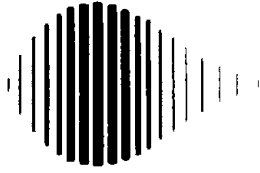


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**NATIONAL CENTER FOR EARTHQUAKE  
ENGINEERING RESEARCH**

State University of New York at Buffalo

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**A Generalization of Optimal Control Theory:  
Linear and Nonlinear Structures**

by

**J. N. Yang, Z. Li and S. Vongchavalitkul**

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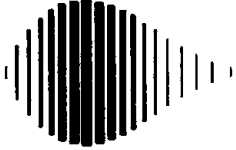
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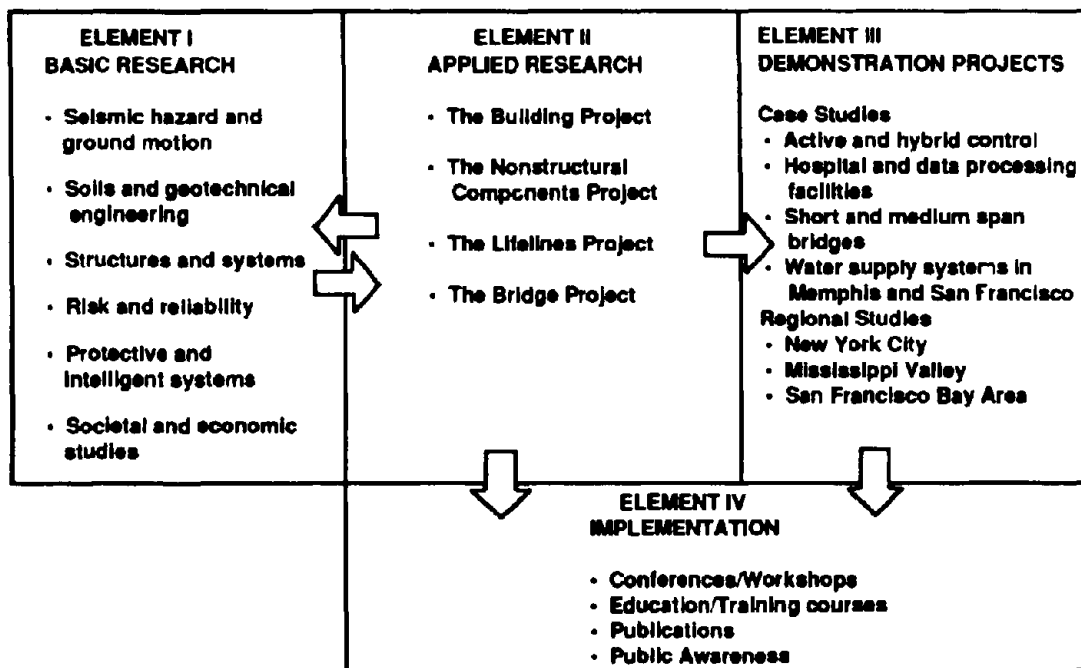
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## PREFACE

The National Center for Earthquake Engineering Research (NCEER) was established to expand and disseminate knowledge about earthquakes, improve earthquake-resistant design, and implement seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures in the eastern and central United States and lifelines throughout the country that are found in zones of low, moderate, and high seismicity.

NCEER's research and implementation plan in years six through ten (1991-1996) comprises four interlocked elements, as shown in the figure below. Element I, Basic Research, is carried out to support projects in the Applied Research area. Element II, Applied Research, is the major focus of work for years six through ten. Element III, Demonstration Projects, have been planned to support Applied Research projects, and will be either case studies or regional studies. Element IV, Implementation, will result from activity in the four Applied Research projects, and from Demonstration Projects.



Research in the **Building Project** focuses on the evaluation and retrofit of buildings in regions of moderate seismicity. Emphasis is on lightly reinforced concrete buildings, steel semi-rigid frames, and masonry walls or infills. The research involves small- and medium-scale shake table tests and full-scale component tests at several institutions. In a parallel effort, analytical models and computer programs are being developed to aid in the prediction of the response of these buildings to various types of ground motion.

Two of the short-term products of the **Building Project** will be a monograph on the evaluation of lightly reinforced concrete buildings and a state-of-the-art report on unreinforced masonry.

The **protective and intelligent systems program** constitutes one of the important areas of research in the **Building Project**. Current tasks include the following:

1. Evaluate the performance of full-scale active bracing and active mass dampers already in place in terms of performance, power requirements, maintenance, reliability and cost.
2. Compare passive and active control strategies in terms of structural type, degree of effectiveness, cost and long-term reliability.
3. Perform fundamental studies of hybrid control.
4. Develop and test hybrid control systems.

*Algorithm development is a major activity at NCEER in active control. This report presents results which are generalizations of the instantaneous optimal control law developed by the first author. The generalizations include the effect of actuator dynamics and a penalty on the acceleration response of the structure. Both linear and nonlinear structures are considered. In the latter case, an optimal nonlinear control law is proposed.*

## ABSTRACT

A systematic generalization of the theory of optimal control for seismic-excited linear, nonlinear and hysteretic structures is presented. The generalized theory includes the effect of actuator dynamics and a penalty on the acceleration response of the structure. In protecting building structures against strong earthquakes or severe wind gusts, both the deformation and the acceleration response of the structure are important quantities to the designer. The proposed generalized performance index includes the acceleration response so that either a simultaneous reduction of the deformation and acceleration or a trade-off between them can be achieved. Experimental results indicate that a significant contribution to the system time delay comes from the actuator response. In this report, the actuator dynamics is explicitly accounted for in the optimization process so that the gain matrix involves actuator characteristics leading to a better control performance. In Part I, the generalization of the optimal control theory for linear structures is presented. Numerical simulation results are obtained to demonstrate the advantages of the generalized optimal control theory. In Part II, an optimal nonlinear control method is proposed for applications to nonlinear and hysteretic structures. The proposed nonlinear control method is based on a generalized performance index. Both the absolute acceleration vector of the structural response and the actuator dynamics are taken into account in the optimization process. The control method using acceleration and velocity feedbacks are also derived. Extensive simulation results indicate that the proposed nonlinear control method is effective for hybrid control of seismic-excited building structures.

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**A GENERALIZATION OF OPTIMAL CONTROL THEORY:  
LINEAR CONTROL**

**PART I**

## SECTION 1 INTRODUCTION

Considerable progress has been made recently in active and hybrid control of civil engineering structures subjected to environmental loads [e.g., 1-26]. Large-scale laboratory experiments and full-scale demonstrations have been conducted [e.g., 4,6,7,10,12]. Further, control of secondary systems housed in the buildings has also been investigated [e.g., 3,5]. Various active control systems have been proposed to protect building structures against earthquakes or strong wind gusts [e.g., 9,10,19]. For safety, integrity and serviceability of the building, both the deformation of each story unit and the absolute acceleration of each floor must be controlled. Structural failure occurs when the deformation of any story unit exceeds a certain ultimate limit. On the other hand, the building may house many secondary systems (or nonstructural components) and the failure of these secondary systems may have serious consequences. For instance, the secondary systems may include sensitive equipments in hospitals, communication centers, computer rooms, vibration-sensitive equipments, etc. These secondary systems may be disrupted or damaged due to a high level of floor accelerations [e.g. 3,5]. This is because the absolute floor acceleration is the input excitation to the secondary systems housed in the building. Thus, control of floor accelerations is also important.

Under strong wind gusts, the safety of tall buildings usually is not an important issue. Rather, the comfort of occupants or tenants is of great concern. It has been shown in the literature that the comfort of occupants is closely related to the floor acceleration and, hence, the acceleration response of tall buildings subjected to strong wind gusts [e.g., 8,14] should be controlled.

Based on the discussion above, both the deformation of each story unit and the absolute acceleration of each floor are important quantities to be controlled. Unfortunately, optimal control theories usually deal with the performance index that involves only the deformation and velocity of the structural response. Although some literatures also include the acceleration response in the performance index, a systematic formulation and approach has not been considered to date. In this report, the optimal control theory will be



generalized to include the absolute acceleration of each floor in the performance index so that a penalty can be imposed on the acceleration response.

A system time delay always exists in active control of engineering structures, in particular when large actuators are required. It has been demonstrated that a system time delay can be detrimental to control of buildings subjected to strong earthquakes [e.g., 6,20]. Practical implementations of active control systems indicate that the major contribution to the entire system time delay comes from the inability of large actuators to react fast enough to the feedback signals [e.g., 7,12]. Hence, the time delay problem can be overcome partially by taking into account the actuator dynamics in the optimization process and the design of controllers. In this report, the optimal control theory is extended to incorporate the actuator dynamics.

Extensive simulation results are obtained to demonstrate the advantages of taking into account both the actuator dynamics and the acceleration response of the structure. Numerical results indicate that (i) with the inclusion of the acceleration response in the performance index, it is possible to substantially reduce the acceleration response of any particular floor which can not be achieved otherwise, and (ii) with the inclusion of the actuator dynamics in the optimization process, the degradation of the control performance, resulting from the actuator response, becomes insignificant. Without accounting for the actuator dynamics, the controlled structure may become unstable unless other compensation methods are used.

## SECTION 2

### EQUATIONS OF MOTION

Consider an  $n$  degrees of freedom linear building structure subjected to a one-dimensional earthquake ground acceleration  $\ddot{X}_0(t)$ . The vector equation of motion is given by

$$\underline{M}\ddot{\underline{X}}(t) + \underline{C}\dot{\underline{X}}(t) + \underline{K}\underline{X}(t) = \underline{H}_1\underline{U}(t) + \underline{\xi}\ddot{X}_0(t) \quad (2.1)$$

in which  $\underline{X}(t)=[x_1, x_2, \dots, x_n]'$  = an  $n$ -vector with  $x_j(t)$  being the deformation of the  $j$ th story unit,  $\underline{U}(t)$  = a  $r$ -dimensional vector consisting of  $r$  control forces,  $\underline{\xi}=-[m_1, m_2, \dots, m_n]'$  = a mass vector. In Eq. (2.1),  $\underline{M}$  is an  $(n \times n)$  mass matrix with the  $i$ - $j$ th element  $M(i,j)=m_i$  for  $j \leq i$  and  $M(i,j)=0$  for  $j > i$ , where  $m_i$  is the mass of the  $i$ th floor.  $\underline{C}$  and  $\underline{K}$  are  $(n \times n)$  band-limited damping and stiffness matrices, respectively, with all elements equal to zero except  $C(i,i)=c_i$ ,  $K(i,i)=k_i$  for  $i=1,2,\dots,n$  and  $C(i,i+1)=-c_{i+1}$ ,  $K(i,i+1)=-k_{i+1}$  for  $i=1,2,\dots,n-1$ , where  $c_i$  and  $k_i$  are the damping coefficient and the stiffness, respectively, of the  $i$ th story unit.  $\underline{H}_1$  is an  $(n \times r)$  matrix denoting the location of  $r$  controllers. In the notation above, an under bar denotes either a vector or a matrix and a prime indicates the transpose of either a matrix or a vector.

In the state vector form, Eq. (2.1) becomes

$$\dot{\underline{Z}}(t) = \underline{A}\underline{Z}(t) + \underline{B}\underline{U}(t) + \underline{W}_1\ddot{X}_0(t) \quad (2.2)$$

in which  $\underline{Z}(t)$  is a  $2n$ -dimensional state vector,  $\underline{A}$  is a  $(2n \times 2n)$  system matrix,  $\underline{B}$  is a  $(2n \times r)$  matrix and  $\underline{W}_1$  is a  $2n$  vector.

$$\underline{Z}(t) = \begin{bmatrix} \underline{X}(t) \\ \dot{\underline{X}}(t) \end{bmatrix}; \underline{B} = \begin{bmatrix} \underline{0} \\ \underline{M}^{-1}\underline{H} \end{bmatrix}; \underline{W}_1 = \begin{bmatrix} \underline{0} \\ \underline{M}^{-1}\underline{\xi} \end{bmatrix}; \underline{A} = \begin{bmatrix} \underline{0} & \underline{I} \\ -\underline{M}^{-1}\underline{K} & -\underline{M}^{-1}\underline{C} \end{bmatrix} \quad (2.3)$$

**SECTION 3**  
**OPTIMAL CONTROL INCLUDING ACTUATOR DYNAMICS**

In this section, the actuator dynamics will be taken into account in the derivation of the optimal control law. For simplicity, the dynamic equations for  $r$  actuators are described by a system of first order differential equations

$$\dot{\underline{U}}(t) + \underline{a}\underline{U}(t) = \underline{b}\underline{q}(t) \quad (3.1)$$

in which  $\underline{a}$  and  $\underline{b}$  are  $(r \times r)$  diagonal matrices with diagonal elements  $a_i$  and  $b_i$  ( $i=1,2,\dots,r$ ).  $a_i$  is a measure of the loop gain or the reaction time of the  $i$ th actuator, and  $b_i$  is a measure of feedback gain or the amplification factor of the  $i$ th actuator. In Eq. (3.1),  $\underline{q}(t)$  is a  $r$ -vector representing the feedback signal (or dynamic input) for generating the required active control vector  $\underline{U}(t)$ . Note that the vector  $\underline{q}(t)$  is proportional to the control vector  $\underline{U}(t)$  and it will be determined later through the optimization process. It should be mentioned that the extension of the optimal control theory in the following is not restricted to the first order differential equation, Eq. (3.1), for the actuator dynamics. Other higher order differential equations can similarly be used.

The dynamic equation of actuators, Eq. (3.1), can be augmented to the state equation of motion, Eq. (2.2), and both can be casted into the following  $(2n+r)$  vector equation

$$\dot{\underline{Z}}_1(t) = \bar{\underline{A}}\underline{Z}_1(t) + \bar{\underline{B}}\underline{q}(t) + \bar{\underline{H}}\ddot{\underline{X}}_0(t) \quad (3.2)$$

in which

$$\underline{Z}_1(t) = \begin{bmatrix} \underline{Z}(t) \\ \underline{U}(t) \end{bmatrix}; \bar{\underline{B}} = \begin{bmatrix} \underline{Q}_{2nr} \\ \underline{b} \end{bmatrix}; \bar{\underline{H}} = \begin{bmatrix} \underline{H}'_1 \\ \underline{Q}_{r1} \end{bmatrix}; \bar{\underline{A}} = \begin{bmatrix} \underline{A} & \underline{B} \\ \underline{Q}_{r2n} & -\underline{a} \end{bmatrix} \quad (3.3)$$

where  $\underline{Q}_{2nr}$ ,  $\underline{Q}_{r2n}$  and  $\underline{Q}_{r1}$  are  $(2n \times r)$ ,  $(r \times 2n)$  and  $(r \times 1)$  zero matrices, respectively.

The following LQR type performance index  $J$  is used

$$J = \int_0^{t_f} [\underline{Z}'_1(t)\bar{\underline{Q}}\underline{Z}_1(t) + \underline{q}'(t)\bar{\underline{R}}\underline{q}(t)] dt \quad (3.4)$$

in which  $t_f$  is a time longer than the duration of earthquakes,  $\bar{\underline{Q}}$  is a  $(2n+r) \times (2n+r)$  positive semidefinite weighting matrix and  $\bar{\underline{R}}$  is a  $(r \times r)$  positive definite weighting matrix, where

$$\bar{Q} = \begin{bmatrix} Q & 0_{2nr} \\ 0_{r2n} & R \end{bmatrix} \quad (3.4a)$$

Following the standard derivation procedures, one can minimize the performance index  $J$  given by Eq. (3.4) subjected to the constraint of the augmented equation of motion given by Eq. (3.2); with the results

$$q(t) = -0.5 \bar{R}^{-1} \bar{B}' \bar{P} Z_1(t) \quad (3.5)$$

where  $\bar{P}$  is an  $(2n+r) \times (2n+r)$  augmented Riccati matrix satisfying the following matrix Riccati equation

$$\bar{A}' \bar{P} + \bar{P} \bar{A} - 0.5 \bar{P} \bar{B} \bar{R}^{-1} \bar{B}' \bar{P} = -2\bar{Q} \quad (3.6)$$

in which the transient part of  $\bar{P}$  has been neglected, since the Riccati matrix establishes its stationary value rapidly.

Thus, the optimal response vector  $Z_1(t)$  that consists of the state vector  $Z(t)$  and the control vector  $U(t)$  can be simulated by substituting Eq. (3.5) into Eq. (3.2) with  $\bar{P}$  being computed from Eq. (3.6)

$$\dot{Z}_1(t) = [\bar{A} - 0.5 \bar{B} \bar{R}^{-1} \bar{B}' \bar{P}] Z_1(t) + \bar{H} \dot{X}_0(t) \quad (3.7)$$

To examine the control operation and the implication of optimal control, we substitute Eq. (3.3) into Eq. (3.5) to obtain the optimal feedback signal  $q(t)$  as follows

$$q(t) = -0.5 \left[ \bar{R}^{-1} \bar{B}' \bar{P}'_{12} Z(t) + \bar{R}^{-1} \bar{B}' \bar{P}'_{22} U(t) \right] \quad (3.8)$$

in which the augmented Riccati matrix  $\bar{P}$  is partitioned into the following submatrices

$$\bar{P} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}'_{12} & \bar{P}_{22} \end{bmatrix} \quad (3.9)$$

where the dimensions of  $\bar{P}'_{12}$  and  $\bar{P}_{22}$  are  $(rx2n)$  and  $(rxr)$ , respectively.

