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Simplified Procedures for Seismic Design of Nonstructural Components and Assessment of Current Code Provisions

by

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) was established to expand and disseminate knowledge about earthquakes, improve earthquake-resistant design, and implement seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures in the eastern and central United States and lifelines throughout the country that are found in zones of low, moderate, and high seismicity.

NCEER's research and implementation plan in years six through ten (1991-1996) comprises four interlocked elements, as shown in the figure below. Element I, Basic Research, is carried out to support projects in the Applied Research area. Element II, Applied Research, is the major focus of work for years six through ten. Element III, Demonstration Projects, have been planned to support Applied Research projects, and will be either case studies or regional studies. Element IV, Implementation, will result from activity in the four Applied Research projects, and from Demonstration Projects.



Research tasks in the Nonstructural Components Project focus on analytical and experimental investigations of seismic behavior of secondary systems, investigating hazard mitigation through optimization and protection, and developing rational criteria and procedures for seismic design and performance evaluation. Specifically, tasks are being performed to: (1) provide a risk analysis of a selected group of nonstructural elements; (2) improve simplified analysis so that research results can be readily used by practicing engineers; (3) protect sensitive equipment and critical subsystems using passive, active or hybrid systems; and (4) develop design and performance evaluation guidelines.

The end product of the Nonstructural Components Project will be a set of simple guidelines for design, performance evaluation, support design, and protection and mitigation measures in the form of handbooks or computer codes, and software and hardware associated with innovative protection technology.

This report documents one part of NCEER's efforts in assessing current seismic code provisions for nonstructural components and recommending possible improvements based on recent research results. The formulas for calculating seismic forces acting on nonstructural components as provided by the 1991 NEHRP Provisions, 1983 Tri-Service Manual, and the 1991 Uniform Building Code are reviewed and recommended revisions to these formulas are made based on either a simplified or a rigorous approach. Also included in the report is a "User Summary" section, which is provided for the convenience of those who are primarily interested in design force calculations and not in analytical details.

ABSTRACT

The detailed seismic design provisions for nonstructural components in buildings were first proposed in the 1978 ATC 03 report. These have been adopted with some minor changes by the 1991 NEHRP Recommended Provisions, which are now being used as the basis of the first generation seismic force provisions for the design of nonstructural components in some recent codes and manuals. Herein, these provisions have been critically evaluated, and improved procedures which incorporate the dynamic characteristics of the supporting structure as well as nonstructural components have been proposed.

The basic format of the proposed procedure for calculating the seismic force is the same as in the NEHRP Provisions, except that the seismic force coefficients are now defined on a more rational basis. In the proposed methods, the coefficients are calculated by modal analysis approaches, similar to the one proposed in Chapter 5 of the 1991 NEHRP Provisions for calculating forces on structural components. Both for architectural components and mechanical equipment, simplified procedures utilizing only the first mode as well as more rigorous procedures using a few first dominant modes are proposed. In all cases, the seismic coefficients are defined by closed-form expressions. The effect of the inelastic behavior of the supporting structure on the seismic coefficients is incorporated through response modification factor R, as is done in the NEHRP Provisions. Simplified methods are also presented to calculate the frequencies, mode shapes and other modal quantities required for calculating the seismic coefficients. The seismic design forces for different architectural components and flexible mechanical equipment, placed in buildings of different heights, periods and ductility, calculated according to the 1991 NEHRP Provisions, Tri-services Manuals and 1991 Uniform Building Code, are compared with the forces calculated by the proposed procedure to demonstrate the importance of various parameters of the structural and nonstructural systems in the calculation of the design forces?

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USER SUMMARY

This summary is provided for the convenience of a user who is primarily interested in calculating the design forces on nonstructural components and not in the details of the analytical background. Both simplified and rigorous approaches for calculating the forces are summarized. More complete details are provided for the simplified approaches. For the rigorous approaches, the reader is directed to specific sections of this report.

The use of the simplified approaches is recommended. They provide a conservative estimate of the force in most cases with the least computations. However, if it is found impractical to design for the calculated force, or if the design of the component calls for a more accurate analysis of the forces, the use of the rigorous approaches is recommended.

1. Architectural Components and Rigidly Connected Mechanical And Electrical Components

The details pertaining to the force formula for these components are given in Section 3.1 of the report.

The force, F_p , is calculated using the following formula, both in the simplified as well as in the rigorous approaches:

$$F_p = A_v C_{cm} P I W_c \tag{1}$$

where:

- A_v = Effective peak velocity related acceleration, Section 1.4.1 of the 1991-NEHRP Provisions.
- P = Performance Criteria Factor specified in this report in Table 3.1 for architectural components and Table 3.2 for mechanical and electrical components.

- I = Importance Factor specified in this report in Table 3.1 for architectural components and Table 3.2 for mechanical and electrical components.
- W =Weight of the component.

 C_{cm} is the Seismic Coefficient for the component placed on floor m. This is defined as follows in the simplified and rigorous approaches.

C_{cm} by Simplified Approach:

The details of this approach are given in Section 3.1.2 of the report. The coefficient C_{cm} is defined as:

$$C_{cm} = C_o + \frac{h_m}{h_N} (C_{cN} - C_o) \tag{2}$$

where:

 $C_o = S/R$

 h_m = The height of the m^{th} floor above the base.

- h_N = The height of the roof or N^{th} mass above the base.
 - S = Site coefficient, Table 3.2 of the 1991-NEHRP Provisions.

R = Response modification factor, Table 3.3 of the 1991-NEHRP Provisions.

 C_{cN} = The seismic coefficient for the roof mass

$$= R_1 \sqrt{2.85p_1^2 - 2.7p_1 + 1.5} \tag{3}$$

$$R_1 = 1.2S/(T_1^{2/3}R) \tag{4}$$

- T_1 = Fundamental period of the building in seconds. This can be estimated by equations (4.4) of the 1991-NEHRP Provisions or by any rational method.
- $p_1 = \gamma_1 \phi_{N1}$ = the product of the fundamental mode participation factor and modal displacement at the roof. It can be calculated by any rational method.

For a uniform building with assumed linear variation of the first mode with height, the coefficient C_{cN} can be more simply defined as follows:

$$C_{cN} = R_1 \frac{\sqrt{15.45N^2 - 2.1N + 1.5}}{(2N+1)^2}$$
(5)

where N = the total number of stories in the building.

C_{cm} by Rigorous Approach

This approach requires that the characteristics of a first few dominant modes of the structure be available. In terms of these modal characteristics, the coefficient C_{cm} is defined by equation (3.2) of this report. The characteristics of the higher modes can be obtained as explained in Section 5 of the report.

2. Flexible or Flexibly Connected Nonstructural Components

The details of the force formula for these components are given in Section 3.2 of the report.

The force, F_p , is calculated using the following formula

$$F_{p} = A_{v}C_{fm}PIW_{c} \tag{6}$$

where A_v, P, I , and W_c are the same as in formula (1).

 C_{fm} is the unit floor response spectrum coefficient which depends upon the dynamic characteristics of: (1) the building, (2) the nonstructural component; and (3) the seismic input. Again, a the simplified single-mode and a rigorous multi-mode approach can be used to calculate this coefficient.

C_{fm} by Simplified Approach

Details of this approach are given in Section 3.2.1 and Appendix B of the report. These are summarized here as follows. For a nonstructural component of natural frequency f placed on the m^{th} floor of a building, C_{fm} is defined as follows:

 $0 < \mathbf{f} \le \mathbf{0.5f_1}$ $C_{fm} = \frac{2f}{f_1} \left\{ R_G + \frac{m-1}{N-1} (R_{max} - R_G) \right\}$

 $0.5f_1 < f \leq f_\ell$

$$C_{fm} = R_G + \frac{m-1}{N-1}(R_{max} - R_G)$$

 $f_\ell < f \leq f_m$

$$C_{fm} = R_G + \frac{m-1}{N-1} \left\{ C_{cN} - R_G + \frac{(f_m - f)}{(f_m - f_\ell)} (R_{max} - C_{cN}) \right\}$$

 $\mathbf{f}_m < \mathbf{f} \leq \mathbf{f}_u$

$$C_{fm} = C_{c1} + \frac{(f_u - f)}{(f_u - f_m)} (R_G - C_{c1}) + \frac{m - 1}{N - 1} \left\{ C_{cN} - C_{c1} - \frac{(f_u - f)}{(f_u - f_m)} (R_G - C_{c1}) \right\}$$

 $\mathbf{f} > \mathbf{f}_{\mathbf{u}}$

$$C_{fm} = C_{c1} + \left(\frac{m-1}{N-1}\right) (C_{cN} - C_{c1})$$

where:

f = natural frequency of the equipment in cps

 f_1 = fundamental frequency of the structure in cps

$$f_{\ell} = \frac{f_N}{2\sqrt{N}}$$

$$f_m = 0.8 \ f_N$$

$$f_u = 1.5 \ f_N$$

$$f_N = sin \frac{\left\{\frac{(2N-1)\pi}{2(2N+1)}\right\}}{sin\left\{\frac{\pi}{2(2N+1)}\right\}} f_1$$

 C_{c1} = defined by formula (2) in this summary. For uniform building, it can also be calculated by equation (B.24) of Appendix B in the report C_{cN} = defined by formula (3) or (5) of this summary R_{max} = defined by equation (B.18) or by equation (B.19) in Appendix B of this report for a uniform building.

$$R_G = 20 \ \frac{S}{R} N^- \sqrt{3}$$

C_{fm} by Rigorous Approach

To use this approach, one needs to know the characteristics of a first few dominant modes. This approach requires significantly more computations. The closed-form formulas to define C_{fm} are provided in Appendix B of this report.

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SECTION 1 INTRODUCTION

In small earthquakes, the majority of damage is due to failures of nonstructural items. Except for economic losses, most of these failures are harmless. In fact, these failures can be easily prevented by simple tying down procedures. However, in some recent moderate size earthquakes, it has been clearly demonstrated that failure of nonstructural items can cause not only unacceptable economic losses but can also pose some serious safety concerns if the nonstructural components and their supports are not properly designed for the expected seismic forces.

The issue of systematic design of these components for earthquake loads was first raised by the 1978 ATC 3-06 Report [1] which prescribed a method to calculate the design forces. These ATC 3-06 provisions were adopted as the 1985 NEHRP Provisions [4] and then later in a slightly modified form as 1991 NEHRP Provisions [5]. A similar approach is also taken by the 1991 Uniform Building Code [8] and the 1982 Tri-services Manual for Seismic Design of Buildings [7] to define the forces on nonstructural components. The latter manual was based on the then SEAOC recommendations [17]. A more rigorous method to calculate forces on nonstructural components is also described by the 1986 Tri-services Manual entitled "Seismic Design Guidelines for Essential Buildings" [11].

Concurrently when these codes were being written and developed, research to obtain the seismic response of equipment more accurately was also being carried out independently with its primary application to equipment in nuclear power plants. This research led to the development of more rational procedures to calculate seismic design forces in nonstructural systems. Although these rational methods were available when the aforementioned code changes were being formulated, they were ignored primarily to simplify the force calculations. This simplification has, of course, led to some compromises in the rationality of these proposed code provisions. To what extent the rationality has been compromised in the provisions is not known. It must be examined before any changes can be proposed.

In Section 2 we first describe three commonly used code provisions: (1) the 1991 - NEHRP Provisions, (2) the Tri-services Manual provisions, and; (3) the Uniform Building Code provisions. The provisions are critically reviewed and their shortcomings are brought out qualitatively. One of the serious drawbacks of these provisions is noted to be that they do not consider dynamic characteristics of the supporting structure for calculating the forces on nonstructural systems. In Section 3 a modal analysis based on a response spectrum approach is proposed to calculate the design seismic forces for architectural components and equipment. The method allows one to incorporate the dynamic properties of the supporting structure explicitly. The approach is similar to the modal analysis approach prescribed in the 1991- NEHRP provisions for calculating displacements, base shear and overturning moments in the supporting structure. For the calculation of forces on architectural components, both a simplified single-mode approach and a more rigorous and accurate multi-mode approach are proposed. The formulas for calculating the forces are provided in closed-form in both approaches. For flexibly supported mechanical and electrical equipment, the current code provisions have more serious problems as they can lead to a gross underestimation of design forces for an equipment whose support frequency is tuned to one of the higher dominant mode frequencies of the supporting building. In such a case, it is necessary that at least a first few dominant modes of the supporting structure be used to capture this tuning effect in calculating the design forces. Here a rigorous response spectrum approach explicitly utilizing the modes of the supporting structure is, therefore, proposed. This is followed by a simpler single mode approach where only the first mode properties are used in the calculation of the force. The numerical results comparing the current code provisions and the proposed approaches are presented in Section 4 for several example problems. The analytical background and the

justification for the single-mode simplified procedures are provided in Appendices A and B.

To implement the proposed response spectrum approach, the modal properties of the supporting structure are required. In Section 5, simple closed-form expressions are provided to calculate the modal frequencies, mode shapes and participation factors for regular building structures with uniform structural properties along their heights. These expressions can still be used for irregular structures in their plans. Applicability of these formulas to calculate the modal properties of somewhat non uniform buildings in their elevations has also been evaluated through numerical examples of several building structures. The effect on the calculated force of neglecting the higher modes as well as the effect of the height at which a nonstructural component is placed in a building have also been examined in this section.

As the building structures designed according to the current code provisions are expected to yield and behave inelastically under a design level earthquake, it is necessary to incorporate this nonlinear effect in calculating the force on nonstructural component as well. This effect is completely ignored in the current code provisions for calculating the forces on nonstructural components although it is included through coefficients like the R-factor while calculating forces in the supporting structure. In the response spectrum methods proposed herein, this effect can also be included through the R-factor. This factor depends upon the type of structural system used in a building. In Section 6, a limited study is conducted to examine the force reduction effect of structural yielding. It is observed that this effect is quite complex and can not be included by using a simple R-factor in all situations. However, for the sake of simplicity in application, the use of R-factor is still recommended. It is also observed that yielding need not cause a reduction in the force of a supported nonstructural component in all situations.

General concluding remarks of this study are summarized in Section 7.

SECTION 2

CODIFIED DESIGN PROVISIONS

In this chapter, we describe the formulas prescribed by the NEHRP Provisions of 1991 [5], the Tri-Services Manuals of 1982 [7] and 1986 [11], and the 1991 Uniform Building Code [8] for calculating the design forces for nonstructural components. The provisions are critically examined and their limitations are brought out. At the risk of some repetition but for the sake of completeness and ready reference, the tables giving the values of various coefficients and factors in these codes and manuals are also reproduced here.

2.1 1991-NEHRP PROVISIONS

The NEHRP provisions consider the architectural components separately from the mechanical and electrical components. The provisions to calculate the design force are as follows:

2.1.1 Architectural Components:

The basic formula for calculating the design force for architectural components is defined as:

$$F_{\mathbf{p}} = A_{\mathbf{v}} C_{\mathbf{c}} P W_{\mathbf{c}} \tag{2.1}$$

where

- F_p = the seismic design force applied at the center of gravity of the component.
- A_v = the seismic input motion coefficient representing the effective peak velocityrelated acceleration of Sec. 1.4.1 of the provisions.
- C_c = the seismic coefficient as defined in Table 8.2.2 for architectural components and reproduced in Table 2.1 in this report. The seismic coefficient C_c varies between 0.6 and 3.0 for different components.

- P = the performance criteria factor given in Table 2.1 (or Table 8.2.2 of the provisions).
- W_c = weight of the component.

The values assigned for the performance criteria factor P are: 0(NR-not required), 0.5, 1.0 and 1.5. The value of 1.0 "is considered the base performance value for most components." The components assigned a value of 1.5 are those which, if damaged, will have more serious consequences than the components which are assigned a value of 1.00. Also the higher the seismic hazard exposure, the higher the value of this factor.

It is observed that:

- (1) The force defined in equation (2.1) does not depend upon the height where a component is placed in a building. That is, a component at the top of a 10 story building is designed for the same force as a similar component in the basement.
- (2) The force does not depend upon the building period.
- (3) Although two equal mass and identically placed component will feel the same seismic force, the Provisions still prescribe different seismic coefficients based on their functions. A component providing a more critical service or function is assigned a higher seismic coefficient value and, in most cases, even a higher seismic performance criteria factor. That is, a more critical or important component (from life safety standpoint) is expected to be designed for a higher force than a less important component.
- (4) The two factors, seismic coefficient C_c and the performance criteria P, could be merged into a single coefficient, but the committee formulating the Provisions chose to keep them separate.

(5) The chosen numerical values of the prescribed coefficients and factors are arbitrary. They are not based on any dynamic response characteristics of either the structure or the component. They represent the collective professional experience and "gut feelings" of experienced engineers.

