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**EFFECTS OF AXIAL FORCE ON FREQUENCY
OF PRESTRESSED CONCRETE BRIDGES**

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ABSTRACT

The purpose of the study presented in this report was to determine if vibration frequencies of a prestressed concrete member can be used to establish prestress losses. A post-tensioned concrete bridge (called the Golden Valley bridge) which had been instrumented for another study was the primary subject of the study. The bridge was a simply-supported, multi-cell box girder. Because the actual stresses in the bridge were known, it was possible to determine if the changes in the measured frequencies of the bridge would correlate with the prestress losses.

The dynamic characteristics of the bridge were measured on days 105, 202, and 455 from the prestressing completion date. The bridge was excited by impact from a heavy truck or by ambient truck traffic. Several data sets were collected in each test, and the frequencies were determined from the free-vibration acceleration data using a fast Fourier transformation method. The theoretical prediction for homogeneous members was that, as the prestress force decreased, the frequency would increase because a reduction in the axial compressive load should stiffen the element. The measured frequencies for the bridge showed an opposite trend. To verify the field observation, a 12-ft. long prestress member was built and tested in the laboratory under different axial loads. The laboratory data showed the same trend as that observed in the field. The difference between the theoretical and measured results was attributed to the fact that, in concrete members with moderate axial loads, prestress forces tend to close shrinkage and other microcracks, and hence stiffen the element.

Based on the measured data for the laboratory specimen, an empirical equation was developed that accounts for the effect of axial force on the rigidity of the element. This expression was attempted for the Golden Valley bridge and led to a reasonable estimate of the fundamental frequencies at different prestress forces.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Non-destructive and non-intrusive methods to determine the condition of existing bridges are the proper means of assessing the need for preventive maintenance, repair, or replacement of the structure. A large number of methods utilizing a variety of equipment and interpretation techniques have been used for this purpose. Some of these methods rely on vibration characteristics of the structure to identify the in-situ condition of the structural components and the system. The measured dynamic data have been used to determine the stiffness of foundation and abutments as well as superstructure elements [1-5, 8-11]. This is generally done by fitting the vibration frequencies and mode shapes to the response of an analytical model and by concluding that the stiffness properties of the best fit analytical model represent the actual stiffness of the structure.

One of the unknown parameters in prestressed concrete bridges which have been in service for some time is the existing prestress force. A substantial difference between the design and the actual prestress force can lead to serviceability and safety problems. Unless the bridge is instrumented at the time of construction, the existing prestress force cannot be directly estimated. In two bridges, for which the existence of an adequate prestress force was in doubt, the vibration characteristics were used to calculate the effective moment of inertia, I , of the superstructure [10]. Based on the value of I , it was determined whether the bridge was cracked

or not. No attempt was made to quantify the actual prestress force. Theoretically, the presence of an axial force in a homogenous beam alters its frequencies of vibration because of the "compression softening" that the member will experience as explained in Section 1.2.

1.2 Theoretical Background

The presence of an axial load on a beam introduces additional terms in the dynamic equilibrium equation of a beam which is vibrating in the lateral direction. For a simply supported prismatic beam, the solution of the equation after the application of boundary conditions leads to Equation 1.1 for the natural frequency of the vibration [12].

$$\omega_n^2 = -\left(\frac{n\pi}{L}\right)^2 \frac{N}{m} + \left(\frac{n\pi}{L}\right)^4 \frac{EI}{m} \quad (1.1)$$

In this equation,

- n= mode number,
- L= span length,
- N= axial compressive force (positive),
- m= beam mass per unit length,
- E= modulus of elasticity, and
- I= moment of inertia for the beam section.

It can be seen in this equation that an increase in the axial compression reduces the frequency and vice versa. This equation can be changed into a non-dimensionalized form of Eq. 1.2.

$$Z = 1 - \frac{1}{n^2} X \quad (1.2)$$

In this equation,

$$Z = Y/Y_0$$

$$Y = \frac{\omega_n^2}{\frac{EI}{mL^4}}$$

$$Y_0 = (n\pi)^4$$

$$X = \frac{N}{\frac{\pi^2 EI}{L^2}}$$

Parameter Z is an index which shows the sensitivity of the square of frequency to the changes in an axial load index. The parameter X is the ratio of the axial load to the buckling load for the beam. It can be observed in Eq. 1.2 that the sensitivity of the frequency to changes in axial loads decreases as the mode number increases. Therefore, only the frequencies for the lower modes (say, the first two modes) can be used to detect changes in the axial load.

1.3 Object and Scope

The primary objective of this study was to attempt to quantify the magnitude of the existing prestress force based on the vibration frequency of prestressed concrete bridges. The fundamental theory prompting this study was that the natural vibration frequencies of solid homogenous beam-column elements are affected by the magnitude of the axial force. As the compressive axial force increases, the frequencies generally decrease [12]. This is due to

"compression softening" of the element. Prestressed concrete members may be considered as beam-columns which are subjected to an axial compressive load which is the prestressed force. As this force changes due to losses and other factors, the vibration frequency is expected to change. A net prestress loss should lead to a higher frequency of vibration.

The study was initially conducted on an actual post-tensioned concrete bridge (called the Golden Valley bridge) located near Reno, Nevada. This bridge was the subject of another study with the aim of measuring prestress losses directly on the tendons. Because the tendons were instrumented from the time of stressing, the magnitude of the actual prestress force in the bridge was known. It was, therefore, possible to correlate the vibration frequencies to the existing prestress force. The dynamic properties of the bridge were measured by instrumenting the superstructure with an array of accelerometers which were triggered after the bridge was excited by a controlled truck impact load or by normal truck traffic on the bridge. The data were collected three times during the first fifteen months after the bridge had been post-tensioned. Because the trend in the measured frequencies was the opposite of what was expected, a post-tensioned concrete beam specimen was constructed and tested in the laboratory under different prestress forces. Laboratory data showed a trend which was similar to that observed in the field. Based on the laboratory data, an empirical expression was developed to determine the effective rigidity of the girder as a function of the prestress force and the geometric and material properties. The validity of the expression was examined and verified for the field data.

CHAPTER 2

DYNAMIC TESTING OF THE GOLDEN VALLEY BRIDGE

2.1 Introduction

The initial part of the study was conducted on a newly constructed post-tensioned concrete highway bridge. This chapter describes the bridge superstructure, the testing schedule, the instrumentation, the method of dynamic testing, and representative measured results.

2.2 Description of the Bridge

The Golden Valley Interchange consists of two parallel 155 ft., simple-span, post-tensioned concrete box girder bridges located on U.S. Highway 395, five miles north of Reno, Nevada. The study presented in this paper was on the bridge carrying the southbound traffic, referred to as the Golden Valley bridge in this report. The structure is 45 ft. wide and has a skew angle of 30 degrees. The cross section of the bridge is shown in Fig. 2.1. An 8 in. thick vertical diaphragm in the transverse direction of the bridge separates the cells at midspan. Each girder has three grout ducts two of which contain thirty-one strands. The third duct contains eighteen or nineteen strands in the exterior and interior girders, respectively. All the tendons consist of 0.5 in. diameter 7-wire strands. Two strands in each of the two western girders (Girders G1 and G2 in Fig. 2.1) were instrumented to measure the actual prestress losses as part of another study [13]. A total of 484 strands were used in the bridge. The tendons were placed in a parabolic profile with eccentricities of 2.5 in. at the ends and 37.5 in. at the midspan of the

bridge. The eccentricities are relative to the centroid of the gross section.

The strands are of Grade 270, Low-Relaxation (ASTM A-416) prestressing steel. The mild steel is of ASTM A-615 Grade 60 reinforcing steel. Based on the test data supplied by the manufacturer of the prestressing steel, the yield stress was 258 ksi and the ultimate strength was 276 ksi. The measured modulus of elasticity was 29,900 ksi. The bridge was air cured.

The specified concrete compressive strength for the post-tensioned concrete in the bridge was 3500 psi at the time of post-tensioning and 4500 psi at 28 days. The measured concrete cylinder strength was 4600 psi and 5300 psi (42.1 MPa), at post-tensioning time and at 28 days, respectively. The shop drawings called for an initial tendon stress of 70% of the specified ultimate strength.

Electrical resistance strain gages were used to monitor the changes in strain in the tendons. Four tendons in the 4¼-inch (108-mm) ducts, two in the interior girder and two in the exterior girder, were instrumented with electrical resistance strain gages. Three strain gages were bonded to three different wires in one strand in each of the instrumented tendons, thus making the total number of gages 12. In addition, two temperature reference gages were bonded to an unstrained piece of prestressing wire. The stressing of the bridge was completed on September 8, 1988, and tendon strains were monitored for a 30-month period. More details about the instrumentation for measuring prestress losses and the prestress loss data are provided in Ref. 13 and 14.

2.3 Dynamic Instrumentation and Testing

Because the majority of stress losses take place during the first year after the post-

tensioning of the bridge, and in order to obtain data for axial forces which are significantly different from each other, it was essential to measure the dynamic properties during the early life of the bridge. Ideally, the measurement of the dynamic properties would need to start immediately after post-tensioning. The earliest the funds for this project could be secured was three month after that time. The dynamic data were collected on day 105, 202, and 455 from the completion of stressing. The corresponding dates were December 23, 1988, March 30, 1989, and December 8, 1989. Only the vertical modes of vibration were of interest. The first two sets of data were collected when the bridge was closed to traffic, whereas the third set was obtained with bridge open to normal traffic. The acceleration data were collected at a rate of 300 samples per second, using a high-speed data acquisition system called DataSeis which was manufactured by Kinometrics. Eight channels of vertical accelerometers were used in the tests. Each channel is equipped with controls for both low-pass filter settings and amplification factors. The filters for all channels were set at 50 Hz. in all tests. The amplification factors were set at different levels which depended on the position of the accelerometers. The Dataseis system allows for review and spectral analysis of the data. This feature was used in the field to spot check the data.

The method of dynamic excitation of the bridge and the arrangement of stations varied depending on whether the bridge was closed (Tests 1 and 2) or open (Test 3) to traffic.

