NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
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Phase Wave Velocities and Displacement Phase Differences in a Harmonically Oscillating Pile

by

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Technical Report NCEER-91-0010

July 8, 1991

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NOTICE

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 3, Lifeline Systems, and more specifically to the study of dams, bridges and infrastructures.

The safe and serviceable operation of lifeline systems such as gas, electricity, oil, water, communication and transportation networks, immediately after a severe earthquake, is of crucial importance to the welfare of the general public, and to the mitigation of seismic hazards upon society at large. The long-term goals of the lifeline study are to evaluate the seismic performance of lifeline systems in general, and to recommend measures for mitigating the societal risk arising from their failures.

In addition to the study of specific lifeline systems, such as water delivery and crude oil transmission systems, effort is directed toward the study of the behavior of dams, bridges and infrastructures under seismic conditions. Seismological and geotechnical issues, such as variation in seismic intensity from attenuation effects, faulting, liquefaction and spatial variability of soil properties are topics under investigation. These topics are shown in the figure below.

Program Elements and Tasks

Dams	Bridges	Infrastructures
• Fragility Curves • Computer Codes • Risk Assessment and Management	• Evaluate and Recommend Response Modification Factor (RMF) • Develop Probabilistic Load and Resistant Factor Design (LRDF) Format	• Inspection, Maintenance and Repair • Non-destructive Tests (NDT) and Inspection • Develop On-line System Identification Techniques (INTELAB) • Evaluate Seismic Effects on Metropolitan New York Transit Facilities

In this report, the authors investigate whether the assumption of synchronous wave emission from an oscillating pile is a reasonable engineering approximation. A second, broader objective of the report is to obtain a more comprehensive physical insight into the nature of wave propagation in a single, harmonically-oscillating pile, embedded in a homogeneous soil.

ABSTRACT

Analytical solutions are developed for the harmonic wave propagation in an axially or laterally oscillating pile embedded in homogeneous soil and excited at the top. Both fixed-head and free-head piles are considered. Pile-soil interaction is realistically represented through a dynamic Winkler model, the "springs" and "dashpots" of which are given values based on results of finite-element analyses with the soil treated as a linear hysteretic continuum. Closed-form expressions are derived for the phase velocities of the generated waves; these are compared with characteristic phase wave velocities in rods and beams subjected to compression-extension (axial) and flexural (lateral) vibrations. The role of radiation and material damping is elucidated; it is shown that the presence of such damping changes radically the very nature of the wave propagation, especially in lateral oscillations where an upward propagating ("reflected") wave is generated even in a semi-infinite head loaded pile. Solutions are also developed for the phase differences between pile displacements at various depths. It is shown that for most piles, such differences would not be significant and, therefore, waves would emanate nearly simultaneously from the periphery of an oscillating pile --- a conclusion useful in analyzing dynamic pile-to-pile interaction, the concequences of which are illustrated in the report.

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SECTION 1

INTRODUCTION

The work to be presented was prompted by the need to develop a deeper understanding of some of the wave-propagation phenomena associated with the dynamic response of piles and pile groups. For instance, it is well known (Kaynia and Kausel 1982, Nogami 1983, Roesset 1984) that two neighboring piles in a group may affect each other so substantially that the overall dynamic behavior of the group could be vastly different from that of each individual pile. This pile-to-pile interaction is frequency-dependent and is a consequence of waves that are emitted from the periphery of each pile and propagate until they "strike" the other pile.

As an example, for a square group of 2 X 2 rigidly-capped piles embedded in a deep homogeneous stratum, Fig. 1-1 portrays the variation with frequency of the vertical and horizontal **dynamic group stiffness and damping factors**, defined as the ratios of the group dynamic stiffness and dashpot coefficients, respectively, to the sum of the static stiffnesses of the individual solitary piles. At zero frequency, the stiffness group factors reduce to the respective static group factors (also called "efficiency" factors) which are invariably smaller than unity.

The continuous curves in Fig. 1-1, adopted from the rigorous solution of Kaynia and Kausel (1982) , reveal that, as a result of dynamic pile-to-pile interaction, the dynamic stiffness group factors achieve values that may far exceed the static efficiency factors, and may even surpass unity. Both stiffness and damping factors exhibit undulations associated with wave interferences, which are not observed in the single-pile response. Specifically, **the peaks of the curves occur whenever waves originating with a certain phase from one pile arrive at the adjacent pile in exactly opposite phase**, thereby inducing an upward displacement at a moment that the displacement due to this pile's own load is downward. Thus, a larger force must be applied onto this pile to enforce a certain displacement amplitude, resulting in a larger overall stiffness of the group, compared to the sum of the individual pile stiffnesses.

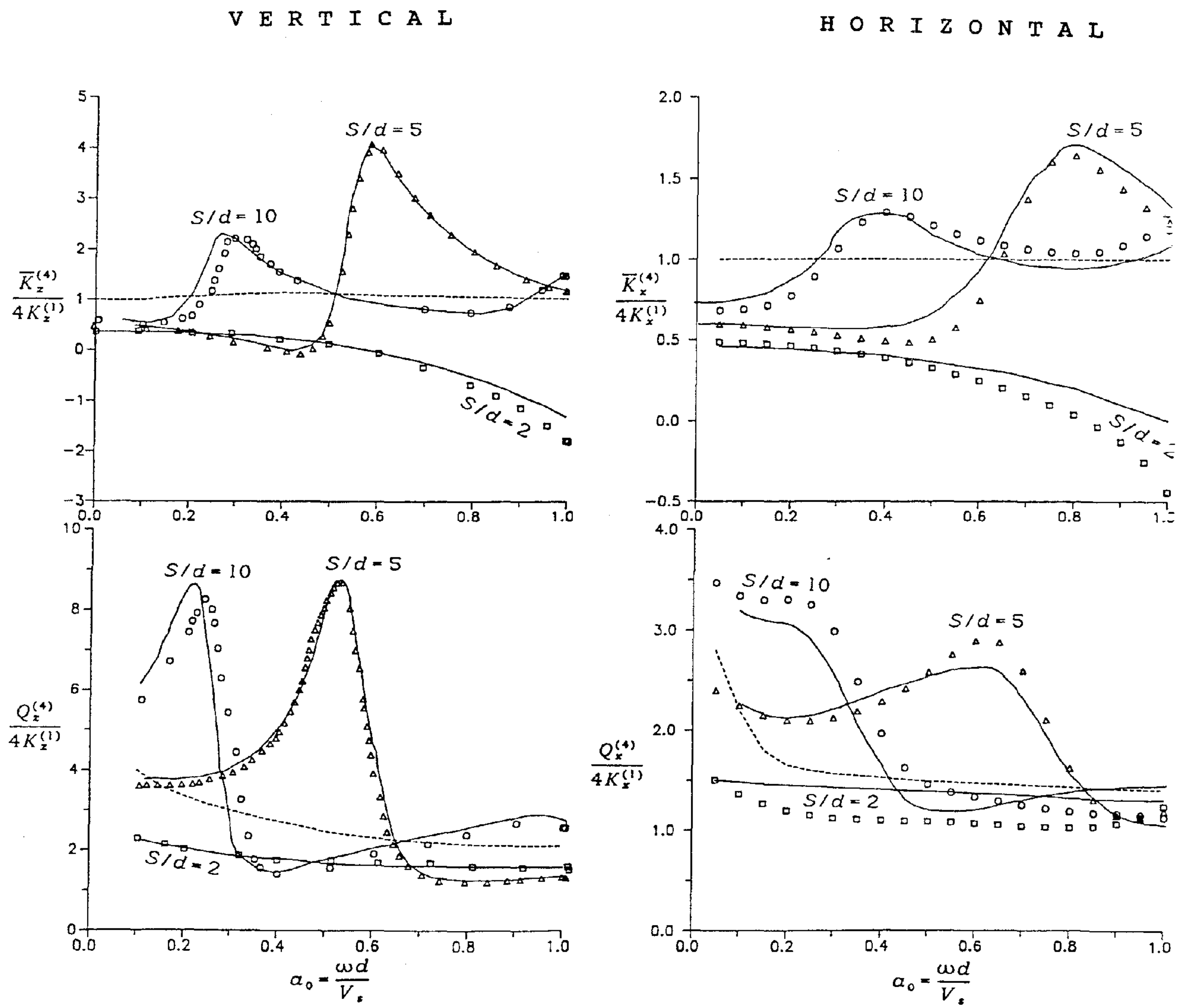

Figure. 1-1 Normalized vertical and lateral impedances of a 2X2 pile group. ($E_p/E_s = 1000$, L/d = 15, $\nu = 0.4$, $\beta = 0.05$). Solid curves: rigorous solution of Kaynia & Kausel (1982) Points: simplified solution of Dobry & Gazetas (1988) left and Makris & Gazetas (1989) right. Impedances are expressed as $\overline{K} + ia_0Q$. Subscripts z or x refer to vertical or horizontal mode. $\overline{K}^{(4)}$ and $Q^{(4)}$ are the total dynamic stiffness and damping of the 4-pile group; and $K^{(1)}$ is the static stiffness of the single (solitary) pile.

Also depicted in Fig. 1-1 as points are the results of a very simple analytical method of solution proposed by Dobry & Gazetas (1988) and further developed by Makris & Gazetas (1989). The method introduces a number of physically-motivated approximations and, in fact, it was **originally intended merely to provide an explanation of the causes of the numerically-observed peaks and troughs** in the dynamic impedences of pile groups. Yet, as is evident from the comparison of Fig. 1-1, the results of the method are remarkably close to the rigorous curves for the three considered pile separation distances of two, five and ten pile diameters. Even some detailed trends in the group response seem to be adequately captured by the simple solution.

The fundamental idea of this method is that the displacement field created along the sidewall of an oscillating pile (in any mode of vibration) propagates and affects the response of neighboring piles. The most crucial of the simplifying assumptions is that **the waves created by an oscillating pile emanate simultaneously** from all perimetric points along the pile length, and hence, for a homogeneous stratum, they form cylindrically-expanding waves that would "strike" an adjacent pile simultaneously at various points along its length. (That is, the arriving waves are all in phase, although their amplitudes decrease with depth.)

The question that arises is whether the satisfactory performance of such a simple method is merely a coincidence (e.g due to cancelation of errors), or is it rather a consequence of fundamentally-sound physical approximations. Answering this question was one of the motives for the work reported herein. Hence, the **first** objective of this paper is to investigate whether the aforementioned key assumption of **synchronous wave emission** from an oscillating pile is indeed a reasonable engineering approximation, and for what ranges of problem parameters.

A **second** broader objective of the paper is to obtain a deeper physical insight into the nature of wave propagation in a single harmonically-oscillating pile, embedded in homogeneous soil. To this end, realistic dynamic Winkler-type models for vertically and horizontally oscillating single piles are developed, from which analytical solutions are derived for the (a) apparent phase velocities of the waves propagating along the pile and (b) for the variation with depth of pile displacements and phase-angle differences. A limited number of rigorous finite-element results are also obtained to substantiate the findings of the Winkler model. It is shown that, indeed, the apparent phase velocities are for typical piles quite large and the displacement phase differences correspondingly small, especially within the upper, most active part of the pile. It is also found that at very high frequencies the phase velocities in a pile embedded in homogeneous soil become assymptotically equal to the wave velocities of an unsupported bar or beam in longitudinal and flexural oscillations.

SECTION 2

PROBLEM DEFINITION

The problem studied in this report refers to a single floating pile embedded in a uniform halfspace and subjected at its head to a harmonic loading of circular frequency ω. The pile is a linearly-elastic flexural beam with Young's modulus E_p, diameter d, cross section area A, bending moment of inertia I, and mass per unit length m. The soil is modeled as dynamic Winkler medium, resisting pile displacements through continuously-distributed linear "springs" (k_x or k_z) and "dashpots" (c_x or c_z), as sketched in Fig. 2-1 for horizontal (x) and vertical (z) motion. The force to displacement ratio of the Winkler medium at every depth defines the complex-valued impedances $k_x + i\omega c_x$ (horizontal motion) or $k_z + i\omega c_z$ (vertical motion), $i = \sqrt{-1}$, where c_x and c_z would, in general, reflect both radiation and material damping in the soil.

