DESIGN PROCEDURE FOR R-FBI BEARINGS

by

N. MOSTAGHEL
J. M. KELLY

Report to the National Science Foundation

COLLEGE OF ENGINEERING
UNIVERSITY OF CALIFORNIA AT BERKELEY
For sale by the National Technical Information Service, U.S. Department of Commerce,

See back of report for up to date listing of EERC reports.

DISCLAIMER
Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation or the Earthquake Engineering Research Center, University of California at Berkeley.
The basic parameters affecting the design of the Resilient–Friction Base Isolator (R–FBI) bearings and their inter–relationship together with a procedure incorporating the SEAONC tentative provisions for the bearing design are presented. The procedure, while providing the isolator with the design displacement capability and the control of the maximum base shear that is transferred through the bearings, also ensures against instability and yields all the dimensions necessary for the fabrication of the R–FBI bearings. As an example it is used to design bearings for a five–story 1/3–scale frame model to be tested on the shaking table at the University of California at Berkeley.
DESIGN PROCEDURE FOR R—FBI BEARINGS

by

N. Mostaghel
Professor of Civil Engineering
University of Utah

and

James M. Kelly
Professor of Civil Engineering
University of California at Berkeley

A Report to the
National Science Foundation

Report No. UCB/EERC-87/18
Earthquake Engineering Research Center
College of Engineering
University of California at Berkeley

November 1987
Abstract

The basic parameters affecting the design of the Resilient–Friction Base Isolator (R–FBI) bearings and their inter–relationship together with a procedure incorporating the SEAONC tentative provisions for the bearing design are presented. The procedure, while providing the isolator with the design displacement capability and the control of the maximum base shear that is transferred through the bearings, also ensures against instability and yields all the dimensions necessary for the fabrication of the R–FBI bearings. As an example it is used to design bearings for a five–story 1/3–scale frame model to be tested on the shaking table at the University of California at Berkeley.
Acknowledgments

The support of the National Science Foundation under grant No. CES-8702724 is gratefully acknowledged.

The authors would also like to thank Mr. A. R. Mortazavi, a graduate student in the Department of Mechanical Engineering, University of Utah, for his help in preparing graphs for the report.
# Table of Contents

Abstract ......................................................................................................................... i

Acknowledgments ........................................................................................................ ii

Table of Contents .......................................................................................................... iii

List of Figures .............................................................................................................. iv

Introduction .................................................................................................................. 2

Design Parameters ....................................................................................................... 4

1. Equivalent Viscous Damping ................................................................................. 4

2. Damping Factors .................................................................................................... 5

3. Design Displacement, Effective and Nominal Periods ......................................... 6

4. Stiffness Reduction Factor ................................................................................... 9

5. Dimensions of the Rubber Core .......................................................................... 12

6. The Sliding Velocity .............................................................................................. 14

7. The Bearing Stress on Teflon ............................................................................. 15

Design Steps .............................................................................................................. 16

Design Example ......................................................................................................... 18

Discussion .................................................................................................................. 20

References .................................................................................................................. 22

Figures ...................................................................................................................... 27
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The R—FBI Bearing</td>
<td>27</td>
</tr>
<tr>
<td>2.</td>
<td>One-Third Scale Structural Model Showing Main Dimensions and Isolation Mounting</td>
<td>28</td>
</tr>
<tr>
<td>3a.</td>
<td>The Effective and Reduced Stiffnesses</td>
<td>29</td>
</tr>
<tr>
<td>3b.</td>
<td>The Isolator Displacement</td>
<td>29</td>
</tr>
<tr>
<td>4.</td>
<td>Variation of Equivalent Viscous Damping with the Friction Force Ratio</td>
<td>30</td>
</tr>
<tr>
<td>5.</td>
<td>Variations of Damping Factor with the Friction Force Ratio</td>
<td>31</td>
</tr>
<tr>
<td>6.</td>
<td>Variations of the Effective Period with the Isolator</td>
<td>32</td>
</tr>
<tr>
<td>7.</td>
<td>Variations of Isolator Displacement with the Isolator</td>
<td>33</td>
</tr>
<tr>
<td>8.</td>
<td>Variations of Nominal Period with the Friction Force Ratio</td>
<td>34</td>
</tr>
<tr>
<td>9.</td>
<td>Variations of R—FBI Displacement with the Friction Force Ratio</td>
<td>35</td>
</tr>
<tr>
<td>10.</td>
<td>Variations of $\beta$ with the Friction Force Ratio</td>
<td>36</td>
</tr>
<tr>
<td>11.</td>
<td>Variations of $\beta$ with the Friction Force Ratio</td>
<td>37</td>
</tr>
<tr>
<td>12.</td>
<td>Variations of $\lambda$ with the Friction Force Ratio</td>
<td>38</td>
</tr>
<tr>
<td>13.</td>
<td>Variations of Rubber Diameter with the Friction Force Ratio</td>
<td>39</td>
</tr>
<tr>
<td>14.</td>
<td>Variations of Total Velocity with the Friction Force Ratio</td>
<td>40</td>
</tr>
<tr>
<td>15.</td>
<td>Section Through the Isolator</td>
<td>41</td>
</tr>
<tr>
<td>16.</td>
<td>Sliding Plates and Teflon Rings</td>
<td>42</td>
</tr>
<tr>
<td>17.</td>
<td>Base Plates and Shear Keys</td>
<td>43</td>
</tr>
<tr>
<td>18.</td>
<td>Cover Plates</td>
<td>44</td>
</tr>
<tr>
<td>19.</td>
<td>Rubber Core and Steel Rod</td>
<td>45</td>
</tr>
</tbody>
</table>
DESIGN PROCEDURE FOR R—FBI BEARINGS
Introduction