2.3.1 Tests 1 and 2 - The location of the stations for the first two tests is shown in Fig. 2.2. The stations were located at approximately 2 ft. from the edge of the deck and 1.3 ft. from the abutments. Because only 8 accelerometers were used while there were 22 stations, the accelerometers had to be moved to sweep all the stations and hence, two reference stations had

to be added to allow for the normalization of the data. The location of the reference stations was selected so that at least one of the reference accelerometers would register significant motion during the excitation of the first few modes. Two accelerometers were placed at the reference points and remained stationary in each day of testing. The other six were moved to different stations as testing proceeded.

To excite the bridge, wooden blocks of approximately 4 in. depth were placed on the deck and a relatively heavy truck was driven over the block to apply an impact load. In the first test, the impact was applied at two locations one at approximately 40'-6" (truck position 1) and the other at about 54'-8" (truck position 2) from the south abutment. This test indicated that the impact location at 40'-6" provides sufficient data and, hence, in Test 2 only this location was used. The impact loading was repeated for a number of times until the moving accelerometers scanned all the stations on the bridge deck.

2.3.2 Test 3 - Because the bridge had been opened to traffic by the time this test was conducted, a change in the accelerometer locations and the excitation method had to be made. To accommodate the prestress measurement study, an access hole to the left cell in the superstructure had been provided. Eight accelerometers were installed on the bottom slab inside the cell as shown in Fig. 2.3. Station 1 was located at 10 in. from the center of the span. There was no need for reference stations and there were no moving accelerometers. The vertical vibration of the bridge was measured several times. In order to obtain amplitudes which were comparable to those in Tests 1 and 2, the data were collected when relatively large trucks were passing the bridge at normal speed (say 50 mph or higher).

2.4 Measured Accelerations

Sample acceleration results for different truck impact positions are shown in Figs. 2.4 and 2.5. The acceleration data are in terms of g , gravity acceleration. The stations for which the data are shown are the reference Stations 1 and 2 (Fig. 2.2). Note that, for truck position 1, higher amplitudes were observed at Station 2 when compared to the amplitude of Station 1 (Fig. 2.4). The impact in Fig. 2.5 was applied at near Station 1. This led to comparable peak amplitudes in Stations 1 and 2. The measured accelerations at other stations generally had similar characteristics of those of the reference stations and are not presented in this report. The measured data in March 1989 (Fig. 2.6) were also similar to those of December 1988 in terms of wave forms. Note that only the impact at position 1 was utilized in this test. The curves present what is typically expected in measuring the free vibration response of simple structures.

The results for Test 3 are somewhat different than the rest of the results (Fig. 2.7). Figure 2.7 (a) shows that higher modes of vibration were excited in Station 1 (Fig. 2.3). The data in Fig. 2.7 (b) shows the response at Station 2 under a different traffic. The relatively large amplitudes during the initial part of the response indicates that the bridge was excited by another vehicle before it was vibrated by a large truck.

2.5 Measured Frequencies

Using the fast Fourier transformation (FFT) method [15 and 16], the frequencies for the first several natural modes of vibration were extracted for the bridge. The computer software used for this purpose was developed by Kinometrics. The program allows for using a window of the data to prevent "leakage" due to discontinuity at the beginning and end of the record. A

Hanning type window was used in the analysis. A sample result for the record measured at Reference Station 1 for Event No. 7 in March 1989 is shown in Fig. 2.8. The first number in the digitized peaks represents the frequency. Because the bridge had a 30-degree skew, the torsional mode of vibration was expected to be excited by the impact. The fundamental bending mode frequency is indicated by the peak at approximately 2 Hz. The second peak represents the first torsional mode while the third peak shows the frequency for the second bending mode. It was possible to identify these modes based on the response at different stations.

The first two bending modes were excited reasonably well at Reference Stations 1 and 2 in all the tests. They were also clearly present in the response of many of the other stations. An examination of the FFT results for many of the other stations showed that the response at the reference stations is representative and can be used to establish the frequencies. Tables 2.1 to 2.3 show the extracted frequencies for the lower two bending modes. The data in Table 2.3 are for Stations 1 and 3 (Fig. 2.3) which are approximately at the same locations as Reference Stations 1 and 2 in Fig. 2.2 relative to the end of the bridge.

The measured data indicated a prestress force of 12026, 11685, and 11115 kips on the days of Tests 1 through 3, respectively. Note that the average first mode frequencies reduced as the prestress force decreased. No consistent trend in the second mode frequency can be observed. The reduction in the first mode frequency as a result of the loss of prestress force was opposite of the theoretical prediction for a homogenous uncracked beam element as discussed in Chapter 1.

CHAPTER 3

EXPERIMENTAL STUDY OF PRESTRESSED CONCRETE BEAM MODEL

3.1 Introduction

Because the trend in the observed frequencies of the Golden Valley bridge was the opposite of that predicted by Eq. 1.2, it was decided to extend the study to a simpler specimen which can be tested under controlled conditions. A simply-supported prestressed concrete beam was designed and constructed for this purpose. Both free vibration and static lateral loading tests were conducted. The characteristics which were determined were the natural frequencies, mode shapes, and beam stiffness for different prestress force levels. The objectives of these tests were to assess the variation of these mechanical characteristics as a function of the prestressing force and to develop a simple method for evaluating the dynamic characteristics of prestressed concrete bridge structures. This chapter describes the specimen, the tests, and the results.

3.2 Description of the Specimen

A sketch of the specimen is shown in Fig. 3.1. The beam was a 12-ft. long simply-supported element with constant cross section. The beam was reinforced longitudinally with two #3 Grade 60 deformed bars on top and bottom. The stirrup size was #2 bar with a spacing of 12 inches. The stirrups were used to facilitate the positioning of the top bars; they were not designed to resist shear. A Grade 250 seven-wire straight strand with a diameter of 0.5 inch was used as the prestressing steel. The strand was placed in a 1-inch diameter duct which

remained ungrouted. The center of the duct coincided with the centroid of the beam.

The concrete was made of type II portland cement and aggregates with 1/2 in. maximum size. The 28-day compressive strength of concrete was 2,950 psi. The specimen developed a small crack at mid span under its own weight. The prestressing arrangement is illustrated in Fig. 3.2. One end of the beam was a hinge support and the other a roller support to allow the beam to freely deform in the horizontal direction when the prestress force was applied. The post tensioning cable was anchored to one end of the beam and was attached to a prestressing jack at the other end..

3.3 Instrumentation and Test Set Up

For the free vibration tests, the specimen was instrumented with seven equally-spaced accelerometers which measured the vertical acceleration response of the beam (Fig. 3.3). The instruments were connected to the same multi-channel data acquisition system which had been used in the testing of the Golden Valley bridge (Chapter 2). In the static tests, the only transducer was a mechanical gage which was mounted at the center of the beam to measure vertical deflection. A load cell was attached to the end of the prestressing cable to measure the axial force for both the static and dynamic tests, and the data were read on a single channel strain indicator.

The specimen was set into free vibration by vertical impact from a hammer. Each dynamic test was carried out after the desired prestress force had been applied and the cable had been anchored. The jack in all experiments was disconnected from the beam in order to avoid the influence of the jack weight on the response of the beam. The static load was applied by

slowly placing metal disks of known weights on the beam.

3.4 Dynamic Testing Procedure

The purpose of the dynamic tests was to determine the axial load effect on the natural frequency of the beam. The prestress force was varied from zero to a relatively large level as shown in Table 3.1. In the experiments, the axial force was increased from zero to the maximum value and then decreased to zero. The maximum axial force was 29,510 lbs which corresponds to a compressive stress ratio of $N/f_c'A = 0.500$, where N , f_c' , and A are the axial force, the concrete strength and the gross area of the beam section, respectively. The estimated axial buckling load for the beam was 39,300 lbs which was based on gross section moment of inertia. It can be observed that the maximum applied load was an upper bound of the expected prestress force. The relatively large maximum prestress force allowed the study of the beam for a wide range of axial loads.

Four sets of free vibration data were collected for each axial force, two with impact applied at mid span and the other two at the quarter point. All the frequencies above 50 Hz were filtered. Different impact locations were used to insure that, at least, the first two natural modes are excited. The duplicated data for each impact location allowed for some redundancy in the measured response.

3.5 Dynamic Results

A typical free-vibration response record is shown in Fig. 3.4. The data represent the motion at mid span due to an impact next to the middle accelerometer. The fast Fourier

transformation technique (FFT) was employed for the analysis of the frequencies and modal amplitudes. Figure 3.5 shows a typical result of the FFT analysis. A review of mode shapes showed that the two peaks in the figure represent the frequencies of the first and second modes. The second mode frequency could be observed only when the impact location was at the quarter point of the beam. A detailed discussion of the results is presented in the next chapter.

3.6 Static Displacement Response

The static vertical displacement of the beam for different prestress levels were measured in order to obtain the effective stiffness of the beam for different axial forces. The beam was loaded by a nearly concentrated load at mid span. Two relatively small load values were used in the tests: 28.4 lbs and 55.5 lbs. The variation of displacement as a function of the axial load is shown in Fig. 3.6. It can be observed that, as the axial load increased, the displacements decreased, thus indicating that the beam stiffened. The rate of increase in stiffness slowed after the axial load reached approximately 16,000 lbs. The increase in stiffness is attributed to the closing of the cracks by the prestress force. As the axial force increased more microcracks and the crack at mid span closed. The closing of the crack increased the moment of inertia and stiffened the beam.

The measured data are listed in Table 3.2. The comparison of displacements for the two different lateral loads at any given prestress force shows that, by increasing the load by a factor of 1.95 (from 28.4 to 55.5 lbs.), the displacements increased by a factor ranging from 2.06 to 2.26. The fact that the latter factors are larger than the load ratio indicates a slight degree of nonlinearity in the response of concrete which is expected.

CHAPTER 4

ANALYSIS OF EXPERIMENTAL RESULTS

4.1 Introduction

The static results presented in Chapter 3 indicated that the beam stiffened as the axial load increased. The frequencies extracted from the measured data confirmed this observation for both the Golden Valley bridge and the beam specimen. This chapter presents the results of the fast Fourier transformation (FFT) analysis for the beam. In addition, a formula is presented for calculating the effective rigidity of prestressed concrete members based on experimental results. The formula includes the effect of axial force as well as the material and geometric properties of the element.