Frequency-dependent values are assigned to these "spring" and "dashpot" coefficients, using the following algebraic expressions developed by fitting parametric results from dynamic finite-element analyses, for a Poisson's ratio of 0.40 (Roesset & Angelides, 1979; Blaney, Kausel & Roesset, 1976; Gazetas & Dobry, 1984)

For the problem of axial vibration we use:

$$k_z \approx 0.6E_s\left(1 + \frac{1}{2}\sqrt{a_0}\right) \tag{2.1a}$$

$$c_z \approx (c_z)_{radiation} + (c_z)_{hysteresis}$$

$$\approx 1.2\ a_0^{-1/4}\pi d\rho_s V_s \quad + \quad 2\beta\frac{k_z}{\omega} \tag{2.1b}$$

For the problem of lateral vibration the spring coefficient depend on the boundary condition of the pile head. Accordingly we use:

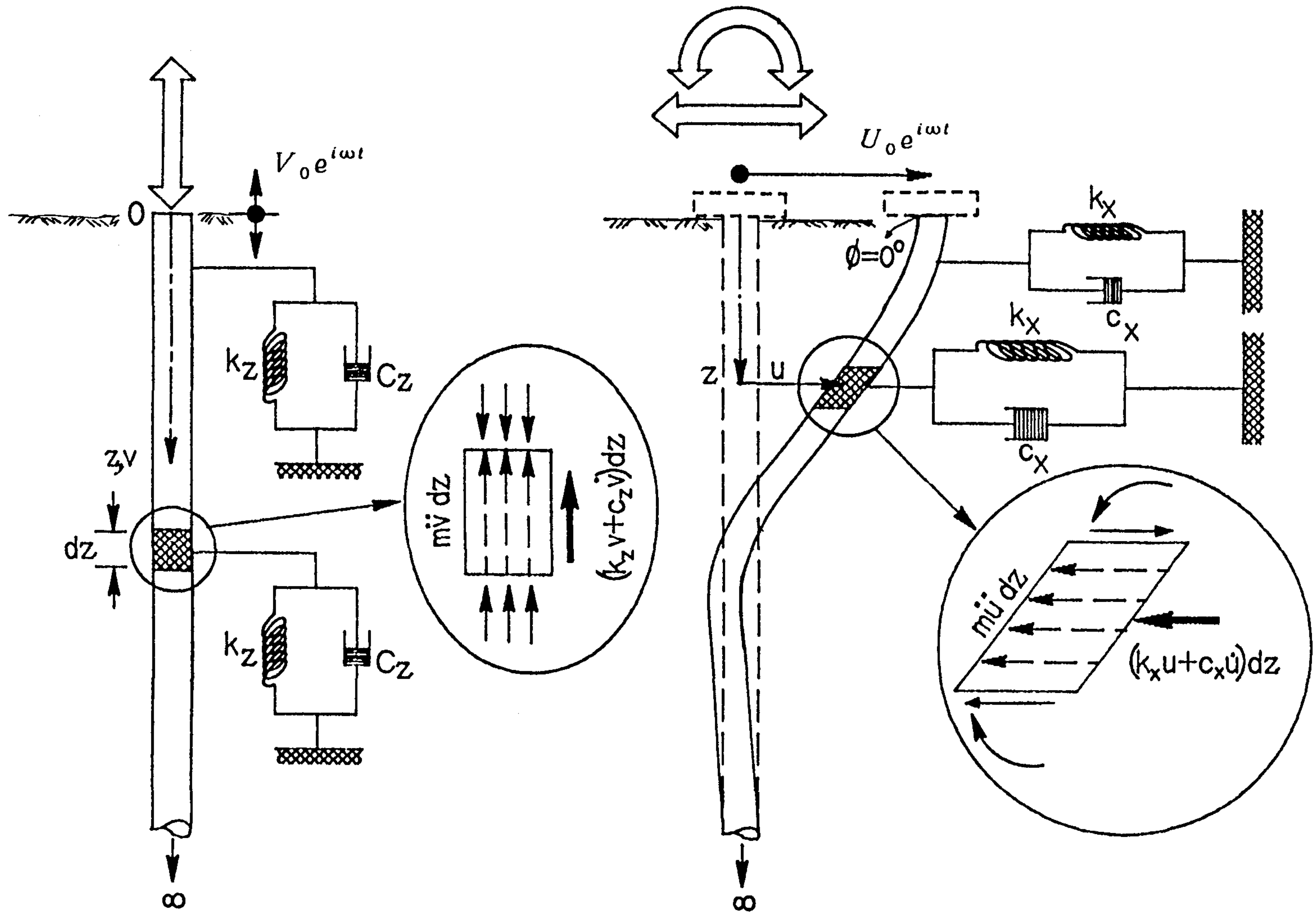

Figure 2-1 Dynamic Winkler model for axially and laterally oscillating pile.

$$k_x \approx 1.2E_s \qquad \textit{fixed} \quad \textit{head} \tag{2.2a}$$

$$k_x \approx 2.1E_s \qquad \textit{free} \quad \textit{head} \tag{2.2b}$$

$$c_x \approx (c_x)_{radiation} + (c_x)_{hysteresis}$$

$$\approx 6\, a_0^{-1/4} d\rho V_s \quad + \quad 2\beta\frac{k_x}{\omega} \tag{2.2c}$$

in which: β, ρ_s, E_s and V_s = hysteretic damping, mass density, Young's modulus and S-wave velocity of the soil, respectively; and

$$a_0 = \frac{\omega d}{V_s} \tag{2.3}$$

The c_x values obtained from Eq. 2.2-c apply in reality only for frequencies ω above the stratum cutoff frequency ω_s, which is essentially identical to the natural frequency, $(\pi/2)V_s/H$, in horizontal (shear) vibrations of the soil stratum; for $\omega < \omega_s$ radiation damping is vanishingly small, in function of the material damping; one may then state: $c_x \approx (c_x)_{hysteresis}$. Similarly, the c_z expression in Eq. 2.1-b applies only for $\omega > \omega_c$, i.e for frequencies beyond the stratum cut-off frequency in vertical compression-extension vibration; for $\omega < \omega_c$, $c_z \approx (c_z)_{hysteresis}$.

It is emphasized that the above expressions have been determined so that the horizontal and vertical stiffness at the head of the pile embedded in the **Winkler** soil model would be essentially **identical** to the respective of the pile head stiffnesses in the **continuum** (finite-element) soil model. Thus, the pile-soil models used in this study (Fig. 2-1 and Eqns. 2.1, 2.2) constitute reasonably accurate representations of a pile in a deep homogeneous soil stratum (see also : Liou & Penzien 1979).

SECTION 3

AXIAL VIBRATION

3.1 Governing equations and solution

For relatively very short (L/d < 10) and stiff (E_p / E_s > 8000) piles, the basic validity of the simplifying assumption of synchronous wave emission is self-evident, since such piles respond essentially as rigid bodies to axial loading (static or dynamic). For the other extreme case, of long and flexible piles, the pile is considered herein as an infinite elastic "thin" rod (i.e., lateral-inertia effects are ignored-- in accordance with classical rod theory). The deflected state of such a pile and the forces acting on an element are sketched in Fig. 2-1. For harmonic steady-state oscillations, the vertical displacement $v(z,t)$ of a point on a cross section of the pile at depth z and time t can be written as

$$v(z,t) = v(z)e^{i\omega t} \tag{3.1}$$

and dynamic equilibrium yields

$$E_p A \frac{d^2 v(z)}{dz^2} - (k_z + i\omega c_z - m\omega^2) v(z) = 0 \tag{3.2}$$

Solutions are obtained separately for each of the two possible cases: $\omega < \overline{\omega}_z$ and $\omega \geq \overline{\omega}_z$ where $\overline{\omega}_z$ is the characteristic frequency:

$$\overline{\omega}_z = (k_z/m)^{1/2} \tag{3.3}$$

First: $\omega < \overline{\omega}_z$. In view of Eq. 2.3, this inequality translates approximately to $a_0 < 1.5$, which is the usual range of practical interest in foundation problems. Eq. 3.2 can be written as

$$\frac{d^2 v(z)}{dz^2} - \lambda^2 v(z) = 0 \tag{3.4}$$

where λ^2 is a complex number with

$$Re\{\lambda^2\} = \frac{k_z - m\omega^2}{E_p A} \quad > \quad 0 \tag{3.5a}$$

$$Im\{\lambda^2\} = \frac{\omega c_z}{E_p A} \quad > \quad 0 \tag{3.5b}$$

By applying De Moivre's formula λ takes the form

$$\lambda = R\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right) \tag{3.6}$$

in which

$$R = \left\{\frac{(k_z - m\omega^2)^2 + (\omega c_z)^2}{(E_p A)^2}\right\}^{1/4} \tag{3.7}$$

$$\theta = Arctan\left(\frac{\omega c_z}{k_z - m\omega^2}\right), \qquad 0 < \theta < \frac{\pi}{2} \tag{3.8}$$

The solution to Eq. 3.4 is

$$v(z,t) = A_1 e^{R\cos\frac{\theta}{2}z} e^{i\left(\omega t + R\sin\frac{\theta}{2}z\right)} + A_2 e^{-R\cos\frac{\theta}{2}z} e^{i\left(\omega t - R\sin\frac{\theta}{2}z\right)} \tag{3.9}$$

For the displacement to remain finite as z tends to infinity A_1 must vanish. Calling V_0 the displacement amplitude at the pile head (z = 0) leads to

$$v(z,t) = V_0 e^{-R\cos\frac{\theta}{2}z} e^{i\left(\omega t - R\sin\frac{\theta}{2}z\right)} \tag{3.10}$$

This equation represents a traveling wave of amplitude decreasing exponentially with depth and of phase velocity

$$C_\alpha = \frac{\omega}{R \sin\frac{\theta}{2}} \qquad \textit{dispersion} \quad \textit{relation} \tag{3.11}$$

in which both R and θ are functions of the frequency (ω) and damping (c_z).

Second: $\omega \geq \bar{\omega}_z$. This inequality translates to approximately $a_0 > 1.5$, a frequency range of lesser interest, but which is nevertheless examined herein as providing insight into the asymptotic behavior at high frequencies. The solution now takes the form

$$v(z,t) = V_0 e^{R \sin\frac{\theta}{2} z} e^{i\left(\omega t - R \cos\frac{\theta}{2} z\right)} \tag{3.12}$$

where

$$R = \left\{ \frac{(m\omega^2 - k_z)^2 + (\omega c_z)^2}{(E_p A)^2} \right\}^{1/4} \tag{3.13}$$

and

$$\theta = Arctan\left(\frac{-\omega c_z}{m\omega^2 - k_z} \right), \qquad -\frac{\pi}{2} < \theta \leq 0 \tag{3.14}$$

Equation 3.12 represents a traveling wave with amplitude decreasing exponentially with depth and phase velocity

$$C_\alpha = \frac{\omega}{R \cos\frac{\theta}{2}} \qquad \textit{dispersion} \quad \textit{relation} \tag{3.15}$$

3.2 Characteristics of the results

From the dispersion relation of Eq. 3.11, the ratio of the pile phase velocity to the soil S-wave velocity is obtained (see Appendix A) :

$$\frac{C_\alpha}{V_s}=\frac{a_0\sqrt{\frac{\pi}{4}s_2}}{\left\{\left(f_1-\frac{\pi}{4}s_1s_3a_0^2\right)^2+f_2^2\right\}^{1/4}\sin\left\{\frac{1}{2}Arctan\left(\frac{f_2}{f_1-\frac{\pi}{4}s_1s_3a_0^2}\right)\right\}} \tag{3.16}$$

where

$$s_1=\frac{G_s}{E_s},\quad s_2=\frac{E_p}{E_s},\quad s_3=\frac{\rho_p}{\rho_s} \tag{3.17}$$

$$f_1=0.6\,(1+0.5\sqrt{a_0}),\qquad f_2=1.2\ \pi s_1 a_0^{3/4}+2\beta f_1 \tag{3.18}$$

This ratio, C_α/V_s, is plotted versus a_0 in Fig. 3-1 for two different characteristic values of relative pile stiffness $s_2=E_p/E_s$ =1000 and 5000, and for a low (0.4) and a high (0.6) value of the product s_1s_3. Note that in the frequency range of greatest practical interest, i.e for $0.2< a_0 < 0.8$, the ratio C_α/V_s attains relatively high values, of the order of 40 for E_p/E_s = 1000 and 85 for E_p/E_s = 5000. As a result, phase differences introduced by waves travelling down the pile would be negligible compared with the phase differences due to S-waves travelling in the soil from one pile to another. Thus, for example, with a pile of L=20d and $\rho_p = 1.4\rho_s$ the error committed by assuming "synchronous" wave emission would be of the order of 8% (for E_p/E_s = 1000) and 3% (for E_p/E_s = 5000).

