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VIBRATION PROPERTIES OF BUILDINGS DETERMINED FROM RECORDED EARTHQUAKE MOTIONS

by

RAKESH K. GOEl ANll K. CHOPRA

A Report on Research Conducted Under Grant No. CMS-9416265 from the National Science Foundation

COLLEGE OF ENGINEERING

UNIVERSITY OF CALIFORNIA AT BERKELEY

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by

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> > December, 1997

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ABSTRACT

Most seismic codes specify empirical formulas to estimate the fundamental vibration period of buildings. Developed first in this investigation is a database on vibration properties period and damping ratio of the first two longitudinal, transverse, and torsional vibration modes - of buildings "measured" from their motions recorded during eight California earthquakes, starting from the 1971 San Fernando earthquake and ending with the 1994 Northridge earthquake. To this end, the natural vibration periods of 21 buildings have been measured by system identification methods applied to the motions of buildings recorded during the 1994 Northridge earthquake. These data have been combined with similar data from the motions of buildings recorded during the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Springs, 1987 Whittier, 1989 Lorna Prieta, 1990 Upland, and 1991 Sierra Madre earthquakes reported by several investigators. The "measured" fundamental periods of moment-resisting frame and shear wall buildings, extracted from the database, are then used to evaluate the empirical formulas specified in present US codes. It is shown that although current code formulas provide periods of moment-resisting frame buildings that are generally shorter than measured periods, these formulas can be improved to provide better correlation with the measured period data. The code formulas for concrete shear wall buildings are, however, inadequate. Subsequently, improved formulas are developed by calibrating the theoretical formulas against the measured period data through regression analysis. The theoretical formula for moment-resisting frame buildings is developed using Rayleigh's method whereas that for shear wall buildings is developed using Dunkerley's method. Finally factors to limit the period calculated by a "rational" analysis, such as Rayleigh's method or computer-based eigen-analysis, of both types of buildings are recommended for code applications.

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ACKNOWLEDGMENTS

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PREFACE

This work on vibration properties of buildings determined from recorded earthquake motions is divided into three parts. Part I is concerned with the code formulas for the fundamental periods of reinforced concrete (RC) and steel moment-resisting frame buildings, whereas Part II is focused on the code formulas for the fundamental period of concrete shear wall (SW) buildings. Appendices containing details that could not be included in the Part I and II are presented in Part **III.**

Each of the Part I and II first presents a brief introduction followed by the measured period database for the type of buildings under consideration. The fundamental period formulas in current US codes are reviewed and evaluated next. Subsequently, theoretical formulas are developed and calibrated against the measured period data using regression analysis techniques. Each part ends with recommendations for improved formula to estimate the fundamental period of a building and a factor to limit the period calculated by a "rational" analysis, such as Rayleigh's method.

Part III contains several appendices where detailed information on several aspects of the project is presented. Appendix· A presents the complete database on vibration properties of buildings determined from recorded earthquake motions. The database was compiled using Microsoft Access 2.0; the database is available electronically from the Earthquake Engineering Research Center at the University of California at Berkeley via their web page at www.eerc.berkeley.edu. Described is the format of the database followed by the techniques that can be used to add and extract data from the database. Although only the fundamental period data is used in Parts I and II of this report, the database includes the vibration period and damping ratio of the first two longitudinal, transverse, and torsional vibration modes.

Appendix B presents the theoretical background, along with examples, for various system identification techniques used in this investigation. Appendix C summarizes results of system identification for twenty-two buildings, conducted as part of this investigation. Appendix D develops theoretical formula and associated assumptions, which form the basis for empirical formulas in current US codes to estimate the fundamental period. Appendix E presents development of the fundamental period formula for moment-resisting frame buildings using Rayleigh's method. Appendix F describes the regression analysis method. Appendix G presents detailed development of theoretical formulas for fundamental period of concrete shear wall buildings using Rayleigh's and Dunkerley's methods. These formulas are used in Part IT of this report for developing improved period formula for shear wall buildings. Appendix H presents sketches of structural plans and information on shear wall dimensions for nine buildings (this information was not available for the remaining seven shear wall buildings) followed by the computations for the fundamental period using the code empirical formula and the proposed formula involving shear wall dimensions. Finally, Appendix I includes an exhaustive list of references on system identification techniques, data on period and damping ratio values for buildings, their recorded motions, and other relevant publications.

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PART I: MOMENT-RESISTING FRAME BUILDINGS

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INTRODUCTION

The fundamental vibration period of a building appears in the equation specified in building codes to calculate the design base shear and lateral forces. Because this building property can not be computed for a structure that is yet to be designed, building codes provide empirical formulas that depend on the building material (steel, R/C, etc.), building type (frame, shear wall etc.), and overall dimensions.

The period formulas in the 1997 UBC *(Uniform Building Code,* 1997) and the 1996 SEAOC recommendations *(Recommended Lateral Force Requirements,* 1996) are derived from those developed in 1975 as part of the ATC3-06 project *(Tentative Provisions,* 1978), based largely on periods of buildings "measured" from their motions recorded during the 1971 San Fernando earthquake. However, motions of many more buildings recorded during recent earthquakes, including the 1989 Loma Prieta and 1994 Northridge earthquakes, are now available. These recorded motions provide an opportunity to expand greatly the existing database on the fundamental vibration periods of buildings. To this end, the natural vibration periods of 21 buildings have been measured by system identification methods applied to the motions of buildings recorded during the 1994 Northridge earthquake (Goel and Chopra, 1997). These data have been combined with similar data from the motions of buildings recorded during the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Springs, 1987 Whittier, 1989 Loma Prieta, 1990 Upland, and 1991 Sierra Madre earthquakes reported by several investigators (an exhaustive list of references is available in Appendix I).

The objective of this investigation is to develop improved empirical formulas to estimate the fundamental vibration period of reinforced-concrete (RIC) and steel moment-resisting frame (MRF) buildings for use in equivalent lateral force analysis specified in building codes. Presented first is the expanded database for measured values of fundamental periods of MRF buildings, against which the empirical formulas in present US codes are evaluated. Subsequently, regression analysis of the measured data is used to develop improved formulas for estimating the fundamental periods of R/C MRF buildings and of steel MRF buildings. Finally, factors to limit the period calculated by a rational analysis, such as Rayleigh's method, are recommended.

 \mathcal{A}

PERIOD DATABASE

The data that are most useful but hard to come by are from structures shaken strongly but not deformed into the inelastic range. Such data are slow to accumulate because relatively few structures are installed with permanent accelerographs, and earthquakes causing strong motions of these instrumented buildings are infrequent. Thus, it is very important to investigate comprehensively the recorded motions when they do become available, as during the 1994 Northridge earthquake. Unfortunately, this obviously important goal is not always accomplished, as indicated by the fact that the vibration properties of only a few of the buildings whose motions were recorded during post-1971 earthquakes have been determined.

Available data on the fundamental vibration period of buildings measured from their motions recorded during several California earthquakes have been collected (Appendix A). This database contains data for a total of 106 buildings, including twenty-one buildings that experienced peak ground acceleration, $\ddot{u}_{g0} \ge 0.15g$ during the 1994 Northridge earthquake. The remaining data comes from motions of buildings recorded during the 1971 San Fernando earthquake and subsequent earthquakes *(Tentative Provisions,* 1978; Bertero et aI., 1988; Cole et aI., 1992; Hart and Vasudevan, 1975; Goel and Chopra, 1997).

Shown in Tables 1 and 2 is the subset of this database pertaining to MRF buildings including 37 data points for 27 R/C MRF buildings, and 53 data points for 42 steel MRF buildings; buildings subjected to $\ddot{u}_{g0} \ge 0.15g$ are identified with an asterisk (*). "C", "U", and "N" denote buildings instrumented by the California Strong Motion Instrumentation Program (CSMIP), United States Geological Survey (USGS), and National Oceanic and Atmospheric Administration (NOAA); "ATC" denotes buildings included in the ATC3-06 report *(Tentative Provisions,* 1978). The number of data points exceeds the number of buildings because the period of some buildings was determined from their motions recorded during more than one earthquake, or was reported by more than one investigator for the same earthquake.

No.	Location	\mathbf{D} Number	No. of Stories	Height (f _t)	Earthquake	Period T (sec)	
						Longitudinal	Transverse
$\mathbf{1}$	Emeryville	NA	30	300.0	Loma Prieta	2.80	2.80
$\overline{2}$	Los Angeles	NA	$\overline{9}$	120.0	San Fernando	1.40	1.30
$\overline{3}$	Los Angeles	\overline{NA}	14	160.0	San Fernando	1.80	1.60
$\overline{\mathbf{4}}$	Los Angeles	NA	13	166.0	San Fernando	$\overline{1.90}$	2.40
$\overline{5}$	Los Angeles	ATC_12	10	137.5	San Fernando	1.40	1.60
$\overline{6}$	Los Angeles	ATC_114	$\overline{7}$	61.0	San Fernando	0.90	1.20
$\overline{7}$	Los Angeles	ATC_2	$\overline{\tau}$	68.0	San Fernando	1.00	1.00
$\overline{8}$	Los Angeles	ATC_3	12	159.0	San Fernando	$\overline{\bf SW}$	1.33
$\overline{9}$	Los Angeles	ATC_5	19	196.8	San Fernando	2.15	2.22
10	Los Angeles	ATC_6	11	124.0	San Fernando	1.43	1.60
11	Los Angeles	ATC_7	$\overline{22}$	204.3	San Fernando	1.90	2.20
$\overline{1}2$	Los Angeles	ATC_9	16.	152.0	San Fernando	1.10	1.80
$\frac{1}{13*}$	Los Angeles	C24236	14	148.8	Northridge	NA	2.28
$14*$	Los Angeles	C24463	5	119.0	Northridge	1.46	1.61
$15*$	Los Angeles	C24463	$\overline{5}$	119.0	Whittier	1.40	1.30
$\overline{16}$ *	Los Angeles	C24569	15	274.0	Northridge	3.11	3.19
$17*$	Los Angeles	C24579	$\overline{9}$	141.0	Northridge	1.39	1.28
$18*$	Los Angeles	N220-2	20	196.8	San Fernando	$2.\overline{27}$	2.09
$19*$	Los Angeles	N220-2	20	196.8	San Fernando	2.27	2.13
$20*$	Los Angeles	N220-2	20	196.8	San Fernando	2.24	1.98
$21*$	Los Angeles	N446-8	22	204.3	San Fernando	1.94	2.14
$\overline{22*}$	Los Angeles	N446-8	$\overline{22}$	$\overline{204.3}$	San Fernando	1.84	$\overline{2.17}$
$23*$	North Hollywood	C24464	20	169.0	Northridge	2.60	2.62
24	North Hollywood	C ₂₄₄₆₄	20	169.0	Whittier	2.15	2.21
25	Pomona	C23511	$\overline{2}$	30.0	Upland	0.28	0.30
26	Pomona	C ₂₃₅₁₁	$\overline{2}$	30.0	Whittier	0.27	0.29
27	San Bruno	C58490	6	78.0	Loma Prieta	0.85	1.10
$\overline{28}$	San Bruno	C58490	$\overline{6}$	78.0	Loma Prieta	0.85	1.02
$\overline{29}$	San Jose	NA	$\overline{\mathbf{s}}$	65.0	Morgan Hill	0.83	0.83
30	San Jose	C57355	10	124.0	Loma Prieta	1.01	SW
$\overline{31}$	San Jose	C57355	10	124.0	Morgan Hill	0.91	$\overline{\bf SW}$
$\overline{32}$	San Jose	C57355	10	124.0	Mount Lewis	0.91	$\overline{\text{SW}}$
$\overline{33*}$	Sherman Oaks	ATC_4	13	124.0	San Fernando	1.20	1.40
$34*$	Sherman Oaks	C24322	13	184.5	Whittier	1.90	2.30
$35*$	Sherman Oaks	C24322	13	184.5	Whittier	\overline{NA}	2.44
36	Van Nuys	ATC_1	$\overline{\mathcal{I}}$	65.7	San Fernando	0.79	0.88
$37*$	Van Nuys	C24386	$\overline{\tau}$	65.7	Whittier	1.40	1.20

Table 1. Period data for R/C MRF buildings.

* Denotes buildings with $\ddot{u}_{g0} \ge 0.15$ g.

NA Indicates data not available.

SW Implies shear walls form the lateral load resisting system.

Number followed by "C" or "N" indicates the station number, and by "ATC" indicates the building number in ATC3-06 report.

No.	Location	\mathbf{D} Number	No. of Stories	Height (f _t)	Earthquake Name	Period T (sec)	
						Longitudinal	Transverse
$1*$	Alhambra	U482	13	198.0	Northridge	2.15	2.20
$2*$	Burbank	C ₂₄₃₇₀	$\overline{6}$	82.5	Northridge	1.36	1.38
$\overline{3}$ *	Burbank	C24370	6	82.5	Whittier	1.32	1.30
4	Long Beach	C14323	7	91.0	Whittier	1.19	1.50
5	Los Angeles	ATC 1	19	208.5	San Fernando	3.00	3.21
$\overline{6}$	Los Angeles	ATC_10	39	494.0	San Fernando	5.00	4.76
7	Los Angeles	ATC_11	15	202.0	San Fernando	2.91	2.79
8	Los Angeles	ATC_12	31	336.5	San Fernando	3.26	3.00
$\overline{9}$	Los Angeles	ATC_13	\overline{NA}	102.0	San Fernando	1.71	1.62
$\overline{10}$	Los Angeles	ATC 14	\overline{NA}	158.5	San Fernando	2.76	2.38
11	Los Angeles	ATC_15	41	599.0	San Fernando	6.00	5.50
12	Los Angeles	ATC_17	NA.	81.5	San Fernando	1.85	1.71
13	Los Angeles	ATC_3	NA	120.0	San Fernando	2.41	2.23
14	Los Angeles	ATC_4	27	368.5	San Fernando	4.38	4.18
15	Los Angeles	ATC_5	19	267.0	San Fernando	3.97	3.50
16	Los Angeles	ATC_6	17	207.0	San Fernando	3.00	2.28
$\overline{17}$	Los Angeles	ATC_7	NA	250.0	San Fernando	4.03	3.88
18	Los Angeles	ATC_8	32	428.5	San Fernando	5.00	5.40
$\overline{19}$	Los Angeles	ATC_9	NA	208.5	San Fernando	3.20	3.20
$20*$	Los Angeles	C24643	19	270.0	Northridge	3.89	BF
21	Los Angeles	N151-3	15	202.0	San Fernando	2.84	2.77
22	Los Angeles	N157-9	39	459.0	San Fernando	4.65	NA
23	Los Angeles	$N163 - 5$	41	599.0	San Fernando	6.06	$\overline{5.40}$
$24*$	Los Angeles	N172-4	$\overline{31}$	336.5	San Fernando	3.38	2.90
$25*$	Los Angeles	N172-4	31	336.5	San Fernando	3.42	2.94
26	Los Angeles	N184-6	27	398.0	San Fernando	4.27	4.26
27	Los Angeles	N184-6	27	398.0	San Fernando	4.37	4.24
$28*$	Los Angeles	N187-9	19	270.0	San Fernando	3.43	3.41
29	Los Angeles	N428-30	32	443.5	San Fernando	4.86	5.50
30	Los Angeles	N440-2	17	207.0	San Fernando	2.85	3.43
$31*$	Los Angeles	N461-3	$\overline{19}$	231.7	San Fernando	3.27	3.34
$32*$	Los Angeles	N461-3	19	231.7	San Fernando	3.02	3.30
$33*$	Los Angeles	N461-3	19	231.7	San Fernando	3.28	3.34
$34*$	Los Angeles	U5208	$\overline{6}$	104.0	Northridge	0.94	0.96
$35*$	Los Angeles	U5233	$\overline{32}$	430.0	Northridge	3.43	4.36
$36*$	Norwalk	U5239	7	96.0	Whittier	1.54	1.54
$37*$	Norwalk	U5239	$\overline{7}$	98.0	Whittier	1.30	1.22

Table 2. Period data for steel MRF buildings (continues ...).

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* Denotes buildings with $\ddot{u}_{go} \ge 0.15g$.

NA Indicates data not available.

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BF Implies braced frame and EBF means eccentric braced frame form the lateral load resisting system.

Number followed by "C", "N", or "U" indicates the station number, and by "ATC" indicates the building number in ATC3-06 report.

No.	Location	\mathbf{D}	No. of	Height	Earthquake	Period T (sec)	
		Number	Stories	(f _t)	Name	Longitudinal	Transverse
$38*$	Palm Springs	C12299	4	51.5	Palm Springs	0.71	0.63
39	Pasadena	ATC_2	9	128.5	San Fernando	1.29	1.44
40*	Pasadena	C ₂₄₅₄₁	6	92.3	Northridge	2.19	1.79
41	Pasadena	N267-8	9	130.0	Lytle Creek	1.02	1.13
42	Pasadena	$N267-8$	9	130.0	San Fernando	1.26	1.42
43	Richmond	C58506	3	45.0	Loma Prieta	0.63	0.74
44	Richmond	C58506	3	45.0	Loma Prieta	0.60	0.76
45	San Bernardino	C ₂₃₅₁₆	3	41.3	Whittier	0.50	0.46
46*	San Francisco	C58532	47	564.0	Loma Prieta	6.25	EBF
$47*$	San Francisco	C58532	47	564.0	Loma Prieta	6.50	EBF
48	San Francisco	NA	60	843.2	Loma Prieta	3.57	3.57
49*	San Jose	C57357	13	186.6	Loma Prieta	2.22	2.22
$50*$	San Jose	C57357	13	186.6	Loma Prieta	2.23	2.23
51	San Jose	C57357	13	186.6	Morgan Hill	2.05	2.16
52	San Jose	C57562	3	49.5	Loma Prieta	0.67	$0.69 -$
53	San Jose	C57562	3	49.5	Loma Prieta	0.69	0.69

Table 2. Period data for steel MRF buildings (... continued).

* Denotes buildings with $\ddot{u}_{go} \ge 0.15$ g.

NA Indicates data not available.

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BF Implies braced frame and EBF means eccentric braced frame form the lateral load resisting system.

Number followed by "C", "N", or "U" indicates the station number, and by "ATC" indicates the building number in ATC3-06 report.

CODE FORMULAS

The empirical formulas for the fundamental vibration period of MRF buildings specified in US building codes -- UBC-97 *(Uniform Building Code,* 1997), ATC3-06 *(Tentative Provisions,* 1978), SEAOC-96 *(Recommended Lateral Force Requirements,* 1996), and NEHRP-94 *(NEHRP,* 1994) -- are of the form:

$$
T = C_t H^{3/4} \tag{1}
$$

where H is the height of the building in feet above the base and the numerical coefficient C_i = 0.030 and 0.035 for R/C and steel MRF buildings, respectively, with one exception: in ATC3-06 recommendations $C_t = 0.025$ for R/C MRF buildings.

Equation (1), which first appeared in the ATC3-06 report, was derived using Rayleigh's method (Chopra, 1995) with the following assumptions: (1) equivalent static lateral forces are distributed linearly over the height of the building; (2) seismic base shear is proportional to $1/T^{2/3}$; and (3) deflections of the building are controlled by drift limitations (Appendix D). While the first two assumptions are evident, the third assumption implies that the height-wise distribution of stiffness is such that the inter-story drift under linearly distributed forces is uniform over the height of the building. Numerical values of $C_t = 0.035$ and 0.025 for steel and R/C MRF buildings were established in the ATC3-06 report based on measured periods of buildings from their motions recorded during the 1971 San Fernando earthquake. The commentary to SEAOC-88 *(Recommended Lateral Force Requirements,* 1988) states that "... data upon which the ATC3-06 values were based were re-examined for concrete frames and the 0.030 value judged to be more appropriate." This judgmental change was adopted by other codes.

The NEHRP-94 provisions also recommend an alternative formula for R/C and steel MRF buildings:

$$
T = 0.1N \tag{2}
$$

in which N is the number of stories. The simple formula is restricted to buildings not exceeding 12 stories in height and having a minimum story height of 10 ft. This formula was also specified in earlier versions of other seismic codes before it was replaced by Eq. (1).

UBC-97 *(Uniform Building Code,* 1997) and SEAOC-96 codes specify that the design base shear should be calculated from:

$$
V = CW \tag{3}
$$

in which W is the total seismic dead load and C is the seismic coefficient defined as

$$
C = \frac{C_{\nu}}{R} \frac{I}{T}; \quad 0.11 \, C_a \, I \le C \le \frac{2.5 \, C_a}{R} \, I \tag{4}
$$

and for seismic zone 4

$$
C \ge \frac{0.8Z N_v I}{R} \tag{5}
$$

in which coefficients C_v and C_a depend on the near-source factors, N_v and N_a , respectively, along with the soil profile and the seismic zone factor Z; *I* is the important factor; and the *R* is the numerical coefficient representative of the inherent overstrength and global ductility capacity of the lateral-load resisting system. The upper limit of $2.5C_aI + R$ on C applies to very-short period buildings, whereas the lower limit of $0.11C_aI$ (or $0.8ZN_vI \div R$ for seismic zone 4) applies to very-long period buildings. These limits imply that C becomes independent of the period for very-short or very-tall buildings. The upper limit existed, although in slightly different form, in

previous versions of UBC and SEAOC blue book; the lower limit, however, appeared only recently in UBC-97 and SEAOC-96.

The fundamental period T , calculated using the empirical Eq. (1), should be smaller than the "true" period to obtain a conservative estimate for the base shear. Therefore, code formulas are intentionally calibrated to underestimate the period by about 10 to 20 percent at first yield of the building *(Tentative Provisions,* 1978; *Recommended Lateral Force Requirements, 1988).*

The codes permit calculation of the period by a rational analysis, such as Rayleigh's method, but specify that the resulting value should not be longer than that estimated from the empirical formula (Eq. 1) by a certain factor. The factors specified in various US codes are: 1.2 in ATC3-06; 1.3 for high seismic region (Zone 4) and 1.4 for other regions (Zones 3, 2, and 1) in UBC-97; and a range of values with 1.2 for regions of high seismicity to 1.7 for regions of very low seismicity in NEHRP-94. The restriction in SEAOC-88 that the base shear calculated using the rational period shall not be less than 80 percent of the value obtained by using the empirical period corresponds to a factor of 1.4 (Cole et aI., 1992). These restrictions are imposed in order to safeguard against unreasonable assumptions in the rational analysis, which may lead to unreasonably long periods and hence unconservative values of base shear.

EVALUATION OF CODE FORMULAS

For buildings listed in Tables 1 and 2, the fundamental period identified from their motions recorded during earthquakes (subsequently denoted as measured period) is compared with the value given by the empirical code formula (Figures 1 to 4, part a); the measured periods in two orthogonal lateral directions are shown by solid circles connected by a vertical line, whereas code periods are shown by a single solid curve because the code formula gives the same period in the two directions if the lateral resisting systems are of the same type. Also included are curves for 1.2T and I.4T representing the limits imposed by codes on the rational value of the period for use in high seismic regions like California. Also compared are the two values of the seismic coefficient for each building calculated according to Eqs. (4) and (5) with $I=1$ for standard occupancy structures; $R = 3.5$ for ordinary concrete moment-resisting frames or $R = 4.5$ for ordinary steel moment-resisting frames; and $C_v = 0.64$ and $C_a = 0.44$ for seismic zone 4 with $Z = 0.4$, soil profile type S_D , i.e., stiff soil profile with average shear wave velocity between 180 and 360 m/s, and $N_v = N_a = 1$. The seismic coefficients corresponding to the measured periods in the two orthogonal directions are shown by solid circles connected by a vertical line, whereas the value based on the code period is shown by a solid curve.

RIC MRF Buildings

The data shown in Figure 1 for all RIC MRF buildings (Table 1) permit the following observations. The code formula is close to the lower bound of measured periods for buildings up to 160 ft high, but leads to periods significantly shorter than the measured periods for buildings in the height range of 160 ft to 225 ft. For such buildings, the lower bound tends to be about 1.2 times the code period. Although data for R/C MRF buildings taller than 225 ft is limited, it appears that the measured period of such buildings is much longer than the code value. The

measured periods of most R/C MRF buildings fall between the curves for $1.2T$ and $1.4T$, indicating that the code limits on the period calculated from rational analysis may be reasonable for high seismic regions like California; improved limits are proposed later. Data on measured periods of buildings in regions of low seismicity are needed to evaluate the much higher values of 1.7T permitted in NEHRP-94 to reflect the expectation that these buildings are likely to be more flexible (Commentary for NEHRP-94). The seismic coefficient calculated from the code period is conservative for most buildings because the code period is shorter than the measured period. For very-short (H less than about 50 ft) or very-tall (H more than about 250 ft) buildings, measured and code periods lead to the same seismic coefficient as C becomes independent of the period.

Since for design applications, it is most useful to examine the periods of buildings that have been shaken strongly but did not reach their yield limit, the data for buildings subjected to $\ddot{u}_{g0} \ge 0.15g$ (denoted with an * in Table 1) are separated in Figure 2. These data permit the following observations. For buildings of similar height, the fundamental period of strongly shaken buildings is longer compared to less strongly shaken buildings because of increased cracking of RIC that results in reduced stiffness. As a result the measured periods are in all cases longer than their code values, in most cases much longer. The lower bound of measured periods of strongly shaken buildings is close to 1.2 times the code period. Thus the coefficient $C_t = 0.030$ in current codes seems to be too small, and a value like 0.035, as will be seen later from the results of regression analysis, may be more appropriate. Just as observed from the data for all buildings, the seismic coefficient value calculated using the code period is conservative for most strongly shaken buildings and the conservatism is larger; exception occurs for very-short or verytall buildings for which the seismic coefficient is independent of the period.