4.2 Frequencies and Mode Shapes

Using the fast Fourier transformation technique (FFT), the first three frequencies and mode shapes were obtained for the beam (Table 4.1). The frequencies in the table are the average values of the results measured at all the seven channels of accelerometers. Because the measured response for impact at the quarter point of the beam included all the first three modes, the FFT results were determined for responses corresponding to this impact point only.

The data in the table clearly indicate that the first natural frequency increased as the axial force was raised. The increase in frequency is attributed to the stiffening of the beam which is caused by the closing of shrinkage and a flexural crack noted at mid span. It appears that the

increase in the prestress force led to the closing of more cracks which in turn increased the effective moment of inertia of the beam. The second and third mode frequencies also generally increased depending on the axial force but they were not as sensitive as the first frequency. This conclusion is in line with the theoretical prediction described in Chapter 1. The somewhat erratic trend in the second mode frequency for higher axial loads is due to the fact this mode is insensitive to the closing of the crack at midspan because the center of the beam forms a stationary node when the beam vibrates in its second mode only.

Figure 4.1 shows the fundamental mode shapes for different axial forces. The innermost curve is for zero axial load and the outermost curve is for the maximum axial load. It can be seen from the figure that the axial force only slightly affected the mode shapes of the beam.

4.3 Effect of Axial Force on Frequencies of Beam

To attempt Eq. 1.1 for the beam specimen, it may seem reasonable to use the effective moment of inertia of the beam using the equation specified in the American Concrete Institute Code [19]. Because the maximum dead load moment in the beam is less than the cracking moment (by approximately 33 percent, based on the measured 28-day concrete strength), the gross moment of inertia controls. Using a unit weight of 150 pcf and the measured concrete strength, Eq. 1.1 leads to the frequencies listed in Table 4.2 for the axial loads that were applied in the experiments. It can be seen that, as the axial force changes from zero to the maximum value, the theory predicts a nearly 40 percent reduction in the first mode frequency. The measured data showed a 32 percent increase in the first mode frequency (Table 4.1). The relationships between the measured frequencies and the axial forces are shown in Fig. 4.2, 4.3,

4.4 for the first, the second, and the third modes, respectively. It can be seen that the rate of increase in frequency reduces at higher axial loads.

The conflict between the measured and calculated results is attributed to the fact that the element is a concrete specimen with initial microcracks which are primarily due to shrinkage. The axial force enhances the stiffness by closing these cracks. The theory based on which Eq. 1.1 is developed does not account for this effect. To reproduce the experimental results, one simple approach is to develop a relationship for the rigidity, EI , of prestressed concrete members that is a function of the prestress force.

4.4 Effective Rigidity of Beam under Axial Force

The results of the FFT analysis were used to determine the effective rigidity of the beam which would result in the same frequency as the measured data when used in Eq. 1.1. This was done by substituting the measured fundamental frequencies for different axial loads in Eq. 1.1 and calculating EI . The results are listed in the right column in Table 4.1. It can be seen that, as the prestress force was raised, the effective EI increased considerably because rigidity is a function of the square of frequency. A graphical display of the effect of axial load is shown in Fig. 4.5.

A linear trend can be seen in the data. A linear regression analysis of the data was conducted to develop an equation for the effective EI . The data for prestress force of 4 kips or less were excluded because (1) they introduced significant nonlinearity in the data and (2) the average prestress force for these points was small (less than 200 psi). In place of these data, an artificial data point was added at $(EI)_e = (EI)_g$ for zero prestress force. The least-square best

fit line was forced to pass through this point. The result was a line with a slope of 1.77. This value was rounded down to the nearest 0.05. A comparison of the modified line and the measured data is shown in Fig. 4.6. The equation shown in this figure accounts for the gain in stiffness as a result of an increase in the axial force. The effective EI from this equation needs to be used in place of EI in Eq. 1.1.

4.5 Application to the Golden Valley Bridge

To examine the effect of using the proposed expression for $(EI)_e$ on the calculated frequencies of the Golden Valley bridge, the fundamental mode frequencies of the bridge were found. The effective prestress forces used in the calculations were based on the measured strains on the tendons plus the loss due to relaxation of the prestressing steel. A separate account of relaxation losses was necessary because these losses are not associated with strain changes and hence cannot be obtained from the tendon strain data. The bridge utilized low-relaxation type of steel. The relaxation losses for the days of testing were estimated using a time-step prestress loss calculation method which is described in detail in Ref. 20. The measured tendon stresses, relaxation losses, and the total prestress force for this bridge on different days of testing are shown in Table 4.3. The total area of prestress steel in the bridge is 74.05 in².

The modulus of elasticity for concrete was found using the ACI code relationship for normal weight concrete [19] and a measured 28-day concrete compressive strength of 5,300 psi. Using the axial forces shown in Table 4.3, and the effective EI from Fig. 4.6, Eq. 1.1 leads to the frequencies which are listed in Table 4.4. It can be seen that the calculated results are within six percent of the measured data and that, as prestress forces are reduced, the calculated

frequencies decrease at about the same rate as the measured values.

4.6 Discussion of the Feasibility of the Method

The proposed expression for the effective rigidity of prestressed concrete members as a function of the axial force may be used to arrive at a reasonable estimate of the frequency of vibration for the members. The data for the Golden Valley bridge showed that, for practical ranges of prestress force, the change in natural frequencies of prestressed concrete members is very small even for the fundamental mode of vibration. Because small changes in structural parameters such as degree of fixity at the supports, mass, section properties, etc. can affect the frequencies and introduce variations in the results which are in the order of the measured changes in frequency, it does not appear feasible to use vibration data to determine the actual prestress losses. As a relative measure, however, the dynamic signature of the bridge may be obtained after prestress losses have occurred (say, after three years) and periodic monitoring of the dynamic characteristics may be used to determine if a substantial prestress loss has occurred. Such monitoring, in combination with visual inspection of the bridge, may be useful in assessing the existing condition of the bridge.

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary

The free vibration testing of bridges subjected to relatively low amplitudes of vibration may be viewed as a non-destructive method to determine the in-situ condition of the bridge system. The study presented in this report was an attempt to determine if vibration frequencies of prestressed concrete members can be used to establish prestress losses. A post-tensioned concrete bridge (called the Golden Valley bridge) which had been instrumented for another study was the primary subject of the study presented in this report. The bridge was a simply-supported, multi-cell box girder. Because the actual stresses in the bridge were known, it was possible to determine if the changes in the measured frequencies of the bridge would correlate with the prestress losses.

The dynamic characteristics of the bridge were measured on days 105, 202, and 455 from the prestressing completion date. At least eight channels of accelerometers were used on each day of testing. The bridge was excited by impact from a heavy truck or by ambient truck traffic. Several data sets were collected in each test, and the frequencies were determined from the free-vibration acceleration data using a fast Fourier transformation method. The theoretical prediction for homogenous members was that, as the prestress force decreased, the frequency would increase because a reduction in the axial compressive load should stiffen the element. The measured frequencies for the bridge showed an opposite trend. To verify the field

observation, a 12-ft. long prestress member was built and tested in the laboratory under different axial loads. This beam was excited by impact from a hammer. The laboratory data showed the same trend as that observed in the field. The difference between the theoretical and measured results was attributed to the fact that, in concrete members with moderate axial loads, prestress forces tend to close shrinkage and other microcracks. As the prestress force is reduced some of these microcracks reopen and hence the stiffness is reduced. A reduction in stiffness leads to a reduction in the natural frequency of vibration.

Based on the measured data for the laboratory specimen, an empirical equation was developed that accounts for the effect of axial force on the rigidity of the element. This expression was attempted for the Golden Valley bridge and led to a reasonable estimate of the fundamental frequencies at different prestress forces.

5.2 Conclusions

Based on the study presented in this report, the following observations and conclusions were made.

- 1) Because the sensitivity of vibration frequencies to changes in the axial force diminishes at higher modes, only the first mode has the potential of being useful in studying the correlation between the frequencies and prestress forces.
- 2) The measured field data showed a reduction in frequency as prestress forces reduced. This trend was the opposite of the theoretical prediction for homogenous elements. The same trend was observed in testing of a prestressed concrete beam in the laboratory. The discrepancy between the theory and experiment is believed to be due to the closing of microcracks by the

prestress forces. This effect is not accounted for in the theoretical formulation of vibration of beams with distributed mass subjected to an axial load.

3) The relatively simple empirical expression which was developed to determine the increase in the effective moment of inertia due to the axial load led to good estimates of the measured fundamental frequencies of the Golden Valley bridge.

4) Because small changes in structural parameters such as the degree of fixity at the supports, mass, section properties, etc. can affect the frequencies and introduce variations in the results which are in the order of the measured changes in frequency, it does not appear feasible to use vibration data to determine the actual prestress losses.

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Table 2.1 - Frequencies for the Lower Two Flexural Modes on 12/23/88
(UNIT: Hz)

EVENT-STATION	MODE 1	MODE 2
6-R1	1.969	7.801
6-R2	2.026	7.801
9-R1	2.139	7.575
9-R2	2.087	7.612
10-R1	2.012	7.410
10-R2	1.969	7.405
12-R1	2.017	7.584
12-R2	2.003	7.533
AVG.	2.028	7.590
STANDARD DEVIATION	0.058	0.280

Table 2.2 - Frequencies for the Lower Two Flexural Modes on 3/30/89
(UNIT: Hz)

EVENT-STATION	MODE 1	MODE 2
2-R1	1.983	7.741
2-R2	1.969	7.688
3-R1	2.191	7.584
3-R2	2.124	7.522
5-R1	2.003	7.591
5-R2	1.969	7.688
7-R1	1.983	7.741
7-R2	1.969	7.688
AVG.	2.024	7.655
STANDARD DEVIATION	0.085	0.080

Table 2.3 - Frequencies for the Lower Two Flexural Modes on 12/8/89
(UNIT: Hz)

EVENT-STATION	MODE 1	MODE 2
6-1	2.108	7.137
6-3	2.108	7.131
7-1	2.040	6.940
7-3	1.875	6.941
8-1	1.996	7.122
8-3	1.940	7.106
AVG.	2.011	7.063
STANDARD DEVIATION	0.093	0.095