2.1.2 Mechanical and Electrical Components

The basic formula to calculate the design forces on mechanical and electrical components is as follows:

$$F_p = A_v \ C_c \ P \ a_c \ W_c \tag{2.2}$$

The only difference between this formula and the formula for the architectural component defined in equation (2.1) is the introduction of the response amplification factor a_c for mechanical components. Also the values of the coefficient C_c are different, as shown in Table 2.2 (or Table 8.3.2a in the provisions). These values now range from a low of 0.67 to a high of 2.00. The value of 2.00 is used for those components, whose damage is likely to have more severe consequences, such as fire protection equipment and systems, emergency and stand-by electrical systems, boilers, furnaces, water heaters, or equipment using combustible energy source or high temperature energy source, chimneys, flues and smokestacks, communication systems, electrical ducts and cable trays, control center equipment, reciprocating and rotating equipment, tanks, heat exchangers, pressure vessels, utility interface, gas and high hazard piping systems, and fire suppression piping.

Again, as discussed before, the differences in the values of this factor for different components is not due to any dynamic considerations of the structure or the component. They just represent the relative importance of the component and the consequences of its failure. This factor could have been included with the performance criteria factor P, but the committee chose to separate the two. As was the case with architectural components, the performance criteria factor P again has four values: 0, 0.5, 1.0, and 1.5. These values are assigned to each component for the three seismic hazard exposure groups. The higher the exposure group and more critical the components, the higher its performance criteria factor P.

The amplification factor, a_c , is introduced to account for the possibility of support motion amplification due to the flexibility of the component support. For components which are directly connected or fixed, the supports are considered to be rigid and thus the acceleration of the component is the same as the acceleration of the floor on which it is supported. In such a case, the amplification factor is equal to 1.0. When the period of the component is within the vicinity of the fundamental period of the building, some resonance effect can be expected. To account for this resonance the amplification factor shown in Figure 2.1 is used. T_c and T used in Figure 2.1 are the fundamental periods of the components and building, respectively.

It is noticed that this amplification factor is quite different from the classical dynamic amplification commonly shown in the text books on mechanical and structural vibrations for purely harmonic inputs. Although not clearly stated in the provisions, the main reason for the difference between the proposed amplification factor and classical amplification factor is that the actual support motions are not truly harmonic and persistent to cause a steady state resonant response. Around the resonance period ratio of 1.0, that is between the period ratios of 0.6 and 1.4, the amplification curve in Figure 2.1 does not have the usual sharp peak but it is flat. This flatness is introduced, primarily, to account for the uncertainties involved in the estimation of the periods. Also for higher T_c/T ratio, the classical amplification usually diminishes very fast, but here for simplicity this has been taken to be 1.0. In any case, the higher period ratio range $(T_c/T > 1.4)$ is of little practical interest as it will occur only when the component support is highly flexible. As was the case with architectural components, the seismic force does not depend upon the height at which the component is located in a building. In the 1985version of the NEHRP Provisions, a factor equal to $(1 + h_x/h_n)$ was included to incorporate the effect of height in the building. However, this factor unrealistically increased the forces by a factor of 2 even for a single story building. Although this discrepancy could have been easily corrected by a simple adjustment of this factor, the provision formulating committee decided to delete this completely from the 1991version. The commentary to Chapter 8 of the Provision [6], states that this effect was "not considered significant because of the manner in which the values were assigned to C_c and P, the relatively light weight of typical components or systems (as compared to the building weights) and the desire to maintain a simple form of" equation (2.2). These reasons are, however, hardly convincing. A more rational approach to include the effect of location of the component is presented later.

2.2 1983-TRI-SERVICES MANUAL PROVISIONS

There are two manuals prepared for the seismic design of defense buildings and installations. One of them is for ordinary buildings and components and it is entitled "Seismic Design for Buildings," Reference 7. The second one is called the "Seismic Design Guidelines for Essential Buildings," Reference 11. This latter reference provides a more rigorous approach to define the design motion and more rigorous methods of analysis based on the principles of dynamics. These two manuals will be referred to as Manual I and Manual II, respectively, in this report.

Both of these manuals have separate chapters on nonstructural components. In Manual I, the forces on nonstructural elements are defined as static forces by simplified formulas, somewhat similar to those prescribed in the NEHRP provisions or in the UBC. Manual II defines these forces in terms of floor response spectra. This procedure considers the dynamic properties of a nonstructural component and its supporting structure. The input ground motion in this approach is defined by site dependent response spectra. The method to obtain a floor response spectrum is provided and illustrated by numerical examples.

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Herein, we will only provide the seismic design force provisions given in Manual I, although for comparison of various provisions the method described in the manual for essential buildings has also been used to obtain numerical results for an example problem.

As in the NEHRP Provisions, Manual I for ordinary buildings also prescribes forces separately for architectural components and for mechanical and electrical elements. These are described as follows.

2.2.1 Architectural Components

For the architectural components the formula to define the force is:

$$F_p = Z \ I \ C_p \ W_p \tag{2.3}$$

where

- Z = Zone factor. The value of which depends upon the seismic zone. There are five zones denoted as 0, 1, 2, 3, and 4 and the corresponding zone factors are 0, 3/16, 3/8, 3/4, and 1, respectively.
- I = Importance factor. It depends upon the type of occupancy which are essential,
 high risk and all others. For these three occupancies the factor values are 1.5,
 1.25 and 1.00, respectively.

 C_p = Component weight.

 W_p = Seismic force coefficient.

Normally, the value of C_p is 0.3. For parapets, ornamentations and appendages which are more likely to fall and cause injury, this value is increased to 0.8. Table 3-4 of the manual, which is reproduced here as Table 2.3, provides the C_p values for various elements of a structure or nonstructural components. The footnotes to the tables have been omitted; they describe some special situations. Special provisions are also defined for the design of non-load bearing panels and their connections to the structure.

As was the case in NEHRP Provisions, here also the forces do not vary with the height of the building, and do not depend upon the dynamic characteristics of the building.

2.2.2 Mechanical and Electrical Components

The forces on mechanical and electrical components are defined in Chapter 10 of Manual I [7]. These forces are for the design of the equipment supports and not the equipment itself. The equipment is supposed to have been designed to withstand the design forces without any malfunction.

The design provision described in this section are only applicable to light equipment. Equipment which are heavier than 20% of the weight of floor at which they are supported or 10% of the weight of the entire weight of the building are not covered by these provisions. Such equipment, being relatively heavy, can appreciably affect the response of the supporting structure due to dynamic interaction. When such dynamic interaction is present, the simplified formula presented here is no longer applicable.

In Manual I, the equipment is divided into two categories: (1) rigid and rigidly mounted equipment, and; (2) flexible equipment or flexibly mounted equipment in buildings. The design force provisions for these two categories of equipment are as follows.

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2.2.3 Rigid and Rigidly Mounted Equipment

Rigid and rigidly mounted equipment are those equipment systems (support and equipment itself) the period of vibration of which is less than 0.05 sec. Some examples of such equipment are: a boiler bolted or securely attached to a concrete pad or the floor of a structure; an electrical panel board securely attached to a solid wall; an electric motor securely bolted to a floor; a flood light with a short step bolted to a wall; a securely anchored heat exchanger, etc. The equivalent static force for the design of the support of such equipment is given by equation (2.3), used earlier for nonstructural components. The value of the coefficient C_p for these equipment is 0.3.

For rigid equipment rigidly connected to a support directly on ground, the equivalent static force is reduced by a factor of 2/3. That is

$$F_{p} = Z I \left(\frac{2}{3}C_{p}\right) W_{p} \tag{2.4}$$

2.2.4 Flexible Equipment or Flexibly Mounted Equipment

The equipment which cannot be classified as rigid or rigidly mounted equipment fall in the category of flexible equipment or flexibly mounted equipment. The equivalent static force on such equipment is defined as

$$F_p = Z I (A_p C_p) W_p \tag{2.5}$$

where Z, I, C_p and W_p are the same as defined for the architectural components. A_p is the amplification factor by which the equipment support motion is amplified because of the relative flexibility of the equipment support. A basic assumption in the definition of A_p , provided below, is that "the equipment responds as a single degree of freedom system to the motion of one of the predominant modes of vibration of the building at the floor level at which the equipment is placed." The Manual I precludes the use of this formula for equipment which cannot be considered to satisfy the above requirements. The amplification factor A_p depends upon the ratio of the equipment period to building period and is shown in Figure 2.2 which has been reproduced from Manual I. Figure 2.2(a) is for the case of buildings with periods less than or equal to 0.5 seconds, whereas Figure 2.2(b) is for more flexible buildings with periods greater than 0.5 seconds.

There can be situations when one does not have information about the equipment or building period. In such cases, the Manual requires that the highest value $(A_p = 5.0$ in Figures 2.2(a) and (b)) be used. This value can be reduced if one has better information about the building period. The larger the building period, the smaller the amplification factor, as the input from the building to the equipment will not have enough cycles in the duration of earthquake to cause resonance. Table 2.4, extracted from the Manual provides these factors.

Although explicit rationale for presenting these values of the amplification factors is not provided, they are said to include in an empirical way the effect of higher building modes, inelastic effects in the building at high response amplitudes, limited duration of earthquake and uncertainties in the calculation of equipment and building periods.

For flexible or flexibly mounted equipment directly on ground, the equivalent static load force is specified as

$$F_p = Z \ I \ (2CS) \ W_p \tag{2.6}$$

where:

$$C = \frac{1}{15\sqrt{T_a}} \tag{2.7}$$

and:

$$S = 1 + \left(\frac{T_a}{T_s}\right) - .5 \left(\frac{T_a}{T_s}\right)^2 \qquad \text{for} \qquad \frac{T_a}{T_s} \le 1.0 \tag{2.8}$$

$$= 1.2 + .6 \left(\frac{T_a}{T_s}\right) - .3 \left(\frac{T_a}{T_s}\right)^2 \qquad \text{for} \qquad \frac{T_a}{T_s} > 1.0 \tag{2.9}$$

in which T_s is the period of the ground soil where the equipment is supported. The product (CS) is, however, limited to a value of 0.14.

Manual I goes into further details for calculating the forces for equipment which can not be considered as single degree of freedom systems. They include piping, stacks or other special structural systems which cannot be considered to response predominantly in a single mode.

2.3 1991 UBC-PROVISIONS

The UBC provides the same formula to define the force on architectural components and on mechanical and electrical components. The total seismic design lateral force, F_p , is defined as [8]:

$$F_p = Z \ I \ C_p \ W_p \tag{2.10}$$

where

- Z = the zone factor given in Table No. 23-I of the code. There are five zones, 1,
 2A, 2B, 3 and 4 for which the corresponding zone factors are 0.075, 0.15, 0.2,
 0.3, and 0.40, respectively.
- I = Importance factor given in Table 23-L of the code. The factor I takes three values: 1.0 for special and standard occupancy structures, 1.25 for essential and hazardous facilities, and 1.5 for machinery and equipment required for life-safety and tanks and vessels containing highly toxic and explosive substance which can pose hazard to public. For panel connectors I is 1.0 for the entire connector.
- C_p = The seismic coefficient as prescribed in Table No. 23-P. This table is reproduced here as Table 2.5 for ready reference. The footnotes of the original table have been omitted here.
- W_p = weight of the equipment or nonstructural component.
The coefficient C_p assumes only two values: 0.75 and 2.0. These values, listed in Table 2.5, are for rigid and rigidly supported equipment (with period < 0.6). For flexible or flexibly supported equipment, the code suggests the use of a rational procedure considering the dynamic properties of both the equipment and the structure which supports it, but the calculated value shall not be less than that listed in Table 2.5. In absence of an analysis or empirical data, the C_p for flexible case can be taken as twice the value listed in Table 2.5 but not to exceed 2.0. For ground supported systems, the coefficient C_p may be taken as 2/3 of the value listed in Table 2.5. This particular provision is similar to the provision in Tri-Services Manual I.

The design forces for exterior panel connection bodies and elements connecting the connection bodies with the structure or the panel are the same as in the Triservices Manual [7]. That is, the connection bodies shall be designed for a force equal to $1\frac{1}{3}$ times the force prescribed by the formula. The anchor elements joining the connection with the frame and the panel shall be designed for a force 4 times the value obtained from equation (2.10).

TABLE 2.1 Seismic Coefficient C_c and Performance Criteria**Factor P For Architectural Components**(Same as Table 8.2.2. of the NEHRP Provisions, Ref. 5)

		P	erform	ance
	a	Criteria Factor (P)		tor (P)
Architectural	Component			
Architectural	Coefficient	- Set	зпис п	azaro Casur
Component	Coefficient	Exp T	osure -	Group
Dutation - and a sing and lla		1 rd	11	
Exterior nonbearing walls	0.9	1.0	1.0	1.0
Interior nondearing walls	1 -	1.0	1 00	
Stair enclosures	1.5	1.0	1.0	1.5
Elevator shalt enclosures	1.5	0.5	0.5	1.5
Other vertical shaft enclosures	0.9	1.0	1.0	1.5
Cantilever elements				
Parapets, chimney, or stacks	3.0	1.5	1.5	1.5
Wall attachments (see Sec. 8.2.3)	3.0	1.5"	1.5"	1.5
Veneer connections	3.0	0.5	1.0 ^g	1.0
Penthouses	0.6	NR	1.0	1.0
Structural fireproofing	0.9	0.5^{f}	1.0 ^c	1.5
Ceilings				
Fire-rated membrane	0.9	1.0	1.0	1.5
Nonfire-rated membrane	0.6	0.5	1.0	1.0
Storage racks more than 8 fee in	e than 8 fee in 1.5 1.0 1.0		1.5	
height (contents included				
Access floors (supported equipment	2.0	0.5	1.0	1.5
included)				
Appendages				
Roofing units	0.6	NR	1.0 ^b	1.0
Containers and miscellaneous	1.5	NR	1.0	1.0
components (free standing)				
Partitions				
Horizontal exits include ceilings	0.9	1.0	1.5	1.5
Public corridors	0.9	0.5	1.0	1.5
Private corridors	0.6	NR	0.5	1.5
Full height area separation	0.9	1.0	1.0	1.5
partitions				-
Full height other partitions	0.6	0.5	0.5	1.5
Partial height partitions	0.6	NR	0.5	1.0

* For superscript, refer to the provisions, Reference 5.

TABLE 2.2 Seismic Coefficient C_c and Performance CriteriaFactor P For Mechanical and Electrical Components(Same as Table 8.3.2a of NEHRP Provisions, Reference 5)

		F	Perforr	nance
		Crite	eria Fa	actor (P)
	Component		_	
Mechanical and	Seismic	Se	ismic	Hazard
Electrical Component or System	Coefficient	Ex	posure	e Group
	$(C_c)^b$	Ι	П	III
Fire protection equipment and systems	2.0	1.5	1.5	1.5
Emergency or standby electrical systems	2.0	1.5	1.5	1.5
Elevator drive, suspension system, and	1.25	1.0	1.0	1.5
controller anchorage				
General equipment				
Boilers, furnaces, incinerators, water				
heater, and other equipment using				
combustible energy sources or high-				
temperature energy sources, chimneys				
flues, smokestacks, and vents	2.0	0.5	1.0	1.5
Communication systems				
Electrical bus ducts, conduit, and				
cable trays ^c				
Electrical motor control centers,	l			
motor control devices, switchgear,				
transformers, and unit substations				
Reciprocating or rotating equipment				
Tanks, heat exchangers, and pressure				
vessels				
Utility and service interfaces				
Manufacturing and process machinery	0.67	0.5	1.0	1.5
Pipe systems				
Gas and high hazard piping	2.0	1.5	1.5	1.5
Fire suppression piping	2.0	1.5	1.5	1.5
Other pipe systems ^d	0.67	NR	1.0	1.5
$HVAC$ and service ducts ^{ϵ}	0.67	NR	1.0	1.5
Electrical panel boards and dimmers	0.67	NR	1.0	1.5
Lighting fixtures ^f	0.67	0.5	1.0	1.5
Conveyor systems (nonpersonnel)	0.67	NR	NR	1.5

* For superscript, refer to the provisions, Reference 5

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TABLE 2.3 Horizontal Force Factor or Seismic Force CoefficientFor Architectural Elements or Elements of Structures(same as Table 3-4 of Reference 7)

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			<u> </u>
	Part or Portion of Structure	Horizontal Direction of Force	Value of C_p^{-1}
	Cantilever Elements:	Normal to	
[a. Parapets	flat surfaces	•••
1.	b. Portion of chimneys or stacks that	Any direction	0.8
	protrude above rigid supports ²		
	All other elements such as wall, parti-		
2.	tions and similar elements-see also	Any direction	0.3
	paragraph 3-3(J)3d. Also includes		
	masonry or concrete fences over 6 feet high.		
3.	Exterior and interior ornamentations and	Any direction	0.8
	appendages. See chapter 9, paragraph 9-3.		
4.	 When connected to, part of, or housed within a building: a. Penthouses b. Anchorage and supports for tanks plus contents c. Rigidly braced chimneys and stacks² d. Storage racks plus contents⁵ e. Suspended ceilings⁶ f. All equipment or machinery 	Any direction	0.3 ^{3,4}
5.	Connections for prefabricated structural elements other than walls, with force applied at center of gravity of assembly	Any direction	0.3 ⁴

*Based on the 1978 SEAOC Revisions.

