

**NATIONAL CENTER FOR EARTHQUAKE  
ENGINEERING RESEARCH**

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**DYNAMIC INTERACTION FACTORS FOR  
FLOATING PILE GROUPS**

by

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## PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

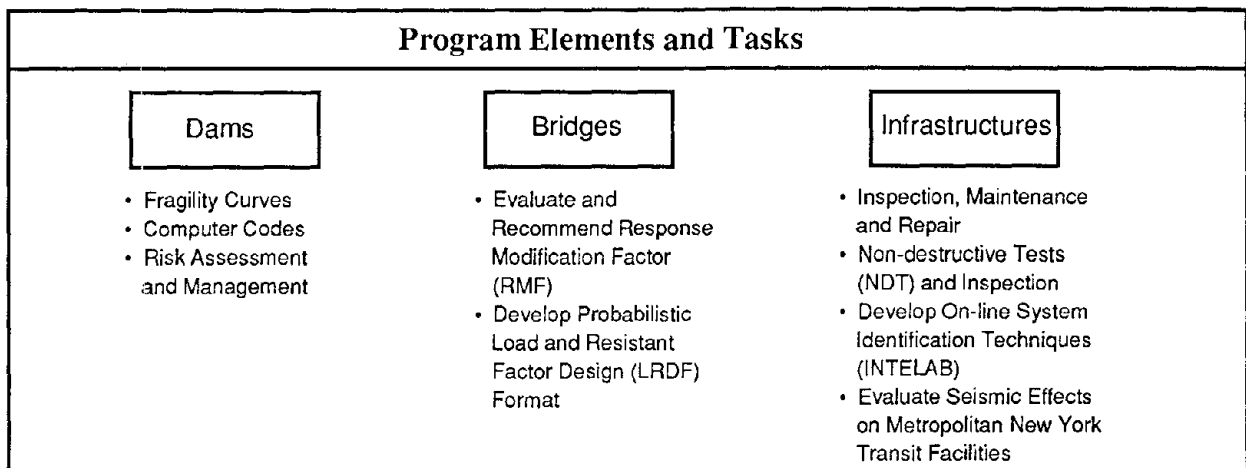
NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 3, Lifeline Systems, and more specifically to the study of dams, bridges and infrastructures.

The safe and serviceable operation of lifeline systems such as gas, electricity, oil, water, communication and transportation networks, immediately after a severe earthquake, is of crucial importance to the welfare of the general public, and to the mitigation of seismic hazards upon society at large. The long-term goals of the lifeline study are to evaluate the seismic performance of lifeline systems in general, and to recommend measures for mitigating the societal risk arising from their failures.

In addition to the study of specific lifeline systems, such as water delivery and crude oil transmission systems, effort is directed toward the study of the behavior of dams, bridges and infrastructures under seismic conditions. Seismological and geotechnical issues, such as variation in seismic intensity from attenuation effects, faulting, liquefaction and spatial variability of soil properties are topics under investigation. These topics are shown in the figure below.



*Many structures and bridges in earthquake prone areas are supported on piled foundations. The seismic response analyses of piled foundations can be performed by superposition of two effects: a kinematic interaction effect, involving the response of pile foundation to base excitation, and an inertial interaction effect, referring to the response of the complete pile-soil-structure system to excitation by D'Alembert forces. In this report, the authors present a comprehensive set of dimensionless graphs of complex-valued dynamic interaction factors versus frequency for vertical, horizontal, and rocking harmonic excitation. These graphs may be readily utilized in practice to perform the inertial interaction analyses in the frequency domain. Thus, this study may be seen as one step in a sequence of investigations carried out by NCEER in the field of seismic design of bridge foundations.*

## ABSTRACT

A comprehensive set of dimensionless graphs of complex-valued **Dynamic Interaction Factors versus Frequency** is presented for vertical, horizontal, and rocking harmonic excitations at the head of individual piles. These readily-applicable graphs have been developed with a rigorous analytical-numerical formulation for two idealized soil profiles (a homogeneous halfspace and a halfspace with modulus proportional to depth) and three pile separation distances (3, 5 and 10 pile-diameters). A wide range of values have been parametrically assigned to pile slenderness and pile-to-soil stiffness ratios. The results are discussed at length to gain valuable insight into the nature of dynamic pile-soil-pile interaction. Geotechnical and earthquake engineers can use the presented graphs exactly as they use the classical interaction factors for static deformation analysis of pile groups.

**KEY WORDS:** Foundation; Piles; Pile group; Dynamic analysis; Interaction factors; Vertical loading; Horizontal loading; Rocking loading; Oscillations; Harmonic vibrations; Waves.





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**SECTION 1**  
**INTRODUCTION**

Under static working loads, the displacements of a pile increase if the pile is located within the deformation field of a neighboring pile. For a pile group, this leads to an **interaction** between individual piles, the consequences of which are: (i) the overall stiffness of the group is smaller than the sum of the individual-pile stiffnesses, and (ii) the sharing among individual piles of the load applied at the pile cap is generally uneven, with the corner piles loaded the most and center piles loaded the least.

In current geotechnical practice, when the displacement of a pile group is of interest, such pile-soil-pile interaction effects are often assessed through the use of interaction factors, by superimposing the effects of two piles at a time (Poulos 1971). An interaction factor,  $\alpha$ , is defined as the fractional increase in deformation (that is, deflection or rotation) at the head of a pile due to the presence of a similarly loaded adjacent pile. Thus, if the stiffness of a single (solitary) pile under a given type of loading is  $K^{(1)}$ , then a load  $P$  will produce a deformation  $u = P/K^{(1)}$ . If two identical piles are **each** subjected to a load  $P$ , then each one will deform by an amount,  $u$ , given

$$u = \frac{P}{K^{(1)}} (1 + \alpha) \quad (1.1)$$

The value of  $\alpha$  depends on the type of loading (axial or lateral), the spacing of the two piles, and the soil and pile material and geometric properties.

The popularity of this superposition method stems from the availability (in published form) of fairly complete sets of

static interaction factors, developed by recourse to integral equation and finite-element formulations (Poulos, 1986, 1971; Butterfield & Banerjee, 1971; Poulos & Davis, 1980), and to simple physically-sound approximations (Randolph & Wroth, 1979).

Unfortunately, the static interaction factors are not applicable to the dynamic analysis of pile groups, except perhaps at very low frequencies of oscillation. Indeed, dynamic studies of pile groups (Wolf & Von Arx, 1978; Waas & Hartmann, 1981; Kaynia & Kausel, 1982; Sheta & Novak, 1982; Nogami, 1983; Kagawa, 1983; Tyson & Kausel, 1983; Roesset, 1984; Dobry & Gazetas, 1988) have demonstrated that the dynamic response of pile groups may differ substantially from their static response, in that the dynamic group efficiency exhibits a strong sensitivity to frequency and may attain values well above unity. Nevertheless, Kaynia & Kausel (1982) have shown that even for dynamic loads Poulos' superposition procedure remains an excellent engineering approximation, provided that **dynamic** interaction factors are used for each frequency of interest. Today, despite the significant progress in understanding dynamic pile group behavior, only a very limited number of dynamic interaction factors have been published in a form readily accessible to practicing geotechnical and earthquake engineers. In response to this apparent need, a comprehensive set of dimensionless graphs of dynamic interaction factors have been developed and are presented in this report as functions of frequency for a practically sufficient range of key material and geometric parameters. These graphs may be readily utilized in practice to obtain realistic estimates of the dynamic response of floating pile groups for soil deposits that could be modeled either as a homogeneous deep stratum, or a deposit with stiffness proportional to depth.

**SECTION 2**  
**DEFINITIONS AND METHOD OF SOLUTION**

Figure 2-1 sketches the system studied: two identical vertical free-head piles, floating in a halfspace with Young's modulus either constant,  $E_s$ , or proportional to depth,  $E_s(z) = E_s(L) z/L$ . The piles, of diameter  $d$  and length  $L$ , are considered to be linear elastic beams with constant Young's modulus,  $E_p$ , and mass density,  $\rho_p$ . The soil is assumed to be a linear hysteretic continuum with constant Poisson's ratio  $\nu_s$ , constant material density  $\rho_s$ , and constant hysteretic damping  $\beta_s$ . Unless otherwise noted, the following typical values were assigned to these three parameters:  $\nu_s = 0.40$ ,  $\beta_s = 0.05$ ,  $\rho_s = 0.70 \rho_p$ . However, the values of:  $\rho_s = 1.2 \rho_p$ , typical of hollow cylindrical piles, and  $\nu_s = 0.48$ , typical for saturated clays, are also given consideration. Finally,  $s$  denotes the axis-to-axis spacing of the piles and  $\theta$  their angle of "departure", i.e. the angle between the line joining the pile centers and the direction of loading (Fig. 2-1).

For two such piles, a frequency dependent dynamic interaction factor,  $\alpha = \alpha(\omega)$ , is defined as:

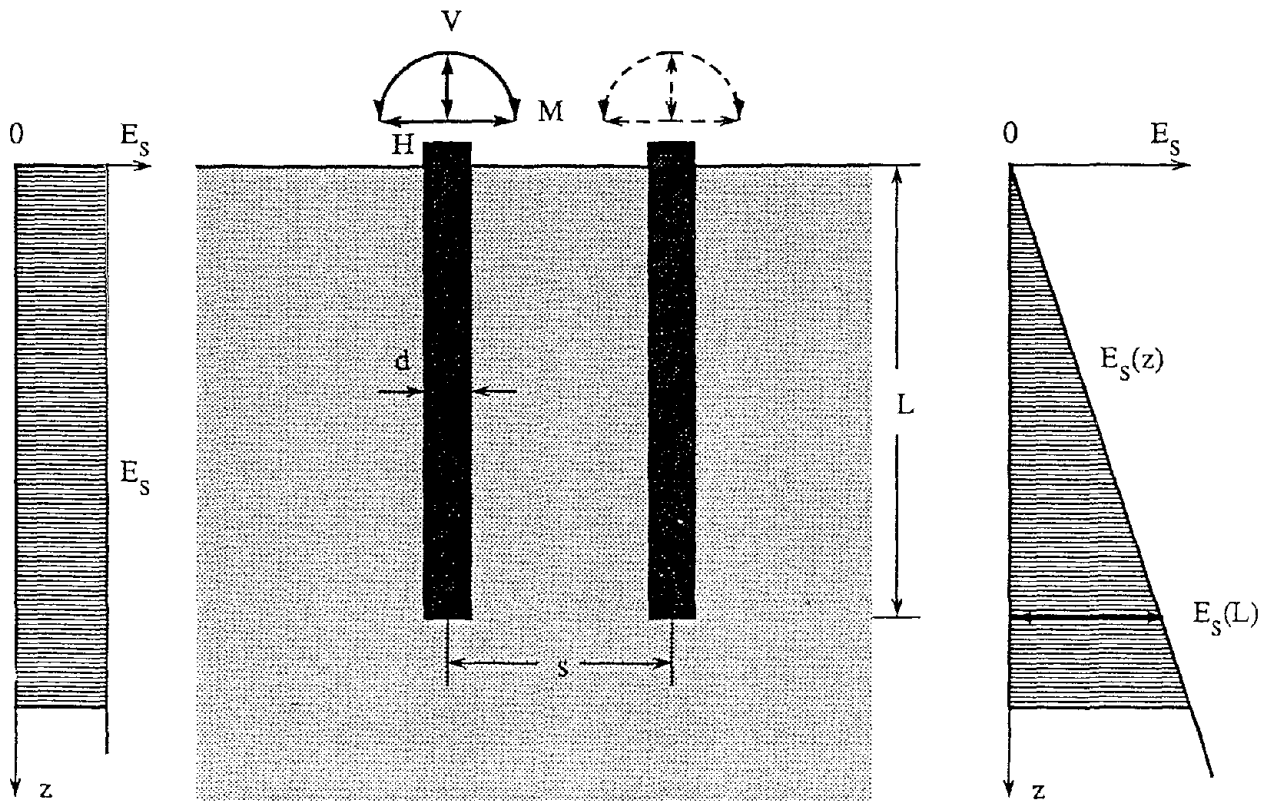
$$\alpha = \frac{\text{dynamic displacement of pile 2 caused by pile 1}}{\text{static displacement of solitary pile 1 due to its own load}} \quad (2.1)$$

in which displacement means translation or rotation. Five different types of dynamic interaction factors are provided in this report, depending on the loading at the pile head and the type of deformation:

- $\alpha_{\mathbf{V}}$ : interaction factor for vertical deflection under vertical loading
- $\alpha_{\mathbf{UH}}$ : interaction factor for horizontal deflection of free-headed piles under horizontal force loading
- $\alpha_{\phi\mathbf{M}}$ : interaction factor for rotation of free-headed piles under moment loading
- $\alpha_{\mathbf{UM}} = \alpha_{\phi\mathbf{H}}$ : interaction factors for horizontal deflection due to moment, or for rotation due to horizontal force of free headed piles.

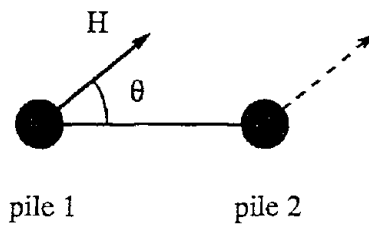
The results presented in the form of graphs of dynamic interaction factors were obtained with a rigorous analytical-numerical formulation developed by Kaynia & Kausel (1982) for dynamic analysis of pile groups in a layered halfspace. In addition, the simplified analytical method of Dobry & Gazetas (1988) and Gazetas & Makris (1990) was utilized in selecting suitable dimensionless problem parameters, in assessing the effects of some of these parameters, and in explaining certain trends observed in the rigorous results.

The Kaynia & Kausel (1982) formulation is in essence a boundary-integral type method in which the Green's Functions, defining the displacement fields due to uniform **barrel** and **disk** loads associated with pile-soil interface tractions, are computed by solving the wave equations through Fourier and Hankel transformations (Kausel, 1981). These functions yield the dynamic soil flexibility matrix which is combined with the analytically-derived pile flexibility matrix, while compatibility of deformations at the pile-soil interface is enforced.



Homogeneous

Nonhomogeneous



PLAN

Fig.2-1 Sketch of the System Studied



### SECTION 3

#### DIMENSIONLESS PROBLEM PARAMETERS

The two key dimensionless parameters that have been shown in the literature to largely control the value of **all** interaction factors are:

- the frequency factor

$$a_o = \frac{\omega d}{V_s^*} \quad (3.1)$$

where  $V_s^*$  a characteristic value of the soil S-wave velocity profile (in this report  $V_s^*$  is taken equal to  $V_s$  for the homogeneous and to  $V_s(L)$  for the inhomogeneous profiles), and

- The pile spacing to diameter ratio (hereafter called spacing ratio)  $s/d$ .

The curves of the interaction factors as functions of the above frequency factor, i.e.,  $\alpha = \alpha(a_o)$ , exhibit peaks and troughs occurring at different locations for different values of the spacing ratio,  $s/d$  (e.g. Kaynia & Kausel, 1982). It becomes clearer when an alternative frequency parameter,

$$b_o = a_o \frac{s}{d} = \frac{\omega s}{V_s^*} \quad (3.2)$$

is used in place of  $a_o$ . Indeed, according to the aforementioned simplified method of Dobry & Gazetas (1988) as a first approximation, interaction factors take the form:

$$\alpha \sim \left(\frac{s}{d}\right)^{-1/2} \exp(-\beta_s \omega s/V) \cdot \exp(-i \omega s/V) \quad (3.3)$$

where  $i = \sqrt{-1}$  and  $V$  is equal to, or a multiple of,  $V_s^*$ , depending on the mode of deformation and the type of soil profile. The first two terms in Eq. 3.3 constitute the amplitude, while the last term controls the undulations with frequency of the interaction factor. It is evident that the amplitude,  $|\alpha|$ , decreases with increasing spacing ratio,  $s/d$ , and with increasing soil hysteretic damping,  $\beta_s$ , and frequency parameter,  $b_o$ . On the other hand, the fluctuations of  $\alpha$  with frequency depend solely on the frequency parameter,  $b_o$ . Hence, the graphs in this report are in the form  $\alpha = \alpha(b_o)$ .

The third most important problem parameter appears to be the angle of "departure",  $\theta$ , for the lateral interaction factors. Following Poulos (1971) and Kausel & Kaynia (1982), results are presented here only for  $\theta = 0^\circ$  and  $\theta = 90^\circ$ . For any other angle, the interaction factors can be obtained with sufficient accuracy from the following relationship:

$$\alpha(\theta^\circ) \approx \alpha(0^\circ) \cos^2\theta + \alpha(90^\circ) \sin^2\theta \quad (3.4)$$

The vertical interaction factor is dependent of  $\theta$ , due to symmetry.

The two other parameters that have been found to have an influence (although rather secondary, for practical purposes) on the interaction factors are:

- the ratio of the "effective" pile modulus to soil Young's modulus

$$\frac{E_p}{E_s} \quad \text{or} \quad \frac{E_p}{E_s(L)}$$

for the homogeneous or inhomogeneous profiles of Fig. 2-1, and



- the pile slenderness ratio  $L/d$ .

Note that the presented graphs, although derived for circular piles of solid cross-section with diameter  $d$ , can also be used for pipe piles and **concrete-filled steel pipe piles** (Gazetas & Dobry, 1984). To this end, an appropriate "effective" modulus,  $E_p$ , is chosen such that:

$$E_p = \frac{(EA)_p}{\pi r_o^2} \quad \text{for axial deformations,}$$

or

$$E_p = \frac{(EI)_p}{\pi r_o^4/4} \quad \text{for lateral deformations} \quad (3.5)$$

where:  $(EA)_p$  and  $(EI)_p$  are the axial and bending rigidities of the **actual** pile section. [For example, for a steel pipe pile:  $(EA)_p = E_{\text{steel}} (\pi r_o^2 - \pi r_i^2)$  and  $(EI)_p = E_{\text{steel}} (\pi r_o^4 - \pi r_i^4)/4$ ;  $r_o = d/2 =$  external radius and  $r_i =$  internal radius of the pipe.]



SECTION 4  
THE GRAPHS

Figures 4-1 to 4-45 portray the real and imaginary parts of the complex-valued dynamic interaction factors as functions of frequency, in the following dimensionless parametric form:

$$\begin{aligned}\alpha &= \text{Real}(\alpha) + i \cdot \text{Imaginary}(\alpha) \\ &= \alpha(b_0; s/d, \theta^0, E_p/E_s \text{ or } E_p/E_s(L), L/d, \rho_s/\rho_p, \nu_s) \quad (4.1)\end{aligned}$$

Table 4-1 explains the organization of the graphs and reveals the considered ranges of problem parameters. Note that  $b_0$  is given values up to 5 which would be sufficiently high for most applications, even when pile spacing equals 10.

The following characteristic trends are worthy of note in the graphs of Figs. 4-1 through 4-45.

#### 4.1 General Trends

1. Dynamic interaction factors are quite different from the respective static interaction factors, to which they converge only at zero frequency. It is apparent that use of static interaction factors in estimating the dynamic response of pile groups must be avoided as it would, in general, worsen rather than improve the prediction.

2. While static interaction factors are invariably positive numbers smaller than unity, the dynamic factors are complex with real and imaginary parts,  $\text{Re}(\alpha)$  and  $\text{Im}(\alpha)$ , that fluctuate with frequency, achieving positive and negative values. As the aforementioned simplified method (Eq. 3.3) had anticipated, this frequency dependence of  $\alpha$  is indeed almost entirely controlled by the frequency parameter  $b_0$ . Of particular significance for the response of pile groups are the negative values of  $\text{Re}(\alpha)$ . Such values arise whenever waves originating from pile "1" with a

certain phase arrive at a neighboring pile "2" in exactly opposite phase, thereby inducing displacements  $v_{12}$  which are opposite to the displacements  $v_{22}$  due to this pile's own load. As a consequence, in a pile group loaded, e.g., through a rigid cap, a larger force must be applied onto pile "2" to enforce a certain displacement amplitude; hence, the dynamic stiffness of the group increases and may achieve values well above the sum of the individual stiffness of each pile ("efficiency" greater than unity, in established geotechnical terminology).

3. The soil Poisson's ratio,  $\nu_s$ , and the soil-to-pile mass density ratio,  $\rho_s/\rho_p$ , have no discernible effect on dynamic interaction factors (Figs. 4-43 to 4-45) On the other hand, the effect of soil hysteretic damping,  $\beta_s$ , can be captured with the simplified expression given in Eq. 3.3:

$$\frac{\alpha(\beta_s)}{\alpha(0.05)} \approx \exp[-(\beta_s - 0.05) \omega s/V] \quad (4.2)$$

where:  $\alpha(0.05)$  is obtained from the provided graphs, while as a first approximation,  $V$  is taken as the average (over depth) shear wave velocity,  $\bar{V}_s$ , or the average "Lysmer's analog" velocity,  $\bar{V}_{La} = [3.4/\pi(1-\nu_s)] \bar{V}_s$ . Specifically,  $V = \bar{V}_s$  for the vertical interaction factor, and for the lateral interaction factors when  $\theta = 90^\circ$ ; while  $V = \bar{V}_{La}$  for the lateral factors when  $\theta = 0^\circ$  (see Dobry & Gazetas 1988). Evidently, the effect of damping may become significant only at high values of the frequency parameter  $b_o$ , i.e. at high frequencies and/or large spacing ratios.

4. Under static and low-frequency conditions, vertical interaction factors are generally greater than lateral interaction factors; and among the latter, interaction factors due to moment loading are smaller than due to horizontal-force loading, while for rotation they are smaller than for

displacement. At intermediate and higher frequencies, however, due to the observed undulations, this general picture may not be true, except perhaps in a general average sense.

#### 4.2 Vertical Interaction Factors

1. Pile spacing ratio,  $s/d$ , affects  $\alpha_v$  in two different ways: by controlling its static and low-frequency amplitude, and by influencing the **frequencies** at which peaks and troughs occur. By contrast, the amplitudes of those peaks and troughs are, for a homogeneous soil hardly influenced by  $s/d$ ; with inhomogeneous soil the effect of  $s/d$  becomes appreciable only for  $s/d$  exceeding 5.

2. The stiffness and slenderness ratios,  $E_p/E_s$  [or  $E_p/E_s(L)$ ] and  $L/d$ , are of a rather secondary importance. Specifically, the effect of  $L/d$  is appreciable at zero and very low frequencies only for relatively-rigid piles (i.e.  $E_p/E_s \approx 10,000$  or  $E_p/E_s(L) \approx 5,000$ ). The effect of  $E_p/E_s$  or  $E_p/E_s(L)$  is noticeable only when these ratios attain very low values (of the order of 200 or 100, respectively) -- i.e., in the lowest range of possible practical interest.

3. Under static and low-frequency conditions, the smallest  $\alpha_v$  values are associated with the inhomogeneous deposit. At moderate and high frequencies, however, inhomogeneity affects mainly the **location and shape** of the peaks and valleys of  $\alpha_v$ , rather than their amplitude.

#### 4.3 Lateral Interaction Factors

1. The spacing ratio,  $s/d$ , affects both the real and imaginary part of the amplitudes of lateral interaction factors **over the whole frequency range** studied. (Contrast with the behavior of  $\alpha_v$ , the amplitude of which is affected by  $s/d$  only at

a zero and very low frequencies.) One of the consequences: for  $s/d = 10$ ,  $\alpha_{uH}$ , the largest of the lateral interaction factors, attains at all frequencies very small values (e.g., less than 0.10 for  $E_p/E_s = 1000$ ); whereas the corresponding vertical factor,  $\alpha_v$ , for frequencies  $b_0$  exceeding 1, achieves for  $s/d = 10$  essentially identical (relatively-high) values with those for  $s/d = 3$ .

2. The effect of the angle of "departure"  $\theta$  is twofold. First, the static and low-frequency amplitudes of all lateral interaction factors decrease as  $\theta$  increases; for instance,  $\alpha_{uH}(90^\circ) \approx 0.60 \alpha_{uH}(0^\circ)$ . Second, at intermediate and high frequencies, increasing the angle  $\theta$  does **not** produce a decreasing amplitude of the peaks and valleys; however, the **rate of fluctuation** (of both real and imaginary parts) of the interaction factor is faster for  $\theta = 90^\circ$  than for  $\theta = 0^\circ$ . To provide a physical explanation of this effect, recall, that according to Eq. 3.3 of the aforementioned simplified model the fluctuations of  $\alpha$  are controlled by  $\omega s/V$ , where  $V$  = the (average) wave velocity of the predominant waves. For  $\theta = 90^\circ$  one pile sends to the other mainly S waves; hence, in the homogeneous deposit, for instance,  $V = V_s$ . For  $\theta = 0^\circ$  the two piles interact through compression-extension, rather than shear, waves; such waves propagate at an apparent phase velocity  $V \approx V_{La}$  ("Lysmer's analog" velocity [Gazetas 1990]). In this case,  $V_{La} = [3.4/\pi(1-\nu)] \cdot V_s \approx 1.80 V_s$ , leading to rates of fluctuation:  $\omega s/V_s$  (for  $\theta = 90^\circ$ ) = 1.80 times  $\omega s/V_{La}$  (for  $\theta = 0^\circ$ ) -- in accord with the observed faster rate of fluctuations for  $\theta = 90^\circ$ .

3. It is well understood that the slenderness ratio,  $L/d$ , plays no role in the lateral response of "flexible" piles, that is, piles whose length  $L$  exceeds a "critical" or "active" length  $\ell_c$  given by the following conservative expression (Gazetas 1990):

$$l \approx 2 d (E_p/E_s)^{0.25} \quad (4.3)$$

applicable for **homogeneous soils**. The part of the pile located below  $l_c$  from the top remains practically idle at all frequencies and, therefore, pile response and pile-to-pile interaction are governed by  $l_c$  and not  $L$ . Most real-life piles, as well as are the piles in our parametric study, are indeed "flexible". For example, with  $E_p/E_s = 1000$ , the "active" length becomes  $l_c \approx 2 d (1000)^{0.25} \approx 11.2 d$ ; hence, only the shortest of the considered piles, with  $L = 10 d$ , falls just below the limit for completely "flexible" behavior.