Table 3.1 - Axial Loads for Different Dynamic Tests

Test No.	Axial Force (lbs)	Compressive Stress Ratio $N/f_c'A$
1	0	0
2	6010	0.101
3	12720	0.215
4	18180	0.308
5	26990	0.457
6	29090	0.493
7	29510	0.500
8	29200	0.494
9	20280	0.342
10	19230	0.326
11	16396	0.278
12	12723	0.215
13	8108	0.137
14	3492	0.059

Table 3.2 - Static Response Results

Axial Force (lbs)	Vert. Load (lbs)	Displacement (in)	Disp. Ratio
0	28.4	0.0173	2.20
	55.5	0.0380	
5589	28.4	0.0136	2.26
	55.5	0.0308	
11045	28.4	0.0123	2.08
	55.5	0.0256	
16291	28.4	0.0097	2.10
	55.5	0.0204	
18704	28.4	0.0095	2.06
	55.5	0.0196	
25733	28.4	0.0088	2.22
	55.5	0.0195	

Table 4.1 Frequencies of the Beam

Axial Force (lbs)	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	$(EI)_c \cdot 10^{-8}$ (lb.in ²)
0	11.41	43.99	--	1.021
6,010	13.47	44.89	53.52	1.549
12,720	14.15	45.71	55.37	1.837
18,180	14.49	45.57	55.67	2.029
26,990	14.72	45.86	56.11	2.266
29,090	14.97	46.10	56.45	2.369
29,510	15.07	45.87	56.28	2.401
29,200	14.78	45.86	56.17	2.327
20,280	14.72	46.20	55.73	2.215
19,230	14.95	46.32	55.54	2.160
16,396	14.72	46.05	55.24	2.044
12,723	13.63	45.42	54.49	1.724
8,108	12.89	44.69	53.12	1.473
3,492	12.09	44.11	52.87	1.220

Table 4.2 Fundamental Frequency of the Beam Based on Eq. 1.1

Axial Force	(Radial Frequency) ²	f(hz)
0	80.59	12.83
6,010	74.75	11.90
12,720	67.65	10.77
18,180	61.26	9.75
26,990	49.23	7.84
29,090	45.90	7.31
29,510	45.21	7.19
29,200	45.72	7.28
20,280	58.62	9.33
19,230	59.95	9.54
16,396	63.42	10.09
12,723	67.64	10.77
8,108	72.60	11.56
3,492	77.25	12.29

Table 4.3 - Prestress Forces in the Bridge

Day of Test	Tendon Stress (ksi)	Relaxation Loss (ksi)	Prestress Force (kips)
105	164.2	1.87	12026
202	159.7	1.90	11685
455	152.1	1.97	11115

Table 4.4 - Measured and Calculated Frequencies for the Golden Valley Bridge

Day of Test	Measured Frequency (Hz)	Calculated Frequency (Hz)	Ratio of Calc. over Meas. Frequency
105	2.028	2.155	1.063
202	2.024	2.149	1.062
455	2.011	2.139	1.064

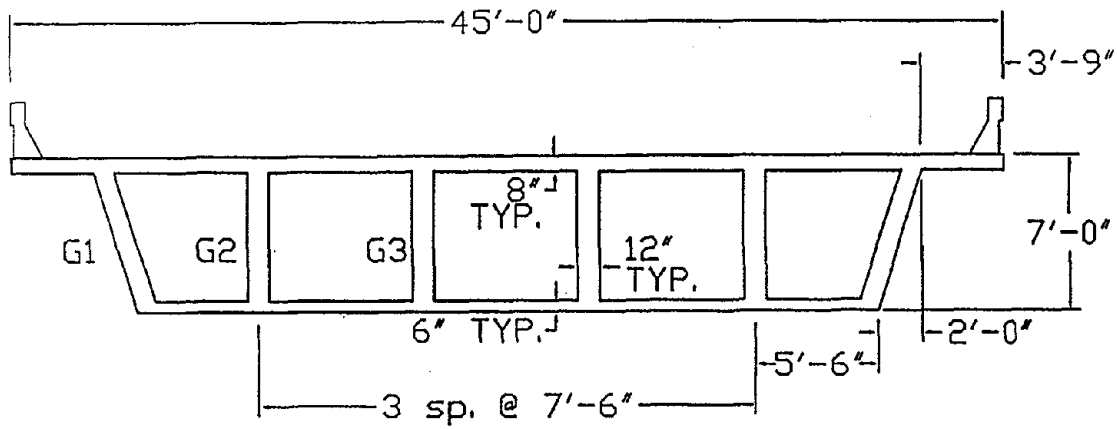


Fig. 2.1 - The Bridge Cross Section.

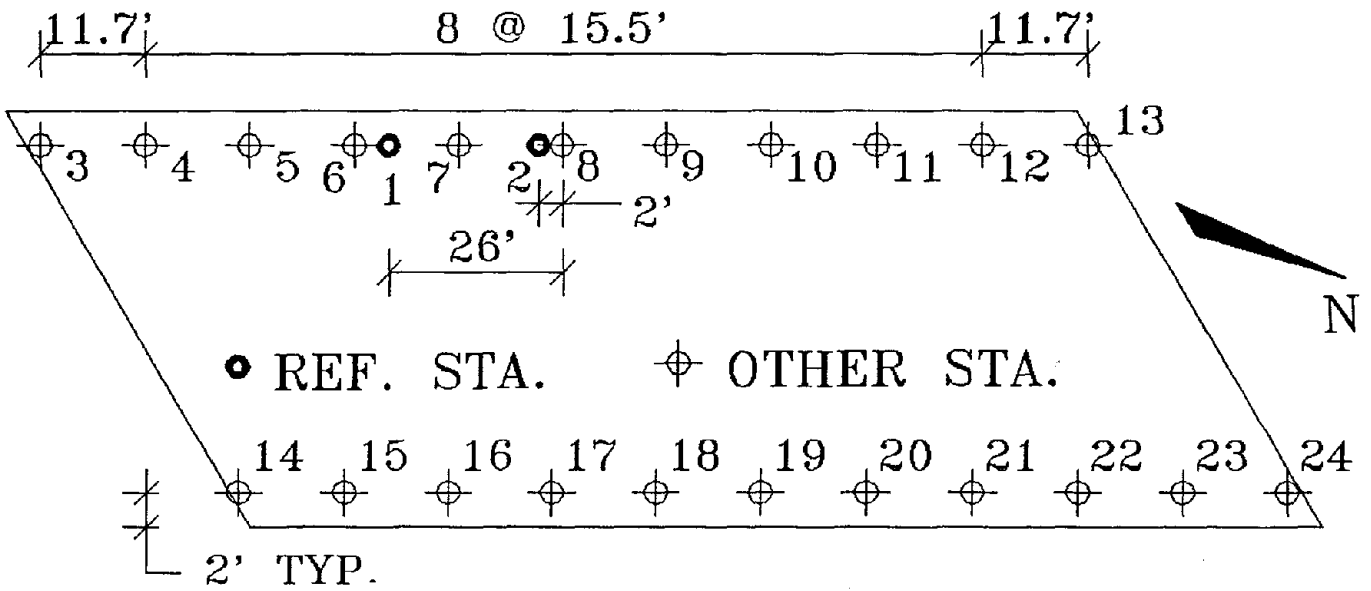


Fig. 2.2 - Station Locations for Tests 1 and 2.

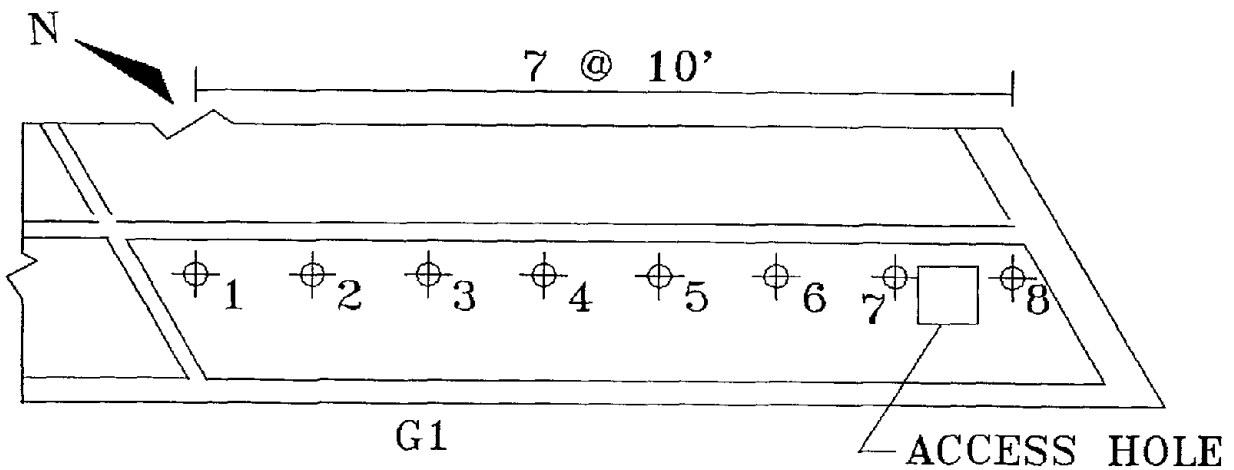


Fig. 2.3 - Station Locations for Test 3.