To see this more clearly, the phase angle from Eq. 3.10 is

$$\phi(z)=\omega t - Rz\sin\frac{\theta}{2} \tag{3.19}$$

where θ and R can be rewritten in dimensionless form as

$$\theta = Arctan\left\{\frac{f_2}{f_1-\frac{\pi}{4}s_1s_3a_0^2}\right\} \tag{3.20}$$

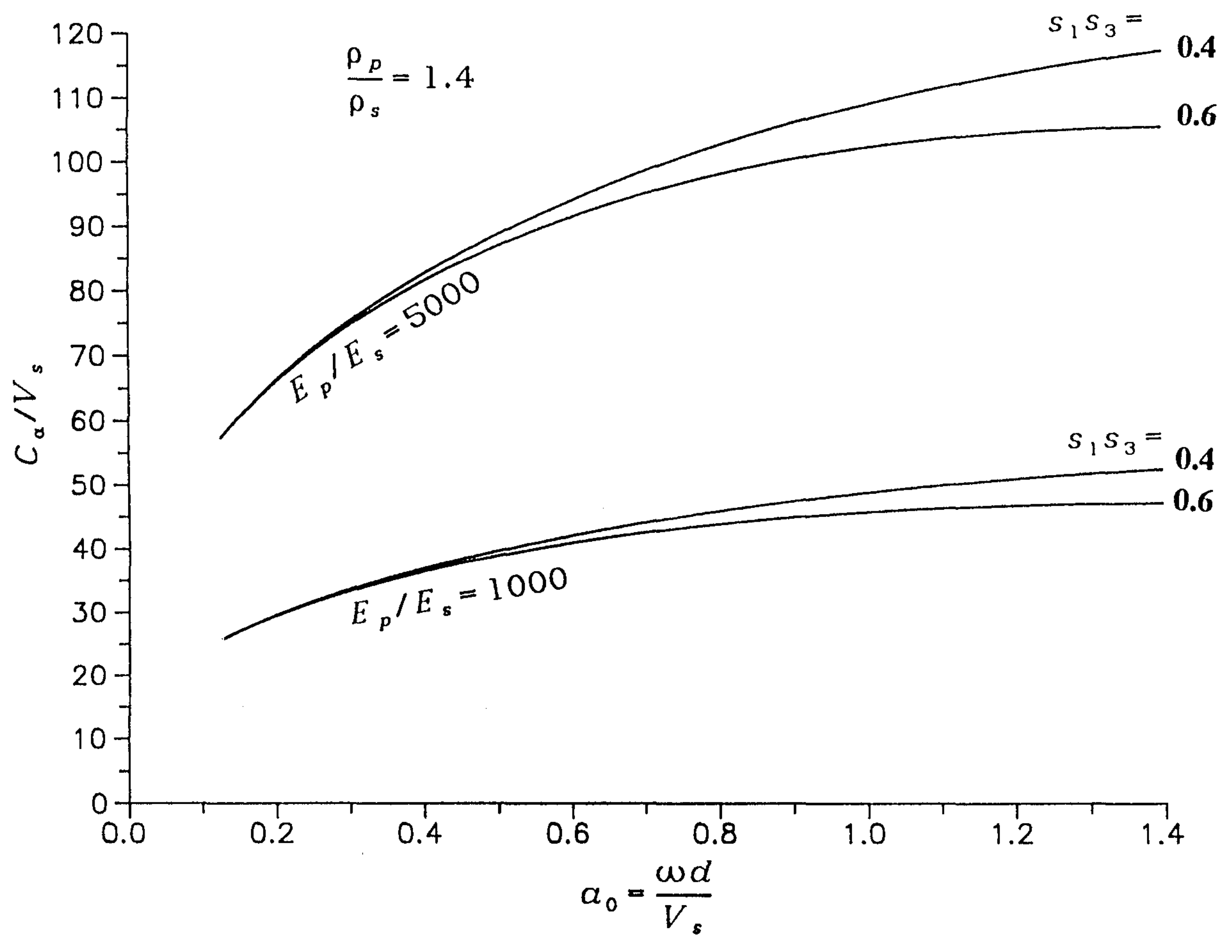

Figure 3-1 Dispersion relationships for phase velocity of waves in an axially vibrating infinitely-long pile, in the frequency range of gratest interest, for two values of pile-to-soil Young's moduli ratio.

$$R = 2d^{-1}\left\{\frac{\left(f_1 - \frac{\pi}{4}s_1 s_3 a_0^2\right)^2 + f_2^2}{(\pi s_2)^2}\right\}^{1/4} \tag{3.21}$$

Fig. 3-2 plots the phase differences $\Delta\phi = \Delta\phi(z)$ between the displacement of a section at depth z and that at the head of the pile, for the two considered values of E_p/E_s (1000 and 5000) and two values of a_0 (0.2 and 0.5). Evidently, even in the case of a relatively flexible pile, the pile at a depth z=20d has a phase difference with the head of only about 15^0. For the stiffer pile: $\Delta\phi \leq 8^0$

These differences are indeed insignificant (within engineering accuracy) and therefore the assumption of synchronous emission is a reasonable approximation.

Since the above results were derived on the basis of an infinitely-long Bar-on-Dynamic-Winkler-Foundation, it is of interest to show their general validity for piles of finite-length supported by a visco-elastic continuum. To this end, a rigorous finite-element study (Blaney et al, 1976) is conducted for a pile of slenderness ratio L/d = 20 embedded in a deep homogeneous stratum and having $E_p/E_s = 1000$ or 5000. Fig. 3-3 plots the distribution along the length of the pile of the real and imaginary parts of the vertical pile displacement, $v = v(z)$, for the same two values (0.2 and 0.5) of the frequency factor a_0. Evidently, the imaginary and real components of the displacement as well as the resulting phase angle remain nearly constant with depth; hence, the phase differences between various points along the pile and its head (also plotted in the figure) are indeed very small, similar to those predicted with the analytical method (Fig. 3-2). Thus, the analytical results and the hypothesis of synchronous wave emission are largely substantiated. However, in much stiffer soils, for which the moduli ratio E_p/E_s may perhaps attain values as low as 400 or less, the apparent phase velocity C_α becomes a smaller multiple of V_s, and then for very slender piles (L>40) phase differences along the pile may at higher frequencies reach 50^0. In such cases the assumption of synchronous emission might not be applicable.

An additional observation on the dispersion relation of Eq. 3.11 deserves a note. While Fig. 3-1 plots C_a for a homogeneous halfspace, in reality, bedrock or at least a stiff rock-like soil layer

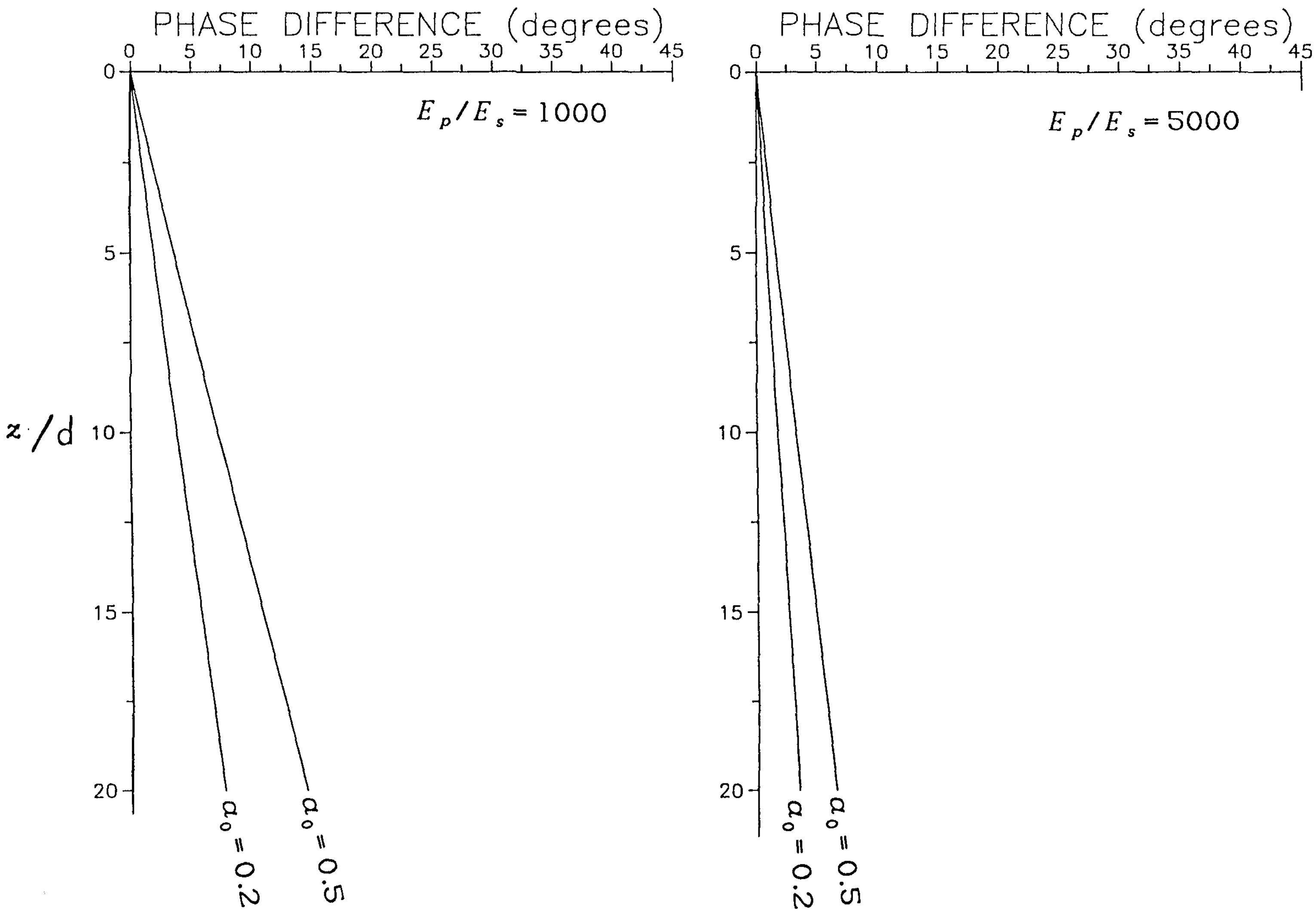

Figure 3-2 Phase difference between displacements at depth z and at pile top for two values of the frequency factor.