It follows from Eq. (3.8) that the input voltage signal  $q(t)$  to the actuator depends not only on the state vector  $Z(t)$  but also on the control vector  $U(t)$ . Hence, both  $Z(t)$  and  $U(t)$  should be measured or estimated. In practical applications, it is not possible to measure both  $Z(t)$  and  $U(t)$  for complex buildings. However, an observer can be established to construct

not only the state vector  $\mathbf{Z}(t)$  but also the control vector  $\mathbf{U}(t)$ .

Substituting Eq. (3.8) into Eq. (3.1), one obtains

$$\dot{\mathbf{U}}(t) + (a + 0.5b\bar{R}^{-1}b'\bar{P}_{22})\mathbf{U}(t) = -0.5b\bar{R}^{-1}b'\bar{P}'_{12}\mathbf{Z}(t) \quad (3.10)$$

Unlike Eq. (3.1) in which the actuator dynamic equations are uncoupled, Eq. (3.10) is coupled. Furthermore, it follows from Eq. (3.10) that, under optimal control, the loop gain or the reaction time of each actuator is modified.

**SECTION 4**  
**GENERALIZED PERFORMANCE INDEX**

In this section, the performance index will be generalized to include the absolute acceleration of each floor, so that a penalty can be imposed on the acceleration response. The generalized performance index for optimal control is expressed as follows

$$J = \int_0^T [Z'(t)QZ(t) + \dot{X}_a'(t)\Omega_a\dot{X}_a + U'(t)RU(t)] dt \quad (4.1)$$

in which  $\dot{X}_a(t)$  is an  $n$  vector consisting of the absolute acceleration of every floor and  $\Omega_a$  is a  $(nxn)$  symmetric positive semidefinite weighting matrix.  $\dot{X}_a(t)$  in Eq. (4.1) can be obtained easily from the state equation of motion, Eq. (2.1), as follows

$$M_0\ddot{X}_a(t) + C\dot{X}_a(t) + KX(t) = HU(t) \quad (4.2)$$

in which  $M_0$  is a  $(nxn)$  diagonal mass matrix with  $m_i$  being the  $i$ th diagonal element. In terms of the state vector  $Z(t)$ , it follows from Eq. (4.2) that

$$\dot{X}_a(t) = A_0Z(t) + B_0U(t) \quad (4.3)$$

in which  $A_0$  is a  $(nx2n)$  matrix and  $B_0$  is a  $(nxr)$  matrix defined as follows.

$$A_0 = [-M_0^{-1}K, -M_0^{-1}C] ; B_0 = M_0^{-1}H \quad (4.4)$$

Substitution of Eq. (4.3) into Eq. (4.1) leads to the following equivalent performance index expressed in terms of  $Z(t)$  and  $U(t)$ .

$$J = \int_0^T [Z'(t), U'(t)] T \begin{bmatrix} Z(t) \\ U(t) \end{bmatrix} dt \quad (4.5)$$

in which  $T$  is a  $(2n+r)$  by  $(2n+r)$  matrix

$$T = \begin{bmatrix} Q + A_0'Q_aA_0 & A_0'Q_aB_0 \\ \hline B_0'Q_aA_0 & R + B_0'Q_aB_0 \end{bmatrix} \quad (4.6)$$

It is observed from Eq. (4.5) that adding the absolute accelerations to the performance index introduces the coupling terms between the response state vector  $Z(t)$  and the control vector  $U(t)$ . Furthermore, the weighting matrix  $T$  appearing in Eq. (4.6) is a symmetric positive semidefinite matrix.

To minimize the performance index,  $J$ , given by Eq. (4.5) subjected to the constraint given by Eq. (2.2), the Hamiltonian  $H$  is formed as follows

$$H = [Z', U'] T \begin{bmatrix} Z \\ U \end{bmatrix} + \lambda' [AZ + BU + W_1 \ddot{X}_0 - \dot{Z}] \quad (4.7)$$

in which the argument  $t$  has been dropped for simplicity.

The necessary conditions for the optimal solution are

$$\frac{\partial H}{\partial \lambda} = 0 \quad ; \quad \frac{\partial H}{\partial U} = 0 \quad ; \quad \frac{\partial H}{\partial Z} + \dot{\lambda}' = 0 \quad (4.8)$$

The first condition  $\partial H/\partial \lambda$  leads to the state equation of motion given by Eq. (2.2). Substituting Eq. (4.7) into the second and third conditions of Eq. (4.8), letting

$$\lambda = \bar{P} Z \quad (4.9)$$

and disregarding  $\ddot{X}_0(t)$ , one obtains the following results

$$U(t) = -0.5 \bar{K}^{-1} (B' \bar{P} + 2B_0' Q_0 A_0) Z(t) \quad (4.10)$$

where  $\bar{P}$  is the Riccati matrix satisfying the following stationary matrix Riccati equation

$$\bar{A}' \bar{P} + \bar{P} \bar{A} - 0.5 \bar{P} B \bar{K}^{-1} B' \bar{P} = -2\bar{Q} \quad (4.11)$$

in which Eq. (4.6) has been used and  $\dot{\bar{P}}$  has been dropped,

$$\begin{aligned} \bar{K} &= R + B_0' Q_0 B_0 \\ \bar{A} &= A - B \bar{K}^{-1} B_0' Q_0 A_0 \\ \bar{Q} &= Q + A_0' Q_0 A_0 - A_0' Q_0 B_0 \bar{K}^{-1} B_0' Q_0 A_0 \end{aligned} \quad (4.12)$$

With the optimal solution  $U(t)$  given by Eq. (4.10), the response state vector  $Z(t)$  can be simulated by substituting Eq. (4.10) into Eq. (2.2) as follows

$$\dot{Z}(t) = [A - 0.5 B \bar{K}^{-1} (B' \bar{P} + 2B_0' Q_0 A_0)] Z(t) + W_1 \ddot{X}_0(t) \quad (4.13)$$

in which  $\bar{K}$  and  $B_0$  are given by Eqs. (4.12) and (4.4), respectively, and  $\bar{P}$  is computed from Eq. (4.11).

When the absolute acceleration response quantities are excluded from the performance index, i.e.,  $Q_a = Q_{nm}$ , it can be shown easily that Eqs. (4.10) to (4.12) reduce to the standard LQR solutions.

**SECTION 5**  
**GENERALIZED OPTIMAL CONTROL**

In this section, the theory of optimal control is generalized to include both the actuator dynamics and the generalized performance index. The vector equation of motion of the building given by Eq. (2.1) and the actuator dynamic equation given by Eq. (3.1) can be combined into a system given by Eq. (3.2),

$$\dot{Z}_1(t) = \bar{A}Z_1(t) + \bar{B}q(t) + \bar{W}\ddot{X}_0(t) \quad (3.2)$$

in which

$$Z_1(t) = \begin{bmatrix} Z(t) \\ U(t) \end{bmatrix}; \bar{B} = \begin{bmatrix} 0_{2n} \\ b \end{bmatrix}; \bar{W} = \begin{bmatrix} W_1 \\ 0_{r,1} \end{bmatrix}; \bar{A} = \begin{bmatrix} A & B \\ 0_{r,2n} & -a \end{bmatrix} \quad (3.3)$$

The generalized performance index  $J$  can be expressed as

$$J = \int_0^T \{ Z_1'(t) \bar{Q} Z_1(t) + \ddot{X}_a'(t) Q_a \ddot{X}_a(t) + q'(t) \bar{R} q(t) \} dt \quad (5.1)$$

in which  $\bar{Q}$  and  $Q_a$  are  $(2n+r) \times (2n+r)$  and  $(n \times n)$  symmetric positive semidefinite weighting matrices, respectively, and  $\bar{R}$  is a  $(r \times r)$  symmetric positive definite weighting matrix. In Eq. (5.1),  $\ddot{X}_a(t)$  is an absolute acceleration vector given by Eq. (4.3), i.e.,

$$\ddot{X}_a(t) = A_0 Z(t) + B_0 U(t) \quad (4.3)$$

Substituting Eq. (4.3) into Eq. (5.1) and using Eq. (3.4a), one obtains the following equivalent generalized performance index

$$J = \int_0^T \{ Z_1'(t) T Z_1(t) + q'(t) \bar{R} q(t) \} dt \quad (5.2)$$

in which the  $T$  matrix is defined in Eqs. (4.4) and (4.6).

Following the same optimization procedures described in the previous sections, the optimal solution is obtained as

$$q(t) = -0.5 \bar{R}^{-1} \bar{B}' \hat{P} Z_1(t) \quad (5.3)$$

in which the  $(2n+r) \times (2n+r)$  Riccati matrix  $\hat{P}$  satisfies the following matrix Riccati equation

$$\bar{A}' \hat{P} + \hat{P} \bar{A} - 0.5 \hat{P} \bar{B} \bar{R}^{-1} \bar{B}' \hat{P} = -2T \quad (5.4)$$

Thus, the optimal response vector  $Z_1(t)$  can be simulated by substituting Eq. (5.3) into Eq.



(3.2); with the result

$$\dot{Z}_1(t) = [\bar{A} - 0.5\bar{B}\bar{R}^{-1}\bar{E}'\hat{P}]Z_1(t) + \bar{W}\dot{x}_0(t) \quad (5.5)$$

The optimal feedback signal,  $q(t)$ , to the actuators given by Eq. (5.3) can be written as

$$q(t) = -0.5[\bar{R}^{-1}b'\hat{P}'_{12}Z(t) + \bar{R}^{-1}b'\hat{P}_{22}U(t)] \quad (5.6)$$

in which Eq. (3.3) has been used and

$$\hat{P} = \begin{bmatrix} \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}'_{12} & \hat{P}_{22} \end{bmatrix} \quad (5.7)$$

where the dimensions of  $\hat{P}'_{12}$  and  $\hat{P}_{22}$  are  $(rx2n)$  and  $(rxr)$ , respectively. The dynamic equations for the actuators are obtained by substituting Eq. (5.6) into Eq. (3.1) as

$$\dot{U}(t) + (a + 0.5b\bar{R}^{-1}b'\hat{P}_{22})U(t) = -0.5b\bar{R}^{-1}b'\hat{P}'_{12}Z(t) \quad (5.8)$$

## SECTION 6 SIMULATION RESULTS

### Example 1: A Six-Story Building Equipped with Active Bracing System

To demonstrate the advantages of taking into account the actuator dynamics, a six-story full-scale linear building recently constructed in Tokyo, Japan by Takenaka Company as shown schematically in Fig. 6-1(a) is considered first [e.g. Refs. 7,12]. The building is equipped with an active bracing system on the first floor. The mass of each floor is identical and is equal to 100 metric tons. The natural frequencies of the building are 0.943, 2.765, 4.876, 7.279, 10.114 and 14.423 Hz and the damping ratio for each vibrational mode is 1%. Detailed structural characteristics are given in Refs. 7 and 12. The El Centro earthquake record scaled to a maximum ground acceleration of 0.3g as shown in Fig. 6-2 is used as the input excitation.

Without any control system, the maximum interstory deformation,  $x_i$  ( $i=1,2,\dots,6$ ) of each story unit and the maximum absolute acceleration  $\ddot{x}_{ai}$  of each floor, in the entire earthquake episode, are shown in columns (2) and (3) of Tables 6.1 and 6.2. With the active bracings system (ABS) in which the angle of inclination  $\theta$  of the active bracing is  $51.5^\circ$ , the building response and the required active control force depend on the weighting matrices.

We first consider the ideal case in which the actuator can respond instantly to the input command without any time delay. No penalty for the acceleration response of the structure is made, i.e.,  $Q_a=0$ . For the present case with only one controller, the  $R$  matrix consists of only one element, denoted by  $R_0$ . The (12x12)  $Q$  matrix in Eq. (4.1) is considered a diagonal matrix with diagonal elements as follows:  $Q(1,1)=10$ ,  $Q(2,2)=5$ ,  $Q(i,i)=1$  for  $i=3,4,\dots,12$ . Time histories of all the response quantities have been computed for  $R_0=7 \times 10^{-8}$ . Within thirty seconds of the earthquake episode, the maximum interstory deformation of each story unit,  $x_i$ , and the maximum acceleration of each floor are presented in columns (4) and (5) of Table 6.1, designated as Case (A). Also shown in Table 6.1 is the maximum active control force  $U=1159\text{kN}$ . To further reduce the response quantities, the diagonal elements of the  $Q$  matrix and  $R_0$  are chosen as follows:  $Q(1,1)=150$ ,  $Q(2,2)=10$ ,  $Q(i,i)=1$  for  $i=3,4,\dots,12$  and  $R_0=10^{-10}$ . The corresponding maximum

response quantities of the building as well as the required maximum control force are presented in columns (4) and (5) of Table 6.2. It is observed from Tables 6.1 and 6.2 that a larger reduction in the response quantities is achieved by use of a larger control force.

We next consider the case in which the actuator is not perfect and its response follows the actuator dynamics given by Eq. (3.1). The actuator dynamics is taken into account in the optimization process as presented in Sections III and V. In the following discussion, we shall use the notations  $\alpha$  and  $\beta$  for  $a_1$  and  $b_1$  in Eq. (3.1), respectively, i.e.,  $a_1 = \alpha$  and  $b_1 = \beta$ . Based on the sinusoidal input-output relation, the amount of the time delay  $\Delta\tau$  due to the actuator response is  $\Delta\tau = \omega^{-1} \tan^{-1}(\omega/\alpha)$  in which  $\omega$  is the sinusoidal frequency in radian per second. Considering the first natural frequency of the building of 0.943 Hz i.e.  $\omega = 5.925$  rad./sec., the time delay  $\Delta\tau$ , expressed in terms of the percentage of the fundamental period of the structure, can be computed as a function of  $\alpha$ . The general trend is that the smaller the  $\alpha$  value the larger the time delay  $\Delta\tau$ . To preserve the same amplitude for the input and output signal,  $\beta$  is set to be equal to  $\alpha$ , i.e.,  $\beta = \alpha$ . The same Q matrix in Case (A) is used, whereas the R matrix, Eq. (3.4a), is zero and the  $\bar{R}$  matrix, Eq. (3.4), is assigned, i.e.,  $R_0 = 0$  and  $\bar{R}_0$  is assigned.

Within thirty seconds of the earthquake episode, the maximum response quantities of the building and the required maximum control force are presented in Tables 6.1 and 6.2, designated as Case (B), for different values of  $\alpha$  and  $\bar{R}_0$ . The corresponding time delay  $\Delta\tau$ , expressed in terms of the percentage of the fundamental period of the structure, is also shown in these two tables for comparison. In Tables 6.1 and 6.2, the values of  $\alpha$  and hence  $\Delta\tau$  represent the characteristics of the actuator. The value of  $\bar{R}_0$  is chosen such that the required maximum control force is approximately the same as that of Case (A) for the purpose of comparing the control performance.

It is observed from Tables 6.1 and 6.2 that the degradation of the control performance due to actuator response is minimal. It is indeed very plausible that even with a time delay of 14% ( $\alpha = 5$ ) of the fundamental period of the structure, the degradation of the control performance is very small as long as the actuator dynamics is taken into account.

We next consider the case in which the actuator dynamics is not accounted for in the optimization process and no penalty for the acceleration response of the structure is made,

i.e.,  $\underline{Q}_a = \underline{Q}$ . In this case, the  $\bar{\mathbf{R}}$  matrix is zero and  $\mathbf{R}_0$  is assigned. Optimal control obtained in Eq. (4.10) is used as the input (command) signal  $q(t)$ , i.e.,  $q(t) = -0.5\mathbf{R}^{-1}\mathbf{B}'\mathbf{P}\mathbf{Z}(t)$ , in which the Riccati Matrix  $\mathbf{P}$  is computed from Eq. (4.11). Then, the control vector  $\underline{\mathbf{U}}(t)$  computed from Eq. (3.1) is applied to the structure. As observed from Eq.(3.1), the control vector  $\underline{\mathbf{U}}(t)$  is approximately equal to the input signal  $q(t)$  if the time delay is small, i.e.,  $\alpha = \beta$  is a large value.

With the  $\mathbf{Q}$  matrix being identical to that used in Case (B) above, the simulation results for the maximum response quantities are presented in Table 6.3 for different values of  $\alpha$  and  $\mathbf{R}_0$ . The controlled structure becomes unstable when the time delay exceeds,  $\Delta\tau$ , 2% of the fundamental period, i.e.,  $\alpha = 50$ , as shown in columns (14) and (15). Table 6.3 indicates that the degradation of the control performance is negligible for a time delay smaller than 2% (or  $\alpha \leq 50$ ) and the controlled structure suddenly becomes unstable when the time delay exceeds 2% of the fundamental period. The same behaviors were observed for the results corresponding to those given in Table 6.2. Time histories of the deformation of the first story unit and the absolute acceleration of the first floor are presented in Fig. 6-3.