Seismic base isolation is increasingly being utilized as a practical and economical way to protect structures and their contents against earthquakes. It has been incorporated into the foundations of a number of new and existing structures [1–6]. There are many proposed systems [7–22]. The ones which have been tested and already implemented into structures are laminated rubber bearings (closely spaced layers of steel and rubber) either with or without lead plugs [23–25] and laminated rubber bearings with a pair of friction plates [1,2]. Design guidelines have also been suggested [26–28]. In all these systems, the rubber carries both the vertical and the lateral loads. A new system, Resilient–Friction Base Isolator (R–FBI), in which the vertical load and the lateral load carrying functions are separated, was proposed in 1983. The system's details and some of its characteristics have already been discussed in literature [29–35]. In summary, an R–FBI bearing (Fig. 1) is composed of a set of stainless steel plates with a teflon sheet bonded to one side, a rubber core (with or without a central steel rod) through the center of the plates, and cover plates. The rubber core distributes the lateral displacement across the height of the isolator and carries no gravity loads. The sliding velocity can be reduced to a desired level by utilizing an appropriate number of sliding plates. The interfacial friction force acts both as the structural fuse and as energy absorber. The bearing will not slide unless the excitations exceed certain levels. As the bearing starts to slide, the rubber deforms, generating the elastic force, which tends to push the system back toward its original position.

Preliminary tests of the bearings together with computer experiments have demonstrated the R–FBI's potential as an effective base isolation system. The analytical procedure for estimating the response of structures supported on R–FBI bearings to earthquake ground motions and a design procedure based on a set of
proposed design spectra have already been discussed [34,36,39]. In this report, a
design procedure for the R—FBI bearing incorporating the tentative provisions
proposed by the Base Isolation Subcommittee of the Seismology Committee of the
Structural Engineers Association of Northern California (SEAONC) [26] is
presented. The procedure, while providing the isolator with the design displace­
ment capability and the control of the maximum base shear that is transferred
through the bearing, also ensures against instability. It yields all the dimensions
necessary for the fabrication of R—FBI bearings. As an example, the procedure is
used to design the bearings for a five-story 1/3-scale steel structure model (Fig. 2).
This model has already been used to check the performance of a number of base
isolation systems on the shaking table at the Earthquake Engineering Research
Center of the University of California at Berkeley. It will also be used to check the
performance of the R—FBI system on the same shaking table in the near future.
The performance of the isolation system for this particular frame has already been
checked through computer experiments [36], and it appears that the R—FBI sys­
tem satisfies the proposed performance criteria given in [37].
Design Parameters

The parameters which control the design of R–FBI bearings are the following:

1. Equivalent Viscous Damping

The energy dissipation capacity of the R–FBI is one of its attractive features, and while the damping is large it is kinetic rather than viscous, and the equivalent value can only be approximated. The total energy dissipated, \( E \), by the system during a full cycle of displacement, \( \delta \), is

\[
E = \left( 4\mu W + 2\pi \zeta_r V_r \right) \delta, \tag{1}
\]

where \( \mu \) is the coefficient of friction, \( W \) is the total weight, \( \zeta_r \) is the equivalent damping of the rubber core and \( V_r \) is the portion of the lateral load taken by the rubber core. The equivalent damping ratio \( \zeta_e \) for the bearing is

\[
\zeta_e = \frac{E}{2\pi K_e \delta^2}, \tag{2}
\]

where \( K_e \) is the effective stiffness (Fig. 3a). The total isolator shear, \( V \), can be represented by

\[
V = \mu W + V_r. \tag{3}
\]

Alternatively it can be represented by

\[
V = K_e \delta, \tag{4}
\]
and also by

\[ V = CW, \quad (5) \]

where \( C \) is the isolator seismic coefficient. Substitution for \( V \) from eq.(5) into eq.(3) yields

\[ V_r = (C - \mu)W. \quad (6) \]

Substitutions from eqs. (1) and (4) to (6) into equation (2) yield

\[ \zeta_e = \zeta_r + (2/\pi - \zeta_r)(\mu/C). \quad (7) \]

The quantity \( \mu/C \) represents the fraction of the total lateral force, \( CW \), which is resisted by the friction force, \( \mu W \). It will be seen that this is the basic parameter controlling the system performance. This parameter will be referred to as the friction force ratio. A plot of the variations of \( \zeta_e \) with \( \mu/C \) for various values of \( \zeta_r \) is given in Fig. 4.

