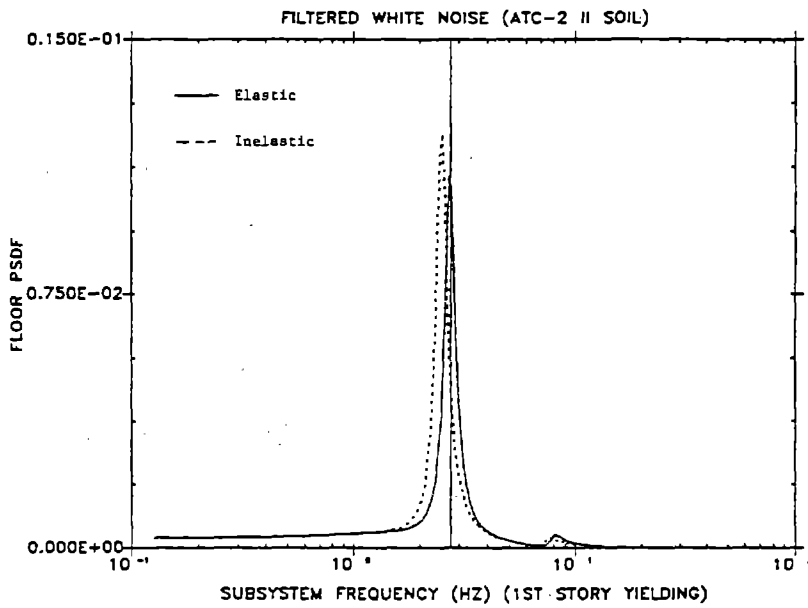


Fig. 5-4 Floor Power Spectral Density Function

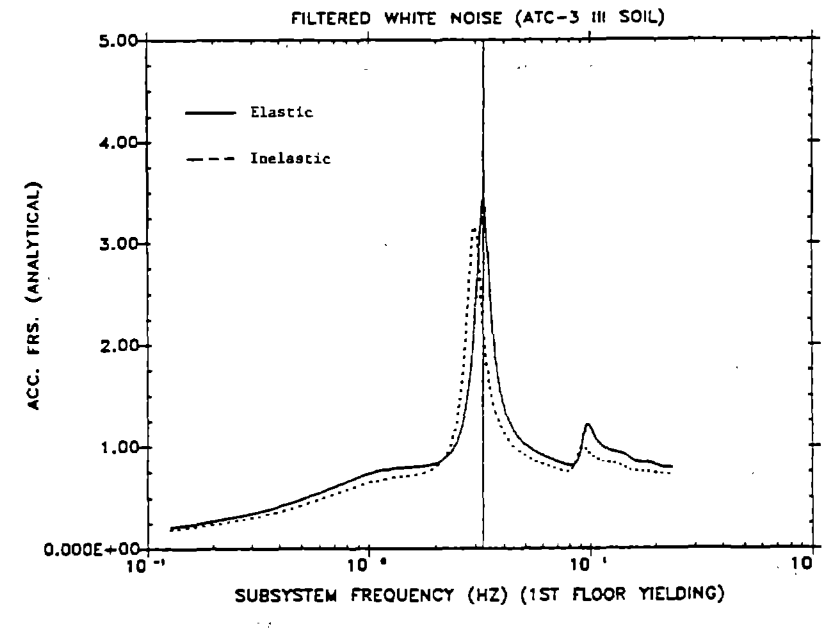


Fig. 5-5 Acceleration Floor Response Spectra

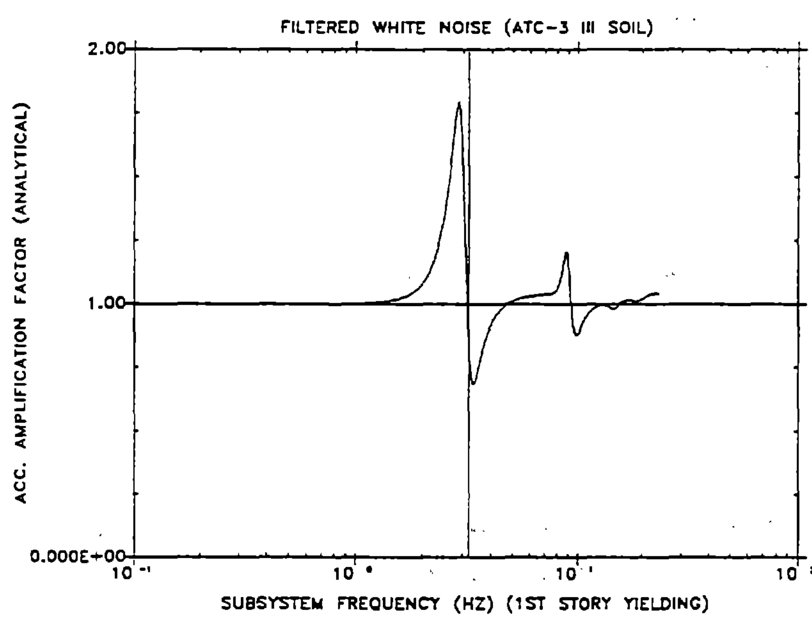


Fig. 5-6 Acceleration Amplification Factor

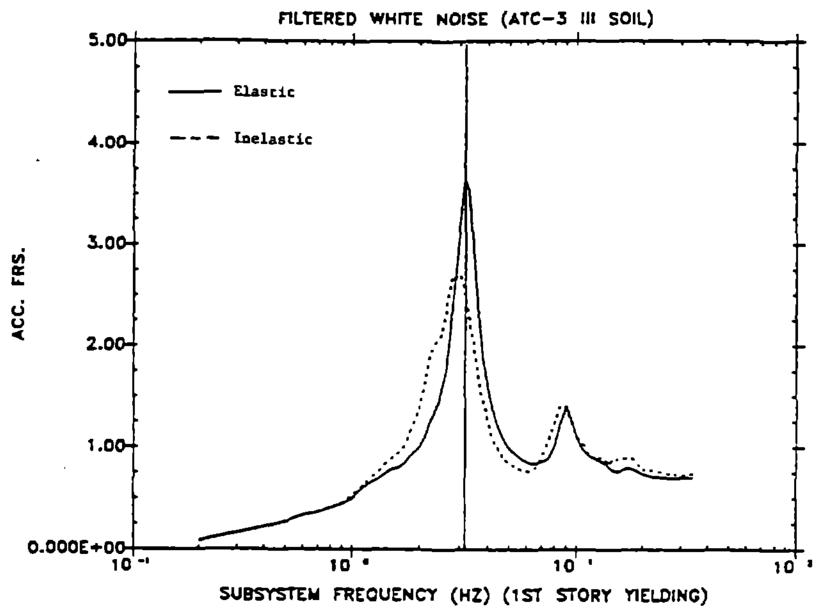


Fig. 5-7 Acceleration Floor Response Spectra

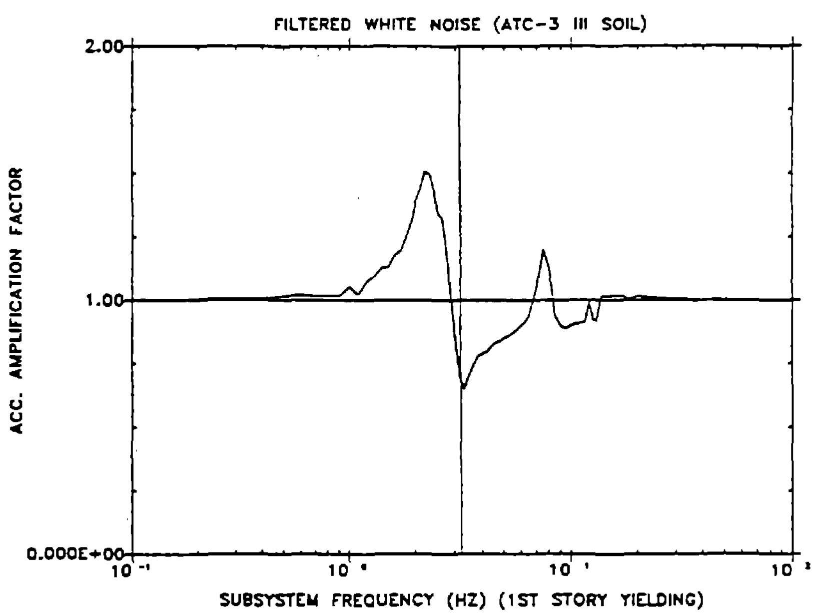


Fig. 5-8 Acceleration Amplification Factor

5.4.2 Internally Induced High Frequency Motion

Sewell and Cornell describe the reason for high frequency response amplification as follows: Upon yielding, the added force excitations internally excite higher modes in the structure, and lead to some "bumps" in the high frequency range of the FRS. Similarly, linear structures mounted on base isolation systems which incorporate hysteretic damping mechanisms (that may result in severe force-difference loadings) may lead to significant (and perhaps unanticipated) high-frequency floor motion[92]. In the above example, if the yielding floor is the fourth floor, the analytical and numerical results are shown in Figs. 5-9, 5-10 and Figs. 5-11, 5-12, respectively. Possible high frequency amplification effects can be seen in the numerical analysis as well as in experimental studies.

Actually, this result can be explained as follows: the change of stiffness and damping at a story close to the ground has a greater effect on the first mode; conversely, the change of these quantities at a story close to the top has a greater effect on the higher frequency modes.

5.4.3 Multiply Supported Secondary Systems

It is easy to understand that primary structural yielding may cause amplifying displacement responses for multiply supported secondary systems, since the primary structures have greater interstory displacement themselves. Two systems are investigated (Fig.5-3, S3-2 and S5-2) subjected to El Centro earthquake. The resulting interstory displacements for both elastic and inelastic cases are shown in Fig. 5-13 (for S3-2) and Fig. 5-14(for S5-2). The amplification effect is clearly demonstrated.