*For superscripts, refer to Manual I, Reference 7.

TABLE 2.4 Amplification Factor A_p For Flexible or Flexibly Mounted Equipment

Building Period	Less than	0.75	1.0	2.0	Greater than
sec.	0.5				3.0
A_p	5.0	4.75	4.0	3.3	2.7

TABLE 2.5 Horizontal Force Factor C_p Of UBC(Same as Table 23-P of Reference 8)

ELEMENTS OF STRUCTURES AND NONSTRUCTURAL	
COMPONENTS AND EQUIPMENT	VALUE OF C_p
	_
I. Part or Portion of Structure	
1. Walls including the following:	
a. Unbraced (cantilevered) parapets	2.00
b. Other exterior walls above the ground floor	0.75
c. All interior bearing and nonbearing walls and partitions	0.75
d. Masonry or concrete fences over 6 feet high	0.75
2. Penthouse (except when framed by an extension of the structural frame)	0.75
3. Connections for prefabricated structural elements other than walls,	
with force applied at center of gravity	0.75
4. Diaphragms	
II. Nonstructural Components	
1. Exterior and interior ornamentations and appendages	2.00
2. Chimneys, stacks, trussed towers and tanks on legs:	
a. Supported on or projecting as an unbraced cantilever above the roof more	
than one half their total height	2.00
b. All others, including those supported below the roof with unbraced	
projection above the roof less than one half its height, or braced or guyed	
to the structural frame at or above their centers of mass	0.75
3. Signs and billboards	2.00
4. Storage racks (include contents)	0.75
5. Anchorage for permanent floor-supported cabinets and book stacks	
more than 5 feet in height (include contents)	0.75
6. Anchorage for suspended ceilings and light fixtures - see also Section 4701(e)	0.75
7. Access floor system	0.75
III. Equipment	
1. Tanks and vessels (include contents), including support systems and anchorage	0.75
2. Electrical, mechanical and plumbing equipment and associated conduit,	
ductwork and piping, and machinery	0.75



FIGURE 2.1: ATTACHMENT AMPLIFICATION FACTOR USED IN NEHRP PROVISIONS.



FIGURE 2.2: AMPLIFICATION FACTOR FOR FLEXIBLE AND FLEXIBLY MOUNTED EQUIPMENT USED IN TRI-SERVICES MANUAL.

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SECTION 3

RESPONSE SPECTRUM APPROACH FOR NONSTRUCTURAL COMPONENTS

Most code provisions used for calculating the story design shear force implicitly assume that the dynamics of a structure can be represented by its fundamental mode. However, in the calculation of forces on nonstructural components, the dynamics of the supporting structure is completely ignored; and it is only partially included with mechanical and electrical components through the use of the first mode frequency in the calculation of the acceleration amplification factor. It is shown later in this report that the errors introduced in the calculated forces by ignoring higher modes can be significant. Therefore, it is desirable to have a method whereby one can include the contributions of the higher modes through simple calculations.

In this section we present modal analysis-based response spectrum approaches which are parallel to the modal analysis procedure in Chapter 5 of the NEHRP Provisions. These approaches allow one to include the effect of the higher modes explicitly, if desired. Simplified approaches which use only the fundamental mode, but incorporate the higher mode effect approximately, are also proposed both for the architectural components and for the mechanical and electrical components.

The proposed response spectrum approach explicitly considers the modal properties (frequencies, mode shapes, participation factors) of the structures, and the period and damping ratio of the equipment. The motion at the base can be defined in terms of the modal seismic design coefficient of the NEHRP Provisions. It can also be defined in terms of ground response spectra such as those defined by the NEHRP Provisions [5] or Manual II [11]. The theoretical background in support of these approaches is given in technical publications elsewhere [Singh, 1975, Singh and Chu, 1976 and Singh, 1980]. The approach was developed based on the assumption that the structure behaves linearly. However, consistent with the provisions of Chapter 5 of the NEHRP Provisions [5] on the use of the modal analysis approach, the nonlinearity of the supporting structure is also included by modifying the elastic input response spectrum values appropriately through the response reduction factor R. For further discussion on the nonlinear effects of the supporting structure, see Section 6 of this report.

The formulas in this approach are presented in the same form as in the NEHRP Provisions. Also, the cases of the architectural components and mechanical and electrical components are treated separately whenever different formulas are necessary.

3.1 ARCHITECTURAL COMPONENT OR RIGIDLY CONNECTED RIGID MECHANICAL OR ELECTRICAL COMPONENT

The force on an architectural component (or a rigidly connected rigid mechanical component) placed in the mth floor is computed by the following formula:

$$F_p = A_v \ C_{cm} \ PIW_c \tag{3.1}$$

where A_v , P, and W_c are the same as those defined by 1991-NEHRP Provisions. In addition to these factors, however, here a factor I called as the importance factor has been introduced. Moreover, the coefficient C_{cm} is now defined such that it includes the dynamic characteristics of the structure explicitly.

The factor I represents the relative importance of the components with respect to each other. All components, whether important or not, will experience some seismic force which is determined by the intensity of the input motion and the dynamic characteristics of the supporting structure and the component. However, some components are more critical than others and this ought to be reflected in determination of the design seismic force. The NEHRP Provisions introduce this importance through the seismic coefficient. This coefficient, however, ignores the dynamic characteristics of the structure. In the formula proposed here, the NEHRP seismic coefficient of a component, normalized by the lowest value of the coefficient, is adopted as the importance factor I. The effect of the dynamics of the system is separately introduced through the seismic coefficient C_{cm} which now explicitly depends upon the dynamic properties of the structure and the component. These two factors P and I are given in Tables 3.1 and 3.2, respectively, for the architectural and for the mechanical and electrical components.

Two methods are proposed to define the seismic coefficient C_{cm} : (a) a more rigorous multi-mode approach, and; (b) a simple first mode approach. In both cases, the coefficient is defined by closed-form formulas. The formulas are expressed in terms of the dynamic properties of the structure (frequencies, mode shapes, participation factors, and damping ratio) and the input response spectrum characteristics.

3.1.1 Multimode Approach

In the multimode approach, where information about, say, the first r modes is available, the seismic coefficient C_{cm} is defined as:

$$C_{cm}^{2} = a_{s}^{2} + \sum_{j=1}^{r} \left\{ \left(1 + 4\beta^{2} \right) \gamma_{j}^{2} \phi_{mj}^{2} + 8\beta^{2} a_{s} \gamma_{j} \phi_{mj} \right\} R_{j}^{2} \ge 1.0$$
(3.2)

Note that the value of this coefficient for any floor level cannot be less than 1.0. Various quantities in the formula for C_{cm} are:

- $a_s = (1 \sum_{j=1}^r \gamma_j \phi_{mj}) =$ mode truncation correction term,
- r = the number of building modes desired to be included in the calculation of force

$$\gamma_j = j^{th}$$
 modal participation factor $= \sum_{k=1}^N W_k \phi_{kj} / \sum_{k=1}^N W_k \phi_{kj}^2$

 $\phi_{mj} = j^{th}$ modal displacement at the m^{th} floor where the nonstructural component is attached

 W_k = weight of the k^{th} floor

 β = structural damping ratio, assumed to be the same for all modes.

 R_j in equation (3.2) is the normalized base input response spectrum value for j^{th} modal frequency ω_j and structural damping ratio β for a 1.0 g maximum ground acceleration, modified for the nonlinearity of the structural system. Although any appropriate site dependent spectrum can be used to define R_j , here to be consistent with the NEHRP Provisions, it is proposed to use the <u>normalized</u> modal seismic design coefficient C_{sj} of the NEHRP Provisions (Eq. 5.3, pp. 60) to define R_j as follows:

$$R_{j} = \frac{C_{sj}}{A_{v}} = \frac{1.2S}{RT_{j}^{\frac{2}{3}}} \le \frac{2.5}{R} \frac{A_{a}}{A_{v}}$$
(3.3)

Exceptions in the NEHRP Provisions, described by equations (5.3a) and (5.3b) of the Provisions also apply.

The variables in equation (3.3) are:

- S = the coefficient for the soil profile characteristics of the site as determined by Table 3.2 of the Provisions.
- $T_j = \text{period of } j^{ih} \text{ mode} = 2\pi/\omega_j, \text{ in seconds.}$
- ω_j = the frequency of j^{th} mode in radians per second.
- R = the response modification factor determined from Table 3.3 of the Provisions. This factor depends upon the type of construction used for the building structure. This is introduced to include the effect of nonlinearity of the structure in the calculation of forces.
- A_a = the seismic coefficient representing the effective peak acceleration as determined in Sec. 1.4.1 of the provisions.

If a building damping ratio other than 5% is considered more appropriate, equation (3.3) can be modified as suggested in the NEHRP Commentary [6].

The formula in equation (3.2) ignores the effect of interaction between various modes. For uniform building structures with well separated structural frequencies, this effect is not important for higher floors. However, for tall and flexible buildings even with well separated frequencies, ignoring this interaction effect can lead to underestimation of the forces on the lower floors. It is for this reason that the minimum value of this coefficient has been limited to 1.0 in equation (3.2). In Appendix A we examine the importance of the interaction between modes as well as the accuracy of the expression proposed to obtain C_{cm} . The modal interaction effect can be important for structures with closely spaced frequencies, for example, in structural systems with torsional modes. The methods to include this effect are also available. See, for example, Singh and Chu (1976) and Singh and Maldonado (1991).

For calculating the modal properties of structural system, required in equation (3.2), see Section 5.

3.1.2 First Mode Approach

In this approach, the terms for $j \ge 2$ which are associated with the higher modes are ignored. However, some correction factors are introduced to include the effect of the dynamics of the neglected higher modes approximately. As it will be shown by numerical examples in Section 5, the effect of the higher modes is to increase the floor acceleration. This increase in the top floor acceleration calculated with only the first mode varied from 12% to 75%. Here we propose to apply a factor of 1.5 to account for this increase. The inclusion of the higher modes also affects the variation of the acceleration with height, as is shown by the numerical results presented in Figures 5.23 to 5.25 in Section 5. Here, to simplify the approach a linear variation of acceleration with floor height is proposed. With these assumptions, the single mode formula for C_{cm} becomes:

$$C_{cm} = C_o + \frac{h_m}{h_N} \left(C_{cN} - C_o \right)$$
(3.4)

where:

 $C_o = \frac{S}{R}$

 $h_m =$ the height of the m^{th} floor above the base

 h_N = the height of the roof or N^{th} mass above the base

It is noted that the coefficient C_o which represents the base motion coefficient at $h_M = 0$ is not equal to 1. It has been modified by the soil factor S and the response modification factor R to incorporate the effect of site soil conditions and the structural ductility in the calculated response.

The coefficient coefficient C_{cN} for the top floor, assuming, a damping ratio $\beta = 0.05$ is defined as:

$$C_{cN}^2 = \left[2.85(\gamma_1\phi_{N1})^2 - 2.7(\gamma_1\phi_{N1}) + 1.5\right]R_1^2$$
(3.5)

wherein

$$R_{1} = 1.2S/(T_{1}^{\frac{2}{3}}R)$$

$$\gamma_{1} = \text{first mode participation factor}$$

$$= \left(\sum_{k=1}^{N} W_{k}\phi_{k1}\right) / \left(\sum_{k=1}^{N} W_{k}\phi_{k1}^{2}\right)$$

 ϕ_{k1} = first modal displacement at the kth floor.

The first mode shape can be calculated by the formula provided in Section 5 for a uniform building, or it can be assumed to be of a simple linear shape.

It is shown in Section 5 that, for buildings with uniform floor mass and story stiffness in which the first mode is assumed to vary linearly with height, the product $\gamma_1 \phi_{N1}$ reduces to:

$$\gamma_1 \phi_{N1} = \frac{3N}{2N+1} \tag{3.6}$$

and the coefficient C_{cN} becomes:

$$C_{cN}^2 = \frac{15.45N^2 - 2.1N + 1.5}{(2N+1)^2} R_1^2$$
(3.7)

3.2 FLEXIBLE OR FLEXIBLY CONNECTED NONSTRUCTURAL COMPONENTS

For flexible nonstructural components which can be represented by a single degree of freedom system or rigid components connected by flexible supports to the main structure, it is necessary to consider the amplification of the floor acceleration due to the flexibility of the system. This amplification effect can be incorporated through a unit floor response spectrum coefficient, C_{fm} , as follows. In terms of the unit floor response spectrum coefficient C_{fm} , the force is calculated as:

$$F_p = A_v \ C_{fm} \ P \ IW_c \tag{3.8}$$

where A_v , P, I and W_c are the same as those in equation (3.1). The values of factors P and I for mechanical and electrical components are given in Table 3.2. The coefficient C_{fm} is the unit floor response spectrum value of a single degree of freedom system of period T_e and damping ratio β_e placed at m^{th} floor of the structure excited by a base motion of 1.0g maximum ground acceleration. This coefficient depends upon the dynamic properties of the structure, the period and damping ratio of the nonstructural component and, of course, the input ground response spectrum. The closed-form formulas are provided in Appendix B to calculate this coefficient. For a more accurate estimate of this coefficient, it is recommended to use this rigorous approach utilizing several dominant modes of the structure.

3.2.1 First Mode Approach

In Appendix B, the development of a simple single-mode approach is also presented. This is a practical approach requiring information only about the fundamental mode. The required computational work is minimal and can be performed with hand calculators. The approach provides a conservative estimate of the forces without a serious compromise in the rationality. The final formulas to calculate the coefficient according to this simple approach are presented next.

For a piece of equipment with natural frequency $f = w_e/2\pi$ placed on the mth floor, the unit floor response spectrum coefficient is defined as follows:

 $0 < f \leq 0.5 f_1$

$$C_{fm} = \frac{2f}{f_1} \left\{ R_G + \frac{m-1}{N-1} (R_{max} - R_G) \right\}$$
(3.9)

$$0.5f_1 < f \le f_\ell$$

$$C_{fm} = R_G + \frac{m-1}{N-1}(R_{max} - R_G)$$
(3.10)

 $f_\ell < f \leq f_{\rm m}$

$$C_{fm} = R_G + \frac{m-1}{N-1} \left\{ C_{cN} - R_G + \frac{(f_m - f)}{(f_m - f_\ell)} (R_{max} - C_{cN}) \right\}$$
(3.11)

$$\mathbf{f_m} < \mathbf{f} \le \mathbf{f_u}$$

$$C_{fm} = C_{c1} + \frac{(f_u - f)}{(f_u - f_m)} (R_G - C_{c1}) + \frac{m - 1}{N - 1} \left\{ C_{cN} - C_{c1} - \frac{(f_u - f)}{(f_u - f_m)} (R_G - C_{c1}) \right\}$$
(3.12)

 $f > f_u$

$$C_{fm} = C_{c1} + \left(\frac{m-1}{N-1}\right)(C_{cN} - C_{c1})$$
(3.13)

where

 $f_1 =$ fundamental frequency of the structure in cps $f_\ell =$ defined by equation (B.20), Appendix B $f_m =$ defined by equation (B.20), Appendix B $f_u =$ defined by equation (B.25), Appendix B $C_{c1} =$ defined by equation (B.24), Appendix B $C_{cN} =$ defined by equation (3.7). $R_{max} =$ defined by equation (B.18) or (B.19), Appendix B $R_G =$ defined by equation (B.23), Appendix B.

