NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo

PB91-179242

Physical Space Solutions of Non-proportionally Damped Systems

by

M. Tong, Z. Liang and G. C. Lee
Department of Civil Engineering
State University of New York at Buffalo
Buffalo, New York 14260

Technical Report NCEER-91-0002

January 15, 1991

This research was conducted at the State University of New York at Buffalo and was partially supported by the National Science Foundation under Grant No. ECE 86-07591.

REPRODUCED BY
U.S. DEPARTMENT OF COMMERCE
NATIONAL TECHNICAL
INFORMATION SERVICE
SPRINGFIELD, VA 22161

NOTICE

This report was prepared by the State University of New York at Buffalo as a result of research sponsored by the National Center for Earthquake Engineering Research (NCEER). Neither NCEER, associates of NCEER, its sponsors, State University of New York at Buffalo, nor any person acting on their behalf:

a. makes any warranty, express or implied, with respect to the use of any information, apparatus, method, or process disclosed in this report or that such use may not infringe upon privately owned rights; or

b. assumes any liabilities of whatsoever kind with respect to the use of, or the damage resulting from the use of, any information, apparatus, method or process disclosed in this report.

Physical Space Solutions of Non-Proportionally Damped Systems

by

M. Tong[1], Z. Liang[2] and G.C. Lee[3]

January 15, 1991

Technical Report NCEER-91-0002

NCEER Project Number 87-1016

NSF Master Contract Number ECE 86-07591

1 Postdoctoral Fellow, Department of Civil Engineering, State University of New York at Buffalo
2 Research Associate, Department of Civil Engineering, State University of New York at Buffalo
3 Professor and Dean of Engineering and Applied Science, Department of Civil Engineering, State University of New York at Buffalo

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo
Red Jacket Quadrangle, Buffalo, NY 14261

PREFACE

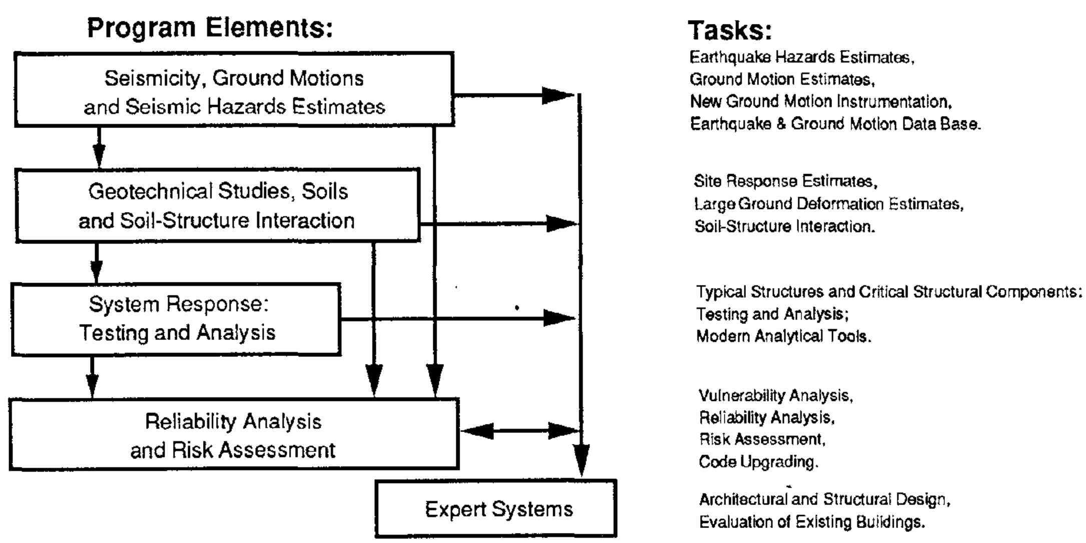
The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 1, Existing and New Structures, and more specifically to system response investigations.

The long term goal of research in Existing and New Structures is to develop seismic hazard mitigation procedures through rational probabilistic risk assessment for damage or collapse of structures, mainly existing buildings, in regions of moderate to high seismicity. The work relies on improved definitions of seismicity and site response, experimental and analytical evaluations of systems response, and more accurate assessment of risk factors. This technology will be incorporated in expert systems tools and improved code formats for existing and new structures. Methods of retrofit will also be developed. When this work is completed, it should be possible to characterize and quantify societal impact of seismic risk in various geographical regions and large municipalities. Toward this goal, the program has been divided into five components, as shown in the figure below:



System response investigations constitute one of the important areas of research in Existing and New Structures. Current research activities include the following:

1. Testing and analysis of lightly reinforced concrete structures, and other structural components common in the eastern United States such as semi-rigid connections and flexible diaphragms.
2. Development of modern, dynamic analysis tools.
3. Investigation of innovative computing techniques that include the use of interactive computer graphics, advanced engineering workstations and supercomputing.

The ultimate goal of projects in this area is to provide an estimate of the seismic hazard of existing buildings which were not designed for earthquakes and to provide information on typical weak structural systems, such as lightly reinforced concrete elements and steel frames with semi-rigid connections. An additional goal of these projects is the development of modern analytical tools for the nonlinear dynamic analysis of complex structures.

One of the major challenges in nonlinear structural analysis of non-classically damped systems under dynamic loading is the development of a reliable computational approach. When excited by dynamic loads, such structural systems not only dissipate energy unevenly, but also show different types of energy transmissions. These special features cannot be handled by the traditional methods used for classically damped systems. This report is intended to provide a sufficient numerical treatment of response calculations for non-classically damped systems. Although the proposed approaches focus on the linear structures at this stage, it has potential capability of handling nonlinear structures with non-classical damping and is superior when applied to classically damped systems.

ABSTRACT

In this report, various numerical methods to obtain the responses of non-proportionally damped systems are compared in terms of their accuracy and efficiency. Based on a theoretical investigation of non-proportional damping mechanisms, a class of new iterative schemes are proposed followed by an analysis of their convergence and their optimal convergent speed. It is found that such iterative schemes possess faster convergent speed than many currently available numerical methods and require much less computer memory space. Moreover, the new schemes converge unconditionally for both classically and non- classically damped systems.

TABLE OF CONTENTS

Preceding page blank

LIST OF FIGURES

Preceding page blank

SECTION 1

INTRODUCTION

Structural design against time-dependent loadings has been a major interest in many engineering disciplines in this half of the century. In mechanical engineering, significant progress has been made by using these designs in control of vibrations. In civil engineering, structures designed against time-dependent loadings are relatively recent and are popular primarily in earthquake-resistant designs which includes system identification and structural control. When dealing with dynamic issues of linear, viscously damped structures, the *fundamental dynamic equation* is often expressed in the following matrix form:

$$\tilde{\mathbf{M}}\,\ddot{\tilde{X}}(t) + \tilde{\mathbf{C}}\,\dot{\tilde{X}}(t) + \tilde{\mathbf{K}}\tilde{X}(t) = \tilde{F}(t) \qquad (1)$$

where the n×n matrices $\tilde{\mathbf{M}}$, $\tilde{\mathbf{C}}$, $\tilde{\mathbf{K}}$ are mass, damping and stiffness matrices respectively. $\tilde{X}$ is the displacement vector. F(t) is a forcing function. The mass matrix is positive definite, so is the stiffness matrix in most civil engineering applications. The damping matrix is allowed to be positive semi-definite. All of these matrices are symmetric.

According to many current designs, a structure may have cetain plastic deformations when excited by strong earthquake ground motions. Such designs mainly consider the stiffness of the structure. A second important approach of earthquake-resistant design of structures is by using the damping term of Equation (1). For many years, analysts and engineers had not paid much attention to understand damping mechanisms. The elementary treatment of damping for single-degree-of-freedom (SDOF) systems has been adapted to multi-degree-of-freedom (MDOF) systems. The assumption of proportional damping, suggested by Rayleigh in 1920s is still used in most situations today. Although non-proportional damping of MDOF systems attracts much attention lately, engineers still deal with their designs by essentially treating the structures as SDOF systems. Namely, the damping effects are treated only as the phenomena of energy dissipation. However, in non-proportionally damped structures, not only energy dissipation but also energy transmission exist, and the latter should be taken into consideration in structural design (see Pollard, (1975), Liang (1985), Liang and Lee (1990)).

There are several unsolved problems concerning the damping mechanisms. A most commonly encountered one in earthquake engineering is the lack of physical explanation for the inability to decouple a non-proportionally damped systems in the physical domain and to obtain the n-modal solutions (see Liang and Lee (1990)). Although many researchers are able to give some mathematical descriptions to this problem, they failed to describe them physically (see Meirovitch (1967), Clough, (1975), Ewins (1986) and Singh (1986)). Several researchers have attempted to solve Equation (1) in the n-dimensional physical domain by using pure proportionally damped systems or by adding numerically computed pseudo-forces (see Thompson et al (1974), Udwadia and Esfandiari (1990)). However, these efforts did not give accurate and unconditionally convergent numerical methods. The authors of this report believe that fundamental analysis is essential in order to completely solve the above problem.

In this report, while our attentions are given to the physical space solutions of Equation (1), some theoretical bases about generally damped structural systems and the damping mechanisms will also be described. It is shown that these theoretical foundations can lead to correct solutions of Equation (1) in the n-dimensional physical space, and provide answers to some problems concerning non-proportionally damped systems as well.

In practice, most civil engineering structural systems are non-proportionally damped. The usual finite element method can easily generate models with thousands of dimensions. In design, identification and control of structures, the determination of the system responses of such non-proportionally damped, large degrees-of-freedom systems should have sufficient accuracy to ensure reliable analyses. Furthermore, the speed of calculation is an important factor in on-line diagnostic and monitoring systems. One example is the real-time active control of buildings subjected to earthquake excitations, (see Soong (1987)). Because non-proportionally damped systems can not be decoupled in the n-dimensional physical space, we have to go through the 2n dimensional state space to calculate their responses. This is generally very time consuming for large order systems. Many attempts in obtaining the response can not yield accurate results or fail to convergence unanimously. This situation makes the task of numerically solving Equation (1) with precise and efficiency appear difficult, though it is a popular topic in today's structural dynamics. In this report we first briefly introduce some rudimentary concepts of complex damping and the ratio of energy transmission over energy dissipation in per mode (TD ratio). Based on the analysis of the modal energy relationship, we will see that if the TD ratios can be technically suppressed down to certain level, then there is a convergent algorithm. This observation lead us then to the invention of a class of unconditionally convergent iterative schemes for solving Equation (1). Both conceptual descriptions of this approach and practical algorithms are included. Mathematical proofs and physical explanations of some of the related theorems are presented. Also numerical examples, including comparisons with recent developments, are given to show the advantages of the suggested methods. Calculation for a real structure is performed to compare the responses from experimental measurements and that from the proposed computation. These results appear well agreeable to each other and show the potential success and development of the proposed approach.

SECTION 2
BRIEF REVIEW OF PREVIOUS AND CURRENT STUDIES

Studies on non-proportionally damped structures are growing rapidly in numbers. Investigations may be classified into two categories: (1) theories that explain the dynamic behaviors of structures and (2) algorithms for computing various responses and parameters. Recent emphases in theoretical development include eigen-systems and modal analysis, criteria of proportional damping, indices of complexity, and complex damping, etc. while the second category gives more attention to solution techniques in practical applications. Few studies involve both fundamental understandings and solution algorithms. In the following, we will focus our review only on certain relevant aspects of dynamic analyses and response calculations.

The criterion for determining whether a system is proportionally damped was developed by Caughey and O'kelly (1965). A proportionally damped system is said to be a system that can be decoupled into n single-degree-of-freedom (SDOF) equations to yield normal (real) modes. Often the methods for solving SDOF systems can be extended to proportionally damped MDOF systems without too many difficulties. Non-proportionally damped systems have complex modes. They can not be decoupled in the n-dimensional physical space. Such a system can be described by a canonical vibration equation:

$$\mathbf{I}_n \ddot{X}(t) + \mathbf{C}\,\dot{X}(t) + \Lambda_k X(t) = F(t) \qquad (2)$$

where I_n is an n×n identity mass matrix, $\mathbf{C} = \mathbf{Q}^T\mathbf{M}^{-1/2}\tilde{\mathbf{C}}\,\mathbf{M}^{-1/2}\mathbf{Q}$, , $\Lambda_k = \mathbf{Q}^T\mathbf{M}^{-1/2}\tilde{\mathbf{K}}\mathbf{M}^{-1/2}\mathbf{Q}$, $X = \mathbf{Q}^T\mathbf{M}^{1/2}\tilde{X}$, $F(t) = \mathbf{Q}^T\mathbf{M}^{1/2}\tilde{F}(t)$, and $\mathbf{Q}$ is the orthonormal eigenvector matrix of $\mathbf{M}^{-1/2}\tilde{\mathbf{K}}\mathbf{M}^{-1/2}$. In a simple, symmetric and oscillatory system, Λ_k is diagonal and positive definite. $\mathbf{C}$ is at least positive semi-definite and $4\Lambda_k - \mathbf{C}^2$ is usually positive definite. Since the system is non-proportionally damped, the damping matrix $\mathbf{C}$ is non-diagonal i.e. $c_{ij} \neq 0$ for some $i \neq j$.

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{12} & c_{22} & \cdots & c_{2n} \\ & \cdots\cdot & & \\ c_{1n} & c_{2n} & \cdots & c_{nn} \end{bmatrix} . \qquad (3)$$

Modal analysis methods, which are based on the eigensystem approach, are powerful tools in studying the dynamic characteristics of such systems (see Clough (1976), Ewins (1986), Michell (1990)). However, using only the eigen-information, it is difficult to understand completely the responses and most likely to overlook the real (physical) meanings of modal parameters. In an attempt to overcome this drawback, Singh (1983), Nair and Singh (1986) suggested a way to describe the complexity of non-proportionally damped systems by introducing indices of non-proportionality. Their approach is intended to describe the nature of a structure and at the same time to estimate certain errors in the response computation. None of the indices developed so far, however, is sufficiently general for most practical applications. This problem is essentially related to the question why a non-proportionally damped system can not be decoupled in the physical domain.

If the damping is non-proportional, then not only energy dissipation but also energy transmission occurs in the vibration system. The energy transmission tangles the modes and makes system decoupling impossible in the physical domain. Based on studies of modal energy relations, Liang and Lee (1990) introduced a theory of complex damping which simultaneously deals with energy dissipation and transmission using complex Rayleigh quotients, $\mathcal{R}_i$'s

$$\mathcal{R}_i = (\mathbf{e}_i \mathbf{C} P_i)/(\mathbf{e}_i P_i) = a_i + j\, b_i \tag{4}$$

where P_i is the i^{th} eigenvector of system (2), and

$$a_i \approx 2\, \xi_i\, \omega_i \quad , \quad b_i \approx 2\, \zeta_i\, \omega_i, \tag{5}$$

with ω_i , ξ_i and ζ_i to be the i^{th} undamped natural frequency, damping ratio (energy dissipation ratio) and energy transmission ratio respectively. The complex quantity $\eta_i = \xi_i + j\, \zeta_i$, which is called the i^{th} complex damping ratio, can be used to characterize various damping effects. If **C** is proportional, then it can be diagonalized and

$$\mathcal{R}_i = 2\, \xi_i \omega_i = \text{real}, \qquad i = 1, \ldots, n. \tag{6}$$

If **C** is non-proportional, Criterion (6) does not hold. With large values of $|\zeta_i|$, great amount of energy will transfer among the system's individual modes. This value of complex damping ratio is an indicator on the degree of difficulty to approximate the responses of a non-proportionally damped system by certain proportionally damped systems. Although it is possible to develop certain other algorithms for the response computation, recent published papers on complex damping theories have not addressed this topic.

The second kind of response calculations, of which Foss (1958) was one of the earliest proponents, considers the solution of the non-proportionally damped structures in a 2n dimensional state-space. Methods generated from this perspective are precise and computer-ready, but computationally intensive. To overcome this drawback, Thompson et al (1974) suggested to calculate the response by using a diagonal n×n matrix $\mathbf{D} = \mathrm{diag}(c_{ii})$ - the diagonal entries of Equation (3), to replace matrix **C**. Following this approach, a great deal of effort was concentrated on understanding the conditions of small errors (see Foss 1958, Warburton et al 1977). But in principle, this type of methods destroys the accuracy and has been criticized by many authors. Among them, Hasselman (1976) first suggested a criterion for judging if the modal coupling can be neglected at certain level. He also pointed out that close space natural frequencies often cause accuracy problem. Warburton et al (1977) and Duncan et al (1979) recognized that errors generated in such response computing are affected by the nature of the forcing function. Yet neither group elaborated further on this observation. In the mean time, Singh (1986) and Voletsos (1986) continued in developing other modal analysis methods which remain to use 2n-dimensional space. Recently Udwadia and Esfandiari (1990) conceptually improved Warburton and Duncan's work in n-dimensional space by introducing an iterative method. The basic idea of their method can be expressed by

$$\ddot{Y}^{(k)}(t) + \mathbf{D}\dot{Y}^{(k)}(t) + \Lambda_k\, Y^{(k)}(t) = F(t) + (\mathbf{D}-\mathbf{C})\dot{Y}^{(k-1)} \qquad (7)$$

where $Y^{(k)}$ denotes the k^{th} iterative result and **D** consists of the diagonal entries of matrix **C**.

The convergence condition for (7) requires the spectral radius of matrix

$T(\omega)$ less than unity for all ω, where

$$T(\omega) = [j \omega (\Lambda_k - \omega^2 I_n + j \omega D)^{-1}](D-C). \quad (8)$$

Unfortunately, this requirement is not generally available for most engineering applications. Indeed, as the order of system increases, the chance of (7) being convergent decreases. This is shown by Theorem 2 of this report.