Figure 1. Comparison of (a) measured and code periods, and (b) UBC-97 seismic coefficients from measured and code periods, for R/C MRF buildings.

Figure 2. Comparison of (a) measured and code periods, and (b) UBC-97 seismic coefficients from measured and code periods, for R/C MRF buildings with $\ddot{u}_{g0} \ge 0.15$ g.

Steel MRF Buildings

The data presented in Figure 3 for all steel MRF buildings (Table 2) permit the following observations. The code formula leads to periods that are generally shorter than measured periods, with the margin between the two being much larger than for R/C MRF buildings (Figure 1a). The code formula gives periods close to the lower bound of measured periods for buildings up to about 120 ft high, but 20-30% shorter for buildings taller than 120 ft; this conclusion is based on a larger data set compared to the meager data for R/C MRF buildings. For many buildings the measured periods exceed 1AT, indicating that the code limits on the period calculated from rational analysis are too restrictive. The seismic coefficient value calculated from the code period is conservative for most buildings and the degree of conservatism is larger compared to RIC buildings; as noted previously for R/C buildings, exception occurs for very-short or very-tall buildings for which the seismic coefficient is independent of the period.

The data for steel MRF buildings subjected to ground acceleration of 0.15g or more (denoted with an $*$ in Table 2) are separated in Figure 4. Comparing these data with Figure 3, it can be observed that the intensity of ground shaking has little influence on the measured period. The period elongates slightly due to stronger shaking but less than for R/C buildings which exhibit significantly longer periods due to increased cracking. Thus period data from all levels of shaking of buildings remaining essential elastic may be used to develop improved formulas for fundamental periods of steel MRF buildings.

Figure 3. Comparison of (a) measured and code periods, and (b) UBC-97 seismic coefficients from measured and code periods, for steel MRF buildings.

Figure 4. Comparison of (a) measured and code periods, and (b) UBC-97 seismic coefficients from measured and code periods, for steel MRF buildings with $\ddot{u}_{g0} \ge 0.15$ g.

THEORETICAL FORMULAS

Although the results presented in the preceding section indicate that the code formulas provide periods that are, in general, shorter than the measured periods, leading to conservative estimates of design forces, these formulas may be improved to provide better correlation with the measured periods. The relation between the period and building height in the improved formulas should be consistent with theoretical formulas presented next.

Using Rayleigh's method, the following relationships for fundamental period of multistory building frames with equal floor masses and story heights have been determined (Housner and Brady, 1963; Appendix E):

$$
T = C_1 \sqrt{H} \quad \text{or} \quad C_2 H \tag{6}
$$

The exponent of H and the numerical values of C_1 and C_2 depends on the stiffness properties, including their height-wise variation.

Another formula for the fundamental period has been derived by Rayleigh's method under the following assumptions: (1) lateral forces are distributed linearly (triangular variation of forces) over the building height; (2) base shear is proportional to $1/T^{\gamma}$; (3) weight of the building is distributed uniformly over its height; and (4) deflected shape of the building, under application of the lateral forces, is linear over its height, which implies that the inter-story drift is the same for all stories. The result of this derivation (Appendix D) is:

$$
T = C_3 H_{2-\gamma}^{\frac{1}{2-\gamma}} \tag{7}
$$

If the base shear is proportional to $1/T^{2/3}$, as in US codes (Eq. 4), $\gamma = 2/3$ and Eq. (7) gives:

$$
T = C_4 H^{3/4} \tag{8}
$$

which is in the ATC3-06 report and appears in current US codes.

The formulas presented in Eqs. (6) to (8) are of the form:

$$
T = \alpha H^{\beta} \tag{9}
$$

in which constants α and β depend on building properties, with β bounded between one-half and one. This form is adopted in the present investigation and constants α and β are determined by regression analysis of the measured period data.

REGRESSION ANALYSIS METHOD

For the purpose of regression analysis, it is useful to recast Eq. (9) as:

$$
y = a + \beta x \tag{10}
$$

in which $y = log(T)$, $a = log(\alpha)$, and $x = log(H)$. The intercept a at $x = 0$ and slope β of the straight line of Eq. (10) were determined by minimizing the squared error between the measured and computed periods, and then α was back calculated from the relationship $a = log(\alpha)$. The standard error of estimate is:

$$
s_e = \sqrt{\frac{\sum_{i=1}^{n} \left[y_i - (a + \beta x_i) \right]^2}{(n-2)}}
$$
(11)

in which $y_i = log(T_i)$ is the observed value (with T_i = measured period) and $(a + \beta x_i) = [\log(\alpha) + \beta \log(H_i)]$ is the computed value of the *i*th data, and *n* is the total number of data points. The *Se* represents scatter in the data and approaches, for large *n,* the standard deviation of the measured periods from the best-fit equation.

This procedure leads to values of α_R and β for Eq. (9) to represent the best-fit, in the least squared sense, to the measured period data. However, for code applications, the fonnula should provide lower values of the period, and this was obtained by lowering the best-fit line (Eq. 10) by s_e without changing its slope. Thus α_L , the lower value of α , is computed from:

$$
\log(\alpha_L) = \log(\alpha_R) - s_e \tag{12}
$$

Since s_e approaches the standard deviation for a large number of samples and y is log-normal, α_L is the mean-minus-one-standard-deviation or 15.9 percentile value, implying that 15.9 percent of the measured periods would fall below the curve corresponding to α_L (subsequently referred to as the best-fit - 1σ curve). If desired, α_L corresponding to other non-exceedance probabilities may be selected. Additional details of the regression analysis method and the procedure to estimate α_L are available elsewhere (Appendix F).

As mentioned previously, codes also specify an upper limit on the period calculated by rational analysis. This limit is established in this investigation by raising the best-fit line (Eq. 10) by s_e without changing its slope. Thus α_U , the value of α corresponding to the upper limit, is computed from:

$$
\log(\alpha_v) = \log(\alpha_R) + s_e \tag{13}
$$

Eq. (9) with α_U and β represents the best-fit + 1 σ curve which will be exceeded by 15.9 percent of the measured periods.

 α
RESULTS OF REGRESSION ANALYSIS

For each of the two categories of MRF buildings -- R/C and steel -- results are presented for the following regression analyses:

- 1. Unconstrained regression analysis to determine α and β .
- 2. Constrained regression analysis to determine α with the value of β from unconstrained regression analysis rounded-off to the nearest 0.05, e.g., $\beta = 0.92$ is rounded-off to 0.90, and $\beta = 0.63$ to 0.65.
- 3. Constrained regression analysis to determine α with β fixed at 0.75, the value in some current building codes (Eq. 1).
- 4. Constrained regression analysis to determine α with β fixed at 1.0, the value which corresponds to the alternative formula specified in NEHRP-94 (Eq. 2).

These regression analyses, implemented using the data from all buildings (Tables 1 and 2), lead to the formulas in Table 3 for R/C MRF buildings and in Table 4 for steel MRF buildings. In order to permit visual inspection, the formulas obtained from the second, third, and fourth regression analyses are presented in Figures 5 and 6 together with the measured period data. In order to preserve clarity in the plots, the formulas from the first regression, which are close to those from the second regression, are not included in these figures. The best-fit curves are labeled as T_R and the best-fit - 1σ curves as T_L .

RIC MRF Buildings

Figure 5 gives an impression of the scatter in the data of the measured periods relative to curves from regression analyses. As expected, the data fall above and below the curve, more or less evenly, and most of the data are above the best-fit - 1σ curve. Observe that, as expected, constrained regression generally implies a larger standard error of estimate, s_e (Table 3), indicating greater scatter of the data about the best-fit curve; s_{ℓ} increases as the value of β deviates increasingly from its unconstrained regression value. However, s_e is insensitive to β in the immediate vicinity of its unconstrained regression value, as evident from nearly identical values (up to three digits after the decimal point) of s_e from the first two regression analyses (Table 3). The value of s_e is significantly larger if $\beta = 0.75$ or 1.0, demonstrating that the period formula with either of these β values, as in present US codes, is less accurate. Thus the best choice is $\beta = 0.90$ with the associated $\alpha = 0.015$.

The values of α and β , determined from all available data, should be modified to recognize that the period of a RIC building lengthens at levels of motion large enough to cause cracking of concrete. The data from buildings experiencing $\ddot{u}_{g0} \ge 0.15g$ are too few (Figure 2) to permit a reliable value of β from unconstrained regression analysis. Therefore, constrained regression analysis of these data with $\beta = 0.90$, determined from the full set of data, was conducted to obtain $\alpha_L = 0.016$ and $\alpha_U = 0.023$ leading to:

$$
T_L = 0.016 H^{0.90}
$$
 (14)

and

$$
T_U = 0.023 \, H^{0.90} \tag{15}
$$

Eqs. (14) and (15) are plotted in Figure 7 together with the measured period data. As expected, very few data fall above the curve for T_u or below the curve for T_L . This indicates that Eq. (14) is suitable for estimating, conservatively, the fundamental period and Eq. (15) for limiting the period computed from rational analysis. This period should not be longer than $1.4T_L$; the factor 1.4 is determined as the ratio 0.023/0.016, rounded-off to one digit after the decimal point.

Steel MRF Buildings

Figure 6 gives an impression of the scatter in the measured period data relative to the best-fit curve. As expected, the data fall above and below the curve, more or less evenly, and most of the data are above the best-fit - 1σ curve. Observe that values of s_e are almost identical for unconstrained regression and constrained regression with rounded-off value of β because this value is close to the regressed value (Table 4); however, s_e increases as the value of β deviates increasingly from its unconstrained regression value. It is larger if $\beta = 0.75$ or 1.0, demonstrating that the period formula with either of these β values, as in present US codes, is less accurate. Thus the best choice is $\beta = 0.80$ with the associated $\alpha_L = 0.028$ and $\alpha_U = 0.045$ leading to:

$$
T_L = 0.028 \, H^{0.80} \tag{16}
$$

and

$$
T_U = 0.045 \, H^{0.80} \tag{17}
$$

Eqs. (16) and (17) are plotted in Figure 8 together with the measured period data. As observed earlier for RIC buildings, Eq. (16) is suitable for estimating, conservatively, the fundamental period and Eq. (17) for limiting the period from rational analysis. The period from rational analysis should not be longer than $1.6T_L$; the factor 1.6 is determined as the ratio 0.045/0.028, rounded-off to one digit after the decimal point. The period formula (Eq. 16) and the factor 1.6, determined from all available data, also apply to strongly shaken buildings because, as observed earlier, the intensity of shaking has little influence on the period of steel MRF buildings, so long as there is no significant yielding of the structure.

Regression Analysis Type	Period Formula				
	Best Fit	Best-Fit -1σ	S_{ρ}		
Unconstrained	$T_{\rm R} = 0.017 \, H^{0.92}$	$T_{I} = 0.014 H^{0.92}$	0.209		
Constrained with $\beta = 0.90$	$T_{R} = 0.018 H^{0.90}$	$T_L = 0.015 H^{0.90}$	0.209		
Constrained with $\beta = 0.75$	$T_{\rm R} = 0.038 H^{0.75}$	$T_L = 0.030 H^{0.75}$	0.229		
Constrained with $\beta = 1$	$T_{R} = 0.011H$	$T_{I} = 0.009H$	0.214		

Table 3. Results from regression analysis: R/C MRF buildings.

Table 4. Results from regression analysis: steel MRF buildings.

Regression Analysis Type	Period Formula				
	Best-Fit	Best-Fit - 1σ	$s_{\scriptscriptstyle e}$		
Unconstrained	$T_{\rm R} = 0.035 H^{0.805}$	$T_{\rm L} = 0.027 \, H^{0.805}$	0.233		
Constrained with $\beta = 0.80$	$T_{\rm p} = 0.035 H^{0.80}$	$T_{\rm r}=0.028H^{0.80}$	0.233		
Constrained with $\beta = 0.75$	$T_R = 0.046 H^{0.75}$	$T_{\rm r}=0.036H^{0.75}$	0.237		
Constrained with $\beta = 1.0$	$T_{R} = 0.013H$	$T_{I} = 0.009H$	0.277		

 $\hat{\mathcal{A}}$

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Figure 5. Regression analysis for R/C MRF buildings: (a) $\beta = 0.90$, and (b) $\beta = 0.75$ and 1.0.

Figure 6. Regression analysis for steel MRF buildings: (a) $\beta = 0.80$, and (b) $\beta = 0.75$ and 1.0.

Figure 7. Recommended period formula and upper limit for the fundamental period of R/C MRF buildings.

Figure 8. Recommended period formula and upper limit for the fundamental period of steel MRF buildings.

CONCLUSIONS AND RECOMMENDATIONS

Based on analysis of the available data for the fundamental vibration period of 27 R/C MRF buildings and 42 steel MRF buildings, measured from their motions recorded during earthquakes, Eqs. (14) and (16) are recommended for estimating, conservatively, the period of R/C and steel buildings, respectively. These formulas provide the "best" fit of Eq. (9) to the available data; the fit is better than possible with $\beta = 0.75$ or 1.0 in current US codes. Furthermore, the period from rational analysis should not be allowed to exceed the value from the recommended equations by a factor larger than 1.4 for R/C MRF buildings or 1.6 for steel MRF buildings. Since these recommendations are developed based on data from buildings in California, they should be applied with discretion to buildings in less seismic regions of the US or other parts of the world where building design practice is significantly different than in California.

Regression analyses that led to the recommended formulas should be repeated periodically on larger data sets. The database can be expanded by including buildings, other than those in Tables 1 and 2, whose motions recorded during past earthquakes have, so far, not been analyzed. Period data should also be developed for additional buildings when records of their motions during future earthquakes become available.

REFERENCES

- Bertero, V. V., Bendimerad, F. M., and Shah, H. C. (1988). *Fundamental Period of Reinforced RIC Moment-Resisting Frame Structures,* Report No. 87, John A. Blume Earthquake Engineering Center, Stanford University, CA, October.
- Chopra, A. K. (1995). *Dynamics of Structures: Theory and Applications to Earthquake Engineering,* Prentice Hall, Upper Saddle River, New Jersey.
- Cole, E. E., Tokas, C. V. and Meehan, J. F. (1992). "Analysis of Recorded Building Data to Verify or Improve 1991 Uniform Building Code CUBC) Period of Vibration Formulas," *Proceedings of SMIP92,* Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, 6-1 - 6-12, May.
- Hart, G. C. and Vasudevan, R. (1975). "Earthquake Design of Buildings: Damping," *Journal of the Structural Division,* ASCE, Vol. 101, No. STl, 11-30, January.
- Housner, G. W. and Brady, A. G. (1963). "Natural Periods of Vibration of Buildings," *Journal of Engineering Mechanics Division,* ASCE, Vol. 89, No. EM4, 31-65, August.
- *NEHRP Recommended Provisions for the Development of seismic Regulations for New Buildings.* (1994). Building Seismic Safety Council, Washington, D. C.
- *Recommended Lateral Force Requirements and Tentative Commentary.* (1988). Seismological Committee, Structural Engineers Association of California, San Francisco, CA.
- *Recommended Lateral Force Requirements and Commentary.* (1996). Seismological Committee, Structural Engineers Association of California, San Francisco, CA.
- *Tentative Provisions for the Development of Seismic Regulations for Buildings.* (1978). ATC3- 06, Applied Technological Council, Palo Alto, CA.
- *Uniform Building Code.* (1997). International Conference of Building Officials, Whittier, CA.

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$

PART II: **SHEAR WALL BUILDINGS**

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

 $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\mathcal{L}^{\text{max}}_{\text{max}}$, where $\mathcal{L}^{\text{max}}_{\text{max}}$

INTRODUCTION

The fundamental vibration period of a building appears in the equation specified in building codes to calculate the design base shear and lateral forces. Because this building property can not be computed for a structure that is yet to be designed, building codes provide empirical formulas that depend on the building material (steel, R/C , etc.), building type (frame, shear wall etc.), and overall dimensions.

The empirical period formulas for concrete shear wall (SW) buildings in the 1997 UBC *(Unifonn Building Code,* 1997) and the 1996 SEAOC bluebook *(Recommended Lateral Force Requirements,* 1996) were derived, by modifying the ATC3-06 formulas *(Tentative Provisions,* 1978), during development of the 1988 SEAOC bluebook to more accurately reflect the configuration and material properties of these systems *(Recommended Lateral Force Requirements,* 1988: Appendix lE2b(I)-T). The period formulas in ATC3-06 *(Tentative Provisions,* 1978) are based largely on motions of buildings recorded during the 1971 San Fernando earthquake. However, motions of many more buildings recorded during recent earthquakes, including the 1989 Loma Prieta and 1994 Northridge earthquakes, are now available. These recorded motions provide an opportunity to expand greatly the existing database on the fundamental vibration periods of buildings. To this end, the natural vibration periods of twenty-one buildings have been measured by system identification methods applied to the motions of buildings recorded during the 1994 Northridge earthquake (Appendix A). These data have been combined with similar data from the motions of buildings recorded during the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Springs, 1987 Whittier, 1989 Loma Prieta, 1990 Upland, and 1991 Sierra Madre earthquakes.

The objective of this investigation is to develop improved empirical formulas to estimate the fundamental vibration period of concrete SW buildings for use in equivalent lateral force analysis specified in building codes. Presented first is the expanded database for "measured" values of fundamental periods of SW buildings, against which the code formulas in present US codes are evaluated; similar work on limited data sets has appeared previously (e.g., Arias and Husid, 1962; Cole et al., 1992; Housner and Brady, 1963; Lee and Mau, 1997). It is shown that current code formulas for estimating the fundamental period of concrete SW buildings are grossly inadequate. Subsequently, an improved formula is developed by calibrating a theoretical formula, derived using Dunkerley's method, against the measured period data through regression analysis. Finally, a factor to limit the period calculated by a "rational" analysis, such as Rayleigh's method, is recommended.

PERIOD DATABASE

The data that are most useful but hard to come by are from structures shaken strongly but not deformed into the inelastic range. Such data are slow to accumulate because relatively few structures are installed with permanent accelerographs and earthquakes causing strong motions of these instrumented buildings are infrequent. Thus, it is very important to investigate comprehensively the recorded motions when they do become available, as during the 1994 Northridge earthquake. Unfortunately, this obviously important goal is not always accomplished, as indicated by the fact that the vibration properties of only a few of the buildings whose motions were recorded during post-1971 earthquakes have been determined.

Available data on the fundamental vibration period of buildings measured from their motions recorded during several California earthquakes have been collected (Appendix A). This database contains data for a total of 106 buildings, including twenty-one buildings that experienced peak ground acceleration, $\ddot{u}_{g0} \ge 0.15g$ during the 1994 Northridge earthquake. The remaining data comes from motions of buildings recorded during the 1971 San Fernando earthquake and subsequent earthquakes (Cole et aI., 1992; Gates et al., 1994; Hart et aI., 1975; Hart and Vasudevan, 1975; Marshall et aI., 1994; MacVerry, 1979; Werner et aI., 1992).

Shown in Table 1 is the subset of this database pertaining to 16 concrete SW buildings (27 data points); buildings subjected to peak ground acceleration, $\ddot{u}_{g0} \ge 0.15g$ are identified with an asterisk (*). "C" and "N" denote buildings instrumented by the California Strong Motion Instrumentation Program (CSMIP) and National Oceanic and Atmospheric Administration (NOAA); "ATC" denotes one of the buildings included in the ATC3-06 report *(Tentative Provisions,* 1978) for which the height and base dimensions were available from other sources, but these dimensions for other buildings could not be discerned from the plot presented in the ATC3-06 report. The number of data points exceeds the number of buildings because the period of some buildings was determined from their motions recorded during more than one earthquake, or was reported by more than one investigator for the same earthquake.

No.	Location	\mathbf{D} Number	No. of Stories	Height (f _t)	Earthquake	Period T (sec)		Width (f _t)	Length (f _t)
						Longi-	Trans-		
						tudinal	verse		
1	Belmont	C58262	$\mathbf{2}$	28.0	Loma Prieta	0.13	0.20	NA	NA
$2*$	Burbank	C24385	10	88.0	Northridge	0.60	0.56	75.0	215.0
$3*$	Burbank	C24385	10	88.0	Whittier	0.57	0.51	75.0	215.0
$\overline{\mathbf{4}}$	Hayward	C58488	$\overline{\mathbf{4}}$	50.0	Loma Prieta	0.15	0.22	NA	NA
$\overline{5}$	Long Beach	C14311	$\overline{5}$	71.0	Whittier	0.17	0.34	81.0	205.0
6	Los Angeles	ATC_3	12	159.0	San Fernando	1.15	MRF	60.0	161.0
$7*$	Los Angeles	C24468	8	127.0	Northridge	1.54	1.62	63.0	154.0
$8*$	Los Angeles	C24601	$\overline{17}$	149.7	Northridge	1.18	1.05	80.0	227.0
9	Los Angeles	C24601	$\overline{17}$	149.7	Sierra Madre	1.00	1.00	80.0	227.0
$10*$	Los Angeles	N253-5	12	161.5	San Fernando	1.19	1.14	76.0	156.0
$11*$	Los Angeles	N253-5	12	161.5	San Fernando	1.07	1.13	76.0	156.0
12	Palm Desert	C12284	$\overline{\mathbf{4}}$	50.2	Palm Springs	0.50	0.60	60.0	180.0
13	Pasadena	N ₂₆₄ -5	10	142.0	Lytle Creek	0.71	0.52	69.0	75.0
$14*$	Pasadena	N ₂₆₄ -5	10	142.0	San Fernando	0.98	0.62	69.0	75.0
$15*$	Pasadena	N264-5	10	142.0	San Fernando	0.97	0.62	69.0	75.0
16	Piedmont	C58334	3	36.0	Loma Prieta	0.18	0.18	NA	NA
17	Pleasant Hill	C58348	$\overline{3}$	40.6	Loma Prieta	0.38	0.46	77.0	131.0
18	San Bruno	C58394	9	104.0	Loma Prieta	1.20	1.30	84.0	192.0
19	San Bruno	C58394	9	104.0	Loma Prieta	1.00	1.45	84.0	192.0
20	San Jose	C57355	$\overline{10}$	124.0	Loma Prieta	MRF	0.75	82.0	190.0
21	San Jose	C57355	10	124.0	Morgan Hill	MRF	0.61	82.0	190.0
22	San Jose	C57355	10	124.0	Mount Lewis	MRF	0.61	82.0	190.0
23	San Jose	C57356	10	96.0	Loma Prieta	0.73	0.43	64.0	210.0
24	San Jose	C57356	10	96.0	Loma Prieta	0.70	0.42	64.0	210.0
25	San Jose	C57356	10	96.0	Morgan Hill	0.65	0.43	64.0	210.0
26	San Jose	C57356	10	96.0	Mount Lewis	0.63	0.41	64.0	210.0
$27*$	Watsonville	C47459	$\overline{4}$	66.3	Loma Prieta	0.24	0.35	71.0	75.0

Table 1. Period data for concrete SW buildings.

* Denotes buildings with $\ddot{u}_{go} \ge 0.15$ g.

NA Indicates data not available.

MRF Implies moment-resisting frames form the lateral load resisting system.

Number followed by "C" or "N" indicates the station number, and by "ATC" indicates the building number in ATC3-06 report.

CODE FORMULAS

The empirical formula for fundamental vibration period of concrete SW buildings specified in current US building codes -- UBC-97 *(Uniform Building Code,* 1997), SEAOC-96 *(Recommended Lateral Force Requirements,* 1996), and NEHRP-94 *(NEHRP,* 1994) -- is of the form:

$$
T = C_t H^{3/4} \tag{1}
$$

where H is the height of the building in feet above the base and the numerical coefficient $C_t = 0.02$. UBC-97 and SEAOC-96 permit an alternative value for C_t to be calculated from:

$$
C_t = 0.1/\sqrt{A_c} \tag{2}
$$

where A_c , the combined effective area (in square feet) of the shear walls, is defined as:

$$
A_c = \sum_{i=1}^{NW} A_i \left[0.2 + \left(D_i / H \right)^2 \right]; \qquad D_i / H \le 0.9
$$
 (3)

in which A_i is the horizontal cross-sectional area (in square feet) and D_i is dimension in the direction under consideration (in feet) of the *ith* shear wall in the first story of the structure; and *NW* is the total number of shear walls. The value of D; / *H* in Eq. (3) should not exceed 0.9.

ATC3-06 *(Tentative Provisions,* 1978) and earlier versions of other US codes specify a different formula:

$$
T = \frac{0.05H}{\sqrt{D}}
$$
 (4)

where D is the dimension, in feet, of the building at its base in the direction under consideration.

UBC-97 and SEAOC-96 codes specify that the design base shear should be calculated from:

$$
V = CW \tag{5}
$$

in which *W* is the total seismic dead load and C is the seismic coefficient defined as

$$
C = \frac{C_v I}{R T}; \quad 0.11 C_a I \le C \le \frac{2.5 C_a}{R} I \text{ and for seismic zone 4 } C \ge \frac{0.8 Z N_v I}{R}
$$
 (6)

in which coefficients C_v and C_a depend on the near-source factors, N_v and N_a , respectively, along with the soil profile and the seismic zone factor Z; *I* is the importance factor; and the *R* is the numerical coefficient representative of the inherent overstrength and global ductility capacity of the lateral-load resisting system. The upper limit of $2.5C_aI + R$ on C applies to very-short period buildings, whereas the lower limit of $0.11C_aI$ (or $0.8ZN_vI+R$ for seismic zone 4) applies to very-long period buildings. These limits imply that C becomes independent of the period for very-short or very-tall buildings. The upper limit existed, although in slightly different form, in previous versions of UBC and SEAOC bluebook; the lower limit, however, appeared only recently in UBC-97 and SEAOC-96.