2. Damping Factor

The SEAONC tentative provisions specify the effects of the damping in the isolation system in terms of a factor \( B \) and give a table of variations of \( B \) with \( \zeta_e \) [26]. Using this table it can be shown that

\[ B = 1.1 + 2\zeta_e \leq 1.9, \text{ For } 17\% < \zeta_e \leq 40\%, \quad (8) \]

\[ B = 1.5 + \zeta_e \leq 2.0, \text{ For } 40\% \leq \zeta_e \leq 50\%. \quad (9) \]
As the equivalent damping in the R–FBI system is expected to be larger than 17%, the value of $\zeta_e$ as given by eq. (7) may be substituted into eqs. (8) and (9). These substitutions yield

\begin{align}
B &= (1.1 + 2\zeta_r) + (4/\pi - 2\zeta_r)(\mu/C) \leq 1.9, \quad (10) \\
B &= (1.5 + \zeta_r) + (2/\pi - \zeta_r)(\mu/C) \leq 2.0. \quad (11)
\end{align}

The values of $B$ as given by the above relations are plotted against the friction force ratio, $\mu/C$, for various values of rubber damping ratio, $\zeta_r$, in Fig. 5.

3. Design Displacement, Effective and Nominal Periods

The design displacement can be estimated by the application of SEAONC [26] tentative provisions. These provisions require that the isolation system have a displacement capacity given by

\begin{equation}
\delta = \frac{10}{B} \frac{ZNS}{T_e}, \quad (12)
\end{equation}

where $Z$ is the effective peak acceleration (EPA), $N$ is a factor ranging from 1.0 to 1.5 to reflect proximity to active fault systems, $S$ is a soil factor that varies from 1.0 to 2.7 over a range of four soil types, $B$ is the damping factor, and $T_e$ is the effective period. From eqs. (4) and (5) it can be shown that

\begin{equation}
\delta = \frac{CW}{K_e}, \quad (13)
\end{equation}

Considering the fact that
substitution for $\delta$ from eq. (13) into eq. (12) yields

$$T_e = \frac{10(2\pi)^2}{g} \frac{1}{C} \left(\frac{ZNS}{B}\right).$$  \hspace{1cm} (15)

Substitution for $T_e$ from the above relation into eq. (12) yields

$$\delta = \frac{10^2(2\pi)^2}{g} \frac{1}{C} \left(\frac{ZNS}{B}\right)^2. \hspace{1cm} (16)$$

For the specified levels of the isolator seismic coefficient, $C$, the above relations can be used to estimate the required effective period $T_e$ and the required minimum displacement capacity $\delta$ for any isolation system. Using $(ZNS/B)$ as a parameter, $T_e$ and $\delta$ are plotted versus $C$ in Figs. 6 and 7.

To include the effects of the R-FBI's friction in the above expressions it is necessary to define the relation between the effective stiffness $K_e$ and the reduced stiffness $K'$. Even though there is no vertical load on the rubber core, the lateral stiffness of the system will be reduced due to the presence of the vertical load $W$. This effect can be represented by a stiffness reduction factor $\beta$. Hence,

$$K' = \beta K, \hspace{1cm} (17)$$

where $K'$ is the reduced stiffness and $K$ is the lateral stiffness of the rubber core when there is no vertical load on the isolator. Therefore, for any displacement $\delta$ the actual lateral force in the rubber core is $K'\delta$. Substitution of this quantity for $V_r$ in
eq.(6) yields

$$\delta = \frac{(C - \mu)}{K'} W . \tag{18}$$

Alternatively eqs. (4) and (5) yield

$$\delta = \frac{CW}{K_e} . \tag{19}$$

Equating these two relations, one obtains

$$K_e = \frac{K'}{1 - \mu/C} . \tag{20}$$

Substitution for $K'$ from eq. (17) into eq. (20) yields

$$K_e = \frac{\beta}{1 - \mu/C} K . \tag{21}$$

Considering the fact that

$$K' = \frac{W}{g} (2\pi)^2 / T^2 , \tag{22}$$

where $T$ is the nominal period, substitution for $K_e$ from eq. (20) into eq. (14) yields the relation between the effective period $T_e$ and the nominal period $T$ as

$$T_e = \sqrt{1 - \mu/C} T . \tag{23}$$

Substituting for $T_e$ from the above relation into eq. (15) and multiplying
both sides of eqs. (15) and (16) by $\mu$ one obtains

$$\mu T = \frac{10(2\pi)^2}{g} \left[ \frac{\mu/C}{1 - \mu/C} \right] (ZNS), \quad (24)$$

$$\mu \delta = \frac{10^2(2\pi)^2}{g} \left[ \frac{\mu}{C} \left( \frac{1}{B} \right)^2 \right] (ZNS)^2. \quad (25)$$

In light of eqs. (10) and (11), the quantities enclosed in the brackets in the above relations are only functions of $\mu/C$ and the rubber damping ratio, $\zeta_r$. For a rubber damping of 5% the quantities $\mu T$ and $\mu \delta$ are plotted versus the friction force ratio, $\mu/C$, for various values of ZNS in Figs. 8 and 9.