TABLE 3.1 Importance Factor, I, and Performance CriteriaFactor (P) For Architectural Components

		P	erform	ance	
		Criteria Factor (P)		tor (P)	
	Component				
Architectural	Importance	Sei	Seismic Hazard		
Component	Factor	Exp	osure	Group	
	(I)	I	п	ш	
Exterior nonbearing walls	1.5	1.5^{d}	1.50	1.5	
Interior nonbearing walls					
Stair enclosures	2.5	1.0	1.0 ^c	1.5	
Elevator shaft enclosures	2.5	0.5 ^e	0.5 ^c	1.5	
Other vertical shaft enclosures	1.5	1.0	1.0	1.5	
Other nonbearing walls	1.5	1.0	1.0	1.5	
Cantilever elements					
Parapets, chimney, or stacks	5.0	1.5	1.5	1.5	
Wall attachments (see Sec. 8.2.3)	5.0	1.5 ^d	1.5	1.5	
Veneer connections	5.0	0.5	1.0 ^g	1.0	
Penthouses	1.0	NR	1.0	1.0	
Structural fireproofing	1.5	0.5^{f}	1.0 ^c	1.5	
Ceilings		-			
Fire-rated membrane	1.5	1.0	1.0	1.5	
Nonfire-rated membrane	1.0	0.5	1.0	1.0	
Storage racks more than 8 fee in	2.5	1.0 1.0 1.5		1.5	
height (contents included	1				
Access floors (supported equipment	2.4	0.5	1.0	1.5	
included)					
Appendages					
Roofing units	1.0	NR	1.0 ^b	1.0	
Containers and miscellaneous	2.5	NR	1.0	1.0	
components (free standing)					
Partitions					
Horizontal exits include ceilings	1.5	1.0	1.5	1.5	
Public corridors	1.5	0.5	1.0	1.5	
Private corridors	1.0	NR	0.5	1.5	
Full height area separation	1.5	1.0	1.0	1.5	
partitions					
Full height other partitions	1.0	0.5	0.5	1.5	
Partial height partitions	1.0	NR	0.5	1.0	

For explanations of the factors with superscript, see NEHRP Provisions.

TABLE 3.2 I	mportance	Factor, I,	and Perfo	rmance Criteria
Factor P ^a	For Mecha	anical and	Electrical	Components

		F	Perform	nance
		Crite	eria Fa	actor (P)
	Component			
Mechanical and	Importance	Se	ismic	Hazard
Electrical Component or System	Factor	Ex	posure	Group
	(I)	Ι	II	III
Fire protection equipment and systems	3.0	1.5	1.5	1.5
Emergency or standby electrical systems	3.0	1.5	1.5	1.5
Elevator drive, suspension system, and	2.0	1.0	1.0	1.5
controller anchorage				
General equipment				
Boilers, furnaces, incinerators, water				
heater, and other equipment using				
combustible energy sources or high-				
temperature energy sources, chimneys				
flues, smokestacks, and vents	3.0	0.5	1.0	1.5
Communication systems				
Electrical bus ducts, conduit, and				
cable trays ^c				
Electrical motor control centers,				
motor control devices, switchgear,				I
transformers, and unit substations				
Reciprocating or rotating equipment				
Tanks, heat exchangers, and pressure				
vessels				
Utility and service interfaces				
Manufacturing and process machinery	1.0	0.5	1.0	1.5
Pipe systems	:			
Gas and high hazard piping	3.0	1.5	1.5	1.5
Fire suppression piping	3.0	1.5	1.5	1.5
Other pipe systems ^d	1.0	NR	1.0	1.5
HVAC and service ducts ^e	1.0	NR	1.0	1.5
Electrical panel boards and dimmers	1.0	NR	1.0	1.5
Lighting fixtures ^f	1.0	0.5	1.0	1.5
Conveyor systems (nonpersonnel)	1.0	NR	NR	1.5

For explanations of the factors with superscript, see NEHRP Provisions.

SECTION 4

١

COMPARISON OF CODE PROVISIONS AND RESPONSE SPECTRUM APPROACH

In this section, we compare the seismic design force provisions of various codes with each other and with the forces calculated by the response spectrum methods proposed in Section 3, both for architectural and for mechanical and electrical components.

Except for the proposed response spectrum approaches, the other three provisions ignore the site characteristics, the type of building system used and the dynamic characteristics of structure such as its frequencies and modes. To examine the effect that these parameters have on the forces calculated for architectural, mechanical and electrical components, several sets of numerical results for two building structures are presented in this section.

The first structure, hereafter referred to as Building 1, is a 10-story shear building with almost uniform mass and stiffness properties. The schematics of this building structure is shown in Figure 4.1 and the frequencies, periods and participation factors are provided in Table 4.1.

The second structure, which will be referred to as Building 2, is a 24-story shear building. The schematics of this structure is shown in Figure 4.2. This structure represents a slight modification of a 24-story concrete frame structure designed by Blume, Newmark and Corning (1961). The mass and stiffness properties of this structure are not uniform along its height, although according to the NEHRP Provisions and Uniform Building Code this structure can still be classified as a regular structure. The modal frequencies, periods and participation factors of Building 2 are provided in Table 4.2. For these two buildings, the seismic design forces calculated according to the 1991-NEHRP Provisions, the 1982-Tri-Services Manuals and the 1991-Uniform Building Code are compared with the seismic design forces calculated with the proposed response spectrum approach to show the differences caused by various factors.

4.1 ARCHITECTURAL COMPONENTS

Herein the design forces on two architectural components calculated by the three code and manual provisions are compared with those obtained via the response spectrum approach. The components chosen are: (1) cantilever parapets for which the code provisions are, perhaps, the most stringent, and; (2) suspended ceilings for which the code provisions are in the normal range. It is assumed that these components are to be designed for the forces in the highest seismic zone. The force equations prescribed in the three provisions for these two components are given in Table 4.3.

In the next set of eight figures we show the force/unit weight, which is the same as the acceleration expressed in g-units, at various floor levels of the two buildings calculated according to the three code provisions and the response spectrum approach. In the first four figures (Figures 4.3 - 4.6) the multi-mode response spectrum approach is used for the comparison whereas in the next four figures (Figure 4.7 - 4.10) the approximate single-mode approach is used. Also, three different site soil conditions as well as three different R-factor values representing three types of structural systems have been used in the calculation of the force/unit weight by the proposed response spectrum approach. A value of R = 8 represents a special moment frame of steel or reinforced concrete, R = 4 represents an intermediate moment frame of reinforced concrete and R = 2 represents an ordinary moment frame of reinforced concrete.

Figures 4.3 and 4.4, respectively, show the distribution of the force/unit weight for parapets and suspended ceilings for Building 1. Similar results for Building 2 are provided in Figures 4.5 and 4.6. The following observations can be drawn from the results in these figures:

- Different code provisions do not agree with each other as they provide different estimates of the forces.
- (2) According to the response spectrum calculations, the force is neither constant nor varies linearly with height. In the 24-story building, the departure from the linear variation is especially severe. Both for parapets and suspended ceilings, the force on top can be 2.5 to 3.5 times the force on the first floor.
- (3) The force varies with the type of structural system used in the building. For buildings designed with no additional strength margin, a component in a less ductile building (a building designed for a smaller ductility ratio) will feel a larger force than a component in a more ductile building. The code provisions do not reflect this effect, whereas it can be conveniently included in the proposed response spectrum approach.
- (4) Compared to the response spectrum approach, the NEHRP provisions are likely to provide a conservative estimate of the force, except for a component in a less ductile building (R = 2). For suspended ceilings, the NEHRP and other code provisions are seen to provide an overly conservative estimate of the forces, especially in more ductile buildings (R = 4 and 8).
- (5) Similar components placed in different types of buildings will have different margins of safety if designed according to the current code provisions.
- (6) The force varies with the type of site soil conditions, although the code provisions do not reflect this.

The next four figures (Figures 4.7-4.10) are similar to the preceding four figures, except that the response spectrum results are now obtained by the proposed simplified single mode approach. The observations made from the previous four figures are still applicable to these figures.

4.2 FLEXIBLY CONNECTED MECHANICAL AND ELECTRICAL COMPONENTS

In this section, we compare the force provisions of various codes with the force computed by the proposed response spectrum approach for flexibly connected components. Since the forces on flexible components depend upon the component frequency and the floor on which it is placed, the floor response spectra of the component force/unit weight (or component acceleration in g-units) are chosen to compare the results. The component's damping ratio is assumed to be 2% for all the floor spectra calculations.

For comparison, the numerical results are again obtained for the same two structures: the 10-story Building 1 and 24-story Building 2. To examine the effect of soil conditions at a site, again three different soil conditions have been considered. To examine the effect of structural ductility on the equipment response, again the buildings designed with three different R-factors of 2, 4 and 8 have been considered.

In Table 4.4, we summarize the force equations used in different approaches for a component falling in the category of general equipment. These equations have been used to define the floor spectra for different approaches which are compared in the following set of figures in this section.

The dominant peaks of the spectra obtained with the proposed approach have also been widened by 15% to account for the uncertainties in the calculation of the modal frequencies.

First in Fig. 4.11 we compare the force spectra calculated according to the three code provisions. We note that since the force in the UBC does not depend upon the

frequency of the component, it is a represented by a horizontal line. The NEHRP and Tri-Services provisions, on the other hand, include the effect of a possible resonance of the equipment with the fundamental period of the building through the amplification factors a_c and A_p , respectively.

In Fig. 4.12 we compare the floor spectra calculated according to various provisions for a component placed on the fifth floor of Building 1, designed for three different R-factors of 2, 4 and 8. The seismic input for a hard site condition S1 is used. Similar results for a component placed on the tenth floor are shown in Figure 4.13. From a comparison of results in these figures, the following observations can be made:

- (1) The force is dependent upon the type of structural system used in a building. A component placed in a less ductile building designed with a smaller R-factor will experience a larger force than a component placed in a more ductile building designed with a larger R-factor.
- (2) When compared with the results of the spectrum approach for a less ductile structure, the UBC and the NEHRP Provisions underestimate the forces.
- (3) As it is evident from Figures 4.12 and 4.13, a floor spectrum can have peaks at the frequencies of the higher modes in addition to the peak at the frequency of the fundamental mode. Thus there can be resonance of the equipment with the higher modes of the supporting structure as well. The current code provisions cannot take into account this effect as they are based on the assumption that resonance can occur only with the first mode.

Figure 4.14 shows the effect of different site soil conditions on the equipment force. It is noted that for an equipment in resonance with the fundamental frequency of the structure, the effect of the site soil condition can be important. However, for other values of the equipment frequency the floor spectrum is practically insensitive to the type of soil.

The following five figures present similar results for the 24-story Building 3. The force spectra obtained by the different methods, for a component placed on floor 6, 12 and 24 are plotted in Figures 4.15, 4.16, and 4.17, respectively. Again to include the effect of building ductility, three buildings designed with R-factor values of 2, 4 and 8 have been considered.

The observations made earlier for Building 1 are also applicable for Building 2. We also note that for the lower floors, the resonance effect at the 2nd and 3rd modal frequencies of the structure can be stronger than that at the frequency of the first mode as the peaks in the floor spectra at these frequencies are higher than the peak at the first mode frequency. Thus ignoring the higher modes underestimates the force acting on equipment tuned to them.

Figures 4.18 shows the effect of the site soil condition on the floor spectra for Building 2. Again this effect can be important, especially if a component is tuned to one of the dominant frequencies of the structure. The code provisions do not have a mechanism for including this site effect as the ground motion intensity parameter used in the code equations does not change with the soil condition.

As mentioned earlier, the Tri-services Manual for Essential Buildings (Manual II) [11] specifically defines the force on mechanical component in terms of a floor spectrum. In Figure 4.19, therefore, we compare the floor spectra obtained by the proposed response spectrum approach and by the provisions of both Tri-services manuals. The differences in the various spectra are noted. It is also noted that the provision in the Tri-services Manual I are higher than the provisions in Manual II. In fact, as noted from the results of Figure 4.20, where we have only plotted the spectra obtained by Manual I and three floor spectra obtained by Manual II, we observe that the method of Manual I seems to provide an upper bound spectra of all the floors. The design of components based on Manual I will, therefore, be relatively more conservative than the design based on Manual II.

In the next seven figures we compare the floor spectra obtained via the simplified single-mode approach with the spectra obtained according to various code provisions. Figures 4.21 through 4.27 provide parallel information to Figures 4.12 through 4.18, except that the spectra in Figures 4.21-27 are obtained via the single mode approach. Figures 4.21 and 4.22 demonstrate the effect of choosing different R-factor on the spectra for Floors 5 and 10 of Building 1. It is noted that except for the building designed for R=8, the code provisions underestimate the force compared to the proposed approach. This difference increases for the higher floors because the code provisions do not change with height whereas the floor spectra obtained by the proposed approach increase for higher floors. Again as seen from Figure 4.23, the effect of soil conditions is important in the proposed approach. The remaining Figures 4.24 - 4.27 are for Building 2 and they show similar results with similar conclusions. From the comparison of Figures 4.21 through 4.27 with the corresponding Figures 4.12 through 4.18, we also note that the single mode approach provides enveloping response values.

Mode No.	Frequency Period		Part. Factor
	[cycles/sec]	[sec]	
1	1.0185	0.9818	-2.9122
2	3.0233	0.3308	-0.9914
3	4.9356	0.2026	-0.6104
4	6.7091	0.1491	-0.4342
5	8.3223	0.1202	0.3128
6	9.7701	0.1024	-0.2157
7	11.0351	0.0906	0.1413
8	12.0793	0.0828	0.0884
9	12.8596	0.0778	-0.0512
10	13.3417	0.0750	-0.0235

TABLE 4.1 Modal Parameters Of Building No. 1

Mode No.	Frequency	Period	Part. Factor
	[cycles/sec]	[sec]	
1	0.5460	1.8316	-383.2827
2	1.3211	0.7569	177.7671
3	2.1311	0.4692	119.5009
4	2.9073	0.3440	-88.3527
5	3.6831	0.2715	61.8244
6	4.4993	0.2223	-49.6557
7	5.2450	0.1907	-48.6557
8	6.0059	0.1665	-37.0350
9	6.5796	0.1520	-34.1981
10	7.2833	0.1373	25.2754
11	7.9588	0.1256	-24.6916
12	8.6424	0.1157	-23.9750
13	9.1663	0.1091	-22.1104
14	9.8032	0.1020	-23.5130
15	10.3392	0.0967	19.4600
16	10.9638	0.0912	18.1303
17	11.8185	0.0846	16.0150
18	12.4525	0.0803	-13.6681
19	13.2201	0.0756	15.9951
20	14.0010	0.0714	-12.9106
21	14.8379	0.0674	8.5423
22	15.6784	0.0638	-17.2519
23	17.0991	0.0585	10.9432
24	18.5490	0.0539	7.8433

TABLE 4.2 Modal Parameters Of Building No.2

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CODES	PARAPETS	SUSPENDED CEILINGS
1. NEHRP $F_p = A_v C_c P W_c$	$(.4)(3)(1.5)W_c$ = 1.80 W_c	$(.4)(.9)(1.5)W_c$ = $.54W_c$
2. Tri-services Ultimate stress $F_{pu} = 1.2(F_p) = 1.2ZIC_pW_p$	$(1.2)(1.0)(1.5)(.8)W_p = 1.44 W_p$	$(1.2)(1.)(1.5)(.3)W_p$ = 0.54 W_p
3. UBC-91 Ultimate stress $F_{pu} = 1.2F_p = 1.2ZIC_pW_p$	$(1.2)(.4)(1.25)(2.)W_p$ = 1.2W _p	$(1.2)(.4)(1.25)(.75)W_p = 0.45 W_p$
4. Response Spectrum $F_p = A_v PIC_{vm} W_c$	$.4(1.5)(5)C_{v_m}W_c = 3 C_{v_m}W_c$	$(.4)(1.5)(1.5)C_{v_m}W_c = .9C_{v_m}W_c$

TABLE 4.3Seismic Design Forces On Parapets And Suspended Ceilings
(Fire Rated) According to Various Provisions

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CODE	GENERAL EQUIPMENT
1. NEHRP $F_p = A_v C_c P a_c W_c$	$(.4)(2.0)(1.5)a_c$ = 1.2 a_c
2. Tri-Services Ultimate stress $F_{p_u} = 1.2(ZIA_pC_pW_p)$	$(1.2)(1.0)(1.5)(.3)A_p$ = .54 A_p
3. UBC-91 Ultimate stress	
$F_{p_u} = 1.2F_p = 1.2ZIC_pW_p$	(1.2)(.4)(1.25)(1.5) = 0.90
4. Response Spectrum Approach $F_p = A_v C_f PIW_c$	$(.4)(3.)(1.5)C_f$ = 1.8 C _f

.

TABLE 4.4Sesimic Design Forces on General Equipment
According to Various Provisions



FIGURE 4.1: TEN STORY SHEAR BUILDING USED IN THE STUDY.