The obtained results (Figs. 4-19 through 4-22) confirm that  $L/d$  has no effect on lateral interaction factors for all but the decisively-"rigid" piles. Only the shortest-and-stiffest of the considered piles  $L/d = 10$  and ( $E_p/E_s = 10,000$ ) belongs in that category, and thereby its  $\alpha_{uH}$  shows some differences from the  $\alpha_{uH}$  of all the other piles.

4. The above conclusion is also in general valid with the nonhomogeneous soil profile. The "active" length in this case is given by the following conservative expression (Gazetas 1990):

$$l_c \approx 2 d (E_p/\tilde{E}_s)^{0.20} \quad (4-4a)$$

$$\text{where } \tilde{E}_s = E_s(d) = E_s(L) \cdot \frac{d}{L} \quad (4-4b)$$

For instance, the typical pile with  $L/d = 20$  and  $E_p/E_s(L) = 500$  has an  $l_c \approx 2 d (500 \times 20)^{0.20} \approx 12.6 d$ ; hence  $L > l_c$  and this pile is "flexible." Almost "flexible" is the  $L/d = 10$  pile, for which  $l_c \approx 2 d (500 \times 10)^{0.20} \approx 11 d$ . Again, only the stiffest-and-shortest of the considered piles [ $E_p/E_s(L) = 5,000$  and  $L/d = 10$ ] is a decisively-"rigid" pile:  $l_c \approx 17.5 d > 10 d$ ; its

interaction factor  $\alpha_{uH}$  shows some (small) differences in peak amplitudes from those of  $\alpha_{uH}$  for the  $L = 20 d$  and  $L = 40 d$  piles.

The small frequency shift observed in Fig. 4-25 (as  $L/d$  decreases, peaks and valleys move towards larger  $b_o$ ) is of absolutely **no significance**, being merely an artifact of plotting versus  $b_o = \omega s/V_s(L)$ ! Indeed, the reader should appreciate that the exact value of  $V_s(L)$  is of no relevance to the interaction of "flexible" piles. If a relevant wave velocity, e.g. that at one diameter depth  $V_s(d) = V_s(L) \cdot (d/L)^{1/2}$ , were to be used instead, this frequency shift would disappear, since the abscissa would change to

$$\frac{\omega d}{V_s(d)} = b_o \cdot (L/d)^{1/2} \quad (4.5)$$

and hence the interaction curves of the longer piles would move farther to the right, thereby "meeting" the corresponding curves of the shorter piles.

5. Increasing the stiffness ratio  $E_p/E_s$  or  $E_p/E_s(L)$  produces an appreciable increase in all lateral interaction factors under static and low-frequency loading. At higher frequencies, however, the increase is somewhat less significant.

Note that, for **static** loading, Randolph (1977) has proposed an approximate expression for  $\alpha_{uH}$ , which for homogeneous soil takes the form:

$$\alpha_{uH} \approx 0.28 \left(\frac{s}{d}\right)^{-1} \left(\frac{E_p}{E_s}\right)^{1/7} (1 + \cos^2 \theta) \quad (4.6)$$

and fits closely the **zero-frequency** results of this study.

6. The two rotational interaction factors,  $\alpha_{uM}$  and  $\alpha_{\phi M}$ , attain very small values for all but the closest possible spacing



ratios ( $s/d \leq 3$ ). For static loading, Randolph (1977) had proposed the following approximations

$$\alpha_{uM} = \alpha_{\phi H} \approx \alpha_{uH}^2 \quad (4.7a)$$

$$\alpha_{\phi M} \approx \alpha_{uH}^3 \quad (4.7b)$$

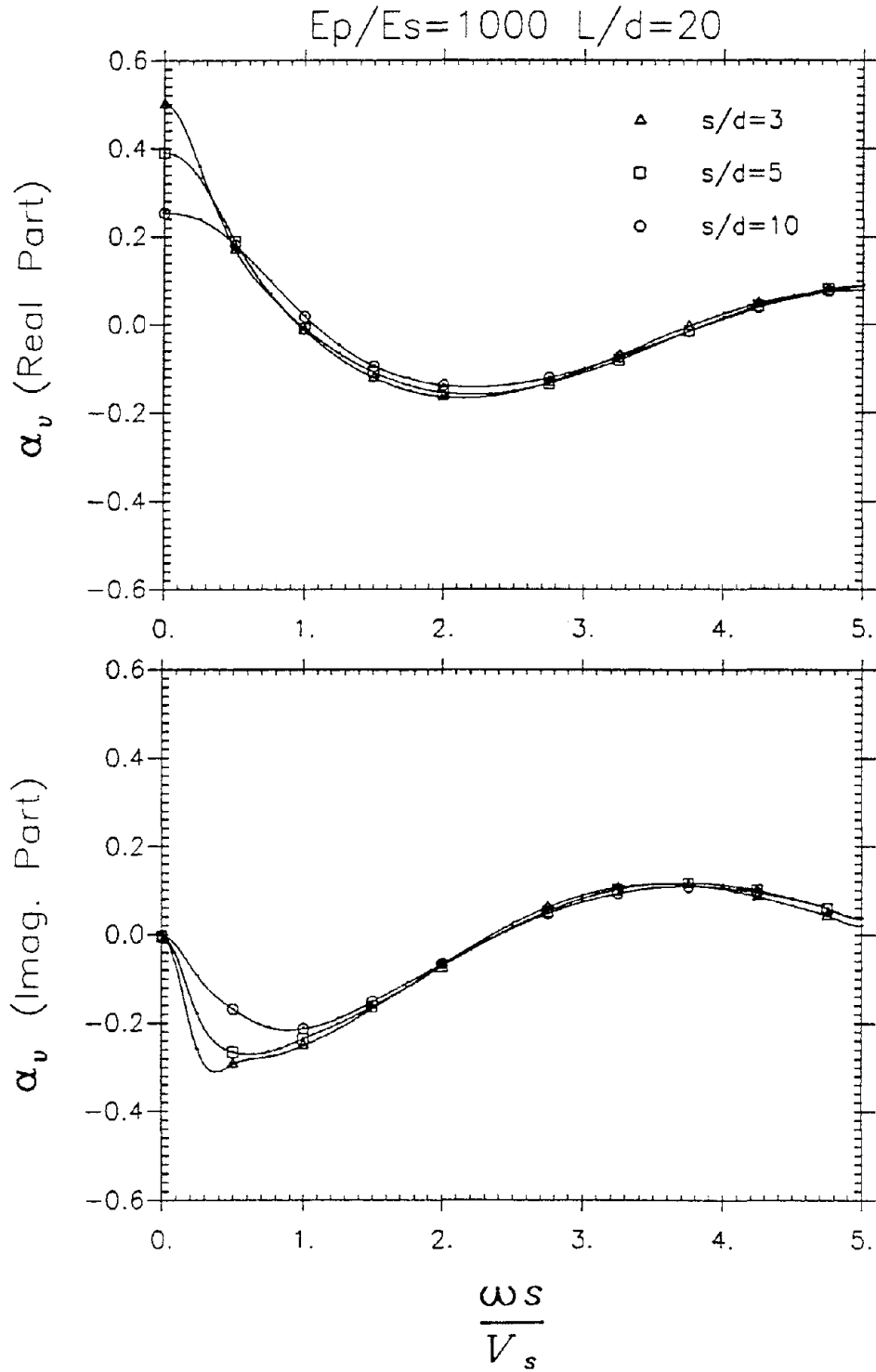
It appears that these relations hold approximately true **even for dynamic** loading, and could be recommended at least in routine practical applications.

Table 4-1

ORGANIZATION OF GRAPHS OF DYNAMIC INTERACTION FACTORS

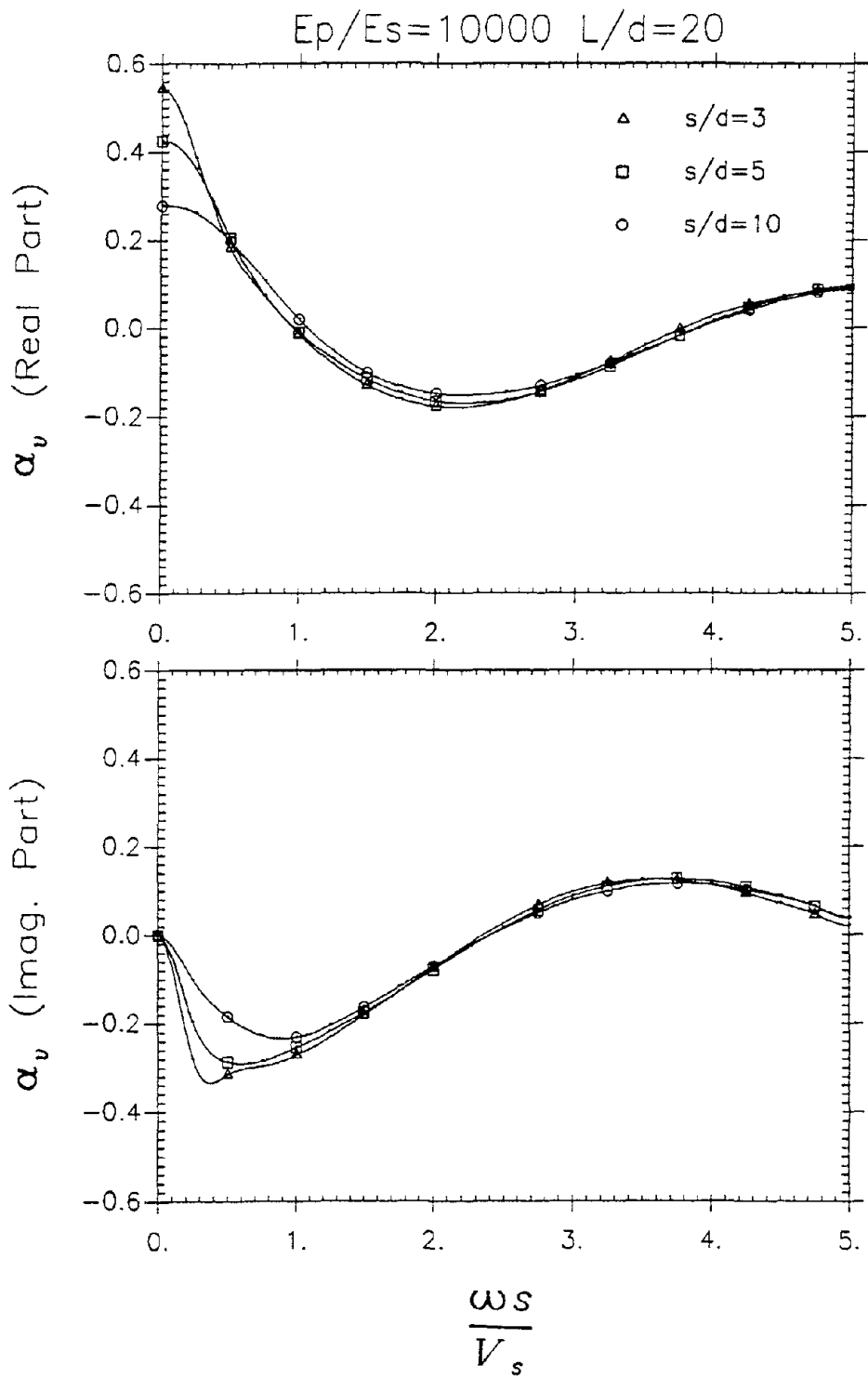
| Interaction                     | Homogeneous Soil Profile                     |  |                      | Non-homogeneous Soil Profile                 |  |                      |
|---------------------------------|--|--|----------------------|--|--|----------------------|
|                                 | $s/d$  | $L/d$  | $E_p/E_s$            | $s/d$  | $L/d$  | $E_p/E_s(L)$         |
| Factor                          | 3,5,10                                       | 10,20,40                                     | 200,1000,10000       | 3,5,10                                       | 10,20,40                                     | 100,500,5000         |
| $\alpha_v$                      | Fig 4-1<br>Fig 4-2                           | Fig 4-5<br>Fig 4-6                           | Fig 4-9              | Fig 4-3<br>Fig 4-4                           | Fig 4-7<br>Fig 4-8                           | Fig 4-10             |
| $\alpha_{uH}$                   | Fig 4-11<br>Fig 4-12<br>Fig 4-13<br>Fig 4-14 | Fig 4-19<br>Fig 4-20<br>Fig 4-21<br>Fig 4-22 | Fig 4-27<br>Fig 4-28 | Fig 4-15<br>Fig 4-16<br>Fig 4-17<br>Fig 4-18 | Fig 4-23<br>Fig 4-24<br>Fig 4-25<br>Fig 4-26 | Fig 4-29<br>Fig 4-30 |
| $\alpha_{uM} = \alpha_{\phi H}$ | Fig 4-31<br>Fig 4-32<br>Fig 4-33<br>Fig 4-34 |  |                      | Fig 4-35<br>Fig 4-36<br>Fig 4-37<br>Fig 4-38 |  |                      |
| $\alpha_{\phi M}$               | Fig 4-39<br>Fig 4-40                         |  |                      | Fig 4-41<br>Fig 4-42                         |  |                      |

All graphs are for  $\nu_s = 0.4$ ,  $\rho_s/\rho_p = 0.7$ ,  $\beta = 0.05$ . Figs.43-45 portray the effects of different  $\nu_s$  and  $\rho_s/\rho_p$  values. The effect of  $\beta$  can be assessed with the help of Eq.3.3.



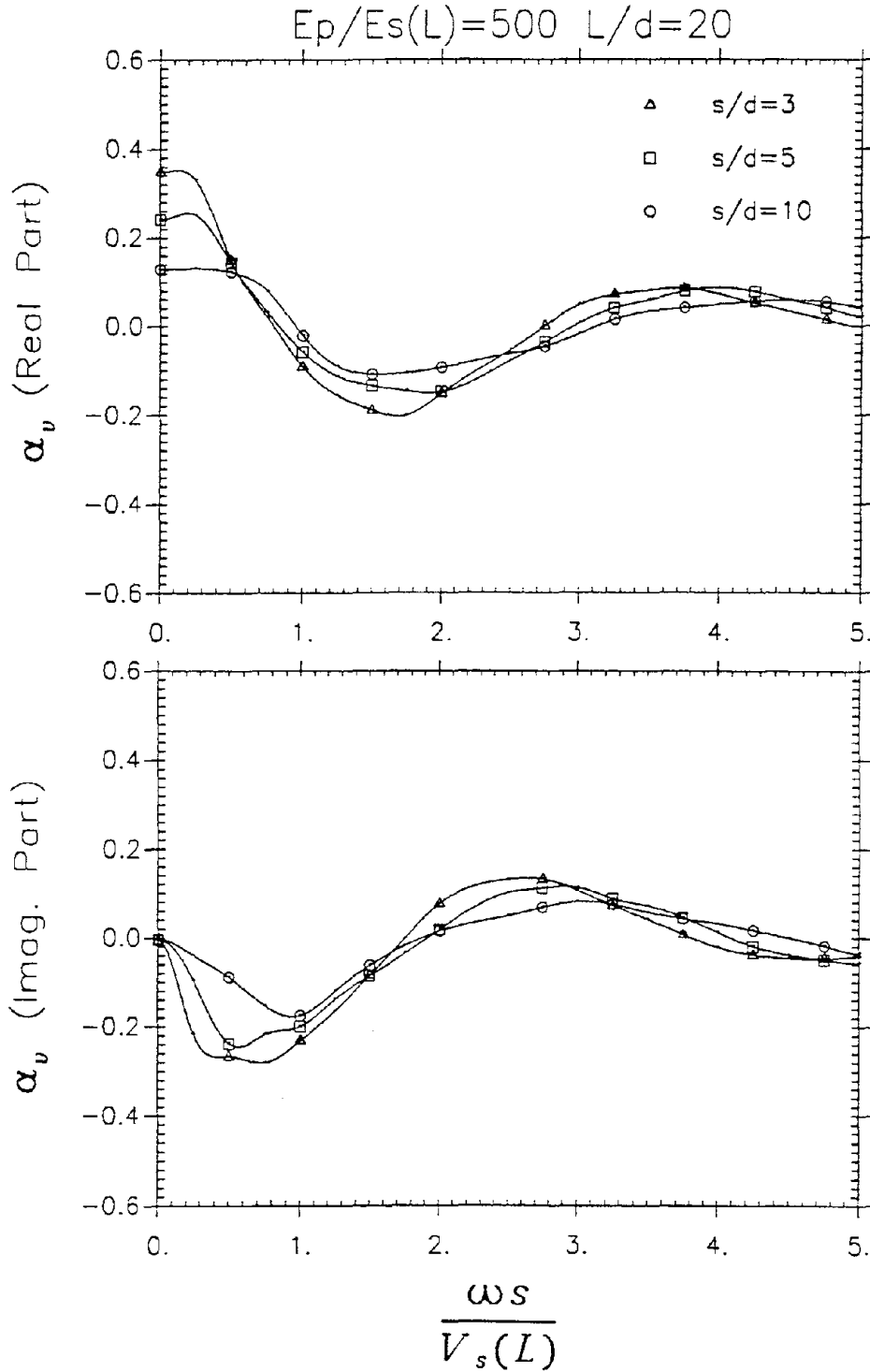
**Fig.4-1 Vertical Interaction Factors for Relatively-Compressible Piles in Homogeneous**

Soil ( $E_p/E_s = 1000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )

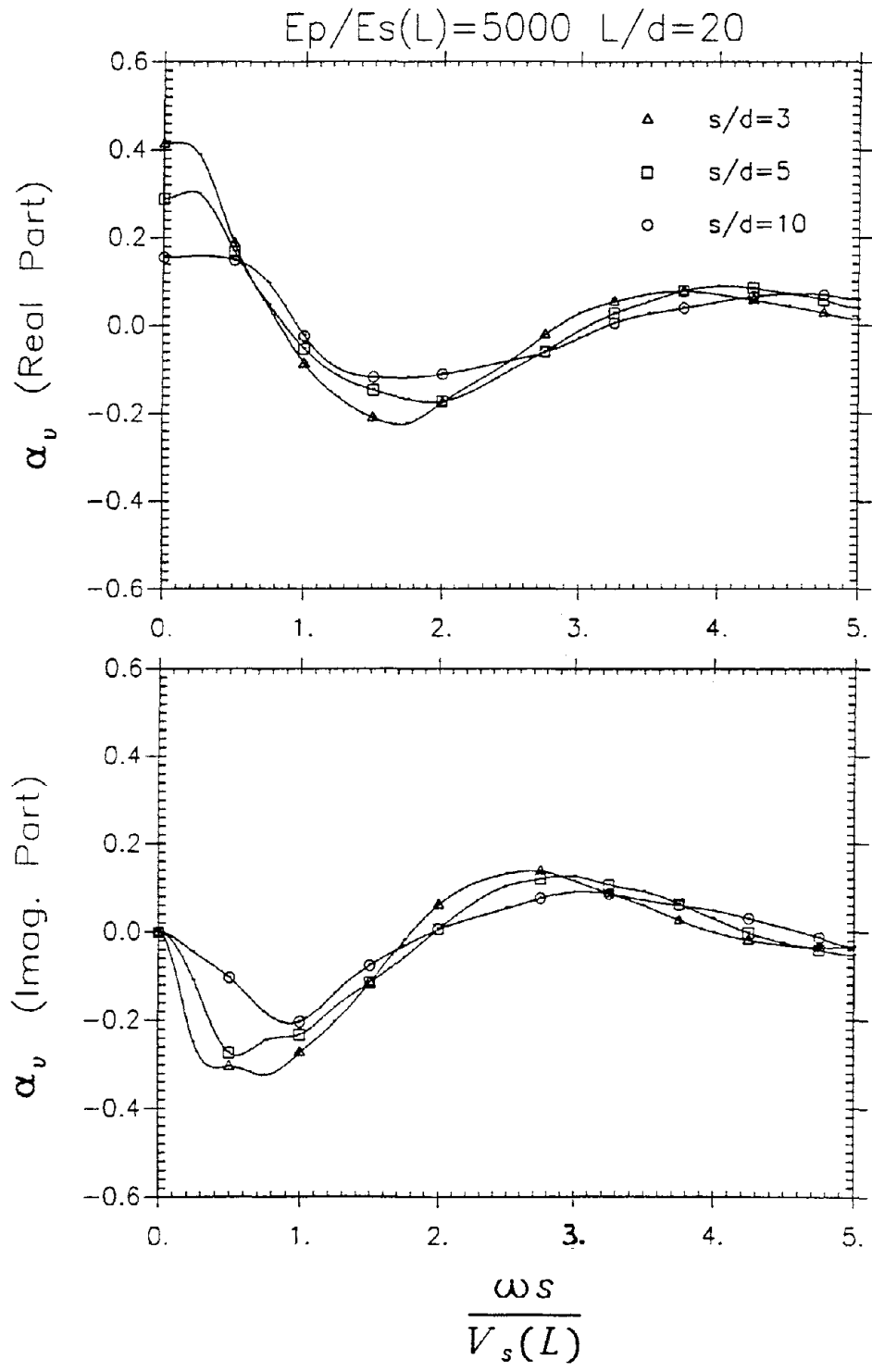


**Fig.4-2 Vertical Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil**

( $E_p/E_s = 10000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )

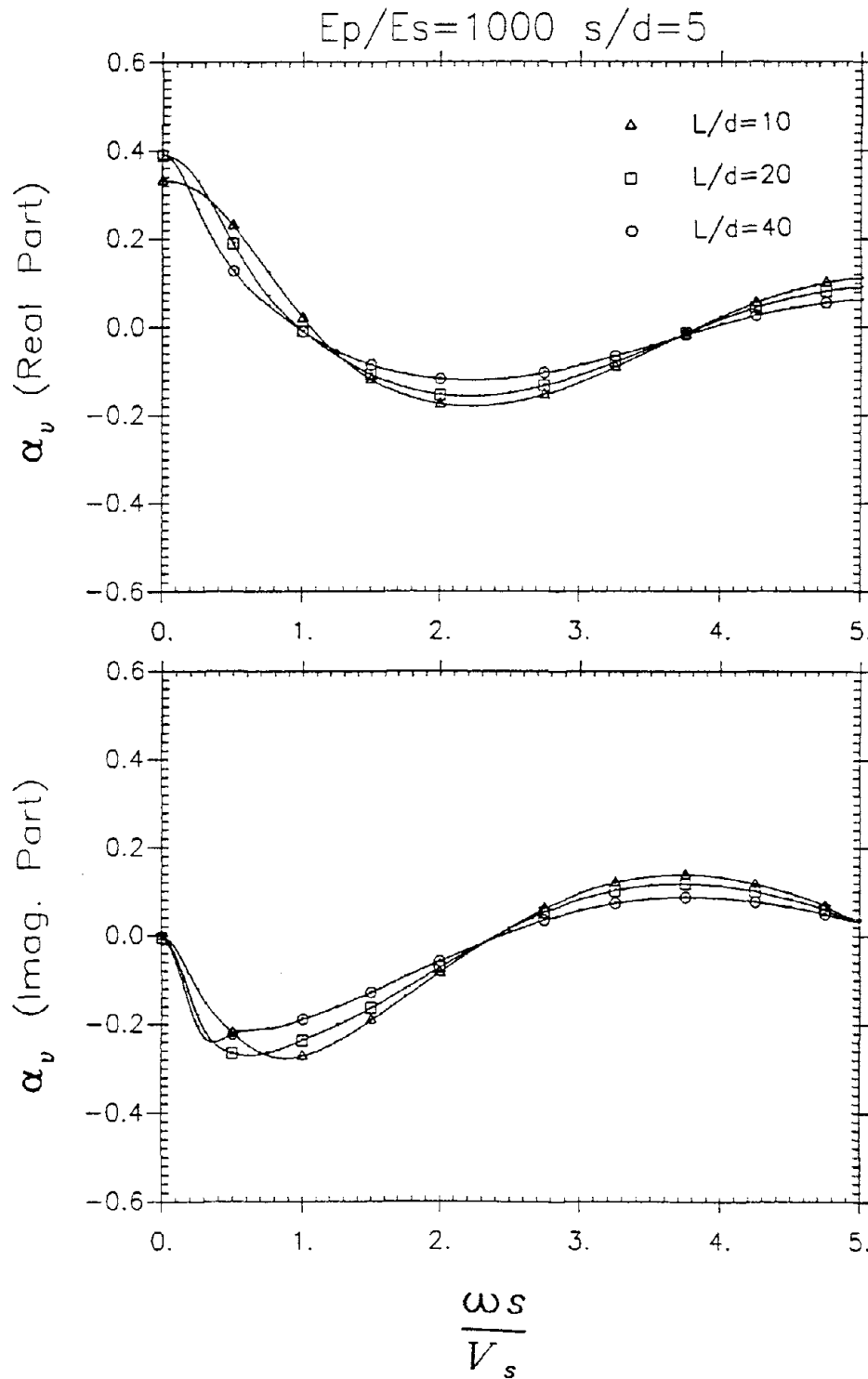


**Fig.4-3 Vertical Interaction Factors for Relatively-Compressible Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )**