4. Stiffness Reduction Factor

As was stated before, even though there will be no vertical load on the rubber core, its lateral stiffness will be reduced in the presence of the vertical load. This reduction is represented by the stiffness reduction factor, $\beta$, in equation (17). Through stability analysis of a set of stacked plates with an elastic core [40], it has been shown that

$$\beta = 1 - \frac{\mu W}{K(1 - \eta)D_{to}}, \quad (26)$$

where $D_{to}$ is the outer diameter of the teflon covers and $\eta$ is defined by

$$\eta = \frac{1}{8} \left[ 1 + \left( \frac{D_{ti}}{D_{to}} \right)^2 \right], \quad 1/8 < \eta < 1/4. \quad (27)$$

Here $D_{ti}$ is the inner diameter of the teflon covers. Substitution for K from eq. (17).
into eq. (26) yields

$$\beta = \frac{1}{1 + \frac{\mu g}{(1-\eta)\Omega^2 D_{to}}}$$ \hspace{1cm} (28)

where $\Omega = \sqrt{K'g/W} = 2\pi/T$. One more equation is needed in order to find both the stiffness reduction factor, $\beta$, and the outer diameter of the teflon rings, $D_{to}$.

By considering Fig. 3b and assuming a shear type behavior (i.e. no rotations), the relation between the applied moment and the resisting moment can be represented by

$$Vh = r(D_{to} - \delta)W$$ \hspace{1cm} (29)

where $h$ is the clear height of the rubber core and $r$ is the moment capacity reduction factor accounting for the presence of the axial load. From the analysis of stability of the R-FBI bearing, it has been shown that [40]

$$r = \beta^2$$ \hspace{1cm} (30)

Considering this relation, substitution for $V$ from eq. (5) into eq (29) yields

$$D_{to} = \left[ 1 + \frac{\mu/\gamma}{\beta^2(\mu/C)} \right] \delta$$ \hspace{1cm} (31)

where $\gamma$ is the shear strain in the rubber and is defined by

$$\gamma = \frac{\delta}{h}$$ \hspace{1cm} (32)
Substitution for \( D_t \) from eq. (31) into eq. (28) yields

\[
\left[ \frac{\mu g}{(1-\eta)\Omega^2\delta} + 1 \right] \beta^3 - \beta^2 + \left( \frac{\mu g}{\mu/C} \right) \frac{\beta - \mu/\gamma}{\mu/C} = 0 .
\] (33)

Considering eqs. (3) and (5) and noting that \( V_r = K' \delta \), the friction force ratio, \( \mu/C \), can be represented by

\[
\mu/C = \frac{1}{1 + \frac{\Omega^2\delta/\mu g}{\mu/C}} ,
\] (34)

implying that

\[
\Omega^2\delta = \left( \frac{1 - \mu/C}{\mu/C} \right) \mu g .
\] (35)

Substitution for \( \Omega^2\delta \) from eq. (35) into eq. (33) yields

\[
\left[ \frac{\mu g}{(1-\eta)(1-\mu/C)} + 1 \right] \beta^3 - \beta^2 + \left( \frac{\mu g}{\mu/C} \right) \frac{\beta - \mu/\gamma}{\mu/C} = 0 .
\] (36)

From eq. (27), it is observed that \( 1/8 < \eta < 1/4 \). Using \( \mu/\gamma \) as a parameter and a \( \eta=1/6 \), the smallest positive root of the above equation is calculated and plotted versus the friction force ratio, \( \mu/C \), in Fig. 10. To show that \( \beta \) is insensitive to variations of \( \eta \), a plot of \( \beta \) versus \( \mu/C \) for \( \eta=1/5 \) and \( \eta=1/7 \) is also presented in Fig. 11.

Considering the fact that for the R–FBI system, \( K_e \geq K \), eq. (21) implies that

\( \beta \geq 1 - \mu/C \). As may be observed from Fig. 11, using
\[ \beta \approx 1 - \frac{\mu}{C} , \quad (37) \]

will yield a conservative value for \( \beta \) for \( \mu / \gamma \geq 0.04 \) independent of \( \eta \). This approximate approach has been utilized recently to formulate a design procedure based on a postulated design spectrum [39].

