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Abstract: The work presented in this report represents one part of the active control implementation project focusing on the development of more efficient control algorithms. Since peak response is closely related to structural safety, considered in this report are full-state optimal polynomial controllers and the corresponding static output polynomial controllers for limiting the peak response of linear and nonlinear civil engineering structures. For linear structures, simulation results demonstrate that the output polynomial control has several advantages over the corresponding linear controller. For control of nonlinear or hysteretic structures, simulation results indicate that the most significant advantage of the static output polynomial controller over the corresponding linear controller is its load-adaptive capability to limit selected peak response quantities of the structure, when the magnitude of the earthquake exceeds its design limit.

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Optimal Polynomial Control for Linear  
and Nonlinear Structures

by

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**Optimal Polynomial Control for Linear  
and Nonlinear Structures**

by

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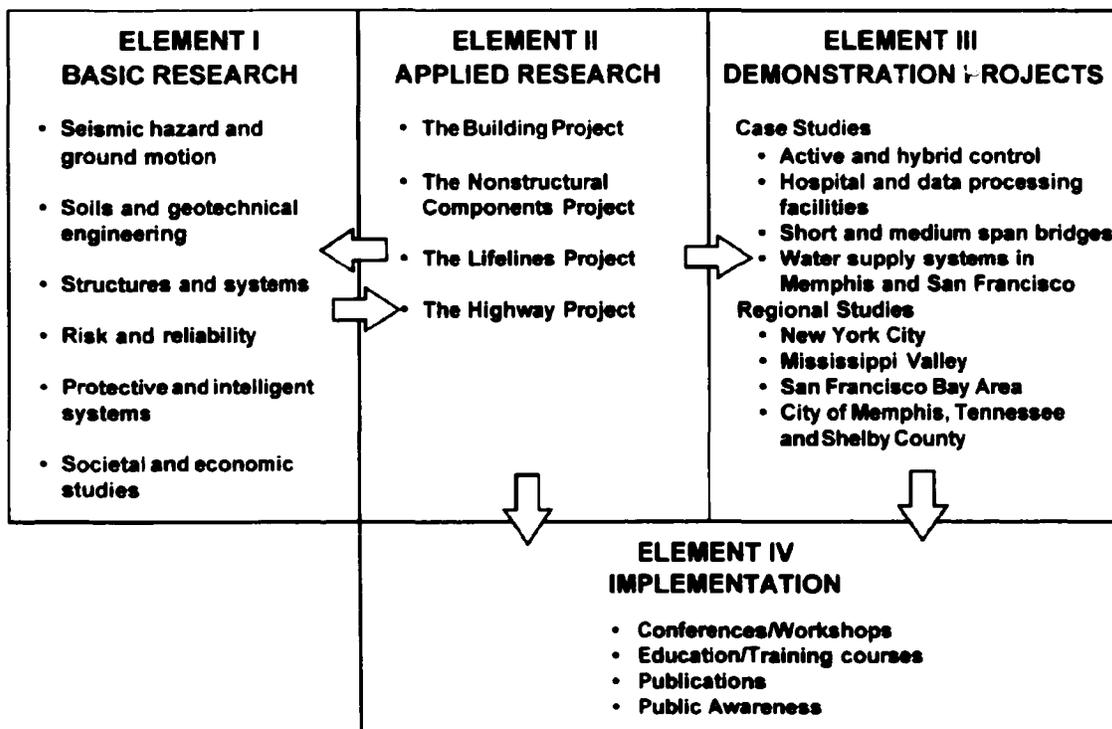
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## PREFACE

The National Center for Earthquake Engineering Research (NCEER) was established to expand and disseminate knowledge about earthquakes, improve earthquake-resistant design, and implement seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures in the eastern and central United States and lifelines throughout the country that are found in zones of low, moderate, and high seismicity.

NCEER's research and implementation plan in years six through ten (1991-1996) comprises four interlocked elements, as shown in the figure below. Element I, Basic Research, is carried out to support projects in the Applied Research area. Element II, Applied Research, is the major focus of work for years six through ten. Element III, Demonstration Projects, have been planned to support Applied Research projects, and will be either case studies or regional studies. Element IV, Implementation, will result from activity in the four Applied Research projects, and from Demonstration Projects.



Research in the **Building Project** focuses on the evaluation and retrofit of buildings in regions of moderate seismicity. Emphasis is on lightly reinforced concrete buildings, steel semi-rigid frames, and masonry walls or infills. The research involves small- and medium-scale shake table tests and full-scale component tests at several institutions. In a parallel effort, analytical models and computer programs are being developed to aid in the prediction of the response of these buildings to various types of ground motion.

## ABSTRACT

To avoid excessive damage under strong earthquakes, the peak response of civil engineering structures should be limited to an allowable level. To this end, active/hybrid control systems have been proposed and investigated for the protection of buildings. In this report, full-state optimal polynomial controllers and the corresponding static output polynomial controllers are proposed for limiting the peak response of linear and nonlinear civil engineering structures. Performance indices, that are quadratic in control and polynomial of an arbitrary order of both the linear or nonlinear states are considered. These performance indices are minimized based on the solution of the Hamilton-Jacobi-Bellman equation using a polynomial function of either linear or nonlinear states, which satisfies all the properties of a Lyapunov function. The resulting controllers are summations of polynomials of different orders of linear or nonlinear states, i.e., linear, cubic, quintic, etc. Gain matrices for different parts of the controllers are calculated easily by solving matrix Riccati and Lyapunov equations. Extensive simulation results indicate that the new optimal polynomial controllers consume less energy in reducing the peak response quantities; however, they may use bigger peak control force than the corresponding linear controllers. Because of the strong dependence on the structural response, the level of response reduction increases for the optimal polynomial controllers with respect to the earthquake intensity. Hence, if the earthquake intensity exceeds the design one, the optimal polynomial controllers are capable of exerting larger control forces thus achieving a higher reduction for the peak structural response. Such response adaptive properties are very desirable for the protection of the integrity of civil engineering structures, because of the inherent stochastic nature of the peak ground acceleration. The proposed optimal polynomial controllers, including the corresponding static output controllers, are viable control strategies, representing valuable additions to available control methods in the literature.

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## TABLE OF CONTENTS

SECTION	TITLE	PAGE
<b>1</b>	<b>INTRODUCTION</b>	1-1
<b>2</b>	<b>OPTIMAL POLYNOMIAL CONTROL FOR NONLINEAR STRUCTURES</b>	2-1
2.1	Problem Formulation and Main Results	2-1
2.1.1	Special Cases	2-3
2.2	Derivation of Optimal Polynomial Controller	2-4
2.3	Simulation Results	2-8
2.3.1	Control of a Scalar System	2-8
2.3.2	Control of a SDOF Structure	2-11
2.3.3	Control of a MDOF Structure	2-17
2.3.4	Summary	2-20
<b>3</b>	<b>OPTIMAL POLYNOMIAL CONTROL FOR NONLINEAR AND HYSTERETIC STRUCTURES</b>	3-1
3.1	Statement of Optimal Control	3-1
3.2	Derivation of Optimal Polynomial Controller	3-3
3.2.1	Constant Gain Matrices	3-7
3.3	Special Optimal Polynomial Controller	3-8
3.4	Response of Hysteretic Structures	3-10
3.5	Other Control Laws for Nonlinear or Hysteretic Structures	3-11
3.6	Simulation Results	3-12
3.6.1	A Base-Isolated Elasto-Plastic Building	3-13
3.6.2	An Elasto-Plastic Building with Active Bracing System	3-20
<b>4</b>	<b>STATIC OUTPUT POLYNOMIAL CONTROL FOR LINEAR AND NONLINEAR STRUCTURES</b>	4-1
4.1	Full-State Polynomial Controllers	4-1
4.1.1	Special Nonlinear Structures	4-1
4.1.2	Linear Structures	4-3
4.2	Derivation of Static Output Polynomial Controller for Linear Structures	4-4
4.3	Derivation of Static Output Polynomial Controller for Nonlinear Structures	4-7
4.4	Iterative Solutions for Static Output Controller	4-8
4.5	Numerical Results	4-9
4.5.1	Three-Story Linear Building	4-9
4.5.2	A Base-Isolated Elasto-Plastic Building	4-13
<b>5</b>	<b>CONCLUSIONS AND DISCUSSION</b>	5-1
<b>6</b>	<b>REFERENCES</b>	6-1

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## LIST OF ILLUSTRATIONS

<b>FIGURE</b>	<b>TITLE</b>	<b>PAGE</b>
2-1	Applied Control Force vs. Relative Displacement for a Scaler System	2-10
2-2	El Centro (NS Component) Earthquake, Scaled to 0.11g	2-10
2-3	Displacement of the Uncontrolled Structure	2-12
2-4	Displacement Based on Linear and Nonlinear Controllers	2-12
2-5	Control Force for Linear and Nonlinear Controllers	2-13
2-6	Control Energy Buildup for Linear and Nonlinear Controllers	2-13
2-7	Peak Displacement Reduction vs. Peak Control Force	2-16
2-8	Peak Displacement Reduction vs. Control Energy	2-16
2-9	Control Energy Buildup (Mexican Earthquake)	2-18
2-10	Peak Displacement Reduction vs. Peak Ground Acceleration for the First Floor of a Three-Story Building	2-18
2-11	Peak Displacement Reduction vs. Peak Ground Acceleration for the Second Floor of a Three-Story Building	2-21
2-12	Peak Displacement Reduction vs. Peak Ground Acceleration for the Third Floor of a Three-Story Building	2-21
2-13	Peak Control Force vs. Peak Ground Acceleration for a Three-Story Building	2-22
2-14	Required Control Energy vs. Peak Ground Acceleration for a Three-Story Building	2-22
3-1	A Base-Isolated Structural Model	3-14
3-2	El Centro Earthquake (NS Component)	3-14
3-3	Hysteresis Loop of Lead-Core Rubber Bearings	3-15
3-4	Time-Histories of Responses and Control Forces: (a) Drift of Rubber Bearings; (b) Control Forces	3-18
3-5	Peak Drift Reduction of Rubber Bearings vs. Peak Ground Acceleration	3-21
3-6	Normalized Peak Control Force vs. Peak Ground Acceleration	3-22
3-7	Normalized Control Energy vs. Peak Ground Acceleration	3-22
3-8	Peak Drift Reduction of Story Units vs. Peak Ground Acceleration; (a) First Story Unit; (b) Sixth Story Unit, and (c) Seventh Story Unit	3-26
3-9	Normalized Maximum Control Force and Control Energy vs. Peak Ground Acceleration; (a) Maximum Control Force, and (b) Control Energy	3-27
4-1	Percentage of Reduction for Peak Interstory Drifts	4-11
4-2	Normalized Peak Control Force and Required Control Energy	4-11
4-3	Response of Control Quantities for the Base-Isolated Building vs. Peak Ground Acceleration; (a) Peak Drift Reduction of Rubber Bearings; (b) Normalized Peak Control Force, and (c) Normalized Control Energy	4-16

## LIST OF TABLES

<b>TABLE</b>	<b>TITLE</b>	<b>PAGE</b>
2-1	Response Quantities for Linear and Cubic Controllers for 60% Reduction in Peak Response	2-14
2-2	Performance of Linear and Nonlinear Controllers with Different Earthquake Input	2-14
3-1	Peak Response Quantities of an Eight-Story Building Equipped with Hybrid Control System	3-19
3-2	Peak Response Quantities of Fixed-Base Eight-Story Building Under 1g El Centro Earthquake	3-19
4-1	Peak Response Quantities for a Three-Story Linear Building Model	4-12
4-2	Peak Response Quantities of an Eight-Story Building Equipped with Hybrid Control System	4-12

## SECTION I

### INTRODUCTION

Intensive research efforts have been made for active/hybrid control of civil engineering structures under severe wind gusts and strong earthquakes. Progress and literature review in the subject area can be found in, e.g., Soong(1990), Soong et al (1991), Housner and Masri (1994), etc. Recently, advanced control theories have been investigated for applications to seismic-excited structures (e.g., Dyke et al (1994), Jabbari et al (1995), Schmitendorf et al (1994), Kose et al (1996), Nagarajaiah et al (1993), Reinhorn et al (1993), Yang et al (1993; 1994b, c, d, e; 1995b, c, d; 1996a, b, c), etc). Under strong earthquakes, the main objective of active/hybrid control is to limit the peak response (e.g., interstory drifts) of the structure to minimize the damage. In this connection, it has been presented by Housner, Soong and Masri (1994) that nonlinear controllers (e.g. Wu et al (1994)) are more effective than the classical linear controllers in reducing the peak response of linear structures. Such evidences were also observed elsewhere ( e.g., Tomasula et al (1994), Agrawal and Yang (1995a), Yang and Agrawal (1995a)).

Control of linear structures by nonlinear controllers was first proposed by Rekasius, Z.V. (1964). He presented a suboptimal solution of the Hamilton-Jacobi-Bellman equation for a general nonquadratic cost function. Since then, many researchers have proposed methods to design different types of nonlinear controllers [e.g., Bass et al (1966), Speyer, J.L. (1976), Sandor et al (1977), Salehi et al (1982), Bernstein, D.S. (1993) etc.]. Their main objective was to derive nonlinear controllers that could respond fast to large peak responses while reacting slowly to small responses. In classical linear control theory, an optimal linear control law for linear structures is derived based on various assumptions, including linear state dynamics subjected to additive Gaussian white noise disturbances, completely accurate system model, quadratic performance index, etc. In practice, however, one or more of these assumptions may not be valid, e.g., the state dynamics may be nonlinear, disturbance may not be additive Gaussian white noise, system model may be inaccurate, etc. Moylan and Anderson (1973) have shown that for linear open-loop plants, the nonlinear optimal control law is more robust than the linear optimal control law for integral performance indices convex in the state. Speyer, J.L. (1976), through the derivation of a cubic controller for stochastic infinite time series

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problem, has shown that the probability of violating a state constraint for the linear optimal controller is much higher than that for the nonlinear controller.

The main objective of active/hybrid control for civil engineering structures is to reduce the peak response quantities of the structure. However, it is extremely difficult to obtain an optimal controller that minimizes the peak response of the structure. Recently, Wu et al (1994) and Tomasula et al (1994) have proposed nonlinear controllers for peak response reduction of seismic-excited linear structures. They have shown advantages of nonlinear controllers over the linear optimal controller for control of linear structures. The controller proposed by Wu et al (1994) is a special cubic order controller obtained by minimizing a nonquadratic performance index and is similar to the cubic controller derived by Speyer, J.L. (1976). Tomasula et al (1994) have proposed a polynomial controller using tensor expansion method for a SDOF structure for a performance index that is quadratic in control and quartic in the states. Agrawal and Yang (1995a, 1996) have proposed an optimal controller that is polynomial of any orders of the state for linear structures.

Aseismic hybrid protective systems, consisting of a combination of active control devices and passive base isolation systems, have been shown to be quite effective. Since the dynamic behavior of most base isolation systems, such as lead-core rubber bearings or frictional-type sliding bearings, is highly nonlinear or inelastic, hybrid protective systems involve control of nonlinear or hysteretic structural systems. Likewise, under strong earthquakes, yielding may occur even if the fixed-base building is equipped with active control systems. As a result, control of nonlinear or hysteretic civil engineering structures has attracted considerable attraction recently. Various control methods have been investigated, including pulse control (Reinhorn et al 1987), polynomial control (Spencer et al 1992), acceleration control (Nagarajaiah et al 1993; Reinhorn et al 1993; Riley et al 1993), instantaneous optimal control (Yang et al 1992a), dynamic linearization (Yang et al 1994b; Reinhorn et al 1993), nonlinear control (Yang et al 1992b; 1994a; Dixon et al 1995), neural network (Krishnan et al 1995), sliding mode control (Yang et al 1993; 1994b, c, d, e; 1995b, c, d; 1996a, b, c), etc. It has been shown by Yang et al (1992b, 1994a) that a controller, having the same nonlinear characteristics as that of the structure, performs better than a linear controller. In fact, the sliding mode controller also has such characteristics (Yang et al 1994c, 1995b). More recently, an optimal controller, that is a polynomial of any order of nonlinear states, has been proposed by Yang & Agrawal (1995a, 1996d) for control of nonlinear and hysteretic structures.