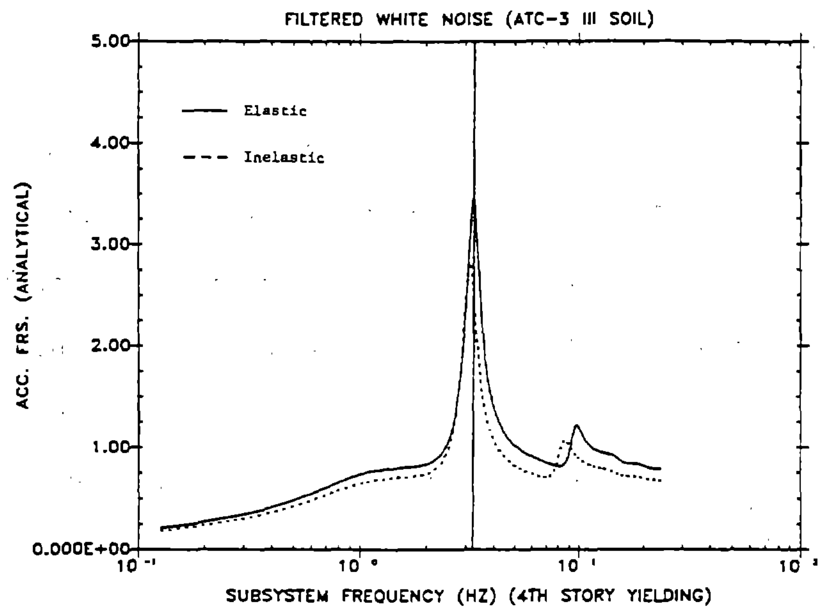


Fig. 5-9 Acceleration Floor Response Spectra

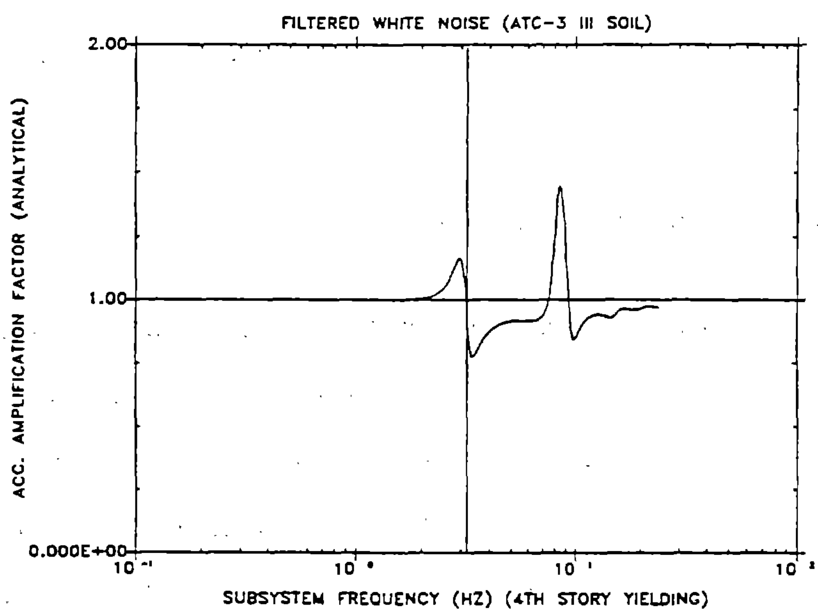


Fig. 5-10 Acceleration Amplification Factor

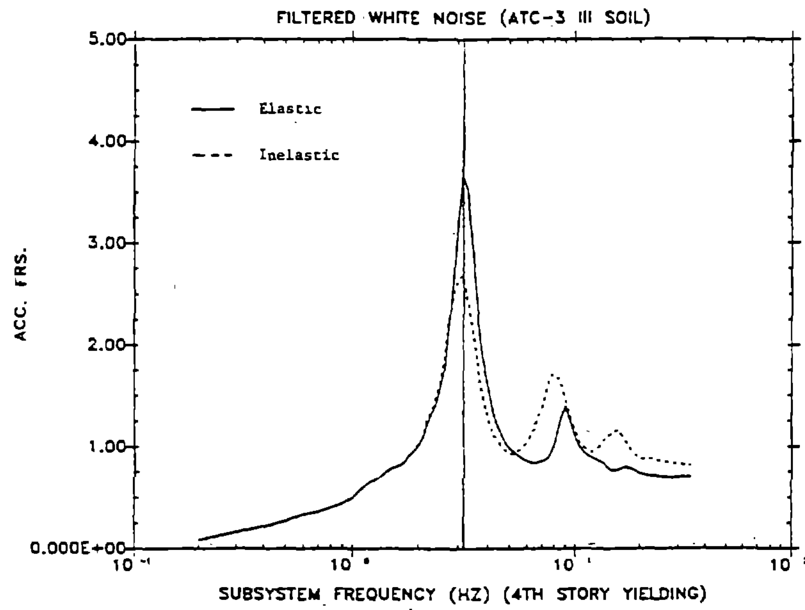


Fig. 5-11 Acceleration Floor Response Spectra

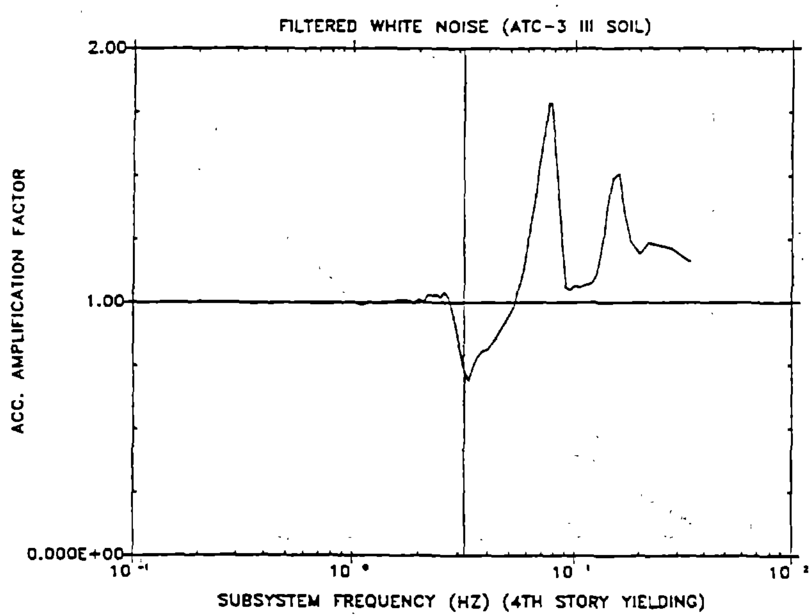


Fig. 5-12 Acceleration Amplification Factor

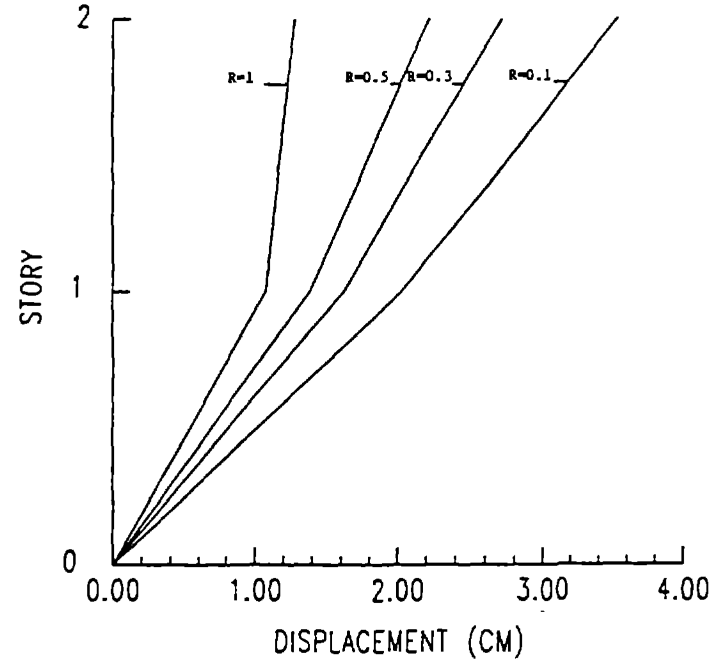


Fig. 5-13 Maximum Displacement for S3-2

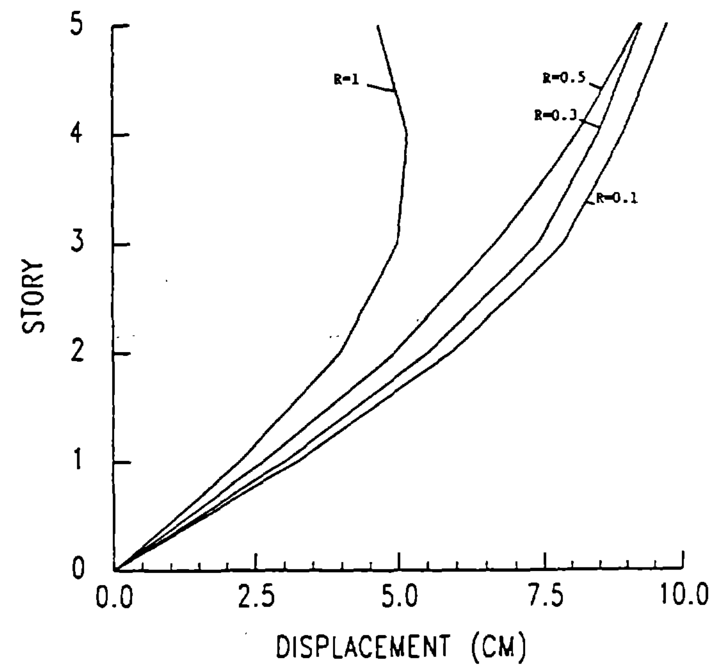


Fig. 5-14 Maximum Displacement for S5-2

5.5 PARAMETRIC EFFECTS

It is desirable to know in which cases the aforementioned three amplifications are likely to occur and in what cases the amplifications are greater. In what follows, the effects of the major parameters are analyzed by time history integration and discussed.

5.5.1 Location of FRS and Yielding Floor

For a given P-S system subjected to El Centro earthquake record, the third-floor response spectral values are investigated for various story yielding. The conclusion as shown in Figs.5-15 to Fig.5-18 is that, if the yielding point is lower than the third floor, the lower frequency subsystem (lower than 2.72 Hz) response may be amplified in the inelastic case. Conversely, if the yielding points are higher than the third floor, the short period subsystem response may be amplified. These results can not be seen in an equivalent SDOF-SDOF system analysis [107].

In addition, a different 10-story structural system (S5-1) having a fundamental frequency of 0.75 Hz is investigated. As shown in Figs. 5-19 to 5-22, the results are similar to those obtained above.

5.5.2 Degree of Yielding

System S4-1 is studied with R= 1, 0.8, 0.5 and 0.1 with different yielding strength ratios ($R = F_y/F_e$) subjected to El Centro earthquake. The results are shown in Figs. 5-23 to 5-26. Generally speaking, the more serious the yielding, the larger is the amplification.