A careful examination of the response behavior for  $\alpha = 50$  indicates that instability comes from negative dampings in the higher modes, i.e., eigen values with negative real parts. This comes from the interaction between the actuator and structural characteristics as can be shown from the augmented equation of motion. This type of instability actually occurs in laboratory experiments as pointed out to us by Professor A.M. Reinhorn of NCEER, Buffalo, where extensive laboratory experiments have been carried out. The situation is expected to be worse when more than one actuator (or controller) is used.

To alleviate such an instability problem, several possible compensation schemes can be used [Refs. 6-7]. One possible approach is to filter out the high frequency components of the resulting control force associated with negative dampings. This approach requires a prior knowledge of the interaction behavior of the actuator and the structure. Another approach is to make some kind of compensations for the time delay as described in Ref. 6. In any case, a definite degradation of the control performance is expected. This clearly indicates the importance of taking into account the actuator dynamics in the design of the controllers as well as the advantage of the optimal control method presented.

### **Example 2: An Eight-Story Building Equipped with an Active Mass Damper**

To illustrate the advantages of the proposed generalized performance index for optimal control, an eight-story building, in which every story unit is identically constructed, is considered. The structural properties of each story unit are:  $m$  = floor mass = 345.6 tons;  $k$  = elastic stiffness of each story unit =  $3.404 \times 10^5$  kN/m; and  $c$  = damping coefficient of each story unit = 2,937 tons/sec, which corresponds to a 2% damping for the first vibrational mode of the entire building [Ref. 17].

The same El Centro earthquake ground acceleration shown in Fig. 6-2 is used as the input excitation. Without any control system, time histories of the structural response quantities have been computed. Within 30 seconds of the earthquake episode, the maximum interstory deformation,  $x_i$ , and the maximum absolute acceleration of each floor,  $\ddot{x}_{ai}$ , for  $i=1,2,\dots,8$ , are presented in columns (2) and (3) of Table 6.4.

An active mass damper is installed on the top floor of the building as shown in Fig. 6-1(b). The properties of the active mass damper system are:  $m_d$  = mass of the damper = 29.63 tons,  $c_d$  = damping of the damper = 25 tons/sec, and  $k_d$  = stiffness of the damper = 957.2 kN/m. Note that the mass  $m_d$  is 2% of the generalized mass associated with the first vibrational mode, and the damping coefficient of the damper is approximately 7.3%. Without the active control force, the mass damper is passive. It has been shown that the passive mass damper is not effective for earthquake hazard mitigations. With the active mass damper, the building response depends on the weighting matrices  $Q$ ,  $Q_a$  and  $R$ . In the present situation, the weighting matrix  $R$  consists of only one element, i.e.,  $R=R_0$ , whereas the dimension of the  $Q$  and  $Q_a$  matrices are (18x18) and (9x9), respectively.

We first consider the case in which no penalty is imposed on the acceleration response, i.e., the acceleration response is not included in the performance index and hence  $Q_a=Q$ . It is assumed that the time delay due to the actuator response is negligible, i.e.,  $\alpha=\beta$  = a large value. The (18x18)  $Q$  matrix is chosen to be a diagonal matrix with  $Q(i,i)=1$  for  $i=1, 2, \dots, 8$  and  $Q(i,i)=0$  for  $i=9,10,\dots,18$ . Within the entire earthquake episode, the maximum response quantities,  $x_i$  and  $\ddot{x}_{ai}$  ( $i=1,2,\dots,8$ ) for  $R_0=4 \times 10^{-10}$  are summarized in the columns (4) and (5) of Table 6.4, designated as Case (A). Also shown in the table is the required maximum control force  $U$ . A significant reduction of the

structural response has been observed from the table.

To reduce the absolute acceleration of each floor in order to protect equipments housed in the building, the (9x9)  $Q_a$  matrix is chosen to be a diagonal matrix with all diagonal elements equal to 2000 except  $Q_a(9,9)=0$  which corresponds to the acceleration of the mass damper. With the same  $Q$  matrix and  $R_0=0.03$ , the maximum response quantities and the required maximum control force within the entire earthquake episode are presented in columns (6) and (7) of Table 6.4, designated as Case (I). As observed from Table 6.4, the required maximum control forces for both Case (A) and Case (I) are about the same, but the acceleration response quantities in Case (I) are smaller than those in Case (A).

Suppose many vibration-sensitive equipments are installed on the sixth floor and the protection of these equipments against earthquakes is most important. In this case the same  $Q$  matrix as in Case (I) is used; however, a large penalty is placed on the acceleration of the sixth floor. As a result, all the diagonal elements of the diagonal matrix  $Q_a$  are zero except  $Q_a(6,6)=2000$ . For  $R_0=0.001$ , the maximum response quantities of the structure and the required maximum control force are shown in columns (8) and (9) of Table 6.4, designated as Case (II).

As observed from Table 6.4, the absolute acceleration of the sixth floor is reduced to 186 cm/sec<sup>2</sup>. Nevertheless, the reduction of the acceleration of the sixth floor is achieved at the expense of the required control force. The ability of the generalized performance index to reduce the acceleration response of a particular floor of the building is clearly demonstrated in Table 6.4.

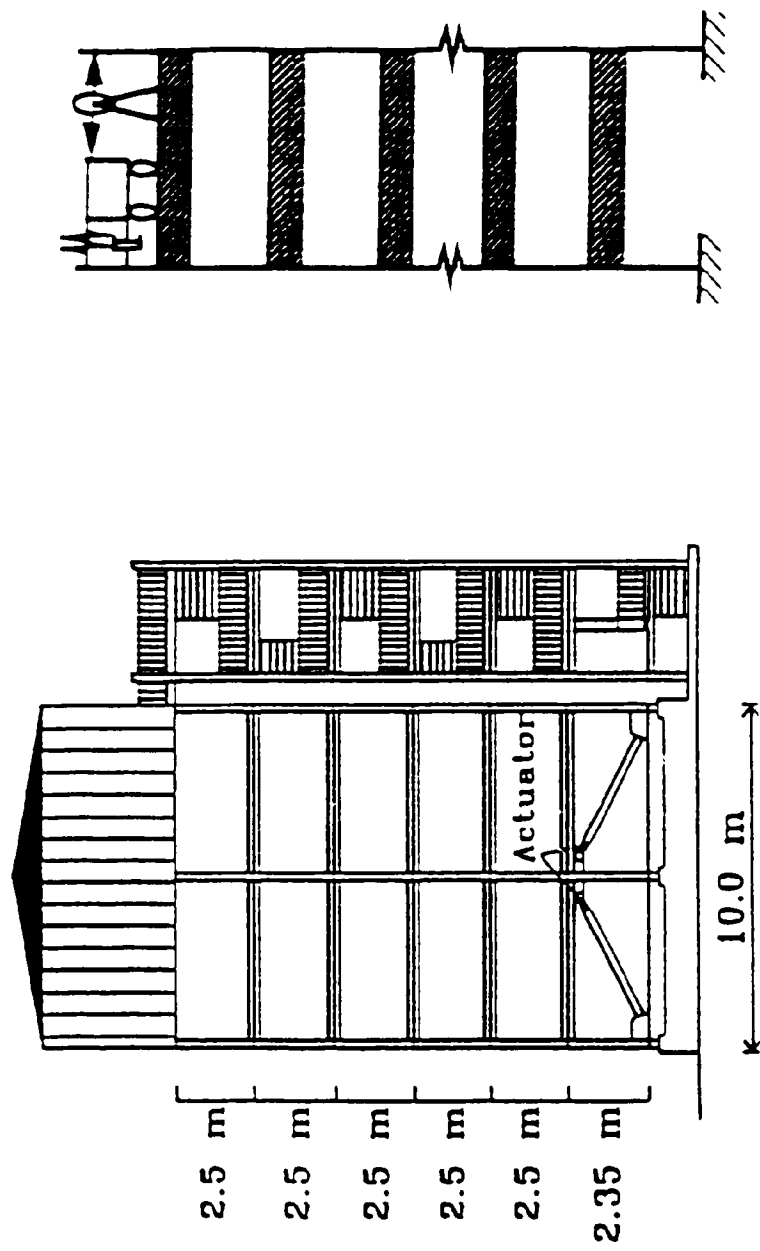
### **Example 3: A Simplified Bridge Model**

Control of seismic-excited bridge structures has attracted attention recently [e.g., Refs. 1, 27]. A long-span bridge deck supported by rubber bearings is modeled as a SDOF system [1] with the mass  $m=1.02 \times 10^6$  kg, stiffness  $k=3.286 \times 10^6$  kg/m and damping coefficient  $c=0.366 \times 10^6$  kg.sec/m. With these properties, the natural frequency is 1.795 rad/sec and the damping ratio is 10%. An actuator can be attached between the bridge deck and the pier to reduce the response of the bridge deck. The same El Centro earthquake

shown in Fig. 6-2, except scaled to a maximum acceleration of 0.2g, is used as the input excitation. Without control, time histories of the bridge response quantities have been computed. The maximum deformation of the rubber bearings,  $x$ , and the maximum absolute acceleration of the bridge deck,  $\bar{x}_a$ , in the entire earthquake episode are presented in Table 6.5, designated as "No Control."

The LQR optimal control is considered first in which the acceleration is not penalized, i.e.,  $Q_a$  is a null matrix. The (2x2)  $Q$  matrix is chosen to be a diagonal matrix with diagonal elements  $Q(1,1)=1.0$  and  $Q(2,2)=0$ . The purpose of such a choice is to reduce the deformation as much as possible. With  $R_0=1.8 \times 10^{-13}$ , the maximum deformation,  $x$ , the maximum absolute acceleration,  $\bar{x}_a$ , and the required maximum control force,  $U$ , are shown in Table 6.5, designated as "Case (A)". As observed from the table, the deformation has been reduced substantially.

The absolute acceleration of the bridge deck is also an important design consideration. To reduce the acceleration, the generalized performance index is used. The  $Q_a$  matrix consists of only one element designated as  $Q_{a0}$ . It is chosen to be 100, i.e.,  $Q_{a0}=100$ . With  $R_0=10^{-3}$  and the same  $Q$  matrix in Case (A), the maximum response quantities are presented in Table 6.5, designated as Case (I). A comparison between the results in Case (A) and that in Case (I) indicates that the acceleration response is reduced significantly using the generalized performance index.



(a)

(b)

**Fig. 6.1 :** Building Structural models ; (a) A Six-Story Building Equipped with Active Bracing System ;  
 (b) Structural Model of 8-Story Building Equipped with Active Mass Damper



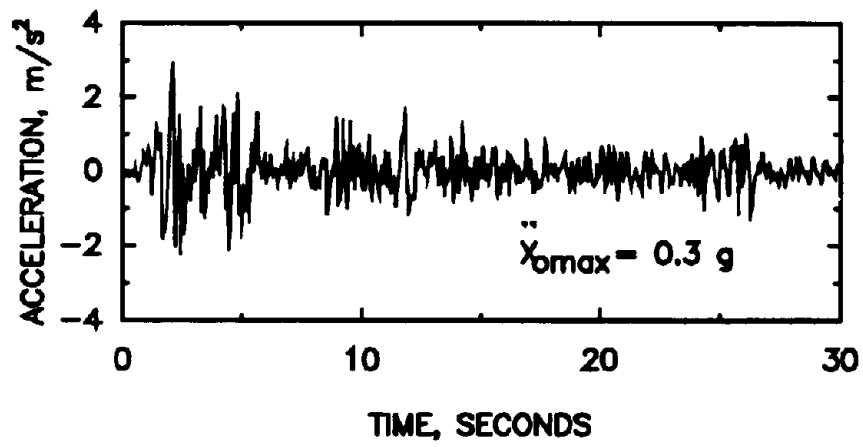
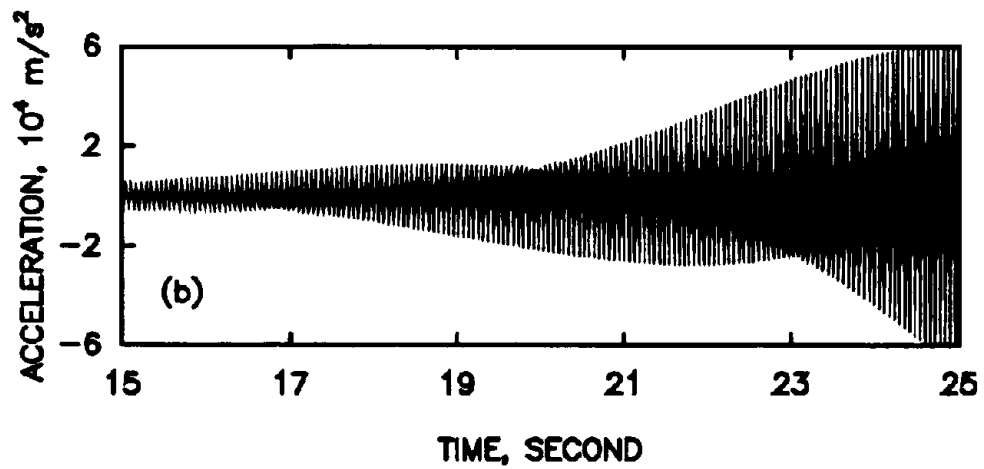
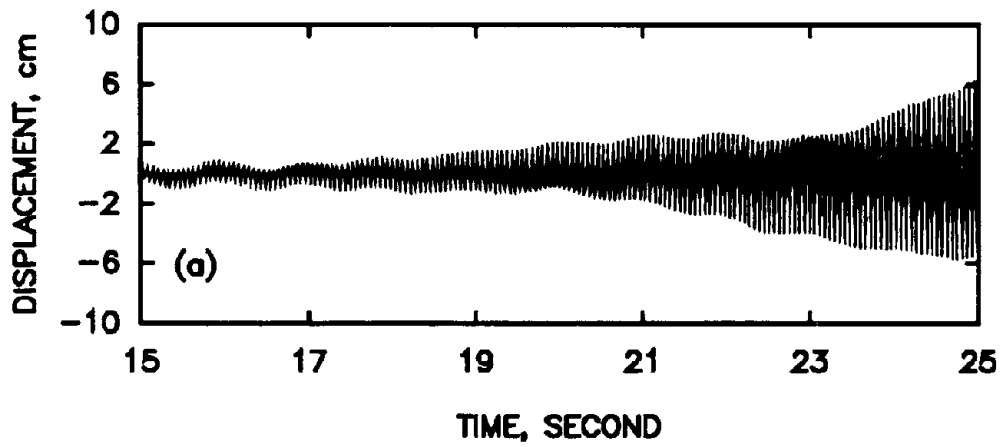


Fig 6.2 : EL Centro Earthquake



**Fig 6.3 : Deformation and Absolute Acceleration Responses**  
**(a) First Story Unit ; (b) First Floor**

Table 6.1 : Maximum Response Quantities of A Six-Story Building : Smaller Control Force

S T O R Y  N O	No Control		CASE (A)		CASE (B)																									
			$R_o = 7 \times 10^{-8}$ $U = 1159$ kN		$\alpha = \beta = 100$		$\alpha = \beta = 50$		$\alpha = \beta = 20$		$\alpha = \beta = 10$		$\alpha = \beta = 5$																	
					$x_i$	$\dot{x}_{ai}$	$x_i$	$\dot{x}_{ai}$	$x_i$	$\dot{x}_{ai}$	$x_i$	$\dot{x}_{ai}$	$x_i$	$\dot{x}_{ai}$	$x_i$	$\dot{x}_{ai}$														
(1)	cm	cm/s <sup>2</sup>	(3)	cm	(4)	cm	(5)	cm/s <sup>2</sup>	(6)	cm	(7)	cm/s <sup>2</sup>	(8)	cm	(9)	cm/s <sup>2</sup>	(10)	cm	(11)	cm/s <sup>2</sup>	(12)	cm	(13)	cm/s <sup>2</sup>	(14)	cm	(15)	cm/s <sup>2</sup>		
1	1.97	316	1.39	256	1.45	299	1.49	306	1.53	302	1.52	303	1.54	328																
2	3.91	623	2.89	308	3.02	343	3.06	358	3.14	371	3.18	376	3.14	382																
3	4.34	744	3.24	381	3.38	418	3.42	447	3.50	491	3.53	523	3.47	538																
4	4.70	753	3.17	457	3.30	498	3.34	512	3.44	525	3.50	535	3.46	550																
5	4.40	885	2.81	524	2.95	556	2.99	567	3.08	593	3.15	596	3.17	598																
6	3.04	970	2.0	654	2.10	687	2.13	703	2.21	730	2.27	748	2.26	744																

Table 6.2 : Maximum Response Quantities of A Six-Story Building : Large Control Force