SECTION 3
TD RATIO AND A GENERAL ALGORITHM

As discussed in the preceding section, a non-proportionally damped system is different from a proportionally damped system essentially in the energy transmission. Although a system with energy transmission in its individual modes can never be decoupled in the n-dimensional physical space, an alternative approach in the n-dimensional physical space can be carried out by separating the damping effect into two portions: dissipation and transmission of energy. Consider the following equation

$$I_n \ddot{Y}^{(k)}(t) + \mathbf{G}\dot{Y}^{(k)}(t) + \Lambda_k Y^{(k)}(t) = F(t) - (\mathbf{C}-\mathbf{G})\dot{Y}^{(k-1)}(t) \qquad (9)$$

where **G** is a diagonal matrix with positive entries. The left side of Equation (9) is in proportional form. If we can find a proper **G** to make (9) a convergent iterative scheme, then the effect of non-proportional damping **C** can be approximated by the effect of **G** as energy dissipation factor and **C** - **G** as energy transmission factor. What will guide us to the proper **G**? Before we explore this question, let us first define a **TD** ratio

$$\gamma_i = \zeta_i / \xi_i = b_i / a_i \qquad (10)$$

which stands for the ratio of energy transmission over energy dissipation in the i^{th} mode (see (4) for the meaning of ζ and ξ , and (5) for b and a). The value of this ratio depends on the system damping and the excitation force. If we also define

$$F_S^{(k)}(t) = F(t) - (\mathbf{C}-\mathbf{G})\dot{Y}^{(k-1)}(t) \qquad (11)$$

as the pseudo force, then the **TD** ratio for each iteration of (9) can be decided by $F_s^{(k)}(t)$ and $\mathbf{G}$.

Using the complex damping theory, we can show that if there is an iterative system (9) with sufficiently small **TD** ratios for each of its iterations, then the response of non-proportionally damped system (2) can be obtained by iterating certain responses of system (9).

Let $\mathbf{G}$ in (9) be the diagonal of matrix $\mathbf{C}$ and subtract (9) from (2), we have

$$\ddot{\eta}^{(k)}(t)+\mathbf{D}\dot{\eta}^{(k)}(t)+\Lambda_k\eta^{(k)}(t) = (\mathbf{D}-\mathbf{C})\dot{\eta}^{(k-1)}(t) \qquad (12)$$

where $\eta^{(k)}(t) = X(t) - Y^{(k)}(t)$. Applying Fourier transform to (12) and denote $\mathscr{F}(\eta^{(k)}(t)) = \hat{\mu}^{(k)}(\lambda)$, we have

$$\hat{\mu}^{(k)}(\lambda) = \mathbf{T}(\lambda)\hat{\mu}^{(k-1)}(\lambda) \qquad (13)$$

where $\mathbf{T}(\lambda)$ is the same formula given in (8) except ω being replaced by λ. Now consider a special case that $\rho(\mathbf{T}(\lambda_i)) = 1$, where λ_i is the i^{th} eigenvalue of system (2). Let P_i be the associated eigenvector for λ_i.

$$\mathbf{T}(\lambda_i)P_i = P_i \qquad (14)$$

Without loss of generality, we calculate the first row of the left hand side of Equation (13), which is

$$-\left[j\lambda_i p_{ii} / \, (k_1 + \lambda_i^2 + jc_{11})\right]\left[(\ 0,\ c_{12},\ c_{13},\ \dots\ c_{1n}) \begin{Bmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{1n} \end{Bmatrix} / p_{11}\ \right].$$

It can be easily shown that the imaginary part in the second square bracket equals b , that is,

$$- \mathrm{Im}\left[(0,\ c_{12},\ c_{13},\ \ldots,\ c_{1n}) \begin{Bmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{1n} \end{Bmatrix} / \ p_{11} \right] = b_i \ . \tag{15}$$

Equation (15) implies that b_i is independent of the choice of **G**. However, by increasing the entries of **G**, the **TD** ratios can be lowered, which eventually leads to a convergent scheme.

In order to find a diagonal matrix **G** to suppress the TD ratio γ, there remains the following questions: How to choose the shape of matrix **G**? Does it possess a simple form? In a later section we will show that the shape of matrix **G** does not influence the convergence of (9), as long as its norm is sufficiently large. Further we will see that by choosing the following

$$\mathbf{G} = g\mathbf{D} \tag{16}$$

with proper scalar g, we may achieve an optimal algorithm.

Denote the largest and smallest eigenvalues of $\mathbf{D}^{-1}\mathbf{C}$ by σ_{max} and σ_{min} respectively. Let

$$g = (\sigma_{max} + \sigma_{min})/2. \tag{17}$$

We now propose our general iterative algorithm in the following block diagram, Figure 3-1.

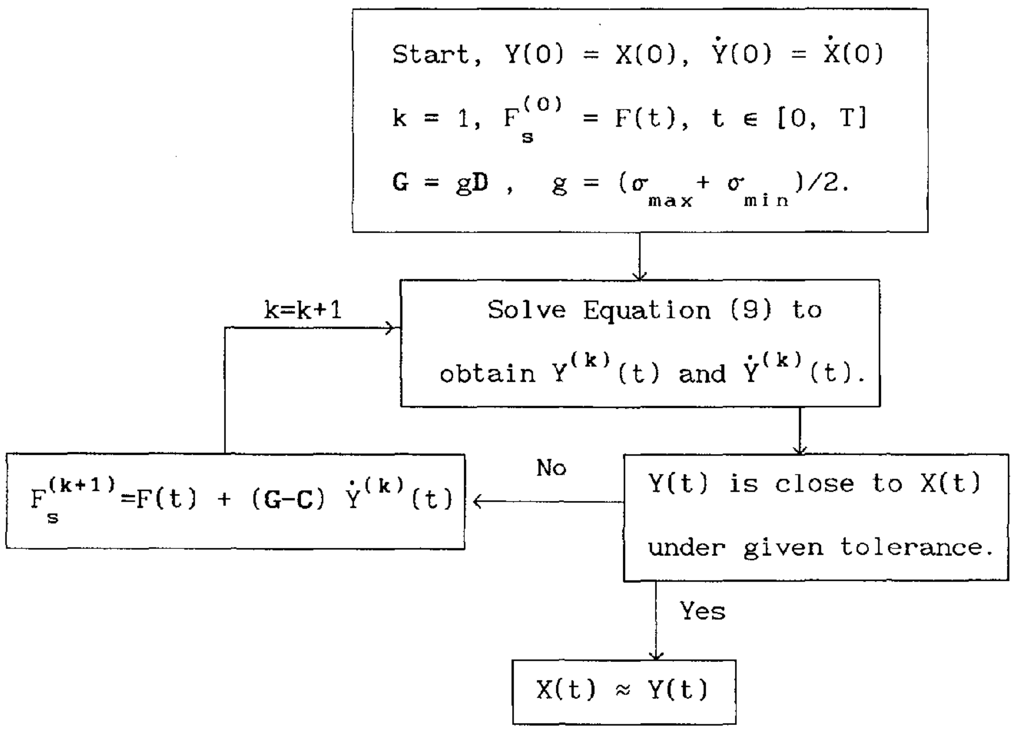


FIGURE 3-1 Block Diagram of a General Algorithm

SECTION 4
CONVERGENCE AND OPTIMAL CONVERGENT SPEED

The iterative equation of the general algorithm can be written as

$$\mathbf{I}_n\ddot{Y}^{(k)}(t) + \mathbf{G}\dot{Y}^{(k)}(t) + \Lambda_k Y^{(k)}(t) = F(t) - (\mathbf{C}-\mathbf{G})\dot{Y}^{(k-1)}(t)$$

where $t \in [0, T]$ and $\mathbf{G}$ is a positive definite diagonal matrix, with the following initial conditions

$Y^{(k)}(0) = x_0 \qquad k = 1, 2, \ldots$

$\dot{Y}^{(k)}(0) = y_0 \qquad k = 1, 2, \ldots$

$\dot{Y}^{(k)}(t) = 0 \qquad t \in [0, T] \quad T < \infty.$

We discuss the convergence of this scheme through an analysis of its Fourier transform. First, we subtract the above equation from the equation

$$\mathbf{I}_n\ddot{X}(t) + \mathbf{G}\dot{X}(t) + \Lambda_k X(t) = F(t) - (\mathbf{C}-\mathbf{G})\dot{X}(t) \qquad t \in [0,T], \tag{18}$$

with initial conditions

$X(0) = x_0,$

$\dot{X}(0) = y_0.$

Let $\eta^{(k)}(t) = X(t) - Y^{(k)}(t)$ for $t \in [0,T]$. We have

$$\mathbf{I}_n\ddot{\eta}^{(k)}(t) + \mathbf{G}\dot{\eta}^{(k)}(t) + \Lambda_k \eta^{(k)}(t) = F(t) - (\mathbf{C}-\mathbf{G})\dot{\eta}^{(k-1)}(t) \tag{19}$$

with

$\eta^{(k)}(0) = 0, \qquad k = 1, 2, \ldots$

$\dot{\eta}^{(k)}(0) = 0, \qquad k = 1, 2, \ldots$

$\dot{\eta}^{(0)}(t) = \dot{X}(t) - \dot{Y}^{(0)}(t) = \dot{X}(t)$.

Here $\eta^{(k)}(t)$ is only defined on [0, T]. In order to perform Fourier integration to (19), we have to extend it to $[0, \infty)$. Note that H(t) is a twice differentiable function on [0, T]. Also the behavior of F(t) beyond T is not of our concern. We therefore may extend H(t) by a smooth function between [T, 2T] such that it satisfies $F(2T) = \dot{F}(2T) = 0$. Furthermore, $F(t) = \dot{F}(t) = 0$ beyond 2T. With a positive definite damping matrix C and the above defined F(t) given on $[0, \infty)$, we see that both X(t) and $Y^{(k)}(t)$ will decay to zero exponentially. Thus $\eta^{(k)}(t) \longrightarrow 0$ as $t \longrightarrow \infty$ and the Fourier integral

$$\hat{\mu}^{(k)}(\lambda)=\int_0^{\infty}\eta^{(k)}(t)e^{j\lambda t}dt$$

can now be performed since the function $\eta^{(k)}(t)$ is defined from 0 to ∞. Now applying Fourier transform to (19), we obtain

$$(-\lambda^2 I_n - j\lambda G + \Lambda_k)\hat{\mu}^{(k)}(\lambda)= j\lambda(C-G)\hat{\mu}^{(k-1)}(\lambda)$$

and
$$\hat{\mu}^{(k)}(\lambda)=\left[(G - j\lambda I_n + j\lambda^{-1}\Lambda_k)^{-1}(C-G)\right]^k \hat{\mu}^{(0)}(\lambda)$$

Denote $U^{-1}(\lambda) = (G - j\lambda I_n + j\lambda^{-1}\Lambda_k)^{-1}$ and $S = (C-G)$. Then we have

$$\hat{\mu}^{(k)}(\lambda) = \left[U^{-1}(\lambda)S\right]^k \hat{\mu}^{(0)}(\lambda).$$

These definitions will be used in many places in the following derivations.

Lemma 1. Suppose $\eta^{(k)}(t)$ is continuous. Then $\eta^{(k)}(t) \longrightarrow 0$ for all $t \in [0, \infty)$ implies $\hat{\mu}^{(k)}(\lambda) \longrightarrow 0$ for all $\lambda \geq 0$.

Proof. According to the description of $\eta^{(k)}(t)$ given before, we can find a number b such that

$$\int_b^{\infty} \eta^{(k)}(t) e^{j\lambda t} dt \longrightarrow 0, \quad \text{for all } \lambda > 0, \quad \text{as } k \longrightarrow \infty.$$

Since we also know that $\eta^{(k)}(t)$ is continuous and $\eta^{(k)}(t) \longrightarrow 0$ for all t, as $k \longrightarrow \infty$, it follows that $\eta^{(k)}(t) \longrightarrow 0$ on $[0, b]$ uniformly. Thus

$$\int_0^{b} \eta^{(k)}(t) e^{j\lambda t} dt \longrightarrow 0, \text{ for all } \lambda, \text{ as } k \longrightarrow \infty.$$

Adding the two integrals together, we have $\hat{\mu}^{(k)}(\lambda) \longrightarrow 0$ for all $\lambda > 0$.

Lemma 2. Let $\rho(U^{-1}(\lambda)S)$ be the spectrum radius of matrix $U^{-1}(\lambda)S$. Suppose $\rho(U^{-1}(\lambda)S) < 1$ for all $\lambda > 0$. Then $\eta^{(k)}(t) \longrightarrow 0$ for almost every $t \geq 0$.

Proof. It is easy to see that for any $\varepsilon > 0$, there is a C_ε such that

$$\int_{C_\varepsilon}^{\infty} |\hat{\mu}^{(k)}(\lambda) e^{jt\lambda}| d\lambda \leq \int_{C_\varepsilon}^{\infty} |[U^{-1}(\lambda)S]^k \hat{\mu}^{(0)}(\lambda)| d\lambda < \varepsilon \text{ for sufficiently large k.}$$

For $\lambda \in [0, C_\varepsilon]$, we have $[U^{-1}(\lambda)S]^k \hat{\mu}^{(0)}(\lambda) \longrightarrow 0$ as $k \longrightarrow \infty$, because $\rho(U^{-1}(\lambda)S) < 1$. Thus

$$\lim_{i \longrightarrow \infty} \int_0^{C_\varepsilon} |\hat{\mu}^{(k)}(\lambda) e^{jt\lambda}| d\lambda = 0$$

It follows that

$$\eta^{(k)}(t) \underset{a.e.}{=} \int_0^\infty \hat{\mu}^{(k)}(\lambda)e^{jt\lambda}d\lambda \longrightarrow 0, \text{ as } k \longrightarrow \infty.$$

Lemma 3. Let C be a positive definite matrix. Then

$$\rho(U^{-1}(\lambda)S) \le \rho(GS)$$

(This is essentially the same Lemma 3 in Udwadia and Esfandiari (1990)).

Proof. Suppose $|\sigma_0|$ is the maximal eigenvalue of $U^{-1}(\lambda)S$ in absolute value. If we take X to be the eigenvector associated with σ_0, then

$$X^H SX = \sigma_0 X^H T^{-1}(\lambda)X = \sigma_0 \left[X^H GX + jX^H(\lambda I_n - \frac{\Lambda k}{\lambda})X\right]$$

Thus

$$|\sigma_0| = \frac{|\ X^H SX\ |}{|\ X^H GX + jX^H(\lambda I_n - \frac{\Lambda k}{\lambda})X\ |} \le \frac{|\ X^H SX\ |}{|\ X^H GX\ |} .$$

On the other hand, the maximal eigenvalue σ_0 of $G^{-1}S$ satisfies

$$|\delta_0| = \underset{X}{\text{Max}} \frac{|\ X^H SX\ |}{|\ X^H GX\ |}.$$

It follows that $|\sigma_0| \le |\delta_0|$.

Let $\|A\| = \underset{z}{\text{Max}} \frac{z^H A^H A\ z}{z^H z}$. It can be checked directly that the following properties are valid for the norm so defined.

(1). $\| \mathbf{A} + \mathbf{B} \| \le \|\mathbf{A}\| + \|\mathbf{B}\|$.

(2). $\|\mathbf{AB}\| \le \|\mathbf{A}\|\|\mathbf{B}\|$.

(3). If $\mathbf{A}$ is symmetric, then $\rho(\mathbf{A}) = \|\mathbf{A}\|$.

Lemma 4. Suppose **G**, **C** are defined as before. Then $\rho(G^{-1}C) \le \rho(G^{-1})\rho(C)$.

Proof. Since G^{-1} is positive definite and $G^{-1}C$ is similar to $R^{-1/2}CR^{-1/2}$, we have $\rho(G^{-1}C) = \rho(G^{-1/2}CG^{-1/2})$. $G^{-1/2}CG^{-1/2}$ is symmetric. Thus $\rho(G^{-1/2}CG^{-1/2}) = \| G^{-1/2}CG^{-1/2}\| \le \| G^{-1/2}\| \|C\| \| G^{-1/2}\| = \rho(G^{-1/2})\rho(C)\rho(G^{-1/2}) = \rho(G^{-1})\rho(C)$.

Theorem 1. There is a positive definite diagonal matrix G such that

$$\rho(G^{-1}S) < 1 .$$

Proof. Since $S = C - G$, it follows that S is negative definite when the norm of G is sufficiently large. We consider the positive definite matrix

$$-G^{-1}S = G^{-1}(G - C) \simeq G^{-1/2}(G-C)G^{-1/2}.$$

Let θ be an eigenvalue of $G^{-1/2}(G-C)\bar{G}^{1/2}$ and X be the associated eigenvector. Then

$$G^{-1/2}(G-C)G^{-1/2}X = \theta X .$$

$$X^H X - X^H G^{-1/2}CG^{-1/2}X = \theta X^H X .$$

Since C is positive,

$$\theta = 1 - \frac{X^H G^{-1/2}CG^{-1/2}X}{X^H X} < 1.$$

On the other hand, using Lemma 4, we have

$$1-\lambda = \frac{X^H G^{-1/2}CG^{-1/2}X}{X^H X} \le \|G^{-1/2}CG^{-1/2}\| = \rho(G^{-1/2}C) \le \rho(G^{-1/2})\rho(C).$$

As $\rho(\mathbf{C})$ is a constant and $\rho(\mathbf{G}^{-1}) = \max_{1\le i\le n} \{\frac{1}{g_i}\}$, we have

$$\lim_{g_i \to \infty} \frac{\rho(\mathbf{C})}{g_i} = 0, \text{ for all } i = 1, 2, \ldots n.$$

Thus there is a $\mathbf{G} = \mathrm{diag}(g_1, g_2, \ldots g_n)$ such that

$$1 - \theta \le \max_{1\le i\le n} \{ \frac{\rho(\mathbf{C})}{g_i} \} < 1.$$

So $0 < \theta < 1$, and $\rho(\mathbf{G}^{-1}\mathbf{S}) < 1$.

Theorem 2. Suppose the system has the order n. Then

$$1 \le \rho(\mathbf{D}^{-1}\mathbf{C}) \le n$$

where $\mathbf{D} = \mathrm{diag}(\mathbf{C})$.