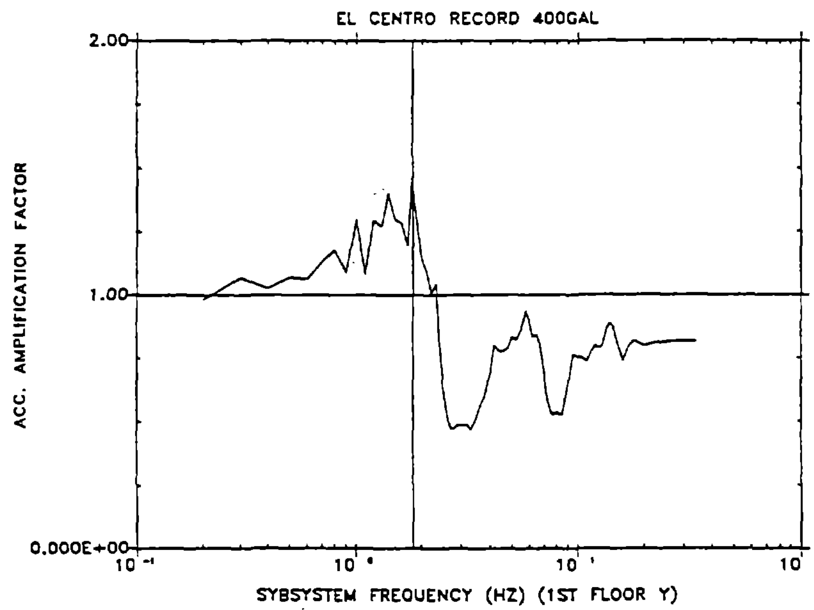


Fig. 5-15 Acceleration Amplification Factor

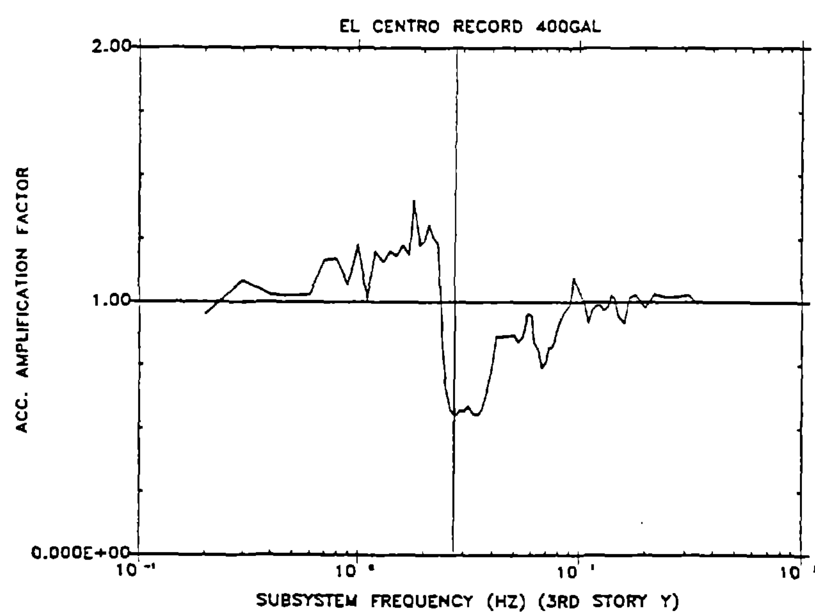


Fig. 5-16 Acceleration Amplification Factor

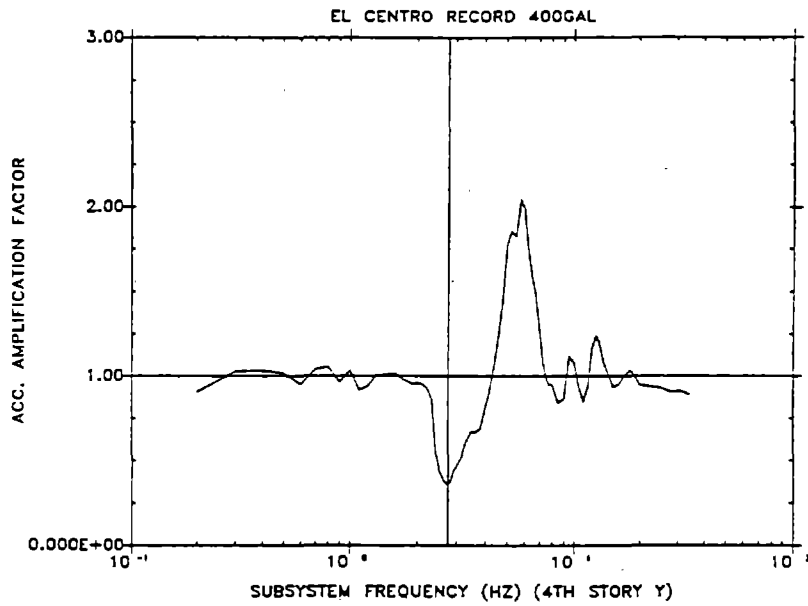


Fig. 5-17 Acceleration Amplification Factor

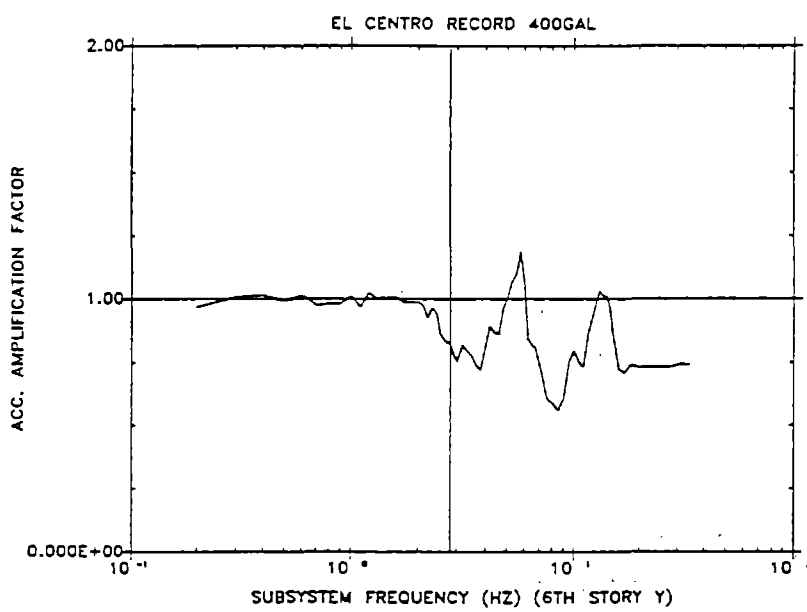


Fig. 5-18 Acceleration Amplification Factor

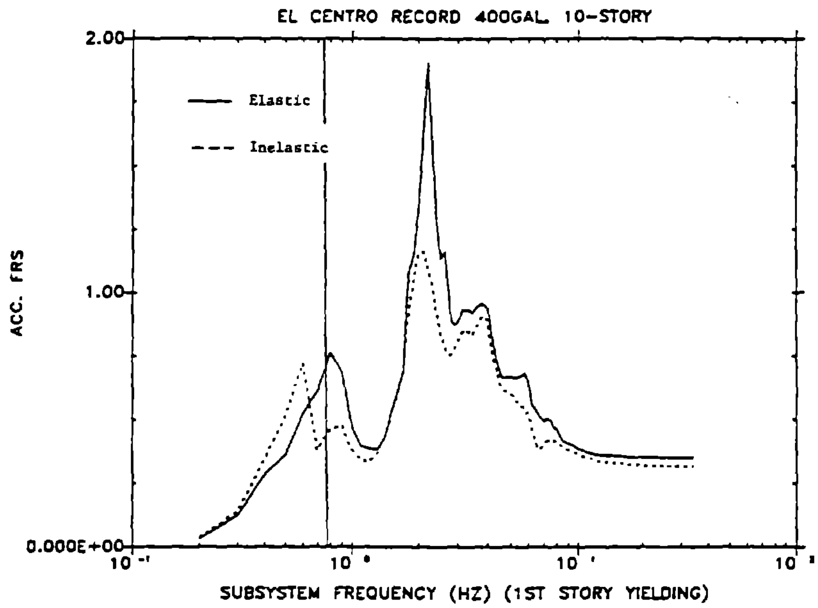


Fig. 5-19 Acceleration Floor Response Spectra

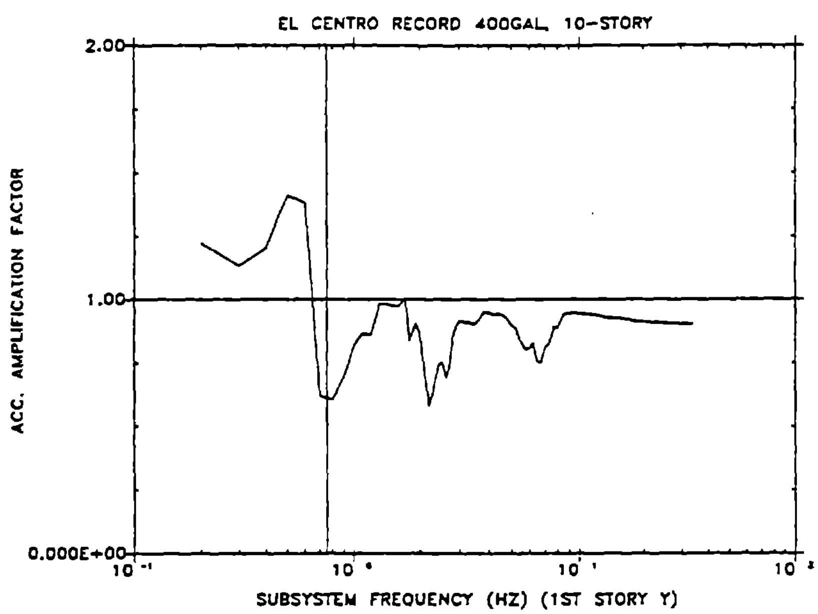


Fig. 5-20 Acceleration Amplification Factor

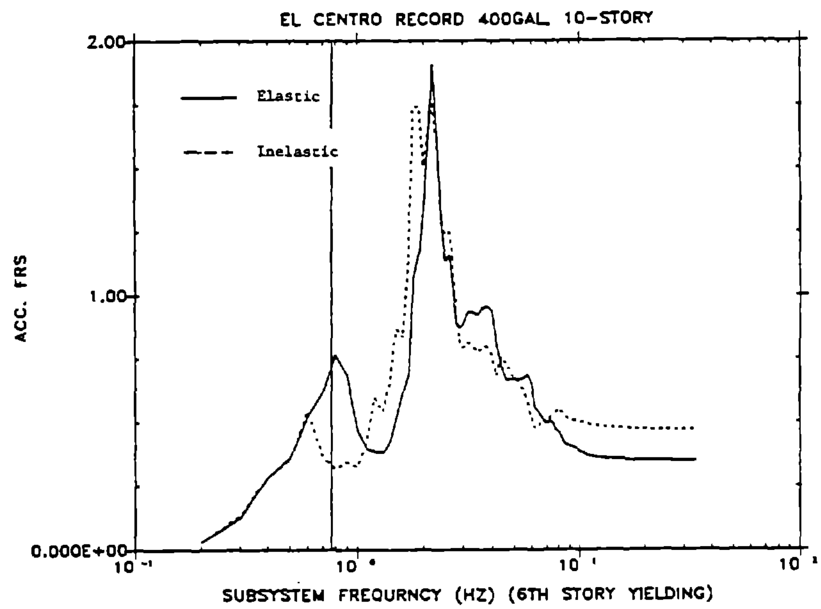


Fig. 5-21 Acceleration Floor Response Spectra

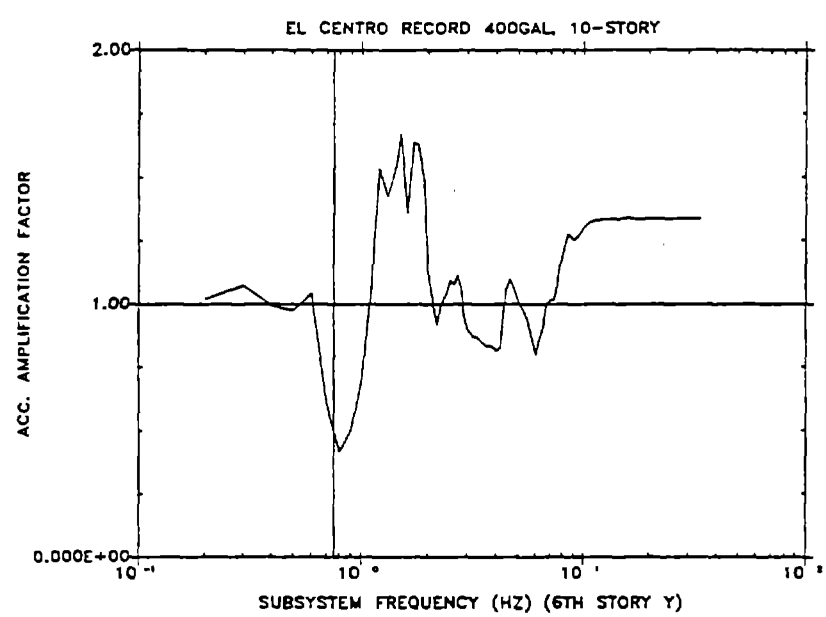


Fig. 5-22 Acceleration Amplification Factor

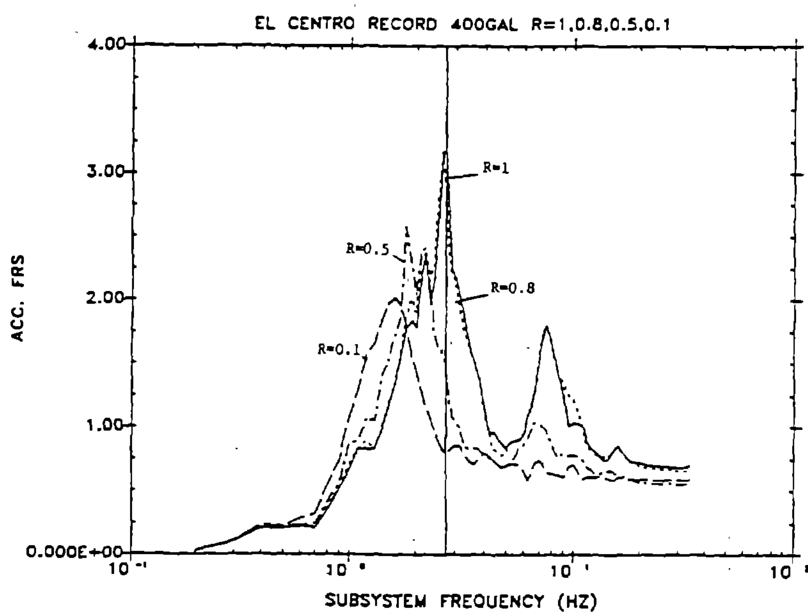


Fig. 5-23 Acceleration Floor Response Spectra

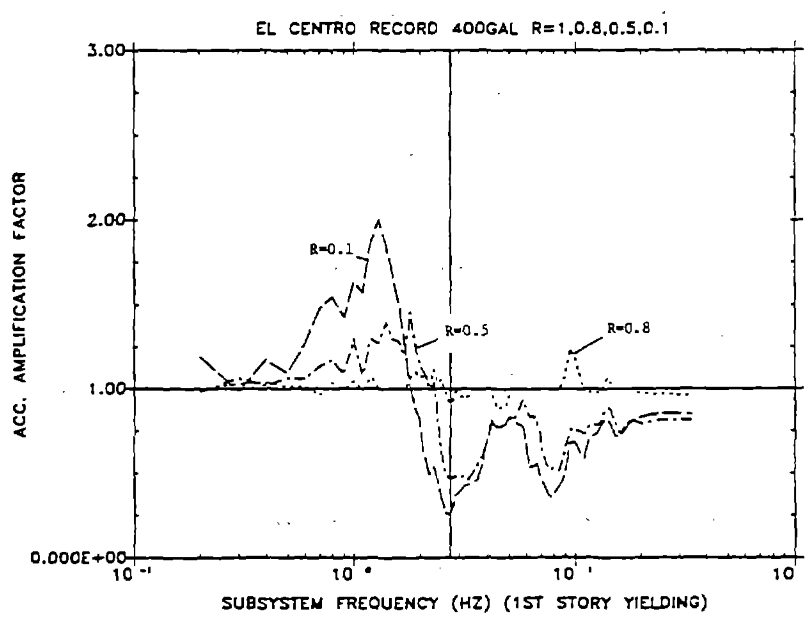


Fig. 5-24 Acceleration Amplification Factor

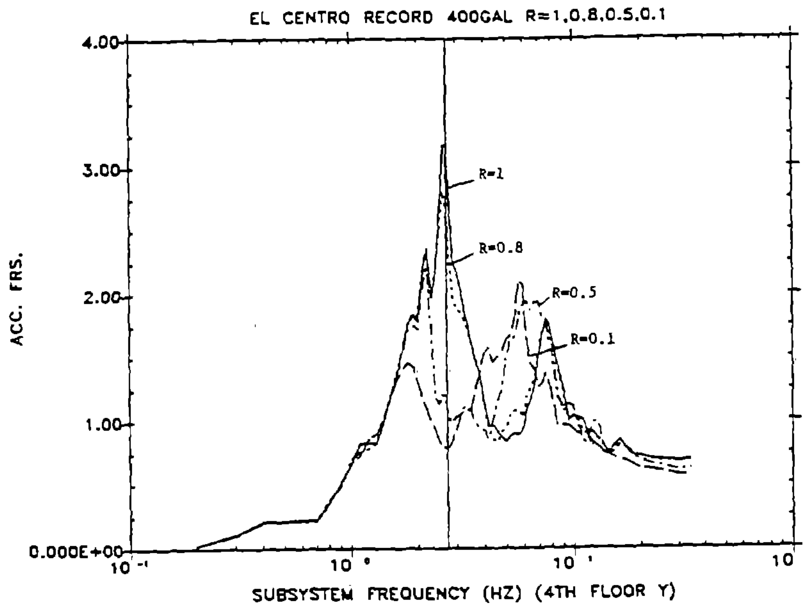


Fig. 5-25 Acceleration Floor Response Spectra

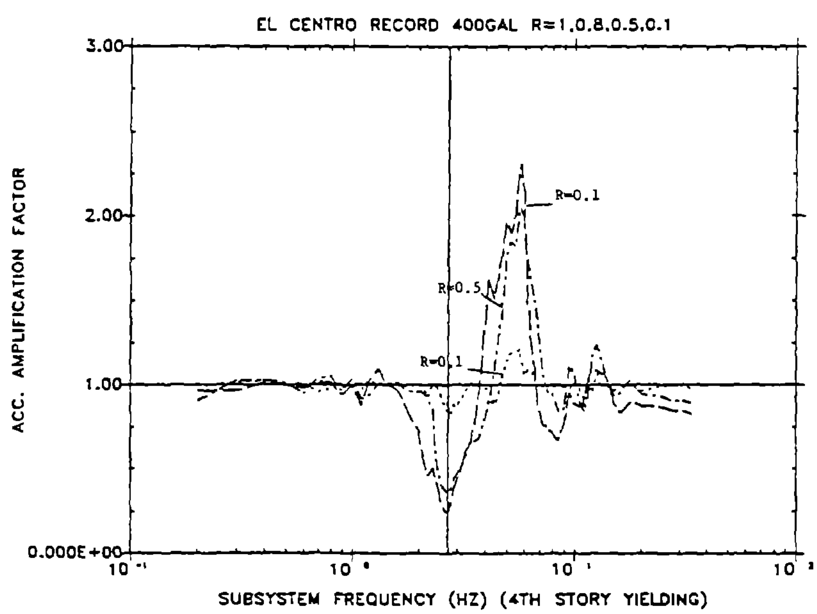


Fig. 5-26 Acceleration Amplification Factor

5.5.3 Ground Motion Parameter

The frequency content of ground motion has an impact on floor response PSDF as indicated in Eq.(5-1), hence they affect the FRS directly. However, the aforementioned three FRS amplification effects with primary system yielding still exist. Only two different earthquake records show this situation herein. The FRS and amplification factor of the above structure subjected to Taft earthquake are shown in Figs. 5-27 to 5-30, and to Mexico earthquake in Figs. 5-31 to 5-34. It is seen that, although Mexico earthquake record causes a significant low frequency response (since there exist more low frequency contents in the record), the major influence of primary system yielding is in the vicinity of structural modal frequencies.

5.6 SUMMARY AND COMMENTS RELATED TO DESIGN

1) A simple approximate approach to obtain the FRS from PSDF of ground motion has been presented. Generally speaking, the results using this method are in good agreement with numerical results.

2) The results using both the proposed analytical method and numerical methods show uniform conclusion, namely, the following three factors may cause a response amplification effect due to primary system yielding.

a) The yielding of the stories which are lower than the floor of interest of the primary structure often causes response amplification at lower frequencies.

b) Nonlinearities at higher stories of the primary systems often cause response amplification at higher frequencies. This result is of importance as applied to equipment which often oscillate at higher frequencies.