The fundamental period T , calculated using the empirical Eqs. (1) or (4), should be smaller than the "true" period to obtain a conservative estimate for the base shear. Therefore, code formulas are intentionally calibrated to underestimate the period by about 10 to 20 percent at first yield of the building *(Tentative Provisions,* 1978; *Recommended Lateral Force Requirements, 1988).*

The codes permit calculation of the period by established methods of mechanics (referred to as "rational" analyses in this investigation), such as Rayleigh's method or computer-based eigen-value analysis, but specify that the resulting value should not be longer than that estimated from the empirical formula (Eqs. 1 or 4) by a certain factor. The factors specified in various US codes are: 1.2 in ATC3-06; 1.3 for high seismic region (Zone 4) and 1.4 for other regions (Zones 3, 2, and 1) in UBC-97 and SEAOC-96; and a range of values with 1.2 for regions of high

seismicity to 1.7 for regions of very low seismicity in NEHRP-94. The restriction in SEAOC-88 that the base shear calculated using the "rational" period shall not be less than 80 percent of the value obtained by using the empirical period corresponds to a factor of 1.4 (Cole et aI., 1992). These restrictions are imposed in order to safeguard against unreasonable assumptions in the "rational" analysis, which may lead to unreasonably long periods and hence unconservative values of base shear.

EVALUATION OF CODE FORMULAS

For buildings listed in Table 1, the fundamental period identified from their motions recorded during earthquakes (subsequently denoted as "measured" period) is compared with the values given by the code empirical formulas (Figures 1 to 3, part a). Also compared are the two values of the seismic coefficient for each building calculated according to Eq. (6) with $I = 1$ for standard occupancy structures; $R = 5.5$ for concrete shear walls; and $C_v = 0.64$ and $C_a = 0.44$ for seismic zone 4 with $Z = 0.4$, soil profile type S_D , i.e., stiff soil profile with average shear wave velocity between 180 and 360 m/s, and $N_v = N_a = 1$ (Figures 1 to 3, part b).

Code Formula: Eq. (1) With $C_t = 0.02$

For all buildings in Table 1, the periods and seismic coefficients are plotted against the building height in Figure 1. The measured periods in two orthogonal directions are shown by circles (solid for $\ddot{u}_{g0} \ge 0.15g$, open for $\ddot{u}_{g0} < 0.15g$) connected by a vertical line, whereas the code period is shown by a solid curve because the code formula gives the same period in the two directions if the lateral-force resisting systems are of the same type. Also included are the curves for 1.2T and 1.4T representing the limits imposed by codes on a "rational" value of the period for use in high seismic regions like California. The seismic coefficients (Eq. 6) corresponding to the measured periods in the two orthogonal directions are also shown by circles connected by a vertical line, whereas the value based on the code period is shown by a solid curve.

Figure 1 leads to the following observations. For a majority of buildings, the code formula gives a period longer than the measured value. In contrast, for concrete and steel moment-resisting frame buildings, the code formula almost always gives a period shorter than the measured value (Part I). The longer period from the code formula leads to seismic coefficient smaller than the value based on the measured period if the period falls outside the flat portion of the seismic coefficient spectrum; otherwise the two periods lead to the same seismic coefficient. For most of the remaining buildings, the code formula gives a period much shorter than the measured value and seismic coefficient much larger than the value based on the measured period. Since the code period for many buildings is longer than the measured period, the limits of *1.2T* or $1.4T$ for the period calculated from a "rational" analysis are obviously inappropriate.

The building height alone is not sufficient to estimate accurately the fundamental period of SW buildings because measured periods of buildings with similar heights can be very different, whereas they can be similar for buildings with very different heights. For example, in Table 1 the measured longitudinal periods of buildings 4 and 12 of nearly equal heights differ by a factor of more than three; the heights of these buildings are 50 ft and 50.2 ft whereas the periods are 0.15 sec and 0.50 sec, respectively. On the other hand, measured longitudinal periods of buildings 13 and 23 are close even though building 13 is 50% taller than building 23; periods of these buildings are 0.71 sec and 0.73 sec, whereas the heights are 142 ft and 96 ft, respectively. The poor correlation between the building height and the measured period is also apparent from the significant scatter of the measured period data (Figure 1a).

Alternate Code Formula: Eq. (1) With C_t From Eqs. (2) and (3)

Table 2 lists a subset of nine buildings (17 data points) with their A_c values calculated from Eq. (3) using shear wall dimensions obtained from structural drawings; for details see Appendix H. These dimensions were not available for the remaining seven buildings in Table 1.

In Figure 2 the alternate code formula for estimating the fundamental period is compared with the measured periods of the nine buildings. The code period is determined from Eqs. (1) to (3) using the calculated value of A_c and plotted against $H^{3/4} \div \sqrt{A_c}$. This comparison shows that

the alternate code formula almost always gives a value for the period that is much shorter than the measured periods, and a value for the seismic coefficient that is much higher than from the measured periods. The measured periods of most buildings are longer than the code imposed limits of *1.2T* and 1.4*T* on the period computed from a "rational" analysis. Although the code period formula gives a conservative value for the seismic coefficient, the degree of conservatism seems excessive for most buildings considered in this investigation.

No.	ID No.	Height (f _t)	Measured Period		A_c (Sq. ft)		\overline{A}_{e} (%)	
			Longi-	Trans-	Longi-	Trans-	Longi-	Trans-
			tudinal	verse	tudinal	verse	tudinal	verse
$1*$	C ₂₄₃ 85	88.0	0.60	0.56	83.5	92.1	0.1642	0.1677
$2*$	C24385	88.0	0.57	0.51	83.5	92.1	0.1642	0.1677
$3*$	C ₂₄₄₆₈	127.0	1.54	1.62	13.8	34.2	0.0265	0.0345
$4*$	C ₂₄₆₀₁	149.7	1.18	1.05	63.3	106.9	0.0635	0.0939
5	C ₂₄₆₀₁	149.7	1.00	1.00	63.3	106.9	0.0635	0.0939
6	C12284	50.2	0.50	0.60	21.5	17.7	0.0537	0.0550
7	C58334	36.0	0.18	0.18	26.2	26.2	0.1311	0.1311
8	C58348	40.6	0.38	0.46	26.6	12.2	0.1118	0.0501
9	C58394	104.0	1.20	1.30	21.5	22.2	0.0330	0.0189
10	C58394	104.0	1.00	1.45	21.5	22.2	0.0330	0.0189
11	C57355	124.0	MRF	0.75	MRF	104.5	MRF	0.2751
12	C57355	124.0	MRF	0.61	MRF	104.5	MRF	0.2751
13	C57355	124.0	MRF	0.61	MRF	104.5	MRF	0.2751
14	C57356	96.0	0.73	0.43	60.7	84.5	0.1280	0.1547
15	C57356	96.0	0.70	0.42	60.7	84.5	0.1280	0.1547
16	C57356	96.0	0.65	0.43	60.7	84.5	0.1280	0.1547
17	C57356	96.0	0.63	0.41	60.7	84.5	0.1280	0.1547

Table 2. Measured periods and areas of selected concrete SW buildings.

* Denotes buildings with $\ddot{u}_{g0} \ge 0.15$ g.

MRF Implies moment-resisting frames form the lateral load resisting system. Number followed by "C" indicates the station number.

ATC3·06 Formula

In Figure 3, the ATC3-06 formula for estimating the fundamental period is compared with the measured periods of all buildings listed in Table 1. The code period is determined from Eq. (4) using the H and D dimensions of the building (Table 1) and plotted against $H \div \sqrt{D}$. This comparison demonstrates that Eq. (4) significantly underestimates the period and

considerably overestimates the seismic coefficient for many buildings and the ATC3-06 imposed limit of *1.2T* is too restrictive.

The ratio $H \div \sqrt{D}$ is not sufficient to estimate accurately the fundamental period of concrete SW buildings because measured periods of buildings with similar values of this ratio can be very different, whereas they can be similar for buildings with very different values of $H \div \sqrt{D}$. For example, in Table 1 the measured transverse period of building 18 and measured longitudinal period of building 27 -- two buildings with similar values of $H \div \sqrt{D}$ -- differ by nearly a factor of five; $H \div \sqrt{D} = 7.51$ and 7.87, and measured periods = 1.30 sec and 0.24 sec, respectively. On the other hand, the measured longitudinal and transverse periods of building 9 are the same, equal to 1 sec, even though the values of $H \div \sqrt{D}$ in the two directions are 16.7 and 9.93. The poor correlation between the ratio $H \div \sqrt{D}$ and the measured periods is also apparent from a large scatter of the measured period data (Figure 3a).

Figure 1. Comparison of (a) measured and code periods, and (b) UBC-97 seismic coefficients from measured and code periods.

Figure 2. Comparison of (a) measured and code periods, and (b) UBC-97 seismic coefficients from measured and code periods; code periods are calculated from the alternate formula.

Figure 3. Comparison of (a) measured and code periods, and (b) UBC-97 seismic coefficients from measured and code periods; code periods are calculated from the ATC3-06 formula.

THEORETICAL FORMULAS

The observations in the preceding section clearly indicate that the current code formulas for estimating the fundamental period of concrete SW buildings are grossly inadequate. For this purpose, equations for the fundamental period are derived using established analytical procedures. Based on Dunkerley's method (Jacobsen and Ayre, 1958: pages 119-120 and 502- 505; Inman, 1996: pages 442-449; Veletsos and Yang, 1977), the fundamental period of a cantilever, considering flexural and shear deformations, is:

$$
T = \sqrt{T_F^2 + T_S^2} \tag{7}
$$

in which T_F and T_S are the fundamental periods of pure-flexural and pure-shear cantilevers, respectively. For uniform cantilevers T_F and T_S are given by (Chopra, 1995: page 592; Timoshenko et aI., 1974: pages 424-431; Jacobsen and Ayre, 1958: pages 471-496):

$$
T_F = \frac{2\pi}{3.516} \sqrt{\frac{m}{EI}} H^2
$$
\n⁽⁸⁾

$$
T_s = 4\sqrt{\frac{m}{\kappa G}} \frac{1}{\sqrt{A}} H
$$
\n(9)

In Eqs. (8) and (9), *m* is the mass per unit height, *E* is the modulus of elasticity, *G* is the shear modulus, *I* is the section moment of inertia, *A* is the section area, and κ is the shape factor to account for nonuniform distribution of shear stresses $(= 5/6$ for rectangular sections). Combining Eqs. (7) to (9) and recognizing that $G = E + 2(1 + \mu)$, where the Poison's ratio $\mu = 0.2$ for concrete, leads to:

$$
T = 4\sqrt{\frac{m}{\kappa G}} \frac{1}{\sqrt{A_e}} H
$$
 (10)

with

$$
A_e = \frac{A}{\left[1 + 0.83 \left(\frac{H}{D}\right)^2\right]}
$$
(11)

where D is the plan dimension of the cantilever in the direction under consideration. Comparing Eqs. (10) and (11) with Eq. (9) reveals that the fundamental period of a cantilever considering flexural and shear deformations may be computed by replacing the area A in Eq. (9) with the equivalent shear area *Ae* given by Eq. (11).

The period *T* from Eq. (10) normalized by T_F is plotted in Figure 4 against the ratio $H \div D$ on a logarithmic scale. Also shown is the period of a pure-shear cantilever and of a pureflexural cantilever. Eq. (10) approaches the period of a pure-shear cantilever (Eq. 9) as $H \div D$ becomes small and the period of a pure-flexural cantilever (Eq. 8) for large values of $H \div D$. For all practical purposes, the contribution of flexure can be neglected for shear walls with $H \div D < 0.2$ whereas the contribution of shear can be neglected for shear walls with $H \div D > 5$; the resulting error is less than 2%. However, both shear and flexural deformations should be included for shear walls with $0.2 \leq H \div D \leq 5$.

Equation (10), based on Dunkerley's method, provides a highly accurate value for the true fundamental period of a shear-flexural cantilever. This can be demonstrated by recognizing that the exact period is bounded by the periods obtained from Dunkerley's and Rayleigh's methods; Dunkerley's method gives a period longer than the exact value (Jacobsen and Ayre, 1958: pages 113-120; Inman, 1996: pages 442-449), whereas Rayleigh's method provides a shorter period (Chopra, 1995: page 554). Also shown in Figure 4 is the period determined by Rayleigh's method using the deflected shape due to lateral forces varying linearly with height, considering both shear and flexural deformations; details are available in Appendix G. The resulting period is very close to that obtained from Eq. (10), derived using Dunkerley's method; the difference between the two periods is no more than 3%. Since the exact period lies between the two approximate values, Eq. (10) errs by less than 3%.

Figure 4. Fundamental period of cantilever beams.

Now consider a class of symmetric-plan buildings -- symmetric in the lateral direction considered -- with lateral-force resisting system comprised of a number of uncoupled (i.e., without coupling beams) shear walls connected through rigid floor diaphragms. Assuming that the stiffness properties of each wall are uniform over its height, the equivalent shear area, *Ae,* is given by a generalized version of Eq. (11) (details are available in Appendix G):

$$
A_{\epsilon} = \sum_{i=1}^{NW} \left(\frac{H}{H_i} \right)^2 \frac{A_i}{\left[1 + 0.83 \left(\frac{H_i}{D_i} \right)^2 \right]}
$$
(12)

where A_i , H_i , and D_i are the area, height, and dimension in the direction under consideration of the *ith* shear wall, and NW is the number of shear walls. With *Ae* so defined, Eq. (10) is valid for a system of shear walls of different height.

Equation (10) is now expressed in a form convenient for buildings:

$$
T = 40 \sqrt{\frac{\rho}{\kappa G}} \frac{1}{\sqrt{\overline{A}_e}} H
$$
\n(13)

where ρ is the average mass density, defined as the total building mass $(= mH)$ divided by the total building volume $(=A_BH - A_B$ is the building plan area), i.e., $\rho = m/A_B$; and \overline{A}_e is the equivalent shear area expressed as a percentage of A_B , i.e.,

$$
\overline{A}_e = 100 \frac{A_e}{A_B} \tag{14}
$$

Equation (13) applies only to those buildings in which lateral load resistance is provided by uncoupled shear walls. Theoretical formulas for the fundamental period of buildings with coupled shear walls are available in Rutenberg (1975), and for buildings with a combination of shear walls and moment-resisting frames in Heiderbrecht and Stafford-Smith (1973) and Stafford-Smith and Crowe (1986). It seems that these formulas can not be simplified to the form ofEq. (13).

Sozen (1989) and Wallace and Moehle (1992) also presented a formula for the fundamental vibration period of SW buildings. Their formula was developed based on pureflexural cantilever idealization of SW buildings and ignored the influence of shear deformations. Furthermore, the numerical constant in their formula was determined based on assumed material properties and effective member stiffness equal to half its initial value. In contrast, the formula developed in this investigation (Eq. 13) includes both flexural and shear deformations and the numerical constant is determined directly from regression analysis of measured period data as described in the following sections.

REGRESSION ANALYSIS METHOD

Although $\overline{C} = 40\sqrt{\rho/\kappa G}$ in Eq. (13) can be calculated from building properties, it is determined from regression analysis to account for variations in properties among various buildings and for differences between building behavior and its idealization. For this purpose, it is useful to write Eq. (13) as:

$$
T = \overline{C} \frac{1}{\sqrt{\overline{A}_e}} H
$$
 (15)

and recast it as:

$$
y = a + x \tag{16}
$$

in which $y = log(T)$, $a = log(\overline{C})$, and $x = log(H + \sqrt{\overline{A}_{e}})$. The intercept a at $x = 0$ of the straight line in Eq. (16) was determined by minimizing the squared error between the measured and computed periods, and then \overline{C} was back-calculated from the relationship $a = \log(\overline{C})$. The standard error of estimate is:

$$
s_e = \sqrt{\frac{\sum_{i=1}^{n} [y_i - (a + x_i)]^2}{(n-2)}}
$$
(17)

in which $y_i = \log(T_i)$ is the observed value (with T_i = measured period) and $(a + x_i) = log(\overline{C}) + log((H + \sqrt{\overline{A}_e}))$ is the computed value of the *i*th data, and *n* is the total number of data points. The s_e represents scatter in the data and approaches, for large *n*, the standard deviation of the measured period data from the best-fit equation.

This procedure leads to the value of \overline{C}_R for Eq. (15) to represent the best-fit, in the least squared sense, to the measured period data. However, for code applications the formula should provide a lower value of the period and this was obtained by lowering the best-fit line (Eq. 16) by S_e without changing its slope. Thus \overline{C}_L , the lower value of \overline{C} , is computed from:

$$
\log(\overline{C}_L) = \log(\overline{C}_R) - s_e \tag{18}
$$

Since s_e approaches the standard deviation for large number of samples and *y* is log-normal, \overline{C}_L is the mean-minus-one-standard-deviation or 15.9 percentile value, implying that 15.9 percent of the measured periods would fall below the curve corresponding to \overline{C}_L (subsequently referred to as the best-fit - 1σ curve). If desired, \overline{C}_L corresponding to other non-exceedance probabilities may be selected. Additional details of the regression analysis method and the procedure to estimate \overline{C}_L are available in Appendix F.

As mentioned previously, codes also specify an upper limit on the period calculated by a "rational" analysis. This limit is established in this investigation by raising the best-fit line (Eq. 16) by s_e without changing its slope. Thus \overline{C}_v , the upper value of \overline{C} corresponding to the upper limit, is computed from:

$$
\log(\overline{C}_U) = \log(\overline{C}_R) + s_e \tag{19}
$$

Eq. (15) with \overline{C}_U represents the best-fit + 1 σ curve which will be exceeded by 15.9 percent of the measured periods.

Regression analysis in the log-log space (Eq. 16) is preferred over the direct regression on Eq. (15) because it permits convenient development of the best-fit – 1σ and best-fit + 1σ curves; both regression analyses give essentially identical values of \overline{C}_R .

RESULTS OF REGRESSION ANALYSIS

The formula for estimating the fundamental period of concrete SW buildings was obtained by calibrating the theoretical formula of Eq. (15) by regression analysis of the measured period data for nine concrete SW buildings (17 data points) listed in Table 2. For each building, the equivalent area \overline{A}_e was calculated from Eqs. (12) and (14) using dimensions from structural plans (Appendix H); for shear walls with dimensions varying over height, Ai and *Di* were taken as the values at the base. Regression analysis gives $\overline{C}_R = 0.0023$ and $\overline{C}_L = 0.0018$. Using these values for \overline{C} in Eq. (15) give T_R and T_L , the best-fit and best-fit - 1 σ values of the period, respectively.

Figure 5, Results of regression analysis: all buildings.

These period values are plotted against $H \div \sqrt{A_e}$ in Figure 5, together with the measured periods shown in circles; the measured periods of a building in the two orthogonal directions are

not joined by a vertical line because the ratio $H \div \sqrt{\overline{A}_e}$ is different if the shear wall areas are not the same in the two directions. Figure 5 permits the following observations. As expected the measured period data falls above and below (more or less evenly) the best-fit curve. The best-fit equation correlates with measured periods much better (error of estimate s_e = 0.143) than formulas (Eqs. 1 to 3) in UBC-97 (s_e = 0.546). It is apparent that the form of Eq. (15) includes many of the important parameters that influence the fundamental period of concrete SW buildings.

In passing, observe that the value of $\overline{C}_R = 0.0023$ for concrete buildings with $E =$ 3.1×10^6 psi (21.4 × 10³ MPa) and μ = 0.2 corresponds to $\rho \approx 0.47$ lb -sec²/ ft⁴ = 240 Kg/m³ or unit weight = 15 pcf, implying approximately 10% solids and 90% voids in the building, which seem reasonable for many buildings.

The values of \overline{C} determined from all available data, should be modified to recognize that the period of a concrete building lengthens at moderate to high levels of ground shaking. Regression analysis of the data from buildings experiencing peak ground acceleration $\ddot{u}_{g0} \ge 0.15g$ (denoted with $*$ in Table 2) gives:

$$
T_L = 0.0019 \frac{1}{\sqrt{\bar{A}_e}} H
$$
 (20)

$$
T_U = 0.0026 \frac{1}{\sqrt{\overline{A}_e}} H
$$
\n(21)

Eqs. (20) and (21) are plotted in Figure 6 with the measured period data. As expected, very few data fall above the curve for T_U or below the curve for T_L , indicating that Eq. (20) is suitable for estimating, conservatively, the fundamental periods and Eq. (21) for limiting the period computed from "rational" analysis. Thus the period from "rational" analysis should not be longer than $1.4T_L$; the factor is determined as the ratio 0.0026÷0.0019, rounded-off to one digit after the decimal point.

Figure 6. Results of regression analysis: buildings with $\ddot{u}_{g0} \ge 0.15$ g.

In using Eq. (12) to calculate A_{ϵ} for nonuniform shear walls, A_i and D_i should be defined as the area and the dimension in the direction under consideration, respectively, at the base of the wall. To provide support for this recommendation, consider the building identified as C57356 in Table 2. The thickness of the shear walls in this ten-story building is 11 inch (30 cm) in the first story, 9 inch (23 cm) in second to fourth stories, 8 inch (20 cm) in fifth to eighth stories, and 7 inch (18 cm) in ninth and tenth stories. Calculating \overline{A}_e by using $D_i = 11$ inch (at the base), 8 inch (at mid height) and 7 inch (at the roof), and substituting in Eq. (20) gives period values 0.36 sec, 0.42 sec, and 0.45 sec, respectively. Although the mid-height-value of *Di* gives the period value close to the "measured" period (0.41 to 0.43 sec, by different investigators), the base value of *Di* provides a shorter period, leading to a conservative value of base shear. This
recommendation is consistent with the current codes *(Uniform Building Code, 1997; Recommended Lateral Force Requirements, 1996).*

 \sim

 $\sim 10^{-11}$

 $\ddot{}$

 $\sim 10^{-10}$

CONCLUSIONS AND RECOMMENDATIONS

Based on the analysis of the available data for the fundamental vibration period of nine concrete SW buildings (17 data points), measured from their motions recorded during earthquakes, Eq. (20) with \overline{A}_e calculated from Eqs. (12) and (14) using wall dimensions at the base is recommended for conservatively estimating the fundamental period of concrete SW buildings. This formula provides the "best" fit of Eq. (15) to the available data; the fit is better than possible from formulas (Eqs. I to 3) in current US codes. Furthermore, the period from "rational" analysis should not be allowed to exceed the value from the recommended equation by a factor larger than 1.4. Since these recommendations are developed based on data from buildings in California, they should be applied with discretion to buildings in less seismic regions of the US or other parts of the world where building design practice is significantly different than in California.

Regression analyses that led to the recommended formulas should be repeated periodically on larger data sets. The database can be expanded by including buildings, other than those in Tables 1 and 2, whose motions recorded during past earthquakes have, so far, not been analyzed. Period data should also be developed for additional buildings when records of their motions during future earthquakes become available.

REFERENCES

- Arias, A. and Husid, R. (1962). "Empirical Formula for the Computation of Natural Periods of Reinforced Concrete Buildings with Shear Walls," Reinsta del IDIEM, 39(3).
- Chopra, A. K. (1995). *Dynamics of Structures: Theory and Applications to Earthquake Engineering,* Prentice Hall, Englewood Cliffs, N.J.
- Cole, E. E., Tokas, C. V. and Meehan, J. F. (1992). "Analysis of Recorded Building Data to Verify or Improve 1991 Unifonn Building Code (UBC) Period of Vibration Fonnulas," *Proceedings of SMIP92*, Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, 6-1 - 6-12.
- Gates, W. E., Hart, G. C., Gupta, S. and Srinivasan, M. (1994). "Evaluation of Overturning Forces of Shear Wall Buildings," *Proceedings of SMIP94*, Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, 105-120.
- Hart, G. C. and Vasudevan, R. (1975). "Earthquake Design of Buildings: Damping," *Journal of the Structural Division,* ASCE, 101(STl), 11-30.
- Hart, G. C., DiJulio, R. M. and Lew, M. (1975). "Torsional Response of High-Rise Buildings," *Journal of the Structural Division, ASCE, 101(ST2), 397-416.*
- Heidebrecht, A. C. and Stafford-Smith, B. (1973). "Approximate Analysis of Tall Wall-Frame Structures," *Journal of Structural Division*, ASCE, 99(ST2), 199-221.
- Housner, G. W. and Brady, A. G. (1963). "Natural Periods of Vibration of Buildings," *Journal ofEngineering Mechanics Division,* ASCE, 89(EM4), 31-65.

Inman, D. J. (1996). *Engineering Vibrations,* Prentice Hall, Englewood Cliffs, New Jersey.

Jacobsen, L. S. and Ayre, R. S. (1958). *Engineering Vibrations,* McGraw-Hill, New York.