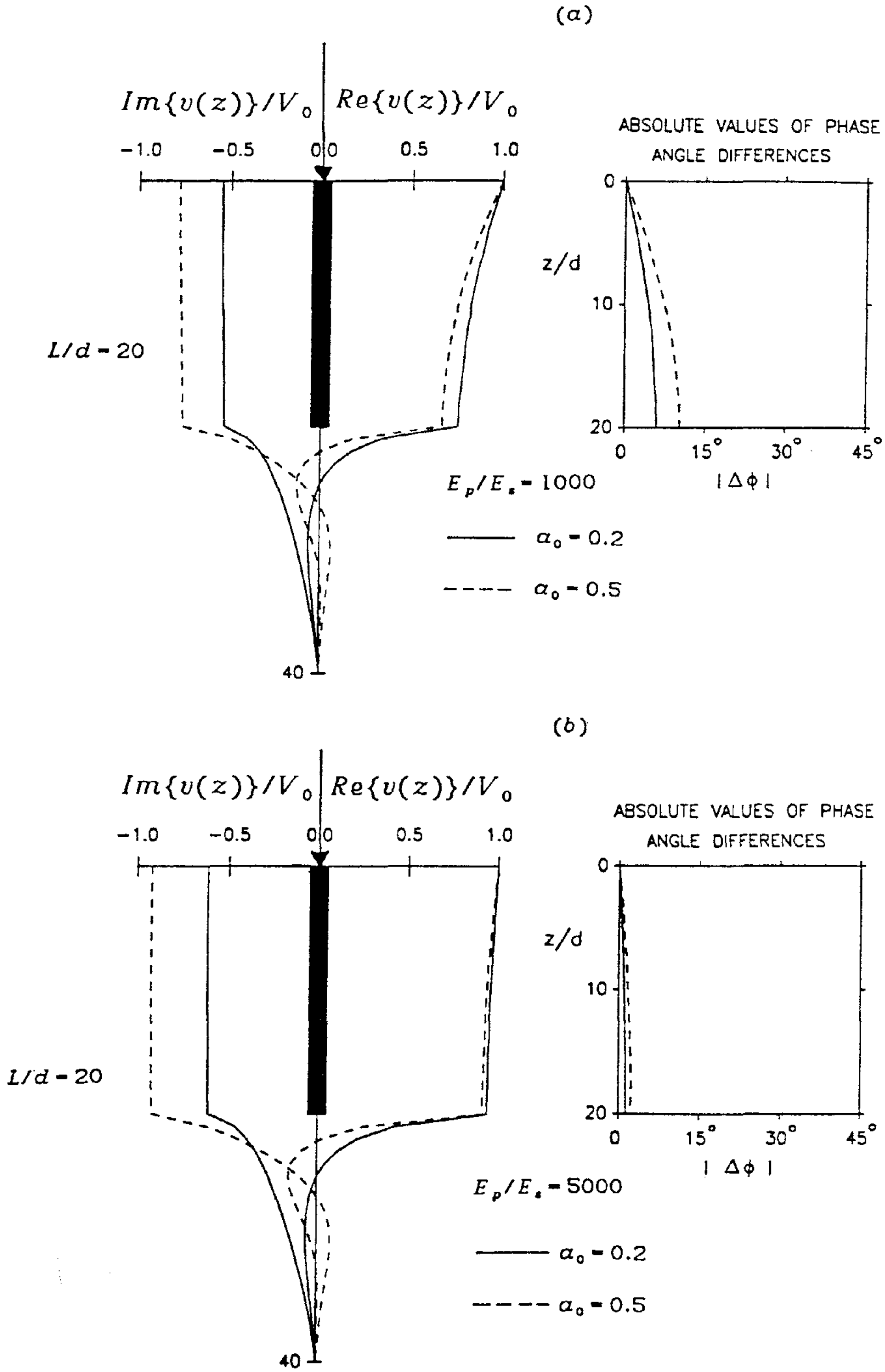

Figure 3-3 Distribution with depth of normalized vertical pile displacements (Imaginary part and Real part) and pile-displacement phase differences for an L/d = 20 pile in a deep homogeneous soil with (a) $E_p/E_s = 1000$ and (b) $E_p/E_s = 5000$. Displacements of soil below the pile are also plotted. Results were obtained with a dynamic finite element formulation (Blaney et al 1976) for the two shown values of the frequency factor.

is likely to exist at some depth below the ground surface. Then the soil deposit is a stratum rather than a halfspace. Below the stratum cutoff frequency, ω_c, the pile-soil system radiates very little energy, and c_z essentially reflects only the hysteretic material damping in the soil. Without material damping $c_z = 0$, and the solution reduces to the case discussed by Wolf (1985, 1988), in which the phase velocity is indeed infinite (since $\theta = 0$). Therefore, as a first approximation, for

$$\omega < \omega_c: \qquad C_\alpha - \rightarrow \infty \tag{3.22}$$

In general, however, the phase velocity is finite as long as there exist a mechanism of energy dissipation along the pile (radiation or material damping).

It is also of interest to study the complete evolution of the phase wave velocity over an extreme range of frequencies ($0 < a_0 < 10$). This is done in Fig. 3-4 for a pile with $E_p/E_s = 5000$ and two different pile mass densities: $\rho_p = 1.4\rho_s$ and $0.7\rho_s$. The solid curve represents the developed dispersion relation; it is obvious that Eq. 3.11 and Eq. 3.21 give the same value for both C_a and dC_a/da_0 at the characteristic frequency, $\overline{\omega}_z$. Also plotted in Fig. 3-4 are the dispersion relations of two other simpler associated systems:

- a semi-infinite rod on Elastic-Winkler foundation, and
- a semi-infinite unsupported rod

These two systems have been studied extensively in the wave-propagation literature (e.g. Graff 1975, Achenbach, 1976), and are obviously particular cases of the pile system studied herein (Fig. 2-1). The phase velocity, C_E for the rod on elastic foundation is recovered from Eqs. 3.11 and 3.21 by setting $c_z = 0$, at all frequencies. As discussed by Wolf (1985) and mentioned earlier herein, C_E becomes infinite at and below the characteristic frequency $\overline{\omega}_z$. Therefore:

$$C_E = \infty, \qquad \omega \leq \overline{\omega}_z \tag{3.23a}$$

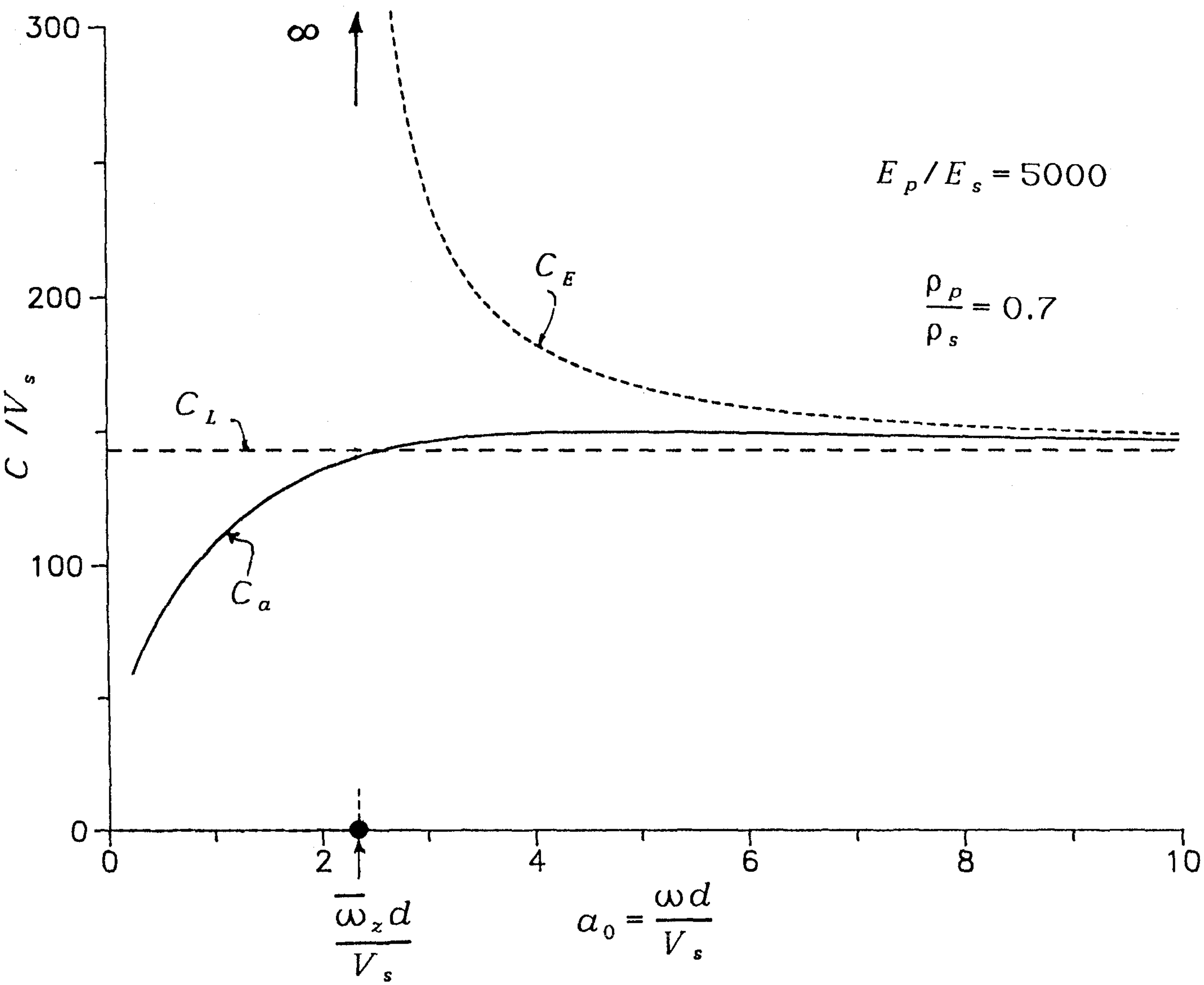

Figure 3-4 Comparison of dispersion relations for three longitudinal phase wave velocities: C_α, for a pile supported on axial "springs" and "dashpots" (modeling embedment in halfspace); C_E, for a bar on axial "springs"; and C_L for an unsupported bar. Two different pile mass densities.