S T O R Y  N O	No Control		CASE (A)		CASE (B)									
	$x_j$ cm (2)	$\ddot{x}_{aj}$ cm/s <sup>2</sup> (3)	$x_j$ cm (4)	$\ddot{x}_{aj}$ cm/s <sup>2</sup> (5)	$\alpha=\beta=100$		$\alpha=\beta=50$		$\alpha=\beta=20$		$\alpha=\beta=10$		$\alpha=\beta=5$	
					$\Delta\tau = 1\%$	$\overline{R}_0 = 9 \times 10^{-11}$	$U = 3039$ kN	$x_j$	$\ddot{x}_{aj}$	$x_j$	$\ddot{x}_{aj}$	$x_j$	$\ddot{x}_{aj}$	$x_j$
1	1.97	316	1.40	302	1.43	258	1.44	244	1.47	259	1.49	263	1.53	266
2	3.91	623	1.80	198	1.83	201	1.84	201	1.88	203	1.92	208	2.03	221
3	4.34	744	1.98	248	2.00	256	2.00	257	2.04	263	2.07	266	2.15	287
4	4.70	753	1.96	355	1.99	358	1.99	356	2.03	371	2.06	383	2.13	408
5	4.40	885	1.90	312	1.93	316	1.94	315	1.97	323	2.01	329	2.06	355
6	3.04	970	1.57	541	1.61	555	1.61	558	1.65	572	1.68	581	1.74	605

Table 6.3 : Maximum Response Quantities of A Six-Story Building : Without Considering Actuator Dynamics

S T O R Y	N O	CASE (C)													
		No Control		CASE (A) $R_0=7 \times 10^{-8}$ $U=1159$ kN		$\alpha=\beta=100$ $\Delta\tau=1\%$ $R_0=7 \times 10^{-8}$ $U=1172$ kN		$\alpha=\beta=80$ $\Delta\tau=1.25\%$ $R_0=7 \times 10^{-8}$ $U=1175$ kN		$\alpha=\beta=60$ $\Delta\tau=1.66\%$ $R_0=7 \times 10^{-8}$ $U=1155$ kN		$\alpha=\beta=55$ $\Delta\tau=1.81\%$ $R_0=7 \times 10^{-8}$ $U=1174$ kN		$\alpha=\beta=50$ $\Delta\tau=2\%$ $R_0=7 \times 10^{-8}$ $U=1 \times 10^5$ kN (unstable)	
		$x_j$ cm	$\dot{x}_{ai}$ cm/s <sup>2</sup>	$x_j$ cm	$\dot{x}_{ai}$ cm/s <sup>2</sup>	$x_j$ cm	$\dot{x}_{ai}$ cm/s <sup>2</sup>	$x_j$ cm	$\dot{x}_{ai}$ cm/s <sup>2</sup>	$x_j$ cm	$\dot{x}_{ai}$ cm/s <sup>2</sup>	$x_j$ cm	$\dot{x}_{ai}$ cm/s <sup>2</sup>	$x_j$ cm	$\dot{x}_{ai}$ cm/s <sup>2</sup>
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1	1.97	316	744	1.39	256	1.39	309	1.39	318	1.39	439	1.39	430	25.1	$2.8 \times 10^5$
2	3.91	623	744	2.89	308	2.92	336	2.93	353	2.94	357	2.95	337	35.5	$1.1 \times 10^5$
3	4.34	744	744	3.24	381	3.29	410	3.30	423	3.32	447	3.33	446	15.0	$0.5 \times 10^5$
4	4.70	753	744	3.17	457	3.21	481	3.22	486	3.24	497	3.25	503	6.6	$1.8 \times 10^5$
5	4.40	885	744	2.81	524	2.85	543	2.86	548	2.88	555	2.89	557	2.9	4630
6	3.04	970	744	2.0	654	2.04	669	2.05	672	2.06	677	2.07	679	2.1	814

**Table 6.4 : Maximum Response Quantities of An 8-Story Building**

S T O R Y N O (1)	No Control		CASE (A) U= 830 kN		CASE (I) U= 854 kN		CASE (II) U= 1267 kN	
	$x_i$ cm (2)	$\ddot{x}_{ai}$ cm/s <sup>2</sup> (3)	$x_i$ cm (4)	$\ddot{x}_{ai}$ cm/s <sup>2</sup> (5)	$x_i$ cm (6)	$\ddot{x}_{ai}$ cm/s <sup>2</sup> (7)	$x_i$ cm (8)	$\ddot{x}_{ai}$ cm/s <sup>2</sup> (9)
1	3.24	277	1.54	261	1.40	282	1.28	280
2	3.23	402	1.44	291	1.33	241	1.14	336
3	3.12	444	1.35	309	1.23	227	1.00	318
4	2.92	469	1.30	278	1.17	246	1.02	276
5	2.60	522	1.15	300	1.06	272	1.00	231
6	2.13	619	0.91	362	0.89	298	0.93	186
7	1.53	723	0.64	384	0.69	330	0.84	308
8	0.80	790	0.48	429	0.55	364	0.77	443

**Table 6.5 : Maximum Response Quantities of  
A Bridge Deck**

	Max. Deformation $x$ , cm	Max. Acceleration $\ddot{x}_g$ , cm/s <sup>2</sup>	Max. Control Force $U$ , kN
No Control	11.36	39.1	0
Case (A)	6.35	40.1	239
Case (I)	7.80	11.5	249

## SECTION 7 CONCLUSIONS

A generalization of the theory of optimal control for seismic-excited linear structures has been presented. The generalized theory takes into account the absolute acceleration response of the structure and the actuator dynamics. The incorporation of the actuator dynamics in the optimization process removes the adverse effect of the system time delay resulting from the actuator response behavior. The inclusion of the acceleration response of the structure in the generalized performance index provides a possibility to penalize or to minimize the acceleration response.

Simulation results are obtained to demonstrate the advantages of taking into account both the actuator dynamics and the acceleration response of the structure. Simulation results indicate that (i) with the inclusion of the acceleration response in the performance index, it is possible to substantially reduce the acceleration response of any particular floor which can not be achieved otherwise, and (ii) with the inclusion of the actuator dynamics in the optimization process, the degradation of the control performance, resulting from the actuator response, becomes insignificant. Without considering the actuator dynamics, the controlled structures may become unstable, unless other compensation methods are used.



**SECTION 8**  
**REFERENCES**

1. Feng, Q., Fujii, S. and Shinozuka, M. (1990), "Use of a Variable Damper for Hybrid Control of the Bridge Response Under Earthquake", Proc. U.S. National Workshop on Structural Control Research, pp. 107-112, U.S.C, CA.
2. Inaudi, J.A. and Kelly, J.M. (1991), "A Simple Active Isolation Scheme", in Dynamics and Control of Large Structures, edited by L. Meirovitch, VIP&SU Press, Blacksburg, VA, pp. 219-230.
3. Inaudi, J.A. and Kelly, J.M. (1993), "Hybrid Isolation Systems for Equipment Protection", paper to appear in Journal of Earthquake Engineering and Structural Dynamics.
4. Kobori, T. and Kamagata, S. (1991), "Dynamic Intelligent Buildings - Active Seismic Response Control", in Intelligent Structures - 2, edited by Y.K. Wen, pp. 279-282.
5. Lai, M.L. and Soong, T.T (1991), "Seismic Design Considerations for Secondary Structural System", Journal of Structural Engineering, Vol. 117, No. 2.
6. Reinhorn, A.M., Soong, T.T., et al (1989), "1:4 Scale Model Studies of Active Tendon Systems and Active Mass Dampings For Seismic Protection", National Center For Earthquake Engineering Research, Technical Report NCFER-89-0026.
7. Reinhorn, A.M. and Soong, T.T. (1990), "Full-Scale Implementation of Active Bracing for Seismic Control of Structures", in Structural Control Research, USC press, pp. 201-208.
8. Samali, B., Yang, J.N. and Yeh, C.T. (1985), "Control of Lateral-Torsional Motion of Wind-Excited Buildings", Journal of Engineering Mechanics, ASCE, Vol. 111, No. 6, pp. 777-796.
9. Soong, T.T. (1988), "State-of-Art Review: Active Structural Control in Civil Engineering", Engineering Structures, Vol. 10, pp. 74-84.
10. Soong, T.T. (1990), Active Structural Control : Theory and Practice, Longman Scientific and Technical, New York.
11. Soong, T.T., Masri, S.F. and Housner, G.W. (1991), "An Overview of Active Structural Control Under Seismic Loads", Earthquake Spectra, Vol. 7, pp.483-505.

12. Soong, T.T. and Reinhorn, A.M. (1991), "Full-Scale Implementation of Active Structural Control", in Intelligent Structures - 2, edited by Y.K. Wen, pp. 252-263, Elsevier Applied Science.
13. Suhardjo, Spencer Jr., B.F. and Sain, M.K. (1990), "Feedback-Feedforward control of Structures Under seismic Excitation", Journal of Structural Safety, Vol. 8, pp. 69-89.
14. Suhardjo, J., Spencer, B.F. and Kareem, A. (1992), "Frequency Domain Optimal Control of Wind Excited Buildings", to appear in Journal of Engineering Mechanics, ASCE.
15. Talbot, M.E. and Shinozuka, M. (1990), "Active Isolation for Seismic Protection of Operating Rooms", Technical Report, NCEER-90-0010, National Center for Earthquake Engineering Research, Buffalo, New York.
16. Yang, J.N., Akbrapour, A., Ghaemmaghami, P. (1985), "Optimal Control Algorithms for Earthquake-Excited Building Structures", Structural Control, edited by H.H.E. Leipholz, Martinus Nijhoff Publishers, pp. 748-761.
17. Yang, J.N., Akbrapour, A., Ghaemmaghami, P. (1987), "New Optimal Control Algorithms for Structural Control", Journal of Engineering Mechanics, ASCE, Vol. 113, No. 9, pp. 1369-1368.
18. Yang, J.N., Long, F.X. and Wong, D. (1988), "Optimal Control of Nonlinear Structures", Journal of Applied Mechanics, ASME, Vol. 55, pp. 931-938.
19. Yang, J.N. and Soong, T.T. (1988), "Recent Advancement in Active Control of Civil Engineering Structures", Journal of Probabilistic Engineering Mechanics, Vol. 3, No. 4, pp. 179-188.
20. Yang, J.N., Akbarpour, A. and Askar, G. (1990), "Effect of Time Delay on Control of Seismic-Excited Buildings", Journal of Structural Engineering Mechanics, ASCE, Vol. 304, No. 10, pp. 179-188.
21. Yang, J.N. and Li., Z. (1991), "Instantaneous Optimal Control for Linear, Nonlinear and Hysteretic Structural Systems--Stable Controllers", Technical Report, NCEER-91-0026, National Center for Earthquake Engineering Research, Buffalo, New York.
22. Yang, J.N. and Li., Z. (1991), "Active and Hybrid Control of Civil Engineering Structures", in Frontier R&D of Constructed Facilities, edited by A. H.S. Ang, pp. 45-59; Proc. of US-Korea-Japan Trilateral Seminar on R&D Frontier for Constructed Facilities, Bookcrafters, October, Honolulu.

23. Yang, J.N., Li, Z., Danielians, A. and Liu, S.C. (1992), "Hybrid Control of Nonlinear and Hysteretic System I", Journal of Engineering Mechanics, ASCE, Vol. 118, No. 7, pp. 1423-1440.
24. Yang, J.N., Li, Z., Danielians, A. and Liu, S.C. (1992), "Hybrid Control of Nonlinear and Hysteretic System II", Journal of Engineering Mechanics, ASCE, Vol. 118, No. 7, pp. 1441-1456.
25. Yang, J.N., Li, Z. and Liu, S.C. (1992), "Stable Controllers for Instantaneous Optimal Control", Journal of Engineering Mechanics, ASCE, Vol. 118, No. 8, pp.1612-1630.
26. Yang, J.N., Li, Z. and Liu, S.C. (1992), "Control of Hysteretic System Using Velocity and Acceleration Feedbacks", Journal of Engineering Mechanics, ASCE, Nov. (in press).
27. Yang, J.N., "Some Thoughts on Hybrid Control of Bridge Structures", Proc. 1st US-Japan Workshop on Earthquake Protective Systems for Bridges, Buffalo, New York (in press).

**A GENERALIZATION OF OPTIMAL CONTROL THEORY:  
NONLINEAR CONTROL**

**PART II**

## SECTION 1 INTRODUCTION

A combined use of active and passive control systems, referred to as the hybrid control system, has been demonstrated to be very effective for seismic-excited civil engineering structures. Various hybrid control systems have been investigated for applications to protect building and bridge structures [e.g., 2-5, 7-8, 13-22 and Refs. in Part I]. However, the application of hybrid control systems involves active control of nonlinear or hysteretic structures, since most passive control devices, such as lead-core rubber-bearing isolation systems, behave either nonlinearly or inelastically.

Control laws can either be linear or nonlinear. Linear control theories for linear structures have been available in the literature. However, control theories for nonlinear structures are limited and intensive research efforts have been made. Recently, instantaneous optimal control has been proposed for applications to nonlinear and hysteretic structures successfully [e.g., 18-21]. In particular, the stable controllers are obtained by use of the Lyapunov direct method [e.g., 20-21]. Basically, the control law proposed in these previous works is the linear control law. Various control laws for discrete pulse control, that is nonlinear in nature, have been proposed for applications to nonlinear civil engineering structures [e.g., 6, 7].

Another control method proposed for applications to buildings equipped with frictional-type sliding isolation systems is the method of dynamic linearization [Ref. 22]. The method of dynamic linearization is to synthesize the control vector so that the response of the controlled structure matches that of a specified system, referred to as the template system [e.g., 1, 9], whereas the response characteristics of the template system is known. This control method has been applied successfully to seismic-excited buildings equipped with a frictional-type sliding isolation system [Ref. 22]. However, for applications to other types of nonlinear structures, the major difficulty is to find a suitable template system such that the response of the nonlinear structure can be matched easily to that of the template system. Because of such a difficulty, the application of the dynamic linearization method is limited to a certain class of base-isolated buildings. The control law associated with the method of

dynamic linearization is usually nonlinear [Ref. 22]. Other control methods have also been proposed for structures equipped with frictional-type sliding isolation systems [e.g., 2, 5, 8].

The polynomial control law has been suggested in Refs. 11-12 for applications to nonlinear structures. It was shown that the control performance of the nonlinear polynomial control law is better than that of the linear control law. However, the main disadvantage of the polynomial control law is that the computations of the gain matrices for higher order control terms are rather cumbersome.

In this report, an optimal nonlinear control law is proposed for applications to nonlinear and hysteretic structures. The proposed nonlinear control method is based on a generalized performance index. The resulting optimal control law resembles the nonlinear characteristics of the structure to be controlled. The absolute acceleration vector of the structural response is included in the generalized performance index, and the actuator dynamics is also taken into account in the optimization process. Likewise, control laws using acceleration and velocity feedbacks are derived in Section 7.

An extensive simulation study has been conducted. Simulation results indicate that (i) the proposed nonlinear control method is effective for hybrid control of some types of seismic-excited building structures, and (ii) the performance of the optimal nonlinear control method is better than that of the linear control method proposed in Refs. 18-21.

**SECTION 2**  
**OPTIMAL LINEAR CONTROL FOR NONLINEAR STRUCTURES**

An instantaneous optimal control method was proposed for nonlinear and hysteretic structural systems [e.g., 18-21]. In the previous works, a discretization of the equations of motion was made to obtain an approximate solution leading to a linear control law. Then, the stability of the controllers is guaranteed by use of the Lyapunov direct method [e.g., 20-21]. Using the Lyapunov direct method, a Riccati-type equation and a Lyapunov equation were obtained for the determination of stable controllers. In this section, we shall derive the same linear control law using the LQR performance index. The nonlinear control law will be proposed in the next section.

Consider an n degrees of freedom nonlinear or hysteretic building structure subjected to a one-dimensional earthquake ground acceleration  $\ddot{X}_0(t)$ . The vector equation of motion is given by

$$\underline{M}\ddot{\underline{X}}(t) + \underline{E}_D[\dot{\underline{X}}(t)] + \underline{E}_s[\underline{X}(t)] = \underline{\xi}\ddot{X}_0(t) + \underline{H}_1\underline{U}(t) \quad (2.1)$$

in which  $\underline{X}(t) = [x_1, x_2, \dots, x_n]'$  is an n-vector with  $x_j(t)$  being the deformation of the jth story unit,  $\underline{U}(t)$  is a r-dimensional vector consisting of r control forces,  $\underline{\xi} = -[m_1, m_2, \dots, m_n]'$  is a mass vector. In Eq. (2.1),  $\underline{M}$  is a (nxn) mass matrix with the i-jth element  $M(i,j) = m_i$  for  $j \leq i$  and  $M(i,j) = 0$  for  $j > i$ , where  $m_i$  is the mass of the ith floor.  $\underline{E}_D[\dot{\underline{X}}(t)]$  and  $\underline{E}_s[\underline{X}(t)]$  are nonlinear damping and stiffness vectors, respectively, and  $\underline{H}_1$  is a (nxr) matrix denoting the location of r controllers. In the notation above, an under bar denotes either a vector or a matrix and a prime indicates the transpose of either a matrix or a vector.

