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ALTERNATIVES TO STANDARD MODE SUPERPOSITION FOR ANALYSIS OF NON-CLASSICALLY DAMPED SYSTEMS

by

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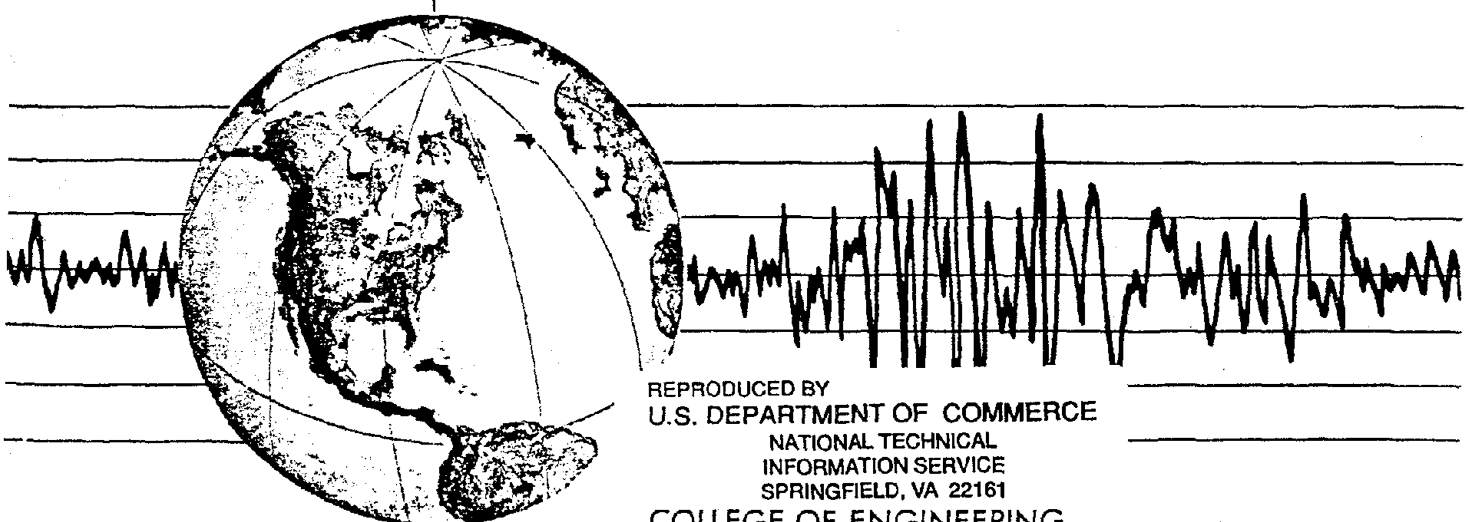

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A 2-DOF system is solved in closed form, using the standard method and each of the three alternatives. The analysis is extended to MDOF systems: the appropriate conditions for the approaches to be valid are obtained. Using the different procedures, the response of a 9-story building undergoing a single sine wave impulse in the basement is calculated.

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ALTERNATIVES TO STANDARD MODE SUPERPOSITION FOR ANALYSIS OF NON-CLASSICALLY DAMPED SYSTEMS

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ABSTRACT

The dynamic response of systems with nonclassical damping may be solved exactly by mode superposition, using the complex mode shapes derived from the damped eigenproblem to uncouple the equations of motion. However, to reduce the computational effort, an approximate procedure that avoids the complex mode solution often is preferred, in which the undamped mode shapes based on the system mass and stiffness matrices are applied as generalized coordinates, and the resulting coupling terms in the generalized damping matrix are merely ignored. In this presentation two other approximate procedures that parallel this are described: (1) the mass matrix is transformed using eigenvectors based on the stiffness and damping matrices, and (2) the stiffness matrix is transformed using eigenvectors based on the damping and mass matrices; the transformed equation sets are then uncoupled by ignoring the coupling coefficients of the generalized mass and stiffness matrices, respectively. A third procedure also is presented that involves the use of corrected diagonal terms in the transformed property matrices, taking into account contributions based on the off-diagonal terms.

To demonstrate the accuracy of these approximations, a 2-DOF system is solved in closed form, using the standard method and each of the three alternatives; results are compared with the exact solution obtained using the complex eigenvectors.

The behaviour of general 2-DOF damped systems under various conditions is considered with respect to the conventional and proposed decompositions. Free vibration decay and forced vibrations of an assumed "structure-equipment" system are studied as illustrative examples.

The analysis is extended to MDOF systems; the appropriate conditions for the approaches to be valid are obtained. The response of a 9-story building undergoing a single sine wave impulse in the basement are calculated using the different approximate procedures.

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1. INTRODUCTION

The discrete equations of motion expressing the behaviour of a dynamically loaded structure may be written as follows:

$$M\ddot{V} + C\dot{V} + KV = Q \,, \tag{1}$$

in which V is the displacement vector, M and K are the mass and stiffness matrices, respectively, Q is the applied load vector, and C represents the viscous damping matrix. In most cases it may be assumed that the system has classical damping, in which case the coordinate transformation based on the undamped system eigenvectors will lead to diagonalization of the generalized coordinate damping matrix in the same way that it produces diagonalized generalized mass and stiffness matrices. Thus, for a classically damped system, the modal coordinate transformation leads to a set of independent modal equations; the dynamic response then may be obtained by solving separately these single-degree-of-freedom modal equations and superposing the modal responses to obtain the total response.

In some situations, however, it is not reasonable to assume that the system is classically damped; then the damping matrix will not be diagonalized by the undamped modal coordinate transformation. In such cases, an exact solution may be obtained by mode superposition if the damped eigenproblem is solved for the complex mode shapes. The orthogonality properties of these damped mode shapes are such that they serve to diagonalize the mass, damping and stiffness matrices when they are utilized in a coordinate transformation. Thus the modal response equations are uncoupled, and the total response may be obtained by solving the independent equations and superposing the results [1-3]. In principle this complex modal coordinate procedure will provide the exact solution for any nonclassically damped structure; however, it has the major disadvantages that the order of the eigenproblem to be solved to get the mode shapes is doubled, and that the mode shapes contain imaginary as well as real terms.

One alternative to this complex mode shape solution is to use the undamped mode shapes in transforming to modal coordinates the nonclassically damped equations of motion. This leads to the same diagonalization of the mass and stiffness matrices as is obtained with a classically damped structure; however, the damping matrix is not diagonalized and the off-diagonal terms in the generalized damping matrix provide coupling between the modal response equations. These coupled modal equations can then be solved simultaneously by standard step-by-step dynamic analysis procedures [4].

In most cases this coordinate transformation is very useful in that it permits a great reduction in the number of equations to be solved simultaneously even though it does not lead to uncoupled equations; the reduction is possible because usually only the first several modal coordinates contribute significantly to the response. However, a major disadvantage of this approach is that the coupled modal equations cannot be solved by the response spectrum method. For this reason an approximate solution sometimes is obtained by uncoupling the modal response equations by neglecting the off-diagonal coefficients in the generalized damping matrix. Omitting these terms introduces an error in the dynamic response results, but it has been found in many cases that the error is small enough to be acceptable [5].

The purpose of this paper is to describe some alternatives to the approximate decoupling procedure explained above. It is apparent in Eq.(1) that there are two possible equivalents to the undamped eigenproblem which results from setting $C = 0$: the zero mass eigenproblem obtained by setting $M = 0$ and the zero stiffness eigenproblem obtained if $K = 0$. Eigenvectors calculated from each of these eigenproblems can be used to perform a coordinate transformation of the equations of motion (Eq.1), leading to diagonalization of the generalized damping and stiffness matrices in the first case and of the generalized damping and mass matrices in the second case. Off-diagonal coefficients remain in the generalized mass and stiffness matrices in the two cases, respectively, and an approximate mode superposition solution can be obtained in each case by neglecting these coupling coefficients. An additional approximation procedure also will be discussed briefly, in which the off-diagonal terms of the generalized damping matrix are used to derive modified diagonal elements of the approximately uncoupled damping matrix.

In the following, the general principles of such approximate modal uncoupling procedures will be explained first; then the concepts will be applied to a general 2-DOF system for which closed-form solutions can be derived for the error resulting from neglect of the generalized modal coupling coefficients. Finally, the approximate procedures will be applied to a multi-degree-of-freedom system to give a general example of the type of results that may be obtained.

2. APPROXIMATE MODAL UNCOUPLING

A-Superposition

For convenience in referring to the alternative uncoupling procedures, the standard approximate mode superposition analysis based on neglecting modal damping coupling coefficients will be denoted the A-Superposition method. Neglecting the damping matrix, the free vibration eigenproblem associated with Eq.(1) may be written

$$-\Lambda_a \mathbf{V} = \mathbf{A}\mathbf{V} \tag{2}$$

where

$$\mathbf{A} = \mathbf{M}^{-1}\mathbf{K} \quad \text{and} \quad -\lambda_{ja} = \omega_{ja}^2 \ .$$

The minus sign is used with the eigenvalue λ_{ja} for generality in describing this and the subsequent approximations.

As is well known, the matrix A of Eq.(2) may be decomposed into its eigenvalues and eigenvectors, thus

$$\Phi_a \Lambda_a \Phi_a^{-1} = -\mathbf{A} \tag{3}$$

where the eigenvectors Φ_a have been normalized so that

$$\begin{aligned} \Phi_a^T \mathbf{M} \Phi_a &= \mathbf{I}\ , \\ \Phi_a^T \mathbf{K} \Phi_a &= -\Lambda_a \end{aligned} \tag{4}$$

in which Λ_a is a diagonal array of the eigenvalues.

Now using the mode shapes Φ_a to transform the coordinates

$$\mathbf{V} = \Phi_a \mathbf{a} \tag{5}$$

in which a is the vector of modal coordinate amplitudes, Eq.1 may be transformed to the set of modal equations

$$\ddot{\mathbf{a}} + \mathbf{C}_a \dot{\mathbf{a}} - \Lambda_a \mathbf{a} = \mathbf{Q}_a \tag{6}$$

where the generalized load vector Q_a is given by

$$\mathbf{Q}_a = \Phi_a^T \mathbf{Q}$$

and the generalized damping matrix is given by

$$\mathbf{C}_a = \Phi_a^T \mathbf{C} \Phi_a \quad . \tag{7}$$

Because the damping matrix C is assumed to be nonclassical, this modal damping matrix includes off-diagonal coefficients that couple the modal equations, Eq.6.

As was described above, an approximate uncoupled set of equations may be obtained by neglecting the off-diagonal coefficients in $\mathbf{C}_a$. The corresponding modal frequencies ω_{ja}^o and damping ratios ξ_{ja}^o may then be expressed as follows:

$$\omega_{ja}^o = i\sqrt{\lambda_{ja}} \tag{7a}$$

$$\xi_{ja}^o = \frac{C_{jja}}{2\omega_{ja}^o} \quad . \tag{7b}$$

Finally, the approximate response of the system in physical coordinates is given by

$$\mathbf{v} = \Phi_a \mathbf{a}^o \tag{7c}$$

where $\mathbf{a}^o$ is the vector of solutions of the uncoupled modal equations.

B-Superposition

The corresponding B-Superposition formulation is developed by omitting the mass matrix from the homogeneous equations of motion with the result

$$-\Lambda_b \mathbf{V} = \mathbf{B}\mathbf{V} \tag{8}$$

where

$$\mathbf{B} = \mathbf{C}^{-1}\mathbf{K} \, .$$

Following the standard procedure, the B matrix now may be decomposed into its eigenvalues and eigenvectors , thus

$$\Phi_b \Lambda_b \Phi_b^{-1} = -\mathbf{B} \tag{9}$$

in which the eigenvectors are normalized so that

$$\Phi_b^T \mathbf{C} \Phi_b = \mathbf{I}$$

$$\Phi_b^T \mathbf{K} \Phi_b = -\Lambda_b \tag{10}$$

where Λ_b is the diagonal eigenvalue array.

Now using the coordinate transformation

$$\mathbf{V} = \boldsymbol{\Phi}_b \mathbf{b} \tag{11}$$

the equation of motion in terms of the generalized coordinates **b** becomes

$$\mathbf{M}_b \ddot{\mathbf{b}} + \dot{\mathbf{b}} - \boldsymbol{\Lambda}_b b = \mathbf{Q}_b \ , \tag{12}$$

in which the generalized load vector is

$$\mathbf{Q}_b = \boldsymbol{\Phi}_b^T \mathbf{Q}$$

and the non-diagonal generalized mass matrix is

$$\mathbf{M}_b = \boldsymbol{\Phi}_b^T \mathbf{M} \boldsymbol{\Phi}_b \ . \tag{13}$$

Eq.(12) may now be approximately uncoupled by neglecting the off-diagonal coefficients of $\mathbf{M}_b$; then the approximate values of the "modal" eigenfrequencies and modal damping ratios can be expressed as follows:

$$\omega_{jb}^o = i\sqrt{M_{jjb}^{-1}\lambda_{jb}} \tag{13a}$$

$$\xi_{ib}^o = \frac{1}{2M_{jjb}\,\omega_{jb}^o} \tag{13b}$$

Consequently, the final approximate response is

$$\mathbf{v} = \boldsymbol{\Phi}_b \, b^o \tag{13c}$$

where b^o is a vector of solutions of Eq.12 neglecting the off-diagonal terms of the M_b matrix.

D-Superposition

The D-Superposition is performed similarly by omitting the stiffness matrix from Eq.(1), which leads to

$$-\Lambda_d \mathbf{V} = \mathbf{D}\mathbf{V} \tag{14}$$

in which

$$\mathbf{D} = \mathbf{M}^{-1}\mathbf{C} \ .$$

D then is decomposed into its eigenvalues Λ_d and eigenvectors Φ_d

$$\Phi_d \Lambda_d \Phi_d^{-1} = -\mathbf{D} \tag{15}$$

with the vectors normalized so that

$$\begin{aligned} \Phi_d^T \mathbf{M} \Phi_d &= \mathbf{I} \\ \Phi_d^T \mathbf{C} \Phi_d &= -\Lambda_d \quad . \end{aligned} \tag{16}$$

Now the coordinate transformation $\mathbf{V} = \Phi_d \, \mathbf{d}$ transforms Eq.1 to the following generalized coordinate form

$$\ddot{\mathbf{d}} - \Lambda_d \dot{\mathbf{d}} + \mathbf{K}_d \mathbf{d} = \mathbf{Q}_d \tag{17}$$

where the generalized load vector is

$$\mathbf{Q}_d = \Phi_d^T \mathbf{Q}$$

and the non-diagonal generalized stiffness matrix is

$$\mathbf{K}_d = \Phi_d^T \mathbf{K} \Phi_d \quad . \tag{18}$$

The approximate uncoupling is achieved by ignoring the off-diagonal coefficients of this matrix. Eigenfrequencies and corresponding modal damping ratios are evaluated according to the approximations

$$\omega_{jd}^o = \sqrt{K_{jjd}} \tag{18a}$$

$$\xi_{jd}^o = -\frac{\lambda_{jd}}{2\omega_{jd}^o} \quad . \tag{18b}$$

Finally, the total response vector of the system in physical coordinates is obtained from contributions of all modes

$$\mathbf{v} = \Phi_d \mathbf{d}^o \tag{18c}$$

where $\mathbf{d}^o$ is the vector of modal responses obtained from Eq.17 when the off-diagonal terms in the transformed stiffness matrix are neglected.

R-Superposition.

A modification of the A-Superposition method that has been applied in some cases [6,7], is to compensate for neglecting the off-diagonal terms of the transformed damping matrix by making adjustments to the diagonal terms. For a two-degree-of-freedom system, eigenfrequencies of the A - Superposition are used as the basis for the eigenfrequency calculation; these eigenfrequencies then give a better approximation of the damping ratios. This concept will be called R-Superposition; it will be demonstrated in the following as a modification of A-Superposition, but in principle the same idea could be applied with each of the approximation methods.

3. CLOSED FORM SOLUTION FOR 2-DOF SYSTEM

The error that results when the A-Superposition approximation is applied has been the subject of many investigations [4,6-11]. A major purpose of the present work is to evaluate similarly the other proposed methods of mode superposition and to estimate the relative approximation errors in all the methods under consideration.

The analysis of a two-degree-of-freedom (2-DOF) system is used for a demonstration example because it permits a closed form solution. In the following, comparisons will be made between the exact analysis obtained by solving the complex eigenproblem, and results derived from the various approximation procedures. Quantities that are compared include first the free vibration parameters and second the amplitudes of forced harmonic response. Finally, a specific numerical example is presented.