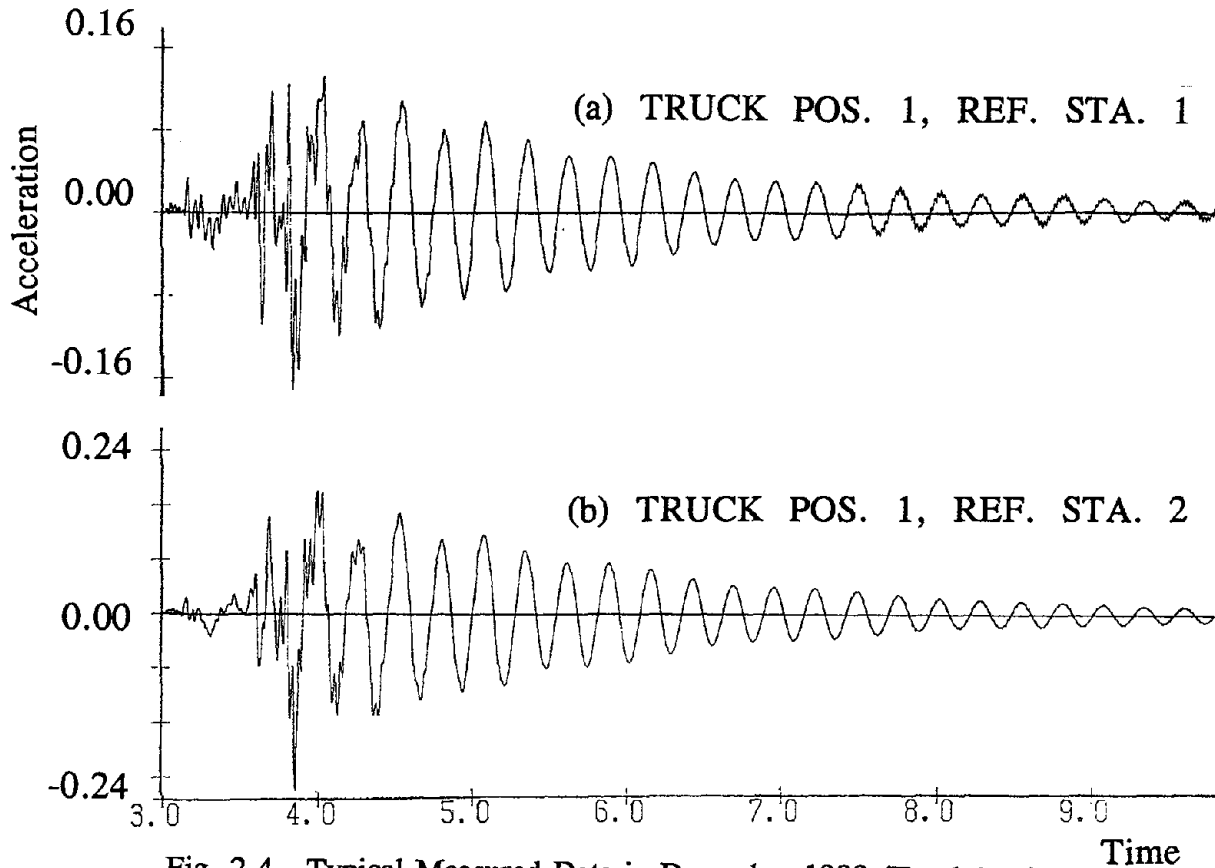


Fig. 2.4 - Typical Measured Data in December 1988 (Truck Position 1).

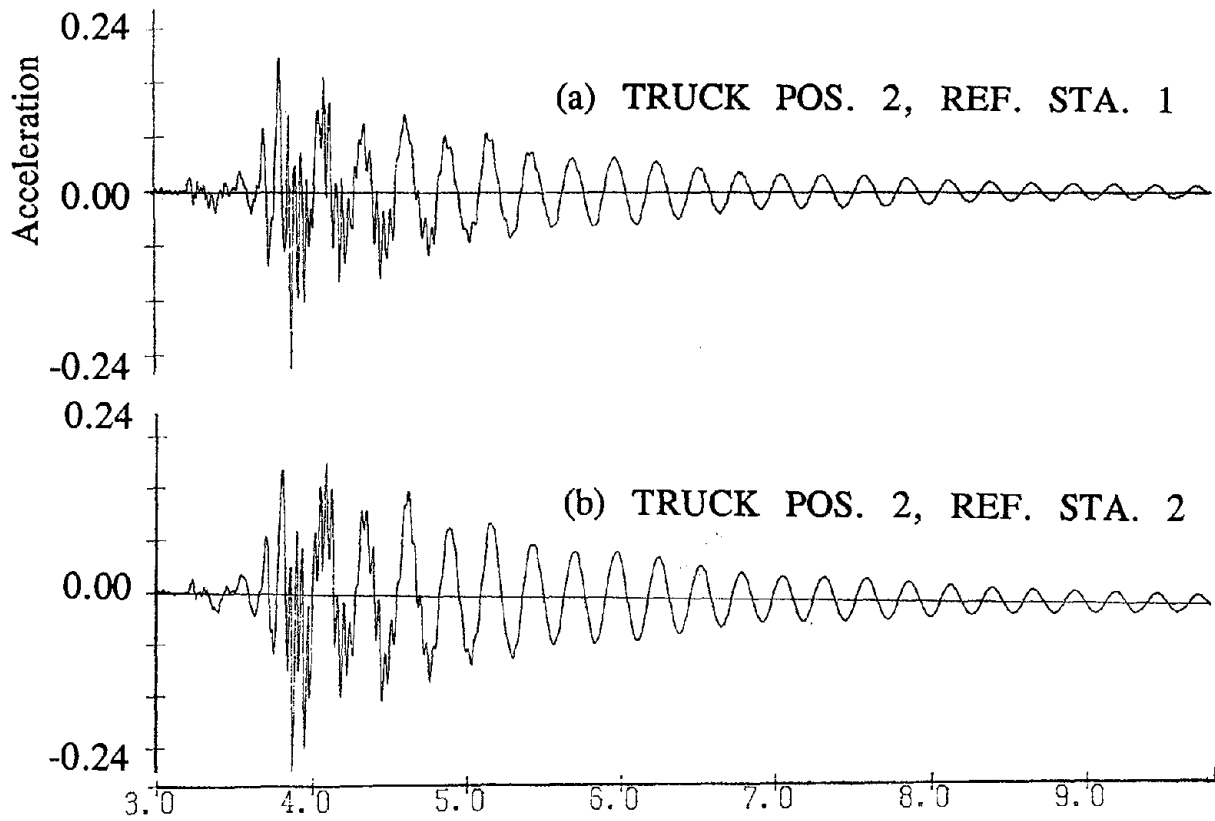


Fig. 2.5 - Typical Measured Data in December 1988 (Truck Position 2).

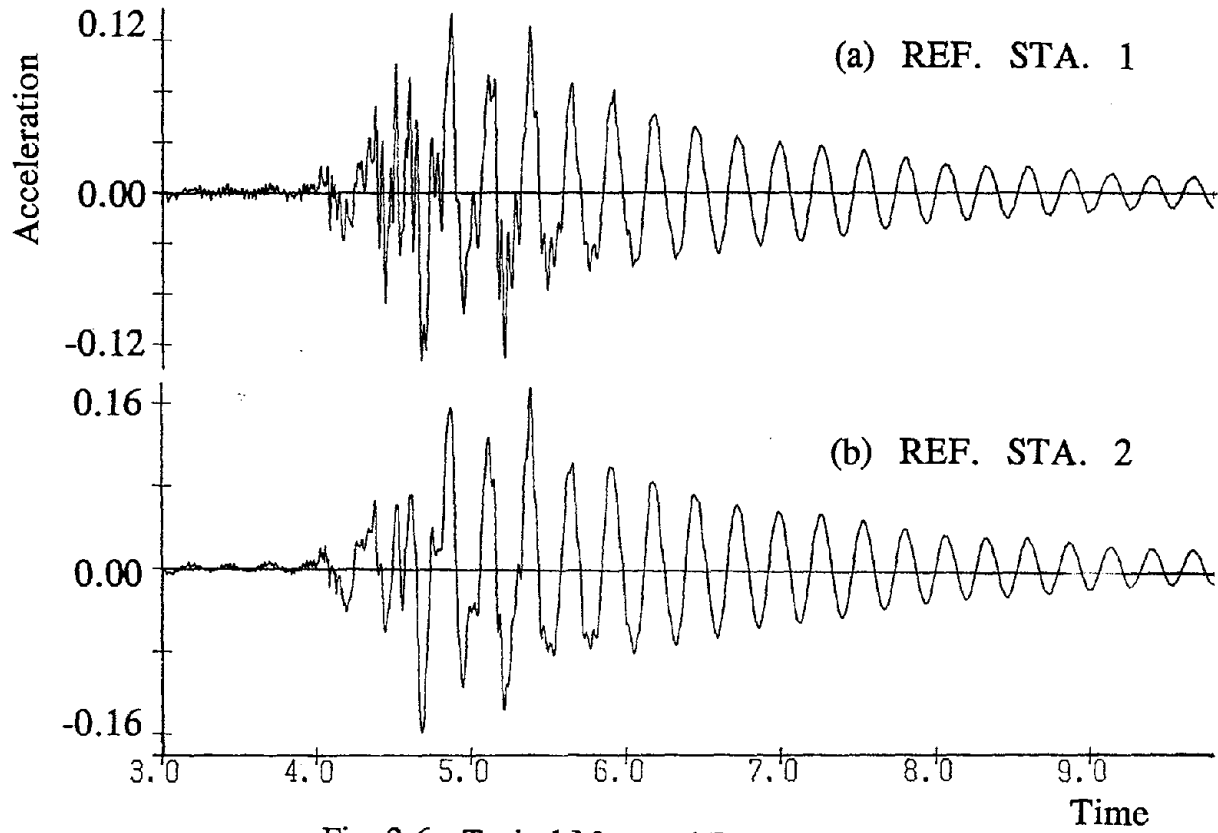


Fig. 2.6 - Typical Measured Data in March 1989.