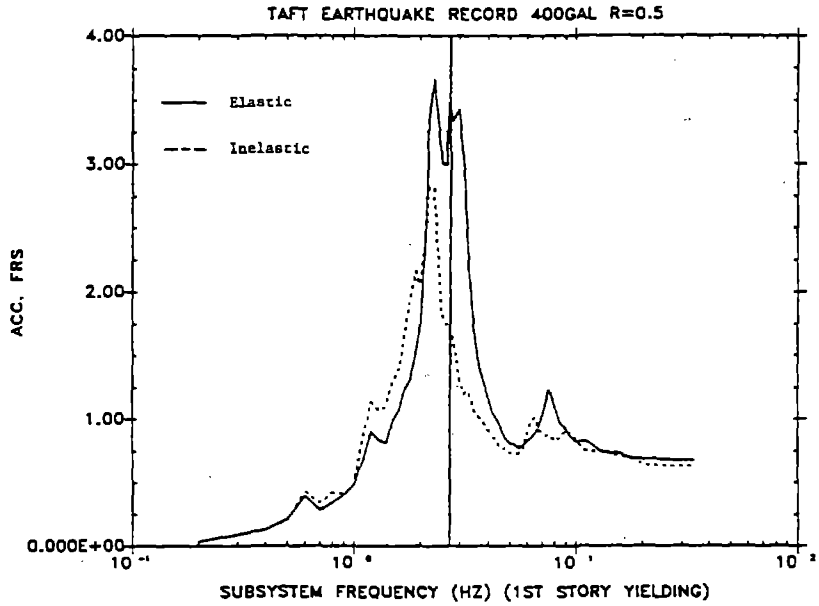


Fig. 5-27 Acceleration Floor Response Spectra

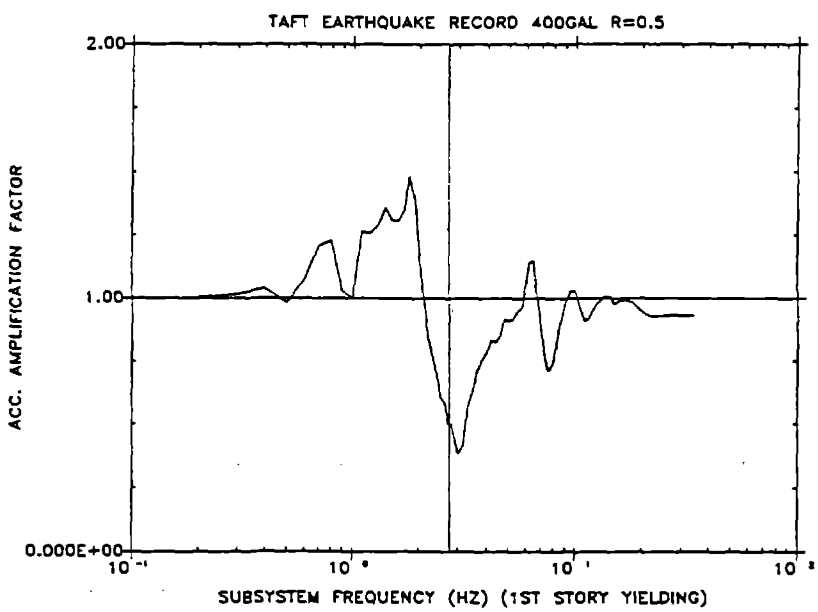


Fig. 5-28 Acceleration Amplification Factor

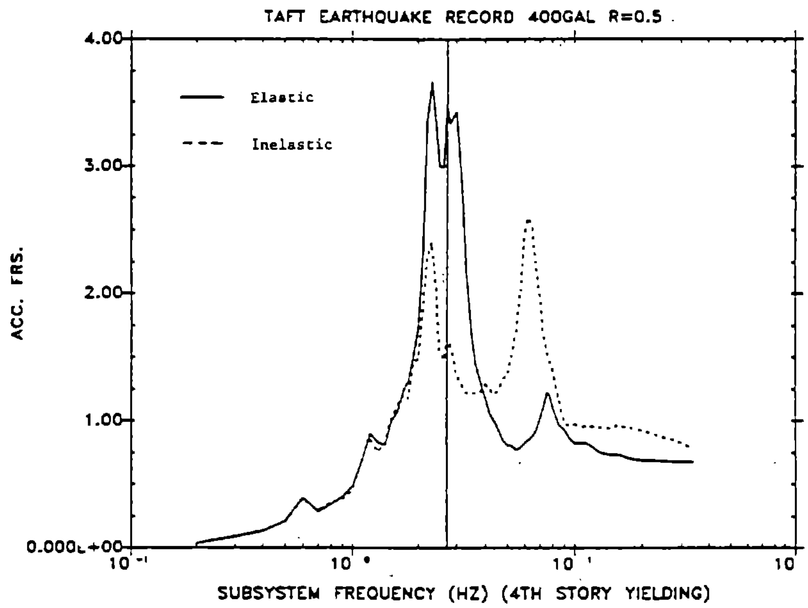


Fig. 5-29 Acceleration Floor Response Spectra

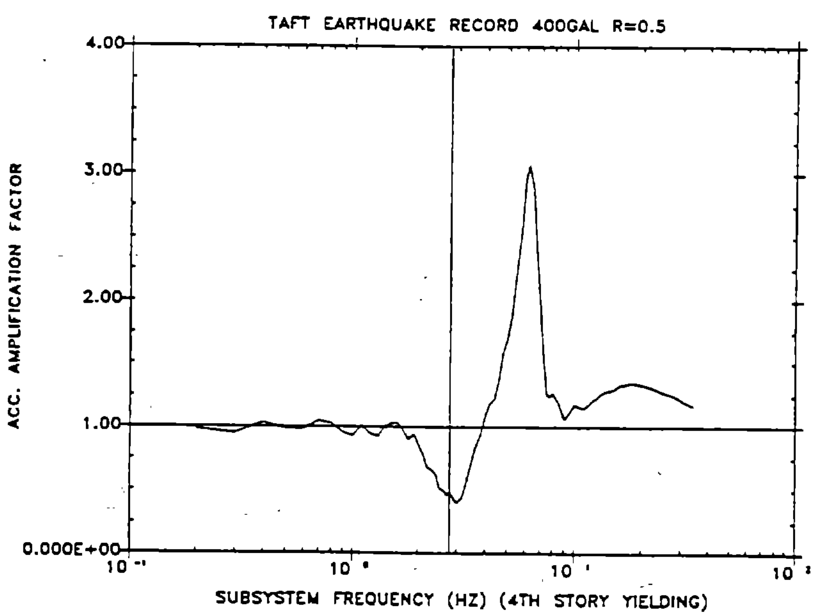


Fig. 5-30 Acceleration Amplification Factor

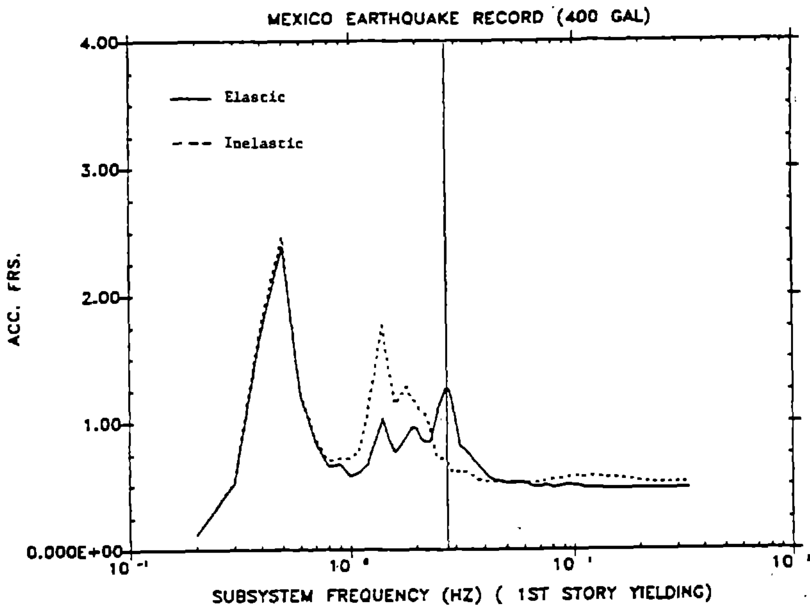


Fig. 5-31 Acceleration Floor Response Spectra

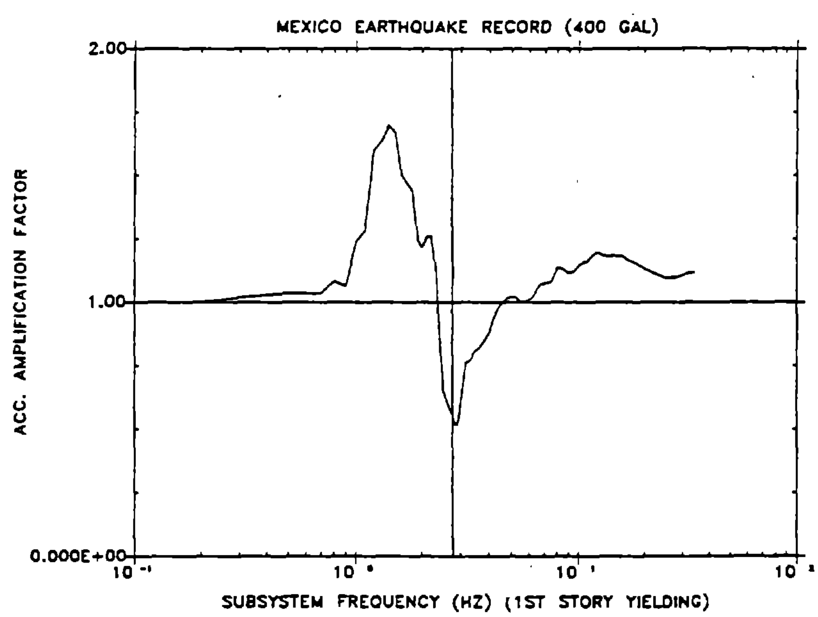


Fig. 5-32 Acceleration Amplification Factor

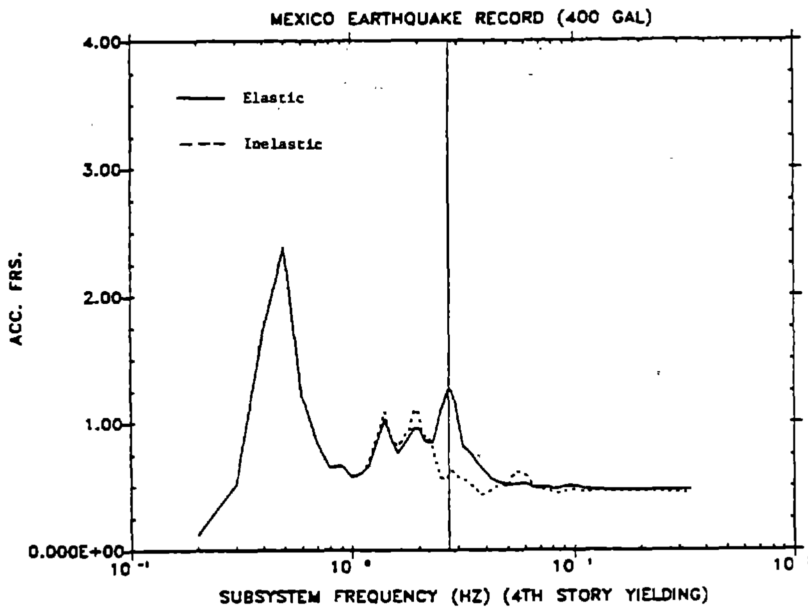


Fig. 5-33 Acceleration Floor Response Spectra

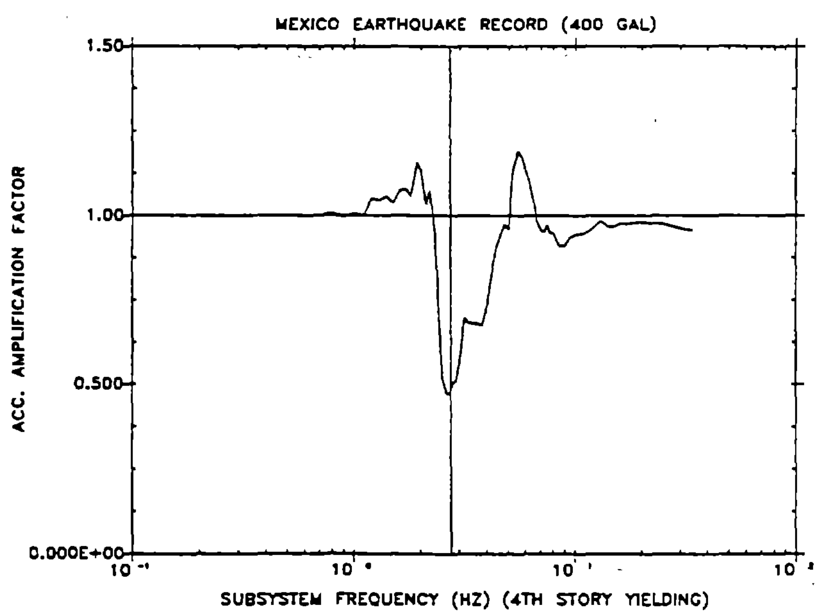


Fig. 5-34 Acceleration Amplification Factor

c) Inelastic deformation may easily cause response amplification in the multiply supported secondary systems.

3) Numerical results not only confirm the above conclusions but also provide insight into the response behaviour of secondary systems.

SECTION 6

CONCLUSIONS

The work reported in this report is aimed at developing a greater understanding of the dynamic behavior of secondary systems in a seismic environment and in developing practical criteria and procedures for the analysis and design of secondary systems.

This report has provided a comprehensive review and assessment of the state-of-the-art in the area of research and practice on seismic response of secondary systems. Included are an appraisal of current engineering practice and design, an update of recent advances, and a discussion of possible future research directions.

A more comprehensive sensitivity analysis has been conducted. Sensitivity factors based on spectral moments are defined for the study of response sensitivity of structural systems to parameter uncertainties under random excitations. The use of spectral moments is suggested as peak response sensitivity indices which provide a quantitative and consistent measure of relative importance of parameter uncertainties in the design of secondary systems. The sensitivity measures derived for eigenvalues and eigenvectors are useful in the modification of floor response spectrum. These results can be particularly useful to designers in order to evaluate relative importance of parameter uncertainties in the primary structure and to determine the desired dynamic characteristics of secondary systems.

In design and performance issues, a more rational quantitative procedure based on the sensitivity study has been proposed which determines the amount of variation of the floor response spectrum due to structural parameter variations. The interaction between the primary structure and secondary systems and nonclassical damping are taken into account as well. Moreover, the effect of nonlinear primary structural behavior has also been considered using the method of equivalent linearization.

In the last part of this report, an approximate formulation for determining the floor response spectrum for both linear and nonlinear primary structures has been presented, which is based on stochastic analysis and the substitute elastic structure method. Then, using this approach and a numerical time history integration method, the response of the primary-secondary system with a yielding primary structure has been investigated. The emphasis is placed on the relationship between secondary system responses before and after yielding occurs in the primary structure. In order to establish a safe design criterion, focus is placed on the analysis of response amplification upon primary structural yielding. The following conclusion can be drawn:

a) The yielding of the stories which are lower than the floor of interest of the primary structure often causes response amplification at lower frequencies.

b) Nonlinearities at higher stories of the primary systems often cause response amplification at higher frequencies. This result is of importance as applied to equipment which often oscillate at higher frequencies.

c) Inelastic deformation may easily cause response amplification in the multiply supported secondary systems.

SECTION 7
REFERENCES

[1] American Society of Mechanical Engineers, ASME boiler and Pressure Vessel Code, *ANSI/ASME BPV-III-1-A Section III, Rules of Construction of Nuclear Power Plant Components*, Division 1, Appendix N, July 1981.

[2] Amin, M. et al. 'Earthquake Response of Multiple Connected Light Secondary Systems by Spectrum Methods', *Proc. ASME 1st Congr. Press. Vess. Pip.*, San Francisco, 1971.

[3] Arora, J.S. and Nguyen, D.T.,' Eigensolution for Large Structural Systems with Substructures', *Int. J. Numer. Methods Engng.*, 15, 333-341, 1980.

[4] ASCE, Structural Analysis and Design of Nuclear Plant Facilities, *Manuals and Reports on Engineering Practice*, No.58, 1980.

[5] ASCE Working Group on Quantification of Uncertainties,' Uncertainty and Conservatism in the Seismic Analysis and Design of Nuclear Facilities', New York, 1986.

[6] Asfura, A. and Der Kiureghian, A.,'A New Floor response Spectrum Method for Seismic Analysis of Multiply Supported Secondary System', *Report No. UCB/EERC-84/04*, University of California, Berkeley, CA.1984

[7] Asfura, A. and Liu, T.H.' A new Combination Rule for Seismic Analysis of Piping Systems', *Proc. the 1985 Pressure Vessels and Piping Conf.* PVP-98-3, 1985.

[8] Asfura, A. and A.Der Kiureghian,' Floor response Spectrum Method for Seismic Analysis of Multiply Supported Secondary System', *J. Earthquake Eng. Struct. Dyn.*, 14, 245-265, 1986.

[9] Aziz, T.S. et. al.,'Seismic Interaction Effects for Steam Generators in CANDU 600

MWe Nuclear Power Plants', *Proc. 5th Int. Conf. on SMiRT*, Berlin, West German, K11/7 (1-8)p, Aug. 1979.

[10] Aziz, T.S.' Coupling Effects For Secondary Systems in Nuclear Power Plants', *ASME Pressure Vessel and Piping Conf.*, Orlando, FL, USA 27 Tun, 1982.

[11] Aziz T.S. and Ghobarah A.,' Equipment Seismic Design: Future Directions', *Proc. of 9th WCEE* Vol. VI, 261-264, Tokyo-Kyoto. Japan, 1988.

[12] Bajan, R.L. and Feng, C.C.,' Free Vibration Analysis by the Modal Substitution Method', *Paper 68-8-1, Space Projections from the Rocky Mountain Region*, American Astronautical Society. Denver, Colorado, 1968.

[13] Baldwin, J.F. and Hutton, S.G.,'Natural Modes of Modified Structures', *AIAA J.*, *23(11)*, 1737-1743, 1985.

[14] Bandyopadhyay, K.K. et al.,' Seismic Fragility of Nuclear Power Plant Components' (Phase II), *NUREG/CR - 4659 BNL - NUREG - 52007*, Vol.2 AN, RM, 1987.

[15] Bathe, K.J. and Gracewski, S.,' On Nonlinear Dynamic Analysis Using Substructuring and Mode Superposition', *Computers Struct.*, 13, 699-707, 1981.

[16] Bellman, R., *Introduction to Matrix Analysis*, 2nd Ed., McGraw-Hill, NY, 1970.

[17] Benfield, W.A., Bodley, C.S. and Morosow, G.,'Modal Synthesis Method', *Symp. Substr. Testing and Synth.*, NASA Marshall Space Flight Center, 1972.

[18] Bessey, R.L. and Kana, D.D.' Some Research Needs for Improved Seismic Qualifications Tests of Electrical and Mechanical Equipment', *J. Nucl. Eng. Design*, 50, 71-82, 1978.

[19] Biggs, J.M. and Roesset, J.M.'Seismic Analysis of Equipment Mounted on a Massive Structure', *in Seismic Design of Nuclear Power Plats*, (ed. R.J. Hansen), pp. 319-343. M.I.T. Press, 1970.