3.1. Free Vibration Parameters

The general 2-DOF system has property matrices of the following form

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \qquad (19)$$

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} .$$

<u>Complex mode superposition method</u> (<u>exact solution</u>)

The characteristic equation of this system may be written in Foss's form [12]

$$\begin{vmatrix} \mathbf{M}\lambda + \mathbf{C} & \mathbf{K} \\ \mathbf{K} & -\mathbf{K}\lambda \end{vmatrix} = 0 \qquad (20)$$

and after certain simplifications may be represented by the fourth order polynomial equation

$$\Delta_m \lambda^4 + \Delta_{mc} \lambda^3 + (\Delta_c + \Delta_{mk}) \lambda^2 + \Delta_{ck} \lambda + \Delta_k = 0 \qquad (21)$$

where

$$\Delta_m = \det \mathbf{M} , \quad \Delta_k = \det \mathbf{K} , \quad \Delta_c = \det \mathbf{C} ,$$
$$\Delta_{mc} = \det (\mathbf{M} + \mathbf{C}) - \Delta_m - \Delta_c ,$$
$$\Delta_{mk} = \det (\mathbf{M} + \mathbf{K}) - \Delta_m - \Delta_k ,$$
$$\Delta_{ck} = \det (\mathbf{C} + \mathbf{K}) - \Delta_c - \Delta_k .$$

This is a linear algebraic equation with constant coefficients which has two pairs of conjugate roots in the complex plane

$$\begin{aligned} \lambda_{1,2} &= \alpha_1 \pm i \beta_1 , \\ \lambda_{3,4} &= \alpha_2 \pm i \beta_2 . \end{aligned} \qquad (22)$$

These may be expressed in terms of the traditional free vibration parameters, undamped frequency ω and modal damping ratio ξ , by transforming the complex plane from rectangular coordinates to polar coordinates as follows:

$$\begin{aligned} \beta &= \omega \cos\rho , \\ \alpha &= \omega \sin\rho , \end{aligned} \qquad (23)$$

where the polar angle is related to the damping ratio as

$$\rho = -arcsin \, \xi .$$

The negative sign in the last expression is explained in complex eigenfunction theory.

Then, the real and imaginary parts of the characteristic equation, in terms of the polar parameters reduce to

$$\omega^4 \Delta_m (7 - 6\xi^2) + \omega^2 \left\{ (\Delta_c + \Delta_{mk}) + 2\xi\omega\Delta_{mc} \right\} + \Delta_k = 0 ,$$
$$(24)$$
$$2\omega^3 (1 - 2\xi^2) \Delta_m + \omega^2 (1 - 4\xi^2 \Delta) \Delta_{mc} - 2\xi\omega (\Delta_c + \Delta_{mk}) + \Delta_{ck} = 0 .$$

If we take into account that for many engineering problems

$$\xi^2 \ll 0.25 \,,$$

the last pair of equations may be rewritten

$$\omega^4\Delta_m + \omega^2 \left\{ (\Delta_c + \Delta_{mk}) + 2\xi\omega\Delta_{mc} \right\} + \Delta_k = 0 \,, \qquad (25)$$
$$2\omega^3\Delta_m + \omega^2\Delta_{mc} - 2\xi\omega(\Delta_c + \Delta_{mk}) + \Delta_{ck} = 0 \,.$$

Two levels of approximation

In the evaluation of approximation errors that result from the neglect of off-diagonal terms in the generalized property matrices we note that two levels of approximation may be identified due to the fact that the uncoupled equation system has two free vibration parameters in each mode - frequency and damping ratio. In the simplest approach or first level of approximation, both frequencies and damping ratios are evaluated directly from the approximately uncoupled equations, as was described in Chapter 2. Thus, in the standard method (A-Superposition) the first level of approximation for frequencies and damping ratios is given by Eqs.7a and 7b, respectively; i.e.

$$\omega_{1,2} = \omega^o_{1,2} \,, \qquad (26)$$

and

$$\xi_{1,2} = \xi^o_{1a,2a} \qquad (27)$$

where superscript o is used for designating the approximate value of those parameters.

In the second level of approximation for A-Superposition, the eigenfrequencies still are given by Eq.26, but the damping ratios are calculated from the diagonal elements of the uncoupled damping matrix together with these approximate frequency values, as was mentioned in Chapter 2 in the description of the R-Superposition method.

A-Superposition (standard approximate solution)

The equations of motion (Eq.1) describe vibrations of a corresponding conservative system if we set C = 0, Q = 0. In order to determine eigenfunctions of the matrix A it is necessary to consider first Eq.2, from which the following characteristic equation is obtained:

$$\Delta_m \lambda_a^4 + \Delta_{mk} \lambda_a^2 + \Delta_k = 0 \quad . \tag{28}$$

Eq.28 can also be derived from Eq.21, if

$$\Delta_c = \Delta_{mc} = \Delta_{ck} = 0 \, .$$

Since the property matrices are positive definite, the biquadratic equation (28) has a discriminant

$$D_a = \Delta_{mk}^2 - 4 \Delta_m \Delta_k =$$

$$= m_{12}^2 k_{12}^2 \left\{ (m_2^{r^{-1}} k_1^{r^{-1}} - m_1^{r^{-1}} k_2^{r^{-1}})^2 + 4 (m_1^{r^{-1}} - k_1^{r^{-1}}) (m_2^{r^{-1}} - k_2^{r^{-1}}) \right\} \tag{29}$$

which is non-negative and less than the square of the second coefficient. The squares of the roots of the equation are real and negative, so the roots themselves are imaginary. In the above expressions, the following notation was adopted for the ratios of the off-diagonal terms to the diagonal terms of the matrices

$$m_j^r = \frac{m_{12}}{m_{jj}} , \quad c_j^r = \frac{c_{12}}{c_{jj}} , \quad k_j^r = \frac{k_{12}}{k_{jj}} \quad (j = 1,2) \, .$$

Calculation of the eigenvalues and eigenvectors, corresponding to matrix A, is carried out using the standard algorithm. If A-Superposition is used to change the equation of motion in physical coordinates (Eq.1) to the general coordinate form (Eq.6), then the characteristic equation to obtain the complex eigenvalues becomes

$$(\lambda^2 - \lambda_{1a}^2 + C_{11a} \lambda) (\lambda^2 - \lambda_{2a}^2 + C_{22a} \lambda) - C_{12a} C_{21a} \lambda^2 = 0 \tag{30}$$

where C_{11a}, C_{22a} are the elements of the principal diagonal, and C_{12a}, C_{21a} are the off-diagonal elements of the transformed damping matrix; also λ_{1a} , λ_{2a} are eigenvalues shown in the matrix (Eq.4).

Note that Eq.30 is identical to Eq.25, thus exact values of the eigenfrequencies and damping ratios are obtained by solving the latter equation.

After some calculation, the characteristic equation (Eq.30), which has a complex argument of the form (23), can be written as

$$\omega^4 (1 - 4\xi^2) - \omega^2 \left\{ \omega_{1a}^2 + \omega_{2a}^2 - 2\xi\omega (C_{11a} + C_{22a}) - \Delta_c \right\} + \omega_{1a}^2 \omega_{2a}^2 = 0 \tag{31}$$

As may be concluded from the expression in the brackets, approximation (26) is valid if the following conditions are imposed

$$\begin{aligned} \xi^2 &<< 0.25 \\ \Delta_c^a + 2\xi\ \omega (C_{11a} + C_{22a}) &<< \omega_{1a}^2 + \omega_{2a}^2 \end{aligned} \tag{32}$$

in which the determinant of the transformed damping matrix is denoted as

$$\Delta_c^a = \det \mathbf{C}_a = C_{11a} C_{22a} - C_{21a} C_{12a} \ .$$

Finally, the error of the frequency calculations is introduced as a new parameter, defined as follows:

$$\left| \mu_{ja} \right| = \left| \frac{C_{jja}^2 - C_{12a} C_{21a}}{\omega_{1a}^2 + \omega_{2a}^2} \right| << 1 , \quad (j = 1, 2) \ . \tag{33}$$

If the first expression (Eq.26) is precise enough for engineering calculation, the damping ratios are easily calculated from

$$\xi_{1a,2a}^r = \frac{C_{11a} C_{22a} - C_{12a} C_{21a}}{2\omega_{1a,2a} (C_{11a} + C_{22a})} \tag{34}$$

But the second expression (Eq.27) implies the damping ratio expressions

$$\xi_{1a,2a}^o = \frac{C_{11a} C_{22a}}{2\omega_{1a,2a} (C_{11a} + C_{22a})} \ . \tag{35}$$

Hence, the damping ratio errors that result from using the first level of approximation may be evaluated according to the formula

$$\zeta_a = \frac{\xi_a^r - \xi_a^o}{\xi_a^o} = c_1^r c_2^r \tag{36}$$

where ratios of the off-diagonal and the diagonal terms of the transformed damping matrix are denoted as

$$c_1^r = \frac{C_{21a}}{C_{11a}} , \quad c_2^r = \frac{C_{12a}}{C_{22a}} \ . \tag{37}$$

B-Superposition

The first alternative eigenproblem (8) applied to the case of a 2-DOF system leads to the quadratic characteristic equation

$$\Delta_c \lambda_b^2 + \Delta_{ck} \lambda_b + \Delta_k = 0\,. \tag{38}$$

Two eigenvalues and two corresponding eigenvectors are determined by solving this equation, in which the discriminant is

$$D_b = c_{12}^2 k_{12}^2 \left\{ (c_1^{r-1} k_2^{r-1} - c_2^{r-1} k_1^{r-1})^2 + 4\,(c_1^{r-1} - k_1^{r-1})\,(c_2^{r-1} - k_2^{r-1}) \right\}. \tag{39}$$

The same notation as used in the Eq.29 is adopted here. Also the same syllogism applied before leads to the same conclusion about the negative roots of Eq.(38).

According to the new proposed approach, the free vibration parameters are to be defined from the equation of motion transformed to the B -coordinates as shown by Eq.(12), while the off-diagonal terms of M_b (which induce coupling of the equations) are neglected. Thus, the simplified formulas for determination of frequency and damping ratio are as follows:

$$\omega_{jb}^o = i\ \lambda_{jb} M_{jjb}^{-1}\ ,$$
$$\xi_{jb}^o = (\ 2M_{jjb}\,\omega_{jb}\ )^{-1} \qquad (j=1,2) \quad (40)$$

An effective tool for verifying the accuracy of the method is to compare the values obtained with these from the complex mode solution. The transformation to the B-domain or, in another words, from source equations of motion to transformed ones, does not change the roots of the characteristic equation. Hence, as in the standard method, the complex roots in polar coordinates can be determined from the transformed characteristic equation

$$\omega^4 \left\{ (1 + 4\xi^2)\Delta_m^b - 8\xi^2(1-\xi^2) M_{11b} M_{22b} \right\}$$
$$+ \omega^2 \left\{ (M_{11b}\lambda_{2b} + M_{22b}\lambda_{1b}) + 2\xi\omega\,(\,M_{11b} M +_{22b}\,) -1 \right\} + \lambda_{1b}\lambda_{2b} = 0\,.$$

The assumption that the approximate eigenfrequencies satisfy the last equation leads to two conditions: the first is the same as (Eq.32), and the second is

$$\left|\mu_{jb}\right| = \left|\frac{1}{M_{jjb}^2\ (\ \omega_{1b}^2\ + \omega_{2b}^2\ )}\right| << 1\ . \qquad (41)$$

The values of the second level approximation damping ratios for each of the modes then are derived from the formulas

$$\xi_{jb}^r = \frac{1 + \omega_{jb}^2 M_{21b} M_{12b}}{2\omega_{jb}\ (\ M_{11b}\ + M_{22b}\ )}\ , \qquad (\ j = 1,2\ )\ . \qquad (42)$$

On the other hand, if the non-diagonal terms of the transformed mass matrix are assumed to be zero, for the first level approximation the damping ratios are

$$\xi_{jb}^o = \frac{1}{2M_{jjb}\,\omega_{jb}} \qquad (\ j = 1,2\ )\ . \qquad (43)$$

Thus, the errors for the less refined assumption are evaluated by

$$\varsigma_{jb} = \frac{\xi_{jb}^r - \xi_{jb}^o}{\xi_{jb}^o} = M_{12b} M_{21b}\, \omega_{jb}^2\ . \qquad (44)$$

D-Superposition

Considering now the D-Superposition type of eigenproblem (Eq.14), one obtains another pair of basis vectors by solving the characteristic equation

$$\Delta_m \lambda_d^4 + \Delta_{mc} \lambda_d^3 + \Delta_c \lambda_d^2 = 0\ , \qquad (45)$$

Retaining only the non-singular roots, this equation may be rewritten as a quadratic equation with a discriminant

$$D_d = (\ m_{12} c_{12}\ )^2 \left\{ (\ m_{11}^{r^{-1}} c_{22}^{r^{-1}} - m_{22}^{r^{-1}} c_{11}^{r^{-1}}\ )^2 + 4\ (\ m_{11}^{r^{-1}} - c_{11}^{r^{-1}})(\ m_{22}^{r^{-1}} - c_{22}^{r^{-1}}\ ) \right\}. \qquad (46)$$

Eigenvalues of the D matrix are again negative because of the positive definite property matrices, as was indicated above.

According to the D-Superposition approach, the original equation (Eq.1) is transformed to the form given by Eq.(17); then neglecting the off-diagonal terms of the transformed stiffness matrix allows the separate solution of the uncoupled modal equations. Now, it is relatively easy to obtain free vibration parameters from each of the uncoupled transformed equations as follows

$$\omega_{jd}^{o} = \sqrt{k_{jj}} \qquad \xi_{jd}^{o} = -\frac{\lambda_{jd}}{2\omega_{jd}} \qquad (j = 1,2) . \tag{47}$$

The exact solution of the transformed equations by the complex mode method gives a characteristic equation (45) in the D-superposition general coordinates of the form

$$\omega^4 (1 - 4\xi^{2)} + \omega^2 \Big\{ 2\alpha (\lambda_{1d} + \lambda_{2d}) - \lambda_{1d}\lambda_{2d} - K_{11d} - K_{22d} + \xi^2 (\lambda_{1d} + \lambda_{2d} - \lambda_{1d}\lambda_{2d}) \Big\} + \Delta_k^d = 0 \tag{48}$$

where

$$\Delta_k^d = K_{11d}K_{22d} - K_{21d}K_{12d} .$$

To verify the first assumption (26) we note that the eigenfrequencies of the simplified system will satisfy the characteristic equation (42) only when conditions (32) and the following expression

$$\left| \mu_{jd} \right| = - \left| \frac{4\xi_{jd}^2 \omega_{jd}^2}{\omega_{1d}^2 + \omega_{2d}^2} \right| << 1 , \quad (j = 1,2) . \tag{49}$$

are satisfied. The corresponding damping ratios are given by

$$\xi_{jd}^{r} = - \frac{\lambda_{1d}\lambda_{2d}\omega_{jd}^2 + K_{12d}K_{21d}}{2\omega_{jd}^3 (\lambda_{1d} + \lambda_{2d})} , \quad (j = 1,2) . \tag{50}$$

Finally, the parameter to evaluate the second assumption (damping ratio) errors, analogous to those from the previous approximation procedures, is expressed by

$$\varsigma_{jd} = \frac{\xi_{jd}^{r} - \xi_{jd}^{o}}{\xi_{jd}^{o}} = \frac{K_{12d}K_{21d}}{\lambda_{1d}\lambda_{2d}\omega_{jd}^2} , \quad (j = 1,2) .$$

R-Superposition

As was emphasized above, a revised superposition method could be derived from any of the approximation methods (A-, B-, or D - Superposition) by extending it to the second level of approximation. In this study , only the extension based on the standard method will be considered because it has been used most widely in engineering practice. Thus, R - Superposition here is nothing more than the second level of approximation of A - Superposition, as described above. In this case, it is convenient to express the second level damping ratio as a correction of the first level approximation, as expressed by Eqs.34 and 35, respectively. The improved damping ratio approximations thus are given by

$$\xi_{ja}^{c} = \xi_{ja}^{o} \left\{ 1 - \frac{C_{12a}C_{21a}}{C_{11a}C_{22a}} \right\} \quad . \tag{51}$$

Remarks

For further development of the 2DOF system response to various loadings, the following conclusions from this preliminary general analysis may be useful:

(1) The accuracy of the approximately determined eigenfrequencies (33) obtained by the standard (A-Superposition) method depends on the terms which are neglected in the transformed damping matrix, whereas the accuracy of the eigenfrequencies derived from the B-and D-transformations (41) and (49) do not depend directly on the corresponding terms.

(2) The errors in damping ratios do not depend on the eigenfrequencies for the standard method, but they increase as the frequency increases for the B - Superposition method and they decrease with frequency for the D- Superposition method. Consequently, it may be recognized that the first of the newly proposed methods gives approximate parameters which match better to the correct values for the lower harmonics ,while the second proposed method gives better results for the higher harmonics;

(3) All the methods could be improved by using two levels of approximation, that is, by using the frequencies from the first level to calculate the damping ratios for the second level (as illustrated by R - Superposition based on Eq.34).

3.2. Forced Harmonic Vibration Parameters.

For completeness of the comparative analysis it is desirable to formulate the frequency response characteristics of all the simplified systems, because it is rather difficult to predict the role of the neglected terms in the final behaviour of a system from the free vibration parameters only. The paper by Duncan P.E. and Taylor R.E. [10] has investigated 2DOF systems already transformed by the standard method and with very a specific damping property. That analysis shows the possibility of significant errors in determination of the second harmonic mode.

The estimation of the degree of modal coupling causing the errors in the different approximate procedures is most precisely carried out in general coordinates. In this way, it is possible to observe the contribution error from the neighbour mode and to get simpler closed form expressions for the mode amplitudes. Hence, the purpose of the analysis herein is to plot amplitude functions of the two principal modes in the two harmonic regimes for all of the considered methods.

A-Superposition.