Proof. We consider $\mathbf{D}^{-1/2}\mathbf{C}\mathbf{D}^{-1/2}$, which is similar to $\mathbf{D}^{-1}\mathbf{C}$. Since $\mathbf{C}$ is positive definite and $\mathbf{D} = \mathrm{diag}(\mathbf{C})$, we have $c_{ij} \le c_{ii}c_{jj}$. Thus

$$\mathbf{D}^{-1/2}\mathbf{C}\mathbf{D}^{-1/2} = [\ a_{ij}]_{n\times n}$$

where $a_{ij} = c_{ij} / \sqrt{c_{ii}c_{jj}} \le 1$.

By using Gerschgorin Disk Theorem (see Inman, 1989 for instance), the eigen values of this matrix satisfy

$$|\sigma-1| \le \sum_{\substack{j=1\\ j\ne i}}^{n} c_{ij}/ \sqrt{c_{ii}c_{jj}} \le \sum 1 = n-1.$$

It follows that $\sigma \le n$. Hence $\rho(\mathbf{D}^{-1}\mathbf{C}) \le n$. Also

$$\mathrm{Trace}(\mathbf{D}^{-1/2}\mathbf{C}\mathbf{D}^{-1/2}) = \sum_{i=1}^{n} a_{ii} = n = \sum_{i=1}^{n} \zeta_i .$$

Therefore, at least one of the eigenvalues is greater than 1. So $\rho(D^{-1}C) \geq 1$.

The above result provides a gauge on the degree of the complexity of dynamic systems with respect to the convergence speed of the proposed method. According to Udwadia and Esfandiari (1990), if a system has $\rho(D^{-1}C) < 2$, then their algorithm will converge. Using the proposed method, we can obtain convergent iterative solutions for all systems. However, the convergent speed varies. This can be seen from the examples presented in the next section.

Lemma 5. Suppose $G = \text{diag}(g_1, g_2, \dots g_n)$ is positive definite. Let $C_1 = GCG$ and $D_1 = \text{diag}(C_1)$, the diagonal matrix of C_1. Then $D_1^{-1}C_1 \simeq D^{-1}C$. In particular, $\rho(D_1^{-1}C_1) = \rho(D^{-1}C)$.

Proof. Since G is diagonal and $C_1 = GCG$, we have

$$D_1 = \text{diag}(C_1) = \text{diag}(m_1^2 c_{11},\ m_2^2 c_{22},\ \dots\ m_n^2 c_{nn}) = GDG.$$

It follows that

$$D_1^{-1}C_1 \ = \ G^{-1}D^{-1}G^{-1}GCG = G^{-1}D^{-1}CG \simeq DC.$$

Hence $\rho(D_1^{-1}C_1) = \rho(D^{-1}C)$.

Theorem 3. The convergence speed provided by matrix gD described in the previous section is invariant of **L** operation. (i.e. **LCL** with **L** a positive definite diagonal matrix).

Proof. From Lemma 5, we know $[\text{diag}(LCL)]^{-1}[LCL] \simeq [\text{diag}(C)]^{-1}[C]$. The two matrices have the same eigenvalues $\sigma_1, \sigma_2, \dots \sigma_n$. Thus

$$g = \frac{\mathrm{Max}(\sigma_i) + \mathrm{Min}(\sigma_i)}{2}$$

can be obtained from both **LCL** and **C**. So g is invariant of **L** operation.

Theorem 4. Let $\bar{\mathbf{G}} = g\mathbf{D}$ with the same g as mentioned above. Then the iterative scheme

$$I_n \ddot{\eta}^{(k)}(t) + \bar{G}\dot{\eta}^{(k)}(t) + \Lambda_k \eta^{(k)}(t) = H(t) - (C-\bar{G})\dot{\eta}^{(k-1)}(t)$$

converges. And for any number n > 0, replacing $\bar{\mathbf{G}}$ by n**D** will result in a slower or even divergent scheme.

Proof. We examine the spectral radius of $\mathbf{I} - \bar{\mathbf{G}}^{-1/2}\mathbf{C}\bar{\mathbf{G}}^{-1/2}$. Let σ_{min} σ_{max} be the smallest and largest eigenvalues of $\mathbf{D}^{-1/2}\mathbf{C}\mathbf{D}^{-1/2}$ where $\mathbf{D}$ = diag(**C**). Then the minimum and maximum eigenvalues of $\bar{\mathbf{G}}^{-1/2}\mathbf{C}\bar{\mathbf{G}}^{-1/2}$ satisfy

$$0 < g\sigma_{min} = \frac{2\sigma_{min}}{\sigma_{min} + \sigma_{max}} < 1$$

$$1 < g\sigma_{max} = \frac{2\sigma_{max}}{\sigma_{min} + \sigma_{max}} < 2 .$$

Thus $\rho\ (\mathbf{I} - \bar{\mathbf{G}}^{-1/2}\mathbf{C}\bar{\mathbf{G}}^{-1/2}) < 1$.

It is easy to see that $1 - m\sigma_{min} = m\sigma_{max} - 1$. Since

$$\rho(\mathbf{I} - n\mathbf{D}^{-1/2}\mathbf{C}\mathbf{D}^{-1/2}) = \max \{ 1 - n\sigma_{min},\ n\sigma_{max} - 1 \},$$

if we do not take n equal to g, then a larger $\rho(\mathbf{I} - n\mathbf{D}^{-1/2}\mathbf{C}\mathbf{D}^{-1/2})$ will result. Thus $\bar{\mathbf{G}}$ is optimal.

This last result shows that the chosen value of the quantity g in the general algorithm is optimal.

SECTION 5

THE ANALYSIS OF ROUND-OFF ERRORS

From the previous considerations of convergence, it is shown that we can have theoretically convergent solutions of Eq.(1) by using the iterative algorithm described in Figure 3-1. This result is obtained when there is no computational errors or the so-called round-off errors involved. In general, however, these errors do exist. In fact, when a large number of iterative steps are required for a solution, the round-off errors are usually not negligible. This is especially true in ill-conditioned situations. In the next section, a few examples will be given to illustrate the problems involving computational errors.

In the remainder of this section, we consider the computational procedure with the presence of round-off errors. The iterative equation may now be written as

$$I_n \ddot{Y}^{(k)} + G\,\dot{Y}^{(k)} + \Lambda_k Y^{(k)} = F - (C-G)\dot{Y}^{(k-1)} + \gamma^{(k)}$$

Similarly, Equation (18) can be written as

$$I_n\,\ddot{X} + G\,\dot{X} + \Lambda_k X = F + (C-G)\,\dot{X} + \beta$$

where $\gamma^{(k)}$ and β are computational round-off errors in the k^{th} iteration and in the calculation of (18), respectively. In such a case, Equation (19) becomes

$$\ddot{\eta}^{(n)} + G\,\dot{\eta}^{(n)} + \Lambda_k \eta^{(n)} = (G-D)\,\dot{\eta}^{(n-1)} + \alpha^{(n)}$$

where $\alpha^{(n)} = \beta - \gamma^{(n)}$. It is easy to see that the following equation

(20) holds,

$$\hat{\mu}^{(n)} = \mathbf{V}^{n} \hat{\mu}^{(0)} + \mathbf{V}^{n-1} \mathbf{T} \alpha^{(1)} + \ldots + \mathbf{T} \alpha^{(n)} \qquad (20)$$

where $\mathbf{T} = [i\lambda \mathbf{G} + \Lambda_{\mathbf{k}} - \mathbf{I}\lambda^{2}]^{-1}$and $\mathbf{V} = [\mathbf{U}^{-1}(\lambda)\mathbf{S}]$ ($\mathbf{U}$ and $\mathbf{S}$ are defined in the preceding section).

In Equation (20), when $\mathbf{V}^{n} \rightarrow 0$ rapidly, we can obtain the convergent solution. However, with the presence of round-off errors, we have

$$\hat{\mu}^{(n)} \rightarrow \sum_{i=1}^{n} \mathbf{V}^{n-i}\mathbf{T}\alpha^{(i)} \neq 0 \qquad (21)$$

which shows that we may not obtain the exact convergent solution.

The above inequality suggests that large errors may result if the value of any one of $\mathbf{V}$, $\mathbf{T}$ or $\alpha^{(i)}$ is large. To reduce large round-off errors, we consider three possibilities. First, we try to reduce the spectrum radius of matrix $\mathbf{V}$ to as small a value as possible. Since $\mathbf{V} = [\mathbf{U}^{-1}\mathbf{S}]$, we need

$$\|\mathbf{U}\| = \| \mathbf{G} - j\lambda \mathbf{T}_{n} + j\lambda^{-1}\Lambda_{k} \| \qquad (22)$$

to have large value. At the same time we must also keep the value of $\|\mathbf{S}\| = \| \mathbf{C} - \mathbf{G}\|$ small. From the consideration of value of $\|\mathbf{U}\|$, a $\mathbf{G}$ with large value of $\|\mathbf{G}\|$ should be selected. From the consideration of the value of $\|\mathbf{S}\|$, a $\mathbf{G}$ close to $\mathbf{C}$ is required. These two conditions conflict with each other at a certain time. Therefore, reducing round-off errors from this approach can only have limited success. The second approach to improve the round-off errors is to reduce the value of the term

$$\| T \| = \| [i\lambda G + \Lambda_k - I\lambda^2]^{-1} \| . \quad (23)$$

It can be seen that in order to reduce the value of the norm of matrix **T**, a **G** with large $\|G\|$ value is preferred. On the other hand, the choice of a proper matrix **G** directly influences the convergent speed. This implies that there is not much room in varying **G** to reduce the round-off errors. The third approach is to reduce the round-off error $\alpha^{(i)}$ in each iteration. This approach is obviously helpful but often requires more computations and computer memory space. In terms of algorithm design, small time steps are needed when large round-off errors are confronted. In any case, the reduction of round-off errors remain to be an important research area of numerical methods for non-proportionally damped linear systems. The authors are still working on this issue.

SECTION 6

NUMERICAL EXAMPLES

The following example shows the various convergent speeds that may be achieved by using different **G** in the general iterative algorithms described in this report.

Example 1. Suppose we have a non-proportionally damped system

$$\mathbf{M}\ddot{X}(t) + \mathbf{C}\dot{X}(t) + \mathbf{K}X(t) = 0$$

where

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 2.40 & -1.98 & 1.32 \\ -1.98 & 4.04 & -1.56 \\ 1.32 & -1.56 & 3.18 \end{bmatrix}, \text{ and } \mathbf{K} = \begin{bmatrix} 15.7 & 0. & 0. \\ 0. & 65.3 & 0. \\ 0. & 0. & 214.8 \end{bmatrix}.$$

Using iterative algorithm (9) with the given **G**, we have

	1	**2**	**3**
$\mathbf{G} = m \times$	$\begin{bmatrix} 2.40 & & \\ & 4.04 & \\ & & 3.18 \end{bmatrix}$	$\begin{bmatrix} 3.84 & & \\ & 3.84 & \\ & & 3.84 \end{bmatrix}$	$\begin{bmatrix} 1.75 & & \\ & 6.28 & \\ & & 0.98 \end{bmatrix}$
Optimal m-value	1.23	1.18	2.26
Optimal spectrum radius	0.69	0.73	0.87
Errors after 10 steps	2%	5%	20%

Figure 6-1 shows the convergent curves under three different **G** matrices. The segments of the curves below the horizontal line of ordinate one describe how the convergent speeds of these iterative schemes vary with

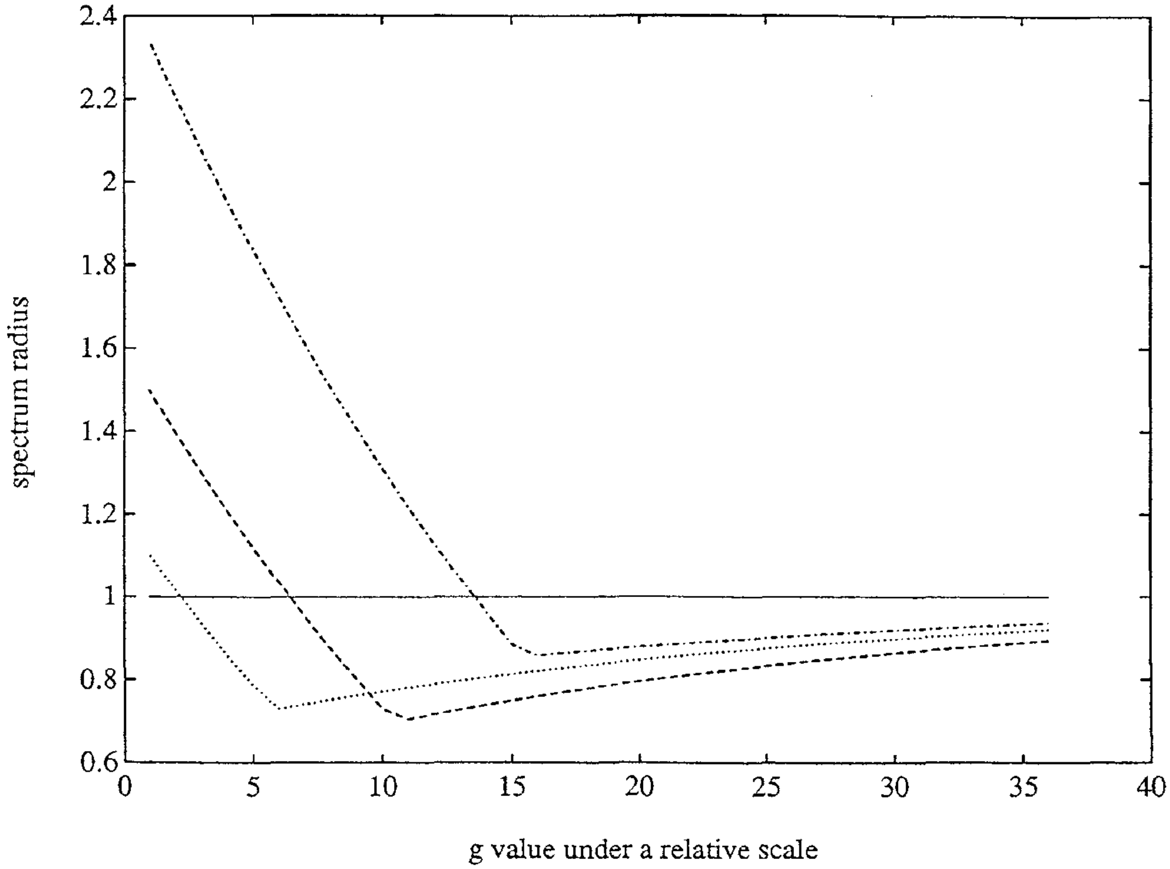


FIGURE 6-1 Convergence Curves

respect to different g values. It is seen that when g is sufficiently large, all these schemes converge. However, the curves approach the asymptotic value of one as g increases. This means that without choosing g properly, we may have to sacrifice the convergent speed rather considerably.

Under the excitation F = [1, 0, 0], we computed the response of the system by using the algorithm described in Figure 3-1 with 10 iterations. Figure 6-2(a), (b) and (c) show the iterated results of the displacements of the first, second and third lumped masses of the system. In order to examine the accuracy of the above results, corresponding analytical solutions are also calculated (see the solid curves in Figure 6-2(a), (b) and (c)), by using Foss's state space method (note that Foss's method only works in 2n dimensional state space. Although it can produce accurate results, it needs more computations than n dimensional physical space methods and is therefore not suitable for large DOF systems). Summarizing the numerical results presented in Figure 6-2 (a), (b) and (c), two comments can be offered. First, the proposed iteration scheme converges and produces solutions that correspond to the results calculated by using Foss's method (1958). Secondly, since the system is loaded at the first lumped mass with unit impulse, the displacement of the first mass vibrates with the largest amplitude. Under such circumstance, the iterated solution and the analytical solution for the first equation of the system are the closest pair among the three pairs. This phenomena has been observed in all other examples given in this report. The reason is that when the vibration amplitude is very small at certain mass point, round-off errors become a severe problem. In this example, when the time step is

