

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH

State University of New York at Buffalo

Hybrid Control of Seismic-Excited Nonlinear and Inelastic Structural Systems

by

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> REPRODUCED BY u.s. DEPARTMENT OF COMMERCE NATIONAL TECHNICAL INFORMATION SERVICE SPRINGFIELD, VA 22161

Technical Report NCEER-91-0020

August I, 1991

This research was conducted at the University of California at Irvine and was partially supported by the National Science Foundation under Grant No. ECE 86-07591.

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Technical Report NCEER-91-0020

NCEER Project Number 90-2202

and

NSF Grant Numbers NSF-BCS-89-04524 and BCS-91-96089

NSF Master Contract Number ECE 86-07591

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 2, Secondary and Protective Systems, and more specifically, to protective systems. Protective Systems are devices or systems which, when incorporated into a structure, help to improve the structure's ability to withstand seismic or other environmental loads. These systems can be passive, such as base isolators or viscoelastic dampers; or active, such as active tendons or active mass dampers; or combined passive-active systems.

In the area of active systems, research has progressed from the conceptual phase to the implementation phase with emphasis on experimental verification. As the accompanying figure shows, the experimental verification process began with a small single-degree-of-freedom structure model, moving to larger and more complex models, and finally, to full-scale models.

The theory of hybrid control, a combined active/passive control mechanism, isfurther developed in this report, which includes a refined version of the instantaneous optimal control algorithm. The potential applicability of such a control strategy is illustrated through several examples involving a combination of sliding isolators or lead-core rubber bearings with active devices such as actuators or active mass dampers.

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ABSTRACT

It has been shown recently that a combined use of active and passive control systems, referred to as the hybrid control system, is more effective, beneficial and practical, in some cases, for reducing the building response under strong earthquakes. However, the use of hybrid control systems involves active control of nonlinear or inelastic structural systems. In Part I, a refined version of the instantaneous optimal control algorithms for nonlinear or inelastic structural systems is proposed. The main advantage of the proposed control algorithms is that the control vector is determined directly from the measured response state vector without a necessity of tracking a timedependent system matrix as proposed previously [30-31]. The optimal algorithm is simplified considerably for the time-variant linear system, which is a special case of the nonlinear system. A method of simulating the controlled response of the hysteretic system using the present control algorithm is presented. Likewise, a variation of the proposed control algorithms, utilizing the measured acceleration and velocity responses as feedbacks for the determination of the optimal control vector, is also presented. Applications of the proposed algorithm to various types of aseismic hybrid control systems are demonstrated. These include a combination of sliding isolators or lead-core rubber bearing isolators and active devices, such as actuators, active mass dampers, etc. The performances of various control systems are evaluated and compared numerically and the advantages of the aseismic hybrid control systems are demonstrated. It is shown that the proposed optimal control algorithm is simple and reliable for on-line operations and it is effective for practical applications.

In Part II, instantaneous optimal control for nonlinear and inelastic systems is formulated incorporating the specific hysteretic model of the system. The resulting optimal control vector is obtained as a function of the total deformation, velocity and the hysteretic component of the structural response. The hysteretic component of the structural response can be estimated from the measured structural response and the hysteretic model used. It is shown that the optimal control vector satisfies not only the necessary conditions but also the sufficient condition of optimality. Specific applications of the optimal algorithm to two types of hybrid control systems are demonstrated. These include (i) active control of base-isolated buildings using frictional-type sliding base isolators, and (ii) active control of base-isolated buildings using lead-core rubber bearings. Numerical examples are worked out to demonstrate the applications of the proposed control algorithm. It is shown that the performance of such an optimal algorithm improves over that of the algorithm without considering the hysteretic components in the determination of the control vector.

ACKNOWLEDGEMENTS

This research is partially supported by the National Science Foundation Grant Nos. NSF-BCS-89-04524 and BCS-91-96089 and National Center for Earthquake Engineering Research Grant No. NCEER-90-2202. Partial support by the University of California, Irvine through an allocation of computer time is acknowledged.

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 $\label{eq:2.1} \begin{split} \mathcal{L}_{\text{max}}(\mathbf{r}) & = \frac{1}{2} \sum_{i=1}^{N} \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \\ & = \frac{1}{2} \sum_{i=1}^{N} \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf$ $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

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HYBRID CONTROL OF SEISMIC-EXCITED STRUCTURAL SYSTEMS

PART I

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SECTION 1 INTRODUCTION

In recent years, intensive research efforts have been made in the application of passive and/or active control systems to reduce the response and damage of civil engineering structures caused by earthquakes and strong winds [e.g., 3-14, 16-18, 20-24, 26-41]. The active control system differs from the passive one in that it requires the supply of external energy. In the area of passive control systems, much progress has been accomplished in base isolation systems [e.g., 5-8, 11, 17-18, 24], such as elastomeric bearings, sliding systems, etc., and different types of mechanical energy dissipators, such as viscoelastic dampers, friction dampers and other devices. In the area of active control systems, active mass dampers, active tendon systems, variable stiffness bracings and others have been developed and tested in the laboratory [e.g., 3,9,20,23, 29] and in a few cases installed in prototype full-scale buildings [e.g., 9,12-14].

More recently, it has been shown that a combined use of active and passive control systems, referred to as the hybrid control system, is more effective, beneficial and practical in some cases $[e.g., 3, 9, 10, 21-22, 34-36, 39-41]$. The idea of hybrid control systems is to utilize the advantages of both the passive and active control systems to extend the range of applicability of both control systems to protect the integrity of the structure. In particular, under extreme environments, such as strong earthquakes, hybrid control systems are superior.

It is well known that most passive control systems behave either nonlinearly, such as sliding isolation systems, or inelastically (hysteretically), such as lead-core rubber bearing isolation systems. As a result, active control of nonlinear or inelastic (hysteretic) structural systems is the major issue of hybrid control. Unfortunately, control theories for nonlinear or hysteretic systems are very limited [e.g., 10, 16,21,30,31]. Recently, instantaneous optimal control algorithms proposed by Yang, et al for linear structures [26-28] have been extended to nonlinear and hysteretic structures [30-31]. In the development of the optimal algorithms for nonlinear structures, the Wilson- θ numerical procedures were used in Refs. 30-31, and the resulting optimal control vector $U(t)$

depends on the feedback response and the estimation or tracking of a system matrix. For elastic nonlinear structures, this does not present any problem. For hysteretic systems whose responses are history-dependent, however, the tracking of the system matrix $A₂$ may be subjected to limitations.

The purpose of this report is to propose a refined version of instantaneous optimal control algorithms, which is simple and reliable for on-line operations, for nonlinear or hysteretic structural systems. Emphasis is placed on applications to aseismic hybrid control systems. In the present approach, the Fourth-Order Runge-Kutta numerical method is used to obtain the transition matrix equation, i.e., the system response at time t is expressed in terms of the system response at the previous times t- Δt and t-2 Δt . The resulting optimal control vector, $U(t)$, at time t depends only on the measured response at time t, thus eliminating the necessity of tracking a history-dependent system matrix. The optimal algorithm is simplified considerably for a time-variant linear system, that is a special case of hysteretic systems. A method of simulating the controlled response of the hysteretic structural system using the proposed control algorithm is also presented.

Recent laboratory experiments indicated some difficulties in measuring the displacement response of the structure subjected to earthquake ground motions. This is because both the ground and the structure are moving during earthquakes, so that there is no absolute reference for the determination of the displacement response. Further, the displacement response obtained by numerically integrating the measured velocity response is not satisfactory due to serious error accumulations and noise pollution. On the other hand, however, the acceleration response can easily be measured by installing acceleration sensors. In this report, an alternate version of the instantaneous optimal control algorithms for nonlinear or inelastic structures, utilizing the acceleration response rather than the displacement response, is proposed following the approach presented by Yang, et al [37-38] for linear structures.

Finally, applications of the proposed algorithms to various types of aseismic hybrid control systems are demonstrated. These include various combinations of active control systems, such as actuators, active mass dampers, etc., and passive base isolation systems, such as sliding isolators and lead-core rubber bearing isolators. The performances of various hybrid control systems are evaluated and compared numerically, and the advantages of aseismic hybrid control systems are demonstrated. It is shown that the proposed optimal control algorithm is effective and simple for practical applications.

SECTION 2 FORMULATION

2.1 General Nonlinear or Hysteretic Structures

For simplicity of presentation, consider a shear-beam lumped-mass nonlinear or hysteretic building structure implemented by an active control system as shown in Fig. 2.1. The structure is idealized by an n-degree-of-freedom system and subjected to a one-dimensional earthquake ground acceleration $\ddot{X}_0(t)$. The matrix equation of motion of the entire structural system can be expressed as

$$
\underline{M}\ddot{X}(t) + \underline{F}_p[\dot{X}(t)] + \underline{F}_s[X(t)] = -\underline{M}\xi\ddot{X}_o(t) + \underline{H}\underline{U}(t) \qquad (2.1)
$$

in which $X(t) = [x_1, x_2, ..., x_n]' =$ an n vector denoting the deformation of each story unit, $\underline{M} =$ an (nxn) mass matrix, $\xi = [1, 1, ..., 1]$ ' = an n vector, $\underline{U}(t) =$ a r-dimensional control vector and $\underline{H} =$ an (nxr) location matrix denoting the location of r controllers. For the notations above, an under bar denotes a vector or matrix and a prime indicates the transpose of a vector or matrix. In Eq. (2.1), $E_D[\dot{X}(t)]$ is an n vector denoting the nonlinear damping force that is assumed to be a function of $\dot{\underline{X}}(t)$ and $\underline{F_s}[\underline{X}(t)]$ is an n vector denoting the nonlinear (or hysteretic) stiffness restoring force that is assumed to be a function of $\underline{X}(t)$.

Introducing the 2n state vector $Z(t)$,

$$
Z(t) = \left[\frac{X(t)}{\dot{X}(t)}\right]
$$
 (2.2)

one can convert the second-order nonlinear matrix equation, Eq. (2.1), into a first order nonlinear matrix equation as follows:

$$
\underline{\dot{Z}}(t) = g[\underline{Z}(t)] + \underline{B}\underline{U}(t) + \underline{W}_{1}\ddot{X}_{0}(t) \qquad (2.3)
$$

where $g[\underline{Z}(t)]$ is a 2n vector which is a nonlinear function of the state vector $\underline{Z}(t)$

FIGURE 2.1 Structural Model of a Multi-Story Building with Active Control Systems: (a) Active Tendon Control System; (b) Active Mass Damper

t(O (2.4) *g[Z(t)]*

and \underline{B} and \underline{W}_1 are 2n vectors

$$
\underline{B} = \begin{bmatrix} 0 \\ -\frac{\rho}{-M^{-1}H} \end{bmatrix} \qquad ; \qquad \underline{W}_1 = \begin{bmatrix} 0 \\ -\frac{\rho}{-K} \end{bmatrix} \tag{2.5}
$$

The first-order nonlinear matrix equation, Eq. (2.3), can be solved step-by-step numerically using the Fourth-Order Runge-Kutta method as follows (see Appendix I):

$$
Z(t) = Z(t - 2\Delta t) + \frac{1}{6} [A_0 + 2A_1 + 2A_2 + A_3]
$$
 (2.6)

in which Δt is the integration time step, and \underline{A}_0 , \underline{A}_1 , \underline{A}_2 , and \underline{A}_3 are 2n vectors

$$
A_0 = 2\Delta t \{ g[Z(t-2\Delta t)] + \underline{B} U(t-2\Delta t) + \underline{W}_1 \ddot{X}_0 (t-2\Delta t) \}
$$

\n
$$
A_1 = 2\Delta t \{ g[Z(t-2\Delta t) + 0.5\underline{A}_0] + \underline{B} U(t-\Delta t) + \underline{W}_1 \ddot{X}_0 (t-\Delta t) \}
$$

\n
$$
A_2 = 2\Delta t \{ g[Z(t-2\Delta t) + 0.5\underline{A}_1] + \underline{B} U(t-\Delta t) + \underline{W}_1 \ddot{X}_0 (t-\Delta t) \}
$$

\n
$$
A_3 = 2\Delta t \{ g[Z(t-2\Delta t) + \underline{A}_2] + \underline{B} U(t) + \underline{W}_1 \ddot{X}_0 (t) \}
$$
\n(2.7)

As observed from Eq. (2.7), \underline{A}_0 , \underline{A}_1 , \underline{A}_2 , and \underline{A}_3 are functions of t-2 Δt , t- Δt and t. For simplicity of presentation, the arguments, t-2 Δt , t- Δt and t, have been omitted.

To simplify the mathematical operation, all the vectors in Eq. (2.6) which are functions of τ < t are grouped into one term, denoted by \underline{D} (t-2 Δt , t- Δt), as follows:

$$
\underline{D}(t - 2\Delta t, t - \Delta t) = \underline{Z}(t - 2\Delta t) + \frac{1}{3}\Delta t \{g[\underline{Z}(t - 2\Delta t)] + \underline{B}\underline{U}(t - 2\Delta t) + \underline{W}_1 \ddot{X}_0(t - 2\Delta t) + 2g[\underline{Z}(t - 2\Delta t) + 0.5\underline{A}_0] + 4\underline{B}\underline{U}(t - \Delta t)
$$
\n
$$
+ 4\underline{W}_1 \ddot{X}_0(t - \Delta t) + 2g[\underline{Z}(t - 2\Delta t) + 0.5\underline{A}_1] + g[\underline{Z}(t - 2\Delta t) + \underline{A}_2])
$$
\n(2.8)

Then, Eq. (2.6) can be expressed as

$$
Z(t) = D(t - 2\Delta t, t - \Delta t) + \frac{\Delta t}{3} [B U(t) + W_1 \ddot{X}_0(t)] \qquad (2.9)
$$

It should be mentioned that for the regular Fourth-Order Runge-Kutta method, the integration time step is $\Delta \tau$ that is twice of Δt used in Eqs. (2.6)-(2.9), i.e., $\Delta \tau = 2\Delta t$ or $\Delta t = \Delta \tau/2$. In other words, the integration step size Δt used in the current numerical scheme is one-half of that used in the regular Fourth-Order Runge-Kutta method [Appendix I]. Because the required control vector $\underline{U}(t)$ is computed from the measurement of the response vector, a smaller time step Δt is used for the numerical solution.

As observed from Eqs. (2.2) and (2.4), the 2n vector $g(Z)$ is a function of X and $\dot{\underline{X}}$ which are elements of the vector \underline{Z} . Thus, the g vector can be expressed as a function of the response state vector Z , i.e., $g = g(Z)$. To evaluate $g(Z)$, the argument vector $Z = [\,\mathbf{X}', \dot{\mathbf{X}}']'$, Eq. (2.2), is computed first and then its components, \mathbf{X} and $\dot{\mathbf{X}}$, are substituted into the right hand side of Eq. (2.4). For the $g[\mathbb{Z}(t-2\Delta t) + 0.5 \underline{A}_1]$ vector appearing in Eq. (2.8), the argument vector (2n-dimensional), denoted by S ,

$$
\underline{S} = \underline{Z}(t - 2\Delta t) + 0.5\underline{A}_1 \qquad (2.10)
$$

is computed first from Eq. (2.10). Then, S is partitioned in the same form as the vector Z , i.e., $S = [S_1', S_2']'$, where S_1 and S_2 are n-dimensional vectors. Finally, the $g[Z(t 2\Delta t$) + 0.5 \underline{A}_{1}] vector is obtained from Eq. (2.4) by replacing \underline{X} and \underline{X} , respectively, by S_1 and S_2 . Likewise, the other two vectors $g[Z(t-2\Delta t) + 0.5 \Delta_0]$ and $g[Z(t-2\Delta t) +$ Δ_2] appearing in Eq. (2.8) are similarily defined.

2.2 Instantaneous Optimal Control

The instantaneous optimal control algorithms proposed by Yang, et al for linear structures [26-28] was extended later to nonlinear structures [30-31]. Instantaneous optimal control is formulated as follows.

The time dependent quadratic objective function J(t) proposed in Refs. 26-28 is given by

$$
J(t) = Z'(t) Q Z(t) + U'(t) R U(t)
$$
 (2.11)

in which Q is a (2nx2n) positive semi-definite matrix and \underline{R} is a (rxr) positive definite matrix, respectively, representing the relative importance of the response vector $Z(t)$ and the control vector $\underline{U}(t)$. The implication of minimizing the objective function given by Eq. (2.11) is that the performance index J(t) is minimized at every time instant t [26-28]. The optimal control vector obtained by minimizing J(t), subjected to the constraint of the equations of motion, is called the instantaneous optimal control algorithm by Yang, et al [Refs. 26-28, 30-31].