- Li, Y. and Mau, S. T. (1997). "Learning from Recorded Earthquake Motion of Buildings," *Journal of Structural Engineering, ASCE, 123(1), 62-69.*
- Marshall, R. D., Phan, L. T. and Celebi, M. (1994). "Full-Scale Measurement of Building Response to Ambient Vibration and the Lorna Prieta Earthquake," *Proceedings of Fifth U. S. National Conference of Earthquake Engineering, Vol. II, 661-670.*
- McVerry, G. H. (1979). *Frequency Domain Identification ofStructural Models from Earthquake Records,* Report No. EERL 79-02, Earthquake Engineering research Laboratory, California Institute of Technology, Pasadena, CA, October.
- *NEHRP Recommended Provisions for the Development of seismic Regulations for New Buildings.* (1994). Building Seismic Safety Council, Washington, D. C.
- *Recommended Lateral Force Requirements and Commentary.* (1988). Seismological Committee, Structural Engineers Association of California, San Francisco, CA.
- *Recommended Lateral Force Requirements and Commentary.* (1996). Seismological Committee, Structural Engineers Association of California, San Francisco, CA.
- Rutenberg, A. (1975). "Approximate Natural Frequencies for Coupled Shear Walls," *Journal of Earthquake Engineering and Structural Dynamics,* 4(1), 95-100.
- Sozen, M. A. (1989). "Earthquake Response of Buildings with Robust Walls," *Proceedings of the Fifth Chilean Conference on Seismology and Earthquake Engineering,* Association Chilean de Sismologia e Ingenieria Antisismica, Santiago, Chile.
- Stafford-Smith, B. and Crowe, E. (1986). "Estimating Periods of Vibration of Tall Buildings," *Journal ofStructural Engineering,* ASCE, 112(5), 1005-1019.
- *Tentative Provisions for the Development of Seismic Regulations for Buildings.* (1978). ATC3- 06, Applied Technological Council, Palo Alto, CA.
- Timoshenko, S., Young, D. H., and Weaver, W. Jr. (1974). *Vibration Problems in Engineering,* Wiley, New York.

Uniform Building Code. (1997). International Conference of Building Officials, Whittier, CA.

- Veletsos, A. S. and Yang, J. Y. (1977). "Earthquake Response of Liquid Storage Tanks," *Advances in Civil Engineering Through Engineering Mechanics,* Proceedings of the Engineering Mechanics Division Specialty Conference, Rayleigh, North Carolina, 1-24.
- Wallace, J. W. and Moehle, J. P. (1992). "Ductility and Detailing Requirements of Bearing Wall Buildings," *Journal of Structural Engineering*, ASCE, 118(6), 1625-1643.
- Werner, S. D., Nisar, A. and Beck, J. L. (1992). *Assessment of UBC Seismic Design Provisions Using Recorded Building Motion from the Morgan Hill, Mount Lewis, and Loma Prieta Earthquakes,* Dames and Moor, Oakland, CA, April.

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PART III: APPENDICES

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APPENDIX A: DATABASE

The vibration periods and modal damping ratios of about twenty-five buildings have been identified from their recorded motions during the Northridge earthquake using system identification techniques. These results are combined with similar data available from the 1971 San Fernando earthquake and other recent earthquakes to form a comprehensive set of data. This data set includes the period and damping values along with information on other factors that could influence these vibration properties. These data are compiled on an electronic database that can be easily updated after every major earthquake. This appendix describes the information included in the database followed by the procedures to update the database and to extract the relevant information for further evaluation.

Database Format

A master database containing information on the vibration properties and other relevant information is compiled using Microsoft Access 2.0, a commercially-available database management software package. Contents of this database are arranged into the following five broad categories:

- 1. General information
- 2. Structure characteristics
- 3. Excitation characteristics
- 4. Recorded motion characteristics
- 5. Response characteristics

For each of these general categories, a series of individual parameters are defined. These parameters are defined in more details in subsequent sections. This information is extracted for each of the building considered in this study and entered into the master database. A separate set

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of data, or "record" is established for each different building. In addition, a separate record is created for data obtained for the same building but for different earthquakes or by different investigators for the same earthquake. If the information on a particular parameter in a record is not available, that parameter is left blank.

General Information

The following general information is included for building identification purposes and for future reference.

Location: This parameter indicates the city where the structure is located.

Identification Number: Each building is assigned a unique identification number. This parameter includes a single character followed by a number. The character indicates the agency that instrumented the building. For example, "C" indicates that this building was instrumented by the California Strong Motion Instrumentation Program (CSMIP). Similarly, "U" and "N" indicate that the building was instrumented by the United States Geological Survey (USGS) or the National Oceanographic and Atmospheric Agency (NOAA), respectively. The number followed by the first character indicates the station number or the instrument identification numbers assigned by the instrumenting agency. For buildings instrumented by the CSMIP or the USGS, this number represents the station number assigned by the respective agency. For example, a number "58262" following the character "C" indicates the station number assigned to the building by the CSMIP. Similarly, a number "482" following the character "U" implies the station number assigned to the building by the USGS. However, for buildings instrumented by the NOAA, the number represents the identification numbers of the instruments used to measure motions of the buildings. For example, a number "151-3" followed by the character "N" indicates that the motions of the building were recorded by the instruments numbered 151 to 153.

Occupancy: This parameter indicates the building usage, for example, office, residential, hospital, or hotel.

Name: This parameter lists the name, if available, of the building. For example, building with identification number "N151-3" is known as the "Kajima International Building".

Address: Several buildings, especially those instrumented by the NOAA, are identified by their address. Thus, this information is also included in the database.

Reference: This parameter identifies the source of the vibration data if these data were not identified in this study. If the properties are identified in this study, "Goel and Chopra" is entered in the database.

Structure Characteristics

The following quantitative and qualitative parameters associated with characteristics of the building are included in the database:

Height: This parameter lists the height in feet from base level of the building. This height is used for estimating the fundamental vibration period of the building from code formulas.

Width: Width is the base dimension in feet along the longitudinal direction of the building.

Length: Length is the base dimension in feet along the transverse direction of the building.

Number of Stories: This parameter represents the number of stories above the ground level.

Number of Basements: This parameter represents the number of stories below the ground level (or the number of basements).

Base to Roof Height: Base to roof height lists the height in feet between the locations of instruments recording the input ground accelerations and the output ground accelerations. This parameter is used to estimate the average drift of the building during an earthquake. Since the instrument measuring the input ground acceleration may not always be located at the base of the

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building or the instrument measuring the output acceleration may not be located at the top-most floor level, this height can be different than the height of the building described previously.

Material: The construction materials encountered in the building data included: Reinforced concrete (R/C); Steel; Masonry (MS); Unreinforced Masonry (URM); mixed materials such as Steel and R/C, Steel and URM, and R/C and URM.

Longitudinal Resisting System: This parameter categorizes the lateral system used in the longitudinal direction of the building for resisting the earthquake loads. Following is a list of lateral load resisting systems encountered in the building data:

- Moment-Resisting Frame (MRF): This category identifies a structure constructed of an assemblage of beams and columns that resist earthquake loads by frame action. The frames may be constructed of steel or reinforced concrete.
- Shear Wall (SW): This category identifies reinforced concrete or masonry systems that resist lateral loads by walls loaded in their own plane.
- Braced Frame (BF): This category identifies a structure whose frames are braced by diagonal members such that lateral loads are resisted by axial forces in the members.
- Eccentric Braced Frame (EBF): This system transfers the load through axial forces in members as well as dissipates earthquake energy through inelastic deformations in the shear links formed by the eccentric bracing.
- Moment Frames and Shear Walls (MRF/SW): This category is assigned to buildings having dual shear wall and moment frame systems or shear wall buildings with moment frames that contribute significantly to the earthquake resistance.

Transverse Resisting System: Similar to the longitudinal resisting system, this parameter indicates the lateral load resisting system of the building in the transverse direction.

Excitation Characteristics

This category includes the following two parameters to indicate the information on the earthquake for which the building vibration properties are identified:

Earthquake Name: This parameter indicates the name of the earthquake, usually indicating the location of the earthquake epicenter.

Earthquake Date: This is the date on which the earthquake occurred.

Recorded Motion Characteristics

This category includes the information on the building motions that is either recorded or derived directly from the recorded motions. This information is useful for estimating the intensity of the earthquake shaking at the building site or deformations of the building itself.

Longitudinal Base Acceleration: This parameter indicates the base peak acceleration recorded in the longitudinal direction of the building.

Longitudinal Roof Acceleration: This parameter indicates the roof peak acceleration recorded in the longitudinal direction of the building.

Transverse Base Acceleration: This parameter indicates the base peak acceleration recorded in the transverse direction of the building.

Transverse Roof Acceleration: This parameter indicates the roof peak acceleration recorded in the transverse direction of the building.

Longitudinal Roof Deformation: The longitudinal roof deformation, specified in feet, is calculated as the difference between the displacements at the roof level and the base level. The displacements used are obtained by double integrating the corrected accelerations.

Transverse Roof Deformation: Similar to longitudinal roof deformation, this parameter indicates the roof deformation in the transverse direction of the building.

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Longitudinal Drift: Measured in percentage (%), this value indicates the average drift over the height of the building in the longitudinal direction. This drift is obtained by dividing the longitudinal roof deformation by the base to roof height of the building.

Transverse Drift: Similar to the longitudinal drift, this value is the average drift of the building in the transverse direction.

Vibration Properties

This category lists the vibration properties of the building identified from its motions recorded during the earthquake. The information includes the vibration period (T) and modal damping ratio (ξ) for the first two modes of the building in each of the following directions: longitudinal, transverse, and torsional.

Database Manipulation

The database for vibration properties should be amenable to easy update after each earthquake to include data for vibration properties of additional buildings. Furthermore, it should also be possible to easily extract the relevant data required for further studies. This section describes how the data can be added to or extracted from the database.

Adding Data

The data for each building can be entered in the database using two alternative approaches. These approaches are described next.

Tables: In Microsoft Access 2.0, the entire database can be viewed in the form of a table. Each row of this table contains data for a record. The various data properties (or fields), mentioned previously, are arranged in columns of this table. Thus, to enter data for each additional building, one can enter the appropriate row and column of the table and type in value of that field. For data records containing a large number of fields, this may become cumbersome because it may not be possible to view the entire record on the computer screen. In such situations, forms, described next, offer a more attractive alternative.

Forms: Forms present the data for all fields of one record in an organized and attractive manner. In order to enter information on various fields of a record, one can navigate the form by using tab key of the keyboard and typing in the appropriate data. Furthermore, data in forms can also be organized differently compared to the tables, with category titles and more descriptive field names. The form designed for entering the vibration property database and used in this study is shown in Figure A.I.

Extracting Data

One of the primary advantages of organizing the data in databases is that it is easy to extract portions of the data, arrange various fields in a manner different compared to the original database, and sort them in ascending or descending order. The process of selectively extracting a subset of data from the larger database is facilitated by the queries.

Queries: Queries are used to extract data selectively from a larger data set stored in the database. To extract this data, the following options may be used:

- The data fields in the records that are to be extracted. Using this information, only selected data fields may be extracted and they may be arranged in a manner different than the main database.
- The order in which the records are to be sorted. The database stores the records in their order of entry. Using this information, the data may be sorted either in ascending or descending order of one or more fields.
- Whether the field is to be displayed. Using this option, a field may be hidden from the final printout but all other options and criteria may be applied to this field.

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• The criteria to be used to extract the data. This criteria in usually in the form of Boolean operators and may be specified on more than one field simultaneously.

Example: The following example illustrates the use of queries in selectively extracting data from the main database. The fields to be extracted are: (1) Location, (2) Identification Number, (3) Height, (4) First Longitudinal Period, and (5) First Transverse Period. The data is to be extracted only for the Northridge earthquake and the steel buildings; the earthquake name or the material should not be printed. Furthermore, the records extracted are arranged in ascending order by the location and the height. The query designed to extract this data is shown in Figure A.2 and the extracted data in Figure A.3.

Figure A.I. Form designed for the vibration properties database.

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Figure A.2. Example query design in Microsoft Access 2.0.

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Figure A.3. Results of example query.

APPENDIX B: SYSTEM IDENTIFICATION METHODS

System identification techniques are used to identify the vibration periods and modal damping ratios of buildings from their motions recorded during earthquakes. Although, a number of parametric and nonparametric system identification techniques are available (Appendix I), this appendix describes the following three more commonly used techniques for system identification of the vibration properties from recorded motion: (a) the transfer function method, (b) the modal minimization method, and (c) the autoregressive modeling method. Described first is the theory for each of these techniques followed by an example.

Transfer Function Method

The transfer function method is a nonparametric system identification approach designed to work for linear and time-invariant systems. This approach involves determining the transfer function, $H(i\omega)$, from the Fourier transforms of input and output accelerations. The modal frequencies are estimated from the locations of the resonant peaks in the absolute value, $|H(i\omega)|$, of the transfer function; and the modal damping ratios are determined from their half-power bandwidths.

The transfer function method usually works well for steady-state harmonic test data or ambient vibration data where the level of noise in the data is small. When applied to earthquake data, difficulties arise because the transfer function is characterized by extreme variability. Furthermore, the transfer function exhibits numerous peaks that are not related to the resonant peaks but are a function of the noise in the data and the model error. Smoothing of the transfer function can reduce the variability and thus make the resonant peaks more apparent, but leads to a loss of information that can result in the damping ratios being overestimated. Generally, past work suggests that the only vibration properties that can be reliably estimated from the transfer

function are the frequencies of the first few modes and possibly the damping ratio in the fundamental vibration mode. Since the goal of this study is to estimate the vibration frequencies as well as the modal damping ratios for at least first two modes, the transfer function method is used only to obtain initial estimates that are to be supplied to the parametric system identification techniques described in the next two sections.

Figure B.1. Smoothed transfer function and initial frequency estimates.

Figure B.l shows the smoothed transfer function determined from the base and roof accelerations of one of buildings considered in this study. The frequencies of the first and second modes of vibration estimated from this transfer function are 0.3175 Hz and 0.8667 Hz, respectively. Although two other peaks at 1.3672 Hz and 2.1851 Hz are also identified, these can not be reliably associated with higher modes of the building. Since the resonant peaks are very sharp, it is not possible to estimate the damping ratio even for the first mode of vibration; the initial guess for modal damping ratios for these modes would be 2% and 3% of the critical damping value.

Modal Minimization Method

The modal minimization method is a parametric system identification approach for estimating the vibration properties of a linear system. This approach involves minimizing the error between the output acceleration and the input acceleration, in the least square sense, for the various vibration parameters of the building (Beck, 1978; Li and Mau, 1971). This section describes the theoretical background for the modal minimization method followed by an example. The approach presented is valid for multiple input and multiple output. The single input and single output approach adopted in the present study is a simplified version of this more general approach.

Theoretical Background

Using modal superposition for linear systems, the output acceleration, a_i , of the building can be expressed in terms of its modal acceleration responses, \ddot{u}_j , through the following equation:

$$
a_i = \sum_j \phi_{ij} \ddot{u}_j \tag{B-1}
$$

in which ϕ_{ij} is the mode shape component of the *j*th mode at location *I*. The modal accelerations are defined by the second order differential equation:

$$
\ddot{u}_j + 2\xi_j \omega_j \dot{u}_j + \omega_j^2 u_j = \sum_k p_{jk} a_{gk}
$$
 (B-2)

where ξ_j and ω_j are the modal damping ratio and the natural frequency, respectively, of the *j*th mode, and p_{jk} is the participation factor of the *j*th mode with respect to the *k*th input motion. Since the input accelerations, a_{gk} , are linear within each time step Δt , the acceleration response,

 \ddot{u}_j for each mode can be computed using the exact solution to the piecewise linear excitation (Chopra, 1995); this computation requires the initial modal velocity, $\dot{u}_i(0)$, and modal displacement, $u_j(0)$ for each mode. Thus, the unknown modal parameters are: ω_j , ξ_j , p_{jk} , $\dot{u}_j(0)$, $u_j(0)$, and ϕ_{ij} .

 \bullet

The modal minimization method seeks to iteratively minimize the squared error:

$$
J = \sum_{i} \sum_{s} [a_{oi}(s\Delta t) - a_i(s\Delta t)]^2
$$
 (B-3)

between the recorded output acceleration, a_{oi} , and the output acceleration, a_i , of the linear model with respect the unknown modal parameters. A relative error corresponding to J of Eq. (B-3) is defined as:

$$
E = \sqrt{\frac{\sum\limits_{i} \sum\limits_{s} [a_{oi}(s\Delta t) - a_{i}(s\Delta t)]^{2}}{\sum\limits_{i} \sum\limits_{s} [a_{oi}(s\Delta t)]^{2}}}
$$
(B-4)

This modal minimization technique implemented on computer by Beck (1978) and Li and Mau (1991) generally follows the above-described methodology with minor differences in the minimization procedures; this study used the program developed by the latter authors. The iterative minimization procedure requires initial estimates of the unknown modal parameters. The initial estimates for the modal frequencies and damping ratios may be obtained from the transfer function approach described previously. Obtaining estimates of the other parameters requires some judgment.

The system identification process using the modal minimization method involves increasing the number of modes until the difference in the relative error E (Eq. B-4) is less than a certain value; a value of 3% is selected in this study. Although the last included mode often contributes little to the output by itself, it improves the error estimate somewhat. This procedure is illustrated with an example next.

Example

The problem solved in the present study is that of single input at the building base and single output at the building roof. For such a problem, the unknown modal parameters are: ω_i , ξ_j , p_j , $\dot{u}_j(0)$, $u_j(0)$, and ϕ_j . Since one component of the mode shape can always be set arbitrarily equal to one, only the first five of these six parameters for each mode need to be identified. The various steps involved in the system identification process are:

- 1. Obtain initial estimates of the modal frequencies and damping ratios from the transfer function. Figure B.2 shows the transfer function between roof and base accelerations in the N-S direction of the 15-story Government Office Building in Los Angeles. Initial estimates of the modal frequencies are shown on this figure. Since it was not possible to obtain estimates of the modal damping ratio, the initial values for the first two modes is selected as 2% and remaining modes as 5% based on judgment.
- 2. Obtain initial estimates of the remaining modal parameters by judgment. The relative magnitude of the participation factor can be judged from the relative peak heights at the modal frequencies. Furthermore, initial modal velocities and displacements can be selected as zero. This selection is reasonable because the recorded motion used in the identification process starts at time zero when the initial motions of the building are for all practical purposes zero.
- 3. Identify the modal parameters using the modal minimization procedure. Start with first two modes and increase the number of modes until the difference between two successive values of the relative error is within 3%. For the example building, four modes are needed to satisfy

the convergence criteria. The results of the system identification are summarized in Table **B.1.**

4. Validate the identified model by comparing (a) the transfer functions, and (b) the time histories of the recorded motion and the motion of the identified model. Such comparisons are presented in Figures B.3 and BA for the example building. These comparisons also assist in reliably estimating the true modal frequencies and damping ratios of the building. For example, while the match in the time history is very good, only the first two peaks of the transfer function are matched well by the system identification procedure. Therefore, modal parameters for these two modes are considered to be reliable estimates of the true vibration properties of the building.

Mode	Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.3220	0.0241	$-6.99E - 00$	$-3.78E-04$	$2.24E-03$	$1.06E + 00$	1st Mode
2	0.8667	0.0390	$1.05E-00$	4.76E-05	7.97E-04	3.28E-01	2nd Mode
	1.4200	0.0887	$-3.89E-01$		$2.11E-05$ $-3.86E-05$	1.28E-01	
	2.1877	0.0358	$1.06E-01$	$4.05E-06$	$-4.07E-0.5$	4.89E-02	

Table B.1. Results of system identification in N-S direction.

Relative Error = 0.326 and Absolute Error = 0.683

Auto-Regressive Modeling Method

The auto-regressive modeling method is also a parametric system identification approach for estimating the vibration properties of a linear system. Similar to the modal minimization procedure, this approach involves minimizing the error between the output acceleration and the input acceleration, in the least squares sense, for the various parameters of the autoregressive model (Ljung, 1987). This section describes briefly the theoretical background for the autoregressive modeling method followed by an example; details of this method can be found elsewhere (Safak, 1988, 1989a, 1989b, 1991).

Figure B.2.. Initial frequency estimates from transfer function in N-S direction.

Roof N-S Acceleration, 15-Story Govt. Office Bldg., Los Angeles

Figure B.3. Comparison of empirical transfer functions: recorded motions and calculated motions.

Figure BA. Comparison of time-histories: recorded motions and calculated motions. *Theoretical Background*

The set of second order differential equations governing the earthquake response of a linear system may be written as:

$$
y(t) + \sum_{j=1}^{n_c} a_j y(t - j\Delta) = \sum_{k=1}^{n_b} b_k x(t - k\Delta - n_c\Delta)
$$
 (B-5)

in which $y(t)$ is the system output, $x(t)$ is the input, Δ is the sampling interval, a_j and b_k are the parameters of the autoregressive model numbering n_a and n_b , respectively, and n_c is the time delay, in terms of the number of discrete time steps, between the input and the output. The n_a and n_b are called the order of the auto-regressive model. The same equation can also be written in the frequency domain as:

$$
Y(\omega) = H(i\omega)X(\omega) \tag{B-6}
$$

where $Y(\omega)$ and $X(\omega)$ are the Fourier transforms of the output, $y(t)$, and the input $x(t)$, respectively, and $H(i\omega)$ is the transfer function defined as:

$$
H(i\omega) = \frac{\sum_{k=1}^{n_b} b_k e^{-i\omega(k+n_c)\Delta}}{1 + \sum_{j=1}^{n_a} a_j e^{-i\omega j\Delta}}
$$
(B-7)

in which $i = \sqrt{-1}$. The denominator of equation seven contains information about the dynamic properties of the building. The frequencies and the damping ratios are computed from the poles, p_j , of the transfer function which are determined as the roots of the following denominator polynomial:

$$
p^{n_a} + a_1 p^{a_a-1} + \cdots + a_{n_a-1} p + a_{n_a} = 0
$$
 (B-8)

For stable structures, i.e., structures with non negative damping, the poles are in complexconjugate pairs. The modal frequencies, $\omega_j = 2\pi f_j$, and the damping ratios, ξ_j , are computed from these poles as:

$$
\xi_j = \frac{\ln\left(\frac{1}{|p_j|}\right)}{\left[\nu_j + \ln\left(\frac{1}{|p_j|}\right)\right]^{1/2}}
$$
\nB-9)
\n
$$
f_j = \frac{\ln\left(\frac{1}{|p_j|}\right)}{2\pi \xi_j \Delta}
$$
\n(B-10)

where v_j is argument of p_j . The frequencies and damping ratios corresponding to the complexconjugate pairs are identical. Therefore, n_a poles result in $n_a/2$ distinct frequencies and damping ratios.

To compute the contribution of each mode to the response, the transfer functions are expanded into partial fractions and the pairs of terms corresponding to pairs of complexconjugate poles are combined:

$$
H(i\omega) = \sum_{j=1}^{n_a/2} \left(\frac{r_j z^{-(n_c+1)}}{1 - p_j z^{-1}} + \frac{r_j^* z^{-(n_c+1)}}{1 - p_j^* z^{-1}} \right) \text{ with } z = e^{i\omega\Delta}
$$
 (B-11)

where r_j denotes the residue for the jth transfer function corresponding to the jth pole. The contribution of the jth mode to the response is obtained by multiplying the input by the corresponding modal transfer function.

The autoregressive modeling method seeks to minimize the error between the output and the input in the least squared sense with respect to the parameters a_j and b_k . The next section describes the various step of the system identification using this method and illustrates these steps with an example.

Example

Various steps of this procedure are illustrated next with an example. For this purpose, the vibration properties of a 6-story commercial building located in Burbank are determined in the N-S direction from its roof and base accelerations recorded during the Northridge earthquake. Various steps of this procedure are implemented conveniently in the System Identification Toolbox of the commercially available software MATLAB. The steps are as follows:

1. Estimate the time delay, *nc,* between the input and output. Time delay compensates the synchronization and discretization errors as well as errors introduced by considering only a limited number of modes. To determine the time delay, a single-mode system is assumed, i.e., $n_a = 2$ and $n_b = 1$, and variation of the squared error between the recorded output and the model output is investigated. The time delay that minimizes the error is selected as the time

delay for further system identification. Figure B.Sa shows the variation for the example building where the error is minimized in the neighborhood of n_c =10. The time delay selected for the system identification is 9.

2. Determine the model order n_a . Similar to the time delay, the model order is also selected by investigating the error with increasing model orders. Generally the error decreases with increasing *na* with sharp decrease at the beginning and gradual flattening with further increase. The beginning of the flat region indicates the optimal model order. Figure B.Sb shows variation of the error for $n_a = 2$ to 30 assuming $n_b = n_a$ and $n_c = 10$. This figure suggests that $n_a = 4$, i.e., two modes, would be sufficient as the model order. However, the model verification tests described next, showed that a much higher value of $n_a = 32$ is required for satisfactory identification.

Figure B.S. Time delay and model order for the example building.

3. Select the parameter n_b . The error is not sensitive to the parameter n_b . This selection is based on zero-pole cancellation criteria rather than the minimum error criteria. Normally, the accuracy of the identification increases with increasing n_b . However, if n_b becomes too high, one or more zeros may become equal to the poles of the transfer function. This results in a zero-pole cancellation eliminating some of the modes of the building. By trying several values, $n_b = 16$ is found to be appropriate for the example building. As shown in Figure B.6, it does not result in any zero-pole cancellation.