For practical implementations of active/hybrid control systems in complex civil engineering structures, it may not be possible to install sensors at all degrees of freedom to measure the full-state vector. An observer, however, may require excessive on-line computations due to a large number of degrees of freedom involved, thus resulting in a system time-delay. As a result, static output feedback control methods, which utilize only the information measured from a limited number of sensors without an observer, are highly desirable for practical implementations of control systems. Recently static output polynomial controllers, which are the extension of the optimal polynomial controllers proposed previously for control of linear and nonlinear structures have been proposed by Agrawal & Yang (1995b). Such static output controllers have been shown to be plausible.

In section II, we present a class of optimal polynomial controllers of various orders for control of linear structures. The performance index to be minimized is quadratic in control and is polynomial of an arbitrary order of the states. This specific polynomial performance index belongs to a general class for which an exact optimal solution can be determined easily. Based on the optimality conditions derived by Bernstein, D.S. (1993) for nonlinear optimal control problems, the performance index is minimized and the general polynomial control law is obtained analytically. The gain matrices for different parts of the controller are calculated easily from matrix Riccati and Lyapunov equations. Our optimal polynomial controller reduces to the controller presented by Wu et al (1994) for a specific choice of weighting matrices. Numerical simulations have been conducted for both the SDOF and MDOF systems to investigate the performance of the optimal polynomial controller with respect to various control objectives, including the peak response reduction, peak control force and required control energy.

In section III, we present a class of optimal polynomial controllers for the peak response reduction of seismically excited nonlinear or hysteretic structures. The performance index to be minimized is quadratic in control and polynomial of any order in nonlinear states. The performance index is minimized using the Hamilton-Jacobi-Bellman equation, and the resulting optimal control law is a summation of polynomials of different orders in nonlinear states, i.e., linear, cubic, quintic, etc. Gain matrices for different parts of the controller are computed easily from Riccati and Lyapunov matrix equations. Numerical simulations have been conducted for control of a base-isolated building using lead-core rubber bearings and a fixed-base yielded building to investigate the performance of the optimal polynomial controller with respect to various control objectives, including the peak response

## SECTION II

### OPTIMAL POLYNOMIAL CONTROL FOR LINEAR STRUCTURES

In this section, we derive analytically an optimal polynomial controller of various orders for linear structures, in which the gain matrices are computed easily from matrix Riccati and Lyapunov equations. The performance and advantages of such an optimal controller are demonstrated by the simulation results for SDOF and MDOF systems.

#### 2.1 PROBLEM FORMULATION AND MAIN RESULTS

Consider an  $n$  degree-of-freedom linear building structure subjected to a one-dimensional earthquake ground acceleration  $\ddot{x}_0(t)$ . The vector equation of motion is given by

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = \bar{H}U(t) + \eta\ddot{x}_0(t) \quad (2.1)$$

in which  $X(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$  is an  $n$  vector with  $x_i(t)$  being either the interstory drift or the displacement of the  $i$ th floor with respect to the ground;  $U(t)$  is a  $r$  vector consisting of  $r$  control forces; and  $\eta$  is an  $n$  vector denoting the influence of the earthquake excitation. In Eq.(2.1),  $M$ ,  $C$  and  $K$  are  $(n \times n)$  mass, damping and stiffness matrices, respectively, and  $\bar{H}$  is a  $(n \times r)$  matrix denoting the location of  $r$  controllers. In the state space, Eq.(2.1) becomes,

$$\dot{Z}(t) = AZ(t) + BU(t) + E(t) \quad (2.2)$$

where  $Z(t)$  is a  $2n$  state vector,  $A$  is a  $(2n \times 2n)$  system matrix,  $B$  is  $(2n \times r)$  controller location matrix, and  $E(t)$  is a  $2n$  excitation vector, respectively, given by

$$Z(t) = \begin{bmatrix} X(t) \\ \dot{X}(t) \end{bmatrix}; A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; B = \begin{bmatrix} 0 \\ M^{-1}\bar{H} \end{bmatrix}; E(t) = \begin{bmatrix} 0 \\ M^{-1}\eta\ddot{x}_0(t) \end{bmatrix} \quad (2.3)$$

A general performance index  $J$  can be expressed as follows

$$J = J(Z_0, U(t), t_0) = S(Z_T, T) + \int_{t_0}^T L[Z(t), U(t), t] dt \quad (2.4)$$

where  $Z_T = Z(T)$  is the terminal state,  $S(Z_T, T)$  is the terminal cost and  $L[Z(t), U(t), t]$  is a nonquadratic non-negative cost function. For infinite time regulator problem, the performance index

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (2.8)$$

$$M_i(A - BR^{-1}B^T P) + (A - BR^{-1}B^T P)^T M_i + Q_i = 0, \quad i = 2, 3, \dots, k \quad (2.9)$$

As can be seen from Eq.(2.7), the nonlinear part of the controller is the sum of polynomials of various orders in terms of the states of the system. Matrices P and  $M_i$ 's in Eqs.(2.8) and (2.9) can be solved using any well-known numerical algorithms or using functions available in MATLAB.

Another optimal polynomial controller has also been derived and the result is identical to the form of Eq.(2.7), except that  $M_i$  ( $i=2,3,\dots,k$ ) are determined from the matrix Riccati equation, i.e.,

$$M_i(A - BR^{-1}B^T P) + (A - BR^{-1}B^T P)^T M_i - M_i BR^{-1}B^T M_i + Q_i = 0, \quad i = 2, 3, \dots, k \quad (2.10)$$

In this case, the performance index J used to be minimized is given by Eq.(2.5) with

$$\bar{h}(Z) = \bar{h}_2(Z) = \bar{h}_1(Z) - \sum_{i=2}^k (Z^T M_i Z)^{i-1} Z^T M_i BR^{-1}B^T M_i Z \quad (2.11)$$

where  $\bar{h}_1(Z)$  is given by Eq.(2.6).

### **2.1.1 SPECIAL CASES**

The optimal polynomial controller derived in Eq.(2.7) reduces to different special cases in the following.

(i) **Linear Controller:** For  $k=0$  or  $Q_i = 0, i = 2, 3, \dots, k$ , the controller presented in Eq.(2.7) becomes,

$$U(t) = -R^{-1}B^T PZ \quad (2.12)$$

which is the well-known result of linear quadratic optimal control (LQR).

(ii) **Special Cubic Controller:** For a special case in which  $k=2$  and  $Q_2$  is chosen to be  $Q_2 = Q + PBR^{-1}B^T P$ , the solution of Eq.(2.9) yields  $M_2 = P$ . Hence, the special cubic controller becomes

$$U(t) = -R^{-1}B^T(1 + Z^T PZ)PZ \quad (2.13)$$

This controller is precisely the one presented by Wu et al (1994) recently. Hence, their controller is a special case of our general polynomial controller.

(iii) **Scalar Control:** For control of a scalar system with  $Z(t)=z(t)$  being a scalar,

$$\dot{z}(t) = az(t) + bu(t) \quad (2.14)$$

the nonlinear controller for this system is given by,

in which  $V=V(Z)$  is the optimal cost function, and  $H(Z, U, V', t)$  is the Hamiltonian function defined as

$$H(Z, U, V', t) = L(Z, U, t) + [V']^T f(Z, U, t) \quad (2.20)$$

where a prime indicates the differentiation with respect to  $Z$ , i.e.,  $V' = \partial V / \partial Z =$  a  $2n$  vector. The necessary condition for the minimization of the right hand side of Eq.(2.19) is,

$$\frac{\partial H(Z, U, V', t)}{\partial U} = \frac{\partial L(Z, U, t)}{\partial U} + \frac{\partial f(Z, U, t)}{\partial U} V' = 0 \quad (2.21)$$

in which Eq.(2.20) has been used. The solution,  $U(t)=\phi(Z)$ , of Eq.(21) will be the minimum control if the second derivative of  $H$  is non-negative, i.e.,  $\partial^2 H(Z, \phi(Z), V', t) / \partial U^2 \geq 0$ . Then, the terminal condition for the optimal cost function will be the same as that of the performance index in Eq.(2.4), i.e.,  $V(Z_T, T) = S(Z_T, T)$ . For the optimal control  $U(t)=\phi(Z)$ , the H-J-B equation in Eq.(2.19) can be written as

$$\frac{\partial V}{\partial t} + H(Z, \phi(Z), V', t) = 0 \quad (2.22)$$

The following theorem derived in Bernstein (1993) will be used.

**Theorem:** For an optimal cost function  $V(Z)$  that satisfies all the properties of a Lyapunov function, if there exists a minimum control  $U=\phi(Z)$  which satisfies Eqs.(2.19) to (2.22), then the closed-loop system is asymptotically stable and the minimum value of the performance index in Eq.(2.4) is given by  $J(Z_0, \phi(Z), t_0) = V(Z_0)$ . Furthermore, the optimal feedback control  $U=\phi(Z)$  minimizes  $J(Z_0, U, t_0)$  in Eq.(2.4) in the sense that,  $J(Z_0, \phi(Z), t_0) = \min_{U \in \Omega} [J(Z_0, U, t_0)]$ . The asymptotic stability of the closed-loop system is automatically guaranteed through the Lyapunov theorem of stability, i.e.,  $\dot{V}(Z) \leq 0$ .

Based on the theorem above, an optimal polynomial controller can be derived in the following manner. A comparison of the system dynamics in Eq.(2.18) with that in Eq.(2.2) leads to

$$f(Z, U, t) = AZ + BU \quad (2.23)$$

in which the external excitation has been neglected.

We choose the nonquadratic cost function  $L(Z, U)$  and a Lyapunov function  $V(Z)$  as follows

$$L(Z, U) = Z^T QZ + U^T R U + h(Z) \quad (2.24)$$

$$g(Z) = \sum_{i=2}^k \frac{1}{i} (Z^T M_i Z)^i \quad (2.33)$$

such that

$$g'(Z) = 2 \sum_{i=2}^k (Z^T M_i Z)^{i-1} M_i Z \quad (2.34)$$

where  $k$  is any integer greater than 2 indicating the order of the multinomial  $g(Z)$ , and  $M_i$ 's are positive-definite matrices. It should be noted that Eq.(2.25) with  $g(Z)$  in Eq.(2.33) satisfies all the properties of a Lyapunov function [Anderson and Moore (1990)]. Substituting Eq.(2.34) into Eq.(2.28), we obtain the optimal polynomial controller as,

$$U = -R^{-1} B^T P Z - R^{-1} B^T \sum_{i=2}^k (Z^T M_i Z)^{i-1} M_i Z \quad (2.35)$$

Note that for any value of  $k$  greater than 2, the maximum order of the controller in terms of  $Z$  is  $(2k+1)$ . With  $g(Z)$  and  $g'(Z)$  given by Eqs.(2.33) and (2.34), respectively, Eq.(2.31) can be written as,

$$\begin{aligned} -\sum_{i=2}^k (Z^T M_i Z)^{i-1} Z^T \dot{M}_i Z &= \sum_{i=2}^k (Z^T M_i Z)^{i-1} Z^T [M_i (A - BR^{-1} B^T P) + (A - BR^{-1} B^T P)^T M_i] Z \\ &+ h(Z) - \left[ \sum_{i=2}^k (Z^T M_i Z)^{i-1} Z^T M_i \right] BR^{-1} B^T \left[ \sum_{i=2}^k (Z^T M_i Z)^{i-1} M_i Z \right] \end{aligned} \quad (2.36)$$

Now, let us choose  $h(Z)$  as follows

$$h(Z) = \sum_{i=2}^k (Z^T M_i Z)^{i-1} (Z^T Q_i Z) + \bar{h}(Z) \quad (2.37)$$

in which  $\bar{h}(Z) = \bar{h}_1(Z)$  is given by Eq.(2.6). Substituting Eq.(2.37) into Eq.(2.36), we obtain  $M_i$  ( $i=2,3,\dots,k$ ) as follows

$$-\dot{M}_i = M_i (A - BR^{-1} B^T P) + (A - BR^{-1} B^T P)^T M_i + Q_i, \quad i = 2,3,\dots,k \quad (2.38)$$

Equation (2.38) is the well-known matrix Lyapunov equation. For time invariant system with constant matrices  $A$  and  $B$  such that  $\dot{M}_i \rightarrow 0$  as  $t \rightarrow \infty$ ,  $M_i$  can be determined from the algebraic Lyapunov equation

$$M_i (A - BR^{-1} B^T P) + (A - BR^{-1} B^T P)^T M_i + Q_i = 0, \quad i = 2,3,\dots,k \quad (2.39)$$

(2.16), which minimizes the performance index  $J$  given in Eq.(2.17). It should be mentioned that the optimal controller for the scalar system given by Eq.(2.14) can be derived analytically for a general class of nonlinear performance index in the steady-state form given by

$$J = \int_0^{\infty} [Z^T Q Z + U^T R U + h(Z)] dt \quad (2.41)$$

in which  $h(Z)$  can be any general nonquadratic part of the performance index. For this general case, one can solve  $g'(Z)$  from Eq.(2.31) for the steady-state condition  $\partial g(Z)/\partial t = 0$ . Then, a substitution of the resulting  $g'(Z)$  into the optimal controller given by Eq.(2.28) leads to the exact solution

$$u_c(t) = -\frac{a}{b}z - \frac{1}{b} \sqrt{(a^2 + \frac{qb^2}{\phi}) + \frac{b^2 h(z)}{\phi}} \quad (2.42)$$

where  $u_c(t)$  is the unique optimal solution. Note that such a unique optimal solution is possible only for the scalar system. For other systems with a general class of function  $h(Z)$ , it is not possible to obtain the optimal solution analytically. Furthermore, the solution for higher order systems is not unique; namely, there are multiple solutions.

Recall that, in the controllers given by Eqs.(2.15) and (2.42),  $\phi$  is the scalar form of the control weighting matrix  $R$ , and  $q$  and  $q_i$  ( $i=2,3,\dots,k$ ) are scalar forms of the weighting matrices  $Q$  and  $Q_i$  ( $i=2,3,\dots,k$ ). For illustrative purpose, we consider a control system with  $a=-0.025$ ,  $b=1$ ,  $\phi=1$  and  $q=q_i=1.5$ ,  $i=2,3,\dots,k$ . Plots of the control force  $u(t)$  versus relative displacement  $z(t)$  given by Eqs.(2.15) and (2.16) for controllers of order one to eleven are shown in Fig. 2-1. A controller of the  $i$ th order consists of all the odd order controllers from 1 to  $i$ . Although Eq.(2.42) is an irrational polynomial in  $z(t)$ , it is found that plots of polynomial controllers in Eq.(2.15) exactly coincide with those of the unique optimal controller  $u_c(t)$  in Eq.(2.42) for the case where  $h(Z)$  is given by Eq.(2.37). It is observed from Fig. 2-1 that all the nonlinear controllers behave like linear controllers for small displacement  $z$ . However, as the displacement  $z$  increases, the control force,  $u(t)$ , increases rapidly for all the nonlinear controllers. Polynomial controllers presented in this paper are exact solutions that minimize a class of performance index in Eq.(2.5).