To minimize the objective function, $J(t)$, the Hamiltonian H is obtained by introducing a 2n-dimensional Lagrangian multiplier vector $\lambda(t)$

$$
H = \underline{Z}'(t) \underline{Q} \underline{Z}(t) + \underline{U}'(t) \underline{R} \underline{U}(t) + \underline{\lambda}'(t) \underline{\zeta}(t)
$$

-
$$
\underline{D}(t - 2\Delta t, t - \Delta t) - \frac{\Delta t}{3} [\underline{B} \underline{U}(t) + \underline{W}_1 \underline{\ddot{X}}_0(t)]
$$
 (2.12)

The necessary conditions for minimizing $J(t)$, subjected to the constraint of Eq. (2.9), are

$$
\frac{\partial H}{\partial \underline{Z}} = 0, \qquad \frac{\partial H}{\partial \underline{U}} = 0, \qquad \frac{\partial H}{\partial \underline{\lambda}} = 0 \tag{2.13}
$$

Substituting Eq. (2.12) into Eq. (2.13), one obtains the optimal control vector $U(t)$ in the following manner (see Appendix II): (i) when the control vector $U(t)$ is assumed to be regulated by the response state vector $Z(t)$, the instantaneous optimal closed-loop (feedback) control algorithm is obtained as

$$
U(t) = -\frac{\Delta t}{3} R^{-1} B^{t'} QZ(t)
$$
 (2.14)

(ii) when the control vector $\underline{U}(t)$ is regulated by the measurement of the earthquake ground acceleration $\ddot{X}_0(t)$ without a feedback response state vector $Z(t)$, the instantaneous optimal open-loop (feedforward) control algorithm is obtained, and (iii) the instantaneous optimal closed-open-loop (feedforward and feedback) control algorithm is obtained by assuming that the control vector $U(t)$ is regulated by the measurements of both the response state vector $Z(t)$ and the earthquake ground acceleration $\ddot{X}_0(t)$ (see Appendix B). Although the control operations for the three optimal algorithms are different, the resulting state vector $Z(t)$ and control vector $U(t)$ remain identical under ideal control environments.

With the optimal control vector given by Eq. (2.14) , the response state vector $Z(t)$ is obtained by substituting Eq. (2.14) into Eq. (2.9) as follows

$$
Z(t) = \left[I + \left(\frac{\Delta t}{3}\right)^2 B R^{-1} B' Q\right]^{-1} \left[D(t - 2\Delta t, t - \Delta t) + \frac{\Delta t}{3} W_1 \ddot{X}_0(t)\right] (2.15)
$$

in which I is a (2nx2n) identity matrix.

Although the optimal control vector $\underline{U}(t)$, Eq. (14), is derived from the necessary conditions, it was shown in Ref. 41 that Eq. (14) also satisfies the sufficient condition of optimality.

2.3 Time-Variant Linear Systems

The instantaneous optimal control algorithm developed previously for the general nonlinear or hysteretic structural systems is directly applicable to time-variant linear systems. In fact, the time-variant linear system is a special case of the general nonlinear system presented above and the instantaneous optimal control algorithm for such a system is presented in this section.

The matrix equation of motion for a time-variant structural system can be written as

$$
\underline{M}\ddot{X}(t) + \underline{C}(t)\dot{X}(t) + \underline{K}(t)\underline{X}(t) = -\underline{M}\underline{\xi}\ddot{X}_0(t) + \underline{H}\underline{U}(t) \qquad (2.16)
$$

in which $C(t)$ and $K(t)$ are time dependent (nxn) damping and stiffness matrices. With the introduction of the state vector $Z(t)$ given by Eq. (2.2), the second order matrix equation of motion given by Eq. (2.16) can be converted into a first order matrix equation of motion as follows

$$
\underline{\dot{Z}}(t) = \underline{A}(t) \, \underline{Z}(t) + \underline{B} \, \underline{U}(t) + \underline{W}_1 \ddot{X}_0(t) \tag{2.17}
$$

in which \underline{B} and \underline{W}_1 are given by Eq. (2.5) and

$$
\mathbf{A}(t) = \begin{bmatrix} \mathbf{0} & I \\ -\mathbf{1} & -\mathbf{1} & -\mathbf{1} \\ -M^{-1}K(t) & -M^{-1}C(t) \end{bmatrix}
$$
(2.18)

Again, using the Fourth-Order Runge-Kutta method, Eq. (2.17) can be solved numerically as follows:

$$
Z(t) = Z(t - 2\Delta t) + \frac{1}{6} \Big[A_0 + 2A_1 + 2A_2 + A_3 \Big]
$$
 (2.19)

in which

$$
A_0 = 2\Delta t \{ \Delta(t)Z(t - 2\Delta t) + \underline{B} U(t - 2\Delta t) + \underline{W}_1 \ddot{X}_0(t - 2\Delta t) \}
$$

\n
$$
A_1 = 2\Delta t \{ \Delta(t)[Z(t - 2\Delta t) + 0.5\Delta_0] + \underline{B} U(t - \Delta t) + \underline{W}_1 \ddot{X}_0(t - \Delta t) \}
$$

\n
$$
A_2 = 2\Delta t \{ \Delta(t)[Z(t - 2\Delta t) + 0.5\Delta_1] + \underline{B} U(t - \Delta t) + \underline{W}_1 \ddot{X}_0(t - \Delta t) \}
$$

\n
$$
A_3 = 2\Delta t \{ \Delta(t)[Z(t - 2\Delta t) + \Delta_2] + \underline{B} U(t) + \underline{W}_1 \ddot{X}_0(t) \}
$$
\n(2.20)

A comparison between Eqs. $(2.6)-(2.7)$ and Eqs. $(2.19)-(2.20)$ indicates that Eqs. (2.19)-(2.20) can be obtained from Eqs. (2.6)-(2.7) by replacing $g[Z(t)]$ by $A(t) Z(t)$. Thus, the instantaneous optimal control vector $\underline{U}(t)$ is identical to Eq. (2.14).

$$
U(t) = - \frac{\Delta t}{3} R^{-1} B' Q Z(t)
$$
 (2.21)

and the response state vector $Z(t)$ with instantaneous optimal control is given by Eq. (2.15) in which \underline{D} (t-2 Δt , t- Δt) is as follows

$$
\underline{D}(t-2\Delta t, t-\Delta t) = \underline{Z}(t-2\Delta t) + \frac{1}{3}\Delta t \left\{ 6\underline{A}(t) \underline{Z}(t-2\Delta t) + \underline{A}(t) \left[\underline{A}_0 + \underline{A}_1 + \underline{A}_2 \right] \right. \\ \left. + \underline{B} \underline{U}(t-2\Delta t) + 4\underline{B} \underline{U}(t-\Delta t) + \underline{W}_1 \ddot{X}_0(t-2\Delta t) + 4\underline{W}_1 \ddot{X}_0(t-\Delta t) \right\} \tag{2.22}
$$

2.4 Instantaneous Optimal Control with Acceleration and Velocity Feedback

The optimal control vector derived above depends on the feedback response state vector $Z(t)$ that consists of the displacement response $X(t)$ and the velocity response $\dot{X}(t)$. For seismic-excited structural systems, the displacement response may not be easy to measure. This is because during strong earthquakes, both the ground and the structure are moving so that there is no absolute reference to measure the displacement. Consequently, it may not be desirable to measure the displacement vector $X(t)$. On the other hand, the acceleration responses can easily be measured by installing acceleration sensors.

In this section, an optimal control algorithm is presented so that the control vector U(t) is computed from the measurements of the acceleration and velocity responses. We follow the approach proposed by Yang, et al [37-39] for linear structures. In this approach, an alternate time-dependent performance index is given by

$$
J^{*}(t) = \underline{\dot{Z}}'(t)Q^{*}\underline{\dot{Z}}(t) + \underline{U}'(t)R\underline{U}(t)
$$
 (2.23)

in which Q^* is a (2nx2n) positive semi-definite matrix and \underline{R} is a (rxr) positive definite matrix. It is mentioned that although the displacement response vector $X(t)$ does not appear in the performance index $J^*(t)$, Eq. (2.23), it is expected that a minimization of $J^*(t)$ will also reduce $X(t)$.

To solve the general nonlinear equation of motion given by Eq. (2.3), the state vector $Z(t)$ is expressed by a backward finite difference form, i.e., $Z(t) = Z(t-\Delta t)$ + $\Delta t \dot{\mathbb{Z}}$ (t- Δt). Then, Eq. (2.3) can be written as

$$
\underline{\dot{Z}}(t) = g[\underline{Z}(t-\Delta t) + \Delta t \underline{\dot{Z}}(t-\Delta t)] + \underline{B} \underline{U}(t) + \underline{W}_{1} \ddot{X}_{0}(t) \qquad (2.24)
$$

with zero initial conditions, i.e., $\dot{Z}(0) = Z(0) = 0$.

The Hamiltonian $H^*(t)$ is obtained from Eqs. (2.23) and (2.24) as
$$
H^{*}(t) = \frac{\dot{Z}'(t)Q^{*}\dot{Z}(t) + \underline{U}'(t)RU(t) + \underline{\lambda}'(\dot{Z}(t) - g[Z(t - \Delta t) + \Delta t] \Delta t \dot{Z}(t - \Delta t)] - B \underline{U}(t) - \underline{W}_{1} \ddot{X}_{0}(t))}
$$
(2.25)

The necessary conditions for minimizing the objective function, $J^*(t)$, subjected to the constraint of Eq. (2.24) are as follows

$$
\frac{\partial H^*(t)}{\partial \underline{\dot{z}}} = 0 \qquad ; \qquad \frac{\partial H^*(t)}{\partial \underline{U}} = 0 \qquad ; \qquad \frac{\partial H^*(t)}{\partial \underline{\lambda}} = 0 \qquad (2.26)
$$

The optimal closed-loop control vector $U(t)$ is obtained by substituting Eq. (2.25) into Eq. (2.26); with the result

$$
\underline{U}(t) = -\underline{R}^{-1}\underline{B}'\underline{Q}^*\underline{Z}(t) \qquad (2.27)
$$

Substituting Eq. (2.27) into Eq. (2.3), one obtains the nonlinear equation of motion with the instantaneous optimal control vector given by Eq. (2.27)

$$
\underline{\dot{Z}}(t) = \left(I + BR^{-1} B' Q^* \right)^{-1} g [Z(t)] + \left(I + BR^{-1} B' Q^* \right)^{-1} \underline{W}_1 \ddot{X}_o(t) \quad (2.28)
$$

The response state vector $Z(t)$ can be simulated numerically from Eq. (2.28) using the Fourth-Order Runge-Kutta method described previously.

2.5 Simulation of Controlled Response for Hysteretic Structural System

One objective of this report is to propose control algorithms suitable for applications to aseismic hybrid control systems. In order to evaluate the effectiveness of the proposed optimal algorithm and to compare the performance of various hybrid control systems, it is necessary to simulate the controlled response of the structure. To facilitate the simulation of the controlled structural response, the hysteretic component of the total deformation will be separated. With the instantaneous optimal control algorithm developed in the previous section, a method of simulation for the controlled response of the hysteretic structural system is presented in this section.

Various hysteretic models for the restoring force of an inelastic system have been developed in recent years [e.g., 25]. In this study, the differential equation model available in the literature [e.g., 1,2,15,19,25] will be used. The stiffness restoring force for a structural member or a story unit, $F_s(x)$, is expressed as

$$
F_{s}(x) = \alpha k x + (1-\alpha) k D_{v} v \qquad (2.29)
$$

in which $x =$ interstory deformation, $k =$ elastic stiffness, $\alpha =$ ratio of post-yielding to pre-yielding stiffness, D_v = yield deformation = constant, and v is a non-dimensional variable introduced to describe the hysteretic component of the deformation, with $|v| \le$ 1, where

$$
D_{\mathbf{v}}\dot{\mathbf{v}} = A\dot{\mathbf{x}} - \beta |\dot{\mathbf{x}}| |\mathbf{v}|^{n-1} \mathbf{v} - \gamma \dot{\mathbf{x}} |\mathbf{v}|^n \qquad (2.30)
$$

In Eq. (2.30), parameters A, β and γ govern the scale and general shape of the hysteresis loop, whereas the smoothness of the force-deformation curve is determined by the parameter n.

The hysteretic model given by Eq. (2.29) and (2.30) provides an explicit mathematical expression with many parameters flexible enough to reflect various hysteretic behaviors of inelastic systems. Likewise, Eq. (2.30) appears to be capable of describing the hysteretic behaviors of different base isolation systems, such as rubber bearings and sliding systems, which are the main concern of this study.

It follows from Eq. (2.29) that the stiffness of the ith story unit of the structure, denoted by $F_{si}(x)$, can be separated into an elastic component α_i k_i x_i and a hysteretic component $(1-\alpha_i)$ k_i D_{yi} v_i, i.e., $F_{si}(x) = \alpha_i k_i x_i + (1-\alpha_i)k_i D_{yi}v_i$. The non-dimensional hysteretic variable v_i follows from Eq. (2.30) as

$$
\dot{\mathbf{v}}_i = D_{yi}^{-1} \left[A_i \dot{x}_i - \beta_i | \dot{x}_i | \left| v_i \right|^{n_i - 1} v_i - \gamma_i \dot{x}_i |v_i|^{n_i} \right] = f_i(\dot{x}_i, v_i)
$$
 (2.31)

Consequently, the matrix equation of motion for a hysteretic structural system with a linear viscous damping model can be expressed as

$$
\underline{M}\ddot{X}(t) + \underline{C}\dot{X}(t) + \underline{F}_s[X(t)] = -\underline{M}\xi \ddot{X}_o(t) + H\underline{U}(t) \qquad (2.32)
$$

in which

$$
E_{s}[X(t)] = K_{e}X(t) + K_{I}Y(t)
$$
\n(2.33)

where $\underline{V}(t) = [v_1(t), v_2(t), \dots, v_n(t)]' =$ an n vector denoting the hysteretic variable, v_i , of each story unit given by Eq. (2.31). In Eq. (2.33), \underline{K}_{e} is an (nxn) band-limited elastic stiffness matrix with all elements equal to zero, i.e., $K_e(i,j) = 0$, except $K_e(i,i) = \alpha_i k_i$ for $i = 1, 2, ..., n$ and $K_e(i, i+1) = -\alpha_{i+1}k_{i+1}$ for $i = 1, 2, ..., n-1$. Similarly, K_i is an (nxn) bandlimited hysteretic stiffness matrix with all elements equal to zero, i.e., $K_I(i,j) = 0$, except $K_1(i,i) = (1-\alpha_i)k_iD_{y_i}$ for $i = 1,2,...,n$ and $K_1(i,i+1) = -(1-\alpha_{i+1})k_{i+1}D_{y_i+1}$ for $i = 1,2,...,n-1$. Matrices \underline{M} , \underline{K}_e and \underline{K}_I should be modified appropriately when an active mass damper is installed.

By introducing a 3n state vector $\underline{\tilde{Z}}(t)$,

$$
\underline{\tilde{Z}}(t) = \begin{bmatrix} X \\ Y \\ \dot{X} \end{bmatrix}
$$
 (2.34)

the second order nonlinear matrix equation of motion, Eqs. (2.32) and (2.33), can be converted into a first order matrix equation as follows

$$
\underline{\tilde{Z}}(t) = g[\tilde{Z}(t)] + \underline{B}U(t) + \underline{W}_1 \ddot{X}_o(t) \qquad (2.35)
$$

in which \underline{B} and \underline{W}_1 are 3n vectors

$$
\underline{B} = \left[\begin{array}{c} \underline{0} \\ \underline{0} \\ \underline{M}^{-1} \underline{H} \end{array} \right] \qquad ; \qquad \underline{W}_1 = \left[\begin{array}{c} \underline{0} \\ \underline{0} \\ -\underline{\xi} \end{array} \right] \qquad (2.36)
$$

and $g[\tilde{Z}(t)]$ is a 3n vector consisting of nonlinear functions of components of the vector $\tilde{Z}(t)$,

gri(t)] = *I(X, 1')* **----------------** -M-¹ (.ex ⁺ KeX ⁺ Kr:f) (2.37)

where $f(\dot{x},v)=[f_1(\dot{x}_1,v_1), f_2(\dot{x}_2,v_2),..., f_n(\dot{x}_n, v_n)]' =$ an n vector with the ith element, $f_i(\dot{x}_i, v_i)$, given by Eq. (2.31).