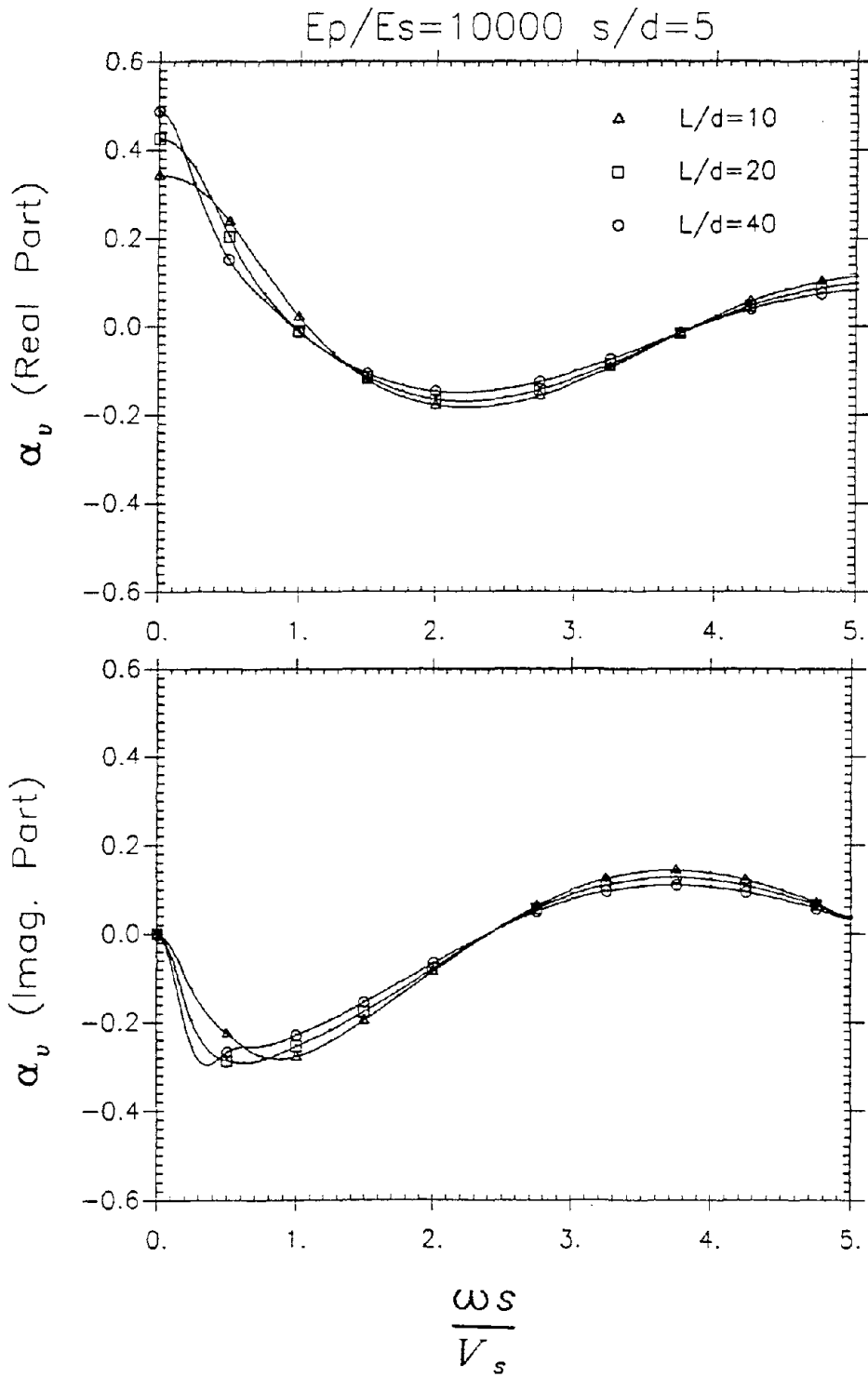
**Fig.4-4 Vertical Interaction Factors for Relatively-Rigid Piles in Nonhomogeneous Soil**

$(E_p/E_s(L) = 5000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05 \text{ and } \nu_s = 0.4)$



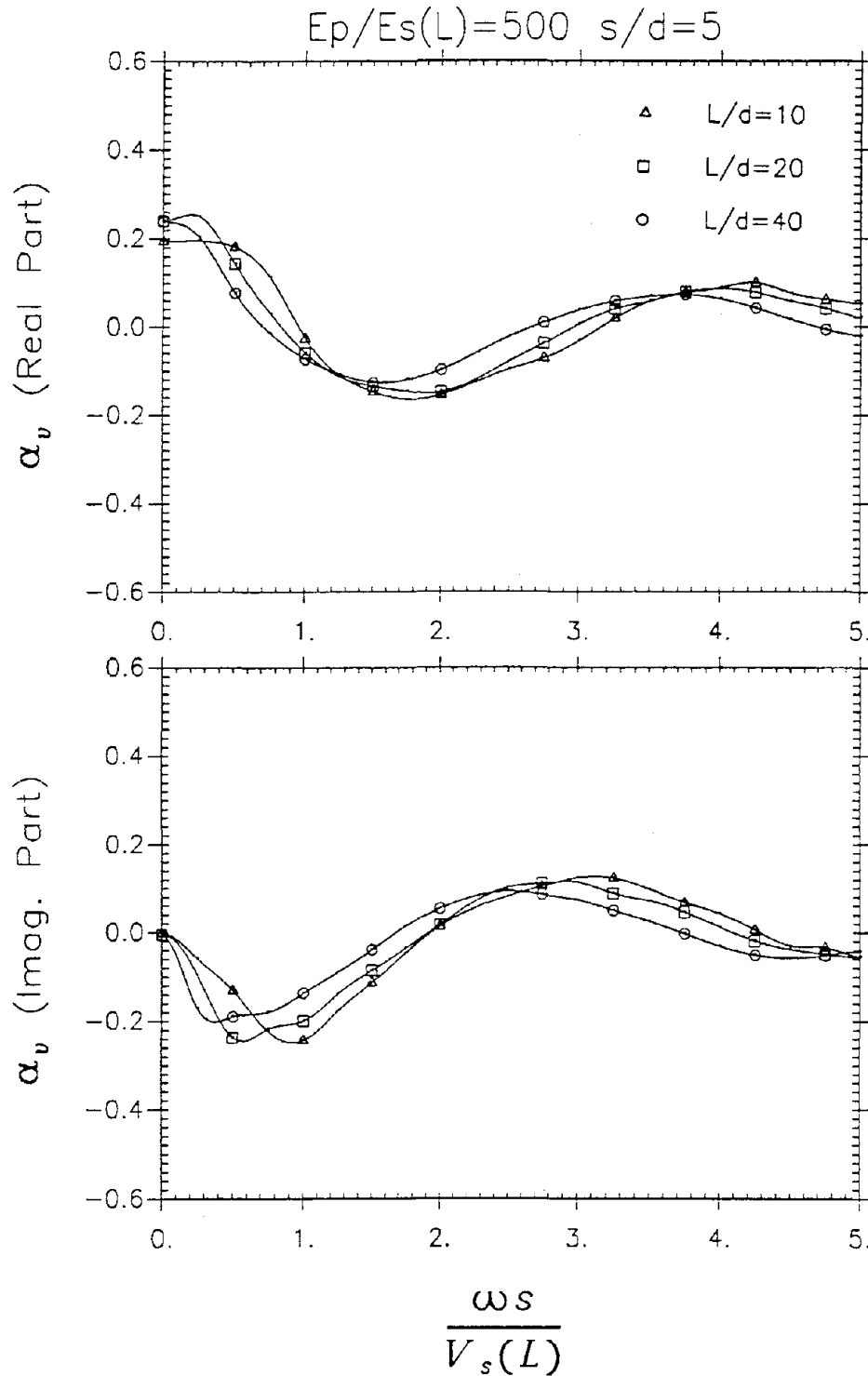
**Fig.4-5 Effect of  $L/d$  on Vertical Interaction Factors for Relatively-Compressible Piles**

in Homogeneous Soil ( $E_p/E_s = 1000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )

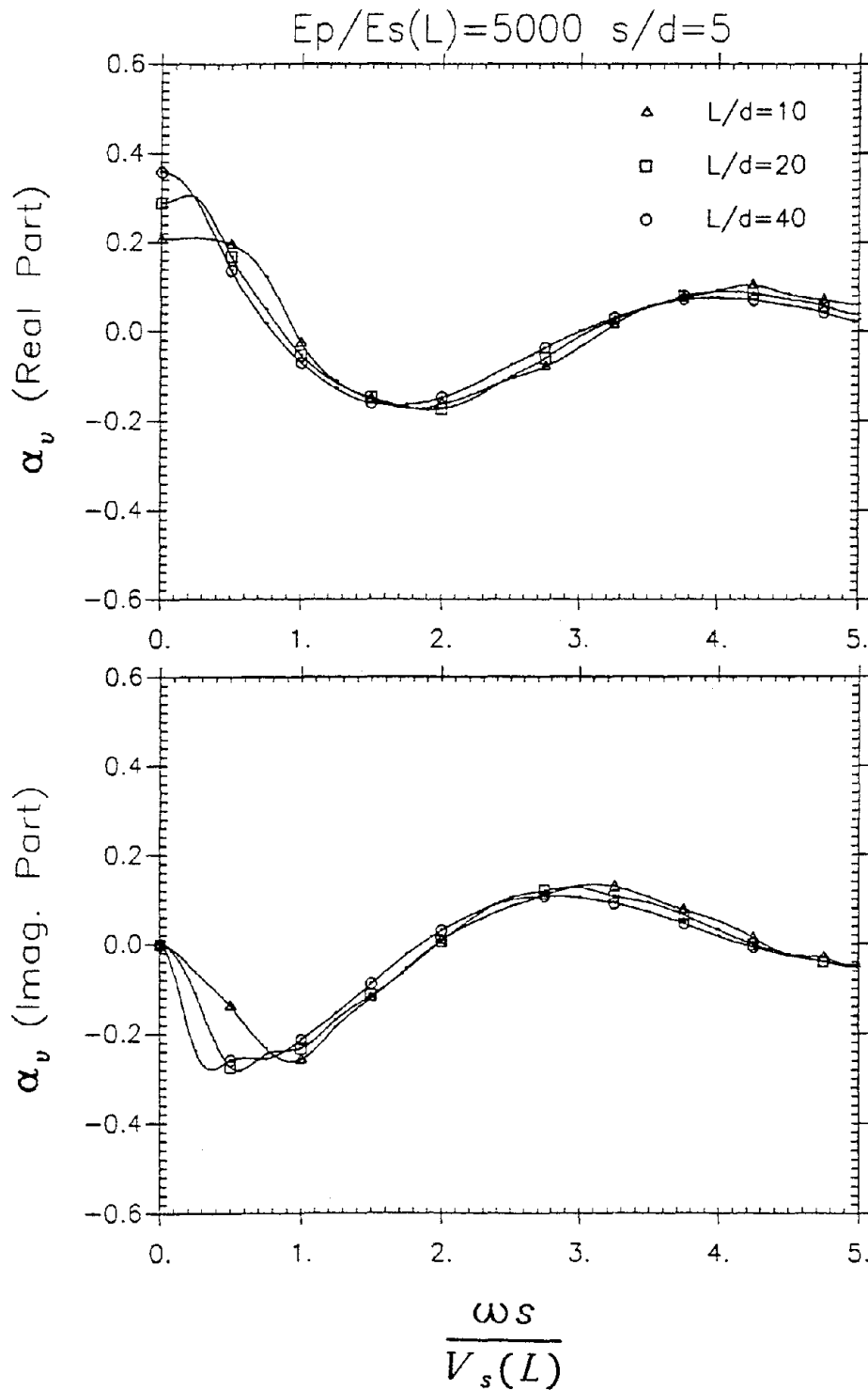


**Fig.4-6** Effect of  $L/d$  on Vertical Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil ( $E_p/E_s = 10000$ ,  $s/d = 5$ ,  $\rho_s/\rho_p = 0.7$ ,  $\beta = 0.05$  and  $\nu_s = 0.4$ )

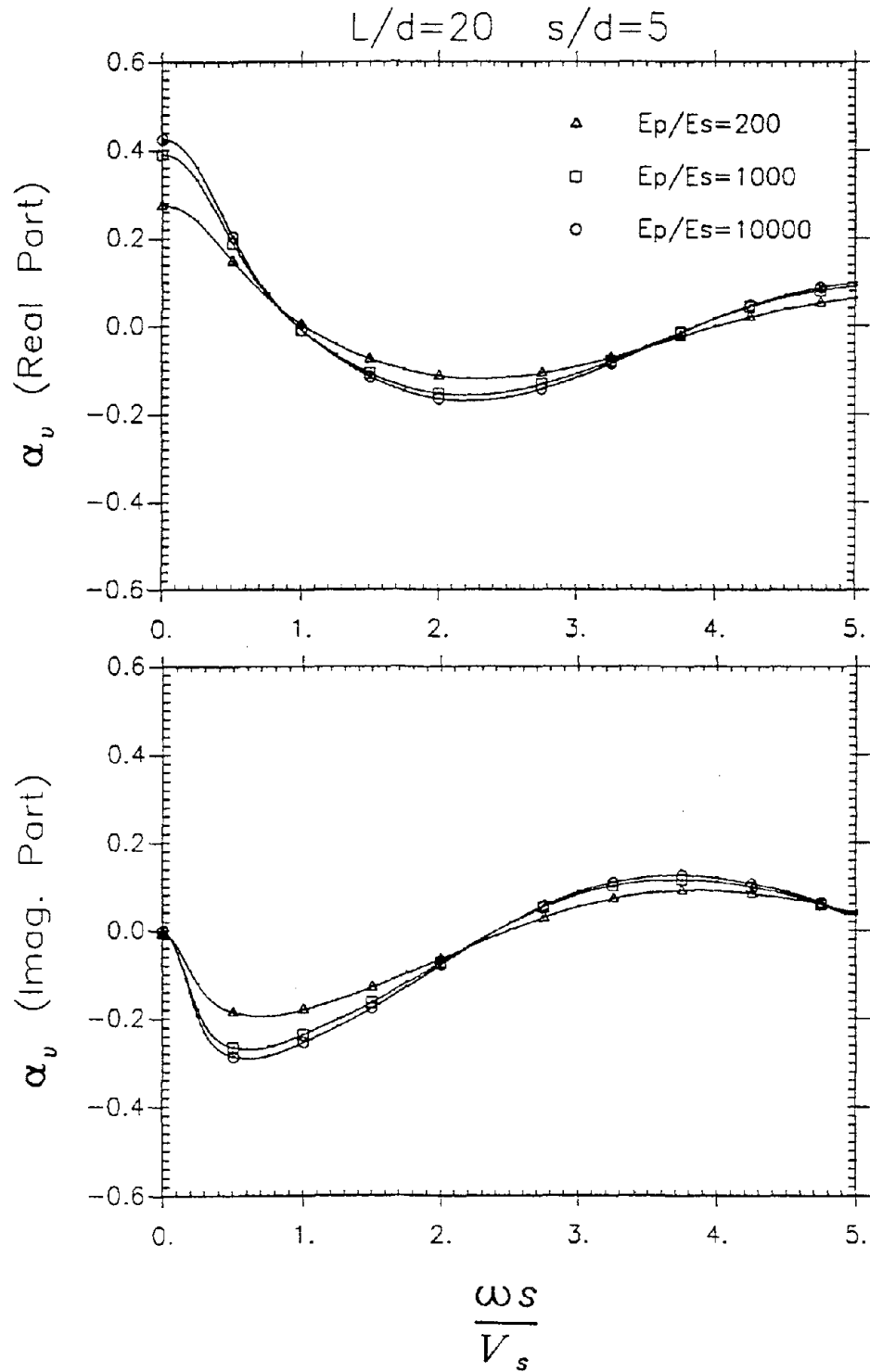




**Fig.4-7 Effect of  $L/d$  on Vertical Interaction Factors for Relatively-Compressible Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )**

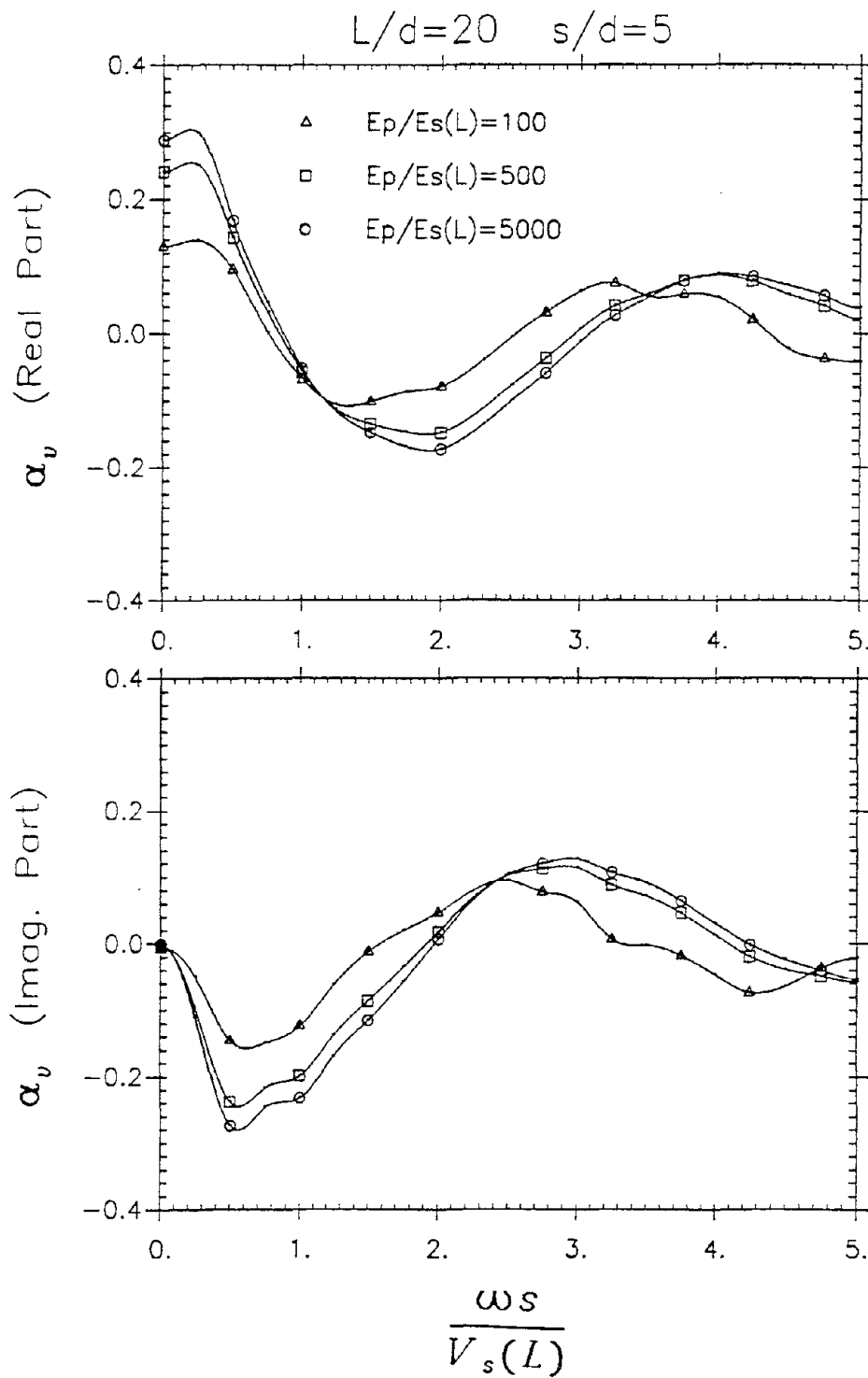


**Fig.4-8 Effect of  $L/d$  on Vertical Interaction Factors for Relatively-Rigid Piles in Non-homogeneous Soil ( $E_p/E_s(L) = 5000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )**



**Fig.4-9 Effect of  $E_p/E_s$  on Vertical Interaction Factors for Piles in Homogeneous Soil**

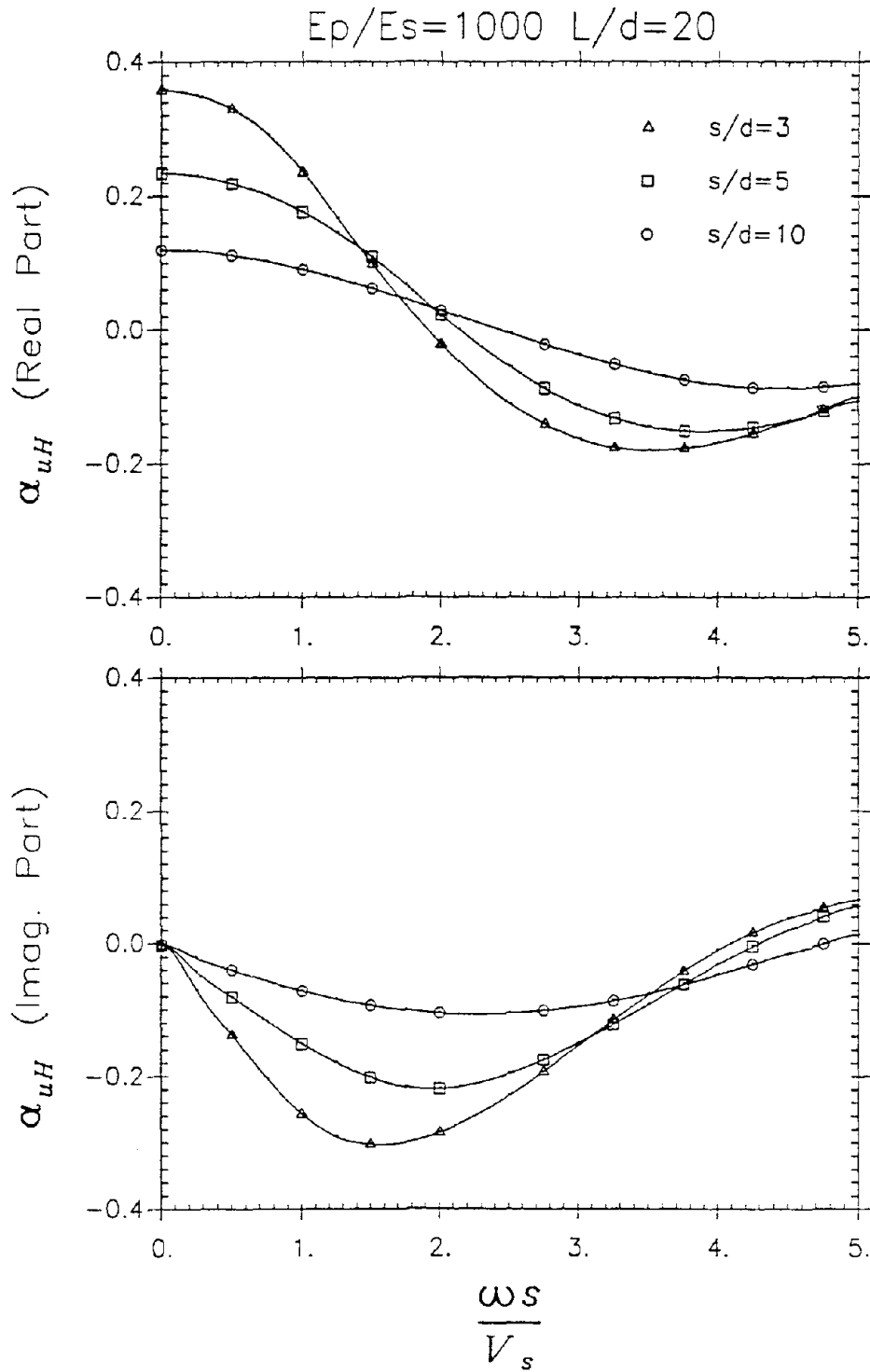
( $L/d = 20, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )



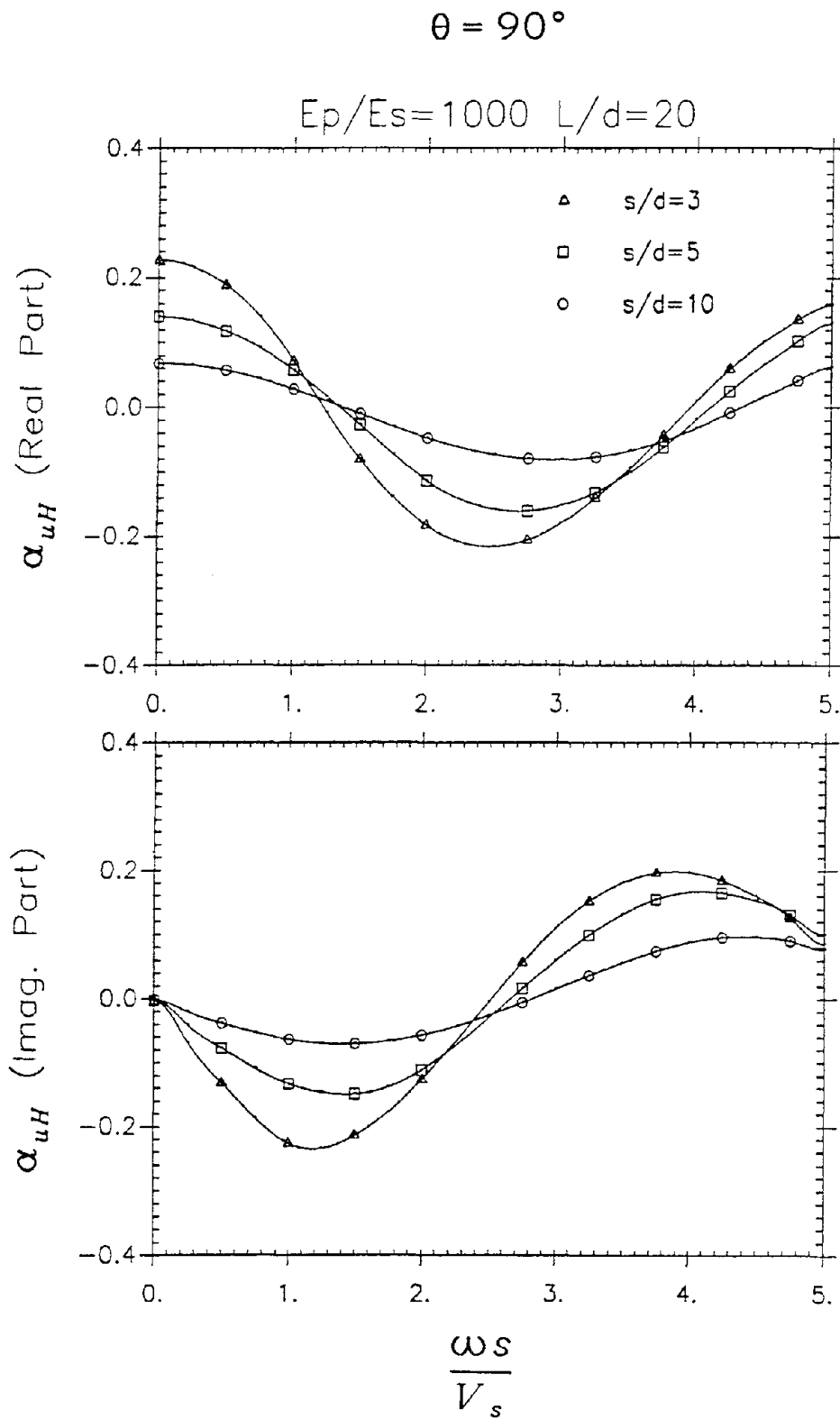
**Fig.4-10 Effect of  $E_p/E_s(L)$  on Vertical Interaction Factors for Piles in Nonhomogeneous**