### **2.3.2 CONTROL OF A SINGLE-DEGREE-OF-FREEDOM (SDOF) STRUCTURE**

A SDOF structure equipped with an active tendon control system is used for the investigation of various characteristics of optimal polynomial controllers. Structural parameters of the SDOF system are: mass  $m=2.942$  metric tons; natural frequency = 4.1 Hz; and damping ratio= 2.62%. Active tendons with stiffness = 385.3 kN and angle of inclination =  $36^\circ$  are used. The N-S component of 1940 El Centro earthquake with a peak ground acceleration (PGA) of 116.68 gal is used for the input excitation, where the time axis has been scaled down by 1/2 as shown in Fig. 2-2. The peak displacement and peak acceleration of the uncontrolled structure are, respectively, 0.475 cm and  $314 \text{ cm/sec}^2$ . The time-history of the displacement of the uncontrolled structure is shown in Fig. 2-3.

The peak response quantities are relevant to the safety of the structure, whereas the peak control force is a measure of the capacity of the actuator required. In addition, the mean square (MS) control force  $\overline{U^2}$ , that is directly related to the total required energy of the actuator during the earthquake episode, is of practical importance, i.e.,

$$\overline{U^2} = \int_0^{T_f} U^T U dt \quad (2.43)$$

in which  $T_f$  is the duration when the control force is required. For civil engineering applications, accumulators may be needed to provide the control energy for the actuators as the stand-by system, since power outage may occur during the earthquake episode.

**(i) Same Level of Response Reduction:** For the problem considered, the weighting matrix  $R$  for the controller consists of only one element, denoted by  $R_0$ , whereas the dimension of the response weighting matrices  $Q$  and  $Q_i$  ( $i=1,2,\dots,k$ ) is  $2 \times 2$ . Suppose the objective of control is to achieve a 60% reduction for the peak displacement. Based on the linear controller, i.e.,  $Q_i$  ( $i=2,3,\dots,k$ ) are null matrices, this objective can be met by using  $R_0 = 2269.81$ ,  $Q(1,1)= 1952.46$ , and all other elements of  $Q$  are zero. To obtain the same level of the response reduction by a cubic controller, we use  $R_0 = 4380505$ ,  $Q(1,1)=1952.46$  and  $Q_2(1,1)= 9762$ , and all other elements of  $Q$  and  $Q_2$  are zero. Various response quantities for the two controllers above are shown in Table 2-1. Time-histories for the displacement and control force for these two cases are shown in Figs. 2-4 and 2-5. It is observed from the Table 2-1 and Figs. 2-4 to 2-5 that, while the peak control force for the cubic controller

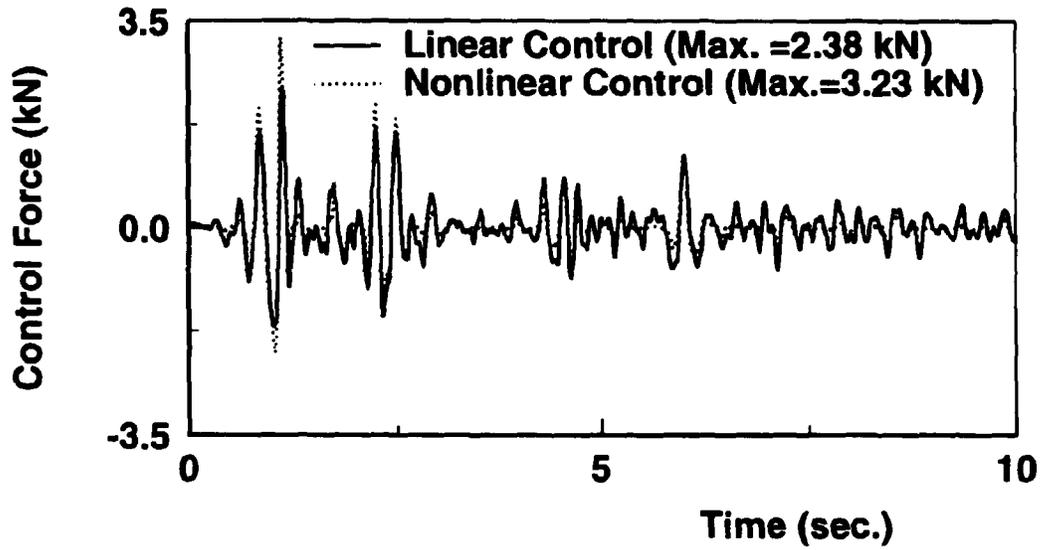


Fig. 2-5: Control Force for Linear and Nonlinear Controllers

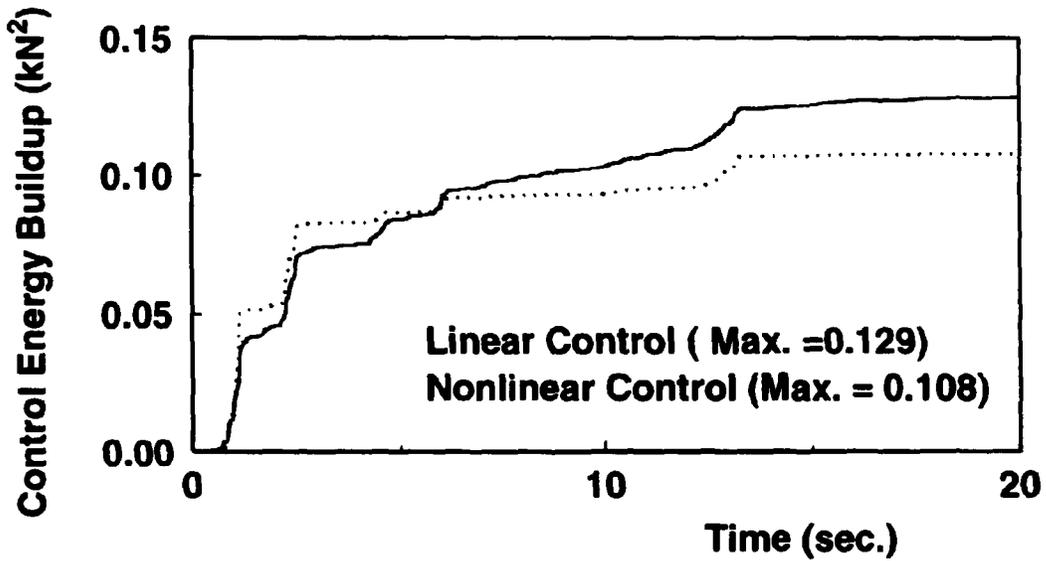


Fig.2-6: Control Energy Buildup for Linear and Nonlinear Controllers

(3.228 kN) with respect to linear controller (2.383 kN) increases by 35.4%, the mean square (MS) control force, which is related to the total control energy, decreases by 16.1%. These results remained unchanged when the peak ground acceleration of the earthquake was increased to 500 gal. The control energy in case of the polynomial controller decreases because the control force from the nonlinear controller at instants other than the peak response is smaller than that of the linear controller as shown in Fig. 2-5. This is reflected in the increase of RMS displacement and acceleration of the structure as shown in Table 2-1. Fig. 2-6 shows the cumulative energy build-up during the earthquake episode for these two controllers. The total energy buildup for the cubic controller is much smaller than that of the linear controller. It is worthwhile to mention that electrohydraulic actuators usually have high control force capacity, whereas stand-by accumulators for a large control energy requirement may pose some practical problems.

To investigate the performance of linear and polynomial controllers for various levels of displacement reductions, numerical simulations were conducted for: (i) linear controller with  $Q(1,1)=1952.5$ , (ii) cubic controller I with  $Q(1,1)=Q_2(1,1)=1952.5$  (iii) cubic controller II with  $Q(1,1)=1952.5$ ,  $Q_2(1,1)=5Q(1,1)$ , and (iv) quintic controller with  $Q(1,1)=Q_2(1,1)=Q_3(1,1)=1952.5$ , by varying the control weighting element  $R_0$ . All other elements of  $Q$ ,  $Q_2$  and  $Q_3$  matrices in above cases were zero. The peak response reduction in % vs. the peak control force and mean square control force are shown in Figs. 2-7 and 2-8. It is observed from Fig. 2-7 that, for peak displacement reductions below 80%, the peak control force required by nonlinear controllers is always higher than that required by the linear controller. The peak control forces required by different nonlinear controllers varies in a nonlinear fashion, because the closed-loop system is no longer linear. The peak control force increases as the order of the polynomial controller is increased. However, for the peak reductions greater than 80%, the peak control forces for all the four cases in Fig. 2-7 coincide, because all nonlinear controllers at small displacements behave like the linear controller. It is observed from Fig. 2-8 that the required control energy for all the nonlinear controllers is smaller than that of the linear controller. It is further observed from Fig. 2-8 that the cubic controller with  $Q_2 = Q$  yields the best performance in terms of the required control energy. This case is even better than the quintic controller, and hence, it may be sufficient to use cubic controllers.

The results presented above were obtained using the El Centro earthquake. Since, earthquakes are stochastic in nature, the effect of random earthquake ground motions on the performance of nonlinear controllers will be investigated. Simulations results using six different earthquakes are shown in Table 2-2. In Table 2-2, the peak ground acceleration (PGA) of each earthquake input is shown in Column(2); the percentage of reduction for the peak displacement is shown in Columns (3) and (6); the peak control force is shown in Columns (4) and (7); and the mean square control force is shown in Columns (5) and (8). The linear controller and the cubic controller II described above were used, and the results are designated as "Linear Controller" and "Nonlinear Controller", respectively. In Table 2-2, the level of reduction for the peak displacement is kept to be the same for both linear and nonlinear controllers. Columns (9) and (10) of Table 2-2 are comparisons for the results of the nonlinear controller in columns (6)-(8) with respect to that of the linear controller in columns (3)-(5). It is observed from Table 2-2 that, although the peak control force for the nonlinear controller is higher than that of the linear controller, the required control energy for the nonlinear controller is smaller. Of particular interest are the results using Mexico N90W earthquake for a 65.3 % peak displacement reduction (last row of Table 2-2). In this case, the peak control force for the two controllers is about the same, but the required control energy in the case of nonlinear controller decreases by 43.6 %. However, as the level of the response reduction increases, the difference between the control energy required by the two controllers decreases, because the nonlinear controller starts to behave like a linear controller. Cumulative build-up of the required control energy based on the Mexico N90W earthquake excitation is shown in Fig. 2-9.

### **2.3.3 CONTROL OF A MULTI-DEGREE-OF-FREEDOM (MDOF) STRUCTURE**

The same three-story building equipped with an active tendon control system on the first floor, investigated by Wu et al (1994), is considered herein. The properties of the building are: mass of each floor = 981 kg; natural frequencies of three modes = 2.34, 7.42 and 12.30 Hz; and the corresponding damping ratios  $\zeta$  (%) = 1.28, 0.54 and 0.42, respectively. Other relevant structural properties of the model can be found in Wu et al (1994). Active tendons with stiffness = 411.55 kN/m have been installed at an inclination of 36° from the horizontal floor. Numerical simulations have been conducted using the El Centro NS (1940) earthquake considered previously.

A linear optimal controller has been designed for a PGA of 300 gal to achieve approximately 66 %, 62 % and 64 % reductions, respectively, for the peak displacements of the first, second and third floors with respect to the ground. This is achieved using  $R_0=2$  and  $Q(1,1)=3.0$ , and all other elements of  $Q$  are zero. To obtain a similar level of response reductions, two optimal polynomial controllers of cubic order have been designed using: (i)  $R_0=2$ ,  $Q(1,1)=1.7$ ,  $Q_2(1,1)=7.0$ , and other elements of  $Q$  and  $Q_2$  are zero; and (ii)  $R_0=3$ ,  $Q(1,1)=4.0$ ,  $Q(2,2)=0.01$ ,  $Q(3,3)=0.001$ ,  $Q_2=Q$ , and all other elements of  $Q$  and  $Q_2$  are zero. These two controllers are designated as “Nonlinear 1” and “Nonlinear 2”, respectively. In a similar manner, the special cubic controller proposed by Wu et al (1994) has been designed using:  $R_0=2$ ,  $Q(1,1)=2.45$ , and all other elements of  $Q$  are zero. Note that for the special cubic controller presented by Wu et al (1994),  $Q_2$  is related to  $Q$  through  $Q_2 = Q + PBR^{-1}B^TP$ . As a result, the cubic controller presented in this paper is more flexible, since  $Q_2$  can be chosen arbitrary to achieve various objectives.

Numerical simulations have been carried out by varying the peak ground acceleration (PGA) of the El Centro earthquake from 100 to 600 gals to examine the performance of linear and three cubic controllers for a wide range of earthquake intensities. Figs. 2-10 to 2-12 present the reduction (%) for the peak displacement of the first, second and third floors, respectively, as a function of PGA. As expected, the peak displacement reduction by the optimal linear controller remains constant for different PGA. On the other hand, the peak response reduction by the three nonlinear controllers increases with the increase of PGA. As mentioned previously, the peak response reduction at the design PGA (i.e., 300 gal) is about the same for all controllers. Figs. 2-10 to 2-12 indicate that (i) for PGA smaller than the design one, i.e.,  $PGA < 300$  gal, the peak response reduction for cubic controllers is smaller than that of the linear controller, and (ii) for the PGA greater than the design one, i.e.,  $PGA > 300$  gal, the peak response reduction for the cubic controllers is higher than that of the linear controller. The latter behavior is very desirable, since a larger reduction for the peak response is needed when the actual earthquake intensity exceeds the design one. Such a response adaptivity to stochastic earthquakes is very beneficial for practical implementations of the control system.