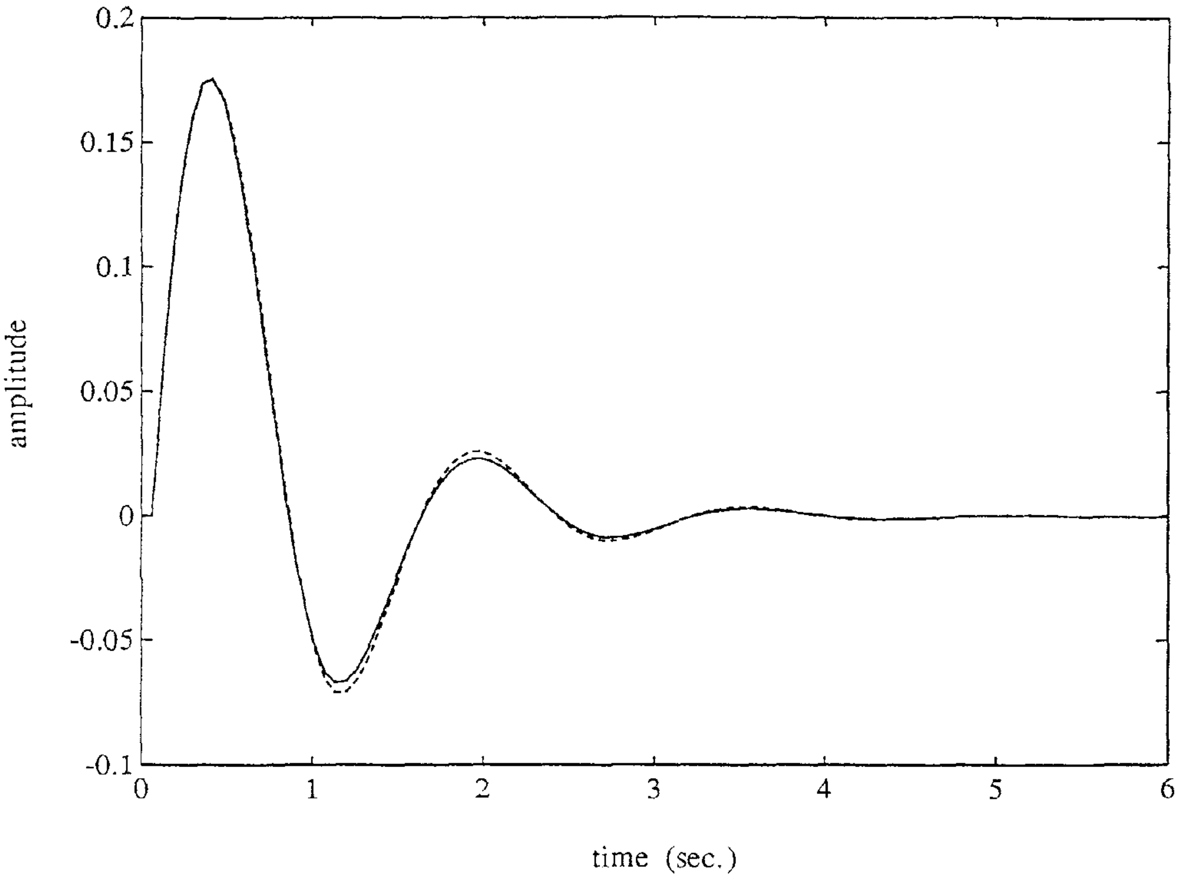


FIGURE 6-2a Analytical and Iterated Solutions, x_1

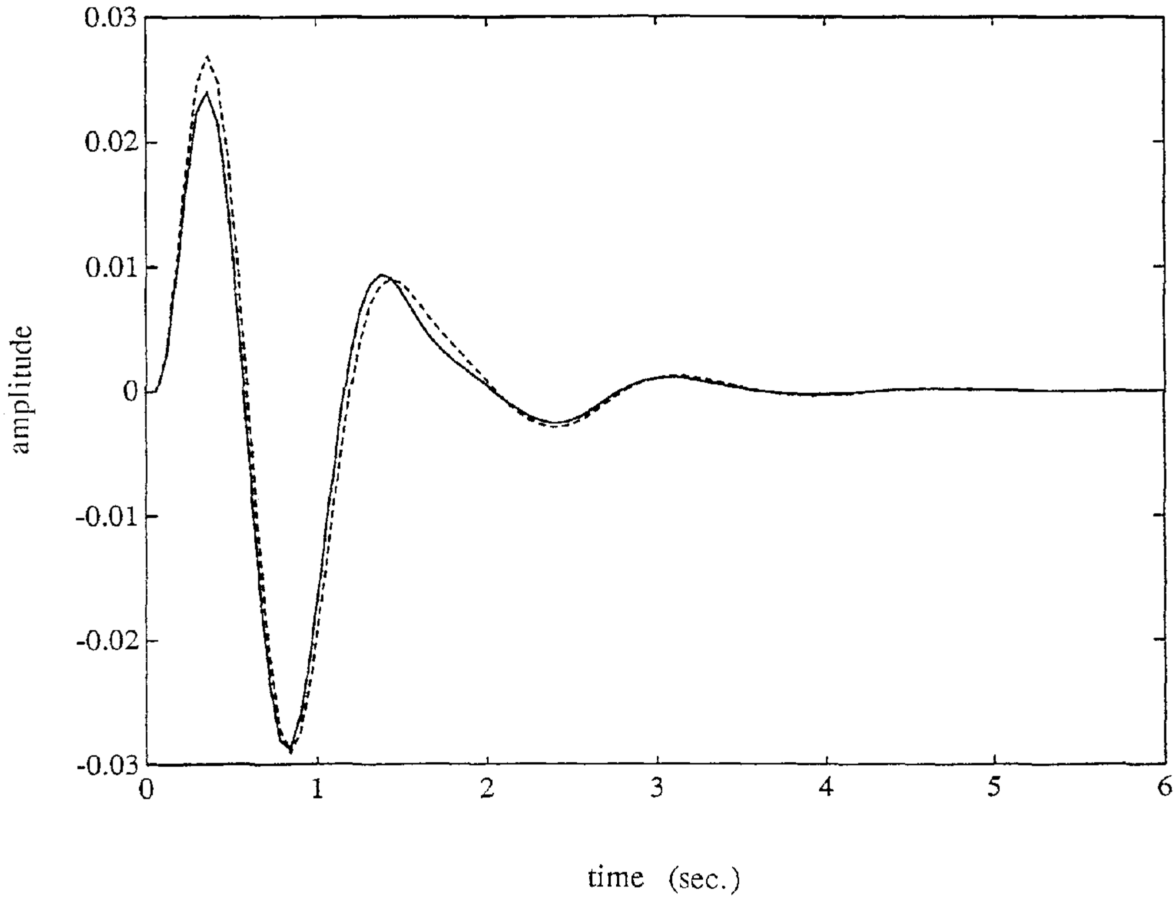


FIGURE 6-2b Analytical and Iterated Solutions, x_2

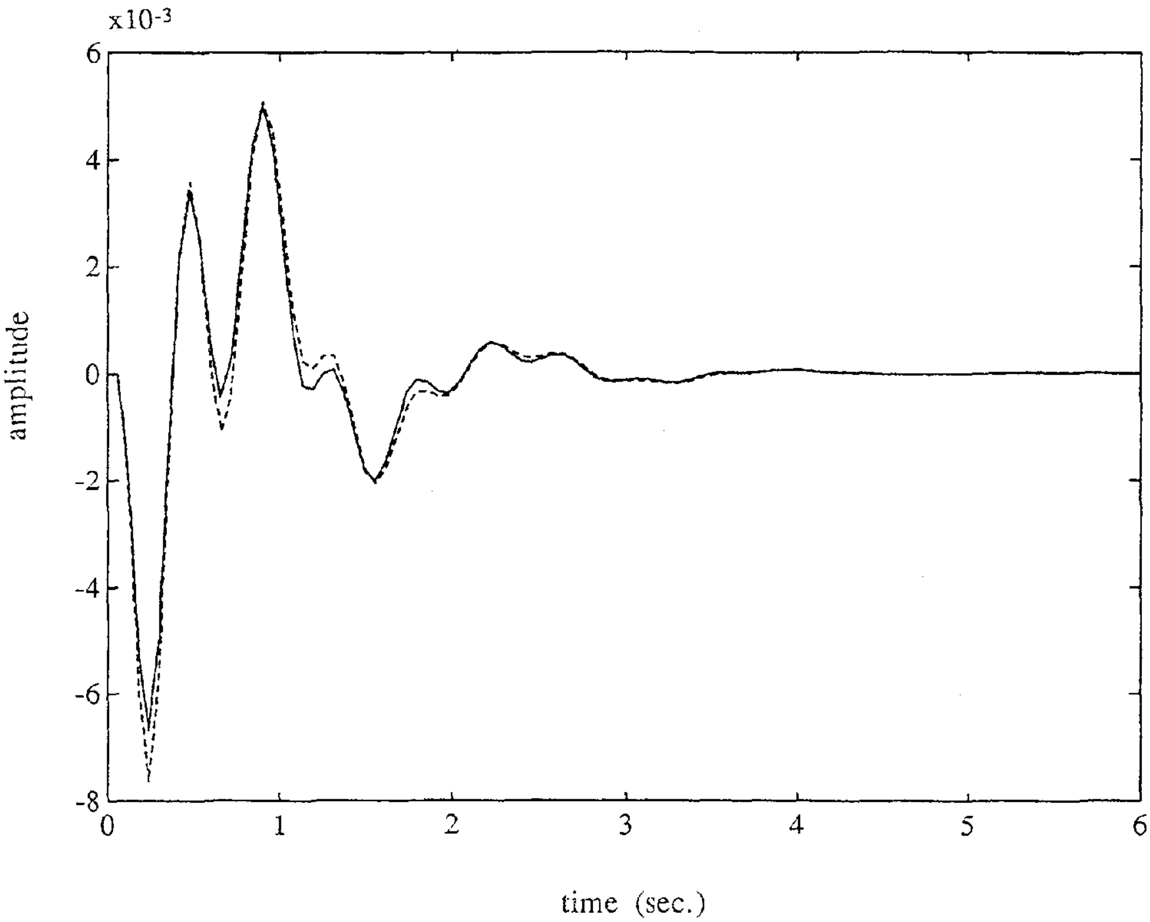


FIGURE 6-2c Analytical and Iterated Solutions, x_3

set to a smaller scale (0.1 sec.), the round-off errors are significantly reduced. Some results in this regard are compared in Figure 6-3(a), (b), (c), where the dotted curves correspond to the improved solutions.

Example 2. In this example, we compare our algorithm with the algorithm suggested by Udwadia and Esfandiari (1990). The system under consideration is 4-DOF with $\mathbf{M} = \mathbf{I}_4$,

$$\mathbf{C} = \begin{bmatrix} 1.3610 & 1.3202 & 1.1658 & 1.7105 \\ & 1.3413 & .9939 & 1.6969 \\ & & 1.4569 & 1.3592 \\ & & & 2.3544 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 40.0 & & & \\ & 40.1 & & \\ & & 40.2 & \\ & & & 40.3 \end{bmatrix}$$

This system has natural frequencies

6.3406, 6.3390, 6.3380 and 6.3279

and complex damping ratios

0.0424 - 0.0012j, 0,0075 - 0.0002j, 0.4634 + 0.0008j, 0.0006 + 0.0005j

The spectrum radius of matrix $(\mathbf{D}^{-1}\mathbf{C})$ is 3.5904. This result indicates that the system does not satisfy the sufficient condition of convergence for Udwadia and Esfandiari method. In Figure 6-4(a), an excitation

$$F = [\ \sin 2t,\ 0.7\sin 5t,\ 0,\ 0\ ]$$

is given. Figure 6-4(b), (c), (d) and (e) show the comparisons between the solutions by the proposed method presented in this report and the one

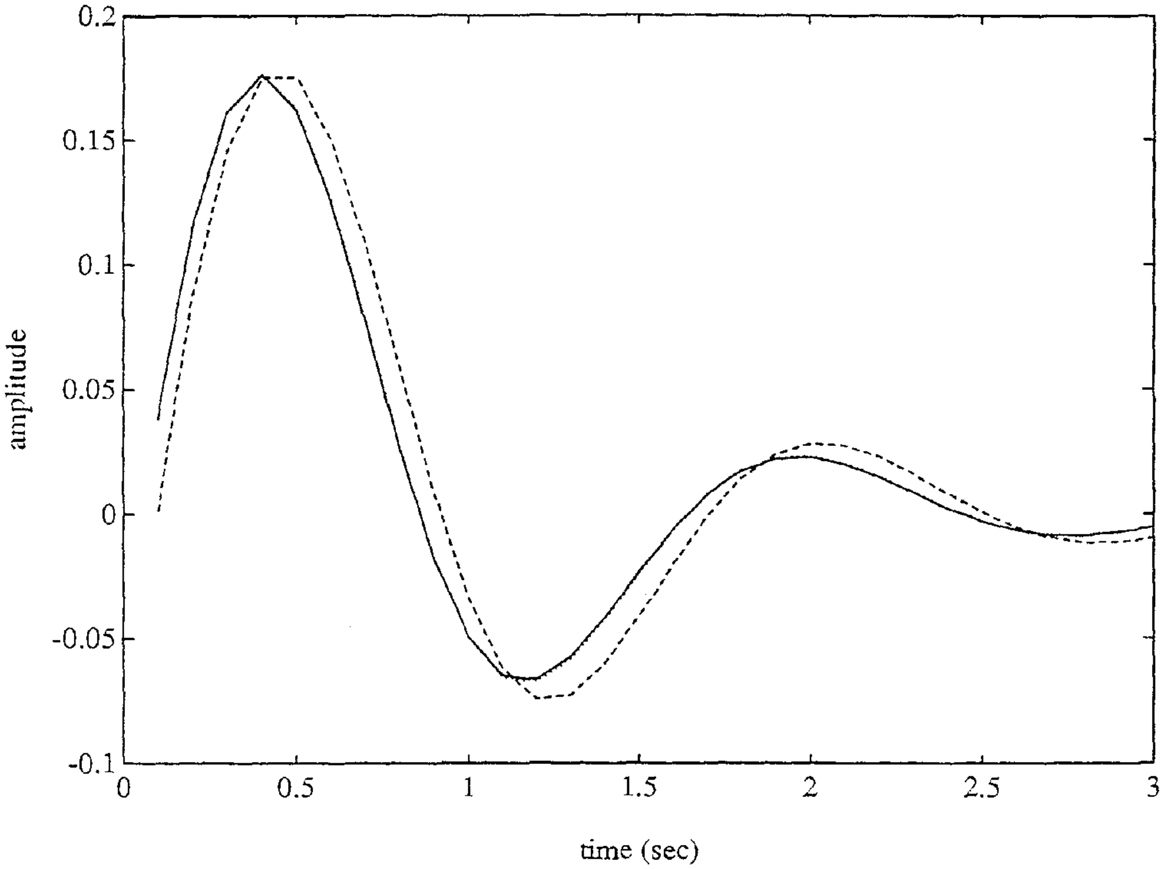


FIGURE 6-3a Improvement for Round-off Errors, x_1

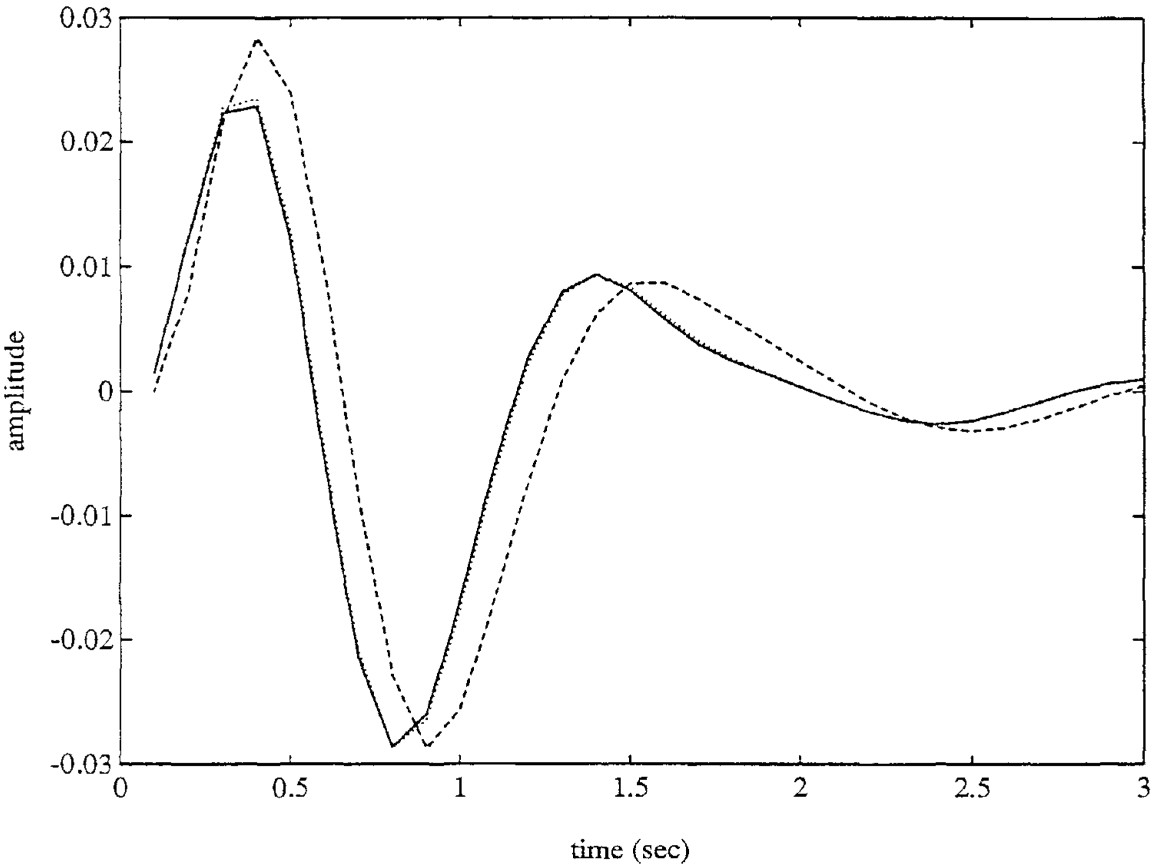


FIGURE 6-3b Improvement for Round-off Errors, x_2

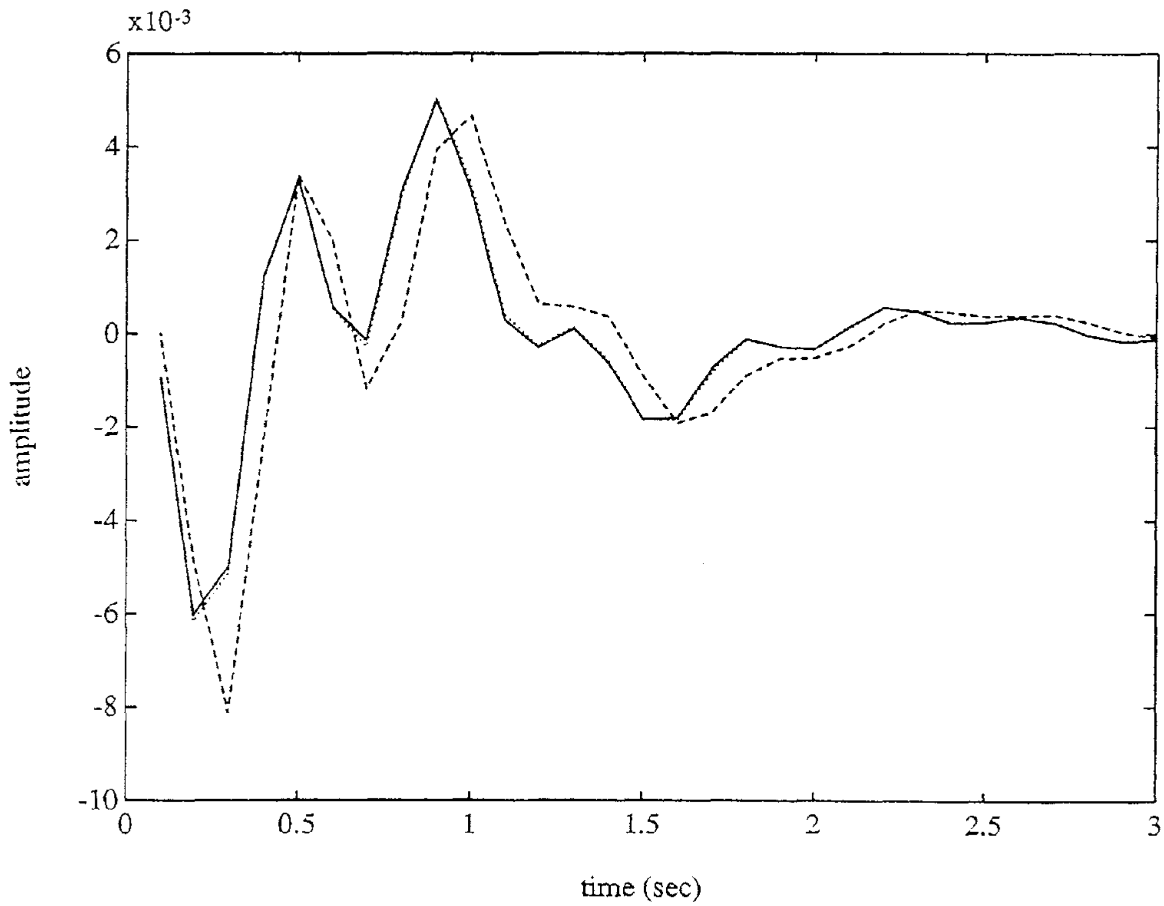


FIGURE 6-3c Improvement for Round-off Errors, x_3

suggested by Udwadia and Esfandiari. The dash curves in the upper frames of Figure 6-4(b), (c), (d) and (e) correspond to the results from Udwadia and Esfandiari method after 10 iterations. Since the results diverge away from the analytical solutions (the solid curves), the scales are set relatively larger than that in the lower frames. It can be seen from the curves in the lower frames of Figure 6-4(b), (c), (d) and (e), that the iterated solutions (dash curves) based on the method proposed in this report agree well to the analytical solutions (solid curves) within a tolerance of 10^{-2}. In this example, five iterations were carried out for the convergent solutions. It is noted that visible errors still exist in Figure 6-4(c) and (d). For large DOF systems, the convergent speeds are expected to be slower. Hence more iterations are necessary. As a comparison, if we take radius $(D^{-1}C) < 2$ as a criterion for the convergence of Udwadia and Esfandiari method, then by Theorem 2 there is only 100/(n-1)% chance that the method will converge, while the method proposed here converges unconditionally.