The optimal control vector $\underline{U}(t)$ is given by Eq. (2.14), which can be expressed as

$$
\underline{U}(t) = -\frac{\Delta t}{3} \ \underline{R}^{-1} \underline{B}' \underline{Q} \begin{bmatrix} \underline{X} \\ - - - - - \\ \underline{X} \end{bmatrix} = \underline{G}_2 \ \underline{X}(t) + \underline{G}_1 \ \underline{\dot{X}}(t) \tag{2.38}
$$

Substitution of Eq. (2.38) into Eq. (2.35) leads to the 3n matrix equation of motion for a hysteretic system under optimal control as follows

$$
\dot{\tilde{Z}}(t) = g[\tilde{Z}(t)] + \tilde{G}\tilde{Z}(t) + \underline{W}_{1}\ddot{X}_{o}(t) \qquad (2.39)
$$

in which

 \bar{z}

$$
\tilde{G} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ M^{-1}H G_2 & 0 & M^{-1}H G_1 \end{bmatrix}
$$
 (2.40)

With optimal control given by Eq. (2.14) or Eq. (2.38), the response for the hysteretic structural system can be simulated by solving Eq. (2.39) numerically using the Fourth-Order Runge-Kutta method.

SECTION 3 NUMERICAL EXAMPLE

To illustrate the application of the instantaneous optimal algorithm developed in this report to nonlinear and inelastic structural systems, several examples are presented in this section. Emphasis is placed on aseismic hybrid control systems. These aseismic hybrid control systems consist of active control devices, such as actuators or active mass dampers and base isolation systems, such as elastomeric bearings or sliding systems. The hysteretic behavior of elastomeric bearings is quite different from that of the sliding systems as will be described later.

An eight-story building that exhibits bilinear elasto-plastic behavior as shown in Fig. 3.1(a) is considered. The stiffness of each story unit is designed such that yielding occurs simultaneously for each story unit. The properties of the structure are as follows: (i) the mass of each floor is identical with $m_i = m = 345.6$ metric tons; (ii) the preyielding stiffnesses of the eight-story units are k_{11} (i=1.2,...,8) = 3.4x10⁵, 3.26x10⁵, 2.85x10⁵, 2.69x10⁵, 2.43x10⁵, 2.07x10⁵, 1.69x10⁵ and 1.37x10⁵ kN/m, respectively, and the postyielding stiffnesses are $k_{i2} = 0.1 k_{i1}$ for $i=1,2,...,8$; and (iii) the viscous damping coefficients for each story unit are $c_i = 490, 467, 410, 386, 348, 298, 243$ and 196 kN.sec/m, respectively. The damping coefficients given above result in a classically damped structure with a damping ratio of 0.38% for the first vibrational mode. The natural frequencies of the unyielded structure are 5.24, 14.0, 22.55, 30.22, 36.89, 43.06, 49.54 and 55.96 rad./sec. The yielding level for each story unit varies with respect to the stiffness; with the results, $D_{yi} = 2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.7,$ and 1.5 cm. The bilinear elasto-plastic behavior can be described by the hysteretic model, Eqs. (2.29) and (2.31), with A_i=1.0, β_i =0.5, n_i=95 and γ_i =0.5 for i=1.2,...,8. A simulated earthquake with a maximum ground acceleration of 0.3g as shown in Fig. 3.2 is used as the input excitation.

With the eight-story building structure described above and the earthquake ground acceleration shown in Fig. 3.2, time histories of all response quantities have been computed. Within 30 seconds of the earthquake episode, the maximum interstory

FIGURE 3.1 Structural Model of a Multi-Story Building with Active Control Systems: (a) Active Tendon Control System; (b) Active Mass Damper

A Simulated Earthquake Ground Acceleration FIGURE 3.2

deformation, x_{mi} , and the maximum absolute acceleration of each floor, a_i , are shown in Table 3.1. The time history of the first floor deformation, $x_1(t)$, is presented in Fig. 3.3(a). Further, the hysteresis loops of the shear force-deformation relation for the first, fourth, and eighth story units are displayed in Fig. 3.4(a) in which "i" signifies the ith story unit. As observed from Fig. 3.4(a) and Table 3.1, the deformation of the unprotected building is excessive and that yielding takes place in each story unit. These results also correspond to that obtained by Yang, et al [30-31].

Example 1: Building Equipped with Active Mass Damper on Top Floor

Consider that the building is equippped with an active mass damper on the top floor as shown in Fig. 3.1(b). The mass, stiffness and damping coefficient of the active mass damper are as follows: m_d =36.3 metric tons; k_d =957.5 kN/m; c_d=27.97 *kN.sec.lm.* Hence, the mass ratio of the damper with respect to the first generalized mass is 10.5%, the damping ratio of the damper is 7.5% and the frequency of the damper is 98% of the fundamental frequency of the unyielded building.

With the active mass damper, the building response depends on the active control force that is determined by the weighting matrices Q and R . The weighting matrix R consists of only one element, denoted by R_0 , since only one controller is installed on the top floor. The weighting matrix Q is (18x18) and it is partitioned as follows

$$
Q = \alpha \left[- \frac{Q}{Q_{21}} - \frac{Q}{Q_{22}} - \frac{Q}{Q_{22}} - \frac{Q}{Q_{22}} \right]
$$
 (3.1)

in which Q_{21} and Q_{22} are (2x9) matrices and 0 is a (16x9) matrix with all elements equal to zero.

It is mentioned that the Q matrix is symmetric. Since the active control forces from the actuator are acting on the mass damper and the top floor, only elements in two rows of the Q matrix are relevant to the control vector $\underline{U}(t)$, Eq. (2.21). In the present case, only elements in the first two rows of Q_{21} and Q_{22} matrices are related to the control vector $\underline{U}(t)$. As a result, all elements of the \underline{O} matrix which have no effect on

of ith story unit; a=maximum absolute acceleration of ith floor; U_m=maximum active of ith story unit; ai=maximum absolute acceleration of ith floor; Um=maximum active X_d =maximum relative displacement of the mass damper; x_{mi}=maximum deformation x_d =maximum relative displacement of the mass damper; x_{mi} =maximum deformation TABLE 3.1: Maximum Response Quantities of Building Equipped with Active Mass Damper: TABLE 3.1: Maximum Response Quantities of Building Equipped with Active Mass Damper: control force control force

 $AMD = Active Mass Damper$ AMD = Active Mass Damper

FIGURE 3.3 Deformation of First Story Unit Using Passive Control Systems: (a) Without Control; (b) With Active Mass Damper; (c) With Base Isolation System alone; (d) With Base Isolation System and Passive Mass Damper ($m_d =$ 50%mj); (e) With Base Sliding System

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the control vector $U(t)$ are set to be zero for convenience.

For illustrative purposes, $R_0 = 10^{-3}$ and Q_{21} and Q_{22} are given as follows

n [-1, -1, -1, -1, -1, -1, -1.59, -2.68, 0.0125] ~1 ⁼ -1, -1, -1, -1, -1, -1, -1.59, -2.68, 0.0107 (3.2) *n* [20, 20, 20, 20, 20, 20, 20, 20, 0.107] ^X *10-*¹ ~2 = 1.73, 1.73, 1.73, 1.73, 1.73, 1.73, 1.73, 1.73, 0.107

Time histories of all the response quantities were computed for an α/R_0 ratio of 4.05x10⁸. The deformation of the first story unit, $x_1(t)$, is plotted in Fig. 3.3(b) and the hysteresis loops for the first, fourth and eighth story unit are displayed in Fig. 3.4(b). Within 30 seconds of the earthquake episode, the following quantities are summarized in Table 3.1: (i) the maximum interstory deformation, x_{mi} , (ii) the maximum acceleration, a_i , of each floor, (iii) the maximum relative displacement of the mass damper with respect to the top floor, \bar{x}_d , and (iv) the maximum active control force U_m . A value of $\Delta t = 0.15x10^{-3}$ secwas used in the numerical computation. The time history of the required active control force is shown in Fig. 3.5.

For comparison, the response quantities for an α/R_o of 4.05x10⁹ are also shown in Table 3.1. It is observed from Table 3.1 that the active mass damper is capable of reducing the building response and that the response quantities reduce as the active control force increases. All the response quantities are within the elastic range for the case of α/R_0 =4.05x10⁹. A further comparison of the results in Table 3.1 with that in Ref. 30 indicates that the present formulation may be more efficient.

Example 2: Building Equipped With Rubber-Bearing Base Isolation System

To reduce the structural response, a lead-core rubber-bearing isolation system is implemented as shown in Fig. 3.6(a). The restoring force of the lead-core rubberbearing system is modeled as

$$
F_{sb} = \alpha_b k_b x_b + (1 - \alpha_b) k_b D_{yb} v_b \tag{3.3}
$$

in which $\nu_{\rm b}$ is given by Eq. (2.31) with i=b. The mass of the base isolation system is $m_b = 450$ metric tons and the viscous damping coefficient is assumed to be linear with

FIGURE 3.6 Structure Model of a Multi-Story Building: (a) With Base Isolation System; (b) With Base Isolation System and Actuator

 $c_b = 26.17$ kN.sec/m. The restoring force of the base isolation system given by Eq. (3.3) is not bilinear elasto-plastic and the parameter values are given as follows: $k_b = 18050.0$ kN/m, α_b =0.6 and D_{yb} =4cm. The parameters governing the scale and general shape of the hysteresis loop of the isolation system, Eq. (2.31), were assumed to be $A_b = 1.0$, β_b =0.5, n_b=3 and γ_b =0.5. The hysteretic characteristics of such a base isolation system, i.e., the hysteretic component v_b versus the displacement x_b , is shown in Fig. 3.8(a). With the base isolation system, the 9 natural frequencies of the preyielding structure are 2.21, 9.31, 17.29, 25.18, 32.19, 38.29, 44.12, 50.37, and 56.74 rad/sec. The damping ratio for the first vibrational mode is 0.15 %. It is observed that the fundamental frequency is reduced significantly by the implementation of the base isolation system. The response vector $\underline{X}(t)$ is given by $\underline{X} = [x_b, x_1, \ldots, x_8]'$.

Time histories of all the response quantities were computed. The time history, $x_1(t)$, of the deformation of the first story unit is depicted in Fig. 3.3(c) and that of the base isolation system, $x_b(t)$, is presented in Fig. 3.9(a). Further, the hysteresis loop for the shear force of the base isolation system is shown in Fig. 3.1O(a). The maximum response quantities of the structure in 30 seconds of the earthquake episode are shown in Table 3.2, in which the row designated by "B" indicates the response of the base isolation system. As observed from Table 3.2 and Fig. 3.3(c), the interstory deformation and the floor acceleration are drastically reduced. The advantage of using a base isolation system to protect the building is clearly demonstrated. However, the deformation of the base isolation system shown in row B of Table 3.2 may be excessive.

Example 3: Building Equipped With Passive Hybrid Control System

To protect the safety and integrity of the base isolation system, a passive mass damper was proposed [Refs. 34-36] to be connected to the base isolation system as shown in Fig. 3.7(a). This is referred to as the passive hybrid control system [Refs. 35- 36]. The properties of the mass damper are as follows. The mass of the mass damper, m_d , is expressed in terms of the ϕ percentage of the floor mass m_i , i.e. $m_d = \phi m_i$, and it will be varied to examine the effect of the mass ratio ϕ . The natural frequency of the mass damper is the same as the first natural frequency of the base isolated building, *i.e.*

type Sliding System

FIGURE 3.9 Deformation of Rubber Bearing Base Isolation System: (a) Without Control; (b) With Passive Mass Damper $(m_d = 50\%m_i)$; (c) With Actuator; (d) With Actuator (Limited No. of Sensors); (e) With Active Mass Damper (f) With Active Mass Damper (Limited No. of Sensors)

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TABLE 3.2: Maximum Response Quantities of Building Equipped with Base Isolation System and Passive Mass TABLE 3.2: Maximum Response Quantities of Building Equipped with Base Isolation System and Passive Mass Damper : X_d=maximum relative displacement of the mass damper; x_{mi}=maximum deformation of
ith floor; a_l=maximum absolute acceleration of ith floor Damper: $\boldsymbol{\mathsf{x}}$ _{d=}maximum relative displacement of the mass damper; $\boldsymbol{\mathsf{x}}_{\mathsf{m}}$ i=maximum deformation of ith floor; ai=maximum absolute acceleration of ith floor

2.21 rad/sec. The damping ratio of the mass damper is 10%. With such a passive hybrid control system, the maximum deformation of each story unit, x_{mi} , and the maximum acceleration of each floor, a_i , within 30 seconds of the earthquake episode are presented in Table 3.2 for different mass ratio, ϕ , of the mass damper. Also shown in row B of Table 3.2 is the maximum deformation of the base isolation system. The maximum deformations for the base isolation system and the first story unit are plotted in Fig. 3.11 as function of the mass ratio $\phi = m_d/m_i$. The maximum relative displacement of the mass damper, denoted by \bar{x}_d , is also shown in Table 3.2. Time histories of the deformation of both the first story unit and the base isolation system are shown in Figs. 3.3(d) and 3.9(b), respectively, for $\phi = 0.5$. It is observed from Table 3.2 and Figs. 3.3 and 3.9 that the passive mass damper is capable of reducing not only the deformation of the base isolation system but also the response of the building; the bigger the passive mass damper, the better the performance of the passive hybrid control system.

Example 4: Building Equipped With Rubber-Bearing Isolation System and Active Devices

Instead of using a passive mass damper, an active mass damper has been proposed in Refs. 34-36 for protecting the base isolation system. It is referred to as the active hybrid control system in Refs 34-36. For the active mass damper, a mass ratio of 50% is considered, i.e., $m_d = 0.5m_i$. With the active mass damper, the structural response depends on the weighting matrices \underline{R} and \underline{Q} . For this example, the weighting matrix \underline{R} consists of only one element, denoted by R_0 , whereas the dimension of the Q matrix is (20x20). R₀ is chosen to be 10⁻³ for simplicity and the Q matrix is partitioned as shown in Eq. (3.1), in which Q_{21} and Q_{22} are (10x10) matrices.

Note that the weighting matrix Q is symmetric. Because there is only one actuator installed on the basement floor, see Fig. 3.6(b), only elements in one row of the Q matrix are relevant to the control vector $\underline{U}(t)$, Eq. (2.21). For the present hybrid control system, only elements in the first row of the Q_{21} and Q_{22} matrices are related to the

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control vector $U(t)$. As a result, all elements of the Q matrix which have no effect on the control vector $U(t)$ are set to be zero for convenience.

First, we consider the case in which displacement and velocity sensors are installed on all the degrees of freedom of the structure. For illustrative purposes, the following values are assigned for Q_{21} (1,j) and $Q_{22}(1,j)$ for j=1,2,...,10.

$$
Q_{21}(1,j) = [102 - 2362 - 11 - 840 - 1397 - 1649 - 1645 - 1433 - 1057 - 561] (3.4)
$$

\n
$$
Q_{21}(1,j) = [67 \t 731 \t 646 \t 621 \t 584 \t 529 \t 457 \t 366 \t 257 \t 133] (3.4)
$$

and $Q_{21}(i,j)=Q_{22}(i,j)=0$ for $i=2,3,...,10$ and $j=1,2,...,10$.

Time histories of all the response quantities have been computed. In particular, the time histories of the deformation of the base isolation system and the first story unit are shown in Fig. 3.9(e) and 3.12(b), respectively, for $\alpha/R_0 = 50000$ and $\Delta t = 0.375 \times 10^{-2}$. The hysteresis loop for the shear force of the base isolation system is plotted in Fig. 3.10(c) and the required active control force is displayed in Fig. 3.13.

Within 30 seconds of the earthquake episode, the maximum interstory deformations, x_{mi} (i=B,1,2,...,8), the maximum acceleration of each floor, a_i , the maximum active control force U_m and force rate \dot{U}_m , as well as the maximum relative displacement of the mass damper, \bar{x}_d , are summarized in the columns designated as "BIS&AMD¹" of Table 3.3.