**Soil( $L/d = 20, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )**

$$\theta = 0^\circ$$

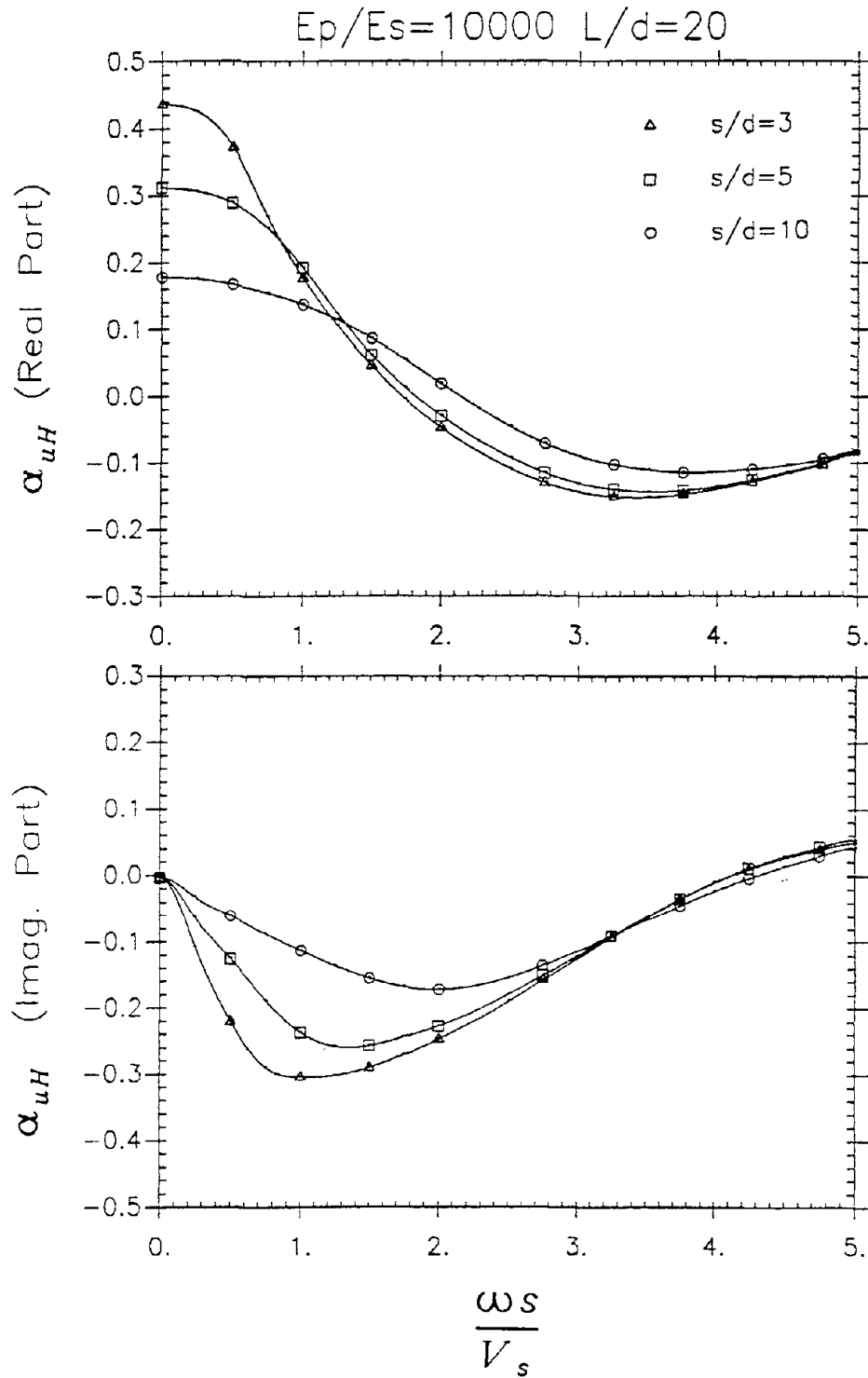


**Fig.4-11 Horizontal Interaction Factors for Relatively-Compressible Piles in Homogeneous Soil ( $E_p/E_s = 1000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$**



**Fig.4-12 Horizontal Interaction Factors for Relatively-Compressible Piles in Homogeneous Soil ( $E_p/E_s = 1000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$**

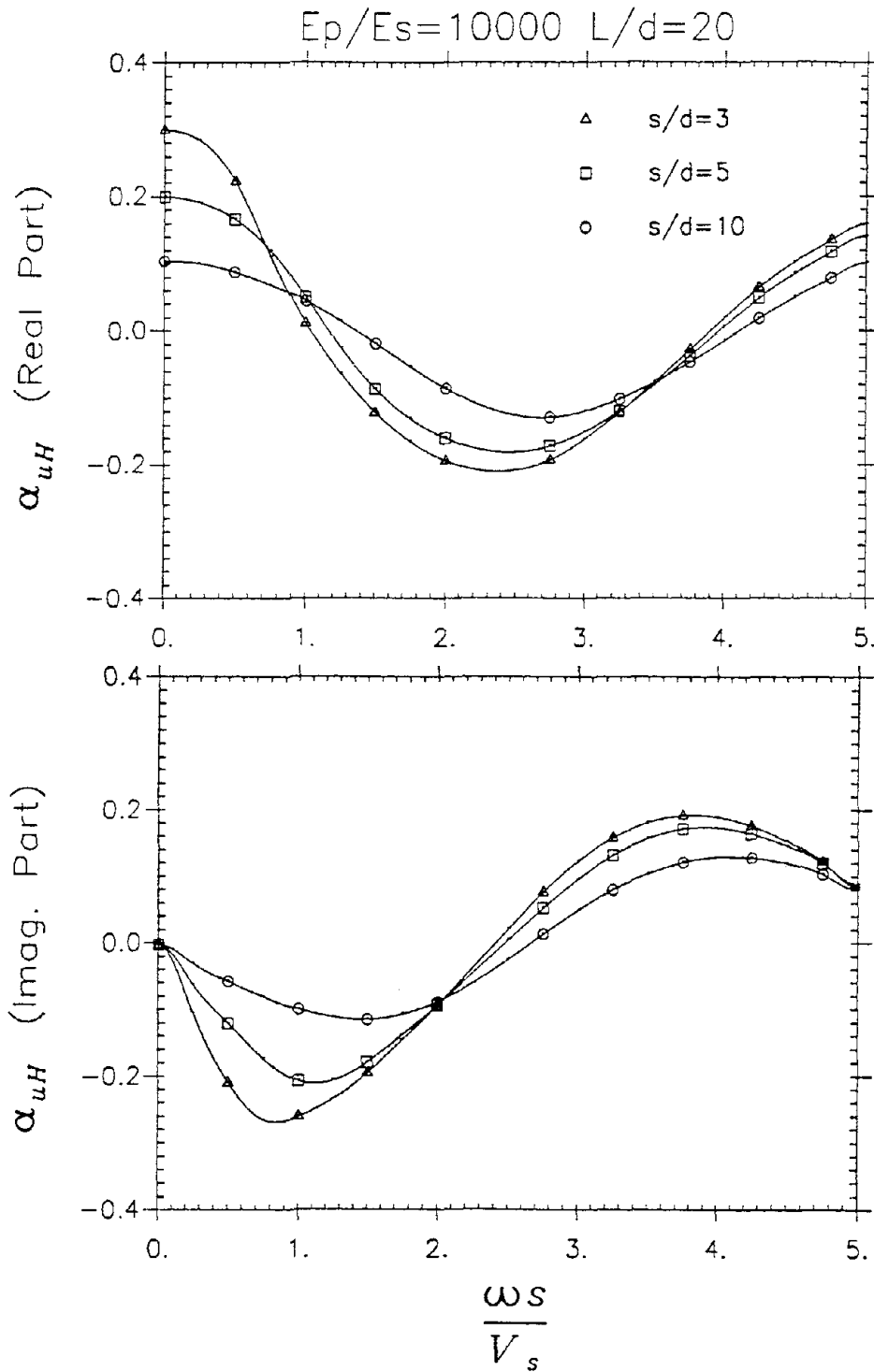
$$\theta = 0^\circ$$



**Fig.4-13 Horizontal Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil**

$(E_p/E_s = 10000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05 \text{ and } \nu_s = 0.4) \theta = 0^\circ$

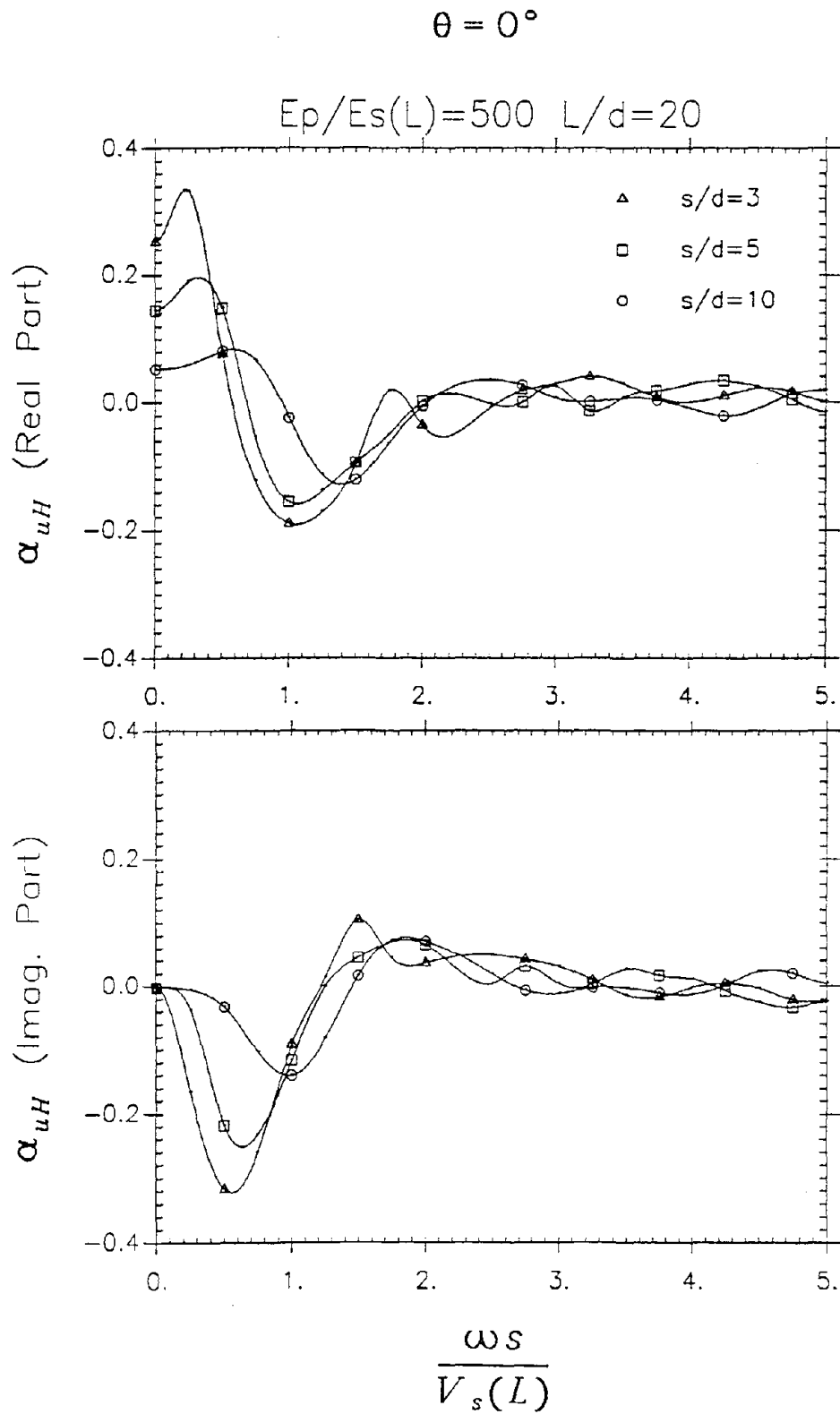
$$\theta = 90^\circ$$



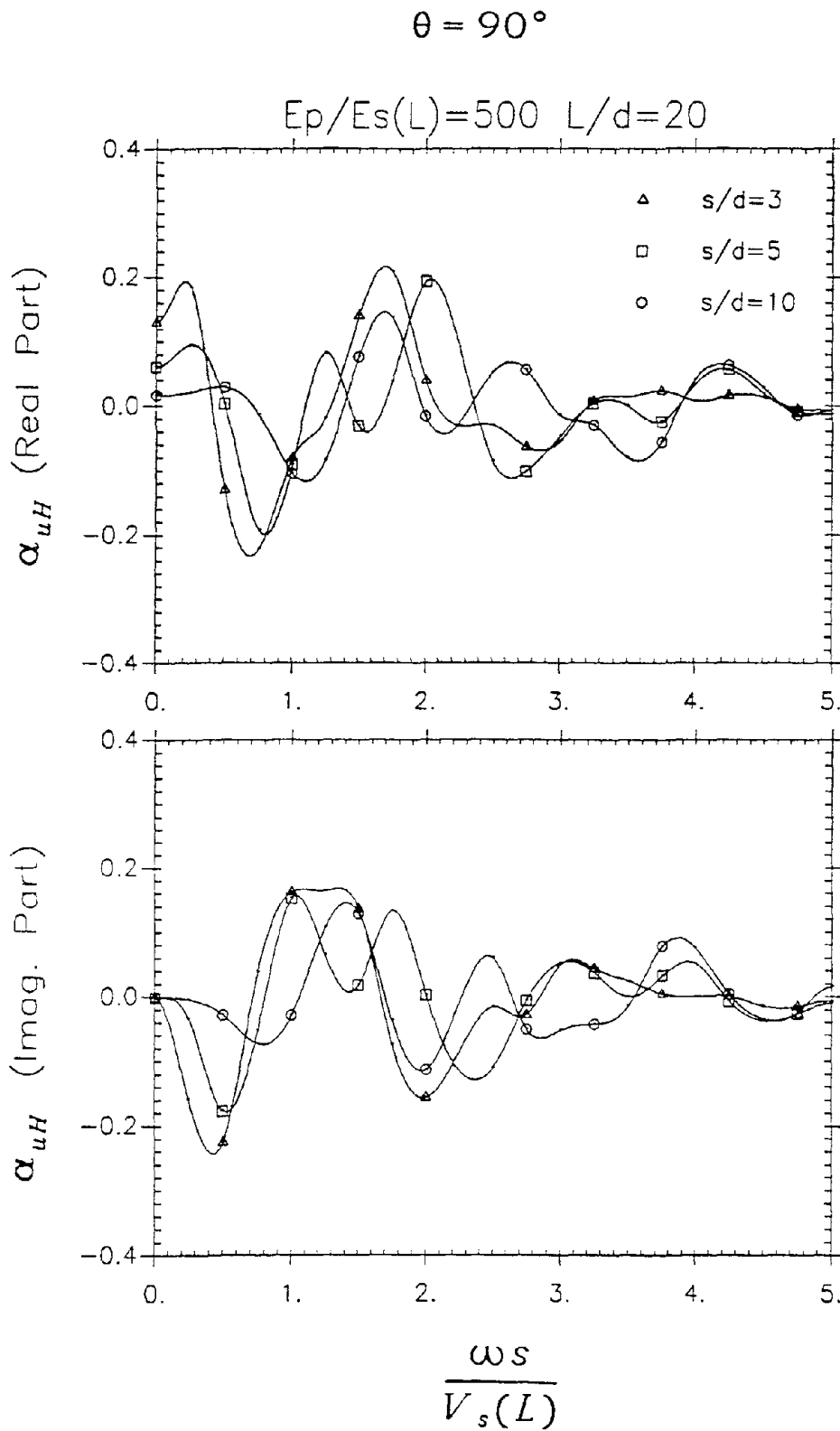
**Fig.4-14 Horizontal Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil**

$(E_p/E_s = 10000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05 \text{ and } \nu_s = 0.4) \theta = 90^\circ$

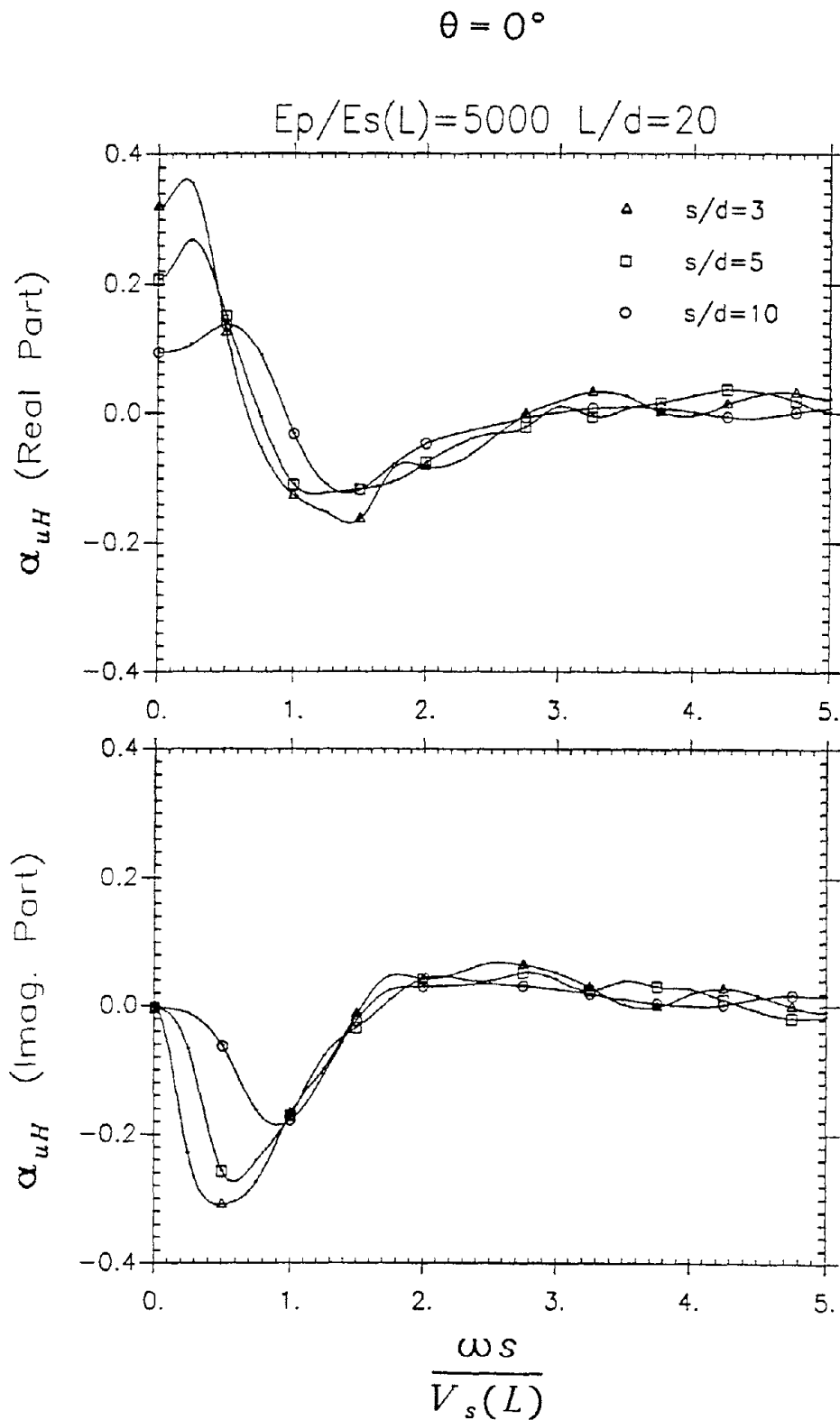




**Fig.4-15 Horizontal Interaction Factors for Relatively-Compressible Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$**



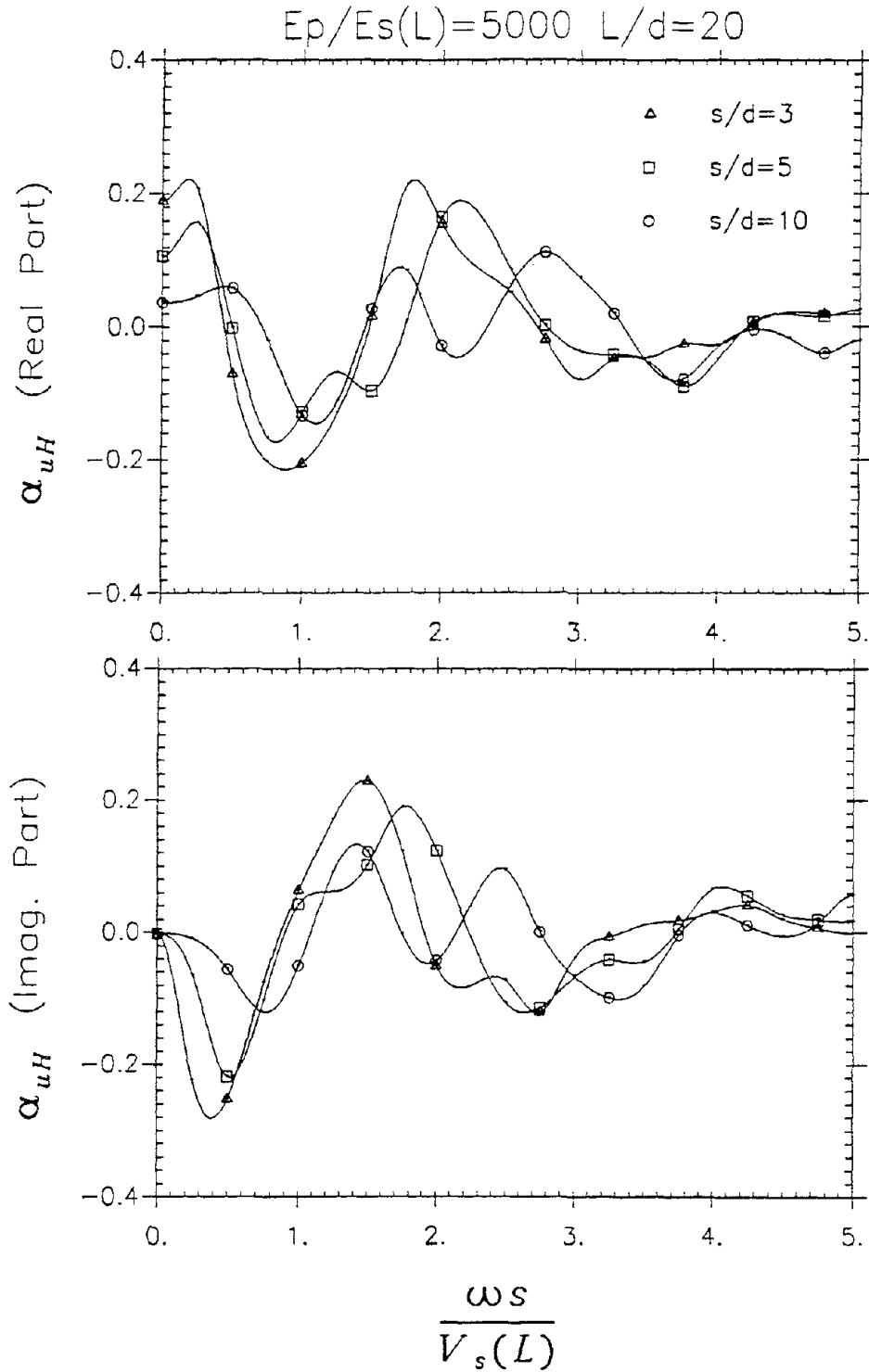
**Fig.4-16 Horizontal Interaction Factors for Relatively-Compressible Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$**