Figs.2-13 and 2-14 show the corresponding normalized peak control force and normalized control energies vs. PGA, respectively. Peak control forces and control energies for all the controllers have been normalized by the corresponding quantities required by the linear optimal controller at a

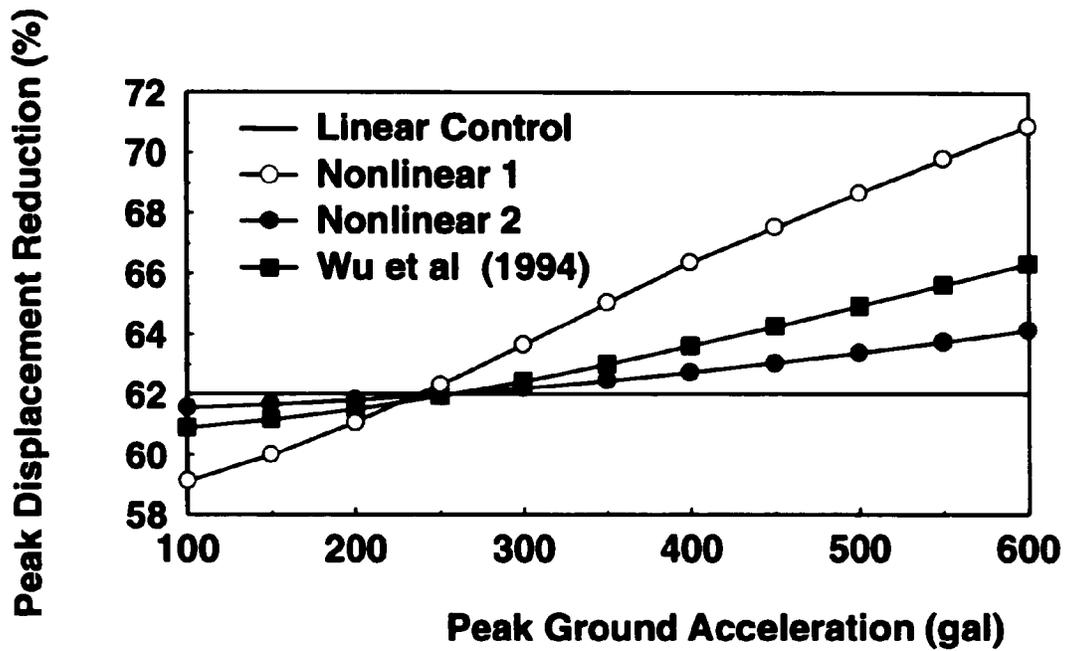


Fig. 2-11: Peak Displacement Reduction vs. Peak Ground Acceleration for the Second Floor of a 3-Story Building

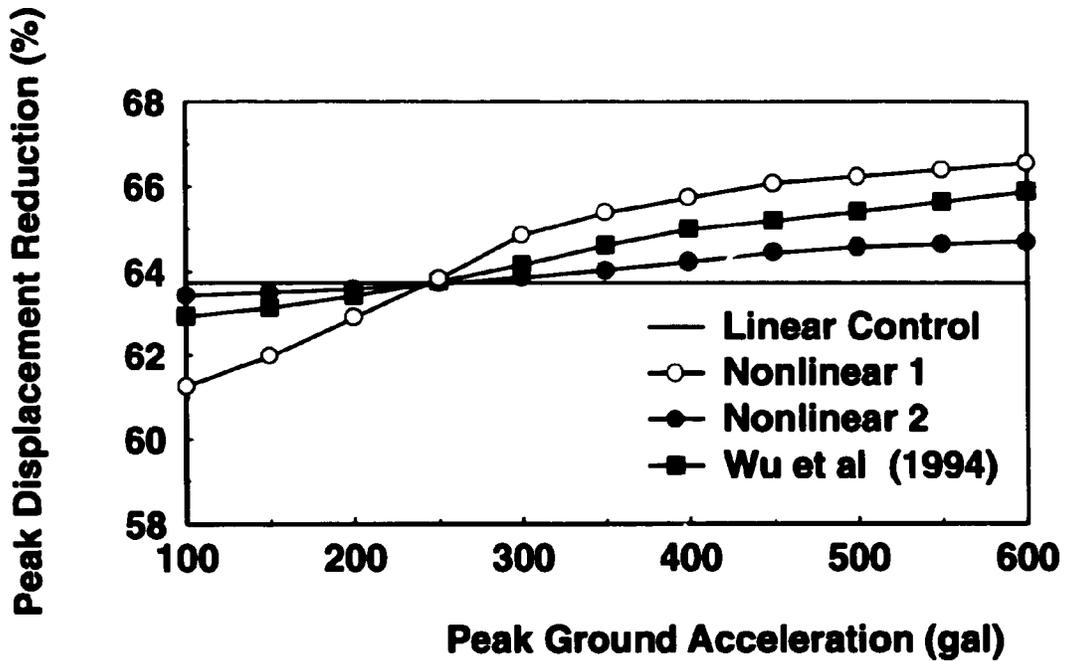


Fig. 2-12: Peak Displacement Reduction vs. Peak Ground Acceleration for the Third Floor of a 3-Story Building

## SECTION III

### OPTIMAL POLYNOMIAL CONTROL FOR NONLINEAR AND HYSTERETIC STRUCTURES

In this section, we derive analytically an optimal polynomial controller for seismically excited nonlinear or hysteretic structures. A performance index that is quadratic in control and polynomial of any order in nonlinear states is minimized based on the Hamilton-Jacobi-Bellman equation. Gain matrices for different parts of the controller are computed easily from Riccati and Lyapunov matrix equations. For the special case in which the damping and stiffness of the structure can be separated into linear and nonlinear parts, a special optimal polynomial controller is also derived. Such an optimal controller reduces to the one derived in Section II, when the structure becomes linear, i.e., the nonlinear parts of damping and stiffness are zero. The performance of the optimal polynomial controller derived in this section is demonstrated by simulation results for a base-isolated building and a bilinear elasto-plastic fixed-base building with a large ductility.

#### 3.1 STATEMENT OF OPTIMAL CONTROL

Consider an  $n$  degree-of-freedom nonlinear building structure subjected to a one-dimensional earthquake ground acceleration  $\ddot{x}_0(t)$ . The vector equation of motion is given by

$$M\ddot{X}(t) + F_c[\dot{X}(t)] + F_s[X(t)] = HU(t) + \eta\ddot{x}_0(t) \quad (3.1)$$

in which  $X(t) = [x_1, x_2, \dots, x_n]^T$  is an  $n$  vector with  $x_i(t)$  being the drift of a designated  $i$ th story unit;  $U(t) = [u_1, u_2, \dots, u_r]^T$  is a  $r$ -vector consisting of  $r$  control forces; superscript  $T$  denotes the transpose of a vector or a matrix; and  $\eta$  is an  $n$ -vector denoting the influence of the earthquake excitation. In Eq.(3.1),  $M$  is a  $(n \times n)$  mass matrix;  $H$  is a  $(n \times r)$  matrix denoting the location of  $r$  controllers;  $F_c[\dot{X}(t)] = F_c$  is an  $n$ -vector denoting the nonlinear damping force; and  $F_s[X(t)] = F_s$  is an  $n$ -vector denoting the nonlinear stiffness which is assumed to be a function of  $X(t)$ . In the state space, Eq.(3.1) becomes

$$\dot{Z}(t) = q(Z(t)) + BU(t) + E(t) \quad (3.2)$$

functions of the weighting matrices  $Q_i$ ,  $i=2,3,\dots, k$ . The relation between  $M_i$  and  $Q_i$  will be defined later.

Generally, a factor of 1/2 should be multiplied to the performance index in Eq.(3.4). However, the optimal solution ( or control law) does not depend on any constant factor multiplied to the performance index. For simplicity of presentation, the constant factor of 1/2 has been dropped.

### **3.2 DERIVATION OF OPTIMAL POLYNOMIAL CONTROLLER**

The penalty for  $\ddot{X}_a(t)$  has been included in the performance index, Eq.(3.4), in order to reduce the absolute acceleration of each floor to an acceptable level. From the equation of motion, Eq.(3.1), the absolute acceleration vector  $\ddot{X}_a(t)$  can be expressed as,

$$\ddot{X}_a(t) = -LM^{-1}[F_c(\dot{X}) + F_s(X)] + LM^{-1}HU(t) \quad (3.7)$$

in which  $L$  is a  $(n \times n)$  transformation matrix. For a shear-beam type building,  $L(i,j) = 1$  for  $j \leq i$  and  $L(i,j) = 0$  for  $j > i$ . Substituting Eq.(3.7) into Eq.(3.4), one obtains a transformed performance index as follows (Yang et al 1992b)

$$J = \int_0^{\infty} \left[ q^T \bar{Q} q + \bar{U}^T \bar{R} \bar{U} + \sum_{i=2}^k (q^T M_i q)^{i-1} q^T Q_i q + \bar{h}(q) \right] dt \quad (3.8)$$

where  $\bar{R}$ ,  $\bar{Q}$  and  $\bar{U}$  are

$$\bar{T}_a = \begin{bmatrix} 0 & 0 \\ 0 & L^T Q_a L \end{bmatrix}; \quad \bar{R} = R + B^T \bar{T}_a B; \quad \bar{Q} = Q + \bar{T}_a - \bar{T}_a B \bar{R}^{-1} B^T \bar{T}_a \quad (3.9)$$

$$\bar{U} = U + \bar{R}^{-1} B^T \bar{T}_a q(Z) \quad (3.10)$$

Substituting Eq.(3.10) into Eq.(3.2), one obtains the transformed state equation as,

$$\dot{Z} = \bar{A}q + B\bar{U} + E(t) \quad (3.11)$$

where

$$\bar{A} = [I - B\bar{R}^{-1}B^T\bar{T}_a] \quad (3.12)$$

The minimization of the performance index in Eq.(3.8) by classical conditions of optimality is very difficult and hence an alternative approach has been developed. This approach is based on the solution of the Hamilton-Jacobi-Bellman (H-J-B) equation (Anderson and Moore 1990) using a

sense that  $J(Z_0, \phi(Z), t_0) = \min_{\bar{U} \in \Omega} [J(Z_0, \bar{U}, t_0)]$ . The asymptotic stability of the closed-loop system is guaranteed through the Lyapunov theorem of stability, i.e.,  $\dot{V}(Z) \leq 0$ .

A comparison of the state equation in Eq.(3.11) with the general state equation in Eq.(3.13) leads to

$$f(Z, \bar{U}, t) = \bar{A}q(Z) + B\bar{U}(t) \quad (3.19)$$

Now, we consider a cost function  $L(Z, \bar{U})$  and a Lyapunov function  $V(Z)$  as follows

$$L(Z, \bar{U}) = q^T \bar{Q} q + \bar{U}^T \bar{R} \bar{U} + h(q) \quad (3.20)$$

$$V(Z) = q^T P q + g(q) \quad (3.21)$$

where  $g(q)$  is some positive definite multinomial of  $q$ . Our aim is to determine the nonquadratic cost function,  $h(q)$ , such that simple analytical solution for the optimal control law  $\bar{U}$  can be derived. Substituting Eqs.(3.19) - (3.21) into Eq.(3.16), one obtains the Hamiltonian function

$$H(q, \bar{U}, V', t) = q^T \bar{Q} q + \bar{U}^T \bar{R} \bar{U} + h(q) + [2q^T P \Lambda + g'(q)^T] (\bar{A}q + B\bar{U}) \quad (3.22)$$

in which the derivative matrix  $\Lambda = \Lambda(Z)$  is given by Eq.(3.6). Substitution of Eq. (3.22) into the necessary condition in Eq.(3.17) leads to

$$2\bar{R} \bar{U} + 2B^T \Lambda^T P q + B^T g'(q)^T = 0 \quad (3.23)$$

From Eq.(3.23), one obtains the optimal nonlinear controller,  $\bar{U}(t)$ , as

$$\bar{U}(t) = -\bar{R}^{-1} B^T \Lambda^T P q(Z) - \frac{1}{2} \bar{R}^{-1} B^T g'(q) \quad (3.24)$$

It can be verified easily that  $\partial^2 H(Z, \bar{U}, V', t) / \partial \bar{U}^2 = 2\bar{R} > 0$ , since  $\bar{R}$  is a positive-definite matrix. Substituting Eqs.(3.19)-(3.21) and Eq.(3.24) into the H-J-B equation in Eq.(3.18), and separating quadratic terms in  $q$  and terms containing  $g'(q)$ , one obtains,

$$-P = P \Lambda \bar{A} + \bar{A}^T \Lambda^T P - P \Lambda B \bar{R}^{-1} B^T \Lambda^T P + \bar{Q} \quad (3.25)$$

$$-\frac{\partial g(q)}{\partial t} = h(q) - \frac{1}{4} g'(q)^T B \bar{R}^{-1} B^T g'(q) + g'^T (\bar{A} - B \bar{R}^{-1} B^T \Lambda^T P) q \quad (3.26)$$

in which the scalar identity  $2q^T P \Lambda \bar{A} q = q^T P \Lambda \bar{A} q + q^T \bar{A}^T \Lambda^T P q$  has been used to obtain Eq.(3.25). Equation (3.25) is the well-known Riccati matrix equation.

To express the controller in Eq.(3.24) as an explicit functionale of multinomials in  $q(Z)$ , we

then it follows from Eq.(3.26) that  $M_i$ 's are determined from the following matrix Riccati equation,

$$-\dot{M}_i = M_i \Lambda (\bar{A} - \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda^T P) + (\bar{A} - \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda^T P)^T \Lambda^T M_i - M_i \Lambda \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda^T M_i + Q_i \quad (3.34)$$

It is observed from Eqs.(3.21), (3.25), (3.27) and (3.31) that the function  $V(Z)$  satisfies all the properties of the Lyapunov function.

### **3.2.1 CONSTANT GAIN MATRICES**

Since the derivative matrix,  $\Lambda = \Lambda(Z)$ , in Eqs.(3.25), (3.31) or (3.34) is a nonlinear function of  $Z$ , Eq.(3.6), gain matrices  $P$  and  $M_i$  for the polynomial controller, Eq.(3.32), can not be calculated off-line. Hence,  $P$  and  $M_i$  will be determined by linearizing  $\Lambda(Z)$  at the initial equilibrium point  $Z=0$ , which is stable for civil engineering structures. For many civil engineering structures, the stable initial point  $Z=0$  is the only equilibrium point. With such a premise, we linearize the derivative matrix  $\Lambda(Z)$  at  $Z=0$ , i.e.,  $\Lambda_0 = \Lambda(Z)|_{Z=0}$ . Replacing  $\Lambda(Z)$  by the linearized form at  $Z=0$ , i.e.,  $\Lambda(Z) = \Lambda_0$ , one obtains from Eqs.(3.25), (3.31) and (3.34) for the steady-state Riccati and Lyapunov matrix equations, respectively,

$$\hat{P} \Lambda_0 \bar{A} + \bar{A}^T \Lambda_0^T \hat{P} - \hat{P} \Lambda_0 \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda_0^T \hat{P} + \bar{Q} = 0 \quad (3.35)$$

$$\hat{M}_i \Lambda_0 (\bar{A} - \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda_0^T \hat{P}) + (\bar{A} - \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda_0^T \hat{P})^T \Lambda_0^T \hat{M}_i + Q_i = 0, \quad i=2, 3, \dots, k \quad (3.36)$$

$$\hat{M}_i \Lambda_0 (\bar{A} - \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda_0^T \hat{P}) + (\bar{A} - \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda_0^T \hat{P})^T \Lambda_0^T \hat{M}_i - \hat{M}_i \Lambda_0 \bar{B} \bar{R}^{-1} \bar{B}^T \Lambda_0^T \hat{M}_i + Q_i = 0 \quad (3.37)$$

in which  $\hat{P}$  is the constant Riccati matrix, Eq.(3.35), and  $\hat{M}_i$  is either a constant Lyapunov matrix, Eq.(3.36), or a constant Riccati matrix, Eq.(3.37). Consequently, the controller in Eq.(3.32) can be written as,

$$U(t) = -\bar{R}^{-1} \bar{B}^T (\bar{T}_a + \Lambda^T \hat{P}) q - \bar{R}^{-1} \bar{B}^T \sum_{i=2}^k (q^T \hat{M}_i q)^{i-1} \Lambda^T \hat{M}_i q \quad (3.38)$$

The optimal controller derived in Eq.(3.38) is a polynomial of nonlinear states  $q$  with gain matrices  $\hat{P}$ ,  $\hat{M}_i$  ( $i=2, 3, \dots, k$ ) and  $\Lambda = \Lambda(Z)$ .  $\hat{P}$  and  $\hat{M}_i$  are constant gain matrices determined by linearizing  $\Lambda$  at  $Z=0$ , Eqs.(3.35)-(3.37). However, the gain matrices  $\Lambda = \Lambda(Z)$  in the controller, Eq.(3.38), which is the derivative matrix, is a nonlinear function of the state  $Z$ .