- 4. Identify the building. Using $n_a = 32$, $n_b = 16$, and $n_c = 9$, the modal frequencies and damping ratios identified for the example building are presented in Table B.2. For this building the first mode is identified with a frequency of 4.607 Hz and damping ratio of 4.0%; the second mode values are 13.475 Hz and 5.3%, respectively. Figure B.7 and B.8 show the contribution of various modes to the output response.
- 5. Verify the identified model. The final step in the system identification using auto-regressive modeling is verification of the identified model. This is accomplished by ensuring the transfer function (Figure B.9) and time history (Figure B.IO) of the auto-regressive model match closely with those of the recorded output motion. In addition, the autocorrelation and the cross correlation of the residuals are checked with the input (Figure B.l1). For a valid identification, the residuals should be close to white noise and independent (uncorrelated) with the input. For this purpose, the dotted lines are also plotted in Figure B.11 for 95% confidence limits for whiteness in the autocorrelation and for independence in the cross correlation. The autocorrelation plot show that the residuals of the identified model satisfy the whiteness criteria. Although the cross correlation exceeds the confidence limit as a few places, the amplitude is generally small. Thus even the autocorrelation criteria is acceptable for the identified model.

Figure B.6. Zero-pole cancellation for the example building.

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j	Frequency	Period	Damping	Pole	Comments
	(Hz)	(sec)		Magnitude	
1	4.607	1.364	0.040	0.996	1st Mode
$\overline{2}$	4.607	1.364	0.040	0.996	
3	13.475	0.466	0.053	0.986	2nd Mode
$\overline{\bf 4}$	13.475	0.466	0.053	0.986	
5	36.071	0.174	0.043	0.969	
6	36.071	0.174	0.043	0.969	
$\overline{7}$	23.180	0.271	0.094	0.957	
8	23.180	0.271	0.094	0.957	
9	18.357	0.342	0.127	0.954	
10	18.357	0.342	0.127	0.954	
11	9.871	0.637	0.239	0.954	
12	9.871	0.637	0.239	0.954	
13	71.118	0.088	0.036	0.950	
14	71.118	0.088	0.036	0.950	
15	59.051	0.106	0.054	0.938	
16	59.051	0.106	0.054	0.938	
17	64.668	0.097	0.051	0.937	
18	64.668	0.097	0.051	0.937	
19	47.817	0.131	0.069	0.937	
20	47.817	0.131	0.069	0.937	
21	27.343	0.230	0.121	0.936	
22	27.343	0.230	0.121	0.936	
23	36.592	0.172	0.101	0.929	
24	36.592	0.172	0.101	0.929	
25	74.201	0.085	0.053	0.924	
26	74.201	0.085	0.053	0.924	
$\overline{27}$	56.639	0.111	0.071	0.922	
28	56.639	0.111	0.071	0.922	
29	48.909	0.129	0.090	0.916	
30	48.909	0.129	0.090	0.916	
31	31.807	0.198	0.184	0.890	
32	31.807	0.198	0.184	0.890	

Table B.2. Results of system identification for the example building.

Roof N-S Acceleration, 6-Story Commercial Building, Burbank

Figure B.7. Modal contributions of autoregressive model; modes 1 to 8.

Figure B.8. Modal contributions of autoregressive model; modes 9 to 16.

Figure B.9. Comparison of empirical transfer functions: recorded motions and calculated motions of autoregressive model.

Figure B.lO. Comparison of time-histories: recorded motions and calculated motions of autoregressive model.

Figure B.II. Residuals for autoregressive model.
APPENDIX C: RESULTS OF SYSTEM IDENTIFICATION

Table C.l. List of identified buildings.

1. Los Angeles - 7-Story UCLA Math-Science Building, CSMIP Station No. 24231

Los Angeles - 7-story UCLA Math-Science Bldg.
(CSMIP Station No. 24231) **SENSOR LOCATIONS** 60 6" Seismic Joint 10 $12,$ Roof 11 Classrooms
and Offices 7_{th} $3013 - 6$ 6th 6" Selamic Join Roof Plan 3rd Floor Plan 5th $15 - 11$ 4th $13 - 5$, 12 3rd $N_{\rm ref}$ ł З $2nd$ $\frac{1}{2}$ 題 lst 5th Floor Plan 1st Floor Plan **W/E Elevation Structure Reference** Orientation; $N = 0$

Figure C.1.1b. Sensor locations in UCLA Math Science Building, CSMIP Station No. 24231

Table C.1.1. Results of system identification in E-W (longitudinal) direction by autoregressive modeling.

		Period			
j	Frequency		Damping	Pole	Comments
	(Hz)	(sec)		Magnitude	
	0.902	1.108	0.071	0.992	1st Mode
$\overline{2}$	0.902	1.108	0.071	0.992	
3	11.648	0.086	0.015	0.978	
4	11.648	0.086	0.015	0.978	
$\overline{5}$	9.430	0.106	0.025	0.971	
6	9.430	0.106	0.025	0.971	
$\overline{7}$	3.136	0.319	0.088	0.966	
8	3.136	0.319	0.088	0.966	
9	6.557	0.153	0.043	0.965	
10	6.557	0.153	0.043	0.965	
11	7.741	0.129	0.043	0.960	
12	7.741	0.129	0.043	0.960	
13	1.393	0.718	0.241	0.959	
14	1.393	0.718	0.241	0.959	
15	4.028	0.248	0.107	0.947	
16	4.028	0.248	0.107	0.947	
17	10.591	0.094	0.042	0.946	
18	10.591	0.094	0.042	0.946	
19	12.046	0.083	0.040	0.942	
20	12.046	0.083	0.040	0.942	
21	10.528	0.095	0.046	0.941	
22	10.528	0.095	0.046	0.941	
23	1.654	0.605	0.304	0.939	
24	1.654	0.605	0.304	0.939	
25	2.531	0.395	0.202	0.938	
26	2.531	0.395	0.202	0.938	
27	4.180	0.239	0.129	0.935	
28	4.180	0.239	0.129	0.935	
29	8.260	0.121	0.066	0.934	
30	8,260	0.121	0.066	0.934	
$\overline{31}$	<u>5.764</u>	0.174	0.116	0.920	
32	5.764	0.174	<u>0.116</u>	<u>0.920</u>	
33	<u>6.161</u>	0.162	0.131	0.904	
34	6.161	0.162	0.131	0.904	
35	5.896	0.170	0.137	0.903	
36	5.896	0.170	0.137	0.903	
37	9.345	0.107	0.103	0.886	
$\overline{38}$	9.345	0.107	0.103	0.886	

Figure C.1.2. Zero-pole cancellation for autoregressive model in E-W direction.

Figure C.I.3. Residuals for autoregressive model in E-W direction.

Figure C.IA. Comparison of empirical transfer functions: recorded motion and calculated motion of autoregressive model in E-W direction.

Figure C.1.5. Comparison of time-histories: recorded motion and calculated motion of autoregressive model in E-W direction.

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Roof E-W Acceleration, 7-Story UCLA Math-Science Bldg, Los Angeles

Figure C.l.6. Modal contributions of autoregressive model in E-W direction.

Table C.l.2. Results of system identification in N-S (transverse) direction by autoregressive modeling. j Frequency Period Damping Pole Comments (Hz) (sec) Magnitude

	(HZ)	(sec)		Magnitude	
1	1.184	0.845	0.172	0.975	1st Mode
\overline{c}	1.184	0.845	0.172	0.975	
$\overline{3}$	2.696	0.371	0.123	0.959	
$\overline{\mathbf{r}}$	2.696	0.371	0.123	0.959	
$\overline{5}$	3.583	0.279	0.100	0.956	
6	3.583	0.279	0.100	0.956	
	4.235	0.236	0.093	0.952	
$\overline{8}$	4.235	0.236	0.093	0.952	
9	5.703	0.175	0.070	0.951	
10	5.703	0.175	0.070	0.951	
11	1.756	0.569	0.236	0.949	
12	1.756	0.569	0.236	0.949	
13	6.105	0.164	0.071	0.947	
14	6.105	0.164	0.071	0.947	
15	7.669	0.130	0.058	0.946	
16	<u>7.669</u>	0.130	0.058	0.946	
17	1.337	0.748	0.331	0.946	
18	1.337	0.748	0.331	0.946	
19	11,485	0.087	0.041	0.943	
20	11.485	0.087	0.041	0.943	
21	4.235	0.236	0.121	0.938	
22	4.235	0.236	0.121	0.938	
23	9.839	0.102	0.053	0.937	
24	9.839	0.102	0.053	0.937	
25	8.938	0.112	0.059	0.936	
26	8.938	0.112	0.059	0.936	
$\overline{27}$	7.396	0.135	0.072	0.935	
28	7.396	0.135	0.072	0.935	
29	10.671	0.094	0.063	0.919	
30	10.671	0.094	0.063	0.919	
31	11.961	0.084	0.066	0.906	
32	11.961	0.084	0.066	0.906	
33	8.521	0.117	0.095	0.904	
$\overline{34}$	8.521	0.117	0.095	0.904	
$\overline{35}$	10.442	0.096	0.462	0.545	
$\overline{36}$	10.442	0.096	0.462	0.545	

Figure C.1.7. Zero-pole cancellation for autoregressive model in N-S direction.

Figure C.1.8. Residuals for autoregressive model in N-S direction.

Figure C.1.9. Comparison of empirical transfer functions: recorded motion and calculated motion of autoregressive model in N-S direction.

Figure C.I.IO. Comparison of time-histories: recorded motion and calculated motion of autoregressive model in N-S direction.

Roof N-S Acceleration, 7-Story UCLA Math-Science Bldg, Los Angeles
na = 36 nb = 10 nc = 8

Figure C.1.11. Modal contributions of autoregressive model in N-S direction.

2. Burbank - 10-Story Residential Building, CSMIP Station No. 24385

Figure C.2.1a. Details of 10-story residential building, CSMIP Station No. 24385

Figure C.2.1b. Sensor locations in 10-story residential building, CSMIP Station No. 24385

	Frequency	Period	Damping	Pole	Comments
	(Hz)	(\sec)		Magnitude	
	1.673	0.598	0.056	0.988	<u>1st Mode</u>
$\overline{2}$	1.673	0.598	0.056	0.988	
3	7.199	0.139	0.073	0.936	2nd Mode
4	7.199	0.139	0.073	0.936	
5	3.147	0.318	0.290	0.892	
6	3.147	0.318	0.290	0.892	
7	9.995	0.100	0.094	0.888	
8	9.995	0.100	0.094	0.888	
9	6.263	0.160	0.162	0.880	
10	6.263	0.160	0.162	0.880	
11	12.029	0.083	0.128	0.824	
12	12.029	0.083	0.128	0.824	
13	9.212	0.109	0.174	0.818	
14	9.212	0.109	0.174	0.818	
15	5.127	0.195	0.502	0.724	
16	5.127	0.195	0.502	0.724	
17	9.538	0.105	0.514	0.540	
18	9.538	0.105	0.514	0.540	

Table C.2.1 Results of system identification in E-W (longitudinal) direction by autoregressive modeling.

Figure C.2.2. Zero-pole cancellation for autoregressive model in E-W direction.

Figure C.2.3. Residuals for autoregressive model in E-W direction.

Figure C.2.4. Comparison of empirical transfer functions: recorded motions and calculated motions of autoregressive model in E-W direction.

Figure C.2.5. Comparison of time-histories: recorded motions and calculated motions of autoregressive model in E-W direction.

Figure C.2.6. Modal contributions of autoregressive model in E-W direction.

	Frequency	Period	Damping	Pole	Comments
	(Hz)	(sec)		Magnitude	
	l.789	0.559	0.075	0.983	1st Mode*
2	1.789	0.559	0.075	0.983	
3	7.619	0.131	0.082	0.924	2nd Mode
4	7.619	0.131	0.082	0.924	
$5\overline{)}$	10.474	0.096	0.062	0.922	
6	10.474	0.096	0.062	0.922	
7	2.977	0.336	0.338	0.881	
8	2.977	0.336	0.338	0.881	
9	5.492	0.182	0.205	0.868	
10	5.492	0.182	0.205	0.868	
11	11.376	0.088	0.105	0.861	
12	11.376	0.088	0.105	0.861	
13	8.494	0.118	0.155	0.848	
14	8.494	0.118	0.155	0.848	
15	3.664	0.273	0.486	0.800	
16	3.664	0.273	0.486	0.800	
17	7.554	0.132	0.297	0.755	
18	7.554	0.132	0.297	0.755	

Table C.2.2 Results of system identification in N-S (transverse) direction by autoregressive modeling.

* Mode not used due to poor match in empirical transfer function

Figure C.2.7. Zero-pole cancellation for autoregressive model in N-S direction.

Figure C.2.8. Residuals for autoregressive model in N-S direction.

Roof N-S Acceleration, 10-Story Residential Bldg, Burbank

Figure C.2.9. Comparison of empirical transfer functions: recorded motions and calculated motions of autoregressive model in N-S direction.

Figure C.2.10. Comparison of time-histories: recorded motions and calculated motions of autoregressive model in N-S direction.

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Figure C.2.II. Modal contributions of autoregressive model in N-S direction.

Mode	Frequency	[Damping]	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	1.4379	0.016	$7.06E-02$	$-6,74E-03$	$5.66E-02$	1.28E-021	
$\overline{2}$	1.7393	0.026	$-2.58E-01$	$-3.14E-02$	7.24E-02	1.55E-01	
3	1.8023	0.133	$-1.73E-00$	5.33E-02	$-4.04E-01$	7.24E-01	1st Tran. Mode*
4	1.8497	0.011	$-3.55E-03$	2.49E-03	2.48E-01	1.75E-01	1st Tor. Mode
5	7.5531	0.015	2.40E-02	$-6.03E-06$	$-4.05E-05$	5.23E-03	
6	8.1820	0.183	2.49E-01	$1.75E-04$	$2.64E-02$	4.85E-02	
7	10.0890	0.576	$-1.05E-00$	$3.53E-03$	$-1.74E-01$	1.16E-01	
8	12.3968	0.894	2.59E-00	$-3.03E-03$	9.94E-02	8.42E-02	
9	22.6437	0.098	4.14E-02	1.66E-05	2.10E-04	9.10E-05	

Table C.2.3. Results of system identification in N-S (transverse) direction by WPCMIMO.

* Damping is not reliable

Relative Error = 0.223 and Absolute Error = 2.406

Figure C.2.12. Initial frequency estimates from transfer function in N-S direction.

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Figure C.2.13. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.2.14. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

3. Sylmar - 6-Story County Hospital, CSMIP Station No. 24514

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	2.8439	0.057	$-1.67E-01$	4.39E-04	$-4.17E-02$	1.96E-01	1st Mode
$\overline{2}$	3.3200	0.088	$-1.44E-01$	$-3.85E-03$	3.11E-02	1.08E-01	
3	5.1094	0.020	$-3.88E-02$	$5.65E-05$	8.66E-03	7.01E-02	
4	6.0289	0.11	$-7.94E-02$	1.84E-04	$2.27E-02$	2.79E-02	
	3.9917	0.011	$2.60E-02$	2.63E-04	6.78E-03	3.64E-02	
6	4.5870	0.009	$-1.76E-02$	$-5.24E - 05$	6.25E-03	2.90E-02	
	7.5857	0.003	$1.37E-02$	6.81E-05	2.22E-03	1.37E-02	
8	7.9046	0.005	4.43E-03	2.03E-05	$-7.73E-03$	4.17E-03	
	6.9689	0.002	1.97E-03	1.98E-04	4.66E-04	1.20E-02	

Table C.3.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.657 and Absolute Error = 7.180

Figure C.3.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.3.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.3.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	2.5134	0.045	$-2.07E-01$	3.36E-03	$-2.51E-02$	3.29E-01	<u>1st Mode</u>
	2.8562	0.124	$-3.24E-01$	$-5.45E-03$	$-2.61E-02$	2.87E-01	
٩	7.2613	0.039	$4.66E-02$	$1.61E-04$	$-2.11E-02$	1.26E-02	
	7.7771	0.010	1.33E-02	$-2.96E-04$	$4.79E-04$	4.71E-03	
	0.7825	0.900	$1.56E-00$	$-7.50E-03$	$2.46E-02$	$2.11E-02$	
6	7.0997	0.266	$-2.74E-01$	$-5.52E-04$	$-1.89E-02$	4.51E-02	
	8.1373	0.230	1.13E-01	$-4.98E-04$	6.23E-02	1.02E-02	
	13.1001	0.696	1.10E-01	$-2.07E-04$	3.68E-02	2.59E-03	

Table C.3.2. Results of system identification in N-S (longitudinal) direction by WPCMIMO.

Relative Error = 0.343 and Absolute Error = 4.751

Figure C.3.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.3.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.3.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

4. Burbank - 6-Story Commercial Building, CSMIP Station No. 24370

Figure C.4.1a. Details of 6-story commercial building, CSMIP Station No. 24370

Figure C.4.1b. Sensor locations in 6-story commercial building, CSMIP Station No. 24370

j	Frequency	Period	Damping	Pole	Comments
	(Hz)	(\sec)		Magnitude	
1	4.558	1.378	0.041	0.996	1st Mode
$\overline{2}$	4.558	1.378	0.041	0.996	
$\overline{\mathbf{3}}$	13.525	0.465	0.065	0.983	2nd Mode
4	13.525	0.465	0.065	0.983	
$\overline{5}$	10.668	0.589	0.166	0.965	
$6 \overline{6}$	10.668	0.589	0.166	0.965	
$\overline{7}$	24.553	0.256	0.073	0.965	
8	24.553	0.256	0.073	0.965	
9	34.228	0.184	0.061	0.959	
10	34.228	0.184	0.061	0.959	
11	38.662	0.163	0.064	0.952	
12	38.662	0.163	0.064	0.952	
13	64.509	0.097	0.041	0.948	
14	64.509	0.097	0.041	0.948	
15	18.997	0.331	0.148	0.946	
16	18.997	0.331	0.148	0.946	
17	29.556	0.213	0.112	0.936	
18	<u>29.556</u>	0.213	0.112	0.936	
19	52.053	0.121	0.065	0.935	
20	52.053	0.121	0.065	0.935	
$\overline{21}$	52.023	0.121	0.066	0.933	
22	52.023	0.121	0.066	0.933	
23	60.611	0.104	0.066	0.923	
$\overline{24}$	60.611	0.104	0.066	0.923	
$\overline{25}$	43.751	0.144	0.092	0.922	
26	43.751	0.144	0.092	0.922	
27	74.606	0.084	0.059	0.916	
28	74.606	0.084	0.059	0.916	
29	72.439	0.087	0.080	0.890	
30	72.439	0.087	0.080	0.890	

Table *CA.!.* Results of system identification in E-W (longitudinal) direction by autoregressive modeling.

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Figure C.4.2. Zero-pole cancellation for autoregressive model in E-W direction.

Figure C.4.3. Residuals for autoregressive model in E-W direction.

Figure CAA. Comparison of empirical transfer functions: recorded motions and calculated motions of autoregressive model in E-W direction.

Figure CA.5. Comparison of time-histories: recorded motions and calculated motions of autoregressive model in E-W direction.

Roof E-W Acceleration, 6-Story Commercial Building, Burbank

Figure C.4.6. Modal contributions of autoregressive model in E-W direction; modes 1 to 8.

Roof E-W Acceleration, 6-Story Commercial Building, Burbank

Figure *CA.7.* Modal contributions of autoregressive model in E-W direction; modes 9 to 16.

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Table C.4.2. Results of system identification in N-S (longitudinal) direction by autoregressive modeling.

Figure C.4.8. Zero-pole cancellation for autoregressive model in N-S direction.

Figure C.4.9. Residuals for autoregressive model in N-S direction.

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Figure C.4.10. Comparison of empirical transfer functions: recorded motions and calculated motions of autoregressive model in N-S direction.

Figure CA.!!. Comparison of time-histories: recorded motions and calculated motions of autoregressive model in N-S direction.

Roof N-S Acceleration, 6-Story Commercial Building, Burbank

Figure C.4.12. Modal contributions of autoregressive model in N-S direction; modes 1 to 8.

Figure CA.13. Modal contributions of autoregressive model in E-W direction; modes 9 to 16.

5. Los Angeles - 19-Story Office Building, CSMIP Station No. 24643

Los Angeles - 19-story Office Bldg. (CSMIP Station No. 24643)

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.2571		0.022 $-8.63E-00$	$-1.57E-02$	$-4.13E-03$	6.84E-01	1st Mode
	0.7141	0.043	$1.42E - 00$	$-2.93E-04$	$1.87E-03$	3.25E-01	2nd Mode
	1.2481	0.051	$-5.52E-01$	$-1.14E-04$	$1.60E-03$	1.03E-01	
	1.8057	0.078	$2.82E-01$	$-2.59E-05$	$1.50E-03$	$7.41E-02$	
	2.3037	0.047	$-1.55E-01$	1.94E-05	$7.05E-04$	$5.42E-02$	
	2.7331	0.036	$2.14E-02$	2.24E-05	1.80E-04	$1.60E-03$	

Table C.S.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.203 and Absolute Error = 1.232

Figure C.S.2. Initial frequency estimate from transfer function in E-W direction.

Figure C.S.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.S.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.2877	0.029	<u>-8.57E-00 </u>	$-6.61E-03$	1.39E-03	3.20E-01	1st Mode
	1.2139	0.040	1.14E-00	9.30E-05	$-1.06E-03$	4.47E-01	2nd Mode
	2.5687	0.107	$-7.64E-01$	$-4.30E-05$	$-1.00E-03$	4.14E-01	
	2.9114	0.244	5.86E-01	$-7.32E - 05$	2.47E-03	1.27E-01	
	3.6779	0.030	9.09E-02	$-1.37E-05$	1.03E-04	1.64E-02	
	3.8857	0.036	8.51E-02	1.56E-05	2.83E-04	$1.62E-02$	
	4.5257	0.053	$-1.48E-01$	1.04E-05	$-4.62E-05$	8.37E-02	

Table C.5.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.295 and Absolute Error = 3.953

Figure C.5.5. Initial frequency estimate from transfer function in N-S direction.

Figure C.S.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.S.? Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

6. Los Angeles - Hollywood Storage Building, CSMIP Station No. 24236

Plan Shape: Rectangular
Base Dimensions: 217' x 51' Typical Floor Dimensions: 217' x 51' Design Date: 1925

Lateral Force Resisting System: Reinforced concrete frames in both directions. Foundation Type: Concrete piles.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.5907	0.0827	$-4.82E - 02$	$.69E-03$	$6.29E-03$	$2.82E - 04$	1st Mode*
	1.2645	0.0085	$-3.90E-02$	$-2.92E-03$	$1.38E - 02$	9.53E-03	
	1.2758	0.0964	$-1.34E-00$	$5.98E-03$	$-2.31E-02$	8.41E-01	2nd Mode
	4.7198	0.09001	$-3.03E-02$		$4.92E-05$ -2.84E-03	3.12E-03	
	6.1129	0.0659	$2.55E-02$		$-2.86E-05$ $-1.52E-03$	4.48E-03	
	7.8347	0.1198	$3.57E-02$		$-3.46E-05$ $-5.27E-04$	$3.13E-03$	

Table C.6.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

* 1st mode not visible in transfer function

Relative Error = 0.271 and Absolute Error = 0.703

Figure C.6.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.6.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.6.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency ¹	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.4397	0.0472	$-7,61E-00$	$-9,23E-02$	$-5.59E-01$		1.49E+00 1st Trans, Mode
	0.4500	0.0436	$2.53E-00$	$6.06E-02$	5.89E-01	3.39E-01	1st Tor. Mode
3	1.5361	0.0801	5.39E-01	$-1.08E-03$	$1.36E-02$		$2.72E-01$ 2nd Tran. Mode
	1.8336	0.0437	$1.24E-01$	5.26E-04	4.02E-03		$4.39E-02$ 2nd Tor. Mode
	5.8838	0.0236	3.17E-01	$-3.72E-04$	$-2.12E-02$	2.28E+00	
6	7.7576	0.0757	$6.66E-02$	4.11E-05	1.74E-04	$1.77E-02$	
	5.9003	0.0299	$-3.81E-01$	4.32E-04	2.48E-02	$2.67E + 00$	
8	4.2006	0.0918	1.14E-01	$-4.18E - 05$	2.68E-03	$7.04E-02$	

Table C.6.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.478 and Absolute Error = 3.981

Figure C.6.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.6.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.6.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

7. North Hollywood - 20-Story Hotel, CSMIP Station No. 24464

North Hollywood - 20-story Hotel

Figure C.7.1b. Sensor locations in 20-story hotel building, CSMIP Station No. 24464

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.3851	0.0587	$-5.79E - 00$	3.35E-03	$-9.30E - 03$	6.86E-01	1st Mode
റ	1.1353	0.1578	8.28E-01	$-7.65E-03$	$-2.71E-02$	1.49E-01	
3	1.2817	0.0315	2.00E-01	1.80E-03	3.05E-02	6.31E-02	
4	2.9555	0.4005	$-11.28E-0$	1.09E-02	1.66E-01	7.45E+01	
5	2.9632	0.3350	9.49E-00	$-9.24E-03$	$-1.34E-01$	7.03E+01	
6	5.1417	0.0114	$-2.11E-02$	$-1.19E-04$	1.25E-03	2.58E-02	
7	5.4144	0.2175	7.15E-01	$-8.31E-04$	$-4.32E-02$	9.02E-01	
8	5.8070	0.0823	$-1.79E-01$	5.22E-04	1.68E-02	1.13E-01	
Q	5.3661	0.0036	$-1.15E-03$	$-4.82E - 05$	5.83E-03	7.59E-03	

Table C.7.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.433 and Absolute Error = 1.055

Figure C.7.2. Initial frequency estimates from transfer function in E-W direction.