The building equipped with a base isolation system tends to behave like a rigid body since the interstory deformation is small compared to that of the base isolation system. As a result, sensors may not be needed for the building. Consider the second case in which displacement and velocity sensors are installed on the active mass damper and the base isolation system only, i.e. no sensor is installed on the building. In this case, all the elements of Q_{21} and Q_{22} matrices are zero except Q_{21} (1,1), $Q_{21}(1,2)$, $Q_{22}(1,1)$, and $Q_{22}(1,2)$. For simplicity, the following values are assigned: $Q_{21}(1,1)=102$, $Q_{21}(1,2)=2363$, $Q_{22}(1,1)=67$, and $Q_{22}(1,2)=731$. The maximum response quantities for α/R_0 =60000 and Δt =0.375x10⁻² have been computed and summarized in the columns designated as "BIS&AMD²" of Table 3.3. Time histories

FIGURE 3.12 Deformation of First Story Unit: (a) Without Control; (b) With Base Isolation System and Active Mass Damper; (c) With Base Isolation System and Active Mass Damper (Limited No. of Sensors); (d) With Base Isolation System and Actuator; (e) With Base Isolation System and Actuator (Limited No. of Sensors); (f) With Base Sliding System and Actuator

Required Active Control Force for Building With Base Isolation System and Active Mass Damper FIGURE 3.13

x_{mi}=maximum deformation of ith story unit; a_i=maximum absolute acceleration of ith x_{ml} =maximum deformation of ith story unit; a_i=maximum absolute acceleration of ith Maximum Response Quantities of Building Equipped with Base Isolation System and TABLE 3.3: Maximum Response Quantities of Building Equipped with Base Isolation System and floor; U_m=maximum active control force; U_m=maximum active control force rate floor; U_m=maximum active control force; U_m=maximum active control force rate Active Mass Damper : X_d=maximum relative displacement of the mass damper; Active Mass Damper : X_{d=}maximum relative displacement of the mass damper; TABLE 3.3:

BIS & AF ⁴		U _{m=4490} .6kN/s	\vec{a}	$\frac{\text{cm}}{s^2}$	117	118	123	122	116	125	117	131	129
	U _m =808.6kN		χ	\overline{c}	711	0.86	0.82	0.83	0.74	0.66	0.58	0.48	0.33
BIS & AF ³		$U_m = 4883.1$ kN/s	\vec{a}	(cm/s ²)	$\frac{8}{1}$	$\frac{9}{1}$	123	121	112	124	112	126	$\frac{1}{3}$
	$U_{m} = 855.4kN$		\times ri	\overline{c} m)	24.85	0.82	0.79	0.80	0.71	0.63	0.56	0.47	0.33
BIS & AMD ²	U_{m} =234.4kN	$U_m = 1218.9kM/s$	\vec{a}	$\frac{\text{cm/s}}{2}$	118	$\frac{2}{11}$	109	105	$\frac{4}{9}$	$\overline{0}$	$\frac{1}{1}$	$\frac{9}{11}$	142
	χ _d =136cm		\bar{x}	\overline{c} m)	24.21	0.69	0.69	0.72	0.68	0.63	0.59	0.53	0.36
	χ _d =135cm	$U_{\rm m}$ =984.8kN/s U_{m} =225.5kN	$\bar{\sigma}$	$\frac{(cm/s^2)}{s}$	118	$\frac{2}{11}$	108	105	ჭ თ	102	$\frac{1}{1}$	$\frac{1}{2}$	141
BIS & AMD			\overline{x}	$\left(\begin{matrix} 1 \ 0 \end{matrix} \right)$	24.80	0.71	0.70	0.73	0.69	0.63	0.59	0.53	0.36
Yield	Displ.		تم	(cm)	$\frac{1}{4}$	र २.	ن ب	2.i	$\frac{1}{2}$	o. N	$\frac{1}{1}$	$\overline{1}$.	$\frac{5}{1}$
u.	\circ o	Œ	z d		B		$\mathbf{\tilde{c}}$	ణ	4	ပာ	ဖ		∞

BIS = Base Isolation System; AMD¹ = Acitve Mass Damper with full sensors; AMD² = Acitve Mass BIS = Base Isolation System; AMD¹ = Acitve Mass Damper with full sensors; AMD² = Acitve Mass Damper with limited No. of sensors; AF³ = Active Control Force with full sensors; AF⁴ = Active Damper with limited No. of sensors; AF³ = Active Control Force with full sensors; AF⁴ = Active Control Force with limited No. of sensors Control Force with limited No. at sensors for the deformation of the base isolation system and the first story unit are shown in Fig. 3.9(t) and 3.12(c), respectively. The hysteresis loop for the shear force of the base isolation system is plotted in Fig. 3.10(d). A comparison of the results in this case with those of the first case in which sensors are installed on all floors of the building indicates that installations of displacement and velocity sensors on the building are not necessary. This conclusion is very important because it is expensive to install sensors. It is further observed from Table 3.3 that a significant reduction of the building response, in particular the response of the base isolation system, can be achieved using an active mass damper with a small active control force and force rate.

Instead of using an active mass damper described above, the base isolation system can be connected directly to an actuator. Such an active hybrid control system will be considered. Consider the first case in which displacement and velocity sensors are installed on every degree of freedom. The $(18x18)$ Q matrix is partitioned as shown in Eq. (3.1) in which Q_{21} , and Q_{22} are (9x9) matrices. For the isolation system connected to an actuator, the control force $\underline{U}(t)$ depends on the difference between the first two rows of the Q_{21} and Q_{22} matrices, i.e., $Q_{21}(1,j)-Q_{21}(2,j)$ and $Q_{22}(1,j)-Q_{22}(2,j)$. For simplicity, elements of $Q_{21}(2,j)$ and $Q_{22}(2,j)$ rows will be assigned to be zero. The following values are assigned to the elements of $Q_{21}(1,j)$ and $Q_{22}(1,j)$ rows for illustrative purposes:

$$
Q_{21}(1,j) = [0.25 \t 5.5 \t 4.5 \t 3.4 \t 2.4 \t 1.4 \t 1.3 \t 1.3 \t 1.3]
$$

\n
$$
Q_{22}(1,j) = [3.5 \t 0.5 \t 0.5 \t 0.5 \t 0.5 \t 0.4 \t 0.4 \t 0.3 \t 0.3]
$$
\n(3.5)

and $Q_{21}(i,j)=Q_{22}(i,j)=0$ for $i=2,\ldots,9$. The weighting matrix R consists of only one element denoted by R_0 . R_0 is chosen to be 10^{-3} . Time histories of all the structural response quantities have been computed. In particular, the time histories of the deformation of the base isolation system and the first story unit are shown in Fig. 3.9(c) and 3.12(d), respectively, for $\alpha/R_0 = 1.55 \times 10^8$ and $\Delta t = 0.357 \times 10^{-2}$. The hysteresis loop for the shear force of the base isolation system is plotted in Fig. 3. 10(e) and the required active control force is displayed in Fig. 3.14.

Within 30 seconds of the earthquake episode, the maximum interstory deformations, x_{mi} , the maximum acceleration, a_i , of each floor, the maximum active control force, U_m , and force rate, \dot{U}_m , are summarized in the columns designated as "BIS&A F^3 " of Table 3.3.

Again, for convenience of instrumentation, we consider the second case in which displacement and velocity sensors are installed on the base isolation system only, i.e., no sensor is installed on the building. In this case, all the elements of Q_{21} and Q_{22} matrices are zero except $Q_{21}(1,1)$ and $Q_{23}(1,1)$. For simplicity, the following values are assigned: $Q_{21}(1,1)=0.9$ and $Q_{23}(1,1)=3.4$. The maximum response quantities for α/R_0 =1.5x10⁸ have been computed and summarized in the columns designated as "BIS&AF⁴" of Table 3.3. Time histories of the deformation of the base isolation system and the first story unit are shown in Fig. 3.9(d) and 3.12(e), respectively. The hysteresis loop for the shear force of the base isolation is plotted in Fig. 3.1O(f). A comparison of these results with those of Case 1 using 18 sensors indicates that installing only the displacement and velocity sensors on the base isolation system is as good as installing displacement and velocity sensors on all degrees of freedom of the structural system.

It is observed from the results presented in Table 3.3 that the performance of the actuator is about the same as that of an active mass damper. However, the required active control force, U_m , and the force rate, \dot{U}_m , are much bigger using an actuator alone. This has been expected because the idea of using a mass damper is to absorb energies from the base isolation system, thus reducing the capacity of the actuator. The passive hybrid control system considered in the previous example demonstrates very well such a conclusion.

Example 5: Building Equipped With Sliding-Type Base Isolator and Actuator

Instead of using a rubber-bearing base isolation system, a frictional-type sliding base isolation system, as shown schematically in Fig. 3.15(a), is considered herein. This type of isolators allows greater resistance to damage by permitting the building to slide on its foundation during severe earthquakes. This isolation system decouples the building

Base Sliding System; (b) With Base Sliding System and Actuator

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from its foundation with nearly frictionless teflon on stainless steel sliding plates that have very low frictional resistance [e.g., 5-8, 17-18].

When the sliding system slides, the frictional force developed in the sliding system, $F_{\rm sh}$, is given by

$$
F_{sb} = \mu \bar{m}g \; sgn(\dot{x}_b) \tag{3.6}
$$

in which mg=w is the weight of the building system above the sliding bearings and μ is the coefficient of friction. When the sliding system sticks to the structure, the frictional force F_{sb} is smaller than the one given by Eq. (3.6). In the dynamic analysis of the sliding system for computing the time history of the structural response, a judgement is needed at every time instant t to determine whether the system is sticking or sliding. This is because the nonlinear frictional force varies from time to time. As a result, such an approach is quite tedious in the numerical computation of the structural response and it does not provide a systematic approach for the control problem.

To overcome the problem described above, the highly nonlinear frictional force is represented by the following analytical function

$$
F_{sb} = \mu \, \bar{m} \, g \, v_b \tag{3.7}
$$

in which v_b is a nondimensional hysteretic quantity described by Eq. (2.30) for i=b. It is mentioned that during the sliding phase (yielding), v_b takes a value of either 1 or -1. During the sticking phase (elastic behavior), the absolute value of v_b is less than unity, i.e., $|v_b| \le 1$. The conditions of sticking and sliding are accounted for by Eq. (2.30) automatically. The parameters governing the scale and general shape of the hysteresis loop of the sliding system are $A_b = 1.0$, $\beta_b = 0.5$, $n_b = 2$, $v_b = 0.5$ and $D_{vb} = 0.012$ cm. A hysteresis loop for this set of parameters, i.e., v_b vs. x_b , representing the hysteretic behavior of the sliding system is schematically shown in Fig. 3.8(b). The mass of the sliding system is $m_b = 450$ metric tons and the coefficient of friction is $\mu = 10\%$. The response vector is given by $\underline{X}=[x_b, x_1, \ldots, x_8]'$. The integration time step Δt is 0.03x10⁻² sec.

Time histories of all the response quantities have been computed for the building equipped with the sliding base isolation alone. The time history, $x_1(t)$, of the first story deformation is depicted in Fig. 3.3(e) and that of the sliding system, $x_b(t)$, is presented in Fig. 3.16(a), where $x_b(t)$ denotes the relative displacement of the teflon and the stainless steel plate. Further, the hysetersis loops for the frictional force of the sliding system and the shear force of the first, fourth, and eighth story units are shown in Figs. 3.17(a) and 3.4(c), respectively. The maximum response quantities of the structure within 30 seconds of the earthquake episode are shown in the columns designated as "With SIS" of Table 3.4. As observed from Table 3.4 and Figs. 3.3(e) and 3.4(c), the interstory deformations and floor accelerations are significantly reduced. The advantage of using a sliding type base isolation system to protect the building is clearly demonstrated. However, the deformation of the eighth story unit is still in the inelastic range.

To further reduce the structural response and to bring the deformation of the 8th story unit into the elastic range, an actuator is connected to the sliding system as shown in Fig. 3.15(b). The actuator will apply the active control force directly on the sliding system. The basic idea of the active force, in this case, is to counteract the frictional force in order to maintain the system in the sliding condition as much as possible, so that the transmission of the earthquake ground motion to the building can be kept to a minimum.

With the active control force, the structural response depends on the weighting matrices \underline{R} and \underline{Q} . Again, the weighting matrix \underline{R} consists of only one element, denoted by R₀, whereas the dimension of the Q matrix is (18x18). R₀ is chosen to be 10⁻³ for simplicity and the Q matrix is partitioned as shown in Eq. (3.1) .

It is noticed that the deformation of the building is in-phase with the frictional force and the velocity of the building is out of phase of the frictional force. As a result, $Q_{22} = Q$ and only the elements in the Q_{21} matrix which correspond to the deformation of the building are needed. These elements are assigned in the following for illustrative purposes:

FIGURE 3.16 Relative Displacement of Base Sliding System: (a) Without Control; (b) With Actuator

Hysteresis Loop for Frictional Force in Base Sliding FIGURE 3.17 System: (a) Without Control; (b) With Actuator

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 $\hat{\mathcal{L}}$

x_{mi}=maximum deformation of ith story unit; a_i=maximum absolute acceleration of ith x_{min} =maximum deformation of ith story unit; a_i=maximum absolute acceleration of ith TABLE 3.4: Maximum Response Quantities of Building Equipped with Sliding-type Isolation System and Actuator : R_d=maximum relative displacement of the mass damper; TABLE 3.4: Maximum Response Quantities of Building Equipped with Sliding-type Isolation System and Actuator : R _d=maximum relative displacement of the mass damper; floor; U_m=maximum active control force floor; Um=maximum active control force

SIS = Sliding-type Isolation System; AF = Active Control Force SIS = Sliding-type Isolation System; AF = Active Control Force

$$
Q_{21}(1,j) = [0, -30000, -3000, -300, -30, -3, -0.3, -0.03, -0.003]
$$
 (3.8)

All other elements of the Q_{21} matrix are set to be zero.

Time histories of all the structural response quantities have been computed. In particular, the time histories of the deformation of the sliding system and the first story unit are shown in Fig. 3.16(b) and 3.12(f), respectively, for $\alpha/R_0 = 2.25 \times 10^8$. The hysteresis loop for the sliding system is plotted in Fig. 3. 17(b). The required active control force is displayed in Fig. 3.18. Within 30 seconds of the earthquake episode, the maximum interstory deformations, $x_{mi}(i=b,1,2,...,8)$, the maximum floor acceleration, a_i , and the maximum active control force, U_m , are summarized in the columns designated as "SIS&AF" of Table 3.4.

It is observed from Table 3.4 and Figs. $3.16(b)$ and $3.12(f)$ that the actuator connected to the sliding system is capable of reducing the response of the building significantly.

Example 6: Hybrid Control System Consisting of a Base Isolation System and an Actuator With Velocity and Acceleration Feedbacks

The instantaneous optimal control algorithm using velocity and acceleration feedbacks has been developed, Eq. (2.27). The application of such an algorithm and its performance will be demonstrated using an eight-story bilinear elasto-plastic building implemented by a hybrid control system. The hybrid control system consists of a rubberbearing isolation system connected to an actuator, see Fig. 3.6(b), identical to that presented in Example 4. In the present case, however, instead of using the displacement and velocity feedbacks, i.e., $\underline{Z}(t)$, the velocity and acceleration feedbacks, $\underline{Z}(t)$, will be used, Eq. (2.27).

The structural response $\underline{X}(t)=[x_b,x_1,\ldots,x_8]'$ depends on the weighting matrices \underline{Q}^* and \underline{R}^* . Since there is only one actuator, the \underline{R}^* matrix consists of only one element, denoted by R_0^* . The dimension of the Q^* matrix is 18x18 and it is partitioned as follows

$$
Q^* = \alpha \left[- \frac{0}{Q_{21}^*} - \frac{0}{Q_{22}^*} - \frac{0}{Q_{
$$

in which Q_{21}^* and Q_{22}^* are (9x9) matrices.

Again, the Q^* matrix is symmetric. Since there is only one actuator installed on the basement floor, see Fig. 3.6(b), the control vector $U(t)$ depends only on the difference between the first two rows of the Q_{21}^* and Q_{22}^* matrices, Eq. (2.27). For convenience, only elements in the first row of the Q_{21}^* and Q_{22}^* are assigned with values and all other elements are set to be zero.

With a base isolation system, the building usually behaves like a rigid body, since its deformation is relatively small compared to that of the base isolation system. As a result, the performance of the control system will not be affected adversely if sensors are not installed on the building as demonstrated in Example 4. Since the installation of sensors is quite expensive, only a velocity sensor and an acceleration sensor will be installed on the base isolation system and the foundation. Consequently, all the elements in Q^*_{21} and Q^*_{22} matrices are zero except $Q^*_{21}(1,1)$ and $Q^*_{22}(1,1)$. For illustrative purpose, the following values are assigned to these two elements: $Q_{21}^*(1,1)=35$ and $Q^*_{22}(1,1) = 0.5$.

Within 30 seconds of the earthquake episode, the following quantities are presented in the last two columns designated as "BIS&AF²" of Table 3.5 for α =18,750 and $R_{0}^{*}=1$: (i) the maximum interstory deformations, x_{mi} , (ii) the maximum floor accelerations a_i and (iii) the maximum active control force U_m . Also presented in the columns designated as "BIS&AF¹" of Table 3.5 for comparison are the corresponding results using $Z(t)$ feedback taken from Table 3.3. The structural response quantitites without active control are also shown in Table 3.5, which are taken from Tables 3.1 and 3.2.