After the A-transformation (standard method) has been applied, the source equation of motion (Eq.1) for harmonic loading becomes

$$\ddot{\mathbf{a}} + C_a \dot{\mathbf{a}} - \Lambda_a \mathbf{a} = q_a e^{i\omega t} \tag{52}$$

where

$$\mathbf{q}_a = \left\{ q_{ja} \right\} .$$

The solution of this equation with a full transformed damping matrix gives an amplitude vector of the form

$$A_a = \begin{Bmatrix} A_{1a} \\ A_{2a} \end{Bmatrix} \tag{53}$$

in which the elements are determined according to the conventional Gauss procedure with determinants of the form

$$\Delta_a = \begin{vmatrix} (-\omega^2 + iC_{11a}\omega - \lambda_{1a}^2) & -iC_{21a}\omega \\ -iC_{21a}\omega & (-\omega^2 + iC_{22a}\omega - \lambda_{2a}^2) \end{vmatrix}$$

$$\Delta_{ja} = -q_{ja}\,(\,\omega^2 + \lambda_{3-j}^2) + i\omega\,(\,q_{ja}C_{3-j,3-j,a} - q_{3-j,a}C_{12a}) \;.$$

For convenience let us define the dimensionless parameters

$$A_{ja}^r = \frac{A_{ja}^o}{A_{ja}}, \quad c_1^r = \frac{C_{21a}}{C_{11a}}, \quad c_2^r = \frac{C_{12a}}{C_{22a}}, (\,j = 1,2\,)$$
$$\eta_a = \frac{q_{2a}}{q_{1a}}, \quad \beta_a = \frac{\omega_{2a}}{\omega_{1a}} \qquad (54)$$

where A_{ja}^o is derived from A_{ja} when $K_{12d} = 0$.

The points of main interest in the frequency domain are, of course, the resonant peaks. Therefore, the resonant amplitudes for the two modes of vibration (first subscript indicates number of the mode, second shows number of the resonance) are estimated herein as follows

$$A_{11a}^r = \left\{\frac{1 + G_{2a}^{-1}R_c}{G_{2a}^{-1}H_{2a}}\right\}^{1/2}, \qquad A_{21a}^r = \left\{\frac{1 + G_{2a}^{-1}R_c\,G}{1 + H_{1a}}\right\}^{1/2},$$
$$A_{12a}^r = \left\{\frac{1 + G_{1a}^{-1}R_c}{1 + H_{2a}}\right\}^{1/2}, \qquad A_{22a}^r = \left\{\frac{1 + G_{1a}^{-1}R_c}{G_{1a}^{-1}H_{1a}}\right\}^{1/2} \qquad (55)$$

where

$$G_{ja} = 1 + \left(\frac{1 - \beta_a^2}{2\xi_{ja}\beta_a}\right)^2 \qquad H_{2a} = \eta_a c_2^r\,(\,\eta_a c_2^r - 2\,)$$
$$H_{1a} = \eta_a^{-1}c_1^r\,(\,\eta_a^{-1}c_1^r - 2\,) \qquad R_c = c_1^r c_2^r\,(\,c_1^r c_2^r - 2\,) \;. \qquad (56)$$

<u>B-Superposition</u>

The transformed equations of motion in the B-coordinates are derived similarly from the source equation (Eq.1) by transformation with respect to the B-eigenvectors, with the result:

$$\mathbf{M}_b \ddot{\mathbf{b}} + \dot{\mathbf{b}} - \Lambda_b \mathbf{b} = \mathbf{q}_b e^{i\omega t} \tag{57}$$

where

$$\mathbf{q}_b = \left\{ q_{jb} \right\} .$$

The vibration mode amplitudes in this case have the form

$$A_b = \begin{Bmatrix} A_{1b} \\ A_{2b} \end{Bmatrix}$$

in which the elements are

$$A_{jb} = \frac{\Delta_{jb}}{\Delta_b}, \quad (j = 1,2) .$$

Here the expressions for the main and secondary determinants of the transformed system of equations have the form

$$\Delta_b = \begin{vmatrix} (- M_{11b}\omega^2 + i\omega - \lambda_{1b}) & - M_{12b}\omega^2 \\ - M_{21b}\omega^2 & (- M_{22b}\omega^2 + i\omega - \lambda_{2b}) \end{vmatrix} \tag{58}$$

$$\Delta_{jb} = -q_{jb} \left(M_{jjb}\omega^2 + \lambda_{3-j} \right) + q_{3-j,b} M_{12b}\omega^2 + iq_{jb}\omega , \quad (j = 1,2) .$$

Hence, the resonances are given by

$$A^r_{11b} = \left(\frac{1 + G_{2b}^{-1} R_{1m}}{1 + G_{2b} H_{2b}} \right)^{1/2} , \quad A^r_{21b} = \left\{ \frac{1 + G_{2b}^{-1} R_{1m}}{1 + \left(\eta_b^{-1} \omega_{1b} M_{12b} \right)^2} \right\}^{1/2} ,$$

$$A^r_{12b} = \left\{ \frac{G_{1b}^{-1} R_{2m} \beta_b^4}{1 + \left(\eta_b^{-1} \omega_{2b} M_{12b} \right)^2)} \right\}^{1/2} , \quad A^r_{22b} = \left\{ \frac{1 + G_{1b}^{-1} R_{2m} \beta_b^4}{G_{1b}^{-1} H_{1b} \beta_b^4} \right\}^{1/2} \tag{59}$$

in which

$$G_{1b} = 1 + \left\{ \frac{1 - \beta_b^2}{2\xi_{1b}\beta_b} \right\}^2 \qquad G_{2b} = 1 + \left\{ \frac{1 - \beta_b^{-2}}{2\xi_{2b}\beta_b} \right\}^2$$

$$H_{1b} = \eta_b^{-1} m^r_1 \left[\eta_b^{-1} m^r_1 - 2 \left(1 - \beta_b^{-2} \right) \right]$$

$$H_{2b} = \eta_b m^r_2 \left[\eta_b m^r_2 - 2 \left(1 - \beta_b^2 \right) \right] \tag{60}$$

$$R_{1m} = \frac{m^r_1 m^r_2 \left(m^r_1 m^r_2 + 8\xi_{1b}\xi_{2b}\beta_b \right)}{\left(2\xi_{1b}\xi_{2b}\beta_b \right)^2}$$

$$R_{2m} = \frac{m^r_1 m^r_2 \left(m^r_1 m^r_2 + 8\xi_{1b}\xi_{2b}\beta_b^{-1} \right)}{\left(2\xi_{1b}\xi_{2b}\beta_b^{-1} \right)^2} .$$

In the last expressions, the same parameters (54) are used as in the standard method with the only differences being the replacing of c_j^r by m_j^r and of the subscript a by b.

D-Superposition

Exactly the same procedure can be used to obtain the frequency characteristics for the second of the proposed methods. Applying the D -eigenvectors for the transformation of the equation of motion (Eq.1) to the general D-coordinates, one obtains

$$\ddot{\mathbf{d}} - \Lambda_d \dot{\mathbf{d}} + K_d = \mathbf{q}_d e^{i\omega t} \quad . \tag{61}$$

The vector of modal amplitudes then has the form

$$\mathbf{A}_d = \begin{Bmatrix} A_{1d} \\ A_{2d} \end{Bmatrix} \tag{62}$$

where

$$A_{jd} = \frac{\Delta_{jd}}{\Delta_d} , \qquad (j = 1,2) \ . \tag{63}$$

In this case the relevant determinant expressions are

$$\Delta_d = \begin{vmatrix} (-\omega^2 - i\lambda_{1d}\omega + K_{11d}) & K_{12d} \\ K_{21d} & (-\omega^2 - i\lambda_{2d}\omega + K_{22d}) \end{vmatrix} \tag{64}$$

$$\Delta_{1d} = q_{jd} \, (-\omega^2 + k_{3-j,3-j}) - q_{3-j,d} K_{12d} - i q_{jd} \lambda_{3-j,d} \omega \quad .$$

Thus, the resonant amplitudes for the D-Superposition will take the form

$$A^r_{11d} = \left\{ \frac{1 + G_{2d}^{-1} R_{1k} \beta_d^4}{1 + G_{2d}^{-1} H_{2d}} \right\}^{1/2} , \qquad A^r_{21d} = \left\{ \frac{1 + G_{2d}^{-1} R_{1k} \beta_d^4}{1 + \ (2\eta_d \xi_{1d} \omega_{1d} K_{12d}^{-1} \ )^{-2}} \right\}^{1/2}$$

$$A^r_{12d} = \left\{ \frac{1 + G_{1d}^{-1} R_{2k}}{1 + [2\eta_d^{-1} \xi_{2d} \omega_{2d} K_{12d}^{-1} \ )^{-2}]} \right\}^{1/2} , \qquad A^r_{22d} = \left\{ \frac{1 + G_{1d} R_{2k}}{1 + G_{1d}^{-1} H_{1d} \beta_d^4} \right\}^{1/2} \tag{65}$$

in which

$$G_{1d} = 1 + \left\{ \frac{1 - \beta_d^2}{2\xi_{1d}\beta_d} \right\}^2 \qquad\qquad G_{2d} = 1 + \left\{ \frac{1 - \beta_d^{-2}}{2\xi_{2d}\beta_d} \right\}^2$$

$$H_{1d} = \eta_d^{-1} k_1^r \, [\, \eta_d^{-1} k_1^r - 2\,(\,1 - \beta_d^2\,)\,]$$

$$H_{2d} = \eta_d k_2^r \, [\, \eta_d k_2^r - 2\,(\,1 - \beta_d^{-2}\,)\,] \tag{66}$$

$$R_{1k} = \frac{k_1^r k_2^r \,(\, k_1^r k_2^r + 8\xi_{1d}\xi_{2d}\beta_d^{-1}\,)}{(\,2\xi_{1d}\xi_{2d}\beta_d^{-1}\,)^2}$$

$$R_{2k} = \frac{k_1^r k_2^r \,(\, k_1^r k_2^r + 8\xi_{1d}\xi_{2d}\beta_d\,)}{(\,2\xi_{1d}\xi_{2d}\beta_d\,)^2} \quad .$$

In the last expressions, dimensionless parameters are used similar to Eq.54, replacing c_j^r by k_j^r and the subscript a by d.

R-Superposition

All frequency response characteristics related to the R-Superposition method are the same as those derived for the standard method (Eq.56) except for the parameters c_j^r (j=1,2) that are defined in Eq.54. In the R - Superposition these terms should be replaced by the corresponding revised damping ratios as given by Eq.51.

3.3. Examples

3.3.1. Vibration of a 2 lumped mass system

As a 2-DOF example, we will consider a simple two lumped mass system as shown in Fig.1. This might represent many different specific applications, but here it is assumed to be a structure with equipment mounted on it. Many papers have been written about the dynamic behaviour of such systems, for example [9,11,13-15]. If an eigenfrequency of the added equipment alone is the same as that of the structure alone, the "structure-equipment" system is called highly tuned. It was shown [14] that a tuned system can represent a distinctly non-classically damped structure, while a detuned system may have classical damping. For such a system, usually the first (top) mass is less than the second one, and, correspondingly, the stiffness of the connection between the masses is smaller than that between the second mass and the base.

Parameters that will be useful in this discussion are (1) the given equipment mass, stiffness and damping coefficients, respectively,

$$m_1 , \quad k_1 , \quad c_1 \tag{67}$$

and the derived frequency and damping ratio of the equipment

$$\omega_1 = \sqrt{m_1^{-1} k_1} , \qquad \xi_1 = \frac{c_1}{2\sqrt{m_1 k_1}} \tag{68}$$

(2) the given structure mass, stiffness and damping coefficients, respectively,

$$m_2 , \quad k_2 , \quad c_2 \tag{69}$$

and the structure derived frequency and damping ratio

$$\omega_2 = \sqrt{m_2^{-1} k_2} , \qquad \xi_2 = \frac{c_2}{2\sqrt{m_2 k_2}} \tag{70}$$

(3) the combined system natural frequencies and damping ratios; respectively,

$$\omega_{jx} , \quad \xi_{jx} \tag{71}$$

where j is mode number 1 or 2 and x = a,b or d corresponding to the specific transformation being applied.

Property matrices in the physical coordinates and in all the generalized coordinates, as well as the characteristic equations related to the A-, B-and D-Superposition methods, are listed in Appendix A. The eigenvalues and eigenvectors, in this case can be expressed in closed form as shown below.

For the A-Superposition the eigenvalues are , for $j = 1,2$;

$$\lambda_{ja} = -\frac{1}{2}\left\{\omega_1^2(1+\frac{m_1}{m_2}) + \omega_2^2 \pm \left(\left\{\omega_1^2\,(1+m_1/m_2) + \omega_2^2\right\}^2 - 4\omega_1^2\omega_2^2\right)^{1/2}\right\}, \tag{72}$$

and the mode shape matrix normalized with respect to the first degree of freedom is:

$$\Phi_{1a} = \begin{bmatrix} \Psi_{11a} & \Psi_{12a} \\ \Psi_{21a} & \Psi_{22a} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 - \dfrac{\omega_{1a}^2}{\omega_1^2} & 1 - \dfrac{\omega_{2a}^2}{\omega_1^2} \end{bmatrix}$$

Also the mode shape matrix normalized with respect to the mass is

$$\Phi_a = \Phi_{1a} \mathbf{M}_a^{-0.5} = \begin{bmatrix} (m_1 + m_2 \Psi_{21a}^2)^{-0.5} & (m_1 + m_2 \Psi_{22a}^2)^{-0.5} \\ \Psi_{21a} (m_1 + m_2 \Psi_{21a})^{-0.5} & \Psi_{22a} (m_1 + m_2 \Psi_{22a})^{-0.5} \end{bmatrix} \quad (73)$$

where M_a is the mass matrix transformed by the Φ_{1a} matrix; its elements are shown in Appendix A (A.3).

For <u>B-Superposition</u> the eigenvalues are

$$\lambda_{jb} = - \frac{\omega_{jb}}{2\xi_{jb}}, \quad (j = 1,2) \quad (74)$$

and the mode shape matrices, normalized with respect to the first degree of freedom and to the mass, respectively, are

$$\Phi_{1b} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad \Phi_b = \begin{bmatrix} c_1^{-1/2} & c_2^{-1/2} \\ 0 & c_2^{-1/2} \end{bmatrix} \quad (75)$$

For <u>D-Superposition</u>, the eigenvalues are obtained from

$$\lambda_{jd} = - \left\{ \omega_1 \xi_1 (1 + \frac{m_1}{m_2}) + \omega_2 \xi_2 \pm \left(\left\{ \omega_1 \xi_1 (1 + m_1/m_2) + \omega_2 \xi_2 \right\}^2 - 4\omega_1 \omega_2 \xi_1 \xi_2 \right)^{1/2} \right\}, \quad (j = 1,2) \quad (76)$$

Then the mode shape matrix normalized with respect to the first elements of normal vectors is:

$$\Phi_{1d} = \begin{bmatrix} \Psi_{11d} & \Psi_{12d} \\ \Psi_{21d} & \Psi_{22d} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 - \dfrac{\xi_{1d}\omega_{1d}}{\xi_1 \omega_1} & 1 - \dfrac{\xi_{2d}\omega_{2d}}{\xi_1 \omega_1} \end{bmatrix}$$

and normalized with respect to the mass it is:

$$\Phi_d = \Phi_{1d} \mathbf{M}_d^{-0.5} = \begin{bmatrix} (m_1 + m_2 \Psi_{21d}^2)^{-0.5} & (m_1 + m_2 \Psi_{22a}^2)^{-0.5} \\ \Psi_{21d} (m_1 + m_2 \Psi_{21d})^{-0.5} & \Psi_{22d} (m_1 + m_2 \Psi_{22d})^{-0.5} \end{bmatrix} \quad (77)$$

where M_d is the generalized mass matrix transformed by the Φ_{1d} mode shapes; its elements are shown in Appendix A (A.10).

The first conclusion that results from consideration of the transformed mass matrix (A.6) is that the B-Superposition method is not applicable to the analysis of a two lumped mass system, because the off-diagonal terms of the transformed matrix are the same as a first diagonal term; thus, great errors can be expected when these terms are neglected. For this reason the B - Superposition method will not be applied in the analysis of the "structure-equipment" system that follows.

Taking into account the specific expressions shown in Appendix A, the behaviour of the approximate solutions with respect to the mass, stiffness and damping properties of the two masses has been analyzed with results as shown in Figs.2-13. Two groups of parameters were investigated and corresponding curves were plotted for the eigen frequencies (Figs.2,6,10) and for the modal damping ratios (Figs.3,7,11). According to the Eqs.55, 59 and 65 the amplitude errors were plotted (Fig.4,5,8,9,12,13) in comparison with exact solutions to estimate errors of approximations defined by

$$\theta_{ij}^{x} = \frac{A_{ij}^{xo} - A_{ij}^{x}}{A_{ij}^{x}} = A_{ij}^{r} - 1 \ , \qquad (\, i,j = 1,2 \, ; \ x = a,b,d \,) \qquad (78)$$

where A_{ij}^{xo} and A_{ij}^{x} , respectively, represent approximate and exact amplitudes of the i-th mode in the j-th resonance; A_{ij}^{r} are relative amplitudes obtained from expressions (55),(59),(65). It is obvious that in the first resonance the main contribution is given by the first mode, and in the second resonance, correspondingly, by the second mode. Let us also define the acceptable limit of the approximation error as 6% for many engineering problems.