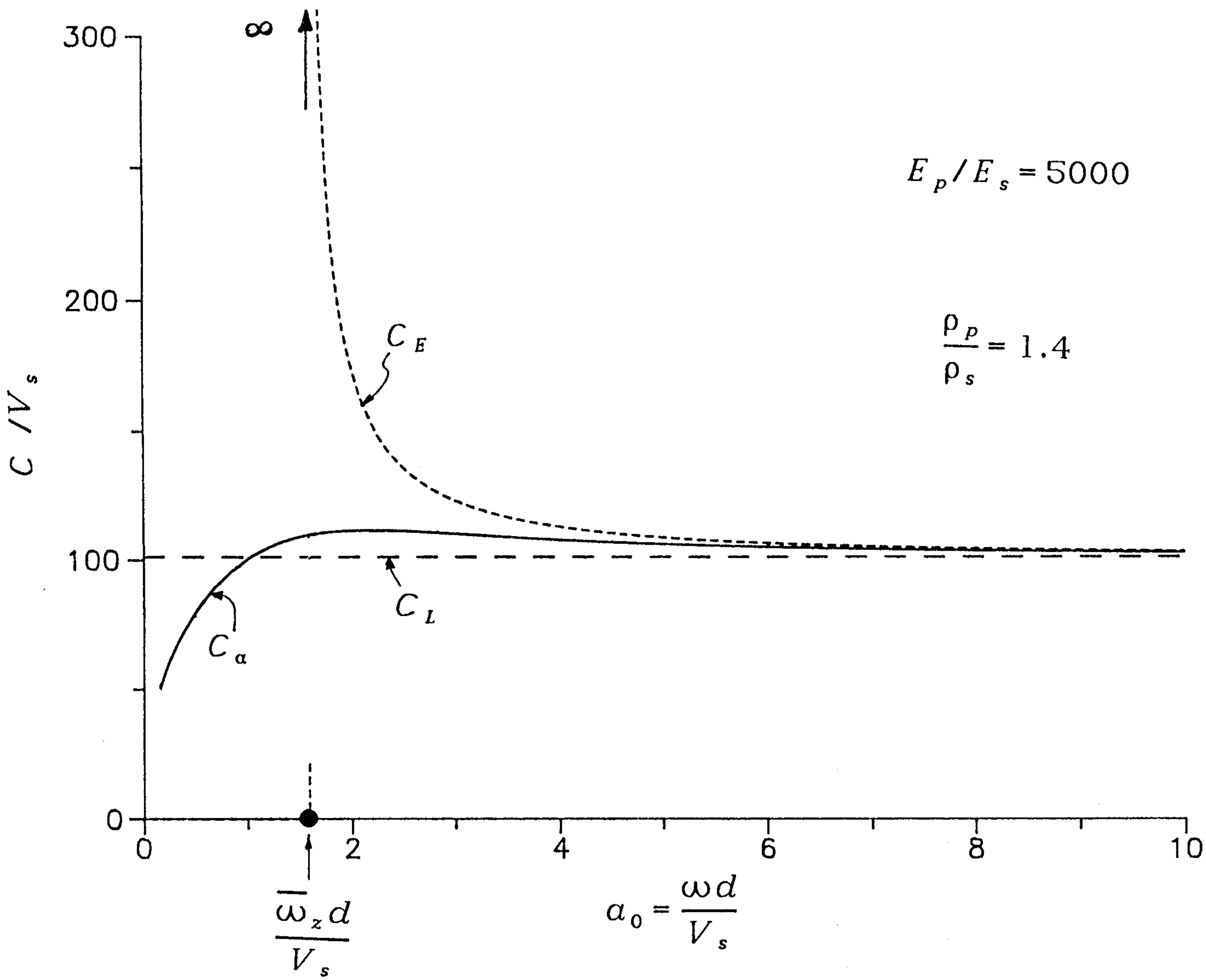

Figure 3-4 Comparison of dispersion relations for three longitudinal phase wave velocities: C_α, for a pile supported on axial "springs" and "dashpots" (modeling embedment in halfspace); C_E, for a bar on axial "springs"; and C_L for an unsupported bar. Two different pile mass densities.

$$C_E = V_s \frac{a_0}{2}\left(\frac{s_2\pi}{\frac{\pi}{4}s_1 s_3 a_0^2 - f_1}\right)^{1/2}, \qquad \omega > \overline{\omega}_z \tag{3.23b}$$

The phase velocity C_L for longitudinal waves in an unsupported rod (called "bar" or "rod" wave velocity) is equal to $\sqrt{E_p/\rho_p}$ only when lateral-inertia effects are ignored. However, for the frequency range studied, $a_0 < 10$, the decline of C_L with frequency ("Pochhammer" effect) is indistinguishable in the scale of the figure.

Fig. 3-4 reveals an interesting feature: all three phase wave velocities, C_α, C_E and C_L, reach **identical asymptotic values at high frequencies.** It appears that at such high frequencies pile inertia effects dominate, while the resistance of the supporting "springs" and "dashpots" becomes negligibly small, in comparison.

SECTION 4

LATERAL VIBRATION

4.1 Active Length of Vibrating Pile

For lateral excitation the assumption of an infinitely-long pile is quite appropriate even for stiff piles, since their "active" length is usually smaller than the total pile length. Indeed, for a fixed head pile on Winkler foundation, the " active" length below which the pile deformations are negligible is given by Randolph (1981):

$$l_c = 4\left(\frac{E_p I}{k_x}\right)^{1/4} \tag{4.1}$$

where the expressions for k_x for fixed head pile is given from Eq. 2.2a. For the typical values of $E_p/E_s = 1000$ and 5000 the "active" length from the above expression is only about **10d** and **15d**, respectively. For a free head pile the active length is even smaller and therefore in most cases, piles respond as infinitely long beams.

4.2 Fixed-Head Pile

4.2.1 Governing equation and solution

The pile is modeled as an Euler-Bernoulli beam (i.e the effects of rotatory inertia and shear distortion are ignored). The deflected state of the pile and the forces acting on an element are sketched in Fig. 2-1, with $u(z,t)$ denoting the horizontal displacement at depth z and time t. Zero slope is imposed at pile head to account for the shape of deformation induced by a horizontally-translating rigid pile cap ("fixed-head" pile, in geotechnical terminology). For a harmonic steady-state excitation

$$u(z,t) = u(z)e^{i\omega t} \tag{4.2}$$

and dynamic equilibrium gives

$$E_pI\frac{d^4u(z)}{dz^4}+(k_x+i\omega c_x-m\omega^2)u(z)=0 \tag{4.3}$$

The solution to Eq. 4.3 is sought separately for the two cases of $\omega<\overline{\omega}_x$ and $\omega\geq\overline{\omega}_x$, where $\overline{\omega}_x$ the characteristic frequency is now:

$$\overline{\omega}_x=(k_x/m)^{1/2} \tag{4.4}$$

First: $\omega<\overline{\omega}_x$. This is again the usual range of greatest interest in foundation dynamics , corresponding approximately to $a_0<1$. Eq. 4.3 reduces to

$$\frac{d^4u(z)}{dz^4}+4\lambda^4u(z)=0 \tag{4.5}$$

with

$$4\lambda^4=\frac{k_x+i\omega c_x-m\omega^2}{E_pI} \tag{4.6}$$

It is convenient to apply the Laplace transform in order to directly accommodate the boundary conditions:

$$L\left\{\frac{d^4u(z)}{dz^4}\right\}+4\lambda^4L\{u(z)\}=0 \tag{4.7}$$

Denoting by $\overline{u}(s)=L\{u(z)\}$ the Laplace transform of $u(z)$ and using standard Laplace-transform properties, Eq. 4.5 becomes an algebraic equation in the transformed space:

$$\overline{u}(s)=u'''(0)\frac{1}{s^4+4\lambda^4}+u''(0)\frac{s}{s^4+4\lambda^4}+u(0)\frac{s^3}{s^4+4\lambda^4} \tag{4.8}$$

where $u'(0)=0$ and

$$u''(0) = \frac{d^2u(z)}{d^2z}\Big|_{z=0} \tag{4.9a}$$

$$u'''(0) = \frac{d^3u(z)}{d^3z}\Big|_{z=0} \tag{4.9b}$$

By applying the inverse Laplace transform leads to the following solution, with the boundary conditions at z=0 incorporated as unknowns:

$$u(z) = u'''(0)\frac{1}{4\lambda^3}(\sin\lambda z.\cosh\lambda z - \cos\lambda z.\sinh\lambda z)$$

$$+u(0)(\cos\lambda z.\cosh\lambda z) + u''(0)\frac{1}{2\lambda^2}(\sin\lambda z.\sinh\lambda z) \tag{4.10}$$

Alternatively, using Euler's complex notation, the expression of the pile displacement becomes

$$u(z) = -ie^{\lambda z}(e^{i\lambda z} - e^{-i\lambda z})\left(\frac{u'''(0)}{16\lambda^3} + \frac{u''(0)}{8\lambda^2}\right) - ie^{-\lambda z}(e^{i\lambda z} - e^{-i\lambda z})\left(\frac{u'''(0)}{16\lambda^3} - \frac{u''(0)}{8\lambda^2}\right)$$

$$+e^{\lambda z}(e^{i\lambda z} + e^{-i\lambda z})\left(-\frac{u'''(0)}{16\lambda^3} + \frac{u(0)}{4}\right) + e^{-\lambda z}(e^{i\lambda z} + e^{-i\lambda z})\left(\frac{u'''(0)}{16\lambda^3} + \frac{u(0)}{4}\right) \tag{4.11}$$

Expressing λ in polar coordinates in the complex plane,

$$\lambda = R\left(\cos\frac{\theta}{4} + i\sin\frac{\theta}{4}\right) \tag{4.12}$$

with

$$R = \left\{\frac{(k_x - m\omega^2)^2 + (\omega c_x)^2}{(4E_pI)^2}\right\}^{1/8} \tag{4.13}$$

$$\theta = Arctan\left(\frac{\omega c_x}{k_x - m\omega^2}\right), \qquad 0 < \theta < \frac{\pi}{2} \tag{4.14}$$

introducing the positive real quantities

$$a = \cos\frac{\theta}{4} + \sin\frac{\theta}{4} \qquad b = \cos\frac{\theta}{4} - \sin\frac{\theta}{4} \tag{4.15}$$

and carrying out the algebra, leads to

$$\begin{aligned} u(z) = & -i(e^{iRaz}e^{Rbz} - e^{-iRbz}e^{Raz})\left(\frac{u'''(0)}{16\lambda^3} + \frac{u''(0)}{8\lambda^2}\right) \\ & -i(e^{iRbz}e^{-Raz} - e^{-iRaz}e^{-Rbz})\left(\frac{u'''(0)}{16\lambda^3} - \frac{u''(0)}{8\lambda^2}\right) \\ & +(e^{iRaz}e^{Rbz} + e^{-iRbz}e^{Raz})\left(-\frac{u'''(0)}{16\lambda^3} + \frac{u(0)}{4}\right) \\ & +(e^{iRbz}e^{-Raz} - e^{-iRaz}e^{-Rbz})\left(\frac{u'''(0)}{16\lambda^3} + \frac{u(0)}{4}\right) \end{aligned} \tag{4.16}$$

To ensure a finite displacement amplitude as z tends to infinity:

$$\frac{u'''(0)}{16\lambda^3} + \frac{u''(0)}{8\lambda^2} = 0 \tag{4.17 }$$

$$-\frac{u'''(0)}{16\lambda^3} + \frac{u(0)}{4} = 0 \tag{4.18 }$$

Substituting Eqs. 4.17 into Eq. 4.16 and re-introducing $e^{i\omega t}$ yields:

$$u(z,t) = \frac{U_0}{2}\{(1+i)e^{-Rbz}e^{i(\omega t - Raz)} + (1-i)e^{-Raz}e^{i(\omega t + Rbz)}\} \tag{4.19}$$

where $U_0 = u(0)$ is the displacement amplitude at the pile head.

The first term in the bracket corresponds to a downward propagating wave and the second term to an upward propagating wave, both with amplitude dacaying exponentially at large z. The two waves have different phase velocities, given by the following dual "dispersion" relation:

$$C_\alpha^{\downarrow} = \frac{\omega}{R\left(\cos\frac{\theta}{4} + \sin\frac{\theta}{4}\right)} \tag{4.20a}$$

$$C_\alpha^{\uparrow} = \frac{\omega}{R\left(\cos\frac{\theta}{4} - \sin\frac{\theta}{4}\right)} \tag{4.20b}$$

Second: $\omega \geq \overline{\omega}_x$, which translates approximately to $a_0 > 1$, a frequency range of lesser practical interest, which is examined herein as providing insight into the asymptotic behavior at high frequencies. Dynamic equilibrium gives

$$\frac{d^4u(z)}{dz^4} - \lambda^4 u(z) = 0 \tag{4.21}$$

where now

$$\lambda^4 = \frac{m\omega^2 - k_x - i\omega c_x}{E_p I} \tag{4.22}$$

Following a similar procedure as the one outlined above, we finally obtain the following solution:

$$u(z,t) = \frac{U_0}{2}\{(1+i)e^{-Rqz}e^{i(\omega t - Rpz)} + (1-i)e^{-Rpz}e^{i(\omega t + Rqz)}\} \tag{4.23}$$

where

$$R = \left\{\frac{(m\omega^2 - k_x)^2 + (\omega c_x)^2}{(E_p I)^2}\right\}^{1/8} \tag{4.24}$$

$$p = \cos\frac{\theta}{4}, \qquad q = -\sin\frac{\theta}{4} \tag{4.25}$$

are real and positive, and

$$\theta = Arctan\left(\frac{-\omega c_x}{m\omega^2 - k_x}\right), \qquad -\frac{\pi}{2} < \theta \le 0 \tag{4.26}$$

Again two different waves emerge, one propagating downward and one propagating upward, with respective phase velocities

$$C_\alpha^{\downarrow} = \frac{\omega}{R\cos\frac{\theta}{4}} \tag{4.27a}$$

$$C_\alpha^{\uparrow} = \frac{\omega}{-R sin\frac{\theta}{4}} \tag{4.27b}$$

In the completely hypothetical case of c_x=0, $\theta = 0$, p=1 , q=0, $R = \left(\frac{m\omega^2 - k_x}{E_p I}\right)^{1/4} = \lambda$ and Eq. 4.24

reduces to

$$u(z,t) = \frac{U_0}{2} = \{(1-i)e^{-\lambda z}e^{i\omega t} + (1+i)e^{i(\omega t - \lambda z)}\} \tag{4.28}$$

In this case indeed only down-going waves exist, as the term corresponding to incoming waves reduces to a decaying exponential.