STIFFNESS [kips/in]

MASS [kips-sec2/in]

	(24)	
4400.0	(23)	7. 9 7
5300.0	(22)	7.67
7200.0	(21)	7.68
7300.0	(20)	7.64
8700.0	(19)	7.68
10100.0	(18)	8.19
10100.0	(17)	8.19
13300.0	(16)	8.31
14100.0	(15)	8.42
14100.0	(14)	8.42
16300.0	(13)	8.52
19500.0	(12)	8.88
19500.0	(12)	8.88
21000.0	(11)	8.95
21000.0		9.03
21000.0		9.03
22000.0	(8)	9.19
23200.0	(7)	9.87
31800.0	(6)	9.87
31900.0	(5)	9.94
33400.0	(4)	10.59
39800.0	(3)	10.59
39500.0	(2)	11.29
42400.0	(1)	11.98
42400.0		17////
		41114

FIGURE 4.2: TWENTY FOUR STORY SHEAR BUILDING USED IN THE STUDY.



Building No. 1 - Cantilever Parapets

FIGURE 4.3: COMPARISON OF FORCES ON CANTILEVER PARAPETS CALCULATED BY VARIOUS CODE PROVISIONS AND MULTI-MODE RESPONSE SPECTRUM APPROACH.

0.80 0.70 R=2 S3 S2 0.60 NEHRP & Tri-Serv 0.50 Force/Unit Weight **S**1 UBC 0.40 R=4 \$3 0.30 **S**1 0.20 R=8 S3 **S**2 **S**1 0.10 0.00 2 3 5 1 7 8 9 10 4 6 **Floor Level**

Building No. 1 - Suspended Ceilings

FIGURE 4.4: COMPARISON OF FORCES ON SUSPENDED CEILINGS CALCULATED BY VARIOUS CODE PROVISIONS AND MULTI-MODE RESPONSE SPECTRUM APPROACH.





FIGURE 4.5: COMPARISON OF FORCES ON CANTILEVER PARAPETS CALCULATED BY VARIOUS CODE PROVISIONS AND MULTI-MODE RESPONSE SPECTRUM APPROACH.


Building No. 2 - Suspended Ceilings

FIGURE 4.6: COMPARISON OF FORCES ON SUSPENDED CEILINGS CALCULATED BY VARIOUS CODE PROVISIONS AND MULTI-MODE RESPONSE SPECTRUM APPROACH.



Building No. 1 - Cantilever Parapets

FIGURE 4.7: COMPARISON OF FORCES ON CANTILEVER PARAPETS CALCULATED BY VARIOUS CODE PROVISIONS AND SINGLE-MODE RESPONSE SPECTRUM APPROACH.



Building No. 1 - Suspended Ceilings

FIGURE 4.8: COMPARISON OF FORCES ON SUSPENDED CEILINGS CALCULATED BY VARIOUS CODE PROVISIONS AND SINGLE-MODE RESPONSE SPECTRUM APPROACH.



Building No. 2 - Cantilever Parapets

FIGURE 4.9: COMPARISON OF FORCES ON CANTILEVER PARAPETS CALCULATED BY VARIOUS CODE PROVISIONS AND SINGLE-MODE RESPONSE SPECTRUM APPROACH.



Building No. 2 - Suspended Ceilings

FIGURE 4.10: COMPARISON OF FORCES ON SUSPENDED CEILINGS CALCULATED BY VARIOUS CODE PROVISIONS AND SINGLE-MODE RESPONSE SPECTRUM APPROACH.

Code Provisions - Building No. 1



FIGURE 4.11: COMPARISON OF FORCES SPECTRA FOR GENERAL EQUIPMENT ACCORDING TO NEHRP PROVISIONS, TRI-SERVICES MANUAL AND UBC.

Building No. 1 - Floor 5



FIGURE 4.12: COMPARISON OF FLOOR 5 SPECTRA CALCULATED BY VARIOUS PROVISIONS AND THE PROPOSED RESPONSE SPECTRUM APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 1 - Floor 10



FIGURE 4.13: COMPARISON OF FLOOR 10 SPECTRA CALCULATED BY VARIOUS PROVISIONS AND THE PROPOSED RESPONSE SPECTRUM APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 1 - Floor 5 - R=4



FIGURE 4.14: EFFECT OF SOIL TYPE ON THE FORCE ON GENERAL EQUIPMENT PLACED ON FLOOR 5.

...

Building No. 2 - Floor 6



FIGURE 4.15: COMPARISON OF FLOOR 6 SPECTRA CALCULATED BY VARIOUS CODE PROVISIONS AND THE PROPOSED RESPONSE SPECTRUM APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 2 - Floor 12



FIGURE 4.16: COMPARISON OF FLOOR 12 SPECTRA CALCULATED BY VARIOUS CODE PROVISIONS AND THE PROPOSED RESPONSE SPECTRUM APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 2 - Floor 24



FIGURE 4.17: COMPARISON OF FLOOR 24 SPECTRA CALCULATED BY VARIOUS CODE PROVISIONS AND THE PROPOSED RESPONSE SPECTRUM APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 2 - Floor 6 - R=4



FIGURE 4.18: EFFECT OF SOIL TYPE ON THE FORCE ON GENERAL EQUIPMENT PLACED ON FLOOR 6.

Building No. 2 - Floor 6



FIGURE 4.19: COMPARISON OF FORCE SPECTRA CALCULATED BY THE TWO TRI-SERVICES MANUALS AND THE PROPOSED RESPONSE SPECTRUM APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 2



FIGURE 4.20: COMPARISON OF FORCE SPECTRA OBTAINED BY THE TWO TRI-SERVICES MANUALS.

Building No. 1 - Floor 5



FIGURE 4.21: COMPARISON OF FLOOR 5 SPECTRA CALCULATED BY VARIOUS PROVISIONS AND THE PROPOSED SIMPLIFIED APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 1 - Floor 10



FIGURE 4.22: COMPARISON OF FLOOR 10 SPECTRA CALCULATED BY VARIOUS PROVISIONS AND THE PROPOSED SIMPLIFIED APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 1 - Floor 5 - R=4



FIGURE 4.23: EFFECT OF SOIL TYPE ON THE FORCE ON GENERAL EQUIPMENT PLACED ON FLOOR 5.

Building No. 2 - Floor 6



FIGURE 4.24: COMPARISON OF FLOOR 6 SPECTRA CALCULATED BY VARIOUS CODE PROVISIONS AND THE PROPOSED SIMPLIFIED APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 2 - Floor 12



FIGURE 4.25: COMPARISON OF FLOOR 12 SPECTRA CALCULATED BY VARIOUS CODE PROVISIONS AND THE PROPOSED SIMPLIFIED APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 2 - Floor 24



FIGURE 4.26: COMPARISON OF FLOOR 24 SPECTRA CALCULATED BY VARIOUS CODE PROVISIONS AND THE PROPOSED SIMPLIFIED APPROACH. GENERAL EQUIPMENT - SOIL S1

Building No. 2 - Floor 6 - R=4



FIGURE 4.27: EFFECT OF SOIL TYPE ON THE FORCE ON GENERAL EQUIPMENT PLACED ON FLOOR 6.

SECTION 5

MODAL PROPERTIES

In Section 3, response spectrum methods were presented for calculating the design forces for nonstructural components. To use these methods, one needs to know the modal properties (modal frequencies, mode shapes, and participation factors). In the simplified single-mode approach, the characteristics of only the fundamental mode are required. To calculate the frequency of the fundamental mode, the formulas provided in the NEHRP Provisions [5], UBC [8] and Tri-services Manual [6] can be used, whereas for the mode shape one can assume a linear variation with height without introducing much error. However, if one wants to use the more rigorous multi-mode approach to calculate a more accurate value of the force then the characteristics of the higher modes are also required. In this section, therefore, we present some simple formulas to obtain the modal properties of the higher modes including the fundamental mode. The formulas are applicable to regular buildings which have no severe mass or stiffness irregularities in their elevations.

The modal properties can be calculated using any standard computer program. Such programs are quite commonly available now. To obtain these modal properties, these programs solve the following eigenvalue problems, stated in the matrix form for a structure as

$$[K]\{\phi_j\} = \omega_j^2[M]\{\phi_j\}$$
(5.1)

where [K] and [M], respectively, are the stiffness and mass matrices of the structure, $\{\phi_j\}$ is the modal displacement vector for the j^{th} mode where j can be 1,2,...,N, and ω_j is the j^{th} mode frequency. For structures with more than 4 floors, the solution of equation (5.1) by hand becomes very involved. It is then best solved by a computer program. The participation factors γ_j for a mode can also be easily calculated by carrying out the following matrix multiplications

$$\gamma_j = \frac{\{\phi_j\}^T [M]\{r\}}{\{\phi_j\}^T [M]\{\phi_j\}}$$
(5.2)

where $\{r\}$ is the vector containing the displacements of each mass in its degrees of freedom when the base of the structure is moved by a unit distance in the direction of base excitation. This vector will usually have the values of 1 and 0 as entries.

It becomes necessary to use the above eigenvalue analysis procedure if a building configuration is strongly irregular in its elevation. However, for buildings with some minor irregularity in elevation, it is possible to avoid the matrix manipulation and calculate frequency values, mode shapes and participation factors by closed-form formulas as described below. These calculations can be easily performed by using a simple hand calculator.

In the following sections, we provide the closed form formulas for the cases of: (a) a uniform shear building in plan and elevation with identical stories and one degree of freedom per floor, and; (b) a general building uniform in elevation with three degrees-of-freedom per floor.

5.1 MODAL PROPERTIES OF A UNIFORM SHEAR BUILDING

For a perfectly uniform shear building in its plan and elevation with equal floor masses and story stiffnesses, the frequencies, mode shapes and participation factors can be defined as:

$$\omega_j = 2\sqrt{\frac{K}{M}} \sin\left\{\frac{(2j-1)}{(2N+1)}\frac{\pi}{2}\right\}$$
(5.3)

$$\phi_{mj} = \sin\left\{\frac{(2j-1)}{(2N+1)}m\pi\right\}$$
(5.4)

$$\gamma_{j} = \frac{\sum_{m=1}^{N} \sin\left\{\frac{(2j-1)}{(2N+1)}m\pi\right\}}{\sum_{m=1}^{N} \sin^{2}\left\{\frac{(2j-1)}{(2N+1)}m\pi\right\}}$$
(5.5)

where the index j is the mode number, m = the floor number, N = the total number of floors, K = average stiffness of stories $= (K_1 + K_2 + ... + K_N)/N, M =$ average floor mass $= (M_1 + M_2 + ... + M_N)/N = \frac{(W_1 + W_2 + ... + W_n)}{gN}, M_k =$ mass of floor k, K_k = stiffness of story k, and W_k = weight of floor k.

These closed-form formulas are based on a simple analysis, described by W. T. Thompson (1993).

Linearly Varying Fundamental Mode

It is often assumed that the variation of the first mode shape with height is a straight line. To demonstrate this, we expand in power series the sine function in the expression for ϕ_{mj} for j = 1 and keep only the first term to provide:

$$\phi_{m1} \simeq \frac{\pi}{2N+1}m \; ; \; m = 1, 2, ..., N$$
 (5.6)

which corroborates the usual assumption that for regular buildings the first modal shape can be approximated by a straight line.

Using the same approximation, the first mode participation factor becomes:

$$\gamma_1 = \frac{2N+1}{\pi} \frac{\sum_{m=1}^N m}{\sum_{m=1}^N m^2} = \frac{2N+1}{\pi} \frac{\frac{N(N+1)}{2}}{\frac{N(N+1)(2N+1)}{6}} = \frac{3}{\pi}$$
(5.7)

and hence the product $\gamma_1 \phi_{N1}$ required to calculate the floor acceleration in the formulation introduced in the previous chapter reduces to:

$$\gamma_1 \phi_{N1} = \frac{3N}{2N+1} \tag{5.8}$$

To demonstrate the accuracy of the formulas in equations (5.3)-(5.5), the modal properties calculated by these formulas are compared with those calculated by the matrix eigenvalue analysis. For a perfectly regular building these formulas are exact. However, for slightly irregular buildings, these formulas can provide reasonably accurate values for the modal parameters. To show this, we have chosen three buildings with some irregularities to assess the accuracy of the formulas in these cases. Buildings 1 and 2 are the same as those used in Section 4. Building 1 has a slight irregularity in the sense that the first story stiffness and first floor mass values are slightly higher than the remaining stories. Building 2 has more irregularity in its mass and stiffness properties along its height. Building 3 is a four story building with a soft first story. This building is shown in Figure 5.1, and its modal parameters are given in Table 5.1.

In Figure 5.2(a) and (b) we compare the numerical values of the periods of the modes and modal participation factors of Building 1 calculated according to equations (5.3)-(5.5), herein called "approximate" values, with the exact values calculated by the matrix eigenvalue analysis. It is noted that both the periods and participation factor values are almost the same for all the modes. In Figure 5.3, we compare the first and seventh mode shapes obtained by the exact and approximate methods. It is noticed that the first mode is virtually identical, but there are some differences in the seventh mode. Although the results are not presented here, the same trend was observed for the other lower and higher modes.

The results of a similar comparison between the exact and approximate periods, participation factors, and mode shapes for Building 3 are presented in Figures 5.4 and 5.5. Qualitatively, the results of Building 3 are similar to those of Building 1, in the sense that the frequency and participation factor values calculated by equations (5.3)-(5.5) match fairly well with the values calculated by the matrix eigenvalue analysis. The modal vectors are also similar but differ relatively more in this case than in the case of Building 1.

Next, we compare the results obtained for Building 2. In this building the k/m ratio of various stories is increasing almost linearly from top to bottom, as is shown in

Figure 5.6. It is increasing at the rate of about 7% of the average k/m ratio per story. From Figures 5.7(a) and (b), we observe that the natural periods and participation factors calculated by equations (5.3)-(5.5) are reasonably accurate. However, as it is shown in Figures 5.8(a) and (b) the mode shapes obtained by the two approaches are quite different, especially so for the higher modes. It will be shown later that this difference in the mode shapes can lead to quite different results in the calculation of forces on architectural and mechanical components.

It is also observed that although Buildings 2 and 3 will qualify to be as regular buildings in the elevation according to the criteria prescribed by the NEHRP Provisions in Table 3.4.2, they are not regular enough to permit the use of equations (5.3)-(5.5) for the calculation of the modal properties. For a tall structure, with linearly varying k/m ratio from bottom to top, equations (5.3)-(5.5) can be used if the change in k/m ratio per story of the building is less than about 2% of the average k/mratio. In other situations one should use the matrix eigenvalue analysis to calculate frequencies and mode shapes.

In the next set of figures we show the effect of using the approximate modal quantities on the calculated response. Figures 5.9, 5.10 and 5.11 show respectively, the maximum floor accelerations obtained by using the approximate and exact modal quantities for Buildings 1, 2, and 3. Since the approximate and exact modal properties of Buildings 1 and 3 were similar, their acceleration values calculated with the two set of properties are also similar. This is, however, not the case for Building 3 as shown in Figure 5.11. Thus, the larger the departure of a building from the uniformity, the greater the difference.

In Figures 5.12, 5.13 and 5.14, we compare the floor response spectrum coefficients C_{fm} calculated for $A_v = 0.4$ at different floors of the three buildings using the approximate and exact modal properties. Again, we notice that although the

results for Building 1 are acceptable, the differences in the response for Building 3, and especially Building 2, are significant. It can also be observed that the differences are more pronounced at the lower floors. In general, it is felt that if a building is irregular in elevation, one should use the matrix eigenvalue analysis to obtain modal properties and response more accurately.

5.2 MODAL PROPERTIES OF A BUILDING IRREGULAR IN PLAN BUT UNIFORM IN ELEVATION

In the previous section we provided formulas for a perfectly uniform building in its plan and elevation. Each floor mass had only one degree of freedom and the story stiffness was represented by a single spring coefficient. These formulas can be generalized to a case where each floor mass can have all possible six degrees of freedom which are coupled to each other such that the story stiffness is represented by a stiffness matrix instead of a single coefficient. This permits a completely arbitrary and irregular layout in the plan of the building.

If the same plan layout is repeated at each story and the floor masses and story stiffness are the same then one can develop closed-form expressions for the modal properties of the complete structure, defined in terms of the modal properties of a single story. The analytical development of these expressions is given elsewhere. Here, only the final expressions are provided for a torsional system such as the one shown in Figure 5.20, where each floor has three degrees of freedom: two translational and one rotational.