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Figure C.7.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.7.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Model	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.3821		0.0650 $-5.58E-00$	$-9.03E-03$	$5.69E-02$	$1.01E + 001$	1st Mode
	1.2939	0.1325		$5.02E-01$ -1.10E-03	$9.80E-03$	3.30E-01	
	1.1051	0.0298	$9.54E-02$	$-1.78E-04$	$-1.18E-02$	$1.81E-02$	
	3.8197	0.0015	7.49E-03	$-1.24E-04$	$-1.24E-03$	$2.25E-03$	
	3.9666	0.0083	8.38E-03	$6.73E-06$	$3.09E-03$	$1.71E-03$	
	4.2187	0.0024	$-1.25E-03$	$-3.81E-05$	$1.88E-03$	$7.20E-04$	

Table C.7.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.360 and Absolute Error = 0.891

Figure C.7.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.7.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.7.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

8. Milikan Library - NOAA Station No. N264-265

Table C.8.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.443 and Absolute Error = 2.800

Figure C.8.1. Initial frequency estimates from transfer function in E-W direction.

Figure C.8.2. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.8.3. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	1.5897	0.0605	$-1.24E-00$	$-1.58E-03$	$4.89E-02$	6.32E-01	1st Mode
$\overline{2}$	1.7578	0.0267	$-2.08E-01$	3.50E-03	$-2.18E - 02$	3.58E-02	
3	6.4101	0.1372	$6.96E-02$	$1.02E-04$	$3.51E-03$	9.78E-03	
	8.1378	0.0250	$3.64E-02$	$-2.58E-0.5$	$-1.11E-03$	6.00E-03	
	7.8556	0.0021	1.94E-02	$2.43E-05$	$8.60E-05$	6.51E-03	
6	7.2776	0.0186	$2.15E-02$	$1.22E-05$	$1.01E-04$	1.44E-03	
	7.2881	0.6551	$-2.04E-01$	2.46E-04	$-1.65E-02$	5.68E-03	
x	8.4907	0.0000	$-2.72E-03$	$1.72E-05$	$-5.05E-04$	7.72E-04	

Table C.8.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.279 and Absolute Error = 0.718

Figure C.8.4. Initial frequency estimates from transfer function in N-S direction.

Figure C.8.5. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.8.6. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

9. Los Angeles - 5-Story Warehouse, CSMIP Station No. 24463

Los Angeles - 5-story Warehouse
(CSMIP Station No. 24463) **SENSOR LOCATIONS**

Figure C.9.1a. Sensor locations in 5-story warehouse, CSMIP Station No. 24463

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.6868				0.0552 -3.16E-00 -1.25E-03 -3.26E-03	$8.13E-01$	1st Mode
	2.1240	0.0656		$4.22E-01$ -1.34E-05	7.84E-04	2.87E-01	2nd Mode
	3.6467	0.0569	$-1.11E-01$		$-1.66E-06$ $-3.18E-04$	$7.71E-02$	
	8.0107	0.0055	$6.75E-02$	$-4.44E-04$	$-3.73E-02$	5.76E-01	
	8.0102	0.0061	$-7.24E - 02$	4.38E-04	$3.92E - 02$	5.83E-01	
6	7.9110	0.0028	$3.03E-04$		$1.76E-05$ -1.56E-03	$1.14E-03$	

Table C.9.I. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.314 and Absolute Error = 0.635

Figure C.9.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.9.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.9.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.6230		0.0363 $-3.25E-00$		4.71E-03 5.75E-03	9.31E-01	1st Mode
	1.8555	0.0721	$-3.25E-00$	$4.01E-03$	$-3.11E-02$	$1.51E + 01$	2nd Mode
	1.8597	0.0731	$3.62E-00$	$-4.01E-03$	$2.88E-02$	$1.85E + 01$	
4	3.4478	0.0697	$-9.83E-02$	$2.03E-05$	$-2.86E-05$	4.77E-02	
	4.7759	0.0609	$2.40E-02$	$-1.58E-0.5$	$-8.12E-04$	$2.91E-03$	

Table C.9.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.164 and Absolute Error = 0.230

Figure C.9.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.9.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.9.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

10. Los Angeles - 3-Story Commercial Building, CSMIP Station No. 24332

Figure C.10.1a. Details of 3-story commercial building, CSMIP Station No. 24332

Foundation Type:

caissons.

Spread footings and drilled bell

156

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	1.7203	[0.2343]	$-1.39E-00$	$-3.00E - 03$	4.76E-03	4.15E-01	
	1.9897	0.0484	$-5.94E-01$	8.50E-04	$9.96E-03$	3.26E-01	1st Mode
	2.6335	0.5218	$1.53E-00$	1.91E-03	$-1.30E-02$	2.96E-01	
	4.4917	0.0362	$5.82E-02$	$-8.27E-05$	$2.22E-03$	1.87E-02	
	5.4994	0.1164	$-1.82E-01$	$-2.52E-05$	$-5.42E - 04$	4.95E-02	
	6.5720	0.0600	$1.94E-02$	$-7.30E-06$	1.39E-03	8.91E-04	

Table C.10.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.250 and Absolute Error = 1.995

Figure C.10.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.lO.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.lOA. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	<u>1.8938 </u>		0.4290 $-328.2E-1$	$-1.98E-01$	8.43E-01	$1.59E + 02$	
	1.8066	0.0071	$-5.95E-02$	$3.04E-03$	$-1.64E-02$	$3.13E-02$	1st Mode*
	3.6306	0.0903	$1.44E-01$	$4.51E-04$	$-1.64E-02$	$6.54E-02$	
	2.0896	0.3859	$2.39E+01$	$1.24E - 01$	$-6.05E-01$	$1.16E + 02$	
	1.4502	0.3263	5.05E-00	$6.08E-02$	$-1.73E-01$	$2.92E + 00$	
	4.0497	0.0475	$6.91E-02$	$-3.50E-04$	$-5.49E-03$	$4.10E-02$	

Table C.1O.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

* Damping is not reliable

Relative Error = 0.366 and Absolute Error = 4.888

Figure C.1O.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.IO.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.I0.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

11. Los Angeles - 17-Story Residential Building, CSMIP Station No. 24601

Los Angeles - 17-story Residential Bldg.
(CSMIP Station No. 24601)

Figure C.11.1b. Sensor locations in 17-story residential building, CSMIP Station No. 24601

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.8493		0.0401 $-3.18E-00$ $-1.47E-03$		$6.31E-03$	8.09E-01	1st Mode
	4.1504	0.1069	$1.12E + 01$	$-1.41E-03$	$-3.66E-02$ 9.39E+01		2nd Mode*
	4.1576		0.1080 -1.10E+01	$1.38E-03$	$3.78E-02$	8.94E+01	
	6.1290		0.0506 -7.70E-03	2.58E-06	$1.40E-04$	9.50E-05	
	12.3674	0.0136	$9.62E-04$	$2.47E-07$	$3.07E-09$	$2.00E-06$	
	8.2835	0.1204	$-1.18E-01$	4.55E-06	1.86E-04	3.57E-03	

Table C.II.I. Results of system identification in N-S (longitudinal) direction by WPCMIMO.

Not reliable

Relative Error = 0.248 and Absolute Error = 3.265

Figure C.ll.2. Initial frequency estimates from transfer function in N-S direction.

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Figure C.II.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.11.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.9506	0.0331		$-2.43E-00$ $-1.79E-03$ $-3.31E-03$		8.30E-01	1st Mode
	4.5000	0.0115	$3.34E-02$	$-4.18E-06$	7.07E-04	2.74E-03	
	4.7540	0.0090	$3.60E-02$	7.18E-06	$-4.78E-04$	$4.35E-03$	
	5.3400	0.0902	$9.89E-02$	$-1.43E-05$	$6.67E-04$	5.76E-03	
	10.3040	0.1082	$-6.32E-02$	4.36E-06	$3.53E-04$	1.29E-03	
	7.3071	0.0468	$-1.03E-02$	$-1.39E-07$	3.23E-04	$2.02E-04$	

Table C.ll.2. Results of system identification in E-W (transverse) direction by WPCMIMO.

Relative Error = 0.352 and Absolute Error = 6.348

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Figure C.ll.5. Initial frequency estimates from transfer function in E-W direction.

Figure C.11.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.11.? Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

12. Los Angeles - 15-Story Govt. Office Building, CSMIP Station No. 24569

Los Angeles - 15-story Govt. Office Bldy.
(CSMIP Station No. 24569)

Figure C.12.1. Sensor locations in 15-story office building, CSMIP Station No. 24569
Mode	Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.3220	0.0241	$-6.99E - 00$	$-3.78E-04$	$2.24E-03$	$1.06E + 00$	1st Mode
∼	0.8667	0.0390	1.05E-00	$4.76E-05$	7.97E-04	$3.28E-01$	2nd Mode
	1.4200	0.0887	$-3.89E-01$		$2.11E-0.5$ -3.86E-0.5	1.28E-01	
	2.1877	0.0358	$1.06E-01$	$4.05E-06$	$-4.07E-05$	$4.89E - 02$	

Table C.12.1. Results of system identification in N-S (longitudinal) direction by WPCMIMO.

Relative Error = 0.326 and Absolute Error = 0.683

Figure C.12.2. Initial frequency estimates from transfer function in N-S direction.

Figure C.12.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.12.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

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Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.3133	[0.0154]	$-6.47E-00$	$-7.86E-04$	$-1.40E - 03$	5.59E-01	1st Mode
	0.8555	0.0321	1.01E-00	$-6.49E-04$	9.07E-04	3.84E-01	2nd Mode
	1.9356	0.3273	$-1.43E-00$	$-7.41E-04$	6.63E-03	7.01E-01	
	2.1726	0.1557	7.93E-01	$5.30E-04$	5.61E-04	6.00E-01	
	2.3915	0.0172	$1.11E-00$	$-3.02E - 04$	$-3.29E - 02$	$6.12E + 00$	
	2.3915	0.0153	$-1.00E-00$	$2.70E-04$	3.07E-02	$5.19E + 00$	
	2.9069	0.0598	$-1.58E-01$	$-1.95E-05$	$-9.07E-04$	7.50E-02	
g	3.6051	0.0517	1.41E-01		$-6.22E - 06$ $-2.31E - 04$	1.14E-01	

Table C.12.2. Results of system identification in E-W (transverse) direction by WPCMIMO.

Relative Error = 0.310 and Absolute Error = 0.556

Figure C.12.5. Initial frequency estimates from transfer function in E-W direction.

Figure C.12.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.12.? Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

13. Los Angeles - 52-Story Office Building, CSMIP Station No. 24602

Figure C.13.1a. Details of 52-story office building, CSMIP Station No. 24602

Figure C.13.1b. Sensor locations in 52-story office building, CSMIP Station No. 24602

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.1612		0.0065 -1.41E+01	$-4.20E-03$	$-4.36E-04$	1.92E-01	1st Mode
2	0.5371	0.0146	2.34E-00	$-2.01E-03$	$-9.67E-03$	3.84E-01	2nd Mode
3	1.0332	0.0177	$-9.09E-01$	$-3.24E-04$	3.89E-03	3.07E-01	
4	1.5220	0.0218	5.08E-01	1.10E-04	$2.83E-04$	1.41E-01	
5	1.9951	0.0303	$-3.70E-01$	5.04E-06	$2.68E-04$	8.11E-02	
6	2.4731	0.0198	1.52E-01	1.94E-05	$-2.38E-04$	3.37E-02	
	3.2628	0.0194	7.60E-02	1.38E-05	4.45E-04	1.20E-02	
8	3.6766	0.0208	$-4.04E-02$	1.10E-05	1.75E-04	9.38E-03	
9	2.9215	0.0251	$-1.58E-00$	$-4.53E-04$	$-1.54E-02$	$1.67E + 00$	
10	2.9367	0.0245	1.48E-00	$3.46E-04$	1.61E-02	$1.52E + 00$	

Table C.13.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.245 and Absolute Error = 1.071

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Figure C.13.2. Initial frequency estimates from transfer function in E-W direction.

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Figure C.13.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.13.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.1673	0.0303	$-2.13E+01$	$-2.25E-03$	$-1.07E - 03$	3.17E-01	1st Mode
$\overline{2}$	0.5371	0.0509	4.16E-00	$-3.91E-03$	$-6.53E-03$	2.70E-01	2nd Mode
3	1.0178	0.0131	$-7.33E-01$	$-1.96E-04$	8.26E-05	1.97E-01	
4	1.4625	0.0228	4.77E-01	$-7.38E - 05$	$-4.71E-05$	2.47E-01	
5	1.8992	0.0320	$-3.61E-01$	$-1.52E-04$	6.39E-04	8.10E-02	
6	3.1560	0.0300	$9.21E-02$	9.60E-05	$-1.92E-03$	1.80E-02	
7	2.3602	0.0113	1.18E-01	$1.54E-04$	$6.99E-04$	2.13E-02	
8	2.6214	0.0259	$-1.54E-01$	$-2.56E-05$	2.85E-03	2.22E-02	
9	2.2931	0.0463	1.91E-01	$-4.20E-04$	2.86E-03	2.42E-02	
10	2.9765	0.0059	2.16E-02	$9.52E-05$	9.84E-04	1.27E-03	
	2.7816	0.0052	$-8.66E-02$	1.00E-04	1.75E-03	7.16E-03	

Table C.13.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.448 and Absolute Error = 4.326

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Figure C.13.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.13.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.13.? Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

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14. Los Angeles - 9-Story Office Building, CSMIP Station No. 24579

SENSOR LOCATIONS $\overline{10}$ $\overline{14}$ Roof $13'$ $15'$ B th $14'$ Unrainforced 128 15 4.013 Gth .
Masonry Walls $4th$ $14'$ $2nd$ \mathbf{H}^{18} 12 Ľ \mathbf{A}^{13} $20'$ $11 +$ $16₄$ $13'$ **Basement** 5th Floor Plan Roof Plan **W/E Elevation** \mathbf{R}_{ref} $155'$ - 1 <u>51'</u> J. $\overline{2}$ 3 Structure Reference Orientation: $N = 40^{\circ}$ 155 Concrete Wall (basement only) $\frac{1}{5}$ A 8 7 $9-$ 5. 2nd Floor Plan **Basement Plan**

Los Angeles - 9-story Office Bldg.
(CSMIP Station No. 24579)

Figure C.14.1. Sensor locations in 9-story office building, CSMIP Station No. 24579

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Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.0143	0.0042	$6.12E + 01$	$.23E-02$	7.92E-03	5.51E-02	
2	0.6226		0.0136 -1.94E+01	$-8.82E-01$	$-2.31E-00$	1.37E+02	
3	0.7172	0.0830	$-2.96E-00$	$-3.44E-03$	1.99E-02	6.83E-01	1st Mode
4	0.8163	0.0135	$-3.30E-01$	$-6.42E-04$	$-2.90E-04$	3.24E-02	
5.	0.6224	0.0134	1.87E+01	8.82E-01	2.28E-00	1.34E+02	
6	0.9218	0.0005	5.40E-03	$-2.21E-04$	5.60E-03	1.77E-03	
⇁	2.4321	0.0833	1.71E-01	7.72E-05	4.92E-03	8.40E-02	
8	2.8442	0.0018	1.92E-02	7.01E-05	$-2.14E-03$	8.65E-03	
9	2.7451	0.0044	1.41E-02	1.11E-05	$-5.29E-04$	4.79E-03	
10	3.8004	0.0097	$-1.38E-02$	$-5.96E-06$	$-1.26E-04$	5.02E-03	
	4.2073	0.0715	$-5.66E-02$	4.28E-07	4.26E-04	1.55E-02	

Table C.14.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.340 and Absolute Error = 1.352

Figure C.14.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.14.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.14.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.6408	0.0060	$-1.44E-01$	$-7.83E - 04$	$-1.28E - 02$	1.97E-02	
2	0.7813	0.0690	$-1.70E-00$	3.97E-03	$-1.08E-01$	2.87E-01	1st Mode
٦	0.7588	0.0000	$-6.78E-02$	3.74E-03	4.90E-02	1.02E-01	
4	1.1144	0.6307	$-7.89E - 00$	$-3.46E - 02$	6.57E-02	$2.53E+00$	
	1.3357	0.4244	$4.14E-00$	8.52E-03	$6.82E-03$	$1.76E + 00$	
	2.7573	0.1304	4.18E-01	4.58E-04	1.93E-03	1.80E-01	
	2.4292	0.0501	9.99E-02	1.65E-08	1.61E-06	2.73E-02	
	3.0762	0.0500	9.99E-02	l.34E-08	$-1.28E-06$	3.20E-02	

Table C.14.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error $= 0.502$ and Absolute Error $= 3.267$

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Figure C.14.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.14.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.l4.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

15. Los Angeles - 8-Story CSULA Administration Building, CSMIP Station No. 24468

SENSOR LOCATIONS

Figure C.15.1b. Sensor locations in 8-story administration building, CSMIP Station No. 24468

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.6479		0.0360 -3.08E-00	$-4.72E-03$	$1.56E-02$		$8.78E-01$ 1st Long. Mode
	0.7263	0.0304	$-2.57E-01$	5.14E-03	$-4.06E-03$	8.95E-03	1st Tor. Mode
	1.8301	0.0724	$3.87E-01$	$2.19E-04$	$3.65E-03$	$8.07E-02$	
	1.9892	0.0030	$2.50E-02$	$2.47E-04$	$-2.01E-03$	3.59E-03	
	4.5171	0.0449	$-2.23E-02$	$-1.05E-05$	$-2.67E-04$	$3.49E-03$	
	3.6944	0.0462		$-1.33E-02$ $-7.75E-06$ $-5.72E-04$		$7.17E-04$	

Table C.15.I. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.240 and Absolute Error = 0.421

Figure C.15.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.I5.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.I5.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.6188	0.0370	$-4.27E-00$	$2.91E-03$	$2.07E-02$	9.18E-01	1st Mode
	1.9653	0.0582	$4.56E-01$	$2.26E-04$	$2.80E-03$	$2.51E-01$	2nd Mode
	5.3866	0.4742	$-1.72E-01$		$3.77E-04$ -1.74E-02	$1.51E-02$	
	5.5919	0.2183	$7.03E-02$	$-6.67E-0.5$	8.51E-03	$1.08E - 02$	

Table C.15.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.253 and Absolute Error = 0.412

Figure C.15.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.15.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.15.? Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

16. Pasadena - 6-Story Office Building, CSMIP Station No. 24541

Figure C.16.1. Sensor locations in 6-story office building, CSMIP Station No. 24541

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.4564	0.0542	$-4.47E-00$		<u>3.13E-03L -3.78E-03L</u>	6.59E-01	1st Mode
	1.8677	0.0736	$3.77E-01$	8.35E-05	$1.67E-04$	4.13E-01	2nd Mode
	3.5074	0.1201	$-1.63E-01$	$1.61E-04$	$-2.09E-03$	$9.63E-02$	
	4.6060	0.0658	$4.87E-02$	$-2.48E-0.5$	$-3.34E-03$	$2.22E-02$	
	5.1310	0.00001	$-3.38E-03$	$-5.25E-05$	$1.44E-03$	6.37E-03	
	6.5609	0.0627	$7.58E-03$	$1.14E-06$	$1.51E-05$	$3.41E-04$	

Table C.16.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.379 and Absolute Error = 0.268

Figure C.16.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.16.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.16.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.4760	0.2218	0.337229	0.001893	0.001131		$3.62E-02$ 1st Tran. Mode*
	0.5595	0.0908	$-2.36E-00$	$-2.36E-04$	3.57E-03	4.76E-01	1st Tor. Mode*
	0.7813	0.0744	$-9.89E-01$	1.51E-03	1.69E-03	2.22E-01	
	2.0093	0.1908	3.76E-01	2.09E-04	$-3.00E-03$		$3.37E-01$ 2nd Tran. Mode*
	4.9434	0.0264	$-4.12E-03$	$-5.85E-05$	$9.34E-04$	1.89E-03	
	4.4202	0.0159	$-3.96E-03$	$-8.64E-06$	$-7.25E-04$	1.13E-03	
	3.8488	0.0996	$-5.86E-02$	$3.35E-05$	$-1.98E-03$	$3.62E-02$	

Table C.16.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

* Damping is unreliable

Relative Error = 0.231 and Absolute Error = 0.100

Roof N-S Acceleration, 6-Story Office Bldg., Pasadena.

Figure C.16.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.16.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.16.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

17. Whittier - 8-Story Hotel, CSMIP Station No. 14606

¹st Floor Plan

Figure C.17.1. Sensor locations in 8-story hotel, CSMIP Station No. 14606

Mode	Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	.4441				$[0.0519] -1.71E-00$ -3.20E-05 2.82E-04	8.03E-01	1st Mode
	5.5298	0.1125	$1.60E-01$	$-4.79E-08$	$3.30E-05$	1.01E-01	2nd Mode
	12.2209	0.0912	$-2.82E-02$	$5.32E-08$	$9.89E-06$	$1.66E-03$	
	20.9184	0.0570	$-1.15E-02$	$2.68E-08$	$2.47E-06$	4.10E-05	

Table C.17.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

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Relative Error = 0.283 and Absolute Error = 1.077

Figure C.17.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.I?.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.1?.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	1.5956		0.1163 -1.11E-00	$3.11E-06$	4.20E-04		$5.91E-01$ 1st Tran. Mode*
	1.7822	0.0846	$-2.41E-01$	$1.79E-05$	$-4.13E-04$		4.96E-02 1st Tor. Mode $*$
	3.2949	0.5232	$2.56E-00$	$-5.63E-05$	8.72E-04	$2.22E+00$	
4	3.2605	0.3913	$-1.96E-00$	$4.19E-05$	$-8.58E-04$	$2.23E+00$	
	4.8955	0.0918	$-4.75E-02$	$6.26E-07$	$-4.26E-05$	$1.25E-02$	
6	5.5757	0.0438	$-1.70E-02$		$-1.27E-06$ $-1.58E-05$	$2.52E-03$	
	9.5854	0.0329			$1.09E-02$ -6.25E-08 -1.94E-06	9.35E-04	

Table C.17.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

* Damping is not reliable

Relative Error = 0.355 and Absolute Error = 1.595

Figure C.17.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.17.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.17.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

18. Los Angeles - 6-Story Office Building, CSMIP Station No. 24652

Mode	Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	1.1116						$[0.0394]$ -1.42E-00 -1.01E-03 3.61E-03 8.18E-01 1st Long, Mode
	3.8208	0.0768		$1.67E-01$ -9.39E-07			1.87E-04 2.22E-02 2nd Long. Mode
	1.2847	0.0367	$-3.43E-01$		$4.85E-04$ -1.03E-03		$4.24E-02$ 1st Tor. Mode
	6.1724	0.1110	$-4.45E-02$	3.25E-06	$1.44E-04$	$1.79E-03$	

Table C.18.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

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Relative Error = 0.272 and Absolute Error = 2.695

Figure C.18.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.18.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.18.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	1.1584		0.0358 -1.80E-00 -1.44E-04		$5.19E-03$	$8.65E-01$	1st Mode
	3.8818	0.0740		$1.95E-01$ 8.08E-06	$1.23E-04$	5.60E-02	2nd Mode
	6.6710	0.0976	$-4.49E-02$		$3.17E-06$ $-4.19E-05$	$2.66E-03$	
	9.0754	0.08141			$9.35E-03$ $-3.48E-07$ $-5.24E-05$	$1.94E-04$	

Table C.18.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.250 and Absolute Error = 1.518

Figure C.18.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.18.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.18.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

19. Alhambra: 900 South Fremont Street - 13-Story Steel Building, USGS Station No. 482

Figure C.19.1 Sensor locations in 13-story building, USGS Station No. 482

	Mode Frequency Damping		Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	3.2271			$[0.1869] -2.00E-01$ -1.95E-04	$4.44E-03$	1.05E-01	
	0.4650	0.0119	$-4.44E-00$		$-9.64E-03$ $-8.32E-03$	$1.02E + 00$	1st Mode
	1.3052	0.0493	$4.13E-01$	7.99E-05	$2.61E-03$	1.79E-01	2nd Mode
	3.8401	0.0721	1.17E-01	8.23E-05	4.56E-04	1.27E-01	

Table C.I9.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.236 and Absolute Error = 0.687

Figure C.19.2. Initial frequency estimates from transfer function in E-W direction.