In the state vector form, Eq. (2.1) becomes

$$\dot{\underline{Z}}(t) = \underline{g}[\underline{Z}(t)] + \underline{B}\underline{U}(t) + \underline{H}_1\ddot{X}_0(t) \quad (2.2)$$

in which  $\underline{g}[\underline{Z}(t)]$  is a 2n vector which is a nonlinear function of the state vector  $\underline{Z}(t)$  and

$$\underline{Z}(t) = \begin{bmatrix} \underline{X}(t) \\ \dot{\underline{X}}(t) \end{bmatrix}; \underline{B} = \begin{bmatrix} \underline{0} \\ \underline{I} \end{bmatrix}; \underline{H}_1 = \begin{bmatrix} \underline{0} \\ \underline{M}^{-1}\underline{\xi} \end{bmatrix}; \underline{g}[\underline{Z}(t)] = \begin{bmatrix} \dot{\underline{X}}(t) \\ -\underline{M}^{-1}[\underline{E}_D + \underline{E}_s] \end{bmatrix} \quad (2.3)$$

The LQR performance index is given by

$$J = \int_0^t [Z'(t)QZ(t) + U'(t)RU(t)] dt \quad (2.4)$$

in which  $Q$  is a  $(2n \times 2n)$  symmetric positive semidefinite weighting matrix and  $R$  is a  $(r \times r)$  positive definite weighting matrix.

To minimizing the objective function,  $J$ , given by Eq. (2.4) subjected to the constraint of the state equation of motion, Eq. (2.2), the Hamiltonian  $H$  is constructed by introducing a  $2n$ -dimensional Lagrangian multiplier vector  $\lambda(t)$ ,

$$H = Z'QZ + U'RU + \lambda'[g(Z) + BU + W_1\dot{X}_0 - \dot{Z}] \quad (2.5)$$

in which the argument  $t$  has been dropped for simplicity.

The necessary conditions for minimizing  $J$  given by Eq. (2.4) are

$$\frac{\partial H}{\partial \lambda} = 0 \quad ; \quad \frac{\partial H}{\partial U} = 0 \quad ; \quad \frac{\partial H}{\partial Z} + \dot{\lambda} = 0, \quad (2.6)$$

The first condition  $\partial H/\partial \lambda = 0$  leads to the state equation of motion given by Eq. (2.2). The second and third conditions are obtained as follows

$$U = -0.5R^{-1}B'\lambda \quad (2.7)$$

and

$$2QZ + \Delta'(Z)\lambda + \dot{\lambda} = 0 \quad (2.8)$$

in which

$$\Delta(Z) = \partial g(Z)/\partial Z \quad (2.9)$$

is a  $(2n \times 2n)$  derivative matrix.

Let

$$\lambda = PZ \quad (2.10)$$

in which  $P$  is a  $(2n \times 2n)$  matrix to be determined. Substitution of Eq. (2.10) into Eqs. (2.7) and (2.8) yields

$$U(t) = -0.5R^{-1}B'PZ \quad (2.11)$$

$$2QZ + \Delta'(Z)PZ + \dot{P}Z + P\dot{Z} = 0 \quad (2.12)$$

Substituting  $\dot{Z}$  given by Eq. (2.2) into Eq. (2.12), using Eq. (2.11) and neglecting the



earthquake ground acceleration  $\ddot{X}_0(t)$ , one obtains

$$\dot{P}Z + \Delta'(Z)PZ + Pg(Z) - 0.5PBR^{-1}B'PZ = -2QZ \quad (2.13)$$

Equation (2.13) should be solved backwards from the terminal point  $t_f$ , i.e.,  $P(t_f)=0$ . However, since the earthquake ground acceleration  $\ddot{X}_0(t)$  is not known, i.e.,  $Z(t)$  is not known, Eq. (2.13) can not be solved. Consequently, an approximation using the equivalent linearization technique is used.

One possible approach is to linearize the structural system at the initial equilibrium point  $Z=Q$  that is stable for civil engineering structures. Hence  $g(Z)$  and  $\Delta(Z)$  are approximated by

$$\begin{aligned} g(Z) &= \Delta_0 Z \\ \Delta(Z) &= \Delta_0 \end{aligned}$$

and Eq. (2.13) becomes

$$\dot{P} + \Delta_0'P + P\Delta_0 - 0.5PBR^{-1}B'P = -2Q \quad (2.14)$$

in which  $P$  is the Riccati matrix where

$$\Delta_0 = \Delta(Z) \Big|_{Z=Q} \quad (2.15)$$

In earthquake engineering applications, it has been shown [e.g., 15, 16] that the time dependent Riccati matrix establishes its stationary values rapidly such that  $\dot{P}=0$  is an excellent approximation. As a result, Eq. (2.15) can be approximated by the matrix algebraic equation

$$\Delta_0'P + P\Delta_0 - 0.5PBR^{-1}B'P = -2Q \quad (2.16)$$

Equation (2.16) was also derived based on the Lyapunov direct method for instantaneous optimal control in Refs. 20-21. Furthermore, since the term  $PBR^{-1}B'P$  is positive semidefinite, Eq. (2.16) can also be approximated by

$$\Delta_0'P + P\Delta_0 = -2Q \quad (2.17)$$

which is known as the Lyapunov equation. From Eq. (2.17), various approximate solutions have been proposed in Ref. 20 for control of linear, nonlinear and hysteretic structures.

**SECTION 3**  
**OPTIMAL NONLINEAR CONTROL FOR NONLINEAR STRUCTURES**

In the previous section, the LQR performance index is used and a linear control law is derived for nonlinear structures where the derivative matrix  $\Delta(\mathbf{Z})$  is evaluated at the initial equilibrium point  $\mathbf{Z}=\mathbf{0}$ . The same solution was obtained in Ref. 20 using the Lyapunov direct method. Such an approach works well when yielding of inelastic structures is not quite serious. As the ductility becomes large, the control performance of the linear control law presented in the previous section will be examined later. In this section, two nonlinear control laws are proposed for control of nonlinear structures.

A performance index is proposed as follows

$$J = \int_0^T [\mathbf{g}'(\mathbf{Z})\mathbf{Q}\mathbf{g}(\mathbf{Z}) + \mathbf{U}'(t)\mathbf{R}\mathbf{U}(t)] dt \quad (3.1)$$

The performance index  $J$ , proposed in Eq. (3.1), is quite different from that of LQR, Eq. (2.4), since  $\mathbf{g}(\mathbf{Z})$  is a nonlinear function of  $\mathbf{Z}$ , see Eq. (2.3), which is the nonlinear characteristic of the hysteretic system.

To minimize the performance index  $J$  subjected to the constraint of the matrix equation of motion, the Hamiltonian  $H$  is expressed as

$$H = \mathbf{g}'(\mathbf{Z})\mathbf{Q}\mathbf{g}(\mathbf{Z}) + \mathbf{U}'(t)\mathbf{R}\mathbf{U}(t) + \lambda'[\mathbf{g}(\mathbf{Z}) + \mathbf{B}\mathbf{U} + \mathbf{W}_1\ddot{\mathbf{x}}_0(t) - \dot{\mathbf{Z}}] \quad (3.2)$$

The necessary conditions for the optimal solution are

$$\frac{\partial H}{\partial \lambda} = 0 \quad ; \quad \frac{\partial H}{\partial \mathbf{U}} = 0 \quad ; \quad \frac{\partial H}{\partial \mathbf{Z}} + \dot{\lambda}' = 0 \quad (3.3)$$

The first condition above leads to the state equation of motion given by Eq. (2.2). Substitution of Eq. (3.2) into the condition  $\partial H/\partial \mathbf{U}=0$ , yields

$$\mathbf{U}(t) = -0.5\mathbf{R}^{-1}\mathbf{R}'\lambda \quad (3.4)$$

Substituting Eq. (3.2) into the third condition,

$$\frac{\partial H}{\partial \mathbf{Z}} + \dot{\lambda}' = 0 \quad (3.5)$$

one obtains

$$2\Delta'(Z)Qg(Z) + \Delta'(Z)\lambda + \dot{\lambda} = 0 \quad (3.6)$$

in which  $\Delta(Z)$  is the derivative matrix

$$\Delta(Z) = \partial g(Z)/\partial Z \quad (3.7)$$

The first nonlinear control law is obtained by setting

$$\lambda = Pg(Z) \quad (3.8)$$

in which  $P$  is a  $(2n \times 2n)$  matrix to be determined. Substitution of Eq. (3.8) into Eq. (3.4) leads to the following control law

$$U(t) = -0.5R^{-1}B'Pg(Z) \quad (3.9)$$

Substituting Eq. (3.8) into Eq. (3.6), using the matrix equation of motion for  $\dot{Z}(t)$  and neglecting the external load  $\ddot{X}_0(t)$ , one obtains

$$\dot{P} + \Delta'(Z)P + P\Delta(Z) - 0.5P\Delta(Z)BB^{-1}B'P = -2\Delta'(Z)Q \quad (3.10)$$

Again, an equivalent linearization technique is used for the determination of the  $P$  matrix. The nonlinear structure is linearized at the initial equilibrium point  $Z=Q$  that is stable, i.e.,

$$\Delta(Z) = \Delta(Z)|_{Z=Q} = \Delta_0 \quad (3.11)$$

and the transient part of the  $P$  matrix is neglected, i.e.,  $\dot{P}=0$ . Then, Eq. (3.10) becomes

$$\Delta_0'P + P\Delta_0 - 0.5P\Delta_0BB^{-1}B'P = -2\Delta_0'Q \quad (3.12)$$

To facilitate the solution of the constant matrix  $P$ , the following transformation is made,

$$P = \Delta_0'P_1 \quad (3.13)$$

in which  $P_1$  is a  $(2n \times 2n)$  constant matrix to be determined. Substituting Eq. (3.13) into Eq. (3.12) and premultiplying the resulting equation by  $(\Delta_0')^{-1}$ , one obtains

$$\Delta_0'P_1 + P_1\Delta_0 - 0.5P_1\Delta_0BB^{-1}B'\Delta_0'P_1 = -2Q \quad (3.14)$$

Equation (3.14) is the matrix Riccati equation from which the Riccati matrix,  $P_1$ , can be determined.

Thus, the control vector given by Eq. (3.9) becomes

$$\underline{U}(t) = -0.5 \underline{R}^{-1} \underline{B}' \underline{\Delta}'_0 \underline{P}_1 \underline{g}(\underline{Z}) \quad (3.15)$$

in which  $\underline{P}_1$  is the (2nx2n) Riccati matrix to be computed from Eq. (3.14).

The nonlinear control law derived above, Eqs. (3.14) and (3.15), is referred to as the first nonlinear control law. The second nonlinear control law is obtained by setting

$$\underline{\lambda} = \underline{\Delta}'(\underline{Z}) \underline{P} \underline{g}(\underline{Z}) \quad (3.16)$$

Substitution of Eq. (3.16) into Eq. (3.4) leads to the control vector  $\underline{U}(t)$  as follows

$$\underline{U}(t) = -0.5 \underline{R}^{-1} \underline{B}' \underline{\Delta}'(\underline{Z}) \underline{P} \underline{g}(\underline{Z}) \quad (3.17)$$

The condition for determining the  $\underline{P}$  matrix is obtained by substituting Eq. (3.16) into Eq. (3.6), using the matrix equation of motion, Eq. (2.2), for  $\dot{\underline{Z}}(t)$  and neglecting  $\ddot{\underline{X}}_0(t)$ ; with the result,

$$\underline{\Delta}'(\underline{Z}) \underline{P} + \underline{\Delta}'(\underline{Z}) [\underline{\dot{P}} + \underline{\Delta}'(\underline{Z}) \underline{P} + \underline{P} \underline{\Delta}(\underline{Z}) - 0.5 \underline{P} \underline{\Delta}(\underline{Z}) \underline{B} \underline{B}^{-1} \underline{B}' \underline{\Delta}'(\underline{Z}) \underline{P} + 2 \underline{Q}] = \underline{0} \quad (3.18)$$

Again, an equivalent linearization at the initial equilibrium point  $\underline{Z}=\underline{0}$ , Eq. (3.11), is used such that  $\underline{\dot{\Delta}}_0=0$ , and the transient part of the  $\underline{P}$  matrix is neglected, i.e.,  $\underline{\dot{P}}=\underline{0}$ . Then Eq. (3.18) becomes

$$\underline{\Delta}'_0 \underline{P} + \underline{P} \underline{\Delta}_0 - 0.5 \underline{P} \underline{\Delta}_0 \underline{B} \underline{B}^{-1} \underline{B}' \underline{\Delta}'_0 \underline{P} = -2 \underline{Q} \quad (3.19)$$

which is exactly the matrix Riccati equation. It can easily be observed that  $\underline{P}_1$  in Eq. (3.14) is identical to  $\underline{P}$  in Eq. (3.19). Thus, the first nonlinear control law, Eq. (3.15), is a special case of the second nonlinear control law, Eq. (3.17), in which  $\underline{\Delta}'(\underline{Z})$  is replaced by the constant matrix  $\underline{\Delta}_0$ .

It should be mentioned that the control laws proposed in Eqs. (3.15) and (3.17) are nonlinear, because  $\underline{g}(\underline{Z})$  is a nonlinear function of the state vector  $\underline{Z}$  given by Eq. (2.3).

**SECTION 4**  
**GENERALIZED NONLINEAR CONTROL**

In order to protect the equipments housed in the building, the absolute acceleration of the floor response must be reduced to an acceptable level. This can be accomplished by including the acceleration response in the performance index. A generalized performance index is proposed in the following

$$J = \int_0^t \left[ \mathbf{g}'(t) \mathbf{Q} \mathbf{g}(t) + \dot{\mathbf{X}}_a'(t) \mathbf{Q}_a \dot{\mathbf{X}}_a(t) + \mathbf{U}'(t) \mathbf{R} \mathbf{U}(t) \right] dt \quad (4.1)$$

in which  $\mathbf{Q}_a$  is an  $(n \times n)$  symmetric positive semidefinite weighting matrix and  $\dot{\mathbf{X}}_a(t)$  is the absolute acceleration vector for all floors. It follows from the matrix equation of motion, Eq. (2.1), that the absolute acceleration,  $\dot{\mathbf{X}}_a(t)$ , can be expressed as

$$\begin{aligned} \dot{\mathbf{X}}_a(t) &= -\mathbf{M}_0^{-1} [\mathbf{E}_D(\dot{\mathbf{X}}) + \mathbf{E}_r(\mathbf{X})] + \mathbf{M}_0^{-1} \mathbf{H} \mathbf{U}(t) \\ &= -\mathbf{L} \mathbf{M}^{-1} [\mathbf{E}_D(\dot{\mathbf{X}}) + \mathbf{E}_r(\mathbf{X})] + \mathbf{L} \mathbf{M}^{-1} \mathbf{H} \mathbf{U}(t) \end{aligned} \quad (4.2)$$

in which  $\mathbf{M}_0$  is a diagonal mass matrix with  $m_i$  being the  $i$ th diagonal element, and

$$\mathbf{L} = \mathbf{M}_0^{-1} \mathbf{M} \quad (4.3)$$

is an  $(n \times n)$  transformation matrix with  $L(i,j)=1$  for  $j \leq i$  and  $L(i,j)=0$  for  $j > i$ .

Substituting Eq. (4.2) into Eq. (4.1) and rearranging, one obtains

$$J = \int_0^t \left[ \mathbf{g}'(\mathbf{Z}), \mathbf{U}'(t) \right] \mathbf{T} \begin{bmatrix} \mathbf{g}(\mathbf{Z}) \\ \mathbf{U}(t) \end{bmatrix} dt \quad (4.4)$$

in which  $\mathbf{T}$  is a  $(2n+r) \times (2n+r)$  generalized weighting matrix

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{12}' & \mathbf{T}_{22} \end{bmatrix} \quad (4.5)$$

In Eq. (4.5),  $\mathbf{T}_{11}$  is a  $(2n \times 2n)$  matrix,  $\mathbf{T}_{12}$  is a  $(2n \times r)$  matrix and  $\mathbf{T}_{22}$  is a  $(r \times r)$  matrix given in the following

$$T_{11} = Q + \begin{bmatrix} Q_m & 0_m \\ 0_m & T_s \end{bmatrix}; \quad T_{12} = \begin{bmatrix} 0_m \\ T_s M^{-1} H \end{bmatrix} \quad (4.6)$$

$$T_{22} = R + (M^{-1}H)'T_s(M^{-1}H)$$

where  $T_s$  is a  $(n \times n)$  transformed matrix of  $Q_s$ , i.e.,

$$T_s = L'Q_sL \quad (4.7)$$

To minimize the performance index given by Eq. (4.4), the Hamiltonian  $H$  is formed as follows

$$H = [\mathbf{g}', \mathbf{U}'] T \begin{bmatrix} \mathbf{g} \\ \mathbf{U} \end{bmatrix} + \lambda' [\mathbf{g} + \mathbf{B}\mathbf{U} + \mathbf{W}_1 \mathbf{X}_0 - \dot{\mathbf{Z}}] \quad (4.8)$$

in which the arguments  $\mathbf{Z}$  and  $t$  have been dropped for simplicity.