Example 3. In the third example, the response of a real structure (see Figure 6-5(a) is computed. This steel rigid frame has five floors with a total height of 23.1m (see further details in George Chao-chih Yao (1991)). If east-west, south-north and torsional motions are considered, the structure has 18-DOF. The spectrum radius of matrix $\mathbf{D}^{-1}\mathbf{C}$ is 2.9433.

Figure 6-5(b) shows the east-west displacement of the second floor under a random ground motion excitation. Figure 6-5(c) gives the calculated response which corresponds well to the measured response. In general, for large DOF structures, the proposed method can result in considerable time and memory saving.

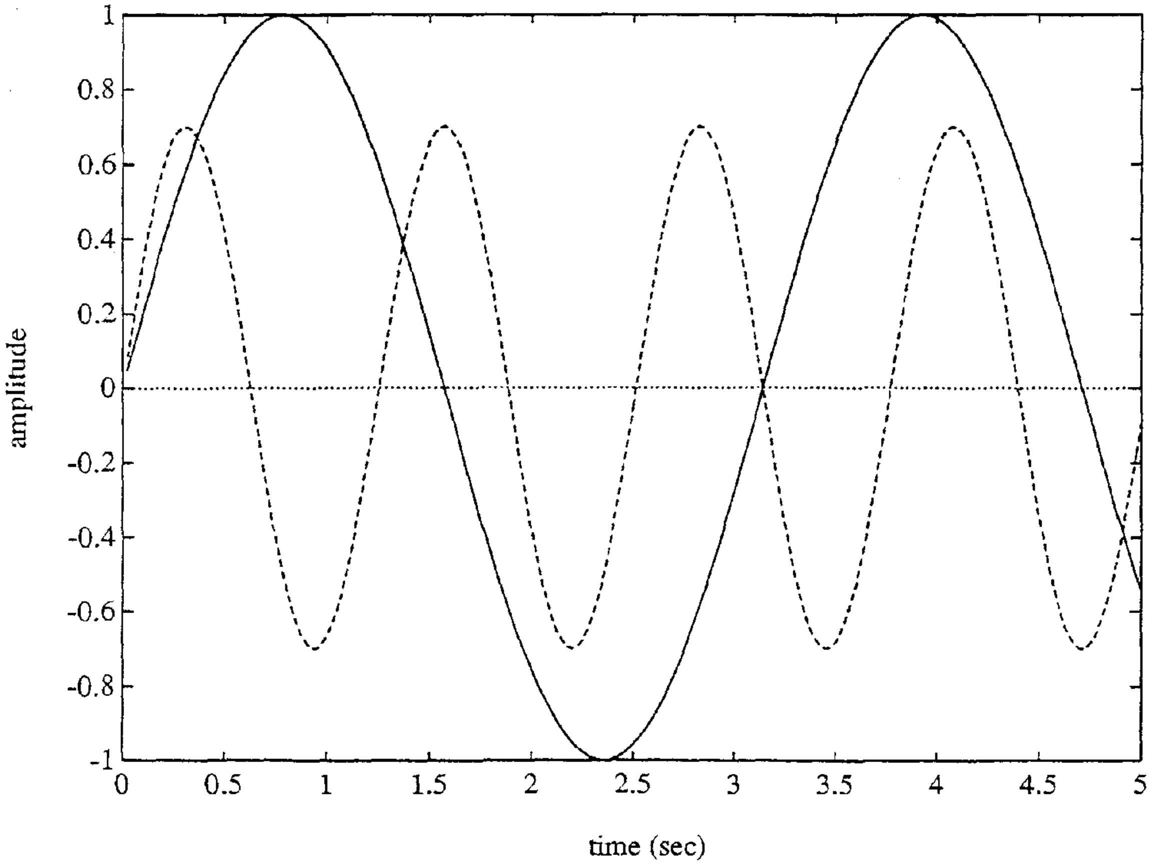


FIGURE 6-4a Driving Force

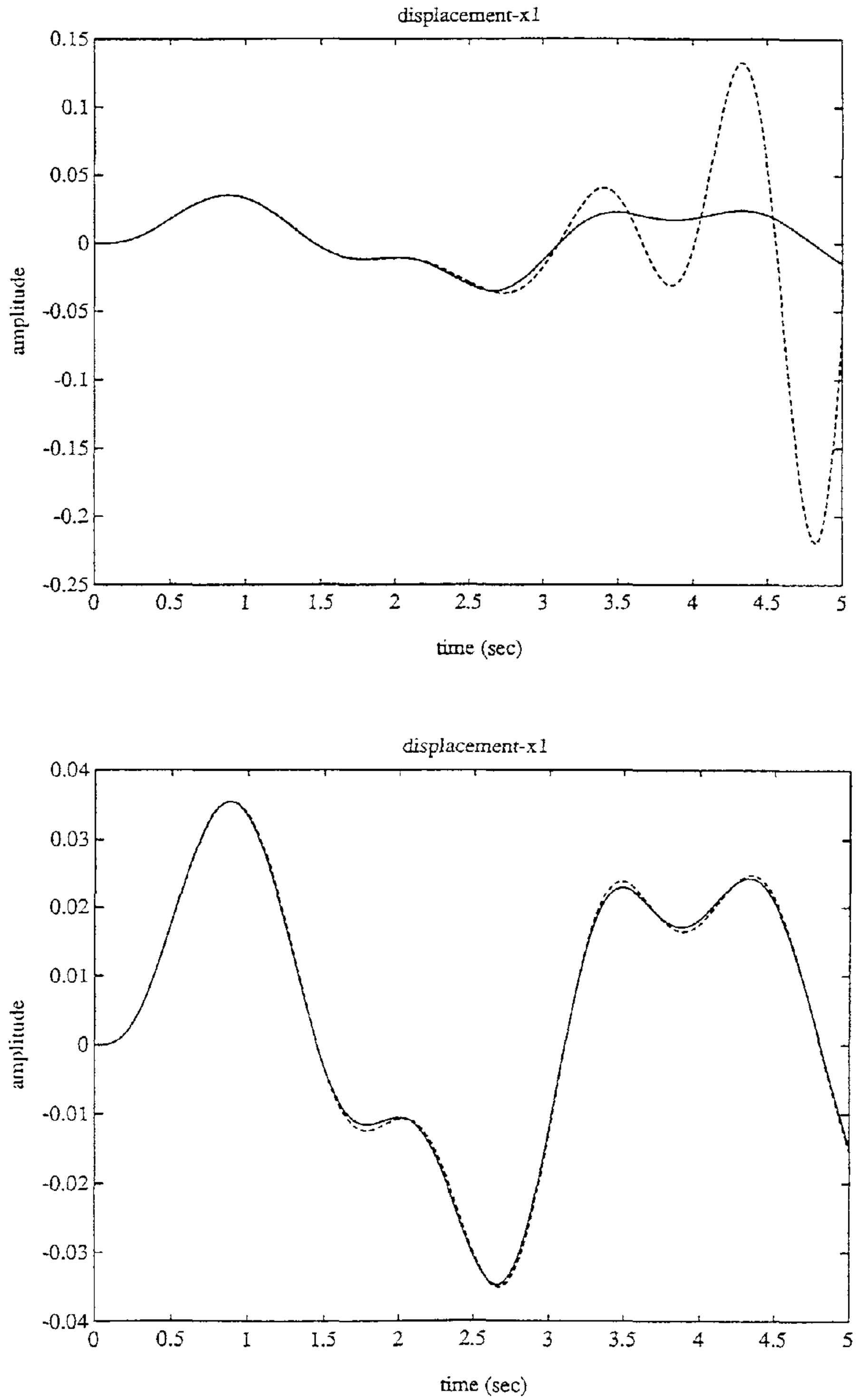


FIGURE 6-4b Response Comparison, x_1

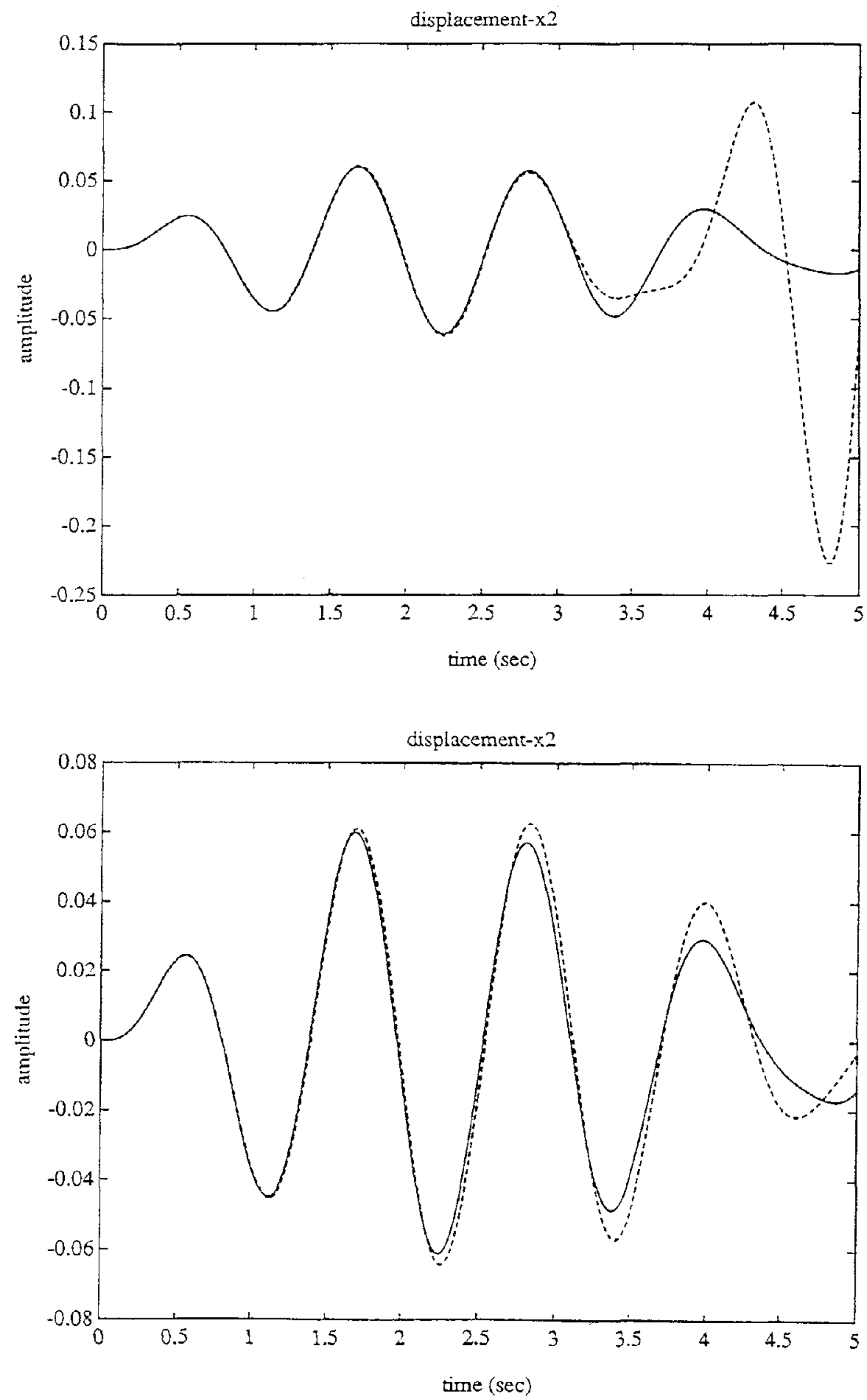


FIGURE 6-4c Response Comparison, x_2

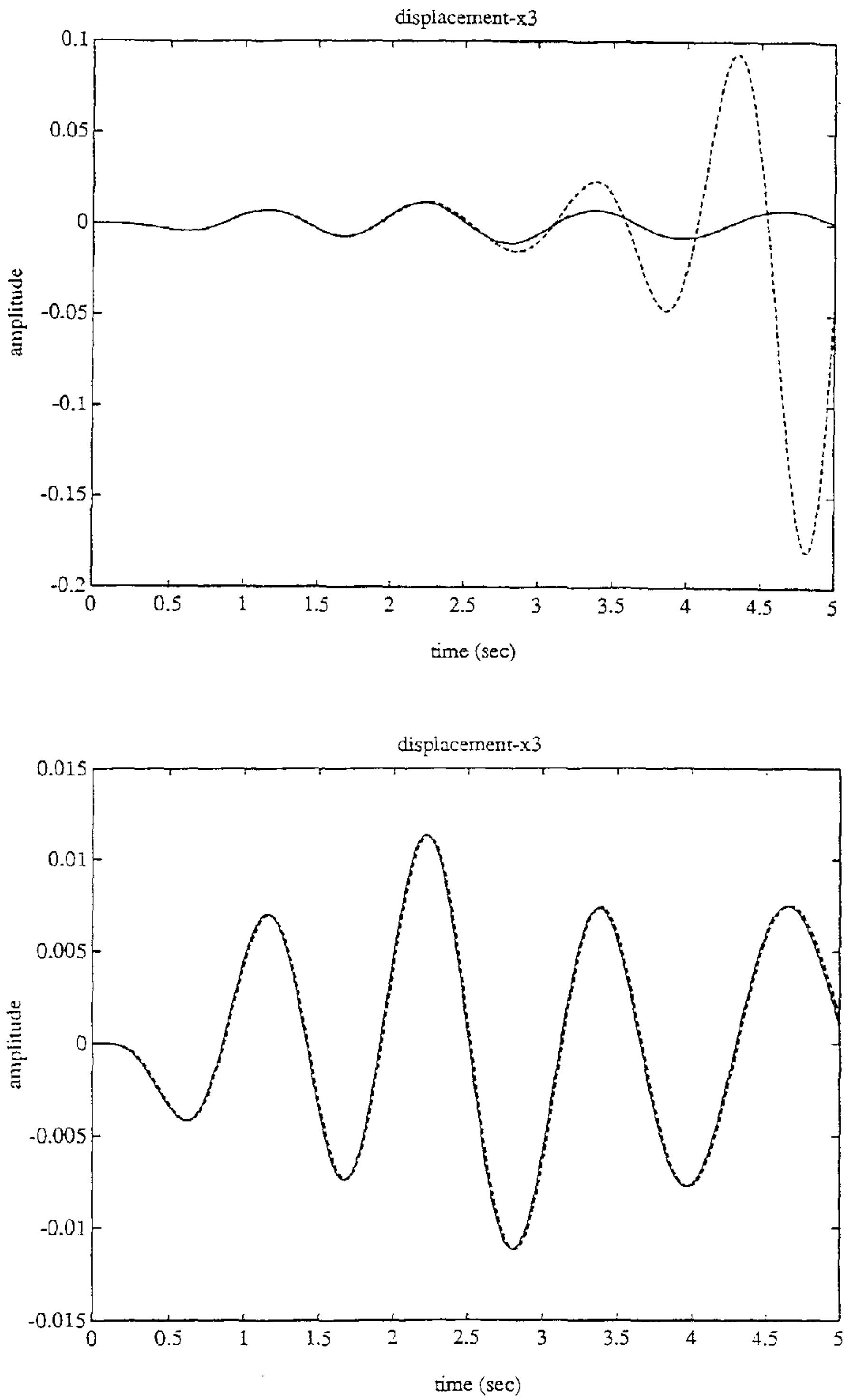


FIGURE 6-4d Response Comparison, x_3

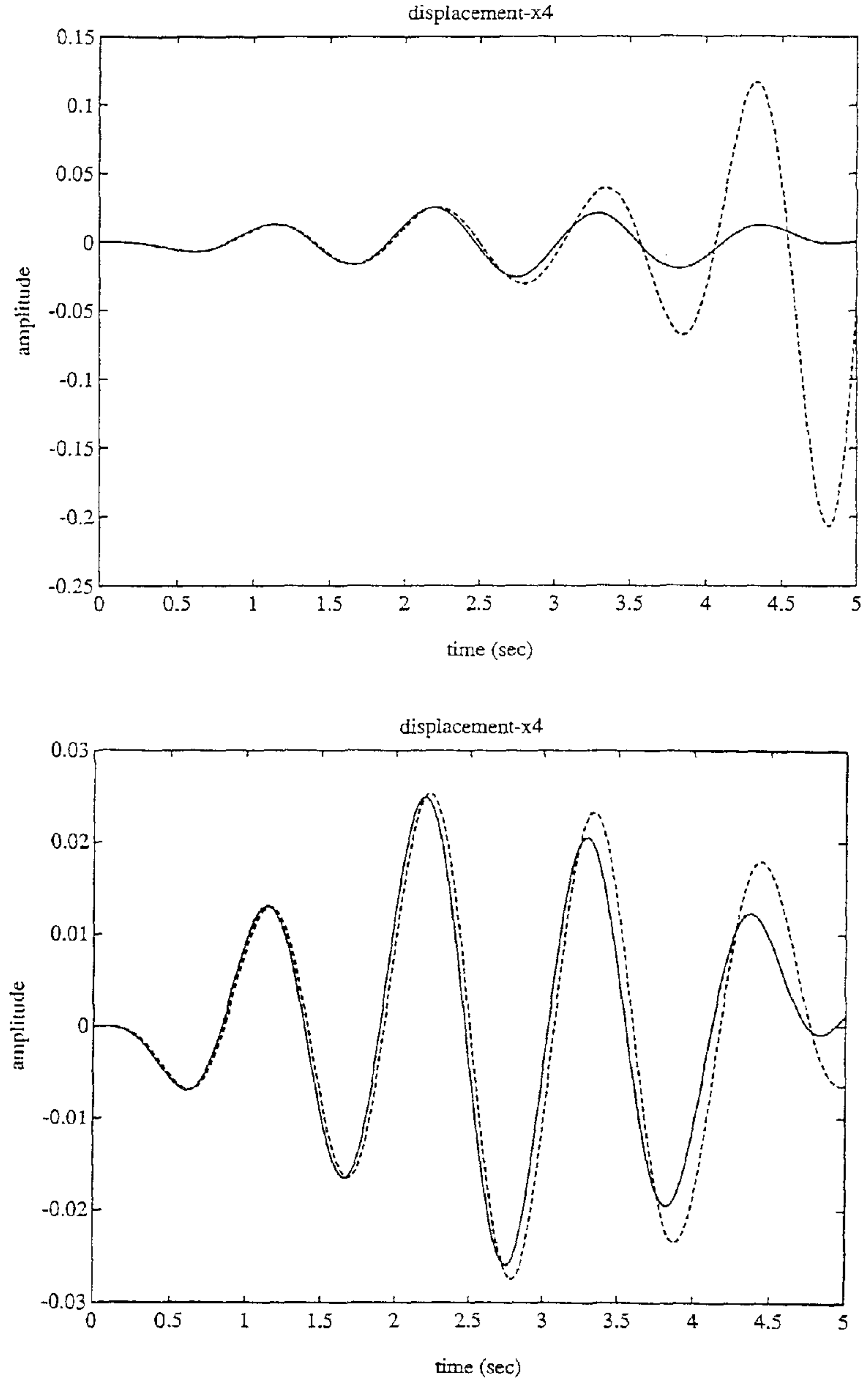


FIGURE 6-4e Response Comparison, x_4

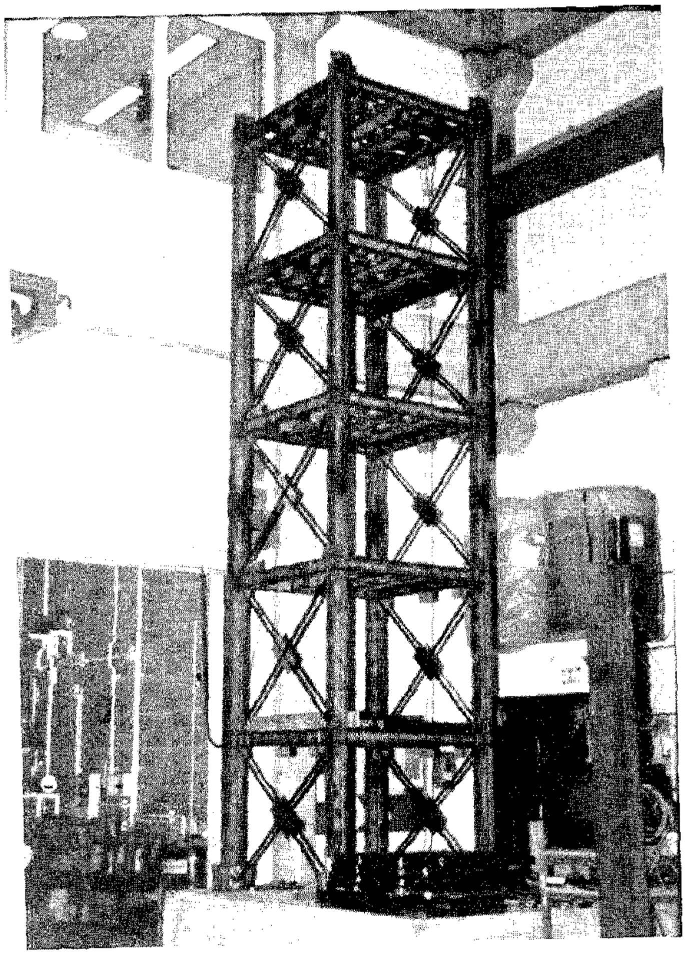


FIGURE 6-5a A Five-Floor Steel Frame

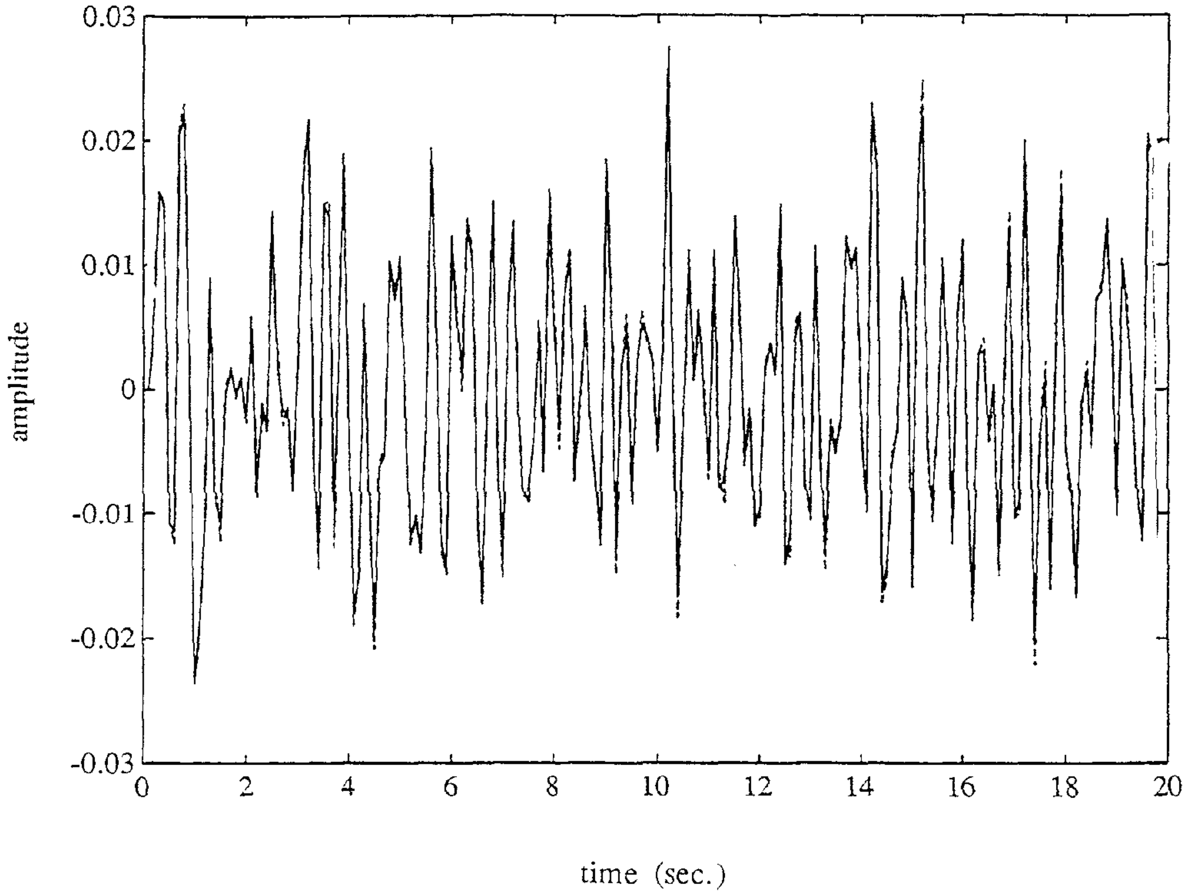


FIGURE 6-5b A Measured Response

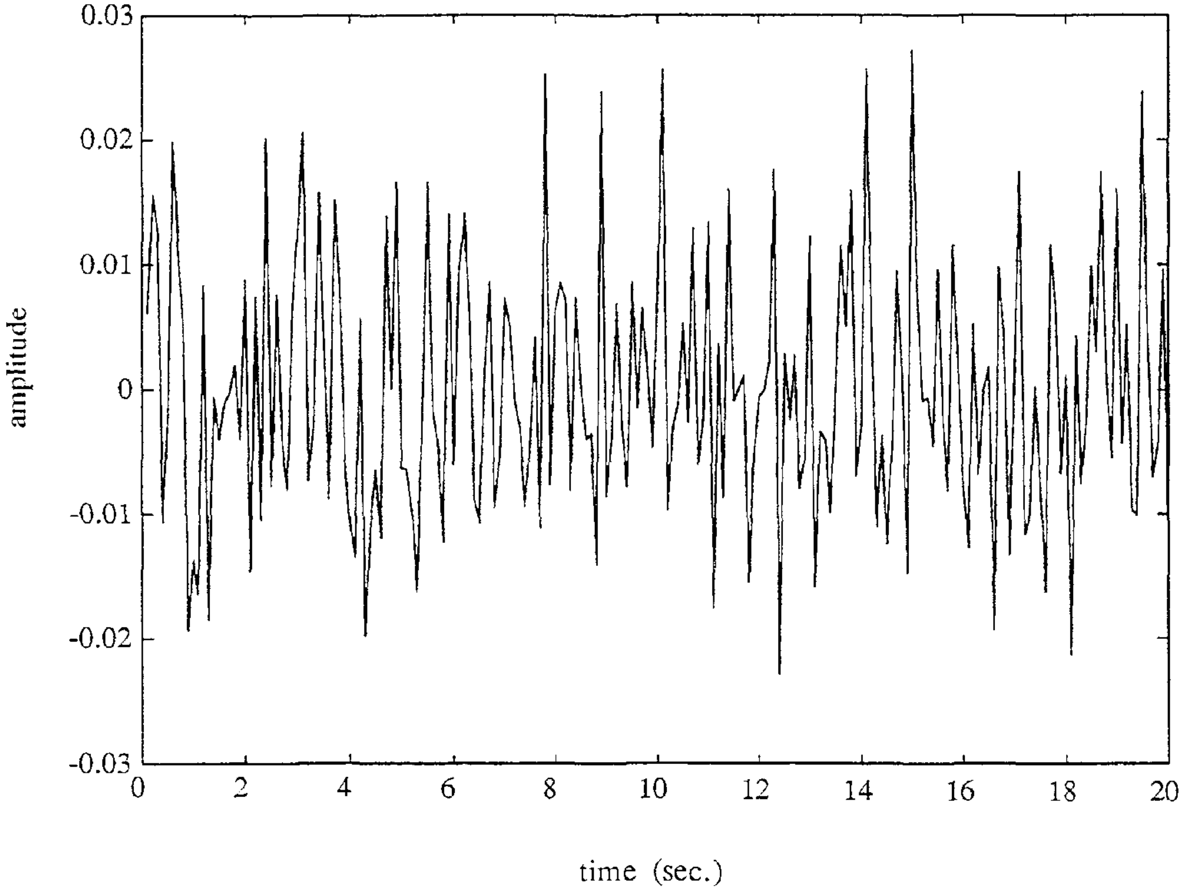


FIGURE 6-5c A Calculated Response

SECTION 7

DISCUSSION AND CONCLUSION

A general iterative algorithm for computing the responses of linear non-proportionally damped systems is presented. The method works in physical space, and has many advantages over traditional state space approaches. Several important features are summarized in the following.

1. This proposed algorithm converges unconditionally. There were no published methods that offer such a complete range of convergence.

2. A simple formula is provided for choosing the optimal convergence point under a given standard. Such a quick optimization may accelerate convergence speed several times in usual cases. Also, approaches to reduce the round-off errors are discussed.

3. In this report, we pointed out the connection between the response of a structure or a system and its internal dynamic actions such as the damping mechanisms.

4. This report presents the theoretical foundations for the numerical approach of computing responses of linear, non-proportionally damped systems. It is anticipated that the fundamental complex damping theory may offer more insights into other characters of non-proportionally damped systems.

6. There are several possible improvements to accelerate further the

convergent speed and reduce the round-off errors and computing memory requirements. The authors have investigated some of these alternatives and obtained some progress. The development towards establishing engineering applicable software is now under the authors considerations.

SECTION 8

REFERENCES

1. Beliveau, J.G., "Identification of Viscous Damping in Structures from Modal Information", J. of Appl. Mech., Vol. 43(2), pp 335-339, 1976.

2. Bellman, R., "Introduction to Matrix Analysis", 2nd ed., New York, McGraw-Hill, 1970.