It is observed from Table 3.5 that the performance of the instantaneous optimal control algorithm using velocity and acceleration feedbacks is as good as that using the

Actuator Using 2 feedback : x_{mi}=maximum deformation of ith story unit; a=maximum Actuator Using Z feedback : $x_{min}=$ maximum deformation of ith story unit; a $=$ maximum TABLE 3.5: Maximum Response Quantities of Building Equipped with Base Isolation System and TABLE 3.5 : Maximum Response Quantities of 8uilding Equipped with Base Isolation System and absolute acceleration of ith floor; U_m=maximum active control force absolute acceleration of ith floor; U_m=maximum active control force

BIS = Base Isolation System; $AF' = Aciive$ Control Force with \leq feedback; $AF^2 = Aciive$ Control BIS = Base Isolation System; AF^1 = Active Control Force with Z feedback; AF^2 = Active Control Force with 2 feedback Force with \dot{z} feedback

displacement and velocity feedbacks. Further, the active control system using the instantaneous optimal algorithms presented herein is capable of significantly reducing both the responses of the building and the base isolation system.

Example 7: Hybrid Control System Consisting of A Base Isolation System and An Active Mass Damper With Velocity and Acceleration Feedbacks

The control algorithm using velocity and acceleration feedbacks, Eq. (2.27), will be used for a hybrid control system consisting of a base isolation system and an active mass damper presented in Example 4. In this case the structural response vector is $X(t) = [x_d, x_b, x_1, \dots, x_g]'$. The weighting matrix R^* consists of only one element, denoted by R_0^* , and the (20x20) weighting matrix Q^* is partitioned as shown in Eq. (3.9), where Q_{21}^* and Q_{22}^* are (10x10) matrices. For illustrative purposes, velocity and acceleration sensors are installed on every degree of freedom of the structural system, i.e., a total of 20 velocity and acceleration sensors are installed. The elements of $Q_{21}^*(1,j)$ and Q_{22} ^{*}(1,j) are assigned in the following for illustrative purpose

 $Q_{21}^*(1,j) = [102, -1353, 11, 840, 1397, 1649, 1645, 1433, 1057, 561]$ $Q_{22}^*(1,j) = [67, 731, 646, 621, 584, 529, 457, 366, 257, 133]$ (3.10)

where $Q_{21}^*(i,j) = Q_{22}^*(i,j) = 0$ for $i = 2,3,...,10$,

With R_0^* = 1 and α = 300, Eq. (3.9), the following maximum response quantitites in 30 seconds of the earthquake episode are presented in the columns designated as "I(B)" of Table 3.6: (i) the maximum interstory deformation x_{im} , (ii) the maximum floor acceleration a_i , and (iii) the maximum active control force U_m . Also presented in the columns designated as "I(A)" of Table 3.6 for comparison are the corresponding results using the displacement and velocity feedbacks, $Z(t)$, taken from Table 3.3.

We now consider the case in which sensors are not installed on the building, i.e., sensors are installed only on the base isolation system and on the mass damper. In this case all the elements of the $Q_{21}^*(1,j)$ and $Q_{22}^*(1,j)$ matrices are zero, except $Q_{21}^*(1,1)$, $Q^*_{21}(1,2)$, $Q^*_{22}(1,1)$ and $Q^*_{22}(1,2)$. For illustrative purpose, the following values are

and Active Mass Damper Using \mathbf{Z}_1 feedback : \mathbf{x}_{mi} =maximum deformation of ith story and Active Mass Damper Using Z feedback : $x_{mi}=maximum$ deformation of *ith* story TABLE 3.6: Maximum Response Quantities of Building Equipped with Base Isolation System unit; a_i=maximum absolute acceleration of ith floor; U_m =maximum active control TABLE 3.6: Maximum Response Quantities of Building Equipped with Base Isolation System unit; a_i=maximum absolute acceleration of *ith* floor; U_m =maximum active control force; $I = full$ Sensors; $II = Limited$ No. of Sensors; $(A) = with Z$ feedback; $(B) =$ force; $I = full$ Sensors; $II = Limited$ No. of Sensors; $(A) = with Z$ feedback; $(B) =$ with \overline{z} feedback with Z feedback

assigned: $Q^*_{21}(1,1) = 102$, $Q^*_{21}(1,2) = -1353$, $Q^*_{22}(1,1) = 67$ and $Q^*_{22}(1,2) = 731$.

Within 30 seconds of the earthquake episode, the maximum response quantities are presented in the last two columns of Table 3.6 designated as "II(B) ". For comparison, the corresponding results using the response state vector $Z(t)$ as feedback are shown in the columns designated as " $II(A)$ " of Table 3.6. These results were taken from Table 3.3.

Although the control algorithm using the \dot{Z} feedback is capable of reducing the structural response quantities, the required active control force is about one order of magnitude bigger than that using the Z feedback as observed from Table 3.6. With the same order of magnitude of the control force, i.e., $U_m = 225.5$ kN, and the \dot{Z} feedback algorithm, the response quantities are much larger than those obtained using the control algorithm with the Z feedback, Case (A) of Table 3.6.

Consequently, it is concluded that the control algorithm using the \dot{Z} feedback is not suitable for the base-isolated building connected to an active mass damper. The applicability and limitation of the control algorithm using the velocity and acceleration feedbacks for different types of aseismic protective systems will be presented in a different report.

SECTION 4 CONCLUSION

A refined version of the instantaneous optimal control algorithm for nonlinear or inelastic structural systems is presented. The optimal algorithm is simple and reliable for on-line operations without tracking any system characteristics. Applications of the optimal algorithm to various types of hybrid control systems have been demonstrated. These include (i) an active mass damper installed on the top floor of the building, (ii) active control of base-isolated buildings using lead-core elastomeric bearings, and (iii) active control of base-isolated buildings using sliding type isolation systems. The building itself considered is bilinear elasto-plastic.

A method of simulating the seismic response of inelastic (hysteretic) systems using the proposed optimal algorithm is presented. This method has been used to evaluate and compare numerically the performance of various control systems. The advantages of aseismic hybrid control systems are clearly demonstrated. It is shown that the optimal algorithm proposed is effective for practical applications.

Because of possible difficulties involved in measuring the displacement response of the building during an earthquake, it is desirable not to use the displacement response as feedbacks. A variation of the instantaneous optimal algorithm utilizing the acceleration response rather than the displacement response as feedback is also presented. Numerical examples demonstrate that the performance of the control algorithm using the velocity and acceleration feedbacks is at least as good as that of the optimal algorithm using the displacement and velocity feedbacks for the hybrid control systems considered.

Another contribution of this report is the analysis methodology presented for evaluating the performance of aseismic hybrid control systems using frictional-type sliding isolation systems. The highly nonlinear frictional force is described by an analytical function of the equivalent hysteretic component, thus facilitating the analysis of controlled response of the entire structural system.

SECTION 5

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APPENDIX A

FOURTH-ORDER RUNGE-KUTTA METHOD

The Runge-Kutta method has been used extensively to solve the first order nonlinear differential equation numerically. In this method, the solution at time $t + \Delta \tau$ is approximated by a truncated Taylor's series around the solution at the previous time 1. Let

$$
\dot{y}(t) = f[y(t), t] \tag{A-1}
$$

be a first order nonlinear differential equation. Given the solution $y(t_n)$ at time t_n , the solution $y(t_n + \Delta \tau)$ at $t_n + \Delta \tau$ is obtained using the Runge-Kutta method as

$$
y(t_n + \Delta \tau) = y(t_n) + \frac{1}{6} \Delta \tau (A_0 + 2A_1 + 2A_2 + A_3)
$$
 (A-2)

where

$$
A_0 = f [y(t_n), t_n]
$$

\n
$$
A_1 = f [y(t_n) + 0.5A_0, t_n + 0.5 \Delta \tau]
$$

\n
$$
A_2 = f [y(t_n) + 0.5A_1, t_n + 0.5 \Delta \tau]
$$

\n
$$
A_3 = f [y(t_n) + A_2, t_n + \Delta \tau]
$$
\n(M-3)

The matrix equation of motion given by Eq. (2.3) can be written as

$$
\underline{\dot{Z}}(t) = f\left[Z(t), t\right] \tag{A-4}
$$

in which

$$
f[Z(t),t] = g[Z(t)] + BU(t) + W_1 \ddot{X}_0(t) \qquad (A-5)
$$

Equation (A-4) is analogy to Eq. (A-1). Hence, using Eqs. (A-2) and (A-3) with t_n + $\Delta \tau =$ t and $\Delta \tau = 2\Delta t$, the solution for $Z(t)$ can be expressed as

$$
Z(t) = Z(t - 2\Delta t) + \frac{1}{6}(A_0 + 2A_1 + 2A_2 + A_3)
$$
 (A-6)

 λ

in which \underline{A}_0 , \underline{A}_1 , \underline{A}_2 and \underline{A}_3 are defined as Eq. (2.7).

APPENDIX B

INSTANTANEOUS OPTIMAL CONTROL ALGORITHMS

Substituting Eq. (2.12) and Eq. (2.13), one obtains the following three matrix equations

$$
2QZ(t) + \underline{\lambda}(t) = 0 \tag{B-1}
$$

$$
2RU(t) - \frac{\Delta t}{3} B' \lambda(t) = 0
$$
 (B-2)

$$
Z(t) = \underline{D}(t-2\Delta t, t-\Delta t) + \frac{\Delta t}{3} \left[\underline{B} \underline{U}(t) + \underline{W}_1 \ddot{X}_0(t) \right]
$$
 (B-3)

(i) Instantaneous Optimal Open-Loop Control (Feedforward)

\nLet the control vector
$$
U(t)
$$
 be regulated by the earthquake excitation alone, i.e.

$$
\underline{\lambda}(t) = q(t) \tag{B-4}
$$

Then equations (B-1), (B-2) and (B-3) become

$$
2QZ(t) + q(t) = 0 \tag{B-5}
$$

$$
2RU(t) - \frac{\Delta t}{3} B'q(t) = 0
$$
 (B-6)

$$
Z(t) = \underline{D}(t-2\Delta t, t-\Delta t) + \frac{\Delta t}{3} \left[\underline{B} \underline{U}(t) + \underline{W}_1 \ddot{X}_0(t) \right]
$$
 (B-7)

The unknown vectors $\underline{U}(t)$, $\underline{q}(t)$, and $\underline{Z}(t)$ can be solved from Eqs. (B-5)-(B-7) as follows. The vector $Z(t)$ is eliminated be substituting Eq. (B-7) into Eq. (B-5) resulting In

$$
2Q = D(t-2\Delta t, t-\Delta t) + \frac{\Delta t}{3} \left[B U(t) + W_1 \ddot{X}_0(t) \right] + q(t) = 0 \quad (B-8)
$$

Premultiplying Eq. (B-8) by $\Delta t/3$ \underline{B}' and adding to Eq. (B-6), one obtains the control vector $\underline{U}(t)$ in the following

$$
\underline{U}(t) = \underline{L}\underline{G}(t) \tag{B-9}
$$

in which

$$
L = \left[\left(\frac{\Delta t}{3} \right)^2 B' Q B + R \right]^{-1}
$$
 (B-10)

$$
G(t) = \frac{\Delta t}{3} B' Q D(t - 2\Delta t, t - \Delta t) - \left(\frac{\Delta t}{3}\right)^2 B' Q W_1 \ddot{X}_0(t) \qquad (B-11)
$$

It is observed from Eqs. (B-9)-(B-11) that the control vector $\underline{U}(t)$ at time t is regulated by the earthquake excitation $\ddot{X}_0(t)$ at time t and $\underline{D}(t-2\Delta t, t-\Delta t)$, that consists of quantities at t- Δt and t- $2\Delta t$. The response state vector $\underline{Z}(t)$ is obtained by substituting Eq. (B-9) into Eq. (B-7).

$$
Z(t) = \underline{D}(t-2\Delta t, t-\Delta t) + \frac{\Delta t}{3} \left[\underline{B} \underline{L} \underline{G}(t) + \underline{W}_1 \ddot{X}_0(t) \right]
$$
 (B-12)

(ii) Instantaneous Optimal Closed-Loop Control (Feedback)

Let the control vector $\underline{U}(t)$ be regulated by the feedback response state vector $\underline{Z}(t)$ alone, i.e.

$$
\underline{\lambda}(t) = \underline{\Lambda} \underline{Z}(t) \tag{B-13}
$$

L.

Then, substitution of Eq. $(B-13)$ into Eq. $(B-1)$ yields

$$
(2Q + \Delta) Z(t) = 0
$$
 (B-14)

The unknown matrix Δ is obtained, for $Z(t) \neq 0$, as

$$
\underline{\Lambda} = 2Q \qquad (\text{B-15})
$$

The control vector $U(t)$ is obtained by substituting Eq. (B-15) into Eq. (B-13) and then into Eq. (B-2) as follows

$$
U(t) = -\frac{\Delta t}{3} R^{-1} B' Q Z(t) \qquad (B-16)
$$

Substituting Eq. (B-16) into Eq. (B-3) and simplifying the resulting expression, one obtains the response vector $Z(t)$ under instantaneous optimal closed-loop control

$$
Z(t) = \left[I + \left(\frac{\Delta t}{3} \right)^2 BR^{-1} B'Q \right]^{-1} \left\{ D \left[(t - \Delta t), (t - 2 \Delta t) \right] + \frac{\Delta t}{3} W_t \ddot{X}_0(t) \right\}
$$
 (B-17)

(iii) Instantaneous Optimal Closed-Open-Loop Control (Feedforward-Feedback)

Let the control vector $\underline{U}(t)$ or $\underline{\lambda}(t)$ be regulated by both the feedback response state vector $Z(t)$ and the measured earthquake ground acceleration $\ddot{X}_0(t)$, i.e.

$$
\underline{\lambda}(t) = \underline{\Lambda}_1 Z(t) + q(t) \tag{B-18}
$$

The control vector $U(t)$ can be eliminated by substituting Eq. (B-2) into (B-3)

$$
Z(t) = D(t-2\Delta t, t-\Delta t) + \frac{\Delta t}{3} \left[\frac{\Delta t}{6} BR^{-1} B' \lambda(t) + \underline{W}_1 \ddot{X}_0(t) \right]
$$
 (B-19)

Equation (B-1) can be rewritten as

$$
Q [Z(t) + Z(t)] + \lambda(t) = 0
$$
 (B-20)

Substituting Eq. (B-19) for the second term of $Z(t)$ in Eq. (B-20) and simplifying the resulting expression, one obtains

$$
\{ Q + \left[\frac{(\Delta t)^2}{18} QBR^{-1}B' + I \right] \underline{\Lambda}_1 \} Z(t) + Q \{ D(t - 2\Delta t, t - \Delta t) + \frac{\Delta t}{3} H_1 \ddot{X}_0(t) \} + \left(\frac{(\Delta t)^2}{18} QBR^{-1}B' + I \right) \tilde{q}(t) = 0
$$
 (B-21)

For $Z(t) \neq 0$ and $q(t) \neq 0$, the solution of unknown matrix, Δ_1 , and vector, $q(t)$, are obtained as follows

$$
\Delta_1 = -\left[\frac{(\Delta t)^2}{18} QBR^{-1}B' + I\right]^{-1} Q \qquad (B-22)
$$

$$
q(t) = \Delta_1 \left[D(t - 2\Delta t, t - \Delta t) + \frac{\Delta t}{3} W_1 \ddot{X}_0(t) \right]
$$
 (B-23)

Thus, the control vector $\underline{U}(t)$ and the response state vector $\underline{Z}(t)$ with instantaneous optimal closed-open-Ioop control are determined in the following:

$$
U(t) = \frac{\Delta t}{6} R^{-1} B' \left[\Delta_1 Z(t) + q(t) \right]
$$
 (B-24)

$$
Z(t) = \left[I - \frac{(\Delta t)^2}{18} BR^{-1} B' \Delta_1 \right]^{-1} \{ D(t - 2\Delta t, t - \Delta t) + \frac{(\Delta t)^2}{18} BR^{-1} B' q(t) + \frac{\Delta t}{3} W_1 \ddot{X}_0(t) \}
$$
(B-25)

in which Δ_1 and $q(t)$ are given in Eqs. (B-22) and (B-23).