The necessary conditions for the optimal solution are

$$\frac{\partial H}{\partial \lambda} = 0 \quad ; \quad \frac{\partial H}{\partial \mathbf{U}} = 0 \quad ; \quad \frac{\partial H}{\partial \mathbf{Z}} + \dot{\lambda}' = 0 \quad (4.9)$$

The first condition  $\partial H / \partial \lambda$  leads to the state equation of motion given by Eq. (2.2). Substitution of Eq. (4.8) into the second and third conditions of Eq. (4.9) leads to the following relations

$$\mathbf{U}(t) = -0.5 T_{22}^{-1} [\mathbf{B}' \lambda + 2 T_{12}' \mathbf{g}(\mathbf{Z})] \quad (4.10)$$

$$2 \Delta'(\mathbf{Z}) T_{11} \mathbf{g}(\mathbf{Z}) + 2 \Delta'(\mathbf{Z}) T_{12} \mathbf{U}(t) + \Delta'(\mathbf{Z}) \lambda + \dot{\lambda} = 0 \quad (4.11)$$

in which

$$\Delta(\mathbf{Z}) = \partial \mathbf{g}(\mathbf{Z}) / \partial \mathbf{Z} \quad (4.12)$$

is the system derivative matrix identical to Eq. (3.7).

Let

$$\lambda = \Delta'(\mathbf{Z}) \mathbf{P} \mathbf{g}(\mathbf{Z}) \quad (4.13)$$

Substituting Eq. (4.13) into Eqs. (4.10) and (4.11), one obtains

$$U(t) = -0.5T_{22}^{-1}[B'\Delta'(Z)P + 2T_{12}' ]g(Z) \quad (4.14)$$

$$\begin{aligned} \Delta'(Z)Pg(Z) + \Delta'(Z)[\dot{P} + \bar{\Delta}'(Z)P + P\bar{\Delta}(Z) - 0.5P\Delta(Z)BT_{22}^{-1}B'\Delta'(Z)P \\ + 2(T_{11} - T_{12}T_{22}^{-1}T_{12}') ]g(Z) = 0 \end{aligned} \quad (4.15)$$

in which

$$\bar{\Delta}(Z) = \Delta(Z)[I - BT_{22}^{-1}T_{12}'] \quad (4.16)$$

At this point it is necessary to linearize the system in order to obtain a constant  $P$  matrix. Again, we linearize  $\Delta(Z)$  at the initial equilibrium point  $Z(t)=Q$  such that

$$\Delta(Z) = \Delta_0 \text{ and } \dot{\Delta}(Z) = 0 \quad (4.17)$$

and neglect the transient part  $\dot{P}$ . Then, Eq. (4.15) becomes

$$\bar{\Delta}_0'P + P\bar{\Delta}_0 - 0.5P\Delta_0BT_{22}^{-1}B'\Delta_0'P = -2(T_{11} - T_{12}T_{22}^{-1}T_{12}') \quad (4.18)$$

in which  $\bar{\Delta}_0$  follows from Eqs. (4.16) and (4.17) as

$$\bar{\Delta}_0 = \Delta_0[I - BT_{22}^{-1}T_{12}'] \quad (4.19)$$

Equation (4.18) is the matrix Riccati equation from which the constant Riccati matrix  $P$  can be determined.

The control law presented in Eqs. (4.14) and (4.19) corresponds to the second nonlinear control law presented in the previous section. It can be shown easily that  $T_{11}=Q$  and  $T_{12}=Q$  for  $Q_1=Q$ . Then, Eqs. (4.14) and (4.19) reduce to Eqs. (3.17) and (3.19), respectively. Furthermore, if  $\Delta(Z)$  appearing in Eq. (4.14) is linearized by  $\Delta_0$ , one obtains

$$U(t) = -0.5T_{22}^{-1}[B'\Delta_0'P + 2T_{12}']g(Z) \quad (4.20)$$

which is the first nonlinear control law.

**SECTION 5**  
**GENERALIZED NONLINEAR CONTROL INCLUDING**  
**ACTUATOR DYNAMICS**

In this section, the actuator dynamics will be taken into account in the derivation of the generalized nonlinear control law. For simplicity, the dynamic equations for  $r$  actuators are described by a system of first order differential equations [23]

$$\dot{U}(t) + aU(t) = bq(t) \quad (5.1)$$

in which  $a$  and  $b$  are  $(rxr)$  diagonal matrices with diagonal elements  $a_i$  and  $b_i$  ( $i=1,2,\dots,r$ ).  $a_i$  is a measure of the loop gain or the reaction time of the  $i$ th actuator, and  $b_i$  is a measure of feedback gain or the amplification factor of the  $i$ th actuator. In Eq. (5.1),  $q(t)$  is a  $r$ -vector representing the feedback signal (or dynamic input) for generating the required active control vector  $U(t)$ . Note that the vector  $q(t)$  is proportional to the control vector  $U(t)$  and it will be determined later through the optimization process. It should be mentioned that the extension of the optimal control theory in the following is not restricted to the first order differential equation, Eq. (5.1), for the actuator dynamics. Other higher order differential equations can similarly be used.

The dynamic equations of actuators, Eq. (5.1), can be augmented to the state equation of motion, Eq. (2.2), and both can be casted into the following  $(2n+r)$  vector equation

$$\dot{Z}_1(t) = S h(Z_1) + \bar{B}q(t) + \bar{H}X_0(t) \quad (5.2)$$

in which

$$Z_1(t) = \begin{bmatrix} Z(t) \\ U(t) \end{bmatrix}; h(Z_1) = \begin{bmatrix} g(Z) \\ U(t) \end{bmatrix}; S = \begin{bmatrix} I & | & B \\ \hline \Omega_{2n} & | & -a \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} \Omega_{2nr} \\ \hline b \end{bmatrix}; \bar{H} = \begin{bmatrix} H_1 \\ \hline \Omega_{r1} \end{bmatrix} \quad (5.3)$$

where  $\Omega_{2nr}$ , and  $\Omega_{r1}$  are  $(2nxr)$  and  $(rx1)$  zero matrices, respectively.

The generalized performance index  $J$  can be expressed as



$$J = \int_0^T \left[ \mathbf{g}'(\mathbf{Z}) \mathbf{Q} \mathbf{g}(\mathbf{Z}) + \mathbf{U}'(t) \mathbf{B} \mathbf{U}(t) + \ddot{\mathbf{X}}_e'(t) \mathbf{Q}_e \ddot{\mathbf{X}}_e(t) + \mathbf{g}'(t) \bar{\mathbf{R}} \mathbf{g}(t) \right] dt \quad (5.4)$$

in which  $\mathbf{Q}_e$  is a (n<sub>e</sub>n<sub>e</sub>) symmetric positive semidefinite weighting matrix,  $\bar{\mathbf{R}}$  is a (r<sub>e</sub>r<sub>e</sub>) symmetric positive definite weighting matrix, and  $\mathbf{Q}$  and  $\mathbf{R}$  have been defined previously.

In Eq. (5.4),  $\ddot{\mathbf{X}}_e(t)$  is the absolute acceleration vector given by Eq. (4.2)

$$\ddot{\mathbf{X}}_e(t) = -\mathbf{L} \mathbf{M}^{-1} [\mathbf{E}_D(\dot{\mathbf{X}}) + \mathbf{E}_S(\mathbf{X})] + \mathbf{L} \mathbf{M}^{-1} \mathbf{H} \mathbf{U}(t) \quad (5.5)$$

Substituting Eqs. (5.5) into Eq. (5.4), one obtains the following equivalent generalized performance index

$$J = \int_0^T \left[ \mathbf{h}'(\mathbf{Z}_1) \mathbf{T} \mathbf{h}(\mathbf{Z}_1) + \mathbf{g}'(t) \bar{\mathbf{R}} \mathbf{g}(t) \right] dt \quad (5.6)$$

in which the  $\mathbf{T}$  matrix is defined in Eqs. (4.5) to (4.7).

Following the same optimization procedures described in the previous sections, the optimal solution is obtained as

$$\mathbf{g}(t) = -0.5 \bar{\mathbf{R}}^{-1} \bar{\mathbf{B}}' \hat{\Delta}'(\mathbf{Z}_1) \hat{\mathbf{P}} \mathbf{h}(\mathbf{Z}_1) \quad (5.7)$$

in which  $\hat{\Delta}'(\mathbf{Z}_1)$  is the derivative matrix of  $\mathbf{h}(\mathbf{Z}_1)$  with respect to  $\mathbf{Z}_1$ ,

$$\hat{\Delta}'(\mathbf{Z}_1) = \frac{\partial \mathbf{h}(\mathbf{Z}_1)}{\partial \mathbf{Z}_1} \quad (5.8)$$

The condition for determining the  $\hat{\mathbf{P}}$  matrix is as follows

$$\begin{aligned} \hat{\Delta}'(\mathbf{Z}_1) \hat{\mathbf{P}} + \hat{\Delta}'(\mathbf{Z}_1) [\hat{\mathbf{P}} + \mathbf{S}' \hat{\Delta}'(\mathbf{Z}_1) \hat{\mathbf{P}} + \hat{\mathbf{P}} \hat{\Delta}(\mathbf{Z}_1) \mathbf{S} \\ - 0.5 \hat{\mathbf{P}} \hat{\Delta}(\mathbf{Z}_1) \bar{\mathbf{B}} \bar{\mathbf{R}}^{-1} \bar{\mathbf{B}}' \hat{\Delta}'(\mathbf{Z}_1) \hat{\mathbf{P}} + 2\mathbf{T}] = 0 \end{aligned} \quad (5.9)$$

Again, an equivalent linearization at the initial equilibrium point  $\mathbf{Z}_1 = 0$  is used such that  $\hat{\Delta}_0 = 0$ , and the transient part of the  $\hat{\mathbf{P}}$  matrix is neglected, i.e.,  $\dot{\hat{\mathbf{P}}} = 0$ . Then Eq. (5.9) becomes

$$(\hat{\Delta}_0 \mathbf{S})' \hat{\mathbf{P}} + \hat{\mathbf{P}} (\hat{\Delta}_0 \mathbf{S}) - 0.5 \hat{\mathbf{P}} \hat{\Delta}_0 \bar{\mathbf{B}} \bar{\mathbf{R}}^{-1} \bar{\mathbf{B}}' \hat{\Delta}_0' \hat{\mathbf{P}} = -2\mathbf{T} \quad (5.10)$$

which is exactly the matrix Riccati equation and

$$\hat{\Delta}_0 = \hat{\Delta}'(\mathbf{Z}_1)|_{\mathbf{Z}_1 = 0} \quad (5.11)$$

Equation (5.7) is the second optimal nonlinear control law. The first optimal nonlinear control law is identical to Eq. (5.7) except that  $\hat{\Delta}'(\mathbf{Z}_1)$  is replaced by  $\hat{\Delta}_0'$ .

**SECTION 6**  
**SIMULATION OF STRUCTURAL RESPONSE**

In order to evaluate the effectiveness and performance of the proposed optimal nonlinear control method, it is necessary to simulate the response of the controlled structure. A method of simulation for hysteretic structures is presented in the following. For simplicity, the damping of the structure is considered as linear viscous damping, i.e.,  $E_D[\dot{X}(t)] = \underline{C} \dot{X}$ , where  $\underline{C}$  is the damping matrix.

The following hysteretic model will be used for both the structures and passive protective systems. The stiffness restoring force,  $F_{si}(t)$ , of the  $i$ th story unit is given by

$$F_{si}(t) = \alpha_i k_i x_i + (1 - \alpha_i) k_i D_{yi} v_i \quad (6.1)$$

in which  $k_i$  = elastic stiffness of the  $i$ th story unit,  $\alpha_i$  = ratio of the post-yielding to pre-yielding stiffness,  $D_{yi}$  = yield deformation = constant, and  $v_i$  is a nondimensional variable introduced to describe the hysteretic component of the deformation, with  $|v_i| \leq 1$ , where

$$\dot{v}_i = D_{yi}^{-1} [A_i \dot{x}_i - \beta_i |\dot{x}_i| |v_i|^{n_i-1} v_i - \gamma_i \dot{x}_i |v_i|^{n_i}] = f_i(\dot{x}_i, v_i) \quad (6.2)$$

In Eq. (6.2), parameters  $A_i$ ,  $\beta_i$  and  $\gamma_i$  govern the scale and general shape of the hysteresis loop, whereas the smoothness of the force-deformation curve is determined by the parameter  $n_i$ .

The state equation of the motion, Eq. (2.1), can be expressed as

$$M \ddot{X}(t) + \underline{C} \dot{X}(t) + \underline{K}_e X(t) + \underline{K}_1 Y(t) = E \ddot{X}_0(t) + H_1 U(t) \quad (6.3)$$

in which  $Y(t) = [v_1(t), v_2(t), \dots, v_n(t)]^T$  = an  $n$  vector denoting the hysteretic component  $v_i$  of each story unit given by Eq. (6.2). In Eq. (6.3),  $\underline{C}$ ,  $\underline{K}_e$  and  $\underline{K}_1$  are  $(n \times n)$  band-limited damping matrix, elastic stiffness matrix and hysteretic stiffness matrix, respectively. All elements of  $\underline{C}$ ,  $\underline{K}_e$  and  $\underline{K}_1$  are zero, except  $C(i,i) = c_i$ ,  $K_e(i,i) = \alpha_i k_i$ ,  $K_1(i,i) = (1 - \alpha_i) k_i D_{yi}$  for  $i = 1, 2, \dots, n$  and  $C(i, i+1) = -c_{i+1}$ ,  $K_e(i, i+1) = -\alpha_{i+1} k_{i+1}$ ,  $K_1(i, i+1) = -(1 - \alpha_{i+1}) k_{i+1} D_{yi+1}$  for  $i = 1, 2, \dots, n-1$ , where  $c_i$  is the damping coefficient of the  $i$ th story unit. The expressions given above for matrices  $\underline{C}$ ,  $\underline{K}_e$  and  $\underline{K}_1$  hold for a base-isolated building connected to an actuator at the base isolation system, Fig. 7-1(a). When the arrangement of the control system is different, the matrices  $\underline{C}$ ,  $\underline{K}_e$  and  $\underline{K}_1$  should be modified appropriately.

By introducing a  $3n$  state vector  $\tilde{Z}(t)$ , a  $(3n \times r)$  matrix  $\tilde{B}$  and a  $3n$  vector  $\tilde{W}_1$

$$\tilde{Z}(t) = \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ \dot{\mathbf{X}} \end{bmatrix} ; \quad \tilde{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}^{-1} \mathbf{H}_1 \end{bmatrix} ; \quad \tilde{W}_1 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}^{-1} \mathbf{f} \end{bmatrix} \quad (6.4)$$

the second-order nonlinear vector equation of motion, Eq. (6.3), can be converted into a first order vector equation as follows

$$\dot{\tilde{Z}}(t) = \tilde{g}[\tilde{Z}(t)] + \tilde{B}U(t) + \tilde{W}_1 \ddot{X}_0(t) \quad (6.5)$$

in which  $\tilde{g}[\tilde{Z}(t)]$  is a  $3n$  vector consisting of nonlinear functions of components of  $\tilde{Z}(t)$ ,

$$\tilde{g}[\tilde{Z}(t)] = \begin{bmatrix} \dot{\mathbf{X}} \\ \text{-----} \\ \mathbf{f}(\mathbf{X}, \mathbf{Y}) \\ \text{-----} \\ -\mathbf{M}^{-1}(\mathbf{C} \dot{\mathbf{X}} + \mathbf{K}_s \mathbf{X} + \mathbf{K}_1 \mathbf{Y}) \end{bmatrix} \quad (6.6)$$

where  $\mathbf{f}(\mathbf{X}, \mathbf{Y}) = [f_1(x_1, v_1), f_2(x_2, v_2), \dots, f_n(x_n, v_n)]'$  is an  $n$  vector with the  $i$ th element,  $f_i(x_i, v_i)$ , given by Eq. (6.2).

The vector equation of motion given in Eq. (6.5) can be augmented by the actuator dynamics, Eq. (5.1), as follows

$$\dot{Z}_2(t) = \tilde{\mathcal{S}} \tilde{h}(Z_2) + \tilde{B}^* q(t) + \tilde{W}^* \ddot{X}_0(t) \quad (6.7)$$

in which  $Z_2(t)$ ,  $\tilde{h}(Z_2)$  and  $\tilde{W}^*$  are  $(3n+r)$  vectors,  $\tilde{\mathcal{S}}$  is a  $(3n+r) \times (3n+r)$  matrix and  $\tilde{B}^*$  is a  $(3n+r) \times r$  matrix,

$$Z_2(t) = \begin{bmatrix} \tilde{Z}(t) \\ \text{-----} \\ U(t) \end{bmatrix} ; \quad \tilde{h}(Z_2) = \begin{bmatrix} \tilde{g}(\tilde{Z}) \\ \text{-----} \\ U(t) \end{bmatrix} ; \quad \tilde{\mathcal{S}} = \begin{bmatrix} \mathbf{I} & | & \tilde{B} \\ \text{-----} & | & \text{-----} \\ \mathbf{0}_{r \times 2n} & | & -\mathbf{a} \end{bmatrix}$$

$$\tilde{B}^* = \begin{bmatrix} \mathbf{0} \\ \text{-----} \\ \mathbf{b} \end{bmatrix} ; \quad \tilde{W}^* = \begin{bmatrix} \tilde{W}_1 \\ \text{-----} \\ \mathbf{0} \end{bmatrix} \quad (6.8)$$

With the optimal nonlinear control law,  $q(t)$ , derived in Eq. (5.7), the response for the hysteretic structural system can be simulated by solving Eq. (6.7) numerically using the Fourth-Order Runge-Kutta method [ e.g., 18-21].