The main conclusions that may be drawn from the plotted curves are:

(1) influence of the mass properties (Fig.2-5); The greater the mass ratio (which is equivalent to increasing equipment mass in comparison with structure mass), the greater the divergence between the exact and the approximately calculated frequencies and damping ratios. The exact first eigenfrequency is closer to that obtained by R-Superposition while the exact second eigenfrequency almost coincides with the one obtained by D-

Superposition (Fig.2). As for damping ratios (Fig.3), the standard method gives the closest value in the first mode, while the exact second mode damping ratio is located between those calculated by the standard and R-Superposition method.

Analyzing the amplitude calculation errors, it is seen (Figs.4,5) that all the approximate methods give an acceptable error (about 2-6%) in the range with very small mass of the equipment ; hence, for the detuned "structure-equipment" system all the approximations are valid as has been suggested by the results of previous research [14]. On the other hand, for mass and stiffness ratios that are close to each other or are close to 0.1 (or in other words, for a highly tuned system) errors can be as large as 7% . Fig.4 shows the range (m_1/m_2 = 0.16 - 0.3 if k_1/k_2 = 0.1 and ξ_1/ξ_2 = 3) where the standard method and the R-Superposition methods are not valid because they give hundred times magnified error in the second mode contribution to the response in the first resonance. In this case, with respect to the fundamental mode contribution in the first resonance, the D-Superposition method gives the more accurate solution.

Further, as the mass ratio increases (i.e. the system is detuned), the D-Superposition method error grows significantly, A- and R-Superposition errors vary similarly to each other; they are almost constant (about 6%) in the first resonance and gradually increase in the second resonance starting at 3% with a mass ratio equal 0.4. R-Superposition leads to higher accuracy than the standard method in both resonance amplitudes.

When the equipment response is evaluated, it is apparent that if the mass ratio is greater than 0.44, even the best of the of approximate methods gives an error greater than 6%; in this case the exact solution is recommended.

(<u>2</u>) <u>influence of stiffness properties</u> (<u>Fig</u>.<u>6</u>-<u>9</u>). Eigenfrequencies and damping ratios obtained using the different approaches are different only when stiffness ratio varies in the range up to 0.2, including the tuned system range (Figs.5,6). All conclusions derived from mass ratio considerations can be transfered almost identically to the stiffness ratio range. For example, the standard method gives a high deviation from the exact solution in the first resonance (Fig.7) because of second component error for the parameter values m_1/m_2 = 0.1, k_1/k_2 = 0.8 - 1.4, ξ_1 / ξ_2 = 3 which also includes the tuned system range. D-Superposition has about the same error as the A-& R-Superposition methods in first mode amplitude, but only one-hundredth as large an error in the second mode amplitude. For the second resonance (Fig.8), the main contribution comes from the second mode. For the tuned "structure-equipment" system, R-Superposition gives a maximum of 16% error, D-Superposition gives up to 26% error and the standard method gives up to 30% error.

Generally speaking, although the performances are similar, the stiffness ratio has less effect on the approximation error than does the mass ratio. The exact solution (complex mode superposition) need be applied for the equipment response calculation only in the stiffness ratio range of 0.6 - 1.4.

(3) influence of damping properties (Fig.10-13). From the preceding analysis, we have chosen the case of the highly tuned "structure-equipment" system ($m_1/m_2 = k_1/k_2$ = 0.1), because it displays almost the same discrepancies for all the approximate procedures. While the ratio of damping in the equipment and the structure (which we will call a damping index) is increasing, the modal damping ratios obtained by the standard and the R - Superposition methods are also smoothly increasing (Fig.11). But those obtained by D-Superposition diverge gradually: the first mode damping is almost exact in the damping index range greater than one, while the second mode damping error is increasing rapidly. As the damping index increases, the relative value of the modal frequencies is reversed. This effect was also emphasized in Veletsos and Ventura's paper [1].

As we can see, there is a significant point in all the amplitude error curves when the damping index for this tuned system is unity (which means that the damping ratios in both structure and equipment have the same value). In this case all methods give the exact solution because there is no modal coupling. In the amplitude analysis of systems with equipment damping less than structure damping (Figs.12,13) we would recommend the R-Superposition as it leads to less error (maximum 6 % error in the first resonance, and maximum 8% error in the second resonance) than the standard method (correspondingly, 13% and 20%). D-Superposition is not recommended for such a system. If the damping index is greater than one but less than 2.7, the following conclusions may be drawn: (1) in first resonance (Fig.12) R - Superposition is best, the accuracy of the A-and D-Superposition varies depending on the second mode contribution ; (2) in the second resonance (Fig.13) R-Superposition is best (error < 13%), the standard method is not so good (error < 20%), and D-Superposition is worst (error > 20%). But when the equipment damping ratio is high enough (damping index greater than 2.7), D-Superposition becomes definitely better than the standard method; R-Superposition is still better (error < 4% in the first resonance, and error is 13% in second resonance).

The complex mode solution is required for equipment response determination when the damping index is in the range 0.3-0.5 or greater than 1.5; however, it is not needed in the structure response analysis for any range of damping index.

3.3.2. Different test loadings.

For numerical evaluation of the validity of the approximation methods for "structure-equipment" system, we have chosen two types of loadings: harmonic excitation and single sine wave impulse.

Harmonic excitation (Fig.14)

A highly tuned "structure-equipment" system has been analysed which has non-classical damping in the traditional sense; damping in the connection between equipment and structure is 3 times greater than that in the structure itself. The external load (P_1 = 4 Ton) is applied harmonically to the equipment. The harmonic amplitude functions of both degrees of freedom for a wide frequency range demonstrate quite different behaviour and resonance values when the standard (A), revised standard (R) and D-Superposition methods are used in comparison with the exact

D-Superposition methods are used in comparison with the exact (complex eigenvalues) results.

The standard A-Superposition method underestimates the structure vibrations and gives a lower value for the structure eigenfrequency, while it overestimates amplitudes of the equipment vibrations. For this reason, the R-and D-Superposition methods are recommended in structure design problems. As for the equipment, the revised standard method gives almost twice the exact value for the resonance amplitude, so, it would be better to calculate equipment response using the D-Superposition method or the complex mode technique (for lower damping values).

Sine wave impulse (Fig.15)

A simple single sine wave acceleration impulse is applied at the base of the structure. The amplitude of the excitation is 100 sm/s^2 and the impulse frequency (ω = 96 rad/s) is close to the fundamental system frequency.

Analysing the curves presented in Fig.15, it may be concluded that the D-Superposition method is quite reliable for analysis of a "structure-equipment" system, especially in evaluation of the structure behaviour. The equipment displacement decay obtained with D-Superposition is similar to the exact behaviour but with slightly modified frequency. The standard method is not recommended for analysis of tuned "structure-equipment" system.

Remarks

The conclusion derived from this analysis that a high error resulted from the standard superposition method applies only to cases with high damping in the system. For small damping, recent work by H.-C. Tsai and J.M. Kelly [16], showed that standard method gives only small errors in structure response results; the method is not recommended for equipment response analysis even for low damping level.

4. APPLICATION OF THE APPROXIMATE MODE SUPERPOSITION METHODS TO MDOF DAMPED SYSTEMS

For analysis of the vibrations of multi-degree-of-freedom systems, the approximation procedures described above are still valid, but further study must be given to the errors that are produced by their application. Two types of approximation errors should be distinguished: (1) those due to ignoring the off-diagonal terms in the transformed property matrices; and (2) those that result from modal truncation, as is usually done with finite element formulations involving a large number of degrees of freedom [17].

In the following, only the first type of approximation error is considered; specific criteria are derived for the standard (A), the B-and the D-Superposition methods applied to MDOF systems subjected to harmonic loading. Numerical results are shown for the specific case of a 9-story building subjected to a sine wave displacement history applied at the base. The R-Superposition method is not discussed because it can be evaluated by closed form analysis, but those results are still under investigation.

4.1. Criteria for validity of approximation methods

<u>Previous proposals</u>

In 1976, T.K. Hasselman [8] suggested the following as a criterion to indicate the validity of ignoring the modal coupling terms in the standard A - Superposition approximation:

$$\left\{ \frac{2\xi_i}{\left\{ \dfrac{\omega_i}{\omega_j} \right\}^2 - 1} \right\}^{1/2} << 1 \qquad (79)$$

in which ξ_i is the i-th mode damping ratio, ω_i is the higher eigenfrequency and ω_j is the lower one. A more general criterion was proposed by Warburton and Soni [9] in 1977 using a different approach. In our notations, their criterion for neglecting modal coupling is

$$\xi_i << D_i \tag{80}$$

in which the criterion parameter D_i is given by

$$D_i = \varsigma \left| \frac{C_{ii}}{C_{ij}} \left\{ \frac{\omega_j^2}{\omega_i^2} - 1 \right\} \right| \Bigg|_{\min j} \tag{81}$$

using the error parameter $\varsigma = 0.05$. In contrast with the Hasselman criterion, the parameter D_i takes into account the ratio of the diagonal to the off-diagonal terms in the transformed damping matrix. Both criteria imply looking through the whole set of i and j pairs to find the extreme value, but in most practical cases " i " may be taken as unity (i.e. fundamental mode) while only " j " varies. Thus the criterion parameter (81) considers the minimum value with respect to " j " index.

In our opinion, the Warburton and Soni criterion is the more rational,and also includes the Hasselman criterion as a special case. Hence, in the following, the equivalent approach will be used to obtain suitable criteria for each of the approximation procedures.

A-Superposition

Starting with the transformed equations of motion (Eq.6), they can be rewritten taking account of the fact that only the transformed damping matrix is non-diagonal. Thus the i-th equation of the full set of n coupled equations may be written

$$\ddot{a}_i + \sum_{j=1}^{n} C_{ija}\, \dot{a}_j + \omega_{ia}^2\, a_i = \sum_{s=1}^{n} \phi_{is}^a\, Q_{sa} \quad . \tag{82}$$

If a harmonic load is applied, both the loading and the response may be written as follows

$$Q_{sa} = Q_{sa}^o\, e^{i\omega t} \ , \qquad a_i = a_i^o e^{i\omega t} \tag{83}$$

so Eq.82 becomes

$$(\ \omega_{ia}^2 - \omega^2\ )\ a_i^o + i\,\omega \sum_{j=1}^{n} C_{ija}\, a_j^o = \sum_{s=1}^{n} \phi_{is}^a\, Q_{sa}^o \quad . \tag{84}$$

Now considering the i-th resonance, Eq.84 takes the form

$$i\,\omega_{ia} \sum_{j=1}^{n} C_{ija}\, a_j^o = \sum_{s=1}^{n} \phi_{is}^a\, Q_{sa}^o \quad . \tag{85}$$

According to the A-Superposition method, it is assumed that the off-diagonal damping coefficients c_{ija} are negligible. In this case, Eq.85 can be simplified to

$$i\,\omega_{ia}\, C_{iia}\, a_i^o = \sum_{s=1}^{n} \phi_{is}^a\, Q_{sa}^o \quad . \tag{86}$$

Now we consider the exact modal equation (Eq.84) in the alternative form

$$i\,\omega_{ia}\, C_{iia}\, a_i^o = \sum_{s=1}^{n} \phi_{is}^a\, Q_{sa}^o - i\,\omega_{ia} \sum_{j=<i>}^{n} C_{ija}\, a_j^o \tag{87}$$

where j=<i> means all values of the " j " index from 1 to n except " i ". In order to evaluate the last term on the right-hand side of this equation we have to express the j-th general displacement from the corresponding equation for the j-th coordinate in the i-th resonance

$$(\, \omega_{ja} - \omega_{ia}^2 \,)\, a_j^o + i\,\omega_{ia} \sum_{i=1}^{n} C_{jia}\, a_i^o = \sum_{s=1}^{n} \phi_{js}^a\, Q_{sa}^o \quad . \tag{88}$$

Until now we have followed the procedure used by Warburton and Soni. However, they neglected all the damping terms in the j-th modal equation imposing their small effect on the j-th general displacement in the i-th resonance, i.e. the third summation in Eq.88. Such an approach seems to us to be too approximate. It is a good idea to retain at least the diagonal terms of the transformed damping matrix in the last equation. Then, we have

$$(\, \omega_{ja} - \omega_{ia}^2 \,)\, a_j^o + i\,\omega_{ia}\, C_{jja}\, a_j^o = \sum_{s=1}^{n} \phi_{js}^a\, Q_{sa}^o \tag{89}$$

and the solution of this equation becomes

$$a_j^o = \frac{\sum_{s=1}^{n} \phi_{js}^a\, Q_{sa}^o}{\omega_{ja}^2 - \omega_{ia}^2 + i\,\omega_{ia}\, C_{jja}} \quad . \tag{90}$$

After substitution of term (90) into the exact modal equation of motion (84) and interchanging the summation signs, the i-th modal equation becomes

$$i\,\omega_{ia}\,C_{iia}\,a_i^o = \sum_{s=1}^{n}\left(\phi_{is}^a Q_{sa}^o - i\,\omega_{ia} \sum_{j=<i>}^{n} \frac{C_{ija}\,\phi_{js}^a}{\omega_{ja}^2 - \omega_{ia}^2 + i\,\omega_{ia}\,C_{jja}} \right) Q_{sa}^o \qquad (91)$$

Now, if we compare the last equation (which is the exact modal equation of i-th generalized coordinate in the i-th resonance) with the approximate one (Eq.86), we can conclude that the approximation is valid if a following inequality is satisfied:

$$\phi_{is}^a >> \sum_{j=<i>}^{n} mod \left| i\,\omega_{ia} \frac{C_{ija}\,\phi_{js}^a}{\omega_{ja}^2 - \omega_{ia}^2 + i\,\omega_{ia}\,C_{jja}} \right|$$

where mod means the absolute value of the complex quantity. Considering the maximum term among all those under the summation sign after multiplication by the number of series member, the right and the left hand side expressions become even more unequal

$$\phi_{is}^a >> 2\,(\,n-1\,)\,\xi_{ia} \left| \frac{C_{ija}\,\phi_{js}^a}{(\,\omega_{ja}^2 - \omega_{ia}^2\,)^2 + \omega_{ia}^2\,C_{jja}^2} \right|_{\max j} \qquad (92)$$

or, taking into account Eq.7b,

$$\phi_{is}^a >> 2\,(\,n-1\,)\,\xi_{ia} \left| \frac{C_{ija}}{C_{iia}} \frac{\phi_{js}^a}{iia \left(\frac{\omega_{ja}^2}{\omega_{ia}^2} - 1 \right)^2 4\xi_{ia}^2 \frac{C_{jja}^2}{C_{iia}^2}} \right|_{\max j} \qquad (93)$$

After converting this inequality with respect to ξ_{ia} we come to the final condition for validity of the A-Superposition

$$\xi_{ia} << \frac{1}{2\,(\,n-1\,)} \left| \frac{C_{iia}}{C_{ija}} \frac{\phi_{is}^a}{\phi_{js}^a} \left\{ \left(\frac{\omega_{ja}^2}{\omega_{ia}^2} - 1 \right)^2 + 4\xi_{ia}^2 \frac{C_{jja}^2}{C_{iia}^2} \right\} \right|_{\min j} \qquad (94)$$

If this condition is satisfied, the modal displacements may be found from Eq.90 as

$$a_i^o = \frac{\sum_{s=1}^{n} \phi_{is}^a Q_{sa}^o}{i\,\omega_{ia}\,C_{iia}} \qquad (95)$$

and the final displacements in the physical coordinates are defined according to (7c).

Thus, in a more general sense with only a minor assumption, the approximation criterion is formulated for the i-th mode vibrating under the s-th component of the external load expansion, evaluating all of the modal contributions to find the minimum one. It is interesting to compare criterion (Eq.94) to that formulated by Warburton and Soni. Neglecting the second term in the brackets in the denominator (which apparently is not always valid because of the possibly high ratio of the diagonal to the off-diagonal terms in the transformed damping matrix) and also assuming that the ratio of normal vector elements is unity, one comes to the condition

$$\xi_{ia} << \frac{1}{n-1} \left| \frac{\dfrac{\omega_{ja}^2}{\omega_{ia}^2} - 1}{2\dfrac{C_{ija}}{C_{iia}}} \right|_{\min j}$$

which is equivalent to Warburton and Soni's condition, but assuming their error parameter is given by $\zeta = 1/(n-1)$; of course its value is 0.05 in the case of a 21 degree-of-freedom system.