**Fig.4-17 Horizontal Interaction Factors for Relative-Rigid Piles in Nonhomogeneous**

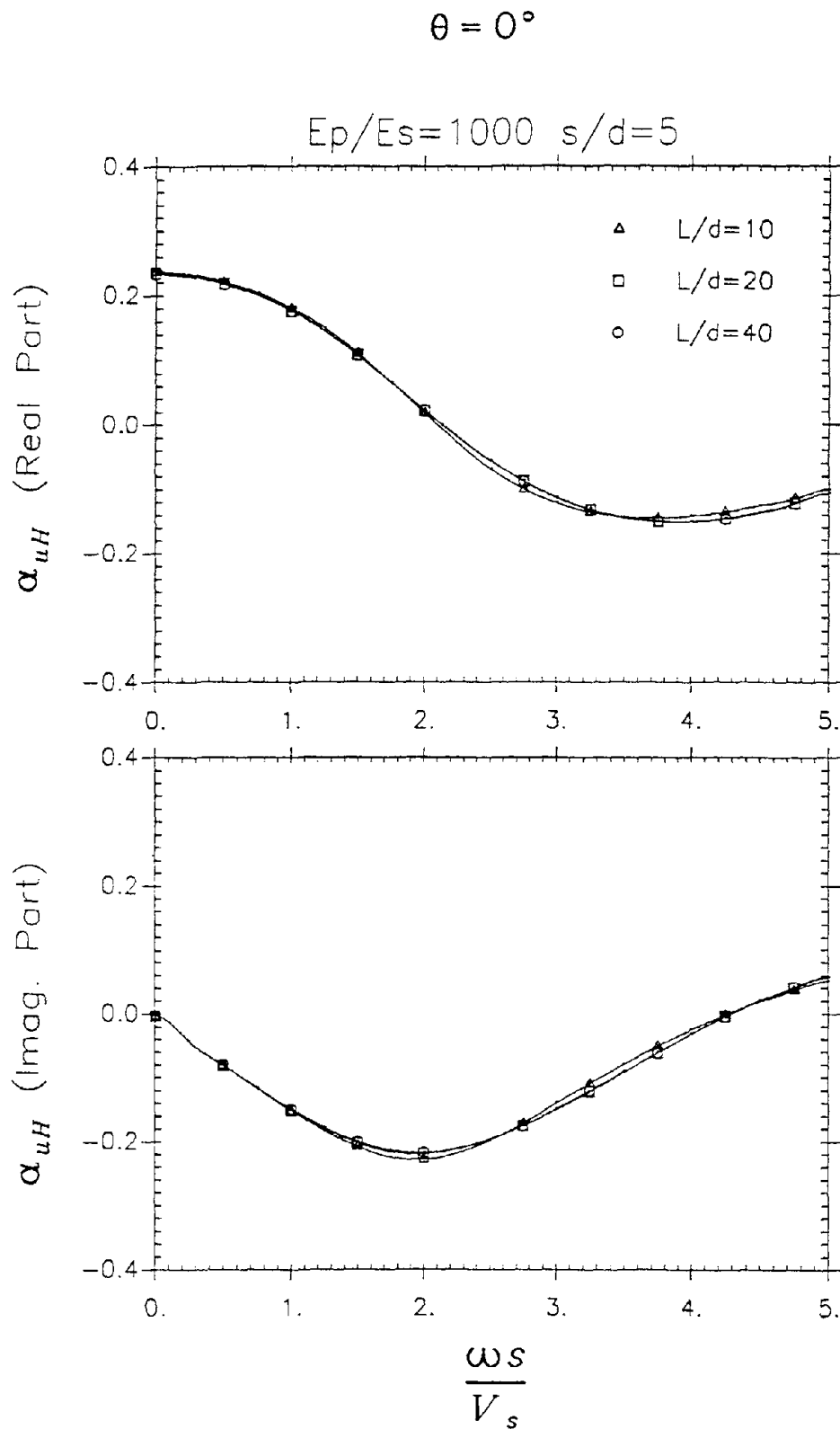
Soil ( $E_p/E_s(L) = 5000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$

$$\theta = 90^\circ$$



**Fig.4-18 Horizontal Interaction Factors for Relatively-Rigid Piles in Nonhomogeneous**

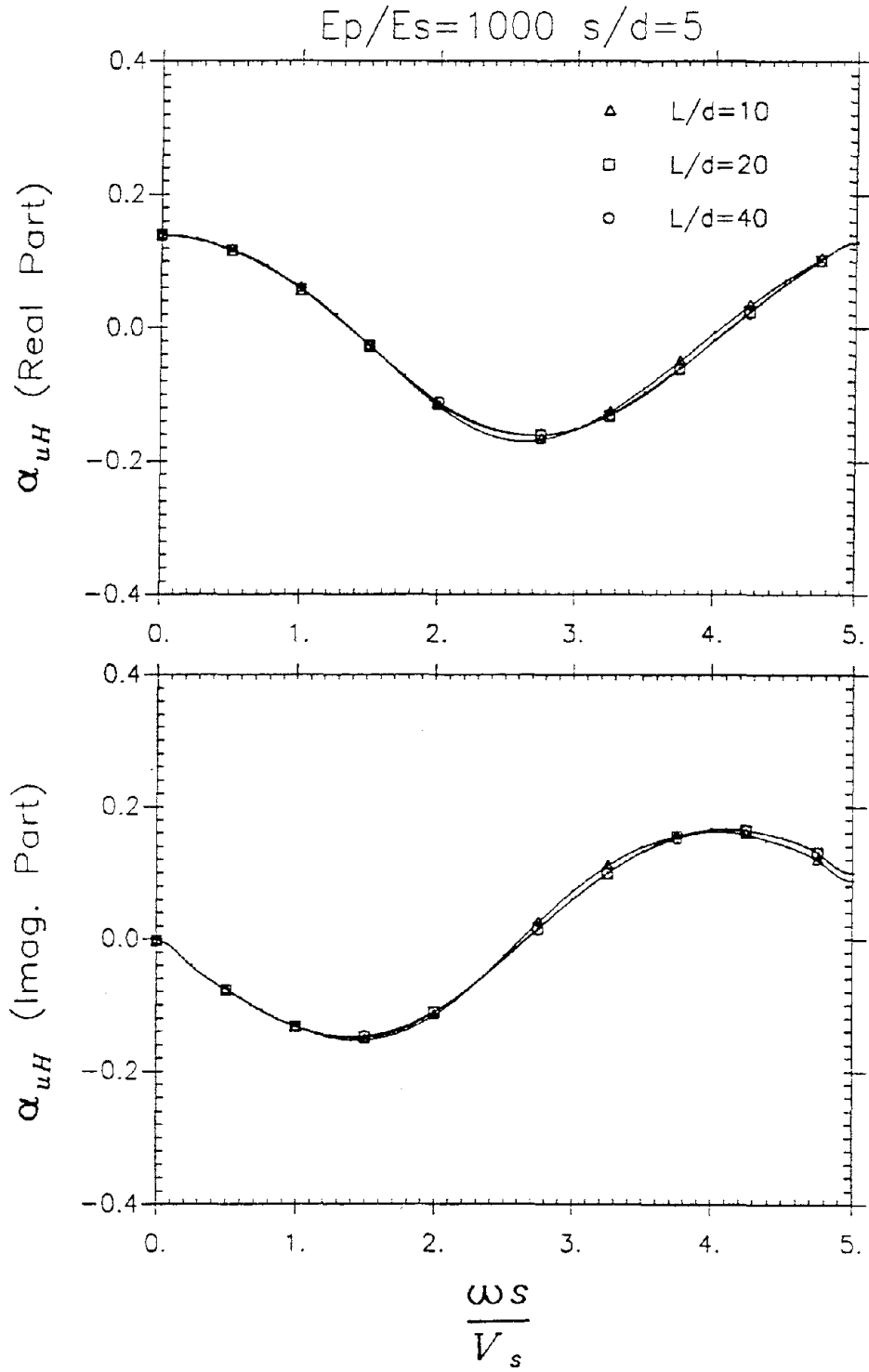
Soil ( $E_p/E_s(L) = 5000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$



**Fig.4-19 Effect of  $L/d$  on Horizontal Interaction Factors for Relatively-Compressible**

**Piles in Homogeneous Soil ( $E_p/E_s = 1000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$**

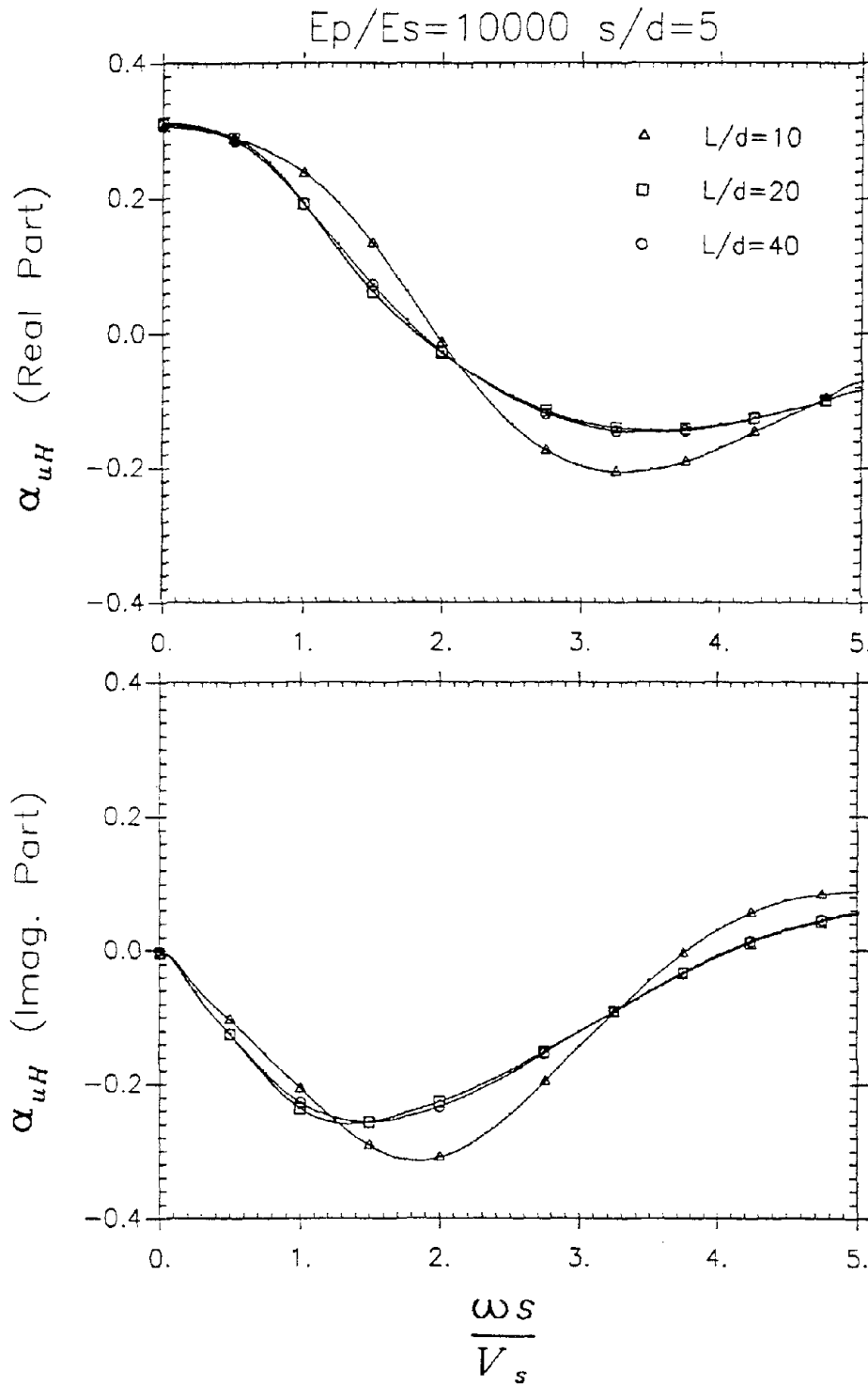
$$\theta = 90^\circ$$



**Fig.4-20 Effect of  $L/d$  on Horizontal Interaction Factors for Relatively-Compressible**

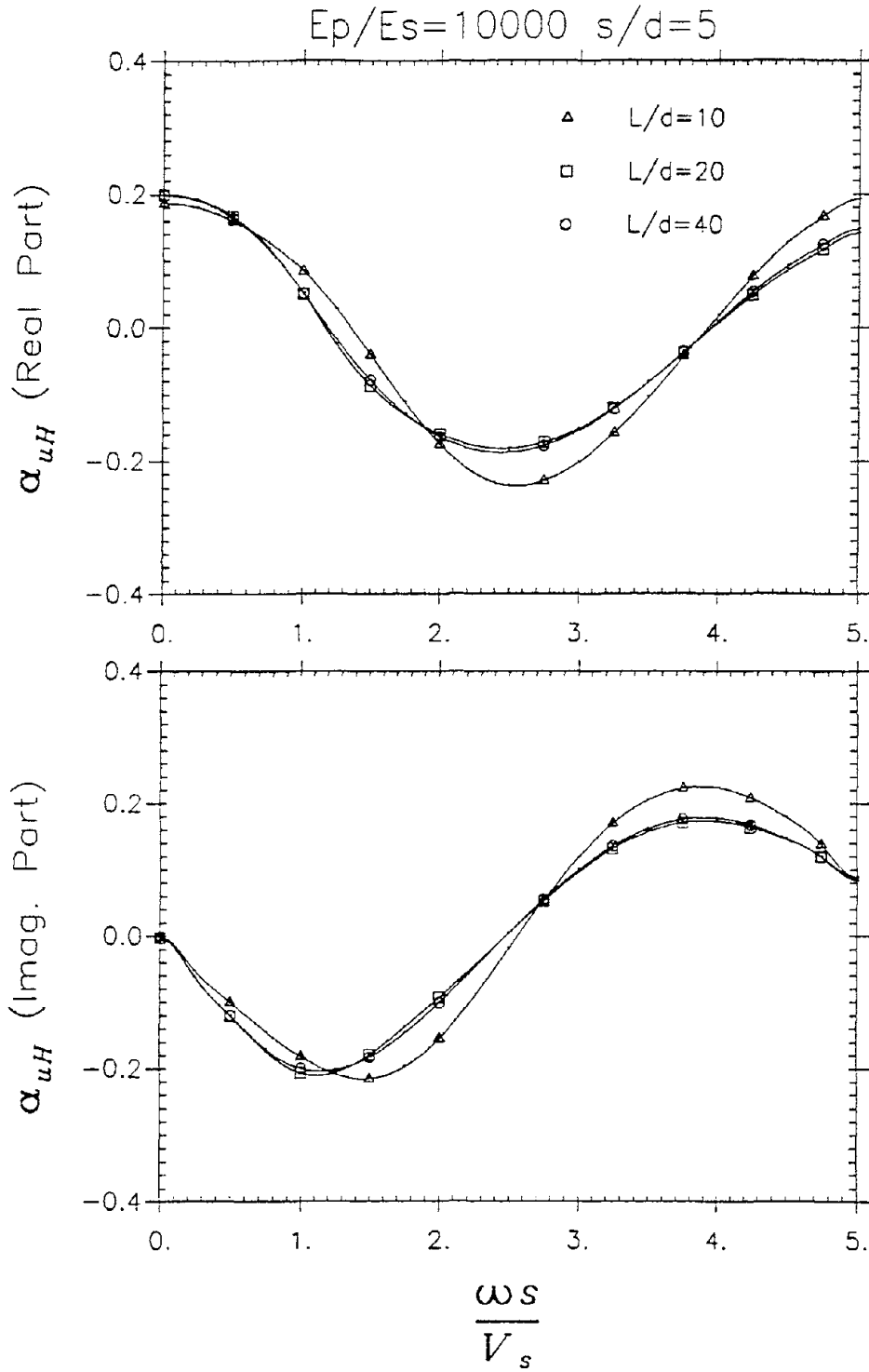
**Piles in Homogeneous Soil ( $E_p/E_s = 1000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$**

$$\theta = 0^\circ$$



**Fig.4-21 Effect of  $L/d$  on Horizontal Interaction Factors for relatively-Rigid Piles in Homogeneous Soil ( $E_p/E_s = 10000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$**

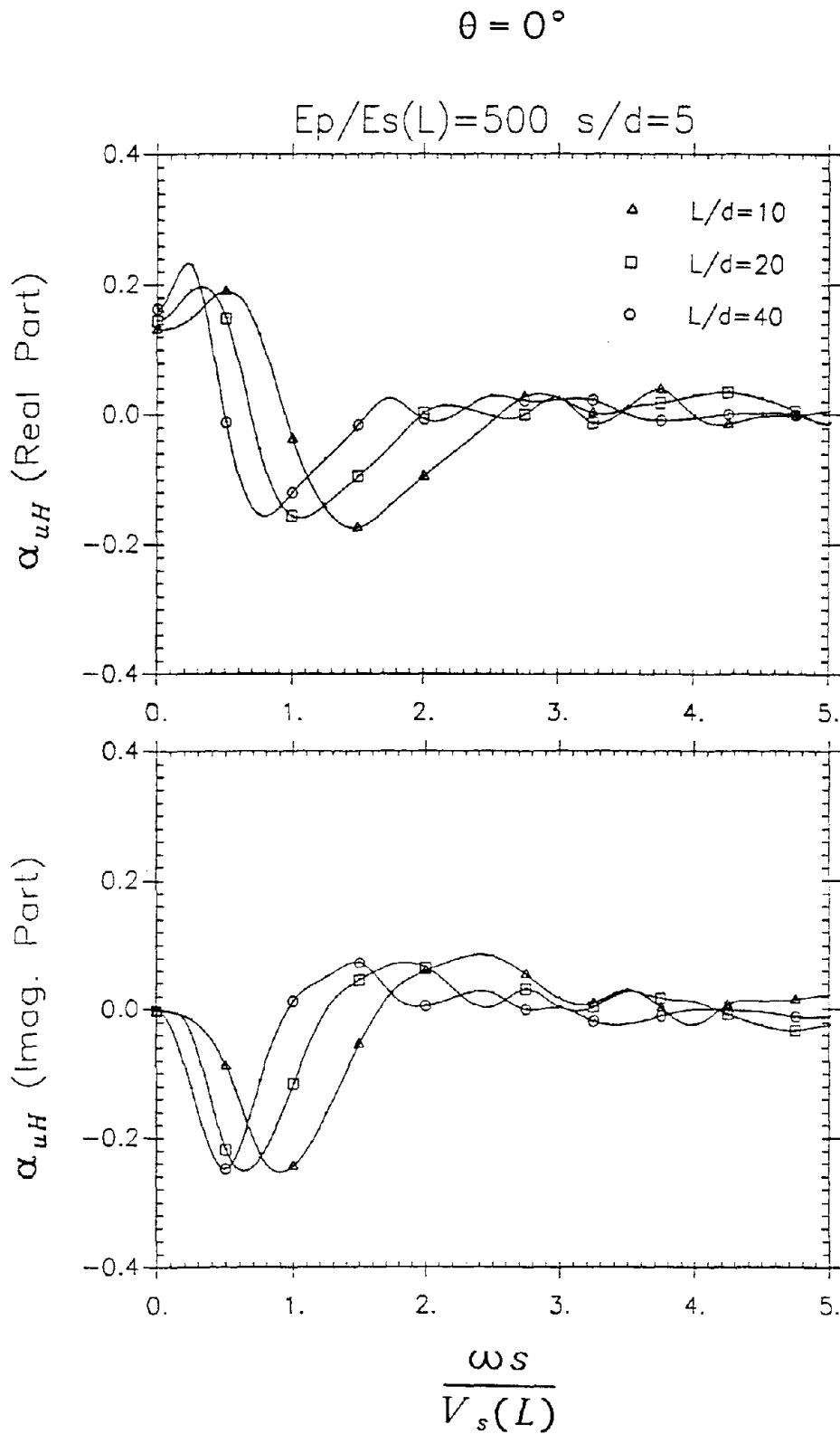
$$\theta = 90^\circ$$



**Fig.4-22 Effect of  $L/d$  on Horizontal Interaction Factors for Relatively-Rigid Piles in**

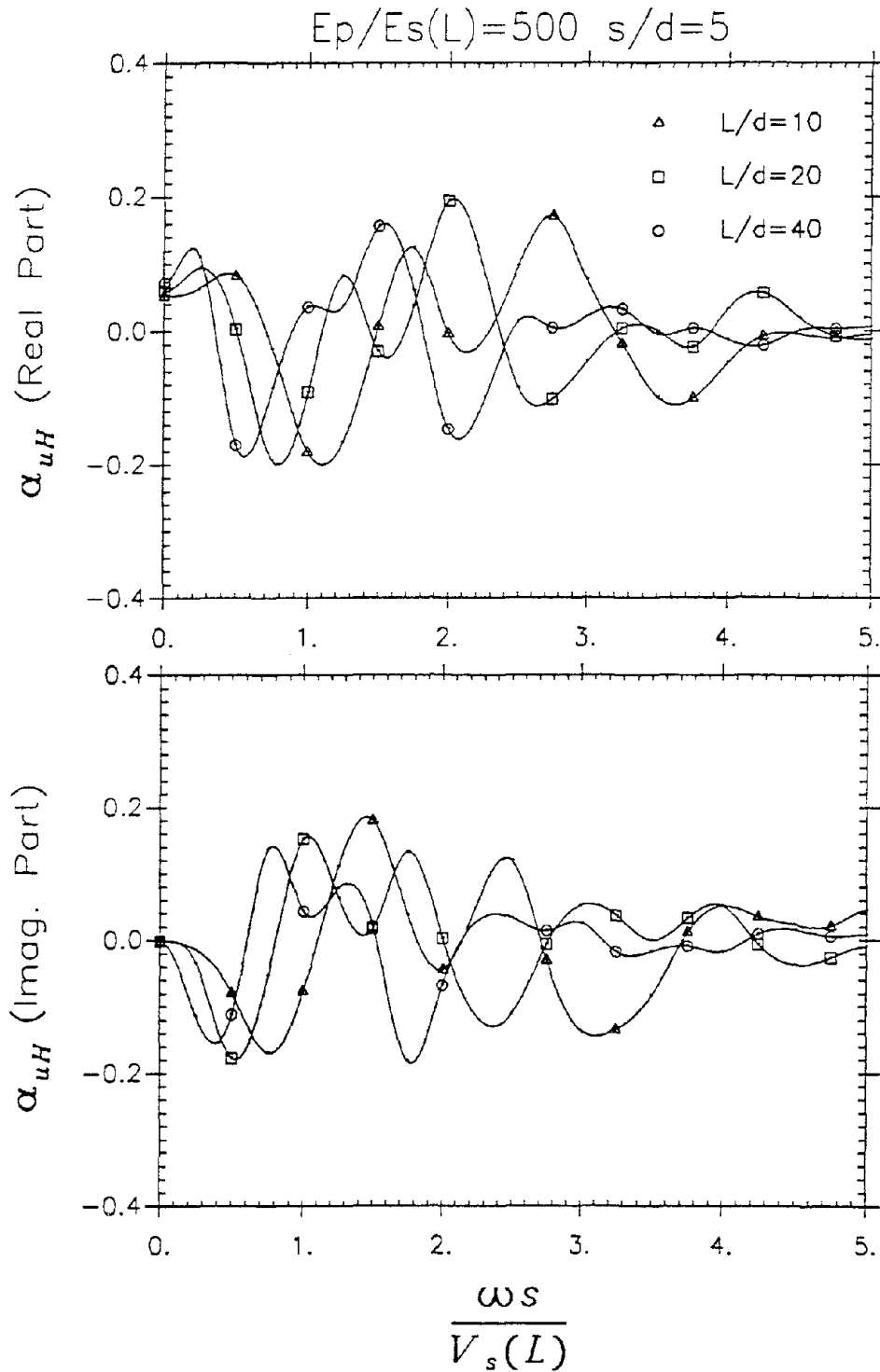
**Homogeneous Soil( $E_p/E_s = 10000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$**





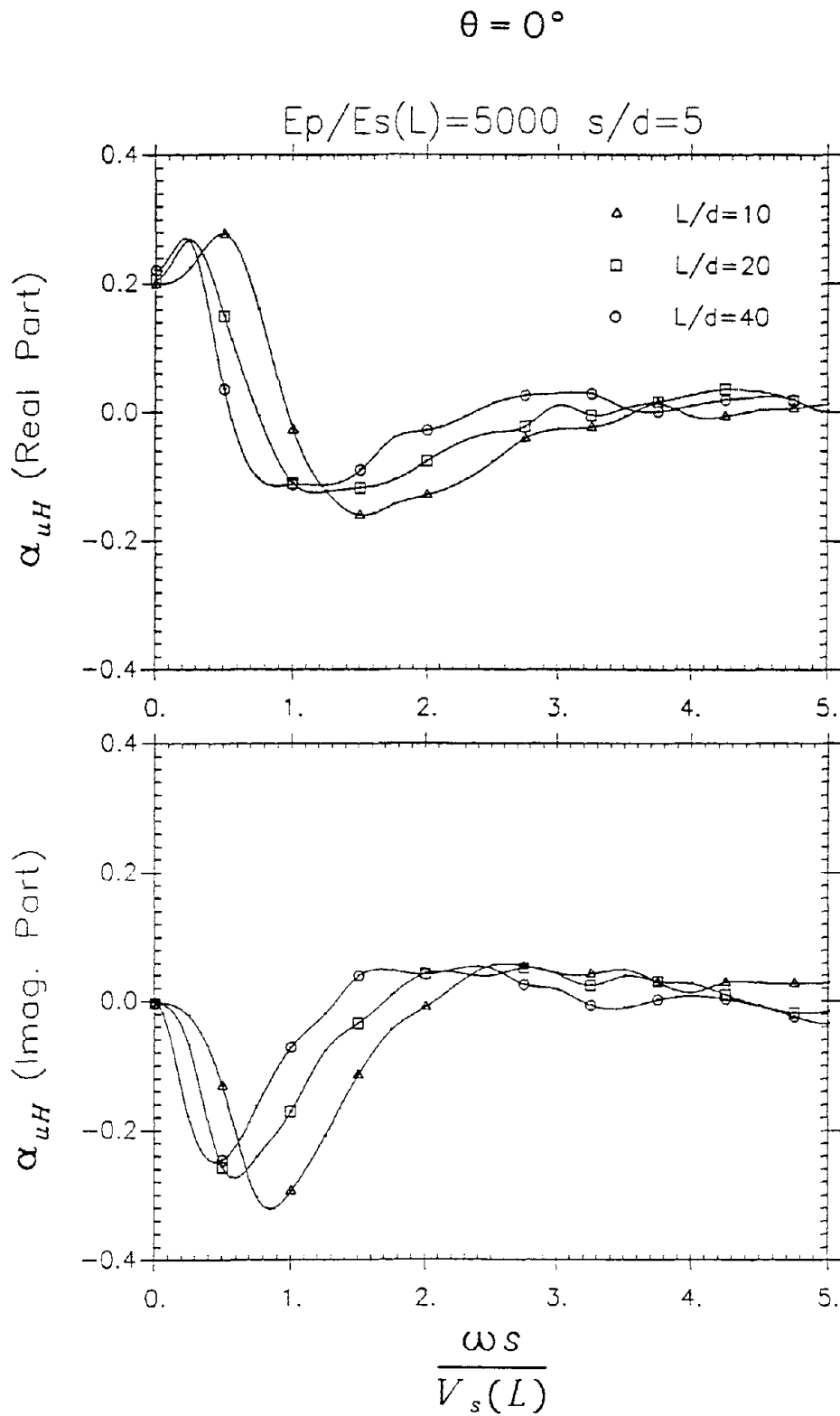
**Fig.4-23 Effect of  $L/d$  on Horizontal Interaction Factors for Relatively-Compressible Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )**

$$\theta = 90^\circ$$



**Fig.4-24 Effect of  $L/d$  on Horizontal Interaction Factors for Relatively-Compressible**

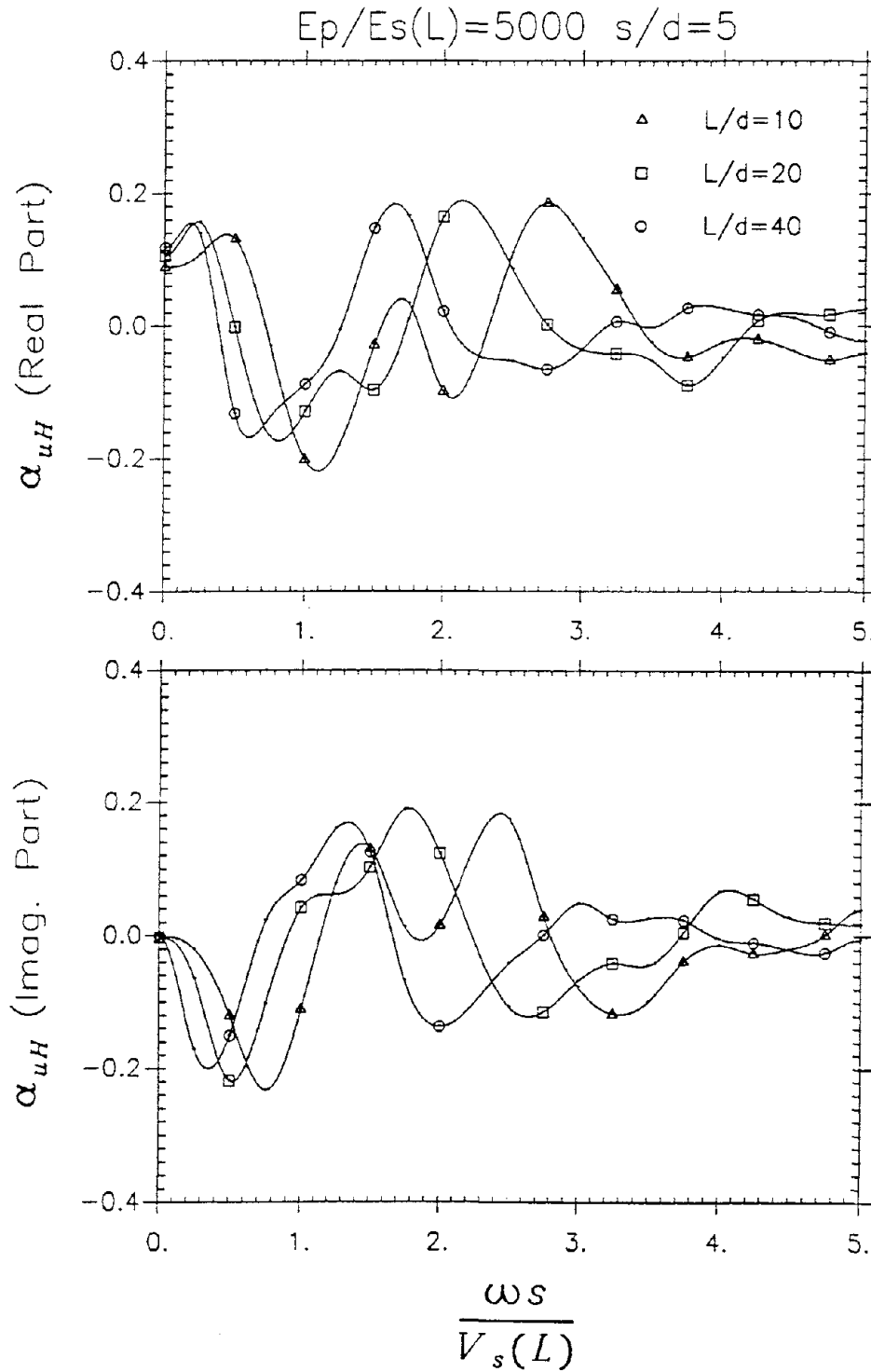
**Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )**