The derivative matrix  $\hat{\Delta}(Z_1)$  appearing in the control law, Eq. (5.7), is given by

$$\hat{\Delta}(Z_1) = \begin{bmatrix} \Delta(Z) & | & Q_{2nr} \\ \hline & & \\ Q_{r2n} & | & I_{rr} \end{bmatrix} \quad (6.9)$$

in which  $I_{rr} = a$  (rxr) identity matrix and

$$\Delta(Z) = \begin{bmatrix} Q_m & | & I_m \\ \hline & & \\ -M^{-1}[K_s + K_f \frac{\partial Y}{\partial X}] & | & -M^{-1}C \end{bmatrix} \quad (6.10)$$

where  $\partial Y/\partial X$  is a diagonal matrix with the  $i$  diagonal element  $\partial v_i/\partial x_i$  given as follows

$$\frac{\partial v_i}{\partial x_i} = \frac{\partial f_i(\dot{x}_i, v_i)}{\partial x_i} = D_{f_i}^{-1} [A_i - \beta_i \text{sgn}(\dot{x}_i) |v_i|^{n_i-1} v_i - \gamma_i |v_i|^{n_i}] \quad (6.11)$$

The constant derivative matrix  $\hat{\Delta}_0$  is given also by Eq. (6.9) except that  $\Delta(Z)$  is replaced by  $\Delta_0$  where

$$\Delta_0 = \begin{bmatrix} Q & | & I \\ \hline & & \\ -M^{-1}K & | & -M^{-1}C \end{bmatrix} \quad (6.12)$$

## SECTION 7

### OPTIMAL CONTROL USING ACCELERATION AND VELOCITY FEEDBACKS

Both linear and nonlinear control laws presented previously require the feedback of the state vector  $\mathbf{Z}(t)$  that should be either measured or estimated using an observer. Frequently, it may be easier to measure the acceleration response than the displacement response [e.g., 20, 21]. Control laws using the acceleration and velocity feedbacks were suggested in Refs. 20 and 21. These control laws will be derived in this section using the LQR type formulation.

Consider the following LQR type performance index

$$J = \int_0^T [\dot{\mathbf{Z}}'(t) \mathbf{Q} \dot{\mathbf{Z}}(t) + \mathbf{U}'(t) \mathbf{R} \mathbf{U}(t)] dt \quad (7.1)$$

in which  $\dot{\mathbf{Z}}(t)$  is the time derivative of the state vector, which consists of the velocity and acceleration responses. To minimize the objective function  $J$  subjected to the constraint of the state equation of motion, Eq. (2.2), the Hamiltonian  $H$  is introduced

$$H = \dot{\mathbf{Z}}'(t) \mathbf{Q} \dot{\mathbf{Z}}(t) + \mathbf{U}'(t) \mathbf{R} \mathbf{U}(t) + \lambda [\mathbf{g}(\mathbf{Z}) + \mathbf{B} \mathbf{U}(t) + \mathbf{W}_1 \dot{\mathbf{X}}_0(t) - \dot{\mathbf{Z}}(t)] \quad (7.2)$$

The necessary conditions for the optimal solution are

$$\frac{\partial H}{\partial \lambda} = 0 ; \quad \frac{\partial H}{\partial \mathbf{U}} = 0 ; \quad \frac{\partial H}{\partial \mathbf{Z}} - \frac{d}{dt} \left[ \frac{\partial H}{\partial \dot{\mathbf{Z}}} \right] = 0 \quad (7.3)$$

in which the general form has been used for the third condition. Substitution of Eq. (7.2) into the last two conditions yields

$$\mathbf{U}(t) = -0.5 \mathbf{R}^{-1} \mathbf{B}' \lambda \quad (7.4)$$

$$\Delta'(\mathbf{Z}) \lambda + 2 \mathbf{Q} \dot{\mathbf{Z}}(t) + \dot{\lambda} = 0 \quad (7.5)$$

in which  $\Delta(\mathbf{Z})$  is the derivative matrix

$$\frac{\partial \mathbf{g}(\mathbf{Z})}{\partial \mathbf{Z}} = \Delta(\mathbf{Z}) \quad (7.6)$$

At this point, we shall linearize the equation of motion at the initial equilibrium point  $\mathbf{Z}=0$ ,

such that

$$\begin{aligned}\Delta(Z) &= \Delta(Z) |_{z=0} = \Delta_0 \\ g(Z) &= \Delta_0 Z\end{aligned}\tag{7.7}$$

Let

$$\lambda = -(\Delta_0^{-1})' P \dot{Z}\tag{7.8}$$

in which  $P$  is a constant matrix to be determined. Substituting Eq. (7.8) into Eqs. (7.4) and (7.5) and neglecting the external excitation, one obtains

$$U(t) = 0.5 B^{-1} B' (\Delta_0^{-1})' P \dot{Z}\tag{7.9}$$

$$-P \dot{Z} + 2Q' \dot{Z} - (\Delta_0^{-1})' P \dot{Z} = 0\tag{7.10}$$

Substituting Eq. (7.9) into the linearized state equation of motion, taking the time derivative of the resulting equation, and substituting the resulting equation into Eq. (7.10), one obtains the following matrix Riccati equation for the determination of the  $P$  matrix,

$$P \Delta^* + (\Delta^*)' P - 0.5 P B B^{-1} B' (\Delta^*)' P + 2Q' = 0\tag{7.12}$$

in which

$$\Delta^* = \Delta_0^{-1} ; B^* = \Delta_0^{-1} B\tag{7.13}$$

If the equation of motion is linear, i.e.,

$$g(Z) = AZ ; \Delta_0 = A\tag{7.14}$$

then, the control law given by Eqs. (7.9) and (7.12) is the exact optimal control which was presented in Ref. 20. However, if the equation of motion is nonlinear, the control law given by Eqs. (7.9) and (7.12) is an approximation which was proposed in Ref. 21.

## SECTION 8 NUMERICAL SIMULATION

To demonstrate the performance of the proposed nonlinear control method and to compare it with that of the linear control method, numerical examples are worked out in this section. Two cases are considered in the following; namely, a moderate earthquake (0.3g) and a strong earthquake (1g).

### Example 1: A Base-Isolated Elasto-Plastic Building

An eight-story building that exhibits bilinear elasto-plastic behavior is considered, Fig. 7.1 [e.g., 18-20]. The properties of the building are as follows : (i) the mass of each floor is identical with  $m_i = m = 345.6$  metric tons; (ii) the preyielding stiffnesses of the eight-story units are  $k_{i1}$  ( $i=1,2,\dots,8$ ) =  $3.4 \times 10^5$ ,  $3.26 \times 10^5$ ,  $2.85 \times 10^5$ ,  $2.69 \times 10^5$ ,  $2.43 \times 10^5$ ,  $2.07 \times 10^5$ ,  $1.69 \times 10^5$  and  $1.37 \times 10^5$  kN/m, respectively, and the postyielding stiffnesses are  $k_{i2} = 0.1 k_{i1}$  for  $i=1,2,\dots,8$ , i.e.,  $\alpha_i = 0.1$  and  $k_i = k_{i1}$ ; and (iii) the viscous damping coefficients for each story unit are  $c_i = 490, 467, 410, 386, 348, 298, 243$  and  $196$  kN.sec/m, respectively. The damping coefficients given above result in a damping ratio of 0.38% for the first vibrational model. The fundamental frequency of the unyielded building is 5.24 rad./sec. The yielding level for each story unit varies with respect to the stiffness; with the results,  $D_{yi} = 2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.7,$  and  $1.5$  cm. The bilinear elasto-plastic behavior can be described by the hysteretic model, Eqs. (6.1) and (6.2), with  $A_i = 1.0$ ,  $\beta_i = 0.5$ ,  $n_i = 95$  and  $\gamma_i = 0.5$  for  $i=1,2,\dots,8$  [Ref. 18]. The same El Centro earthquake with a maximum ground acceleration of 0.3g as shown in Fig. 6.2 of Part I is used as the input excitation.

Time histories of all the response quantities have been computed. Within 30 seconds of the earthquake episode, the maximum interstory deformation,  $x_i$ , and the maximum absolute acceleration of each floor,  $\ddot{x}_{ai}$ , are shown in columns (3) and (4) of Table 7.1, designated as "No Control". As observed from Table 7.1, the deformation of the unprotected building is excessive and that yielding takes place in the upper five story units.

To reduce the structural response, a lead-core rubber-bearing isolation system is

implemented as shown in Fig. 7.1(a). The restoring force of the lead-core rubber-bearing isolation system is modeled by Eq. (6.1) with  $F_{ib} = \alpha_b k_b x_b + (1 - \alpha_b) k_b D_{yb} v_b$  in which  $v_b$  is given by Eq. (6.2) with  $i = b$ . The mass of the base isolation system is  $m_b = 450$  metric tons and the viscous damping coefficient is assumed to be linear with  $c_b = 26.17$  kN sec/m. The restoring force of the base isolation system given above is not bilinear elasto-plastic and the parameter values are given as follows:  $k_b = 18050$  kN/m,  $\alpha_b = 0.6$ ,  $D_{yb} = 4$ cm,  $A_b = 1.0$ ,  $\beta_b = 0.5$ ,  $n_b = 3$  and  $\gamma_b = 0.5$ , Eq. (6.2). The hysteresis loop of such a base isolation system, i.e.,  $x_b$  versus  $v_b$ , is shown in Fig. 7.2. For the building with the base isolation system alone, the first natural frequency of the preyielded structure is 2.21 rad/sec and the damping ratio for the first vibrational mode is 0.15%. The response vector  $\underline{X}(t)$  is given by  $\underline{X} = [x_b, x_1, \dots, x_8]^T$ .

The maximum response quantities of the building within 30 seconds of the earthquake episode are shown in columns (5) and (6) of Table 7.1 designated as "With BIS". As observed from Table 7.1, the interstory deformation and the floor acceleration are drastically reduced. However, the deformation of the base isolation system shown in row B of Table 7.1 should be reduced.

Since the effect of actuator dynamics has been demonstrated in Part I, it is not necessary to present similar results. It is mentioned that the degradation of the control performance due to the actuator response is minimal as long as the actuator dynamics is taken into account. In what follows, we shall assume that the time delay due to the actuator response is negligible, i.e.,  $\alpha = \beta$  is a large value, so that  $q(t) = \underline{U}(t)$ .

To protect the safety and integrity of the base isolation system, an actuator is connected to the base isolation system as shown in Fig. 7.1(a). With the actuator applying the active control force  $\underline{U}(t)$  to the base isolation system, the structural response depends on the weighting matrices  $\underline{Q}$ ,  $\underline{Q}_a$  and  $\underline{R}$  where  $\underline{\tilde{R}} = \underline{Q}$ . For this example, the weighting matrix  $\underline{R}$  consists of only one element, denote by  $R_0$ .

We first consider the linear control law given by Eqs.(2.11) and (2.16). The (18x18)  $\underline{Q}$  matrix is considered a diagonal matrix with all the diagonal elements equal to 1.0, i.e.,  $Q(i,i) = 1$  for  $i = 1, 2, \dots, 18$ , and  $R_0 = 10^{-7}$  is used. Time histories of all the response quantities have been computed. Within 30 seconds of the earthquake episode, the maximum



response quantities and the required maximum control force,  $U$ , are shown in columns (7) and (8) of Table 7.1. As observed from the table, the deformation of the base isolation system is reduced by 50%, where the response quantities of the superstructure reduce slightly except that of the top story unit.

The first nonlinear control law presented in Eq.(4.20) is considered in which the Riccati matrix  $P$  is computed from Eq.(4.18). In this case, the (18x18) diagonal  $Q$  matrix is assigned as follows :  $Q(i,i)=2$  for  $i=1,2,\dots,9$  and  $Q(i,i)=0$  for  $i=10, 11, \dots, 16$ . The (9x9)  $Q_a$  matrix that corresponds to the acceleration response is considered a diagonal matrix with all the diagonal elements equal to 1.0, i.e.,  $Q_a(i,i)=1$  for  $i=1,2,\dots,9$ , and  $R_0=2 \times 10^{-7}$  is used. The maximum response quantities and the required maximum control force are presented in columns (9) and (10) of Table 7.1, designated as "Nonlinear Control I". It is observed that all the response quantities and the active control force are smaller than those associated with optimal linear control, columns (7) and (8). In particular, the acceleration response quantities improve significantly because of the use of the generalized performance index.

We next consider the second nonlinear control law in Eq. (4.14) and use the same matrices  $Q$ ,  $Q_a$  and  $R$  in the first nonlinear control law. The corresponding results are shown in columns (11) and (12) of Table 7.1, designated as "Nonlinear Control II". It is observed from the table that all the response quantities and the required active control force are slightly smaller than those associated with the first nonlinear control law, columns (9) and (10). As a result, the performance of the second nonlinear control law is slightly better. It should be mentioned that for the second nonlinear control law the system derivative matrix,  $\Delta(Z)$ , is not linearized, Hence, the system derivative matrix  $\Delta(Z)$  should be computed on-line, resulting in an increase of the on-line computational efforts.

Suppose the same base-isolated building used above is subjected to the El Centro earthquake shown in Fig. 6.2 of Part I but scaled uniformly to a maximum ground acceleration  $1g$ . With such a strong earthquake input, the maximum response quantities of the building with and without a base isolation system are presented in columns (3)-(6) of Table 7.2, designated as "No Control" and "With BIS", respectively. It is observed that (i) Without a base isolation system, the ductility of the building response is quite high, and (ii)

With a base isolation system the response of the rubber bearing is too large, whereas the response quantities of all the story units are close to the yield limit  $D_y$ .

With the same active control devices and the same weighting matrices,  $\mathbf{Q}$ ,  $\mathbf{Q}_a$  and  $\mathbf{R}$  used previously, the maximum response quantities for the corresponding control methods are presented in Table 7.2. The same conclusions described previously are obtained from Table 7.2: (i) the control performance of the two nonlinear control methods is better than that of the linear control method, and (ii) the control performance of the second nonlinear control method is slightly better than that of the first one.

#### **Example 2: An Elasto-Plastic Building With Active Bracing Systems**

The same eight-story elasto-plastic building considered in Example 1 is subjected to the same El Centro earthquake with a maximum ground acceleration of  $1g$ . However, instead of using a rubber-bearing isolation system, an active bracing system is installed on every floor. The angle of inclination of the bracings with respect to the floor is  $25^\circ$ . Hence, the dimensions of the weighting matrices  $\mathbf{Q}$ ,  $\mathbf{Q}_a$  and  $\mathbf{R}$  are  $(16 \times 16)$ ,  $(8 \times 8)$  and  $(8 \times 8)$ , respectively. These weighting matrices will be assigned as diagonal matrices in the following. Also, the time delay due to the actuator response is assumed to be negligible, i.e.,  $\alpha = \beta =$  a large value.

For the linear control law given by Eqs. (2.11) and (2.16),  $Q(i,i) = 1$  for  $i = 1, 2, \dots, 16$  and  $R(i,i) = 10^{-9}$  for  $i = 1, 2, \dots, 8$  are used. For nonlinear control laws, we choose  $Q(1,1) = 4000$ ,  $Q(i,i) = 1000$  for  $i = 2, 3, \dots, 8$ ,  $Q(i,i) = 0$  for  $i = 9, 10, \dots, 16$ , and  $Q_a(i,i) = 1$ ,  $R(i,i) = 10^{-8}$  for  $i = 1, 2, \dots, 8$ .

Within 30 seconds of the earthquake episode, the maximum response quantities are presented in Table 7.3, including the maximum interstory deformation,  $x_i$ , the maximum absolute acceleration,  $\ddot{x}_{ai}$ , and the maximum control force  $U_i$ . It is observed from Table 7.3 that the control performance of all control laws is quite satisfactory. Further, the second nonlinear control law seems to perform slightly better than the first one.