B-Superposition

Keeping the same algorithm, one can rewrite the transformed equations of motion in the B-general coordinates (Eq.12) as follows

$$\sum_{j=1}^{n} M_{ijb}\, \ddot{b}_j + \dot{b}_i - \lambda_{ib}\, b = \sum_{s=1}^{n} \phi_{is}^{b}\, Q_{sb} \ , \quad (j = 1 , n) . \tag{96}$$

For a harmonic load of the type of (Eq.83), the i-th physical displacement has an analogous form

$$b_i = b_i^o e^{i\omega t} \tag{97}$$

and i-th modal equation will be

$$-\omega^2 \sum_{j=1}^{n} M_{ijb}\, b_j^o + i\,\omega b_i^o - \lambda_{ib}\, b_i^o = \sum_{s=1}^{n} \phi_{is}^{b}\, Q_{sb}^o \ . \tag{98}$$

At the i-th resonance it becomes

$$-\omega_{ib}^2 \sum_{j=1}^{n} M_{ijb}\, b_j^o + i\,\omega_{ib}\, b_i^o = \sum_{s=1}^{n} \phi_{is}^{b}\, Q_{sb}^o \tag{99}$$

If, according to the B-Superposition method, the off-diagonal terms in the transformed mass matrix are neglected, the i-th modal equation takes the simple but approximate form

$$i\,\omega_{ib}\,b_i^o = \sum_{s=1}^{n} \phi_{is}^b\,Q_{sb}^o \quad . \tag{100}$$

Let us rewrite the exact i-th modal equation (109) in correspondence to the approximate form

$$i\,\omega_{ib}\,b_i^o = \sum_{s=1}^{n} \phi_{is}^b\,Q_{sb}^o + \omega_{ib}^2 \sum_{j<i>}^{n} M_{ijb}\,b_j^o \tag{101}$$

An expression for the modal contributions included in the second term in the right-hand side (101) can be determined by considering the j-th modal equation in the i-th resonance and neglecting off-diagonal terms in the transformed mass matrix. Hence, the mentioned equation becomes

$$-\omega_{ib}\,M_{jjb}\,b_j^o + i\,\omega_{ib}\,b_j^o - \lambda_{jb}\,b_j^o = \sum_{s=1} \phi_{js}^b\,Q_{sb}^o \ . \tag{102}$$

Solving this equation with respect to the term b_j^o and substituting the result in Eq.101, after interchanging the summation signs we obtain the exact modal equation of motion in j-th general coordinates

$$i\,\omega_{ib}\,b_i^o = \sum_{s=1}^{n} \left\{ \phi_{is}^b + \omega_{ib}^2 \sum_{j<i>}^{n} \frac{M_{ijb}\,\phi_{js}^b}{-\ (\lambda_{jb} + \omega_{ib}^2 M_{jjb}\ ) + i\,\omega_{ib}} \right\} Q_{sb}^o \tag{103}$$

Consequently, the condition for the two equations (100) and (101) to be equal is

$$\phi_{is}^b >> \sum_{j=<i>}^{n} mod \left| \frac{\omega_{ib}^2 M_{ijb}\,\phi_{is}^b}{-(\lambda_{jb} + \omega_{ib}^2 M_{jjb}\ ) + i\,\omega_{ib}} \right| \tag{104}$$

or, considering only the largest term in this expansion,

$$\phi_{is}^b >> (\ n-1\ ) \left| \frac{\omega_{ib}^2 M_{ijb}\,\phi_{is}^b}{(\lambda_{jb} + \omega_{ib}^2 M_{jjb}\ )^2 + \omega_{ib}^2} \right|_{\max j} \tag{105}$$

Applying relations (13a) and (13b) between approximate eigenfrequencies and modal damping ratios from one side, using the elements of the transformed property matrices from the other side, and solving the last inequality with respect to the i-th modal damping ratio, one can estimate the validity of the B-Superposition by the condition

$$\xi_{ib} << \frac{\xi_{ib}}{n-1} \left| \frac{M_{iib}}{M_{ijb}} \frac{\phi_{is}^{b}}{\phi_{js}^{b}} \frac{M_{jjb}^{2}}{M_{iib}^{2}} \left\{ \frac{\omega_{jb}^{2}}{\omega_{ib}^{2}} - 1 \right\}^{2} + 4\xi_{ib}^{2} \right|_{\min j} \qquad (106)$$

If this condition is satisfied, we can use the approximate solution derived from (100)

$$b_i^o = \frac{\sum_{s=1}^{n} \phi_{is}^{b} Q_{sb}^{o}}{i\,\omega_{ib}} \qquad (107)$$

Then the final displacements in the physical coordinates are found according to formula (13c).

D-Superposition method

In order to get an equivalent expression related to the D-Superposition, we will repeat the procedure using equations of motion derived by the D-transformation (Eq.17). For the i-th general displacement we have

$$\ddot{d}_i - \lambda_{id}\,\dot{d}_i + \sum_{j=1}^{n} K_{ijd}\, d_j = \sum_{s=1}^{n} \phi_{is}^{d} Q_{sd} \qquad (108)$$

In the harmonic regime, when

$$d_i = d_i^o e^{i\omega t} \qquad (109)$$

the i-th modal equation becomes

$$-\omega^2 d_i^o - i\,\omega\lambda_{id}\, d_i^o + \sum_{j=1}^{n} K_{ijd}\, d_j^o = \sum_{s=1}^{n} \phi_{js}^{d} Q_{sd}^{o}$$

and in the i-th resonance ($\omega = \omega_{jd}$) it reduces to the

$$-i\,\omega_{id}\lambda_{id}\, d_i^o + \sum_{j=<i>}^{n} K_{ijd}\, d_j^o = \sum_{s=1}^{n} \phi_{is}^{d} Q_{sd}^{o} \qquad (110)$$

According to the idea of the D-Superposition method, the off-diagonal terms of the transformed stiffness matrix are neglected, and the equation is simplified to the form

$$-i\,\omega_{id}\lambda_{id}\, d_i^o = \sum_{s=1}^{n} \phi_{is}^{d} Q_{sd}^{o} \qquad (111)$$

It is possible to express the contribution from the other modes to the i-th modal displacement, similarly, by considering their behaviour in the i-th resonance and retaining only the diagonal terms in the transformed stiffness matrix

$$-\omega_{id}^2 d_j^o - i\,\omega_{id}\lambda_{jd}\, d_j^o + K_{jjd}\, d_j^o = \sum_{s=1}^{n} \phi_{js}^d Q_{sd}^o \qquad (112)$$

Thus, substitution of the approximate displacement d_j^o into the properly rewritten i-th modal equation results in the following

$$-i\,\omega_{id}\lambda_{id}\, d_i^o = \sum_{s=1}^{n} \left\{ \phi_{is}^d - \sum_{j=<i>}^{n} \frac{K_{ijd}\,\phi_{js}^d}{(\,K_{jjd} - \omega_{id}^2\,) - i\,\omega_{id}\lambda_{jd}} \right\} Q_{sd}^o \qquad (113)$$

So, Eqs.111 and 113 would be similar if the condition is met

$$\phi_{is}^d >> \sum_{j<i>} mod \left| \frac{K_{ijd}\,\phi_{js}^d}{(\,K_{jjd} - \omega_{id}^2\,) - i\,\omega_{id}\lambda_{jd}} \right| \qquad (114)$$

or in terms of the maximum value of this expression

$$\phi_{is}^d >> (\,n-1\,) \left| \frac{K_{ijd}\,\phi_{js}^d}{(\,K_{jjd} - \omega_{id}^2\,)^2 - \omega_{id}^2\lambda_{id}^2} \right|_{\max j} \qquad (115)$$

Substituting the expressions for the approximate eigenfrequency and damping ratio (18a),(18b), the approximation condition is obtained in the form

$$\xi_{id} << \frac{\xi_{id}}{n-1} \left| \frac{K_{iid}}{K_{ijd}} \frac{\phi_{is}^d}{\phi_{js}^d} \left\{ \frac{\omega_{jd}^2}{\omega_{id}^2} - 1 \right\}^2 + 4\xi_{jd}^2 \frac{K_{jjd}}{K_{iid}} \right|_{\min j} \qquad (116)$$

If condition (116) is satisfied, one can determine the modal response from the simple equation (111) as

$$d_i^o = - \frac{\sum_{s=1}^{n} \phi_{is}^d Q_{sd}^o}{i\,\omega_{id}\lambda_{id}} \qquad (117)$$

and the final response from formula (13c).

4.2. Vibration of 9 story building with external damper

<u>Response to an impulse load</u>

As an example of application of the approximation methods to MDOF systems we will consider a 9 story lumped mass building with uniform mass, stiffness and internal damping distribution but with alternative positions of additional interstory dampers. Two cases were studied: (1) the added damper is installed in the eighth story (Fig.16a); (2) the added damper is installed in the third story (Fig.16b). The following values of the physical properties of a typical floor are assumed for the numerical analysis: floor mass m = 0.1 Ton, interfloor stiffness k = 10^4 KN/m, and interfloor viscous damping coefficient c = 5 Ns/m. The viscous coefficient for the added damper is three times larger, i.e. C = 15 Ns/m. The resulting transformed property matrices and eigenfrequencies are shown in Appendix B. In Table 2 eigenfrequencies and corresponding modal damping ratios obtained from all the approximation procedures are given for both cases of external damper location.

Absolute displacements of the 9th and 6th floors of the 9 story building subjected to a basement motion of the single sine wave impulse type were calculated using the Cal 86 program. The input basement acceleration can be expressed as follows:

$$H(t) = \begin{cases} 100\, sin\, 96t\ , & 0 < t < 0.26s. \\ 0 & t > 0.26s. \end{cases}$$

The exact solution was calculated by direct time integration using the Wilson θ - method. Fig. 17 presents the time history response of the specified floors when the added damper is in the 8th story. It is clear that the D - Superposition method gives a better approximation of the top displacement than does the standard method. But notice that the accuracy of D-Superposition deteriorates when a lower floor displacement is considered. If we install the added damper at the third floor instead of the 8th floor (Fig. 18), the standard method is more accurate for calculating response of 9th floor, but for the 6th floor, both the standard and D-Superposition methods give almost the same error (approximately 7%) in the maximum displacement evaluation. As was noted previously for the 2 lumped mass system, the B-Superposition method can not describe the behaviour of the lumped mass system because of the regular tri-diagonal structure of the damping and stiffness matrices.

4.3. Analytical and numerical comparison

Usually in practical problems, a point of major interest is the response of the structure in its first resonance. The dominant contribution in such a case will be from the fundamental mode. That is why in the following, we will consider first mode, involving the first row in the non-diagonal transformed matrix. Based on the criteria derived in the previous section and the transformed property matrices from Appendix B, it is possible to conclude which of the highest modes has the greatest influence on the fundamental mode. For example, taking i=1 we observe the maximum contributions: for the standard method - from the eighth mode (j=8) and for the B-and D-Superposition methods - from ninth mode (j=9). Note that the diagonal terms in the transformed matrices are increasing according to the number of the mode; therefore, it is necessary to keep in mind the high value of the ratios C_{jja}/C_{iia}, M_{jjb}/M_{iib} and K_{jjd}/K_{iid} that are contained in the corresponding criteria.

Formally comparing the criteria (94), (106) and (116) shows that the accuracy of the proposed B-and D-Superposition procedures should increase as the damping in the system increases. For complicated systems with well spaced eigenfrequencies the last mentioned ratios of diagonal terms are expected to be higher, and since for the standard method (94) this ratio is squared, one can expect deteriorating accuracy when this approximation is used. In the other cases, the error of the A-Superposition approximation is expected to be lower than the errors of B- or D-Superposition. In cases where the off-diagonal terms are small compared with those in the diagonal, and where the last diagonal term is large in comparison with the first one could expect less error when the B-Superposition is used (Eq.106). But for the chain type structures such as the examples considered here (Appendix B) we can note that B-Superposition gives a final off-diagonal term that is 2 to 3 times bigger than the first diagonal term. Hence, for such systems certainly, B-Superposition method is not recommended.

5. CONCLUSIONS

The following may be stated as the principal general conclusions drawn from the research described in this report:

1. The method of decoupling the equations of motion that generally is used in dynamic analysis of structural systems with non-proportional damping employs a coordinate transformation based on eigenvectors derived from the original system mass and stiffness matrices; this is referred to as the standard or A-Superposition method in this report. Results of this study show that A-Superposition should be used in the analysis of any lumped mass 2-DOF detuned system (i.e. a system in which the frequencies of the individual components are distinctly different). It also is recommended for multistory lumped mass shear buildings if the damping in the lower stories is greater than or equal to that in the upper stories. On the other hand, this method leads to rather large approximation errors in the second mode for tuned lumped 2-DOF systems, as well as in all response quantities for multistory shear building with greater damping in the upper stories than in the lower.

Thus it is clear that the standard method may give unexpected equipment response errors in analysis of 2-DOF equipment-structure system. The calculated response of the structure is more reliable than that of the equipment, but still is subjected to discrepancy for systems with high average damping due to the large approximation errors in the second mode contribution to the structure motion. In general, this work has confirmed previous research results that show the error sensitivity of the standard method (A-Superposition) to variations in the damping coefficients - - either by increasing of average damping value or by increasing variation of the damping coefficients within the structure. Usually the response error is more sensitive to changes in damping of equipment than of the structure.

2. The first alternative to the standard decoupling method employs a coordinate transformation based on eigenvectors derived from the original system stiffness and damping matrices; it is referred to here as the B-Superposition method. As would be

expected, this method leads to errors that are more sensitive to mass variation than to damping and stiffness variation.

In the lumped parameter systems considered in this study, the original stiffness and damping matrices are of tri-diagonal form while the original mass matrix is diagonal; the generalized mass matrix resulting from the coordinate transformation then often has off-diagonal terms equal to or greater than those on the diagonal. In this case, it is clear that neglect of the diagonal terms may lead to unacceptable errors. Also, the general analysis of MDOF systems shows that B-Superposition gives better results for the lower modes than the higher ones, but that the A- and D-Superposition methods are preferable in all respects.

3. The coordinate transformation employed in the second alternative to the standard method, called D-Superposition, is based on eigenvectors derived from original system damping and mass matrices. In contradiction to what might be expected, the approximation error in this method is not as sensitive to variation of the stiffness coefficients as is the standard method to the damping coefficients. In the analysis of a well detuned equipment-structure system, results from D-Superpositon are quite acceptable for cases in which the ratio of equipment to structure stiffness is considerably greater than the corresponding mass ratio. D-Superpositon also is preferred to the standard method for tuned systems if the equipment damping is at least three times greater than that of the structure. In general, D-Superposition is especially recommended for analysis of the structure response, and is somewhat less reliable in predicting the equipment response. In cases where the equipment damping is equal to or less than the structure damping, D-Superposition can lead to large approximation errors.

The D-Superposition method also is recommended for analysis of MDOF lumped mass shear buildings if the damping of the upper stories is greater than in the lower stories.

4. The revised standard method (R-Superposition) is a modification of the standard method and is based on the same coordinate transformation. The eigenfrequencies are calculated from the uncoupled equations of motion obtained by neglecting the

off-diagonal coefficients of the transformed damping matrix, as in the standard method, but uses these frequencies in calculating the modified modal damping ratios. The procedure can be applied directly only for 2- or 3-DOF systems, and was used in this work only for study of lumped equipment-structure systems. The results show that R-Superposition is most effective in reducing the equipment response amplitude error as compared with the error given by the standard method.

In principle, similar improvements could be proposed for B- and D-Superposition applied to equipment-structure systems, but the concept has not been explored in this work. Also, no analyses have yet been done of MDOF systems using a revised standard (R-Superposition) method.

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Appendix A. Dynamic characteristics of the two lumped mass system.

A.1. The properties matrices in the physical coordinates.

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \qquad \text{(A.1)}$$

$$\mathbf{K} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix}$$

A.2. Transformed properties matrices in the general A -coordinates.

$$\mathbf{M}_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{K}_a = \begin{bmatrix} \lambda_{1a} & 0 \\ 0 & \lambda_{2a} \end{bmatrix} \qquad \text{(A.2)}$$

$$\mathbf{C}_a = \begin{bmatrix} \dfrac{c_{11}}{M_{1a}} & \dfrac{c_{12}}{M_{1a}} \\ \dfrac{c_{12}}{M_{2a}} & \dfrac{c_{22}}{M_{2a}} \end{bmatrix}$$

where

$$c_{jj} = c_j \beta_{ja}^4 + c_{3-j} \left(1 - \beta_{ja}^2 \right)^2$$
$$c_{12} = 2\xi_2 \omega_2 m_1 \left(\varsigma^{-1} \beta - 1 \right) \qquad \text{(A.3)}$$
$$M_{ja} = m_1 + m_2 \left(1 - \beta_{ja}^2 \right)$$

with the dimensionless parameters

$$\beta_{ja} = \frac{\omega_{ja}}{\omega_1}, \quad (j = 1,2) \qquad \beta = \frac{\omega_2}{\omega_1}, \qquad \varsigma = \frac{\xi_2}{\xi_1} \, . \qquad \text{(A.4)}$$

The characteristic equation in the A-Superposition modal coordinates

$$\lambda^4 + \left\{ \frac{k_1 + k_2}{m_2} + \frac{k_1}{m_1} \right\} \lambda^2 + \frac{k_1 k_2}{m_1 m_2} = 0 \qquad \text{(A.5)}$$

A.3. Transformed properties matrices in the general B -coordinates.

$$\mathbf{M}_b = \begin{bmatrix} \dfrac{m_1}{c_1} & \dfrac{m_1}{c_1} \\ \dfrac{m_1}{c_2} & \dfrac{m_2}{c_2} \end{bmatrix}$$

$$\mathbf{C}_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{K}_b = \begin{bmatrix} k_1/c_1 & 0 \\ 0 & k_2/c_2 \end{bmatrix} \qquad \text{(A.6)}$$

The characteristic equation in the B -basic coordinates

$$c_1 c_2 \lambda^2 + (k_1 c_2 + k_2 c_1) \lambda + k_1 k_2 = 0 \qquad \text{(A.7)}$$

of which roots are

$$\lambda_{jb} = - \frac{k_j}{c_j} \qquad \text{(A.8)}$$

A.4. Transformed properties matrices in the general D -coordinates.