3. Berman, A. and E.Y. Nagy, "Improvement of a Large Analytical Model Using Test Data", AIAA J., Vol. 21, pp 1168-1173, 1983.

4. Ben-Israel, S. and T.N.E. Greville, "Generalized Inverse", 1973.

5. Buhariwala, K.J. and J.S. Hansen, "Construction of a Consistent Damping Matrix", J. of Appl. Mech., Vol. 55, pp 443-447, June 1988.

6. Caughey, T.K. and M.J. O'kelly, "Classical Normal Modes in Damped Linear Dynamic Systems", ASME J. of Appl. Mech., Vol. 32, pp 583-588, 1965.

7. Caravani, P. and W.T. Thomson, "Identification of Damping Coefficients in Multidimensional Linear Systems", ASME J. of Appl. Mech., Vol. 41, pp 379-392, 1974.

8. Clough, R.W. and J. Penzien, "Dynamics of Structures", McGraw-Hill, 1975.

9. Clough, R.W. and S. Mejtahedi, "Earthquake Response Analysis Considering Non-proportional Damping", Earth. Eng. & Struc. Dyn., Vol. 4, pp 489-496, 1976.

10. Clough, R.W., "Finite Element Method After Twenty-Five Years: A Personal View", 46 Refs., Eng. Appl. of the Finite Element Method. Int. Conf., pp 1.1-1.34, 1979.

11. Deblauwe, F.G., D.L. Brown and H. Vold, "Some Concepts for Spatial Sine Testing Parameter Identification" Proc. of IMAC-7, pp 1174-1178, Jan. 1989.

12. Duncan, P.E. and R.E. Taylor, "A Note on the Dynamic Analysis of Non-Proportionally Damped Systems", Earth. Eng. & Struc. Dyn., Vol. 7, pp 99-105, 1979.

13. Ewins, D.J., "Modal Testing, Theory and Practice", Research Studies Press LTD, England, 1986.

14. Foss, K. A., "Co-ordinates That Uncouple the Equations of Motion of Damped Linear Dynamic Systems", ASME J. of Appl. Mech. Vol. 25, pp 361-363, 1958.

15. Fuh, J., S. Chen and A. Berman, "System Identification of Analytical Models of Damped Structures", AIAA/ASME 25th Structures, Structural Dynamics and Materials Conf., Paper no. 84-0926, pp 122-116, 1984.

16. Gohberg, L. et al, "Matrix Polynomials", Academic Press, 1982.

17. Golla, D.F. and P.C. Hughes, "Dynamics of Viscoelastic Structures -- A Time-Domain, Finite Element Formulation", J. of Appl. Mech., Vol. 52, pp 897-906, Dec. 1985.

18. Golub, G.H. et al, "Matrix Computations", John Hopkins Univ Press, 1985.

19. Golub, G.H. and C.F. Van Loan, "Matrix Computations", John Hopkins University Press, 1985.

20. Hall, B.M. et al, "Linear Estimation of Structural Parameters for Dynamic Test Data", AIAA/ASME 11th Structures, Structural Dynamics and Materials Conf., pp 193-197, 1970.

21. Hanagud, S.M., J.I. Craig; et al, "Identification of Structural Dynamic Systems with Non-proportional Damping", AIAA/ASME 25th Structures, Structural Dynamics and Materials Conf., Paper no. 84-0093, pp 283-291, 1984.

22. Hasselman, T.K., "Modal Coupling in Lightly Damped Structures", AIHA J., 14, pp 1627-1628, 1976.

23. Hollowell, W.T., W.D. Pilkey and W.M. Sieveka, "System Identification of Dynamic Structures", J. of Finite Element in Analy. and Des., The Int. J. of Appl. Finite Element, Vol. 4, pp 65-77, 1988.

24. Ibrahim, S.R. and E.C. Mikulcik, "A Method for the Direct Identification of Vibration Parameters from the Free Response", Shock & Vib. Bulletin, Vol.47, Sept. 1977.