Let Ω_1 , Ω_2 and Ω_3 be the frequencies of a story with corresponding eigenvectors as $\{b^{(1)}\}$, $\{b^{(2)}\}$, and $\{b^{(3)}\}$, respectively. These story eigenproperties are obtained from the solution of the following 3×3 matrix eigenvalue problem:

$$\Omega_s^2[\bar{M}]\{b^{(s)}\} = [\bar{K}]\{b^{(s)}\}; \quad s = 1, 2, 3$$
(5.9)

For a diagonal mass matrix $[\overline{M}]$, these story eigenproperties can be defined by closedform expressions in terms of the floor mass, floor radius of gyration, eccentricity between the mass and stiffness center and story stiffness parameters. These expressions are given in Appendix C.

In terms of these story eigenproperties, the frequencies and mode shapes of the entire structure are defined as:

$$\begin{cases} \omega_1 \\ \omega_2 \\ \omega_3 \end{cases}_k = 2 \begin{cases} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{cases} \sin\left(\frac{\Lambda_k}{2}\right)$$

$$\left\{ \begin{cases} a_{1k}^{(s)} \\ \vdots \end{cases} \right\}$$

$$\left\{ \begin{cases} a_{1k}^{(s)} \\ \vdots \end{cases} \right\}$$

$$(5.10)$$

$$\left\{ \phi_{k}^{(s)} \right\} = \frac{1}{C_{k}} \left\{ \begin{array}{c} \left\{ a_{2k}^{(s)} \right\} \\ \vdots \\ \left\{ a_{Nk}^{(s)} \right\} \end{array} \right\}$$
(5.11)

where s = 1, 2, 3; k = 1, ..., N, and

$$\{a_{jk}^{(s)}\} = \{b^{(s)}\}\sin(j\Lambda_k)$$
(5.12)

$$\Lambda_k = \frac{\pi(2k-1)}{(2N+1)}$$
(5.13)

$$C_k = \text{normalization constant} = \sum_{j=1}^{N} \sin^2(j\Lambda_k)$$
 (5.14)

The frequencies calculated by equation (5.7) need not be in increasing order even for Ω_1 , Ω_2 , and Ω_3 arranged in their increasing orders of magnitude. So if one wants to have frequencies in increasing order, they will have to be rearranged.

The participation factor corresponding to each modal vector can be defined by

$$\gamma_k^{(s)} = \gamma^{(s)} \sum_{j=1}^N \sin(j\lambda_k) \tag{5.15}$$

where $\gamma^{(s)}$ is the participation factor for a story defined as

$$\gamma^{(s)} = \{b^{(s)}\}[m]\{r\}$$
(5.16)

in which $\{r\}$ is the vector of floor influence coefficient. This vector will have an entry of 1 in the degree of freedom along the excitation and zero otherwise.

To determine if these expressions can provide sufficiently accurate results for buildings with small irregularities, the six story 18-degree of freedom structure shown in Figure 5.15 was used for comparison purposes. The stiffness properties of the stories have been changed slightly. These stiffness properties are also shown in Figure 5.15.

In Figures 5.16 we compare the modal periods obtained using the above closedform expressions and those obtained by a matrix eigenvalue analysis. Similarly, in Figures 5.17(a) and (b) we compare the first and third modal shape vectors obtained by the two approaches. From these figures, it can be seen that the closed-form expressions can be conveniently used to calculate the modal properties of torsional systems if the mass and stiffness properties do not change significantly from one story to another.

5.3 EFFECT OF NEGLECTING HIGHER MODES ON THE FORCE CALCULATED FOR NONSTRUCTURAL COMPONENTS

It was pointed out before that the design forces prescribed by almost all code provisions are based on the assumption that a regular structure responds primarily in its first mode. Usually the effect of the higher modes is approximately included in the distribution of the base shear along the building stories. For calculating the forces in architectural and mechanical component, however, no higher modes effect is considered. In fact, the design forces defined for the nonstructural components do not consider any modal characteristic of the structure on which the component is supported, except for the mechanical components where the effect of the first modal period is considered only in calculating the amplification factor.

Figures 5.18 and 5.19 and 5.20 show the effect of including an increasing number of modes in the calculation of maximum acceleration in g-units (or the force per unit weight) at all the floors of the three buildings. Also shown in these figures is the floor acceleration obtained by assuming the first mode to be varying linearly with height. In addition, the accelerations calculated according to the simplified single mode approach presented in Section 3 are plotted. For the 10-story Building 1 and 4-story Building 3, the top floor response calculated with all modes is about 12 to 16% higher than the response calculated only with the first mode. For the 24-story Building 3, on the other hand, the top floor response with all 24 modes is 75% higher than the response calculated with only one mode. From these figures, we note that the use of only the first mode neither provides accurate response nor represents its variation with height properly. The simplified single mode approach proposed in Section 3 is seen to provide an enveloping response for the lower floors. On the top floor, however, there can be some underestimation of response calculation by this approach in some cases, as is shown by Figure 5.20.

In Figures 5.21 and 5.22 we show the effect of including an increasing number of modes on the floor response spectrum coefficient $(C_{f_m}A_v)$ for Buildings 1 and 2. It is clear that the floor response spectrum coefficients for the lower floors are dramatically affected by the higher modes. For the higher floors, the dominant first mode peak is not affected by the higher modes. However, if higher modes are not included, the peaks in the floor spectrum coefficient at the higher modal frequencies are likely to be missed.

5.4 EFFECT OF FLOOR HEIGHT ON FLOOR ACCELERATION CO-EFFICIENT AND FLOOR SPECTRUM COEFFICIENT

It was mentioned in Section 2 that none of the current code provisions recognize the possibility of getting floor accelerations which vary with height in a building. That is, a nonstructural component placed on the first floor of a building is designed for the same force level as a component placed on the top floor. To show how much variation can occur in the floor acceleration coefficients and floor spectrum coefficients from floor to floor, we present Figures 5.23 and 5.24. Figure 5.23 shows the variation of the floor acceleration as a function of the floor level for the three buildings. It is noted that the acceleration of the top floor can be as high as 2.5 times the acceleration of the first floor. This ratio seems to increase with the building height. Although there is no common trend for this variation, it is evidently nonlinear and varies from building to building.

In Figures 5.25 and 5.26 the floor spectrum coefficient at three floors of Building 1 and 2 are compared. Again the variation of floor spectra with height is clearly evident, confirming that mechanical components placed at different floors of a building can experience quite different levels of force.

Mode No.	Frequency	Period	Part. Factor
	[cycles/sec]	[sec]	
1	1.0342	0.9669	-0.8896
2	2.8413	0.3520	0.3163
3	4.2740	0.2340	-0.1786
4	5.2420	0.1908	0.1327

TABLE 5.1 Modal Parameters Of Building No.3



FIGURE 5.1: FOUR STORY SHEAR BUILDING USED IN THE STUDY.

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FIGURE 5.2: COMPARISON OF EXACT AND APPROXIMATE MODAL PERIODS AND PARTICIPATION FACTORS. BUILDING NO. 1



FIGURE 5.3: COMPARISON OF EXACT AND APPROXIMATE MODAL SHAPE FOR MODE 1 AND MODE 7. BUILDING NO. 1


FIGURE 5.4: COMPARISON OF EXACT AND APPROXIMATE MODAL PERIODS AND PARTICIPATION FACTORS. BUILDING NO. 3





FIGURE 5.5: COMPARISON OF EXACT AND APPROXIMATE MODAL SHAPE FOR MODE 1 AND MODE 4. BUILDING NO. 3.



FIGURE 5.6: VARIATION OF K/M RATIO WITH HEIGHT FOR BUILDING NO. 2.



Mode Number



Mode Number

FIGURE 5.7: COMPARISON OF EXACT AND APPROXIMATE MODAL PERIODS AND PARTICIPATION FACTORS. BUILDING NO. 2



Floor Level



Floor Level

FIGURE 5.8: COMPARISON OF EXACT AND APPROXIMATE MODAL SHAPE FOR MODE 1 AND MODE 6. BUILDING NO. 2



FIGURE 5.9: COMPARISON OF MAXIMUM FLOOR ACCELERATIONS CALCULATED WITH EXACT AND APPROXIMATE MODAL PROPERTIES. BUILDING NO. 1



FIGURE 5.10: COMPARISON OF MAXIMUM FLOOR ACCELERATIONS CALCULATED WITH EXACT AND APPROXIMATE MODAL PROPERTIES. BUILDING NO. 3.



FIGURE 5.11: COMPARISON OF MAXIMUM FLOOR ACCELERATIONS CALCULATED WITH EXACT AND APPROXIMATE MODAL PROPERTIES. BUILDING NO. 2.



FIGURE 5.12: COMPARISON OF FLOOR SPECTRA FOR FLOORS 1 AND 10 CALCULATED WITH EXACT AND APPROXIMATE PROPERTIES. BUILDING NO. 1



FIGURE 5.13: COMPARISON OF FLOOR SPECTRA FOR FLOORS 1 AND 4 CALCULATED WITH EXACT AND APPROXIMATE PROPERTIES. BUILDING NO. 3.



FIGURE 5.14: COMPARISON OF FLOOR SPECTRA FOR FLOORS 1 AND 24 CALCULATED WITH EXACT AND APPROXIMATE PROPERTIES. BUILDING NO. 2



FIGURE 5.15: SIX STORY TORSIONAL SYSTEM USED IN THE STUDY.



FIGURE 5.16: COMPARISON OF APPROXIMATE AND EXACT MODAL PERIODS OF THE TORSIONAL SYSTEM IN FIGURE 5.15



FIGURE 5.17: COMPARISON OF APPROXIMATE AND EXACT MODAL SHAPE OF MODE 1 OF THE TORSIONAL SYSTEM IN FIGURE 5.15



FIGURE 5.18: COMPARISON OF APPROXIMATE AND EXACT MODAL SHAPE OF MODE 3 OF THE TORSIONAL SYSTEM IN FIGURE 5.15



FIGURE 5.19: FLOOR ACCELERATIONS CALCULATED WITH INCREASING NUMBER OF MODES. BUILDING NO. 1



FIGURE 5.20: FLOOR ACCELERATIONS CALCULATED WITH INCREASING NUMBER OF MODES. BUILDING NO. 3



FIGURE 5.21: FLOOR ACCELERATIONS CALCULATED WITH INCREASING NUMBER OF MODES. BUILDING NO. 2





FIGURE 5.22: FLOOR SPECTRA FOR FLOORS 1 AND 10 CALCULATED WITH INCREASING NUMBER OF MODES. BUILDING NO. 1





FIGURE 5.23: FLOOR SPECTRA FOR FLOORS 1 AND 24 CALCULATED WITH INCREASING NUMBER OF MODES. BUILDING NO. 2



FIGURE 5.24: FIGURE SHOWING THE VARIATION OF FLOOR ACCELERATIONS OF THREE BUILDINGS WITH FLOOR LEVEL.



FIGURE 5.25: VARIATION OF FLOOR SPECTRA WITH FLOOR LEVEL. BUILDING NO. 1



FIGURE 5.26: VARIATION OF FLOOR SPECTRA WITH FLOOR LEVEL. BUILDING NO. 2

SECTION 6

EFFECT OF STRUCTURAL NONLINEARITY

Consistent with Chapter 5 of the NEHRP Provisions, in Section 3 we proposed the use of the R-factor for including the nonlinear inelastic effects. Depending upon the type of building, the inclusion of the R-factor reduces the calculated force proportionately. For example, for a special moment resistant steel frame, this will reduce the elastically calculated forces by a factor of 8.

This procedure clearly implies that inelastic effects will always reduce the force. This is true for the forces in the supporting structure. However, for the force in nonstructural components this is not always true. For example, in the studies by Lin and Mahin (1985) and Sewell et al. (1986), it was found that in some instances the calculated accelerations and forces acting on the nonstructural subsystems were greater in the nonlinear case than in the linear case.

To investigate the effect of structural nonlinearity on the response of nonstructural components and equipment here we present numerical results obtained for the maximum floor accelerations and the floor response spectra for two inelastic structures. The nonstructural components are assumed to remain elastic in all the examples. The first structure considered here is the same as the 4-story structure (Building 3) considered in the previous sections. The stiffness and inertia properties of this structure are shown in Figure 6.1. Its strength properties are such that it goes into inelastic range in all stories when a strong seismic input with maximum ground acceleration of $\frac{1}{3}$ g is applied. That is, this structure has a distributed ductility. The second structure considered here is a regular 10-story shear building with uniform mass = 1 kips - sec²/in and stiffness = 4908 kips/in in each story. Its story strengths are such that it yields only in its first story for a maximum ground acceleration level of about 0.5g. This represents the case of a localized ductility. The inelastic constitutive law

proposed by Bouc (1967) and Wen (1976), known in the literature as the Buoc-Wen model, is used to describe the behavior of the force-displacement relationship of both structures.

Figure 6.2 compares the floor acceleration response spectra, obtained for the elastic and inelastic cases, at the fourth floor of the structure 4-story for a 2% equipment damping ratio. The seismic input for these results was defined by 50 artificially generated acceleration time histories of broad-band type with average maximum acceleration of $\frac{1}{3} g$. The spectra shown in the figure are the mean of the spectra obtained for the 50 input time histories. The ductility level in the four stories ranged from 2.5 in the first story to 1.5 in the top story.

The response spectrum value at very high frequencies represent the maximum floor acceleration value. It is noted that the values of the inelastic spectrum are significantly lower than the elastic response spectrum value in the entire frequency range. The ratio of inelastic to elastic spectrum values at different frequencies is plotted in Figure 6.3. Here the results for other equipment damping ratios have also been included. It is noted that the maximum reduction in the elastic response occurs at the peaks of the spectra. The maximum acceleration is also reduced as is shown by the horizontal line near the 100 Hz-frequency. This reduction becomes more pronounced as the earthquake level increases. For those cases, such as this example, when there is a reduction in the response, the higher the ductility, the higher the reduction.

In the next figure, Figure 6.4, we show the R-factor calculated for the dominant peaks of the floor spectra at various floors of the structure. In this case the R-factor is defined as the ratio between the elastic and inelastic absolute accelerations of the subsystem. It can be seen that the first peak, which is usually the highest except for the lower floors, is reduced the most. Next we show the flow spectra results of the 10-story structure for the elastic and inelastic cases. In this particular case, and contrary to what one would normally expect, the floor acceleration for the inelastic case can be higher than the acceleration for the elastic structure.

The floor spectra values for this example represent the average response obtained again for 50 synthetic acceleration time histories with broad band characteristics. The average maximum acceleration of the time histories was 0.489 g. The first story had a ductility ratio of 2.73 whereas all the higher stories were still in the elastic range.

Figure 6.5 compares the elastic and inelastic spectra for the lowest floor. It is noted that in the high frequency range the inelastic spectrum values are higher than the elastic spectrum values. This is more clearly shown in Figure 6.6, where the ratio of inelastic to elastic values has been plotted. This ratio is now seen to be higher than 1 in the high frequency range. The horizontal line at the end of the spectra near 100 Hz clearly shows the amplification of the floor acceleration value in the inelastic range.

It is also seen that for both the elastic and inelastic structures, the peak corresponding to the first structural frequency is not the highest. In this case, the peaks at the second and third structural frequencies are higher than the peak at the first frequency. Thus the higher mode effects for the lower floors can be important even in the nonlinear case.

The amplification of the high frequency spectrum values in the inelastic case becomes less pronounced as we move to the top floors. This can be observed in the floor spectra and in the ratio of inelastic-to-elastic spectra for the 10th floor displayed in Figures 6.7 and 6.8. It is seen that there is still some amplification at higher frequencies but it is not important. Also the floor acceleration in the inelastic case is now smaller than the elastic case. The amplification of inelastic response in the high frequency range is affected by several factors. One of the possible factors is the modal interaction or internal resonance of the modes of nonlinear systems (Nayfeh and Mook, 1979). Although the phenomenon always occurs, it seems to be more important when the higher modes are odd multiples of one of the dominant lower modes which is also predominantly excited by the input. This seems to be happening in our present case. It can be observed from the natural frequencies for this particular structure listed in Figures 6.5 and 6.7. The second and third mode frequencies are about 3 and 5 times the first mode frequency. If this structure is excited by an input with significant energy at the frequency of the first mode, it is likely to initiate the internal resonance between the first, second and third modes. To demonstrate this we choose as the seismic input the ground motion recorded in Parkfield, CA in 1966. This motion has its frequency content concentrated near the first frequency of the structure, as is shown by its ground response spectrum presented in Fig. 6.9.

The floor response spectra for floor 1 of the 10-story building subjected to the Parkfield ground motion for elastic and inelastic cases are plotted in Figure 6.10. The ratio of these two spectra is plotted in Figure 6.11. It is seen from these results that the higher frequencies receive a significant amount of energy from the first mode. The floor spectra for the lowest floor of the elastic and inelastic structure are plotted in Figure 6.12. For this structure, the amplification of the high frequency response at the higher frequency in the inelastic case is not as dramatic as it is at floor 1. Similar results have also been reported earlier by Sewell et al. (1986) for a six story structure. Although it is not mentioned in their report, the structural parameters of their example were such that internal resonance could have played an important role in the observed amplification of the response in their results.