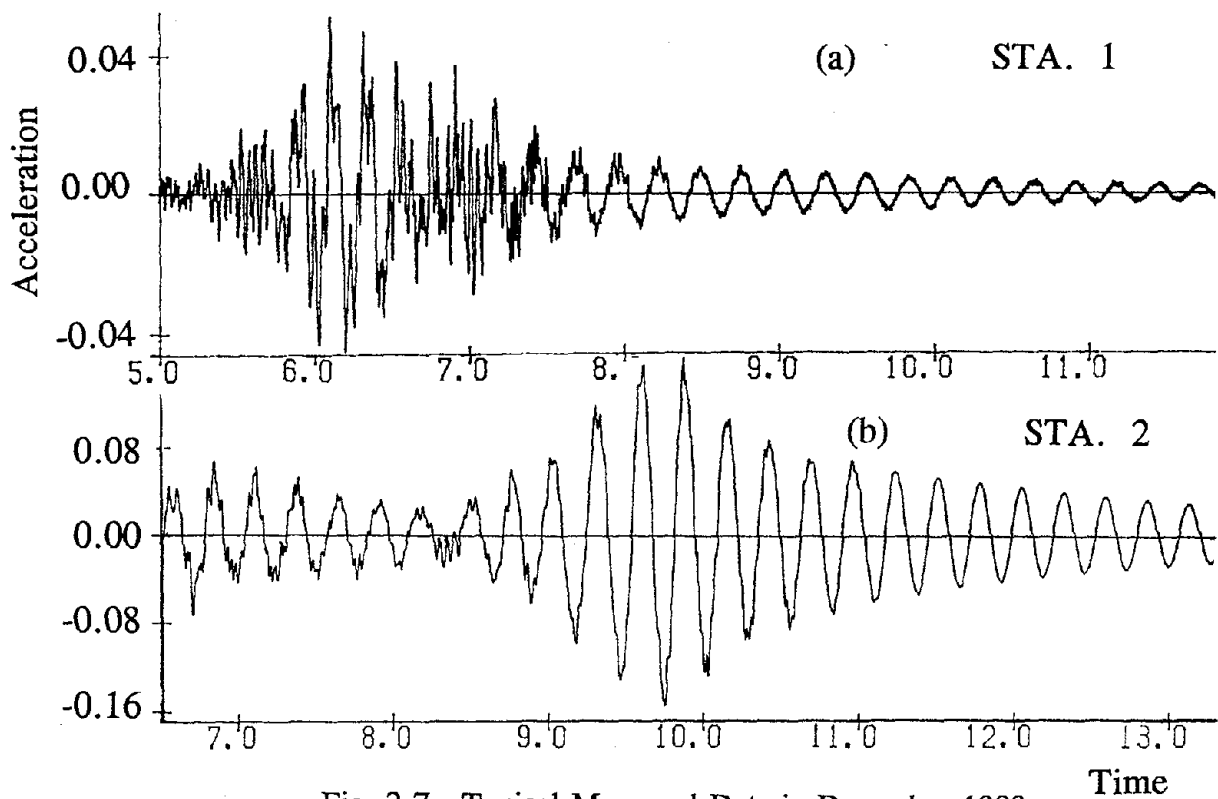


Fig. 2.7 - Typical Measured Data in December 1989.

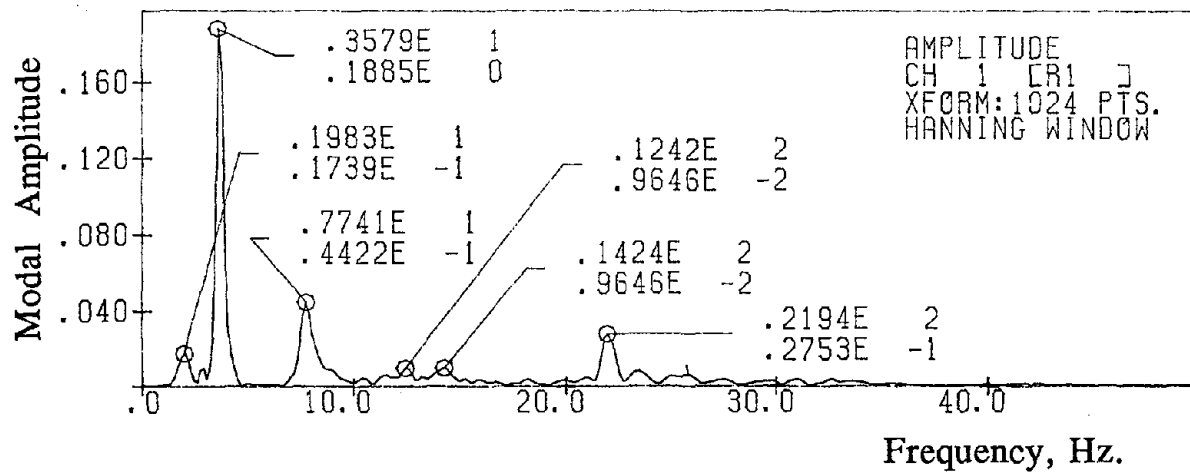


Fig. 2.8 - Sample Frequency Analysis Result.

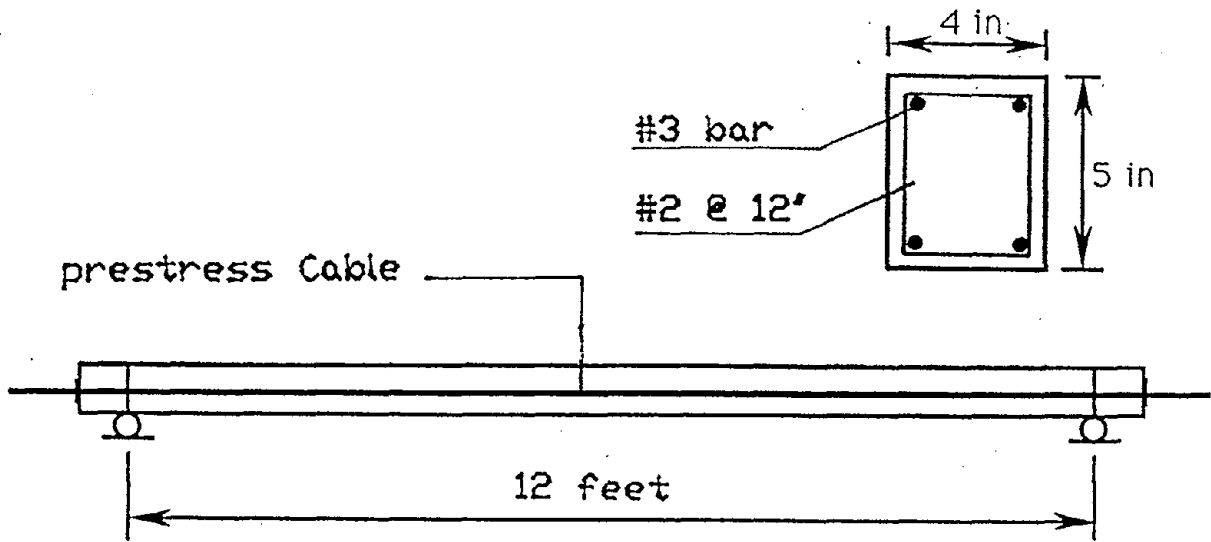


Fig. 3.1 - Details of the Beam Specimen.

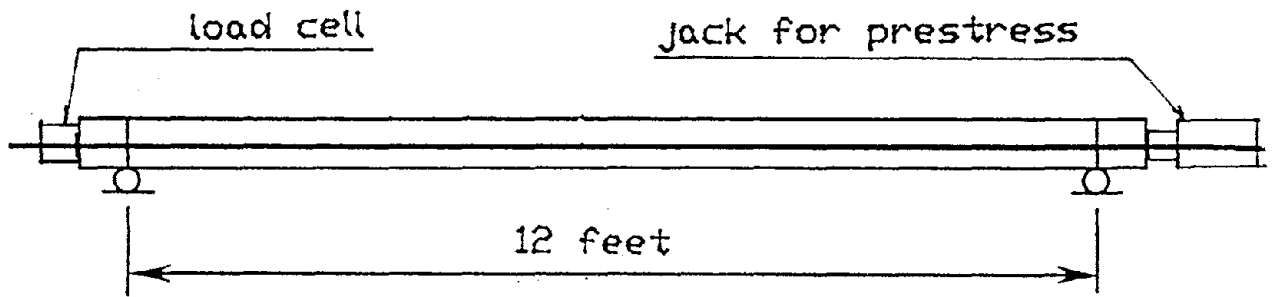


Fig. 3.2 - Prestress Load Set Up.

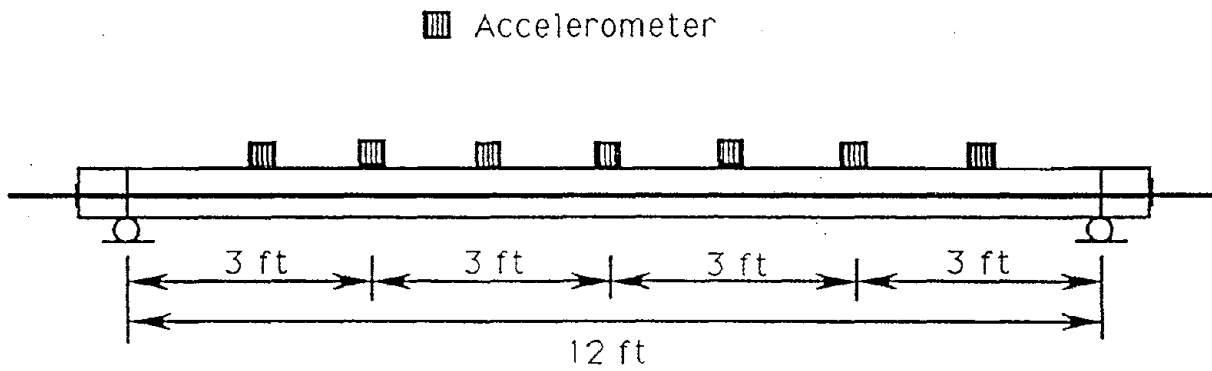


Fig. 3.3 - Location of the Accelerometers.

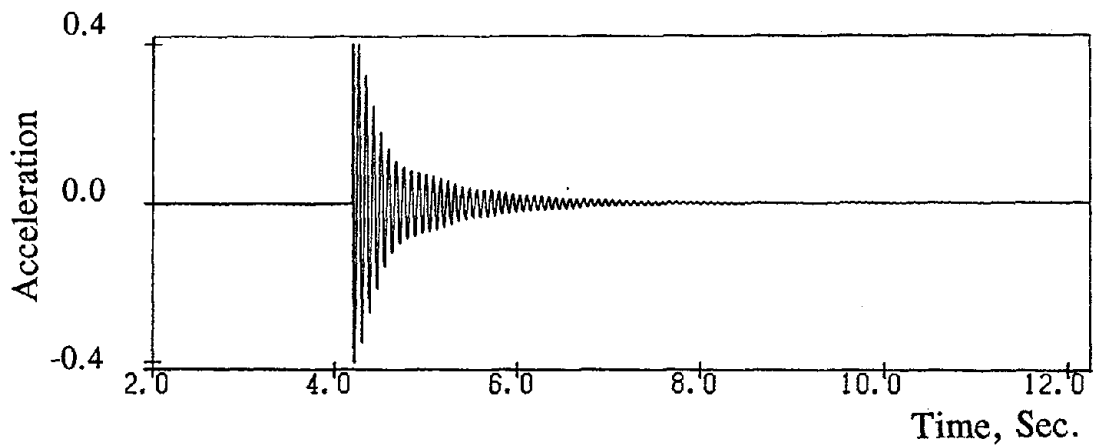


Fig. 3.4 - Typical Free-Vibration Response for the Beam Specimen.