Once \( \beta \) is known, the outer diameter of the teflon rings, \( D_{to} \), can be found from eq. (31). This equation may be rewritten as

\[ D_{to} = \lambda \delta , \quad (38) \]

where

\[ \lambda = 1 + \frac{\mu / \gamma}{(\mu / C) \beta^2} . \quad (39) \]

Using the values of \( \beta \) given in Fig. 10, the values of \( \lambda \) are evaluated and plotted in Fig. 12. This figure is for \( \eta = 1/6 \). Since \( \beta \) is not sensitive to variations of \( \eta \), this figure may be used to estimate \( \lambda \) for all values of \( \eta \). This figure also shows, for various values of \( \mu / \gamma \), the existence of minimum values for \( \lambda \) which may be used for optimization purposes.

5. Dimensions of the Rubber Core

Assuming the behavior of the rubber in shear is linear, the stiffness of the rubber core can be expressed by

\[ K = \frac{A_r G}{h} , \quad (40) \]
where $A_r$ is the cross-sectional area of the rubber core and $G$ is its shear modulus. Considering the engineering definition for shear strain as given by eq. (32), the cross-sectional area for the rubber core can be represented by

$$A_r = \frac{K\delta}{\gamma G} \quad (41)$$

Considering eqs. (17) and (18), it can be shown that

$$K\delta = \left(\frac{1}{\beta}\right)\left[\frac{1 - \mu/C}{\mu/C}\right](\mu W) \quad (42)$$

Substitution for $K\delta$ from eq. (42) into eq. (41) yields

$$A_r = \left(\frac{\mu W/\gamma G}{\beta}\right)\left(\frac{1 - \mu/C}{\mu/C}\right) \quad (43)$$

The cross-sectional area of the rubber core as given above can also be defined by

$$A_r = (\pi/4)\left[1-(\frac{d_{ri}}{d_{ro}})^2\right]d_{ro}^2 \quad (44)$$

where $d_{ri}$ is the inner diameter of the rubber core, and $d_{ro}$ is its outer diameter. Substitution for $A_r$ from eq. (43) into eq. (44) yields

$$d_{ro} = \sqrt{\frac{(F_1/\beta)}{\frac{1 - \mu/C}{\mu/C}}} \quad (45)$$

where
Using $F_1/\beta$ as a parameter, the required outer diameter of the rubber core, $d_{ro}$, is calculated and plotted in Fig. 13.

6. The Sliding Velocity

The maximum total sliding velocity, $\dot{\delta}$, for design purposes may be estimated from

\[ \dot{\delta} = \frac{2\pi}{T_e} \frac{\delta}{\bar{B}}. \]  

(47)

Substitution for $\delta/T_e$ from eq. (12) into the above relation yields

\[ \dot{\delta} = 10(2\pi) \frac{ZNS}{\bar{B}}. \]  

(48)

Considering relations (8) and (9), a plot of this total sliding velocity versus the friction force ratio, $\mu/C$, is presented in Fig. 14. The interfacial velocity $\dot{\delta}_p$ is given by

\[ \dot{\delta}_p = \frac{\dot{\delta}}{N+1}, \]  

(49)

where $N$ is the number of sliding plates.

The clear height of the bearing, $h$, can be represented by
\[ h = N t_p + (N+1)t_t , \]  

where \( t_p \) and \( t_t \) are the thicknesses of the sliding plates and the teflon covers respectively. Once the final values of \( h \) and \( d_{ro} \) are selected, the nominal period can be calculated through eqs. (17), (22), and (43) as

\[ T = 2\pi \sqrt{\frac{W}{gK'}} = 2\pi \left[ \frac{1}{\beta} \frac{W}{A_r G}(h/g) \right] . \]  

7. The Bearing Stress on Teflon

The outer diameter of the teflon covers, \( D_{to} \), is given by eq. (38). Once the outer diameter of the rubber core, \( d_{ro} \), is known, the inner diameter of the teflon covers, \( D_{ti} \), can be estimated by

\[ D_{ti} \geq d_{ro} + \frac{\delta}{N+1} . \]  

Also the lateral dimension of the bearing, \( D_b \), can be estimated by

\[ D_b \geq D_{to} + \frac{\delta}{N+1} . \]  

These relations ensure that the inner and the outer edges of the steel rings will not travel over the teflon covers. Having \( D_{to} \) and \( D_{ti} \), the bearing stress on the teflon, \( \sigma_t \), can be estimated from

\[ \sigma_t = \frac{W}{(\pi/4)(D_{ti}^2 - D_{to}^2)} . \]
Design Steps

Given $W$, the total weight on the bearing, $\mu$, the friction coefficient, $\gamma$, the allowable shear strain in the rubber, and the postulated seismic intensity, ZNS, the various dimensions of the bearing can be estimated through the following steps:

1. Decide on the value of the isolator seismic coefficient $C$ and find the friction force ratio, $\mu/C$. By selecting $C$, the designer at the outset decides how much force he is allowing to be transferred through the isolator.

2. Use the given ZNS value together with the value of $\mu/C$ found in step 1 to find $\mu T$ and $\mu \delta$ from Figs. 8 and 9, respectively. Then use the given value for $\mu$ to find the nominal period $T$ and the required isolator displacement capacity $\delta$.