 $\overline{}$

HYBRID CONTROL OF SEISMIC-EXCITED STRUCTURAL SYSTEMS

PART II

SECTION 1 INTRODUCTION

It has been shown recently that a combined use of active and passive control systems, referred to as the hybrid control system, is more effective, beneficial and practical, in some cases, for reducing the building response under strong earthquakes [e.g., 3,9,23,24,28,29]. However, the use of hybrid control systems involves active control of nonlinear or inelastic structural systems, since passive control systems, such as base isolation systems, usually behave nonlinearly or inelastically under dynamic loads. Instantaneous optimal control algorithms for linear, nonlinear or inelastic structures were proposed in Refs. 19-22. Recently, a refined version of such optimal control algorithms was presented in Refs. 28-29, with emphasis placed on applications to aseismic hybrid control systems. An aseismic hybrid control system consists of active control devices, such as an active mass damper, and passive base isolation systems. In Refs. 28-29, the optimal control vector $\underline{U}(t)$ was obtained from the measured 2n state vector $Z(t)$.

For inelastic structural systems, such as base-isolated buildings using either 1eadcore rubber bearing isolators or sliding-type isolations [e.g. ,4-8, 10, 12-14], the total deformation consists of an elastic component and a hysteretic component. Various mathematical models to describe the behavior of the hysteretic component have been proposed in the literature $[e.g., 1,2,4-8, 11-15, 17-18]$. To-date, the hysteretic component of the structural response has not been incorporated in the formulation of the optimal control problem, although the hysteretic model has been used in the simulation of the controlled response of buildings equipped with aseismic hybrid control systems [28-29].

Based on the concept of instantaneous optimal control [19-22,28-29], the optimal control problem is formulated incorporating the specific hysteretic model in this report. This is accomplished by including the hysteretic component explicitly in the equations of motion, which are used as the constraints for the determination of the optimal control vector. For an n-degree-of-freedom system, the dimension of the state vector $Z(t)$ used in Refs. 28-29 is 2n. Including the hysteretic component explicitly in the present formulation, the dimension of the new state vector $\tilde{Z}(t)$ becomes 3n. The optimal control vector D(t) is obtained by minimizing a time dependent quadratic performance index [16,19-21] exactly in the same manner as that described in Refs. 19-22 and 28-29. In this report, we also prove that the optimal control vector satisfies not only the necessary conditions but also the sufficient condition of optimality. Since the control vector $U(t)$ is obtained as a linear function of the 3n state vector $\vec{Z}(t)$, it depends not only on the measured deformation and velocity but also on the hysteretic component of the structural response. The hysteretic component of the structural response can be estimated from an observer using the measured structural response and the specific hysteretic model used. A simple method of constructing the observer for the estimation of the hysteretic component is also presented.

Specific applications of the present control algorithm to two important types of aseismic hybrid control systems are demonstrated. These include (i) a frictional-type sliding isolation system connected to an actuator, and (ii) a lead-core rubber bearing isolation system connected to either an actuator or an active mass damper. These two types of aseismic hybrid systems are highly nonlinear and hysteretic in nature. Theoretically, the performance of the present optimal algorithm should be better than that proposed in Refs. 28-29, because more information is used in the determination of the control vector. Several examples have been worked out and the performance of the present optimal algorithm has been investigated, evaluated and compared numerically with that of the previous algorithm [Ref. 28-29]. It is shown numerically that the performance of the present optimal algorithm is indeed better than that of the previous one for the two types of aseismic hybrid control systems investigated herein.

SECTION 2

OPTIMAL CONTROL OF INELASTIC STRUCTURAL SYSTEMS

2.1 Nonlinear Hysteretric Model for Inelastic Systems

Various hysteretic models for the restoring force of an inelastic system have been developed in recent years [e.g. 1,2,4-8,11-15,17-18]. In this study, the differential equation model available in the literature [e.g., 1,2,11,15,18] will be used for both structures and passive protective systems. The stiffness restoring force for a structural member or a story unit, $F_s(x,t)$, is expressed as

$$
F_{\rm g}(x,t) = \alpha k x + (1-\alpha) k D_{\rm g} v \qquad (2.1)
$$

in which x=deformation, k=elastic stiffness, α = ratio of post-yielding to pre-yielding stiffness, D_v = yield deformation = constant, and v is a nondimensional variable introduced to describe the hysteretic component of the deformation, with $|v| \leq 1$, where

$$
D_{\mathbf{v}}\dot{\mathbf{v}} = A\dot{\mathbf{x}} - \beta |\dot{\mathbf{x}}| |\mathbf{v}|^{n-1} \mathbf{v} - \gamma \dot{\mathbf{x}} |\mathbf{v}|^{n}
$$
 (2.2)

In Eq. (2.2), parameters A, β and γ govern the scale and general shape of the hysteresis loop, whereas the smoothness of the force-deformation curve is determined by the parameter n. It is noticed that D_y is the measure of the yielding level if the hysteresis loop is not bilinear. In this case, however, D_v can be chosen as a large number such that $|v| \leq 1$ as will be described later.

The hysteretic model given by Eqs. (2.1) and (2.2) is phenomenological in nature; however, it does provide an explicit mathematical expression with many parameters flexible enough to reflect various hysteretic behaviors of inelastic systems. Likewise, Eq. (2.2) appears to be capable of describing the hysteretic behaviors of different base isolation systems, such as rubber bearings and sliding systems, which are the main concern of this study.

2.2 Equations of Motion

For simplicity, consider a base-isolated one-dimensional inelastic building

implemented by a hybrid control system as shown in Fig. 2.1(a). The structure system is idealized by an n-degree-of-freedom system and subjected to a one-dimensional earthquake ground acceleration $\ddot{X}_0(t)$. Let $x_i(t)$ be the relative displacement between the ith floor and the i-1th floor, i.e., the deformation of the ith story unit. The stiffness restoring force, $F_{si}(t)$, of the ith story unit is given by

$$
F_{s_i}(t) = \alpha_i k_i x_i + (1 - \alpha_i) k_i D_{y_i} v_i
$$
 (2.3)

in which the subscript i represents the quantity associated with the ith story unit and

$$
D_{y_i} \dot{v}_i = A_i \dot{x}_i - \beta_i |\dot{x}_i| |\dot{v}_i|^{n_i - 1} v_i - \gamma_i \dot{x}_i |\dot{v}_i|^{n_i}
$$
 (2.4)

The matrix equation of motion of the entire building system can be expressed as follows

$$
\underline{M}\ddot{\underline{X}}(t) + \underline{C}\dot{\underline{X}}(t) + \underline{K}_{\underline{y}}\underline{X}(t) + \underline{K}_{\underline{y}}\underline{V}(t) = H\underline{U}(t) + \underline{F}\ddot{\underline{X}}_0(t) \qquad (2.5)
$$

in which a linear viscous damping model has been assumed, $\underline{X}(t)=[x_1,x_2,\ldots,x_n]' =$ an n vector denoting the deformation of each story unit, and $\underline{V}(t)=[v_1, v_2, ..., v_n]' =$ an n vector denoting the hysteretic variable of each story unit. For the notations above, an under bar denotes a vector or a matrix and a prime indicates the transpose of a vector or a matrix.

In Eq. (2.5), M is an (nxn) mass matrix with the i-jth element $M(i,j) = m_i$ for $i=1,2,...,n$ and $j=1,2,...,i$ and $M(i,j)=0$ for $j>i$, where m_i is the mass of the ith floor, C is an (nxn) band-limited damping matrix with all elements equal to zero except $C(i,j) = c_i$ for $i = 1,2,...,n$ and $C(i,i+1) = -c_{i+1}$ for $i = 1,2,...,n-1$, where c_i is the damping coefficient of the ith story unit. K_e and K_1 are (nxn) band-limited elastic stiffness matrix and hysteretic stiffness matrix, respectively. All elements of \underline{K}_{e} and \underline{K}_{I} are zero except $K_e(i,i) = \alpha_i k_i, K_1(i,i) = (1-\alpha_i)k_i D_{vi}$ for $i = 1,2,...,n$ and $K_e(i,i+1) = -\alpha_{i+1} k_{i+1}, K_1(i,i+1) = -\alpha_{i+1} k_{i+1}$ $(1-\alpha_{i+1})k_{i+1}$ D_{yi+1} for i=1,2,...,n-1. H is an (nxr) matrix denoting the location of r controllers and E is an n vector denoting the influence of the earthquake ground acceleration. The matrices \underline{M} , \underline{C} , \underline{K}_e and \underline{K}_I described above hold when the base-isolated

(a)

(b)

 $II-5$

building is connected to an actuator as shown in Fig. 2.1(b). These matrices should be modified appropriately when the base-isolated building is connected to an active mass damper.

From Eq. (2.4), the vector \underline{V} appearing in Eq. (2.5) is given by

$$
\dot{\underline{V}} = f(\dot{\underline{X}}, \underline{V}) \tag{2.6}
$$

in which the ith element of $\dot{\underline{V}}$ or $\underline{f}(\dot{\underline{X}}, \underline{V})$, denoted by \dot{v}_i or $f_i(\dot{x}_i, v_i)$, is given by

$$
\dot{\nu}_i = f_i(\dot{x}_i, v_i) = D_{yi}^{-1} \left[A_i \dot{x}_i - \beta_i | \dot{x}_i | |v_i|^{n_i - 1} v_i - \gamma_i \dot{x}_i |v_i|^{n_i} \right]
$$
(2.7)

By introducing a 3n state vector $\tilde{Z}(t)$,

$$
\tilde{Z}(t) = \begin{bmatrix} X \\ Y \\ \dot{X} \end{bmatrix}
$$
 (2.8)

the second-order nonlinear matrix equation of motion, Eq. (2.5), can be converted into a first order matrix equation as follows:

$$
\dot{\tilde{Z}}(t) = g[\tilde{Z}(t)] + \underline{B}U(t) + W_1 \ddot{X}_0(t) \qquad (2.9)
$$

in which \underline{B} and \underline{W}_1 are 3n vectors

$$
\boldsymbol{B} = \begin{bmatrix} \mathbf{Q} \\ \mathbf{Q} \\ \mathbf{M}^{-1} \mathbf{H} \end{bmatrix} \qquad ; \qquad \boldsymbol{W}_{1} = \begin{bmatrix} \mathbf{Q} \\ \mathbf{Q} \\ \mathbf{M}^{-1} \mathbf{F} \end{bmatrix} \tag{2.10}
$$

and $g[\tilde{Z}(t)]$ is a 3n vector consisting of nonlinear functions of components of the vector $\tilde{Z}(t)$,

g[Z(t)] ⁼ I(X,Y) (2.11)

The first-order nonlinear matrix equation, Eq. (2.9), can be solved step-by-step

numerically using the Fourth-Order Runge-Kutta method as follows [Ref.29]:

$$
\tilde{Z}(t) = \tilde{Z}(t - 2\Delta t) + \frac{1}{6} [A_0 + 2A_1 + 2A_2 + A_3]
$$
 (2.12)

in which Δt is the integration time step, and \underline{A}_0 , \underline{A}_1 , \underline{A}_2 , and \underline{A}_3 are 3n vectors

$$
A_0 = 2\Delta t \{g[\tilde{Z}(t - 2\Delta t)] + \underline{B} U(t - 2\Delta t) + \underline{W}_1 \ddot{X}_0 (t - 2\Delta t)\}
$$

\n
$$
A_1 = 2\Delta t \{g[\tilde{Z}(t - 2\Delta t) + 0.5\hat{A}_0] + \underline{B} U(t - \Delta t) + \underline{W}_1 \ddot{X}_0 (t - \Delta t)\}
$$

\n
$$
A_2 = 2\Delta t \{g[\tilde{Z}(t - 2\Delta t) + 0.5\hat{A}_1] + \underline{B} U(t - \Delta t) + \underline{W}_1 \ddot{X}_0 (t - \Delta t)\}
$$

\n
$$
A_3 = 2\Delta t \{g[\tilde{Z}(t - 2\Delta t) + \underline{A}_2] + \underline{B} U(t) + \underline{W}_1 \ddot{X}_0 (t)\}
$$
\n(2.13)

As observed from Eq. (2.13), \underline{A}_0 , \underline{A}_1 , \underline{A}_2 , \underline{A}_3 are functions of t-2 Δt , t- Δt and t. For simplicity of presentation, the arguments, t-2 Δt , t- Δt and t, have been omitted.

To simplify the mathematical operation, it is desirable to group all vectors in Eq. (2.12), which are functions of $\tau <$ t, into one term, denoted by \underline{D} (t-2 Δt , t- Δt), as follows:

$$
\underline{D}(t - 2\Delta t, t - \Delta t) = \tilde{Z}(t - 2\Delta t) + \frac{1}{3}\Delta t \{g[\tilde{Z}(t - 2\Delta t)] + B\underline{U}(t - 2\Delta t) + \underline{W}_1 \tilde{X}_0(t - 2\Delta t) + 2g[\tilde{Z}(t - 2\Delta t) + 0.5\underline{A}_0] + 4\underline{B}\underline{U}(t - \Delta t)
$$
\n
$$
+ 4\underline{W}_1 \tilde{X}_0(t - \Delta t) + 2g[\tilde{Z}(t - 2\Delta t) + 0.5\underline{A}_1] + g[\tilde{Z}(t - 2\Delta t) + \underline{A}_2]
$$
\n(2.14)

Then, Eq. (2.12) can be expressed as

$$
\tilde{Z}(t) = D(t - 2\Delta t, t - \Delta t) + \frac{\Delta t}{3} [B U(t) + W_1 \ddot{X}_0(t)] \qquad (2.15)
$$

It is mentioned that for the regular Fourth-Order Runge-Kutta method, the integration time step is $\Delta \tau$ that is twice of Δt used in Eqs. (2.12)-(2.15), i.e., $\Delta \tau = 2\Delta t$ or $\Delta t = \Delta \tau/2$. In other words, the integration step size Δt used in the current numerical scheme is one-half of that used in the regular Fourth-Order Runge-Kutta method.

As observed from Eqs. (2.6), (2.7) and (2.11), the g vector is a function of \underline{X} , \underline{V} and $\dot{\underline{X}}$ which are elements of the vector $\tilde{\underline{Z}}$. Thus, the g vector can be expressed as a function of the response state vector \tilde{Z} , i.e., $g = g(\tilde{Z})$. To evaluate $g(\tilde{Z})$, the argument vector $\tilde{Z} = [\,\underline{X}', \underline{V}', \underline{X}']'$ is computed first and then its components, $\underline{X}, \underline{V}$ and \underline{X} , are substituted into the right hand side of Eq. (2.11). For the $g[\tilde{Z}(t-2\Delta t) + 0.5 \Delta_1]$ vector appearing in Eq. (2.14), the argument vector (3n-dimensional), denoted by S ,

$$
\underline{S} = \underline{\tilde{Z}}(t - 2\Delta t) + 0.5\underline{A}_1 \qquad (2.16)
$$

is computed first from Eq. (2.16) . Then, S is partitioned in the same form as the vector \tilde{Z} , i.e., $S = [S_1', S_2', S_3']'$, where S_1, S_2 and S_3 are n-dimensional vectors. Finally, the g[\tilde{Z} (t-2 Δt) + 0.5 \underline{A}_1] vector is obtained from Eq. (2.11) by replacing \underline{X} , \underline{V} and \underline{X} , respectively, by S_1 , S_2 and S_3 . Likewise, the other two vectors $g[\tilde{Z}(t-2\Delta t) + 0.5 \Delta_0]$ and $g[\tilde{Z}(t-2\Delta t) + A_2]$ appearing in Eq. (2.14) are similarily defined.

2.3 Instantaneous Optimal Control

The instantaneous optimal control algorithm proposed by Yang, et al for linear structures [19-20] was extended later to nonlinear structures [21,28-29]. It is used herein for applications to control of inelastic systems.

The time dependent quadratic objective function J(t) is given as follows

$$
J(t) = \underline{\tilde{Z}}'(t) Q \tilde{Z}(t) + \underline{U}'(t) R \underline{U}(t)
$$
 (2.17)

in which Q and R are (3nx3n) and (rxr) weighting matrices, respectively, representing the relative importance of the response vector $\tilde{Z}(t)$ and the control vector $U(t)$. The optimal control vector obtained by minimizing the time-dependent objective function, J(t), is referred to as the instantaneous optimal control algorithm [e.g., 16,19-22].