$$\mathbf{M}_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{matrix} \{1 \\ [0 \end{matrix} \quad \mathbf{C}_d = \begin{bmatrix} \lambda_{1d} & 0 \\ 0 & \lambda_{2d} \end{bmatrix}$$

$$\mathbf{K}_d = \begin{bmatrix} \dfrac{k_{11}}{M_{1d}} & \dfrac{k_{12}}{M_{1d}} \\ \dfrac{k_{12}}{M_{2d}} & \dfrac{k_{22}}{M_{2d}} \end{bmatrix} \qquad \text{(A.9)}$$

where

$$\begin{aligned} k_{jj} &= c_j \beta_{jd}^4 + c_{3-j} (1 - \varsigma_{jd} \beta_{jd})^2 \\ k_{12} &= \omega_2^2 m_1 (\varsigma \beta^{-1} - 1) \\ M_{jd} &= m_1 + m_2 (1 - \varsigma_{jd} \beta_{jd})^2 \end{aligned} \qquad \text{(A.10)}$$

with the dimensionless parameters of the form

$$\beta_{jd} = \frac{\omega_{jd}}{\omega_1}, \qquad \varsigma_{jd} = \frac{\xi_{jd}}{\xi_1} \qquad \text{(A.11)}$$

The characteristic equation in the D -basic coordinates.

$$\lambda^4 + \left\{ \frac{c_1 + c_2}{m_2} + \frac{c_1}{m_1} \right\} \lambda + \frac{c_1 c_2}{m_1 m_2} = 0 \qquad \text{(A.12)}$$

Appendix B. Transformed property matrices for the 9 story building

Transformed damping matrix (A-Superposition method):

$$\mathbf{C}_a = \begin{bmatrix} 1.424 & & & & & & & & \\ -0.464 & 15.609 & & & & & & & \\ 0.904 & -6.928 & 45.767 & & & & & & \\ 0.908 & -6.964 & 13.565 & 73.466 & & & SYMM & & \\ 0.252 & -1.930 & 3.758 & 3.777 & 92.789 & & & & \\ -0.848 & 6.500 & -12.662 & -12.727 & -3.525 & 136.428 & & & \\ 1.820 & -13.938 & 27.151 & 27.292 & 7.560 & -25.474 & 209.320 & & \\ 2.070 & -15.868 & 30.909 & 31.070 & 8.607 & -29.000 & 62.185 & 249.706 & \\ 1.368 & -10.48 & 20.423 & 20.530 & 5.68 & -19.162 & 41.090 & 46.777 & 225.490 \end{bmatrix}$$

Transformed mass matrix (B-Superposition method):

$$\mathbf{M}_b = \begin{bmatrix} .0133 & & & & & & & & \\ .0330 & .3434 & & & & & & & \\ -.0055 & -.0745 & .0339 & & & & & & \\ -.0081 & -.0456 & .0136 & .0190 & & & SYMM & & \\ -.0173 & -.1038 & .0223 & .0179 & .0447 & & & & \\ .0116 & .0286 & -.0048 & -.0070 & -.0150 & .0200 & & & \\ .0360 & .2464 & -.0528 & -.0411 & -.0872 & .0312 & .2103 & & \\ .0037 & .0930 & -.0118 & -.0021 & -.0223 & .0032 & .0551 & .0488 & \\ .0306 & .1670 & -.0417 & -.0352 & -.0638 & .0265 & .1549 & .0250 & .1403 \end{bmatrix}$$

Transformed stiffness matrix (D-Superposition method):

$$\mathbf{K}_d = \begin{bmatrix} .27E4 & & & & & & & & \\ .98E2 & .26E5 & & & & & & & \\ -.21E3 & .15E4 & .71E5 & & & & & & \\ .25E3 & -.19E4 & .40E4 & .13E6 & & & SYMM & & \\ -.62E2 & .46E3 & -.98E3 & .12E4 & .18E6 & & & & \\ .12E3 & -.91E3 & .19E4 & -.24E4 & .58E3 & .25E6 & & & \\ -.16E3 & .12E4 & -.25E4 & .31E4 & -.75E3 & .15E4 & .33E6 & & \\ .11E3 & -.80E3 & .17E4 & -.21E4 & .51E3 & -.10E4 & .13E4 & .38E6 & \\ -.23E4 & .17E5 & -.35E5 & .44E5 & -.11E5 & .21E5 & -.27E5 & .18E5 & .33E6 \end{bmatrix}$$

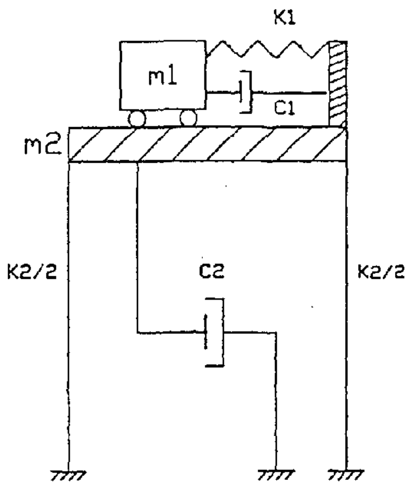


Figure 1. 2-DOF "structure-equipment" model

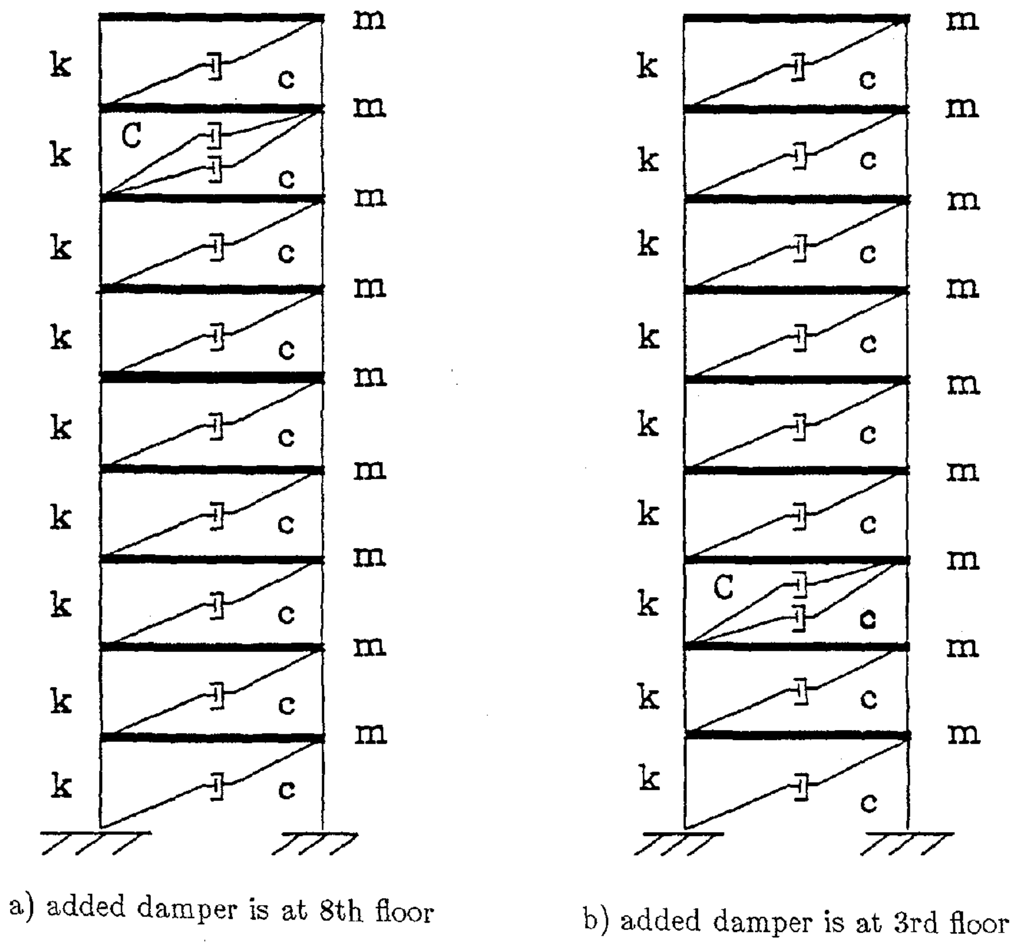


Figure 16. 9-DOF structural model

Parameters:

$$\frac{k_1}{k_2} = 0.25 \qquad \frac{\xi_1}{\xi_2} = 3$$

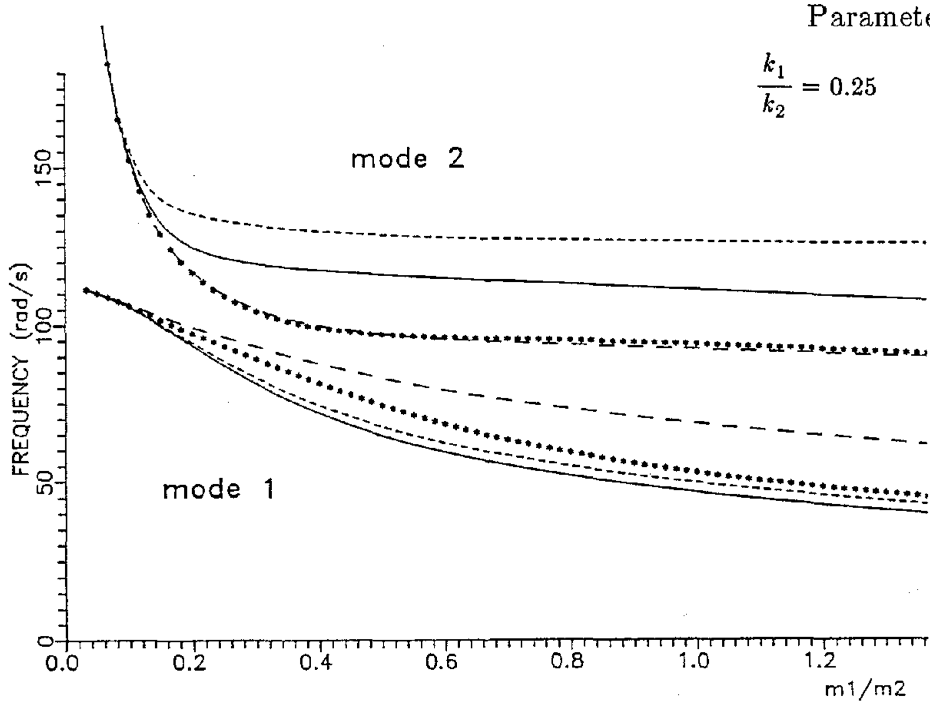


Figure 2. 2-DOF system eigenfrequencies v/s mass ratio

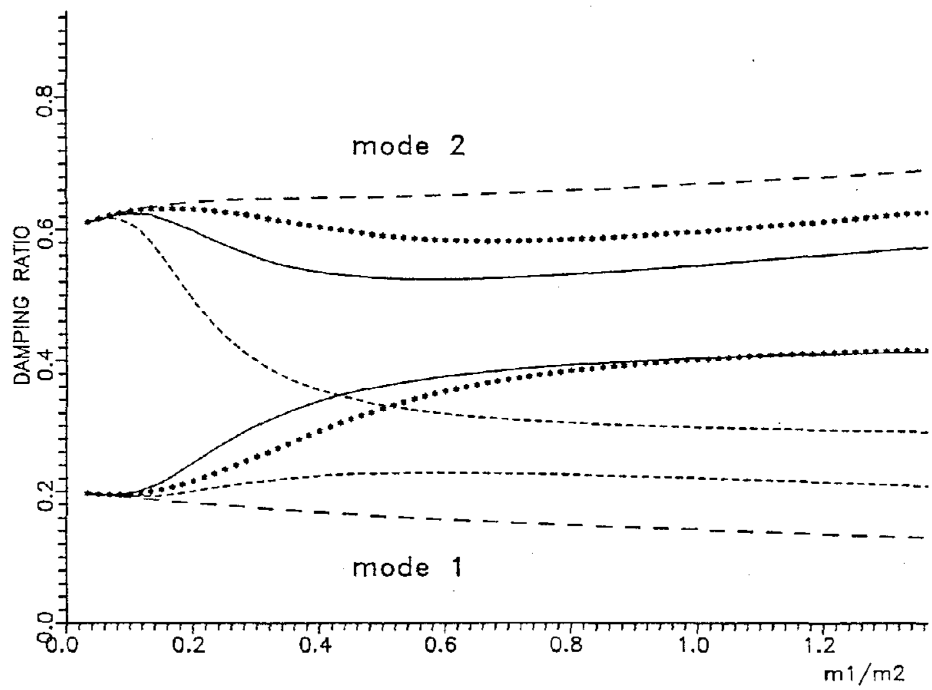


Figure 3. 2-DOF system damping ratios v/s mass ratio

* * *	exact solution
———	A-Superposition
— — —	D-Superposition
- - - -	R-Superposition

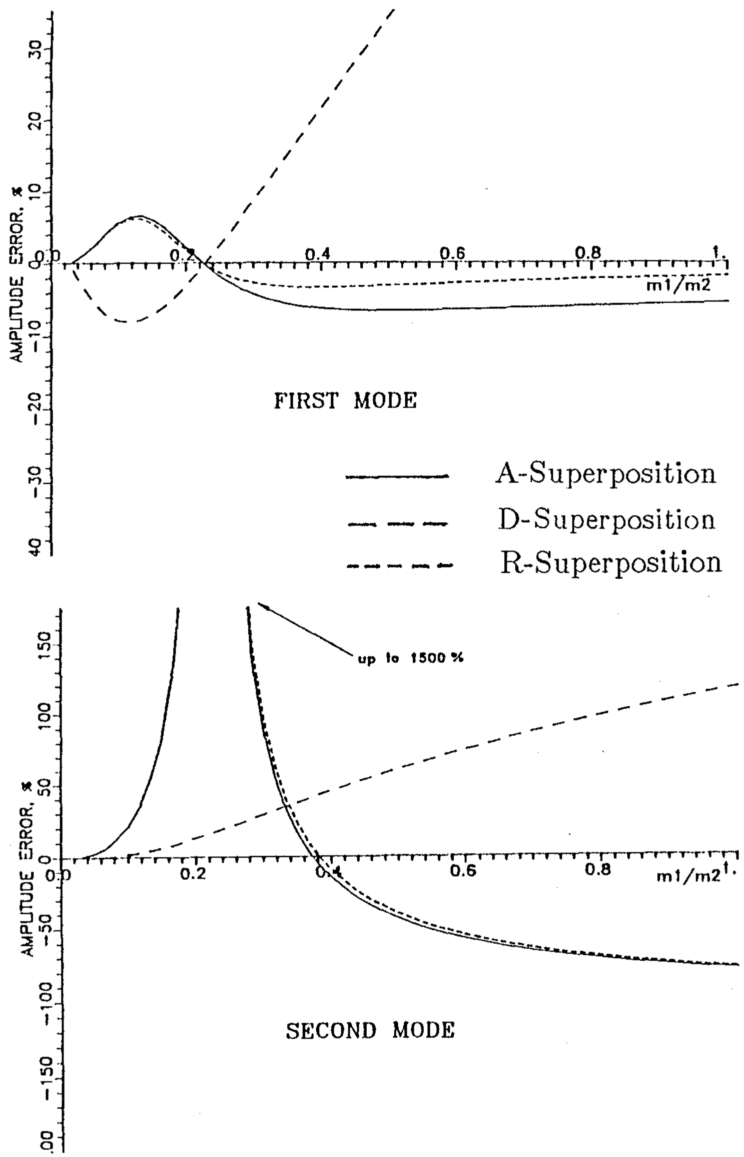


Figure 4. 2-DOF system amplitude approximation errors v/s mass ratio in first resonance

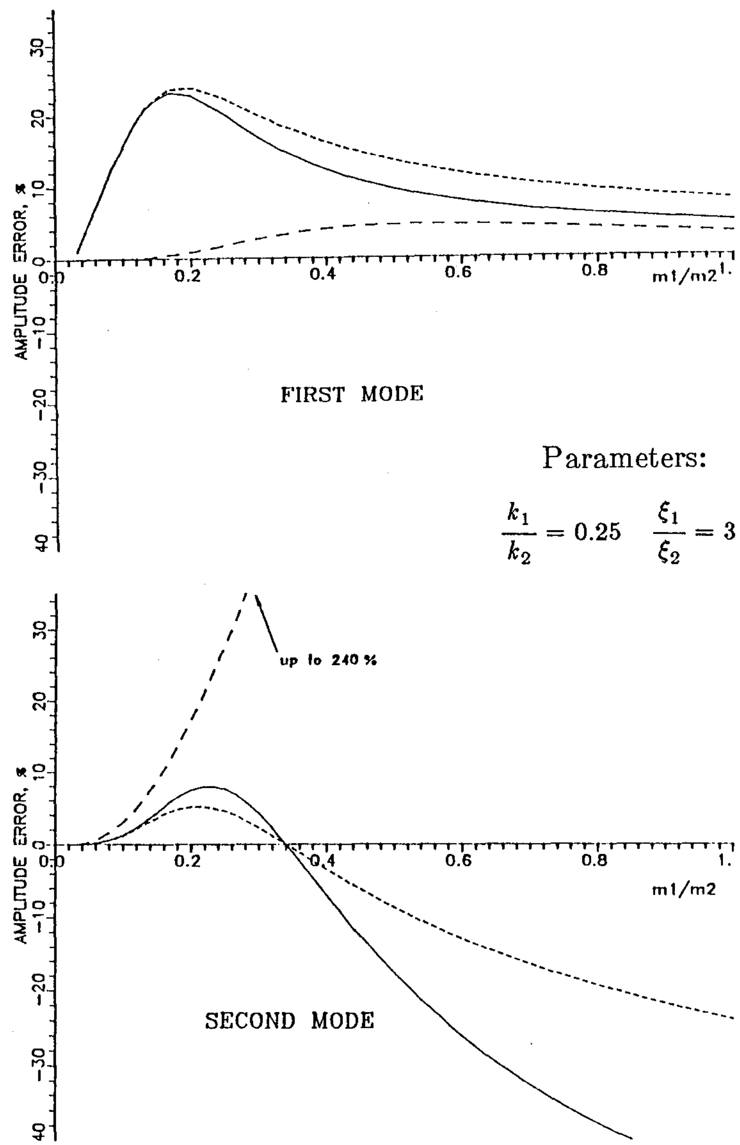


Figure 5. 2-DOF system amplitude approximation errors v/s mass ratio in second resonance