damping and stiffness can be separated into linear and nonlinear parts as follows

$$F_c[\dot{X}(t)] = C \dot{X}(t) + F_{nc} ; F_s[\dot{X}(t)] = K X(t) + F_{ns} \quad (3.43)$$

in which  $C$  and  $K$  are  $(n \times n)$  linear damping and stiffness matrices, respectively, and  $F_{nc} = F_{nc}[\dot{X}(t)]$  and  $F_{ns} = F_{ns}[X(t)]$  are  $n$ -vectors representing the nonlinear parts. Thus, the state equation of the system can be expressed as

$$\dot{Z}(t) = A\bar{q}(Z) + BU(t) + E(t) \quad (3.44)$$

in which  $\bar{q}(Z) = A^{-1}q(Z)$  is the nonlinear state vector given by

$$\bar{q}(Z) = Z + A^{-1}\tilde{f}(Z) \quad (3.45)$$

where  $\tilde{f}(Z)$  is the nonlinear part of  $\bar{q}(Z)$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; \quad \tilde{f}(Z) = \begin{bmatrix} 0 \\ -M^{-1}[F_{nc} + F_{ns}] \end{bmatrix} \quad (3.46)$$

For the performance index in Eq.(3.4) with  $q$  being replaced by  $\bar{q}$ , one has

$$J = \int_0^{\infty} \left[ \bar{q}^T Q \bar{q} + \ddot{X}_a^T Q_a \ddot{X}_a + U^T R U + \sum_{i=2}^k (\bar{q}^T M_i \bar{q})^{i-1} \bar{q}^T Q_i \bar{q} + \bar{h}(\bar{q}) \right] dt \quad (3.47)$$

where  $\bar{h}(\bar{q}) = \bar{h}_1(\bar{q})$  is given by Eq.(3.5) with  $q$  being replaced by  $\bar{q}$ , i.e.,

$$\bar{h}(\bar{q}) = \bar{h}_1(\bar{q}) = \left[ \sum_{i=2}^k (\bar{q}^T M_i \bar{q})^{i-1} \bar{q}^T M_i \Lambda \right] \bar{R}^{-1} B^T \left[ \sum_{i=2}^k (\bar{q}^T M_i \bar{q})^{i-1} \Lambda^T M_i \bar{q} \right] \quad (3.48)$$

Following the same derivations presented in the preceding subsection, the optimal controller is obtained as

$$U(t) = -\bar{R}^{-1} B^T (\bar{T}_a + \Lambda^T \hat{P}) \bar{q} - \bar{R}^{-1} B^T \sum_{i=2}^k (\bar{q}^T \hat{M}_i \bar{q})^{i-1} \Lambda^T \hat{M}_i \bar{q} \quad (3.49)$$

where  $\bar{T}_a$  and  $\bar{R}$  are given by Eq.(3.9), i.e.,

$$\bar{T}_a = \begin{bmatrix} 0 & 0 \\ 0 & L^T Q_a L \end{bmatrix}; \quad \bar{R} = R + B^T \bar{T}_a B \quad (3.50)$$

and the derivative matrix  $\Lambda$  is given by,

$$\Lambda = \frac{\partial \bar{q}}{\partial Z} = I + A^{-1} \frac{\partial \tilde{f}(Z)}{\partial Z} \quad (3.51)$$

For most civil engineering structures, the nonlinear part  $\tilde{f}(Z)$  of  $\bar{q}(Z)$  and its derivative  $\partial \tilde{f}(Z)/\partial Z$  are zero around the equilibrium point  $Z=0$ . Consequently,  $\Lambda_0 = \Lambda|_{Z=0} = \partial \bar{q}/\partial Z|_{Z=0} = I$ . Matrices

$$\dot{v}_i = D_{yi}^{-1} \left[ A_i \dot{x}_i - \beta_i |\dot{x}_i| |v_i|^{n_i-1} v_i - \gamma_i \dot{x}_i |v_i|^{n_i} \right] = f_i(\dot{x}_i, v_i) \quad (3.59)$$

In Eq.(3.59),  $A_i$ ,  $\beta_i$ ,  $\gamma_i$  and  $n_i$  are parameters characterizing the hysteresis loop of the inelastic behavior of the  $i$ th story unit. Substituting Eq.(3.58) into Eq.(3.1) with  $F_c(\dot{X}) = C\dot{X}$ , one obtains the vector equation of motion as follows

$$M\ddot{X} + C\dot{X} + K_e X(t) + K_f \bar{V}(t) = HU(t) + \eta \ddot{X}_0(t) \quad (3.60)$$

where  $K_e$  and  $K_f$  are the elastic and inelastic stiffness matrices, assembled for each story unit according to Eq.(3.58);  $\bar{V}(t) = [v_1, v_2, \dots, v_n]^T$  is an  $n$  vector denoting the hysteretic component of each story unit given by Eq.(3.59). The derivative matrices  $\Lambda(Z) = \partial q(Z)/\partial Z$ , Eq.(3.6), and  $\Lambda_0 = \Lambda(Z)|_{Z=0}$  appearing in the control law, Eqs.(3.35)-(3.38), are given by

$$\Lambda(Z) = \begin{bmatrix} 0_{nn} & I_{nn} \\ -M^{-1}[K_e + K_f \frac{\partial \bar{V}}{\partial X}] & -M^{-1}C \end{bmatrix}; \quad \Lambda_0 = \begin{bmatrix} 0_{nn} & I_{nn} \\ -M^{-1}K_e & -M^{-1}C \end{bmatrix} \quad (3.61)$$

in which  $0_{nn}$  and  $I_{nn}$  are  $(n \times n)$  null and identity matrices, respectively, and  $\partial \bar{V}/\partial X$  is a diagonal matrix with the  $i$ th diagonal element  $\partial v_i/\partial x_i$  given by,

$$\frac{\partial v_i}{\partial x_i} = \frac{\partial \dot{v}_i}{\partial \dot{x}_i} = D_{yi}^{-1} \left[ A_i - \beta_i \text{sgn}(\dot{x}_i) |v_i|^{n_i-1} v_i - \gamma_i |v_i|^{n_i} \right] \quad (3.62)$$

The linearized constant matrix  $\Lambda_0$ , that is required for the calculation of the feedback gain matrices in Eqs.(3.35)-(3.37), is obtained from  $\Lambda(Z)$  by setting  $Z=0$  or  $\partial \bar{V}/\partial X=0$ , see Eq.(3.61). For numerical simulations of the structural response, the hysteretic vector  $\bar{V}$  can be augmented in Eqs.(3.59) and (3.60) so that the state vector  $Z = [X^T, \dot{X}^T, \bar{V}^T]^T$  has a  $3n$ -dimension. A detailed description of the procedures for numerical simulations of the response of hysteretic structures can be found in Yang et al (1992a).

### **3.5 OTHER CONTROL LAWS FOR NONLINEAR OR HYSTERETIC STRUCTURES**

The performance of the controller presented in this paper will be compared with some other controllers available in the literature. These control methods include the LQR method based on

control system consisting of rubber-bearing isolators and actuators, Fig. 3-1. The performance of the proposed controller will be compared with that of various controllers described previously.

### **3.6.1 A BASE-ISOLATED ELASTO-PLASTIC BUILDING**

An eight-story building that exhibits bilinear elasto-plastic behavior is considered. The properties of the building are as follows: (i) the mass of each floor is identical with  $m_i = 345.6$  metric tons; (ii) preyielding stiffnesses  $k_i (i=1,2..8)$  of eight-story units are 340400, 325700, 284900, 268600, 243000, 207300, 168700 and 136600 kN/m, respectively, and postyielding stiffnesses are  $0.1 k_i$  for  $i=1,2,....,8$ , i.e.,  $\alpha_i = 0.1$  in Eq.(3.58); and (iii) the linear viscous damping coefficients for each story unit are  $c_i = 490, 467, 410, 386, 348, 298, 243$  and  $196$  kN.sec/m, respectively. The damping coefficients result in a damping ratio of 0.38 % for the first vibrational mode. The fundamental frequency of the unyielded building is 5.24 rad/sec. The yielding level for each story unit varies with respect to the stiffness; with the results,  $D_{y_i} = 2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.7$  and  $1.5$  cm, Eq.(3.59). The bilinear elasto-plastic behavior can be described by the hysteretic model, Eq. (3.59), with  $A_i = 1.0, \beta_i = 1.0, n_i = 95$  and  $\gamma_i = 1.0$  for  $i=1,2..8$ . The El Centro NS (1940) earthquake with a peak ground acceleration of 0.3g, referred to as the design earthquake as shown in Fig. 3-2, is used for the input excitation.

Without any control system, it has been observed that the deformation of the unprotected building is excessive and that yielding takes place in the upper five stories (Yang et al 1992b, 1994a). Hence, a lead-core rubber bearing isolation system is used to reduce the response of the building. The stiffness of the lead-core rubber-bearing is modelled by Eq.(3.58) with  $F_{i_b} = \alpha_b k_b x_b + (1 - \alpha_b) k_b D_{y_b} v_b$  in which the subscript b stands for the base-isolation system. The hysteretic component,  $v_b$ , is modelled by Eq.(3.59). Properties of the base-isolation system are:  $m_b = 450$  metric tons, stiffness  $k_b = 18050$  kN/m, damping  $c_b = 26.17$  kN.sec/m,  $\alpha_b = 0.6, D_{y_b} = 4$  cm,  $A_b = 1.0, \beta_b = 0.5, n_b = 3$  and  $\gamma_b = 0.5$ , Eq.(3.59). The hysteresis loop of such a base-isolation system, i.e.,  $x_b$  versus  $v_b$ , is shown in Fig. 3-3.

For the building with the base-isolation system, the first natural frequency of the preyielded structure is 2.21 rad/sec and the damping ratio for the first vibrational mode is 0.16 %. Within 30

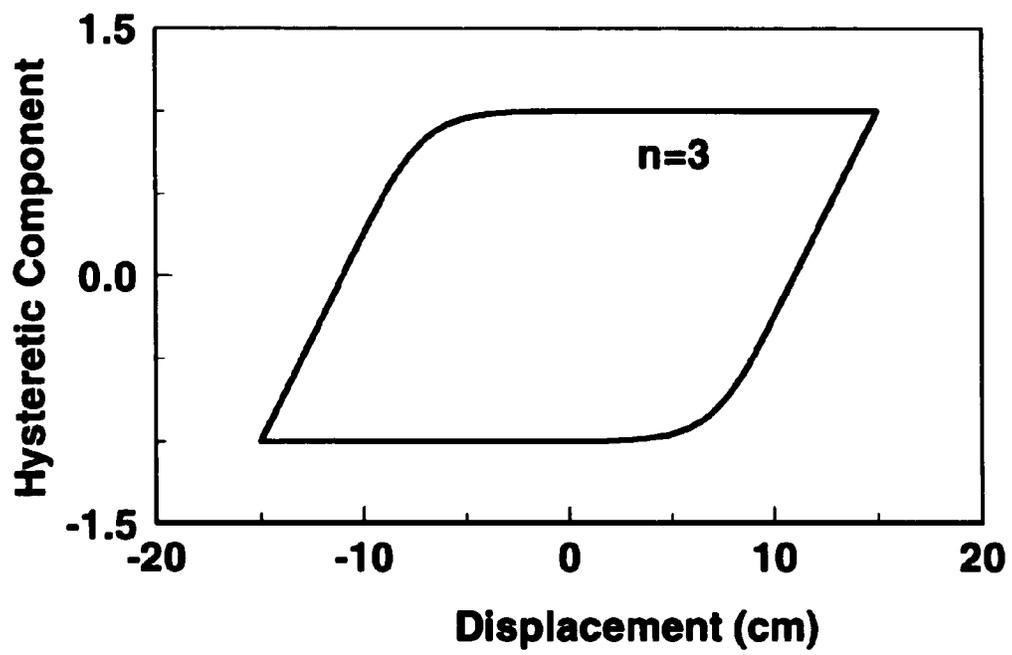


Fig. 3-3: Hysteresis Loop of Lead-Core Rubber Bearings

are chosen to be  $Q_a(1,1) = 10$  and  $Q(1,1)=130$  for the second case. With  $R = 7.3 \times 10^{-11}$ , the peak response quantities are presented in Columns (7) and (8), respectively, of Table 3-1, designated as “Linear 2”. It is observed that, although the drift of the rubber bearing and the peak control force are reduced, the building response quantities increase.

For the controller, Eq.(3.38), presented in this study, we first consider the special case in which  $Q_i = 0, i=2, 3, \dots, k$ . Such a special controller was proposed by Yang et al (1992b, 1994c). In this case, we choose  $R=1.0 \times 10^{-7}$ , and  $Q_a$  and  $Q$  are diagonal matrices as follows:  $Q_a(i,i)=[10, 15, 15, 20, 20, 30, 50, 50, 50]$ ,  $Q(1,1)=100$ ,  $Q(i,i)=10$  for  $i=2, 3, \dots, 9$ , and  $Q(i,i)=0$  for  $i=10, 11, \dots, 18$ . The peak response quantities based on this controller are shown in Columns (9) and (10) of Table 3-1, designated as “Nonlinear 1”. We observe that the overall performance of this controller is slightly better than that of linear controllers. Next, we consider a general case, where  $Q_2 \neq 0$  and  $Q_i = 0$  for  $i=3, 4, \dots, k$ . Diagonal weighting matrices are chosen as follows:  $R=1.0 \times 10^{-4}$ ,  $Q_a(i,i)=[4, 6, 6, 8, 8, 12, 35, 35, 35] \times 10^3$ ,  $Q(1,1)= 2$ ,  $Q(i,i)= 1$  for  $i= 2, 3, \dots, 9$ ,  $Q(i,i)= 0$  for  $i= 10, 11, \dots, 18$ ,  $Q_2(1,1)=8$ ,  $Q_2(i,i)=6$  for  $i=2, 3, \dots, 6$ ,  $Q_2(i,i)=1$  for  $i=7, 8, 9$  and  $Q_2(i,i) =0$  for  $i=10, 11, \dots, 18$ . The peak response quantities based on this controller are shown in Columns (11) and (12), respectively, of Table 3-1, designated as “Nonlinear 2”. It is observed that, while the overall performance is similar to that of “Nonlinear 1”, the peak control force has been decreased by 5%. In particular, the overall performance of “Nonlinear Controller 2” is comparable with that of the sliding mode controller.

Time histories for the drift of rubber-bearings are shown in Fig. 3-4(a), in which the response without actuator is shown by the solid curve. The dashed and dash-dotted curves represent the responses using Nonlinear 2 and Nonlinear 1 controllers, respectively. The required control forces for both controllers are shown in Fig. 3-4(b). As observed from Fig. 3-4 and Table 3-1, hybrid control is quite effective and the performance of both Nonlinear 1 and Nonlinear 2 controllers are comparable.