To minimize the objective function, Eq. (2.17), subjected to the constraint given by Eq. (2.15), the Hamiltonian H is constructed by introducing a 3n-dimensional Lagrangian multiplier vector $\lambda(t)$

$$
H = \underline{\tilde{Z}}'(t) \underline{Q} \underline{\tilde{Z}}(t) + \underline{U}'(t) \underline{R} \underline{U}(t) + \underline{\lambda}'(t) \{\underline{\tilde{Z}}(t) - \underline{D}(t - 2\Delta t, t - \Delta t) - \frac{\Delta t}{3} [\underline{B} \underline{U}(t) + \underline{W}_1 \underline{\tilde{X}}_0(t)] \}
$$
(2.18)

The necessary conditions for minimizing $J(t)$, subjected to the constraint of Eq. (2.15), are

$$
\frac{\partial H}{\partial \underline{\tilde{Z}}}=0, \qquad \frac{\partial H}{\partial \underline{U}}=0, \qquad \frac{\partial H}{\partial \underline{\lambda}}=0 \qquad (2.19)
$$

Substituting Eq. (2.18) into Eq. (2.19) , one obtains the optimal control vector U(t) in the following manner: (i) when the control vector $\underline{U}(t)$ is assumed to be regulated by the response state vector $\tilde{Z}(t)$, the instantaneous optimal closed-loop (feedback) control algorithm is obtained

$$
\underline{U}(t) = -\frac{\Delta t}{3} R^{-1} \underline{B}' Q \underline{\tilde{Z}}(t) \qquad (2.20)
$$

(ii) when the control vector $U(t)$ is regulated by the measurement of the earthquake ground acceleration $\ddot{X}_0(t)$ without a feedback response state vector $\tilde{Z}(t)$, the instantaneous optimal open-loop (feedforward) control algorithm is obtained, and (iii) the instantaneous optimal closed-open-loop (feedback and feedforward) control algorithm is obtained by assuming that the control vector $U(t)$ is regulated by the measurements of both the response state vector $\tilde{Z}(t)$ and the earthquake ground acceleration $\ddot{X}_0(t)$ [see Ref. 29]. Although the control operations for the three optimal algorithms are different, the resulting state vector $\tilde{Z}(t)$ and control vector $\underline{U}(t)$ remain the same under ideal control environments.

With the control vector given by Eq. (2.20), the response state vector $\tilde{Z}(t)$ is obtained by substituting Eq. (2.20) into Eq. (2.15) as follows

$$
\tilde{Z}(t) = \left[I + \left(\frac{\Delta t}{3}\right)^2 BR^{-1}B'Q\right]^{-1} \left[D(t - 2\Delta t, t - \Delta t) + \frac{\Delta t}{3}W_t \ddot{X}_0(t)\right] (2.21)
$$

in which \underline{I} is a (3nx3n) identity matrix.

2.4 Sufficient Condition For Optimal Control

 $\hat{\boldsymbol{\theta}}$

In the original derivations for the instantaneous optimal control algorithms, such as Refs. 19-21,28 and 29, the optimal control vector $U(t)$ was obtained from the necessary conditions, such as Eq. (2.19), i.e., $\partial H/\partial \tilde{Z}=0$, $\partial H/\partial \underline{U}=0$ and $\partial H/\partial \underline{\lambda}=0$. In fact, the derived optimal control vector $U(t)$ also satisfies the sufficient condition of optimality as will be proved in the following.

The sufficient condition for the optimal solution is given by [Ref. 31]

$$
\left[\begin{array}{cc} \partial \tilde{Z}', \ \partial \underline{U}' \end{array}\right] \left[\begin{array}{ccc} \frac{\partial^2 H}{\partial \tilde{Z}^2} & \frac{\partial^2 H}{\partial \tilde{Z} \partial \underline{U}} \\ \frac{\partial^2 H}{\partial \underline{U} \partial \tilde{Z}} & \frac{\partial^2 H}{\partial \underline{U}^2} \end{array}\right] \left[\begin{array}{cc} \partial \tilde{Z} \\ \partial \underline{U} \end{array}\right] > 0 \tag{2.22}
$$

Taking derivatives of H in Eq. (2.18), one obtains

$$
\frac{\partial H}{\partial \tilde{Z}} = 2Q\tilde{Z} - \lambda ; \quad \frac{\partial H}{\partial U} = 2RU - \frac{\Delta t}{3} B'\lambda
$$
\n
$$
\frac{\partial^2 H}{\partial \tilde{Z}^2} = 2Q ; \quad \frac{\partial^2 H}{\partial U^2} = 2R ; \quad \frac{\partial^2 H}{\partial \tilde{Z}\partial U} = \frac{\partial^2 H}{\partial U \partial \tilde{Z}} = 0
$$
\n(2.23)

Substitution of Eq. (2.23) into the left hand side of Eq. (2.22) leads to the following expression

$$
2\left[\begin{array}{cc} \partial \tilde{Z}' \end{array}, \frac{\partial U'}{\partial} \right] \left[\begin{array}{cc} Q & Q \\ 0 & R \end{array}\right] \left[\begin{array}{cc} \partial \tilde{Z} \\ \partial U \end{array}\right] = 2 \left(\frac{\partial Z'}{Q} \frac{\partial Z'}{Q} + \frac{\partial U'}{R} \frac{\partial U}{\partial U}\right) > 0 \qquad (2.24)
$$

Since Q is a positive semi-definite matrix and R is a positive definite matrix, Eq. (2.24) is greater than zero. Thus, the sufficient condition for the optimal solution, Eq. (2.22), is satisfied.

2.5 Estimation of Hysteretic Component Vector \underline{V}

The optimal control vector $\underline{U}(t)$ given by Eq. (2.20) is regulated by the 3n response state vector $\tilde{Z}(t) = [\underline{X}'(t), \underline{Y}'(t), \dot{X}'(t)]'$. The displacement and velocity response vectors, $\underline{X}(t)$ and $\underline{X}(t)$, are available from measurements using sensors. However, the hysteretic component vector $\underline{V}(t)=[v_1(t),v_2(t),...,v_n(t)]'$ is not measurable and hence it should be estimated. An approach to estimate $v_i(t)$ for $i=1,2,...,n$ is by use of a reduced order nonlinear state observer. **In** the present case, the observer can be constructed easily, since $v_i(t)$ is only a function of the interstory velocity $\dot{x}_i(t)$ as shown in Eq. (2.7). As a result, the estimate of the hysteretic component $v_i(t)$, denoted by $\hat{v}_i(t)$, follows from Eq. (2.7) as

$$
\dot{\hat{v}}_i(t) = f_i(\dot{x}_i, \hat{v}_i) = D_{yi}^{-1} \left[A_i \dot{x}_i - \beta_i \mid \dot{x}_i \mid \theta_i \mid \hat{v}_i - \gamma_i \dot{x}_i \mid \hat{v}_i \mid \big|^{n_i} \right]
$$
(2.25)

in which the argument t for $\dot{x}_i(t)$ and $\dot{v}_i(t)$ has been omitted for simplicity and $\dot{x}_i(t)$ is obtained from measurements.

Given the measured $\dot{x}_i(t)$, Eq. (2.25) can be integrated on-line to obtain the estimate $\hat{v}_i(t)$ for $v_i(t)$ without any difficulty. It is mentioned that when the response level is within the elastic limit, the hysteretic component v_i is very closed to the interstory deformation x_i , Eq. (2.1). As in the case of any observer, the accuracy of the estimates $\hat{v}_i(t)$ for $i = 1, 2, ..., n$, depends on the accuracy of the hysteretic model representing the behavior of the system as well as the corresponding parameters, such as A_i , β_i , γ_i and n_i . This subject, however, is beyond the scope of this report.

SECTION 3 NUMERICAL EXAMPLE

To illustrate the application of the optimal control algorithm developed in this report for inelastic structural systems, several examples are presented in this section. Emphasis is placed on aseismic hybrid control systems. The same eight-story building that exhibits binlinear elasto-plastic behavior used in Refs. 28 and 29 is considered. The stiffness of each story unit is designed such that yielding occurs simultaneously for each story unit. The properties of the structure are as follows: (i) the mass of each floor is identical with $m_i = m = 345.6$ metric tons; (ii) the preyielding stiffnesses of the eightstory units are k_{11} (i=1.2, ...,8) = 3.4x10⁵, 3.26x10⁵, 2.85x10⁵, 2.69x10⁵, 2.43x10⁵, 2.07x10⁵, 1.69x10⁵ and 1.37x10⁵ kN/m, respectively, and the postyielding stiffnesses are $k_{i2} = 0.1$ k_{i1} for $i=1,2,...,8$; and (iii) the viscous damping coefficients for each story unit are $c_i = 490, 467, 410, 386, 348, 298, 243$ and 196 kN.sec/m, respectively. The damping coefficients given above result in a classically damped structure with a damping ratio of 0.38% for the first vibrational mode. The natural frequencies of the unyielded structure are 5.24, 14.0, 22.55, 30.22, 36.89, 43.06, 49.54 and 55.96 rad./sec. The yielding level for each story unit varies with respect to the stiffness; with the results, $D_{yi} = 2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.7,$ and 1.5 cm. The bilinear elastoplastic behavior can be described by the hysteretic model, Eqs. (2.3) and (2.4), with $A_i=1.0$, $\beta_i=0.5$, $n_i=95$ and $\gamma_i=0.5$ for $i=1.2,\dots,8$. A simulated earthquake with a maximum ground acceleration of 0.3g as shown in Fig. 3.1 is used as the input excitation.

With the eight-story bilinear elasto-plastic building structure described above and the earthquake ground acceleration shown in Fig. 3.1, time histories of all the response quantities have beeri computed. Within 30 seconds of the earthquake episode, the maximum interstory deformation, x_{mi} , and the maximum absolute acceleration of each floor, a_i , are shown in Table 3.1. Other response quantities have been presented in Ref. 29. As observed from Table 3.1, the deformation of the unprotected building is excessive and that yielding takes place in each story unit.

A Simulated Earthquake Ground Acceleration FIGURE 3.1 :

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TABLE 3.1: Maximum Response Quantities of Building Equipped with Base Isolation System and Active Mass Damper (BIS & AMD) : TABLE 3.1: Maximum Response Quantities of Building Equipped with Base Isolation System and Active Mass Damper (BIS & AMD) \bar{x}_d =maximum relative displacement of the mass damper; x_{mi} =maximum deformation of ith story unit; a=maximum \bar{x} d=maximum relative displacement of the mass damper; x_{min} =maximum deformation of *ith* story unit; a =maximum absolute acceleration of ith floor; U_m =maximum active control force absolute acceleration of *ith* floor; Um=maximum active control force

Limited No. of Sensors BIS & AMD	$U_m = 196kN$ $\widehat{\mathbf{e}}$	$\frac{\text{cm/s}}{s^2}$ ä	$\frac{8}{18}$	111	108	106	56	$\frac{8}{9}$	106	117	130
	\bar{x} _d =110cm	ϵ m \bar{x}_{mi}	23.3	0.66	0.65	0.68	0.64	0.59	0.54	0.49	0.33
		(cm/s^2) ä	118	112	109	105	$\frac{4}{3}$	$\overline{101}$	$\frac{11}{11}$	119	142
Full Sensors BIS & AMD	$U_m = 234.4kN$ \bar{x}_{d} =136cm ઉ	(cm) x_{mi}	24.21	0.69	0.69	0.72	0.68	0.63	0.59	0.53	0.36
	U_{m} =201.5kN	$\left(\frac{cm}{s^2}\right)$ äj	117	112	109	107	$\frac{8}{2}$	$\mathbf{6}$	106	116	$\frac{13}{1}$
	\bar{x} _d =110cm $\widehat{\mathbf{B}}$	$\binom{cm}{ }$ x_{mi}	23.2	0.67	0.66	0.69	0.64	0.60	0.54	0.49	0.33
	$U_m = 225.5kN$	$\left(\frac{cm}{s^2}\right)$ ä	118	112	108	105	$\frac{4}{3}$	102	$\frac{11}{11}$	121	141
	\bar{x} _d =135cm $\widehat{\mathcal{E}}$	(cm) x_{mi}	24.80	0.71	0.70	0.73	0.69	0.63	0.59	0.53	0.36
		$\left(\frac{cm}{s^2}\right)$ ä	158	155	152	157	164	177	185	198	206
with BIS		(cm) x_{mi}	42.70	1.31	1.25	1.28	1.17	1.07	0.97	0.82	0.52
		m/s^2 ä ୍ର		486	456	571	529	558	720	580	617
without control		$\overline{\text{cm}}$ x_{mi}		3.05	2.80	3.14	2.95	2.70	2.91	2.90	2.08
Yield Displ	تحط	cm)		2.4	2.3	2.2	2.1	2.0	$\frac{9}{1}$	1.7	1.5
zo \circ \approx \overline{a}			ρη		Ν	ω	4	5	৩	ᠵ	∞

⁽A) = without \underline{V} feedback; (B) = with \underline{V} feedback (A) = without V feedback; (B) = with V feedback

Example 1: Building Equipped With Rubber-Bearing Base Isolation System and Active Mass Damper

To reduce the structural response, a lead-core rubber-bearing isolation system connected to an active mass damper is implemented as shown in Fig. 3.2(a). The restoring force of the lead-core rubber-bearing system is modeled as

$$
F_{sb} = \alpha_b k_b x_b + (1 - \alpha_b) k_b D_{vb} v_b \tag{3.1}
$$

in which $\nu_{\rm b}$ is given by Eq. (2.4) with i=b. The mass of the base isolation system is $m_b = 450$ metric tons and the viscous damping coefficient is assumed to be linear with c_b =26.17 kN.sec/m. The restoring force of the base isolation system given by Eq. (3.1) is not bilinear elasto-plastic and the parameter values are given as follows: $k_b = 18050$ kN/m, $\alpha_b = 0.6$ and D_{yb} =4cm. The parameters governing the scale and general shape of the hysteresis loop of the isolation system, Eq. (2.4), were assumed to be $A_b = 1.0$, β_b =0.5, n_b=3 and γ_b =0.5. The hysteretic characteristics of such a base isolation system, i.e., x_b versus v_b , are shown in Fig. 3.3(a). For the building with the base isolation system without an active mass damper, the 9 natural frequencies of the preyielding structure are 2.21, 9.31, 17.29, 25.18, 32.19, 38.29, 44.12, 50.37, and 56.74 rad/sec. The damping ratio for the first vibrational mode is 0.15%. It is observed that the fundamental frequency is reduced significantly by the implementation of the base isolation system. The response vector $\underline{X}(t)$ is given by $\underline{X}=[x_b,x_1,\ldots,x_8]'$.

Without an active mass damper, time histories of all the response quantities were computed and presented in Ref. 29. The maximum response quantities of the structure in 30 seconds of the earthquake episode are shown in the column of Table 3.1 designated as "With BIS". As observed from Table 3.1, the interstory deformation and the floor acceleration are drastically reduced. However, the deformation of the base isolation system shown in row B of Table 3.1 may be excessive.

To protect the safety and integrity of the base isolation system, an active mass damper was proposed in Refs. 23-24 to be connected to the base isolation system as shown in Fig. 3.1(a). The properties of the mass damper are as follows. The mass of

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the mass damper, m_d , is equal to 50% of the floor mass, i.e., $m_d = 0.5m_i$ and the natural frequency of the mass damper is the same as the first natural frequency of the base isolated building, i.e. 2.21 rad/sec. The damping ratio of the mass damper is 10%. With the active mass damper, the response vector is given by $\underline{X}(t) = [x_d, x_b, x_1, \dots, x_8]'$ and the structural response depends on the weighting matrices \underline{R} and \underline{Q} . For this example, the weighting matrix **R** consists of only one element, denoted by R_0 , whereas the dimension of the Q matrix is (30x30). R_0 is chosen to be 10⁻³ for simplicity. The Q matrix is partitioned as follows

Q ⁼ *a* Q Q Q **(3.2)**

in which Q is a (20x10) matrix with all elements equal to zero, and Q_{21} , Q_{22} and Q_{23} are $(10x10)$ matrices.

Note that Q is a symmetric matrix. Because of the fact that there is only one actuator installed on the basement floor as shown in Fig. $3.2(a)$, only the elements in one row of the Q matrix are relevant to the control vector $U(t)$, see Eq. (2.20). In the present case, only the elements in the first row of Q_{21} , Q_{22} and Q_{23} matrices are related to the control vector $U(t)$. As a result, all elements of the Q matrix which have no effect on the control vector $U(t)$ are set to be zero for convenience.

It is mentioned that matrices Q_{21} , Q_{22} and Q_{23} are associated with the feedbacks of $\underline{X}(t)$, $\underline{V}(t)$ and $\underline{X}(t)$, respectively. For instance, setting all the elements of Q_{21} equal to zero, the control vector $\underline{U}(t)$ given by Eq. (2.20) is no longer a function of $\underline{X}(t)$. Hence, the main difference between the current control algorithm and that presented in Refs. 28 and 29 lies in the matrix Q_{22} . If all elements of Q_{22} are zero, there is no feedback of $Y(t)$ and the results will be identical to those presented in Refs. 28 and 29.