3. Calculate $\mu/\gamma$ and use it together with $\mu/C$ found in step 1 to find $\beta$ and $\lambda$ from Figs. 10 and 12, respectively.

4. Substitute the values of $\delta$ and $\lambda$ from steps 2 and 3 into eq. (38) to find the outer diameter of the teflon covers, $D_{to}$.

5. Find the value of $F_1$ from eq. (46). To find $F_1$, one needs an estimate of $d_{ri}/d_{ro}$. As $d_{ri}$ is the inner diameter of the rubber core it is equal to the diameter of the central steel rod. This rod is positioned inside the rubber core to prevent strain concentration at the sliding interfaces. There is, as yet, no analytical way of estimating $d_{ri}/d_{ro}$. Its value ranges from zero to perhaps $1/2$. Use $d_{ri}/d_{ro}=1/4$ together with the values of $\mu/\gamma$ and $\beta$ to find $F_1$ and $F_1/\beta$. Use this value of $F_1/\beta$ together with the value of $\mu/C$ to find $d_{ro}$ the outer diameter of the rubber core.
6. Use eqs. (52) to (54) to estimate the inner diameter of the teflon cover, \( D_{ti} \), the lateral dimension of the bearing, \( D_b \), and the bearing stress on teflon, \( \sigma_t \).

7. Use the values of ZNS and \( \mu/C \) to find the total velocity \( \dot{\delta} \) from Fig. 14.

The number of the sliding plates, \( N \), should be chosen such that the maximum interfacial velocity \( \dot{\delta}_p \), as given by eq. (49), will be sufficiently small. This is necessitated by the fact that the teflon's friction coefficient increases with sliding velocity and decreases with the bearing stress. For the given values for \( \mu \) and the bearing stress found in step 6, estimate \( \dot{\delta}_p \) either from direct tests or available experimental results \([38,41]\). Once \( \dot{\delta}_p \) is known, then \( N = [\dot{\delta} / \dot{\delta}_p] - 1 \) and the clear height of the bearing, \( h \), can be found from eq. (50). In using this equation one should use sliding plates thick enough to prevent any damage to the plates due to handling or operations.
Design Example

The procedure described above will be used to design the R-FBI bearings for the frame shown in Fig. 2. This frame is a 1/3-scaled model which has been used to check the performance of several isolation systems on the shaking table at the Earthquake Engineering Research Center of the University of California at Berkeley [23-25]. It will also be used to check the performance of the R-FBI system on the same shaker table. The total weight of the model frame is 80 kips. Four bearings will be used to support the frame. The following values will be assumed for the purpose of this example:

\[ \begin{align*}
W &= \text{total weight on one bearing} = 20,000 \text{ lbs} \\
\mu &= \text{friction coefficient} = 0.06 \\
\gamma &= \text{allowable shear strain in rubber} = 100\% \\
\zeta_r &= \text{damping of the rubber core} = 5\% \\
G &= \text{effective shear modulus of the rubber} = 150 \text{ psi} \\
ZNS &= \text{postulated seismic intensity} = 0.4
\end{align*} \]

Step 1. Assuming the lateral force to be transmitted to the superstructure to be 10% of the structure's weight, the isolator seismic coefficient \( C = 0.1 \). Therefore, \( \mu/C = 0.6 \).

Step 2. For \( ZNS = 0.4, \mu/C = 0.6 \) and \( \mu = 0.06 \), Figs. 8 and 9 yield a nominal period \( T = 3.3 \text{ sec.} \) and a displacement \( \delta = 4.5 \text{ inches} \). The effective period is calculated from eq. (23) to be \( T_e = 2.1 \text{ sec.} \).

Step 3. For \( \mu/C = 0.6 \) and \( \mu/\gamma = 0.06 \), Figs. 10 and 12 yield \( \beta = 0.45, \lambda = 1.5 \).

Step 4. For \( \delta = 4.5 \text{ inches} \) and \( \lambda = 1.5 \) eq (38) yields \( D_{to} = 6.7 \text{ inches} \). Use \( D_{to} = 7.5 \text{ inches} \).
Step 5. For the given values of $W$, $\mu$, $\gamma$, $G$, assuming $d_{ri}/d_{ro}=1/4$, eq. (46) yields $F_1=10.86$. Therefore, $F_1/\beta=10.86/0.45=24.13$. Considering this quantity and $\mu/C = 0.6$, Fig. 13 yields $d_{ro} = 4.0$ inches.

Step 6. For $d_{ro} = 4.0$ inches, considering relation (52), assume $D_{ti} = 4.5$ inches. For $D_{to} = 7.5$ inches, considering relation (53), assume $D_{bo} = 8.0$ inches. For $W=20,000$ lbs. and $D_{to} = 7.5$ inches eq. (54) yields the bearing stress on teflon, $\sigma_t = 707$ psi.