**Fig.4-25 Effect of  $L/d$  on Horizontal Interaction Factors for Relatively-Rigid Piles in**

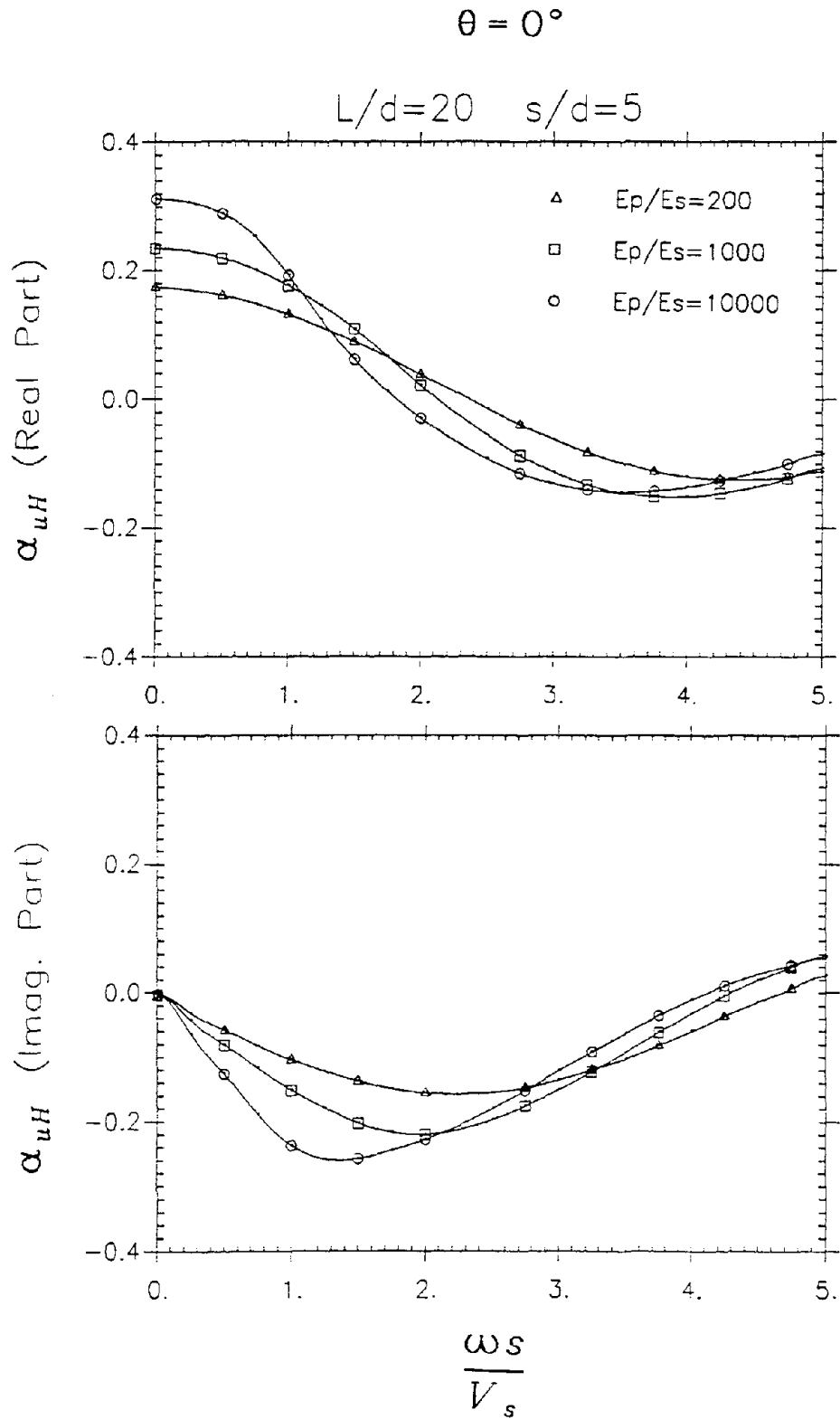
**Nonhomogeneous Soil ( $E_p/E_s(L) = 5000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$**

$$\theta = 90^\circ$$



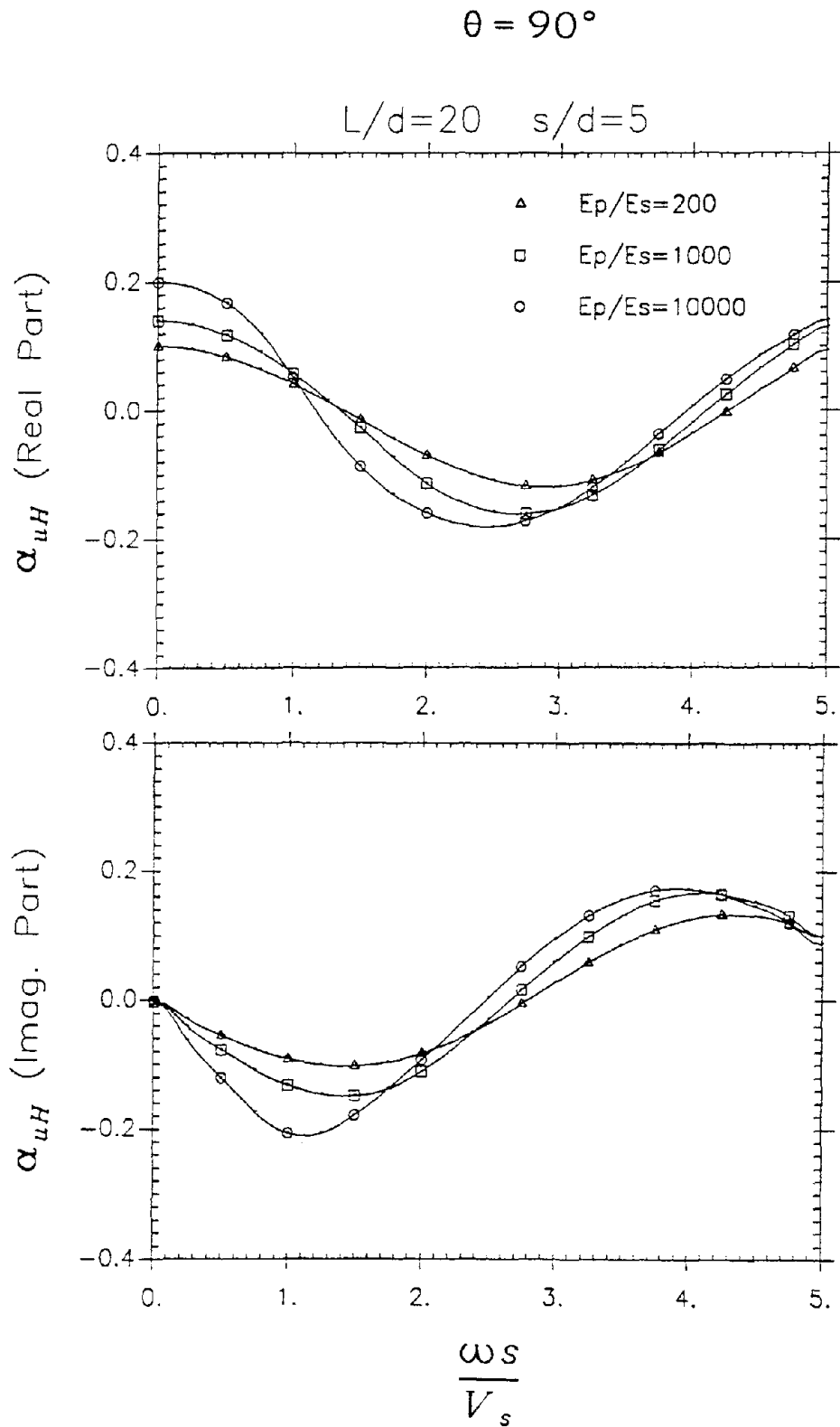
**Fig.4-26 Effect of  $L/d$  on Horizontal Interaction Factors for Relative-Rigid Piles in**

**Nonhomogeneous Soil( $E_p/E_s(L) = 5000, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$**



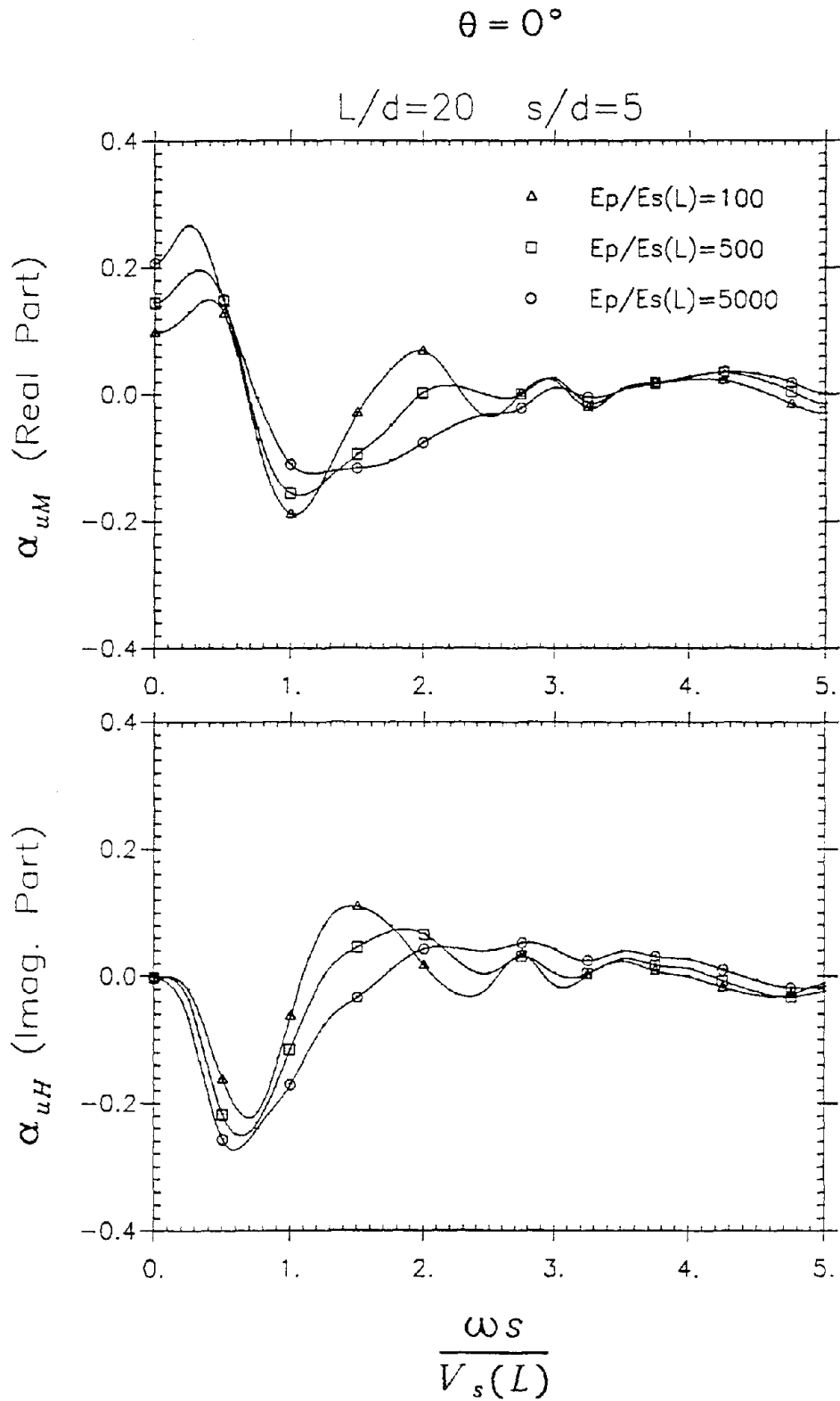
**Fig.4-27** Effect of  $E_p/E_s$ , on Horizontal Interaction Factors for Piles in Homogeneous

Soil( $L/d = 20, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ ) $\theta = 0^\circ$



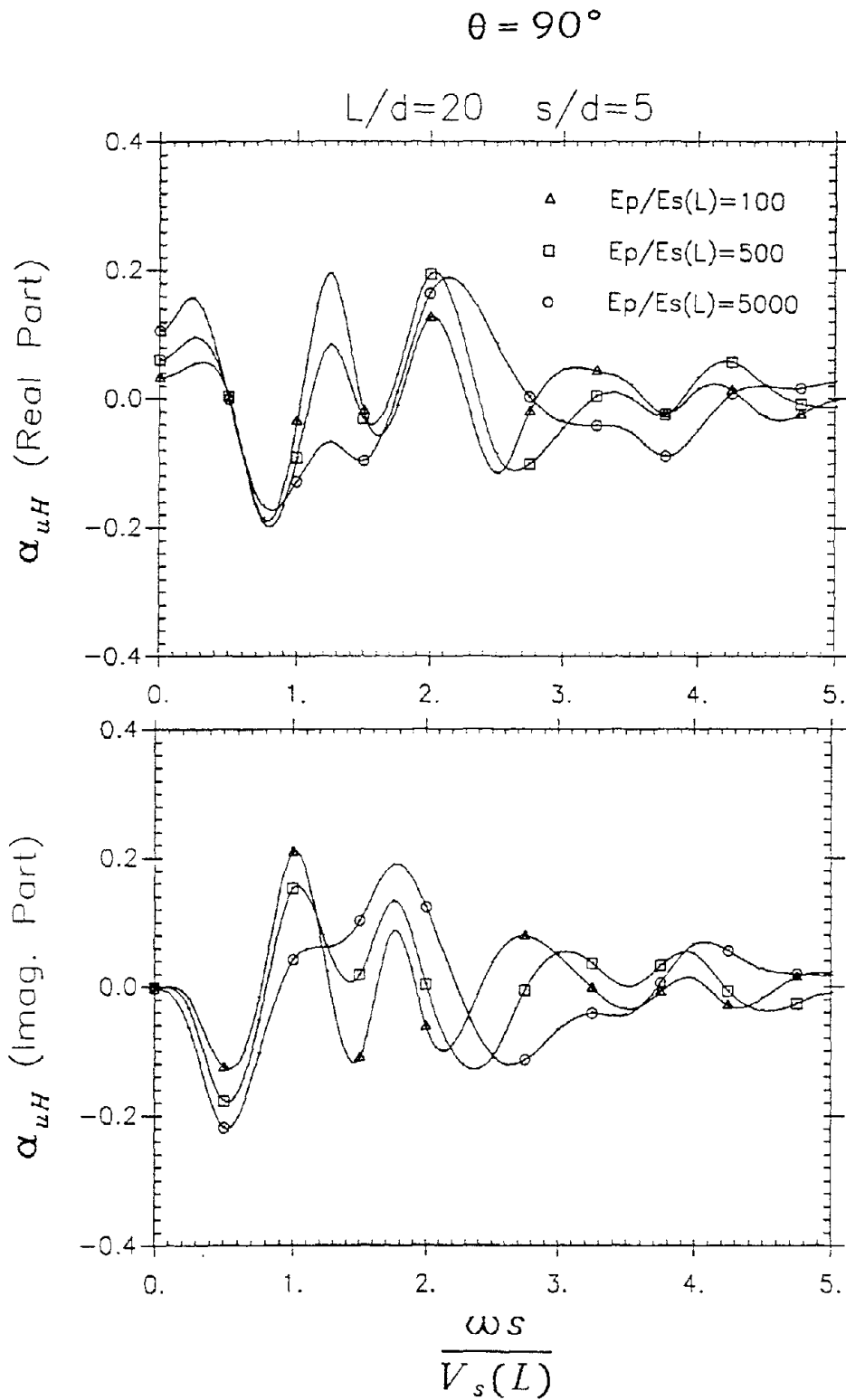
**Fig.4-28** Effect of  $E_p/E_s$  on Horizontal Interaction Factors for Piles in Homogeneous

Soil( $L/d = 20, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$



**Fig.4-29 Effect of  $E_p/E_s$  on Horizontal Interaction Factors for Piles in Nonhomogeneous**

Soil( $L/d = 20, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ ) $\theta = 0^\circ$



**Fig.4-30 Effect of  $E_p/E_s$  on Horizontal Interaction Factors for Piles in Nonhomogeneous**

**Soil( $L/d = 20, s/d = 5, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ ) $\theta = 90^\circ$**



$$\theta = 0^\circ$$

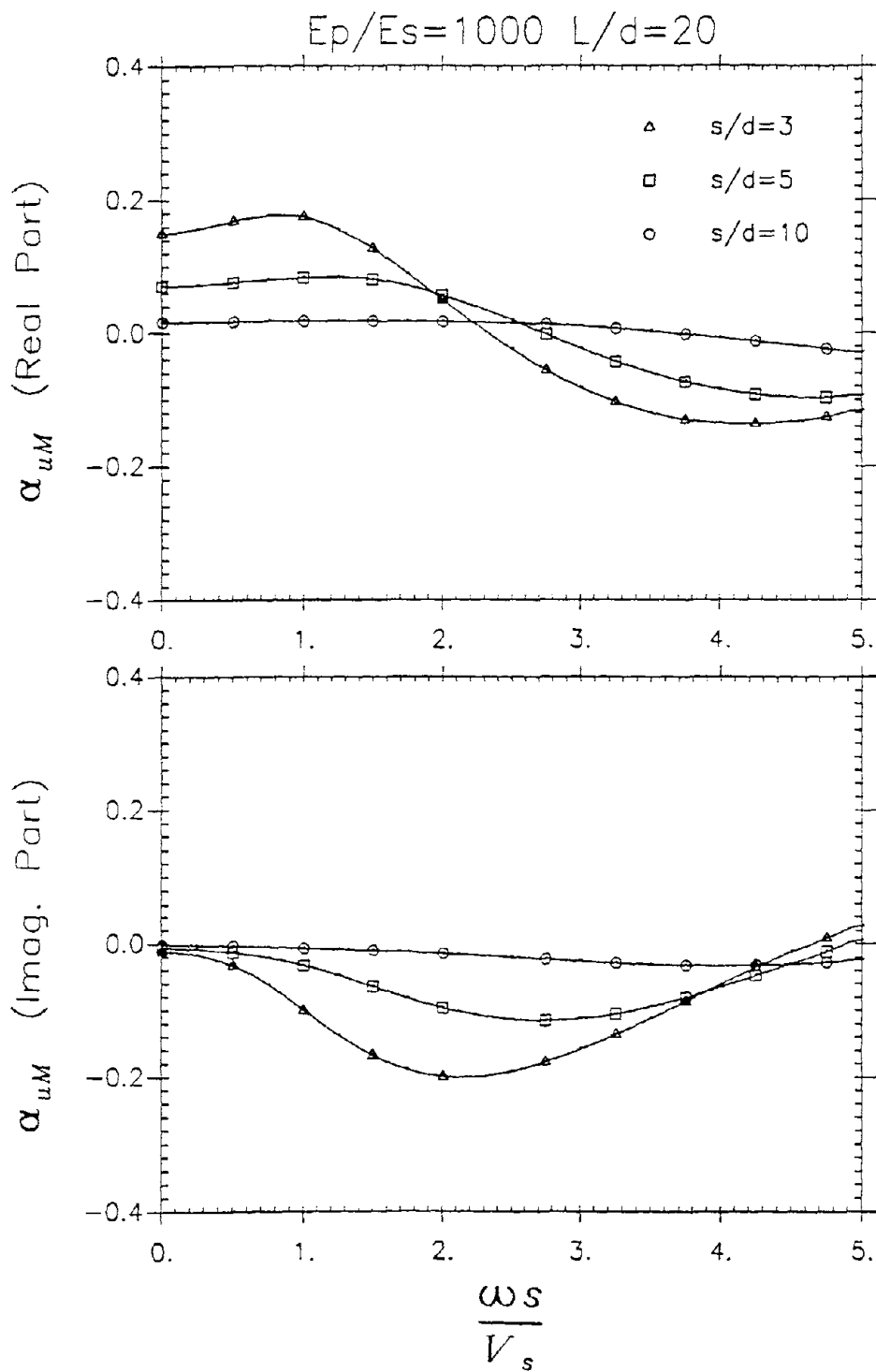
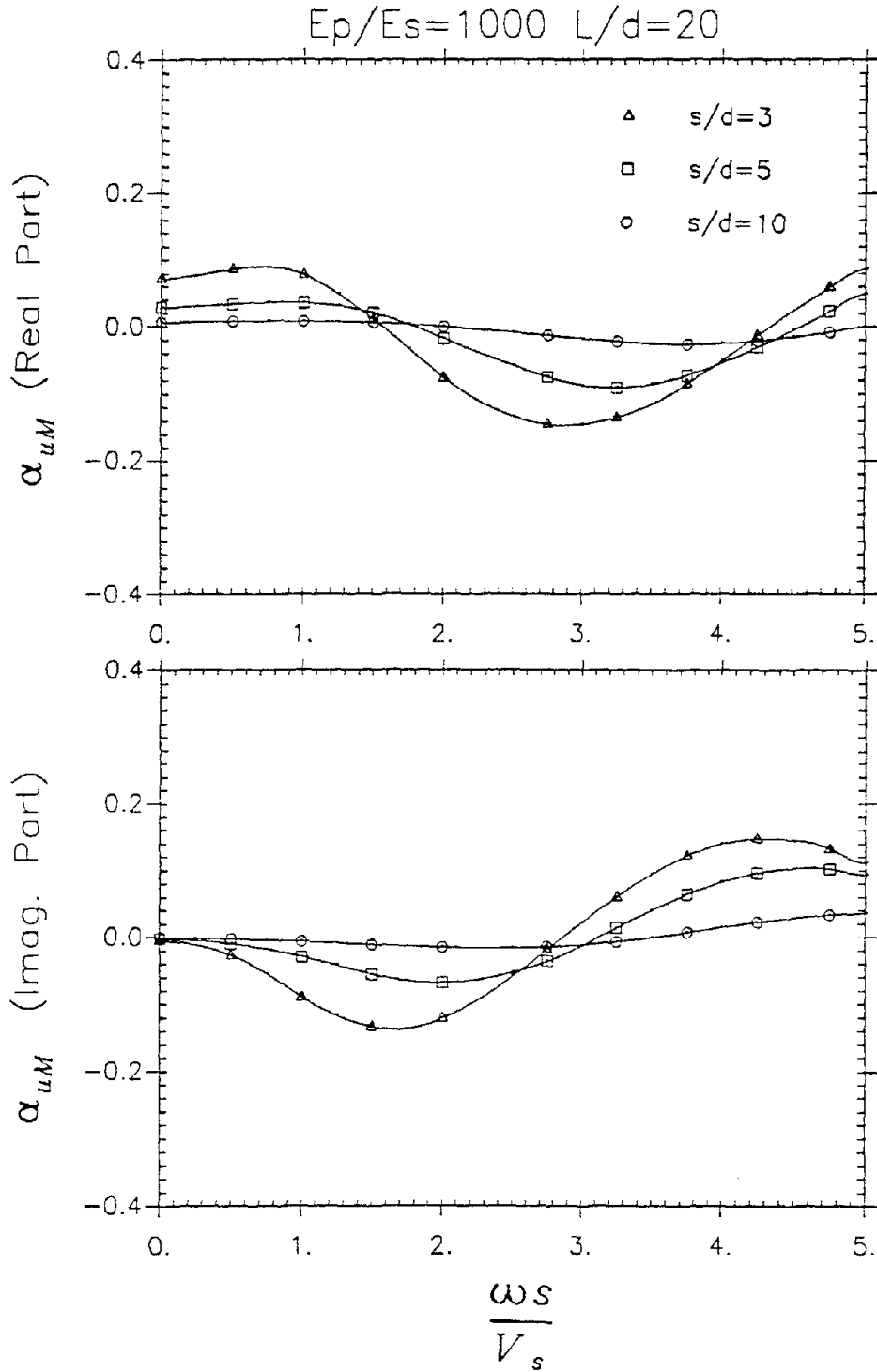
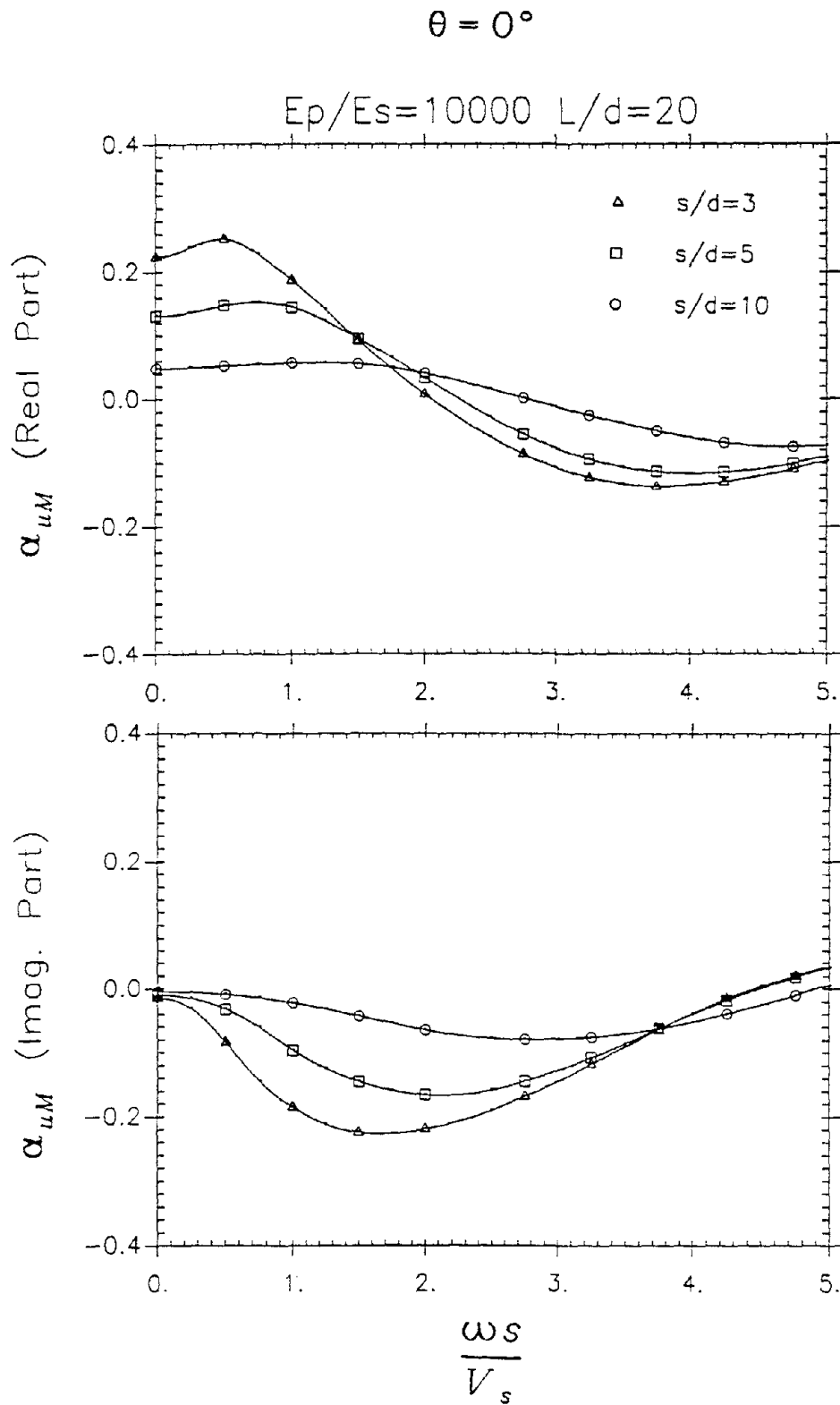