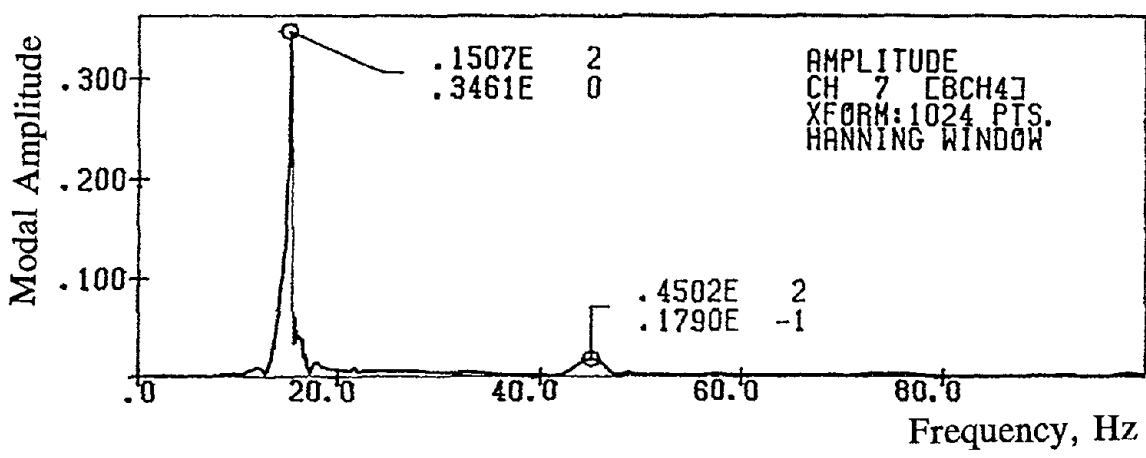


Fig. 3.5 - Typical FFT Result for Impact at Quarter Point.

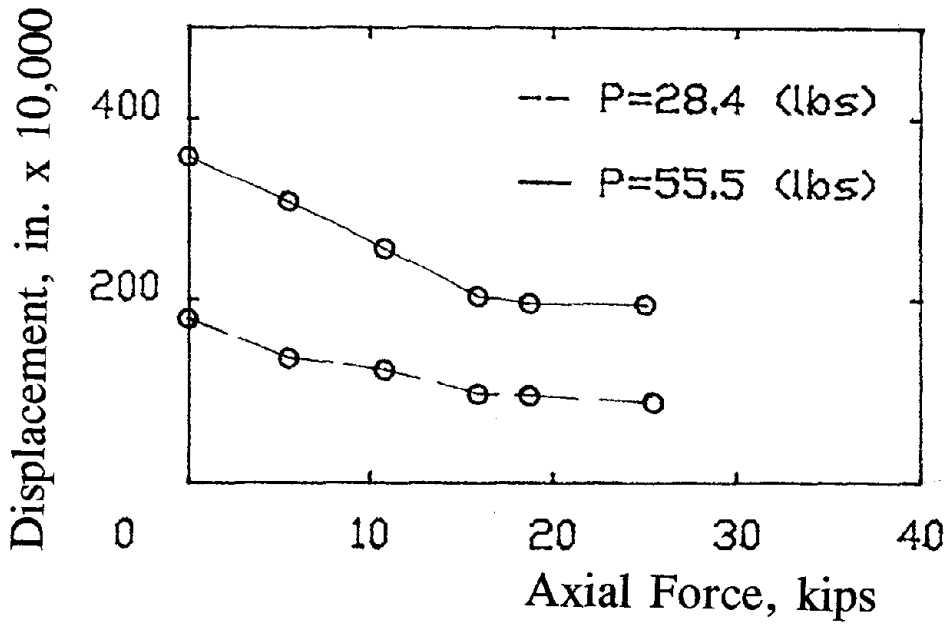


Fig. 3.6 - Variation of Mid Span Deflection in Terms of Prestress Force.

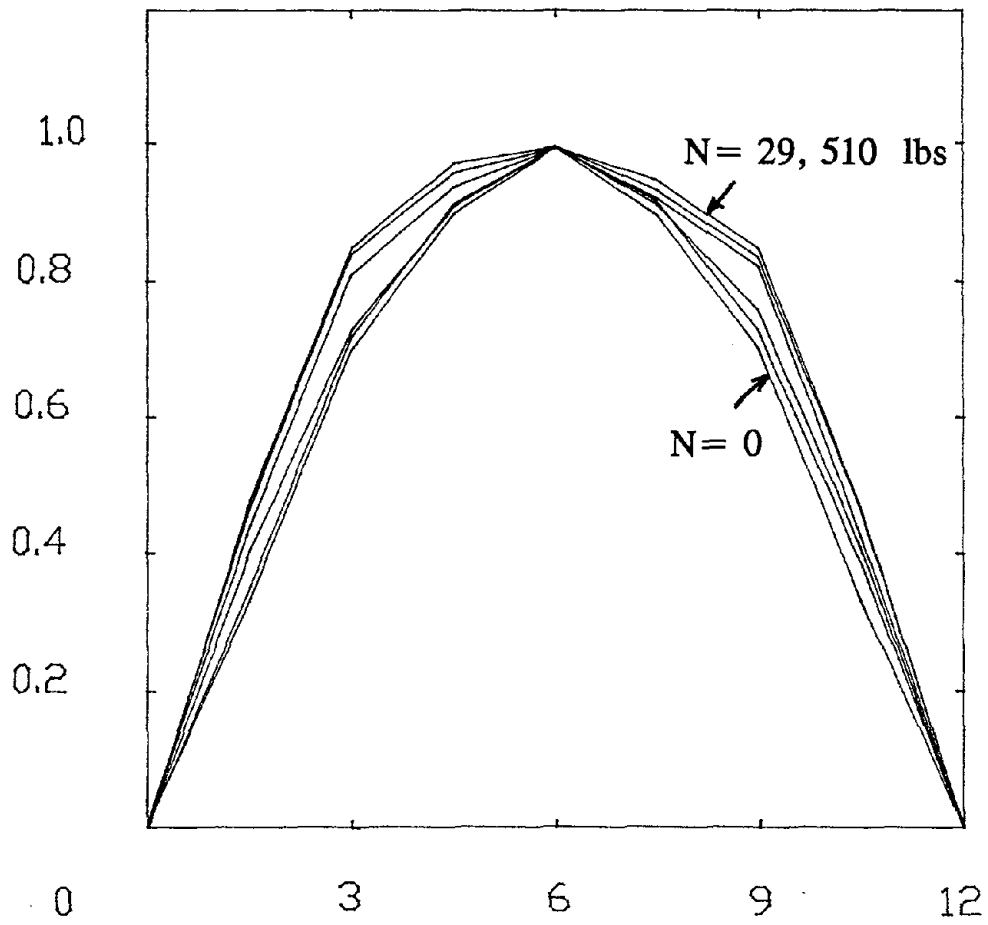


Fig. 4.1 - Effect of Axial Load on the Fundamental Mode Shape of the Beam.

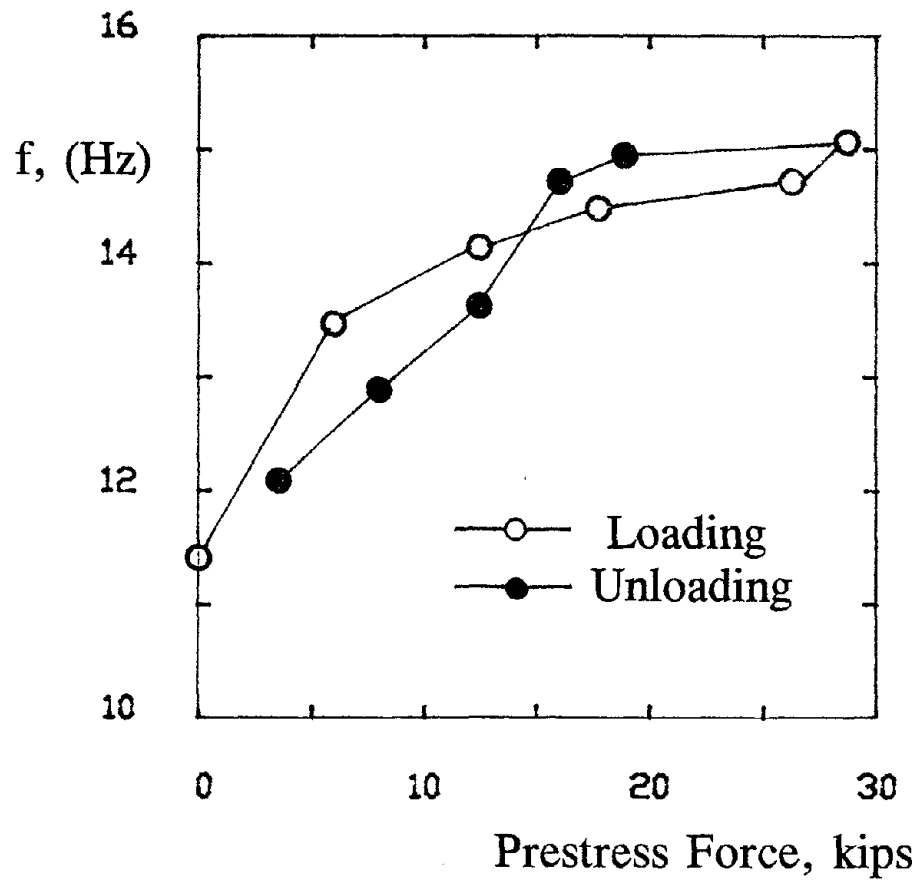


Fig. 4.2 - Effect of Axial Load on the First Mode Frequency of the Beam.