Step 7. For $ZNS=0.4$ and $\mu/C = 0.6$ Fig. 14 yields $\dot{\delta}=13.2$ in./sec.$=66$ ft./min.. Using $N=24$ sliding plates the interfacial velocity is calculated from eq. (49) to be $\dot{\delta}_p = 2.6$ ft./min. In this range of velocity and the bearing stress of 707 psi the value of the friction coefficient $\mu$ for teflon is about 0.06 [41]. Using steel plates with thickness $t_p = 1/8$ inches and teflon with thickness $t_t = 1/16$ inches, eq. (50) yields the clear height $h = 49/16$ inches.

The shop drawings incorporating the above dimensions with a 1/32 inches tolerance are given in Figs. 15 to 19.
Discussion

The parameters which affect the design of the R-FBI bearings are: the isolator normal load $W$, the friction coefficient $\mu$, the allowable shear strain, $\gamma$, the damping, $\zeta_r$, and the effective shear modulus, $G$, for the rubber core, the isolator seismic coefficient, $\mu/C$, and the postulated site intensity, $ZNS$. As the central steel rod may have some contribution to the lateral stiffness of the rubber core, the effective shear modulus may be larger than the actual shear modulus for the rubber. Its value should be determined through bearing tests. Once the value of the reduced stiffness, $K'$, is found experimentally, then using relations (17) and (40) one can show that the effective shear modulus is given by $G = K'h/\beta A_r$. As may be observed from Figs. 8 to 14, the isolator seismic coefficient, $\mu/C$, is the main variable which controls the response of and the various dimensions of the R-FBI bearing. It represents the fraction of the total lateral force which is transferred to the super-structure by friction. The rest of the lateral force is transferred by the rubber core. As expected, for given values of $ZNS$ and $\mu$, Fig. 9 shows that the smaller the $\mu/C$, the smaller is the required displacement capacity of the bearing, while Fig. 13 shows that the diameter of the rubber core increases with the reduction of $\mu/C$.

Figure 12 shows that for a given value of $\mu/\gamma$ there exists a $\mu/C$ for which $\lambda$ or equivalently the outer diameter of the teflon ring $D_{to}$ attains its minimum value. For a given structure and a postulated seismic intensity, it may be possible to use an optimum value for $\mu/C$. The object of the optimization would be to minimize the sliding base displacement and acceleration and at the same time use a realizable value for $\mu$. This value of $\mu$ should be large enough to prevent sliding under low level excitations and wind.

The friction coefficient $\mu$ is a function of the bearing stress, the interfacial
velocity \( \dot{\delta}_p \), and the smoothness of the steel surface. But \( \dot{\delta}_p \) is also a function of \( \mu \), \( C \), and the postulated seismic intensity \( ZNS \). Therefore, it is necessary to check the realizability of the friction coefficient. In the example design the value \( \mu = 0.06 \) was assumed. For the bearing stress of \( \sigma_L = 707 \) psi and the calculated interfacial sliding velocity \( \dot{\delta}_p = 2.6 \) ft./min., the value of \( \mu \approx 0.06 \) [41].

The diameter of the rubber core for the designed bearing (with 1/32 inches of tolerance) is 31/32 inches. This was based on rubber with an effective shear modulus of 150 psi. By replacing the rubber core with softer or harder rubber, one can change the dynamic properties of the isolator. Therefore, by stacking plates with a limited number of geometric dimensions and rubber cores with compatible dimensions and various properties, one can design an R–FBI bearing with almost any desired dynamic properties by merely using appropriate numbers of the proper size sliding plates together with a geometrically compatible rubber core which has the requisite properties. This implies the promise of R–FBI bearings as an off-the-shelf item.
References


Fig. 1. The R-FBI Bearing
Fig. 2. One Third Scale Structural Model Showing Main Dimensions and Isolation Mounting
Fig. 3a. The Effective and Reduced Stiffnesses

Fig. 3b. The Isolator Displacement
Fig. 4 Variation of Equivalent Damping with the Friction Force Ratio

\[ \xi = \{ 10\%, 7\%, 5\% \} \]

\( \xi, \text{ Equivalent Damping} \)
Fig. 5. Variations of Damping Factor with the Friction Force Ratio
Fig. 6. Variations of the Effective Period with the Isolator Seismic Coefficient
Fig. 7. Variations of Isolator Displacement with the Isolator Seismic Coefficient
Fig. 8. Variations of Nominal Period with the Friction Force Ratio

\[ \xi_r = 5\% \]

\[ ZNS = 2, 3, 4, \ldots, 1.2 \]
Fig. 9. Variations of R-FBI Displacement with the Friction Force Ratio
Fig. 10. Variations of $\beta$ with the Friction Force Ratio

$\eta = \frac{1}{6}$

$\frac{\mu}{\gamma} = .02, .04, .06, \ldots, .20$
Fig. 12. Variations of $\lambda$ with the Friction Force Ratio