Fig.4-31 Coupling Interaction Factors for Relatively-Compressible Piles in Homogeneous Soil ( $E_p/E_s = 1000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$

$$\theta = 90^\circ$$



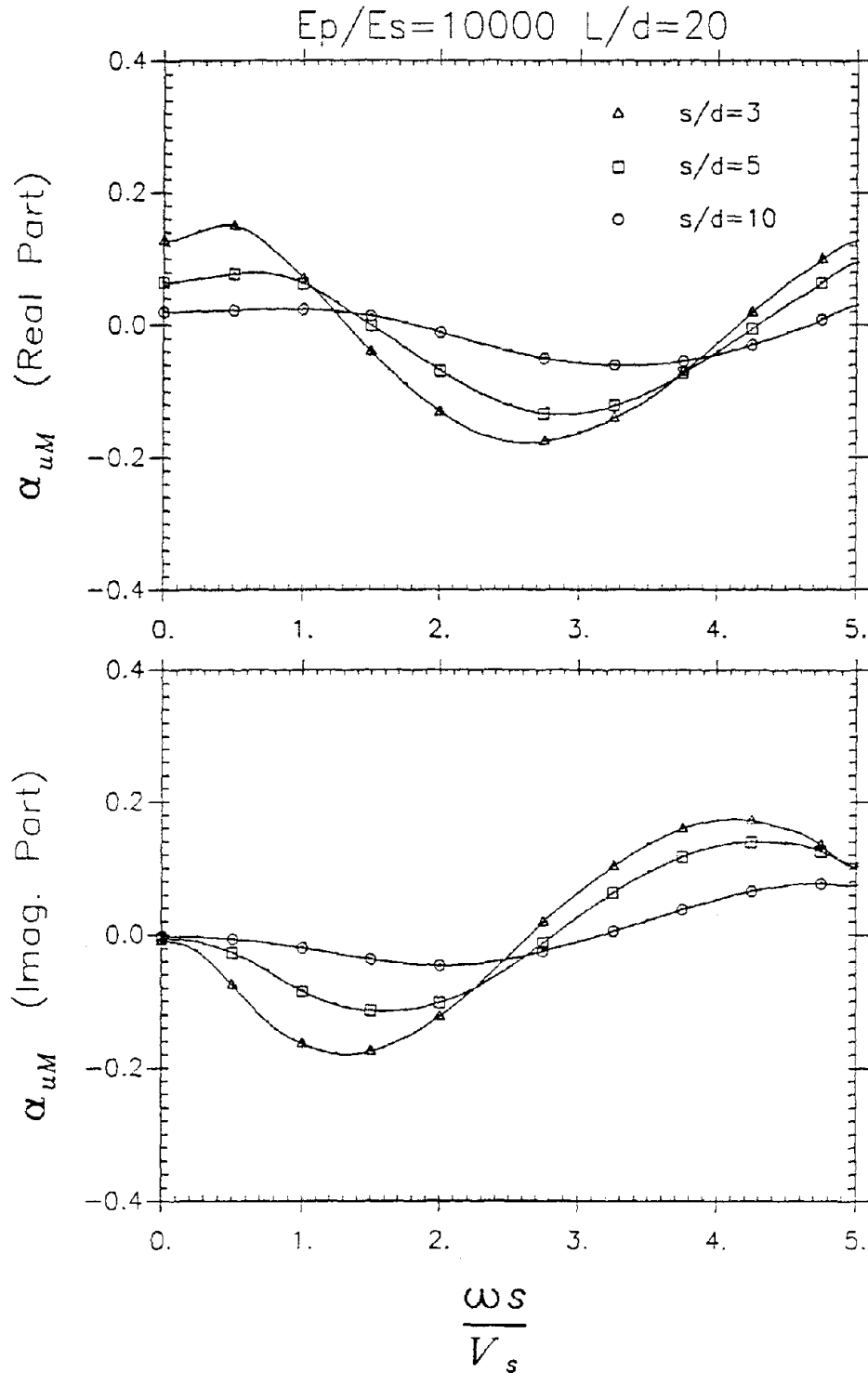
**Fig.4-32** Coupling Interaction Factors for Relatively-Compressible Piles in Homogeneous Soil ( $E_p/E_s = 1000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$



**Fig.4-33 Coupling Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil**

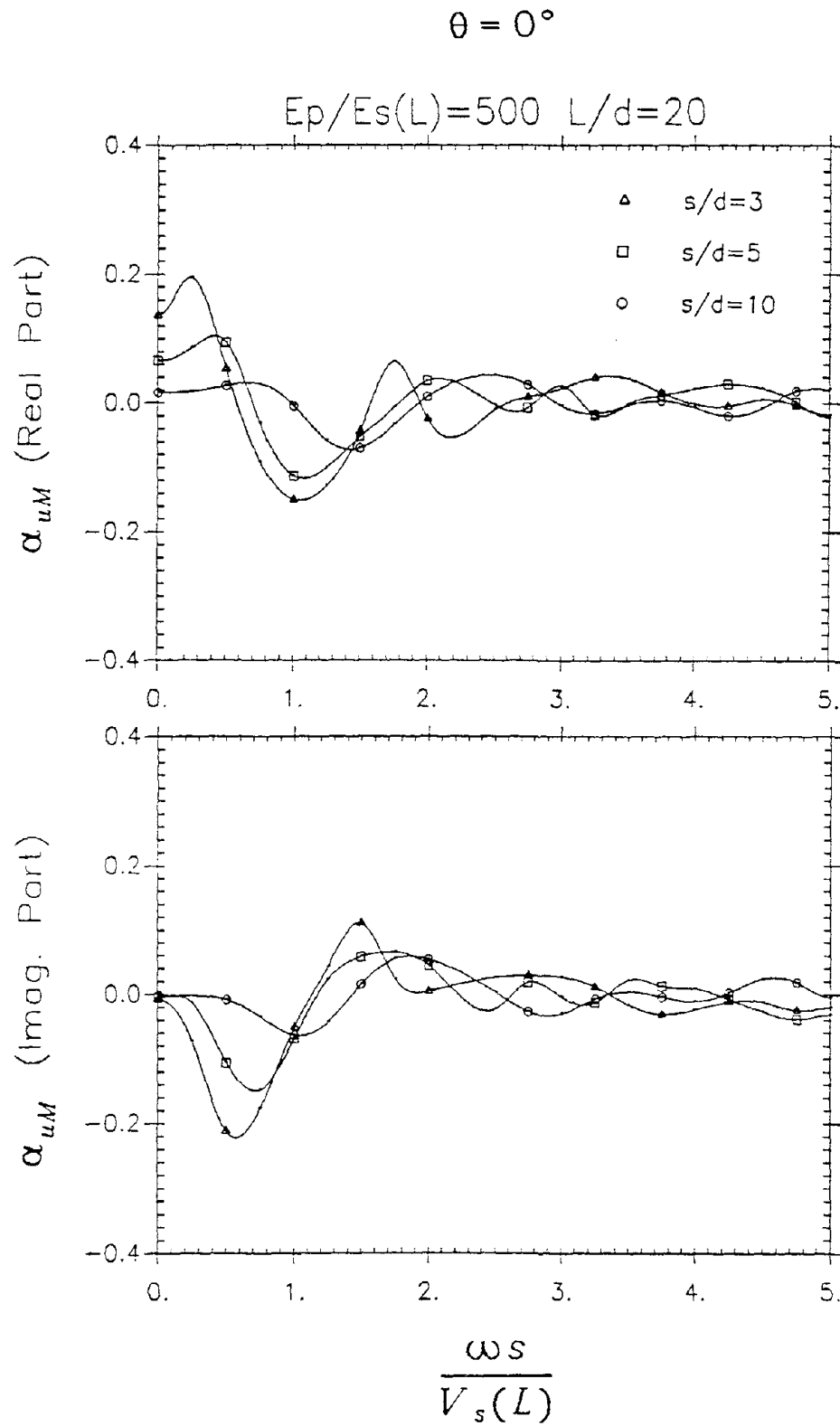
$(E_p/E_s = 10000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05 \text{ and } \nu_s = 0.4) \theta = 0^\circ$

$$\theta = 90^\circ$$



**Fig.4-34 Coupling Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil**

( $E_p/E_s = 10000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$



**Fig.4-35** Coupling Interaction Factors for Relatively-Compressible Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$

$\theta = 90^\circ$

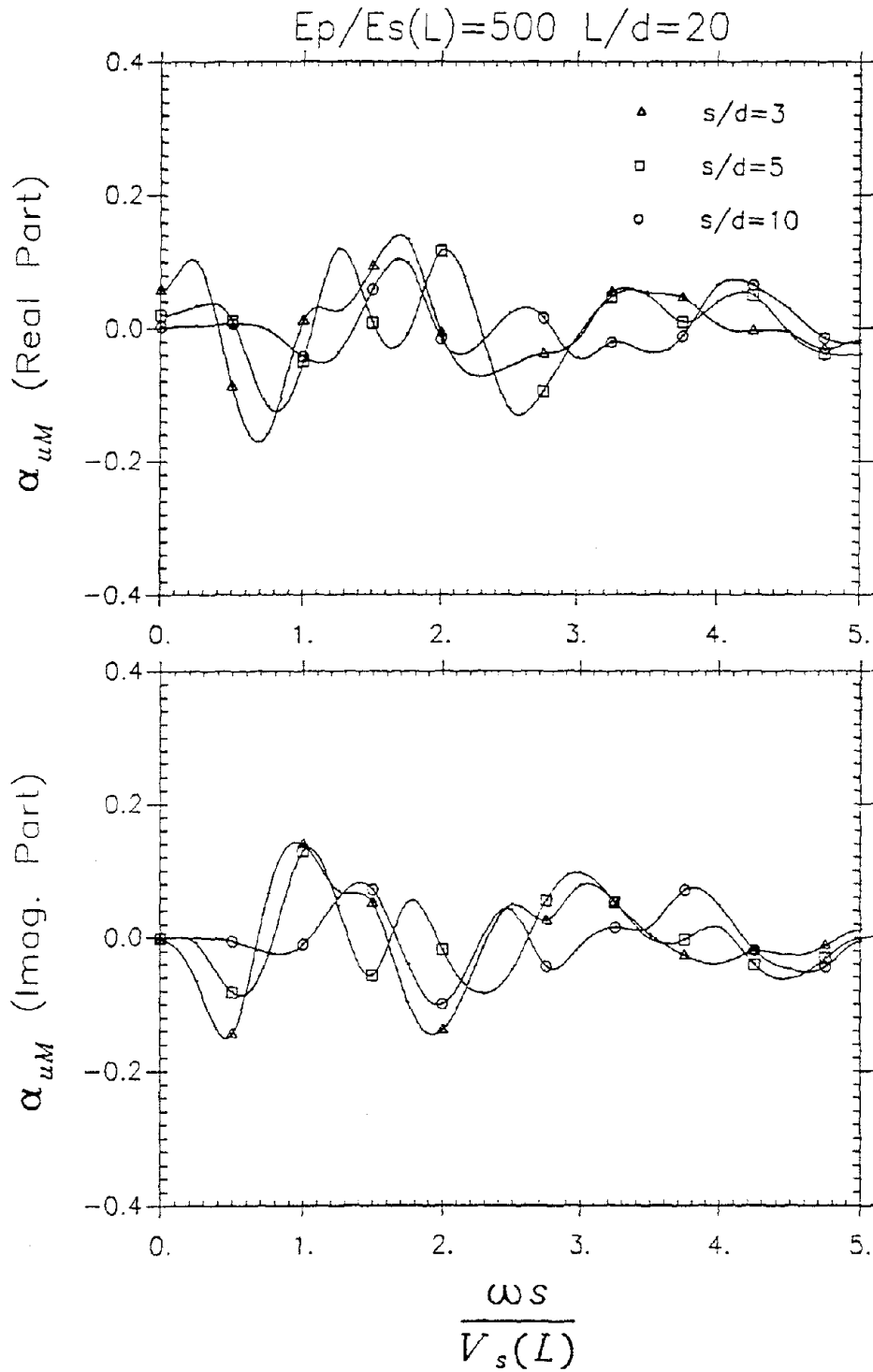
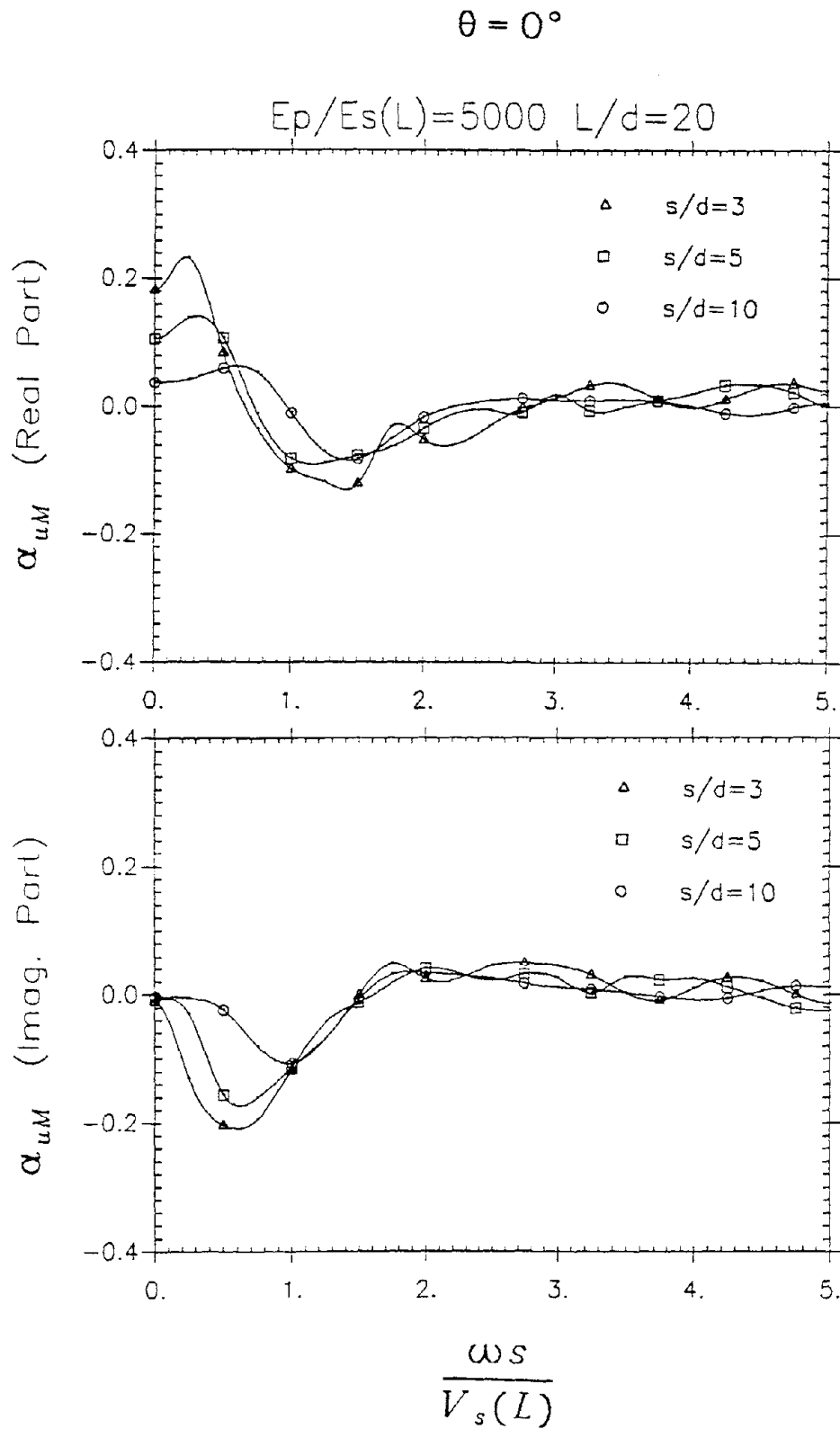


Fig.4-36 Coupling Interaction Factors for Relatively-Compressible Piles in Nonhomogeneous Soil ( $E_p/E_s(L) = 500, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$



**Fig.4-37 Coupling Interaction Factors for Relatively-Rigid Piles in Nonhomogeneous**

Soil ( $E_p/E_s(L) = 5000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$

$\theta = 90^\circ$

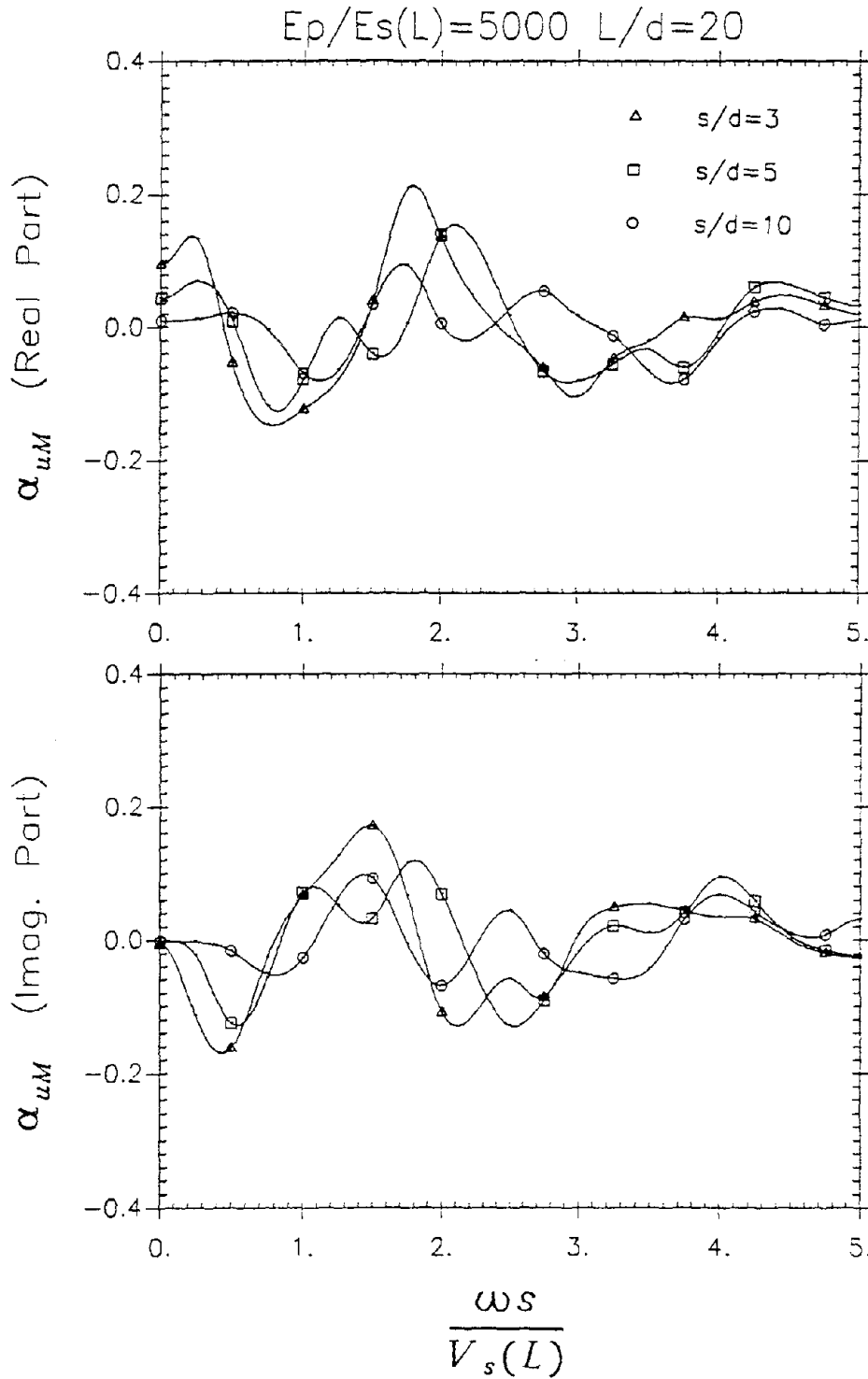
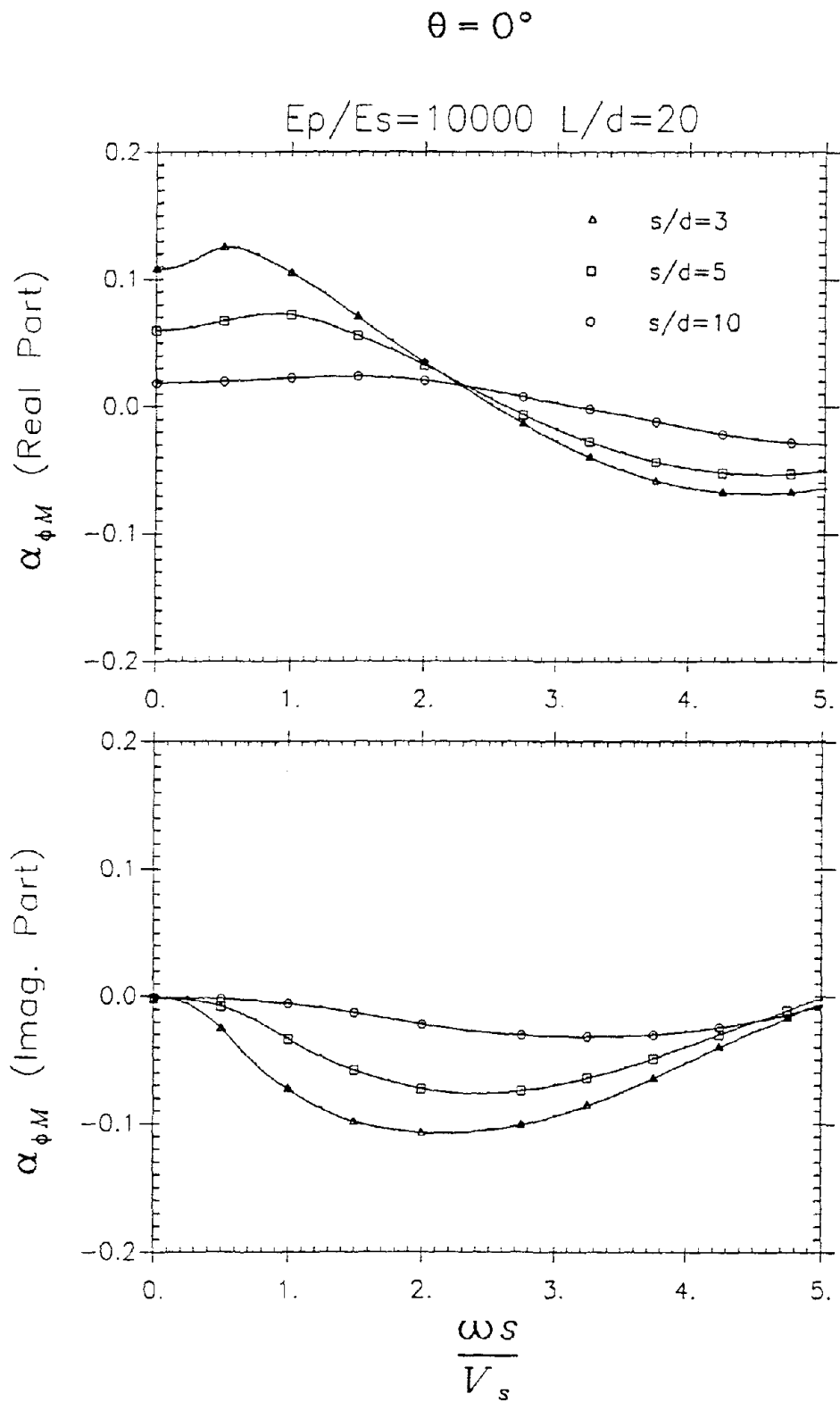