With hybrid control, the building response quantities are well within the elastic range except the drift of rubber bearings. Hence, the reduction for the drift of rubber bearings will be compared for different controllers in the following. The results presented in Table 3-1 are based on the design earthquake, i.e., El Centro earthquake with a peak ground acceleration (PGA) of 0.3g. Since the PGA is stochastic in nature, numerical simulations have been conducted for the same earthquake with different PGA. Based on the same design for various controllers presented in Table 3-1, simulation

Table 3-1: Peak Response Quantities of an Eight-Story Building Equipped with Hybrid Control System

F L O O R	D <sub>y</sub>	With BIS		Linear 1 U = 1491 kN (4.73%) $\overline{U^2} = 1047 \text{ kN}^2$		Linear 2 U = 1031 kN (3.27%) $\overline{U^2} = 226 \text{ kN}^2$		Nonlinear 1 U = 1437 kN (4.56%) $\overline{U^2} = 689 \text{ kN}^2$		Nonlinear 2 U = 1350 kN (4.29%) $\overline{U^2} = 711 \text{ kN}^2$		Sliding Mode U = 1494 kN (4.74%) $\overline{U^2} = 651 \text{ kN}^2$		
		x <sub>i</sub> cm	$\ddot{x}_{ai}$ cm/s <sup>2</sup>	x <sub>i</sub> cm	$\ddot{x}_{ai}$ cm/s <sup>2</sup>	x <sub>i</sub> cm	$\ddot{x}_{ai}$ cm/s <sup>2</sup>	x <sub>i</sub> cm	$\ddot{x}_{ai}$ cm/s <sup>2</sup>	x <sub>i</sub> cm	$\ddot{x}_{ai}$ cm/s <sup>2</sup>	x <sub>i</sub> cm	$\ddot{x}_{ai}$ cm/s <sup>2</sup>	
N O	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
B	4.0	21.35	130	14.4	45	10.7	70	10.7	38	10.7	46	10.8	77	
1	2.4	0.62	123	0.15	43	0.22	71	0.20	39	0.18	48	0.14	42	
2	2.3	0.59	113	0.16	40	0.25	66	0.20	36	0.18	43	0.14	37	
3	2.2	0.65	111	0.19	33	0.29	53	0.22	30	0.21	34	0.16	38	
4	2.1	0.63	102	0.21	29	0.30	46	0.23	30	0.22	30	0.15	31	
5	2.0	0.65	91	0.22	32	0.30	49	0.22	38	0.21	40	0.14	38	
6	1.9	0.65	103	0.23	39	0.31	66	0.22	46	0.20	46	0.18	39	
7	1.7	0.60	135	0.22	50	0.34	68	0.20	47	0.20	51	0.20	42	
8	1.5	0.41	163	0.16	64	0.27	105	0.15	60	0.16	64	0.15	60	

Table 3-2: Peak Response Quantities of a Fixed-Base 8 Story Building Under 1g El Centro Earthquake

F L O O R	D <sub>y</sub>	Without Control	Linear Control U <sub>max</sub> = 5475 kN (20.21%) $\overline{U^2} = 5608 \text{ kN}^2$		Nonlinear Control 1 U <sub>max</sub> = 4966 kN (18.33%) $\overline{U^2} = 5368 \text{ kN}^2$		Nonlinear Control 2 U <sub>max</sub> = 5192 kN (19.16%) $\overline{U^2} = 5510 \text{ kN}^2$		Nonlinear Control 3 U <sub>max</sub> = 5683 kN (20.97%) $\overline{U^2} = 11602 \text{ kN}^2$		
		x <sub>i</sub> cm	x <sub>i</sub> cm	u <sub>i</sub> kN	x <sub>i</sub> cm	u <sub>i</sub> kN	x <sub>i</sub> cm	u <sub>i</sub> kN	x <sub>i</sub> cm	u <sub>i</sub> kN	
N O	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	2.4	4.88	4.35	4961	4.32	4966	4.25	5192	4.32	5683	
2	2.3	4.10	4.36	4893	4.35	4803	4.26	5010	4.30	5451	
3	2.2	5.38	4.24	5475	4.36	4911	4.28	5068	4.32	5428	
4	2.1	5.47	4.21	4613	4.29	4345	4.23	4438	4.08	4731	
5	2.0	6.87	3.97	4013	4.09	3853	3.98	3891	3.72	4098	
6	1.9	8.48	3.79	3355	3.86	3358	3.65	3372	3.22	3502	
7	1.7	10.64	3.82	2303	3.60	2617	3.80	2441	2.79	2542	
8	1.5	4.61	3.65	573	3.69	1306	3.53	1212	3.50	1231	

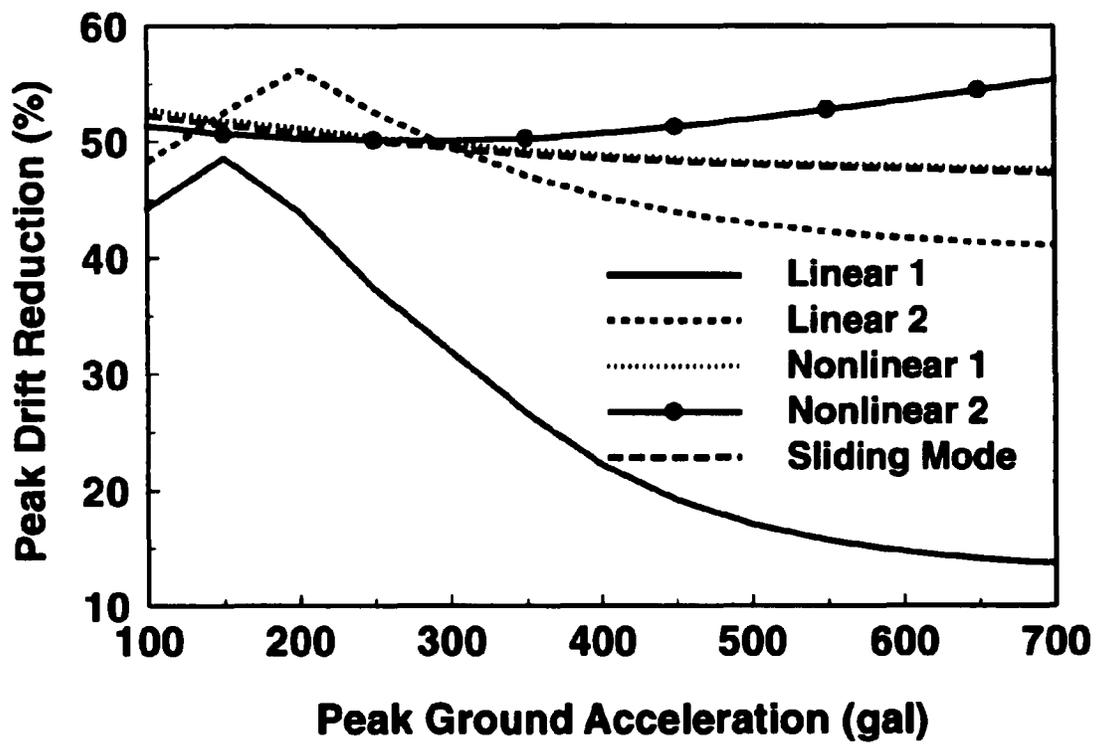


Fig. 3-5: Peak Drift Reduction of Rubber Bearings vs. Peak Ground Acceleration

particular, the ductilities of the fifth, sixth and seventh story units are very large. Hence, it is important to install controllers at every floor to apply control forces effectively.

The objective for the control design is to prevent a collapse of the building by reducing the ductility of each story unit to be smaller than 2.5 cm. We first consider the LQR control law, Eq.(3.64), in which the structural system is linearized first at  $Z=0$ . In this case, we choose  $R(i,i) = 5.0 \times 10^{-5}$ ,  $i=1, 2, \dots, 8$ ,  $Q_a(i,i) = 0.1$ ,  $i=1, 2, \dots, 8$  and  $Q(i,i) = [15, 16, 22, 18, 15, 11, 6, 1, 0, 0, 0, 0, 0, 0, 0, 0] \times 10^3$ . All other elements of the matrices above are zero. The peak interstory drifts and the peak control force for each controller are shown in columns (4) and (5), respectively, of Table 3-2, designated as "Linear Control". Also shown in the table are the maximum of peak control forces,  $U_{max}$ , and the maximum required control energy,  $\overline{U^2}$ , among all actuators. The maximum control force,  $U_{max}$ , has also been expressed in paranthesis as the percentage of the total building weight that is 2764.8 metric tons. The peak control force is about 20.21% because the earthquake has a PGA of 1g.

For the polynomial controller, we consider the case when  $Q_i = 0$ ,  $i = 2, 3, \dots, k$ . The diagonal weighting matrix  $Q$  is chosen as  $Q(i,i) = [15.3, 15.5, 17.6, 16.1, 14.2, 11.9, 9, 3.3, 0, 0, 0, 0, 0, 0, 0, 0] \times 10^3$ , whereas weighting matrices  $R$  and  $Q_a$  are the same as the linear controller above. The peak interstory drifts and the peak control forces are shown in columns (6) and (7), respectively, of Table 3-2, designated as "Nonlinear Control 1". It is observed that the peak control force and the maximum required energy are smaller than those of the linear controller for a similar response reduction of the building. Next, we consider a general case, where  $Q_2 \neq 0$  and  $Q_i = 0$  for  $i=3, 4, \dots, k$ . The weighting matrices  $Q_a$ ,  $R$  and  $Q$  are kept to be the same as in the case of Nonlinear Control 1, except that  $Q(7,7)=7000$  and  $Q(8,8) = 2500$ .  $Q_2$  is chosen to be a diagonal matrix with diagonal elements  $Q_2(i,i) = [0.9, 0.9, 0.9, 0.8, 0.8, 0.8, 0.8, 0.5, 0, 0, 0, 0, 0, 0, 0, 0]$ . The peak interstory drifts and the peak control forces are shown in columns (8) and (9), respectively, of Table 3-2, designated as "Nonlinear Control 2". It is observed from Table 3-2 that the performance of both nonlinear controllers is better than that of the linear controller. A comparison between the results for both nonlinear controllers indicates that a better reduction for the interstory drifts is achieved by Nonlinear Control 2 at the expense of increased peak control force and control energy.

as good as that of the cubic-order controller, in the sense that for the same level of the peak response reduction, these higher-order control laws require a bigger peak control force and a larger control energy. In fact, the performance for different orders of control laws depends heavily on the nature of nonlinearity of the structure considered. For the hysteretic-type nonlinearity considered in both examples above, (i.e., hysteretic rubber bearings and yielded building), the cubic control law has the best performance.

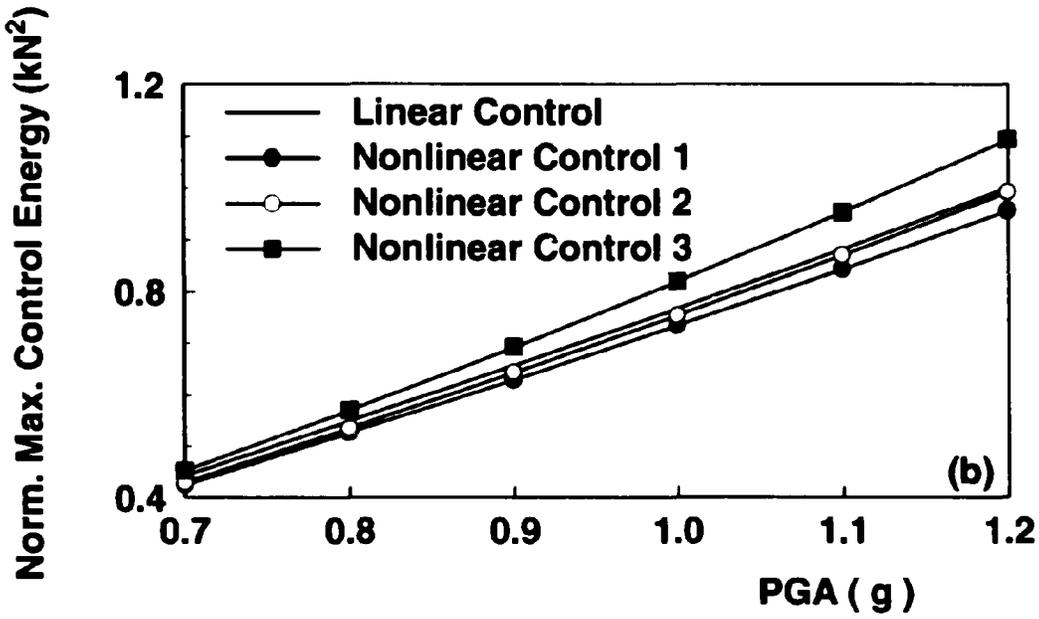
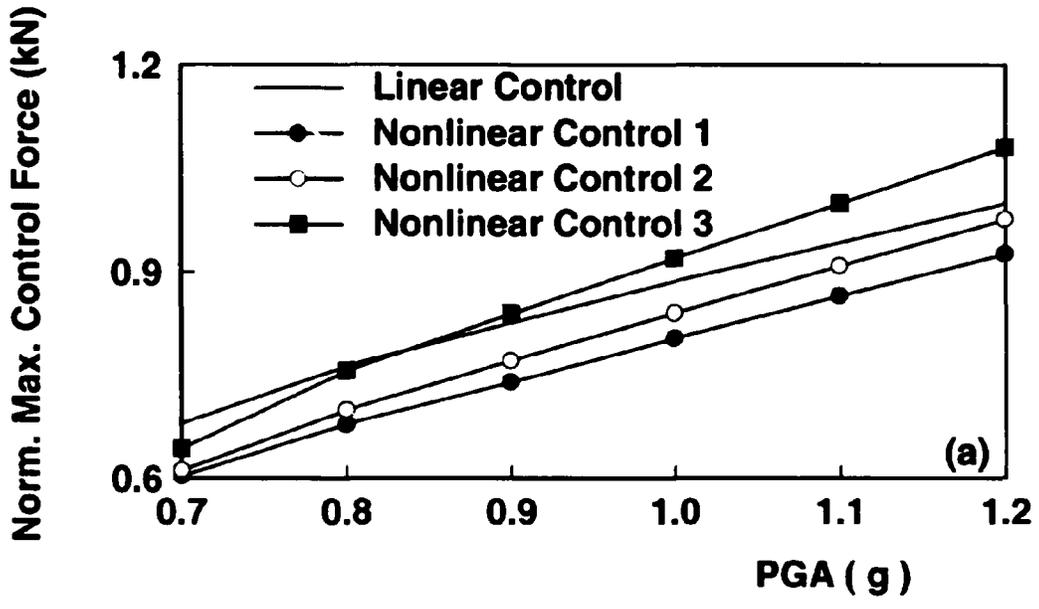


Fig. 3-9: Normalized Maximum Control Force and Control Energy vs. Peak Ground Acceleration; (a) Maximum Control Force, and (b) Control Energy

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## SECTION IV

### STATIC OUTPUT POLYNOMIAL CONTROL FOR LINEAR AND NONLINEAR STRUCTURES

The optimal polynomial controllers proposed in sections II and III, respectively, for linear and nonlinear structures, require full-state feedbacks. In this section, these optimal controllers are extended to static output feedback controllers, that utilize only the information measured from a limited number of sensors installed at critical locations without an observer. The derivations are based on a similar algorithm for static output feedback linear controller proposed by Levine and Athans (1970). The performance of these static output controllers is demonstrated by simulation results for: (i) active control of a 3-story linear building, and (ii) hybrid control of a base-isolated 8-story building using rubber-bearing isolators.

#### 4.1 FULL-STATE POLYNOMIAL CONTROLLERS

##### 4.1.1 SPECIAL NONLINEAR STRUCTURES

Consider an  $n$  degree-of-freedom nonlinear structure subjected to a one-dimensional earthquake ground acceleration,  $\ddot{x}_0(t)$ . The vector equation of motion is given by,

$$M\ddot{X}(t) + F_c[\dot{X}(t)] + F_s[X(t)] = HU(t) + \eta\ddot{x}_0(t) \quad (4.1)$$

in which  $X(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$  is an  $n$  vector with  $x_i(t)$  being the drift of the  $i$ th designated story unit;  $U(t) = [u_1, u_2, \dots, u_r]^T$  is a  $r$ -vector consisting of  $r$  control forces; and  $\eta$  is an  $n$  vector denoting the influence of the earthquake excitation. The superscript  $T$  above indicates the transpose of a matrix or a vector. In Eq.(4.1),  $M$  is a  $(n \times n)$  mass matrix;  $H$  is a  $(n \times r)$  matrix denoting the location of  $r$  controllers;  $F_c[\dot{X}(t)]$  is an  $n$ -vector denoting the nonlinear damping force; and  $F_s[X(t)]$  is an  $n$ -vector denoting the nonlinear stiffness force.