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Figure C.19.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.19.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.4204	0.0000	2.89E-00	7.57E-03	$-2.01E-02$	3.12E-01	
	0.4505	0.0194	$-7.09E-00$	$-1.03E-02$	$-7.15E-03$	$2.30E + 00$	1st Mode
	1.2728	0.0643	$4.04E-01$	$-2.80E-04$	$3.08E-03$	1.96E-01	2nd Mode
	2.9829	0.0788	$-1.22E-01$	$3.05E-05$	1.38E-03	7.49E-02	
	3.7287	0.0383	$3.67E-02$	$-3.29E-05$	9.86E-04	$2.74E-02$	
	4.0595	0.0092	5.18E-03	$6.12E-05$	3.87E-04	1.58E-03	

Table C.19.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.323 and Absolute Error = 1.035

Figure C.19.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.19.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.19.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

LOS ANGELES

1100 WILSHIRE BLVD

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Figure C.20.1. Sensor locations in 32-story building, USGS Station No. 5233

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.5907	0.0329	1.59E+01	$-5.70E-01$	4.58E-00	$9.92E + 01$	2nd Tor. Mode
2	0.2808	0.0413	$-1.43E + 01$	1.07E-01	$-8.97E-02$	1.31E+00	1st Tor. Mode
3	-0.0062	0.0055	.22E+01	$-1.65E + 01$	3.38E-01	1.84E-03	
$\overline{4}$	0.2296	0.0090	$-1.01E + 01$	6.15E-02	7.36E-02	4.69E-01	
5	0.2919	0.0165	1.43E+01	$-6.31E-02$	1.08E-02	1.44E+00	1st Long. Mode
6	0.5881	0.0355	$-1.90E + 01$	6.34E-01	$-4.95E-00$		$1.09E+02$ 2nd Long. Mode
7	0.5548	0.0030	3.17E-00	$-4.42E-02$	2.69E-01	1.00E+00	
8	1.0561	0.0167	$-2.36E-01$	1.20E-03	$-2.72E-03$	4.38E-02	
9	1.0867	0.2598	6.71E-01	$-1.12E-02$	$7.21E-03$	5.39E-02	
10	1.4649	0.0066	$-5.09E-03$	9.33E-04	3.07E-02	3.99E-02	
11	1.5024	0.0076	7.23E-02	1.02E-03	$-2.97E-02$	2.82E-02	

Table C.20.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.377 and Absolute Error = 2.521

 \sim

Figure C.20.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.20.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMlMO in E-W direction.

Figure C.20.4. Comparison of time-histories: recorded motions and calculated motions from WPCMlMO in E-W direction.

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.6689	0.0294	<u>-1.87E-00 </u>	$-1.01E-02$	$-3,99E-02$.29E-01	
2	0.2295	0.0177	$-1.11E+01$	$4.36E-02$	$-5.32E-02$	2.83E-01	1st Mode
3	0.5538	0.0299	$2.70E-00$	$2.25E-03$	$-7.37E-02$	4.54E-01	2nd Mode
4	1.0586	0.0012	$-2.26E-01$	$-4.65E-03$	$-6.55E-03$	2.23E-02	
5	1.2360	0.0184	4.39E-01	$-2.17E-03$	2.84E-02	6.80E-02	
6	2.2550	0.0532	1.94E-01	$-6.09E - 04$	2.38E-02	5.80E-02	
	1.6792	0.0076	9.09E-02	$1.24E-03$	1.30E-02	1.18E-02	
8	1.8565	0.0355	$-2.39E-01$	4.89E-04	$-1.31E-02$	5.24E-02	
9	2.2729	0.0000	$.68E-02$	$2.26E-04$	$-1.12E-02$	4.02E-03	

Table C.20.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Relative Error = 0.419 and Absolute Error = 10.187

 $\bar{\mathcal{A}}$

Figure C.20.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.20.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.20.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.21.1. Sensor locations in 6-story hospital building, USGS Station No. 5082

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	1.0648		0.0516 -2.58E-00		$4.38E-03$ 2.34E-02	7.04E-01	1st Mode
	3.1250	0.3305	$1.58E-00$		$3.29E-03$ $-4.25E-02$	6.30E-01	
	3.1434	0.0400			$7.21E-02$ -1.56E-04 -1.55E-04	$1.16E-02$	
	3.6626	0.1114	$-3.90E-01$		$-6.76E-04$ $-5.05E-03$	1.58E-01	
	4.3789	0.1321		$-3.86E-01$ $-4.69E-04$	8.74E-03	2.00E-01	

Table C.21.1. Results of system identification in E-W (longitudinal) direction by WPCMIMO.

Relative Error = 0.343 and Absolute Error = 8.968

Figure C.21.2. Initial frequency estimates from transfer function in E-W direction.

Figure C.21.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure C.21.4. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Table C.21.2. Results of system identification in N-S (transverse) direction by WPCMIMO.

Mode	$ $ Frequency $ $	Damping	Part.	Initial	Initial	Modal	Comments
No.	Hz		Factor	Disp.	Velo.	Cont.	
	1.0376				0.0427 -2.31E-00 -1.43E-03 5.53E-03 9.68E-01		1st Mode
	3.2715	0.0835		$1.80E-01$ - $1.11E-04$	9.02E-03	$4.32E-02$	

Relative Error = 0.261 and Absolute Error = 5.600

Figure C.2l.5. Initial frequency estimates from transfer function in N-S direction.

Figure C.21.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.21.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

22. Los Angeles - 13-Story Office Building, CSMIP Station No. 24567

Figure C.22.1. Sensor locations in 13-story office building, CSMIP Station No. 24567

Mode	Frequency	Damping	Part.	Initial	Initial	Modal	Comments
No.	(Hz)		Factor	Disp.	Velo.	Cont.	
	0.3803	0.0802	$-8.22E-01$	8.05E-04	2.55E-04	7.85E-01	1st Mode
$\overline{2}$	1.0986	0.1489	1.95E-01	$5.89E-05$	$-3.26E-04$		$5.88E-01$ 2nd Mode*
3	2.0581	0.1332	$-5.07E-02$	$1.78E-05$	$-2.18E-04$	2.03E-01	
4	3.5180	0.1559	4.62E-01	$1.98E - 04$	$-2.17E-03$	$3.05E + 01$	
	3.5545	0.1579	$-4.50E-01$	$-2.00E-04$	1.79E-03	$2.90E + 01$	
6	5.7913	0.0472	$2.70E-03$	$-6.38E-07$	$5.25E-06$	4.19E-03	

Table C.22.1 Results of system identification in E-W (transverse) direction by WPCMIMO.

* Damping is not reliable

Relative Error = 0.466 and Absolute Error = 3.341

Figure C.22.2 Initial frequency estimates from transfer function in E-W direction.

Figure C.22.3. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in E-W direction.

Figure *C.22A.* Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in E-W direction.

Mode No.	Frequency (Hz)	Damping	Part. Factor	Initial Disp.	Initial Velo.	Modal Cont.	Comments
	0.4356	0.0927	$-4.77E-01$	$-4.80E - 05$ 4.93E-04		5.92E-01	1st Mode
2	1.1353	0.2527	1.17E-01	3.74E-05	5.01E-05	$2.65E-01$	
3	2.2378	0.1570	$-4.82E-02$ 9.98E-06		$-6.84E-05$ 3.87E-01		
4	3.2740	0.1390	1.44E-02	$12.22E-06$	$-1.40E-04$ 6.69E-02		
	4.4041	0.0463	$-3.95E-03$	$-1.37E-06$	$-6.89E-05$ 1.56E-02		
6	5.0297	0.0000	$-1.43E-04$	$-2.26E-06$	14.22E-05	7.78E-04	

Table C.22.2 Results of system identification in N-S (longitudinal) direction by WPCMIMO.

Relative Error = 0.535 and Absolute Error = 2.324

Figure C.22.5 Initial frequency estimates from transfer function in N-S direction.

Figure C.22.6. Comparison of empirical transfer functions: recorded motions and calculated motions from WPCMIMO in N-S direction.

Figure C.22.7. Comparison of time-histories: recorded motions and calculated motions from WPCMIMO in N-S direction.

APPENDIX D: BASIS FOR CODE FORMULA

The period formula specified in current US codes is based on the following assumptions:

- 1. Equivalent lateral forces are distributed linearly over height of the building.
- 2. Base shear is proportional to $1/T^{\alpha}$.
- 3. Weight of the building is distributed uniformly over its height.
- 4. Deflection of the building due to the equivalent lateral force distribution is linear over its height implying that the inter-story drift, Δ , is constant over height of the building.

For these assumptions, period of a building, idealized as a cantilever with uniform mass, may be estimated using the Rayleigh's method as follows:

$$
T = 2\pi \sqrt{\left(\int m \delta^2(x)\right) + \left(g \int f(x) \delta(x) dx\right)}
$$
 (D-1)

in which $\delta(x)$ is the deflected shape of the building due to the equivalent lateral force $f(x)$. Utilizing assumption (1) to (3) regarding distribution of lateral forces leads to:

$$
f(x) = 2mg\frac{C}{T^{\alpha}}\frac{x}{H}
$$
 (D-2)

in which C is the constant related to the seismic coefficient, and H is total height of the building, and assumption (4) for deflected shape of the building gives:

$$
\delta(x) = \Delta x \tag{D-3}
$$

Utilizing Eqs. $(D-2)$ and $(D-3)$ in $(D-1)$ leads to:

$$
T = \left[(2\pi)^2 \frac{\Delta}{2gC} \right]^{\frac{1}{2-\alpha}} H_{\frac{1}{2-\alpha}}
$$
 (D-4)

For $\alpha = 0$, which corresponds to base shear independent of the period, Eq. (D-4) gives:

$$
T = 2\pi \sqrt{\frac{\Delta}{2gC}} \sqrt{H}
$$
 (D-5)

For $\alpha = 1$, implying base shear proportional to 1/T, Eq. (D-4) leads to:

$$
T = (2\pi)^2 \frac{\Delta}{2gC} H
$$
 (D-6)

For $\alpha = 2/3$, indicating base shear proportional to $1/T^{2/3}$, Eq. (D-4) results in:

 $\ddot{}$

$$
T = \left[(2\pi)^2 \frac{\Delta}{2gC} \right]^{3/4} H^{3/4}
$$
 (D-7)

Eq. (D-7) is consistent with the formula $T = C_t H^{3/4}$ in US seismic codes that specify base shear proportional to $1/T^{2/3}$ in the velocity-controlled region of the design spectrum.

 \mathbf{v}

 $\bar{\gamma}$

APPENDIX E: THEORETICAL FORMULA FOR MRF BUILDINGS

Derived in this appendix are the theoretical formulas for fundamental period of frame buildings using the Rayleigh's method. The formulas are derived for the shear buildings, that is buildings with rigid girders (or beams).

Using Rayleigh's method, the fundamental period of a shear building is:

$$
T = 2\pi \sqrt{\frac{\sum_{j=1}^{N} m_j \Psi_j^2}{\sum_{j=1}^{N} k_j (\Psi_j - \Psi_{j-1})^2}}
$$
(E-1)

in which m_j is the mass and Ψ_j is the deflection at jth floor of the building; and, k_j is the stiffness of the jth story. For equal mass at all floors, i.e., $m_j = m$ for $j = 1 - N$, and linear deflected shape of the building, i.e., $\psi_j = j/N$, Eq. (E-1) leads to:

$$
T = 2\pi \sqrt{\frac{\frac{m}{N^2} \sum_{j=1}^{N} j^2}{\sum_{j=1}^{N} k_j \left(\frac{j}{N} - \frac{j-1}{N}\right)^2}} = 2\pi \sqrt{\frac{m \sum_{j=1}^{N} j^2}{\sum_{j=1}^{N} k_j}}
$$
(E-2)

Buildings with Uniform Stiffness Over Height

For uniform building stiffness over height, i.e., $k_j = k$ for $j = 1 - N$, Eq. (E-2) simplifies

to:

$$
T = 2\pi \sqrt{\frac{\frac{m}{N^2} \sum_{j=1}^{N} j^2}{k \sum_{j=1}^{N} 1}} = 2\pi \sqrt{\frac{m}{k} \frac{\sum_{j=1}^{N} j^2}{N}}
$$
(E-3)

Eq. (E-3) can be further simplified by utilizing the following result:

$$
\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}
$$
 (E-4)

to obtain

$$
T = 2\pi \sqrt{\frac{m}{k} \frac{(N+1)(2N+1)}{6}} = 2\pi \sqrt{\frac{m}{k} \frac{2N^2 + 3N + 1}{6}}
$$
(E-5)

For N > 6, i.e., building with more than 6 stories, the N^2 term in Eq. (E-5) would dominate and the following relationship may be used to estimate the fundamental period:

$$
T = C_1 N \tag{E-6}
$$

in which the coefficient C_1 depends on the unit mass and stiffness properties among other constants in Eq. (E-5). For buildings with uniform story height, Eq. (E-6) may also be written in terms of the total building height, H , as:

$$
T = \overline{C}_1 H \tag{E-7}
$$

Buildings with Linearly Decreasing Stiffness Over Height

For linearly decreasing stiffness with height,

$$
k_j = k_N(N-j+1) \tag{E-8}
$$

in which k_N is the stiffness at the top story level. This leads to

$$
\sum_{j=1}^{N} k_j = k_N \sum_{j=1}^{N} (N+1-j) = k_N (N+1) \sum_{j=1}^{N} 1 - k_N \sum_{j=1}^{N} j = k_N (N+1) N - k_N \frac{(N+1)N}{2} = \frac{1}{2} k_N
$$
 (E-9)

Utilizing Eqs. (E-9) and (E-4) in Eq. (E-2) leads to:

$$
T = 2\pi \sqrt{\frac{m (2N+1)}{k_N 3}}
$$
 (E-10)

For $N > 3$, Eq. (E-10) may be approximated as:

$$
T = C_2 \sqrt{N} \tag{E-11}
$$

or

Buildings with Linear Deflection Due toTriangular Load

For linear deflection under the triangular loading, the stiffness required at the jth story is given as:

$$
k_j = \frac{k_N}{N} \sum_{i=j}^{N} i
$$
 (E-13)

which leads to:

$$
\sum_{j=1}^{N} k_j = \frac{k_N}{N} \sum_{j=1}^{N} \sum_{i=j}^{N} i = \frac{k_N}{N} \frac{N(N+1)(2N+1)}{6}
$$
 (E-14)

Utilizing Eqs. (E-4) and (E-14) in Eq. (E-2) leads to:

$$
T = 2\pi \sqrt{\frac{m}{k_N}} N
$$
 (E-15)

which can obviously be written as:

$$
T = C_3 \sqrt{\mathcal{N}} \tag{E-16}
$$

or

$$
T = \overline{C}_3 \sqrt{H} \tag{E-17}
$$

APPENDIX F: REGRESSION ANALYSIS METHOD

The fundamental period of a building can be expressed as:

$$
T = \alpha \, \widetilde{H}^{\beta} \tag{F-1}
$$

For MRF buildings $\overline{H} = H$ and α and β are the numerical constants to be determined from regression analysis. For SW buildings $\overline{H} = H \div \sqrt{A_e}$, β is fixed at one, and α is the numerical constant to be determined from regression analysis. Eq. (F-l) may be recast as:

$$
y = a + \beta x \tag{F-2}
$$

where $y = log(T)$, $a = log(\alpha)$, and $x = log(\overline{H})$. Equation (F-2) represents a straight line with intercept *a* and slope β . Therefore, the power relationship of Eq. (F-1) becomes a linear relationship in the log-log space as shown in Figure F.l.

Figure F.I. Conceptual explanation of regression analysis.

In the regression analysis, the intercept a and slope β of the line in Figure F.1 were determined by minimizing the squared error between the measured and computed periods, and then the numerical constant α was back calculated from the relationship $a = log(\alpha)$.

The goodness of the fit is represented by the standard error of estimate defined as:

$$
s_e = \sqrt{\frac{\sum_{i=1}^{n} \left[y_i - \left(a + \beta x_i \right) \right]^2}{\left(n - 2 \right)}}
$$
(F-3)

in which $y_i = log(T_i)$ is the observed value (with T_i = measured period) and $(a + \beta x_i) = \log(\alpha) + \beta \log(\overline{H})$ is the computed value of the ith data, and *n* is the number of data points. The s_e represents scatter in the data and approaches, for large *n*, the standard deviation of the measured period data from the best-fit equation.

This procedure leads to the value of α_R for Eq. (F-1) to represent the best-fit, in the least squared sense, to the measured period data. However, for code applications the formula should provide a lower value of the period and this was obtained by lowering the best-fit line (Eq: F-2) by s_e without changing its slope (Figure F.1). Thus α_L , the lower value of α , is computed from: $log(\alpha_L) = log(\alpha_R) - s$ _c (F-4)

Since s_e approaches the standard deviation for large number of samples and if y is assumed to be log-normal, α_L is the mean-minus-one-standard-deviation or 15.9 percentile value, implying that 15.9 percent of the measured periods would fall below the curve corresponding to α_L (subsequently referred to as the best-fit - 1σ curve). If desired, α_L corresponding to other nonexceedance probabilities may be selected.

As mentioned previously, codes also specify an upper limit on the period calculated by a "rational" analysis. This limit is established in this investigation by raising the best-fit line (Eq. F-2) by s_e without changing its slope (Figure F.1). Thus α_v , the upper value of α corresponding to the upper limit, is computed from:

$$
\log(\alpha_{U}) = \log(\alpha_{R}) + s_{e} \tag{F-5}
$$

Eq. (F-1) with α_U represents the best-fit + 1 σ curve which will be exceeded by 15.9 percent of the measured periods.

Regression analysis in the log-log space (Eq. F-2) is preferred over the direct regression on Eq. (F-1) because it permits convenient development of the best-fit - 1σ and best-fit + 1σ curves. Also note that β is fixed at one for SW buildings and only α is determined from the regression analysis.

APPENDIX G: THEORETICAL FORMULAS FOR SW BUILDINGS

Rayleigh's Method

For a cantilever with uniformly distributed mass and stiffness properties, the fundamental period may be calculated from:

$$
T = 2\pi \sqrt{\frac{\int m(x)\delta^2(x)dx}{\int f(x)\delta(x)dx}}
$$
 (G-1)

in which $m(x)$ = mass per unit length, $f(x)$ = applied force, and $\delta(x)$ = deflection due to the applied force at height $= x$ from base of the cantilever. For a cantilever with uniform mass $m(x) = m$ and a triangular distribution of applied force $f(x) = f_0 x/H$, Eq. (G-1) becomes:

$$
T = 2\pi \sqrt{\frac{m\int \delta^2(x)dx}{\int H} \int x\delta(x)dx}}
$$
(G-2)

The triangular distribution of forces over the height of the cantilever is similar to that specified in building codes. For such a height-wise distribution of forces, variations of shear force and bending moment with height of the cantilever are given as:

$$
V(x) = \frac{1}{2} \frac{f_o}{H} (H^2 - x^2)
$$
 (G-3)

$$
M(x) = \frac{1}{6} \frac{f_o}{H} \left(2 H^3 - 3 H^2 x + x^3 \right)
$$
 (G-4)

The deflection at location x can be calculated by the principle of virtual work as:

$$
\delta(x) = \int_{0}^{x} \frac{\nu(\zeta)}{\kappa GA} V(\zeta) d\zeta + \int_{0}^{x} \frac{\overline{m}(\zeta)}{EI} M(\zeta) d\zeta = \delta_{S}(x) + \delta_{F}(x)
$$
\n(G-5)

in which $v(\zeta)$ and $\overline{m}(\zeta)$ are the shear force and bending moment, respectively, at location ζ due to a virtual unit force at location x ; E and G are the Young's modulus and shear modulus, respectively; and κ is the shape factor for the cantilever. The first term in Eq. (G-5) is the contribution due to shear deformation whereas the second term is the contribution due to flexural deformation. For the cantilever with triangular loading, the deflections due to shear and flexure are:

$$
\delta_S(x) = \int_0^x \frac{1}{\kappa GA} \frac{1}{2} \frac{f_o}{H} \left(H^2 - \zeta^2 \right) d\zeta = \frac{1}{6} \frac{f_o}{H} \frac{1}{\kappa GA} \left(3 H^2 x - x^3 \right) \tag{G-6}
$$

$$
\delta_F(x) = \int_0^x \frac{(x-\zeta)}{EI} \frac{1}{6} \frac{f_o}{H} \left(H^3 - H^2 \zeta + \zeta^3 \right) d\zeta = \frac{1}{120} \frac{f_o}{H} \frac{1}{EI} \left(20 H^3 x^2 - 10 H^2 x^3 + x^5 \right) \tag{G-7}
$$

Deflection due to Shear Alone

Considering deflections due to shear alone (Eq. G-6), the numerator and denominator in Eq. (G-2) are:

$$
\int_{0}^{H} m \delta_{S}^{2}(x) dx = \int_{0}^{H} m \left[\frac{1}{6} \frac{f_{o}}{H} \frac{1}{\kappa GA} \left(8H^{3} x - x^{3} \right) \right]^{2} dx = \frac{17}{315} \left(\frac{f_{o}}{\kappa GA} \right)^{2} m H^{5}
$$
\n
$$
(G-8)
$$
\n
$$
\int_{0}^{H} m \delta_{S}^{2}(x) dx = \int_{0}^{H} m \left(6H^{3} \frac{1}{\kappa GA} \right) \left(8H^{3} \frac{1}{\kappa GA} \right)^{2} dx = \frac{17}{315} \left(\frac{f_{o}}{\kappa GA} \right)^{2} m H^{5}
$$
\n
$$
(G-9)
$$

$$
\int_{0}^{H} f(x) \delta_{S}(x) dx = \int_{0}^{H} \frac{f_o}{H} x \left[\frac{1}{6} \frac{f_o}{H} \frac{1}{\kappa GA} \left(3H^2 x - x^3 \right) \right] dx = \frac{2}{15} \frac{f_o^2}{\kappa GA} H^3
$$
\n(G-9)

Utilizing Eqs. G-8 and G-9 in Eq. G-2 leads to:

$$
T = 2\pi \sqrt{\frac{17}{315} \left(\frac{f_o}{\kappa GA}\right)^2 mH^5} = 2\pi \sqrt{\frac{17}{42} \frac{m}{\kappa G}} = 3.997 \sqrt{\frac{m}{\kappa G}} \frac{H}{\sqrt{A}}
$$
(G-10)

This formula compares well with the following exact solution for a shear cantilever:

$$
T = 4\sqrt{\frac{m}{\kappa G}} \frac{H}{\sqrt{A}}
$$
 (G-11)

Deflection due to Flexure Alone

 \bar{z}

Considering deflections due to flexure alone (Eq. G-7), the numerator and denominator in Eq. $(G-2)$ are:

$$
\int_{0}^{H} m \delta_{F}^{2}(x) dx = \int_{0}^{H} m \left[\frac{1}{120} \frac{f_{o}}{H} \frac{1}{EI} \left(20H^{3} x^{2} - 10H^{2} x^{3} + x^{5} \right) \right]^{2} dx = \frac{21128}{9979200} \left(\frac{f_{o}}{EI} \right)^{2} m_{i}
$$
 (G-12)

$$
\int_{0}^{H} f(x) \delta_F(x) dx = \int_{0}^{H} \frac{f}{H} x \left[\frac{1}{120} \frac{f}{H} \frac{1}{EI} \left(20 H^3 x^2 - 10 H^2 x^3 + x^5 \right) \right] dx = \frac{11}{420} \frac{f^2}{EI} H^5
$$
\n(G-13)

Utilizing Eqs. G-12 and G-13 in Eq. G-2 leads to:

$$
T = 2\pi \sqrt{\frac{\frac{21128}{9979200} \left(\frac{f_o}{EI}\right)^2 m H^9}{\frac{11}{420}}}=2\pi \sqrt{\frac{2641}{32670} \frac{m}{E}} \frac{H^2}{\sqrt{I}} = 1.786 \sqrt{\frac{m}{E}} \frac{H^2}{\sqrt{I}} \tag{G-14}
$$

which also compares well with the following exact solution for a flexural cantilever:

$$
T = \frac{2\pi}{3.516} \sqrt{\frac{m}{E}} \frac{H^2}{\sqrt{I}} = 1.787 \sqrt{\frac{m}{E}} \frac{H^2}{\sqrt{I}}
$$
 (G-15)

Recognizing that $I = A D^2 / 12$, in which *A* is the area and *D* is the dimension of shear wall along the direction under consideration, Eq. G-14 and G-15 may be re-written as:

$$
T = 1.786 \sqrt{\frac{m}{E}} \frac{H^2}{\sqrt{AD^2}} = 6.188 \sqrt{\frac{m}{E}} \frac{H}{D} \frac{1}{\sqrt{A}} H = \overline{C}_5 \frac{H}{D} \frac{1}{\sqrt{A}} H
$$
(G-16)

$$
T = 1.787 \sqrt{\frac{m}{E}} \frac{H^2}{\sqrt{AD^2}} = 6.190 \sqrt{\frac{m}{E}} \frac{H}{D} \frac{1}{\sqrt{A}} H
$$
(G-17)

Deflections Due to Shear and Flexure

Considering deflections due to shear as well as flexure, the numerator and denominator in Eq.

$$
(G-2) are:\nH0\nH0 m0 S2(x) dx = H0 m (δS(x) + δF(x))2 dx\n= H0 mδS2(x) dx + H0 mδF2(x) dx + 2H0 mδS(x) δF(x) dx\n= H0 315 $\left(\frac{f_o}{\kappa GA}\right)^2 m H^5 + \frac{21128}{9979200} \left(\frac{f_o}{EI}\right)^2 m H^9 + \frac{467}{22680} \frac{f_o}{\kappa GA EI} m H^7$
\n^H₀
\n<
$$

Utilizing Eqs. (G-18) and (G-19) in Eq. (G-2) gives:

$$
T = 2\pi \sqrt{\frac{\frac{17}{315} \left(\frac{f_o}{\kappa GA}\right)^2 m H^5 + \frac{2641}{1247400} \left(\frac{f_o}{EI}\right)^2 m H^9 + \frac{467}{22680} \frac{f_o}{\kappa GA}}{15 \frac{f_o}{\kappa GA} H^3 + \frac{11}{420} \frac{f_o^2}{EI} H^5}
$$

\n
$$
= 2\pi \sqrt{\frac{m}{E}} \frac{H}{\sqrt{A}} \sqrt{\frac{\frac{17}{315} \left(\frac{E}{\kappa G}\right)^2 + \frac{2641}{1247400} \left(\frac{12}{D^2}\right)^2 H^4 + \frac{467}{22680} \frac{E}{\kappa G} \frac{12}{D^2} H^2\right]}}{\left[\frac{2}{15} \frac{E}{\kappa G} + \frac{11}{420} \frac{12}{D^2} H^2\right]}
$$

\n
$$
= 2\pi \sqrt{\frac{m}{E}} \frac{H}{\sqrt{A}} \sqrt{\frac{\frac{17}{315} \left(\frac{E}{\kappa G}\right)^2 + \frac{5282}{17325} \left(\frac{H^2}{D^2}\right)^2 + \frac{467}{1890} \frac{E}{\kappa G} \frac{H^2}{D^2}}}{\left[\frac{2}{15} \frac{E}{\kappa G} + \frac{11}{35} \frac{H^2}{D^2}\right]}
$$
(G-20)

Dunkerley's Method

Single Shear Wall

Based on Dunkerley's method, the fundamental period of a cantilever considering flexural and shear deformations, can be computed from:

$$
\frac{1}{\omega^2} = \frac{1}{\omega_F^2} + \frac{1}{\omega_S^2}, \text{ or}
$$

$$
\frac{1}{f^2} = \frac{1}{f_F^2} + \frac{1}{f_S^2}, \text{ or}
$$

$$
T^2 = T_F^2 + T_S^2
$$
 (G-21)

in which $T_F = 1/f_F = 2\pi/\omega_F$ and $T_s = 1/f_s = 2\pi/\omega_s$ are the periods of the fundamental periods of pure-flexural and pure-shear cantilevers, respectively. For uniform cantilevers, these periods are given by:

$$
T_F = \frac{2\pi}{3.516} \sqrt{\frac{m}{EI}} H^2
$$
 (G-22)

and

$$
T_s = 4\sqrt{\frac{m}{\kappa G}} \frac{H}{\sqrt{A}}
$$
 (G-23)

In Eqs. (G-22) and (G-23), *m* is the mass per unit height, E is the modulus of elasticity, G is the shear modulus, I is the section moment of inertia, A is the section area, and κ is the shape factor to account for nonuniform distribution of shear stresses $(= 5/6$ for rectangular sections). Utilizing Eqs. (G-22) and (G-23) in (G-21) gives:

$$
T = \left[\frac{4\pi^2}{3.516^2}\frac{m}{EI}H^4 + 16\frac{m}{\kappa GA}H^2\right]^{1/2} = 4\sqrt{\frac{m}{\kappa G}}\left[0.2\frac{\kappa G}{E}\frac{H^2}{I} + \frac{1}{A}\right]^{1/2}H
$$
 (G-27)

Since $I = A D^2 + 12$, Eq. (G-27) can be further simplified to:

$$
T = 4\sqrt{\frac{m}{\kappa G}} \left[\frac{1}{A} \left\{ 1 + 2.4 \frac{\kappa G}{E} \left(\frac{H}{D} \right)^2 \right\} \right]^{1/2} H
$$
 (G-28)

Recognizing that $G = E \div 2(1 + \mu)$, where the Poison's ratio $\mu = 0.2$ for concrete, leads to:

$$
T = 4\sqrt{\frac{m}{\kappa G}} \frac{1}{\sqrt{A_{\epsilon}}} H
$$
 (G-29)

with

$$
A_e = \frac{A}{\left[1 + 0.83 \left(\frac{H}{D}\right)^2\right]}
$$
(G-30)

where D is the plan dimension of the cantilever in the direction under consideration.