[20] Biggs, J.M. 'Seismic Response Spectra for Equipment Design in Nuclear Power Plants', *Proc. 1st Int. Conf. on SMiRT*, Berlin, West German, K4/7, Sep. 1971.

[21] Bumpus, S., LLL/DOR Seismic Conservatism Program,: Investigations of the Conservatism in the seismic Design of Nuclear Power Plants - Part IX: Nonlinear Structural response, Lawrence Livermore Laboratory *Report No. UCID-18100.*

[22] Burdisso, R.A. and Singh, M.P.' Multiply Supported Secondary Systems Part I: Response Spectrum Analysis', *J. Earthquake Eng. Struct. Dyn.* 15 53-72, 1987.

[23] Cartwright, D.E. and Longuet-Higgins, M.S." The Statistical Distribution of Maxima of a random function", *Proc. Roy. Soc.* London, Ser. A, 237, 212-232, 1956.

[24] Chakravarti, M. K. and Vanmarke, E.H. 'Probabilistic Seismic Analysis of Light Equipment Within Buildings,' *Proc. Fifth World Conf. on Earthquake Eng.*, II, Rome, Italy, 1973.

[25] Chakravorty, M.K., Wong, A.Y.C. and Stetson, M.B.,'Frequency Domain Approach to Seismic Analysis of Multiple Supported Secondary Systems', *Proc. the Int. Meeting on Fast Reactor Safety Technology*, 294-303, 1979.

[26] Chen, Y.Q. and Gong, S.L., " An Approximate Method for Determination of Reliability for Seismic Responses by Time Integration ", *Fourth International Conference on Structural Safety and Reliability*, Japan, Kobe 1985.

[27] Chen, Y.Q. and T.T. Soong,T.T.,' Seismic Response of Secondary System', *Journal of Engineering structures*, 10, 218-228, 1988.

[28] Chowdhury, A.H. ' Seismic Qualification of Interior Mechanical Systems', *Proc. Third U.S.A. National Conf. on Earthquake Engineering*, II, 1613-1623, 1986.

[29] Chiatti, G. and A. Sestieri,' Solution of Quasi-harmonic Equations Through the Combined Finite Element-Transfer Matrix Method', *Int. J. Numer. Anal. Eng.*, 17, 1161-1175, 1981.

[30] Citerley, R. C. and Ball, R.E.,'Nonlinear Seismic Analysis of the Reinforced Concrete Nuclear Reactor Containment Vessel', *J. Nucl. Eng. Design*, 17, 341-344, 1971.

[31] Clough, R.W. and Penzien,"*Dynamics of Structures*" McGraw-Hill, Inc., NY, 1975.

[32] Collins, J.D. and Thomson, W.T.,'The Eigenvalue Problem for Structural Systems with Statistical Properties', *AIAA J.*, 7, 642-648, 1969.

[33] Craig, Jr. R.R. and Bampton, M.C.C. 'Coupling of Substructures for Dynamic Analysis,' *AIAA J.*, 6, 1313-1319, 1968.

[34] Craig, Jr. R.R. and Chang, C-J. 'On the Use of Attachment Modes in Substructure Coupling for Dynamic Analysis', *Paper 77-405, AIAA/ASME 18th Struct., Struct. Dyn., and Materials Conf.*, San Diego, California, 1979.

[35] Craig, J. and Goodno, B.J.'Response Measurements for Glass Cladding Panels', *ASCE J. Struct. Div.*, 7, 2199-2214, 1981.

[36] Craig, J.I., Leistikow, R. and Fennell, C.J.,' Experimental Studies of the Performance of Precast Cladding Connections', *Proc. of 9th WCEE*, Vol. VI, 201-206, Tokyo-Kyoto, Japan 1988.

[37] Dawe, D.J. and Morris, J.R.,'Vibration of Curved Plate Assemblies Subjected to Membrane Stresses', *Sound Vib.*, 81, 229- 237, 1982.

[38] Der Kiureghian, A., Asfura, A., Sackman, J. L. and kelly, J. M.'Seismic Response of Multiply Supported Piping Systems', *Proc. Seventh Int. Conf. on SMiRT*, Chicago, Illinois, 1983.

[39] Der Kiureghian, A. , Sackman, J.L. and Nour-Omid, B. 'Dynamic Analysis of Light Equipment in Structures:Response to Stochastic Input', *J. Eng. Mech. Div.*, 109, No. EM1, 90-110, Feb, 1983.

[40] Der Kiureghian, A.,' Effect of Modeling on Seismic Response of Secondary System', *Proc. of 9th WCEE*, Vol. VI, 219-224, Tokyo-Kyoto, Japan, 1988.

[41] Dermizakis, S.N. and Mahin, S.A.' Development of Substructuring Techniques for On-line Computer Controlled Seismic Performance Testing', *UcB/EERC-85/04*, Berkeley, 1985.

[42] Dowell, E. H.'On Some General Properties of Combined Dynamical Systems', *J. Appl. Mech.*, 46, 206-209, 1979.

[43] Esfandiari, S., Hoy, A.J. and Jordan, G.W., 'Practical Applications of Dynamic Feedback Between Primary and Secondary Systems', *Transactions of the 9th int. Conf. on SMiRT*, K2, 833-838, Boston, 1987.

[44] Fisher E.G. and Dause, W.M. 'Combined Analysis and Test of Earthquake Resistant Circuit Breakers', *J. Earthquake Eng. Struct. Dyn*, V, 231-243, 1976.

[45] Frank, P.M., *Introduction to System Sensitivity Theory, Academic Press*, NY. 1978.

[46] Gasparini, D. A., Tsiatas G. and Sun, W.J.' Random Vibration of Cascaded Secondary Systems', *Proc. 7th international Conf. on SMiRT K(a)*, 445-451, Chicago, IL, 1983.

[47] Geering, H.P.,' New Methods in Substructuring', *21st Struct. Dyn. Matl. Conf.*, Pt. 2, Paper 80-0799, 801-808, AIAA (CP 801), New York, 1980.

[48] Gerdes, L.D,'Dynamic Structural Analysis of Uncoupled Subsystems,: *Proc. the 4th Int. Conf. on SMiRT*, San Francisco, k(6), 1977.

[49] Goodno, B.J., Will, K.M. and Palsson, H.'Effect of Cladding on Building Response to Moderate Ground Motion', *Proc. 7th WCEE*, Instanbul, Turkey, 449-456, 1980.

[50] Goodno, B.J., M. Meyyappa and S. Nagarajaiah,' A Refind Model for Precast Cladding and Connections', *Proc. 9th WCEE*, Vol. VI, 195-200, Tokyo-Kyoto, Japan, 1988.

[51] Goyder, H.G.D.,' Methods and Application of Structural Modeling from Measured Structural Frequency Response Data', *Sound Vib.*, 68, 209-230, 1980.

[52] Greif and L. Wu,' Substructure Analysis of Vibrating System', *The Shock and Vibration Digest*, 15, 17-24, 1983.

[53] Gupta, A.K. and Kordero, K.' Combination of modal Responses', *Proc. 6th Int. Conf. on SMiRT*, Paris, Paper No. K7/5, 1981.

[54] Gupta, A.K. and Tembulkar, J.M.' Dynamic Decoupling of Secondary Systems', *J. Nucl.*

Eng. Design, 81, 359-373, 1984.

[55] Gupta, A.K. and Tembulkar, J.M.' Dynamic Decoupling of Multiply Connected MDOF Secondary System', *J. Nucl. Eng. Design*, 81, 375- 383, 1984.

[56] Gupta, A.K. and Chen,'Comparison of Modal Combination methods', *J. Nucl. Eng. Design*. 78, 53-68, 1984.

[57] Gupta, A.K., 'Seismic Response of Multiply Connected MDOF Primary and Secondary Systems', *J. Nucl. Eng. Design*, 81, 385- 394, 1984..

[58] Gupta, A.K. and Jaw, J.W. 'CREST, A Computer Program for coupled Response Spectrum Analysis of Secondary Systems', *User's Manual* Depart. of Civil Eng., North Carolina State University, Raleigh, June, 1985.

[59] Gupta, A.K. and Jaw, J.W. 'Seismic Response of Nonclassically Damped Systems', *J. Nucl. Eng. Design*, 91, 153-159, 1986.

[60] Gupta, A.K. and Jaw, J.W.'Response Spectrum Method for Nonclassically Damped Systems', *J. Nucl. Eng. Design*, 91, 161- 169, 1986.

[61] Gupta, A.K. and Jaw, J.W.' Complex Modal Properties of Coupled Moderrately Light Equipment-structure Systems', *J. Nucl. Eng. Design*. 91, 171-178, 1986.

[62] Gupta, A.K. and Jaw, J.W.' Coupled Response Spectrum Analysis of Secondary Systems using Uncoupled modal Properties', *J. Nucl. Eng. Design*, 92, 61-68, 1986.

[63] Gupta, A.K. and Jaw, J.W.' A New Instructure Response Spectrum (IRS) Method for Multiply Connected Secondary Systems With Coupling Effects', *J. Nucl. Eng. Design*, 96, 63-80, 1986.

[64] Hadjian, A.H. and Hamilton, C.W.,' Probabilistic Frequency Variations of Concrete Structures', *2nd Int. Conf. on SMiRT*, Berlin, 1973.

[65] Hadjian, A.H. ' On the Decoupling of Secondary Systems for Seismic Analysis', *6th WCEE* 3286-3291, New Delhi, India, 1977.

[66] Hadjian, A.H. and Ellison, B.' Decoupling of Secondary Systems for Seismic Snslysis', *ASME Pressure Vessel and Piping Conf.*, San Antonio, TX, USA 17. Jun 1984.

[67] Haftka, R.T., Martinovic, Z.N. and Hallauer, W.L.' Enhanced Vibration Controllability by Minor Structural Modifications', *AIAA J.* 23, August 1985.

[68] Hale, A.L. and Meirovitch,' A General Substructure Synthesis Method for the Dynamic Simulation of Complex Structures', *Sound Vib.*, 69, 309-326, 1980.

[69] Hamilton, C.W. and Hadjian, A.H.,"Probabilistic Frequency Variations of Structure-Soil Systems", *J. of Nuclear Engineering and Design* 38, 303-322, 1976.

[70] Hernried, A.G.and Sackman, J.L.' Response of Secondary Systems in Structures Subjected to Transient Excitation', *J. Earthquake Eng. Struct. Dyn.* 12, 737-748, 1984.

[71] Hernried, A.G. and Jeng, Haw ' Dynamic Response of Non structural Componts', *Proc. Third U.S.A. National Conf. on Earthquake Eng.*, II,1601-1612, 1986.

[72] HoLung, J. Cai and Lin, Y.K. 'Frequency Response of Secondary Systems under Seismic Excitation', *Tech. Report NCEER- 87-0013*, State University of New York at Buffalo, Buffalo, NY, July, 1987.

[73] Housner, G.W. and Jennings, P.C." Generation of Artificial Earthquakes"' *J. Engng Mech. Div. ASCE*, 90, EM 1, 113-150, 1964.

[74] Hurty, W.C.,'Dynamic Analysis of Structural Systems Using Component Modes', *AIAA J.*, 3, 678-685, 1985.

[75] IEEE, Standard 344-1975, IEEE Recommended Practice for Seismic Qualifications of Class IE Equipment for Nuclear Power Generating Stations, STD. 344, 1975.

[76] Igusa, T. and Der Kiureghian, A. , 'Dynamic Analysis of Multiple Tuned and Arbitrarily Supported Secondary Systems', *Report No. UCB/EERC-83/07*, University of California, Berkeley, CA, 1983.

[77] Igusa, T. and Der Kiureghian, A. and Sackman, J.L.,'Modal Decomposition Method

for Stationary Response of Non-classically Damped Systems', *J. Earthquake Eng. Struct. Dyn.*, 12, 121-136, 1984.

[78] Igusa, T. and Der Kiureghian, A.,'Dynamic Characterization of Two-Degree-of-Freedom Equipment-Structure System', *J. Eng. Mech.*, III, No.1, 1-19, Jan. 1985.

[79] Igusa, T. and Der Kiureghian, A. , 'Dynamic Response of Multiple Supported Secondary Systems', *J. Eng. Mech.*, III, No.1, 20-41, Jan. 1985.

[80] Igusa, T. and Der Kiureghian, A. , 'Generation of Floor Response Spectra Including Oscillator-Structure Interaction', *J. Earthquake Eng. Struct. Dyn*, 13 661-676, 1985.

[81] Igusa, T. and Der Kiureghian, A.,'Reliability of Secondary Systems with Uncertain Tuning', *Proceedings of ICOSSAR'85*, I-191-200, Kobe, 1985.

[82] Igusa, T. and Der Kiureghian, A.,' Response of Uncertain Systems to Stochastic Excitation', *J. of Eng. Mech. ASCE*, 114(5), 812-832, 1988.

[83] Igusa, T. and Der Kiureghian, A.,'Interaction in Primary Systems' *Proceeding of the 1988 ASME Pressure Vessels and Piping Conference*, Pittsburgh, Pennsylvania, 45-50, June 19-23, 1988.

[84] Ingham, T.J. and Sackman, J. L.' Some Methods for the Analysis of Equipment-Structure Interaction', *J. Earthquake Eng. Struct. Dyn.* 12, 583-601, 1984.

[85] Ishac, M.F. ' Effect of Estimated Torsional Ground Motion on Lateral and Rotational Floor Response Spectra', *6th Int. Conf. on SMiRT*, K10/6, Paris, France 17, Aug. 1981.

[86] Jing-Wen, Jaw and Gupta, A.K.,'Special Issues in The Floor Response Spectrum Method', *Proc. Third U.S.A. National Conf. on Earthquake Eng.*, II, 1589-1600, 1986.

[87] Kana, D.D and Huzar, S.,'Synthesis of Shuttle Vehicle Damping Using Substructure Test Results', *J. Spacecraft and Rockets*, 10, 790-797, 1973.

[88] Kapur, K. K. and Shao, L. C.' Generation of Seismic Floor Response Design', *Proc. Specialty Conf. on Structural Design of Nuclear Plant Facilities*, ASCE, Chicago, 1973.

[89] Kaul, M.K.,"Stochastic Characterization of Earthquakes Through Their Response Spectrum." *J. EESD* vol 6, 497-509, 1978.

[90] Kawakatsu, T. et al, "floor Response Spectra Considering Elasto-Plastic Behaviour of Nuclear Power Facilities", *5th SMiRT*, Paper K9/4, Berlin, 1978.

[91] Kawakatsu, T. ' Floor Response Spectra Considering Elasto- Plastic Behaviour of Nuclear Power Facilities', *Proc. The 5th Int. Conf. of SMiRT*. Paris, France, August 1981.

[92] Kelly, T.E., "Floor Response of Yielding Structures", *Bulletin of the New Zealand National Society for Earthquake Engineering*, Vol. 11, No. 4, 1978.

[93] Kelly, J.M. 'The Influcece of Base Isolation on the Seismic Response of Light Secondary Equipment', *UCB/EERC-81/17*, Berkeley, 1982.

[94] Kelly, J.M. and Tsai, H. ' Seismic Response of Light Equipment in base-isolated Structures', *J. Earthquake Eng. Struct. Dyn.*, 13, 711-732, 1985.

[95] Klein, L.R. and Dowell, E.H.,'Analysis of Modal Damping by Component Modes Methods Using Lagrange Multipliers', *J. Appl. Mech.*, 41, 527-528, 1974.