In general, it is felt that incorporation of inelastic effects in the calculation of force

response by a single R-factor is not entirely possible. As it is shown in the numerical examples, there can be some cases where the calculated force will be underestimated. Thus for important equipment in critical facilities it may be necessary to conduct a more detailed analysis to obtain accurate response and ensure safety. Simple analyses, such as those in the existing codes or even the spectrum analysis proposed in this study, may not provide an adequate design for critical components. However, for normal equipment, the use of the R-factor provides a practical and simple approach to include the effect of yielding in the calculation of forces.

Finally, it is evident that to fully assess the effect of the nonlinear behavior of the structure on equipment, a more comprehensive study than the one presented here is needed. The objective of this section was to briefly examine by means of a few numerical examples the appropriateness of using a response reduction factor to define the seismic design force for equipment and nonstructural components. A more complete study on this topic is currently underway.



FIGURE 6.1: FOUR STORY SHEAR BUILDING USED IN THE STUDY.



FIGURE 6.2: AVERAGE FLOOR RESPONSE SPECTRA OF FLOOR 4 FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO 50 ACCELEROGRAMS

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FIGURE 6.3: RATIO OF INELASTIC TO ELASTIC SPECTRA OF FLOOR 4

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FIGURE 6.4: R-FACTOR FOR FIRST FOUR PEAK RESPONSES



FIGURE 6-5: AVERAGE FLOOR RESPONSE SPECTRA OF FLOOR 1 FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO 50 ACCELEROGRAMS



FIGURE 6.6: RATIO OF INELASTIC TO ELASTIC SPECTRA OF FLOOR 1



FIGURE 6.7: AVERAGE FLOOR RESPONSE SPECTRA OF FLOOR 10 FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO 50 ACCELEROGRAMS


FIGURE 6.8: RATIO OF INELASTIC TO ELASTIC SPECTRA OF FLOOR 10



FIGURE 6.9: GROUND RESPONSE SPECTRA FOR PARKFIELD EARTHQUAKE (1966, CHOLAME-SHANDON ARRAY #2, N65E)



FIGURE 6-10: FLOOR RESPONSE SPECTRA OF FLOOR 1 FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO THE PARKFIELD ACCELEROGRAM



FOR PARKFIELD ACCELEROGRAM



FIGURE 6.12: FLOOR RESPONSE SPECTRA OF FLOOR 10 FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO PARKFIELD ACCELEROGRAM

SECTION 7 CONCLUDING REMARKS

The commonly used code provisions for calculating the seismic forces for the design of nonstructural components are first critically evaluated. Although the nonstructural components receive seismic motion filtered through the supporting structure, the code formulas defined for calculating the seismic force do not depend upon the dynamic characteristics of the supporting structure. Also the three code provisions examined in this study are not consistent with each other as there are significant differences in the seismic forces calculated according to these codes.

Rational methods based on the theory of structural dynamics are proposed for calculating the forces on nonstructural components. The proposed methods still defines the force on the component in the same basic format as in the NEHRP and other code provisions.

The performance criteria factor P as presented in the NEHRP Provisions is still used. In addition, the normalized seismic coefficient of the NEHRP Provisions is used as the importance factor I. As more experience is gained, it may be necessary to update the values of these factors.

The methods explicitly consider the dynamic characteristics of the supporting structure, defined in terms of the modal frequencies, mode shapes and participation factors. The nonlinear effect of yielding of the supporting structure is included in essentially the same way as is done by the modal analysis procedure suggested in the 1991 NEHRP Provisions. Simplified practical procedures, requiring the characteristics of only the fundamental mode, are also proposed to obtain a conservative estimate of the forces. In all cases closed form formulas are provided to calculate the forces. Comparisons of the forces calculated by the three provisions with the forces calculated by the proposed procedures are also carried out to examine the consistency of the current code provisions and to evaluate the importance of various structural and nonstructural parameters.

In order to include the dynamic characteristics of the supporting structure to calculate the forces more rationally, the proposed method requires more calculations than those required in the current code provisions. However, since the forces in the proposed approaches are defined by closed-form formulas, these calculations can be easily performed with modern hand calculators in most situations.

SECTION 8

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APPENDIX A

MAXIMUM ACCELERATION COEFFICIENT

In Section 3 we provided a formula in equation (3.2) for calculating the seismic coefficient C_{cm} . This formula requires the knowledge of frequencies, mode shapes, etc. for the higher modes. Later, a simplified formula requiring information only about the first mode was also proposed. In this Appendix, we provide a justification for these simplified formulas.

Following the work of Singh and Maldonado (1991) it can be shown that the maximum acceleration of a floor, or the seismic coefficient C_{cm} , expressed in terms of the input response spectrum for a unit ground acceleration is:

$$C_{cm}^{2} = A_{s}^{2} + \sum_{j=1}^{r} \left\{ (1+4\beta^{2})\gamma_{j}^{2}\phi_{mj}^{2} + 8\beta^{2}a_{s}\gamma_{j}\phi_{j} \right\} R_{j}^{2} + 2\sum_{j=1}^{r} \sum_{k=j+1}^{r-1} (\gamma_{k}\phi_{mj})(\gamma_{k}\phi_{mk}) \left[(A_{jk} + B_{jk})R_{j}^{2} + (C_{jk} + D_{jk})R_{k}^{2} \right]$$
(A.1)

where, except for coefficients A_{jk} , B_{jk} , C_{jk} and D_{jk} which are defined below, other quantities are defined in Section 3 after equation (3.2).

$$A_{jk} = [(u_2 - u_1)z_1 - (v_2 - v_1)z_2] /\Delta$$

$$B_{jk} = [(v_1u_2 - u_1v_2)z_2 - (v_1 - v_2)z_1] /\Delta$$

$$C_{jk} = (w_4 - v_2A_{jk})/v_1$$

$$D_{jk} = (w_1 - B_{jk})$$
(A.2)

where

$$\Delta = (u_2 - u_1)(v_1 u_2 - u_1 v_2) + (v_1 - v_2)^2$$
$$r_{jk} = \frac{\omega_j}{\omega_k}$$

$$u_{1} = -2r_{jk}^{2}(1 - 2\beta^{2}) ; u_{2} = -2(1 - 2\beta^{2})$$

$$v_{1} = r_{jk}^{4} ; v_{2} = 1$$

$$z_{1} = v_{1}w_{3} - v_{1}^{2}w_{1} - u_{1}w_{4} ; z_{2} = v_{1}w_{2} - w_{4} - u_{1}v_{1}w_{1}$$

$$w_{1} = 4\beta^{2}r_{jk}$$

$$w_{2} = r_{jk}^{2} \left\{ 1 - 8\beta^{2} + 16\beta^{4} \right]$$

$$w_{3} = -(1 - 4\beta^{2})(1 + r_{jk}^{2})r_{jk}^{2}$$

$$w_{4} = r_{jk}^{4}$$

$$(A.3)$$

The double summation term (or the cross term) in equation (A.1) represents the effect of correlation between various modes. Although contribution of this term is relatively small compared to the first term, its calculation is somewhat more involved than the first term. In this study, the effect of neglecting this term has been examined for several example problems.

Nine buildings of different heights with number of stories varying from 2 to 24 have been considered. All buildings were uniform in the elevation. Thus, their modal properties could be simply calculated by closed-form formulas provided in Section 5. The floor mass and story stiffness of these buildings were selected such that the fundamental period of the buildings calculated according to equation (5.3) in Section 5 was the same as the period given by equation (4.4) of the 1991-NEHRP Provisions for steel buildings:

$$T_1 = 0.035 h_N^{\frac{3}{4}} \tag{A.4}$$

where h_N is the height in feet above the base to the highest level of the building. For a story height of 12 feet, the value of h_N is equal to 12N.

The floor accelerations for these nine buildings were calculated using equation (A.1) with and without the cross terms. These floor accelerations are plotted in

Figure (A.1). From these results, it is noted that the cross terms seem to make the most difference in the lower floors of the taller structures. For higher floors, neglecting the cross terms is seen to provide a more conservative estimate of the floor accelerations. It is also noted that the taller buildings have smaller acceleration; in fact, the accelerations at the lower levels can be even smaller than the maximum ground acceleration. The fact that the accelerations of the taller buildings are smaller than those of the shorter buildings is primarily due to the reason that the ground acceleration spectrum value at the higher periods reduces with the period according to equation (3.3).

These results indicate that neglecting the cross terms can produce an unconservative estimate of the forces on the lower floors. However, by limiting the acceleration calculated with no cross terms to be not less than the maximum ground acceleration, acceptable results can be obtained for all buildings. This is the reason for limiting the value C_{cm} in equation (3.2) to be greater or equal to 1.0. Figure (A.2) compares the floor acceleration calculated with this limitation but without the cross terms with the floor acceleration calculated with cross terms. It is seen that in most cases, the results calculated without the cross terms are now acceptable. In Figure (A.3) we present similar comparisons of accelerations calculated with equation (3.2) and equation (A.1) for the three buildings considered in Section 4. From these results it can be concluded that the proposed equation (3.2) provides acceptable values of the coefficient when compared with the more rigorous and accurate (but cumbersome) formula in equation (A.1).

The first mode formula proposed in equation (3.5) is a direct application of equation (3.2). All higher mode terms are ignored, and the acceleration calculated with the first mode is increased by 50% to account for the contribution of the higher modes. Also, the damping ratio of the structure is assumed to be 5%. Substitution for $a_s = 1 - \gamma_1 \phi_{N1}$ and $\beta = 0.05$ in the first term of equation (3.1) and amplification of this value by 50%, led to equation (3.5). This equation can be expressed simply in terms of the number of stories N if the first mode is assumed to vary linearly with height. This simplified expression is given by equation (3.7).

To show how the accelerations calculated by this first mode approximation compare with the accelerations calculated by the more rigorous and accurate formula provided by equation (A.1), we present Figures (A.4) and (A.5). Figure (A.4) is for nine uniform buildings of different heights and Figure (A.5) is for the example buildings considered in Section 4, referred to as Building 1, 2, and 3. It is seen that the first mode method provide a conservative estimate of the floor acceleration in all cases, except for the higher floors of Building 2 (Figure A.5).



FIGURE A.1: COMPARISON OF FLOOR ACCELERATION CALCULATED WITH AND WITHOUT CROSS TERMS FOR UNIFORM BUILDINGS OF DIFFERENT HEIGHTS.



FIGURE A.2: COMPARISON OF FLOOR ACCELERATION CALCULATED WITH CROSS TERMS AND PROPOSED APPROACH FOR UNIFORM BUILDINGS OF DIFFERENT HEIGHTS.



FIGURE A.3: COMPARISON OF FLOOR ACCELERATION CALCULATED WITH CROSS TERMS AND PROPOSED APPROACH FOR BUILDINGS 1,2 AND 3.



FIGURE A.4: COMPARISON OF FLOOR ACCELERATION CALCULATED WITH CROSS TERMS AND FIRST LINEAR MODE APPROACH FOR UNIFORM BUILDINGS OF DIFFERENT HEIGHTS.



FIGURE A.5: COMPARISON OF FLOOR ACCELERATION CALCULATED WITH CROSS TERMS AND FIRST LINEAR MODE APPROACH FOR BUILDINGS 1,2 AND 3.

APPENDIX B

UNIT FLOOR SPECTRUM COEFFICIENT

In this appendix we examine a method which can be used to calculate the unit floor response spectrum coefficient C_{fm} and its possible simplifications.

The coefficient C_{fm} represents the floor acceleration response spectrum value for a unit acceleration ground response spectrum. References 13 and 14 describe a method for calculating the floor acceleration spectrum value for an equipment of frequency w_e and damping ratio β_e . Here, a slightly modified version of this method which reduces the error due to truncation of modes is presented. The accuracy of this method has been verified by several numerical simulation studies involving time history analysis for ensembles of ground motion time histories.

In its most complete form, the expression defining the factor C_{fm} can be written as follows.

$$C_{fm}^{2} = a_{s}^{2}R_{e}^{2} + 2a_{s}\sum_{j=1}^{r} p_{j}A_{j} + \sum_{j=1}^{r} p_{j}^{2}B_{j} + \sum_{j=1}^{r-1} \sum_{k=j+1}^{r} p_{j}p_{k}(X_{jk} + Y_{jk})$$
(B.1)

In this expression, the first term represents the correction for the truncated higher modes, the single summation term represents the contributions of the individual modes to the coefficient and the double summation terms represent the effect of correlation between different modes. Various quantities in this expression are defined as:

r = number of modes $a_s =$ correction factor $= 1 - \sum_{j=1}^{r} p_j$ $p_j = \gamma_j \phi_{mj}$ $\gamma_j = j^{th}$ modal participation factor $\phi_{mj} = j^{th} \text{modal displacement of } \mathbf{m}^{th} \text{ floor}$

$$R_e = 1.2F_e S/(T_e^{\frac{2}{3}}R)$$

S =soil factor

$$T_e = \text{equipment period} = 2\pi/w_e$$

$$w_e =$$
 equipment frequency in rad/s

- R = response modification factor
- F_e = factor by which the input ground spectrum value

should be modified to account for the difference in the damping

ratios of the equipment and the structure. For converting

a 5% input response spectrum value to a 2% spectrum value,

this factor is 1.25 according to the Commentary of the NEHRP Provisions [6].

The coefficients A_j , B_j and X_{jk} are different for the resonance case (when $w_e = w_j$) and the nonresonance case (when $w_e \neq w_j$) whereas the coefficient Y_{jk} is the same for the resonance and nonresonance cases. These coefficients are defined as follows:

Resonance Case:

If one of the structural frequencies, say ω_{ℓ} , is equal to ω_{e} , the corresponding coefficients A_{ℓ} , B_{ℓ} and $X_{\ell k}$ become:

$$A_{\ell} = (1-a)R_{\ell}^2 + aR_{\ell}^2 \tag{B.2}$$

$$B_{\ell} = (1-b)R_{\ell}^2 + bR_e^2 \tag{B.3}$$

$$X_{\ell k} = (1 - c)R_{\ell}^2 + cR_e^2 \tag{B.4}$$

where:

$$a = 1 + \frac{4\beta^2 \beta_e^2}{\beta^2 - \beta_e^2}$$

$$b = \frac{1 + 4\beta^2 + 16\beta_e^2 \beta^2}{4(\beta^2 - \beta_e^2)}$$

$$c = \frac{Y_2 + Y_3 - (1 + x)Y_4}{4(\beta^2 + \beta_e^2)}$$
(B.5)

in which β_{ϵ} and β are the damping ratios of the equipment and structure, respectively. The coefficients x, Y_2, Y_3 , and Y_4 are defined in equations (B.13) and (B.14).