Several Shear Walls

Now consider a class of symmetric-plan buildings -- symmetric in the lateral direction considered -- with lateral-force resisting system comprised of a number of uncoupled (i.e., without coupling beams) shear walls connected through rigid floor diaphragms. Assuming that the stiffness properties of each wall are uniform over its height, Eq. (G-27) may be written as:

$$
T = \left[\sum_{i=1}^{NW} \left\{\frac{4\pi^2}{3.516^2} \frac{m}{E I_i} H_i^4 + 16 \frac{m}{\kappa G A_i} H_i^2\right\}\right]^{1/2}
$$

= $4 \sqrt{\frac{m}{\kappa G}} \left[\sum_{i=1}^{NW} \left(\frac{H_i}{H}\right)^2 \frac{1}{A_i} \left\{1 + 2.4 \frac{\kappa G}{E} \left(\frac{H_i}{D_i}\right)^2\right\}\right]^{1/2} H$ (G-31)

in which *A;, Hi,* and *Di* are the area, height, and dimension in the direction under consideration of the *ith* shear wall, and NW is the number of shear walls. Eq. (G-31) may be written in the same form as Eq. (G-29) if A*^e* is defined as:

$$
A_{\epsilon} = \sum_{i=1}^{NW} \left(\frac{H}{H_i} \right)^2 \frac{A_i}{\left[1 + 0.83 \left(\frac{H_i}{D_i} \right)^2 \right]}
$$
(G-32)

Equation (G-29) can be expressed in a form convenient for buildings:

$$
T = 40 \sqrt{\frac{\rho}{\kappa G}} \frac{1}{\sqrt{\overline{A}_e}} H
$$
 (G-33)

where ρ is the average mass density, defined as the total building mass $(= mH)$ divided by the total building volume $(= A_B H - A_B$ is the building plan area), i.e., $\rho = m/A_B$; and \overline{A}_e is the equivalent shear area expressed as a percentage of *As,* i.e.,

$$
\overline{A}_e = 100 \frac{A_e}{A_B} \tag{G-34}
$$

1. Burbank - 10 Story Residential Building, CSMIP Station No. 24385

Figure H.1.1. Sketch of shear walls in CSMIP Station No. 24385

Direction

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Transverse 192.07 0.2994 0.2019 0.3721

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2. Los Angeles - 8 Story Administration Building, CSMIP Station No. 24468

Figure H.2.1. Sketch of shear walls in CSMIP Station No. 24468

 $\hat{\mathcal{A}}$

3. Los Angeles - 10 Story Residential Building, CSMIP Station No. 24601

Figure H.3.1. Sketch of shear walls in CSMIP Station No. 24601

4. Palm Desert - 4 Story Medical Building, CSMIP Station No. 12284

Figure H.4.1. Sketch of shear walls in CSMIP Station No. 12284

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5. Piedmont - 3 Story School Building, CSMIP Station No. 58334

Figure H.5.1. Sketch of shear walls in CSMIP Station No. 58334

 $\bar{\mathcal{A}}$

 $\bar{\alpha}$

 $\bar{\bar{z}}$

6. Pleasant Hill - 3 Story Commercial Building, CSMIP Station No. 58348

CSMIP STATION NO. 58348 PLEASANT HILL

Figure H.6.1. Sketch of shear walls in CSMIP Station No. 58348

7. San Bruno - 9 Story Office Building, CSMIP Station No. 58394

CEMIP STATION NO. 58394 SAN BRUNO - \mathbf{H} \mathbf{r} $f: T, CH)$
A= 18₁ σ $\overline{1}$ j 16.7_M $\pmb{\mathsf{u}}$ q is
S $161₅$ M \mathbf{u} Ń İ $1617M$ ្តុ \bullet ¢, Δ $612M$ \mathbf{v} Ô \blacktriangleleft $\overline{B^2}$ 7 $\overline{S^2}$ \overline{S} $\frac{1}{2}$ 1 kO.

Figure H.7.1. Sketch of shear walls in CSMIP Station No. 58394

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8. San Jose - 10 Story Commercial Building, CSMIP Station No. 57355

Figure H.8.1. Sketch of shear walls in CSMIP Station No. 57355

 $\bar{\gamma}$

 $CSMIP$ STATION NO. 57356 San Jose 10- story Residential Bldg. \mathbf{I} I 59 ~-- - 4 $\frac{1}{\sqrt{2}}$ $12 - N$ ą $\frac{1}{100}$ 75.56 ัั่รู้
ร $\overline{2}$ $\frac{1}{93}$ $\frac{1}{2}$ (74) \mathbf{o} $92 - M$ $\overline{57}$ \hat{t} BOULDARY
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11- <mark>ب</mark> $\frac{1}{\sqrt{2}}$ $75 - 10$ $\frac{1}{2}$ -- مِّ مَعْ مَاهَ مَا يَجْ تَبْ ĨЬ $-225M$ WALL THICKNESS ROOF \mathbf{S} \mathbf{r} \mathbf{I} ø ő. C'I_._~ - ."1:=.... . I *,"'lO* ~N\ \mathbf{S} --'-9 ,Li. -3 $\frac{1}{\sqrt{100}}$
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 $\frac{1}{\sqrt{100}}$ $, Lz \neq M$ \overline{a} tJ d \overline{a} \overline{a}

Figure H.9.1. Sketch of shear walls in CSMIP Station No. 57356

Location Occupancy Station ID Name Height Building Plan Area San Jose Residential 57356 Town Park Tower Apartment 96 ft 13440 sq. ft

l,

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APPENDIX I: BIBLIOGRAPHY

System Identification

- Beck, J. L. (1978). *Determining Models of Structures from Earthquake records,* Report No. EERL 78-01, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, CA, June.
- Beck, R. T. and Beck, J. L. (1985). *Comparison Between Transfer Function and Modal Minimization Methods for System Identification,* Report No. EERL 85-06, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, CA, November.
- Beck, J. L. and Jennings, P.C. (1980). "Structural Identification Using Linear Models and Earthquake records," *Earthquake Engineering and Structural Dynamics,* 8, 145-160.
- Genatios, C., Lafuente, M. and Lopez, O. (1994). "On the Identification of Structural Dynamic Properties: Parametric Evaluation of the Experimental Techniques," *Proceedings of Fifth U.* S. *National Conference ofEarthquake Engineering,* Vol. IT, 743-752, July.
- Hart, G. C. and Yao, J. T. P. (1977). "System Identification in Structural Dynamics," *Journal of Engineering Mechanics,* 103(EM6), 1089-1104.
- Hong, K.-S. and Yun, C.-B. (1993). "Improved Method for Frequency Domain Identification of Structures," *Engineering Structures,* 15(3), 179-188.
- Imai, H., Yun, C. B., Maruyama, 0., Shinozuka, M. (1989). "Fundamentals of System Identification in Structural Dynamics," *Probabilistic Engineering Mechanics,* 4(4),162-173.
- Li, Y. and Mau, S. T. (1991). "A Case Study of MIMO System Identification Applied to Building Seismic Records," *Earthquake Engineering and Structural Dynamics,* 20, 1045- 1064.
- Ljung, L. (1987). *System Identification: Theory for the User,* Prentice Hall, Englewood Cliffs, NJ.
- McVerry, G. H. (1979). *Frequency Domain Identification of Structural Models from Earthquake Records,* Report No. EERL 79-02, Earthquake Engineering research Laboratory, California Institute of Technology, Pasadena, CA, October.
- Safak, E. (1989). "Adaptive Modeling, Identification, and Control of Dynamic Structural Systems. I. Theory," *Journal of Engineering Mechanics*, ASCE, 115(11), 2386-2405.
- Safak, E. (1989). "Adaptive Modeling, Identification, and Control of Dynamic Structural Systems. II. Applications," *Journal of Engineering Mechanics*, ASCE, 115(11), 2406-2426.
- Safak, E. (1988). *Analysis of Recordings in Structural Engineering: Adaptive Filtering, Prediction, and Control,* Open-file Report 88-647, U.S. Geological Survey, Menlo Park, CA.
- Safak, E. (1991). "Identification of Linear Structures Using Discrete-Time Filters," *Journal of Structural Engineering,* ASCE, 117 (10), 3064-3085.
- Safak, E. and Celebi, M. (1991). "Seismic Response of Transamerica Building. II: System Identification," *Journal of Structural Engineering*, ASCE, 117(8), 2405-2425.
- Safak. E. and Celebi, M. (1992). "Recorded Seismic Response of Pacific Park Plaza. II. System Identification," *Journal of Structural Engineering*, ASCE, 118(6), 1566-1589.
- Werner, S. D., Beck J. L. and Levine, M. B. (1987). "Seismic Response Evaluation of Meloland Road Overpass Using 1979 Imperial Valley Earthquake Records," *Earthquake Engineering and Structural Dynamics,* 15,249-274.
- Werner, S. D., Nisar, A, and Beck, J. L. (1992). *Assessment of UBC Seismic Design Provisions Using Recorded Building Motions from Morgan Hill, Mount Lewis, and Loma Prieta Earthquakes,* Dames and Moor, Oakland, California, April.

Periods and Damping

- Beck, J. L. (1978). *Determining Models of Structures from Earthquake records,* Report No. EERL 78-01, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, CA, June.
- Beck, J. L. and Jennings, P.C. (1980). "Structural Identification Using Linear Models and Earthquake records," *Earthquake Engineering and Structural Dynamics,* 8,145-160.
- Bendimerad, F. M., Shah, H. C., and Hoskins, T. (1991). *Extension of Study on Fundamental Period ofReinforced Concrete Moment-Resisting Frame Structures,* Report No. 96, John A. Blume Earthquake Engineering Center, Stanford University, CA, June.
- Bertero, V. V., Bendimerad, F. M., and Shah, H. C. (1988). *Fundamental Period of Reinforced Concrete Moment-Resisting Frame Structures, Report No. 87, John A. Blume Earthquake* Engineering Center, Stanford University, CA, October.
- Boroschek, R. L., Mahin, S. A. and Zeris, C. A. (1990). "Seismic Response and Analytical Modeling of Three Instrumented Buildings," *Proceedings of Fourth U.* S. *National Conference of Earthquake Engineering, Vol. II, 219-228, May.*
- Celebi, M. (1992). "Highlights of Lorna Prieta Responses of Four Tall Buildings," *Proceedings ofTenth World Conference on Earthquake Engineering, 4039-4044.*
- Celebi, M. (1993). "Seismic Response of Eccentrically Braced Tall Building," *Journal of Structural Engineering,* ASCE, 119(4), 1188-1205.
- Celebi, M. (1993). "Lorna Prieta Response of an Eccentrically Braced Tall Building," *Proceedings of the Structures Congress* '93, Vol. 2, 1545-1550, April.
- Celebi, M. (1993). "Seismic Response of Two Adjacent Buildings. I. Data and Analyses," *Journal of the Structural Engineering, ASCE, 119(8), 2461-2476.*
- Celebi, M. (1993). "Seismic Response of Two Adjacent Buildings. ll. Interaction," *Journal ofthe Structural Engineering,* ASCE, 119(8),2477-2492.
- Celebi, M. (1994). "Seismic Response of A Thirteen-Story Building," *Proceedings of Fifth* U. S. *National Conference ofEarthquake Engineering,* Vol. **I,** 87-96, July.
- Celebi, M. and Safak, E. (1991). "Seismic Response of Transamerica Building. I: Data and Preliminary Analyses," *Journal of Structural Engineering*, ASCE, 117(8), 2389-2404.
- Celebi, M. and Safak, E. (1992). "Seismic Response of Pacific Park Plaza. I. Data and Preliminary Analyses," *Journal of Structural Engineering*, ASCE, 118(6), 1547-1565.
- Cole, E. E., Tokas, C. V. and Meehan, J. F. (1992). "Analysis of Recorded Building Data to Verify or Improve 1991 Uniform Building Code (UBC) Period of Vibration Formulas," *Proceedings of SMIP92,* Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, 6-1 - 6-12, May.
- Coats, D. W. (1989). *Damping in Building Structures During Earthquakes,* Report No. NUREG/CR-3006, U.S. Nuclear Regulatory Commission, Washington, DC, January.
- De la Llera, J. C. and Chopra, A. K. (1992). "Evaluation of Code-Accidental Torsion Provisions Using Earthquake Records from Three Nominally Symmetric-Plan Buildings," *Proceedings*

of SMIP92, Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, May.

- *Earthquake Investigations in California* 1934-1935. (1936). Special Publication No. 201, U. S. Government Printing Office, Washington.
- Fenves, G. L. (1990). "Evaluation of Lateral Force Procedures for Buildings," *Proceedings of SMIP90,* Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, 6-1 - 6-10, July.
- Foutch, D. A., Housner, G. W., Jennings, P. C. (1975). *Dynamic Responses of Six Multistory Buildings During the San Fernando Earthquake,* Report No. **EERL** 75-02, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, CA, October.
- Freeman, S. A. and Honda, K. K. (1973). *Response of Two Identical Seven-Story Structures to the San Fernando Earthquake of February* 9, 1971, John A. Blume & Associates, San Francisco, CA, October.
- Gates, W. E., Hart, G. c., Gupta, S. and Srinivasan, M. (1994). "Evaluation of Overturning Forces of Shear Wall Buildings," *Proceedings of SMIP94,* Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, 105-120, May.
- Hart, G. C. and Vasudevan, R. (1975). "Earthquake Design of Buildings: Damping," *Journal of the Structural Division,* ASCE, 101(ST1), 11-30.
- Hart, G. c., DiJulio, R. M. and Lew, M. (1975). "Torsional Response of High-Rise Buildings," *Journal of the Structural Division, ASCE, 101(ST2), 397-416.*
- Hashimoto, P. S., Steele, L. K., Johnson, J. J., and Mensing, R. W. (1993). *Review of Structure Damping Value for Elastic Seismic Analysis of Nuclear Power Plant,* Report No. *NUREG/CR-6011,* U.S. Nuclear Regulatory Commission, Washington, DC.
- Hashimoto, P. S., Tiong, L. W., Steele, L. K., Johnson, J. J., and Beck, J. L. (1993). *Stiffness and Damping Properties of a Low Aspect Ratio Shear Wall Building Based on Recorded Earthquake Responses,* Report No. *NUREG/CR-6012,* U.S. Nuclear Regulatory Commission, Washington, DC.
- Haviland, R. (1976). *A Study ofthe Uncertainties in the Fundamental Translational Periods and Damping Values for Real Buildings,* Publication No. R76-12, Department of Civil Engineering, Massachusetts Institute of Technology, Mass., February.
- Housner, G. W. and Brady, A. G. (1963). "Natural Periods of Vibration of Buildings," *Journal ofEngineering Mechanics Division,* ASCE, 89(EM4), 31-65.
- Jeary, A. P. (1986). "Damping in Tall Buildings A Mechanism and A Predictor," *Earthquake Engineering and Structural Dynamics,* 14(5), 733-750.
- Li, Y. and Mau, S. T. (1991). "A Case Study of MIMO System Identification Applied to Building Seismic Records," *Earthquake Engineering and Structural Dynamics,* 20, 1045- 1064.
- Lin, B. C. and Papageorgiou, A. S. (1989). "Demonstration of Torsional Coupling Caused by Closely Spaced Periods - 1984 Morgan Hill Earthquake Response of the Santa Clara County Building," *Earthquake Spectra,* 5(3), 539-556.
- Marshall, R. D., Phan, L. T. and Celebi, M. (1994). "Full-Scale Measurement of Building Response to Ambient Vibration and the Loma Prieta Earthquake," *Proceedings of Fifth U.* S. *National Conference ofEarthquake Engineering,* Vol. II, 661-670, July.
- McVerry, G. H. (1979). *Frequency Domain Identification ofStructural Models from Earthquake Records,* Report No. EERL 79-02, Earthquake Engineering research Laboratory, California Institute of Technology, Pasadena, CA, October.
- Papageorgiou, A. S. and Lin, B. C. (1989). "Influence of Lateral-Load-Resisting System on the Earthquake Response of Structures-A System Identification Study," *Earthquake Engineering and Structural Dynamics,* 18(6), 799-814.
- Safak, E. and Celebi, M. (1991). "Seismic Response of Transamerica Building. II: System Identification," *Journal of Structural Engineering*, ASCE, 117(8), 2405-2425.
- Safak. E. and Celebi, M. (1992). "Recorded Seismic Response of Pacific Park Plaza. II. System Identification," *Journal of Structural Engineering*, ASCE, 118(6), 1566-1589.
- Sedarat, H., Gupta, S. and Werner, S. (1994). *Torsional Response Characteristics of Regular Buildings Under Different Seismic Excitation Levels,* Data Utilization Report CSMIP/94-01, Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, January.
- Werner, S. D., Nisar, A. and Beck, J. L. (1992). *Assessment of UBC Seismic Design provisions Using Recorded Building Motion from the Morgan Hill, Mount Lewis, and Loma Prieta Earthquakes,* Dames and Moor, Oakland, CA, April.
- Wood, J. H. (1972). *Analysis of the Earthquake Response of a Nine-Story Steel Frame Building During the San Fernando Earthquake,* Report No. EERL 72-04, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, CA, October.

Recorded Motions

- Etheredge, E. C. and Porcella, R. L. (1987). *Strong-Motion Data fom the October* 1, 1987 *Whittier Narrows Earthquake,* Open-File Report 87-616, U.S. Geological Survey, Menlo Park, CA.
- Hudson, D. E., Editor. (1971). *Strong-Motion Instrumental Data on the San Fernando Earthquake of Feb.* 9, 1971, Earthquake Engineering Research Laboratory, California Institute of Technology and Seismological Field Survey, NOAA, February.
- Porcella, R. L., Etheredge, E. c., Maley, R. P., and Acosta, A. V. (1994). *Accelrograms Recorded at USGS National Strong-Motion Network Stations During the Ms=6.6 Northridge, California Earthquake of January* 17, 1994, Open-File Report 94-141, U.S. Geological Survey, Menlo Park, CA.
- Shakal, A. F. and Huang, M. J. (1986). "Torsional Response of Three Instrumented Buildings During the 1984 Morgan Hill Earthquake," *Proceedings of Third* U. S. *National Conference ofEarthquake Engineering, 1639-1650.*
- Huang, M. et al. (1986). *CSMIP Strong-Motion Records from the Palm Springs, California Earthquake of* 8 *July,* 1986, Report No. OSMS 86-05, California Department of Conservation, Division of Mines and Geology, Office of Strong Motion Studies, Sacramento, CA.
- Shakal, A. et al. (1987). *CSMIP Strong-Motion Records from the Whittier, California Earthquake of* 1 *October,* 1987, Report No. OSMS 87-05, California Department of Conservation, Division of Mines and Geology, Office of Strong Motion Studies, Sacramento, CA.
- Huang, M. et al. (1989). *CSMIP Strong-Motion Records from the Santa Cruz Mountains (Loma Prieta), California Earthquake of* 17 *October* 1989, Report No. OSMS 89-06, California Department of Conservation, Division of Mines and Geology, Office of Strong Motion Studies, Sacramento, CA.
- Shakal, A. et al. (1990). *Quick Report on CSMIP Strong-Motion Records from the February 28, 1990 Earthquake near Upland, California,* Report No. OSMS 90-02, California Department of Conservation, Division of Mines and Geology, Office of Strong Motion Studies, Sacramento, CA.
- Shakal, A. et al. (1991). *CSMIP Strong-Motion Records from the Sierra Madre, California Earthquake of* 28 *June* 1991, Report No. OSMS 91-03, California Department of Conservation, Division of Mines and Geology, Office of Strong Motion Studies, Sacramento, CA.
- Shakal, A. et al. (1994). *CSMIP Strong-Motion Records from the Northridge, California Earthquake of January* 17, 1994, Report No. OSMS 94-07, California Department of Conservation, Division of Mines and Geology, Office of Strong Motion Studies, Sacramento, CA.
- Shakal, A. F., Huang, M. J. and Darragh, R. B. (1994). "Some Implications of Strong-Motion Records from the 1994 Northridge Earthquake," *Proceedings of SMIP94,* Strong Motion Instrumentation Program, Division of Mines and Geology, California Department of Conservation, Sacramento, May.

General

Chopra, A. K. (1995). *Dynamics of Structures: Theory and Applications to Earthquake Engineering,* Prentice Hall, Englewood Cliffs, NJ.

Housner, G. W. and Jennings, P.C. (1982). *Earthquake Design Criteria*, Engineering Monograph, Earthquake Engineering Research Institute. CA.

MATLAB User's Guide. (1992). The MathWorks, Inc., Natick, Mass.

- *NEHRP Recommended Provisions for the Development of seismic Regulations for New Buildings.* (1994). Building Seismic Safety Council, Washington, D. C.
- Newmark, N. M. (1967). *Design Criteria for Nuclear Reactors Subjected to Earthquake Hazards,* Urbana, lllinois, May.
- Newmark, N. M., and Hall, W. J. (1978). *Development of Criteria for Seismic Review of Selected Nuclear Power Plants,* Nuclear Regulatory Commission Report *NUREG/CR-0098* , Washington, DC.
- Newmark, N. M. and Hall, W. J. (1982). *Earthquake Spectra and Design,* Engineering Monograph, Earthquake Engineering Research Institute. CA.
- Priestley, M. J. N. and Hart, G. C. (1989). *Design Recommendations for the Period of Vibration of Masonry Wall Buildings,* Report No. *SSRP-89/05,* Structural Systems Research Report, University of California at San Diego and Los Angeles, CA, November.
- *Recommended Lateral Force Requirements and Commentary.* (1990). Seismological Committee, Structural Engineers Association of California, San Francisco, CA.
- *Tentative Provisions for the Development of Seismic Regulations for Buildings.* (1978). ATC3- 06, Applied Technological Council, Palo Alto, CA.
- *User's Guide: System Identification Toolbox for Use with MATLAB.* (1993). The Math Works, Inc., Natick, Mass.

Uniform Building Code. (1997). International Conference of Building Officials, Whittier, CA.

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Kayen, R.E., Seed, R.B., Nishio, S. and Stokoe II, K.H., April 1994,

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 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$

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 $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$