4.2.2 Characteristics of the results

Using Eqs 4.20 and 4.27, the phase velocities $C_a^{\downarrow}$ and $C_a^{\uparrow}$ of the direct (down-going) and "reflected" (up-coming) waves are portrayed as solid lines in Fig. 4-1, over a very wide range of the frequency factor $0 < a_0 < 10$. Also plotted in this figure are the frequency-dependent phase velocities of:

- a semi-infinite beam on Elastic-Winkler foundation (C_W)

- a semi-infinite unsupported flexural beam (C_F)

These two cases are recovered from the developed formulation for $c_x = 0$ and $k_x = c_x = 0$, respectively. The corresponding phase velocities are:

$$C_W = \infty \quad , \qquad \omega < \overline{\omega}_x \tag{4.29a}$$

$$C_W = V_s \frac{a_0}{2} \left(\frac{s_2 \frac{\pi}{4}}{\frac{\pi}{4} s_1 s_3 a_0^2 - f_1} \right)^{1/4} , \qquad \omega \geq \overline{\omega}_x \tag{4.29b}$$

and

$$C_F = V_s \frac{\sqrt{a_0}}{2} \left(\frac{s_2}{s_1 s_3} \right)^{1/4} \tag{4.30}$$

where now

$f_1 = 1.2 \qquad fixed-head$

$$f_2 = 6 s_1 a_0^{3/4} + 2\beta f_1 \tag{4.31}$$

$f_1 = 2.1 \qquad free-head$

The following trends are worthy of note in Fig. 4-1 :

1. The presence of material and geometric damping in the pile-soil system has a very significant effect on the nature of propagating waves and the respective phase velocities. As already mentioned, an upward propagating ("reflected") wave is generated only in the damped system. Moreover, at the low frequency range of usual interest ($a_0 < 1$), while the phase velocity becomes infinite in the undamped case (C_W) , both $C_\alpha^{\downarrow}$ and $C_\alpha^{\uparrow}$ achieve very small values and, in fact, tend to zero with decreasing frequency. Hence the presence of a rigid soil layer or rock at a shallow depth that

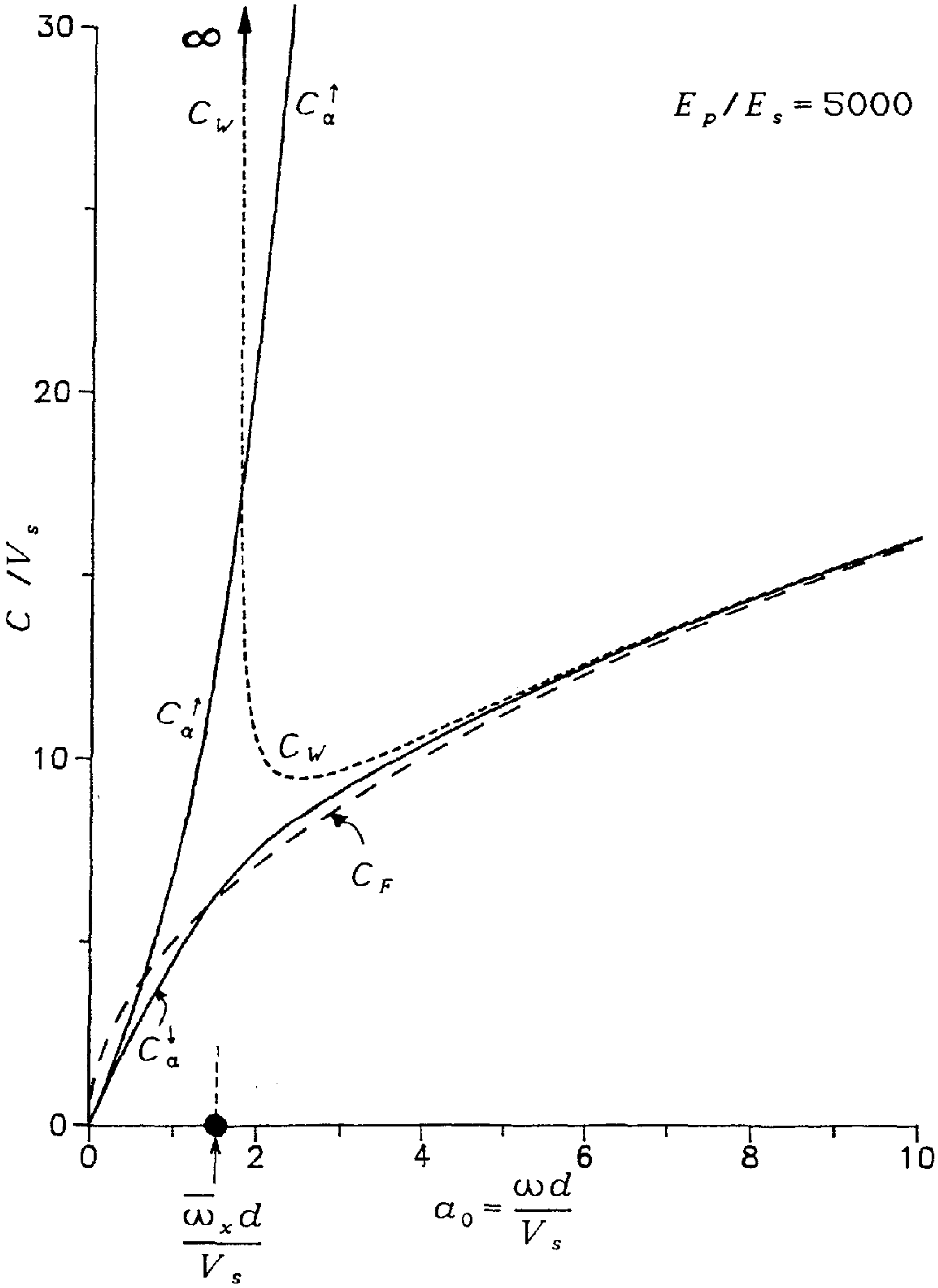

Figure 4-1 Phase wave velocities of beams in lateral harmonic oscillations. The two solid lines are for the up- and down-going waves in a pile on lateral "springs" and "dashpots" (modeling embedment in Halfspace). C_W is for a flexural beam on lateral "springs", and C_F is for an unsupported flexural beam.

would create a cutoff frequency, ω_s, below which radiation damping diminishes deserves attention. In such a case, if soil and pile material damping were ignored, then $c_x=0$, $\theta = 0$, $a=b=1$, $R = \left(\frac{k_x - m\omega^2}{4E_pI}\right)^{1/4} = \lambda$(real number), and Eq. 41 simplifies to

$$u(z,t) = U_0 e^{-\lambda z}(\sin\lambda z + \cos\lambda z)e^{i\omega t} \tag{4.32}$$

which describes a **standing wave** and which is identical in form with the static solution (Scott, 1981). Hence, in this case there are **no propagating waves** (infinite apparent phase velocity) and all points move in phase, although with an amplitude decreasing exponentially with depth, in accord with the aforesaid behavior of the elastically restrained beam (Eq. 4.29a)

2. The phase velocity $C_\alpha^{\downarrow}$ of the downward propagating wave in the pile remains very close to the velocity C_F of the (unsupported) flexural beam for all, but the very low, frequencies. Nevertheless, it is perhaps surprising that $C_\alpha^{\downarrow}$ is much closer to C_F than C_W. Hence, neglecting radiation and material damping might affect adversely the nature of the solution

3. The phase velocities of the three downward propagating waves, namely, $C_a^{\downarrow}$ in the pile, C_w in the elastically-restrained beam, and C_F in the flexural beam, converge to a single curve at high frequencies (say $a_0 > 3$), and tend to infinity by growing in proportion to $\sqrt{\omega}$. On the other hand, the velocity $C_\alpha^{\uparrow}$ of the "reflected" wave in the pile emerges at low frequencies being equal to the velocity $C_a^{\downarrow}$ of the "direct" wave, but then it soon diverges significantly and tends to infinity as a power of ω. That the phase velocities grow without limit with increasing frequency is an inaccuracy attributed to neglecting rotatory-inertia and shear-distortion effects. Such effects must be included in the formulation if more correct values are to be obtained for phase velocities at very high frequencies.

4. No clear conclusions can be drawn from Fig. 4-1 regarding the assumption of "synchronous" wave emission from a laterally oscillating pile. Both $C_\alpha^{\downarrow}$ and $C_\alpha^{\uparrow}$ attain relatively small values,

of about $2V_s$ to $5V_s$, in the frequency range of greatest interest --- even for a relatively stiff pile ($E_p/E_s = 5000$). It seems that the only way to assess the significance of such wave velocities is by examining the phase differences among lateral displacements along the pile.

To this end, the phase of the motion at a particular depth z in time t is computed from

$$\phi(z,t) = Arctan\left(\frac{Im(u(z,t))}{Re(u(z,t))}\right) \tag{4.33}$$

and for z equal to zero the phase becomes

$$\phi(0,t) = Arctan\left(\frac{Im(u(0,t))}{Re(u(0,t))}\right) \quad = \quad \omega t \tag{4.34}$$

The phase difference between the motion at depth z and the motion at the head of the pile

$$\Delta\phi = \omega t - \phi(z,t) \tag{4.35}$$

is plotted in Figs. 4.2 and 4.3 as a function of z/d for two values of the dimensionless frequency a_0 (0.2 and 0.5) for $E_p/E_s = 1000$ and 5000. It is clear that phase differences remain quite small up to a certain depth, beyond which they increase rapidly, especially at higher frequencies. The same figures also show the normalized amplitude of pile displacements versus z/d. It is evident that strictly speaking the assumption of simultaneous emission is not valid. Nevertheless, it is also clear that **phase differences become substantial only at relatively great depths where the displacement amplitude has decreased significantly;** thus waves emitted from such depths would have a negligible amplitude and their phase differences would be of little, if any, consequence to adjacent piles. Hence, the error introduced by assuming "synchronous" wave emission along the pile would in most cases be acceptable. This may explain the successful performance of the method developed by Dobry and Gazetas (1988) and Makris & Gazetas (1989), as already illustrated with Fig. 1.1