For many civil engineering structures, such as inelastic or hysteretic structures, the nonlinear damping and stiffness can be separated into linear and nonlinear parts as follows

$$F_c[\dot{X}(t)] = C\dot{X}(t) + F_{nc}; \quad F_s[X(t)] = KX(t) + F_{ns} \quad (4.2)$$

in which  $C$  and  $K$  are  $(n \times n)$  linear damping and stiffness matrices, respectively, and  $F_{nc} = F_{nc}[\dot{X}(t)]$  and  $F_{ns} = F_{ns}[X(t)]$  are  $n$ -vectors representing the nonlinear parts. Let us introduce a  $2n$  nonlinear state vector  $\bar{q}(Z)$

$$\bar{q}(Z) = Z + A^{-1} \tilde{f}(Z) \quad (4.3)$$

in which  $Z = [X^T(t), \dot{X}^T(t)]^T$  is a  $2n$  state vector;  $A$  is a  $(2n \times 2n)$  elastic system matrix; and  $\tilde{f}(Z)$  is a  $2n$  nonlinear vector

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; \quad \tilde{f}(Z) = \begin{bmatrix} 0 \\ -M^{-1}(F_{nc} + F_{ns}) \end{bmatrix} \quad (4.4)$$

Then, the equation of motion, Eq.(4.1), in the state space can be expressed as

$$\dot{Z}(t) = A\bar{q}(Z) + BU(t) + E(t) \quad (4.5)$$

in which  $B$  is  $(2n \times r)$  matrix for controller locations; and  $E(t)$  is a  $2n$  excitation vector, respectively, given by

$$B = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}; \quad E(t) = \begin{bmatrix} 0 \\ M^{-1}\eta\ddot{x}_0(t) \end{bmatrix} \quad (4.6)$$

For such special nonlinear structures, an optimal full-state polynomial controller was derived in Eq.(3.49) of section III as follows

$$U(t) = -R^{-1}B^T \Lambda^T \hat{P} \bar{q}(Z) - R^{-1}B^T \sum_{i=2}^k (\bar{q}^T \hat{M}_i \bar{q})^{i-1} \Lambda^T \hat{M}_i \bar{q}(Z) \quad (4.7)$$

in which  $\Lambda = \Lambda(Z)$  is the derivative matrix of  $\bar{q}(Z)$ ,

$$\Lambda = \Lambda(Z) = \partial \bar{q}(Z) / \partial Z = I + A^{-1} \partial \tilde{f}(Z) / \partial Z \quad (4.8)$$

and positive-definite matrices  $\hat{P}$  and  $\hat{M}_i$ ,  $i=2,3,\dots,k$ , are obtained by solving Riccati matrix equations, Eqs. (3.52) and (3.55)

$$\hat{P}A + A^T \hat{P} - \hat{P}BR^{-1}B^T \hat{P} + Q = 0 \quad (4.9)$$

$$\hat{M}_i \tilde{A} + \tilde{A}^T \hat{M}_i - \hat{M}_i BR^{-1}B^T \hat{M}_i + Q_i = 0 \quad (4.10)$$

where

$$\tilde{A} = A - BR^{-1}B^T \hat{P} \quad (4.11)$$

The controller in Eq.(4.7) has been obtained by minimizing the performance index, Eq.(3.47)

$$J = \int_0^{\infty} \left\{ \bar{q}^T Q \bar{q} + U^T R U + \left[ \sum_{i=2}^k (\bar{q}^T M_i \bar{q})^{i-1} \bar{q}^T Q_i \bar{q} \right] + \bar{h}(\bar{q}) \right\} dt \quad (4.12)$$

where [see Eq.(3.48) and (3.56)]

$$\begin{aligned} \bar{h}(\bar{q}) = & \left[ \sum_{i=2}^k (\bar{q}^T M_i \bar{q})^{i-1} \bar{q}^T M_i \right] B R^{-1} B^T \left[ \sum_{i=2}^k (\bar{q}^T M_i \bar{q})^{i-1} \bar{q} \right] \\ & - \sum_{i=2}^k (\bar{q}^T M_i \bar{q})^{i-1} \bar{q}^T M_i B R^{-1} B^T M_i \bar{q} \end{aligned} \quad (4.13)$$

Note that the second term in Eq.(4.12) is quadratic in control, whereas the first term and the third term in summation are polynomials in  $\bar{q}(Z)$  of different orders. Weighting matrices  $Q$ ,  $R$  and  $Q_i$  ( $i=2,3,\dots,k$ ) can be chosen by the designer to penalize the selected response quantities. However, matrices  $M_i$  ( $i=2,3,\dots,k$ ) are implicit function of the weighting matrices  $Q_i$  ( $i=2,3,\dots,k$ ) defined by Eq.(4.10).

#### **4.1.2 LINEAR STRUCTURES**

For linear structures in which the nonlinear part  $\tilde{f}(Z)$  in Eq.(4.3) is zero, i.e.,  $\bar{q}(Z)=Z$  and  $\Lambda(Z)=I$ , the state equation of motion, Eq.(4.5), becomes

$$\dot{Z}(t) = AZ(t) + BU(t) + E(t) \quad (4.14)$$

and the optimal polynomial controller in Eq.(4.7) becomes

$$U(t) = -R^{-1} B^T P Z(t) - R^{-1} B^T \sum_{i=2}^k (Z^T M_i Z)^{i-1} M_i Z \quad (4.15)$$

The gain matrices  $P$  and  $M_i$  ( $i = 2, 3, \dots, k$ ) are determined from the Riccati equations, Eqs.(4.9)-(4.11), as follows

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (4.16)$$

$$M_i \tilde{A} + \tilde{A}^T M_i - M_i B R^{-1} B^T M_i + Q_i = 0 \quad (4.17)$$

where

$$\tilde{A} = A - B R^{-1} B^T P \quad (4.18)$$

The optimal polynomial controller in Eq.(4.15) minimizes the polynomial performance index

$$J = \int_0^{\infty} \left\{ Z^T Q Z + U^T R U + \left[ \sum_{i=2}^k (Z^T M_i Z)^{i-1} Z^T Q_i Z \right] + \bar{h}(Z) \right\} dt \quad (4.19)$$

As a result, closed-loop systems  $\tilde{A} = A - BR^{-1}B^T P$ , Eq.(4.18), and  $(\tilde{A} - BR^{-1}B^T M_i)$  for  $i=2, 3, \dots, k$  are stable. The stability of the closed-loop systems in Eqs.(4.21) and (4.23) ensures the stability of the system in Eq.(4.14) for the full-state feedback controller in Eq.(4.15).

Consider a  $m$ -dimensional output vector  $y(t)$ ,

$$y(t) = cZ(t) \quad (4.25)$$

where  $c$  is a  $(m \times 2n)$  observation matrix and  $m$  is the number of sensors installed on the structure. We construct an output polynomial controller as follows

$$U(t) = -R^{-1}B^T N y - R^{-1}B^T \sum_{i=2}^k (y^T c K_i c^T y)^{i-1} S_i y \quad (4.26)$$

in which  $N$  and  $S_i$  are  $(2n \times m)$  output gain matrices to be determined, and  $K_i$  is related to  $S_i$ . Such an output controller reduces to the state controller in Eq.(4.15) when  $c$  is a  $(2n \times 2n)$  identity matrix as will be shown later on.

A substitution of  $y(t)$  in Eq.(4.25) into Eq.(4.26) leads to the following

$$U(t) = -R^{-1}B^T N c Z(t) - R^{-1}B^T \sum_{i=2}^k (Z^T \tilde{c} K_i \tilde{c} Z)^{i-1} S_i c Z(t) \quad (4.27)$$

where  $\tilde{c} = c^T c$ . Note that the positive definite multinomial  $Z^T M_i Z$  in the second term of the full-state controller in Eq.(4.15) has been replaced by the positive definite multinomial  $Z^T \tilde{c} K_i \tilde{c} Z$  in the static output controller, Eq.(4.27). Consequently, the static output controller in Eq.(4.27) will stabilize the system in Eq.(4.14) if we can find the gain matrices  $N$  and  $S_i$  such that closed-loop systems  $A - BR^{-1}B^T N c$  and  $A - BR^{-1}B^T (N + S_i) c$  for  $i=2,3,\dots,k$ , are stable. Methods to obtain  $N$  and  $S_i$ , such that these closed-loop systems are stable, are presented as follows.

A static output feedback controller for linear systems, Eq.(4.21), was proposed by Levine and Athans (1970) by minimizing a performance index,  $\hat{J}_1 = E(J_1)$ , where  $E(J_1)$  is the stochastic average of the performance index  $J_1$  in Eq.(4.22); with the result

$$U(t) = -F y(t) \quad (4.28)$$

The feedback gain matrix  $F$  is obtained from

$$F = R^{-1}B^T K_p L_p c^T [c L_p c^T]^{-1} \quad (4.29)$$

where the closed-loop system of Eq.(4.35),  $\bar{A}_1$ , is stable.

$$\bar{A}_1 = A - BR^{-1}B^T Nc - BF_1c \quad (4.39)$$

Again, equating the feedback gain matrix  $F_1$  above to the feedback gain matrix in the second part of the controller of Eq.(4.27), i.e.,  $R^{-1}B^T S_1c$ , one obtains

$$F_1 = R^{-1}B^T S_1c \quad (4.40)$$

Then,  $S_1$  is obtained by substituting Eq.(4.40) into Eq.(4.36) as

$$S_1 = K_1 L_1 c^T [c L_1 c^T]^{-1} \quad (4.41)$$

Substituting Eq.(4.40) into Eq.(4.39), the stable closed-loop system  $\bar{A}_1$  becomes  $\bar{A}_1 = A - BR^{-1}B^T(N + S_1c)$ . Thus, we have derived  $N$  and  $S_1$  such that closed-loop systems  $\bar{A}_1 = A - BR^{-1}B^T Nc$  and  $\bar{A}_i = A - BR^{-1}B^T Nc - BF_1c$  for  $i = 2, 3, \dots, k$  are stable. It should be noted that  $K_1$  in Eq.(4.26) is related to  $S_1$  through Eq.(4.41).

For the case of full-state feedback, the observation matrix  $c$  is an identity matrix, and Eqs.(4.30) and (4.37) reduce to,

$$K_p A + A^T K_p - K_p B R^{-1} B^T K_p + Q = 0 \quad (4.42)$$

$$K_1 \bar{A} + \bar{A}^T K_1 - K_1 B R^{-1} B^T K_1 + Q_1 = 0 \quad (4.43)$$

respectively. Consequently, one has  $P = N = K_p$ ,  $M_1 = S_1 = K_1$  and the quadratic term  $Z^T \tilde{c} K_1 \tilde{c} Z$  reduces to  $Z^T M_1 Z$ . Hence, the static output controller in Eq.(4.27) reduces exactly to the optimal full-state feedback controller in Eq.(4.15) as a special case.

### **4.3 DERIVATION OF STATIC OUTPUT POLYNOMIAL CONTROLLER FOR NONLINEAR STRUCTURES**

For nonlinear structures, the system equation is given by Eq.(4.5). The full-state polynomial controller is given by Eqs. (4.7)-(4.8) and the gain matrices  $\hat{P}$  and  $\hat{M}_1$  should be determined from Eqs.(4.9)-(4.10). For many civil engineering structures, either nonlinear or hysteretic,  $Z=0$  is the only equilibrium point that is stable. Further, for these civil engineering structures  $\tilde{f}(Z)|_{Z=0} = \partial \tilde{f}(Z)/\partial Z|_{Z=0} = 0$  and hence  $\Lambda_0 = \Lambda(Z)|_{Z=0} = I$ . Based on these premises, constant

reliable and useful. The simulation results presented in the next section are based on such a method using IMSL double precision subroutine “DUMING”. Due to space limitations, details for the iteration procedures are not presented.

## **4.5 NUMERICAL RESULTS**

### **4.5.1 THREE-STORY LINEAR BUILDING:**

Consider a three-story linear building model equipped with an active bracing system in the first story unit. The mass, stiffness and damping coefficient of each story unit are  $m_i = 1$  metric ton,  $k_i = 980$  kN/m and  $c_i = 1.407$  kN.s/m, respectively, for  $i=1, 2$  and  $3$ . The El-Centro (NS component) earthquake with a peak ground acceleration of  $0.3g$ , shown in Fig. 3-2, is used as the input excitation. Only the interstory drift and velocity of the first story unit are measured and used for the design of static output controllers. The performance of the third order (cubic) static output controller in Eq.(4.26), i.e.  $Q_2 \neq 0, Q_1=0, i = 3, 4..k$ , will be compared with that of the linear static output controller in Eq.(4.28) for three levels of response reductions. For the linear controller, a diagonal state weighting matrix  $Q=[10^5, 10^4, 10^3, 1, 1, 1]$  is used. Since there is only one controller, the control weighting matrix  $R$  is a scalar. Three cases of response reductions corresponding to three different  $R$  values, i.e.,  $R = 0.901 \times 10^{-9}$ ,  $0.1698 \times 10^{-7}$  and  $0.7095 \times 10^{-7}$ , have been considered. The peak interstory drifts,  $x_i$ , and the peak absolute acceleration of floors,  $\ddot{x}_i$ , for  $i = 1, 2$  and  $3$  are shown in Columns (4)-(5), (8)-(9) and (12)-(13) of Table 4-1. The maximum control force  $U$  and the required control energy  $\overline{U^2}$  in 20 seconds of the earthquake episode are also shown in Table 4-1. The control energy  $\overline{U^2}$  is computed as the integration of the square of the control force  $U(t)$  over 20 seconds. The building response quantities without control are shown in Columns (2) and (3) for comparison. To obtain similar levels of response reductions using the third order controller, the following weighting matrices for three cases are chosen:  $Q=[10^4, 10^3, 10^2, 1, 1, 1]$ ,  $Q_2=[8000, 400, 10, 0, 0, 0]$ ,  $R=0.1698 \times 10^{-7}$ ;  $Q=[10^4, 10^3, 10^2, 1, 1, 1]$ ,  $Q_2=[4200, 1, 1, 0, 0, 0]$ ,  $R=0.1698 \times 10^{-6}$ ; and  $Q=[10^3, 10^2, 10, 1, 1, 1]$ ,  $Q_2=[2200, 150, 10, 0, 0, 0]$ ,  $R=0.1698 \times 10^{-5}$ , respectively. The simulation results for the peak response quantities are shown in Columns (6)-(7),