25. Ibrahim, S.R., "Advantages in Time Domain Modal Identification and Modeling of Structure", 2nd Int. Symposium on Aerospace & Structure Dynamics, April 1986.

26. Inman, D.J and S.K. Jha, "Identification of the Damping Matrix for Tires", Proc. of IMAC-4, pp 1078-1080, 1986.

27. Itoh, T., "Damped Vibration Mode Superposition Method for Dynamic Response Analysis", Earth. Eng. & Struc. Dyn., Vol. 2, pp 47-57, 1973.

28. Juang, J.N. and R.S. Pappa, "An Eigensystem Realization Algorithm (ERA) for Modal Parameter Identification and Model Reduction", AIAA J. of Guidance, Control and Dynamics, Vol.8, No.5, pp 620-627, Sept.-Oct. 1985.

29. Liang, Z., "A Possible Error of Impulse Test and the Way of its Improving", Proc. of IMAC-III, pp 802-808, 1985.

30. Liang, Z., "On Modal Testing in the Time Domain", Ph.D. Dissertation, SUNY at Buffalo, Feb. 1987.

31. Liang, Z. and D.J. Inman, "Rank Decomposition Methods in Modal Analysis", Proc. of IMAC-6, pp 1176-1179, 1988.

32. Liang, Z. and G.C. Lee, "On Complex Damping of MDOF Systems", Proc. of IMAC-8, pp 1176-1179, 1990.

33. Liang, Z. and G.C. Lee, "Damping of structures - Part 1: Complex Damping Theory", NCEER Report 91-0004, 1991 (in press).

34. Liang, Z. and G.C. Lee, "Representation of the Damping Matrix", J. of Eng. Mech., ASCE., May 1991 (to appear).

35. Liang, Z., M. Tong and G.C. Lee, "A Strong Criterion for Testing Proportionally Damped Systems", Proc. of Damping '91, Feb. 13-15, 1991.

36. Liang, Z., G.C. Lee and M. Tong, "On a Linear Property of Lightly Damped Systems", Proc. of Damping '91, Feb. 13-15, 1991.

37. Mitchell, L.P., "Complex Modes: A Review", Proc. of IMAC-8, pp 891-899, 1990.

38. Mau, S.T., "A Subspace Modal Superposition Method for Non-classical Damped Systems", Earth. Eng. & Struc. Dyn., Vol. 16, pp 931-942, 1988.

39. Mau, S.T., "System Identification Using Classical and Non-Proportional Normal Modes", Proc. 8th World Conf. Earthquake Eng., San Francisco, Vol. 4, pp 403-410, 1984.

40. Meirovitch, L., "Analytical Methods in Vibrations", MacMillan Comp., 1967.

41. Nair, S.S. and P. Singh, "Examination of the Validity of Proportional Damping Approximations With Two Further Numerical Indices", J. of Sound and Vibration, Vol. 104, pp 348-350, 1986.

42. Ortega, J.M., "Matrix Theory", 2nd ed., Plenum Press, New York, 1987.

43. Pollard, H., "Sound Waves in Solids", McGraw-Hill, 1975.

44. Rajaram, S. and J.L. Junkins, et al, "Identification of Vibrating Flexible Structures", AIAA J. of Guidance, Control and Dynamics, Vol. 8, no.4, pp 463-470, July.-Aug. 1985.

45. Singh, M.P., "Seismic Design Response by an Alternative SRSS Rule", Earth. Eng. & Struc. Dyn., Vol. 11, pp 771-783, 1983.

46. Singh, M.P. and M. Ghafory-Ashtiany, "Modal Time History of Non-classically Damped Structures for Seismic Motions", Earth. Eng. & Struc.

Dyn., Vol. 14, pp 133-146, 1986.

47. Singh, M.P., "Seimic Response By SRSS for Non-proportional Damping", J. of Eng. of Mechanics Division, ASCE, Vol. 106, pp 1405-1419, 1986.

48. Soong, T.T.; A.M. Reinhorn and J.N. Yang, "A Standardized Model for Structural Control Experiments and Some Experimental Results", Active Control, Martinus Nijhoff Pub., Amsterdam, pp 669-693, 1987.

49. Thompson, W.T.; T. Calkins and P. Caravani, "A Numerical Study of Damping", Earth. Eng. & Struc. Dyn., Vol. 3, pp 97-103, 1974.

50. Thoren, A.R. "Derivation of Mass and Stiffness Matrices from Dynamic Test Data", AIAA/ASME/SAE 13th Structures, Structural Dynamics and Materials Conf., San Antonio, TX, Paper no.72-346, 1972.

51. Udwadia, Firdaus E.and Ramin S. Esfandiari, "Nonclassically Damped Systems: An Iterative Approach", J. of Appl. Mech., pp 423-433, June 1990.

52. Veletsos, A.S. and C.E. Ventura, "Modal Analysis of Non-classically Damped Linear Systems", Earth. Eng. & Struc. Dyn., Vol. 14, pp 217-243, 1986.

53. Vold, H. and G. Rocklin, "The Numerical Implementation of a Multi-input Modal Estimation Method for Mini-computer", Proc. of IMAC-1, 1982.

54. Vold, H. and R. Williams, "Multiphase-Step-Sine Method for Experimental Modal Analysis", Sound and Vibration Mag., June 1987.

55. Warburton, G.B. and R. Soni, "Errors in Response Calculations for Non-Proportional Damping", Earth. Eng. & Struc. Dyn., Vol. 5, pp 365-376, 1977.

56. Yao, G.C., "Diagnostic Studies of Steel Structures Through Vibrational Signature Analysis", Ph.D. Dissertation, Dept. of Civil Engineering, SUNYAB, 1991.

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
LIST OF TECHNICAL REPORTS

The National Center for Earthquake Engineering Research (NCEER) publishes technical reports on a variety of subjects related to earthquake engineering written by authors funded through NCEER. These reports are available from both NCEER's Publications Department and the National Technical Information Service (NTIS). Requests for reports should be directed to the Publications Department, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Red Jacket Quadrangle, Buffalo, New York 14261. Reports can also be requested through NTIS, 5285 Port Royal Road, Springfield, Virginia 22161. NTIS accession numbers are shown in parenthesis, if available.

NCEER-87-0001 "First-Year Program in Research, Education and Technology Transfer," 3/5/87, (PB88-134275/AS).

NCEER-87-0002 "Experimental Evaluation of Instantaneous Optimal Algorithms for Structural Control," by R.C. Lin, T.T. Soong and A.M. Reinhorn, 4/20/87, (PB88-134341/AS).

NCEER-87-0003 "Experimentation Using the Earthquake Simulation Facilities at University at Buffalo," by A.M. Reinhorn and R.L. Ketter, to be published.

NCEER-87-0004 "The System Characteristics and Performance of a Shaking Table," by J.S. Hwang, K.C. Chang and G.C. Lee, 6/1/87, (PB88-134259/AS). This report is available only through NTIS (see address given above).

NCEER-87-0005 "A Finite Element Formulation for Nonlinear Viscoplastic Material Using a Q Model," by O. Gyebi and G. Dasgupta, 11/2/87, (PB88-213764/AS).

NCEER-87-0006 "Symbolic Manipulation Program (SMP) - Algebraic Codes for Two and Three Dimensional Finite Element Formulations," by X. Lee and G. Dasgupta, 11/9/87, (PB88-219522/AS).

NCEER-87-0007 "Instantaneous Optimal Control Laws for Tall Buildings Under Seismic Excitations," by J.N. Yang, A. Akbarpour and P. Ghaemmaghami, 6/10/87, (PB88-134333/AS).

NCEER-87-0008 "IDARC: Inelastic Damage Analysis of Reinforced Concrete Frame - Shear-Wall Structures," by Y.J. Park, A.M. Reinhorn and S.K. Kunnath, 7/20/87, (PB88-134325/AS).

NCEER-87-0009 "Liquefaction Potential for New York State: A Preliminary Report on Sites in Manhattan and Buffalo," by M. Budhu, V. Vijayakumar, R.F. Giese and L. Baumgras, 8/31/87, (PB88-163704/AS). This report is available only through NTIS (see address given above).

NCEER-87-0010 "Vertical and Torsional Vibration of Foundations in Inhomogeneous Media," by A.S. Veletsos and K.W. Dotson, 6/1/87, (PB88-134291/AS).

NCEER-87-0011 "Seismic Probabilistic Risk Assessment and Seismic Margins Studies for Nuclear Power Plants," by Howard H.M. Hwang, 6/15/87, (PB88-134267/AS).

NCEER-87-0012 "Parametric Studies of Frequency Response of Secondary Systems Under Ground-Acceleration Excitations," by Y. Yong and Y.K. Lin, 6/10/87, (PB88-134309/AS).

NCEER-87-0013 "Frequency Response of Secondary Systems Under Seismic Excitation," by J.A. HoLung, J. Cai and Y.K. Lin, 7/31/87, (PB88-134317/AS).

NCEER-87-0014 "Modelling Earthquake Ground Motions in Seismically Active Regions Using Parametric Time Series Methods," by G.W. Ellis and A.S. Cakmak, 8/25/87, (PB88-134283/AS).

NCEER-87-0015 "Detection and Assessment of Seismic Structural Damage," by E. DiPasquale and A.S. Cakmak, 8/25/87, (PB88-163712/AS).

NCEER-87-0016 "Pipeline Experiment at Parkfield, California," by J. Isenberg and E. Richardson, 9/15/87, (PB88-163720/AS). This report is available only through NTIS (see address given above).

NCEER-87-0017	"Digital Simulation of Seismic Ground Motion," by M. Shinozuka, G. Deodatis and T. Harada, 8/31/87, (PB88-155197/AS). This report is available only through NTIS (see address given above).
NCEER-87-0018	"Practical Considerations for Structural Control: System Uncertainty, System Time Delay and Truncation of Small Control Forces," J.N. Yang and A. Akbarpour, 8/10/87, (PB88-163738/AS).
NCEER-87-0019	"Modal Analysis of Nonclassically Damped Structural Systems Using Canonical Transformation," by J.N. Yang, S. Sarkani and F.X. Long, 9/27/87, (PB88-187851/AS).
NCEER-87-0020	"A Nonstationary Solution in Random Vibration Theory," by J.R. Red-Horse and P.D. Spanos, 11/3/87, (PB88-163746/AS).
NCEER-87-0021	"Horizontal Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by A.S. Veletsos and K.W. Dotson, 10/15/87, (PB88-150859/AS).
NCEER-87-0022	"Seismic Damage Assessment of Reinforced Concrete Members," by Y.S. Chung, C. Meyer and M. Shinozuka, 10/9/87, (PB88-150867/AS). This report is available only through NTIS (see address given above).
NCEER-87-0023	"Active Structural Control in Civil Engineering," by T.T. Soong, 11/11/87, (PB88-187778/AS).
NCEER-87-0024	Vertical and Torsional Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by K.W. Dotson and A.S. Veletsos, 12/87, (PB88-187786/AS).
NCEER-87-0025	"Proceedings from the Symposium on Seismic Hazards, Ground Motions, Soil-Liquefaction and Engineering Practice in Eastern North America," October 20-22, 1987, edited by K.H. Jacob, 12/87, (PB88-188115/AS).
NCEER-87-0026	"Report on the Whittier-Narrows, California, Earthquake of October 1, 1987," by J. Pantelic and A. Reinhorn, 11/87, (PB88-187752/AS). This report is available only through NTIS (see address given above).
NCEER-87-0027	"Design of a Modular Program for Transient Nonlinear Analysis of Large 3-D Building Structures," by S. Srivastav and J.F. Abel, 12/30/87, (PB88-187950/AS).
NCEER-87-0028	"Second-Year Program in Research, Education and Technology Transfer," 3/8/88, (PB88-219480/AS).
NCEER-88-0001	"Workshop on Seismic Computer Analysis and Design of Buildings With Interactive Graphics," by W. McGuire, J.F. Abel and C.H. Conley, 1/18/88, (PB88-187760/AS).
NCEER-88-0002	"Optimal Control of Nonlinear Flexible Structures," by J.N. Yang, F.X. Long and D. Wong, 1/22/88, (PB88-213772/AS).
NCEER-88-0003	"Substructuring Techniques in the Time Domain for Primary-Secondary Structural Systems," by G.D. Manolis and G. Juhn, 2/10/88, (PB88-213780/AS).
NCEER-88-0004	"Iterative Seismic Analysis of Primary-Secondary Systems," by A. Singhal, L.D. Lutes and P.D. Spanos, 2/23/88, (PB88-213798/AS).
NCEER-88-0005	"Stochastic Finite Element Expansion for Random Media," by P.D. Spanos and R. Ghanem, 3/14/88, (PB88-213806/AS).
NCEER-88-0006	"Combining Structural Optimization and Structural Control," by F.Y. Cheng and C.P. Pantelides, 1/10/88, (PB88-213814/AS).
NCEER-88-0007	"Seismic Performance Assessment of Code-Designed Structures," by H.H-M. Hwang, J-W. Jaw and H-J. Shau, 3/20/88, (PB88-219423/AS).

NCEER-88-0008 "Reliability Analysis of Code-Designed Structures Under Natural Hazards," by H.H-M. Hwang, H. Ushiba and M. Shinozuka, 2/29/88, (PB88-229471/AS).

NCEER-88-0009 "Seismic Fragility Analysis of Shear Wall Structures," by J-W Jaw and H.H-M. Hwang, 4/30/88, (PB89-102867/AS).

NCEER-88-0010 "Base Isolation of a Multi-Story Building Under a Harmonic Ground Motion - A Comparison of Performances of Various Systems," by F-G Fan, G. Ahmadi and I.G. Tadjbakhsh, 5/18/88, (PB89-122238/AS).

NCEER-88-0011 "Seismic Floor Response Spectra for a Combined System by Green's Functions," by F.M. Lavelle, L.A. Bergman and P.D. Spanos, 5/1/88, (PB89-102875/AS).

NCEER-88-0012 "A New Solution Technique for Randomly Excited Hysteretic Structures," by G.Q. Cai and Y.K. Lin, 5/16/88, (PB89-102883/AS).

NCEER-88-0013 "A Study of Radiation Damping and Soil-Structure Interaction Effects in the Centrifuge," by K. Weissman, supervised by J.H. Prevost, 5/24/88, (PB89-144703/AS).

NCEER-88-0014 "Parameter Identification and Implementation of a Kinematic Plasticity Model for Frictional Soils," by J.H. Prevost and D.V. Griffiths, to be published.

NCEER-88-0015 "Two- and Three- Dimensional Dynamic Finite Element Analyses of the Long Valley Dam," by D.V. Griffiths and J.H. Prevost, 6/17/88, (PB89-144711/AS).

NCEER-88-0016 "Damage Assessment of Reinforced Concrete Structures in Eastern United States," by A.M. Reinhorn, M.J. Seidel, S.K. Kunnath and Y.J. Park, 6/15/88, (PB89-122220/AS).

NCEER-88-0017 "Dynamic Compliance of Vertically Loaded Strip Foundations in Multilayered Viscoelastic Soils," by S. Ahmad and A.S.M. Israil, 6/17/88, (PB89-102891/AS).

NCEER-88-0018 "An Experimental Study of Seismic Structural Response With Added Viscoelastic Dampers," by R.C. Lin, Z. Liang, T.T. Soong and R.H. Zhang, 6/30/88, (PB89-122212/AS).

NCEER-88-0019 "Experimental Investigation of Primary - Secondary System Interaction," by G.D. Manolis, G. Juhn and A.M. Reinhorn, 5/27/88, (PB89-122204/AS).

NCEER-88-0020 "A Response Spectrum Approach For Analysis of Nonclassically Damped Structures," by J.N. Yang, S. Sarkani and F.X. Long, 4/22/88, (PB89-102909/AS).

NCEER-88-0021 "Seismic Interaction of Structures and Soils: Stochastic Approach," by A.S. Veletsos and A.M. Prasad, 7/21/88, (PB89-122196/AS).

NCEER-88-0022 "Identification of the Serviceability Limit State and Detection of Seismic Structural Damage," by E. DiPasquale and A.S. Cakmak, 6/15/88, (PB89-122188/AS).

NCEER-88-0023 "Multi-Hazard Risk Analysis: Case of a Simple Offshore Structure," by B.K. Bhartia and E.H. Vanmarcke, 7/21/88, (PB89-145213/AS).

NCEER-88-0024 "Automated Seismic Design of Reinforced Concrete Buildings," by Y.S. Chung, C. Meyer and M. Shinozuka, 7/5/88, (PB89-122170/AS).

NCEER-88-0025 "Experimental Study of Active Control of MDOF Structures Under Seismic Excitations," by L.L. Chung, R.C. Lin, T.T. Soong and A.M. Reinhorn, 7/10/88, (PB89-122600/AS).

NCEER-88-0026 "Earthquake Simulation Tests of a Low-Rise Metal Structure," by J.S. Hwang, K.C. Chang, G.C. Lee and R.L. Ketter, 8/1/88, (PB89-102917/AS).

NCEER-88-0027 "Systems Study of Urban Response and Reconstruction Due to Catastrophic Earthquakes," by F. Kozin and H.K. Zhou, 9/22/88, (PB90-162348/AS).

NCEER-88-0028 "Seismic Fragility Analysis of Plane Frame Structures," by H.H-M. Hwang and Y.K. Low, 7/31/88, (PB89-131445/AS).

NCEER-88-0029 "Response Analysis of Stochastic Structures," by A. Kardara, C. Bucher and M. Shinozuka, 9/22/88, (PB89-174429/AS).

NCEER-88-0030 "Nonnormal Accelerations Due to Yielding in a Primary Structure," by D.C.K. Chen and L.D. Lutes, 9/19/88, (PB89-131437/AS).

NCEER-88-0031 "Design Approaches for Soil-Structure Interaction," by A.S. Veletsos, A.M. Prasad and Y. Tang, 12/30/88, (PB89-174437/AS).

NCEER-88-0032 "A Re-evaluation of Design Spectra for Seismic Damage Control," by C.J. Turkstra and A.G. Tallin, 11/7/88, (PB89-145221/AS).