Non Resonance Case:

$$A_{j} = (1 - C_{j} - D_{j}/r_{j}^{2})R_{j}^{2} + (C_{j} + D_{j})R_{e}^{2}$$
(B.6)

$$B_j = (1 - C'_j - D'_j / r_j^2) R_j^2 + (C'_j + D'_j) R_e^2$$
(B.7)

$$X_{jk} = (P_1 + Q_1)R_e^2 + \left\{ (Y_4 - v_1P_1)/r_j^4 - Q_1/r_j^2 \right\} R_j^2 \tag{B.8}$$

$$Y_{jk} = (P_2 + Q_2)R_e^2 + \left\{ (Y'_4 - v_2 P_2)/r_k^4 - Q_2/r_k^2 \right\} R_k^2$$
(B.9)

where the coefficient R_j , R_k are the modal spectral coefficients:

$$R_j = \frac{1.2S}{T_j^{\frac{2}{3}}R} \quad ; \quad R_k = \frac{1.2S}{T_k^{\frac{2}{3}}R}$$

 and

$$T_j = \frac{2\pi}{\omega_j}$$
 = period of the jth mode ; $T_k = \frac{2\pi}{\omega_k}$ = period of the kth mode $\omega_j = j^{th}$ mode frequency ; $\omega_k = k^{th}$ mode frequency

The remaining coefficients in equations (B.6)-(B.9) are:

$$r_j = \frac{\omega_j}{\omega_e} ; \ r_k = \frac{\omega_k}{\omega_e}$$
 (B.10)

$$C_{j} = \left[(1 - v_{1})(w_{2} - w_{4}) + (u_{1} - x)(w_{3} - xw_{4}) \right] / \Delta_{1}$$

$$C'_{j} = \left[(1 - v_{1})(w'_{2} - w_{4}) + (u_{1} - x)(w'_{3} - xw_{4}) \right] / \Delta_{1} \qquad (B.11)$$

$$D_{j} = \left[(u_{1} - xv_{1})(w_{2} - w_{4}) - (1 - v_{1})(w_{3} - xw_{4}) \right] / \Delta_{1}$$

$$D'_{j} = \left[(u_{1} - xv_{1})(w'_{2} - w_{4}) - (1 - v_{1})(w'_{3} - xw_{4}) \right] / \Delta_{1}$$

$$P_{1} = \frac{(u_{1} - x)(Y_{3} - xY_{4}) + (1 - v_{1})(Y_{2} - Y_{4})}{\Delta_{1}}$$

$$P_{2} = \frac{(u_{2} - x)(Y_{2}' - xY_{4}') + (1 - v_{2})(Y_{2}' - Y_{4}')}{\Delta_{2}}$$

$$Q_{1} = \frac{(u_{1} - xv_{1})(Y_{2} - Y_{4}) - (1 - v_{1})(Y_{3} - xY_{4})}{\Delta_{1}}$$

$$Q_{2} = \frac{(u_{2} - xv_{2})(Y_{2}' - Y_{4}') - (1 - v_{2})(Y_{3}' - xY_{4}')}{\Delta_{2}}$$
(B.12)

The coefficients C_j , C'_j , etc. are defined in terms of the following factors:

$$x = -2(1 - 2\beta_e^2)$$

$$u_1 = -2r_j^2(1 - 2\beta^2) ; u_2 = -2r_k^2(1 - 2\beta^2)$$

$$v_1 = r_j^4 ; v_2 = r_k^4.$$

$$w_2 = -4r_j^2\beta_e^2(1 - 4\beta^2)$$

$$w_3 = r_j^2(-1 + 4\beta^2 + 4\beta_e^2r_j^2)$$

$$w_4 = r_j^4$$

$$w_2' = 16\beta_e^2\beta^2r_j^2$$

$$w_3' = 4(\beta^2 + \beta_e^2r_j^2)r_j^2$$
(B.13)

$$Y_{2} = 4\beta_{e}^{2}E_{2}$$

$$Y_{3} = 4\beta_{e}^{2}E_{1} + E_{2}$$

$$(B.14)$$

$$Y_{4} = E_{1}$$

$$Y_{2}' = 4\beta_{e}^{2}E_{4}$$

$$Y_{3}' = 4\beta_{e}^{2}E_{3} + E_{4}$$

$$Y_{4}' = E_{3}$$

$$E_{1} = \frac{(u_{2} - u_{1})(v_{1}z_{3} - v_{1}^{2}z_{1} - u_{1}z_{4}) - (v_{2} - v_{1})(v_{1}z_{2} - z_{4} - u_{1}v_{1}z_{1})}{\Delta_{3}}$$

$$E_{2} = \frac{(v_{1}u_{2} - u_{1}v_{2})(v_{1}z_{2} - z_{4} - u_{1}v_{1}z_{1}) - (v_{1} - v_{2})(v_{1}z_{3} - v_{1}^{2}z_{1} - u_{1}z_{4})}{v_{1}\Delta_{3}}$$

$$E_{3} = \frac{(z_{4} - v_{2}E_{1})}{v_{1}}$$

$$E_{4} = z_{1} - E_{2}$$
(B.15)

$$z_{1} = 4\beta^{2}r_{j}r_{k}$$

$$z_{2} = r_{j}^{2}r_{k}^{2}(1 - 4\beta^{2})^{2}$$

$$z_{3} = -(1 - 4\beta^{2})(r_{j}^{2} + r_{k}^{2})r_{j}^{2}r_{k}^{2}$$

$$(B.16)$$

$$z_{4} = r_{j}^{4}r_{k}^{4}$$

$$\Delta_{1} = (1 - v_{1})^{2} + (u_{1} - x)(u_{1} - xv_{1})$$

$$\Delta_{2} = (1 - v_{2})^{2} + (u_{2} - x)(u_{2} - xv_{2})$$

$$\Delta_{3} = (v_{1} - v_{2})^{2} + (u_{2} - u_{1})(v_{1}u_{2} - u_{1}v_{2})$$
(B.17)

One immediately notes that the expressions for calculating the double summation terms or cross terms, which represent the effect of modal correlation, are more complicated than those of the single summation terms. The question immediately arises whether one could neglect these terms to simplify the calculations. However, it is observed that even for structures with well separated frequencies, these terms cannot be neglected, especially for calculating the coefficient for the lower floors. To examine the importance of the cross terms we present floor response spectra results for four uniform and nonuniform buildings in Figures B.1 through B.6. Figure B.1 compares the floor spectra for floors 1 and 6 of a uniform 24-story building. The importance of the cross terms can be clearly seen in the spectrum for the first floor. For higher floors, however, the effect of these cross terms is not important as seen from Figure B.2. Also it is observed that this effect is not very important for shorter structures which are also stiffer, as it is seen from the results in Figure B.3 for the 10-story Building No. 1 (see Figure 4.1) and Figure B.6 for the 4-story Building No. 3 (see Figure 5.1). Figures B.4 and B.5 display the floor response spectra for floors 1, 6, 12 and 24 of the nonuniform 24-story Building No. 2 (see Figure 4.2). Here again the cross terms in equation (B.1) contribute significantly to the spectra for the lower floors.

Thus, if one wants to calculate a more realistic and accurate value of the response, it is necessary to include the cross terms in equation (B.1). Moreover, as observed from the results presented in Section 5, it may also be necessary to consider several modes, especially for calculating the equipment response at the lower floors. At the top floor, however, one can still calculate the peak response (which usually happens at the fundamental frequency) by using only the fundamental mode. We make use of this observation to propose a simple but conservative approach for the to calculation of the floor spectra using only the first mode.

Single Mode Approach

In this approach, we first calculate the peak floor response spectrum value for a unit ground acceleration at the top floor using either the characteristics of the fundamental mode calculated by eigenvalue analysis or the estimated first mode frequency and mode shape. This peak response value can be calculated taking r = 1 in equation (B.1):

$$R_{max} = \sqrt{a_s^2 R_e^2 + 2a_s p_1 A_1 + p_1^2 B_1} \tag{B.18}$$

If it is assumed that the first mode varies linearly with height, the coefficient $p_1 = \gamma_1 \phi_{N1}$ (and the correction factor a_s) is a function of the total number of stories of the building N, according to equation (3.6). Assigning the values $\beta = 0.05$ and $\beta_e = 0.02$ for the damping ratios to calculate the coefficients A_1 and B_1 from equations (B.2) and (B.3) and realizing that at resonance $R_e = F_e R_1$ with $F_e = 1.25$, it can be shown that R_{max} becomes:

$$R_{max} = \sqrt{\frac{390.3N^2 + 4N + 1}{2N + 1}} R_e \tag{B.19}$$

The corresponding floor spectrum at the top floor is then defined as shown in Figure B.7.

The remaining parameters that define the top floor spectrum are C_{cN} , f_{ℓ} and f_m . The coefficient C_{cN} is defined by equation (3.7) in Section 3. It represents the maximum acceleration of the top floor per unit ground acceleration, estimated using only the first mode. The frequency parameters f_{ℓ} and f_m are defined in terms of the estimated highest structural frequency f_N in cps according to the following expressions:

$$f_{\ell} = \frac{f_N}{2\sqrt{N}}$$
; $f_m = 0.8f_N$ (B.20)

The value of f_N can be estimated by the following formula

$$f_N = f_1 \frac{\sin\left\{\frac{(2N-1)\pi}{2(2N+1)}\right\}}{\sin\left\{\frac{\pi}{2(2N+1)}\right\}}$$
(B.21)

Equation (B.20) is exact for perfectly uniform buildings in plan and elevation. For other buildings, it provides only an estimate of the highest frequency.

We next define the unit floor response spectrum at the first floor as shown in Figure B.7. To define the amplitude R_G of the plateau in the spectrum one could, in

principle, try the approach used to define R_{max} , *i.e.*, use the resonance formulas with a single mode. However, in this case this method will not yield reliable estimates of the peak floor spectrum values because these values are strongly influenced by the higher modes. Therefore, a different approach is required. The heavier line in Figure B.8 represents the peak values of the floor response spectra for uniform buildings of increasing height using as input a ground spectrum with unit maximum acceleration and S = R = 1. These values were calculated using the full formulation presented earlier in this Appendix including all the modes and cross terms. The curve was then approximated by the following analytical expression:

$$R_G = 20N^{-\frac{1}{\sqrt{3}}} \tag{B.22}$$

In order to use this expression to define the floor response spectrum, it is corrected by the soil and response modification factors S and R as follows:

$$R_G = 20 \frac{S}{R} N^{-\frac{1}{\sqrt{3}}}$$
 (B.23)

The other parameters required to define the first floor spectrum are f_m , f_u , and C_{c1} . The coefficients C_{c1} represents the maximum acceleration of the first floor of the building when it is subjected to a ground motion with a unit peak amplitude. It can be calculated as explained in Section 3 according to equation (3.4). It is straightforward to show that, using equations (3.4) and (3.7) for a regular building in which $h_N = Nh_1$, the coefficient C_{c1} is given by the following expression:

$$C_{c1} = \frac{(2N^2 - N - 1)C_o + \sqrt{15.45N^2 - 2.1N + 1.5} R_1}{N(2N + 1)}$$
(B.24)

where $C_o = S/R$.

The frequency f_u is defined in terms of f_N as:

$$f_u = 1.5 f_N \tag{B.25}$$

The corner frequencies are defined to cover most cases of practical interest as well as simplify the description of the spectrum of the first floor.

To obtain the unit floor response spectrum values at an intermediate floor, a linear interpolation is made. The expressions for the floor response spectrum coefficient C_{fm} , defined according to this linear interpolation are given by equations (3.9) through (3.13) in Section 3.

Floor response spectra obtained by the simplified method based on equations (3.9)-(3.13) are also plotted as Figures B.1 through B.6 for various buildings and different floors. It is observed that, in most cases, the approximate approach using only the first mode provides conservative estimates of the equipment response. At the first floor levels, the approximate spectra are somewhat smaller than the spectra obtained by the exact approach. However, these small differences should not lead to unconservative design of important equipment in view of the importance factor I and performance criteria factor P used in equations (3.8).





FIGURE B.1: COMPARISON OF FLOOR RESPONSE SPECTRA FOR FIRST AND SIXTH FLOOR OF A UNIFORM 24-STORY BUILDING.



FIGURE B.2: COMPARISON OF FLOOR RESPONSE SPECTRA FOR TWELFTH AND TOP STORY OF A UNIFORM 24-STORY BUILDING.



FIGURE B.3: COMPARISON OF FLOOR RESPONSE SPECTRA FOR FIRST AND TOP FLOOR OF BUILDING NO. 1.



FIGURE B.4: COMPARISON OF FLOOR RESPONSE SPECTRA FOR FIRST AND SIXTH FLOOR OF BUILDING NO. 2.



FIGURE B.5: COMPARISON OF FLOOR RESPONSE SPECTRA FOR TWELFTH AND TOP FLOOR OF BUILDING NO. 2.





FIGURE B.6: COMPARISON OF FLOOR RESPONSE SPECTRA FOR FIRST AND TOP FLOOR OF BUILDING NO. 3.



FIGURE B.7: FLOOR SPECTRA AT FIRST AND TOP FLOOR FOR SIMPLIFIED SINGLE MODE APPROACH.


FIGURE B.8: PEAK FLOOR RESPONSE SPECTRUM VALUES FOR THE FIRST FLOOR OF UNIFORM BUILDINGS.

APPENDIX C

MODAL PROPERTIES OF A TORSIONAL BUILDING

To define the closed-form expressions for the modal shapes and frequencies of a torsional building represented in Section 5, we need to obtain the solution of the following associated eigenvalue problem:

$$[\bar{K}]\{b^{(s)}\} = \bar{\lambda}_s[\bar{M}]\{b^{(s)}\} ; \ s = 1, 2, 3 \tag{C.1}$$

where the stiffness matrix $[\bar{K}]$ and mass matrix $[\bar{M}]$ are:

$$[\bar{k}] = \begin{bmatrix} k_x & 0 & -k_x e_y \\ 0 & k_y & k_y e_x \\ -k_x e_y & k_y e_x & k_t \end{bmatrix}$$
(C.2)

$$[\bar{M}] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_o \end{bmatrix}$$
(C.2)

The coefficients k_x , k_y and e_x , e_y are, respectively, the story stiffness coefficients and eccentricities in the x and y directions. The coefficient k_t is the torsional stiffness of the story. The coefficients m and I_o are, respectively, the mass of the floor and its mass moment of inertia with respect to an axis perpendicular to x and y axes passing through the mass center.

In the following, we define the closed-form expressions for the eigenvalues and eigenvectors of equation (C.1).

The characteristic polynomial associated with the eigenproblem (C.1) is:

$$\bar{\lambda}^3 - I_1 \bar{\lambda}^2 + I_2 \bar{\lambda} - I_3 = 0$$
 (C.4)

where:

$$I_1 = a_{11} + a_{22} + a_{33} \tag{C.5.a}$$

$$I_2 = a_{11}a_{22} + a_{22}a_{33} + a_{11}a_{33} - a_{13}a_{31} - a_{23}a_{32} \tag{C.5.b}$$

$$I_3 = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{31}a_{13}a_{22} \tag{C.5.c}$$

and the coefficients a_{ij} are:

$$a_{11} = \frac{k_x}{m}$$
; $a_{22} = \frac{k_y}{m}$; $a_{33} = \frac{k_t}{I_o}$ (C.6.a)

$$a_{13} = -\frac{k_x}{m}e_y$$
; $a_{31} = -\frac{k_x}{I_o}e_y$ (C.6.b)

$$a_{23} = \frac{k_y}{m} e_x$$
; $a_{32} = \frac{k_y}{I_o} e_x$ (C.6.c)

The roots (eigenvalues) of the characteristic polynomial are:

$$\bar{\lambda}_1 = \Omega_1^2 = \frac{2}{\sqrt{3}} \alpha \sin(\theta + \frac{2}{3}\pi) + \frac{I_1}{3}$$
 (C.7.a)

$$\bar{\lambda}_2 = \Omega_2^2 = \frac{2}{\sqrt{3}}\alpha \sin(\theta) + \frac{I_1}{3} \qquad (C.7.b)$$

$$\bar{\lambda}_3 = \Omega_3^2 = \frac{2}{\sqrt{3}} \alpha \sin(\theta + \frac{4}{3}\pi) + \frac{I_1}{3}$$
 (C.7.c)

The parameters α and θ are:

$$\alpha = \sqrt{\frac{1}{3}(I_1^2 - 3I_2)} \tag{C.8}$$

$$\theta = \frac{1}{3} \operatorname{arc\,sin}\left(\frac{\sqrt{3}}{2}\frac{\beta}{\alpha^3}\right) \tag{C.9}$$

with:

$$\beta = I_1 I_2 - \frac{2}{9} I_1^3 - 3I_3 \tag{C.10}$$

The eigenvectors are obtained solving the homogeneous system of equations:

$$\begin{bmatrix} a_{11}\bar{\lambda}_{s} & 0 & a_{13} \\ 0 & a_{22} - \bar{\lambda}_{s} & a_{23} \\ a_{31} & a_{32} & a_{33} - \bar{\lambda}_{s} \end{bmatrix} \begin{bmatrix} b_{1} \\ b_{2} \\ b_{3} \end{bmatrix}_{s} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; s = 1, 2, 3 \qquad (C.11)$$

The eigenvectors $\{b^{(s)}\}$ normalized with respect to the mass matrix $[\bar{M}]$ becomes:

$$\{b^{(s)}\} = \frac{1}{\Delta_s} \begin{cases} C_1^{(s)} \\ C_2^{(s)} \\ C_3^{(s)} \end{cases}$$
(C.12)

where the coefficients Δ_s and $C_i^{(s)}$ are:

$$\Delta_s = \sqrt{m(C_1^{(s)})^2 + m(C_2^{(s)})^2 + I_o(C_3^{(s)})^2}$$
(C.13)

$$C_1^{(s)} = \bar{\lambda}_s^2 - \bar{\lambda}_s(a_{22} + a_{33}) + a_{22}a_{33} - a_{23}a_{32} \tag{C.14.a}$$

$$C_2^{(s)} = a_{23}a_{31} \tag{C.14.b}$$

$$C_3^{(s)} = (\bar{\lambda}_s - a_{22})a_{31} \tag{C.14.c}$$

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