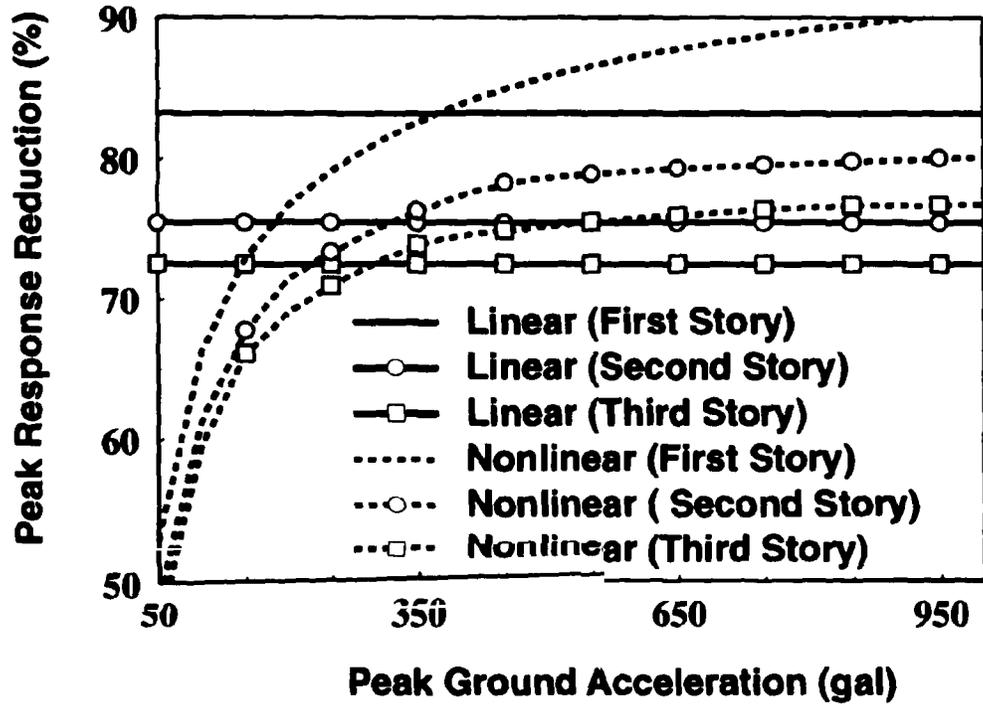


Fig. 4-1: Percentage of Reduction for Peak Interstory Drifts

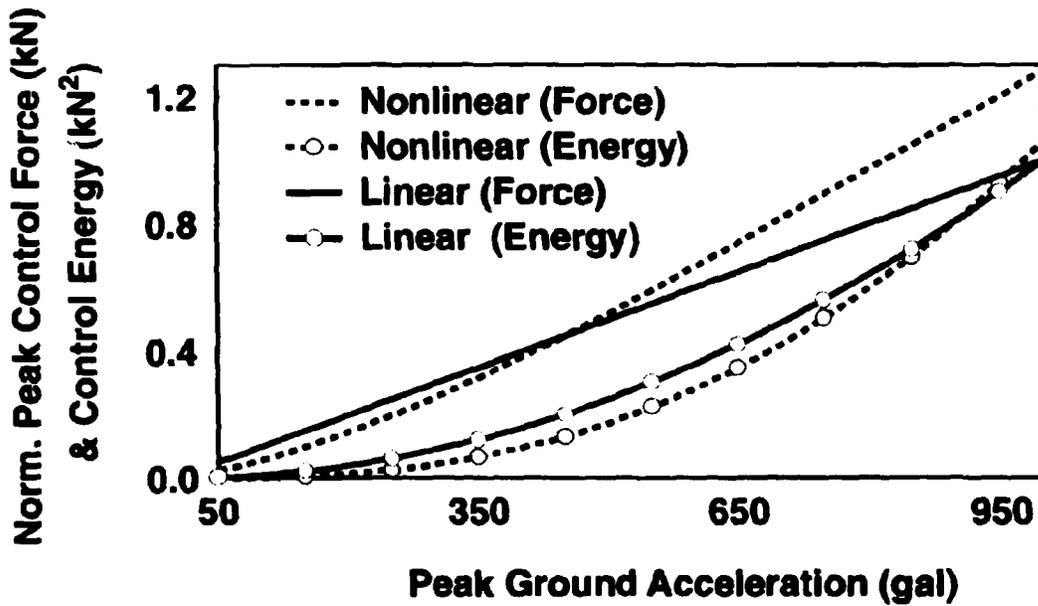


Fig. 4-2: Normalized Peak Control Force and Required Control Energy

magnitude of the  $R$  matrix can be designed with a larger value. This eliminates the convergence problems in the iteration process quite effectively. Hence, the proposed static output polynomial controller has a computational advantage over the linear controller.

#### **4.5.2 A BASE-ISOLATED ELASTO-PLASTIC BUILDING**

An eight-story building that exhibits bilinear elasto-plastic behavior is considered. The mass of each floor is identical with  $m_i = 345.6$  metric tons. The  $i$ th element,  $F_{s_i}[x_i(t)]$ , of the nonlinear stiffness force,  $F_s[X(t)]$ , is modelled as

$$F_{s_i}[x_i(t)] = \alpha_i k_i x_i + (1 - \alpha_i) k_i D_{y_i} v_i \quad (4.46)$$

in which  $k_i$  = preyielding elastic stiffness of the  $i$ th story unit,  $\alpha_i$  = ratio of the post-yielding to preyielding stiffness,  $D_{y_i}$  = yield deformation = constant, and  $v_i$  is a nondimensional hysteretic component of the deformation, with  $|v_i| \leq 1$ , where

$$\dot{v}_i = D_{y_i}^{-1} \left[ A_i \dot{x}_i - \beta_i |\dot{x}_i| |v_i|^{n_i-1} v_i - \gamma_i \dot{x}_i |v_i|^{n_i} \right] \quad (4.47)$$

It should be noted that the model of the nonlinear stiffness force,  $F_s[X(t)]$ , in Eq.(4.46) is similar to the one expressed by Eq.(4.2). Preyielding stiffnesses,  $k_i$  ( $i=1,2..8$ ) in Eq.(4.46), of eight-story units are 340400, 325700, 284900, 268600, 243000, 207300, 168700 and 136600 kN/m, respectively, and postyielding stiffnesses are  $0.1 k_i$  for  $i=1,2,...,8$ , i.e.,  $\alpha_i=0.1$  in Eq.(4.46). Linear viscous damping is assumed such that  $F_{nc}$  in Eq.(4.2) is zero. The linear viscous damping coefficients for each story unit are  $c_i=490, 467, 410, 386, 348, 298, 243$  and  $196$  kN.sec/m, respectively, where  $c_i$  is the  $i$ th diagonal element of the  $C$  matrix in Eq.(4.2). The damping coefficients given above result in a damping ratio of 0.38 % for the first vibrational mode. The fundamental frequency of the unyielded building is 5.24 rad/sec. The yielding level for each story unit varies with respect to the stiffness; with the results,  $D_{y_i} = 2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.7$  and  $1.5$  cm, Eq.(4.47). The bilinear elasto-plastic behavior can be described by the hysteretic model in Eq.(4.47) with  $A_i=1.0$ ,  $\beta_i=1.0$ ,  $n_i=95$  and  $\gamma_i=1.0$  for  $i=1,2,...,8$ . The El Centro NS (1940) earthquake with a peak ground acceleration of 0.3g, as shown in Fig. 3-2, is used for the input excitation.

total building weight, that is 3,214.8 metric tons. It is observed that the drift of rubber bearings has been reduced by 50%.

For the nonlinear output controller presented in this study, i.e., Eq.(4.45), we first consider the case in which  $Q_i = 0$ ,  $i=2, 3, \dots, k$ . In other words, we only consider the first term of the controller in Eq.(4.45). In this case, we choose  $R=8.4 \times 10^{-6}$ , and all elements of the Q matrix are zero except  $Q(10,10)=99$ . Again, only  $x_b$  and  $\dot{x}_b$  were measured. The peak response quantities based on this controller are shown in Columns (7) and (8) of Table 4-2, designated as “Nonlinear 1”. We observe that the overall performance of this controller is slightly better than that of the Linear 1 controller. Next, we consider a more general case, in which  $Q_2 \neq 0$ , and  $Q_i = 0$  for  $i=3, 4, \dots, k$ . Weighting matrices are chosen as follows:  $R=8.4 \times 10^{-6}$  and all the elements of Q and  $Q_2$  are zero except  $Q(10,10)=70$  and  $Q_2(10,10)=0.3$ . The peak response quantities based on this nonlinear controller are shown in Columns (9) and (10) of Table 4-2, designated as “Nonlinear 2”. It is observed that, while the overall performance is similar to that of the Nonlinear 1 controller, the peak control force  $U$  has been increased by 18.3 % and the required control energy,  $\overline{U^2}$ , has decreased slightly.

With hybrid control, the building response quantities are well within the elastic range except the drift of rubber bearings. Hence, only the reduction for the drift of rubber bearings will be compared for different controllers. The results presented in Table 4-2 are based on the El Centro earthquake with a peak ground acceleration (PGA) of 0.3g. Based on the same design for various controllers presented in Table 4-2, simulation results for the percentages of reduction for the peak drift of rubber bearings as a function of PGA are shown in Fig. 4-3. It is observed from Fig. 4-3(a) that the percentages of the peak drift reduction for both Linear 1 and Nonlinear 1 controllers are quite similar and remain almost constant with the increase of PGA. On the other hand, the percentage of the peak drift reduction for rubber bearings for Nonlinear 2 increases as the PGA increases. Because of such a load-adaptive property, Nonlinear 2 controller is more effective in limiting the peak response of rubber bearings when the earthquake intensity exceeds the design one (i.e., 0.3g). It should be mentioned that the trend for the percentage of the response reduction for the superstructure is quite different from that for rubber bearings. In fact, as PGA increases, the percentage of the response

reduction for the superstructure decreases for all these controllers. However, since these response quantities are well within the elastic range, they are not presented.

The required peak control force,  $U$ , and the control energy,  $\overline{U^2}$ , are presented in Figs. 4-3(b) and 4-3(c), respectively. These quantities have been normalized, respectively, by the corresponding results for Linear 1 controller subjected to a 700 gals of PGA input. As observed from Figs. 4-3(b) and 4-3(c), the peak control force and the control energy required by Linear 1 and Nonlinear 1 controllers are almost the same. These quantities are significantly higher for the Nonlinear 2 controller. Consequently, the increase in the percentage of the response reduction for rubber bearings for Nonlinear 2 controller, as shown in Fig. 4-3(a), is achieved at the expense of the increase of the peak control force and total control energy, as shown in Figs. 4-3(b) and 4-3(c).

In summary, for active control of a 3-story linear building, it is shown that the static output polynomial controller has significant advantages over the linear static output controller in terms of the peak response reduction and the required control energy. For linear controller, the percentage of the peak response reduction remains constant for all level of the peak ground acceleration (PGA). However, for the static output polynomial controller, the percentage of reduction for the peak response increases as the PGA increases. In the case of hybrid control of a base-isolated building, the static output polynomial controller has advantage in terms of the peak response reduction only for the rubber-bearing isolators. In addition, the design of the static output polynomial controller has a computational advantage in terms of the rate of numerical convergence.

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## SECTION V

### CONCLUSIONS AND DISCUSSION

In section II, a new optimal controller, that is polynomial of any order in terms of the states of the system, is proposed for applications to seismically excited linear structures. A performance index, that is quadratic in control and polynomial of any order of the states is considered. The minimization of the performance index is based on the solution of the Hamilton-Jacobi-Bellman equation using a polynomial function of the states, which satisfies all the properties of a Lyapunov function. The optimal polynomial controller is derived analytically and the gain matrices are computed easily by solving matrix Riccati and Lyapunov equations. Such an optimal polynomial controller provides more degrees of freedom for the designer to penalize (or reduce) different response quantities. The performance of the controller has been investigated through numerical simulations for a wide range of earthquake intensities and different earthquake records.

In the case of SDOF structures, the optimal polynomial controller requires larger peak control force but smaller control energy in order to achieve the same level of peak response reductions as the LQR controller. For the earthquake ground motions which are predominantly harmonic, such as the Mexico earthquake, the optimal polynomial controller requires the same peak control force as that of the LQR controller, but using a significantly lower level of control energy, for the same level of the peak response reduction. In the case of MDOF structures, the differences in the peak control force and the control effort are quite small for both the optimal polynomial controller and the LQR controller, for the same level of the peak response reduction at the given design earthquake intensity. The percentage of the peak response reduction for the LQR controller remains constant with respect to the peak ground acceleration (PGA). However, the peak response reduction for the optimal polynomial controller increases with the increase of PGA. As a result, when the intensity of the actual earthquake is smaller than the design one, the level of peak response reductions, the required peak control force and the control effort are smaller for the optimal polynomial controller. However, as the actual earthquake intensity exceeds the design one, the optimal polynomial controller is capable of exerting a larger control force and control effort to achieve a higher level of peak response reductions.

The most important advantage of the optimal polynomial controller for linear structures is its strong dependence on the structural response. As a result, the polynomial controller is capable of reacting fast to unexpected strong earthquakes to achieve a higher level of the peak response reduction. Likewise, the optimal polynomial controller possesses some kind of adaptivity to the stochastic nature of earthquakes. These properties of the optimal polynomial controller are significant for practical applications of control systems to protect the integrity of civil engineering structures.

In section III, we propose an optimal controller for peak response control of seismically excited nonlinear and hysteretic structures. A performance index, that is quadratic in control and polynomial of any order of nonlinear states, is minimized based on the solution of the Hamilton-Jacobi-Bellman equation using a polynomial function of nonlinear states, which satisfies all the properties of a Lyapunov function. The resulting optimal controller is polynomial in nonlinear states of the system. Gain matrices for different parts of the controller are computed easily by solving Riccati and Lyapunov matrix equations.

Numerical simulations have been conducted for (i) a fixed-base elasto-plastic eight-story building equipped with active control systems and subjected to a strong earthquake, and (ii) the same building equipped with a hybrid control system consisting of actuators and lead-core rubber bearings. Simulation results indicate that the performance of the optimal polynomial controller presented is quite reasonable. For the building equipped with a hybrid control system, the main advantage of such a controller is its ability to increase the percentage of reduction for the peak response of rubber bearings with the increase of the earthquake intensity. Such a load-adaptive capability is desirable in protecting the base isolation system, in particular when the magnitude of earthquakes exceeds the design one. For the fixed-base elasto-plastic building subjected to a 1g design earthquake, the purpose of control is to prevent a catastrophic failure by reducing the ductility of the building to be smaller than 2.5. Simulation results indicate that the optimal polynomial controller presented has a slightly better capability than the linear controller for reducing the building ductility.

In section IV, we propose two static output polynomial controllers corresponding to optimal polynomial controllers for linear and nonlinear structures presented in Sections II and III, respectively. The static output controllers utilize only the information measured from a limited number of sensors installed at strategic locations without an observer, thus facilitating practical implementations of

active/hybrid control systems on civil engineering structures. For linear structures, the output controller is a polynomial of the states, whereas the output controller is a polynomial of nonlinear states for nonlinear and hysteretic structures. These static output controllers reduce to the optimal polynomial controllers when the full-state vector is measured. Simulation results indicate that the static output polynomial controllers proposed are viable control strategies for active/hybrid control of seismically excited civil engineering structures.

For active control of linear structures, simulation results demonstrate that the output polynomial controller has some advantages over the corresponding linear controller. These advantages include (i) a less requirement for the control energy, (ii) a load-adaptive capability to limit the peak response quantities of the structure when the magnitude of the earthquake exceeds the design one, and (iii) less difficulty involved in the design of the controller in terms of numerical convergence. For control of nonlinear or hysteretic structures, simulation results indicate that the most significant advantage of the static output polynomial controller over the corresponding linear controller is its load-adaptive capability to limit selected peak response quantities of the structure, when the magnitude of the earthquake exceeds the design one.

Finally, the optimal polynomial controllers and the corresponding static output polynomial controllers, for linear, nonlinear and hysteretic structures, proposed in this report are viable control strategies. These new controllers represent additions to control methods available in the literature for active/hybrid control of seismically excited civil engineering structures.

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## SECTION VI

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