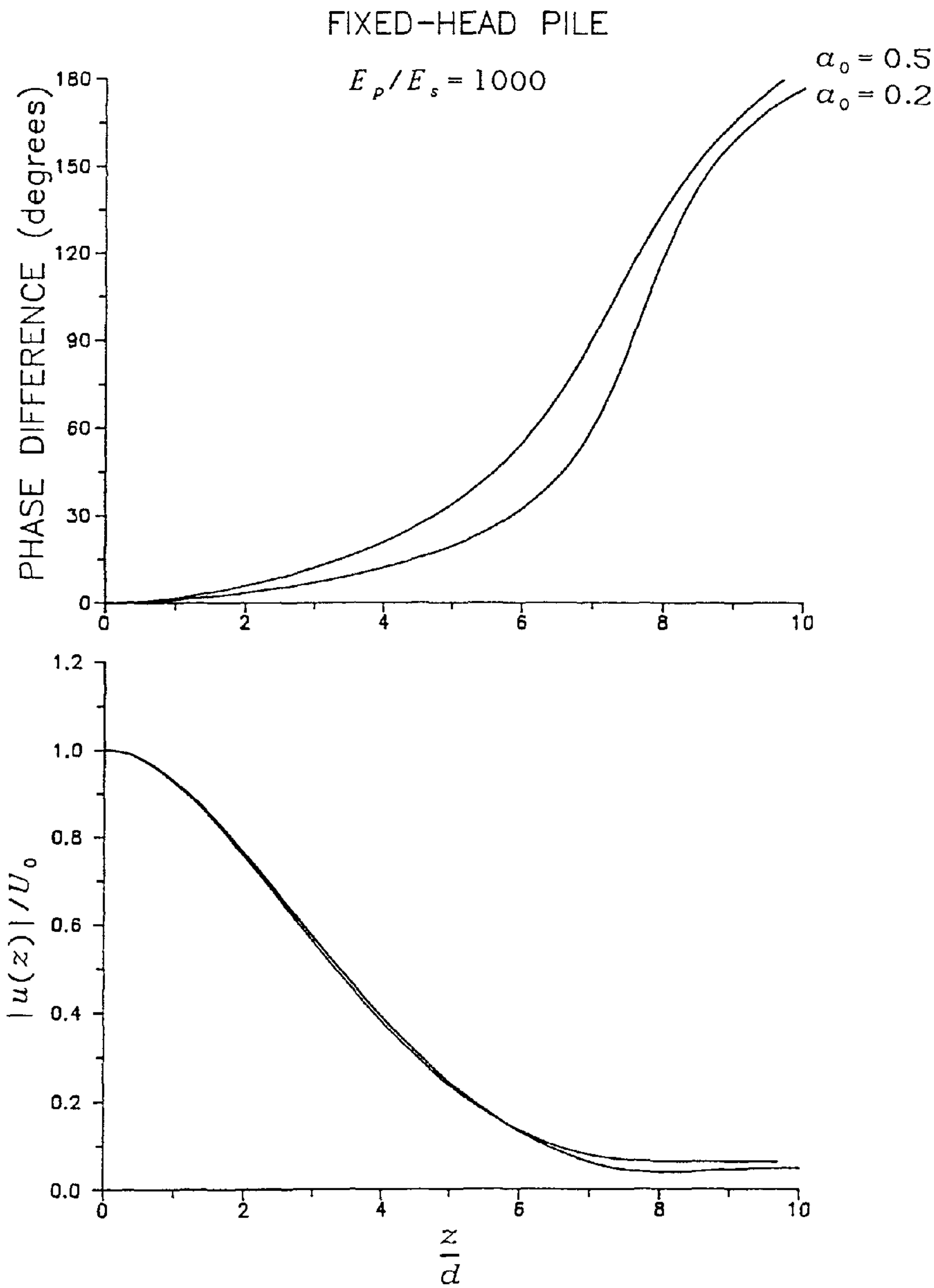

Figure 4-2 Variation with depth of phase differences and normalized lateral deflection amplitudes for a fixed-head pile $E_p/E_s = 1000$, at two frequency factors.

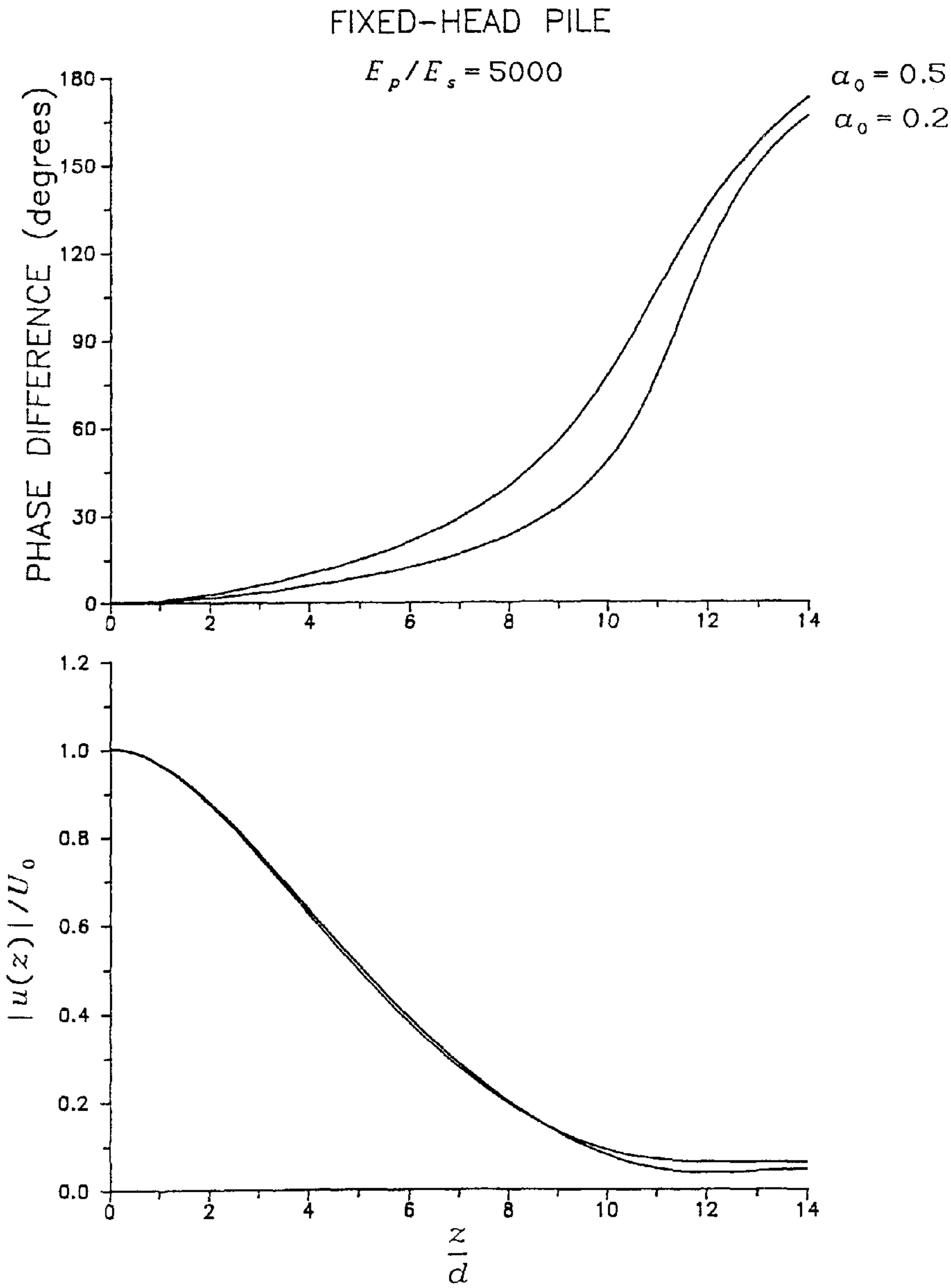

Figure 4-3 Variation with depth of phase differences and normalized lateral deflection amplitudes for a fixed-head pile $E_p/E_s = 5000$, at two frequency factors.

4.3 Free-Head Pile

4.3.1 Governing equation and solution

The only difference in this case from the previous is the boundary conditions at the pile head. For a free head pile the moment at the pile head is zero.

First: $\omega < \overline{\omega}_x$.

The Laplace transform of Eq. 4.7 gives

$$\overline{u}(s) = u'''(0)\frac{1}{s^4+4\lambda^4} + u'(0)\frac{s^2}{s^4+4\lambda^4} + u(0)\frac{s^3}{s^4+4\lambda^4} \tag{4.36}$$

where now $u''(0) = 0$

Applying the inverse Laplace transform leads to the following solution, with the boundary conditions at z=0 incorporated as unknowns:

$$u(z) = u'''(0)\frac{1}{4\lambda^3}(\sin\lambda z.\cosh\lambda z - \cos\lambda z.\sinh\lambda z)$$

$$+u(0)(\cos\lambda z.\cosh\lambda z) + u'(0)\frac{1}{2\lambda}(\sin\lambda z.\cosh\lambda z + \cos\lambda z.\sinh\lambda z) \tag{4.37}$$

Expressing λ in polar coordinates in the complex plane with R, θ, a, and b given by Eqs 4.13, 4.14 and 4.15 and carrying out the algebra one obtains

$$u(z) = e^{Rbz}e^{iRaz}\left(\frac{u(0)}{4} + (1-i)\frac{u'(0)}{8\lambda} - (1+i)\frac{u'''(0)}{16\lambda^3}\right)$$

$$+e^{-Raz}e^{iRbz}\left(\frac{u(0)}{4} - (1+i)\frac{u'(0)}{8\lambda} + (1-i)\frac{u'''(0)}{16\lambda^3}\right)$$

$$+e^{Raz}e^{-iRbz}\left(\frac{u(0)}{4} + (1+i)\frac{u'(0)}{8\lambda} - (1-i)\frac{u'''(0)}{16\lambda^3}\right)$$

$$+e^{-Rbz}e^{-iRaz}\left(\frac{u(0)}{4}-(1-i)\frac{u'(0)}{8\lambda}+(1+i)\frac{u'''(0)}{16\lambda^3}\right) \tag{4.38}$$

To ensure a finite displacement amplitude as z tends to infinity

$$\frac{u(0)}{4}+(1-i)\frac{u'(0)}{8\lambda}-(1+i)\frac{u'''(0)}{16\lambda^3}=0 \tag{4.39a}$$

$$\frac{u(0)}{4}+(1+i)\frac{u'(0)}{8\lambda}-(1-i)\frac{u'''(0)}{16\lambda^3} \tag{4.39b}$$

The solution of the above system gives

$$u'(0)=\lambda u(0) \tag{4.40a}$$

$$u'''(0)=2\lambda^3 u(0) \tag{4.40b}$$

Substituting Eq. 4.40 into Eq. 4.38 and re-introducing $e^{i\omega t}$ yields:

$$u(z,t)=\frac{U_0}{2}\{e^{-Rbz}e^{i(\omega t-Raz)}+e^{-Raz}e^{i(\omega t+Rbz)}\} \tag{4.41}$$

Note that the only difference between the solution for the free head pile (Eq. 4.41) and the solution for the fixed head pile (Eq. 4.19) is that the coefficients of the down going and up-coming waves are equal to one instead of 1-i and 1+i. Accordingly the dispersion relation is identical for the cases given by Eqs. 4.20. The phase differences though would not be the same since they depend on the ratio of the imaginary to the real part of the total displacement.

For the particular case where $c_x=0$, we have a=b=0, and $R=\lambda$ and the solution reduces to

$$u(z,t)=u(0)e^{-\lambda z}\cos\lambda z e^{i\omega t} \tag{4.42}$$

which is again a standing wave similar to the solution given by Eq. 4.32.

Second: $\omega \geq \overline{\omega}_x$

The solution of Eq. 4.21 for this case is

$$u(z,t)=\frac{U_0}{2}\{e^{-Rqz}e^{i(\omega t-Rpz)}+e^{-Rpz}e^{i(\omega t+Rqz)}\} \tag{4.43}$$

where R, p, q and θ are given by Eqs 4.24, 4,25, and 4.26. Again, the only difference between the solutions for the free and fixed-head pile is that the coefficients of the down-going and up-coming waves are equal to one instead of 1+i and 1-i.