Consider the case in which there is no feedback of $\underline{V}(t)$, i.e., all elements of \underline{Q}_{22} are zero. Since there is only one actuator, all the elements of Q_{21} and Q_{23} are set to be zero, except $Q_{21}(1,j)$ and $Q_{23}(1,j)$ for $j=1,2,\ldots,10$. For illustrative purpose, the following values are assigned.

$$
Q_{21}(1,j) = \begin{bmatrix} 102 & -2362 & -11 & -840 & -1397 & -1649 & -1645 & -1433 & -1057 & -561 \end{bmatrix} (3.3)
$$

\n
$$
Q_{22}(1,j) = \begin{bmatrix} 67 & 731 & 646 & 621 & 584 & 529 & 457 & 366 & 257 & 133 \end{bmatrix} (3.3)
$$

and $Q_{21}(i,j)=Q_{23}(i,j)=0$ for $i=2,3,...,10$ and $j=1,2,...,10$.

Time histories of all the structural response quantities have been computed for α/R_0 =50000 and Δt =0.375x10⁻². Within 30 seconds of the earthquake episode, the maximum interstory deformations, x_{mi} (i=B,1,2,...,8), the maximum acceleration of each floor, a_i , the maximum active control force, U_m , and the maximum relative displacement of the mass damper, \bar{x}_d , are summarized in column (A) of Table 3.1 designated as "Full sensors". Time histories of many response quantities were plotted and presented in Ref. 29.

We consider now the case in which the hysteretic component $Y(t)$ is also a feedback to the control vector $U(t)$. In this case, elements of $Q_{21}(1,j)$, $Q_{22}(i,j)$ and $Q_{23}(1,j)$ are assigned in the following for illustrative purposes

$$
Q_{21}(1,j) = [102 -3363 -11 840 1397 1649 1645 1433 1057 561]
$$

\n
$$
Q_{22}(1,j) = [0, -100, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$$

\n
$$
Q_{23}(1,j) = [67 931 646 621 584 529 457 366 257 133]
$$

and all other elements are zero. Eq. (3.4) indicates that only the hysteretic component of the base isolation system is used as a feedback.

Within 30 seconds of the earthquake episode, all the maximum response quantities are summarized in column (B) of Table 3.1 designated as "Full sensors" for comparison. It is observed from Table 3.1 that (i) the performance of the present optimal algorithm with an additional feedback of the hysteretic component is slightly better than that of the optimal algorithm without a feedback of the hysteretic component, and (ii) the aseismic hybrid control system using a lead-core rubber bearing isolation system and an active mass damper is very effective in protecting building structures against strong earthquakes.

The base-isolated building tends to behave like a rigid body and the interstory deformation is small compared with that of the base isolation system. As a result,

sensors may not be needed for the building. Consider the case in which displacement and velocity sensors are installed on the active mass damper and the base isolation system only, i.e., no sensor is installed on the building. For the first case in which the hysteretic component $Y(t)$ is not accounted for in the determination of $U(t)$, one has $Q_{22} = Q$. Further, all elements of Q_{21} and Q_{23} are zero, except $Q_{21}(1,1)$, $Q_{21}(1,2)$, $Q_{23}(1, 1)$ and $Q_{23}(1, 2)$. For simplicity, the following values are assigned; $Q_{21}(1, 1) = 102$, $Q_{21}(1,2) = -2363$, $Q_{23}(1,1) = 67$ and $Q_{23}(1,2) = 731$. The maximum response quantities for α/R_0 =50000 and Δt =0.375x10⁻² sec. were computed and summarized in column (A) of Table 3.1 under the heading of limited number of sensors.

We next consider the case in which the hysteretic component $\underline{V}(t)$ is used for the determination of $\underline{U}(t)$. In this case, the following elements are assigned: $Q_{21}(1,1)=102$, $Q_{21}(1,2) = -3363$, $Q_{22}(1,1) = 0$, $Q_{22}(1,2) = -100$, $Q_{23}(1,1) = 67$, and $Q_{23}(1,2) = 931$. All other elements of the Q matrix are zero. The maximum response quantities for α/R_0 =50,000 and Δt =0.375x10⁻² sec. are summarized in the last two columns of Table 3.1 designated as "limited number of sensors. "

A comparison of the results for the case using a limited number of sensors with those of the case in which sensors are installed on all floors of the building indicates that installations of displacement and velocity sensors on the building are not necessary. This conclusion is very important because it is expensive to install sensors. It is further observed from the last four columns of Table 3.1 that the performance of the present control algorithm is slightly better than that of the previous algorithm without considering the hysteretic component $\underline{V}(t)$ as a feedback.

Example 2: Building Equipped with Rubber Bearing Isolation System and Actuator

Instead of using an active mass damper described above, the base isolation system can be connected directly to an actuator. With such a hybrid control system, the $(27x27)$ Q matrix is partitioned as shown in Eq. (3.1), where Q_{21} , Q_{22} and Q_{23} are (9x9) matrices. From the conclusions above, it is not necessary to install sensors on the baseisolated building. Hence, we consider only the case in which displacement and velocity sensors are installed on the base isolation system only, i.e., no sensor is installed on the building. When $\underline{V}(t)$ is not considered as a feedback for the determination of $\underline{U}(t)$, all elements of Q_{21} , Q_{22} and Q_{23} are zero, except $Q_{21}(1,1)$ and $Q_{23}(1,1)$. For simplicity, the following values are assigned: $Q_{21}(1,1)=0.9$ and $Q_{23}(1,1)=3.4$. The maximum response quantities for $\alpha/R_0 = 1.55 \times 10^8$ and $\Delta t = 0.375 \times 10^{-2}$ sec. are summarized in the columns designated as "BIS&AF¹" of Table 3.2 [Ref. 29].

When $Y(t)$ is used in the determination of $U(t)$, one additional non-zero element from the Q_{22} matrix is assigned. For illustrative purposes, the following values are assigned: $Q_{21}(1,1)=0.1$, $Q_{22}(1,1)=0.5$ and $Q_{23}(1,1)=3.5$, and all other elements of the Q matrix are zero. The maximum response quantities in 30 seconds of the earthquake episode are presented in the columns designated as "BIS&A $F²$ " of Table 3.2 for comparison. It is observed from Table 3.2 that the performance of the present control algorithm is slightly better than that of the previous algorithm without considering the feedback of $\underline{V}(t)$.

Example 3: Building Equipped With Sliding Base Isolation System and Actuator

Instead of using a rubber-bearing base isolation system, a frictional-type sliding base isolation system, as shown schematically in Fig. 3.2(b), is considered for the same eight-story bilinear elasto-plastic building described in Example 1. This type of isolators allows greater resistance to damage by permitting the structure to slide on its foundation during severe earthquakes. This isolation system decouples the structure from its foundation with nearly frictionless teflon on stainless steel sliding plates that have very low frictional resistance.

When the sliding system slides, the frictional force, F_{sb} , developed in the sliding system is given by

$$
F_{sb} = \mu \bar{m}g \; sgn(\dot{x}) \tag{3.5}
$$

in which $mg=w$ is the weight of the structural system above the sliding bearings and μ

BIS = Base Isolation System; AF^1 = Active Force without Y feedback
 AF^2 = Active Force with Y feedback BIS = Base Isolation System; $AF¹ = Active Force without V feedback$ $AF^2 = Accive$ Force with V feedback

is the coefficient of friction. When the sliding system sticks, the frictional force F_{sb} is smaller than the one given by Eq. (3.5) . In the dynamic analysis and active control of the sliding system, the highly nonlinear frictional force is represented in this report by the following analytical function

$$
F_{sb} = \mu \bar{m} g v_b \tag{3.6}
$$

in which v_b is a nondimensional hysteretic quantity described by Eq. (2.4) with i=b. It is mentioned that during the sliding phase, v_b takes a value of either 1 or -1. During the sticking phase, the absolute value of v_b is less than unity, i.e., $|v_b| \le 1$. The conditions of sticking and sliding are accounted for by Eq. (2.4) automatically. The parameters governing the scale and general shape of the hysteresis loop of the sliding system are $A_b = 1.0$, $\beta_b = 0.5$, $n_b = 2$, $\gamma_b = 0.5$ and $D_{yb} = 0.012$ cm. The hysteresis loop of the hysteretic component of such a sliding system is schematically shown in Fig. 3.3(b). The mass of the sliding system is $m_b=450$ metric tons and the coefficient of friction is $\mu=10\%$. The response vector is given by $\underline{X}=[x_b, x_1, \ldots, x_8]'$.

Time histories of all the response quantities were computed. The time history, $x_1(t)$, of the first story deformation and that of the sliding system, $x_b(t)$, were presented in Ref. 29, where $x_b(t)$ denotes the relative displacement of the teflon and the stainless steel plate. Further, the hysetersis loops for the shear force of the sliding system, the first, fourth, and eighth story units were also given in Ref. 29. The maximum response quantities of the structure within 30 seconds of the earthquake episode are shown in Table 3.3 in the columns designated as "With SIS". As observed from Table 3.3, the interstory deformations and floor accelerations are drastically reduced using a sliding type base isolation system. However, the deformation of the eighth story unit is still in the inelastic range. The time history of the deformation of the first story unit, $x_1(t)$, without the base sliding system is shown in Fig. 3.4(a). The corresponding result when the building is equipped with a base sliding system alone is presented in Fig. 3.4(b).

FIGURE 3.4 :

Deformation of First-Story Unit : (a) Without Control; (b) With Sliding Isolation System; (c) With Sliding Isolation System and Actuator without V feedback; (d) With Sliding Isolation System and Actuator with V feedback

TABLE 3.3: Maximum Response Quantities of Building Equipped with Sliding Isolation System and
Actuator (SIS & AF): x_{mi} =maximum deformation of ith story unit; a_i =maximum TABLE 3.3: Maximum Response Quantities of Building Equipped with Sliding Isolation System and Actuator (SIS & AF) : $x_{mi}=maximm$ deformation of *ith* story unit; a_i=maximum absolute acceleration of ith floor; U_m =maximum active control force absolute acceleration of *ith* floor; Um=maximum active control force

	$\left(\frac{\text{cm}}{\text{s}^2}\right)$ \ddot{a}	380.	214	239	230	217	242	298	218	344
$U_m = 2321.3kN$ $\widehat{\mathbf{B}}$	(cm) x_{mi}	44.86	0.50	0.50	0.56	0.68	0.78	0.74	0.93	0.87
	$\frac{\text{cm}}{\text{s}^2}$ \ddot{a}	688	325	342	379	300	379	412	341	391
$U_m = 3074.7kN$ \mathfrak{A}	(cm) x_{mi}	18.13	0.56	0.76	0.91	0.83	0.89	1.00	1.09	0.99
	$\left(\text{cm/s}^2\right)$ \ddot{a}	658	370	547	531	404	424	462	541	601
with SIS	(cm) x_{mi}	11.54	1.30	1.23	1.38	1.40	1.50	1.72	1.82	1.64
	$\frac{\text{cm}}{\text{s}^2}$ ä	ı	486	456	571	529	558	720	580	617
without control	(cm) x_{mi}		3.05	2.80	3.14	2.95	2.70	2.91	2.90	2.08
Yield Displ	${\rm D_{yi}}$ $\binom{cm}{ }$	ı	2.4	2.3	2.2	$\overline{2.1}$	2.0	1.9	1.7	
100K	z0	$\mathbf{\Xi}$		\mathbf{c}	\mathfrak{c}	4	5	\circ		∞

(A) = without Y feedback; (B) = with Y feedback (A) = without \overline{Y} feedback; (B) = with \overline{Y} feedback

To further reduce the structural response and to bring the deformation of the 8th story unit into the elastic range, an actuator is connected to the sliding system as shown in Fig. 3.2(b). The actuator will apply the active control force directly on the sliding system. The basic idea of the active force, in this case, is to counteract the frictional force in order to maintain the system in the sliding condition as much as possible, so that the transmission of the earthquake ground motion to the building can be kept to a minimum.

With the active control force, the structural response depends on the weighting matrices \underline{R} and \underline{Q} . Again, the weighting matrix \underline{R} consists of only one element, denoted by R₀, whereas the dimension of the Q matrix is (27x27). R₀ is chosen to be 10⁻³ for simplicity and the Q matrix is partitioned as shown in Eq. (3.2), where Q_{21} , Q_{22} and Q_{23} are (9x9) matrices and Q is a (18x9) zero matrix.

First, consider the case in which the hysteretic component vector \underline{V} is not a feedback quantity for the determination of the control vector $U(t)$. In this case, all the elements of the Q_{22} matrix are zero, i.e., $Q_{22}=Q$. Note that the deformation of the building is in-phase with the frictional force and the velocity of the building is out of phase of the frictional force. As a result, we set $Q_{23} = Q$. As described in Part I, for the base-isolated building connected to an actuator, the control force $\underline{U}(t)$ depends on the difference between the first two rows of the Q_{21} and Q_{22} matrices, i.e., $Q_{21}(1,j)-Q_{21}(2,j)$ and $Q_{22}(1,j)-Q_{22}(2,j)$. For simplicity, elements of $Q_{21}(2,j)$ and $Q_{22}(2,j)$ rows will be assigned to be zero. The elements of $Q_{21}(1,j)$ are assigned in the following for illustrative purposes:

$$
Q_{21}(1,j) = [0, -30000, -3000, -300, -30, -3, -0.3, -0.03, -0.003]
$$
 (3.7)

All other elements of the Q_{21} matrix are set to be zero.

Time histories of all the structural response quantities were computed for α/R_0 =2.25x10⁸ and Δt =0.03x10⁻² sec. Time histories of the deformations of the sliding system and the first story unit as well as that of the required active control force were shown in Ref. 29. Hysteresis loops for the sliding system were also plotted in Ref. 29. Within 30 seconds of the earthquake episode, the maximum interstory deformations,

Within 30 seconds of the earthquake episode, the maximum interstory deformations, x_{mi} (i=b,1,2,...,8), the maximum floor acceleration, a_i , and the maximum active control force, U_m , are summarized in the column (A) of Table 3.3. The time history of the deformation of the first story unit, $x_1(t)$, is shown in Fig. 3.4(c).

Secondly, the hysteretic component vector \underline{V} is used for the feedback to determine the control vector $\underline{U}(t)$ and hence $Q_{22} \neq Q$. Recall that the basic idea of installing an actuator is to overcome the frictional force in the sliding system in order to minimize the sticking phase of the motion during earthquakes. In other words, the active control force is intended to keep the sliding phase of the motion as much as possible. It is observed from Eq. (3.6) that the frictional force is proportional to v_b of the sliding system. Hence, a feedback of v_b will be sufficient to reduce the frictional force. Consequently, all elements of the Q_{21} and Q_{23} matrices are set to be zero. Further, since v_b of the sliding system is reflected only by the element $Q_{22}(1,1)$, all elements of the Q_{22} matrix are also set to be zero except $Q_{22}(1,1)$ that is chosen to be -10.258, i.e. $Q_{22}(1,1)$ = -10.258.

Within 30 seconds of the earthquake episode, the maximum response quantities for $\alpha/R_0 = 3.39x10^9$ and $\Delta t = 0.03x10^{-2}$ sec. are summarized in the columns (B) of Table 3.3 for comparison. The time history of the deformation of the first story unit, $x_1(t)$, is presented in Fig. 3.4(d). It is observed from Table 3.3 and Fig. 3.4 that (i) the performance of the present optimal algorithm with Y feedback is superior to that of the optimal algorithm without considering Y feedback, and (ii) the hybrid control system consisting of a frictional-type sliding system and actuators is quite effective, although the required maximum active control force is large.

SECTION 4 CONCLUSIONS

Optimal control for nonlinear and inelastic structural systems has been formulated incorporating the specific hysteretic model of the system. This is accomplished by including the hysteretic components of the structural system in the equations of motion which serve as constraints in the optimization process. The resulting control vector is obtained as a function of the deformations, velocities and hysteretic components of the structural system. The hysteretic components of the structural response can be estimated easily from a simple observer using the measured structural response quantities.

In this report, the optimal control vector has been shown to satisfy not only the necessary conditions but also the sufficient condition of optimality. Specific applications of the optimal algorithm to two types of aseismic hybrid control systems have been demonstrated. These include (i) a lead-core rubber bearing isolation system connected to either an actuator or an active mass damper, and (ii) a frictional-type base sliding isolation system connected to an actuator. Numerical results indicate that (i) the performance of the present optimal algorithm is better than that of the algorithm without considering the feedback of hysteretic components of the structural response, (ii) the present optimal algorithm is simple for on-line operations and practical applications, (iii) the two types of aseismic hybrid control systems considered are effective for practical applications, and (iv) for active control of base-isolated buildings, the installation of sensors on the building is not necessary.

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{0}^{\infty}\frac{1}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{2\pi}}\right)^{2\alpha} \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{1}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{2\pi}}\right)^{\alpha} \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}$

SECTION 5

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