The results shown in Table 7.3 indicate that yielding still occurs in some story units. To bring the response of each story unit into the elastic range, larger control forces are needed. For the linear control law, the  $\mathbf{Q}$  matrix remains the same but each diagonal

element of the  $\mathbf{R}$  matrix is changed to  $10^{-10}$ . For the nonlinear control laws, the  $\mathbf{Q}_a$  and  $\mathbf{R}$  matrices remain the same, but  $Q(1,1)=4000$ ,  $Q(i,i)=1000$  for  $i=2,3,\dots,8$  and  $Q(i,i)=0$  for  $i=9,10,\dots,16$  are used. The corresponding maximum response quantities are presented in Table 7.4. Again, the control performance of all the control laws is very satisfactory. When all the response quantities are within the linear elastic range, there is no difference between the first and the second nonlinear control laws as evidenced by the results shown in Table 7.4.

Finally, we would like to point out that control of hysteretic buildings, in which a large ductility is involved, requires more controllers. For a perfectly linear elastic building, either an active mass damper installed on the top floor or an active bracing system (ABS) installed on the first floor is enough to control the entire building. However, for the elasto-plastic eight-story building subjected to a 1g earthquake considered in this example, either an active mass damper or an active bracing system alone is not capable of controlling the building response. The reason is that once a story unit yields with a large ductility, controllers installed in the other story unit cannot effectively exert the control force through the given load path. As a result, eight controllers are used in this example.

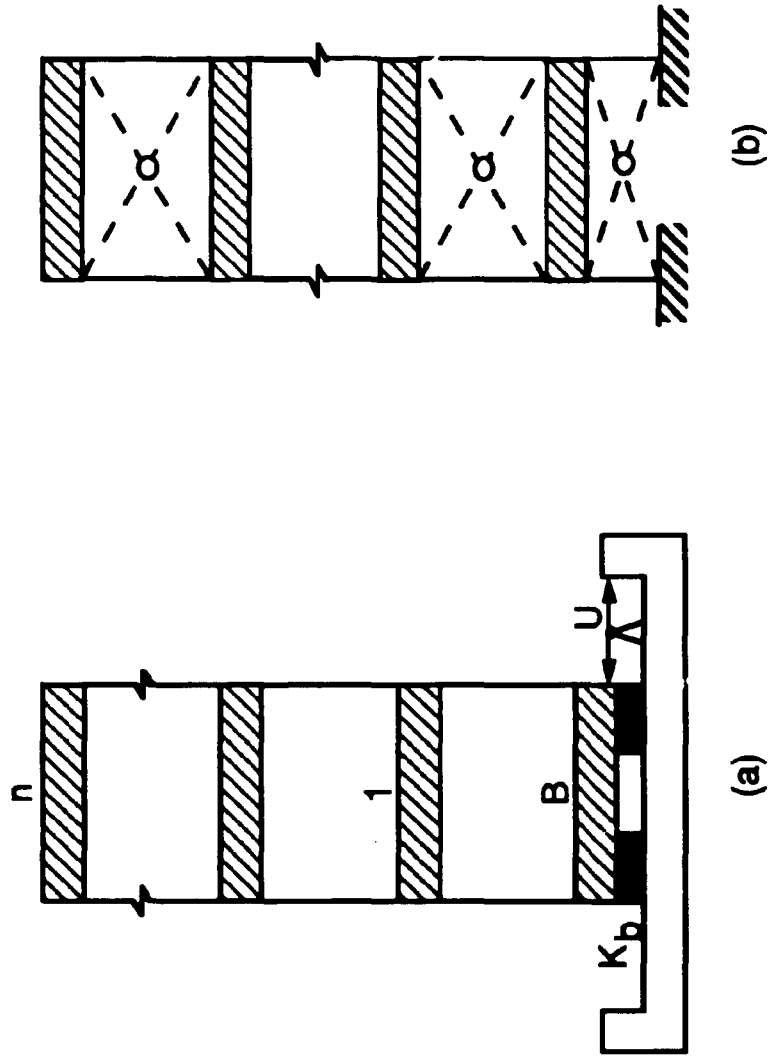


Fig. 7.1 : Structural Model of a Multi-Story Building : (a) With Rubber Bearing Isolation System and Actuator ; (b) With Active Bracing System

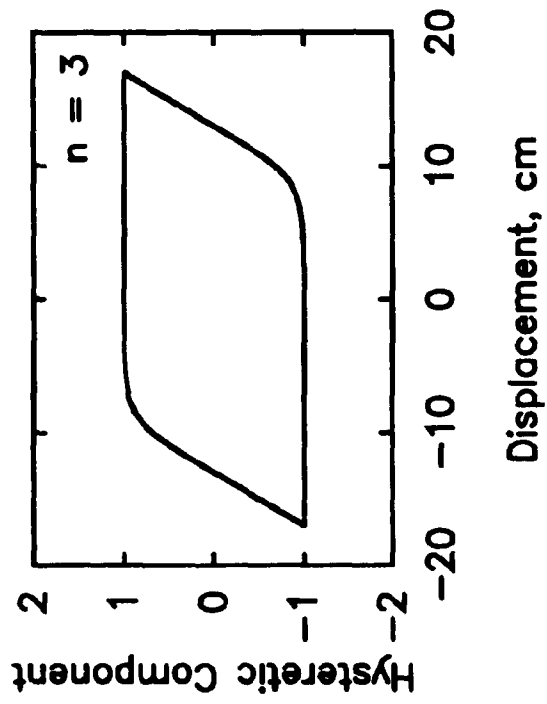


Fig. 7.2 : Hysteresis Loop of Lead-Core Rubber Bearing

Table 7.1 : Maximum Response Quantities of A Base-Isolated Building ( 0.3g Earthquake)

S T O R Y	D <sub>y</sub> cm (2)	No Control		WITH BIS		LINEAR CONTROL		NONLINEAR CONTROL I		NONLINEAR CONTROL II	
		X <sub>i</sub> cm (3)	$\dot{x}_i$ cm/s <sup>2</sup> (4)	X <sub>i</sub> cm (5)	$\dot{x}_i$ cm/s <sup>2</sup> (6)	X <sub>i</sub> cm (7)	$\dot{x}_i$ cm/s <sup>2</sup> (8)	X <sub>i</sub> cm (9)	$\dot{x}_i$ cm/s <sup>2</sup> (10)	X <sub>i</sub> cm (11)	$\dot{x}_i$ cm/s <sup>2</sup> (12)
B	-	-	-	21.36	117	10.14	124	8.31	65	8.06	63
1	2.4	2.05	381	0.62	112	0.51	124	0.38	65	0.37	63
2	2.3	2.08	434	0.59	112	0.52	115	0.38	61	0.37	59
3	2.2	2.16	500	0.65	111	0.60	98	0.40	58	0.39	57
4	2.1	2.40	460	0.63	101	0.61	86	0.38	52	0.36	50
5	2.0	2.68	559	0.63	91	0.58	101	0.35	62	0.34	60
6	1.9	3.16	446	0.64	103	0.56	105	0.34	64	0.33	62
7	1.7	4.37	594	0.60	130	0.59	120	0.32	70	0.31	67
8	1.5	1.94	614	0.41	163	0.46	184	0.22	88	0.22	85

**Table 7.2 : Maximum Response Quantities of A Base-Isolated Building (1g Earthquake)**

S T O R Y	D <sub>y</sub> cm (1)	No Control		WITH BIS		LINEAR CONTROL		NONLINEAR CONTROL I		NONLINEAR CONTROL II	
		x <sub>i</sub> cm (3)	$\ddot{x}_{ij}$ cm/s <sup>2</sup> (4)	x <sub>i</sub> cm (5)	$\ddot{x}_{ij}$ cm/s <sup>2</sup> (6)	x <sub>i</sub> cm (7)	$\ddot{x}_{ij}$ cm/s <sup>2</sup> (8)	x <sub>i</sub> cm (9)	$\ddot{x}_{ij}$ cm/s <sup>2</sup> (10)	x <sub>i</sub> cm (11)	$\ddot{x}_{ij}$ cm/s <sup>2</sup> (12)
B	-	-	-	67.61	351	29.69	316	25.97	212	24.95	231
1	2.4	5.04	1032	1.92	350	1.48	303	1.21	213	1.16	216
2	2.3	4.24	1150	1.95	325	1.44	312	1.21	200	1.16	181
3	2.2	5.31	1070	2.13	304	1.52	282	1.27	183	1.22	166
4	2.1	5.48	1171	2.07	277	1.48	255	1.19	165	1.15	147
5	2.0	6.76	1203	1.95	304	1.33	278	1.12	197	1.08	167
6	1.9	8.67	1000	1.81	345	1.28	348	1.06	208	1.01	184
7	1.7	10.36	805	1.59	374	1.44	356	0.99	219	0.96	222
8	1.5	4.62	726	1.07	421	1.14	449	0.75	298	0.72	285

**Table 7.3 : Maximum Response Quantities of An Elasto-Plastic Building With Active Bracing System**

S T O R Y N O (1)	Dy cm (2)	No Control		Linear Control			1St Nonlinear Cntr			2ndNonlinear Cntr		
		$x_i$ cm (3)	$\ddot{x}_m$ cm/s <sup>2</sup> (4)	$x_i$ cm (5)	$\ddot{x}_m$ cm/s <sup>2</sup> (6)	$U_i$ kN (7)	$x_i$ cm (8)	$\ddot{x}_m$ cm/s <sup>2</sup> (9)	$U_i$ kN (10)	$x_i$ cm (11)	$\ddot{x}_m$ cm/s <sup>2</sup> (12)	$U_i$ kN (13)
1	2.4	5.04	1032	3.33	838	9167	2.06	901	11605	2.05	897	11558
2	2.3	4.24	1151	3.18	717	8629	3.73	758	12152	3.71	755	12085
3	2.2	5.31	1070	3.34	621	8176	2.15	666	10230	2.14	664	10190
4	2.1	5.48	1172	2.80	543	7315	1.54	599	8518	1.53	597	8483
5	2.0	6.76	1204	2.24	494	6398	1.12	545	6727	1.11	543	6700
6	1.9	8.67	1001	1.85	529	5236	0.79	506	4934	0.79	504	4914
7	1.7	10.36	805	1.41	559	3761	0.52	479	3221	0.51	477	3208
8	1.5	4.62	726	0.79	576	1968	0.26	466	1588	0.26	464	1582



**Table 7.4 : Maximum Response Quantities of An Elasto-Plastic Building With Active Bracing System  
(Large Control Force)**

S T O R Y  N O (1)	Dy cm (2)	No Control		Linear Control			1St Nonlinear Cntr			2nd Nonlinear Cntr		
		$x_i$ cm (3)	$\ddot{x}_m$ cm/s <sup>2</sup> (4)	$x_i$ cm (5)	$\ddot{x}_m$ cm/s <sup>2</sup> (6)	$U_i$ kN (7)	$x_i$ cm (8)	$\ddot{x}_m$ cm/s <sup>2</sup> (9)	$U_i$ kN (10)	$x_i$ cm (11)	$\ddot{x}_m$ cm/s <sup>2</sup> (12)	$U_i$ kN (13)
1	2.4	5.04	1032	1.72	912	18969	0.85	954	21589	0.85	954	21568
2	2.3	4.24	1151	1.54	853	16791	1.43	906	17929	1.43	905	17911
3	2.2	5.31	1070	1.39	811	14736	0.96	874	16010	0.96	873	15994
4	2.1	5.48	1172	1.19	792	12453	0.69	851	13498	0.69	851	13485
5	2.0	6.76	1204	0.99	790	10132	0.51	835	10871	0.51	834	10861
6	1.9	8.67	1001	0.78	789	7727	0.36	824	8197	0.36	824	8190
7	1.7	10.36	805	0.54	790	5222	0.23	818	5491	0.23	817	5486
8	1.5	4.62	726	0.28	791	2637	0.12	814	2756	0.11	814	2753

## **SECTION 9**

### **CONCLUSIONS**

An optimal nonlinear control method is proposed for applications to seismic-excited nonlinear or hysteretic building structures. Emphasis is placed on hybrid control of base-isolated hysteretic buildings. Both the absolute acceleration response of the building and the actuator dynamics have been accounted for in the optimization process. Control laws using acceleration and velocity feedbacks are also derived. Simulation results indicate that (i) the proposed nonlinear control method is effective for hybrid control of seismic-excited buildings isolated by rubber-bearing isolators, and (ii) the performance of the proposed nonlinear control method is better than that of the linear control method proposed previously.

## SECTION 10

### REFERENCES

1. Brogan, W.L. (1991), Modern Control Theory, Prentice Hall, Third Edition, NJ.
2. Feng, Q., Shinozuka, M. and Fujita, T. (1991), "Hybrid Isolation Systems Using Friction-Controllable Sliding Bearings", Dynamics and Control of Large Structures, edited by L. Meirovitch, VPI&SU Press, Blacksburg, VA, pp. 207-218.
3. Inaudi, J.A. and Kelly, J.M. (1991), "A Simple Active Isolation Scheme", Dynamics and Control of Large Structures, edited by L. Meirovich, VPI&SU Press, Blacksburg, VA, pp. 219-230, 1991.
4. Inaudi, J.A. and Kelly, J.M. (1993), "Hybrid Isolation Systems for Equipment Protection", paper to appear in J. Earthquake Engr. and Structural Dynamics.
5. Kawamura, M., Shinozuka, M., Fuji, S. and Feng, Q. (1991), "Hybrid Isolation System Using Frictional Controllable Sliding Bearings", Intelligent Structures -2, edited by Y.K. Wen, pp. 264-278, Elsevier Applied Science.
6. Masri, S.F., Bekey, G.A., and Caughey, T.K. (1981), "On-Line Control of Nonlinear Flexible Structures", J. Applied Mechanics, ASME, Vol. 49, No. 4, pp. 871-884.
7. Reinhorn, A.M., Manolis, G.D., and Wen, C.Y. (1987), "Active Control of Inelastic Structures", J. Engr. Mechanics, ASCE, Vol. 113, No. 3, pp. 315-332.
8. Riley, M.A., Reinhorn, A.M. and Constantinou (1991), "Active Control of Absolute Motion in Sliding Systems", Dynamics and Control of Large Structures, edited by L. Meirovitch, VPI&SU Press, Blacksburg, VA, pp.243-254.
9. Slotine, J.J.E. and Li, W. (1991), Applied Nonlinear Control, Prentice Hall, NJ.
10. Soong, T.T. (1990), Active Structural Control: Theory and Practice, Longman Scientific and Technical, NY.
11. J. Suhardjo, B.F. Spencer, Jr. and M.K. Sain (1992), "Nonlinear Optimal Control of a Duffing System", International Journal of Nonlinear Mechanics, Vol. 27, No. 2, pp. 157-172.
12. J. Suhardjo, B.F. Spencer, Jr., M.K. Sain and D. Tomasula (1992), "Nonlinear Control of a Tension Leg Platform", Innovative Large Span Structures - Vol. 1, (N.K. Srivastanva, A.N. Sherbourne and J. Roorda, Editors), Canadian Society for Civil Engineering, Toronto, Canada, pp. 464-474.

13. Tadjbakhsh, I.G. and Rofooei, F. (1992), "Optimal Hybrid Control of Structures Under Earthquake Excitation", J. Earthquake Engr. and Structural Dynamics, Vol. 21, pp. 233-252.
14. Tabot, M.E. and Shinozuka, M. (1990), "Active Isolation for Sismic Protection of Operating Rooms", Technical Report, NCEER-90-0010, National Center for Earthquake Engineering Research, Buffalo, New York.
15. Yang, J.N, Akbrapour, A. and Ghaemmaghmi, P. (1987), "New Optimal Control Algorithms for Sturctural Control", J. Engr. Mechanics, ASCE, Vol. 113, No. 9, pp. 1369-1368.
16. Yang, J.N., Long, F.X. and Wong, D. (1988), "Optimal Control of Nonlinear Structures", J. of Applied Mechanics, ASCE, Vol. 55, pp. 931-938.
17. Yang, J.N., Danielians, A., and Liu, S.C. (1991), "Aseismic Hybrid Control Systems for Building Structures", J. Engr. Mechanics, ASCE, Vol. 117, No. 4, April, pp. 836-853.
18. Yang, J.N., Li, Z., Danielians, A., and Liu, S.C. (1992), "Hybrid Control of Nonlinear and Hysteretic System I" J. Engr. Mechanics, ASCE, Vol. 118, No. 7, April, pp. 1423-1440.
19. Yang, J.N., Li, Z., Danielians, A., and Liu, S.C. (1992), "Hybrid Control of Nonlinear and Hysteretic System II" J. Engr. Mechanics, ASCE, Vol. 118, No. 7, April, pp. 1441-1456.
20. Yang, J.N, Li, Z. and Liu, S.C. (1992), "Stable Controllers for Instantaneous Optimal Control", J. Engr. Mechanics, ASCE, Vol. 118, No. 8, pp. 1612-1630.
21. Yang, J.N., Li, Z. and Liu, S.C. (1992), "Control of Hysteretic System Using Velocity and Acceleration Feedbacks", J. Engr. Mechanics, ASCE, Nov.
22. Yang, J.N. and Li, Z. (1992), "Control of Base-Isolated Building Structures", Innovative Large Span Structures - Vol. 1, (N.K. Srivastanva, A.N. Sherbourne and J. Roorda, Editors), Canadian Society for Civil Engineering, Toronto, Canada, pp. 475-498.

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