$\eta = \frac{1}{6}$

$\frac{\mu}{\gamma} = 0.02, 0.04, 0.06, \ldots, 0.20$
Fig. 13. Variations of Rubber Diameter with the Friction Force Ratio

\[
\frac{F_1}{\beta} = 5, 10, 25, 50, 75, \ldots, 500
\]
Fig. 14. Variations of Total Velocity with the Friction Force Ratio

\[ \xi_r = 5\% \]

\[ ZNS = 0.2, 0.3, 0.4, \ldots, 1.2 \]

Total Velocity, \( \dot{v} \) (in./sec.)
Fig. 15. Section Through The Isolator

(24 x 2/16" + 25 x 1/16" = 4 9/16")
Fig. 16. Sliding Plates and Teflon Rings
All Welds Similar
1/2" End Return

9/16" drill four holes

Stainless steel plate

Weld

A 36 steel plate

*Note: these dimensions reflected on all sides.

Polished finish
3/16" weld

Round edges

1/4"
3/4"

Shear Key

Fig. 17. Base Plates and Shear Keys
Cover Plate

Fig. 18. Cover Plates
Fig. 19. Rubber Core and Steel Rod

Rubber Core

Steel Rod (1)

Rubber Caps (2)

Round ends

A 36 steel
EARTHQUAKE ENGINEERING RESEARCH CENTER REPORT SERIES

EERC reports are available from the National Information Service for Earthquake Engineering (NISEE) and from the National Technical Information Service (NTIS). Numbers in parentheses are Accession Numbers assigned by the National Technical Information Service; these are followed by a price code. Contact NTIS, 5285 Port Royal Road, Springfield Virginia, 22161 for more information. Reports without Accession Numbers were not available from NTIS at the time of printing. For a current complete list of EERC reports (from EERC 67-1) and availability information, please contact University of California, EERC, NISEE, 1301 South 4th Street, Richmond, California 94804.


UCB/EERC-80/03 "Optimum Inelastic Design of Seismic-Resistant Reinforced Concrete Frame Structures," by Zagajeski, S.W. and Bertero, V.V., January 1980, (PB80 164 635)A06.


UCB/EERC-80/05 "Shaking Table Research on Concrete Dam Models," by Niwa, A. and Clough, R.W., September 1980, (PB81 122 368)A06.


UCB/EERC-80/14 "2D Plane/Axisymmetric Solid Element (Type 3-Elastic or Elastic-Perfectly Plastic)for the ANSR-II Program," by Mondkar, D.P. and Powell, G.H., July 1980, (PB81 122 350)A03.


UCB/EERC-80/22 "3D Solid Element (Type 4-Elastic or Elastic-Perfectly-Plastic)for the ANSR-II Program," by Mondkar, D.P. and Powell, G.H., July 1980, (PB81 123 242)A03.

UCB/EERC-80/23 "Gap-Friction Element (Type 5) for the ANSR II Program," by Mondkar, D.P. and Powell, G.H., July 1980, (PB81 122 285)A03.

UCB/EERC-80/24 "U-Bar Restraint Element (Type 11) for the ANSR II Program," by Oughourlian, C. and Powell, G.H., July 1980, (PB81 122 293)A03.


UCB/EERC-81/08 "Unassigned," by Unassigned, 1981.


UCB/EERC-83/14 "Shaking Table Tests of Large-Panel Precast Concrete Building System Assemblages," by Oliva, M.G. and Clough, R.W., June 1983, (PB85 110 210/A01).


UCB/EERC-83/16 "System Identification of Structures with Joint Rotation," by Dimsdale, J.S., July 1983, (PB84 192 210/A06).


UCB/EERC-83/19 "Effects of Bond Deterioration on Hysteretic Behavior of Reinforced Concrete Joints," by Filippou, F.C., Popov, E.P. and Bertero, V.V., August 1983, (PB84 192 020/A10).


UCB/EERC-83/23 "Local Bond Stress-Slip Relationships of Deformed Bars under Generalized Excitations," by Eligehausen, R., Popov, E.P. and Bertero, V.V., October 1983, (PB84 192 848/A09).


UCB/EERC-84/05 "Earthquake Simulation Tests and Associated Studies of a 1/10th-scale Model of a 7-Story R/C Frame-Wall Test Structure," by Bertero, V.V., Aktan, A.E., Charney, F.A. and Sause, R., June 1984, (PB84 239 409/A09).

UCB/EERC-84/06 "R/C Structural Walls: Seismic Design for Shear," by Aktan, A.E. and Bertero, V.V., 1984.


UCB/EERC-84/08 "Experimental Study of the Seismic Behavior of a Two-Story Flat-Plate Structure," by Mochle, J.P. and Diebold, J.W., August 1984, (PB86 122 533/A012).


UCB/EERC-84/16 "Simplified Procedures for the Evaluation of Settlements in Sands Due to Earthquake Shaking," by Tokimatsu, K. and Seed, H.B., October 1984, (PB85 197 887/A03).