NCEER-88-0033 "The Behavior and Design of Noncontact Lap Splices Subjected to Repeated Inelastic Tensile Loading," by V.E. Sagan, P. Gergely and R.N. White, 12/8/88, (PB89-163737/AS).

NCEER-88-0034 "Seismic Response of Pile Foundations," by S.M. Mamoon, P.K. Banerjee and S. Ahmad, 11/1/88, (PB89-145239/AS).

NCEER-88-0035 "Modeling of R/C Building Structures With Flexible Floor Diaphragms (IDARC2)," by A.M. Reinhorn, S.K. Kunnath and N. Panahshahi, 9/7/88, (PB89-207153/AS).

NCEER-88-0036 "Solution of the Dam-Reservoir Interaction Problem Using a Combination of FEM, BEM with Particular Integrals, Modal Analysis, and Substructuring," by C-S. Tsai, G.C. Lee and R.L. Ketter, 12/31/88, (PB89-207146/AS).

NCEER-88-0037 "Optimal Placement of Actuators for Structural Control," by F.Y. Cheng and C.P. Pantelides, 8/15/88, (PB89-162846/AS).

NCEER-88-0038 "Teflon Bearings in Aseismic Base Isolation: Experimental Studies and Mathematical Modeling," by A. Mokha, M.C. Constantinou and A.M. Reinhorn, 12/5/88, (PB89-218457/AS).

NCEER-88-0039 "Seismic Behavior of Flat Slab High-Rise Buildings in the New York City Area," by P. Weidlinger and M. Ettouney, 10/15/88, (PB90-145681/AS).

NCEER-88-0040 "Evaluation of the Earthquake Resistance of Existing Buildings in New York City," by P. Weidlinger and M. Ettouney, 10/15/88, to be published.

NCEER-88-0041 "Small-Scale Modeling Techniques for Reinforced Concrete Structures Subjected to Seismic Loads," by W. Kim, A. El-Attar and R.N. White, 11/22/88, (PB89-189625/AS).

NCEER-88-0042 "Modeling Strong Ground Motion from Multiple Event Earthquakes," by G.W. Ellis and A.S. Cakmak, 10/15/88, (PB89-174445/AS).

NCEER-88-0043 "Nonstationary Models of Seismic Ground Acceleration," by M. Grigoriu, S.E. Ruiz and E. Rosenblueth, 7/15/88, (PB89-189617/AS).

NCEER-88-0044 "SARCF User's Guide: Seismic Analysis of Reinforced Concrete Frames," by Y.S. Chung, C. Meyer and M. Shinozuka, 11/9/88, (PB89-174452/AS).

NCEER-88-0045 "First Expert Panel Meeting on Disaster Research and Planning," edited by J. Pantelic and J. Stoyle, 9/15/88, (PB89-174460/AS).

NCEER-88-0046 "Preliminary Studies of the Effect of Degrading Infill Walls on the Nonlinear Seismic Response of Steel Frames," by C.Z. Chrysostomou, P. Gergely and J.F. Abel, 12/19/88, (PB89-208383/AS).

NCEER-88-0047 "Reinforced Concrete Frame Component Testing Facility - Design, Construction, Instrumentation and Operation," by S.P. Pessiki, C. Conley, T. Bond, P. Gergely and R.N. White, 12/16/88, (PB89-174478/AS).

NCEER-89-0001 "Effects of Protective Cushion and Soil Compliancy on the Response of Equipment Within a Seismically Excited Building," by J.A. HoLung, 2/16/89, (PB89-207179/AS).

NCEER-89-0002 "Statistical Evaluation of Response Modification Factors for Reinforced Concrete Structures," by H.H-M. Hwang and J-W. Jaw, 2/17/89, (PB89-207187/AS).

NCEER-89-0003 "Hysteretic Columns Under Random Excitation," by G-Q. Cai and Y.K. Lin, 1/9/89, (PB89-196513/AS).

NCEER-89-0004 "Experimental Study of 'Elephant Foot Bulge' Instability of Thin-Walled Metal Tanks," by Z-H. Jia and R.L. Ketter, 2/22/89, (PB89-207195/AS).

NCEER-89-0005 "Experiment on Performance of Buried Pipelines Across San Andreas Fault," by J. Isenberg, E. Richardson and T.D. O'Rourke, 3/10/89, (PB89-218440/AS).

NCEER-89-0006 "A Knowledge-Based Approach to Structural Design of Earthquake-Resistant Buildings," by M. Subramani, P. Gergely, C.H. Conley, J.F. Abel and A.H. Zaghw, 1/15/89, (PB89-218465/AS).

NCEER-89-0007 "Liquefaction Hazards and Their Effects on Buried Pipelines," by T.D. O'Rourke and P.A. Lane, 2/1/89, (PB89-218481).

NCEER-89-0008 "Fundamentals of System Identification in Structural Dynamics," by H. Imai, C-B. Yun, O. Maruyama and M. Shinozuka, 1/26/89, (PB89-207211/AS).

NCEER-89-0009 "Effects of the 1985 Michoacan Earthquake on Water Systems and Other Buried Lifelines in Mexico," by A.G. Ayala and M.J. O'Rourke, 3/8/89, (PB89-207229/AS).

NCEER-89-R010 "NCEER Bibliography of Earthquake Education Materials," by K.E.K. Ross, Second Revision, 9/1/89, (PB90-125352/AS).

NCEER-89-0011 "Inelastic Three-Dimensional Response Analysis of Reinforced Concrete Building Structures (IDARC-3D), Part I - Modeling," by S.K. Kunnath and A.M. Reinhorn, 4/17/89, (PB90-114612/AS).

NCEER-89-0012 "Recommended Modifications to ATC-14," by C.D. Poland and J.O. Malley, 4/12/89, (PB90-108648/AS).

NCEER-89-0013 "Repair and Strengthening of Beam-to-Column Connections Subjected to Earthquake Loading," by M. Corazao and A.J. Durrani, 2/28/89, (PB90-109885/AS).

NCEER-89-0014 "Program EXKAL2 for Identification of Structural Dynamic Systems," by O. Maruyama, C-B. Yun, M. Hoshiya and M. Shinozuka, 5/19/89, (PB90-109877/AS).

NCEER-89-0015 "Response of Frames With Bolted Semi-Rigid Connections, Part I - Experimental Study and Analytical Predictions," by P.J. DiCorso, A.M. Reinhorn, J.R. Dickerson, J.B. Radziminski and W.L. Harper, 6/1/89, to be published.

NCEER-89-0016 "ARMA Monte Carlo Simulation in Probabilistic Structural Analysis," by P.D. Spanos and M.P. Mignolet, 7/10/89, (PB90-109893/AS).

NCEER-89-P017 "Preliminary Proceedings from the Conference on Disaster Preparedness - The Place of Earthquake Education in Our Schools," Edited by K.E.K. Ross, 6/23/89.

NCEER-89-0017 "Proceedings from the Conference on Disaster Preparedness - The Place of Earthquake Education in Our Schools," Edited by K.E.K. Ross, 12/31/89, (PB90-207895).

NCEER-89-0018 "Multidimensional Models of Hysteretic Material Behavior for Vibration Analysis of Shape Memory Energy Absorbing Devices, by E.J. Graesser and F.A. Cozzarelli, 6/7/89, (PB90-164146/AS).

NCEER-89-0019 "Nonlinear Dynamic Analysis of Three-Dimensional Base Isolated Structures (3D-BASIS)," by S. Nagarajaiah, A.M. Reinhorn and M.C. Constantinou, 8/3/89, (PB90-161936/AS).

NCEER-89-0020 "Structural Control Considering Time-Rate of Control Forces and Control Rate Constraints," by F.Y. Cheng and C.P. Pantelides, 8/3/89, (PB90-120445/AS).

NCEER-89-0021 "Subsurface Conditions of Memphis and Shelby County," by K.W. Ng, T-S. Chang and H-H.M. Hwang, 7/26/89, (PB90-120437/AS).

NCEER-89-0022 "Seismic Wave Propagation Effects on Straight Jointed Buried Pipelines," by K. Elhmadi and M.J. O'Rourke, 8/24/89, (PB90-162322/AS).

NCEER-89-0023 "Workshop on Serviceability Analysis of Water Delivery Systems," edited by M. Grigoriu, 3/6/89, (PB90-127424/AS).

NCEER-89-0024 "Shaking Table Study of a 1/5 Scale Steel Frame Composed of Tapered Members," by K.C. Chang, J.S. Hwang and G.C. Lee, 9/18/89, (PB90-160169/AS).

NCEER-89-0025 "DYNA1D: A Computer Program for Nonlinear Seismic Site Response Analysis - Technical Documentation," by Jean H. Prevost, 9/14/89, (PB90-161944/AS).

NCEER-89-0026 "1:4 Scale Model Studies of Active Tendon Systems and Active Mass Dampers for Aseismic Protection," by A.M. Reinhorn, T.T. Soong, R.C. Lin, Y.P. Yang, Y. Fukao, H. Abe and M. Nakai, 9/15/89, (PB90-173246/AS).

NCEER-89-0027 "Scattering of Waves by Inclusions in a Nonhomogeneous Elastic Half Space Solved by Boundary Element Methods," by P.K. Hadley, A. Askar and A.S. Cakmak, 6/15/89, (PB90-145699/AS).

NCEER-89-0028 "Statistical Evaluation of Deflection Amplification Factors for Reinforced Concrete Structures," by H.H.M. Hwang, J-W. Jaw and A.L. Ch'ng, 8/31/89, (PB90-164633/AS).

NCEER-89-0029 "Bedrock Accelerations in Memphis Area Due to Large New Madrid Earthquakes," by H.H.M. Hwang, C.H.S. Chen and G. Yu, 11/7/89, (PB90-162330/AS).

NCEER-89-0030 "Seismic Behavior and Response Sensitivity of Secondary Structural Systems," by Y.Q. Chen and T.T. Soong, 10/23/89, (PB90-164658/AS).

NCEER-89-0031 "Random Vibration and Reliability Analysis of Primary-Secondary Structural Systems," by Y. Ibrahim, M. Grigoriu and T.T. Soong, 11/10/89, (PB90-161951/AS).

NCEER-89-0032 "Proceedings from the Second U.S. - Japan Workshop on Liquefaction, Large Ground Deformation and Their Effects on Lifelines, September 26-29, 1989," Edited by T.D. O'Rourke and M. Hamada, 12/1/89, (PB90-209388/AS).

NCEER-89-0033 "Deterministic Model for Seismic Damage Evaluation of Reinforced Concrete Structures," by J.M. Bracci, A.M. Reinhorn, J.B. Mander and S.K. Kunnath, 9/27/89.

NCEER-89-0034 "On the Relation Between Local and Global Damage Indices," by E. DiPasquale and A.S. Cakmak, 8/15/89, (PB90-173865).

NCEER-89-0035 "Cyclic Undrained Behavior of Nonplastic and Low Plasticity Silts," by A.J. Walker and H.E. Stewart, 7/26/89, (PB90-183518/AS).

NCEER-89-0036 "Liquefaction Potential of Surficial Deposits in the City of Buffalo, New York," by M. Budhu, R. Giese and L. Baumgrass, 1/17/89, (PB90-208455/AS).

NCEER-89-0037 "A Determinstic Assessment of Effects of Ground Motion Incoherence," by A.S. Veletsos and Y. Tang, 7/15/89, (PB90-164294/AS).

NCEER-89-0038 "Workshop on Ground Motion Parameters for Seismic Hazard Mapping," July 17-18, 1989, edited by R.V. Whitman, 12/1/89, (PB90-173923/AS).

NCEER-89-0039 "Seismic Effects on Elevated Transit Lines of the New York City Transit Authority," by C.J. Costantino, C.A. Miller and E. Heymsfield, 12/26/89, (PB90-207887/AS).

NCEER-89-0040 "Centrifugal Modeling of Dynamic Soil-Structure Interaction," by K. Weissman, Supervised by J.H. Prevost, 5/10/89, (PB90-207879/AS).

NCEER-89-0041 "Linearized Identification of Buildings With Cores for Seismic Vulnerability Assessment," by I-K. Ho and A.E. Aktan, 11/1/89.

NCEER-90-0001 "Geotechnical and Lifeline Aspects of the October 17, 1989 Loma Prieta Earthquake in San Francisco," by T.D. O'Rourke, H.E. Stewart, F.T. Blackburn and T.S. Dickerman, 1/90, (PB90-208596/AS).

NCEER-90-0002 "Nonnormal Secondary Response Due to Yielding in a Primary Structure," by D.C.K. Chen and L.D. Lutes, 2/28/90.

NCEER-90-0003 "Earthquake Education Materials for Grades K-12," by K.E.K. Ross, 4/16/90.

NCEER-90-0004 "Catalog of Strong Motion Stations in Eastern North America," by R.W. Busby, 4/3/90.

NCEER-90-0005 "NCEER Strong-Motion Data Base: A User Manuel for the GeoBase Release (Version 1.0 for the Sun3)," by P. Friberg and K. Jacob, 3/31/90.

NCEER-90-0006 "Seismic Hazard Along a Crude Oil Pipeline in the Event of an 1811-1812 Type New Madrid Earthquake," by H.H.M. Hwang and C-H.S. Chen, 4/16/90.

NCEER-90-0007 "Site-Specific Response Spectra for Memphis Sheahan Pumping Station," by H.H.M. Hwang and C.S. Lee, 5/15/90.

NCEER-90-0008 "Pilot Study on Seismic Vulnerability of Crude Oil Transmission Systems," by T. Ariman, R. Dobry, M. Grigoriu, F. Kozin, M. O'Rourke, T. O'Rourke and M. Shinozuka, 5/25/90.

NCEER-90-0009 "A Program to Generate Site Dependent Time Histories: EQGEN," by G.W. Ellis, M. Srinivasan and A.S. Cakmak, 1/30/90.

NCEER-90-0010 "Active Isolation for Seismic Protection of Operating Rooms," by M.E. Talbott, Supervised by M. Shinozuka, 6/8/9.

NCEER-90-0011 "Program LINEARID for Identification of Linear Structural Dynamic Systems," by C-B. Yun and M. Shinozuka, 6/25/90.

NCEER-90-0012 "Two-Dimensional Two-Phase Elasto-Plastic Seismic Response of Earth Dams," by A.N. Yiagos, Supervised by J.H. Prevost, 6/20/90.

NCEER-90-0013 "Secondary Systems in Base-Isolated Structures: Experimental Investigation, Stochastic Response and Stochastic Sensitivity," by G.D. Manolis, G. Juhn, M.C. Constantinou and A.M. Reinhorn, 7/1/90.

NCEER-90-0014 "Seismic Behavior of Lightly-Reinforced Concrete Column and Beam-Column Joint Details," by S.P. Pessiki, C.H. Conley, P. Gergely and R.N. White, 8/22/90.

NCEER-90-0015 "Two Hybrid Control Systems for Building Structures Under Strong Earthquakes," by J.N. Yang and A. Danielians, 6/29/90.

NCEER-90-0016 "Instantaneous Optimal Control with Acceleration and Velocity Feedback," by J.N. Yang and Z. Li, 6/29/90.

NCEER-90-0017 "Reconnaissance Report on the Northern Iran Earthquake of June 21, 1990," by M. Mehrain, 10/4/90.

NCEER-90-0018 "Evaluation of Liquefaction Potential in Memphis and Shelby County," by T.S. Chang, P.S. Tang, C.S. Lee and H. Hwang, 8/10/90.

NCEER-90-0019 "Experimental and Analytical Study of a Combined Sliding Disc Bearing and Helical Steel Spring Isolation System," by M.C. Constantinou, A.S. Mokha and A.M. Reinhorn, 10/4/90.

NCEER-90-0020 "Experimental Study and Analytical Prediction of Earthquake Response of a Sliding Isolation System with a Spherical Surface," by A.S. Mokha, M.C. Constantinou and A.M. Reinhorn, 10/11/90.

NCEER-90-0021 "Dynamic Interaction Factors for Floating Pile Groups," by G. Gazetas, K. Fan, A. Kaynia and E. Kausel, 9/10/90.

NCEER-90-0022 "Evaluation of Seismic Damage Indices for Reinforced Concrete Structures," by S. Rodríguez-Gómez and A.S. Cakmak, 9/30/90.

NCEER-90-0023 "Study of Site Response at a Selected Memphis Site," by H. Desai, S. Ahmad, G. Gazetas and M.R. Oh, 10/11/90.

NCEER-90-0024 "A User's Guide to Strongmo: Version 1.0 of NCEER's Strong-Motion Data Access Tool for PCs and Terminals," by P.A. Friberg and C.A.T. Susch, 11/15/90.

NCEER-90-0025 "A Three-Dimensional Analytical Study of Spatial Variability of Seismic Ground Motions," by L-L. Hong and A.H.-S. Ang, 10/30/90.

NCEER-90-0026 "MUMOID User's Guide - A Program for the Identification of Modal Parameters," by S. Rodríguez-Gómez and E. DiPasquale, 9/30/90.

NCEER-90-0027 "SARCF-II User's Guide - Seismic Analysis of Reinforced Concrete Frames," by S. Rodríguez-Gómez, Y.S. Chung and C. Meyer, 9/30/90.

NCEER-90-0028 "Viscous Dampers: Testing, Modeling and Application in Vibration and Seismic Isolation," by N. Makris and M.C. Constantinou, 12/20/90.

NCEER-90-0029 "Soil Effects on Earthquake Ground Motions in the Memphis Area," by H. Hwang, C.S. Lee, K.W. Ng and T.S. Chang, 8/2/90.

NCEER-91-0001 "Proceedings from the Third Japan-U.S. Workshop on Earthquake Resistant Design of Lifeline Facilities and Countermeasures for Soil Liquefaction, December 17-19, 1990," edited by T.D. O'Rourke and M. Hamada, 2/1/91.

NCEER-91-0002 "Physical Space Solutions of Non-Proportionally Damped Systems," by M. Tong, Z. Liang and G.C. Lee, 1/15/91.