[96] Konig, G. and Worner J.D.,' Component-Structure-Interaction During Strong Dynamic Excitation', *Proc. of 9th WCEE* Vol. VI, 231-236, Tokyo-Kyoto, Japan, 1988.

[97] Kurosaki, A. and Kozeki, M., ' Spectrum Analysis with Quadratic Participation for Seismic Design of Light Secondary System', *Transactions of the 9th int. Conf. on SMiRT*, K2, 857-862, Boston, 1987.

[98] Lazzeri, L and Viti, G.,' Heavy Interaction Problems in FRS', *Transactions of the 9th int. Conf. on SMiRT*, K2, 845-848, Boston, 1987.

[99] Lazzeri, L,' On the Design of Piping in non Linear Conditions under Dynamic Loads, *Transactions of the 9th int. Conf. on SMiRT*, K2, 875-882, Boston, 1987.

[100] Lee, M.C and Penzien, 'Stochastic Seismic Analysis of Nuclear Power Plants and Piping Systems subjected to Multiple Support Excitations', *Report Number UCB/EERC-80/19*,

University of California, Berkeley, Calif., June 1980.

[101] Leimbach, K. R. and Sterkel, J.P. ' Comparison of Multiple Support Excitation Solution Techniques for Piping Systems', *Proc. 5th Int. Conf. on SMIRT*, K10/2, Berlin, Germany, August 13-17, 1979.

[102] Leung, Y-T.,' An Accurate Method of Dynamic Substructuring with Simplified Computation', *Int. J. Num. Methods Eng.*, 14, 1241-1256, 1980.

[103] Li, D.F. and Gunter, E.J.,' A Study of the Mode Truncation Error in the Component Mode Analysis of a Dual- Rotor Systems', *J. Eng. Power*, 104, 525-532, 1982.

[104] Lin, C. W. and Liu, T.H.,' A Discussion of coupling and Resonance Effects for Integrated Systems Consisting of Subsystems', *Proc. Extreme Load Conditions and Limit Analysis Procedures for Structural Reactor Safeguards and Containment Structures*, Paper U3/3, Berlin, September 1975.

[105] Liu, L.K., Child, C.L. and Nowotny, B.,' Effect of Parameter Variation on Floor Spectra', *Specialty Conf. on Structural Design of Nuclear Power Plant Facilities*, Chicago, December 1973.

[106] Lin, Y.K.,"*Probabilistic Theory of Structural Dynamics*", Huntington, New York, 1976.

[107] Lin, J. and Mahin, S., "Seismic Response of Light Subsystems on Elastic Structures", *J. Stru. Eng. ASCE*, ST2, Vol. 111., 400-417, 1985.

[108] MacNeal, R.H.,' A Hybrid Method of Component Mode Synthesis', *Comp. and Struct.*, 1, 581-601, 1971.

[109] Manolis, G.D. and Juhn, G.,' Substructuring Techniques in the Time Domain for Primary-Secondary Structural Systems', *Technical Report NCEER-88-0003*, Buffalo, New York, 1988.

[110] Manolis, G.D., Juhn, G. and Reinhorn, A.M.,' Experimental Investigation of Primary-Secondary System Interaction', *Technical Report NCEER-88-0019*, Buffalo, New York, 1988.

[111] Mau, S.T.,‘ A Subspace Modal Superposition Method for Non-classically Damped System’, *J. Earthquake Eng. Struct. Dyn.*,vol. 16, 931-942, 1988.

[112] Mayers, R.L. et. al.,‘Evaluation of Methods Used for the Direct Generation of Response Spectra’, *Proc. 7th Int. Conf. on SMiRT* K(a) Chicago, 1983.

[113] Meirovititch, L. and Hale, A.L.,‘ On the Substructure Synthesis Method ’, *AIAA J.*, 19, 940-947, 1981.

[114] Mehrain, M., Clayton, M. and Stuart, J,‘ Seismic Testing and Design of Raised Computer Floors’, *Proc. of 9th WCEE* Vol. VI, 225-230, Tokyo-Kyoto, Japan, 1988.

[115] Mizuno H., IIBA, M. and Yamaguchi N.,‘ Seismic Behavior and Hazaed Evaluation of Medical Equipment with Casters’, *Proc. of 9th WCEE*, Vol. VI, 255-260, Tokyo-Kyoto, Japan, 1988.

[116] Nagamatsu, A. and Okuma, M.,‘analysis of Vibration by Component Mode Synthesis Method; Part 1’, *Bull. JSME*, 24, 1148-1453, 1981.

[117] Nakhata, T., Newmark, N.M. and Hall, W. J.‘Approximate Dynamic Response of Light Secondary Systems’, *Struct. Research Series Report No. 396*, Civil Engineering Studies, University of Illinois, Urbana, Ill., 1973.

[118] Natke, H.G.,‘ Angenaherta Fehleremittlung fur Modal Synthese- Ergebnisse Innerhalb der Systemanalyse und Systemidentifikation’, *Z. Angew. Math. Mech.*, 61, 41-53, 1981.

[119] Nelson, R.B.,‘Simplified Calculation of Eigenvector Derivatives’, *AIAA J.* 14(9), 1201-1205, 1976.

[120] Newmark, N.M. ‘Earthquake Response Analysis of Reactor Structures’, *J. Nucl. Eng. Design*, 20, no. 2, 303-322, 1972.

[121] Newmark, N.M. ‘A Response Spectrum Approach for Inelastic Seismic Design of Nuclear Reactor Facilities’, *Proc. the 3th Int. Conf. on SMiRT*, k5/1, London, 1975.

[122] Newmark, N.M. ‘ Inelastic Design of Nuclear Reactor Structures and Ith Implications

on Design of Critical Equipment', *Proc. the 4th Int. Conf. on SMiRT*, k4/1, 1977.

[123] Newmark, N.M. and Kennedy, R.P., Final Report - Evaluation of a Typical PWR Reactor Building Subjected to a High Acceleration, Short Duration Earthquake, Report Prepared for Electricite de France under Contract *No. 7B031*, August 1979.

[124] Nigam, N.C. and Bains, G.P.S.,' Optimum Aseismic Design of Equipment Support-Structure Systems', *Proc. of 9th WCEE*, Vol. VI, 243-248, Tokyo-Kyoto, Japan, 1988.

[125] U.S. Nuclear Requlatory Commission, Standard Review Plan, *NUREG-75/087*, Nov. 1975.

[126] U.S. Nuclear Requlatory Commission, Subsystem Response Review, *NUREG/CR-1700*, Jan. 1981.

[127] U.S. Nuclear Regulatory Commission, Structural Building Response Review, *NUREG/CR-1423*, Vol.I-II, May, 1980.

[128] U.S. Nuclear Regulatory Commission, Regulatory Guide, Revision 1.122, Feb. 1978.

[129] Nuclear Power Engineering Test Center of Japan, Seismic Proving Tests on Reliability for Large Components and Equipment of Nuclear Power Plants, Tadotsu Engineering Laboratory, Tadotsu, Japan, 1982.

[130] Ogawa, N. 'A Vibration Test of Earthquake Induced Hydraulic Effects of Liquid-Filled Pipelines', *8 WCEE*, VI, San Francisco, 1984.

[131] Okada, T. et al., 'Nonlinear Earthquake Response of Equipment Anchored on R/C Building Floor', *Bull. of ERS Research Center, No. 3*, Institute of Industrial Science, Univ. of Tokyo, 1980.

[132] Palsson, H., Goodno, B.J., Craig, J.I. and Will, K-M 'Cladding Influence on Dynamic Response of Tall Buildings', *J. Earthquake Eng. Struct. Dyn.*, 12, 215-228, 1984.

[133] Passon, H and Goodno, B.J.,' Influence of Interstory Drift on Cladding Panels and Connections', *Proc. of 9th WCEE* Vol. VI, 213-217, Tokyo-Kyoto, Japan, 1988.

[134] Pekau, O.A. and Huttelmaier, H.P.,' Analysis of Plate Vibrations Using Superelements', *J. Mech. Des.*, 104, 439-444, 1980.

[135] Penzien, J. and Chopra, A.K.' Earthquake Response of Appendage on a Multi-Story Building', *Proc. Third World Conf. on Earthquake Eng.*, New Zealand, 1965p, 1973.

[136] Peters, K. A., Schmitz, D. and Wagner, U. 'Determination of Floor Response Spectra on the Sasis of the Response Spectrum Method', *J. Nucl. Eng. Design*, 44, pp. 255-262, 1977.

[137] Pickel, T.W. ' Evaluation of Nuclear System Requirements for Accommodating Seismic Effects', *Proc. The 1th Int. Conf. on SMiRT*, K1/2, Berlin, Germany, Sep. 1971.

[138] Plaut, R.H. and Huseyin, K.,'Derivatives of Eigenvalues and Eigenvectors in Non-Self-Adjoint Systems', *AIAA J.*, 11(2), 250-251, 1973.

[139] RDT Standard F9-2T, Seismic Requirements for Design of Nuclear Power Plants and test Facilities, Jan. 1974.

[140] Reed, J.W. 'Response Spectrum Analysis for Molti-Supported Subsystems', *Proc. The 7th Int. Conf. on SMiRT*, K7/6, Chicago, August 1983.

[141] Reich, M., et. al,' Elastic and Inelastic Methods of Piping System Analysis', *BNL* 19763, 1974.

[142] Rosenblueth, E. and Bustamante, J.E.,"Distribution of Structural Response to Earthquakes", *J. Engng Mech. Div., ASCE*, 88,EM3, 75-106, 1962.

[143] Rubin, S.,'Improved Component-Mode Representation for Structural Dynamic Analysis', *AIAA J.*, 13, 995-1006, 1975.

[144] Rudisill, C.S. and K. G. Bhatia, K.G.,' Second Derives of the Flutter Velocity and the Optimization of Aircraft Structures', *AIAA J.* 10(12), 1569-1572, 1972.

[145] Rudisill, C.S.,'Derivatives of Eigenvalues and Eigenvectors for a General Matrix', *AIAA J.*, 12(5), 721-722, 1974.

[146] Ruzicka, G. C. and Robinson, A.R.'Dynamic Response of Tuned Secondary Systems', *Report Number UILU-ENG-80-2020*, Department of Civil Engineering, University of Illinois, Urbana, ILL., Nov. 1980.

[147] Rihal, S.S,' Earthquake Resistance and Behavior of Heavy Facades/cladding & Connections in Medium-Rise Steel-Framed Buildings', *Proc. 9th WCEE*, VI, 207-212, Tokyo-kyoto, Japan, 1988.

[148] Sackman, J. L. and Kelly, J.M.' Seismic Analysis of Intal Equipment and Components in Structures', *J. Eng. Struct.*, 1, no. 4. 179-190. July. 1979.

[149] Der Kiureghian, A. , Sackman, J.L. and Nour-Omid, B. 'Dynamic Analysis of Light Equipment in Structures: Modal Properties of the Combined System', *J. Eng. Mech. Div.*, 109, No. EM1, Feb, 1983.

[150] Scanlan, R.H. and Sachs, K. 'Earthquake Time Histories and Response Spectra,' *J. Eng. Mech.* Div. 100, no. EM4, 635-655, August, 1974.

[151] Schneider, S., Lee, H. and Godden, W. 'Behavior of a Piping System under Seismic Excitation', *Report No. UCB/EERC-82/03*, University of California, Berkeley, CA. 1982.

[152] Sewell, R.T., Cornell, C.A., Toro, G.R. and McGuire, R.K.,' A Study of Factors Influencing Floor Response Spectra in Nonlinear Multi-Degree-of-Freedom Structures', *Report of The John A. Blume Earthquake Engineering Center, No. 82*, Stanford University, 1986.

[153] Sewell, R.T. and Cornell, C.A. et. al.," Factors Influencing Equipment Response in Linear and Nonlinear Structures", *Transactions of the 9th int. Conf. on SMiRT*, K2, 849-856, Boston, 1987.

[154] Sharma A.M. and Singh, M.P.,' Direct Generation of Seismic Floor Response Spectra for Classically and Non-classically Damped Structures', *Report No. VPI-E-83-84*, Virginia Polytechnic Institute and State University, Blacksburg,VA, 1984.

[155] Shaw, D.E.' Seismic Structural Response Analysis for Multiple Support Excitation',

Proc. 3rd Int. Conf. on SMiRT, K5/2, London, England, September 1-5, 1975.

[156] Shibata, H., et al., 'Response Analysis of a Piping System in Three Story Building on Shaking Table', *Bull. of ERS Research Center, No. 5*, Institute of Industrial Science, Univ. of Tokyo, 1971.

[157] Shibata, A. and Sozen, M.A.,' Substitute-structure Method for Seismic Design in R/C', *J. Struct. Div. ASCE*, 102, 1-18, 1976.

[158] Shibata, H., et al.,' An Experimental Study of damping Characteristics with Emphasis on Insulation for Nuclear Power Plant Piping System (Seismic Damping Ratio Evaluation Program)', *Transactions 5th Int. Conf. on SMiRT*, K6/5, Paris, France, August 1981.

[159] Shibata, H., Shigeta, T. and Sone, A.'Seismic Response and Reliability of Mechanical Systems Effects of Uncertainty of Ground motions', *Proc. the US-Southeast Asia Symposium on Eng. for Natural Hazards Protection*, Manila, 1977.

[160] Shinozuka, M. and Astill, C.J.,'Random Eigenvalue Problems in Structural Analysis', *AIAA J.*,10, 456-462, 1972.

[161] Singh, A.K., 'A Stochastic Model for Predicting Maximum Response of Light Secondary Systems', *Thesis presented to the University of Illinois at Urbana*, Illinois, in 1972, in partial fulfillment of the requirements for the degree of Doctor of Philosophy, 1972.

[162] Singh, A.K. and Ang, A.H.S.,'Stochastic Prediction of Maximum Seismic Response of Light Secondary System', *J. Nucl. Eng. Design*, 29, 218-230, 1974.

[163] Singh, M.P.,'Generation of Seismic Floor Spectra', *J. Eng. Mech. Div.*, 101, no. EM5,593-607, Oct. 1975.

[164] Singh, M.P.'Generation of Seismic Floor Response Spectra for Equipment Design', *Proc. Specialty Conf. on Struct. Design of Nucl. Plant Facilities, ASCE*, Chicago, 1973.

[165] Singh, M.P.,'Seismic Design Input for Secondary Structures', *J. Struct. Div.*, 106, no. ST2. 505-517, Feb. 1980.

[166] Singh, M.P., 'Seismic Response by SRSS for Nonproportional Damping', *J. Eng. Mech. Div.*, 106, no. EM6, 1405-1419, Dec. 1980.

[167] Singh, M.P.,' Seismic Floor Spectra by Mode Acceleration Approach', *J. Eng. Mech.*, III, No.11, 1402-1419, Nov. 1985.

[168] Singh, M.P. and Suarez, L.E. ' A Perturbation Analysis of the Eigenproperties of Equipment- Structure System', *J. Nucl. Eng. Design* 97 167-185, 1986.

[169] Singh, M.P. and Ghafory-Ashtiany, M. ' Modal Time History Analysis of N0n-classical Damped Structures for Seismic Motions', *J. Earthquake Eng. Struct. Dyn.*, 11, 133-146, 1986.

[170] Singh, M.P. and Burdisso, R.A.' Multiply Supported Secondary Systems Part II: Seismic Inputs', *J. Earthquake Eng. Struct. Dyn.*, 15. 73-90, 1987.