Parameters:

$$\frac{m_1}{m_2} = 0.1 \qquad \frac{\xi_1}{\xi_2} = 3$$

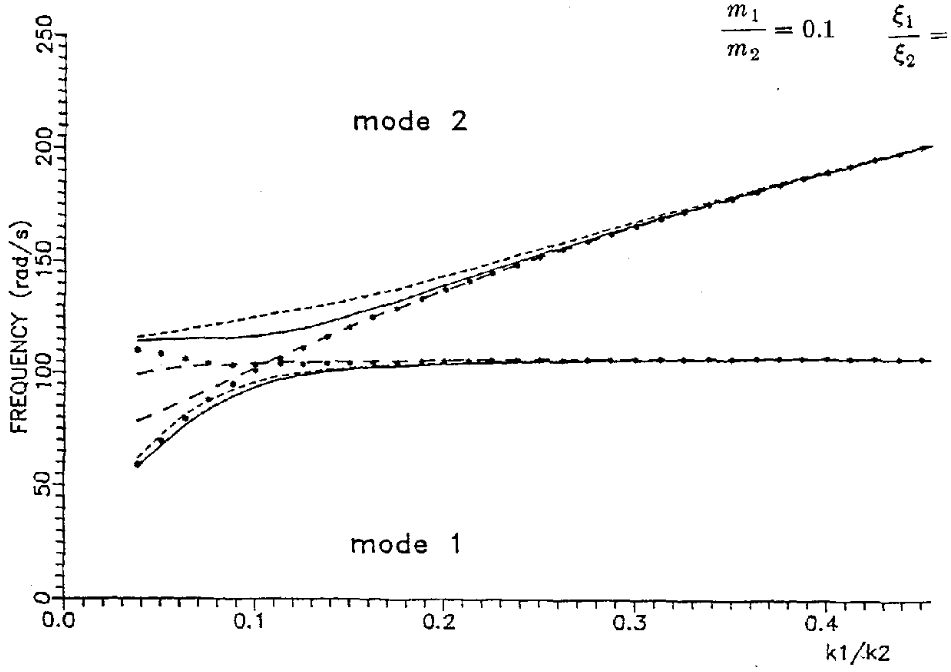


Figure 6. 2-DOF system eigenfrequencies v/s stiffness ratio

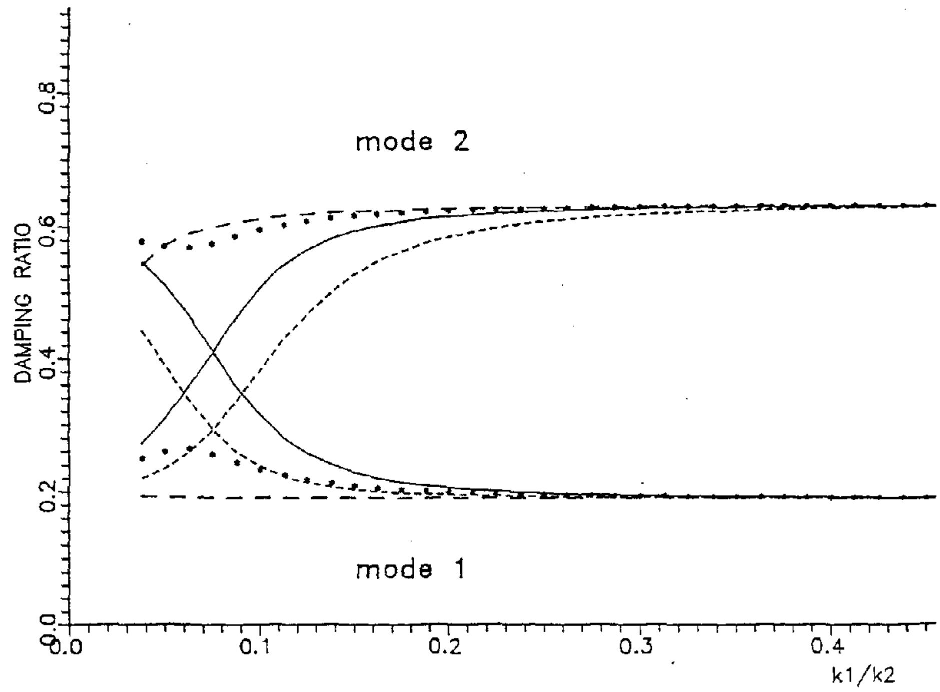


Figure 7. 2-DOF system damping ratios v/s stiffness ratio

* * *	exact solution
———	A-Superposition
— — —	D-Superposition
- - - - -	R-Superposition

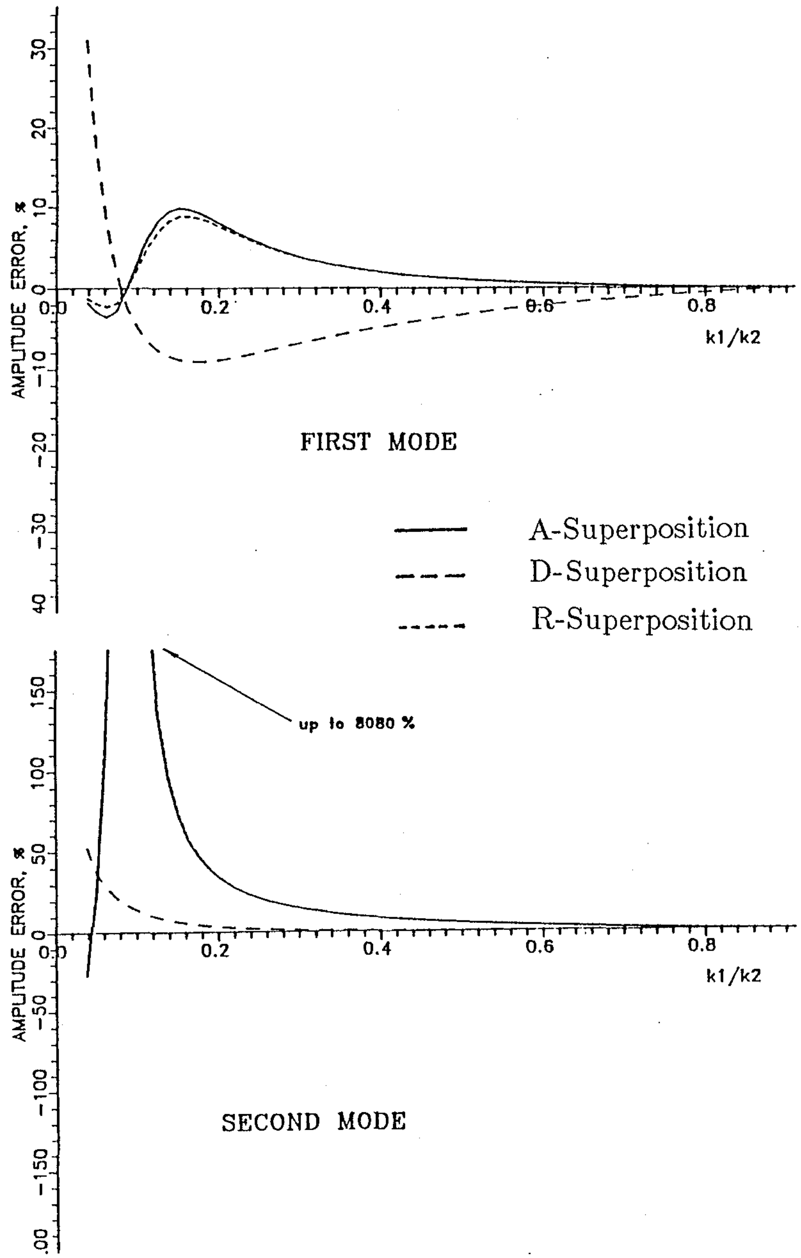


Figure 8. 2-DOF system amplitude approximation errors v/s stiffness ratio in first resonance

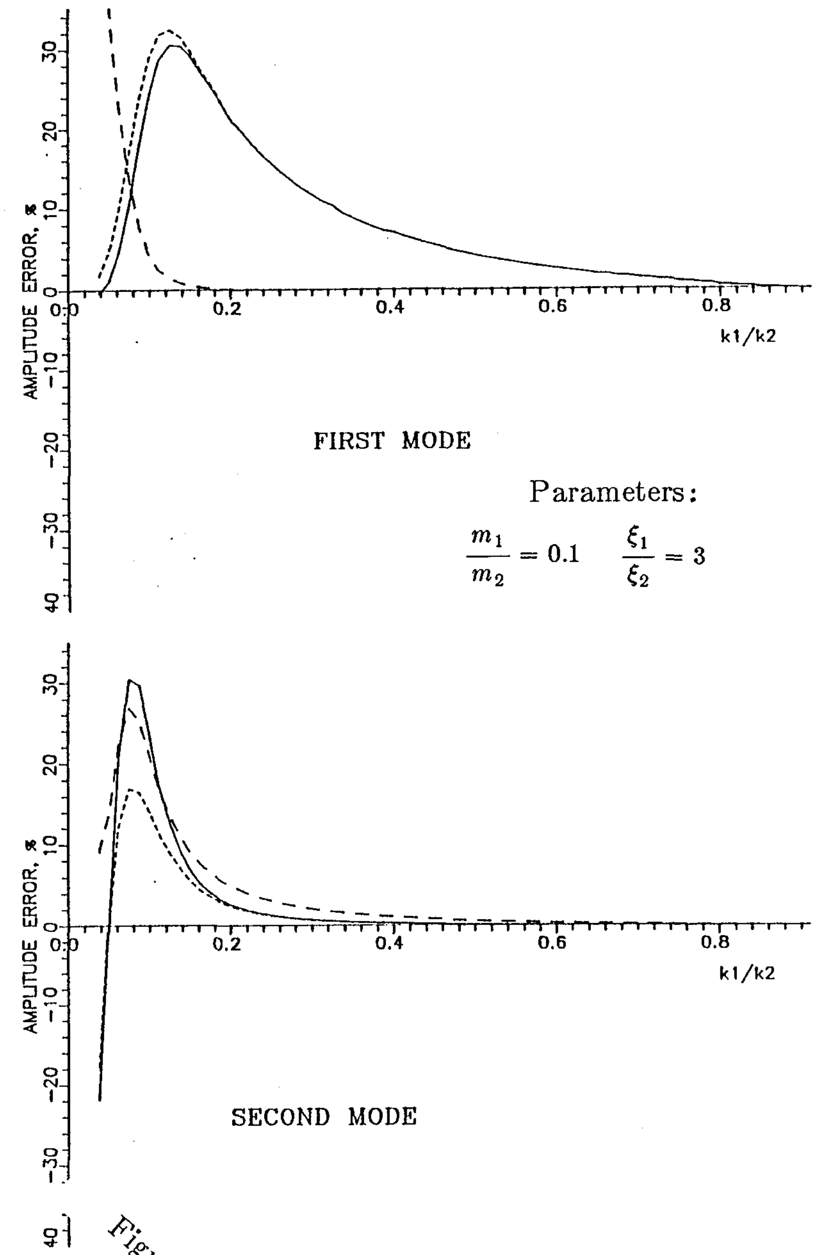


Figure 9. 2-DOF system amplitude approximation errors v/s stiffness ratio in second resonance

Parameters

$\frac{m_1}{m_2} = 0.1 \qquad \frac{k_1}{k_2} = 0.1$

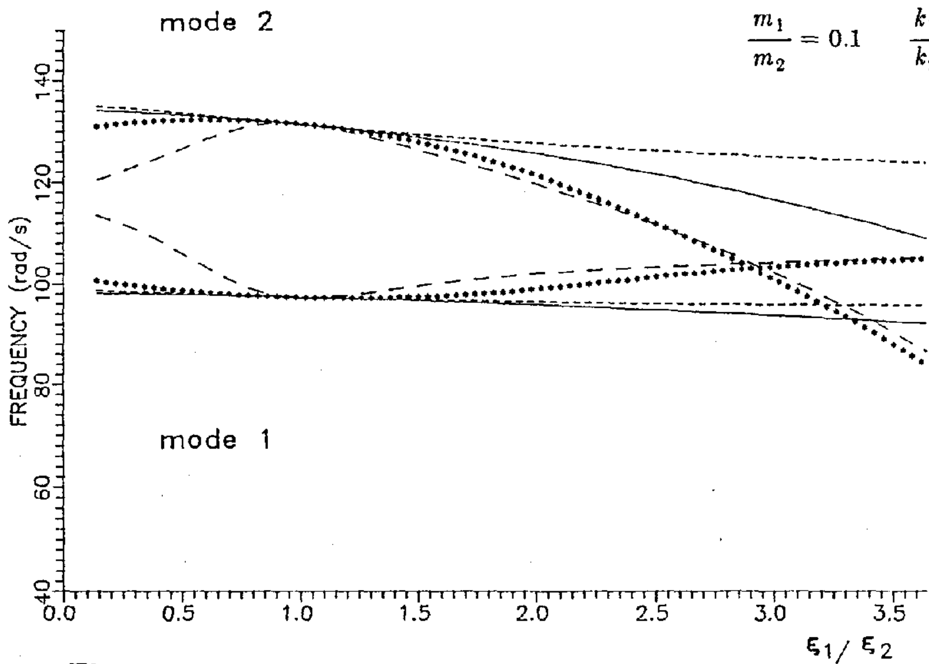


Figure 10. 2-DOF system eigenfrequencies v/s damping ratio

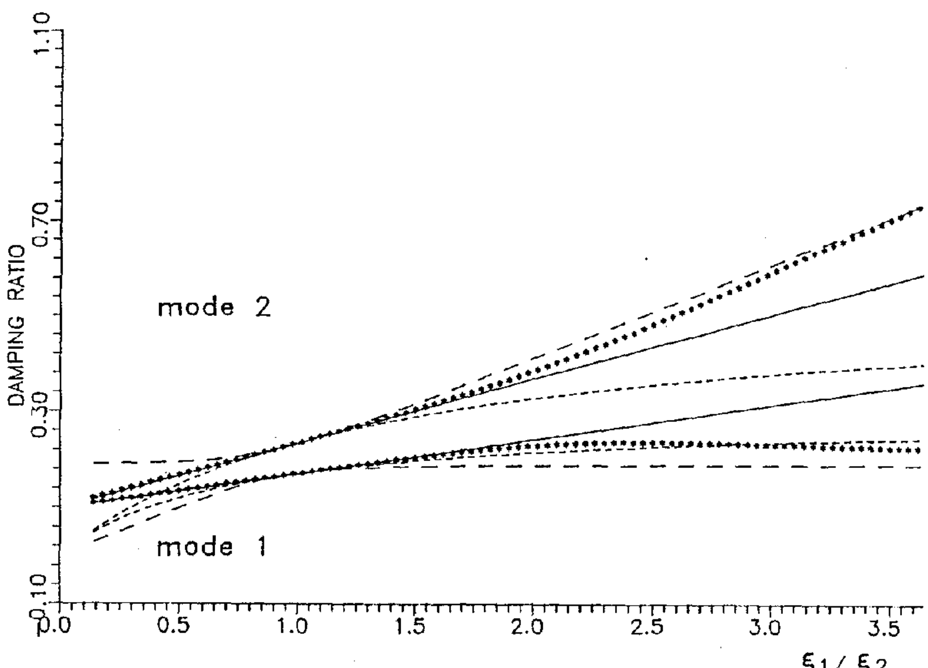


Figure 11. 2-DOF system damping ratios v/s damping ratio

* * *	exact solution
———	A-Superposition
— — —	D-Superposition
- - - - -	R-Superposition