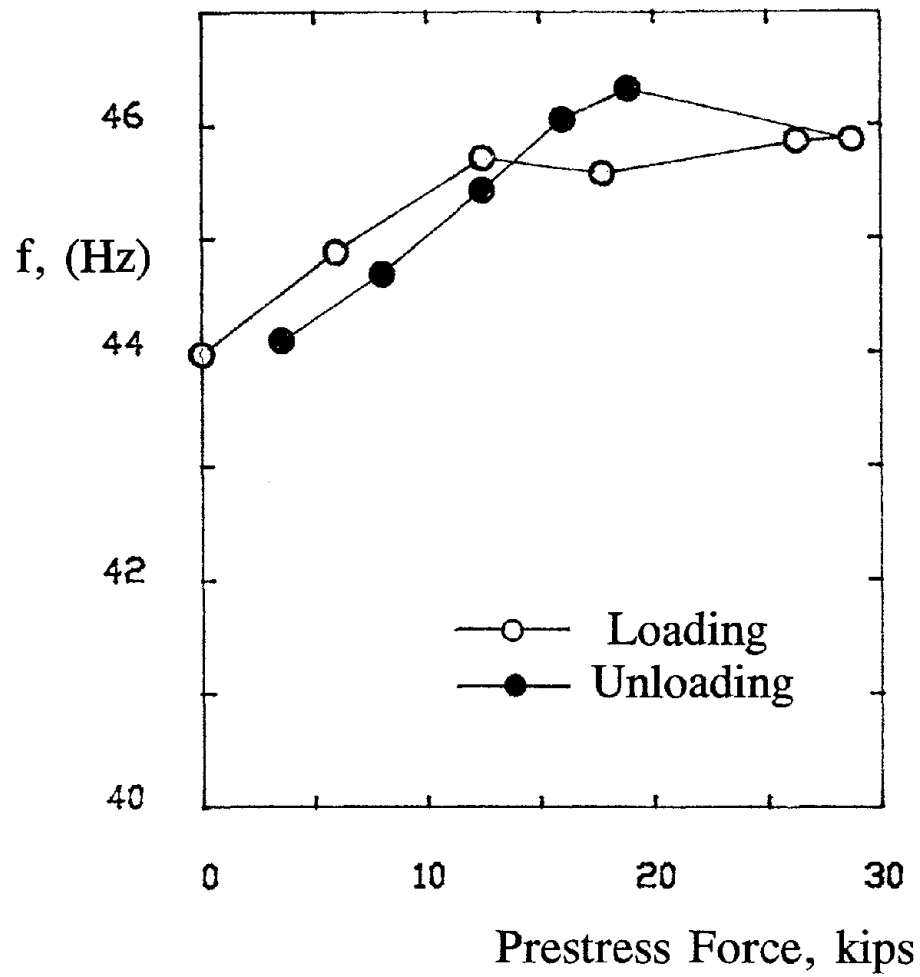


Fig. 4.3 - Effect of Axial Load on the Second Mode Frequency of the Beam.

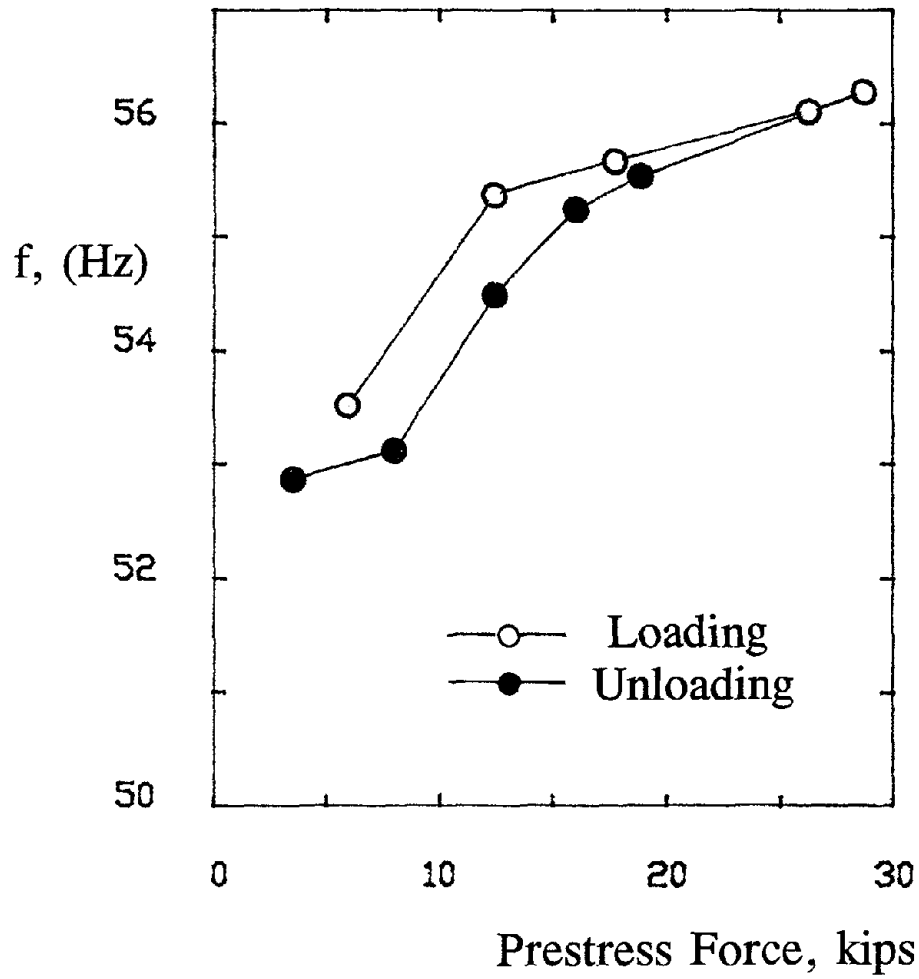


Fig. 4.4 - Effect of Axial Load on the Third Mode Frequency of the Beam.

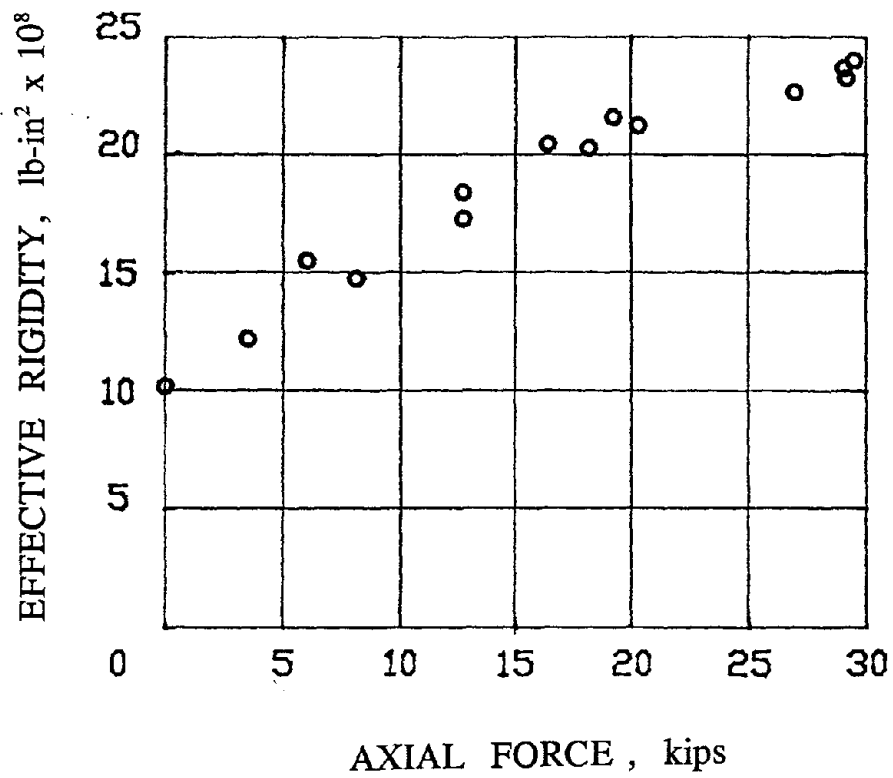


Fig. 4.5 - Variation of Rigidity in Terms of Prestress Force.

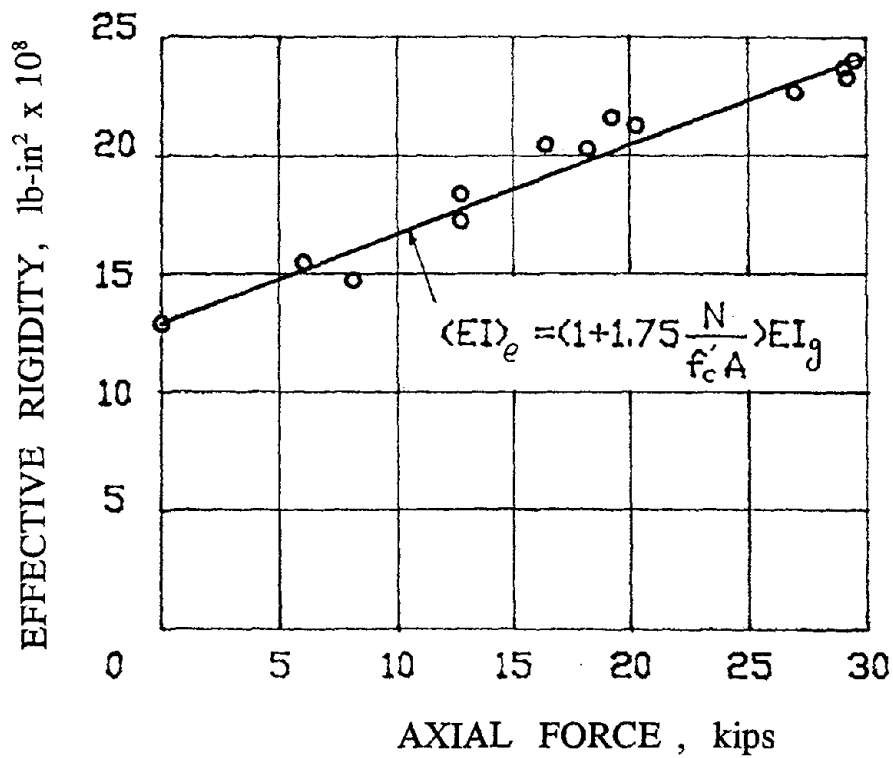


Fig. 4.6 - Least Square Fit of Rigidity in Terms of Prestress Force.

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