[171] Singh, M.P. and Suarez, L.E.' Seismic Response Analysis of Structure-Equipment Systems with Non-classical Damping Effects', *J. Earthquake Eng. Struct. Dyn.*, 15, 871-888, 1987.

[172] Singhal, A., Lutes, L.D. and Spanos, P.,' Iterative Seismic Analysis of Primary-Secondary Systems', *Technical Report NCEER-88-0004* Buffalo, New York, 1988.

[173] Smeby, W. ' Stochastic Analysis of Seismic Response of Multiply Supported Secondary Systems', *Mec., Mater., Electr.* (France) V 389-391. May-July 1982.

[174] Smith, P.W.,' Random Response of Identical, One-dimensional Coupled Sub-systems', *Sound Vib.*, 70, 343-353, 1980.

[175] Socha, L.' The Sensitivity Analysis of Stochastic Non-linear Dynamic System, *J. Sound and Vibration*, 110(2), 271-299, 1986.

[176] Soong, T.T.,'Pharmacokinetics with Uncertainties in Rate Constants-II:Sensitivity Analysis and Optimal Dosage Control', *J. Mathematical Bioscience* 13, 391-396, 1972.

[177] Soong, T.T., *Random Differential Equations in Science and Engineering*, Academic Press, New

York and London, 1973.

[178] Soong, T.T.,'A note on Expectation of a Quadratic Form, Stoch. Analy. Appl.', 2(3), 295-298, 1984.

[179] Soong, T.T. and Chung, L.L.' Seismic Performance of Secondary Systems under Primary Structural Uncertainties', *Presented at 1986 Structural Congress, ASCE*, New Orleans, Sep. 1986.

[180] Soong, T.T. and Chen, Y.Q.,"Stochastic Response Sensitivities of Secondary Systems to Primary Structural Uncertainties", *Symposium on Stochastic Structural Dynamics*, Urbana, Il, Oct. 1988.

[181] Sponos P.D. etc.,' Decoupled Dynamic Analysis of Combined Systems by Iterative Determination of Interface Accelerations', *J. Earthquake Eng. Struct. Dyn.*, 16, 491-500, 1988.

[182] Stavrinidis, C.P. ' A procedure for Coupling Dynamical Equations', *Computer Methods Appl. Mech. Eng.*, 20, 1-7, 1979.

[183] Stiemer, S. and Godder, W. ' Shaking Table Tests of Piping Systems with Energy Absorbing Restrainer', *Report No. UCB/EERC- 80/34*, University of California, Berkeley, CA, 1980.

[184] Suarez, L.E. and Singh, M.P.,' Floor Response Spectra with Structure-Equipment Interaction Effects by a Mode Synthesis Approach', *J. Earthquake Eng. Struct. Dyn.*, 15 141-158, 1987.

[185] Suarez, L.E. and Singh, M.P.,' Eigenproperties of Non- classically Damped Primary Structure and Equipment Systems by a Perturbation Approach', *J. Earthquake Eng. Struct. Dyn.*, 15, 565-583, 1987.

[186] Suarez, L.E. and Singh, M.P.,' An Exact Component Mode Synthesis Approach', *J. Earthquake Eng. Struct. Dyn.*, 16, 293-310, 1988.

[187] Subudhi, M. and Bezler, P.' Alternate Procedures for the Seismic Analysis of Multiply Supported Piping Systems', *Proc. The 8th Inta. Conf. on SMiRT*. Belgium 19, August, 1985.

[188] Sun, W.' Secondary System Piping Analysis Including Seismic and a loss of coolant accident', *ASME. Pressure Vessels and Piping Conf.* Portland, OR, USA 19, Jun. 1983.

[189] Swan, S.W., Miller, D.D. and Yanev, P.I. ' The Morgan Hill Earthquake of April 24 1984 -Effects on Industrial Facilities', *Earthquake Spectra*, 1, 1985.

[190] Tanaka, K., Kikuchi, K. and Okmura, A.,' Improvements of Computation Accuracy in Transfer Matrix Method Vibration Analysis by Using a Branching Process', *Sound Vib.*, 76, 587-590, 1981.

[191] Tolani, S.K. and Rocke, R.D.,'Modal Truncation of Substructures Used in Free Vibration Analysis', *J. Eng. Indus.*, 98, 827-834, 1976.

[192] Traill-Nash, R.W.,' Modal Methods in the Dynamics of Systems with Non-classical Damping', *J. Earthquake Eng. Struct. Dyn.*, 9, 153-169, 1981.

[193] Tsai, N.C., ' Combination of Responses from Closely Spaced modes', *7th Int. Seminar on Computational Aspects of the Finite Element Method*, Chicago, IL,USA 29, Aug. 1983.

[194] Tsai, N.C., ' Spectral Response Analysis of a Coupled System', *ASME Pressure Vessel and Piping Conf.* Orlando, FL, USA 27, June 1982.

[195] Tsai, H.C.,'Non-classical Damping in Dynamic Analysis of Base-isolated Structures with Internal Equipment', *J. Earthquake Eng. Struct. Dyn.* 16, 29-43, 1988.

[196] Vaidya, N.R. and Bazan-Zurita, E.,' On the Effects of Floor Flexibility on Floor Response Spectra', *Transactions of the 9th int. Conf. on SMiRT*, K2, 863-868, Boston, 1987.

[197] Vanmarcke, E.H.,' structural Response to Earthquake', section 8 of *Seismic Risk and Engineering Decisions*, Elsevier Scientific Publishing Company, Amsterdam. 1976.

[198] Vanmarke, E.H. ,' A Simplified procedure for Predicting Amplified Response Spectra

and Equipment Response', *Proc. 6th WCEE*, III, New Delhi, India, 1977.

[199] Vashi, K.M. ' Seismic Spectral Analysis of Structural Systems Subjected to Nonuniform Excitation at Supports', *Proc. Second ASCE Specialty Conf.*, 1-A, pp. 188-211, New Orleans, Louisiana, December, 1975.

[200] Villalobos, A.O.,' Effective Response Spectra for the Dynamic Qualification of Equipment and Subsystems', *Transactions of the 9th int. Conf. on SMiRT*, K2, 869-874, Boston, 1987.

[201] Villaverde, R.,' Simplified Seismic Analysis of Secondary Systems', *J. Struct. Eng.*, 112, No. 3, March, 1986.

[202] Villaverde, R., ' Simplified Approach for The Seismic analysis of Equipment Attached to Elastoplastic Structures ', *J. Nucl. Eng. Design*, 103, 267-279, 1987.

[203] Villaverde, R. Approximate Seismic Analysis of Piping of Equipment mounted on Elastoplastic Structures', *Transactions of the 9th int. Conf. on SMiRT*, K2, 827-832, Boston, 1987.

[204] Villaverde, R. and Newmark, N.M.' Seismic Response of Light Attachments to Buildings', *Structural Research Series No. 469, University of Illinois*, Urbana, Illinois, Feb. 1980.

[205] Villaverde, R.,' Rosenblueth's Modal Combination Rule for Systems with Non-Classical Damping', *J. Earthquake Eng. Struct. Dyn.*, 16, 315-328, 1988.

[206] Wang, Y.K., Subudhi, M. and Bezler, P.' Comparison Study of Time History and Response Spectrum - Responses for Multiply Supported Piping Systems', *Proc. the 5th Inter. Conf. on SMiRT*. k7/5, Paris, France, Aug. 1981.

[207] Warburton, G.B and Soni, S.R.' Errors in Response Calculation of Non-classically Damped Structural Dynamics', *J. Earthquake Eng. Struct. Dyn.*, 5, 363-375, 1977.

[208] Wen, Y.K.,"Stochastic Seismic Response Analysis of Hysteretic Multi-Degree-Of- Freedom Structures" *J. EESD* 7, 181-191, 1979.

[209] Wen, Y.K.," Equivalent Linearization for Hysteretic Systems under random Excitation", *J. of Applied Mechanics*, 47, 150-154, 1980.

[210] Wilson, E.L., Der Kiureghian, A. , and Bayo, E.P.' A Replacement for the SRSS Method in Seismic Analysis', *J. Earthquake Eng. Struct. Dyn.*, 9, 187-194, 1981.

[211] Wu, R.W., Hussain, F.A. and Lin, L.K.' Seismic Response Analysis of Structural System Subjected to Multiple Support Excitations', *J. Nucl. Eng. Design*, 47, 273-282, 1978.

[212] Yamahara, H.,' Present Situation of the Development of the Base-Isolated Building in Japan ', *Report Shimizu Construction* Co.,LTD Sep. 1987.

[213] Yang, J.N., Sarkani, S. and Long, F.X.,'Modal Analysis of Non- classically Damped Structure Systems Using Canonical Transformations, *Report No. NCEER-88-0014*, Buffalo, New York, 1988.

[214] Yang, J.N., Sarkani, S. and Bushlaibi, A., "Response of Yielding Primary- Secondary Systems", *Symposium on Stochastic Structural Dynamics, University of Illinois at Urbana-Champaign*, October 30-November 1, 1988.

[215] Yong, Y. and Lin, Y,K.' Parametric Studies of Frequency Response of Secondary Systems under Ground-Acceleration Excitations', *Technical Report NCEER-87-0012*, June, 1987.

[216] Zhu, D.,' Improved Substructure Method for Structural Dynamic Analysis', *AIAA Paper 81-0622, Presented at AIAA Dynamics Specialists Conference*, Atlanta, Georgia, 1981.

APPENDIX A

STRUCTURAL CHARACTERISTIC MATRICES

Only shear beam systems are discussed in this report. Assuming that both the multi-degree-of-freedom primary system and the multi-degree-of-freedom secondary system are of the shear beam type with one or more connecting points, the characteristic matrices associated with a P-S system can be easily established.

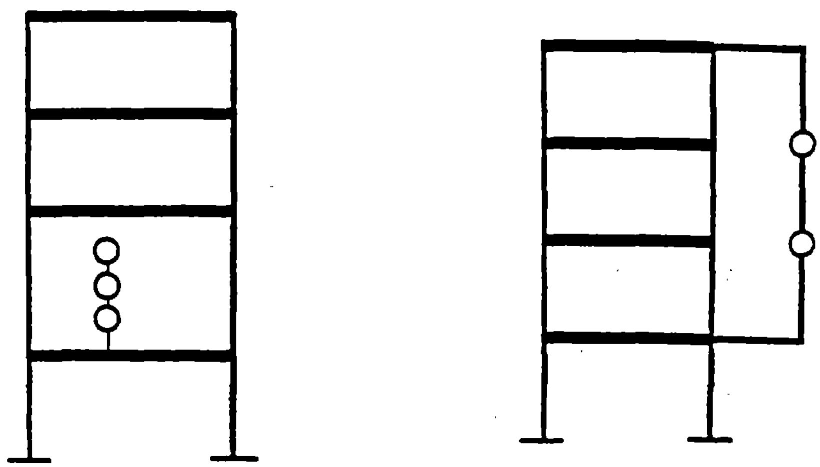
For example, for SA-1 P-S system as shown in Fig. A-1, the displacement of the combined system is

$$\underset{\sim}{X} = \begin{pmatrix} X_{p1} \\ X_{p2} \\ X_{p3} \\ X_{p4} \\ X_{s1} \\ X_{s2} \\ X_{s3} \end{pmatrix}$$

with mass matrix

$$M = \begin{pmatrix} M_{p1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{p2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{p3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{p4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{s1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{s2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{sn} \end{pmatrix}$$

Since there is only one connecting point in this system, the stiffness matrix is



SA-1 SA-2

Fig. A-1 P-S Systems

$$K = \begin{pmatrix} K_{p1}+K_{p2} & -K_{p2} & 0 & 0 & 0 & 0 & 0 \\ -K_{p2} & K_{p2}+K_{p3}+k_{s1} & -K_{p3} & 0 & -k_{s1} & 0 & 0 \\ 0 & -K_{p3} & K_{p3}+K_{p4} & -K_{p4} & 0 & 0 & 0 \\ 0 & 0 & -K_{p4} & K_{p4} & 0 & 0 & 0 \\ 0 & -k_{s1} & 0 & 0 & k_{s1}+k_{s2} & -k_{s2} & 0 \\ 0 & 0 & 0 & 0 & -k_{s2} & k_{s2}+k_{s3} & -k_{s3} \\ 0 & 0 & 0 & 0 & 0 & -k_{s3} & k_{s3} \end{pmatrix}$$

The damping matrix is formed by the following procedure: First, the damping matrices of both primary and secondary systems are established using the Rayleigh assumption, i.e., they are linear combinations of their respective mass and stiffness matrices as given by

$$C = \alpha M + \beta K$$

Thus, based on the same rule used in forming the stiffness matrix, the damping matrix can be obtained from the damping matrices of the primary and secondary systems. Non-classical damping exists when the primary system and the secondary system possess different damping ratios. The damping matrix of the combined P-S system takes the form

$$C = \begin{pmatrix} C_{p1}+C_{p2} & -C_{p2} & 0 & 0 & 0 & 0 & 0 \\ -C_{p2} & C_{p2}+C_{p3}+c_{s1} & -C_{p3} & 0 & -c_{s1} & 0 & 0 \\ 0 & -C_{p3} & C_{p3}+C_{p4} & -C_{p4} & 0 & 0 & 0 \\ 0 & 0 & -C_{p4} & C_{p4} & 0 & 0 & 0 \\ 0 & -c_{s1} & 0 & 0 & c_{s1}+c_{s2} & -c_{s2} & 0 \\ 0 & 0 & 0 & 0 & -c_{s2} & c_{s2}+c_{s3} & -c_{s3} \\ 0 & 0 & 0 & 0 & 0 & -c_{s3} & c_{s3} \end{pmatrix}$$

For the SA-2 P-S system, the stiffness and damping matrices are

$$K = \begin{pmatrix} K_{p1}+K_{p2}+k_{s1} & -K_{p2} & 0 & 0 & -k_{s1} & 0 \\ -K_{p2} & K_{p2}+K_{p3} & -K_{p3} & 0 & 0 & 0 \\ 0 & -K_{p3} & K_{p3}+K_{p4} & -K_{p4} & 0 & 0 \\ 0 & 0 & -K_{p4} & K_{p4}+k_{s3} & 0 & -k_{s3} \\ -k_{s1} & 0 & 0 & 0 & k_{s1}+k_{s2} & -k_{s2} \\ 0 & 0 & 0 & -k_{s3} & -k_{s2} & k_{s2}+k_{s3} \end{pmatrix}$$

$$C = \begin{pmatrix} C_{p1}+C_{p2}+c_{s1} & -C_{p2} & 0 & 0 & -c_{s1} & 0 \\ -C_{p2} & C_{p2}+C_{p3} & -C_{p3} & 0 & 0 & 0 \\ 0 & -C_{p3} & C_{p3}+C_{p4} & -C_{p4} & 0 & 0 \\ 0 & 0 & -C_{p4} & C_{p4}+c_{s3} & 0 & -c_{s3} \\ -c_{s1} & 0 & 0 & 0 & c_{s1}+c_{s2} & -c_{s2} \\ 0 & 0 & 0 & -c_{s3} & -c_{s2} & c_{s2}+c_{s3} \end{pmatrix}$$