Fig.4-38 Coupling Interaction Factors for Relatively-Rigid Piles in Nonhomogeneous

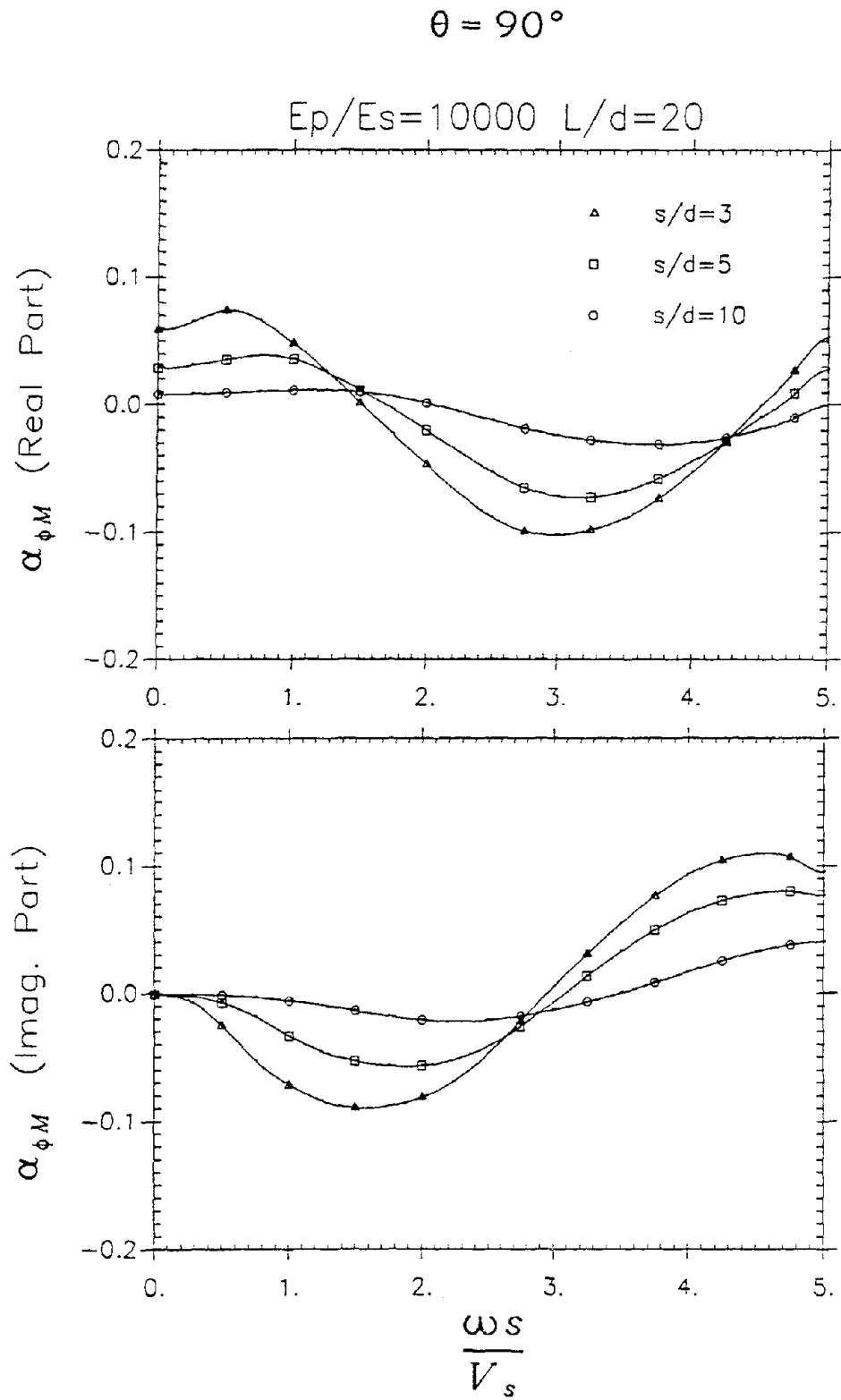
Soil ( $E_p/E_s(L) = 5000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$





**Fig.4-39 Rotational Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil**

$(E_p/E_s = 10000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05 \text{ and } \nu_s = 0.4) \theta = 0^\circ$



**Fig.4-40 Rotational Interaction Factors for Relatively-Rigid Piles in Homogeneous Soil**

$(E_p/E_s = 10000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05 \text{ and } \nu_s = 0.4) \theta = 90^\circ$

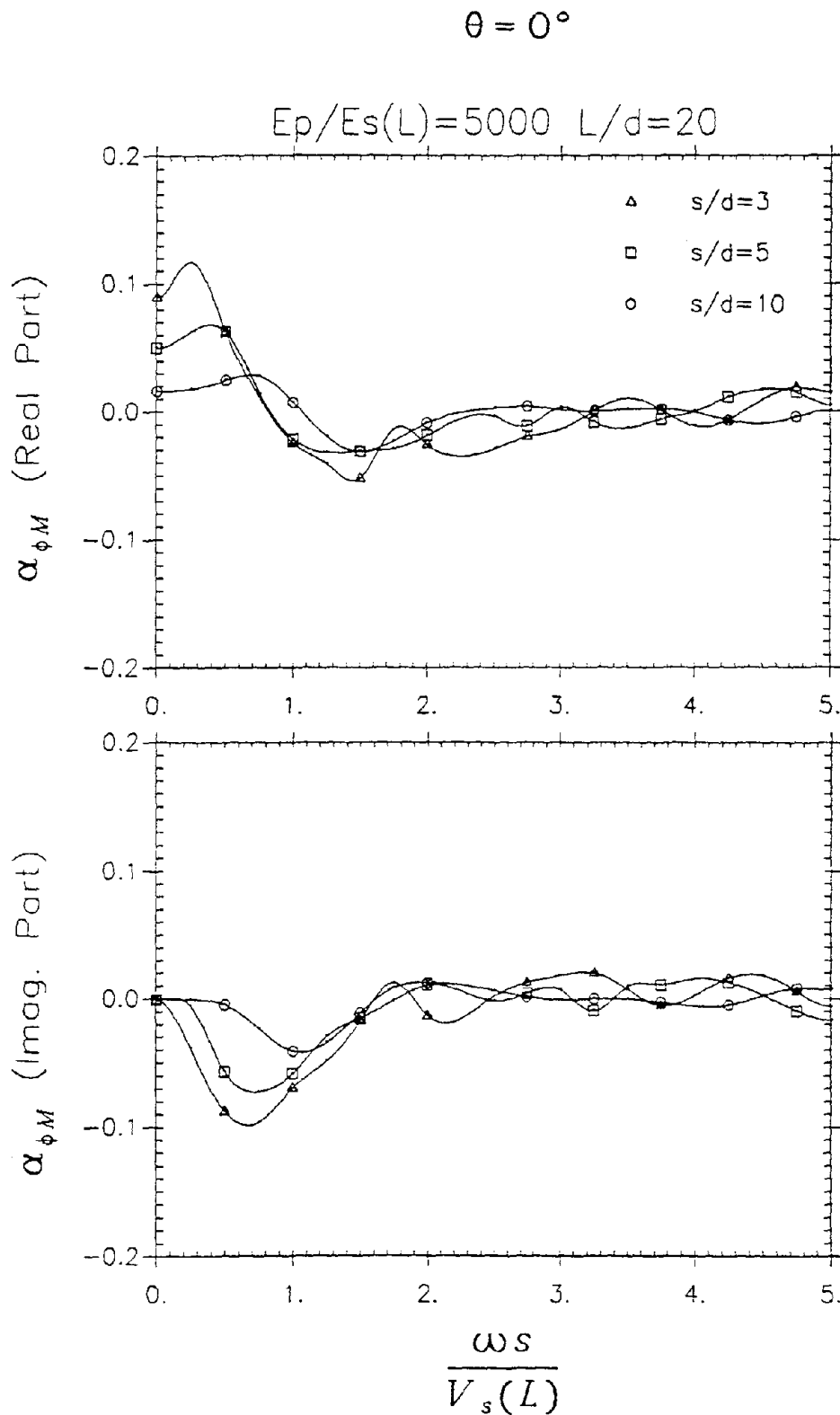
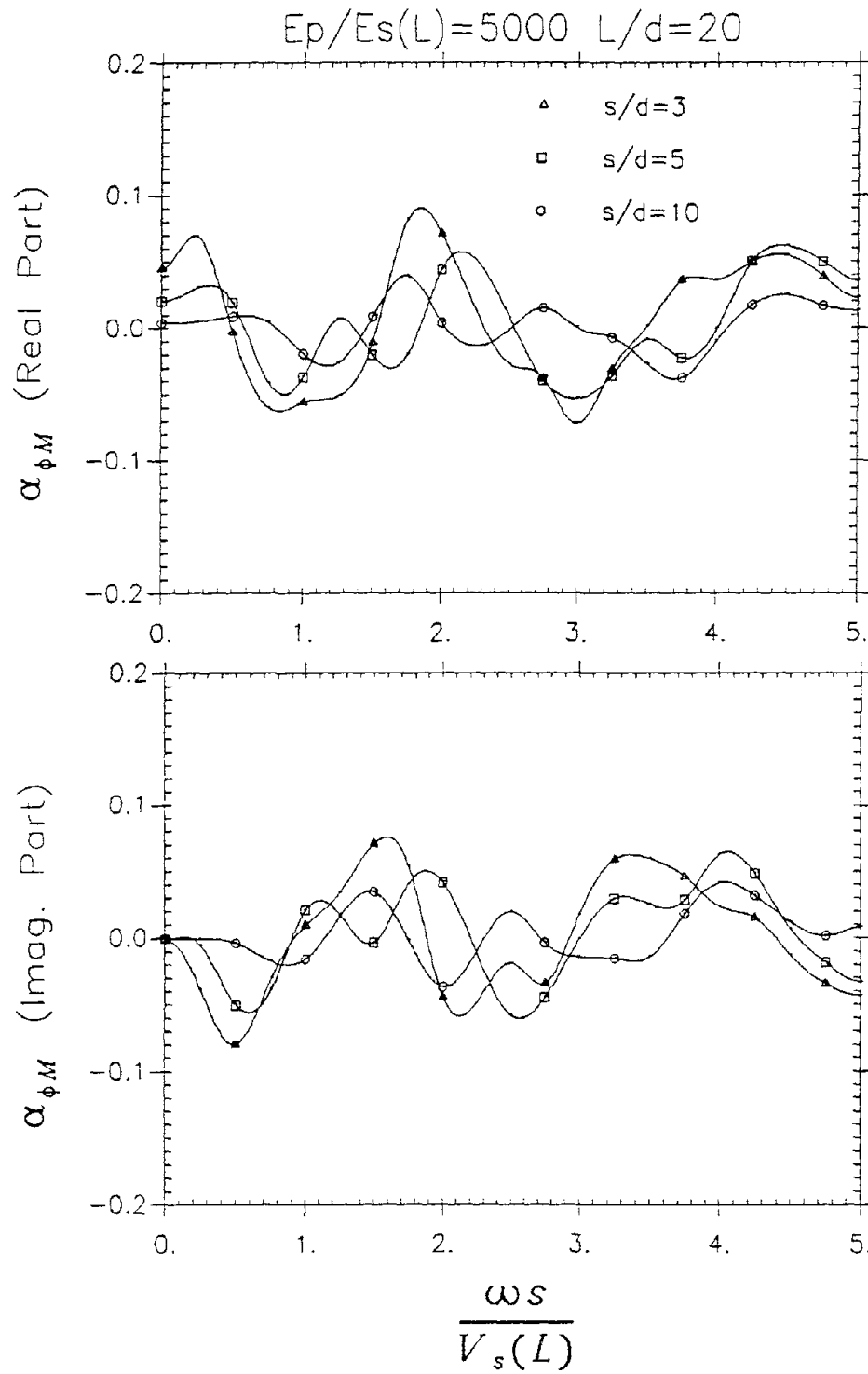


Fig.4-41 Rotational Interaction Factors for Relatively-Rigid Piles in Nonhomogeneous

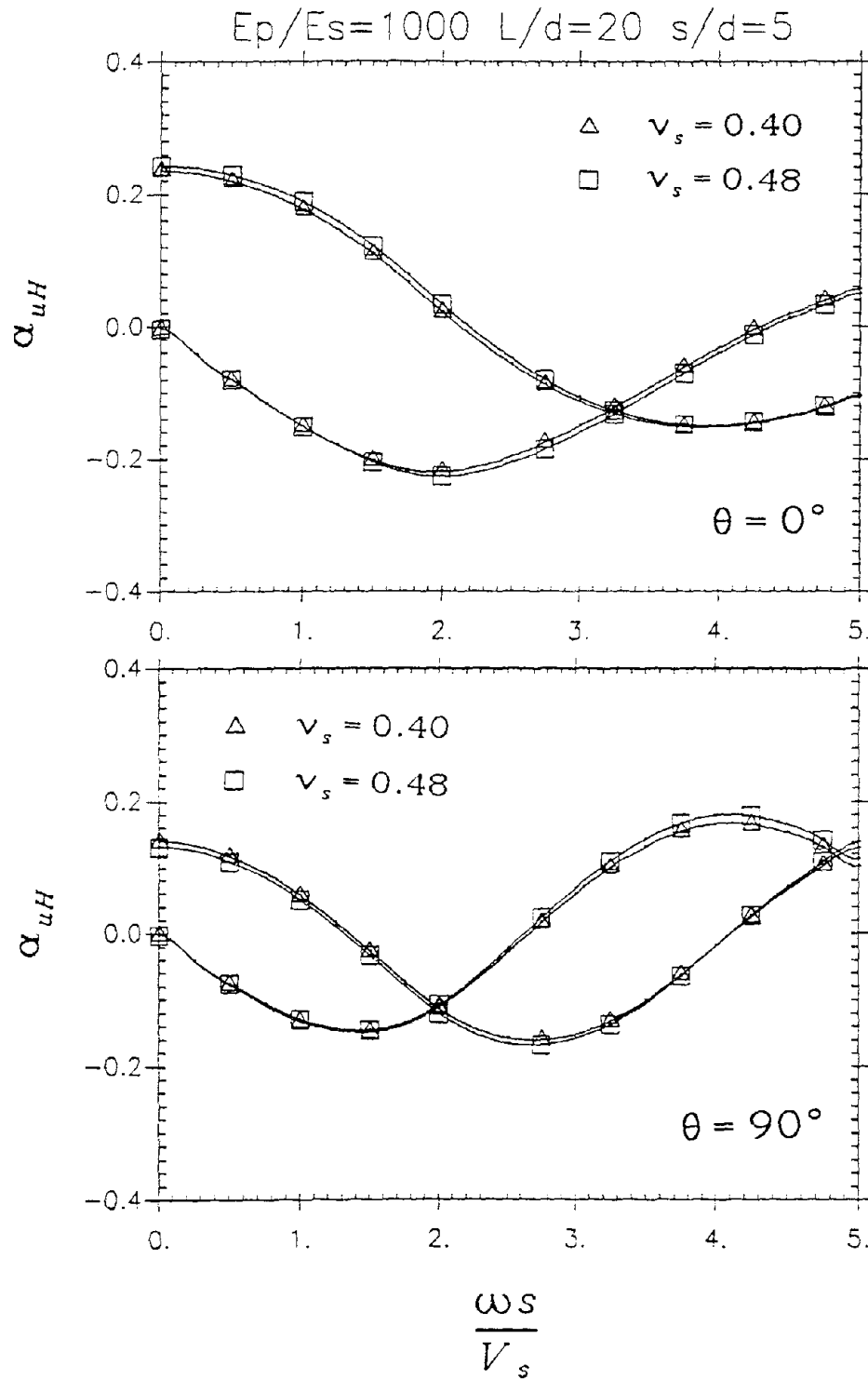
Soil ( $E_p/E_s(L) = 5000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 0^\circ$

$$\theta = 90^\circ$$

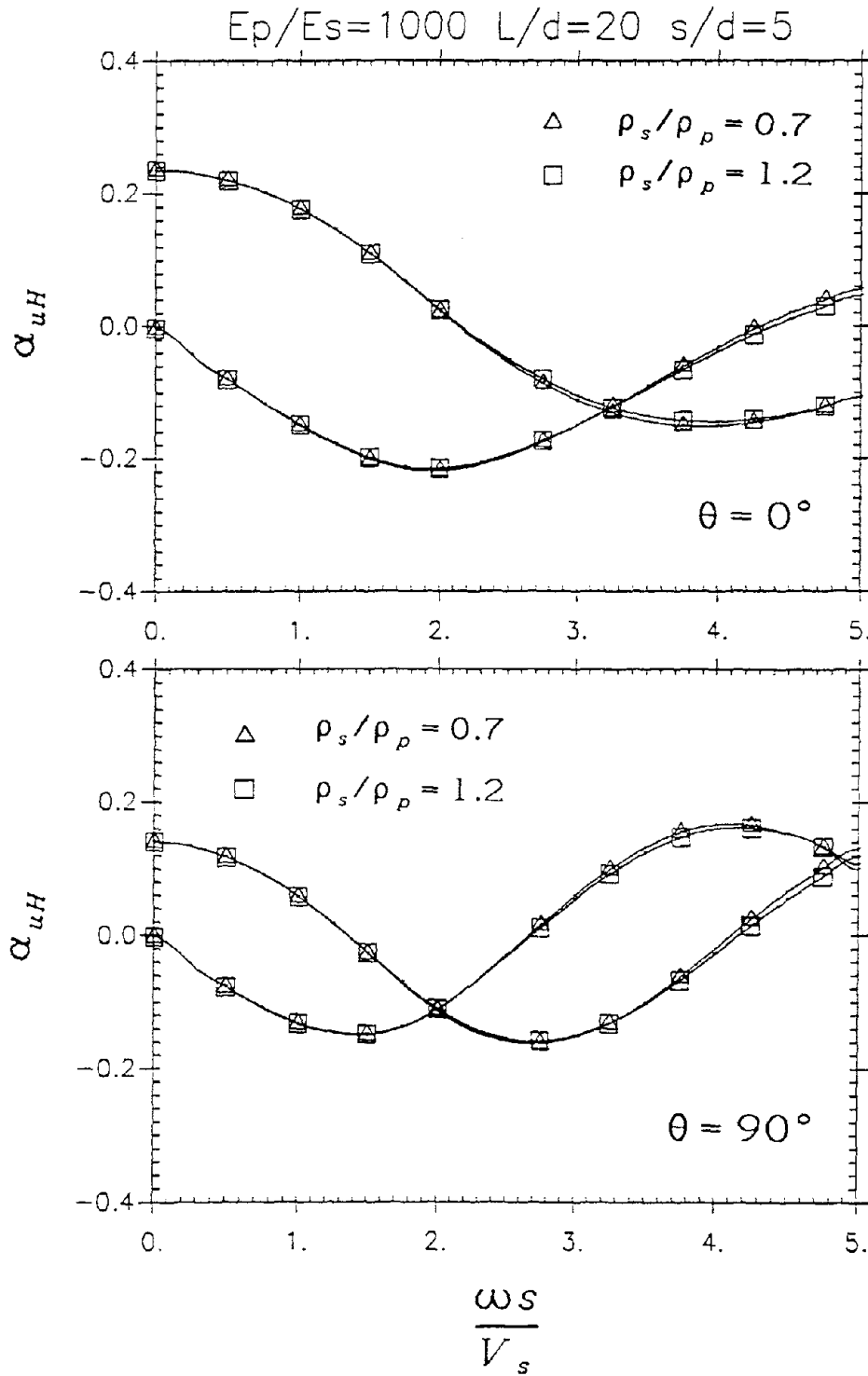


**Fig.4-42 Rotational Interaction Factors for Relatively-Rigid Piles in Nonhomogeneous**

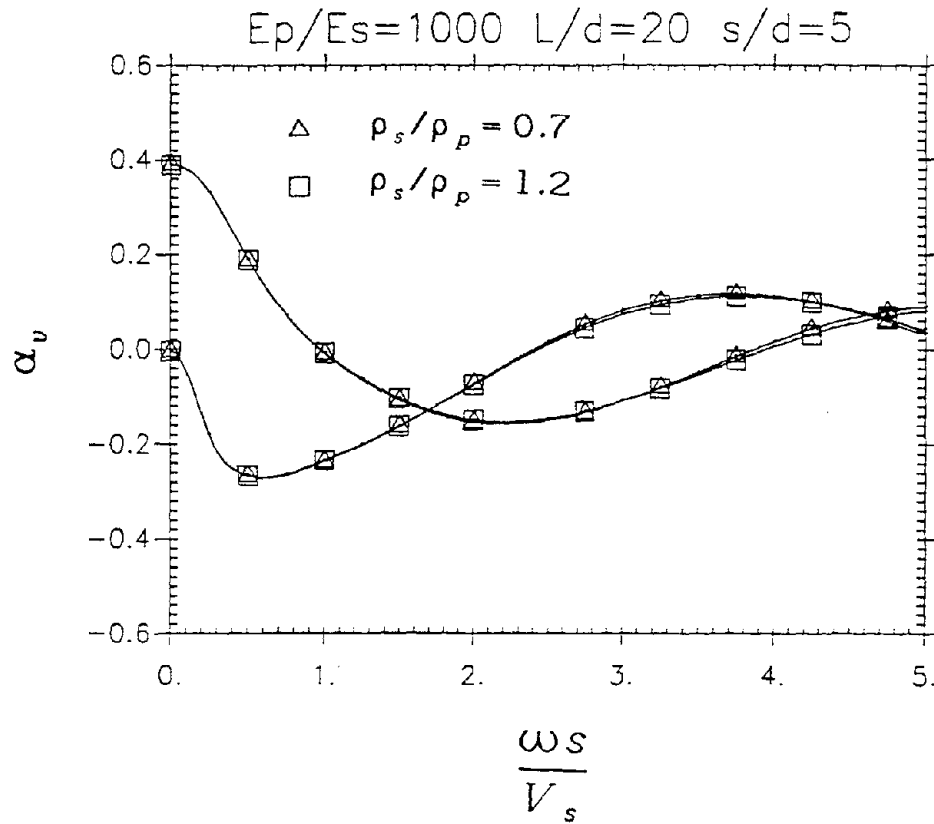
Soil ( $E_p/E_s(L) = 5000, L/d = 20, \rho_s/\rho_p = 0.7, \beta = 0.05$  and  $\nu_s = 0.4$ )  $\theta = 90^\circ$



**Fig.4-43 Effect of Poisson's Ratio  $\nu_s$  on Horizontal Interaction Factors for Relatively-Compressible Piles in Homogeneous Soil ( $E_p/E_s = 1000, L/d = 20, s/d = 5, \beta = 0.05$  and  $\rho_s/\rho_p = 0.7$ )**



**Fig.4-44 Effect of Mass Density Ratio  $\rho_s/\rho_p$  on Horizontal Interaction Factors for Relatively-Compressible Piles in Homogeneous Soil ( $E_p/E_s = 1000, L/d = 20, s/d = 5, \beta = 0.05$  and  $\nu_s = 0.4$ )**



**Fig.4-45 Effect of Mass Density Ratio  $\rho_s/\rho_p$  on Vertical Interaction Factors for Relatively-Compressible Piles in Homogeneous Soil ( $E_p/E_s = 1000, L/d = 20, s/d = 5, \beta = 0.05$  and  $\nu_s = 0.4$ )**





**SECTION 5**  
**CONCLUSION**

Graphs of dynamic interaction factors for vertical and horizontal displacements and rotations of free-head piles embedded in homogeneous and nonhomogeneous halfspace have been presented. These results should be of practical value in seismic design of pile foundations and in the seismic analysis of soil-structure interaction. The presented graphs can be readily applied by engineers already familiar with the use of static interaction factors in the design of pile groups.



## SECTION 6

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