For the particular case where $c_x = 0$, we have p=1, q=0, and $R = \lambda$ and the solution reduces to

$$u(z,t)=\frac{U_0}{2}=\{(e^{-\lambda z}e^{i\omega t}+e^{i(\omega t-\lambda z)}\} \tag{4.44}$$

As in the fixed head case, only the down-going wave is a propagating one while the term corresponding to up-coming wave reduces to a decaying exponential.

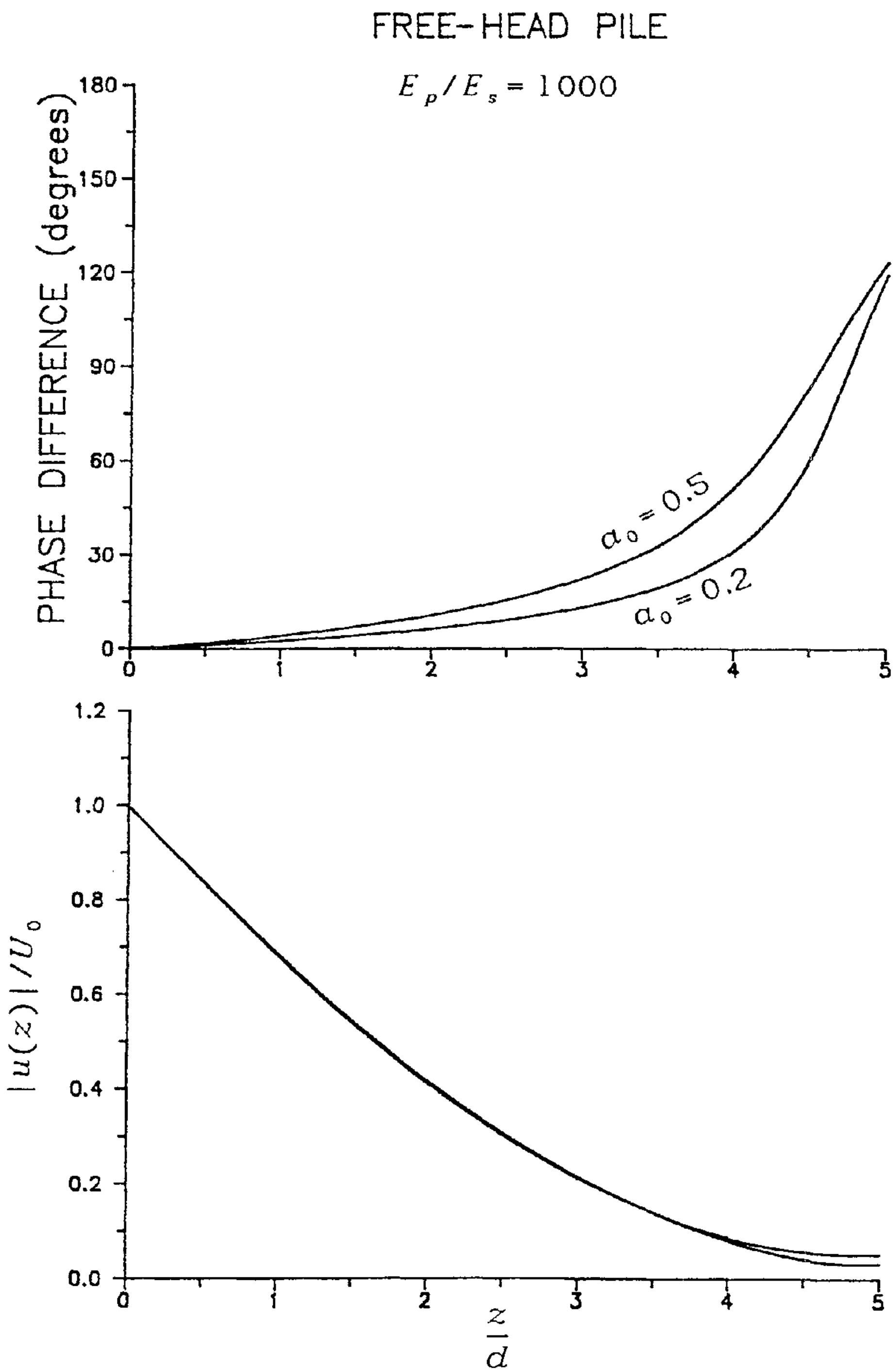

Figure 4-4 Variation with depth of phase differences and normalized lateral deflection amplitudes for a free-head pile $E_p/E_s = 1000$, at two frequency factors.

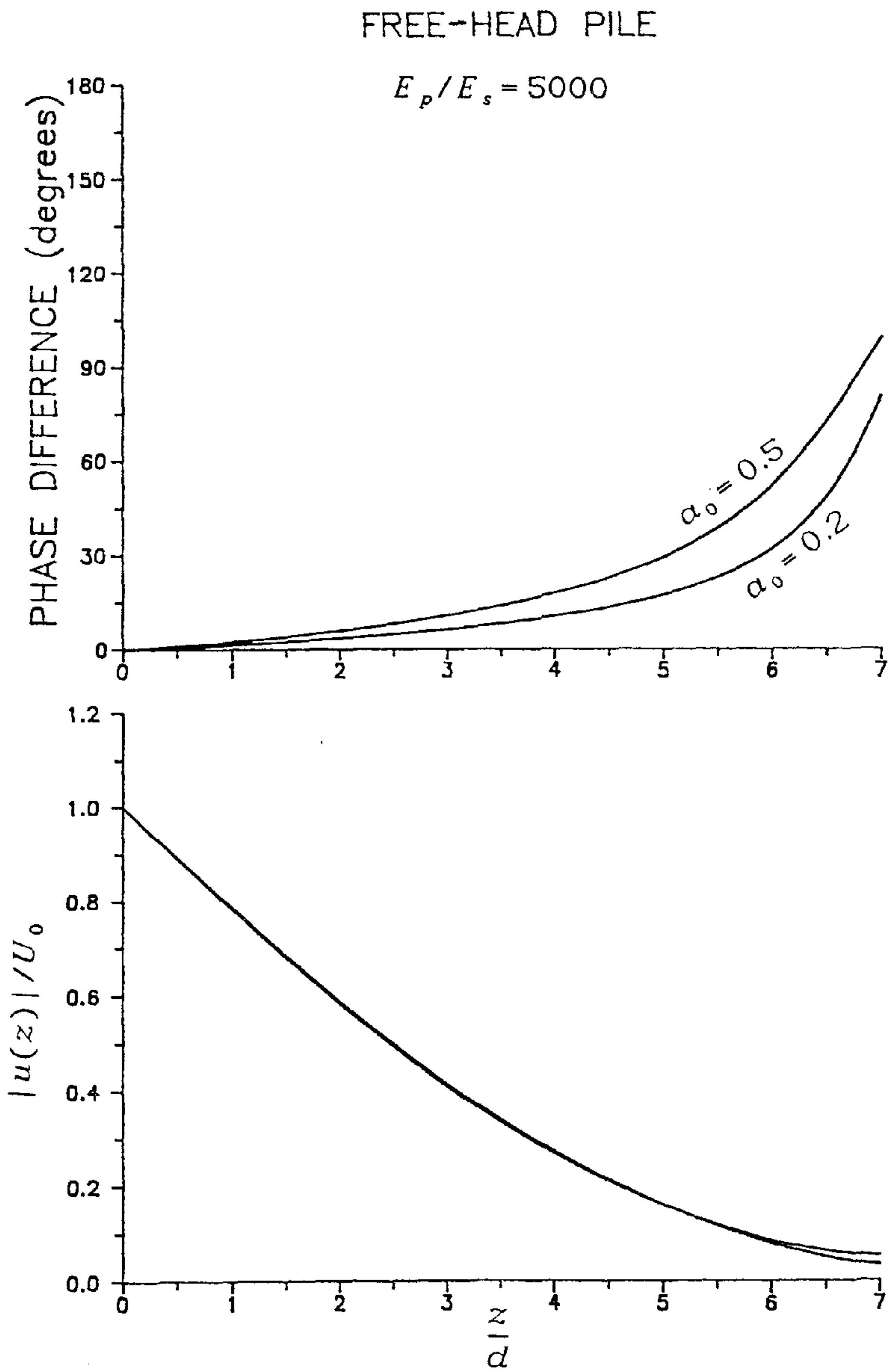

Figure 4-5 Variation with depth of phase differences and normalized lateral deflection amplitudes for a free-head pile $E_p/E_s = 5000$, at two frequency factors.

SECTION 5

CONCLUSIONS

5.1 Regarding Axial Vibrations

1. When an infinitely-long pile, embedded in a realistic Dynamic-Winkler model of a homogeneous halfspace, is subjected to **axial** harmonic head loading, it undergoes steady-state oscillations due to a compression-extension wave that propagates downward with amplitude exponentially decaying with depth, and a frequency-dependent phase velocity, C_α (dispersive system).

2. In the frequency range of greatest interest in foundation dynamics ($0.2 \le a_0 \le 0.8$), C_α increases monotonically with frequency and for typical real-life piles achieves quite large values compared to the S-wave velocity in soil, V_s. As a result, phase differences between displacements along the oscillating pile are very small and can be neglected in approximate studies of through-soil interaction between two adjacent piles -- a conclusion for which additional (direct and indirect) supporting evidence is provided in the paper.

3. In the aforementioned frequency range of interest, the wave velocity C_E , of a bar elastically restrained solely by Winlker springs, is infinite. On the other hand, C_α could only approach infinity at frequencies below a possible stratum cutoff frequency (when radiation damping vanishes) if all material hysteretic damping were ignored.

4. At high frequencies ($a_0 \approx 5 - 10$), C_α , C_E, and the (unsupported) "bar" wave velocity C_L, reach the same asymptotic value, equal to, about $\sqrt{E_p}/\rho_p$ (lateral inertia -- Pochhammer -- effects are not as yet distinguishable).

5.2 Regarding Lateral Vibrations

1. During lateral steady-state oscillation under harmonic "fixed-head" horizontal leading, two

waves develop in the pile: a downward propagating ("direct") wave with phase velocity $C_\alpha^{\downarrow}$, and an upward propagating ("reflected") wave with a different phase velocity $C_\alpha^{\uparrow}$ -- both with amplitude decaying exponentially with depth.

2. The two phase velocities, $C_\alpha^{\downarrow}$ and $C_\alpha^{\uparrow}$, increase monotonically with frequency, the latter at a much faster rate. In the frequency range of greatest interest they both attain very low values, only a few times larger than V_s in the soil, but smaller that C_F, i.e. the phase velocity of an unsupported flexural beam.

3. By contrast to the "spring-and-dashpot" supported pile, only **one** downward propagating wave develops in a beam supported solely on springs. Moreover, the phase velocity in the latter, C_W, is infinitely large below the characteristic frequency $\overline{\omega}_x = \sqrt{k_x/m}$ -- i.e. in the frequency range of greatest interest. Therefore, ignoring the material and, especially, the radiation damping generated by the soil-pile system would change the very nature of the wave propagation in laterally oscillatilng piles.

4. Despite the relatively low values of $C_\alpha^{\downarrow}$ and $C_\alpha^{\uparrow}$ at $0 < a_0 < 1$, the two waves ("direct" and "reflected") combine in such a way that phase differences between pile deflections at various depths remain quite small along the upper, most active part of the pile. Such differences increase considerably at greater depths, but this has only a minor effect on how wave-energy is radiated from a pile -- an observation of significance in the behavior of pile groups.

5. The phase velocities of the three downward propagating waves, C_α, C_W and C_F, converge to a single curve at high frequencies ($a_0 >$ 3), while growing in proportion to $\sqrt{\omega}$.

SECTION 6

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