APPENDIX B

DERIVATION OF TRANSFER FUNCTION

With initial conditions $\underline{X}(0) = 0$ and $\dot{\underline{X}}(0) = 0$, the Fourier transforms of Eq.(3-1) and Eq.(3-16) give

$$[-\omega^2 M + i\omega C + K][\underline{X}(\omega)] = -M\underline{r}\ddot{x}_o(\omega) \qquad (B-1)$$

Therefore

$$H(\omega) = [-\omega^2 M + i\omega C + K]^{-1} \qquad (B-2)$$

Since

$$HH^{-1} = 1 \qquad (B-3)$$

$$\frac{\partial H}{\partial \beta}H^{-1} + H\frac{\partial H^{-1}}{\partial \beta} = 0$$

and, from Eq.(B-2),

$$\frac{\partial H}{\partial \beta} = -H[-\omega^2\frac{\partial M}{\partial \beta} + i\omega\frac{\partial C}{\partial \beta} + \frac{\partial K}{\partial \beta}]H \qquad (B-4)$$

In the above, M, K and C are linear functions of m_i, k_i and c_i (if we choose them as variables). Hence, the second-order derivatives of these matrices with respective to these variables are zero, giving

$$\frac{\partial^2 H}{\partial \beta^2} = -\frac{\partial H}{\partial \beta}[-\omega^2\frac{\partial M}{\partial \beta} + i\omega\frac{\partial C}{\partial \beta} + \frac{\partial K}{\partial \beta}]H - H[-\omega^2\frac{\partial M}{\partial \beta} + i\omega\frac{\partial C}{\partial \beta} + \frac{\partial K}{\partial \beta}]\frac{\partial H}{\partial \beta} \qquad (B-5)$$

It follows from Eq.(3-17) that

$$\frac{\partial \underline{g}}{\partial \beta} = -[\frac{\partial H}{\partial \beta} M + H \frac{\partial M}{\partial \beta}]\underline{r} \qquad (B-6)$$

$$\frac{\partial^2 \underline{g}}{\partial \beta^2} = -[\frac{\partial^2 H}{\partial \beta^2} M + 2\frac{\partial H}{\partial \beta}\frac{\partial M}{\partial \beta}]\underline{r} \qquad (B-7)$$

$$\frac{\partial T}{\partial \beta} = (\frac{\partial \underline{g}}{\partial \beta})^* \underline{g}^T + \underline{g}^* (\frac{\partial \underline{g}}{\partial \beta})^T \qquad (B-8)$$

and

$$\frac{\partial^2 T}{\partial \beta^2} = (\frac{\partial^2 \underline{g}}{\partial \beta^2})^* \underline{g}^T + 2(\frac{\partial \underline{g}}{\partial \beta})^* (\frac{\partial \underline{g}}{\partial \beta})^T + \underline{g}^* (\frac{\partial^2 \underline{g}}{\partial \beta^2})^T \qquad (B-9)$$

APPENDIX C

DERIVATION OF EIGENVALUES AND EIGENVECTORS

C.1 Complex Eigenvalues and Eigenvectors

Consider free vibration of an n-degree-of-freedom system characterized by

$$M\ddot{\underline{X}}(t) + C\dot{\underline{X}}(t) + K\underline{X}(t) = 0 \qquad (C-1)$$

Let

$$\underline{Y}(t) = \begin{bmatrix} \dot{\underline{X}}(t) \\ \underline{X}(t) \end{bmatrix} \quad and \quad \dot{\underline{Y}}(t) = \begin{bmatrix} \ddot{\underline{X}}(t) \\ \dot{\underline{X}}(t) \end{bmatrix}$$

The state space equation is

$$A'\dot{\underline{Y}}(t) + B\underline{Y}(t) = 0 \qquad (C-2)$$

with

$$A' = \begin{pmatrix} 0 & M \\ M & C \end{pmatrix} \quad and \quad B = \begin{pmatrix} -M & 0 \\ 0 & K \end{pmatrix} \qquad (C-3)$$

where A' and B are $2n \times 2n$ symmetrical matrices. Using Fourier transform, one obtains

$$i\Omega A'\dot{Y}(\Omega) + BY(\Omega) = 0 \qquad (C-4)$$

$$A = iA' \qquad (C-5)$$

$$[B + \Omega A]Y(\Omega) = 0 \qquad (C-6)$$

The eigenvalues are the solutions of

$$det[B + \Omega A] = 0 \qquad (C-7)$$

The normalized eigenvector $\underline{Y}$ satisfies

$$Y^T A Y = 1 \qquad (C-8)$$

Thus the complex eigenvalues and eigenvectors can be obtained.

C.2 First-Order Derivatives of Eigenvalues

Assume existence of the derivatives of eigenvalues and eigenvectors as they are in most engineering problems. For i^{th} mode, differentiation of Eq.(C-6) with respect to the structural parameter β gives

$$[\frac{\partial B}{\partial \beta} + \frac{\partial \Omega_i}{\partial \beta} A + \Omega \frac{\partial A}{\partial \beta}]\underline{Y}_i + [B + \Omega_i A]\frac{\partial \underline{Y}_i}{\partial \beta} = 0 \qquad (C-9)$$

Since $(B + \Omega A)\underline{Y}_i = 0$, premultiplying $\underline{Y}_i^T$ gives

$$\frac{\partial \Omega_i}{\partial \beta}(\underline{Y}_i^T A \underline{Y}_i) = -\underline{Y}_i^T[\frac{\partial B}{\partial \beta} + \Omega_i \frac{\partial A}{\partial \beta}]\underline{Y}_i \qquad (C-10)$$

Hence,

$$\frac{\partial \Omega}{\partial \beta} = -\underline{Y}_i^T[\frac{\partial B}{\partial \beta} + \Omega_i \frac{\partial A}{\partial \beta}]\underline{Y}_i \qquad (C-11)$$

C.3 Derivatives of Eigenvectors

The derivatives of the eigenvectors can be written as

$$\frac{\partial \underline{Y}_i}{\partial \beta} = \sum_{k=1}^{n} C_k \underline{Y}_k = Y\underline{C} \qquad (C-12)$$

$$\frac{\partial \underline{Y}_i^T}{\partial \beta} = \sum_{k=1}^{n} C_k \underline{Y}_k^T \qquad (C-13)$$

From Eq.(C-9), one obtains

$$(B + \Omega A) Y \underset{\sim}{C} = -[\frac{\partial B}{\partial \beta} + \frac{\partial \Omega_i}{\partial \beta} A + \Omega \frac{\partial A}{\partial \beta}] \underline{Y}_i \qquad (C-14)$$

and, upon premultiplying Eq.(C-14) by Y^T,

$$Y^T (B + \Omega A) Y \underset{\sim}{C} = -Y^T F(\Omega) \underline{Y}_i$$

$$F(\Omega) \equiv \frac{\partial B}{\partial \beta} + + \frac{\partial \Omega_i}{\partial \beta} A + \Omega \frac{\partial A}{\partial \beta} \qquad (C-15)$$

For the k^{th} component, $k = 1, \ldots, 2n$, one obtains

$$(\Omega_k - \Omega_i) C_k = \underline{Y}_k^T F(\Omega) \underline{Y}_i \qquad (C-16)$$

$$C_k = \frac{\underline{Y}_k^T F(\Omega) \underline{Y}_i}{\Omega_k - \Omega_i} \qquad k \neq i \qquad (C-17)$$

Therefore

$$\frac{\partial \underline{Y}_i}{\partial \beta} = \sum_{k=1, k \neq i}^{n} \frac{\underline{Y}_k^T F(\Omega) \underline{Y}_i}{\Omega_k - \Omega_i} \underline{Y}_k + C_i \underline{Y}_i \qquad (C-18)$$

Differentiation of

$$\underline{Y}_i^T A \underline{Y}_i = 1 \qquad (C-19)$$

with respect to β gives

$$\frac{\partial \underline{Y}_i^T}{\partial \beta} + \underline{Y}_i^T \frac{\partial A}{\partial \beta} \underline{Y}_i + \underline{Y}_i^T A \frac{\partial \underline{Y}_i}{\partial \beta} = 0 \qquad (C-20)$$

$$\sum_{k=1}^{n} C_k[\underline{Y}_k^T A \underline{Y}_i + \underline{Y}_i^T A \underline{Y}_k] + \underline{Y}_i^T \frac{\partial A}{\partial \beta} \underline{Y}_i = 0 \qquad (C-21)$$

Since Y_i and Y_k are orthogonal, one obtains

$$C_i = -\underline{Y}_i^T \frac{\partial A}{\partial \beta} \underline{Y}_i / 2 \qquad (C-22)$$

$$\frac{\partial \underline{Y}_i}{\partial \beta} = \sum_{k=1, k\neq i}^{n} \frac{\underline{Y}_k^T F(\Omega) \underline{Y}_i}{\Omega_k - \Omega_i} \underline{Y}_k - \frac{1}{2} \underline{Y}_i^T \frac{\partial A}{\partial \beta} \underline{Y}_i \underline{Y}_i \qquad (C-23)$$

C.4 Second-Order Derivatives of Eigenvalues

Premultipling Eq.(C-9) by $\underline{Y}_i^T$ gives

$$\underline{Y}_i^T F(\Omega) \underline{Y}_i = 0 \qquad (C-24)$$

and, upon differentiation with respect to β, one obtains

$$\frac{\partial \underline{Y}_i^T}{\partial \beta} F(\Omega) \underline{Y}_i + \underline{Y}_i^T [\frac{\partial^2 B}{\partial \beta^2} + \frac{\partial^2 \Omega_i}{\partial \beta^2} A + 2 \frac{\partial \Omega_i}{\partial \beta} \frac{\partial A}{\partial \beta} + \Omega_i \frac{\partial^2 A}{\partial \beta^2}] \underline{Y}_i + \underline{Y}_i^T F(\Omega) \frac{\partial \underline{Y}_i}{\partial \beta} = 0 \qquad (C-25)$$

$$\frac{\partial^2 \Omega_i}{\partial \beta^2} = -\underline{Y}_i^T [\frac{\partial^2 B}{\partial \beta^2} + 2 \frac{\partial \Omega_i}{\partial \beta} \frac{\partial A}{\partial \beta} + \Omega_i \frac{\partial^2 A}{\partial \beta^2}] \underline{Y}_i - \underline{Y}_i^T F(\Omega) \frac{\partial \underline{Y}_i}{\partial \beta} + \frac{\partial \underline{Y}_i^T}{\partial \beta} F(\Omega) \underline{Y}_i \qquad (C-26)$$

APPENDIX D

UNIFORM SEISMIC LOAD

In order to compare different analytical methods of structural response analysis, such as response spectrum method (widely used in current design code), time history integration method, and stochastic analysis, a special filtered white noise is used to establish a uniform load. The following procedure is used:

1) In stochastic vibration analysis, the response spectrum for a set of ground motion records is related to the power spectral density function of the ground motion. The simple formula Eq.(5-16) for this relationship is given by Kaul[89].

2) It is noted that both forms of mean power spectral density function of the accelerograms and the PSDF compatible with standard response spectra in aseismic design code are approximations of the filtered white noise spectral density described by Kanai-Tajimi:

$$S_g(\omega) = \frac{1 + 4\zeta_g^2(\frac{\omega}{\omega_g})^2}{[1 - (\frac{\omega}{\omega_g})^2]^2 + 4\zeta_g^2(\frac{\omega}{\omega_g})^2} S_o \qquad (D-1)$$

where ω_g is the filter natural frequency, ζ_g is the filter natural damping ratio, and S_g is strength of white noise excitation.

For a set of ω_g, ζ_g and S_0, $S_g(\omega)$ can be obtained. Further, a corresponding acceleration response spectrum resulted from Eq.(D-1) can be obtained as well. It is not difficult to find a filter which is compatible with a target response spectrum. As a result of this procedure, a special filtered white noise, i.e., the SAS-filtered white noise compatible with ATC-3 400-gal design acceleration response spectra is found (duration is 10 seconds).

Table D.1 gives a list of the filter parameters. A comparison between the design spectra and analytical results substituting the parameters to Eq.(D-1) is shown in Fig. D-1.

3) Using these SAS filtered white noise models, the associated artificial ground motions can be generated by following the simulation model

$$Y(t) = \sum_{k=1}^{N} A_k(\omega_k) Cos(\omega_k + \phi_k) \qquad (D-2)$$

where ϕ_k are the uniformly distributed random variables between $0-2\pi$, ω_k are the discrete frequencies, and A_k are the amplitudes, which are related to the PSDF by

$$A_k(\omega_k) = \sqrt{2S(\omega_k)\Delta\omega} \qquad (D-3)$$

Thus, a set of uniform seismic loads, a design response spectrum, a power spectral density function and the number of artificial ground motions have been established.

Table D.1 *SAS-Filtered white noise parameters*

Matched R.S.	S_0	T_g	ζ_g
ATC-3, I-soil	0.00016	0.34	0.7
ATC-3, II-soil	0.00031	0.54	0.85
ATC-3, III-soil	0.000421	0.9	0.8

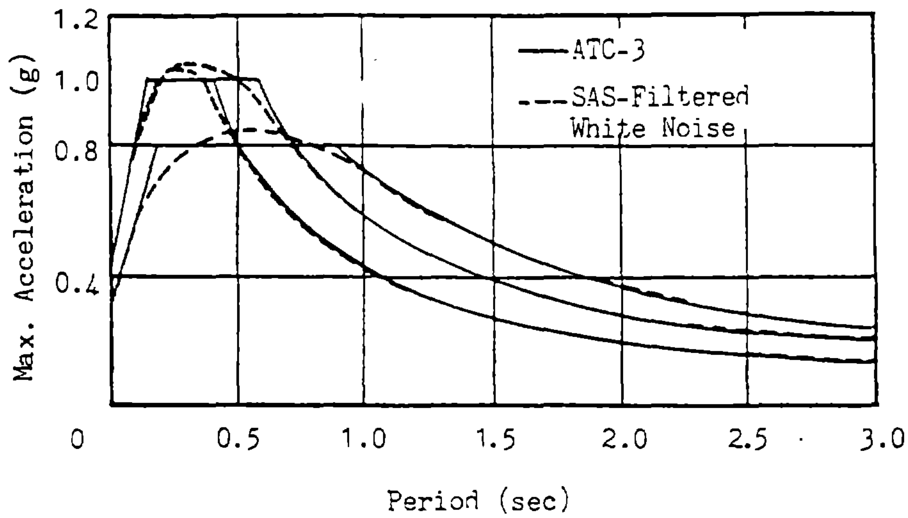


Fig. D-1 ATC-3 Design and Analytical Response Spectra

COMPUTER PROGRAM

Name	Computation function	Usage range
RSPS	**Response sensitivity**	P-S system
	Uncertain structural parameters	
	including nonlinear uncertainty	
ESPS	**Eigenvector/value sensitivity**	P-S system
	Uncertain structural parameters	
	including nonlinear uncertainty	
RCEE	**Real and complex eigensystem analysis**	P-S system
	Real and complex model analysis	
THPS	**Elasto-plastic time history response**	P-S system
	including energy analysis and damage assessment	
FDRA	**Frequency response function**	P-S system
	Response covariance function	
AFKK	**Analytical floor response spectrum**	P-S system
	using Kaul's method	
AFVV	**Analytical floor response spectrum**	P-S system
	using Vanmarcke's method	
THBI	**Elasto-plastic time history response**	P-S system
	For base isolation system	
ASEW	**Generation of artificial ground Motions**	Input PSDF
		or response spectrum
RRSS	**Response spectrum**	By code requirement

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