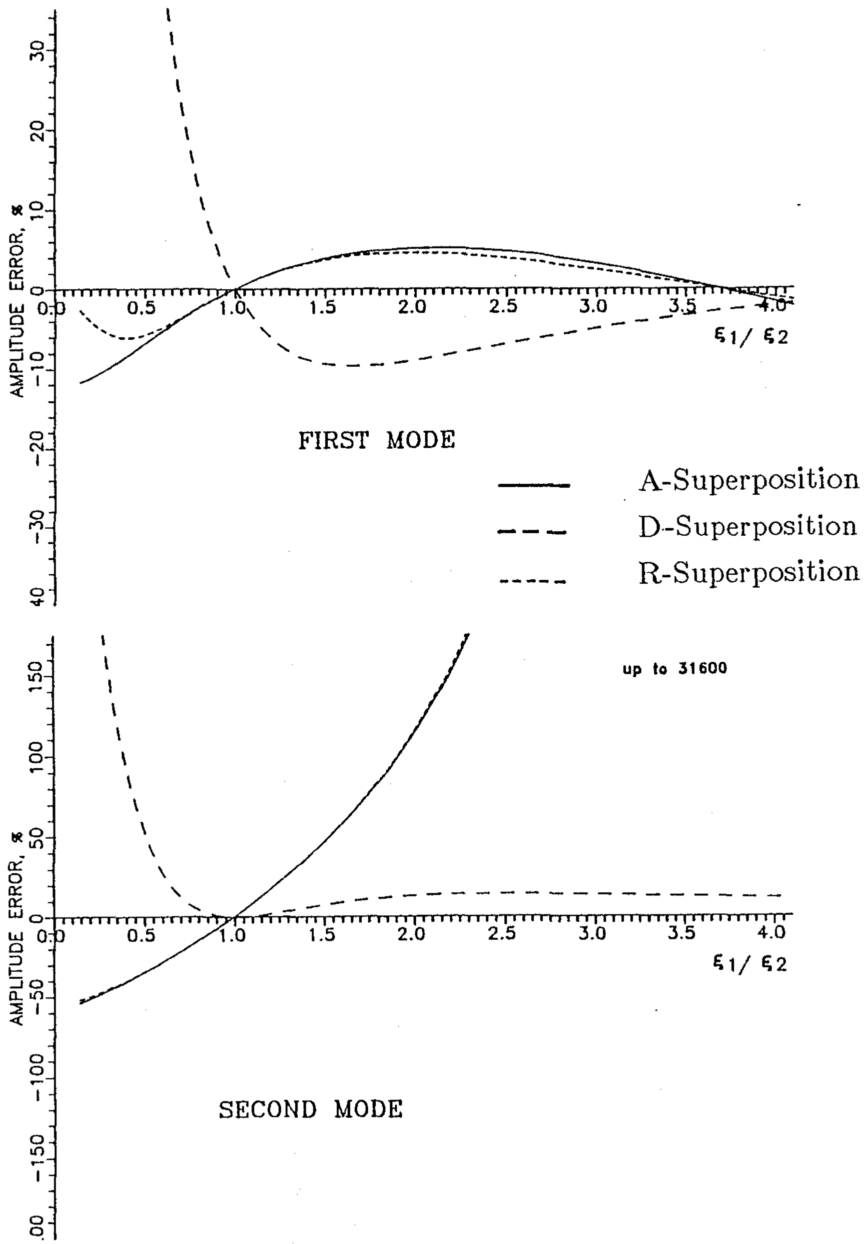


Figure 13. 2-DOF system amplitude approximation errors v/s damping ratio in first resonance

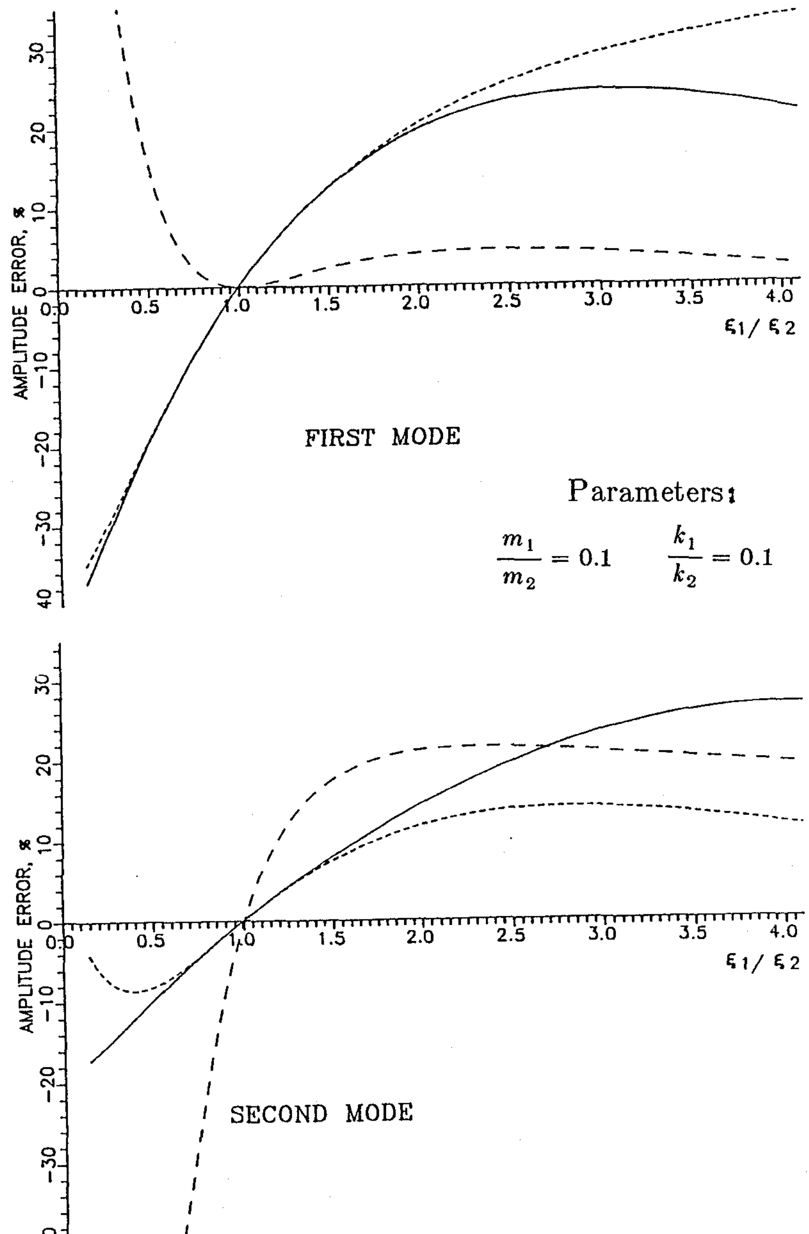


Figure 12. 2-DOF system amplitude approximation errors v/s damping ratio in second resonance

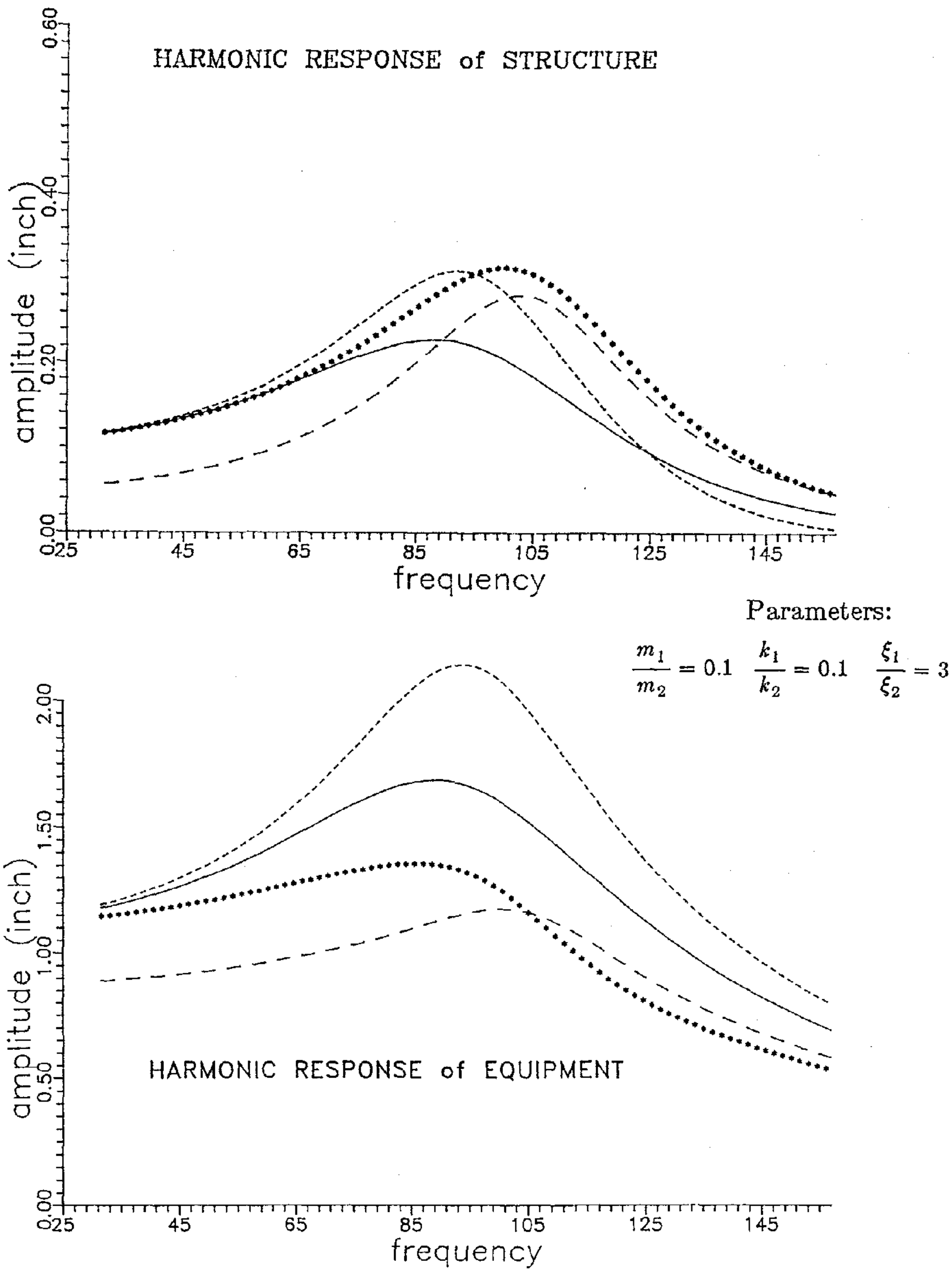


Figure 14. Harmonic response of "structure-equipment" system

* * *	exact solution
———	A-Superposition
— — —	D-Superposition
-----	R-Superposition

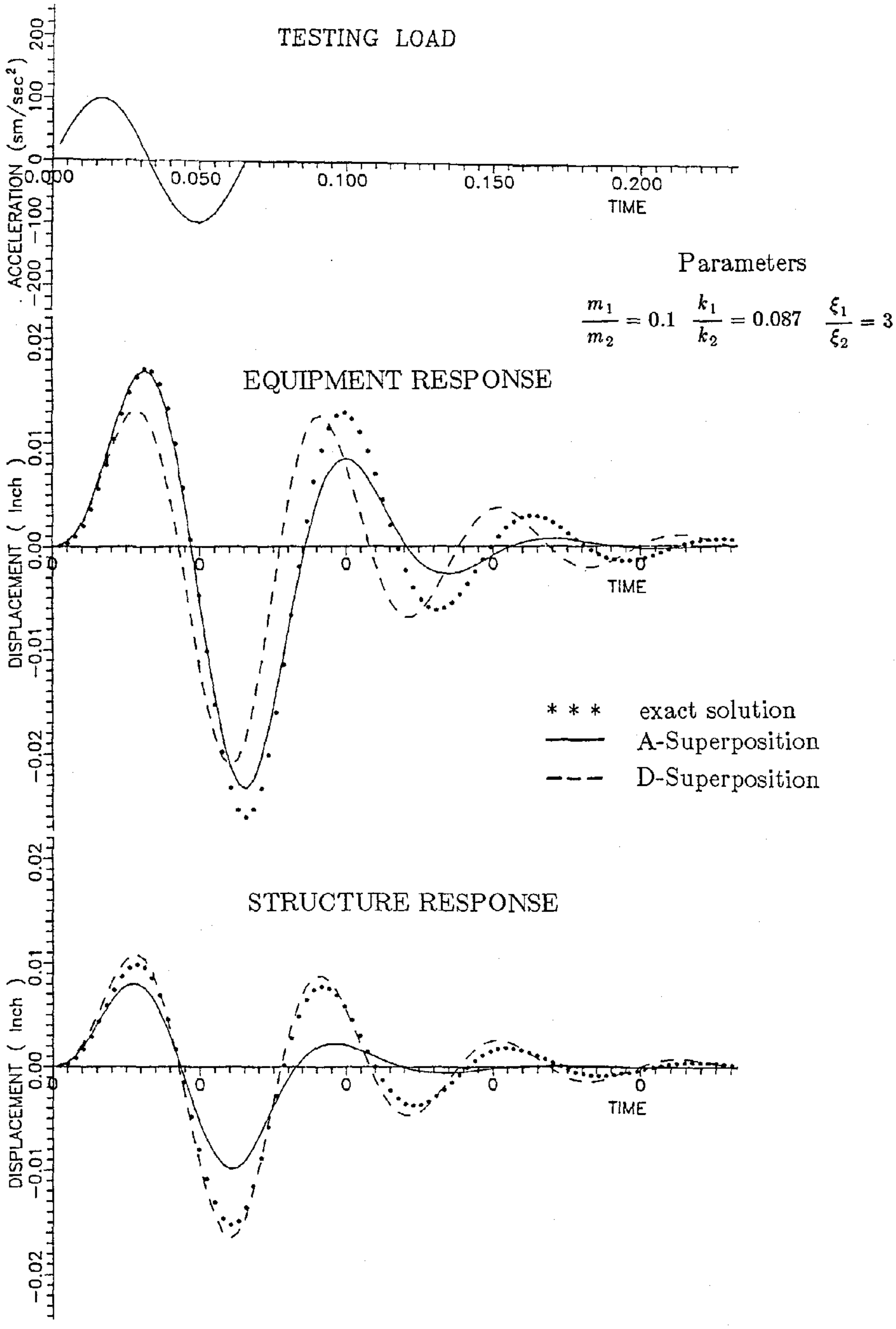
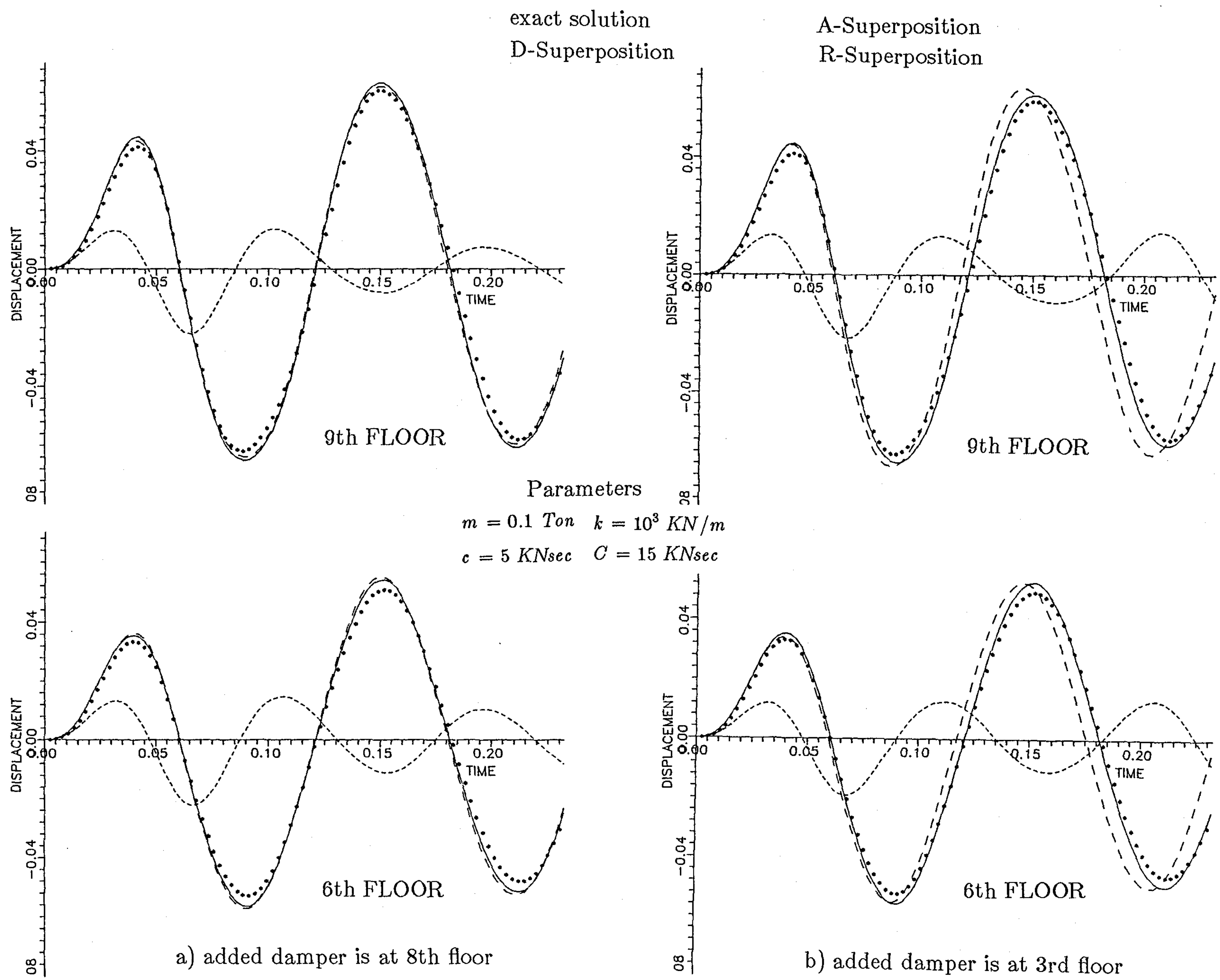


Figure 15. Time response of "structure-equipment" system



a) added damper is at 8th floor

b) added damper is at 3rd floor

Figure 17. 9-DOF system time response under sine wave load

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